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The different sections of the Transactions are designated by symbols, the following being used: AER (Aeronautics), APM (Applied Mechanics), FSP (Fuels and Steam Power), HYD (Hydraulics), IS (Iron and Steel), MAN (Management), MSP (Machine-Shop Practice), OGP (Oil and Gsa Power), PME (Petroleum Mechanical Engineering), WDI (Wood Industries), RP (Research Papers), and RI (Record and Index). In this Index, except for references to the Record and Index Section, three numbers follow the symbol for the section; the first is the volume number, the second the paper number, and the third the page number. Thus AER-54-6-41 signifies that the material is found in the Aeronautics Section, vol. 54, paper no. 6, on page 41. In the case of the Record and Index Section of Transactions, there is no paper number, the entire section being numbered consecutively from page 1 to the end, so that the first number refers to the volume and the second to the page (RI-54-24).

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Foreword

VOLUME 54 of the Transactions of The American Society of Mechanical Engineers contains 27 individual sections published during 1932 under the sponsorship of the Society's 15 professional divisions, as well as its annual Record and Index. The technical papers and reports that make up this volume represent the Society's annual contribution to the permanent record of mechanical-engineering achievement. Most of these papers and reports were presented at meetings of the Society and of its professional divisions and local sections, and were issued originally in pamphlet form. These pamphlets appear in sections, each section being sponsored by one of the Society's professional divisions. Other papers published by the Society but not included in the Transactions may be located by means of the Record and Index which forms a part of this volume.

All sections of the Transactions are bound together at the end of the year for the convenience of libraries and of engineers who wish all of the papers in permanent form. Copies of the bound Transactions will be found in depositories located in selected engineering, university, and public libraries throughout the world. A complete list of these depositories will be found on pages 89–91 of the Record and Index. Copies of the bound Transactions have also been set aside for sale.

The Record and Index of the A.S.M.E. for 1932 which forms a part of these Transactions, is the permanent record of the Society's activities for the year. It includes lists of committees, reports of meetings and committee activities, memorial notices of deceased members, as well as indexes of special A.S.M.E. publications and of *Mechanical Engineering*.

In this volume the papers of each section are grouped together and are numbered serially. The sections are arranged alphabetically. Each paper is designated by a symbol composed of key letters followed by the volume number and paper number. The letters refer to the section of the Transactions to which the paper is assigned. Thus AER refers to the Aeronautic section, IS to the Iron and Steel section, etc. The arrangement of the index and designations for locating material are explained at the beginning of the index.

THE PUBLICATIONS COMMITTEE.

AER-54-1

Transmission of Visible, Infra-Red, and Ultra-Violet Radiation Through Fog

With Reference to Developing Aids in Fog Aviation

By S. HERBERT ANDERSON,[1] OCEANPORT, N. J.

Fog menaces all forms of transportation, particularly air travel. To study the visibility of beacons through fog, a laboratory method of producing fog was developed. Fogs of two types have been made with a range of densities corresponding to natural fogs in which the visibility varies from 20 ft. to 2000 ft. Transmission of radiation through these fogs has been measured, covering a wave-length range of 0.30 to 0.40 micron in the ultra-violet, 0.40 to 0.75 micron in the visible, and 0.75 to 2.6 microns in the infra-red parts of the spectrum. The conclusion is that the penetration of infra-red through fog is many times that of visible light, the ratio depending upon the distance and the nature of the fog, and consequently infra-red gives promise in developing aids for fog landing, especially in localizing a fog-obscured airport.

FOG, menacing and impeding, still hangs over all forms of transportation. On land, on sea, and in the air its risks still challenge the ingenuity of man. All of his skill is needed if he is to make travel more secure. Especially is this true of travel in the air. Here the pilot requires superior visibility to enable him to distinguish the landmarks, beacons, and signals on which the proper guidance of his plane depends. Here the menace of fog is directly proportional to the lack of that visibility so necessary for safe operation. With the development of aviation and the growth of interest in it, has come a corresponding recognition of the need for the study of fog penetration, with a view to diminishing its dangers. The last three years have witnessed a remarkable advance in these directions.

Prior to the investigation under discussion in this paper (autumn of 1928) only one paper dealing with measurements of fog penetration had appeared. Utterback[2] in 1919 measured the transmission of light through a laboratory-made fog. He used 11 bands of color between $\lambda = 0.46\mu$ and $\lambda = 0.63\mu$, and found a maximum of transmission for a wave-length range between 0.53μ and 0.59μ (corresponding to a color range from green to yellow). Granath and Hulburt,[3] of the Naval Research Laboratory, in 1929 reported their measurements through a natural fog for a range from 0.40μ (violet) to 3.00μ (infra-red). Their data showed the transmission to increase with wave length, thereby agreeing with King's[4] equation

$$I = I_o e^{-(a + b\lambda^{-4})x} \qquad [1]$$

in which I_o is the intensity of radiation in a clear atmosphere, I the intensity after passing through a distance x of fog, λ is wave length, and a and b are parameters. More recently Houghton and Straton,[5] of Massachusetts Institute of Technology, have presented experimental and theoretical investigations covering a range from 0.40μ to 0.70μ. They found a small maximum of transmission at 0.49μ. The work of Benford[6] and of Karrer and Tyndall[7] contributed to information on atmospheric transmission but very little on the specific problem of fog penetration. The results of these investigators will be discussed later.

A knowledge of the nature of fog enables one to make certain predictions regarding its effects on a beam of radiation projected from a searchlight or an airway beacon. Fog particles are minute spheres of water ranging in diameter from 1 to 10 microns (1 micron, μ, $= 10^{-6}$ meters $= 0.001$ mm.). The diameters of the most numerous particles in fogs occurring along seacoasts and the radiation fogs of interior lowlands are from 6 to 8 microns. Particles which make up smoke and haze are much smaller, 0.001 to 0.1 micron in diameter. In the "black" fogs found over industrial centers, both smoke particles and fog particles are present. In such fogs the particles are smaller than in sea fogs.

The attenuation of a beam of radiation in passing through a layer of fog may be attributed to four factors:

1. True absorption, including selective absorption
2. Scattering
3. Diffusion
4. Diffraction.

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[1] Chief, Light Section, Signal Corps Laboratory, Fort Monmouth. Physicist for the Daniel Guggenheim Fund for the Promotion of Aeronautics, Inc., 1928–1930. Mr. Anderson received his Ph.D. from the University of Illinois, 1912. Member of the staff of the Department of Physics, University of Washington, 1912 to 1918 and 1919 to 1928. Commissioned First Lieutenant, Air Service, March, 1918. Assigned to Science and Research Section, Air Service, March, 1918, to March, 1919, and to Engineering Division, Air Service, March, 1919, to September, 1919. Was engaged in development and testing work of airplane instruments. Instructor in Air Corps Engineering School, McCook Field, Dayton, Ohio, during the summers of 1920 and 1926. Research Physicist for the Daniel Guggenheim Fund for the Promotion of Aeronautics, July, 1928, to July, 1930. Assigned to Wright Field, Dayton, Ohio, to work with the engineers of the Air Corps, U. S. Army, in developing aids for fog avigation. Physicist in charge of Light Section, Signal Corps Laboratories, Fort Monmouth, N. J., October, 1930, to date. Published papers on photoelectric phenomena, acoustics, X-rays, electric-arc characteristics; prepared the first editions of the Instruction Manual on Airplane Instruments and Manual of Aerial Navigation for the Air Corps, U. S. Army; preliminary report for the Daniel Guggenheim Fund for the Promotion of Aeronautics on the "Penetration of Light Through Fog," published in *Aviation*, May, 1930.

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Utterback, Trans. Illum. Eng. Soc., 1919.

[3] Granath and Hulburt, *Phys. Rev.*, vol. 34, p. 140, 1929.

[4] King, Phil. Trans. Royal Society (London), vol. A-212, p. 375, 1913.

[5] Houghton and Straton, *Phys. Rev.*, vol. 37, p. 466, 1931.

[6] Benford, *Gen. Elec. Rev.*, vol. 29, pp. 728 and 873, 1926.

[7] E. Karrer and Tyndall, Bureau of Standards, Sci. Paper no 389, 1920.

In *absorption*, the energy of radiation incident on the fog droplets is changed into some other form. It consequently disappears as radiant energy.[8] In *scattering*, *diffusion*, and *diffraction*, the radiation incident to the fog particles is turned aside or given a new direction. Thus the intensity of the beam suffers a loss. The physical processes by which this is accomplished in the three cases are quite distinct. In *scattering*, the particles become radiators, emitting light in all directions. The theory of scattering was set forth by Lord Rayleigh.[9] He showed the intensity of the scattered light to be given by

$$I_s = \frac{Kd^6}{\lambda^4} \qquad [2]$$

where K is a constant, d the diameter of the particle, and λ the wave length. The derivation of this formula assumed the diameter of the particle to be small compared with the wave length of incident light. Wave lengths of visible radiation range from 0.40μ to 0.75μ, and the most useful infra-red from 0.75μ to 2.0μ. Since the average diameter of droplets in a true fog is about 7μ—large as compared with the radiation wave lengths—Rayleigh scattering could not be expected to be important in the attenuation of radiation through fog. However, in haze, smoke, and small-particle fogs, where the particles are less than 1μ in diameter, the Rayleigh scattering must be considered a significant factor.

In *diffusion*, radiation is deflected from its original path by refraction and reflection. This is probably the most important cause for loss of energy in the transmission of radiation through fog.

In *diffraction*, there is a deviation of a ray of light, which is a function of particle size and wave length. The diffraction pattern produced by a group of fog particles of uniform size is a ring of rainbow colors, blue on the inside and red on the outside, about the source of light. This is called a corona. One sometimes observes it about the sun or moon, due to thin clouds in the high atmosphere. Very rarely is it seen in a ground fog; probably due to the fact that the great variety of sizes of fog particles causes an overlapping in the diffraction patterns. Under these conditions the effect of diffraction would be the same as diffusion.

The actual volume of water in a cubic meter of dense foggy air is extremely small, being 6 × 10^{-7} cu. m. or 0.6 cc. per cu. m. In order to obtain a layer of water 1 cm. thick (of any desired area), fog particles from a layer 16.7 km. thick would be required. Such a thickness of water would be opaque to infra-red radiations longer than 1.4μ, but transparent to visible and ultra-violet. Hence it must be concluded that true absorption is a negligible factor up to 1.4μ.

It appears, then, that the principal factors in the attenuation of visible radiation through fog are diffusion (due to reflection and refraction by fog particles) and possibly diffraction. There is no reason for believing these should show selective effects. Consequently, the problem of fog penetration with visible radiation did not appear a very promising one to the physicist.

Opposing these arguments, however, were persistent reports that neon lights and incandescent lights with yellow and orange filters exhibited greater visibility in fog than did white lights. It became desirable to secure, if possible, quantitative information on transmission of radiation through fog. Should any color be especially effective in fog penetration, this fact should be known and used in the designing of aviation lighting.

As the work progressed, the investigation was extended into the ultra-violet and infra-red regions, the total range covered being from 0.3μ to 2.6μ.

The unreliability of nature for furnishing fogs in the quantity and of the quality needed for such an investigation suggested the use of a laboratory method to expedite the research. Any information thus gained would be a valuable guide for field tests in natural fogs.

Design of Laboratory Fog Equipment

Principles of Fog Production. The design of laboratory equipment for the production of fog was made with a view to simulating as nearly as possible the processes in natural fogs. Hence, a brief statement of the principles involved will be given.

The essential conditions for fog formation have been known since the pioneer work of J. J. Thomson[10] and C. T. R. Wilson.[11] These are:

1 The presence in the air of condensation nuclei
2 A humid condition of the atmosphere.

The first condition is the more important, since the degree of saturation or supersaturation required for volume condensation depends upon the nature of the condensation nuclei. Wilson showed that in filtered air a fog could not be formed until a supersaturation of eightfold (800 per cent) was produced. The explanation of this is found in the fact that vapor pressure over a convex surface is greater than over a flat surface. The excess pressure is given by

$$\Delta p = \frac{2Td_v}{R(d_w - d_v)} \qquad [3]$$

in which

Δp = excess vapor pressure over a convex surface
T = surface tension of the liquid
R = radius of curvature of the surface
d_w = density of the liquid (water)
d_v = density of the vapor.

The foregoing equation shows that the excess pressure is inversely proportional to the radius. This means that if a droplet of water (for example, a fog particle) is present in the air, there is a tendency for the droplet to evaporate; the smaller the drop, the greater this tendency. If a drop is to be stable, the tendency to evaporate must be neutralized by some other force. A condensation nucleus supplies this neutralizing force.

The following table shows the magnitude of excess vapor pressure of water due to curvature:

Diameter of particle	Excess vapor pressure
1 × 10^{-3} cm.	2.8 dynes per sq. cm.
5 × 10^{-5} cm.	56.0 dynes per sq. cm.
2.8 × 10^{-8} cm.	100,000.0 dynes per sq. cm.

The pressure of saturated water vapor for a temperature of 10 deg. cent. (50 deg. fahr.) is about 12,250 dynes per sq. cm. The diameter of a large fog particle is about 1 × 10^{-3} cm. For a particle of this size, the excess vapor pressure, 2.8 dynes per sq. cm., is small compared with the vapor pressure of the space around it. Consequently, the tendency to evaporate is not large. However, if a particle has molecular dimensions (2.8 × 10^{-8} cm.), the excess vapor pressure is very large, 100,000 dynes per sq. cm. This is a little greater than eight times the saturated vapor pressure at 10 deg. cent. The foregoing explains the necessity for raising the supersaturation to eightfold in order to produce condensation in air free from dust, hygroscopic particles, and ions. Under such conditions, the volume vapor pressure must equal or exceed the vapor pressure of particles of molecular dimensions.

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[8] Since heat radiation occurs at all temperatures, the fog particles will emit radiant energy. However, this will be so feeble and of such long wave length that it can be neglected as compared with the incident radiations.

[9] *Phil. Mag.*, vol. 41, p. 107, 1871.

[10] J. J. Thomson, "Conduction of Electricity Through Gases," 2d ed., p. 149 et seq.

[11] C. T. R. Wilson, Phil. Trans. of the Royal Society (London), vol. A-189, p. 265, 1897; vol. A-192, p. 403, 1899.

In an admirable paper Willet[12] has shown that fogs and clouds are formed in nature when the saturation is less than 100 per cent, and rarely when more than 107 per cent. Condensation with this relatively low vapor pressure is possible through the presence in the air of condensation nuclei. These may be:

1 Electric ions
2 Inert, irregularly shaped particles (smoke or dust)
3 Hygroscopic particles.

Electric ions act as condensation nuclei because the electric charge tends to neutralize the excess vapor pressure caused by curvature of the surface. It can be shown[13] that a particle having a unit ionic charge has a critical diameter, 1.26×10^{-7} cm., so that, should the diameter of the particles be equal to or larger than this, condensation will occur. If the particle be smaller, condensation will not occur. However, it was shown that saturation must be between fourfold and sixfold for condensation on electric ions. Since this is much greater than found in nature, it must be concluded that electric ions are not important in the formation of natural fogs.

Inert particles of dust and smoke act as condensation nuclei because of (1) flat areas over which the vapor pressure is equal to the saturated-vapor pressure of the air and (2) concave areas over which the vapor pressure is less than that of saturated-vapor pressure.

Hygroscopic particles have an attraction for water molecules. This force of attraction is opposite from the vapor pressure of the convex surface of the drop. Such particles are very effective condensation nuclei. Some such substances found in the air are sodium chloride (NaCl), magnesium chloride ($MgCl_2$), and sulphuric anhydride (SO_3). NaCl and $MgCl_2$ occur in the air over the ocean and along the seacoasts. These substances furnish the condensation nuclei of fogs for these localities. SO_3 is formed by the oxidation of SO_2. The latter is found generally wherever coal is burned. Around industrial centers other products of combustion and industrial processes form numerous hygroscopic particles.

Condensation nuclei are present in the air at all times in greater or less numbers. When a volume of air of high moisture content is cooled to the dewpoint or lower, a fog may occur. According to Willet,[14] the processes by which this is accomplished form a basis for classification. Only two types, more prevalent than others, will be mentioned here, viz., (1) radiation fogs and (2) advection fogs.

Radiation fogs occur most often at night, along river bottoms, in valleys, in cities, and around industrial centers. The conditions for the formation are:

1 A previous relatively high humidity, caused by daytime evaporation of water
2 A cooling of the lower strata of air at night by (*a*) radiation and (*b*) by contact with ground cooled by radiation after sundown; this results in a shallow temperature inversion
3 A fairly static condition of the atmosphere. Radiation fogs do not usually form with wind greater than 5 m.p.h. On the other hand, extremely calm air permits precipitation as dew. A slight turbulence bringing a large volume of air in contact with the ground slowly enough to permit cooling and rapidly enough to prevent complete precipitation is necessary.

The second type, advection fogs, occur (*a*) when a warm, humid body of air drifts over a cold surface and (*b*), more rare, when a warm, humid body of air comes in contact with a cold layer of air. Such fogs occur in the early spring and summer along seacoasts, i.e., along the New England Coast and the Pacific Coast. A further excellent example is the fogs off Newfoundland. Here the air from over the Gulf Stream drifts over the Labrador Current.

In the laboratory production of fog, condensation nuclei were introduced into an enclosure containing air saturated with water vapor. The size of the fog particles formed was controlled to some extent by the nature of the nuclei and the degree of supersaturation produced. The details will be described later.

The Fog Chamber. The fog chamber as finally developed and used is shown in Fig. 1. *C-C* is a length of standard steel steam pipe, 6 in. in diameter and 20 ft. long. It was closed at each end by a piece of plate glass. (For most of the tests Pyrex glass was used, since it has a uniform high transmission from 0.36 microns in the ultra-violet to 2.5 microns in the infra-red. For measurements in the ultra-violet and infra-red, plates of fused quartz were used.) The pipe was lined on the inside with a helical coil of copper tubing, $^1/_4$ in. in diameter, 12 turns to the foot. Through this coil hot or cold water could be circulated and by this means change quickly the temperature of the air in the pipe. The pressure could be increased or decreased by connections to an air-pressure line and a vacuum tank, respectively. From a water tank *W*, fitted with an electric heating unit, a 3-in. pipe *P-P* runs to each end of the fog chambers. Warm, humid air above the water could be circulated through the fog chamber by a small electric fan hung on the inside of the water-tank cover. A second electric fan was located inside the fog chamber. When this latter was running, a turbulence, similar to that in a natural fog, was produced. Valves *V-V*, at each end of the 3-in. circulating pipe, could be closed, thus shutting off the fog chamber from the water tank. The temperature of the air within the fog chamber was indicated by three thermocouples, one at each end and one in the middle. A fourth thermocouple, soldered to the exterior of the outside wall of the fog chamber, gave the temperature of the wall. A fifth thermocouple in the water tank indicated the temperature of the water.

Special precautions were taken to screen the light-detecting instrument from light reflected from the inside walls of the fog chamber. In the first place, the reflecting power of the inside walls was made very low. After assembling the equipment, the interior was sprayed with optical-black lacquer. While this was still sticky, soot from burning camphor gum was deposited on it. This produced a mat-black surface of very low reflecting power. In the second place, baffle rings were built into the temperature-control unit and so located as to prevent light reflections from reaching the observing end of the fog chamber.

Condensation nuclei were introduced into the fog chamber by a smoke injector *S*. Tobacco, burned in this injector, formed satisfactory nuclei. For large-particle fog magnesium chloride was mixed with the tobacco.

At the middle section of the fog chamber were three glass windows, 1 in. in diameter. Two of these were at opposite ends of a diameter and the third was at an angle of 30 deg. to the line of the other two. A beam of light from a small projector could be directed through the two windows opposite each other. Through the third window a microscope of low magnifying power

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[12] Willet, *Monthly Weather Review*, vol. 56, p. 435, 1928 (a report prepared for the Daniel Guggenheim Fund for the Promotion of Aeronautics).

[13] J. J. Thomson, loc. cit.

[14] Willet, loc. cit.

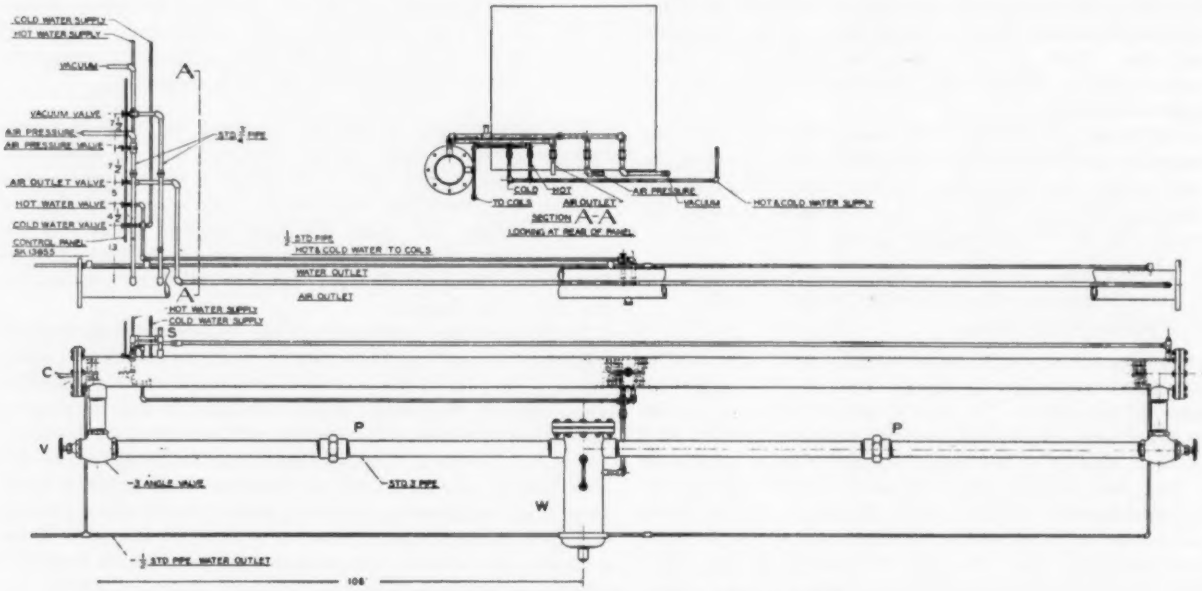

FIG. 1 DIAGRAM OF LABORATORY FOG CHAMBER

was directed on the fog particles illuminated by the light beam. Thus it was possible to make a qualitative observation on the number and size of the fog particles.

PRODUCTION OF FOG

Two kinds of fog were used in this investigation, which will be called (1) "small-particle" fog and (2) "large-particle" fog. The diameters of the most numerous particles in the small-particle fog were about 6 microns and in the large-particle fog about 8 microns.

The first step in the formation of both types of fog was the same, viz., the circulation of warm, moist air from the water tank *W* through the fog chamber. Condensation on the small windows at the center section indicated a saturated condition of the air. The end windows were kept slightly warmer than the rest of the fog chamber to prevent condensation there.

Small-Particle Fog. In the formation of small-particle fog, a large number of condensation nuclei were introduced by burning tobacco. The valves *V-V* were closed, the stirring fan was started, and the temperature was adjusted to 20 deg. cent. Shortly the testing light was completely obscured, indicating a change in the size of the particles. If the fog were watched through the microscope at the center section, the growth of these particles was easily observed. At first they were too small to be resolved by the low-powered microscope, but as the fog became stable the individual particles were distinguishable. Even though smoke particles were used as condensation nuclei, this fog must not be considered a smoke haze. It was definitely a fog in which the bulk of each particle was largely liquid water. The condensation took place from about 100 per cent saturation. This type of fog is probably similar to the black fogs found over industrial centers.

Large-Particle Fog. To produce large-particle fog, magnesium chloride was mixed with the tobacco, thus introducing hygroscopic nuclei of $MgCl_2$. The valves *V-V* were then closed, the stirring fan was started, and the temperature was adjusted to 20 deg. cent. In order to increase the moisture content of the air the pressure in the fog chamber was increased to 1.15 atmospheres. When a stable condition was reached, the air was suddenly expanded by opening the valve of the vacuum tank. An adiabatic expansion and cooling resulted in which the saturation reached about 150 per cent. A very dense fog was quickly formed, completely obscuring the testing light.

Both types of fog were very dense at first. They thinned out slowly, lasting from two to three hours, during which time measurements were made of the transmission of various bands of radiation.

METHOD OF MEASURING THE TRANSMISSION OF RADIATION THROUGH FOG

Source of Light. For the investigations in the visible and infra-red portions of the spectrum the source of light was a 200-watt, 6-volt, Mazda C incandescent lamp. The filament was specially concentrated, a single helix of seven turns, well adapted for projecting a narrow beam. By means of a lens projector a beam of radiation, limited by a circular diaphragm $2^1/_2$ in. in diameter, was directed through the fog chamber. The image of the filament was focused at the position of the light-detecting instruments, producing an illumination of 170 foot-candles and a corresponding beam candlepower of 70,000. The lamp was operated on a 15-volt d.-c. circuit, with a storage battery floating on the line. The current was kept at 28 ($\pm$ 0.1) amperes by a rheostat. Operating below the rated power of the lamp made possible a very constant source of light. The temperature of the filament, measured with an optical pyrometer, was 3000 deg. kelvin.

For the ultra-violet portion of the spectrum, the Mazda C lamp was replaced by a "sun lamp" *S-1*. With this type a mercury vapor arc about $^1/_4$ in. long was formed between electrodes of tungsten. The bulb of Corex-D glass transmits little or no radiation shorter than 0.28μ. However, it was better adapted to this work than a quartz mercury arc lamp, for it was possible to project a narrow beam from it. The glass projecting lenses were replaced by one of fused quartz.

Radiation Measuring Instruments. The intensity of the light emerging from the fog chamber was determined by photoelectric cells for the ultra-violet, visible, and near-infra-red parts of the spectrum, and for the infra-red of longer wave lengths by a thermoelement. Photoelectric currents were measured by a Leeds & Northup galvanometer with a sensitivity of 0.0005 microampere per mm. scale deflection. By using a shunt with the galvanometer, currents could be measured over a range from 1 to 1500.

Thus it was possible to measure transmissions of light somewhat smaller than 0.1 per cent. Preliminary tests showed that if the photoelectric cells were operated on a comparatively low voltage, the photo currents were proportional to the light flux on the cell for the range of illuminations used. (Subsequent to these tests the same results were published by Koller.[15])

A Hilger D-34 infra-red spectrometer, fitted with a linear thermopile, was used for isolating various bands of infra-red. The galvanometer was a Leeds & Northrup type HS, with a sensitivity of 20-mm. deflection per microvolt.

The comparison of a set of measurements taken at one time with those taken at another necessitated some means of measuring the transparency of fog. An arbitrary method was utilized. The photoelectric current (using the band of near infra-red 0.85μ) as obtained from cell 3, type UX-868, with Wratten filter 88A in front of it, was taken as a measure of fog transparency. Galvanometer deflections for all other bands of radiation were read at the same time as the deflections obtained with this band.

Bands of Radiation Used. Fourteen bands of radiation covering a total range from 0.30μ to 2.60μ were used in this investigation. Two were in the ultra-violet, four in the visible, and eight in the infra-red parts of the spectrum, respectively. The selection of these bands was governed by three conditions:

1. Each band should be as narrow as possible
2. One band should not overlap another
3. The radiation energy of each band should be sufficiently large to render possible an accurate measurement.

The first two conditions were imposed by the very nature of the problem. In order to determine the variation in transmission of various colors (bands of radiation) through fog, the measurements should be made for pure colors. Otherwise the results obtained would give transmission coefficients which might be misleading. It would be possible to secure pure colors (narrow bands) by using a spectrometer or monochrometer. However, should this be done, the third condition might be violated—the energy in each band would be too small for reliable measurement. It was found possible to use a spectrometer for isolating bands in the infra-red, but filters had to be used in the visible and the ultra-violet parts of the spectrum.

In Table 1 is given a list of the bands of radiation used, the width of each band, and the means of isolating each. In Fig. 2 the bands isolated by filters are shown graphically. The energies of the 0.3235μ, 0.360μ, and 0.4275μ bands were due largely to lines of the mercury-arc spectrum. The bands of the visible and near infra-red were cut out from the continuous spectrum of the Mazda C lamp.

The isolation of these latter bands was secured not only by the selective transmission of filters, but also by the spectral sensitivity of photoelectric cells. As much difference was found in the long wave-length limits of individual cells of the same type as existed between individual cells of different types. For example, the limit of cell No. 3, type UX-868, was 1.15μ and of cell No. 1, same type, was 0.825μ. This type had caesium sub-oxide as the active element. The limit of PJ-15, a pure caesium cell, was 0.70μ and of the Burt cell (sodium cathode) was 0.56μ. Practically all red and yellow filters had a high transmission (90 per cent or more) in the dark-red and infra-red to 2.5 microns. If the elimination of this radiation of long wave length was desired, some other filter had to be used with the yellow or red filter. One that would serve was a liquid filter of copper sulphate solution, cutting off radiations longer than 0.70μ. Such a filter, however, was not convenient. The proper selection of a photoelectric cell made possible a more simple accomplishment of the same result. For the band in the near infra-red (0.70μ to 1.15μ) the long wave-length limit was given by the cut-off of cell No. 3, UX-868. For the red band (0.60μ to 0.825μ), cell No. 1 of the same type gave the long wave-length limit at 0.825μ.

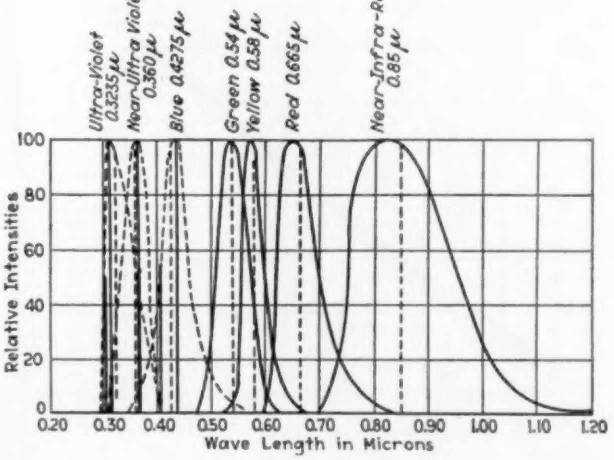

FIG. 2 BANDS OF COLOR IN ULTRA-VIOLET AND VISIBLE RADIATION ISOLATED BY FILTERS

Nearly every blue and violet filter had a transmission band in the red or infra-red. Such was the case with the theater blue (dark shade) and the red-purple ultra used to isolate the blue and near-ultra-violet bands, respectively. If such filters were used with a cell of the type UX-868, a large part of the photoelectric current would be due to the near-infra-red transmitted by these filters. In this work these filters were used with a PJ-15 (pure caesium) cell having a cut-off at 0.70μ. Hence the near infra-red was ineffective.

Isolation of only two bands in the infra-red by filters was found possible, one from 0.70μ to 1.15μ and the other from 1.10μ to 2.75μ. Preliminary observations showed that the

TABLE 1 BANDS OF RADIATION USED, WITH WIDTH OF BAND AND MEANS OF ISOLATING

Color	Center of gravity of band	Wave-length limit	Filter	Light detector
Ultra-violet......	0.3235μ	0.295μ–0.400μ	No. 1 Corex red-purple + 2 cm. $NiSO_4$ concentrated solution	Burt quartz photoelectric cell (sodium)
Near-ultra-violet.	0.360μ	0.305μ–0.41μ	No. 2 red-purple ultra	G. E. PJ-15 (caesium)
Blue............	0.4275μ	0.35μ –0.55μ	No. 3 theater blue, dark shade	G. E. PJ-15 (caesium)
Green...........	0.540μ	0.475μ–0.625μ	No. 4 Wratten No. 61	G. E. PJ-15 (caesium)
Yellow..........	0.580μ	0.525μ–0.675μ	No. 5 h.r. yellow-orange shade + pale blue green	UX 868 No. 1 (caesium suboxide)
Red.............	0.665	0.60μ –0.825μ	No. 6 h.r. signal red 155%	UX 868 No. 1 (caesium suboxide)
Near-infra-red ...	0.85μ	0.70μ –1.15μ	No. 7 Wratten No. 88A	UX 868 No. 3 (caesium suboxide)
Infra-red........	1.90μ	1.10μ –2.75μ	No. 8 Corning h.t. + blue-green medium	Moll & Burger single-junction thermo-element
Infra-red........	1.00μ	0.975μ–1.025μ	Infra-red spectrometer	Hilger linear thermopile
Infra-red........	1.30μ	1.25μ –1.35μ	Infra-red spectrometer	Hilger linear thermopile
Infra-red........	1.40μ	1.335μ–1.465μ	Infra-red spectrometer	Hilger linear thermopile
Infra-red........	1.60μ	1.505μ–1.695μ	Infra-red spectrometer	Hilger linear thermopile
Infra-red........	1.94μ	1.82μ –2.06μ	Infra-red spectrometer	Hilger linear thermopile
Infra-red........	2.30μ	2.165μ–2.435μ	Infra-red spectrometer	Hilger linear thermopile
Infra-red........	2.50μ	2.36μ –2.64μ	Infra-red spectrometer	Hilger linear thermopile

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[15] Koller, Jour. Optical Soc. of Am. and Rev. Sci. Instr., vol. 19, p. 135, 1929.

transmission through fog was much greater for the long wave-length band than for any others. Since this band was a wide one (1.65 microns), it seemed advisable to investigate in greater detail the transmission of the infra-red than would be possible by using filters. An infra-red spectrometer was tried and found suitable.

Method of Making Observations

As previously pointed out, the photoelectric current was proportional to the light flux incident on the cell. With a thermo-element the thermoelectric current was likewise proportional to the radiation flux incident on the element. Consequently, the percentage of transmission through fog of a certain band of color necessitated only the finding of the ratio between galvanometer deflections (for that band) with fog and with no fog.

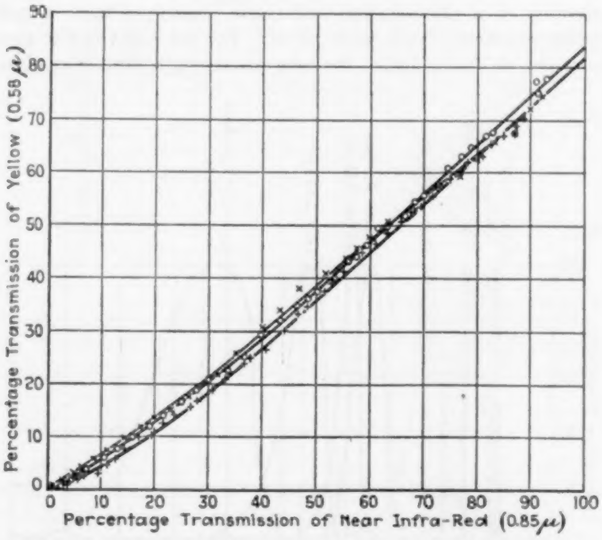

Fig. 3 Transmission of Yellow Light Through Fog

Three set-ups were required for each type of fog. In the first, observations were made on the ultra-violet band (0.3235μ), the near-ultra-violet (0.360μ), the blue (0.4275μ), and the near-infra-red (0.85μ). Part of the beam of light emerging from the fog chamber was concentrated by a quartz lens on the Burt cell. With filter No. 1 this isolated and responded to the ultra-violet band (0.3235μ). Another portion of the beam was directed on cell PJ-15 by means of a right-angled prism. With filter No. 2 before this cell, the near-ultra-violet band (0.360μ) was detected. With filter No. 3 the blue band (0.4275μ) was detected. A sliding filter frame made possible the quick changing of filters. By the use of a double-throw switch, either the Burt cell or PJ-15 could be connected to the galvanometer. A third part of the beam was directed by a second right-angled prism on cell UX-868 No. 3, with filter No. 7 before it. This cell was connected to a second galvanometer. Reference band 0.85μ was thus measured.

At the beginning of each series of readings the galvanometer deflection was taken for each band with the fog chamber clear. This deflection was called 100 per cent transmission. The fog was then produced. At first the fog was so dense that the galvanometer deflections were too small to be read. (This meant a transmission of less than 0.1 per cent.) As the fog slowly cleared, the deflections became readable, and the series was started. At a signal from one observer, the deflection due to the 0.85μ band was read simultaneously with the deflection due to the blue band (0.4275μ), then with the deflection due to the near-ultra-violet (0.360μ), and next with the ultra-violet (0.3235μ). This procedure was repeated again and again until the transmission through the fog reached about 95 per cent. Each series of readings consisted of from 75 to 250 individual observations. For each type of fog and each set-up at least three sets of readings were taken, and in some cases as many as five.

The second set-up was similar to the first. Filters Nos. 5 and 6 were used with cell UX-868 No. 1 to isolate the yellow (0.580μ) and red (0.665μ) bands, respectively. Filter No. 4 before cell PJ-15 gave the green band (0.540μ). The method of taking the readings was identical with the first set-up.

In the third set-up the greater portion of the beam was focused by a quartz lens upon the slit of the infra-red spectrometer. A small part of the beam was directed on the cell for detecting the reference band (0.85μ). As with the first set-up, the deflection due to the 0.85μ band was read simultaneously with that of each of the bands isolated by the infra-red spectrometer.

Interpretation of Measurements

Graphical Representation of Data. From each set of readings the transmissions were computed for each band of color and plotted against the corresponding transmissions of the near-infra-red band (0.85μ). Fig. 3 illustrates this procedure. Measurements with three fogs are represented, each curve being plotted from data obtained with a single fog. This figure indicates also the success attained in reproducing fogs of a given type.

Average transmissions were computed for each color from a set of curves such as Fig. 3 and plotted against those of the near-infra-red (0.85μ). These average curves for small-particle and large-particle fog are shown in Figs. 4 and 5, respectively. Since the transmission of the near-infra-red band is taken as a measure of fog transparency, the abscissas of these curves represent fog transparencies, dense fogs being at the left and light fogs at the right. All points on a vertical line (for example, at 5 per cent) represent a fog of given transparency. The points where the various curves cut this line show the relative transmissions of the different colors. Hence each set of curves is a graphical representation of (1) the transmissions of the various colors as a function of fog transparency and (2) the relative transmissions of the various colors for a fog of any given transparency.

The variation of transmission with wave length is shown to better advantage in Figs. 6 and 7. The curves of these figures were derived from the curves of Figs. 4 and 5, respectively. Here, each curve corresponds to a fog of given transparency. This is indicated at the right end of each curve, where the distance required to reduce the intensity of the near-infra-red (0.85μ) band to 0.1 per cent is given.

Discussion of Curves. The most noticeable feature of the curves of Figs. 6 and 7 is the greater transmission of infra-red radiation compared with that of other portions of the spectrum. This is in agreement with previous predictions, but is now shown for the first time in quantitative form. Consider, for example, in Fig. 7 the third curve from the bottom. This represents transmissions through a dense fog in which the intensity of the near-infra-red (0.85μ) was reduced to 0.1 per cent in 101 ft. The wave length for which the transmission is a minimum is 0.58μ, corresponding to yellow light. The transmission of this color through 20 ft. of fog is 15 per cent, while the transmission of infra-red of wave length 2.6μ is 87 per cent. The infra-red has 5.8 times the transmission of the yellow. Through 100 ft. of the same kind of fog, the contrast is more striking, the ratio of the transmission of the infra-red to yellow being 6500 to 1.

In small-particle fog the color of smallest penetrating ability is blue-green (0.475μ). Taking the third curve of Fig. 6, it is seen that the transmission through 20 ft. of fog is only 13.5 per cent, compared with 89 per cent for infra-red of wave length 2.6μ. The penetration of the latter is 6.6 times that of the blue-green. Through 100 ft. of fog of the same kind the ratio of transmission of infra-red (2.6μ) to blue-green (0.475μ) is 11,000 to 1.

The second feature of these curves attracting attention is the increase in transmission from the minimum in the visible to 0.30μ in the ultra-violet. This is contrary to the views generally held and is very surprising. However, a well-known fact tends to verify this—viz., that it is possible for a person to acquire a sunburn in a fog so dense that the sun is not visible. Since sunburn is caused only by radiations between 0.25 and 0.33 micron, ultra-violet must penetrate fog. Referring again to curve 3 of Fig. 7, it will be noticed that the transmission at 0.30μ is about the same as at 0.85μ, and about twice that of the yellow band (0.58μ) through 20 ft. of fog. Through 100 ft. of fog the ultra-violet (0.30μ) has a transmission 14 times that of the yellow (0.58μ). This fact suggests the possible value of ultra-violet in photography through fog and in signaling through fog.

The curves for small-particle fog, Fig. 6, do not show such a marked difference between the transmissions of the ultra-violet and the visible. Hence, in smoky fogs and hazes the ultra-violet would probably prove of little value.

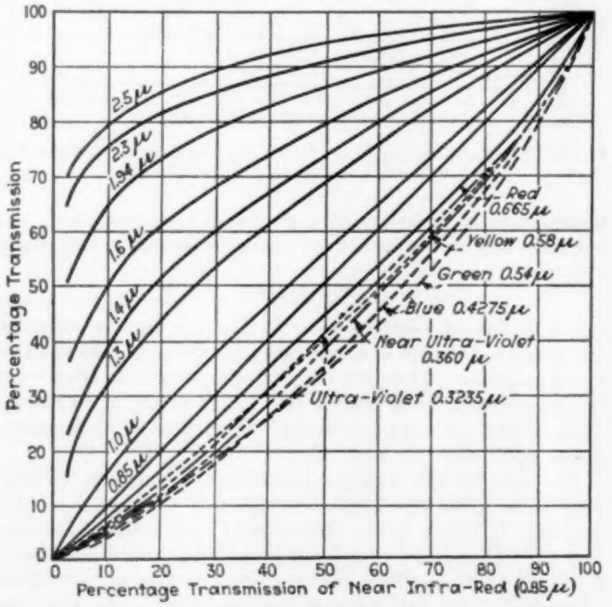

FIG. 4 TRANSMISSION OF RADIATION THROUGH SMALL-PARTICLE FOG

A general comparison of the curves of Figs. 6 and 7 shows that the transmissions of infra-red radiation through small-particle fog are somewhat greater than through large-particle fog. The difference is not great at any point, being largest for the most dense fogs. The ratio of transmissions is the largest for 1.4 microns.

In general, the transmission of the visible region is considerably less than in the infra-red or ultra-violet, the minimum occurring at some point in the visible. There is a difference in the position of this minimum in the two types of fogs. For the small-particle fog it occurs at a wave length of 0.475μ, corresponding to a blue-green color. For the large-particle fog it is at 0.58μ, or yellow color. This indicates a critical relation between the size of the particles and the wave length for which maximum absorption occurs. Further investigation in different types of fogs should be carried on to establish this relationship.

Law of Absorption of Radiation by Fog. If there could be established a law of absorption of radiation by fog which would enable us to compute the percentage transmission for a given band of radiation and a given distance, this would be of considerable value in the design of beacons for fog penetration. It is generally assumed that the attenuation of radiation by an absorbing medium is expressed by

$$I = I_o e^{-kx} \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

in which k is the coefficient of absorption, and the other symbols have the same meaning as in Equation [1]. The data presented by the curves of Figs. 6 and 7 show that in a fog the absorption coefficient k must depend upon the wave length, particle size, and concentration of fog particles. A satisfactory equation must take account of all these factors. In the King equation, which Granath and Hulbert found to agree with their data,

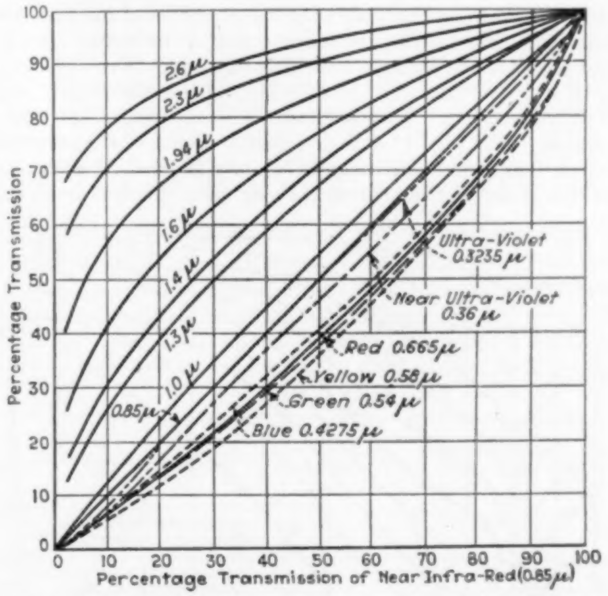

FIG. 5 TRANSMISSION OF RADIATION THROUGH LARGE-PARTICLE FOG

$$k = a + b\lambda^{-4} \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

This means that the attenuation is due to two factors: (1) an absorption, independent of wave length, expressed by the term a and (2) a scattering of radiation, expressed by the second term $b\lambda^{-4}$. This equation was derived by King to account for the absorption of the radiation of the sun by the earth's atmosphere. The parameters a and b have different values for clear and for dust-filled air. Furthermore, dust particles scatter radiation of wave lengths longer than 0.61μ, but not shorter. This critical wave length is probably a function of particle size.

Equation [5] has been tested, using data taken from the curves of Fig. 7. Coefficients of absorption k were computed from the transmissions and plotted against wave length. (Data for wave lengths shorter than 0.6μ were not used, as it is apparent the law of absorption must be different in the region $\lambda < 0.6\mu$ from what it is for $\lambda > 0.6\mu$.) Curves 1 and 3 of Fig. 8 were plotted from the coefficients of absorption computed from the curves 3 and 9, respectively, of Fig. 7. Then, values of the constants a and b were computed using the values of k corresponding to wave lengths 0.6μ and 2.6μ, respectively. Curve 2 is the theoretical curve corresponding to curve 1, and curve 4 corresponding to curve 3. The difference between the experimental and theoretical curves is large. Consequently it must be concluded that Equation [5] does not give the law of absorption in fog. In other words, the Rayleigh law of scattering does not apply for the attenuation of radiation in fog. The law was derived on the assumption that the scattering particles were small compared with the wave length of radiation. As has been already pointed out, fog particles have a diameter from 2 to 30 times that of the radiation used in this work. Hence the lack of applicability of the law is not surprising.

This conclusion is not in agreement with that reached by Granath and Hulburt. There may be three reasons for the difference. In the first place, Granath and Hulburt made their observations in very thin fogs (visibility 0.6 km.). This fact would indicate, not only that the concentration of fog particles was much less than in the fog chamber of the author's work, but also that the particles were of smaller size. Secondly, they used only two bands in the infra-red, with peaks at 1.4μ and 3.0μ, respectively. Both of these bands were broad and overlapped. (See Fig. 1 of their paper.) Hence the absorption which they found and assigned to wave lengths corresponding to the peaks of their curves may be quite different from the absorption of narrow bands located at the same wave lengths. The reliability of their measurements in the visible region is not questioned. However, the range covered (0.4μ to 0.6μ) is much too small to form a basis for any judgment. In the third place, a source of error which they point out is the absorption of water vapor. There are five absorption bands within the region in which their bands of radiation were located. The amount of water vapor in the air on clear days (used for determining 100 per cent transmission) was unquestionably much less than on foggy days. Hence, their absorption coefficients include the effect of water vapor as well as fog particles. In the author's work this source of error was eliminated. Measurements through the fog chamber were taken with the air saturated at 20 deg. cent. at all times. The transmission through the air clear of fog (which was taken as 100 per cent transmission with respect to fog) included the absorption of water vapor. Consequently, the absorption coefficients are for fog only.

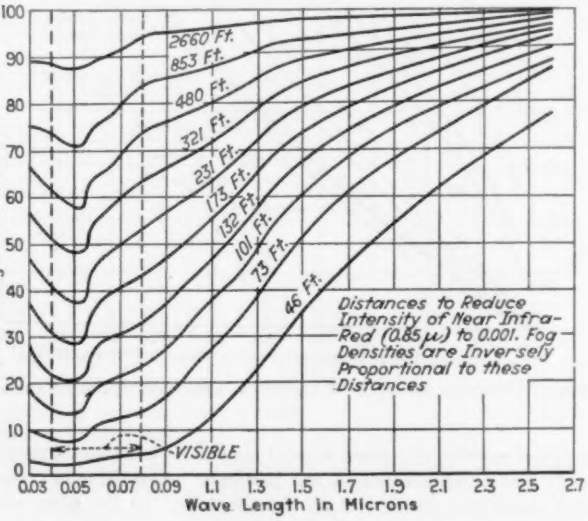

FIG. 6 TRANSMISSION OF RADIATION THROUGH SMALL-PARTICLE FOG AS A FUNCTION OF WAVE LENGTH (COLOR)

The most recent contribution to the theory of absorption of radiation by fog is that of Houghton and Straton.[16] Their equation for transmission is:

$$I = I_o e^{-2\pi a^2 NKx} \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

in which a is the radius of the fog particle, N the concentration (number of particles per unit volume), and K is a function of a and λ which oscillates about the value 1 for $a/\lambda < 6$ and becomes asymptotic to 1 for $a/\lambda > 6$. This equation has also been tested by the author's data. The lack of agreement is fully as great as with the King equation. Hence, it must be concluded that at present there is no satisfactory equation by which transmission of radiation through fog can be computed, and one must rely on empirical data such as presented in this paper.

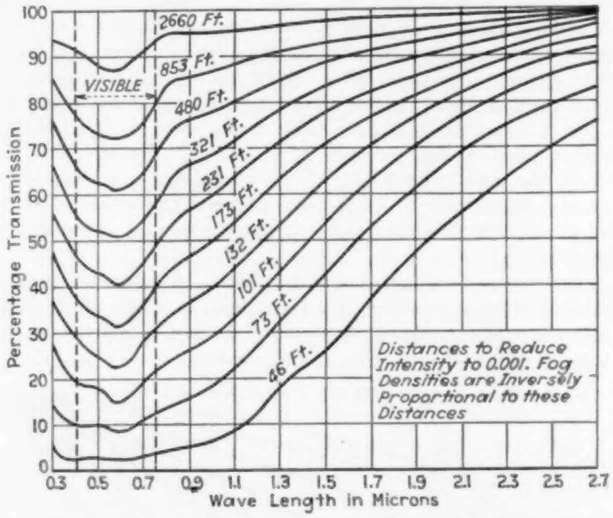

FIG. 7 TRANSMISSION OF RADIATION THROUGH LARGE-PARTICLE FOG AS A FUNCTION OF WAVE LENGTH (COLOR)

APPLICATIONS

Use of Colored Light in Fog. In the introduction attention was called to the need of investigating the value of colored lights in fog avigation. Does any band of color, especially effective in fog penetration, exist within the visible range of radiation? This question can now be answered. Unfortunately, the measurements made show no such color.

The basis for this negative answer is well shown in Table 2. This table gives the respective distances through dense fog necessary to reduce the transmissions of various bands of radiation to $^1/_{10}$ per cent. For large-particle fog, the values are 83 ft., 78 ft., 73 ft., and 79 ft. for blue, green, yellow, and red lights, respectively. No one color is predominantly better than the others. With small-particle fog, the distances are 69 ft., 71 ft., 81 ft., and 88 ft. for blue, green, yellow, and red lights, respectively. With this type of fog the red is somewhat more effective than the other colors, penetrating a distance 1.27 greater than the blue. This advantage is not large enough to be of any great value. In the lighter fogs, the differences in transmissions for the various colors are less than in dense fogs. Hence it may be concluded that there is no band of color of visible light which is especially effective in fog penetration.

This conclusion is in agreement with that of observers whose data were obtained in natural fogs. Granath and Hulburt found the distance required to reduce the transmission of red light (0.65μ) and of blue light (0.45μ) to $^1/_{10}$ per cent to be 1.4 km. and 1.2 km., respectively. The ratio of these distances is $1.4/1.2 = 1.165$. Their observations were made through an exceedingly light fog (probably small-particle fog), visibility 0.6 km. Using data from the uppermost curve of Fig. 6 of this paper (representing a thin, small-particle fog), the corresponding distances are 1350 ft. and 1050 ft. for red and blue lights, respectively. The ratio is 1350/1050 = 1.287. The difference between this ratio and that of Granath and Hulburt is not large and can be accounted for by the difference in fog densities.

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[16] Houghton and Straton, loc. cit.

Breckenridge and Nolan,[17] of the Bureau of Standards, made a study of the "Relative Visibility of Luminous Flashes From Neon Lamps and From Incandescent Lamps With and Without Red Filters." Their observations were on relative visibilities rather than intensities. However, their conclusions are the same. They found that, under any weather condition, the neon lamp and the red-filtered incandescent lamp had no greater range of visibility than white light.

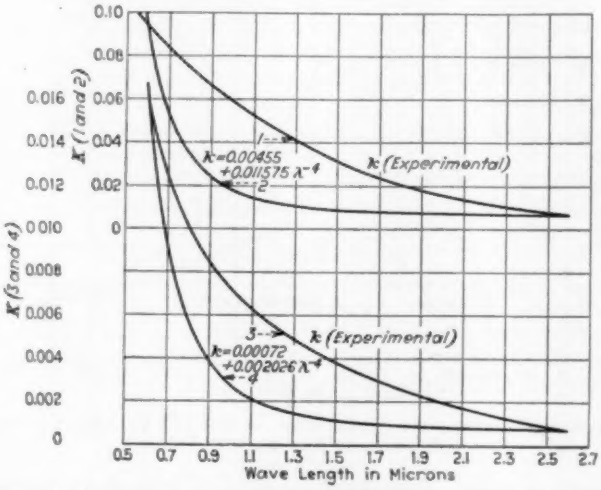

FIG. 8 COEFFICIENT OF ABSORPTION AS FUNCTION OF WAVE LENGTH

Other factors than the percentage of intensity transmitted determine the visibility of light, such as these:

1 The relative sensibility of the eye for various colors
2 The effect of contrast.

In Fig. 9 is shown the relative sensibility of the eye for monochromatic radiation (pure spectral colors). This figure shows that the normal eye is most sensitive to yellow-green light (0.555μ) and that the sensibility falls off rapidly toward each end of the spectrum. The eye is approximately 10 times more sensitive to the yellow light (0.58μ) used in this investigation than to the red (0.665μ) and 25 times more sensitive than to the blue (0.4275μ). This means that in order to have red, yellow, and blue light sources which would appear of equal brightness to the eye, the intensity of the red would have to be 10 times that of the yellow and the blue 25 times that of the yellow. Since the data in Table 2 show little variation in the energy transmission of blue, yellow, and red light through fog, one must conclude that the eye sensibility is a more important factor than the relative transmissions of visible colors.

The matter of contrast may be just as important. The value of neon lights for advertising signs is well recognized. This value is probably due to the contrast of the neon color with the white lights of the streets. Breckenridge and Nolan found it somewhat easier to pick up the flashes from red and neon lights than from the white light beacon, especially when other white lights were in the field of view. The factor of contrast as applied to aviation lighting is one that should be given careful study. Contrast is more important for low intensities of light when the visibility is near the threshold.

It must be concluded from present information that white light beacons are just as effective as colored beacons. If future studies based on eye sensibility and contrast show that certain colors have decided advantages, it will be necessary to develop a light source emitting those colors so that the efficiency and intrinsic brilliancy are comparable with those of the tungsten incandescent light.

Use of Ultra-Violet Radiation in Fog Photography. As previously pointed out, the transmission of the ultra-violet between 0.30 and 0.40 micron is greater than that of the visible, and about the same as that of the near-infra-red between 0.70 and 0.80 micron. This is true for large-particle fog, but not for small-particle fog. The near-infra-red region has been used very effectively in long-distance photography. By using "extreme red sensitive" film (sensitized with kryptocanine) and a Wratten 89A filter, Captain Stevens secured good negatives showing objects 270 miles distant. The band of radiation used with the combination extends from 0.68μ to 0.79μ. Since the transmission of the ultra-violet band (0.30μ to 0.40μ) through fog was about the same as that of this near-infra-red, the idea of using ultra-violet in photography through fog was suggested.

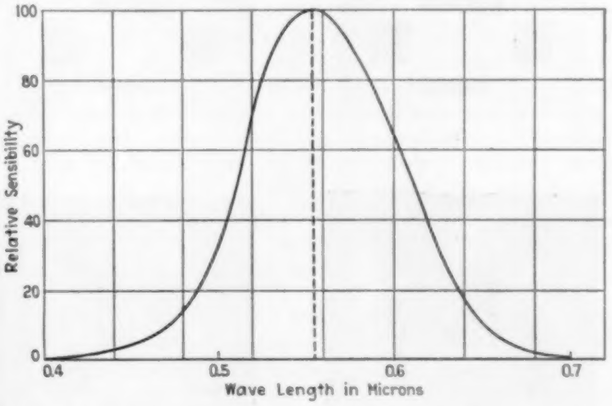

FIG. 9 RELATIVE SENSIBILITY OF THE EYE FOR MONOCHROMATIC RADIATION (PURE SPECTRAL COLORS)

In a limited way this was tested. As natural fog was not available, a laboratory fog was used. A target 22 in. by 28 in., made by pasting squares of black paper on white cardboard, was

TABLE 2 RESPECTIVE DISTANCES THROUGH DENSE FOG NECESSARY TO REDUCE THE TRANSMISSIONS OF VARIOUS BANDS OF RADIATION TO 0.1 PER CENT

Color	Center of gravity band	Coefficient of absorption: Large-particle fog	Coefficient of absorption: Small-particle fog	Distance in feet to reduce transmission to 0.1% (threshold visibility): Large-particle fog	Distance in feet to reduce transmission to 0.1% (threshold visibility): Small-particle fog
Ultra-violet	0.3235μ	0.0683	0.0886	101	78
Near-ultra-violet	0.360μ	0.0766	0.09355	90	74
Blue	0.4275μ	0.0838	0.09975	83	69
Green	0.540μ	0.0886	0.09725	78	71
Yellow	0.580μ	0.0948	0.08545	73	81
Red	0.665μ	0.08715	0.0785	79	88
Near-infra-red	0.850μ	0.0683	0.0683	101	101
Infra-red	1.00μ	0.0615	0.0562	112	123
Infra-red	1.30μ	0.0412	0.03638	168	190
Infra red	1.40μ	0.03629	0.02926	190	234
Infra-red	1.60μ	0.02689	0.02208	257	313
Infra-red	1.94μ	0.01684	0.01372	436	504
Infra-red	2.30μ	0.0101	0.00920	684	752
Infra-red	2.60μ	0.00707	0.00679	978	1017

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[17] Breckenridge and Nolan, Research Paper no. 78, Bureau of Standards Journal of Research, vol. 3, July, 1929.

illuminated by a quartz mercury arc and a sun lamp. Photographs of this target were taken through the fog with a band of ultra-violet light (0.25μ to 0.40μ) and a band of visible light (0.42μ to 0.50μ). Fig. 10 shows results from four negatives. Print *A* is a picture of the target with visible light, no fog; *B* was taken with ultra-violet light, no fog; *C*, visible with fog; *D*, ultra-violet with fog. The black areas in *B* are not quite as intense as in *A*. This evidently is due to some reflection of ultra-violet from the black paper. *D* is distinctly sharper than *C* and shows less general fogging of the negative. This photographic test appears to be a satisfactory confirmation of the results obtained with the fog chamber. In penetration of large-particle fogs, ultra-violet surpasses visible light.

It is not suggested that ultra-violet will give the same results in photography through fog that the near-infra-red gives through a hazy atmosphere. Conditions in the two cases are not the same. In the latter the particles are very small, so that the light is scattered according to the Rayleigh inverse fourth-power law. When no filter is used, this scattered light fogs the negative and obscures the desired image. The deep-red filter absorbs this scattered light and transmits the long wave lengths emanating directly from the object photographed. In large-particle fog absorption is due to other factors. Hence an ultra-violet filter is not as effective in fog as a deep-red filter in haze. However, the ultra-violet filter does isolate a band penetrating fog better than visible light, so that sharper negatives are obtained with it.

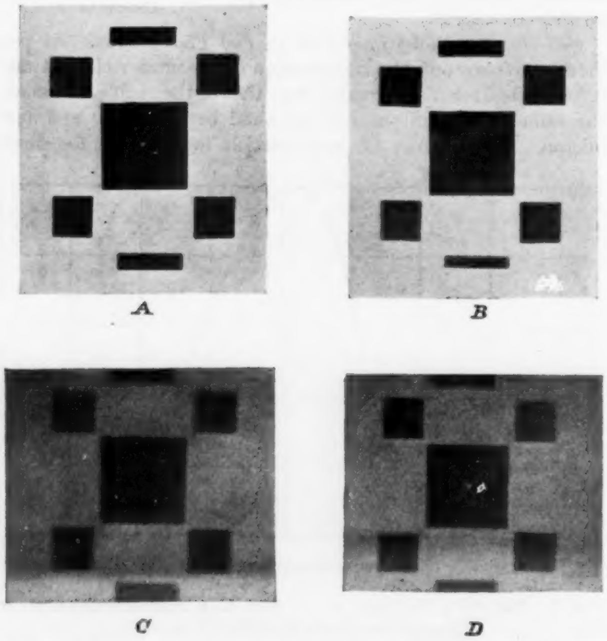
A B C D

FIG. 10 PHOTOGRAPHS THROUGH FOG WITH VISIBLE AND ULTRA-VIOLET LIGHT

(*A*, visible light, no fog; *B*, ultra-violet light, no fog; *C*, visible light, fog; *D*, ultra-violet light, fog.)

Utilization of Infra-Red Radiation. As indicated previously, the most important result of this investigation is the discovery of the relatively large transmission of infra-red radiation compared with that of other colors. This suggests the use of infra-red for localizing a fog-obscured airport. The comparative quantities of infra-red and visible radiations penetrating fog are shown in Fig. 11. The upper curve shows the energy radiated from an ideal black body at 3000 deg. kelvin as a function of wave length. This represents approximately the energy distribution from a 1-kw. tungsten lamp (as used in an airway beacon). If an ordinate is drawn at $\lambda = 0.75\mu$, the area under the curve between 0.40μ and 0.75μ represents the energy radiated in the visible region, and the area under the curve between 0.75μ and 2.6μ represents the infra-red radiation. (If the lamp is mounted in a beacon projector, the actual proportions radiated will be somewhat different due to reflection coefficients of the mirror and absorption of the lens.) The ratio of infra-red radiated to visible is about 7 to 1. The lower curve is obtained by multiplying the ordinates of the upper curve by the respective transmission coefficients obtained from curve 3 of Fig. 7. The lower curve represents the relative amounts of energy penetrating a fairly dense fog. From this curve, the ratio of infra-red to visible is found to be 20 to 1. That is, from a standard airway beacon, 20 times as much infra-red as visible penetrates fog. Since the eye is not sensitive to infra-red, this is not utilized. However, if an infra-red receiver having the same relative sensitivity as the human eye can be evolved, the possibility of picking up the radiation from a beacon in a fog is at least 20 times greater than at present. This can be increased by the use of a beacon specially designed for infra-red radiation, such a beacon to be used in foggy weather.

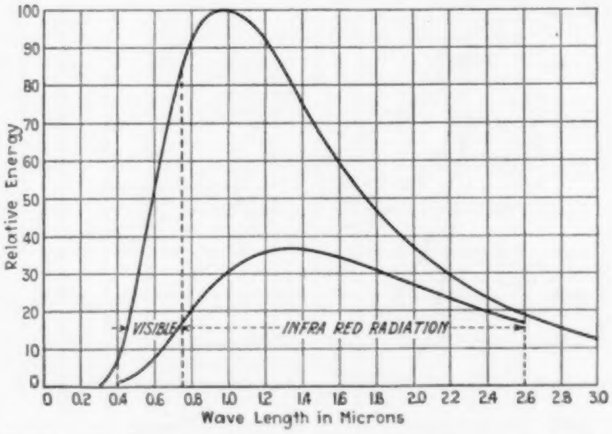

FIG. 11 COMPARATIVE QUANTITIES OF INFRA-RED AND VISIBLE RADIATIONS PENETRATING FOG

The author has designed and built such a receiver and has given it a limited test. The infra-red radiation can be detected by this instrument when the fog is so dense that it completely obscures the visible light from the projector.

An infra-red beacon has certain unmistakable advantages for localizing purposes, such as these:

1 The same methods of projection can be used as with light beacons and floodlights; thus there will be no difficulty in projecting intersecting beams or sheets of infra-red
2 Such infra-red beams will not be affected by local electrical disturbances, and there will be no interference with radio waves broadcast from nearby stations
3 The power source for infra-red beacons consists of standard electrical equipment, transformers, or motor-generator sets, which require a minimum of maintenance.

CONCLUSION

A laboratory method for the production of fog has been developed in which processes of formation of natural fogs were closely simulated. With this equipment fogs of two types have been made with a range of densities corresponding to natural fogs in which the visibility varies from 20 ft. to 2000 ft. Transmission of radiation through these fogs has been measured, covering a wave-length range of 0.30 to 0.40 micron in the ultra-violet, 0.40 to 0.75 micron in the visible, and 0.75 to 2.6 microns in the infra-red parts of the spectrum. From this investigation definite conclusions can be drawn regarding the value of the various regions of radiation in developing aids for fog avigation.

Conclusions.

1 Visible light is less effective than ultra-violet and infra-red radiation in fog penetration

2 The transmission of various colors of visible light through large-particle fogs is about the same for all colors. In small-particle fogs, the transmission of red light is about 25 per cent higher than other colors. This advantage is not large enough to be of any great value. In general there is no color of visible light especially effective in fog penetration. Hence colored beacons are of no greater value in marking airways than are white light beacons, as judged from the relative transmissions

3 In dense, large-particle fogs ultra-violet has a transmission greater than that of visible light. This has been verified by securing better photographs through fog with ultra-violet radiations than with visible light. The use of ultra-violet for photography through fog should be further investigated

4 The penetration of infra-red radiation through fog is many times that of visible light, the ratio depending upon the distance and the nature of the fog. Consequently, infra-red gives promise of great value in developing aids for fog landing, especially in localizing a fog-obscured airport.

Discussion

F. C. Breckenridge.[18] Dr. Anderson has covered a greater spread of wave length, a greater variety of fogs, and made more numerous observations than any other investigator in this field. The data he has accumulated will prove especially valuable in testing theoretical discussions of the problem.

It is to be regretted that Dr. Anderson has not given more definite data on the size of his fog droplets and the number of droplets per unit volume in his fogs. He mentions qualitative observations on the number and size of the fog particles and gives 6 microns and 8 microns as the approximate diameters of the droplets in his small- and large-particle fogs, respectively. In testing theoretical formulas it is important to know how much confidence may be placed in these estimates.

From the standpoint of fog flying, the significance of Dr. Anderson's work hinges entirely upon the degree to which his laboratory fogs reproduced natural fogs. More specifically, it depends upon the size of the fog particles, since a change in the number of drops per unit volume may be offset by an inverse change in the length of the light path. The particle size estimated agrees well with values determined by the meteorologists. If these estimates correctly represent the prevailing particle size, then the results should apply to natural fogs, and there is a conflict between these fog-chamber results and the results obtained with natural fogs by Granath and Hulburt. If, however, it is possible to assume that the average particle size in these laboratory fogs was smaller than those in Granath and Hulburt's natural fogs, then we can harmonize the results of the two researches, but in that case we are compelled to conclude that infra-red radiation less than 3.0 microns in wave length does not penetrate natural fog sufficiently for practical use. Evidently further data for infra-red transmission are needed.

In applying Dr. Anderson's remarks in regard to the sensibility of the eye, it is important to remember that he is measuring his radiation in energy units. If the radiation is measured in light units, as is usually the case in illuminating engineering problems, then the sensibility of the eye to radiation of different wave lengths enters into the rating of the light source and does not require further consideration.

E. O. Hulburt.[19] Dr. Anderson, as a result of his measurements of the transmission of light through artificial fog in a pipe 20 ft. in length, has concluded "that the penetration of infra-red radiation through fog is many times that of visible light, the ratio depending upon the distance and nature of the fog." This conclusion, if correct, would be of immense practical consequence. It is not, however, in agreement with our experiments (referred to in Dr. Anderson's paper). We experimented with the passage of light through about $^1/_4$ mile of natural fog along the Potomac River near Washington; the experiments were continued for nearly a year during various conditions of fogginess. We found, disappointingly enough, no appreciable increase in the transparency of the fog as the wave length of the light was increased from the visible red to about 3μ in the infra-red.

It seems probable that the discrepancy between our experiments and Anderson's is to be attributed to the fact that we used natural fog and he used artificial fog. Further experiments would be necessary to find out about it. The writer should like to emphasize very strongly that one cannot with safety extrapolate measurements on artificial fogs in a 20-ft. pipe to derive conclusions about the passage of light through 1000 ft. or more of natural fog. Furthermore, the facts that the sun is sometimes red at sunset, that distant lights are sometimes reddened, and that photography with light of the longer wave lengths often gives pictures free from the veiling effects of haze indicate that in haze the longer wave lengths are often transmitted more favorably than the shorter ones. But, again, one cannot with safety take these facts to mean that the same thing is true of fog, for this involves the doubtful assumption that the optical characteristics of haze are the same as those of fog.

Wesley L. Smith.[20] Dr. Anderson's paper very ably confirms the observations of those of us who have flown through all of the varying weather conditions. As early as 1925 the writer had decided that lights were of little use under severe weather conditions.

No mention is made of the type of detector that it may be necessary to use with ultra-violet or infra-red radiation. The writer would like very much to hear from Dr. Anderson on these possibilities.

Author's Closure

Mr. Breckenridge and Dr. Hulburt both have raised the question as to whether measurements of the transmission of radiation through a laboratory-made fog can be applied to a natural fog. Such a procedure is legitimate if the size of the particles in the natural fog is the same as in the laboratory-made fog. This is apparent from the following consideration. The transmission of radiation through any attenuating medium can be expressed by

$$T = \frac{I}{I_0} e^{-kx} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [4]$$

The coefficient of absorption k is a function of the radius of the particles a, the wave length of radiation λ, and the number of particles per unit volume N. So far, no satisfactory formula for this function has been obtained. However, all formulas suggested involve N to the positive first power. This is the simplest and most reasonable assumption. Hence, kx in Equation [4] can be replaced by

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[18] Photometric Section, Bureau of Standards, Washington, D. C.

[19] Superintendent, Heat and Light Division, Naval Research Laboratory, Washington, D. C.

[20] Chief Engineer, National Air Transport, Inc., Washington, D. C.

$$kx = k'Nx \qquad [6]$$

where

$$k = f(a,\lambda) \qquad [7]$$

Hence we may write Equation [4] in the form

$$T = e^{-k'Nx} \qquad [4a]$$

From this it is apparent that the transmission for a given wave length will be the same through two fogs of *different* lengths and concentration of particles, provided the particles are of the same size and the product Nx is the same for the two fogs.

An example will clarify this point. It is possible to compute the number of fog particles per cubic centimeter formed in the large-particle fog (8 microns in diameter), using the mass of excess water vapor per cubic centimeter resulting from adiabatic cooling and the mass of an individual fog particle. This comes out to be 40,000 particles per cc. Using $x = 20$ ft. (length of the fog chamber),

$$Nx = 800{,}000$$

For a distance of 1000 ft. through a fog of the same particle size and having the same transmission, $N = 800$. This is a reasonable value for a dense natural fog, judging from the meager data available. Observations by Wells and Thuras (see bulletin No. 5, 1916, U. S. Coast Guard) give a value for N in a heavy sea fog of 1200 particles per cc.

The author agrees with Mr. Breckenridge that definite information of the size and number of droplets per unit volume would add materially to the value of this investigation. Unfortunately such information is extremely difficult to obtain. The values for the diameters of the fog droplets (6 microns for the small-particle fog and 8 microns for the large-particle fog) were secured in the following manner. Through one of the small windows at the middle section of the fog chamber a small metal rod was inserted. This was insulated from the walls and charged to a high voltage. A few fog particles were attracted to this rod and lodged on it. With a micrometer microscope the diameters of such particles were measured. The number of particles observed was infinitesimal compared with the total number. Hence, this method gives only an approximation of particle size.

Possible explanations of the discrepancy between this work and Dr. Hulburt's have already been given. As he suggests, the only satisfactory way to settle the questionable points is by further measurements.

The author is in complete agreement with Dr. Hulburt in his contention that we cannot apply the laws of transmission through *haze* to *fog*. Fog particles are of a different order of magnitude, being 100 to 1000 times larger than haze particles. The phenomena of absorption and scattering are entirely different in these two kinds of disperse media.

Replying to Mr. Smith's query as to the type of detector for infra-red radiation, we have laboratory instruments (such as a thermopile and sensitive galvanometer) capable of measuring very minute quantities of infra-red radiation. But the design and construction of a robust, reliable instrument suitable for airplane installation is a difficult matter. The most feasible method seems to be the use of a photoelectric cell of infra-red sensitivity coupled to a vacuum-tube amplifier. The limitation to the development of an instrument of this type is the small infra-red sensitivity of photoelectric cells which are available at present. Much progress has been made in this direction within the past year, and hence the ultimate development of a satisfactory infra-red detector seems not far off.

A Radio System for Flying and Landing Aircraft in Fog[1]

By H. DIAMOND[2] and F. W. DUNMORE,[3] WASHINGTON, D. C.

While scheduled air transportation has been immeasurably aided by the provision of radio-direction facilities on the fixed airways, interruption of scheduled flying is still the rule whenever the landing field lies in an area completely inclosed by fog. The results secured by the development of instrument flying and of radio navigational aids to point-to-point flying are then nullified through the lack of means for safe landing under adverse conditions of visibility. The rigorous maintenance of scheduled flying by day or night requires the removal of this last great hazard to the reliability of air travel and transportation. This paper describes a radio system of blind landing aids, developed by the Aeronautics Research Division of the Department of Commerce at the National Bureau of Standards, which gives good promise for the solution of this difficult problem. The results already obtained with this system indicate that it will soon be ready for use under the severe conditions encountered in commercial air transportation. The system has been developed to be adaptable for use in conjunction with the radio navigational aids already being provided for point-to-point flying on the civil airways of the United States, and for this reason requires a minimum of additional equipment both on the ground and on the airplane.

A GENERAL idea of the operation and use of this system for flying and landing aircraft in fog may be had by reference to Fig. 1, which is a three-dimensional view showing the location of the ground transmitting equipment. Referring to Fig. 1, the 2-kw. directive radio beacon with large loop antennas, shown at *A*, is the main radio range beacon provided for point-to-point flying on the fixed airways. This beacon is normally located just off the airport (so that the loop antennas may not constitute an obstruction to flying), and serves to direct an incoming airplane to the vicinity of the airport. This beacon may be either of the aural or the visual type. A medium frequency receiving set is used on board the airplane for receiving the beacon course indications. One of the most important functions of the beacon is its use as a homing device. It not only directs an airplane along the airway route, but, in addition, indicates when the airplane is passing directly over the beacon tower, by means of a zero-signal zone effect. Utilizing this zero-signal zone, it is possible to locate the beacon to within 100 to 1000 ft., depending upon the altitude of the airplane.

Before reaching the beacon tower, the pilot has learned the wind direction at the landing field, either through the Government weather broadcast or by two-way communication with the ground. Upon receiving the zero-signal indication when directly over the tower of the main beacon, the pilot retunes his medium-frequency receiving set to the frequency of a low-power (200-watt) runway localizing beacon shown at *B*. This localizing beacon operates on the same principle as the visual-type radio range beacon for the airways. Using small loop antennas so that it may be located at one edge of the field without constituting an obstruction, it directs a course along the runway most suitable for landing (under the particular wind conditions then existing).[4] Knowing the geography of the field, i.e., the location of the main beacon relative to the desired runway, the pilot then uses his navigational instrument (such as the magnetic compass, etc.) for orienting himself along that runway. The course indications received from the runway localizing beacon greatly facilitate this maneuver.

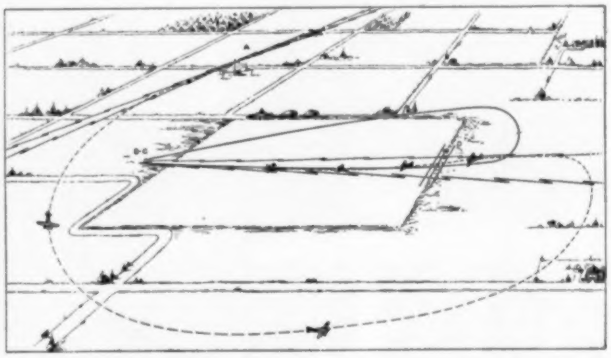

Fig. 1 Three-Dimensional View Showing Radio System for Flying and Landing Aircraft in Fog

Lateral position is thus given to the pilot by means of the runway localizing beacon *B* for the purpose of keeping the airplane directed to and over the desired landing-field runway. It is now necessary to provide means for vertical guidance which will direct him along a safe landing path, and also for longitudinal guidance to inform him that he has arrived at the boundary of the landing field. Vertical guidance is given by an inclined ultra-high-frequency radio-beam transmitter located at *C*. This landing beam operates on a frequency of the order of 100,000

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[1] Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

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[3] Bureau of Standards. Born January 24, 1891, at Haverhill, Mass. Commercial radio operator, summers, 1911–13. B.S. degree, Pennsylvania State, 1915. Student engineering course, General Electric Company, 1915–17; research department, American Radio and Research Corporation, 1918; radio laboratory staff, Bureau of Standards, 1918 to date, specializing in research work on the radio direction finder, directional transmission, and radio aids to marine and air navigation.

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] At a landing field having two runways, perpendicular to each other, two localizing beacons are necessary so that the pilot may land approximately into the wind. But one runway beacon will, however, be operated at a given time.

kilocycles and is directed at a small angle above the horizontal. It is used in such a way as to provide a very convenient gliding path for the landing airplane. Longitudinal position is given by a boundary marker beacon located at the edge of the landing field (*D*, Fig. 1) and operating on the same radio frequency as the runway localizing beacon. When crossing the boundary of the landing field, the pilot receives a distinctive aural signal from the marker beacon.

Description and Operation of Ground and Airplane Equipment

Before going into a detailed analysis of the use of the system by a pilot in order to effect a landing in dense fog, it is necessary to describe briefly the ground and airplane equipment employed and the manner of operation of this equipment.

Runway Localizing Beacon. This is a 200-watt, double-modulation, visual-type beacon, using modulation frequencies of 65 and 86.7 cycles, respectively. The beacon apparatus is housed in a two-story shed located on the border of the landing field and in line with the runway along which the course is to be directed. (See Fig. 2.) The transmitting set and machinery are shown on the first floor and the transmitting loop antennas on the second floor. These antennas consist of seven turns of wire wound on wooden frames 6 ft. high by 8 ft. in length.

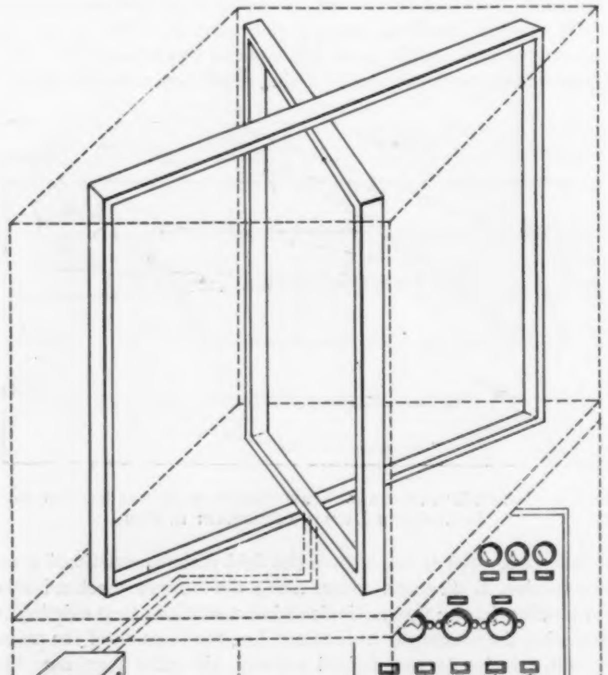

Fig. 2 Runway-Localizing Beacon Set-Up, Showing Location of Loop Antennas, Transmitting Set, and Power Equipment

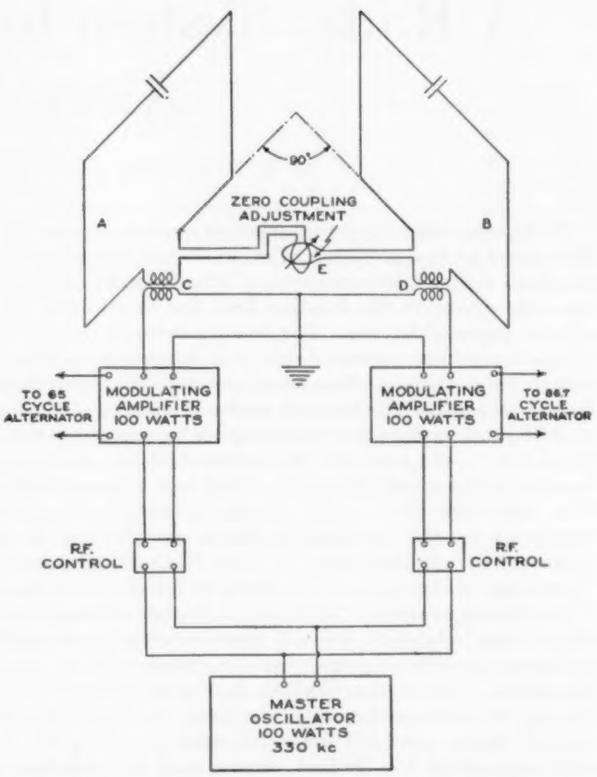

Fig. 3 Transmitting Circuit Arrangement for Runway-Localizing Beacon

The transmitting circuit arrangement is shown in Fig. 3. To simplify apparatus, the transmitter is coupled to the antenna system through two coupling transformers, *C* and *D*, rather than by means of the goniometer employed in the usual radio range beacon set-up. The variometer *E* serves to neutralize any residual inductive coupling between loop antennas *A* and *B*.

Referring to Fig. 3, the two loop antennas, crossed at 90 deg., carry currents of the same radio frequency but modulated to two different low frequencies, 65 and 86.7 cycles, respectively. These antennas are so oriented that the vertical plane containing the axis of the desired runway bisects the angle between the two antennas. An airplane flying in this plane therefore receives equal signals from the two antennas. On either side of this plane the signal received from one antenna is greater than from the other. On the airplane, a visual course instrument indicates to the pilot the relative received signals from the two antennas and, consequently, the relative position of the airplane with respect to the runway.

Boundary Marker Beacon. The boundary marker beacon operates on the same radio frequency as the runway localizing beacon. It employs a horizontal open antenna consisting of a horizontal wire, approximately 6 ft. above the ground and 200 ft. long, extending across the runway at the edge of the landing field. The pilot receives an aural signal beginning at about 100 ft. on one side of the boundary line and ending the same distance on the other side. Since a vertical-pole antenna is employed on the airplane, this signal is interrupted for a brief interval directly over the landing-field boundary line because of the zero-signal zone effect.

The transmitting set employs two low-power transmitting tubes excited alternately from a 500-cycle plate-supply transformer and feeding a common oscillatory circuit, coupled to the horizontal antenna. The tube filaments are also supplied from the 500-cycle source through a suitable winding on the 500-cycle transformer. Due to the alternate excitation of the transmitting tubes, the resultant modulation has a frequency of 1000 cycles, which is the aural signal heard by the pilot.

Receiving System for Localizing and Marker Beacon Signals. The receiving system on the airplane is as shown in Fig. 4. The medium-frequency receiving set normally employed for the reception of the weather broadcast and radio-beacon services is also used for receiving the signals from the runway localizing and marker beacons. The localizing beacon, being of the visual type, permits the use of automatic volume control in its reception. This is quite essential, since the pilot, in making a landing, is concerned with so many things that the burden of close manual adjustment of receiving-set sensitivity must be eliminated.

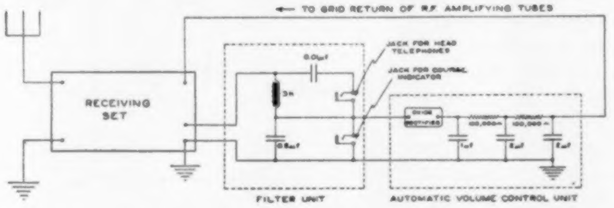
← TO GRID RETURN OF R.F. AMPLIFYING TUBES
RECEIVING SET
3H
JACK FOR HEAD TELEPHONES
JACK FOR COURSE INDICATOR
OXIDE RECTIFIER
100,000Ω
100,000Ω
1μf
2μf
2μf
FILTER UNIT
AUTOMATIC VOLUME CONTROL UNIT

FIG. 4 CIRCUIT ARRANGEMENT FOR RECEIVING RUNWAY-LOCALIZING BEACON AND MARKER BEACON SIGNALS

The volume-control device operates through rectification of the alternating voltage across the course-indicator terminals and application of the rectified voltage, after filtering, as a negative biasing voltage on the grids of the radio-frequency amplifying tubes of the receiving set. An increase in the input voltage to the receiving set normally tending to increase the voltage across the indicator terminals is therefore accompanied by an increase in the negative bias on the radio-frequency amplifying tubes, and consequently by a reduction in the receiving-set sensitivity such as to maintain substantially constant voltage across the course indicator.

The provision of this automatic volume-control device makes possible the addition of an important instrument on the airplane instrument board, giving the pilot information as to his approximate distance from the localizing beacon transmitter. Since increasing input voltage to the receiving set is accompanied by increasing negative biasing voltage on the grids of the radio-frequency amplifying tubes, the deflection of a direct-current milliammeter connected in the plate circuit of these tubes will decrease as the airplane approaches the beacon transmitter. This meter may therefore be calibrated directly in approximate distance from the transmitter. Its operation is of considerable assistance to the pilot in maintaining a constant radius when circling the localizing beacon (during the process of finding the desired runway), and also in giving the pilot sense of approach to the landing field, once he has oriented himself along the runway.

A filter circuit is connected in the receiving set output to direct the aural signals (1000 cycles) received from the boundary marker beacon to the pilot's head telephones, and the double-modulation signal received from the runway localizing beacon to the visual course indicator. The automatic volume control is connected across the course-indicator terminals, and consequently is actuated only by the localizing beacon signals. The 1000-cycle marker beacon signal is therefore permitted to vary through a minimum even though the receiving-set output voltage (in so far as the course indicator is concerned) is maintained substantially constant.

The course indicator may be either the reed indicator type, in which case the runway course indications received by the pilot are as shown in Fig. 5 or the zero-center pointer instrument type shown in Fig. 6. In the latter arrangement, the pointer remains at the zero mark when the airplane is on the course and swings to right or left as the airplane deviates to the right or left of the course. This instrument is operated by a reed converter, i.e., a reed indicator in which, besides the set of driving coils for operating each reed from the received beacon signals, a set of pick-up coils is provided for each reed. The motions of the two reeds induce alternating voltages in the two pick-up coils, which are proportional to the reed-vibration amplitudes, and consequently to the intensities of the two low frequencies constituting the received beacon signal. The induced alternating voltages are rectified by means of copper-oxide rectifiers and the rectified voltages applied in opposition to the terminals of the zero-center instrument which the pilot sees. The pointer-type instrument is preferable for landing purposes, since it offers the advantage of combining the runway course indicator with the landing beam indicator into a single convenient landing instrument which will be described below.

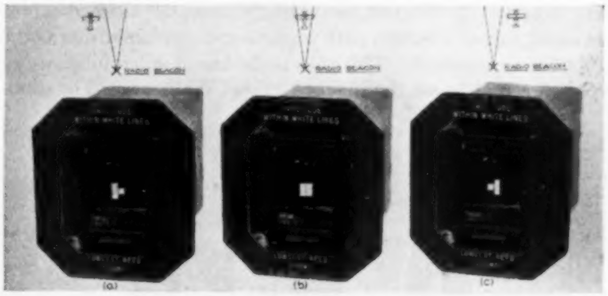

FIG. 5 COURSE INDICATIONS AS RECEIVED ON VIBRATING-REED COURSE INDICATOR

(*a*, unequal reed amplitudes, airplane off course to left; *b*, equal reed amplitudes, airplane on course; *c*, unequal reed amplitudes, airplane off course to right.)

The Landing Beam. The function of the landing beam is to furnish a pilot with the necessary vertical guidance to permit landing under conditions of no visibility. It consists essentially of a horizontally polarized beam directed at a small angle above the horizontal, this angle and the degree of directivity being so adjusted that a predetermined line of constant field intensity on the under side of the beam will mark out just the proper gliding path, clearing all obstructions and convenient for landing. In the set-up at College Park, Md., the beam is transmitted on a frequency of 93,700 kc. (3.2 m.) and is oriented in the same horizontal direction as the course of the runway localizing beacon. On the airplane, a horizontal dipole antenna feeding a detector-amplifier-rectifier unit is employed for receiving the landing beam

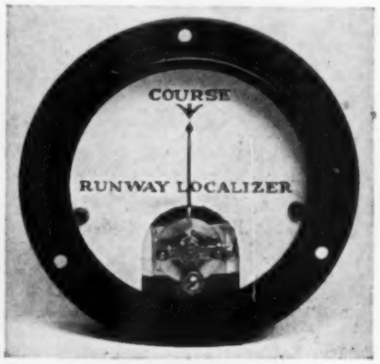

FIG. 6 POINTER-TYPE LOCALIZING-BEACON COURSE INDICATOR

signal. As will be evident below, the receiving equipment constitutes essentially a vacuum-tube voltmeter arrangement for exploring the field intensity in different portions of the landing beam.

(1) Method of Using the Landing Beam. Reference to Fig. 7 will show how suitable indication of absolute height above ground is secured by means of the landing beam. The vertical space pattern of the inclined ultra-high-frequency landing beam is clearly indicated in this figure. The airplane is readily directed approximately into the vertical plane containing the axis of the beam by means of the course indications from the runway localizing beacon. It does not, however, fly along the inclined axis of the beam, but on a curved path whose curvature diminishes as the ground is approached. This path is the line of equal intensity of received signal below the inclined axis of the beam. The diminution of intensity as the airplane drops below the inclined axis is compensated by the increase of intensity due to approaching the beam transmitter. Thus by flying the airplane along such a path as to keep the received signal intensity constant, as observed on a microammeter on the instrument board, the pilot comes down to ground on a curved line suitable for landing. If the airplane rises above this line of equal intensity of received signal, the microammeter deflection increases, while if it drops below this line the microammeter deflection decreases.

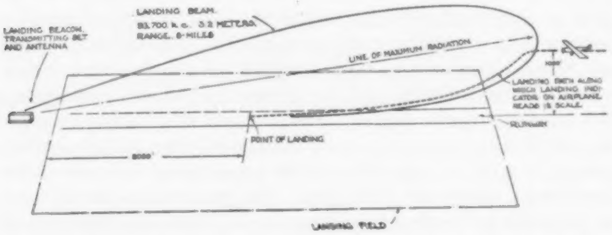
LANDING BEACON, TRANSMITTING SET AND ANTENNA
LANDING BEAM. 93,700 K.C. 3.2 METERS RANGE, 8 MILES
LINE OF MAXIMUM RADIATION
LANDING PATH ALONG WHICH LANDING INDICATOR ON AIRPLANE READS 15 SCALE
RUNWAY
POINT OF LANDING
LANDING FIELD

FIG. 7 VERTICAL DIRECTIVE CHARACTERISTIC OF THE 93,700-KC. LANDING BEAM, SHOWING LANDING GLIDE PATH

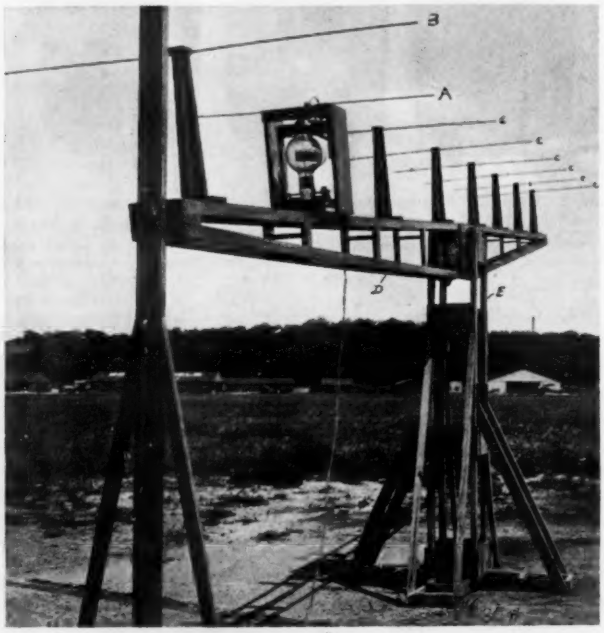

FIG. 8 LANDING-BEAM TRANSMITTING SYSTEM, SHOWING ELECTRON-TUBE OSCILLATOR AND DIRECTIVE ANTENNA ARRAY

Several important advantages obtain for this method of furnishing altitude indication. The landing path may be so directed as to clear all obstructions. The pilot following the landing path is automatically kept above obstructions and no longer needs a thorough knowledge of the terrain in order to effect a safe landing. Secondly, the landing path may be of different shape to suit different landing fields. This is of particular importance in getting into a small field. A third advantage lies in the fact that in the act of following the landing path, the pilot automatically "levels off," thereby facilitating a normal landing, albeit somewhat fast. A fourth advantage is that the landing path may begin at any desired altitude within a rather wide range (500 to 5000 ft.).

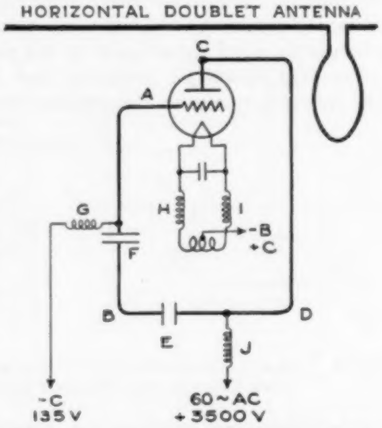

FIG. 9 CIRCUIT ARRANGEMENT OF 93,700-KC. LANDING-BEAM OSCILLATOR

(2) Directive Transmitting Antenna System. An ultra-high frequency was chosen for the landing-beam transmitting system in order to secure the attendant reduction in size and simplicity of equipment. Of a number of directive antenna systems studied, the arrangement developed by Unda and Okabe, and described by Yagi, for the production of horizontally polarized directed beams, was found most suitable. A photograph of the directive antenna array, as set up at College Park, is shown in Fig. 8. This is housed in a shed for protection against weather. The ultra-high-frequency source (93,700 kc.) is coupled to the horizontal doublet *A* (made of $^1/_4$-in. copper tubing), which serves as the radiating antenna and is accurately tuned to the frequency of the source. About 0.8 m. behind the radiating antenna is placed a reflecting antenna *B*, also a horizontal doublet, tuned to a frequency somewhat lower than the frequency of the source. At approximately every meter in front of the radiating antenna, horizontal doublet directing antennas *C* are placed. These are tuned to a frequency somewhat higher than that of the source. This array of antennas is supported on a horizontal wooden structure *D*, approximately 2.75 m. above the ground, and pivoted on the vertical support *E*. To obtain the vertical directive characteristic shown in Fig. 7, the wooden structure *D* is tilted approximately 8 deg. above the horizontal.

(3) The Ultra-High-Frequency Transmitting Circuit Arrangement. The necessary power output on the high frequency used is secured through the use of a 500-watt three-element tube (General Electric ZP-2) in the oscillatory circuit shown in Fig. 9. The small air condenser *E* is of a capacity about equal to the interelectrode grid-to-plate capacity of the tube. The plate inductance *CD* and the grid inductance *AB* are placed between this condenser and the tube, the leads making the shortest possible connections between these elements of the circuit constituting the inductances. Condenser *F* is of relatively large capacity and is necessary only as a safety measure in case of flashover of condenser *E*. *G*, *H*, *I*, and *J* are suitable choke coils to keep the oscillatory current out of the power-supply leads.

Fig. 10 shows the 500-watt tube and associated apparatus, including the horizontal doublet radiating antenna. The tube is operated with 3500 volts of 60-cycle frequency on the plate and a negative grid bias of about 135 volts. The tube circuit is tuned

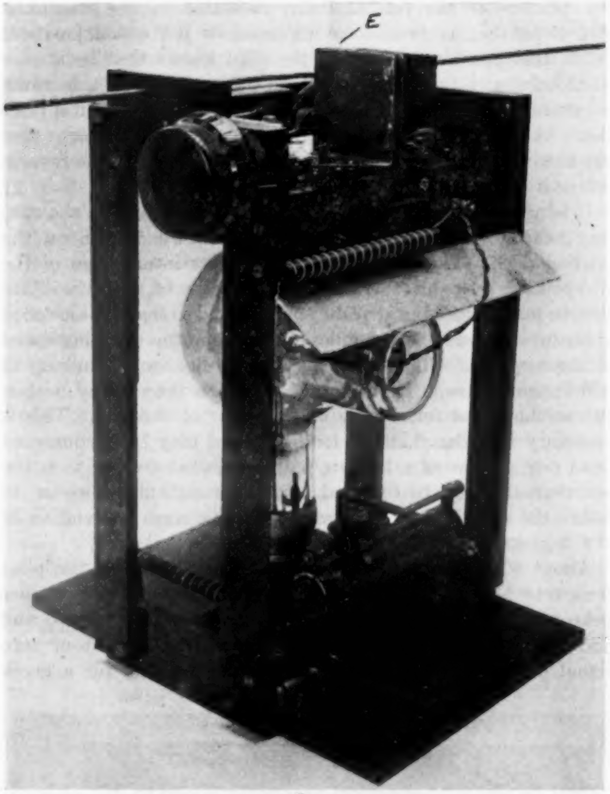

FIG. 10 ELECTRON-TUBE OSCILLATOR AND RADIATING ANTENNA FOR THE 93,700-KC. LANDING BEAM (AS SHOWN AT *A*, FIG. 8)

to the desired frequency within narrow limits by moving one of the plates of condenser *E* by means of an insulated adjustment screw. The frequency has proved sufficiently stable for use without external means for frequency control, due in part to the fact that but one broadly tuned circuit is used at the receiving end. The 60-cycle modulation facilitates audio-frequency amplification at the receiving end.

(4) The Receiving System. The receiving circuit arrangement (see Fig. 11) uses only two tubes without regeneration. This receiving circuit requires no adjustments on the part of the pilot. The tuning is fixed. Even the volume control is dispensed with, since the path followed during the use of the receiving set constitutes a line of constant field intensity of the directed beam.

A 224 heater-type screen-grid tube is employed for the detector, to afford the necessary high amplification without undue microphonic noises. To obtain good efficiency it is necessary to connect the detector tube directly in the center of the horizontal doublet antenna. The radio-frequency portion of the circuit (i.e., the detector) is confined to the section above the four radio-frequency chokes. The four leads running from the lower side of these chokes carry either direct current or the received audio modulation. The detecting portion of the receiving circuit is external to the airplane, being mounted in a streamline weatherproof box about 14 in. above the top wing. The doublet antenna is in the form of two copper rods housed in wooden streamlined supports projecting from the streamlined detector box. The streamlined detector box and antenna system are arranged to plug in electrically to the supporting upright on the wing, a five-terminal plug making the necessary connections. The supporting upright also makes a plug connection to a socket in the wing. The rest of the apparatus, which includes the audio amplifying tube and transformer, oxide rectifier, *A* and *B* batteries, and indicating instrument, are located within the airplane.

Fig. 12 at *A* shows an inside view of the streamlined detector box. The amplifier-rectifier unit is shown at *B*. The oxide rectifier shown eliminates one tube, and has been found perfectly stable in its operation at the low frequency employed.

The rectified output current from this receiving set is fed to a direct-current microammeter constituting the landing beam indicator. (See Fig. 13.) To facilitate its use, this instrument is mounted on its side so that the pointer moves vertically instead of horizontally. During landing, the deflection, to be kept constant (corresponding to the desired glide path), is chosen at half-scale reading, the instrument pointer being then in horizontal position. A rise of the pointer above this position indicates that the airplane is above the proper landing path, while the reverse is true if the pointer falls below its horizontal position. The indications of the position of the airplane relative to the landing path are thus made readily comprehensible.

(5) Combined Landing Instrument. The use of the landing system described herein has been greatly facilitated by the combination of the runway-course-indicating instrument shown in Fig. 6 with the beam-indicating instrument shown in Fig. 13 into a single instrument, as shown in Fig. 14. Two perpendicular reference lines are provided on the face of the combined instru-

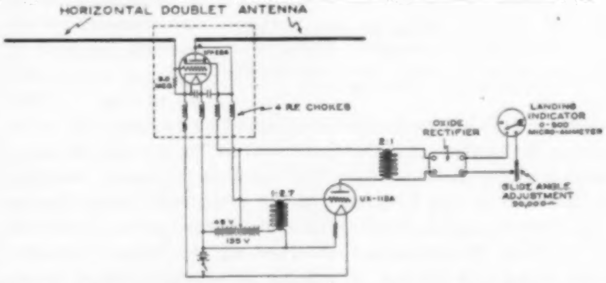
HORIZONTAL DOUBLET ANTENNA
UY-224
3.0 MEG
4 RF CHOKES
OXIDE RECTIFIER
2:1
LANDING INDICATOR 0-500 MICRO-AMMETER
GLIDE ANGLE ADJUSTMENT 50,000 ~
1:2.7
UX-112A
45 V
135 V

FIG. 11 CIRCUIT DIAGRAM OF SPECIAL ULTRA-HIGH-FREQUENCY LANDING-BEAM RECEIVING SET

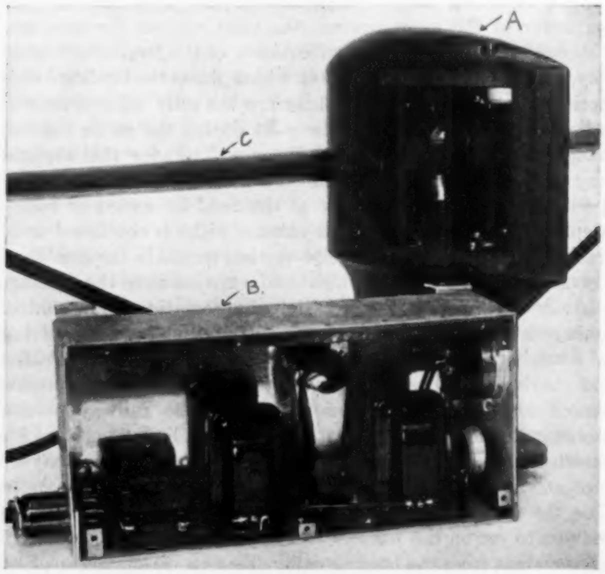

FIG. 12 SPECIAL RECEIVING SET FOR RECEIVING LANDING-BEAM SIGNALS

(The detector is shown at *A*, the amplifier rectifier unit at *B*, and the horizontal doublet antenna at *C*.)

ment, the vertical reference line corresponding to the position of the runway and the horizontal reference line to the proper landing path. The pointers of the runway-course indicator and the landing-path indicator are arranged so that they cross each other, the former moving to the right or left of the vertical reference line and the latter above or below the horizontal reference line. The position of the point of intersection of the two pointers thus gives, through a single reading, the position of the airplane with respect to the runway and proper landing path.

(6) Complete Radio Equipment Required on the Airplane. The complete radio equipment required on the airplane by the radio system of blind flying and landing aids described in this paper is indicated by Fig. 15, which is self-explanatory.

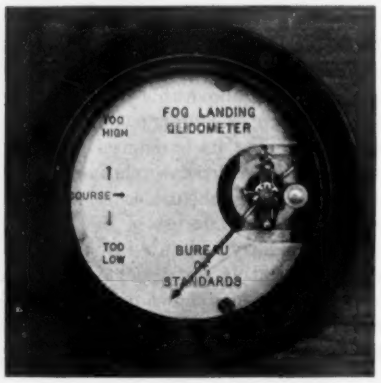

Fig. 13 Landing-Beam Indicator Used on Pilot's Instrument Board

(This is to show the position of the airplane relative to the proper landing path.)

Use of the Radio System of Blind Landing Aids

Method of Making a Landing. Let us now consider the operations required on the part of the pilot in making a blind landing. It is assumed that the information necessary for maintaining the airplane in stable flight is available through the use of suitable flight instruments. The medium-frequency receiving set is tuned to the frequency of the main radio range beacon (say, 280 kc.) and indications as to the correct course leading to the vicinity of the airport observed on the course indicator. Since automatic control of volume is provided, these course indications are available to the pilot without manipulation on his part.

Upon receiving the zero-signal indication when directly over the tower of the main beacon, the pilot retunes the medium-frequency receiving set to the frequency of the localizing beacon (say, 330 kc.) and throws a switch which places the landing-beam receiving set in operation. These are the only adjustments of radio equipment required of the pilot during the entire landing maneuvers. By watching the distance indicator the airplane may be made to circle the field on a fixed radius. An altitude specified by the control officer at the field by means of radiotelephony is maintained. This circular flight is continued until orders are received to land. The vertical needle in the combined instrument, Fig. 14, now furnishes information as to the landing-field runway direction, the provision of automatic volume control again relieving the pilot of the necessity for adjusting the receiving set sensitivity for variations in the distance between the airplane and the localizing beacon transmitter. The pilot next orients himself along the desired runway, using the runway beacon course indications, together with his compass indications and his knowledge of the geography of the field. The airplane may be kept at any altitude within, say, 500 to 5000 ft. From time to time the pilot glances at the rough-distance indicator and endeavors to get on the runway course at a point approximately 5 miles distant from the landing field. He now directs his airplane along the runway course and in the direction of the landing field, by navigating until the vertical needle of the combined instrument, Fig. 14, is over the central circle on the instrument face. When approximately half the distance to the field is covered, the pointer of the landing-beam indicator in the combined instrument begins to rise to midscale, or horizontal position. When this position is reached, the pilot knows that he is at a point on the gliding path. To follow the gliding path he must maneuver the airplane to keep this pointer in a horizontal position. A pointer position above the central circle indicates that the airplane is above the proper landing path, while the reverse is true if the pointer swings below this circle.

It is necessary now that the pilot keep accurately on the runway beacon course, at the same time following down on the landing path. This is done by holding the intersection of the two pointers over the central circle. (See Fig. 14.) To facilitate this, he may reduce the airplane engine speed so that the air-speed indicator registers about 5 miles above the normal landing speed of the airplane. This also helps during the actual landing, as will appear below. The accuracy to which the runway beacon course should be followed is of the order of ±3 deg. This is necessary in order that the landing beam may be encountered head on; otherwise, a landing path somewhat steeper than the one desired would be followed. It is not difficult, however, to follow the course to the necessary accuracy, since a deviation of ±1 deg. may be observed on the combined instrument.

About 100 ft. from the landing-field boundary line the pilot begins to hear a 1000-cycle signal from the boundary marker beacon. This signal increases rapidly, decreases to zero, and then passes through another maximum. The instant of zero signal defines the landing-field boundary line, and for a given field informs the pilot that he is at a definite distance above ground. He may now throttle down his engine to landing speed and maneuver his airplane to follow the landing path accurately to the point of landing.

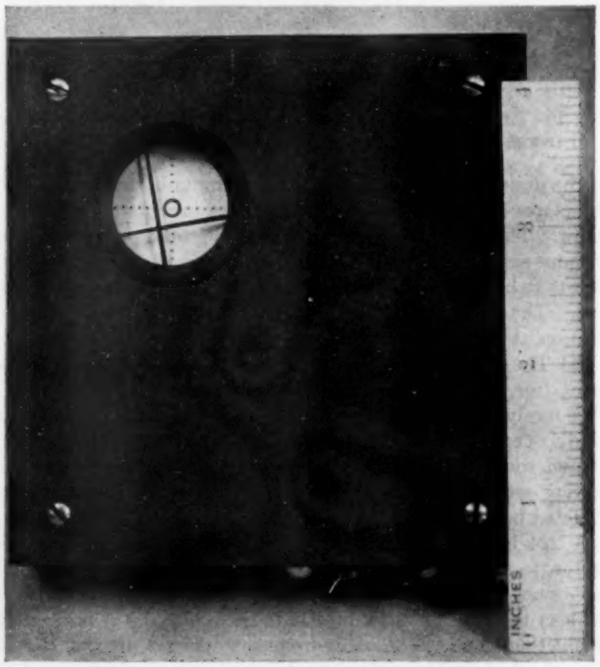

Fig. 14 Combined Instrument Giving Indication of the Position of the Airplane Relative to Both the Runway and the Landing-Beam Path

(The position of the intersection of the pointers shows the location of the airplane, the vertical dotted line represents the runway, and the horizontal dotted line the proper landing path. In the photograph the indication is that the airplane is to the left of the runway and below the proper landing path.)

Expansion of System to Take Care of Wind Direction. A number of possible arrangements of the system ot blind landing aids outlined above, whereby wind direction may be taken into account, are available. The problem is to determine the most economic arrangement; for example, it has been suggested that the localizing beacon and landing-beam transmitters may be buried in the center of the field and oriented into the wind at all times by remote control from the operations office, or even automatically. Aside from the technical problems involved, the provision of a sufficiently large waterproof room with sturdy roof construction level with the surface of the ground, so that an airplane might roll over it, seems economically unfeasible. In addition to the consideration of costs, the arrangement noted is open to the objection that the landing-beam transmitter would require special adjustment in each new position of orientation. The shape of the landing path should be convenient for landing and at the same time insure the clearance of all obstructions. Since the obstructions are quite different when coming into the average landing field from different directions, the need for the adjustment of the landing-beam transmitter is apparent.

Perhaps the most feasible arrangement is the one shown in Fig. 16. The transmitting equipment for both the localizing beacon and landing beam is so simple that the duplication shown appears warranted. It is estimated that the commercial cost of the complete installation would not exceed $15,000. An obvious advantage is that the equipment once set up remains in permanent adjustment.

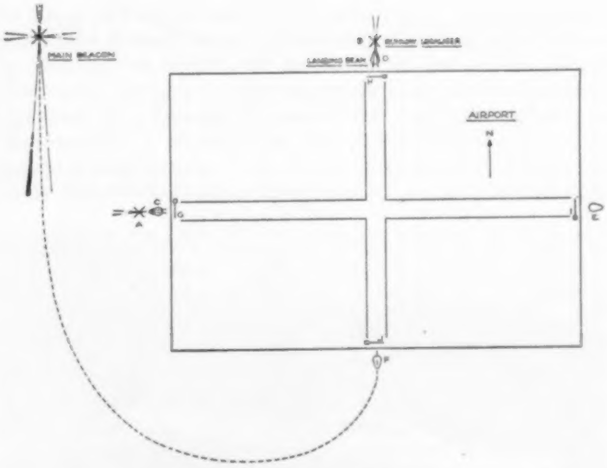

FIG. 16 SYSTEM OF BLIND LANDING AIDS NECESSARY TO TAKE CARE OF WIND DIRECTION ON A TWO-RUNWAY FIELD

Referring to Fig. 16, the beacon arrangement shown is for a two-runway field. Two runway localizing beacons, *A* and *B*, and four landing beam transmitters, *C*, *D*, *E*, and *F*, are required. Four boundary marker beacons, *G*, *H*, *I*, and *J*, are necessary. At any given time but one localizing beacon, one landing-beam transmitter, and one boundary-marker beacon are in operation, being so chosen as to require landing into the wind.

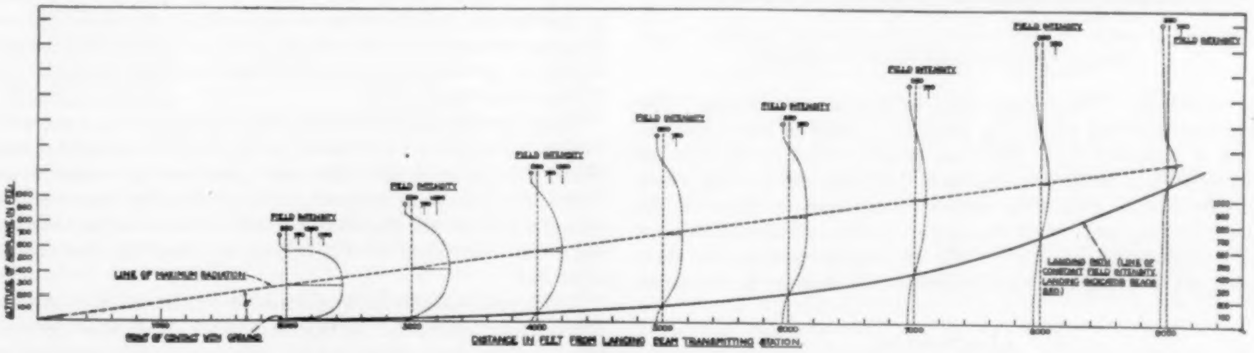

FIG. 17 LANDING GLIDE PATH

In Fig. 16 the wind is from the north, and the airplane is shown landing into the wind, using runway-localizer beacon *B*, landing-beam beacon *D*, and boundary-marker beacon *J*.

Practical Applications. The practicability of the radio system of blind-landing aids described has been partially demonstrated in numerous test flights on the Aeronautics Branch experimental airplanes. Fig. 17 shows the landing glide path as found by actual flight tests at College Park, Md. No difficulty has been experienced in following both the runway-localizing beacon and the landing-beam indications simultaneously. Arrangements are now being made for actual blind-landing tests, the results of which it is felt will completely demonstrate the feasibility of the system.

Even in its simplest arrangement (see Fig. 1), which does not take care of wind direction, this system should have numerous applications to scheduled flying. It should successfully prevent practically all interruptions to scheduled service due to dense fog, since fog is rarely accompanied by strong winds. Under special conditions of low visibility (for example, when the airport has a

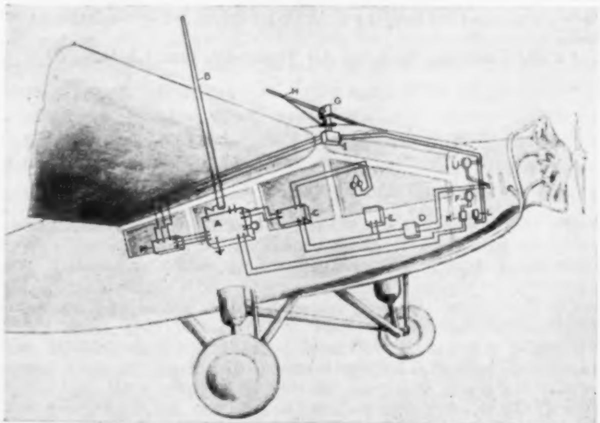

FIG. 15 COMPLETE AIRPLANE INSTALLATION SHOWING RADIO EQUIPMENT REQUIRED TO MAKE USE OF THE RADIO SYSTEM OF BLIND FLYING AND LANDING AIDS

(*A*, medium-frequency receiving set for voice and beacon signals; *B*, 6-ft. vertical-pole antenna; *C*, filter unit for separating voice and beacon signals; *D*, tuned-reed converter; *E*, automatic volume control for beacon signals; *F*, rough distance indicator; *G*, landing-beam receiving-set detector unit; *H*, horizontal doublet antenna for landing-beam receiving set; *I*, landing-beam receiving-set amplifier unit; *J*, combined landing instrument; *K*, pilot's control unit for medium-frequency receiving set; *L*, pilot's control and test unit for landing-beam receiving set; *M*, *A*, *B*, and *C*, batteries.)

ceiling of 100 ft. or more) the system may be used as an aid to landing regardless of wind conditions. A particularly interesting application consists of the use of this system for complete or partial blind landing on seadromes or on airplane carriers, where the landing deck can at all times be oriented into the wind. Such a proposed seadrome, with main beacon *A*, runway-localizer beacon *B*, fog-landing beam *C*, and boundary-marker beacon *D*, is shown in Fig. 18. A large number of conditions exist where a partial use of the system described appears practicable. The runway-localizing beacon, in particular, seems to have possibilities of extensive use. This was demonstrated in the tests at Mitchel Field, under the auspices of the Guggenheim Fund, where a special landing altimeter was employed in conjunction with the localizing beacon. With the expansion of the system to take care of wind direction, as shown in Fig. 16, it is felt that the last great obstacle to flying under any conditions of weather is overcome.

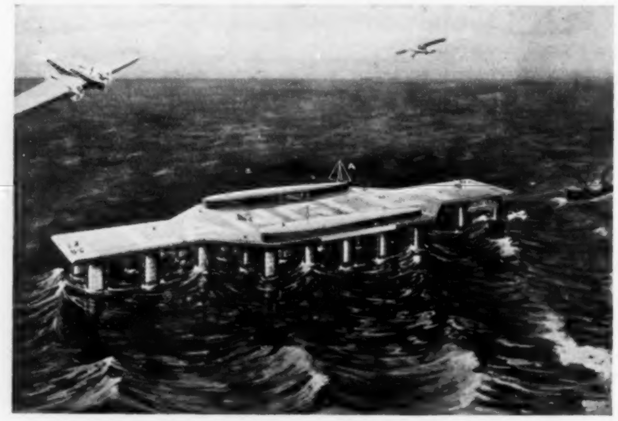

FIG. 18 METHOD OF APPLYING BLIND FLYING AND LANDING RADIO AIDS TO PROPOSED ARMSTRONG SEADROMES

Discussion

H. N. WILLETS.[5] The Bureau of Standards is to be complimented on the energetic steps it is taking to further the use of radio equipment now being flown by transport lines so that in addition to the two-way radio-telephone service the pilot may have the advantage of directional aids.

The present two-way radio-telephone system provides full control of the plane by the ground personnel, and the installed beacon transmitters of the aural type lay out a definite radio path that the pilot may follow. The recently developed anti-static headphones enable the pilot to use the full sensitivity of his equipment without being subjected to the discomfort of static shocks. There still seems to be a question as to whether the reed type of indication is preferable to the aural type, but in recent tests where visual transmission was uninterrupted when the voice transmission was broadcast this would indicate a desirable development.

The present installed two-way radio-telephone equipment on the transport lines will lend itself to visual indication when this system is perfected.

It may not be generally appreciated, but within the last year the air-transport companies have installed over a hundred of their own ground radio-telephone transmitters and the investment for this apparatus is over $1,800,000.

WESLEY L. SMITH.[6] This radio landing beam seems to supply the missing link that is necessary for the regular scheduled operation of airplanes through any weather conditions. With the aid of the present flight instruments and the radio range beacons along the airways, we have been able to fly our planes on schedule whenever we could be assured of some visibility beneath the clouds at the destination.

There are still some problems of coordination of this aid with the other aids that we now have, but having flown on this landing beam, the writer is sure that these problems are simple ones. This system can be developed to be more accurate than a pilot's eyesight and he will therefore be able to make better landings using the beam than he will be able to when the visibility is unlimited.

It is hoped that this landing beam can be installed soon at some commercial airport, such as that at Newark, N. J., where ground fogs are prevalent, so that as many pilots as possible can have an opportunity to try the beam. It is only by putting these things into regular use that we get them fully developed, and there is a very great need for landing aids under ground-fog conditions.

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AER-54-3

General Maintenance and Suggestions for Lowering Overhead

By H. C. DOWNEY,[1] MITCHELL FIELD, N. Y.

When an airplane crashes, it is easy to set fire to it and write it off, but the author contends that half of the wrecked planes, except where they have been badly burned, may be salvaged and rebuilt. He gives many details of where and how this may be done. The value of constant inspection is emphasized, and the paper describes in detail many points sometimes overlooked. Special tools for difficult repair jobs are outlined.

PLANES crash occasionally, as all well know. The first impulse in the case of an airplane crash is to junk the wrecked machine. It is so easy to set fire to a wrecked plane, with gasoline on it, that no doubt many planes have been burned up, with no salvage, which could have been rebuilt at a small percentage of the cost of a new machine. It would probably be good news to the insurance companies if more of them were repaired. With the exception of a plane that has been consumed by fire, there are few wrecks that cannot be rebuilt. The question is of course whether they can be rebuilt economically. The author has several times rebuilt planes successfully that other people thought advisable to junk. At least half of the total number of wrecked airplanes can be rebuilt without undue cost and at a profit to the operator.

It is not intended to infer that people have not been conscientious in the matter of salvaging wrecks. On the contrary, they have been overconscientious. They seem to feel that, even though a member does look all right, there may be some hidden defect that will fail later on, whereas if they knew more about materials they would not be under this impression.

It can be said without qualification that inspection is the most important function in the operation of airplanes other than the actual piloting. During actual flying operations, all parts cannot be seen, but during overhaul everything must be scrutinized for cracks or wear. Control cables should be replaced. Electrical wiring also should be replaced. Care should be taken that control parts do not rub. Welds must be inspected for cracks. Bent members should be replaced or trued up. Bearings must be checked for clearance. Rigging must be accurate. Corroded parts must be replaced or otherwise repaired. Propellers must be balanced and checked for blade angle. Landing gears must be trued, and if of the oleo type, the oleo chambers must be inspected.

Those who have been concerned with the overhaul of aircraft have very often noted that the old wooden-frame airplanes were capable of taking shocks that would have caused failure in a metal structure. Wood is capable of sustaining great loads for a short time with no more damage usually than elongated bolt holes. In the same circumstances steel would probably have failed altogether. Now that we have oleo landing gears, the airplanes are not so likely to be subjected to great shock.

The reason that wood stands up so well is that, for the same strength and weight as steel, wood presents more bulk and consequently more surface to take up the load. This is also true of duralumin, but to a lesser extent.

In the service there is a large turnover in aircraft and a great amount of salvage, but the services are concerned with obsolescence as well as with depreciation. Indeed, obsolescence is a more serious problem than depreciation, since the military services must constantly encourage manufacturers to improve performance and to keep abreast of the developments in other parts of the world.

In commercial service, obsolescence of type would of course enter into the matter, but not nearly so much as in the fighting service. It is thought therefore that a much greater amount of repair and working would be done. There appears to be no reason, other than obsolescence, why aircraft should not be reworked and overhauled many times during the life of the craft, up to perhaps as much as ten years. There have been cases in the service where airplanes were in operation for as much as seven or eight years and in some isolated instances as much as ten years. During this time, naturally many parts were changed, and possibly different engines were installed.

When an engine is overhauled several times, the custom in the service is for the engine to retain its identity under a certain number until such time as the upper half of the crankcase is cracked or otherwise damaged and worn, when it is written off.

Rigging

One of the most interesting phases of overhaul work, and perhaps the least understood and appreciated, is the rigging department. In spite of the attempts of designers to eliminate the rigging feature and its attendant troubles, it still is with us, since even on a rigid-wing monoplane we must rig our controls if the airplane is to handle properly. We probably shall always have a certain amount of rigging to do on aircraft, since adjustments to controls are indispensable and inevitable.

Some rather peculiar things happen to ailerons. The author recalls that once in Panama we set up a DH at the depot, with new wings and ailerons out of stock, and proceeded to test-fly it. Test pilots reported that it had most abominable aileron control. The author flew it and found that they were right.

Rigging was checked from one end to the other, but no error

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[1] Lieutenant Downey was born on Oct. 4, 1894, at Dobbs Ferry, N. Y. He was educated at Dobbs Ferry High School and New York University. Was graduated from Air Corps Engineering School, at McCook Field, Dayton, Ohio. Worked as mechanic, draftsman, and engineer for Gallaudet Engine Corp., 1915; Aeromarine Plane and Motor, 1915; Standard Aircraft Corp., 1916; Chas. F. Willard, 1916. Designed and flew flying boat, 1916–1917; entered Army Air Service, 1917; commissioned as Second Lieutenant, Air Service, 1918; worked for Curtiss Aeroplane and Motor Corp., 1919; participated in bombing of German naval vessels off Virginia Capes in 1921; made several long flights testing earth inductor compass while at McCook Field, 1924; Engineer Officer, France Field, Canal Zone, 1928–1929; Assistant Engineering Officer, Mitchell Field, 1931. Has flown 3000 or more hours on all types.

Presented before a meeting of the Metropolitan Section, New York, N. Y., Feb. 25, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

was found anywhere. The usual trouble in a case like this is due to warped trailing edges on ailerons, but these were found perfectly straight.

This airplane caused a great amount of trouble for awhile, until finally the difficulty was located. There were two kinds of DH wings in stock. One kind had little filler strips between the ribs at the rear spar where the aileron was attached, while the other did not. Likewise, the ailerons in one case had these strips, and in another type they did not. These strips were only $^{3}/_{16}$ in. thick, and there was hardly any noticeable difference in the wings and ailerons when assembled. Nevertheless when the wings without the strips were assembled with ailerons that had the strips, or vice versa, the resulting difference in the air flow was so great that one could not bank the plane without using both hands on the control.

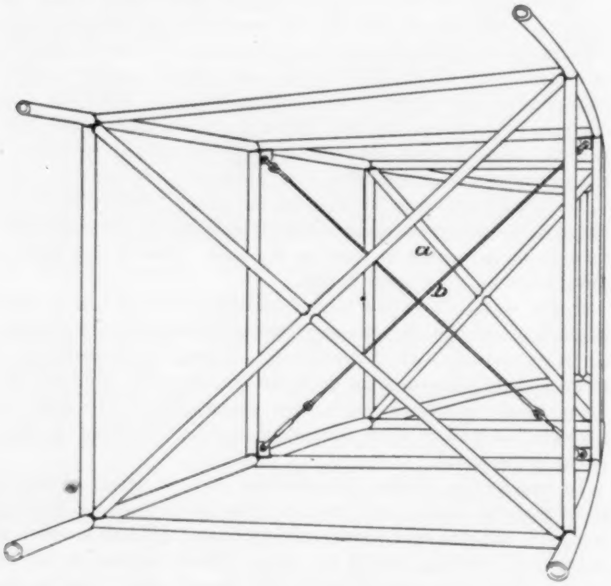

Fig. 1 Method of Reenforcing Bomber Fuselage Against Torsion
(*a*, X-brace of tubing; *b*, X-brace of aircraft cable.)

In another case a problem was encountered on a certain amphibian plane. In the early stages of this plane the wings were constructed with plain dural ribs and wooden spars. These dural ribs corroded rapidly to such an extent that the depot was forced to rebuilt the wings with wooden ribs. These were computed so that their design load factor was as high as before with the metal ribs.

We struck one snag, however. The metal cap strip on the original ribs was very thin where it passed over the spars. The wooden cap strip, in order to have the same strength, was $^{3}/_{16}$ in. thick. This of course changed the contour of the wing considerably. It was feared that the performance of this plane would be adversely affected by such a change.

Much thought was put on this matter. It was finally decided to go ahead with the job. The contour of the wing was changed so as to allow of this thickness at the spars. The rest of the contour was faired in accordingly. Upon completion of the change it was found that no difference whatever in the performance could be noted even with an increase of weight of about 60 lb. The reworked airplane was flown alongside of one with the original wings on it, and no loss of performance could be noted, although there may have been some slight loss.

Again, on a certain three-motored plane, it has been found that the balance, cruising speed, and general performance were greatly improved by moving the wing engines forward about 2 ft. Although there were more powerful engines installed, it is believed that some of the increase in performance was due to the elimination of interference.

The engines were originally under the wing. On another three-motored plane, it was found that the speed was stepped up considerably by the installation of Townend rings on the wing engines and the raising of the wing about 9 in. Here again the improvement was no doubt due to the elimination of interference.

All this indicates, as has been proved in the wind tunnel, that any obstruction or abrupt change in section on a wing or other part will change the flow of air and cause power-consuming drag. In other words, no wing section in the light of our present knowledge is any good that does not give perfectly smooth air flow over its surfaces.

All of our practical experience to date appears to indicate that we must have rigidly braced, smooth wing surfaces that will allow of no flutter or flexibility.

However, it seems from various tests with slots at different places in the surface that these at certain large angles of attack appreciably increase the lift and may prevent the burble or stall.

It seems, therefore, that if we are to attain really large speed ranges that will make flying an easily acquired art, instead of a profession for highly trained men, we must study the bird's wing.

The bird appears to have a flexible wing that is variable in area as well as contour and camber, also allowing a number of slots in various places between the feathers. With this arrangement they are able to fly at high speeds and land lightly at very low velocities. Whether we will ever be able to duplicate this mechanically is a question, since it is a very difficult problem to build such a device light and strong enough.

Methods of Repair

To illustrate some of our other problems, the author will digress for a moment. On a certain bombing airplane of which we were operating a number, or attempting to, we had a series of exasperating troubles. At first, pilots complained that motors cut out badly in bumpy weather and also in banks. It was found that the upper or high engine always caused the miss in turns. It was therefore decided to equip all motors on these planes with fuel pumps. The system in use was a gravity system.

After this was corrected, it was found that the whole tail assembly vibrated and shimmied so badly that the pilots did not want to fly them any longer.

Therefore, after studying the problem, it was decided to stiffen up the fuselage in a certain manner, as well as reenforce the tail surfaces.

It is inadvisable to make a metal fuselage too rigid, for then synchronous vibration may cause resonance, which is worse than shimmying of the tail surfaces, for it causes a pound in the members that will drive a pilot to distraction.

In order to get stiffness and at the same time prevent resonance, the system shown in Fig. 1 was adopted. At every other station of the fuselage a rigid X-brace of steel tubing was welded in. At intermediate stations an X-brace of steel cable was installed. The effect of this was that the steel braces stopped the twist, but the cables at the intermediate stations damped out the vibration, so that resonance was not developed. The explanation is that the cables have a greatly different period of vibration from the steel members, and since they do not take loads in compression anyway, they naturally destroy the beat or rhythm of vibration and tend to quiet the structure. It was then found that landing-gear fittings on the fuselage were weak, and these were reenforced.

It also was found that pins on dural mechanisms did not have sufficient bearing surface. These bearings were bushed up with steel bushings and were given more bearing surface.

Other things developed. The windshields were of such design that, instead of sheltering the pilot, at certain times he was almost blown out of the cockpit. A new windshield was designed and fitted to these planes. It also was found that the impeller of the earth inductor compass was blanketed so that it would not generate. It was necessary to move this farther back on the fuselage to a position where it would be operated by the slipstream of the engines. Such are a few of the problems faced by the maintenance engineer.

Recently a certain anti-knock fuel was introduced to the service. This is apparently a very good commercial product and is quite popular, but as soon as we attempted to use it our spark-plug life dropped from about 100 hours to 40 hours, and we began experiencing corroded and stuck valves, broken valve springs, ruptured coils and condensers on magnetos, and various other difficulties. It appears that in the presence of the high temperatures developed in aircraft engines the anti-knock fluid shows a corrosive action which may be troublesome.

It may not be apparent how this could affect magnetos, but it is thought that the fluid corroded the spark-plug electrodes so much that there was a consequent overloading of the coils and condensers, eventually causing rupture of the insulation.

ENGINES

We find that we have to test our spark plugs very thoroughly, especially since we started using this special fuel. It is necessary to clean them quite often and to adjust clearances very closely. We then put them in a pressure cartridge and fire them with the aid of a booster device under from 80 to 125 lb. pressure, in order to insure that they will fire under compression in the engine; since increased pressure raises the resistance at the spark gap and may cause the plug to short-circuit through the mica insulation. This device is shown in Fig. 2.

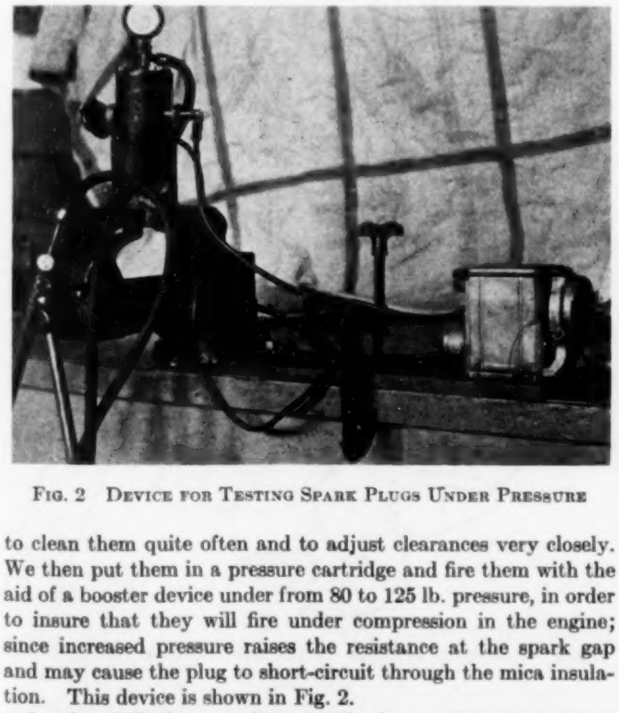

FIG. 2 DEVICE FOR TESTING SPARK PLUGS UNDER PRESSURE

Another difficulty experienced with the use of medicated fuel is sticking valves. A deposit of carbonized material collects on the valves stems after a time. There appears to be no good way to remove this unless the engine is taken down. In this connection it is thought that it might be advantageous to have a sliding sleeve incorporated over each valve stem, in order to insure operation if this condition occurs.

Sticking valves very seldom cause forced landings, but they are troublesome, since they cause an engine to miss, and unless corrected, this may necessitate the replacement of the valve because of burning. Altogether, it is an unmitigated nuisance.

It is suggested that to prevent this the aircraft-engine designers use a sliding sleeve with a flange at the bottom, as shown in Fig. 3, so that the valve will have over twice the bearing surface, and further, in case any particles of carbon collect on the valve stem, they will not prevent the operation of the valve, because it will ride on the outer guide. Further, with this arrangement, the protruding flange will prevent particles from collecting on the guide walls. Also it is thought that the clearance could be reduced, which would in itself prevent particles from collecting.

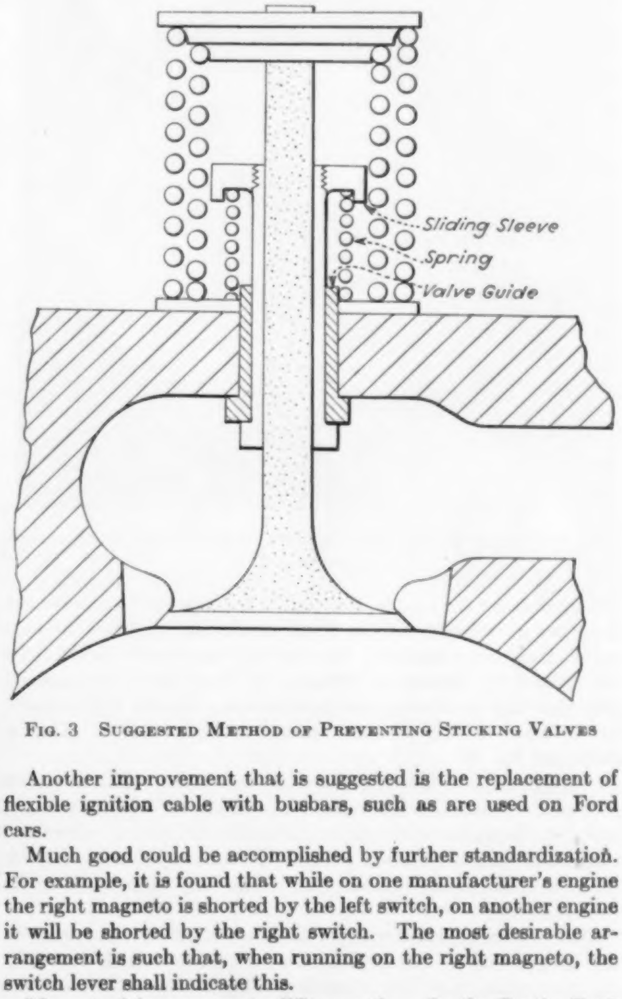

FIG. 3 SUGGESTED METHOD OF PREVENTING STICKING VALVES

Another improvement that is suggested is the replacement of flexible ignition cable with busbars, such as are used on Ford cars.

Much good could be accomplished by further standardization. For example, it is found that while on one manufacturer's engine the right magneto is shorted by the left switch, on another engine it will be shorted by the right switch. The most desirable arrangement is such that, when running on the right magneto, the switch lever shall indicate this.

Magneto drives appear to differ greatly. On the Curtiss D-12 we have the Oldham coupling, which is quite satisfactory, but on other engines we have various other devices, which means that magnetos are not interchangeable, even on engines of the same number of cylinders and same power.

Propeller hubs likewise are found to be of different types, and in cases where the hub is splined, the number of splines will differ. All this is disturbing, since one is required to stock a great number of parts, which means increased cost of operation.

Metal propellers must be checked carefully, especially when any vibration is present during flight. Very often these propellers will show good static balance and will track properly, but have faulty dynamic balance. This can be detected by balancing each blade separately over a knife edge. The center of gravity of each blade should of course be located at the same point in order to get proper weight distribution. If there is a difference in the location of the center of gravity between the two blades, there will be large unbalanced forces set up when the blades are in motion, hence the term "dynamic balance." Faulty dynamic balance may also be caused by a small difference in blade section between the two blades at like stations. This difference may not be apparent except under close measurement; nevertheless it will cause very rough operation.

Very often one wishes to check the timing of an engine in a plane. It may be found that the manufacturer has supplied a timing disk for the engine, but that it is necessary to remove the propeller in order to use the disk, and having removed the propeller, there is no means of turning the engine over slowly. To get around this annoying difficulty, we have made up a special disk that can be fitted to the front of the propeller hub in order to facilitate timing without removing the propeller. This is shown in Fig. 4.

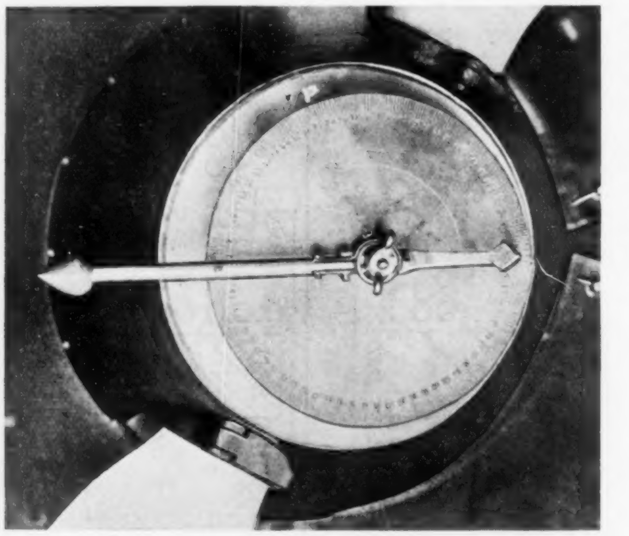

FIG. 4 TIMING DISK THAT CAN BE USED WITHOUT REMOVING PROPELLER

A suggestion for indexing the crankcase of the engine is shown in Fig. 5. With this arrangement it would be necessary to remove the propeller or to use a timing disk for setting valves and magnetos. An adjustable pointer is clamped to the propeller blade so that settings can be indicated directly from markings on the crankcase.

One of the greatest worries that maintenance men have is the danger of fire. An airplane is an AAA fire hazard, especially on the ground, for then there is no slipstream to blow the fire out if it does start. Further, when on the ground, men are usually working on them, removing or replacing batteries, wiring, or other parts. It is very easy to get an arc from a battery by grounding a pair of pliers between the battery post and the metal framework. There is always gasoline present in liquid, or worse yet, in vapor form. For this reason batteries should be located away from the engine compartment and the fuel lines.

Regulations against smoking should be rigidly enforced, for it is believed that 90 per cent of all aircraft fires are caused by someone dropping a lighted match in a pool of gasoline.

Fire extinguishers should be carried on all airplanes. Also portable extinguishers should be easily accessible in all hangars. All Army aircraft carry the pressure type, which sprays Foamite or some other similar product over the carburetors and the engine compartment.

In airplanes everything possible is done to prevent fluttering or vibration, and cable is especially susceptible to this, with resultant fraying and rupture of insulation. These cables must, however, be heavily insulated and also shielded, so that no radiation will be set up that will interfere with radio operation.

Many things can be done to increase accessibility and to reduce maintenance on aircraft engines. For example, if both spark plugs on vee engines could be placed on the outside of the cylinder, it would be a great help to the mechanics, for these spark plugs must be removed quite often for inspection and cleaning. Also it would be helpful if carburetors were placed on the outside of the blocks. At present on certain engines it is difficult to get at the carburetors, even for the simplest adjustments, and very often it is easier to remove the whole assembly than it is to attempt to adjust or to synchronize them while on the engine.

Magnetos are usually tucked away under other protruding parts where it is difficult to get at them. All this probably makes for lightness, but it also is conducive of delay in repair.

If accessories could be arranged as shown in Fig. 6, it would be a great aid to mechanics and might reduce the overhead for the operator. The magnetos are shown moved out somewhat so as to give the mechanic more room to get at them in adjusting them.

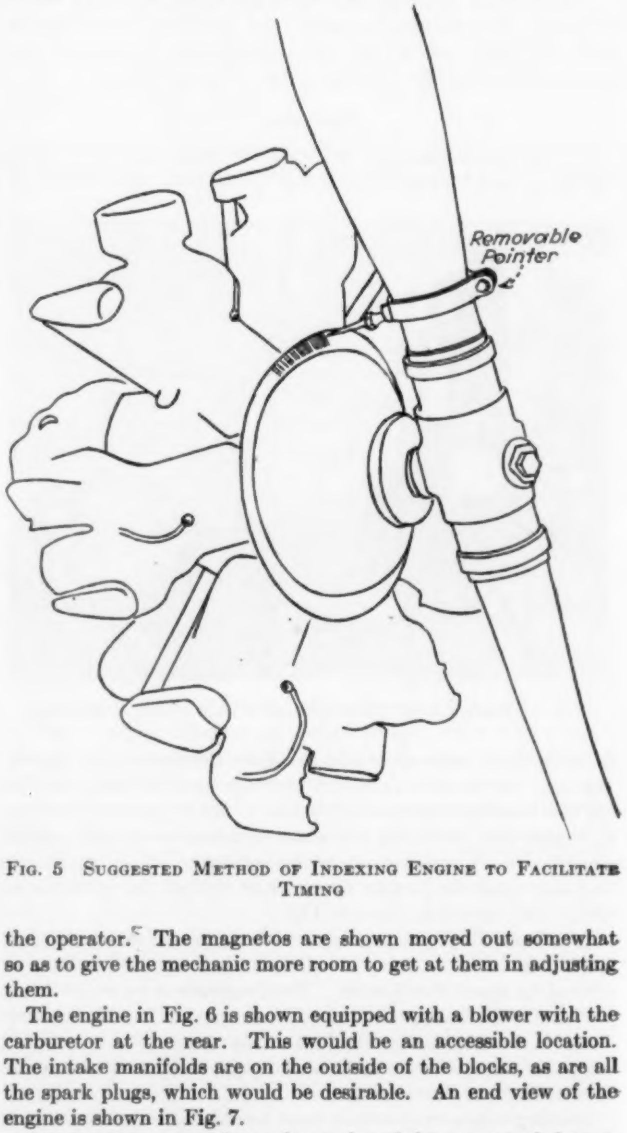

FIG. 5 SUGGESTED METHOD OF INDEXING ENGINE TO FACILITATE TIMING

The engine in Fig. 6 is shown equipped with a blower with the carburetor at the rear. This would be an accessible location. The intake manifolds are on the outside of the blocks, as are all the spark plugs, which would be desirable. An end view of the engine is shown in Fig. 7.

At present on a certain service engine of the water-cooled type, it takes fully four man-hours to change a set of spark plugs, or one-half day's work for one man. This is because the carburetors are on the inside of the vee, and they and their manifolding make it difficult to get at the plugs. From a military point of view this is not desirable; neither is it in a commercial sense, since it is most desirable for a military plane to be quickly serviced and also very desirable to have a commercial plane with low maintenance.

The suggested engine also has its starter and water, oil, and gasoline pumps at the rear, as well as the generator, at the rear between the blocks. These accessories would all be run off the same gear train at the vertical drive shaft from the crankshaft master gear. The magnetos are driven from the front crankshaft gear, and the propeller shaft through reduction gearing. The magnetos should run much cooler in this position.

There are of course other considerations which govern the location of these accessories. It is desirable to have plugs on the opposite sides of the cylinders to get good flame propagation, but it is thought that with proper combustion-chamber design they could all be located on the outside. There is the objection of exhaust gases entering the cockpits with exhausts led up from the inside of the vee, but it should be easy enough to conduct these gases off to each side over the tops of the cylinders.

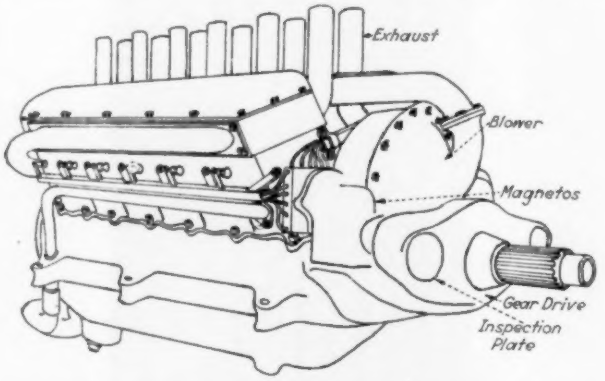

Fig. 6 Suggestions for Improving Accessibility
(Starter, carburetor, and generator are at the rear end; spark plugs are on outside block.)

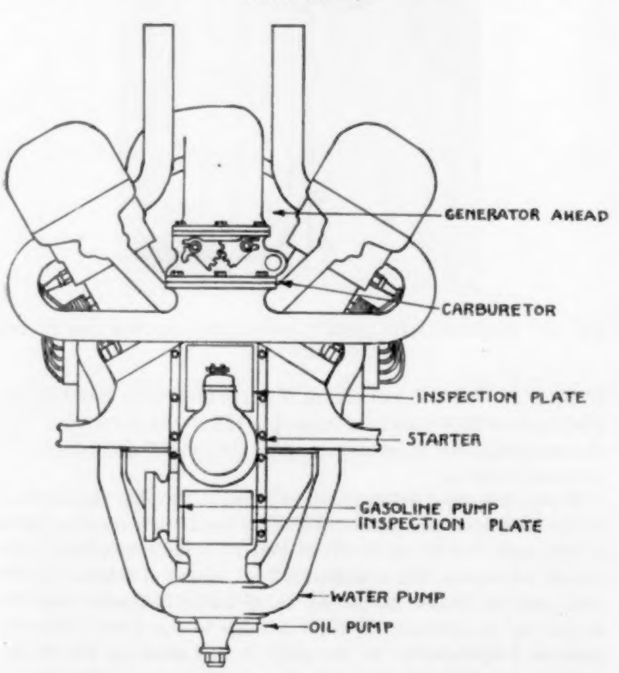

Fig. 7 End View of Suggested Engine

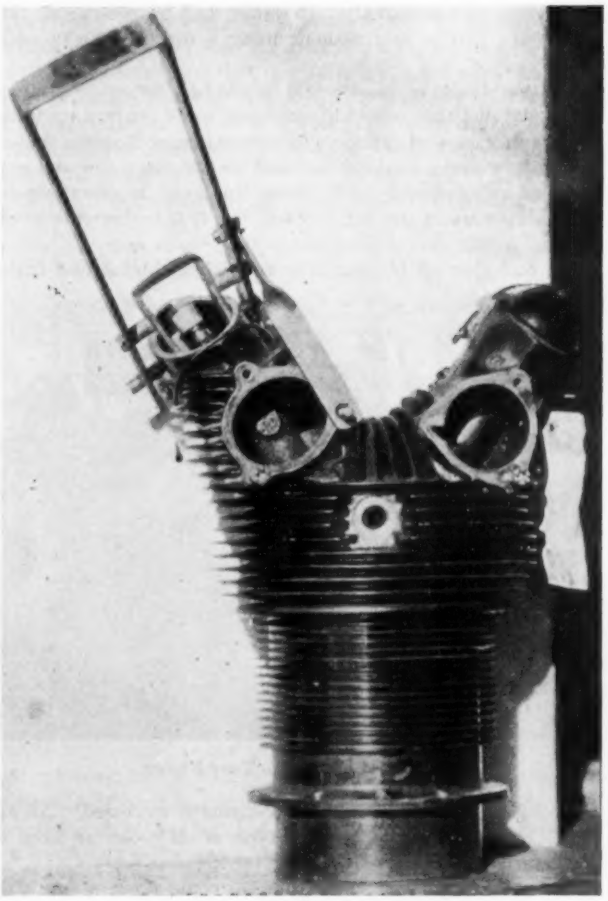

Fig. 8 Valve-Spring-Removal Tool

There are several inspection plates shown on the suggested engine. These are highly desirable when replacing parts or timing engines, in order to check gear backlash or shaft end play.

It is estimated that the maintenance time on such a design might be at least cut in half over the time on the present engines of the vee type.

It is true that the radial engine widely used in commercial service is very accessible, but it is thought that the vee-type engine will be used more in commercial work in liquid-cooled engines, as the speed of commercial aircraft is stepped up.

Present figures on service-type ships show that there is from two to four hours of labor required for every hour of flying. This figure, although it is much less than it was five years ago, is still thought to be much too high.

Many special tools are needed for maintenance of aircraft engines, but unfortunately it is found only too often that the tools supplied by the manufacturer are unsatisfactory. For example, we were forced to manufacture a valve-spring removal tool for a certain well-known air-cooled radial engine, since the one made by the factory was almost worthless. (See Fig. 8.)

A test bench for testing electrical accessories is shown in Fig. 9.

Another thing which some one might devise is a device which could be screwed into the cylinder to indicate the position of the No. 1 piston throughout the stroke with reference to the angular rotation of the crankshaft. It would then be possible to time the engine in the field without using a disk on the propeller hub.

Another device of value would be one to blow out or drain off water and dirt that collect in carburetors and get in the nozzles, thus causing several cylinders to cut out; since it seems that no matter how many strainers are used on the gas, water and sediment get in occasionally. No doubt this water is sometimes due to condensation in the engine itself, but it is bothersome nevertheless.

We find that oil is constantly running at higher and higher temperatures as the power of the engine is increased. All aircraft engines should have some type of oil cooler to keep oil temperatures down.

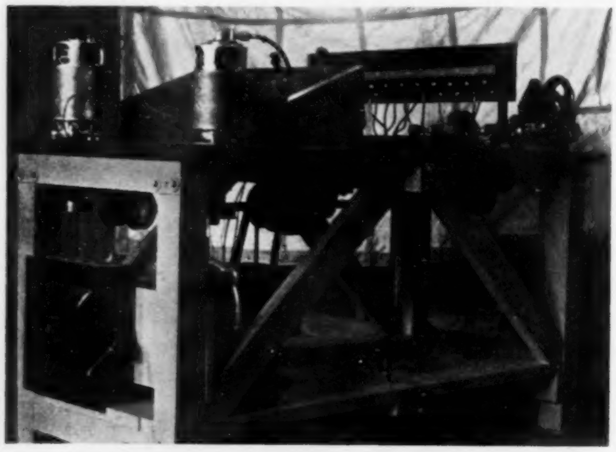

FIG. 9 ELECTRICAL TEST BENCH

An interesting development that has come along of late is the electric oil and water heater installation now used on Army aircraft. Hangars are difficult and also expensive to heat. Furthermore, there often are cases where it is necessary to land service planes in the winter time where there are no hangars. These heating units are inserted in the water manifolds, and also the oil tank can be coupled with an ordinary bayonet socket to a lighting circuit. They rapidly raise the temperature of the water and oil to about 60 deg. fahr. and hold it there. The water is heated by two 600-watt units, while the oil tank is heated by one 300-watt unit. We do not as yet know what maintenance troubles will be encountered with this installation, but the idea is promising. (See Fig. 10.)

Among the useful articles developed by the author at the Panama Air Depot was a tool for checking the main bearings on the Curtiss D-12 and other engines during the boring operation on these bearings.

Bearings on aircraft engines are either reamed or bored, and close measurements must be taken during the boring or reaming operations.

The inside micrometer is a delicate thing to use and liable to error. Therefore, the gage shown in Fig. 11 was designed, with the following points in view: First, it was desired to have a measuring device which would eliminate the inside micrometer; second, a device that would not be likely to get cocked in the bearing during the measuring operation (a fault which the inside micrometer is liable to), and, third, a device that would take only the minimum or high-spot measurement. It was also desired to have an instrument that would be more robust than the inside micrometer. The device shown can be inserted in the bearing and allowed to extend. The outside micrometer can then be used on it, or if desired the measurements can be taken directly from graduations on the gage itself. The gage may be made to read to 0.001 in.

It is an expanding device with two outside blocks machined to a curvature such that it will slip into the bearing in question. Between these outside blocks there is a sliding vee block which is operated by a knurled screw with graduations on the head. This device turned out to be very useful in the boring operations of Curtis D-12 main bearings.

Many other interesting problems come up in the overhaul and repair of aircraft.

For example, at the Panama Air Depot it was desired to replace some sections of longerons on the NBS-1, or Martin bomber, since the wood rotted away rapidly in that climate.

Reenforcement would be necessary at the splice, bearing in mind that glue in such a damp climate was a doubtful proposition.

The original longeron was of spruce, 2 in. square in section. We had some chrome-molybdenum steel strip available luckily, about the proper size, namely, 0.090 in. by 2 in., also some 0.125 in. by 2 in. We also had plenty of Army and Navy standard aircraft bolts of 2330 steel heat treated to 125,000 lb. per sq. in. The chrome-molybdenum steel was good for 90,000 lb. per sq. in. non-heat-treated.

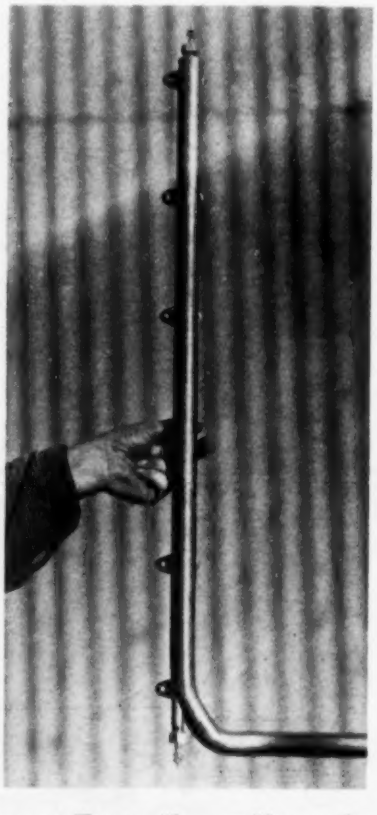

FIG. 10 ELECTRICAL HEATING UNIT FOR HEATING OIL AND WATER ON AIRPLANE

It was therefore determined, in order to develop the strength of the solid longeron at the splices, to reenforce with two plates of the steel, bolted as shown in Fig. 12. We found from competent references that a longeron 2 in. square of selected spruce was good for 10,000 lb. per sq. in. ultimate in tension and 750 lb. per sq. in. ultimate in shear parallel to the grain. In compression perpendicular to the grain it was good for 840 lb. per sq. in.

This problem is not quite as simple as might appear. The generally accepted best practice is to scarf the joint with a 1-in-12 slope, as shown in Fig. 12. Also the best practice is to stagger the bolts as shown. Since the longeron will be drilled for $^1/_4$-in. bolts, as shown, its strength in tension can be computed as follows:

$$[2(2) - 2(0.25)]\ (10{,}000) = 35{,}000 \text{ lb.}$$

Since the shearing strength for the bolts is given by competent authority as 3570 lb., it will need 10 bolts at each end of the scarf. The strength of the plates in tension will be:

$$2(2 - 0.25)\ (90{,}000)\ (0.09) = 28{,}350 \text{ lb.}$$

This is low. Ten bolts staggered and spaced 2 in. apart, as shown, should give a strength in longitudinal shear as follows:

$$[18 \text{ in.} - 10\ (0.25)]\ 1.5(2) = 34{,}875 \text{ lb.}$$

This is enough. A factor of 1.5 is used here, since it is thought the condition does not quite approach double shear.

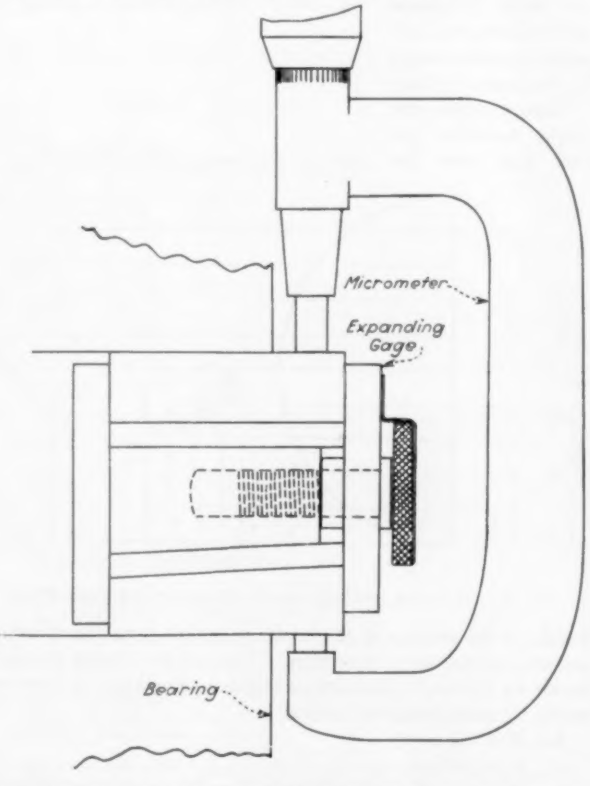

FIG. 11 METHOD OF MEASURING BEARINGS WITH AID OF EXPANDING GAGE

The straps will develop a strength on longitudinal shear as follows:

$$2[182 - 10(0.25)]\ 1.5\ (0.09)\ (60{,}000) = 251{,}110 \text{ lb.}$$

This is ample. As regards bearing strength of the steel plate against the bolts, we have the following: Value given for $^1/_4$-in. bolts against 0.09 plate is about 2200 lb. We have, then, 10 bolts in double bearing, which gives 10(2200) 2 = 44,000 lb.

As for shear perpendicular to the grain, it can be seen that the strength is far over the requirements.

It appears, then, that every case has been covered except compression and torsion. It is not thought necessary in this case to analyze for torsion, since the scarf is placed close to a strut point. As for compression parallel to the grain, it is known without going further that the steel is over-strength.

It is apparent, then, that it will be advisable to increase the thickness of the plate to $^1/_8$ in. in order to develop the strength in torsion. In another case it was found that the engine of a certain airplane could not be installed without the removal of some of its magnetos.

Since this was a nuisance and likely to throw the engine out of time, it was decided to install a removable section instead of the fixed member, which was in the way. This happened to be a member in tension supporting the engine bed. The method of reworking this is shown in Fig. 13. The strength of this material in tension non-heat-treated is about 90,000 lb. per sq. in. We installed two pairs of male and female clevises, which were welded and riveted as illustrated. The clevis pin was a tapered affair. Clevis and clevis pins were made of S.A.E. 2330 steel, also good for about 90,000 lb. per sq. in. in tension. Here we should develop the strength of the tube in tension on the clevis ends, also in double shear on the rivets and pins. This material machines very well too, and can be welded if handled properly.

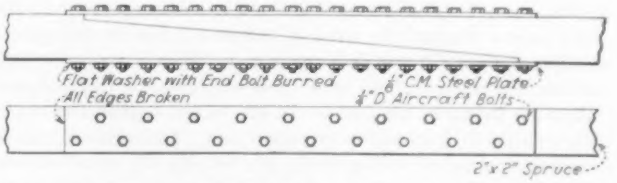

FIG. 12 METHOD OF SPLICING WOODEN LONGERON WHERE GLUE IS UNRELIABLE

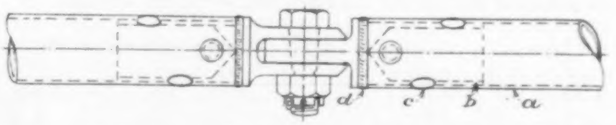

FIG. 13 METHOD OF INSTALLING CLEVISES ON REMOVABLE SECTION

(*a*, $1^1/_8$-in. outside diameter, 0.028-in. wall, chrome-molybdenum steel tube; *b*, 0.065-in. wall clevis of 2330 steel; *c*, mild-steel rivets, $^1/_4$ in. diameter; *d*, weld.)

To analyze this case, these clevises could either be welded or pinned, but a weld in tension as this would be a doubtful proposition, so it was decided to use pins and rivets, in connection with the weld. The cross-sectional area of the tube, if we used $^1/_4$ in. diameter pins, would be 0.19442 sq. in. = 2(0.25 × 0.058), for two drill holes for pins or 0.165 sq. in., which gives a strength in tension of 14,850 lb. It is probable that $^1/_4$-in. mild-steel rivets will be low in shear and bearing strength against 0.058 wall tubing, but we will depend on the weld to develop the rest of the strength. Of course, we could weld a sleeve on the tube to develop shear and bearing strength, but this operation would anneal the tube and reduce its strength in tension.

The clevis pin of course must develop a double shearing strength of about 1500 lb. An undercut taper pin will be used so that the job can be drawn up securely when finished, for it is undesirable to have any play in a member supporting an engine, as it may set up a heavy vibration.

Its average cross-section, then, must be 0.0165/2(9/6), since a shearing value of 60,000 lb. per sq. in. will be used. This works out as 0.124 sq. in., or close to $^3/_8$ in. in diameter.

Three mild-steel rivets will be used in each clevis, since this is a convenient arrangement. We will space these 120 deg. apart, as shown, because we do not want them in line, for then they may tear out more easily. Three rivets in double shear will develop a shearing strength of 6(1719), or 10,314 lb., according to competent references. This, of course, is low, but we will depend upon the welding to fill it out.

It is convenient to space these rivets down the tube $^1/_2$ in., 1 in., $1^1/_2$ in. This gives a mean of 7 to 8 in. of material in longitudinal shear. Then then develops as follows: 2(0.875) 1.5 (60,000)–15,750 lb., using the factor of 1.5 since, as stated, this condition is not considered quite equal to double shear. It follows, of course, that the walls of the clevis inside should be at least 0.065 in. in thickness, but that they should not be much thicker, since it will add weight and will cause unequal expansion and contract during the welding operation, due to the difference in weight of the metals being joined. This is a very important consideration in light structures such as airplane fuselages.

Another factor to consider is the size of the lugs at the clevis ends which will hold the large taper pins. The cross-sectional area from the end of the lug to the pin hole should be as follows: $A(1.5)\ (60{,}000) - 14{,}850$, or $A = 0.165$ sq. in. If we have a lug thickness of $^3/_8$ in. then we must have about $^7/_{16}$ in. of material between the pin hole and the end of the lug. Therefore, the radius of the lug end will be $^7/_{16}$ in. + $^3/_{16}$ in. = $^5/_8$ in.

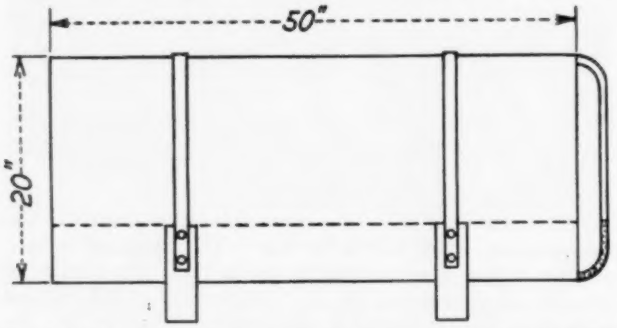

FIG. 14 METHOD OF INSTALLING AUXILIARY TANK, LAID ON ITS SIDE

We find, however, that the bearing strength of a $^3/_8$-in. pin against $^3/_8$-in. metal in thickness is somewhat low, about 13,360 lb., so we will increase the pin size to $^1/_2$ in., where we will get a value of 17,812 lb., from competent references. It appears, then, that we can safely use a pin with a mean diameter of $^7/_{16}$ in.; that is, $^1/_2$ in. at one end and $^3/_8$ in. at the other.

The rivets have a bearing strength of only 6(1378) lb., which works out as 8268 lb., which is, as was expected, low. We believe, however, that we can depend upon the weld to develop the necessary strength not available in the rivets. The matter of bearing strength depends largely upon the fit between the sleeve of the clevis and the inside of the tube. If this is a good fit, a high bearing strength will be developed. If it is not, the job will be very low in bearing strength. Therefore, we will make this job an especially good fit.

In a job like this, we do not deem it advisable to heat-treat the parts unless it is difficult to develop the strength without it. Heat-treated parts are likely to be brittle and to crack where subjected to vibration.

It is highly important, however, to incorporate fillets, as shown in Fig. 13, where there is a break in section, since sharp corners are likely to localize stresses and start cracks at these points.

Good workmanship, however, is really the prime requisite in things of this sort, since a job that has been properly calculated may easily fail because of sloppy fits and tool marks, or the parts may be overstressed in working.

Rivets must be fitted snugly, and such parts as the male and female clevises shown must match nicely, so that they will go together without forcing.

It may have been noticed in the foregoing computations that particular attention was paid to bearing stresses in the case of bolts, rivets, and pins. This is because failures of parts due to lack of sufficient bearing surface are common. Indeed, they are probably the most common of failures on aircraft.

In the construction and operation of airplanes and motor boats, one very often runs into very knotty problems that defy solution by ordinary means. Suppose, for example, that one desires to put in an auxiliary fuel tank for a long-distance flight, since the standard fuel tank will not suffice. Suppose further that there is sufficient room to install a cylindrical tank 20 in. in diameter and 50 in. long, which will be laid on its side as shown in Fig. 14. Again suppose it is important to know the supply of fuel with respect to height in the tank. In fact, it is very important in an airplane or even motor boat to know this, in order to estimate probable cruising range at any time.

Now, of course one might measure the fuel and note the height of the column of gas in the gage, but then this is laborious and amateurish in operation. This problem cannot be solved easily by the usual processes of trigonometry, but can be solved easily by integrating as follows:

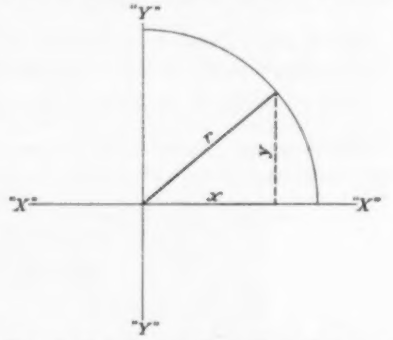

FIG. 15 DIAGRAM OF FUEL-TANK PROBLEM

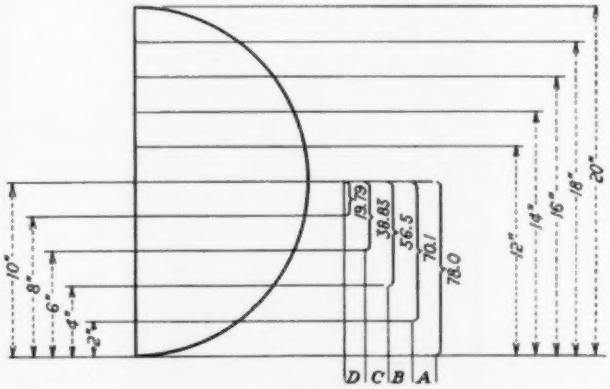

FIG. 16 METHOD OF CALCULATING CAPACITY OF FUEL TANK

Let D = diameter
r = radius
x = any distance subtended by a perpendicular y dropped to the X axis, as in Fig. 15.

Then:

$$y = \sqrt{(r^2 - x^2)}$$

We wish to know first the cross-sectional area of the tank by increments of heights of 2 in. After that, it will be easy to compute volume of the tank for every 2 in. of height. Therefore, we will integrate the area of one-quarter of the cross-sectional area, as shown in Fig. 16, between where x is zero and the various values of x.

Any table of integrals will show that the integral of $y = \sqrt{(r^2 - x^2)}$ is as follows:

$$\int \sqrt{(r^2 - x^2)}dx = \frac{x\sqrt{(r^2 - x^2)}}{2} + \frac{r^2}{2}\left(\sin^{-1}\frac{x}{r}\right) + c$$

The theory of the operation is that we compute the value of the integral for two values of x and take the difference, during which operation, of course, the constant c drops out, but in this case one value of $x = 0$, so that the subtraction is not actually carried out.

Substituting $x = 10$ in the following formula, we get:

$$\left[\frac{10\sqrt{(100 - 100)}}{2} + \frac{r^2}{2}\left(\sin^{-1}\frac{x}{r}\right)\right] = \frac{100}{2}\ (\sin^{-1} 1)$$

Substituting $x = 0$, we get zero, of course. Therefore, the area between $x = 0$ and $x = 10$ is 50(1.56), or 78.57 sq. in.

Substituting $x = 8$ and $x = 0$, we have

$$\left[\frac{8\sqrt{(100 - 64)}}{2} + 50(0.922)\right] - 0, \text{ or } 70.1 \text{ sq. in.}$$

Substituting $x = 6$ and $x = 0$, we get:

$$\left[\frac{6\sqrt{(100 - 36)}}{2} + 50(0.65)\right] - 0, \text{ or } 56.5 \text{ sq. in.}$$

Substituting $x = 4$ and $x = 0$, we get:

$$\left[\frac{4\sqrt{(100 - 16)}}{2} + 50(0.40)\right] - 0, \text{ or } 38.83 \text{ sq. in.}$$

Substituting $x = 2$ and $x = 0$, we get:

$$\left[\frac{2\sqrt{(100 - 4)}}{2} + 50(0.2)\right] - 0, \text{ or } 19.79 \text{ sq. in.}$$

Thus we have the areas computed as shown in Fig. 16. It is very simple, then, to determine what our gas reserve is for every 2 in. of height in the tank. Of course, the tank being symmetrical about its center lines, the volumes will be the same for each half.

We must now compute the increments in order by subtracting as follows:

$A = 78.54 - 70.1 = 8.44$ sq. in. for 2 in. of height
$B = 78.54 - 56.5 = 22.04$ sq. in. for 4 in. of height
$C = 78.54 - 38.83 = 39.71$ sq. in. for 6 in. of height
$D = 78.54 - 19.79 = 58.75$ sq. in. for 8 in. of height

Volumes, then, are as follows:

For 2 in. of height 2(8.44)50/231 = 3.65 gal.
For 4 in. of height 2(22.04)50/231 = 9.54 gal.
For 6 in. of height 2(39.71)50/231 = 17.19 gal.
For 8 in. of height 2(58.75)50/231 = 25.43 gal.
For 10 in. of height 2(78.54)50/231 = 34.00 gal.
For 12 in. of height 2(19.79)50/231 + 34.00 = 42.56 gal.
For 14 in. of height 2(38.83)50/231 + 34.00 = 50.81 gal.
For 16 in. of height 2(56.60)50/231 + 34.00 = 58.46 gal.
For 18 in. of height 2(70.1)50/231 + 34.00 = 64.35 gal.
For 20 in. of height 2(34) = 68 gal.

In the same manner, tanks of other curvatures can be integrated to determine their contents by increments of height.

Discussion

Walter C. Thee.[2] Lieutenant Downey clearly shows in his paper how overhead can be decreased, but in doing so, he showed also the absolute necessity of an officer in charge of maintenance to be well qualified for the position. A great aid in maintenance is to have a good supervisor that knows the fundamental principles of machine-shop practice and can apply them successfully and economically. He also must have a working knowledge of metallography, strength of materials, and dynamics of internal-combustion engines, as well as the fundamental principles of engines, ignition systems, propeller design, airplane rigging, and the application of fabrics and woodwork.

A good maintenance superintendent will increase the efficiency of an organization a great amount and will reduce the unit cost of maintenance so that salvaging of wrecks or reclamation of component parts will be economical.

The author stresses the importance of inspection. The functionalization of inspection has grown in our industries mainly because it has proved itself to be profitable. Inspection in regard to the operation of airplanes may be called an official examination of unit assemblies or parts to detect errors or flaws and to determine quality, accuracy, finish, and interchangeability. To accomplish this it is necessary to have a definite knowledge of particular facts obtained and shown to be correct by scientific research. When definite knowledge is established, a record should be made for the purpose of accumulating and preserving all data under one form so that a standard practice is established.

THE IMPORTANCE OF STANDARDS

The value of standard practice applied to the maintenance of airplanes is that it brings about a coordination of information and uniformity of instruction for the maintenance of one type of equipment in a single manner. Standard clearances, tolerances, and adjustments are specified and used as a basis to detect errors when inspections are made of airplanes or their unit assemblies and parts.

If we look at inspection in the light of Gantt's philosophy, it is evident that without standards there can be no inspection, for no one can see that a quality order is carried out if no quality order is issued, and a quality order to be such must contain standards quality. Furthermore, if standards for inspections are specified, definite information is furnished the inspector. An inspector's or any man's judgment is no better than his information.

OBSOLESCENCE, DEPRECIATION, AND AMORTIZATION DEFINED

In regard to the author's views on obsolescence and depreciation, the writer is heartily in accord with him. The principal factor that will determine when an airplane should be discontinued from service is obsolescence. Obsolescence can be divided into two major classifications: Obsolescence through change in design of product, which does not reflect on the value of the machine as a useful piece of equipment, but merely that the part which it was bought to produce was found to be of wrong design or to have short life; and obsolescence through an economic replacement.

The terms depreciation, obsolescence, and amortization are sometimes confused and used indiscriminately. Depreciation has been defined as the act of lessening or seeking to lessen the price or value, the reduction of worth, the gradual wearing out of equipment. Depreciation can be calculated in advance; it deals with the anticipated life of the equipment to produce for a given number of hours.

Amortization is the extrication of a debt, usually by means of a sinking fund. Wear and abuse do not necessarily enter into this meaning, but rather an anticipated life or fixed time in which to set aside funds to pay the debt.

Obsolescence is a measure of progress that reduces the value of the tool or machine on hand when improvements have been made in design or manufacturing methods that increased their efficiency or reduced operating costs or hazards. Obsolescence cannot be calculated like depreciation in accounting practice, as the former depends on research and can be divided into two major classifications:

1 Obsolescence through change in design of product
2 Obsolescence through an economic replacement.

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[2] Captain, Quartermaster Corps; Commanding Officer, Second Corps Area Motor Repair Shops, Fort Hancock, Sandy Hook, N. J.

MAINTENANCE VERSUS ENGINEERING DESIGN

The paper is not confined strictly to airplane maintenance, as at times the author has described problems involving the computation of design load factor relative to rebuilding wings with wooden or metal ribs and changing the position of the engines in order to improve the balance, cruising speed, and general performance. These problems should be charged to the engineering division and worked out by them rather than by the maintenance division. Placing such responsibility upon a division that specializes on problems involving design will accelerate the establishment of standard practice and centralize all records when definite knowledge is received.

Obsolescence is usually that lessening in value that is brought about by developments, inventions, and changes in the art that render airplanes, any part of its equipment, or whole sections of it uneconomical of use as compared with newer types available of greater efficiency.

Obsolescence has been defined as "going out of use," which definition represents a falling off in value of usefulness of a thing from causes outside of the thing itself as distinguished from the effect of wear or physical deterioration.

Wear and tear is a form of depreciation, and the one regularly recognized in accounting practice is that due to the elements, to abrasion, accidents, accelerated usage, and the like. Wings can be repaired, struts replaced, new bearings installed, cylinders reground, and bushings replaced. Experience has shown that where obsolescence has not set in or is operating and the equipment is kept in reasonably good operating condition for the work that it is called upon to regularly perform, it is then maintained at from 80 to 85 per cent of its original functioning condition. The principal factor which will determine when an airplane or any of its units should be discontinued from service is obsolescence.

The causes of obsolescence that will have a tendency to affect maintenance and the continued use of airplanes are:

1 The development of an extensive change in the method of manufacture
2 The making of a substitution of materials
3 Newer planes of the latest type will give the same or better performance more economically
4 A newer or modern plane may reduce (*a*) the hazard to the operator and passengers or (*b*) the hazard of fire
5 More rigid equipment may give greater required safety.

SIMPLIFICATION OR SPECIALIZATION REDUCES OVERHEAD

Maintenance of airplanes can be simplified only by specializing on the various routine repair operations that are required in service, applying the fundamental principles of industrial organization now used by the manufacturers to produce airplanes in mass quantities, and observing the laws of manufacturing management, as described by L. P. Alford.[3] In order to apply these laws or fundamental principles of industrial organization and management to the maintenance of airplanes, repairs must be made in mass quantities. Small local service stations should confine their work to the facilities, qualified personnel, and adequate equipment available. By this it is not meant that small local service stations should have a large force of expensive skilled mechanics and a complete assortment of expensive shop equipment, unless they can be economically operated and the volume of work for each machine warrants its purchase and installation.

HOW MASS PRODUCTION CAN BE APPLIED TO MAINTENANCE

The fundamental principle of the organized system of maintenance of airplanes is based upon qualified personnel and adequate facilities with which to do the work. In order to keep overhead down to an absolute minimum, to hire qualified personnel, and to operate adequate facilities economically, maintenance should be simplified and divided into five phases or zones of maintenance in order to make specialization possible. In each of these phases or zones there are three distinctly different elements; namely, personnel, the tool equipment, and supplies.

ORGANIZATION OF AIRPLANE MAINTENANCE

The zones of supplies should extend from spare parts carried on airplanes to the centralized stockage in airplane maintenance depots. The zone of maintenance functions to extend from the work of the pilot or individual airplane mechanic to the major operations of reconstruction shops or factories where the airplane or equipment is manufactured.

The work of airplane maintenance and handling of aeronautical supplies will be so graded in the zones of maintenance as to remove all maintenance facilities—i.e., service station and warehouse operations—from the field of operations of airplanes and especially of large fleet operators of transport planes. The most extensive repair operations, requiring skilled mechanics, always limited in number, the expensive and elaborate shop equipment, and large stocks of airplane supplies should be centralized in airplane repair depots or factory service stations.

THREE ESSENTIAL ELEMENTS OF MAINTENANCE

There are five natural zones or phases of maintenance. In each of these zones there are the following three distinctly different elements: (1) The personnel, (2) the shop and tool equipment, and (3) airplane supplies. Throughout the five zones, the three elements, personnel, equipment, and supplies, will be graded in accordance with the character of maintenance to be performed.

FIVE ZONES OF MAINTENANCE AND THE FUNCTIONS OF EACH

The functions and responsibilities of each of the five phases or zones of maintenance should be as follows:

(*a*) *First Zone—Airplane Pilots' or Airplane Mechanics' Maintenance.* This embraces cleaning, waxing, greasing, servicing, tightening (bolts and screws), making ordinary running repairs and minor adjustments, such as oil changed, pushrods greased, cleaning strainers, gas, and oil lines, changing plugs, and adjusting tappets, that can be accomplished with the tool kit and the spares usually carried on the airplane. It is in this phase or zone of airplane maintenance that the little things are done from day to day in the way of maintaining the plane and engine. It is at this time that preventative maintenance is applied, and if done correctly it will do more to lengthen the life of the plane and engine than will any other one thing. This phase of maintenance also includes the keeping of up-to-date accurate log books in which are recorded for each engine the total number of hours since overhaul of the engine then in service. With this data, there is also recorded for each engine the time it has run since the several routine operations required in service have been performed, such as oil changed, valves checked, pushrods lubricated, magnetos checked, and strainers cleaned.

(*b*) *Second Zone—Periodic Inspection.* This will embrace periodic and systematic inspection to insure thorough and constant maintenance, with enforcement and correction of the first phase of maintenance. The first and second phases or zones, together, will form "preventative maintenance," for which the official in charge of the airplane or fleet of airplanes is directly responsible. The repairs undertaken will be limited to those that can be accomplished economically and satisfactory by local personnel, repair facilities, and stock of spares that may be available. It is at this phase of maintenance that log books for both plane and engine are carefully checked. This system of maintenance should require each plane to carry an inspection form on which is recorded in abstract form the total operating time of the plane and all engines, with any other special information the log books contain on that particular ship or its engines. A careful check should be made on this inspection form to see that it contains the time each engine has run since the several routine operations required in service have been performed, such as oil changed, valves checked, pushrods lubricated, magnetos checked, and strainers cleaned.

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[3] "Laws of Manufacturing Management," Trans. A.S.M.E., 1926, p. 393. Abridged in *Mechanical Engineering*, April, 1927, p. 301.

(*c*) *Third Zone—Unit Replacement.* This contemplates a removal of an unserviceable unit assembly in which a sub-assembly or part needs a repair and immediate replacement thereof by a similar serviceable unit assembly held in reserve of the third phase or zone stock or stocks of spare units. Unit replacement will be practiced in all cases unless it is unmistakably obvious that a minor repair or replacement of a sub-unit assembly or part is all that is required and that this repair or replacement can be made without removal of the unit assembly from the plane or without disassembly of the unit, and that the supplies required are on hand or quickly and readily obtainable.

The third phase or zone of maintenance further contemplates organization of unit replacement repair shops, in all airports or at replacement centers established in a territory relative to a number of very small airports.

In undertaking or authorizing third zone service station repairs where such repairs are not contemplated in the regular set-up, an airfield manager or service superintendent should consider the following:

(1) The mechanical personnel must be capable of performing the contemplated work without damage to the plane or assemblies.

(2) The service station and tool equipment required for the work must be on hand at the location where the repairs are to be made.

(3) The spare unit assemblies, spare sub-assemblies, or spare parts required in the work contemplated must be in immediate stock at the place where the work is to be done, or easily and quickly obtainable by shipment from the next higher zone of aeronautical supply vicinity. This availability must be established as certain before removal of an unserviceable unit assembly from a plane for the purpose of repair is justified.

(*d*) *Fourth Zone—Unit Repair.* This will embrace the "tear down" and repair of any or all unit assemblies. The fourth phase or zone work contemplates a balanced personnel organization composed of the various types of mechanics classified and qualified in accordance with the various trades, as electricians, welders, engine repairers, ship builders and carpenters, painters, and upholsterers, required to repair the various unit assemblies of the airplane involved. It contemplates the organization, operation, and maintenance of departments of the various types used in the trades referred to, and it also contemplates the extensive supply of spare unit assemblies, sub-assemblies, and spare parts required in unit repair operations. In all fourth zone service stations the three essential elements must be in balance, i.e., trade classified personnel, the service station and tool equipment, and the required supplies.

As a rule, these fourth zone shops will be concerned only with the repair of unit assemblies and sub-assemblies. When the repair of an airplane or airplanes is imposed upon the fourth zone shop, it will remove and replace unserviceable unit assemblies in accordance with the third zone principles and practice, absorbing the unit repair work in its normal operations. There should always be, however, physical separation between third and fourth zone operations because of the distinctly different objectives involved. The objective of a third zone service station is to return airplanes to service in a serviceable condition as quickly as possible. The objective of a fourth zone service station is excellent and efficient shop organization, a smooth-running program with work of first-class quality, and a high rate of production in view, measured in terms of the number of unit assemblies repaired and returned to stock each month.

Fourth zone shops reclaim all serviceable unit assemblies, sub-assemblies, and spare parts from airplanes and aeronautical equipment that are not considered economically repairable.

In the operation of fourth zone service stations, the work of repair of unit assemblies and sub-assemblies will be balanced with the territorial requirements on the one hand and the capacity of fourth zone service stations to turn out such required repairs on the other hand. In this connection, fourth zone service stations should evacuate work beyond their capacity of fifth zone repair depots to which they may be assigned for supply.

(*e*) *Fifth Zone—Reconstruction.* This covers three subdivisions, as follows: (1) This will embrace repair work that is really reconstruction and requires special expensive single-purpose tool equipment and special tool operators who regrind crankshafts and cylinders, remove and replace valve seats properly in the aluminum heads of the modern air-cooled cylinders, renew crankshaft bearings, rebuild instruments, carburetors, magnetos, and wings, and reset pitch of propeller blades and balance same. In addition to repair and manufacture, the fifth zone of maintenance embraces the reclamation of and salvage of airplanes, unit assemblies and sub-assemblies in order to replenish stocks from uneconomically repairable planes, that nevertheless constitute a source of low cost supply of serviceable or economically repairable unit assemblies, sub-assemblies, and parts. It also includes assembling of units, sub-units, and planes.

(2) Whereas a large percentage of supplies of the fourth zone is in spare unit assemblies and sub-assemblies, the supplies of the fifth zone must embrace every individual part of such assemblies and sub-assemblies and the raw materials, i.e., steel, copper, alloys, wood, cloth, leather, wire, paints, etc., essential to aeronautical remanufacture.

(3) The fifth zone repair depots will cooperate in every way possible with fourth and third zone service stations assigned to them for supply so as to accomplish, between the three zones, the work of unit repair and unit replacement. Fifth zone repair depots will endeavor to maintain a smooth flow of supplies and reserve stocks through both the fourth and third zones. Whenever fourth and third zone work is imposed upon the fifth zone repair depots, such work will be performed in accordance with the principles and practices of third and fourth zone service stations. A physical separation, however, between third zone work of unit replacement and the higher zones of unit repair and unit reconstruction should always be maintained to protect fourth and fifth zone shops against the loss of man labor hours that always attends the repair of specific airplanes.

RELATION OF MAINTENANCE ELEMENTS TO PHASES OR ZONES OF MAINTENANCE

The three elements of maintenance, that is, personnel, shop or tool equipment, and supplies, should be graded throughout the five zones in accordance with the requirements of operating planes and fleets on the one hand, and on the other hand the necessity for maintenance organization, which requires the centralization of limited elements. The highly trained personnel required for the fourth and fifth zone work is always very limited. The shop and tool equipment used by such personnel is likewise limited; first, because of the large investment necessary in maintaining a fireproof building, in having specially designed equipment, in maintaining a large corps of technically trained mechanics, recoverers, and ship builders, and then in the need for carrying costly inventories of parts and supplies. The third element, "airplane supplies," embraces articles that have a high mortality and should be distributed in such amounts as will prevent exhaustion of stocks and dissipation of supplies that can be sent where needed and when needed only from centralized stocks. In the use and distribution of these three elements, incident to setting up the five zones of maintenance, the following resources are available:

(*a*) *Element of Personnel.* The first zone should include pilots, assisted by individual airplane mechanics, line or ground crews, who are qualified to care for airplanes. The prescribed functions being regular cleaning, waxing, lubricating, servicing, air, oil, water, tightening of bolts or screws, and emergency repairs.

The second zone should include field managers, flight superintendents, and especially selected line and ground crews trained and qualified to operate and maintain small fleets of airplanes of transport companies. The prescribed functions should be regular inspections, correction of errors or neglect in first zone work, assistance to airplanes that made forced landings, etc.

The third zone should include small maintenance organizations of general aircraft mechanics under a service superintendent qualified in airplane maintenance. The prescribed functions should be the removal and replacement of unit assemblies and exchange normally with fourth zone unit repair shops, and exchange with fifth zone shops whenever practicable.

The fourth zone maintenance should be limited to authorized factory service stations and will include large maintenance organizations of various trade classifications in the airplane industry. The prescribed functions should be repair of unserviceable unit assemblies in shop programs, based on "quantity production" methods and a quick return of repaired unit assemblies to reserve stock for third zone use. The personnel, in order to function properly and efficiently as well as economically in fourth zone work, will be balanced as to trade qualifications, in accordance with well-established fundamental principles of industrial organization and management.

The fifth zone maintenance should be limited entirely to factory personnel who may be augmented by the hire of highly skilled personnel qualified to reconstruct and manufacture airplanes, unit assemblies, sub-assemblies, and parts, or by unskilled personnel to be trained. The personel will consist of extensive manufacturing and airplane supply organizations of all trade classifications in airplane industry. The prescribed function should be reclamation of units and parts; salvage and remanufacture of airplanes, units and parts; extensive warehousing, storage, and issue.

(*b*) *Elements of Shop and Tool Equipment.* The first and second phases or zones of airplane maintenance should be limited to small kits of hand tools which come originally with every plane upon its leaving the factory, augmented in the second zone by such extras as may be carried at various airports.

The third phase or zone should be limited to such hand tools as are required to remove and replace unit assemblies and to such simple repair equipment as may be installed at airports, which is required for minor repairs incident to ordinary flying operation. In addition, the third phase or zone may have added work benches, vises, air compressor, paint-spray machine, high-pressure lubricating devices, hoisting jacks, traveling crane, electric grinders, drill press, welding outfit, riveting machine, and a limited addition of small hand tools, such as taps and die sets, airplane repair, and wood-working tools.

The fourth phase or zone should include the machinery and equipment required in service stations of the various trades connected with the airplane industry required to repair unit assemblies.

The fifth phase or zone should include similar but more extensive equipment required for the reconstruction and remanufacture of planes, units and parts, i.e., all machinery, tools, and facilities pertaining to the airplane industry.

(*c*) *Element of Airplane Supplies.* The first and second zone supplies should be limited to the few spare parts intended to be carried on planes and spare planes of transport companies or small fleets.

The third zone supplies should consist of a reasonable proportion of reserve stock of spare unit assemblies and sub-assemblies, and of a small stock of frequently required spare parts that do not require the disassembling of unit assemblies to install. Supplies of the third zone should be located in every large city where aeronautical service stations are established and where aeronautical activity is most prevalent.

The fourth zone supplies should include reserve stocks of spare unit assemblies and sub-assemblies and such stocks of spare parts required to repair unit assemblies and sub-assemblies.

The fifth zone supply will consist of centralized stocks of aeronautical supplies, including raw materials, spare unit assemblies and spare parts, stocks of machinery, equipment, tools, and accessories required to maintain airplanes.

NECESSITY OF A SUCCESSFUL STUDY AND STANDARDIZATION OF REPAIR OPERATIONS

An airplane mechanic should not be required to be able to rebuild an entire plane, any more than a factory mechanic should be required to build each unit and assemble an entire airplane. Prior to 1900, automobiles were built on the plan of having one man assemble the engine, and when he was through with it he proceeded to the axles and so on. Today there is a minute specialization in the manufacture of automobile[4] and airplane parts. Airplane repair operations should be studied more thoroughly and standardized so that the work can be simplified by specializing on unit repair operations with the resultant benefit to the airplane user.

The principal objective of the suggested scheme of maintenance as described is to make possible repairs of unserviceable material in mass quantities, thereby securing satisfactory work at a reasonable cost.

SUGGESTED LOCATIONS FOR ZONES OF MAINTENANCE

Until the volume of repairs warrants the establishment of fifth zone repair depots in various parts of the United States and foreign countries, reconstruction work, such as regrinding crankshafts, renewing main bearings, regrinding and honing cylinder barrels, rebuilding carburetors, magnetos, starters, generators, and instruments, should be done at the respective factories that produced them, where practically all of the necessary tools, equipment, supplies, and personnel are already available.

The major service stations for fourth zone maintenance and supplies should be located in the vicinity of such cities as New York, Cleveland, Chicago, St. Louis, Los Angeles, Fort Worth, and Atlanta.

The minor service stations for third zone maintenance and supplies should be located in every large city where aeronautical activity is most prevalent.

The first and second zone maintenance should be located and applied at every airport or flying field.

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[4] John Younger, "The Automotive Industry," *Mechanical Engineering*, April, 1930, pp. 491–492.

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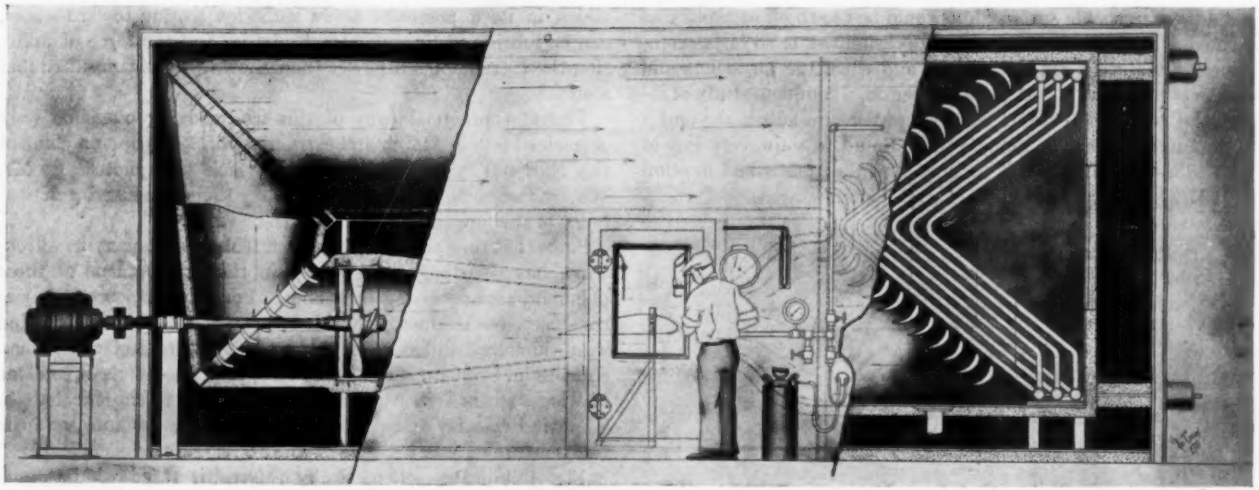
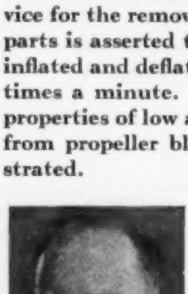


FIG. 1 REFRIGERATED WIND TUNNEL

The Ice Hazard on Airplanes

BY WILLIAM C. GEER,[1] ITHACA, N. Y.

The problem of the formation of ice on airplane wings and propeller blades and of its prevention or quick removal is discussed in the paper. The most practical device for the removal of ice on the leading edges of airplane parts is asserted to be a rubber overshoe capable of being inflated and deflated with an air pump at the rate of three times a minute. A special oil mixture gives the rubber properties of low adhesion toward ice. The removal of ice from propeller blades has not been successfully demonstrated.



THE fact that there is an ice hazard need not be discussed in this presence. Every pilot who flies a mail route over the Alleghany Mountains or northwest of Chicago need not be told that over a considerable portion of the year he faces grave risk due to the accumulation of ice upon the leading edges of wings, struts, and other parts of his plane.

Nor does one need to enter into a discussion of the causes of the formation of this ice. Such men as Andrus,[2] Smith,[3] Scott[4] and Barnes,[5] Carroll and McAvoy,[6] and others have given far more thought to this problem than has the author and have published their conclusions.

A number of different chemical methods have been proposed and tried, the object being to prevent the formation of ice on airplanes. Of all of these methods, only one of them has given any indication of success. In principle this one consists of the use of a coating upon the airplane, which coating is composed of a material which, in contact with water, either solid or liquid, will lower its freezing point. This means that when a freezing rain comes in contact with such a coating, the freezing point will be lower and the rain will remain as water and not freeze as solid.[7]

However, the chemical principles involved indicate that a quantity of them must be present, and because the water will wash off appreciable amounts from minute to minute, this quantity must be large. Also, it is obvious that a plane may fly through a warm rain before it passes into an ice-forming area; and therefore either such soluble material may be removed before the need arises or some means must be provided under the control of the pilot by which he may replenish the material upon the surfaces where the ice may form. In the case of biplanes these surfaces are so numerous that an intricate system of pipes, tubes, or other equipment would be required, leading to an increase in weight of the plane and to greater complexity of the pilot's work.

Many attempts have been made to provide solid materials in the form of paint or varnish, the physical nature of these being such that the adhesion tension between a film of them and water, in solid or liquid form, might be so low that ice as formed would

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[1] Research chemist. Mr. Geer was born in Ogdensburg, N. Y., and obtained secondary education in the normal school at Potsdam, N. Y., going then to Cornell University, where he took an A.B. degree in 1902 and a Ph.D. in 1905. At Cornell he specialized in chemistry and physics; then remained one year as instructor in chemistry, afterward joining the U. S. Forest Service as a specialist in the utilization of waste wood, also investigating extraction of turpentine and rosin from the waste pine wood of the Southern States. In 1907, he joined the staff of the B. F. Goodrich Company as chief chemist, remaining until 1925; he was a member of the Board of Directors and the Executive Committee and was Vice-President in charge of development and research. Retiring on account of health, after a period of travel and rest he built a small research laboratory and has since been engaged in private research, primarily in the rubber field.

[2] *Monthly Weather Review*, vol. 58, p. 22 (1930).

[3] *Monthly Weather Review*, vol. 57, p. 503 (1929).

[4] *Journal of the Franklin Institute*, vol. 210, p. 537 (1930).

[5] National Research Council, Canada, 1929.

[6] N.A.C.A. Technical Note no. 293 (1928); N.A.C.A. Technical Note, no. 313 (1929).

[7] Knight and Clay, N.A.C.A. Technical Note, no. 339 (1930).

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

not adhere to such a surface, but would be blown off as rapidly as formed. Without doubt, there are possibilities in so far as zero-adhesion materials are concerned; and when the physicists and chemists have conducted, if they ever do, a complete study of interfacial adhesion tensions between water as a substance and a large number of solids, many will be found to show very low or zero adhesion. It was this thought that the author had in mind in 1927 when his interest in this ice problem first arose.[8]

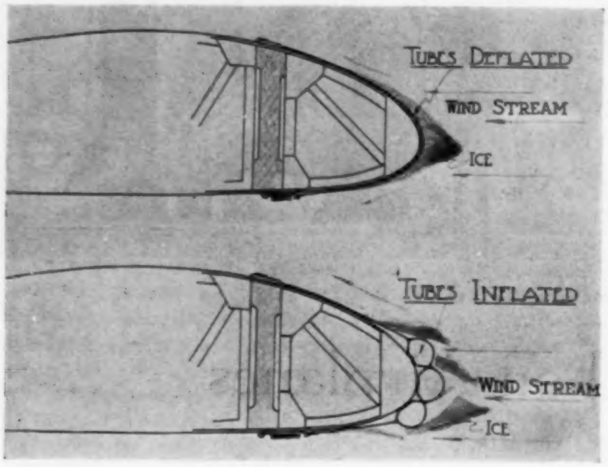

FIG. 2 GENERAL PLAN OF PNEUMATIC RUBBER OVERSHOE FOR ICE REMOVAL

However, the work of the author and Dr. Merit Scott, which was sponsored by the Daniel Guggenheim Fund for the Promotion of Aeronautics, Inc., and which was done through the cooperation of the Department of Physics at Cornell University in the Fall of 1929, has shown that even though low or nearly zero adhesion be obtained, the ice is not blown off, but because it shapes itself about the leading edges of airplane parts, movement of it is essential if it be removed. A discussion of this subject has already been published.[9] These studies proved conclusively in the laboratory that not only was a low adhesion surface necessary, but that some mechanical device was required through the operation of which the ice after formation could be removed.

The most practical of such devices proved to be a rubber overshoe attached to the leading edge of each of the exposed parts and capable of inflation by means of air pressure. The surface of the overshoe was formed of a thin sheet of vulcanized rubber into which was soaked a mixture of oils, the freezing points of which were so low that they would, under any practical conditions, be still liquid. Thus the rubber, which in itself possesses a low adhesion tension toward water, can be lubricated by such an oil mixture and the force of adhesion therefore kept still lower than shown by vulcanized rubber alone.

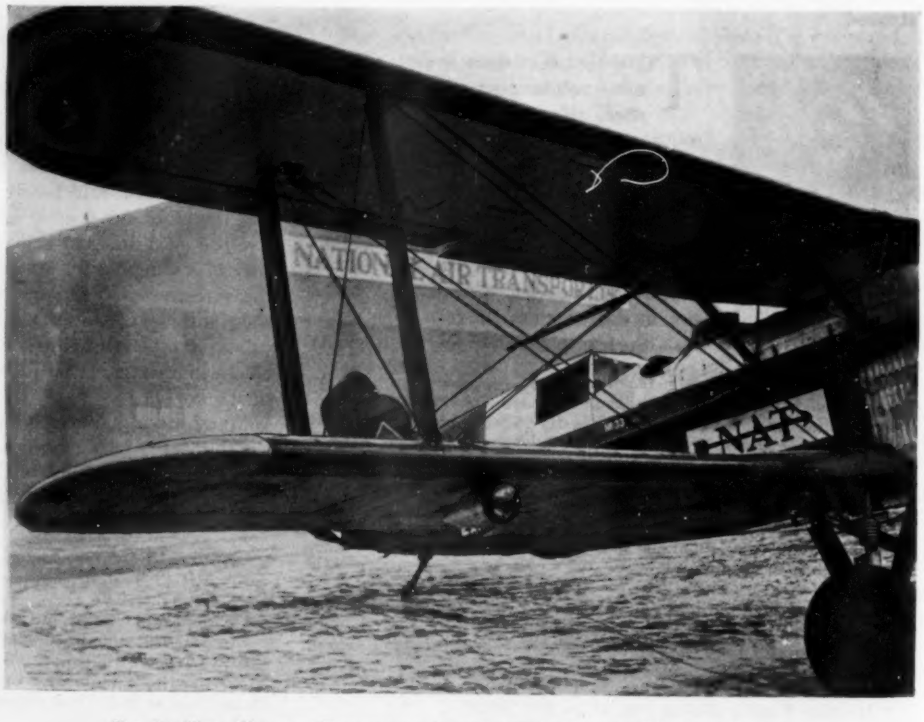

FIG. 3 NEAR VIEW OF PNEUMATIC OVERSHOES APPLIED TO WING OF PLANE

The first practical tests of this ice-removing overshoe were performed late in March and early in April, 1930, upon a plane of the National Air Transport, Inc., which was piloted by Mr. Wesley L. Smith, and indicated clearly the success and practicability of this pneumatic rubber-overshoe idea.

Since that time the B. F. Goodrich Rubber Company at Akron, Ohio, has been engaged actively in the development of these pneumatic ice-removing overshoes. For this purpose there was constructed at the factory of this company the largest refrigerated wind tunnel which has ever been made, the throat of which measures 18 in. by 36 in. This tunnel has permitted tests of sections of wings, struts, and wires in full size, although short in length. The wind velocity attainable is up to 85 miles per hour, and the temperature may be brought down to 0 deg. fahr.

Fig. 1 indicates schematically this refrigerated wind tunnel. The part to be tested is set up in position as shown, and precooled water is sprayed into the wind stream ahead of the leading edge of the part to be tested. Temperature, wind velocity, and other data are of course taken. The design of this wind tunnel was worked out through the cooperation of the engineers of the Langley Memorial Laboratory at Langley Field.

For the protection and removal of ice from wings, struts, radio masts, and other of the larger parts, the design of this pneumatic overshoe consists essentially of a layer of flexible fabric covered with rubber, in the leading edge of which are placed one, two, or three tubes composed of flexible fabric covered and lined with rubber. The general plan of it may be seen in Fig. 2. The principles involved are those of gaining strength and uniformity of inflation by means of flexible fabric, and air tightness on the inside of the tubes and contact with the atmosphere on the outside by the use of a proper rubber mixture.

The overshoe is made and vulcanized in the rubber factory

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[8] *Ind. Eng. Chem.*, vol. 19, p. 1098 (1927).

[9] Geer and Scott, N.A.C.A. Technical Note, no. 345 (1930).

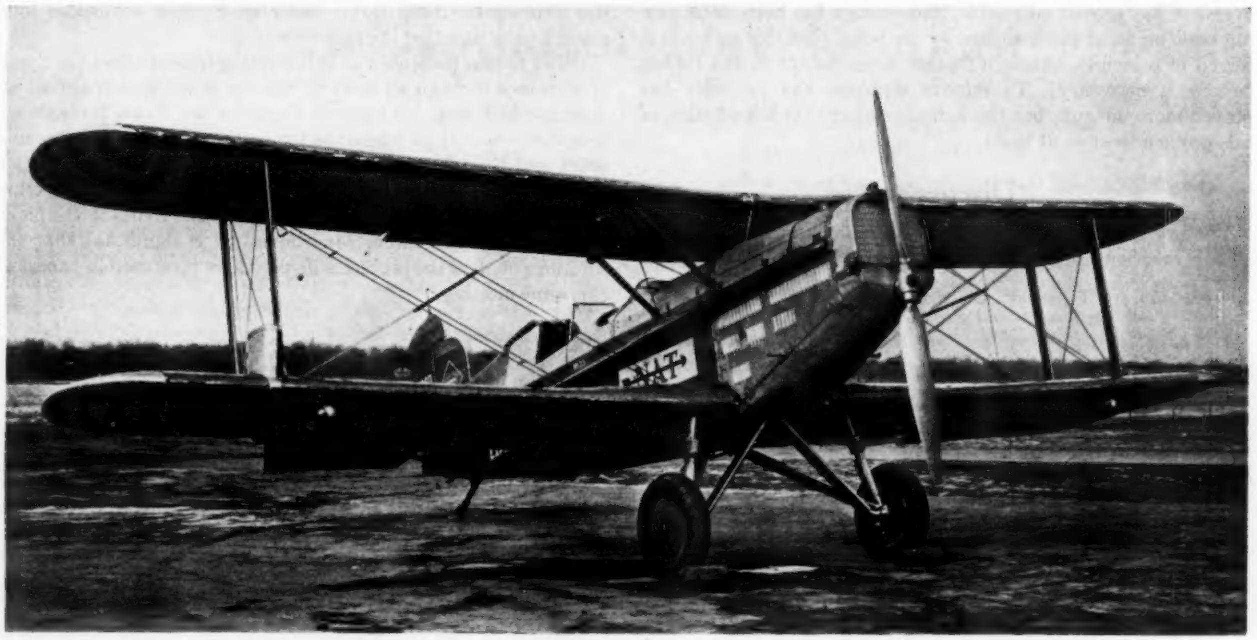

FIG. 4 FULL VIEW OF PLANE EQUIPPED WITH ICE-REMOVING DEVICE

before application to the plane. It is then attached to the leading edge as far back as the maximum diameter of wing or strut. Pressure tubes of small size are attached to the inflation tubes of the overshoe by means of nipples vulcanized in place. These tubes are placed inside the wings and are carried to a distribution valve in the fuselage.

Thus, as indicated in Fig. 2, this part upon the leading edge lies flat, with the tubes deflated, offering relatively little wind resistance, and a small change in curvature of the airplane part.

After the overshoe has been applied to the plane and attached by grommets or any other simple means, there is applied upon this outer rubber surface an oil mixture which soaks into the rubber and gives to it the properties of low adhesion toward ice.

This oil mixture is a combination which swells the rubber but little, and yet when the temperature is reduced to that of ice formation, develops upon its surface a thin lubricating film of oil which effectively lowers the adhesion of the ice. A near view of the pneumatic overshoes applied to the wings of a plane is shown in Fig. 3, and a full view of the plane in Fig. 4. These figures indicate how small these overshoes are relative to the wing surface of the plane.

On this particular plane installation the weight per foot of length of the wing overshoe was 0.526 lb., or a total of 36 lb. for the 68.5 ft. of covering. The wire overshoes, which are of a slightly different design, the details of which are not yet fully completed, would weigh less than 0.03 lb. per ft. The total weight involved to cover struts and wires is not given, because the construction of airplanes varies so greatly that a total-weight figure would be meaningless. It is, however, safe to say that ice may be removed by this device without seriously decreasing the useful load capacity of any given airplane.

In the operation of the plane it is necessary to provide an air pump, a capacity tank, and a rotary valve. These have been designed. The details need not be entered into because the scheme is a simple one of engineering in the field of handling air. However, if one assumes the mail plane of any type equipped with these overshoes, he would find tubes running from proper nipples attached to the overshoes to a rotary valve driven by an electric motor at a speed such that the tube in each overshoe when in use would be inflated and deflated three times per minute.

This rate has been found to be the most practical one after a long series of refrigerated wind-tunnel studies. In a completely equipped plane the pilot when running into an ice-forming area would need but to throw a switch to start his automatic rotating valve in operation and to open a valve connected with his air-storage tank. From then on the tubes will inflate and deflate at the rate of three times per minute and the ice be blown off before any of it has become thick enough to impede the maneuverability of the plane or to weight it down seriously. So far as the larger parts, such as the wings and struts, are concerned, the problem is solved and frequent practical tests have so proved.

However, the problem of wires and that of the propeller are far more difficult. The wires are so small, and the vibration of them when any body, either the overshoe or ice, is attached is so serious, that it has been a very difficult matter to find an article such that in itself the protecting device would not increase the vibration and yet be efficient enough to remove the ice as desired. Such a design has been made and has been tested enough to lead one to believe that it will be successful.

The propeller presents difficulties of even a more serious character because the ice, even if removed, is thrown off the propeller blades irregularly, and therefore unbalances it, leading to serious vibration of the plane. Several schemes have been tried with some success, but the work has not yet progressed far enough to warrant a statement that any one of them can be applied with certainty in actual practice.

Cooperation has been enjoyed with the laboratory of the National Advisory Committee for Aeronautics. In this laboratory tests of the effect of the overshoes on lift and drag have been made, and publications of the tests will doubtless be forthcoming.

To sum up, therefore: the removal of ice from airplanes has definitely been demonstrated in practical flying tests on full scale in so far as wings and struts are concerned by means of a pneumatic rubber overshoe applied to the leading edges of these parts. The removal of ice from wires has been studied on a 3-ft. scale in the refrigerated wind tunnel where a very small rubber overshoe has proved successful, and enough has been done at a full scale on an airplane to lead to the belief that this problem is solved in principle, although further development in the rubber factory is necessary. To remove ice from the propeller has proved more difficult, but the indications are that the solution of this problem is close at hand.

Discussion

WESLEY L. SMITH.[10] Since this paper has been written, complete protection from ice has been provided for a monoplane, and the airplane has been flown under ice-forming conditions long enough to be sure that the system is perfect.

Other papers presented at this meeting indicate that the flying of airplanes through all sorts of weather conditions is almost an accomplished fact. Whenever airplanes are flown through atmospheric conditions where the temperature is below the freezing point and the relative humidity is greater than 90 per cent, ice will form on the various parts of the airplane. Ice protection then becomes a part of the general problem of flying the airplane regardless of the weather conditions. It is significant that the solutions of all of the problems involved are presented in papers at this same meeting.

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[10] National Air Transport, Chicago, Ill.

Wing Radiator Development

By G. A. LUBURG,[1] GARDEN CITY, N. Y.

This general paper on the subject of wing radiators contains a summary of experimental data of the Curtiss company and information of much original work. It should be of interest and help to airplane designers. The author explains apparently contradictory results by stating the comparatively few installations of this type and the usual accompanying secrecy. The use of wing radiators for general service is questionable, as they are expensive to build and easily subject to damage.

During 1920 the Curtiss Aeroplane and Motor Company built a Racer to compete for the Gordon Bennett Cup in France, and it was in connection with the development of this design that the idea of wing radiators originated. There was not sufficient time to develop the idea to an extent that the radiators could be relied upon for the race. Investigations continued, however, and an experimental radiator was installed in a Curtiss Oriole during 1921. The installation was not satisfactory due to defective construction, bad circulation, and leaks, and aside from this, the radiator itself was too small. Increased speed of the machine, however, indicated the advantage of the installation. A second radiator was made early in 1922 with better header construction and larger cooling area. This radiator was of sufficient size so that real test flights could be made. Results were so satisfactory and installation construction difficulties were so far overcome that wing radiators were included in the 1922 Curtiss Army Racer design. A Navy Racer a year before that had a top speed of 186 miles an hour when equipped with a Lamblin radiator was also fitted with wing radiators, with a speed increased to over 200 miles an hour. These wing radiators were all constructed of a flat sheet which lay directly on the wing cover and to which was soldered a corrugated sheet, these corrugations forming the water passages. Headers at the leading and trailing edges distributed the circulating water through the corrugated sheets.

Until 1928 the main construction details changed very little, although a number of installations were made by the Curtiss Company for the various Pulitzer-Schneider Cup Racers. All of them gave good performance, and some of the installations proved very rugged and reliable from a maintenance standpoint. Wing radiators were incorporated in the PW-8 Pursuit, a one-place type for the U. S. Army, and the service test of these machines indicated that wing radiators could be manufactured to give satisfactory service for general use.

For the 1926 Schneider Cup Race at Hampton Roads, the Italians sent over their first machine equipped with wing radiators. The author does not know the details of the construction of these radiators except that they were built up from separate flat tubes running fore and aft. The British machines that competed in the 1927 Races at Venice were equipped with wing radiators, the Supermarine having smooth skin radiators and the Glosters with a corrugated type, very much on the same style as the Curtiss type. The Mercury Racer built by Lieut. Alford J. Williams and his associates for the 1927 Schneider Cup Race had wing radiators constructed of flattened tubes. During test flights considerable trouble was experienced due to leaks and air locks.

In 1929 the Curtiss Company started an investigation of a separate-tube type radiator, and in 1930 built and installed a wing radiator of this design on the Marine Corps Racer flown by Captain Page.

Types and Construction

Fig. 1 shows a section of the Curtiss-design radiators of the corrugated type. The 0.005-in. thick brass for the corrugated sheet was 14 in. wide, so that after running this sheet through the corrugating rollers it became 9 in. wide. The lower flat sheet was also therefore made 9 in. wide and was tinned. The tops of the corrugations, which had to be soldered to this flat sheet, were tinned by a rolling process, and the unit was then sweated together by running through rollers that were heated to the proper temperature. It can easily be seen that this work required the utmost skill, as the sweating of these two sheets together had to be practically perfect. The rollers had to be exactly with the right temperature, and the speed at which the radiator section was fed through these rollers had to be governed to close limits. On racing installations, where maintenance was a secondary consideration, the radiator sections on the upper and lower surfaces of the wing were attached to single headers at the leading and trailing edges. Radiators of this type therefore had to be slid like an envelope over the wing panel. The ones designed for the PW-8 Army Pursuit were, however, constructed with separate header tanks for the radiators attaching to the lower and upper surfaces of the wing panel, so that these radiators were much more readily removable. The cooling system on this pursuit type was, moreover, fitted with a bypass valve by which the proportion of water going through the bypass as compared with that going through the radiator could be regulated, and therefore regulate the water temperature in the engine.

The new Curtiss-type separate-tube radiator as installed on the Page Racer is made up of individual tubes terminating at the leading and trailing edges in headers. A cross-section of these tubes is shown in Fig. 2. These tubes at intervals are sweated to brass tie strips, which are in turn attached to the surface of the wing. These radiators are probably more expensive to construct, but maintenance costs are very much lower than with the other type.

One type used on the Williams Racer consisted of tubes rolled flat to a dimension $^{3}/_{32}$ in. by $^{11}/_{32}$ in. This type naturally ap-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

proaches a smooth-skin condition with correspondingly reduced drag characteristics.

The smooth-skin types used by the Supermarine Company are built on the wing skeleton and help take the wing drag and wing torsion loads. Fig. 3 shows the design of this type. It should be noted that these radiators are made from sheet copper instead of brass. The material is approximately twice as thick and the weight is correspondingly increased. However, the heat dissipation through the copper radiator is very much higher than through the brass.

Heat Dissipation

The American radiators are all constructed of brass approximately 0.005 in. thick, which has a heat-transfer coefficient of 0.284 calorie per degree centigrade per second per square centimeter per centimeter thick. The British, according to reports, used copper and in some cases duralumin, which have heat transfer coefficients of 0.72 and 0.36, respectively. The copper is heavier than brass, but the number of square feet needed to transfer a given amount of heat will be less. Table 1 shows comparative data of British radiators (data taken from R. & M. Report No. 1311).

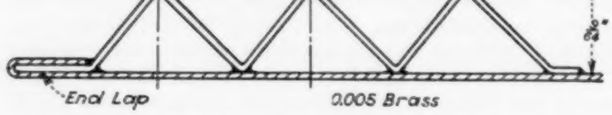

Fig. 1 Original Curtiss Section

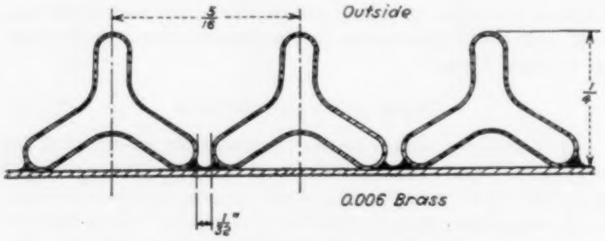

Fig. 2 Section of Tube Type

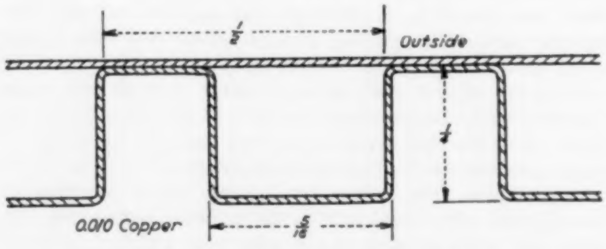

Fig. 3 Section of Supermarine Type

TABLE 1 COMPARISON OF BRITISH RADIATORS

	Drag per hp. at 90 m.p.h., in lb. per hp.	Lb. per sq. ft. cooling area	Heat dissipation at 90 m.p.h. per sq. ft. cooling area, in hp.[a]
Supermarine	0	1.70	0.91
Gloster	0.034	0.92	0.75
Average British Honeycomb	0.160	0.41	0.87

[a] 1 hp. = 42 B.t.u. per min.

Characteristics

For the heat dissipation per pound of drag the wing type is considerably more efficient than the best core type. The smooth outside skin used on the Supermarine is naturally the most efficient because of the complete absence of parasite drag, assuming that the power required to support the weight of each type of radiator in the air is approximately the same. In Table 1 the weights per square foot of cooling area of the wing radiators are based on the weight complete including water, whereas the weight of the honeycomb type is based on a section having no headers nor side walls nor water. The weight of the average American honeycomb for a 100 to 120 m.p.h. plane is between 0.50 and 0.60 lb. per sq. ft., which weight includes the complete radiator and filled with water, but exclusive of the supporting cradle. Fig. 4 shows a number of heat-dissipation curves of various types of radiators. All curves except curve 1 are calculated from actual flight tests and tunnel-test results. Curve 1 is standard up to 120 m.p.h. Above this velocity the curve was approximated from flight-test data on various high-speed honeycomb-radiator-equipped planes. In studying curves 1 and 2, it is found that at high speeds the corrugated type dissipates more heat than the honeycomb, which is probably due to increased wiping effect over the wing. Also the mass of air that flows through the honeycomb does not rise directly with the increase of the airplane speed.

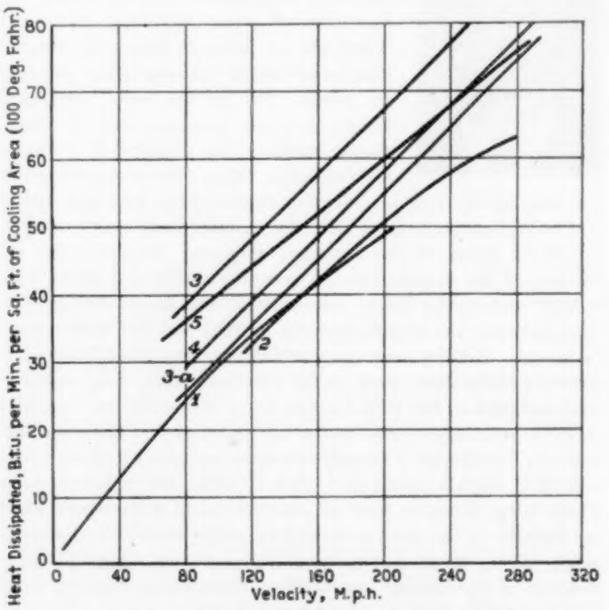

Fig. 4 Heat-Dissipation Curves of Various Radiator Types
(Curve 1, honeycomb; 2, Curtiss brass corrugated; 3, Curtiss tube type, 3-a, Curtiss tube type; 4, copper corrugated; 5, flat-plate type.)

Curve 3 represents the heat dissipated by the Page Racer, based on the area of the outside of the section shown in Fig. 2. However, the lower surface is also exposed to a considerable air flow due to the tubes being soldered to the attaching strips. This allows clearance under each tube for air to circulate. A correction for this additional area gives curve 3-a, which is close to what should be expected.

Curve 4 indicates how a corrugated type (Gloster) made of copper dissipates more heat per square foot than does the corresponding type made of brass. This seems reasonable in view of the fact that copper has a heat-transfer coefficient three times as great as brass.

Curve 5 represents the flat-plate Supermarine type. At high speeds the heat dissipation falls below that of the Gloster corrugated type shown in curve 4. This may be due to the sluggish moving boundary-air layer between the corrugations

TABLE 2 CHARACTERISTICS OF VARIOUS WING-RADIATOR INSTALLATIONS

Ship	Total cooling area, sq. ft.	Hp.	Maximum velocity	Total weight complete with water, lb.	Weight of radiator skin only, lb.	Weight of water, except in engine	Total weight per sq. ft., lb.	Total weight per hp., lb.	Total cooling area per hp., sq. ft.
Oriole, 1922	162	185	116	92.5	48.5	14.5	0.57	0.50	0.875
Army Racer, 1922	244	460	224	221.0	148.0	48.0	0.907	0.48	0.53
Navy Racer, 1922	244	460	200	226.0	145.0	45.0	0.925	0.49	0.53
Schneider Racer, 1923–4	254	500	247	225.0	145.0	45.0	0.885	0.45	0.508
Hawk PW-8, 1924	220	460	172	226.0	135.0	57.0	1.02	0.49	0.478
Pulitzer Racer, 1925	265	600	263	228.0	138.0	57.0	0.86	0.38	0.442
Racer, 1926	266	705	255	232.0	138.0	57.0	0.875	0.33	0.376
XO-13A, 1927	425	720	177	382.0	235.0	90.0	0.90	0.53	0.595
Capt. Page Racer, 1930	285	770	240	327.0	180.0	128.0	1.19	0.425	0.37
Supermarine smooth-skin, 1927	284	875	287	480.0	...	..	1.70	0.55	0.325[a]
Gloster corrugated skin, 1927	284	875	277	261.0	...	..	0.92	0.30	0.325[a]

[a] All areas' weights are approximate.

reducing the efficiency of the radiator at slower speeds, while at high speed this boundary layer is kept cleared out, especially at the forward section of the wing, where most of the heat dissipation takes place. A report made by the Washington Navy Yard showed that the smooth-skin radiator dissipated about 32 per cent more than the old Curtiss corrugated type. Radiator cooling area per horsepower is higher for honeycomb than for wing radiators, showing the correspondingly higher heat-dissipation efficiency of the wing type.

Also the corrugated Gloster wing radiator has 44 per cent more cooling area per square foot of projected area than does the Supermarine type, but the added heat dissipation is only 19 per cent.

At high incidence the heat dissipation of the upper surface increases about 15 to 20 per cent, while that of the lower surface remains the same. The increase in the upper surface is due to more air turbulence as the angle of incidence increases.

Aluminum paint even over copper gives a slight increase in heat dissipated. Also a highly polished surface will radiate more heat, and, conversely, a dirty surface will greatly decrease the rate of heat dissipation.

Due to impact pressure and wiping effect, the section from the leading edge back 25 per cent of the chord dissipates five times the heat dissipated by the rest of the wing. These tests were made in England on a thin airfoil section. It is therefore safe to assume that on thick airfoils the effect will be very much more pronounced because of increased turbulence and wiping effect.

Table 1 gives a fairly good comparison of the relative drags of three types of English radiators. The Gloster corrugated radiator increases the drag of that part of the wing to which it is attached by 24 per cent. The Supermarine smooth-skin type therefore more than makes up the increased weight by the elimination of this increased drag.

On the Williams fin-tube type of wing radiator it was found that the drag of that part of the wing covered was doubled and that the lift was slightly reduced (N.A.C.A. Report 318). This type used by Williams was similar to the Curtiss type except that the fins on the Williams type were considerably higher.

Recent unofficial reports from the Army show that a plane having 73 sq. ft. total cove-radiator area had a speed of 160 m.p.h. The plane was converted to a type having a smooth-skin wing radiator of 60 sq. ft., and with the new installation the top speed became 165 m.p.h. This may be considered a very reasonable speed increase for a revamped ship. It should, however, be remembered that the largest gain in reduced drag does not necessarily come from the use of wing radiators but from the possible cleaner fuselage lines.

Another example of how a considerable speed increase was obtained by changing from the Lamblin type to the wing type is mentioned in the first paragraph of this paper. In this case the increase was 14 m.p.h., which is considerable for a higher speed ship. The English have found that at speeds over 300 m.p.h. a change from the corrugated-skin to the flat-skin type of wing radiator results in a gain in speed of $1^1/_2$ per cent.

Prestone is now being tried out as a cooling medium in place of water. Due to the much higher difference in temperature between the coolant and the air, as well as a larger proportion of the heat being dissipated through the exhaust, actual tests show that a Prestone radiator may have as low as 36 to 40 per cent cooling area as compared with an equivalent water-cooled installation. This decrease in size proportionately decreases parasite resistance due to the radiator. The advantages of the wing-type radiator are also correspondingly reduced. The Prestone-cooled wing radiators would have to be soldered with a special compound having a high melting point due to the high temperature in a Prestone system.

SUMMARY

Table 2 summarizes the general characteristics of various wing-radiator installations. Other general characteristics can be listed as follows:

1 The corrugated or tube wing radiator gives from 4 to 7 per cent speed increase at 200 m.p.h. over the honeycomb type.

2 The smooth skin gives further reduction of drag over the other wing types, but at an increase in weight.

3 Corrugated and tube types have been built as light as the honeycomb type.

4 The use of aluminum paint or even the polishing of the radiator increases the rate of heat radiation.

5 The leading-edge part of the present types dissipates approximately 80 per cent of the heat.

6 The upper and lower surfaces dissipate the same quantity of heat at low angles of attack; at high angles the upper surface increases its rate of heat dissipation.

7 The weight, installed and filled with water, runs from 0.33 to 0.53 lb. per hp.

8 The number of square feet of cooling surface per horsepower is approximately 0.51 at 200 m.p.h. and 0.44 at 250 m.p.h. for the tube type, and 0.36 at 300 m.p.h. for the smooth-skin type.

CONCLUSION

This paper intends to correlate and summarize such data as are available on wing-radiator design and construction. Due to the comparatively few installations that have been made and the secrecy that usually accompanies these installations, the data are rather meager, and in many cases are not based upon the same assumption or conditions. The results, therefore, to a certain extent, contradict each other. In new designs it is best to use data that have been obtained from an installation of the same type as well as judgment from past experience.

The use of wing radiators for general service is questionable. They are very expensive to build, and due to the thin material of which they are made and their exposed location, they are subject to damage. The use of Prestone reduces the radiator efficiency to about one-third of the water-cooled type, so that the advantage of gain in speed from a wing radiator over the core-type installation is cut to a very small amount. In this case, its use will probably be confined to planes of very high speed.

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McCook Field Report, Serial No. 2326, "Cooling Tests on Curtiss Wing-Radiator Section."

Washington Navy Yard Report No. 375, "Heat Dissipation and Pressure Drop for Plate Type and Curtiss and Williams Wing Radiators."

R. & M. Report No. 1311, "Wind-Tunnel Test on Gloster and Supermarine Wing Radiator."

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AER-54-6

FIG. 1 LOCKHEED "VEGA" SIX-PASSENGER SINGLE-ENGINE PLANE

Supercharging the Airplane Engine and Increasing Speed With Altitude

BY ROBERT J. NEBESAR,[1] DETROIT, MICH.

On longer distances a high cruising speed is a necessity. As the speed increases with the cube root of the horsepower, high-speed transportation would not be economical. Flying at higher altitudes with supercharged engines would greatly increase the efficiency. A simple mathematical equation for close estimation of the increased speed in altitude is developed, and the merits of the size of an efficient plane are discussed.

THE problem of high-speed transportation by airplane has many important factors. The desired speed depends primarily on the given distance and on the cost, that is, the efficiency of such transportation. For short distances a surplus of speed would not be justified at a higher cost; for long distances, where the questions of comfort and higher safety of daylight travel enter into the problem, a higher cruising speed is a necessity.

In this paper the author proposes to analyze the best available means of increasing speed and to obtain a criterion of efficiency which, as will be demonstrated, even for greater speeds can be kept reasonably high.

The following nomenclature is used:

NOMENCLATURE

b = span of wings, ft.
$C_{D\text{ind.}}$ = coefficient of induced drag
$C_{D\text{prof.}}$ = coefficient of profile drag
$C_{D\text{par.}}$ = $C_{D\text{prof.}} + C_{D\text{struc.}}$ = coefficient of total parasite drag in the polar diagram (recalculated on unit area of wings)
C_L = coefficient of lift
$\varphi = \dfrac{C_{D\text{par.}}}{C_{D\text{prof.}}}$ = ratio of total parasite drag to profile drag
h = a certain altitude, ft.
$\lambda = b^2/S$ = aspect ratio of wing
L_B = power loading, lb. per hp.
L_w = wing loading, lb. per sq. ft.
ρ_0 = standard density of air at sea level
ρ_h = density of air at altitude h
$T_{HP\text{av.}}$ = thrust horsepower available
$T_{HP\text{req.}}$ = thrust horsepower required
S = wing area, sq. ft.
V_0 = speed at sea level, miles per hour
V_h = speed at altitude h, miles per hour
W = gross weight of airplane, lb.

I—THE ENGINE AND ITS EFFICIENCY IN THE AIRPLANE

One of the main factors to be considered in flying is weight. The weight of an airplane consists of four parts:

1 Weight of the *structure and equipment* of the plane, that is, wings, fuselage, landing gear, tail surfaces, flying controls, flying

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 COMPARISON OF TWO IDENTICAL PLANES WITH DIFFERENT-SIZED POWER PLANTS

1 Gross weight, lb.	2 Pay load, lb.	3 Total rated hp.	4 Range at cruising speed, miles	5 Range in hours	6 High speed, m.p.h.	7 Cruising speed, m.p.h.	8 Fuel capacity, gal.	9 Fuel consumption at cruising speed, gal. per hr.	10 Take-off distance at sea level, ft.	11 Climb at sea level, ft. per min.	12 Wing load, lb. per sq. ft.	13 Power load, lb. per hp.
4500	1100	420	700	4.6	180	152	100	21.7	700	1350	16.35	10.70
4500	1500	300	700	5.2	160	135	81.5	15.7	980	850	16.35	15.00
4100	1100	300	700	5.2	160	135	81.5	15.7	890	950	14.90	13.65

instruments, seats, electrical installations, gasoline, and oil tanks.

2 Weight of the *power plant*, including engine mount, engine controls, engine instruments, propeller with hub, starter, gasoline and oil installations, and water radiator, water and its installation in the case of water-cooled engines.

3 *Useful load*, which includes pilots and attendants, gasoline, and oil.

4 *Pay load*, that is, passengers, baggage, mail, etc., which is often included in the useful load.

The size and type of engine determines the weight of the power plant and also a part of the useful load—that is, the weight of gasoline and oil—which weight varies directly with the specific consumption of the particular type of engine. The difference in weight of tankage will be considered negligible.

Therefore, from the viewpoint of weight, two engines will be considered equivalent if they will enable us to obtain the horsepower per pound of combined weight of power plant and fuel for the same number of hours of flying.

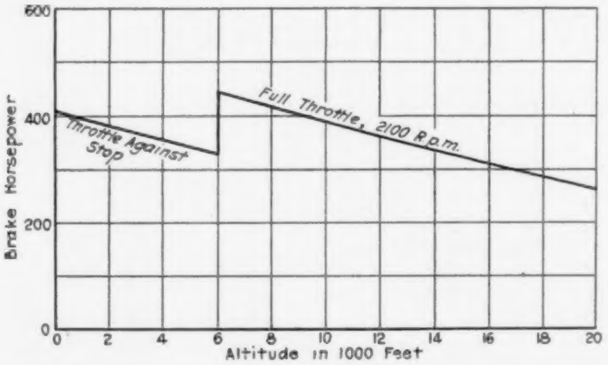

FIG. 2 POWER AT ALTITUDES—PRATT & WHITNEY "WASP" SERIES SC ENGINE
(Compression ratio, 5.25:1; impeller gear ratio, 10:1.)

From the standpoint of speed, flying range, and transport capacity, this would be much more complicated, and no definite relation could be determined. The frontal area of the power plant, its adaptability for efficient cowling, would have to be considered. Also, the speed of the engine would be of great importance due to propeller efficiency.

The best way of comparing two different engines would be to test them out on a given plane, e.g., a six-passenger single-engined machine having a pay load of about 1100 lb. and a range of 700 miles. Let us therefore compare two identical planes of a well-known aerodynamically highly efficient type —the Lockheed Vega (Fig. 1)—one equipped with a 420-hp. engine and the other with a 300-hp. See Table 1.

In order to have comparative results let us bring up the 300-hp. plane to the same gross weight as the 420-hp. one, the saving in weight resulting from decreased horsepower being taken care of in increased pay load.

We see then that the pay load of the 420-hp. plane would be 24.5 per cent of the gross load, and that of the 300-hp. plane, 33.3 per cent of the gross, that is, an increase of 36 per cent. But the cruising speed of the 300-hp. plane would decrease 12.5 per cent and all its flying qualities would be very unfavorably affected. There would be a decrease of 59 per cent in the rate of climb combined with a 40 per cent increase in the take-off distance. Even in bringing the pay load down to the value of the 420-hp. plane (as shown in the third line of Table 1), the flying qualities would not be much improved: climb would be still 42 per cent lower, and take-off distance 27 per cent longer, while crusing speed would remain at its low value.

Now let us see what would happen if we went in the other direction, namely, increase the horsepower.

The higher horsepower would increase the speed and improve the flying qualities of the plane to an unnecessary extent, but the pay load would decrease. In order to keep the pay load at the same percentage of gross weight, we should have to employ a larger-sized plane—a problem which will be considered later.

For the case in question it will be seen that the combination of wing and power assumed is the most desirable. A landing speed of 62 m.p.h. for a plane with all-around good control at low speeds is within reasonable limits.

Flying at altitude with a standard engine would decrease the speed and also the cruising range, owing to the lower efficiency of such an engine at altitude. This will be shown in the next section. However, there is a decided advantage in altitude flying with a supercharged engine.

The weight of a supercharged engine delivering a constant horsepower at a certain altitude would not be practically increased, and the specific fuel consumption would not be much

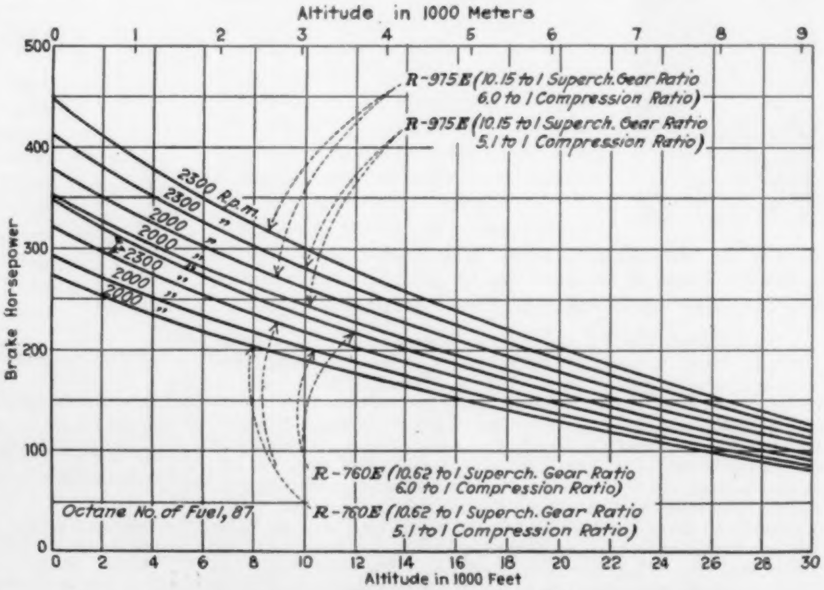

FIG. 3 ESTIMATED POWER AT ALTITUDE—WRIGHT "WHIRLWIND" ENGINES

higher, because of the higher compression ratio and therefore the greater thermodynamic efficiency of such an engine.

As the speed of the plane would be increased and the combined power-plant and fuel weight per horsepower would be constant, the range of the plane would increase with the speed for the same pay load; and as the good flying qualities, landing speed, take-off, and climb would be maintained, we should have a more efficient, more desirable airplane.

Fig. 2 shows the performance at altitude of a Pratt & Whitney "Wasp" Series SC engine, which has a 5.25:1 compression ratio, and a 10:1 impeller gear ratio, and develops 450 hp. at 6000 ft. altitude.

Fig. 3 shows the altitude performance of Wright "Whirlwind" J-6 engines and Fig. 4 that of English "Jupiter" engines.

The Diesel engine would seem to offer many advantages for altitude flying. As it has a high compression ratio and excess of air and oxygen, the power at altitude decreases slower than with the gasoline engine. This has been verified by different flight tests, where the top speed increased to a maximum value at an altitude of 8000 ft. Further, the lower specific fuel consumption will of course extend the cruising range.

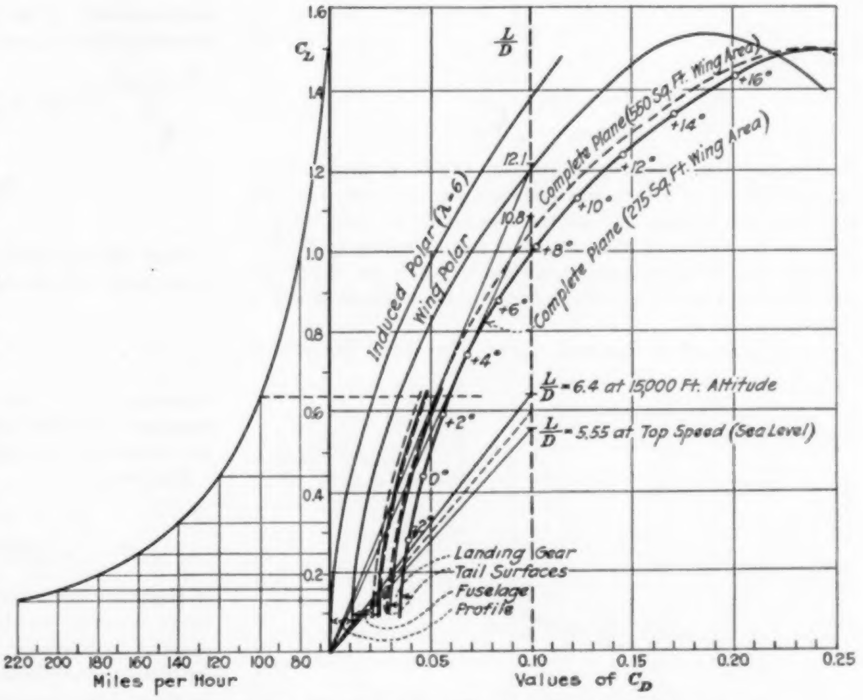

FIG. 6 POLAR CURVES OF LOCKHEED "VEGA" HIGH-WING MONOPLANE

The power at altitude of a 225-hp. Packard Diesel engine is given in Fig. 5.

II—SPEED AT ALTITUDE

Before developing the mathematical equation for high speed at altitude, let us consider the problem for a certain plane, the polar curve of which has been established from the test results as shown in Fig. 6.

The maximum lift coefficient (C_L = 1.5) and wing loading (16.35 lb. per sq. ft.) determine the stalling speed of 65 m.p.h. Due to the ground effect, the landing speed is few miles lower. To any lift coefficient there corresponds a certain speed at sea level—shown by means of the curve at the left in Fig. 6. The efficiency of flying at any speed, or any angle, can be determined by projecting that point corresponding to the desired speed, or angle of attack, on the polar curve from zero point on the vertical line through C_D = 0.10, where we find the value of L/D.

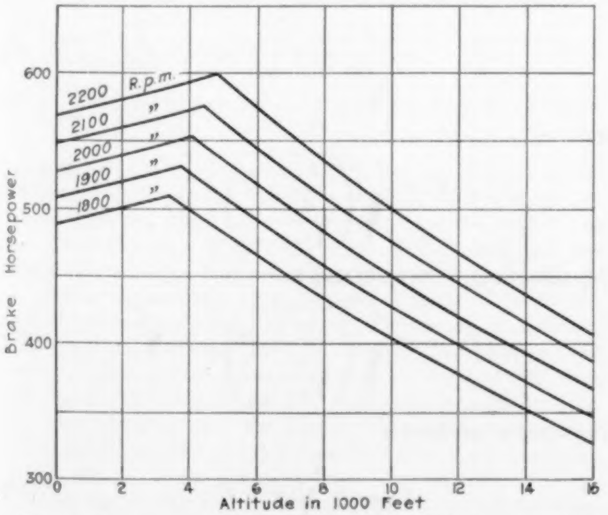

FIG. 4 POWER AT ALTITUDE—"JUPITER" XFAM AND XFAB ENGINES

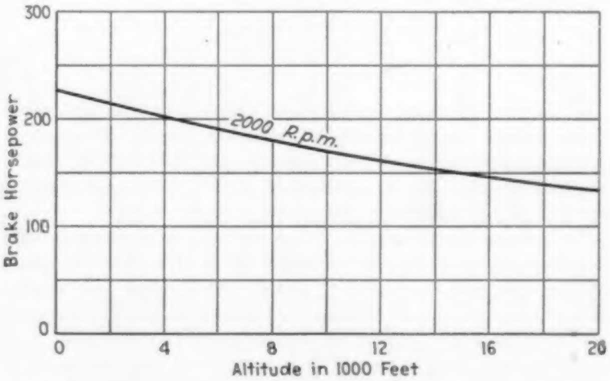

FIG. 5 POWER AT ALTITUDE—225-HP. PACKARD DIESEL ENGINE

The efficiency, as determined by the L/D ratio, depends very much on the wing loading, and therefore on speed. For planes with low wing loading and with large wing area, a higher value of L/D max. could be reached, but the speed would be much lower. For increasing speed the ratio L/D would decrease.

In the present case the maximum L/D is 10.8, and is reached at an angle of attack of +3 deg. for a speed of 95 m.p.h. For the high speed of 179 m.p.h., corresponding to an angle of —3 deg., the value of L/D is only 5.55.

Let us now transform our polar curve into the regular required thrust-horsepower curve, as shown in Fig. 7, and see what happens at altitude. To any point on the required thrust-horsepower curve at sea level would correspond a point in altitude where the speed, as well as the thrust horsepower required, would increase in the ratio of the square root of the air densities:

$$\frac{V_h}{V_0} = \sqrt{\frac{\rho_0}{\rho_h}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots [1]$$

$$\frac{T_{HP\ \text{req.}\ h}}{T_{HP\ \text{req.}\ 0}} = \sqrt{\frac{\rho_0}{\rho_h}} \quad\ldots\ldots\ldots\ldots\ldots\ldots [2]$$

If we neglect the variation in slipstream effects on wing and tail, the given angle of attack will maintain its L/D ratio and will shift on a line passing through the zero point; and the value of L/D can be read on a horizontal line corresponding to $W/375$. Plotting the thrust-horsepower-available curve we check the high speed of 179 m.p.h. corresponding to the angle of attack of —3 deg.

For the case of a standard engine, the thrust horsepower available at constant speed would decrease in the ratio given in the empirical formula:

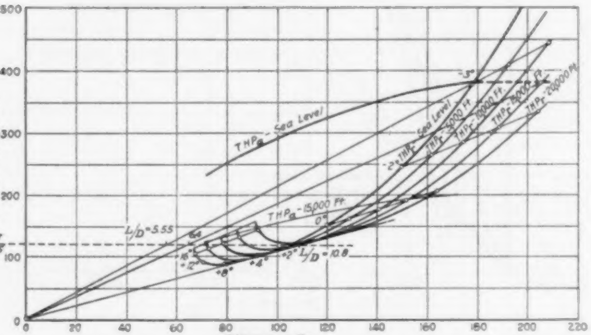

FIG. 7 THRUST-HORSEPOWER CURVES OF LOCKHEED "VEGA" HIGH-WING MONOPLANE

$$\frac{T_{HP\ ah}}{T_{HP\ a0}} = \left(\frac{\rho_h}{\rho_0}\right)^{1.35} \quad\ldots\ldots\ldots\ldots\ldots\ldots [3]$$

In order to visualize the decrease of speed at altitudes, some of the altitude curves have been plotted in Fig. 7. These show that the speed decreases very rapidly and the angle of attack increases rapidly. For an altitude of 15,000 ft. the high speed is only 152 m.p.h., and the angle of attack about 0 deg.

For a supercharged engine, if we neglect the increase in efficiency of the propeller due to higher value of V/ND, the speed will increase and the angle of attack will increase but slowly. Nevertheless a better L/D value for higher speed will be reached. For an altitude of 15,000 ft. the speed would be 206 m.p.h., L/D would increase in same ratio, from 5.55 to 6.4, and the angle of attack would increase to about —2$^1/_2$ deg.

If we replot the change in the angle of attack on the polar diagram, Fig. 6, a mathematical expression for the increase in speed at altitudes may be developed.

Since the equation for the thrust horsepower required for steady horizontal flight at sea level is:

$$T_{HP\text{req.}\,0} = \frac{1}{375 \times \pi \times 0.002558} \times S \times \left(\frac{W}{S}\right)^2 \times \frac{S}{b^2} \times \frac{1}{V_0} + \frac{0.002558}{375} \times S \times C_{D\text{par.}} \times V_0^3 \ldots\ldots [4]$$

for top-speed flight the thrust horsepower required at that speed must equal the maximum thrust horsepower available.

As stated in Equation [2], the thrust horsepower required for a given angle of attack increases at altitude as the square root of the air density. Then, since at an altitude h (density ρh) the thrust horsepower required equals the maximum thrust horsepower available at sea level, the corresponding thrust horsepower at sea level for a given angle of attack would be:

$$\frac{T_{HP\text{av.max.}}}{\sqrt{\frac{\rho_0}{\rho_h}}} = \frac{1}{375 \times \pi \times 0.002558} \times S \times \left(\frac{W}{S}\right)^2 \times \frac{S}{b^2} \times \frac{1}{V_0} + \frac{0.002558}{375} \times S \times C_{D\text{par.}} \times V_0^3 \ldots\ldots [5]$$

From this equation we can determine the corresponding speed at sea level, and the top speed at the altitude h will be

$$V_h = V_0 \times \sqrt{\frac{\rho_0}{\rho_h}} \quad\ldots\ldots\ldots\ldots\ldots\ldots [6]$$

Equation [5] is of the fourth degree and to solve it for speed V would be somewhat difficult, but if we analyze its components we can obtain very simple results.

The portion

$$\frac{1}{375 \times 0.002558 \times \pi} \times S \times \left(\frac{W}{S}\right)^2 \times \frac{S}{b^2} \times \frac{1}{V_0}$$

of Equation [5] is the induced thrust horsepower required and varies inversely as the speed. For usual wing loadings W/S is about 16 lb. per sq. ft. and the aspect ratio S/b^2 about 6, and for high-speed, well-designed planes its value at top speed would be about 5 to 10 per cent of the total thrust horsepower required. Its variation with speed with the supercharged engines up to 30,000-ft. altitudes is small, as may be seen from the polar curve.

The portion

$$\frac{0.002558}{375} \times S \times C_{D\ \text{par}} \times V_0^3$$

of Equation [5] is the parasite thrust horsepower required. It depends primarily on the aerodynamical fineness of the design, the value $C_{D\text{par.}} = \varphi \times C_{D\text{prof.}}$, being measured in terms of the profile drag coefficient. For the range of speeds considered, this portion of the drag can be assumed constant, that is, $C_{D\text{par.}}$ is parallel to $C_{D\text{ind.}}$ and increases as the cube of the speed.

Neglecting the variation of the induced thrust horsepower required for a range of speeds up to 30,000 ft. altitude, as previously stated, we can, for a given plane, by substituting the constants A, B, and C, transform Equation [5] as follows:

$$\frac{A}{\sqrt{\frac{\rho_0}{\rho_h}}} = B + C \times V_0^3 \quad\ldots\ldots\ldots\ldots [7]$$

and

$$V_0 = \sqrt[3]{\left(\frac{A}{\sqrt{\frac{\rho_0}{\rho_h}}} - B\right) \times \frac{1}{C}} \quad\ldots\ldots\ldots\ldots [8]$$

Top speed at altitude h will then be

$$V_{h\max.} = \sqrt[3]{\left(\frac{A}{\sqrt{\frac{\rho_0}{\rho_h}}} - B\right)\frac{1}{C}} \times \sqrt{\frac{\rho_0}{\rho_h}} \quad\ldots\ldots\ldots [9]$$

Top speed at sea level is

$$V_{0\max.} = \sqrt[3]{(A - B)\,\frac{1}{C}} \quad\ldots\ldots\ldots\ldots [10]$$

and if we omit the constant B, whose influence on top speed at

TABLE 2 EFFECT OF INCREASING SIZE OF PLANE

(Single-engined plane; speed, 100 m.p.h.)

		S = 275 sq. ft.; W = 4500 lb.				S = 550 sq. ft.; W = 9000 lb.			
		Area or cross-section, in sq. ft. (or size)	Drag, lb.	Per cent of wing profile drag	Thrust hp. required	Area or cross-section, in sq. ft. (or size)	Drag, lb.	Per cent of wing profile drag	Thrust hp. required
	Wing induced	275	153	.....	40.8	550	306	.....	81.6
Total parasite	Wing profile	275	88	.....	23.4	550	176	.....	46.8
	Fuselage group, including windshield, engine, and cowl	18	94	106.8	25	28	146	83	39
	Tail surfaces—horizontal and vertical	70	40	45.5	10.6	99	56.5	32.1	15
	Landing-gear group, including wheels, struts, and tail wheel	32 × 6 wheels	46	52.3	12.2	36 × 8 wheels	66	37.5	17.6
	TOTAL	...	421	204.6	112	...	750.5	152.6	200
	$\varphi = \frac{C_{D\,par.}}{C_{D\,prof.}}$	...	...	3.046	...	...	...	2.526	...
	L/D	...	$\frac{4500}{421} = 10.7$	.....	...	...	$\frac{9000}{750.5} = 12$	.....	...

sea level is approximately the same as at altitude, we obtain a simple equation

$$V_{h\max.} = V_{0\max.} \sqrt{\frac{1}{\sqrt{\frac{\rho_0}{\rho_h}}}} \times \sqrt{\frac{\rho_0}{\rho_h}} = V_{0\max.} \times \sqrt[3]{\frac{\rho_0}{\rho_h}} \quad \ldots\ldots [11]$$

or the speed at altitude increases as the cube root of the air-density ratio.

For the altitude of 30,000 ft. the difference between the speed obtained with this simplified method and the theoretical value calculated for a given case will be little over 2 per cent, and the increase of speed will be only 36 per cent instead of 39 per cent as given by Equation [11].

Curves showing the increase of top speed at different altitudes are given in Fig. 8.

III—EFFECT OF SIZE OF PLANE

The size of the plane is of primary importance in obtaining high speed and flight efficiency.

In Fig. 6 there is shown the polar curve of a six-passenger plane with 275 sq. ft. wing area. The wing loading is 16.35 lb. per sq. ft. and the power loading 10.7 lb. per hp. The maximum L/D is 10.8, and for a top speed of 179 m.p.h. L/D = 5.55.

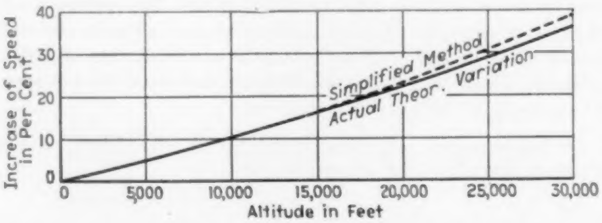

FIG. 8 INCREASE IN TOP SPEED AT ALTITUDE DUE TO CONSTANT POWER

From different flight and wind-tunnel tests the author has determined the drag of different parts of the plane and has plotted the corresponding values in the polar diagram, Fig. 6, and in the thrust-horsepower diagram, Fig. 9. Due to the angle of incidence of wing and to the interference, it has been found that the drag coefficients for fuselage, tail surfaces, and especially landing gear decrease as the low angle of attack is approached. The wing-profile drag-coefficient variation is taken from data obtained with the Langley Memorial Aeronautical Laboratory high-density wind tunnel (N.A.C.A. Tech. Note No. 367, Fig. 6).

The values of drag and thrust horsepower required for these different items at 100 m.p.h. speed are tabulated in the left half of Table 2.

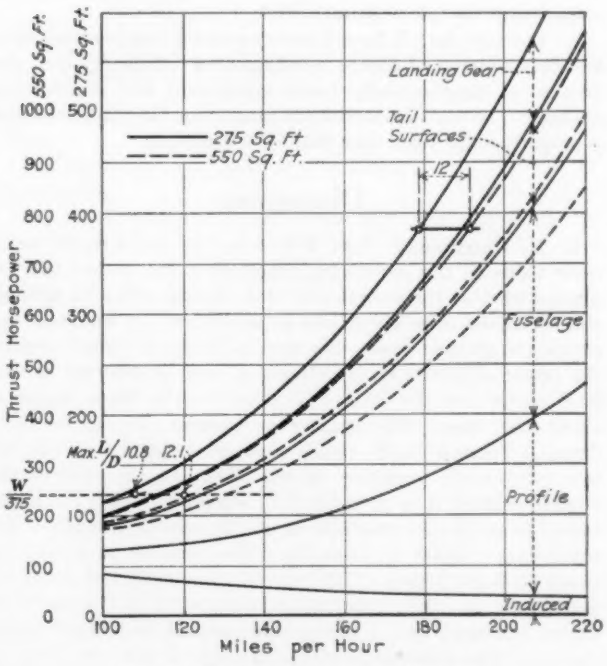

FIG. 9 THRUST-HORSEPOWER CURVES

The efficiencies of the different parts of the plane are compared with the efficiency of the wing on the basis of the ratio of the drag of each part to the profile drag, expressed as a percentage. φ = 3.046, and L/D at 100 m.p.h. = 10.7.

Let us now consider a larger plane—twice the size—of similar design. The wing and power loading will be kept the same, so that the landing speed, take-off, and climb will not be changed.

The drag and thrust horsepower required at 100 m.p.h. for this plane of 550 sq. ft. wing surface are given in the right half of Table 2. We can accordingly visualize how the corresponding parasite drags decrease in their ratio to the wing drag.

The fuselage drag coefficient is only 77.6 per cent of that of the smaller plane; the tail surfaces drag coefficient, 70.6 per cent, and the landing-gear drag coefficient, 71.8 per cent.

The ratio of total parasite drag to wing profile drag, or φ, is decreased to 2.526, and L/D at 100 m.p.h. is increased to 12.

Corresponding values are plotted in the polar diagram, Fig. 6, and shown by broken lines.

The thrust-horsepower curves are shown by broken lines in Fig. 9 to half-scale, so that the induced and profile thrust-horsepower curves for 275 and 550 sq. ft. wing are identical. We can check up an increase of 12 m.p.h. in speed, with increased efficiency. *L/D* at high speed will be = 6, as compared with 5.55 for the 275-sq. ft. plane.

Flying at altitudes would still improve the efficiency, and the speed would increase quite considerably. At 20,000 ft. altitude the top speed of 233 m.p.h. could be easily attained.

Apart from the aerodynamical advantage obtained, the improvements will be as follows:

1 Weight saving; a more efficient and lighter structure could be built for a larger unit.

2 More comfort for passengers could be provided because of a larger and roomier fuselage.

3 Better inherent stability could be obtained, because of the easier problem of weight distribution and the increased damping moment of a larger airplane. It has been shown by Professor Klemin that the disturbing moment increases with the cube of the dimensions of the airplane, while the damping moment increases with the fourth power.

4 The airplane will have a lower operating cost per passenger, not only because of higher aerodynamical efficiency, but also because of proportionally lower investment and depreciation charges. The cost of material and production for a large machine is proportionally lower than that for a small one.

Discussion

H. K. Cummings.[2] Figs. 2 to 5 in the paper would be of more value to the power-plant engineer if the author had explained whether the variation of brake horsepower with altitude was estimated from flight data or determined by means of approximate altitude tests. For each of the four Wright engines the power at 20,000 ft. is between 45 and 48 per cent of sea-level power and the rate of decrease for the Wasp engine is about the same. The engines in question are considerably "ground boosted" and require a superior quality of fuel for safe full-throttle operation at sea level. In the case of the Jupiter engines it is probable that sea-level pressure is maintained in the intake manifold up to the critical altitude of the supercharger (3300 to 4800 ft.). The decrease in power between 6500 and 16,000 ft. (28 per cent) is slightly less than that determined at the Bureau of Standards for a Curtiss D-12 engine equipped with a low-altitude geared centrifugal supercharger. The decrease in power of only 40 per cent between sea level and 20,000 ft. shown by the Packard Diesel engine is indeed noteworthy, being about two-thirds that experienced by conventional aircraft engines.

The supercharged engine which will deliver a specified horsepower at a particular altitude may weigh much less than the unsupercharged engine required to deliver equal power at the same altitude, but the engine equipped with a high-altitude geared centrifugal supercharger will not compare as favorably with the unsupercharged engine at low altitudes, on account of the power required to drive the supercharger. Although the built-in centrifugal supercharger is much used at low altitudes, the turbo-centrifugal supercharger and the Roots-type supercharger are better adapted to the problem of maintaining approximately sea-level power up to considerable altitudes. If the author contemplates engines supercharged up to 30,000 ft., as is implied in Section 2, the use of a compressor of several stages as proposed recently by Schey[3] might be considered.

With reference to advantages 1 and 4 mentioned at the end of the paper, it will probably be found necessary to supercharge the cabin of a passenger transport operating at high altitudes.

Author's Closure

The author wishes to thank Mr. Cummings for his comparison of given altitude curves with the Bureau of Standard's experiences. The altitude performances of the engines, as shown in Figs. 2 to 4, are determined by means of approximate altitude tests, keeping constant r.p.m., and the author purposely gave these three different types of operating them:

Fig. 2, where the throttle would be only partly open until the altitude of safe full-throttle operation is reached.

Fig. 3, where the boosted sea-level output is permissible (throttle fully open), provided a fuel of 87 octane value is used.

Fig. 4, where the safe intake-manifold pressure is maintained, and the throttle gradually opened up to the critical altitude.

It is believed that up to the altitude of 20,000 ft., passenger flying could be organized on existing airways, the engines having built-in centrifugal superchargers and no special equipment or supercharging of the cabin. The increase on the cruising speed would be quite noteworthy, considering that the engines would then, with wide-open throttle, deliver at that altitude the same power as at present cruising speeds at sea level. At 20,000-ft. altitude the cruising speed would be higher than the top speed at sea level, as we have it nowadays.

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[2] Chief, Automotive Power Plants Section, Bureau of Standards, Washington, D. C.

[3] "Superchargers and Supercharging," O. W. Schey, *S.A.E. Journal*, May, 1931, vol. 28, pp. 524–534.

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	Franklin Institute
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	Engineers' Society of Western Pennsylvania
	Carnegie Institute of Technology
	Carnegie Library (Schenley Park)
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AER-54-7

Study on Airplane Weight Complex

By A. A. GASSNER,[1] HASBROUCK HEIGHTS, N. J.

By deciding on the type of engine and the overall dimensions of an airplane, the weight becomes quite well fixed. To obtain good take-off and climb characteristics the gross weight per horsepower should be kept low. Such items as engines, starters, propellers, generators, wheels, tires, and instruments have a weight which cannot be altered by the airplane designer, so his problem is quite difficult. In this paper the author groups the various equipment and parts of an airplane into three classes: (1) Those with weights outside the designer's direct influence, (2) those with weights over which the designer has limited influence, and (3) those with weights which can be controlled by the designer. Design data are also given for three Fokker airplanes. These are the Super-Universal, the F-10-A, and the F-32.

ONE OF THE essentials of airplane design is the treating of the weight problem. To design a plane around a given engine with a minimum of dead weight and a maximum of useful load, at the same time fulfilling demands for strength of all structural and control parts, is a problem. The finished airplane should, of course, exceed in performance all ships of the same class previously built by the same concern, and must at least equal contemporary competitive designs.

Commercial planes can be classified into three groups: (1) Mailplanes carrying only the pilot and mail or express matter, (2) training and sport planes of the open-cockpit type, and (3) passenger-transport planes of the cabin type. In this paper only the latter type will be considered in detail, although the basic principles are of course alike for all kinds of airplanes.

In treating scientific or technical problems one is most always inclined to generalize. However, one should, as far as possible, accumulate a large number of particular instances before attempting a generalization, and then attempt to discover the reason that underlies that particular collection of similar instances.

In the field of airplane design most designers are very free in giving information as to wing areas, control-surface areas, number and horsepower of engines, and such facts. With but a single exception, C. Dornier, the author has never seen statements given out by any designer on detailed weights of their ships. Magazines and sales literature contain data on the weight of empty planes, but as a rule this data means little, as information as to what equipment and furnishings are included in this weight empty is lacking.

As the author has available only the data on his own previously constructed ships, and has, unfortunately, to generalize on a rather limited number of particular cases, it is hoped that the question brought up here will have a stimulating influence in persuading other airplane manufacturers to publish their experiences with this problem.

The automobile industry is in a different position as to such trade secrets. There are none existing. Every concern buys and tries out models of the other manufacturers and the results prove profitable to every one. Unfortunately the financial question prohibits the same methods for the airplane industry, and the airplane engineer is not able to check up on his competitors to the same degree as is the automobile engineer.

Three items influence airplane weight: (1) The dimensioning and appointments of the fuselage or cabin necessary for the comfort of the passengers; (2) the demands on fuel and oil capacity of the tanks, and on cooling and operative facilities as made when the engine type and the cruising range are given; and (3) the necessary structural safety requirements. An endeavor will be made to show that the weight of a plane is predetermined to a very large degree when the engine type and the overall dimensions of the plane are chosen, and that great care has to be taken in this respect. This fact explains why new ships of designers having long experience most always prove to be successful, and that a ship specially designed around a given engine is most always better in general performance than a ship primarily designed for a different type of engine and later equipped with a more powerful motor.

It is essential to choose the right kind of material and design details for all parts of the ship. Today, one cannot say that all-metal construction is superior for the one type and the composite design with steel-tube fuselage is better for the other. Ease of production and adaptability to changes have a great influence on this selection. It can, however, be said that one special type of design of tanks, engine mounts, floors, etc., is basically superior to another.

To the airline operator the plane is merely another and new form of transportation vehicle. He manufactures and sells transportation, and the fundamental principles of all manufacturing business have to be taken care of. He can sell his transportation only when the public derives a material benefit by buying it in preference to older methods, and if the saving in time, which is the one advantage of airplane traveling, is not derived by a disregard of comfort and safety. Comfort is a rather indefinite term, but the traveling public seems to have accepted the facilities as offered by the railroads in Pullman cars as satisfactory. People want more than necessities, they demand luxuries.

The airline operator expects, therefore, that his planes are comparable to busses and railroad Pullman cars so far as the interior equipment and arrangement are concerned. Large windows which afford unobstructed view, good heating and ventilating facilities, and large, comfortably upholstered chairs and ample leg room and sufficient aisle space are needed. The

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 GROUP A—WEIGHTS OUTSIDE OF DESIGNER'S DIRECT INFLUENCE

	Super-Universal		F-X-A		F-32	
Engines with accessories	1 Wasp C	720	3 Wasps C	2160	4 Hornets B	3360
Propellers	9 ft. 6 in. 2 blade	90	8 ft. 6 in. 3 blade	309	(2, 10 ft. 2 blade) (2, 9 ft. 3 blade)	542
Starter installation	Hand inertia	20	Hand inertia	60	Electric inertia	160
Generator assembly		..		..	2 × 15 amp.	34
Starting booster	Magneto	10	Magneto	10	Coils	8
Ignition switches		1		4		5
Ammeter, light switches		2		3		4
Cabin dome lights		2		4		12
Battery and box	35 amp. per hr.	40	70 amp. per hr.	70	2–70 amp. per hr.	140
Starter switches		..		..		6
Starter cables		..		..		30
Wheels with brakes	32 × 6	80	44 × 10	185	54 × 12	239
Tires and tubes	36 × 8	66	44 × 10	130	58 × 14	312
Tail wheel with tire		..		..		24
Flight instruments		15		15		19
Engine instruments		10		32		38
Starter crank and shaft extension		4		10		12
Hand fire extinguishers		15		15		15
First-aid kit		4		4		8
Safety belts		6		14		32
Radio shielding of engines		..		..		50
Total group A, lb		1085		3025		5050
Weight empty, lb		3250		7780		14900
Group A, per cent of weight empty		33.4		38.9		33.9

TABLE 2 GROUP B—WEIGHTS ON WHICH THE DESIGNER HAS LIMITED INFLUENCE

	Super-Universal		F-X-A		F-32	
Fuel tanks with supports	2 × 65 gal	130	2 × 180 gal.	140	4 × 175 gal.	380
Oil tanks with supports		10		35		65
Fuel piping		20		40		65
Oil piping		10		20		33
Fuselage and nacelle cowling		55		150		210
Exhaust system		45		140		250
Cabin heating system		15		20		60
Cabin and cockpit floors		60		150		350
Cabin windows and frames		50		130		150
Pilots' cockpit enclosure		40		60		122
Pilots' seats and supports		20		30		33
Cabin interior walls and doors		35		80		252
Fuselage outside doors		15		30		58
Electric light installation		10		20		40
Baggage racks, cabin		..		20		22
Toilet equipment		20		20		40
Cabin wall paneling		20		35		120
Cabin sound insulation		..		70		127
Cabin upholstery		10		25		120
Brake control assembly		10		10		25
Instrument board less inst		5		15		18
Linoleum (floor)		10		30		50
Pass. chairs with cushions	6	60	12	170	30	380
Engine controls		15		60		105
Total group B, lb		655		1500		3075
Weight empty, lb		3250		7780		14900
Group B, per cent of weight empty		20.2		19.3		20.6

larger type planes have to be equipped with lavatories, pantries, folding tables, thermos bottles for drinking water, ice boxes, and other equipment of this nature.

The designer or manufacturer of the commercial transport plane has to solve a difficult problem. He must provide all this luxurious equipment which the modern traveler demands, and still keep the weight of the plane as low as possible to ensure good performance, a problem which does not exist to any such extent in other branches of manufacturing of transportation vehicles.

The designer has less freedom in handling this weight problem than one might first suppose. Of course, all purchasers, be they private individuals, business corporations, or operators of commercial airlines, consider the disposable load which can be carried by the plane as one of its most useful characteristics.

Weight empty plus disposable load makes the gross weight of the ship. The author's experience shows that the gross weight of a modern commercial plane as used in this country should be between 10 and 15 times the rated horsepower for multi-engined ships, and between 13 and 18 times the rated engine horsepower for single-engined ships. The gross weight for the same total horsepower must be lower for multi-engined ships as this class has to be able to continue level flight with full load if one engine ceases to function.

To obtain good performance, especially good take-off and climb characteristics, the lower factor of the ones given should be used. As the gross weight has to be kept within rather narrow limits, it is necessary to reduce the weight empty of the plane if one wishes to increase the disposable load.

There are a number of items on each plane which have a prefixed weight which cannot be influenced by the designer of the airplane. This group consists of engines, starters, propellers, generators, wheels, tires, instruments, etc.

The type and horsepower of the engine are in most cases given before the first sketches of the plane are made. Sometimes a different design of engine is substituted for the old one during the manufacturing of the type, but in both cases the airplane designer has very little choice, as the engine selection depends on a number of factors, as for instance, price, horsepower, method of cooling, service facilities, and reputation of the engine-manufacturing company.

Selection of the engine circumscribes the gross-weight range of the airplane, and determines the type and thereby the weight of starter, wheels and tires, propellers, engine instruments, ignition switches, etc. In most of the cases the purchaser of the plane gives specifications as to the electrical equipment to be used. This means that one must consider the weight of generator, batteries, electric-light equipment, and electric inertia starters with cables and switches as fixed, and cannot be influenced by airplane designers. The Department of Commerce regulations provide that all commercial planes must be equipped with safety belts for passengers, and with first-aid kits and hand fire extinguishers, the weights of this equipment also being fixed.

In a second group are items on which the designer has a limited influence as far as weight is concerned, because he has but a limited choice of material and has to consider very definite requirements made by the sales department or by the purchaser of the ship. This second group consists of fuel and oil tanks, fuel and oil pipe lines, fuselage and nacelle cowlings, exhaust system, cabin and cockpit floors, cabin windows, pilots' and passenger seats, cabin interior walls and doors, upholstery, baggage racks, toilet equipment, cabin sound insulation, cabin floor carpet or linoleum, electric-light installation, and engine controls.

The third group consists of weights of items on which the designer has direct influence. These are all structural parts, such as wings, fuselage, tail surfaces, landing-gear structure, nacelle structure (outboard engine mount), and flying control group.

In Tables 1 to 4 these weights and their percentage of weight empty are given for three entirely different types of ships—the Fokker Super-Universal, the Fokker F-10-A, and the Fokker F-32. All these ships are well known in this country and their specifications are given in Table 5. Although these ships are so entirely different from each other in engines, loads carried, size, wing, and power loading, it is surprising how closely the

TABLE 3 GROUP C—WEIGHTS DIRECTLY WITHIN DESIGNER'S INFLUENCE

	Super-Universal		F-X-A		F-32	
		Total		Total		Total
Wing assembly		870		1800		3525
Front spar	168		480		950	
Rear spar	145		410		810	
Ribs	83		160		353	
Misc. woodwork	60		75		230	
Covering 3-ply	270		425		810	
Fittings	41		60		134	
Fixed equipment	45		60		61	
Ailerons	28		72		105	
Paint finish	30		58		72	
Fuselage structure, fairing battens, covering		315		660		1570
Fuselage structure	265		520		1300	
Fairing battens	18		80		135	
Covering, doped	32		60		135	
Nacelle structure		...		120		450
Tail-surface assembly		80		170		380
Stabilizer	32		62		135	
Elevator	21		52		86	
Fin	5		12		39	
Rudder	14		21		55	
Brace struts	8		23		65	
Landing gear less wheels		210		395		680
Axle assembly	64		155		290	
Axle brace struts	22		36		76	
Shock-absorber strut assembly	72		135		136	
Landing-gear cowling	20		26		89	
Brake control	8		11		16	
Tail-wheel structure	24		32		78	
Flying-controls assembly		35		110		170
Elevator-control column	7		32		36	
Rudder pedals	6		23		22	
Elev. and rudder-control cables	9		25		41	
Aileron-control cables	5		12		25	
Stab.-adjusting mechanism	8		18		46	

TABLE 4 SUMMARY

	Super-Universal		F-X-A		F-32		Average for these 3 types
	Per cent W.E.	Lb.	Per cent W.E.	Lb.	Per cent W.E.	Lb.	
Group A							
Weight outside designer's influence	33.4	1085	38.9	3025	33.9	5050	35.4
Group B							
Weight on which designer has limited influence	20.2	655	19.3	1500	20.6	3075	20
Group C							
Weight directly within designer's influence	46.4	1510	41.8	3255	45.5	6775	44.6
Weight empty		3250		7780		14900	
Useful load	41.4	2300	40.6	5320	38.6	9350	
Gross weight		5550		13100		24250	

TABLE 5 DESIGN DATA OF THREE FOKKER SHIPS

	Super-Universal	F-X-A	F-32
Length overall	36 ft. 6 in.	50 ft.	69 ft. 10 in.
Span	50 ft. 8 in.	79 ft. 3 in.	99 ft.
Engines	1 Wasp "C" 425 hp.	3 Wasp "C" 1275 hp.	4 Hornet "B" 2300 hp.
Wing area total	387 sq. ft.	854 sq. ft.	1322 sq. ft.
Weight empty	3250 lb.	7780 lb.	14900 lb.
Useful load	2300	5320	9350
Gross weight	5550	13100	24250
Wing loading	14.35 lb. per sq. ft.	15.3 lb. per sq. ft.	18.3 lb. per sq. ft.
Power loading	13.1 lb. per hp.	10.3 lb. per hp.	10.5 lb. per hp.
Top speed	143 m.p.h.	154 m.p.h.	146 m.p.h.
Cruising speed	120	126	122
Passenger and crew	8	14	32
Fuel capacity gal., normal	120	360	500
Fuel cons. per hr., cruising	20	60	110
Range at cruising speed	6 hr.—720 miles	6 hr.—755 miles	4½ hr.—550 miles
Factor: $\frac{\text{No. passengers and crew} \times \text{range at cruising}}{\text{Fuel (gal.)}}$	48.0	29.4	35.2

percentages of weight empty of groups A, B, and C come. The average percentage of weight empty is 35.4 per cent for group A, 20 per cent for group B, and 44.6 per cent for group C.

The fact that about 55 per cent of the weight empty of any landplane of the commercial cabin type is either entirely outside of the plane designer's influence or can be influenced only to a very limited degree, is also indicated by a comparison of ships built by different manufacturers around the same engines for the same general purpose, but of different structural design.

As already mentioned, the weights of empty planes as published do not always contain a list of items of equipment included in this weight, nevertheless the variation between these figures is small.

Operators of air lines and most private purchasers have in mind a fixed program as to where the ship is to be used. The distance between refueling fields plus a generous margin for delays due to weather conditions is specified as the range at cruising speed. If the engine has been pre-selected, as is usually the case, it is necessary to divide the required range by the cruising speed which has been roughly estimated, and one obtains thereby the duration of flight without refueling at cruising speed and engine revolutions of about 85 per cent of maximum design revolutions.

The power and fuel-consumption curves of the engine give the fuel consumption in gallons per hour, and as the duration of flight without refueling in hours is known, one can easily calculate the required tank capacity. Fuel tanks have a weight of 0.75 to 0.95 lb. per gal. capacity if constructed of brass, and a weight of 0.45 and 0.55 lb. per gal. capacity if made of aluminum or aluminum-alloy sheet.

There are left to the designer but two possibilities of influencing the fuel-tank weight: One is to increase the cruising speed, and with a given distance between refueling, thereby reduce the amount of fuel which has to be carried at take-off. The second possibility is to use lighter material for the tank construction. Most ships built during the last few years are, however, using aluminum tanks of either riveted or welded construction, and further weight reduction in this type is not very probable on account of serviceability and maintenance problems.

More promising is the way of increasing cruising speed. The average range to be flown without refueling is about 500 miles. If the ship has a cruising speed of 120 miles and is equipped with a single Wasp engine running at about 1600 r.p.m., the fuel consumption is 19 gal. per hr., the duration of flight is 4.16 hr., and the necessary fuel capacity therefore 79 gal. If the tank is of aluminum with a weight of 0.5 lb. per gal. capacity, the tank weighs 40 lb.

When the cruising speed is increased by 10 per cent to 132 m.p.h., the duration becomes 3.79 hr., the fuel required is 72 gal., and the tank weight is 36 lb. In this particular case an increase of cruising speed of 10 per cent means, therefore, a saving on tank weight of 10 per cent.

The fuel-tank weight is, however, only between 2 and 4 per cent of the weight empty as has been found for a great number of transport planes, and a saving of 10 per cent of the tank weight is comparatively negligible by itself, as it would mean only a saving of 0.2 to 0.4 per cent of the weight empty. However, these possibilities of small individual savings repeat themselves many times during the designing of new planes, and are well to be remembered as they may total up to 5 or 8 per cent of the weight empty.

Increase of cruising speed by 10 per cent from 120 to 132 m.p.h. reduces the amount of fuel to be carried at take-off for a range of 500 miles from 79 gal. to 72 gal. This means a weight reduction of 7 × 6 = 42 lb. for the fuel and 4 lb. for the fuel tanks.

The total saving of 46 lb. can be either carried as pay-load or the wing can be reduced in area. For the single Wasp-engined ship which is considered here, a wing loading of around 13 lb. per sq. ft. would be appropriate. The wing area could therefore be reduced by about 3 sq. ft., and if the wing weight is 1.8 lb. per sq. ft. another 5.4 lb. is saved.

For this example the weight empty has been reduced by the 4 lb. as calculated for the fuel tank and 5.4 lb. for the wing, i.e., a total of 9.4 lb. The useful load has been reduced by 42 lb. of fuel, and the gross weight thereby by 51.4 lb. This single Wasp ship would have a gross weight of about 5000 lb., and by all the careful consideration and the assumption of an increase of cruising speed of 120 m.p.h. by a full 10 per cent, only 1 per cent of the gross weight of the ship was saved.

The oil tanks cannot be made smaller by decreasing the fuel-tank capacity, because of the heating up of oil and because of the reserve capacity necessary. Tanks of a capacity of less than 3 gal. for each 100 b.hp. of the engine cannot be installed. The only way to save weight in oil-tank construction is by the use of aluminum or aluminum-alloy tanks instead of brass or terne-plate tanks which have been previously used.

The fuel and oil lines again are more or less determined by the choice of the engine, i.e., at least as far as their inside diameter is concerned. Copper pipes are used for fuel lines, as aluminum in the required small diameters is too easily damaged and more apt to crystallize. Oil lines, however, are made of aluminum tube in most of the larger ships. With the size of the pipe lines are also given the size and weight of all pipe fittings and connections as well as of the fuel cocks, strainers, drains, etc.

The only possibility for the designer to save weight is therefore by decreasing the length of the pipe lines. For the oil lines this can be done by placing the tank close to the engine. The fuel tanks are now carried in the wing in most of the ships of the class under discussion, and the distance to the carburetor is determined by a number of other considerations, as for instance, weight distribution and balance, and cannot be changed for the sake of saving a few feet of pipe. The weight of the fuel and oil pipe lines is determined to 70 per cent or more by the choice of the engine, and only the remaining 30 per cent or less can be influenced by the designer.

Cowling for air-cooled engines is determined by cooperation between engine and plane manufacturer, and in many cases the difficulty is to keep the engine warm enough, contrary to the common conception. Weight of cowling, which is of course always made of aluminum or aluminum-alloy sheet, is depending therefore to a very large percentage on size and power of the engines. The same is true for the exhaust system, with the additional requirement of means of silencing, which is necessary for the comfort of pilot and passengers. On some engines the exhaust system has to be made so that exhaust gases can be used for heating up the fuel mixture or so that air, preheated by exhaust, can be supplied to the carburetor. Weights of such devices should be actually added to the engine weight, as it is most always impossible to operate the engine without them.

Closely connected with the exhaust problem is the cabin-heating question, as practically all planes are now heated by fresh air which passes over the hot exhaust pipes or mufflers. Cabin size and shape have, of course, great influence on design and size of the cabin heater, and efficient heating, with ventilation, is one of the many cases where consideration of passenger comfort requires additional or increased devices, and therefore weight increase.

Cabin floor space is determined by the number of passengers carried. For each passenger a floor space of at least $4^{1}/_{2}$ sq. ft. area is required, and the larger multi-engined ships provide 6 to 7 sq. ft. These figures include the floor space under chairs and benches, but not floors in pilot's cockpit, entrance compartments, toilets, galleys, and baggage compartments. This requirement of floor space and the necessary strength requirements are determining the floor weights to a large degree.

Large cabin windows must be provided for the passengers' comfort, and window panes and frames must be sufficiently strong to resist the air pressures. The use of non-shatterable glass is advisable on account of its safety value. It has, however, the disadvantage of weight increase.

The weight of window per passenger decreases with the increase in size of the plane, as it becomes necessary to arrange three or four seats side by side and two passengers use one window.

Cabin interiors, with walls, doors, bulkheads, upholstery, sound-proofing, baggage racks, chairs, cushions, and a number of minor items, have to be designed for passenger comfort, and become a rather intricate engineering problem with increase of size of the ship. Many pounds can be saved or wasted on these items, but the demands of operators and sales departments are justly concentrated on satisfying the traveling public.

There remains therefore only the group of structural parts on which the designer's influence on weights is not limited, and this group makes approximately 45 per cent of the weight empty. If the designer is able to save as much as 10 per cent of the weight of this group, he will be saving only $4^{1}/_{2}$ per cent of the weight empty or around $2^{1}/_{2}$ per cent of the gross weight. A lightening of the structural parts by 10 per cent can only be accomplished by using the most efficient design, and the best available materials.

From all this it can be seen that only close cooperation and understanding between all parties concerned, and closest control of weights of all parts of the ship will have the desired effect of reducing the weight of the empty plane. A logical procedure in starting a layout of a new type of ship would therefore be to determine first the weights as definitely given by choosing the engine and accessories, and by determining weights of the group upon which the designer has limited influence. During this stage, careful study is necessary to make sure that all the items of equipment and furnishing are really absolutely essential, and that none of them could be spared in the particular design. Every effort has to be made to keep the size of the ship, for a given motor, as small as possible, and as commensurate to the comfort and safety of passengers and crew.

Discussion

T. P. Wright.[2] The writer is in entire agreement with the author on the importance of weight control in airplane development. It is one of the items existing in airplane design which is different from those found in other branches of engineering as regards importance and, therefore, one which is rather novel in nature, and on which there are no precedents to work.

Weight charts have been prepared similar to those appearing in the paper, but applicable to widely varying types; namely, the Condor, an 18-passenger transport; the Carrier Pigeon, a 2000-lb. payload mailplane; and the Fledgling, a 2800-lb. training plane. (See Tables 6 to 10.) It is interesting to note that the average percentages of weight empty given in the paper for the three groups listed compare as follows, giving a very close check:

	Fokker	Curtiss
Group A	35.4	35.1
Group B	20.0	21.8
Group C	44.6	43.0

The author states that his experience indicates that the gross weight of a modern commercial plane should be between 10 and 15 times the rated horsepower for multi-engined ships and 13 and 18 times for single-engined ships. On the Condor, this ratio becomes 14.9, on the Carrier Pigeon 12.7, and for the Fledgling 11.9, which do not follow the statement above. The author shows that a 10 per cent increase in cruising speed gains only 1 per cent saving in weight. Assuming that the 10 per cent increase in cruising speed is obtained by greater r.p.m. of the engine, should not the fuel consumption per hour also increase?

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[2] Chief Engineer, Curtiss Aeroplane and Motor Co., Inc., Buffalo, N. Y. Mem. A.S.M.E.

TABLE 6 GROUP A—WEIGHTS OUTSIDE DESIGNER'S DIRECT INFLUENCE

Items	Condor	Carrier Pigeon II	Fledgling (J-6-7)
Engines	1724	862	449
Engine accessories (less stacks)	13	5	9
Propellers	539	266	81
Starting system	108	44	35
Generators	50	26	..
Booster magneto	11	9	..
Ignition switches	3	1	2
Electrical instruments	2	1	..
Lights (nav., landing, interior)	14	28	..
Battery and container	81	78	..
Switches	5	3	..
Wheels (incl. tires, etc.)	454	145	80
Flight and engine instruments	56	31	25
Fire extinguishers	31	15	19
First-aid kit	2	1	3
Safety belts	29	2	5
Water-cooling system	288	146	..
Fuel hand pump, valves, strainers, cocks, etc.	35	15	8
Radio equipment	100	40	..
Flare equipment	50	46	..
Tool kit	..	..	4
Total group "A"	3595	1764	720
Weight empty	11818	4236	2153
Group A, per cent of weight empty	30.4	41.6	33.4

TABLE 7 GROUP B—WEIGHTS ON WHICH DESIGNER HAS LIMITED INFLUENCE

Items	Condor	Carrier Pigeon II	Fledgling (J-6-7)
Fuel tanks with supports	178	95	30
Oil tanks with supports	29	14	26
Fuel piping, etc.	82	18	7
Oil piping, cooler, etc.	50	26	23
Fuselage and nacelle cowling	669	201	133
Exhaust system	121	97	34
Cooling system (less water)	378	187	..
Cabin heating system	70	12	..
Cabin and cockpit floors and supports	348	5	46
Cabin windows and frames	171	..	..
Lavatory equipment	118	..	..
Pilots' seats and supports	28	28	14
Passenger chairs and cushions	405	..	14
Cabin interior walls and doors	280	..	..
Cabin sound insulation and upholstery	142	21	..
Electrical wiring, conduits, etc.	56	20	..
Baggage racks and compartments	120	19	12
Brake controls	18	15	9
Instrument boards	8	3	4
Engine controls	51	10	14
Misc. cabin furnishings	64	18	16
Total group "B"	3395	789	391
Weight empty	11818	4236	2153
Group B, per cent of weight empty	28.7	18.6	18.2

TABLE 8 GROUP C—WEIGHTS DIRECTLY WITHIN DESIGNER'S INFLUENCE

Items	Condor	Carrier Pigeon II	Fledgling (J-6-7)
Wing group	2612	861	559
Tail group	245	136	81
Fuselage structure	827	323	170
Nacelle structure	380	..	..
Engine-mount structure	94	58	31
Landing-gear chassis	348	187	88
Tail-wheel chassis	70	22	21
Surface controls	252	96	92
Total group C	4828	1683	1042
Weight empty	11818	4236	2153
Group C, per cent of weight empty	40.9	39.7	48.5

TABLE 9 SUMMARY

	Condor		Carrier Pigeon II		Fledgling (J-6-7)		Average per cent for 3 types
	Per cent W.E.	Lb.	Per cent W.E.	Lb.	Per cent W.E.	Lb.	
Group A							
Weight outside designer's influence	30.4	3595	41.6	1764	33.4	720	35.1
Group B							
Weight on which designer has limited influence	28.7	3395	18.6	789	18.2	391	21.8
Group C							
Weight directly within designer's influence	40.9	4828	39.7	1683	48.5	1042	43.0
Weight empty		11818		4236		2153	
Max. useful load	34.0	6082	44.3	3364	24.4	696	
Max. gross weight		17900		7600		2849	

TABLE 10 DESIGN CHARACTERISTICS

	Condor	Carrier Pigeon II	Fledgling
Length overall	57 ft. 6 in.	34 ft. 6 in.	28 ft. 2 in.
Span	91 ft. 8 in.	47 ft. 5 in.	39 ft. 5 in.
Engines	(2) GV-1570	(1) GV-1570	J-6-7
Wing area total	1510	550	365
Weight empty	11818	4236	2153
Useful load	6082	3364	696
Gross weight	17900	7600	2849
Wing loading	11.9	13.8	7.8
Power loading	14.9	12.7	11.9
High speed	138.9	150.0	116
Cruising speed (0.85 max.)	118	128.0	92
Range at cruising speed (miles)	824	650	315
Fuel cons. (lb. per hr.) cruising speed	382	205	70
Passengers and crew	20	1	2
Fuel capacity, max. (gal.)	444	175	40

In conclusion, the most important and significant fact brought out by this discussion is that the designer of an airplane has a very narrow margin within which he is able to exercise his ability of efficient design from the weight standpoint. This limited margin averages 28.5 per cent of the gross weight of the three types chosen for the discussion. The matter of weight control is of extreme importance in airplane design, and the paper is of interest to all aeronautical engineers.

C. T. Porter.[3] The paper is extremely well expressed and very timely. Although the material presented is well known to the experienced designer, nevertheless a careful study of the weight ratios as given should do a great deal to discourage the enthusiasts who hope by revolutionary methods of construction to make impossible savings in weight, and to get great increases in performance.

It seems to the writer that inasmuch as so much of the weight of an airplane is outside the control of the designer, the real saving is to come from better aerodynamic design, which, as the author shows, will have immediate effect on the weight of fuel carried per passenger mile. It would seem that he might have elaborated a little more on the saving in weight through reduction of power required per passenger with increased aerodynamic performance, as this seems to be the direction in which all development should point. Weight tables and general

TABLE 11 GROUP A—WEIGHTS OUTSIDE DESIGNER'S DIRECT INFLUENCE

Item	Pup	Commuter	Loening Amphib.	Keystone Bomber	Patrician
Engines	515	545	960	1620	2322
Engine accessories (less stacks)	12	7	8	13	15
Propellers	80	82	119	268	378
Starting system	27	31	42	94	71
Generators	..	15	38	44	29
Generator control boxes	..	3	..	..	..
Booster magneto	9	..	..	..	..
Ignition switches	3	2	2	3	2
Lights.*	..	2	2	26	30
Battery and container	..	40	43	80	66
Switches	..	..	2	3	1
Wheels and tires	57	62	102	273	273
Flight, engine and electrical instruments	21	14	28	48	45
Fire extinguisher	16	7	16	47	17
First-aid kit	4	2	..	..	2
Fuel pump, valves, strainers	2	8	11	28	15
Radio equipment	..	..	..	10	..
Flare equipment	..	..	..	..	43
Tool kit	4	..	..	..	20
Safety belts	4	7	6	14	25
Armament installation	..	..	..	107	..
Total group A	754	827	1379	2678	3354
Group A per cent W.E.	38.1	28.3	34.7	34.5	33.3
Average per cent for group A	33.8				

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[3] Chief Engineer, Keystone Aircraft Corp., Bristol, Pa. Mem. A.S.M.E.

TABLE 12 GROUP B—WEIGHTS ON WHICH DESIGNER HAS LIMITED INFLUENCE

Item	Pup	Commuter	Loening Amphib.	Keystone Bomber	Patrician
Fuel tanks with supports	63	49	52	282	191
Oil tanks with supports	12	13	23	26	32
Fuel piping, etc.	7	20	6	75	48
Oil piping, etc.	6	3	4	14	15
Oil cooler	..	..	28	..	..
Fuselage and nacelle cowl and covering	85	298	434	483	466
Exhaust system	9	35	13	115	211
Cabin heating system	..	..	..	..	20
Cabin and cockpit floors	25	50	18	131	161
Windows and windshields	2	24	5	29	172
Lavatory equipment	..	..	..	..	104
Seats, supports and cushions	38	39	24	48	256
Cabin interior furnishings	..	25	..	..	200
Cabin sound insulation	..	..	..	..	130
Electrical wiring	2	25	53	85	82
Baggage compartment	9	2	2	7	14
Brake controls	..	..	..	6	6
Instrument boards	7	3	2	10	8
Engine controls	12	18	13	40	50
Miscellaneous furnishings	..	33	20	22	300
Total group B	277	637	697	1373	2466
Group B per cent W.E.	14.2	22.1	17.5	17.7	24.6
Average per cent for group B	19.2				

characteristics of airplanes manufactured by the Keystone Aircraft Corporation are given, which fall in the classifications adopted by the author. (See Tables 11 to 14.)

TABLE 13 GROUP C—WEIGHTS DIRECTLY WITHIN DESIGNER'S INFLUENCE

Item	Pup	Commuter	Loening Amphib.	Keystone Bomber	Patrician
Wing group	516	613	791	2280	2546
Tail group	65	123	128	240	277
Fuselage structure	194	..	183	640	827
Engine-mount structure	23	84	35	125	77
Hull	..	390	511	..	..
Wing-tip floats	..	61	57	..	..
Landing-gear chassis	71	123	131	285	356
Tail-wheel chassis	8	14	17	58	40
Surface controls	52	55	59	91	107
Total group C	929	1463	1912	3719	4230
Group C per cent W.E.	47.7	49.6	47.8	47.8	42.1
Weight empty	1960	2927	3988	7770	10050
Average per cent for group C	47.0				

Lessiter C. Milburn.[4] There is one point on which the writer would like to comment, namely, that the company with which the writer is connected has been building service-type vibration-tested, riveted aluminum-alloy gasoline tanks in capacities varying from 50 to 350 gal. with unit weights of about 0.32 lb. per gal. as compared to the minimum figure of 0.45 which the author mentions. This represents a direct saving of 0.1 lb. per gal. or 50 lb. on an airplane of 500-gal. capacity.

TABLE 14 SUMMARY

Item	Pup		Commuter		Loening Amphibion		Keystone Bomber		Patrician		Average per cent for 5 ships
	Lb.	Per cent W.E.	Lb.	Per cent W.E.	Lb.	Per cent W.E.	Lb.	Per cent W.E.	Lb.	Per cent W.E.	
Group A—Weights outside designer's influence	746	38.1	827	28.3	1379	34.7	2678	34.5	3354	33.3	33.8
Group B—Weights on which designer has limited influence	277	14.2	637	22.1	697	17.5	1373	17.7	2466	24.6	19.2
Group C—Weights on which designer has direct influence	929	47.7	1463	49.6	1912	47.8	3719	47.8	4230	42.1	47.0
Weight empty	1952	....	2927	....	3988	....	7770	....	10050	....	....
Useful load	708	26.6	1223	29.5	1367	25.5	5230	40.2	5950	37.2	....
Gross weight	6260		4150		5355		13000		16000		....

TABLE 15 DESIGN DATA

Item	Pup	Commuter	Loening Amphibion	Keystone Bomber	Patrician
Length overall	26 ft. 11 in.	32 ft. 4 in.	34 ft. 10 in.	48 ft. 9 in.	61 ft. 7 in.
Span	37 ft. & 32 ft.	40 ft.	45 ft.	74 ft. 9 in.	86 ft.
Engines	1—J-5	1—J-6	1 Tornado	2 Hornets	3 Cyclones
Wing area, total sq. ft.	324	467	504	1137	930
Weight empty (lb.)	1952	2927	3988	7770	10050
Useful load (lb.)	708	1223	1367	5230	5950
Gross weight (lb.)	2660	4150	5355	13000	16000
Wing loading (lb. per sq. ft.)	8.2	8.9	10.6	11.5	17.2
Power loading (lb. per sq. ft.)	12.1	13.8	11.1	12.4	10.1
Top speed (m.p.h.)	116	105	125	116	140
Cruising speed (m.p.h.)	93	84	100	93	112
Passengers and crew	2	4	2	5	20
Fuel capacity (gal.)	40	70	140	535	550
Fuel consumption, gal. per hr. cruising	11	15	24	52.5	79
Range at cruising speed	330	390	580	950	780

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[4] Vice-President and Chief Engineer, The Glenn L. Martin Co., Baltimore, Md.

Altitude Laboratory Tests of Aircraft Engines[1]

By H. K. CUMMINGS[2] and E. A. GARLOCK,[3] WASHINGTON, D. C.

This paper gives a concise description of the altitude laboratory and its operation, followed by a discussion of available reports on altitude laboratory testing at the Bureau of Standards during the past 13 years and a summary of conclusions drawn as to engine performance under altitude conditions. Experimental data are presented on a modern 12-cylinder aircraft engine to illustrate (1) the effect of pressure and temperature on power and (2) some theoretical and practical advantages from supercharging. Additional problems to be studied with the present equipment and the need of an altitude laboratory for testing air-cooled aircraft engines are mentioned. Approximate altitude performance data on engines of the latter type are already being obtained by the Navy Department.

THE power delivered by an internal-combustion engine operating a mile above sea level is reduced about 20 per cent by the decrease in air density, while the loss in power at an elevation of two miles is at least 30 per cent. Consequently, the designer of aircraft power plants is vitally concerned with the effects of altitude on engine power and performance. Complete information on this subject cannot be obtained from flight tests, for the change in air density affects the air resistance and the propeller efficiency, as well as the performance of the power plant.

H. K. Cummings

E. A. Garlock

Early in the World War, tests of the Liberty engine and of centrifugal superchargers were made on Pike's Peak, although the equipment which could be transported to the altitude of 14,000 ft. permitted only crude tests. The Bureau of Standards, therefore, in cooperation with the National Advisory Committee for Aeronautics, undertook in 1917 the design and construction of an altitude laboratory (1)[4] where complete tests of aircraft-engine performance could be made under the precise conditions of temperature and pressure which prevail at various high altitudes.

The Altitude Laboratory

This laboratory was the first of its kind in the world, although a similar laboratory has since been built in France, and altitude laboratories are contemplated or under construction in Italy and Japan. The original testing plant (2) has been modified from time to time to provide increased capacity, better control of test conditions, and improved safety (30) in operation. The present altitude laboratory (36) comprises an altitude chamber, air-conditioning apparatus, a battery of exhaust pumps, and power-absorbing equipment, together with apparatus for metering the amounts of air, fuel, oil, and water supplied to the engine and means for measuring engine temperatures and pressures.

Referring to Figs. 1 and 2, the altitude chamber is a room about 15 ft. long, 7 ft. wide, and 7 ft. high inside, with walls of reinforced concrete capable of withstanding an external pressure of 1 ton per sq. ft. This room contains refrigerating coils and the stand on which engines are mounted. Opposite the engine stand are doors opening outward into safety tunnels which lead to the outside of the building. When the chamber is exhausted for an altitude test, the doors are held tightly closed against rubber gaskets by the differential pressure, but should an explosion occur within the chamber, they are free to open, relieving the pressure without damage to the chamber. No one is permitted in the tunnels when a test is in progress.

In the insulated air-cooling box above the altitude chamber, the carburetor air is passed successively over four banks of refrigerating coils which cool it to a low temperature and remove most of its moisture as frost. The air then enters a chamber of large cross-section where any entrained snow tends to settle out, due to the slower speed. Heating grids for final temperature adjustment are next encountered by the cold, dry air. The insulated pipe from the air-cooling box to the altitude chamber contains a 6-in. throttling valve, with a 3-in. valve in parallel. The latter is used for final adjustment of the carburetor air pressure. The air flow is measured by means of a calibrated venturi in the pipe line between the throttling valves and the carburetor. The refrigerating plant includes a positive-displacement booster pump which takes ammonia vapor from the coils at an absolute pressure of 5 lb. per sq. in. and delivers it at atmospheric pressure to the two 25-ton ammonia compressors. This equipment makes it possible to supply engines using up to 1000 cu. ft. of air per minute with practically dry air at temperatures down to —40 deg. cent.

The two direct-driven water-sealed vacuum pumps shown in Fig. 1 are used to remove the engine exhaust gases. The engine exhausts into a special water-jacketed manifold, and a water spray is injected to cool the gases. The quantity of water used and its rise in temperature serve to measure the heat dissipated in the exhaust. By suitably controlling the pump suction a pressure corresponding to the desired altitude is maintained at the exhaust ports. Two smaller vacuum pumps in an adjacent building are connected to the altitude laboratory by means of a 10-in. main. One of these pumps is usually sufficient to reduce the pressure of the air in the altitude chamber to the desired figure. The minimum pressure which can be maintained at the exhaust ports corresponds to an altitude of about 30,000

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[1] Publication approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

[2] Chief, Automotive Power Plants Section, Bureau of Standards.

[3] Assistant Physicist in charge of altitude testing, Bureau of Standards.

[4] Numbers in parentheses refer to Bibliography at end of paper.

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

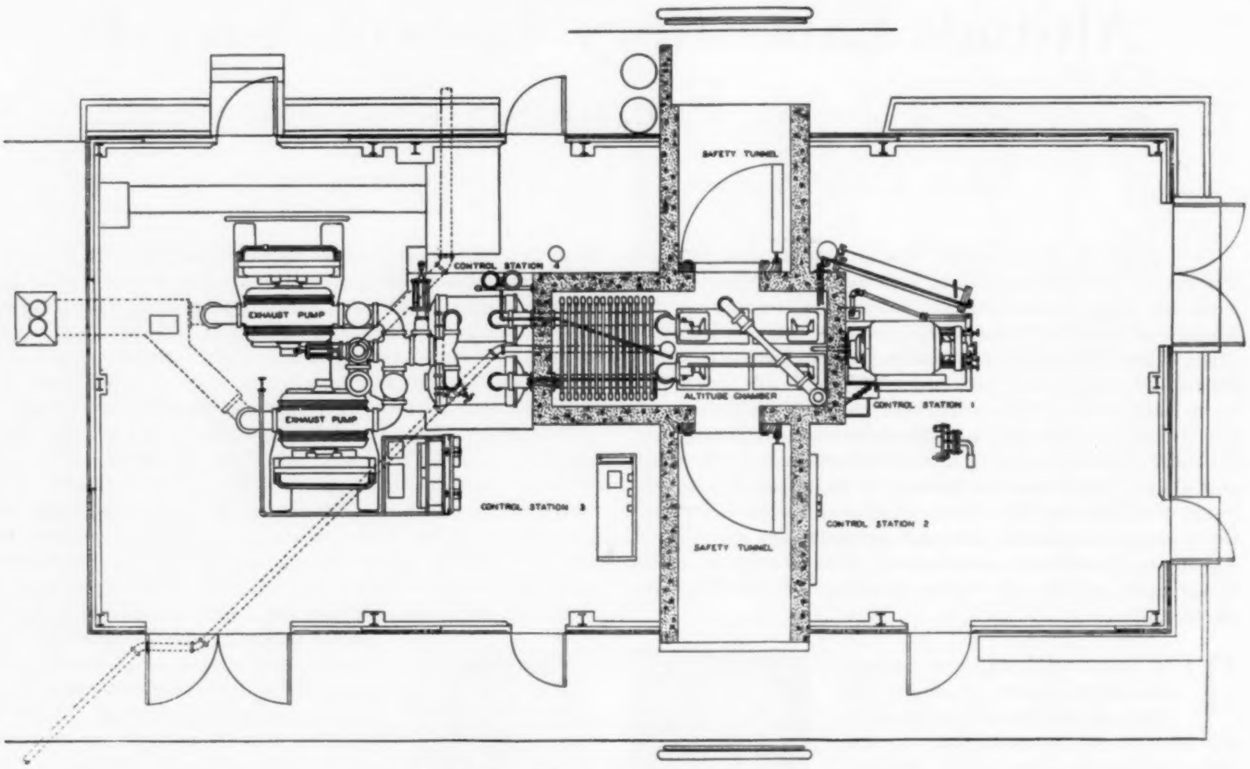

FIG. 1 PLAN VIEW OF ALTITUDE LABORATORY

ft., but the air pressure at the carburetor and that surrounding the engine can be held at a lower value if desired.

At the first control station are located the usual engine controls and instruments. The combined electric dynamometer and water brake, coupled to the engine by means of a shaft carried through the altitude chamber wall, is capable of absorbing at least 700 hp. The temperatures of jacket water and engine oil are regulated by circulating them through tanks which are maintained at constant temperature by admitting steam or cold water as required. From the inlet and outlet jacket-water temperatures and the quantity of water circulated, the heat dissipated in the jacket water is computed.

At the second control station are barometers and manometers for reading absolute pressures and pressure differences. The engine exhaust and chamber pressures are adjusted by hydraulically operated valves and the carburetor air pressure by valves controlled by hand wheels. The venturis used to measure carburetor air, jacket water, and exhaust water are read here.

All recorded temperatures are obtained at the third control station, using thermocouples connected to a potentiometer through selector switches. From this station the heating grids which determine the temperature of the carburetor air are controlled.

Fuel consumption is determined at the fourth control station by timing the flow from a calibrated measuring tank. These tanks are filled by air pressure from fuel drums kept outside the building. During an altitude test the pressure on the fuel in the measuring tank supplying the engine is adjusted with respect to the air pressure at the carburetor entrance.

To insure simultaneous observations, annunciators and signal lights are installed at the pressure, temperature, and fuel stations. The operators also can communicate by telephone without leaving their posts. The staff consists of a test engineer assisted by an airplane-engine mechanic and three observers.

Reports on Altitude Tests

The tests conducted in the altitude laboratory during the past 13 years have been of three general kinds: (*a*) Performance tests of a particular engine under specified conditions, such as those laid down by the Army (9) in 1920. (*b*) Comparative tests of a given engine under various conditions or with various compression ratios. (*c*) Tests for the purpose of comparing fuels, lubricants, carburetors, methods of carburetor control, induction systems, and various gear-driven superchargers.

Tests of the first kind were made for the Army or the Navy on many representative engines, and the results obtained on the following engines have been released for publication: Liberty 12 (13), 300-hp. Hispano-Suiza (14), 300-hp. Maybach (20), and 185-hp. B.M.W. (21). Tests of the second kind were made specifically on Liberty, Hispano-Suiza, Packard, and Curtiss engines, but much data obtained on other tests with these engines are also available for use. The first two reports issued in printed form considered (*a*) compression ratio, pressure, and temperature (3) and (*b*) heat losses, friction, and engine speed, (4) and were based on tests of the 150-hp. and 180-hp. Hispano-Suiza engines. A second analysis by Gage (16) considered tests of the 300-hp. Hispano-Suiza and Liberty 12 engines. The effect of temperature (24), compression ratio (25), and friction (31) were later considered individually by Sparrow, who also presented a general discussion of aviation-engine performance (29) before the Franklin Institute in 1925. Tests of the third kind are represented by six reports on fuels or carburetion: two in 1918 (5, 6), two in 1920 (10, 11), one in 1924 (23), and one in 1925 (28).

In connection with many of the reports cited, it should be pointed out that the precise relationship between altitude, pressure, temperature, and density known as the standard atmosphere (27, 33) was not used in the altitude laboratory prior to 1924. The actual relation used is shown graphically in most reports.

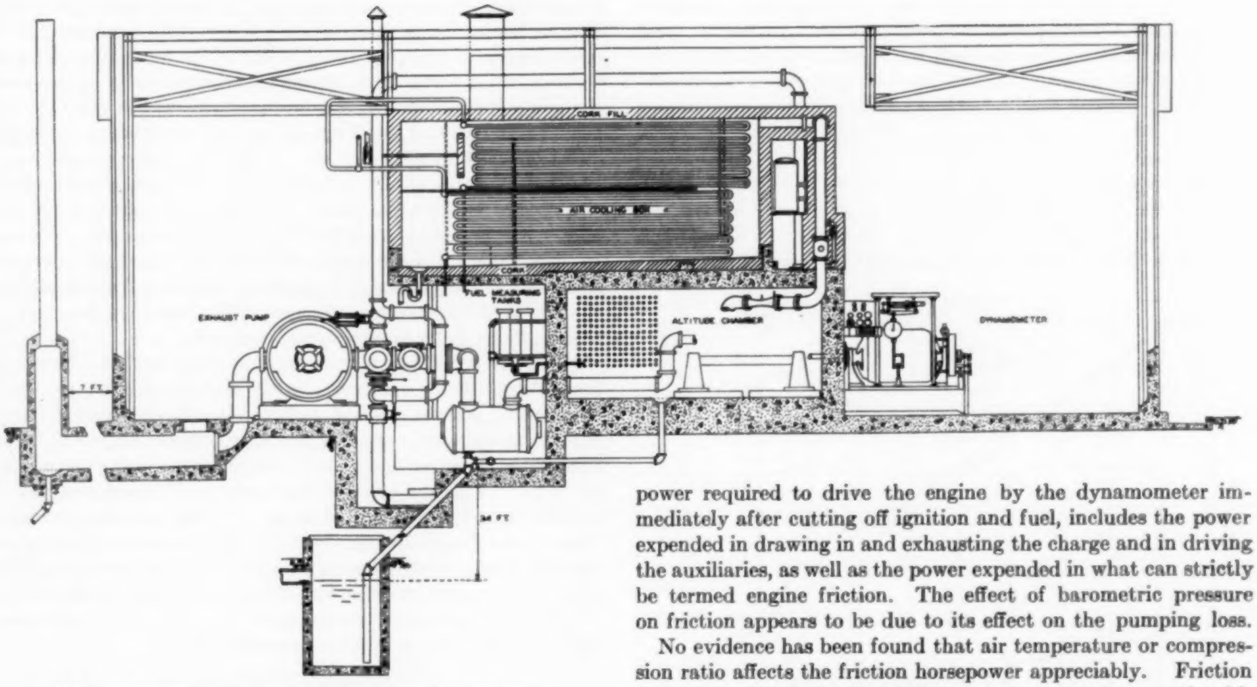

Fig. 2 Sectional Elevation, Altitude Laboratory

In some cases the change of altitude or air density means only a change of pressure.

Engine Performance at Altitude

Indicated Horsepower. The altitude laboratory tests appear to be most satisfactorily compared on the basis of indicated horsepower, defined as the sum of brake horsepower and friction horsepower. The difficulty of obtaining satisfactory indicator cards of high-speed engine performance at sea level is well known, and while the balanced diaphragm indicator (15) developed at the Bureau of Standards makes it possible to obtain indicator cards at any altitude, the variations in pressure shown by the shaded areas in Fig. 3 make such cards only qualitative. These particular cards were obtained on a 300-hp. Hispano-Suiza engine operated at 1800 r.p.m. and have been discussed in *Automotive Industries* (22).

The indicated horsepower of water-cooled aircraft engines appears to vary directly as the pressure and inversely approximately as the square root of the absolute temperature. A priori, the indicated horsepower should depend on the weight of charge received by the engine per cycle. Hence the volumetric efficiency, defined as the ratio between the volume of charge received per cycle measured at the temperature and pressure existing at the carburetor entrance and the piston displacement, must vary approximately as the square root of the absolute temperature, because the density of the air supplied to the carburetor varies inversely as the absolute temperature. It should be remarked at this point that the foregoing discussion applies only to comparisons of power at the same engine speed and fuel-air ratio. The effect of increasing compression ratio (i.e., expansion ratio) is to increase the indicated thermal efficiency approximately in proportion to the air-cycle efficiency, provided the character of the fuel is such that neither detonation nor preignition occurs at either compression ratio (28). The increase in thermal efficiency is accompanied by a decrease in the relative heat losses to the water jacket and in the exhaust.

Friction Horsepower. Friction horsepower, measured by the power required to drive the engine by the dynamometer immediately after cutting off ignition and fuel, includes the power expended in drawing in and exhausting the charge and in driving the auxiliaries, as well as the power expended in what can strictly be termed engine friction. The effect of barometric pressure on friction appears to be due to its effect on the pumping loss.

No evidence has been found that air temperature or compression ratio affects the friction horsepower appreciably. Friction decreases with increasing jacket-water temperature and with decrease in barometric pressure. For all engines tested, the average decrease in friction mean effective pressure (f.m.e.p.) was found to be 0.1 lb. per sq. in. for 1 cm. of mercury decrease in barometric pressure. In the normal operating range, the f.m.e.p. increases with engine speed, but in most instances the percentage increase is less than the percentage increase in speed. The increase in friction resulting from a decrease in throttle opening is also due to change in pumping loss. For a group of representative engines, the change in f.m.e.p. accompanying a change in manifold suction of 1 in. of mercury was found to range from 0.2 lb. per sq. in. at an engine speed of 1200 r.p.m. to 0.4 lb. at 1800 r.p.m. The estimated value at 2200 r.p.m. is 0.5.

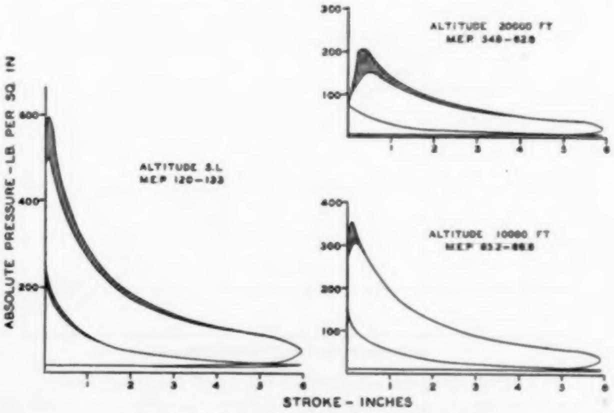

Fig. 3 Specimen Indicator Cards

(These cards were obtained during the altitude test of an eight-cylinder, water-cooled V-type engine having a 5.51-in. bore, 5.91-in. stroke, and a compression ratio of 5.6 to 1. The speed during this test was 1800 r.p.m.)

Brake Horsepower. From the foregoing discussion, it is evident that, while brake horsepower is a function of barometric pressure and air temperature and is not uniquely defined by specifying the air density, the brake horsepower should not be assumed to vary inversely as the absolute temperature or as the square root of the absolute temperature. When it is not practicable to make an altitude test, the mechanical efficiency may be determined at sea level and its variation with altitude estimated (29) or the empirical formula given in the early report on the "Effect of Temperature on Power" (3) and quoted in various textbooks may be used.

Fuel-Air Ratio. It is the practice of the Bureau of Standards to obtain the maximum power value in any engine test from a mixture-ratio run rather than to rely upon the correctness of a single carburetor adjustment for maximum power. This has given a large amount of data from which to estimate the effect of various factors on the fuel-air ratios giving maximum power and maximum economy. The following conclusions were stated by Sparrow (23): (*a*) With gasoline as a fuel, maximum power is obtained with fuel-air mixtures of from 0.07 to 0.08 lb. of fuel per pound of air; (*b*) maximum power is obtained with approximately the same ratio over the range of pressures and temperatures encountered in flight; (*c*) nearly minimum specific fuel consumption is secured by decreasing the fuel content of the charge until the power is 95 per cent of its maximum value. These conclusions are not affected by differences in engine speed or compression ratio, but faulty distribution to the various cylinders is likely to result in the use of unnecessarily rich mixtures.

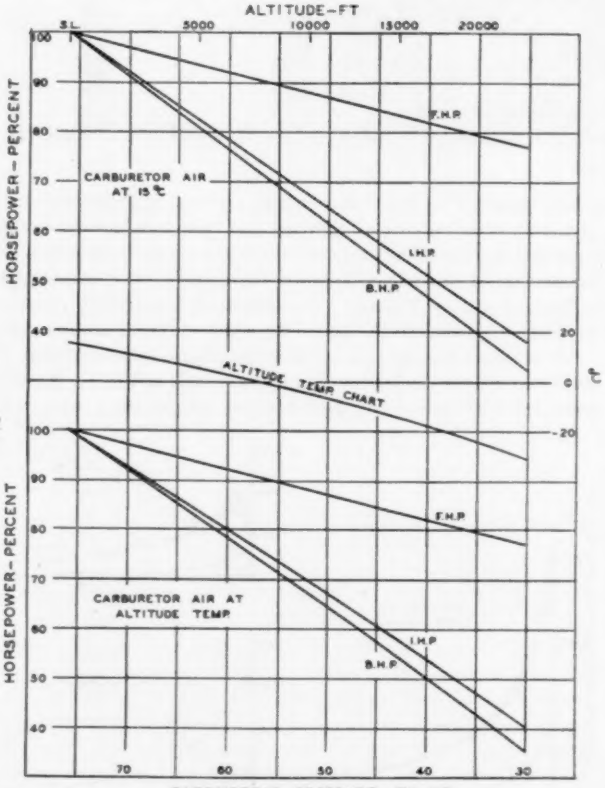

FIG. 4 CHANGE OF ENGINE POWER WITH ALTITUDE

(Showing the effect of pressure alone and the combined effect of pressure and temperature on indicated, friction, and brake horsepower. These curves are based on altitude tests of a modern twelve-cylinder, water-cooled, V-type engine having a 4.5-in. bore, 6-in. stroke, and a compression ratio of 5.3 to 1. The speed during this test was 2000 r.p.m.)

Air Humidity. Recent tests in the altitude laboratory have showed that indicated horsepower is decreased by air humidity at altitude in the ratio of the absolute humidity to the barometric pressure. Humidity not only decreases power, but changes the effective mixture ratio and the optimum spark advance. The correction of engine tests for humidity is discussed in a recent research paper by Brooks (35). The maximum effect of humidity under standard atmosphere conditions is to decrease the power 1.7 per cent at sea level, 0.7 per cent at 5000 ft., 0.4 per cent at 10,000 ft., and 0.2 per cent at 15,000 ft. so that the direct effect of humidity of the air used in altitude tests is not important above 10,000 ft. However, a more serious effect of moisture in the air is observed when air of high relative humidity is supplied to the carburetor at temperatures at or below 10 deg. cent. Erratic performance and a marked decrease in power are likely to result from the formation and accumulation of snow in the carburetor. Similar difficulties may be encountered in flight under some circumstances (18).

Vapor Lock. Vapor lock is another flight hazard which was first identified in connection with altitude laboratory tests. When using volatile gasoline under high altitude pressure conditions at temperatures above 0 deg. cent., erratic engine performance was experienced and traced to interruption of the fuel supply by vapor. This difficulty is now quite well recognized and is familiar to many operators of motor vehicles as well as to pilots. Fuel-system design frequently limits the volatility of the gasoline which may safely be used, especially in warm weather. This boiling of the gasoline in the fuel lines would be less likely to occur at altitude if provision were made to have the fuel in the main tank promptly reach altitude temperature.

EXPERIMENTAL DATA

Below are presented hitherto unpublished results of tests on a well-known 12-cylinder water-cooled aircraft engine having a $4^1/_2$-in. bore and a 6-in. stroke. The compression ratio was 5.3 and the operating speed 2000 r.p.m. in all tests.

Effects of Pressure and Temperature on Power. In the upper part of Fig. 4 is shown the relative change in horsepower with barometric pressure or altitude when the carburetor-air temperature is held constant. Under these conditions the variation of indicated, friction, and brake horsepower with barometric pressure is linear and the decrease in indicated horsepower is almost directly proportional to the change in pressure (at B = 39 cm. Hg, the indicated horsepower is reduced to one-half its sea-level value).

In the lower part of Fig. 4 is shown the relative change in horsepower with barometric pressure when the carburetor-air temperature varies according to the standard atmosphere. Since friction horsepower does not vary with air temperature, this curve is unchanged. In both cases the friction horsepower has 79.5 per cent of its sea-level value at 20,000 ft. Both the indicated- and brake-horsepower lines are now slightly convex upward. At 20,000 ft. the indicated horsepower is reduced to 47.5 per cent of its value at sea level and the brake horsepower to 43 per cent, as against 44.5 per cent and 39.7 per cent, respectively, at 15 deg. cent. carburetor-air temperature.

The effect on indicated horsepower of varying carburetor-air temperature at constant barometric pressure is illustrated in Fig. 5. Sparrow's discussion of the temperature correction (24) was based on 1600 measurements lying chiefly between —20 and +40 deg. cent. Here the range from —38 to 100 deg cent. is covered, and only the lowest point departs as much as 2 per cent from the curve which represents the inverse square-root relation. Gardiner and Schey (32) reported tests of a Liberty 12 engine over the range from 7 to 82 deg. cent. in which the variation of indicated power was approximately linear and intermediate between the inverse square-root relation and the inverse first-power relation. On the contrary, a test of the same engine at the Bureau of Standards in 1926 showed agreement with the inverse square-root relation within 2 per cent from 15 to 106 deg. cent.

Supercharging. A brief paper on the "Value of Supercharging" (3) by Dickinson and Anderson (originally issued as Aeronautic Power Plants Report No. 9) presents a family of curves showing the relative change with altitude in the normal sea-level brake horsepower of an engine when the air pressure at the carburetor entrance is maintained constant as exhaust pressure is reduced. These curves were obtained to show what gain in horsepower may reasonably be expected through supercharging and were used by Kemble (12) in calculating performance curves for an aircraft engine equipped with a centrifugal supercharger. A reexamination of the data on which the curves were based suggested that they might be redrawn as straight lines, giving a chart similar to Fig. 6, which was obtained with the 12-cylinder engine already mentioned, operated at a speed of 2000 r.p.m. with the carburetor air at 15 deg. cent.

Lines *A-B* and *A-C* in Fig. 7 are taken from Fig. 6 and represent respectively, the relative brake horsepower of the unsupercharged engine at various altitudes and the gross brake horsepower of the same engine with sea-level pressure maintained at the carburetor entrance at all altitudes. Line 1–1–1 shows the net brake horsepower when the engine is supercharged by means of a gear-driven Roots-type supercharger having a rated altitude of 7500 ft. Line 2–2–2 shows the net brake horsepower when the engine is supercharged by means of a gear-driven centrifugal supercharger installed between the carburetors and the engine. Line 3–3–3 is a calculated curve taken from Fig. 11 in Chenoweth's paper (34) on "Supercharged Engine Performance" and shows the performance that is to be expected of the engine when it is equipped with a centrifugal supercharger of this rating.

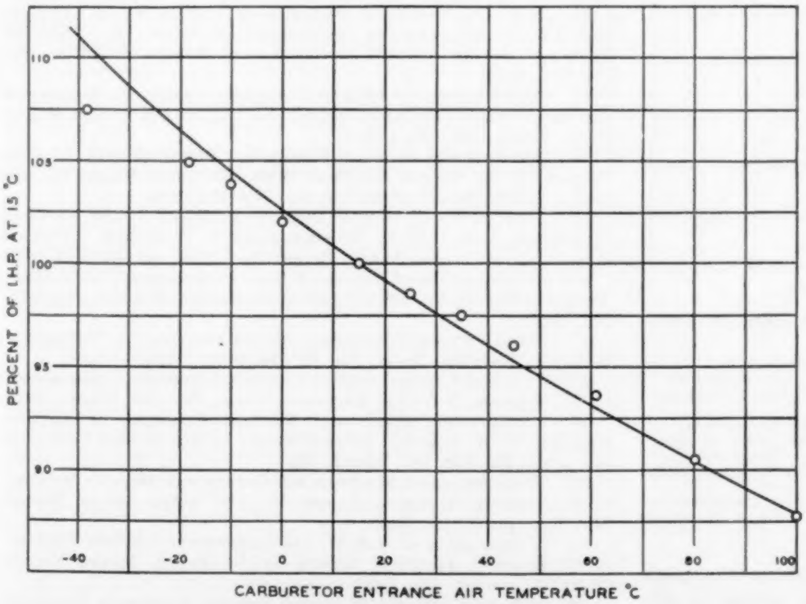

FIG. 5 CHANGE OF INDICATED HORSEPOWER WITH TEMPERATURE

(Full line represents change of horsepower inversely proportional to the square root of the absolute temperature, based on a standard temperature of 15°C. Circles indicate experimental values. For engine specifications, see Fig. 4. Speed, 2000 r.p.m.)

Supercharging has been a major problem in the altitude laboratory for several years past, and reports covering this work in detail are being prepared for publication by the National Advisory Committee for Aeronautics, at whose request this investigation was originally undertaken.

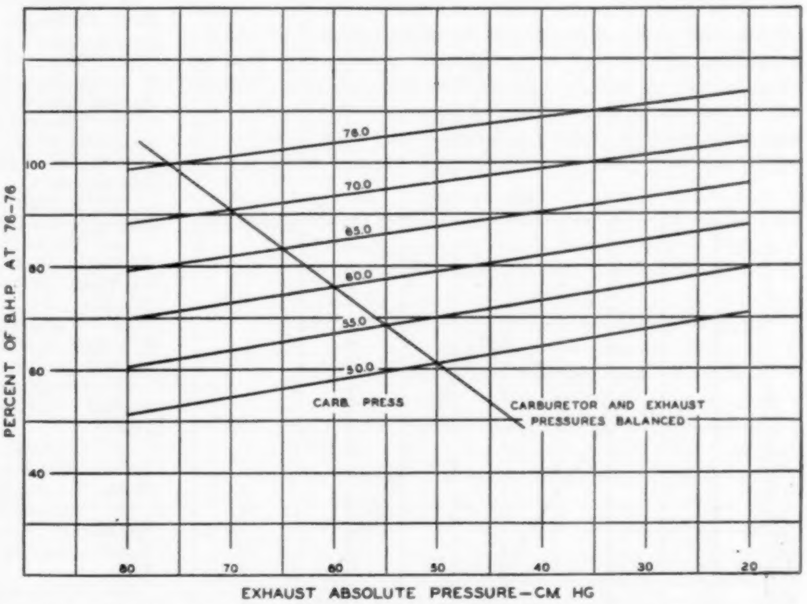

FIG. 6 CHANGE OF BRAKE HORSEPOWER WITH EXHAUST ABSOLUTE PRESSURE

(Referred to condition where both carburetor entrance and exhaust pressures are 76 cm. Hg. One line is for equal carburetor entrance and exhaust pressures, while the remaining lines are for various constant carburetor entrance pressures. Carburetor air temperature, 15° C. For engine specifications, see Fig. 4. Speed, 2000 r.p.m.)

ALTITUDE LABORATORY PROGRAM

A further research program of altitude testing is now being formulated. This will probably include, in addition to work on supercharging, a study of the performance characteristics of modern aviation fuels and tests of engines cooled by liquids other than water to determine the relative efficiency of high cylinder-wall temperatures. Work is in progress to ascertain the reasons for the wide variations in explosion pressure from cycle to cycle obtained with the balanced diaphragm indicator. When means of obtaining more satisfactory indicator cards have been developed, the study of pressures and temperatures in aircraft engines will be extended to include altitude conditions.

Beyond the immediate program for the present altitude laboratory, consideration is being given to the design of a laboratory in which modern air-cooled engines can be tested under altitude conditions. The provision of means for adequately cooling such engines with air at reduced pressure offers a problem surpassing in difficulty any involved in the design of a laboratory for testing water-cooled engines.

However, it is believed that altitude tests of air-cooled engines will be of sufficient value to justify the cost of such equipment.

Approximate Altitude Testing

As pointed out by DuBois (26) in 1924, approximate altitude performance data can be obtained without an altitude chamber if facilities are available (*a*) for conditioning the carburetor air as regards temperature, pressure, and humidity and (*b*) for maintaining the desired low pressure at the exhaust ports of the engine. In making such tests of unsupercharged engines, care must be taken to prevent leaks into the air horn and carburetor, as these will render the air measurements inaccurate. Air-bleeds to the carburetor must be inclosed and connected to the air horn. In regular altitude tests of supercharged engines, when the air surrounding the engine is at low pressure, any leakage takes place from the induction system outward, reducing volumetric efficiency. Thus an approximate altitude test of a supercharged engine may give results which are high owing to the absence of such leakage.

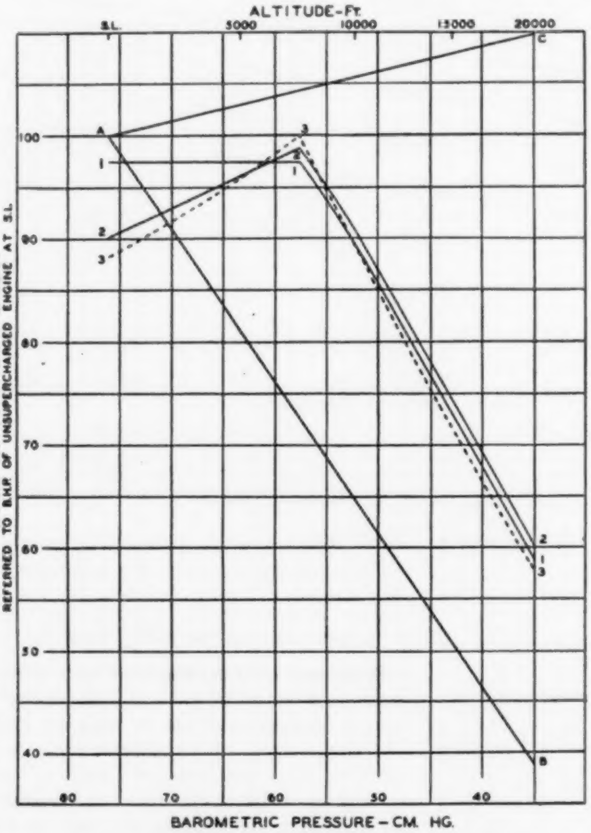

FIG. 7 CHANGE OF BRAKE HORSEPOWER WITH BAROMETRIC PRESSURE

Referred to normal power of unsupercharged engine at sea level.

Curve	Conditions of test	Air temperature
A–B	Altitude test, no supercharging	15° C. (at carb.)
A–C	Sea-level pressure independently maintained at carburetor entrance	15° C. (at carb.)
1–1–1	Gear-driven Roots-type supercharger in advance of carburetors	Alt. temp. at supercharger entr.
2–2–2	Gear-driven centrifugal supercharger between carburetors and engine	Alt. temp. at carb. entr.

Curve 3-3-3 shows relative brake horsepower calculated by Chenoweth for the engine with geared-driven centrifugal supercharger. For engine specifications see Fig. 4. Speed, 2000 r.p.m.

The Navy Department has installed equipment for making approximate altitude tests at the Aeronautical Engine Laboratory in Philadelphia and has made such tests on several air-cooled aircraft engines. In these tests the carburetor air was not conditioned as to temperature and humidity. The Army Air Corps is understood to be designing equipment for the same purpose at Wright Field. Data thus obtained will be of undoubted value although somewhat approximate.

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Discussion

Sanford A. Moss.[5] As the authors point out, existing data indicate that variation of engine power depends upon the square root of absolute temperature, instead of according to the law which would be naturally expected, variation with the first power of the absolute temperature. However, this law possibly applies primarily only in a region where the pressure and temperature are about the atmospheric values. Is it known that the same law holds when the pressure is above atmospheric, as in the case of modern engines with supercharging or ground boosting at sea level or when a supercharger is used to restore sea-level pressure at altitude, in which case the temperature is nearly the adiabatic value corresponding to the pressure ratio, and much above the usual atmospheric value? Perhaps some additional testing work would be required in these regions. The authors mention a possible reason why variation is with the square root of the absolute temperature, instead of with the absolute temperature, as being a matter of volumetric efficiency. Why should the volumetric efficiency be affected by the temperature?

P. M. Sartell.[6] The authors make the statement that the Aeronautical Engine Laboratory at the Naval Aircraft Factory, Philadelphia, Pa., does not condition the carburetor air as to temperature and humidity. The writer feels that the authors have been misinformed. As to temperature of the carburetor air, it is cooled to —60 deg. fahr. in the air refrigerator and then heated to the desired value by means of electrical heaters. Regulation is kept to within ±2 deg. fahr. of the desired temperature.

As to humidity, the air is dry due to its previous cooling to —60 deg. fahr. In a conversation with Mr. Cummings it was found that the Bureau of Standards also uses dry air.

Donald L. Bower.[7] The writer has never seen any figures on the effect of supercharging on the fuel rate and thermal efficiency of aircraft engines, and would therefore like to ask the authors for these figures on engine performance, both at sea level and at high altitudes, giving a comparison of these quantities with engine operating supercharging and non-supercharging.

Authors' Closure

The second question asked by Dr. Moss is discussed briefly by Sparrow in references (24) and (29). If the intake valve acted as a simple orifice, the velocity of flow would vary as the square root of the absolute temperature. The problem is complicated by cooling or heating of the charge in the induction system and cylinder during the intake stroke, but there is adequate experimental evidence that volumetric efficiency as defined in this paper varies with the square root of the absolute temperature, at least in the case of unsupercharged, water-cooled engines. The indicated horsepower curves in Fig. 4 and the experimental points in Fig. 5 show that the inverse square-root temperature correction is not limited to sea-level conditions nor to a narrow range of air temperatures. The correction should be directly applicable to cases where the air pressure and temperature at the carburetor entrance have been raised above atmospheric values by means of a supercharger. As Dr. Moss suggests, additional tests are required to determine the effect on power of pressure and temperature changes at the carburetor entrance when there is a supercharger between the carburetor and the engine cylinders. Studies also should be made to ascertain the effect of atmospheric conditions when the carburetor is replaced by fuel-injection apparatus.

In reply to Mr. Sartell, the particular reports of approximate altitude tests at the Aeronautical Engine Laboratory available when this paper was written specified only control of carburetor and exhaust pressures. We are glad to learn that the Navy now uses dry air and controls its temperature within ±2 deg. fahr.

In regard to the thermal efficiency and specific fuel consumption of aircraft engines, Mr. Bower will find reports of engine tests listed in the bibliography which show the effect of altitude on these quantities in the absence of supercharging. In general, indicated thermal efficiency and specific fuel consumption in pounds per indicated horsepower-hour change little with altitude. Brake thermal efficiency tends to decrease and fuel consumption in pounds per brake horsepower-hour tends to increase with altitude, owing to the decrease in mechanical efficiency. We have no data available comparing the thermal efficiency and economy of supercharged and unsupercharged engines, but a report is in preparation which will compare the performance of an unsupercharged water-cooled engine (for specifications see Fig. 4) with the performance of similar engines equipped, respectively, with Roots-type and centrifugal-type superchargers. Increasing compression ratio, as pointed out in the paper, increases the indicated thermal efficiency, but increasing compression by supercharging without a corresponding increase in expansion ratio reduces the indicated thermal efficiency. In the case of single-cylinder engine tests the variation in mechanical efficiency may be such as to give little or no decrease in brake thermal efficiency due to supercharging. Thus recent tests at the Langley Memorial Aeronautical Laboratory on the N.A.C.A. universal test engine at various compression ratios and at "boosts" up to 10 in. of mercury showed a marked improvement due to increasing the compression ratio and also a very slight improvement in fuel economy when boosted.

It is believed that a brief description of the new Isotta Fraschini altitude laboratory will be of interest. During 1931 a laboratory for the testing of air-cooled aircraft engines under altitude conditions was completed at Milan, Italy. The altitude chamber consists of a double, closed-throat wind tunnel in the form of a vertical figure eight. Air at the desired temperature and pressure is drawn over the engine by means of an eight- or ten-blade propeller, and is subsequently cooled by passage over ammonia coils. It is reported that pressures and temperatures corresponding to altitudes up to 7 km. (23,000 ft.) can be maintained and that the cooling-air velocity at the engine is about 90 m.p.h. The power of the engine is absorbed by means of a water brake located outside of the altitude chamber. The publication of a full description of this laboratory, with test data on air-cooled engines, will be awaited with great interest.

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AER-54-9

Sonic Altimeter for Aircraft

By CHESTER W. RICE,[1] SCHENECTADY, N. Y.

The author describes the development of an aircraft altimeter which works on the sound-echo principle now commonly employed for depth finding at sea. The problem was to devise means and equipment to send a powerful sound from the airplane and receive the echo back from the ground through the intense masking sounds produced by the engine and propeller. It is believed that the use of the sonic altimeter and sonic boundary markers will make fog flying reasonably safe.

IT IS presumed that all are familiar with the excellent results that are obtained with the sound-echo method of depth finding used on shipboard; so much so that many doubtless have thought of making an aircraft altimeter on the same principle. Most of these, however, have rejected the idea when they stopped to think about the large amount of engine noise through which they would have to listen. Roughly, the problem is to hear a distant echo in the noise of a hundred boiler factories, where to be understood one has to shout into the other fellow's ear.

Before proceeding to the experimental work, it will be well to review some of the fundamentals of ear-response, the masking effect of noise, and the sound output from various familiar sources. In Fig. 1 there are given a family of curves[2] showing the sound-pressure, frequency characteristics of the average ear. The lower boundary of the area represents the threshold of audibility at the particular frequency. The upper boundary is the threshold of feeling. The intermediate lines give the values of the sound pressures, which are 10, 100, 1000, etc. times the threshold values. The author has added to the ordinate scale (which gives the sound pressure at the ear in dynes per square centimeter or bars) the approximate value of the sound pressure corresponding to familiar places, such as a quiet office, department store, etc. This helps to place the magnitude of various sound pressures.

From Fig. 1 it is seen that the average threshold for hearing is approximately 0.0006 dyne per sq. cm., between 1000 and 4000 cycles, and that the ear is relatively much more sensitive to sound pressures in this region than for low frequencies, where a much larger sound pressure is required to reach the threshold. The threshold of feeling is reached at sound pressures of the order of 3000 dynes per sq. cm.

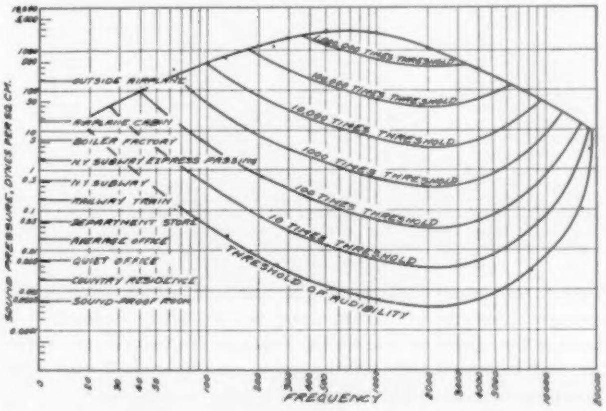

Fig. 1 Sound-Pressure Frequency Characteristics of Normal Ear

TABLE 1 AMOUNT OF NOISE IN TYPICAL PLACES AND APPROXIMATE MASKING EFFECT ON AVERAGE AND LOUD SPEECH

Typical place	Amount of threshold shift for average speech, in decibels	Ratio of sound intensity to the threshold intensity	Ratio of sound pressure to the threshold sound pressure	R.M.S.[a] sound pressure, dynes per sq. cm.	Maximum distance for hearing average speech, ft.	Maximum distance for hearing when speaking as loudly as possible, ft.
Soundproof booth	0	1	1	0.0006	1000	10,000
Country residence	10	10	3.17	0.0019	316	3,160
Quiet office	20	10^2	10	0.006	100	1,000
Average office	30	10^3	31.7	0.019	31.6	316
Department store	40	10^4	100	0.06	10	100
Railway train	50	10^5	317	0.19	3.16	31.6
New York subway	60	10^6	1,000	0.6	1	10
New York subway (express passing)	70	10^7	3,170	1.9	0.316	3.16
Boiler factory	80	10^8	10,000	6.0	0.1	1
In airplane cabin	90	10^9	31,700	19.0	......	0.316
	100	10^{10}	100,000	60.0	......	0.1
Outside airplane, 18 ft. from propeller	110	10^{11}	317,000	190.0	......	......
	120	10^{12}	1,000,000	600.0	......	......
Threshold of painful sound	130	10^{13}	3,170,000	1900.0	......	......

[a] Root mean square.

In Table 1 there are given some approximate figures[3] on the masking effect on speech which is due to the noise present in various familiar places. In column 2 the masking effect of the interfering noise is expressed as the amount of threshold shift produced by the extraneous noise measured in decibels. Column 3 gives the ratio of the required speech intensity to the threshold intensity under different conditions of noise, and column 4 gives the ratio of the required sound pressure to the threshold value. Column

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[1] Engineering Department, General Elec. Co. Born Dec. 16, 1888, in Lynn, Mass., the son of Edwin Wilbur Rice, Jr. Moved to Schenectady with his parents after consolidation of Thomson-Houston and Edison General Electric companies. Early schooling at private academies and the Albany Academy. Entered Harvard University in 1906; received S.B. degree in 1910 and M.E.E. in 1911. Entered the Testing Department, General Electric Company, in 1911. Consulting Engineering Department in 1912, where he worked under Dr. Charles P. Steinmetz. Transferred to the Research Laboratory in 1917, and to the Engineering General Department in 1927, as assistant to the vice-president in charge of engineering.

[2] Fletcher, "Speech and Hearing," p. 141.

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Fletcher, "Speech and Hearing," p. 187, and "Airplane Noise," from R. H. Galt, *Journal Acoustical Society*, vol. 2, p. 43, July, 1930. Unfortunately the data on airplane noise given in Fig. 1 and Table 1 were not available before the preliminary work described in the present paper was completed.

TABLE 2 SOUND OUTPUT FROM VARIOUS SOURCES

Sound source	Power radiated, watts		R.m.s. sound pressure at 3-ft. bars	Reference
Speech:				
Loud as possible	1000×10^{-6}	Avg.	2.0	(1)
Average conversational	10×10^{-6}	Avg.	0.2	
Weak without whispering	0.1×10^{-6}	Avg.	0.02	
Very soft whisper	0.001×10^{-6}	Avg.	0.002	
Bass drum	24.6	Peak	316	(2)
Trombone	6.4	Peak	161	
Trumpet	0.314	Peak	36	
French horn	0.053	Peak	15	
Clarinet	0.050	Peak	14	
Piccolo	0.084	Peak	19	
Piano	0.267	Peak	33	
75-piece orchestra	13.8	Peak	236	
Pipe organ	12.6	Peak	226	
Mouth-blown whistle (4000 cycles)	0.06	Avg.	16	(3)
Sonic altimeter whistle	152	Avg.	783	
Foghorn (diaphone)	42 (sound measurement)	Avg.	400	(4)
	1680 (temp. measurement)	Avg.	2600	
Steamship whistle	270	Avg.	1040	(5)
Police whistle	0.08	Avg.	18	
Riveter	60	Avg.	493	
Automobile horn	6.6	Avg.	163	
Church bells	8.0	Avg.	180	
Thunder	2470	Avg.	3160	
Blast of explosion	98	Avg.	630	
Sawing wood	0.03	Avg.	11	

(1) H. Fletcher, "Speech and Hearing," pp. 67, 68.
(2) L. J. Sivian, H. K. Dunn, and S. D. White, *Jour. of Acous. Socy.*, vol. 2, p. 369, Jan., 1931.
(3) Tests conducted in development of sonic altimeter.
(4) L. V. King, *Phil. Trans. of Royal Soc.*, series A, vol. 218, pp. 211–293, 1919.
(5) Rogers H. Galt, *Jour. of Acous. Socy.*, vol. 2, p. 42, July, 1930.

5 gives the sound pressure in the different places, assuming an average threshold of 0.0006 dyne per sq. cm. Column 6 indicates the distance at which average conversational speech can just be heard through the noise, and column 7 the corresponding distance when talking as loudly as possible.

In Table 2 there is given the sound output in watts for various familiar sources, together with the corresponding sound pressure in dynes per square centimeter at 3 ft. from the source. The sound pressure is calculated from the relation

$$P_{\text{r.m.s.}} = \sqrt{\frac{415\,W \times 10^{6}}{4\pi R^{2}}} \text{ dynes per sq. cm.} \ldots\ldots [1]$$

where W equals sound radiated in watts and R is the distance from the source in centimeters. It will be noted from the equation that it has been assumed that the radiation is to take place equally in all directions through a spherical surface. When sound is radiated near the surface of the ground and the listener is at a considerable distance, a more correct assumption is that the sound is radiated through a hemisphere which corresponds to multiplying the sound pressure obtained from Equation [1] by $\sqrt{2}$. Since the values given in Table 2 are merely to indicate orders of magnitude, the spherical assumption has been used throughout for the sake of simplicity. It is seen from Table 2 that familiar sounds range from 10^{-9} watts, for a very soft whisper, to 1000 watts, for whistles and foghorns.

EARLY EXPERIMENTS

During the summer of 1929, while spending week-ends at camp on Honnedaga Lake, in the Adirondacks, New York State, the author decided to find out whether it was not possible to get through the airplane noise with the aid of a directive transmitter and receiver, when supplemented with an acoustic filter.

A $1\frac{1}{2}$-hp. outboard-motor boat was used in place of an airplane, and the echoes from the shore represented the ground. Some idea of the noise made by the outboard-motor boat is obtained from the fact that a person seated 4 ft. from the driver had to speak very loudly to be understood. If reference is made to Table 1, it will be noted that this corresponds roughly to a sound pressure of one or two bars.

Preliminary experiments, in which the audibilities of firecrackers of various sizes and whistles of different pitches were compared indicated that a small mouth-blown high-pitched whistle (3600 cycles) was much more readily distinguished through the engine noise than either firecrackers or any of the low-pitched whistles.

Considerable improvement was obtained by using a large megaphone (12 in. diameter to 2 in. diameter in 30 in.), both on sending and receiving, as shown by tests in which a member of the author's family blew the whistle on the dock while the author ran the motor boat out and back. The results were as follows:

1 Unaided ear—whistle drowned by engine noise at approximately 120 ft.

2 Megaphone held loosely to ear—whistle drowned at 500 ft.

3 Unaided ear on boat, with whistle blown into megaphone at dock—whistle drowned by engine noise at 480 ft.

These rough experiments show that a megaphone, used either as transmitter or receiver, will give approximately four times the range. Echo experiments at a spot on the lake called "Rocky Point" gave distinct echoes up to approximately 225 ft., both with the megaphone placed to the ear or by blowing the small whistle into the megaphone and listening with the unaided ear. The echoes at Rocky Point were obtained from a group of large flat-faced rocks rising almost perpendicularly from the water, approximately 30 ft. high by 60 ft. long. The fact that echoes were obtained up to 225 ft., which is approximately half the distance at which the whistle was drowned out by the engine noise in the previous test, shows that the reflection was approximately specular. Similar echo experiments made along the wooded shores gave good echoes up to 150 to 200 ft.

Some tests were made with a pair of stethoscope earpieces attached to the small end of the megaphone by a small cone. When this connection was made tightly, the value of the megaphone was practically destroyed due to the masking effect of the low-pitched horn resonance. By leaking or venting the junction between the small end of the megaphone and the small cone, the resonance was eliminated, and good results were obtained.

These experiments led to the belief that a Stewart high-pass acoustic filter[4] would be ideal for this job. Subsequent trials with acoustic high-pass filters which had a calculated cut-off around 2300 to 2500 cycles gave excellent results and practically eliminated the low-pitched noises.

Early in the progress of this work it became evident that the high-pass filters should be enclosed to prevent the entrance of sound and the disturbing effect of wind blowing over the holes. This was accomplished by enclosing them in a soundproof damped cylindrical box.

Multiple-unit broadside-directive devices, of the type proposed by Wilson, Zeleny, and Mason,[5] showed fine directive properties, but the one constructed by the author, which is shown in Fig. 2, was weak in intensity due to the crudity of the elements and high attenuation in the long lengths of small-diameter rubber tubing.

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[4] G. W. Stewart, *Physical Review*, vol. 20, p. 529, 1922. For recent work see "Acoustics," by Stewart and Lindsay.

[5] *Physical Review*, vol. 15, p. 178, 1920. See also Rayleigh Sci. Papers, vol. 5, p. 129, and Max Mason, U. S. patent 1,422,876, filed June, 1919.

Some preliminary tests using the binaural effect as a level indicator for aircraft showed considerable promise. Two large megaphones, spaced from 2 to 6 ft. apart, as shown in Fig. 3, were used in these experiments. High-pass filters were connected at the small end between the megaphones and the stethoscopes. With this arrangement sharp echoes were obtained from Rocky Point at from 150 to 225 ft., as well as a good sense of direction. By swinging the megaphones to locate the direction of the echo wave front and by comparing this with the angle of the boat, it could be determined whether the boat was parallel or at an angle to the shore. The ability to swing the megaphones helped greatly in finding the position. If, instead, the boat had to be turned, the time element reduced the effect of the observation; thus a compensator[6] or scheme for turning the pick-up device seems desirable for the application of the echo method to a sonic vertical keeper.

METHODS OF MEASURING ECHO TIME

Fig. 4 shows the set-up. The phonograph with a small projection on the turntable operated the small balanced air valve, which blew the whistle once each revolution. An air tank supplied from one of the engine cylinders through a check valve served as the source of compressed air. Two methods were tried with this apparatus, as follows:

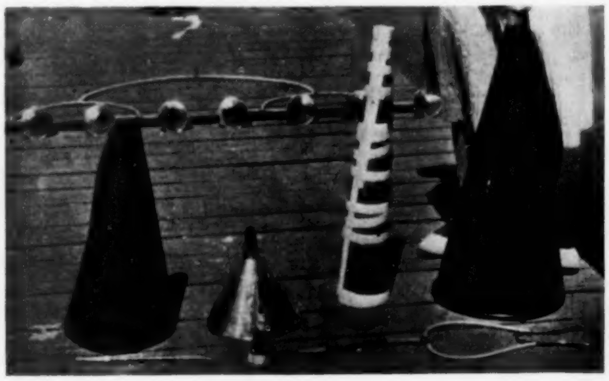

FIG. 2 PICK-UP DEVICES USED IN EARLY EXPERIMENTAL WORK

First, the position of the pointer on the turntable was estimated at the time the echo was received, by moving the finger around the periphery until the position of the finger and pointer coincided when the echo was heard. Tests with this method indicated an error of approximately 5 per cent in estimating distances from 80 to 500 ft.

The second method consisted in using the sound box of the phonograph to introduce a click into the listening system by making mechanical contact with the pointer. Here the procedure is to move the sound box around until the echo and the click appear to be simultaneous. A few trials with this method made it appear more difficult to operate than the visual one. The trouble seemed to be that there were too many things for the mind to coordinate easily. Besides the original source of sound, there is an echo and then a click.

COMPARISON OF AIRPLANE AND MOTOR-BOAT NOISES

Dr. A. R. Stevenson, Jr., and the author made the following tests at the Schenectady Airport, using his Stinson, Jr., plane, which is powered with a 165-hp. J-6 Wright engine.

The plane was held and the engine was run at full throttle, 1700 r.p.m. With the unaided ear the small mouth-blown whistle could be distinctly heard when sounded at 30 ft. perpendicular to the plane. When using the 12-in. diameter 30-in. long megaphone, fitted with high-pass filter and stethoscope earpieces, the limit was approximately 50 ft. The need of the high-pass filter when using a megaphone in the vicinity of an airplane was amply evident from these tests, for the sound was painfully loud if the stethoscopes were put in the ears without the high-pass filter.

In the motor-boat tests cited, the limit using the megaphone was approximately 500 ft. In other words, there is a ratio of 10 to 1 in the distance at which the whistle is just audible. This corresponds to a 10-to-1 ratio in sound pressures; and since the power output is proportional to the square of the sound pressure, there results 100 to 1 as the ratio of airplane to motor-boat noise. It is interesting to note that the powers of the two engines stand approximately in the same ratio.

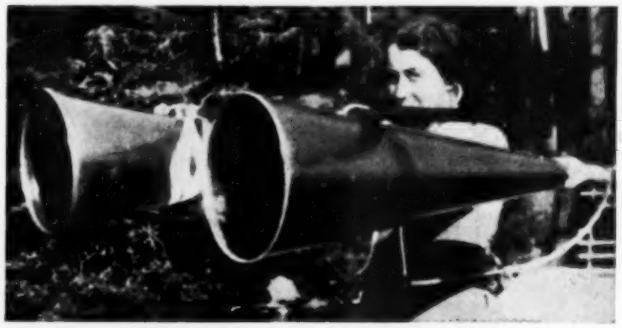

FIG. 3 EARLY EXPERIMENTS IN USE OF BINAURAL EFFECT

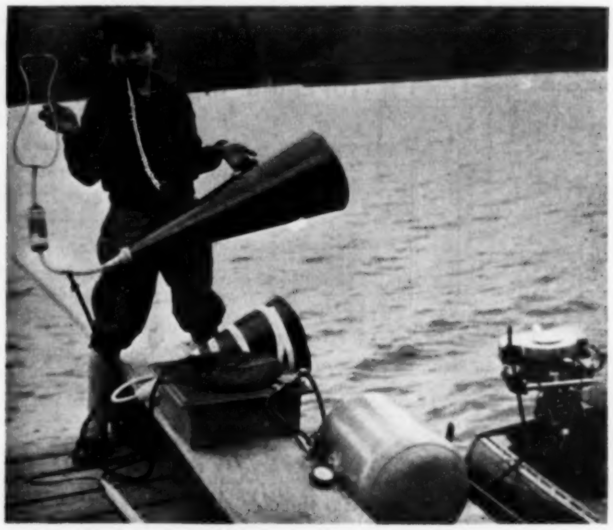

FIG. 4 EARLY SONIC ALTIMETER, WITH ECHO-TIMING DEVICE

While at Honnedaga Lake the motor boat was run away from the dock until it was estimated that it sounded about as loud as an airplane would when flying overhead at the normal height of 3000 ft., and in this way it was found that the motor boat needed to be about 300 ft. distant. This again gave a 10-to-1 ratio in distance for the same sound pressure, which corresponds to a 100-to-1 ratio in noise power. Reference to the data of Table 1 shows that these estimates were not far from the mark.

It was concluded from these tests that if a sound source could be made which would give roughly 50 times the sound pressure of the small mouth-blown whistle, a successful sonic altimeter could be made. Since a pressure of about 1 lb. per sq. in. is exerted when blowing the small whistle, it seemed reasonable to assume that a suitable whistle designed for a pressure of 50 to 100 lb. per sq. in. would meet the need.

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[6] "Acoustics," Stewart and Lindsay, p. 275. See also Max Mason, U. S. patent 1,682,706, filed June, 1919.

Experiments With Various Sound Sources

Some time later Dr. Stevenson and Messrs. J. H. Clough and J. S. Apperson joined the author in making further listening tests and in comparing the effectiveness of different types of sound sources.

TABLE 3 DISTANCES AT WHICH SOUNDS COULD BE RECOGNIZED

	Unaided ear, ft.	High-filter megaphone, ft.	Ratio of col. 3 to col. 2
Engine Full Throttle on Ground			
Whistle	270	570	2:1
Revolver, 32 caliber	240	390	1:6
Cannon, 10 gage	105	170	1:6
Engine Throttle Corresponding to Sustaining Speed			
Whistle	720	1200	1:7
Engine Full Throttle on Ground			
Small mouth-blown whistle	30	51	1:7
Early Motor-Boat Tests			
Small mouth-blown whistle	120	480	4:0

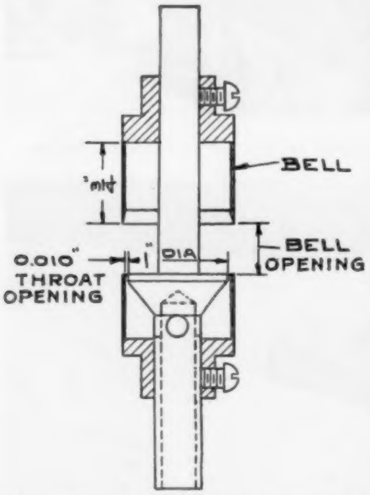

Fig. 5 Experimental 1-In. 3000-Cycle Whistle

In the meantime the small whistle shown in Fig. 5 had been built. It had a bell $^{3}/_{4}$ in. long by 1 in. in diameter and was supported on a $^{3}/_{8}$-in. diameter central rod. The annular opening for the air jet was 10 mils, and the distance between the annulus and the lower edge of the bell was found to give the best output at approximately $^{1}/_{2}$ in. The air pressure available at the whistle was approximately 50 lb. per sq. in. gage.

The distances at which the various sounds could just be distinctly recognized are summarized in Table 3. For these ground tests the author sat in the open doorway of the airplane cabin and listened while the various sound sources receded at right-angles to the plane.

That the ratios of distances heard with the megaphone plus the high-pass filter to those with the ear alone are worse for the airplane-engine experiments than with the motor boat is probably due to the fact that the acoustic filter was improperly shielded for the high-intensity airplane sounds. Better shielding was used in the later work.

At the conclusion of the ground tests, Messrs. Stevenson and Clough flew over the airport and listened to the whistle which was blown from the ground. The first time over, both heard the whistle loudly at 225 ft. The second time, they came over at 500 ft., and the whistle was distinctly heard by Mr. Clough, but not by Dr. Stevenson. Mr. Clough was leaning out of the cabin window.

Under these conditions the plane was making approximately 60 miles per hour, which corresponds to an engine speed somewhat above that necessary to sustain it.

Preliminary Calculations on the Probable Heights to Which Echoes May Be Received in an Airplane

The sound-pressure tests which are described in Appendix 1 show that the 1-in. 3000-cycle whistle, when operating at 50 lb. per sq. in. gage, produces a mean spherical sound pressure of 783 dynes per sq. cm. at 3 ft. For radiation into a hemisphere, assuming that the presence of the ground does not affect the power output of the source, multiply the mean spherical sound pressure by $\sqrt{2}$ and obtain 1100 dynes per sq. cm. at 3 ft. The sound pressure at 500 ft., which Mr. Clough could just distinguish through the engine noise, is therefore,

$$P_{500} = 1100 \times \frac{3}{500} = 6.6 \text{ dynes per sq. cm} \ldots\ldots [2]$$

Tests given in Appendix 2 on the transmission characteristics of the horn and the high-pass filter show that the system gives approximately a threefold sound-pressure amplification at 3000 cycles and that there will be approximately a tenfold reduction in the low-frequency masking sounds from the engine.

The sound-pressure curve for the 1-in. whistle in the 12-in. by 30-in. megaphone, given in Appendix 1, shows that the mean sound pressure over the 12-in. opening of the horn is approximately 4400 dynes per sq. cm. Therefore, by putting the horn on the whistle, there is obtained four times the sound pressure at 3 ft. from the source. These findings may be summarized as follows:

1 Airplane in flight (60 miles per hour), with ear alone, the 1-in. whistle could be heard at 500 ft. Sound pressure at ear 6.6 bars.

2 If receiving megaphone and high-pass filter were installed on plane, there would be obtained approximately three times this distance, or 1500 ft.

3 With the horn added to the whistle, the distance should be multiplied by approximately four, giving 6000 ft.

4 If specular reflection is assumed at the ground, one should divide this height by two and obtain as the limiting altitude 3000 ft. Obviously the assumption of specular reflection is seldom realized when one considers that the wave length of the 3000-cycle note is approximately 4 in. A slight roughness of this order of magnitude will cause the sound to be diffusely reflected. In the case of trees and shrubbery this effect will be still further enhanced by a relatively poor coefficient of reflection.

In Appendix 3 it is shown that the intensity ratio of specular to diffuse reflection is given by

$$\frac{I^1_R}{I_R} = \frac{1}{4K \tan^2 \alpha} \ldots\ldots\ldots\ldots [3]$$

where K is the coefficient of reflection and α the semi-angle of the conical beam sent out by the whistle in the megaphone. For $K = 1$ and $\alpha = 10$ deg.; $I^1_R/I_R = 8$. This corresponds to a sound-pressure ratio of 2.8 to 1. If $K = 0.5$, the intensity ratio becomes approximately 16 to 1 and the pressure ratio 4 to 1. For $K = 0.25$, the intensity ratio becomes 32.8 to 1 and the pressure ratio 5.75 to 1.

5 Assuming diffuse reflection with a reflection coefficient near to unity, divide the height obtained under the assumption of specular reflection by approximately three and obtain as the limiting altitude 1000 ft.

These results convinced the author that a practical sonic altimeter for airplanes was possible.

First Flight Tests

Mr. Ray Stearns, engineer of this company's Aeronautics and Marine Department, became interested in the project and provided further facilities to try out the sonic altimeter by actual flight tests. Arrangements were made with Mr. V. A. Rickard, manager of the Schenectady Airport, to equip his Stinson-Detroiter six-place cabin plane, which was powered with a 225-hp. J-5 Wright engine, with suitable equipment.

The 1-in. whistle was installed in a 12-in. by 30-in. megaphone just back of the baggage compartment. This brings the center line of the whistle approximately 17 ft. from the propeller. A motor-operated whistle valve and air-storage tank were installed in the baggage compartment. The storage tank was supplied with CO_2 gas from a high-pressure tank, through a reducing valve.

A pair of listening megaphones were installed approximately 6 ft. back of the whistle megaphone. The distance between the centers of the listening megaphones across the ship was approximately 18-in. Tubing 1 in. in diameter, with rubber-hose connections, properly insulated from the fuselage by felt attachments, was brought forward from the listening megaphones to the pilot's seat in the front of the cabin, a distance of approximately 16 ft. Various sizes of acoustic high-pass filters and other frequency selective devices were then inserted between the 1-in. speaking tube and the stethoscopes in the listener's ears. The object of the two receiving megaphones was to permit experiments with the binaural effect and allow changes to be made in one listening line while keeping the other fixed for comparison purposes. Intensity controls were introduced in the 1-in. listening line by having clamps which would close off a 1-ft. section of 1-in. felt-lined hose, to form a gradual restriction and subsequent expansion. The object of the intensity control was to vary the total intensity of signal and noise over a wide range in as nearly as possible a non-frequency selective manner.

In order to measure the time interval between the outgoing signal and the echo, a timer was arranged in front of the listener, driven from the motor-operated whistle valve through a flexible shaft. With this arrangement a short blast of the whistle went out every time the pointer passed zero on the scale. The motor speed was adjusted to send out a blast approximately every 2 seconds. At this speed the pointer on the dial makes one revolution in 2 seconds. Taking the velocity of sound as 1000 ft. per sec., as a round figure, the time interval between outgoing signal and echo for an altitude of 1000 ft. is 2 seconds; for 500 ft., 1 second; for 50 ft., $^{1}/_{10}$ second; and for 5 ft., $^{1}/_{100}$ second.

The height of the plane above the ground is then obtained by noting the approximate position of the pointer, as it moves around the dial, when the echo is heard. The numbers on the dial are marked so as to correspond directly with the height above the ground in feet.

It will be noted that the time intervals for moderate heights are not excessively short, and experience shows that with a little practice the altitude can be estimated within ± 5 per cent. If the aviator is flying toward a mountain, the rising ground is indicated by a reduction in the echo time interval. With a little practice the pilot also can estimate quite accurately his height without using the dial.

It is important to select a suitable frequency for the whistle, and 3000 cycles has been selected as an appropriate value. At this frequency one cycle has a duration of $^{1}/_{3000}$ of a second; and in order that the blast shall have a musical character, which is important, to allow of differentiating it from the extraneous engine noises, it is necessary that the blast have a duration of the order of 10 to 30 cycles. This means that with 3000 cycles as the frequency, a blast of 30 cycles lasts $^{1}/_{100}$ of a second, which corresponds to an altitude of 5 ft. above the ground.

If 300 cycles is used, 1 cycle equals $^{1}/_{300}$ of a second, and 30 cycles equals $^{1}/_{10}$ of a second, which corresponds to 50 ft. in altitude. It is evident from this comparison that a low frequency of 300 cycles is not favorable for altitude work.

It also is well known that the masking effect of a low-frequency sound is very much greater for a sound of similar pitch than it is for a sound having a much higher pitch.

Another reason for selecting the high frequency is that the depth of penetration into woods will be less with high frequency than with a low frequency. In other words, by using a high frequency, a distinct echo is received from the tops of trees, which would not be the case with a low frequency. Here the inverse fourth-power law of scattering comes to the rescue.

On the high-frequency side it is preferable not to go above 4000 cycles because the ear begins to fall off in sensitivity in this vicinity and also the turbulence around the plane and in the atmosphere will probably become more troublesome by scattering the sound and reducing the intensity of the echo.

Still another object in keeping the frequency between 2000 and 4000 cycles is to allow the acoustic filter to differentiate easily the sound of the signal from the low-frequency noises produced by the engine and the propeller. The engine and propeller noises probably have their maximum intensity between 100 and 300 cycles.

A further advantage of high frequency is that the wave length is small, and one can make highly directive devices of reasonable dimensions.

The foregoing reasons for using a high-frequency sound source may be summarized as follows:

1 Many cycles available in a short blast.

2 Reduced masking effect from low-frequency engine noise.

3 Improved scattering.

4 Maximum ear sensitivity.

5 Wide separation between engine and signal frequencies permits effective filtering or tuning.

6 Directive transmitters and receivers of moderate size.

Conclusions From Early Flight Tests

A great many variations were made in the test equipment during the early flight experiments, a few of which are listed as follows:

1 Variation of height at which echoes could be heard for different positions of the whistle in the sending megaphone.

2 Tipping the megaphone forward, backward, and sidewise.

3 Duration of the whistle blast.

4 Effect of covering listening megaphones with thin fabric.

5 Varying the intensity of signal with the intensity-control clamps.

6 Leaking the listening megaphones at the small end with various sized holes, to reduce the low-frequency response.

7 Various types of acoustic tuning devices at the receiving end of the speaking tube.

8 Effect of climb and bank on received echoes, using both horns connected binaurally and monaurally to the stethoscope earpieces.

9 Some preliminary work on electrical pick-up devices.

10 Observations on height at which echoes can be heard over various types of terrain, such as woods, shrubbery, unkempt fields, airport ground, Mohawk River, ice on Ballston Lake, 6 in. of rather hard snow, bare trees and trees with leaves, ravines, and hillsides.

11 Tests on still days, windy days, and bumpy days.

Any one who has tried to do experimental work in an airplane will realize the difficulties encountered in obtaining anything but qualitative results. Better facilities and a great many more hours available for flight would be required to obtain reliable answers to the various questions raised in the tests outlined. It therefore seems desirable at this time to record only what might be termed the overall results, as follows:

1 With engine running at cruising speed, reliable echoes were obtained over all types of terrain at 800 ft. and below.

2 With engine throttled for gliding flight, 1600 ft. and below.

3 The echo is lost at 800 ft. when bank or climb angle reached approximately 30 deg.

4 Echoes at 800 ft. were clearer and sharper over flat ground, water, and ice than over trees and fields.

5 When flying over mountainous country, the rise or fall of the ground is easily detected by the time interval between the outgoing and the received signal, as measured by the pointer on the altitude indicator or as judged by the ear alone.

6 When flying parallel to a steep hillside, the echo was noticeably long and drawn out by reflection from the varying distances.

7 When coming into a landing, the approach of the ground was very evident due to the closing in of the time interval between signal and echo, and when the plane reached approximately 5 or 10 ft. from the ground, the outgoing signal and echo blended into a continuous reverberation. This provided a definite notification point which it was felt would allow the pilot to make successful blind landings.

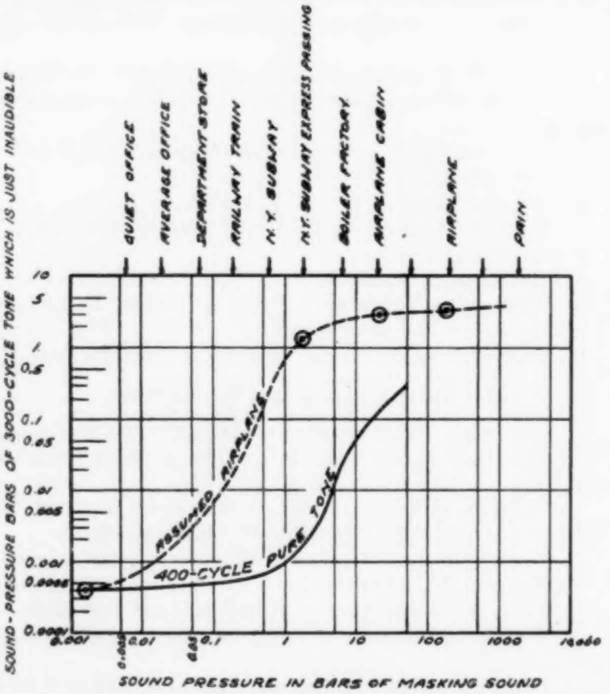

FIG. 6 PURE-TONE AND ASSUMED AIRPLANE-MASKING CURVES

During these early tests the author was ably assisted by Messrs. A. H. Brown, Jr., A. L. Ruiz, and H. L. Bojer, who were assigned to the work from time to time.

After the project was well under way, Mr. H. V. Marvin assigned Mr. A. J. Muchow to the work. He took full charge of the subsequent construction and installation of apparatus for the Douglas plane, as well as making sound measurements and calculations.

REVIEW OF EARLY CALCULATIONS IN LIGHT OF SUBSEQUENT FLIGHT TESTS AND DEDUCTION OF AIRPLANE-MASKING CURVE

It is interesting to note that the altitudes for satisfactory echoes predicted from the early motor-boat and airport-ground tests were confirmed by the subsequent flight tests in the Stinson-Detroiter.

In Fig. 6 there is shown in the full curve the masking effect of a 400-cycle pure tone on a 3000-cycle signal for different sound pressures of the 400-cycle masking tone. This curve is obtained from the values given by Fletcher[7] on the assumption of a threshold of 0.0016 bar for the 400-cycle tone and 0.00042 bar for the 3000-cycle tone. The curve shows that moderate values of the 400-cycle sound pressure produce very little interference with the reception of a 3000-cycle tone. As the sound pressure of the masking tone increases above approximately one bar, the masking effect on the 3000-cycle tone increases very rapidly. For loud sounds—that is, after passing 10 bars—the masking curve shows a tendency to flatten out again. The flattening out of masking curves at high sound pressures is more evident from some of Fletcher's other curves where the masking tone is nearer the frequency of the signal.

The upper dotted curve is a hypothetical masking curve which has been drawn to represent the masking effect of airplane noise on the reception of a 3000-cycle signal. The general shape and position of this curve have been deduced from the sonic altimeter experiments. For example, starting on the assumption that the airplane noise at the mouth of the pick-up megaphone is roughly 190 bars, the flight tests show that the necessary sound pressure received at the ear must be approximately six bars to have the signal distinctly heard above the engine noise. If this value of sound pressure is divided by 2, there is obtained three bars as the first point on the hypothetical masking curve. This assumes that a ratio of 2 to 1 in sound pressure is sufficient to make the signal audible.

The fact that the 10-to-1 reduction in sound pressure for the low frequencies, which is produced by the filter, does not appreciably affect the calculation of the height at which echoes may be heard, indicates that the masking curve is flat in this region. The ordinate for the second point, which goes with the 19-bar noise, has therefore been taken only slightly below the ordinate for the 190-bar noise.

If it is assumed that throttling the engine for gliding reduces the engine-sound pressure to approximately $^1/_{100}$ its normal value, there is obtained 1.9 bars as the masking-sound pressure when gliding. Under these conditions the flight observations give roughly twice the height for good echoes, and therefore approximately 1.5 bars have been taken as the ordinate on the masking curve. From there down there has merely been drawn a smooth curve similar in form to the full-line pure-tone masking curve.

If the imaginary masking curve is correct, it indicates that small reductions in airplane noise or improvements in the low-frequency characteristics of the pick-up system will add very little to the heights at which one can receive satisfactory echoes. On the other hand, any improvement in the efficiency of the 3000-cycle pick-up or 3000-cycle sound source will add almost directly to the echo altitudes. The fact that the echo heights obtained on the 225-hp. Stinson were practically the same as those obtained with the installation on the 450-hp. Douglas plane, as well as the listening tests in Dr. Stevenson's plane, which was powered with a 165-hp. engine, tends to confirm the foregoing conclusions.

ARMY TESTS

At this point of the development it was felt that the experiments had demonstrated the feasibility of the sonic altimeter and that the next step would be to induce somebody to make a trial installation and thereby determine the actual merit of the sonic altimeter as applied to practical aviation.

Messrs. Ray Stearms, A. H. French, C. E. Clark, and J. Bosway took the matter up with the Army officials. Shortly thereafter Major Hugh Mitchell and Lieut. Albert F. Hegenberger came to Schenectady and tried out the crude equipment which had been installed on the Stinson-Detroiter.

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[7] Fletcher, "Speech and Hearing," p. 117.

Blind-landing projects come under Lieutenant Hegenberger at Wright Field. Shortly after the aforementioned trip, arrangements were made to equip Lieutenant Hegenberger's two-passenger, open-cockpit Douglas plane, which is powered with a Pratt & Whitney 450-hp. Wasp engine, with the sonic altimeter.

It was felt that for this trial installation the simplest and most rugged arrangement should be selected as determined from the tests on Mr. Rickard's Stinson-Detroiter. The installation is shown diagrammatically in Fig. 7, with the principal dimensions.

In this case the air supply for the whistle was obtained by bleeding one of the engine cylinders through a suitably designed cooler and check valve. The need of a cooler in front of the check valve was suggested by Mr. H. I. Becker, after much difficulty had been experienced in trying to make hot check valves work. The author is also indebted to Messrs. R. W. Ayers and C. L. Heisler for valuable suggestions regarding the check-valve design.

In place of the mechanically driven timer used on the Stinson, there was substituted a spring-driven timer in which a contact-operated solenoid released the pointer electrically each time the motor-operated whistle valve sent out a blast. Under these conditions the altitude, as indicated by the timer, is independent of the speed of the motor which drives the whistle valve. The sending megaphone and air-storage tank were mounted in the front compartment of the Douglas plane, as shown in Fig. 8. Here the check valve and cooler may be clearly seen. Fig. 9 is a close-up, looking into this compartment from the other side of the plane.

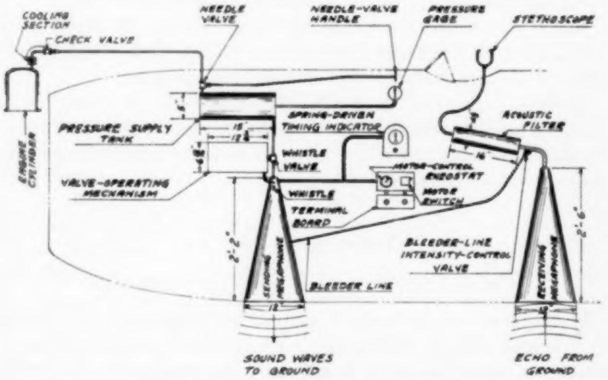
COOLING SECTION
CHECK VALVE
NEEDLE VALVE
NEEDLE-VALVE HANDLE
PRESSURE GAGE
STETHOSCOPE
ENGINE CYLINDER
PRESSURE SUPPLY TANK
VALVE-OPERATING MECHANISM
SPRING-DRIVEN TIMING INDICATOR
ACOUSTIC FILTER
WHISTLE VALVE
MOTOR-CONTROL RHEOSTAT
MOTOR SWITCH
WHISTLE
TERMINAL BOARD
BLEEDER-LINE INTENSITY-CONTROL VALVE
BLEEDER LINE
SENDING MEGAPHONE
RECEIVING MEGAPHONE
6"
15"
12"
16"
2'-2"
2'-6"
12"
12"
SOUND WAVES TO GROUND
ECHO FROM GROUND

FIG. 7 DIAGRAM OF SONIC ALTIMETER INSTALLATION

A single receiving megaphone was mounted in the tail of the ship, approximately 18 ft. from the sending megaphone. Aluminum tubing of 1 in. diameter, with suitable breaks, connected by rubber tubing, to reduce the sound transmission, and flexible attachments to the structure of the plane were provided to bring the sound from the megaphone to a point between the two seats. Here a reducing cone was provided to make an impedance fit between the 1-in. speaking tube and the $^5/_{16}$-in. diameter high-pass filter and stethoscopic earpieces.

Preliminary flight tests with this equipment showed that the outgoing signal was about equal to the echo at 600 ft. altitude, which is not loud enough to give a natural echo effect. It was difficult to judge heights without looking at the timer, and the effects on coming in to land were not nearly as pleasing as they had been in the Stinson. Accordingly there was installed a 1-in. speaking tube to convey an additional signal from the whistle megaphone to the listening line. The sound was inserted into the listening system ahead of the acoustic high-pass filter, after passing through a reducing cone and intensity-control valve. The listener can adjust the intensity of the outgoing signal blast to give the best psychological effect both in flight and for landing. It was found that a single adjustment was satisfactory for both conditions.

The sonic altimeter has been in practically continuous use without showing any important mechanical or acoustical defects since its installation. The average of results by several listeners indicates the following:

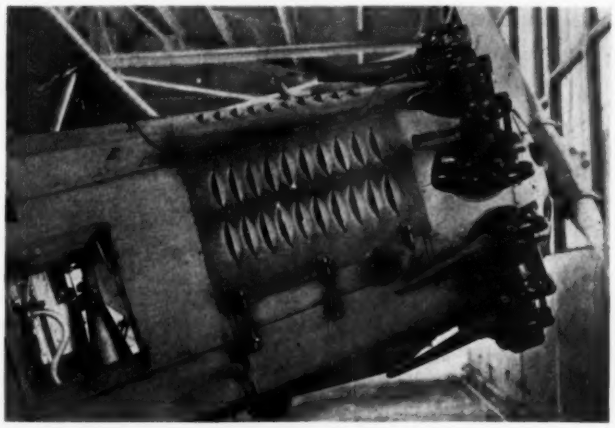

FIG. 8 INSTALLATION OF SONIC ALTIMETER ON ARMY PLANE

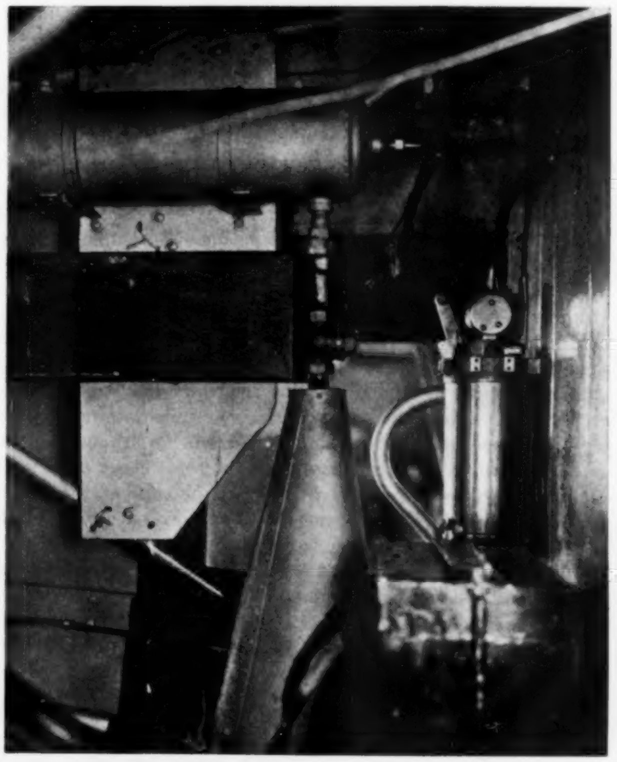

FIG. 9 TRANSMITTING EQUIPMENT FOR SONIC ALTIMETER INSTALLED ON ARMY AIRPLANE

1 Engine running at cruising speed, reliable echoes at 800 ft. and below.

2 Engine idling for a glide, reliable echoes at 1400 ft. and below.

Numerous blind landings have been made by Lieutenant

Hegenberger, in which he finds he can make use of the blending of outgoing signal and echo in the vicinity of 5 ft. as a point from which to pull back the stick and allow the plane to settle to the ground.

The total weight of the apparatus as installed is approximately 45 lb. In this connection it should be understood that no special stress has been put on reducing either the weight or the volume of the apparatus to a minimum.

USES OF THE SONIC ALTIMETER AND SONIC BOUNDARY MARKERS

In conclusion, it may be interesting to review in a general way how the sonic altimeter and other aeronautical devices may be employed to make fog flying safe.

If an aviator is flying cross-country and is caught in a fog, it is highly desirable to have a means that will allow him to determine approximately his height above ground. In the first place, the aviator wishes to know that he is at a safe height to clear buildings, transmission-line towers, mountains, etc., and may proceed without danger of collision. Supposing that he is bound for the Schenectady Airport. He can steer the plane by compass or use his directive radio receiver to keep him headed for radio station WGY at Schenectady. The sonic altimeter keeps him at a safe height from the ground if he wishes to fly low. If he wishes to fly high, he may come down to 1000 ft. or so from the ground and check his barometric altimeter with the sonic, as proposed by Dr. E. F. W. Alexanderson, and then climb again to a high altitude.

When he reaches the vicinity of Schenectady, he is supposed to pick up some sort of short-wave radio or other type of airport marker located at the Schenectady Airport. This will bring him in proximity to the landing field. In addition to the radio or other beacon which locates the field, it is generally conceded that it will be desirable to have some sort of field-boundary or runway marking. Several schemes using ultra-short-wave radio, sound, or light have been proposed. It is felt that at present the most simple and rugged means of accomplishing this result would be to mark the field by sending up properly located and coded beams of sound. The present work on the sonic altimeter has shown that relatively cheap and rugged apparatus can be used to send up sound beams which may be picked up at approximately 3000 to 6000 ft.

After locating the extent of the field and his position with respect to it by some of the methods outlined, the pilot will cut his engine and start gliding down. As he approaches the ground the times between signal and echo come closer and closer together. At approximately 5 ft. from the ground the signal and echo blend in a reverberation. The pilot then pulls back the stick and allows the plane to settle to the field.

A great advantage of the sonic altimeter is that it puts the ear to work and relieves the eye to watch the various instruments and obtain the first glimpse of the ground in making a fog landing. Judging from observation there are few fogs where it will not be possible to see the ground before actually setting the plane down.

There are other cases where an aviator may wish to attempt a fog landing on an airport or field which is not provided with radio beacons or other landing aids. In this case he must be able to feel his way down through the fog until he is able to see the ground. With the sonic altimeter he can safely approach the ground without danger of collision, and if he finds he is in one of those exceptionally heavy fogs where vision is impossible at a few hundred feet, his only alternative is to fly on and look for an opening in the fog in some other direction. If, after feeling his way down to 200 ft. in some other locality, he is able to pick up the ground by sight, as is usually the case, he may then fly at this altitude and use his sight to pick out a suitable landing place.

Since fog is the exception rather than the rule, the sonic altimeter or any other type of altimeter will be used relatively seldom. It is therefore essential that the device be light, simple to operate, and cheap and reliable; otherwise it would not pay to carry the apparatus for the infrequent times when it is needed.

ACKNOWLEDGMENTS

It is desired to acknowledge the great assistance rendered by Mr. Ray Stearns' enthusiastic support throughout the progress of the work.

The author also is greatly indebted for the fine cooperation obtained from Mr. V. A. Rickard of the Schenectady Airport.

Special mention also should be made of the able assistance which Mr. A. J. Muchow gave to all phases of the development.

Lieutenant Albert F. Hegenberger's wide personal experience and skill as a flyer, combined with his enthusiasm and vision, were essential factors in the success of the enterprise.

The hearty cooperation obtained from Capt. H. M. McClelland and others of the Army staff at Washington and Wright Field was also a large factor in the success of the project.

If space permitted, the author would like to cite a long list of General Electric men, besides those mentioned in the text, whose special knowledge was freely given to assist in overcoming many difficult phases of the development.

Appendix No. 1

CHARACTERISTICS OF SOME SOUND SOURCES

A STUDY of the sound output and efficiency of various sound sources constituted an important part of the sonic altimeter development. A summary of the most interesting results are collected in Table 4. The efficiencies in Table 4 are the ratios of sound output of the source to the power available from the adiabatic expansion of the air. It will be noticed that a corrected value of sound output for the diaphone (foghorn) is given. In the original article[8] the expression for the sound output is given as:

TABLE 4 CHARACTERISTICS OF SOME SOUND SOURCES

Sound source	Frequency, cycles per sec.	Air pressure, lb. per sq. in. gage	Air flow, cu. ft. per min.	Mean spherical sound pressure at 3-ft. bars	Sound output, watts	Efficiency, per cent
1-in. whistle, 1/2-in. bell	3600	50	37	461	52	2.4
1-in. whistle, 3/4-in. bell	3000	50	37	783	152	7.0
1-in. whistle, 1-in. bell	2000	50	37	610	92	4.3
2-in. whistle, 0.010-in. throat	3000	81	107	1185	346	4.6
2-in. whistle, 0.010-in. throat	3000	50	69	946	221	5.4
2-in. whistle, 0.020-in. throat	3000	72	192	1600	630	4.8
2-in. whistle, 0.040-in. throat	3000	55	306	1790	790	4.2
Allan Cunningham horn (size 3)	350	50	30	1660	560	32.4
Small siren	3000	50	32	201	10	0.5
Siren with resonator, built by E. W. Kellog	200	50	49	500	62	2.0
Diaphone (foghorn) from tests by L. V. King	175	19.9	800	..	..	..
By temperature measurement	..	..	...	2600	1190[a] 1680[b]	5.6[c]
By sound-pressure measurements	..	..	...	400[d]	42[d]	..

[a] King.
[b] Corrected.
[c] This point is one taken from test No. 1 in L. V. King's article and represents average condition.
[d] Should be considered as an estimate only because of difficulty in determining distribution of sound pressure.

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[8] L. V. King, Phil. Trans. of Roy. Soc., series A, vol. 218, p. 224, 1919.

$$\text{Sound output} = MC_v\Delta T \quad \text{[4]}$$

where
M = mass of air consumed per second
ΔT = temperature drop through diaphone
C_v = specific heat of air at constant volume
C_p = specific heat of air at constant pressure

The correction consists in replacing C_v by C_p, so that

$$\text{Sound output} = MC_p\Delta T \quad \text{[5]}$$

The need for this correction was pointed out by Mr. W. L. Knaus, who has assisted greatly by making all of the calculations for this Appendix.

In the original article the expression for efficiency is correct, as the same error was made in deriving the expression for available energy, and therefore the quotient of sound output and available energy, which is the efficiency, remains unchanged.

Table 4 shows that the sirens, at least those which the author built, consumed a large quantity of air and gave a relatively poor sound output. The 1-in. whistle with the $^3/_4$-in. bell gave the highest efficiency of any of the whistles, consumed a moderate amount of air, and gave a good sound output. The Allan Cunningham horn (size 3) shows a remarkably high efficiency and large sound output for a very moderate air consumption.

It would be interesting to try to build a practical high-frequency horn on this principle. Briefly, the horn consists of a flat diaphragm which is pressed against the small end of a short flaring horn. The clamping ring which supports the diaphragm is joined to the horn an inch or so from the small end, in such a way as to form a closed annular chamber. The air from the high-pressure line is fed into this annular chamber through a small constriction. The action is presumably somewhat as follows:

The air pressure builds up in the annular space and then escapes into the horn by forcing the diaphragm back off the closed small end. When the contents of the chamber are emptied, the diaphragm closes the small end of the horn, and the pressure builds up again in the annular chamber through the constriction.

WHISTLE DIRECTIVE CURVES

Fig. 10 gives the polar sound-pressure curve of the 1-in. 3000-cycle whistle at 3 ft. from the throat, for an air pressure of 50 lb. per sq. inch gage. The measurements were taken in a soundproof room 17 ft. by 8 ft. by $9^1/_2$ ft. high, using a condenser microphone and suitable amplifier. The experimental points are given by circles, and a smooth curve was drawn through the points for the purpose of obtaining the mean spherical sound pressure. Obviously more points are required to justify the smooth curve, but this would call for more money than was available. At 3000 cycles the sound pressures as indicated by the condenser microphone were divided by three to correct for pressure doubling and cavity resonance, as described in Appendix 2.

In what follows, the mean spherical sound pressure of a source is taken to be that which, if exerted uniformly over the surface of a sphere, would result in the same power output as the actual source when radiating through the surface of the same sphere. The following method of calculation applies only to those types of sources which have a distribution that may be represented by a figure of revolution. All that is necessary, then, is a polar distribution curve taken in a plane through the axis of symmetry.

Referring to Fig. 10, consider the sound output through a zone such as zone 3.

If r = radius of sphere and h = height of zone, the area of the zone is $2\pi rh$.

The mean sound pressure P_3 over the zone is taken to be that along a radius such as r_3. The power radiated through the zone is

$$W_3 = 2\pi rh\left(\frac{P_3^2}{425}\right) \times 10^{-6} \text{ watts} \quad \text{[6]}$$

If the sphere is divided into n equal zones, the expression for the total power is given by the following summation:

$$W = \sum_{K=1}^{K=n} W_K = \sum_{K=1}^{K=n}\left[2\pi r \times \frac{2r}{n}\left(\frac{P_K^2}{425}\right) \times 10^{-6}\right]$$

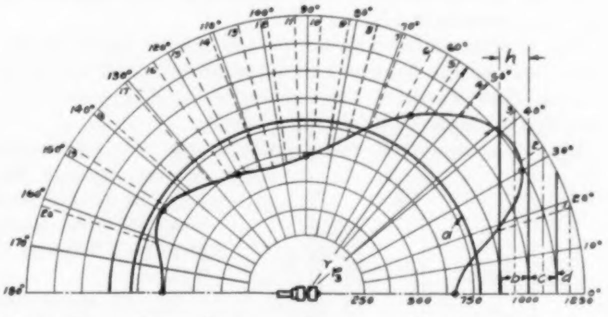

FIG. 10 POLAR SOUND-PRESSURE CURVE AT 3 FT. FROM THROAT OF WHISTLE

(1-in. whistle, 3000 cycles, $^{15}/_{32}$-in. bell opening, 0.01-in. throat, 50 lb. per sq. in. gage pressure. a, mean spherical sound pressure, 783 bars, bars at 3 ft. from source; d, zone 1; c, zone 2; b, zone 3.)

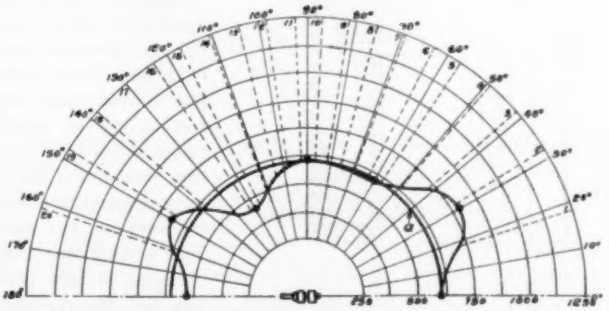

FIG. 11 POLAR SOUND-PRESSURE CURVE AT 3 FT. FROM THROAT OF WHISTLE

(1-in. whistle, 2000 cycles, $^{15}/_{16}$-in. bell opening, 0.01-in. throat, 50 lb. per sq. in. gage pressure. a, mean spherical sound pressure, 610 bars, bars at 3 ft. from source.)

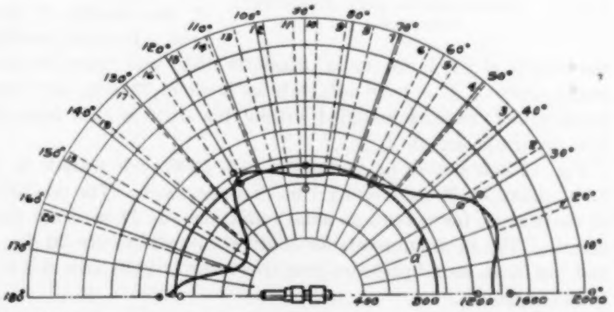

FIG. 12 POLAR SOUND-PRESSURE CURVE AT 3 FT. FROM THROAT OF WHISTLE

(2-in. whistle, 3000 cycles, $^{15}/_{32}$-in. bell opening, 0.01-in. throat, 50 lb. per sq. in. gage pressure. a, mean spherical sound pressure, 946 bars, bars at 3 ft. from source.)

$$= \frac{\Sigma P_K^2}{n} \times \frac{10^{-6}}{425} \times 4\pi r^2 \text{ watts} \quad \text{[7]}$$

From this it is seen that the first term, $\frac{\Sigma P_K^2}{n}$, is the square of the mean spherical sound pressure P_m, or

$$P_m = \sqrt{\frac{\Sigma P_K^2}{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [8]$$

In Fig. 10 n has been taken as 20. It is evident from the foregoing that a simple average of uniformly spaced readings will not give a correct result.

The present calculation is similar to that of obtaining the mean spherical candlepower of a light source, and is explained in almost any illumination text.[9]

In Fig. 10 the calculated mean spherical sound pressure is 783 bars at a distance of 3 ft.

Fig. 11 gives similar data for the 1-in. whistle with 1-in. bell, which lowered the frequency to approximately 2000 cycles per second. The calculated mean spherical sound pressure is 610 bars at 3 ft., as shown by the heavy circle.

Fig. 12 gives corresponding data for the 2-in., 3000-cycle whistle shown in Fig. 13. The calculated mean spherical sound pressure is 946 bars at 3 ft.

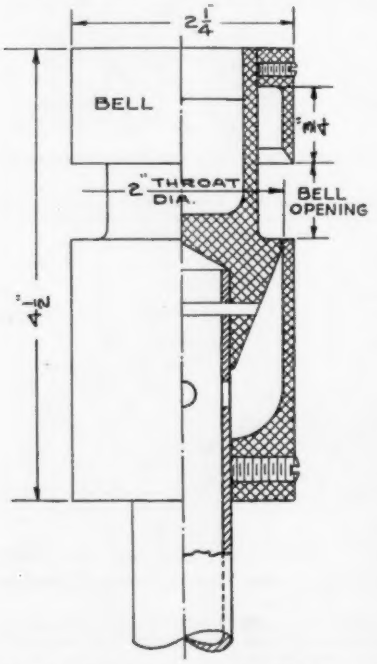

Fig. 13 Experimental 2-In. 3000-Cycle Whistle

Fig. 14 shows the effect on the sound-pressure distribution obtained by putting the 1-in. 3000-cycle whistle in a 12 in. diameter by 14 in. long by 1 in. diameter conical horn. The whistle was moved back and forth along the axis of the cone until the position was found which gave maximum sound pressure on the axis. The whistle was then clamped in this position for the directive curve readings. It will be noted that the microphone was held at 3 ft. from the center of the mouth of the horn. In other words, the mouth of the horn (12 in. diameter disk) was taken as the sound source. This horn subtends an angle of 43 deg., and the mean sound pressure included within this angle is 1290 bars at 3 ft. from the mouth.

Fig. 15 is a similar curve for the 1-in. 3000-cycle whistle in a 12 in. diam. by 30 in. long by 1 in. diameter horn. The position of the whistle for maximum axial radiation was 21 in. from the mouth. The horn subtends an angle of approximately 20 deg., and the mean sound pressure over this angle is 4400 bars at 3 ft. from the mouth.

Effect of Horn on Power Output of Whistle, and Calculated Concentration Factors

The concentrating efficiencies of the horns are brought out in the following comparisons:

(1) The ratio of sound pressure and power density which would exist with a fixed amount of power distributed over a sphere and over the solid angle subtended by the horn.

A_s = area of sphere
A_h = area subtended by horn angle
W_s = power density over the sphere
W_h = power density over the area subtended by the horn angle
P_s = sound pressure over sphere
P_h = sound pressure over the area subtended by the horn angle

Since the amounts of power distributed over the sphere and over the area subtended by the horn are the same, $W_sA_s = W_hA_h$.

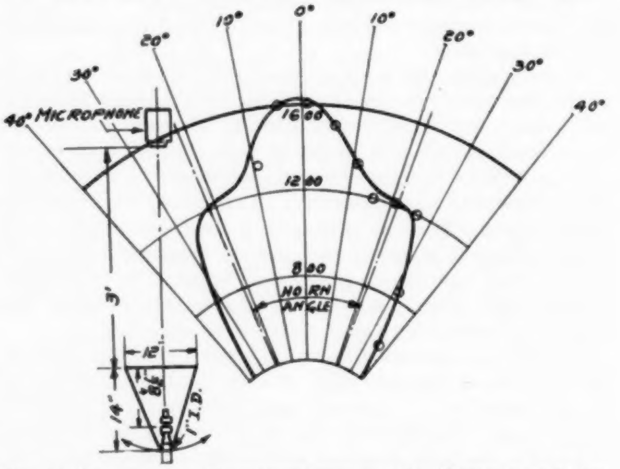

Fig. 14 Directional Characteristic of 1-In. Whistle in 14-In. Horn

(1-in. whistle, $^1/_2$-in. bell opening, 50 lb. per sq. in. gage pressure, mean sound pressure 1290 bars; bars at 3 ft. from source.)

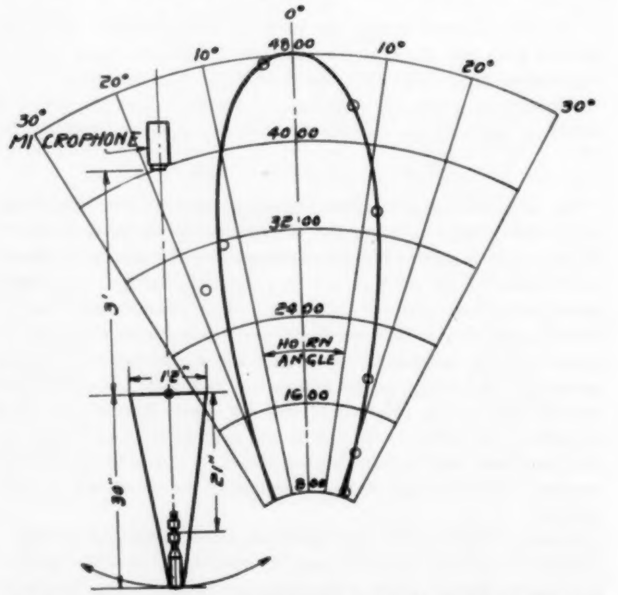

Fig. 15 Directional Characteristic of 1-In. Whistle in 30-In. Horn

(1-in. whistle, $^1/_2$-in. bell opening, 50 lb. per sq. in. gage pressure, mean sound pressure 4400 bars; bars at 3 ft. from source.)

$$\frac{W_h}{W_s} = \frac{A_s}{A_h} = \frac{\text{(watts per sq. cm. in solid angle of horn)}}{\text{(watts per sq. cm. in } 4\pi \text{ steradians)}} \ldots [9]$$

The ratio A_s/A_h can be easily found. Referring to Fig. 16, which is a section through the center of the sphere,

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[9] "A Text Book of Illumination," William Kunerth, pp. 68–77, or "Illuminating Engineering," Cady and Dates.

$$\frac{A_s}{A_h} = \frac{4\pi r^2}{2\pi r h} = \frac{2}{1 - \cos\left(\dfrac{\alpha}{2}\right)} \ldots\ldots\ldots\ldots [10]$$

For the long horn, $\alpha = 20.8$ deg.

$$\frac{W_h}{W_s} = 122 \ldots\ldots\ldots\ldots [11]$$

The ratio of sound pressures is:

$$\frac{P_h}{P_s} = \sqrt{\frac{W_h}{W_s}} = 11 \ldots\ldots\ldots\ldots [12]$$

For the short horn, $\alpha = 42.8$ deg.

$$\frac{W_h}{W_s} = 29 \text{ and } \frac{P_h}{P_s} = 5.4 \ldots\ldots\ldots\ldots [13]$$

(2) The ratio of sound pressure and power density, actually measured, for the 1-in. 3000-cycle whistle operating in and out of the horns. For the long horn, $P_h = 4400$ bars at 3 ft., so that

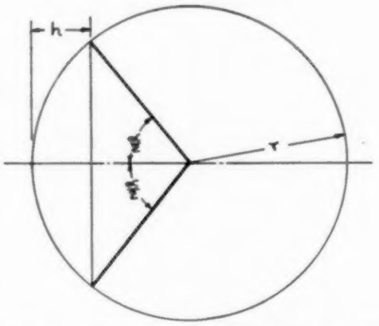

FIG. 16 SECTION SHOWING SPHERICAL ANGLE OF HORN

$$\frac{P_h}{P_s} = \frac{4400}{783} = 5.6 \ldots\ldots\ldots\ldots [14]$$

$$\frac{W_h}{W_s} = \frac{\text{(watts per sq. cm. observed for whistle in horn)}}{\text{(watts per sq. cm. observed with whistle in the open)}}$$

$$= \left(\frac{P_h}{P_s}\right)^2 = 31.5 \ldots\ldots [15]$$

For the small horn, $P_h = 1290$ bars at 3 ft.

$$\frac{P_h}{P_s} = \frac{1290}{783} = 1.65 \ldots\ldots\ldots\ldots [16]$$

$$\frac{W_h}{W_s} = (1.65)^2 = 2.7 \ldots\ldots\ldots\ldots [17]$$

(3) It was found by calculation from the polar distribution curve, Figs. 14 and 15, that the amount of power radiated was greatly reduced by placing the whistle in the horn. In the following, the ratio of sound pressures and power densities is found on the assumption that the whistle maintained the *same* output in and out of the horn.

In the case of the long horn it was found that the total output was 86 watts. With the whistle in the open, the total output was 152 watts. Therefore, if the outputs were equal, the ratio of W_h/W_s would not be that found in (2), but

$$\frac{W_h}{W_s} = \frac{\text{(watts per sq. cm. which would be obtained in cone if horn did not reduce output)}}{\text{(watts per sq. cm. with whistle in open)}}$$

$$= 31.5 \times \frac{152}{86} = 55.7 \ldots\ldots\ldots\ldots [18]$$

$$\frac{P_h}{P_s} = \sqrt{55.7} = 7.5 \ldots\ldots\ldots\ldots [19]$$

With the whistle in the short horn, the measured output was 29 watts. Equalizing, as in the foregoing, there is obtained:

$$\frac{W_h}{W_s} = 2.70 \times \frac{152}{29} = 14.2$$

$$\frac{P_h}{P_s} = \sqrt{14.2} = 3.8 \ldots\ldots\ldots\ldots [20]$$

A comparison of the power-density ratios found in (3) with those found in (1) gives a measure of the horn concentration. This may be called the "concentration efficiency."

For the long horn, this ratio is:

$$\frac{\left(\dfrac{W_h}{W_s}\right)_{(3)}}{\left(\dfrac{W_h}{W_s}\right)_{(1)}} = \frac{55.7}{122} = 45 \text{ per cent} \ldots\ldots\ldots [21]$$

For the short horn, this ratio is:

$$\frac{\left(\dfrac{W_h}{W_s}\right)_{(3)}}{\left(\dfrac{W_h}{W_s}\right)_{(1)}} = \frac{14.2}{29.0} = 49 \text{ per cent} \ldots\ldots\ldots [22]$$

The subscripts refer to the method of computing the power-density ratio.

MEASURED AND CALCULATED AIR FLOWS

The circles in Fig. 17 give the observed air flows for the 1-in. whistle and the 2-in. whistle with 0.01-in. throats. The smooth curves drawn through the points indicate calculated values obtained from the following familiar equations for adiabatic flow:[10]

$$W = A\sqrt{2g\,\frac{r}{r-1}\,\frac{P_1}{V_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{r}} - \left(\frac{P_2}{P_1}\right)^{\frac{r+1}{r}}\right]} \text{ lb. per sec.} \ldots [23]$$

where

W = rate of discharge
A = area of throat in square feet
g = acceleration of gravity in feet per second^2
$r = \dfrac{\text{specific heat of the gas at constant pressure}}{\text{specific heat of the gas at constant volume}}$
= 1.40 for air
P_1 = tank pressure in lb. per sq. ft. abs.
V_1 = specific volume of air in tank in cu. ft. per lb.
P_3 = pressure outside of orifice in lb. per sq. ft. abs.
P_2 = pressure in throat

$$P_2 = P_3 \text{ if } P_3 > P_1\left(\frac{2}{r+1}\right)^{\frac{r}{r-1}} = 0.53\,P_1$$

and

$$P_2 = P_1\left(\frac{2}{r+1}\right)^{\frac{r-1}{r}} \text{ if } P_3 < 0.53\,P_1$$

If the velocity of the jet is destroyed without doing external work or removing energy in the form of heat, the expansion is said to be one of constant total heat; and it can be shown that the initial temperature T_1 and the final temperature T_2 are the same except for a negligible difference due to the Joule-Thomson effect.

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[10] For example, see "Technical Thermodynamics," by Zeuner, translated by Klein, vol. 1, p. 236.

If V_2 = specific volume in cu. ft. per lb. of the air after expansion to pressure P_2 and absolute temperature T_2,

WV_2 = discharge in cu. ft. of free air per second

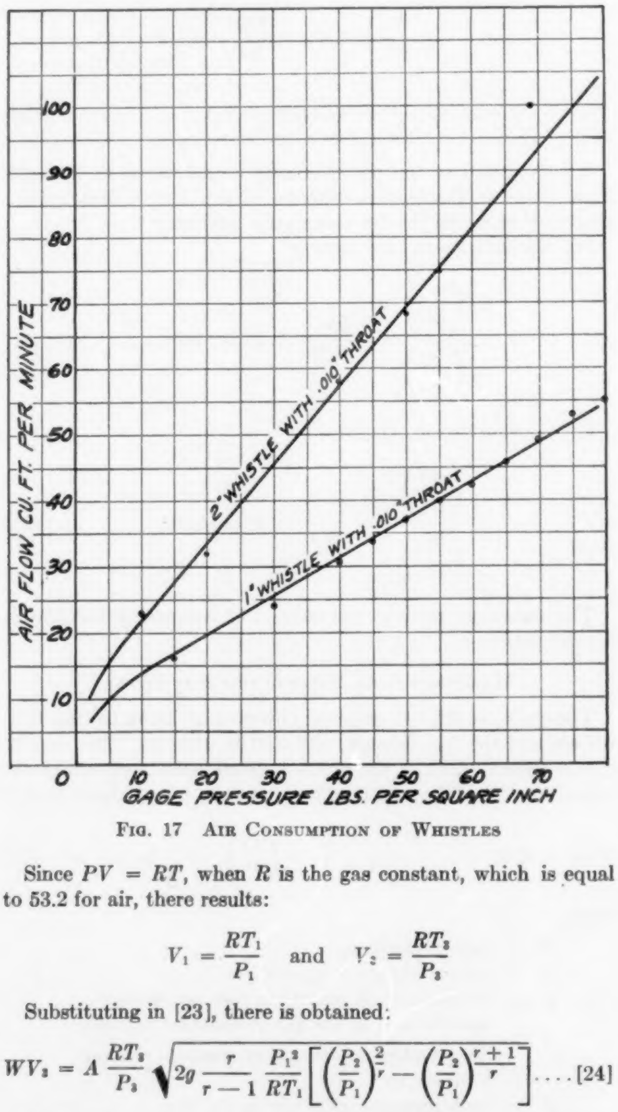

Fig. 17 Air Consumption of Whistles

Since $PV = RT$, when R is the gas constant, which is equal to 53.2 for air, there results:

$$V_1 = \frac{RT_1}{P_1} \quad \text{and} \quad V_2 = \frac{RT_2}{P_2}$$

Substituting in [23], there is obtained:

$$WV_2 = A\,\frac{RT_2}{P_2}\sqrt{2g\,\frac{r}{r-1}\,\frac{P_1^2}{RT_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{r}} - \left(\frac{P_2}{P_1}\right)^{\frac{r+1}{r}}\right]} \ldots . [24]$$

Taking $T_1 = T_2 = 68$ deg. fahr. = 528 deg. fahr. abs.,

P_2 = (14.7 × 144) lb. per sq. ft.

a = throat area in sq. in.

$$\text{The rate of flow} = 1048a\left(\frac{P_1}{P_2}\right)\sqrt{\left(\frac{P_2}{P_1}\right)^{1.425}\left[1 - \left(\frac{P_2}{P_1}\right)^{0.287}\right]}$$

cu. ft. per min.......[25]

In Fig. 18 there is given for ready reference the flow through a 0.1-sq. in. orifice for different tank pressures in lb. per sq. in. gage.

The velocity of the jet issuing from the throat of a simple orifice is:

$$v = \sqrt{2g\,\frac{r}{r-1}\,P_1V_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}\right]} \text{ ft. per sec.} \ldots . [26]$$

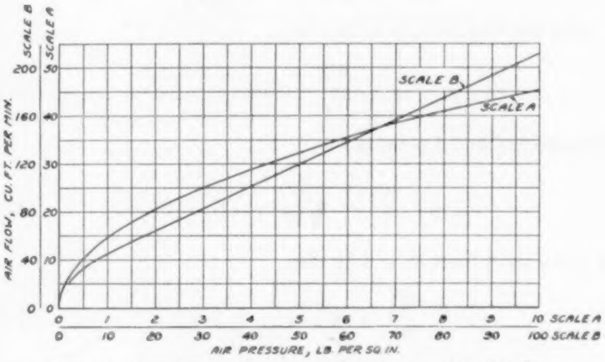

Fig. 18 Air Flow Through Orifice of 0.1 Sq. In.

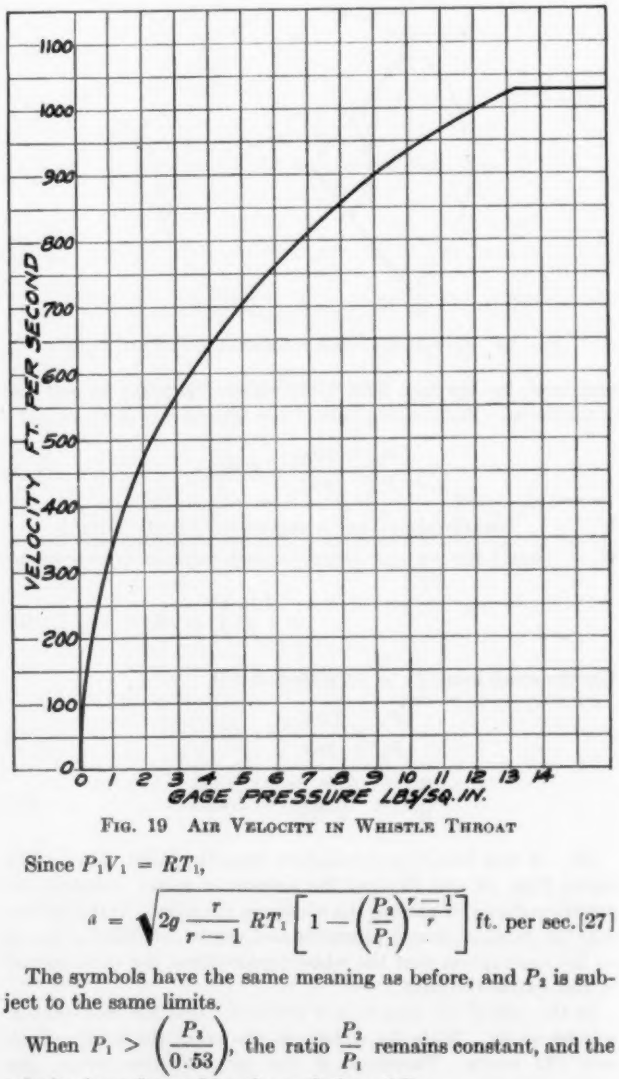

Fig. 19 Air Velocity in Whistle Throat

Since $P_1V_1 = RT_1$,

$$a = \sqrt{2g\,\frac{r}{r-1}\,RT_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}\right]} \text{ ft. per sec.} [27]$$

The symbols have the same meaning as before, and P_2 is subject to the same limits.

When $P_1 > \left(\frac{P_2}{0.53}\right)$, the ratio $\frac{P_2}{P_1}$ remains constant, and the velocity is unchanged by the variation of P_1.

With $R = 53.2$ and $T_1 = 528$ deg. fahr. abs., as before,

$$v = 2510\sqrt{1 - \left(\frac{P_2}{P_1}\right)^{0.287}} \text{ ft. per sec.} \ldots\ldots\ldots [28]$$

The calculated velocities from Equation [28] are given in Fig. 19.

The foregoing equations for velocity apply only to simple orifices with frictionless flow. With expanding nozzles of proper design, greater velocities than the maximum shown in Fig. 19 can be obtained.

Power for Isothermal and Adiabatic Compression and Power Available in Adiabatic Expansion

The values given in Table 5 and plotted in Fig. 20 were calculated from the following well-known relations:

The work per cycle of an ideal isothermal compressor is

$$w = P_3 v_3 \log_e \frac{P_1}{P_3} \text{ ft-lb.} \ldots\ldots\ldots\ldots\ldots [29]$$

v_3 = volume of intake per cycle in cu. ft.

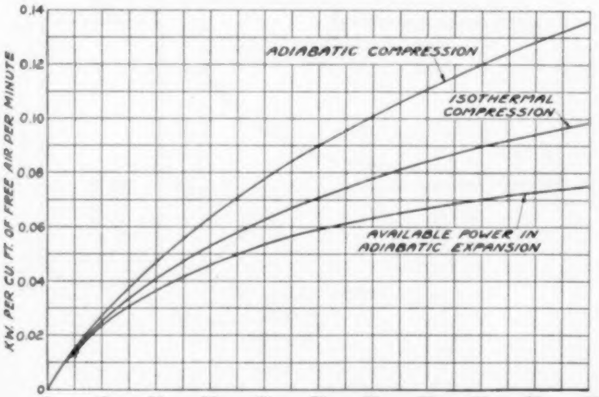

Fig. 20 Power Required for Compression and Available Power in Expansion of Air

Taking $P_3 = (14.7 \times 144)$ lb. per sq. ft., there is obtained the following expression for the power required to compress 1 cu. ft. of free air per minute:

$$\text{Kw.} = 0.110 \log_{10} \left(\frac{P_1}{P_3}\right) \ldots\ldots\ldots\ldots\ldots [30]$$

P_1 and P_3 are the tank and external pressures, respectively, as before.

The work per cycle of an ideal adiabatic compressor is:

$$w = \frac{r}{r-1} P_3 v_3 \left[\left(\frac{P_1}{P_3}\right)^{\frac{r-1}{r}} - 1\right] \text{ ft-lb.} \ldots\ldots\ldots [31]$$

The power required to compress 1 cu. ft. of free air per minute becomes:

$$\text{Kw.} = 0.166 \left[\left(\frac{P_1}{P_3}\right)^{0.287} - 1\right] \ldots\ldots\ldots\ldots [32]$$

TABLE 5

Gage pressure, lb. per sq. in.	Kilowatt for isothermal compression of 1 cu. ft. of free air per minute	Kilowatt for adiabatic compression of 1 cu. ft. of free air per minute	Kilowatt available in adiabatic expansion of 1 cu. ft. of free air per minute
0.1	0.00033	0.00033	0.00033
0.2	0.00065	0.00065	0.00065
0.5	0.00160	0.00162	0.00160
1.0	0.00316	0.00319	0.00313
2.0	0.00613	0.00624	0.00603
5.0	0.0139	0.0148	0.0136
10.0	0.0248	0.0272	0.0234
20.0	0.0412	0.0468	0.0365
40.0	0.0626	0.0778	0.0534
60.0	0.0775	0.101	0.0631
80.0	0.0895	0.120	0.0700
100.0	0.0985	0.136	0.0750

In computing the energy available in the expansion for 1 cu. ft. of free air, it has been assumed that the air in the tank cools to room temperature before being expanded.

If v_1 = volume of air at pressure P_1 and room temperature (T_1 or T_3) having the same weight as 1 cu. ft. of free air (pressure P_3), the work done in adiabatically expanding the volume v_1 to pressure P_3 is given by:

$$w = \frac{r}{r-1} P_1 v_1 \left[1 - \left(\frac{P_3}{P_1}\right)^{\frac{r-1}{r}}\right] \text{ ft-lb.} \ldots\ldots\ldots [33]$$

Since $T_1 = T_3$, $P_1 v_1 = P_3\,(1.00)$ by Boyle's law:

$$v_1 = \frac{P_3}{P_1}$$

Substituting this in [33], there is obtained:

$$w = \frac{r}{r-1} P_3 \left[1 - \left(\frac{P_3}{P_1}\right)^{\frac{r-1}{r}}\right] \text{ ft-lb.} \ldots\ldots\ldots [34]$$

It may be readily shown that the ratio of Equation [34] to Equation [31], where v_3 in [31] is 1 cu. ft., is given by

$$\left(\frac{P_1}{P_3}\right)^{\frac{r-1}{r}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [35]$$

The values in column 4 of Table 5 were accordingly obtained by multiplying column 3 by Equation [35].

Efficiency of 1-In. 3000-Cycle Whistle at Different Supply Pressures

The efficiency curve given in Fig. 21 is based on a curve showing the sound pressure output of the 1-in. 3000-cycle whistle at different gage pressures, which was taken in the early part of the development. The conviction is now reached that the absolute values of the sound pressures given by this early curve were wrong, but that the shape of the curve was correct. The mean spherical sound pressure (783 bars at 3 ft. with 50 lb. per sq. in. gage) of Fig. 10 was accordingly used to obtain a correction factor.

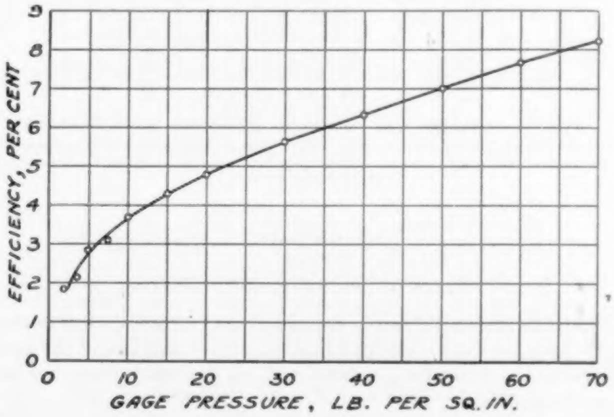

Fig. 21 Efficiency Curve of 1-In. 3000-Cycle Whistle for Various Gage Pressures

As previously stated, the author has consistently divided the sound pressures indicated by the condenser microphone, for frequencies in the vicinity of 3000 cycles, by 3 to allow for pressure doubling and cavity resonance. On this basis the efficiency of the 1-in. 3000-cycle whistle was 7 per cent at 50 lb. per sq. in. gage pressure. If no such allowance had been made, the efficiency would be nine times as high, or 63 per cent. This would bring the whistle up to approximately twice that of the Allan Cunningham horn.

Variation of Sound Pressure With Distance

The data plotted in Fig. 22 give the results of some outdoor sound-pressure measurements. The whistle was pointed vertically upward and was approximately 8 ft. above the ground. The point marked by the large circle at 3 ft. is the 90-deg. point from the booth measurements of Fig. 10. The average line drawn through the points shows that the sound pressure is inversely proportioned to the distance from the source.

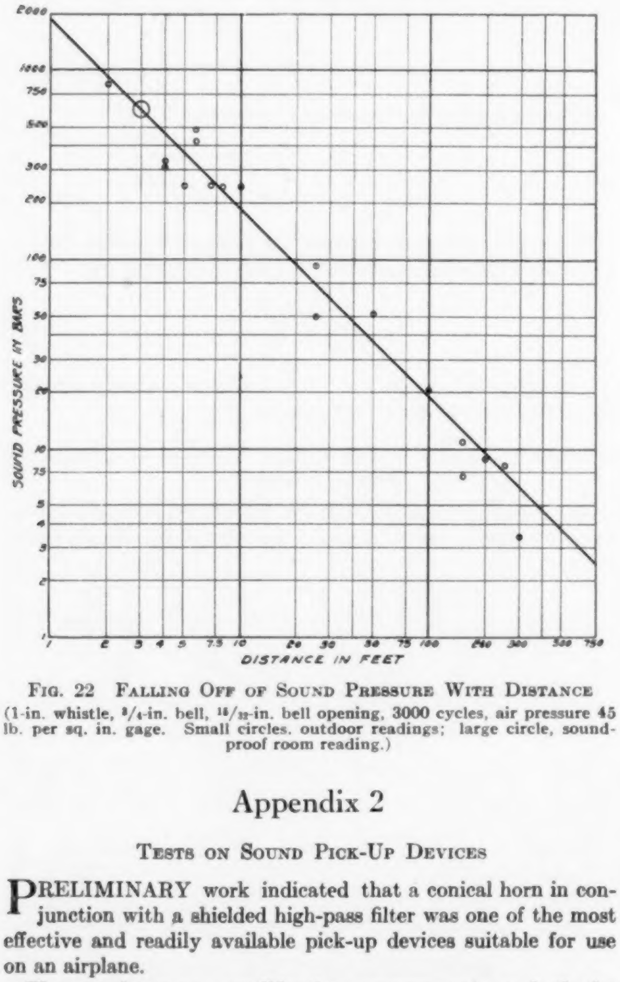

Fig. 22 Falling Off of Sound Pressure With Distance

(1-in. whistle, $^3/_4$-in. bell, $^{15}/_{32}$-in. bell opening, 3000 cycles, air pressure 45 lb. per sq. in. gage. Small circles, outdoor readings; large circle, soundproof room reading.)

Appendix 2

Tests on Sound Pick-Up Devices

PRELIMINARY work indicated that a conical horn in conjunction with a shielded high-pass filter was one of the most effective and readily available pick-up devices suitable for use on an airplane.

The sound-pressure amplification measurements on both the conical horn (*A*, Fig. 23) and the parabolic mirror (*G*, Fig. 23) as well as a 30-in. by 1-in. diameter tube, were made in a 17-ft. by 8-ft. by $9^1/_2$-ft. soundproof booth.

The apparatus set up for the conical horn is shown in Fig. 24. A similar set-up for the parabolic mirror is shown in Fig. 25.

The sound-pressure amplification measurements on the 30 in. long by 1 in. diameter tube were made with a set-up similar to that of Fig. 24, except that the horn was replaced by the tube.

The sound-pressure amplification factors of the horn, parabolic mirror, and 1 in. diameter by 30 in. long pipe were measured in the following manner: Constant sound pressure was assumed to be maintained at the mouth of the horn by holding constant sound pressure on a condenser microphone which was placed ahead of the pick-up device under test, as shown in Figs. 24 and 25. The output sound pressure of the pick-up combination under test was measured by another condenser microphone. This latter condenser microphone was fitted with a plug which left a clearance of approximately $^1/_{32}$ in. between the diaphragm and the plug, the sound being admitted to this cavity through a $^5/_{16}$ in. diameter tube. The voltage output of this condenser microphone was measured after passing through an electric wave analyzer which it is believed eliminated the effects of any harmonics present in the original sound. After running the test, the absolute value of the sound pressure in the plane of the pick-up device was determined by taking the measuring microphone out of its box, removing the plug, and placing it in the position previously occupied by the pick-up device under test.

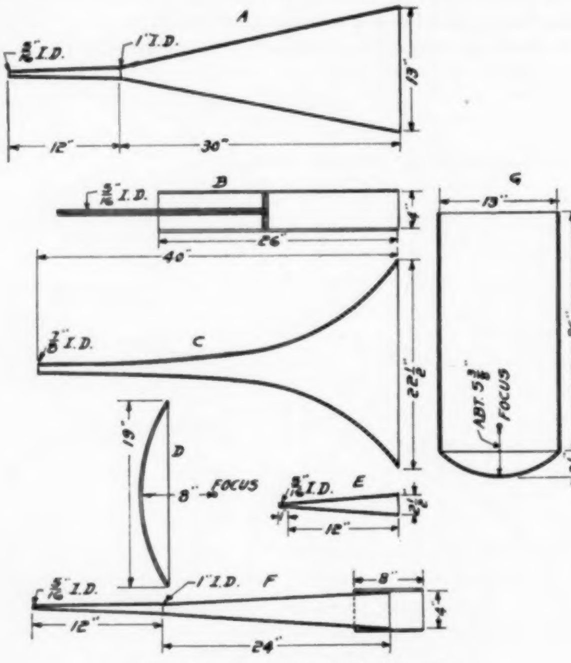

Fig. 23 Sound Pick-Up Devices

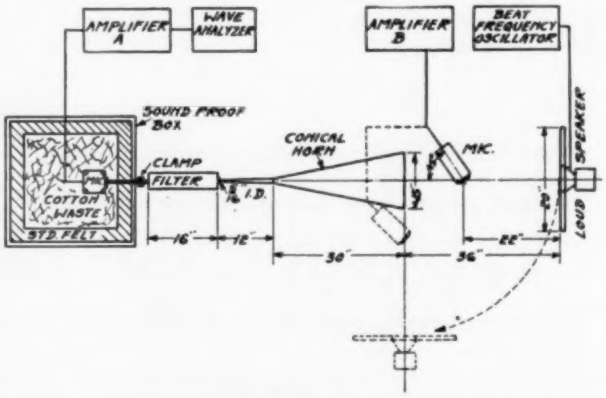

Fig. 24 Apparatus Set-Up for Sound-Pressure Measurements on High-Pass Filter and Conical Horn

The input sound pressures were corrected for pressure doubling and cavity resonance by the curve shown in Fig. 26, which is taken from the work of Ballentine.[11]

Fig. 27 shows the end-on characteristics of the horn with and

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[11] Stuart Ballentine, "Effect of Cavity Resonance on Frequency Response Characteristic of Condenser Microphone," Inst. Radio Engrs., vol. 18, p. 1206, July, 1930.

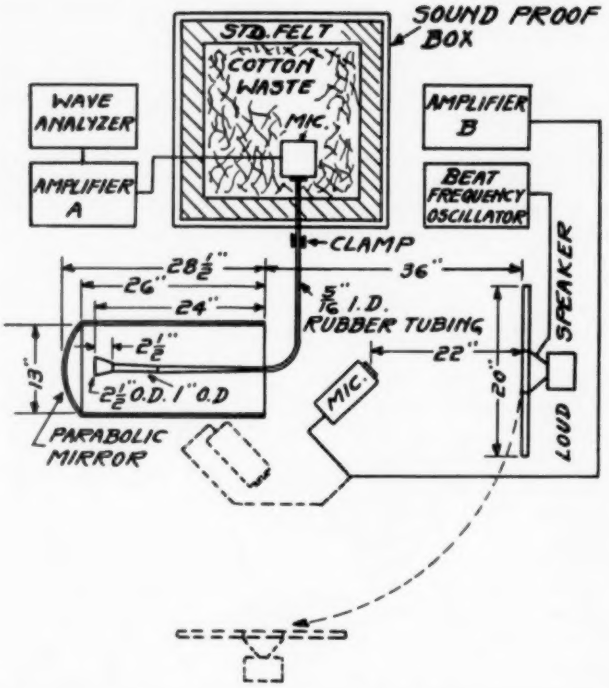
SOUND PROOF BOX
STD. FELT
COTTON WASTE
MIC.
WAVE ANALYZER
AMPLIFIER A
AMPLIFIER B
BEAT FREQUENCY OSCILLATOR
CLAMP
28½"
26"
24"
36"
5/16" I.D. RUBBER TUBING
2½"
2½" O.D. 1" O.D
13"
22"
20"
MIC.
LOUD SPEAKER
PARABOLIC MIRROR

FIG. 25 APPARATUS SET-UP FOR SOUND-PRESSURE MEASUREMENTS ON PARABOLIC MIRROR

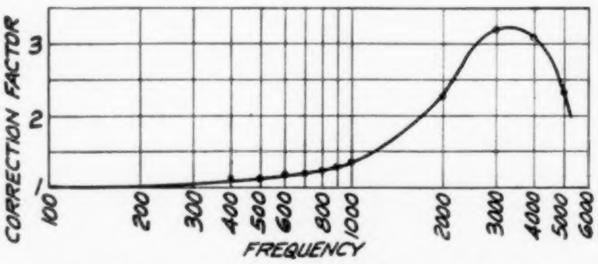

FIG. 26 CORRECTION CURVE FOR CONDENSER MICROPHONE

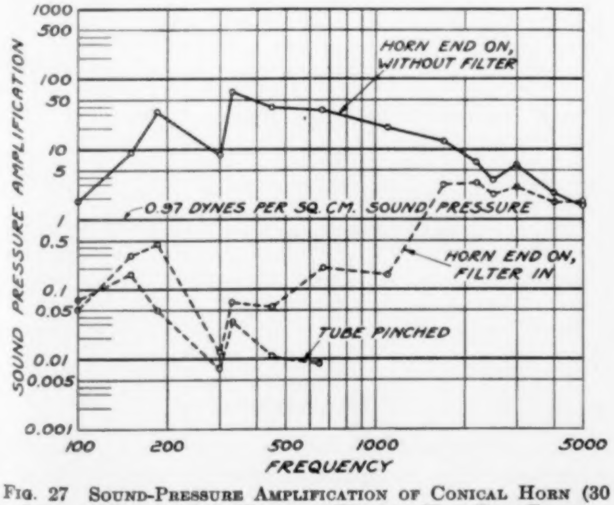

FIG. 27 SOUND-PRESSURE AMPLIFICATION OF CONICAL HORN (30 IN. BY 13 IN. BY 1 IN.), WITH AND WITHOUT HIGH-PASS FILTER

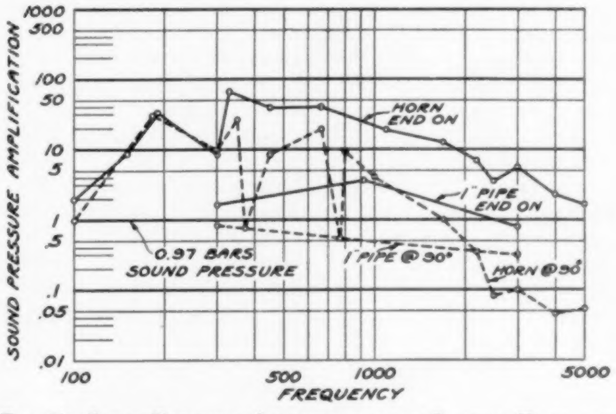

FIG. 28 SOUND-PRESSURE AMPLIFICATION OF CONICAL HORN AND 1-IN. DIAMETER PIPE, END ON AND SIDEWISE

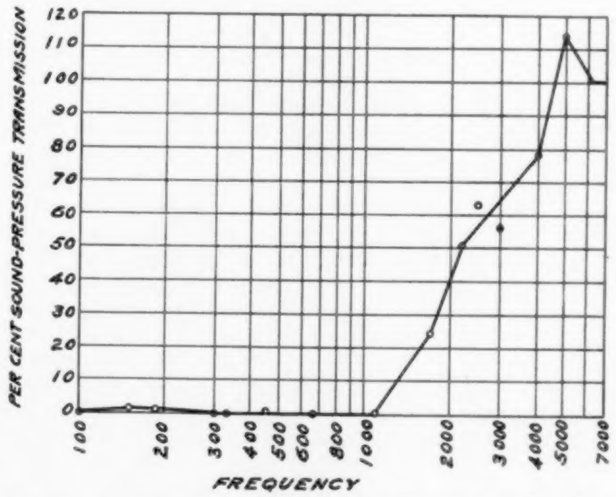

FIG. 29 SOUND-PRESSURE TRANSMISSION CURVE FOR HIGH-PASS FILTER

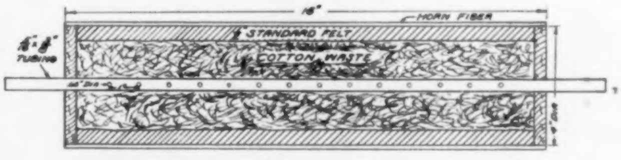
16"
HORN FIBER
STANDARD FELT
COTTON WASTE
TUBING

FIG. 30 SHIELDED HIGH-PASS ACOUSTIC FILTER

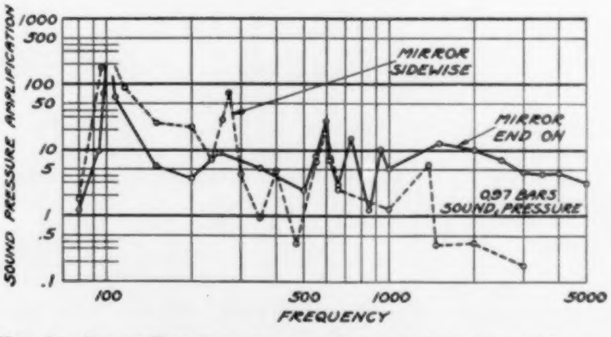

FIG. 31 SOUND-PRESSURE AMPLIFICATION OF PARABOLIC MIRROR, END-ON AND SIDEWISE

without the high-pass filter, and also the effect of mechanical sound transmission. It is seen from this figure that there is obtained approximately a threefold amplification at 3000 cycles for the horn and filter combination. In the low-frequency region the effects of horn resonance are eliminated, and there also is obtained a 10-to-1 reduction in the sound pressure by using the acoustic high-pass filter.

Fig. 28 shows the end-on and sidewise sound-pressure amplification for the large horn and 1 in. diameter pipe. It is seen from this that the horn is not particularly directive at the low frequencies, but gives a ratio of approximately 50 to 1 at 3000 cycles, and is therefore very effective in cutting down the high-frequency components of the engine noise which arrive directly from the engine.

Fig. 29 gives the sound-pressure transmission characteristics of the shielded acoustic high-pass filter, which is shown in Fig. 30. This curve is obtained from Fig. 27 as the ratio of sound pressure with filter to sound pressure without filter at various frequencies. The curve marked "Tube Pinched," which gives the mechanical transmission of sound, was subtracted from the one marked "Filter In" before the ratio was taken. This curve indicates only 65 per cent transmission at 3000 cycles, and it is evident therefore that the filter should be redesigned for a somewhat lower cutoff to obtain maximum 3000-cycle signal.

Fig. 31 shows the end-on and sidewise sound-pressure amplification of the parabolic mirror set-up of Fig. 25. The parabolic mirror was placed at the end of a 26 in. long by 12 in. diameter pipe with the hope of improving the sidewise directivity and to shield the small focal pick-up horn from the slipstream. It is evident from the figure that practically nothing is gained in low-frequency directivity by the large cylindrical shield. At 3000 cycles the tests indicate a sound amplification of approximately 5 to 1 and end-on to sidewise directivity ratio of 25 to 1. Dr. H. F. Olson and Dr. Irving Wolff in their recent paper on "Sound Concentrators for Microphones"[12] obtained a 2-to-1 ratio in favor of their parabolic concentrator at 3000 cycles as compared with their horn. In view of their results it is hoped at some future date to make further tests on parabolic devices.

Comparative listening tests using a 3000-cycle tone in the booth indicated that sound pressures at the ear from the various pick-up devices shown in Fig. 23 varied approximately as the diameters of the pick-up areas. These rough observations, using the ear also, indicated that the useful angle of pick-up for a 13 in. diameter by 30 in. long horn was approximately 10 deg. on either side of the axis.

Appendix 3

DIFFUSE AND SPECULAR REFLECTION AT THE GROUND

If the sound intensity at the small distance d near the source is I_D ergs per sq. cm. per sec., the intensity at the ground, which is at a distance D, will be

$$I_D = I_d\left(\frac{d}{D}\right)^2 \text{ ergs per sq. cm. per sec.} \quad [36]$$

In case it is desired to consider that the sound is assumed to be confined by the megaphone to a cone having a semi-angle α as shown in Fig. 32, the sound intensity over the circular patch on the ground will then be

$$I_D \cos\alpha = I_d\left(\frac{d}{D}\right)^2 \cos\alpha \quad [37]$$

The semi-angle (α = 10 deg.) is small, and one can therefore take $\cos\alpha = 1$. The total sound flux falling on the "illuminated" circle of area A will then be

$$J = I_d\left(\frac{d}{D}\right)^2 A = I_d\left(\frac{d}{D}\right)^2 \pi D^2 \tan^2\alpha \text{ ergs per sec.} \quad [38]$$

The reflected intensity on the axis right near the surface is equal to the incident intensity divided by π. The factor π takes care of the scattering accompanying diffuse reflection. The reflected intensity at a distance R on the axis will then be

$$I_R = K\,\frac{I_d\left(\frac{d}{D}\right)^2 A}{\pi R^2} \text{ ergs per sq. cm. per sec.} \quad [39]$$

where K is the coefficient of reflection of the surface.

To be exact, R in Equation [39] should be measured from the edge of the circle. A good approximation for the intensity off the axis, provided the distance R is large compared with the diameter of the "illuminated" circle, is obtained by multiplying Equation [39] by $\cos\beta$, where β is the angle between the axis and the line joining R with the center of the circle.

For the echo received back at the airplane, substitute $R = D$ and $A = \pi D^2 \tan^2\alpha$ in Equation [39], and obtain

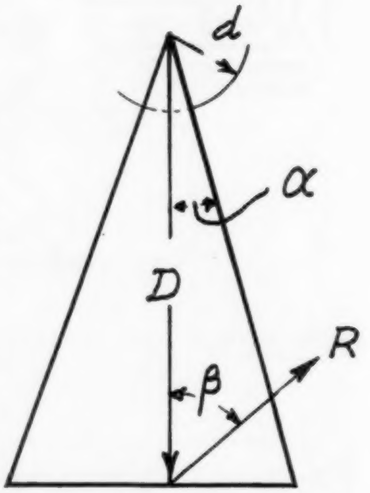

FIG. 32 DIFFUSE REFLECTION OF SOUND

$$I_R = \frac{KI_d d^2 \tan^2\alpha}{D^2} \text{ ergs per sq. cm. per sec.} \quad [40]$$

In the case of perfect specular reflection, there is obtained

$$I'_R = I_d\left(\frac{d}{2D}\right)^2 \text{ ergs per sq. cm. per sec.} \quad [41]$$

The ratio of [41] to [40] gives:

$$\frac{I'_R}{I_R} = \frac{1}{4K\tan^2\alpha} \quad [42]$$

For $K = 1$ and α = 10 deg., there is obtained

$$\frac{I'_R}{I_R} = 8 \quad [43]$$

This corresponds to a sound-pressure ratio of 2.8 to 1. If $K = 0.5$, the intensity ratio becomes approximately 16 and the pressure ratio 4 to 1. For $K = 0.25$, the intensity ratio becomes $7.95/0.25 = 32.8$ and the pressure ratio 5.75.

The author is indebted to Dr. F. A. Benford for the results obtained in the foregoing.

Discussion

WESLEY L. SMITH.[13] The effective range of this instrument is less than 1000 ft., so that its use seems to be limited to that of an aid to landing. While its weight is a trifle high, it no doubt can be consolidated with other instruments to form a very useful part of the landing aids that are needed. A combination of the radio landing beam, the sensitive aneroid barometer equipped with a barometric correction scale, and the sonic altimeter should provide all that is needed to land under conditions of no visibility.

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[12] *Journal of Acoustical Society of America*, vol. 1, p. 310, April, 1930.

[13] Chief Engineer, National Air Transport, Inc., Chicago, Ill.

It would seem possible to use the gas pressure provided by the sonic-altimeter equipment to drive the gyroscopes in the artificial horizon, the directional gyro, and the automatic pilot, and also to use this same gas pressure to operate the ice overshoes. If this could be done, part of the weight that is necessary for the sonic altimeter could also serve other purposes.

AUTHOR'S CLOSURE

Mr. Wesley Smith has correctly summed up the present status of the sonic altimeter. As stated in the text, the weight of the installation on the Army Douglas plane was 45 lb.; the latest design, of which three have been sold to the Army, has a total installed weight of 28 lb. Further reductions which are immediately possible will bring this down to less than 25 lb.

A recent flight test with the sonic altimeter on the *Los Angeles* demonstrated that good height determinations may be made from 3000 ft. and below.

The immediate applications of the sonic altimeter should be to lighter-than-air craft and to flying boats. In the case of airships the increased weight is amply compensated for by the increased safety should the ship get caught out in a fog. A similar situation exists in the case of flying boats, because water landings will be rendered practical using the sonic altimeter alone, without radio beacons or other special landing aids.

AER-54-10

The Tail Spinning of Aircraft

By GILBERT G. BUDWIG,[1] WASHINGTON, D. C.

Tail spinning of aircraft is a subject of interest to manufacturers and operators alike. There is much to be learned in the application of corrective measures. The problem was investigated by the Department of Commerce, and it was determined that there were four factors which in most cases will determine if an airplane will have satisfactory spinning characteristics: It should be stable about all three axes; the center of gravity should be well forward with respect to the center of pressure; the wing loading should be kept below a certain maximum figure depending mostly upon the characteristic of the particular airfoil; the weights of airplane and cargo should be grouped as closely as possible around the center of gravity. One or more of these characteristics were involved in every airplane that spun unsatisfactorily, and the correction of this factor brought about the desired result.

THE tail spinning of aircraft, together with the structural characteristics relating thereto, is a subject of vital interest to manufacturers and operators alike. Generally speaking, there is yet much to learn about spinning, and this is particularly true in the application of corrective measures to aircraft which inherently have undesirable spinning characteristics.

The problem was forcibly brought to the attention of the Department of Commerce during the latter part of 1928. It became evident from our accident statistics, which were by that time fairly comprehensive, that in certain makes of aircraft the proportion of tail-spin accidents to the total accidents was inconsistent with other makes of aircraft. In light of the fact that four or five, or perhaps more, airplanes were crashing due to tail spins, with a frequency far exceeding other makes, we were led to believe that the cause to a great extent must have been due to peculiar flight characteristics in the airplanes involved. We therefore set out to determine what these characteristics were and how best the corrective measures could be instituted.

First, we spin-tested the airplanes which were apparently causing the trouble. Some rather interesting results were obtained. One of these airplanes, when put into a spin, would continue to spin indefinitely even though the controls were moved back to neutral, recovery being effected only after the controls had been reversed to the extreme position, and then the recovery followed an additional four turns. Another airplane, when put into a spin, developed so much back pressure on the stick that it was necessary to use both hands to force it to neutral, and recovery was possible then only after three additional turns.

Another airplane, when spun to the left, would continue spinning to the left even though extreme right rudder was applied, recovery being effected by reversing the controls and awaiting approximately four additional turns. We also found in a number of cases that, when the controls were moved to the extreme position to commence the spin, they would, if released, remain there, and a considerable amount of physical force would be necessary to return them to neutral.

When this condition was discovered, it became apparent that some test procedure would necessarily be imperative to eliminate the existing hazard. In light of the fact that all of the airplanes tested which had undesirable spinning characteristics were longitudinally unstable, it became obvious that a stability and spin requirement should be made a part of the Airworthiness Requirements.

After considerable study, such a minimum standard was determined and put into effect about Jan. 1, 1929. This standard provided for a determination of longitudinal, lateral, and directional stability and for a measurement of spinning characteristics. It seemed to the department at that time that these standards were not only absolutely necessary, but were also practicable and could well be applied without materially adding to the cost or decreasing the performance of airplanes. The procedure involved was comparatively simple.

As previously, manufacturers continued to submit complete drawings, stress analysis, and other technical data in support of their application for an Approved Type Certificate for a new design of airplane. After these technical data were examined and were adjudged satisfactory, an inspector for the Department of Commerce was assigned to flight-test the airplane in question. These inspectors were carefully chosen for their ability as pilots and for their comprehensive knowledge of airplanes, with particular reference to flight characteristics. They were the best men available for that type of work, and their objective was to determine the stability and spinning characteristics of the airplanes, as well as the workmanship, materials, and methods involved in their fabrication.

A brief outline of the flight-test procedure which was then promulgated and which is still effective follows:

The airplane is tested for balance and for stability longitudinally, laterally, and directionally, and if found satisfactory, the spin test is conducted. The airplane is first spin-tested by the factory test pilot, and the results are confirmed by our engineering inspector on the succeeding tests.

This procedure has since been used continuously in checking all airplanes in certain weight classifications which have been submitted for approval, and this has yielded two years of very valuable and interesting experience. During this time, the principles underlying inherent stability have become generally recognized and well enough understood so that it is a comparatively simple matter to predict the stability of an airplane. The question of spinning is, however, considerably more complicated, and no entirely satisfactory method has yet been evolved

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[1] Director of Air Regulation, Department of Commerce. Born Nov. 17, 1895, at Cleveland, Ohio. Connected with commercial aviation since 1916 and employed during the World War as civilian instructor stationed at Chanute Field, Rantoul, Ill.; Kelly Field, San Antonio, Texas; Rockwell Field, San Diego, and March Field, Riverside, Calif. Flew the air mail immediately after the war, and then did two years of barnstorming. Later, did a substantial amount of free-lance test piloting, and owned and operated the Budwig Radio Company at Glendale, Calif., for approximately five years. Became affiliated with the Aeronautics Branch of the Department of Commerce in June, 1927, as aeronautical inspector; appointed chief of the inspection service in May, 1928; appointed director of air regulation on Oct. 4, 1929.

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

for theoretically determining the spinning characteristics of an airplane. A tremendous amount of work has been done on this subject, actively stimulated and made necessary by the Department of Commerce regulations pertaining thereto. Our own experience allows us to state certain general principles which will be of interest even though they are not quantitative and not absolute. The first of these general principles is that bad spinning characteristics are not confined to any one particular type of airplane. We have found them in conventional biplanes, low-wing monoplanes, and high-wing monoplanes.

Another common experience is that many airplanes will develop two or more different types of spins, depending upon the attitude from which the spin is started. Usually we have found that such airplanes have satisfactory characteristics if they are spun in the usual manner from a stalled position, but that they will develop an uncontrollable spin if it is started from some less conventional attitude. One rather serious and important fact in this connection is that biplanes which have entirely satisfactory spinning characteristics when properly rigged have a tendency to develop dangerous spins when allowed to get out of rig even by a small amount. Since many operators of these smaller airplanes are not too careful about maintaining their airplanes in exactly their original rigging condition, we are frequently encountering spinning difficulties of this nature.

Perhaps a brief outline of some of our experiences in conducting these tests will be of interest. One airplane of the high-wing, two-place type, when tested, disclosed that at both cruising speed and full power the plane was unstable longitudinally and directionally under all conditions. The left spin, with light load (half a tank of gasoline, pilot, and parachute), was irregular in that the third, fifth, seventh, and ninth turns were flat. The ship would whip into a steep spin and gradually flatten out until the end of the third turn; the time-elapse in the third turn was 3 seconds; time of the second turn was $1^{1}/_{2}$ seconds. After the third turn, the ship would again whip into the steep spin and gradually flatten out until it would whip off at the end of the fifth turn. The stabilizer position varied from neutral to maximum negative.

With full load, the ship spun about the same, but increased control forces were necessary to force the stick forward. If power was used in the flat turn, the ship would recover immediately; if used in the steep turn, the spin tightened and became flat, making recovery impossible unless the motor was cut.

To correct these spin characteristics, the motor was moved forward 4 in., the wing raised 2 in. and moved back 2 in., and the lift struts were replaced with streamline struts. The ship was again tested and proved satisfactory.

One of the unaccountable spinning characteristics encountered by one of our engineering inspectors in spin-testing was with a three-place low-wing monoplane. One of these airplanes had been reported to have come down in a flat spin with a 245-lb. pilot and a parachute weighing some 20 lb. in the rear cockpit. This case was taken up with the manufacturer's test pilot, who reported that the ship would flat-spin under adverse loading conditions, but would recover if the controls were released. He offered to demonstrate the fact. The same type of aircraft was loaded with 3 gal. of gas ahead of the center of gravity, with the pilot, the parachute, and the sandbags, totaling 215 lb., in the rear cockpit. In putting this plane into a left spin, the first turn appeared normal, but gradually the nose came up during the next two turns until the plane was in an attitude of a flat spin. It was held in this flat-spinning attitude for three turns, after which both the rudder and the stick controls were released. The plane seemed to be swinging considerably ahead and slightly to the left in the left spin of the longitudinal axis of the plane. Considerable centrifugal pressure was noted, tending to throw the pilot against the back of the seat, but not sufficient to prevent him from leaving the ship if necessary. After eight turns the stick control was released and the feet were removed from the rudder. The stick traveled to the inside rear corner, where it remained for about one turn. The nose then started down with the stick moving forward at the same time. The airplane recovered from this spin in three and one-half or four turns after the release of the controls. The spin was then repeated to the right with identical results.

The foregoing case is one of a very few instances of a controllable flat-spin condition, especially since the plane recovered without forcing the controls.

Another very interesting case experienced by one of the inspectors was with a five-place cabin-type airplane. Before this test was started, the plane was loaded with weight equivalent to four passengers. Three attempts were made to spin the plane to the right, but each time the plane would make about a three-fourth turn of a spin and would then spiral out. A left spin was then attempted. This was started by stalling the plane with the nose up at an angle of 45 deg. and by banking to the left at an angle of 50 deg. This was done at an altitude of about 5500 ft. As the plane started falling to the left, the left rudder was kicked sharply and the stick control pulled back to the right in the rearmost position. The plane fell into a spin immediately, and before it had completed one full turn there developed a very severe back pressure on the stick. The rudder was immediately reversed, and an attempt was made to push the stick forward, first with one hand, then with both. It was only possible to get the stick forward about 3 in. back of neutral position, the inspector having insufficient strength to move it any farther. The plane immediately assumed a flat-spinning attitude. After four turns had been made in this position with every attempt made to reverse the controls, the throttle was opened. This caused the nose to rise to a higher position, which also increased the speed of rotation. The motor was immediately throttled, and opposite aileron and rudder control with stick forward were attempted, this being done for six turns with no apparent effect. Then forward stick and opposite rudder with no aileron was attempted for six turns, with no result. The speed of rotation and attitude of the plane remained unchanged. Then several different combinations of controls were tried, none of which had the slightest effect. The plane had then reached an altitude of 2500 ft., and the inspector unfastened his belt to determine whether there were any centrifugal forces which would prohibit him from jumping. None were experienced, so he decided to continue his attempt to regain control of the airplane. At 1100 ft. the airplane for the first time broke from its rhythmic slow turning and appeared to do one-half turn of a slow normal spin, the nose dropping almost to a normal angle. However, reaching this position, it immediately swung back to the same even, slow flat spin. At that time the inspector was using right rudder and forward stick so severely that he unintentionally applied slight left aileron. This apparently released some of the back pressure on the stick, and it advanced slightly forward. He then tried hard left aileron, forward stick, and right rudder. With the controls in this position, the airplane slightly banked to the left to about a 45-deg. angle. The nose then fell slowly, which automatically released the back pressure on the stick, letting it gradually go forward. From this position, the airplane went into a left bank at 800 ft., and was then brought to normal flying attitude. The peculiar spinning characteristic of this airplane was apparently due to the excessive load placed behind the center of gravity and from improper rigging. It completed 28 turns before recovery was accomplished.

It would be possible to continue, almost indefinitely, outlining in detail the experiences encountered in the spin-testing of aircraft. Space, however, prohibits this, and no doubt the foregoing will, to a measure at least, outline the interesting problems involved.

It has been our experience that in all of these types there are four very important factors which will, in most cases, determine whether or not an airplane will have satisfactory spinning characteristics. These four factors are as follows: (*a*) It is necessary that the airplane be stable about all three axes; (*b*) it is necessary that the center of gravity be well forward with respect to the center of pressure, thus producing a nose-heavy moment which is balanced by a down-load on the stabilizer; (*c*) the wing loading should be kept below a certain maximum figure which depends, among other things, upon the characteristics of the particular airfoil; (*d*) the weights of the airplane and its cargo should be grouped as closely as possible around the center of gravity of the airplane.

It has been found in general that one or more of these factors were involved in the case of every airplane which spun unsatisfactorily and that the correction of this particular factor brought about the desired result. Even this statement, however, is not absolute, for in a few cases in which all of these factors were apparently satisfactory, the airplane still continued to spin unsatisfactorily. There are air forces effective in these cases which we do not as yet understand and which will require a great amount of research to thoroughly comprehend.

It will be seen from this rather brief summary that the problem is comprehensive, and while much has been accomplished, there still remains the necessity for further investigation. Perhaps our most valuable contribution in this connection is the impetus which we have given to intensive spin study by focusing a great deal of attention on that particular maneuver. So far as the theory of flight is concerned, additional research work will be of tremendous assistance.

In an effort to determine the actual value of spin-testing from the standpoint of safety, we have made a general study of spinning accidents for the entire period over which we have any record. In addition to this, we have made a detailed study of the year 1929, believing that year to be of particular interest in view of the fact that it was the first year during which spin tests were conducted.

The following table lists the important facts for this four-year period covered by our records. The percentage figures are determined by dividing the total number of spin crashes for a given period by the total number of airplanes which our records indicate were flying at that time.

Date	Number of airplanes	Total spin crashes	Percentage
All of 1927	2759	96	3.5
First half 1928	4284	127	3.0
Second half 1928	5654	186	3.3
First half 1929	7781	149	1.9
Second half 1929	9922	206	2.1
First half 1930	9773	193	2.0
Second half 1930	9863	196	2.0

It will be noted that during the first half of 1929 the percentage of spin accidents dropped very sharply and that from then to the present time the percentage has remained approximately the same. The decrease in accidents amounts to 1.4 per cent for each half year. Since there were some 10,000 airplanes in existence during the last half of 1929 and all during 1930, it is conservative to say that 140 accidents were prevented each half year, or 280 accidents per year, by the introduction of the spin-test procedure. It is reasonable to suppose that the 2 per cent figure indicated will decrease considerably as soon as the older types of airplanes pass out of existence.

The detailed study of the year 1929 shows even more striking results. The accident reports which we have charge 236 crashes during that year to spins. Of these 236 crashes during the entire year, only 11 occurred in licensed airplanes which had been spin-tested by the department. Of these 11 there were only 6 which could even remotely be charged to the airplane, because 5 of them were caused by incompetent pilots or acrobatics at low altitudes. Thus we arrive at the very remarkable conclusion that only 2$^1/_2$ per cent of all the spin accidents which occurred during 1929 were attributable to airplanes spin-tested by Department of Commerce inspectors. If you compare these 6 accidents with the fact that there were 9922 airplanes in existence at the conclusion of 1929, and recall at the same time that most of these so-called spin crashes were the result of stalled turns at low altitudes, you will realize that the department's flight-test procedure was very effective in preventing fatalities during the year.

Of the total of 236 spin crashes, 39 per cent were unlicensed or war-surplus airplanes, and 56$^1/_2$ per cent were licensed airplanes which had not been spin-tested before approval.

A significant fact is that these accidents cannot be charged against any small number of manufacturers nor any particular type of airplane. There were 10 airplane manufacturers whose airplanes were involved in three or more spin accidents during each half year of 1929, in addition to a number of others who had less than three accidents.

The foregoing seems to indicate that no matter how much attention is paid to the design and the manufacture of aircraft, occasionally undesirable spinning characteristics will develop and apparently for reasons at present but little understood.

Our experience has forced us to the conclusion that there is a definite scarcity of good test pilots in this country, and as a result, it is also quite noticeable that many airplanes are insufficiently test-flown and that many undesirable characteristics other than spinning could be corrected to the ultimate advantage of every one concerned. It is becoming increasingly more significant that merely because a manufacturer will meet our minimum requirements, is no criterion that the product is salable. Indications are that airplane manufacturers will shortly face the necessity of manufacturing a product which will more readily serve the purpose for which it was intended, rather than to meet any minimum requirement which the Department of Commerce may establish. The economic problem involved in the utilization of aircraft will necessitate the manufacture of a product which is the best that modern design and construction will permit. It is becoming increasingly evident that aircraft falling below the highest possible standard will, due to competition, be forced into the background.

AER-54-11

Water-Recovery Apparatus for Airships

By C. P. BURGESS,[1] WASHINGTON, D. C.

In order to operate efficiently, an airship must maintain an approximate equilibrium of its weight and buoyancy. Since the former is progressively reduced when liquid fuel is burned, some other means than valving the costly lifting gas, helium, must be employed to compensate for the weight of the fuel consumed. The author states five alternative means, and then briefly outlines the work done in developing one of them—recovering water for ballast from the engine exhaust, following which he sets forth the theory of such water recovery, accompanied by illustrative examples. The desirable features of a water-ballast collector are then dealt with, and the paper concludes with brief notes on the principles of design and construction of the apparatus and on recent developments in ballast collectors.

THE gross weight of an airship burning liquid fuel, and having no means of acquiring ballast during flight, is progressively reduced by the weight of the fuel consumed. In order to fly efficiently, and to make landings, an airship must be maintained in an approximate equilibrium of weight and buoyancy. Prior to the use of helium in American airships, it was the custom to maintain static equilibrium by valving the lifting gas to the outside atmosphere as required to compensate for the progressive diminution of the load carried. The lifting gas, hydrogen, being comparatively cheap, this method of operation was practicable although not entirely satisfactory. The introduction of helium, which was at first very expensive and obtainable only in severely limited quantities, made it imperative to find some means of compensating for the weight of fuel consumed without valving helium.

Alternative Means of Maintaining Equilibrium

The possible methods of maintaining static equilibrium without loss of lifting gas may be classified as follows:

a Condensing and recovering from the exhaust gases the water formed by combustion of the hydrogen contained in the fuel

b Taking water from the atmosphere by means of a hygroscopic material such as calcium oxide

c Lifting water from the sea or other bodies of water on the surface of the earth

d Burning a gaseous fuel of approximately atmospheric density, instead of liquid fuel

e Burning hydrogen in combination with liquid fuel in the proper proportions to maintain equilibrium of weight and buoyancy.

All of these methods have actually been tried in airships, although not always in conjunction with helium as the lifting gas.

A perforated container filled with calcium hydroxide was carried on some of the early flights of the U.S.S. *Shenandoah*. Its weight was found to increase about 50 per cent by moisture absorbed from the atmosphere when flying on days of high humidity. This method of adding ballast during flight is impracticable because of the large initial load of hygroscopic material which must be carried, the slow rate of absorption of water in dry weather, and the corrosive nature of the hydroxide.

During the World War, small quantities of water ballast were sometimes picked up by non-rigid airships operating at sea by the simple expedient of lowering a canvas bucket over the side of the car. This procedure is obviously of no practicable value except on very small airships, intended for carrying not more than three or four persons. Shortly after the war, some experiments were undertaken by the U. S. Navy and also by the Goodyear Tire and Rubber Company in picking up water ballast through a trailing hose. Two different methods were tried for lifting the water to the airship. The Navy depended upon the velocity head resulting from towing a bell-mouth hose through the water; while the Goodyear Company had a more elaborate apparatus using a water turbine driven by towing through the water, and in turn driving a rotary pump which forced water through the hose into the car of the airship. These experiments demonstrated the possibility of picking up water in small airships; but it would be impracticable to bring an airship of any considerable size sufficiently close to the surface to carry out such an operation.

Gaseous fuel of approximately atmospheric density is used with great success on the airship *Graf Zeppelin*. Hydrogen is used as the lifting gas on that airship; and it is doubtful if there would be any advantage in using helium for inflating an airship exposed to the fire risk resulting from carrying a combustible gas in separate cells. If the fuel gas could be carried in the inner portions of double cells, completely surrounded by helium, the combined use of fuel gas and helium might be justified; but the practical difficulties of constructing and handling such double cells are very great. At the present time the U. S. Navy is constructing an experimental non-rigid airship having fuel gas contained in a balloonet surrounded by helium.

The use of hydrogen as a lifting gas to be burned or valved to maintain the equilibrium in conjunction with helium is open to the same objection as fuel gas in that it largely nullifies the safety advantage of helium, unless it can be entirely surrounded by that inert gas; and even then the fire hazard might be considerable.

Hydrogen is not particularly suitable as a fuel gas, but it has been used for that purpose. In 1919, one engine of a U. S. Navy non-rigid airship was equipped with a hydrogen mixing valve, as well as a gasoline carburetor. This airship made a successful

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[1] Bureau of Aeronautics, Navy Department. Charles P. Burgess was born in Boston, Mass., April 9, 1888. He comes from a noted family of naval architects. He studied naval architecture at the University of Glasgow, Scotland, graduating in 1910. He entered the U. S. Navy Department during the war, and specialized in airship design, which at that time was a new art in the United States, almost without scientific basis. Mr. Burgess applied the older science of naval architecture to building up a new science of airship design. His book on "Airship Design" is recognized as the standard work on its subject. His experience has not been confined to the theoretical side of his profession. He was on the *Shenandoah* when she broke away from her mooring mast in a winter gale, and he has flown across the ocean on the *Graf Zeppelin*.

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

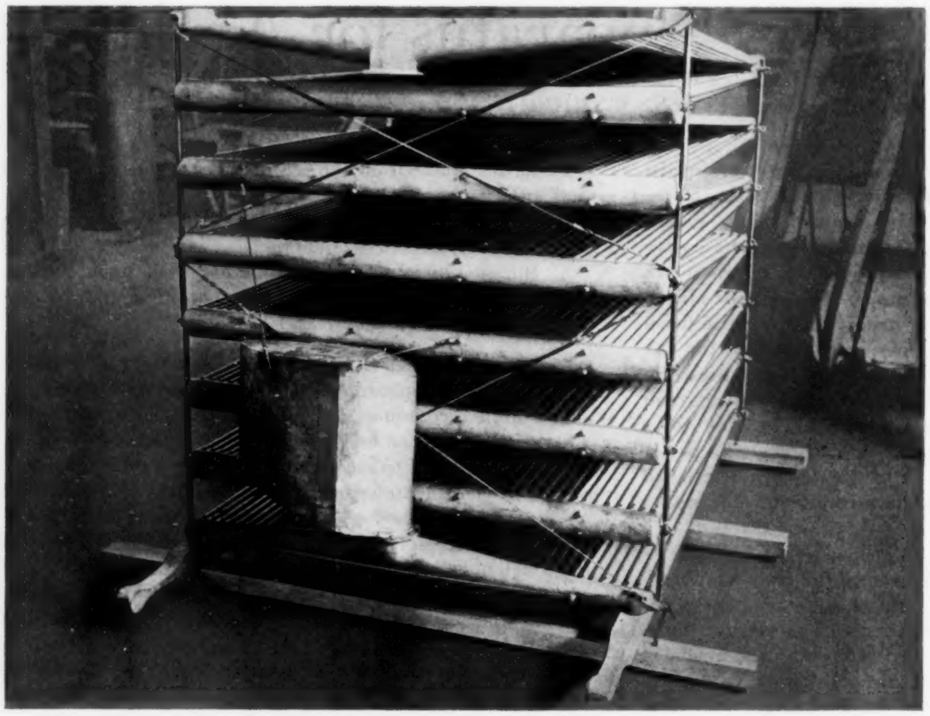

Fig. 1 Water-Ballast Collector for U.S.S. "Shenandoah"

flight from New York to Newfoundland, burning gasoline and hydrogen alternately.

Finally there is the method of maintaining equilibrium by recovery of water from the engine exhaust. When 100 lb. of ordinary aviation gasoline is burned, about 135 lb. of water is produced; and of this 135 lb., eight-ninths, or 120 lb., consists of oxygen taken from the air, and only 15 lb. is hydrogen contained in the original 100 lb. of fuel. Besides its convenience for maintaining equilibrium without loss of buoyant gas, recovered water ballast has other important advantages. The most valuable one is the great gain in safety resulting from having a large amount of ballast available to drop in case of loss of gas from damage to the cells when the ship is far from her base or destination. The extra ballast may also prevent disaster when landing; and it permits fuel or other loads to be taken aboard the airship at an intermediate or emergency landing in exchange for ballast, without any addition of buoyant gas, which might not be readily obtainable.

A disadvantage of the exclusive use of water recovery for maintaining static equilibrium is that the maximum altitude which can be reached on a voyage, without valving helium, is limited by the percentage fullness of the gas cells at the start of the voyage. In order to overcome this difficulty, it has been proposed to use small fuel-gas cells which would be filled at the start of a long voyage and would occupy the space which must be provided to allow for the expansion of the helium in order to rise to any desired altitude. The fuel gas would first be consumed without recovery of ballast; and after that, the liquid fuel would be used in conjunction with the recovery apparatus.

History of Water Recovery

The first known attempt to provide a water-ballast-recovery apparatus was on the British rigid airship *Mayfly*, constructed in 1910. As this punningly named airship did not fly, the apparatus was never put to a practical test. It consisted of about 400 ft. of thin metal pipe led from the exhaust manifold aft along the keel and then forward again to the engine car, where it was fitted with a water separator.

The English aviation magazine *Flight* for June 10, 1911, published a photograph and a short description of "The Zimmerman Ballast Gainer for Airships." This was a rather compact apparatus composed of flattened tubes with fins. It was constructed for a 40-hp. engine; it weighed 97 lb., and is said to have recovered water equal to 90 per cent of the weight of the fuel consumed. So far as is known, the apparatus was never given a practical test on an airship.

U. S. Patent No. 1,071,180, taken out by A. A. Remington, of Birmingham, England, in 1913, is for a water-recovery apparatus in which the exhaust gases are passed through a coiled pipe, cooled by the air flow. The claims for the patent are sweeping and fundamental; but apparently nothing came of it.

In 1917, an experimental water-recovery apparatus was constructed and tested at Kingsnorth, Scotland. The apparatus consisted of a honeycomb radiator of 1-in. tubes, 10 in. long, arranged to take the exhaust gas in the spaces between the tubes. It is said that the recovery of water was satisfactory while the tubes were clean, but decreased seriously after ten hours' run.

At the end of the war, the prospect of adopting helium for the airships of the U. S. Navy made the development of water-recovery apparatus essential. Experiments were carried out at the Washington Navy Yard, using the principle of condensation by injecting water into the exhaust, and cooling the mixture of injected and recovered water in an ordinary radiator. The weight of the apparatus was prohibitive; and little was accomplished by the experiments except to show what not to do. The trouble with cooling the gas by the injection of water is that all the heat taken from the gas must be transferred from the water to the atmosphere at a very low temperature gradient, so that there is no ultimate advantage in the rapid and efficient initial transfer of the heat from the gas to the injected water. Furthermore, deposits of soot and oil in the water-cooling radiators are much more destructive of efficiency than similar deposits in the comparatively large tubes of an apparatus designed for direct transfer of heat from the gas to the atmosphere.

The next step in the development of water recovery was made by the U. S. Army Air Service, based on the results of experiments at the Bureau of Standards. An apparatus was constructed in which the exhaust gases passed through long aluminum tubes, 1 in. in diameter, parallel to the outside air flow. It was installed in the non-rigid airship *D-3* in 1923. The apparatus recovered water reasonably well, but was structurally weak owing to the great length of the tubes.

The Air Service did not continue its work; and the Navy again took up the problem, employing the engineer-physicist who had developed the partially successful apparatus used on the *D-3*. The result was the construction for the U.S.S. *Shenandoah* of a compact apparatus in which each unit consisted of

45 parallel rows of 1-in.-diameter aluminum tubes arranged in ten banks connected by cast aluminum headers, as shown in Fig. 1. This was the first really successful water-recovery apparatus. All engines of both the *Shenandoah* and the *Los Angeles* were ultimately equipped with similar units. The weight was approximately one pound per horsepower, and in average weather conditions, water equal in weight to about 105 per cent of the weight of fuel consumed could be recovered from it.

ATTEMPTED IMPROVEMENTS

In spite of the generally satisfactory performance of the water recovery apparatus developed for the U.S.S. *Shenandoah*, and subsequently installed also on the U.S.S. *Los Angeles*, it is open to the grave objection of having very high air resistance, amounting to nearly 10 per cent of the total resistance of the airship. It is also difficult to remove soot deposited by the exhaust gases inside the tubes; and considerable trouble has been experienced from freezing of the water in the apparatus in very cold weather. The first attempt to reduce the air resistance was the construction of a two-stage apparatus in which the first stage consisted of a nest of duralumin tubes, parallel to the air flow, for cooling the gas to near the initial condensation temperature of about 53 deg. cent., and a second stage, or condensing element, which was virtually a large, flat box constructed of doped outer-cover cloth stretched on a duralumin frame, and occupying four panels of the surface of the hull. Experimental installations of this type were tried on both the *Shenandoah* and the *Los Angeles*. Neither experiment was successful; the amount of water recovered was disappointing; gas leaked badly into the ship; and the pulsations of the exhaust against the fabric produced a terrific drumming noise.

A water-recovery apparatus known as Mark III was tried unsuccessfully on the U.S.S. *Los Angeles*. Tubes with their axes at right angles to the air flow were used for the initial cooling of the gases down to near the condensation temperature; and an element constructed of corrugated plates was used for the remainder of the cooling. The plates cracked on trial after less than one hour's operation. In spite of this disappointing result, it is possible that failure was due to insufficient support of the apparatus against vibratory forces, and the use of corrugations of too great pitch and too long radius of curvature, rather than to a mistaken principle.

FINS FOR COOLING SURFACES

Fins on the air side of radiators designed for extracting heat from liquids greatly increase the efficiency of such apparatus; and it has been proposed to use fins in a similar manner for water recovery on airships. The objection to this proposal is that the problem of transferring heat from gas to air is quite different from transferring it from a liquid to air or other gas. Heat flows with comparative ease from a liquid to the walls of a metal container; but a much greater resistance is encountered in the transfer between gases and metal surfaces. It follows that in cooling a liquid, much larger surface areas are required on the air side than on the liquid side. Fins on the air side of the container afford a ready means of supplying this difference of area. On the other hand, the transfer of heat between air and gases requires equally large areas on both the gas and air side. It is possible to provide these areas in the form of fins; but the heat would be required to flow a much greater distance through metal than in the case of equal areas of thin plates or tube walls directly in contact with gas on one side and air on the other, so that for a given area, the direct method is more effective than fins. An exception occurs in that part of the apparatus which is close to the engine. In order to avoid damaging the metal by high temperatures, it is desirable to expose more area to the air than to the gas, and this additional area is most conveniently provided by fins.

COOLING BY EXHAUST TURBINES

It has been proposed to cool the exhaust gases and, at the same time, make use of their energy by passing them through an exhaust turbine designed to convert their heat into mechanical energy. The scheme is not very promising, because the mechanical equivalent of the heat to be removed from the gas considerably exceeds the total energy output of the main engines; and it has hitherto been impossible to construct an exhaust turbine which, without causing excessive back pressure, would deliver more than about 10 per cent of the horsepower of the engine to which it is attached.

A possibility along this line is to use an exhaust-gas turbine of the type designed by C. Lorenzen, of Berlin, in which the blower is within and concentric with the rotor. The turbine blades are hollow and are cooled by the air from the blower passing through them. It is possible that enough air could be delivered by the blower to cool the exhaust gases, and condense the water in a compact apparatus contained wholly within the hull of the airship. The Lorenzen turbine as hitherto constructed delivers a quantity of air only about equal to the amount used by the main engine. A volume of air many times greater than this would be required for cooling the exhaust gases; and it is by no means certain that the Lorenzen turbine and blower could be adapted to deliver such large quantities of air without having a separate blower unit. Experience with exhaust turbines having more than a few per cent of the power of their main engines has not been very promising.

DEFINITIONS

The term "water-recovery apparatus," which has hitherto been used, is extremely cumbersome. "Ballast gainer," the name given by Zimmerman to his apparatus in 1911, is better; but it is believed that "ballast collector" is better still.

Primary Cooler. The process of cooling the gases and condensing the water vapor may be carried out continuously in one apparatus, or separated into two or more stages. The exhaust gases issue from the engine at temperatures between 650 and 750 deg. cent., which is above the melting point of aluminum. These very hot gases must be conducted through steel pipes until the temperature is reduced to about 500 deg. cent. before they can safely be injected into a cooling element constructed of aluminum. The steel pipes for the initial cooling are sometimes called the "pre-cooler." It is considered that "primary cooler" more correctly describes this part of the apparatus, and is a better term.

Secondary Cooler. From 500 to 53 deg. cent. no condensation of the water vapor occurs, and if this stage of the ballast collector is separated from the primary cooler it may appropriately be called the "secondary cooler."

Condenser. From about 53 deg. cent. to the final minimum temperature of perhaps 35 deg. cent., the water vapor is condensed, so that the part of the apparatus working within that temperature range is actually the condensing element, and may be called the "condenser."

There are, of course, no sharply defined points in the passage of the gases through the ballast collector at which the various temperatures mentioned in the foregoing definitions are reached.

Water Separator. The condensed water is carried along by the exhaust gas in the form of spray, which must be separated from the gas if it is not to be lost. The separator most commonly used consists of an aluminum box containing baffle plates between which the exhaust gas passes in an irregular course.

The drops of water strike the baffle plates and run down small vertical collecting gutters into a sump at the bottom of the separator. The efficiency of this type of collector depends to a very great extent on the velocity of the gas through it. A velocity of about 600 ft. per min. has proved most satisfactory. It has been found that if the velocity is less than 500 ft. per min., the water is not thrown out from the gas; while at more than 900 ft. per min. velocity, the drops of water deposited on the baffles are again torn off, and escape with the gas.

Recently a rotary centrifugal type of separator known as the "Hagan separator" has been tried out with encouraging results.

Theory of Water-Ballast Recovery

The quantity of steam or water vapor in the engine exhaust is only about 8.5 to 9 per cent by weight of the total quantity of gas. Condensation will not even commence until the gas is cooled to the temperature at which it is saturated by this amount of moisture. The saturation temperature is about 55 to 53 deg. cent., depending upon the initial amount of water vapor in the air and the ratio of air to fuel in the mixture delivered to the engine. To condense a weight of water equal to the weight of fuel consumed, the final temperature must be brought down to the neighborhood of 35 deg. cent. These temperatures are emphasized because it is often thought that it is only necessary to reduce the temperature to the boiling point of water, namely, 100 deg. cent., in order to condense the water.

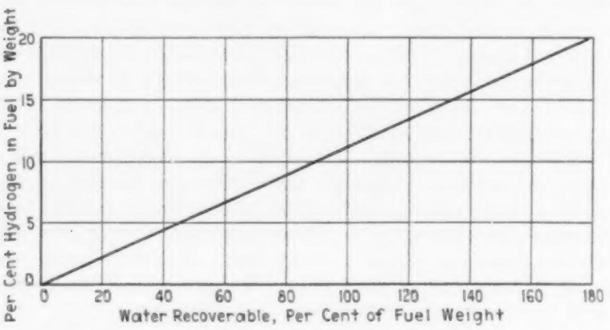

Fig. 2 Effect of Hydrogen Content of Fuel on Weight of Water Recoverable

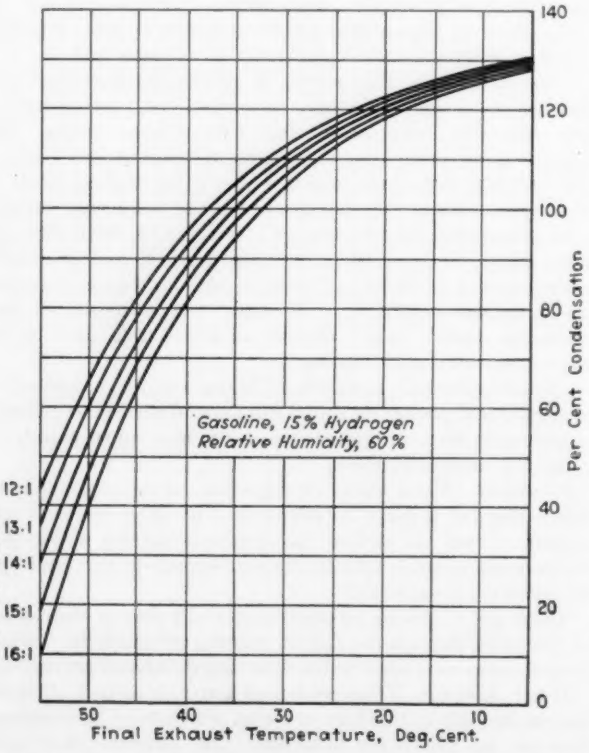

Fig. 3 Effect of Mixture Ratio on Weight of Water Recoverable

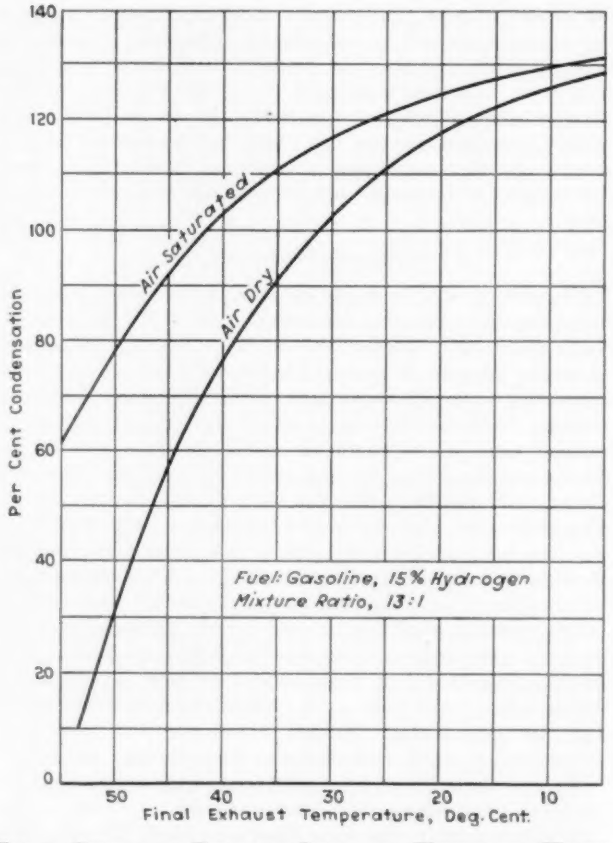

Fig. 4 Effect of Humidity Ratio on Weight of Water Recoverable

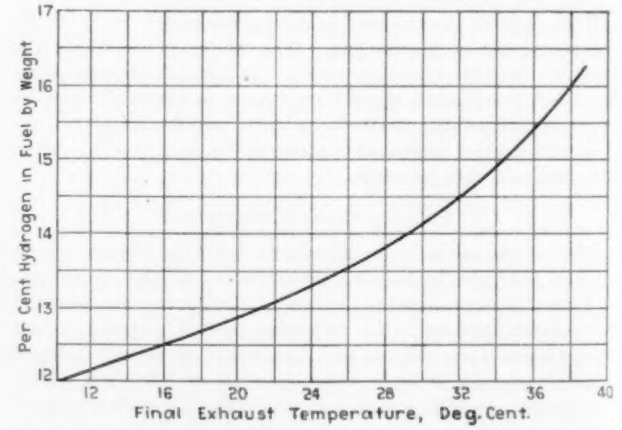

Fig. 5 Final Temperature to Which Exhaust Gas Must Be Cooled to Condense to 1 Lb. of Water per Lb. of Hydrocarbon Fuel Burned

(Assumptions: Air temperature 15 deg. below final exhaust temperature; mixture ratio; theoretical combining ratio; relative humidity, 60 per cent.)

It has been proposed to cool the exhaust gases to this temperature by injecting cold air into them from the outside atmosphere. Such an injection of air would so dilute the water vapor that saturation could hardly be reached at all, except on days of high humidity.

Four variables determine the amount of water which can be condensed from the exhaust per pound of fuel consumed. These variables are:

1 The hydrogen content of the fuel
2 The mixture ratio of fuel to air
3 The humidity of the air, and
4 The final exhaust temperature.

The effects of these variables are shown by the curves in Figs. 2 to 5, inclusive.

Ordinary aviation gasoline is about 15 per cent hydrogen.

Fuel oils suitable for aircraft Diesel engines contain about 12.5 per cent hydrogen by weight. Satisfactory water recovery with Diesel engines will depend upon not having too high a ratio of air to fuel. This may prove a stumbling block, especially with two-cycle engines and at reduced power.

SURFACE AREA REQUIRED

The theory of heat transmission from gas to gas through metal walls is not yet sufficiently comprehensive to permit of accurate calculation of the surface area required for any given conditions. The theory of heat transfer between gases flowing in tubes of circular cross-section has been described in a paper on "Heat Transfer in the Condensation of Water From Engine Exhaust Gas," by Robert F. Kohr and Lucinthia Butler, presented before the Division of Industrial and Engineering Chemistry at the 67th Meeting of the American Chemical Society, in Washington, D. C., 1924.

The theory is summarized here, and certain consequences of it are considered. It is assumed that: (1) the surface temperature of the pipes is the same inside and outside at any given point, and is uniform around the circumference at any point in the length of the pipe; (2) the cooling air flows with uniform velocity in a direction parallel to the axis of the pipe, and is turbulent. On these assumptions, the necessary area of cooling surface is given by the following equations:

For counterflow:

$$S = \frac{\log_e \dfrac{T_a - t_b}{T_b - t_a}}{\dfrac{q_h q_c}{q_h + q_c}\left(\dfrac{1}{H_h} - \dfrac{1}{H_c}\right)}$$

For parallel flow:

$$S = \frac{\log_e \dfrac{T_a - t_a}{T_b - t_b}}{\dfrac{q_h q_c}{q_h + q_c}\left(\dfrac{1}{H_h} + \dfrac{1}{H_c}\right)}$$

where S = total surface area of the tubes, sq. cm.
T_a = initial exhaust temperature, deg. cent.
T_b = final exhaust temperature, deg. cent.
t_a = initial air temperature, deg. cent.
t_b = final air temperature, deg. cent.

The terms H_h and H_c are given by:

$$H_h = M_h C_h$$

$$H_c = M_c C_c$$

where M_h = mass flow of gas through the system, grams per second
M_c = mass flow of air through the system, grams per second
C_h = specific heat of gas at constant pressure
C_c = specific heat of air at constant pressure.

The terms q_h and q_c are derived from the relation between surface cooling and skin friction established by Lanchester and Stanton (Technical Report No. 94 of the British Advisory Committee for Aeronautics, 1913), combined with Lee's equation for skin friction to give the following expression:

$$q = Cm\left[0.0765\left(\frac{\mu}{MD}\right)^{0.35} + 0.0009\right]$$

where m = unit mass flow along the pipe, grams per sec. per sq. cm.
μ = viscosity of the air or gas in poises
D = diameter of the pipe in cm.
C = specific heat at constant pressure in calories per gram per deg. cent.

According to Kohr, the following mean values may be assumed for C and μ:

	Exhaust gas above 53 deg. cent.	Exhaust gas below 53 deg. cent.	Outside air
C	0.26	2.2	0.24
μ	2.4×10^{-4}	2.0×10^{-4}	1.85×10^{-4}

The high specific heat of the exhaust gas below 53 deg. cent. is due to the release of the latent heat of condensation of the water vapor.

The foregoing expression for q is strictly applicable only to tubes of circular cross-section. In most types of ballast collector the air does not pass through circular passages; and in some types the air stream may have no outer boundary surface at all, so that D_c is infinite. Kohr has suggested that when the air passes between the tubes, D_c should be calculated by the following expression:

$$D_c = \sqrt{\frac{4ab}{\pi} - D_h^2}$$

where a and b are the vertical and transverse spacings of the tubes.

A similar difficulty occurs in calculating M_c, since it is not always clear how much of the air is effective in absorbing the heat from the gas.

A curious consequence of the theory is that apparently the highest rate of cooling is not obtained when the air stream is unbounded. Theoretically there is an optimum size of the air stream which should not be exceeded. This curious anomaly follows from the theoretical conclusion that $q_c = 0.0009\ C_c m$ when $D_c = \infty$. It is believed that the actual value of q_c is considerably greater than this, and consequently the theory is inaccurate when D_c is large. It is also inapplicable when the flow of air is across the tubes.

NUMERICAL EXAMPLES

An example will show the method of applying the theory to the computation of the surface area required.

Find the surface area necessary for a ballast collector designed for a 400-hp. engine at 60 knots speed, given that the outside air temperature is 15 deg. cent., and the gas is to be cooled to 35 deg. cent. It is also given that the engine consumes 0.45 lb. of aviation gasoline per horsepower-hour with a 15-to-1 mixture ratio.

$$M_h = 0.45 \times 16 \times 400 = 2880 \text{ lb. per hr.}$$
$$= 363 \text{ grams per sec.}$$

It has been found that m_h may equal 1.6 grams per sq. cm. per sec., without causing excessive back pressure in a collector built of 1-in. tubes with 10 turns of 180 deg. Undoubtedly, higher velocities may be employed in conjunction with larger tubes or fewer turns; but it will be assumed in the present problem that $m_h = 1.6$ grams per sq. cm. per sec.

Let $D_h = 1$ in. $= 2.54$ cm.

Let $D_c = 6$ in. $= 15.24$ cm.

If the density of the air is 0.076 lb. per cu. ft., and flows through the collector at a velocity of 60 knots, or 101.5 ft. per sec.,

$$M_c = 101.5 \times 0.076 = 7.72 \text{ lb. per sq. ft. per sec.}$$
$$= 3.78 \text{ grams per sq. cm. per sec.}$$

Since $D_c = 6\ D_h$, it will be assumed that the aggregate area of the air passages is 36 times that of the gas passages; and consequently,

$$M_c = \frac{36 \times m_c \times M_h}{m_h}$$
$$= \frac{36 \times 3.78 \times 363}{1.6}$$
$$= 30{,}800 \text{ grams per sec.}$$

For the gas above 53 deg. cent.,

$$q_h = 0.26 \times 1.6 \times \left[0.0765 \times \left(\frac{2.4 \times 10^{-4}}{1.6 \times 2.54}\right)^{0.35} + 0.0009\right] = 0.00141$$

Below 53 deg. cent.,

$$q_h = 2.2 \times 1.6 \times \left[0.0765 \times \left(\frac{2.0 \times 10^{-4}}{1.6 \times 2.54}\right)^{0.35} + 0.0009\right] = 0.01160$$

For the air,

$$q_c = 0.24 \times 3.78 \times \left[0.0765 \times \left(\frac{1.85 \times 10^{-4}}{3.78 \times 15.24}\right)^{0.35} + 0.0009\right] = 0.00163$$

Above 53 deg. cent.,

$$\frac{q_h q_c}{q_h + q_c} = \frac{0.00141 \times 0.00163}{0.00304} = 0.00076$$

Below 53 deg. cent.,

$$\frac{q_h q_c}{q_h + q_c} = \frac{0.0116 \times 0.00163}{0.0132} = 0.00143$$

Above 53 deg. cent.,

$$H_h = 363 \times 0.26 = 94.5$$

Below 53 deg. cent.,

$$H_h = 363 \times 2.2 = 800$$
$$H_c = 30{,}800 \times 0.24 = 7400$$

Let the apparatus be designed for parallel flow between the engine exhaust pipe and the beginning of condensation.

$T_a = 650$ deg. cent.; $T_b = 53$ deg. cent.; $t_a = 15$ deg. cent.

The heat taken up by the air equals that lost by the exhaust gas. Whence:

$$\frac{t_b - t_a}{T_a - T_b} = \frac{H_h}{H_c} = \frac{94.5}{7400} = \frac{1}{78}$$

$$\frac{t_b - 15}{650 - 53} = \frac{1}{78}; \quad t_b = 22.6 \text{ deg. cent.}$$

$$\frac{T_a - t_a}{T_b - t_b} = \frac{650 - 15}{53 - 22.6} = 20.9$$

$$S = \frac{\log_e 20.9}{0.00076 \times \left(\frac{1}{94.5} + \frac{1}{7400}\right)} = 374{,}000 \text{ sq. cm.}$$
$$= 403 \text{ sq. ft.}$$

That part of the apparatus in which condensation takes place is designed for counterflow. The conditions here are as follows:

$T_a = 53$ deg. cent.; $T_b = 35$ deg. cent.; $t_a = 15$ deg. cent.

$$\frac{t_b - t_a}{T_a - T_b} = \frac{H_h}{H_c} = \frac{800}{7400} = \frac{1}{9.25}$$

Whence: $t_b = 17$ deg. cent.

$$\frac{T_a - t_b}{T_b - t_a} = \frac{53 - 17}{35 - 15} = 1.80$$

$$S = \frac{\log_e 1.80}{0.00143 \times \left(\frac{1}{800} - \frac{1}{7400}\right)} = 298{,}000 \text{ sq. cm.}$$
$$= 321 \text{ sq. ft.}$$

The total surface area required is 403 + 321 = 724 sq. ft. or 1.81 sq. ft. per hp.

It may be noted that in the course of the calculations it was necessary to make some assumptions which are scarcely justifiable without experimental evidence. One of the doubtful assumptions is that the velocity of the cooling air through the collector is the same as the ship's air speed. Another is that the entire mass of cooling air is effective in absorbing heat from the gas. Both of these assumptions are on the dangerous side, and if incorrect, would lead to underestimating the area required; and yet experience indicates that a cooling surface of 1.81 sq. ft. per hp. is ample for a 60-knot airship. Clearly, there is much room for improvement in the theory of heat transfer. However, refinement in theory is likely to lead to extremely cumbersome calculations taking account of the variations of specific heat, viscosity, etc., with temperature; and even then there will probably remain insurmountable difficulties in determining by calculation the velocity of the air flow in various parts of the apparatus; so that the practical designer is likely to continue to depend largely on experience and experiment in the design of water-ballast collectors.

In the numerical example, the total cross-sectional area of the exhaust passages must be equal to $M_h/m_h = 363/1.6 =$ 227 sq. cm. = 35.2 sq. in. = 45 tubes of 1 in. internal diameter. The total perimeter of these tubes is 11.8 ft., and to obtain the surface area of 724 sq. ft., the length of the exhaust passage must be 724/11.8 = 61 ft.

Desirable Features in a Ballast Collector

The ideal ballast collector should have the following characteristics:

a One hundred per cent recovery of water under average operating conditions

b Low air resistance
c Light weight
d Small back pressure
e Easily cleaned from deposits of oil and soot
f Compactness and ruggedness
g Freedom from freezing in winter.

While the first of these requirements may be regarded as the most important one, it does not require much consideration here, because it is mainly a matter of providing sufficient cooling area. Some forms of ballast collector may require less area than others, but that is not an advantage in itself unless it is reflected in less weight and air resistance. It generally happens that forms designed for maximum cooling per unit area have a high total resistance.

Low air resistance is obviously a matter of prime importance, and is one which has proved most difficult of solution. Since a certain amount of area is necessary, the goal to be sought by the designer is an apparatus having no more resistance than the frictional resistance of its surface. The ultimate ideal is to use the surface of the hull for the cooling area, so that there will be no net resistance at all due to the collector; but there is enormous room for improvement without going to that ultimate refinement. The apparatus at present used on the U.S.S. *Los Angeles* has about fifteen times more resistance than the skin friction of an equal surface.

Tubes running in an approximately longitudinal direction have very little resistance, except skin friction; but they must be connected by headers, and unless these are carefully designed, their air resistance may be large. One solution is to place the headers within the hull; but that is likely to involve structural difficulties, and may cause overheating of the headers. If longitudinal tubes are used, they must not be so clustered together that they obstruct the flow of air between them. They must also be supported at close intervals, usually about two and a half feet, in order to avoid deformation by the heat and vibration. The necessity for closely spaced supports greatly augments the difficulty of eliminating all resistance but skin friction.

Placing the tubes with their longitudinal axes at right angles to the air flow facilitates obtaining an apparatus which is strong and compact, with headers and supports presenting little resistance; but having very great resistance from the tubes if their cross-sections are circular. Tubes of streamlined section would probably not help matters very much unless they were spaced so far apart as to involve excessively large and heavy headers.

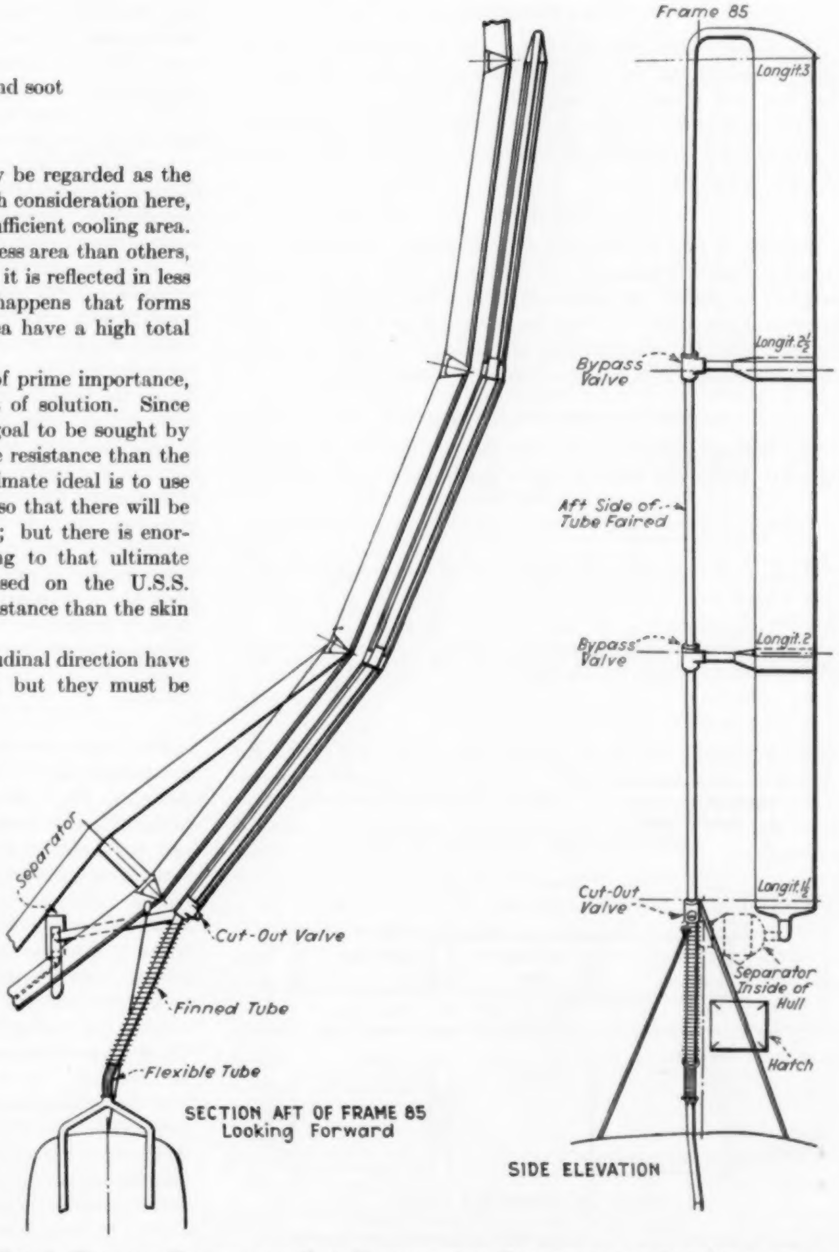

FIG. 6 VERTICAL SECTION AND SIDE ELEVATION OF CONDENSER FOR CAR No. 3 OF THE U.S.S. "LOS ANGELES"
(Scale, 1/80 full size.)

Transverse tubes may be flattened and placed as shown in Fig. 6, so as to present substantially flat surfaces parallel to the air flow. There is at present no experimental evidence to show whether or not tubes may be safely flattened in this manner without encountering the troubles which arise whenever an attempt is made to use flat plates in a ballast collector.

FACTORS DETERMINING WEIGHT

For average speed and operating conditions in modern rigid-airship practice, about 1.5 sq. ft. of cooling surface per horsepower is required for the ballast collectors. Assuming that 1-in.-diameter aluminum tubes with a wall thickness of 0.02 in. are used, the weight of the tubes is 0.29 lb. per sq. ft., or 0.435 lb. per hp. The total weight of the collector is fully two or three times greater, depending upon the design of the headers, supports, etc. In order to avoid the troubles resulting from the use of flat plates, it seems necessary to have aluminum castings for the headers. These are the largest single items of weight in a ballast collector; and their weight varies greatly, depending upon the arrangement of tubes and the general features of the design.

After the headers, the structure required to support the apparatus is the most important variable among the weights. A compact design lends itself to a low weight of supporting structure.

Back Pressure

The ballast collectors in the U.S.S. *Los Angeles* have ten sharp turns, of nearly 180 deg. each, in the gas passages. In spite of this, the back pressure is negligible, so that it is not likely to become an important factor in any ordinary type of apparatus. It is customary to provide gas passages having an area of about 0.08 sq. in. per hp.

Cleaning

Deposits of soot and oil in the tubes reduce the efficiency of the apparatus. It would be advantageous to have the headers designed to permit the insertion of brushes to sweep out the interiors of the tubes; but that has never yet been done. Cleaning can only be accomplished at present by blowing steam through the apparatus, or else completely disassembling it.

Compactness and Ruggedness

The ballast collectors now installed in the *Los Angeles* are probably about the most compact and rugged type which can be designed. Progress toward types of less resistance is likely to involve spreading the apparatus along the sides of the hull, which will certainly involve a loss of compactness. Ruggedness or strength may be obtained with the less compact collectors by more numerous and shorter connections to the ship's structure. Probably additional supporting structure will be required within the hull. The weight of this structure must be regarded as the price paid for better aerodynamic efficiency.

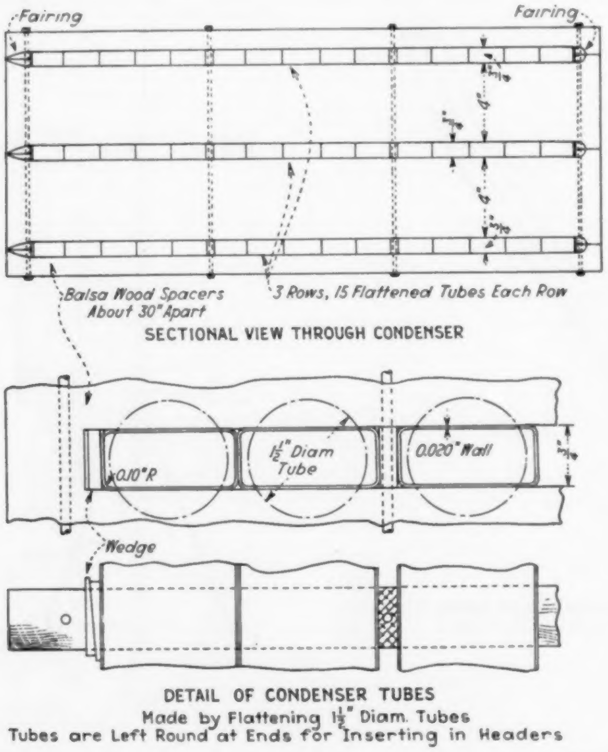
Fairing
Fairing
Balsa Wood Spacers About 30" Apart
3 Rows, 15 Flattened Tubes Each Row
SECTIONAL VIEW THROUGH CONDENSER
1½" Diam Tube
0.020" Wall
0.10"R
Wedge
DETAIL OF CONDENSER TUBES
Made by Flattening 1½" Diam. Tubes
Tubes are Left Round at Ends for Inserting in Headers

Fig. 7 Sectional View of Condenser Shown in Fig. 6 (Above) and Detail of Condenser Tubes (Below)

Protection Against Freezing

The problem of preventing freezing of ballast collectors in winter time presents one of the greatest difficulties to the designer seeking the ideal type. The difficulty has been overcome in the *Los Angeles* by closing the sides of the collector with a fabric curtain, and placing ordinary radiator shutters over the front face. Such an arrangement is permissible in an apparatus which already has nearly as much air resistance as a flat plate of the same frontal area; but it would completely spoil the advantages of the low-resistance types which it is hoped to develop.

A collector having fore-and-aft tubes might be provided with shutters having their surfaces and axes of rotation parallel to the air flow, without causing excessive resistance when either open or closed; but they must be very strong to avoid fluttering in the air stream, and their weight is likely to be excessive. An alternative to the use of shutters is to reduce the cooling surface in cold weather by bypassing part of the apparatus. In order to keep the total cross-sectional area of the gas passages constant, the reduction in cooling area should be accomplished by reducing the length of the gas passage; but this is likely to require some rather large and heavy intermediate headers, so that it may be better to keep the length of the passage constant, and cut out some of the tubes in parallel; even at the cost of increasing the back pressure as a result of the reduced cross-sectional area. It may be noted that the total cross-section provided in the *Los Angeles* type of collector is about three times greater in area than the engine exhaust pipe, so that it is probable that many of the tubes could be cut out without causing excessive back pressure, particularly in a collector designed to have fewer turns in the gas passages than the present type.

Principles of Design and Construction

The primary cooler must be constructed of steel pipes, because the temperature is too high for aluminum to sustain without damage. The rate of cooling is approximately proportional to the difference between the temperatures of the gas and the air, and consequently the gas is very quickly cooled through the range between about 700 and 500 deg. cent. This part of the ballast collector requires only about 7 per cent of the total surface area; and it is not even necessary to use small-diameter tubes to increase the cooling area. It is quite satisfactory to pass the gas through a single steel pipe of the same dimensions as the exhaust pipe from the engine. The portion of this pipe nearest the engine may be finned on the outside, so that the temperature of the metal will be nearer to the air than to the gas temperature. A cut-out valve is provided between the engine and the primary cooler, or between the primary and secondary elements. This valve is exposed to very high temperatures, and must be of a type that will withstand great heat without sticking or becoming inoperative. A steel flap valve with an asbestos seat is the only type which has been successful.

The secondary cooler and the condenser are usually constructed as a single unit; and they will be so considered in this discussion. The smaller the tubes of this element, the greater their cooling efficiency; but the greater also are the difficulties of supporting them, and preventing loss of efficiency through clogging with carbon deposits. Amid the conflicting requirements for large and small tubes, experience indicates that aluminum tubes about 1 in. in diameter, with walls 0.02 in. thick, are a satisfactory compromise.

The portion of the collector in which condensation occurs should slope downward so that condensed water will not gather in pockets or flow back into the hotter gases and be re-evaporated.

In the type of ballast collector shown in Fig. 1, the headers are aluminum castings, with sockets to receive the tube ends. These sockets are very accurately machined with a slight taper to fit over the tubes. The tube walls are too thin for screw connections or satisfactory welding. The joints are kept tight by wires which pull the opposite headers together. In order to allow for the effect of temperature variations, the wires are made of monel metal, having half the coefficient of expansion of aluminum, and are placed within the tubes so that they will be at the gas temperature, while the tubes are at temperatures about midway between the gas and air.

RECENT DEVELOPMENTS IN BALLAST COLLECTORS

A novel type of ballast collector is contemplated for the new 6,500,000-cu. ft. airship *Akron*, now under construction for the U. S. Navy. The collector consists of 14 banks of longitudinal tubes about 8 ft. in length, having elliptical cross-sections and placed close against the hull of the airship. The ends of the tubes fit over elbows which are welded to headers inside the ship. These headers connect the successive banks of tubes, and being inside the hull, they add nothing to the ship's air resistance.

The apparatus should have a high rate of cooling in proportion to its surface area; and the air resistance will be very low. The chief difficulties to be overcome are mechanical, since welds and flat surfaces are involved, and these features have hitherto proved stumbling blocks in other ballast collectors.

A very simple ballast collector is being constructed for trial on the U.S.S. *Los Angeles*. It comprises four rows of aluminum tubes, 3 in. in diameter by 0.025 in. wall thickness, extending aft and upward along the side of the ship for a length of 50 ft., and then turning forward and down for an equal distance. The tubes are inclined at about 10 deg. to the longitudinal meridian lines of the ship. At this angle their air resistance is but slightly greater than that of longitudinal tubes and is only about 2 per cent as much as transverse tubes. Their comparatively large diameter, 3 in. instead of the usual 1 in., tends to reduce their cooling efficiency, but this is compensated by higher gas velocity, which is considered to be permissible because the directness of the gas passages should more than offset the effect of the increased velocity in building up the back pressure.

The principal advantages of this type of apparatus are its simplicity and ruggedness and the elimination of headers. It is not new; since as noted in the historical summary, a collector of this type was installed, but never put to the test of service, in the British airship *Mayfly* in 1910. The keel condensers, formerly much used in small steamboats, were similar in principle.

Another type of ballast collector once tried in Great Britain is to be given a retrial in the *Los Angeles*. This is a honeycomb radiator, constructed as a high and narrow unit with air passages consisting of 1-in. tubes, 36 in. in length. It has the merits of compactness and great facility of temperature control. The resistance is not particularly low; and the ill-success attained with an apparatus of this type in tests at Kingsnorth in 1917 indicates that there may be trouble from too rapid a deposit of soot upon the outside of the tubes. On the other hand, the cleaner exhaust obtainable with modern engines may eliminate this trouble.

Discussion

T. G. W. SETTLE.[2] It is of great value to the profession to have the subject of water-recovery apparatus so fully and concisely covered. The following comments are offered more in the way of elaboration upon some of the author's material than as disagreement with him.

At the beginning of the paper, two additional methods of maintaining equilibrium might well have been listed:

(*f*) Collecting atmospheric moisture (rain, fog, etc.) in scuppers on top of the ship. This method has been developed by the British, and was used in both the *R-100* and *R-101*. It is a rather haphazard and uncertain method, of course. It is often available when not needed and unavailable when badly needed. However, it is very cheap in weight and drag, and it should be considered as possibly a valuable secondary means of obtaining water.

(*g*) Valving hydrogen from "trimming" cells submerged inside the main helium cells. It is understood that the newest German ship, the *L.Z. 129*, will utilize this method.

Brief comment will now be made on the other methods of maintaining equilibrium, as listed by the author.

(*a*) *Condensation of Water Vapor in Engine Exhausts.* This is the only method that has been developed to what may be termed a practical service state, except for the isolated example of the *Graf Zeppelin* and her gaseous fuel. This method will be discussed further in a following paragraph.

(*b*) *Hygroscopic Materials.* Impracticable, as indicated by the author.

(*c*) *"Picking Up" Sea Water.* The author dismisses this possible method with a few remarks on partially successful tests by non-rigid ships in early years. It is significant that this method has never been tried by modern airships (rigids) so far as is known. On the *Akron* class, with ample electrical power available for winches to hoist large "bags" of water, it is believed that this method has at least enough theoretical merit to warrant a full-scale trial. The "spy" winch and cable could be utilized. Sea-water ballast would have the added rather great advantage that it would be anti-freezing in character in all except extremely low temperatures, thus minimizing the quantity of alcohol to be carried on board. The corrosiveness of sea water on structure would have to be accepted and guarded against.

(*d*) *Gaseous Fuel.* Used very successfully by the *Graf Zeppelin*. However, it is regarded as a "passing phase" in airship development; in fact, it may be considered as having already "passed," since the *L.Z. 129* will not employ this means. Probably the biggest consideration against gaseous fuel is the fire hazard involved, especially in a helium ship. However, power-plant factors enter also, some in favor of and some opposed to gaseous fuels. Many independent considerations seem to indicate future power plant evolution toward compression-ignition engines of large unit horsepower ratings, using fuel oils, rather than toward electric-ignition engines using more volatile fuels.

(*e*) *Burning Hydrogen With Liquid Fuel.* Not practicable, principally because of tendencies toward pre-ignition and detonation when using hydrogen, especially in high-compression-ratio engines.

Condensing Apparatus. In designing this equipment several somewhat mutually antagonistic factors enter into the problem, and the best that can be hoped for is a fair compromise. Some of these factors are heat transfer, aerodynamic characteristics, structural integrity, fouling of gas-side surfaces, temperature control, and weight. The only type that has been used successfully in routine service conditions to date is the so-called *Los Angeles* type, a "block" type, having transverse tubes arranged in zigzagged banks between headers. This type is very poor aerodynamically and has other disadvantages, as pointed out by the author, but it has served its purpose admirably in the absence of anything better, since 1924, on the *Shenandoah* and the *Los Angeles*. Many substitutes and attempted improvements have been tried, none of which has yet proved itself. These have failed for various reasons, but probably the most common cause of failure has been structural weakness. It is hoped and believed that one or more practical substitutes, comprising improvement in one or more of the essential characteristics, may be available shortly. The paper covers the detailed design and principles of construction of cooling and condensing units very thoroughly, and no comment thereon is possible except a "well done."

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[2] Lieutenant, U. S. Navy; Inspector of Naval Aircraft, Goodyear-Zeppelin Corporation, Akron, Ohio.

General Considerations as to Exhaust Recovery Systems. Present operational practice in this country is wholly premised on "100 per cent recovery" ability in usual operating conditions. While it is believed that some exhaust-recovery equipment will be necessary and desirable for a long time, perhaps indefinitely, it is at the same time believed that the "100 per cent recovery" ability will not persist. The advent of heavy-oil engines, which is looked upon as a certain eventuality, will force attention to other means of maintaining static equilibrium, even if such other means are not perfected prior to that time. This is because the exhaust from a heavy-oil engine has far less "relative humidity" than that from a gasoline engine; so much less as to make a 100 per cent recovery standard impracticable with heavy-oil engines.

It is believed that the two possible alternatives will be picking up sea water and hydrogen trimming cells submerged in helium. It is further believed and recommended that airship designers should not wait to have their "hand forced" in this matter by the advent of Diesel engines, but should thoroughly investigate the two apparent alternatives without delay. It would appear that a water pick-up technique, if successfully developed, would have many arguments in its favor for replacing a portion of the present 100 per cent exhaust-recovery equipment on airships.

AUTHOR'S CLOSURE

Lieutenant Settle's discussion calls for no reply, except thanks for his appreciation of my efforts. I agree with him that picking up sea water remains a possibility for maintaining equilibrium in airships operating at sea. Experiment along that line is well worth while, although it is a difficult problem to devise means of taking large quantities of water at a time without danger of imposing sudden shocks upon the airship.

AER-54-12

Engine Cowl Rings

By PAUL M. BOYD,[1] BUFFALO, N. Y.

The radial air-cooled engine is a popular power plant. Its reputation with the public can be said to date from the summer of 1927, when it demonstrated its reliability and ruggedness in the hazardous long-distance flights of that year. Since then many refinements and improvements have been brought forth, but the drag of the radial engine has always been high. Large frontal areas are inherent with the radial engine, and as the power output of the present radial has increased, so has its frontal area. Until recently it did not seem likely that the radial engine could ever successfully compete with the liquid-cooled type as the power plant for use in high-speed airplanes. However, with the development of the N.A.C.A. type of cowling and the engine cowl ring by various aerodynamical laboratories, the big gap existing in actual ship's performance between these two widely divergent power plants has been eliminated.

A STUDENT of history is often impressed with the way old ideas have of recurring under slightly different guise as something entirely new. One generation conceives an idea, develops it, employs it for a while, and then discards it for something else which is considered superior; a succeeding generation being pressed for the solution of a problem often works out an answer practically identical with that obtained before, but none the less original despite its similarity to earlier work. So it is with the present refined cowlings for the radial engine. Fifteen years ago airplanes were equipped with practically the identical cowling that is being heralded today as the savior of the radial engine, the main difference being not in the cowling itself, but in the reasons for its use. Today the purpose is to reduce drag. Fifteen years ago the purpose was to catch oil thrown by the engine.

The designers of the first air-cooled engines were confronted with many difficult problems. Perhaps the most difficult was that of obtaining satisfactory cooling of the power plant. It was soon found that an air-cooled power plant that rotated cooled itself satisfactorily. The rotary engine was therefore developed and remained the most popular airplane power plant to the end of the war. The carburetion problem in the rotary engine developed some rather drastic lubrication systems. It was necessary to use castor oil, and the rotary engine was capable of slinging this unpleasant substance quite as effectively as a pinwheel throws sparks. The designers of the early tractor airplanes soon came to the conclusion that this situation was too much for the pilots of even those hardy days. One of the first attempts to minimize this fire hazard and source of filth was made in 1911 by Blériot, who installed a square, box-like cowl, open at the front, over the engine.

Blériot's design was soon improved, and by 1916 the majority of military airplanes were equipped with engine cowlings similar to those shown in Figs. 1, 2, and 3. The cooling air entered the cowl ring through a large annular opening at the front, was carried around by the engine, and passed out through a hole in the under side of the cowling. These cowl rings did not have the annular discharge openings around the complete circumference as do the present designs, but they did possess that external symmetrical smoothness which is so favorable to good propeller efficiency, and lessened turbulence and drag. Fig. 3, particularly, represents a power plant faired into the fuselage lines in a manner which can be compared favorably with present design conceptions. Fig. 4 illustrates a bi-motored Caudron with rotary engines covered with cowlings which remind one at once of the present N.A.C.A. type. Fig. 5 illustrates an American design, an Elias airplane constructed in 1924. The engine cowl rings shown in place could be termed exact duplicates of present-day design. It is doubtful whether the designers fully appreciated the aerodynamic benefits obtained through the use of this type of cowling; more likely they looked upon it more as a necessary expedient required for cleanliness. That this was the case is borne out by the fact that when the radial engine replaced the rotary, the engine cowl rings were forgotten.

The rotary engine had many undesirable features—limited power output, objectionable flying qualities resulting from the gyroscopic torque, dirtiness, etc. Engine designers meanwhile had advanced the radial type of engine by 1919 to the point where it gave satisfactory service. Figs. 6 and 7 illustrate military airplanes built just after the war, with fixed radial engines. The cowling of these engines is not unlike that in use at present, as the various types of engine cowl rings in vogue are not as yet employed on all airplanes.

The poor performance, particularly in high speed, obtained with airplanes powered with air-cooled engines showed that there was room for great improvement. Late in 1927 the National Advisory Committee for Aeronautics started a comprehensive research program for the development of suitable low-drag cowling for the air-cooled engine. The work was carried on with a full-scale model in the 20 ft. diameter wind tunnel. At about this time the Engineering Division of the U. S. Army Air Corps began working on the same problem at Wright Field. Through these efforts was developed an engine cowl ring with a long skirt, extending in some cases for nearly 2 ft. aft of the line of engine cylinders. This type of ring is now known as N.A.C.A. or Wright Field cowling.

At practically the same time the National Physical Laboratory in England had undertaken a similar experimental program. The engine cowl ring developed by the N.P.L. has a short chord and does not extend to the rear of the engine cylinders. Abroad this cowl is styled the Townend ring, H. C. H. Townend being the man who carried through the development work.

The illustrations show that both these cowlings are adaptations

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[1] Curtiss Airplane and Motor Co., Inc. Mr. Boyd was born in St. Paul, Minn., in 1900. He learned to fly in 1922, and subsequently was a pilot with the Curtiss Northwest Airplane Company until 1924. In that year he received the degree of B.S. in M.E. from the University of Minnesota. He took post-graduate work in aeronautical engineering at M.I.T. in 1924–1925, receiving the degree of M.S. in 1926. In June, 1925, he joined the Curtiss Company as a draftsman. At present he is devoting his attention to airplane power-plant installation design and to test flying.

Presented at the National Aeronautic Meeting, Baltimore, Md., May 12 to 14, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

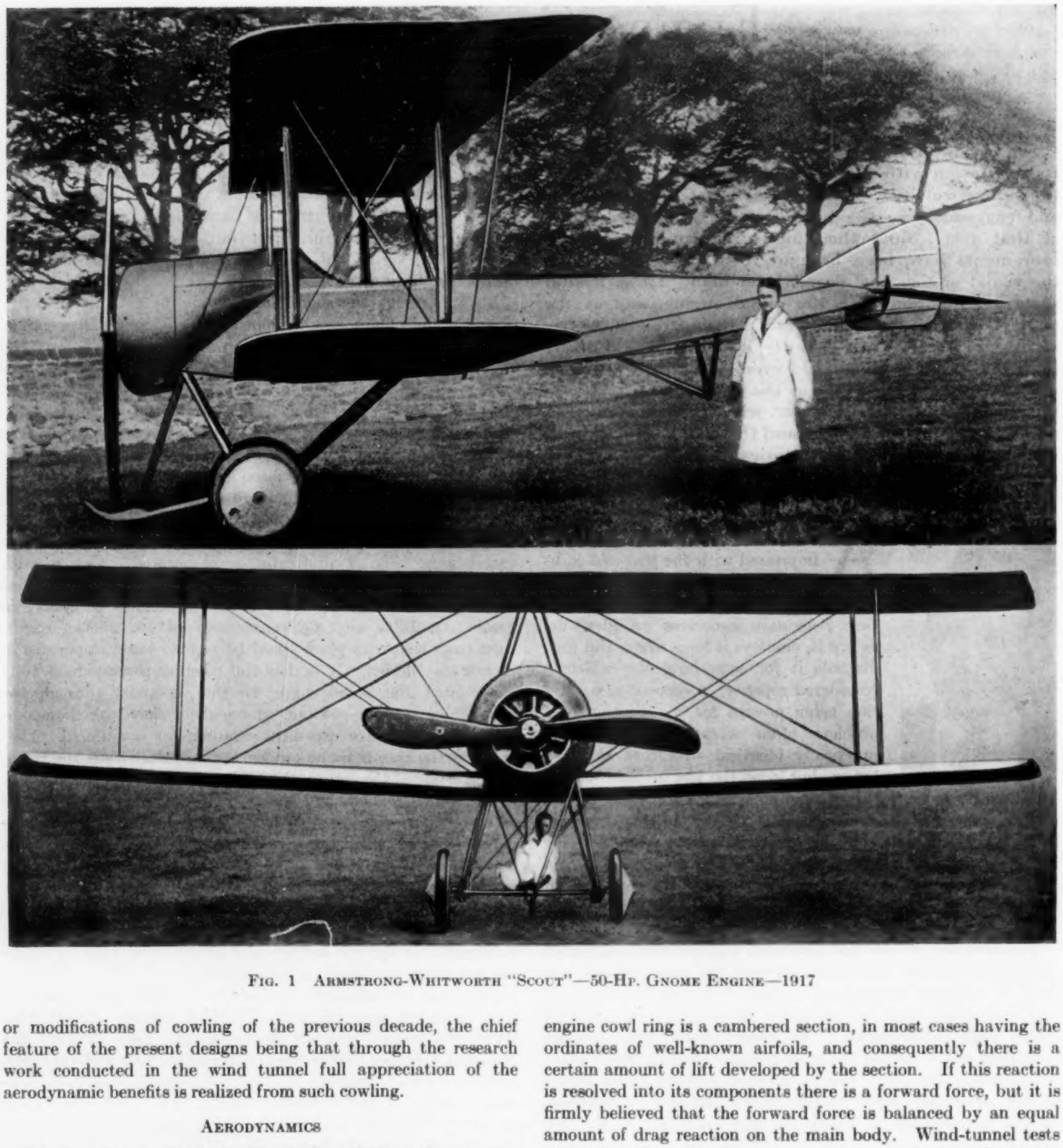

FIG. 1 ARMSTRONG-WHITWORTH "SCOUT"—50-HP. GNOME ENGINE—1917

or modifications of cowling of the previous decade, the chief feature of the present designs being that through the research work conducted in the wind tunnel full appreciation of the aerodynamic benefits is realized from such cowling.

AERODYNAMICS

Advancement in aviation is largely dependent on the progress made in eliminating drag or improving the L/D ratio. The new types of radial-engine cowling are the latest contribution toward this end. Considerable credence has been given by some, particularly in regard to the small-ring type of cowling, to the belief that the benefit derived from these new forms of cowling is due to the forward component of the lift forces acting on the ring, the idea being that the effective thrust is increased by an amount equal to the forward component. This idea appears to be fundamentally wrong, since it seems to violate the fundamental principles of mechanics, and suggests perpetual motion. "Appears to be" is said advisedly, because it is rash to state that any alleged aerodynamic phenomenon is impossible. The engine cowl ring is a cambered section, in most cases having the ordinates of well-known airfoils, and consequently there is a certain amount of lift developed by the section. If this reaction is resolved into its components there is a forward force, but it is firmly believed that the forward force is balanced by an equal amount of drag reaction on the main body. Wind-tunnel tests show that the total drag of a cowl-ring and streamline-body combination is greater than that for the body alone.

When the nose of a body, however, is covered with protuberances such as the cylinders of a radial air-cooled engine, there is a different answer. This condition results in turbulence, and turbulence means drag to the airplane designer. Here lies the answer to the source of the improvement secured through the use of engine cowl rings. An uncowled radial engine excites a turbulent flow of air whose cross-section may have a diameter two or three times as great as the diameter of the body to which the engine is attached. When this same combination is fitted with a cowl ring, the downwash flow from the cambered ring section contracts this region of turbulence, and the lines

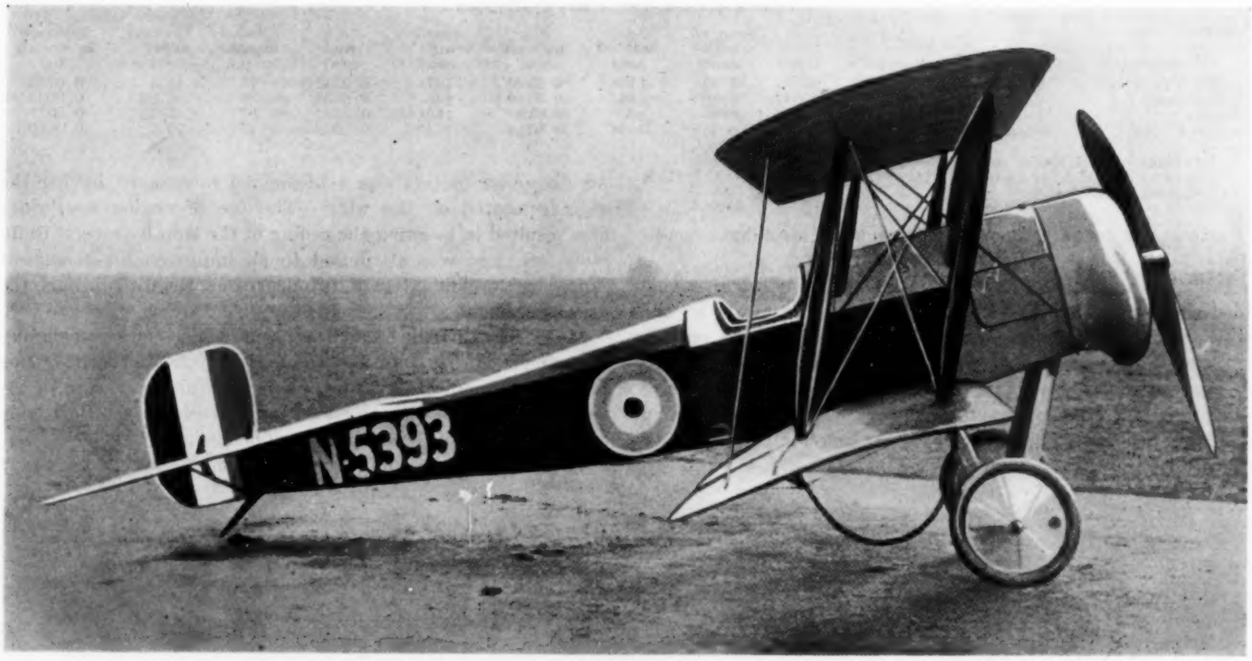

FIG. 2 BRISTOL "BULLET"—130-HP. CLERGET—1918

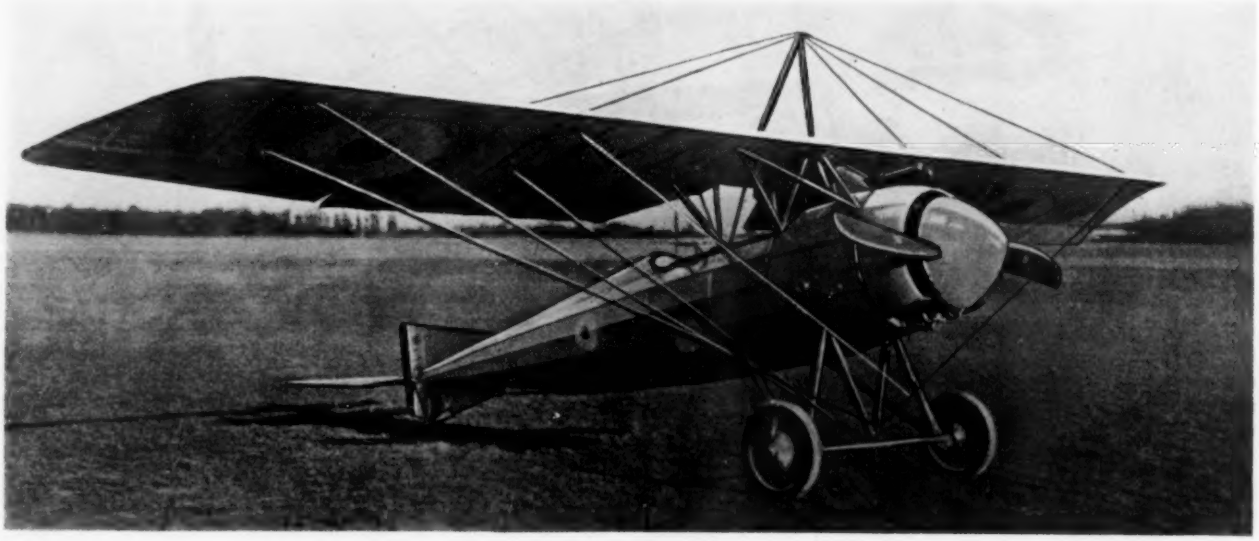

FIG. 3 TYPE P MORANE-SAULNIER—110-HP. LE RHONE—1918

of flow then closely parallel the body. This is borne out by the following facts:

1 The benefit derived from a cowl ring becomes markedly improved as the number of cylinders in the engine increases, the optimum results being obtained on a nine-cylinder engine.

2 An engine cowl ring can be installed on an engine with such a large negative angle of attack that no benefit whatsoever is realized from the downwash of the ring. In fact, it may be so attached that the resultant speed is even slower than that obtained with no cowl ring.

3 The use of the cowl ring brings about more effective tail-control surfaces, indicating that the air passing over them is without turbulence. There is also usually a slight increase in elevator and rudder control loads.

In some cases it has been found that the adoption of engine cowl rings has had an adverse effect on the stability of the ship on which it has been installed. This is far from being the usual result, and when encountered, the change in stability is usually not excessive, and may be due to several causes. As already stated, the use of engine cowl rings tends to make the tail surfaces more effective. Therefore, less stabilizer movement will be required to trim the ship for any condition of level flight. Once the ship is disturbed, however, from this state of equilibrium, it is found that the stabilizer does not build up the proper righting moment until the oscillations of the disturbance have been carried to greater extremes than was necessary before. This results from the fact that the cowl ring, in the majority of cases, is beneficial only throughout a limited range of angles

TABLE 1 N.A.C.A. RING COWLING

Ship	Engine	No. of cyl.	Hp.	Prop. eff. with cowl[1]	Speed without cowl	K_D without cowl	Speed with cowl	K_D with cowl	Speed increase, m.p.h.	Per cent speed increase	Decrease in overall K_D
Curtiss, AT5-A.	Whirlwind	9	200	0.83	118[3]	0.0379	137[2]	0.0243	19	16.1	0.0136
Curtiss Hawk........	Whirlwind	9	300	0.83	140	0.0340	146	0.0301	6	4.28	0.0039
Fleetster............	Hornet "B"	9	615	0.84	168[2]	0.0409	180[3]	0.0332	12	7.15	0.0077
Lockheed Vega.......	Wasp	9	450	0.84	165[3]	0.0315	182[3]	0.0236	17	10.3	0.0079

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[1] Efficiency with N.A.C.A. cowling.
[2] Tested by N.A.C.A. at Langley Field.
[3] Indicated air speeds.

of attack, the range corresponding to normal level-flight conditions.

Probably another factor contributing to the effect of engine cowl rings on stability is the influence the ring exerts on the position of the center of pressure of the fuselage. On the average long-nosed, well-streamlined body the center of pressure is ahead of the center of gravity, this combination giving a negative pitching moment at low angles of attack and positive pitching moments at high angles, in general making for poor stability characteristics. Most radial-engine-powered ships, however, have shorter fuselages, so that the center of pressure lies aft of the center of gravity. The moments resulting from any disturbance are therefore opposite to those of the previous case and conduce to good stability characteristics. When an engine cowl ring is added, a situation analogous to that of a slotted wing is created. The center of pressure of an airfoil with an open slot is well forward. The center of pressure of a fuselage with an engine cowl ring is likewise brought forward, probably in some cases ahead of the center of gravity, with a resulting detriment to dynamic stability characteristics.

It has been mentioned that the engine cowl ring is of benefit only in level flight. This fact is hard to accept. One would normally assume that any great reduction in overall drag would be also reflected in improved climb and ceiling. Generally this is not the case, these particular performance characteristics remaining unchanged by the addition of engine cowl rings. This falling short of expectations is due to a change in the flow of the relative air over the ring when the ship is in a climbing attitude. The normal ring possesses a constant angle of attack throughout its entire circumference. At the best climbing angle for the ship the relative wind strikes the top arc of the ring at a greatly increased angle. The ring being an airfoil, this portion of it becomes stalled, and the effect of infinite aspect ratio obtained in level flight is lost. The resulting drag becomes practically equivalent to that existing when no ring is in place. An exception to this was a bi-engined monoplane having the engines located on the wing. The use of engine cowl rings here resulted in boosting the ceiling of the ship by several thousand feet, and was attributed to an improved lift coefficient caused by the elimination of turbulence over the wing through the use of the rings.

Much more could be written concerning the aerodynamics of engine cowl rings, but it would carry beyond the scope of this paper. The subject is more capably and fully gone into in the various reports of the American and British aeronautical laboratories. A list of these reports is appended.

Table 1 illustrates improvements in high speed obtained on several ships with the N.A.C.A. type of cowling The differentials obtained with the two cantilever monoplanes illustrate a comparable improvement in each case. The differentials in the case of the two Curtiss Hawks are far from checking as closely. The AT5-A was a Curtiss Hawk powered with a 200-hp. J-5 engine, and was credited with an official top speed of 122 m.p.h. by the Army at Wright Field. If this value is taken and the cube root of the power ratio applied it is found that the 300-hp. J-6 Whirlwind Hawk should make 140 m.p.h., which is the figure with which it is actually credited. The AT5-A was provided with an N.A.C.A. cowl and the speed of that particular ship was increased 19 m.p.h. An identical ship with the exception of the greater power was equipped with an N.A.C.A. type of cowl by other parties with an increase in speed of only 6 m.p.h. While it is not expected that the higher-powered ship would give an equivalent differential, one would naturally anticipate more than the 6 m.p.h. actually obtained. This comparison serves to point out the sensitiveness of the N.A.C.A. type of cowling. The development of a successful cowling of this type calls for considerable research on each installation. Apparently no hard and fast rules can be established, and each different application must assume the proportions of an entirely new research problem.

Table 2 shows the increases in high-speed performance obtained through the use of the small cowl ring. The improvements

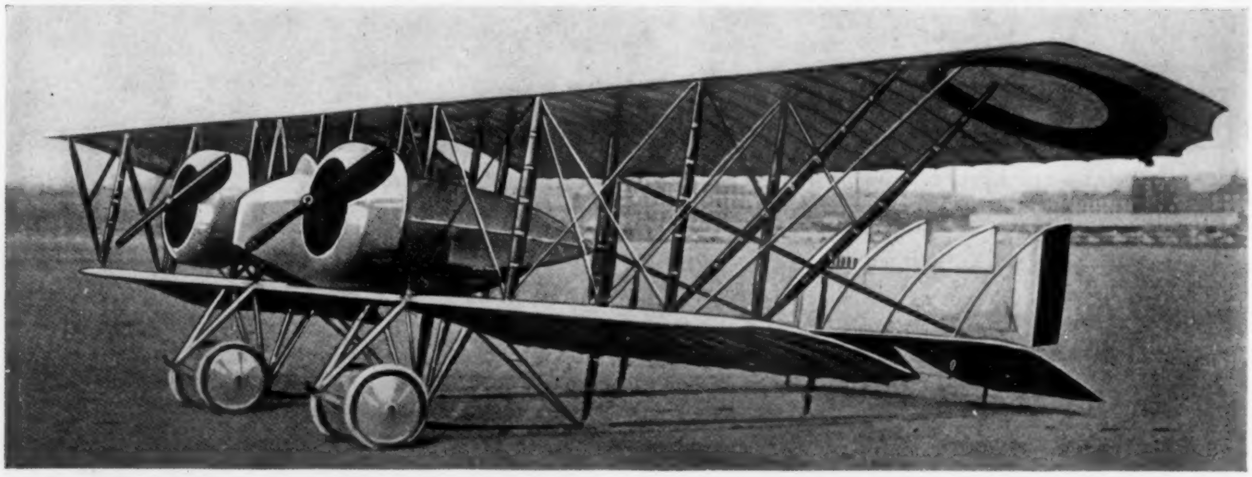

FIG. 4 GA TYPE CAUDRON—80-HP. LE RHONE—1918

TABLE 2 SMALL-ENGINE RING COWLING[1]

Ship	Engine	No. of cyl.	Hp without ring	Prop. eff.[2]	Speed without ring	K_D without ring	Hp. with ring	Speed with ring	K_D with ring	Speed increase, m.p.h.	Per cent speed increase	Decrease in overall K_D
1[3]	Cyclone	9	615	0.84	167	0.0417	625[4]	186	0.0307	19.0	11.4	0.0110
2[3]	Wasp	9	450	0.82	141.9	0.0486	475[4]	148	0.0451	6.1	4.3	0.0036
3[3]	Wasp	9	450	0.82	141	0.0492	450	146	0.0443	5.0	3.5	0.0049
4[3]	Cyclone	9	615	0.84	163.5	0.0442	625[4]	176	0.0361	12.5	7.6	0.0081
5[3]	Whirlwind	9	400	0.84	160	0.0308	400	182	0.0209	22.0	13.8	0.0099
6[3]	Two Hornets	9	1230	0.79	121	0.2055	1230	123.7	0.1928	2.7	2.2	0.0127
Falcon mailplane	Cyclone	9	615	0.84	147.2	0.0608	625[4]	155.4	0.0525	8.2	5.6	0.0083
Stearman C-3R	Whirlwind	7	240	0.82	135[5]	0.0301	240	140[5]	0.0268	5.0	3.7	0.0033
Fairchild cabin, model 100	Hornet "B"	9	615	0.78	139.6[5]	0.0661	615	145[5]	0.0590	5.4	3.9	0.0071

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[1] Improvements shown in this table were obtained with use of Curtiss anti-drag ring.
[2] Efficiency as calculated when cowl ring is in place.
[3] U. S. Army and Navy military airplanes.
[4] Increase of power due to supercharging effect obtained through use of ring.
[5] Indicated air speeds, all others actual timed speeds over speed courses.

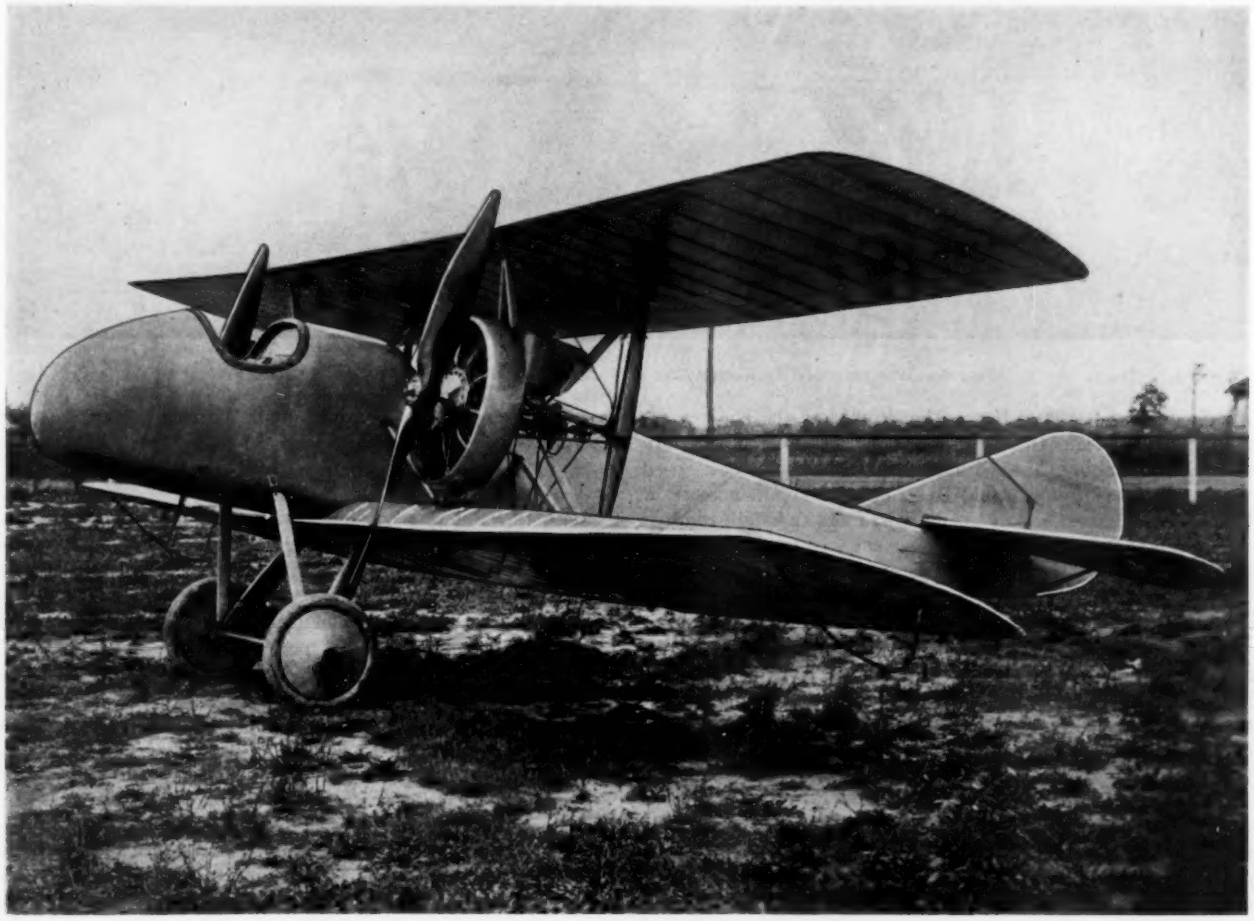

FIG. 5 ELIAS ES-1—LE RHONE ENGINES—1924

were all obtained through the use of the Curtiss anti-drag ring. The design on this ring is standardized. All the rings employed in these tests had the same chord and airfoil section, approximately the same angle of attack, and approximately the same location in reference to engine cylinders. The ships on which these rings were tested range from a twin-engined bomber to a single-seat pursuit ship. Because a number of the military ships are still in the experimental stage they are referred to by number. Ships Nos. 1, 4, and 5, on which particularly gratifying results were obtained through the use of a cowl ring, all had what one might refer to as complete streamlining. The wheels were incased in spats, landing-gear knuckles were faired, strut and wire terminals were covered by cuffs, and the tail wheel was streamlined. Thus the fuselage with its radial-engined nose remained as the single source of most of the drag. For example, wind-tunnel tests on ship No. 5 showed that its engine-fuselage combination without a cowl ring had a K_D of 0.0166, which amounts to 53.9 per cent of the total K_D of 0.0308 for the ship. The addition of the engine cowl ring reduced the K_D for the entire ship to 0.0209. For all practical purposes, it can be stated that the saving was all accomplished on the engine-fuselage unit, though some of the reduction may be due to lessened interference. This example establishes the conditions which indicate how much benefit can be expected from the use of engine cowl rings. The bigger the percentage of the total K_D caused by the engine-fuselage unit, the greater will be the improvement realized through the use of engine cowl rings. With conventional cowling the K_D for an engine of the size of the Hornet

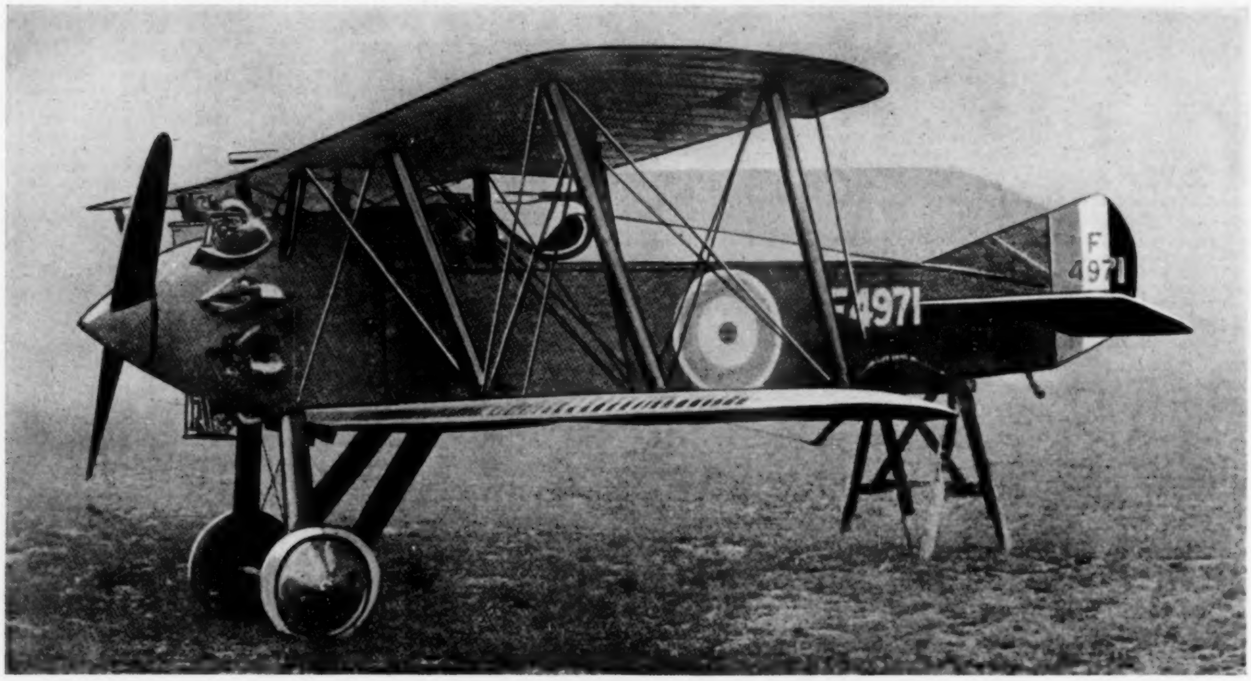

Fig. 6 Armstrong-Whitworth "Ara"—320-Hp. A. B. C. "Dragonfly"—1919

is estimated at 0.0090. If one then considers the engines entirely removed from this ship and the nacelles of a perfectly streamlined airship shape, with the same power, the best speed that could be expected would be 124.8 m.p.h., an increase of 3.8 m.p.h. The use of the engine cowl rings therefore resulted in securing 71 per cent of the improvement that was theoretically possible. This case serves to indicate that the improvement to be expected in a large ship, where the drag of the nacelles is a small percentage of the total, will be small.

Test-stand experiments brought forth the fact that the engine cowl ring in addition to its principal function of reducing turbulence, in some cases also contributed a certain amount of external supercharging, the amount depending on the location of the carburetor air scoop. Tests indicated that 10–15 hp. was realized in the case of the Cyclone engine with its regular air scoop. On ship No. 2 a special air scoop was constructed to replace the unit provided on the engine. Its use gave a speed increment of about 2.7 m.p.h., which indicates a power increase of about 25 hp. If special air scoops had been provided on the twin-engined bomber, ship No. 6, it is quite likely the speed increment due to the engine cowl rings would have been increased to 3.5–4 m.p.h.

The tabulated results indicate that, on a clean military ship, an average speed increase of approximately 10 per cent can be realized. For the slower commercial ship having a top speed of around 135 m.p.h. a 5 per cent improvement can be expected. For slower ships in the 110-m.p.h. class 3 per cent is more likely to represent the improvement to be gained. It must be recognized that these results were all obtained with the same design of ring, and that the optimum condition did not necessarily exist in any case. The results are those obtained by adding an engine cowl ring over the engine, retaining the original engine cowl without alteration. Experimenting with the angle of attack of the ring or the ring's position in each case might have led to better results. The angle of attack is not a particularly critical characteristic, however, as the N.A.C.A. found that with a Curtiss XF7C-1 the speed did not vary more than a mile from the best results obtained with the ring set in different positions through an angular range of 6 deg.

Relative Merits

Both types of ring cowling have certain fields of application in which the one excels the other. The N.A.C.A. cowling has shown its ability to give wonderful results as evidenced by the large increase in speed secured when it has been employed on various familiar cantilever monoplanes. The field of application of the N.A.C.A. cowling, however, is much more limited than that of the smaller cowl ring. The relationship between the cowling itself and the fuselage lines is much more critical than is the case with the smaller cowling. Most of the successful adaptations of the N.A.C.A. cowl to date have been on large, smoothly faired fuselages. The smaller cowl will give a consistent improvement in performance on practically any fuselage. This fact is established by the speed increase obtained on numerous airplanes with the Curtiss anti-drag ring. All these rings have the same airfoil section and angle of attack, and give worthwhile improvements in speed. It is appreciated that the installation might not be the best possible in some cases, and that greater performance might be obtained with a modified ring, but these changes would mean that the production program now possible by standardizing on one style ring would be lost.

In regard to accessibility, weight, and cooling, the small cowl ring is definitely superior. In the case of the Curtiss anti-drag ring the loosening of two clamping turnbuckles is the only step necessary to remove the ring. The ring comes off complete in one unit, leaving the uncowled engine clear for ready servicing.

The N.A.C.A. type of cowling is not as readily disposed of when it becomes necessary to service an engine.

The weights of the small cowl ring are also proportionately lower than other types of radial-engine high-speed cowling. The following figures illustrate the weights of Curtiss anti-drag rings for various engines:

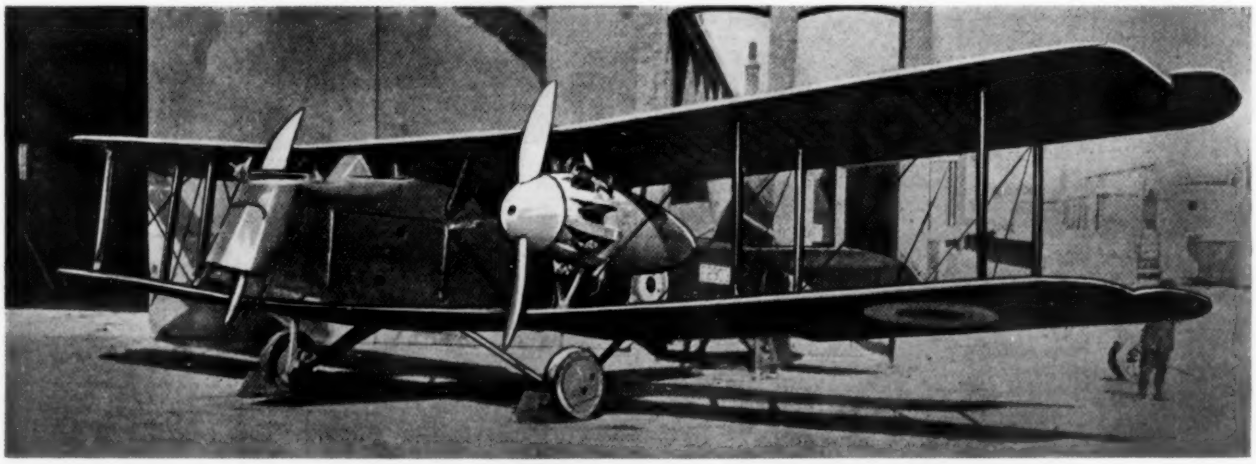

FIG. 7 BOULTON AND PAUL "BOURGES" BOMBER—A. B. C. ENGINES—1919

J-6-7	Whirlwind	16 lb.
J-6-9	Whirlwind	$17^{1}/_{2}$ lb.
	Wasp	$17^{1}/_{2}$ lb.
	Cyclone	$19^{1}/_{2}$ lb.

The weight of the normal engine cowling must, of course, be added to the above, but the figures quoted are a small penalty for the superior performance obtained.

The small cowl ring has no adverse effect of any consequence in regard to engine temperatures. There is a slight increase in head temperatures in some instances, but it is so slight as to cause no concern. With the N.A.C.A. cowling, it is generally necessary to have a rather elaborate baffling system to keep engine temperatures within the allowable limits.

Both types greatly enhance the outward appearance and general lines of whatever aircraft they happen to be installed upon.

DETAILS OF CONSTRUCTION

The Curtiss anti-drag ring is a cowl of the small-ring type constructed from a single cambered sheet of 0.051 aluminum bumped to conform to the upper surface of the Curtiss C-72 airfoil. It has a chord of 12 in. and a uniform angle of attack of —8 deg. A $^{3}/_{8}$-in. outside diameter by 0.035-in. steel tube is rolled into the leading edge as a reinforcement, while the trailing edge is stiffened with a $^{1}/_{8}$-in.-diameter steel wire. The ring is split at the bottom, permitting it to be lifted over the propeller for installation on the engine, and is held in place by clamping it over the engine rocker-arm housings by means of two steel turnbuckles. Fig. 8 illustrates the details of construction. For every rocker-arm housing on the engine a support pad is provided. These supports are made from dural, heat treated, and have two pieces of $^{1}/_{4}$-in. felt pinched into place. These units are then riveted to the cambered shell so that each shall rest on a rocker-arm housing. The ring is reinforced locally where the turnbuckles are attached by 0.064 dural sheets $2^{1}/_{2}$ in. in width. The turnbuckle trunnions engage in heat-treated steel hooks. Lateral rotation of the ring, which would permit the supporting pads to slip off the rocker-arm housings, is prevented by steel fittings having prongs which extend down, and rest against the sides of two different housings. The fabrication of the entire assembly is expedited by the use of Thompson rivets.

The support pads are located as far as possible toward the trailing edge of the ring as, in general, best results are obtained with the ring in the most forward position. This puts the trailing edge approximately at the rear of the rocker-arm housings. Figs. 9 and 10 illustrate one of these rings in place on a Curtiss Helldiver powered with a Cyclone engine. It has been found possible to use practically the same design of support pad for all types of engines, the length in some cases having to be varied. Where the design of the rocker-arm housing was such as to make for poor support, it was found desirable to furnish stampings of sheet brass which could be attached to the engine with the regular inspection-plate screws, this step permitting the use of the standardized support pad.

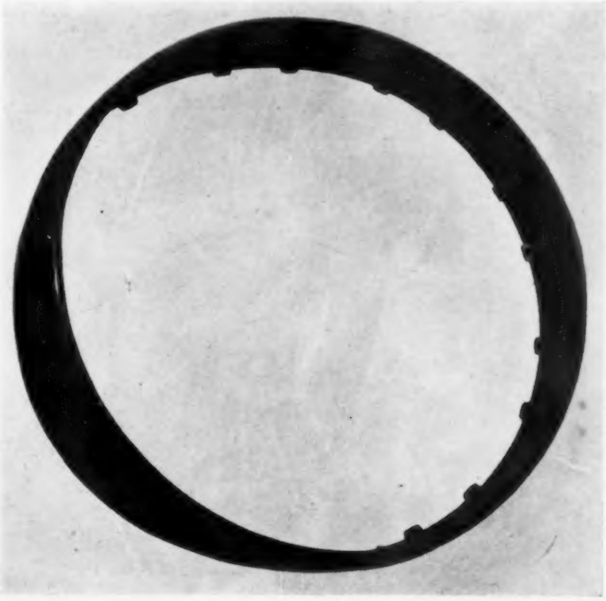

FIG. 8 CURTISS ANTI-DRAG RING

The only precaution to observe in placing the ring in position is not to tighten up too heavily on the turnbuckles. The ring needs only to be snugly fitted, too much tightening tending to distort it from a true circle. The two thicknesses of felt provide plenty of "give" for whatever increase in overall diameter of the engine takes place under operating conditions and at the same time insure room for additional take-up at some future date.

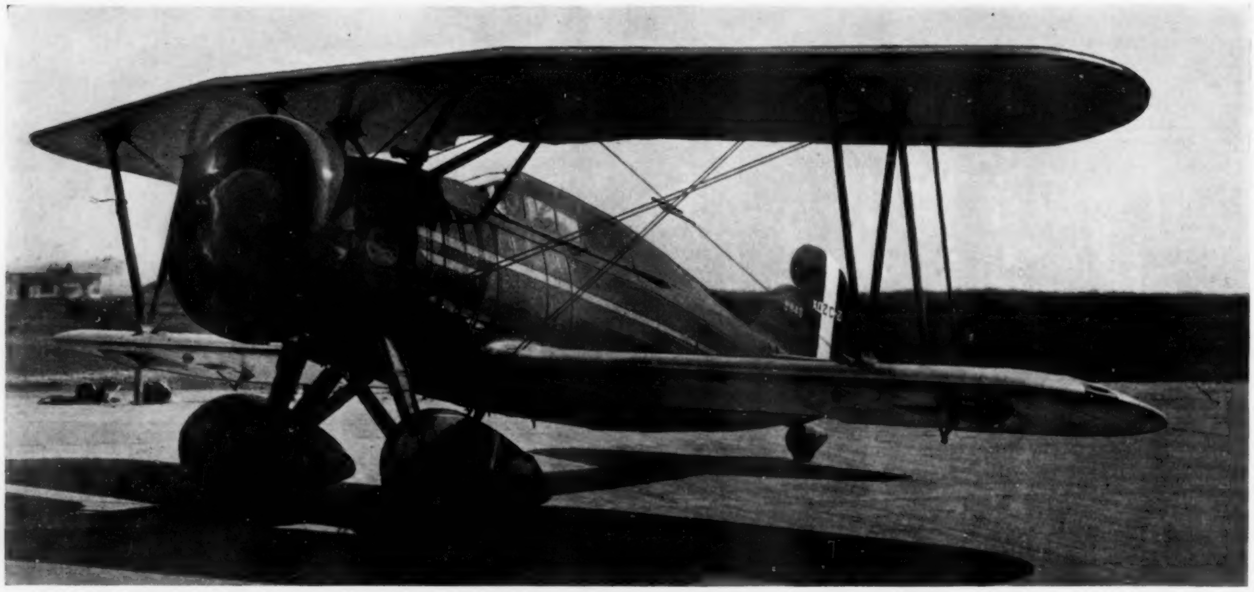

FIG. 10 CURTISS ANTI-DRAG RING INSTALLED ON NAVY HELLDIVER

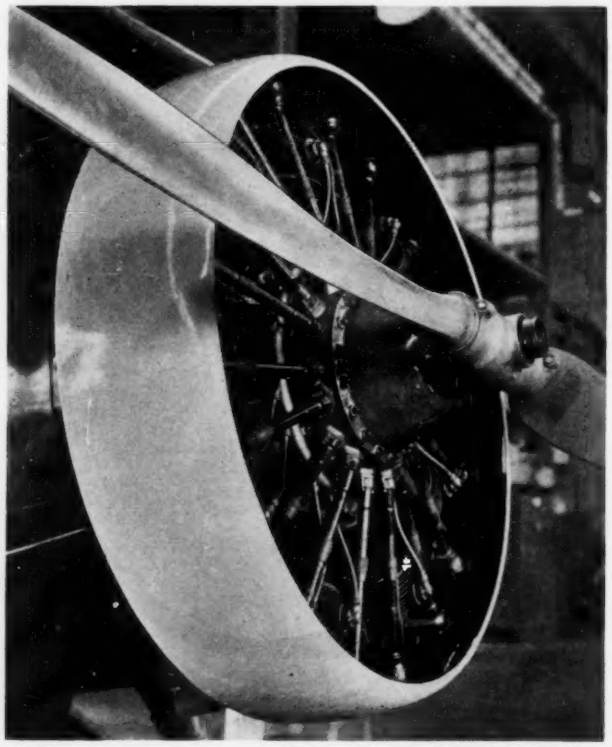

FIG. 9 CURTISS ANTI-DRAG RING INSTALLED ON CYCLONE ENGINE

ECONOMICS

The economic benefits to be derived through the use of engine cowl rings are easily appreciated. For the air-line operator and the private owner doing much cross-country flying their use results in worth-while savings. Higher cruising speeds may be realized, resulting in better schedules and lower gas consumption with reduced fuel costs if the original cruising r.p.m. is retained. Or, on the other hand, if the original cruising speed met the existing requirements satisfactorily the same schedule can be retained at lower engine r.p.m., resulting in reduced fuel costs and longer time intervals between engine overhaul. The realization of a 5 per cent speed improvement in a 135-m.p.h. ship powered with a 240-hp. engine through the installation of an engine cowl ring is equivalent for level-flight conditions to having a power plant of 275 hp.

In the military field the use of engine cowl rings results in increased speeds and greater radius of action, the relative weight of the improved characteristics depending on the particular mission on which the ship is employed.

REFERENCES

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N.A.C.A. Report No. 314—Drag and Cooling With Various Forms of Cowling for a "Whirlwind" Radial Air-Cooled Engine—II, by Fred E. Weick.

R. & M. No. 1267—Reduction of Drag of Radial Engines by the Attachment of Rings of Airfoil Section, by H. C. H. Townend.

Technical Note No. 334—Comparative Performance Obtained With XF7C-1 Airplane Using Several Different Engine Cowlings, by Oscar W. Schey, Ernest Johnson, and Melvin N. Gough.

Technical Note No. 355—Effect on the Angular Position of the Section of a Ring Cowling on the High Speed of an XF73-1 Airplane, by Melvin N. Gough.

Discussion

KARL D. WOOD.[2] The author's statement of the conditions under which the drag saving due to a ring cowl is important may be graphically represented as shown in Fig. 11. Using data from Table 2, the per cent increase in maximum speed of the airplane due to the ring cowling has been plotted against the maximum speed without the cowling. The points scatter somewhat, but exhibit a marked trend, and justify the statement that the faster the airplane, the greater the effect of the ring.

It is worth noting that the author's specific statement regarding the power saving due to the use of a ring may be generalized

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[2] Assistant Professor of Mechanics of Engineering, Cornell University, Ithaca, N. Y.; Engineer, Consolidated Aircraft Corp. Mem. A.S.M.E.

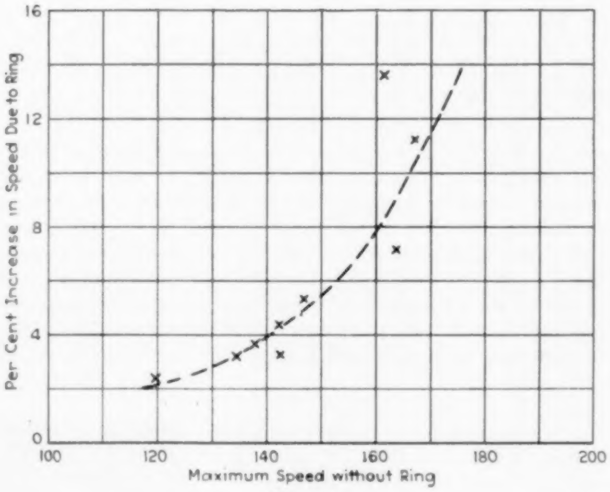

FIG. 11

by saying that 5 per cent increase in top speed permits 15 per cent reduction in power if the speed is not increased. This is consistent with C. B. Millikan's equation for maximum speed $V_{max} = 48.6 \sqrt[3]{(Thp/f)}$, where f is the flat plate equivalent of parasite and profile drag. It can moreover be shown that for most airplanes 15 per cent increase in power will permit carrying 30 per cent more weight at cruising speed. Mr. Boyd's statement that 20 lb. ring weight is small penalty may thus be amplified by saying that for a 2000-lb. airplane of top speed 150 m.p.h., the ring would save power even if it weighed 30 per cent × 2000 = 600 lb. This argument is of course for a given airplane weight and wing area. For a given landing speed the maximum permissible ring weight is more conservative. A general study of this sort, giving an equation for the maximum permissible increase in weight for a given percentage reduction in drag, is presented by the writer in the *Sibley Journal of Engineering* for December, 1931.

G. W. LEWIS.[3] One fact that is distinctly brought out as a result of papers such as this is that some type of cowling for a radial air-cooled engine is desirable. The type of cowling best suited for a given design depends entirely upon the character of the installation, the degree of maintenance necessary, and the desired improvement in the high-speed characteristics of the aircraft.

The National Advisory Committee for Aeronautics, in the investigation of the cowling and cooling of radial air-cooled engines, endeavored to obtain a cowling which would give the minimum of drag, with satisfactory cooling of the engine. The resulting value of N.A.C.A. cowling does not in itself consist of the cowling alone, but of the afterbody of the fuselage or engine nacelle structure. The N.A.C.A. cowling, to give the best results, is necessarily a part of a streamline body. When this type of cowling is used without a streamline body, the decrease in drag is not greater than that obtained with the ring-type cowling.

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[3] Director of Aeronautical Research, National Advisory Committee for Aeronautics, Washington, D. C. Mem. A.S.M.E.

AER-54-13

Engine Ring Cowlings

By W. H. EVERS,[1] BALTIMORE, MD.

The fairing of engines, either with the N.A.C.A. or the Townend type of rings, is becoming more and more the general practice with newly designed or revamped planes. There is a possibility of higher speed without increase of power, combined with a reduction of operating cost. The author points out the differences in the two types of cowling, and gives a description of test results and construction and suggestions for new experiments. Latest changes on several engines show accessories that affect installation of ring cowlings.

THE fairing of radial engines, either with N.A.C.A.- or Townend-type ring cowlings, is becoming more and more the general practice with newly designed and revamped planes, because the possibility of higher speed without increase in power, combined with a reduction in operating cost, is very tempting to every operator and designer.

Although the theory of the N.A.C.A. cowling and that of the Townend-type ring differ, the aim and ultimate result are the same. The reduction of drag, with resulting increase in speed, gives a more efficient plane for the commercial operator and the private owner. The theory of both types of cowlings is most likely known to all aeronautical engineers from publications of the N.A.C.A. and the British Aeronautical Research Committee, etc.

For general information the author will point out briefly the differences between the two types of cowlings before going into a description of test results, construction, and suggestions for new experiments.

The early type of N.A.C.A. ring cowling is designed to fair an engine with a fuselage in a very efficient way and at the same time to guide the air flow for cooling the engine around the cylinders and back into the outside flow by means of a circular opening in front and a slot between the trailing edge and the fuselage at the rear.

The fairing or streamlining of engine and fuselage is producing the major part of the drag decrease, while the minor part is due to the decrease of turbulent area back of the engine, by smoothing out the air flow disturbed by projecting cylinders and other parts, as illustrated in Figs. 1 and 5.

The combined factors give a considerable reduction of parasite drag, which results in the difference of speed between a cowled and an uncowled ship.

Later type N.A.C.A. cowlings are somewhat modified, and very promising results have been obtained with the wide exhaust collector-ring cowling according to T.N. No. 334.

The Townend-type ring cowling differs from the N.A.C.A. cowling in respect to fairing of the engine with the fuselage. The ring cowling is not a fairing, but due to the ring location in the area of divergent and disturbed flow has the same effect as the N.A.C.A. cowling in smoothing out the air flow back of the engine and thereby reducing the turbulent area. This, in turn, produces the minor part of drag decrease, while the major part is due to the difference between the drag of the ring and the forward component of the resulting force created by the high lift of the circular airfoil or ring, as shown in Fig. 4.

Fig. 5 illustrates the air flow over an uncowled engine as observed and measured during test flights. The angle between the thrust line and the direction of air flow between the cylinders and back of the engine was approximately 20 deg. The air flow over an engine with ring cowling is shown in Fig. 6 for comparison with Fig. 5.

The history of these cowlings goes back to the early days of aviation, although at that time the cowlings served primarily as oil-collector rings on rotary engines; but the thought of reducing parasite drag by fairing engine with fuselage entered as far back as 1912, as the author remembers distinctly; otherwise a plain oil-collector ring instead of a complete cowling would have served the purpose.

In 1915, Prof. Hugo Junkers filed a patent which reminds one of a ring cowling, as Fig. 7 shows. Instead of an engine, a radiator was enclosed in an airfoil-shaped ring to reduce the drag, as the description states.

Lincoln Beachey's plane with pusher-engine installation was also equipped with a Townend-type ring cowling, as shown in Fig. 12. The cowling of course was acting as oil guard, but probably increased the efficiency of the plane without Beachey or Curtiss knowing it.

The experimental N.A.C.A. cowlings used for comparison tests with Townend-type rings are shown in Figs. 1, 2, and 3.

Cowling 3 gave the most promising results on a Wasp-powered plane in respect to speed and engine temperature, but the cost of production runs very high on account of the exhaust manifold being a part of the cowling.

Experimental flight tests with N.A.C.A. cowlings 1 and 2 were made in Hornet- and Wasp-powered ships, and cowling 1 proved to be superior in speed to cowling 2. The cause of the difference in drag between the two cowlings is thought due to the fact that in cowling 1 the cooling air is guided back into the outside flow parallel to the outlines of the fuselage without an outward component or deflection of the air flow as observed on cowling 2, which causes a turbulent area at the point of outlet.

Several times the writer has been asked about the superiority of N.A.C.A. or ring cowlings, but the answer depends entirely on the type of plane and the purpose for which the ship is used.

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Presented at the Fifth National Aeronautic Meeting, Baltimore, Md., June 12–14, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

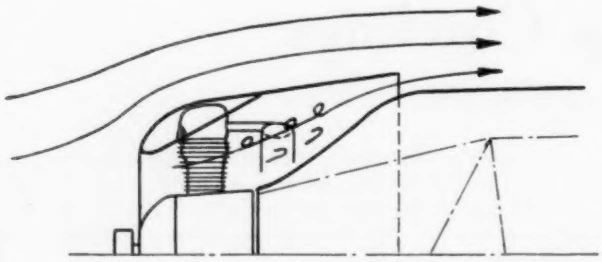

FIG. 1

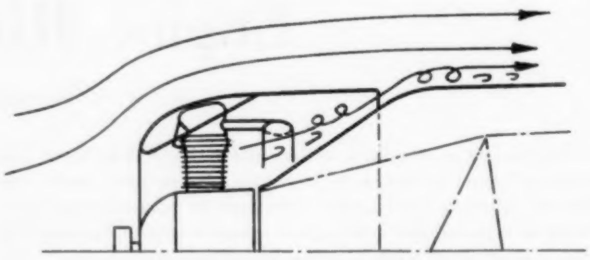

FIG. 2

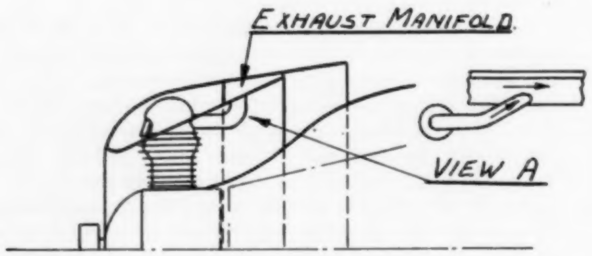

FIG. 3

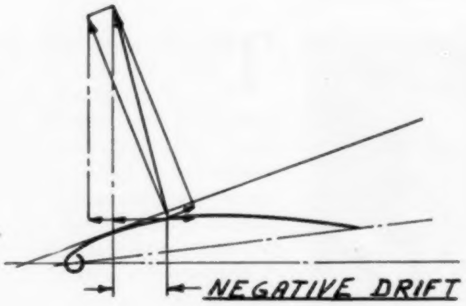

FIG. 4

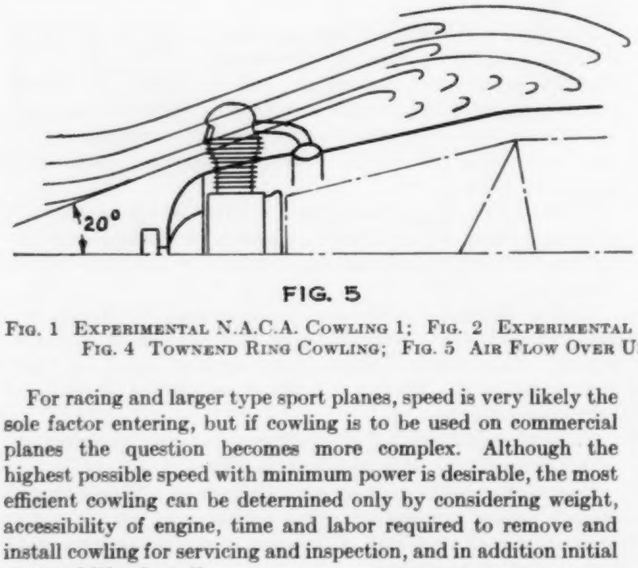

FIG. 5

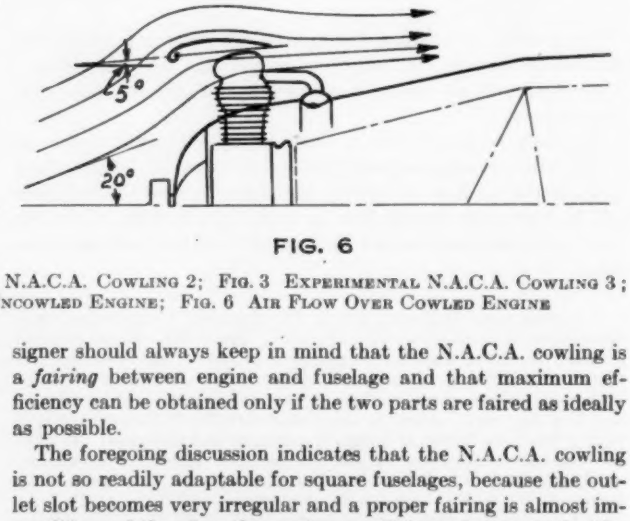

FIG. 6

FIG. 1 EXPERIMENTAL N.A.C.A. COWLING 1; FIG. 2 EXPERIMENTAL N.A.C.A. COWLING 2; FIG. 3 EXPERIMENTAL N.A.C.A. COWLING 3; FIG. 4 TOWNEND RING COWLING; FIG. 5 AIR FLOW OVER UNCOWLED ENGINE; FIG. 6 AIR FLOW OVER COWLED ENGINE

For racing and larger type sport planes, speed is very likely the sole factor entering, but if cowling is to be used on commercial planes the question becomes more complex. Although the highest possible speed with minimum power is desirable, the most efficient cowling can be determined only by considering weight, accessibility of engine, time and labor required to remove and install cowling for servicing and inspection, and in addition initial cost and life of cowling.

Comparison tests with the same ship equipped with N.A.C.A. and ring cowling showed that the increase in speed is greatest with the N.A.C.A. cowling. Both cowlings have the same characteristics—the higher the speed of the plane without cowling the greater will be the speed increase with engine cowling, because the drag reduction is more pronounced on a high-speed ship, on account of the larger ratio of engine drag to total drag.

The front part of the fuselage should be cylindrical or as close to it as possible for N.A.C.A. cowling installation, because this will not only improve the lines of the ship and will please the eye, but will also improve the efficiency of the cowling. The designer should always keep in mind that the N.A.C.A. cowling is a *fairing* between engine and fuselage and that maximum efficiency can be obtained only if the two parts are faired as ideally as possible.

The foregoing discussion indicates that the N.A.C.A. cowling is not so readily adaptable for square fuselages, because the outlet slot becomes very irregular and a proper fairing is almost impossible, and therefore the maximum efficiency is not obtainable. If N.A.C.A. cowling is to be adopted for a new ship, the fuselage should be designed for the cowling to avoid disappointments.

The Townend-type ring cowling on the other hand is readily adaptable for any type and shape of fuselage, and if the ring is designed and located right, the increase in cruising speed should be about 70 to 80 per cent of the speed increase obtainable with the N.A.C.A. cowling. In a series of tests made with N.A.C.A. and ring cowlings, it was found impossible to equal or better the increase in performance obtained with N.A.C.A. cowlings with Townend-type rings.

This proves, at least for the types of ships tested, that the N.A.

C.A. cowling is superior to the Townend-type ring if speed is considered alone, which of course is limited to a few types of planes built for special purposes only.

If weight has to be considered in addition to speed, the superiority of the N.A.C.A. cowling is considerably reduced in favor of the ring cowling. The average weight of a N.A.C.A. cowling for a 400- to 500-hp. engine is about 80 lb., including brackets and braces, as compared to 20 lb., the weight of a complete ring cowling.

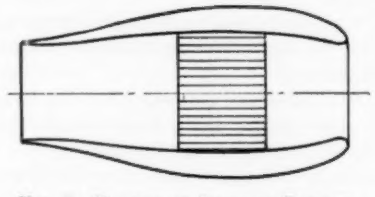

FIG. 7 DESIGN OF JUNKERS PATENT

The ring cowling is therefore 80 — 20 = 60 lb. or 60 ÷ 80 = 75 per cent lighter.

Only under exceptional conditions is it necessary to take into consideration the extra cowling weight, added to boost the speed of the plane, because a reduction in payload of revamped planes is not necessarily the consequence, as the following discussion will show.

In calculating operating cost, the payload is generally estimated to be about 60 per cent of the actual payload, because very seldom is the ship loaded to full capacity, and the weight of the cowling within certain limits is therefore of little consequence. Besides, an increase in empty weight of about 2 per cent of the gross weight is permissible, and in most cases the added weight is within this limit. In addition, the fuel consumption for a given distance is reduced on account of the increase in speed without increase in power, and therefore the amount of fuel in the tanks can be reduced at the starting point, if the load on board should make this necessary.

Almost every run between landings on our air lines is shorter than the maximum range of the plane, and a decrease of $3^1/_2$ gal. of fuel for the ring and 13 gal. for the N.A.C.A. cowling as equivalent weight is possible, without reducing the necessary margin of safety. On new planes originally equipped with engine cowlings or rings, the weight is included in the empty weight of plane.

A positive point in favor of the ring cowling is the accessibility of the engine, which is very poor on N.A.C.A.-cowling installations. Of course this condition can be improved considerably by eliminating the inside bell and hinging the skirt of the cowling so that it will open up like an automobile hood, as shown in Fig. 13.

The time required to remove and install the cowling for engine inspection and servicing is another factor in favor of the ring cowling, as will be shown.

Other features in favor of the Townend-type ring are initial cost and lower depreciation, because the life of the cowlings is considered to be the same.

It is therefore the opinion that the N.A.C.A. cowling is the favorite where speed alone is essential, but for commercial planes, which are flown and thoroughly inspected daily, the ring cowling is surely to be preferred.

The ring cowling hides nothing from the view of the mechanic and inspector, and really has to be removed only when servicing the rocker-arm boxes, which of course means daily, on most of our commercial lines, and these should require as little time and effort as possible.

For revamping of ships in service the ring cowling is in every way superior to the N.A.C.A. cowling, because the low weight does not change the balance. Reworking of the fuselage to suit the ring is not necessary, and therefore the only cost to boost the speed at a reduced cost of operation is the price of the cowling, which again pays for itself in about 100 hours of flying time with a cowling life of at least 1000 hours.

An important question the operators generally ask is about the return on the investment for cowlings.

Let us consider two different ships, a single engine and a trimotor job, to answer the question satisfactorily.

The single-engine ship is equipped with a Hornet and the trimotor job with three Wasp engines.

Performance at cruising speed will be considered only, because it is the speed the commercial operator is interested in.

The single-engine ship equipped with Hornet engine and N.A. C.A. or ring cowling has the following characteristics:

Average cruising speed with conventional cowl, m.p.h.	115.0
Average cruising speed with N.A.C.A. cowl, m.p.h.	120.0
Average cruising speed with ring cowling, m.p.h.	119.0
Average gain with N.A.C.A. cowling m.p.h.	5.0
Average gain with ring cowling m.p.h.	4.0
Payload of ship, lb. m.p.h.	1600
Operating cost per plane per hr. incl. 35% overhead m.p.h.	$42.00[a]
Cost per plane mile (conventional cowling) m.p.h.	$0.365[a]
Cost per pound payload per hour m.p.h.	$0.04375[a]
Cost per pound payload per mile m.p.h.	$0.00038[a]
Initial cost of N.A.C.A. cowling. m.p.h.	$600.00
Initial cost of ring cowling m.p.h.	$130.00
Cowling life in hours of flying time m.p.h.	1000
Depreciation per hour flying time of N.A.C.A. m.p.h.	$0.60
Depreciation per hour flying time of ring m.p.h.	$0.13
Weight of N.A.C.A. cowling, lb. m.p.h.	80.0
Weight of ring cowling, lb. m.p.h.	20.0

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[a] Based on 115-m.p.h. cruising speed and 60 per cent average payload = 960 lb.

The saving per hour of flying time is approximately as follows:

(a) *Equipped with N.A.C.A. Cowling.*		
Saving due to increase in speed = 5 miles at 36.5 cents per plane mile		$1.83
Depreciation per flying hour	$0.60	
Additional labor for servicing engine	0.10	0.70
Total saving per hour of flying time		$1.13
(b) *Equipped with Ring Cowling.*		
Saving due to speed increase = 4 miles at 36.5 cents per plane mile		$1.46
Depreciation per flying hour	$0.13	
Additional labor for servicing engine	0.05	0.18
Total saving per hour flying time		$1.28

At a first glance the saving in operating cost and time may seem very small, but if daily operation of several planes is considered, the matter begins to look different.

Now let us consider a trimotor ship equipped with three Wasp engines and with three ring cowlings of the following characteristics:

Average cruising speed with conventional cowl, m.p.h.	119.5
Average cruising speed with ring cowling, m.p.h.	126.0
Average gain with ring cowling, m.p.h.	6.5
Payload of ship, lb. m.p.h.	2700
Operating cost per plane per hr. incl. 35% overhead m.p.h.	$87.00[a]
Cost per plane mile (conventional cowling) m.p.h.	$0.73[a]
Cost per pound of payload per hour m.p.h	$0.0537[a]
Cost per pound of payload per mile	$0.00045[a]
Initial cost of three ring cowlings	$375.00
Cowling life in hours of flying time	1000
Depreciation per hour flying time	$0.375
Weight of three ring cowlings, lb.	60.00

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[a] Based on 119.5-m.p.h. cruising speed and 60 per cent average payload = 1620 lb.

The savings per hour of flying time is approximately as follows:

Saving due to increase in speed = 6.5 miles at 73 cents per plane mile		$4.75
Depreciation per flying hour,	$0.38	
Additional labor for servicing engine	0.15	0.53
Total saving per hour of flying time		$4.22

These figures, based on actual test results, show that ring cowlings on single-engine and trimotor ships pay for themselves in about 100 flying hours. After that the operator really gets something for nothing, because the increase in speed is connected with a decrease in operating cost of from one to four dollars per flying hour.

It is probably interesting to know that Western Air Express ships equipped with ring cowlings lowered the flying time between Kansas City and Los Angeles by one hour on the 14-hour trip.

The life of the ring cowling in the foregoing calculation is estimated very conservatively and is based on inspection of cowlings more than 600 flying hours in service, which are still without any sign of fatigue.

Another point of interest, at least to operators, is the fact that by cruising against headwinds the reduction in speed is decreased on cowling-equipped planes, due to the higher initial speed, as the following figures show:

Cruising speed with conventional cowling, m.p.h.	119.5
Cruising speed with ring cowling, m.p.h.	126.0
Headwind, m.p.h.	20.0

The speed reduction of the first plane is 20 ÷ 1.195 = 16.7 per cent, as compared with the decrease on the second plane of only 20 ÷ 1.26 = 15.8 per cent.

The cooling of the engine is not affected by ring-cowling installations; in fact, on several test flights and certain cowling locations a slight temperature reduction was observed; but there is one exception, and that is a 2-to-1-geared radial engine of 500 hp. which was used on several experimental flights.

With ring cowling installed, the temperatures increased very rapidly, and several wide-open throttle flights had to be discontinued after 4 to 5 minutes' climb on account of excessive temperatures. After removing the cowling the temperatures could be held within certain limits, although higher than on straight-drive radial engines.

The foregoing experience was most likely due to the wrong location of the cowling in relation to the propeller, because the cowling was designed for a straight-drive engine on which the leading edge of the cowling is much closer to the propeller. But this conclusion could not be checked by moving the ring forward, because the test with this engine had to be discontinued for other reasons.

During ring tests and experiments with N.A.C.A. and ring cowlings it was found absolutely essential to have a properly designed temperature control of the mixture to prevent high cylinder temperatures on days with outside temperatures below 60 deg. fahr.

With hot spot in the "on" position and cylinders 8 and 9 exhausting, through the hot spot, the cylinder temperatures dropped about 30 deg. fahr. on head and 10 deg. fahr. on base. At the same time the engine began to smooth out and the vibrations disappeared, because the evaporation of fuel and distribution of gas improved, and with it the irregularity in mixture. If the carburetor preheater was closed in addition to the hot spot, keeping the engine r.p.m. constant, the temperatures again dropped an additional 20 deg. on head and 20 deg. on base within 2 min.

Cowling tests on six different ships from 300 to 2300 hp. made with special attention to flight characteristics showed no changes in static or dynamic stability, and no change on controllability, either in stall or dive. According to information received, it seems that certain ships have become unstable after ring-cowling installation, probably due to change in air flow, and therefore the Department of Commerce requires a flight test on every type of ship to which a ring cowling is added, after issue date of A.T.C.

The ring cowlings used for comparison tests with the N.A.C.A. cowlings were designed on lines following the investigation of Townend.

The airfoil-shaped rings had a chord length of 12 and 14 in., set at an angle of incidence of —5 deg. and —8 deg., equal to an angle of attack of about 15 deg., the stalling point of the profile used being at about 17 deg.

In selecting the airfoil for the ring cowling, it was kept in mind that the most efficient section is the one which has a high L/D at the stalling point.

The angle of incidence was calculated after the angle at which the airflow over the engine diverges had been determined by tests and measurements.

No difference in efficiency between 12-in.- and 14-in.-wide ring cowlings was noted during tests; therefore the 12-in. section is to be preferred on account of lighter weight.

For maximum efficiency it is not advisable to use one type of ring cowling for different types of ships, but a good average speed increase can be obtained by using a ring of about 12-in. chord and —5- to —8-deg. angle of incidence, as experimental test flights indicate.

Another factor of importance is the location of the ring cowling in relation to the propeller trailing edge, which seems to vary with the speed of the plane and the shape of the nose.

The gap between the trailing edge of the propeller blade and the leading edge of the ring cowling should be between 4 and 5 in. for the best performance.

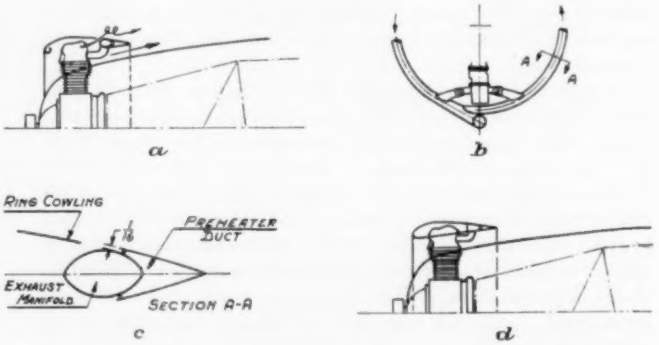

Fig. 8 Type of Ring Cowling as Designed in 1929 and as It Might Be Reworked

In positions with decreased and increased gap, a drop in speed up to 2 per cent was observed.

For new designs the location and the most efficient angle of incidence of the ring cowling should be determined in the wind tunnel to get maximum performance.

The same airfoil section can be used on all cowlings, but it should be remembered that the most efficient wing profile is not always the best for the ring cowling, as has been stated before.

It is not possible to give complete data of all tests and observations made within the limited space of this paper, but the following experience is most likely of general interest.

On some of the experimental flights it was quite annoying when exhaust gases were forced into the closed cockpit after ring installation. This phenomena was caused by the convergence of air flow after the ring was installed, and therefore leaky exhaust manifolds should not be tolerated, for the benefit of the pilot, whenever the plane is equipped with ring cowling.

Following is a brief description of the construction of our ring cowling, developed from flight tests and observations made in the field and shop and in handling cowling for engine servicing.

Fig. 14 shows a close-up of a Hornet ring cowling of 12-in. chord. The ring itself is quickly detachable, and can be removed in about 5 min. for servicing rocker-arm boxes and inspection of engine. Reinstallation of ring takes from 6 to 8 min., including time for safety wiring, by one man without extra help. The construction of the ring in three sections makes complete removing very easy. The parts to handle are small and weigh only 5 to 6 lb. each, depending on size of engine.

Our first experimental rings were made in one piece, and on account of the large diameter were very awkward to handle, especially in the field. When not completely removed, the engine was partly covered, which made servicing much more difficult than with the present three-piece ring.

To remove the ring the three wing nuts are backed up and the steel straps are thrown forward. After removing the self-locking nut, each section can be removed individually, and the cowling is completely out of the way.

Figs. 8*A*, 8*B*, and 8*C* show another type of ring cowling designed in 1929 and tested for comparison with other cowlings described before.

The exhaust manifold with fairing is part of the ring. The fairing forming the trailing edge of the cowling is used to supply preheated air to the carburetor as shown in Figs. 8*B* and 8*C*. The front part of the cowling is not connected to the manifold, as can be seen from Fig. 8*A*.

Part of the cooling air escapes through the slot to circulate around and cool the manifold. The speed increase with this type of cowling was about the same as with plain-ring cowlings used on previous tests. It was observed during flight tests that the cooling air leaving the forward slot has an outward component which creates a large turbulent area all around the ring.

If the ring is reworked, as in Fig. 8*D*, the flaring out of the air flow is eliminated, and the performance should be considerably better. The dissipation of the heat of the exhaust manifold should be sufficient, because more than half of the surface is exposed to the cooling air. Of course the speed of the ship must improve considerably to compensate for the higher weight and the cost of this cowling compared with the plain ring; otherwise the installation is not warrantable, even on a newly designed ship.

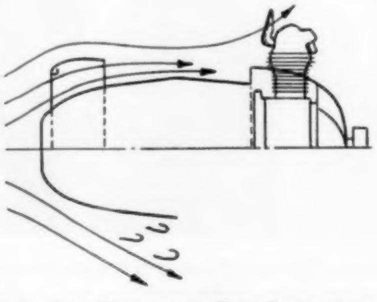

FIG. 9 RING COWLING TO TEST PERFORMANCE INCREASE OF PUSHER INSTALLATION

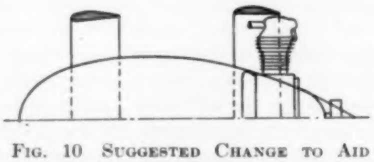

FIG. 10 SUGGESTED CHANGE TO AID COOLING EFFICIENCY

To gain data on the possible performance increase of a pusher installation, tests and experimental flights have been made with a ring cowling installation as shown in Fig. 9. Most nacelles on pusher installations are generally not perfectly streamlined on account of structural interference, oil-tank installation, etc., which prevent the designer from getting the most efficient finesse ratio of cowling.

Preliminary investigation of the nacelle cowling used for test flights showed that the air flow at the cowling nose diverged at an angle of about 20 deg. from the normal direction, creating a turbulent area ahead of the engine as shown in the lower half of Fig. 9.

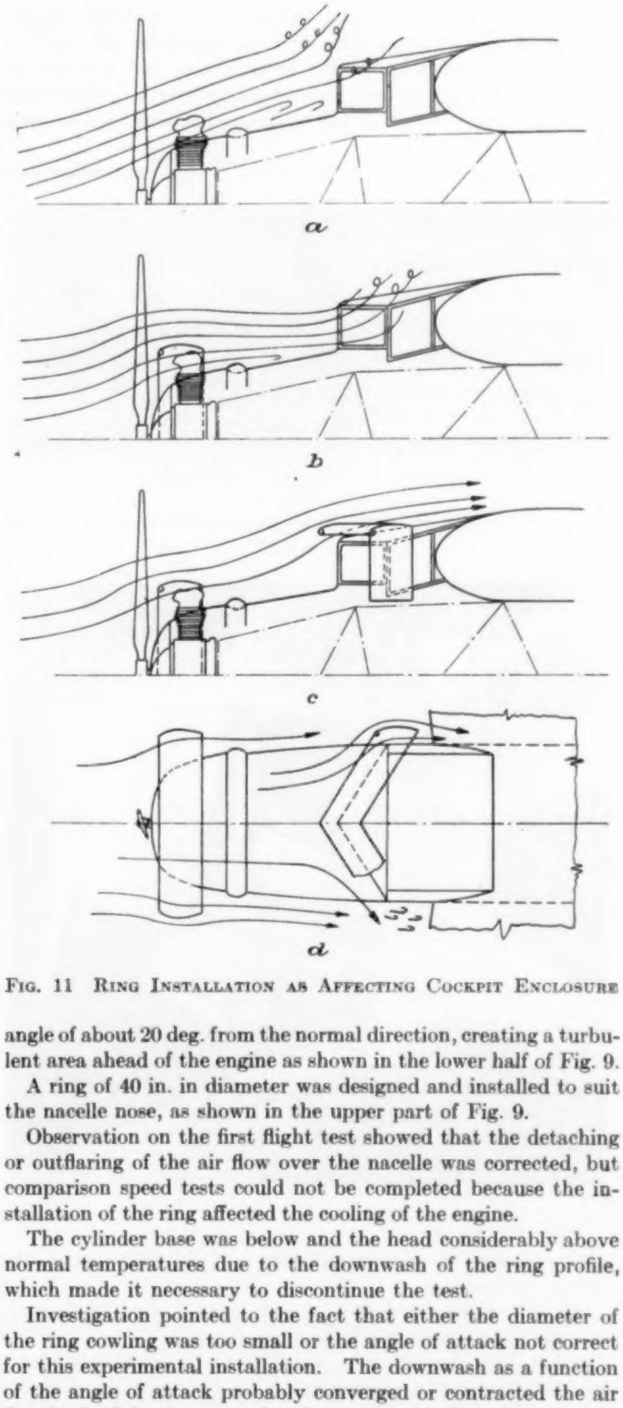

FIG. 11 RING INSTALLATION AS AFFECTING COCKPIT ENCLOSURE

A ring of 40 in. in diameter was designed and installed to suit the nacelle nose, as shown in the upper part of Fig. 9.

Observation on the first flight test showed that the detaching or outflaring of the air flow over the nacelle was corrected, but comparison speed tests could not be completed because the installation of the ring affected the cooling of the engine.

The cylinder base was below and the head considerably above normal temperatures due to the downwash of the ring profile, which made it necessary to discontinue the test.

Investigation pointed to the fact that either the diameter of the ring cowling was too small or the angle of attack not correct for this experimental installation. The downwash as a function of the angle of attack probably converged or contracted the air flow back of the ring to such a degree that it affected the cooling of the cylinder heads as observed during test flights.

Another cause for overheating of cylinder heads is illustrated in the upper part of Fig. 9.

The outer layers of the air flow around the nacelle cowling, between the ring cowling and the engine, probably detaches from the inner layer and flares out again due to the propeller action, and thereby decreases the cooling efficiency. A change of this angle of incidence or the installation of a second ring for cooling purposes only, just forward of cylinders, as shown in Fig. 10, probably would have corrected the trouble.

A check has been made in the wind tunnel which shows that the installation of two rings with different angles of incidence is superior to one ring on pusher installations, but conclusions cannot be reached until tests have been terminated. Flight tests will be made soon to check the actual performance and cooling efficiency.

The experiments with a ring installation on the nacelle nose point the way to another problem spot on cabin ships, the cockpit enclosure.

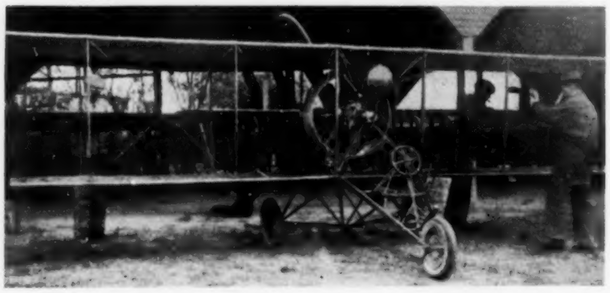

FIG. 12 "CURTISS PUSHER," LINCOLN BEACHEY'S PLANE, WITH ENGINE RING

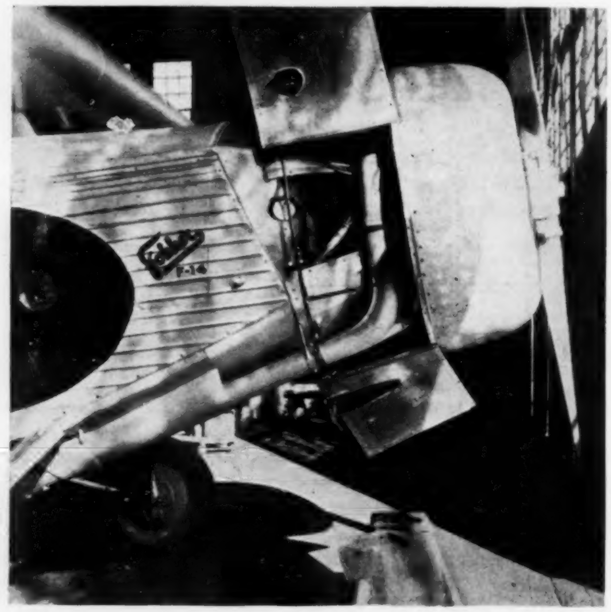

FIG. 13 EXPERIMENTAL N.A.C.A. COWLING FOR HORNET ENGINE

Fig. 11*A* shows the air flow over the front part of the fuselage, with the cockpit enclosure in a typical location.

In Fig. 11*B* the probable direction of the air flow is shown after installation of a ring cowling. The flow over the nose and the engine has improved, but diverges again on top of the fuselage by the vertical or sloped windshield of the cockpit.

By installing an airfoil-shaped cowling over the cockpit, as shown in Figs. 11*C* and 11*D*, it is likely that the same results can be obtained as with a ring around an engine or pusher nacelle nose. Of course the general direction of air flow over the windshield of cockpit enclosure would have to be determined first, either in a tunnel or actual flights, before it would be possible to fix the most efficient angle of incidence of the top and side airfoil.

It is likely that the angles of incidence of the two airfoils will be different depending entirely on the angle of outflare in relation to normal flow, parallel to the thrust line.

If the performance increase on actual flight tests warrants the adoption of this type of cowling, a transparent material can be used for manufacturing the airfoils to prevent impairing the visibility of the pilot upward or to the sides. Actual tests with this type of cowling will be made soon.

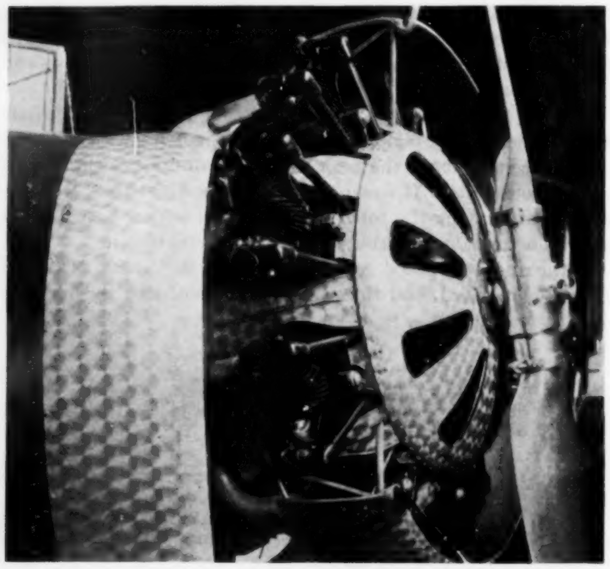

FIG. 14 RING COWLING FOR HORNET B ENGINE, WITH ONE SECTION REMOVED

The value of engine-ring cowlings on new planes or as additional equipment to planes in service has been proved in experimental tests and in flights over our airlines, as test data show.

Latest changes on several engines show accessories that almost prevent installation of ring cowlings or at least make installation complicated and reduce the efficiency considerably.

The interfering parts, such as exhaust manifolds, carburetor heaters, etc., should be designed by the engine manufacturers for ring-cowling installation, because it is very difficult to design a ring to clear these parts and still maintain maximum efficiency and rigidity combined with quick detachable features.

It is hoped that the foregoing suggestions will guide engine designers and manufacturers in the future when changes of parts are contemplated, interfering at present with ring-cowling installation.

AER-54-14

Some Effects of Corrosion Fatigue on Streamline Wire for Aircraft[1]

By HUGH E. HAVEN,[2] ANNAPOLIS, MD

Several years ago aircraft designers began using what is commonly known now as "streamline wire" in the construction of tension members of aircraft. Wires of approximately elliptical section were adopted for the purpose of reducing parasite resistance. These wires are subjected to vibrations of varying frequencies, believed to be mainly of the torsional type. They are also under a pure tension stress when the plane is in dead air or on the ground. In addition to the aforementioned stresses, the wires are always subject to the very deleterious effects of corrosion. Corrosion, obviously, is a serious factor where naval or sea planes are concerned. Some of the effects of the combination of these conditions are discussed in this paper, in which the author presents data thus far obtained in an investigation at the Engineering Experiment Station, Annapolis, Md.

THE ALTERNATING-torsion machines used in this investigation were developed at the Engineering Experiment Station. Essentially, the machines are modifications of the Upton-Lewis alternating-bend machines. The initial purpose of the modified machines was to subject cylindrical specimens to alternating torsion. Later, changes were made so that streamline wire of approximately elliptical cross-section could be tested. For this purpose grips were designed to secure the specimen in proper alignment with the axis of the horizontal driving shaft of the machine. The rotation of the specimen is resisted by opposing springs, and a torsional moment is thus applied to the specimen. Since this torsional moment is proportional to the strength of the springs and their displacement, the nominal maximum stress on the specimen can be calculated.

Fig. 1 shows the machine. In this the arm *C* is caused to oscillate by means of an adjustable eccentric attachment to the rotating wheel *A*, thus rocking the shaft *D*. The driving grip *E* is rigidly attached to the end of the shaft *D*, and the specimen *F* is clamped in it and in the driven grip *G*. Alternating rotation of the shaft *D* is thus transmitted through the specimen to the spring arm *O*. Since this rotation is resisted by the action of the springs *J* on the arm *O*, an alternating torsional stress is applied to the specimen which can be varied by altering the throw of the eccentric attachment on wheel *A*. The range of stress can be varied by means of the turnbuckle *B*, and in the present investigation lay between a maximum positive value and an equal maximum negative value.

The machine is driven by a constant-speed motor at the low speed of 360 r.p.m. through the belt *M*. The large flywheel on the shaft helps to maintain a constant speed. A pencil record of the stress is made by the mechanism shown at *L*. The total number of cycles is counted on the revolution counter *N*. Fig. 2 is an enlarged detail of the grips and specimen. No attempt is made to polish the specimens, and they are tested in the "as-received" condition.

Determination of Stress

In this test the torsional stress depends on two variables: (1) the torsional moment due to the action of the springs on a lever arm, and (2) the moment due to inertia of the oscillatory parts. The stress due to the former will be designated S and that due to the latter S_1. The stresses in a wire of elliptical cross-section are calculated by the following formula, which holds within the elastic limit:

$$T_1 = \frac{\pi S a b^2}{2} \qquad [1]$$

where T_1 = torque in in-lb.
S = maximum shearing stress in lb. per sq. in.
a = semi-major axis in inches
b = semi-minor axis in inches.

As alternating torsional stresses are applied to the specimen due to the action of a lever arm on the springs and also as a result of inertia of the oscillating parts of the machine, the torsional moment T due to the latter, in absolute units, is expressed by the equation:

$$T = \frac{4\pi^2 I \tan\theta}{t^2} = 4\pi^2 I n^2 \tan\theta$$

where I = moment of inertia of oscillating parts
θ = amplitude of angular oscillation in degrees
n = frequency of oscillation = $1/t$
t = time in seconds for one revolution of wheel A.

To express the value of the torsional moment T in inch-pounds, the equation must be multiplied by 1/384, which expresses the relationship between absolute units and inch-pounds.

Then:

$$T_1 = \frac{\pi^2 I n^2 \tan\theta}{96} \qquad [2]$$

where T_1 = torsional moment in in-lb.

Equating [1] and [2], and replacing S by S_2 where S_2 = stress

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[1] Opinions or assertions contained in the article are the private ones of the writer and are not to be construed as official or reflecting the views of the Navy Department or the Naval Service at large.

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Presented at the National Aeronautic Meeting, Baltimore, Md., May 12 to 14, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers, are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 RESULTS OF TENSION TESTS

Material letters	Original dimensions, in.	Ultimate strength, lb.	Tensile strength	Yield point	Johnson's limit	Proportional limit	Elongation	Reduction of area
			lb. per sq. in.				per cent	
LCA	0.185 × 0.750	15,355	145,900	133,900	109,250	61,400	8.2	36.2
KXA	0.185 × 0.750	20,670	188,200	185,900	.....	....	6.5	46.9
KZA	0.162 × 0.640	12,250	151,200	131,300	107,700	53,250	7.5	39.3
LXA	0.190 × 0.690	23,250	225,900	179,700	149,800	66,000	7.8	41.7
LXA 17.5	0.190 × 0.690	11,500	111,600	35,000	25,900	21,300	50.3	63.6
LYA	0.185 × 0.748	17,800	161,300	155,700	.....	....	7.7	40.8

Material LXA 17.5 was heat-treated to 1750 deg. fahr., held for 1/2 hr., and allowed to cool in the furnace.

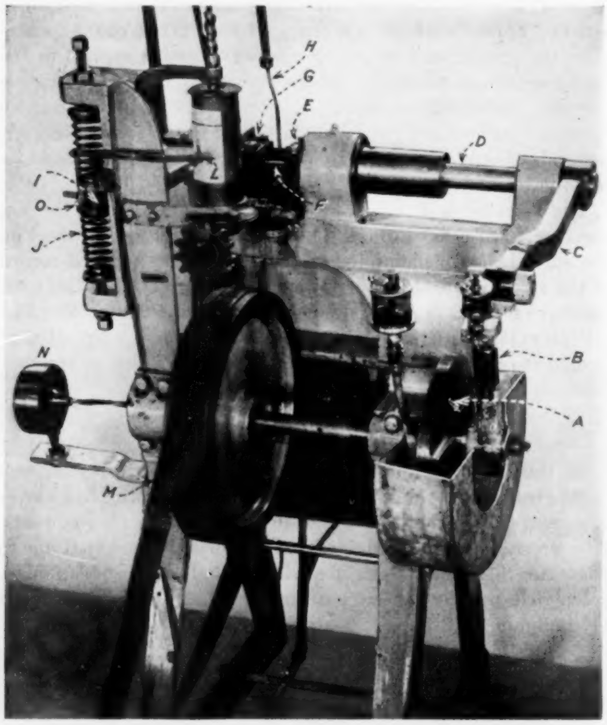

FIG. 1 MODIFIED UPTON-LEWIS ALTERNATING TORSION TESTING MACHINE

A,	Adjustable eccentric drive wheel.	*H*,	Corroding-medium supply line.
B,	Turnbuckle adjustment of stress range.	*I*,	Spring support.
C,	Horizontal driving arm.	*J*,	Calibrated springs.
D,	Horizontal driving shaft.	*L*,	Pencil recording mechanism.
E,	Right-hand specimen grip.	*M*,	Main drive from constant speed motor.
F,	Specimen.	*N*,	Revolution counter.
G,	Left-hand specimen grip.	*O*,	Arm.

in an elliptical specimen in pounds per square inch due to the inertia of the oscillating machine parts:

$$S_2 = \frac{\pi I n^2 \tan\theta}{48\, ab^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [3]$$

Since the stresses in the specimen due to inertia are proportional to the square of the frequency, it is imperative that these stresses be kept small in comparison with the stresses produced by the action of the springs. A slight increase in the speed of the machine greatly increases the stress in the specimen. The present machines are not adapted for testing the smaller sizes of streamline rods due to the large inertia stresses.

TABLE 2 CHEMICAL COMPOSITION OF MATERIALS TESTED

Material letters	C	S	P	Mn	Si	Ni	Cu	Cr	Fe
LC	0.09	0.012	0.007	0.45	0.16	0.53	0.09	12.63	[b]
KX	0.13	0.009	0.008	0.35	0.33	8.45	0.14	18.83	[b]
KZ	0.08	0.008	0.006	0.008	1.86	68.13	29.34	[a]	0.55
LX	0.10	0.012	[a]	0.51	0.56	[a]	[a]	18.34	[b]
LY[c]	0.48	0.020	[a]	0.63	0.14	0.01	[a]	0.058	[b]

[a] Not determined.
[b] Remainder.
[c] Electroplated with cadmium.

Admittedly, the section of a streamline wire is not a true ellipse, but the foregoing formulas are given to show how the stresses which have been plotted on the graphs, Fig. 3, are computed. Detailed information as to these stress computations may be obtained upon application to the Engineering Experiment Station, Annapolis, Md.

MATERIALS

The tension results of the materials tested to date are given in Table 1. To facilitate their identification, each material has been arbitrarily assigned letters. In each case "A" after the letters indicates the material in the streamline-wire form.

In tensile tests for a few of the sizes of wire it was impossible to use the extensometer. In no case was it possible to release the load and return to zero, due to the fact that grips were used to hold the specimens.

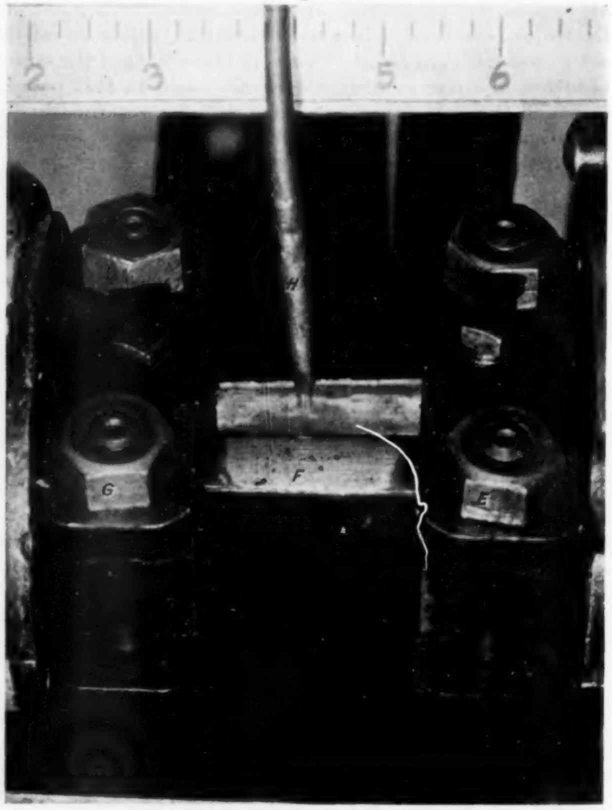

FIG. 2 GRIPS AND SPECIMEN

(*E*, Right-hand specimen grip; *F*, Specimen; *G*, Left-hand specimen grip; *H*, Corroding-medium supply line.)

The chemical compositions of the various materials are given in Table 2.

ALTERNATING TORSION TESTS

Graphs showing the results of tests of streamline wire in alternating torsion are shown in Fig. 3. In these graphs the ordinates represent the half-stress range and the abscissas represent cycles on a logarithmic scale. Each graph has its own scale of abscissas, which are indicated in numerals in the vicinity of the respective plotted points. The decimal figures at the bottom of each graph indicate the average dimensions of the individual specimens in inches.

Under alternating torsion the directions of the principal tensile and compression stresses make an angle of 45 deg. with the axis of the specimen. The directions of the principal shear stress, however, are longitudinal and transverse. Examination of the surfaces of fracture of failed streamline-wire specimens has shown the region of maximum stress to be at the end of the minor axis, as would be expected.

In Fig. 3 the results of ordinary endurance tests in air are indicated by the small circles, and the stress-cycle relationship by the curved solid lines.

The results of corrosion fatigue in carbonate water are indicated by triangles. The fatigue graphs are typical ones, as they do not differ in any particular way from results obtained with rotating-cantilever specimens. They consist of curves descending toward a horizontal asymptote, the ordinate of which represents the endurance limit.

The alternating-torsion endurance limit for the LCA material is approximately 36,500 lb. per sq. in. The alternating-torsion corrosion-fatigue limit for the same material is about 18,000 lb. per sq. in. In this material considerable scatter of the experimental points of failure of the specimens under alternating-torsion corrosion fatigue will be noted.

Although material LXA has the highest physical properties of any of the groups reported in this paper, it appears to have a materially lower torsional-endurance strength. Too few specimens of this material have been tested to make it possible to determine definitely the torsional-endurance or corrosion-fatigue limits, but the former is estimated to be somewhat above 20,000 lb. per sq. in. Since the alternating-torsion endurance limit depends on the transverse as well as the longitudinal strength of the metal, it is probable that the abrupt descent of the graph of material LXA is due to the lack of transverse strength. Tensile results have shown that the material has high strength longitudinally.

The graph in the upper right-hand corner of Fig. 3 presents the endurance and corrosion-fatigue results of material LXA annealed at 1750 deg. fahr., all strain effects of cold working being thus relieved. The stresses represented in this graph are only nominal, since working due to the testing process itself has undoubtedly increased the elastic values of the material over its primitive ones. However, these results seem to indicate that some relief of the severe working strains found in material LXA is desirable for sections subjected to alternating torsion. Additional specimens of this material, drawn at 1200 deg. fahr., are to be tested.

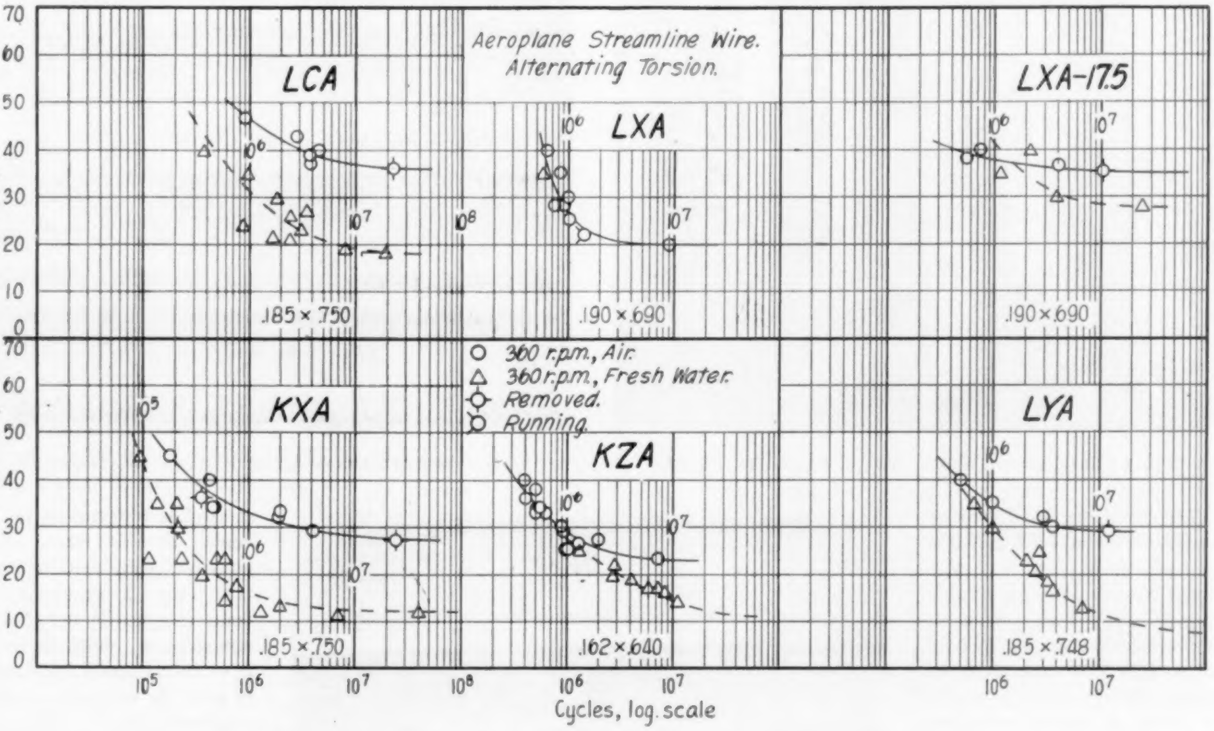

FIG. 3 RESULTS OF TESTS OF STREAMLINE WIRE IN ALTERNATING TORSION

The torsional-endurance limit for material KXA is 27,500 lb. per sq. in. The torsional corrosion-fatigue limit is 12,000 lb. per sq. in. Attention is called to the scatter of the experimental points of failure of the specimens.

Material KZA has a torsional-endurance limit of 23,000 lb. per sq. in. and its corrosion-fatigue limit is 11,000 lb. per sq. in. It will be noted that there is much less scatter of the experimental points than for the LCA and LXA materials.

TABLE 3 RESULTS OF PRINCIPAL PHYSICAL AND FATIGUE TESTS

Material letters	Ultimate strength, lb.	Tensile strength	Yield point	Johnson's limit	Proportional limit	Torsional endurance limit	Torsional corrosion-fatigue limit	Elongation	Reduction of area
		per sq. in.						per cent	
LCA	15,355	145,900	133,900	109,250	61,400	36,500	18,000	8.2	36.2
KXA	20,670	188,200	185,900	.....	....	27,500	12,000	6.5	46.9
KZA	12,250	151,200	131,300	107,700	53,250	23,000	11,000	7.5	39.3
LXA	23,250	225,900	179,700	149,800	66,000	[a]	[a]	7.8	41.7
LXA 17.5	11,500	111,600	35,000	25,900	21,300	35,000	27,500	50.3	63.6
LYA	17,800	161,300	155,700	.....	....	29,000	7,500	7.7	40.8

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[a] Not yet determined.

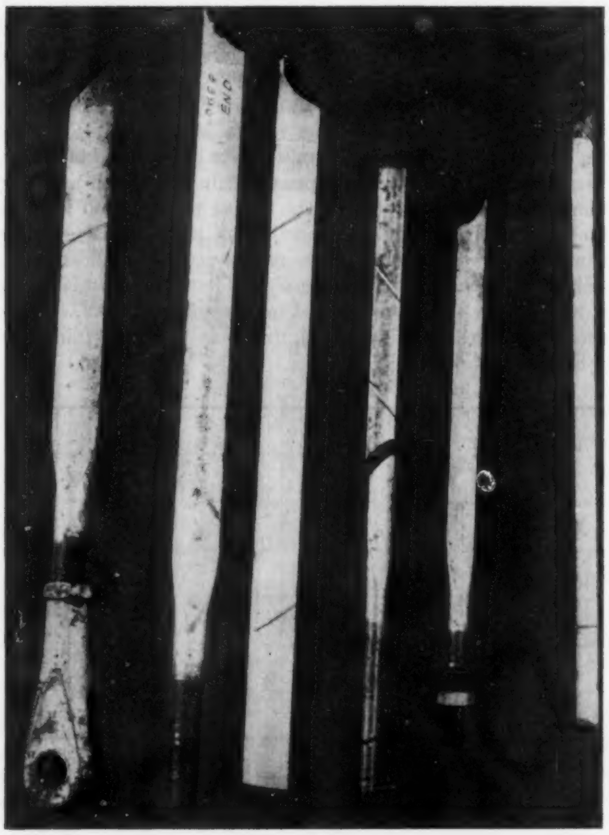

FIG. 4 STREAMLINE WIRES WHICH FAILED IN SERVICE

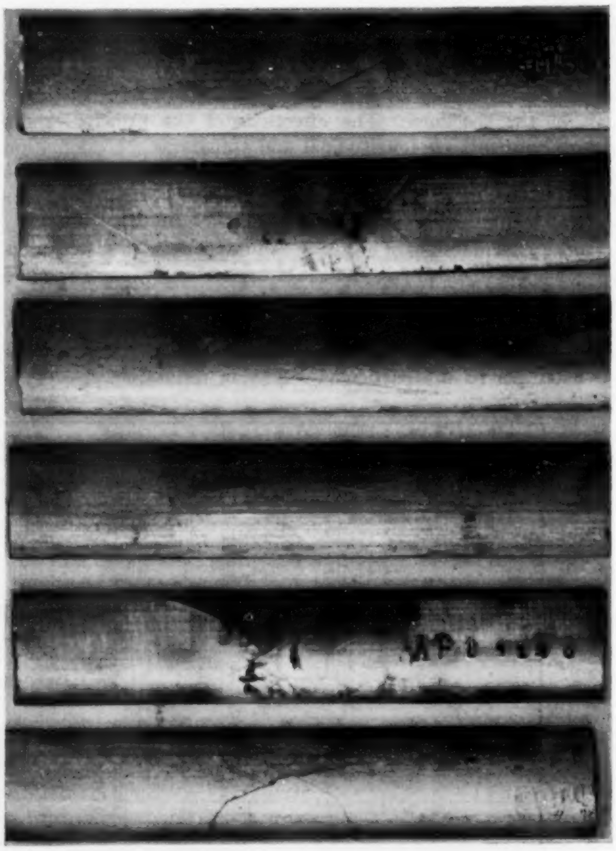

FIG. 5 TEST SPECIMENS AFTER FAILURE—MATERIAL LCA

The torsional-endurance limit for material LYA is 29,000 lb. per sq. in., and the torsional corrosion-fatigue limit is comparatively low, having an estimated value of only 7500 lb. per sq. in. This material shows little or no scatter in its points of failure in either endurance or corrosion fatigue. In Table 2 the LYA material is seen to be a carbon steel of approximately 0.5 carbon. The surface of the elliptical section electroplated with cadmium, but this coating has given little or no protection to the material in corrosion fatigue in alternating torsion.

Table 3 gives a ready means of comparing the physical and fatigue properties of these materials. As seen from this table, a high ultimate or tensile strength does not indicate a high torsional-endurance or corrosion-fatigue limit.

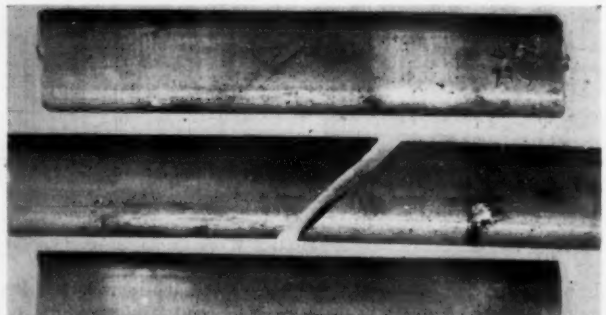

FIG. 6 TEST SPECIMENS AFTER FAILURE—MATERIAL KXA

As no experiments have been made with round rods, the effects of cold working on the streamline wire cannot be discussed at this time. Since, however, cold working raises both the longitudinal and transverse tensile and shearing strengths of sheet materials, it would naturally be expected to raise the alternating-torsion endurance limits. The only property, longitudinal or transverse, that is not increased by cold working is ductility; instead it is especially decreased by cold working. Ductility seems to affect the alternating-torsion endurance and corrosion fatigue limits very materially, as shown by a comparison of the two materials LXA 17.5 and KZA.

From the work so far completed it appears that high tensile strength has been accompanied by a reduction in the shear strength of the materials. It is believed that a correct balance between the amount of cold working and heat treatment of the material will appreciably increase its resistance to alternating torsion. An investigation of the effects of heat treatment is indicated by the marked change in properties between materials LXA and LXA 17.5.

The wide scatter of points of failure in torsional fatigue of materials LCA and KXA as compared with the little or no scatter in materials KZA and LYA, is believed to be caused by lack of uniformity of material, even in the same wire, and to the presence of internal strains in the wire due to cold working.

Several samples of streamline wire which have failed in service are shown in Fig. 4. The material in these samples is a high-strength carbon steel, coated with aluminum paint. It should be noted that the failures in all cases occur at an angle of approximately 45 deg. with the longitudinal axis of the wire.

Several failures of test specimens of materials LCA, KXA, and KZA are shown in Figs. 5, 6, and 7, respectively. In these test specimens the failures have occurred at an angle of 45 deg. to the axes, as is found in the actual failures shown in Fig. 4.

CONCLUSIONS

No definite conclusions can be given as yet, because only a small portion of the investigation has been completed. The few conclusions which have been drawn, as well as the graphs presented, are only tentative and are subject to change as the work progresses. Considerable work remains to be done, particularly in the investigation of the effect of cold working and of heat treatment on the torsional-fatigue properties of streamline wire.

ACKNOWLEDGMENT

The author wishes to acknowledge the encouragement received from Captain H. R. Greenlee, U. S. Navy, Officer-in-Charge, Engineering Experiment Station, and the assistance received from W. C. Stewart, Associate Metallurgist, Engineering Experiment Station. Suitable acknowledgment will be made in a later paper on the subject to the manufacturers who have furnished material for this investigation.

Discussion

D. J. McADAM, JR.[3] The investigation described was under my supervision from January to October, 1930, it being the outgrowth of an extended investigation of stress corrosion of metals that the writer has described in a number of papers. As the inertia of the alternating-torsion machine cannot be estimated accurately, the speed has been limited to 360 r.p.m. With higher speeds the inertia stress would be so large that accurate estimates of the total stress (even with accurate speed regulation) would be impossible. Even at 360 r.p.m., with the smaller streamline wires, the machine cannot be used, because the inertia stress would be too large a part of the entire stress.

As the austenitic steels and monel metal could not have been strengthened by heat treatment, the degree of cold working received by alloys KXA, LXA, and KZA can be obtained by comparing the tensile properties with those of fully annealed material. The degree of cold working received by the wires of stainless iron LC and carbon steel LY could be roughly estimated in the same way, if we knew the tensile properties of the bars from which the wires were made. Assuming that these steels had been strengthened considerably by heat treatment before cold working, it seems probable that the metals used in this investigation had been strengthened by cold working approximately as follows: LCA, 25 per cent; KXA, 50 per cent; KZA, 80 per cent; LXA, more than 100 per cent; LYA, 40 per cent.

The torsional-endurance limit is normally not much less than one-fourth the tensile strength. (By "endurance limit" is here meant the fatigue limit obtained with specimens as free as possible from stress concentration due to surface defects or to corrosion.) The fatigue limit obtained with stainless iron LCA probably is little, if any, below the normal endurance limit. For the other materials "as received," the torsional fatigue limits seem to be abnormally low. The greater the probable amount of cold working the greater is the deficiency in the fatigue limit obtained with these wires.

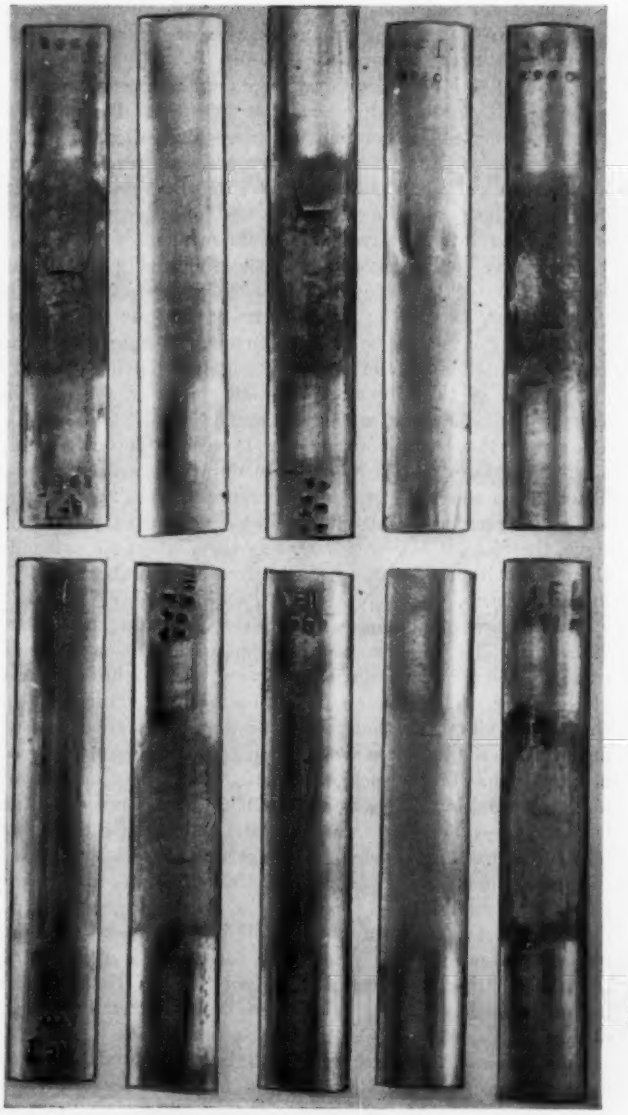

FIG. 7 TEST SPECIMENS AFTER FAILURE—MATERIAL KZA

The "corrosion fatigue limit," unlike the endurance limit, is not an absolute limit. Nevertheless, the estimated corrosion-fatigue limit is of considerable value in comparing the behavior of metals under stress and corrosion. The corrosion fatigue limits obtained with the stainless iron LCA and carbon steel LYA are about normal. (The corrosion fatigue limit for stainless iron normally is about twice the limit for carbon steel.) For the high-chromium steel LXA, Allegheny metal KXA, and Monel metal KZA, the corrosion fatigue limits are abnormally low.

The low torsional-fatigue limits probably are due in part to internal stresses. As shown by the writer and by others, internal stresses may have great effect on the fatigue limit. Even in the form of round rod, cold-worked alloys may have high internal stresses. In the form of streamline wire, the internal stresses are probably even greater.

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[3] Metallurgist, Bureau of Standards, Dept. of Commerce, Washington, D. C.

Cold-worked Monel metal may be practically freed from internal stresses by prolonged annealing at 800 to 900 deg. fahr. It has been asserted by some manufacturers that Allegheny metal cannot be freed from internal stresses without great loss of the strengthening effects of cold working. It seems probable, however, that long annealing at the lowest effective temperature would reduce internal stress without important loss of strength.

There may be another cause for the abnormally low fatigue limits of some of these alloys. The deficit may be due in part to "notch effect" caused by irregularities or flaws on the surface. The lowering of the fatigue limit due to notch effect increases with increase in hardness of the metal. Even blunt notches may take away some of the advantage of high tensile strength. The sharper the notch, the less is the advantage of high strength. The greater the strength of the metal, the greater is the damaging effect of surface irregularities due to normal manufacture, and hence the greater is the probable difference between the ideal and actual fatigue limits. As streamline wires are subject to fatigue, therefore, it is doubtful whether there is any advantage in the use of wire with tensile strength greater than about 150,000 lb. per sq. in.

Most failures of streamline wires in service are due to torsional vibration. The greatest amplitude of oscillation is at mid-length. The twist is clockwise at one end and counter-clockwise at the other. The nodes, at which failures occur, are near the ends. The vibrations that cause failure probably are due to repeated torsional impulses synchronizing with the natural period of oscillation. Under such conditions, failure may occur even if the material is of satisfactory quality. The number of cases of failed streamline wire received at the Naval Experiment Station and at the Bureau of Standards suggests the idea that most of these failures may be due to excessive vibrations rather than to defective material.

There are two possible methods of diminishing the synchronous torsional vibration: (*a*) increasing the stiffness of the wire, (*b*) providing means of damping the vibrations. If the second method cannot be used, it would seem desirable that the design of the wire be changed so as to increase the stiffness.

To increase the stiffness, and thus raise the natural frequency and decrease the amplitude of vibration, it is not necessary or desirable that the cross-section should be increased at the mid-length. As the torsional stress near the mid-length is very small, the cross-section need not be increased here. In fact, the vibrational stress will be less if the inertia of the middle portion is kept small. The increase in cross-section is needed at and near the ends.

The ideal streamline wire to resist torsional vibration therefore would have largest cross-section at the ends and would decrease in cross-section from the ends toward the mid-length. Manufacture of such wires would be more difficult than manufacture of the wires now in use. Wires now in use, however, are designed chiefly to resist tensile stress, whereas the frequent torsional-fatigue failures seem to indicate that wires should be designed especially to resist torsional fatigue as well as tensile stress.

ROBERT WORTHINGTON.[4] The writer would like to ask how it happens that deflection of streamline wire is of a twist nature rather than an entirely simple bend and whether failures are not related to very excessive and avoidable whipping.

The materials were tested in the as-received condition and, as the author suggests, probably carried irregular cold-working strains. It has been our experience that these strains in Monel metal may be relieved without a lowering of strength by a low-temperature annealing treatment at 550 to 600 deg. fahr. for 15 to 30 min. This treatment, rather than softening the metal, results in a marked increase in strength and a slight increase in ductility. In one case, for instance, the proportional limit of rather severely cold-worked rod was increased by this stress-relief annealing from 36,000 lb. per sq. in. to 105,000 lb. per sq. in. and the tensile strength from 131,000 lb. per sq. in. to 140,000 lb. per sq. in. At the same time the elongation improved from 9 per cent to a value of 11 per cent. It seems likely that release of these strains would improve the transverse strength in streamline wire even more than the tensile strength.

It is surprising to see that the greater degree of cold work gave a lower degree of torsional strength in the case of stainless iron. For Monel metal we have found that torsional strength runs parallel to tensional in respect to improvement with cold work.

Has the degree of stressless corrosion damage for cadmium plate been determined? This would seem to be very important, as so much of the damage to steels, as pointed out by Dr. D. J. McAdam on reports of his work at Annapolis, is due to stressless corrosion. It is interesting that it required only 14 days to reduce the torsional fatigue strength of this material from 29,000 to 7500 lb. per sq. in., and it would be still more significant if nearly as much damage should result from stressless corrosion.

Are salt-water tests being made? Dr. McAdam has shown that salt-water exposure leads to considerably greater damage to fatigue strength of steels and chromium steels than fresh, whereas in the case of Monel metal salt water is at least no more damaging than fresh.

The author called attention to the scatter of points on the corrosion fatigue graphs of the chromium alloys, and the relative lack of scatter on the graph for the Monel metal. Since the scatter is much more marked in the corrosion fatigue determinations than the air fatigue, the cause of the scatter would seem to lie in variations in corrosion behavior rather than in the lack of uniformity in the metal or the presence of internal strains, as suggested by Lieutenant Haven.

AUTHOR'S CLOSURE

In answer to the questions by Dr. D. J. McAdam: It was pointed out in the paper that the alternating-torsion machines described were not adapted for testing the smaller sizes of streamline rods due to the large inertia stresses occasioned in the specimens by the oscillating machine parts. This condition was understood, and at the time the paper was presented plans were being considered for eliminating this difficulty as far as possible. Two additional machines have been constructed in order to test the smaller size tierods. These machines are designed so that the oscillating parts on the spring side of the machines are much lighter than the corresponding parts of the machines described. In this way the stresses due to inertia are reduced.

The tensile values for the round wires from which the elliptical rods LXA and LYA were made are not available. The tensile values for the other three materials in the round form are as follows: LCA, 107,000 lb. per sq. in.; KXA, 123,000 lb. per sq. in.; KZA, 135,000 lb. per sq. in.

Tensile tests of the round rods in the fully annealed condition have not been made.

Attention was called by Dr. McAdam to the possibility of freeing cold-worked Monel metal and Allegheny metal from internal stresses by a prolonged annealing. It has been the practice at this station to anneal cold-worked Monel metal in this way before making fatigue tests. For practical reasons, it was considered advisable to make the initial tests of the streamline wires in the condition used in actual service before introducing heat-treatment of the samples. Likewise, no attempts were made to remove the surface irregularities for the same reason.

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[4] Development and Research Department, International Nickel Company, New York, N. Y.

Undoubtedly, much can be done by the way of rod design to reduce synchronous torsional vibration. Nevertheless, the torsional-fatigue properties are an important index in the selection of the most suitable material for streamline wires. This investigation has shown that the tensile strength is not a dependable index as to the resistance of streamline wires to torsional fatigue.

In answer to the questions by Robert Worthington: The examination of service fractures of streamline tierods has indicated that the failures were occasioned by stresses set up by torsional vibration. The fractures examined were not those of simple bending. Frequently failure occurs in the threaded ends of the tierods, where stress concentration is present and the torsional stress is high.

It was previously mentioned that, ordinarily, it is the practice to relieve the internal stresses of cold-worked Monel metal by a low-temperature anneal. However, for practical reasons, it was considered advisable to make the initial tests of the rods in the condition as fabricated for actual service.

The degree of stressless damage for cadmium plate has not been determined. The determination of damage due to stressless corrosion involves two-stage tests—a corrosion stage and a fatigue stage.

So far no alternating-torsion tests of streamline wire using salt water have been made. The present program includes tests made in Severn River water. This water has a salt content about one-sixth that of sea water. Monel metal shows good resistance to salt-water corrosion fatigue.

Considerable scatter of experimental points is usually encountered on making fatigue and corrosion-fatigue tests of the stainless irons and steels. However, the tendency of the material to show scatter is intensified by the presence of internal stress. Doubtless the surface condition of the rods, referred to as lack of uniformity, was responsible for some of the scatter.

AER-54-15

Some Design Factors Which Influence Aviation Insurance Rates

By JEROME LEDERER,[1] NEW YORK, N. Y.

The five principal factors which insurance underwriters consider when they are asked to estimate aviation hazards are: (1) the aircraft itself, (2) operational management, (3) the pilot, (4) the geographical limits, and (5) the uses for which the aircraft will be insured. The relative importance of these factors depends on the coverage being considered, and the connection of various aircraft design factors with these coverages is set forth.

ENGINEERS, generally speaking, regard aviation insurance as an obscure business, managed by a band of uninformed pessimists, who, realizing that commercial aviation would be impossible without insurance, dictate rates out of all reason with the risks involved. Notwithstanding this opinion, aviation underwriters of the leading companies have had a long experience in aviation. Some of them are well-known war "aces;" others were flying and operating aircraft several years before the war. Such experience is an essential qualification for intelligent underwriting. Underwriters know a good airplane when they see one, and the rates they promulgate are about as sound as the aircraft they insure.

The various coverages which insurance companies offer are: fire on the ground only; fire under all circumstances; accidental damage including fire during flight and/or following a crash; tornado; theft; cargo; public liability; passenger liability; property damage; personal accident; and workmen's compensation. The present cost of full protection will generally aggregate from 15 to 20 per cent of the value of the aircraft for the first five coverages. The cost of public liability, passenger liability, and property damage will vary with the limits desired by the insured. In public liability, for example, the coverage with limits of $5000 for any one individual and $10,000 for any one accident will cost from $86 to $105, and the coverage with limits of $10,000 for any one individual and $20,000 for any one accident will cost from $103 to $120. Passenger liability with a $10,000 limit per passenger costs from $60 to $150 per seat. Property damage, $5000 limit, costs from $86 to $110.

The five main factors which underwriters consider when they are asked to estimate aviation hazards are: the aircraft, operational management, the pilot, the geographical limits, and the uses for which the aircraft will be insured. Their relative importance depends on the coverages being considered. The connection of various aircraft design factors with these coverages follows.

Aircraft Performance—Loadings

Landing Speed. Wing loading and power loading give important indications to the underwriter. Knowing the wing loading, the underwriter can estimate the stalling and consequently the landing speed of an airplane. The landing speed is considered important, because the amount of damage sustained by aircraft on the ground will generally vary with this speed. Devices which reduce the landing speed and the landing run, such as slots, rotors, flaps, and brakes, are favorably regarded by underwriters if the experience with them is satisfactory.

Climb. Wing loading in combination with power loading indidicates take-off and climbing ability—important in getting out of small fields, clearing obstacles at the ends of fields, and making emergency ascents. Wing and power loadings in the vicinity of ten are considered normal. However, as power-plant reliability increases, underwriters may look upon higher loadings without undue apprehension where good landing fields are available.

Angle of Descent. Aircraft which, because of high aerodynamic efficiency, land at a relatively flat angle or which have a tendency to float on landing, receive the disfavor of underwriters. In this connection, the attempt of a British manufacturer to increase the angle of descent by mechanically increasing the drag of certain undercarriage struts is worth remembering.

Stability. Stability, stalling characteristics, controllability, and ground maneuverability are usually considered satisfactory if the aircraft carries an approved-type certificate, unless experience with the aircraft proves otherwise. On aircraft used for scheduled operations the stability should be considerably greater than on the average aircraft. For satisfactory longitudinal stability on such airplanes the center of gravity should be located at about 28 per cent of the chord on a wing where the most forward position of the center of pressure is 30 per cent.

Ground Loop. The tendency of some aircraft to ground loop, generally because of insufficient rudder control in the three-point position, poor tail-wheel castering, or defects in the design of the undercarriage, is strongly disfavored, especially on large fast-landing aircraft where a ground loop generally results in heavy damage.

Multi-Engined Aircraft. Multi-engined aircraft are regarded with suspicion unless they prove their ability to maintain level flight with authorized flying load and one or more engines out of commission. The authorized load is assumed to be the load allowed under the approved-type certificate unless the operator reduces it. A further consideration is the altitude of the terrain over which the multi-engined aircraft will fly. A ceiling of, say, 5000 ft. on two out of three engines would be dangerous matter for an aircraft operating out of Cheyenne where the terrain would be higher than the ceiling with two engines.

Marine Aircraft. Landing speed, take-off, climb, and gliding

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[1] Chief Engineer, Aero Engineering and Advisory Service, Inc. Assoc-Mem. A.S.M.E. The author was born in New York City. He attended New York University and received the M.E. degree from the College of Engineering, receiving the David Orr prize in mechanical engineering upon graduation. While in college, Mr. Lederer specialized in aeronautical engineering and was assistant instructor for one year after graduation, devoting considerable time to wind-tunnel research. Following this, he investigated in Europe the application of certain aluminum-silicon alloys, and then served as aeronautical engineer with the U. S. Air Mail Service. This experience was supplemented by acting as consultant in designing and stress analyzing airplanes in connection with obtaining approved type certificates from the Department of Commerce.

Presented at a meeting of the Metropolitan Section of the A.S.M.E., New York, Feb. 25, 1931.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of the authors, and not those of the Society.

angle are not so important on marine aircraft because of the larger landing areas, fewer obstructions, and the assumption that the aircraft will always fly over water. However, stability, stalling characteristics, and controllability receive the usual consideration.

Small Aircraft. Small aircraft with a high degree of maneuverability induce acrobatics, and for this reason are often regarded with diffidence by underwriters.

THE POWER PLANT—RELIABILITY

The first consideration in the power plant is reliability. This not only includes the reliability of the engine, but also the fuel system, the location and construction of tanks, the location of carburetor intake, and the general reputation of the engine. An approved-type certificate or acceptance by the military services is not regarded as proof of power-plant reliability. However, such acceptance is generally prerequisite because it indicates sound design and manufacture. Reliability can be proved only by a good experience covering continuous service under actual operating conditions.

New Engines. Among new engines, those made by established and reliable manufacturers will receive more favorable consideration than others. Changes in the design of a good engine, such as using different materials in the crankcase, put it in the category of a new engine.

ENGINE TYPES

In-Line Engines. So far as accidental damage is concerned, the in-line or V-type engine will generally suffer less severely in a crash than other types. In case of an accident such as nosing over, the crankcase of an in-line engine is occasionally cracked, front cylinders ruined and the crankshaft sprung, yet the rear cylinders, pistons, rods, valves, and valve-actuating mechanism often remain unharmed. Also accessory material, drives, carburetors, magnetos, and starters, being in protected spots, remain undamaged.

Radial Engines. Air-cooled radial engines generally suffer more damage than in-line engines because the crankshaft is more likely to be sprung, the crankcase cracked, and the lower part of the engine damaged beyond repair.

Inverted Engines. Worse than the radial engine, from the point of view of damage sustained in accidents, is the in-line inverted engine. All the cylinders, valve-actuating mechanisms, and accessories are more exposed to injury by reason of their location. However, this is indirectly compensated to some extent by the apparently greater ease of inspection and maintenance.

Air-Cooled Vs. Liquid-Cooled Engines. Considering air-cooled engines in general as against liquid-cooled engines, underwriters would prefer to limit the operation of liquid-cooled engines to organizations which are prepared and trained to take care of the additional maintenance and inspection, because of the fragile construction of radiators and the additional plumbing at present.

Materials. Materials are considered for cost of replacement and susceptibility to damage and corrosion. Some crankcases can be submerged in sea water for a considerable time and put back into service without trouble, while others will deteriorate rapidly and require replacement. These factors are considered when seaplanes or flying boats come up for insurance.

The tendency of castings or other parts of the engine to warp and crack when suddenly cooled, sometimes occasioned by the immersion of an aircraft, is also considered by underwriters. Most engines can withstand this test with only slight damage, requiring a good overhauling and perhaps a few new parts. The loss in the temper of parts due to the heat-treating effect seems to be remote as the operating temperatures are below the point where quick immersion would affect the temper of either steel or aluminum alloy.

Location. The location of an engine in a well-protected position, such as is commonly found on amphibians, is regarded with favor. Twin-engined aircraft, with an engine on each side, are preferred to the three-engined type with one engine in the nose. Pusher installations where the engine is protected by easily repaired parts of the fuselage or wings would also be favorably looked upon, although the passenger liability rates would probably be affected adversely. The cooling and performance of pusher or tandem installations are regarded suspiciously until experience proves the reliability of the installation.

Fire Hazards. The inverted engine probably has the smallest fire hazard because the chances of fuel or oil dripping on the ignition system or hot parts of the engine are reduced, the hot exhaust stacks are farther away from gasoline tanks, and the general tendency of the fluids is to drop away from the aircraft rather than on to the fire wall and the fuselage. The upright engine with the carburetor near the ignition system and the fuel lines in the V offer a greater fire hazard in the air. The radial engine has an advantage over the in-line engine in quicker cooling of cylinder heads and exhaust manifolds after power-plant failure, important for fire following crash coverage.

Fuel Systems. Gravity fuel systems are preferred from the point of view of accidental damage because of their simplicity and lower maintenance requirements. However, the more complicated systems using hand and engine pumps appear to be as reliable as the gravity system if properly maintained. Here, as with water-cooled engines, underwriters prefer to have the gravity system on aircraft operated by private owners, possibilities of poorer maintenance being present. Wabble pumps, engine pumps, relief valves, and the additional connections on pump systems require constant attention, especially with connections on the suction part of the line. The inverted engine because of the low position of the carburetor and cylinders appears to have the advantage here as the gravity system can almost always be used with this type of engine.

Air Locks. A large number of accidents are due to power-plant failure immediately after take-off. It is believed that such failures are frequently due to an air lock caused by locating the shut-off valves close to the tanks and allowing a large length of tubing between the valve and the carburetor. It is customary to close these valves after the aircraft is landed, permitting the engine to run on the gasoline which remains in the lines and carburetor until all the gasoline in the lines is drained, thus leaving an air or vapor space between the valve and the carburetor. When the valve is opened, preparatory to taking off, the gasoline must force this air or vapor out of the lines before a satisfactory flow is obtained. Very often the fuel will trickle down to the carburetor in a sufficient amount to operate the engine on the ground while it is being warmed up at slow running speed, and enough will collect in the carburetor bowl to take the aircraft off the ground. However, soon after take-off, when the demand for a full flow of gasoline is continuous, the carburetor will suck in the air or vapor which remained in the lines, the engine will miss, and a forced landing will often result, sometimes with disastrous consequences.

This danger can be avoided by placing the valves close to the carburetor so that the distance in which air or vapor can accumulate is very small.

Screen strainers, sudden changes in the diameter of the fuel lines, and certain types of flexible tubes will tend to collect air and create air locks.

In addition to air locks, vapor locks will often cause a forced landing. When an engine is running through the air, the carburetor and plumbing are cooler than the engine. When the engine is stopped, and no cooling takes place, the heat flows from the engine to the carburetor and plumbing. When the gasoline is allowed to flow again, the pipes and carburetor may be hot enough to vaporize the gasoline, causing vapor locks and consequent trouble. Fuel lines which are located close to the hot parts of the engine may also give trouble. A good example of this is the location of the fuel lines in the V of a V-type engine.

Aircraft equipped with fuel-pump systems where there is considerable lift of gasoline from the tank to the pump may work satisfactorily at sea level, but at higher altitudes will also experience vapor-locking troubles due to lower atmospheric pressure if the temperature is not sufficiently reduced at higher altitudes.

Generally speaking, fuel systems which have been accepted by the military services are preferred to those which the average commercial manufacturer installs. There should be more cooperation between the airplane and the engine manufacturers in fuel-system design.

Lubrication. It is believed that putting the oil tank behind the firewall, as some engineers have suggested, might place the tank inconveniently far from the engines. An oil tank so located may reduce fire hazards and the longer return line might retard oil foaming. However, in the conventional airplane the fumes from the oil would reach the pilot and passenger more readily; it would increase the difficulties of cooling the oil in hot weather and keeping it warm in cold weather, especially in the lines. But most important of all is the longer suction line which would be required, with greater danger of developing leaks from the added vibration and therefore chances of losing oil pressure. Of course, if a satisfactory installation can be made behind the firewall, so much the better.

The oil tank should be high enough to keep the pump primed and located so as to be easily cooled when necessary. Installations are found where oil tanks receive very little of the cooling air due to poor location of louvers in the cowling.

Tanks. Fuel tanks are not sufficiently rugged on many aircraft. The force with which fuel is ejected from the hose, especially where the nozzle slightly constricts the flow, is sometimes responsible for damage to shallow wing tanks. Many tanks are made of rather thin material, not too well baffled, and repeated refueling with the fuel striking the tank bottom with unnecessary force is often responsible for leaking seams and loosened baffles. The manufacturer would do well to arrange a screen or deflector in such a manner that the full force could not be directed against the bottom of the tank. A coarse-mesh screen should be placed in each filler neck also. The tank supports are often found to give way, permitting the tank to sag, with the possibility of breaking fittings and affecting performance.

Where a multiplicity of tanks is used, it would be well to have some device which warns the pilot when the tank is almost empty. The ordinary fuel gage is not sufficiently accurate. Most pilots depend upon watches and the r.p.m. of the engine. If they permit a tank to run dry, an air lock is likely to form before they can switch over to another tank.

The tanks should be installed so that in case of fire in the air the gasoline can be dumped rather than to have droppable tanks. This is because pilots seldom care to embarrass themselves by losing parts of the aircraft. However, they will be less hesitant in disposing of the fuel. The danger of public injury and property damage must also be considered with droppable tanks. The dump valves should be located so that the contents will not pour over the engine or other parts of the aircraft.

The location of tanks in the wing of high-wing aircraft increases the fire hazard following a crash because the tanks generally fall on top of the engines.

The capacity of fuel tanks should be in considerable excess of minimum requirements for any particular route.

Batteries and Ignition. The propeller on an airplane is always a source of danger, and this is especially true when it is possible to accidentally start the engine, as, for example, when a plane equipped with a booster magneto is left unattended without the gas being shut off. Two recent accidents resulting in painful though not fatal injuries were sustained by persons who knew better than to touch the engine switch but who could not resist finding out what the booster lever was for. As a safety measure, the booster switch should be grounded through the engine switch, so that it cannot be used unless the engine switch is on. This is considered from the point of view of workmen's compensation and public liability.

Batteries should be installed in such a way as to prevent the danger of fire due to dripping fuel or oil. Often fittings which hold batteries in place are not strong enough for the job, resulting in loose terminals and dangerous sparking.

Engine Controls. The control systems employing steel tubes and bellcranks or Arens controls are preferred to the Bowden cable type. Controls and valves should be located where passengers cannot disturb them. Valves located near the floor where they can be unwittingly kicked are especially dangerous.

Strainers. Strainers should be located so that they will be very accessible and so that the draining fluid will not run over the aircraft. It would be well to have them installed so that they could be drained in flight.

Starters. From the point of view of workmen's compensation, underwriters prefer aircraft equipped with starters.

Exhaust System. The most frequent cause of fire following a crash is the contact of fuel or oil with hot exhaust manifolds. Any means by which exhaust manifolds are cooled are very favorably regarded by underwriters.

The seams of exhaust pipes should be turned away from the fuselage so that, if they open, the flame will not strike the fabric. The ends of the exhaust manifolds or stacks should not be too close to holes in the bottom of the fuselage through which any fuel can drip, and they should be well away from any part of the fabric. Stainless-steel exhaust manifolds are preferred if ethyl gasoline is used.

Fire Extinguishers. The ordinary hand fire extinguisher is not considered very useful for fires in the air unless they occur in the cockpit. It is not unusual to find the hand type of extinguisher locked in the luggage compartment. Pressure fire extinguishers should be installed so that the fire-extinguishing medium covers the more dangerous parts of the engine, especially the carburetor. In the event of a crash, a pilot is supposed to shut off the gasoline valve, cut his switch, and turn on the pressure fire extinguisher. The last two operations could be reduced to one by arranging the switch so that by turning it further, the pressure extinguisher would automatically be put into operation. Types of extinguishers which operate automatically in a crash are approved. The National Fire Protection Association, 16 Batterymarch Street, Boston, Mass., has a pamphlet entitled "Fire and Life Safety in Aviation" which thoroughly covers the fire-extinguishing requirements for aircraft.

Fire Wall. The bottom of the fire wall should have a small sloping channel or other means to guide the oil which splashes against the fire wall or drips from the oil tank away from the bottom of the fuselage, possibly to a drain on the landing gear.

THE AIRCRAFT—GENERAL TYPES

Were it not for the mooring risks and difficulties in salvaging, marine aircraft would be considered more favorably by the underwriters than the average land aircraft. Unfavorable metacentric heights, danger of waves piling over the wings, puncturing of hulls and pontoons, chances of drifting against obstructions or of being hit by other watercraft while moored, must be considered.

Fire following crash is not so seriously considered on marine aircraft as on other types, but the chances of fire on the water and during flight are as great as on land planes. Consideration should be given to a means of preventing fuel and oil from overflowing on to the water during refueling and drainage operations.

High-wing monoplane types are preferred because of better seaworthiness and less chance of corrosion in the wings.

There is room for considerable development in making marine aircraft more seaworthy. In present designs there are no provisions for heaving to, for keeping way on, and for steerage way. A ship out of control in the trough of the sea is quite likely to have a beam sea climb aboard. Eight- or ten-foot swells would be disastrous to the toughest of sailing craft if they were as thoroughly out of control as any seaplane would be once down on the water. One of the first things that would happen to most flying boats in any kind of sea would be the abolition of the glasswork of the pilot's cockpit and the flooding of that compartment. This would have her down by the head. From then on it would only be a question of time. But if there were any sort of control in the water, there would be a much better chance. The following precautions are suggested:

1 Facilities for relentless engine inspection
2 Marine equipment including:
  a Water rudder or steerage device
  b Means for keeping way on—small sail or outboard motor
  c A sea anchor for heaving to
3 Emergency radio and antenna
4 Generous signal equipment
5 Life preservers for moral effect as well as safety
6 Sextant, charts, log tables
7 Chronometer.

Amphibians are favorably regarded except for the proximity of the hulls to the ground and danger of landing on the ground with the wheels up or on the water with wheels down.

Fuselage Construction. The conventional steel-tubular construction or that using duralumin tubes or angles is easier to repair in the event of damage than are most monocoque fuselages. Riveted duralumin with a metal skin is difficult to repair after distortion, and at the end of about 1500 hours of service the rivets may be rather loose and affect the flying qualities. Wooden monocoque fuselages are likely to suffer from the following defect: Wherever there are concentrated loads coupled with strong vibration, as in the vicinity of the engine-mount fittings, the wood is likely to crack and split. This condition is very difficult to repair and is aggravated by the entrance of oil and water, which softens the wood and may eventually lead to structural failure. Other points where failures in the monocoque construction are likely to occur are at the landing-gear attaching points and tail skid. At these points there is little vibration, but the loads are highly concentrated and unless properly distributed may cause failure. Another trouble is the accumulation of dust behind the bulkheads, which holds moisture against the wood shell and rots it. If these failures do not occur after four or five hundred hours of service, it may be assumed that the aircraft is satisfactory for coverage against accidental damage. However, in a crash wooden monocoque fuselages break up more easily than metal; the resulting hazards to the passengers due to smaller absorption of energy by the fuselage structure and the increasing danger from splinters are evident.

Metal monocoque fuselages seem to suffer from corrosion when used in the vicinity of salt water. If, through an accident, such fuselages are totally submerged in salt water, it is almost impossible to prevent corrosion in the seams. Naturally the loss is quite large under these circumstances, and such construction is reflected in the rates unless special means are taken to prevent corrosion. The repair of some metal monocoque fuselages is very expensive, necessitating large replacements for only a small amount of damage. In other types of construction, this is not so true. It is very difficult to realign metal monocoque fuselages once out of alignment.

Baggage compartments, cargo space, lavatories, etc., should be placed in front of the fuselage rather than in the rear to protect passengers in case of crash. This eliminates the danger of a large load coming forward on to them.

For flying boats and amphibians, metal is preferred to wooden construction if for no other reason than that there is more salvage after an accident or submergence.

Fire. The front of the fuselage should be covered by metal for a short distance in back of the engine mount to reduce the fire hazard. The fabric should be made to clear the primary structure by a fairing so that in the event of a flash fire, which burns off the fabric, the main structure can be saved. An all-metal fuselage with a Diesel engine would probably receive the best consideration in rates for fire in the air and fire following crash.

Wings. The conventional braced wooden wing is preferred by underwriters because it is easy to repair and cheap to replace. Full-cantilever wings are not so well considered as braced wings, because in the event of damage to, say, a wing tip, entire wing replacement is necessary, whereas on a braced wing, repairs could be easily made. At least such was the ruling in a typical case by the Department of Commerce.

The high wing is preferred to the low wing from the point of view of damage on the ground. However, the visibility offered by the low-wing type is considered of sufficient advantage to counteract this. Furthermore, in the event of a crash, the low wing offers more protection to the passengers. For operations over forests and barren lands, the high wing is preferred to the low wing because of smaller danger of damage on bad fields and also because the wing is out of the way while working around the airplane.

The center section of the wings or the fuselage should have suitable fittings for hoisting the aircraft. This would hasten salvage operations after an accident and prevent considerable damage.

Certain types of construction are preferable for use in countries where salvage operations are difficult or where the plane is likely to be far from repair bases. For example, there is the case of an airplane forced down in a remote part of Canada by a broken engine mount. Temporary repairs were made by welding the handle of a frying pan to the engine-mount strut, and the aircraft was then flown to the nearest repair base.

Undercarriage. A husky undercarriage with oversized wheels, good brakes, good tread, and universal joints at the fuselage attaching points is preferred. The universal joints lower the damage sustained by the fuselage when the undercarriage is wiped off. Where castings are used, manganese bronze or similar tough material is preferred. Plenty of bearing area in the hinges is recommended. Retractable undercarriages should have indicators showing the position of the wheels. The idea of leaving part of the wheels exposed to prevent damage in case of landing with the undercarriage retracted is favorably regarded.

Control Surfaces. Braced tail surfaces made in independent sections, so that if one side is injured the other side remains unharmed, are favored. Fins which are built integral with the fuselage will cause more damage to the fuselage in an accident than detachable ones. Oversized hinges are desirable on all control surfaces, especially the ailerons.

Control System. The control system should have plenty of bearing areas, or have bearings that may be easily replaced. It takes almost no time to get considerable undesirable looseness in the control system. The designer should not place too great dependence on lubrication for ease of operation, because in cold weather the oil or grease will harden and make operation more difficult. It is important to be able to make quick and easy inspections of all linkages in the control system, especially where the stabilizer control mechanism attaches to the stabilizer spar. Bearings here should be extra large. Bearings, in general, should be designed for impact and wear rather than for the theoretical bearing loads.

Cabin. The cabin should be lined with materials which are fireproof. The chairs should be well anchored, fitted with safety belts, and padded on the backs. Doors should be large, and if possible so built that they cannot jam in case of an accident. Windows should be large enough to permit the exit of a normal person. Cigarette receivers should be very evident.

Cockpit. The cockpit should be roomy and comfortable. The instrument board should not be too close to the pilot, making it necessary for him to change the focus of his eyes considerably when glancing at it. With a distant instrument board the pilot will be able to take in more at one glance, and there is less danger that he will hit his head against it in a crash.

The windows should be made to open easily in all kinds of weather. They should be designed so as to eliminate reflection of lights in night flying.

Soft padding should be placed wherever the head of the pilot can come in contact with the cockpit. There should be nothing hard located above the pilot's head for a distance of about six inches to eliminate the danger of striking his head in bumpy weather.

The influence of the air-mail pilot is evident in some recent designs of single-engined passenger planes. The pilot is located in an open cockpit aft of the cabin instead of directly behind the engine. One occasionally hears an opinion that the pilot should be located in the front so that he will exercise more care in guiding his ship by virtue of his danger in event of a crash. However, it is believed that the pilot should be located in the best position to guide his craft rather than in the most dangerous position. If his location aft of the cabin gives him greater ease of mind and therefore better control of his ship, so much the better. This is analogous to marine practice. The pilot of a ship is situated in the most favorable position for guiding his craft rather than in the most dangerous position in the event of a crash.

Instruments. Underwriters like to see good instruments, easy to maintain and properly installed. Air-mail planes as a rule are well arranged in this respect, but the average commercial job is poor. One prominent transport company operating over large stretches of water supplies two compasses so that the pilot has a check on the correctness of his bearings. The installation of recording tachometers would receive very favorable consideration by the underwriters, especially in scheduled operations. Radio-beacon equipment without weather-broadcasting facilities tends to make the pilot fly in bad weather without knowledge of conditions at his terminal airport. Hence such an installation is not received favorably. However, with receiving and broadcasting equipment, the radio-beacon installation is given considerable credit. Horizon indicators are regarded with approval, but mechaviators are looked upon with suspicion.

Wiring. The wiring system should be easy to inspect and the danger of short-circuiting eliminated as far as possible. Certain sound-deadening materials used in aircraft are very inflammable and the wiring should be protected from them. This is important as it affects the hazards of fire on the ground as well as fire in the air.

Lights. On aircraft using more than one rudder, the tail light should not be placed on the fuselage in front of the fin because the outboard rudder will blanket it. Larger and more visible navigation lights are strongly recommended. Celluloid cases of tail lights are likely to burst into flames on the ground because of heat generated by the lamp. This has been proved by actual tests. Glass cases should be used wherever possible.

Flares. Flares should fall free of the aircraft; the danger of hooking on to the tail skid or tail surfaces should be entirely eliminated. Chances of coming loose on the ground or in flight should also be avoided as the fire hazard is large.

Folding Wings. Folding wings are favorably regarded for tornado and windstorm coverage. With the wings folded back, there is less chance of damage on the ground under storm conditions.

Propeller Clearance. On multi-engined aircraft the clearance between the propeller and the fuselage should be great enough to permit the pilot to swing his hand out of the window without being hit by a propeller.

Tail Skids. The tail skid proper is generally of sufficient strength, but the anchorages and bearings are not as a rule. Quite often anchorages are pulled out or bearings soon become loose and shaky.

Special Designs. Aircraft built for special services such as crop dusting, forest surveying, hauling machinery, etc., receive special consideration. Crop dusting, for example, is considered an especially dangerous undertaking because of the chance of fire due to ignition of dust particles inside the fuselage, danger of corrosion due to chemicals, and low flying. Dual-engined aircraft, capable of climbing on one engine, would be preferable for such work.

Miscellaneous Considerations. Steps and grips used by mechanics in inspecting engines or other parts of the aircraft should be built considerably stronger than is commonly the case. The walkways on wings should be skid-proof, especially in the vicinity of the propeller. For operation in cold climates, grips should be provided in the tail of high-wing full-cantilever aircraft so that skis frozen in the ground can be shaken loose. This is not necessary on braced wings because the brace struts can be used. Incidentally, after the skis are loose and the aircraft starts forward, mechanics find it difficult to run from the tail to the door of the fuselage on full-cantilever jobs. However, on braced-wind types they can cling to the brace struts and work their way to the door.

Salvage. Contrary to popular opinion, the insurance companies do not care to salvage the defective parts of an aircraft. Although they might be able to lower the amount of the claim by doing this, such defective salvage may be reemployed to the loss of the insurance company.

Structural Failures and Other Accidents. The accident report of the Department of Commerce for the year 1929 and the first six months of 1930 indicates a serious increase in the number and percentage of accidents due to structural failure during flight. From January to June, 1929, there were eight failures out of 744 accidents; from July to December, 1929, there were seventeen failures out of 924 accidents; from January to June, 1930, there were thirty-eight failures out of 930 accidents. Furthermore the fire losses in 1930 were three times those in 1929, amounting to about $2,000,000.

While the number of accidents has decreased in scheduled flying, the opposite is true of non-scheduled flying. The increase in the total number of accidents in non-scheduled operations, for the first six months of 1930, was 24 per cent over that for the same period in 1929. The increase in fatal accidents was 22 per cent. In addition to this depressing knowledge, the selection of risks is against the insurance companies; operators try to foist doubtful propositions on underwriters. These facts justify a strong degree of skepticism. It is evident that aviation underwriters, instead of being a band of greedy pessimists, are among the world's great optimists. This optimism is largely based on the continued development of better aircraft structures, engines, and accessories, for which the engineer is responsible.

Discussion

F. S. Hubbard.[2] In regard to the angle of descent, the ability of a ship to glide either at a flat or a steep angle to the horizontal is desirable. The ability to glide under control at or near maximum lift will of course depend largely in the case of a conventional airplane on the pilot's skill. In order to insure real safety some form of lateral control should be provided that will not be affected by the burbling of the wing at this angle.

In locating the center of gravity of the airplane in reference to the chord of the wing, the writer does not believe that a fixed rule can be established, as the best position will be dependent on the type of wing cellule used. For example, the best center-of-gravity position for a monoplane will be quite different from that for a highly staggered biplane wing cellule.

The writer is thoroughly in agreement with the author on his discussion of multi-engined aircraft, as he believes this type of aircraft should maintain flight with one or more engines out of commission, and unless they do so should not be approved for passenger-carrying work.

The author has covered very thoroughly the power-plant question, which includes both the reliability of the engine and the satisfactory installation of the different power-plant items.

In the discussion of general types of aircraft the statement is made that in the event of a crash the low-wing monoplane offers more protection to the passengers than the high-wing. This is not true if the ship should nose over and go over on its back, as under this condition the high-wing monoplane would be preferable, as the wing would offer some protection to the cabin or cockpit, which would not be true in the case of the low-wing monoplane.

T. P. Wright.[3] Although the author's opening statement may in general be true, the writer nevertheless feels that aeronautical engineers are becoming more and more cognizant of the necessity of creating a product which the hard-headed businessman, in which class are placed the insurance representatives, can consider safe and can therefore both use and insure on the same basis as maintains for other means of transportation. In other words, the writer admits the great need for improving the safety of airplanes under all conditions of operation before it is reasonable to expect great reductions in insurance rates.

The following notes may be of interest:

Landing Speed. The trend is more or less toward higher landing speeds at the present time, which is unfortunate, as the writer is quite convinced that landing speeds, certainly in the case of airplanes for general pilot-owner use, must be reduced rather than increased.

Multi-Engined Aircraft. The present Department of Commerce regulations take care of the performance item of maintaining reasonable altitude under full load when one power unit of the total used is out of commission. This is believed to be a proper regulation, and assuming that it is being enforced in all cases, and there is no reason to suspect it is not, it then appears that there is a very decided improvement in safety and reliability when using multi-engined aircraft.

Engine Location. The writer is pleased to note the importance which has been placed on engine location, as the removal of the engine from the fuselage is an extremely important step in the design of multi-engined aircraft toward improving safety.

Fuselage Construction. The writer believes that the criticism of the so-called monocoque fuselage on the score of maintenance is made too soon after the appearance of this type of construction in service, as certain information at hand, particularly from the military service, indicates an actual decrease in maintenance cost when using monocoque construction.

Wings. The difficulty of making definite statements on the best wing arrangement for all uses is clearly brought out by the author, as each arrangement has distinct advantages and disadvantages. Probably the best that one can do is to attempt to solve each problem on its own merits, admitting the impossibility of setting down any one arrangement of wings as representing the best for all cases.

Instruments. There might have been added the necessity for installing crash pads, which in the cases of quite minor accidents will frequently save the pilot from very painful injuries.

Structural Failures and Other Accidents. It is disappointing to note the quoted statistics on increase in structural failures when comparing 1929 and 1930. In all probability this was brought about by the inflation of the industry, with the consequent putting out on the market of a great number of types made by manufacturers of short experience, coupled with the great mass of detailed work in the Department of Commerce, because of which design errors may not have been discovered. All in all, however, the structural failures that have occurred are quite small in proportion to the total causes of accidents and will surely be greatly reduced from this point on.

Author's Closure

The author agrees with Mr. Hubbard's remarks concerning the impracticability of a fixed rule for locating the center of gravity. Probably the statement should have been qualified by saying that the suggested location at 28 per cent of the chord was found satisfactory on an unstaggered biplane with a Clark Y airfoil. This airplane was very successful in the United States Air Mail Service and subsequent private air-mail operations.

Low-wing airplanes are more dangerous than high-wing ones in the event of nosing over completely, but this does not always occur in accidents. Furthermore, it is believed the Department of Commerce insists on designing for this condition.

Mr. Wright's comments on high landing speeds are in agreement with the underwriters. There are indications abroad that the slot and flap will soon play a more prominent part in the reduction of landing speeds.

Regarding the performance of multi-engined aircraft, the Department of Commerce makes no distinction as to the altitude of the terrain over which the airplane is to operate. The performance data are generally measured at above sea level, not the operating terrain. Furthermore, gradual increases in weight by reinforcement of cowling, landing gear, etc. often make serious changes in the performance of aircraft which are not often considered.

Regarding monocoque construction, the author's original comments have been reinforced by a gear of additional experience (this is written Jan. 17, 1932). However, he believes that some one will evolve a type of construction far superior to the present.

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Stresses in Rigid Airships: The Effect of Indeterminateness on Their True Value

By W. WATTERS PAGON,[1] BALTIMORE, MD.

Part I of the paper discusses statical determination as applied to a rigid airship and cites the difficulty of computing the actual stresses in a redundant structure of high degree. Part II treats the subject of statical stresses in an unbraced shell of varying longitudinal profile, advancing from a simple prismatic tube, through such a tube with a conical nose, to a symmetrical form of elliptical profile. Part III deals, in the way of a progress report, of the effect of introducing internal bracing, beginning with a single wire in a complete shell, prismatic or curved, and leading up to a more complex system in two prismatic bays.

The conclusions tentatively show that unless there is a considerable degree of rigidity in the bracing systems the shell stresses cannot be determined with even fair accuracy; that the change to loose wired ring frames requires careful attention to stresses induced in the shell by reason of the lowered rigidity of the ring frames as compared with tight wired frames; that there should be local reinforcement of members near concentrations of load; that keels and corridors have a marked influence on the stress distribution in the shell and that their effect on these stresses is as yet undetermined; that ships of curved profile differ so from those of prismatic form, heretofore studied, that the approach to St. Venant's Principle is different and probably less certain; that there is some similarity to the stresses in a semi-rigid; that redundancy of the crossed diagonals in the facets of the shell is of rather localized effect; that even joint rigidity causes a secondary effect that is of importance; that some points are deserving of further study, such as the "keel effect" adjacent to loaded longitudinals; and the rigidity of the longitudinals between main frames; that there is evidence of a stress system underrunning the systems calculated by the common theories; that as the number of sides of the ship increases, from triangular to rectangular (square), hexagonal and octagonal, the stresses agree less and less well with the common theories, progressively, and since the trend is large, the effect should be studied further; that further study is needed to determine if, and how much, additional braced frames reduce the discrepancy just noted; finally, attention is called to the fact that the discrepancies discussed apply only to part of the stress in any one member, and that the discrepancy from the total stress will be a lesser percentage.

FOREWORD

MUCH effort has been devoted to this subject in the United States, England, and Germany; many able minds have considered it from many angles; but no one has found successfully a solution on that basis which is the only one used in the structural design of bridges and buildings. (See R. & M. 1057, p. 22, also R. & M. 800.) The author determined as a matter of pure research to see how far he could carry this tried method, i.e., the method of finding the determinate stresses—which are unique—and adding to them by some reasonable device the effects of redundancies. As a complete answer the results attained thus far fall very short of a complete solution; hence this paper must be treated somewhat as a progress report, submitted at this time in order that some one may make ground tests in order to check the mathematical conclusions. It is believed that the investigation of stresses is justified by the marked increase in size of ships now building, and the radical alteration of the bracing system and the longitudinal curvature. The topic has long been in the author's mind, but was emphasized by the discovery of excessive flexibility in the rings of a design prepared by him as the basis for a competitive bid in the 1928 Competition for U. S. ships *ZR-4* and *ZR-5*.

The structure of lighter-than-air ships still offers a field for extensive study. Only the rigid type will be considered.[2]

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[1] Consulting Engineer. His degrees are: A.B., 1905, Johns Hopkins University, in mathematics and physical course; S.B. in C.E., 1907, Massachusetts Institute of Technology; M.C.E., 1910, Harvard University. His experience includes the following: 1907–1909, Baltimore Bridge Company (now Baltimore Warehouse, Carnegie Steel Company), as detailer, checker, estimator, field superintendent, on bridges and buildings; 1910–1917, with J. E. Greiner, later member of J. E. Greiner & Co., consulting engineers, in charge of design and construction of bridges, coal piers, etc.; 1913–1914, resident engineer, Washington Street Bridge, Norwalk, Conn.; 1918, in private practice on structural design and as Civilian First Assistant to Construction Quartermaster, Camp Meade, Md., then Captain, Construction Division of the Army, First Assistant to Construction Quartermaster (later Construction Quartermaster), Curtis Bay Ordnance Depot, Anne Arundel County, Md.; 1919 to date, consulting engineer in charge of design and construction of bridges, factories, etc. Consultant to U. S. Navy on U.S.S. *Shenandoah*, and to U. S. Army on semi-rigid *RS-1*. Co-author of "Pressure Airships," Ronald Press Co. Won $5000 prize on new type of rigid airship in 1928 Navy competition. Submitted competitive design for Navy ships *ZR-4* and *ZR-5* in 1929 competition, as chief engineer, Rigid Airship Division, American Brown Boveri Electric Co., and vice-president and chief engineer, Dirigible Engineering Co., Camden.

[2] A semi-rigid comprises a fabric envelope (but cv. metalclad) stiffened below by a metal keel, which has generally been of triangular cross-section, statically determinate as regards its internal forces, but indeterminate as to the percentage of the total bending moment and shear which it carries. The design basis has assumed full bending carried by the keel, but theory and experiment show that this assumption is not realized, that the envelope may carry nearly one-half, and that there may even be interaction between envelope and keel due to variations of pressure ("breathing") that have no origin in the external loads.

See "Pressure Airships," by Blakemore and Pagon, also "Adjustment of the Elastic Properties of a Model Keel, U. S. Army Airship *RS-1*," Wm. Hovgaard, *Journal of Mathematics and Physics*, vol. V, no. 1, Dec., 1925.

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

A rigid is in principle a full structural frame, which carries all loads and, ultimately, all gas-lift forces, wind pressure, and reactions from the control surfaces. The gas and air forces are seldom applied at the joints alone, but usually act more or less uniformly as local bending loads on the frame members. C. P. Burgess has well covered the computation of these loads in "Airship Design," and for the purpose of this paper they will be assumed as accurately known in direction, position, and amount. Reference should also be made to R. V. Southwell's paper R. & M. 1057. Numerous expedients have been ingeniously devised to minimize the local bending stresses, and the author will assume that these forces are applied at the joints and that by superposition the total stresses can be found in each member.

Even with these assumptions it will be found that the rigid is a highly indeterminate structure, and the author proposes in this paper to address himself to the effect of indeterminateness on the true value of the stresses. A review is in order of the meaning of "indeterminateness," and will be found in Appendix No. 1.

I—A SHIP AS A DETERMINATE STRUCTURE

A determinate structure may be:

1 A plane truss formed only of triangular areas
2 A three-dimensional truss formed only of tetrahedrons
3 A "space framework" consisting of a closed surface that is formed of triangular areas (facets), without, however, any internal members crossing the enclosed space.

A ship of small size might be so framed as to conform to the third type, but ships as built previously have had tight-wire bracing at intervals, serving as bulkheads for the gas cells; and now there is a trend in large ships toward the use of rigid ring bracing instead, completely unwired. See Appendix No. 6.

It is clear that many degrees of indeterminancy are produced by even only one transverse frame. For instance, *each* such frame in an 8-, 16-, or 32-sided ship will cause the following degrees:

a Transverse wiring, in one plane, and statically determinate in itself in that
plane, $n - 3$ = 5 deg. for 8 sides
(or counting symmetry = $^1/_2 n - 1$, or less) = 13 deg. for 16 sides
= 29 deg. for 32 sides
(N.B. As usually built the number is two or more times these figures)
b Rigid ring, of triangular cross-section, $2n$ = 16 deg. for 8 sides
(or counting symmetry = $n + 3$, or less) = 32 deg. for 16 sides
= 64 deg. for 32 sides

A ship with (say) 13 cells would have 12 times these numbers; hence clearly the effect of redundancy in common practice is large.

COMPUTATION OF INDETERMINATE STRUCTURES

Structures that are indeterminate can be solved only by (1) a composite method, or (2) semi-empirical methods.

(1) By eliminating all redundancy, in any desired manner, the statically determinate stresses can be computed, and then by invoking the laws of elastic deformation the redundant members can be considered and their effect superposed. This method is exceedingly laborious in many cases, and impossible in fact when 6 to 10 or more redundancies occur; hence it is customary in airship practice to design on semi-empirical methods.

(2) Semi-empirical methods comprise:

a Method of moments
b Method of shears
c Method of inverse ratio.

These methods are well described by Burgess in the book quoted, and reference should also be made to R. & M. 1057. Many writers have shown that by studying the deflection of a frame, based on the assumptions that the transverse frames do not distort from a plane, and that in that plane the radial and tangential positions of the joints do not change; based on uniformity of size of the longitudinals, of the diagonals, and of the ring members, respectively, the methods *a* and *b* give identical results. Burgess has shown (Trans. A.S.M.E., 1929) that each is accordant with the principle of least work. But any variation of size destroys the agreement, as does the introduction of any variant, such as a keel or corridor; hence one or the other (or both) must in general be incorrect. In N.A.C.A. TN 140, the author has shown (pp. 19–21) that for the frames to remain plane the counter wires must all be acting, in addition to the above requirement of uniformity of size.

COMMENTS ON PREVIOUS WORK

Method *a* has been considered of late to be closer to the truth than the others, but discrepancies between the theoretical stresses and those found in model tests have not been explained. Southwell (R. & M. 737, 790, 791, 819, 1057) has shown that for the assumption of plane bending to hold, the loads applied to any transverse must be placed in one, singular conformation. He and Pippard make frequent reference to the reservations which St. Venant himself held regarding the accuracy of the theory when applied to cross-sections that are proportionately near to the loaded section. Pippard has shown (R. & M. 977) by tests that for an unbraced, prismatic tube the assumption of plane bending is an impossible one, and this point the author discusses theoretically in the present paper; he also shows (R. & M. 948) that unless the bracing is highly rigid as compared with the rest of the structure, there are wide discrepancies in those frame spaces near the loaded frame. See Appendix No. 5.

But substantially all work that has been done theoretically has been based on parallel body alone, and Pippard has shown (R. & M. 948) by experiment that even the introduction of a nose cone at the end of a prismatic model has a marked effect on the stresses. Tests by the Bureau of Aeronautics, U. S. Navy, of a model of the *Shenandoah*, see Appendix No. 3, showed also the effect of applying a nose cone, in that the secondary longitudinals became stressed to approximately the full value that they would have on the plane-bending assumption, although there were no shear wires attached to them whatsoever, and their entire stress came from the nose cone. Redundancies in this model, however, largely obscured the external load stresses. Recorded strains in the *Los Angeles* during flight cannot be correlated with any of the theories with reasonable accuracy, though perhaps they are concordant with the moment theory to some degree. (N.A.C.A. TR 325.)

Prof. Wm. Hovgaard in two papers, *Journal of Mathematics and Physics*, Vol. II, No. 4, December, 1923, "A New Proof of the Theory of Ordinary Bending and Its Extension to Beams of Non-Homogeneous Materials," and Vol. IV, No. 2, April, 1925, "Determination of the Stresses in a Beam by Means of the Principle of Least Work," discusses interestingly the stresses in a beam. A few quotations follow:

"If then we accept this principle" [i.e., Least Work], "it follows that a beam in bending will have an inherent tendency to conform to the law expressed in Equation [1]." [I.e., $M/I = p/y$.] "The question arises: to what extent is this result achieved in practice? Saint-Venant has shown that when the length of a beam is large in comparison with its transverse dimensions, the state of stress and strain created by terminal couples is practically independent of the mode in which these couples are applied, and conforms to the above-stated law, except quite locally near the ends. This applies probably also to non-homogeneous girders with the qualification that here the adjustment of strains and stresses will extend to a greater distance from the ends before an ideal régime is established. Practical experience, as well as large- and small-scale experiments, has shown that the law holds with great accuracy even close to the points of application of the terminal forces, and this is true not only of homogeneous but also of non-homogeneous girders such as beams of reinforced concrete. It is not here asserted that the ideal condition will be attained in any beam under all circumstances, but by means of the Principle of Least Work it is shown that the tendency to do so must always exist..." And, again:

"It is not asserted that the ideal state of stress and strain defined by Equation [16]" [i.e., a modified form of the beam formula, for non-homogeneous beams] "actually exists in any ordinary girder, but it is believed to be shown that in accordance with the Principle of Least Work there will be a tendency for the stresses and strains to so adjust themselves as to approach to this condition. This tendency agrees with and confirms the Principle of Saint-Venant." He then applies the modified formula to find the stresses in a semi-rigid, and shows that the envelope would carry "more than one-third of the entire bending moment," the remainder being carried by the keel.

In the second paper he states that the solution "gives assurance that the state of stresses to which it leads is one to which the beam must tend, even although it may not in practice conform to it near the points of application of the external forces." He works from certain assumptions, such as: "All the external forces as well as their distribution over the end sections are assumed to be known..." "No external forces act on the lateral bounding surfaces..." "It is assumed that the distribution of the external forces over the terminal section at A" [i.e., the free-end section] "is such as to conform to the yet unknown stress distribution required by the Principle of Least Work. The same assumption is made for the reactions over the section at the fixed end." "As in the solutions given by Saint-Venant and Clebsch it is further assumed that there are no normal transverse stresses, that is, the longitudinal fibers, of which we may imagine the body to be made up, do not exert on each other any pressures or tensions normal to their direction." Finally, "we have assumed that the forces at the free end and the reactions at the fixed end[3] are distributed over the terminal sections in accordance with the stress distribution subsequently determined by the Principle of Least Work. In practice these conditions are never fulfilled, but as there is an inherent tendency of the stresses to conform to that principle, we are justified in asserting that at a certain distance from the terminals the stresses will adjust themselves in accordance therewith. Saint-Venant has shown that this adjustment actually takes place very close to the terminals (Saint-Venant's Principle)."

In "The Mathematical Theory of Elasticity," by Love, second edition, Par. 89, is the following: "In the problem of Article 87, the tractions, of which the bending moment EI/R is the statical equivalent, are distributed over the terminal sections in the manner of tensions and pressures on the elements of area, these tensions and pressures being proportional to the distance from the neutral plane. But the practical utility of the solution is not confined to the case where this distribution of terminal traction is exactly realized. The extension to other cases is made by means of a principle, first definitely enunciated by Saint-Venant, and known as 'the principle of the elastic equivalence of statically equipollent systems of load.' According to this principle, the strains that are produced in a body by the application, to a small part of its surface, of a system of forces statically equivalent to zero force and zero couple, are of negligible magnitude at distances which are large compared with the linear dimensions of the part. In the problem in hand, we infer that, when the length of the bar is large compared with any diameter of its cross-section, the state of stress and strain set up in its interior by the terminal couple is practically independent of the distribution of the tractions, of which the couple is the resultant, in all the portions of the bar except comparatively small portions near its end."

In Par. 95, for the bending of a narrow rectangular beam, of width $2h$ and depth $2c$, by a terminal load, he shows that the "terminal tractions, of which W is the resultant, are distributed over the end so as to be proportional to $c^2 - y^2$. As in Art. 89, the distribution of the load is important near the ends only, if the length of the beam is great in comparison with its depth."

In Par. 245 he shows that the deflection of a beam due to a central load of W is increased by the local stress at the load. "Either part, AC or AB, of the beam might be treated as a cantilever, fixed at A and bent by terminal load $^1/_2W$ acting upward at the other end; but Saint-Venant's solution would not be strictly applicable to the parts AB or AC, for the cross-sections are distorted into curved surfaces which would not fit together at A.[4] In Saint-Venant's solution of the cantilever problem the central part of the cross-section at A is vertical, and the tangent to the central-line at A makes with the horizontal a certain small angle S_0.... The correction of the central deflection which would be obtained in this way would be equivalent, in the case of a narrow rectangular beam, to increasing it by the fraction $45d^2/16L^2$ of itself, where L is the length of the span, and d is the depth of the beam. The correction is therefore not very important in a long beam." However, if the length/depth ratio be small, for instance, 5, this fraction amounts to one-ninth, and for a ratio of 4, to three-seventeenths.

Reference should also be made to R. & M. 821, "On Castigliano's Theorem of Least Work, and the Principle of St. Venant," R. V. Southwell. See also Appendix No. 3.

In Trans. A.S.M.E., 1929, Burgess determines the stresses in a cantilever, parallel-sided space frame of 8 sides, with keel, upon the methods of moments, shear, inverse ratio, least work, etc. Underlying the computation of stresses is an assumption, expressed as, "....all longitudinals and diagonals are pin-connected to an absolutely rigid wall," and, again, "The frame at the right-hand end is rigidly cross-braced," and, again, "The internal work in all transverse members, including the cross-bracing of the right-hand end frame, is negligible. This is equivalent to assuming that EA is infinite for these members in the least-work and inverse ratio calculations."

The excellent report of the Airship Stressing Panel (R. & M. 800) gives methods for computation of stress. Certain assumptions and qualifications are of note:

1 The shear in any facet is assumed to be equally divided between the two diagonals.

2 Appendix IV clearly states (p. 33), in the discussion of the assumption that the frames remain plane, "It should be emphasized that the alternative solutions will be neither more nor less exact than the solutions found previously: their differences corresponding to a difference in the implied distribution of the external loads and it must be left for subsequent investigation to decide which are the more representative of actual conditions."

3 The loaded frames are radially wired, and it is shown in Appendix XI that such a system has greater rigidity than other types, hence the frames are considered to be truly rigid.

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[3] Cv. Appendix No. 3, stresses at Frame No. 70, upper and lower longitudinals.

[4] Cv. Appendix No. 3, Frame No. 70, longitudinals D, E, F, G, and H.

4 Appendix VIII shows that when all members are considered flexible there is a considerable difference in the stresses as computed by least work from those that result by considering the frames rigid. It also shows marked variation of stresses in all shell members for different systems of loading of a given radially wired frame.

5 There is a basic assumption that the supplementary load system will be resisted entirely by the frame itself. The following statements are made on p. 60: "Since there is no practicable method of allowing for these local stress effects due to the bulkhead elasticity, *it is important to design the bulkheads as rigid as possible in order to eliminate them*" (italics the author's), also, "The effect of the elasticity of the bulkheads is therefore a local disturbance, the amount of the disturbance depending on the exact method of application of the load. After the second bulkhead, the stresses in the longitudinals as we pass from bay to bay, are practically the same as those for a tube in which the loads are applied through a rigid bulkhead. The difference between the stresses in the longitudinal members in the two cases does not increase after the second bulkhead, but is in the nature of a constant initial difference, and the percentage difference decreases from bay to bay in geometrical progression."

6 Another effect, which the author will not consider in this paper, is the reduction in modulus of elasticity, mentioned in Appendix X.

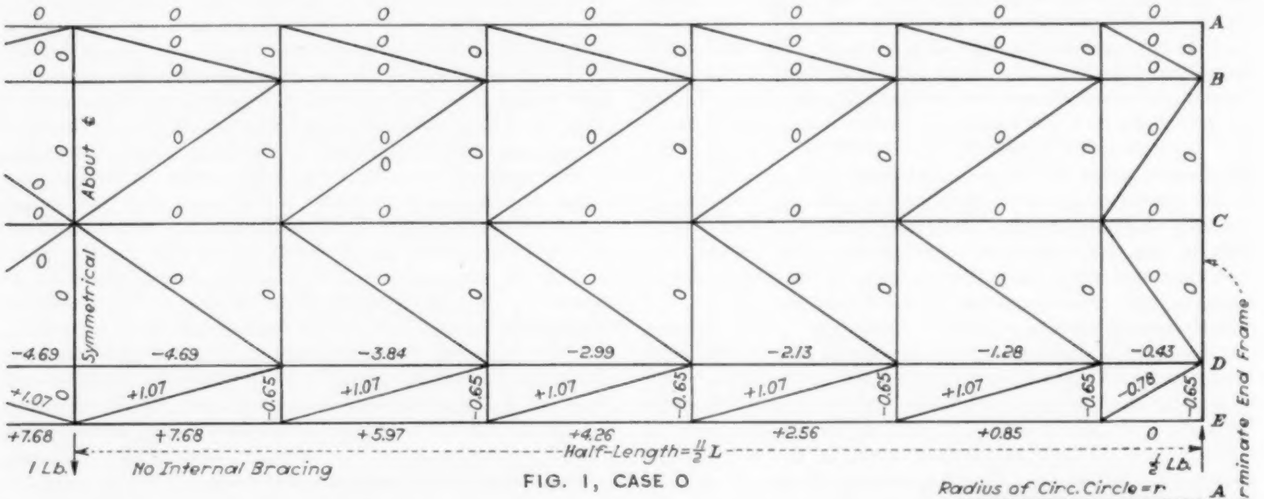
A
B
C
D
E
About ℄
Symmetrical
0
−4.69
−4.69
−3.84
−2.99
−2.13
−1.28
−0.43
+1.07
−0.65
−0.78
+7.68
+7.68
+5.97
+4.26
+2.56
+0.85
0
1 Lb.
No Internal Bracing
Half-Length = 11/2 L
1/2 Lb.
Statically Determinate End Frame

FIG. 1, CASE O

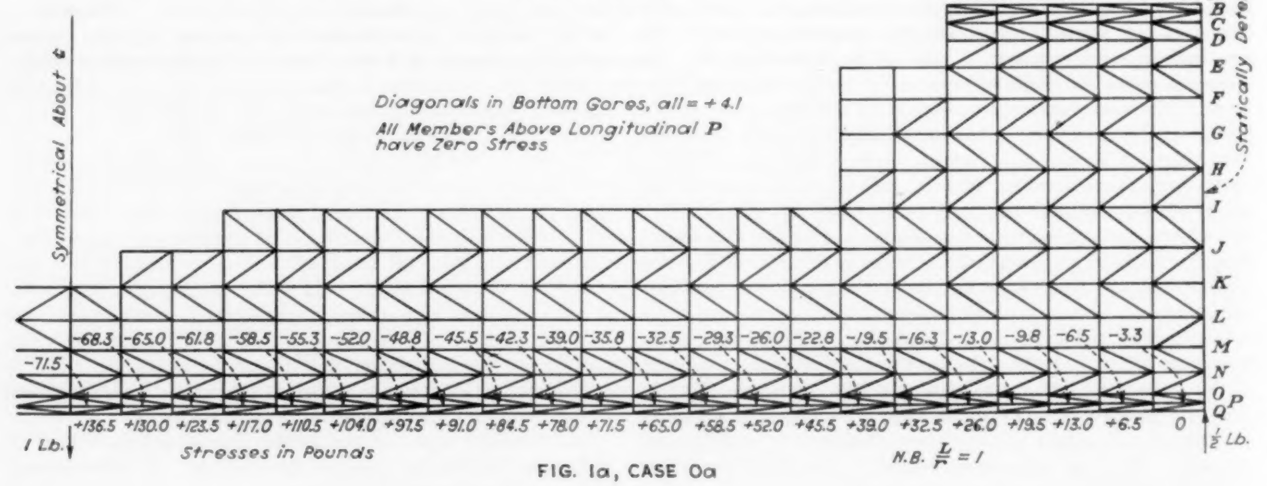
Radius of Circ. Circle = r
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
Symmetrical About ℄
Diagonals in Bottom Gores, all = +4.1
All Members Above Longitudinal P have Zero Stress
−71.5
−68.3
−65.0
−61.8
−58.5
−55.3
−52.0
−48.8
−45.5
−42.3
−39.0
−35.8
−32.5
−29.3
−26.0
−22.8
−19.5
−16.3
−13.0
−9.8
−6.5
−3.3
+136.5
+130.0
+123.5
+117.0
+110.5
+104.0
+97.5
+91.0
+84.5
+78.0
+71.5
+65.0
+58.5
+52.0
+45.5
+39.0
+32.5
+26.0
+19.5
+13.0
+6.5
0
1 Lb.
Stresses in Pounds
N.B. L/r = 1
1/2 Lb.

FIG. 1a, CASE Oa

FIGS. 1 AND 1a

7 And the ineffectiveness of the shell diagonals in stiffening the longitudinals, given in Appendix XII.

8 Attention is called, however, to studies in Appendix XIII of the rigidifying effect of the shell caused by initial tensioning of the diagonal wires.

In the other R. & M. reports mentioned will be found much excellent and intensive work on the degree of accuracy to be expected of computations based on the assumption of plane bending, or upon the assumption of planar transverse frames. Southwell shows (R. & M. 819, 790, etc.) that a given loading can be divided into one which will carry along the structure in St. Venant's fashion, and one which ("supplementary") will be self-equilibrating in the loaded region. The theory implies an inextensible frame, i.e., infinite rigidity. But Pippard shows that unless this frame be in fact inextensible, i.e., unless its rigidity be in fact very large compared with that of the shell, the stresses found in the shell do not compare well with "St. Venant stresses."

All bases for the presupposition that St. Venant's methods will give acceptable results have involved the assumptions of parallel body, great elongation ratio, and great rigidity of the loaded frames. The authors quoted repeatedly stress this fact. Until recently the facts did not differ widely from these assumptions, in that the main frames were cross-wired rigidly and in an indeterminate (and hence rigid) fashion, and considerable parallel body was used. But the introduction of ring frames, the reduction of the elongation ratio, and the elimination of parallel body so alter the facts that the author believes that the assumption of plane bending, *without further proof*, is unjustified. It is of course certainly true that the range over which any local stress extends is limited, but every loaded frame is a group of focal centers for such stresses. Or, speaking in terms of Southwell's analysis, correction stresses must be considered for every loaded frame. Every keel, or corridor, is also an irregularity.

St. Venant considered conditions that appear in a long prismatic beam, at sections well removed from the point of loading. Local stresses at such point inherently prohibit local deformation according to planar distribution. An airship of variable diameter throughout, having an elongation ratio in modern designs of 4 to 6, having flexible bracing so that the initial positions in their plane of the joints are not closely maintained, must vary quite a little from the fundamental assumptions.

II—TENTATIVE RESULTS

Determinate Structures. Tacitly, the previous work quoted begs the question, from the standpoint of this paper. The manner of application of the load to these frames, and their rigidity, influences the stresses throughout the ship, as evidenced by the papers of Southwell and Pippard. For instance, consider a structure, similar to the ones already studied, but without keel, and with statically determinate end frames. See Fig. 1. The determinate stresses shown, which must be true—unless the joints are not frictionless—differ widely from all of the theories, as shown in Table 3, Appendix No. 4. This table has been computed according to the recognized formulas and by determinate methods.

The first reaction to this table may probably be expressed: "But Fig. 1 does not even approximate to an actual ship, having no nose and no internal bracing." At most it states that this is one of an infinite variety of designs of a given contour; that it differs widely from the common theories; that therefore the theories cannot be universally true. That the determinate stresses listed are not erroneous may easily be confirmed by reference to Table 4, in Appendix No. 5, which gives comparison of theoretical stresses and of stresses actually found by experiment for a similar case. (Cv. R. & M. 977.) There is a marked agreement with the statical stresses, and a marked disagreement with the stresses as computed by other methods.

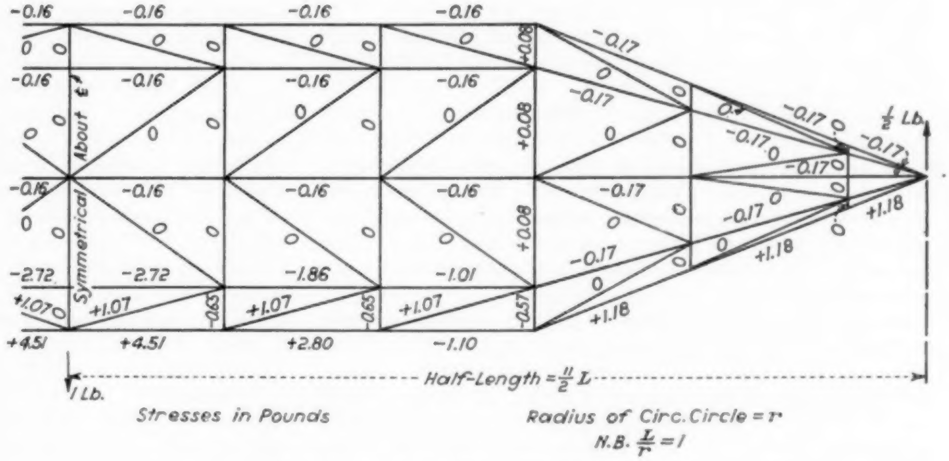
About ℄
Symmetrical
$\frac{1}{2}$ Lb.
1 Lb.
Half-Length = $\frac{11}{2}L$
Stresses in Pounds
Radius of Circ. Circle = r
N.B. $\frac{L}{r}=1$

FIG. 2 CASE 1—NO INTERNAL BRACING

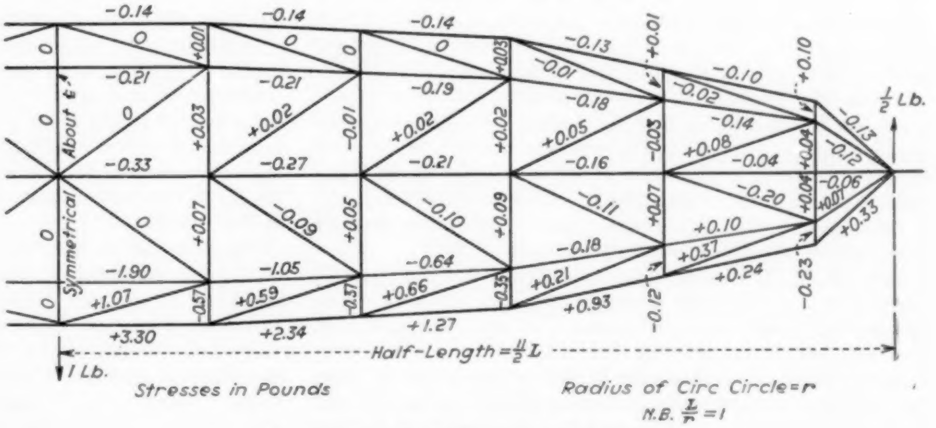
About ℄
Symmetrical
$\frac{1}{2}$ Lb.
1 Lb.
Half-Length = $\frac{11}{2}L$
Stresses in Pounds
Radius of Circ Circle = r
N.B. $\frac{L}{r}=1$

FIG. 3 CASE 3—NO INTERNAL BRACING

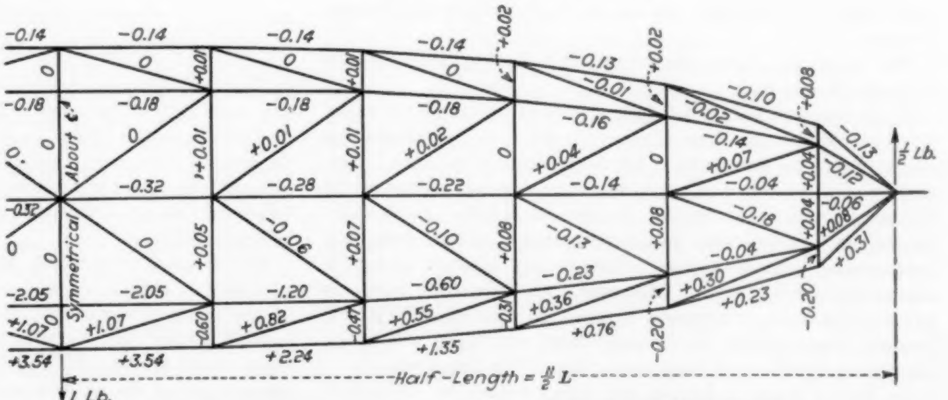
About ℄
Symmetrical
$\frac{1}{2}$ Lb.
1 Lb.
Half-Length = $\frac{11}{2}L$
Stresses in Pounds
Radius of Circ. Circle = r
N.B. $\frac{L}{r}=1$

FIG. 4 CASE 5—NO INTERNAL BRACING

In Fig. 2 will be found a somewhat closer approximation to an actual ship, in that there is now a blunt nose similar to that used in all rigids built in the early part of this century—but still no internal bracing. Here again the stresses do not resemble in any way those determined by the common theories. The effect of the nose, however, is marked.

Figs. 3 and 4 all differ from the common theories, though they show an interesting trend that is a function of the longitudinal curvature.

One fact is clearly expressed by these designs, all of which are determinate, namely, that a load applied at a joint will cause high stress in those gores (a "gore" meaning a longitudinal group of facets) whose common edge passes through the loaded joint, provided there is no internal bracing. This is also equally true even if redundant wires are present in every facet. How far the internal bracing modifies this statement remains to be proved.

Another fact expressed is that at joints where "kinks" occur in the longitudinal profile, stress in any gore is partially transferred to the neighbor gore; and only at such joints can transfer take place unless there is internal bracing. The designs shown are all for an 8-sided ship, in which, therefore, all gores become stressed in some part of their length after 4 kinks have been passed, even without a nose cone. The stress, in other words, spreads spirally around the shell. Note, however, that in a 32-sided ship 16 kinks would be required, meaning that a load on a central frame could only just encircle the ship if a total of 31 frame spaces are provided, and no nose cone.

Again, the amount of stress transmitted to the neighbor gores is a function of the number of sides. For instance, comparing 32 sides with 8 sides, the stress transfer will be 3.9 times as much. Also it is a function of the kink angle in the longitudinal profile, a sharp kink giving greater transmission.

Discussion of Statical Stresses. Let us see what may be learned from this statical sequence by further study.

First, in Fig. 1, Case 0, which has no kinks but is all parallel body, with a determinate frame at each end, the whole being symmetrical, it will be noted that a load, say 1 lb., applied at any joint, can stress only the two adjacent gores—as already stated. All members above the *D* longitudinals have no stress for the load indicated, and it makes no difference where the end reactions are placed except in so far as the stresses in the end frames themselves are concerned. Two gores carry all moments and shears, and because they are of shallow depth the stresses are high.

The figure also shows the stresses in a 32-sided ship. These are more excessive.

These cases and all which follow involve of course the ratio, L/r, or (length of frame space)/(radius of circumscribing circle), which for convenience has been assumed as unity. Any change of this value will alter only the numerical values of the stresses, proportional to the changes of length. Each facet shown can be subdivided into any number of shorter ones, still maintaining determinateness, without any change except in numerical stress value. Moreover, all diagonal members can be doubled, or increased n times, so that each facet will become indeterminate *in its own plane*, and there will be no transfer of stress to any members outside of the two gores.

In Fig. 2, Case 1, having one kink, we see an immediate change. The end reaction of $^1/_2$ lb. is now carried by two systems. There is, first, the system of the two gores that adjoin the load of 1 lb., whose stresses are now much reduced; also, second, there is a system that resembles, somewhat, a reinforced concrete girder, or more nearly a semi-rigid airship (see footnote 2). This latter consists of a "top flange," located at the ship's axis and having an area equal to that of *all* eight longitudinals, and a "bottom flange" which consists of the one bottom longitudinal (where frequently the ship has been reinforced with an actual keel). The first system carries all shears in the parallel body; in the nose no diagonals have any stress, all shear being carried by the longitudinals. Sub-facets may be inserted here as before, if desired, without change except in numerical stress values. Any facet may be made indeterminate, as before, except those which are contiguous to the kink.

In Fig. 3, Case 3, having 3 kinks, the stress from the two gores has been still further reduced and distributed to the other gores, so that now the whole stress system has changed materially. The bottom longitudinal has less stress than before, but it still is affected to a much larger degree than any of the others. The stresses increase downward as the two gores are approached.

In Fig. 4, Case 5, having 5 kinks, the process has gone further but there is not much change (in percentage) from Fig. 3. The profile is now almost elliptical, with two parallel bays amidships. In a rough way, Cases 3 and 5 are almost identical in stress, and even Cases 1 and 3 do not differ as greatly as any one case differs from any of the common theories.

There has not been sufficient time to compute the statical stresses for designs of more than 8 sides, because although simple methods have been devised, there is still considerable work involved. But for the purpose of this paper it is shown that the transverse bracing that is always used is called upon to perform—on the assumptions that underlie the common theories—a multifold duty. Every one of the stress systems shown differs from every other, and from the common theories, hence the transverse bracing must be able to convert any one of an infinite number of stress systems into a system of just one single type, i.e., a system where the stresses in the longitudinals are linear functions of the distance from the neutral axis and the stresses in the shear wires are functions of the angle of inclination of the facet planes.

The next step to be taken is the determination of the effect of a single intermediate transverse frame, determinate within itself, but of necessity causing indeterminateness in the aggregation of transverse frame and shell. Once the effect of a single frame is determined, there comes then the determination of additional frames. The redundancies caused by only one frame are somewhat formidable, even for an 8-sided prismatic shell, and the author must content himself with these in this paper.

III—INTERNAL BRACING

Effect of a Single, Self-Determinate, Transverse Frame

The high order of redundancy of an actual ship precludes the possibility of an exact solution, as has already been said. To find the best solution, even though approximate, there must be extensive study of various methods. Even then the result can only be obtained by slow steps and painstaking care. The following discussion outlines the steps which the author has taken thus far.

One Single-Wire Bracing Member, in a Complete Shell. Let us apply to Figs. 1, 1a, and 4 a single wire on the vertical axis of the plane of symmetry, with loads of 1 lb. at the top and bottom joints. Let the area of all longitudinals and all transverse struts of the shell be equal; and the area of all wire diagonals and of the single bracing member be equal to one-tenth of the area of the longitudinals. The variation of each case for other sizes of the bracing wire may be given by formula. Let the wires be steel, the other members duralumin, and the ratio of moduli of elasticity 3.

The most direct path for the stress is through the wire, but its elastic stretch will cause strain in the shell, hence the portions carried by the two systems will be inversely proportional to their relative rigidities. The hull stresses due to either load are given in the figures, and those for the mate will be symmetrical. The elongation ratio of these figures is 5.5, but other ratios are tabulated below.

Division of Load Between Bracing Wire and Shell

		Parallel Body, 8 Sides		
Elongation ratio	=	5.5	4.5	3.5
Per cent in wire	=	99.4	99.0	98.2
Per cent in shell	=	0.6	1.0	1.8
		Parallel Body, 32 Sides		
Elongation ratio	=	5.5	4.5	3.5
Per cent in wire	=	99.997	99.994	99.988
Per cent in shell	=	0.003	0.006	0.012
		Curved Body, 8 Sides		
Elongation ratio	=	5.5	4.5	......
Per cent in wire	=	97.0	93.1	......
Per cent in shell	=	3.0	6.9	......

Note that in the curved body the shell receives a much larger proportion of the force, because of greater rigidity relatively, also that 32 sides cause lowered rigidity relatively.

One Single-Wire Bracing Member, in a Length of Two Bays. Let us consider the effect of the single wire and the same load system on the shell in a length of two main bays between braced end frames consisting also of a single, vertical wire. In effect the structure has an elongation ratio of 2.

		Parallel Body, 2 Main Bays Long 8 Sides	 32 Sides
Elongation ratio	=	2.0	2.0
Per cent in wire	=	88.3	99.6
Per cent in shell	=	11.7	0.4

Note that the shorter shell is much more rigid, but the 32 sides again cause lowered rigidity.

Self-Determinate Braced Frames in a Length of Two Main Bays

Parallel Body. Let us consider an octagonal structure such as is shown in Fig. 5, having a central and two end frames, but for the moment let the bracing of these frames consist only of a single vertical wire. Let there be a single load of 1 lb. at the central frame resisted by loads of $^1/_2$ lb. at the end frames. This simulates a case where the frame loading exceeds the gas lift at that frame, which may actually occur at heavy concentrations, and which has some resemblance to certain aerodynamic systems. The moduli are the same as before, also the areas, but the ratio of Ea for the wires of the transverses divided by EA for the longitudinals is denoted by k.

Then the stresses in the shear wires of the shell will be as follows:

Member	Statical stress, no intermediate frame	Least-work stress for various values of k 0.03	 0.30	 0.60	 3.0	 ∞	Stress by method of shears	Stress by method of moments
A-B	0	+0.455	+0.480	+0.482	+0.484	+0.484	+0.079	+0.079
B-C, C-D	0	0	0	0	0	0	+0.190	+0.190
D-E	1.07	+0.615	+0.590	+0.588	+0.586	+0.586	+0.079	+0.079

Let us now consider the structure as shown in Fig. 5, with the self-determinate frames shown. To avoid the complexity of a crossed diagonal, consider that the members may resist compression as well as tension.

There are 5 redundant members, but symmetry reduces this number to 3, and these have been chosen as the shear wires *A-B*, *B-C*, and *C-D* in any panel of the ship. The work, or deflection, computations are too voluminous to give here, but the results are tabulated in Table 1.

In Fig. 8 is a graph showing the variation given in Table 1. It should be noted that a different type of bracing would produce stresses of differing values, but the trends would be similar with change of rigidity. It is evident that when k is less than $^1/_4$ the stresses vary considerably from the common theories. If $k = 0.10$ be considered to represent a customary ratio in ships as built, then the discrepancy is roughly 33 per cent as an average, and increases very rapidly for lower values.

A difficulty arises in applying the methods of moments and shears to a structure with only a single system of wiring in the facets, which cannot be symmetrical above and below the axis. For instance, E_1E_2 cannot have a stress in the octagonal design, although by these methods it should have a stress equal to that in A_1A_2. Similarly in every panel of the cases shown in the figures the upper longitudinals will have one more increment of stress than the respective lower ones.

TABLE 1 STRESSES IN SHELL OF LENGTH EQUAL TO 2 MAIN BAYS, FOR VARIATION OF RIGIDITY OF INTERNAL BRACING OF A GIVEN TYPE (SEE FIG. 5)

Octagonal Case

Member	Statical stress, no intermediate frame	Least-work stress for various values of k 0.01	 0.10	 0.20	 1.0	 ∞	Stress by method of shears[1]	Stress by method of moments[1]
A_1A_2	0	−0.379	−0.157	−0.135	−0.116	−0.111	−0.125	....
A_2A_3	0	−0.758	−0.314	−0.270	−0.232	−0.222	−0.250	....
A_3A_4	0	−1.137	−0.470	−0.405	−0.348	−0.333	−0.375	−0.375
B_1B_2	0	−0.100	−0.123	−0.135	−0.147	−0.150	−0.151	....
B_2B_3	0	−0.011	−0.167	−0.203	−0.237	−0.245	−0.240	....
B_3B_4	0	+0.078	−0.211	−0.271	−0.327	−0.341	−0.328	−0.265
C_1C_2	0	−0.009	−0.148	−0.150	−0.152	−0.153	−0.151	....
C_2C_3	0	+0.082	−0.173	−0.166	−0.156	−0.155	−0.151	....
C_3C_4	0	+0.173	−0.198	−0.181	−0.160	−0.157	−0.151	0
D_1D_2	−0.85	−0.400	−0.124	−0.098	−0.074	−0.067	−0.063	....
D_2D_3	−1.71	−0.790	−0.100	−0.045	+0.004	+0.018	+0.026	....
D_3D_4	−2.56	−1.180	−0.075	+0.009	+0.082	+0.103	+0.114	+0.265
E_1E_2	0	0	0	0	0	0	0	....
E_2E_3	+1.71	+0.800	+0.248	+0.195	+0.148	+0.135	+0.125	....
E_3E_4	+3.42	+1.600	+0.495	+0.390	+0.295	+0.270	+0.250	+0.375
A-B	0	+0.237	+0.098	+0.084	+0.073	+0.070	+0.079	+0.079
B-C	0	+0.126	+0.154	+0.169	+0.185	+0.189	+0.190	+0.190
C-D	0	+0.012	+0.185	+0.188	+0.190	+0.191	+0.190	+0.190
D-E	+1.07	+0.502	+0.155	+0.122	+0.093	+0.084	+0.079	+0.079

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[1] For formulas used, see Appendix No. 4, also note at end of appendix.

The method of shears is based upon the relative deflection of two parallel frames of infinite rigidity, hence as the rigidity of the frames approaches infinity the tabulated stresses would approach those by the common methods as a limit, if there were crossed diagonals in the facets. Actually they do show a close approximation, though there are slight variations. It is of great interest to note that on either side of the value, $k = 0.10$, there is a change of sign in some longitudinals. For very small values of k the stresses should approach the statical stresses as a limit, and longitudinals *D* and *E* do so. But *A*, *B*, and *C* do not, because of the fact that the wire-bracing size in the end bulkheads was changed as well as in the central bulkhead, as would be the case in an actual ship, and therefore for $k = 0$ the structure would be unstable unless the $^1/_2$-lb. reactions had been applied at the joints E_1. These three members, therefore, become large, as should be true. In order to find the trend with number of sides, a hexagonal but similar case has been computed. See Fig. 6. It might be noted, in passing, that the methods used give results agreeing within 1 per cent with Professor Pippard's computed stresses for a hexagon with a keel and with rigid frames. The shear-wire stresses are tabulated herewith, and the longitudinals may easily be computed from the increments of stress due to the shear wires.

STRESSES IN SHELL OF LENGTH EQUAL TO 2 MAIN BAYS, FOR VARIATION OF RIGIDITY OF INTERNAL BRACING OF A GIVEN TYPE (SEE FIG. 6)

HEXAGONAL CASE

Member	Statical stress, no intermediate frame	Least-work stresses for various values of k					Stress by method of shears[1]	Stress by method of moments[1]
		0.03	0.30	0.60	3.0	∞		
A-B	0	+0.177	+0.119	+0.113	+0.107	+0.106	+0.118	+0.118
B-C	0	+0.146	+0.222	+0.228	+0.234	+0.236	+0.236	+0.236
C-D	0.707	+0.239	+0.144	+0.137	+0.131	+0.129	+0.118	+0.118

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[1] For formulas used, see Appendix No. 4.

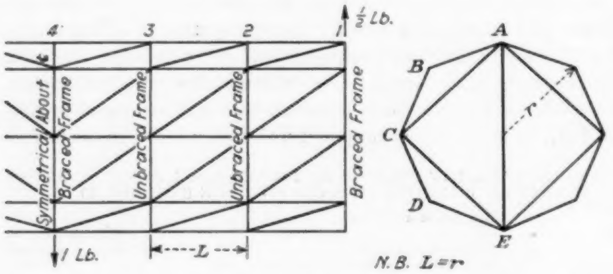

FIG. 5 OCTAGON

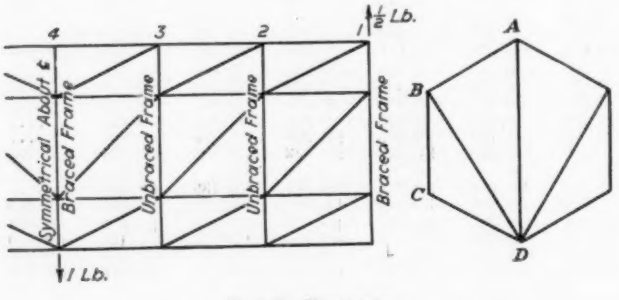

FIG. 6 HEXAGON

The two cases just cited are extreme ones in respect of the distance between main frames as compared with the diameter of the shell. To show the trends more broadly the following case deals with a hexagonal shell in which the distance between main frames has been reduced to a length equal to the radius of the shell.

STRESSES IN SHELL SIMILAR TO PRECEDING, WITH SPACING OF MAIN FRAMES REDUCED BY ONE-THIRD (SEE FIG. 7)

HEXAGONAL CASE

Member	Statical stress, no intermediate frame	Least-work stresses for various values of k					Stress by method of shears[1]	Stress by method of moments[1]
		0.03	0.30	0.60	3.0	∞		
A-B	0	+0.223	+0.160	+0.137	+0.110	+0.101	+0.118	+0.118
B-C	0	+0.054	+0.171	+0.197	+0.224	+0.233	+0.236	+0.236
C-D	0.707	+0.374	+0.201	+0.173	+0.147	+0.139	+0.118	+0.118

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[1] For formulas used, see Appendix No. 4.

Fig. 8 shows graphically the results of the tabulations. It will be noted that the ratio of the tabular stresses to those computed by the common theories is plotted against the value of k. Clearly the discrepancy between the two is far greater in the case of the octagon than in the case of the hexagon.

Curved Body. Amidships the curved body will approximate to parallel body and the previous discussion applies. Fore and aft there will be a difference. This effect has not yet been studied. It is true that most of the heavy concentrations of load, where the lift is less than the local loading, lie near midlength. At any frame the lift is, of course, distributed over the top and sides of the frame, and the modern tendency is to concentrate the loads near the 45-deg. points of the lower portion of the frames.

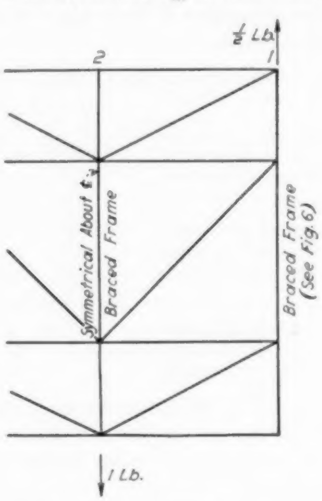

FIG. 7 HEXAGON

Unwired Ring Frames. In Appendix No. 6 is a rough computation of the relative rigidity of unwired and tight-wired frames, indicating the approximate value of $k = 0.142$ which is marked *A* on Fig. 8. The degree of redundancy of the unwired frame is such that at present the effect on the shell can only be approximated. Within the accuracy of this computation, it is apparent that frames having a depth of one-eighth of the radius, or less, will cause serious discrepancy; and that even for deeper rings great care should be paid to the stresses induced in the shell. Much further study is needed.

ALL MAIN FRAMES BRACED, WHETHER SELF-DETERMINATE OR HIGHLY REDUNDANT

The only information at hand at present is that afforded by the brief study made by Burgess in Trans. A.S.M.E., 1929, and by the experiments of Pippard. See Appendix No. 5. It is evident that even a multiplicity of frames are not locally effective, unless the loaded frames are themselves *very* rigid. The *Shenandoah* model seemingly points to the same conclusion. It should be noted that Pippard's frames, especially those with $^5/_8$-in. rods, were very rigid. See point *C*, Fig. 8.

SUMMARY

The foregoing discussion in reality deals with two effects; the local "supplementary" load system, and the general bending of the hull. These can never, in the author's belief, be considered singly because of the limited elongation ratio and of the multiplicity of supplementary systems. Some system of stresses as shown by a series of influence lines seems to be the solution of this general design problem, the influence lines, for instance, being drawn for loads at each longitudinal. Each influence line will show the full effect of a load at any point, i.e., will cover the general stress condition as well as the local stress system. The author has found no general argument which will even approximately describe the effect of a change in any one of the many variables which enter this problem, because the interrelation of the variables is too complex. The shear and moment theories seemingly give rough approximations, and may be satisfactory for general design purposes; but for determining the actual size of members, or the actual stress in any given member they seem, at this writing, to be too uncertain.

The curves of Fig. 8 are the nearest approach to a statement that can at present be made as to the effect of the number of sides; but some small part of the effect shown is a function of the angle, ϕ, the angle between the longitudinals and the facet diagonals, and for a given length of panel the tan ϕ is a function of the number of sides. Of course as ships have grown larger and larger in diameter, the spacing of main frames has increased so that now there may be 3 intermediate, unbraced frames in a bay where before there were 2 or only 1, necessitated by the increasing number of longitudinals and the need for an optimum value of the angle, ϕ. The stresses for Fig. 7 show the effect of reducing the spacing of main frames, and have been computed for two reasons: (1) to show the stresses for main bays of length approximating to that of actual ships, and (2) to show, in part, the effect of the angle, ϕ, since the shortening of a panel to increase the angle, ϕ, will increase the rigidity of the shell and therefore produce a curve lying higher on the chart than before, i.e., with greater discrepancies.

The author has attempted to find and apply all checks that were available on the arguments and results of this paper. Only by extended study can it be determined whether the arguments will be borne out in fact in the actual ship. The paper points toward certain tentative conclusions, that must be confirmed or denied before they are accepted. The most serious bending and shear stresses in a ship result from (1) running into a gust and (2) operation of the controls. The latter applies force to a rigid frame, which creates inertia "forces" that are pretty uniformly distributed, except as regards the concentrated loads near the bottom of the shell. The former applies forces that probably are quite concentrated on the windward side of the nose, which produce the same kind of inertia "forces." Such a force system is far from being applied as St. Venant requires, and much more closely according to the forces discussed in Fig. 5, for instance. Perhaps this paper contains a germ for a theory of the cause which resulted in the breaking away of the control car of the U.S.S. *Shenandoah*, with heavy loss of life. For some years past the author has had this belief.

The following conclusions, therefore, are offered tentatively and in the hope that opportunity will arise for checking and amplifying them, or disproving them.

TENTATIVE CONCLUSIONS

All evidence points to the facts:

1 That unless there is great rigidity in the transverse bracing, especially at points of loading, the shell stresses cannot be determined with even fair accuracy by the common methods.

2 The change to loose-wired frames requires careful attention to stresses induced in the shell by reason of the lowered rigidity of the ring frames as compared with highly redundant, tight-wired frames.

3 The arguments of this paper and the results of experiments point to the conclusion that with curved profile the approach to the theoretical stresses of St. Venant's principle is very different and probably less certain, than in parallel body.

4 That there should be local reinforcement of members near concentrations of load.

5 That keels and corridors have a marked influence on the stress distribution in the shell, and that their effect on these stresses is as yet undetermined.

6 That the stresses in a rigid have some similarity to those in a semi-rigid

7 That redundancy of the crossed diagonals in the facets of the shell is of rather localized effect.

8 That joint rigidity causes a secondary effect that is of importance.

9 That there is evidence of a stress system underrunning the systems calculated by the common theories.

10 That some points are deserving of further study, such as the "keel effect," or increased stressing near load points, and such as the rigidity of the longitudinals between main frames and the effect of altered modulus of elasticity in curved members on conditions of bending.

11 That the stresses in an octagonal prismatic body compare much less favorably with those stresses computed by the common theories than do those in a hexagonal prismatic body; and that seemingly this shows as a trend that there will be increased differences as the number of sides increases; that this point deserves further study, especially in curved body, to establish the trend further; and that the order of difference is sufficiently large to be of importance in the design of ships.

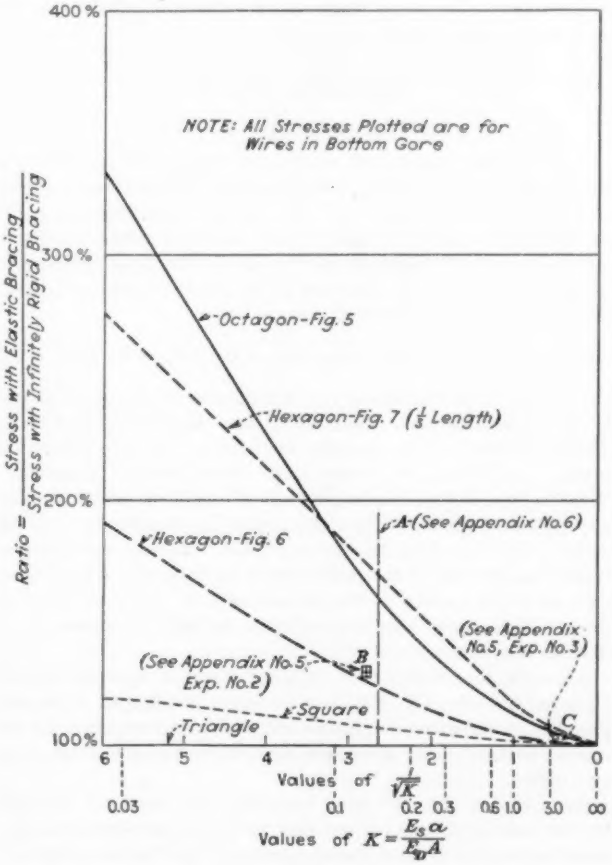

FIG. 8

12 That further study is required to determine how far the presence of additional braced frames may diminish the effect of the preceding conclusions, although there is already experimental evidence that for sizes of bracing used in current practice there is little diminution.

13 It should be noted that this discussion deals with that portion of the stress in any member that results from each localized group of loads; that other portions of the stress in such a member that are caused by loads more distant from it may show less discrepancy; that therefore the discrepancy in

the total stress may be a smaller percentage of the total stress; but that for such load systems as that caused by a sudden large gust, all localized groups of loads are of the same character, probably causing considerable discrepancy even in the total stress.

Acknowledgment

The author wishes to acknowledge the helpful assistance of his associate, Mr. Willard W. Troxell, who has contributed many valuable suggestions and criticisms, and who has carried out much of the extremely tedious computations on which the paper is based; to the Bureau of Aeronautics, United States Navy, for the use of data obtained from the photoelastic model of the U.S.S. *Shenandoah;* to the writers quoted for the masterly studies which they have made and for the full presentation of their experimental data.

Appendix No. 1

STATICAL DETERMINATION

A (statically) determinate structure is one in whose members the stresses due to known joint loads can be found by the laws of statics alone. These laws tell us that a stationary body (or one moving with balanced forces) can have neither resultant force nor resultant moment on it in any of the three component directions. As usually expressed, if the forces X are longitudinal, forces Y lateral, and forces Z vertical, then

$$\Sigma X = 0;\ \Sigma Y = 0;\ \Sigma Z = 0;\ \Sigma M_x = 0;\ \Sigma M_y = 0;\ \Sigma M_z = 0$$

the summations expressing the component forces or moments in the three directions. Be it noted here, in passing, that these directions need not be mutally rectangular, but they must be mutally exclusive, i.e., must have some relative angularity. What is true of the structure as a whole is of necessity true for any portion segregated from the rest by an imaginary dividing surface, and therefore true for any joint. In such portions the truss members cut by the surface must be replaced by their total internal stress applied at the cut end as if in reality an external force. These equations are sufficient to find all stresses in a determinate frame.

Maxwell determined that in a structure of n members and j joints, $3j - 6 - n = 0$, is a necessary condition. It is not, however, a sufficient condition, and the members must be arranged so that there is neither superfluity nor insufficiency at any point.

An *indeterminate structure* (omitting the word "statically" for the sake of brevity) is one that has more members than can be solved by the laws of statics alone, and the degree of indeterminacy is measured directly by the number of surplus, or "redundant," bars. In other words, $3j - 6 - n$ is positive.

An *unstable structure* is one which is deficient in the number of its members, and the degree of instability is likewise measured by the deficiency of members. In other words, $3j - 6 - n$ is negative. Such structures may or may not be generally unstable according as the deficiency is general or localized.

The term *quasi-determinate* has been applied to such instances as that of the facet with two diagonals, each capable of withstanding tension only, because one such member (unless initially tensioned) must become slack and therefore, in principle, disappear. ("Facet" is used to mean any plane, exterior surface bounded by members.) Pippard (R. & M. 1057, Par. 36) has emphasized that when a ship is stressed near to its point of failure, most of such wires, even if initially tensioned, must have become inactive, so that at such time the ship may have become in fact determinate. A little study will show that *one* such redundant wire causes indeterminancy only in its own facet, in a determinate ship, so that if half of the redundant wires in a ship, arranged in checkerboard fashion, become inactive, the ship has become determinate in general, though there will still be local indeterminancy in each still-redundant facet. Pippard has shown by experiment (R. & M. 977) that in a rough way the shear in a facet is divided equally between the two wires, if initially stressed, of course. Southwell suggested (R. & M. 1057) the "device of the separate panel" as a means of avoiding the difficulties caused by such quasi-determinancy. Pippard has also shown that as compared with the effect of bulkhead bracing the effect of these redundancies is small. In any event this paper will exclude all such wires for the present. The Airship Stressing Panel in R. & M. 800 consider that the shear in a facet is divided equally. This is common practice in bridge design.

Appendix No. 2

OVALING

When an externally loaded closed tube, a tubular column, or a tubular beam has an excessive length-diameter ratio, collapse may occur by "ovaling" of the section at some point in the length. In boiler tubes collapse rings, similar to transverse bracing, have been used to prevent collapse. This effect will not enter into consideration in this paper. It has been studied by Bryan, Southwell, and Frazier (R. & M. 1081). The theory has not been extended, however, to cover a frame structure to the author's knowledge. Reference is also made to Appendix XII of R. & M. 800.

Of collateral interest is an excellent study of pipe bends in "The Elastic Deformation of Pipe Bends," by Wm. Hovgaard, *Journal of Mathematics and Physics*, Vol. VI, No. 2, November, 1926. This paper stresses the fact that the assumption that the distance from the neutral axis to any fiber remains constant does not in fact always hold true, may lead to serious discrepancy. After allowance is made in the paper for variation of this distance, the theoretical results agree very closely with experiment, but differ from the "St. Venant stresses" by from 100 per cent to 400 per cent, and similarly for deflections.

Reference may be made also to R. & M. 800, Appendix I, p. 67, and R. &. M. 792, "The Reduction of the Effective Value of Young's Modulus in Flexible Compression Members," by A. J. S. Pippard.

Appendix No. 3

NOTES ON DESIGN MEMORANDUM NO. 57, BUREAU OF AERONAUTICS, U. S. NAVY, BY C. P. BURGESS

Photoelastic Experiments on Lengthened Model of U.S.S. "Shenandoah"

"Design Memorandum No. 28 was devoted to the analysis and interpretation of the photoelastic experiments upon a celluloid model representing seven main frame bays of the *Shenandoah.* It was shown that the distribution of stresses was much more in acccord with the shear theory than with the bending theory. On the other hand, strain-gage experiments on the actual ship in flight showed the stress to be in accordance with the bending theory rather than with the shear theory. It was thought that the great discrepancy between the model and the full-scale experiments was due to the difference in end conditions. These conditions were very different because both ends of the model were open, allowing considerable distortion of the end frames out of their planes; whereas the closed bow and stern structures of the airship hull very much restricted such distortions."

The model was therefore lengthened by five additional bays,

TABLE 2 COMPARISON OF OBSERVED AND CALCULATED STRESSES IN MODEL OF U.S.S. "SHENANDOAH"

	Frame space 70–80				Frame space 80–90			
		Stresses calculated by				Stresses calculated by		
Longitudinal	Observed stress	Theory of moments	Theory of shears	Theory of inverse ratio	Observed stress	Theory of moments	Theory of shears	Theory of inverse ratio
A	+67.5	+70.5	+78.9	+95.6	+62.8	+63.9	+71.4	+86.5
B	20.5	20.6	—	—	16.4	18.6	—	—
C	64.9	61.6	73.1	86.0	53.6	55.8	66.2	77.9
D	11.7	16.1	—	—	—	—	—	—
E	17.4	41.0	67.6	64.3	—	—	—	—
F	4.6[1]	8.3	—	—	—	—	—	—
G	—1.8[1]	+12.3	+7.4	+7.8	—	—	—	—
H	—6.1[1]	—1.6	—	—	—	—	—	—
I	—20.5	—17.9	—104.2	—119.0	—	—	—	—
J	—16.6	—15.8	—	—	—	—	—	—
K	—44.0	—41.8	—51.5	—48.6	—30.0	—37.9	—46.5	—44.0
L	—30.2	—25.5	—	—	—21.7	—23.1	—	—
M	—63.2	—53.0	—195.0	—47.0	—52.2	—48.0	—176.0	—42.5
N	+4.9	—26.0	+326.0	+17.4	—3.1	—23.6	+295.0	+1.6
O	—	—20.0	—	—	—	—	—	—
P	—2.7	—16.7	—	—	—4.3	—15.2	—	—
	Frame space 90–100				Frame space 100–110			
A	+45.1	+57.1	+63.9	+77.4	+41.5	+50.3	+56.3	+68.2
B	13.7	16.7	—	—	10.8	14.7	—	—
C	40.7	50.0	59.2	69.6	29.7	44.0	52.2	61.4
K	—23.1	—34.0	—41.6	—39.4	—19.9	—29.9	—36.7	—34.7
L	—16.5	—20.7	—	—	—15.7	—18.2	—	—
M	—42.0	—43.0	—158.0	—38.0	—30.2	—37.8	—139.0	—33.5
N	—4.9	—21.1	+265.0	+14.1	—12.4	—18.6	+232.5	—12.4
P	—4.3	—13.8	—	—	—4.5	—12.0	—	—
	Frame space 110–120				Frame space 120–130			
A	+28.6	+43.6	+48.8	+59.1	+18.6	+37.0	+41.2	+50.0
B	6.9	12.8	—	—	6.9	10.8	—	—
C	20.8	38.2	45.3	53.2	13.6	32.3	38.3	45.0
K	—18.8	—25.9	—31.9	—30.1	—15.2	—22.0	—27.0	—25.5
L	—11.1	—15.8	—	—	—9.4	—13.4	—	—
M	—23.4	—32.8	—120.5	—29.0	—19.0	—27.8	—102.0	—24.6
N	—9.9	—16.1	+202.0	+10.8	—7.4	—13.7	+170.8	+9.1
P	—5.4	—10.4	—	—	—4.5	—8.8	—	—
	Frame space 150–160							
A	+11.1	+19.8	+22.2	+24.9	—	—	—	—
B	2.0	5.8	—	—	—	—	—	—
C	6.4	17.3	20.6	23.2	—	—	—	—
K	—6.5	—11.8	—14.1	—14.9	—	—	—	—
L	—5.4	—7.2	—	—	—	—	—	—
M	—13.3	—14.9	—54.8	—13.4	—	—	—	—
N	—3.7	—7.3	+91.5	+5.9	—	—	—	—
P	—2.4	—4.7	—	—	—	—	—	—

[1] Stresses in *F*, *G*, and *H* are too small to be of value.

"the longitudinals of which were curved to a radius of about eleven feet and terminate in a celluloid cone or cap." "The lengthened model was secured to a wall as a cantilever, with the new closed end outward; and was tested in two loading conditions." First an upward load of 43 lb. was applied at the rear of the end cap, i.e., at Frame 185, and then this was replaced by a downward load of 26 lb., making a reversal of 69 lb. Stresses in the celluloid members were taken photoelastically and those in the wires by a small Larsen tensiometer. The actual stresses were of little use, "because of the rather erratic initial stresses which appear to be unavoidable in such a model. The important data are the changes of stress in passing from one load condition to the other."

"The secondary longitudinals had no shear wires secured to them, and hence should have no load according to the method of shears." In applying the inverse-ratio method, "it is assumed that the counter shear wires are slack. In panels having both main and secondary shear wires, the load is divided between the two systems of wiring according to the inverse ratio of the internal work in the two systems of wires."

Under the head of Conclusions: "The photoelastic investigation shows actual stresses much more closely in accord with the simple bending theory than with any of the other theories. The principal discrepancy is in the keel where the top (*N*) girder had usually the sign of stress called for by the theory, but of much less magnitude, and there was a corresponding increase in the load carried by the *M*-girders. It was remarkable that the secondary longitudinals which were entirely unconnected to the shear wires, and therefore should have had no load, according to the shear and inverse ratio theories, were actually stressed to almost the full intensity called for by the bending theory."

"The result indicates that for future design work the simple bending theory should give satisfactory results if the shear wires are reasonably proportioned to the loads to be transmitted. In the primary strength calculation, the internal members of the keel should not be counted as contributing to the resistance of the hull against bending; but in designing these internal keel members, allowance should be made for their possibly receiving end loads up to 25 per cent or even 50 per cent of the load calculated for them on the assumption that they are fully effective in the primary bending of the hull."

The numerous tables given have been collected into Table 2. In a few cases there were small discrepancies between corresponding longitudinals on opposite sides of the model, such as *B* and *B′*, etc., and the table eliminates these discrepancies by averaging all such members, from *B* to *M*. Longitudinal *A* is the top member, and *M*, *M′*, and *N* form the keel, with box girder *P* inserted in the keel to carry concentrated loads. Only in frame space 70–80 are all stresses given, hence this bay is the only one listed fully in Fig. 9.

It will first be noted that taking all results into consideration the observed results agree much more closely with the theory of moments than with the other theories; but even so there are considerable discrepancies. Of these discrepancies, the most notable characteristic is that whereas in frame space 70–80 their percentage value is small,[5] still as the successive frame spaces are studied their percentage value increases steadily as the load is approached. The accompanying diagram, Fig. 9, shows a plotting of a ratio expressing the discrepancy for each individual longitudinal. This ratio is the quotient obtained by dividing the stress obtained by use of the method of moments by the actual observed stress, using the values listed in Table 2. For instance, for longitudinal *A*, the top member, the theoretical stress is always greater than the actual, and from frame 120 to frame 160 it is nearly twice. If the member were designed for the theoretical stress in this region it would be nearly 100 per cent too heavy in cross-section. Other members show a similar trend, so that roughly speaking there is a discrepancy which increases in a linear fashion to 100 per cent at frame 120 and continues to increase to frame 160. The members that form the keel differ considerably from the others. The apex member, *N*, shows a stress of opposite sign at frame 70, so that between 70 and 80 the ratio is infinite, the box girder, *P*, has a ratio of 6 at frame 70; then both decline rapidly to about frames 110 and 120, after which they agree somewhat well with the top longitudinals. Members *L* and *M* agree with the top longitudinals at first, but then—probably because of the change in *N* and *P*—show a declining tendency that is contrary to the top longitudinals.

At frame 70 the assumption of a rigid transverse was clearly realized, and the actual stresses agree closely with the moment theory.[5] As the load is approached the error increases progressively. The order of variation of the discrepancy is somewhat remarkable, when consideration is given to the fact that every main transverse was rigidly wired in the style that was standard when the *Shenandoah* was built. All theoretical considerations that are quoted in this paper, and the experimental results obtained by Pippard, point to a rapid diminution of the discrepancy after the first main frame from the load has been passed.

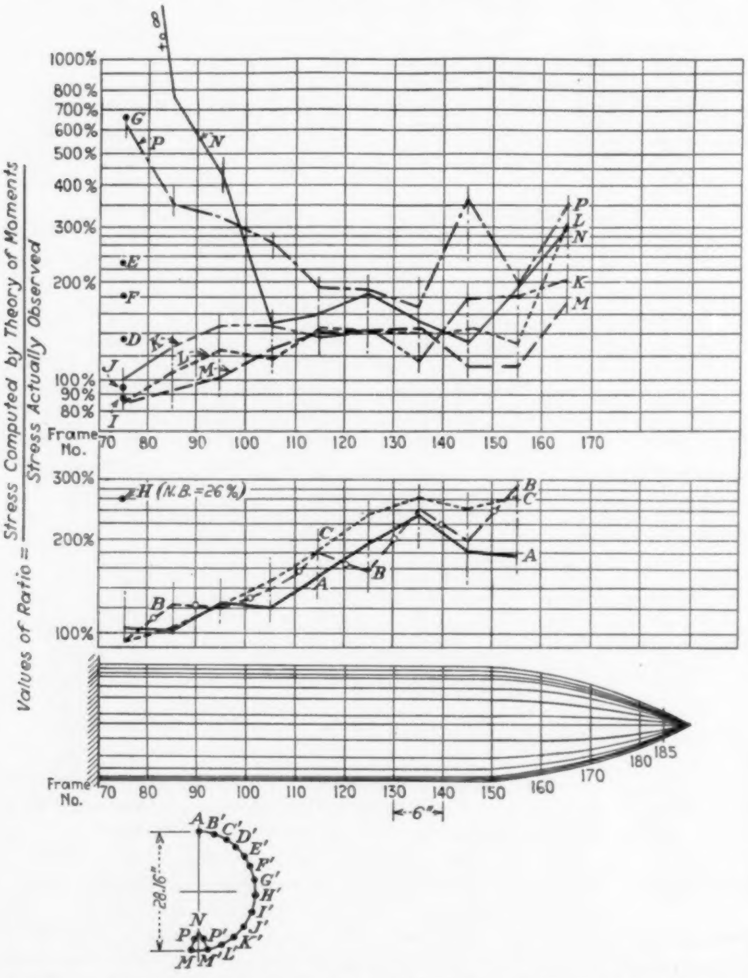

FIG. 9 RATIO OF STRESS COMPUTED BY THEORY OF MOMENTS TO STRESS ACTUALLY OBSERVED IN "SHENANDOAH" MODEL

The point at which the trend in the ratio alters is somewhat near the point at which the parallel body changes to curved body, and this point is frame 140, where the extension was joined to the original model.

That the discrepancy persists for so many bays, and does not become zero until the very rigid supporting wall was reached, is remarkable, when it is recalled that this model had in it all of the double diagonals of the primary shear wiring; the double diagonal system of secondary wiring; the rigidity of the joints that results from the very thick celluloid truss members that had of course, to be used; the highly redundant wiring of the main transverses, which occurred at every second frame; every one of which conditions is essential for the obtaining of correct results from the theory of moments. But in spite of all these favorable elements, the discrepancy is large, and it extends through 12 bays.

A point of interest is that the nose cap[6] produced marked stresses in the secondary longitudinals, which could otherwise get no stress because of the absence of any shear wire connections. This paper explains why stresses developed; it has not yet gone sufficiently far for the author to be able to compute, in so complex a structure, just what the stresses should be. Moreover, the stresses in the longitudinals of the model are a function of the reactions at each of the pin joints at the wall, because frame 70 was a wired frame and probably caused, therefore, shearing stresses on some of the joints. Unless these reactions were known it would not be possible to compute the stress.

It is evident that *N* and *P* have composite stresses; first, a stress of the same sign and proportionally about the same value as *L* and *M;* second, a stress of opposite sign resulting from cantilever action of the keel from its connections to the rigid wall support. Since the keel is much more flexible than the shell, this effect disappears in about four frame spaces, beyond which the members *N* and *P* act very much like the adjacent longitudinals.

Little can be said about members *D, E, F, G, H, I,* and *J* because they were recorded only for frame space 70–80. Their values are plotted in Fig. 9 in the same manner as the others, but of course only as single points. All that can be said of them is that passing around the perimeter of the section from *A* to *M* the ratio increases progressively (excepting that *E* and *F* are reversed) until *G*[7] is reached, i.e., to about mid height, and then there is an abrupt change to a very small value for *H*, after which the ratio again increases steadily to about unity at the bottom longitudinals. In a general way the ratio has a value that approximates to the cotangent of the central angle between the longitudinal in question and the horizontal, transverse axis of the cross-section.

Appendix No. 4

All longitudinals are of equal area; all diagonals, of equal

TABLE 3 (SEE FIG. 1) OCTAGONAL, PRISMATIC SPACE FRAME, WITH DETERMINATE END FRAMES, SUBJECTED TO A CENTRAL-LOAD OF 1 LB. AND REACTIONS OF ½ LB. EACH, ALL APPLIED AT "E" LONGITUDINAL. STRESSES IN SIXTH AND FIFTH BAYS FROM EITHER END, ALL COMPUTED

(Tension is +)

	Sixth Bay				Fifth Bay		
Member	Statical	Method of shear	Method of moments	Member	Statical	Method of shear	Method of moments
A_0A_1	0	—0.568	—0.568	A_1A_2	0	—0.443	—0.443
B_0B_1	0	—0.489	—0.489	B_1B_2	0	—0.400	—0.400
C_0C_1	0	0.	0.	C_1C_2	0	0	0
D_0D_1	—4.69	+0.489	+0.489	D_1D_2	—3.84	+0.400	+0.400
E_0E_1	+7.68	+0.568	+0.568	E_1E_2	+5.97	+0.443	+0.443
A_0B_1	0	—0.079	—0.079	A_1B_2	0	—0.079	—0.079
C_0B_1	0	+0.190	+0.190	C_1B_2	0	—0.190	—0.190
C_0D_1	0	—0.190	—0.190	C_1D_2	0	+0.190	+0.190
E_0D_1	+1.07	+0.079	+0.079	E_1D_2	+1.07	+0.079	+0.079

---

[5] For top and bottom longitudinals, but not for side longitudinals and keel members.

[6] There were also such stresses without the nose cap.

[7] The ratio is negative at *G*.

area. Length of bay equal to radius of circumscribing circle. The methods of moments and shear assume infinitely rigid bulkheads at the loaded frames. These methods give identical results. The stresses have been computed from the customary formulas,

$$S = \frac{F}{\sin\phi}\,\frac{\sin\theta}{\Sigma\sin^2\theta}$$

$$S = \frac{Fl\,\Sigma y}{2\Sigma y^2}$$

where ϕ = angle between diagonal and longitudinal
θ = angle of facet to horizontal
F = $^1/_2$ lb.
l = length of diagonal
y = distance from any longitudinal to neutral axis (which is the center line)

The summation of y applies to all longitudinals above (outside of) the diagonal in question.

Note that the stresses in the longitudinals do not prove to be planar, but that on a zig-zag section they are planar (i.e., on a section, for instance, through A_0A_1, B_1B_2, C_0C_1, D_1D_2, E_0E_1). In other words, the total horizontal stress at the joints of any frame is planar. This is true regardless of the direction of the diagonals.

Appendix No. 5

NOTES FROM R. & M. 948, 971, AND 977, BY PROF. A. J. SUTTON PIPPARD

In these papers Professor Pippard has given an excellent series of experiments on a hexagonal, parallel-body tube, with and without crossed diagonals, with and without transverse bracing, and with and without a conical nose piece. The study was made to determine the accuracy of the design methods advocated by the Airship Stressing Panel, R. & M. 800, of which he and R. V. Southwell were members.

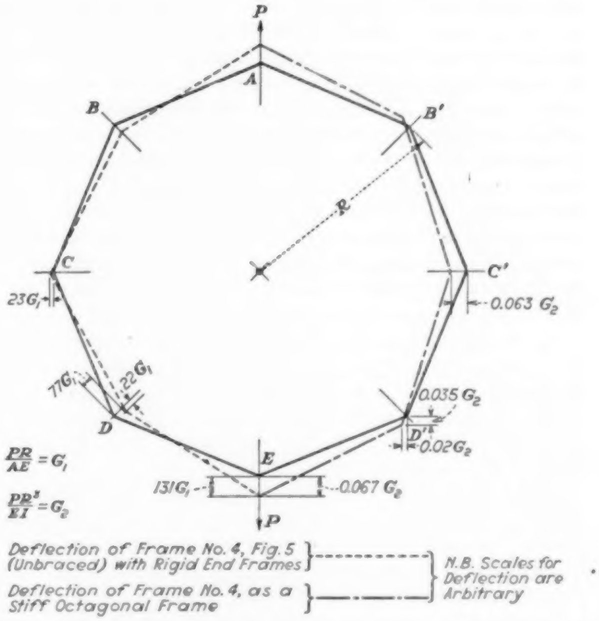
P
A
B
B'
R
C
C'
$23G_1$
$0.063\ G_2$
$77G_1$
$22G_1$
$0.035\ G_2$
D
D'
$0.02G_2$
E
$131G_1$
$0.067\ G_2$
P
$\frac{PR}{AE} = G_1$
$\frac{PR^3}{EI} = G_2$
Deflection of Frame No. 4, Fig. 5 (Unbraced) with Rigid End Frames
Deflection of Frame No. 4, as a Stiff Octagonal Frame
N.B. Scales for Deflection are Arbitrary

FIG. 10 COMPARATIVE DEFORMATIONS OF RIGID, UNWIRED RING FRAME, AND UNBRACED PRISMATIC SHELL

A = area of longitudinal.
E = modulus of longitudinal and strut.
I = moment of inertia of transverse strut.

TABLE 4

	Load System No. 6			Load System No. 7			
Member	Measured R. & M. 977	Computed by statics	Ratio, per cent	Measured R. & M. 977	Computed by statics	Ratio, per cent	
C_1D_1	Nil	0	0	—228.4	—240	105	
C_2D_2	—2.8	0	0	85.6	120	140	
C_3D_3	102.6	120	118	199.0	240	120	
C_4D_4	—211.6	—240	114	—424.2	—480	113	119 avg.
			avg. 116				
C_1D_2	3.6	0	0	—125.4	—156	124	
C_2D_1	Nil	0	0	148.8	156	105	
C_2D_3	10.6	0	0	27.6	0	0	
C_3D_2	Nil	0	0	14.0	0	0	
C_3D_4	137.6	156	113	276.0	312	113	
C_4D_3	—144.0	—156	108	—286.0	312	109	113 avg. 116 avg.
			avg. 111				
B_1C_1	5.6	0	0	—578.4	—720	125	
B_2C_2	15.0	0	0	300.0	360	120	
B_3C_3	263.4	360	137	530.0	720	136	
B_4C_4	—633.2	—720	114	—1268.8	—1440	123	126 avg.
			avg. 126				
B_1C_2	—5.2	0	0	—128.0	—156	122	
B_2C_1	Nil	0	0	102.4	156	152	
B_2C_3	32.6	0	0	64.0	0	0	
B_3C_2	—7.4	0	0	25.6	0	0	
B_3C_4	130.0	156	120	263.2	312	118	
B_4C_3	—104.6	—156	149	—204.4	—312	152	136 avg. 131 avg.
			avg. 136				
A_1B_1	—2.4	0	0	—918.8	—1200	131	
A_2B_2	41.4	0	0	481.2	600	125	
A_3B_3	378.8	600	158	812.0	1200	148	
A_4B_4	—1001.2	—1200	120	—1996.2	—2400	120	132 avg.
			avg. 136				
A_1B_2	12.6	0	0	—132.6	—156	118	
A_2B_1	Nil	0	0	93.6	156	167	
A_2B_3	62.4	0	0	104.0	0	0	
A_3B_2	11.0	0	0	63.2	0	0	
A_3B_4	134.6	156	116	269.6	312	116	
A_4B_3	—89.8	—156	174	—181.6	—312	172	143 avg. 137 avg.
			avg. 145				
		Final average	128		Final average	128	

These papers afford an excellent basis for studying the effect of variations in redundancy. Only in two respects are the experiments disappointing: that they deal only with a hexagon, and that they are confined entirely to parallel body, with a conical nose piece, from which must be drawn conclusions as to ships of 32, 36, or more sides, and of continuously curved profile. In spite of these shortcomings, they are a valuable contribution.

Let us study Table 4, which is based on his set No. 1, R. & M. 977, Load Systems Nos. 6 and 7. In the calculated stresses the crossed-wire redundancy has been met by *assuming equal division of shear between the wires*, although this is inconsistent with the actual observed distribution. See Fig. 10 for diagram of members.

In this table several interesting points appear. There are actually stresses in the side gores (2–3) where statically none can develop; clearly therefore there was considerable rigidity in the joints of the transverse frames, due to the use of screwed studs of small diameter (p. 4). As an average, for bay *CD*, the statical stresses are 116 per cent of the observed, indicating that approximately 14 per cent of the load has been lost from the upper and lower gores; for bay *BC*, 131 per cent and 23 per cent; for bay *AB*, 137 per cent and 27 per cent, respectively; and these figures hold very closely for both load systems. There is wide variation between the wires of each diagonal pair, and since in the quasi-statical computation the redundancy was met by assuming equal division of stress, the ratios show equally erratic values. It is interesting to note that the variation from equality increases progressively from bay to bay, and, perhaps strange to say, in the same manner for both load systems in almost every case. In R. & M. 800, Appendix V, it is stated that the fictitious bar effect must produce such a variation, which is not the same in successive bays; but the fictitious bars cannot act in a single parallel gore because they lie at the center line of the gore, and therefore can have an effect only on that part of the load that is carried by a secondary stress system.

The longitudinals do not all differ from the computed stresses by a constant percentage, but if the diagonals, instead of being assumed equal in stress, are altered to the same ratio as that of the experimental results, the longitudinals are found to be somewhat more consistent (i.e., about 5 per cent better in bay *BC* and 10 per cent in *AB*).

Comparison of the two load systems, Nos. 6 and 7, shows that doubling of the load at D_4 causes a doubling of the stresses in the two top gores within an error of 3 per cent (except A_3B_3 which is about 7 per cent). The secondary effect (to be discussed later) is evidently also doubled, as might be expected.

On the other hand, the lower gores for system No. 7 should agree closely with the upper gores of system No. 6, because they are caused by identical loads. But in fact, this agreement is not close, and moreover, a change takes place between the end bay and the other two. For instance a rearrangement of Table 4, as given in Table 5, brings out the condition more clearly.

TABLE 5

Member	System No. 6	Member	System No. 7	Ratio, per cent	Averages	
C_3D_3	102.6	C_2D_2	85.6	83.4		
C_4D_4	—211.6	C_1D_1	—228.4	108	95.7	
C_3D_4	137.6	C_2D_1	148.8	108		
C_4D_3	—144.0	C_1D_2	—125.4	87	97.5	96.6
B_3C_3	263.4	B_2C_2	300.0	114		
B_4C_4	—633.2	B_1C_1	—578.4	91.5	102.7	
B_3C_4	130.0	B_2C_1	102.4	78.8		
B_4C_3	—104.6	B_1C_2	—128.0	122.5	100.6	101.6
A_3B_3	378.8	A_2B_2	481.2	127		
A_4B_4	—1001.2	A_1B_1	—918.8	91.5	109.2	
A_3B_4	134.6	A_2B_1	93.6	69.7		
A_4B_3	—89.8	A_1B_2	—132.6	147	108.3	108.8

It is evident that the first and fourth lines of each group are fairly consistent in themselves, and the second and third lines, in themselves. These pairs consist of a longitudinal and the diagonal of its facet that causes its stress increment, showing that the source of variation lies in the division of stress between the crossed diagonals. The average of the four ratios of each group is approximately 100 per cent, but increases slightly toward the wall support, and the averages of the pairs of ratios in each group of four are almost equal.

For both load systems, Nos. 6 and 7, there is a resultant net shear on the whole cross-section (upward on the right) of 200 lb., hence that portion of the observed stresses that results from rigidity of the joints has a common element of the same sign. This seems to be a sufficient explanation of the lack of agreement noted in the second paragraph above.

It may be concluded from the experimental results that the load is carried by two means; first, a statical system that agrees well with the statical conditions, and, second, a secondary system that carries an increasing percentage of the load as the rigidity of the joints of each successive frame has opportunity to act. (But see later discussion of R. & M. 971, in which the shear carried by each gore remains almost constant.)

The method of support on the wall at the left end is undoubtedly influential in the inducing of secondary stresses (which can perhaps be said to have a trend toward St. Venant's principle), and if a symmetrical structure were tested, having a reaction at A_1 and another at A_4, similar to Fig. 1, the secondary effect might be greatly different.

Professor Pippard clearly sets forth the limitations of his apparatus, citing the "substitution of partially fixed joints for pin joints," the "elastic bulkhead in the plane of the transverse frame at which the load is applied, instead of applying the load in a special manner in order to prevent collapse." He found that "initial tensioning plays an important part in stress distribution." In his first experiment he found it necessary to apply corrections of from 2 per cent to 12 per cent in order to show equilibrium at the joints, and all of the correction was applied to the longitudinals because of their relatively large size.

Experiments Nos. 2 and 3 are similar to No. 1, except that "great care was exercised in getting the cross-bracing wires and the bulkhead bracing very tight," and that in No. 3 "the radial members in the end bulkhead, which had previously been 4 B. A. swaged rods, were replaced with $^5/_8$-in.-diameter steel rods." In Experiment No. 2 "there are appreciable differences between the measured and theoretical loads in the members, the characteristic being that the loads in the wires of the top panels, that is, C_4D_5, C_5D_4, etc., are larger than calculated, while the loads in the remaining wires are smaller." In Experiment No. 3 "it is clear that the elasticity of the end bulkhead is an appreciable factor in the distribution of loads in the structure, and care is needed to insure that such bulkhead bracing is effective if the theoretical results are to be applied to the calculation of strength." In this experiment the maximum discrepancy is about 3 per cent, as compared with 31 per cent in experiment No. 2.[8] Clearly the very rigid $^5/_8$-in. bars can be considered almost infinitely rigid,[9] so that the results should check the theory for such a condition; but the wires of Experiment No. 2 are such as to give a considerable degree of rigidity, and yet there are discrepancies of 12, 13, 14, 29, and 31 per cent in C_4D_5, B_6C_1, B_4C_5, C_5D_4, and A_5B_4. It is of interest to note that the last of this list is in the wall bay, furthest from the load.[8]

In Experiment No. 4 all bulkheads were braced with 4 B. A. swaged rods but there was no other change. "It will be seen that in the bay between bulkheads *C* and *D* there is a serious difference in all members between the actual and calculated loads, the panel $C_4D_4C_5D_5$ carrying much more than is indicated by theory and the other panels correspondingly less. Once the bulkhead *C* is passed, however, the measured and calculated loads agree well."

In Experiment No. 5 the end bulkhead was braced with $^5/_8$-in.-diameter rods and the others as in Experiment No. 4. The maximum discrepancy here, in member A_4B_5, is about 4 per cent.

In Experiments Nos. 7, 8, and 9 a conical nose piece was applied and the stresses measured. The top longitudinal was also a $^7/_8$-in. solid "keel" bar. In No. 7 there was bracing in the end frame only; in No. 8 all frames were braced with 4 B. A. rods; in No. 9 there was bracing in the end frame only, using $^5/_8$-in. rods. A comparison of No. 7 (with nose) with No. 6 (without nose), and with the calculated stresses for No. 6, shows that in 14 cases out of 18 the measured stresses of No. 6 agree more closely than those of No. 7. This indicates that the addition of the nose piece causes a wider discrepancy from the calculated stresses. A comparison of the ratios of calculated stress to measured stress in No. 8 and 7 shows that there is better agreement in some members in the latter case. Even in No. 9, with its heavy bracing in the end frame, combined with the nose piece, there are discrepancies of 23, 25, 27, and 32 per cent. It is clear that the nose structure serves only to disturb the approach toward agreement with the planar stresses. The addition of radial bracing in the intermediate frames of No. 8 only serves to magnify the error in some of the members. Professor Pippard himself states, referring to the experiments with the nose piece, "the redistribution of stresses requires a long tube in which to become operative" (p. 18); "when the transverse was approximately rigid in its own plane, the loads induced in the members of the nose piece were sufficient to cause some of them to bow" (p. 20); "for effective restraint a very stiff nose structure is necessary, as well as a very nearly rigid transverse frame" (p. 20); and, finally, under the head of Conclusions:

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[8] See point *B*, Fig. 8.

[9] See point *C*, Fig. 8.

"It appears essential, however, that both the transverse frames and the nose structure must be very stiff if anything approaching a close approximation is to be obtained.[9] Even under these conditions, the stresses in the members appear to require a considerable length of structure to adjust themselves to the theoretical values. This may be well appreciated when it is realized that the nose members introduced alternate tensile and compressive axial forces to the joints of the transverse frame to which they were attached, and although the system gives zero resultant axial force, this distribution of load is one which clearly requires some length of tube before the high local stresses imposed can be equalized."

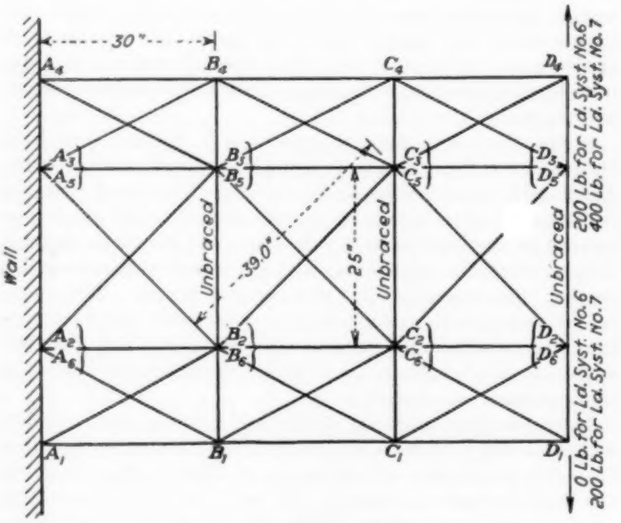
30"
A4
B4
C4
D4
A3
A5
B3
B5
C3
C5
D3
D5
A2
A6
B2
B6
C2
C6
D2
D6
A1
B1
C1
D1
Wall
Unbraced
39.0"
25"
Unbraced
Unbraced
200 Lb. for Ld. Syst. No. 6
400 Lb. for Ld. Syst. No. 7
0 Lb. for Ld. Syst. No. 6
200 Lb. for Ld. Syst. No. 7

FIG. 11 (SEE R. & M. 977)

In Experiments Nos. 10, 11, and 12 he shows that torsional stresses of a braced tube agree well with theory, but in Experiment No. 13 where all bracing was omitted, the gores adjacent to the two applied forces of the couple, i.e., 2–3 and 5–6, carry almost the entire shear, in accordance with statics.

In R. &. M. 977 he concludes (p. 12): "it is evident that redundant bracing when in the plane of the bulkhead is far more efficient than when placed in the panels, in conducing to an equalization of stresses;" "unless efficient bracing is provided in the plane of loading to act as an initial distributor of the external load, even if the tube exhibits a high degree of redundancy in other planes, the process of equalization is a very slow one and the stresses, even at a distance from the plane of loading, are dependent to a considerable extent upon the arrangement of the load system;" "it is important that effective bracing should be provided in the plane of the load system. Unless this is done there is a very serious liability of error."

In R. & M. 971, Table 2, are given the measured loads in the structure when the end frame is wired with 4 B. A. wires, but all wires acting downward to the right are omitted. It can be seen that the bracing system serves to distribute the load of 100 lb. at D_4 to the lower gores, which carry their shearing force almost constant to the wall, with little apparent change due to the secondary system. The general conclusion reached is that the "device of the separate panel" is a good approximation, i.e., that in effect, each redundancy of the crossed wires is largely confined to the members of the facet containing the redundant member.

* * * * * * * *

It may be concluded from Pippard's experimental results that the crossed-wire redundancies are of somewhat local nature; that very rigid bracing is required at all loads; that after a second braced frame is passed St. Venant's principle may be effective in *parallel body if the bracing is very rigid*, but that in curved body (i.e., when there is a conical nose) there is still a serious discrepancy even after 3 braced frames have been passed.

Appendix No. 6

RELATIVE RIGIDITY OF TIGHT-WIRED AND UNWIRED FRAMES

The modern type of ring frame is in general formed of three polygonal rings braced by a Warren system in three planes to form a complete ring of triangular section. For the purpose of this paper it is sufficient to reduce such a structure to a planar ring having an outer and an inner chord, braced together by a single system. The relative sections of the chords may differ, but it is assumed herein that they are of equal area.

It is simpler to treat them also as being circular instead of polygonal. Professor Pippard, in R. & M. 913, and previous papers, has derived formulas for their change of shape under load. The formulas are complex, but fortunately the author has found that they may be reduced to simple approximate terms. Applying a correction, then, to the formula for the deflection of a circular ring due to two axial loads of P,

$$\Delta_d = \frac{2PR^3}{EI}\left(\frac{\pi}{8} - \frac{1}{\pi}\right)\left(\frac{\sin\theta}{\theta}\right)^3 \cos^2 \tfrac{1}{2}\theta$$

where Δ_d = the axial elongation of the polygonal ring of $2N$ sides
N = one-half the number of sides
θ = one half the central angle subtended by one side
R = radius of center line of the ring
E,I = modulus and moment of inertia of the ring.
Let r = outside radius of the ring
qr = radial depth of the ring
A = sectional area of either chord
a = area of section of the bracing wire of previous studies

Then $\Delta_d = \dfrac{2Pr}{aE_s} = \dfrac{0.15P(r - \frac{1}{2}qr)^3}{\frac{1}{2}E_A q^2 r^2}\left(\dfrac{\sin\theta}{\theta}\right)^3 \cos^2 \tfrac{1}{2}\theta,$

and since $\dfrac{aE_s}{AE_A} = k,$

$$k = 6.65\,\frac{q^2}{(1 - \frac{1}{2}q)^3}\left(\frac{\theta}{\sin\theta}\right)^3 \sec.^2 \tfrac{1}{2}\theta$$

If $q = {}^1/_8$, in an 8-sided ship
then $k = 0.142$, which is the value marked A in Fig. 8.

This approximation is correct with 1 per cent, in so far as the formula is concerned. The assumption, however, that the deflection of an unwired ring is the same in its effect on the shell stresses as the stretch of a single vertical wire, cannot be correct, because in the former case the ring members in part belong to both systems, shell and ring, and do work in both. Or to look at the problem in a different light, the change of shape of the transverse frame (unbraced) of the shell will not be the same as the change of shape of this frame when acting as a ring, hence there will be reactions not only at the two joints where the loads P act, but reactions also at *every joint* of the frame.

To determine roughly whether the discrepancy of these two deflection shapes would be great, the shape of the two is shown in Fig. 10. They do not agree, showing that a more accurate result must still be obtained for the effect of the unbraced ring.

The self-determinate frame with vertical central wire if loaded alone would transmit all stress through the central wire, and would be similar to the single wire, hence this computation is a rough approximation to such a frame.

Discussion

Karl Arnstein.[10] The paper is a very interesting outline of strength calculation as applied to rigid-airship design. The data used in this analysis have been obtained from existing publications dealing with stress analysis and from some of the author's calculations on assumed structures.

Although the method of approach is of interest, the conclusions cannot be considered as sufficiently applicable to warrant the passing of judgment upon the airship art, particularly in view of the fact that the papers published summarize only a portion of the work which has been done in the study of stress analysis for airships.

It also is to be regretted that the assumed airship structures have been so "simplified" that they are considerably removed from the types of structure being employed and that the concentrated-load condition considered in this study has such a small bearing on the basic-load distributions experienced in airship design that the results of such an investigation should not be used for the comparison of airship types.

Even in the early Zeppelins attempts were made to avoid concentrated loadings. In those cases where concentrated loads were unavoidable, special provisions were made for getting these loads into the frames. In the case of the bottom keel, which may well serve as an example, the lower section of the frames was considerably stiffened by framing in order to distribute the load over a larger portion of the frame.

On these early ships it also was the practice to use auxiliary framing at those points on the hull where inward radial loads could be expected. In the U. S. Navy airship *Akron* the distribution of both types of loading to the frames is efficiently carried out by means of (*a*) the two lower longitudinal keels and (*b*) the use of inherently stiff main frames.

It should be remembered that with the increase in size of airships the resistance to static loads is becoming a smaller factor in the overall strength of the ship, while the aerodynamic loads are becoming more severe.

Fortunately these aerodynamic loads are far from concentrated, their effect being felt over large portions of the ship's surface. Most of these loads are experienced near the bow and the stern of the ship, where the structure is extremely rigid, due to the small diameter of the frames and the domelike character of the structure.

It also should be recalled that since the bending moments may be practically the same all around the cross-section, a linear stress distribution (St. Venant) is by no means ideal. In fact, a distribution giving a somewhat reduced maximum fiber stress is more economical, even though the stresses nearer the neutral axis may depart considerably from a straight-line distribution.

Comparative calculations, using the method of least work, have demonstrated that the character of the stress distribution cannot be wholly charged to the rigidity of the main frames. The character of the distribution is primarily dependent upon the relative rigidity of the main frames and shell.

Charles P. Burgess.[11] The author has presented an interesting paper on the interaction of longitudinal and transverse stresses in rigid airships. The bulk of the paper is presented as a progress report, and criticism is forestalled as it is not clear whither the progress is leading.

It has long been known that the shear in the neighborhood of large local keel loads is not distributed in accordance with the usual fairly simple theories used in the calculation of airship stresses. In modern rigid airships this departure of local stress distribution from theory is of small importance because the static loads are so distributed that they do not cause large shear forces or bending moments. The aerodynamic forces are responsible for the greater part of the shearing stress, and these forces are so distributed around the cross-section that they do not cause enough transverse distortion to have more than negligible influence upon the longitudinal stress distribution. Large local aerodynamic forces occur only upon the tail surfaces, and extremely strong transverse frames are used to transmit these loads into the hull of the ship.

Observations with strain gages on the U. S. Navy's airships *Shenandoah* and *Los Angeles* in flight have shown that the stress distribution approximates to the bending theory and that the departures from theory are on the safe side. In other words, the stresses in the longitudinals vary somewhat less than as their distances from the neutral axis, so that the maximum stresses are smaller than calculated by the bending theory. The longitudinals near the neutral axis take a considerably larger percentage of the total bending moment than calculated; but since the stresses in these members are small, the departure from theory is of no practical importance.

It is to be hoped that the author will continue his researches and give airship designers the benefit of his conclusions regarding the exact distribution of local stresses. Here, again, a reservation must be made in regard to the use of the word "exact" in airship stress calculations. Such calculations may be mathematically exact, but practically they never can be so, because the real distribution of stress from any given set of forces is locally very much influenced by unpredictable conditions involved in the construction of the ship and the initial tensioning of the vast maze of wires which take the shear and gas-pressure forces.

A. J. S. Pippard.[12] The problem of stress determination in such a complex structure as that of a rigid airship is so difficult that any work bearing upon its solution is to be welcomed, perhaps more especially when such work is of an arithmetical nature. It is comparatively easy to suggest possible methods of attack, but without the check of arithmetic these will not make a very forcible appeal to the engineer responsible for the design of such a structure.

The author indicates that he is endeavoring to find a solution by reducing his structure to a statically determinate frame, the stresses in which will then be corrected for the effects of redundancies.

The typical airship structure has such a large number of redundancies that treatment by exact methods is impossible on account of the labor involved, and while the author's proposal is, theoretically at least, one method of attack, one feels that treatments which in the first place assume the structure to be completely redundant and then apply corrections on the results obtained are much more hopeful.

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[10] Vice-President, in charge of engineering, Goodyear-Zeppelin Corporation, Akron, Ohio. Mem. A.S.M.E.

[11] Bureau of Aeronautics, Navy Department, Washington, D. C.

[12] Professor of Civil Engineering, University of Bristol, England.

Stress analysis of a statically determinate structure is a simple matter, since the elasticity of the members does not affect the result. The introduction of even a single redundancy, however, changes the character of the problem, since the elasticity of the members then enters into the calculations. As the number of redundancies increases, the stress distribution will usually depart further from that of the simple frame. The two problems are different, and on this account the writer is doubtful whether the author will obtain a general method of analysis of sufficient accuracy for airship work by the method which he is suggesting.

Coming to the tentative conclusions of the paper, the writer agrees with a number of the points in principle.

It is essential that the transverse bracing shall have a considerable degree of rigidity if the standard methods which the author quotes are to give a high degree of accuracy, but the writer thinks the error in the total stress at any point will not be considerable with the degree of rigidity obtainable in a modern airship. Possibly the author is overanxious as to the loss of rigidity of ring frames when these are loose-wired, since experience of the British airships showed that such frames can be made remarkably rigid.

It was pointed out in the report of the Airship Stressing Panel (R. & M. 800) that any local variation in the section of a ship such as that introduced by a keel or a corridor would cause departure from the stresses calculated by the methods set out in that report, and the writer agrees with conclusions No. 4 and 5. He also agrees with conclusion No. 7, that redundancy due to crossed diagonals in the facets has a rather localized effect, and the successful "device of the separate panel" suggested by Professor Southwell for dealing with such redundancies is based on this fact.

With regard to conclusion No. 8, it is clear that joint rigidity may in some cases be of importance; continuous or rigid joints do in effect introduce extra redundancies of a sort which apply secondary stresses to the longitudinal members. The magnitude of such secondary stresses is a problem which requires considerably more analysis, but it is a difficult one and peculiar to each design.

In conclusion the writer would like to thank the author for the kind references to his experimental work and to say that he regrets that circumstances did not allow him to continue the work upon structures more nearly approximating the shape of a modern airship.

Author's Closure

Since the paper was printed, an attempt has been made to evaluate the effect of providing bracing in *all* frames, instead of only those frames where loads were concentrated. This computation was carried out for the case of a ship six bays long and of square cross-section. The results are shown in Fig. 12 by the curved marked "Square—all frames braced." Comparison of this curve with the one marked "Square," which is for a ship similar in all respects, except that only those frames where loads occur are braced, shows that with all frames braced the stresses deviate even more markedly from the infinitely rigid case.

The very able discussions by Dr. Arnstein, Mr. Burgess, and Professor Pippard bring out clearly the complexity of the problem confronting an airship designer. It is so complex that, as Mr. Burgess points out, "exact solutions are practically unobtainable." After a large number of ships have been built, of different sizes, there will, it is hoped, be developed satisfactory empirical or semi-empirical rules for design. But until such rules are developed, and proved to be satisfactory in successful ships, the art must depend upon such analyses and tests which those interested can perform, even upon simplified structures. While such analyses and tests may not give a complete answer, they are nevertheless necessary to point the way and to set up warning signs at pitfalls that may beset it. It was from this point of view that the author started his investigations.

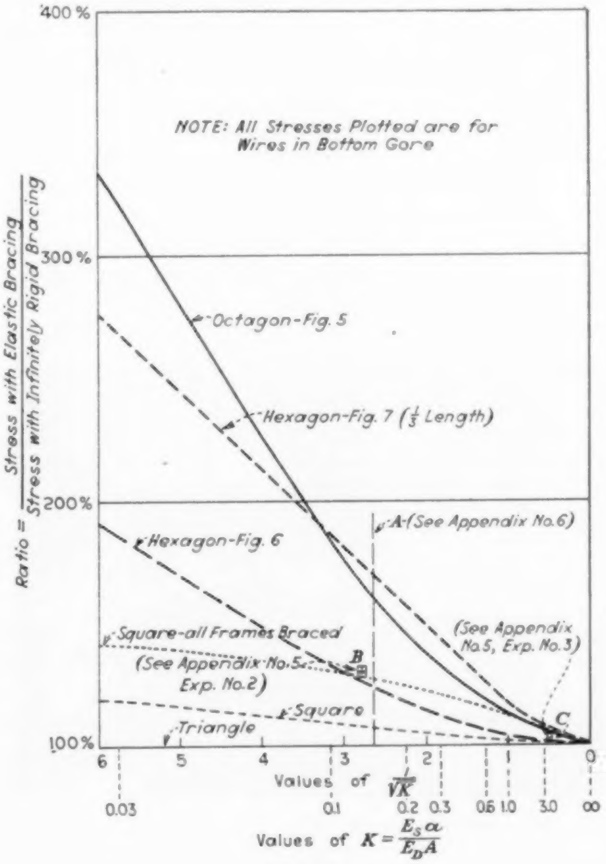

FIG. 12

(This is Fig. 8 of the paper, with a curve added for "Square—all frames braced.")

The author wishes to express his thanks to those who have so interestingly discussed the paper and also to acknowledge his indebtedness to the National Advisory Committee for Aeronautics for a large mass of data made available to him in this research.

AER-54-17

Special Methods of Testing Aircraft Materials

By DAVID M. WARNER,[1] DAYTON, OHIO

This paper presents a study of special methods employed at Wright Field in the mechanical testing of characteristic aircraft materials and construction, together with a practical exposition of the special equipment developed. The testing of welded joints, riveted joints and fittings, and of parts such as axles, wheels, and brakes has been discussed, together with the combined loading of struts, columns, and beams. The proportional loading of wing ribs, the elimination of low-strength steel tubes, the reverse bending of streamline wires, the superiority of preformed cable, the determination of an economic pulley-cable diameter ratio, and the fatigue testing of streamline wires and of extra-flexible control cable are taken up in detail. The advisability of testing aircraft-engine materials over the temperature ranges to which they are subject in service is emphasized by the tension and Brinell results obtained on typical light piston alloys. The importance of simulating service conditions, especially in all tests involving endurance in any form, is heavily stressed.

THE mechanical testing of aircraft structural materials for the purpose of determining their physical properties and characteristics is done in accordance with standard practice and methods of procedure as recognized by the American Society for Testing Materials and other engineering societies. However, a relatively high percentage of the testing done in the aircraft laboratory is on fabricated parts, accessories, and assemblies of great variety whose strength and other characteristics depend quite as much upon design and fabrication as upon the qualities of the materials of which they are constructed. The variety of shapes, forms, and combinations which find their way to the Air Corps laboratory constitute a representative cross-section of the whole field of aircraft design and construction, both commercial and military. The methods of procedure and the equipment required for doing the work are governed by the data to be derived. For making most of the simpler tests it is usually possible to get along with improvised set-ups in the universal standard testing machines, but for those involving special conditions, such as elevated temperatures, combined loading, friction, fatigue, or endurance, it is necessary to provide special jigs and set-ups, and for some tests special machines must be developed.

In these tests the specific conditions of service are simulated in such manner as to make it possible to derive empirical data in a relatively short time, which otherwise could be obtained only from actual service, with the attendant loss of time and with the added uncertainty of the influence of accidental factors.

After a brief review of some of the more characteristic tests, a few special methods and developments will be presented in detail.

Special Tests With Jigs

The general use of gas and electric welding as a method of joining steel tubes in the construction of fuselage, landing gears, and similar parts makes it necessary to test samples of the various types of joints from time to time in order to insure the maintenance of a high standard of workmanship. The cluster joint is cut out of the fuselage, with the various members left long enough for holding in the tension machine. The joint is then jigged up for support in such position as will insure a direct loading on the member under test. The efficiency of the joint in each case is reported as a percentage of the original full strength of the member. The values obtained in all tests requiring improvised jigs and set-ups are largely dependent upon the experience and judgment of the testing engineer.

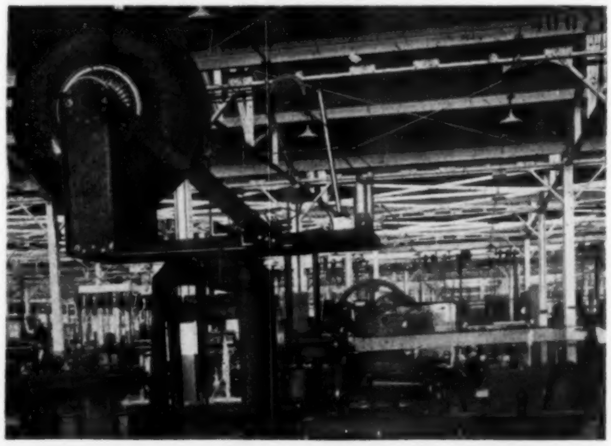

Fig. 1 Brake Test Set-Up on Universal Testing Machine for Friction and Proof Loading Tests

Riveted joints are tested in the same manner. The size, number, and locations of rivets will be reported with the materials data, and in the case of welded bases the quality and appearance of the welds also will be noted.

Fittings and parts are tested in accordance with their assembly requirements as to supports and direction of loading. Fittings are usually secured to an appropriate jig or mock-up to simulate service conditions, and mounted in the testing machine in such manner as will insure direct loading on the part under test, a new set-up being required for each lug or other member. Cantilever axles are mounted on a base beam, on which they are supported by means of their regular lugs, while the loads are

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Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

applied perpendicular to the wheel bearing at its middle point in a universal testing machine. Deflections for curves may be taken from a projecting arm attached to the spindle and extending parallel to its axis.

Struts and columns are usually given only axial loading, but occasionally combined column and bending data are desired, in which case the member is mounted in a suitable jig and tested in a standard compression machine. When tested for independent loading, the transverse load is usually applied by means of a dynamometer, but when proportional loading is desired, one end of the column is supported by a lever which converts a percentage of the column load into transverse load, which is applied to the side of the column by means of cable and pulley or other mechanical arrangement. In either case the column is given lateral support to prevent displacement by the transverse load. Deflection data are taken by means of either scales or dial indicators as desired.

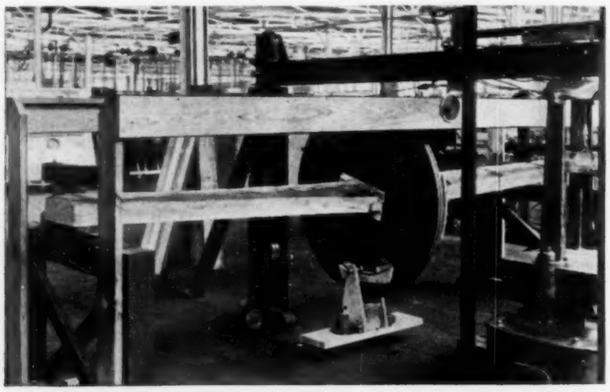

Fig. 2 Rig for Testing Beams in Combined Torsion and Bending Assembled With Universal Testing Machine (R. L. Hankinson)

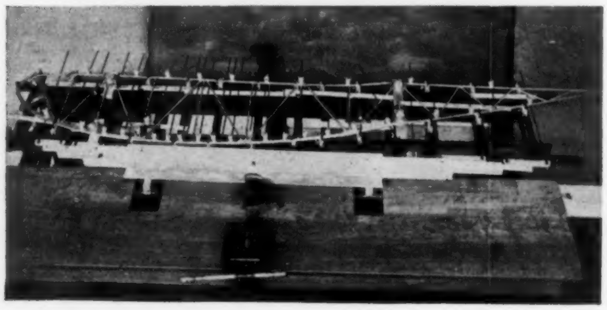

Fig. 3 Wing Rib Testing Machine for Deformation and Maximum Strength Tests (Sedam, Fuller, and Hankinson)

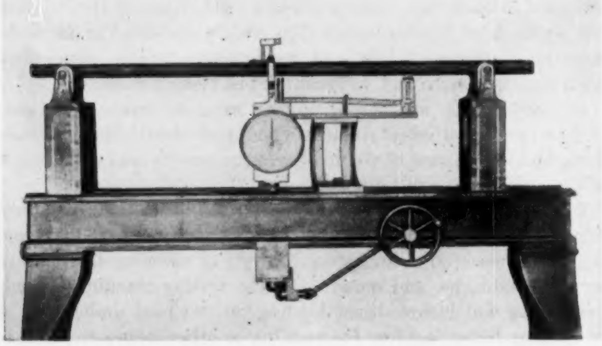

Fig. 4 Tube Bending Machine for Inspection Tests for Minimum Yield Point (R. R. Moore)

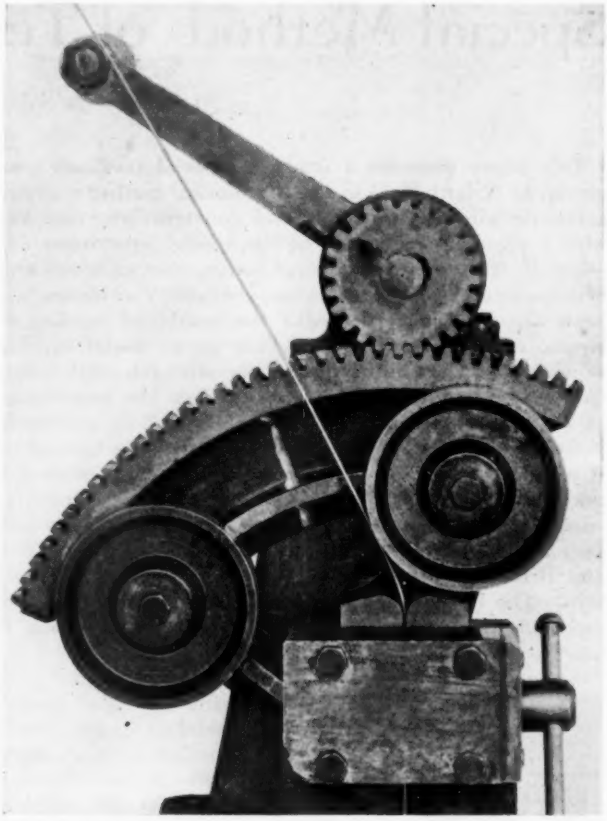

Fig. 5 Reverse Bending Machine for Testing Wire, Especially Streamline Wire (D. M. Warner)

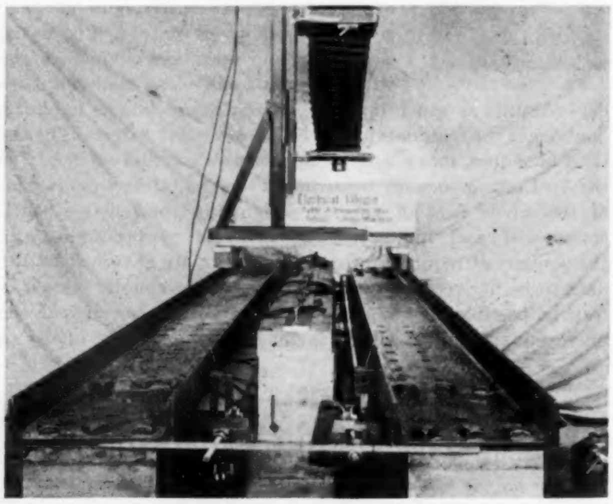

Fig. 6 Fatigue Machine for Vibrating Streamline Wire Under Tension, With Camera Set-Up for Measurements (D. M. Warner)

A series of experimental wing beams of different designs and constructions was tested in combined torsion and bending for the purpose of comparison. A special ring was built in connection with an Olsen 50,000-lb. testing machine for the work (Fig. 2).

It consists of two end-anchor or fulcrum units, a central torque unit, and a connection unit through which the torque moment is applied and weighed by the tension-testing machine. The ends of the beam were secured in the end units, which are fitted with roller-mounted knife edges to resist rotation and to prevent longitudinal binding. The torque moment was applied by means of a large disk at the center, through which the beam was passed and carefully centered. Two tension cables passing off in opposite directions from the top and bottom of the disk establish a couple tending to rotate the beam. The two cables were brought together through the connecting unit overhead and passed down through the weighing head of the machine to the power head. The transverse loads were suspended from a ball bearing which rides in a concentric slot in the lower quadrant of the disk. The deformation data were taken at regular intervals along the beam by means of Wisler dial indicators operated by fine wires from quadrant arms. The results obtained with this set-up were quite satisfactory.

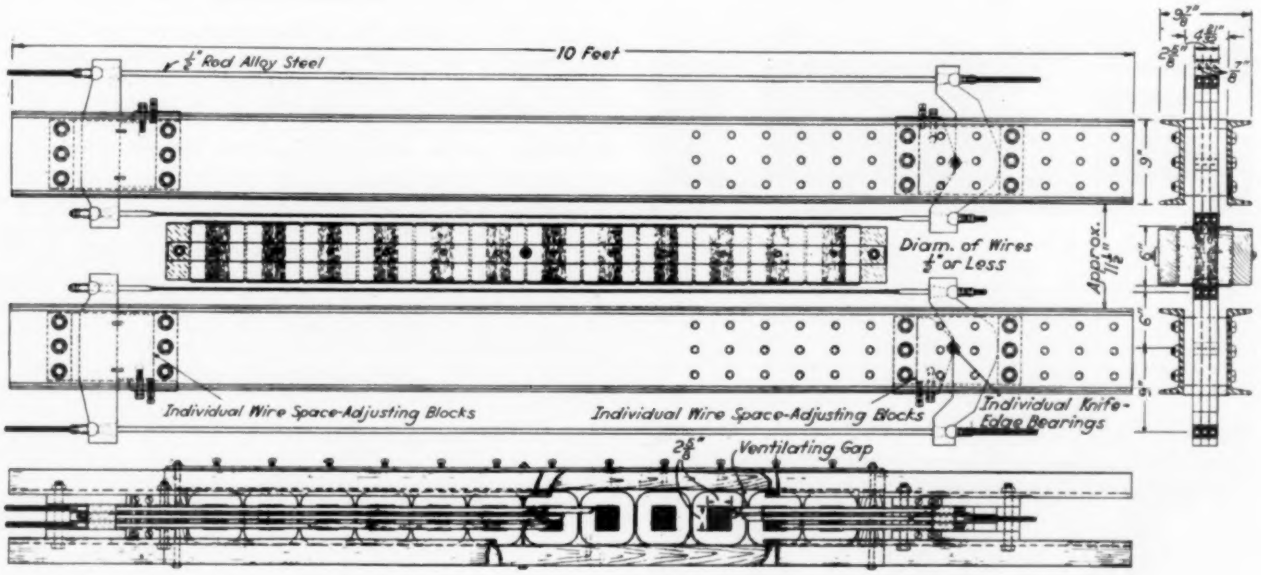
10 Feet
½" Rod Alloy Steel
Diam. of Wires ¼" or Less
Approx. 11½"
9"
6"
9"
Individual Wire Space-Adjusting Blocks
Individual Wire Space-Adjusting Blocks
Individual Knife-Edge Bearings
2⅝"
Ventilating Gap

FIG. 7 DIAGRAM OF FATIGUE TESTING MACHINE FOR VIBRATING STREAMLINE WIRE UNDER TENSION

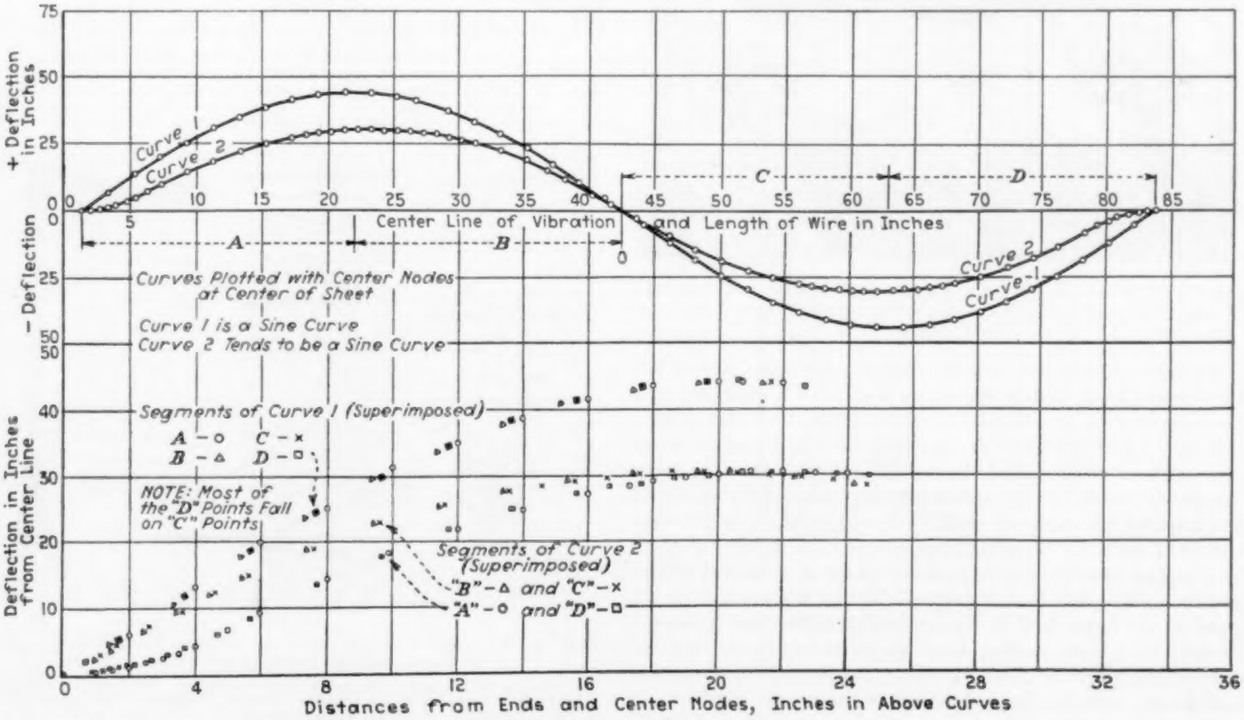

FIG. 8 CURVES DEVELOPED BY STREAMLINE WIRES VIBRATED UNDER TENSION

(Showing behavior with both plain and knife-edge terminals. Curve 1, streamline wire with knife-edge terminal fittings; curve 2, same wire with plain round trunnion strap-end fittings. Data from photographs of vibrating wire.)

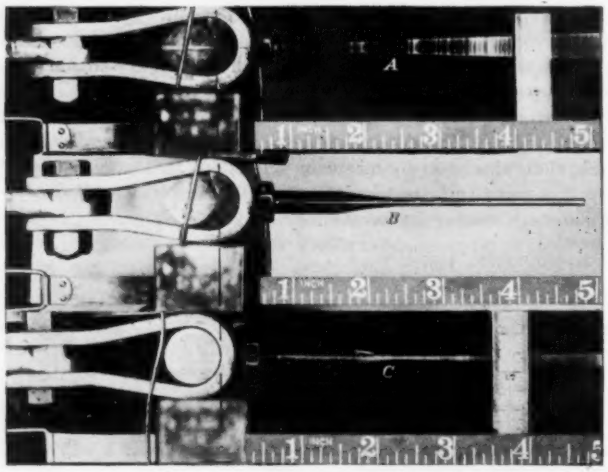

FIG. 9 BEHAVIOR OF STREAMLINE WIRES WITH PLAIN AND KNIFE-EDGE TERMINALS

(Showing both lateral and longitudinal vibrations under identical conditions of test. *A*, No. 5 streamline wire vibrating at 60 v.p.s. under tension of 500 to 600 lb. No failures after test run of two months. *B*, No. 5 streamline wire with knife-edge fitting. Mounted in fatigue machine for combined tension and vibration test. *C*, No. 5 streamline wire vibrating at 60 v.p.s. under tension of 500 to 600 lb. Failures in threaded section at trunnion after 3 to 6 hours' test run.)

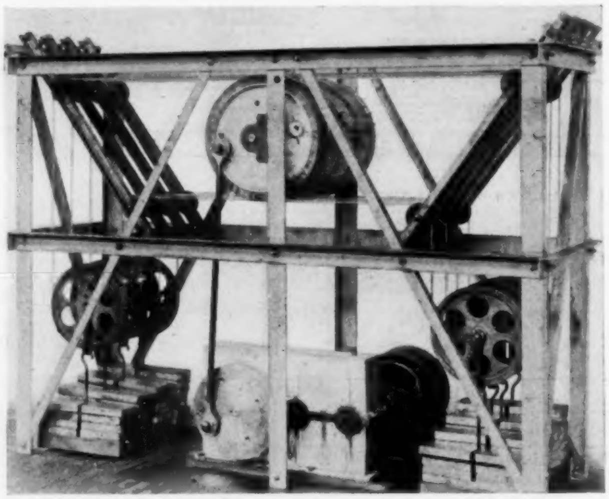

FIG. 10 CABLE AND PULLEY FATIGUE MACHINE FOR TESTING CONTROL CABLES, PULLEYS, AND ANTI-FRICTION PULLEY BEARINGS (D. W. WARNER)

Airplane wheels are usually tested with tires assembled. For direct radial loading the wheel stands on a sand bed in a box mounted on top of an Olsen 100,000-lb. universal testing machine, while the loads are applied through a pair of slings dropped from either end of the spindle shaft to a crossbar under the power head. Radial deflections are taken both vertically and laterally by means of special extensometers. For side-load testing a wheel is assembled horizontally on a vertical shaft in a strong base jig which is mounted on a universal testing machine. The side load is applied through a system of straps glued to the upper half of the tire, which collect and pass over its edge to a cable leading down to the power head. By this method the wheel is given cantilever loading at its assembled radius limit with the effect of the roll of the tire included.

Brakes are tested while the wheel is assembled in a vertical position free to rotate on a shaft in a cantilever jig, which is mounted on top of a large universal testing machine (Fig. 1). The brake is applied by means of a spring dynamometer attached to the hand lever, while torque is delivered to the wheel through a strap passing over the periphery of the inflated tire and down through to the power head. The brake friction moment is read from the machine, while the corresponding hand force for its application is indicated by the dynamometer. Although this method is not true to service conditions, the parts are tested for strength, and the friction data serve for comparison with other brakes.

SPECIAL METHODS AND EQUIPMENT

Wing ribs are tested as beams supported by spar sections and loaded proportionally over practically the full length of the chord. A special machine was built to accommodate the various sizes and shapes of ribs (Fig. 3). It consists of a table

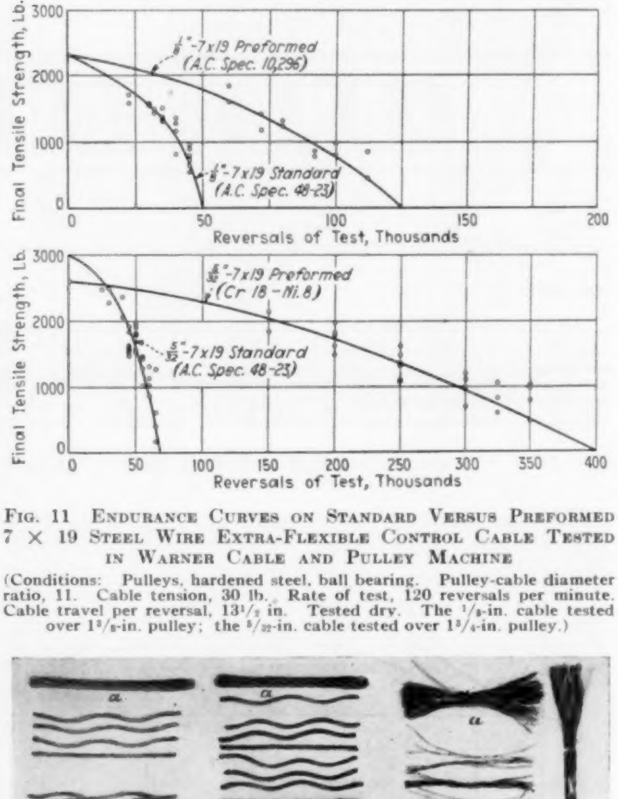

FIG. 11 ENDURANCE CURVES ON STANDARD VERSUS PREFORMED 7 × 19 STEEL WIRE EXTRA-FLEXIBLE CONTROL CABLE TESTED IN WARNER CABLE AND PULLEY MACHINE

(Conditions: Pulleys, hardened steel, ball bearing. Pulley-cable diameter ratio, 11. Cable tension, 30 lb. Rate of test, 120 reversals per minute. Cable travel per reversal, $13^1/_2$ in. Tested dry. The $^1/_8$-in. cable tested over $1^3/_8$-in. pulley; the $^5/_{32}$-in. cable tested over $1^3/_4$-in. pulley.)

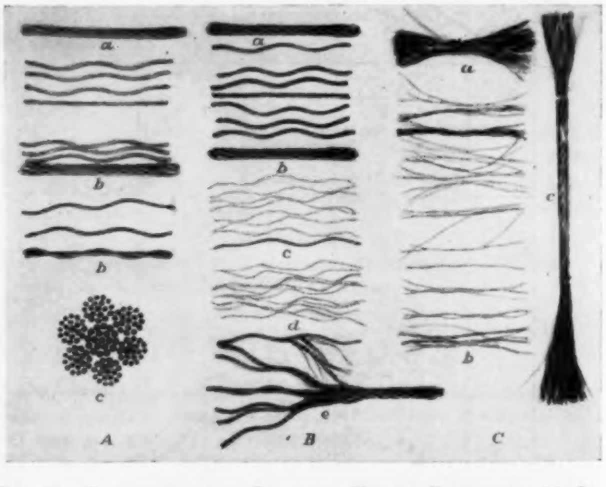

FIG. 12 CONSTRUCTION OF STANDARD VERSUS PREFORMED $^5/_{32}$-IN. 7 × 19 EXTRA-FLEXIBLE AIRCRAFT CONTROL CABLE

(*A*, behavior of Tru-Lay cable cut without binding or soldering; *a*, original, as cut; *b*, reassembled; *c*, Tru-Lay section, magnified. *B*, behavior of preformed Enduro cable cut without binding; *a*, original, as cut; *b*, reassembled; *c*, core, outer strand; *d*, outer wires, outer strand; *e*, reverse twist in center strand. *C*, standard cable cut without binding or soldering; *a*, original, as cut; *b*, cannot reassemble; *c*, standard cable sweat-soldered for test.)

bearing a frame for mounting the rib, an adjustable proportional loading-lever system, and a dynamometer and hand screw for applying the loads. The rib is held by means of cross-section blocks fitted into the spar ports, with center holes to fit the two anchor posts, which are adjustable for incidence. The loads are applied along the chord by means of a series of drawbars with adjustable lugs to engage the rib profile. These drawbars lead to the compound levers which distribute the load in accordance with the loading schedule. Wisler dials are mounted at convenient points for taking deflections.

Since tubes are important structural elements in an airplane and the presence of a single low-strength tube might cause disaster, a non-destructive bend test has been devised for the routine inspection of each individual tube. The testing machine consists of a stand bearing a long base on which is mounted a support at either end equipped with balanced knife-edge V-bearings to receive the tube as a beam, and a loading unit at the center with a dynamometer attached (Fig. 4). The tube is mounted as a beam over a constant span and loaded at the middle to produce a stress and deflection representative of the specified minimum yield limit for the grade of material in question. The required loads for the different cross-sectional dimensions are obtained from tables previously calculated for

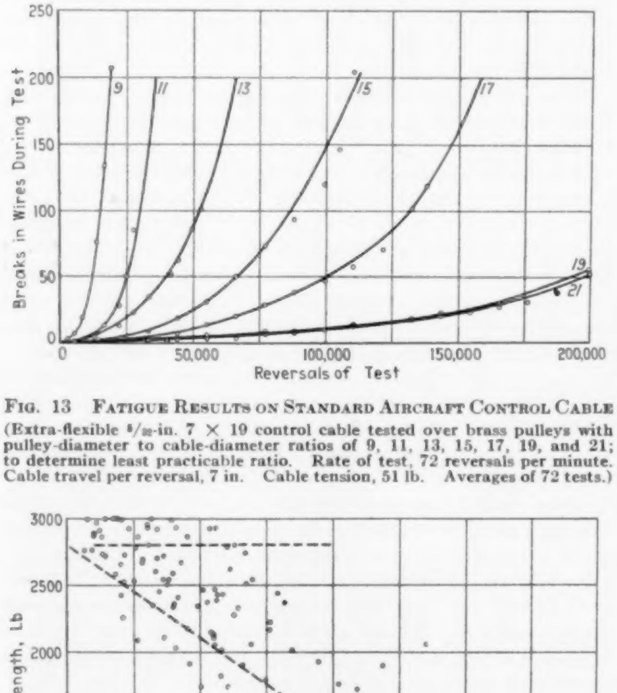

FIG. 13 FATIGUE RESULTS ON STANDARD AIRCRAFT CONTROL CABLE

(Extra-flexible $^5/_{32}$-in. 7 × 19 control cable tested over brass pulleys with pulley-diameter to cable-diameter ratios of 9, 11, 13, 15, 17, 19, and 21; to determine least practicable ratio. Rate of test, 72 reversals per minute. Cable travel per reversal, 7 in. Cable tension, 51 lb. Averages of 72 tests.)

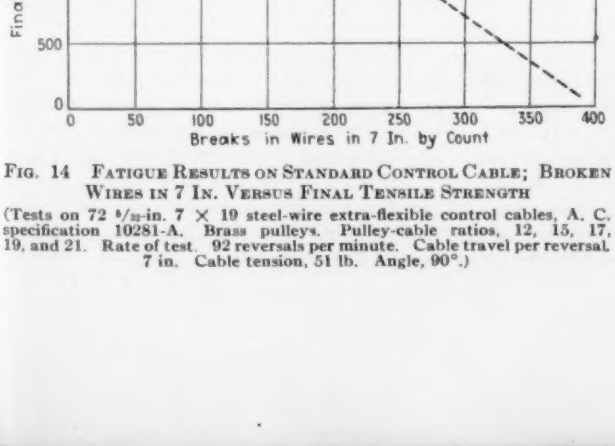

FIG. 14 FATIGUE RESULTS ON STANDARD CONTROL CABLE; BROKEN WIRES IN 7 IN. VERSUS FINAL TENSILE STRENGTH

(Tests on 72 $^5/_{32}$-in. 7 × 19 steel-wire extra-flexible control cables, A. C. specification 10281-A. Brass pulleys. Pulley-cable ratios, 12, 15, 17, 19, and 21. Rate of test. 92 reversals per minute. Cable travel per reversal, 7 in. Cable tension, 51 lb. Angle, 90°.)

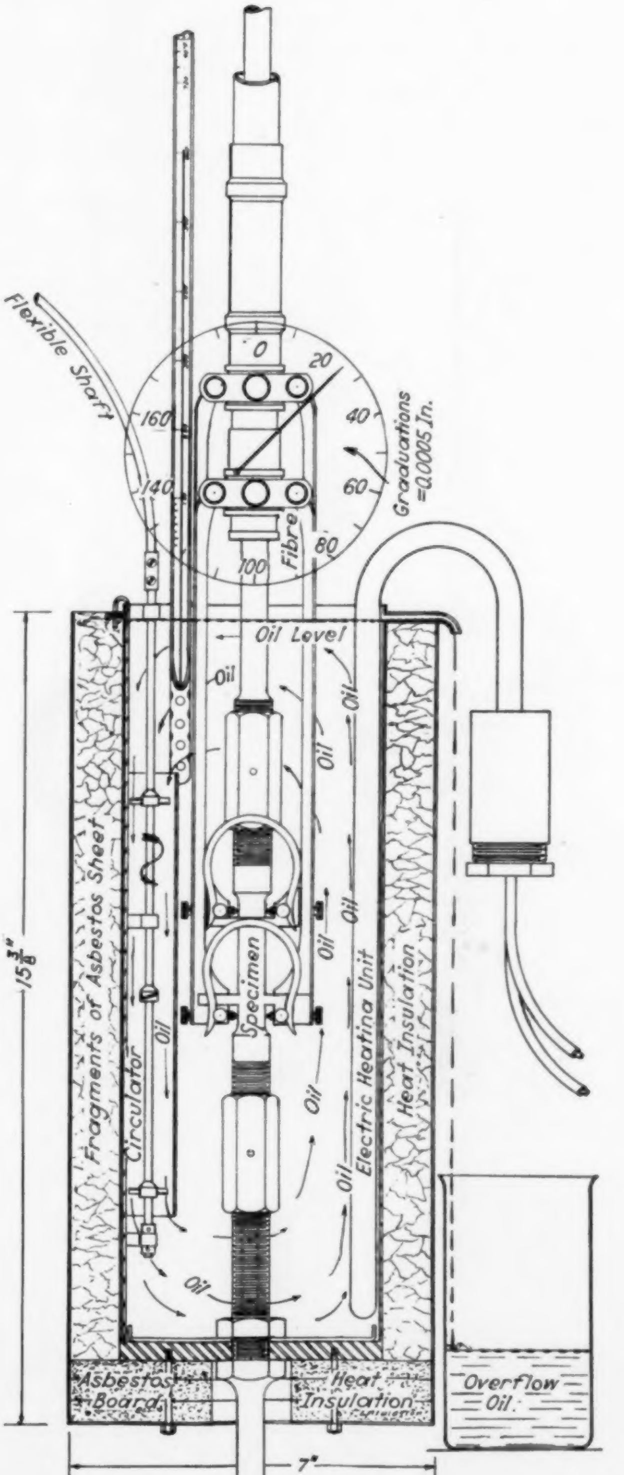

FIG. 15 DIAGRAM OF FURNACE AND EXTENSOMETER SET-UP FOR TENSION TESTS AT ELEVATED TEMPERATURES (D. M. WARNER)

(Showing positions of heating unit, oil circulator, thermometer, and specimen.)

the various grades of tubing. While this test does not determine the actual strength of the material, it serves to eliminate the tube with low yield point or low modulus of elasticity.

Ductility is an essential requirement of tension members manufactured from wire, since most of them are subjected to vibration. The measurement of percentage elongation in the tension test has not proved to be satisfactory; hence, the reverse bend test has been substituted. One end of the wire is gripped in a vise with jaws rounded to specified radii, while the other is bent back and forth through an angle of 180 deg. to failure. Under these conditions the wire is required to withstand a specified number of 90-deg. bends. When made by hand in a bench vise with rounded jaws, the number of bends not only increase with wire diameter, but the results obtained by different operators often vary by as much as 50 per cent, especially on the larger wires. To eliminate these variables, a reverse bending machine was developed (Fig. 5). It consists of a vise with a pair of radius adapters to accommodate each standard size of wire, a quadrant lever unit which bends the wire back and forth over the adapters through 180 deg., and a crank and pinion for driving the quadrant.

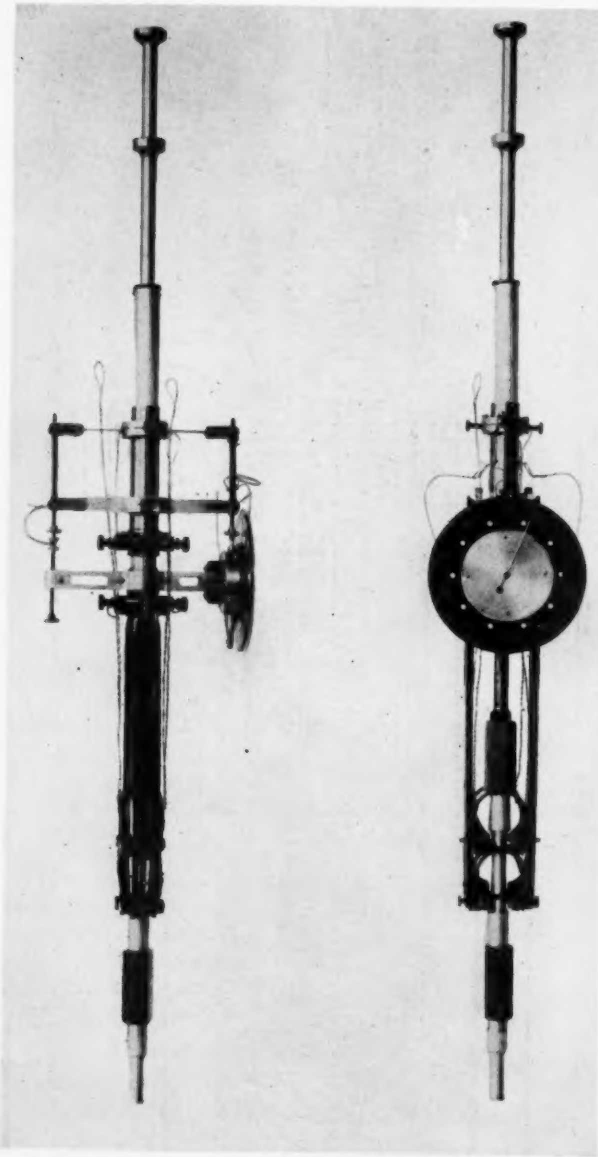

FIG. 16 EXTENSOMETER AND SPECIMEN ASSEMBLY FOR TENSION TESTS AT ELEVATED TEMPERATURES (D. M. WARNER)

While the wire is gripped in the vise one of the two 4-in. rollers on the lever unit bends it across and forces it down over the one adapter, then the other brings it back in the same manner to conform to the other side, and so on to failure. The jaw adapters automatically center the different sizes of wires and regulate the clearance under the rollers, thus making conditions constant for all operators. When the tests were made by hand, the Army requirements on streamline wire ranged from 7 bends on the No. 1 size to 13 on No. 8, but since the machine gives the same number on all wires of like quality, regardless of size, Army specifications now call for 9 bends for all sizes. Streamline wires are bent over radii of three times the thickness, while round wires are usually bent over a radius of $^{3}/_{16}$ in.

For the purpose of determining the cause of failures of streamline wires in service, a series of fatigue tests were made in which the wires were vibrated under tension, until failures occurred, in a special machine (Fig. 6). It consisted essentially of two heavy tension units, each holding three wires under the desired tensions, and a central power unit consisting of a battery of twelve electromagnets to vibrate them. Provision was made for the independent adjustment of each wire for tension and for spacing to or from the magnets at either end, as well as for individual test (Fig. 7).

It was found that failures were due to fatigue from reverse bending induced by the refusal of the terminal fitting to respond to the vibration of the wire. Accordingly, a special fitting with a knife-edge bearing was designed which permitted free vibration and stopped the failures. Check tests were then run on four new No. 5 wires, three assembled with service terminals at one end and the new fittings at the other, while the fourth had knife-edge fittings at both ends. When tested under identical conditions and vibrated to the greatest amplitude that could be maintained, which produced angles of from 4 to $4^{1}/_{2}$ deg. at the knife-edge terminals, the first three wires failed after $5^{1}/_{2}$, $2^{1}/_{2}$, and $6^{1}/_{2}$ hours, respectively, while the fourth, with knife-edge fittings at both ends, continued for over two months without failure. The tests were run under loads of between 500 and 600 lb., representing only about 10 per cent of their rated strength.

A panoramic photograph was taken over the full length of the vibrating wires by means of a camera mounted on a sliding frame, as shown in Fig. 6. From the resulting mosaic, data were obtained for the curves plotted in Fig. 8, which show the behavior of the whole wire, including the damping at the service terminal which caused the failures. The behavior of the two types of terminals is contrasted in Fig. 9. Considerable longitudinal vibration is shown by the concave crosslines.

Steel-wire control cables which operate over pulleys through angles of 90 deg. or less under loads probably rarely exceeding 10 per cent of their tensile strength, fail by fraying in the short sections that pass back and forth over pulleys or guides, with the result that they are replaced when a few broken wires are noticed.

In order to determine the cause and importance of this fraying in service and to determine the relative merits of cables of different materials, construction, and manufacture, it was necessary to make endurance tests on specimens under tension over the standard pulleys and to make a study of the cable and pulley relationship.

Accordingly, a fatigue machine was developed for testing both cables and pulleys (Fig. 10). This machine consists of a rigid steel frame having these features:

(a) A drum located in mid-section for driving the cables
(b) A set of standard control pulleys for the cable to pass over
(c) A set of suspension loading pulleys
(d) A power unit with crank and connecting rod to oscillate the drum through the desired angle.

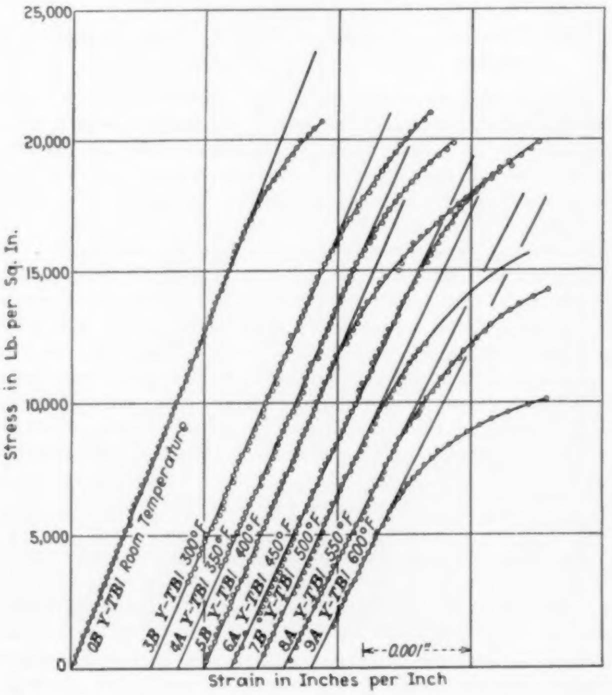

FIG. 17 STRESS-STRAIN CURVES FROM TENSILE TESTS ON A PISTON ALLOY AT ELEVATED TEMPERATURES

(Y-alloy; composition: 4 Cu, 2 Ni, 1.5 Mg, remainder, Al. Heat treatment, none. Specimens, A. S. TB1. Skin on as cast. Threaded. Size, 1/2 in. diameter by 2 in. gage length.)

Average Test Results

Temperature, deg. fahr.	Proportional limit	Ultimate strength	Modulus of elasticity
Room	14,460	26,620	10,554,000
300	15,700	26,940	9,332,000
350	13,140	27,570	9,342,000
400	9,310	25,200	9,485,000
450	12,040	26,680	8,018,000
500	12,250	22,090	7,819,000
550	7,490	19,940	8,084,000
600	5,300	15,080	8,011,000

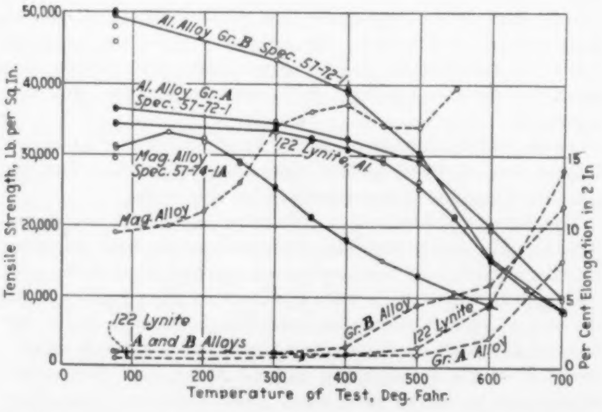

FIG. 18 TENSION AND ELONGATION RESULTS ON FOUR PISTON ALLOYS AT ELEVATED TEMPERATURES

(Tensile results on cast alloys. Specimens, A. C. TB1; gage, 1/2 in. by 2 in.; cast in size. Tested in oil bath after 30 min. at temperature. Olsen 20,000-lb. machine used.)

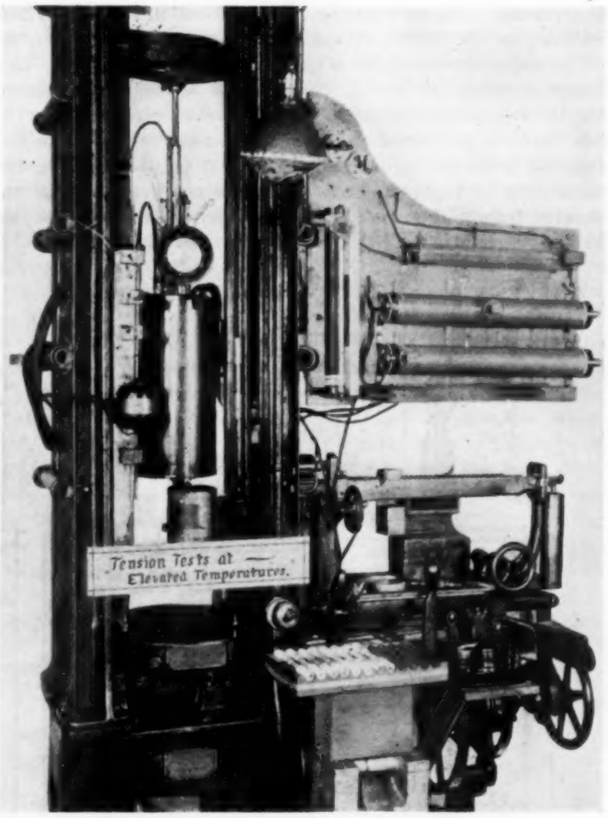

FIG. 19 FURNACE AND EXTENSOMETER SET-UP IN UNIVERSAL TESTING MACHINE FOR TENSION TESTS AT ELEVATED TEMPERATURES (D. M. WARNER)

It has a capacity of eight cable specimens, four at one end and four at the other, and sixteen control pulleys. The pulleys are mounted in pairs, one up and one down, in members set at an angle, to adjust for different diameters.

The cable specimens, made up to standard lengths with eyes at either end, hook on the back of the drum and pass over the top horizontally to the first test pulley, then over through an angle of 90 deg. and down under the loading pulley and up again through 90 deg. over the second test pulley, to return under the drum and up to the hook on the back. The drum eliminates the constant jerking of the load which is inevitable when cables are driven direct from cranks. It drives the cable back and forth over the two test pulleys under a tension of half the suspended load, including the weight of pulley and pan. Any desired cable tension may be applied. A counter on the crankshaft records the revolutions of the test run, or half the reversals. Two independent endurance tests are made on each cable at points several feet apart where it operates over the pulleys. At the conclusion of the endurance run the cable is tested for final strength at both worn sections. These results are recorded, together with the conditions and the number of reversals of test. The original tensile strength is obtained on an undamaged section of the cable. When removed from the machine, the cable is dipped into a solder bath at points several inches back on either side of each worn section to make tension-test lengths of approximately 30 in. The solder prevents the worn strands from shifting and makes it possible to hold the cable in V-grips to make the tension tests.

When a curve is desired, it is made from the endurance results obtained on a series of cable specimens run to regular intervals of reversals. The results are plotted as shown in Fig. 11. These curves show the preformed cable, Air Corps specification 10,296, to be quite superior to the standard cable conforming to Air Corps specification 48-23. Still greater superiority is shown by the preformed cable made of stainless-steel wire. Due to the fact that the preformed and the standard cables look alike and that the latter shows the higher original tensile strength, the superiority of the preformed cable was not evident until endurance tests were made. The chief difference between the two kinds of cable is in their construction, as shown in Fig. 12.

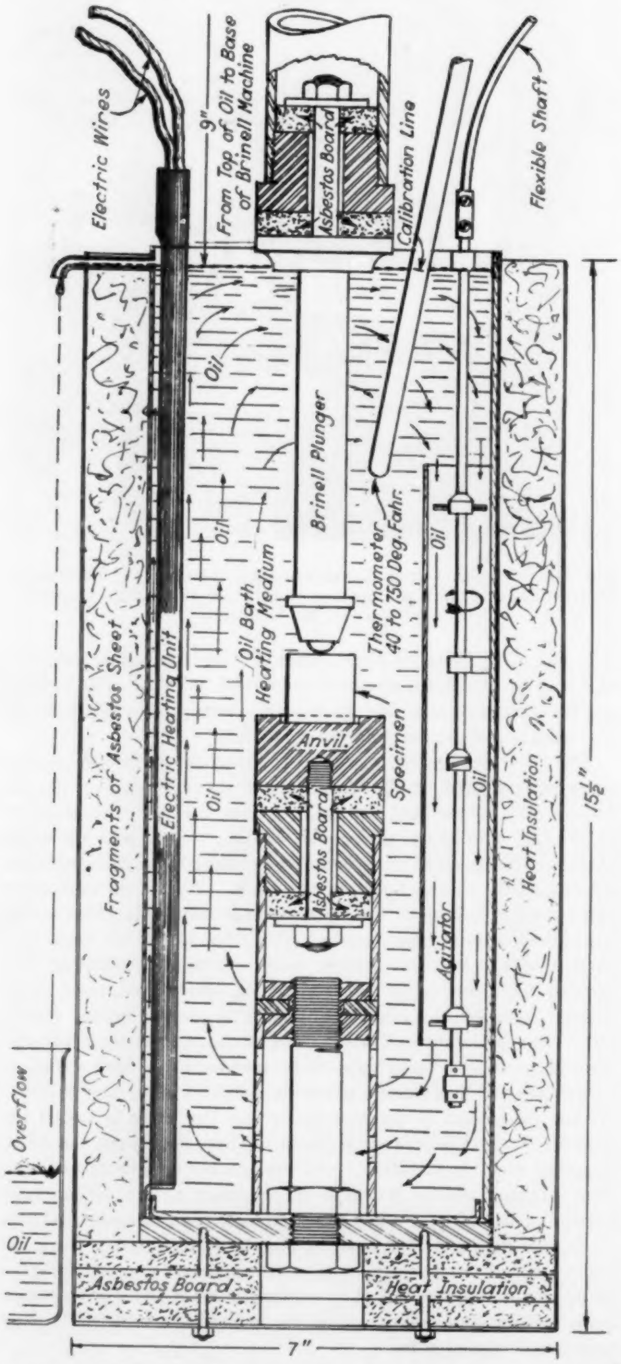

FIG. 20 DIAGRAM FOR FURNACE AND BRINELL SET-UP FOR HARDNESS TEST AT ELEVATED TEMPERATURES (D. M. WARNER)

In addition to the usual tests for materials and assembly strength, it is necessary to resort to the endurance test on control pulleys to determine these factors:

(a) The least practical diameter ratio of pulley to cable
(b) The most desirable type of sheave
(c) The ability of the sheave to resist cable wear
(d) The effect of the sheave on the cable
(e) The relative merits of different types and makes of bearings.

Since control cables and pulleys are always tested together, the same fatigue machine is used for making endurance tests on both, the data being taken in accordance with the purpose of the test. A series of tests was made, on an earlier machine, to determine the least practical ratio of pulley diameter to cable.

Standard $^5/_{32}$-in. 7 × 19 steel-wire cable (Air Corps specification 10 281-A) was run over a series of pulleys machined from cast brass with diameters ranging from 21 down to 9 times that of the cable. The specimens were run over the pulleys through an angle of 90 deg. under a tension of 51 lb. at the rate of 92 reversals per minute. At fairly regular intervals of run the machine was stopped while the broken wires in each cable were counted as closely as possible without removing the specimen, as a measure of the rate of breakdown. The averages of the progressive counts on several cables over each diameter of pulley were plotted against the corresponding average of reversals of test in Fig. 13, to show the rate of cable failure over the different diameters of pulleys.

The final tensile results were plotted against the total count of broken wires in each in Fig. 14. Although the curves would be different under different conditions of test, and especially for the better preformed cable, it is probable that the general relationship between the several diameter ratios would be practically the same.

Endurance tests under constant conditions are run on control pulleys fitted with anti-friction bearings of different types and manufacture to determine their relative merits. At present the size AN-210-4 pulleys so equipped are required to satisfy specified endurance tests under a cable tension of 500 lb.

TESTS AT ELEVATED TEMPERATURE

Because of the high temperatures at which aircraft engines operate it is necessary to test materials to be used in pistons and similar parts for hardness and tensile properties over the entire range of elevated temperatures to which the parts are subject in service, in order to determine their suitability for this purpose and also to determine the relative merits of different materials and treatments. To make such tests the specimens had to be heated to the desired temperature, held constant long enough to reach equilibrium, and then tested at the same temperature.

An electrically heated oil-bath furnace, with mechanical circulator, and electrical heating unit, a special extensometer, and a Brinell adaptation were developed for this work.

The furnace consists of a cylindrical steel tank for the oil, with a light outer jacket filled with asbestos for heat insulation. For the tension tests the furnace is assembled around the lower pulling bolt where, as a unit, it is free to align itself with the upper pulling bolt when the specimen is coupled in for test. The oil is heated by the electrical unit which extends down in one side of the furnace and is kept at an even temperature throughout by forced circulation. The heated oil rises around the heating unit and is driven back down on the opposite side of the tank by the mechanical circulator through a well extending to within 3 in. of the bottom, while the specimen is located midway where the temperature averages practically constant

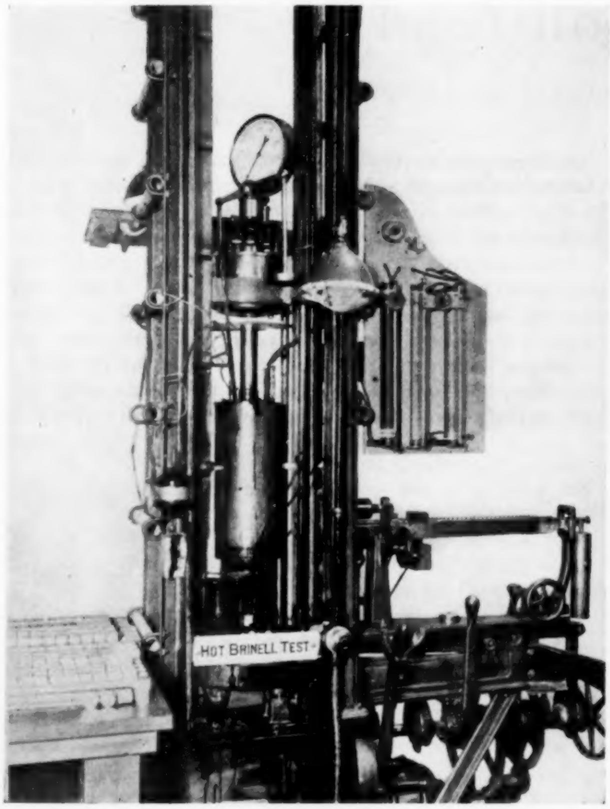

Fig. 21 Furnace and Brinell Set-Up in Universal Testing Machine for Hardness Tests at Elevated Temperatures
(D. M. Warner)

(Fig. 15). The tank is kept full to the overflow pipe by adding a little oil from time to time. The temperature of the bath is indicated by a thermometer (2-deg. graduations) which hangs suspended to an even depth throughout the test.

By preliminary experimentation with thermocouples placed on the outside of the specimen at the end and middle point of the half-inch gage section and in the center of the shoulder a half-inch from the gage section, it was found that the temperature of the specimen came to within 3 deg. fahr. of that of the bath, as indicated by the thermometer, after a maximum of from 15 minutes at 300 deg. fahr. to 8 minutes at 500 deg. fahr. for aluminum alloy. In the tests reported all specimens were held in the furnace for a period of 30 minutes.

By adjusting the thermostats the temperature was held constant at any desired point within the limits of the oil used. Standard threaded specimens with 2-in. gage length were used in these tests. The strain data were taken over the 2-in. gage length by means of the specially developed extensometer consisting of a telescoping unit on the upper pulling bolt with extension bars running down into the bath, where they were secured to the specimen by means of C-springs which were removed before breaking the specimen. The indicator reads direct to 0.00025 in. on large graduations on a 5-in. dial (Fig. 16).

From the tension tests the proportional limit, ultimate strength, per cent elongation, and modulus of elasticity were determined at regular intervals from room temperature up to 700 deg. fahr. A set of representative stress-strain curves obtained on grade A aluminum alloy, Air Corps specification 57-72-1, are given in Fig. 17, and tension and elongation results obtained on this and three other alloys are shown in Fig. 18. These will suffice to show the advisability of testing materials at the temperatures required of them in service. The tests were made in an Olsen 20,000-lb. universal testing machine, as shown in Fig. 19. In order to make all results comparable, the tests at room temperature were made with the same set-up but without the oil bath.

The Brinell tests at elevated temperatures were made in the same furnace, as described, with the pulling bolt replaced by an anvil post for supporting the specimen midway in the center of the bath, as shown in Fig. 20. In this case the furnace was firmly supported on the weighing table of the 20,000-lb. Olsen testing machine, while an Alpha Brinell testing machine unit was assembled in the movable head, which served as a vehicle for rough adjustment only (Fig. 21). A special head was cast and bored to accommodate the Brinell unit. By substituting a long insulated plunger, the ball was extended down into the oil to the specimen, where it remained throughout the tests, being slightly withdrawn to permit the exchange of specimens, which were handled by means of tongs. An asbestos baffle plate on the plunger prevented the oil in the Brinell cylinder from being heated by the fumes from the furnace. The test load was applied by the Brinell unit, as in standard practice, thus insuring true Brinell loading conditions.

These tests were made on a series of specimens 1 in. in diameter by 1 in. high. First, they were all tested on one end at room temperature (in the same set-up without the oil), and then one or more were tested on the other end at each of the regular intervals of temperature over the desired range. After cooling they were all tested again at room temperature to determine the permanent effect of the heating. All Brinell impressions were measured at room temperature by means of a specially mounted microscope with lateral travel graduated to 0.01 mm. A comparison of the tensile strength and Brinell hardness results obtained on a representative aluminum alloy at temperatures up to 700 deg. fahr. is shown graphically in Fig. 22.

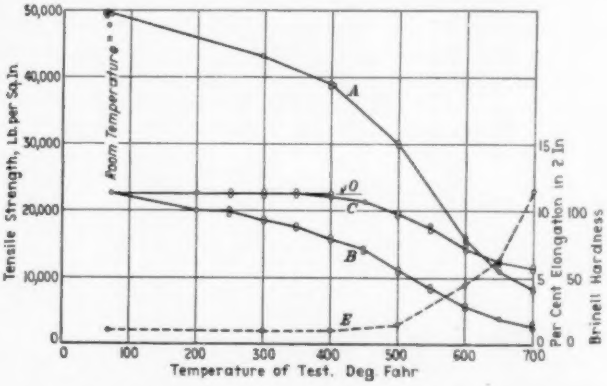

Fig. 22 Tensile and Brinell Results on a Representative Piston Alloy at Elevated Temperatures

(Aluminum alloy, grade B, A. C. specification 57-72-1. *A*, tensile strength over temperature range; *B*, Brinell hardness over temperature range; *C*, final Brinell hardness at room temperature; *O*, original Brinell hardness at room temperature; *E*, per cent elongation in 2 in. at elevated temperatures. Composition: Cu, 3.0; Ni, 0.5; Fe, 1.5; Si, 0.5; Mg, 0.5; Al, difference. Note on *C*: After finishing the Brinell tests at elevated temperatures, the same specimens were again tested at room temperature to determine the permanent effects of the 30-minute heating. The results are shown in curve *C*. Thus each specimen was tested first at room temperature, then at elevated temperature, then again at room temperature. Different specimens were used at each station.)

At elevated temperatures the Brinell test is more economical than the tensile test, since the specimens are easier to prepare and the testing conditions are more readily controlled; hence since the Brinell results practically parallel those of tensile strength, the Brinell test is preferred for exploratory work.

The Autogiro Rotor

By HERACLIO ALFARO,[1] PHILADELPHIA, PA.

The rotor of the Autogiro is described, as some authorities have considered that this revolving feature has added much in the way of increased safety and utility to the heavier-than-air craft. In the latest designs the Pitcairn Aircraft blade spars have been made of chrome-molybdenum steel. The necessity of attaining rigidity in torsion has determined in some designs the size of the blade main tubular spar. Research during the year has been devoted to mastering this structural requirement. The chief features are flexibility in the plane of lift, rigidity in the plane of rotation, and rigidity in torsion.

THE Autogiro has been presented to this Society in a general way on various occasions in the past. Mr. Cierva himself, inventor of this novel aircraft, read a paper in Cleveland introducing his invention and describing it. Later papers by Mr. Agnew Larsen and Mr. Laurence LePage have very well covered practically all of its general aspects. I therefore decided to discuss to a greater extent in this paper one of its component units only.

Of all parts in the Autogiro the most striking one is undoubtedly the rotor. This odd revolving affair of "flimsy looking" blades is taken by many aeronautical authorities as the outstanding development in the art of flying and is considered as bringing increased safety and utility to the heavier-than-air aircraft industry of today.

It looks odd because of its marked departure in appearance from natural fliers, and it seems flimsy because of its hinged and flexible blades, which feature is nevertheless established as one of the greatest assets of the Autogiro.

Much as is the case with the wheel, which did not follow the example of nature for ground transportation, the Autogiro rotor departs entirely from nature's flying models—the flapping-wing and soaring birds. This does not keep the Autogiro from approaching nearer to accomplishing what birds do than does the stationary-wing aircraft, and I dare say it goes farther than nature would go itself, should it decide to raise flapping and soaring birds of very large sizes.

For reasons of structural strength only, nature could not produce satisfactory flapping or soaring birds of the size of the present-day airplane without evolving the use of substances of greater structural efficiency than those of which birds of today are made.

Not a Windmill

Can the rotor of an Autogiro be called a windmill? Qualitatively the forces and behavior of both are the same. Quantitatively they are so different that they are well worthy of a different name.

In a windmill practically the only thing wanted is a great torque. In the Autogiro rotor the torque is zero, and the blades should be allowed to revolve as freely as possible, with the double purpose of obtaining great parachute effect when in vertical descent and great lift-over-drag ratio when in horizontal flight.

An Autogiro rotor can be distinguished from a regular windmill by its longer and narrower blades, fewer in number, more flexible, and set at slight positive angles to their plane of rotation. Also, for most purposes, Autogiro rotors are likely to have hinged blades—a feature that seldom, if ever, will be justified in a windmill.

Autorotation

"What makes the rotor spin?" This is a popular question, not only of the average non-technical man but also of many engineers. Explanations of autorotation have been given often, but they have failed at times to satisfy a great many people; so that a further attempt at an explanation appears needful.

The Autogiro rotor spins for the same reason that a windmill spins. The apparent difference is in that the Autogiro rotor has a positive angle of pitch in the blades, which would indicate that it could not rotate the way it does when in operation. A windmill has its blades set at a negative angle. This difference is due, as said before, to the different requirements of the Autogiro rotor and the windmill. The rotor is not expected to furnish any torque, but it is intended to give a high lift.

To further explain autorotation, Fig. 1 must be considered, which represents a symmetrical airfoil (similar to those used in the Autogiro) placed at 90 deg to an air stream such as would meet the blades of a rotor in vertical descent, before starting rotation. The flow will probably behave as shown. Due to the streamlined shape of the airfoil, the flow will go around the leading edge easier than around the trailing edge, and produces[2] a forward force sufficient to start rotation. As the rotational speed increases, the accelerating rotational moment increases; later, however, when rotation is fast, it decreases again, to become zero as soon as the rotor reaches its normal rotational speed.

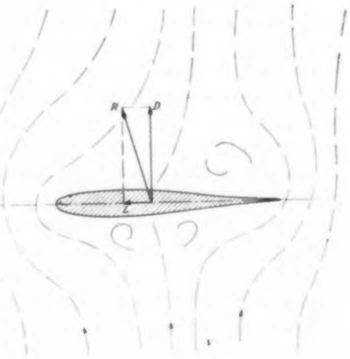

Fig. 1

The N.A.C.A. M-1 symmetrical airfoil of 6.16 per cent of chord thickness, placed at 90 deg to the air stream, gives a lift coefficient $C_L = 0.05$, which in the present case applies to the forward component L, shown in Fig. 1. If the air velocity V is of 20 mph and the area of the airfoil 10 sq ft, the forward component becomes $L = 0.05 \times 0.00256 \times 10 \times 20^2 = 0.512$ lb, which is sufficient to start rotation. In this airfoil the forward component L becomes zero when the angle of attack is 93 deg. It is estimated that with airfoils of 12 per cent of chord

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[1] Formerly with Autogiro Company of America.

Presented at the Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12 to 14, 1931.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] N.A.C.A. Tech. Report No. 317, "Wind Tunnel Tests on a Series of Wing Models Through a Large Angle of Attack Range, Part 1."

thickness $L > O$ at all angles below 95 deg. The Clark-Y has a zero value of L at about $95^1/_2$ deg. The Göttingen 429 at 90 deg would have approximately an L component = 0.83 lb if the area was, as before, 10 sq ft and the wind velocity 20 mph.

Once the velocity of rotation reaches a certain value, conditions become such as are shown in Fig. 2, where the velocity V_1 is the resultant of adding rotational speed V_2 to the previous velocity V. The usual resultant R of the air forces can be resolved into the lift L and drag D when referred to the direction of motion p-p and its perpendicular z-z. However, when referred to the plane of rotation x-x and the axis y-y, the same resultant R is resolved into T, which is sensibly equal to L, and into F, which is opposite to D and promotes rotation.

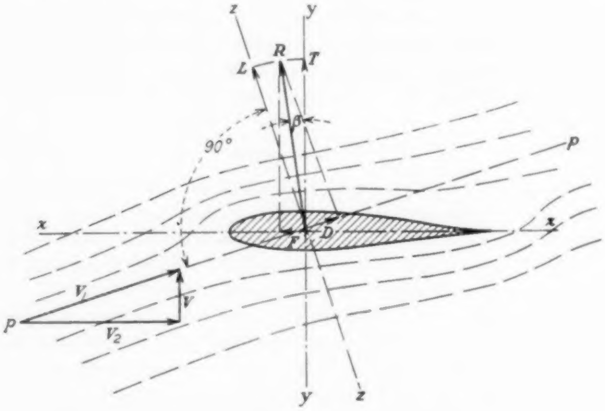

FIG. 2

As the rotational velocity increases, the angle β between the resultant R and the y-axis diminishes. V_2 becomes constant when β equals zero.

It has been found that the best lift is obtained when the blades have a slight positive angle of pitch. In this case, the rotor may have difficulty in getting started since the L-component with the blades at rest would be very small. There is a critical angle of pitch which is the best for a high lift, and beyond it, in a matter of a few degrees, autorotation is altogether impossible even with an initial start.

So far, the Göttingen 429 section has been used for the blades almost exclusively, due primarily to its stationary center of pressure, which is a very desirable feature. There are other sections that are likely to give better results. The matter is now under careful investigation and may result in some improvement.

SELECTION OF AIRFOIL[3]

From an aerodynamical viewpoint, the best airfoil for Autogiro blades can be selected from a group about which the characteristics of each individual airfoil are known.

Consider an element ΔS of a blade of the rotor of an Autogiro in a steady vertical descent, Fig. 3. The axis of rotation is parallel to the y-axis, and therefore rotation may occur in the plane of the x-axis. The air under the compound velocities of descent V and of rotation V_2 will meet the blade element with a resultant velocity V_1 at an angle β with the plane of rotation. These velocities are vectorially represented in the figure. Steady velocity of descent implies a steady rotational velocity. Steady velocity of rotation can only occur if the resultant R of all air reactions in the blade is parallel to the y-axis. Deviation of R fore and aft from the perpendicular to the plane of rotation would result in an acceleration or a deceleration of the rotational speed.

From the foregoing, the following relations can be written:

$$R_1 = L\cos\beta + D\sin\beta$$

$$R_{c1} = L_c\cos\beta + D_c\sin\beta$$

These coefficients refer to the component true velocity V_1 of the blade element.

$$V_1 = \frac{V}{\sin\beta} = V\csc\beta$$

also

$$R = R_c\,(\Delta S)V_1^2 = (L_c\cos\beta + D_c\sin\beta)\,(\Delta S)V^2\csc^2\beta$$

and

$$R_c = \frac{R}{(\Delta S)V^2} = (L_c\cos\beta + D_c\sin\beta)\csc^2\beta$$

and also

$$\frac{L}{D} = \frac{\cos\beta}{\sin\beta} = \cot\beta$$

In practice, the values of β are always very small, and therefore $\cos\beta$ is nearly equal to 1 and $D_c\sin\beta$ is negligible. It therefore can be written:

$$R_c\text{ approx.} = L_c\csc^2\beta = L_c + L_c\cot^2\beta$$

$$(\csc^2\beta = 1 + \cot^2\beta)$$

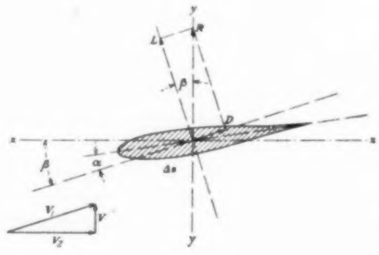

FIG. 3

The first term of this expression is also very small, and can also be neglected finally, so that it may be written:

$$R_c\text{ approx.} = L_c\cot^2\beta = L_c\left(\frac{L}{D}\right)^2 \quad\text{[1]}$$

The rotational-path speed is:

$$V_2 = \frac{V}{\tan\beta} = V\frac{L}{D} \quad\text{[2]}$$

Fig. 4 is a chart where the values of $L_c(L/D)^2$ for a number of airfoil sections have been plotted. The Sloane 105 section seems to be the best from the point of view of parachute effect in an Autogiro. It must be remembered that structural or other considerations may advise a different selection. Equations [1] and [2], however, furnish good guidance and point out the advantage of airfoils of high L-over-D ratio.

COEFFICIENT OF RESISTANCE

Mr. Cierva in his theory shows that the parachute effect or resistance of an Autogiro rotor in vertical descent is several times greater than that of a parachute of the same diameter. At first, it seems that, since a parachute or round disk of equal size obstructs the flow entirely, any other device which would let part of the flow go through would produce a smaller resistance. On second thought, however, it can be seen that the velocity imparted to the column of air handled by the Autogiro rotor in its descent may be much greater than that imparted by a disk or parachute of the same diameter under similar conditions.

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[3] From notes on the Autogiro at Massachusetts Institute of Technology in 1925, by H. Alfaro, reproduced in part later in a thesis by Weihmiller and Hosch.

This can be better understood by comparing the proposed type of flow pattern shown in Fig. 5, which refers to an Autogiro descending vertically, and the flow pattern around a flat disk under similar conditions, as shown in Fig. 6. In the latter the flow builds some sort of an invisible dome in front of the plate and a streamlined cap behind it. These bodies are composed of eddies and dead or stationary (relatively so) air and go with the plate. The rest of the flow therefore goes around with relative smoothness, which results in a somewhat low resistance.

The flow pattern of an Autogiro rotor, however, does not give a chance to the air to go around to the same extent. The intermittent action of the blades avoids the formation of the dead-air caps mentioned, with the result of greater interference of the flow and greater resistance.

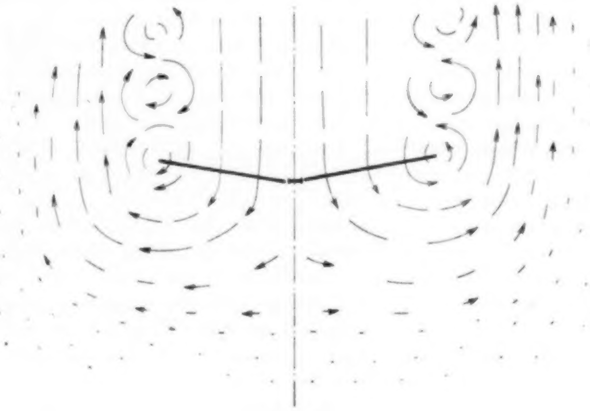

Fig. 5

Control

The ailerons are not strictly necessary, but in practice they must be used because they increase maneuverability to an enormous extent. An aircraft cannot turn any faster than it can roll. If a turn is attempted by applying the rudder alone, the first result is a side slip, resulting in a bank and then in a turn. Coming out of it with the rudder action alone is achieved also through a side slip inside the turn. The ailerons enable the pilot to make a more perfect turn, and therefore a quicker and tighter turn.

A peculiar faculty of the rotor and wing arrangement as used in the Autogiro is the effectiveness of the ailerons at high angles. This does not mean that the Autogiro has as snappy a lateral control at low speeds as at high ones. At low speeds the ailerons respond slowly but effectively and without the characteristic feeling of stalled control of airplanes at high angles.

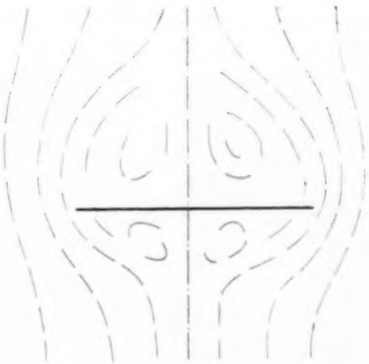

Fig. 6

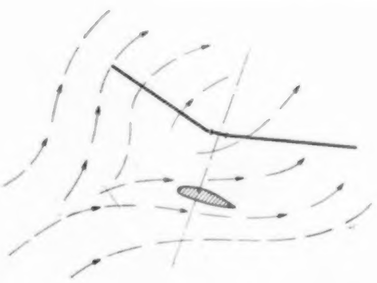

Fig. 7

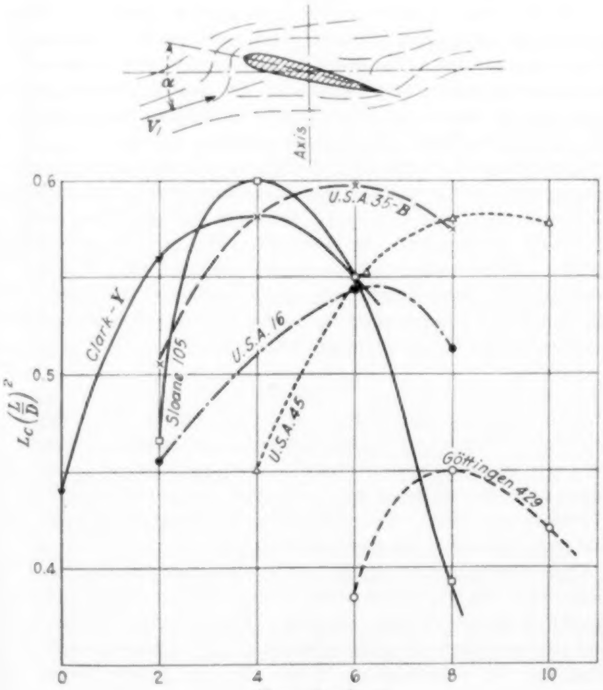

Fig. 4

This can be better understood through Fig. 7, where the flow of air is shown around a rotor and fixed-wing combination at large angles. The direction of the flow where the wing is located is such that no stalling occurs for the fixed wings. This is again one of the many fortunate findings of the Autogiro, which one is prompted to attribute to the extreme ingenuity of its inventor rather than to mere chance.

There are so many solutions like this one in the conception of the Autogiro that it is difficult to believe that any man could deliberately study and find out and then combine all these things harmoniously.

Articulation of the Blades

Autogiro experts claim that the most striking feature of this invention lies in the articulated connection of the blades to their hub. It serves so many purposes that engineers consider it as one of its greatest assets, since it simplifies many problems entering its design.

These hinges allow the blades to move freely up and down in the plane of lift, and also they provide freedom in the plane of rotation. The last is restricted by rubber blocks at the root of the blade to about ±5 deg either side from normal, while in the plane of lift the blades can move some 35 deg above horizontal and about 7 to 9 deg below it. Shock absorbers and "interbracing cables" are provided between the blades to give additional restriction to the freedom of the blades in the plane of rotation. The blades as they rotate change from driver to driven once in each revolution, which tends to produce a sort of pendular action and to start roughness. The shock absorbers used at present are of the hydraulic type and allow slow self-centering of the blades, while they restrain them from sudden jerks.

The rubber blocks not only help the self-centering of the blades, but they resist the load due to the mechanical starter torque transmitted to the rotor.

The advantages in connection with the articulation of the blades are numerous. It eliminates the gyroscopic effect of the rotor; without it the controlling ability of this aircraft would be practically impossible. As it now is, when the machine is nosed up, the blades in the fore-and-aft position will remain momentarily in their original plane of rotation. The blades have a total freedom in every direction except in torsion, and consequently the blades on the port and starboard sides will have increased or decreased their angle of attack (according to the direction of rotation of the rotor) in the same amount as the alteration produced by the nosing up of the machine. These blades, therefore, will climb or dive out of their original plane of rotation in order to maintain their original position relative to the axis of rotation. This self-aligning of the blades happens so quickly that the naked eye cannot detect any lag of the blades in following the maneuvers of the aircraft. Furthermore, since the power required for the blades to follow into a new attitude is taken from the surrounding air (at the expense of a little added drag), no damping or sluggishness in the response to the controls is observed. This is of the utmost importance for the practical side of flying.

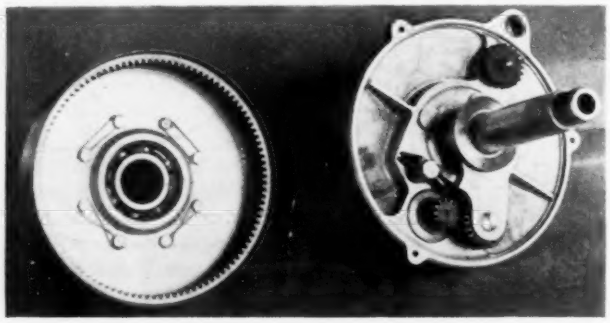

FIG. 8

Another advantage attributable to the hinged blades is the stability of the Autogiro in bumpy air. The blades give up and down slightly in the presence of gusts or pockets. The added smoothness has been observed time and again during flight and is valuable from the point of view of the airline operator in carrying passengers.

Stability in vertical descent is also much improved by the articulated arrangement.

Structurally, the hinge has at least two distinct advantages. One is the elimination of any bending moment at the root of the blade, which cuts down the structural weight to enormous extent; and the other is the reduction of the maximum possible overloads to a small figure. Mr. Cierva proves in his theory that, in a bad dive recovery, under conditions seldom if ever possible to find in practice, the maximum overload in the blades is 4.4 times the normal. This applies to a rotor of the normal proportions as used today.

But the main advantage of the articulated blade is that it makes the Autogiro possible. It is rather an essential instead of merely an advantage, since it makes workable the use of a single rotor, and it was with this in view that Mr. Cierva evolved this hinged arrangement.

ROTOR STARTER

Again, a usual question of many is, "Are the blades driven by the motor at any time?" No, is the answer, although it is not perhaps legally a correct answer. There is a power transmission that is sometimes connected with the rotor, but never during flight. It is used only to simplify and shorten the take-off. Taxying takes too long to give sufficient speed to the blades for a take-off. Many devices have been studied to take care of this. Only two have been used with any success. The first one was the "deflector tail"—a sort of box-kite tail that could be arranged to deflect up into the rotor the slipstream from the propeller when on the ground. By so doing the blades could be given quite an amount of initial rotation on the ground, and the take-off could then be much shortened. The structure of this tail, however, presented additional parasite drag, which handicapped performance in other lines. A mechanical connection with the motor had been studied in England, but was thought to be too heavy. A lighter design had been worked out on paper in this country by Mr. Joseph S. Pecker, of the Pitcairn engineering group. The general thought was to get rid of that clumsy-looking tail. At that time I had on my hands the problem of designing and building an Autogiro for Pitcairn Aircraft in Cleveland, and I had my own ideas regarding a starter. I thought that to play safe I had to arrange a limiting torque device between the engine and the transmission of power to the rotor. I figured that only 15 hp was needed to transmit ample rotation to the rotor, while the engine had over 100 available. Part of this was required to spin the front propeller on the ground, but I felt that if I were to make a foolproof device I either had to design the transmission for the worst condition of load or to use my torque-limiting device. I remained conservative, and used it, but it was thought later by Mr. Pecker that this precaution was not necessary, because some care in the application of the clutch could be expected on the part of the pilot. I went through with my idea, however. This was the first Autogiro mechanical starter ever made. Figs. 8 and 9 show this starter. The present mechanical starter is not a development resulting from this one. It was developed independently by Mr. Pecker, and is used as standard by the Autogiro Specialties Corporation in various sizes.

In all rotors a brake is installed for at least two reasons: One is to stop the blades from revolving when taxying with strong winds, that, if allowed to meet the revolving rotor, might lift or overturn the machine. The other is to avoid approaching groundcrew people from being hit by the blades.

Suggestions have been made as to folding blades that will reduce the space required to store an Autogiro, and it is thought of as a probable future development.

STRESSES

The blades are subject to a very high tensile stress due to centrifugal force, which naturally depends on the rate of rotation. This rate increases with altitude in the proportion of $\sqrt{(\rho_0/\rho_h)}$, where ρ_0 is the density of air at sea level and ρ_h that of altitude. Therefore the critical design condition to investigate should be that at the ceiling of the particular machine.

These blades also are subject to bending, and the more so the higher their rigidity in the plane of lift. That is why the blades should be flexible in that direction. It can be well understood that the centrifugal-force distribution along the blade is different from the lift distribution. While the one follows nearly a straight line of almost constant slope, the other follows a curve whose maximum is nearer the tip than the root of the blade.

much as in regular propellers. When in horizontal flight, this lift curve changes also according to the momentary position of the blade in its circular path. This may cause fatigue unless, as is the case, the bending stresses are rather low. If the blade was sufficiently flexible, it would acquire by itself a shape following the rate of change of the combination of tension and bending, and the bending stresses would be zero. But since it is to some extent rigid, some bending actually must be faced. Mr. Cierva has evolved a relatively simple method of figuring the bending for various blade-plan forms. He also has shown in his work that for almost any of the plan forms used the maximum stresses were met in the middle of the blade—that is, at a point one-half of the radius away from the center.

As mentioned before, there is also shearing due to torsion. This is the smallest of all stresses.

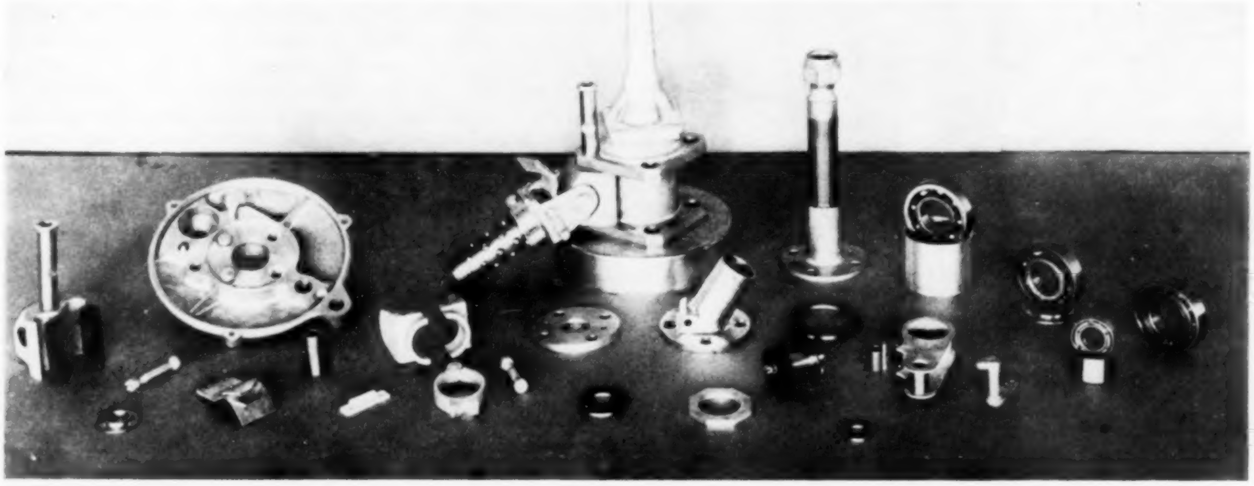

FIG. 9

The three fiber stresses should be computed for the worst condition of operation and then combined. A conservative margin of safety should be found. Mr. Cierva advises as a typical maximum-load condition the recovery from a dive where the velocity would have reached $1^{1}/_{4}$ times its horizontal-flight maximum. He assumes this to be done at the ceiling, where the rotational speed is the highest, and he considers it possible to bring the plane of the rotor instantly into a position normal to the direction of motion.

It has been found by Autogiro pilots that this assumed condition is really an ideal maximum. The computed fiber stresses under these conditions correspond to the rotor furnishing a normal force 4.4 times greater than the weight of the aircraft for a rotor of the proportions normally used. Mr. Cierva recommends a load factor of 6, so that the margin is really higher than that provided for in the regular airplane. This is due, as said before, to the hinge arrangement of the blades.

As a typical example, some figures corresponding to an Autogiro built in Cleveland by the author for the Pitcairn Aircraft, Inc., are given in Table 1.

Many practical tests and theoretical studies have been conducted, and the results indicate that this procedure is conservative and safe.

Various other points along the blades are investigated for stresses, as for instance at the root, where a suitable connection must be designed, and also at the attachment of the ribs to the main blade spar. This member in the latest designs has been made of chrome-molybdenum steel, heat treated to about 150,000 lb per sq in. Fatigue tests of spars also have been conducted under severe conditions with gratifying results, confirming the present procedure of Autogiro engineers.

TABLE 1 CHARACTERISTICS OF TYPICAL AUTOGIRO

Diameter of rotor, ft	38
Maximum chord, in	18
Weight of each blade complete, lb	58
Maximum bending moment at 0.5 R, in-lb	1,313
Share of the bending taken by the bakelite covering of the blades (found empirically), per cent	30
Fiber stress due to bending, lb per sq in	8,750
Centrifugal tension at 0.5 R, lb	2,950
Fiber stress due to tension, lb per sq in	14,730
Torsional moment at 0.5 R, in-lb	806
Shearing stress due to torsion, lb per sq in	4,465
Combined tensile stress, lb per sq in	25,240
Combined shearing stress, lb per sq in	13,000
Maximum design stresses in tension, lb per sq in	151,700
Yield point of poorest blade-tube specimen, lb per sq in	170,000
Margin of safety	1.12

The necessity of attaining rigidity in torsion had determined in some designs the size of the blade main tubular spar. It is believed that blades insufficiently rigid in this respect originate oscillations that may become dangerous and that at least are very unpleasant. Much of the research work conducted last year by Autogiro engineers was devoted to mastering this important structural requirement, which now is well under control.

To close this subject, it may be said that the three major structural requirements for the design of a blade are:

1 Flexibility in the plane of lift
2 Rigidity in the plane of rotation
3 Rigidity in torsion.

The second is easy to obtain, but the first and third are difficult to combine and demand the exercising of considerable engineering ingenuity.

The Autogiro is no mystery; it is just another engineering development. It is appealing because it deals with flying. It is doubly fascinating because it comes to solve a long-sought problem by adding substantial elements of safety, utility, and comfort to the engineering knowledge gathered in the aeronautical field.

AER-54-19

Design Problems of Controllable-Pitch Propellers

By W. B. HEINZ,[1] SCHENECTADY, N. Y.

This paper presents analytical studies of the most common problems that are encountered in the design of mechanisms to rotate airplane propeller blades about their axes. It includes (1) a brief discussion of centrifugal thrust-bearing loads, (2) an analysis of the torque created by centrifugal force, (3) an analysis of lateral bearing loads from which their frictional torques can be determined, and (4) an investigation of the centrifugal torque of multiple masses by means of which a counterbalance can be designed to neutralize to any desired extent, at all settings, the centrifugal blade torque. The analyses are carried further than in previous published work, and results are reached by which quantitative design data can be obtained with minimum numerical and graphical labor.

THE problem of controlling the pitch of airplane propellers while in flight has been studied for years.[2] The need of some reliable mechanism for this purpose is widely recognized as the result of expansion in military and commercial aeronautics. Practically all serious attempts to design and construct controllable-pitch hub mechanisms have been confined to devices by which the blades can be rotated in the hub about their axes. It can be demonstrated by aerodynamic considerations that a reasonable approach to ideal pitch control is accomplished by this means and that major improvements in economy and performance are to be expected in airplanes equipped with successful propellers of this type. A number of separate designs have been constructed and tested. Several have received extensive flight tests and have justified those sanguine hopes aroused by model and flight tests of propellers with blades adjustable at standstill.

Since pitch control is receiving widespread attention at this time, it is believed that the publication of several analytical solutions of certain design problems may prove of value to the industry. Hence there are presented in this paper analytical studies of the most common problems encountered in the design of mechanisms to rotate the blades about their axes. Each blade must be held in place in the hub by some means which will allow rotation about its axis, and there must be provided some arrangement by which the necessary torques can be applied about the blade axes to cause this rotation.

There are here considered (1) the centrifugal loads upon the blade thrust bearings and (2) the operating torques involved.

Design Problem

The design of a controllable-pitch hub mechanism centers about two broad problems: First, means of retaining the blades in the hub structure, which will be capable of resisting the high centrifugal forces exerted and which will readily allow each blade to be turned about its own axis and, second, a mechanism by which the blades can be turned about their axes in the manner desired.

Means which have been conceived to accomplish the first result include flexible supports, which allow limited blade rotation through flexure; hydraulic cylinders; mercury float arrangements; paraffin-saturated fiber bearings; and especially designed ball and roller thrust bearings. Because of recent developments it now seems safe to assume that a bearing of the latter type can be obtained which will stand up satisfactorily under the extremely high loads encountered.

As an example, the method of support which is used must restrain the blades of a 525-hp. propeller against a centrifugal force which may rise to 100,000 lb. or more per blade. The centrifugal force on a hollow steel propeller blade, which is lighter than a corresponding one of aluminum alloy, has been calculated by well-known processes of graphical integration. For this propeller of 9 ft. 5 in. diameter the centrifugal force per blade amounts to 70,300 lb. at 2500 r.p.m. This force increases as the square of the r.p.m., which may reach 3000 in a power dive.

Once the centrifugal loading of the thrust bearing has been calculated and a bearing has been obtained which can be expected to satisfactorily meet the necessary requirements, there must be determined the operating forces exerted by a hub mechanism.

The problem is primarily that of determining the torque required to turn each blade in its socket under the most extreme operating conditions. The most important components are the following:

(*a*) The aerodynamic torque created in the blade
(*b*) The torque created by centrifugal force
(*c*) The frictional torque in the thrust bearing
(*d*) The frictional torque in the lateral blade bearings
(*e*) Miscellaneous frictional torques in the operating mechanism itself.

Aerodynamic Torque

In a well-designed propeller blade the sections are all arranged so that their centers of gravity lie on a straight line, which is generally radial. The sections are not symmetrical about their centers of gravity, however, and the pressure distribution

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[2] "The Value of Controllable Pitch Propellers to the Aeronautics Industry," by W. B. Heinz, A.S.M.E. Annual Aeronautical Meeting, Baltimore, Md., May 12 to 15, 1931 (with bibliography).

Contributed by the Aeronautic Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

over each one is such that there is exerted an aerodynamic moment tending to increase its angle. Analysis of this torsion has been performed, based upon assumed ideal flow conditions. This assumption in itself is so far in error that the results are not quantitatively accurate, but they are nevertheless sufficiently representative of the true values that one important conclusion can be drawn. That is, the aerodynamic torsion is so small in relation to the centrifugal and frictional moments that it need not be given quantitative consideration.

Centrifugal Torsion

Fig. 1 shows a propeller blade section, rotating with constant velocity about the *Y*-axis parallel to the *X*-*Z* plane. The forces to be considered are analyzed at a certain instant of time, when the section is passing through the position in which it is shown. As indicated, the radius from the axis of rotation to the center of gravity of the section is z. The distances x and y locate an elemental mass dm in the section, which has acting on it the centrifugal force

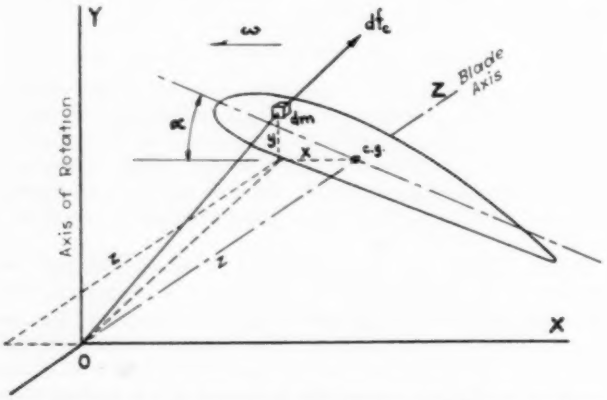

Fig. 1 Creation of Torsion in a Propeller Blade by Centrifugal Force

$$df_c = \omega^2 dm\sqrt{x^2 + z^2} \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

This analysis is based upon the assumption that all forces are carried from a point at the intersection of the blade and engine axes. On this assumption it is apparent that the force df_c tends to swing dm on the extremity of the line *O*-dm downward into the *X*-*Z* plane of rotation. All other elements are similarly urged toward this plane, but are prevented from going there by the internal resisting moment of the blade structure. The blade is thus placed in a state of torsional stress by centrifugal force.

The elemental force expressed by Equation [1] acts at the lever arm y to swing dm downward at the extremity of its radius $\sqrt{(x^2 + z^2)}$. The resisting structure must apply an upward force to balance this moment by its action about the lever arm $\sqrt{(x^2 + z^2)}$. If we term this equivalent force f_e, we can equate moments as follows:

$$y\omega^2 dm\sqrt{(x^2 + z^2)} = f_e\sqrt{(x^2 + z^2)} \ldots\ldots\ldots\ldots [2]$$

The torsion created in the blade by its exertion of the force f_e is

$$dQ_c = xf_e$$

from which

$$f_e = \frac{dQc}{x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [3]$$

Equation [2] now becomes

$$y\omega^2 dm = \frac{dQ_c}{x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [4]$$

From Equation [4] we obtain for the elemental centrifugal torsion created by the mass dm, the expression

$$\underline{dQ_c = xy\omega^2 dm} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

The torsion thus created by any complete blade section is obtained by integrating Equation [5] over the section area:

$$Q_c = \rho\omega^2 \sum_{\text{area}} xy\, dx\, dy \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

In Equation [6] there has been substituted for the mass dm its equivalent $\rho dxdy$. The section is taken as of unit thickness along the *Z*-axis, and ρ represents the mass density of the material.

It is interesting to note from Equation [6] that the centrifugal torsion created by any section is independent of its distance z from the axis of rotation.

It is well known that the product of inertia (U) of any section is

$$U = \sum_{\text{area}} xy\, dx\, dy \ldots\ldots\ldots\ldots\ldots\ldots [7]$$

We can thus write for Equation [6] the following:

$$Q_c = \rho U\omega^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [8]$$

In terms of moments of inertia,

$$\begin{aligned} U &= (J_{\text{maj.}} - J_{\text{min.}}) \sin\alpha\cos\alpha \\ &= {}^1/_2(J_{\text{maj.}} - J_{\text{min.}}) \sin 2\alpha \ldots\ldots\ldots\ldots [9] \end{aligned}$$

where α is the angle between the major and minor axes of the section, respectively, and the coordinate axes about which U is taken.

Equation [8] now becomes:

$$Q_c = \frac{\rho\omega^2}{2}(J_{\text{maj.}} - J_{\text{min.}}) \sin 2\alpha \ldots\ldots\ldots\ldots [10]$$

To obtain the centrifugal torque created by an entire blade we must sum up all the individual section torques of Equation [10] from the blade root to its tip. This summation may be written

$$Q_c(\theta) = \frac{\rho\omega^2}{2} \sum_0^{\text{tip}} J(z) \sin 2\alpha(z, \theta) \ldots\ldots\ldots [11]$$

In Equation [11] we have written $J(z)$ instead of $(J_{\text{maj.}} - J_{\text{min.}})$, the letter z in parentheses calling attention to the fact that J varies along the blade radius. Similarly, $Q_c(\theta)$ implies that the total torque varies with the blade setting angle θ; and $\alpha(z, \theta)$ means that α is a function of both z and θ.

Equation [11] now represents the summation of torques for all sections along the blade as it varies with changes in the setting angle θ. The term for each section (Equation [10]) if written as a function of θ becomes

$$Q_c(\theta) = \frac{\rho\omega^2}{2} J(z) \sin 2\alpha(z, \theta) \ldots\ldots\ldots\ldots [12]$$

Inspection of Equation [12] discloses the fact that it represents any one of a group of sine waves of equal double frequency, whose coordinates are Q_c and α and whose amplitude and phase both change along z. Their summation (Equation [11]) is then another sine wave:

$$Q_c(\theta) = Q_m \sin 2(\theta + \lambda) \ldots\ldots\ldots\ldots [13]$$

In Equation [13] there are two new symbols. Q_m is the amplitude of this new sine wave, it being physically the maximum total torque which can be obtained in a given blade by changing its setting. The symbol λ represents the "phase angle" of the wave, which in reality is that negative angle at which the blade must be set for the total centrifugal torque to be zero.

We can now readily obtain the entire curve of $Q_c(\theta)$ for any blade by (*a*) graphically solving Equation [11] for any two values of θ, and then by (*b*) solving Equation [13] for Q_m and λ.

After we have performed step (*a*) for any two angles θ_1 and θ_2, we can place in Equation [13] the two torques obtained for $Q_c(\theta)$. Designating them Σ_1 and Σ_2, there result

$$\begin{aligned} \Sigma_1 &= Q_m \sin 2(\theta_1 + \lambda) \\ \Sigma_2 &= Q_m \sin 2(\theta_2 + \lambda) \ldots\ldots\ldots\ldots [14] \end{aligned}$$

Solution of Equation [14] gives for the phase angle:

$$\tan 2\lambda = \frac{\Sigma_1 \sin 2\theta_2 - \Sigma_2 \sin 2\theta_1}{\Sigma_2 \cos 2\theta_1 - \Sigma_1 \cos 2\theta_2} \ldots\ldots\ldots [15]$$

NUMERICAL EXAMPLE

The solution of Equation [13] has been worked out by this method for a hollow steel blade, whose characteristics are given in Fig. 2, rotating at 2000 r.p.m. For simplicity the values 0 deg. and 45 deg. were chosen for θ_1 and θ_2, respectively, when Equation [15] became

$$\tan 2\lambda = \frac{\Sigma_1}{\Sigma_2} \ldots\ldots\ldots\ldots [16]$$

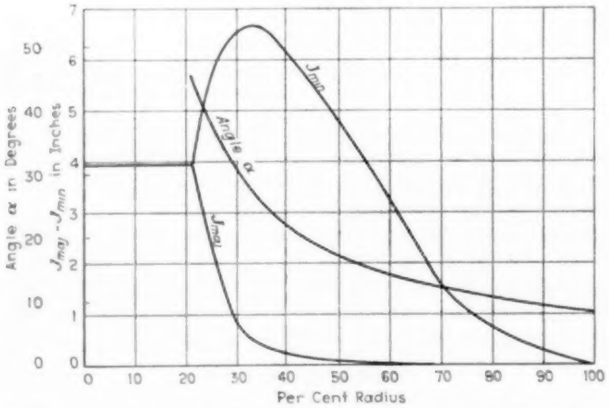

FIG. 2 MOMENTS OF INERTIA AND SECTION ANGLES OF PROPELLER BLADE

Thus by graphical integration for only two points we obtain the complete centrifugal torque-curve equation:

$$Q_c(\theta) = 1953 \sin 2(\theta + 9.875 \text{ deg.}) \text{ pound-inches} \ldots [17]$$

The curve is readily plotted, as shown in Fig. 3.

This fundamental Equation [13] or [17] represents the total centrifugal torque created in a blade of a rotating propeller. The coefficient Q_m is the maximum value of this torque which could be obtained at any one speed by varying the setting angle θ. The angle λ is that negative setting at which the total torque would be zero. Both of these terms can be determined for any particular case by graphical solution of Equation [11] for any two values of θ, followed by simultaneous solution of the two equations obtained from [13], when these two sets of values are substituted therein.

We have now a simple method by which the centrifugal torque created in any propeller blade can be determined for all angular settings. Such information is valuable in the design of a counterbalance to free the operating mechanism from all or part of this centrifugal component. It points out the way to so proportion a counterbalance and its angular position that the blade torque will be exactly neutralized at all settings, or that any desired net torque in either direction can be obtained.[3]

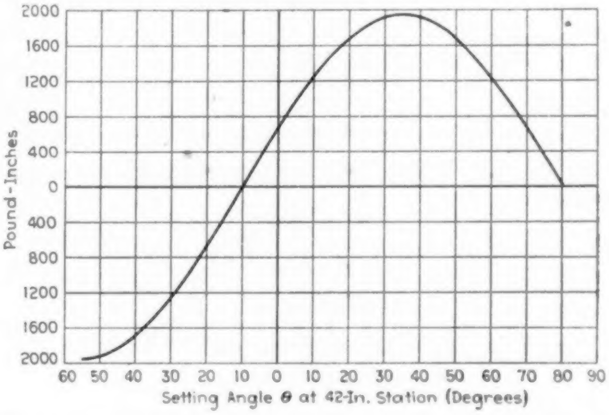

FIG. 3 TOTAL CENTRIFUGAL BLADE TORQUE VERSUS SETTING ANGLE

(For blade of 525-hp. propeller at 2000 r.p.m.)

THRUST-BEARING FRICTION

Thrust bearings of several different types are being developed for use in controllable-pitch propellers. Their characteristics are all different, and the manufacturer of any particular type should supply load friction data. Then by the application of known centrifugal loadings the required turning moments can be calculated.

LATERAL BEARING FRICTION

The fourth component of operating torque is that occasioned by friction in the lateral blade bearings. Friction coefficient data will be obtained from the bearing manufacturers, but in order to make intelligent use of it the bearing loads must be known.

In this analysis only the following three forces are considered as affecting the lateral bearing loads:

1 Blade thrust, which propels the airplane
2 Blade drag, which resists the engine torque
3 Centrifugal force acting on every blade section.

It is assumed that the first two forces can be added as vectors at 90 deg., and that the resultant always acts in the direction of least blade stiffness. The deflection produced is assumed to be perpendicular to the plane of rotation. All vibratory stresses are neglected.

The method of analysis here outlined is not rigorously correct, but it seems to be the best available, and the assumptions upon which it is based are close enough for all practical purposes. Each propeller blade is mounted in the hub as a cantilever beam, in which the resultant thrust produces deflections as indicated in Fig. 4.

In Fig. 4,

δ_1 = the deflection at radius z which would be created by aerodynamic thrust and drag alone

δ_2 = the restoring deflection which would be created by centrifugal force if acting alone at deflection δ

δ = the resultant deflection which occurs from the combination of δ_1 and δ_2.

---

[3] See Appendix 1, Centrifugal Torque of Multiple Masses.

The centrifugal restoring moment acting alone would create a deflection equal and opposite to that from an equivalent distributed load T_c shown in Fig. 5. The forces F_c and T_c are shown acting upon a blade section of area A and unit length, it being necessary for equilibrium that the equivalent force be:

$$T_c = F_c \frac{\delta}{z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [18]$$

The centrifugal force is:

$$F_c = \rho A z \omega^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots [19]$$

Hence

$$T_c = \rho \delta A \omega^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots [20]$$

By ordinary beam theory the equivalent centrifugal deflection is:

$$\delta_2 = \frac{1}{E} \int\int \frac{1}{J_{\text{maj.}}} \int\int T_c \, \overline{dz^4}$$

$$= \frac{\rho\omega^2}{E} \int\int \frac{1}{J_{\text{maj.}}} \int\int A \delta \overline{dz^4} \ldots\ldots\ldots [21]$$

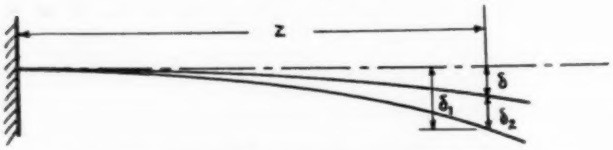

FIG. 4 COMPONENT DEFLECTIONS OF A PROPELLER BLADE

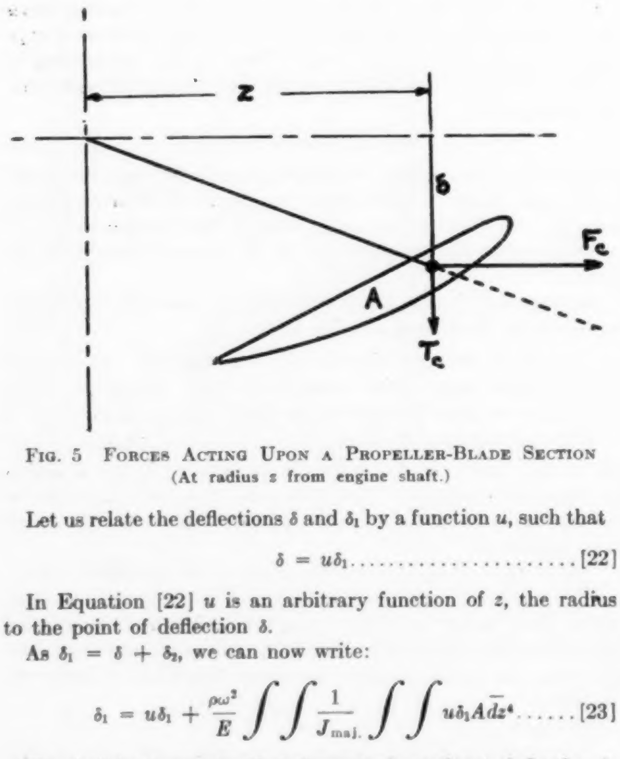

FIG. 5 FORCES ACTING UPON A PROPELLER-BLADE SECTION
(At radius z from engine shaft.)

Let us relate the deflections δ and δ_1 by a function u, such that

$$\delta = u\delta_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots [22]$$

In Equation [22] u is an arbitrary function of z, the radius to the point of deflection δ.

As $\delta_1 = \delta + \delta_2$, we can now write:

$$\delta_1 = u\delta_1 + \frac{\rho\omega^2}{E} \int\int \frac{1}{J_{\text{maj.}}} \int\int u\delta_1 A \overline{dz^4} \ldots\ldots [23]$$

We shall now assume that the actual resultant deflection is everywhere proportional to the deflection which would occur with zero centrifugal force. Therefore u is constant, and Equation [23] reduces to the form:

$$\frac{1-u}{u} = \frac{1}{\delta_1} \left[\frac{\rho\omega^2}{E} \int\int \frac{1}{J_{\text{maj.}}} \int\int \delta_1 A \overline{dz^4} \right] \ldots\ldots [24]$$

In Equation [24] the form $(1 - u)/u$ is given instead of the direct solution for u, because the graphical evaluation is simplified, and the term $(1 - u)/u$ can readily be determined. From it a series of values of u can be found and plotted against z. If the assumption is valid that u is constant, a straight horizontal line will result.

Bending moments created by aerodynamic forces alone can be computed, and from them the corresponding deflections. Multiplication by the factor u will then give the resultant bending moments and deflections.

In order to study the bearing loads we can represent the forces as in Fig. 6, in which the bending moment resulting from thrust and centrifugal force has been analyzed above. The moment created by blade drag can be computed aerodynamically, and the two combined vectorially.

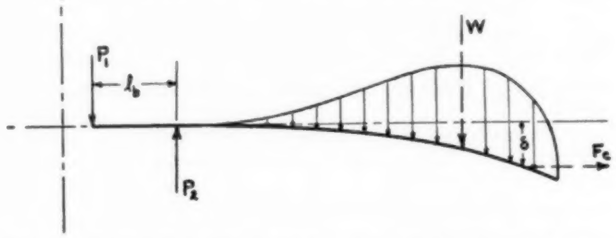

FIG. 6 LOADS AND BEARING REACTIONS UPON A PROPELLER BLADE

Thus:

$$P_1 = \frac{1}{l_b} \sqrt{({}_cM_t^2 + M_d^2)} \ldots\ldots\ldots\ldots\ldots [25]$$

$$P_2 = P_1 + W$$

$$= \frac{1}{l_b} \sqrt{({}_cM_t^2 + M_d^2)} + \sqrt{(\text{thrust}^2 + \text{drag}^2)} \ldots\ldots [26]$$

In Equations [25] and [26] the symbol ${}_cM_t$ represents the resultant bending moment at the blade root from combined thrust and centrifugal force as calculated above; M_d is the bending moment created by blade drag.

By the use of these two equations we can calculate approximately the forces experienced by the two lateral bearings of each blade. Having found the bearing loads, we can apply the proper friction coefficients, and thus obtain these moments which resist turning of the blades about their axes.

NUMERICAL EXAMPLE

The aerodynamic forces, moments, and deflections have been computed for a hollow steel blade of 9 ft. diameter, at 1800 r.p.m. and an assumed 140 m.p.h. forward speed. Curves showing pertinent data appear in Fig. 7. It will be noted that the curve of u versus z is flat over the region of highest deflection, and the assumption of constant u is thus shown to be reasonable within practical limits.

In one hub design which has been investigated the distance l_b is $7\frac{1}{4}$ in. Aerodynamic analysis indicates that the following values are representative:

$$\sqrt{({}_cM_t^2 + M_d^2)} = 6000 \text{ lb-in.}$$

$$W = 370 \text{ lb.}$$

From Equations [25] and [26] we obtain for the bearing loads in this design:

$$P_1 = \frac{6000}{7.25} = 828 \text{ lb.}$$

$$P_2 = 828 + 370 = 1198 \text{ lb.}$$

CONCLUSIONS

There have been presented in this paper two analytical studies the results of which can be applied directly to any particular design. First, a simple means has been developed by which the centrifugal torque in a given propeller blade can be readily determined for all angular settings, and, second, straightforward equations have been derived which make it possible to compute approximately the lateral bearing loads imposed by a blade under combined thrust, drag, and centrifugal force. Equations [11] and [13] are sufficient for the determination of centrifugal torque, while an aerodynamic analysis of blade deflections and the solution of Equations [24], [25], and [26] are necessary to compute lateral bearing loads.

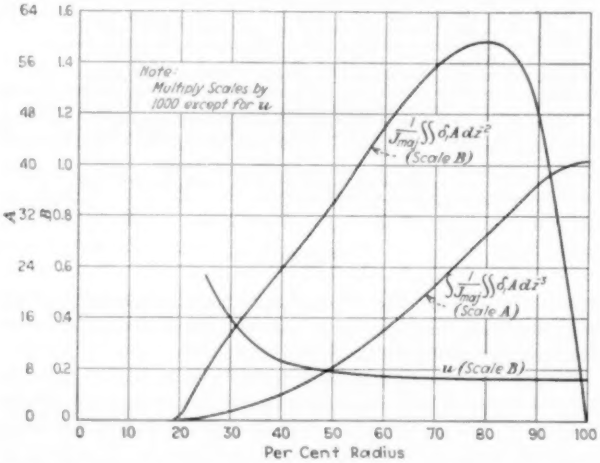

FIG. 7 STEPS IN GRAPHICAL INTEGRATION FOR THE SOLUTION OF FUNCTION u

The practical value of the analysis presented here lies in its reduction of the labor which has heretofore been necessary to compute a centrifugal torque-setting angle curve for a given propeller blade. Assurance can easily be gained that safe stresses are not exceeded in operating mechanisms, and in addition a systematic method is indicated for the design of a counterbalance to reduce the net centrifugal blade torque to any desired value at all settings. Such a method is developed in Appendix 1.

The method given here for the calculation of resultant lateral bearing loads with deflected blades is the first that the author has seen. He will appreciate critical examination of the process and will look for further experimental checks of the results.

It is hoped that this material will prove of practical worth to designers of pitch-control mechanisms and that a clearer understanding of phenomena involved may be gained from it by other readers.

Appendix 1

CENTRIFUGAL TORQUE OF MULTIPLE MASSES

This is an analysis of the combined centrifugal torque characteristics of a known propeller blade and two concentrated masses.

INTRODUCTION

The operating mechanisms of certain types of controllable-pitch propeller hubs include as common features an arm which is attached to the blade root and a connecting link by which the required turning moment is applied about the blade axis. It is desired to develop an analytical means for designing a counterweight to balance out, at all angular blade settings, the combined centrifugal torque of the blade, its arm, and connecting link.

ANALYSIS

Fig. 8 represents the projection upon a plane through the engine shaft (X-X axis) of a propeller blade section (hereafter referred to as "the blade") at the 42-in. station, where setting angles are customarily measured; a mass m_1 consisting of the operating arm and link referred to above; and a mass m which counterbalances the blade and mass m_1 in centrifugal torque. The actual distance of these members from the plane of projection has no bearing upon the subsequent analysis, and they can be represented as lying in the common plane shown. Rotation is about the X-X axis.

The angle θ represents the setting of the propeller blade and is the independent variable of the problem. It fixes the position of the entire assembly, which rotates together relative to the coordinate axes as the blade setting is varied by the pitch-controlling mechanism.

R_1 and m_1 are known constants, as is ϕ_1, the angle between the arm and the blade, while ψ_1 and ψ are angles which depend upon θ. The unknowns are m_1, R, and ϕ, independent of θ, to be determined for a balanced condition.

By analogy with Equation [13], the centrifugal torques of the masses m_1, m, and the blade are, respectively,

$$Q_1 = -{}_{m}Q_1 \sin 2\psi_1 \qquad [27]$$
$$Q = -{}_{m}Q \sin 2\psi \qquad [28]$$
$$Q_c = Q_m \sin 2(\theta + \lambda) \qquad [29]$$

For balance of moments,

$$Q = Q_c + Q_1 \qquad [30]$$

and, using geometrical equivalents for ψ_1 and ψ,

$$-{}_{m}Q \sin 2\left(\theta + \phi - \frac{\pi}{2}\right) = Q_m \sin 2(\theta + \lambda) - {}_{m}Q_1 \sin 2\left(\theta + \frac{\pi}{2} - \varphi_1\right) \qquad [31]$$

The right side of Equation [31] is a sine wave

$$y = Q_m \sin 2(\theta + \lambda) - {}_{m}Q_1 \sin 2(\theta + \beta) \qquad [32]$$

where

$$\beta = \frac{\pi}{2} - \phi_1$$

The maximum value of this wave occurs when $dy/d\theta = 0$, when $\theta = \theta_3$ Thus

$$2Q_m \cos 2(\theta_3 + \lambda) - 2{}_{m}Q_1 \cos 2(\theta_3 + \beta) = 0 \qquad [33]$$

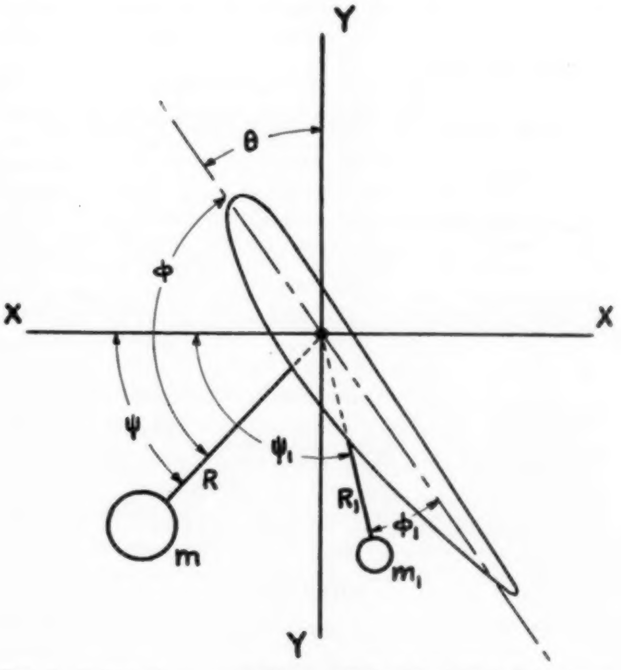

FIG. 8 THREE COMPONENT MASSES WHICH CREATE CENTRIFUGAL TORQUE ABOUT A BLADE AXIS

$$\tan 2\theta_3 = \frac{\cos 2\lambda - \frac{{}_{m}Q_1}{Q_m}\cos 2\beta}{\sin 2\lambda - \frac{{}_{m}Q_1}{Q_m}\sin 2\beta} \qquad [34]$$

Now Equation [34] can be solved for θ_3, which when placed in Equation [32] will give the maximum value of the wave. This maximum is evidently also the value of $-{}_{m}Q$ in Equation [31].

The intersection with the zero axis will be $\theta = \theta_3 - \frac{\pi}{4}$. Therefore (see Equation [31])

$$\phi - \frac{\pi}{2} = -\left(\theta_3 - \frac{\pi}{4}\right)$$

$$\phi = \frac{3\pi}{4} - \theta_3, \qquad [35$$

We have now derived relations by which the angle ϕ can be determined, at which a counterbalancing mass should be set for complete neutralization of centrifugal blade torque at all settings. A method for proportioning the mass and its lever arm will be presented in the following numerical solution of a test case.

NUMERICAL EXAMPLE

A tentative design of an operating arm, pin, and link for a typical mechanism indicates a total equivalent weight of 0.399 lb., at a radius of 2.86 in. from the blade axis.

The maximum centrifugal moment of this weight occurs when $\psi_1 = 45$ deg. Thus

$$\begin{aligned} {}_mQ_1 &= (\text{mass})\ (xy\omega^2) \\ &= 0.00422\omega^2 \text{ lb-in.} \\ &= 186 \text{ lb-in. at 2000 r.p.m.} \end{aligned}$$

The unbalanced moment of each of these component masses about its center of gravity is neglected.

For the blade considered in the body of this paper, we saw that

$$\begin{aligned} Q_m &= 1953 \\ {}_mQ_1 &= 186 \\ \lambda &= 9.875 \text{ deg. (use 10 deg.)} \\ \beta &= \frac{\pi}{2} - \phi_1 \end{aligned}$$

We observe that through the angle β the maximum torque depends upon ϕ_1, which in turn is different for various propeller applications. In order to investigate the effect of ϕ_1 upon θ_3, four computations were made:

ϕ_1	β	θ_3
7.5 deg.	82.5 deg.	36.0 deg.
12.5 deg.	77.5 deg.	36.8 deg.
17.5 deg.	72.5 deg.	37.1 deg.
22.5 deg.	67.5 deg.	37.35 deg.

The above results show that only a small error is introduced by using for design purposes average figures of about

$$\begin{aligned} \phi_1 &= 16 \text{ deg.} \\ \theta_3 &= 37 \text{ deg.} \\ \phi &= \frac{3\pi}{4} - \theta_3 \\ &= 98 \text{ deg.} \\ \beta &= \frac{\pi}{2} - \phi_1 \\ &= 74 \text{ deg.} \end{aligned} \qquad [36]$$

Equation [32] now becomes

$$\begin{aligned} y = {}_mQ &= 1953 \sin 94 \text{ deg.} - 186 \sin 222 \text{ deg.} \\ &= 2074 \text{ lb-in.} \end{aligned}$$

The torque created by the counterweight must be, from Equation [31]

$$\begin{aligned} Q &= -{}_mQ \sin 2\left(\theta + \phi - \frac{\pi}{2}\right) \\ &= -2074 \sin 2(\theta + 8 \text{ deg.}) \end{aligned} \qquad [37$$

The maximum value will be at $\psi = 45$ deg. Thus

$${}_mQ = \frac{mR^2\omega^2}{24} = \frac{mR^2}{24}(210)^2 = 2074 \qquad [38]$$

Ref. Equation [5]

Equation [38] can now be solved for the product mR^2, from which the required weight and radius can be related as follows:

$$W = \frac{36.4}{R^2} \text{ lb.}$$

where R = radius to center of gravity of counterbalance in inches

$W = mg$

For the above design, where $R = 3^1/_2$ in., $W = 2.98$ lb. total, including the equivalent weight of the arm which carries it.

The angle between this weight and the operating arm should be

$$\pi - (\phi + \phi_1) = 66 \text{ deg.}$$

NOTE: In a previous calculation, in which the operating arm and link were not considered, the weight was 2.66 lb. at 61 deg.

Appendix 2

LIST OF SYMBOLS

A = area of blade section at radius z
E = modulus of elasticity
F_c = centrifugal force on section of area A and unit thickness
$J_{\text{maj.}}$ = section moment of inertia about major axis
$J_{\text{min.}}$ = section moment of inertia about minor axis
$J(z)$ = $(J_{\text{maj.}} - J_{\text{min.}})$ expressed as a function of z
M_d = bending moment at root created by blade drag
${}_cM_t$ = bending moment at root created by combined thrust and centrifugal force
P_1 = inner radial bearing load
P_2 = outer radial bearing load
Q = centrifugal torque of counterbalance
Q_1 = centrifugal torque of operating arm and link
${}_mQ_1$ = maximum value of Q_1
Q_c = centrifugal torque of blade
Q_m = maximum value of Q_c
${}_mQ$ = maximum value of Q
R = radius from blade axis to c.g. of counterbalance
R_1 = radius from blade axis to c.g. of operating arm and link
T_e = equivalent distributed force creating same bending moments as centrifugal force
U = product of inertia
W = weight of counterbalance
X, Y, Z = coordinate axes
f_c = centrifugal force
f_e = equivalent force acting on an elemental mass to resist the centrifugal moment
l_b = distance between inner and outer radial blade bearings
m = effective mass of counterbalance and arm
m_1 = effective mass of operating arm and link
u = a function of z
x, y, z = rectangular coordinates
α = angle between the principal section axes and the coordinate axes about which U is taken
β = $\pi/2 - \phi_1$
δ = resultant deflection from the combination of δ_1 and δ_2
δ_1 = static deflection at radius z which would be created by aerodynamic thrust and drag alone
δ_2 = restoring deflection which would be created by centrifugal force if acting alone at deflection δ
θ = blade-setting angle, measured at 42-in. radius
θ_1, θ_2 = arbitrarily chosen angles for solution of Equation [11]
θ_3 = setting angle for maximum centrifugal blade torque
λ = negative setting angle for zero centrifugal blade torque
ρ = mass density of blade material
Σ_1, Σ_2 = total centrifugal blade torque at settings θ_1 and θ_2
ϕ = angle between projections on a plane perpendicular to blade axis, of counterbalance and major axis of 42-in. section
ϕ_1 = angle between projections on a plane perpendicular to blade axis, of effective radius of operating arm and major axis of 42-in. section
ψ = angle between counterbalance and X-axis
ψ_1 = angle between effective radius of operating arm and X-axis
ω = angular velocity of propeller.

Discussion

FRANK W. CALDWELL.[4] The paper summarizes some of the propeller-design problems that have furnished the basis of engineering discussion for a good many years.

The method of calculating the centrifugal torsion is described in the writer's paper, "American Progress in Controllable Pitch Propellers," presented to the International Congress for Aeronautics in London in 1923. It was earlier given in a different form by Drzwiecke in his book, "L'Hélice," published in Paris in 1920.

The method of applying the counterweight to balance the centrifugal moments was developed by the writer with the assistance of Mr. D. A. Dickey, now chief of the Propeller Branch at Wright Field, and Mr. E. G. McCauley. This work was done at McCook Field about 1920.

A method of calculating the bending stresses in the propeller blades, taking account of the deflection and resulting restoring moment, was developed by the writer and was published by the National Advisory Committee for Aeronautics about 1921 in a paper written jointly by Mr. N. S. Clay, of the Westinghouse Company, and the writer.

While these forces and stresses form a valuable background for the propeller designer, there are additional factors that must be taken into account if the design is to be successful. Two factors that have a profound influence on the design are the losses due to operation at high tip speed and the vibration stresses.

In a paper, "Physical Basis of Air Propeller Design," presented at the St. Louis Meeting of The American Society of Mechanical Engineers in 1920, Mr. E. N. Fales and the writer published some new data obtained in the wind tunnel at McCook Field on the characteristics of airfoils at high speeds. It was pointed out at that time that the old conception that propeller efficiency was a function only of V/nD would have to be revised. Airfoil and propeller tests during the past few years have made it apparent that the losses due to high tip speeds are a predominant consideration in propeller design. At the present time the only method known for minimizing these losses is through the use of thin sections, which can be most readily accomplished with the solid drop-forged blades.

The importance of the vibration stresses in propeller design is not reduced by the fact that they are difficult to calculate. While the vibration stresses are superimposed on the static stresses, they form the predominant stress in the propeller, and practically all failures in service are the result of vibration. In fact, it is very easy to break a propeller by means of vibration in the absence of any other stresses.

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[4] Mt. Lebanon, Pittsburgh, Pa.

Some very valuable work on propeller vibrations was recently done by Mr. John E. Younger at Wright Field. Mr. Felix Pawlowski, also, while at Wright Field has collected some valuable data on the torsional characteristics of propeller sections for the purpose of calculating torsional characteristics.

The effect of vibration on the propeller-pitch-control mechanism is of the very greatest importance, and the most difficult problem in the design is provision for resistance to the destructive effects of vibration.

Fred E. Weick.[5] This paper, with its consideration of the various components of the centrifugal force and air loads on propeller blades, should be a great help to the large number of inventors of controllable and automatically variable-pitch propellers. After many discussions with the inventors about their propeller mechanisms, it would seem that very few understand the problem fully, particularly in regard to the torsional or twisting components.

One conclusion was made by the author which, while satisfactory for the purpose chosen, is not entirely correct, considering all the factors involved. This statement is that the aerodynamic torsion is so small in relation to the centrifugal and frictional moments that it need not be given quantitative consideration. This is probably satisfactory from the point of view of the moments required to turn the whole blade at the root. It is interesting, however, to consider the effect on the twist of the blade throughout its length and the resultant effect on the pitch. It happens that, at the outer portions of the blade where it is thin and flexible in torsion, the aerodynamic twisting moments are large compared to the twisting moments due to centrifugal force. In operation, therefore, the outer or working portion of the blade usually has a higher pitch than when at rest, even though the blade as a whole has a tendency to turn in the hub in such a manner as to reduce the pitch. This increase in pitch of the working portion of the blade is as much as 1 or 2 deg. with engines of high power. It is greater for conditions of take-off and climb than for high speed and cruising flight, which is exactly the opposite of the variation desired in a controllable-pitch propeller. This increases the difficulty of designing a self-operating propeller which automatically deflects to give the proper pitch change, but gives no particular difficulty in connection with the mechanically controlled type, in which it merely makes it necessary to use a slightly greater pitch change.

Author's Closure

The discussion contributed by Mr. Weick is constructive and to the point. It displays the intimate familiarity with propeller theory and practice which he has attained through his extensive experience in laboratory and flight investigations.

Much of the work which Mr. Caldwell has done in propeller development is widely known and acknowledged. As he points out, in 1923 he followed Drzwiecke with a paper which developed the centrifugal torque upon a single-blade section. In April, 1929, Messrs. Wood and Perring published another analysis of the same problem, in R. and M., No. 1274 (London). Four months later Mr. Caldwell included a repetition of his previous study as part of a paper entitled "Variable-Pitch Propellers," presented in 1929 before the Cleveland Aeronautic Meeting of S.A.E. In none of these earlier works was the analysis carried further than a single-blade section, and in applying their results it has always been necessary to perform a series of graphical computations to arrive at a torque-angle curve for an entire blade. Mr. Caldwell published such laboriously calculated curves in N.A.C.A. Technical Note No. 201 (1924) and in the foregoing paper. It might be suspected from both curves that their author did not recognize the centrifugal torque upon an entire propeller blade as a sine curve of 90-deg. wave length.

The differences between these curves and correct ones are of negligible numerical importance. The author wishes to point out, however, that they could have been computed more accurately with far less work by means of Equation [13] in the paper. In addition, this equation presents a rigorous analytical relation for the centrifugal torque on an entire blade, which is useful in further studies of propeller problems.

Such an application of the equation has been demonstrated in Appendix 1, where the torque relations of counterbalancing devices are analyzed. It is of course generally recognized that such devices are old in conception and use, Mr. Caldwell and his assistants having worked with them as early as 1920. The author has not located any analytical studies of their action in the literature, however, and it is probable that designs have heretofore been made upon elementary and trial-and-error bases.

The method of calculating the bending stresses in propeller blades referred to by Mr. Caldwell was published in 1924 in N.A.C.A. Technical Note No. 201, pp. 6, 7. In this study the resisting moment of the blade structure was completely neglected, and an attempt was made only to balance the bending moments from thrust and centrifugal force against each other. There was used for this purpose an assumption (p. 7, lines 11–13) which is erroneous in that it holds true only for a loaded string with zero moment of inertia which is given a straight-line deflection. An actual deflected propeller blade does not approach this condition.

"A rough approximation of the elastic stress" obtained on the basis of the foregoing assumption is too low, and since the error in the assumption depends upon the material used, the method cannot safely be used as a "means of comparison of propellers of different materials."

The author's paper deals with the problem in a more directly analytical manner, and it takes into account the simultaneous effects of thrust and centrifugal bending moments, as well as the internal resisting moment of the blade itself.

The particular propeller blade taken for the numerical examples was similar in plan form to the Navy No. 4350, but of hollow steel construction instead of solid aluminum alloy. In order to test the equations presented by application to a well-known standard blade, the centrifugal torque Equation [13] has recently been worked out for a Navy propeller No. 3792, 10-ft. diameter. The torque on this blade is expressed by an equation similar in form to Equation [17] of the paper:

$$Q_c(\theta) = 3333 \sin 2(\theta + 9.95 \text{ deg.}) \text{ lb-in. at 2000 r.p.m.}$$

Since the angle at which the torque becomes zero is —9.95 deg. as compared with —9.875 deg., the only practical difference in these equations is that the centrifugal torque in blade No. 3792 is 1.7 times that in the hollow steel blade previously calculated.

The time required for this additional calculation was about two hours for one man with a sliderule and a planimeter. A planimeter is convenient, but not essential.

The final four paragraphs of Mr. Caldwell's discussion are somewhat irrelevant in that they deal with problems which, although of recognized importance, have not been taken up in the present paper.

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AER-54-20

Airplane Landing Gears

By FREDERICK KNACK,[1] MOLINE, ILL.

The landing gear is an important part of an airplane, for although failure may not be as dangerous to life as failure of another part, it is bound to be costly. Five things to look for in a landing gear are strength, satisfactory shock absorption, low drag, light weight, and stable taxying characteristics, and these are not necessarily antagonistic one to another nor difficult of attainment. Landing-gear failures are seemingly more frequent than other failures in spite of high load factors, possibly because taxying loads are neglected in specifying design loads. Small planes are often not able to make use of a retractable chassis, and the one-wheel chassis and tandem-wheel truck are suggested as possible methods of reducing chassis drag. Mention is made of the evolution of the Monocoupe landing gear toward decreased weight and resistance and greater dependability.

THE principal components of the airplane are five: the wings, fuselage, tail surfaces, power plant, and chassis. And the last of these five, the chassis, comprising landing wheels and tail skid, the only one of the five that does not perform a useful function when the airplane is in flight, is the one perhaps most likely to give trouble. The landing gear, used for only a few minutes at the start and at the end of each flight, a hindrance rather than a help to the performance of the airplane in its natural element, is a potent source of trouble, as most pilots and maintenance men will attest. Its duty is an important one, and while failure of the gear may not be as likely to result in serious injury or death to the occupants of an aircraft as would failure of the other principal parts, it may so result; and in any case expensive repairs, not only to the gear itself, but to the rest of the vehicle, are almost certain to be necessary. So it is well for designers to give considerable attention to this adjunct of the airplane, and indeed much that is worth while has been done in the way of improvement, both in aerodynamic and in strength properties.

The purpose of a landing gear is twofold. First, it functions to permit the transition from ground to air and from air to ground, and, second, it enables the aircraft to maneuver on the ground. Since it cannot aid the flight of a plane, the gear should be so made as to hinder it as little as possible; the wind resistance must be a minimum. To sum up, the characteristics to be sought in a good landing gear are:

(1) Strength
(2) Satisfactory shock absorption
(3) Low drag
(4) Light weight
(5) Stable taxying characteristics.

Adequate strength is necessarily a prime requisite of the landing gear and need not be difficult of achievement if the loads it will be called upon to resist be determined with fair accuracy. Current practice is to design for the loads specified by the Department of Commerce for the conditions of level landing, with and without side load, and three-point landing. In level landing the thrust line is assumed horizontal, and the ground reaction applied at the center of the wheel has a vertical component equal to the weight of the airplane (less chassis) multiplied by the required load factor, and a horizontal or drag component of not more than 25 per cent of this value, depending on the position of the wheels with respect to the center of gravity of the airplane. In the other principal loading condition, three-point landing, the vertical reaction is the same as in the first condition, but no drag component is specified. It is a question, however, whether these loading conditions cover the most severe stresses imposed on the undercarriage. True, they would seem to cover all possible attitudes of the airplane (an auxiliary loading condition is provided to take care of a one-wheel or side-slip landing), and the load factor would seem to be high enough considering that except in most unusual circumstances the airplane makes contact with the ground at such a speed and attitude that most of its weight is carried by the wings. Yet it is a fact that landing gears that show ample margins of safety when analyzed for the foregoing conditions frequently fail in ordinary use. Searching for a cause, one is struck by the thought that very likely the gears suffer much more by striking obstructions in the field during the landing or take-off run or while taxying than at the instant of landing. The obstruction might be nothing more than just the surface roughness of a poor field or a small ditch, a stone, or something of the sort. If this be the case, then it is apparent that the rules need revision, and a larger value of the drag component is indicated. Perhaps if a drag component were added to the three-point landing reaction, sufficient to cause the resultant to pass through the center of gravity, with the tail skid assumed to be just clear of the ground, this might more truly represent actual conditions. Or another loading condition might be worked out to care for the situation. Better landing fields will of course bring their own cure.

If the loads acting on the chassis are known, it is simple enough to design the members to have sufficient strength, but if there is an element of doubt it is well to provide against it by some such provision as pinning the landing gear to the fuselage structure through universal joints. Then in the event of a failure, the structure would fold up or be wiped off without, probably, a great deal of damage to the plane. If universal joints are not used, failure of a member of the gear usually means that fittings will be twisted and torn and fuselage members also twisted and bent, resulting in a sizable repair bill.

In the ideal gear, all members carrying load would meet in a point, and that point would coincide with the point at which the external load is applied. But such an ideal condition does not exist although it may be approached. In the gear shown in Fig. 1, the struts are so arranged that only the straight lower portion of the axle carries any bending. Consider the effect of the

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[1] Chief Engineer, The Monocoupe Corporation. The author was born on April 11, 1906, in New York City. He was educated in the public schools and entered New York University in 1922, receiving the degree of B.S. in M.E. in 1926 and the degree of A.E. in 1928. Was instructor, 1926 and 1927, in the Daniel Guggenheim School of Aeronautics and operated the wind tunnel there. Joined Aerotech, Inc., consulting aeronautical engineers (Streator and Moline, Ill.), in 1927. Took his present position in 1930.

Contributed by the Aeronautic Division and presented at the Kansas City Meeting, Kansas City, Mo., Sept. 7 to 9, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

load component V in front elevation. If the oleo leg and axle meet in a point which is on the line of action of V, then obviously the loads induced in them will be purely axial except in the lower part of the axle, which must carry bending as well. Similarly, in plan view, the struts which resist the drag component D will have only axial loads produced in them, if, as shown, they intersect on the line of action of the external force. These two, V and D, are the principal loads to consider. There is also the side component H, which acts at the ground. Obviously the gear could not be designed to bring the intersection of the oleo and axle to coincide with the point of contact of wheel and ground, as this would result in a long lower axle having an unsightly appearance, and would move the oleo fitting so far from the center of the wheel as to cause excessive bending moments under the vertical and drag components, V and D. However, the load factor for this condition is only half as great as in the normal landing, so the stresses due to eccentricity of the side load should not prove critical for the structure. The brake load, shown in side elevation, must also produce bending, which will be resisted by the axial load in the rear strut, which intersects the axle below the center line, and being in compression due to the other loads acting in braked landing, sets up a counter torque. In this particular landing gear, the torque may be resisted in another way, so as not to stress the axle and drag strut.

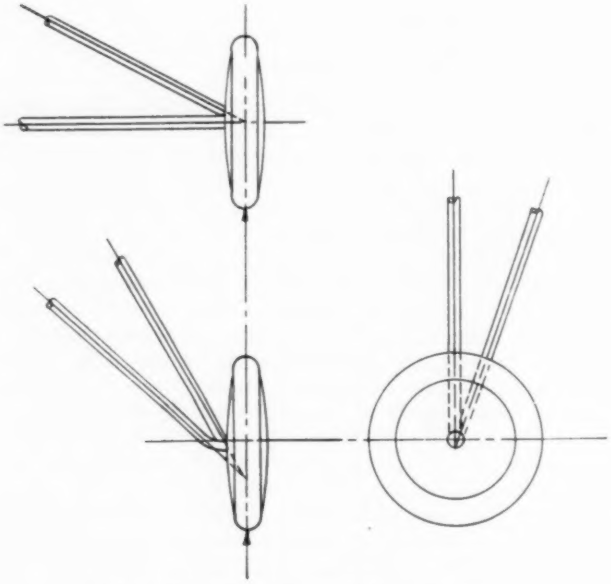

FIG. 1 IDEAL STRUCTURAL ARRANGEMENT

Heat treatment plays an important part in the building of the landing gear. The usual heat treatment leaves the metal, 4130 chrome-molybdenum steel, with an ultimate tensile strength of 180,000 lb. per sq. in. In this condition the material is tough and hard, but not brittle. Care must be exercised in the formation of pieces to be heat treated. In one instance it was found that the weld between an 18-gage tube and a 13-gage tube often cracked after heating, although the wall thickness of the one was only twice as great as the other, a condition well within the bounds of ordinary welding limits. It was necessary to substitute 16-gage material in the lighter member to cure the trouble. In another instance a 1025-steel strap was welded over the upper end of an axle as part of the fitting, and after heat treatment cracks often developed in the axle just below the strap. A cure was effected by using a 4130-steel strap and so placing it on the axle that the weld did not occur at the same place on both sides (Fig. 2).

A second requirement of the Department of Commerce is that the shock-absorbing device must absorb the energy of a free fall of 18 in. without subjecting the structure to loads great enough to cause failure. This seems a reasonable value and is undoubtedly a greater test than the absorber will be asked to pass in service. The passing of the old straight-axle type of landing gear was closely followed by the elimination of rubber cord as the shock-absorbing medium. Cord was objectionable for many reasons, such as the fact that its properties varied with age, that it deteriorated rapidly, and was not smooth acting. Practically all planes are now equipped with either oildraulic or pneumatic shock struts, which take the impact on oil and air. Both types operate satisfactorily, although the former may frequently break the rebound spring and the other requires attention to the air pressure, which must be kept at the specified figure and at the same value for both struts.

Another function which the shock strut is called upon to perform in certain types of undercarriage is to resist the brake loads. In the gear shown in Fig. 3 it is a question as to whether, when the brakes are applied, the resulting torque is taken through the axle or vee struts or by the oil strut. The oil strut will do the trick providing there is no opportunity for play in the fitting at the wheel. This assumption is borne out by the fact of breakage on many Monocoupes of the $^5/_{16}$-in. nickel-steel bolt in this fitting. This particular bolt had ample strength except in the condition where brake torque is assumed to be resisted by the shock strut, where the margin of safety was small; so that it seems fairly evident that breakage was due to that cause, especially as the bolt in the upper end of the strut never failed. The breakage was not always immediately evident, and it happened occasionally that a bolt, previously broken but unnoticed, permitted the wheel to drop down while the plane was in flight, resulting in a dangerous condition. To overcome this, a piece of $^5/_8$-in. outside diameter 16-gage tubing was welded to the lower end of the shock strut, and

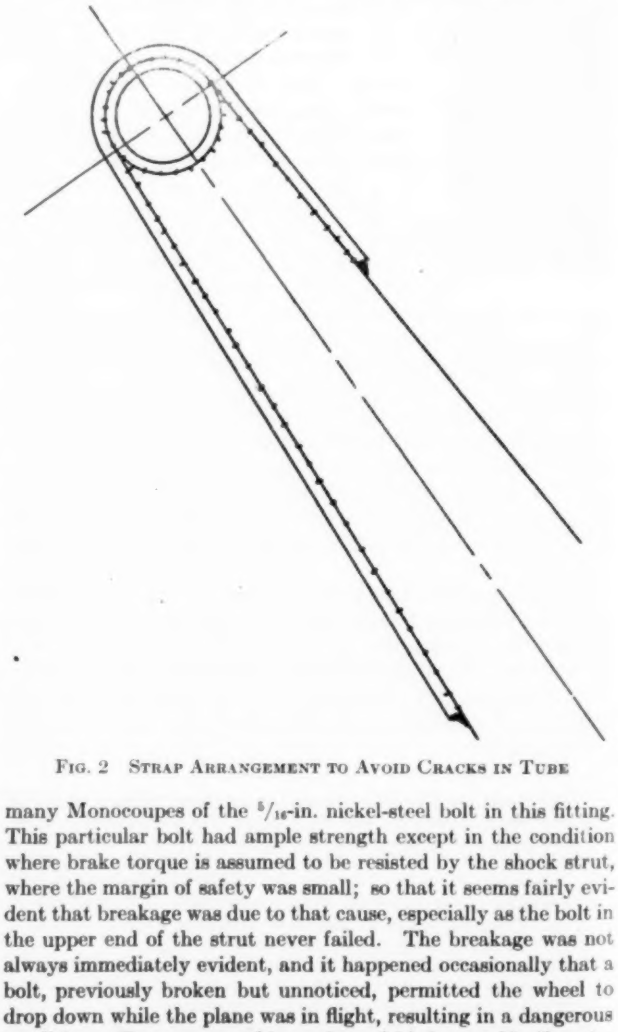

FIG. 2 STRAP ARRANGEMENT TO AVOID CRACKS IN TUBE

a $^3/_{16}$-in. bolt was passed through it and the fitting (Fig. 4). This bolt is normally never in contact with the strut and interferes with it in no way; but should the main bolt fail, it will prevent the shock strut from separating from the rest of the gear, and permit a landing, the landing loads passing directly from the shoulder of the shock strut to the fitting and not through the $^3/_{16}$-in. bolt. In fact, this condition (of a broken main bolt) might exist indefinitely and the plane be flown with safety, but it is almost certain that the pilot would be warned by the rattling of the gear and would effect the necessary repairs. The $^5/_{16}$-in. bolts were of course replaced by $^3/_8$ after proving too light in service, and since making the change no failures have been reported, but the safety-bolt feature is incorporated in all landing gears and is recommended as a simple precaution against failure and damage to the plane in an instance of this kind.

Sought as much as strength in a gear is low wind resistance, and the two factors are not necessarily antagonistic. The strong gear will probably have short members, for one thing. Earlier the point was brought out that curving the lower end of the axle reduced the bending loads in the members and resulted in a stronger gear. But this is not the only advantage gained, as it is evident that by so doing the interference between axle and shock strut is lessened and the drag lowered, thus adding an aerodynamic to a structural reason for adopting this procedure where the design is such that it can be used. Recently wheel streamlines, known as wheel pants, have come into considerable favor. There is no question that well-designed pants improve the performance and appearance of an airplane, but they also introduce another problem. They add considerable keel area forward of the center of gravity and thereby disturb the directional stability of the airplane. On small planes, at least, it is enough to place the airplane, normally having satisfactory stability, in a condition of rather poor stability in yaw, having in this respect almost as great an adverse effect as the addition of pontoons. Greater fin area seems to be indicated in such cases.

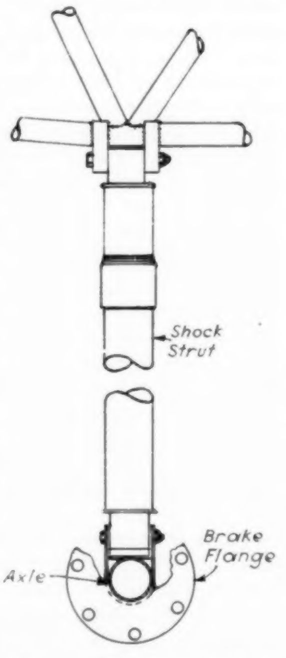

Fig. 3 Shock Strut Resisting Brake Torque

Retractable landing gears are receiving much attention, as they should, because by this means the ideal of no landing-gear drag at all is achieved. But however desirable it may seem, this solution of the problem is not practical for small planes, particularly if they are not of the low-wing cantilever type, which affords a fairly large space for the folded wheels. As this paper is written, two promising solutions to the small-plane problem are under investigation. The first is a one-wheel landing gear. This sounds fantastic, but is not necessarily so; a gear of this type was used with fair success a year ago on a plane entered by Hosler in the Cirrus Derby. O. A. Wheelon, of Curtiss-Wright, has been working along these lines, and the following paragraphs are taken from his notes.

They are the result of a series of model tests and calculations on the directional stability of a monowheel chassis, and although the investigation is not complete, the results are encouraging.

The monowheel chassis is essentially a central wheel conventionally located slightly ahead of the center of gravity. Directional control is maintained by a steerable tail wheel, a very important feature. Tip skids are used to prevent damage to the wing tips and to maintain a nearly level keel in taxying.

Reasons for its use are simplicity of structure in the case of low-powered planes or amphibions, and when used with retracting gear in a high-performance biplane exceptional speed characteristics would be obtained.

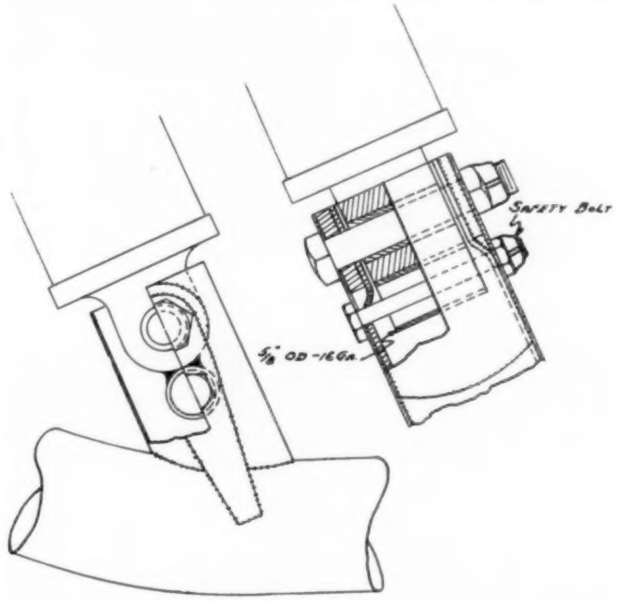

Fig. 4 Safety Bolt Arrangement

In Germany, as well as in this country, single-wheel and single-skid gliders have met with greater success than two-wheel or two-skid layouts. Some have explained this by the instantaneous sense of lateral balance before take-off and follow-through on landing. The mental hazard perhaps offers the greatest opposition. Most pilots believe they would have no difficulty with a single-wheel job but have horrors of a "dub" pilot trying it. Perhaps if we could eliminate the dub pilot from the "hot" plane classification and settle him with the low-powered category whose characteristics are so similar to the gliders that his still-less-experienced friends fly every day with one wheel and no difficulty, our viewpoint will change.

The degree of directional stability considered from ground reactions alone depends upon three factors. First, the fore-and-aft location of the center of gravity; that is, the more weight carried on the tail wheel, the greater degree of stability. The second is the tip angle, or the angle of clearance between the tip and the ground; the smaller this angle, the greater the degree of stability. The third factor is the height of the center of gravity above the ground; the lower the center of gravity, the greater the degree of stability and of course the lighter the tip loads. The last two are very closely connected, and might be jointly considered as the percentage of the weight carried at the tip when down.

In all cases the system is considered stable when the yawing moment capable of being developed by the tail wheel exceeds that present due to the yawing moment of the tip skid.

The foregoing considerations neglect all air controls which in themselves should be sufficient for control.

Other factors governing the characteristics are the relative amount of fin area fore and aft of the center of gravity and the proportions of control surfaces. The ground angle should be less than the burble point to facilitate take-off with the tail down.

The degree of stability desirable in any plane is best determined by varying the fore-and-aft location of the front wheel, keeping the tip angle as small as possible and still not having both tips drag. Theoretical considerations show that it is possible to develop tip load factors several times greater than that on the tail and still to maintain directional control.

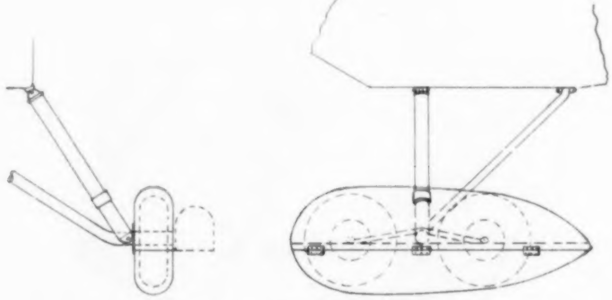

FIG. 5 TANDEM WHEEL GEAR

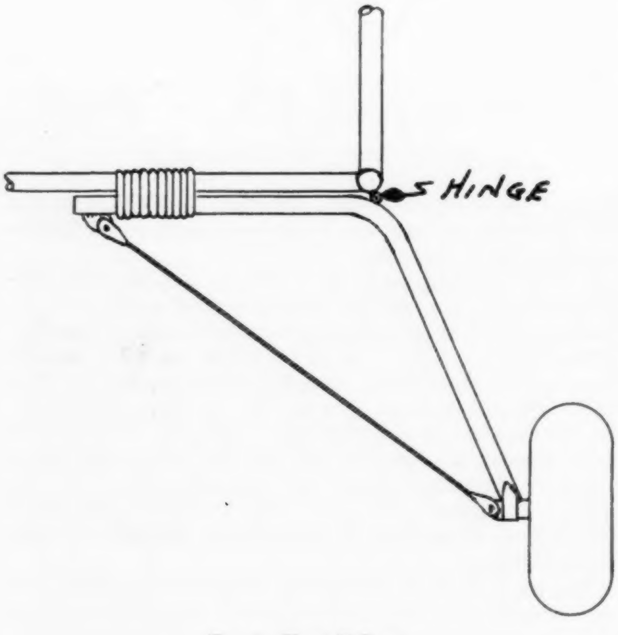

FIG. 6 TYPE F GEAR

The only time when the plane is directionally unstable is in the case of a tail-high landing with a tip down, and at such an attitude the rudder should still be effective. The duration of bumps is so short that it seems unlikely that angular acceleration could take place before restoring conditions are present.

From a common-sense viewpoint it would seem desirable to use an oversize unsprung air wheel with moderate wing loadings. In higher performance ships it would seem highly desirable to have a long throw oleo and taxi on the air wheel.

Structurally the arrangement should prove less likely to encounter failure as is so evident in narrow-tread gears at present where the entire load factor is at times developed on one wheel, causing failure which need not be anticipated with a one-wheel design. It has the added attractiveness of being less expensive than a two-wheel chassis, and even on a small high-wing monoplane it is probable that space could be found in which to stow the retracted single wheel.

The other gear which is under test is of a more practical nature (Fig. 5). It consists of two small wheels mounted in tandem on a truck which is slid on the axle in place of the conventional wheel, and can be used on any size airplane, although naturally it is more useful on planes to which a retractable undercarriage cannot be adapted. The whole thing can be enclosed by pants which, because of the size of the wheels relative to the size of a single wheel to carry the same load, will have only about one-quarter the frontal area of pants for the single wheel, with an even greater reduction in keel area. Because the pants are so small, it is possible to so arrange that in flight the bottom of the wheels, as well as the top, may be covered, still further decreasing the drag. The frame or truck is free to move on the axle through a limited angle, this being necessary so that in taxying over rough ground the wheels are enabled to climb over obstacles without disturbing the position of the fuselage by causing it to pitch. The pants for this tandem truck are made in two halves divided on a plane which is substantially parallel to the ground and slightly below the center of the wheels. The upper half is attached rigidly to the frame, and the lower is hinged to it along one side. While the airplane is in flight, the case entirely surrounds the wheels. When the airplane is ready to land, the pilot by a suitable mechanism causes the lower half to open up, exposing the lower part of the wheels, and permitting the plane to land, and to take-off or taxi on the ground. Some of the advantages of this gear are (1) lighter weight, (2) less drag, (3) lessened tendency to nose over when brakes are applied, (4) softer taxying, as in moving over rough ground only half the vertical displacement of each wheel is transmitted to the fuselage.

As this paper is written, the tandem-wheel arrangement has not yet been tried on an airplane, but it is expected that by the time of the meeting data on the flight tests will be available.

The airplane spends as little time as possible on the ground, but it should be able to move about on it easily at the pilot's will and not its own caprice, so that attention must be given to the taxying qualities of a landing gear. The shock absorber must of course be able to level out the inequalities of the surface so that riding will not be unpleasant, but more than that it must be amenable to guidance. The tread must not be so wide nor the wheels so far forward as to make it possible for ground looping to occur, nor should the tread be so narrow that the plane may tip over, nor the wheels so far back that it is in danger of nosing over. Brakes now so generally used complicate the problem. A delicate adjustment of the position of the wheels must be made to place them at just the point where the plane will not nose over when brakes are applied or ground loop, and the allowed variation is not great. Experience seems to show that the angle between a perpendicular to the ground and a line passed through the center of gravity and the axle, with thrust line horizontal, should be greater than 14 deg. but less than 17 deg.

If the shock strut is so placed that, as it shortens under the load, it permits the wheel to move forward, an unstable condition exists. Should the plane start to ground loop to the right, for example, the greater load is thrown on the left wheel, which then moves forward with respect to the right, and the tendency is for a still sharper turn in the same direction. It is better that the wheels move straight out from the fuselage as the struts deflect, although perhaps a slight rearward movement would not be undesirable.

It may be of interest to consider the evolution of the landing gear on one plane, the Monocoupe. This, a two-place, high-wing monoplane, has not changed greatly through the years, except to be constantly refined. The first model, built in 1926, had the conventional straight-axle landing gear common at that time, with rubber-cord shock absorbers. The next year, following the trend away from the straight axle, which often fouled grass and obstructions, a switch was made to a divided-axle type, with the shock strut running to the top longeron and a vee strut to the center of the fuselage. Rubber cord was retained as the shock-absorbing material. This gear, adopted in deference to the agitation against the straight axle, has disadvantages of its own, such as increased weight and an increased drag, which resulted in a top speed lowered by, roughly, 3 miles an hour. Shortly the rubber-cord shock strut was abandoned in favor of an oildraulic strut, which measurably improved the performance of the gear, but did not help the weight and drag factors any.

In 1929 this gear was modified by running the shock strut to the lower longeron, instead of to the upper, with a great reduction in weight and a return of the wind resistance to about the same figure as for the straight axle. This gear was first tried out and developed on a racing-model Monocoupe. The principal source of trouble was the spring in the oildraulic strut; if too flexible, it compressed too easily in taxying, and if the loads on the wheels were slightly unbalanced, it encouraged ground looping by letting one side of the plane drop much lower than the other; if too stiff, taxying was very rough. In either case, it was common to have the spring break after short usage, and often this resulted in a failure of some other part of the landing gear. As the spring comes into play in taxying, it serves to bear out the contention made earlier that taxying is more severe on the chassis than landing. The latest development in the Monocoupe gear is one in which a tierod replaces one of the tubular members, and the other two are narrowly spaced and may be faired together (Fig. 6). This has resulted in an improvement in top speed of 5 miles per hour over the previous gear. The structure is cantilever in a sense.

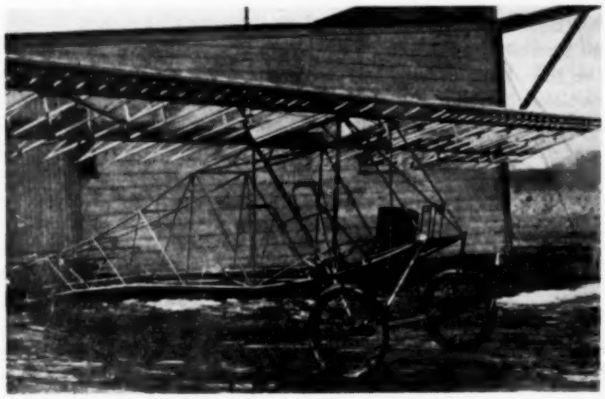

FIG. 7 AIRPLANE LANDING GEAR—STRAIGHT AXLE CHASSIS

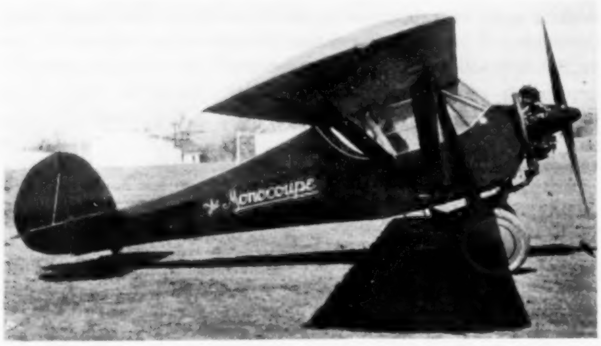

FIG. 8 AIRPLANE LANDING GEAR—DIVIDED AXLE, LONG STRUT

The axle is a continuous member, running from the wheel to a hinged fitting on the lower longeron and thence horizontally into the center of the fuselage. This horizontal portion is wrapped with shock cord, thus returning to one of the earlier evils, but the advantages gained are felt to overcome this disadvantage. To relieve the axle of the enormous bending loads it would carry as a full cantilever, its two extremities are joined by a streamlined tierod. The drag component of the load is taken by the strut which lies in the plane of the vertical portion of the axle. This landing gear, while only recently tried out, has so far behaved excellently. At present optional equipment, it may soon be made standard. It is both cheaper and lighter than the old gear.

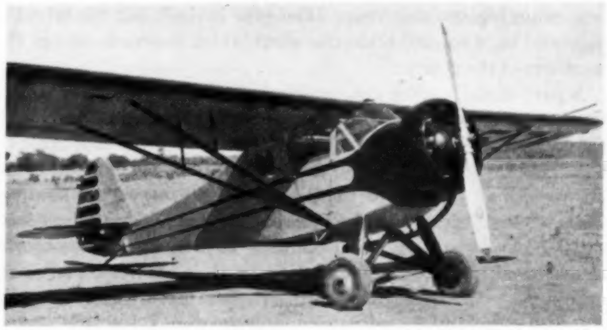

FIG. 9 AIRPLANE LANDING GEAR—DIVIDED AXLE, SHORT STRUT

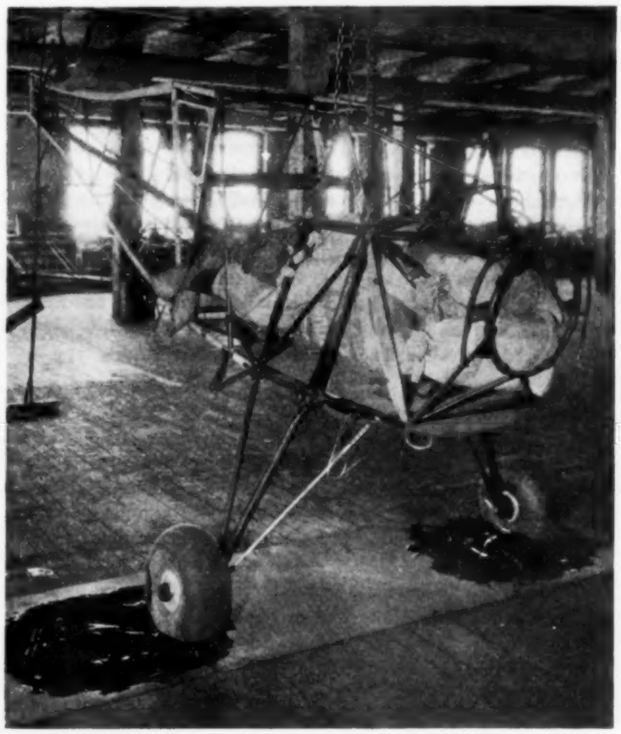

FIG. 10 AIRPLANE LANDING GEAR—DIVIDED AXLE, NEWEST GEAR

Many manufacturers are working toward the improvement of the landing gear. When balloon wheels were introduced by Goodyear a few years back, it was thought they would make it possible to eliminate other shock absorbers and permit the use of a rigid gear. Such hopes proved vain, for experiments have proved that a shock unit in addition to the wheels is desirable and that a combination of balloon wheel and shock absorber gives excellent performance under all conditions, although we prefer the smaller and lighter semi-balloon to the full balloon tires. Even so, the rigid chassis is not an impossibility, as Waco demonstrated on its new cabin biplane this year. The gear is rigid, but the wheel is mounted on a splined oildraulic strut which moves in one of the members of the gear.

A part of the landing gear which does not always receive the attention it deserves is the tail skid. Its design depends largely upon whether or not brakes are used. If not, then the skid must be a rigid affair or the plane will prove uncontrollable in a wind, and no wheel may be used or the plane will roll forever on landing. Many types of skids have been used, but the leaf-spring type is as satisfactory as any on small planes. It is inexpensive, absorbs shocks well, requires no attention, and is easily replaced, if necessary. The question of a suitable shoe is more difficult. No material seems able to withstand long the abrasive action of concrete runways, or even cinder or gravel. It may be Stellited, but this process is far more expensive than the cost of the shoe itself. After tests on Monel metal and Illium, two alloys noted for their resistance to abrasion, it was found that ordinary malleable cast iron had greater resistance. Shoes of this material can be had for 6 cents apiece. Stelliting would add a dollar to their cost; so the policy of this company is to equip with the cast-iron shoes and make them cheap enough so that the owner can replace them as often as necessary.

On a plane equipped with brakes, a tail wheel is possible as well as desirable. One difficulty with wheels having a plain journal bearing is that dirt and grit quickly get in and wear the axle, or stop the wheel from turning, with the result that it is subjected to the same wear as a skid shoe. If the wheel is movable, it should of course be castered; but on small planes, even with the help of brakes, it is often difficult to taxi in a wind if the tail wheel is pivoted. A steerable wheel is much the more desirable type.

Discussion

WELLWOOD BEALL.[2] Landing-gear problems are a source of great annoyance to every engineer. Landing-gear failures are prevalent at every airport. Hence, as the author has suggested, engineers should study this problem more deeply.

Several points were mentioned that are very important. The center lines of all the struts in the landing gear should meet at a common point somewhere in the wheel. The lower this intersection is, or the closer it is to the ground, the better. The disadvantage of having this point close to the ground is the large bend that is required in the axle. However, a compromise can be made. The importance of this consideration is little appreciated by many designers, but if many of the present landing gears were redesigned at the lower end and the fitting was made so that the axes of the struts intersect at a common point, many of the troubles in the landing gear would be eliminated.

Since, as has been mentioned, the forces that tend to break down the landing gear are usually induced from taxying, rather than from landing impacts, the method of fabrication of the fittings and the struts must be carefully considered. In other words, fatigue set up by rough taxying must be compensated for in improved design; hence, the author's remarks concerning welding practice on the landing gear.

Another source of trouble in landing gears in which brakes are used is the method of absorbing brake torque. There have been several methods used to absorb this torque, but at the present time most installations take the torque through the pin joint to the bottom of the oleo strut and through the walls of the oleo strut. It has been found that oleo struts are much more effective when they do not have to carry this torque which induces a bending moment in the strut. One method of taking this brake torque was used on a Ford tri-motor, where the torque was taken through an auxiliary strut attached to the brake disk and to a point above the oleo strut. This, however, was not entirely satisfactory, as the action of the oleo tended to rotate the brake around the axle, thus giving uneven deceleration. This sensation was rather uncomfortable. On the later Fords, this long auxiliary strut has been connected to the bottom, or the unsprung, portion of the oleo, but the disadvantage of this is that they are carrying bending moment through the oleo gear.

The safety bolt used in the Monocoupe landing gear, as described, is ingenious. However, it is the writer's opinion that the landing gear described in Fig. 6 will not prove as satisfactory as is expected. The reasons for this are that the rubber shock-absorbing unit has never proved itself satisfactory and that the landing gear as designed will carry considerable shock to the fuselage through the hinge point. This shock is carried to the fuselage from the wheel directly up the strut and through the hinge joint, since the design is of semi-cantilever construction. This type of landing gear and spring suspension has been tried many times before, but it has been found that with the shock-absorbing unit on the opposite side of the hinge point from where the shock is applied, a considerable amount of the shock is transmitted to the hinge point without ever reaching the shock-absorbing unit.

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AER-54-21

A Study of Light Signals in Aviation and Navigation

By I. LANGMUIR[1] and W. F. WESTENDORP,[2] SCHENECTADY, N. Y.

I. Langmuir

W. F. Westendorp

Laboratory experiments have been devised to make measurements of the visibility of light signals under conditions essentially similar to those encountered by the aviator or the navigator. Data have been collected on the direct visibility of flashing-point sources of light of different colors, flash lengths, and intervals, against different backgrounds. The time taken to locate a visible beacon was studied as a function of the beacon intensity and frequency of flashing.

The threshold candlepower required for visibility of a point source against a background is given by an empirical equation. Colored point sources were not found to be useful except in the case of red lights with background intensities above moonlight. For an airplane approaching a beacon it is advantageous to use frequencies of flashing as high as 12 to 30 per min., although with exceptionally clear atmosphere, lower frequencies may be better. In a study of the visibility of flashes of diffused light superimposed on a steady white background, white-light flashes gave the best results. A selective differential photoelectric receiver is described which detects signals of modulated diffused light of low intensity. This sensitivity is independent of the steady background brightness up to 100 times moonlight, and is from 6 to 13,000 times as great as that of the eye in the range of background intensity from darkness up to moonlight. The greatest difficulty in transmission of light signals through fog lies in the loss of advantages of the point source. Dense fog may increase the distances at which diffused-light signals may be detected. The range depends to a considerable extent on the reflectivity of the ground. A theoretical treatment of the diffusion of light through fog, based on the scattering of the light rays by fog particles, indicates that airplanes can be guided through fog at distances of several miles by means of diffused modulated light acting on a differential photoelectric receiver.

In all forms of traffic, whether it be navigation, automobiling, or simply walking, we depend on the use of our eyes; but most strikingly is this true in aviation. At present the pilot depends on direct vision for guiding his plane, and hence "visibility" is a vital factor in aviation. Methods have been developed recently by which "blind" flying is possible and practical for definite commercial routes. The extent to which these methods will prove useful cannot be predicted, but it is probable that light beacons and direct visibility will continue to be of great importance to the aviator.

Very little systematic experimental work seems to have been done previously on the visibility of light signals as used in aviation. The purpose of this paper is to present some results that have recently been obtained by the authors.

The most obvious method to obtain data on the visibility of light beacons is to set up different types of beacons and to observe them from an airplane under various conditions. However, this method lacks a fundamental quality of most scientific investigation—that of arbitrary control of all conditions; the weather would be different on different nights, and furthermore, even during a single night the background illumination due to moonlight or starlight would vary.

The plan was therefore adopted of making measurements on visibility in a dark room in the laboratory where there is complete control over the illumination of the background, the location,

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Dr. Langmuir has been the recipient of distinguished honors in the fields of chemistry, physics, and engineering. Among the awards made to him have been the Nichols medal, given by the N. Y. Section of the American Chemical Society and awarded to him in 1915 and again in 1920; the Hughes medal, awarded by the Royal Society of London, 1918; the Rumford medal by the American Academy of Arts and Sciences, 1921; the Cannizzaro prize, awarded by the Royal National Academy of the Lincei, Rome, Italy, 1925; the Perkin medal by the American Section of the Society of Chemical Industry, January, 1928; the School of Mines medal, Columbia University, 1929; the Chandler medal, 1929; and the Willard Gibbs medal, in May, 1930.

Dr. Langmuir has received the following honorary degrees: University of Edinburgh, LL.D., 1921; Northwestern University, D.Sc., 1921; Union College, D.Sc., 1923; Columbia University, D.Sc., 1925; Kenyon College, D.Sc., 1927; Technische Hochschule, Berlin-Charlottenburg, D.Ing., 1929.

[2] Research Laboratory, General Electric Company. Mr. Westendorp was born in Amsterdam, Holland, in 1905. He was graduated from the University of Delft in 1928, coming to the United States in July of that same year. He became associated with the General Electric Company in the radio engineering department, and is a graduate of the "Advanced Course in Engineering" of that company. In September, 1929, Mr. Westendorp was transferred to the Research Laboratory.

Contributed by the Aeronautic Division, cooperating with the American Institute of Physics, and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

the color, and the flashes of the beacon or other light source. It has been the aim to duplicate as far as possible in the laboratory the conditions of visibility encountered by the aviator or navigator, especially those characteristic of poor visibility.

In reasonably clear weather, when the direct rays from a beacon can reach the eyes of the aviator, he perceives the signal as a flash of light from a point source. The visibility of such point sources depends on many factors, such as the candlepower of the sources, the distance from the observer, the absorption or scattering of light by the intervening atmosphere, the color of the light, the diffused background illumination (starlight, moonlight, or sunlight), the duration and the frequency of the flashes, the presence of other disturbing point sources of light, and finally upon the qualities of the observer—his eyesight, his mental condition, and especially his attentiveness.

Under conditions of poor visibility, particularly those due to the presence of fog, the light from a beacon or other source may be scattered to such an extent that the direct rays are not visible, although diffused light from the source may still reach the aviator in perceptible amount.

The visibility of such diffused light signals depends primarily upon the brightness of illumination in the fog surrounding the aviator produced by the signal, the color of the light, the intensity of any steady illumination of the fog (starlight, moonlight, etc.), and the duration and frequency of the flashes of the signal.

It will be shown that the eye is relatively insensitive to diffused light as compared to that from point sources. On the other hand, it has been made possible by use of a photoelectric cell with amplifier equipment to detect diffused light many thousands of times too faint to see with the eye.

Any estimate of the distance to which diffused light signals of this kind can be transmitted involves a study of the laws of the scattering of light by fog. It will be shown that under certain conditions signals may be transmitted to relatively great distances through fog.

The aviator will be particularly interested in the time required to detect or locate a visible light signal. If a point source is of such low intensity (threshold intensity) that it is barely visible when pointed out to the observer, it takes an indefinitely long time for the observer to locate the source if it is in an unknown position in a large and uniformly illuminated background.

The threshold intensity for a flashing point source is that which is required to render the flash barely visible when occurring at regular intervals and in a known position. If the flashes are then made to come at irregular intervals or in unknown positions, a relatively long time may be required to become aware of them.

A useful signal to the aviator must be recognizable within a very short time, and must therefore be of an intensity appreciably greater than the threshold.

Visibility of Point Sources. When the light from a steady point source falls on the eye of the observer, it is focused by the lens of the eye upon a very small area of the retina. The amount of light so focused is proportional to the area of the pupil opening of the eye and to the light flux density F in lumens per square centimeter.

In clear air, without scattering or absorption,

$$F = C/D^2 \qquad [1]$$

where C is the candlepower of the source in the direction toward the observer and D is the distance from the source to the observer.

In the presence of haze or fog the direct light flux from a point source decreases exponentially with increasing distance, so that in general we have:

$$F = (C/D^2)e^{-D/\lambda} \qquad [2]$$

where $1/\lambda$ is the scattering or absorption coefficient characteristic of the fog or haze and λ may be looked upon as the mean free path of the light rays.

A discussion of the magnitude of λ under various conditions of haze, for light of different colors, has been given by F. Benford.[3] It is of the same order of magnitude as the distance termed "visibility" by the aviator, and thus except under conditions of fog is ordinarily to be measured in miles.

In their laboratory experiments the authors have not studied the effect of haze in decreasing the light flux received from a source. They have, however, determined under various conditions the visibility of light sources which produce known flux intensities F at the eye of the observer. These data are obviously applicable to the conditions encountered by the aviator if λ is known or if the exponent D/λ is small compared to unity—that is, if λ is large compared to the distance D.

The visibility of a point source depends to a very great extent upon the brightness of illumination of the background. The aviator may encounter background intensities which range from that produced by bright sunlight on snow down to that of starlight on a cloudy night with forest-covered ground.

When light from a point source, giving a flux intensity F in accord with Equation [1], falls perpendicularly upon a plane surface of a perfect diffuse reflector, the brilliancy of illumination H of the surface in candles per square centimeter is

$$H = F/\pi \qquad [3]$$

In Table 1 are given values of F and H characteristic of various sources. These light intensities[4] are those that would be observed at sea level when the point source is at the zenith of an exceptionally clear sky. Stars of the sixth magnitude corresponding to $F = 10^{-12}$ lumens per sq. cm. are the faintest that can ordinarily be seen with the unaided eye.

TABLE 1

(Light flux density F and brightness of illumination H produced by various sources)

Source	F (lumens per sq. cm.)	H (candles per sq. cm.)	Brightness of source (candles per sq. cm.)
Sun	10.4	3.3	150,000
Full moon	2.2×10^{-5}	7.1×10^{-6}	0.37
Venus (max. brightness)	1.1×10^{-8}	3.5×10^{-9}	..
1st-magnitude star	8.5×10^{-11}	2.7×10^{-11}	..
6th-magnitude star	8.5×10^{-13}	2.7×10^{-13}	..
Average starlight	1.8×10^{-7}	5.8×10^{-8}	5.8×10^{-9}
1 candle at 1 km	10^{-10}	3.2×10^{-11}	..

Effect of Flashing. If the light source consists of separate flashes of candlepower C, each of which lasts for a time t_o, the candlepower required for threshold visibility will in general be greater than that required for a steady light.

Blondel and Rey[5] have found that the threshold intensity F required for visibility of a light of duration t_o seconds is given by

$$Ft_o = F_\infty (t_o + 0.21) \qquad [4]$$

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[3] F. Benford, *Gen. Elec. Rev.*, vol. 29, p. 873 (1926).

[4] These values have been calculated from data given in Russel, Dugan, and Steward's book, "Astronomy," vol. 2, pp. 613, 625, 626, Ginn & Co. (1927). The value of H for starlight is based upon the statement that Rhijn found that the total light of the sky on a clear dark night is six times the light from all the stars and that the latter is equivalent to that of 1440 stars of the first magnitude. Thus the brightness H corresponds to that produced by 2160 first-magnitude stars at the zenith.

[5] A. Blondel and J. Rey, *Jour. de Phys. et de le Radium*, vol. 1, pp. 530–551 (1911).

where F_∞ is the threshold intensity for a steady light. Thus when a flash lasts several seconds, so that 0.21 is negligible, the threshold is the same as for a steady light, but for flashes of very short duration ($t_o \ll 0.2$) the visibility depends only upon the total quantity of light received—i.e., the product Ft_o, which must then be equal to 0.21 F_∞.

The aviator flying in clear weather in full moonlight is not in great need of light beacons as the natural illumination is sufficient to make landmarks discernible. For this reason the authors in their experiments confined the background brightness to an upper limit of full moonlight falling perpendicularly on a white diffusing surface, corresponding to a brightness of 7×10^{-6} cp. per sq. cm.

Experiments on the Visibility of Flashing Point Sources

The following arrangement was used. The observer was placed at distances ranging from 1.5 to 6 m. from a vertical white screen 1.5 m. by 1.5 m. in size, with its center about the height of the eye. Lamps, behind the observer and screened from his direct view, illuminated the screen so that its brightness could be varied from that of moonlight down to zero. These low brightnesses were measured indirectly by photometering the brightness of a similar surface at one-tenth the distance from the lamps and dividing the value by 100.

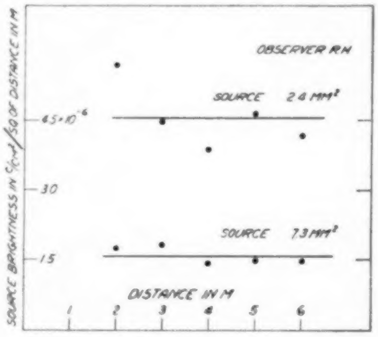

Fig. 1

(Visibility of small flashing illuminated areas as affected by their size and their distance from the observer. Screen is 1.5 m. square with steady illumination of moonlight brightness. The ordinates represent threshold values.)

In the screen there were 150 small round holes ranging from 2 mm.² to 66 mm.² area and one square hole of 1 sq. cm. area. The area of the latter could be varied by means of a plane diaphragm so that the opening always remained square and could easily be measured. All the holes were covered on the front by white paper 0.11 mm. thick, so that they were invisible unless illuminated from behind. When so illuminated by a small lamp, the surface brightness could be varied either by regulating the current or by changing the distance of the lamp. Flashing was produced by interrupting the current or by intercepting the light by means of a pendulum shutter. These illuminated spots of light served as point sources.

The threshold value of the intensity of the point source was determined by gradually decreasing the intensity until the observer lost track of the flashes. This method eliminates all uncertainty as to the location of the light source. In the experiments on threshold intensity, the observer remained in a dark room for a half hour for dark adaptation; then the flashing light was increased until he could easily see it, and the observer was asked to signal by tapping at every flash. By reducing the light until his signaling became irregular or stopped, the threshold was determined. The values given in Figs. 1 to 7 are the average of three, or more often of ten, observations. This "threshold" will later be used as a unit of light intensity in the determination of the time to locate a flashing light.

If an illuminated area is of sufficiently small size, its visibility will depend upon the total light from it—i.e., upon the product of its area and its surface brightness. If an area is so large that it is clearly perceived as a surface instead of a point, its effect depends rather upon the contrast in brightness between it and the surrounding surface. Whether an area is recognized as a point or as an extended surface depends upon the visual acuity, and this in turn varies with the brightness of the background illumination. With starlight intensity, or less, a relatively large area is equivalent in its effect to a point source. For an illuminated surface surrounded by a completely black background, Ricco's law states that the total light (area × brightness) determines the visibility if the angle subtended by the source is less than about 1 deg. With higher background brightness the total light will determine visibility only for areas which subtend much smaller angles.

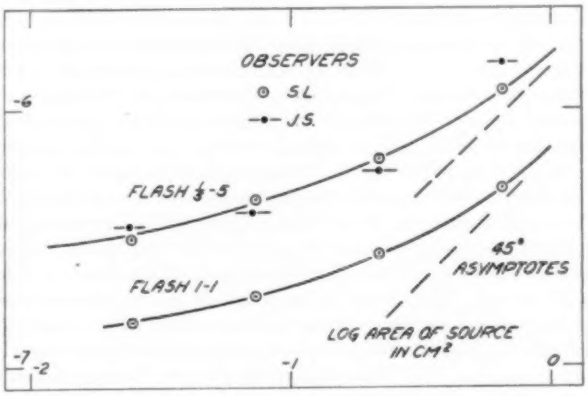

Fig. 2

(Effect of the size of the illuminated area and the duration of flashes on visibility. Screen of moonlight brightness at 1.5 m.)

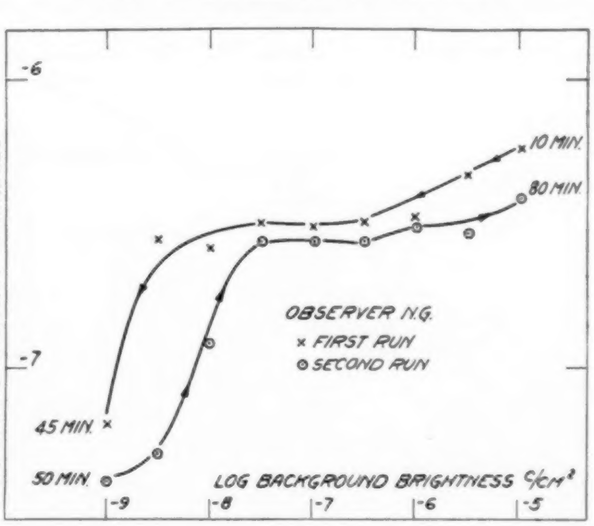

Fig. 3

(Effect of background brightness and adaptation on the visibility of flashing point sources. Flashes are 1 sec. on and 1 sec. off. Observer is 3 m. from screen. The arrows indicate the sequence of the observations.)

It was therefore of interest to determine in the experiments the conditions under which the illuminated areas were effectively point sources. It also was desired to know whether the apparent size of the large screen was of importance in determining the threshold intensity of the point sources.

Fig. 1 represents the results of a series of observations in which the brightness of the screen was kept constant at 8×10^{-6} cp. per sq. cm. and the distance of the observer from the screen was varied from 2 to 6 m., while the square point source was flashed for 1/5 sec. every 5 sec. By division of the surface brightness of the source at threshold intensity by the square of the distance to the observer, a value independent of the distance would be obtained, if the source is effectively a point source and if the screen area is without effect. This appears to be valid within a range of variation that can be accounted for by fatigue of the eye or other personal factors.

It also should be noted that when the area of the source is cut down to one-third, the surface brightness for threshold visibility must be increased threefold. A square source of 7.3 sq. mm. area at a distance of 2 m., which subtends an angle of 5 min. of arc, is small enough to act as a point source with a background of moonlight brightness.

In the experiment, which gave the data shown in Fig. 2, the observer was 1.50 m. from the screen, having moonlight brightness. The common logarithm of the threshold candlepower is plotted against the logarithm of the area of the source. The visibility of small sources depends only on the candlepower, but with large areas the threshold candlepower increases in proportion to the area, indicating that a constant contrast is required for threshold visibility. From the position of the 45-deg. asymptotes it is calculated that the additional brightness to make sources visible that have a subtended angle of more than 0.3 deg. is 20 per cent for the 1 sec. on and 1 sec. off flashes and 40 per cent for the 0.2-sec. flashes. Areas less than about 3 sq. mm. which subtend an angle of about 5 min. are approximately equivalent to point sources. In further experiments the illuminated holes having an area of 2.2 sq. mm. were used as point sources.

Effect of Background Brightness on the Visibility of Flashing Point Sources. Fig. 3 gives some data on the threshold values of the candlepower of flashing point sources as influenced by the brightness of the background illumination. Observations were started after only 10 min. for dark adaptation; after these the screen brightness was decreased, and again four observations were taken, etc. Every cross on the graph represents the average of four observations. After 50 min. the screen brightness was increased again, and after 80 min. the original brightness was reached. Apparently a continued dark adaptation stronger than any effects of fatigue must account for the lower values of the second series. Figs. 4 and 5 show curves for two observers at 3 and 6 m. distance and a 1 sec. on and 1 sec. off flash after half an hour for dark adaptation. The asymptotic values for zero background brightness are indicated by the arrows and are of the same order as those for 10^{-9} cp. per sq. cm.

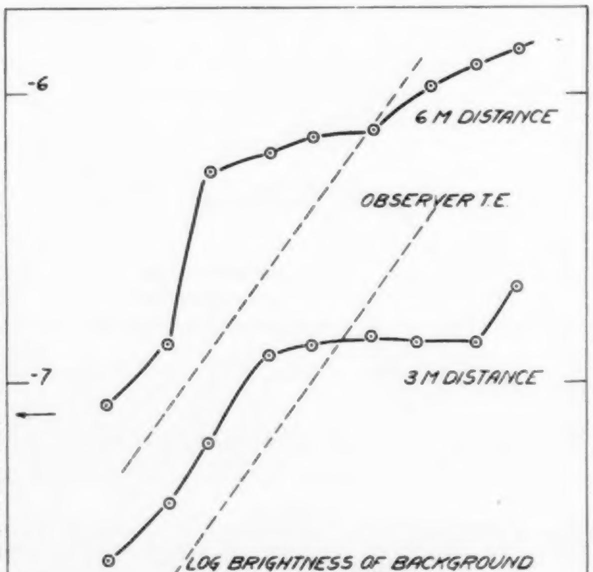

FIG. 4

(Effect of background brightness and distance on the visibility of flashing point sources. The broken lines correspond to Equation [5].)

The curves obtained by different observers differ considerably, but all agree in showing that the threshold value of candlepower increases much slower than in proportion to the background brightness. Starlight ($H = 6 \times 10^{-8}$) is sufficient to require an increase of fivefold to tenfold in the threshold candlepower of a point source.

It is interesting to compare these data with astronomical observations on the visibility of stars. Ordinarily a star of the sixth magnitude ($F = 8.5 \times 10^{-13}$ lumens per sq. cm.) is the faintest that is visible to the unaided eye against the background of starlit sky ($H = 6 \times 10^{-8}$). This corresponds to 7.6×10^{-8} cp. at a distance of 3 m. This point has been plotted in a circle in Fig. 5 and is seen to be in fairly good agreement with the observations of point sources at 3 m. distance.

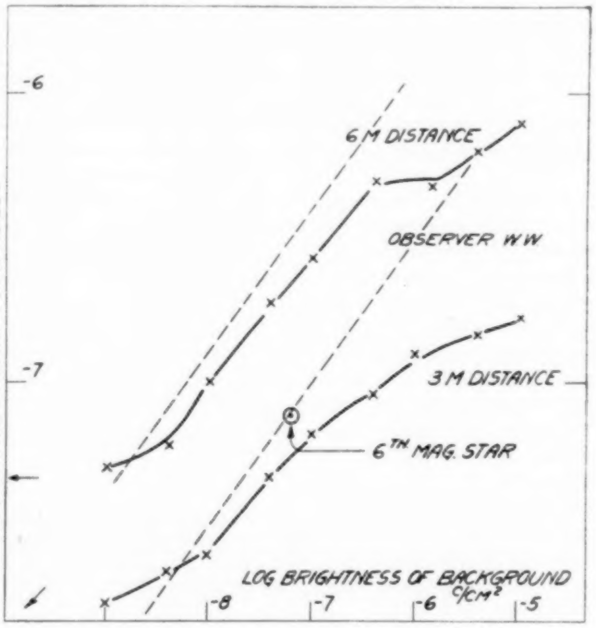

FIG. 5

(Effect of background brightness and distance on the visibility of flashing point sources. The broken lines correspond to Equation [5].)

Russell[6] finds, however, that stars of magnitude 8.5, for which $F = 8.5 \times 10^{-14}$, are recognizable if the light of the neighboring parts of the sky are screened off. This is in reasonable agreement with the values of F ranging from 1.3 to 2.3×10^{-13} obtained from the threshold candlepowers observed with zero background intensity (see arrows in Figs. 4 and 5) by dividing by D^2, D being the distance in centimeters.

Venus, when of maximum brightness (magnitude —4.3), can be seen very readily at noontime on a clear day if its exact location in the sky is pointed out, although once lost it may be difficult to find again. Its brightness, which corresponds to $F = 1.1 \times 10^{-6}$ lumens per sq. cm., is distinctly above the threshold value of visibility for a point source in a clear sky at noon which has a brightness of the order of $H = 1$ cp. per sq. cm.

The following empirical equation seems to be in reasonable agreement with these data:

$$F = C/D^2 = 3.5 \times 10^{-9} H^{1/2} \text{ lumen per sq. cm.} \ldots\ldots \quad [5]$$

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[6] H. N. Russell, *Astrophys. Jour.*, vol. 45, p. 60 (1917).

where F and C are threshold values for a steady point source in a background of brightness H, expressed in candles per square centimeter.

TABLE 2

(Comparison of observed and calculated threshold values for point sources with backgrounds of various brightness)

Object	$F_{obs.}$	H	$F_{cal.}$
8.5th-mag. star.....	8.5×10^{-14}	10^{-9} (darkness)	1.1×10^{-13}
6th-mag. star.......	8.5×10^{-13}	5.8×10^{-8} (starlight)	8.5×10^{-13}
4.1-mag. star.......	....	2×10^{-6} (moonlit sky)	5.0×10^{-12}
1st-mag. star.......	....	5.8×10^{-4} (twilight)	8.5×10^{-11}
Venus (max.).......	1.1×10^{-8}	1.0 (daylight)	3.5×10^{-9}

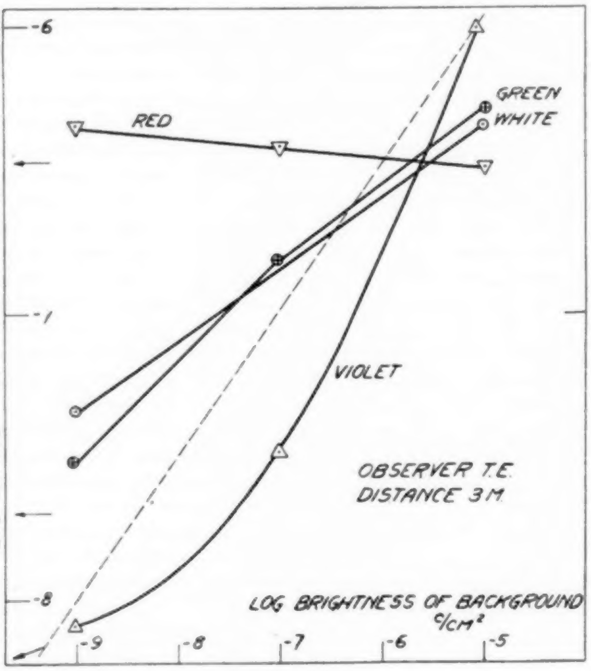

FIG. 6

(Visibility of colored flashing point sources as affected by the brightness of a background illuminated by white light. Flashes are 1 sec. on and 1 sec. off. Screen is at 3 m. The broken line corresponds to Equation [5].)

The second column of Table 2 gives the light flux density F from various heavenly bodies. The third column gives the background brightness H under which it is observed. The value of F calculated from H by Equation [5] is given in the last column.

The empirical square-root relation seems to give an approximately correct value for the visibility of steady light sources over an extremely wide range of values of H from 10^{-9} to 1.

According to Blondel and Rey's law, as given by Equation [4], the threshold brightness for flashes of 1 sec. duration is only 21 per cent greater than for a steady light. Thus the square-root relation found for steady sources should be roughly applicable to the experiments with 1-sec. flashes.

The broken lines in Figs. 4 and 5 give the threshold candlepower calculated by Equation [5]. These straight lines give values of F which agree with the observed within a factor of about 3. The discrepancies appear in large part to be individual characteristics of the observers.

In clear air the visibility of point sources is extremely high. Thus by Equation [4], or Table 2, at a distance of 10 km., a light of 0.11 candle is visible with a completely dark background, 0.85 candle with starlight, 5 candles with moonlight, 85 candles with twilight (stars just visible), and 8500 candles in daylight. These values are to be regarded as lower limits. For signaling purposes it will be shown that candlepowers about 30 times greater than these values should be used.

Colored Point Sources. Similar measurements were made with red, green, and violet sources. The color filters used were Wratten No. 70 red-pass beyond 6400Å, No. 74 green-pass from 5100 to 5700Å, and No. 76 violet-pass from 3100 to 4800Å. For calibration of the source the colors were matched with white in a photometer with an accuracy of about ±20 per cent. Fig. 6 shows the result of measurements at 3 m. distance with 1 and 1 sec. flashes. The visibility of red point sources is independent of the white background brightness; violet point sources are more than 100 times as visible against an absolutely dark background as against a moonlight background. The high visibility of violet at low intensities is of doubtful practical value, for it can be obtained only if the observer does not look straight at the source, the center of the retina being less sensitive for violet. Green light acts like white. The remarkable difference between red and violet is due to the Purkinje effect. It is accounted for by a shift in the visibility curve toward the blue end of the spectrum as the brightness of object and surroundings decreases. Houstoun[7] has plotted relative visibility against the logarithm of the wave length at high and low brightnesses and finds two curves with a shift of 480Å, closely resembling probability curves.

Time to Locate a Flashing Point Source of Light. In aviation, we are not interested in the intensity at which we lose sight of a beacon, but we do want to know when we can pick it up. Thus as a first requirement it may be stated that, in order to pick up a light, it must have an intensity above the one determined as threshold in the experiments. If this requirement is fulfilled, we can see the light, and next we want to know the length of time necessary to locate it.

To determine this experimentally, the authors used 30 holes in the screen distributed irregularly over an area of 50 cm. height and the full width of 150 cm., leaving 50 cm. above and 50 cm. below without holes; these blank areas correspond to the sky and the foreground. The flashing light (0.2 sec. on and 4.8 sec. off) could be placed behind any of the holes. This adjustment of the flashing source could be done behind the screen unseen by the observer who sat at 1.5 m. in front of the screen. The area of each of the holes was 2.24 sq. mm., and the brightness of the screen was moonlight. The measurement itself consisted merely in measuring the time from the instant the flasher circuit was

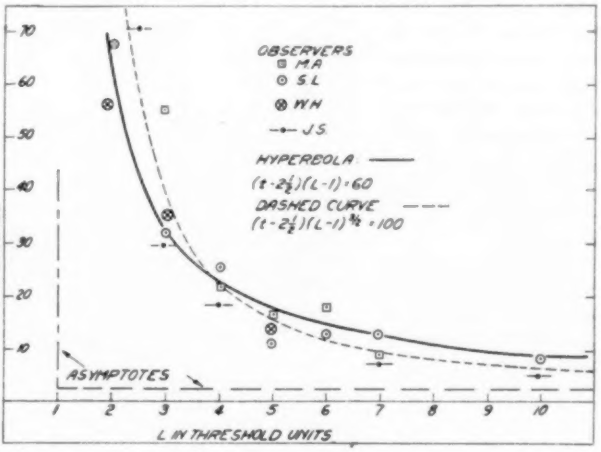

FIG. 7

(Time needed to locate a flashing point source in an unknown position within a field subtending a horizontal angle of 56 deg. and a vertical angle of 19 deg. of moonlight brightness, $H = 7 \times 10^{-6}$ cp. per sq. cm. The abscissas give the brightness of the source in threshold units. Flashes are 0.2 sec. on and 4.8 sec. off.)

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[7] R. A. Houstoun, *Phil. Mag.*, vol. 10, pp. 416–432 (1930); see especially p. 420.

closed and the observer was told to look for the light, until the instant he signaled that he had located the flash.

Fig. 7 shows the time as a function of the intensity of the flash expressed in threshold units, each point being an average of ten observations; the use of the threshold unit gives a common scale in spite of different observers having different threshold intensities. An interesting result is that a flash equal to ten times the threshold is seen practically always at the first flash, while it takes three flashes before the light is located if it is only five times the threshold. Naturally the threshold is an asymptote; so is also the 2.5-sec. line, since the average is half the time interval between flashes if the first flash is seen.

This suggests an hyperbola as an appropriate empirical equation. The full-line curve in Fig. 7 is a plot of the equation

$$(t - 0.5T)\ (L - 1) = 60$$

where T, the period of the flashes, is 5 sec. A better approximation is obtained by using an exponent 3/2 for the factor $L - 1$. The broken line curve also plotted in Fig. 7 has the equation

$$(t - 0.5T)\ (L - 1)^{3/2} = 100 \ldots\ldots\ldots\ldots [6]$$

The fact that it takes a relatively long time to find flashing point sources only a few times brighter than threshold proves that they are observed only by a searching process during which the eye scans the whole field in which the point source may occur. Unless the eye happens to be looking at or the attention is fixed upon a small area which contains the light at the time of the flash, the flash will not be seen. For flashes of low intensity and short duration, the average finding time should thus be proportional to the period T and to the solid angle φ in which the flash is to be sought.

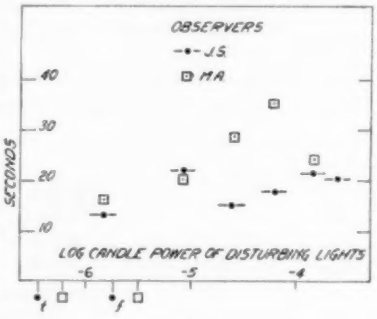

FIG. 8

(Time required to locate a flashing point source of five times threshold intensity in presence of 30 disturbing lights of intensities up to 500 times threshold. Flashes are every 5 sec.)

In the foregoing experiments the flashes were located within an area 50 × 150 cm. at a distance of 150 cm., so that the solid angle was $\varphi = 0.30$.

We may thus write Equation [7] in the following form:

$$(t - 0.5T)\ (L - 1)^{3/2} = 67\ \varphi\ T \ldots\ldots\ldots\ldots [7]$$

This equation, which is now generally applicable, gives the average time t required to detect the presence of light flashes of short duration and of period T in an unknown position within a known area which subtends the solid angle φ; L measures the light intensity of the flashes in threshold units—that is, in terms of the threshold light intensity of similar flashes (the same duration and period) in known positions. The experiments indicate that this equation is valid for background brightnesses ranging from zero up to that of moonlight, the effect of the background being only to change the magnitude of the threshold unit. The equation is probably applicable only when T is greater than one or two seconds.

To obtain a signal which can be detected within two or three flashes, say within an average of $t = 2.5T$ we see by Equation [7] that

$$L > 1 + 10\ \varphi^{2/3}$$

Thus with solid angles, such as those within which the aviator must seek beacon signals, the signals should be at least 10 times threshold.

For flashes of 1 sec. duration one may safely take as a threshold a value three times that given by Equation [5], and therefore may conclude that a flashing white light source to be useful as a signal to an aviator (visible on first or second flash) should be of an intensity such that it produces an illumination F at the position of the observer at least as great as

$$F = C/D^2 = 10^{-7}\ H^{1/2} \text{ lumen per sq. cm.} \ldots\ldots [8]$$

For signals to be transmitted 10 km. in clear weather with a background of bright moonlight on a snow-covered surface, a point source should then have at least 250 candlepower.

The Effect of Disturbing Lights on the Visibility of Point Sources. To find the effect of disturbing lights among which the aviator has to find a flashing beacon, there were placed 30 constant lights behind the holes in the screen. The lamps were all connected in series so that simultaneously their intensity could be adjusted up to 500 times the threshold of visibility. The flashing light (5 sec. period) was adjusted at five times the threshold for each observer and for each observation placed in an unknown position behind the screen as in the previous experiment. The finding times were determined with the disturbing lights at different intensities ranging from 4 times up to 500 times the threshold. The results were very surprising, as can be seen from Fig. 8, where the finding time is plotted against the logarithm of the candlepower of the disturbing lights. Apparently the influence of the disturbing lights is very small, the time being less than doubled if the 30 disturbing lights are made 100 times as strong as the flashing light when this is adjusted at five times the threshold. The marks t at the lower margin indicate the thresholds for the two observers and f the candlepower at which the flashing source was adjusted. Every point in the plot represents the average of 10 observations.

The disturbing lights in the foregoing experiments were paper-covered holes of 2.24 sq. mm. area illuminated to various brightnesses.

In a later series of experiments, disturbing lights having still greater intensities were produced by illuminating holes having areas 7.3, 22, and 66 sq. mm., the surface brightness in each case being 5×10^{-3} cp. per sq. cm., the candlepowers thus being 0.00036, 0.0011, and 0.0033, respectively. The screen was at a distance of 1.5 m.

The observer J. S., as an average of 30 observations with disturbing lights of 0.0033 cp., required a finding time of only 12 sec. for a flashing light of five times threshold intensity. With lights of 0.0011 cp., the time was 11 sec. as an average of 32 observations. Notwithstanding the greater intensities of these disturbing lights, the finding times are considerably less than those shown in Fig. 8. It appears that the observer J. S., as a result of continued training, had developed skill in locating the flashing light among the disturbing lights.

The holes in the screen which were used as light sources in these experiments were rather uniformly distributed over its surface. No hole was at a distance of less than 8 cm. from its nearest neighbor, so that the flashing light had in no case been within less than 3 deg. from the nearest steady disturbing light. It was thought that this might be the reason for the small influence of the disturbing lights upon the finding time.

Several holes were therefore made at distances of 2 cm. from disturbing lights corresponding to angular distances of 0.75 deg.

Among a set of 60 observations by J. S. were interspersed 28 in which the flashing light was only 2 cm. from the nearest disturbing light, while in the other 32 the distance was 8 cm. or more.

In both cases the flashing lights were of 1.7×10^{-6} cp. (five times threshold), while the disturbing lights were 3.6×10^{-4} cp. (1100 times threshold). The average finding times were 31 and 6.3 sec., respectively. It may be concluded from this experiment that steady lights at an angular distance of 3 deg. from the beacon have very little effect on its visibility, but that if they are at 1 deg. or less from the beacon, they may increase by many fold the time needed to locate it.

Effect of Duration and Frequency of Flashes on Visibility of Point Sources. If the source is a steady light of definite candlepower, one may wish to know the best way of interrupting the light to make it most conspicuous. The two variables are T, the flashing period, and t_o, the duration of each flash in seconds.

In the measurements the authors have used as a criterion for conspicuousness the average time to find the flashing light. The apparatus consisted of a small tubular projector with which a starlike spot of light approximately 1 sq. mm. in area could be focused on any point of the surface of a white screen 150 cm. by 150 cm. In front of this projector an interruptor disk was rotated with a speed adjustable between 6 and 240 r.p.m.; the disk was cut out so as to make the light pass during $^3/_4$ or $^1/_2$, $^1/_4$, $^1/_8$, $^1/_{16}$, $^1/_{32}$, or $^1/_{64}$ of the revolution. In front of the disk a shutter was placed which could be opened by pushing a button. Thus at any given instant a series of flashes could be projected on the screen, for which T could be between 0.25 and 10 sec. and t_o/T between $^3/_4$ and $^1/_{64}$. The observer sat 2 m. in front of the screen with his eyes about 50 cm. below the center. At the instant he heard the shutter click he started looking for the light, and with a stopwatch the time elapsing between opening the shutter and the observer's signal of locating the light was measured. By taking the average of 10 to 30 such observations, there was determined the average finding time of a flashing light having a definite T, t_o, and intensity.

The experiments showed that for moonlight, starlight, or dark background and with an interrupted source of fixed candlepower the light is most conspicuous when T lies between 0.5 and 1.0 sec. and t_o/T is between 0.5 and 0.75 sec. Decreasing T or increasing t_o/T decreases the flashing or flickering appearance and makes recognition difficult. Decreasing t_o/T below 0.5 increases the average finding time (with values of T as low as 1 sec.) because of the lowered visibility of short flashes in accordance with Equation [4]. When T is increased above 1 sec., the finding time is increased, because the eye, in wandering over the screen in search of the light, may pass by it during the intervals between flashes, whereas it would not do so with shorter intervals.

When T is greater than 1 sec., there is also another cause which increases the finding time. Even for bright flashes there is a theoretical lower limit for the average finding time which can be verified by experiment. This is determined as follows: The probability of opening the shutter while the light is shut off by the interrupter disk is $1 - (t_o/T)$; if the shutter is opened during the dark part of the cycle, the average time elapsing before the light comes on will be $0.5\ (1 - t_o/T)T$. For the cases that the shutter opens while the flash is on, the minimum locating time is zero; therefore the average finding time of a flashing light is at least $0.5\ (1 - t_o/T)^2T$. For rotating beacons t_o/T is very small, e.g., 5/360 deg., and the minimum average finding time or waiting time is $0.5 \times T$—that is, one-half of 10. This was checked experimentally, as the finding time for a strong light, and T larger than 1 sec. is very little different from the theoretical limit.

The relative visibility of flashes of different durations with moonlight background brightness was also determined in connection with these experiments. Results in reasonable agreement with Blondel and Rey's relation, Equation [4], were found. For example, with a flashing point source in a known location, the threshold intensity for flashes lasting 1 sec. was only 0.65 of that required for flashes lasting 0.2 sec. This result is in reasonable agreement with Blondel and Rey's Equation [4], according to which the threshold intensity for a 1-sec. flash should be 0.59 as great as for a flash of 0.2 sec.

Rotating Beacons. When a rotating beacon is used to produce light flashes, the conditions are in some ways essentially different from those just considered. The lamp used has a definite output in lumens, but the beam candles projected toward the aviator vary with the angular horizontal beam width θ_H, while t_o, the duration of the flash, varies in proportion to $T\theta_H$, T being the period of rotation.

Let it be assumed that the beacon projects a beam of rectangular cross-section with vertical and horizontal angles θ_V and θ_H, respectively, measured in radians. Then if I is the total luminous output of the beacon in lumens, the candlepower within the beam is $I/\theta_V\theta_H$. By Equation [2] it is found that the light flux density within the beam at a distance D from the beacon is

$$F = \frac{I}{D^2\theta_V\theta_H}\, e^{-D/\lambda} \ldots\ldots\ldots\ldots \text{[9]}$$

The duration of the flash is:

$$t_o = (\theta_H/2\pi)T \ldots\ldots\ldots\ldots \text{[10]}$$

If the value of F in Equation [9] is to be equal to the threshold value, we may combine Equations [9], [10], and [4] and obtain:

$$Ft_o = F_o(t_o + 0.21) = (IT/2\pi\theta_V D^2)e^{-D/\lambda} \ldots\ldots \text{[11]}$$

Since F_o is constant (independent of t_o), it thus appears that to obtain the most efficient use of the light (minimum I), t_o and therefore θ_H should be made as small as possible, preferably at least as small as 0.1 sec. With a 10-sec. period this requires a horizontal beam width of not over 3.6 deg.

Let these equations be applied to a typical airport beacon of the Department of Commerce design. The concentrated filament lamp is of 1000 watts and gives 22,500 lumens. The parabolic mirror subtends an angle of 120 deg. from the lamp filament, and thus receives light from a solid angle $\varphi = \pi$, or 25 per cent of the total. Allowing for intercepted light and imperfect reflection, it may be assumed that 20 per cent of the 22,500 lumens is utilized in the beam, so that $I = 4500$ lumens. Measurements show that the effective beam widths θ_H and θ_V are both 3.5 deg., or 0.061 radians. Thus the average beam candles should be $4500/(0.061)^2 = 1.2 \times 10^6$, which agrees well with experimental determinations with this beacon.

Substituting these values of I, θ_V, and θ_H in Equations [10] and [11] and eliminating t_o, we find

$$F_oD^2e^{D/\lambda} = \frac{5.6 \times 10^4\ T}{1 + 0.046\ T} \ldots\ldots\ldots\ldots \text{[12]}$$

To calculate the maximum distance at which this beacon can be seen, we should put F_o equal to the threshold value for steady sources, which by Equation [5] is approximately $F_o = 10^{-11}$ lumens per sq. cm. for moonlight background brightness. Thus if D_1 is the distance at which the intensity corresponds to the threshold value ($L = 1$), we have from Equation [12], after taking the square root,

$$D_1e^{D_1/2\lambda} = 7.5 \times 10^7\ [T/(1 + 0.046\ T)]^{1/2} \ldots\ldots \text{[13]}$$

If the atmosphere were absolutely clear so that $\lambda = \infty$, the light intensity would vary according to the inverse-square law, and in this case the beacon could theoretically be seen at a distance of 1.96×10^8 cm. (1960 km.) if the period of rotation is $T = 10$ sec., or 1.51×10^8 cm. (1510 km.) if $T = 5$ sec.

TABLE 3

(Pick-up of Department of Commerce type of rotating beacon by approaching airplane moving with velocity v = 5000 cm. per sec. = 180 km. per hour = 112 miles per hour; $\varphi = 0.5$; T = period of rotation in seconds)

λ (cm.)	Visibility, 2.3λ (km.)	Maximum range D_1 (cm.) $T = 5$	$T = 10$	$T = 20$	Additional distance for pick-up Δ (cm.) $T = 5$	$T = 10$	$T = 20$	Effective range $D_1 - \Delta$ (cm.) $T = 5$	$T = 10$	$T = 20$
10^7	230.0	3.18×10^7	3.49×10^7	3.76×10^7	3.49×10^6	4.64×10^6	5.96×10^6	2.83×10^7	3.03×10^7	3.16×10^7
10^6	23.0	6.35×10^6	6.78×10^6	7.10×10^6	8.8×10^5	1.12×10^6	1.37×10^6	5.47×10^6	5.66×10^6	5.73×10^6
10^5	2.3	1.00×10^6	1.05×10^6	1.09×10^6	1.83×10^5	2.24×10^5	2.70×10^5	8.2×10^5	8.3×10^5	8.2×10^5
5×10^4	1.15	5.60×10^5	5.83×10^5	6.00×10^5	1.12×10^5	1.36×10^5	1.68×10^5	4.48×10^5	4.47×10^5	4.32×10^5
2×10^4	0.46	2.54×10^5	2.66×10^5	2.71×10^5	5.8×10^4	7.40×10^4	1.00×10^5	1.96×10^5	1.92×10^5	1.71×10^5
10^4	0.23	1.39×10^5	1.45×10^5	1.48×10^5	3.72×10^4	5.03×10^4	7.58×10^4	1.02×10^5	9.5×10^4	7.2×10^4
5×10^3	0.12	7.6×10^4	8.0×10^4	8.1×10^4	2.52×10^4	3.80×10^4	6.29×10^4	5.1×10^4	4.2×10^4	1.8×10^4

The absurdity of this result indicates that the maximum range of such bright light sources at night is always determined by the scattering or absorption coefficient, so that it is not justifiable to put $\lambda = \infty$. The distance in which the intensity of a parallel beam falls to one-tenth because of scattering or absorption is 2.3λ, and this may be roughly taken to be equal to the distance denoted by the term "visibility" in aeronautical weather reports.

The columns headed D_1 in Table 3 give values of the maximum range D_1 for a series of different visibilities and for three rates of rotation of the beacon corresponding to periods of 5, 10, and 20 sec., respectively. These data have been calculated from Equation [13] by plotting curves giving λ as a function of D_1 (for each of the three values of T).

It should be noted that with more rapid rotation the amount of light received in each flash decreases. If the visibility were really unlimited ($\lambda = \infty$), the ranges would be in the ratio 1 to 1.30 to 1.59 for T = 5, 10, and 20 sec., respectively.

Examination of the data of Table 3 shows that the effect of the limited visibility is to bring these ratios far closer to unity. For example, with $\lambda = 10^5$ (1 km.), the ratios are 1 to 1.05 to 1.09. Thus the decreased flash intensity caused by rapid rotation becomes of minor importance even in determining the maximum range.

Pick-Up of Beacon by an Approaching Airplane. We have seen in the derivation of Equation [7] that a considerable time interval is necessary to "pick up" a flashing source of light if its visibility is only little above the threshold value. A rapidly moving airplane may thus approach within a distance D, which is far less than the maximum range D_1, before the signal will be seen. To estimate the effective range of a beacon we want to determine this average distance D.

If, as before, L denotes the intensity of the beacon flashes in threshold units, at any distance D, we have, by Equation [11],

$$L = (D_1/D)^2 e^{(D_1 - D)/\lambda} \qquad [14]$$

Consider an airplane moving directly toward a beacon. When it reaches a point corresponding to D_1, the value of L is unity. Let x measure the distance along the path of the plane from this point so that $x = D_1 - D$. Taking the logarithm of Equation [14] and differentiating at the point $x = 0$, we have

$$\frac{d \ln L}{dx} = \frac{1}{\lambda} + \frac{2}{D_1}$$

Thus for reasonably small values of x the variation of L with x is given by

$$L = \exp\left[x\left(\frac{1}{\lambda} + \frac{2}{D_1}\right)\right]$$

or which may be written

$$L = e^{x/\lambda'} \qquad [15]$$

where

$$\frac{1}{\lambda'} = \frac{1}{\lambda} + \frac{2}{D_1} \qquad [16]$$

Inspection of Table 3 shows that D_1 is considerably larger than λ. The variation of L due to absorption is thus usually much greater than that due to the inverse square law at distances such that the beacon is barely visible.

Equation [7] gives us the average time t required to pick up a flashing light having flashes of uniform intensity. In the present case the successive flashes are of increasing intensity. The reciprocal of t is the probability per second of picking up the light. The average distance Δ which the observer must travel toward the beacon before seeing it is thus the distance over which the time integral of this probability is unity. Thus

$$\int_0^{\Delta} \frac{dx}{vt} = 1$$

where v is the velocity of the plane toward the beacon. Substituting into this the value of t given by Equation [7] and the value of L from Equations [15] and [16], we obtain

$$\varphi T v/\lambda' = \int_0^{\Delta/\lambda'} \frac{du}{\frac{1}{2\varphi} + 67\,(e^u - 1)^{-3/2}} \qquad [17]$$

When Δ/λ' is sufficiently small, the term $^1/_{2\varphi}$ can be neglected, and we have

$$\left.\begin{aligned} 67\,\varphi T v/\lambda' &= (2/3)p^{3/2} - 2p^{1/2} + 2\tan^{-1} p^{1/2} \\ \text{where} \qquad p &= e^{\Delta/\lambda'} - 1 \end{aligned}\right\} \qquad [18]$$

When $\Delta/\lambda' < 0.1$, this reduces to

$$67\,\varphi T v/\lambda' = (2/5)\,(\Delta/\lambda')^{5/2} \qquad [19]$$

For large values of Δ/λ', $(e^u - 1)^{-3/2}$ may be replaced by $e^{-3u/2}$ and integration then gives

$$T v/\lambda' = 2\,[(\Delta/\lambda') + (2/3) \ln (1 + 134\,\varphi e^{-3\Delta/2\lambda'}) - K] \qquad [20]$$

where K is an integration constant.

In applying these results to the beacon we are considering let us place $\varphi = 0.5$. This corresponds to a field of view, in which the beacon is to be sought, measuring 20 deg. vertically and 82 deg. horizontally. With this value of φ, Equation [18] is found to be reasonably accurate for all values of $\Delta/\lambda' < 1$; and Equation [20], which holds accurately for $\Delta/\lambda' > 3$, reduces to

$$\varphi T v/\lambda' = \Delta/\lambda' - 2.897 + 0.285 \log_{10}(1 + 67\, e^{-3\Delta/2\lambda'}) \qquad [21]$$

TABLE 4

(Table of values of the integral of Equation [17] for the case $\varphi = 0.5$. Below about $\Delta/\lambda' = 1$, these values are independent of φ)

Δ/λ'	$\varphi T v/\lambda'$	Δ/λ'	$\varphi T v/\lambda'$	Δ/λ'	$\varphi T v/\lambda'$
0.1	1.98×10^{-5}	1.4	0.0310	5.0	2.124
0.2	1.21×10^{-4}	1.8	0.0737	6.0	3.10
0.4	7.75×10^{-4}	2.2	0.1516	8.0	5.10
0.6	2.36×10^{-3}	2.6	0.281	10.0	7.10
0.8	5.25×10^{-3}	3.0	0.470	20.0	17.10
1.0	1.070×10^{-2}	4.0	1.204		

For $\Delta/\lambda' > 6$; $\varphi T v/\lambda' = \Delta/\lambda' - 2.90$.

In the range from 1 to 3, numerical integration was resorted to.

Table 4 gives some typical values of Δ/λ' and $\varphi Tv/\lambda'$ from which a curve may be prepared.

In Table 3 are given values of Δ calculated for a plane moving toward the beacon with a velocity of 5000 cm. per sec. (180 km. per hour, or 112 miles per hour). This represents the average additional distance the plane must travel from the point where threshold visibility is reached before the beacon is seen within a large field which subtends a solid angle $\varphi = 0.5$. These values of Δ were calculated by means of a curve prepared from the data of Table 4. The values of λ' were first calculated by Equation [16] from λ and D_1, and then $\varphi Tv/\lambda'$ was obtained; the corresponding value of Δ/λ' from the curve multiplied by λ' then gave Δ.

It should be noted that Δ is relatively large. For example, with a "visibility" of 23 km. and with a beacon rotating once in 10 sec., the plane will travel on the average a distance of 11.2 km. from the point where the threshold is reached before the beacon is seen. It takes the plane 224 sec. to travel this distance, so that an average of 22.4 flashes above threshold value are needed before the beacon is observed. At the end of this time the intensity of the light from the beacon is 4.25 threshold units. The effective range, which is perhaps the best measure of the value of a beacon under given conditions of visibility, is thus $D_1 - \Delta$, as given in the last three columns of Table 3. It should be observed that, with the highest visibility, slightly greater effective ranges are obtained with $T = 20$ sec. than with 10 or 5 sec. But with a "visibility" of 2.3 km., the range is greater for $T = 10$ sec. than for either 5 or 20. With still poorer visibility the maximum range shifts gradually to the beacon with $T = 5$. Similar calculations show that with λ as low as 10^4 the maximum effective range occurs with T about 2 sec.

These results indicate that under conditions of poor visibility considerably more rapid rates of rotation than the standard 10-sec. period are desirable. The lowering of the light effect of each flash, which causes D_1 to decrease with T, is more than offset by the greater ease and rapidity with which the more frequent flashes can be detected in unknown positions within the large field of view presented to the aviator.

VISIBILITY OF FLASHES OF DIFFUSED LIGHT

The light from cities reflected from the lower surfaces of high stratus or cirrus clouds can frequently be seen from the ground at distances as great as 50 miles on clear, dark nights. Similarly an aviator flying high above clouds covering a city can see an illuminated area in the upper surface of the clouds at great distances. In these cases recognition of the source of light is facilitated by the contrast in intensity between the illuminated area and its surroundings.

If the aviator flies at night within a cloud over a city, he may have difficulty recognizing the increased intensity of diffused light because of its uniformity and steadiness. Charles A. Lindbergh has stated to one of the authors that under such conditions the flash of green light produced by a trolley was visible when large steady flares at a nearby airport were not recognizable.

Useful signals can thus be transmitted through fog by sudden flashes of a light source which is producing a general illumination of the fog.

A rotating beacon immersed in a fog or just below low clouds produces a nearly steady or only gradually changing illumination of the fog. Definite interruptions by means of a shutter mechanism may increase the visibility under these conditions, and the best results could then be obtained if the beacon were pointed toward the zenith.

With a desire to overcome the loss of efficiency due to interrupting a steady source and to obtain flashes of short duration and great intensity, a special flashing arc beacon was constructed. The flash is produced by making and breaking a contact between an iron-rod cathode and a copper block as anode. A large reactor stores up energy during a short circuit period and delivers it to the arc when the electrodes are separated. This increases the duration and intensity of the arc and reduces the maximum current and voltage required. The device is fed by direct-current line or from a 9-kw. 150-v. direct-current generator or mercury-rectifier outfit.

The maximum arc current is about 200 amp. The surge resembles a half-cycle of sinusoidal wave and is of a total duration of about 0.2 sec. The device is usually adjusted to give a flash every 2 sec. A reflector is used to throw the light which would strike the ground up into the sky. The light output per flash is about 50,000 lumens per sec., and the effective duration is about 0.1 sec., and thus, according to Equation [4], the visibility of these flashes should be the same as those of an interrupted steady source of $F_\infty = 160{,}000$ lumens.

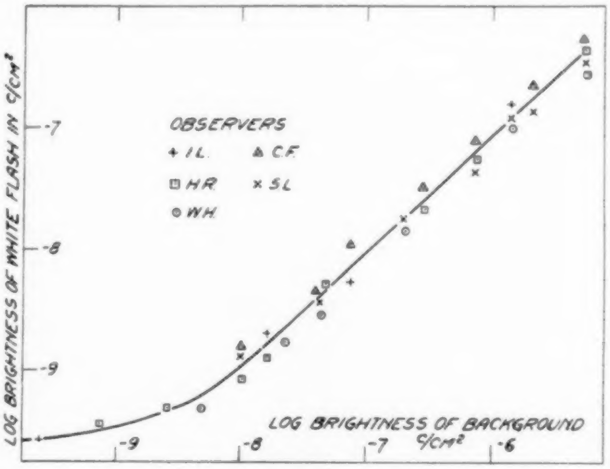

FIG. 9

(Effect of steady background brightness on the visibility of diffused flashes of white light. The ordinates represent the intensities needed for the detection of one-half the flashes.)

The range of visibility of signals of this kind depends primarily on the ability of the observer to distinguish the diffused flash from the steady background illumination. Many experimental investigations have been made of the minimum change of brightness required to produce a perceptible effect. The results are summarized in Weber's law that the smallest detectable change is a constant fraction of the total illumination. Most observers find that a 2-per cent change in brightness is observable. These measurements, however, have practically all been made with far higher background intensities than those that interest us, and the brightness of only part of the field was changed, leaving the remainder steady.

In order to study this problem from the point of view of the aviator, Mr. C. C. Steffens has conducted a series of measurements in this laboratory. His results are summarized in Figs. 9 and 10.

A white sheet 2 m. square was hung up in a dark room 1.8 m. from the observer. The sheet was uniformly illuminated by four tungsten lamps placed behind the observer opposite the four quarters of the screen. Each lamp was mounted inside a small box, which was open at one end and which contained two ground-glass diffusing screens, between which diaphragms of various openings could be interposed. The intensity of the screen illumination was varied by changing the current through the lamp filaments, but the lamp was never operated at below one-tenth normal candlepower in order not to change the color too greatly. Greater candle power variations than 10 to 1 were obtained by using the diaphragms whose diameters ranged from 4 down to 0.03 cm. The lamps and larger diaphragms were calibrated by photometering the screen, using reasonably high brightness obtained by placing the lamps at suitable distances from the screen. Tests showed that the variation of color by the 10 to 1 change in filament brightness did not cause appreciable effects, for the same results were obtained by changing the brightness by means of the diaphragms.

Flashes of light were superimposed upon the steady screen illumination by turning on and off lamps in four other similar boxes. Shutters were avoided, for the sound produced by them could lead to psychological errors in detecting the flash. The currents were actually turned on and off silently by means of a thyratron circuit in which the time interval could be adjusted.

After the observer had remained in the dark room long enough for dark adaptation, the steady-screen brightness was adjusted to any desired value from zero up to that of full moonlight (7×10^{-6} cp. per sq. cm.).

Flashes were then made at irregular intervals, starting at intensities high enough to be easily seen, decreasing rapidly until no flashes were seen, and then increasing the intensity slowly until about 50 per cent of the flashes were seen. This kind of effective threshold proved to be surprisingly definite and reproducible.

Fig. 9 gives data obtained by five observers with 1-sec. flashes of white light. The full line on the logarithmic plot which best represents the data is practically straight and of a slope 0.95 in the range of background brightnesses between 10^{-8} and 7×10^{-6} cp. per sq. cm. Below 10^{-8} the threshold flash rapidly becomes a larger fraction of the background brightness, until with a background of 2.7×10^{-10} the flash must be equal to the background in intensity.

The relation of the intensity of the flash to the background is thus given in Table 5. At brightnesses ranging from 10^{-4} to 1, Weber's law holds, and the ratio remains constant at about 0.02.

TABLE 5

(Relation between brightness of a 1-sec. flash of white light which is 50 per cent visible and the white background brightness)

Background (cp. per sq. cm.)	Flash to background
2.7×10^{-10}	1.000
10^{-9}	0.240
10^{-8}	0.112
10^{-7}	0.095
10^{-6}	0.080
7×10^{-6}	0.073

These flash intensities are such that on the average one-half are noted by the observer. With flashes close to these threshold values the observer often expresses doubt as to whether a flash that is noted is real, but actually, with white flashes, it is an extremely rare occurrence for the observer to report a flash if one did not actually occur. On the other hand, with threshold flashes of red light, 30 or even 40 per cent of the reported flashes were imaginary.

When the intensity of the flashes is made twice that of the threshold, the flashes are unmistakable, even though the observer is not particularly attentive. Every flash is seen under these conditions.

With a background intensity as high as moonlight (7×10^{-6}), a 1-sec. threshold flash is of such duration that its beginning and end are separately recognizable. With background brightnesses greater than 1.6×10^{-8} (0.4 of starlight), the flash brightness needed for threshold visibility is the same for 1-sec. and for 5-sec. flashes. With a background brightness less than about 6×10^{-9} (0.1 starlight), threshold flashes produce the sensation that the light gradually increases in brightness and slowly fades away. With a background at 2.7×10^{-10} cp. per sq. cm., a 1-sec. threshold flash is seen only after a period of from 2 to 4 sec. after its beginning, and the flash seems to persist for 6 to 10 sec. after it has really ended. With a flash of twice threshold intensity, both of these time lags drop to about 1 sec.

With these low background intensities (less than 0.1 starlight), it was observed that a 5-sec. flash needed to be only 0.5 as bright as a 1-sec. flash for 50-per cent visibility. This seems to indicate that at these low intensities the numerical constant in the Blondel-Rey Equation [4] should be 1.7 sec. instead of 0.21. At intensities above starlight, however, one is probably justified in using the unchanged Equation [4] to calculate the visibility of flashes of very short duration. Thus the cp-sec. per sq. cm. required for a threshold instantaneous flash may be obtained by multiplying the flash brightness, as given by Fig. 9 or Table 5, by 0.17 sec. (i.e., 0.21 ÷ 1.21). For very low intensities, however, this factor should be 0.63 (i.e., 1.7 ÷ 2.7).

Very interesting changes in the appearance of the illuminated screen were observed as its brightness was gradually decreased. At brightnesses from 1 to 3×10^{-8} cp. per sq. cm., the screen appeared to be remarkably uniform in intensity and perfectly steady, and seemed to be of a gray-green color, resembling the appearance of a window opening out on a vast open space on a starlit night. At intensities above 3×10^{-8} the resemblance to an open window disappeared, and it appeared to be a screen.

As the intensity was decreased, a remarkable change in quality occurred quite critically, usually at about 8×10^{-9} cp. per sq. cm. The appearance of the open window vanished and was replaced by an area of non-uniform intensity of yellowish color. If the eyes were directed at the center of the screen, the central part appeared to be darker than the edges. The central part showed fluctuations of intensity, both in time and space, while the edges appeared steady. The edges seemed to be even brighter than when the actual brightness was greater. Thus as the light intensity was deliberately decreased through the critical regions, the center of the screen appeared to grow darker, but the edges seemed to grow brighter. The fluctuations became slower and of larger size as the intensity decreased in the range below the critical value. Undoubtedly these effects result from fluctuations in the number of quanta of light striking the cones in the central part of the retina. At a background brightness of 3×10^{-10} cp. per sq. cm., areas subtending 5 to 10 deg. appeared to increase rapidly in brightness and gradually (in about 5 sec.) to fade away. The apparent size seemed to correspond roughly with that of the smallest black object that could be recognized against the black screen (visual acuity).

Since the aviator will rarely have to deal with background intensities less than 0.1 starlight (6×10^{-9} cp. per sq. cm.), effects of these kinds will not often be observed.

In connection with these experiments a study was made of the ability of the observer to detect gradual changes in light intensity such as might occur when an aviator flies at night in fog toward a city. Slow, steady changes in brightness, increasing or decreasing at about 3 per cent per second, were produced, beginning at unknown times, and the observer was asked to report any change and also to describe the brightness in terms of the original brightness.

With moonlight background an increase was first noticed when the brightness has increased an average of 45 per cent. With 0.1 moonlight 33 per cent is seen, while with starlight a 20-per cent change is needed. The results were, however, extremely irregular. For example, in some cases an increase in brightness was not noted until the brightness had changed in the ratio 1 to 2. Often an increase or decrease was reported to continue for a considerable time after it had stopped. Estimates of absolute intensities were approximately proportional to the $^3/_2$-power of the true intensities with an average error of ± 40 per cent.

Roughly, therefore, we can conclude that gradual changes in light intensity are not noticeable with certainty until the intensity has changed in the ratio 1 to 2 or even 1 to 3.

Studies also were made of the visibility of colored flashes. These data are summarized in Fig. 10, on which, for comparison, the curve for white light from Fig. 9 is reproduced. The red light examined by a spectroscope showed a band lying mostly between 5900 and 6900, the green lamp gave a band mostly between 5000 and 5700, while the blue gave 4500 to 4900. The lamps were photometered only at high intensities to avoid the Purkinje effect.

The data of Fig. 11 were obtained with a neon lamp. The lower curve agrees well with the curve for red light of Fig. 10.

With red flashes, the threshold value remains nearly constant at about 10^{-7} cp. per sq. cm., while the white-background brightness decreases from moonlight down to about 0.5 starlight (2×10^{-8} cp. per sq. cm.). In this range the threshold flash was observed as a change in color, but not as a change in intensity, of the light on the screen. With background intensities less than this, the brightness of a 1-sec. red flash needs to be about 30 times that of the white background, and the flash then does not produce any appreciable change in color of the background, but appears as a change in its brightness.

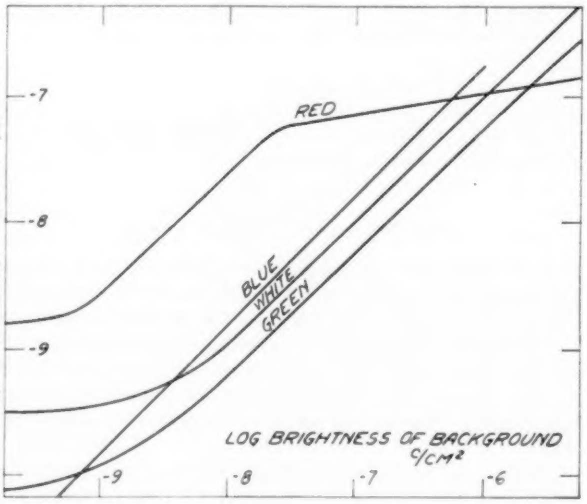

FIG. 10

(Visibility of diffused flashes of colored light as affected by the brightness of steady illumination by white light. The ordinates are similar to those of Fig. 9.)

Flashes of diffused red light are thus far less advantageous for the transmission of visible signals than are flashes of white or green. At high background intensities, such as moonlight, however, less red light than white light would be needed.

The foregoing data may be used to calculate the visibility of the flashes produced on a white diffusing surface, such as that of a cloud, by the light from a flashing point source at a distance D from the surface.

The data given in Table 6 have been calculated by means of Equations [1] and [3], using the factors in Table 5, but multiplying them by 2 in order to have flashes that can be seen with certainty. It has been assumed that the light from the source falls at normal incidence on the white surface which reflects 100 per cent of the light according to Lambert's cosine law. If we have a surface which reflects the fraction R and if the incident light beam makes an angle θ with the normal to the reflecting surface, then the candlepowers in Table 6 are to be divided by $R \cos \theta$.

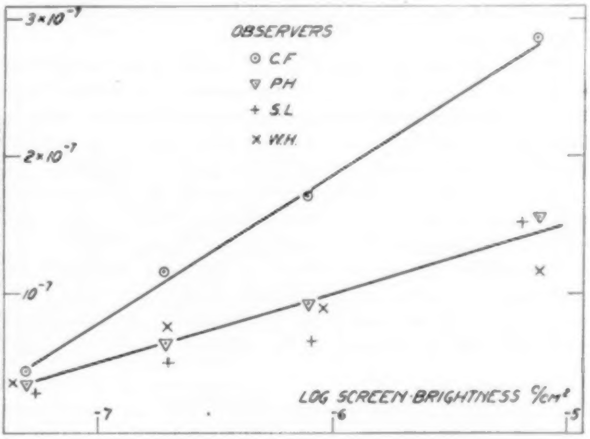

FIG. 11

(Visibility of flashes of light from a neon arc as affected by white-background brightness.)

On a dark, cloudy night when $H = 10^{-8}$ (0.17 starlight), it is thus possible to see the light flashes reflected down from clouds at a height of 1000 m. from a flashing point source of only 71 cp. on the ground; but if the cloud surface were illuminated by full moonlight, it would be necessary to increase the point source from 71 to 32,000 cp. to produce an equally visible flash at the same distance.

With flashes of very short duration, we have seen that the candle-seconds needed are only about $^1/_6$th of the candles of the steady light needed to produce 1-sec. flashes. The rotating beacon which we have considered gives 1.2×10^6 beam candles for 0.1 sec. in each flash, and thus by Table 6 visible diffused flashes should be observed when its beam at a distance of 4.7 km. from the beacon falls at normal incidence on a cloud illuminated by full moonlight. On a dark night, with absolutely clear air and $H = 10^{-9}$, such flashes should be visible at 100 km.

The flashing arc beacon giving 50,000 lumen-sec. (or 8000 candle-seconds in upper hemisphere) should thus produce visible flashes at 1.25 km. on clouds exposed to bright moonlight or at 26 km. when the steady light on the clouds is 10^{-8}.

Comparative Visibility of Diffused Light and Light From Point Sources. In the second column of Table 7 are given, for a series of steady background brightnesses, the values of light flux den-

TABLE 6

(The candlepower of a point source needed to give a distinctly visible 1-sec. flash [2 × threshold] on a white diffusing surface [cloud or snow] at distance D. H = steady illumination of the surface in cp. per sq. cm.)

H	D = 1 km. (cp.)	D = 2 km. (cp.)	D = 5 km. (cp.)
10^{-9}	15	60	375
10^{-8}	71	284	1,770
10^{-7}	600	2,400	15,000
10^{-6}	5,000	20,000	125,000
7×10^{-6}	32,000	128,000	800,000

TABLE 7

(Comparative visibility of diffused light and light from point sources, both giving 1-sec. flashes)

Background brightness H (cp. per sq. cm.)	Light flux density F for threshold visibility of point source	Light flux density F for threshold visibility after diffusion from white surface	—Ratio of sensitivity— Point source versus diffused source	Photocell versus visual for diffused light
10^{-9}	1.1×10^{-13}	7.5×10^{-10}	6,800	6
10^{-8}	3.5×10^{-13}	3.5×10^{-9}	10,000	28
10^{-7}	1.1×10^{-12}	3.0×10^{-8}	27,000	240
10^{-6}	3.5×10^{-12}	2.5×10^{-7}	72,000	2,000
7×10^{-6}	9.2×10^{-12}	1.6×10^{-6}	170,000	13,000

sity F from a flashing point source needed for threshold visibility, as calculated by Equation [5]. For comparison with these data we have, in the third column, the values of F which are visible according to the data of Table 5 when rendered diffuse by falling on a perfectly white diffuse reflector. Here we have made use of the relation between F and H given by Equation [3].

It is seen that the power of the eye to focus the light from a point source upon a small part of the retina makes it from 7000 to 170,000 times more sensitive to light from point sources than to diffused light which cannot be so concentrated. This advantage increases roughly in proportion to the square root of the background brightness, probably largely as a result of the increasing acuity of vision.

In considering the values of the ratios given in the fourth column of Table 7, it should be recognized that the time needed to find a flashing point source is an important factor, while with flashing diffused lights the average time needed to detect the signals will not exceed one-half of the period of flashing. Thus, as we have seen, the flashing point sources for practical reasons need to be 10 or 20 times the threshold, while a factor of 2 is sufficient to make diffused flashes immediately visible.

Photoelectric Cell as a Detector of Diffused Modulated Light. Although the human eye is extremely sensitive to the light from point sources, it becomes many thousands of times less sensitive when the light is diffused, as for example by fog. The photoelectric cell on the other hand, gives a response which is determined by the total light that falls upon it, and therefore it should possess particular advantages as compared to the eye for detecting diffused light.

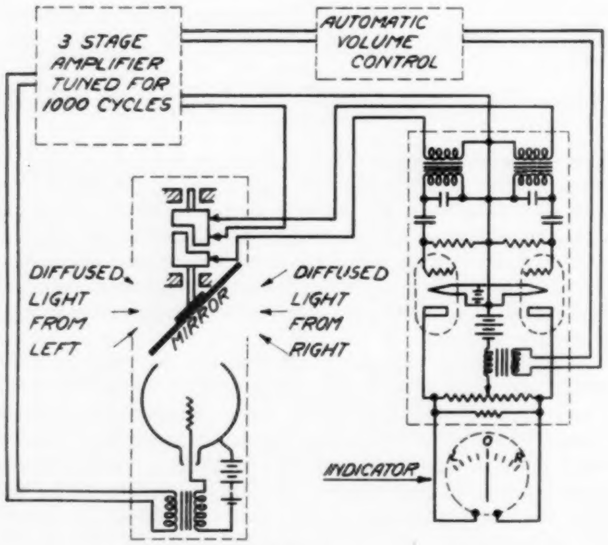
3 STAGE AMPLIFIER TUNED FOR 1000 CYCLES
AUTOMATIC VOLUME CONTROL
DIFFUSED LIGHT FROM LEFT
MIRROR
DIFFUSED LIGHT FROM RIGHT
INDICATOR
0

FIG. 12
(Photoelectric receiver for modulated light.)

We have seen how the flashing of diffused light signals for visual observation gives them the characteristics needed to distinguish them from the steady diffused light such as moonlight or the light from cities. In using the photoelectric cell, however, it is better to modulate the light at a definite frequency, such as 1000 per sec., and thus be able to distinguish the signal light from all others by using a tuned amplifier for the photo-cell current.

In order to determine how well such a system may be used to detect diffused light, the authors have constructed a special photo-cell and amplifier equipment shown diagrammatically in Fig. 12.

The photoelectric cell should be of large light-gathering power. The authors used a PJ-19 argon-filled caesium photo cell which has been developed for television outfits. The spherical bulb was 19 cm. diameter and had a window opening 15 cm. in diameter, or 175 sq. cm. area. It was operated with 45 volts on the anode and gave a sensitivity of 10 microamperes per lumen for white light and for the light from a mercury arc and about 8 microamperes per lumen for light from a neon lamp.

In front of the window of the photo cell was a mirror which could be rotated slowly (200 to 300 r.p.m.) about an axis inclined at 45 deg. to the mirror surface. In this way the diffused light from the right and left sides (of the airplane) could be thrown alternately and periodically into the photo cell.

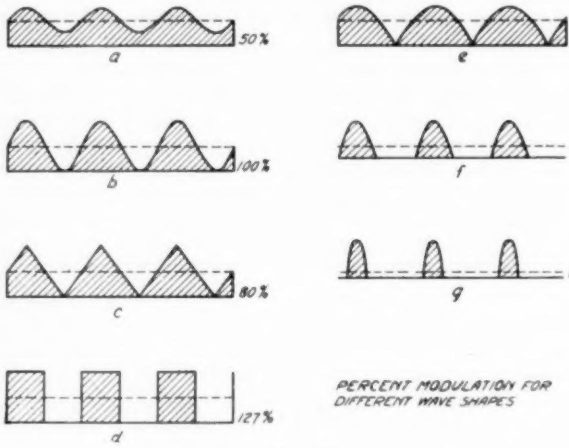

FIG. 13
(Effectiveness of various types of modulation.)

The alternating-current component from the photo cell passed to an amplifier consisting of three stages of screen-grid amplification (UY-224 tubes), the first two stages being tuned for 1000 cycles. The amplifier was thus highly selective for the modulated light signals. The voltage amplification factor was about 2,000,000.

On the shaft of the rotating mirror was a commutator through which output of the amplifier was passed. This acted as a single-pole double-throw switch which threw the amplified 1000-cycle current alternately to two detector tubes (UX-112A) in push-pull arrangement, as shown at the right-hand side of Fig. 12.

In this way the modulated light that entered the photo cell from the right tended to produce a deflection to the right of the needle of the direct-current indicating instrument, which had a sensitiveness of 150 microamperes for full scale.

The indicator reading thus gave a measure of the difference in the diffused light intensities on the two sides of the photo cell. The frequency of rotation of the mirror was immaterial so long as it was more rapid than the indicator needle could follow.

This differential arrangement was adopted so that the direction of the light source could be determined even in the presence of fog.

When desired, the light intensity in either direction could be determined by properly fixing the position of the mirror after stopping its rotation, or more conveniently by rendering one or the other of the detecting circuits inoperative.

When using the differential arrangement, it is usually desirable to have an indication which measures the relative rather than the absolute difference in the intensities in the two directions.

This was accomplished by utilizing the automatic volume control illustrated in Fig. 12. In this device the 1000-cycle component of the output of the detector-tube circuit was rectified by a

UX-112A tube, and this rectified voltage was used to bias the grid of the first tube in the amplifier. A filter circuit eliminated the fluctuations caused by the mirror rotation so that the average of the intensities of the amplified 1000-cycle current produced by the light from the two sides was kept nearly constant as the actual light intensity increased. The difference in the intensities from the two sides was, however, given by the indicating instrument.

To test the sensitivity of this apparatus a source of modulated light of known intensity was constructed by using a shutter operated by a synchronous motor. The light intensity was made to increase and decrease linearly with time (as shown by diagram C in Fig. 13) with a frequency of 1000 per sec. The intensity varied from zero at the minimum to 0.5 cp. at the maximum so that the average candlepower was 0.25. The amplitude A_1 of the 1000-cycle component in a Fourier series expansion, $I = A_1 \sin \omega t$, corresponds to 0.2 cp. (i.e., 80 per cent of the average 0.25).

Two large sheets of white cardboard were mounted in vertical planes at right angles to one another and placed at a distance of 40 m. from the described source of modulated light, so that the light fell on each plane at an angle of incidence of 45 deg. The brightness of illumination of these sheets was thus 2.8×10^{-9} cp. per sq. cm. (i.e., $0.2 \times 0.71/\pi(4000)^2$) in terms of the amplitude of the 1000-cycle component.

The photo-cell outfit was mounted symmetrically between the sheets, so that when the mirror was rotated the cell was exposed alternately to the diffused light from the two sheets.

By making a slight adjustment in the inclination of one of the sheets, with respect to the light rays from the source, the needle of the indicator instrument was brought to zero (see Fig. 12). A change of the inclination of either sheet by as little as 0.9 deg. then produced a distinct deflection of the needle (about 15 microamperes). This corresponds to a change in brightness of 1.5 per cent of the average brightness (2.8×10^{-9}). Thus the photo-cell-amplifier outfit was sensitive to a change in the absolute brightness of only 4×10^{-11} cp. per sq. cm., which is only about one-sixth of the threshold of the eye for flashing lights with a completely dark background (see the data of Fig. 9).

A particular advantage of the photo-cell equipment is its complete insensitiveness to the steady background illumination. Thus the sensitiveness to modulated light was not appreciably altered by letting light from incandescent lamps fall on the sheets up to a brightness of 10^{-8} cp. per sq. cm. (about 100 times moonlight). With full daylight or sunlight, however, the photo cell was overloaded, and the sensitiveness to modulated light decreased greatly. Undoubtedly special cells and circuits can be devised that would make it possible to detect very weak signals of diffused modulated light even in the presence of sunlight.

It is interesting to compare this observed sensitiveness of the photo-cell apparatus with the corresponding visual data given in Tables 6 and 7.

As before, we may assume that a reliable signal must produce a brightness of twice threshold value. Thus for the photoelectric detector we need a light flux density $F = 2.5 \times 10^{-10}$ lumens per sq. cm. This corresponds to 2.5 cp. at 1 km. The last column in Table 7 gives a factor which measures the relative advantage of the photo-cell detector over visual observation for diffused light signals. This factor is obtained by dividing the values of F in the third column by the threshold value for the photoelectric detector (1.26×10^{-10}). In full moonlight the photo cell can detect a diffused modulated light of an intensity only 1/13,000 of that of a diffused flashing light just visible to the eye.

Sources of Modulated Light. The instantaneous intensity I of modulated light may be expressed as a function of time t by a Fourier expansion:

$$I = A_o + A_1 \sin \omega(t - t_1) + A_2 \sin 2\omega(t - t_2) + \text{etc.}$$

The effectiveness of modulated light when used with a tuned receiver such as has been described depends upon the amplitude A_1 of the fundamental component, and therefore this component should be made as large as possible compared to A_o, A_2, etc.

One may define the degree of modulation as the ratio A_1 to A_o—that is, the ratio of the amplitude of the fundamental component to that of the average light. This ratio for a series of different wave shapes is given in Fig. 13. The maximum possible degree of modulation is 200 per cent, which is approached if the light consists of separate flashes of very short duration.

An efficient source of modulated light has been developed[8] in the form of a self-modulating neon lamp which operates on the thyratron principle and which produces the light flashes illustrated in diagram g of Fig. 13, giving a degree of modulation as high as 188 per cent.

DIFFUSION OF LIGHT IN FOG

The aviator encounters several conditions that cause poor visibility. Haze and smoke usually are due to the presence of extremely small particles in the air—particles of diameters less than the wave length of light. According to Rayleigh's law, the amount of light scattered by such particles is inversely proportional to the 4th power of the wave length of the light. Thus, white light transmitted through haze or smoke becomes red, while the light scattered is mainly blue. Scattering of this kind accounts for the blue color of the sky and of distant mountains and for the red color of the setting sun.

Besides scattering light, smoke, and smoky fog obviously absorb a considerable amount of light.

In districts far from the sources of industrial smoke, however, the aviator encounters clean white fog, either as ground fog or as clouds. Such fog consists of small water drops which range in diameter[9] from about 2.2×10^{-3} to about 10^{-2} cm., so that they are 40 to 200 times larger than the wave length of ordinary light. The color effects characteristic of haze and smoke are thus absent in clean fog and in clouds.

The question is often raised whether certain colors of light such as red or blue are not transmitted through fog far better than white light.[10] The large size of the drops indicates that the scattering of light by fog should be the same throughout the visible and ultra-violet spectrum, and in the infra-red a decrease in scattering should occur only when wave lengths of about 20 microns are reached. In the visible spectrum this conclusion is confirmed by the fact that the disk of the sun when faintly discernible through clean fog or cloud appears colorless; also by the fact that the illumination on cloudy days is not of distinctly different color from that on clear days.

The scattering of light by fog and clouds is thus due to the interception of the light rays by the spherical droplets of water. Let n be the number of droplets per unit volume, each of diameter d; then w, the weight of the drops per unit volume, is:

$$w = (\pi/6)nd^3 \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

If a beam of light of intensity I passes a distance dx through the cloud, the decrease in the intensity of the direct beam is $-dI = I(\pi/4)nd^2\, dx$, since $(\pi/4)d^2$ is the area intercepted by each drop. The scattering coefficient is thus $(\pi/4)nd.^2$ The reciprocal of this, or

$$\lambda_o = 4/(\pi nd^2) = \frac{2}{3}\frac{d}{w} \ldots\ldots\ldots\ldots\ldots\ldots [23]$$

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[8] This modulated light source will be described in a forthcoming paper by W. F. Westendorp.

[9] "Handbuch d. Expl. Physik," 25 (1), p. 51, 1928.

[10] See "Fog Penetration," by L. H. Anderson, U. S. Air Services, July, 1931, pp. 20 to 25. (This is to appear in Trans. A.S.M.E., Aeronautical Engineering.)

is the "free path" of the light rays through the cloud; i.e., the average distance that the rays can travel before the intensity of the direct beam falls to $1/e$th value. This free path is entirely analogous to that employed in the kinetic theory of gases.

Experimental determinations of w in rain clouds have given 1×10^{-6} to 2×10^{-6} grams per cu. cm. An estimate of the maximum possible values of w can be made from the water-vapor content of air.

The air rising into a rain cloud is saturated when it reaches the level of the bottom of the cloud. Under summer conditions the temperature at this point (1000 m. elevation) may be taken to be 20 deg. cent. The total water-vapor content is then 17.2×10^{-6} grams per cu. cm. If this saturated air rises another 1000 m., it would cool by adiabatic expansion (allowing for the heat of condensation of the water vapor to 15.8 deg., and the water-vapor content would then be 13.3×10^{-6} grams per cu. cm. The difference between these quantities, i.e., $w = 3.9 \times 10^{-6}$, must be the volume of the droplets per cubic centimeter in the cloud at an altitude of 2000 m. A similar calculation for winter conditions with a temperature of —5 deg. cent., for the base of the cloud gives a temperature of —11.6 deg. at an altitude of 2000 m., which gives at this level $w = 1.4 \times 10^{-6}$. These values of w are in good agreement with the experimentally found values.

The shortest free paths which may occur in clouds or clean fogs thus correspond roughly to $w = 2 \times 10^{-6}$ and $d = 3 \times 10^{-3}$ which give by Equation [23] $\lambda_e = 1000$ cm., or 10 m. According to Equation [22], there are only 140 droplets per cu. cm. in a fog of this character.

In the less dense clouds and fogs usually encountered by the aviator, the free paths λ_e will range from 30 to 100 m.

The light which is intercepted by a droplet is scattered in various directions, but not uniformly, for the larger part is scattered in a forward direction. This means that the forward component of the light originally in a parallel beam does not decrease as rapidly as does the intensity of the unscattered component of the beam. This phenomenon corresponds exactly to that of the persistence of velocities treated in the kinetic theory of gases.[11]

The distance at which the sharp outlines of an object can be seen through a fog depends primarily upon the distance that a ray of light can travel without appreciable change in direction. Thus the free path λ_e that we have calculated from the diameter of the droplets is applicable to such problems.

However, if we are to consider the amount of diffused light transmitted through or reflected from a cloud layer of given thickness, we need to consider a different kind of free path λ—i.e., the average distance which the light from a parallel beam can travel before its forward component has fallen to $1/e$th value. This free path will probably be two or three times as great as λ_e.

The transmission of light through fog is governed by laws similar to those of the diffusion of one gas into another and is analogous to the flow of heat or electricity in a conducting solid. The intensity of the light at any point in a fog is approximately the same in all directions and is measured by H, the brightness of illumination in candles per square centimeter.

Corresponding to this average brightness H, there is a random flux density $F = \pi H$ of light flowing across an imaginary plane inclined in any direction. However, because H is not uniform throughout the fog, there will be a drift flux density F_d which represents the net flow across an imaginary plane surface. This is essentially a vector quantity, and its component in any direction is proportional to the gradient of H in that direction.

The coefficient of self-diffusion D in a gas may be calculated from the mean free path λ and the average molecular velocity v, giving the result $D = (1/3)\lambda v$. An analogous calculation for the diffusion of light leads to the following expression for F_x, the component of the drift flux in the direction parallel to the x-axis:

$$F_x = -(4\pi/3)\lambda \frac{\partial H}{\partial x} \ldots\ldots\ldots\ldots\ldots\ldots [24]$$

Thus the quantity $4\lambda\pi/3$ may be termed the luminous conductivity of the fog. The brightness H is a scalar quantity which may be regarded as the luminous potential and is entirely analogous to electric potential, to temperature in heat conduction problems, or to concentration in diffusion problems.

In deriving Equation [24] it was assumed that the free path λ was small compared to the dimensions of the cloud.

With this point of view, let us now calculate the distribution of brightness in certain typical simple idealized cases.

1 *Point Source at Center of Spherical Cloud:*

Consider a source of L lumens at the center O of a cloud of radius r_o. Since we are assuming that the light is scattered, but not absorbed, the total flux across any spherical surface having its center at O is L. Thus the light flux density F_r in a radial direction at any distance r from the center is:

$$F_r = L/4\pi r^2 \ldots\ldots\ldots\ldots\ldots\ldots [25]$$

Combining this with Equation [24] and integrating, we find

$$H = \frac{3\,L}{16\pi^2\lambda}\left(\frac{1}{r} + K\right)$$

where K is the integration constant. Since we have assumed that λ is small compared to r_o, we may put $H = 0$ when $r = r_o$ (the outer surface of the cloud), and thus find $K = -1/r_o$, so that

$$H = \frac{3\,L}{16\pi^2\lambda}\left(\frac{1}{r} - \frac{1}{r_o}\right) \ldots\ldots\ldots\ldots\ldots [26]$$

Let us examine more closely these boundary conditions. As seen from a distance, the outside surface of the cloud will appear to have a brightness H_o which may be calculated by Equation [3] from the flux density F given by Equation [25] when $r = r_o$. Thus:

$$H_o = L/4\pi^2 r_o^2 \ldots\ldots\ldots\ldots\ldots\ldots [27]$$

An observer located at the outside surface of the cloud would see this brightness H_o if he faces the interior of the cloud, but finds the brightness $H = 0$ if he faces away from the center. Thus at this surface H is no longer a scalar quantity, as it is in the interior of the cloud, but depends on the direction of observation. If we put in Equation [26] $H = H_o$, as given by Equation [27], we find that

$$r_o - r = (4/3)\lambda \ldots\ldots\ldots\ldots\ldots\ldots [28]$$

In other words, the brightness at a distance $(4/3)\lambda$ inside of the outer surface of the cloud has a value H_o which will account for the light flux emanating from the cloud. This result appears reasonable when we remember that the light that escapes from the cloud passes through a layer comparable with λ in thickness without appreciable scattering.

In general, therefore, in calculating the brightness distribution within any cloud mass, we may place $H = 0$ at the outer surface. The apparent brightness as seen from a point outside is then the brightness which is calculated for a surface lying at a depth $(4/3)\lambda$ inside of the true surface.

It is interesting to compare the brightness H inside of an illuminated cloud with the brightness which would be obtained at the same distance in the absence of the fog if the light source should shine directly on a white diffusing surface. Comparing

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[11] See Jeans, "Dynamical Theory of Gases."

H as given by Equation [26] with $H = L/4\pi^2r^2$, which would apply in the absence of fog, we see that the effect of the fog is to increase the brightness in the ratio

$$3r(r_o - r) : 4\lambda r_o \ldots\ldots\ldots\ldots [29]$$

As r increases, this ratio reaches a maximum value $3r_o/160\lambda$ when $r = 0.5\ r_o$. Thus in a spherical cloud having a radius of 1000 m. and a free path $\lambda = 30$ m., a 5000-cp. light ($L = 60{,}000$ lumens) at the center would give a brightness $H = 3.8 \times 10^{-6}$ cp. per sq. cm. at a distance of 500 m. from the source. This is over 6 times the brightness (0.63×10^{-6}) that would be obtained if the light from the source at this distance fell directly with normal incidence on a white diffusing surface. The fog, because of its poor luminous conductivity, thus tends to prevent the escape of the light and so increases the brightness within the cloud. The brightness of the outside surface of the cloud, by Equation [27], would be $H_o = 1.5 \times 10^{-7}$ cp. per sq. cm., which is about 4000 times the threshold sensitiveness of the photoelectric receiver.

Except by means of rockets or flares we are not ordinarily able to produce light sources at the centers of large clouds, so the foregoing case may seem to be rather academic. However, if the ground reflects nearly all the light that falls on it, as for example when it is snow-covered, then with a thick layer of fog over the ground and a light source on the ground the drift flux distribution is twice that given by Equation [25], and therefore Equation [26] gives the brightness distribution if its right side is multiplied by 2. We shall see that imperfect reflection from the ground modifies these conditions.

Directional Effect in Fog. The drift-flux density F_x in a fog as given by Equation [24] may be regarded as the difference of two fluxes flowing in opposite directions. Any small plane white surface (of dimensions small compared to λ) placed in the fog in a plane perpendicular to the x-axis will be illuminated differently on its two sides, the difference of brightness ΔH_x being

$$\Delta H_x = \frac{F_x}{\pi} \ldots\ldots\ldots\ldots [30]$$

Substituting into this the value of F_r from Equation [25] and dividing by H as given in Equation [26], we find for the directional ratio

$$\frac{\Delta H_r}{H} = 4\lambda/3r(1 - r/r_o) \ldots\ldots\ldots\ldots [31]$$

The minimum value of this quotient, $16\lambda/3r_o$, occurs when $r = 0.5\ r_o$. Thus in the example we have just considered the smallest value of $\Delta H_r/H$ is 0.16. The differential photoelectric receiver, which has been described, can detect differences of only 1.5 per cent, and should therefore be able to indicate the direction from which the light comes.

If such a device were used in an airplane, the plane could be turned toward the light of greatest intensity until the two intensities balance; straight flight would then bring the plane to the source.

2 *Layer of Cloud Uniformly Illuminated From Above or Below:* Let us consider a layer of cloud of uniform thickness h, the upper surface of which is illuminated by sunlight or moonlight. If F_s is the flux density produced by the source, as given for example in Table 1, then if the upper surface of the cloud were a perfect diffuse reflector, its brightness would be

$$H_s = (F_s/\pi)\sin\alpha \ldots\ldots\ldots\ldots [32]$$

where α is the angular altitude of the source above the horizon. Let H_T be the actual brightness of the top of the cloud layer, which is less than H_s because of light lost by transmission through the cloud; let H_B be the brightness of the bottom of the cloud layer as seen from the ground; and let R be the average reflection coefficient of the ground. The corresponding flux densities ($F = \pi H$) across horizontal planes are F_T and F_B. The light flux reaching the ground is thus F_B, while that returning from the ground is RF_B, and the loss of light caused by absorption by the ground is $F_B(1 - R)$. This loss must be equal to the drift flux F_x through the cloud layer and must also be equal to $F_s \sin\alpha - F_T$, the net light input at the top of the cloud layer. Thus by Equations [24] and [32] we have

$$H_B(1 - R) = \frac{4\lambda}{3h}(H_T - H_B) \ldots\ldots\ldots\ldots [33]$$

and

$$H_B(1 - R) = H_s - H_T \ldots\ldots\ldots\ldots [34]$$

Separating the two variables H_T and H_B, we obtain the equations

$$\frac{3h}{4\lambda} = \frac{H_T}{H_s - H_T} - \frac{1}{1 - R} \ldots\ldots\ldots\ldots [35]$$

and

$$\frac{3h}{4\lambda} = \frac{H_s - H_B}{H_B(1 - R)} - 1 \ldots\ldots\ldots\ldots [36]$$

Photometric measurements of H_T and H_B could readily be made from an airplane, and by simultaneous determinations of h, the values of λ for various types of cloud layers could be found. Measurements from the ground alone, however, give some useful data. Published tables of photographic exposure show that on days described as "very dull" the exposure needs to be increased only about tenfold. If under these conditions the upper surface of the cloud layer reflects 90 per cent of the incident light, there can be no appreciable loss by absorption within the cloud layer. The effect of a cloud layer sufficient to make a very dull day, which we may assume is 1000 to 2000 m. thick, is thus to change the light flux on the ground in the ratio $H_B : H_s = 1 : 10$. From Equation [36] this gives

$$\frac{3h}{4\lambda} = \frac{8 + R}{1 - R}$$

Photometer measurements which the authors have made from a tower at an airport have shown that the reflectivity R of the ground is ordinarily very low; for example, the airport surface varied from 9 to 21 per cent reflectivity. Tree-covered land (without leaves) gave only 5 per cent. The foregoing expression gives values of h/λ which are not very sensitive to R; thus for $R = 0.1$ and $R = 0.2$, we find h/λ is 12 and 13.7, respectively. This gives $\lambda = 80$ m. with a 1000-m. layer or $\lambda = 160$ m. with a 2000-m. layer.

If we have a layer of cloud uniformly illuminated from below, as for example by a series of regularly spaced lights, the problem differs from that just considered in that the light reflected downward F_B from the bottom of the cloud layer is partly reflected back by the ground and contributes to the illumination of the cloud. Equation [34] is still applicable, but in place of Equation [33] we have

$$H_T = \frac{4\lambda}{3h}(H_B - H_T) \ldots\ldots\ldots\ldots [33a]$$

and instead of Equations [35] and [36] we obtain

$$1 + \frac{3h}{4\lambda} = \frac{H_s - H_T}{H_T(1 - R)} \ldots\ldots\ldots\ldots [35]a$$

and

$$\frac{3h}{4\lambda} = \frac{H_B(2 - R) - H_s}{H_s - H_B\,(1 - R)} \ldots\ldots\ldots\ldots [36a]$$

where H_s is the brightness which the bottom of the cloud would have if it were perfectly reflecting and the ground were completely non-reflecting. If F_s is the average vertical flux density from the light sources on the ground, then $H_s = F_s/\pi$. It should be noted (by Equation [36a]) that if the cloud layer is thick or λ is very small, so that $3h/4\lambda$ becomes very large, H_B approaches the value

$$H_B = H_s/(1 - R)$$

and thus may become much greater than H_s if the ground has high reflectivity.

3 *Point Source in Fog Close to Ground With Poor Reflectivity:* We have already considered in case 1 an example where the ground is a good reflector. Let us now consider a case where the ground is completely black ($R = 0$) and the light source is located at a point B at a height b above the ground.

Since the ground absorbs all the light that falls on it, $H = 0$ at this surface, just as at the outside surface of the fog. We can simplify the problem by assuming that the fog is infinitely thick. The problem is then similar to that of the potential distribution around a point charge at a distance b from a plane conducting surface, and may be readily solved by the method of images. In the present case the distribution is the same as if the black ground were replaced by a completely transparent plane and in addition to the light source at B we postulate a light sink (negative source) at B' which is the image of B in the plane. The brightness H at any point at a distance r from the light source ($r \gg \lambda$) and at an angular height α above the horizon (as seen from the source) is

$$H = \frac{3L_u b}{4\pi^2 \lambda r^2} \sin\alpha \ldots\ldots\ldots\ldots\ldots [37]$$

In this equation L_u represents the light flux in lumens emitted by the source in an upward direction, i.e., in the upper hemisphere. This distinction is necessary, since rays emitted downward would be absorbed directly by the ground. In practice the efficiency of the light source can be nearly doubled by placing a hemispherical reflector below the lamp to convert the downward rays into the useful upward light. In this case L_u is approximately equal to L, the total luminous output of the lamp.

If the light source were actually at the level of the ground ($b = 0$), Equation [37] would give $H = 0$. When we consider that the light radiating from the source reaches on the average a hemispherical surface of radius λ before being scattered, we see that the effective height of a point source on the ground is roughly $(^2/_3)\lambda$. If the light were focused vertically upward, its effective height[12] would more probably be λ.

Thus b in Equation [37] may be taken to be $b_o + (^2/_3)\lambda$ (or $b_o + \lambda$), if b_o is the actual height of the light source.

With $b_o = 0$, so that $b = (^2/_3)\lambda$, Equation [37] thus becomes

$$H = L_u \sin\alpha/2\pi^2 r^2 \ldots\ldots\ldots\ldots\ldots [38]$$

If there were no fog and the light source should shine directly with normal incidence on a white surface at distance r, the brightness would be

$$H = L_u/2\pi^2 r^2 \ldots\ldots\ldots\ldots\ldots\ldots [38a]$$

which is thus the same as the brightness within the fog at points directly over the source. For other values of α the brightness would be less.

Fig. 14 gives data on the range at which a modulated light of intensity $L_u = 60{,}000$ lumens (5000 spherical candles with hemispherical reflector) would produce in an infinite fog a brightness $H = 3 \times 10^{-9}$ cp. per sq. cm., this being about 75 times the threshold value of the photoelectric receiver.

Curve I is for the case that the beacon is at the ground level (effective height $(^2/_3)\lambda$). The range is then independent of the density of the fog. Curves II and III illustrate the effect of placing the beacon on a tower 50 m. high. The range is greatly increased and is now dependent on the density of the fog, being much greater for a dense fog than for one less dense. With no fog at all the range (for $H = 3 \times 10^{-9}$) is represented by curve IV, which by Equation [38a] is a semicircle of radius equal to the maximum height of curve I. Comparison of curves III and IV shows how greatly fog can extend the range of diffused light signals.

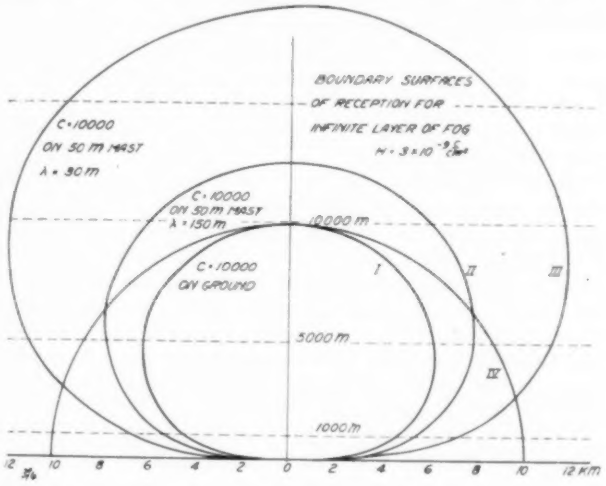

FIG. 14

(Surfaces of equal brightness [$H = 3 \times 10^{-9}$ cp. per sq. cm.] in an infinite fog overlying dark-colored ground [$R = 0$] near which is a light source projecting 60,000 lumens into the upper hemisphere; C is the candlepower in the upward direction, i.e., of the lamp with its hemispherical reflector.)

If we take x as the horizontal distance from the light source ($x = r \cos\alpha$), then by partial differentiation of Equation [37] and combination with Equation [24] we find that the horizontal component of the drift-flux density, at any point, is

$$F_x = \frac{3L_u b}{\pi r^3} \,.\, \sin\alpha \cos\alpha \ldots\ldots\ldots\ldots [39]$$

and thus by Equation [30] the directional ratio is

$$\frac{\Delta H_x}{H} = (4\lambda/r) \cos\alpha \ldots\ldots\ldots\ldots\ldots [40]$$

which is independent of b, the height of the light source.

If the receiver can detect a difference of 1.5 per cent between opposite directions, it should be capable of giving directional effects at distances of about 200λ, or 10 km. (if H is as great as 3×10^{-9}).

4 *Point Sources Within Uniform Layer of Fog or Cloud: Case a: White Ground* ($R = 1$). Consider a point source of light on perfectly reflecting ground which is covered by a layer of fog of thickness h. At the upper surface of the fog $H = 0$. By the method of images the problem is then equivalent to the determination of the potential distribution produced by a series of

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[12] A still greater effective height might be obtained in quiet, foggy weather by building a fire on the ground so as to produce a rising vertical column of clear air up through which a searchlight beam could be projected to a height much greater than λ.

alternate positive and negative charges arranged along a vertical line with a distance $2h$ between adjacent charges. Madelung has solved[13] this and many similar problems in his studies of the electric forces in crystals built up of ions.

The complete solutions are expressible only in terms of Hankel functions, for which, however, tables are available. We are interested mainly in the distribution of brightness at distances from the source which are fairly large compared to h, the thickness of the cloud layer. Under these conditions the equations take much simpler forms. The following equations are modifications of Madelung's results adapted to our calculation of distribution of brightness.

At any point within the fog layer at a height y above the ground and a horizontal distance x from the beacon, the brightness H is

$$H = \frac{3\,L \exp(-\,\pi x/2h)\cdot\cos(\pi y/2h)}{4\pi^2\lambda h^{1/2}x^{1/2}} \quad \text{[41]}$$

Here L is the total lumens output from the light source. Because of the perfect reflection from the ground all of this light escapes from the upper surface of the cloud layer. This equation is accurate for all values of x which are large compared to h. However, for values of x even as small as $(^1/_3)h$ the errors are not greater than about 10 per cent.

A calculation of the directional ratio similar to that which led to Equation [40] gives in the present case for $x \gg h$:

$$\frac{\Delta H_s}{H} = 2\pi\lambda/3h \quad \text{[42]}$$

The brightness H_s of the upper surface of the fog, at a point directly over the beacon, $x = 0$, $y = h$, is (exactly):

$$H_s = 0.09280\,L/h^2 \quad \text{[43]}$$

At points on the upper surface for which $x > h$, the brightness H_s is

$$H_s = \frac{L \exp(-\,\pi x/2h)}{2\pi\, h^{3/2}\, x^{1/2}} \quad \text{[44]}$$

Examination of these results shows that the brightness in the fog and on its upper surface decreases exponentially as the horizontal distance x increases, falling to $1/e$th value in each increment of $0.63h$ in the distance.

As an example of the range at which modulated light signals can be detected, we may assume the following conditions. An airplane flies at a height y of 250 m. in a fog layer $h = 500$ m. thick, with a free path $\lambda = 50$ m. The ground is snow-covered, and on the ground is a modulated light beacon of 5000 cp., $L = 60{,}000$. By Equation [41] we find that the brightness in candlepower per square centimeter at various horizontal distances x from the beacon is 1.2×10^{-6} at 1 km., 3.7×10^{-8} at 2 km., 1.3×10^{-9} at 3 km., and 5×10^{-11} at 4 km. Thus since 4×10^{-11} cp. per sq. cm. can be detected, the signal could be picked up at a distance of 3 to 4 km. from the source. The directional ratio of about 20 per cent should give good directional effects at distances less than 3 km. The brightness H_s at the top of the cloud layer is $3\lambda/h$, or 0.3 of that at the height of 250 m., so that the ranges of detection just found are approximately independent of the height of the plane within the layer.

Case b: Black Ground ($R = 0$). By the method of images, this problem is equivalent to that of finding the potential of a series of dipoles arranged in a vertical line with uniform spacing of $2h$. Each dipole consists of an upper positive source L and a lower negative source $-L$, with a distance $2b$ between them.

If the height of the source above ground is small compared to h, the thickness of the fog layer, then the brightness H at any point (x, y) is:

$$H = \frac{3L_u b}{2^{3/2}\pi\lambda h^{3/2}x^{1/2}} \sin\left(\frac{\pi y}{h}\right) \exp(-\,\pi x/h) \quad \text{[45]}$$

L_u is again the total light thrown upward by the beacon.

As in Equation [37], b is the effective height $b_o + (^2/_3)\lambda$ of the beacon.

Thus if the beacon is practically at the ground level, so that $b = (^2/_3)\lambda$, we have:

$$H = \frac{L_u \sin(\pi\, y/h)\,.\,\exp\,(-\pi\, x/h)}{2^{1/2}\,\pi h^{3/2}\, x^{1/2}} \quad \text{[46]}$$

A calculation of the directional ratio gives for $x \gg h$:

$$\frac{\Delta H_s}{H} = 4\pi\lambda/3h \quad \text{[47]}$$

which is twice as great as that given by Equation [42] for $R = 0$.

The brightness H_s at the top surface of the fog directly over the beacon is:

$$H_s = 0.4262 b L_u/h^3 \quad \text{[48]}$$

At horizontal distances x, large compared to h, the brightness of the top surface is:

$$H_s = \frac{2^{1/2}\, L_u\lambda\, \exp\,(-\,\pi x/h)}{3\; h^{5/2}x^{1/2}} \quad \text{[49]}$$

A rough integration of the total light flux L_s in lumens from the upper surface of the fog gives:

$$L_s = 2\,L_u b/h \quad \text{[50]}$$

It should be noted that the logarithmic decrement in the brightness as x increases is twice as great as in the case that the ground is white.

As an example let us take as before a modulated light beacon of 5000 cp. ($L_u = 60{,}000$). For various values of h and x we have calculated H at the height $y = 0.5\,h$ by Equation [46] for the case that the beacon is close to the ground. Table 8 gives the distances at which the brightness H has the values specified in the first column. With the photoelectric receiver capable of detecting 4×10^{-11} cp. per sq. cm., ranges of 1 to 3 km. are possible. At heights other than $0.5h$ the results are not greatly different.

TABLE 8

(Distances in meters at which the light from a 5000-cp. beacon, with reflector, will produce given brightness of illumination in the interior of a fog layer of thickness h covering dark-colored ground)

H (cp. per sq. cm.)	$h = 250$ m.	$h = 500$ m.	$h = 1000$ m.
10^{-8}	580	950	1500
10^{-9}	760	1300	2180
10^{-10}	930	1640	2870
4×10^{-11}	1000	1780	3140

A tenfold increase in the output of the beacon or a tenfold increase in the effective height b would produce only a 20 or 30 per cent increase in the range at which the light from the beacon could be detected by the photoelectric receiver.

Comparing these data with those calculated for the perfectly reflecting ground, we see that the effect of making the ground black is to reduce the range to about 42 per cent of its value for perfectly reflecting ground.

5 *Reflection From and Transmission Through a Cloud Layer:* The case that we have just considered of a thin layer of fog in contact with the ground is probably the most unfavorable that can be found for light transmission inside of the fog layer. If, however, the aviator flies above the fog layer, he can make use of

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[13] E. Madelung, *Phys. Zeitsch.*, vol. 19, pp. 524–532 (1918).

the spot of light which is transmitted up through the fog according to Equation [50]. If the upper surface of the fog is horizontal, the light flux emitted from this spot in a direction inclined at an angle α above the horizon will be proportional to sin α. The light flux density F at any point which is at the distance r (along inclined line) from the center of the spot and at the height y above the top of the fog layer is, thus:

$$F = L_o y/\pi r^3 = 2\,L_u by/\pi hr^3 \ldots\ldots\ldots\ldots [51]$$

The range of the signals that may be transmitted in this way is far greater than those within the fog layer itself. For example, putting $L_u = 60{,}000$, $b = (^2/_3)\lambda$, $\lambda = 50$ m., $h = 250$ m., we have $F = 5100\ y/r^3$. If we take $y = 2.5 \times 10^4$, (250 m.) and $F = 1.2 \times 10^{-10}$ corresponding to the threshold ($H = 4 \times 10^{-11}$) of the photoelectric receiver, we see that we should be able to detect the modulated light transmitted up through the fog at a distance of 10.2 km. as compared to the distance of only 1 km. within the fog layers, as given in Table 8.

If there are other layers of cloud above the fog on the ground, the light reflected down from them will greatly increase the range and will also make it possible to detect the modulated light within the fog layer at distances far greater than those given in Table 8.

If the bottom of the fog layer is at a height y_o above the ground, then the light L_u from the beacon forms a spot of light on the bottom of the cloud which reflects the greater part of the total light. Thus if the plane flies below the cloud layer, the range may be calculated by the first two members of Equation [51], where L_o is now to be replaced by L_u and y is the height of the ceiling above the plane.

When the cloud layer is definitely off the ground, the transmission of light through it is given by Equations [35*a*] and [36*a*]. The fraction of the light output of the beacon which escapes from the top of the cloud layer is equal to H_T/H_s. By Equation [35*a*] this is:

$$H_T/H_s = 1/\left[1 + (1 - R)\,(1 + 3h/4\lambda)\right] \ldots\ldots [52]$$

When $R = 0$ and h/λ is large, this reduces to the same value as the ratio L_o/L_u given by Equation [50] for the case of the fog lying on black ground. Equation [51] is thus still applicable for the detection of light transmitted through a cloud layer which is at a considerable height above ground.

In this case the brightness within the cloud layer will extend for relatively large horizontal distances from the source. The brightness H at any point may be calculated by linear interpolation between H_B and H_T, which are given by Equations [35*a*] and [36*a*]. The brightness H_s, which is to be used for this purpose, is equal to

$$H_s = L_u\, y_o/2\,\pi^2 r^3 \ldots\ldots\ldots\ldots [53]$$

where r is the inclined distance from the light source to the given point at the bottom of the cloud. In most applications where $r \gg y$, we may put $r = x$, the horizontal distance from the source.

Under these conditions we find by partial differentiation of Equation [53] and combining with Equations [24] and [30] that the directional ratio is

$$\frac{\Delta H_x}{H} = \frac{4\lambda}{x} \ldots\ldots\ldots\ldots [54]$$

which is $3h/\pi x$ times as great as that given by Equation [47] for the ratio inside of a fog layer on the ground.

As an example let us consider a cloud layer of a thickness of $h = 500$ m. whose bottom surface is $y_o = 500$ m. above the ground, which has the reflectivity $R = 0.2$. Let $\lambda = 50$ m. and $L_u = 60{,}000$ lumens. By Equation [53] we find $H_s = 1.5 \times 10^8/r^3$ cp. per sq. cm., and then by Equations [35*a*] and [36*a*] we get $H_T = 1.3 \times 10^7/r^3$ and $H_B = 1.09H_s = 1.65 \times 10^8/r^3$. The directional ratio is $\Delta H_x/H = 2 \times 10^4/x$. Placing $H_T = 4 \times 10^{-11}$, we find that a plane flying near the top of the cloud layer should be able to detect the modulated light at a distance $x = r = 6.9$ km. Near the bottom of the cloud layer the range would be 16.1 km. At distances of 1.65 and 3.8 km. (top and bottom, respectively) the brightness would be 3×10^{-9} (75 times threshold) and the directional ratios would be 0.121 and 0.053, respectively, whereas 0.015 can be observed with this brightness.

Effect of True Absorption of Light Within a Fog. So far we have considered the scattering of light within a clean white fog in which absorption is negligible. It may be of interest to deal briefly with the modifications that occur if there is also an absorption of light such as exists, for example, in a smoky fog.

In this case, in each unit of volume, an amount of light will be absorbed (destroyed as distinguished from scattered) which will be proportional to the light intensity H at that point. The absorption may be measured in terms of a volume-absorption coefficient A, which is the light absorbed per unit volume per unit light intensity. Thus AH lumens of light are absorbed per unit of volume.

The absorbing power of the fog may also be described in terms of a mean free path λ_A. Perhaps the best way of defining this quantity is by means of the relation

$$-\frac{dF}{F} = \frac{dx}{\lambda_A} \ldots\ldots\ldots\ldots [55]$$

where $-dF$ is the decrease in the intensity of a beam of light of flux density F, resulting from absorption, while traveling a distance dx.

It can be shown that

$$\lambda_A = 4\pi/A \ldots\ldots\ldots\ldots [56]$$

Let us now consider three special cases in which H depends only on a single coordinate; thus H depends upon:

(1) the distance r from a fixed plane, $\beta = 0$
(2) the distance r from a fixed line, $\beta = 1$
(3) the distance r from a fixed point, $\beta = 2$.

Here β is a number that has the value 1, 2, or 3 for the three cases.

Then by Poisson's equation we have for all three cases:

$$\frac{d^2H}{dr^2} + \frac{\beta}{r}\frac{dH}{dr} = \frac{3H}{\lambda_A\lambda_s} \ldots\ldots\ldots\ldots [57]$$

where λ_s is the free path for scattering, i.e., the quantity which we have previously designated by λ.

If we impose the boundary condition that $H = 0$ at $r = \infty$, the integration of Equation [57] gives for:

(1) *Plane Case* $\beta = 0$:

$$H = H_o \exp\left[-(3/\lambda_A\lambda_s)^{1/2}\,r\right] \ldots\ldots\ldots [58]$$

where H_o is the brightness of illumination at the fixed plane $r = 0$. Differentiation and substitution into Equation [24] gives the flux density F_o at $r = 0$

$$F_o = 4\pi(\lambda_s/3\lambda_A)^{1/2}\,H_o \ldots\ldots\ldots\ldots [59]$$

(2) *Cylindrical Case* $\beta = 1$:

The exact expression for this case involves Bessel functions. However, in practical cases where $(3/\lambda_s\lambda_A)^{1/2}\,r$ is larger than unity, a sufficiently close approximation is given by

$$H = 0.03618\,\lambda_A^{1/4}\lambda_s^{-3/4}\,r^{-1/2}\,L_1 \exp\left[-(3/\lambda_A\lambda_s)^{1/2}r\right] \ldots [60]$$

where L_1 is the light output from the line source in lumens per unit length.

(3) *Spherical Case* $\beta = 2$:

$$H = (3L/16\pi^2\lambda_s r) \exp [-(3/\lambda_A\lambda_s)^{1/2}r] \qquad [61]$$

where L is the total output from the point source in lumens.

In the absence of absorption, Equation [26] gives

$$H = 3L/16\pi^2\lambda_s r \qquad [62]$$

Comparing Equations [58], [60], and [61] with [62], the combined effect of scattering and absorption is seen to give an effective free path equal to $0.577\ (\lambda_A\lambda_s)^{1/2}$. Because of the presence of the exponential factor in these equations, it is obvious that useful signals can never be transmitted to distances greater than about $10 \times (\lambda_A\lambda_s)^{1/2}$.

We have now considered the various factors which seem to be of greatest importance in determining the transmission of diffused light signals from ground to an airplane. It seems possible to draw the definite conclusion that, with sources of reasonable intensity and with photoelectric detectors which can now be built, signals may be transmitted through dense white fog. The presence of smoke may greatly decrease the range.

Discussion

S. Herbert Anderson.[14] This paper is a very important contribution—if not the most important one yet made—to the use of light signals in aviation. As a result of research and compilation there is gathered here a fund of information which should be of greatest value to the engineer and physicist interested in the problems of signaling through a thick atmosphere. Furthermore, the results obtained are of great value wherever signal lamps are used—in ocean navigation, street traffic, railway signaling, and military and naval signaling. So far as the writer knows, this is the first time that an attempt has been made to determine the illumination or flux density required for threshold visibility of lights of various types (point sources, white and colored, and diffuse sources) over such a wide range of background brightness. Data of this nature are of fundamental importance in the design of beacons for both clear and foggy atmospheres. Of equal importance is the investigation with flashing signals of the threshold illumination as a function of the period of the flash and the time interval of the flash.

In all investigations in which the results are dependent on the reaction of individuals to stimuli to the sense organs, as large a number of observations by as large a number of normal individuals as is feasible should be made, in order to establish relationships which may be used with reasonable assurance of their applicability. It would seem that the importance of some of the conclusions given in the paper warrants a larger number of observations than were possible in this single investigation. Such relations as:

a The threshold flux density from a steady point source as a function of background brightness, Equation [5],

b Relative visibility of various colors with various background brightnesses, Fig. 6,

c The threshold flux density of a flashing point source as a function of the period and duration of the flash, Equation [7],

d The optimum period and duration of the flash for given flux densities and background brightnesses, and

e The visibility of flashes of diffused light, Fig. 9.

are worthy of further investigation and might well be taken up by some such organization as the Bureau of Standards.

From their study of threshold flux density and the effect of duration and period of the flash upon visibility, the authors have computed the effective range of the standard of rotating beacon as a function of the absorption coefficient of the atmosphere and the period of the beacon, as given in Table 3. In order that such data should serve their purpose we must know how the absorption coefficients correspond to the various states of the atmosphere: haze, light fog, moderate fog, and dense fog. An attempt has been made to supply this information by giving the "visibility" for each value of λ used, as 2.3λ. If "visibility" is used in the sense employed by observers of the Weather Bureau, viz., the distance at which dark-colored objects can just be distinguished, it seems to the writer that the factor 2.3 is much too small. This opinion is based on some observations made in an attempt to determine the transmission of a beam of light through a fog when a dark-colored object is just visible. Of course the visibility of an object in a fog depends upon the illumination on the object as well as the absorption of light by the fog. Photometric measurements of daylight illumination on a vertical surface in natural fogs give values ranging from 60 to 1300 foot-candles with an average of about 500 foot-candles. Observations in a laboratory fog chamber using this illumination showed that an object was just visible when the transmission was $^1/_{1000}$th. This gives a value of visibility $= 6.91\lambda$ rather than 2.3λ.

In the airways work of the Weather Bureau the following classification is used.

a Light fog—visibility greater than $^3/_4$ mile

b Moderate fog—visibility greater than $^1/_5$ mile, less than $^3/_4$ mile

c Dense fog—visibility not greater than $^1/_5$ mile.

Using these distances for visibility, the corresponding values of λ can be computed.

Quality of fog	Visibility	λ
Light	1.2 km.	1.74×10^4 cm.
Moderate	0.8 km.	1.15×10^4 cm.
Dense	0.32 km.	4.6×10^3 cm.

According to these calculations, a value of $\lambda = 2 \times 10^4$ cm. indicates a light fog in which the visibility is somewhat greater than 1 km.; and $\lambda = 5 \times 10^3$ cm. indicates a dense fog in which the visibility is about $^1/_3$ km. The corresponding effective ranges of the standard beacon with a 10-sec. period are 1.92 km. for a light fog and 0.42 km. for a dense one. It would be interesting to know how these computed values correspond to those observed by experienced pilots. Visibility $= 6.91\lambda$ may be too large. A value in between this and 2.3λ may be better.

It would be worth while to carry out a series of observations involving simultaneous measurements of transmission of light through fog and distances of visibility, from which a relation between mean free path of light rays and visibility could be established. With this relation Table 3 could be used with considerable confidence.

The most interesting feature of this investigation, and probably the most important, is the development of an extremely sensitive photoelectric receiver for light. A light-sensitive instrument which is from 6 to 13,000 times as sensitive as the eye (depending on the background brightness) is truly remarkable. This should prove a valuable addition to the equipment for fog navigation. Whether or not it will have the range which is claimed for it depends not only on its sensitivity, but upon the way light is diffusely propagated through fog.

This brings us to the most surprising feature of the paper—the theory of diffusion of light in fog. It is well recognized that light is readily propagated in a diffused manner through fog. But it is no less than startling to have it pointed out that under certain conditions (a light source at the center of a large cloud of fog) the flux density may be *six* times as great in a fog as in clear air. Even the practical case of a light on a 50-meter mast and assuming zero reflection from the ground is very surprising, showing a greater flux density than in clear air.

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[14] Signal Corps Laboratories, Fort Monmouth, Oceanport, N. J.

If the fundamental assumptions on which this theory is based are sound, the theory must be correct. As the writer understands the presentation, it is assumed that in a clean water fog there is no loss of light energy in the diffusion by the fog droplets. If a certain quantity of light falls on a droplet, the same quantity leaves it, the only change being in the direction of the light ray.

It seems to the writer that this assumption is questionable. The scattering of light by a fog droplet is by reflection and refraction. In refraction some light passes through the water drop, and this is accompanied by a small absorption. A loss in reflection is more difficult to check on. But there may be a loss here too. If there is a loss of light energy in this diffusion process, the propagation through fog is accompanied by a true absorption. If this is the case, the theory as presented needs some modification. This theory can be and should be tested as pointed out on page 15 ante.

On page 13 the following statement is made: "The large size of the drops (fog drops) indicates that the scattering of light by fog should be the same throughout the visible and ultra-violet spectrum, and in the infra-red a decrease in scattering should occur only when wave lengths of about 20 microns are reached." The work which the writer did on the transmission of radiation through fog[15] indicates that a marked decrease in scattering occurs for wave lengths much shorter than 20 microns. This is shown in Figs. 6 and 7 of the paper referred to. This is also shown well by using the concept of the "mean free path," λ_0, of radiation through fog. Take a fog of such density that $\lambda_0 = 6.1$ m. for light of wave length 0.55μ. For a wave length of 2.6μ the value of λ_0 is 108 m., showing a much larger free path and, hence, less scattering. This work corroborates the statement on page 13 for visible radiation, but not for infra-red.

Authors' Closure

By Mr. Langmuir. A point source of intensity C candles if seen through a thickness x of fog will appear to be of an intensity $C \exp(-x/\lambda)$. Whether or not it is visible at a given distance depends mainly upon the "background brightness," which in this case means the intensity of illumination of the fog either by the given source or by other sources which may be present. Such problems can be readily worked out quantitatively from the principles outlined in this paper.

A *small* white diffused reflecting surface in a fog uniformly illuminated to brightness H will also have the brightness H and is thus invisible, although the light that comes to the observer from the surface corresponds to a brightness $H \exp(-x/\lambda)$. This means that the brightness $H\,[1 - \exp(-x/\lambda)]$ is supplied by diffusion from the fog particles that lie between the observer and the source.

If now the white surface is replaced by a black surface of the same size and position, the *apparent* brightness of its surface is $H\,[1 - \exp(-x/\lambda)]$. It is well known that surface brightnesses which differ less than 1 per cent cannot be distinguished by the eye under favorable conditions, as for example in a Lumner-Brodhun photometer. Thus the black object in a *uniformly illuminated fog* cannot be seen if $\exp(-x/\lambda)$ is less than 0.01 or if $x > 4.6\,\lambda$. It is more likely under ordinary conditions that the intensity difference needed for "visibility" will correspond rather to 10 per cent difference, giving $x < 2.3\,\lambda$.

A white card with a black cross in uniformly illuminated fog will evidently be visible at the same distance as the black cross alone. But if the card is illuminated by a separate source so that its white parts are brighter than the fog, the visibility may be greatly increased.

It is understood that in Mr. Anderson's fog-chamber experiments the object was illuminated, but the fog was not, and therefore the visibility that he observed was far greater than would be observed under more nearly normal conditions.

When light travels in a fog between two points at a distance x apart, the number of "collisions" of the light ray with fog particles is approximately $(x/\lambda)^2$, and if d is the diameter of a droplet, the total distance which the light travels through liquid water will not exceed $(x/\lambda)^2 d$. If we take $\lambda = 30$ m. or 3×10^3 cm. and $d = 3 \times 10^{-3}$ cm. and $x = 1$ km. (10^5 cm.), we find that even in this extreme case the light travels through only 3 cm. of water. Thus if the water of the droplets is *clean*, no appreciable absorption is to be expected.

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[15] "Transmission of Visible, Infra-Red, and Ultra-Violet Radiation Through Fog," by S. Herbert Anderson. Presented at Fifth National Meeting of the A.S.M.E. Aeronautic Division, Baltimore, Md., May 12–14, 1931. Published in Trans. A.S.M.E., vol. 54 (1932), as Paper AER-54-1.

AER-54-22

Airship Factory and Dock of the Goodyear Zeppelin Corporation

By WILBUR J. WATSON,[1] CLEVELAND, OHIO

The paper describes the airship factory and dock at Akron of the Goodyear Zeppelin Corporation, covering its design and construction, a study of wind pressures, a comparison with other hangars, the docking problem, the design of the doors and the roof covering, with the details of the preparation of the site, the substructure, the structural-steel fabrication and the erection methods employed, the mechanical and the electrical equipment, and the floors, the fire protection, and the helium storage.

THE airship factory and dock of the Goodyear Zeppelin Corporation at Akron, Ohio, completed in 1929, was designed and built to construct and house two new airships for the United States Navy, the *ZRS-4* and *ZRS-5*, since named *Akron* and *Macon*. These airships have a length of 785 ft and a diameter of 133 ft and a gas capacity of 6,500,000 cu ft of helium gas, and are much the largest airships ever built. The building, however, has been planned to house an airship considerably larger than these. It can easily accommodate an airship of 10,000,000 cu ft capacity.

The principal purpose of this building is to serve as a factory for airship construction, and it is therefore thoroughly equipped with cranes, runways, elevators, etc. While essentially a factory building, it may be used in the future as a dock for ships in service, and this possibility has been borne in mind by the designers.

Airship Design and Construction

The name Zeppelin is synonymous, in the public mind, with the airship, for to Count Ferdinand von Zeppelin is due the greatest credit for the development of lighter-than-air craft. Zeppelin began the construction of rigid airships about 1900 at Friedrichshafen, on Lake Constance, and at the outbreak of the World War in 1914 had already built many such crafts. Count Zeppelin's first ship was 410 ft long by 38 ft maximum diameter and contained 400,000 cu ft of hydrogen. Compare this with the *Graf Zeppelin*, 776 ft in length by 100 ft diameter and 3,700,000 cu ft capacity.

The war gave a tremendous impetus to the development of these ships, due to their value for scouting and bombing operations. Although about 115 zeppelins had been built at Friedrichshafen at the close of the war, most of them had met various fates largely due to the hazards of warfare. Until Zeppelin undertook the development of the rigid airship, but little advance had been made in lighter-than-air craft since the invention of the hot-air balloon by the brothers Montgolfier and the hydrogen-inflated balloon by Charles in the later years of the 18th century. It was in a montgolfiere that Benjamin Franklin made an ascent in France. The free, or non-rigid, balloon is simply a bag of gas, usually hydrogen, carrying a wicker basket supported from the bag by a network of ropes. A semi-rigid airship has a structural keel which carries the cabin, the gas cells above being enclosed in an envelope which has no supporting framework.

The rigid airship has a complete metallic framework of rigid construction, consisting in the zeppelin essentially of longitudinal girders holding vertical rings, between which are placed the separate gas cells. The structural material used for most of these rigid airships is duralumin, an alloy of aluminum with copper, manganese, and magnesium that possesses approximately the strength of structural steel, but has the advantage of much lighter weight. In the design of structural members every possible way of decreasing weight without sacrifice of strength is resorted to, all unnecessary metal is removed by punching, and all shapes are carefully designed for minimum weight. The fabric for the gas cells used in the earlier ships was gold-beater's skin, mounted upon a cotton base. No gold-beater's skin is used in the *Akron*, the cells being constructed of a strong cotton fabric especially treated to make it gas tight.

The gas used in all countries except the United States is hydrogen, which is an inflammable gas. In this country, however, helium is used. Helium is an inert gas, non-explosive and nonburnable, possessing only about 83 per cent the buoyancy of hydrogen, but is much more safe. It is found at several places in the Western states, in combination with nitrogen. The lifting capacity of helium gas is 0.0656 lb per cu ft, assuming the gas to be 94 per cent pure, 60 deg air humidity, 29.95 in. barometric pressure, and 32 F air temperature.

The method of assembling an airship of the zeppelin type at Akron is, first, to erect the rings in a horizontal position on the floor of the hangar, raising them into vertical position by hanging them from the structural framework of the building, and then assembling the longitudinal girders and other members by means of platforms carried by overhead cranes and supported by movable scaffolding on the hangar floor.

It is not the intention to describe in detail the principles of airship construction in this paper, but only to give a general picture, as it affects the design of the factory and dock. In Table 1 are compared the main features of the larger airships.

Development of the Building Plan

The general shape and plan of the building is the development

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[1] Engineer, Wilbur Watson & Associates. Dr. Watson was born at Berea, Ohio, on April 5, 1871. He received the degrees of B.S. and C.E. from the Case School of Applied Science, Cleveland, Ohio. He was in the engineering department of the Lake Shore & Michigan Southern Railway, 1889–94; design and supervision of construction, Osborn Engineering Company, Cleveland, 1898–1907; private practice as consulting, designing, and supervising engineer for bridges and buildings, Cleveland, including the Cleveland Freight Terminal for the New York Central Railroad. Dr. Watson was one of five engineers selected by the American Institute of Steel Construction to formulate a standard code for structural-steel buildings and was elected an honorary member of the institute in recognition of this service. Recent work executed by the firm includes the airship factory and dock at Akron, Ohio, for the Goodyear Zeppelin Corporation, the Northern Ohio Food Terminal at Cleveland, and the Lorain-Carnegie Bridge (now nearly completed). Dr. Watson is the author of "Bridge Architecture." The honorary degree of Doctor of Engineering was conferred upon him by the Case School of Applied Science in April, 1930.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 PRINCIPAL CHARACTERISTICS OF "LOS ANGELES," "GRAF ZEPPELIN" AND "ZRS-4"

	"Los Angeles"	"Graf Zeppelin"	"ZRS-4"
Nominal gas volume, cu ft	2,470,000	3,710,000	6,500,000
Length overall, ft	658.3	776	785
Maximum diameter, ft	90.7	100	133
Height overall, ft	104.4	113	147
Gross lift, lb	153,000	258,000	403,000
Useful lift, lb	60,000	.....	182,000
Number of engines	5	5	8
Total horsepower	2,000	2,750	4,480
Maximum speed, mi per hr	73	80	84
Range without refueling at 50 knots cruising speed, nautical miles	4,000	6,125	10,580
Fineness ratio	1:7.2	1:7.7	1:5.9

of many years of study on the part of Dr. Karl Arnstein[2] and his assistants, in the endeavor to determine upon the most efficient type of building to meet exacting aeronautical requirements at a minimum cost.

The most difficult part of airship operation at present is the launching and docking of the ship, and it is essential that the building which serves as its shelter should cause the least practicable interference with the normal wind current, in order that the launching and docking operations may not be complicated by cross-currents created by the building itself or by the open doors. It is this consideration that suggested the shape of this building to Dr. Arnstein.

Hangars of similar design, although much smaller, had been built and operated at Dresden and Liegnitz in Germany.

The shape of this building is described in mathematical terms as two semi-paraboloids connected by a parabolic cylinder; that is to say, sections taken across it form parabolas, and its longitudinal section also forms two half-parabolas connected by a straight line.

Its shape has been described as a half of an egg, but a better simile would be a half of a silkworm's cocoon, cut in two the long way.

The length is 1175 ft between center lines of door tracks, width 325 ft center to center of arch pins, and its height $197^1/_2$ ft from center of lower to center of top pins. The height from the floor to the platform at the top is 211 ft.

The floor area is 364,000 sq ft, much the largest single uninterrupted floor area yet built. The volume is approximately 55,000,000 cu ft.

A Study of Wind Pressures

The correct distribution of wind pressures on a given structure, as well as its intensities, is not as definitely determined as one could wish. The usual American practice calls for a wind pressure on buildings of 30 lb per sq ft, although values ranging from 20 lb to 60 lb are often used.

When designs were started for the Akron hangar, early in 1928, no American experiments on buildings of comparable shape had been made to determine the amount of suction forces or negative pressures due to the creation of partial vacuums caused by the deflection of the air currents.

In the design of sheds, or hangars, for airships, the wind pressures become of paramount importance, due to the fact that the stresses produced in the structure by wind pressures are relatively greater than is the case of most buildings and also to the unusual height and exposure of the structures. It becomes, therefore, of great importance to determine as accurately as possible just what these pressures may be.

In order to obtain some experimental data relative to the action of wind on a building of the shape proposed for the Akron hangar, Dr. Arnstein caused a model to be made about 5 ft in length and had a series of tests conducted in the 9-ft wind tunnel of the Daniel Guggenheim School of Aeronautics at New York University. These tests were made early in 1928 by Dr. Klemperer, of the Goodyear Zeppelin Corporation, and Prof. A. Klemin, of the university staff. A velocity of 50 mph was used for most tests. The shell was perforated with 87 orifices 0.09 in. in diameter, and 22 similar holes were drilled into two of the door segments. To these holes were attached rubber tubes which led to the manometers. The model was tested at various angles to the direction of the wind.

In general, the region where the overpressure changed to suction was found to be about 50 to 60 per cent of the height of the building, and at the region of the doors this became as high as 72 per cent. The suction reached its maximum at a point slightly windward of the apex.

It was found that the opening of the doors affects only the pressure distribution on the doors themselves and in their vicinity, but the central regions or those at the other end are influenced but little, if at all.

In general terms, these experiments showed that a current of air impinging upon an obstacle caused positive pressures in front and suction at the sides and rear, and in most cases the maximum pressures are not positive but negative.

The experiments indicated that the outward force should be assumed at greater than one-half the inward, with provision for securing roofing against local outward forces equal to the full direct pressure. American practice in designing buildings to resist wind pressures has heretofore neglected the importance of these suction or outward pressures.

Observations on air currents at the Lakehurst hangar have demonstrated that the currents around the open doors (sliding type) may obtain a velocity twice that of the prevailing wind. As the launching or docking of an airship cannot safely be carried out against a wind greater than a certain maximum for each case, the great importance of eliminating these locally increased currents can readily be seen.

As one writer has expressed it, "What ever flies, roosts;" and up to this time no roosting place has yet been evolved for the airship to take the place of a covered building, called a "hangar" in Germany and France, a "shed" in England, and either a "hangar" or, preferably, a "dock" in America. It is vitally important, therefore, that the ship should have access to its roosting place in the worst possible navigating weather.

Orientation. As it is desirable that airships should head into the wind when landing, the longitudinal axis of the hangar should coincide with the direction of the winds prevailing during flying weather. For the location selected at Akron it has been found that the proper orientation is north 30 deg east by south 30 deg west, approximately.

Comparison With Other Hangars

The original zeppelin shed at Friedrichshafen, built in 1908–1909, is $603^1/_2$ ft (184 m) long, 150.8 ft (46 m) wide, and 65.6 ft (20 m) high. Later a larger hangar was built, having a length of 787.2 ft (240 m), a width of 150.8 ft (46 m), and a height of 114.8 ft (35 m). These hangars, and most others built in Europe, are of structural steel arches, with sliding doors at the ends. The Luftschiffbau Zeppelin Corporation has recently completed a new hangar at Friedrichshafen equipped with cylindrical doors instead of straight sliding doors, a brief description of which is given later.

At Orly, France, two reenforced concrete hangars have been built, having a length of 984 ft, a span of 298 ft, and a height of 194 ft 6 in. These hangars have not yet been equipped with doors. The cross-section is parabolic, similar to the Akron building, and the structure is made up of reenforced concrete ribs connected with reenforced concrete curtains between the rings. The design is by M. Freysennet, a noted structural engineer.

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[2] Vice-President and Chief Engineer, Goodyear Zeppelin Corporation.

All of these hangars have vertical doors that open horizontally, from the center outward. The "orange-peel," or quadri-spherical door, a distinctive feature of the Akron dock, never had been used except for a few much smaller buildings in Germany.

New Friedrichshafen Hangar. As the original sheds at Friedrichshafen, built by the Luftschiffbau Zeppelin, were comparatively small, the latest building constructed (1929) may be taken as representative of the best thought of the original German designers of the dirigible airship.

The length of this latest addition to the Friedrichshafen plant is 815 ft inside, the width 165 ft $3^{3}/_{4}$ in., and the height 157 ft. The floor area is 150,000 sq ft and the cubical contents about 18,000,000 cu ft working space, or a total of 22,000,000 cu ft.

In design, this structure is composed of a series of three hinged arches, with the lower pins only slightly raised above the ground, about 12 ft. The arches, 22 in number, are equally spaced at 10 m (32.8 ft). The windows are about 3 m (10 ft) wide by 10 m (32.8 ft) high and are spaced two to each panel in the vertical sides, which are carried up to a height of about 100 ft. The roof profile is broken.

The distinctive feature of this design is the curved end doors, not spherical, as at Akron, or straight, as at Lakehurst, but circular with straight vertical sides. The great advantage of these doors over the straight doors is in the lessened obstruction to wind. The roofing is of pumice-stone concrete, reenforced, with asphalt-composition roofing. Two stairways and one elevator are installed. Sidewalls are of brick to a height of about 100 ft. Floors are wood and concrete.

Karachi Airship Shed. The airship shed at Karachi, India, completed in 1927, may be taken to represent the latest British practice in the design of airship hangars, as the shed at Cardington, England, is quite similar.

The Karachi shed has a clear length, inside, of 850 ft, a clear width of 200 ft, and a clear height of 170 ft. The trusses are three-hinged arches of 230-ft span and 200-ft rise, with the lower pins at the floor level instead of being supported upon low towers, as are those at Lakehurst and at Scott Field. These arches are spaced 27 ft 5 in. on centers. The overall width is 230 ft.

There is a rolling door at one end only, the other end being closed by a blank wall, and this door differs materially from those at Lakehurst and Scott Field in that the two leaves are supported at the top as well as at the base, the upper support being, however, independent of the building structure. This door has a base 60 ft wide center to center tracks and is counterweighted. The time of opening doors is from 8 to 10 min.

The floor areas is 195,500 sq ft, and the shed is designed to house an airship of 5,000,000 cu ft capacity. It was intended to be used purely as a docking hangar, not for manufacturing purposes.

Scott Field Hangar. The United States Army hangar at Scott Field, near St. Louis, was built in 1922.

This hangar is 810 ft long, 150 ft span, and 150 ft high at center, all clear inside dimensions. It is composed of 28 three-hinged arches and contains 3200 tons of steel.

The covering is asbestos-protected metal for the sides and concrete tiles for the roof. The doors are of the straight type. Each leaf of the doors contains 135 tons of steel and about 500 tons counterweight. About 3 min is required to open the doors, which are operated by four 220-hp, 220-v, d-c motors.

Lakehurst Hangar. The United States Navy hangar at Lakehurst was built in 1920 to house the navy dirigibles. It is 803 ft long, 262 ft wide, and 172 ft high, clear inside dimensions. The end doors are of the counterbalanced sliding type, 350 ft wide and 200 ft high. The frame is of structural steel, and the covering is asbestos sheets for the sides and treated timber for the roof. The framework is composed of three-hinged arches, having a span of 264 ft and rise of 131 ft center to center of the pins. The upper pin is 195 ft above the floor.

The Docking Problem

One of the most serious problems encountered in the development of large rigid airships is that of landing and of handling the ship from the ground.

The hangars at Friedrichshafen are equipped with docking rails, which carry dollies, or "hand-operated trucks, arranged to resist the quite considerable upward and sideward pull on the cables that are attached to the ship's sides and held in quick-release clutches on the dollies.

These docking rails are spaced a distance apart equal to about $1^{1}/_{2}$ times the maximum diameter of the ship, running the entire length of the building, through the doors, and out into the landing field a distance of about twice the ship's length, at both ends of the dock.

As it is very desirable that there be no projections of any kind above the floor of the dock or above the ground level of the landing field, on account of the danger of men tripping, obstruction to airplane maneuvers outside, and interference with manufacturing operations within the building, the accepted form of docking rail has been of the slotted type, the construction being somewhat similar to the center slot of a street tramway. The dollies have wheels both above and below the slot. The objections to this slotted rail are obvious—the difficulty of obtaining a uniform slot, difficulty of maintaining it so, and the accumulation of dirt and ice or snow which must be removed prior to each operation, often with considerable difficulty.

The United States Navy hangar at Lakehurst is equipped with slotted rails of this type. The British airship sheds have no docking rails, dependence being placed upon the use of greater man power for handling the ships.

In the construction of the Akron dock, provision has been made for docking rails of the slotted type, but before the floor was completed, the designers decided to omit the slots for the present, and that part of the floor was finished in such a way that the slots could be put in later, or their place could be taken by standard-gage railroad tracks, which have since been installed.

Mobile stub masts have been built and successfully used at Lakehurst and at Akron, and it seems assured that mechanical methods are destined to supersede man power in handling airships on the ground.

Design of Doors

The most unusual feature of this building is doubtless the spherical doors, with their hinges and supporting trucks, and the operating mechanism for opening and closing them.

If one can visualize a quarter of a half of an orange peel set up on a flat surface, held with a pin at its pointed top and resting on a set of rollers distributed under the bottom edge, and imagine this one-eighth of an orange peel as 202 ft high and 214 ft wide at the bottom, fastened or hinged at the top point with a huge hollow forged pin and resting on 40 wheels at its base, he would have an idea of what one leaf of the two that close one end of the hangar will be like.

Not considering the snow that may accumulate on the doors or the wind pressure against them, the doors alone weigh about 600 tons for each leaf, or 1200 tons for each end of the building.

Obviously the simplest way to mount this heavy leaf so that it can be moved back and forth is on wheels running on a track, similar to railroad equipment. In this case, 40 forged-steel, double-flanged wheels, 27 in. in diameter, running on the two rails, weighing 100 lb per yd each, set like a railroad track and to the standard railroad gage, are used under each leaf of the doors.

The wheels are spaced uniformly in pairs under the long curved structural-steel sill which constitutes the base of the door. Each pair of wheels is set radially so as not to bind against the curved rails which form the track, but each wheel is mounted on a separate short axle, because if the pairs were arranged like car wheels, there would be great loss of force and excessive wear due to the necessity of slipping one wheel continually, as the outer wheel would have to travel a greater distance than the inner wheel to pass around the circular runway.

Because it is not practicable to so build a large structure of this nature that the weight will be equally divided on the 40 wheels, each wheel is separately mounted under bearing boxes which rest against heavy springs. If one wheel, then, receives more weight than its neighbor, its springs will give more until the weight is more evenly distributed.

The radius of the curved standard-gage track on which the doors are carried is 187.69 ft to the center between the rails from a point projected down from the hinge pin above.

To carry this heavy weight on the ground a curved runway built of reenforced concrete, 10 ft wide and deep enough to go below the frost line, is provided, and under this runway is a series of concrete piles driven to solid rock.

The hinge pins to fasten the top points of the doors to the main shell of the building are secured by heavy steel frames to the roof girders, and a separate pin is provided for each leaf of the door, spaced 4 ft apart, side by side.

The pin is required to resist the side pressure against it caused by the door tending to fall inward, but it does not need to carry any vertical weight whatever, because the door is horizontal at the top where it bears against the pin, except when a strong wind blowing through the building (with doors on the other end open) is of sufficient strength to tip it backward, at which time the pin is required to hold the door from being blown out.

It is estimated that the pressure against the pin may reach a maximum of about 550,000 lb inward or 450,000 lb outward, depending on the snow and winds.

The pin cannot be regarded as being always fixed in the same spot, although tons of metal are used to make it fast, because in a building so large as this the sun's heat will cause expansion and contraction that will move the entire roof and wabble the pin around several inches. To provide for this movement of the hinge pins, the horizontal thrust bearing is made a ball-and-socket joint, free to move in any direction within moderate limits. The pin is 17 in. in diameter and about 6 ft long, and the bearing ball is about 30 in. in diameter. The bearing can move up and down on the pin 4 in. each way to provide for the change in height of the door due to expansion and contraction.

It is especially desirable that the floor of the dock and the yard in front of it should be nearly level, as men handle the ships by hand ropes as they are docked or launched. Anything projecting above the yard level would be in the way of the men. This requirement also makes it necessary to keep the circular track that supports the doors flush with the surface.

To still further maintain a level yard, the runways for the doors are buried so that just the tops of the rails are on the yard level, and the bases of the rails and the ties are all concreted in below the floor line.

For these reasons and, further, because of the important fact that these doors must be under control at all times and not subject to starting off by themselves to swing around and cause wreckage in a heavy wind, it was decided to use a rack-drive system for operating them.

These devices consist of a very large gear, called a "bull gear," with heavy, coarse teeth, mounted horizontally on a fixed concrete base just outside of the dock (one at each corner of the building) and set far enough to the side to be in the clear of the open yard in front. This gear is revolved by a train of gears connected to a 125-hp motor through a worm-gear reduction. The worm-gear reduction was selected because of its characteristic of being "self locking," so that as soon as the power of the motor is shut off, the gear stops and cannot be started again in either direction through pressure of the wind.

The large "bull gear" engages a rack built to surround the base of the door at the bottom, just above the wheels near the ground. This rack is rigidly attached to the door in a vertical position. As the motor revolves the "bull gear" slowly, the rack is driven back and forth and pushes the door along with it.

All of the reduction gears, including the worm gear, are of substantial construction and flexibly set up in a cast-steel frame mounted on a reenforced concrete foundation which is built as a part of the circular runway foundations.

The motor is special in that it is capable of being used at two speeds with a constant torque effort. It can be connected up to drive the doors at a maximum speed of travel of 20 ft per min or at 40, as desired. The brake for holding is on the armature shaft of the motor and is a so-called hydroelectric brake, capable of being set with a time limit so that a second or so can be allowed for the motor to finally slow down before the brake begins to stop it too suddenly.

Electric cutout switches are mounted on the runway adjacent to the driving units, so that as projections from the door operate them, the door comes to a slow-down and creeping speed, automatically as the two leaves come together at the center, or as the doors come to the extreme open position. This makes it only necessary for the operator to push a button to start the door, after which it will open and stop, or close and stop, as the case may be, automatically.

It may be noted that the speed of movement of the doors is rather higher than that of the two other similar hangars in this country, which are geared for about 12 to 25 ft per min as against this construction with 20 to 40 ft per min. This device is protected, however, by the automatic-control feature which compels the doors to slow down to a creeping speed as they approach the ends of their travel, and, further, the wind cannot increase the speed materially by its pressure due to the worm gear and the constant-speed motor. They also can be driven at any desirable speed up to 40 ft per min by manual control.

The driving units are set under separate roofs outside the hangar, for the reason that the pressure of the gear against the rack resists the outward pressure against the wheels due to the door weight, and so that there can be no danger of sparks from the control switches in case hydrogen gas is used in the bags of the dirigibles.

Mention of the outward thrust against the track wheels under the door should perhaps be made and also of its tendency to largely increase the effort needed to start the doors in motion. The wheels are made with a contour similar to that of railroad car wheels to counteract this side thrust, and the motor provided with a heavy torque "pull out" capacity for starting. It is estimated that it may require, at times of extreme snow and wind, a pressure of as much as 50 tons to start the doors in motion, but ordinarily much less.

This at once raises the question as to why roller bearings were not used to materially reduce this effort. While roller bearings would without doubt reduce the starting effort to a fraction of the amount required for the plain bearings, they would similarly increase the more serious problem of control in slowing down and stopping. With roller bearings, one would be inclined to figure for a much less travel speed because of the fact that impact varies as the square of the velocity. The friction assists materially to slow down and stop.

For those interested in the details, it might be well to state that the bull wheel is of cast steel with 22 cut teeth, 9-in. face, 9-in.

circular pitch. The driving pinion is shrouded, has 8 teeth, not cut (chipped smooth). The intermediate gear (cast steel) has 60 teeth, cut 60-in., diametral pitch, 9-in. face. The pinion on the worm-gear shaft has 16 teeth. The worm gear has 46 teeth, $2^{1}/_{2}$-in., circular pitch, $5^{1}/_{2}$ in. face; the worm is $6^{1}/_{4}$ in., diametral pitch, $2^{1}/_{2}$-in. face, double thread, and has a 14° 17″ lead angle. The motor is connected by a flexible coupling and is of 125/62 hp, with a synchronous speed of 600/300 and is arranged for a 440-v, three-phase, 60-cycle current with a capacity of 200/100 hp on a 15-min rating. The entire unit is set up on a continuous bedplate.

Adjustment is provided for setting the "bull gear" in the pitch line of the rack by jackscrews and a spring, allowing limited variation for moderate unevenness in construction of the rack.

The rack in which the "bull gear" is engaged is made from a curved (192′ 10″ radii) 14-in. H-beam with 4-in.-diameter pins set on 9-in. pitch. The rack is rigidly attached to the bottom of the doors.

It is estimated that the door can be opened or closed in 5 min.

Expansion and Contraction

One of the features of this design is the entire absence of expansion joints. The three center arches are fixed in position, while all others are carried upon rollers, placed transversely to the axis of the structure, allowing it to expand freely as a whole, from the center to each end. This allows the end arches, supporting the upper door pintles or hinges, to move laterally about 4 in. under the maximum range of temperature, a motion which is taken up partly on the pins themselves and partly transmitted to the doors, and absorbed by the door frames in bending.

It should be borne in mind that the main-arch shoes are horizontal, and subjected to horizontal forces as well as vertical. The horizontal forces are transmitted to the footings through flat plates on the upper and lower shoeplates that engage deep slots in the rollers.

The vertical reaction on each typical arch shoe due to dead load only is 677,000 lb and the horizontal reaction 200,000, a ratio of 1 to 3.38. The horizontal reaction may, under certain conditions of wind load, be reversed and amount to 119,000 lb inward instead of outward.

An interesting detail in connection with the expansion and contraction of the structure is the arrangement of the pins supporting the doors. These pins are 8 in. longer than the bearing of the castings surrounding them, allowing the top of the door to slide up and down on the hinges, in order to take up part of the movement of the pin in this way. Observations made thus far indicate a range of movement of about 2 in.

The computed rise and fall of the upper arch pins under wind may be 1.68 in., while the horizontal movement may be as great as 9.47 in. The corresponding vertical movements of the arch ribs are 5.46 in., and clearances between the main shell of the building and the doors had to be very carefully computed for all conditions of loading in order to make certain that there would be no interference.

Roof Covering

In selecting the external surface for the factory and dock at Akron, the material required was one that would be fire resistant, comparatively light weight, and of such a nature that it would not rupture under the strains caused by the slight motion anticipated in the structure due to high wind velocities. To accomplish this it was decided to use some form of sheet metal attached directly to the steel girts and purlins. The entire external covering was divided into three types, according to location, which for convenience will be called type A, or "exposed surface roofing," type B, or "steep surface roofing," and type C, or "flat surface roofing."

Type A covering was intended for the surface beginning at the grade line and extending up to the top edge of the skylights, on the sides of the structure, and to a line projected therefrom horizontally around the spherical doors at ends of structure. This covering was to consist of metal sheets, pretreated with a protective covering and designed to withstand an ultimate load of 100 lb per sq ft, either from the outside or inside of the structure with limiting deflection of 2 per cent of the span. This area to have no further covering after erecting except painting.

Type B covering was intended for the surface between the line at the top edge of the skylights, as described, and a line located approximately 40 ft from the center line of the structure, on the sides and projected therefrom horizontally around the doors. This covering was to consist of the metal sheets, except that they were to be designed for an ultimate load of 200 lb per sq ft, either from the outside or inside of the structure, with a limiting deflection of 2 per cent of the span. In order to overcome condensation, this type of covering was to be covered with an insulating material 1 in. thick, securely attached to the metal sheets, and then waterproofed with a standard "steep surface" roof covering, guaranteed for 10 years. As built, this section was not insulated.

Type C covering was intended for the surface on the top of the structure, a width of approximately 80 ft. This roof deck and covering, including insulating, is the same as type B except that the waterproofing is standard "flat surface" roof covering, guaranteed for 10 years.

In order to create high visibility of the structure from the air a color scheme was determined upon. The entire area of type C covering is dead black. Type B area is divided into three horizontal strips of equal width: the upper strip to be chrome yellow; the next Goodyear blue, with the words "Goodyear Zeppelin" and the "Wingfoot" trademark painted in chrome yellow with approximately 22-ft-high letters; and the third strip chrome yellow.

A strip of surface bounded by the top and bottom edges of the skylights, a width of approximately 24 ft and extending around the ends of the shell and the spherical doors, was to be dead black. Below the lower edge of the skylights, the surface to be dead black or light gray in color, depending on the kind of type A covering selected. As built, only the sign with its background has been painted.

The entire area was covered with Robertson V-beam roof deck sheets, No. 22 gage core metal for type A surface and No. 20 gage metal for type B and type C surfaces. These sheets are of sheet steel formed into deep angular corrugations with a flat surface where the corrugation comes in contact with structural supporting members (see Fig. 1). This steel core is hermetically enclosed and sealed in three impervious protective coatings—asphalt, asphalt-impregnated asbestos felt, and a heavy waterproofing envelope. The sheets are all fabricated to the size and shape required for the work, before tar protective covering is applied, and delivered at the work ready for erection.

This sheet was developed by the Robertson Company for this work, and due to the V-beam shape has a greater load-carrying capacity and will span wider spacings of purlins than the standard corrugated sheets heretofore used.

The under side of the deck sheets was painted aluminum gray, full finish, at the factory.

Preparation of the Site

The building is located in a level plain between low hills about 4 miles southeast of the business center of Akron. The site is part of the Municipal Airport of the City of Akron, which owns the surrounding property. About 60 acres, on which is located the dock, is the property of the Goodyear Zeppelin Corporation,

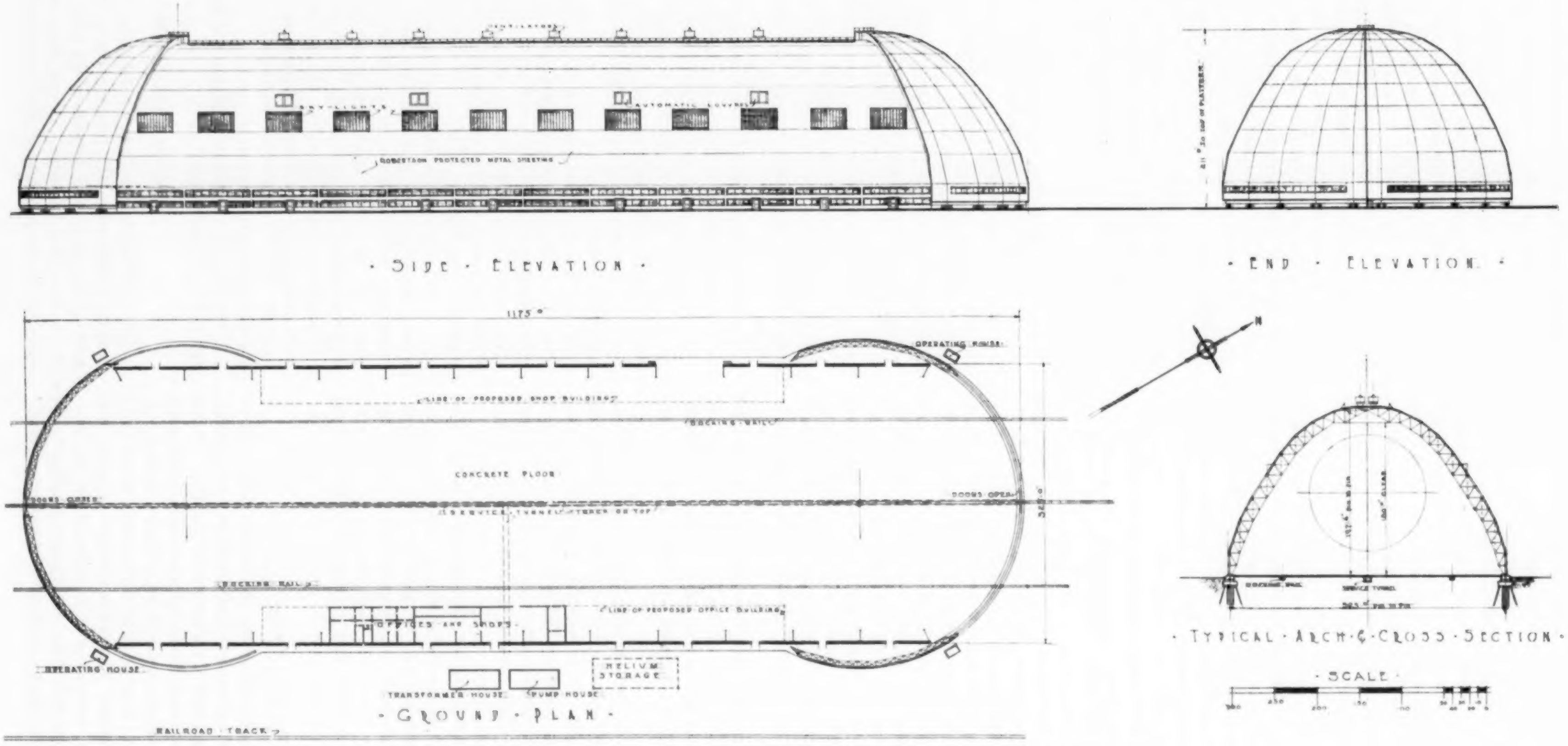

VENTILATORS
SKY-LIGHTS
AUTOMATIC LOUVRES
ROBERTSON PROTECTED METAL SHEETING
· SIDE · ELEVATION ·
211' TO TOP OF PLATFORM
· END · ELEVATION ·
1175' 0"
OPERATING HOUSE
LINE OF PROPOSED SHOP BUILDING
DOCKING RAIL
CONCRETE FLOOR
DOORS CLOSED
DOORS OPEN
SERVICE TUNNEL—TRACK ON TOP
325' 0"
DOCKING RAIL
LINE OF PROPOSED OFFICE BUILDING
OFFICES AND SHOPS
OPERATING HOUSE
HELIUM STORAGE
TRANSFORMER HOUSE
PUMP HOUSE
· GROUND · PLAN ·
RAILROAD TRACK
N
180' CLEAR
DOCKING RAIL
SERVICE TUNNEL
· TYPICAL · ARCH & CROSS · SECTION ·
· SCALE ·
300
250
200
150
100
50
40
30
20
10
0

Fig. 1 Goodyear Zeppelin Corporation Airship Factory and Dock at Akron, Ohio

but the latter has the right to use the surrounding ground for maneuvering and docking purposes.

The preparation of this site required the removal of about 1,500,000 cu yd of earth from the adjacent hills, which was used to fill in the low parts of the site.

The general level of the ground at the building site was about 1038, referred to sea level. The soil consisted of about 2 ft of muck and 28 feet of sand, gravel, and clay, underlaid by a solid sandstone, presenting a fairly level surface. The water level was about 1036, with little chance for drainage below this level. It was deemed best to remove the 2 ft of muck from the site of the building and fill in with about 6 ft of selected gravel and clay from the adjacent hillside, which was deposited in 9-in. layers and rolled. The finished grade of the dock is Elevation 1044.

It is intended to build a mooring mast later as a part of this development. The City of Akron undertook the general preparation of the site, including grading, relocation of the line of the Baltimore & Ohio Railroad through the site, clearing of all buildings, trees, and other obstructions, relocation of highways, providing water and sewers, and carrying out a general drainage scheme.

Substructure of Airship Factory and Dock

The substructure work consists of concrete footings for the arches, carried on vertical and inclined concrete piles driven to rock, concrete ties across the building laid on the sand and clay after removing the muck and heavily reenforced to take the thrust from the arches, concrete door foundations for supporting the rails carrying the doors, which are also carried on concrete piles, a concrete service tunnel the full length and one-half the width of the building, and two lines of concrete docking rail supports. The piles are of the MacArthur type, carrying maximum loads of 30 tons each.

The typical arch foundations are subjected to dead-load reactions of 340 tons vertical and 100 tons horizontal, while the maximum reactions under the worst possible combinations of dead, snow, and wind loads are 472 tons vertical and 168 tons horizontal.

These reactions are taken by reenforced concrete footings 15 ft square and 5 ft 6 in. deep, each of which is carried by 25 piles, of which 10 are battered, the battered piles being the inside and outside lines. The battered piles are inclined about 4 in. to the foot. The footings are tied together by reenforced concrete ties carried entirely across the building, underneath the floor. These ties are 2 ft 3 in. deep by 4 ft wide, and are reenforced with 12 bars $1^1/_4$ in. square.

The tracks for supporting the spherical doors are 10 ft wide by 5 ft 6 in. deep, reenforced with 22 bars $1^1/_4$ in. square and supported on two rows of concrete piles 7 ft apart and 9 ft center to center of piles. The segmental door foundations are tied into the center tunnel and into the docking rail supports at all intersections.

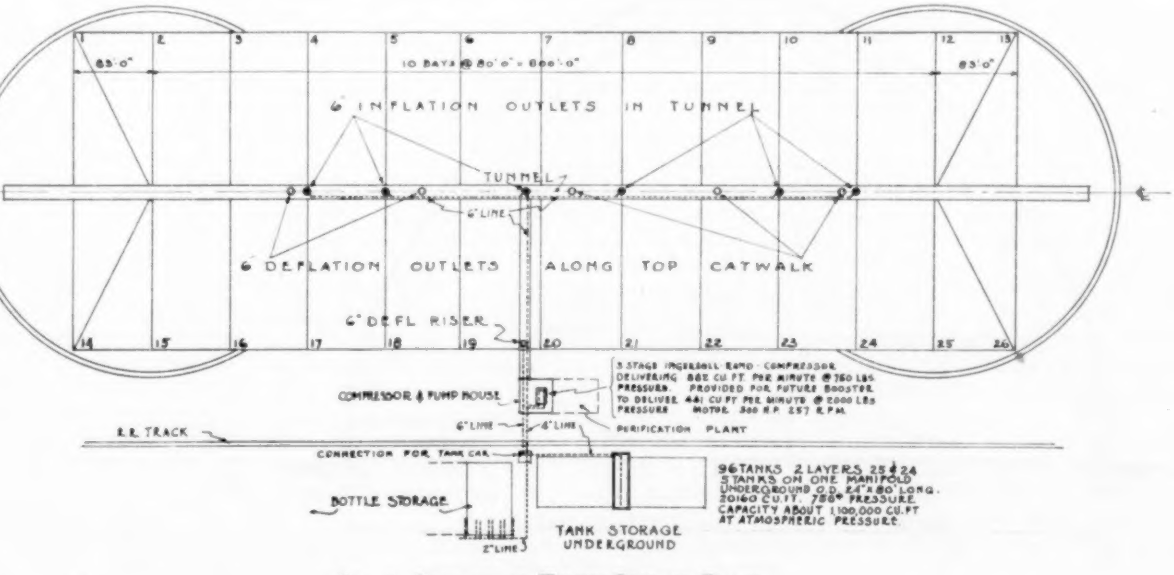
The field control method of proportioning the concrete was used, the contract being based on using six sacks of cement per yard, with provision for adjustment of cost, plus or minus, as the case might be, for more or less cement per yard.

The contractor was instructed to produce a concrete developing a crushing strength of not less than 3000 lb per sq in. in 28 days, using 6 gal of water to the sack of cement. On this basis, a theoretical mix of 1 part cement to 2.96 parts fine aggregate to 3.37 parts of coarse aggregate was established, maintaining these basic proportions of fine and coarse aggregates. Cement content was varied to suit the particular requirements of the different parts of the work. In massive members, slumps as low as 2 in. were used, while in thin members, calling for a very plastic material to properly flow around reenforcement and into forms, slumps as high as 7 in. were used, water control being rigidly maintained in all cases.

Structural-Steel Fabrication

Shopwork on this structure was begun about January 1, 1929, at the Ambridge Plant drawing room of the American Bridge Company, where layouts were prepared and material lists were made. Practically all material was ordered from the mill cut to length. Special care had to be taken in regard to ordering the material for the chord members of the arches, as these were silicon steel, making rectification of mistakes difficult and costly, as silicon steel shapes are rolled only on special orders. After the material was ordered, details were prepared and sent to the shop for fabrication.

In fabricating the arches, these were split up in sections that


1
2
3
4
5
6
7
8
9
10
11
12
13
83'-0"
10 BAYS @ 80'-0" = 800'-0"
83'-0"
6" INFLATION OUTLETS IN TUNNEL
TUNNEL
6" LINE
6" DEFLATION OUTLETS ALONG TOP CATWALK
6" DEFL RISER
14
15
16
17
18
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23
24
25
26
3 STAGE INGERSOLL RAND COMPRESSOR DELIVERING 882 CU FT PER MINUTE @ 760 LBS PRESSURE. PROVIDED FOR FUTURE BOOSTER TO DELIVER 441 CU FT PER MINUTE @ 2000 LBS PRESSURE MOTOR 300 H.P. 257 R.P.M.
COMPRESSOR & PUMP HOUSE
6" LINE
4" LINE
PURIFICATION PLANT
R.R. TRACK
CONNECTION FOR TANK CAR
BOTTLE STORAGE
2" LINE
TANK STORAGE UNDERGROUND
96 TANKS 2 LAYERS 25 & 24
5 TANKS ON ONE MANIFOLD
UNDERGROUND O.D. 24" x 80' LONG.
20160 CU. FT. 750# PRESSURE
CAPACITY ABOUT 1,100,000 CU. FT
AT ATMOSPHERIC PRESSURE

Fig. 2 Diagram of Helium-Storage Plant

were as large as shipping restrictions would permit. All necessary milling, punching, cutting, reaming, assembling, and riveting of these sections was done in the shop and then they were hauled out to the assembly yard. Particular care had to be taken in the matter of the milling of joints in the chord members. To prevent any error the various set-ups were checked several times before milling operation was begun, as any error in the set-up of the milling machine would have affected all arches. No difficulty was experienced with the work on the silicon steel, which is harder than carbon steel. The milling of the silicon-steel members required more time than for carbon steel. The punching, while not requiring more time, is limited to less thickness of material than in carbon steel, the limit of thickness being about $^1/_4$ in. less than the diameter of the punch. Silicon steel, due to its hardness, wears out punches and dies quicker than carbon steel. Reaming requires slightly more time and is also harder on the tools. The shop cost of fabricating silicon steel is estimated to be about $2.50 to $3 per ton higher than for carbon steel. The fabrication of the vertical and horizontal purlins, rafters, etc. was usual shop practice. These parts were assembled and riveted at the shop in as large sections as possible to avoid unnecessary field assembly and riveting.

As the sections composing the arches were received in the yard, they were assembled into half-arches in a horizontal position, about 3 ft above the ground and supported on timber blocking. After the assembly of the half-arches in each case, a wire was stretched from the center of the bottom pinhole to the center of the top pinhole and the distance was measured, which in all cases except one checked within $^1/_2$ in. of the computed distance, which was a little over 255 ft. In the exceptional case, the distance varied $^3/_4$ in. from the computation. Offsets or ordinates from certain panel points on the chords to the wire were measured, and these varied not more than $^5/_{16}$ in. from the computed measurements. The small variations between measured and computed distances was an excellent check and proof of the accuracy of the shopwork. After measurements were taken, the field holes in the connections of the sections, which had been sub-punched in the shop, were reamed to size while in the assembled position, assuring good fit-up in the field. The ease with which the material went together in erection and the extremely small number of holes requiring reaming in the field was splendid proof of the soundness of the shop methods used in the fabrication of this work. After reaming of holes in the connections was completed, the arch ribs were taken apart, painted, and loaded on cars for shipment. The same procedure was followed in the fabrication of the door rib arches.

Shop Paint. "Metalastic," manufactured by the Sherwin-Williams Company, was used for all painting.

All shopwork was fabricated in accordance with the standard specifications of the American Institute of Steel Construction for structural steel for buildings.

Sub-punching and reaming was required for the arch ribs only.

Erection Methods Employed

The designers of this structure had contemplated erecting the steel frame by the use of a traveler which would be supported upon three standard-gage railroad tracks, one on the center line of the building, and one on each side, spaced 100 ft therefrom, on the center line of the proposed docking rails. This traveler would be designed to pass under the trusses with its derricks removed.

The engineers of the erection department of the American Bridge Company deemed this method of erection to be too slow, and developed an alternate plan which was submitted to the designers and approved by them.

This method consisted in erecting the lower sections of a pair of arches about 100 ft in height and 80 ft apart, upon temporary bents, placing all bracing between the arch ribs to this height; then assembling the center portion of the arches on the ground, upon a cradle, and lifting them into position by means of counterweights carried upon the side sections already erected and the drums of the locomotive cranes.

Erection started with the southerly two arches of the three fixed center arches, and proceeded southward to the south doors, which were erected in open position, supported upon the exterior of the main structure, and assembled by using locomotive cranes as high as these could reach and then utilizing a derrick traveler located on top of the arches.

The erection plan involved the use of eight lines of railroad tracks, two outside the building and six inside. Altogether about $1^1/_2$ miles of standard-gage railroad tracks was used at one time, over which were operated seven locomotive cranes, five with 50-ft booms and two with 105-ft booms and goose-necks which gave them a total reach of about 125 ft.

The erection bents for supporting the side sections of the arches consisted of two steel columns, each supported upon a 300-ton jack at its base, the function of the jack being to spread the tops of the lower sections enough to allow the center section to pass by and then to bring them together again in order to effect the closure between the two sections.

At the top of these columns were placed the sheaves for carrying the counterweight boxes, weighing about 72 tons each pair loaded

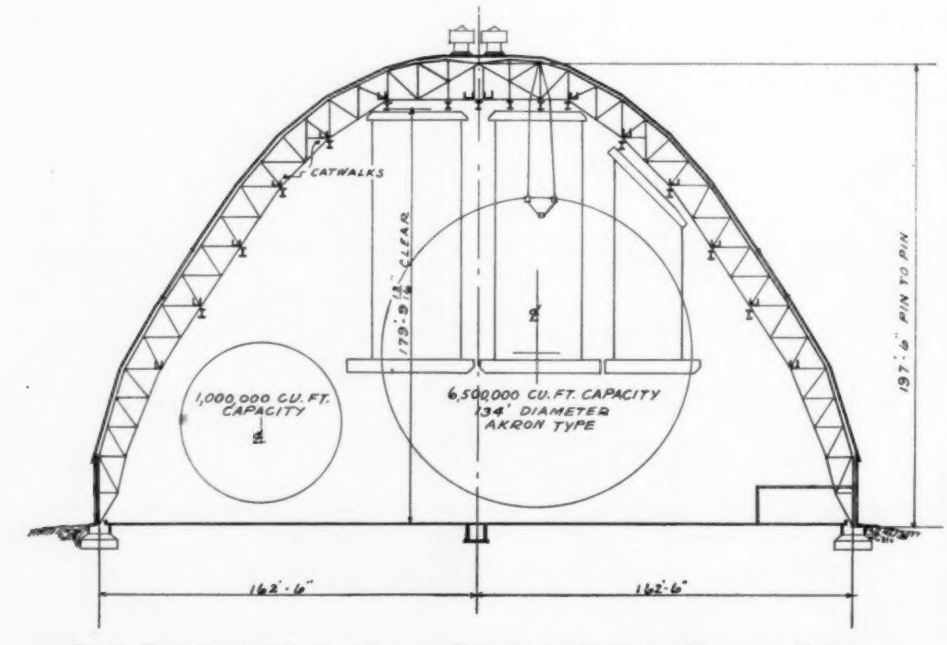

Fig. 3 Typical Cross-Section Showing Position of Airships of Different Capacities

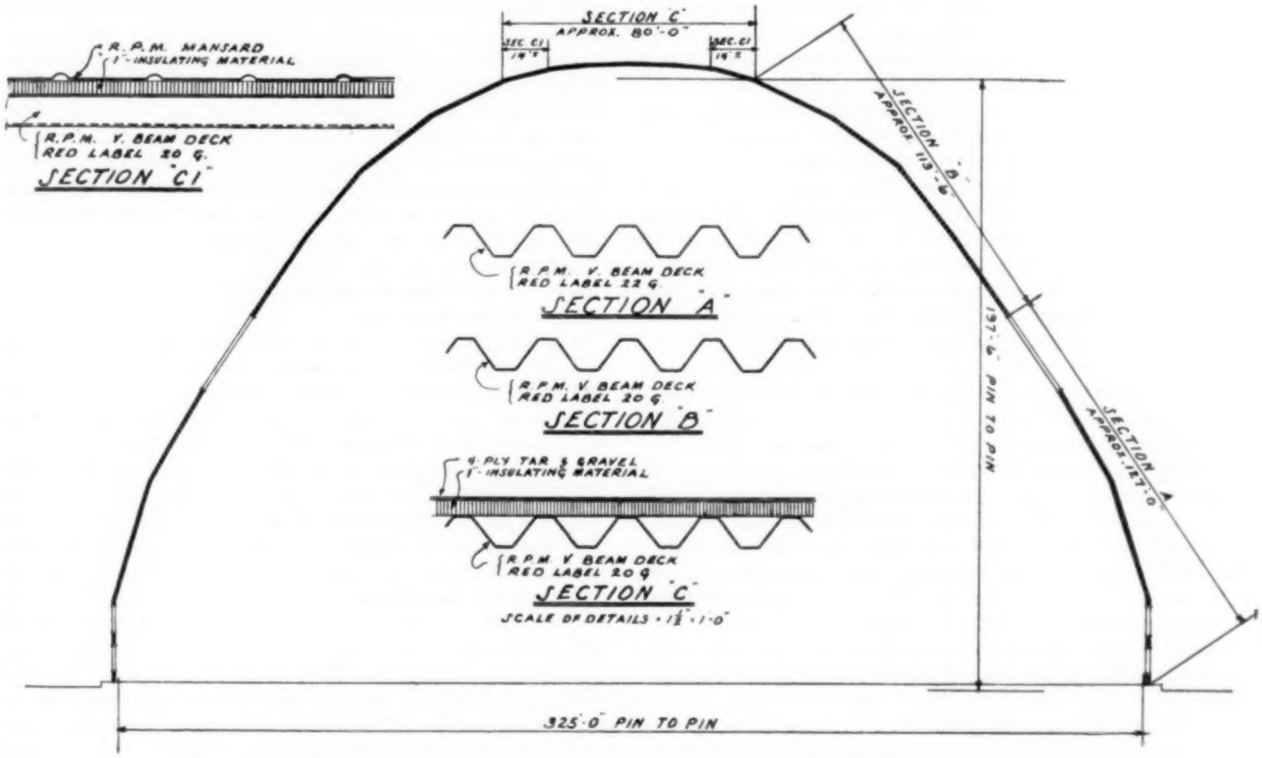

Fig. 4 Sections of Hangar Roof, With Roofing Materials

and equivalent to about 80 per cent of the weight of the center portion of the span, leaving about 20 per cent to be lifted by the cranes. Each center section erected weighed about 360 tons.

The center sections were assembled by the cranes upon cradles which comprised longitudinal girders carrying the ends of the arches and transverse ties to take up the thrust.

During the lifting of the center section of the arch truss, the four corners were kept level by means of weighted piano wires and graduated boards, which served very well as height gages, or telltales.

The traveler used on top of the arches for erecting bracing and the end arches was of the usual type of horizontal frame about 80 ft wide and 80 ft long, carrying two stiff-leg derricks with 90-ft booms. The travelers were mounted on temporary I-beams and moved by hand winches. The derricks were operated from the ground.

For erecting the end diagonal arches and the upper parts of the doors and the cantilever trusses, two additional stiff-leg derricks with 90-ft booms were used, mounted on top of the end arches near the center.

Speed of Erection. Erection of steel was started on April 29, 1929, the entire operation consisting of assembling the arch sections in a horizontal position, ready for hoisting into position by the locomotive cranes. This method was followed for all the arch erection. The first set of arches was raised into position on May 22, 1929, the operation of hoisting and connecting up taking about 6 hours. The second set of arches was raised June 12; the third, July 6; the fourth, August 27; and the fifth, set September 19. The hoisting of the last set was accomplished in 4 hours.

The south-end diagonals were completed July 20, and the erection of the south door was started August 3 and completed September 1. This was moved into closed position on September 16 after being completely riveted up.

The north diagonals were completed October 18, and the north doors were closed November 25.

Mechanical Equipment

Mechanical Equipment of Buildings. As this building is primarily a factory for the construction of airships, it is equipped with a considerable amount of mechanical handling devices. At the center of the building, overhead, there are two crane runways consisting of 12-in. I-beams supported on the trusses. These runways will carry 6-ton electric hoists for handling material.

At each side of the crane tracks are two 12-in. I-beams designed to carry working platforms which can be lowered and raised to any desired height, and on which the workmen engaged in assembling the ship's structure are employed.

Farther down the curved sides of the structure are more I-beams for carrying additional working platforms.

At the center of the structure there are provided fixed working platforms the full length of the building overhead, and at various levels, six on each side, are catwalks 2 ft wide, also running the full length of the structure and located between the chords of the arches. Access to the upper platform is provided by two stairways, one on each side of the building near the center, and by a specially designed inclined railway, intended to transport men and materials to the upper levels.

This inclined railway consists of two counterbalanced cars running on a curved track, but so built that the car itself remains horizontal at all times. The motors and drums for operating this elevator are located on the upper platform, and the car can be operated by a man at the top or from either car, as desired.

At the center of the building, just under the floor, there is a service tunnel, extending the full length of the building and containing electric outlets for both lights and power, water lines, compressed air lines, and the gas lines for inflating the gas cells.

Shops and Offices. The building is 325 ft wide between arch pins, but the necessary width for assembly of the ships is only 240 ft. This leaves a space for shops and offices on each side of the building about 42 ft wide by 640 ft long. Eventually it is expected that shops and offices will occupy all of this space, but at present they are confined to the east side of the building.

On account of the expansion and contraction of the main building frame, it was necessary to build these offices and shops entirely independent of the main building, and it also was necessary to use fireproof construction for them to meet insurance requirements.

Heat. The engineers consider it useless to attempt to heat this structure. The offices and shops are heated by independent gas-steam radiator units, and electric heaters are used on the floor and on the working platforms in cold weather.

Light and Power. Only a small amount of natural light is provided, the designers intending that most operations should require artificial light. There were three reasons for doing this: First, the fact that assembly would probably require continuous work, day and night; second, the difficulty of making skylights watertight and the cost of good skylights; third, the fact that even with ample windows, much of the work would need to be done by artificial light owing to the interference of the assembled ship with the light from the outside.

The interior lighting has proved to be quite satisfactory, due largely to painting the under side of the covering with aluminum paint and the use of a light gray paint for the structural steel, and a large part of the work is actually carried out with only natural lighting.

Electrical Equipment

Electric current is brought to the site at 22,000 volts to a transformer building located adjacent to the main building at about the center, at the east side. This current, as well as all other utilities, is brought to the building underground, as no wires can be tolerated above the ground surface. At the transformers, the current is converted into 460-volt, 60-cycle, three-phase for power, which is delivered to the main switchboard housed in the same building with the transformers. Six secondary transformers are located at convenient distributing points just outside the building walls, designed to receive primary current at 460 volts and distribute at 115 to 230 volts for lighting purposes. Distribution is accomplished through six weathertight main distributing panels located outside of the building, so that switching, with any accompanying arcs, can at no time be the cause of a gas explosion.

The general lighting consists of adjustable projectors of 1000 watt capacity each, located along the catwalks, from 65 to 150 ft above the floor, and seven lines of outlets along the catwalks and upper platform. Additional lighting is also provided at all stair and hoist landings.

Outlets for extension cables are provided at all trusses, 80-ft centers, at both sides of the building and in the tunnel at the center. Most of the manufacturing operations are carried on by lights taken from these outlets. Each of these outlets at trusses consists of a gang of six heavy-duty receptacles, each capable of handling 10 to 12 kw. Those in the tunnel are of approximately 20 kw capacity each. All of these heavy-duty receptacles are housed in gasketed cast-iron boxes so as to be absolutely gas-tight.

Power at 460 volts is carried to a point near the center of the building on the top platform, from where it is distributed to the crane and hoist motors. Separate power feeders are carried to each of the door motors direct from the main switchboard. Each main feeder is protected by an automatic circuit breaker, no fuses of any description being used on the main switchboard. This is a precaution adopted to prevent any extended shutdown due to unreliability of fuses. All main feeders from the switchboard are run underground, using lead-covered cables in fiber ducts laid in concrete, preventing any damage to insulation from water. All other wiring is run in rigid galvanized conduits secured to the building structure.

The usual shop lighting is installed throughout the shop portion of the building, and provisions have been made for ample power supply in the shops.

The door operation is as follows: Pressing the "open" or "close" button will start the equipment on the high-speed motor winding and accelerate the same to full speed. Travel will continue at full speed until the first slow-down limit switch is encountered in that direction. This will insert secondary resistance in the high-speed winding, causing the motor to slow down. When the second slow-down switch is encountered, the motor will become disconnected from the high-speed winding and thrown into slow-speed connections with secondary resistance. This will cause the motor to decelerate to half-speed and below, depending upon the amount of secondary resistance left in the circuit on the low-speed winding. It is expected that, with 80 to 100 per cent normal torque on motor, the final slow speed will be approximately one-third of the top speed. The door will then continue at such slow speed until the final stop, at which time power will be entirely disconnected from the equipment, including the hydraulic-operated brake. It is intended to so arrange the hydraulic-operated brake that, after cutting off power, there will be an inherent time lag of $1^1/_2$ to $2^1/_2$ sec before the brake is finally set. It should be noticed that the setting of the brake is a gradual application. The operation in the reverse direction is the same.

The motors are four type MT-566-Y-BL 200-100-hp General Electric motors, 15-min rating, 600-300-rpm, 440-v, three-phase, 60-cycle, having special long shaft and third outboard bearing and with shaft extension and brake pads at the opposite end for the motor-mounted brake.

The motor-mounted brakes are General Electric type CR-9516-466 hydraulic-operated. These brakes are provided with operators of such capacity that they will provide a retarding torque of approximately 1200 lb-ft, and at the same time will have an inherent time delay of from $1^1/_2$ to $2^1/_2$ sec.

The electrical equipment also includes the necessary panels, switches, contactors, accelerating relays, terminals, interlocks, starting-duty resistors, operating pushbuttons and 24 double-circuit limit switches with spring return and roller lever; 16 of these switches are for slowdown and 8 for opening both sides of line on final stop.

Floors

As the floor of this dock is used as a working platform for laying out delicate work, it was essential that there should be no obstructions, that facilities should be provided for fastening-down jigs, and that as level a surface as practicable should be provided.

It seemed to the designers that a heavy tongue-groove or splined wood floor met these conditions best, and designs were made and bids were received on a floor consisting of 4-in. by 4-in. yellow pine, toe-nailed to 6-in. by 6-in. sleepers laid to 3-ft centers. All flooring was to be No. 1 common dense grade, long-leaf yellow pine, quartered from 8-in. by 8-in. timbers, in order to obtain a near edge-grain face. Sleepers were to be creosoted. All finished floor was to have two heavy coats of linseed oil. The base for this floor was to consist of 6 in. of rolled slag, on which the 6-in. by 6-in. sleepers were to be laid and granulated slag used to fill between them.

There were two serious objections to this timber floor: First, whether such a floor would hold its shape under the severe weather conditions to which it would be subjected, it being impracticable to avoid alternate wetting and drying under the doors when open to admit or to let out ships, and the probability of water dropping from condensation on the steel structure, and second, the much greater cost of such a floor as compared to a concrete floor.

The concrete floor as actually constructed consists of 6 in. of rolled slag, 5 in. of concrete, and 1 in. of finish, placed integral with the 5-in. base. All finish is rough troweled and hardened with Master Builders hardener, 40 lb per 100 sq ft. A wire-mesh reenforcing consisting of No. 6 wires, spaced 6 in. both ways, is used for all concrete floor.

Special floor inserts, designed and made by Donley Bros. Co. of Cleveland, were used throughout the building. These inserts are spaced on 10-ft centers each way. Special holding-down bolts, designed to resist an upward or sideways pull of 5 tons each, were installed in the floor along the center and 50 ft each side of the center, one to each 80-ft panel. These are set in blocks of concrete and are for use in holding down the ship and as anchors for hoisting ropes.

Fire Protection

As no precedent had been presented to the fire underwriters, new rulings were required for this building. The underwriters requested that only incombustible materials be used in the construction of the building, which eliminated wood roofing.

It also was requested that a system of outside and inside fire hydrants be installed and connected to an automatic fire pump located outside the building and capable of boosting the normal pressure of 50 to 60 lb per sq in. to 125 lb per sq in.

Water is brought to the pump house just east of the building by a 12-in. main, and from that point a 10-in. loop is carried entirely around the building. This 10-in. loop supplies 12 outside fire hydrants and 24 inside hydrants.

In addition to these water lines, chemical fire extinguishers are distributed liberally throughout the building, on the main floor, on catwalks, and on the upper platform.

Helium Storage

Gas-Handling and Storage Facilities. As the airships now being built at this plant will use only helium gas, provision for this gas only is being made. Helium gas is found in certain natural-gas fields the gas of which contains 2 to 4 per cent of helium, the remainder of the gas in most cases being nitrogen.

The gas is shipped from the field in small steel containers, called bottles, about 4 ft long and 9 in. in diameter, each bottle containing about 170 cu ft of free gas, under a pressure of about 2000 lb. As helium gas follows rather closely Boyle's law of gases, which states the volume is inversely proportional to the pressure, each cubic foot of gas under 2000 lb pressure is equivalent to approximately 133 cu ft of free gas at atmospheric pressure.

As the ships now being built will require about 6,500,000 cu ft of gas each, it will be seen that 38,000 bottles would be required. The Navy has built special tank cars for transporting helium, each car to carry about 600,000 ft of gas.

To receive this gas, there is installed a permanent railroad track parallel to the building and about 100 ft therefrom. Alongside this track, at about the center of the building, is the pump house, containing a compressor and a purification plant, and adjacent to this are provided underground storage cylinders having a storage capacity of over 1,000,000 cu ft of gas at 750 lb pressure.

Gas is taken directly from the tank cars or bottles into the gas cells of the ship or into the storage cylinders, as desired. The gas is drawn out of the gas cells by the suction pump, which is able to deflate 1,000,000 cu ft of gas in 24 hours and place it into storage at 750 lb pressure, or which may, at a lower rate, compress gas back into the cylinders or tank cars at 2000 lb pressure for reshipment.

The purifying plant is necessary in order to remove air and other impurities which mix with the helium and decrease its lifting power, requiring removal at intervals.

These tanks, 96 in number, are built for a working pressure of 750 lb per sq in. and were tested with a pressure of 1250 lb per sq in. at the shop, corresponding to a unit stress of 28,000 lb per sq in.

The tanks are 24 in. outside diameter, 80 ft $2^1/_4$ in. long over all, with a wall thickness of shell of $^1/_2$ in. and flat heads $2^1/_4$ in. thick, joined to the shell with one girth seam each. The material is open-hearth steel, with a minimum ultimate strength of 60,000 lb and yield point of 38,000 lb.

The high-pressure test is intended to be sufficient to produce a stress equal to 75 per cent of the yield point of the material, in addition to which the vessels are subjected to a hammer test developing a stress of 60 per cent of the yield point. The high-pressure test is equal to $P = 2ts/D$, or $(2 \times {}^1/_2 \times 28{,}000)/23$, which equals 1220 lb per sq in., or 1.62 times the working pressure. The pipes are pickled and one coat of coal-tar primer is applied at the shop and two coats in the field.

These tanks are placed underground, in two layers, supported on three lines of concrete footings, and are covered with 12 in. of gravel. They are in two groups, with a concrete valve pit between. The tanks are valved in groups of five and are connected with 4-in. pipes. The main pipes to the compressor are 6 in. in diameter, as are also the deflation pipes inside the building. The inflation outlets are located in the center tunnel and are 4 in. in diameter.

The compressor is a three-stage Ingersoll-Rand machine especially designed for helium compression and will deliver 882 cu ft per min of 96 per cent pure helium gas at 750 lb per sq in. pressure. A "booster" can be added later which will deliver 441 cu ft per min at 2000 lb pressure, and this will be done when a permanent fixed purification plant is added. The compressor has a direct-connected 300-hp. 257-rpm motor.

Before adopting this method of storing helium gas, careful estimates of cost were made comparing this method with storing in a gas holder at atmospheric pressure and storing at high pressures in large forged cylinders.

The method of storing at an intermediate pressure was found to be the most economical.

AER-54-23

Racing Propellers for Light Airplanes

By FRED E. WEICK,[1] LANGLEY FIELD, VA.

To clear up the apparently puzzling situation in regard to propellers for light airplanes with high-speed engines the author has suggested a design for a racing propeller and then attempted to change its shape systematically in every possible way, noting the effect on the efficiency and the maximum speed. The effect of having the engine deliver its rated power at various revolution speeds is also investigated, as well as running a particular engine at revolution speeds above the rated.

This paper has been prepared for the purpose of clearing up an apparently puzzling situation in regard to propellers for light airplanes with high-speed engines, particularly when the highest possible air speed is sought for racing purposes. It seems that many of the fastest light airplanes have been equipped with propellers which differ considerably in form from the more or less standard propellers of the Army and Navy, which should represent the highest standard of efficiency obtainable. This has naturally led to some question as to what actually is the best form to use. Another point which has been questioned by some is the advisability of making use of such fast-turning engines as seem to be the trend of the times for light airplanes.

The best manner of throwing light on this subject, it seems, will be to design a racing propeller of conventional form for a typical light airplane, and then to change its shape systematically in every possible way and note the effect on the efficiency and on the maximum speed. The effect of having the engine deliver its rated power at various revolution speeds will also be investigated, as well as running a particular engine at revolution speeds above the rated.

It will be assumed that this light airplane has a nicely cowled four-cylinder in-line tractor engine in the nose of the fuselage, and that this engine develops 90 hp. at its rated speed of 2000 r.p.m. (These values represent the average of the most popular engines for light airplanes.) The maximum speed of this light plane fitted with the best high-speed conventional metal propeller is, say, 110 m.p.h.

The standard propeller, which is of the conventional detachable blade, adjustable-pitch, aluminum-alloy type, is defined by the form curves in Fig. 1. This particular propeller has been selected because it has been the subject of a large number of full-scale tests on various fuselages in the N.A.C.A. 20-ft. Propeller Research Tunnel, and the results have been given in convenient form for the selection of the best pitch and diameter for any given purpose. (These results, incidentally, can be found in either of the first two references at the end of the paper.) The appearance of this propeller is shown in Fig. 2, along with the fuselage on which it was tested that most nearly represents a well-cowled, in-line, air-cooled installation.

With the aid of the foregoing data a series of propellers of different pitches have been selected for the conditions of 90 hp., 2000 r.p.m., and 110 m.p.h. The propulsive efficiencies obtained with the various pitches are plotted as a curve in Fig. 3, the various pitches being represented by the blade angles at three-fourths of the tip radius. The propulsive efficiency includes the effect of the body on the propeller and the increase in body drag due to the propeller slipstream; and in obtaining the values given in Fig. 3, as well as all of those used in this paper, due consideration has been given to the effect of relative body size and propeller diameter on the propulsive efficiency. From Fig. 3 it will be noted that the propeller giving the highest propulsive efficiency and consequently the greatest maximum speed has a blade angle at three-fourths of the radius of 20 deg. and a propulsive efficiency of 78.8 per cent. This propeller has a diameter of 6.78 ft. It is also interesting to note that the pitch is not a particularly critical factor, for any propeller of this form between that having a blade angle of 17 deg. and a diameter of 7.23 ft. and that having a blade angle of 25 deg. and a diameter of 6.15 ft. will give a propulsive efficiency within 1.5 per cent and a maximum speed within about 0.7 m.p.h. of the best. Furthermore, a maximum speed within 0.1 m.p.h. of the best can be obtained with any pitch angle within a range of about 3 deg. This is partly due to the body interference for, as the pitch is reduced, the diameter becomes greater and the body interference loss becomes less, compensating to some degree for the lower propeller efficiency accompanying the lower pitches.

High revolution speeds are often associated with high tip speeds and their accompanying efficiency loss, which can be very great if the tip speeds are above the velocity of sound in air. Recent tests in the 20-ft. Propeller Research Tunnel (reference 3) show that for propellers of the standard form the efficiency is unaffected by tip speed up to a critical value of approximately 1000 ft. per sec., but beyond this value it falls off very rapidly. Since the tip speed of the 6.78-ft. propeller turning 2000 r.p.m. at 110 m.p.h. is only 730 ft. per sec., it is well below the critical value and it will have no efficiency loss from this cause. In fact, as long as light planes have engines of about 100 hp. or less there need practically never be any fear of loss of efficiency due to too high a tip speed, for regardless of the revolutions the diameters will be sufficiently small to keep the tip speed under the critical value.

It has been found that the conventional metal propeller having the best pitch and diameter for high speed on this assumed light plane with an engine delivering 90 hp. at 2000 r.p.m. and a maximum speed of 110 m.p.h. has a propulsive efficiency of 78.8 per cent. This means that 21.2 per cent of the brake horsepower being delivered to the propeller is lost, and it will be of interest to examine this in detail to see if there is a reasonable

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

possibility of reducing the loss. It is fundamental that with any form of fluid propulsion a mass of the fluid must be set in motion in a direction opposite to the craft propelled in order to provide a thrust or propelling force, and this rearward moving air represents energy which is lost. The proportion of lost energy depends on the mass of air handled and the velocity imparted to it, which in turn depend primarily on the propeller diameter, the speed of advance, and the thrust. For a given diameter, speed, and thrust, the foregoing loss, which may be termed an induced loss, is a minimum if the thrust is uniform and constant with time over the entire propeller disk. This minimum loss, which represents an ideal condition beyond which it is not possible to go, can be found very easily by means of the simple Froude momentum theory. For the case of the 6.78-ft. propeller absorbing 90 hp. at 110 m.p.h., the ideal efficiency is calculated as 94.2 per cent, giving an unavoidable loss of 5.8 per cent of the engine power with this diameter. Thus the power loss, which there may be some possibility of reducing to give improved performance, is only 21.2 — 5.8 = 15.4 per cent.

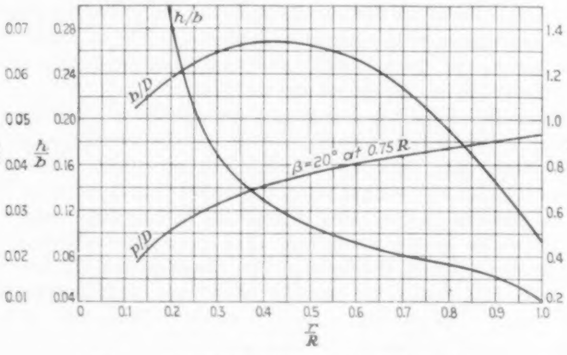

FIG. 1 PROPELLER BLADE FORM CURVES

(D = diameter; b = blade width; k = blade thickness, p = pitch; R = tip radius, $D/2$; r = radius; airfoil sections, R.A.F.-6 type.)

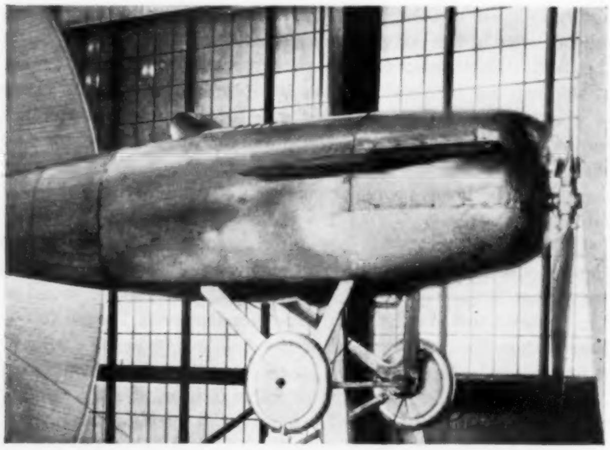

FIG. 2 PROPELLER NO. 4412 ON FUSELAGE NO. 1 IN THE N.A.C.A. PROPELLER RESEARCH TUNNEL

Due to the fact that screw propulsion in accompanied by a torque as well as a thrust, the slipstream is given a certain rotation which also represents a loss. This is usually small, however, and in this case is approximately 0.6 per cent. The rotational energy could be regained by means of guide vanes called a counter propeller, but the drag of the vanes would more than offset the gain obtained. The total loss, which may possibly be reduced somewhat, is therefore cut down to 15.4 — 0.6 = 14.8 per cent. This remaining portion is due to skin friction, turbulence, and body interference.

It is now interesting to investigate every possible variation or change in the form of the propeller which might possibly reduce these losses due to friction, turbulence, and body interference, and allow a closer approach to the ideal efficiency.

From the best experimental data available it seems that the loss due to body interference, i.e., the difference between the efficiency of the propeller alone without a body and the propulsive efficiency with the body present, is in this case approximately 4 per cent. The important items which can influence this body interference loss are the diameter of the propeller, the shape of the body, the fore-and-aft distance between the propeller and the body, and the distribution of the thrust along the propeller blade radially. The last of these, the radial-thrust distribution, is rather important, for the loss can be materially reduced by reducing the thrust near the center of the propeller where the slipstream is thrown against the body. The thrust can be reduced somewhat near the hub by making the blade there as narrow as strength and a low drag section permit, but a much greater effect is obtained by varying the pitch distribution along the radius. A propeller with its pitch reduced in toward the hub in the best possible manner will, with an ordinary body, have a propulsive efficiency about 3 per cent higher than an otherwise similar propeller with uniform pitch. According to the best experimental data available the standard propeller already has approximately the best possible pitch distribution, which eliminates the possibility of an appreciable gain in this direction. The shape of the body and the forward position of the propeller, while of appreciable importance, are out of the control of the propeller designer and so will not be taken up here.

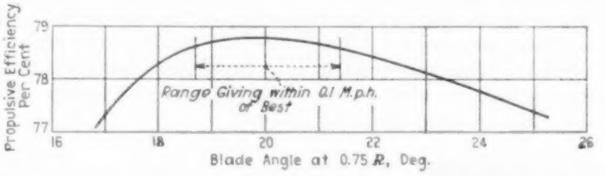

FIG. 3 PROPULSIVE EFFICIENCIES OBTAINED WITH VARIOUS PITCH SETTINGS

The effect on the propulsive efficiency of increasing the propeller diameter, within the limits possible while keeping the same power, revolutions, and air speed, is very small. Since there already has been obtained the best pitch, considering both body interference and propeller efficiency, the only way left which will permit of increasing the diameter appreciably without decreasing the propeller efficiency is to decrease the blade width or change the airfoil section. Changing the airfoil section, in so far as the effect on the diameter is concerned, is about the same as changing the pitch, and since the pitch is already the best pitch, there can be expected no appreciable body interference gain in this direction. This leaves only the possibility of narrowing the blades, and this will also give a slight increase in the efficiency of the propeller itself, not considering the body interference.

Suppose for a moment that it would be possible, considering strength, to cut the blade width in half. (The airfoil sections would not be changed, and consequently the thickness would also be cut to half of its former value, and the cross-sectional area to one-fourth.) The best propeller with these narrow blades will have a diameter of about 8.2 ft. This propeller alone, not considering the body interference, would, as determined by the vortex blade element theory and checked by experimental data, have an efficiency about 1 per cent higher than that of the standard propeller. It is interesting to note here that the ideal efficiency with the 8.2 ft. diameter is 1.7 per cent greater than that with the standard propeller. This accounts for more than the entire actual gain, indicating that the friction and the turbulence losses are slightly greater with the narrower and longer blades. The increased diameter would account for an additional increase of 1.5 per cent due to the effect on the body interference, making the total gain in efficiency 2.5 per cent with the narrow blades and 8.2 ft. diameter.

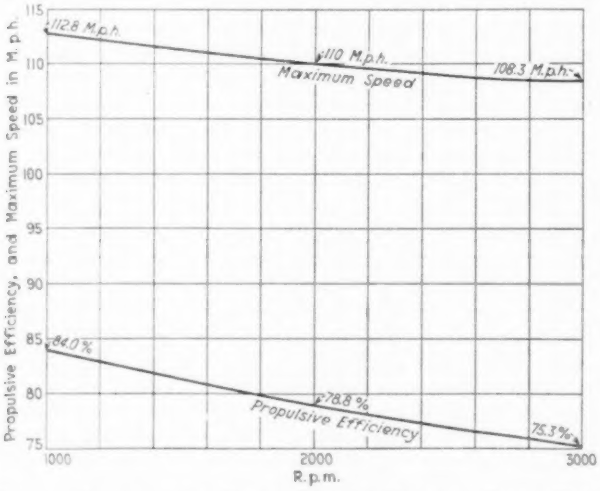

FIG. 4 VARIATION OF EFFICIENCY AND MAXIMUM SPEED WITH PROPELLER REVOLUTIONS

(The horsepower remaining constant with a value of 90.)

The radically narrow and thin propeller assumed as an extreme example is of course out of the question for actual use with present materials and forms of construction. It does not seem likely that it would ever be possible to obtain satisfactory safety, reliability, and smooth operation with an aluminum-alloy propeller having blades less than about three-fourths the width and thickness of those of the standard propeller, especially considering that the diameter with the narrower blades is greater. In this case the total efficiency gain would be just 1 per cent, and the maximum speed of the airplane would be increased by one-half mile per hour.

It has been commented upon that some rather fast light airplanes, particularly British ones, are equipped with propellers having rather wide blades. According to both theory and experiment, this gives them a loss in efficiency as compared with our narrower metal blades, but a very slight one.

The thickness of the blades, or rather the ratio of thickness to width, is directly connected with the shape of the airfoil sections, and these two should be considered together when considering the effect of thickness ratio on the efficiency. Our standard propeller has airfoil sections based on the R.A.F. 6, which have been used in propellers for years and are very good in thin sections for some performance conditions. These sections are very poor, however, in the greater thickness ratios, and their effectiveness increases as the thickness ratio decreases down to at least about 0.04, which is well below the thickness of the sections of this standard propeller. According to full-scale tests on propellers of various thicknesses having these same sections (reference 4), it would be possible for us to get an increase in efficiency of about 0.5 per cent by reducing the blade thickness to about 0.06, a reasonable value which, with the proper design, could be used with safety.

Another form of airfoil section has come into rather wide use in propellers in this country in recent years, this form being based on the Clark Y, and probably about as efficient for high-speed conditions as any likely to be developed in the future. The efficiency of these sections does not vary greatly with thickness ratio up to 0.10, all of the thicknesses giving about the same propeller efficiency as the thinnest R.A.F. 6 type. Thus, if this standard propeller was provided with Clark Y-type sections, one could expect an increase in efficiency of about 0.5 per cent, regardless of the blade thickness, within reasonable limits. It is not likely that this can be appreciably improved upon by changing the airfoil sections.

Another possible variable is the plan form of the blades. In this matter it is advantageous to have a certain amount of taper and rounded tips. The effect is not great, however, for the increase in efficiency by changing from a straight blade with a square tip to a tapered and rounded one such as that of the standard propeller is less than 2 per cent, and there is no probability of an appreciable further increase.

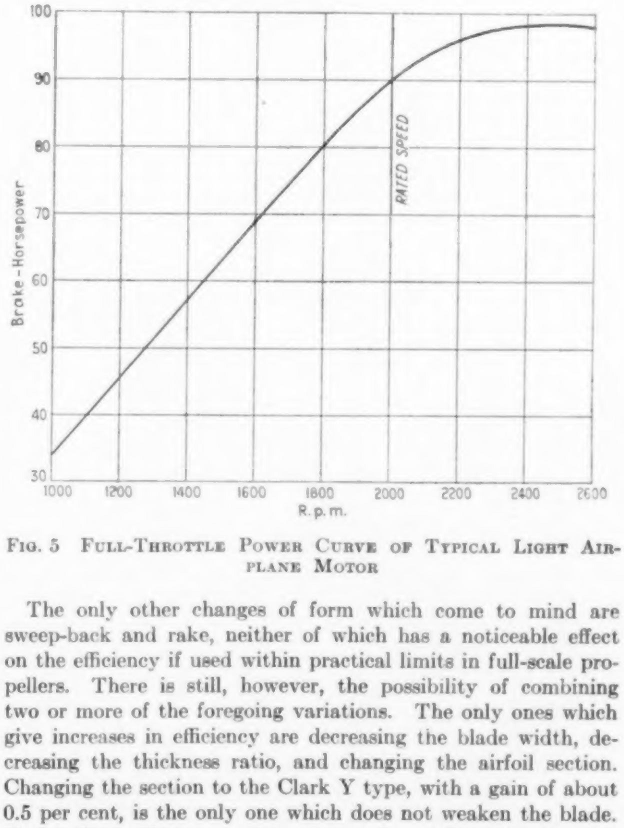

FIG. 5 FULL-THROTTLE POWER CURVE OF TYPICAL LIGHT AIRPLANE MOTOR

The only other changes of form which come to mind are sweep-back and rake, neither of which has a noticeable effect on the efficiency if used within practical limits in full-scale propellers. There is still, however, the possibility of combining two or more of the foregoing variations. The only ones which give increases in efficiency are decreasing the blade width, decreasing the thickness ratio, and changing the airfoil section. Changing the section to the Clark Y type, with a gain of about 0.5 per cent, is the only one which does not weaken the blade. If the Clark Y-type sections are used, another advantage is gained, for then there can be used thickness ratios up to 0.1 or possibly 0.12 without loss of efficiency. The blade width can therefore be decreased, which gives the greatest gain in efficiency, without decreasing the actual thickness, and this will allow a greater reduction in width for the same strength. The width-to-diameter ratio could be reduced in this manner to about two-thirds of the present value before the section thickness ratio would become great enough to produce an appreciable efficiency loss. Any changes in this direction would of course require careful regard to the strength and smooth-running qualities, with suitable overload whirling tests if possible. Assuming, as is quite likely true, that a satisfactory propeller could be developed with two-thirds of the standard blade width ratio and

Clark Y-type sections having the same actual thickness as the present blades, a propulsive efficiency of 80.5 per cent could be expected, as compared with 78.8 per cent with the standard propeller. Thus the most that one can expect to obtain by changing the shape from the standard blade in this case is an increase in efficiency of something under 2 per cent, which means an increase of maximum speed of something under 1 mile per hour.

In getting this slight increase in efficiency there were used a narrower blade and a larger diameter (7.4 ft. instead of 6.78 ft.), and this larger diameter in itself causes a higher ideal efficiency. The ideal efficiency of the "best possible" 7.4-ft. propeller is 95.1 per cent, an increase of 0.9 per cent over that with the standard propeller. Thus, the increase in ideal efficiency is responsible for fully one-half of the entire efficiency gain and, with the best propeller which can be devised, there still will be a loss of 14.2 per cent of brake horsepower to body interference, skin friction, and turbulence.

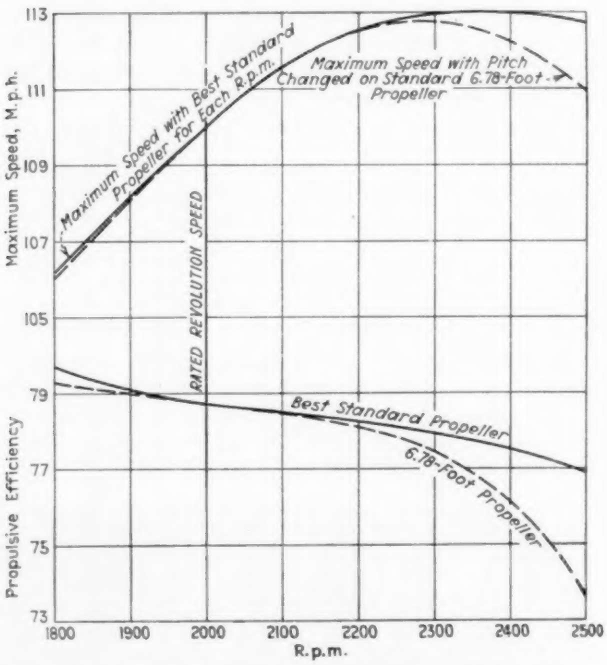

FIG. 6 EFFECT OF RUNNING A GIVEN ENGINE AT VARIOUS FULL-THROTTLE REVOLUTION SPEEDS

From the foregoing it is seen that the propeller designer, having the conditions of body form, engine power, revolutions, and approximate airplane speed all fixed outside of his control, has but a small chance of increasing the maximum speed by even 1 full mile per hour over that with the best standard propeller which can be selected in a few minutes from a chart (reference 1 or 2). It may also be of interest to see what influence the airplane and engine designers and the operators can have on the maximum speed. Apparently body interference is costing about 4 per cent, and it seems likely from test data available that this could be reduced to about 2 per cent without increasing the drag of the body by lengthening the nose and locating the propeller farther forward. This would no doubt entail disadvantages such as extra weight, but in this paper only the speed aspect is being considered.

Another factor which has a large effect on the propeller efficiency is the revolution speed at which it must operate. The efficiency obtained with the best pitch and diameter of the standard form propeller is plotted in Fig. 4 for revolution speeds ranging from 1000 to 3000, the engine in each case delivering 90 hp. The fuselage is the same as before, and with a revolution speed of 2000 r.p.m., a maximum air speed of 110 m.p.h. is obtained. It will be noticed that with 1000 r.p.m. a propulsive efficiency of 84 per cent is obtained, as compared with 78.8 per cent with 2000 r.p.m. and 75.3 per cent with 3000 r.p.m. If the 1000 r.p.m. were obtained by means of reduction gears with a power loss of about 2 per cent, the overall propulsive efficiency would be about 82 per cent, which is just about the same as that obtained with the old OX-5 motor which delivers 90 hp. at 1400 r.p.m. As can be seen from Fig. 4, this would give a gain in maximum speed of about 1.7 m.p.h.; while running the revolutions up to 3000 would cause a loss in maximum speed of the same amount. This comparison, incidentally, shows that, considering the powers and air speeds involved, present light plane motors do not run at excessive revolution speeds for reasonable propeller efficiency. Also, even with 3000 r.p.m. the tip speed would be well under the range where it would cause a loss in efficiency, for the diameter in this case would be less than $5^1/_2$ ft.

The comparisons just made show that the airplane and engine designers control factors which might effect an increase in propulsive efficiency and speed three or four times as great as that within the range of the propeller designer. It is also of some interest to show what effect the operator can have on the maximum speed with a given airplane and engine to start with, by running the engine at higher than the rated revolutions. The power curve of the typical engine, which is rated at 90 hp. at 2000 r.p.m., is shown in Fig. 5, the peak occurring at 2500 r.p.m. The solid curves in Fig. 6 show the propulsive efficiencies and maximum speeds which would be obtained with the best standard propellers if the engine were allowed to turn at full-throttle revolution speeds different from the rated one. It will be noted from Figs. 5 and 6 that, although the power becomes greater as the revolutions go up to a certain value, the propulsive efficiency drops off. The greatest speed, which is 3 m.p.h. higher than that obtained at the rated revolutions, occurs at a revolution speed about 200 r.p.m. below that for the peak of the power curve. The dotted curves in Fig. 6 show that nearly the same gain in speed can be obtained if the original standard 6.78-ft. propeller is used throughout, and the higher revolutions obtained by merely decreasing the pitch of the blades. Thus, the operator also can influence the maximum speed obtained over a short period for a race to a much larger extent than the propeller designer.

It is no doubt evident from this discussion that the propeller, assuming that the design is within reasonable limits of good practice, cannot be expected to have anything but a relatively small effect on the high speed of an ordinary light airplane. (It should be kept in mind, incidentally, that the high speed is the only performance item which has been considered here.) In the case of a racing light plane which has a maximum speed of 150 or 160 m.p.h., variations in the propeller have even less effect.

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AER-54-24

The Design of Practical Light Airplanes

By KARL H. WHITE,[1] ROBERTSON, MO.

Expense has instilled the thought of low-powered airplanes in the minds of airplane manufacturers during the slump, in order that a market can be maintained for their product and an outlet seed planted for later production. The important features in the design of light airplanes are (1) price, (2) safety, and (3) appearance and performance. The manufacturing cost depends upon materials employed and simplicity of design, and also varies with the quantity produced. Curves are plotted showing the cost of materials and direct labor for different quantities of production. Safety is dependent upon (1) visibility, (2) low take-off and landing speeds, (3) controllability at low speed, and (4) fire hazard. Photos show how visibility is obtained in the pusher type of light airplane. Appearance and performance rank equally in third place, and the author speaks of the necessity for "eye appeal." Performance in light airplanes is not measured in terms of high speed and maneuverability, but should be measured in terms of take-off and landing characteristics, together with controllability at low speeds. Wing-cell design is discussed, and charts show comparison of wing weights of light airplanes and normal-powered airplanes.

WHY a light airplane? The light airplane is one logical result of the aftermath of an oversold airplane public. The word "airplane" has been too expensive for the majority to seriously consider—not too dangerous or unsafe, but too expensive. "Normal powered" airplanes have required engines costing from $1500 up. This in itself, not to consider operating expense, has been sufficient to keep the prices of airplanes out of general reach. The airplane manufacturer has therefore groped around for a means of lowering the cost of some of the new models and has resorted to the low-powered engines costing less than $500 as a good start toward the possibility of producing a real low-priced airplane.

Price, then, becomes the most essential feature in the design of a practical light airplane. No matter how safe a light airplane may be, if it is expensive the public cannot afford to buy it, and therefore no business concern will entertain the thought of its manufacture.

The second consideration is safety. Although it is embarrassing to admit that human life trails second to a price consideration, it is nevertheless an actual fact. The airplane business is similar to any other, and the American public today is forced first to look at its pocket book, and then if there is a chance of meeting the requirements financially, it enquires further regarding safety, performance, and other details that may come to mind.

Appearance and performance equally demand third place, but the first consideration of price cannot be used as an alibi to present a crude-looking object to the public. The automobile industry today is bending every effort for good looks and graceful lines, and the commercial airplane designer who omits a certain amount of "eye appeal" in his product will soon be left behind in the struggle for a place in the light-airplane market.

PRICE AS A CONSIDERATION

Experience has shown that the cheapest type of light airplane to build is the monoplane. The biplane has certain definite advantages such as (1) being more compact (2), easier to obtain large wing area per unit weight, and (3) facilitating undercarriage design, but if price heads the list, the monoplane must be selected.

Cost may easily be discerned in parts. The fewer the parts the less the cost. Table 1 gives a comparison of the number of parts required in the wing cell of a conventional open two-place one-bay biplane, the well-known Robin monoplane, and the Curtiss-Wright Junior monoplane, which is in the light airplane class.

TABLE 1 COMPARISON OF PARTS

Name	Two-place one-bay biplane	Robin	Curtiss-Wright Junior
Beams	8	4	4
Ribs	48	38	38
Compression struts	16	10	10
Internal tierods	24[a]	24[b]	24[c]
Drag-wire fittings	32	24	16
Leading edge	4	2	2
Trailing edge	4	2	2
Wing tips	4	2	2
Wing hinges	8	4	4
External fittings	18	12	4
External strut units	19	18	20
External tierods	20	4	4
Tierod terminals	88	56	20
Bolts in wing cell	120	96	50
Total number parts per plane.	413	296	200

[a] single, [b] double, [c] 50 per cent double.

Of course, the relative cost of the parts being considered enters into this analysis, but without going into too much detail, it is readily seen that the monoplanes have the advantage from a price consideration. Wherever possible the number of parts should be cut to a minimum. Separate units should be combined. Fig. 1 is an example of how two units, a fuel and oil tank, on a light

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[1] Design Engineer, Curtiss-Wright Airplane Co. Mr. White was born in Winchester, Mass., Aug. 25, 1897. His parents moved in 1905 to Lawrence, Kan., where Mr. White received his education. He was graduated from the University of Kansas with a B.S. degree in Mechanical Engineering in June, 1921. After graduation he entered the employ of the Aeromarine Plane & Motor Company, Keyport, N. J., as draftsman. He was promoted to Project Engineer and was later Assistant Superintendent of the Airplane Division. In 1924 he went with the Chance Vought Company, Long Island, New York, as Chief Draftsman. In 1926 he became Assistant Chief Engineer, but resigned in 1927 to leave military aviation and to enter the commercial aviation field as Chief Engineer for the Bellanca Aircraft Co., then located on Staten Island, New York. In 1929 Mr. White joined the Moth Aircraft Corp., Lowell, Mass., as Chief Engineer. In 1930 this company merged with the Curtiss-Robertson Airplane Mfg. Co. and the Travel Air Co., now forming the commercial airplane unit of the Curtiss-Wright Corporation, Robertson, Mo. Mr. White is the designer of the recent Curtiss-Wright Junior light airplane.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

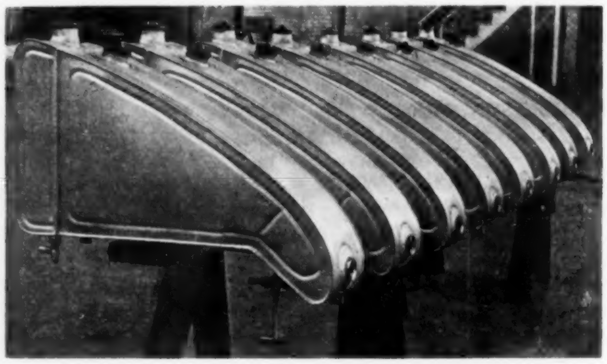

FIG. 1 TWO UNITS, AN OIL AND A FUEL TANK, COMBINED INTO ONE

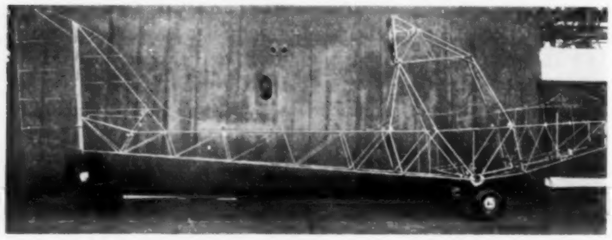

FIG. 2 A WELDED FUSELAGE UNIT

(Note that the fin, cockpit cowling, fairing, seat supports, center section, and controls have been combined into a single unit.)

airplane have been combined into one. This cuts down weight, facilitates handling through the production line, and requires only one support and tie-down arrangement instead of two. This arrangement also provides an oil-cooling medium which has proved its merit in hot-weather flying.

Regarding the type of airplane construction to be employed, some manufacturers have considered duralumin throughout, others duralumin monocoque fuselages with wood wings, and still others have used a combination of welded-steel tubular fuselage structure with wood wings and ailerons. There is a strong desire among airplane designers to go into all-metal construction, but the present-day practical low-cost light airplane, built for the present-day market, of necessity has been a combination of wood wings and steel tubular fuselage and tail structure.

The steel tubular fuselage is not only very strong and safe against injury to occupants in bad landings, but assembles readily and eliminates a great amount of handwork now required in other types of metal construction and also reduces the number of loose fittings to a minimum. This construction permits welding seat supports, surface control bearings, cable guides, engine-control fittings, and many other parts integral with the fuselage structure, and facilitates employing one unit for two or three different purposes. It is particularly well adapted to the light airplane because of the abuse to which it is subjected in the novice market, and because it is easy to repair in the field. Fig. 2 shows how the fin, center section, engine mount, seat supports, cowling, and surface control unit have been combined into one steel welded unit with the body of a light pusher type airplane.

Manufacturing cost also depends in a large degree on the quantity to be produced. Unfortunately we have not yet reached the quantity production period in airplane manufacture where expensive tools, jigs, dies, and fixtures can be afforded, which go hand in hand with sheet-metal work and pressed parts. When this time does come, our present method of assembly will indeed seem very crude, and the picture of cost presented herein will take on an entirely different aspect.

Fig. 3 shows how labor cost varies with the number of light airplanes produced per week. This cost is separated to show the labor comparison between all-steel tubular construction used in conventional fuselages and the wood construction as employed in a wing cell.

Fig. 4 shows how little the material cost is reduced when purchased for larger quantities of production. The material cost in airplane manufacture will have to be lowered considerably before we will be able to produce a real "flivver of the air."

FEATURES THAT MAKE FOR SAFETY

Under safety come the following features, which are listed in their relative importance: (1) Visibility, (2) low take-off and landing speeds, (3) controllability at slow speed, and (4) fire hazard.

Visibility can best be obtained in the pusher-type airplane. The old war-time slogan of "Sit well aft for protection in a crash," is gradually being replaced with the peace-time idea of sitting well forward to prevent a crash. As more airplanes get into the air it will become all the more important to have 100 per cent

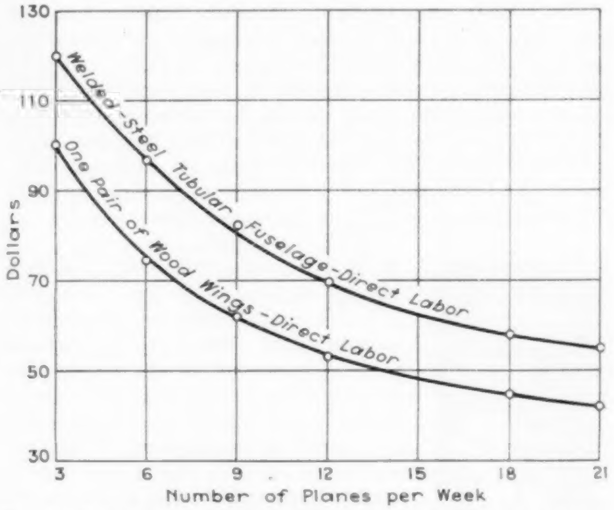

FIG. 3 VARIATION OF LABOR COST WITH NUMBER OF LIGHT AIRPLANES PRODUCED PER WEEK

(Comparison is made of the labor in all-steel construction and wood construction.)

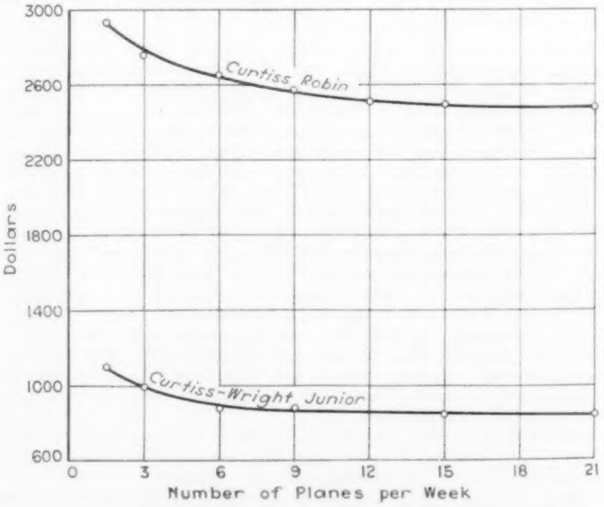

FIG. 4 MATERIAL COST COMPARED WITH NUMBER OF PLANES PRODUCED PER WEEK

visibility. Fig. 5 shows visibility possibilities in a pusher-type light airplane.

Low take-off and landing speeds give the light plane owner, who is usually an amateur, more time for thought during this maneuver. An airplane designed so that it cannot be forced off the ground is safer for a beginner than one which can get off accidentally before flying speed is reached. For this reason a low landing gear is preferable to the high one. Until flying speed is reached, the tail cannot be lowered sufficiently to force the ship into the air. Because of propeller clearance, the pusher type permits low landing gear, which also in turn makes it difficult for the plane to nose over. Nosing over still is a common occurrence among trained fliers, as well as amateurs, especially in the light plane class. Cost again crops up, and money is saved in repair bills. Fig. 6 shows an axle being formed in a press, which constitutes a low-type landing gear used on one of the present light airplanes.

Controllability at slow speed is of vital importance in any amateur airplane. The beginner is not always certain just when he has reached the stall, which often occurs when landing, and effectiveness of surface controls is paramount under these conditions. Any feature which can be incorporated in the design of airplanes to improve present-day control at stalling speeds will do much toward increasing safety of flying. Experiments are being conducted at the present time to determine the practicability of placing a stop in the up elevator control, so that all one has to do to land is to "cut gun" and pull the stick back and let the plane do the rest. A well-designed airplane will permit this maneuver without tendency to fall off in a spin.

Fire is not common in the air with present-day flying, but nevertheless this hazard should receive its share of consideration. Fuel tanks should be so located as to preclude the possibility of fire blowing back into the cockpits in case of accidental fire and forced descent. Many military planes now have the fuel tanks on the sides of the fuselage, while the majority of commercial planes carry fuel in the wings.

APPEARANCE AND PERFORMANCE

Appearance in light-airplane design should be measured in terms of gracefulness or pleasing harmony of outline. In the fighter pursuit class of airplanes, appearance is measured by the number of guns carried or the length of cowling over the engine, but the light airplane should have all the earmarks of grace and ease of contour. It should not necessarily appear light, but on the other hand should instill the thought of strength and ruggedness. The more that ruggedness is brought out in the design, the more pleasing the appearance, providing that gracefulness is still in the picture.

Biplane appearance is usually aided by sweepback, stagger, and dihedral, although all of these characteristics add to the cost. Monoplanes never appear as graceful without dihedral as they do with dihedral. The amount required for good looks is often decided upon by the actual flying characteristics of the airplane, but nevertheless anything less than about 2 deg. gives a flat appearance to the wing cell.

The fuselage need not necessarily be round, square, or triangular, to look right—it may be most any section; but it must be located and arranged in harmony with the rest of the airplane. If a fuselage is round, the tail planes should not be square or triangular in profile, but should conform to and have a general likeness to the body structure. A fuselage triangular in section looks well with a triangular tail unit. Tail units with abrupt changes in contour look well with wing tips of the same stroke. Consider for a moment how an elliptical wing tip would look with a box tail unit. If the box tail unit must be used, then use a square tip with sheared corners. The color schemes employed in the finishing of airplanes can do much to improve or harm a given outline. It does not appear that airplane manufacturers have yet taken full advantage of what the automobile industry has learned in the way of color combinations, but no doubt more time and consideration will be given to this art as the industry becomes more and more stabilized.

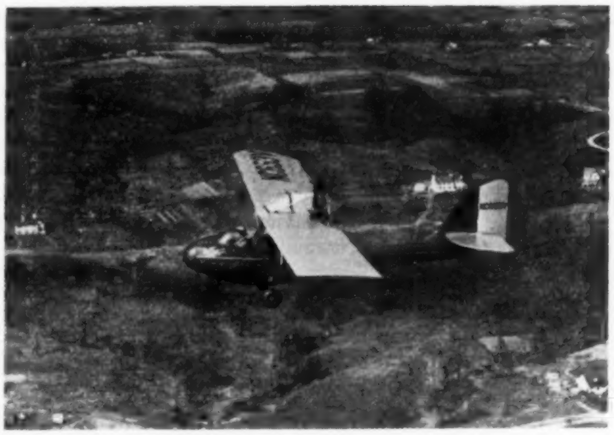

FIG. 5 VISIBILITY POSSIBILITIES IN A PUSHER TYPE OF LIGHT AIRPLANE

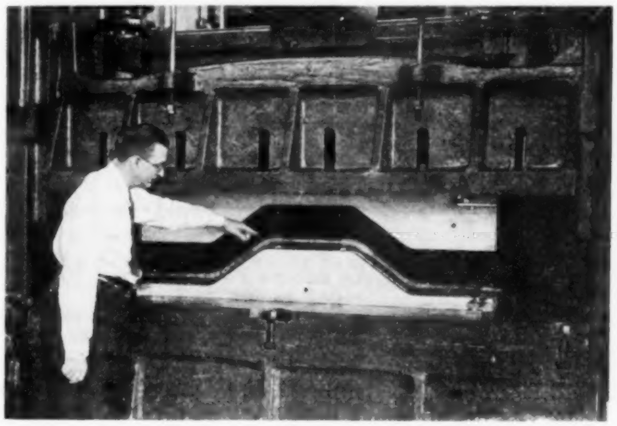

FIG. 6 ONE-PIECE AXLE IN PRESS. THIS TYPE PROVIDES LOW LANDING GEAR

Performance in light airplanes is not measured in terms of high speed and maneuverability, but should be measured in terms of take-off and landing characteristics, together with controllability at low speeds. The light airplane is the step between the glider and the "normal powered" class. Every one knows the necessity for high aspect ratio on successful gliders, and this need is also present in gaining performance with the light airplane. Wing cells with high aspect ratios require particular consideration with regard to structure, because of the weight involved in large spans. A study of high aspect ratio wing weights immediately presents itself as the designer proceeds to select his wing cell.

The first consideration is beam height, which of course is dependent upon airfoil section. The best present-day airfoils for light airplanes restrict the rear beam depth. Because of the large span, the beams should be kept well apart to assure a rigid drag truss, and this again adds to the difficulty of obtaining a satisfactory rear beam. A special airfoil was developed for the Curtiss-Wright Junior which gives a 15 per cent greater rear beam depth than could have been obtained with other airfoils of approximately the same characteristics. A 15 per cent gain in beam depth increases the moment of inertia 52 per cent, which aids materially in keeping down beam weights.

Fig. 7 shows wing weights per square foot plotted against aspect ratio for different types of wing cells. Fig. 8 shows the wing weight per square foot plotted against specified high incidence load factor, while Fig. 9 plots wing weight per square foot against wing loading. These figures show approximately what can be expected in the saving of wing weights for light airplanes as compared to "normal powered" airplanes.

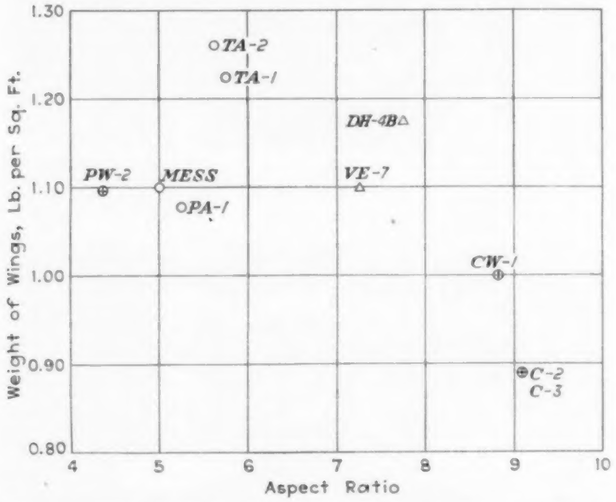

FIG. 7 WING WEIGHTS PER SQUARE FOOT COMPARED WITH ASPECT RATIO

(Legend: ○, single-bay biplane; △, double-bay biplane; ⊕, monoplane braced. Reprinted in part, by courtesy of Air Service, from Inf. Circ. No. 541.)

The names of the airplanes, listed by model designation in Figs. 7, 8, and 9, are as follows:

TA-1.....	G. Elias
TA-2.....	Huff Daland
Mess.....	Verville
DH-4B...	DeHavilland
PW-2.....	Loening Monoplane
PA-1.....	Loening Biplane
VE-7.....	Vought
C-2......	Aeronca One-Place Light Airplane
C-3......	Aeronca Two-Place Light Airplane
CW-1....	Curtiss-Wright Junior Light Airplane

It is regretted that no data are available at the present time on light airplanes other than the ones listed, as this would make a much more interesting study.

Light-airplane design is still young. Many new and interesting models are breaking forth all over the country, and there is every reason to believe that the schoolboy will take up the art of flying because of its sporting nature as soon as price, safety, and performance (within the limits outlined herein) are brought within his reach and understanding.

Discussion

MILTON C. BAUMAN.[3] Mr. White starts out with the question, "Why a light airplane?" This question must be answered before the design can be comprehensively discussed. Has the normal-powered airplane been modified at a sacrifice of speed just to meet a price or are the flying characteristics of the light airplane the real reason for it?

Mr. White leads us to believe that price is the most important factor in the design of these small planes. He then tells us how the cost is reduced. This is done only by virtue of the modified design of the normal airplane. In the first place the light airplane does not require as large a motor as the normal airplane, and in the second place it has been simplified as much as possible. The manufacturing methods are the same for both types. Is this the way the price should be reduced?

The author states that the manufacturing cost also depends on the quantity produced, but that quantity production is not justified. It seems that quantity production is the real solution for a cheap, satisfactory airplane, and that the savings resulting from changes in design of engine and plane are, after all, insignificant as compared to the reduced cost of quantity production. If this is true, then how are we going to get more people to seriously use airplanes? By making airplanes of high performance equipped with the best engines available or by making airplanes of low performance with relatively inferior engines, but selling

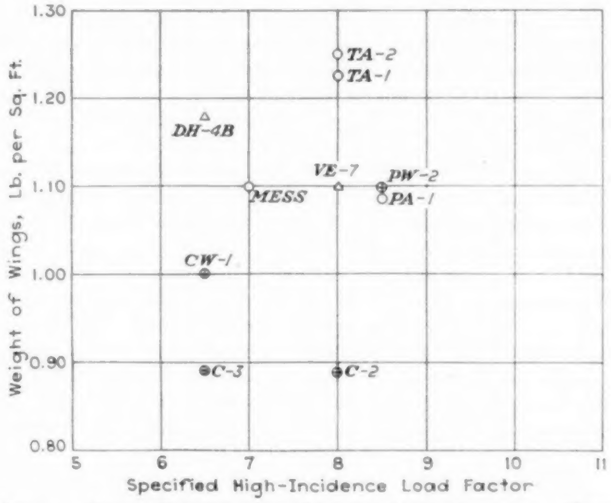

FIG. 8 WING WEIGHTS PER SQUARE FOOT COMPARED WITH SPECIFIED HIGH-INCIDENCE LOAD FACTOR

(Legend: Circle, single-bay biplane; triangle, double-bay biplane; crossed circle, monoplane braced.)

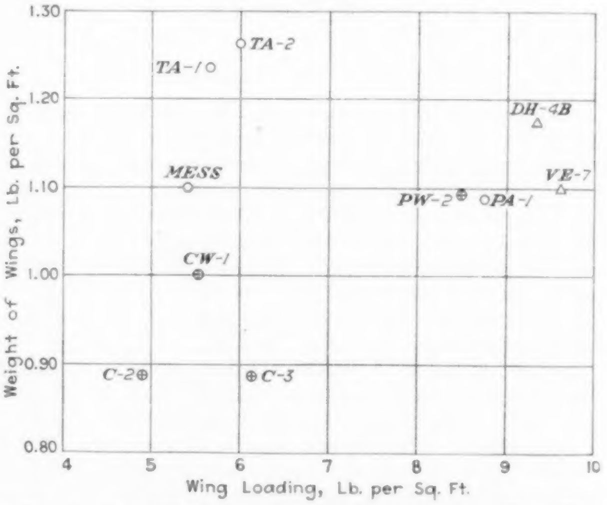

FIG. 9 WING WEIGHTS PER SQUARE FOOT COMPARED WITH WING LOADING

(Legend: Circle, single-bay biplane; triangle, double-bay biplane; crossed circle, monoplane braced.)

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[3] Chief Engineer, Inland Aviation Company, Kansas City, Mo.

at a lower price? Remember, it is one thing to get a person into the air and still another to get that person to use it as a means of transportation, which really sells airplanes.

Wellwood Beall.[3] Mr. White's paper has brought to attention many truths about the features and design of light airplanes and has given much interesting information. The appearance of the airplane has been highly stressed. In order to merchandise any object, it must have eye appeal. Of two objects, all other things being equal, the one with the more pleasing appearance will of course be the more successful. Since most light planes have approached the same performance and since they are constructed almost identically, due to the regulations of the Department of Commerce, it stands to reason that the most important feature in the design of a plane will be its appearance.

However, in certain types of structures and designs, inherent beauty of form cannot be built into the airplane. In this case a great improvement in the appearance can be effected by the use of an intelligent and artistic paint job. The point is that much credit for a pleasing appearance can be given to the paint job. This of course includes the type of finish, whether dull or bright, whether smooth or rough, whether corrugated, or whether free from all joints or cracks; in other words, it also includes the type of covering. Then the color of the paint and the layout of the paint pattern are important. A well-thought-out pattern with an artistic color scheme will do wonders with an ordinary machine, and with a machine that has good lines inherently, a wonderful looking airplane can be produced.

Too little thought is given to this consideration in the aeronautical field. A lesson could be learned from the automobile industry; in later years the biggest battles in the automobile industry have been fought and won in the paint room.

Hugh L. Thompson.[4] Mr. White has given a very excellent résumé of the light airplane. He has stressed a very important point, and that is price. Of course there are other important points, but if people are going to buy the light airplane, it must be sold at a price that is as low as the medium-priced automobile—say, around $1500. Under present conditions this is difficult to do and make a profit. Large-scale production will help. However, there is something else lacking for the light airplane that should be brought out, particularly as so much attention is being given to power. What is needed is a good, inexpensive power plant for this type of airplane. At present aircraft engines run in price around $8 or $9 per horsepower. The larger motors have been developed to a point of great reliability, but the little motors have not reached this stage as yet—probably because they are too new. The motor for the light airplane should not be reduced in horsepower, but it needs to be reduced in weight and price and increased in reliability, and then light airplanes can be built at a price that will sell them and make a profit. Let us have more development and research work along these lines.

Frank M. Smith.[5] Mr. White is laying too much stress on the matter of price. While undoubtedly the flying public want low-priced airplanes, they do not want cheap airplanes. The building of an airplane to satisfy a class that does not consider life worth a few extra dollars is poor psychology and a questionable procedure, and the establishment of a successful business on such a basis is entirely impractical.

Such sales psychology is all wrong, and the day is past when a dumb public will buy airplanes on the recommendation of an airplane pilot or salesman, but prides itself on its knowledge of aviation and is demanding something better than a cheap crate.

A combination of speed, safety, comfort, visibility, economy of operation, and service is what the flying public demand, and when a ship combining these essentials can be produced at a reasonable figure, the responsible element of the public will buy, and not until then. Such a plane must be a cabin-type, all-metal plane of a design different from anything now in the air.

Author's Closure

Referring to Mr. Bauman's discussion, the author thinks that sales statistics will bear him out in the matter of price being at the top of the designer's guide in developing light airplanes. Of course, we cannot think of offering an unsafe airplane to the public, and even if we should, we all know the Department of Commerce would not allow it.

The author heartily agrees with Mr. Beall that no one gives sufficient attention to color schemes. Unfortunately, within the past two years the manufacturers have, of necessity, been using up inventory wherever possible.

It is true, as contended by Mr. Thompson, that the field for engines of around 50 hp. used in light airplanes is new and that experience will in time enable us to greatly improve these power plants.

Mr. Smith is making a big jump in recommending that what we need is something entirely different from anything new in the air. The author agrees that the future airplane will not look much like the old "Jenny," but progress is made slowly, and new types develop gradually from the present conventions. So let us take it easy, at least until the depression is over.

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[4] U. S. Aircraft Engineering School, Inc., Kansas City, Mo.

[5] Lieutenant, U.S.N. With Stout Engineering Laboratories, Dearborn, Mich. Formerly vice-president and general manager, Moth Aircraft Corp., and superintendent, Curtiss-Robertson Co.

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APM-54-1

The Hydraulic Jump and Related Phenomena

By BORIS A. BAKHMETEFF,[1] NEW YORK, N. Y.

In this paper the author presents certain results which were obtained by him in studying the hydraulic jump over a period of years. A general characteristic of the "state" of flow is introduced through the notion of "Kineticity of Flow," expressed numerically by a "Kinetic flow factor." The fundamental relations, referring to the jump, are presented in generalized form. A physical basis is offered to explain the different forms in which the jump occurs. The practical important cases of a jump below a sluice and at the foot of a weir are studied in detail. Formulas are devised and charts given which make it possible to solve different problems of engineering practice.

I—PRELIMINARY

Energy Diagram. In dealing with the hydraulic jump and related problems it has become customary to refer the flow to the "specific-energy diagram" (Fig. 1).[2] Given the shape of the canal and the discharge Q, flowing in parallel movement under varying depths d, the curve $\epsilon = f(d)$ pictures the energy per unit weight

$$\epsilon = d + \frac{v^2}{2g} = d + \frac{Q^2}{2ga^2} \quad\ldots\ldots[1]$$

carried by the flow at each and every depth. In this equation v is the average velocity, and a the cross-sectional area at the depth d.

Point C in the energy diagram (Fig. 1), at which the ϵ-curve passes through a minimum, determines the "critical depth" d_{cr}. Flow at $d = d_{cr}$, being flow with the minimum possible content of energy, is said to be in the "critical state." Following Boussinesq's nomenclature, flow at stages above the critical depth ($d > d_{cr}$) is said to be in the "tranquil" state, while flow at stages below the critical ($d > d_{cr}$) is in the "rapid" state.

Hydraulic Jump. In the jump (Fig. 2), the flow passes abruptly from a rapid to a tranquil state, corresponding in the energy diagram to a sharp change from point (1) on the lower branch to point (2) on the upper branch of the ϵ-curve. The horizontal difference between points (1) and (2), or

$$\epsilon_j = \epsilon_1 - \epsilon_2 \quad\ldots\ldots\ldots\ldots[2]$$

is the specific energy lost in the jump.

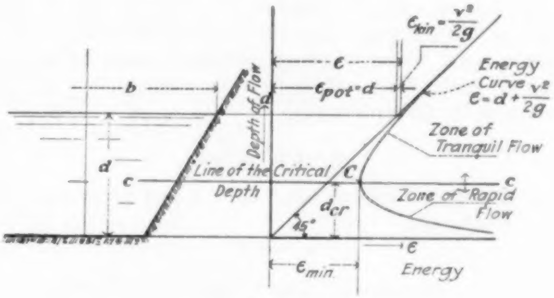

FIG. 1

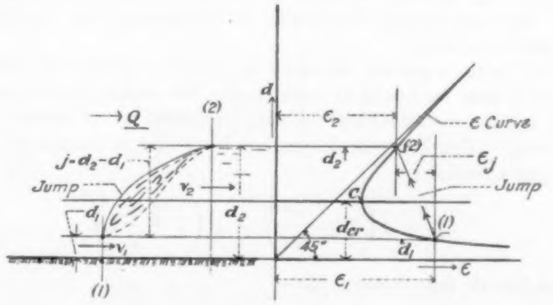

FIG. 2

The depths d_1 and d_2 before and after the jump will be called (Fig. 2) the "conjugated" depths. They are usually determined by means of the momentum principle of Bélanger.

For a canal of rectangular shape,[3] with a discharge per unit width of $q = Q/b$, the conjugated depths are related by the well-known symmetrical equations:

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[1] Consulting Engineer. Mem. A.S.C.E. Boris A. Bakhmeteff was born in 1880 at Tiflis, Caucasus, Russia. In 1898, graduated Classical Gymnasium, Tiflis; 1903, Institute of Engineers of Ways of Communication, St. Petersburg; 1904-1905, research work at Polytechnic Institute, Zurich, Switzerland, and practical work in the United States (New York State Barge Canal and office of Colonel Bogart, consulting engineer); 1911, received degree of "Adjunct Applied Mechanics" corresponding to Doctor of Engineering; academic work at the Polytechnic Institute Emperor Peter the Great, St. Petersburg; 1905–1916, as assistant professor, docent, and professor; is the author of "Lectures on Hydraulics," St. Petersburg, 1912-1913, "Varied Flow of Liquid in Open Channels," Petrograd, 1912, and "Introduction to the Study of Variable Flow of Liquids," 1914; 1907–1915, consulting engineer in St. Petersburg, specializing on water power; was connected with the Dneiper project (300,000–1,000,000 hp.) as chief engineer, the "Volchow" and "Immatra" plants (150,000 hp.) of the "Petrograd Company for Transmission of Hydraulic Power" as chief engineer, and the Rion Plant, Caucasus (50,000 hp.), for the electrification of the Trans-Caucasus Railroad; member of Technical Board of the Russian Reclamation Service; chairman of "Board for Engineering Research" of the Reclamation Service; served as expert on governmental and municipal boards connected with irrigation, internal navigation, sewage disposal, legislation on public utilities, and water rights; with the advent of the war, volunteered first with the Red Cross, subsequently with the Central War Industrial Committee; a public voluntary organization of business and professional men; 1915–1916, Chief Plenipotentiary of the C.W.I. Committee to the United States; member of Anglo-Russian Purchasing Commission; March, 1917, after the fall of the Czar Government, was appointed Under Secretary of State (Vice Minister) for Commerce and Industry of the Provisional (Kerensky) Government; May, 1917, appointed Ambassador to Washington, remaining in that post until 1922, at which time he resigned; since 1923 established in New York as consulting engineer connected with various industrial enterprises.

[2] Bakhmeteff, "On Varied Flow of Liquid in Open Channels" (in Russian), St. Petersburg, 1912 (reprinted 1928); also Proc. A.S.C.E., February, 1931; Hinds, *Eng. News-Record*, 1920; Böss, *Forschungsarbeiten V.D.I.*, No. 284, 1927.

[3] The general case of a canal of any cross-sectional form is dealt with in Appendix No. 1.

Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$$\left.\begin{aligned} d_2 &= \frac{d_1}{2}\left[-1+\sqrt{1+\frac{8q^2}{gd_1^3}}\right] \\ d_1 &= \frac{d_2}{2}\left[-1+\sqrt{1+\frac{8q^2}{gd_2^3}}\right] \end{aligned}\right\} \qquad [3]$$

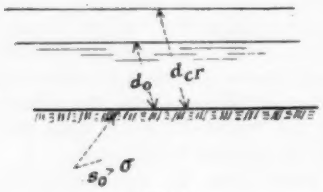

FIG. 3

Critical Depth. For a rectangular canal the critical depth is

$$d_{cr} = \sqrt[3]{q^2/g} \qquad [4]$$

For a canal of any shape with the discharge Q, the critical depth is the depth satisfying the equation

$$\frac{Q^2}{g} = \frac{a^3}{b} \qquad [5]$$

Critical Slope. In uniform flow with the depth d_0, corresponding to a discharge Q, the streaming will be *rapid* if the bottom slope is sufficiently "steep" to cause the depth of uniform movement d_0 to be smaller than the critical depth d_{cr}. (See Fig. 3.)

The slope which makes uniform flow critical ($d_0 = d_{cr}$) is called the "critical slope" and will be designated by σ.

For a canal whose width is sufficiently great to make the hydraulic radius $R = \sim d$, we have, for uniform flow, in accordance with the Chézy formula,

$$q = Cd_0\sqrt{d_0 s_0}$$

For flow in the critical state, $s_0 = \sigma$ and $q_0^2 = gd_0^3$. Thus $gd_0^3 = C^2 d_0^3 \sigma$, and

$$\sigma = g/C^2 \qquad [6]$$

II—KINETICITY. THE KINETIC FLOW FACTOR

The position of a point on the energy curve characterizes the "state" of flow. The higher the point on the upper branch [(2) in Fig. 2], the more tranquil the flow. The lower the point on the lower branch [(1) in Fig. 2], the greater the "rapidity" of the streaming.

To obtain a general standard by means of which the "state" of flow may be qualified numerically, the author makes use of the notion of "kineticity" of flow, measured by a "kinetic flow factor." The latter is designated by κ and is defined by the energy equation:

$$\epsilon = d\left(1+\frac{v^2}{2gd}\right) = d\left(1+\frac{1}{2}\kappa\right) \qquad [7]$$

The kinetic flow factor is thus

$$\kappa = 2\,\frac{v^2/2g}{d} = 2\,\frac{Q^2/2ga^2}{d} \qquad [8]$$

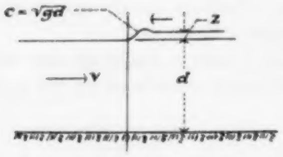

FIG. 4

or twice the ratio of the kinetic-energy head to the depth of flow. Obviously, for a given discharge κ is a function of the depth.

For a rectangular canal, remembering that $q^2/g = d^3_{cr}$ and with $Q^2/a^2 = q^2/d^2$, we have:

$$\kappa = d^3_{cr}/d^3.$$

For critical flow, with $d = d_{cr}$, $\kappa = 1$
For tranquil flow, with $d > d_{cr}$, $\kappa < 1$
For rapid flow, with $d < d_{cr}$, $\kappa > 1$

Kineticity equal to unity, corresponding to the critical depth, is thus the dividing criterion between tranquil and rapid flow. This simple and convenient characteristic explains the particular manner of designation as suggested in [7] and [8].

Celerity of Propagation. The kinetic flow factor provides also for a simple relation between the velocity of flow and the speed of propagation c of an intumescence of small height (Fig. 4).

According to the well-known St. Venant formula for a rectangular canal,

$$c = \sqrt{gd\left(1+\frac{3}{2}\frac{z}{d}\right)} \qquad [9]$$

which for small values of z/d is identical with the Lagrange formula

$$c = \sqrt{gd}$$

By comparison with [8] it is seen that

$$\kappa = \frac{v^2}{c^2} \qquad [10]$$

The velocity w with which the intumescence moves relative to the embankment is

$$w = c \pm v = c(1 \pm \sqrt{\kappa})$$

III—GENERALIZED FORMULA FOR THE JUMP IN RECTANGULAR CHANNELS

Substituting the value of κ from [8] in [3],

$$\left.\begin{aligned} \frac{d_2}{d_1} &= \frac{1}{2}\left[-1+\sqrt{1+8\kappa_1}\right] \\ \frac{d_1}{d_2} &= \frac{1}{2}\left[-1+\sqrt{1+8\kappa_2}\right] \end{aligned}\right\} \qquad [11]$$

where κ_1 and κ_2 are, respectively, the kinetic flow factors before and after the jump.

Solving Equation [11] with respect to κ_1 and κ_2,

$$\left.\begin{aligned} \kappa_1 &= \frac{1}{2}\frac{d_2}{d_1}\left(\frac{d_2}{d_1}+1\right) \\ \kappa_2 &= \frac{1}{2}\frac{d_1}{d_2}\left(\frac{d_1}{d_2}+1\right) \end{aligned}\right\} \qquad [11a]$$

Finally, substituting [11] into

$$\kappa_2 = \left(\frac{d_{cr}}{d_2}\right)^3 = \left(\frac{d_{cr}}{d_1}\right)^3 \cdot \left(\frac{d_1}{d_2}\right)^3 = \kappa_1\left(\frac{d_1}{d_2}\right)^3$$

the following symmetrical relations are obtained:

$$\left.\begin{aligned} \kappa_2 &= \frac{8\kappa_1}{(-1+\sqrt{1+8\kappa_1})^3} \\ \kappa_1 &= \frac{8\kappa_2}{(-1+\sqrt{1+8\kappa_2})^3} \end{aligned}\right\} \qquad [12]$$

The specific energies before and after the jump are, respectively:

$$\epsilon_1 = d_1\left(1+\frac{\kappa_1}{2}\right)$$

$$\epsilon_2 = d_2\left(1+\frac{\kappa_2}{2}\right)$$

Eliminating the depths by means of Equations [11] and [12],

$$\frac{\epsilon_2}{\epsilon_1} = \frac{\frac{1}{2}\left(\sqrt{1+8\kappa_1}-1\right) + \frac{2\kappa_1}{\left(\sqrt{1+8\kappa_1}-1\right)^2}}{1+\frac{1}{2}\kappa_1} \quad \ldots\ldots [13]$$

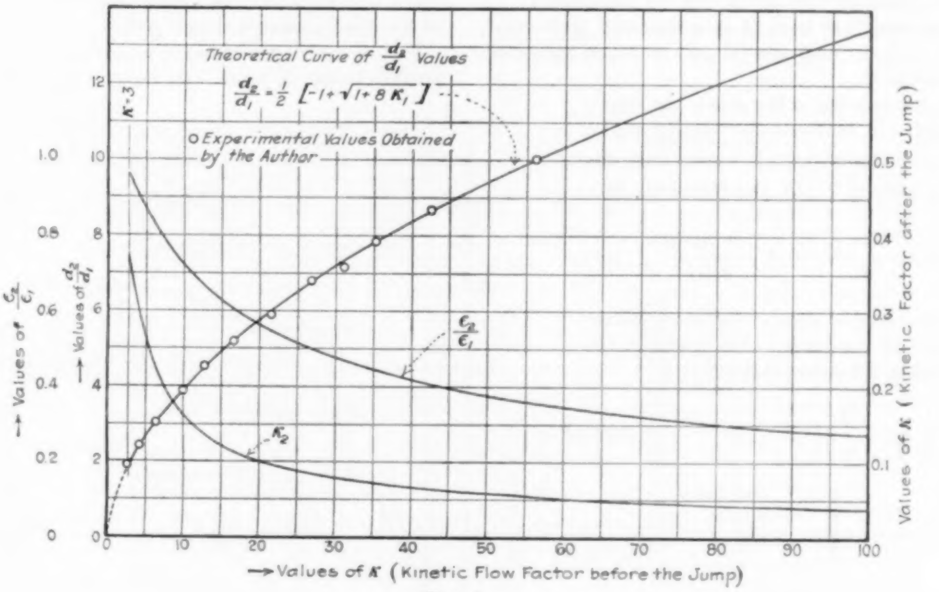

FIG. 5

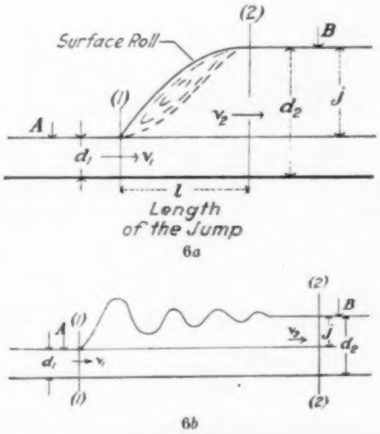

FIG. 6

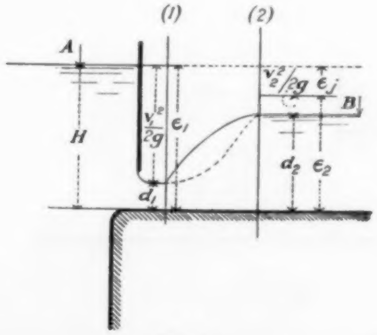

FIG. 7

Equations [11] to [13] present the fundamental relations governing the jump in dimensionless form, in terms of a single variable κ_1 or κ_2.[4] The kinetic flow factor thus constitutes a generalized coordinate. Fig. 5, drawn in accordance with Equations [11] to [13], makes it possible to solve practical problems in a very simple manner, as the following examples show:

Example 1. Assume a flow of 400 cu. ft. per sec. in a rectangular canal 20 ft. wide. Assume $d_1 = 0.8$ ft. Determine the height of the jump and the loss of energy.

With $v_1 = 400/(20 \times 0.8) = 25$ ft. per sec. and $v^2/2g = 9.7$ ft., the kinetic flow factor is $\kappa_1 = 2(9.7/0.8) = 24.2$. The initial energy $\epsilon_1 = 0.8 + 9.7 = 10.5$ ft. From Fig. 5, $d_2/d_1 = 6.45$; $\epsilon_2/\epsilon_1 = 0.52$. Hence $d_2 = 6.45 \times 0.8 = 5.16$ ft.; the height of the jump $j = 5.16 - 0.8 = 4.36$ ft. On the other hand, $\epsilon_2 = 0.52 \times 10.5 = 5.46$ ft.; and the loss of energy is 48 per cent, or $0.48 \times 10.5 = 5.04$ ft.

Example 2. With the same discharge, determine the depth d_1 which will sustain a jump resulting in $d_2 = 4$ ft. The velocity $v_2 = 400/(4 \times 20) = 5$ ft.; $v^2/2g = 0.388$; $\epsilon_2 = 4.39$; $\kappa_2 = 2(0.39/4) = 0.194$. From Fig. 5 the corresponding $\kappa_1 = 7.4$; $d_2/d_1 = 3.36$; $\epsilon_2/\epsilon_1 = 0.794$. Hence $d_1 = 4/3.36 = 1.19$ ft.; and the loss of energy = 20.6 per cent.

IV—FORMS OF THE JUMP. STABILITY

THE JUMP BELOW A SLUICE

The hydraulic jump is known to occur in two distinct forms:[5]

1 The "direct" form, in which the upper stage is reached through *one* single rise [Fig. 6(*a*)].

2 The "undulate" form, where the connection between the two stages presents a series of undulations [Fig. 6(*b*)].

The direct form is accompanied by a "surface roll" covering the underlying continuous streaming. In the "undulate" form, when the jump is small, the whole surface may be smoothly continuous.

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[4] *Annales Inst. Polyt.*, St. Petersburg, 1911. See also Miami Conservancy District Technical Report, 1917, p. 3.

[5] The different forms of jump have been differentiated by Bazin. See also Flamant's, "*Hydraulique*," and Böss, *Forschungsarbeiten V.D.I.*, No. 284.

An explanation of the phenomenon is gained by considering the case of a jump below a sluice (Fig. 7). In what follows the canal will be assumed to have a rectangular section. The slight losses accompanying the efflux will be disregarded. In other words, assuming that level A is maintained permanent, the energy ϵ_1 at the *vena contracta* (1) will be taken to remain constant and equal to H.

By regulating the opening of the sluice, the depth d_1 will be changed. In each case the velocity will be $v_1 = \sqrt{2g(H - d_1)} = \sqrt{H}\sqrt{2g[1 - (d_1/H)]}$; the discharge $q = d_1\sqrt{2g(H - d_1)}$, and the kineticity

$$\kappa_1 = 2 \cdot \frac{v^2/2g}{d_1} = 2\,\frac{H - d_1}{d_1} = 2\,\frac{1 - (d_1/H)}{d_1/H} \quad \ldots [14]$$

By means of [11] to [13] we may further calculate the other elements of the jump. For example, the depth d_2 (substituting [14] into [11]) will be:

$$d_2 = \frac{d_1}{2}\left[\sqrt{16\,\frac{H}{d_1} - 15} - 1\right] \quad \ldots\ldots\ldots [15]$$

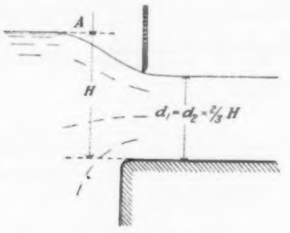

FIG. 8

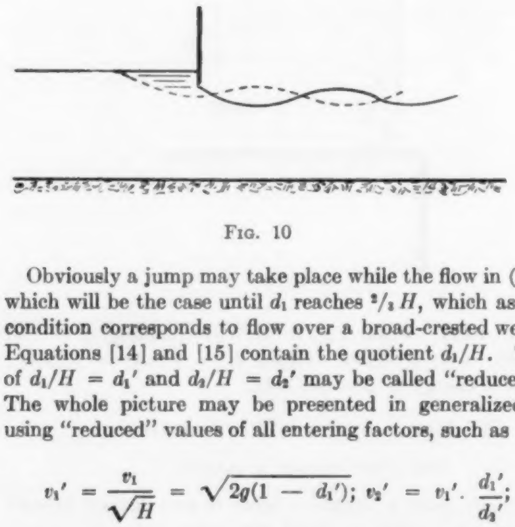

FIG. 10

Obviously a jump may take place while the flow in (1) is rapid, which will be the case until d_1 reaches ${}^2/_3\,H$, which as a limiting condition corresponds to flow over a broad-crested weir (Fig. 8). Equations [14] and [15] contain the quotient d_1/H. The values of $d_1/H = d_1'$ and $d_2/H = d_2'$ may be called "reduced" depths. The whole picture may be presented in generalized form by using "reduced" values of all entering factors, such as

$$v_1' = \frac{v_1}{\sqrt{H}} = \sqrt{2g(1 - d_1')};\ v_2' = v_1' \cdot \frac{d_1'}{d_2'};$$

$$q' = \frac{q}{H\sqrt{H}} = d_1'\sqrt{2g(1 - d_1')};\ d_{cr}' = \frac{d_{cr}}{H}$$

$$= d_1'\sqrt[3]{2\,\frac{1 - d_1'}{d_1'}};\ \epsilon_1' = 1;\ \epsilon_2' = \frac{\epsilon_2}{H};\ \epsilon_j' = 1 - \epsilon_2',\ \text{etc.}$$

The results are presented in chart form in Fig. 9. Obviously, the relations as obtained are not limited to the specific manner by which the jump was produced (Fig. 7). In fact, the chart given in Fig. 9 is a general characteristic of the jump as a function of d_1 under the condition that ϵ_1 is constant.

Referring to Fig. 9, it is seen that the d_2'-curve is ascending for values of $d_1' < 0.4$; having reached a maximum of $d_2' = 0.8$ at $d_1' = 0.4$, with $\kappa = 3$, the depth d_2 diminishes. In other words, $\partial d_2/\partial d_1$ is positive for the zone where $\kappa > 3$, and negative for kineticities smaller than $\kappa = 3$.

This circumstance appears to have a direct bearing on the possible forms of jump. In fact, experiments carried out by the author[6] indicate that in the zone where $\kappa > 3$ the jump occurs in the direct form. Moreover, the phenomenon is *stable*, and the values of d_2/d_1 are in close accord with the theoretical curve (Fig. 5).[7] On the contrary, in the zone of low kineticities ($\kappa < 3$) the surface becomes undulated; the waves increasing as κ declines. With this the whole phenomenon becomes *unstable*. The undulations shift back and forth in the direction of flow, until with a further opening of the sluice the whole efflux becomes subject to a periodical pendulatory movement. A phase such as that indicated in Fig. 10 by the solid line is succeeded by streaming, where the flow tears itself entirely from the edge of the sluice as shown by the broken line.

Within the scope of the author's experiments, with the undulatory form present there was actually no portion of flow

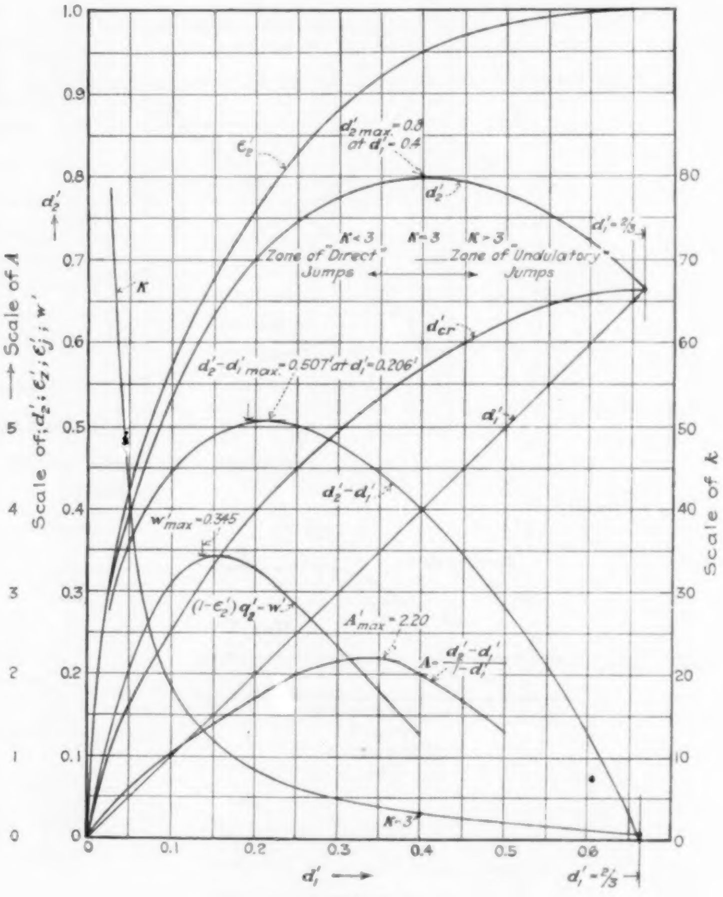

FIG. 9

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[6] *Annales Inst. Polyt.*, St. Petersburg, 1911.

[7] The close accord between observed and theoretical values is characteristic of all experiments where the kineticity was high. (Rehbock; Gibson, Proc. Inst. C. E., 1913–14; Koch-Carstanjen "Bewegung des Wassers," 1928, etc.)

back of the jump where the movement was parallel and to which Equation [3] could be strictly applied. All that can be said is that the tops of the waves were distinctly above the "conjugated" values of d_2 obtained from [11]. The author's work was of a preliminary character, and further detailed experiments are required,[8] in particular to determine the losses in an "undulate" jump. However, attention is drawn to the fact that the loss of energy at the dividing point at $\kappa = 3$ is only 5 per cent of ϵ'. One may reasonably expect that in the undulate zone the relative losses are still smaller.

Most of the earlier experiments (those of Bidone and Bazin) were made on jumps produced by inserting a barrier in a stream flowing uniformly at a rapid rate ($d_0 < d_{cr}$, Fig. 11). One may estimate the range of kineticities obtained in such a manner by

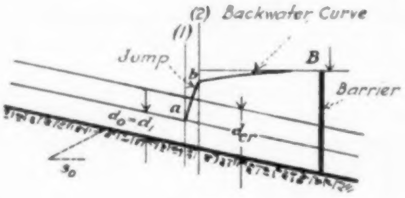

FIG. 11

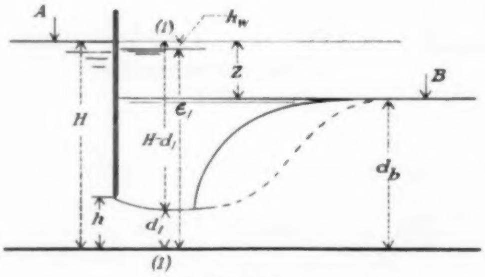

FIG. 12

considering the case of a broad canal, with $R \cong d$. For uniform movement, with $v = C\sqrt{d}\,\sqrt{s_0}$, the kinetic flow factor is

$$\kappa = \frac{v^2}{gd_1} = \frac{C^2 d s_0}{gd} = \frac{C^2}{g} s_0$$

or, referring to [6]

$$\kappa = \frac{s_0}{\sigma} \qquad [16]$$

or the ratio of the actual bottom slope to the critical slope. To obtain $\kappa \geq 3$, the slope would have to be $s_0 \geq 3\sigma$. For a case characterized by an average value of C, namely, 100, this would mean that $s_0 \geq 3 \times 0.0032 = \sim {}^1/_{100}$.

Naturally in most watercourses, where the flow is due to the slope only, the kineticity will be lower and the jump of the undulate type. This will explain the rather confusing results obtained at times through experiments; and also why certain observers (Bélanger in his earlier works, Rühlman, etc.) were led to ignore losses in the jump entirely, determining the relation between the conjugated depths from the Bernoulli equation.

V—The Jump Below a Sluice in Engineering Practice

There are a number of important cases where the chart of Fig. 9 and the related equations may be directly applied. In such practical computations, the head to which the elements of the jump are to be referred when using the chart should be the energy head $\epsilon_1 = d_1 + v_1^2/2g$ at (1) in Fig. 7. This head is less than H, determined by the level A by

$$h_w = (1 - \varphi^2)(H - d_1)$$

where φ is the velocity coefficient in the efflux formula (see Fig. 12).

(a) *Free or Submerged Efflux?* (Fig. 12.) Given the positions of levels A and B, the discharge and all other circumstances will depend on whether the outflow is submerged under a working head Z, or whether, by reason of a jump, the efflux becomes free, with a working head $H - d_1$. The discharge in free outflow is obviously greater. So is the erosive action of the outflowing water. The form of flow will depend on whether d_B as determined by level B is larger or smaller than the depth d_2, conjugated to d_1 through the jump.

(b) *The Jump as an Annihilator of Energy.* In passing excess water through a sluice or over a spillway, the problem often is to annihilate excess energy in order to reduce erosive action in the tailrace. The jump appears to be an efficient means for this purpose.[9] The quantity of energy annihilated in the jump per unit of time is equal to

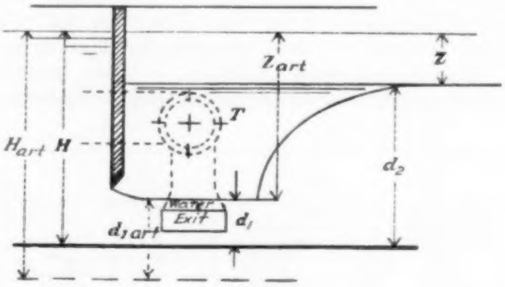

FIG. 13

$$w = \Delta . q . (\epsilon_1 - \epsilon_2) \text{ ft-lb. per sec.}$$

where Δ is the specific gravity of the liquid. By multiplying the respective values of the chart, Fig. 9, one may obtain the reduced value of

$$w' = q'(1 - \epsilon_2') \qquad [17]$$

As shown in the chart, w' maximum is equal to 0.345 and occurs at $d_1' = 0.15$, corresponding to $\kappa_1 = 11.3$. Thus the maximum possible dissipation of energy which may be effected by means of a jump per unit width of flow is $0.345 \times \Delta \times H^{3/2} = 21.5$ $H^{3/2}$ ft-lb. per sec.

(c) *The Jump as a Fall Increaser* (Fig. 13). The jump has been suggested as a "fall increaser," to cope with the reduction of head in power plants during periods of high water.[10] In Fig. 13 the natural fall Z is increased to Z_{art} by letting the turbines T discharge into the pool ahead of the jump.

With $Z = H - d_2$ given, the greatest possible increase of the head will correspond to the maximum value of

$$\frac{d_2 - d_1}{H - d^2} = \frac{d_2' - d_1'}{1 - d_2'} = A' \qquad [18]$$

Fig. 9 shows that this maximum is reached at $d_1' = 0.35$ with $A'_{max} = 2.20$. However, between $0.25 < d_1' < 0.40$, the value of A' is > 2, so that within these limits the fall may be increased at least three times. To obtain the required d_1' =

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[8] It is due to this uncertainty that the curves of Fig. 5 are not extended below $\kappa = 3$.

[9] Technical Reports, Part III, Miami Conservancy District, 1917; Rehbock, Proc. Intl. Congress Appl. Mech., Delft, 1925; *Bau Ingenieur*, 1928; etc.

[10] Saugey, *Z.V.D.I.*, 1906.

$(d_1/H)_{\text{art}}$ with a given Z, it is usually incumbent to lower the sluice bottom.

VI—The Jump at the Foot of a Weir

Fig. 14 refers to the important practical case of a jump at the foot of a weir. Depending on the stage of level B, the tail water may either "cover" the nappe falling from the dam ($B' - b'$), or, if the depth t is small, it may be repelled by a jump $b''B''$ (toe roll). In this case the flow is free. The erosive action may become dangerous.

The question is identical with one investigated by Bazin,[11] who distinguished between "nappes covered by tail water" (*Nappes couvertes par le reflux d'aval*) and "free nappes with a removed jump" (*Nappes libres a ressaut éloigné*). Guided merely by experimental observations, the great hydraulist suggested that the presence of that or another form of flow depends on the value of Z/p, the quotient of the "step" between the levels by the height of the weir. The value of Z/p, which divides the two possible forms, was found by Bazin to be practically constant and equal to $(Z/p)_0 = 0.75$. Steps above $0.75p$ are expected to produce free nappes, independently of whether the weir is "submerged" or not in the customary sense of the word. The opposite is the case for $Z/p < 0.75$.

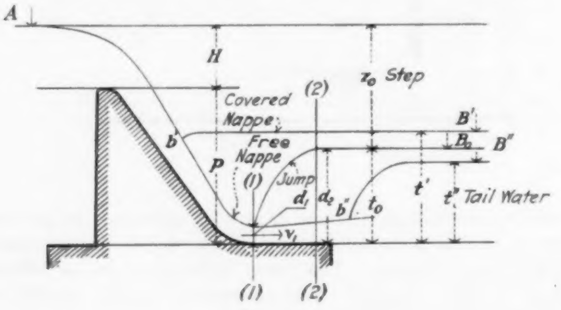

FIG. 14

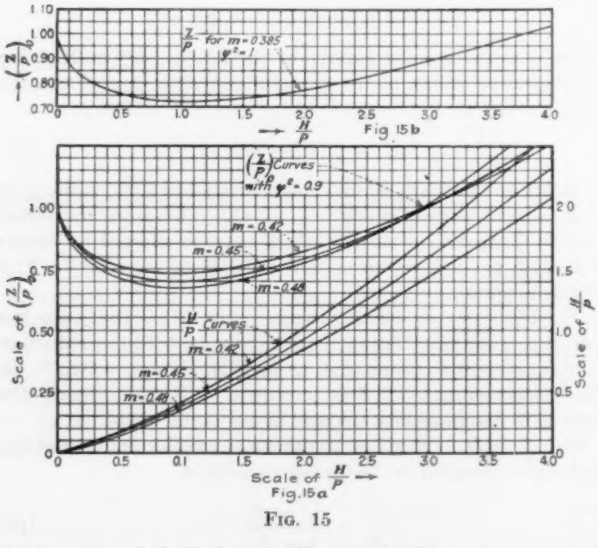

Fig 15b

Fig.15a

FIG. 15

Bazin's deductions are fully substantiated by theoretical analysis. In fact, returning to Fig. 14, it is evident that free or covered flow will depend on whether the tail-water depth t is above or below the depth d_2, conjugated through the jump to the depth d_1, which is the depth of free flow at the foot of the dam.

A more detailed analysis of the case is given in Appendix No. 2. The value of $(Z/p)_0$ is there proved to be dependent on the value of the coefficient of discharge over the weir m and the velocity coefficient φ, which takes account of the losses of energy between the pool A and section (1) at the foot of the structure.[12]

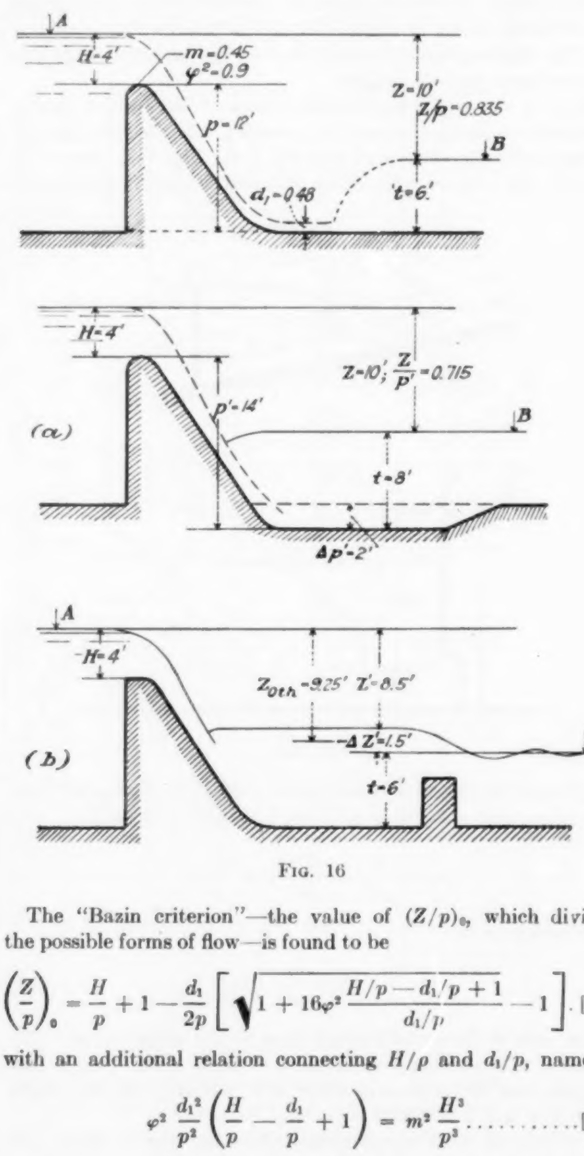

FIG. 16

The "Bazin criterion"—the value of $(Z/p)_0$, which divide the possible forms of flow—is found to be

$$\left(\frac{Z}{p}\right)_0 = \frac{H}{p} + 1 - \frac{d_1}{2p}\left[\sqrt{1 + 16\varphi^2\,\frac{H/p - d_1/p + 1}{d_1/p}} - 1\right]. \quad [19]$$

with an additional relation connecting H/p and d_1/p, namely,

$$\varphi^2\,\frac{d_1^2}{p^2}\left(\frac{H}{p} - \frac{d_1}{p} + 1\right) = m^2\,\frac{H^3}{p^3} \ldots\ldots\ldots\ldots [20]$$

Fig. 15 gives the values of $(z/p)_0$ and of d_1/p for a series of weir coefficients m with an average value of $\varphi^2 = 0.9$ ($\varphi = \sim 0.95$). The range of the m coefficients selected covers most of the cases which occur in practice. In comparing the curves of Fig. 15 with the figures of Bazin, it should be remembered that Bazin experimented with values of H/p between 0.37 and 1.74. For $m = 0.42$, corresponding to a sharp-edged weir, the respective theoretical values for $(Z/p)_0$ are between 0.73 and 0.78. Moreover, the curves in this region are flat, which explains why Bazin was led to suggest a constant value of $(Z/p)_0 = 0.75$, which is in good accord with the theoretical computations. The curves show, however, that for smaller H/p values (high dams), as well as for larger H/p (weirs of very small height), the limiting values of $(Z/p)_0$ are considerably larger.

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[11] "Expériences nouvelles," 1898.

[12] *Annales Inst. Polyt.*, St. Petersburg, 1914.

The following example shows the nature of the problems which may be solved by means of the curves of Fig. 15.

Example. Assume in Fig. 16 that $H = 4$ ft., $m = 0.45$, $p = 12$ ft., and the tail-water depth $t = 6$ ft. The relative step $Z/p = 10/12 = 0.83$, which is much in excess of $(Z/p)_0 = 0.77$, corresponding to $H/p = 4/12 = 0.33$ on the curves. The flow, therefore, will be free with $d_1 = 0.04p = 0.48$ ft. Suppose now that in order to diminish erosion the tail water must be brought to submerge the toe; to achieve such purpose, Z/p must be reduced.

(*a*) One way of doing this is to increase p by providing a basin at the foot of the dam. [Fig. 16(*a*).] An additional depth of $\Delta p = 1$ ft. making $p' = 13$ ft. is theoretically sufficient; in fact, H/p' is now $4/13 = 0.31$, requiring that $(Z/p)_0$ be 0.78, while in the present case $Z/p' = 10/13 = 0.77$.

As an additional safeguard, making $\Delta p' = 2$ ft., we obtain $Z/p' = 10/14 = 0.715$ as against the theoretical $(Z/p)_0 = 0.79$.

(*b*) Another way is to reduce Z by inserting a barrier [Fig. 16(*b*)]. To make Z/p equal to the theoretical value of $(Z/p)_0 = 0.77$, Z must be reduced to $0.77 \times 12 = 9.25$. Raising the tail water 1.5–2 ft. and making $Z' = 8.5$–9 ft. would answer all practical purposes.

Appendix No. 1

THE JUMP IN A CANAL OF ANY CROSS-SECTIONAL FORM

THE author has found it expedient to handle this problem in the following manner. Given a prismatic canal of the cross-sectional form shown in Fig. 17 and the discharge Q. Assume the bottom of the canal to be horizontal. The momentum equation applied to the liquid body between sections (1) and (2) gives:

$$\frac{\Delta Q}{g}(v_2 - v_1) = \Delta(a_1 Z_{01} - a_2 Z_{02}) \qquad \ldots\ldots[21]$$

where a_1 and a_2 are the cross-sectional areas corresponding to the conjugated depths d_1 and d_2, while Z_{01} and Z_{02} are the distances of the respective centers of gravity of the cross-sections below the surface of the flowing liquid.

Substituting $v = Q/a$ in Equation [21], it can be written

$$\frac{Q^2}{ga_2} + a_1 Z_{01} = \frac{Q^2}{ga_2} + a_2 Z_{02} \ldots\ldots\ldots\ldots [22]$$

The symmetrical form of [22] shows that the conjugated depths correspond to two equal values $[m(d_1) = m(d_2)]$ of a single function:

$$m(d) = \frac{Q^2}{ga} + aZ_0 \ldots\ldots\ldots\ldots\ldots\ldots [23]$$

which for a given discharge is obviously a function of the depth d only. The function m has two branches, and, further, it is easily shown that it passes through a minimum at the depth corresponding to

$$\frac{Q^2}{g} = \frac{a^3}{b} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [24]$$

analogous to [5], and determining the critical depth.

In Fig. 18 the $m(d)$ curve is traced for a trapezoidal canal of the section there shown for a discharge of $Q = 300$ cu. ft. per sec.

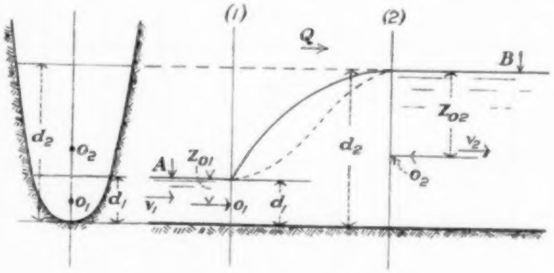

FIG. 17

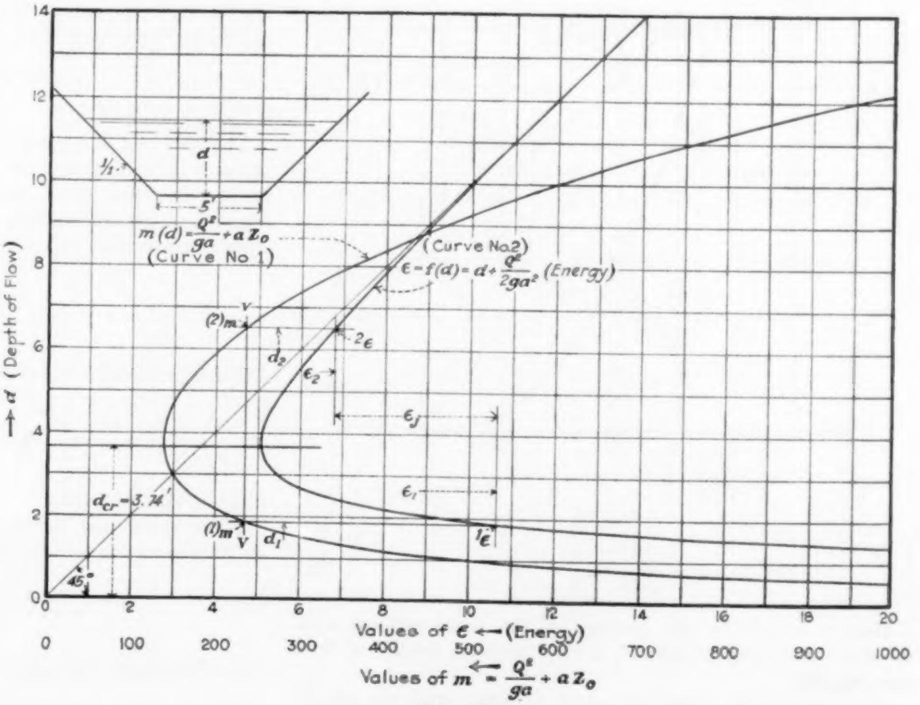

FIG. 18

The points of intersection $(1)_m$ and $(2)_m$ of any vertical v with the two branches of the m-curve determine a pair of conjugated depths d_1 and d_2. By tracing on the same chart the energy curve $\epsilon = f(d)$ (curve No. 2 in Fig. 18), one obtains an idea of the energy balance in the jump. In fact, horizontal lines drawn from $(1)_m$ and $(2)_m$ to intersect with the respective branches of the ϵ-curve determine the specific energies ϵ_1 and ϵ_2 and thus the loss ϵ_j in the jump. Applying the procedure to a series of verticals, one obtains the elements for the curve shown in Fig. 19, which expresses the jump as function of d_1.

It should be noted that the curves given in Fig. 18 and 19 describe the phenomenon for but one given discharge Q. To obtain general characteristics similar to those given for a canal of rectangular section in Figs. 5 and 9, it would be necessary to repeat the procedure for a series of discharges.

The inversion of curvature in the $\frac{\epsilon_1 - \epsilon_2}{\epsilon_1}$ curve in Fig. 19 implies the existence of two zones, similar to Fig. 9, corresponding to a direct and an undulatory form of the jump.

Kinetic Flow Factor. The value of the kinetic flow factor at the critical depth, because of [24], is

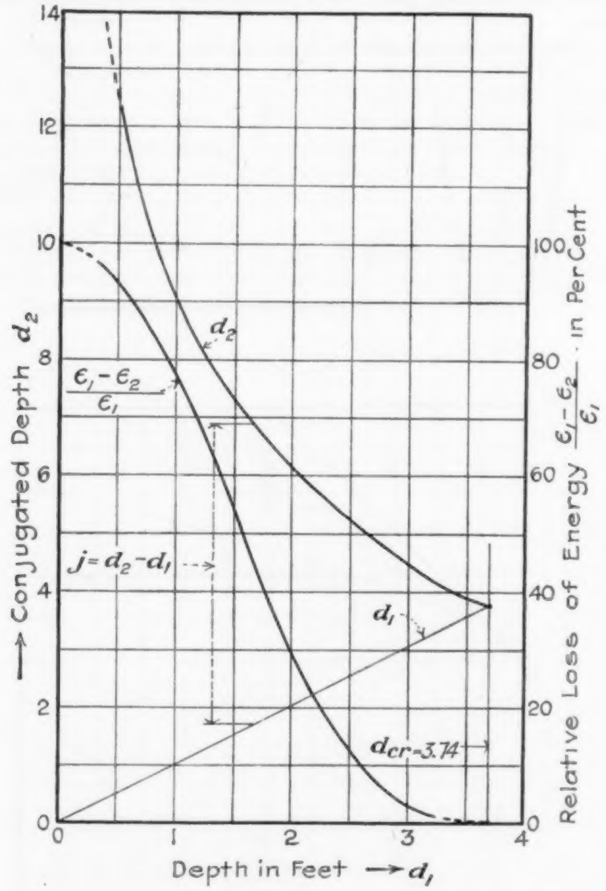

FIG. 19

$$\kappa_{cr} = \frac{Q^2}{ga^2d} = \frac{a}{b}\cdot\frac{1}{d} = \frac{\delta}{d} \quad \text{[25]}$$

Here $\delta = a/b$, or the quotient of the cross-sectional area divided by the top breadth b, is the "average depth" of flow (see Fig. 20).

For open canals, where the width increases with the depth, $\delta = a/b$ is always $< d$; hence the kinetic flow factor at the critical depth is < 1, or less than the kinetic factor in a rectangular canal.

Appendix No. 2

THE JUMP AT THE FOOT OF A WEIR

WITH reference to Fig. 14, assuming that H is the head over the weir, corrected for the velocity of approach, we have for section (1)

$$\left.\begin{aligned} &\text{Velocity:} && v_1 = \varphi\sqrt{2g(H + p - d_1)} \\ &\text{Discharge:} && q = \varphi d_1\sqrt{2g(H + p - d_1)} \end{aligned}\right\} \quad \text{[26]}$$

On the other hand, for the weir

$$q = m\sqrt{2g}\,H^{3/2} \quad \text{[27]}$$

Designating

$$\frac{H}{p} = x; \quad \frac{d_1}{p} = y \quad \text{[28]}$$

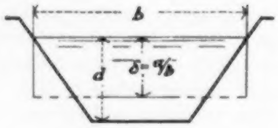

FIG. 20

and eliminating q between [26] and [27],

$$\varphi^2y^2(x - y + 1) = m^2x^3 \quad \text{[29]}$$

which is Equation [20], giving the relation between H/p and d_1/p_1.

The kineticity in section (1) is

$$\kappa_1 = 2\frac{v_1^2/2g}{d_1} = 2\varphi^2\frac{H + p - d_1}{d_1} = 2\varphi^2\frac{x - y + 1}{y} \quad \text{[30]}$$

The depth d_2 conjugated to d_1 through the jump, because of [11], is

$$d_2 = \frac{d_1}{2}\left[\sqrt{1 + 16\varphi^2\,\frac{x - y + 1}{y}} - 1\right] \quad \text{[31]}$$

Obviously the flow at section (1) will be free or covered by the tail water, depending on whether the depth

$$t = H + p - Z \quad \text{[32]}$$

is less or greater than d_2.

The lower the tail-water level (t_2'' in Fig. 14), the farther the distance to the repelled jump. As t increases, the jump moves toward the dam, until with $t_0 = d_2$ the limiting condition is reached. This corresponds to a dividing value of the step Z_0 which answers the requirements of the equation

$$t_0 = H + p - Z_0 = \frac{d_1}{2}\left[\sqrt{1 + 16\varphi^2\frac{x - y + 1}{y}} + 1\right] \quad \text{[33]}$$

Dividing by p and rearranging,

$$\left(\frac{Z}{p}\right)_0 = x + 1 - \frac{y}{2}\left[\sqrt{1 + 16\varphi^2\left(\frac{x - y + 1}{y}\right)} - 1\right] \quad \text{[34]}$$

which is Equation [19].

When computing the curves of Fig. 15, a procedure which avoids solving the cubic equation is to put

$$\frac{x}{y} = \frac{H}{d_1} = \xi \quad \text{and} \quad \frac{1}{y} = \frac{p}{d_1} = \eta$$

which transforms [30] into

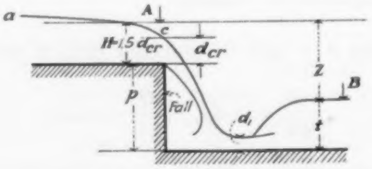

FIG. 21

$$\eta = 1 - \xi + \frac{m^2}{\varphi^2}\xi^3$$

By assuming a series of ξ values, we compute in each case η; then $y = 1/\eta$ and $x = \xi/\eta$.

The curve $m = 0.385$; $\varphi = 1$ [Fig. 15(*a*)], is specifically intended for appraising conditions at the foot of a "fall" (Fig. 21), as used in irrigation and other engineering practice. The surface in the canal approaches the fall by means of a drop curve *ac*, crossing the critical depth in the immediate vicinity of the edge. For a cross-section of rectangular shape the critical depth is $\sqrt[3]{q^2/g}$ and the energy head $\epsilon = 1.5\sqrt[3]{q^2/g}$. The flow over the fall may be treated as a streaming over a broad-crested weir, with the ideal discharge coefficient $m = 0.385$ and a head $H = 1.5\sqrt[3]{q^2/g}$.

Under such assumptions Equations [29] and [34] are directly applicable. The step Z is to be computed as

$$Z = p + H - t = p - t + 1.5\sqrt[3]{q^2/g}$$

By assuming $\varphi = 1$, we disregard the small losses in the falling nappe and add to the safety of the design.

Discussion

O. G. Tietjens.[13] For those who are not so well acquainted with the notation used in hydraulics, it may be pointed out that Equation [1] and the "specific energy diagram" presented in the paper are nothing more than the usual Bernoulli equation. However, it may be emphasized that Equation [1] and all the conclusions derived from it hold only if the curvature of the river bed and of the water surface are small enough so that the centrifugal forces of the water can be neglected; otherwise, the assumption made with respect to the uniformity of the velocity distribution over the cross-section does not hold.

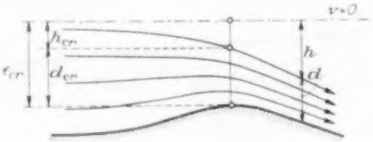

Fig. 22

Figs. 1 and 2 would possibly have been made clearer by giving a picture such as Fig. 22, showing the varying depth d and the corresponding variation of the water surface.

According to the Bernoulli equation, it is simply

$$h = \frac{v^2}{2g}$$

or

$$h + d = \epsilon = d + \frac{v^2}{2g} \quad\text{[35]}$$

This equation is identical with Equation [1] of the paper and shows at the same time the physical meaning of ϵ.

Assuming a rectangular cross-section of the channel, we have

$$Q = bd\sqrt{2gh} \quad\text{or}\quad d = \frac{Q}{b\sqrt{2gh}}$$

where b is the constant width of the channel. This, together with Equation [35], gives

$$\epsilon = h + \frac{Q}{b\sqrt{2gh}} \quad\text{[36]}$$

In the lower part of Fig. 23, ϵ is plotted as a function of h according to Equation [36].

Setting

$$\frac{\partial\epsilon}{\partial h} = 0$$

Equation [36] gives

$$\left.\begin{aligned} \epsilon_{cr} &= \frac{3}{2}\sqrt[3]{\frac{Q^2}{b^2g}} \\ \text{for}\quad h &= \frac{1}{2}\sqrt[3]{\frac{Q^2}{b^2g}} = \frac{d}{2} \end{aligned}\right\} \quad\text{[37]}$$

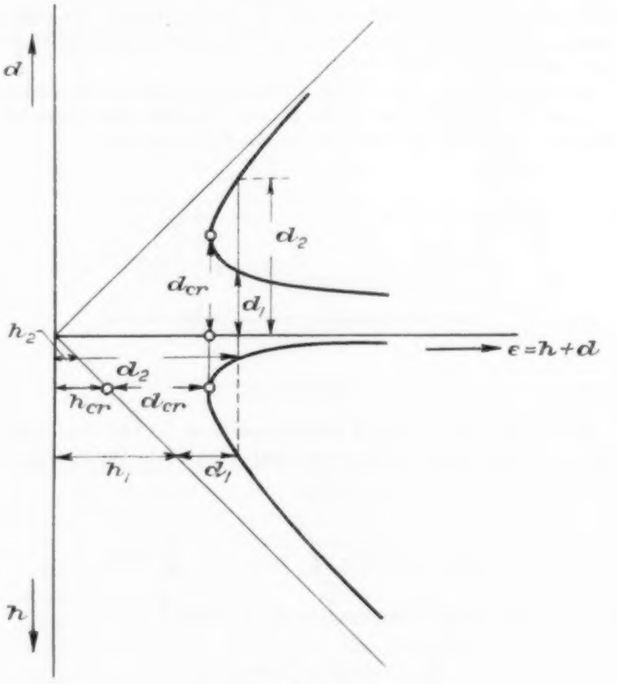

Fig. 23

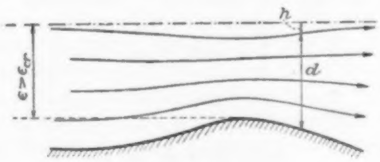

Fig. 24

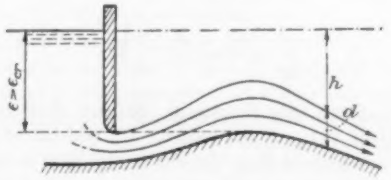

Fig. 25

This equation has the following physical significance: A certain height $\epsilon [= \epsilon_{cr}]$ of the water level over the weir is necessary to carry a certain quantity of water Q through a channel of the width b.

Fig. 1 of the paper can easily be obtained from Fig. 23 by plotting the ordinates ϵ corresponding to certain values of h as a function of $\epsilon - h = d$, as is done in the upper part of Fig. 23.

According to Fig. 23, for a water level higher than the critical as determined by Equation [37], two values of h and d are possible. Two types of flow which are possible in this case are shown in Fig. 24 and Fig. 25. In Fig. 22, the height of the water level ϵ has just the critical (i.e., minimum) value. In this case d is twice as large as h, and therefore the velocity

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[13] Research Laboratories, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

$$v = \sqrt{2gh} = \sqrt{gd}$$

This value is equal to the velocity of propagation of tidal waves of water depth d. However, since any intumescence can penetrate against the direction of flow only if the velocity of the flow is less than the velocity of wave propagation, we see that any intumescence becomes more or less abrupt at the point where the velocity of flow reaches the wave velocity. In other words, a hydraulic jump is only possible where the velocity of flow reaches the wave velocity.

An expression for the loss of energy due to hydraulic jumps is given in Equation [13] of the paper. Another expression for this loss can easily be obtained in the following way:

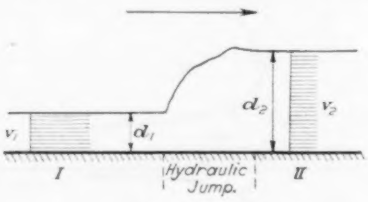

FIG. 26

In Fig. 26, the change of momentum from I to II (i.e., mass times the difference in the velocities) is equal to the resultant force:

$$Q \frac{\gamma}{g} (v_1 - v_2) = \gamma \int_0^{y = d_2} bydy - \gamma \int_0^{y = d_1} bydy$$

or for a rectangular cross-section (b = const.)

$$\frac{Q}{b} (v_1 - v_2) = \frac{g}{2} (d_2^2 - d_1^2)$$

In connection with the equation of continuity:

$$\frac{Q}{b} = v_1 d_1 = v_2 d_2$$

we therefore obtain

$$\left.\begin{aligned} \frac{v_1^2}{2g} &= \frac{d_2 + d_1}{4} \cdot \frac{d_2}{d_1} \\ \text{and} \quad \frac{v_2^2}{2g} &= \frac{d_2 + d_1}{4} \cdot \frac{d_1}{d_2} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots [38]$$

Incidentally, these two Equations [38] are identical with the Equations [3] or [11] of the paper. If, instead of an abrupt transition, it is assumed that the change in level is continuous, the increase of the water depth would be according to the Bernoulli equation

$$d_2' - d_1 = \frac{v_1^2 - v_2^2}{2g}$$

where d_2' is of course larger than d_2. The difference $d_2' - d_2$ thus represents the loss of energy:

$$d_2' - d_2 \equiv d_2' - d_1 - (d_2 - d_1) = \frac{v_1^2 - v_2^2}{2g} - d_2 + d_1$$

or in connection with Equation [38]

$$d_2' - d_2 = \frac{d_2 + d_1}{4} \left(\frac{d_2}{d_1} - \frac{d_1}{d_2}\right) - d_2 + d_1$$

or

$$d_2' - d_2 = \frac{(d_2 - d_1)^3}{4 d_1 d_2} = \text{loss due to hydraulic jump} \ldots [39]$$

A similar expression containing the velocities instead of the water depths can be obtained in the same way:

$$\frac{(v_1 - v_2)^3}{2g(v_1 + v_2)} = \text{loss due to hydraulic jump} \ldots\ldots [40]$$

This derivation of the loss due to hydraulic jumps is not much shorter than that given in the paper. However, this derivation has an advantage in that it expresses the *physical* aspect of the problem in a clearer manner than does the more elegant method used by the author.

In this connection it may be of interest to refer to results which can easily be obtained by a discussion of the well-known banking-curve equation.

Starting from the Eulerian equation

$$\frac{v}{g} \frac{dv}{ds} + \frac{dh}{ds} + \frac{1}{\gamma} \frac{dp}{ds} + \lambda \frac{1}{t} \frac{v^2}{2g} = 0$$

in which the last term is added to take into account the resistance of flow, the equation of the banking curve can easily be derived by assuming that the angle (α) in Fig. 27 is small enough to permit neglecting the difference between dx and ds. In the following figures α is exaggerated. The relation between the changing of the depth t in the direction of flow (i.e., dt/dx) and the slope dy/dx is then given by the equation

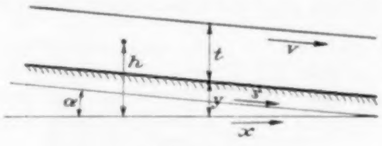

FIG. 27

$$\frac{dt}{dx} \left[1 - \left(\frac{v}{c}\right)^2\right] = - \left[\frac{dy}{dx} + \frac{\lambda}{2} \left(\frac{v}{c}\right)^2\right] \ldots\ldots [41]$$

where $c = \sqrt{gt}$ and $y = f(x)$ is the contour of the river bed.

Assuming a constant slope $-dy/dx = S_0$ and replacing v by Q/bt and c^2 by gt, Equation [41] can be integrated:

$$dt \cdot \frac{1 - \frac{A}{t^3}}{S_0 - \frac{\lambda}{2} \frac{A}{t^3}} = dx$$

where $A = Q/bg$ for the sake of simplicity.

Or

$$\frac{t^3 - A}{S_0 \frac{2}{\lambda} t^3 - A} dt = \frac{\lambda}{2} dx \ldots\ldots\ldots\ldots\ldots [42]$$

Assuming a slope S_0 less than $\lambda/2$, the variation of the depth t with x is shown in Fig. 28. For values less than the critical depth corresponding to a velocity equal to $c = \sqrt{g t_{cr}}$ it may be noted that the depth increases rapidly up to the critical value t_{cr}. The continuation of this line (marked as a dotted line) has no physical meaning. There is one line for each slope S_0 which corresponds to a steady motion (v = const.); this line approaches the line corresponding to the velocity equal c as S_0 approaches $\lambda/2$. For velocities smaller than that of steady flow, the upper line shows the increase of the depth with x. Fig. 29 shows a way in which a water level as determined by Fig. 28 can actually occur.

Assuming a slope S_0 larger than $\lambda/2$ we obtain Fig. 30. It is seen in this figure that the depth of steady flow is less than that corresponding to a velocity equal c. Fig. 31 shows that the flow below a sluice assumes quite a different appearance from that in Fig. 29 if the slope exceeds the critical value $\lambda/2$.

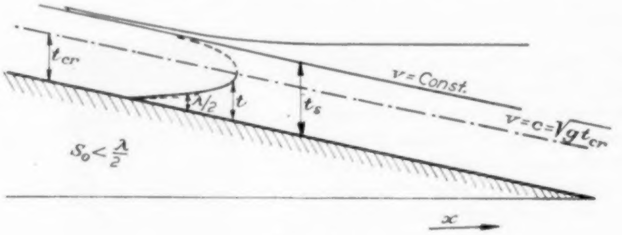

Fig. 28

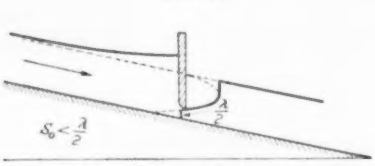

Fig. 29

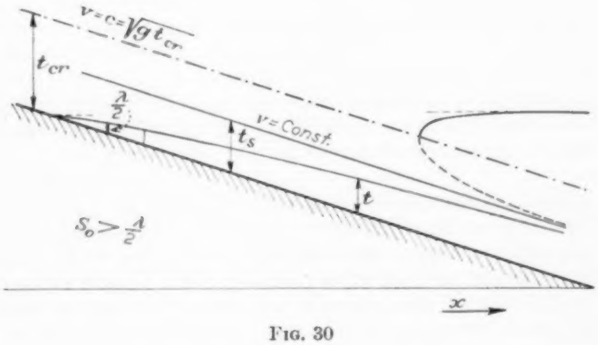

Fig. 30

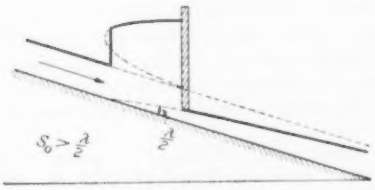

Fig. 31

This method of representing the behavior of the water flow so far as the hydraulic jump is concerned does not give any information about the jump itself. However, it may contribute a little to the understanding of these phenomena.

Finally, it may be of interest to mention the analogy between the hydraulic jump and the compression shock in the flow of gases through nozzles.[14] The critical velocity for water corresponds to the velocity of wave propagation just as the critical velocity of a gas corresponds to the acoustic velocity.

Author's Closure

The profession owes it to Dr. Tietjens for bringing the discussion back to fundamentals.

Naturally the expression of specific energy, as given in Equation [1], is similar in form to the so-called Bernoulli equation; for both the Bernoulli equation and the specific energy Equation [1] represent energy carried by a unit weight of the flowing liquid.

There is, nevertheless, an essential difference. The Bernoulli equation features the principle of conservation of energy applied to a single streamline or, in the case of parallel open flow, to the movement as a whole. The energy in this case is referred to a horizontal datum line *O-O* in Fig. 32, the equation describing the energetic "history" of the moving fluid with regard to such datum line.

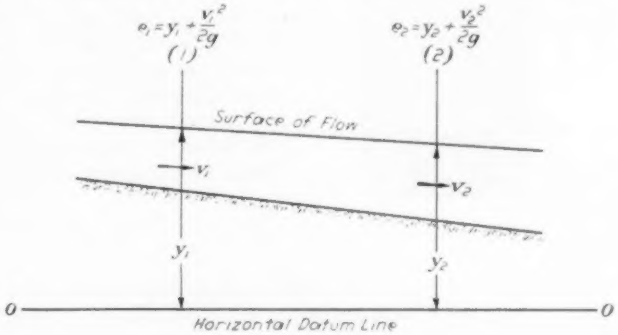

Fig. 32

The purpose of the specific energy diagram (Fig. 1) is to describe the energetic conditions in a cross-section as such. It characterizes the *state* of flow under varying stages. The energy is referred to a datum line which passes through the bottom of the channel cross-section. When the canal is laid with a bottom slope, the position of the datum line in two cross-sections will be at different levels.

The lower part of Fig. 23 is the specific energy diagram, traced as a function of the kinetic energy head. The two possible forms of movement, illustrated by Figs. 24 and 25, may be derived from the specific energy diagram (Fig. 1) by drawing a vertical (ϵ const) to intersect with the two branches of the energy curve. The points of intersection will determine the two depths—one in the zone of tranquil flow and the other in the zone of rapid flow—which will be the two possible ways in which the given discharge may flow with an energy content, determined by the distance of the vertical line from the zero point.

The specific energy diagram (Fig. 1) describes the change of energy for a given discharge Q. Dr. Tietjens' most interesting analysis implies the assumption of ϵ constant. Supplementing his presentation, one may determine the different discharges:

$$q = d \cdot v = d\sqrt{2gh} = d\sqrt{2g(\epsilon - d)} = (\epsilon - h)\sqrt{2gh}$$

which will carry the given ϵ. In general, to every discharge there will correspond two sets of depths and connected velocities, except for the critical depth, at which the discharge will be a maximum. This method has been extensively used by the late Professor Koch of Darmstadt (see "Koch-Carstanjen, "Bewegung des Wassers," Berlin, 1926). Featuring the critical depth as the one at which the discharge for a given energy content is a maximum is customary also in American hydraulic literature.[15]

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[14] O. G. Tietjens, "Flow of Gases at a Rate Exceeding the Acoustic Velocity," Trans. A.S.M.E., Applied Mechanics Division, January-April, 1931, vol. 53, no. 3.

[15] See discussion by Bakhmeteff in Proc. A.S.C.E., February, 1931.

Certain historical remarks might be of interest in connection with the topics referred to in Dr. Tietjens' discussion. The reasoning, leading to the determination of the loss in a hydraulic jump, as presented in Equation [39], was developed by Bélanger in the 1840s' and since has been customary with French hydraulicians; see, for example, Flamant, "Hydraulique," 1923, p. 133.

The different surface curves in varied flow (Figs. 28–31) were first presented in systematic form by the Belgian engineer and professor, M. Boudin. ("Annales Trav. Pub. de Belgique," T. XX, 1861–62.)

The bottom slope designated in Equation [42] by Dr. Tietjens as $\lambda/2$ is the so-called critical slope at which the depth of uniform movement becomes identical with the critical depth. The critical slope divides all possible bottom slopes into "mild" slopes ($s_0 <$ critical) and "steep" slopes ($s_0 >$ critical).

The distinction, with the characteristics of flow in each case, was made clear by St. Venant (1851) and further developed by Boussinesq ("Essai sur la théorie des eaux courantes," Paris, 1877; "Théorie de l'écoulement tourbillonant et tumultueux," Paris, 1897).

APM-54-2

Analysis of Stresses in Circular Plates and Rings

With Application to Cylinders With Rigidly Attached Flat Heads and Flanges

By E. O. HOLMBERG[1] and KARL AXELSON,[2] MILWAUKEE, WIS.

E. O. Holmberg

While the fundamental theory for calculating stresses in circular plates is well known, it has not been employed generally by engineers because of its complexity. In the present paper the authors present a simplified method for calculating the stresses in flat, circular plates, with or without a hole in the center, the loads being applied in any manner provided they are uniformly distributed in circles concentric with the edges. They also discuss the stresses in a hollow cylinder having rigidly attached flat heads, and give a method of calculating stresses in cylinders with rigidly attached flanges.

Karl Axelson

ALTHOUGH the fundamental theory for calculating stresses in circular plates is well known, it has not been employed to any great extent by the engineer, being considered too complicated for practical use. Lately, however, increased attention has been given to this subject and several attempts have been made to simplify the problem.

In May, 1927, in Mechanical Engineering, Prof. E. O. Waters and J. Hall Taylor published a method for determining stresses and deflections in pipe flanges in strict accordance with the theory of elasticity. The deflection formulas were checked by a series of tests.

In developing his formulas Professor Waters made the assumption "that the proportions of commercial flanges are such that the bolt load is applied near the outer edge and the pipe load near the inner edge." This assumption limits the exact application of these formulas to cases where the flanges are loaded as stated above.

Messrs. A. H. Wahl and J. Lobo, Jr., in their paper on stresses in circular plates in Trans. A.S.M.E. Applied Mech. Section, Jan.-April, 1930, in deriving their simplified formulas for various loading conditions make similar assumptions, treating cases where the loads are applied either at the extreme edge of the plate or uniformly distributed over the whole surface thereof.

Part I of the present paper will show a method for calculating the stresses in flat, circular plates with or without hole in the center without making the aforementioned assumptions. The loads may be applied in any manner, the only condition being that they are uniformly distributed in circles concentric with the edges. In Part II are treated the stresses in a hollow cylinder with rigidly attached flat heads. Part III presents a method of calculating stresses in cylinders with rigidly attached flanges.

I—CALCULATION OF STRESSES IN FLAT, CIRCULAR PLATES WITH OR WITHOUT CENTRAL HOLES

Referring to Fig. 1, the following symbols are used:

h = thickness of plate
x = distance from center of plate to point under consideration
φ = slope of plate at radius x due to elastic deformation
z = distance from neutral surface to point under consideration

Further

σ_t = tangential or hoop stress
σ_r = radial stress
$\frac{1}{m}$ = Poisson's ratio
E = modulus of elasticity

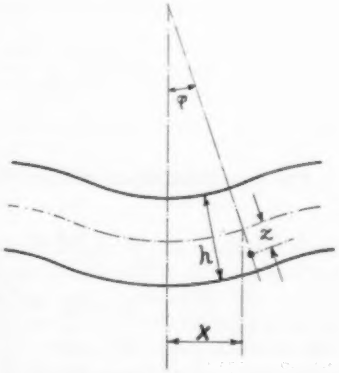

Fig. 1

For various loading conditions of the plate the following differential equations may be set up:

A—For uniformly distributed load (over area a, Fig. 2):

$$\frac{d^2\varphi}{dx^2} + \frac{1}{x}\frac{d\varphi}{dx} - \frac{\varphi}{x^2} + Nx = 0 \quad \ldots\ldots\ldots\ldots [1]$$

where $N = \frac{6(m^2 - 1)}{m^2} \times \frac{p}{Eh^3}$ and p = pressure per unit area.

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[1] Mechanical Engineer, A. O. Smith Corporation. Assoc-Mem. A.S.M.E. Born in 1894 in Lund, Sweden. Graduated from Chalmer's Institute of Technology, Gothenburg, Sweden, as a Mechanical Engineer in 1916. Military training in Swedish Army, 1916–1919. Arrived in the United States in 1919 and obtained a position with the A. O. Smith Corporation as a designer. Since 1925 in charge of a group of designers.

[2] Designer, A. O. Smith Corporation. Born in 1895 in Trollhattan, Sweden. Graduated from Chalmer's Institute of Technology, Gothenburg, Sweden, as a Mechanical Engineer in 1916. After being connected with the Ljungstrom Steam Turbine Company, in Sweden, and the Maffei Schwartzkopf Werke, Germany, came to the United States in 1923. Since 1928 employed by the A. O. Smith Corporation as a designer.

Contributed by the Applied Mechanics Division and presented at the Hartford, Conn., Meeting, June 1–3, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

B—For a single force in center or load uniformly distributed in a circle (over area b in Fig. 2):

$$\frac{d^2\varphi}{dx^2} + \frac{1}{x}\frac{d\varphi}{dx} - \frac{\varphi}{x^2} + \frac{S}{x} = 0 \ldots\ldots\ldots\ldots [2]$$

where $S = \frac{6(m^2 - 1)}{\pi m^2} \times \frac{P}{Eh^3}$ and P = total load.

C—For part of plate where no shearing forces occur (area c in Fig. 2):

$$\frac{d^2\varphi}{dx^2} + \frac{1}{x}\frac{d\varphi}{dx} - \frac{\varphi}{x^2} = 0 \ldots\ldots\ldots\ldots [3]$$

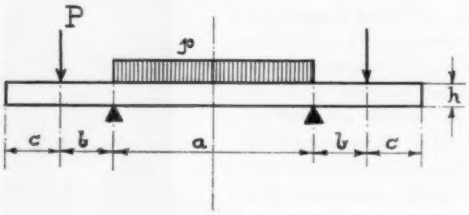

FIG. 2

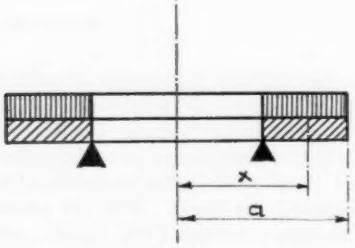

FIG. 3

$$\varphi = C_1x + \frac{C_2}{x} - \frac{N}{8}x^3 + \frac{Na^2}{2}x \log_e x$$

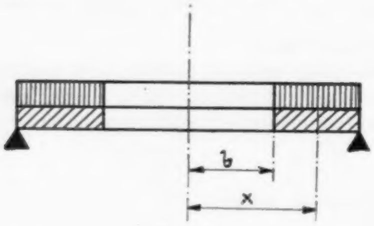

FIG. 4

$$\varphi = C_1x + \frac{C_2}{x} + \frac{N}{8}x^3 - \frac{Nb^2}{2}x \log_e x$$

The general solutions of these differential equations are:

$$\varphi = C_1x + \frac{C_2}{x} - \frac{N}{8}x^3 \ldots\ldots\ldots\ldots [1a]$$

$$\varphi = C_3x + \frac{C_4}{x} - \frac{S}{2}x \log_e x \ldots\ldots\ldots\ldots [2a]$$

$$\varphi = C_5x + \frac{C_6}{x} \ldots\ldots\ldots\ldots [3a]$$

where the integration constants C_1 to C_6, inclusive, may be determined from the conditions at the edges of the plate.

Loading conditions may be assumed for which none of the three equations given holds true. Two such cases are illustrated in Figs. 3 and 4. The respective equations are given in the figures.

If it is desired to find the linear deflections the operation

$$y = \int \varphi dx + C$$

has to be performed and the constant C determined by the conditions at the edges.

While all of the above formulas are well known, they are derived here for completeness and as an introduction to what follows.

The application of these formulas will now be shown for two specific cases.

(a) LOOSE PIPE FLANGE LOADED WITH BOLT LOAD AND GASKET LOAD ONLY

Referring to Fig. 5: From a to a_0, Equation [3a] holds; from a to b, Equation [2a] holds; and from b to d, Equation [3a] holds.

Applying these equations, the six constants are determined by the following conditions:

$\sigma_r = 0$ for $x = a_0$ and $x = d$

φ and $d\varphi/dx$ are continuous for $x = a$ and $x = b$.

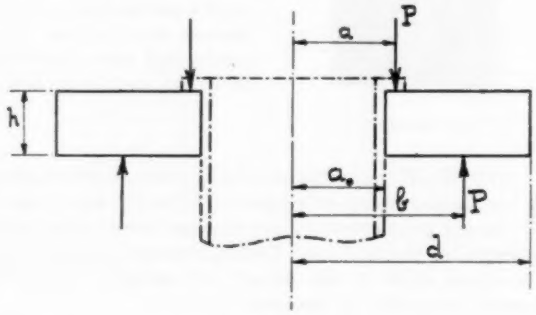

FIG. 5

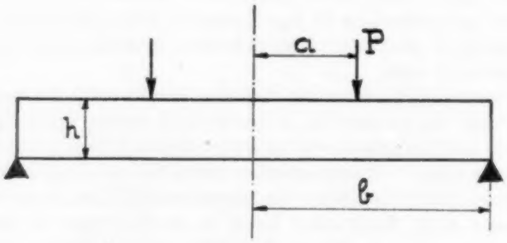

FIG. 6

The maximum stress is a hoop stress, occurs at the corners for $x = a_0$, and is given by

$$\sigma_t = \frac{3}{2\pi} \times \frac{P}{h^2(d^2 - a_0^2)}\left[\frac{m-1}{m}(b^2 - a^2) + \frac{2(m+1)}{m}d^2\log_e \frac{b}{a}\right]$$

or for $m = 4$,[3]

$$\sigma_t = \frac{1.195P}{h^2(d^2 - a_0^2)}\left[0.3(b^2 - a^2) + d^2 \log_e \frac{b}{a}\right] \ldots\ldots [4]$$

Numerical results obtained from this formula check closely with those evaluated by the use of Professor Waters' formula for flat rings[4] if the relation between a_0 and d is such as is commonly used for commercial flanges. Equation [4], however, is correct for any values of a_0 and d. Waters' formula when applied

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[3] See "Determination of Poisson's Ratio, Etc.," by Prof. T. McLean Jasper, *Proc. Am. Soc. Testing Materials*, vol. 24, part II, 1924.

[4] Waters' formula for flat rings:

$$\sigma_t = \frac{5.72P(b - a)}{2h^2(d - a_0)}\left[\frac{d^2 \log_{10} \frac{d}{a_0}}{d^2 - a_0^2} + 0.117\right]$$

to a case where a_0 is small in relation to d gives too high values for σ_t. Equation [4] is therefore far more general and just as simple to apply.

(*b*) FLAT PLATE SUPPORTED ALONG THE OUTER EDGE AND LOADED BY A CONCENTRIC LOAD *P*

Referring to Fig. 6: From $x = 0$ to $x = a$, Equation [3a] holds; and from $x = a$ to $x = b$, Equation [2a] holds.

The equation for the maximum stress is obtained in similar manner as before. This stress occurs in the center of the plate and is

$$\sigma_0 = \frac{0.5975P}{h^2}\left[0.3\,\frac{b^2 - a^2}{b^2} + \log_e \frac{b}{a}\right] \quad\ldots\ldots\ldots [5]$$

If in Equation [4] a_0 is set equal to zero and $d = b$—in other words, if the plate is pierced in the center by a very small hole—the stress becomes

$$\sigma_t = \frac{1.195P}{h^2}\left[0.3\,\frac{b^2 - a^2}{b^2} + \log_e \frac{b}{a}\right] \quad\ldots\ldots\ [4a]$$

A comparison of Equations [5] and [4a] shows that if a plate is pierced by a small hole in the center, the maximum stress in the plate is theoretically twice as large as if the plate had been solid. This may be of importance in case the plate is made of brittle material.

II—STRESSES IN A CYLINDER WITH RIGIDLY ATTACHED FLAT HEADS UNDER INTERNAL PRESSURE

Fig. 7 shows a cylinder with rigidly attached flat head. The head may be either cast in one piece with the cylindrical body or attached thereto by welding. Fig. 8 is an exaggerated picture of this cylinder when subjected to internal pressure.

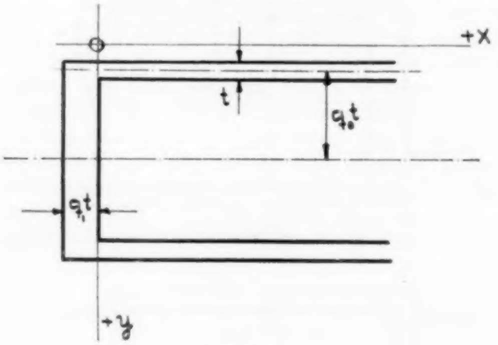

FIG. 7

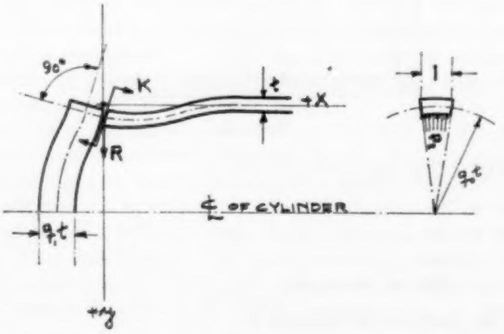

FIG. 8

In deriving the stresses involved, the formulas established in Part I have been used. The cylinder wall has been considered a beam on an elastic foundation. For complete derivation see Appendix No. 2.

Notation:

t = thickness of cylinder wall in inches
q_0t = mean radius of cylinder in inches
q_1t = thickness of head in inches
p = internal pressure, lb. per sq. in.
K = couple acting at junction of head and cylinder per inch of circumference, in in-lb. per inch
R = shearing force at junction of head and cylinder per inch of circumference, lb. per inch.

The stresses influencing design are:

(1) The bending stress in the cylinder wall at the junction with the head:

$$\sigma = \frac{6K}{t^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots [6]$$

(2) The longitudinal tension stress in the wall:

$$\sigma_l = \frac{q_0}{2}\,p \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots [7]$$

σ_l must be numerically added to σ.

(3) The bending stress at the center of the plate:

$$\sigma_{r0} = \sigma_{t0} = \frac{2}{3q_1}\left(\frac{1.83q_0^2}{q_1}p - \frac{9K}{q_1t^2} - 4.5\,\frac{R}{t}\right) \quad\ldots\ldots\ [8]$$

(4) The radial bending stress at the radius q_0t:

$$\sigma_r = -\frac{6}{q_1^2}\left(\frac{K}{t^2} + \frac{q_1}{2}\,\frac{R}{t}\right) \quad\ldots\ldots\ldots\ldots [9]$$

(5) The tangential bending stress at the same radius:

$$\sigma_t = \sigma_r + 0.5625\,\frac{q_0^2}{q_1^2}\,p \quad\ldots\ldots\ldots\ldots [10]$$

(6) To the stresses evaluated under (4) and (5) must be *numerically* added tension stress due to R:

$$\sigma_r{}' = \sigma_t{}' = \frac{R}{q_1t} \quad\ldots\ldots\ldots\ldots\ldots [11]$$

In the foregoing formulas R and K are found from

$$\frac{R}{tp} = \frac{q_0q_1(6.93q_1^3 + 3.89q_0q_1 + 2.97q_0\sqrt{q_0} + 6.85\sqrt{q_0})}{9.13q_1^4\sqrt{q_0} + 20.78q_1^3 + 23.7q_1^2\sqrt{q_0} + 18q_1q_0 + 5.15\sqrt{q_0}} \quad\ldots\ldots\ldots [12]$$

$$\frac{K}{t^2p} = \frac{R/tp(3q_1 + 2.63q_1^2\sqrt{q_0}) - q_0q_1^2 - 0.5625q_0^2}{2\sqrt{3}\;q_1^2 - 4.5} \quad\ldots\ldots\ [13]$$

In the formula for σ the concentration of stress due to the sharp corner has not been taken into account. This stress concentration is localized over a very small area and cannot affect any of the other stresses evaluated above. In practice it is usual to provide a fillet large enough to reduce this stress concentration to within safe values. A fillet will materially reduce the bending stress σ and slightly reduce the tangential stress σ_t at the virtual edge of the plate, but no efforts have been made to accurately evaluate its influence.

The foregoing is also applicable to the flange treated in Part III.

To facilitate the use of the stress formulas given above the values of R/tp and K/t^2p have been plotted in Figs. 9 and 10 for various values of q_0 and q_1. Their application will now be illustrated by numerical examples.

EXAMPLE 1 (Fig. 11):

$q_0 = 10$
$q_1 = 3$
$t = 1$ in.
$p = 1000$ lb. per sq. in.

From Figs. 9 and 10:

$$\frac{R}{tp} = 3.05 \qquad R = 3050 \text{ lb. per in.}$$

$$\frac{K}{t^2p} = 4.3 \qquad K = 4300 \text{ in-lb. per inch}$$

Stresses in cylinder wall:

From Equation [6], $\sigma = 6 \times 4300 = 25{,}800$ lb. per sq. in.
From Equation [7], $\sigma_l = 5 \times 1000 = 5000$ lb. per sq. in.

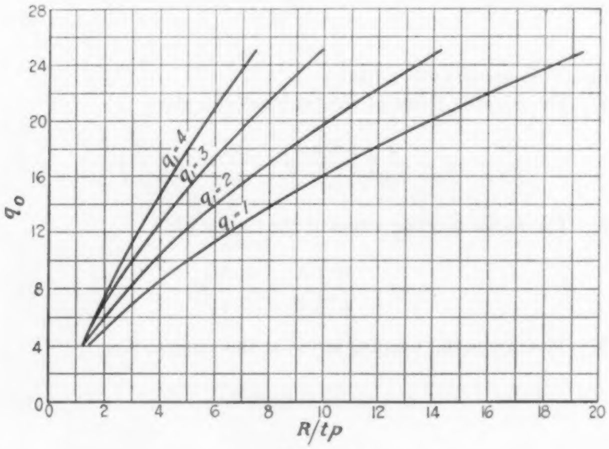

FIG. 9

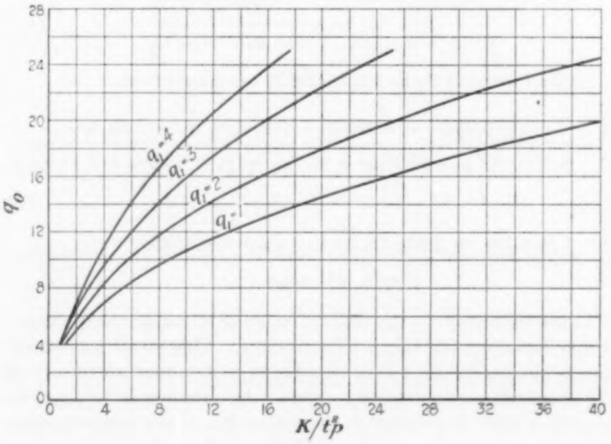

FIG. 10

Total stress in cylinder wall:

$$\sigma + \sigma_l = 30{,}800 \text{ lb. per sq. in.}$$

Stresses in plate:
Bending stress in center. From Eq. [8],

$$\sigma_{r0} = \sigma_{t0} = \frac{2}{3 \times 3}\left(\frac{1.83 \times 10^2 \times 1000}{3} - \frac{9 \times 4300}{3} - \frac{4.5 \times 3050}{1}\right)$$

$$= 7650 \text{ lb. per sq. in.}$$

To this must be added numerically stress due to R. From Eq. [11]:

$$\sigma_r' = \sigma_t' = \frac{3050}{3} = 1016 \text{ lb. per sq. inch, or}$$

Total stress at center of plate:

$$\sigma_{tot} = 8666 \text{ lb. per sq. in.}$$

Bending stresses at virtual edge of plate. From Equations [9] and [10]:

$$\sigma_r = -\frac{6}{3^2}\left(4300 + \frac{3}{2} \times 3050\right) = -5920 \text{ lb. per sq. in.}$$

$$\sigma_t = -5920 + 0.5625 \times \frac{10^2}{3^2} \times 1000 = 330 \text{ lb. per sq. in.}$$

To these must be numerically added as before

$$\sigma' = 1016 \text{ lb. per sq. in.}$$

Total $\sigma_r = 6936$ lb. per sq. in.
Total $\sigma_t = 1346$ lb. per sq. in.

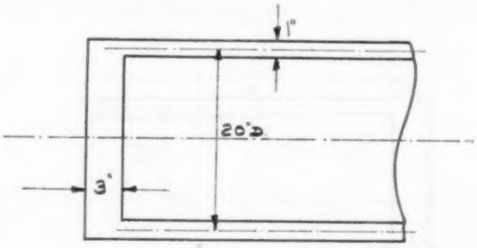

FIG. 11

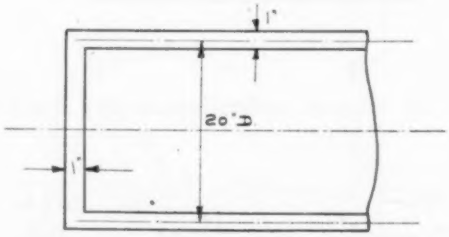

FIG. 12

EXAMPLE 2 (Fig. 12):

$q_0 = 10$
$q_1 = 1$
$t = 1$ in.
$p = 1000$ lb. per sq. in.

From the charts as in Example 1:

$R = 5050$ lb. per inch
$K = 8800$ in-lb. per inch

Stresses in cylinder wall:

$\sigma = 6 \times 8800 = 52{,}800$ lb. per sq. in.
$\sigma_l = 5 \times 1000 = 5000$ lb. per sq. in.

$\sigma + \sigma_l = 57{,}800$ lb. per sq. in.

Stresses in plate:

Bending stresses in center:

$$\sigma_{r0} = \sigma_{t0} = \frac{2}{3}(1.83 \times 10^2 \times 1000 - 9 \times 8800 - 4.5 \times 5050)$$

$$= 50{,}600 \text{ lb. per sq. in.}$$

To this must be added as before

$$\sigma_r' = \sigma_t' = 5050 \text{ lb. per sq. in.}$$

Total stress in center of plate:

$$\sigma_{tot} = 50{,}600 + 5050 = 55{,}650 \text{ lb. per sq. in.}$$

Bending stresses in virtual edge of plate:

$$\sigma_r = -6\left(\frac{8800}{1} + \frac{5050}{2 \times 1}\right) = -68{,}000 \text{ lb. per sq. in.}$$

$$\sigma_t = -68{,}000 + 0.5625 \times 100 \times 1000 = -11{,}700 \text{ lb. per sq. in.}$$

To these stresses must be added numerically as before

$$\sigma' = 5050 \text{ lb. per sq. in.}$$

Total σ_r = 73,000 lb. per sq. in.
Total σ_t = 16,700 lb. per sq. in.

When considering that the tension stress in the cylinder far from the head is only 10,000 lb. per sq. in., the stresses at the junction in both examples are far too high for correct design. This condition may be remedied, as generally is the case in practice, by thickening the wall of the cylinder as the junction is approached. As the bending moment K can be shown to decrease very rapidly with increasing distance from the junction[5] a large fillet will in most cases suffice to decrease the stress to lower values. The examples show that a considerable increase of the thickness of the head also effects a considerable decrease of the bending stress in the cylinder wall. The proportions between thickness of head and cylinder wall, as chosen in Example 1, are evidently fairly close to economical design if a large fillet or a tapering cylinder wall is introduced at the junction. This is particularly important if the cylinder is manufactured of brittle material.

III—STRESSES IN A CYLINDER WITH A RIGIDLY ATTACHED FLANGE

Analysis of the stresses in pipe flanges with high and low hubs have been made during recent years by Professor Waters and others. In the formulas developed, however, the effect of internal pressure has been omitted. With the high pressures sometimes encountered in modern practice it has been felt that their effect has too much influence to be neglected.

The object of the following analysis has not been to develop a simplified formula for calculating pipe flanges, but to furnish a method for figuring extreme cases where large dimensions and high loads make a more accurate formula desirable.

Fig. 13 shows a cylinder with rigidly attached flange which may be either cast integral with or welded to the cylinder. Fig. 14 is an exaggerated picture of the cylinder when subjected to a bolt and gasket load P and an internal pressure p.

Notation:

t = thickness of cylinder wall in inches
q_0t = mean radius of cylinder in inches
q_1t = thickness of flange in inches
q_2t = radius of bolt circle in inches
q_3t = outside radius of flange in inches
p = internal pressure, lb. per sq. in.
P = total bolt load, lb.
K = couple acting at junction of flange and cylinder per inch circumference, in in-lb. per inch
R = shearing force at junction of flange and cylinder per inch circumference, lb. per inch.

The stresses influencing design are:

(1) The bending stress in the cylinder wall at the junction with the flange:

$$\sigma = \frac{6K}{t^2} \quad \text{[14]}$$

(2) The longitudinal tension stress in the cylinder wall σ_l due to internal pressure or external forces. This stress must be *numerically* added to σ.

(3) Maximum radial bending stress in the flange at the virtual inner edge at both faces of the flange. Virtual inner edge = radius q_0t:

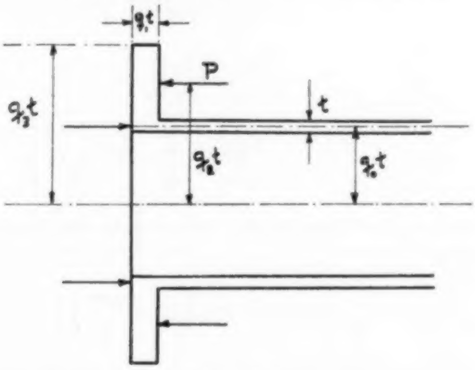

FIG. 13

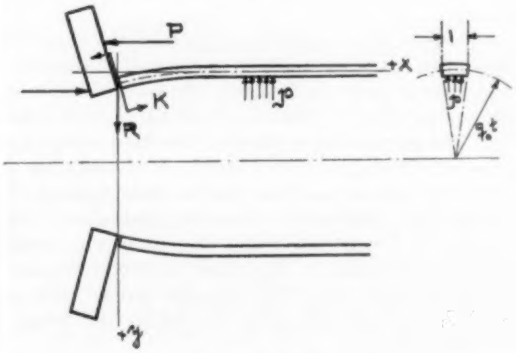

FIG. 14

$$\sigma_{r0} = \frac{6}{q_1^2t^2}\left(K - \frac{q_1t}{2}\dot{R}\right) \quad \text{[15]}$$

(4) To stress σ_{r0} must be *numerically* added stress arising from R and p:

$$\sigma_{rR} = \frac{R}{q_1t} + p \quad \text{[16]}$$

(5) Maximum tangential bending stress, occurring at the same point as σ_{r0}:

$$\sigma_{t0} = \sigma_{r0} + \frac{0.800}{q_1^2t^2(q_3^2 - q_0^2)}\left[q_3^2\left(-15K + 7.5q_1tR\right.\right.$$

---

[5] See Appendix No. 2.

$$\left. + 1.492P \log_e \frac{q_2}{q_0}\right) + 0.4475P\,(q_2^2 - q_0^2)\Bigg] \ldots\ldots [17]$$

(6) To stress σ_{t0} must be *numerically* added tangential stresses arising from radial deformation due to R and p:

$$\sigma_{tR} = \frac{q_1^2}{4}\,T_1 \times \frac{R + q_1 tp}{t} \ldots\ldots\ldots\ldots\ldots [18]$$

In the foregoing formulas R and K are found from

$$R = \frac{\left(q_0 - \dfrac{q_1^3 T_1}{4}\right)(1 + 0.329\sqrt{q_0}\,T_1)\,tp - T_2\left(q_1 + 0.76\sqrt{q_0}\right)\dfrac{P}{t}}{1.315\sqrt{q_0} + T_1\,[q_1^2(1 + 0.0822\sqrt{q_0}\,T_1) + 1.136q_1\sqrt{q_0} + 0.865q_0]} \ldots\ldots\ldots [19]$$

$$K = \frac{(q_1^2 T_1 + 2.63\sqrt{q_0})\,Rt + q_1 T_2 P - \left(q_0 - \dfrac{q_1^3 T_1}{4}\right)t^2 p}{1.5q_1 T_1 - 3.464} . [20]$$

$$T_1 = \frac{3q_0^2 + 5q_3^2}{q_1^3(q_3^2 - q_0^2)} \ldots\ldots\ldots\ldots\ldots\ldots [21]$$

$$T_2 = \frac{3.58}{q_1^3(q_3^2 - q_0^2)}\left(\frac{q_3^2}{3}\log_e \frac{q_2}{q_0} + \frac{q_2^2 - q_0^2}{10}\right) \ldots\ldots [22]$$

The slope of the flange at the virtual inner edge is given by the equation

$$\varphi_0 = \frac{q_0}{Et^2}\left[2PT_2 - 3T_1\left(K - \frac{q_1 tR}{2}\right)\right]$$

An approximate value of the linear deflection of the outer edge of the flange may be obtained by multiplying φ_0 with the width of the flange.

In the foregoing the dimensions of cylinder, thickness of flange, etc., have been given as multiples of the wall thickness t. This has been done in order to facilitate the work of constructing a chart from which the quantities T_1, T_2, and R could be determined directly. The preparation of charts of this kind, covering a great number of conditions, involves a multitude of numerical calculations, and the authors have not had to their disposal sufficient time to perform this work. However, preliminary work has been started, and it is hoped that the charts will considerably facilitate the application of the rather involved formulas.

For a more direct use of the formulas derived they are here given with the actual dimensions of cylinder and flange substituted for the ratios.

Notation:

t = thickness of cylinder wall in inches
a = mean diameter of cylinder in inches
b = diameter of bolt circle in inches
d = outside diameter of flange in inches
h = thickness of flange in inches
p = internal pressure, lb. per sq. in.
P = total bolt load, lb.

$$R = \frac{\left(s^2 - \dfrac{h^3}{2t}T_1\right)(t + 0.2325sT_1)p - 2T_2(h + 0.5377s)P}{1.860st + T_1\left[h^2\left(2 + 0.1160\dfrac{s}{t}T_1\right) + 1.6103sh + 0.866s^2\right]} \ldots\ldots\ldots [19a]$$

$$K = \frac{(h^2 T_1 + 1.860st)R + hT_2 P - 0.500tp\left(s^2 - \dfrac{h^3}{2t}T_1\right)}{1.500hT_1 - 3.464t} [20a]$$

$$s = \sqrt{at} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [23]$$

$$T_1 = \frac{t^3(3a^2 + 5d^2)}{h^3(d^2 - a^2)} \ldots\ldots\ldots\ldots\ldots\ldots [21a]$$

$$T_2 = \frac{3.580t^3}{h^3(d^2 - a^2)}\left[\frac{d^2}{3}\log_e \frac{b}{a} + 0.100(b^2 - a^2)\right] \ldots [22a]$$

$$\sigma_{r0} = \frac{6}{h^2}\left(K - \frac{h}{2}R\right) \ldots\ldots\ldots\ldots\ldots [15a]$$

$$\sigma_{rR} = \frac{R}{h} + p \ldots\ldots\ldots\ldots\ldots\ldots\ldots [16a]$$

$$\sigma_{t0} = \sigma_{r0} + \frac{0.800}{h^2(d^2 - a^2)}\left[d^2\left(-15K + 7.5hR + 1.492P\log_e \frac{b}{a}\right) + 0.4475P(b^2 - a^2)\right] \ldots\ldots\ldots [17a]$$

$$\sigma_{tR} = \frac{h^2}{4t^3}\,T_1(R + hp) \ldots\ldots\ldots\ldots\ldots [18a]$$

Although the foregoing equations have been derived with the assumption of a cylinder of infinite length attached to the flange, they may be applied with a high degree of accuracy to flanges with shorter hubs, provided the length of the hub is

$$l \gtreqless 1.1\sqrt{at} \ldots\ldots\ldots\ldots\ldots\ldots [24]$$

See Professor Timoshenko's discussion of Waters' formula for hubbed flanges.

The use of the foregoing formulas will be illustrated by numerical examples.

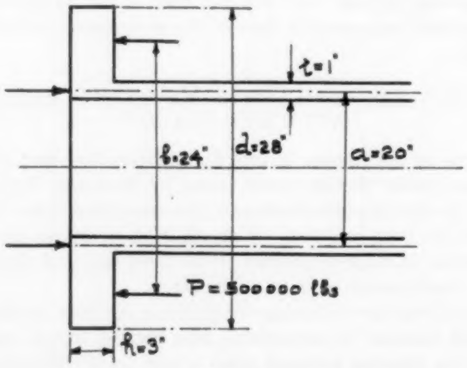

FIG. 15

EXAMPLE 1 (Fig. 15):

In Fig. 15 is shown a cylinder with rigidly attached flange, loaded with bolt and gasket load and subjected to an internal pressure of 1000 lb. per sq. in. The longitudinal load is assumed to be produced by the internal pressure acting over the full area of the cylinder.

Hoop stress and longitudinal stress far from the flange:

σ_h = 9500 lb. per sq. in.
σ_l = 4750 lb. per sq. in.

From Equation [23] $s = \sqrt{20} = 4.472$
From Equation [21a], $T_1 = 0.493$
From Equation [22a], $T_2 = 0.0225$

Inserting values of s, T_1, T_2, and given dimensions and loads into [19a] and [20a], gives

$R = -2700$ lb. per in.
$K = 5910$ in-lb. per in.

Stresses in cylinder wall:

From Equation [14] $\sigma = 35{,}460$ lb. per sq. in.
To this add numerically $\sigma_l = 4750$ lb. per sq. in.

Total stress in cylinder wall:

$\sigma + \sigma_l = 40{,}210$ lb. per sq. in.

Stresses in flange:

Radial:

From Equation [15a], $\sigma_{r0} = 6640$ lb. per sq. in.
From Equation [16a], $\sigma_{rR} = 100$ lb. per sq. in.

Total radial stress:

$\sigma_{r\,tot} = \sigma_{r0} + \sigma_{rR} = 6740$ lb. per sq. in.

Tangential:

From Equation [17a], $\sigma_{t0} = 13{,}320$ lb. per sq. in.
From Equation [18a], $\sigma_{tR} = 330$ lb. per sq. in.

Total tangential stress:

$\sigma_{t\,tot} = \sigma_{t0} + \sigma_{tR} = 13{,}650$ lb. per sq. in.

Minimum length of hub:

From Equation [24], $l = 5$ in.

In the same manner stresses have been calculated when the internal pressure p has been removed. In Example 2, Table 1, all dimensions except thickness of flange have been maintained. The pressure p and bolt load P remain the same. Example 3 is calculated with very high internal pressure. All figures pertaining to these three examples will be found in Table 1.

TABLE 1 CALCULATIONS

		Example 1		Example 2		Example 3	
t,	inches	1		1		2	
a,	inches	20		20		16	
b,	inches	24		24		24	
d,	inches	28		28		32	
h,	inches	3		2		6	
P,	lb.	500,000		500,000		1,500,000	
p,	lb. per sq. in.	1,000	0	1,000	0	5,000	0
R,	lb. per in.	− 2,700	− 3,243	− 3,720	− 4,170	− 9,130	−12,120
K,	in-lb. per in.	5,910	6,108	8,880	8,800	23,750	26,000
σ_l,	lb. per sq. in.	4,750	0	4,750	0	8,750	0
σ_h,	lb. per sq. in.	9,500	0	9,500	0	23,800	0
σ	lb. per sq. in.	35,460	36,650	53,280	52,800	35,630	39,000
$\sigma + \sigma_l$,	lb. per sq. in.	40,210	36,650	58,030	52,800	44,380	39,000
σ_{r0},	lb. per sq. in.	6,640	7,315	18,900	19,460	8,520	10,400
σ_{rR},	lb. per sq. in.	100	1,080	860	2,085	3,480	2,020
$\sigma_{r\,tot.}$	lb. per sq. in.	6,740	8,395	19,760	21,545	12,000	12,420
σ_{t0},	lb. per sq. in.	13,320	11,250	17,770	16,060	18,900	15,800
σ_{tR},	lb. per sq. in.	330	3,400	2,870	6,950	6,670	3,900
$\sigma_{t\,tot.}$	lb. per sq. in.	13,650	14,650	20,640	23,010	25,570	19,700

The stress $\sigma + \sigma_l$ drops considerably with removal of pressure p but this is due to the removal of the longitudinal forces acting on the cylinder and not to a reduced bending moment. The maximum change in σ amounts to about 10 per cent in Example 3 where a high internal pressure was assumed.

In Example 2 where the flange is reduced in thickness to 2 in. the stress σ is materially higher due to the greater flexibility of the thinner flange. The radial and tangential stresses in the flange become higher for the same reason.

In all the examples given the greatest stress by far has been encountered in the cylinder wall at the junction with the flange. This stress may obviously be reduced by introducing a large fillet or a tapering cylinder wall at the junction. The bending moment K can be shown to decrease rapidly with increasing distance from the junction. The tapering section will consequently be effective even if very short. No effort has been made to accurately evaluate this effect due to the mathematical difficulties encountered. Sufficient safety may be obtained by applying above formulas as they stand and using the section modulus obtained from the thickened cylinder wall when calculating the stress σ.

Appendix No. 1

The differential Equations [1], [2], and [3] are derived as follows:

Referring to Fig. 1:

ϵ_t = unit elongation in tangential direction
ϵ_r = unit elongation in radial direction

At the point under consideration

$$\epsilon_t = \frac{z\varphi}{x} \qquad \epsilon_r = z\frac{d\varphi}{dx}$$

From the theory of elasticity,

$$\epsilon_t = \frac{1}{E}\left(\sigma_t - \frac{\sigma_r}{m}\right) \qquad \epsilon_r = \frac{1}{E}\left(\sigma_r - \frac{\sigma_t}{m}\right)$$

From these four equations

$$\sigma_t = \frac{mEz}{m^2-1}\left(m\frac{\varphi}{x} + \frac{d\varphi}{dx}\right)$$

$$\sigma_r = \frac{mEz}{m^2-1}\left(m\frac{d\varphi}{dx} + \frac{\varphi}{x}\right)$$

By means of these expressions a differential equation for the relation between slope φ and distance x may be set up.[6] For different loading conditions this equation will be found to assume the shapes given in Part I.

Appendix No. 2

DERIVATION OF FORMULAS FOR CYLINDER WITH RIGIDLY ATTACHED FLAT HEAD

In Figs. 8 and 16 a system of coordinates is erected with the x-axis coinciding with the center line of the cylinder wall far from the head when the cylinder is expanded by internal pressure. The y-axis passes through the junction between cylinder and head.

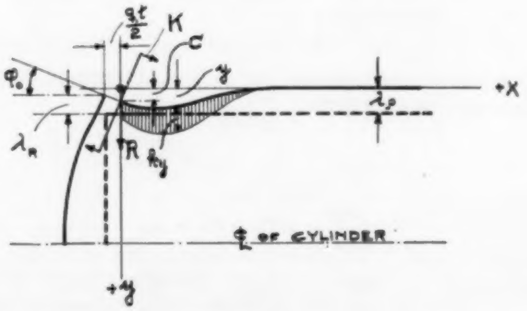

FIG. 16

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[6] Timoshenko and Lessells, "Applied Elasticity." Föppl "Technische Mechanik," vol. III.

In Fig. 16 the heavy broken line represents the center line of the cylinder wall and head before deformation. The heavy solid line represents the same center line after deformation.

A strip of unit width is cut out as shown to the right in Fig. 8. This strip may be treated as a beam on an elastic foundation. When the x-axis is chosen as shown, the y-values are produced by couple K and shearing force R only.

Notation:

t = thickness of cylinder wall in inches
$q_0 t$ = mean radius of cylinder in inches
$q_1 t$ = thickness of head in inches
p = internal pressure, lb. per sq. in.
K = couple acting at junction of head and cylinder per inch circumference, in in-lb. per inch
R = shearing force at junction of head and cylinder per inch circumference, lb. per inch
y = deflection of cylinder wall due to K and R in inches
λ_p = radial deformation of cylinder far from head due to p in inches
λ_R = radial deformation of head due to R in inches
φ_0 = slope of cylinder wall for $x = 0$ equals slope of head at radius $q_0 t$ in radians
E = modulus of elasticity
$\frac{1}{m}$ = Poisson's ratio ($m = 4$)
$I = t^3/12$ = moment of inertia of strip of unit width
k = resistance of elastic foundation for unity deflection $= E/q_0^2 t$

$$\beta = \sqrt[4]{\frac{k}{4IE}} = \frac{1.315}{t\sqrt{q_0}}$$

For a long cylinder the following equations hold:

$$y = e^{-\beta x}(C \cos \beta x + D \sin \beta x) \ldots\ldots\ldots\ldots [25]$$

$$dy/dx = \beta e^{-\beta x}[-C(\cos \beta x + \sin \beta x) + D(\cos \beta x - \sin \beta x)] \ldots\ldots [26]$$

$$d^2y/dx^2 = 2\beta^2 e^{-\beta x}(C \sin \beta x - D \cos \beta x) \ldots\ldots [27]$$

$$d^3y/dx^3 = 2\beta^3 e^{-\beta x}[C(\cos \beta x - \sin \beta x) + D(\cos \beta x + \sin \beta x)] \ldots\ldots [28]$$

(See Timoshenko and Lessells, "Applied Elasticity.")
In these equations

y = deflection
$\frac{dy}{dx}$ = slope
$\frac{d^2y}{dx^2} = \frac{\text{Bending moment}}{IE}$
$\frac{d^3y}{dx^3} = \frac{\text{Shearing force}}{IE}$

At the end of the cylinder, for $x = 0$, Equations [25]–[28] assume the following forms:

$$y = C = \lambda_p - \lambda_R + \frac{q_1 t}{2}\varphi_0 \ldots\ldots\ldots\ldots [25a]$$

$$\frac{dy}{dx} = \beta(D - C) = \varphi_0 \ldots\ldots\ldots\ldots [26a]$$

$$\frac{d^2y}{dx^2} = -2\beta^2 D = -\frac{K}{IE} \ldots\ldots\ldots\ldots [27a]$$

$$\frac{d^3y}{dx^3} = 2\beta^3(C + D) = \frac{R}{IE} \ldots\ldots\ldots\ldots [28a]$$

Fig. 17 shows the head of the vessel after pressure is applied. Couple K and force R are here shown as reaction forces from the cylinder wall acting in opposite direction from those shown in Fig. 16.

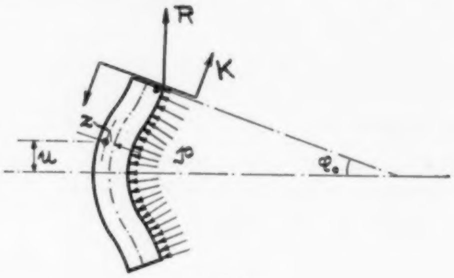

FIG. 17

Referring to Fig. 17,

u = distance from center of plate
z = distance from neutral surface of plate

For a round plate loaded by a uniformly distributed load the slope at a point at distance u from center is

$$\varphi = C_1 u + \frac{C_2}{u} - \frac{N}{8}u^3$$

where

$$N = \frac{m^2 - 1}{m^2} \cdot \frac{6p}{Eq_1^3 t^3} \quad \text{(See Equation [1a])}$$

For this case $C_2 = 0$ because otherwise $\varphi = \infty$ for $u = 0$. The equation becomes

$$\varphi = C_1 u - \frac{N}{8}u^3 \ldots\ldots\ldots\ldots [29]$$

and its derivative

$$\frac{d\varphi}{du} = C_1 - \frac{3N}{8}u^2 \ldots\ldots\ldots\ldots [30]$$

As shown in Appendix 1, the radial stress at a point at radius u at a distance z from the neutral surface of a circular flat plate is

$$\sigma_r = \frac{mEz}{m^2 - 1}\left(m\frac{d\varphi}{du} + \frac{\varphi}{u}\right)$$

For the virtual edge of the plate where $u = q_0 t$ we may write

$$\frac{mE}{m^2 - 1}\left(m\frac{d\varphi}{du} + \frac{\varphi}{u}\right)_{u = q_0 t} = k_1 \ldots\ldots\ldots\ldots [31]$$

or

$$\sigma_r = k_1 z$$

The moment of these σ_r-forces taken over the whole thickness of the plate per unit length of circumference is obviously balanced by the couple K and the moment of the shearing force R.

$$\int_{-\frac{q_1 t}{2}}^{+\frac{q_1 t}{2}} \sigma_r z\,dz = \int_{-\frac{q_1 t}{2}}^{+\frac{q_1 t}{2}} k_1 z^2\,dz = k_1\frac{q_1^3 t^3}{12} = -K - \frac{q_1 t}{2}R \ldots [32]$$

From Equations [29], [30], [31], and [32] the following relation is obtained:

$$C_1 = \frac{6(m - 1)}{mEq_1^3 t^3}\left(pq_0^2 t^2\frac{(3m + 1)}{8m} - 2K - q_1 tR\right)$$

and for $m = 4$,

$$C_1 = \frac{117}{64}\frac{pq_0^2}{Eq_1^3t} - \frac{9K}{Eq_1^3t^3} - \frac{4.5\,R}{Eq_1^2t^2}$$

This value inserted into Equation [29] gives, for $u = q_0t$,

$$\varphi_0 = \frac{9}{8}\frac{q_0^3}{Eq_1^3}p - \frac{9q_0}{Eq_1^3t^2}K - \frac{4.5\,q_0}{Eq_1^2t}R \ldots\ldots\ [33]$$

The radial increase λ_R must now be determined (Fig. 18).

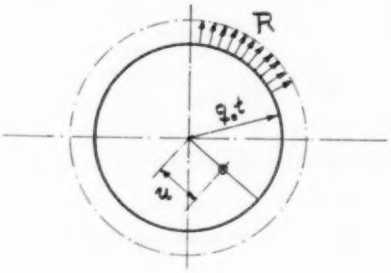

FIG. 18

Radial displacement of a point at radius u is

$$v = Cu + \frac{C_0}{u}$$

C_0 obviously equals zero, which gives

$$v = Cu;\quad \frac{dv}{du} = C$$

The stress due to R at the virtual edge of the plate is

$$\sigma_r' = \frac{R}{q_1t} = \frac{mE}{m^2-1}\left(m\frac{dv}{du} + \frac{v}{u}\right) = \frac{mEC}{m-1}$$

or

$$C = \frac{m-1}{m}\,\frac{R}{Eq_1t}$$

This expression combined with the equation $v_{q_0t} = \lambda_R = Cq_0t$ gives

$$\lambda_R = \frac{m-1}{m} \times \frac{Rq_0}{Eq_1}$$

or, for $m = 4$,

$$\lambda R = \frac{3}{4} \times \frac{Rq_0}{Eq_1} \ldots\ldots\ [34]$$

The increase in radius of the cylinder when under pressure is obviously

$$\lambda_p = \frac{q_0^2tp}{E} \ldots\ldots\ [35]$$

The values for φ_0, λ_R, and λ_p as given by Equations [33]–[35] inserted into Equations [25a]–[28a] give the following four equations:

$$C = \frac{q_0^2tp}{E} - \frac{3}{4}\frac{Rq_0}{Eq_1} + \frac{9}{16}\frac{q_0^3}{Eq_1^2}tp - \frac{4.5q_0}{Eq_1^2} \times \frac{K}{t} - \frac{2.25q_0R}{Eq_1} \ldots\ldots\ [25b]$$

$$D - C = 0.857\ \frac{q_0^3\sqrt{q_0}}{Eq_1^3}tp - \frac{6.85q_0\sqrt{q_0}}{Eq_1^2t}K - \frac{3.425q_0\sqrt{q_0}}{Eq_1^2}R \ldots\ldots\ [26b]$$

$$D = \frac{2\sqrt{3}\,q_0}{E}\,\frac{K}{t} \ldots\ldots\ [27b]$$

$$C + D = \frac{2.63q_0\sqrt{q_0}}{E}R \ldots\ldots\ [28b]$$

from which the values of the unknown quantities C, D, R, and K may be obtained.

$$\frac{R}{tp} = \frac{q_0q_1(6.93q_1^3 + 3.89q_0q_1 + 2.97q_0\sqrt{q_0} + 6.85\sqrt{q_0})}{9.13q_1^4\sqrt{q_0} + 20.78q_1^3 + 23.7q_1^2\sqrt{q_0} + 18q_0q_1 + 5.15\sqrt{q_0}} \ldots\ldots\ [12]$$

$$\frac{K}{t^2p} = \frac{\frac{R}{tp}\left(3q_1 + 2.63q_1^2\sqrt{q_0}\right) - q_0q_1^2 - 0.5625q_0^2}{2\sqrt{3}\,q_1^2 - 4.5} \ldots\ [13]$$

$$D = \frac{2\sqrt{3}\,q_0}{E} \cdot \frac{K}{t} \ldots\ldots\ [36]$$

$$C = \frac{2.63q_0\sqrt{q_0}}{E}R - D \ldots\ldots\ [37]$$

When K is known the bending stress may be found immediately by dividing by the section modulus of the strip. See Equation [6].

The bending moment K_x at a distance x from the junction decreases very rapidly with increasing values for x, following the equation

$$K_x = e^{-\beta x}\left[\left(-\frac{R}{\beta} + K\right)\sin\beta x + K\cos\beta x\right]$$

The stresses in the head may now be evaluated by inserting the values for φ and $d\varphi/du$ as given by Equations [28] and [29] into the equations

$$\sigma_r = \frac{mEz}{m^2-1}\left(m\frac{d\varphi}{du} + \frac{\varphi}{u}\right)$$

$$\sigma_t = \frac{mEz}{m^2-1}\left(m\frac{\varphi}{u} + \frac{d\varphi}{du}\right)$$

using the value

$$C_1 = 1.83\,\frac{pq_0^2}{Eq_1^2t} - \frac{9K}{Eq_1^3t^3} - \frac{4.5R}{Eq_1^2t^2}$$

For the center of the plate, where $u = 0$, these stresses become

$$\sigma_{r0} = \sigma_{t0} = \frac{2}{3q_1}\left(\frac{1.83q_0^2}{q_1}p - \frac{9K}{q_1t^2} - \frac{4.5R}{t}\right) \ldots\ldots\ [8]$$

and for the virtual edge of the plate, where $u = q_0t$,

$$\sigma_r = -\frac{6}{q_1^2}\left(\frac{K}{t^2} + \frac{q_1}{2}\frac{R}{t}\right) \ldots\ldots\ [9]$$

$$\sigma_t = \sigma_r + 0.5625\,\frac{q_0^2}{q_1^2}\,p \ldots\ldots\ [10]$$

These are the *bending* stresses produced by K, R, and p. To these must be added numerically stresses produced by R considered as tension equally distributed along the circumference:

$$\sigma_r^1 = \sigma_t^1 = \frac{R}{q_1t} \ldots\ldots\ [11]$$

Appendix No. 3

DERIVATION OF FORMULAS FOR CYLINDER WITH RIGIDLY ATTACHED FLANGE

Similarly, as in the problem treated in Part II, the point of origin is chosen as indicated in Figs. 14 and 19, so that the x-axis coincides with the center line of the cylinder wall far from the flange when the cylinder is expanded by pressure p. In Fig. 19 the center lines of cylinder wall and flange are shown before and after deformation by heavy broken and heavy solid lines, respectively. A strip of unit width is cut out of the cylinder as shown to the right in Fig. 14. This strip may be treated as a beam on an elastic foundation. When the x-axis is chosen as shown, the y-values are produced by K and R only.

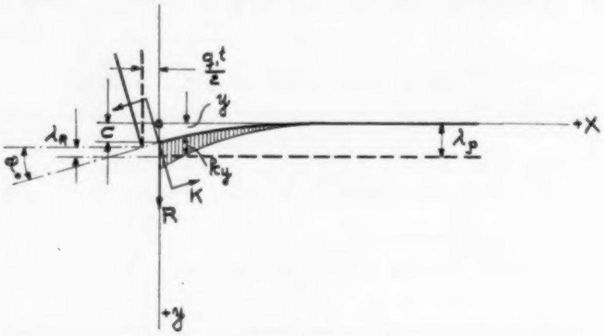

FIG. 19

Notation:

t = thickness of cylinder wall in inches
$q_0 t$ = mean radius of cylinder in inches
$q_1 t$ = thickness of flange in inches
$q_2 t$ = radius of bolt circle in inches
$q_3 t$ = outside radius of flange in inches
p = internal pressure in lb. per sq. in.
P = total bolt load, lb.
K = couple acting at junction of flange and cylinder per inch circumference in in-lb. per inch
R = shearing force at junction of flange and cylinder per inch circumference, lb. per inch
y = deflection of cylinder wall due to K and R in inches
λ_p = radial deformation of cylinder far from flange due to p in inches
λ_R = radial deformation of flange due to R in inches
φ_0 = slope of cylinder wall for $x = 0$ equals slope of flange at radius $q_0 t$ in radians

For further nomenclature see Appendix No. 2.

For the longitudinal strip shown in Fig. 14 Equations [24]–[27] hold. (See Appendix No. 2, II.) For the end of the cylinder, for $x = 0$, these equations assume the following forms:

$$y = C = \lambda_p - \lambda_R - \frac{q_1 t}{2}\varphi_0 \qquad [38]$$

$$\frac{dy}{dx} = \beta(D - C) = -\varphi_0 \qquad [39]$$

$$\frac{d^2y}{dx^2} = -2\beta^2 D = \frac{K}{IE} \qquad [40]$$

$$\frac{d^3y}{dx^3} = 2\beta^3(C + D) = \frac{R}{IE} \qquad [41]$$

For a flange as shown in Fig. 20 the slope is expressed by Equations [2a] and [3a] in part I. Equation [2a] holds for values of u from $q_0 t$ to $q_2 t$, and [3a] from $q_2 t$ to $q_3 t$.

1—From $q_0 t$ to $q_2 t$:

$$\varphi = C_1 u + \frac{C_2}{u} - \frac{S}{2} u \log_e u \qquad \text{where} \qquad S = \frac{1.79\,P}{E q_1^3 t^3}$$

$$\frac{d\varphi}{du} = C_1 - \frac{C_2}{u^2} - \frac{S}{2}(\log_e u + 1)$$

2—From $q_2 t$ to $q_3 t$:

$$\varphi = C_3 u + \frac{C_4}{u}$$

$$\frac{d\varphi}{du} = C_3 - \frac{C_4}{u^2}$$

For the outer edge of the flange where $u = q_3 t$, $\sigma_r = 0$. Therefore

$$m\frac{d\varphi}{du} + \frac{\varphi}{u} = 0 \quad \text{(See Appendix No. 1)}$$

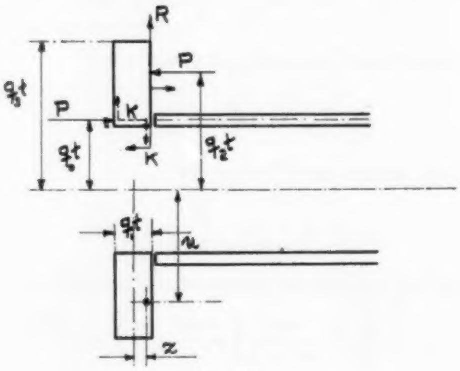

FIG. 20

By inserting the values of $d\varphi/du$ and φ/u for $u = q_3 t$ into this equation it becomes

$$mC_3 - \frac{mC_4}{q_3^2 t^2} + C_3 + \frac{C_4}{q_3^2 t^2} = 0$$

For $u = q_2 t$ two equations are obtained due to continuity.

$$C_1 + \frac{C_2}{q_2^2 t^2} - \frac{S}{2}\log_e(q_2 t) = C_3 + \frac{C_4}{q_2^2 t^2}$$

$$C_1 - \frac{C_2}{q_2^2 t^2} - \frac{S}{2}\left(1 + \log_e(q_2 t)\right) = C_3 - \frac{C_4}{q_2^2 t^2}$$

Eliminating C_3 and C_4 the following relation between C_1 and C_2 is obtained:

$$C_1 = \frac{m-1}{m+1}\frac{C_2}{q_3^2 t^2} + \frac{S}{2}\log_e(q_2 t) + \frac{S}{4}\left(\frac{m-1}{m+1}\frac{q_2^2}{q_3^2} + 1\right)$$

For the virtual inner edge of the flange, where $u = q_0 t$, the moment of the σ_r-forces must be equal to the moment caused by the cylinder wall. This moment is

$$K - \frac{q_1 t}{2} R = \int_{-\frac{q_1 t}{2}}^{+\frac{q_1 t}{2}} \frac{mEz}{m^2 - 1}\left(m\frac{d\varphi}{du} + \frac{\varphi}{u}\right) z\,dz$$

$$= \frac{mE}{m^2 - 1}\left(m\frac{d\varphi}{du} + \frac{\varphi}{u}\right)\frac{q_1^3 t^3}{12}$$

Inserting the values of φ/u and $d\varphi/du$ for $u = q_0t$ gives

$$C_2 = \frac{q_0^2q_3^2t^2}{q_3^2 - q_0^2}\left[-\frac{12(m+1)}{mEq_1^3t^3}\left(K - \frac{q_1t}{2}R\right) + \frac{m+1}{m-1}\frac{S}{2}\log_e\frac{q_2}{q_0} - \frac{S}{4}\frac{q_3^2 - q_2^2}{q_3^2}\right]$$

If the values of C_1 and C_2 are inserted in the equation

$$\varphi_0 = C_1q_0t + \frac{C_2}{q_0t} - \frac{S}{2}q_0t\log_e q_0t$$

and m is set $= 4$, the following expression for the slope results:

$$\varphi_0 = -\frac{3q_0(3q_0^2 + 5q_3^2)}{Eq_1^3t^2(q_3^2 - q_0^2)}\left(K - \frac{q_1t}{2}R\right) + \frac{7.16q_0P}{Eq_1^3t^2(q_3^2 - q_0^2)}\left(\frac{q_3^2}{3}\log_e\frac{q_2}{q_0} + \frac{q_2^2 - q_0^2}{10}\right)$$

The values λ_p and λ_R must next be determined.

$$\lambda_p = \frac{q_0^2tp}{E}$$

The flange may be considered as a thick-walled cylinder loaded by internal pressure of magnitude

$$\frac{R}{q_1t} + p \text{ lb. per sq. in.}$$

The radial deformation for $u = q_0t$ is therefore

$$\lambda_R = \frac{R + q_1tp}{4E}\,\frac{q_0(3q_0^2 + 5q_3^2)}{q_1(q_3^2 - q_0^2)}$$

The expressions for φ_0, λ_p, and λ_R are now inserted into Equations [38] and [39], which, taken together with Equations [40] and [41], give a system of four equations containing the four unknown quantities C, D, K, and R. The solution of this system is

$$R = \frac{\left(q_0 - \frac{q_1^3T_1}{4}\right)(1 + 0.329\sqrt{q_0}\,T_1)tp - T_2(q_1 + 0.76\sqrt{q_0})\frac{P}{t}}{1.315\sqrt{q_0} + T_1[q_1^2(1 + 0.0822\sqrt{q_0}\,T_1) + 1.136q_1\sqrt{q_0} + 0.865q_0]} \quad \ldots\ldots\ldots[19]$$

$$K = \frac{(q_1^2T_1 + 2.63\sqrt{q_0})Rt + q_1T_2P - \left(q_0 - \frac{q_1^3T_1}{4}\right)t^2p}{1.5q_1T_1 - 3.464} \quad \ldots[20]$$

$$T_1 = \frac{3q_0^2 + 5q_3^2}{q_1^3(q_3^2 - q_0^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots[21]$$

$$T_2 = \frac{3.58}{q_1^3(q_3^2 - q_0^2)}\left(\frac{q_3^2}{3}\log_e\frac{q_2}{q_0} + \frac{q_2^2 - q_0^2}{10}\right) \quad \ldots\ldots[22]$$

$$D = -\frac{2\sqrt{3}q_0}{Et}K \quad \ldots\ldots\ldots\ldots\ldots[42]$$

$$C = \frac{2.63q_0\sqrt{q_0}}{E}R + \frac{2\sqrt{3}q_0}{Et}K \quad \ldots\ldots\ldots\ldots[43]$$

When K is known the bending stress in the cylinder wall may be found immediately by division by the section modulus of the strip. (See Equation [14], Part III.)

The bending moment K_x at a distance x from the junction decreases rapidly with increasing values of x, following the equation

$$K_x = e^{-\beta x}\left[\left(\frac{R}{\beta} + K\right)\sin\beta x + K\cos\beta x\right]$$

Compare this with similar expression in Appendix No. 2.

The maximum radial bending stress in the flange at the virtual inner edge at both faces of the flange becomes (see Fig. 21)

$$\sigma_{r0} = \frac{6}{q_1^2t^2}\left(K - \frac{q_1t}{2}\text{R}\right) \quad \ldots\ldots\ldots\ldots\ldots[15]$$

To this stress must be *numerically* added stress arising from the force R distributed over the whole thickness of the flange and also stress due to pressure p.

This stress is $\qquad \sigma_{rR} = \dfrac{R}{q_1t} + p \quad \ldots\ldots\ldots\ldots\ldots[16]$

The maximum tangential bending stress occurs at the same point as σ_{r1} shown in Fig. 21 and is

$$\sigma_{t0} = \sigma_{r0} + \frac{0.800}{q_1^2t^2(q_3^2 - q_0^2)}\left[q_3^2\left(-15K + 7.5q_1tR + 1.492P\log_e\frac{q_2}{q_0}\right) + 0.4475P\left(q_2^2 - q_0^2\right)\right] \quad \ldots[17]$$

To this stress must be *numerically* added tangential stresses arising from radial deformation, λ_R, over the whole thickness of the flange, due to R and p.

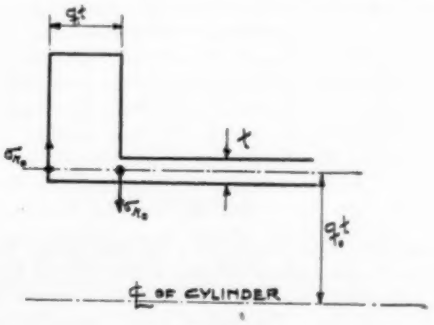

FIG. 21

$$\sigma_{tR} = \frac{q_1^2}{4}T_1\cdot\frac{R + q_1tp}{t} \quad \ldots\ldots\ldots\ldots\ldots[18]$$

Discussion

E. O. WATERS.[7] Four years ago, when the writer, with J. Hall Taylor, presented a paper on "The Strength of Pipe Flanges," there was some question as to whether industry was ready to use a design procedure that involved the elastic theory of flat plates. At that time it was common practice to assume that a flange was a beam and to design it accordingly. Naturally, the authors of that paper wished to avoid all complications and refinements of analysis that would prejudice designers against the use of the flat-plate theory, and therefore made the simplifying assumption, as quoted by Messrs. Holmberg and Axelson, "that the proportions of commercial flanges are such that the bolt load is applied near the outer edge and the pipe load near the inner edge." Furthermore, they made the assumption, in the case of hubbed flanges, that the bending moment at the root of the hub could be computed at the meridian plane of the flange instead of at its inner surface and that the expansion of the hub due to fluid pressure (if such existed) could safely be neglected. A very large number of deflection measurements on flanges, of both the flat and hubbed types, indicated that these assumptions were reasonable as long as the proportions or the loadings were not extreme.

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Since 1927 there has been marked progress, both in the design of standard flanges and of extremely high pressure vessels and fittings, and it is appropriate that engineers should have the more refined methods of analysis brought to their attention, as the authors of the present paper have done in such capable fashion.

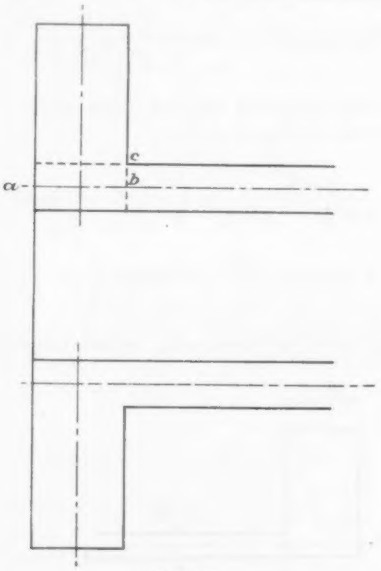

FIG. 22

In deriving their Equation [4] for a flat loose flange, the authors have employed a principle which has a wide field of application, but which apparently has escaped notice on the part of investigators of this subject. The writer called attention to this in his discussion of the paper by Wahl and Lobo on "Stresses and Deflections in Flat Circular Plates with Central Holes," wherein the authors, after giving their simplified formulas for various loading conditions, point out that these loading conditions may be combined, and a combined result obtained, by the method of superposition. Actually, however, the Wahl and Lobo formulas are capable of a still wider range of usefulness, by employing the principle of juxtaposition—i.e., a flat plate with a variety of loadings at different points may be broken down into a series of concentric rings each of which has a simple loading, and the simplified equations may be used and solved by the condition that the slope and radial moment are continuous at the boundary of each ring. This is what the authors have done in deriving Equation [4], and it will be noted that, with three juxtaposed rings, six constants had to be determined. In fact, the number of constants will always be twice the number of rings, and the large number of simultaneous equations that are involved when this method is applied to any but the simplest cases has probably militated against its use. However, the writer has devised a method, similar to the three-moment method for continuous beams, which makes it possible to solve any flat-plate problem with loadings uniformly distributed over circles or annular areas, without recourse to intricate networks of simultaneous equations.

It might be of interest to compare the results of the authors' examples 1, 2, and 3, on the cylinder-flange combination, with calculations based on the Taylor-Waters formulas. The latter give 16,916, 19,810, and 25,230 lb. per sq. in., respectively, for the maximum tangential flange stress, which compare surprisingly well with the more refined solutions, bearing in mind that the Taylor-Waters method takes no account of fluid pressure and the ratio $b-a/d-a$ assumed in the examples is somewhat extreme. Coming to the bending stresses in the cylinder, the agreement is not so close. The Taylor-Waters formulas give 45,430, 67,800, and 69,570 lb. per sq. in. as the theoretical maxima, which when transferred to the boundary between the flange and cylinder (point c, Fig. 22) become 20,150, 41,100, and 14,820. The authors' results, it will be noted, lie about halfway between these extremes. The discrepancy is essentially due to the fact that the authors have assumed that line ab, Fig. 22, remains straight, while the Taylor-Waters formula assumes that it takes the curvature of a crimped tube. The writer believes that the actual state of stress lies somewhere between these two extremes, and would like to see experimental evidence on this point.

In conclusion, the writer would like to ask why the authors have treated the cylinder as made up of strips without any lateral constraint. In the formula for β, just preceding Equation [25], the factor $m^2 - 1/m^2$ seems to have been omitted, which is contrary to the treatment in Timoshenko and Lessells' "Applied Elasticity," the authors' principal reference.

A. M. WAHL.[8] It is gratifying to note that the flat-plate theory is apparently being increasingly applied in practical design. Although the underlying theory, as used by the authors, is well known, nevertheless they deserve credit for demonstrating particular methods of attack for the guidance of practical men. In this connection, the writer would like to mention that he also has used the same general theory in the calculation of rotating thin cylinders with flat ends, which is a case very similar to that of a cylinder with a flat end under internal pressure, as treated by the authors.

In solving problems of this kind involving a cylinder with a flat end, the writer has found that the principle of superposition may often be applied so as to result in considerable simplification of the analytical work. As is well known, the action of a thin-walled cylinder is essentially that of a bar on an elastic foundation, and the results using the theory of a bar on elastic foundation may often be directly applied. By determining the slopes and deflections at the ends of the cylindrical portion, in terms of the unknown moments and reactions, and equating these to the slopes and deflections similarly determined for the plate portion, sufficient equations may be obtained to allow of the determination of the unknown moments, reactions, and deflections. In applying the principle of superposition, the simplified formulas given in a previous paper on circular plates by the writer and G. Lobo, Jr. (Trans. A.S.M.E., 1930, APM-52-3) may also be used. In many cases, when calculating flat plates, a considerable simplification is made possible by using these formulas.

On the third page the authors discuss the question of concentration of stress at the fillet where the plate portion joins the cylindrical portion. This effect might, in the writer's opinion, be determined with sufficient accuracy by the use of photoelastic tests. In case the thickness of the plate portion is much larger than that of the cylindrical portion and the fillet radius is small, a rough approximation may be had by using the stress concentration factors for fillets given by Timoshenko and Dietz (Trans. A.S.M.E., 1925, p. 199), since these factors were found to apply for cases of bending with relatively small fillet radii.

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[8] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Assoc-Mem. A.S.M.E.

W. M. Coates.[9] The contribution made in this paper is a new application of well-established theory concerning the state of stress and strain in thin circular plates and thin cylindrical shells. The formulas given by this theory are not complicated in themselves, although they often have been so regarded. Since they provide a direct attack on a large group of problems, they deserve to be brought into general engineering usage, to provide the possibility of check and possible improvement by experience. The comparative simplicity of rational as against empirical methods is often only a matter of presentation, and the authors of this paper have demonstrated the possibility of a simple rational method of solving their problems. By using for the most part the generally accepted notation, they have made their work more easily followed. However, there are certain points at which over-conciseness in the descriptive matter leads to uncertainty as to their intention. In the following, the discussion is confined to Part II, "Stresses in a Cylinder With Rigidly Attached Flat Heads Under Internal Pressure," and the accompanying Appendix No. 2. The writer's purpose is to bring to the authors' attention the points mentioned, as well as certain others upon which improvement seems possible.

In discussing the thin cylinder, from Equation [6] on, and in Appendix No. 2, on the eighth page, the first column, and on the ninth page, the middle of the second column, there is no consideration of the hoop stress among those which influence the design, although it may be shown to exceed the longitudinal stress in some cases. This may be demonstrated by considering the behavior of both of these stresses for the general case: a thin cylinder under uniform inner pressure p with a head on it which causes the edge loads K and R, as shown in Fig. 16.

At some distance from the junction of cylinder and head, the membrane formulas[10] apply with sufficient accuracy:

$$\text{membrane}\left\{\begin{array}{ll}\text{longitudinal stress} = \dfrac{pq_0}{2} & \text{(See Eq. [7])}\\ \text{hoop stress} = pq_0 & \end{array}\right\} \quad [44]$$

Near the junction, the change in the nature of the shell, from cylinder to head, sets up discontinuity stresses which must be superimposed upon the membrane stresses. The forces which cause these are shown in Fig. 23. They give rise to the stresses[10]

$$\left.\begin{array}{lll}\text{direct} & \text{hoop} & = \sigma_2 = \dfrac{T_2}{t} = -2q_0\beta\,[R\breve{S}_1 - \beta K\breve{S}_4]\\ \text{bending}\left\{\begin{array}{l}\\ \\ \end{array}\right. & \begin{array}{l}\text{longitudinal}\\ \\ \text{hoop}\end{array} & \begin{array}{l}= \sigma_{1\theta} = \dfrac{6K_1}{t^2} = \dfrac{6}{\beta t^2}\,[R\breve{S}_3 - \beta K\breve{S}_2]\\ \\ = \sigma_{2\theta} = \dfrac{6K_2}{t^2} = \mu 6_1\theta\end{array}\end{array}\right\} \quad \ldots\ldots\ldots\ldots [45]$$

in which the notation is as given in the paper, except that, in addition, T_2, K_1, and K_2 are as shown in the accompanying Fig. 23, and

$$\left.\begin{array}{ll}\breve{S}_1 = e^{-\beta x}\cos\beta x & \breve{S}_3 = e^{-\beta x}\sin\beta x\\ \breve{S}_2 = \breve{S}_1 + \breve{S}_3 & \breve{S}_4 = \breve{S}_1 - \breve{S}_3\end{array}\right\} \ldots\ldots [46]$$

$\mu = 1/m$ = ratio of lateral to longitudinal strain in simple tension

$$\beta = \sqrt[4]{\frac{3(1-\mu^2)}{g_0^2t^4}} \cong \frac{1.285}{t\sqrt{g_0}} \qquad \text{for } \mu = 0.3$$

These $\breve{S}$ functions are the only variables. They are made up, it is seen, of damped trigonometric functions; they have been tabulated,[11] and are as easily used as the ordinary trigonometric functions.

The resultant stress, either longitudinal or hoop, is obtained as the algebraic sum of the component stresses in the direction in question. The bending stresses $\sigma_{1\theta}$ and $\sigma_{2\theta}$, given above are at the outer surface, and have their signs reversed at the inner surface.

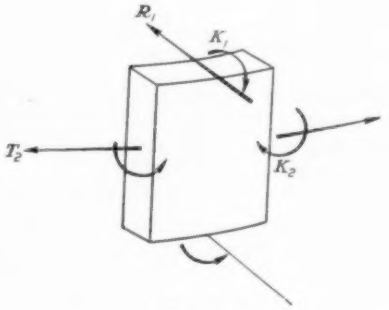

Fig. 23

The discussion of these discontinuity stresses in the paper gives $\sigma_{1\theta}$—see σ in Equation [6] for the edge value and the equation following Equation [37] for the law by which the longitudinal moment K_x (here K_1) varies. The equations which will lead to the others are given on the eighth page without further development except for the edge values, given in Equations [26a] through [28a]. Inasmuch as some of these stresses reach their maxima away from the edge, the presentation of the paper is incomplete. In these Equations [26a] through [28a] the mathematical constants C and D are left to be determined (see Equations [36] and [37]) whereas they are already determined in Equations [45] herein. The physical constants (K, the edge moment, and R the edge shearing force) are determined from the condition that cylinder and head join at their edges with the same radial displacement and the same angular displacement.

In setting up this condition, as well as in setting up the equations already referred to, there are certain inaccuracies which, although perhaps not of great consequence, are still inconsistent, since they amount to a neglect of μ without mention, whereas at other places at which the role is about the same, μ is considered—e.g., in the expression for λR. The corrections are: The expression for β, using μ instead of $1/m$ for algebraic simplicity, should read:

$$\beta = \sqrt[4]{\frac{3\,(1-\mu^2)}{g_0^2t^4}} \cong \frac{1.295}{t\sqrt{g_0}} \qquad \text{for } \mu = {}^1/_4$$

In the explanation following Equation [28], the denominators of the expressions containing the bending moment and the shearing force should be the "flexural rigidity" of the plate, $\dfrac{Et^3}{12\,(1-\mu^2)}$ instead of $\dfrac{Et^3}{12}$.

The radial (normal) displacement of the cylindrical membrane should be $\lambda_p = \dfrac{pq_0^2t}{2E}\,(2-\mu)$, instead of as given by Equation [35].

In Equations [36] and [37], the expressions for the constants of integration C and D should be corrected by the use of the above value of β, instead of the value given in the paper.

There are other inaccuracies which seem more serious, although it is possible that they may be removed by a fuller explanation of the method of attack. The solution for the angular displacement φ in the equation following Fig. 17 is for the case given by Equation [1], with u in place of x. The derivation of this equation hypothesizes no forces applied parallel to the middle surface of the circular plate. However, the forces R of Fig. 17 are exactly of this nature; their presence tends to reduce φ; and in the equation which replaces Equation [1] they introduce an additional term, to give

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[9] Professor of Engineering Mathematics, University of Michigan, Ann Arbor, Mich.

[10] "The State of Stress in Full Heads of Pressure Vessels," W. M. Coates, Trans. A.S.M.E.

[11] "Strength of Materials," S. Timoshenko, vol. 2, p. 405. There, φ gives $\breve{S}_2$, ψ gives $\breve{S}_4$, θ gives $\breve{S}_1$; then S_3 may be computed.

$$\frac{d^2\varphi}{du^2} + \frac{1}{u}\frac{d\varphi}{du} - \frac{\varphi}{u^2} - \frac{2NR}{pq_1t} + Nu = 0$$

where R is assumed to be uniformly distributed across the thickness q_1t of the plate. Then the solution for φ will differ from the value taken in Equation [29], and it is the new value which should be used throughout the application.

It is possible that the authors of this paper propose to regard the effect of R as negligible and to consider an approximate solution to the foregoing equation. In that case, the question warrants some discussion in the presentation. This discussion should also cover the use of R on one side of Equation [32] when it is disregarded on the other, and also the fact that the application of the boundary conditions on φ between cylinder and head will determine a value of R which is not zero.

It might also be mentioned that the C which enters after Fig. 18 is not the C used elsewhere and that its appearance is confined to the derivation of the expression for λ_R. If the effect of R on φ be assumed to be negligible, the logic of introducing λ_R at all is not clear.

In view of these facts, the accuracy of the values determined for the physical constants R and K in Equations [12] and [13] is questioned. If approximations are to be made, they may also be taken more directly, and will yield simpler formulas. Consider as a limiting case a very thick flat head applied to the cylinder, such that it restrains the end as completely as if it were built in. Then the radial and angular displacements are both zero at the edge, and the values of R and K become

$$R = \frac{p(2-\mu)}{2\beta} \cong 0.66\ pt\sqrt{g_0}$$

$$K = \frac{p(2-\mu)}{4\beta^2} \cong 0.26\ pt^2g_0$$

If these are inserted into Equation [45] of this discussion, they become

$$\left.\begin{aligned} 0_2 &= -0.85\ pq_0\check{S}_2 \\ 0_{1\theta} &= -1.55\ pq_0\check{S}_4 \\ 0_{2\theta} &= -0.46\ pq_0\check{S}_4 \end{aligned}\right\} \qquad [45a]$$

Then by taking the algebraic sum of components in Equations [44] and [45], the resultant stresses are:

$$\left.\begin{aligned} \Sigma\sigma_1 &= \text{resultant longitudinal stress} = pq_0\ [0.5 \pm 1.55\ S_4] \\ \Sigma\sigma_2 &= \text{resultant hoop stress} = pq_0\ [1-0.85\ S_2 \pm 0.46\ S_4] \end{aligned}\right\}$$

........[47]

These give the stresses at the outer and inner surfaces at any section of the cylinder, the coordinate x of the section entering the $\check{S}$ functions as given by Equation [46].

These stresses reach their greatest values as follows:

$(\Sigma\sigma_1)_{\max} = 2.05\ pq_0$ occurs at edge on outer surface

$({}_2\Sigma\sigma)_{\max} = 1.07\ pq_0$ occurs at βx 2.65 on inner surface, $x = 2.06\ t\sqrt{g_0}$.

In case the head applied to the cylinder is thicker than the cylinder, but not thick enough to warrant the assumption of full restraint against angular deflection at the end, the approximation that there is no radial displacement will be kept, but the moment K will be cut to n times its previous value, where n is a fraction. Then, running through the same processes, the resultant stresses are:

$$\left.\begin{aligned} \Sigma\sigma_1 &= pq_0\ [0.5 \pm 1.55\ (\check{S}_4 - n\check{S}_1)] \\ \Sigma\sigma_2 &= pq_0\ [1 - 0.85\ (\check{S}_1 + n\check{S}_3) \pm 0.46\ (\check{S}_4 - n\check{S}_1)] \end{aligned}\right\} . [48]$$

This is a more general form of Equations [47], which may be obtained from it for $n = 0$. These stresses may be investigated for various values of n and the results tabulated. Here it is sufficient to consider the case $n = 0.5$; then

$\Sigma(\sigma_1)_{\max.} = 0.86\ pq_0$ occurs at $\beta x = 1.25$, $x = 0.97\ t\sqrt{q_0}$

$\Sigma(\sigma_2)_{\max.} = 1.07\ pq_0$ occurs at $\beta x = 2.32$, $x = 1.81\ t\sqrt{q_0}$

Both of these maxima are at the outer surface.

This demonstrates the point mentioned at the beginning of the discussion, that in some cases it is possible that the hoop stress will exceed the longitudinal stress in the cylinder, and hence it should be considered. Another point which should be kept in mind is that for ductile materials the numerical maxima of the differences between the principal stresses is the criterion for yield. Since the third principal stress is approximately zero, this means that to the consideration of the maxima of $\Sigma\sigma_1$ and $\Sigma\sigma_2$ should be added also that of $|\Sigma\sigma_1 - \Sigma\sigma_2|$ at both surfaces. The distances at which the membrane formulas may be regarded as sufficiently accurate may also be obtained from the equations. It is a matter of interest, and also of possible further investigation, that the maximum resultant hoop stress in both cases considered is about the same, and in each case only about 7 per cent in excess of the membrane stress. Evidently, the fraction n is a function of the thickness of the head, and for each thickness there corresponds an n which may be inserted into Equation [48] to give the complete determination of the stresses at any point on either surface of the cylinder. The limitations on these equations must be kept in mind:

(1) The cylinder is "thin," that is, the ratio $\dfrac{\text{radius}}{\text{thickness}} = q_0$ is above, say, 10, for safety; there is probably reasonably good accuracy for somewhat lower values. If q_0 is too small, the effect of the shearing forces and the displacement of the neutral surface from the middle will introduce error.

(2) The derivation assumed no radial displacement at the end. The accuracy of this assumption is a matter of experience. It probably holds good for thicknesses of the flat circular head down to and somewhat below the thickness of the cylinder.

The state of stress in the flat circular head has not been mentioned in this discussion; however, it is seen that a similar approximation may be made, using Equation [1] and its solution as given by Equation [29], but determining C from the condition that the plate is supported at its edge and carries the uniformly distributed bending moments K. This is handled in Timoshenko, "Strength of Materials," vol. 2, pars. 19 and 22. From the fact that the angular displacement φ is numerically the same at the edges of the circular plate and the cylinder, the value of K may be determined, or what amounts to the same thing, that of n.

The determination of the foregoing limitations, or of the values of n for different head thicknesses, or even of R and K directly, is a matter which might be well referred to experiment. The problem is of such a nature that it may be carried out very nicely and also comparatively inexpensively, by measuring at the outer surface of the cylinder both the resultant strains or the angular displacement due to bending. Such measurements have been made by Huggenberger among others, on slightly different structures, and the instruments and the method of experiment are very completely described in his publications.[12] It is to be hoped that if such experiments are made, the results may be published completely.

T. McLean Jasper.[13] This paper adds to the mathematical precision available for obtaining the theoretical stress values in the problems dealing with plates and rings and is worthy of commendation for the amount of thought which has been expended aiming to approach an exact solution of some of the problems outlined in the title.

The writer is particularly interested in the flange problem and especially in the question of a balanced design for flanges used in high-pressure vessels of considerable size. This problem involves flanges of considerable thickness and also the methods of attaching the flanges to the cylinders. The formulas presented in this paper presume the use of a very rigid material in which the deformations are relatively small and apply most appropriately to flanges made of steel for the conditions in which Hooke's law applies. It does not consider, however, the effect of the sharpness of the corner at the junction between the flange and the cylinder. This is a matter which is now under experimental consideration and which it is hoped can be so adjusted as to approach or even exceed in safety by proper filleting the theoretical stresses presented in this paper. If proper filleting is not carried out, the stresses calculated by the method presented in the paper will be very much too low. The effect of the experimental work will be to influence the coefficient to be applied to the formulas developed by the authors.

The flange problem from many angles is a complicated one, and the adequacy of flanges is dependent on the manner of gasket design and the sealing qualities of gaskets under working conditions, the general intelligence of the people tightening bolts and fitting covers to openings, the proper balance between cylinder and flange stiffness, and the proper bolt sizes and their location.

When one considers the operation of vessels such as are used in the oil and chemical industry, in which the flanged openings are uncovered frequently, and also the class of operator employed on such work whose idea of stopping a leak is simply to pull the bolts tighter with wrenches whose lever arm is increased promiscuously, then the problem presents many very great difficulties indeed.

The authors have presented a very commendable piece of mathematical work which should be fully appreciated by elasticians everywhere.

Arthur McCutchan.[14] The authors are to be complimented for the clear presentation of their analysis of stresses in circular plates and rings. The curves of Figs. 9 and 10 should prove particularly helpful in designing welded pressure vessels.

From a reader's standpoint the addition of a subscript b, or one perhaps more suitable, to the symbol σ for bending stress would be of assistance, as subscripts are used to designate the other stresses, and the lack of one for this particular stress continually raises a question in the reader's mind as to whether the authors are referring to bending stress or some other stress for which the proper subscript might have been omitted in printing.

In the analysis of stresses in a cylinder with a rigidly attached flange, on the fifth page, the authors state that their object is "to furnish a method for figuring extreme cases where large dimensions and high loads make a more accurate formula desirable." In the furtherance of this object numerical examples are worked out which should presumably indicate the correct method of procedure in attacking such problems. An examination of these examples, however, shows several assumptions and conclusions that are not in harmony with the principles usually employed in flange design.

Material presented in the paper deals with broad general principles applicable to the design of welded and flanged structures for pressure vessels and piping. It is possible that, due to the wide scope of the paper, certain principles of practical pipe-flange design have not been given the consideration they deserve. A few of the controlling factors in pipe-flange design are called to attention in the following discussion of bolt loading.

Example 1, sketched in Fig. 15, closely approximates a 20-in. fitting flange for which facing dimensions are available in the A.S.A. Steel Flanged Standard. It is stated that a gasket joint is under consideration, so it appears reasonable that some necessary gasket compression in terms of the internal pressure should be established, as well as the area over which the internal pressure may be considered as acting, rather than arbitrarily placing the total bolt load as that equal to the internal pressure acting over the "full" area of the cylinder.

The actual bolt load on any such joint depends largely on what system of control, if any, is used in making up the joint. A rather comprehensive study of raised-face gasket joints in steam lines operating at 400 lb. pressure and 700 deg. fahr. shows that the practice of sledging up the bolts "until the joint is solid" and then retightening in the same manner after a short period of service makes a thoroughly satisfactory joint at this temperature. Alloy-steel bolting material having a minimum yield point of 70,000 lb. per sq. in. is used in these joints, and numerous tests have shown that stresses in the neighborhood of this yield point are obtained in the bolts by this method of making up pipe joints. The corresponding stresses in the flanges as calculated by the Waters-Taylor formulas would be considerably above the yield points of the material, yet no objectionable deformation of the flanges results. This condition is possible because of the localized nature of the region of maximum stress. Local flow of the material which is stressed beyond the yield point apparently results in a redistribution of stress without evidence of instability of the flange as a whole.

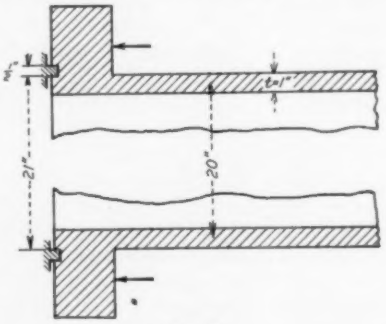

Fig. 24

Experience with flanges subjected to loads which would develop excessively high stresses as calculated by such formulas, together with Professor Waters' statement that the stresses as found by his formula did not have the same practical significance as tension in a bolt, leads one to question the desirability of making fine distinctions as to method of calculating bending stress in flanges.

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[12] "Über die Festigkeit der gewölbten Böden und der Zylinderschale," Höhn-Huggenberger, part 2, p. 184.

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[14] Engineer, Engineering Division, Detroit Edison Company, Detroit, Mich.

In particular, if a small tongue-and-groove joint is assumed for the flange illustrated in Fig. 15, with dimensions corresponding to a 20-in. A.S.A. flanged facing (see Fig. 24), the bolt load may be recalculated as follows. The area over which internal pressure acts, as considered in the A.S.A. Steel Flanged Standards, extends to the outer periphery of the gasket surface.

The force equal to the internal pressure of 1000 lb. given by 1000 × $\pi/4\ (22)^2 = 381{,}000$ lb.

The gasket area of the small tongue-and-groove joint is found as $\pi 21^1/_2 \times {}^1/_2 = 33.8$ sq. in.

The required gasket compression to secure a fluid-tight joint varies with each type and size of gasket and is largely a matter of experience for any given service. In high-pressure steam lines a gasket compression of 6 to 8 times the working steam pressure is generally found to be a satisfactory initial gasket compression for a raised-face joint. In the case of a narrow gasket such as the small tongue-and-groove joint, assumed as applicable to the joint under consideration, a gasket compression of 12 to 16 times the W.S.P. usually is considered a suitable gasket pressure to use in design. Assuming a gasket compression of 12 times the W.S.P., with the pressure acting as being necessary, the bolt load to give this compression may be found as 12 × 1000 × 33.8 = 406,000 lb.

The initial bolt load for which the flange should be designed is the sum of the foregoing forces, or 787,000 lb., rather than the 500,000 lb. assumed by the authors.

In the example of a calculation using very high internal pressure, example 3 of Table 1, the discrepancy between assumed bolt loading and that which experience has shown to be necessary is more apparent. The total bolt load for which the flange should be designed is approximately 1.75 times that used by the authors.

While the differences in magntiude of the bolt loadings used in the examples of the calculation of flange stresses in nowise affect the application of the authors' formulas, they do affect the flanges finally determined as satisfactory.

A little discussion as to the magnitude of the stresses which may be considered as reasonably satisfactory for the particular sections investigated and materials considered would be a useful addition to the authors' paper. As mentioned in connection with the application of the Waters-Taylor formula, the tangential or hoop stress at the inner edge of the flange ring is not related to failure of the material in the same manner as tension in the bolting. An analysis of the true significance of such stresses as affecting the practical design of flanges would be of value.

Authors' Closure

The authors are grateful indeed for the amount of constructive criticism to which their paper has given rise.

The discrepancies in numerical results when the Waters-Taylor formula and the authors' formula are applied are due to different assumptions of the shape of the deflected flange, as pointed out by Professor Waters. The authors agree with Professor Waters that the actual curvature of line *a–b*, Fig. 22, is probably somewhere between a straight line (the authors' assumption) and the shape of a crimped tube (Waters' assumption). Considering the actual proportions of a flange, however, it seems somewhat more probable that the curvature will closely approach a straight line.

Referring to the discussion by Prof. W. M. Coates, the authors acknowledge his suggestions regarding a more accurate value for β and the employment of the modulus of flexural rigidity. The use of these new values causes slight changes in the numerical coefficients of the expressions for R and K. These expressions become:

1 For cylinder with flat head:

$$\frac{R}{tp} = \frac{q_0q_1\left(5.86q_1^3 + 3.77q_0q_1 + 2.912\,q_0\sqrt{q_0} + 6.08\,\sqrt{q_0}\right)}{8.65q^4\sqrt{q_0} + 20.10q_1^3 + 23.26q_1^2\sqrt{q_0} + 17.94\,q_1q_0 + 5.20\,\sqrt{q_0}} \quad \ldots [12b]$$

$$\frac{K}{t^2p} = \frac{\frac{\overline{R}}{tp}\left(3q_1 + 2.58\,q_1^2\sqrt{q_0}\right) - 0.875\,q_0q_1^2 - 0.5625\,q_0^2}{3.35\,q_1^2 - 4.5} \quad \ldots [13b]$$

$$D = \frac{3.35\,q_0}{E} \times \frac{K}{t} \quad \ldots\ldots\ldots\ldots\ldots [36b]$$

$$C = \frac{2.58\,q_0\sqrt{q_0}}{E}R - D \quad \ldots\ldots\ldots\ldots [37b]$$

II For cylinder with flange:

$$R = \frac{\left(q_0 - 0.286\,q_1^3T_1\right)\left(0.875 + 0.303\sqrt{q_0}\,T_1\right)tp - T_2\left(q_1 + 0.772\sqrt{q_0}\right)\frac{P}{t}}{1.295\sqrt{q_0} + T_1\left[q_1^2\left(1 + 0.0862\sqrt{q_0}T_1\right) + 1.16\sqrt{q_0}q_1 + 0.895\,q_0\right]} \quad \ldots [19b]$$

$$K = \frac{\left(q_1^2T_1 + 2.59\sqrt{q_0}\right)Rt + q_1T_2P - 0.875\left(q_0 - 0.286q_1^3T_1\right)t^2p}{1.5\,q_1T_1 - 3.35} \quad \ldots\ldots [20b]$$

$$D = -\left(3.35\,q_0/E\right) \times (K/t) \quad \ldots\ldots\ldots\ldots [42b]$$

$$C = \left(2.59q_0\sqrt{q_0}/E\right) \times (R - D) \quad \ldots\ldots\ldots [43b]$$

These equations supersede Equations [12], [13], [36], [37], [19], [20], [42], and [43].

The influence of these changes on numerical results is very small, in the neighborhood of 1 to 2 per cent, or of the same order as inaccuracies due to an assumed value of Poisson's ratio.

In the discussion of the influence of the radial force R on the angular deformation of the flat head, Professor Coates sets up a differential equation identical with the authors' Equation [1] except for the term $2NR/pq_1t$

This term, however, cannot be accurate, as it is of a different dimension than the other terms of the equation. The influence of the shearing force R on angular deflections varies with the linear deflection of the plate. In all practical applications the linear deflections are of such a magnitude that the influence of R on the *angular deflection* may be safely neglected. The influence of R on the *radial extension* of the plate, however, is of a different magnitude and cannot be neglected.

The authors agree with Professor Coates that the hoop stresses at a certain distance from the junction reach a maximum higher than the hoop stress far from the junction. The difference amounts to only a few per cent.

Prof. T. McLean Jasper brings out the fact that the effect of a fillet has not been adequately treated. The reasons why the authors did not attempt to consider a fillet in their mathematical analysis is simply the tremendous complexity of the problem. It is hoped that the tests now being conducted by Professor Jasper will contribute to a clearer conception of the matter.

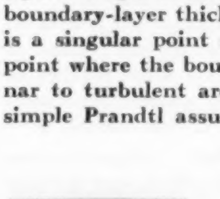
The interesting fact mentioned by Mr. A. McCutchan regarding a localized stress above the elastic limit of the material causing a regional permanent deformation without impairing the flange as a whole, can hold true only for ductile materials. In a brittle material a stress beyond this limit will immediately cause cracking.

The Boundary Layer and Skin Friction for a Figure of Revolution

By CLARK B. MILLIKAN,[1] PASADENA, CALIF.

The boundary-layer equations for a figure of revolution are first derived in the Prandtl form. These are then integrated to give the so-called "integral relation" for the case in question. Power-series expressions for the boundary-layer profiles in laminar and turbulent flow are assumed and substituted into the integral relation. In this way expressions are obtained for the boundary-layer thickness and for the contribution to the skin friction of the laminar and turbulent portions of the boundary layer. In particular a simple expression is given for the boundary-layer thickness at the stagnation point, which is a singular point of the equation. Conditions at the point where the boundary-layer flow changes from laminar to turbulent are discussed in some detail, and the simple Prandtl assumption for the transition-point behavior is adopted as the basis of the further calculations. Numerical values for the N.P.L. International Airship Model are introduced, and theoretical curves are given for the resistance coefficient as a function of the Reynolds number. These are compared with previous theoretical results and with experimental values.

The agreement of the theory with experiment is considered satisfactory in view of the assumptions which had to be introduced into the analysis in order to supplement the present state of knowledge concerning boundary-layer phenomena. The general integral relation developed in the paper can be used to obtain more accurate results in the future, when more is known regarding boundary-layer profiles in curved and diverging (or converging) flows.



1—Introduction

In 1927 Prandtl published a theory giving the skin friction acting on pointed flat plate in a two-dimensional flow parallel to the surface and perpendicular to the leading edge (1).[2] This theory, which gave excellent accordance with experiment, was based on the assumptions that over the forward part of the plate the flow was laminar and followed Blasius' solution of the Prandtl boundary-layer equations, over the rear portion it was turbulent and followed the "$^1/_7$-power law" for velocity distribution, and that the transition point from laminar to turbulent flow followed a definite and simple law. Experiments have since indicated that this theory gives a satisfactory value for the skin friction at the curved surface of a streamlike body in a two-dimensional flow when the radius of curvature is large compared to the boundary layer thickness. Recently Jones (2) and Dryden and Kuethe (3) have further extended the theory so as to give an account of the skin friction acting on a dirigible model in an axial flow. In both of these discussions the dirigible was replaced by an "equivalent flat plate," so that the essential differences between a two-dimensional flow and one about a finite figure of revolution had either to be ignored or, in the latter case, taken into account by means of rather arbitrary assumptions. Hence it seemed desirable to investigate the possibility of a more precise consideration of the problem. The fundamental aim of the present work has therefore been the investigation of the boundary-layer phenomena for a body with axial symmetry about an axis in the direction of flow.

The laminar boundary layer for such a case was treated as long ago as 1908 by Dr. E. Boltze (4). Boltze's dissertation is, however, rather inaccessible, so that the present author has been unable to examine it. From the published abstracts it appears that Boltze gave the laminar boundary-layer equations for the case of axial symmetry and also gave numerical integrations in certain cases. In view of the difficulty of obtaining Boltze's paper, and at the risk of repeating some of his analysis, the author will derive the laminar boundary-layer differential equations for axial symmetry before proceeding to the deduction of the integral relation for this case and its subsequent application to both the laminar and turbulent régimes. The latter two topics are almost certainly not treated in Boltze's paper, since even for plane flow they were first discussed by Karman and Prandtl much later.

2—Derivation of the Laminar Boundary-Layer Equations for Axial Symmetry

We start with the vector form of the Navier-Stokes and continuity equations for the steady motion of a viscous incompressible fluid,

$$\left.\begin{aligned} \boldsymbol{q}\cdot\nabla\boldsymbol{q} + \frac{1}{\rho}\nabla p &= \nu\nabla^2\boldsymbol{q} \\ \nabla\cdot\boldsymbol{q} &= 0 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ [1]$$

where $\boldsymbol{q}$ is the vector velocity, p the pressure, ρ the density, ν the kinematic coefficient of viscosity, ∇ the vector operator "gradient," and the standard Gibbs vector notation is used. We must find the approximate form which these equations take in the neighborhood of the surface of a figure of revolution when the viscosity is very small, i.e., $\nu \longrightarrow 0$. For this purpose it is convenient to introduce an orthogonal, curvilinear coordinate system defined as follows: Let x_1 = constant, x_2 = constant, x_3 =

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[2] Numbers in parentheses refer to items in bibliography at end of paper.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

constant denote the three families of orthogonal surfaces defining the coordinate system. Then we choose x_1 such that one member of the family x_1 = constant coincides with the boundary of our surface of revolution (cf. Fig. 1). x_3 is chosen so that x_3 = constant gives planes through the axis of symmetry, i.e., x_3 is the azimuthal angle θ of Fig. 1. Then the surfaces x_2 = constant are perpendicular to the two other families. If ds is an element of length, then for such an orthogonal system we have in general

$$ds^2 = h_1^2dx_1^2 + h_2^2dx_2^2 + h_3^2dx_3^2$$

where the coefficients h_1, h_2, h_3, are functions of x_1, x_2, x_3. Putting $dx_1 = dx_2 = 0$ we see that $h_3 = r$, where r is the distance of any point (x_1, x_2, x_3) from the axis. In our case we are interested only in a thin region of thickness δ on either side of the boundary. If C_r denotes the "longitudinal" radius of curvature of the boundary then if $\delta \ll C_r$ we may write $h_1 = 1$, $h_2 = 1$. Hence

$$h_1 = 1; \quad h_2 = 1; \quad h_3 = r, \text{ where } \delta \ll C_r$$

We now choose the origin of x_1 at the boundary and that of x_2 at the most upstream point of the boundary (i.e., at the forward stagnation point for a closed body) and write for simplicity

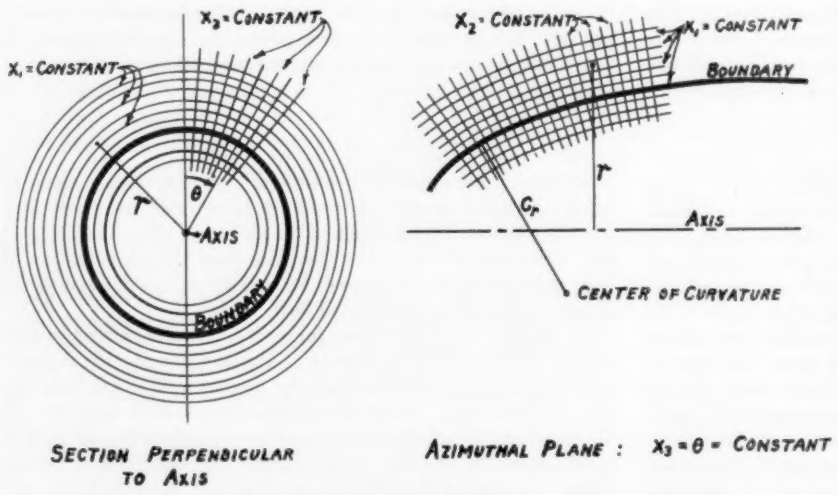

FIG. 1 COORDINATE SYSTEM FOR A FIGURE OF REVOLUTION

$x_1 = y$ = normal distance from the boundary

$x_2 = x$ = distance along the boundary from the leading edge or forward stagnation point.

Hence our curvilinear system is specified by:

$$\left.\begin{array}{lllll} x_1 = y & h_1 = 1 & & \boldsymbol{q} = u\boldsymbol{i}_x + v\boldsymbol{i}_y + q_\theta\boldsymbol{i}_\theta & \\ x_2 = x & h_2 = 1 & & & \\ x_3 = \theta & h_3 = r(x,y) & y \ll C_r & \dfrac{\partial}{\partial\theta} = 0 & q_\theta = 0 \end{array}\right\} \ldots[2]$$

where u, v, q_θ are the x, y, θ components of velocity and $\boldsymbol{i}_x$, $\boldsymbol{i}_y$, $\boldsymbol{i}_\theta$ are unit vectors in the indicated directions. The final two restrictions are imposed by our fundamental assumption of axial symmetry.

We must now express Equations [1] in terms of these curvilinear coordinates. The general expressions for ∇p and $\nabla \cdot \boldsymbol{q}$ are (writing q_1, q_2, q_3 as general velocity components in the directions x_1, x_2, x_3).

$$\nabla p = \frac{1}{h_1}\frac{\partial p}{\partial x_1}\boldsymbol{i}_1 + \frac{1}{h_2}\frac{\partial p}{\partial x_2}\boldsymbol{i}_2 + \frac{1}{h_3}\frac{\partial p}{\partial x_3}\boldsymbol{i}_3$$

$$\nabla \cdot \boldsymbol{q} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial x_1}(h_2h_3q_1) + \frac{\partial}{\partial x_2}(h_3h_1q_2) + \frac{\partial}{\partial x_3}(h_1h_2q_3)\right]$$

Analogous general expressions for $\boldsymbol{q}.\nabla\boldsymbol{q}$, and $\nabla^2\boldsymbol{q}$ have been given by the author (5), but will not be reproduced here in view of their complexity. If the restrictions [2] are introduced into these expressions and the two above, the following expressions follow immediately from [1]:

$$\left.\begin{array}{l} u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = \nu\left[\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial^2 v}{\partial x\partial y} + \left(\dfrac{\partial u}{\partial y} - \dfrac{\partial v}{\partial x}\right)\dfrac{1}{r}\dfrac{\partial r}{\partial y}\right] \\ u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial y} = \nu\left[\dfrac{\partial^2 v}{\partial x^2} - \dfrac{\partial^2 u}{\partial x\partial y} + \left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right)\dfrac{1}{r}\dfrac{\partial r}{\partial x}\right] \\ \qquad\dfrac{\partial(ru)}{\partial x} + \dfrac{\partial(rv)}{\partial y} = 0 \qquad (\text{where } y \ll C_r) \end{array}\right\} \ldots\ldots\ldots[3]$$

In order to get the boundary-layer equations we must let $\nu \rightarrow 0$ in [3]. To get an approximate form of [3] in this case we must use the method of asymptotic integration which requires us to replace y (already assumed small compared with C_r) by a new variable $\eta = y/\sqrt{\nu}$. Then the final (continuity) equation of [3] shows us that as $\nu \rightarrow 0$, v also approaches 0 like $\sqrt{\nu}$. Hence we introduce the new variables

$$\eta = y/\sqrt{\nu} \qquad u = f(x, \eta)$$
$$v = \sqrt{\nu}\,h\,(x, \eta)$$

Substituting these into [3] and neglecting powers of $\sqrt{\nu}$ higher than the first, the two first equations of [3] give

$$f\frac{\partial f}{\partial x} + h\frac{\partial f}{\partial \eta} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial^2 f}{\partial \eta^2} + \frac{\partial f}{\partial \eta}\frac{1}{r}\frac{\partial r}{\partial \eta}$$

$$\frac{\partial p}{\partial \eta} = 0$$

Returning to the original variables we get therefor as the boundary layer equations for axial symmetry,

$$\left.\begin{array}{ll} u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{1}{\rho}\dfrac{dp}{dx} + \nu\left(\dfrac{\partial^2 u}{\partial y^2} + \dfrac{1}{r}\dfrac{\partial r}{\partial y}\dfrac{\partial u}{\partial y}\right) & \\ \qquad p = p(x) & \\ \dfrac{\partial(ru)}{\partial x} + \dfrac{\partial(rv)}{\partial y} = 0 & y \ll C_r \end{array}\right\} \ldots\ldots[4]$$

For a two-dimensional flow $r \rightarrow \infty$, and these equations reduce to the classical ones given by Prandtl for this case. Other interesting special cases are: (*a*) axial flow outside or inside of a circular tube where $r = r_0 \pm y$, r_0 = constant, $\partial r/\partial y = \pm 1$; and (*b*) flow impinging perpendicularly on a flat circular disk where $r = x$, $\partial r/\partial y = 0$.

It appears to be practically impossible to give a general solution of these equations except in very special cases. For given functions $p(x)$ and $r(x, y)$ numerical solution of the equations is of course possible, but the calculations would in general be very tedious. Boltze has apparently given a certain number of such numerical integrations, but for the present work we shall use the alternative method of approximation first given by Karman for two-dimensional flow.

3—The Integral Relation for the Boundary Layer About a Figure of Revolution

Karman's method for the approximate discussion of conditions in a boundary layer is based on the use of the so-called "Karman Integral Relation," which is essentially a first integral of the Prandtl differential equations (6, 7). This method was originally given for two-dimensional flow. In the present section, then, we shall derive the analogous integral relation for axially symmetrical flow.

We define the thickness of the boundary layer by $y = \delta$ and assume that for $y \geq \delta$ the flow is unaffected by viscosity, i.e., is a potential flow. We also assume that the boundary layer is so thin compared to the dimensions of the body that the potential flow at $y = \delta$ is the same as would be given at $y = 0$ by the potential flow of a perfect fluid about the body. If U denotes the velocity at a point x of the boundary which would occur for such a potential flow, then we assume that at $y = \delta$ $u(x, y) = U(x)$, where $U(x)$ is the actual velocity just outside the boundary layer.

Bernoulli's theorem gives for the potential flow

$$\frac{1}{\rho}\frac{dp}{dx} = -U\frac{dU}{dx} = -UU' \qquad \text{(say)}$$

so that the first equation of [4] may be rewritten as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = UU' + \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{1}{r}\frac{\partial r}{\partial y}\frac{\partial u}{\partial y}\right)$$

Let us now multiply this equation by rdy and integrate from 0 to δ, obtaining the following expression:

$$\int_0^\delta ru\frac{\partial u}{\partial x}dy + \int_0^\delta rv\frac{\partial u}{\partial y}dy = UU'\int_0^\delta rdy + \nu\int_0^\delta\left(r\frac{\partial^2 u}{\partial y^2} + \frac{\partial r}{\partial y}\frac{\partial u}{\partial y}\right)dy \qquad [5]$$

Let us consider the terms individually. Discussing the second term first, we obtain by a partial integration

$$\int_0^\delta rv\frac{\partial u}{\partial y}dy = rvu\Big|_0^\delta - \int_0^\delta u\frac{\partial(rv)}{\partial y}dy$$

But at both $y = 0$ and $y = \delta$ the velocity v vanishes, hence the integrated term disappears. Applying the continuity equation (the final equation of [4]) we obtain

$$\int_0^\delta rv\frac{\partial u}{\partial y}dy = +\int_0^\delta u\frac{\partial(ru)}{\partial x}dy$$

and hence the left-hand side of [5] becomes

$$\int_0^\delta ru\frac{\partial u}{\partial x}dy + \int_0^\delta rv\frac{\partial u}{\partial y}dy = \int_0^\delta\left[ru\frac{\partial u}{\partial x} + u\frac{\partial(ru)}{\partial x}\right]dy$$

$$= \int_0^\delta\frac{\partial(ru^2)}{\partial x}dy = \frac{d}{dx}\int_0^\delta ru^2dy - r_\delta U^2\frac{d\delta}{dx}$$

by Leibnitz' rule for the differentiation of an integral with variable limits of integration. Here r_δ is the value of r at $y = \delta$.

Consider an element of the boundary layer (Fig. 2) and let Q be the flux across any section x = constant of the boundary layer. Then

$$Q = 2\pi\int_0^\delta rudy \qquad \frac{dQ}{dx} = 2\pi\frac{d}{dx}\int_0^\delta rudy$$

But now since we have postulated steady motion, the conservation of mass in the element states that the total flux out of it must be zero. Hence, neglecting second-order differentials,

$$2\pi r_\delta U\frac{d\delta}{dx}dx = \frac{dQ}{dx}dx = 2\pi\,dx\frac{d}{dx}\int_0^\delta rudy$$

$$\therefore\ r_\delta U^2\frac{d\delta}{dx} = U\frac{d}{dx}\int_0^\delta rudy$$

so that we have finally for the left-hand side of [5]

$$\int_0^\delta ru\frac{\partial u}{\partial x}dy + \int_0^\delta rv\frac{\partial u}{\partial y}dy = \frac{d}{dx}\int_0^\delta ru^2dy - U\frac{d}{dx}\int_0^\delta rudy$$

For the final integral of [5] we have

$$\nu\int_0^\delta\left(r\frac{\partial^2 u}{\partial y^2} + \frac{\partial r}{\partial y}\frac{\partial u}{\partial y}\right)dy = \nu\int_0^\delta\frac{\partial}{\partial y}\left(r\frac{\partial u}{\partial y}\right)dy = \nu r\frac{\partial u}{\partial y}\Big|_0^\delta$$

$$= -\nu r_0\left(\frac{\partial u}{\partial y}\right)_0$$

where we use a subscript zero to denote conditions at the boundary itself, and the integrated term vanishes at the upper limit because of the fact that we approximate to the known asymptotic approach of $u \rightarrow U$ for $y \rightarrow \delta$ by requiring that $\partial u/\partial y = 0$ at $y = \delta$.

Hence Equation [5] finally becomes

$$\frac{d}{dx}\int_0^\delta ru^2dy - U\frac{d}{dx}\int_0^\delta rudy = UU'\int_0^\delta rdy - \nu r_0\left(\frac{\partial u}{\partial y}\right)_0 \qquad [6]$$

This is the simplest form of the boundary-layer integral relation for the case of axial symmetry. There are two remarks which should immediately be made with reference to this equation. The first is that the relation may also be deduced by elementary methods, using only the "momentum law" and the continuity condition, and without assuming a laminar flow satisfying the Navier-Stokes equations [2]. This latter was the method originally employed by Karman for the two-dimensional case (6; see also 3) so that this alternative derivation will not be given here. If a laminar flow is not assumed the resulting equation differs from [6] in one particular. If τ_0 represents the shearing stress or skin friction per unit area at the wall, then $\nu(\partial u/\partial y)_0$ is replaced by τ_0/ρ. In using [6] in the future we shall use the more general form given by this substitution, remembering that for laminar flow

$$\frac{\tau_0}{\rho} = \nu\left(\frac{\partial u}{\partial y}\right)_0$$

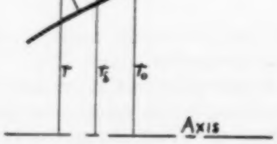

FIG. 2 BOUNDARY-LAYER ELEMENT IN A PLANE THROUGH THE AXIS

The second remark is that [6] can be transformed into another form which is somewhat more complex in appearance but which is more useful for calculations and which also brings out more clearly the essential similarities and differences between the axially symmetrical and two-dimensional integral relations. We proceed to deduce this alternative expression.

Consider a small section of the boundary, Fig. 3(a). Then since $y \ll C_r$ the element P_0P may be taken as a straight line and

$$r = r_0 + y\cos\alpha$$

where α gives the inclination of the boundary to the axis. But from Fig. 3(b)

$$\cos\alpha = \sqrt{1 - r_0'^2} = \beta(x)\ (\text{say}) \qquad \text{[7]}$$

where r_0' denotes dr_0/dx. Hence

$$r(x, y) = r_0(x) + y\beta(x) \qquad \text{[8]}$$

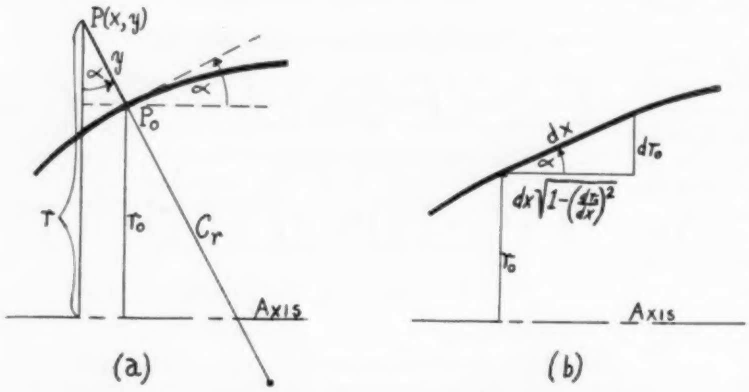

FIG. 3 BOUNDARY ELEMENT IN A PLANE THROUGH THE AXIS

where for any given boundary r_0 and β are known functions of x. Substituting [8] into [6] we are led very simply to

$$\left.\begin{aligned} &\frac{d}{dx}\int_0^\delta u^2dy - U\frac{d}{dx}\int_0^\delta udy + \frac{r_0'}{r_0}\left[\int_0^\delta u^2dy - U\int_0^\delta udy\right] \\ &+\frac{\beta}{r_0}\left[\frac{d}{dx}\int_0^\delta u^2ydy - U\frac{d}{dx}\int_0^\delta uydy\right] + \frac{\beta'}{r_0}\left[\int_0^\delta u^2ydy\right. \\ &\qquad \left. - U\int_0^\delta uydy\right] = UU'\delta\left(1 + \frac{\beta}{2}\frac{\delta}{r_0}\right) - \frac{\tau_0}{\rho} \end{aligned}\right\} \qquad \text{[9]}$$

where $\beta' = d\beta/dx$, and we have replaced $\nu(\partial u/\partial y)_0$ by the more general τ_0/ρ. The point $r_0 = 0$ is a singular point of this equation which must be investigated in more detail for each special case. For a two-dimensional flow we must let $r_0 \rightarrow \infty$, in which case [9] reduces to the simple Karman integral relation. For purposes of practical calculations we may make a further important simplification by assuming $\delta \ll r_0$. Then we get finally:

$$\left.\begin{aligned} &\frac{d}{dx}\int_0^\delta u^2dy - U\frac{d}{dx}\int_0^\delta udy + \frac{r_0'}{r_0}\left[\int_0^\delta u^2dy - U\int_0^\delta udy\right] \\ &\qquad = UU'\delta - \frac{\tau_0}{\rho} \\ &\text{where} \qquad \delta \ll C_r;\ \delta \ll r_0 \end{aligned}\right\} \qquad \text{[10]}$$

The legitimacy of this simplification near the singular point $r_0 = 0$ must later be investigated for any particular boundary $r_0(x)$.

This equation differs from the two-dimensional Karman integral relation only by the term in r_0'/r_0, which vanishes in the limiting case of two-dimensional flow, $r_0 \rightarrow \infty$, and also in the very important case of axial flow inside or outside of a uniform circular tube, r_0 = constant. The method of using [10] is the same as that employed so successfully in the two-dimensional case, i.e., a velocity profile $u(y)$ and a corresponding law of shearing stress $\tau_0(u)$ are assumed and the integrals evaluated. The equation then becomes an ordinary differential equation for $\delta(x)$. From the solution of this differential equation all of the characteristics of the boundary layer may be calculated.

4—APPLICATION OF THE INTEGRAL RELATION TO THE AXIAL FLOW ABOUT A STREAMLINE FIGURE OF REVOLUTION

The initial and fundamental assumption made in applying the previous analysis to the axial flow about streamline bodies of revolution is that the phenomenon of separation does not occur. That this assumption is justified has been verified for several dirigible models both by detailed investigations on the boundary layer and by calculating the form drag from experimentally observed pressure distributions [cf., for example, (3) and (8)]. The form drag for good streamline shapes has in several investigations been found to be zero within the limits of the experimental accuracy, which means that the theoretical potential flow is very closely approximated by the real flow outside of the boundary layer. Hence we may safely assume in general that over the experimentally used range of Reynolds' numbers separation does not occur for moderately or very well-streamlined figures of revolution. This assumption has two important consequences. First, it implies that the Prandtl boundary-layer method of approximating to the real flow is valid (for any Reynolds numbers of usual magnitude or larger), and second, it allows us to equate the skin friction integrated over the surface with the total drag of the body.

The second assumption which we shall make is that the flow in the forward portion of the boundary layer is laminar while that over the rear portion may be turbulent, and that the transition from laminar to turbulent flow occurs instantaneously. In other words, the entire boundary layer upstream from a certain critical value of $x(x_c)$ is laminar, while that downstream from x_c is completely turbulent. We shall consider the conditions at this critical or transition point in detail in section 5.

In accordance with the method of procedure mentioned at the end of the previous section, we must now make definite assumptions for the velocity profile and shearing stress in the laminar and turbulent regions. For the laminar régime the type of profile originally used by Pohlhausen (7) appears to be most convenient. This is

$$\frac{u}{U} = a + b\frac{y}{\delta} + c\left(\frac{y}{\delta}\right)^2 + d\left(\frac{y}{\delta}\right)^3 + \ldots\ldots$$

where the constants $a, b, c, d, \ldots\ldots$ are determined by boundary conditions at $y = 0, \delta$. The analysis is of exactly equal difficulty for a linear and a parabolic profile, and the latter should be the more accurate of the two. For the two-dimensional flow along a flat plate Pohlhausen found that a linear profile gave a skin friction about 13 per cent higher than the Blasius accurate profile, while a parabolic profile gave a value about 6 per cent low. If terms of degree higher than 2 in y/δ are retained, two serious difficulties are introduced. First, the differential equation for δ becomes very awkward in general, requiring a very tedious numerical or graphical integration. Second, the quantity U' occurs throughout all of the calculations. U' can only be obtained (in general) by a graphical or numerical differentiation of the experimentally determined U (from pressure distribution and Bernoulli's equation). Both of these difficulties are fortunately avoided if we cut off the power series for u/U after the second-degree term. Hence in this paper we assume a parabolic laminar velocity profile of

$$\frac{u}{U} = a + b\frac{y}{\delta} + c\left(\frac{y}{\delta}\right)^2$$

To determine the coefficients we have the two obvious boundary conditions furnished by the "zero slip" requirement and by the definition of the boundary-layer thickness

$$y = 0 \rightarrow u = 0; \quad y = \delta \rightarrow u = U$$

We know that actually u approaches U asymptotically as y increases. To get an approximation to this behavior we add as a third boundary condition

$$y = \delta \rightarrow \frac{\partial u}{\partial y} = 0$$

These three conditions determine the constants a, b, c and give

$$\frac{u}{U} = 2\,\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

The shearing stress at the boundary is then

$$\frac{\tau_0}{\rho} = \nu\left(\frac{\partial u}{\partial y}\right)_0 = 2\nu\,\frac{U}{\delta} \ldots\ldots\ldots\ldots\ldots [11a]$$

For a turbulent boundary layer in a flow where $U' = 0$ (flat plate, uniform tube) it is well known that, at least for the range of Reynolds' numbers encountered in normal wind-tunnel tests, the velocity profile may be satisfactorily represented by the approximation formula $u/U = (y/\delta)^{1/7}$. There is at present no theory and there are also very few experimental data giving the velocity profile for cases in which the basic streamlines have either divergence or curvature. However, such experimental results as have been published indicate that a formula of the form $u/U = (y/\delta)^{1/n}$ may fairly generally be used. For pure divergence Nikuradse found $n \backsim 7$ until the divergence became so rapid that separation occurred (9). In similar experiments Doench found $n \backsim 8$ (10). By measuring the velocity distribution in the boundary layers about airship models Simmons found values of n varying between about 3.6 and 4.7, depending on the position along the dirigible (11), while Ower and Hutton found an approximately constant value of about 6 (12). In view of these very incomplete and rather conflicting data it seems that in order to get an approximate idea of the flow processes about a streamline figure of revolution, as satisfactory an assumption as any is to use the constant rectilinear flow value of $n = 7$. Hence in the following calculations we have assumed for the turbulent boundary layer

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [12]$$

By dimensional analysis Karman and Prandtl have shown that in a boundary layer the surface shearing stress is completely determined by the law for velocity distribution (7, 1), and Karman has given the following expression corresponding to the velocity profile [12]:

$$\frac{\tau_0}{\rho} = 0.0225\ U^2\left(\frac{\nu}{U\delta}\right)^{1/4} \ldots\ldots\ldots\ldots [12a]$$

This expression has been verified experimentally and will be used in the succeeding analysis.

For purposes of calculation it is convenient to introduce the notation:

$$w = \frac{u}{U}, \quad z = \frac{y}{\delta}$$

in terms of which [10] takes the form

$$\frac{d}{dx}\left[U^2\delta\int_0^1 w^2dz\right] - U\frac{d}{dx}\left[U\delta\int_0^1 wdz\right]$$
$$+\frac{r_0'}{r_0}U^2\delta\left[\int_0^1 w^2dz - \int_0^1 wdz\right] = UU'\delta - \frac{\tau_0}{\rho}\,..[13]$$

Let us first consider the laminar portion of the boundary layer. From Equation [11]

$$\int_0^1 w^2dz = \frac{8}{15} \qquad \int_0^1 wdz = \frac{10}{15}$$

Introducing these and [11a] into [13] we obtain after a little reduction:

$$\frac{d\delta^2}{dx} + \left(9\,\frac{U'}{U} + 2\,\frac{r_0'}{r_0}\right)\delta^2 = 30\,\frac{\nu}{U} \ldots\ldots\ldots [14]$$

This equation is easy to integrate between $x = 0$ and x, and gives

$$U^9r_0^2\,\delta^2\Big|_0^x = 30\nu\int_0^x U^8r_0^2dx$$

But at $x = 0$, $U = r_0 = 0$, hence

$$\delta^2 = \frac{30\nu}{U^9r_0^2}\int_0^x U^8r_0^2dx$$

It is very convenient to express all quantities as dimensionless ratios, so that we define

L = axial overall length of the streamline body
a = distance along axis measured downstream from the forward stagnation point. From Fig. 3: $da = dx\cdot\sqrt{(1 - r_0'^2)} = \beta dx$
U_0 = axial velocity of flow far upstream from the body
R = U_0L/ν = Reynolds' number of the flow (independent of x, a)
R_δ = $U\delta/\nu$ = boundary-layer Reynolds' number $[R_\delta = R_\delta(a)]$
$()_l \sim ()_{\text{laminar}}$

With this new notation we get very simply

$$\left.\begin{aligned}&\frac{\delta_l}{L} = \frac{\sqrt{30}}{R^{1/2}}M\left(\frac{a}{L}\right); \quad R_{\delta l} = \sqrt{30}\,R^{1/2}\,\frac{U}{U_0}M\left(\frac{a}{L}\right)\\ &M\left(\frac{a}{L}\right) = \frac{1}{\left(\dfrac{U}{U_0}\right)^{9/2}\dfrac{r_0}{L}}\int_0^{a/L}\frac{\left(\dfrac{U}{U_0}\right)^8\left(\dfrac{r_0}{L}\right)^2}{\beta}\,d\left(\frac{a}{L}\right)\end{aligned}\right\} \ldots\ldots\ldots [15]$$

The auxiliary function M has been introduced for the sake of conciseness. [15] gives the laminar boundary-layer thickness as a function of the Reynolds number of the flow and of the position along the model surface. (Assuming a given streamline body, i.e., U/U_0, r_0/L given functions of a/L.)

The total contribution of the laminar boundary layer to the drag is the integral of the axial component of the skin-friction force per unit area from $a = 0$ to $a = a_c$ where the transition to the turbulent boundary layer occurs.

$$(\text{Drag})_l = 2\pi\int_0^{a_c} r_0\tau_0da = 4\pi\nu\rho\int_0^{a_c}\frac{r_0U}{\delta_l}\,da$$

from [11a]. We define the laminar contribution to the drag coefficient by

$$C_{vl} = \frac{(\text{Drag})_l}{\frac{\rho}{2} U_0^2 (\text{Vol})^{2/3}}$$

where (Vol) = the total volume of the body. Expressing everything in the form of dimensionless ratios and introducing [15],

$$C_{vl} = \frac{4.59}{R^{1/2}} \frac{L^2}{(\text{Vol})^{2/3}} \int_0^{\frac{a_c}{L}} \frac{\frac{U}{U_0}\frac{r_0}{L}}{M} d\left(\frac{a}{L}\right) \ldots\ldots [16]$$

Before proceeding to the turbulent layer it will be well to discuss the behavior of the equations at the singular points $r_0 = 0$. We shall first show that at least for a blunt-nosed body, the approximate form [10] of [9] is still valid in the neighborhood of the stagnation point at the nose, even though the condition $\delta \ll r_0$ is not necessarily satisfied in this region. Let O be the forward stagnation point, P a neighboring point on the surface, C_{r0} the radius of curvature at P, and φ the angle between C_{r0} and the axis (Fig. 4). For a blunt-nosed body, since P is close to O, we may take OP as a circular arc, so that C_{r0} is constant and the center of curvature Q is the same for any point P and lies on the axis. x is the length of the arc OP and the angle φ is small. Hence near the singular point O:

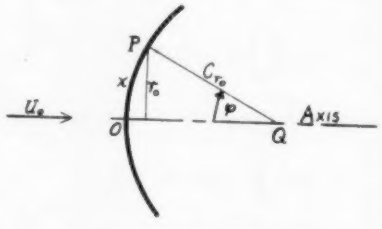

FIG. 4 ELEMENT AT THE NOSE, I.E., AT THE STAGNATION POINT

$$x = C_{r0}\varphi; \quad r_0 = C_{r0} \sin\varphi = C_{r0} \quad \varphi \left(1 - \frac{\varphi^2}{6} + \ldots\ldots\right)$$

$$\therefore r_0' = \frac{dr_0}{dx} = \frac{1}{Cr_0}\frac{dr_0}{d\varphi} = 1 - \frac{\varphi^2}{2} + \ldots\ldots$$

$$\beta = \sqrt{1 - r_0'^2} = (1 - 1 + \varphi^2 + \ldots\ldots)^{1/2} \doteq \pm\varphi$$

$$\therefore \frac{\beta}{r_0} = \pm\frac{1}{C_{r0}}; \quad \beta' = \frac{d\beta}{dx} = \pm\frac{1}{C_{r0}}$$

Hence, considering only absolute values:

$$\frac{\beta}{r_0}\left[\frac{d}{dx}\int_0^\delta u^2 y dy - U\frac{d}{dx}\int_0^\delta uydy\right] = \frac{d}{dx}\int_0^\delta u^2\frac{y}{C_{r0}}dy$$

$$- U\frac{d}{dx}\int_0^\delta u\frac{y}{C_{r0}}dy \ll \frac{d}{dx}\int_0^\delta u^2 dy - U\frac{d}{dx}\int_0^\delta udy;$$

$$\frac{\beta'}{r_0}\left[\int_0^\delta u^2 ydy - U\int_0^\delta uydy\right] = \frac{1}{r_0}\left[\int_0^\delta u^2\frac{y}{C_{r0}}dy\right.$$

$$\left.- U\int_0^\delta u\frac{y}{C_{r0}}dy\right] \ll \frac{r_0'}{r_0}\left[\int_0^\delta u^2 dy - U\int_0^\delta udy\right];$$

$$\frac{\beta}{2}\frac{\delta}{r_0} = \frac{1}{2}\frac{\delta}{C_{r0}} \ll 1$$

since it has already been assumed in [9] that $\delta \ll C_{r0}$. Hence, even at the nose where $r_0 \rightarrow 0$, [10] and the subsequent differential Equation [14] are valid approximations to [9] and may legitimately be used to describe the boundary layer. We use the laminar Equation [14] since for any normal conditions the flow would be expected to be laminar in this region.

If we rewrite [14] in the form

$$\frac{d\delta^2}{dx} = \frac{30\nu - \left(9U' + 2r_0'\frac{U}{r_0}\right)\delta^2}{U}$$

we see that $r_0 = 0$ is a singular point of this equation since there $U = 0$. In order to avoid infinite values of $d\delta^2/dx$ which are physically inadmissible, the numerator of the right side must also vanish with r_0. Hence using subscripts s to denote conditions at the stagnation point $r = 0$, we have

$$\delta_s = \sqrt{\frac{30\nu}{9U_s' + 2r_{0s}'\frac{U_s}{r_{0s}}}} \ldots\ldots\ldots\ldots [17]$$

in order to evaluate [17] we must make some assumption as to the nature of the flow near the stagnation point. We therefore assume that in this region the potential flow is the same as would occur near the stagnation point of a sphere of radius C_{r0} in the uniform flow U_0. This gives the well-known result

$$U = \frac{3}{2}U_0 \sin\varphi \doteq \frac{3}{2}U_0\varphi$$

$$\therefore U_s' = \left(\frac{dU}{dx}\right)_s = \frac{1}{C_{r0}}\left(\frac{dU}{d\varphi}\right)_s = \frac{3}{2}\frac{U_0}{C_{r0}} \quad \text{and} \quad r_{0s}'\frac{U_s}{r_{0s}} = \frac{3}{2}\frac{U_0}{C_{r0}}$$

Hence

$$\delta_s = \sqrt{\frac{60\nu}{33\frac{U_0}{C_{r0}}}} = \sqrt{\frac{60}{33}}\, L \frac{\sqrt{C_{r0}/L}}{\sqrt{\frac{U_0 L}{\nu}}}$$

Therefore at the stagnation point at a rounded nose the laminar boundary layer has a finite thickness given by

$$\frac{\delta_s}{L} \doteq \frac{1.348}{\sqrt{R}}\sqrt{\frac{C_{r0}}{L}} \ldots\ldots\ldots\ldots\ldots\ldots [18]$$

In the neighborhood of the other singular point at the tail of the body, the boundary layer will be relatively thick and the assumptions leading from [9] to [10] are not justified. Hence near the tail our calculations no longer give the true conditions. This means that our determination of δ near the tail may be in considerable error. However, the error introduced into the most interesting characteristic, the total drag coefficient, will probably be very small. This is because the boundary layer in this region is so thick that the skin friction is very small. Hence a considerable error in it will affect the total drag coefficient very little.

Turning now to the turbulent region, we have, from [12],

$$\int_0^1 w^2 dz = 7/9 \qquad \int_0^1 wdz = 7/8$$

and substituting these expressions and [12a] into [13] we obtain for the differential equation for the turbulent-boundary-layer thickness

$$\frac{d\delta^{\frac{5}{4}}}{dx} + \left(\frac{115}{28}\frac{U'}{U} + \frac{35}{28}\frac{r_0'}{r_0}\right)\delta^{\frac{5}{4}} = 0.289\frac{\nu^{\frac{1}{4}}}{U^{1/4}} \ldots [18a]$$

This equation is readily integrated and gives, where we shall in the future use a subscript c to denote conditions at the critical or transition point,

$$U^{\frac{115}{28}} r_0^{\frac{35}{28}} \delta^{\frac{5}{4}} \Big|_{x_c}^{x} = 0.289\, \nu^{\frac{1}{4}} \int_{x_c}^{x} U^{\frac{108}{28}} r_0^{\frac{35}{28}}\, dx$$

or

$$\delta^{\frac{5}{4}} = \frac{0.289\, \nu^{\frac{1}{4}}}{U^{\frac{115}{28}} r_0^{\frac{35}{28}}} \int_{a_c}^{a} \frac{U^{\frac{108}{28}} r_0^{\frac{35}{28}}}{\beta}\, da + \left(\frac{U_c}{U}\right)^{\frac{115}{28}} \left(\frac{r_{0c}}{r_0}\right)^{\frac{35}{28}} \delta_c^{\frac{5}{4}}$$

Introducing the previously used dimensionless parameters we get finally, corresponding to [15],

$$\left.\begin{aligned} \frac{\delta_t}{L} &= \frac{0.370}{R^{\frac{1}{5}}} N\left(\frac{a}{L}, \frac{a_c}{L}\right); \quad R_{\delta t} = 0.370 R^{\frac{4}{5}} \frac{U}{U_0} N\left(\frac{a}{L}, \frac{a_c}{L}\right) \\ N\left(\frac{a}{L}, \frac{a_c}{L}\right) &= \frac{1}{\left(\frac{U}{U_0}\right)^{\frac{23}{7}} \frac{r_0}{L}} \left[\int_{a_c/L}^{\frac{a}{L}} \frac{\left(\frac{U}{U_0}\right)^{\frac{27}{7}} \left(\frac{r_0}{L}\right)^{\frac{5}{4}}}{\beta} \right. \\ &\qquad \left. d\left(\frac{a}{L}\right) + 3.46 R^{\frac{1}{4}} \left(\frac{\delta_{ct}}{L}\right)^{\frac{5}{4}} \left(\frac{U_c}{U_0}\right)^{\frac{115}{28}} \left(\frac{r_{0c}}{L}\right)^{\frac{5}{4}}\right]^{\frac{4}{5}} \end{aligned}\right\} \quad ..[19]$$

where $()_t \backsim ()_{\text{turbulent}}$.

For the contribution of the turbulent-boundary-layer to the skin friction we have

$$(\text{Drag})_t = 2\pi\rho \int_{a_c}^{L} r_0 \frac{\tau_0}{\rho}\, da = 0.045\, \pi\rho\nu^{\frac{1}{4}} \int_{a_c}^{L} \frac{r_0 U^{7/4}}{\delta^{1/4}}\, da$$

or defining

$$C_{vt} = \frac{(\text{Drag})_t}{\frac{\rho}{2} U_0^2 (\text{Vol})^{2/3}}$$

a short calculation gives

$$C_{vt} = \frac{0.3625}{R^{1/5}} \frac{L^2}{(\text{Vol})^{2/3}} \int_{a_c/L}^{1} \frac{\left(\frac{U}{U_0}\right)^{7/4} \frac{r_0}{L}}{N^{1/4}} d\left(\frac{a}{L}\right) \quad \ldots [20]$$

Equations [15], [16], [18], [19], and [20] enable us to calculate the boundary-layer thickness over the entire surface except near the extreme tail, and the complete drag coefficient:

$$C_v = C_{vl} + C_{vt} \ldots\ldots\ldots\ldots\ldots\ldots [21]$$

as soon as the transition conditions have been assumed and made precise.

5—Transition Conditions and Hypotheses

Although experiment and theory agree in indicating that a breakdown may occur from the laminar to the turbulent régime in a boundary layer, the actual conditions in this transition region are still largely in the dark. Experiments with flat-plate boundaries (13, 14) show that the transition is far from instantaneous, and indeed that the distance between the last well-defined laminar profile and the first fully developed turbulent profile may be fairly large. The experiments on the boundary layer about figures of revolution appear to indicate a somewhat smaller transition region, but no very definite conclusions in this regard can be drawn as yet. The assumption which Prandtl made in his theory mentioned at the beginning of this paper was that the transition was instantaneous, and that behind the transition point the turbulent boundary layer had the same thickness and behaved exactly as it would have if the entire boundary had been turbulent from the leading edge. This very simple assumption gives a considerable jump or discontinuity in δ at the transition point, which is not experimentally observed. Before discussing the reason for the good agreement between the Prandtl predictions relative to skin friction, which are based on this assumption and on experiment, let us try to formulate an alternative method of handling the transition region.

For this purpose we consider first a two-dimensional flow over a flat plate so that $r_0 = \infty$, U = constant, and the integral relation [10] reduces to the classical Karman form:

$$\frac{d}{dx}\int^{\delta} u^2 dy - U \frac{d}{dx}\int_0^{\delta} u\, dy = -\frac{\tau_0}{\rho}$$

Let us suppose that at $x = x_1$ we have still a laminar velocity profile while at x_2 the boundary layer has the turbulence characteristics fully developed. Considering the region of the boundary layer bounded by x_1 and x_2, i.e., the transition region, we require the momentum law to be satisfied in it. This means that we require the transition region to satisfy the elementary dynamical laws. Hence integrating the Karman integral relation between the limits x_1 and x_2 we get

$$\int_{x_1}^{x_2} \left[\frac{d}{dx}\left(U^2\delta \int_0^1 w^2 dz\right) - U \frac{d}{dx}\left(U\delta \int_0^1 w\, dz\right)\right] dx$$

$$= -\int_{x_1}^{x_2} \frac{\tau_0}{\rho}\, dx$$

$$\therefore\ U^2\delta \int_0^1 (w^2 - w)\, dz \Big|_{x_1}^{x_2} = -\int_{x_1}^{x_2} \frac{\tau_0}{\rho}\, dx$$

Now writing $\bar{\tau}_0$ = mean value of τ_0 over the transition region

$$\delta \int_0^1 (w^2 - w)\, dz \Big|_{x_1}^{x_2} = -\frac{\bar{\tau}_0}{\rho U^2} (x_2 - x_1)$$

If now we suppose that at x_1 we have our laminar parabolic profile and at x_2 the $1/7$th-power turbulent profile, then a little calculation gives

$$\delta_{t2} = 1.37\delta_{l1} + 10.3 \frac{\bar{\tau}_0}{\rho U^2} (x_2 - x_1) \ldots\ldots\ldots [22]$$

If we had assumed that the profile at x_1 was the true Blasius profile, then this equation would be unchanged except that the coefficient of δ_{l1} would be altered from 1.37 to 1.27.

Unfortunately not enough experimental or theoretical data on the transition region exist to enable us to use [22]. If we assume $x_2 = x_1$, i.e., that the transition is practically instantaneous, then

$$\delta_{tc} = 1.37\delta_{lc} \ldots\ldots\ldots\ldots\ldots\ldots [23]$$

which gives a discontinuity in δ at the transition point. It is easy to show that [23] also holds for an instantaneous transition in the boundary layer around a streamline figure of revolution, if the same velocity profiles as above are assumed. The discontinuity in δ_c given by [23] is in general much less than that given by Prandtl's transition hypothesis, so that it would appear that [23] should more closely represent the true behavior of δ than does the latter hypothesis.

With regard to the skin friction, however, it is not at all certain that [23] is a better assumption than that of Prandtl, for the following reason: For a given boundary-layer thickness the skin friction for a turbulent profile is much larger than for a laminar profile. In the true gradual transition the flow character changes continuously from laminar to turbulent and hence

r_0 also changes smoothly from the laminar value at x_1 to the (in general larger) turbulent value at x_2. If, however, we assume an instantaneous transition then there is a large discontinuity in r_0 unless there is a correspondingly considerable discontinuity in δ_c. From the excellent agreement between Prandtl's theory and experiments on flat plates it would appear that the very large jump from δ_{lc} to δ_{tc} implied by this theory is approximately the correct one in order to give a fairly smoothly increasing value of r_0. If this conclusion is also valid for figures of revolution, then it would seem that if skin friction is the most important quantity under investigation, the Prandtl transition hypothesis is more satisfactory than the dynamically more logical hypothesis given by [23].

Another very great advantage of the Prandtl hypothesis lies in the enormously greater simplicity of the calculations in working out any particular case. For since the turbulent boundary layer behaves as if it had started at the stagnation point, therefore in [19], $U_c = r_{0c} = 0$ and $N(a/L, a_c/L)$ becomes $N(a/L)$, a function of only one variable instead of two. For both of the above reasons we have used the Prandtl transition hypothesis in the numerical calculations of the next section.

Having decided upon the nature of the transition region which will be assumed, there remains the question as to how the position of the transition point is to be chosen. In agreement with the experimental results on the flow in tubes (15) and in boundary layers along flat plates we assume that the laminar flow becomes unstable and turbulent flow sets in at a critical value of the boundary-layer Reynolds' number R_δ. Hence the transition point occurs for $R_\delta = R_{\delta c}$. The numerical value for $R_{\delta c}$ is known to depend on the initial turbulence in the incident air stream for flat plates (13). We therefore carry over this result and assume that $R_{\delta c}$ is a function of the turbulence of the free air stream, increasing as the turbulence decreases. The lowest value of $R_{\delta c}$ which Schiller was able to obtain in tubes (corresponding to the most violent incident turbulence) was 1160. Here R_δ was defined in terms of the radius of the tube. Zijnen found a minimum $R_{\delta c}$ for the boundary layer along a flat plate of about 1650. Tollmien has recently calculated theoretically the stability conditions in a flat-plate boundary layer and obtained a minimum $R_{\delta c}$ of 1320 (16). The upper limit for $R_{\delta c}$ is still uncertain; it may be that if all external disturbances could be eliminated $R_{\delta c}$ maximum might approach infinity. However, the highest values normally obtained with experimentally possible flows are of the order of 12,000. Hence for normal experiments we may expect $R_{\delta c}$ to lie between 1500 and 12,000, and to vary inversely with the degree of initial turbulence in the incident flow.

One further remark should be made with reference to the transition hypotheses. It appears that at very small Reynolds' numbers (R) the transition point lies very near to the tail of a normal airship body. Hence under these conditions practically the entire drag is laminar skin friction, and is hence practically independent of the transition characteristics assumed. Conversely at very large Reynolds' numbers the critical or transition point is very near the nose and the drag is almost entirely turbulent skin friction. In this case the boundary layer is extremely thin at the transition point so that a large variation in the transition hypothesis will have a very small effect on the boundary-layer conditions far behind, and hence on the drag. Therefore at both very large and very small Reynolds' numbers the transition hypothesis is of negligible importance. It affects the total drag considerably only at the intermediate Reynolds' numbers where the transition point occurs in the central region of the airship.

6—Numerical Calculations for the N.P.L. International Long Airship Model

For the purpose of numerical calculation to test the present theory the so-called "N.P.L. long airship model" was chosen since it has been the subject of experimental investigations in many of the aerodynamic laboratories of the world (17). Table 1 gives the data used. r_0/L was taken from Table 4, p. 14, of (3), β was obtained from a graphical differentiation of the curve of r_0/L vs. a/L, U/U_0 was calculated from the experimental pressure distribution given in Table 10 of (8).

TABLE 1

a/L	r_0/L	β	U/U_0
0	0	0	0
0.01	0.0165	0.700	0.920
0.02	0.0230	0.870	0.965
0.03	0.0278	0.920	0.990
0.06	0.0379	0.959	1.040
0.10	0.0471	0.983	1.065
0.15	0.0545	0.993	1.066
0.20	0.0590	0.998	1.064
0.25	0.0609	1.000	1.054
0.30	0.0612	1.000	1.039
0.35	0.0612	1.000	1.032
0.40	0.0612	1.000	1.030
0.45	0.0612	1.000	1.032
0.50	0.0612	1.000	1.037
0.55	0.0610	1.000	1.044
0.60	0.0596	0.999	1.050
0.65	0.0566	0.998	1.049
0.70	0.0525	0.996	1.042
0.75	0.0474	0.994	1.032
0.80	0.0411	0.991	1.019
0.85	0.0334	0.987	1.003
0.90	0.0248	0.983	0.984
0.95	0.0140	0.974	0.95
1.00	0	0.940	0

$L^2/(\text{Vol})^{2/3} = 24.81.$
$L = 87.2$ cm.

The auxiliary functions M, N (Equations [15], [19]), were obtained by graphical integration with a "Coradi" integraph. Since the calculations were made on the basis of the Prandtl transition hypothesis, N is a function of a/L only just as is M. From [15], $R_{\delta l}/R^{1/2}$ was calculated as a function of a/L. For the succeeding calculations it was found convenient to consider this relation as $R_{\delta c}/R^{1/2}$ as a function of a_c/L. Here and in the future $R_{\delta c}$ denotes the value of $R_{\delta l}$ at the transition point. From [16] and [20] $R^{1/2}C_{vl}(a_c/L)$ and $R^{1/5}C_{vt}(a_c/L)$ were determined (again by graphical integration) as functions of a_c/L. These three functions are shown in Fig. 5. A definite value of $R_{\delta c}$ was now assumed and a_c/L was determined as a function of R from Fig. 5. Using Fig. 5 again, C_{vl} and C_{vt} were then determined as functions of R, and finally the total drag coefficient $C_v = C_{vl} + C_{vt}$ was obtained as a function of R. The process was then repeated for a series of assumed values of $R_{\delta c}$.

The results are plotted to a logarithmic scale in Fig. 6. The abscissas here are not $R = V_0L/\nu$ but $R_v = U_0(\text{Vol})^{1/3}/\nu)$ since most of the experimental results are given in terms of this latter

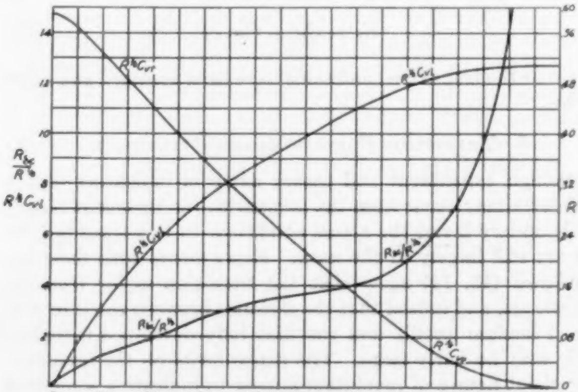

Fig. 5 Curves Giving Laminar and Turbulent Contributions to Drag Coefficient, and Boundary-Layer Thickness at Transition Point, in Terms of Position of Transition Point and the Reynolds Number of the Flow. (For N.P.L. Long Dirigible Model.)

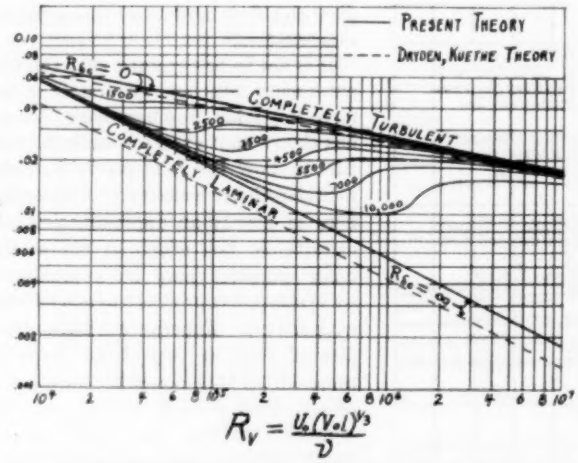

FIG. 6 LOGARITHMIC PLOT OF THEORETICAL TOTAL DRAG COEFFICIENT OF N.P.L. LONG MODEL AS A FUNCTION OF THE REYNOLDS NUMBER OF FLOW (R_v), FOR VARIOUS VALUES OF THE CRITICAL BOUNDARY-LAYER REYNOLDS NUMBER

quantity. For the N.P.L. long model $R_v \doteq {}^1/_5R$. The curves marked laminar and turbulent are limiting cases in which the entire boundary layer is laminar and turbulent, respectively. They are obtained by putting $R_{\delta c} = \infty$ and $R_{\delta c} = 0$. The analytical expressions for these limiting cases are

$$C_{v\text{-laminar}} = \frac{12.70}{R^{1/2}} = \frac{5.68}{R_v^{1/2}}; \qquad C_{v\text{-turbulent}} = \frac{0.590}{R^{1/5}} = \frac{0.428}{R_v^{1/5}}$$

A few curves from Fig. 15 of Dryden and Kuethe's paper (3) have also been plotted for comparison in Fig. 6. We shall discuss these curves a little in the next section.

The laminar and turbulent boundary-layer thickness have been calculated for two values of R_v and are plotted in Fig. 7. The combined δ on the basis of the Prandtl transition hypothesis is indicated for $R_{\delta c} = 4500$ for both values of R_c.

Finally the value of δ_l at the forward stagnation point has been calculated from Equation [18]. From the shape of the nose it was determined that $C_{r0}/L = 0.015$. Hence [18] gives:

$$\frac{\delta_s}{L} = \frac{0.165}{R^{1/2}} = \frac{0.074}{R_v^{1/2}} \ldots\ldots\ldots\ldots\ldots [24]$$

7—COMPARISON WITH EXPERIMENT AND DISCUSSION

A portion of Fig. 6 has been replotted to a linear scale in Fig. 8. The range of Reynolds' numbers included is that which is normally attained in wind-tunnel tests. Six experimentally determined curves have also been plotted. Four of these were obtained at the Bureau of Standards (3), one in the N.A.C.A. variable density tunnel (17), and one in the 10-ft. wind tunnel of the Guggenheim Aeronautics Laboratory at the California Institute of Technology (18). The two limiting curves for pure laminar and pure turbulent flow given by Dryden and Kuethe are also included in the figure.

It is seen that all of the experimental curves lie between the two limiting curves deduced from the present theory. This fact remains true for all of the experimental results on the N.P.L. long model which the author has been able to find. The general shape of the experimental curves is also very similar to the theoretical ones. In particular all the experimental curves approach the "turbulent" limiting curve at large Reynolds' numbers in just the manner prescribed by the theory. For low Reynolds' numbers the experimental results which have been carried far enough to make an estimate possible seem also to be approaching the theoretical limiting values more or less asymptotically. If the critical Reynolds' number for a sphere be taken as a measure of the turbulence in a wind-tunnel stream as suggested by Dryden and Kuethe (3) then the values of $R_{\delta c}$ which must be chosen to give the best theoretical agreement with the various experimental curves of Fig. 8 are qualitatively at least in excellent harmony with the measured turbulence in the various tunnels. Thus the variable density tunnel is known from sphere tests to have had an extremely high degree of turbulence (at least in the condition in which it was when the plotted results were obtained). Hence the very low value of $R_{\delta c}$ of about 1200 which is indicated by Fig. 8 is not surprising. The fact that this value is actually below the lowest experimental values obtained in flat plate or tube tests, and is also slightly lower than Tollmien's lower limit for flat-plate flow, may be due to a real difference between the minimum values of $R_{\delta c}$ possible in essentially rectilinear and in curved flow, or it may occur because of the approximate nature of the velocity profiles assumed in the present theory. Similarly the C.I.T. tunnel has the lowest turbulence, as indicated by sphere tests, of any tunnel which has as yet published data on the subject, and the $R_{\delta c}$ of about 4800 indicated by Fig. 8 is the highest required for any set of results except those obtained in the 3 ft. tunnel at the Bureau of Standards (not reproduced in Fig. 8). The experimental data in the latter case are, however, of quite doubtful accuracy, largely because of the uncertainty as to the wind-tunnel interference effects for this set of experiments.

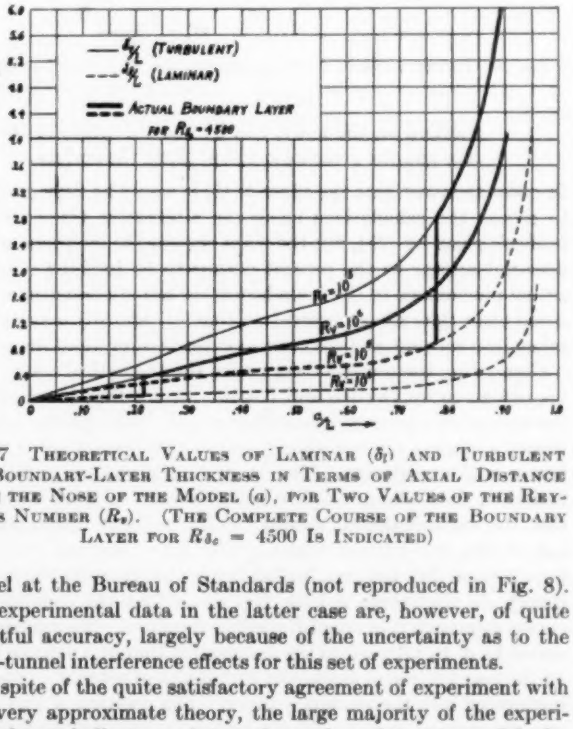

FIG. 7 THEORETICAL VALUES OF LAMINAR (δ_l) AND TURBULENT (δ_t) BOUNDARY-LAYER THICKNESS IN TERMS OF AXIAL DISTANCE FROM THE NOSE OF THE MODEL (a), FOR TWO VALUES OF THE REYNOLDS NUMBER (R_v). (THE COMPLETE COURSE OF THE BOUNDARY LAYER FOR $R_{\delta c}$ = 4500 IS INDICATED)

In spite of the quite satisfactory agreement of experiment with this very approximate theory, the large majority of the experimental tests indicate one interesting and consistent type of deviation from the theory. This deviation may be expressed by stating that the values of $R_{\delta c}$ at low R_v's seem to be higher than those at large R_v's for the same experimental curve. The explanation for this discrepancy may lie in any of the following points:

a The approximate nature of the velocity profiles assumed in the theory

b The over-simplified nature of the Prandtl transition hypothesis employed in making the calculations

c A possible variation in wind-tunnel turbulence with speed with an associated variation in $R_{\delta c}$

d The possibility that $R_{\delta c}$ is not only determined by the turbulence conditions in the incident air, but also by the

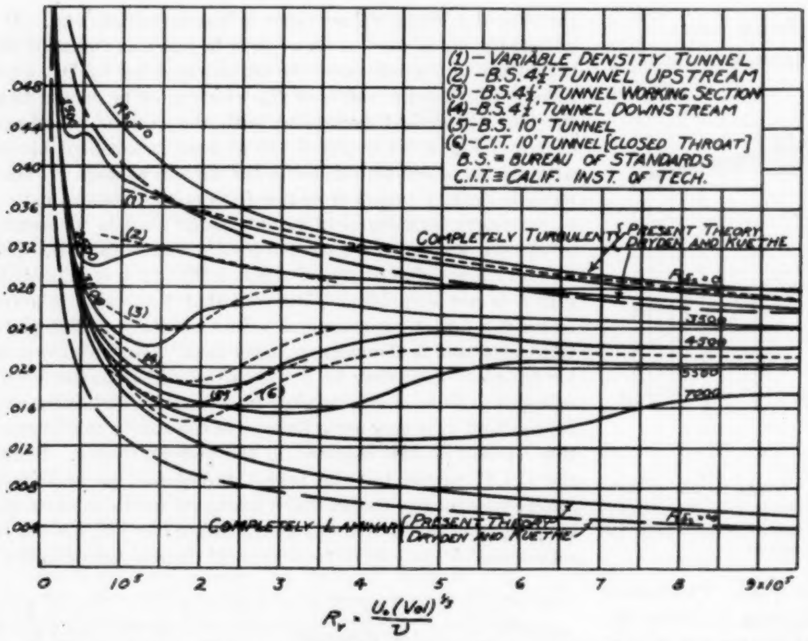

FIG. 8 EXPERIMENTAL CURVES FOR C_v VS. R_v, AND THEORETICAL CURVES FOR VARIOUS VALUES OF R_{δ_c}

position on the body, i.e., by the curvature of the surface and the pressure gradient along the surface.

In order to investigate the effect of (*b*) it would be very desirable to repeat the calculations using the alternative transition hypothesis given by [23], but the work becomes so laborious that it does not appear to the present author to be justified until the actual phenomena in the boundary layer are much better understood than they are at present so that (*a*) and (*d*) may also be taken into account and the corresponding assumptions improved.

Comparing the present theory with the theoretical results previously given by Dryden and Kuethe, we see that the shapes of the transition curves are somewhat similar although the present theory's minima are less abrupt and in this respect agree rather better with experiment. The D. & K. limiting curve for pure turbulent flow falls noticeably beneath ours, so much so indeed that the variable-density-tunnel results lie above it and entirely out of the skin-friction region. The lower limiting curve for pure laminar flow lies even further below that given by the present theory, so that there is little indication that the experimental curves approach it at small Reynolds' numbers. To remedy this defect D. & K. introduced the assumption that at a definite point on the rear portion of the model turbulence of a definite type was introduced into the boundary layer. The position of this point was determined so as to get the best agreement between theory and experiment. The effect of the assumption is that if the transition point occurs near the rear of the model, i.e., at low values of R_v, then $R_{\delta c}$ is smaller than if the transition occurs near the front of the model, as at high R_v's. Hence the theoretical curves are raised for small R_v's and left unchanged at large R_v's. On the basis of the present theory, therefore, this assumption is just the opposite of what is required since it would exaggerate the discrepancies between theory and experiment. It would be very interesting to examine the effect of position along the surface on $R_{\delta c}$, as could be done by testing models of different lengths and seeing whether the discrepancies between theory and experiment occurred at the same values of R in the two cases, or at the same positions of the transition point ac/L on the model. Experiments on the stability of a laminar boundary layer in curved or diverging flow might also throw light on this question.

It is not to be expected that the present theory will hold accurately at very large Reynolds' numbers since Karman has shown that even for essentially rectilinear flow the $^1/_7$th-power turbulent boundary-layer velocity profile assumed here is merely an approximation to a logarithmic law which at higher Reynolds' numbers may be approximated by $1/n$th-power profiles where n increases with R (19). Experiments have also indicated that at very large Reynolds' numbers the $^1/_7$th power becomes a $^1/_8$th power, then a $^1/_9$th power, etc. (20). Hence the limiting "turbulent" curve of Fig. 6 will not remain a straight line but will have an upward curvature at very large Reynolds' numbers, finally approaching a horizontal asymptote. The variable density results actually indicate this phenomenon, the experimental curve curving upward and crossing the "theoretical" straight line at about $R_v = 1.5 \times 10^6$.

The theoretical curves for δ/L given in Fig. 7 have been compared with experiments on other airship models (11, 12) and the general shape and order of magnitude of the curves verified. Near the tail the theory is not valid and gives $\delta \rightarrow \infty$, and at the transition point the discontinuity which we have assumed is, of course, not found in the experimental results. If an accurate knowledge of the boundary layer thickness is desired within the critical range of Reynolds' numbers then it seems that some more rational transition hypotheses than the Prandtl formulation which we have used must be adopted.

8—CONCLUSION

In this paper we have first derived the Prandtl boundary-layer equation for a boundary with axial symmetry and have then obtained a first integral of it giving the "integral relation" for the boundary layer about a figure of revolution. An approximate form of this, valid when the boundary-layer thickness is small compared to the distance from the axis, is obtained as a comparatively simple relation. As an example of the use of this equation, expressions for the laminar and turbulent boundary-layer thickness and skin friction of an airship shape are given, based on simple assumptions relative to the laminar and turbulent boundary-layer velocity profiles. The transition region between laminar and turbulent flow in the boundary layer is discussed and a transition hypothesis alternative to that of Prandtl is given. Finally numerical calculations based on the previous analysis are carried out for a particular airship model and the results compared with experiment and with those of the earlier theory of Dryden and Kuethe. The agreement with experiment is quite satisfactory and leads to the hope that the method will be capable of yielding really accurate results when better approximations to the true velocity distributions and to the transition region become available through basic research on the turbulence problem.

SUMMARY OF FINAL NOTATION EMPLOYED

x = distance along the surface measured from stagnation point
y = distance normal to the surface
a = axial distance from nose of the airship

L = length of airship
r_0 = radius of the airship surface
C_r = longitudinal radius of curvature of airship surface
δ = boundary-layer thickness
U = velocity just outside the boundary layer
U_0 = axial velocity of the flow far from the airship and relative to it
C_v = drag coefficient defined in terms of $(\mathrm{Vol})^{2/3}$
ν = coefficient of kinematic viscosity
$R = U_0L/\nu$
$R_v = U_0\,(\mathrm{Vol})^{1/3}/\nu$
$R_\delta = U\delta/\nu$
$()_l \backsim ()_{\text{laminar}}$
$()_t \backsim ()_{\text{turbulent}}$
$()_c \backsim ()_{\text{critical}}$ (transition point)
$()_s \backsim ()_{\text{stagnation}}$ (forward stagnation point)

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Discussion

O. G. Tietjens.[3] It gives the writer great pleasure, as a former student of Dr. L. Prandtl, to lead the discussion of this excellent paper. The two great and still unsolved problems in aerodynamics and hydrodynamics are the problem of air resistance or, generally speaking, of fluid resistance and the problem of turbulence. Or possibly there is only the one problem of turbulence. The solution of this problem, although now remote, would furnish us, probably, with the means of solving the problem of fluid resistance.

This paper is not only a remarkable step forward in the direction of solving the problem of airship resistance, which is of course of eminent practical importance, but it also shows that those conceptions and theories developed by Dr. Prandtl and his associates—the writer wishes to refer particularly to Dr. von Karman—begin to gain friends also in this country.

Attention may be drawn to a paper on a similar subject published about a year ago by T. Levi-Civita, entitled "General Conclusions from Prandtl's Theory of the Boundary Layer."[4] In this paper the differential equations are derived, not only for a body of axial symmetry, but for a body of any shape. The rate of approximation in Levi-Civita's paper is the same as that used by Dr. Millikan; terms in $\sqrt{\nu}$ having powers higher than the first are neglected. However, due to the much more general derivation of the equation, no solution could be given for any particular case.

The writer has a question referring to Equation [12] according to which the velocity in the boundary layer increases with the $^1/_7$ power of the distance from the boundary. Since tests made by J. Nikuradse show for large Reynolds' number proportionality with the $^1/_8$ or even $^1/_9$ power, the writer would like to ask the author how the different assumptions with respect to this exponent influence the final coefficient of drag $C_{v\text{-turbulent}}$. In connection with this, it should be considered whether the tests with $U^1 = \dfrac{dU}{dx} = 0$ (plate, tube) which give the exponent $^1/_7$, etc., will give the same exponent if $U^1 \neq 0$ as along a rounded body.

The writer would like to make some remarks with respect to the transition from laminar to turbulent flow in the boundary layer. First of all, it is not certain that the measurements of the velocity distribution in the boundary layer show sufficiently clearly whether the transition is instantaneous or whether the distance between the last well-defined laminar profile and the first fully developed turbulent profile is large or small. The gage reading will fluctuate too much at this region to allow an exact measurement of the velocity distribution.

Due to the fact that the critical Reynolds' number $U\delta_c/\nu$ depends largely on the turbulence of the free air stream, which may vary slightly from time to time, the breakdown of the laminar flow in the boundary layer will occur irregularly at larger or smaller δ_c corresponding to a more or less turbulent condition of the free air stream. The writer is of the opinion that the transition does not occur instantaneously, but nevertheless in a comparably short time.

About ten years ago the stability of a laminar boundary layer[5] was investigated. A certain disturbance $e^{i(kx-nt)} \cdot f(y)$ was superposed on the laminar layer. Writing $n = r + is$, we have

$$e^{i(kx-rt)} \,.\, e^{st} \,.\, f(y).$$

If the calculation of the stability of such a disturbance gives a positive s, then there is an increase of the disturbance with the time according to e^{st}, and thus finally turbulence; if the calculation gives a negative s, then the disturbance decreases and the laminar flow is stable. At that time the writer was not able to derive from this calculation a critical Reynolds' number. Later it was shown by W. Tollmien[6] that this was due to the linear velocity distribution assumed in the boundary layer. However, the writer did prove that, if the laminar flow is unstable, the rate of increase of the disturbance (i.e., the exponent s) is proportional to $\sqrt{\frac{\nu U}{\delta^3}}$.

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[3] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

[4] T. Levi-Civita, "Allgemeine Folgerungen aus der Prandtlschen Grenzschichttheorie," *Vorträge aus dem Geb. der Aerod. und verw. Geb., Aachen* (1929), Springer (1930).

[5] Tietjens, O. G., "Beitraege zur Entstehung der Turbulenz," *Zeit. f. ang. Math. u. Mech.*, 5 (1925), s. 200.

If we introduce into the above expression the thickness of the laminar boundary layer, $\delta = \frac{5.5a}{\sqrt{\frac{U \cdot a}{\nu}}}$, we get s proportional to $\frac{U}{a}\sqrt[4]{\frac{U \cdot a}{\nu}}$. Thus it is seen that for large Reynolds' numbers, large velocities, and small values of a, the exponent s becomes large. Although the transition from laminar to turbulent flow will probably be somewhat different in a boundary layer from that in a pipe, it may be of interest to see in Fig. 9 in what short a time the laminar flow changes into the turbulent flow. The Reynolds' number $U\cdot$ diameter/ν is about **16,000**. The number of pictures per second is 400.

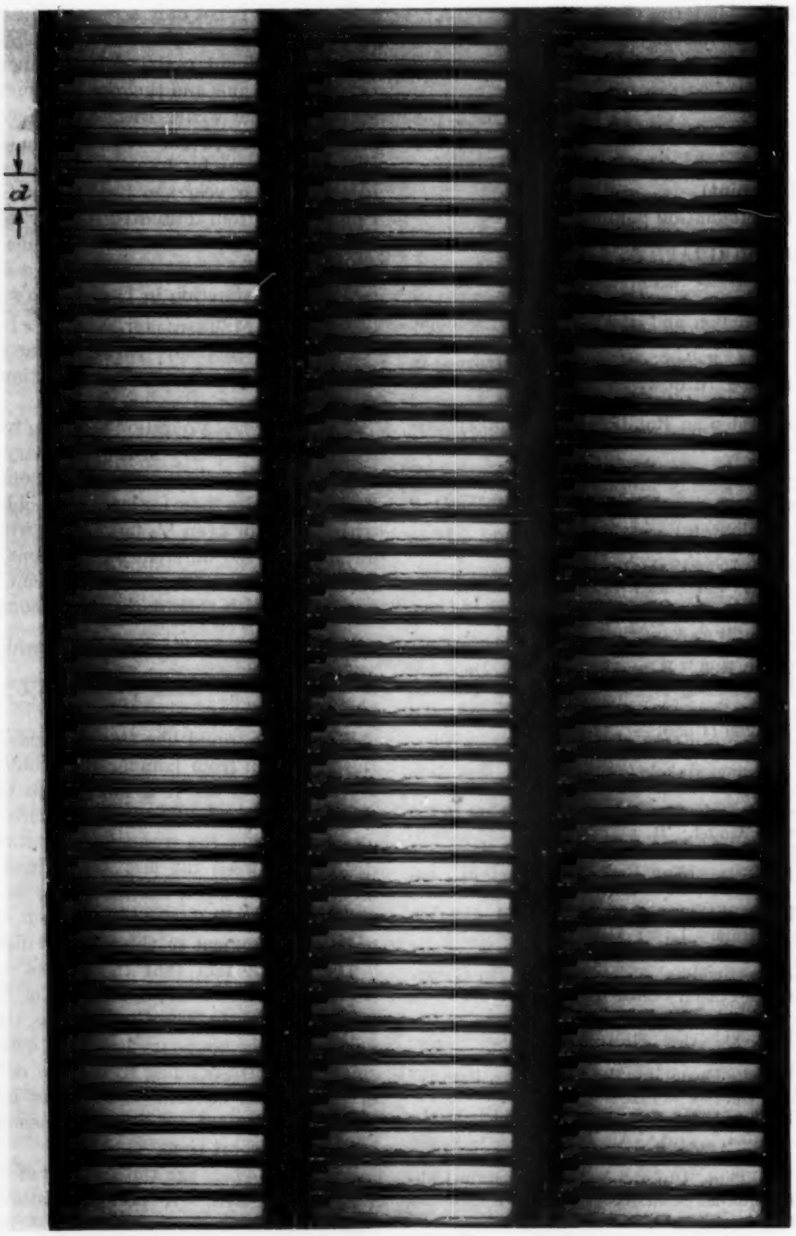

Fig. 9 Transition of Laminar to Turbulent Flow in a Pipe

(High-speed motion pictures taken at 400 per second. Direction of flow is from left to right. $Ud/\nu = 16{,}000$.)

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[6] Tollmien, W., "Ueber die Entstehung der Turbulenz, Part 1," *Nachr. von der Ges. der Wiss. zu Göttingen* (1929).

With regard to the critical Reynolds' number, it does not seem quite correct to refer to the value 1160 obtained by L. Schiller. In this case there is no question of a transition from a laminar to a turbulent flow, since the flow enters the pipe in an extremely disturbed condition and there is no δ_{laminar} or δ_c to which we can refer.

Regarding the minimum $R_{\delta_c} = 1320$ calculated by W. Tollmien, this value should be compared with the highest critical Reynolds' number that can be obtained by experiments, since the calculation of Tollmien is based on infinitely small disturbances. The critical Reynolds' number has been called by Dr. Tollmien the minimum critical Reynolds' number only with respect to the wave length of the disturbance assumed and not with respect to the amplitude of the disturbance; i.e., the calculation gives a smallest critical Reynolds' number for a certain wave length of the disturbance, and for any other wave length a larger critical Reynolds' number. However, as a disturbance of any wave length may occur, the smallest value of the Reynolds' number obtained for any frequency has to be considered as the critical Reynolds' number.

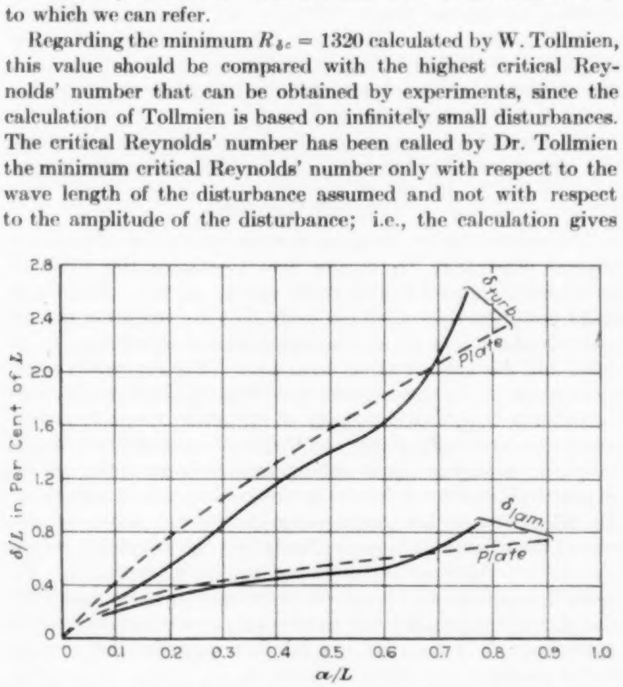

FIG. 10 COMPARISON OF THICKNESS OF BOUNDARY LAYER OF AN AIRSHIP MODEL AND A PLATE

(The laminar and turbulent boundary layers are shown by a solid line; the plate is shown by a broken line. $U.L/\nu = 500{,}000$.)

In order to compare the thickness of the laminar and turbulent boundary layers of an airship hull with those of a plate, Fig. 10 is given. It is interesting to note that both lines are higher for the plate than for the airship hull, which may explain to a certain extent the fact that the curves of the drag coefficient as a function of the Reynolds' number calculated by Dryden and Keuthe are always lower than those calculated by Dr. Millikan. The writer would like to ask Dr. Millikan up to what value of a/L he believes the theory is valid.

W. KLEMPERER.[7] The paper is of signal importance to all aerodynamicists interested in the determination of the drag of round streamlined bodies such as airships and fuselages for airplanes, because it is a successful scientific step toward lifting the puzzling veil from the maze of seemingly conflicting experimental evidence. It has long been a matter of grave concern to the wind-tunnel experimenters that the better a wind-resisting body is streamlined and refined the more vague the theory of drag becomes and the more problematic the direct applicability of small-scale test results grows. Basic theories such as have done so much to clear up the induced-drag problems of wings are still utterly meager in this field of low-parasite-drag problems, because too little is known of how the shape of the solid contour influences the mechanism of air friction. The author has shown how reasonable assumptions about this mechanism, borrowed from other simpler shapes, give a conclusive explanation of the fallacy of airship-model tests made at too low a Reynolds' number and why the great efforts that have been spent in various countries cooperating in elaborate "comparison" tests are very interesting, but failed to solve the mysterious question of which tunnel gives the results truest to full size. The resistance of models whose boundary layer is laminar over a large portion of the hull cannot possibly bear a simple relation to the full-size ship whose boundary layer is undoubtedly turbulent over the major part of the ship. On the other hand, experiments conducted at very high Reynolds' number, and especially on very large models at very high velocity in a wind possessing a fine turbulence texture, may quite well possess drag coefficients which can be correlated to those of full size by proper allowance for the shrinking of the boundary layer with increasing Reynolds' number. As a matter of fact, quite a number of model tests have been made by our company, as well as by others, on models of airships extending into Reynolds' number as high as 3 and even 6 times those of the far end of the author's Fig. 8. The value of these tests appears greatly enhanced in the light of this contribution.

Evidently the most important task for future experimentation is not so much the mere accumulation of further drag measurements on models of arbitrary sizes and surface condition, but it must be accompanied by explorations of the mechanism of surface friction on such bodies, a survey of the phenomena inside the boundary layer. We have made experiments both on large models and aboardship throwing some light on the actual character of the boundary layer. We have evidence that the 7th-root profile is a good approximation in the amidship region, whereas the fractional exponent grows decidedly larger toward the bow and smaller toward the tail. Happily this seems to have only a relatively small influence upon the net results and conclusions drawn by the author. However, it emphasizes the desirability of developing a theory for the variation of the velocity profile normal to the hull as a function of the angle of the meridional tangent and perhaps the local radii of curvature. It may be expected that some correlation can be established with the conditions in conical or wedge-shaped effusors and nozzles. With theory and experiment then forging ahead hand in hand, there is hope that drag-test results of streamline bodies will eventually be amenable to scientific interpretation beyond the vague scope of such obsolete concepts as the "surface friction coefficients" of the hydraulicists.

HUGH L. DRYDEN.[8] The author is to be congratulated for his success in removing one of the assumptions heretofore found necessary for the approximate computation of boundary-layer flow around bodies of revolution. With the many assumptions still remaining, and with the existence of a free constant whose value may be selected to give the best agreement, the theory cannot yet be said to be in a satisfactory position. The major features are undoubtedly correct, and there is great promise, as the author states, that ultimately "the method will be capable of yielding really accurate results."

The remarks on the tenth page, first paragraph, indicate a misunderstanding of a part of the paper by Kuethe and the writer.[9] The assumption that turbulence sets in shortly behind the major cross-section was found necessary, not primarily to explain the shape of the curves at small Reynolds' numbers, but to explain the fact that the effect of turbulence on one model was observed experimentally to be small, although the theory predicts an effect as large as for the other models. In addition, experiments with wire loops on the models confirmed the existence of turbulence behind the maximum cross-section. For further discussion, see page 19 of the paper by Kuethe and the writer.[9]

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[7] Goodyear-Zeppelin Corporation, Akron, Ohio.

[8] Bureau of Standards, Washington, D. C.

[9] See reference (3) in bibliography at end of paper.

Albert F. Zahm.[10] This interesting paper purports to investigate "the boundary-layer phenomena for a body with axial symmetry about an axis in the line of flow." The flow meant is, it is judged, that of an undriven, slightly viscid, incompressible, infinite fluid, initially having uniform speed and pressure at all points far from the immersed body assumed to be fixed. The fluid sticks motionless to the body's surface, has a material shear speed $\partial u/\partial y$ very near it, but a negligible one at a short distance δ, where the actual velocity hardly differs from that of a perfect fluid in steady potential flow. This shear region is the thin boundary layer studied in the last century by Froude, Calvert, Hele-Shaw, Marey, etc., and in the present century by numerous researchers.

Of the "boundary-layer phenomena" to be investigated, the prime one is the velocity $\boldsymbol{q}$ within the layer, whence are derived the rate of shear and the friction. To find $\boldsymbol{q}$ one should solve the Navier-Stokes equation

$$\frac{\partial \boldsymbol{q}}{\partial t} + \boldsymbol{q} \cdot \nabla \boldsymbol{q} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \boldsymbol{q}$$

whose first term is the time rate of change of the flow velocity at any point. Omitting this term leaves the form [1], which is Prandtl's 1904 boundary-layer equation. It is easier to solve, though not rigorously correct. The error commonly is assumed negligible.

One might also retain the first term and drop the next two, obtaining

$$\frac{\partial \boldsymbol{q}}{\partial t} = \nu \nabla^2 \boldsymbol{q}$$

as the writer did in 1903, and Dr. Carlo del Lungo in 1905, to express the flow past a skin-friction plane. Solving this equation gives, at normal distance y, the laminar speed

$$u_2 = \frac{2U_0}{\sqrt{\pi}} \int_0^s e^{-s^2}\, ds$$

where $s = \frac{1}{2} \sqrt{\frac{U_0}{\nu}} \frac{y}{\sqrt{x}}$ and $x = U_0 t$. Compare this with Blasius' solution of Prandtl's equation,

$$u_1 = \frac{U_0}{2} (as - bs^4 + cs^7 \ldots\ldots\ldots)$$

where $a = 1.327$, $b = 0.073372041$, $c = 0.0051000561$, etc. The graphs of u_1 and u_2, here shown, comprise between them a good part of the measured values given in [13] and [14] for $U_0 > 50$ ft-sec., thus indicating that neither formula is a perfect expression of the velocity distribution.

A better graph seems to be the intermediate one whose ordinate is

$$u = u_1 + n\,(u_2 - u_1)$$

where $o < n < 1$. The shown careful measurements of Zijnen, uncorrected for tunnel-wall effect, make n about 0.5 for $U_0 =$ 40 met.-sec., while those of Hansen make $o < n < 0.5$ for $U_0 =$ 16 met.-sec. Perhaps, therefore, $\partial \boldsymbol{q}/\partial t$ is not wholly negligible for such plane flow or for the axial flow treated by the author.

The author's premise that "the theoretical potential flow is very closely approximated by the real flow," all over a well-streamlined model, entails equality of the theoretical and actual pressures all over the model, and hence zero pressure drag in a uniform stream. Accurate measurement shows equality of these pressures at the stem, but not at the stern, of models having excellent streamline forms. Witness, for example, Fuhrman's studies of five airship-hull models; also reports 291 and 335 of the National Advisory Committee for Aeronautics. The pressure drag there given is in no case zero, though it may be as low as one-fourth the friction drag.

These comments are merely casual suggestions for any minute and critical scrutiny of the author's carefully constructed theory.

Author's Closure

With regard to Dr. Tietjens' question as to the effect of exponents other than $^1/_6$ of the final $C_{v\text{-turbulent}}$ the author is unfortunately unable to give a very definite answer. The matter could really be settled satisfactorily only by repeating the numerical calculations for other exponents and comparing the results, and this has not as yet been done. The reasonable agreement between the theory and experiment would, however, seem to indicate that, for the range of Reynolds' numbers investigated experimentally, either the $^1/_7$-power assumption is practically satisfactory or the deviations from this assumption are not of practical importance for the determination of drag coefficients. Dr. Klemperer, in his comments on the present paper, has discussed the effect of a "rounded body" on the exponent, as have the British experiments mentioned in the bibliography. The general conclusion which one can draw appears to be that, while the $^1/_7$th-power profile is certainly not rigorously correct over the entire body, it is nevertheless a fair first approximation over the major portion.

The values of a/L beyond which the theory is definitely not valid could only be determined by careful boundary-layer measurements on the same dirigible shape for which the calculations were made. Furthermore, the allowable limits on a/L will almost certainly depend on the shape of the body and on the Reynolds' number R_v. A comparison of such experimental data as are available (referring, however, to dirigible models somewhat different in shape from that used in the present paper) with the theoretical curves of Fig. 7 lead the author to believe that at least for R_v's of the order of 10^6 or larger the behavior of the equations near the singular point near the tail should not cause practically important deviations for values of a/L less than about 0.90. Discrepancies in this region ($a/L > 0.9$) should have a negligible effect on drag.

The "misunderstanding" to which Dr. Dryden refers was in reality rather a "misstatement" for which the author tenders his apologies to Messrs. Dryden and Kuethe. The sentence in question should more properly have read: "The assumption introduced by D. & K.—namely, that at a definite point on the rear portion of the model turbulence of a definite type was introduced into the boundary layer—tends to remedy this defect."

With regard to Dr. Zahm's comments, the author is unable to understand why the omission of the term ($\partial \boldsymbol{q}/\partial t$ in the Navier-Stokes equations should lead even to a "negligible" error in the present case. The derivation of Equations [4] and [1] is of course only valid for a laminar flow, but with this restriction it is not clear why the assumption of a stationary flow such that $\partial \boldsymbol{q}/\partial t =$ 0 is not rigorously correct for the flow under consideration.

The assumption of zero pressure drag, to which Dr. Zahm has called attention, has been the subject of many experiments which have led to rather contradictory conclusions. Fuhrman's experiments are quite out of line with practically all of the later tests in giving very high pressure drags (of the order of 50 to 60 per cent of the total drag). The investigations reported in

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[10] Chief, Division of Aeronautics, Library of Congress, Washington, D. C.

N.A.C.A. Report 335, which gave pressure drag about 45 per cent of total drag, were on two-dimensional flow past strut forms and do not appear to have any immediate bearing on the present problem, which is concerned entirely with figures of revolution. The results printed in N.A.C.A. Report 291 and in certain of the early British reports give pressure drags of the order of 20 to 30 per cent of total drag. On the other hand, a rather extensive series of British investigations (cf. Reports and Memoranda 600, 801, 1169) in which careful allowance was made for wind-tunnel interference, static pressure drop along the tunnel, etc., indicated pressure drags so low as either to be not measurable or at most to be of the order of 6 or 7 per cent of total drag. The following excerpt is quoted from R. & M. 801: "When plotted . . . for longitudinal force, the same result was derived as was given in R. & M. 600; viz., at zero yaw, the form (pressure) resistance vanishes. . . . It would thus appear not only that the resistance of an airship envelope, neglecting interference of cars and fins, is entirely due to skin friction, but also that the longitudinal force at higher angles of yaw is also due to the same cause." It seemed to the present author that these conclusions furnished some justification for the assumption of zero pressure drag which underlies the present approximate theory, as well as those of B. M. Jones and Dryden and Kuethe.

The author wishes finally to comment briefly upon one point in the paper which, it has occurred to him, might cause some confusion. In section 3 the actual flow is approximated by assuming that outside of the boundary layer the flow is rigorously that given by the potential flow of an inviscid fluid about the body, and further that the boundary layer is so thin that the flow outside is identical with the potential flow which would exist at the surface. From this picture it follows that at $y = \delta$, $v = 0$; i.e., the normal component of velocity vanishes at the outer surface of the boundary layer. It appears to the author that this is an entirely consistent and permissible picture to use as an approximation to the true boundary layer flow. However, it differs somewhat from the picture given by Prandtl, which does not give $v = 0$ at $y = \delta$. It is easy to demonstrate, however, that Equation [6] is obtained unaltered whichever of the pictures is used. It will suffice here to outline briefly the course of the analysis when $v(\delta)$ is not assumed to vanish. Consider the left side of Equation [5] and write:

$$I = \int_0^\delta r\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)dy$$

Then by partial integration

$$I = \int_0^\delta ru\frac{\partial u}{\partial x}dy + rvu\Big|_0^\delta - \int_0^\delta u\frac{\partial(rv)}{\partial y}\,dy,$$

and applying the continuity equation to the final term,

$$I = \int_0^\delta \left(ru\frac{\partial u}{\partial x} - u\frac{\partial(ru)}{\partial x}\right)dy + Ur(\delta)v(\delta)$$

$$= \int_0^\delta \frac{\partial(ru^2)}{\partial x}\,dy + Ur(\delta)v(\delta)$$

Using the continuity equation again,

$$Ur(\delta)r(\delta) = U\int^\delta \frac{\partial(rv)}{\partial y}\,dy = -U\int_0^\delta \frac{\partial(ru)}{\partial x}\,dy$$

Hence

$$I = \int^\delta \frac{\partial(ru^2)}{\partial x}\,dy - U\int^\delta \frac{\partial(ru)}{\partial x}\,dy,$$

and interchanging the differential and integral processes, using Leibniz's rule, the terms in $d\delta/dx$ cancel, and we obtain the left side of [6] just as before.

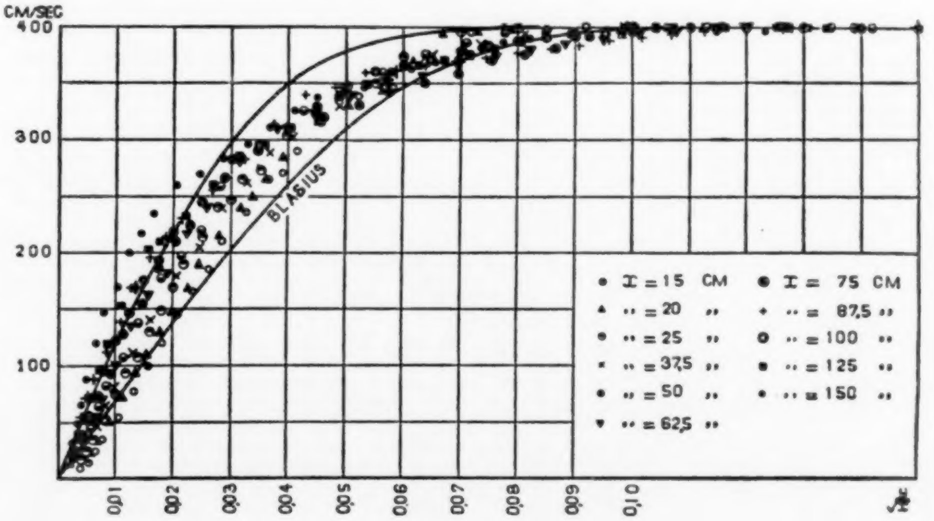

FIG. 11

(Air speed near a friction plane in a wind of 40 meters a second. Upper and lower graphs respectively delineate solutions of $\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2}$ and $u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$; dots denote Zijnen's measurements—see Bibliography reference 13.)

(The illustration accompanies Dr. Zahm's discussion on page 42.)

APM-54-4

On the Subcritical Speeds of the Rotating Shaft

By C. RICHARD SODERBERG,[1] EAST PITTSBURGH, PA.

Disturbances sometimes encountered at speeds above the fundamental whirling speed, having the fundamental frequency, are usually, and correctly, attributed to internal friction. Other disturbances frequently met with at speeds below the fundamental whirling speed, and when the motions also have the fundamental natural frequency, have been thought to be due to minute variations of the angular velocity of rotation caused by the eccentric application of gravity. The present paper discusses the latter group of phenomena, and aims to show that the cause assigned is incorrect.

THE phenomenon of shaft whirling has been extensively treated in the engineering literature of the last two or three decades, and to many who are familiar with the subject it may seem difficult to justify further discussion. There are, however, several phenomena which sometimes may appear mysterious. An important group of these are the disturbances which are sometimes encountered at speeds *above* the fundamental whirling speed, having the fundamental frequency. These are usually, and correctly, attributed to internal friction, but the exact mechanism of the motion is by no means clearly understood. Another group are the disturbances frequently encountered at speeds *below* the fundamental whirling speed, and where the motions also have the fundamental natural frequency. These were observed comparatively early by several investigators, and their cause was assigned to minute variations of the angular velocity of rotation caused by the eccentric application of gravity.[2] The fact that the vertical shaft did not exhibit this phenomenon was taken as a proof of this theory, and at the present time it is often believed to be correct. The present paper is a discussion of the latter group of phenomena, and it is intended to show that this nearly classical explanation is probably incorrect.

The purely academical aspects of the problem are hardly important enough to warrant lengthy discussions, but the problem actually does have an important practical aspect. Certain types of rotating machinery, notably certain groups of d.c. machines, are compelled to operate at or near one-half the fundamental whirling speeds. Many of these machines are sensitive to vibrations in the rotor, and the author has encountered a considerable number of cases where the elimination of these vibrations has presented an important problem.

NOMENCLATURE

Numerical calculations are carried out in the pound-inch-second system, where $g = 386$.

Symbol	Unit	Significance
k	lb. per in.	rigidity
m	lb./g	mass
n, p	......	number, order
r, s	in.	radii
x, y	in.	coordinates
y_0	in.	shaft deflection
ϵ, μ	radian	small angles
ξ, η	in.	components of s
θ, ϕ, ψ	radian	angles
χ	......	small number
ρ	in.	radius of gyration
ω	radians per sec.	angular velocity
ω_c	radians per sec.	critical angular velocity

The notations $\dot{r}$ and $\ddot{r}$ are used for dr/dt and d^2r/dt^2, etc.

CASES OF RESONANCE

The investigation concerns the simple system shown in Fig. 1. This consists of a massless, horizontal shaft in two bearings, on which is mounted a disk of mass m and radius of gyration ρ. If the rigidity (lb. per in.) of the shaft is denoted by k, the "critical speed" of the arrangement is given by the angular velocity

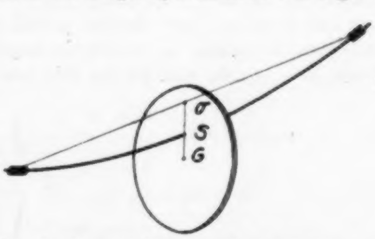

FIG. 1 SIMPLE SYSTEM CONSISTING OF A MASSLESS HORIZONTAL SHAFT RUNNING IN TWO BEARINGS AND CARRYING A SINGLE DISK

$$\omega_c = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{y_0}} \qquad [1]$$

where y_0 is the static deflection under gravity.

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[1] Development Division Engineer, Power Engineering Department, Westinghouse Electric & Manufacturing Co. Mem. A.S.M.E. Mr. Soderberg was born in Sweden, and was graduated from Chalmers' Institute of Technology as a naval architect in 1919. Following this he received a scholarship from the American Scandinavian Foundation for Study of Shipbuilding in America, and took postgraduate work at M.I.T. and University of Michigan during 1919–1920.

He worked in the shipbuilding industry until 1922, when he became connected with the Westinghouse Electric & Manufacturing Co. as a mechanical engineer in its railway-motor engineering department, specializing on industrial vibration problems. He was transferred to the power engineering department in 1924, where he studied the general mechanical problems of large, high-speed rotating machines. He formed a connection with the Swedish General Electric Co. in Sweden during 1928–1930 as designer of turbine generators. At present he is engaged on development problems of large power machinery.

[2] Stodola, "Steam and Gas Turbines," Translation from sixth German edition by Loewenstein, vol. 2, p. 1122.

Gümbel, *Dinglers polytechnische Journal*, 1917, p. 235, and 1918, p. 71.

Föppl, *Zeitschrift f. d. gesamte Turbinenwesen*, nos. 18 and 19, 1918.

Lorentz, *Zeitschrift des Vereines deutscher Ingenieure*, 1919, pp. 240 and 888.

Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Disturbances are thus observed when the arrangement is rotated at the angular velocity ω_c, provided that there is a certain eccentricity of the disk. This type of resonance will be referred to as "fundamental resonance."

Experience shows, however, that measurable disturbances may occur at other speeds as well, particularly at $\omega_c/2$. The intensity of the disturbance depends on some "error" in the mounting of the disk. If the error is eccentricity, the phenomenon will be referred to as "subresonance due to eccentricity." If the error consists of variations in the deflection of the shaft, the phenomenon will be referred to as "subresonance due to variable elasticity." Other types of error, such as inclination of the disk, may also produce resonance phenomena, but these will not be considered in the present connection.

The object of the investigation is to compare the relative intensity of these three cases of resonance. Friction forces are often of importance in cases of resonance, but the inherent tendencies for motion of the system always depend on other properties; friction (unless it is excessive) merely restricts the motion to certain limits. Therefore the friction forces will be neglected in the following.

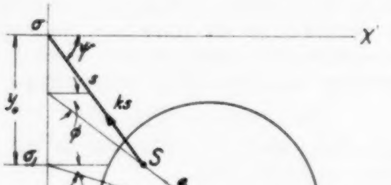

FIG. 2 ECCENTRICALLY MOUNTED DISK

A—FUNDAMENTAL RESONANCE ($\omega = \omega_c$)

The eccentrically mounted disk is investigated in Appendix No. 1. Fig. 2 shows a typical position of the disk. σ is the initial, unstrained position of the shaft center, σ_1 is the position in static equilibrium and S is the position at the moment under consideration. G is the center of gravity of the disk. The position of the disk is given by the coordinates r, θ, and ϕ, the first two locating G and the last giving the direction of $GS = e$, the eccentric radius. $\dot{\phi} = \omega$ is the angular velocity of rotation. When $\omega = \omega_c$, the solution for this case is found to be

$$\left.\begin{aligned}\phi &= \omega t\\ \theta &= \omega t - \frac{\pi}{2}\\ r &= \frac{e}{2}\,\omega t\end{aligned}\right\}\qquad [2]$$

Fig. 3 shows a typical position of the disk during this motion. It is evident that the center of gravity G will describe a spiral, the motion away from the center being linear with time.

The increment in the radius r after n revolutions ($\omega t = 2\pi n$) is

$$\Delta r = \pi n e \qquad [3]$$

B—SUBRESONANCE DUE TO ECCENTRICITY ($\omega = \omega_c/2$)

The solution for this case is also developed in Appendix No. 1. Introducing the small angle

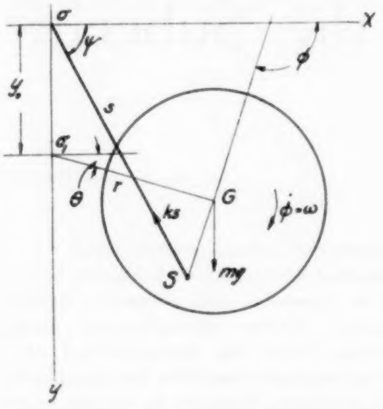

FIG. 3 TYPICAL POSITION OF DISK DURING ROTATION

$$\epsilon = \frac{ge}{\rho^2\omega^2} = 4\,\frac{y_0e}{\rho^2} \qquad [4]$$

the solution is found to be

$$\left.\begin{aligned}\phi &= \omega t - \epsilon\cos\omega t\\ \theta &= \omega t - {}^3/_8\,\epsilon\omega t\sin\omega t\\ r &= {}^4/_3\,e[1 - {}^3/_8\,\epsilon(2\sin\omega t + \omega t\cos\omega t)]\end{aligned}\right\}\qquad [5]$$

This motion is considerably more complicated than that at the fundamental resonance. The variation in angular velocity is small ($\epsilon\omega$), but the angle θ is subjected to periodic variations of increasing amplitude. The increment in the radius r is oscillating in nature, being positive when $\omega t/\pi$ is odd and negative when $\omega t/\pi$ is even. Putting $\omega t = 2\pi n$, the increment in the radius r, accumulated during n revolutions, is

$$\Delta r = \pm\epsilon\pi n e \qquad [6]$$

the plus sign corresponding to $n = 0, 1, 2, 3, \ldots$, and the negative sign to $n = {}^1/_2, {}^3/_2, {}^5/_2, \ldots$.

A similar condition of subresonance may be excited by subjecting the disk to a variable moment. If the moment may be written $M\sin p\omega t$, resonance will occur at the angular velocities $\omega = \omega_c/(p-1)$ and $\omega_c/(p+1)$. The nature of the motion is similar to that demonstrated above; in fact, the above solution is valid for the case that $p = 1$ and $\epsilon = M/(m\rho^2p^2\omega^2)$.

C—SUBRESONANCE DUE TO VARIABLE ELASTICITY ($\omega = \omega_c/2$)

The motion of a disk mounted on a shaft of variable elasticity is investigated in Appendix No. 2. The minimum rigidity of the shaft is denoted by k, the maximum rigidity by $(1+\chi)k$, where χ is a very small dimensionless quantity. The critical speed is still defined as $\omega_c = \sqrt{k/m}$; χ is assumed to be so small that the extension of the resonance range may be neglected. Resonance occurs when $\omega = \omega_c/2$; it is due to the raising and lowering of the center of gravity, which occurs twice every revolution.

Fig. 4 shows a typical position of the disk. The radius r now refers to the shaft center and is measured from the mean static position σ_m. The angle ϕ denotes the direction of the axis of maximum rigidity. When $\omega = \omega_c/2$ the solution is found to be

$$\left.\begin{aligned}\phi &= \omega t\\ \theta &= 2\omega t\\ r &= {}^1/_2\,\chi y_0\omega t\end{aligned}\right\}\qquad [7]$$

where $y_0 = mg/(1+\chi/2)k$ is the mean static deflection. In this case the motion of the shaft center is a spiral, the radius vector rotating with double angular velocity. Putting $\omega t = 2\pi n$, the increment in the radius for n revolutions is given by

$$\Delta r = \chi \pi n y_0 \ldots\ldots [8]$$

COMPARISON OF THE SOLUTIONS

The intensity of these three cases of resonance may now be compared by comparing the number of revolutions required to produce a certain increment of unity in the radius r. Denoting these numbers of revolutions by n_A, n_B, and n_C, respectively, we obtain from Equations [3], [6], and [8]

$$n_A = \frac{1}{\pi e}; \quad n_B = \frac{1}{\epsilon \pi e}; \quad n_C = \frac{1}{\chi \pi y_0} \ldots\ldots [9]$$

showing that

$$n_A : n_B : n_C = 1 : \frac{1}{\epsilon} : \frac{e}{\chi y_0} \ldots\ldots [10]$$

These quantities should be considered as indicating the tendency toward radial motion. The premises upon which the solutions were based may no longer be fulfilled after a large number of revolutions, but the actual number of revolutions required to attain a certain motion will always be greater than that indicated by these results.

Selecting a rotating system of "practical" proportions, we may put

$\rho = 10$ in. $\quad y_0 = 0.030$ in. $\quad \omega_c = 113$ rad./s

$e = 0.001$ in. $\quad \epsilon = 1.2.10^{-6}$ $\quad \chi = 5.10^{-3}$

These figures give

$$n_A : n_B : n_C = 1 : 833{,}000 : 6.7$$

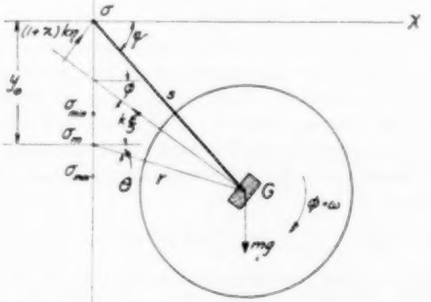

FIG. 4 TYPICAL POSITION OF ROTATING DISK MOUNTED ON A SHAFT OF VARIABLE ELASTICITY

showing that subresonance due to eccentricity is very improbable in comparison with the other two types. This conclusion would be valid even if the variation in angular velocity $\epsilon\omega$ were a thousand times greater than that assumed above. Subresonance due to variable elasticity, on the other hand, may be expected to occur in almost any horizontal arrangement. In the above set of figures it has been assumed that the shaft deflection varies 0.5 per cent, or about 0.00015 in. during a revolution. This amount would be difficult to demonstrate experimentally even by static measurements. Shafts having press fits or keys may frequently have considerably greater variation.

CONCLUSIONS

It has been shown in the foregoing that when subresonance at one-half the critical speed is encountered in rotating shafts, it is most probably due to minute variations in shaft deflection, and not to eccentricity. Therefore the most promising measure of correction of such cases, apart from the elimination of the resonance, is to eliminate the variation in the shaft deflection.

Appendix No. 1

SHAFT WITH ECCENTRIC DISK

THE GENERAL SOLUTION

CONSIDER a disk mounted symmetrically on a massless shaft, Fig. 1. The shaft is rotating in the horizontal position; Fig. 2 shows an instantaneous position. σ is the initial, unstrained position of the shaft center, σ_1 is the position in static equilibrium under the influence of gravity, and S is the actual position at the moment under consideration. G is the center of gravity of the disk. The quantities r, θ, and ϕ are clearly sufficient to specify the position of the disk; the solution required should give these quantities as functions of time and other known quantities. The radius vector r gives the distance of the center of gravity from the static center of rest σ_1, the angle θ gives the direction of this radius vector relative to the x-axis, and the angle ϕ gives the direction of the eccentric radius e relative to the x-axis. The total displacement of the shaft center is s, the resultant elastic force of the shaft is denoted by ks acting in the negative direction of s. This elastic force and the gravity are the only two external forces which will be considered.

Projecting all the forces parallel with and at right angles to the radius r, and taking moments around the point G, the following three differential equations are obtained.

$$m\ddot{r} - mr\dot{\theta}^2 + ks \cos(\psi - \theta) - mg \sin\theta = 0 \ldots\ldots [11]$$

$$mr\ddot{\theta} + 2m\dot{r}\ddot{\theta} + ks \sin(\psi - \theta) - mg \cos\theta = 0 \ldots\ldots [12]$$

$$m\rho^2\ddot{\phi} - kse \sin(\psi - \phi) = 0 \ldots\ldots [13]$$

ρ is the radius of gyration of the disk and m its mass. s and ψ may be eliminated from these equations by the relations

$$s \cos(\psi - \theta) = y_0 \sin\theta + r - e \cos(\phi - \theta) \ldots\ldots [14]$$

$$s \sin(\psi - \theta) = y_0 \cos\theta - e \sin(\phi - \theta) \ldots\ldots [15]$$

$$s \sin(\psi - \phi) = y_0 \cos\phi - r \sin(\phi - \theta) \ldots\ldots [16]$$

Introducing also the notations

$$y_0 = \frac{mg}{k} \ldots\ldots [17]$$

$$\omega_c = \sqrt{\frac{k}{m}} \ldots\ldots [18]$$

these differential equations become, after rearrangement,

$$\frac{d^2r}{dt^2} + r(\omega_c^2 - \dot{\theta}^2) = \omega_c^2 e \cos(\phi - \theta) \ldots\ldots [19]$$

$$\frac{d}{dt}(r^2\dot{\theta}) = \omega_c^2 er \sin(\phi - \theta) \ldots\ldots [20]$$

$$\rho^2 \frac{d^2\phi}{dt^2} + \omega_c^2 er \sin(\phi - \theta) = ge \cos\phi \ldots\ldots [21]$$

On account of the non-linearity of these equations, it is difficult to obtain an exact solution, but due to the fact that e and r are ordinarily very small as compared to ρ (for ordinary practical cases $r/\rho \cong 10^{-3}$), certain approximations are permissible.

Adding the last two equations and dividing by ρ^2 the following equation is obtained.

$$\frac{d}{dt}\left[\dot{\phi} + \frac{r^2}{\rho^2}\,\theta\right] = \frac{ge}{\rho^2}\cos\phi \qquad [22]$$

$\dot{\phi}$ and $\dot{\theta}$ have the same order of magnitude, at least as far as their mean values are concerned. The second term within the brackets may therefore be neglected, and the equation becomes

$$\frac{d^2\phi}{dt^2} - \frac{ge}{\rho^2}\cos\phi = 0 \qquad [23]$$

This is the ordinary differential equation for a mathematical pendulum of length ρ^2/e when ϕ is the angle of inclination to the horizontal. An exact solution would lead to elliptic functions, but due to the greatness of the length of the pendulum, no large error is committed if ϕ in the second term is replaced by ωt, where ω is the mean value of $\dot{\phi}$. This makes it possible to integrate the equation, whence

$$\phi = \omega t - \epsilon\cos\omega t \qquad [24]$$

where

$$\epsilon = \frac{ge}{\rho^2\omega^2} = \frac{y_0 e}{\rho^2}\,\frac{\omega_c^2}{\omega^2} \qquad [25]$$

is a very small angle.

The angular velocity $\dot{\phi}$ is thus nearly constant; the maximum deviation from the mean value being $\epsilon\omega$. It is reasonable to assume that $\dot{\theta}$ is also nearly constant, but this is not so easily proved. The term $\omega_c^2 er\sin(\phi - \theta)$ in [21] is certainly small, but it is not clear directly whether or not it is below the order $\omega_c^2 er$. In order to prove this assumption, put

$$\theta = \omega t + \mu\cos\omega t \qquad [26]$$

where μ is a small angle. This gives

$$\theta = \omega - \mu\omega\sin\omega t \text{ and } \dot{\theta}^2 = \omega^2 - 2\mu\omega^2\sin\omega t \qquad [27]$$

since μ^2 may be neglected besides unity. The angle $\phi - \theta$ now has the value $-(\epsilon + \mu)\cos\omega t$. Being a small angle, its sine may be replaced by the angle itself, and its cosine by unity. Equation [19] now becomes

$$\frac{d^2r}{dt^2} + (\omega_c^2 - \omega^2)r = \omega_c^2 e - 2\mu r\omega^2\sin\omega t \qquad [28]$$

If μ were zero, this equation would have a stationary solution

$$r = r_0 = \frac{\omega_c^2 e}{\omega_c^2 - \omega^2} \qquad [29]$$

a value which may be introduced as a close approximation into the last term. Equation [28] then yields the solution

$$r = r_0\left[1 - 2\mu\,\frac{\omega^2}{\omega_c^2 - 2\omega^2}\sin\omega t\right] \qquad [30]$$

This preliminary solution for r may be introduced into Equation [20], in order to determine the actual value of μ. Neglecting products and squares of the quantities ϵ and μ, the following value for μ is obtained:

$$\mu = -\epsilon\,\frac{(\omega_c^2 - \omega^2)(\omega_c^2 - 2\omega^2)}{\omega_c^2(\omega_c^2 - 4\omega^2)} \qquad [31]$$

The assumption that μ is small is thus correct, except when $\omega = \omega_c/2$. Barring this combination, the solution may be written

$$\phi = \omega t - \epsilon\cos\omega t \qquad [32]$$

$$\theta = \omega t - \epsilon\,\frac{(\omega_c^2 - \omega^2)(\omega_c^2 - 2\omega^2)}{\omega_c^2(\omega_c^2 - 4\omega^2)}\cos\omega t \qquad [33]$$

$$r = e\,\frac{\omega_c^2}{\omega_c^2 - \omega^2}\left[1 + 2\epsilon\,\frac{\omega^2(\omega_c^2 - \omega^2)}{\omega_c^2(\omega_c^2 - 4\omega^2)}\sin\omega t\right] \qquad [34]$$

SOLUTION WHEN $\omega = \omega_c$

The preceding solution fails when $\omega = \omega_c$ because the assumption of $\dot{r} \cong 0$ is no longer justified. The differential equations are

$$\left.\begin{aligned} \ddot{r} + r(\omega_c^2 - \dot{\theta}^2) &= \omega_c^2 e\cos(\phi - \theta) \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} &= \omega_c^2 e\sin(\phi - \theta) \end{aligned}\right\} \qquad [35]$$

Assuming that $\dot{\phi}$ and $\dot{\theta}$ are to remain essentially constant $\cong\omega_c$ it follows that the angle $\phi - \theta$ also is constant. Then the equations can be satisfied only when $\phi - \theta = \pi/2$, and the solution becomes

$$\left.\begin{aligned} \phi &= \omega t \\ \theta &= \omega t - \frac{\pi}{2} \\ r &= \frac{e}{2}\,\omega t \end{aligned}\right\} \qquad [36]$$

when at the time $t = 0$ the point S agrees with σ_1, $\phi = 0$, $\dot{\phi} = \omega_c$ and $\theta = -\pi/2$. Any other set of boundary conditions would entail an intermediary period of adjustment, but there is not much doubt that the motion will ultimately follow Equations [36]. Fig. 3 shows an arbitrary position of the shaft during the resonance range.

SOLUTION WHEN $\omega = \omega_c/2$

The solution given by [32] to [34] fails when $\omega = \omega_c/2$ as well, because now the assumption as to the smallness of μ is found to be erroneous. There is nothing to invalidate the expression [24] for ϕ, however. Equation [33] indicates that when $\omega = \omega_c/2$ the angle θ will oscillate with increasing amplitude about its mean position. This suggests as a possible solution for θ, in place of [26],

$$\theta = \omega t + \chi\omega t\sin\omega t \qquad [37]$$

where χ is a small dimensionless quantity. It is of course necessary to restrict the discussion to values of time which make the angle $\chi\omega t$ small. By this restriction, the angle $\phi - \theta$ remains small, so that its sine may be replaced by the angle itself, and its cosine by unity. From [37] the following expressions are obtained:

$$\theta = \omega + \chi\omega\sin\omega t + \chi\omega^2 t\cos\omega t \qquad [38]$$

$$\ddot{\theta} = 2\chi\omega^2\cos\omega t - \chi\omega^3 t\sin\omega t \qquad [39]$$

$$\theta^2 = \omega^2 + 2\chi\omega^2\sin\omega t + 2\chi\omega^3 t\cos\omega t \qquad [40]$$

when higher powers than the first in χ are neglected. Introducing these values into [19] gives

$$\frac{dr^2}{dt^2} + r(\omega_c^2 - \omega^2) = \omega_c^2 e + 2\chi\omega^2 r[\sin\omega t + \omega t\cos\omega t] \qquad [41]$$

If χ were zero, this equation would have the stationary solution

$$r = r_0 = \frac{\omega_c^2 e}{\omega_c^2 - \omega^2}$$

This value of r may be introduced as a close approximation into the remaining term on the right-hand side of the equation.

The following solution is then easily obtained, when at $t = 0$, $r = r_0$:

$$r = r_0\left[1 + 2\chi\,\frac{\omega^2}{\omega_c^2 - 2\omega^2}\left(\frac{\omega_c^2}{\omega_c^2 - 2\omega^2}\sin \omega t + \omega t \cos \omega t\right)\right] \quad\ldots[42]$$

This gives for $\dot{r}$

$$\dot{r} = 2\chi\omega r_0\,\frac{\omega^2}{\omega_c^2 - 2\omega^2}\left[2\,\frac{\omega_c^2 - \omega^2}{\omega_c^2 - 2\omega^2}\cos \omega t - \omega t \sin \omega t\right]\ldots[43]$$

Introducing the value 2ω for ω_c, these equations become

$$r = r_0\,[1 + \chi(2 \sin \omega t + \omega t \cos \omega t)]\ldots\ldots\ldots[44]$$

$$\dot{r} = \chi r_0\omega[3 \cos \omega t - \omega t \sin \omega t]\ldots\ldots\ldots[45]$$

The constant χ is now to be determined by inserting the values of r and θ thus obtained in Equation [20]. Neglecting higher powers of χ, it is found that

$$\chi = -{}^3/_8\epsilon\ldots\ldots\ldots[46]$$

The complete solution for this case may thus be written, since $r_0 = {}^4/_3 e$.

$$\phi = \omega t - \epsilon \cos \omega t\ldots\ldots\ldots[47]$$

$$\theta = \omega t - {}^3/_8\epsilon\omega t \sin \omega t\ldots\ldots\ldots[48]$$

$$r = {}^4/_3 e[1 - {}^3/_8\epsilon(2 \sin \omega t + \omega t \cos \omega t)]\ldots\ldots\ldots[49]$$

The Case of a Variable External Moment

If the disk is subjected to an external moment of the form $M \cos p\omega t$, the same method of solution may be used with minor modifications. Neglecting gravity and putting

$$\epsilon_1 = \frac{M}{mp^2\omega^2\rho^2}\ldots\ldots\ldots[50]$$

the general solution is found to be

$$\left.\begin{aligned}
\phi &= \omega t - \epsilon_1 \cos p\omega t\\
\theta &= \omega t - \epsilon_1\,\frac{(\omega_c^2 - \omega^2)(\omega_c^2 - (p^2 + 1)\omega^2)}{[\omega_c^2 - (p + 1)^2\omega^2][\omega_c^2 - (p - 1)^2\omega^2]}\cos p\omega t\\
r &= e\,\frac{\omega_c^2}{\omega_c^2 - \omega^2}\left[1 + 2\epsilon_1\,\frac{\omega^2(\omega_c^2 - \omega^2)}{[\omega_c^2 - (p + 1)^2\omega^2][\omega_c^2 - (p - 1)^2\omega^2]}\sin p\omega t\right]
\end{aligned}\right\}\ldots[51]$$

In this case, a condition of resonance will prevail at the speeds

$$\omega_1 = \frac{\omega_c}{p + 1}\ldots\ldots\ldots[52]$$

and

$$\omega_2 = \frac{\omega_c}{p - 1}\ldots\ldots\ldots[53]$$

When $p = 1$, these two subcritical speeds merge into one, and the solution [51] is found to be identical with [32]–[34], when ϵ is replaced by ϵ_1.

The conditions at these subcritical speeds may be investigated by the same methods as those employed in the preceding section, but the results are somewhat lengthy and do not add many features of value beyond those given by [47]–[49].

Appendix No. 2

SHAFT WITH VARIABLE RIGIDITY

CONSIDER a system similar to that shown in Fig. 1, but where the eccentricity is zero and the rigidity of the shaft varies between a minimum value k in one direction and a maximum value $(1 + \chi)k$ in the other direction, where χ is a small positive quantity. Fig. 4 shows the disk in an arbitrary position. The shaft deflection s may be considered as the resultant of the components ξ and η in the directions of minimum and maximum rigidity, respectively. The external forces to be considered are thus $k\xi$, $(1 + \chi)k\eta$ and mg. The coordinates are still denoted by r, θ, and ϕ; their significance is obvious from Fig. 3. Projecting all forces on r, at right angles to r, and taking moments around G, the following equations are obtained.[3]

$$m\ddot{r} - mr\dot{\theta}^2 + k\xi \cos(\phi - \theta) - (1 + \chi)k\eta \sin(\phi - \theta) - mg \sin \theta = 0\ldots[54]$$

$$mr\ddot{\theta} + 2m\dot{r}\dot{\theta} + k\xi \sin(\phi - \theta) + (1 + \chi)k\eta \cos(\phi - \theta) - mg \cos \theta = 0\ldots[55]$$

$$m\rho^2\ddot{\phi} + \chi k\xi\eta = 0\ldots\ldots\ldots[56]$$

Introducing the values of ξ and η,

$$\xi = y_0 \sin \phi + r \cos(\phi - \theta)\ldots\ldots\ldots[57]$$

$$\eta = y_0 \cos \phi - r \sin(\phi - \theta)\ldots\ldots\ldots[58]$$

and putting

$$\omega_c = \sqrt{\frac{k}{m}}\ldots\ldots\ldots[59]$$

$$y_0 = \frac{mg}{\left(1 + \dfrac{\chi}{2}\right)k}\ldots\ldots\ldots[60]$$

these equations become

$$\ddot{r} + r[\omega_c^2(1 + \chi \sin^2(\phi - \theta)) - \dot{\theta}^2] = \frac{\chi\omega_c^2 y_0}{2}\sin(2\phi - \theta)\ldots[61]$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} - \frac{\chi\omega_c^2}{2}r \sin 2(\phi - \theta) = -\frac{\chi\omega_c^2 y_0}{2}\cos(2\phi - \theta)\ldots[62]$$

$$\ddot{\phi} + \frac{\chi\omega_c^2}{2\rho^2}[y_0^2 \sin 2\phi + 2y_0 r \cos\theta - r^2 \sin 2(\phi - \theta)] = 0.\,[63]$$

In order to obtain a reasonably simple solution, it is necessary to restrict the analysis to values of r which are small in comparison with y_0, the mean static deflection. The quantity χ is already assumed to be small. The equations may then be simplified to

$$\ddot{r} + r(\omega_c^2 - \dot{\theta}^2) = \frac{\chi\omega_c^2 y_0}{2}\sin(2\phi - \theta)\ldots\ldots\ldots[64]$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -\frac{\chi\omega_c^2 y_0}{2}\cos(2\phi - \theta)\ldots\ldots\ldots[65]$$

$$\ddot{\phi} + \frac{\chi\omega_c^2}{2}\,\frac{y_0^2}{\rho^2}\sin 2\phi = 0\ldots\ldots\ldots[66]$$

The last equation shows that $\dot{\phi}$, the angular velocity of rotation, will be essentially constant, even for appreciable values of χ.

Considering first the case that $\dot{\phi} = \omega$ is very small, it is obvious that the shaft center will be raised and lowered twice every revolution. An investigation into the statics of the problem will reveal that, when χ is small, the shaft center will describe a circle with radius $\frac{\chi\omega_c^2}{2}y_0$ around the position of static equilibrium. Under these circumstances, it is obvious that the radius r will make two revolutions for every revolution of the disk

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[3] In writing the moment equation, the reactions on the bearing supports must be taken into account.

($\theta = 2\omega$). It is reasonable to assume that the same general conditions will be approximately true at higher speeds, so that we may put in Equation [65] $\ddot{\theta} = 0$ and $\dot{r} = 0$. This gives

$$\theta = 2\omega t - \frac{\pi}{2} \ldots\ldots\ldots\ldots [67]$$

Making $\theta = 2\omega$ in Equation [64], then gives

$$\frac{d^2r}{dt^2} + r(\omega_c^2 - 4\omega^2) = \frac{\chi\omega_c^2}{2}y_0 \ldots\ldots\ldots\ldots [68]$$

This equation gives the following stationary solution:

$$r = \frac{\chi y_0}{2}\,\frac{\omega_c^2}{\omega_c^2 - 4\omega^2} \ldots\ldots\ldots\ldots [69]$$

This solution is restricted to the case that $2\omega \neq \omega_c$, however, because when $2\omega = \omega_c$ the above assumption as to the smallness of $\dot{r}$ is no longer justified.

If it is assumed that $\dot{\phi} = \omega$ and $\dot{\theta} = 2\omega = \omega_c$ are essentially constant, the equations to be satisfied are

$$\left.\begin{aligned} \ddot{r} &= \frac{\chi\omega_c^2 y_0}{2}\sin(2\phi - \theta) \\ 2\dot{r}\,\omega_c &= -\frac{\chi\omega_c^2 y_0}{2}\cos(2\phi - \theta) \end{aligned}\right\} \ldots\ldots [70]$$

which demand that

$$2\phi - \theta = 0 \ldots\ldots\ldots\ldots [71]$$

whence

$$\left.\begin{aligned} \phi &= \omega t \\ \theta &= 2\omega t \\ r &= \chi\,\frac{y_0}{2}\,\omega t \end{aligned}\right\} \ldots\ldots\ldots\ldots [72]$$

is the solution for the special boundary conditions $\phi = \theta = 0$, $r = 0$ when $t = 0$. The rate of increase of r will thus be the same as under the influence of an unbalance corresponding to the eccentricity $\chi y_0/2$.

Discussion

David Robertson.[4] The author has already published several excellent articles on the vibration and whirling of shafts, and a further contribution from him is very welcome.

The effects of the irregularity of angular velocity induced by the gravitational torque have already been investigated by Rodgers,[5] who makes the same fundamental assumptions, but uses rectangular coordinates and leaves his result in that form. However, his result may be simply expressed by saying that the motion of the c. g. is the resultant of two circular motions; one is the steady whirl of the same frequency as the shaft rotation and the other is one of double frequency going the same way round. Relative to the steady whirl, the subsidiary whirl has a frequency equal to the shaft rotation.

This agrees with the author's solution, except that the radius of the subsidiary whirl derived from his equations is four times that obtained from Rodgers' work. The discrepancy is accounted for by an omission on Rodgers' part to allow for the variation of the angular velocity when deducing the coordinates of the elastic center from those of the center of gravity.

Rodgers does not recognize that he is dealing with a very special case and that his results do not apply generally. The author does definitely state that he considers no external forces except those due to the elastic resistance to bending and to the weight of the shaft, but he does not call attention to the consequences of this restriction, which amounts to assuming that there are no elastic forces due to torsion of the shaft. This is the same thing as assuming that the ends of the shaft are not coupled to anything possessing inertia. If either or both ends of the shaft were coupled to a driving motor or load of some sort, the inertia of that apparatus would exert a torque opposing any change of angular velocity which would, as a rule, greatly exceed that due to the elastic resistance to bending, unless the torsional stiffness of the shaft were so small that it would be incapable of transmitting any useful torque and of driving a load. Equation [25] would then be very far wrong.

Thus the whole of this paper would apply roughly to a rotary converter, but it is quite inapplicable to most other machines.

There is no doubt about the smallness of μ as a general rule, but Equation [31] can hardly be regarded as a proof that this is so, for this equation was derived by assuming it to be true.

The solution for $\omega = \omega_c$ has nothing to do with the gravitational torque, but merely shows how the shaft runs out when it is steadily run at the critical speed, with very special initial conditions. Rodgers gives the same result, but expressed in rectangular coordinates, in his Equations [26] and [27], before he begins to allow for any variations of angular velocity. Jeffcott[6] also deals with this problem.

When $\omega = {}^1/_2\omega_c$, Rodgers' form of the solution gives an infinite radius for the secondary whirl. He does not deal with the initial motion corresponding to Equations [47]–[49] of the paper.

Rodgers also studies the shaft with variable rigidity, both without and with unbalance, on page 136 of his paper. With no unbalance, he finds a main whirl and a subsidiary whirl of double frequency, and his value for the radius of the latter agrees with the author's equations.

Rodgers gives a separate solution when there is unbalance, but here he is quite wrong. Unbalance makes no difference.

The writer thinks that the author has not quite expressed his meaning in the statement on the second page about frictional forces, as he surely will agree that the friction is not merely "often," but is *always* important in the neighborhood of the critical speed. In fact, it then becomes the most important factor of all.

J. N. Goodier.[7] In spite of the already extensive literature of the subject, it is clear that this paper is an important one and heartily to be welcomed. The evidence presented for attributing subresonance to minute variations in shaft elasticity is most emphatic, and the author is to be congratulated on the series of judicious approximations which enabled him to extract his conclusions from the differential equations.

Of the two cases of subresonance discussed, it appears that one—that due to eccentricity—is accompanied by angular oscillation (equation for ϕ in [5]), while the other proceeds without disturbing the uniform rotation. If the system were coupled to another, rotating uniformly, subresonance due to eccentricity would accordingly induce torsional vibration. It would be interesting to know whether, in a system of practical proportions, this could be a source of resonant torsional vibration.

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[4] Professor, Electrical Engineering Department, Merchant Venturers Technical College (Faculty of Engineering, University of Bristol), Bristol, England.

[5] C. Rodgers, "On the Vibration and Critical Speeds of Rotors," *Phil. Mag.*, July, 1922, p. 134. (The factor g has been omitted from the second term of Equations [34] and [35].)

[6] H. H. Jeffcott, "The Lateral Vibration of Loaded Shafts in the Neighborhood of a Whirling Speed," *Phil. Mag.*, March, 1919, sec. 7, p. 310.

[7] Commonwealth Fund Fellow, University of Michigan, Ann Arbor, Mich.

J. P. Den Hartog.[3] Two years ago the writer conducted some tests which seem to be a verification of the author's analytical result. A model shaft, carrying a disk in the middle, was run in rigidly supported ball bearings. The disk contained balancing holes, so that the eccentricity e could be varied. The usual critical speed was observed; its amplitude being roughly proportional to the unbalance e. But the resonant amplitude at the subcritical speed proved to be entirely independent of the unbalance. With the best obtainable balance, the amplitude at half critical speed was four times as large as at the critical speed itself. While by changing the unbalance the amplitude at the critical speed was varied from 25 per cent to 200 per cent, the amplitude at half critical speed remained 100 per cent throughout.

A non-uniformity in the rigidity of the shaft was looked for, but could not be detected. At the time it was not appreciated that such a very small amount of non-uniformity could be responsible for the effect. Taking the rate of increase observed in the experiment and substituting it into the author's formula [8], it is roughly estimated that the non-uniformity of the shaft rigidity was $^1/_{400}$; an amount which was too small to be detected with the means used at the time.

It is possible to derive the three main formulas [3], [6], and [8] in a somewhat simpler manner, as follows: In all three cases the shaft is supposed to rotate without any vibration whatever. Then the oscillating force on the disk can be easily calculated. This force is then made to act on the non-rotating disk, giving a linearly increasing vibration.

In the case of fundamental resonance the force acting on the disk is the centrifugal one of the amount $m\omega_c^2e$. Its component in any direction, for instance the vertical one, is $m\omega_c^2e \cos \omega_c t$.

In the case of a non-uniform shaft the gravity mg of the disk is counteracted by the shaft elasticity ky_0. However, this elasticity varies between the values ky_0 and $ky_0 (1 + \chi)$ with the mean value $ky_0 (1 + \chi/2)$. Therefore the pulsating force on the disk in the vertical direction is $^1/_2 ky_0\chi \cos 2\omega t$. At half critical speed $\omega_c = 2\omega$, so that the force is $^1/_2 ky_0\chi \cos \omega_c t$.

In the case B—Subresonance Due to Eccentricity, the conditions are somewhat more complicated. When there is no vibration, the center of gravity G describes a circular path of radius e about σ_1 as center. While G is moving downward, gravity exerts on the disk an accelerating moment; while G is moving upward, the moment is decelerating. From this it is seen at once that this action of gravity on the eccentricity is completely equivalent to the action of a non-uniform driving moment on the shaft. The pulsating force on the disk now is due to the inertia reaction of the motion of the center of gravity.

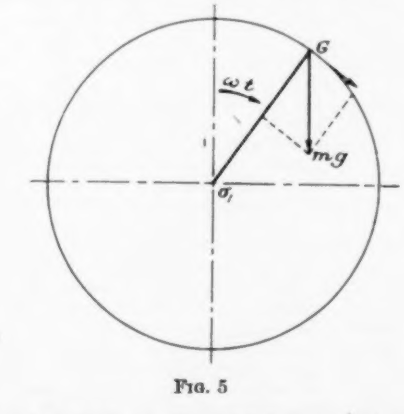

Fig. 5

Fig. 5 shows the tangential component of gravity to be $mg \sin \omega t$; the moment is $mge \sin \omega t$; the angular acceleration $ge \sin \omega t/\rho^2$; the tangential acceleration $ge^2 \sin \omega t/\rho^2$. The vertical component of this is $ge^2 \sin^2 \omega t/\rho^2 = -ge^2 \cos 2\omega t/2\rho^2 +$ const. The inertia reaction of this on the disk is $mge^2 \cos 2\omega t/2\rho,^2$ or at half resonant speed, $mge^2 \cos \omega_c t/2\rho^2$.

In all three cases, therefore, we have a pulsating force acting on the disk of frequency ω_c and of amplitude, say, P. The differential equation $m\ddot{\chi} + k\chi = P \cos \omega_c t$ has the well-known solution:

$$\chi = \frac{P}{2k} \omega_c t \sin \omega_c t$$

from which

$$\Delta\chi = \Delta r = \frac{P}{2k} \cdot \omega_c t.$$

Substituting the expressions for P in this in each of the three cases leads to the results [3], [5], and [8] with the exception of a factor 4 in formula [5], which, however, does not affect the order of magnitude of the result.

Author's Closure

Professor Robertson, in his discussion, concludes that the results of the paper apply only to a system having no connected rotating masses, such as a rotary converter, but would be quite inapplicable to most other machines. If a torsional system were connected with that of Fig. 1, adding one or more degrees of freedom, there are three alternative consequences. First, the connecting shaft may be very rigid, so that the natural frequency of the corresponding mode of motion is very high in comparison with the frequency of rotation. Then, no other modification than a certain increase of the radius of gyration of the disk is required in order to reduce the combined system to that of Fig. 1. Second, the connecting shaft may be very flexible, making the corresponding natural frequencies very low. In this case no appreciable influence is caused by the added torsional inertia. Third, the connecting shaft may be so proportioned that torsional resonance takes place. In general, the result will now be an increase in the effect of torsional impulses in the system; that is, the eccentricity of the disk would appear to be increased. The exact type of motion will now be very complicated, and Professor Robertson's statement may in a sense be correct. In no case can the influences of the unbalance, with regard to lateral amplitudes, be increased very materially, however, since some of the available energy will now go to a building up of the torsional vibrations. No reference has so far been made to the fact that the addition of torsional modes of vibration would also modify the system with regard to the modes of lateral vibrations. Such modifications, if excessive, would of course invalidate the entire arguments upon which the paper is based. It is worth noting, however, that the results obtained will surely hold for more complicated systems, in so far as the fundamental mode of motion is concerned.

Professor Robertson's criticism that Equation [31], which is derived on the assumption that μ is small, cannot be used to prove the same fact is undoubtedly justified. However, the only use of the assumption that μ is small, which is made in Equation [31], is to neglect higher powers than the first of μ and ϵ, the latter being known to be small. If the neglected terms in μ^2 and $\mu\epsilon$ were retained, another more complicated expression for μ would be obtained. It is easily verified that this expression converges toward [31] when ϵ is small.

Mr. Goodier's question is partly answered by the foregoing reply to the discussion by Professor Robertson. Such a state of resonance is theoretically possible, although no such experiments are known to the author. The available energy is so small, however, that no real phenomenon of resonance with the impulses due to gravity appears possible. Certainly, this phenomenon, even if it could be induced under well-controlled laboratory conditions, need not be looked upon as another source of torsional vibrations.

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[3] Research Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Assoc-Mem. A.S.M.E.

The author is much indebted to Dr. Den Hartog for his reference to experiments which seem to verify the conclusions of the paper. These experiments are of the same nature as the practical difficulties encountered in direct-current machines, to which reference has been made in the paper. In both cases, the attempts to eliminate the vibrations by reducing the eccentricity failed.

Dr. Den Hartog's suggestion for arriving at the results in a simpler manner is indeed a good one. This method would have been used but for the fact that the assumption upon which it is based is not so easily seen in problems of the type covered by the paper. The author is referring to the method of evaluating certain inertia forces, under the assumption that no vibrations take place, and then determining the amplitude of vibration by the aid of these (somewhat imaginary) inertia forces. This method is nothing but an application of relative motion, but it was felt that its truth is not directly obvious for systems of the type shown in Fig. 1. It does give the right results, however. In the case of sub-resonance due to eccentricity the disk is assumed to rotate with the point S rigidly held at σ_1. The coordinates (x, y) of G are then:[9]

$$x = e \cos \varphi = e \cos \omega t + e\epsilon \sin \omega t \cos \omega t$$
$$y = e \sin \varphi = e \sin \omega t - e\epsilon \cos^2 \omega t$$

where the angle ϕ is as given in [5] and the quantity ϵ is small in comparison with e.

The second terms of these expressions give inertia forces of the frequency $\omega_e = 2\omega$ and maximum value $^1/_2 me\epsilon\omega_e^2$. The resulting amplitude determined in accordance with the method suggested by Dr. Den Hertog is $^1/_4 e\epsilon\omega_e d$, which agrees with that given by [5] when it is remembered that $\omega = \omega_e/2$.

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[9] Soderberg, "The Vibration Problem in Engineering," *Electric Journal*, 1924, p. 299.

APM-54-5

The Strength of Thin Plates in Compression

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The stability of thin plates has been investigated by many authors. However, in aeronautical structures, thin metal sheets are often used beyond the stability limits, and the load which can be carried by the structure is determined by the ultimate strength in compression. A recent series of experiments by the Bureau of Standards showed the ultimate load to be independent of width and length of the plate and approximately proportional to the square of the thickness. In the present paper an approximate theoretical analysis of this problem is developed, by which the "effective width" and the ultimate strength can be found. The result of this analysis shows the ultimate strength of a plate to be proportional to the square roots of the modulus of elasticity and the yield point of the material, and to the square of the thickness. This result gives a good check with the experiments mentioned.

THE problem of stability of thin plates has been investigated by many authors, S. Timoshenko having been especially successful in obtaining simple results for most of the cases which are of importance in engineering. However, in aeronautical structures, thin metal sheets are often used beyond the stability limits, and the load which can be carried by the structure is determined by the ultimate strength in compression. Recently a series of interesting experiments has been made by the Bureau of Standards, in cooperation with the Bureau of Aeronautics, Navy Department, for the purpose of determining the strength of plates under "edge compression." In these tests the edges parallel to the load were supported and the load was applied by rigid or nearly rigid beams perpendicular to the load. The ultimate load was found to be independent of the width and length of the plate, and approximately proportional to the square of the thickness.

Denoting the ultimate load by P and the thickness by t, the result of these experiments can be expressed approximately by the simple formula

$$P = Kt^2$$

where K is a constant depending on the physical properties of the material. From the theoretical point of view the question occurs how the physical characteristics of the material enter in the constant K, which obviously has the dimension of a stress. The following approximate analysis is a simple method of attacking this question.

Consider a thin sheet of length L and width b under a compressive load P. To simplify the calculations, let it be assumed that the entire load is carried by two strips of width w, one on each side of the sheet, and that the loading is uniform across these strips (Fig. 1). Each strip carries the load $P/2$. Then if σ is the stress in the material,

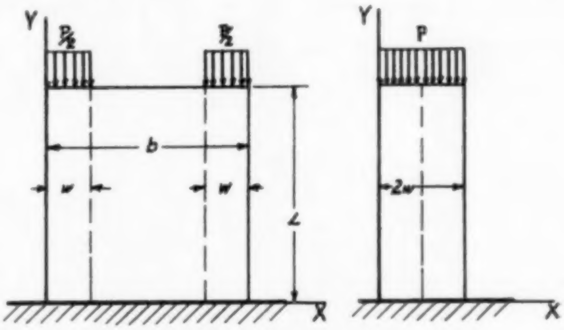

FIG. 1

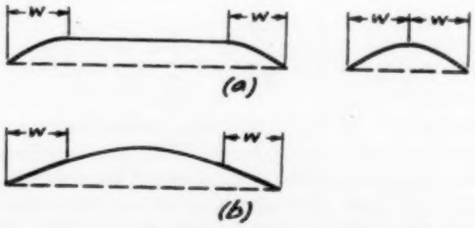

FIG. 2 ASSUMED CROSS-SECTIONS OF BUCKLED PLATES

$$P = 2wt\sigma \qquad \text{[1]}$$

Consider now the plate to be simply supported on the sides, so that the lateral deflection z along the sides is zero. For simplicity assume that the deflection is such that horizontal tangents at the inner edges of the two load-supporting strips are parallel to the X-direction, Fig. 2(a). Then the center of the sheet can be disregarded and the two strips can be handled as if they were together.

The lateral deflection z of such a strip under compression is given by the equation

$$\frac{Et^3}{12(1-\mu^2)}\left(\frac{\partial^4 z}{\partial x^4} + 2\frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4}\right) + \sigma\frac{\partial^2 z}{\partial x^2} = 0 \qquad \text{[2]}$$

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[1] Director, Guggenheim Aeronautical Laboratory, California Institute of Technology. Mem. A.S.M.E. Dr.-Ing. von Kármán received his M.E. at Budapest in 1902 and Ph.D. at Göttingen in 1908; honorary degree of Doctor of Engineering, University of Berlin, 1929. He was Privat Docent, Göttingen, 1910–1913; Professor of Mechanics and Aerodynamics, Director of the Aerodynamical Institute, University of Aachen, 1913; Member of Gesellschaft der Wissenschaften zu Göttingen, 1925; foreign member of the Royal Academy of Sciences, Toronto, 1928; Director of the Graduate School of Aeronautics, California Institute of Technology, 1928.

[2] Instructor in charge of the Department of Engineering Drawing and instructor in Machine Design, California Institute of Technology. Mem. A.S.M.E. E. E. Sechler is a graduate of California Institute of Technology, with B.S. in Mechanical Engineering in 1928, M.S. in Mechanical Engineering in 1929, and M.S. in Aeronautics in 1930. He was in the Engineering Department of Douglas Aircraft Company in the summer of 1929.

[3] Research Fellow in aeronautical structures, Guggenheim Aeronautical Laboratory, California Institute of Technology. Mem. A.S.M.E. L. H. Donnell was born in 1895 in Maine. He is a graduate in Mechanical Engineering, University of Michigan, 1915; Doctor's degree, University of Michigan, 1930. He worked in the engineering departments of automobile companies (Franklin, Packard, Dodge) for seven years, and taught engineering mechanics as Assistant Professor, University of Michigan, for seven years.

Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. (The paper proper has been prepared by Dr. Von Kármán; the Appendix, by Messrs. Sechler and Donnell.)

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

where E is the modulus of elasticity and μ is Poisson's ratio for the material. A solution of this equation is

$$z = z_0 \sin \frac{\pi x}{2w} \sin \frac{\pi y}{l}$$

which satisfies the conditions that $z = 0$ and $\partial^2 z/\partial x^2 = 0$ when $x = 0$ or $2w$, and $z = 0$, $\partial^2 z/\partial y^2 = 0$ when $y = 0$ or any multiple of l. l then is the length of a half-wave of the deformation along the line $x = w$. Substituting this value for z in [2] and dividing through by $\sin \frac{\pi x}{2w} \sin \frac{\pi y}{l}$ gives:

$$\frac{Et^2}{12(1 - \mu^2)} \left(\frac{\pi^4}{16w^4} + \frac{2\pi^4}{4l^2w^2} + \frac{\pi^4}{l^4} \right) - \sigma \frac{\pi^2}{l^2} = 0$$

from which

$$\frac{\sigma}{E} = \frac{\pi^2 t^2}{12(1 - \mu^2)} \left[\frac{l}{4w^2} + \frac{1}{l} \right]^2 \qquad [3]$$

Assume now that the two strips carrying the load are at the limit of stability, corresponding to "the most dangerous wave length." This means that we must find the minimum value of σ/E. Finding the derivative of $\left[\frac{l}{4w^2} + \frac{1}{l} \right]$ with respect to l, and equating it to zero, we find

$$\frac{1}{4w^2} - \frac{1}{l^2} = 0$$

$$l = 2w$$

therefore

$$\frac{\sigma}{E} = \frac{\pi^2 t^2}{12(1 - \mu^2)} \left[\frac{1}{2w} + \frac{1}{2w} \right]^2 = \frac{\pi^2 t^2}{12(1 - \mu^2) w^2} \qquad [4]$$

from which

$$w = \frac{\pi}{\sqrt{12(1 - \mu^2)}} \sqrt{\frac{E}{\sigma}}\, t \qquad [5]$$

and

$$P = 2wt\sigma = \frac{\pi}{\sqrt{3(1 - \mu^2)}} \sqrt{\frac{E}{\sigma}}\, t^2 \sigma = \frac{\pi}{\sqrt{3(1 - \mu^2)}} \sqrt{E\sigma}\, t^2 \qquad [6]$$

Equations [5] and [6] give us the relation between effective width and the load carried by the plate. The equations hold in this form below the elastic limit. Beyond the elastic limit, it is necessary to replace E by the slope of the stress-strain curve E', and determine the maximum of the product $E'\sigma$. However, it is easy to see that this maximum value will not differ greatly from the product of the value of E below the elastic limit and the yield point σ_y. Hence the maximum load that can be sustained by the plate is:

$$P_{\text{ult.}} = \frac{\pi}{\sqrt{3(1 - \mu^2)}} \sqrt{E\sigma_y}\, t^2 \qquad [7]$$

In the experiments mentioned above, the average values of E and σ_y for the duralumin sheets tested were 10,500,000 lb. per sq. in. and 41,000 lb. per sq. in., respectively. Using these values and $\mu = 0.25$ in [7],

$$P_{\text{ult.}} = 1.23 \times 10^6 t^2$$

J. S. Newell[4] gives, as an empirical formula based on these experiments, the value of the maximum load for duralumin as

$$P_{\text{ult}} = 1.2 \times 10^6 t^2 - 3.0 \times 10^6 t^3$$

where the second term represents only a small correction for usual thicknesses. A more exact checking of the experimental results is given in the Appendix by Messrs. Donnell and Sechler.

These results have also been applied to the case of plates supported by longitudinal stiffeners. It is assumed that the stresses in the stiffener and in the effective strip of the plate are the same when buckling takes place. It is then possible to determine the width w of the effective strips on each side of the stiffener and to calculate a strengthening factor, by which the strength of the stiffener alone is multiplied, to obtain the strength of the stiffener with its adjacent effective strips. However, it is desired first to check these results by experiments, and the authors expect to do this very soon. It is felt that the experiments on stiffened sheets mentioned by Newell did not have sufficiently definite end conditions for this purpose.

Stiffeners combined with curved plates can be investigated in the same way. Analyzing the effect of a small curvature $1/R$ of the plate, it is observed that the strengthening factor should increase with the curvature due to the increased stability of the curved strips. However, if the stiffeners are located on the concave side of the plates, as must usually be the case with aeronautical structures, the curvature also has an opposite influence on the strengthening factor, because portions of the curved plates are nearer to the neutral axis and hence contribute less to the resultant moment of inertia than corresponding portions of flat plate.

It is easy to show that for small curvatures, i.e., for large values of R, the increase of the strengthening factor is proportional to $1/R^2$, while the loss due to the last-mentioned effect is proportional to $1/R$. It is therefore concluded that if plates supported by stiffeners are curved, the strength decreases with the curvature at first. This fact, which is rather contrary to what one would expect at first sight, was found experimentally in the tests cited by Mr. Newell.

Appendix

THE formula derived by Dr. von Kármán for the ultimate compressive load on a sheet simply supported at the sides can be written

$$P_{\text{ult.}} = C\sqrt{E\sigma_y}\, t^2 \qquad [8]$$

Experience with other types of stability problems indicates that the ultimate compressive load on a sheet with other edge conditions—such as a sheet clamped along the sides or the portion of a sheet between stiffeners—can be expressed by the same formula [8], but with a different value of the constant C. In making his analysis, Dr. von Kármán, for simplicity, assumed somewhat artificial conditions, and hence it is not to be expected that the value of C given above is exactly correct even for the case of simply supported sides. The purpose of the derivation was to prove that the ultimate load was proportional to $\sqrt{E\sigma_y}t^2$, and it is intended to obtain the correct value of C, for any given edge conditions, from experimental evidence.

It is interesting, however, to compare the value of C found in experiments with the values calculated for certain extreme assumptions.

If a perfectly rectangular flat sheet is freely supported along the sides and compressed between rigid flat plates at the ends, the compressive stress will obviously be uniformly distributed. As the end plates are brought together, this compressive stress will increase, and when it reaches a certain critical value σ_b, lateral buckling deflections will occur all across the sheet, except at the edges, which are constrained from lateral movement. Just as the buckling begins, it can be assumed that the compressive stress is still uniformly distributed across the sheet. This is the classical theory of buckling; there is no reason to suppose anything wrong with it.

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[4] "The Strength of Aluminum Alloy Sheets," *Airway Age*, November, 1930.

The thing that Dr. von Kármán has investigated is what takes place when the end plates are brought still further together. It is known from general experience with buckling that, in the middle part of the sheet, the compressive stresses will remain substantially constant as the buckling deflections increase. On the other hand, at the very edges buckling is prevented by the lateral support, so that the stress there must increase according to Hooke's law until it reaches the yield point of the material σ_y; general experience indicates that the maximum resistance of the sheet will be realized at about this time. At points near the edges the stress will be somewhere between the limits σ_b and σ_y. The distribution of stress along the sheet will now be something like that shown in Fig. 3(*a*). The total area under the curve is a measure of the total force on the sheet, while the shaded area shows the total force according to the classical theory.

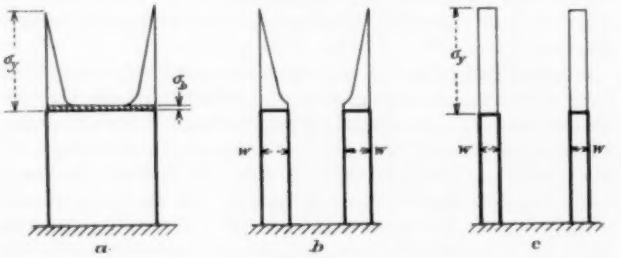

FIG. 3

For very thin sheets such as used in aeronautical work, σ_b is so small compared to σ_y that the middle portion can be neglected entirely. This reduces the problem to that of the buckling of two strips of unknown width w under a stress distribution something like that shown in Fig. 3(*b*). To facilitate calculations Dr. von Kármán makes a further simplification by assuming that the strips are under uniform stress σ_y, as in Fig. 3(*c*). This does not affect the final result as much as might be thought, because the width w is still to be solved for, and with the assumption of uniform distribution we will get a smaller value for w than we would with the distribution of Fig. 3(*b*); consequently the total force will be more nearly the same in the two cases.

We can now calculate w by exactly the same methods that are used in the classical theory to calculate the buckling stress; in that case the stress is unknown, but the width is known, while here the stress is known and the width is unknown. To make this calculation it is necessary to find or assume the shape of the buckled surface. In the derivation given in the main part of the paper it is assumed that buckled cross-sections of the sheet are as shown in Fig. 2(*a*), so that if the two effective portions are joined they form half a sine wave. With this assumption $C = \dfrac{\pi}{\sqrt{3(1-\mu^2)}} = 1.90$ for $\mu = 0.3$.

Another and perhaps more likely assumption would be that buckled cross-sections are as shown in Fig. 2(*b*) such that the portions w can be considered straight. It is shown later that with this assumption the ultimate load is found to be approximately

$$P_{\text{ult.}} = \sqrt{\frac{2}{1+\mu}}\sqrt{E\sigma_y t^2} = 1.24\,\sqrt{E\sigma_y}t^2,\ \text{for } \mu = 0.3 \ldots [9]$$

The actual buckled shape is probably somewhere between these two extremes and so one would expect to find experimental values of C somewhere between 1.90 and 1.24.

The Bureau of Standards experiments[5] previously mentioned were made on sheets of four materials of widely different physical properties: duralumin, stainless steel, monel metal, and nickel. Several widths were tested for each thickness (or nearly the same thickness), all widths giving nearly the same results. Table 1 shows the average ultimate load found for each average thickness, and also values of E and σ_y computed from the stress-strain curve given in the report for each thickness (or the nearest thickness). The yield point σ_y was taken as the stress for which the stress-strain slope is one-third the original elastic slope, as suggested in the report. In the last column of the table the values of $C = \dfrac{P_{\text{ult.}}}{\sqrt{E\sigma_y t^2}}$ (from Equation [8]) are calculated. These values are also shown plotted against the thickness in Fig. 4.

TABLE 1 TESTS MADE ON SHEETS OF VARIOUS MATERIALS

	t	$P_{\text{ult.}}$	$E \times 10^{-6}$	σ_y	$C = \dfrac{P_{\text{ult.}}}{\sqrt{E\sigma_y t^2}}$
Duralumin	0.017	310	10.6	39,000	1.67
	0.030	930	10.2	41,000	1.60
	0.045	2,190	10.6	40,500	1.65
	0.060	3,840	10.5	39,000	1.67
	0.075	5,180	10.6	44,500	1.34
	0.090	7,500	10.6	42,000	1.39
Stainless steel	0.015	400	26.5	52,500	1.51
	0.034	1,990	27.3	52,000	1.45
	0.050	4,000	24.3	42,000	1.58
	0.060	5,650	34.0	45,000	1.27
	0.078	8,340	25.8	26,000	1.71
	0.095	12,400	27.0	35,000	1.41
Monel metal	0.020	520	26.0	36,000	1.34
	0.030	1,080	22.7	34,500	1.35
	0.044	2,250	24.3	27,000	1.43
	0.063	4,650	25.3	27,500	1.41
	0.079	7,350	22.8	46,000	1.15
	0.093	9,400	22.0	34,000	1.26
Nickel	0.020	620	28.4	40,000	1.46
	0.032	1,540	27.6	33,000	1.58
	0.042	2,550	29.0	30,000	1.55
	0.060	4,500	27.5	36,500	1.25
	0.080	5,000	27.6	15,500	1.20
	0.094	7,900	27.0	21,300	1.18

If Equation [8] is correct these values of C should all be the same. Considering the uncertainties of such tests and the fact that the physical properties varied very widely even for the same material and the indefiniteness of the yield point, the values of C obtained show remarkable constancy. It is also notable that there seems to be no systematic difference between the different materials, the average values for duralumin and stainless steel, for instance, being very nearly the same, in spite of the great difference in physical properties. This is convincing evidence of the correctness of Dr. von Kármán's conclusions that the effect of the physical properties of the material is measured by $\sqrt{E\sigma_y}$ (neglecting the slight influence of Poisson's ratio). There seems to be a slight systematic decrease in the value of C with the thickness. However, this might easily be explained by the fact that the flexibility of the testing jig used in the experiments would play a more important part when testing the thicker, and therefore stiffer, sheets.

The average value of C for smaller thicknesses is about 3/2. It is hoped soon to make a similar experimental determination of the value of C for the more important case, in practice, of portions of a plate between stiffeners.

Derivation of Equation 9: If it be assumed that cross-sections of the effective strip remain straight, the lateral deflection can be taken as

$$z = z_0 x \sin \frac{n\pi y}{L} \ldots\ldots\ldots\ldots\ldots\ldots [10]$$

Longitudinal elements of the strip shorten, due to buckling, by the amount $\displaystyle\int_0^L \frac{1}{2}\left(\frac{\partial z}{\partial y}\right)^2 dy$. The external work done by the load during buckling is therefore

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[5] N.A.C.A. Report No. 356.

TABLE 2

Type of stiffener	a	a	a	a		b		c	d
Number of stiffeners	2	3	4	2		2		3	2
Distance between lines of rivets, in.	11.25	5.625	3.75	11.25	10.25	+ 2	at 0.875	5.5	11.0
Sheet thickness	0.019	0.032	0.052	0.019		0.032		0.031	0.020
Length of specimen, in.	12	12	12	6		12		12	12
Allowable stress on sheet	1900	5800	13,400	1900	3250		20000[a]	6000	1850
Allowable load on sheet	434	2230	8280	434	1070	+	1120	2235	445
Load on stiffeners	2600	3900	5200	3100		4700		2790	3400
Predicted maximum load	3034	6130	13,480	3534		6890		5025	3845
Observed maximum load	2960	6100	13,200	3300		6500		5240	3800

[a] Stress on section between rivets attaching sheet to stiffener estimated from Bureau of Standards data on Flat Sheet.

$$V_1 = \int_0^L \int_0^w \sigma t dx \, \frac{1}{2} \left(\frac{\partial z}{\partial y}\right)^2 dy$$

The internal energy of bending and torsion in the strip after buckling is

$$V_2 = \frac{Et^3}{12(1-\mu^2)} \int_0^L \int_0^w \frac{1}{2} \left[\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial^2 z}{\partial y^2}\right)^2 + 2(1-\mu)\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 + 2\mu \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} \right] dx dy$$

Substituting [10] in these equations and setting $V_1 = V_2$,

$$\frac{\sigma}{E} = \frac{t^2}{12(1-\mu^2)w^2} \left[\frac{n^2\pi^2 w^2}{L^2} + 6(1-\mu) \right]$$

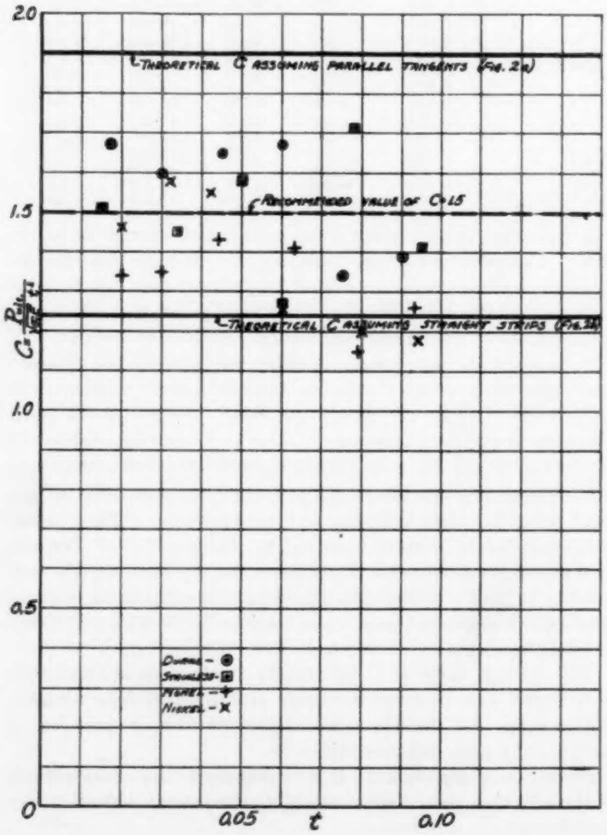

FIG. 4 VALUES OF *C* PLOTTED AGAINST THICKNESS *t*

To make σ/E a minimum, n should evidently have its minimum possible value, namely, unity. The first term in the brackets, $\pi^2 w^2/L^2$, can then be neglected in comparison with the second term if w is small compared to L, giving

$$\frac{\sigma}{E} = \frac{1}{2(1+\mu)} \frac{t^2}{w^2} \quad \text{[11]}$$

Equation [9] can then be obtained from [11] in the same manner that [7] was obtained from [4] in the body of the paper.

Discussion

Joseph S. Newell.[6] This paper is a timely and interesting contribution to the problem of all-metal airplane construction which is so vexing to designers at present. Equation [5] furnishes a rational means for determining the effective width of sheet which functions with the stiffener on a stiffened thin plate and will provide the numerous investigators who are cooperating on this problem a standard method for analyzing their data from this point of view.

It is, however, suggested that another assumption be considered by investigators of this problem of the stiffened thin plate in compression. The assumption is that the load which the combination of sheet and stiffener will carry is equal to the load which the sheet alone will take plus the load which the stiffener alone will take. To determine the load on the sheet one may use the data obtained by the Bureau of Standards[7] to determine the stress at maximum load on a sheet whose width is equal to the clear distance between rows of rivets connecting the sheet to the stiffeners.

This method of attacking the problem obviates the assumption that the stress in the sheet and in the stiffener are equal when maximum load is approached. In view of the fact that the effective EI of an assembly of sheet and stiffener, when tested as a beam, is not equal to but is normally appreciably less than the product of the modulus of elasticity of the material times the moment of inertia of the cross-section of the combined section, it would appear difficult to justify any assumption based on the homogeneity of the composite section. Furthermore, thin sheets normally wrinkle between the rivets attaching them to a stiffener as the load approaches the maximum that the combination will take. Under such conditions the strains of sheet and stiffener are obviously unequal; hence an assumption of equality of stress is hardly justifiable. The assumption of "effective widths" has long been employed in naval architecture and mechanical engineering. It may be the solution to this problem, but in view of the results shown in Table 2 the proposed scheme of taking the strength of the composite section equal to the sum of its parts appears to be worthy of further consideration.

Fig. 5 shows four types of stiffeners used on flat aluminum-alloy sheets for which more or less data are available. All of the tests have been on panels 12 in. wide. Most of them have been 12 in. long, but there are a few at 6 in. The data in Table 2 are a few taken from the results of investigations made at the Massachusetts Institute of Technology. Attention is invited to the agreement between "Predicted" and "Observed" values where the sheet carries about 12 per cent of the load on the combination, as well as where it carries nearly 75 per cent of the load. The agreement is too close to be the result of mere coincidence, hence appears worthy of the closest investigation.

The scheme in Table 2 is readily applicable for use in design, if later investigations justify its use at all, since one may determine

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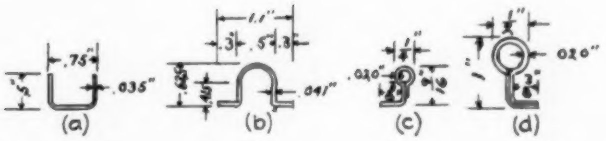
[6] Associate Professor of Aeronautical Structural Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[7] N.A.C.A. report No. 356.



	(a)	(b)	(c)	(d)
Allowable load for 12 in. length, lb	1300	2300	930	1700
Allowable load for 6 in. length, lb	1550	..	...	..

Fig. 5

the strength of a stiffener by test or analysis, add to it the load carried by a sheet of width equal to the distance between attachments of sheet to stiffener, and so obtain the strength of the combination. This strength, or total load, may be divided by the total area to obtain an "average" or "apparent" stress for the combination which may be used in analyzing the composite section.

Determining the maximum load to be taken by a stiffener involves the question of end conditions. Tests to date have been made on flat-ended sections as it has been felt advisable to obtain some working data which engineers having judgment could utilize. They are more in the nature of "guessing points" to be tempered by good judgment than exact scientific data. To obtain the latter means the use of knife edges or similar devices to get a true, pin-ended condition on the combination of sheet and stiffener. Since it is impossible to determine the effective centroid of the composite cross-section analytically, due to non-homogeneity of riveted assemblies, the determination of the axis of loading involves a trial and error method in the testing machine. This is practicable, but slow. Eventually it may be necessary to carry out investigations in which such a method will be used, if it proves difficult to obtain a correlating coefficient between the flat end conditions of these tests and the conditions obtained during static tests of actual airplanes, but the demand at present is for a set of approximate values obtained from tests to serve as values which a designer may modify to suit his problem.

Such values are admittedly temporary, and further investigations over a greater range of specimens may necessitate revising them completely. The present demand is for data with the limitations of the tests carefully stated in each case. Next comes the correlation of the data and the development of rational or empirical methods for predicting strengths of thin plates such as are used in aircraft. The present paper covers a possible method which is worthy of careful consideration, and the authors are to be congratulated on the clarity as well as the timeliness of their presentation.

AUTHORS' CLOSURE[8]

In answering Professor Newell's interesting discussion of the stiffened-sheet problem, it could be pointed out that rivet connections between sheet and stiffener may function quite differently when the sheet is tested in bending and in compression, and that if the rivets are not spaced closely enough to prevent wrinkling when testing in compression, then no simple theory can apply. However, this does not answer the arguments completely, and it is felt that there is not at present enough experimental data to decide what theory will fit most closely the conditions found in practice.

The objection to the experiments mentioned by Professor Newell was that flat-ended columns seem to lie somewhere between columns with perfect end fixity and pin-ended columns; this is also indicated by the results given for compression of the stiffeners alone. As fixed-end columns are supposed to have four times the resistance of corresponding pin-ended ones, it would seem that the effect of the sheet in increasing the width of the end, and hence the degree of end fixity, might be quite large compared to the effect which it is wished to measure. It may be possible to eliminate this factor more easily than by the method mentioned in the discussion, by arranging for a high degree of end fixity which will not be influenced by the presence of the sheet.

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[8] In the absence in Europe of Dr. von Kármán, this was written by L. H. Donnell.

APM-54-6

Flow of Steels at Elevated Temperatures

By F. P. Coffin[1] and T. H. Swisher,[2] Schenectady, N. Y.

Flow tests of steels at elevated temperatures, as carried on in the Research Laboratory of the General Electric Company, are described. The object of these tests is to predict the ability of materials to maintain dimensions under stress at operating temperatures for long periods of time. Various methods of testing are used. Some are for the purpose of obtaining a mass of information in the shortest possible time; here the rate of flow is measured for a series of successively reduced stresses at each of several successively increased temperatures. Others are designed to furnish more fundamental information regarding the nature of flow; here new specimens are subjected to constant stress at a constant temperature. A third is the constant-extension test where the stress is varied with time to maintain a constant total elongation (elastic plus plastic).

F. P. Coffin

T. H. Swisher

FLOW TESTS of steels at elevated temperatures are carried on at the General Electric Company in seven furnaces, five of which are shown in Fig. 1. Each furnace holds four horizontal test specimens. A 12-in. gage length is used. Test specimens are 26 in. long, with extension pieces welded to each end. The test bars are loaded in tension through a 100-to-1 lever system (see Fig. 2). For stresses under 40,000 lb. per sq. in. a 0.505-in.-diameter specimen is used; for stresses over 40,000 lb. per sq. in., a 0.375-in.-diameter specimen is used. Test specimens are supported by springs at each end of the furnace in such a way as to minimize the initial bending moment in the gage length. For a tensile load of 1000 lb. per sq. in., the maximum bending stress, after correction, is calculated to be 60 lb. per sq. in.; for tensile loads of 5000 to 40,000 lb. per sq. in., the maximum bending stress is 35 lb. per sq. in. When plastic flow occurs, the bending stresses decrease. Universal joints are used at the ends of the rods to insure a straight pull.

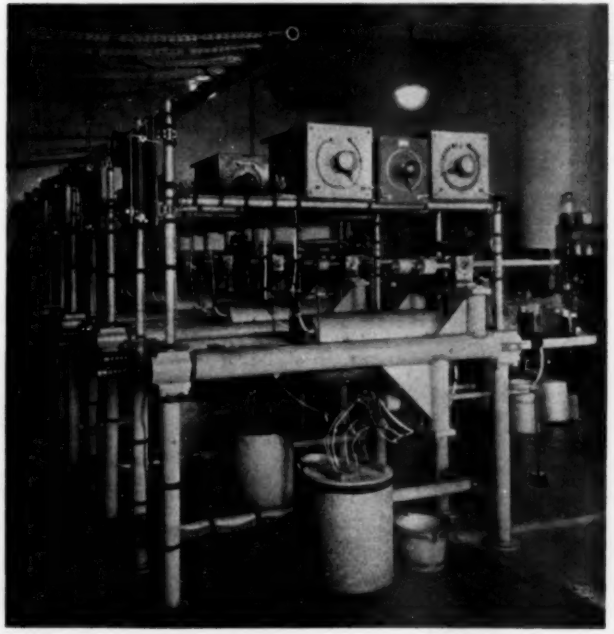

Fig. 1 Five Creep-Stress Testing Machines Having a Total Capacity of Twenty Test Specimens

Extension is measured by Ames dial indicators divided to the ten-thousandth part of an inch. The division marks are 2 mm. apart, and extensions may be read to the hundred-thousandth part of an inch. On a 12-in. gage length this makes it possible to distinguish magnitudes smaller than a millionth part of an inch per inch of length. Movement in the gage length is transmitted to the indicators through quartz rods. A rod of alloy steel is used along the gage length to minimize change in the gage reading due to change in temperature. Fig. 3 shows the universal joints, extensometers, and spring supports. Fig. 4 shows the extensometer clamps. The ends of the clamps are undercut so that they grip the test pieces for a length of $^1/_{16}$ in.

The clamping screws are extra long, and in addition sleeves having the same area as the screws are used to provide enough spring in the clamps to compensate for the decrease in diameter of the test bar caused by extension.

Temperature is controlled by special Leeds & Northrup controllers, the control thermocouples being located in the bottom

Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

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[1] Research Laboratory, General Electric Company. Mr. Coffin was graduated from Harvard University in 1903 in the course in Electrical Engineering. During 1905 he was with the General Electric Company in the Testing Department and since 1906 has been in the Research Laboratory. He has done pioneering work in connection with the development of the high-power mercury-arc rectifier, also in connection with mercury and other vapors for possible use in turbines at high temperatures. For the past few years he has specialized in the study of the flow of steels at elevated temperatures in connection with the application of various types of carbon and alloy steels to the construction of turbines and mercury boilers.

[2] Mechanical Engineer, Research Laboratory, General Electric Company. Jun. A.S.M.E. Mr. Swisher was graduated from the West Virginia University in 1922. He was with the General Electric Test Department, 1923–1925, and has been in the Mechanics Section, Research Laboratory, since 1925, engaged in research and development work in connection with the mercury-vapor process, refrigerators, oil-burner controls, and flow of steels at elevated temperatures.

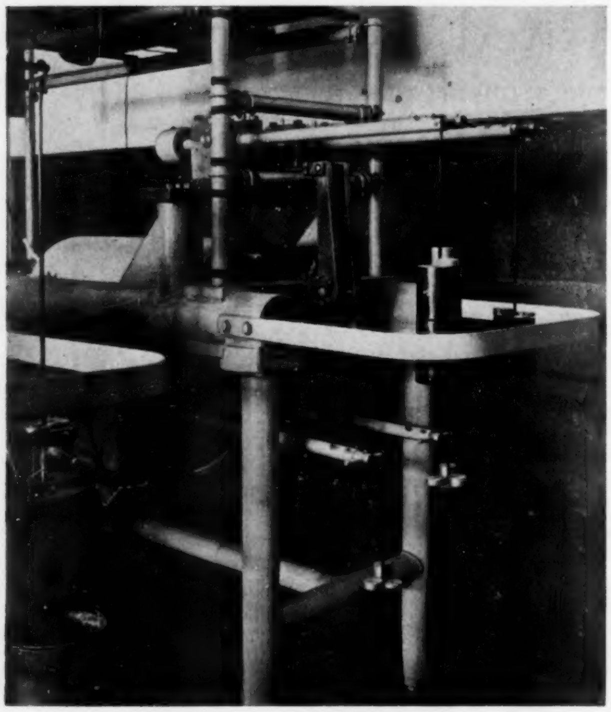


FIG. 2 VIEW SHOWING LEVER ARRANGEMENT

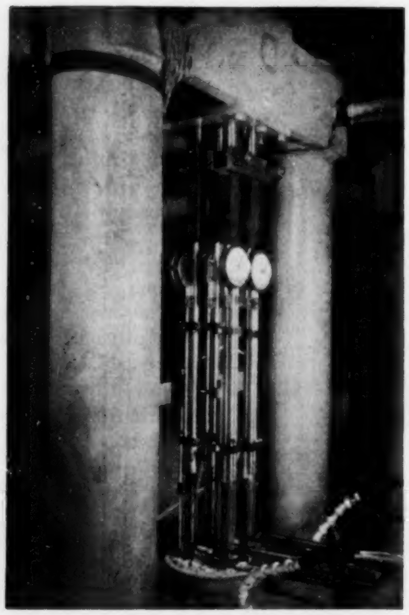


FIG. 3 VIEW SHOWING UNIVERSAL JOINTS, EXTENSOMETERS, AND SPRING SUPPORTS

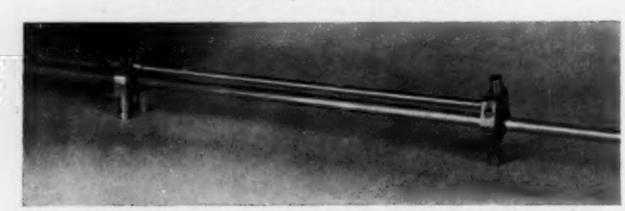
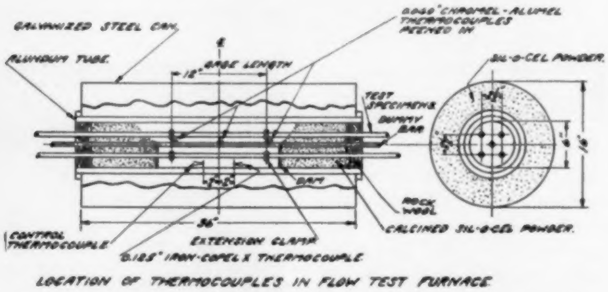


FIG. 4 EXTENSOMETER CLAMPS USED IN CREEP TESTS; 0.375-IN. TEST BAR, 12-IN. GAGE LENGTH

of the furnace 2 in. from the center. Fig. 5 shows the control thermocouple with the auxiliary heater used to obtain a short cycle of about 2 min. duration. During this cycle there is no measurable change of temperature in the test bars. The standard thermocouple is a 0.125-in. iron-copel X located in the bottom of the furnace 2 in. from the center, opposite the control thermocouple. Gage readings are taken when this thermocouple reads within 0.5 deg. cent. of the correct temperature. A $^1/_2$-in. dummy bar is located at the center of the furnace and extends out the ends. This bar has a temperature gradient approximately the same as the test bars. Three 0.040-in. chromel-alumel thermocouples are peened into this bar, one opposite each extensometer clamp and one at the center of the gage length. By means of a system of compensated windings and rheostats, it is possible to keep the temperature of the gage length uniform within 1 deg. cent. The difference in temperature between top and bottom of the furnace is small, of the order of 1 deg. cent. Temperatures are measured with a General Electric precision potentiometer sensitive to 1 millionth of a volt, which corresponds to $^1/_{60}$ deg. cent. The records on the Leeds & Northrup instruments are used for a check on the operation of the temperature control system and to record variations in tem-

FIG. 5 CONTROL THERMOCOUPLE WITH HEATER FOR OBTAINING SHORT TEMPERATURE CYCLE



GALVANIZED STEEL CAN
0.040" CHROMEL-ALUMEL THERMOCOUPLES PEENED IN
ALUNDUM TUBE
GAGE LENGTH
12"
SIL-O-CEL POWDER
TEST SPECIMENS
DUMMY BAR
BAFF
ROCK WOOL
36"
CALCINED SIL-O-CEL POWDER
CONTROL THERMOCOUPLE
EXTENSION CLAMP
0.125" IRON-COPEL X THERMOCOUPLE
16"
LOCATION OF THERMOCOUPLES IN FLOW TEST FURNACE

FIG. 6 LOCATION OF THERMOCOUPLES IN FLOW-TEST FURNACE

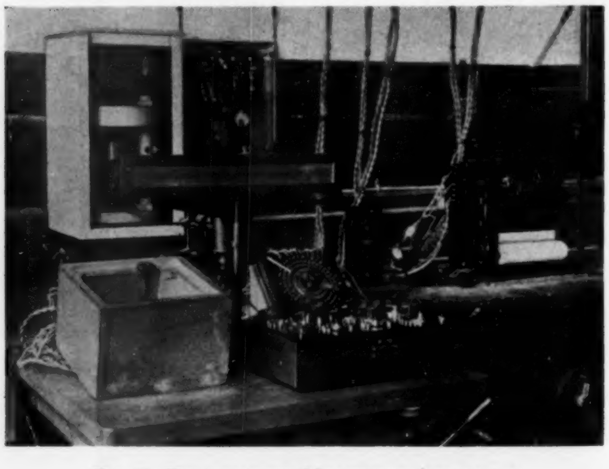


FIG. 7 TEMPERATURE-MEASURING APPARATUS

perature caused by power interruptions or other irregularities. Fig. 6 is a diagram of the furnace arrangement, and Fig. 7 shows the precision potentiometer and one of the Leeds & Northrup instruments.

Method of Conducting Tests

The standard temperatures of testing are:

Deg. cent.	Deg. fahr.
400	752
450	842
500	932
550	1022

Various methods of testing are used. Many flow-rate tests have been made for the purpose of obtaining a bulk of information in the shortest possible time. Here the rate of flow was measured for a series of successively reduced stresses at each of several successively increased temperatures.

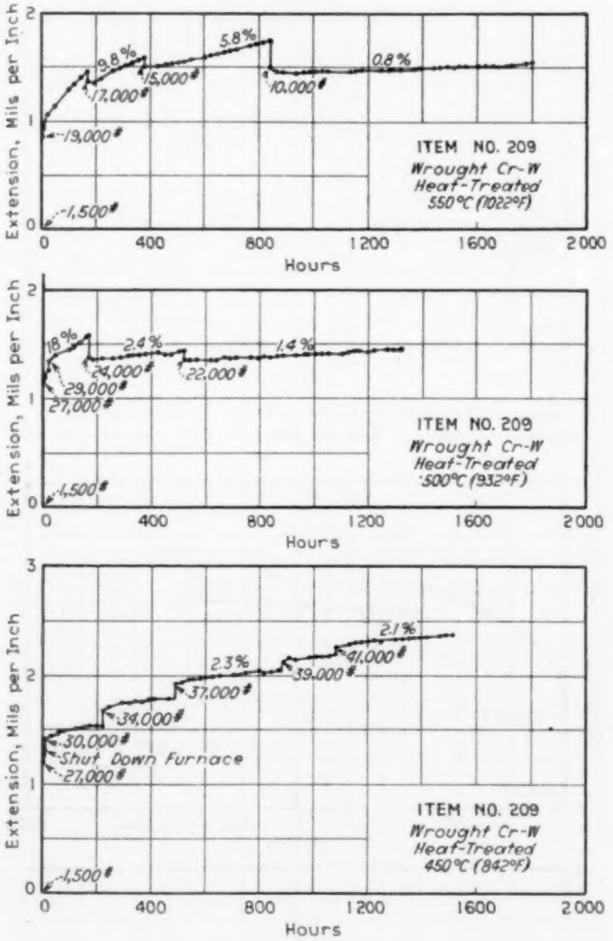


Fig. 8 Flow-Rate Tests of Item No. 209
(Composition: C, 0.36; Cr, 1.38; V, 0.19; W, 1.98; Mn, 0.30; Si, 0.19.)

Tests have been made to study flow under constant extension. The test specimens were loaded until the total elongation (elastic plus plastic deformation) reached a definite amount. As flow took place, the stress was lowered from time to time so as to maintain the total elongation approximately constant. These tests also yielded flow-rate information. The initial part of the tests showed that the stress decreased rapidly at first as flow took place.

In cases where more information of a fundamental nature is wanted on a particular steel, new specimens are subjected to a constant stress at a uniform temperature for long periods of time.

The following procedure was used for the flow-rate tests: The furnaces were brought up to temperature with the bars under a stress of 1000 lb. per sq. in. Then stresses were applied which were expected to produce a flow rate of approximately 10 per cent in 100,000 hours, after a flow of about 1 mil per inch had taken place. It was usually necessary to increase or decrease the stress in order to get a flow rate of the order of 10 per cent. After this flow rate was established by the flow remaining constant, say, for 200 hours, flow rates of about 3 per cent and 1 per cent in 100,000 hours were established. This gave flow rates for different stresses at one temperature. In some cases the same bars were tested at successively higher temperatures. The bars were generally stretched 1 to 1.5 mils per inch at each temperature.

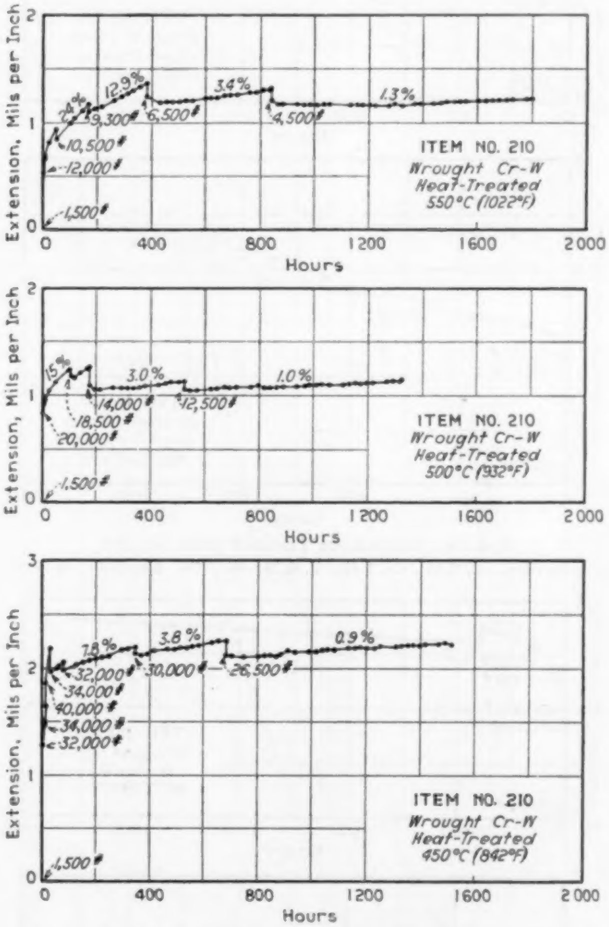


Fig. 9 Flow-Rate Tests of Item No. 210
(Composition: C, 0.42; Cr, 1.98; V, 0.20; W, 2.91; Mn, 0.34; Si, 0.24.)

When stress versus the flow rate, so determined, for a certain temperature, is plotted on logarithmic paper, the points generally fall close to a straight line drawn through them. This straight-line relation helps to extrapolate to the region of tolerable deformation.

It is desirable to conduct tests wherein the stress applied produces a flow rate of the order of magnitude that is permissible in practice; tests have been made as close to this region as time would permit.

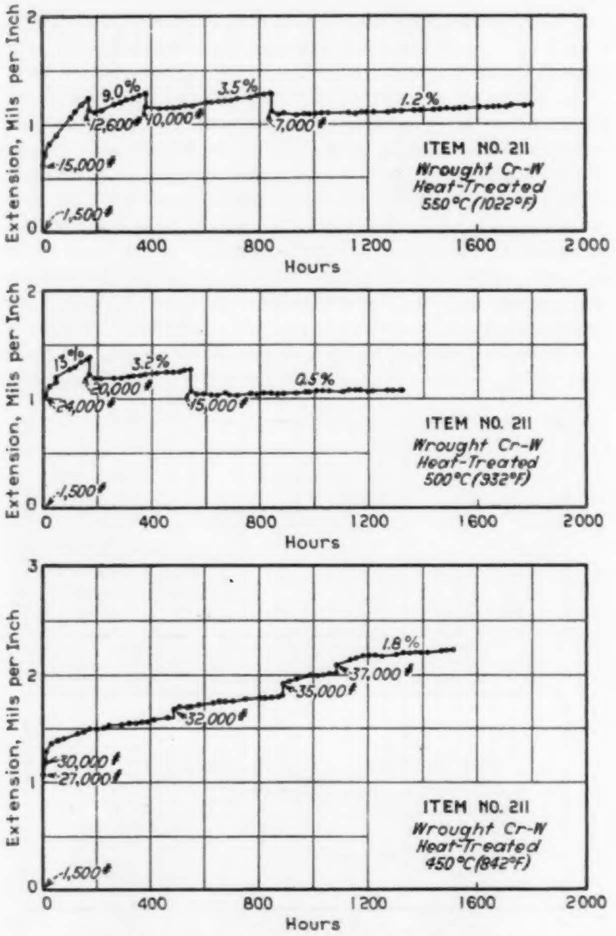


Fig. 10 Flow-Rate Tests of Item No. 211
(Composition: C, 0.41; Cr, 1.47; V, 0.29; W, 2.04; Mn, 0.32; Si, 0.27.)

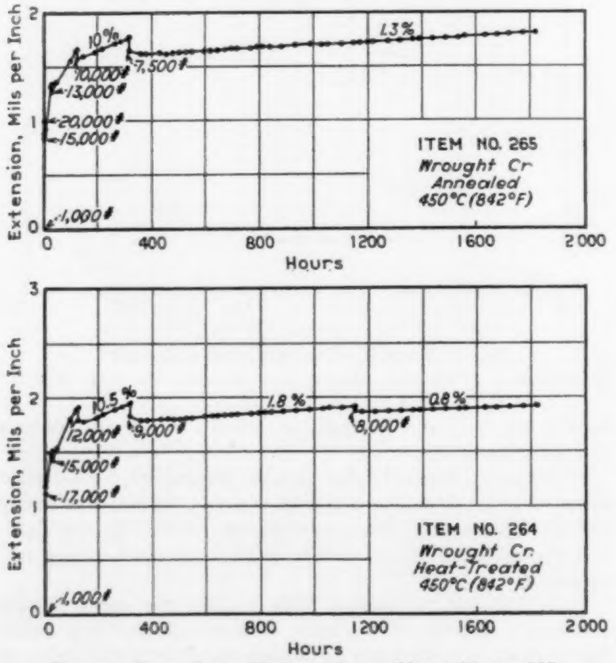


Fig. 11 Flow-Rate Tests of Items Nos. 264 and 265
(Composition: C, 0.45; Cr, 1.0.)

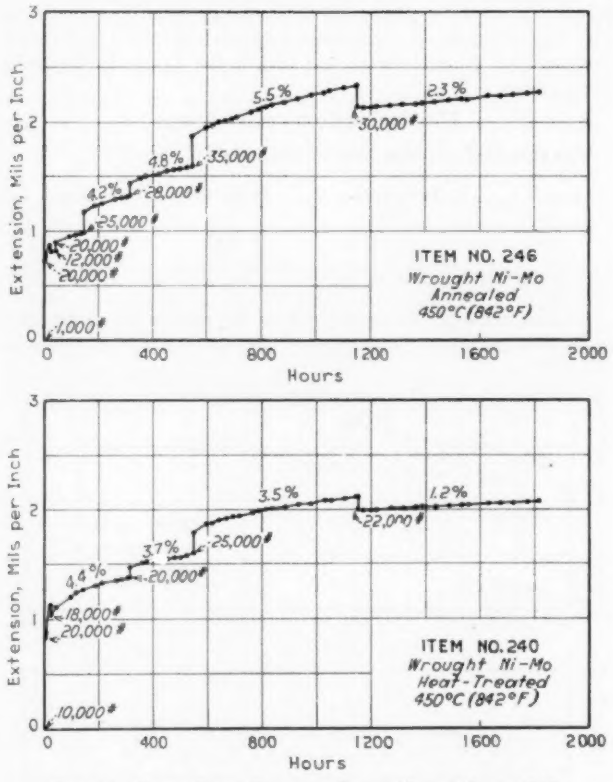


Fig. 12 Flow-Rate Tests of Items Nos. 240 and 246
(Composition: C, 0.48; Ni, 3.38; Cr, 0.08; Mo, 0.32; Mn, 0.63; Si, 0.20.)

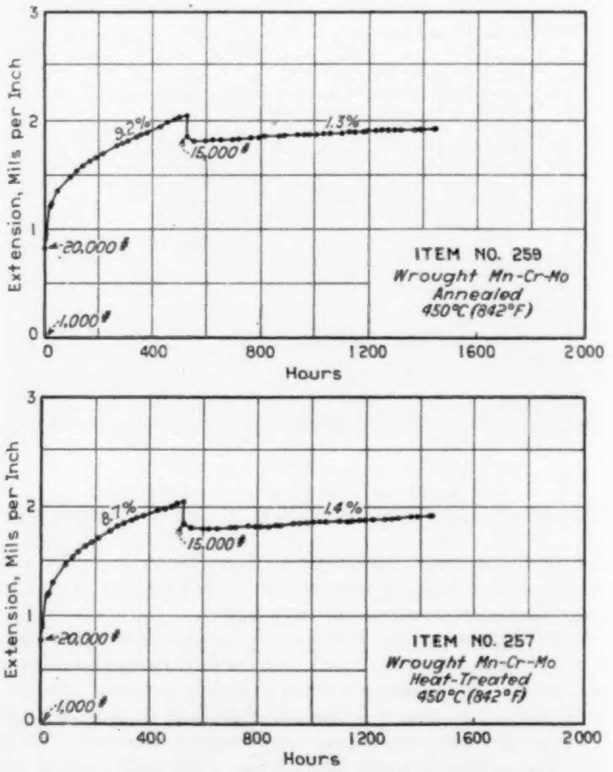


Fig. 13 Flow-Rate Tests of Items Nos. 257 and 259
(Composition: C, 0.54; Cr, 0.51; Mo, 0.22; Cu, 0.06; S, 0.081; Mn, 1.18; Si, 0.23.)

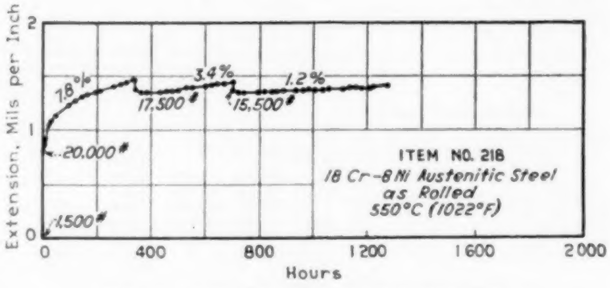


FIG. 14 FLOW-RATE TEST OF ITEM NO. 218
(Composition: C, 0.10; Ni, 8.57; Cr, 17.25.)

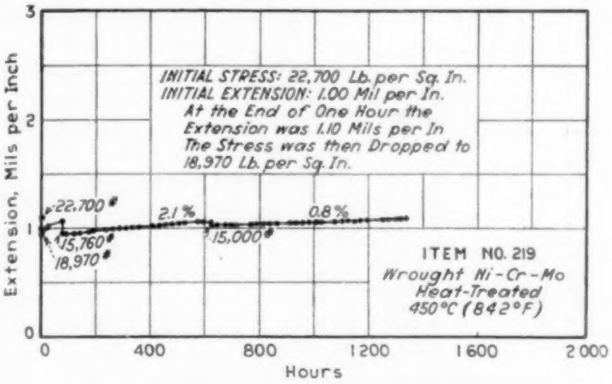


FIG. 15 CONSTANT EXTENSION TEST OF ITEM NO. 219
(Composition: C, 0.34; Ni, 2.76; Cr, 1.09; Mo, 0.78; Mn, 0.80; Si, 0.79.)

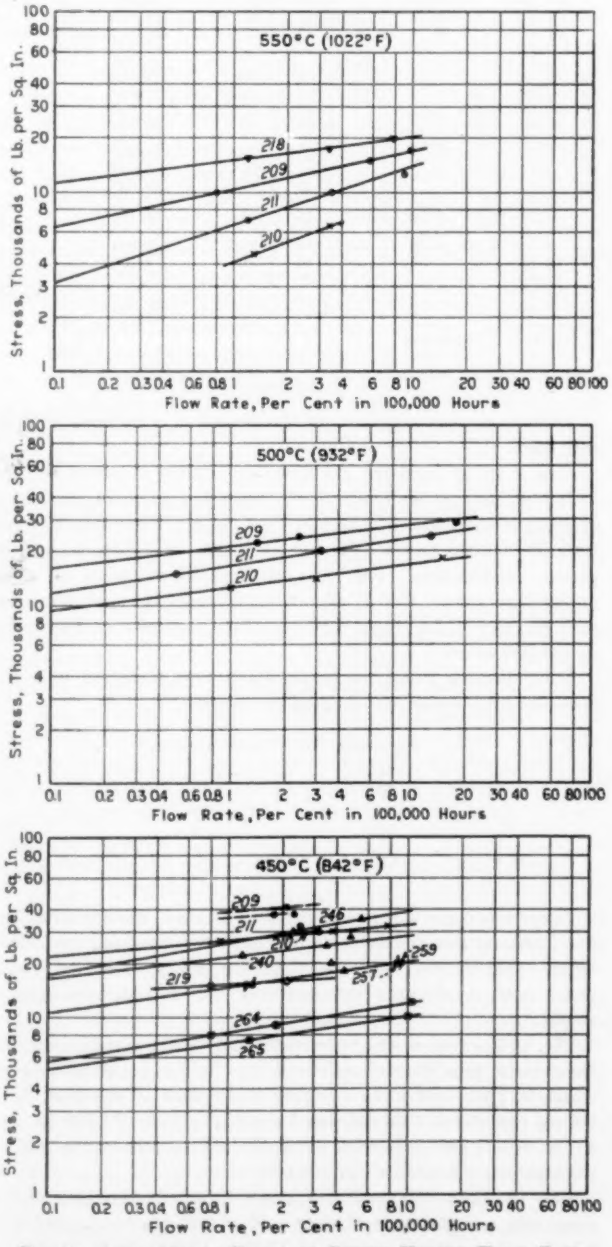


FIG. 16 LOGARITHMIC PLOTS OF STRESS VERSUS FLOW RATE

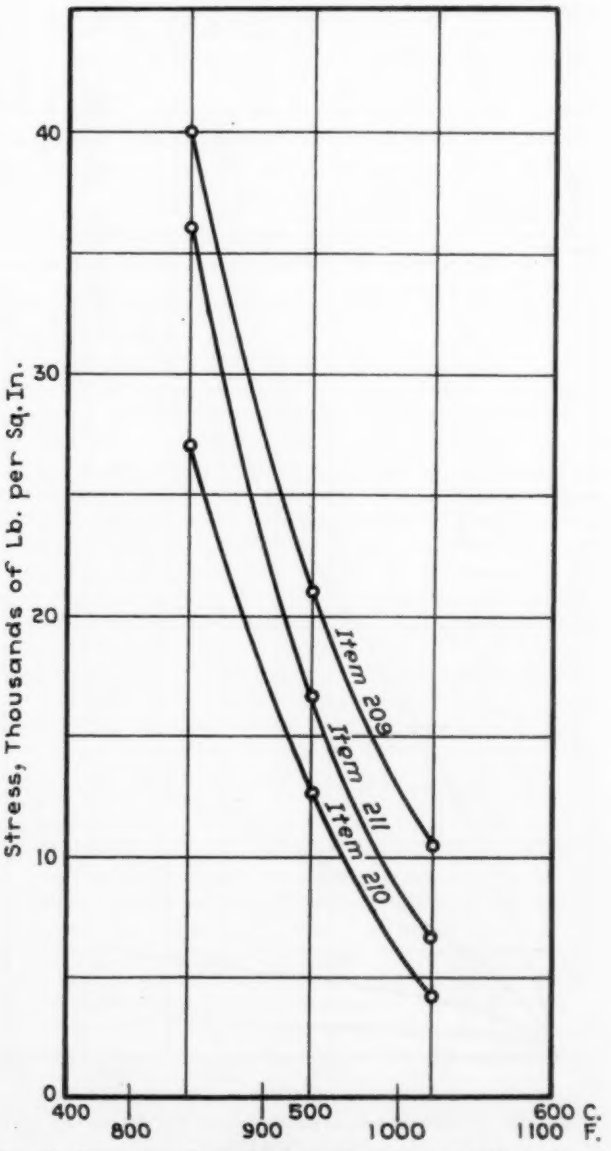


FIG. 17 STRESS FOR FLOW RATE OF 1 PER CENT IN 100,000 HOURS AT VARIOUS TEMPERATURES

TABLE 1 STRESS FOR FLOW RATE OF 1 PER CENT IN 100,000 HOURS

Item	Group	Condition	Heat treatment, deg. cent.	Chemical composition C	Ni	Cr	Mo	V	W	Mn	Si	Stress, lb. per sq. in., 752° F., 400° C.	Stress, lb. per sq. in., 842° F., 450° C.	Stress, lb. per sq. in., 932° F., 500° C.	Stress, lb. per sq. in., 1022° F., 550° C.
108	C	Wrought	900 A.C., 600 F.C.	0.40	....	.....	....	....	....	0.75	....	....	5,900	....	....
117	C	Wrought	800 F.C.	0.44	....	.....	....	....	....	0.75	0.18	1,800	9,000	....	....
119	C	Wrought	900 A.C., 850 O.Q., 625 O.Q., 450 F.C.	0.46	....	.....	....	....	....	0.76	0.24	15,500	2,900	....	....
118	Ni	Wrought	800 F.C.	0.31	3.42	.....	....	....	....	0.83	0.16	12,000	4,600	....	....
120	Ni	Wrought	900 A.C., 850 O.Q., 650 O.Q., 450 F.C.	0.37	3.39	.....	....	....	....	0.81	0.27	6,500	5,300	....	....
103	Mn	Wrought	900 A.C., 600 F.C.	0.36	....	.....	....	....	....	0.98	0.37	....	15,200	....	....
113	Mn	Wrought	800 F.C.	0.50	....	0.05	....	....	....	1.04	0.17	....	13,200	8,500	2,100
114	Mn	Wrought	800 F.C.	0.29	....	.....	....	....	....	1.05	....	....	12,600	7,900	3,200
115	Mn	Wrought	900 A.C., 600 F.C.	0.29	....	.....	....	....	....	1.05	....	....	12,800	10,200	5,300
206	Mn	Wrought	800 F.C.	0.39	0.54	0.08	....	....	....	1.58	....	....	15,300	....	2,300
207	Mn	Wrought	800 F.C.	0.36	....	0.05	....	....	....	1.11	....	....	15,000	9,200	2,200
269	Mn	Wrought	825 F.C.	0.41	....	.....	....	....	....	1.46	0.16	....	15,200	....	....
264	Cr	Wrought	825 O.Q., 650 O.Q., 425 F.C.	0.45	....	1.0	....	....	....	....	....	....	8,300	....	....
265	Cr	Wrought	800 F.C.	0.45	....	1.0	....	....	....	....	....	....	7,200	....	....
104	Ni-Cr	Wrought	900 A.C., 600 F.C.	0.42	1.38	0.62	....	....	....	0.61	0.20	....	6,800	....	....
121	Ni-Cr	Wrought	800 F.C.	0.44	1.19	0.54	....	....	....	0.64	0.16	....	10,300	....	....
221	Ni-Cr	Wrought	825 O.Q., 625 A.C.	0.51	1.65	0.87	....	....	....	....	....	....	5,400	....	....
101	Cr-W	Wrought	900 A.C., 600 F.C.	0.42	....	1.98	....	0.20	2.91	0.34	0.24	....	38,000	....	....
209	Cr-W	Wrought	900 A.C., 700 A.C., 600 F.C.	0.36	....	1.38	....	0.19	1.98	0.30	0.19	....	39,000	21,000	10,500
210	Cr-W	Wrought	900 A.C., 700 A.C., 600 F.C.	0.42	....	1.98	....	0.20	2.91	0.34	0.24	....	26,900	12,500	4,100
211	Cr-W	Wrought	900 A.C., 700 A.C., 600 F.C.	0.41	....	1.47	....	0.29	2.04	0.32	0.27	....	36,000	16,600	6,500
252	Cr-W	Wrought	875 S.C., 675 O.Q., 425 S.C.	0.50	....	0.76	....	....	1.18	....	....	....	11,800	6,550	....
253	Cr-W	Wrought	875 O.Q., 650 O.Q., 425 S.C.	0.50	....	0.76	....	....	1.18	....	....	....	10,900	5,400	....
246	Ni-Mo	Wrought	900 A.C., 800 F.C., 650 F.C.	0.48	3.38	0.08	0.32	....	....	0.63	0.20	....	26,000	....	....
240	Ni-Mo	Wrought	900 A.C., 825 O.Q., 650 O.Q., 425 F.C.	0.48	3.38	0.08	0.32	....	....	0.63	0.20	....	21,400	....	....
111	Ni-Cr-Mo	Wrought	950 A.C., 800 A.C., 650 F.C.	0.34	2.77	1.09	0.78	....	....	0.80	0.29	....	11,700	....	....
112	Ni-Cr-Mo	Wrought	950 A.C., 750 A.C., 650 F.C.	0.34	2.77	1.09	0.78	....	....	0.80	0.29	....	16,000	....	....
122	Ni-Cr-Mo	Wrought	835 O.Q., 625 O.Q., 400 F.C.	0.34	2.74	0.61	0.25	....	....	0.60	....	....	25,600	....	....
123	Ni-Cr-Mo	Wrought	825 F.C., 625 O.Q., 425 F.C.	0.34	2.74	0.61	0.25	....	....	0.60	....	....	13,000	....	....
217	Ni-Cr-Mo	Wrought	950 A.C., 815 A.C., 625 A.C., 600 F.C.	0.39	2.78	0.84	0.48	....	....	0.73	0.24	....	30,600	....	....
219	Ni-Cr-Mo	Wrought	900 A.C., 835 O.Q., 625 O.Q., 400 F.C.	0.34	2.76	1.00	0.78	....	....	0.80	0.29	....	15,100	....	....
257	Mn-Cr-Mo	Wrought	900 A.C., 825 O.Q., 625 O.Q., 425 F.C.	0.54	....	0.51	0.22	Cu 0.06	S 0.081	1.18	0.23	....	14,400	....	....
259	Mn-Cr-Mo	Wrought	900 A.C., 825 F.C., 675 O.Q., 425 F.C.	0.54	....	0.51	0.22	Cu 0.06	S 0.081	1.18	0.23	....	14,400	....	....
106	Stainless Fe	Wrought	950 A.C., 650 F.C. and A.C.	0.11	2.77	12.50	....	....	....	0.46	0.21	....	2,400	....	....
107	Stainless Fe	Wrought	950 A.C., 650 F.C. and A.C.	0.13	Tr.	13.40	....	....	....	0.60	Tr.	....	12,200	....	....
218	Austenitic	Wrought	As rolled	0.10	8.57	17.25	....	....	....	....	....	....	....	....	15,200
102	Cr-Si	Wrought	N. 1 hr. 900 A.C., D. 1 hr., 600 F.C.	0.31	....	3.62	....	....	....	0.34	3.32	....	16,000	....	....

NOTE: N, normalize; A.C., air cool; O.Q., oil quench; F.C., furnace cool; D, draw; S.C., slow cool.

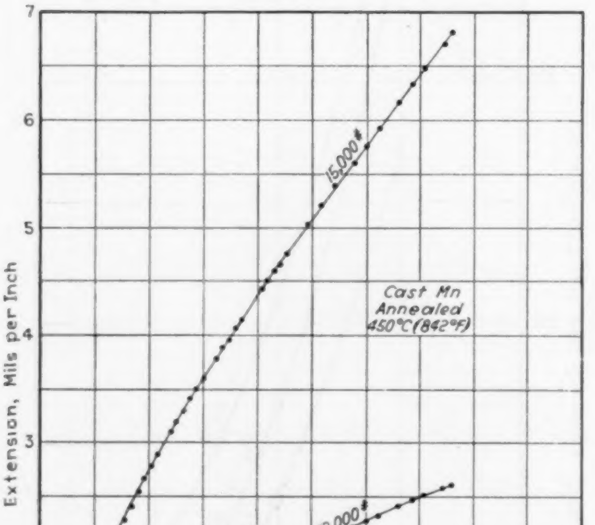


FIG. 18 CONSTANT-STRESS FLOW TEST OF FOUR BARS OF CAST-MANGANESE STEEL

(Composition: C, 0.31; Mn, 1.40; Si, 0.36. Heat treatment: 900°C., 6 hours, F.C.; 850°C., 4 hours, F.C.; 700°C., 4 hours. F.C.)

TEST DATA

Table 1 gives a summary of the results of a number of flow-rate tests.

Figs. 8 to 14 inclusive are flow-time curves of several of the tests summarized in Table 1. Stresses are given in thousands of pounds per square inch, and flow rates are expressed in per cent in 100,000 hours. Elastic extension is shown as well as plastic deformation. Fig. 16 gives logarithmic plots of stress versus flow rate. Fig. 17 shows the relation between temperature and the stress required to produce a flow rate of 1 per cent in 100,000 hours.

The following items are steels which were tested in both the annealed and in the heat-treated condition:

Annealed	Heat treated
265	264
252	253
246	240
123	122
259	257
114	115

These tests, and other tests listed in Table 1, where similar but not identical steels were tested in both the annealed and heat-treated conditions, indicate that frequently annealed steels resist flow at elevated temperatures better than heat-treated steels.

Fig. 15 is a test of a nickel-chrome-molybdenum steel, in which the (elastic plus plastic) extension was held approximately constant at 1 mil per inch. During the course of the test, 1340 hours, the stress was decreased gradually from 22,700 lb. per sq. in. to 15,000 lb. per sq. in., and the flow rate at the end of the test was 0.8 per cent in 100,000 hours.

Fig. 18 shows the results of a test in which four bars of the same cast manganese steel were held at 450 deg. cent. (842 deg. fahr.) with constant stresses of 15,000, 12,000, 9000, and 6000 lb. per sq. in. for 1514 hours (63 days). Tests of this nature provide data on total flow and on variation of flow rate with stress.

Appraisal of Results

The object of flow tests is to predict the ability of materials to maintain dimensions under stress at elevated temperatures for long periods of time, say, 5, 10, or even 20 years. From a family of curves as shown in Fig. 18, and in the light of flow rates established by stepdown loading, predictions may be attempted as to the probable flow in the course of, say, 100,000 hours. The analysis involving frankly a 20-to-1 or 40-to-1 extrapolation should be framed with due respect for metallurgical factors of vital importance. When long-time tests of a steel show a very small flow rate, decreasing as time goes on, it may be inferred with fair confidence that the extension-time curve will not suffer inflection.

Discussion

P. G. McVetty.[3] The demand for reliable creep data is increasing so rapidly that any possible simplification of the apparatus required is of great value. By putting four tests into one furnace the authors have greatly simplified the problem of controlling the temperatures of a large number of tests.

Another important simplification results from the use of the same specimen for several successive tests. There always has been some objection to this procedure because of the effect that strain hardening during a test may have upon those which follow it. The importance of this effect in comparison with that due to unavoidable differences among test specimens can be evaluated only after many more tests have been conducted.

From experience with creep curves, the writer has some doubts as to the reliability of rates determined in a period as short as 200 to 300 hours. If the rate continues to decrease as time goes on, the results obtained in a short test are on the safe side. In some cases, however, sudden increases of length are noted, with the result that the testing time is very much prolonged. It is evident that the analysis of the results depends greatly upon the decision as to when a creep rate representative of the stress has been obtained.

A study of the data given in Table 1 indicates the large effect upon safe working stresses which may result from what appear to be slight changes in analysis or heat treatment. For example, items 209 and 210 have similar analyses and identical heat treatments. The allowable stresses for the latter are appreciably lower in spite of a higher content of chromium and tungsten. Similar changes in results are observed when the same steel has different heat treatments. This leads to the conclusion that internal structure probably has an important effect upon creep phenomena.

The test represented by Fig. 15 is important in many applications. Instead of studying change of length at constant stress, it shows change of stress required to keep constant length. In practice this corresponds to the gradual reduction of stress in bolts due to creep. In the past very little has been done in this field of investigation.

In the concluding paragraph of the paper, the authors make the following statement, "When long-time tests of a steel show a very small flow rate, decreasing as time goes on, it may be inferred with fair confidence that the extension-time curve will not suffer inflection." This conclusion has not been proved, and it is open to some objection, especially at the higher test temperatures. For example, the test of item No. 210 at 550 deg. cent. and 4500 lb. per sq. in. indicates such an inflection. If we look upon creep as the net effect of the opposing actions of strain hardening and annealing and agree that annealing is a function not only of temperature but also of time of exposure, it seems inevitable that inflections will occur. This point can be settled by conducting creep tests over very long periods of time, and it is the opinion of the writer that a large number of very long tests at low stresses are necessary to explain creep phenomena.

F. H. Norton.[4] From what can be observed from the experimental curves it would seem that the precision in the measurement of temperatures and elongation is probably the best obtained by any experimenters on creep in this country. The comparatively long gage length of 12 in. is a distinct advantage in obtaining this precision. However, it is felt that this type of extensometer would not be as satisfactory for the higher temperature tests—that is, 1200 to 1500 deg. fahr.

It is interesting to observe that, where it is possible to make comparisons, the results obtained by the authors agree very well with our results, except for steel No. 209, where with a specimen of approximately the same composition we obtain approximately one-third of their value. Unfortunately we have very few tests at the temperature of 842, where most of the tests were reported in this paper.

Steels 117C and 119C, which are of approximately the same composition, show creep stresses in the ratio of 1.16 to 1.00 at 400 deg. cent., and approximately a three-to-one difference at 450 deg. cent.

The authors have plotted their results on logarithmic paper, and it is observed that in all cases the slope of the resulting curve is one in five, which agrees exactly with the results that we have been obtaining upon steels, although in this paper the results have been extended to somewhat lower rates than we have ordinarily obtained.

C. L. Clark[5] and A. E. White.[6] As each of the values given is the result of tests lasting hundreds of hours, it is evident that an enormous amount of time and effort was expended.

The writers have had occasion to investigate many steels of the type reported, and they are pleased to find that in general these results are in agreement with those obtained by them.

It is interesting to note that of the three chromium-tungsten-vanadium steels considered, Nos. 209, 210, and 211, the one having the lowest alloy content, No. 209, possesses the greatest creep resistance over the entire temperature range considered. This also is in keeping with the writers' findings and furnishes conclusive proof that increases in the alloying content do not necessarily increase a metal's high-temperature stability, as is believed by many, and may often have the opposite effect.

In this same connection, the writers recently prepared a series of nearly 100 heats in which the amounts of alloying elements present were gradually varied. The results of certain of these tests will be presented in a future paper, but it may now be stated that with many of the alloying elements, especially those of the chromium and tungsten types, definite amounts of each must be present in order to obtain the maximum high-temperature properties. Variations either above or below these critical amounts result in a marked decrease in the resulting high-temperature properties, especially in those measured by the proportional limit and the creep tests.

If one examines a large number of the creep values that have been reported for plain carbon steels, he will be surprised at the large variations in them. For example, at 850 deg. fahr., which corresponds fairly well to the 450 deg. cent. (842 deg. fahr.) temperature used by the authors, the writers have found the values to vary from 5000 to 20,000 lb. per sq. in. Many of these differences may be accounted for by variations in such factors as carbon content, test procedure, heat treatment, and method of presentation of results, but often the differences are large even when these various factors are about the same. The writers believe much of the difference in these latter cases may be due to the method and care employed in making the steel which was used in the tests. For example, they recently determined the creep values of two plain carbon steels at 850 deg. fahr., both of which had the same composition and heat treatment, and differed only in that one was made according to alloy-steel practice while the other was the usual run of commercial steel. For the carefully prepared steel they obtained a value of 11,000 lb., while with the ordinary one a stress of only 7000 lb. was required to produce the same creep of 1 per cent per 100,000 hours. For this reason, they believe that in all tables of physical properties, the source of the material—that is, whether it is open-hearth, electric, induction, etc.—should be stated whenever possible.

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It is observed that in all the creep tests reported, either up-step or down-step loading was employed. The writers would like to ask the authors if the same material was ever subjected to both types, and if so, if the results were in agreement; also if single-step loading was ever used, and if so, if the results from this method compared favorably with those from the multiple-step procedure.

An interesting fact to be observed in connection with the down-step loading process is that all of the specimen's recovery does not always occur immediately upon removal of any given increment. We have observed this recovery lasting for periods of as long as 150 hours. The amount which does occur immediately upon removal of the load corresponds fairly well with the elastic deformation corresponding to that increment, while the remaining recovery seems to be at the expense of some of the plastic deformation which has previously occurred.

L. Levine.[7] The object of this discussion is to show the development and behavior of one mathematical expression involving the three variables, total flow, stress, and time, which fits the data given in Fig. 18 and affords some basis for further predictions.

TABLE 2 COMPARISON OF VALUES

t hours	Lb. per sq. in.	y calculated	y observed
100	6,000	0.132	0.13
	9,000	0.267	0.27
	12,000	0.540	0.53
	15,000	1.09	1.1
500	6,000	0.241	0.237
	9,000	0.560	0.57
	12,000	1.33	1.33
	15,000	3.16	3.19
1000	6,000	0.301	0.305
	9,000	0.767	0.77
	12,000	1.97	2.03
	15,000	5.07	5.07
1500	6,000	0.36	0.36
	9,000	0.96	0.95
	12,000	2.61	2.61
	15,000	7.06	6.76

The method of analysis used is "grapho-analytical." Relations between the variables are first found by plotting them on semi-logarithmic and logarithmic coordinates, and these later combined to give the complete expression:

$$y = \epsilon^{0.133\left\{St^{0.135} - \frac{32.4}{t^{0.046}}\right\}}$$

This gives the plastic extension y in terms of the stress S and the duration of stress t. That it is correct in the range covered by the available data is shown by Table 2, which compares the actual observed values for several different stresses and time intervals with values calculated from the foregoing empirical function.

Before commenting on this function, its limitations should be clearly recognized. All that is known is that for this particular steel, at this particular temperature, for stresses between 6000 and 15,000 lb. per sq. in. and a time duration of 1500 hours, the function expresses the behavior of the steel. That it will continue to do so over greater time intervals or for different stresses is not known. Nothing is said about other steels or other temperatures. But having this function, it is a matter of some curiosity to see how it behaves over a wide range. If the steel continues to follow it, well and good; if not, information must be sought in another direction.

1. When stress is zero,

$$y = \frac{1}{\epsilon^{4.3/t^{0.046}}}$$

This points to an increase of length even without stress—but only a slight increase. When t is 100,000, for example, y = 0.09 mil per in. Whether this is due to an error in the function, to actual physical growth of the metal, or to both, cannot be stated. In any case it is of small consequence.

2 When $t = 0$, $y = 0$, regardless of S. This is necessary because all flow is measured from the length at zero time.

3 When t is infinite, y is infinite for all positive values of S. This shows that the function does not approach an asymptote with time for any stress.

4 At a point of inflection the second derivative of y with respect to t is zero.

Some of the values of S which satisfy this relation are as follows:

S lb. per sq. in.	T hours	Y mils per inch
19,100	1,000	19
16,300	5,000	30
14,600	10,000	30
11,200	100,000	45
8,100	1,000,000	46

These results shown graphically would give a family of curves like Fig. 19.

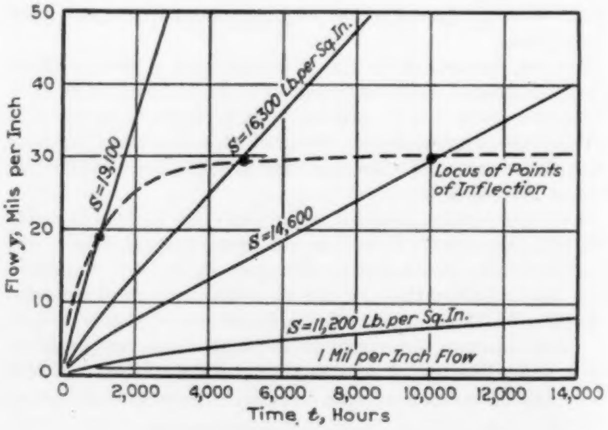

Fig. 19

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[7] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Jun. A.S.M.E.

The form of the equation indicates that there is an inflection in the curve for every stress. For the lower stresses, however, the inflection occurs at a time well beyond the life of any machine. This matter of points of inflection is an interesting one, for such points have been observed by various investigators.

5 Returning to the original flow function, we may note that when y = 1 mil per inch, the exponent of ϵ is zero. Therefore:

$$St^{0.173} = 32.4$$

Here is a matter which might deserve further investigation. If it were to be found that an expression of the form St^n = constant stated a necessary relationship between S and t for a total flow of 1 mil per inch, then by running tests on two or three bars at moderate stresses for a short time, the two empirical constants could be determined and the limiting design stress easily calculated. One cannot, of course, on the basis of this one test draw any conclusions on this subject. It is, however, an interesting subject for speculation.

E. L. Robinson.[6] The manner in which the authors have conducted their tests and presented the results makes them of particular value to designers of high-speed machines interested in maintaining dimensions over long periods of time.

These tests have been made on long test pieces, over long periods of elapsed time, at moderate stresses and extensions. These facts make for confidence in accepting the conclusions for application to actual behavior of machines.

Speed of measurement of flow rate could have been secured by causing the apparatus to magnify minute extensions occurring in a small gage length over a short time. But the results plotted in Fig. 18 show that flow rates based on a short elapsed time do not have the validity of rates based on longer periods of elapsed time.

In fact, with tests covering still longer periods, it may be necessary, for steady stress conditions, as suggested in the discussion by Mr. Levine, to find better methods for analytic interpretation than the logarithmic plots shown in Fig. 16 which characterize flow-rate tests under conditions of stress relief approximating constant extension.

The need for conducting tests showing behavior under different conditions of service is frankly recognized. In fact, the authors have not specifically pointed out that three of the flow-rate tests give curves for stress relief at constant extension similar to that shown in Fig. 15, while Fig. 15 itself constitutes a flow-rate test. Also among the flow-rate tests are three incidental steady stress tests of 500 hours' duration similar to those given in Fig. 18, which would be listed as "long time tests" by many other authors.

The futility of trying to judge relative dimensional stability of materials at high temperature on the basis of short-time strength tests at high stress is well demonstrated by the convergence of the lines in Fig. 16, for the 550 deg. cent. tests. The relative desirability of these materials would be reversed for short applications of high stresses.

Authors' Closure

Mr. McVetty is undoubtedly right in believing that anomalous behavior in creep may escape detection in tests of 200 to 300 hours. The time required to establish a flow rate depends upon the order of the magnitude of the rate. In our apparatus 200 to 300 hours will usually be sufficient time to establish a rate of 10 per cent in 100,000 hours. A run of at least 500 hours, however, is required ordinarily to establish a flow rate of the order of 1 per cent in 100,000 hours.

It also is true that the result of a flow-rate test depends upon the portion of the flow-time curve used to establish the flow rate. Fig. 18 shows that the rate of flow changes as flow takes place and as time goes on. In our usual flow-rate tests it is our practice to stretch the test bars 1 mil per inch before establishing the first flow rate, thereby putting the tests on an equal basis from the standpoint of the amount of flow.

In the last sentence of the paper it was implied that for the small stresses permissible in design the extension-time curve will not suffer inflection during the usual life of a machine. Whether such inflection may occur at some remote time cannot now be determined. Mr. Levine's discussion bears on this point. His equation, which is based on a single test (the one shown in Fig. 18), shows inflection points for small flow rates, but at distant points on the time axis.

Messrs. Clark and White also raise a point which has a bearing upon this matter of inflection. They point out that upon removal of a stress there is an elastic after-effect. We have frequently observed that this effect lasts for several hundred hours with no observable flow. (See item 210 at 550 deg. cent.) Sometimes there may be a partial recovery during this time, followed later by a change to a higher rate of flow. When using the step-down method, we have to use our judgment in regard to the length of time necessary to continue the test in order to be sure that we have passed this point of early inflection. By plotting three different flow rates on the same specimen at each temperature, we obtain a reasonably safe check in this regard.

Mr. F. H. Norton questions the applicability of the dial gage to flow tests at temperatures above 600 deg. cent. We have used them in tests made at temperatures up to 800 deg. cent. without any more difficulty than at lower temperatures.

The falling off in the ratio of the strength of the carbon steel 119 with respect to 117, at 450 deg. cent., is typical of the behavior of some quenched steels when compared with the same steels in the annealed condition. The high value shown for the steel 209 may be attributed to either or both of two factors: (1) The tungsten content may represent an optimum value, being approximately 2 per cent—we have been unable to get equally high values with tungsten contents of from 1.63 to 1.80 per cent, and Mr. Norton's specimen was 1.7 per cent; (2) steel 209 was hot-rolled to 0.5 in. in diameter, while our other specimens were rolled to 0.75 in. or greater in diameter; hence the amount of hot-working received by steel 209 was greater.

Messrs. Clark and White also comment on the optimum percentage of an alloying element such as tungsten. When we raised the tungsten content to 2.9 per cent, in steels 101 and 210 the strength definitely fell off at 500 deg. and at 550 deg. when compared to 209. As to the source of carbon steels we have confined our tests so far to open-hearth steels.

The results of the tests by Messrs. Clark and White, wherein a study of a series of steels of varying alloy content is being made, will be looked forward to with great interest, as systematic tests of this nature will be very useful.

Of the three methods used for loading the specimen, the step-up method is of least interest to our conditions of application, owing to the difficulty in judging the transition from the first stage to the second stage of the flow. When testing new types of steel, the most expeditious method for obtaining stable rates of flow is by first using the step-up method until a preliminary extension is obtained, amounting to about 1 mil per inch. The stress is stepped up with sufficient rapidity to obtain this extension in from three to seven days with flow rates as high as from 30 per cent to 70 per cent in 100,000 hours. The stress is then stepped down to obtain a flow rate of approximately 10 per cent for a sufficient length of time for it to become reasonably stable. It is then stepped down again to obtain, successively, rates of around 2 or 3 per cent, and finally around 1 per cent.

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[6] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

After sufficient preliminary exploration has been done by the step-down method to enable us to select particular types of steel for further investigation, we make confirmatory tests by the constant-stress method, as indicated in Fig. 18 in the paper.

We have run several constant-stress tests for periods of about six months, and then have plotted the final rates of flow against the stress on log-log paper. The four points usually fall fairly near a straight line, the deviations evidently being due to differences in strain hardening between the four bars. The rates of flow indicated by the straight line agree reasonably well with those obtained by the step-down method, provided we give preference to the points which represent the flow rates on the more highly stressed specimens.

The constant-stress tests, however, give information of a different character which is just as useful for design purposes as the flow-rate tests.

The Flexibility of Corrugated Pipes Under Longitudinal Forces and Bending

By L. H. DONNELL,[1] PASADENA, CALIF.

Pipes with circumferential corrugations are being used in expansion bends in steam lines and in other applications where the maximum longitudinal or bending flexibility is required. In the present paper the longitudinal flexibility is calculated for several types of corrugations, and from this a "reduced modulus of elasticity" is derived by which corrugated pipes can be calculated as if they were smooth. It is then proposed to use this same reduced modulus for calculating the bending of corrugated pipes. A set of experiments is described which check these results.

The flexibility of pipes with corrugations cannot be calculated by elementary methods because the change of radius during the bending of the corrugations produces circumferential tensile or compressive stresses which make the bending similar to that of a beam on an elastic foundation. By using the method of internal energy, a "reduced modulus of elasticity" has been computed for pipes with V-shaped corrugations (Fig. 1) and with semicircular corrugations (Fig. 3 when $h = b$), for longitudinal compression or extension of the pipe (see Appendix). A formula has been derived by analogy for the intermediate case of corrugations of parabolic or sine-wave shape (Fig. 2). In using these reduced moduli, calculations are to be made as if the pipe were not corrugated (but having the same average radius, length, and wall thickness) and had the reduced modulus as its modulus of elasticity.

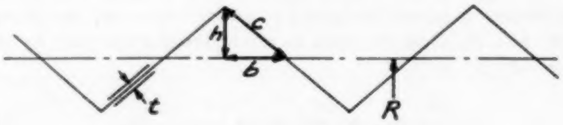

Fig. 1 V-Shaped Corrugations

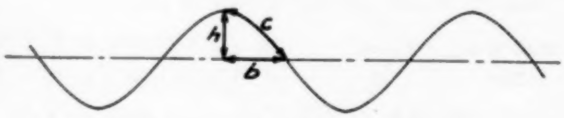

Fig. 2 Parabolic or Sinusoidal Corrugations

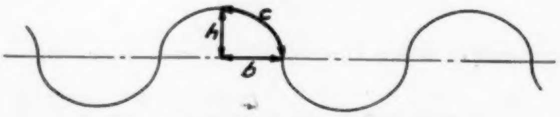

Fig. 3 Semielliptical Corrugations

It is assumed that the same reduced moduli can be used in computing bending deflections of corrugated pipe,[2] the calculations being made as if the pipe were not corrugated. There is no theoretical evidence to support this assumption, but it seemed reasonable, as the stiffness against transverse shearing of corrugated pipe is high, and it checked well with a set of experiments described in the Appendix. In computing deflections of pipe bends by this method it is not known what effect must be expected due to the flattening of cross-sections of the pipe, which flattening has been shown to have considerable influence in the case of plain pipe. It is probable that this flattening has much less effect with corrugated pipe than with plain pipe, as the resistance to flattening is greater and the forces producing it are smaller than with plain pipe.

In presenting the results below, the different types of corrugations have been dimensioned and treated in as similar a manner as possible. This leads to apparently unnecessary complications in some cases, but makes easier the comparison of the different types and the derivation of approximate formulas for intermediate types by analogy.

It is assumed that the depth of the corrguations is small compared with the radius of the pipe, and that the wall thickness is fairly small compared with the radius of curvature of the corrugations. However, the formulas developed should hold approximately if this is not the case.

Symbols

E = modulus of elasticity of the material
E' = "reduced modulus of elasticity," with which a plain pipe of the same length and wall thickness has the same flexibility
t = wall thickness
R = average radius of pipe
b, h, c = dimensions of corrugations as shown in Figs. 1–3
β = $(cb/Rt)^2$
u, v, w = longitudinal, radial, and normal displacements, respectively
s = distance measured along the curve.

The "reduced modulus" is then found to be

$$E' = \frac{E}{1 + \dfrac{4ch^2}{bt^2} K} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where K is a number depending on the corrugations. For V-shaped corrugations (Fig. 1):

$$K = \frac{h+b}{h+b} \, \frac{1}{1 + 2\beta - 0.14\,\beta^{1.5}} = \frac{1}{1 + 2\beta - 0.14\beta^{1.5}} \quad \ldots [2]$$

For parabolic or sinusoidal corrugations (Fig. 2) a very approximate formula is proposed:

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[1] Research Fellow in Aeronautical Structures, Guggenheim Aeronautical Laboratory, California Institute of Technology. Mem. A.S.M.E. L. H. Donnell was born in 1895 in Maine. He is a graduate in Mechanical Engineering, University of Michigan, 1915; Doctor's degree, University of Michigan, 1930. He worked in the engineering departments of automobile companies (Franklin, Packard, Dodge) for seven years, and taught engineering mechanics as assistant professor, University of Michigan, for seven years.

Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of The American Society of Mechanical Engineers. [The work described was carried out at the University of Michigan.]

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] S. Timoshenko suggested that this might be possible.

$$K = \frac{h + 1.6b}{h + b}\,\frac{1}{1 + 1.4\beta - 0.15\beta^{1.4}} \quad\ldots\ldots\ldots[3]$$

For semicircular or semielliptical corrugations (Fig. 3):

$$\left.\begin{aligned} K &= \frac{h + 2b}{h + b}\,\frac{1}{1 + \beta - 0.16\beta^{1.3}} \\ &= \frac{1.5}{1 + \beta - 0.16\beta^{1.3}},\ \text{if } h = b \end{aligned}\right\} \quad\ldots\ldots\ldots[4]$$

This formula has been proved only for the case when $h = b$, but it is probably roughly correct for semielliptical corrugations with h different from b.

Appendix

V-Shaped Corrugations. If h is small compared with R, Fig. 4, and the material has the same properties in tension and compression, we can assume without much error that points such as P and P' do not move radially (*vertically* in the figure), and we can use the average radius R in computing the width and elastic support of an elemental strip, such as that cut out by the planes M–N and M'–N'. Under these assumptions the points P and P' become points of inflection and the problem can be reduced to that of a bent strip (Fig. 5), of uniform width αR (where α is a small angle) under horizontal end loads $\alpha F/2\pi$ and a vertical elastic support proportional to the vertical displacements v (due to tangential stresses). The tangential strain will be the same as the radial strain or approximately v/R, making the tangential stress Ev/R, or a force per unit length of strip Etv/R. The resultant of such forces on each side of the strip will then be a vertical elastic supporting force $(Et\alpha/R)\,v = kv$ per unit length of strip.

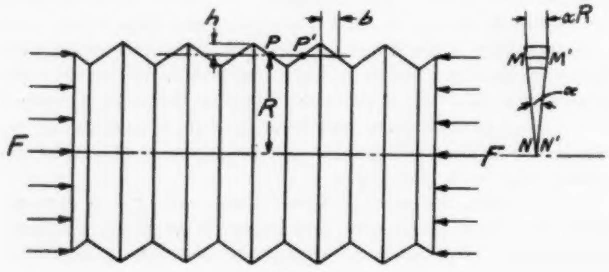

FIG. 4

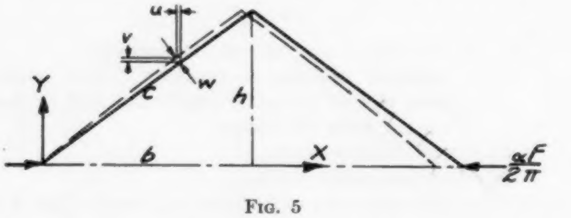

FIG. 5

In Fig. 5, u represents horizontal, v vertical, and w normal deflections, and s distances along the strip. Then:

$$ds = \frac{c}{b}\,dx,\quad dw = \frac{c}{b}\,dv \ \text{(for small deflections)}$$

Hence the bending moment is

$$M = EI\,\frac{d^2w}{ds^2} = \frac{b}{c}\,EI\,\frac{d^2v}{dx^2} \quad\ldots\ldots\ldots\ldots\ldots[5]$$

M and v are zero at the ends of the strip, and these conditions, as well as those of symmetry and continuity at the middle, are satisfied if:

$$v = \Sigma a_n \sin\frac{n\pi x}{2b} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots[6]$$

where n is odd only. Then, using [5]

$$M = -\frac{EI\pi^2}{4cb}\,\Sigma n^2 a_n \sin\frac{n\pi x}{2b} \quad\ldots\ldots\ldots\ldots\ldots[7]$$

The internal energy of bending is then

$$W_1 = \int_0^{2c}\frac{M^2 ds}{2EI} = \frac{EI\pi^4}{32cb^3}\int_0^{2b}\left(\Sigma n^2 a_n \sin\frac{n\pi x}{2b}\right)^2 dx$$
$$= \frac{\pi^4 E\alpha R t^3}{384\, cb^2}\,\Sigma n^4 a_n^2 \ldots[8]$$

The work done against elastic forces is, using [6],

$$W_2 = \int_0^{2c}\frac{kv^2 ds}{2} = \frac{E\alpha tc}{2Rb}\int_0^{2b}\left(\Sigma a_n \sin\frac{n\pi x}{2b}\right)^2 dx = \frac{E\alpha tc}{2R}\,\Sigma a_n^2 \quad\ldots\ldots[9]$$

The horizontal shortening due to bending is, from [6],

$$\Delta = 2(u_{x=b}) = 2\,\frac{h}{b}\,(v_{x=b}) = \frac{2h}{b}\,(a_1 - a_3 + a_5 \ldots) \ldots[10]$$

If the strip is given a small displacement from the position of equilibrium, produced by giving a_n an increment da_n, the change in W_1 and W_2 must be equal to the external work done by the force $\alpha F/2\pi$, or:

$$\frac{\partial W_1}{\partial a_n}\,da_n + \frac{\partial W_2}{\partial a_n}\,da_n = \frac{\alpha F}{2\pi}\,\frac{\partial\Delta}{\partial a_n}\,da_n \quad\ldots\ldots\ldots[11]$$

or, using [8], [9], and [10] and dividing by da_n,

$$\frac{\pi^4 E\alpha R t^3 n^4 a_n}{192cb^2} + \frac{E\alpha t c a_n}{R} = \pm\frac{\alpha F h}{\pi b}\begin{pmatrix} + \text{ for } n = 1, 5, 9 \ldots \\ - \text{ for } n = 3, 7, 11 \ldots \end{pmatrix}$$

$$a_n = \pm\frac{192\,Fhcb}{\pi^5 ERt^3}\,\frac{1}{\left(n^4 + \dfrac{192}{\pi^4}\,\dfrac{c^2b^2}{R^2t^2}\right)} = \pm\frac{2Fhcb}{\pi ERt^3}\,\frac{1}{n^4 + 2\beta}$$

approximately, where $\beta = (cb/Rt)^2$. Hence, using [10],

$$\Delta = \frac{2h}{b}\,(a_1 - a_3 + a_5 - \ldots) = \frac{4Fh^2c}{\pi ERt^3}\left(\frac{1}{1 + 2\beta} + \frac{1}{81 + 2\beta}\right.$$
$$\left. + \ldots\right) = \frac{\pi Fh^2c}{4ERt^3}\left(\frac{1}{1 + 2\beta - 0.14\beta^{1.6}}\right),\ \text{approx.}\ldots[12]$$

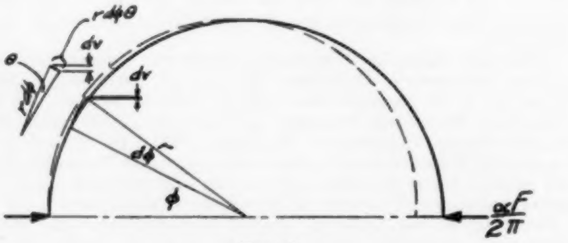

FIG. 6

The expression in the last parenthesis was obtained empirically after calculating values of the series it replaces for values of β from zero to 100.

Semicircular Corrugations. In a similar manner the case of semicircular corrugations may be reduced to that of an arched strip, as shown in Fig. 6. Due to bending of the strip there will be some rotations of cross-sections of the strip as indicated by the angle θ in the figure. This rotation will produce linear displacements of points in the strip. The change in radial displacements in a length $rd\phi$ of the strip, due to the rotation θ is $rd\phi\theta$, and the corresponding change in vertical displacement is:

$$\left.\begin{aligned} dv &= rd\phi\theta \sin\phi \\ \text{whence} \qquad \theta &= \frac{1}{r\sin\phi}\frac{dv}{d\phi} \end{aligned}\right\} \qquad [13]$$

The rotation θ is caused by bending, and the change in θ in an element of length $rd\phi$ (in other words, the relative angular displacement of the two ends of the element) is, just as in the bending of a straight beam:

$$d\theta = \frac{M}{EI}(rd\phi)$$

Hence, using [13],

$$M = \frac{EI}{r}\frac{d\theta}{d\phi} = \frac{EI}{r^2}\frac{d\left(\frac{1}{\sin\phi}\frac{dv}{d\phi}\right)}{d\phi} \qquad [14]$$

The bending moment is zero at the ends of the strip, and we can write

$$M = \frac{EI}{r^2}\Sigma a_n \sin n\phi \qquad [15]$$

where n is odd only, from symmetry. Substituting this in [14] we have

$$\Sigma a_n \sin n\phi d\phi = d\left(\frac{1}{\sin\phi}\frac{dv}{d\phi}\right)$$

Integrating both sides,

$$-\Sigma\frac{a_n}{n}\cos n\phi = \frac{1}{\sin\phi}\frac{dv}{d\phi} + C_1$$

When $\phi = \pi/2$ we have $dv/d\phi = 0$, from which $C_1 = 0$. Then

$$dv = -\Sigma\frac{a_n}{n}\sin\phi\cos n\phi\, d\phi$$

Integrating once more,

$$v = -\frac{a_1}{2}\sin^2\phi + \sum_{n=3}^{n=\infty}\frac{a_n}{2n}\left(\frac{\cos(n+1)\phi}{n+1} - \frac{\cos(n-1)\phi}{n-1}\right) + C_2 \qquad [16]$$

When $\phi = 0$ or π, $v = 0$, from which

$$C_2 = \sum_{n=3}^{n=\infty}\frac{a_n}{n(n^2-1)}.$$

Then from [15] the internal energy of bending is

$$W_1 = \int_0^\pi \frac{M^2 rd\phi}{2EI} = \frac{EI}{2r^3}\int_0^\pi(\Sigma a_n \sin n\phi)^2 d\phi = \frac{E\alpha Rt^3\pi}{48r^3}\Sigma a^2{}_n \qquad [17]$$

The work done against elastic forces is, using [16],

$$W_2 = \int_0^\pi \frac{kv^2rd\phi}{2} = \frac{kr}{2}\int_0^\pi v^2 d\phi$$

$$= \frac{Et\alpha r}{8R}\int_0^\pi\left[-a_1\sin^2\phi + \sum_{n=3}^{n=\infty}\frac{a_n}{n}\left(\frac{\cos(n+1)\phi}{n+1} - \frac{\cos(n-1)\phi}{n-1} + \frac{2}{n^2-1}\right)\right]^2 d\phi$$

$$= \frac{Et\alpha r}{8R}\int_0^\pi\left[\left(-\frac{a_1}{2} + \sum_{n=3}^{n=\infty}\frac{2a_n}{n(n^2-1)}\right) + \ldots\left(\frac{a_n}{n(n+1)} - \frac{a_{n+2}}{(n+1)(n+2)}\right)\cos(n+1)\phi\ldots\right]^2 d\phi$$

$$= \frac{Et\alpha r\pi}{16R}\left[2\left(\frac{a_1}{2} - \sum_{n=3}^{n=\infty}\frac{2a_n}{n(n^2-1)}\right)^2 + \ldots\left(\frac{a_n}{n(n+1)} - \frac{a_{n+2}}{(n+1)(n+2)}\right)^2\ldots\right] \qquad [18]$$

The horizontal shortening due to bending is, using [15],

$$\Delta = \int_0^\pi \frac{Mrd\phi}{EI} r\sin\phi = \frac{\pi}{2}a_1 \qquad [19]$$

Using [11] and [17], [18] and [19] and letting $r^4/R^2t^2 = \beta^1$, we find

$$\left(\frac{1}{\beta^1} + \frac{9}{4}\right)a_1 - \frac{a_3}{4} - 3\sum_{n=3}^{n=\infty}\frac{2a_n}{n(n^2-1)} = \frac{6Fr^3}{\pi ERt^3\beta^1} \qquad [20]$$

$$\left(\frac{1}{3\beta^1} + \frac{8}{n^2(n^2-1)^2} + \frac{2(n^2+1)}{(n-1)^2n^2(n+1)^2}\right)a_n - \frac{2a_1}{n(n^2-1)} + \frac{4}{n(n^2-1)}\sum\frac{2a_p}{p(p_2-1)} - \frac{a_{n+2}}{n(n+1)^2(n+2)} - \frac{a_{n-2}}{n(n-1)^2(n-2)} = 0 \qquad [21]$$

where p has all odd values but 1 or n.

The numbers a_1, a_3, a_5, etc., diminish rapidly in importance. By taking all above a certain one as zero the remainder may be found from [21]. Then using [19] and [20] we find

$$\Delta = \frac{3Fr^3}{ERt^3[1 + 2.25\beta' - 0.2(\beta')^{1.2}]} \qquad [22]$$

The expression in the brackets was obtained empirically after calculating values of Δ for values of β' from 0 to 100.

Formula [22] is put into a form where it is probably approximately applicable to semielliptical corrugations by the following reasoning. The quantity in the brackets of the denominator represents the effect of the elastic support, while the remainder is the formula for Δ if there were no elastic support. The last part can readily be calculated by elementary methods for the case of an ellipse (Fig. 3), and by combining the resulting formula with the parenthesis of [22] we obtain a form which is certainly true for $h = b$ and should be nearly true when h differs somewhat from b. To interpret the meaning of $\beta' = r^4/R^2t^2$ for the case of an ellipse we use an analogy with the previously calculated case of V-corrugations by setting

$$\beta' = \frac{r^4}{R^2t^2} = \frac{4}{\pi^2}\frac{\left(\frac{\pi}{2}r\right)^2 r^2}{R^2t^2} = \frac{4}{\pi^2}\frac{c^2b^2}{R^2t^2} = \frac{4}{\pi^2}\beta$$

All this has been done in deriving Equation [4].

Equations [1], [2], and [4] were obtained by setting the values of Δ in [12] and [22] equal to the value of Δ for a plain pipe with modulus of elasticity E'.

$$\Delta + \frac{F2b}{E2\pi Rt} = \frac{F2b}{E'2\pi Rt}$$

$$E' = \frac{E}{1 + \frac{\pi ERt\Delta}{Fb}}$$

The quantity $F2b/E2\pi Rt$ is the shortening of the corrugated pipe due to direct stress, which must be added to the shortening due to bending, previously calculated. This is of no importance except for very shallow corrugations, and its value then approaches that given.

Formula [3] is derived simply by analogy from [2] and [4], considering that the case of parabolic or sinusoidal corrugations is an intermediate one, about six-tenths of the way between V-shaped and semielliptical corrugations, as elementary calculations show would be the case if there were no elastic support.

Experiments. A piece of corrugated culvert pipe was tested in direct compression and in bending, as shown in Figs. 7 and 8. In the compression test extensometers were placed on both sides of the specimen to eliminate the effects of eccentric loading. In the bending test the pipe was supported at the ends and the load divided between two points equal distances from the ends, so that the center of the pipe was in a condition of pure bending. Deflections in the center portion were measured with the aid of unloaded bars hung from the pipe. Dimensions of the pipe were R=5.33 in.; t=0.065 in.; b=0.670 in.; h=0.235 in.; c=0.735 in. The shape of the corrugations was between the sinusoidal and V-shape—nearer the former. The reduced modulus E' calculated from the results of the direct loading test was 1,510,000 lb. per sq. in.; from the bending test it was 1,450,000 lb. per sq. in. The theoretical value of E' from Formula [2] was 2,250,000 lb. per sq. in.; and from Formula [3] it was 1,200,000 lb. per sq. in.

$E = 3 \times 10^7$

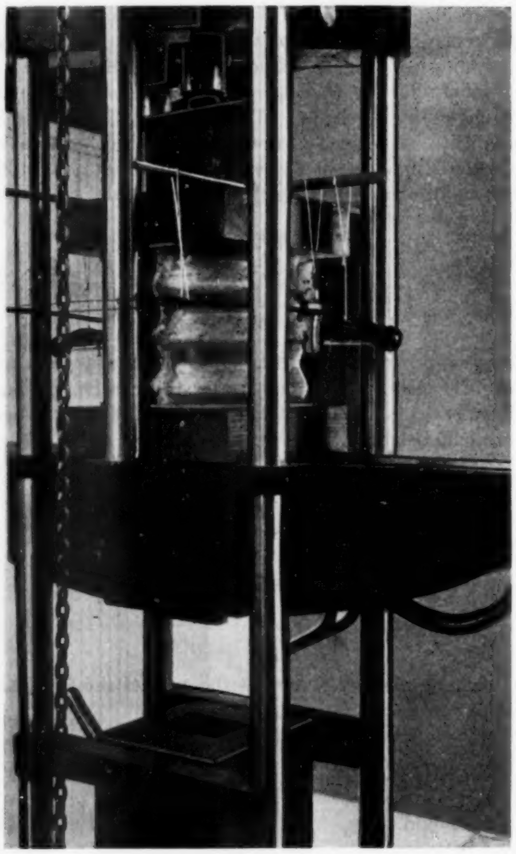

Fig. 7 Direct-Compression Test

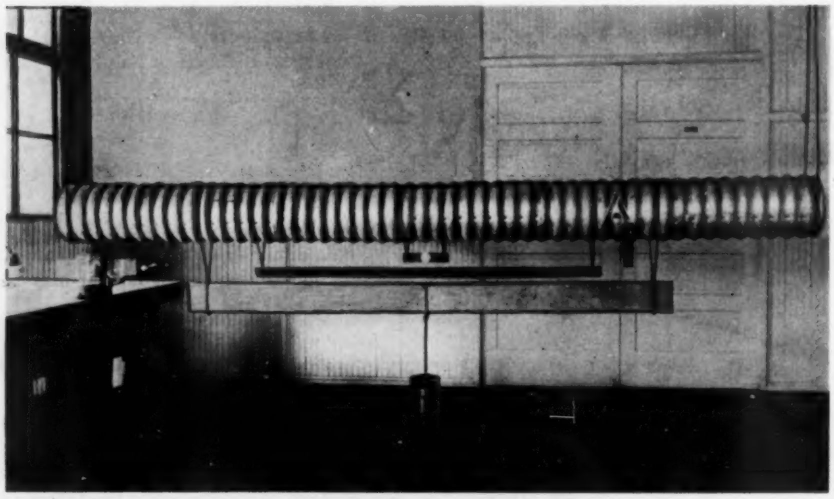

Fig. 8 Bending Test

Discussion

E. T. Cope[3] and E. A. Wert.[4] The author is to be congratulated upon the deduction of an empirical formula for the calculation of K', "reduced modulus of elasticity." Beyond a doubt this form of analysis is applicable to a corrugated pipe in which the profile of the corrugation is symmetrical, either parabolic, sinusoidal, or semicircular, and the thickness of pipe wall is small in comparison with the height of the corrugations. However, the writers are fearful lest the implication contained in the first sentence of the summary may lead to calculated results, when applied to available corrugated steam-pipe members, which are not in accordance with the results of tests.

During the summer of 1930 the writers conducted a series of tests to determine load-deflection relations for ten pieces of high-pressure steam pipe of 10 in. and 14 in. nominal diameter. Three of these pieces of pipe were smooth, five were corrugated, and two were creased. The corrugated pipes were made under the patents of P. Koffoski, of Poland.[5] As far as the writers know, this is the only kind of corrugated pipe now procurable for use in steam-power-plant installations. The results of these tests, together with the mathematical calculations of load-deflection relation for smooth-pipe members of the same dimensions as the corrugated pipe, were published in a paper entitled "Load-Deflection Relations for Large Plain, Corrugated, and Creased Pipe Bends," by the writers.[6]

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[3] Research Department, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

[4] Drafting and Surveying Bureau, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

Figs. 9 and 10 show the corrugated pipe on which tension loading was applied. In Fig. 10 the tension loading was applied only on the straight part of the pipe member. Elongation was measured over the corrugated portion only. Figs. 11 and 12 show dimensioned drawings of these two pipes, and Figs. 13 and 14 the profiles of the corrugations of these pipes, as nearly as it was possible to measure them.

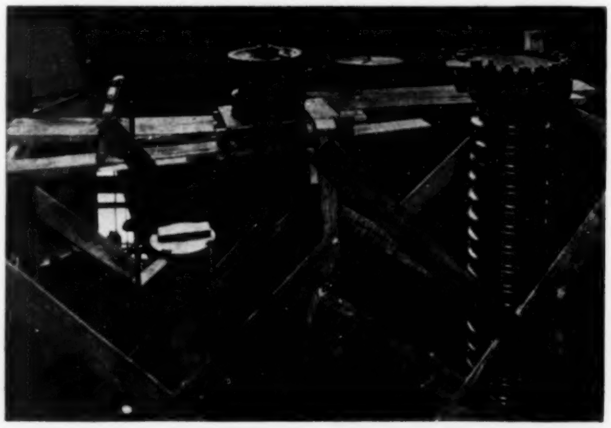

FIG. 9 CORRUGATED PIPE ON WHICH TENSION LOADING WAS APPLIED

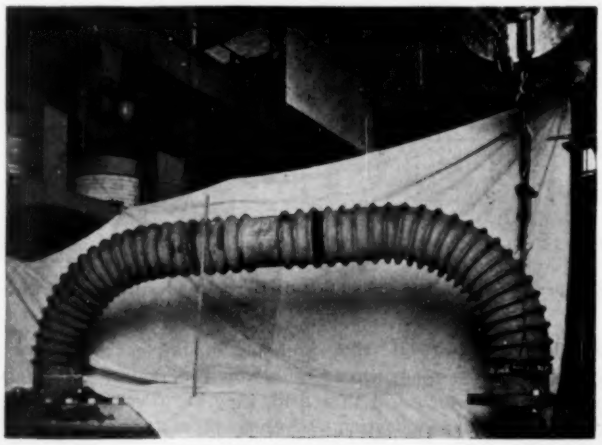

FIG. 10 TENSION LOADING APPLIED ONLY ON STRAIGHT PART OF PIPE MEMBER

It should be remembered that the corrugations on commercial corrugated steam pipe such as those shown are not of exact form. The method of fabrication results in each separate corrugation being an individual. The corrugations are not symmetrical curves such as those pictured in the paper. Again, the wall thickness is likely to be large in comparison with the radius of the corrugation. The 14-in. pipe, it will be noted, had a $^1/_2$-in. wall and the mean radius of the crest of the corrugation was 1 in. It appears that the validity of the empirical formula deduced depends upon the fact that the radius of trough and crest of the corrugations are equal. This makes its application to commercial corrugated steam pipe of questionable value.

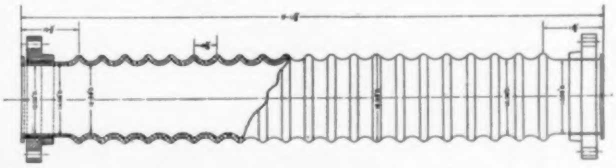

FIG. 11 DIMENSIONAL DRAWING OF 14-IN. CORRUGATED TANGENT PICTURED IN FIG. 9

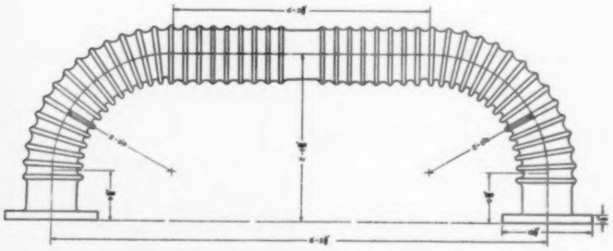

FIG. 12 DIMENSIONAL DRAWING OF 10-IN. CORRUGATED EXPANSION U-BEND PICTURED IN FIG. 10

(Mean outside diameter at ends, 10.36 in.; mean thickness of wall at ends, 0.22 in.; mean inside diameter at ends, 9.22 in.; mean outside diameter of crests of corrugations, 12.11 in.; mean outside diameter of troughs of corrugations, 10.25 in.)

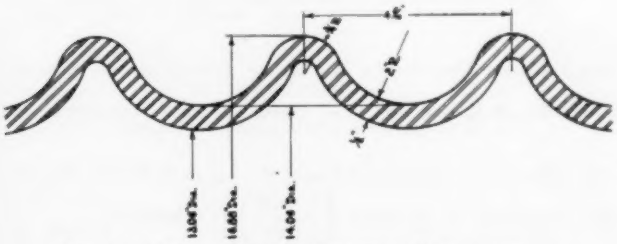

FIG. 13 PROFILE OF PIPE SHOWN IN FIGS. 9 AND 11

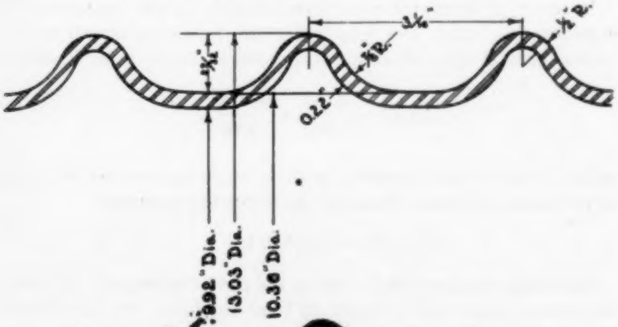

FIG. 14 PROFILE OF PIPE SHOWN IN FIGS. 10 AND 12

The load-deflection relations under axial loading of the two corrugated pipes which the writers have tested are shown in Figs. 15 and 16. On the same sheets are shown the calculated load-deflection relations of smooth-pipe members of the same major dimensions.

Further study showed that the elongation of such a corrugated pipe as either one shown, due to thermal expansion, cannot be taken up by the compression of the pipe unless a large axial restraining force was applied. As an example, consider the corrugated tangent such as the straight portion of Fig. 10. In Fig. 16 the elongation in tension is seen to be 0.005 in. in 19.47 in. of length when an axial load of 9000 lb. was applied. The coefficient of thermal expansion for mild steel is, according to Halborn and Day, equal to $0.006212\left(\frac{t-32}{100}\right) = 0.001623\left(\frac{t-32}{100}\right)^2$, where t is the temperature of the steel in degrees fahrenheit.

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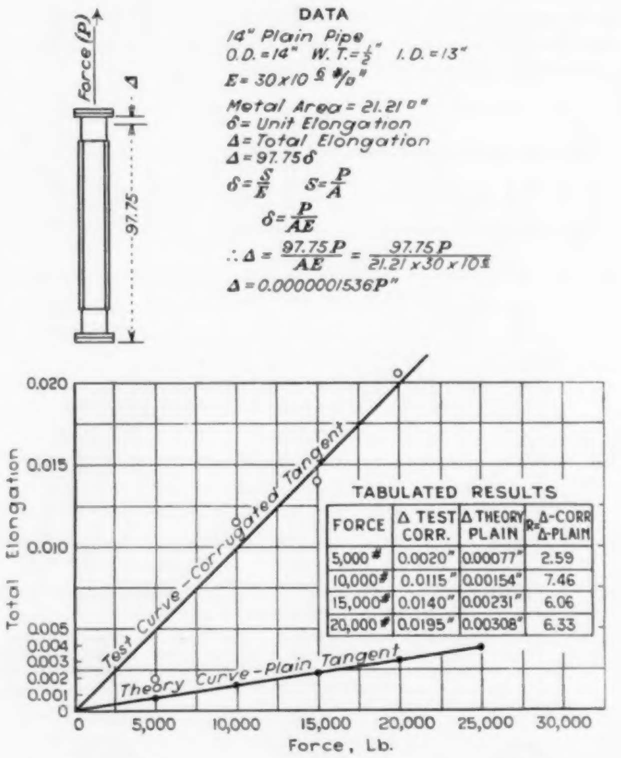
[5] Corrugated pipe, according to the Koffoski patent, is made in England by Aiton & Co., Ltd., Derby, and in Germany by Seiffert & Co., Düsseldorf.

[6] Presented at the A.S.M.E. Hartford, Conn., Meeting, June 1-3, 1931.



TABULATED RESULTS

FORCE	Δ TEST CORR.	Δ THEORY PLAIN	R = Δ-CORR / Δ-PLAIN
5,000#	0.0020"	0.00077"	2.59
10,000#	0.0115"	0.00154"	7.46
15,000#	0.0140"	0.00231"	6.06
20,000#	0.0195"	0.00308"	6.33

FIG. 15 LOAD-DEFLECTION RELATIONS UNDER AXIAL LOADING OF 14-IN. PIPE

(Comparison of elongation of 14-in. smooth pipe tangent having the same major dimensions as a 14-in. corrugated tangent.)

If the pipe temperature (steam, liquid, or air temperature) were 500 deg. fahr., one may write the following equation between thermal elongation and compression due to axial loading:

$$19.47 \times \frac{3.58}{1200} = \frac{0.005}{9000}P$$

where P is the axial thrust in pounds necessary to hold the pipe in its initial position. Solving the foregoing equation:

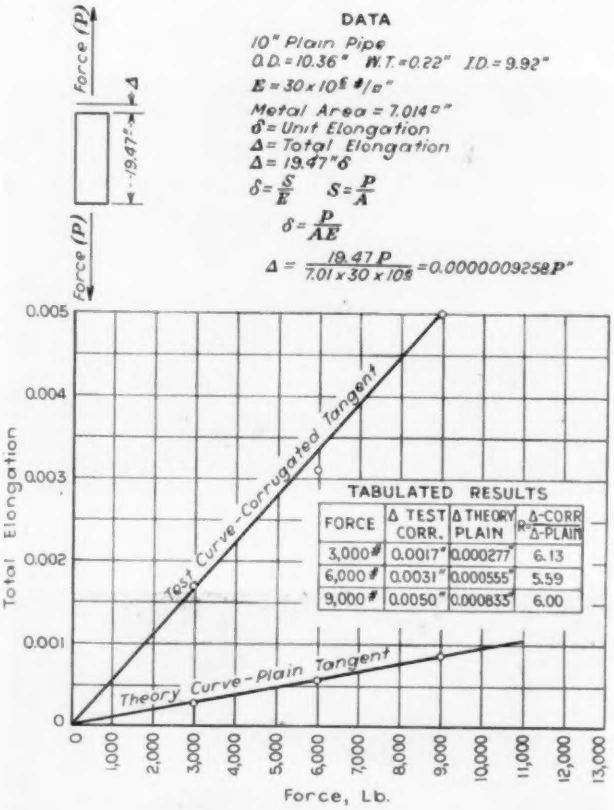
$$P = 104,550 \text{ lb.}$$

Obviously such an axial thrust could not be tolerated. If, now, the steam temperature should be 1000 deg. fahr., the axial thrust would be much larger than that calculated. It is the practice of designers of steam piping [illegible]e tangents in bending rather than in compression. The corrugated tangent in bending is a highly flexible member which should find wide application in piping design.

By way of a summary, it may be remarked that a mathematical formula, either rational or empirical, applicable to the load-deflection relations for corrugated tangents (and bends also) would be of considerable value to the designer of power-plant steam piping. This is particularly true in stations in which higher steam temperatures are being contemplated. It must not be forgotten, however, that the form of corrugation obviously should be that which can be commercially produced at a price within the reach of the prospective user.

AUTHOR'S CLOSURE

It is not quite correct to call the formulas developed in the paper "empirical." The formulas for V-shaped and semicircular corrugations are admittedly approximate, but they are based entirely on theory and not on experiments. This has the important significance that, while they probably do not cover a certain field of application as accurately as would an empirical formula based on experiments in that field, they are presumably quite general in their application—covering all materials, sizes, and proportions, up to such very extreme proportions as would make the approximate assumptions on which the theory is based of major importance. They should therefore be valuable to designers in this new field of engineering, as indicating the general influence and importance of each of the various physical quantities involved, even if the final result is not very exact. And if experiments show serious discrepancies, then at least these formulas should be ideal bases for empirical formulas, using the same general form, but modifying the constants to conform to experiment.



TABULATED RESULTS

FORCE	Δ TEST CORR.	Δ THEORY PLAIN	R = Δ-CORR / Δ-PLAIN
3,000#	0.0017"	0.000277"	6.13
6,000#	0.0031"	0.000555"	5.59
9,000#	0.0050"	0.000833"	6.00

FIG. 16 LOAD-DEFLECTION RELATIONS UNDER AXIAL LOADING OF 10-IN. PIPE

(Comparison of elongation of 10-in. smooth pipe tangent having the same major dimensions as a 10-in. corrugated tangent.)

As to the formulas given for parabolic and elliptical corrugations, their only justification lies in the fact that at the present state of the subject they are probably better than nothing. By giving formulas for these progressively different types of corrugations it was felt that a good basis was given for estimating other intermediate shapes of corrugations.

As to corrugations which are not symmetrical, there is no reason to suppose that a fair estimate cannot be obtained from the formulas by using average values of the dimensions, at least if the asymmetry is not extreme. Thus for the 10-in. and 14-in. corrugated pipes tested by Messrs. Cope and Wert, using one-half the total depth and width of the corrugations for h and b, formulas [1] and [4] for elliptical corrugations give values for E/E' of 8.04 and 7.13, respectively. The results from the experiments with axial loads, found from the slopes of the test curves given in the discussion, were about 8.7 and 6.6.

APM-54-8

Celluloid as a Medium for Photoelastic Investigation

By R. H. G. EDMONDS[1] and B. T. McMINN,[2] SEATTLE, WASH.

R. H. G. Edmonds

This paper deals with the physical properties of celluloid and their bearing on its use in the photoelastic method of stress determination. The authors present curves and test data on American-made celluloid, showing stress-strain and time-creep relations. Their results indicate that the physical constants for this material are not fixed, but have a consistent variation with speed of loading. Their work shows that celluloid is not perfectly elastic under any condition of loading. They point out some important peculiarities in the interrelation between photoelastic color and stress and strain.

B. T. McMinn

THE nitrocellulose compound called celluloid and sold under trade names such as pyralin, viscoloid, xylonite, etc., is one of the most common materials used by photoelastic investigators for the manufacture of models. This material is easily obtained in sheets of uniform thickness and quality and it has many characteristics that recommend it for photoelastic use. It is readily machined and worked into complicated shapes and models. It has a low modulus of elasticity, which permits large strains with moderate loading. Its flexibility and toughness add to the ease with which it can be handled. There are, however, certain peculiarities in the physical and optical properties of this material which, in the opinion of some scientists, render the photoelastic results obtained somewhat inaccurate.

Other investigators are quite firm in their belief that the use of celluloid in photoelastic work is quite satisfactory.

Because of this difference of opinion and because information on American-made celluloid is rather meager, it seemed highly desirable to run a rather extensive series of tests on the physical and photoelastic properties of this material.

This paper contains a brief review of papers already published on the subject, together with a report on tests run by the authors.

For details regarding the photoelastic method or for information regarding other materials that can be used, the reader is referred to the bibliography at the close of this paper.

Review of Previous Work

Very complete data on the physical and optical properties of this material have been published by Coker and Chakko (3).[3] Their tests were made on xylonite of British manufacture. The following information taken from their paper is of interest in connection with the present investigation. The elastic limits are not tabulated in the table of physical constants, but in the text it is found that the values range between 1600 and 2500 lb. per sq. in. The moduli of elasticity range from 250,000 to 350,000. Many data are given regarding the optical properties of this material. These authors state in their conclusions that: "The whole of the evidence appears to show that the transparent celluloid examined obeys a linear stress-optical law which holds up to approximately twice the range of the elastic limit of stress; and that within this range optical determinations of stress distribution may be relied upon."

Filon and Jessop (8) in their interesting paper on "Stress-Optical Effects in Transparent Solids Strained Beyond the Elastic Limit," describe some highly important results. The celluloid (British xylonite) which they examined showed definite optical and strain creep. In their summary of conclusions they say: ". . . Optical creep is comparatively large and may easily amount to 10 per cent of the initial value in a few minutes. . . . This has an important bearing on stress determinations from observations of artificial double refractions in celluloid. . . . The stretch of celluloid follows laws closely similar to those of the optical retardation." The title of their paper would indicate that the writers were dealing with effects beyond the elastic limit, but just what they consider the elastic limit is not made clear. Strain creep apparently begins at much lower stresses than optical creep. They show strain-creep and recovery curves for a specimen loaded to 60 kg. per sq. cm., which is equivalent to 853 lb. per sq. in.; also similar results for a stress of 86 kg. per sq. cm., or 1220 lb. per sq. in. Optical-creep observations were made on specimens at somewhat higher stresses. This indicates the importance of these observations, as it would seem almost impossible to keep stresses within such limits when making photoelastic tests.

The German scientists Ambronn (6) and Wächtler (7), after

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[2] Assistant Professor of Mechanical Engineering, University of Washington. Mem. A.S.M.E. Professor McMinn holds the degrees of B.S. in Mechanical Engineering from Oregon State College and M.S. and M.E. from the University of Washington. His particular interests are in materials and testing, in which he has taught courses for twelve years. He has been engaged in research work in photoelasticity for the last six years.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Numbers in parentheses refer to items in the appended bibliography.

extended experimental investigation, have severely criticized the use of celluloid models for exploring the stresses in engineering structures. Their work was carried out on German celluloid.

As has been already stated, published test data on the properties of American-made celluloid are very meager. In 1923 Heymans and Allis (4) made tests on two different kinds of celluloid. Camphor celluloid (made by du Pont) gave an elastic limit of 3900 lb. per sq. in. and a modulus of elasticity of 363,000. Naphtha celluloid (manufacturer not given) showed an elastic limit of 5400 lb. per sq. in. and a modulus of 365,000. The stress-strain curves were not shown.

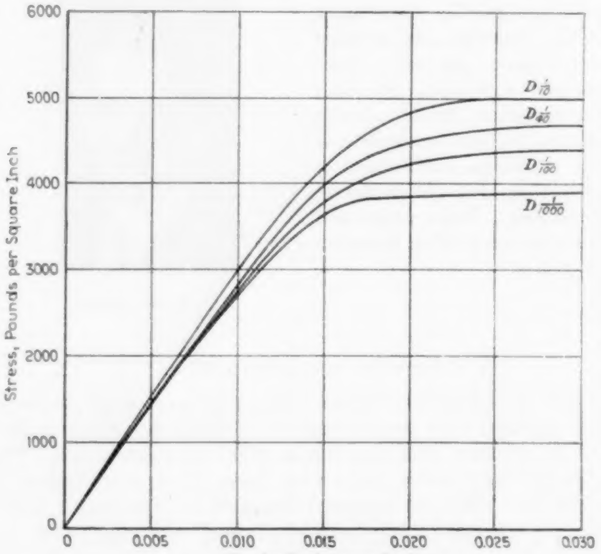

Fig. 1 Stress-Strain Curves—Material "D"
(Thickness of material, 0.250 in.; speeds, 0.001 in. to 0.1 in. per in. per min.)

Timoshenko and Dietz, in connection with their paper on "Stress Concentration Produced by Holes and Fillets" (5), ran some tests on du Pont celluloid and placed the elastic limit at 1700 lb. per sq. in. and Young's modulus at 302,000.

Very often the statement is made that so long as the stresses in the celluloid model are kept within the elastic limit, the results are satisfactory. There seems to be a difference of opinion as to the location of the elastic limit.

Tests on the Physical Properties

Material for these tests was obtained from the du Pont Viscoloid Co. and was of seven different thicknesses, as listed in Table 1. This material was of quite uniform thickness and quality. The purchases were made at five different times over a period of five years.

The test specimens were of a simple type for testing in tension. They were all made of approximately the same width of $^5/_8$ in., and of sufficient length to use a gage length of 10 in.

TABLE 1 PHYSICAL CONSTANTS OF THE MATERIAL TESTED

Material index	Thickness, inches	Speed, inches per inch	Modulus of elasticity	Elastic limit, lb. per sq. in.	Ultimate strength, lb. per sq. in.	Brinell hardness number
1	0.130	0.1	292,000	4200	5400	11.8
A	0.130	0.001	270,000	2900	4200	11.8
B	0.155	0.1	410,000	5800	7460	13.7
B	0.155	0.001	380,000	4600	5900	13.7
C	0.165	0.1	310,000	4000	5500	10.9
C	0.165	0.001	270,000	2900	4400	10.9
D	0.250	0.1	300,000	4100	5050	9.9
D	0.250	0.001	278,000	3400	3900	9.9
E	0.280	0.1	347,000	3900	5200	9.9
E	0.280	0.001	325,000	3000	4100	9.9
F	0.340	0.1	300,000	4400	5500	9.9
F	0.340	0.001	274,000	3060	3720	9.9
G	0.157	0.025	400,000	5500	7300	14.3
G	0.157	0.001	388,000	4300	6000	14.3

A series of tension tests were run, using a pendulum-type machine provided with an autographic arrangement. Early in this work it became apparent that the speed of loading should be taken into account. It was decided that four different speeds, based on the rates of elongation, should be used, namely, 0.100, 0.025, 0.010, and 0.001 in. per in. per min. Sufficient runs were made to get four curves for each thickness and type of celluloid available. Fig. 1 shows one set of these curves and is typical of the whole series. The complete stress-strain diagram is not shown. Each specimen was tested to the breaking point, however. The general tendency was for the stress, after passing the elastic limit, to drop off gradually till just before the point of rupture, when it rose slightly. The total elongation was somewhat less for high speeds than for low.

These stress-strain curves (Fig. 1 and the others taken) indicate definitely that the nominal elastic limit and the ultimate strength increases with an increase in the speed of loading. There also seems to be a change in the modulus of elasticity.

In order to investigate these matters more precisely a mirror extensometer was used, with which it was possible to secure reliable and consistent results, even at light loads. The data ob-

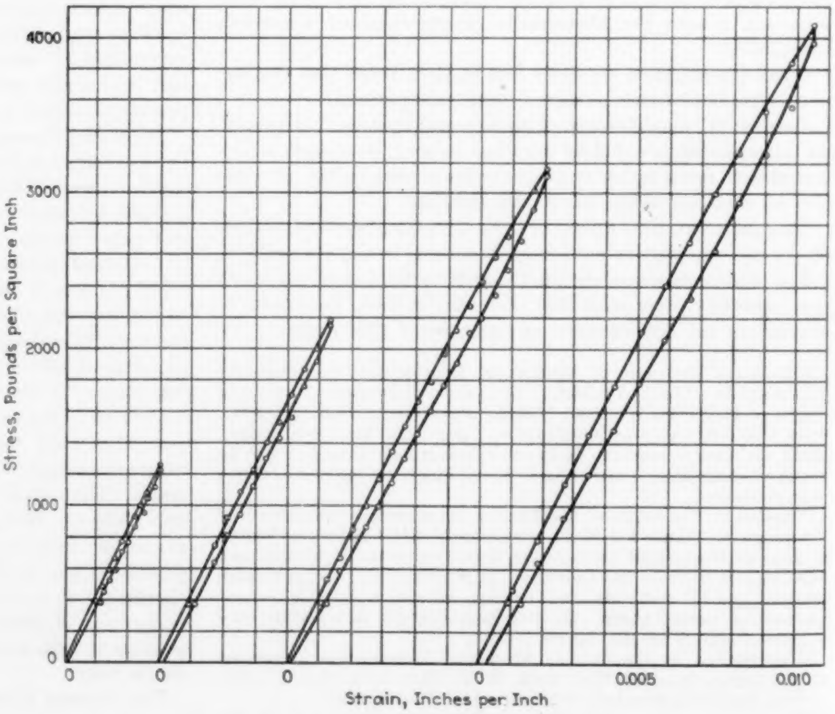

Fig. 2 Hysteresis Loops—Material "G"
(Thickness of material, 0.157 in. Loaded from 0 to maximum stresses of 1000 to 4000 lb. per sq. in. up and down at uniform rate of speed of 0.001 in. per in. per min.)

tained in this way were used to verify the correctness of slope and curvature of the stress-strain curves already obtained on the tension machine.

An additional series of tests were made to show more definitely the effect of time on strain readings. Runs were made at all stated speeds up to certain maximum stresses and back to zero at uniform rates of elongation and recovery. The plotted curves enclosed a definite hysteresis loop, even at low stresses and high speeds. Fig. 2 shows a typical set of these curves. Here again all different samples of the material were tested and showed the same general characteristics. The hysteresis loop, while differing in width, was always present.

The gradual increase in strain as a constant load is applied on a member may be termed "creep." This characteristic was very apparent in the celluloid examined. In order to find how creep varied with the amount of stress and time, readings were taken over a long period. Fig. 3 shows the results graphically for one specimen. It is interesting to note that even for the strongest material and the low stress of 1000 lb. per sq. in. there was a measurable creep in a relatively short period of time.

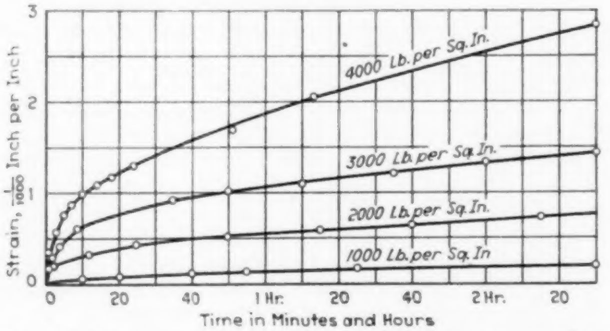

FIG. 3 TIME-CREEP CURVES SHOWING INCREASE OF STRAIN WITH TIME—MATERIAL "G"

(Stress kept constant at values given.)

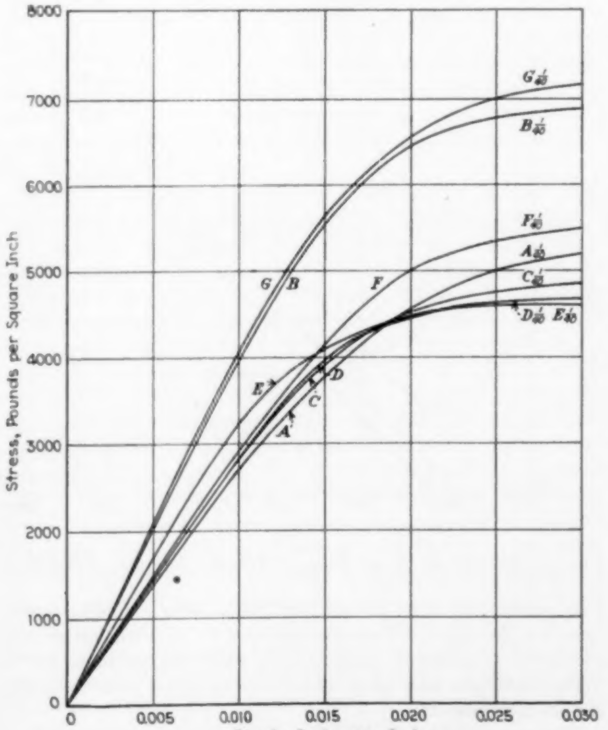

FIG. 4 STRESS-STRAIN CURVES FOR DIFFERENT MATERIALS AT THE SAME SPEED

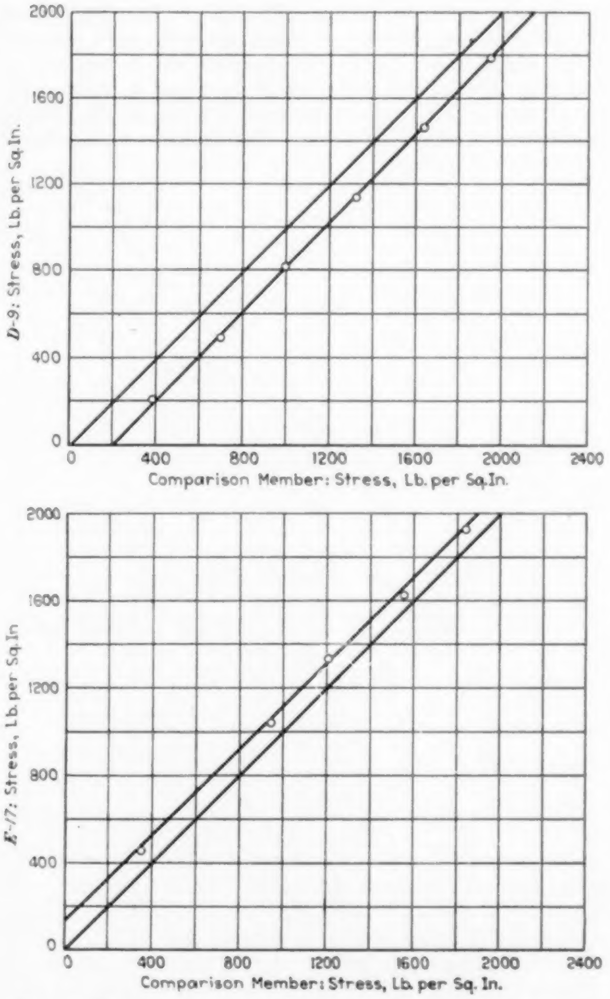

FIG. 5 EFFECT OF INITIAL STRESS DUE TO INITIAL COLOR

Observations on the rate of recovery after the load was removed were not plotted. It was noticed, however, that for low loads the recovery was quite rapid. For stresses approaching the nominal elastic limit, the time of recovery was a matter of days, but unless the stresses were very high the material would recover in time.

A critical examination of Figs. 1–3 indicates that a true proportionality between stress and strain does not exist under any condition of loading. The effect of creep and time of loading is very apparent. The modulus of elasticity, the nominal elastic limit, and the ultimate strength all increase with the speed of loading.

Physical constants for the different thicknesses of material at maximum and minimum speeds of loading are shown in Table 1. The modulus of elasticity was taken as the mean slope of the nominally straight portion of the stress-strain curve. The elastic limit was uncertain. For the purpose of measurement, it was taken as the unit stress at a point on the curve where the slope was two-thirds of that taken for the modulus of elasticity. The ultimate strength was taken from the original load-deformation curves.

Fig. 4 shows the stress-strain curves for all different materials at the same speed of loading. Material G has the greatest strength and the highest modulus of elasticity, with B a close second and E next. These three materials are in the order of their age, G being five years old, B four, and E three. The rest of the curves were well grouped, the age of the material ranging from 6 months to a year. This indicates an increase in strength with age. No other conclusions seem possible. There is no consistent variation with thickness. According to the statement of the manufacturer, no change was made in chemical composition or the method of manufacture.

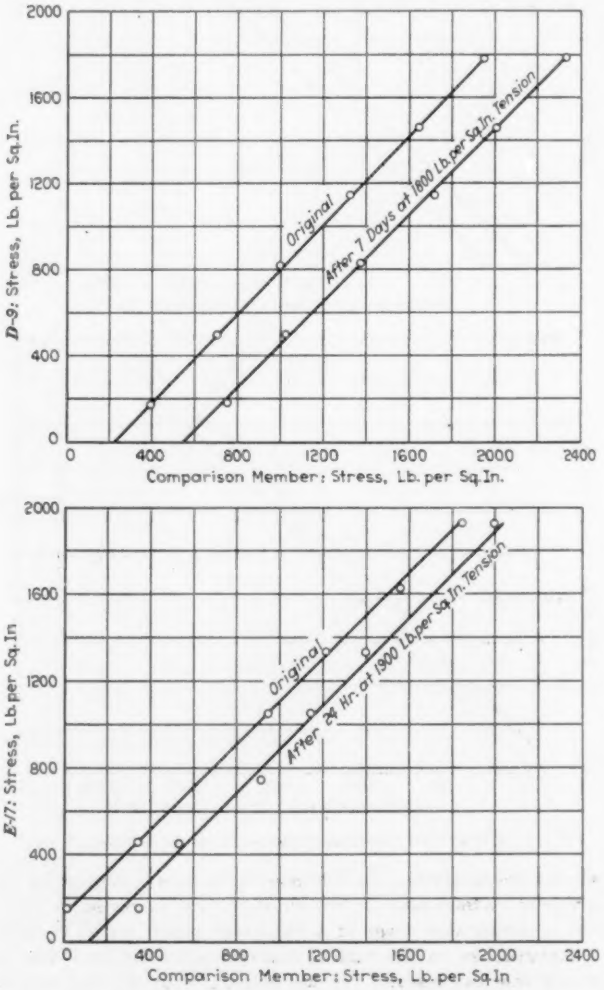

Fig. 6 Effect of Continuous Load on Stress Due to Residual Color

Photoelastic Tests

The materials whose physical properties were investigated in the earlier part of this work were further investigated to ascertain their photoelastic characteristics. The test specimens, with two exceptions, had a nominal width of $^{5}/_{8}$ in., and a 10-in. gage length available for strain measurements. The comparison members, and in two cases the test specimens, had a nominal width of $^{1}/_{4}$ in. and a uniform section for 5 in. of length. The specimens were made with great care and finished by hand, to eliminate as far as possible the strains due to shaping.

The curves of Fig. 5 were obtained by reading the respective dynamometers for crossed tension members with the common image extinguished for loads as shown. For both curves the comparison members were carefully selected so as to be free from initial color. The main member for the upper curve was selected because it had an initial color effect indicative of a simple tensile stress, while the one for the lower curve was selected because it had initial color indicating a simple compressive stress. The lines of 45 deg. slope through the origin were drawn to facilitate comparison of slope and "set-over."

The tests yielding the curves of Fig. 6 were obtained by making observations of dynamometer readings for extinction of the common image of crossed members, for increasing loads to the maximum value shown, then maintaining the load on the specimen by deadweights for the times shown—the comparison member being unloaded during that period—and then making a run of dynamometer readings for extinction up to the previous maximum. The comparison members for both curves were carefully selected so as to be free from initial color. The test specimen for the upper curve was selected to have an initial color indicative of a simple tensile stress, while the specimen for the lower curve was selected so as to have an initial color indicating simple compression.

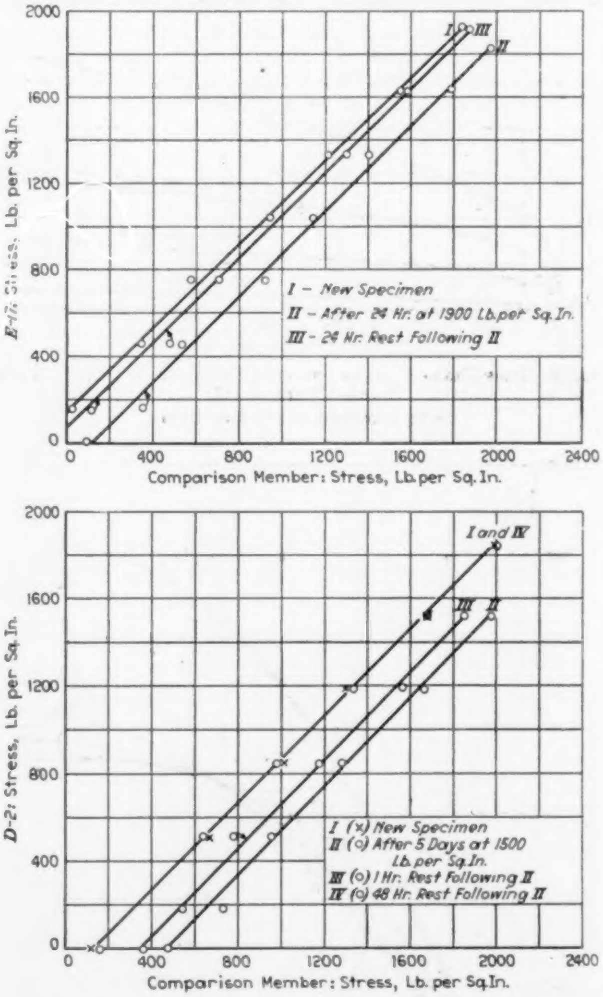

Fig. 7 Effect of Rest Period After Continuous Loading

The tests plotted in Fig. 7 were similar to those of Fig. 6, except that after a period of rest following a continuous load another set of readings of comparison-member stress versus test-specimen stress for extinction was obtained and plotted to the same coordinates. In the lower curve, runs at two intervals of rest are plotted.

Fig. 8 is the result of a test in which simultaneous readings of stress and strain in the test specimen and stress in the comparison member to produce extinction of the common image were measured. The stresses in the comparison member necessary to produce extinction were plotted as ordinates, with abscissa values in one case the measured stress in the test specimen, and in the other, the measured strain in the test specimen.

In all the above-mentioned curves the data were selected from not less than several runs in each case; the values and consistency shown are typical of all the runs made, and it was deemed that the inclusion of more curves of the same types of tests would be but duplication.

It is obvious that if two tension members selected for freedom from initial color be "crossed" and loaded to produce extinction of the common image in suitable polarized light, the plot of the dynamometer readings to coordinates, as in Fig. 5, would be a straight line of 45 deg. slope through the origin. The curves of Fig. 5 deviate from such a curve by constant values of stress, the apparent significance being that the test specimen of the upper curve has an initial tensile stress of 210 lb. per sq. in., and the one of the lower curve an initial compressive stress of 140 lb. per sq. in., to which is algebraically added the applied load at any time to produce the total. This apparent significance requires interpretation. The apparent initial stress is due to the initial color, and it is obvious from considerations of equilibrium that the initial stress cannot be a simple uniform tension or compression. But it is reasonable to assume that the initial color, read by means of a comparison member and expressed in terms of a simple tensile or compressive stress, could represent a complex condition in which different layers of the material are stressed to different degrees, and possibly in different directions. Such a complex condition of initial stress, and corresponding initial strain, is easy to conceive of, in view of the nature of the process used in the production of this material. For convenience of reference hereafter, the photoelastic stress equivalent (stress read from the comparison member when the common image is extinguished) of the initial color will hereafter be called the "apparent initial stress."

While it cannot be true that the apparent initial stress indicated in Fig. 5 is the actual stress, the effect of initial color on later loading and photoelastic observation is obviously the equivalent of the apparent initial stress being algebraically additive to the applied load, as typified by the curves of Fig. 5.

The first part of this work showed that celluloid exhibits a residual strain after a continuous load. The curves of Fig. 6 show a similar phenomenon in terms of residual color, which is read photoelastically in terms of residual stress. In the upper curve of Fig. 6 the test material at the start had an apparent initial tension of 230 lb. per sq. in. After seven days of continued loading at 1800 lb. per sq. in. it showed an apparent initial tension of 550 lb. per sq. in., together with residual strain. In the lower curve of the same figure, the original specimen had an apparent initial compressive stress of 130 lb. per sq. in., while after 24 hours at 1900 lb. per sq. in. tension it had an apparent residual tension of 120 lb. per sq. in. Study of the curves shows that the effect of the apparent residual stress on stress determined photoelastically (by comparison member with extinction of the common image) thereafter, is identical with that of the apparent initial stress; i.e., it is algebraically additive to the applied stress to produce the photoelastically determined stress.

The augmented initial color remaining in a specimen after a cycle of loading, producing the apparent residual stress just discussed, if produced by moderate maximum values of load, disappears gradually, as does the residual strain accompanying it, if the specimen is left unloaded. In the upper curves of Fig. 7, photoelastic tests of the original material produced curve I, showing an apparent initial compression of 150 lb. per sq. in. After a continuous load of 1900 lb. per sq. in. for 24 hours, the curve obtained was II, indicating an apparent residual tensile stress of 120 lb. per sq. in. The specimen was allowed to rest unloaded for 24 hours, and then yielded values given in curve III, showing an almost complete loss of the residual color introduced by the continued load, and an almost complete reassumption of the original condition of initial compression.

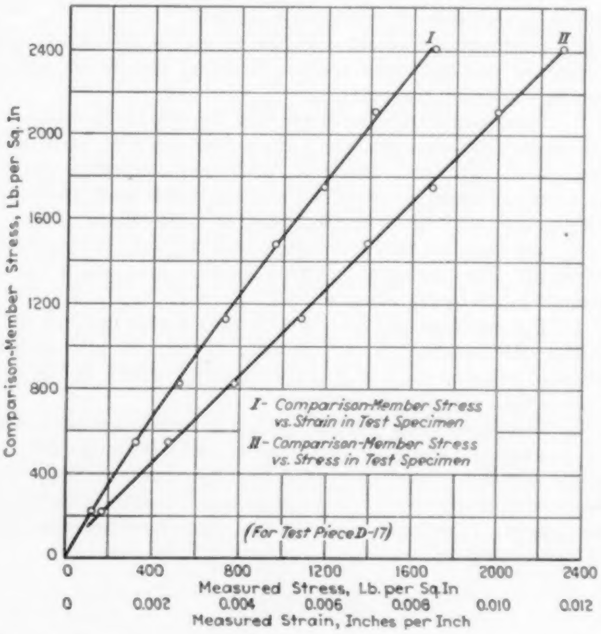

Fig. 8 Comparison-Member Stress Vs. Strain and Stress in Test Specimen

In the lower figure of Fig. 7, similar relative changes are exhibited, except that the specimen started with an apparent initial tensile stress of 110 lb. per sq. in., and readings were made after periods of rest of one hour and 48 hours. Curve IV shows complete recovery from the effects of the continued load.

The logical inference from a study of these curves is that for tests immediately following the acquisition of apparent *residual* color and stress, they may be considered as affecting the results as do apparent *initial* stress and color, but that *residual* stress and color from a cycle of loading are temporary in nature and disappear as a function of time, while the *initial* color and apparent stress are of a more permanent and stable nature.

The series of tests from which the data for Fig. 8 were selected were run to demonstrate that, for the maximum loads chosen, the creep characteristic of celluloid, developed in the first part of this work, does not vitiate the proportionality between stress and photoelastic color. In Fig. 8 the stress equivalents of photoelastic color, as obtained by use of a comparison member, are plotted as ordinate values, while measured stresses and strains in the test specimen are plotted as abscissa values. Curve I is similar to the stress-strain curves of the first part of this work, showing a slight curvature and indicating the same lack of perfect proportionality between photoelastically measured stress and strain as was previously shown for measured stress versus strain.

Curve II of Fig. 8 is of the same nature as those shown in the several previous figures and is of 45 deg. slope, exclusive of a slight observational error, and has a slight "set-over" due to initial color.

CONCLUSIONS

On the basis of these tests the following conclusions can be drawn in regard to the celluloid examined:

1 The physical constants for celluloid are not fixed but have a consistent variation with speed of loading, the higher speeds having the higher constants.

2 There seems to be evidence that for celluloid a true proportionality between stress and strain does not exist under any condition of loading. Therefore, while one may call a certain point on the strain-strain curve a nominal elastic limit, there is a very grave doubt as to the existence of a true elastic limit.

3 Age seems to have a bearing on the physical properties of celluloid. The undesirable characteristics were much more marked in the newly made material.

4 The results of the photoelastic tests herein reported lead to the conclusion that, within the range of the tests, photoelastic color in celluloid models is proportional to stress, but is not strictly proportional to strain.

5 The photoelastic method applied to celluloid models may be expected to yield satisfactory results where proportionality of stress and strain have no critical bearing on the problem, but where such proportionality is an important factor the results will be in error. This error is due to the complicated interrelation of stress, creep, and time, and may reach a high value.

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Discussion

Max M. Frocht.[4] This paper deals with the mechanical properties of American-made celluloid and aims at a reexamination of the fundamental stress-optic law of a material which made photoelastic investigations practicable.

The present tendency is away from celluloid and toward more sensitive materials such as bakelite. It would thus be of basic importance to conduct such investigations as reported in this paper also for bakelite. Should such an investigation be undertaken, it is suggested that more sensitive methods be used to determine the stress-optic law. Instead of the tension compensator, a Babinet compensator might be used, and the phase difference directly determined against the known loads. It would also be of interest to examine somewhat more complicated stress systems than those in pure tension, such as a disk under diametral compression. The photoelastic results could then be compared with the values of the available theoretical solutions. The writer has worked with both celluloid and bakelite, and in his opinion the readings from the tension compensator with celluloid may vary 5 per cent or more. Fig. 9 shows the relative optical sensitivities of celluloid and bakelite. The models were of the same dimensions and under identical loading. (See Fig. 10.) It will be observed that for the one fringe in the celluloid beam there are five in the one of bakelite, which means an advantage of 5 to 1 in favor of bakelite.

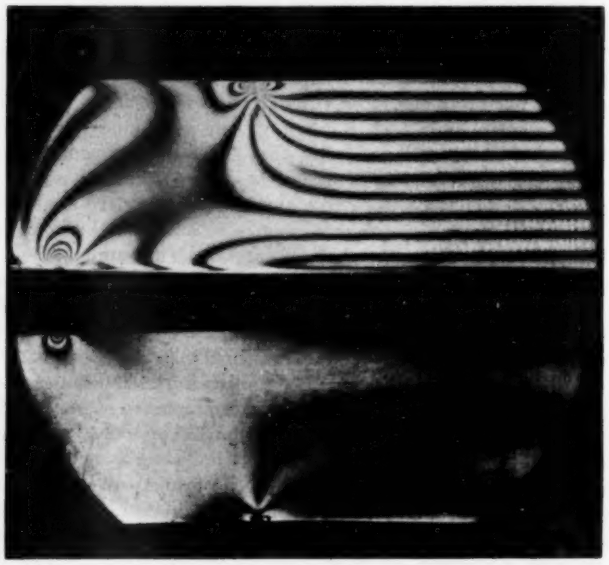

FIG. 9 RELATIVE OPTICAL SENSITIVITIES OF CELLULOID AND BAKELITE

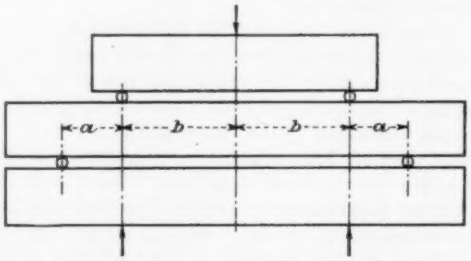

FIG. 10 ARRANGEMENT TO PRODUCE IDENTICAL LOADING

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APM-54-9

Kinematography in Photoelasticity

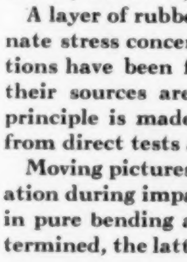
By MAX M. FROCHT,[1] PITTSBURGH, PA.

Photoelasticity is one of several methods for stress analysis. Kinematography in photoelasticity was made practicable by the introduction of the monochromatic method and should be found useful in the study of gear action, tool cutting, impact stresses, and in the phenomenon of failure.

Moving pictures of 15 models in 29 set-ups are presented which show a variety of stress-distribution patterns and their formations. Stress concentration can spring from various sources such as holes, grooves, notches, sharp curves, scratches, and the edges of flat steel dies.

A layer of rubber, lead, or wood has been found to eliminate stress concentration due to a die. Stress concentrations have been found to be mutually destructive when their sources are many and close. Application of this principle is made to the design of a thread and results from direct tests are shown which substantiate it.

Moving pictures are presented showing the stress fluctuation during impact, and from these the increase of stress in pure bending and the time of stress oscillation are determined, the latter being in good agreement with theory.



THERE are several ways in which to study stress distribution. There is the purely mathematical method, which starts with the laws of equilibrium and the assumption of continuity and derives by logical processes conditions for stress functions. There is also the purely experimental method, which aims to measure local stresses directly by means of refined and sensitive strain gages. In the field of two-dimensional stress systems there is a third method, namely, the optical or photoelastic method, whose singular merit consists of giving a definite visual stress pattern to each stress system, from which the shear stresses can always be read directly. The photoelastic method has until recently been based upon the color effect which a stressed element of a corresponding celluloid or Bakelite model produces. These color effects are obtained when white light is taken as a source. In recent years, however, it has been found desirable to substitute monochromatic light. This change has numerous merits, one of which is that, when combined with a sensitive material such as Bakelite or phenolite, it makes kinematography of stress formation practicable.

Moving pictures of stress formation were made by Dr. Ziro Tuzi, of the Physical Chemical Institute, Tokyo, Japan, and were shown by him at the International Congresses of Applied Mechanics and Metallic Structures held last year at Stockholm and Liége, respectively.

The application of kinematography to photoelasticity carries interesting possibilities. It provides an excellent means of recording the history of stress formation and of bringing these results to the classroom or lecture hall, thus being of educational value. Kinematography should also prove useful in the study of gear action, and will probably add to our understanding of tool cutting. The pictures of group 5 (Figs. 43–45) show that kinematography has potential possibilities in the study of impact stresses and the phenomenon of failure.

It is the object of this paper to present moving and static pictures of stress-distribution patterns and their formations in a variety of models and under diverse loading which will bring home to the reader a concentrated demonstration of the things one sees in the polariscope, and to interpret these pictures non-mathematically from the point of view of stress concentration. It also deals with fluctuations of stresses during impact, and shows that there are possibilities in that direction.[2]

THE APPARATUS USED

It is assumed that the reader is familiar with the essentials of photoelasticity.[3] The apparatus used and its optical arrangement are shown in Figs. 1 and 2. The polariscope is one made by the Winkel-Zeiss firm, of Göttingen, Germany, and uses Ahrens instead of Nicol prisms. The loading, except in impact, was continuous and uniformly increasing, and was obtained by connecting a rubber hose to a nearby faucet and allowing a steady stream of water to fall into a large bucket suspended from the lever of the straining frame. Loading by means of flowing water has decided merits over deadweights. One can sit at the screen and leisurely watch the fringe formation as the load increases. This can be repeated as many times as necessary in order to determine the fringe order for any particular model. All pictures except some in impact were taken at the rate of 16 per second, this being the fixed speed of the DeVry camera which was used throughout the work. The models were all of Bakelite and were annealed by the author's method, which is discussed in another paper.[3] The monochromatic light was obtained from a mercury-vapor lamp and the use of a No. 77 Wratten filter.

MOVING PICTURES

Moving pictures were taken of 15 models in 29 different setups. For convenience these were divided into five groups, as follows:

1 Central compression
2 Bending
3 Die on a large plate
4 Angles, disks, and square plates
5 Experimental impact pictures.

In interpreting these pictures it is well to bear in mind three points: first, that a fringe is a locus of points of equal maximum shear; second, that the crowding of fringes accompanied by

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Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] The work described in the paper was carried out in the Photoelastic Laboratory of the Mechanics Department in the University of Michigan, Ann Arbor, Mich.

[3] For a brief statement of these essentials, see author's paper on "Recent Advances in Photoelasticity," Trans. A.S.M.E., 1931.

sharp curvatures is evidence of stress concentration; and third, that the points where fringes start such as the four dots in Fig. 39 are points where yielding begins, and this may be of significance in ductile materials since these are points of maximum shear. With this in mind we are able to make certain general and at times useful observations from the mere inspection of the photographs.

We see that a single concentrated load, even if it is small, produces large local stresses in the immediate vicinity of the load (Fig. 3). We also see that in a long prismatic bar compressed by two concentrated forces (Fig. 4) the ends are under high and variable stresses, even though the middle portion be under uniform and moderate stress. The center of the bar in Fig. 5 is of a uniform degree of darkness or brightness, showing a uniform distribution of stresses, whereas the ends show evidence of stress concentration.

Figs. 6, 7, 8, and 9 show rather complicated fringe patterns which are evidence of a variable stress distribution across a horizontal section of symmetry. Fig. 10, however, shows a remarkable exception to this group. It will be observed that the central portion across nearly the whole length is uniformly dark, and is thus under a uniform distribution of stress except in the immediate proximity of the notches. Thus stress concentrations at close intervals are mutually destructive, and the effect of many is much smaller than the effect of one. We shall return to this point in group 2. Referring now to group 2 (Figs. 12–18) we first of all observe the beam in pure bending, Fig. 12. Here the fringes are straight, parallel, and equidistant, from which it is comparatively easy to deduce the well-known linear law of stress distribution. This is true everywhere except in the neighborhood of the points of application of the loads and reactions. In the small intervals between the loads there is no neutral axis as is assumed in the elementary engineering theory.

Fig. 14 shows graphically why Bakelite and not celluloid was chosen as the material for models. It will be seen that there are about five times as many fringes in the Bakelite beam (upper halves of pictures) as there are in the corresponding celluloid beams. It must be remembered that the two beams are of the same dimensions and under identical loads. This has been accomplished by using the set-up shown in Fig. 11.

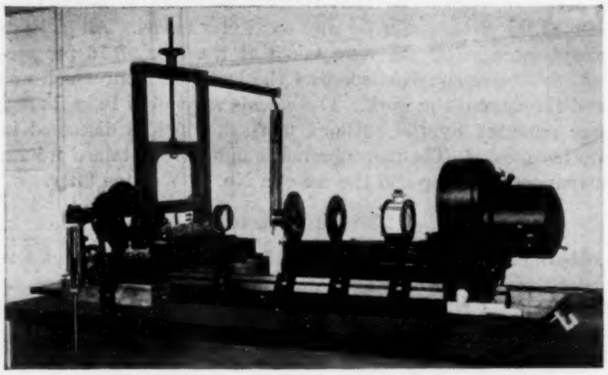

FIG. 1 POLARISCOPE AND STRAINING FRAME

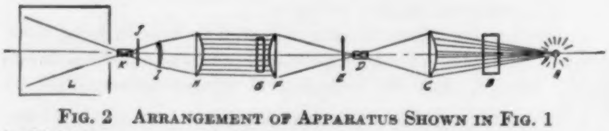

FIG. 2 ARRANGEMENT OF APPARATUS SHOWN IN FIG. 1

A—Light source
B—Water cooler
C—Lens; $F = 100$ mm.
D—Polarizer; 10-mm. Ahrens prism
E—Quarter plate
F—Lens; $F = 150$ mm.
G—Test model
H—Lens; $F = 150$ mm.
I—Lens
J—Quarter plate
K—Analyzer; 10-mm. Ahrens prism
L—Camera or screen

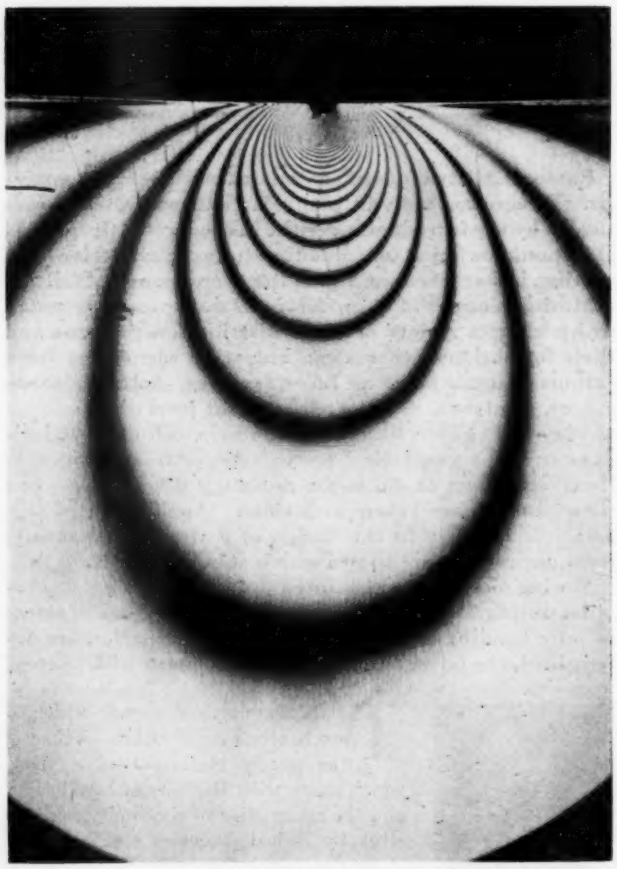

FIG. 3 EFFECT OF CONCENTRATED LOAD

Fig. 13 shows the stress concentrations due to large semicircular grooves. It will be observed that the disturbances due to such grooves are felt throughout the length and depth of the whole beam, and that in such a case the assumption of linear distribution is undoubtedly at variance with actual conditions.

Attention is next called to Fig. 23, which vividly shows the effect of the size of the fillet upon stress concentration. This should impress the designer with the need for avoidance and elimination of sharp corners wherever possible.

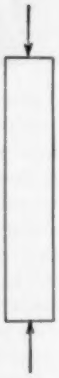

FIG. 4

Figs. 20, 21, and 22 are particularly interesting. Here, as in group 1, it is seen that stress concentrations emanating from points close together mutually destroy each other, or that one notch has a greater weakening effect than many. This is of importance in the design of screws. In this connection the author wishes to quote from a report by Prof. S. Timoshenko and W. P. Irwin to the Westinghouse Electric and Manufacturing Company, who, in making an investigation on the relative strengths of two systems of threads, came to the conclusion that "if instead of a single groove a thread such as that shown in Fig. 19 be taken, the weakening effect will be much less than for the case of a single groove." To reduce stress concentrations at the extremities of the thread, fairly large fillets were employed as shown. Their conclusions were arrived at on the basis of a series of fatigue tests using a cantilever fatigue-testing machine. Some fatigue tests for establishing the weakening effect of V-grooves were also made by the Aeronautical Research Committee, England,[4] which established the following formula for

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[4] See paper No. 860 by Norman Thomas.

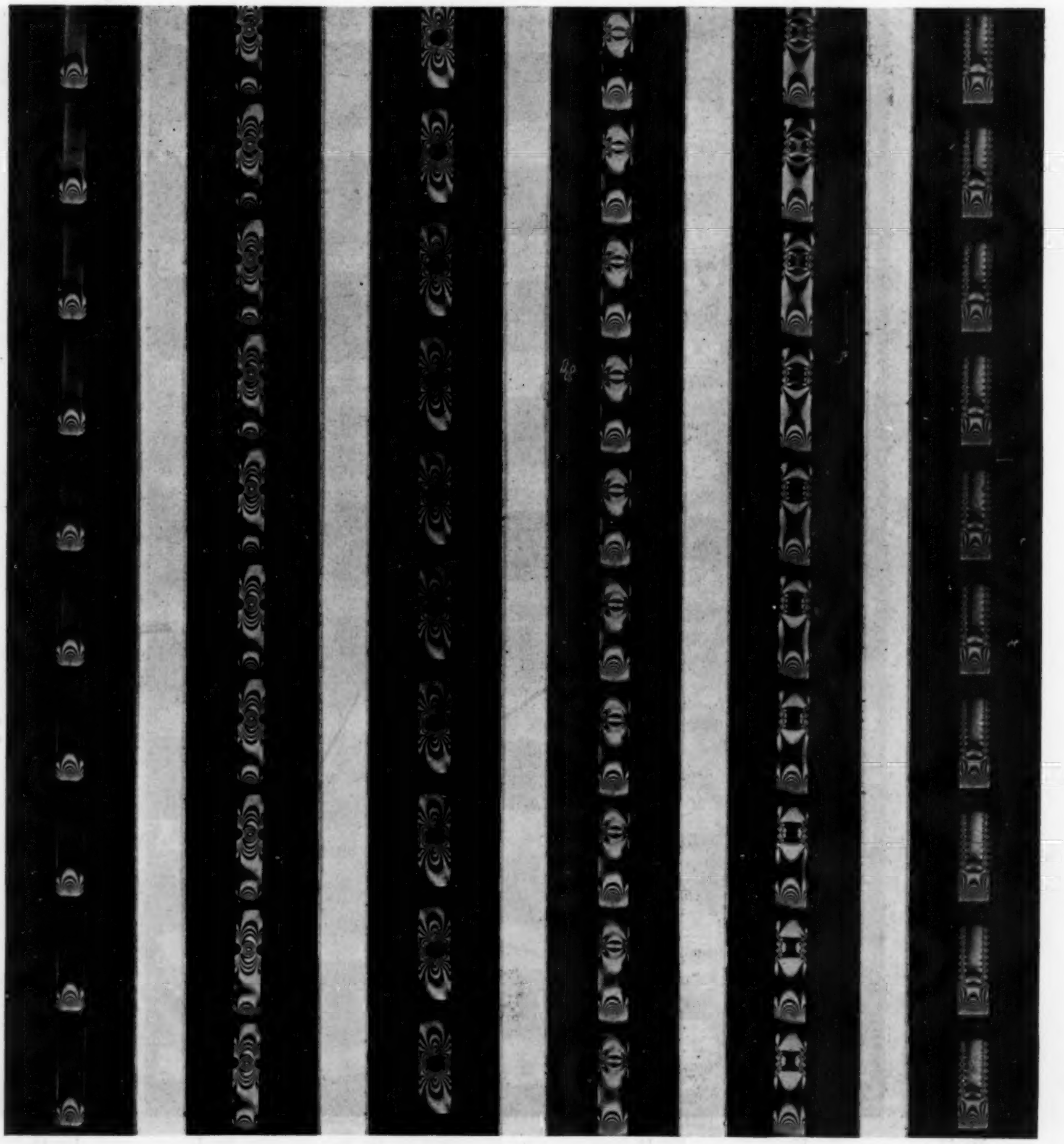

Fig. 5 Fig. 6 Fig. 7 Fig. 8 Fig. 9 Fig. 10

Group 1—Central Compression

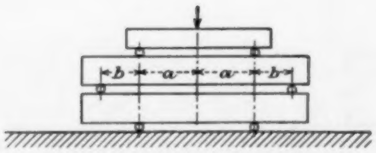

Fig. 11 Set-Up Used for Fig. 14

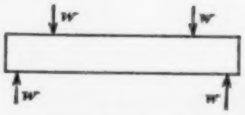

Fig. 12a Pure Bending

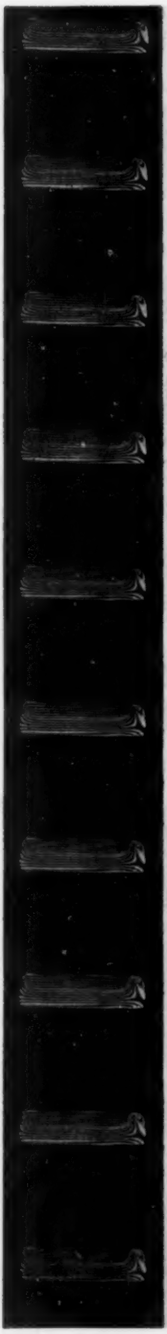

Fig. 12

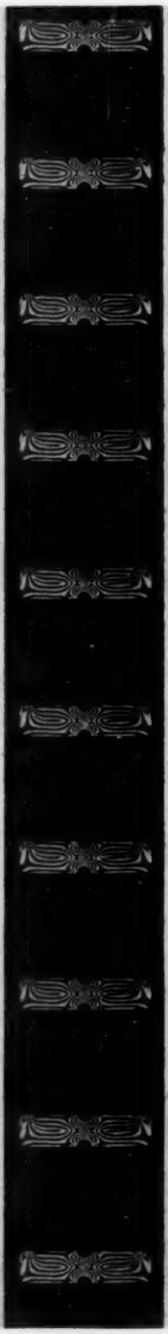

Fig. 13

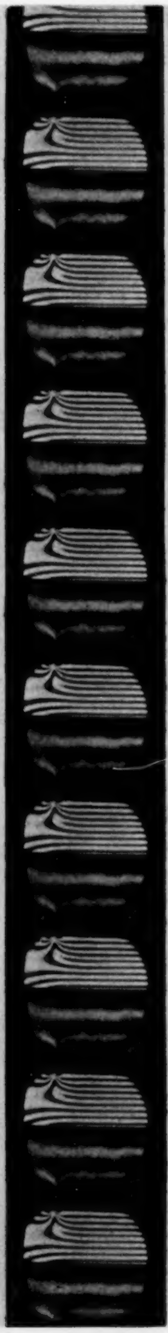

Fig. 14

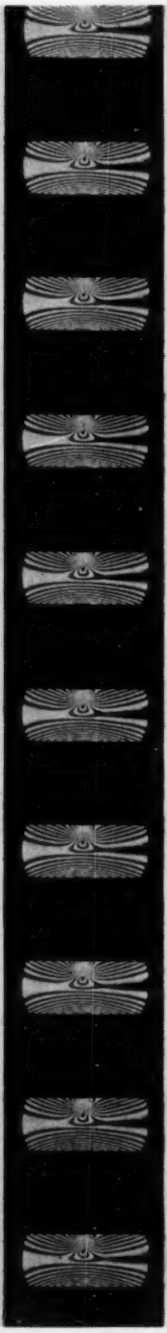

Fig. 15

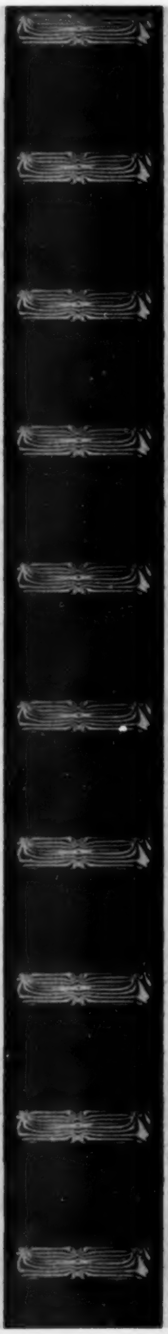

Fig. 16

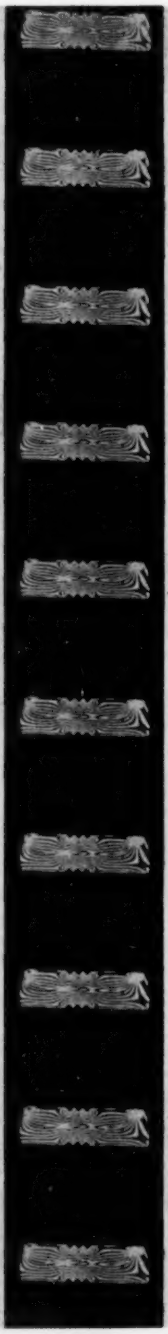

Fig. 17

Fig. 18

Group 2—Pure and Central Bending

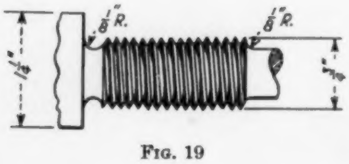

Fig. 19

calculating the weakening effect of a single groove:

$$k = 1 + 0.45\sqrt{\frac{d}{r}}$$

in which d is the depth of the groove, r the radius at the bottom of the groove, and k the factor of stress concentration. This formula gives results in good agreement with those of the above-quoted report. A point of considerable interest here is that the photoelastic conclusions drawn from a two-dimensional system are also true in the corresponding three-dimensional case. There are other cases in which this is true, so that the usefulness of photoelasticity extends beyond the boundaries of the two-dimensional system.

As a corollary to the principle of mutual destruction of stress concentrations we conclude that a scratch on a polished surface is of greater potential danger than one on a rough surface, and that a deep scratch on a polished surface may be more detrimental than an entirely rough surface.

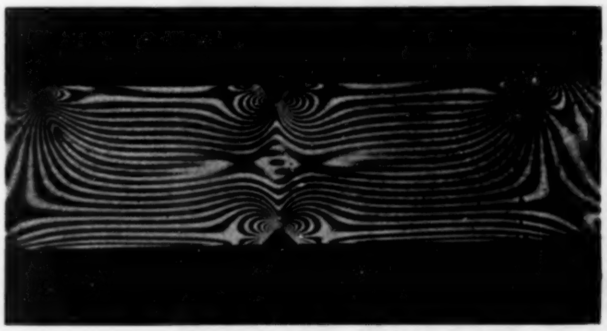

FIG. 20 BEAM WITH ONE NOTCH. PURE BENDING

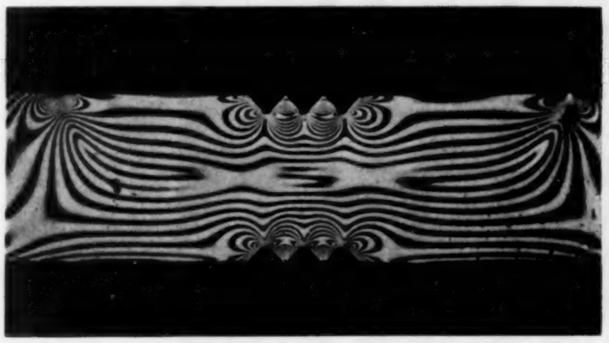

FIG. 21 BEAM WITH THREE NOTCHES. PURE BENDING

Turning to group 3 (Figs. 24–28), which deals with stress concentrations caused by dies,[5] let us first examine Fig. 29. It is seen that the pressure under the die is not uniform and that there are pronounced stress concentrations at the edges.[6] Fig. 30 shows the effect of a layer of rubber placed between the die and the plate. Two things stand out: first, the total vanishing of the stress concentrations and a nearly uniform pressure distribution; and second, a shifting of the maximum shear zones from points A and B to region C.

Similar results can be obtained with lead and wood, as is shown in Figs. 26 and 28. These photographs furnish practical suggestions to engineers. To the civil engineer they suggest the danger of placing a flat-end steel column on a concrete floor. To the mechanical engineer and shop men it suggests that the same danger exists in the mounting of machinery on foundations. In all these cases local stresses can be eliminated and the life and safety of the foundation increased by placing blocks of wood or lead between the steel and the concrete.

At the suggestion of Professor Timoshenko the steel die was coated with a layer of paraffin, thereby reducing the friction forces between the die and the plate. It was found that under small loads the action of the paraffin is the same as that of rubber, lead, or wood, but that under larger loads its action is negligible. Rounding or chamfering the corners of the die proved of no help in eliminating stress concentration.

Fig. 27 shows the action of a Bakelite die on a Bakelite plate. It suggests the possibility of investigating photoelastically the stress phenomenon in the die as well as in the plate.

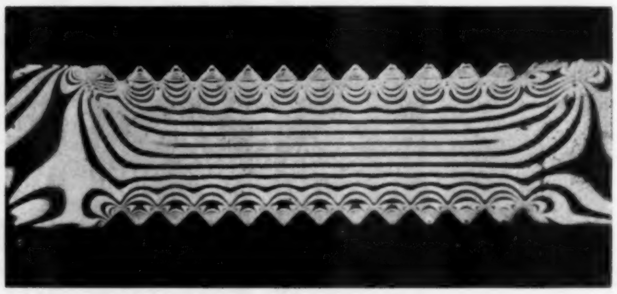

FIG. 22 BEAM WITH MANY NOTCHES. PURE BENDING

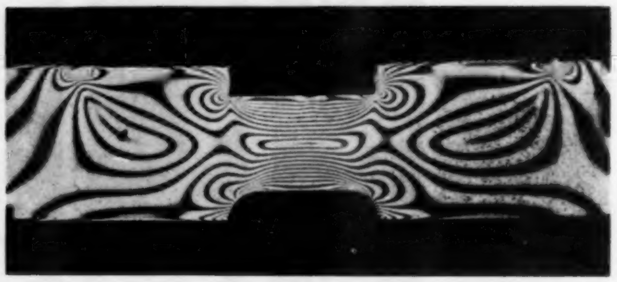

FIG. 23 COMPARISON OF STRESS CONCENTRATIONS DUE TO SHARP CORNERS AND ROUNDED FILLETS. PURE BENDING

Fig. 31 shows the action of a steel die with a single V-notch in the center. It is seen that the notch becomes an additional source of stress concentration although of smaller magnitude than the edges. However, when the number of notches is increased to cover the whole die, the principle of mutual destruction again operates and no additional stress concentrations are set up. On the contrary, it seems that they may even tend to counteract the bad effects at the edges. (See Fig. 32.)

In group 4, miscellaneous set-ups are shown. The two angles of Fig. 36 are of the same size and under identical loads, and again demonstrate the influence of the size of the fillet upon stress concentration. It will be seen that the angle with the sharp corners has more fringes than the corresponding angle with the large fillet, and that they are of greater curvature.

In the solid disk shown in Fig. 38 the stress-distribution pattern is fairly simple, and the first fringe seems to be approaching the boundary without ever reaching it, the boundary acting as a neutral curve of zero stress except in the vicinity of the concentrated loads.

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[5] The theoretical aspects of this problem have been gone into by several investigators: namely, J. H. Mitchell (London Math. Soc., Proc., vol. 32, 1900, p. 35), M. Sadowski (*Zeit. f. angew. Math. u. Mech.*, vol. 5, no. 8, 1928, p. 107), and Gustav Mesmer (Doctor's Dissertation, Georg-August-Universität, Göttingen, 1929).

[6] For a good celluloid photograph substantiating these results see Gustav Mesmer's Dissertation mentioned in the preceding footnote.

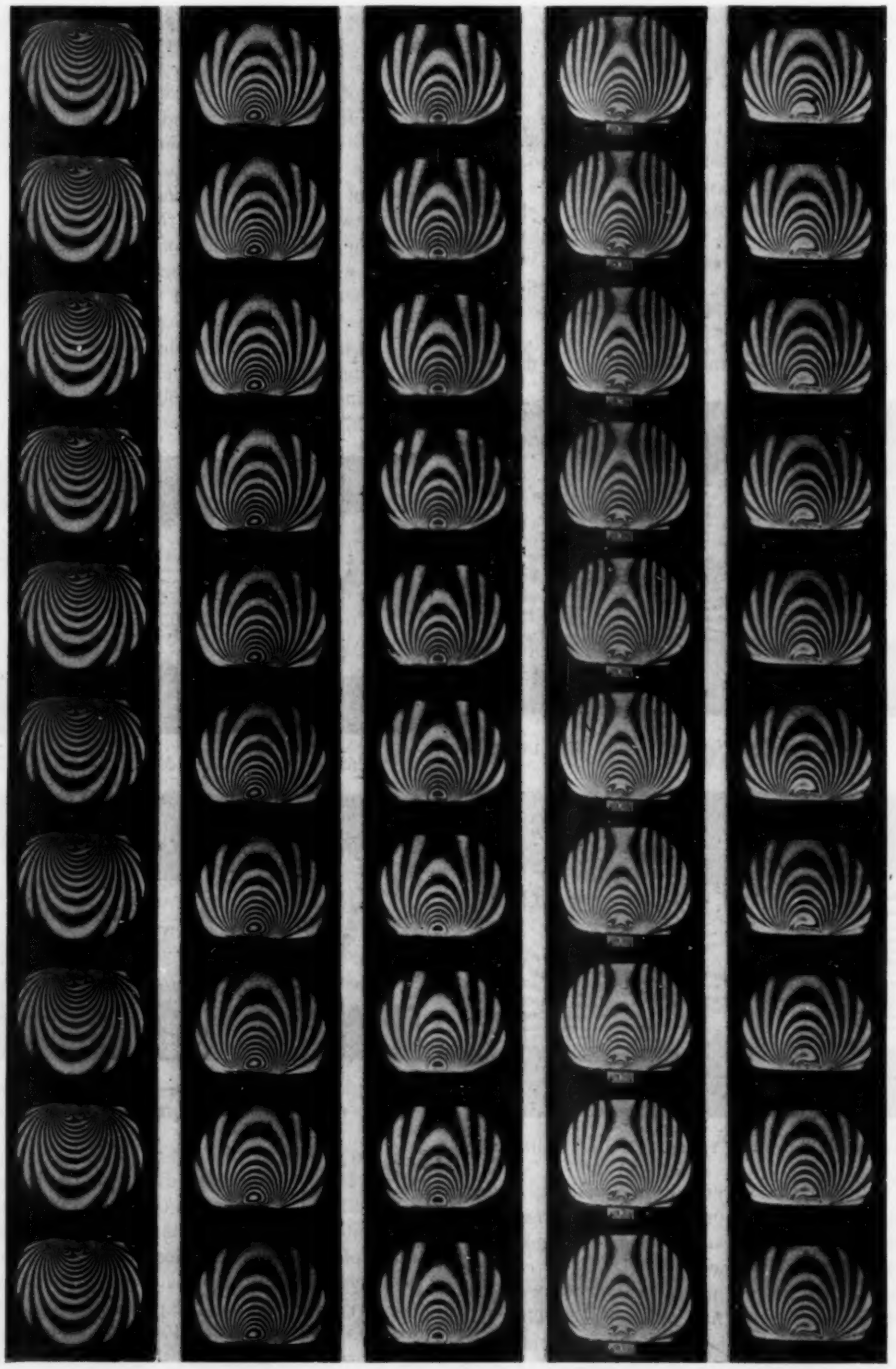

Fig. 24 Steel Die

Fig. 25 Rubber

Fig. 26 Lead

Fig. 27 Bakelite

Fig. 28 Wood

Group 3—Die on a Large Plate

Fig. 39 shows the pronounced disturbances produced by drilling a hole in the center of the disk.[7]

Group 5 (Figs. 43–45) deals with stress fluctuations during impact. Here the kinematographic method is of primary importance as static pictures of stress fluctuations are evidently impossible. This is a pioneer field in which the equipment and technique are still to be developed. To the best of the author's knowledge the pictures shown in this group are the first of their kind. Three models were photographed: a large plate under a concentrated load, a beam in pure bending, and a disk under vertical loads. Due to the limitations of the camera it was decided to start the work with slow impact, and this was accomplished by inserting a spring in the loading arrangement (see Fig.

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[7] This problem has recently been investigated by Dr. Billewicz. See his dissertation for the Doctor's Degree at the University of Michigan, 1931.

40). Most of the energy of impact was thus diverted to the elastic system of the apparatus and only a small part of the energy went toward increasing the stresses in the models. Photographs were taken at the rate of 16, 32, and 64 exposures per second, the latter being the limiting speed of the camera available.[8] Many of the photographs show the fringes due to impact quite clearly. Fig. 41 shows the stress fluctuations in pure bending due to the drop of a 5-lb. weight about 2 in. The data for this curve were taken from a strip of film which was projected upon the screen and the fringes counted.

The curve shows that the maximum stress due to impact of the falling weight is at least $6^1/_2$ times larger than the static stress caused by the same weight.[9] From the photographs it was found that the disturbances due to the shock were felt for a little over one second with the spring, and about half that time without the spring. To check the reliability of the photoelastic results, the time of one stress oscillation was computed from the known value of the spring constant and compared with that furnished by Fig. 41.[10]

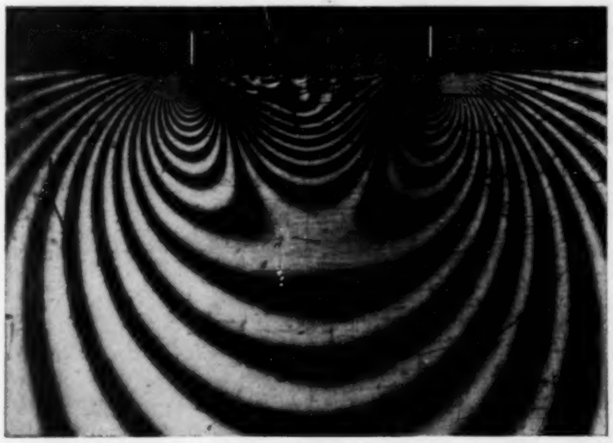

Fig. 29 Steel Die

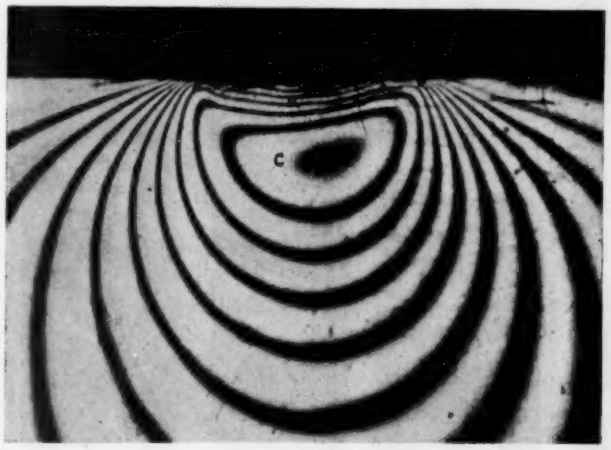

Fig. 30 Steel Die Acting Through Layer of Rubber

By direct experiment the spring constant was found to be 90 lb. The rod and platform were found to weigh 5 lb., so that when the impact weight of 5 lb. was dropped the spring was oscillating under a 10-lb. load.

Let τ be the period of oscillation of the spring and δ_{st} the statical deflection caused by the oscillating weight, then[11]

$$\tau = 2\pi\sqrt{\frac{\delta_{st}}{g}}$$

This gives

$$\tau = 2\pi\sqrt{\frac{10 \times 1}{90 \times 12} \times \frac{1}{32}} = 0.106 \text{ sec.}$$

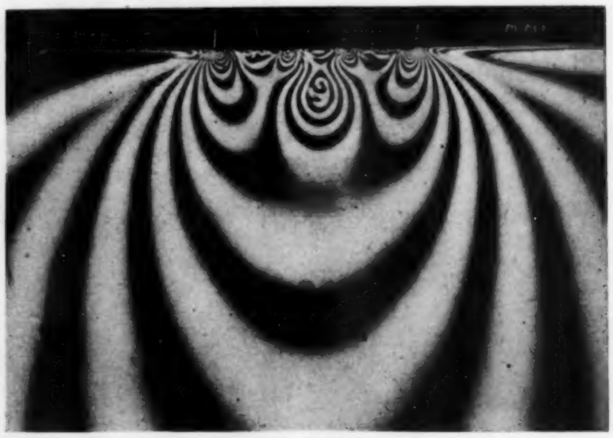

Fig. 31 Steel Die With V-Notch in Center

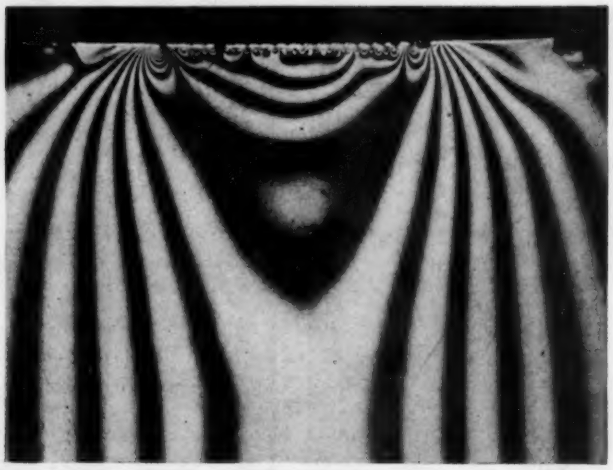

Fig. 32 Steel Die With Many Notches

Fig. 41, which is based on a film taken at the rate of 64 pictures per second, shows an average of 7 pictures per stress oscillation. This gives for the period of oscillation

$$\tau = 7 \times \frac{1}{64} = 0.119 \text{ approx.}$$

which is in good agreement with the theoretical value—especially so since the deflection of the lever has been neglected in the former computation.

To utilize Bakelite to limits beyond its maximum strength, it is suggested that the model be subjected to a negative initial

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[8] A Victor camera was used for pictures taken at 32 and 64 exposures per second.

[9] From the curve it is seen that the initial stress was 4 fringes, the maximum stress $10^1/_2$ fringes, and the final stress 5 fringes. The falling weight therefore caused a static stress of $5 - 4 = 1$ fringe, and an impact stress of $10^1/_2 - 4 = 6^1/_2$ fringes.

[10] This was suggested by Professor Timoshenko.

[11] See "Vibration Problems in Engineering," by Timoshenko, p. 2.

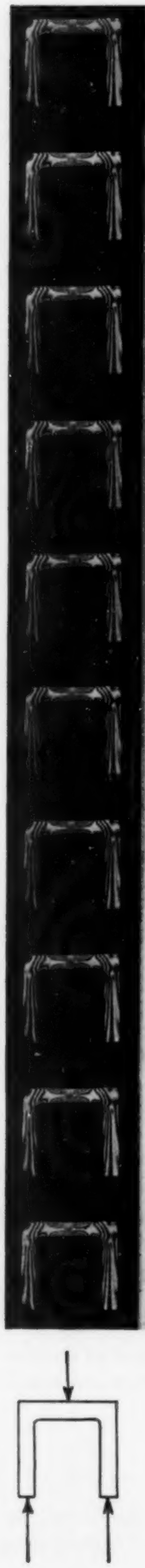

Fig. 33

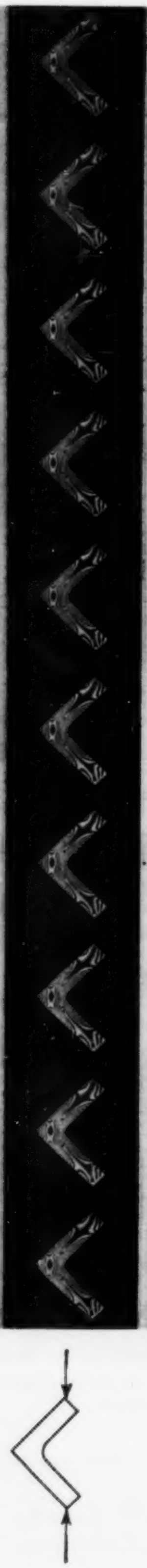

Fig. 34

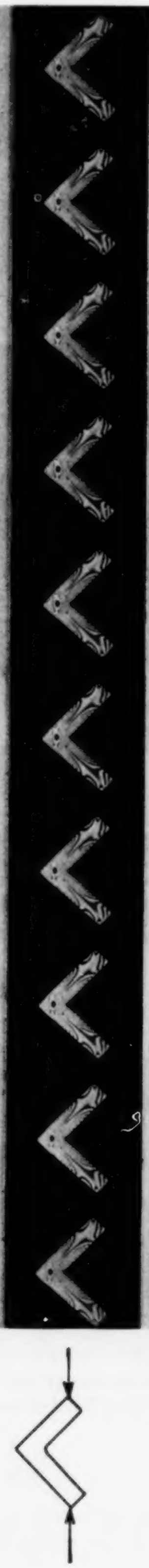

Fig. 35

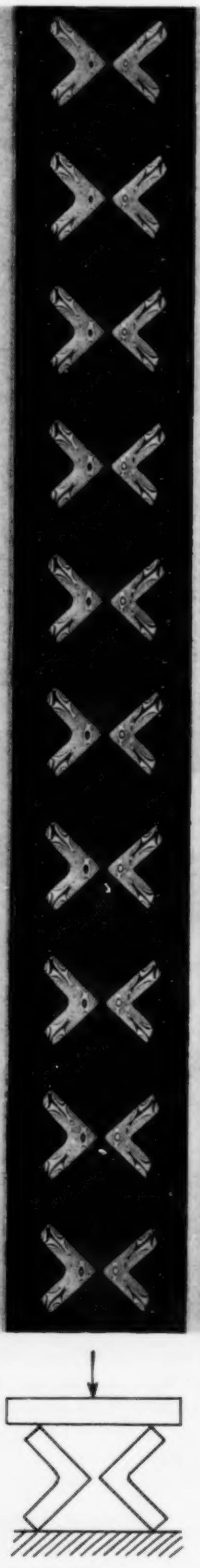

Fig. 36

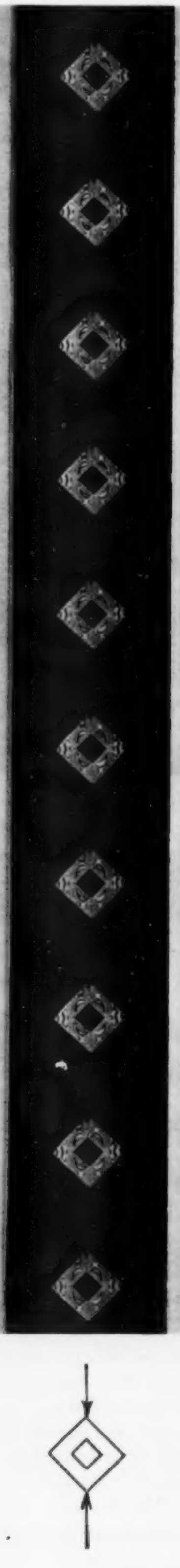

Fig. 37

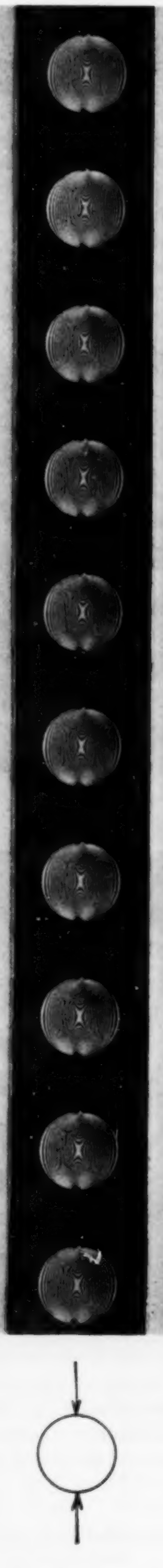

Fig. 38

Fig. 39

Group 4—Angles, Disks, Square Plates

bending moment, thereby doubling the range of the possible stress due to impact.

The time of direct impact has been measured at various times and found to be of the order of 0.001 sec. This suggests the need of a high-speed camera capable of taking several thousand pictures per second. When such high-speed equipment becomes available the problem of stress fluctuation during impact, as well as the phenomenon of failure, will lend itself to photoelastic investigation.

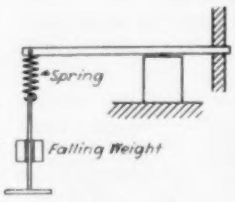

FIG. 40 SKETCH SHOWING LOADING FOR SLOW IMPACT

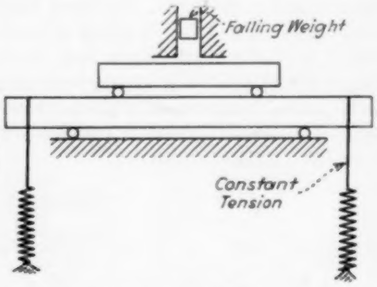

FIG. 42 SUGGESTED LOADING FOR IMPACT

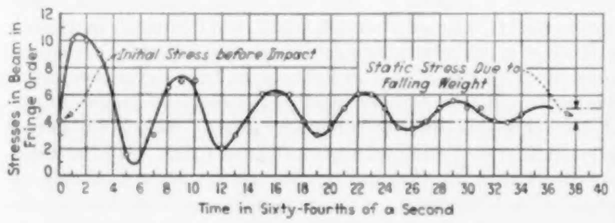

FIG. 41 FLUCTUATION OF STRESS IN PURE BENDING UNDER IMPACT (64 exposures per second.)

ACKNOWLEDGMENT

The author wishes to express his indebtedness to Prof. S. Timoshenko for the interest shown in this work and for valuable advice. It was he who suggested the study of impact stresses. The author is especially grateful for permission to quote from his unpublished report. The author also wishes to thank Prof. E. L. Ericksen, Head of the Department of Mechanics at the University of Michigan, for cooperating in the work by providing funds for supplies and materials; Prof. O. W. Boston, Director of Engineering Shops of the University of Michigan, for allowing him the use of the DeVry camera; and Dora Lipkin-Frocht for valuable laboratory assistance during the filming of the pictures.

Discussion

RAYMOND J. ROARK.[12] The writer, after reading papers on photoelasticity and the application of the mathematical theory of elasticity, regrets that the results secured are not more definitely correlated with the results of tests to destruction on the actual materials of construction. Both the general theory of elasticity and an experimental procedure such as is employed in photoelasticity are essentially techniques; both have been carried to a high state of development by their exponents, and both are powerful and useful tools. But they are alike in being strictly limited by the assumptions upon which they are based, and the results which they yield are for that reason not infrequently of less significance than might be supposed. The elastician and the experimenter are of course quite aware of these limitations, but the average engineer may not be; and impressed by mathematical wizardry or experimental ingenuity, he is likely to accept too literally the apparent implications of such investigations.

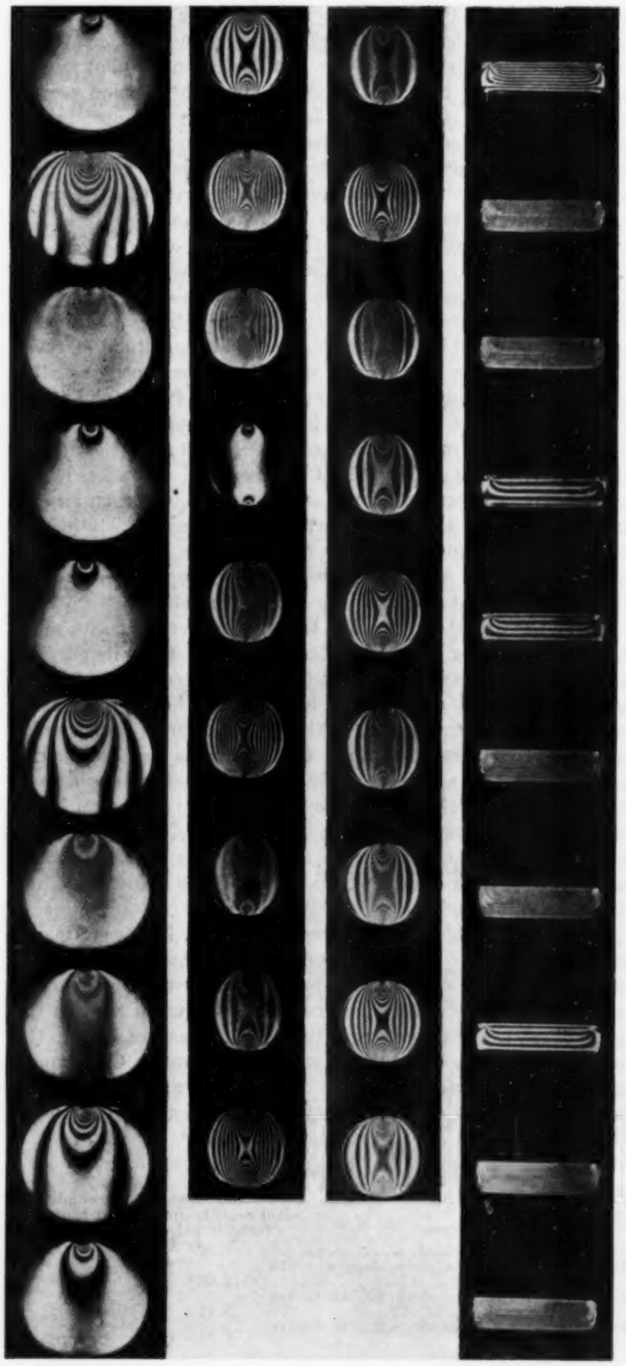

Fig. 43 (a) Fig. 44 (b) Fig. 45

[(b) is a continuation of (a).]

GROUP 5—FLUCTUATION OF STRESS DURING SLOW IMPACT

For example, as Mr. Frocht and others have shown, the presence and intensity of stress concentrations due to peculiarities of form are beautifully demonstrated by photoelasticity. For many cases the theory of elasticity can also be effectively applied. Now the conditions indicated by either technique are such as to lead the inexpert to an exaggerated opinion of the seriousness of such stress concentrations. Even for cases involving fatigue of ferrous metals, where little if any plastic action can be counted on, it is well known that the weakening effect of holes, notches, scratches, etc., is much less than might be surmised from the photoelastic indications. For cases of static loading, where some plastic action inevitably occurs before failure in even the most brittle materials, the effect of such stress raisers may reasonably be expected to be even less. Indeed, it appears to be an open question as to whether the relative ultimate strength of elements of various forms can be reliably predicted from a study of the intensity and distribution of elastic stresses alone.

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[12] Professor, University of Wisconsin, Madison, Wis.

There are many structural elements, especially castings, the form of which is necessarily such as to lead to stress concentrations of greater or less severity. Photoelasticity provides a means of ascertaining the distribution and magnitude of these stresses within the elastic limit; but does it necessarily afford a criterion for judging as to the actual margin of security against failure, and does it conclusively indicate the most advantageous among a number of available designs? Is it, to put the matter differently, safe to assume that if an element *A* shows less intensity of stress concentration within the elastic limit than an element *B* of different form, *A* is better fitted to resist a static load or has a greater factor of safety? The writer is inclined to be skeptical of this. Perhaps a few definite examples will serve to illustrate the point and be of interest in themselves.

The writer, together with Mr. R. S. Hartenberg,[13] has for some time been carrying on an investigation which has for its purpose the study of stress concentration (especially as concerns effect on ultimate strength under static loading) by the use of plaster models and the correlation of the results secured with results obtained when cast iron and other materials are used. The plaster used gives an almost straight-line stress-strain curve up to the ultimate, and might therefore be expected to be as sensitive to the effects of stress raisers, or to be as "tender," as any material of construction. Of the many different tests that have been made and that are contemplated, a few will be selected to illustrate the point made.

Beams 1 in. wide, $1^1/_2$ in. deep, and 14 in. long were made; some of these were left as molded, some were notched and some were drilled. All were tested to failure, and the nominal modulus of rupture of each form was calculated in the usual way, on the basis of the net cross-section at the point of fracture. From the results so secured, the factor of stress concentration *C* for conditions at failure was determined. The results are given in Table 1, together with the calculated factors of stress concentration for elastic conditions.

TABLE 1

Description of beam	*C* for conditions within elastic limit (calculated)	*C* for conditions at rupture (experimental)
Plain rectangle as molded	1.00[a]	1.00
One 72° V-notch $^1/_4$ in. deep, sharp as could be made	<11.00[a]	2.24
One 72° V-notch $^1/_4$ in. deep, 0.05 in. radius at point	5.47[a]	2.02
One 72° V-notch $^1/_4$ in. deep, 0.15 in. radius at point	2.55[a]	1.76
One saw cut $^1/_4$ in. deep	...[c]	2.34
Many saw cuts $^1/_4$ in. deep, $^1/_4$ in. apart	...	1.62
One vertical hole 0.0625 in. diameter	2.82[b]	1.23
One vertical hole 0.500 in. diameter	2.00[b]	1.21

[a] By Inglis formula: $C = 1 + 2\sqrt{(d/\rho)}$.
[b] By Coker's formula: $C = 3K/(K + 1)$.
[c] Coker found, by photoelastic analysis, a stress-concentration factor of 2.78 for a rectangular notch with rounded corners.

It is not the fact that the experimentally determined coefficients are much smaller than the calculated coefficients that is to be remarked; such a disparity is to be expected, because even with the brittle material used, some plastic action is present. What is noteworthy is the fact that the difference in the strength of the several types of specimen is so much less than the difference in intensity of elastic stress would lead one to expect—whether determined by mathematical analysis or photoelasticity. Only a small number of tests have so far been made on cast-iron beams of this sort, but these showed smaller coefficients and less difference in coefficients than the plaster beams.

Again, circular plates, 12 in. in diameter and $^1/_2$ in. thick, were tested with edge support and concentrated center loading, the diameter of the circular area over which the load was applied being varied from 0.2 in. (the least that could be used without local crushing) to 1 in. Table 2 gives the relative magnitude of the loads that caused failure and, for comparison, the relative magnitude of the loads which, according to the theory of elasticity, would have caused a given stress within the elastic limit. Again it appears that elastic stress is not alone a sufficient index to relative ultimate strength.

TABLE 2

Diameter of loaded area, in.	Relative strength (from test to failure), per cent	Relative load to produce a given elastic stress (calculated), per cent
1.00	100.0	100.0
0.50	87.5	82.2
0.20	89.2	66.7

In general, it appears probable that any analysis of stress within or even slightly beyond the elastic limit, whether mathematical or photoelastic, is likely to lead to exaggerated ideas of the importance of stress raisers, and to indicate, at least to the inexpert, a greater disparity in static strength as between one design or form of element and another than actually exists. This is not meant to imply that such analyses are not of great value. It is intended to suggest that, by themselves, they do not always provide an adequate basis for design and that it is important that they should be correlated with the record of materials in service and in tests to failure.

F. L. EVERETT.[13a] A field of study considerably greater than has previously been known is revealed by the paper. The presentation of further possibilities of stress analysis by the use of photoelasticity such as a study of impact and of failure is particularly interesting.

The photoelastic method can very profitably be extended beyond that suggested up to the present time to investigate the failure of materials due to fatigue. Until now the precise manner in which fatigue cracks are produced finds explanation based only on observations of the final fracture. The fractured surface consists of two regions, one at the point of greatest stress, which is believed to form relatively slowly, and the other represented by the remainder, which breaks suddenly and as a brittle material due to the inability of the reduced section to sustain the entire load.

It is of interest to study the history of the fatiguing action, particularly during the gradual formation and growth of the initial cracks. The usual design of fatigue specimen does not lend itself to study by photoelastic means, since the method requires a two-dimensional system. A two-dimensional type of fatigue specimen, however, together with a schematic drawing of a suitable apparatus designed to give a constant maximum bending moment over the length of the specimen, is shown in Fig. 46.

In applying this apparatus, the axis of the polarized beam would pass through the model in a direction normal to the plane of the drawing. The source of light could be a Neon lamp operated as a stroboscope, synchronized with the apparatus producing the fatigue stress, or a monochromatic of polychro-

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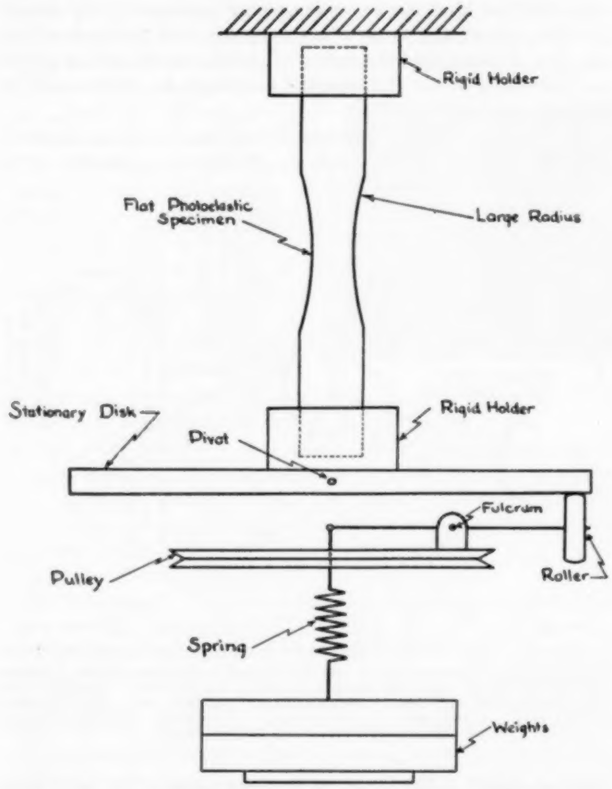

FIG. 46 FATIGUE APPARATUS FOR PHOTOELASTIC STUDY

matic light and mechanical stroboscope may be used. Stress concentration resulting from various-shaped notches in fatigue specimens could of course be readily studied.

The resulting behavior could be observed at any desired magnification by throwing the picture on a screen or preserved for future study, according to the scheme described and used by the author, by utilizing the motion-picture camera.

ARSHAG G. SOLAKIAN.[14] To the knowledge of the writer, this is the third attempt ever made in applying kinematography to the photoelastic study of stresses produced by moving loads. It is very pleasing to see the author's success in obtaining excellent patterns of stress fringes, developed under the action of dynamic stresses.

Of all the possible applications of kinema-photoelasticity, the study of dynamic stresses is of the greatest importance, and it is hoped that in the near future photoelasticity will greatly facilitate the formulation of a comprehensive and satisfactory theory concerning the disputed phenomenon of failures of materials under moving loads, such as is the case in gear action, tool cutting, centrifugal forces, repeated stresses, impact stresses, etc. Of all these different problems, the writer wants to emphasize the importance of photoelasticity in the study of impact stresses.

In the investigation of the behavior of materials in impact tests, the abrupt change of cross-section producing a non-uniform distribution of stress across a section of the specimen or near that region has not yet been considered to its full merit and cared for in the design of the elements of machines or structures. The ordinary mechanical tests register only the total energy required for causing failure of the material, usually represented graphically by the area under a complete stress-strain curve. The final tabulation of the results representing the resistance of the material against dynamic stresses is usually in terms of the amount of energy absorbed per unit area of cross-section in failure or unit volume of material withstanding the impact. The determination of the total effective volume of material by mechanical means is an impossible proposition, and the results obtained by various ingenious methods should be looked upon very cautiously with regard to their accuracy of principles. Only photoelasticity can tell us something definite and of convincing character through the study of stress fringes, all over the region affected by impact.

In static-stress problems, isochromatic or monochromatic fringe photographs of stress concentrations have positively demonstrated the fact that stress distribution across a section in the neighborhood of holes, notches, or similar discontinuities of boundary lines, etc. is of a varying character. Coker and Heymans have demonstrated in 1922 that, with specimens 10 mm. square in cross-section, notched halfway through with notches having different radii at the bottom, the stress at the bottom of the notch increases from two to six times the average stress, as the radius of the notch is decreased from 10 to 0.6 mm. Such stress concentrations as a matter of fact exist also during the application of the moving loads and are of seriously harmful character, on account of their being mostly responsible for the initial starting of a crack line or yielding of the material at the most stressed point.

The various impact machines used in the past are of either of pendulum type (Izod, Charpy, Amsler, Olson, Russell), or drop-hammer (Page, Fremont, Hatt-Turner), or rotating flywheel (Guillery), or repeated-impact (Stanton, Natsumura). The pressure produced by impact in the specimen is determined by time-position, or deflection, or deformation, or elastic-stretch methods. None of these testing machines with their elaborate accessories is adapted for the determination of the internal stresses and strains produced during the action of the pressure. On the other hand, total energy measurements are not of a conclusive character in telling the whole story of impact phenomena. Besides, there are numerous factors influencing greatly the accuracy of the results obtained in a mechanical test, such as the effect of the size and velocity of the moving mass on the resilience of the specimen, the amount of the kinetic energy of the mass lost before impact in producing vibrations and overcoming inertia and frictional resistances, etc. That is why impact tests in general have been, on certain occasions, of opposite nature in their conclusions, depending upon the kind of the machine, the type and size of the specimen, and the notch adopted by different investigators.

Photoelasticity is fortunately free of these complications. It is simple in principle and operation. The recent introduction of stress-cured bakelite for models and the use of isochromatic or monochromatic fringes of stresses, combined with the further development of kinematrography for high-speed exposures, will considerably facilitate the solution of a great many theoretically indeterminate problems in general and the investigation of dynamic stresses in particular.

J. P. DENHARTOG.[15] The beautifully sharp pictures obtained by the author lend themselves very well to an experimental study of the location of the so-called "singular points" of the two-dimensional stress distribution. This is especially striking when seeing the loading gradually applied, as in the moving picture.

Though the printed illustrations give but an imperfect idea of the beauty of the moving picture, it will be attempted to describe the point in hand with the aid of Fig. 23 of the paper. In the middle, between the two fillets at the right (one sharp, one round), there are two black curves just touching each other. The detail of this is shown schematically in Fig. 47*a*. With increasing loading, the moving picture passes through the stages *a*, *b*, *c*, *d*, then *a* again, etc. The intersection point of the two curves of Fig. 47*d* is a "singular point." The theory of these points has recently been published by Prof. L. Föppl, of Munich, Germany.[16] It is apparent that for a study of this important phenomenon, the monochromatic method of the author is superior to the usual polychromatic one.

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[15] Westinghouse Research Laboratories, East Pittsburgh, Pa.

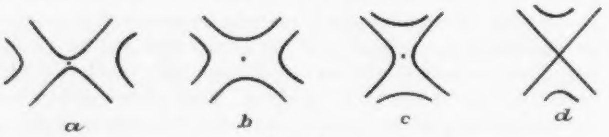

FIG. 47

On the subject of the impact pictures taken by the author, the writer cannot be quite as enthusiastic as on the rest of the paper. It is not seen what possible use can be derived from such tests or from results such as those shown in Fig. 41. The stress multiplication due to impact can be made as large as desirable by dropping the weight from a sufficient height.

Ziro Tuzi.[17] On reading the paper the writer is pleased to know that the fringe method which he once recommended[18] has been adopted for practical use with such splendid results. He desires to pay his respects to the technical skill of Dr. Frocht in getting such beautiful films as shown in Figs. 5 to 39. They seem to be wonderfully clear and excellent for demonstration purposes.

The problem of the material available for photoelastic work is always rather difficult on account of the lack of uniformity of the sensitive materials, and the author was very fortunate in being able to obtain such good bakelite free from persistent initial stress. About this problem the editor of *Engineering* (London) states in the articles[19] discussing the writer's fringe method that Mr. H. T. Jessop could eliminate the initial stress of bakelite successfully. Mr. I. Arakawa[20] also states that he has succeeded fairly well in annealing bakelite specimens. The writer has repeated these methods with various kinds of bakelite, made in our country, the majority of which resulted unsuccessfully. Phenolite is practically the same material as bakelite, with only a difference in the process of production. Manufacturing it by ourselves lately, we have had much difficulty in obtaining uniformity, free from obstinate mottle, though very recently we found a sure way to obtain good ones. Upon these experiences the writer is of the opinion that the problem of annealing depends not on the method but on the material plate itself. It seems that there are two kinds of initial stresses in the baked plate of such organic compounds: one is similar to the stream lines in the optical glass, displaying discontinuous mottle, the other being a mere continuous tint. So far as the writer has experienced, the former can never be removed by any means, while the latter is very easily eliminated by a simple annealing process in an air bath of 65 to 90 deg. cent., with slow cooling, as mentioned in the writer's previous publication, and it is better to embed the sample in refractory powder. Therefore in case the raw material belongs to the second kind, though accidentally, the elimination of the initial stress is rather easy, whether it is bakelite or phenolite, but with the first kind the annealing seems hopeless by any means. For the present, the writer is not quite sure how to obtain a good one, but it seems that the purity of the chemicals such as phenol or formalin is one of the essential conditions to obtain optically uniform plates.

The results of the experiments of the falling weight, as shown in Figs. 40 to 45, are very interesting. Really the application of the photoelasticity to the study of dynamic stress is the most striking problem. Concerning this, the very ingenious photoelastic results of Mr. K. Sekiguti have been introduced in Dr. J. Obata's paper[21] read at the World Engineering Congress, Tokyo. In his experiment the fringes of a section of a bakelite beam under vibration were photographed with a falling plate camera parallel to the reaction force measured by a piezoelectric dynamometer, as reproduced in Figs. 48 and 49. The writer recently has also tried the same kind of experiment (not published), endeavoring to shorten the time of exposure down to 1/2500 sec., with a film running at a speed of 2.5 m. per sec. and a slit of 1 mm. Fig. 50

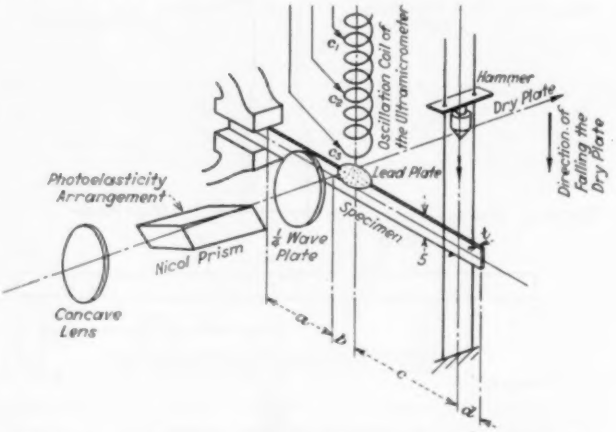

FIG. 48 TEST BY DR. J. OBATA OF VIBRATION OF BAKELITE BEAM
(Experimental arrangement for the simultaneous recording of the vibration due to an impact and the change in stress distribution. Specimen is a bakelite bar 235 mm. long. Measurements: *a*, 20.5 mm.; *b*, 113.0 mm.; *c*, 58.5 mm.; *d*, 10.0 mm.; *s*, 11.92 mm.; *t*, 5.98 mm. Hammer, distance of fall, 176 mm.; weight, 34.39 gr.)

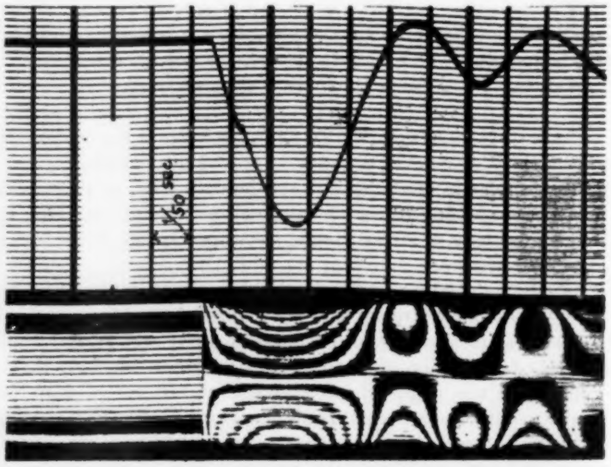

FIG. 49 RECORD OF THE VIBRATION OF A BAKELITE BAR
(Due to an impact and the stress distribution, or photoelastic fringes, in it. Obtained by K. Sekiguti.)

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[16] *Abh. Bay. Akad. Wiss.*, 1930.

[17] Member of the Institute of the Physical and Chemical Researches, Tokyo, Japan.

[18] "Photographic and Kinematographic Study of Photoelasticity," Z. Tuzi, Sci. Paper Inst. Phys. Chem. Researches, Tokyo, vol. 8, pp. 247–267 (1928). "On the Development of Experimental Method of Photoelasticity," Z. Tuzi, Ebenda, vol. 12, pp. 21–36 (1929).

[19] *Engineering*, Jan. 27, p. 116, and Feb. 13, pp. 193–196 (1931).

[20] (3) I. Arakawa, Proceedings of Physico-Mathem. Soc., Japan, ser. 3, vol. 7, no. 11, p. 161 (1925).

[21] J. Obata, Proc. World Eng. Congress, Tokyo, vol. 5, p. 91.

shows the variation of fringes at the central section of a curved beam when struck by a hammer. From the photograph it can be concluded that the exposure of 1/2500 sec. is not short enough to analyze such a change. To catch the elastic wave in a solid is no doubt a very interesting problem; but assuming the Young's modulus and the density of phenolite to be $E = 35{,}000$ kg. per sq. cm. and $\rho = 1.26$ gr. per cu. cm., respectively, the sound velocity in phenolite can be calculated to be about 1650 m. per sec., with the formula $U = \sqrt{(E/\rho)}$. It seems the exposure in photographing the wave front should be, at most, shorter than 1/1650000 sec., admitting for the displacement of 1 mm. of a fringe during the exposure. Such technique, however, seems rather difficult for the present, and yet the writer thinks that such a problem is not only very interesting, but very important and worthy of future efforts.

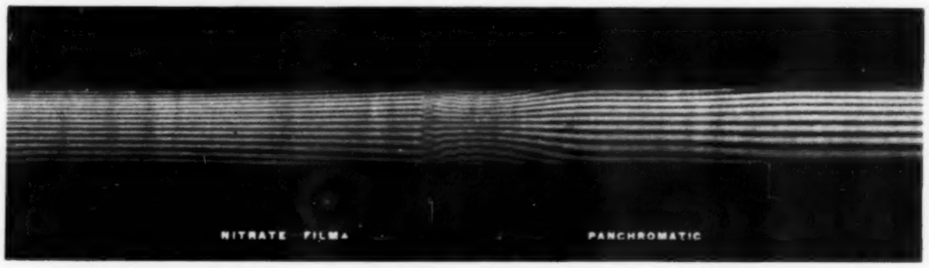

Fig. 50 Variation of Fringes at Center of Curved Beam Struck by a Hammer

H. F. Moore.[22] Professor Frocht's demonstration that stress concentration due to several notches close together is less than that due to a single notch is very interesting. This lessening of effective stress concentration in the case of screw threads was shown by R. R. Moore,[23] and his work was checked by tests at the University of Illinois. Such a lessening of stress concentration finds a rough analogy in the bending stresses in rails under closely spaced wheel loads as shown by Talbot[24] and by Timoshenko and Langer.[25]

In connection with the photoelastic method of stress determination and other non-mathematical methods, such as the soap-film method, the brittle-material method, the endurance-limit method, the "yield point by scaling" method, etc.[26] it is well to note that the theoretical stress concentration is always found to be worse than the effective stress concentration as shown by tests to failure and that various materials differ widely in the degree of effective stress concentration developed by a given notch, hole, or other "stress raiser."

Author's Closure

Broadly speaking, there are three aspects to the science of strength of materials: one dealing with stress analysis, another with mechanical properties of materials and tests to destruction or failure, and a third with theories of failure.

Professor Roark's comments pertain to the fact that to date we have not evolved a satisfactory, comprehensive theory to explain the failure of materials under stress. He does not question the correctness or validity of the results of stress analysis, but rather their significance and bearing on the question of behavior of materials at failure.

He calls attention to the importance of correlating stress data with records of materials in service and with tests to failure. There is no doubt that the designer cannot shape or dimension his machine parts and structural members from a knowledge of stress distributions alone. The mechanical properties of the materials, their degree of homogeneity and isotropy, the kind of loading, and the rate of loading must be known as well, in order to make a rational design. The author also agrees that under static conditions of loading the significance of stress concentration is rather small.

Professor Roark goes, however, beyond the limits justified by available experimental evidence when he says: "The seriousness of stress concentration is exaggerated even for cases involving fatigue of metals where little if any plastic action can be counted on." On this point the author wishes to quote an eminent authority. Professor S. Timoshenko says: "Under the action of reversal of stress, small cracks often start at points of stress concentration, and the majority of fractures of machine parts in service can be attributed to such progressive cracks."[27]

The destructive effect of stress concentration at a round hole can also be vividly seen from Fig. 51,[28] which shows a disastrous failure of one disk, and another on the verge of such failure.

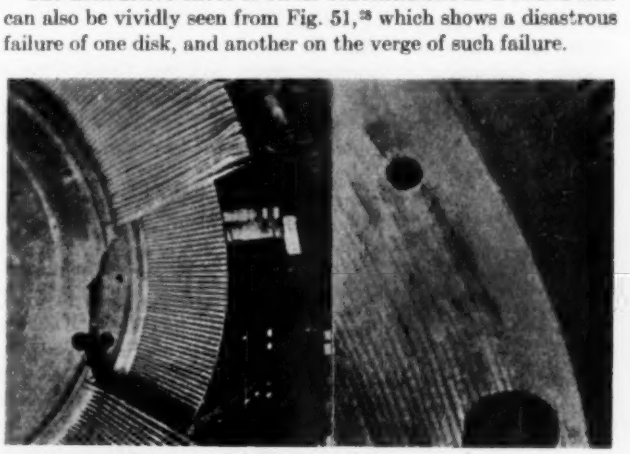
a b

Fig. 51

The investigation which Professor Roark reports should prove interesting and useful. It is hoped that materials other than cast iron will be used. It has already been shown that this material is less affected by discontinuities than, say, high-carbon steel.

Table 2, it would seem, provides as much data against as for Professor Roark's contention that there is a large disparity between actual failing loads and those predicted by the mathematical theory of elasticity.

Professor Roark seems to have set out to caution the inexpert or average engineer "against a too literal acceptance of the apparent implications of stress concentration factors," and to reassure him that "the seriousness of these stress raisers has been exaggerated." The experimental evidence available today shows, however, that greater harm may come from attempts to lull the average engineer into a false sense of security than is likely to result from the meager efforts to call attention to the existence, value, and significance of stress concentration.

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[22] Research Professor of Engineering Materials, University of Illinois, Urbana, Ill.

[23] Proc. Am. Soc. Test. Matls., vol. 26, part 2, pp. 265 and 269 (1926).

[24] Trans. A.S.C.E., vol. 82 (1918), Proc. Amn. Ry. Eng. Assn., vol. 19 (1918).

[25] "Stresses in Railroad Track," paper at December, 1931, meeting of A.S.M.E.

[26] See "Resistance of Materials," by Seely, Appendix 3 (pub. by Wiley).

[27] See "Strength of Materials," by S. Timoshenko, vol. 2, p. 615.

[28] From "The Protection of Steam-Turbine Disk Wheels From Axial Vibration," by W. Campbell, *Trans. A.S.M.E.*, 1924, vol. 46.

Dr. DenHartog points out that photoelasticity can be successfully applied to the study of "singular points." This point might, perhaps, be better illustrated by referring to Fig. 39, instead of Fig. 23, which clearly shows two permanently dark spots on the horizontal diameter near the large circle. The dark point shown in Fig. 47 is not found in the corresponding Fig. 23, and the erroneous impression was undoubtedly gained during the hurried showing of the film.

Dr. DenHartog cannot see what possible use can be derived from impact tests. Numerous uses have been mentioned in the paper, and others have been added by Drs. Everett, Solakian, and Tuzi. In addition to these, other impact problems might be added such as impact of air, steam, or water on the opening of a valve.

The study of "singular points" is undoubtedly interesting, but is it more useful than the problem of stress intensity during impact? Being an engineer himself, the author appreciates and shares the desire for practical application of all knowledge. Nevertheless, he would like to see greater freedom given to the research worker and lesser demands made to justify his work by immediate useful application. The history of science abounds in evidence of pure research finding uses and application totally unforeseen at the time when the work was done.

The author was particularly pleased to hear from Dr. Tuzi. If there is an initial stress in bakelite which cannot be removed by any annealing process, the author was fortunate not to have encountered such in his work. He has worked with a variety of samples containing pronounced green and red bands. Some difficulty was encountered with the material close to the edges. Aside from that, the material responded to treatment. It is, however, likely that the manufacturing process should have an influence on the problem of annealing.

The author was glad to learn of the interesting work by Messrs. Sekiguti and Obata. He shares Dr. Tuzi's opinion that it would be instructive to photograph the elastic wave, and wishes him success in any such attempts.

In an oral discussion Dr. Nádai criticized the author's work and method. (1) He called attention to the fact that the curves of Fig. 3 are ovals or ellipses and not circles, as they should be theoretically, and criticized the author for not obtaining a closer agreement between photoelasticity and theory. (2) He similarly criticized Fig. 30, claiming that under uniform pressure the fringes must be circles passing through the edges of the die (points A and B in the figure). (3) He further said that while the author was unsuccessful in obtaining such circles, Dr. Mesmer, of Göttingen, did obtain the circles in question. (4) And lastly he said that in 1929 Dr. Mesmer had discovered a process of annealing of bakelite and criticized the author for not acknowledging his work.

In reply to (1), the curves in question are evidently not circles. It is also true that under certain conditions the fringes should be circles. Dr. Nádai has failed, however, to mention these conditions. Only in the case of a load acting on an infinite plate could the fringes be expected to be circular. No claim, however, is made in this paper that Fig. 3 refers to such a plate. Furthermore, no claim was made that Fig. 3 was intended as a photoelastic corroboration of any mathematical prediction. The only comment on this figure in the text was that a single concentrated load, even if small, produces large local stresses in the immediate vicinity of the load. The model in this case was a fairly oblong plate about 4 in. by 6 in., and the purpose of Fig. 3 was to furnish photographic evidence of stress concentration. If it is desired to compare the shape of the fringes for this plate with those of an infinite plate, it would be necessary to restrict ourselves to the fringes in the immediate vicinity of the concentrated load. If that is done, then the fringes are seen to be as close to circles as those obtained by Dr. Mesmer.[29]

The situation is similar in regard to Fig. 30. It is true that for an infinite plate and under uniform pressure the fringes may be expected to be circular. But the author did not claim to be working with such a plate, nor to have corroborated any mathematical theory, nor to have secured *perfectly* uniform pressure under the die. The only claim made was that the effect of a layer of rubber is to remove the stress concentrations, to produce a *nearly* uniform pressure distribution, and to shift the maximum shear zone from points A and B to point C. These claims are believed to be warranted by the stress pattern shown in Fig. 30.

Referring now to the circles obtained by Dr. Mesmer, it is the author's opinion that they prove exactly nothing. It is interesting to note how these circles were obtained. Dr. Mesmer tells us that he produced the isochromatics for this case by preparing a special die of celluloid with two carefully rounded corners which was pressed against the test piece.[30] In other words, the circles were not obtained by applying a known uniform pressure and then observing the formation and shape of the isochromatics, but by carefully filing and rounding the edges of an initially flat die until after numerous trials approximately circular isochromatics were obtained. Such results are a bit forced. The author fails to see how manipulations of this kind establish the relation between the isochromatics and uniform pressure which is claimed by Dr. Nádai. It is evident that by continuing this process of "carefully rounding" the edges of the die an infinite number of other isochromatics can be obtained as well.

The author regrets that in his paper on "Recent Advances in Photoelasticity" he failed to mention Dr. Mesmer's attempts to anneal bakelite. At the time of writing the paper he was not familiar with Dr. Mesmer's valuable work. Proper acknowledgment was made to the sources that were known to the author. He thus acknowledged the work of Mr. Jessop,[31] who it was claimed had discovered a process of annealing bakelite as far back as 1923. Such unintentional omissions unfortunately occur. We thus find G. Mesmer, working under Dr. Nádai in 1929, failing to acknowledge Mr. Jessop's work of 1923.

With respect to Dr. Mesmer's annealing method, the author regrets to have to quote from memory, as he was unsuccessful in obtaining a copy of his paper in Pittsburgh. His annealing experiments were carried out in paraffin oil and not in an air bath and at a temperature much lower than that used by the author. This process left stresses which Dr. Mesmer estimated as 35 kg. per sq. cm., which in our units amounts to about 500 lb. per sq. in. In terms of fringes for bakelite $^1/_4$ in. thick, such as used by the author, this is equivalent to about 1.76 fringes, the stress value of fringe No. 1 being 284 lb. per sq. in. Bakelite models with such large initial stresses could not be used for photoelastic research, and this may perhaps be the reason why Dr. Mesmer used celluloid in most of his investigations.

The author was very much interested in the remarks of Mr. Solakian and in the problem suggested by Dr. Everett.

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[29] See cit. on p. 5, or Fig. 304, p. 245, in "Plasticity," by A. Nádai and A. M. Wahl.

[30] See dissertation by G. Mesmer, loc. cit., p. 5, or "Plasticity," by A. Nádai and A. M. Wahl, pp. 248, 249.

[31] See footnote 3, p. 137, *Trans. A.S.M.E.*, APM-53-11.

APM-54-10

Photoelastic Study of Shearing Stresses in Keys and Keyways

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A. G. Solakian

The technique of photoelastic determination of stresses in structural members using bakelite as the transparent medium is described. The mechanical properties of this material and the process of annealing it for the removal of the internal stresses are given. The shearing stresses in keys and keyways are investigated. It is shown that it is desirable to round the corners of the keyway, with a corresponding chamfering of the key. The use of color fringes called isochromatics for the determination of shearing stresses has been found very helpful and is highly recommended.

G. B. Karelitz

1 Introduction

The work described in this paper has been carried on as a research problem supported by the Ernest Kempton Adams Fellowship of Columbia University, New York. The laboratories and facilities of the Physics Department of this university were used. The problem assigned consisted of working out a simple technique for the photoelastic determination of stresses in a two-dimensional stressed state and of applying this method for the investigation of a practical case. A large variety of similar mechanical and structural problems might, however, be analyzed, giving quite interesting results.

The design of a key fitting into a keyway in order to transmit a given torque is more or less based on empirical formulas or "standard practice." St. Venant's classical formulas for the distribution of stresses in the different fibers of shafts of various cross-sections are hardly applicable to those ordinary cases where the introduction of a keyway causes discontinuity of the boundary lines. Very valuable tests were carried on in 1909 by Prof. Herbert F. Moore[3] in an investigation of the effects of keyways on the torsional strength of shafts, and some have tried to apply many ingenious methods for the measurement of stress intensities in shafts with keyways, such as the use of strain gages, Prandtl's membrane analogy, Luder's lines, etc. Yet these methods presume a torque applied to the shaft without due consideration to the rôle played by the key in transmitting the resulting stresses to its adjacent sides, producing heavy concentrations of stresses, an exact knowledge of which is of utmost importance for a balanced design of the elements and for the combined safety of the entire mechanism.

The photoelastic method is by far the most convenient and direct method of observation of stresses in this case. The magnitudes, as well as the directions of shearing and principal stresses, can be determined with sufficient accuracy. Meantime a colored or black and white photographic record of stress concentrations can be taken for future references, if so desired.

During the last 20 years a great many interesting problems have been handled by the photoelastic method. Practically most of these cases were of tension, compression, or bending, which could be analyzed mathematically with more or less difficulty and the results compared with those obtained by photoelasticity. The models used were of glass or xylonite (celluloid). Recently the possibilities of bakelite and phenolite were brought out.[4,5,6] Bakelite, known as C25 bakelite,[7] has been adopted for the work described in this paper, on account of its high optical sensitiveness, resulting in several orders of color fringes under moderate stresses.

The model tested is a compound piece consisting of three different parts—namely, the key, the shaft, and the hub or

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[2] Associate Professor, Department of Mechanical Engineering, Columbia University. Mem. A.S.M.E. Mr. Karelitz is also Division Engineer, Transportation Appliances Division, South Philadelphia Works, Westinghouse Elec. & Mfg. Co., Philadelphia, Pa. He was born in Russia in 1895 and received degrees in naval architecture and mechanical engineering from the Polytechnic Institute at Petrograd in 1918. For three years he was special engineer with the Waterways of Communication in Russia. He came to the United States in 1921 and has since been connected with the Westinghouse Elec. & Mfg. Co. Prior to his recent appointment at the South Philadelphia Works he had been manager of the mechanics division of the Pittsburgh Research Laboratory. He began his work at Columbia in 1930.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] "The Effect of Keyways on the Strength of Shafts," by Herbert F. Moore, Bulletin no. 42, Publications of the Engineering Experiment Station of the University of Illinois.

[4] "On the Graphical Determination of Stresses From Photoelastic Observations," by Prof. L. N. G. Filon, *Engineering*, Oct. 19, 1923, p. 511.

[5] "A New Material for the Study of Photoelasticity," by Z. Tuzi, Scientific Papers of the Institute of Physical and Chemical Research, no. 112, October, 1927, Tokyo.

[6] "Further Development in Photoelasticity," by R. V. Baud, Journal of the Optical Society of America, vol. 18, no. 5, May, 1929.

[7] Supplied by the Apex Specialties Corporation, of Brooklyn, N. Y., distributor for Bakelite Corporation of America.

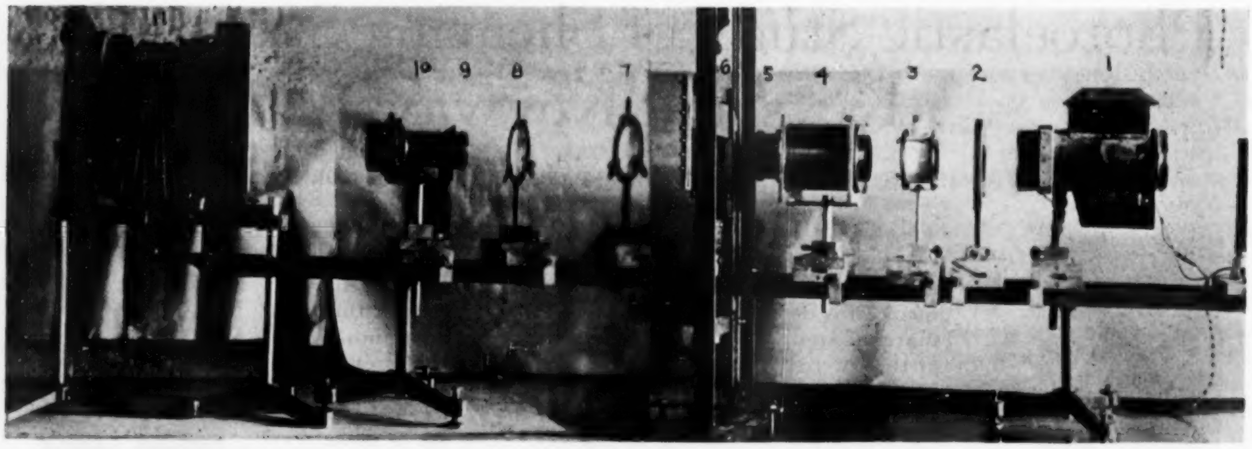

FIG. 1 OPTICAL SET OF COLUMBIA UNIVERSITY

1, point-source light; 2, condenser; 3, water cell; 4, polarizer; 5, quarter-wave mica plate; 6, tension frame; 7 and 8, convex lenses; 9, quarter-wave mica plate; 10, analyzer; 11, screen and plate holder.)

pulley. The results of the tests are therefore dependent on the accuracy with which the parts are machined. Another peculiarity of the problem is the application of a constant torque to the shaft with the object of reproducing the actual conditions existing in practice. In order to eliminate the internal stresses caused during the machining process, the models were annealed after fitting the different pieces together. The establishment of a proper annealing process involved a large amount of experimentation.

2 THE PHOTOELASTIC METHOD

(*a*) *Historical.* The phenomena of double refraction of rays of light in Iceland spar was known to E. Bartholin in 1670. Due to the discoveries and further investigations of Seebeck, Brewster, Wertheim, Neumann, and others, it has been found that an isotropic material, such as glass, produces accidental double refraction of plane-polarized rays of light when stressed in tension or compression.

It was Prof. A. Mesnager[8] who first applied the double refracting property of glass under load into practical use in connection with the determination of stresses in the members of a glass model of an arch bridge. To Dr. E. G. Coker[9] is due the greatest credit of introducing celluloid as a material for models, the perfection of an optical set, and of the application of photoelasticity to the solution of a great many mechanical and structural problems. Recently Dr. H. Favre[10] developed a purely optical method of determination of principal stresses by measuring the absolute retardation of the refracted rays of light through a stressed piece of glass.

(*b*) *Principal Stresses and Maximum Shear.* A complete discussion of the details of a two-dimensional stressed state may be found in any standard book on the strength of materials. For the purpose of this paper it will only be mentioned that the principal stresses at a point being p and q, the maximum shearing stress will be $(p - q)/2$ and will act in a plane at 45 deg. with the planes of the principal stresses. Also it may be noted that when the two principal stresses are equal and of the same sign ($p = q$), the maximum shearing stress will be zero in any direction at this point.

The photoelastic method allows of observing directly the values of the maximum shearing stresses. Let a transparent stressed plate be illuminated by plane-polarized light and projected on a screen. When a ray of this light strikes the plate at a stressed point, the ray will be decomposed into two plane-polarized rays, vibrating in the planes of the principal stresses. These two rays travel through the plate with different velocities, proportional to the magnitude of the principal stresses, so that when they emerge out of the plate, the phase of one of the rays is retarded with respect to the other. The amount of retardation for a given wave length and thickness of material is proportional to the optical constant of the material and the difference of the two principal stresses, or to the intensity of the maximum shearing stress as given by $(p - q)/2$. An analyzer placed in the path of the two refracted rays unites the planes of vibrations of the two rays into one. However, on account of the existing phase difference between the two rays, interference takes place. If a monochromatic light be used—i.e., the green light of a mercury-vapor lamp of a wave length of 5460.7 angstroms, the field will be darkened for those points of the plate where the retardation is equal to $^1/_2$, $^3/_2$, $^5/_2$, etc., wave lengths. There will be in general a whole line of points producing the same retardation due to continuity of the stress function. The field will therefore be covered with dark fringes. Fringes of the first, second, third, etc., orders will correspond to retardations of $^1/_2$, $^3/_2$, $^5/_2$, etc., wave lengths. Since the retardation is proportional to the maximum shearing-stress intensity, each fringe line is a contour of equal shearing stress. The values of the shearing stresses corresponding to the different fringe lines can be determined easily by testing a calibrating beam in flexure, the material of the beam being the same as that of the model.

However, by using white light instead of the monochromatic light, a succession of colored fringes will appear in the order of yellow, red, and green. Indeed, whenever the green light is extinguished by interference, as described, the complementary reddish color will appear, and instead of the dark fringes, identical fringes of reddish color will come into appearance on the screen on which the plate model is projected. For each value of retardation another color will appear, and the field will have a multicolored appearance.

3 PHOTOELASTIC APPARATUS OF COLUMBIA UNIVERSITY

(*a*) *The Optical Set.* The authors were very fortunate in being provided with two large Iceland spar Nicols from the Physics Laboratories of Columbia University. This enabled them to design a very simple optical set with the addition of only three convex lenses. The loss of light therefore was small, and the introduction of optical troubles due to the presence of a great many lenses was avoided. A general assembly of the set is shown in Fig. 1. It is made up of a 500-watt and 110-volt alternating current point-source incandescent lamp operated through a switch and a rheostat, a condensing lens, a water cell, a large polarizing prism, a quarter-wave mica plate, the torsion or bending frame, two large convex lenses, another quarter-wave mica plate, an analyzing prism, and a screen with a photographic plate holder. The functions of each of these will be given later. The loading frame will now be described.

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[8] "Mesures des efforts intérieurs dans les solids et applications," by A. Mesnager, Budapest Congress of the International Association for Testing Materials, 1901.

[9] E. G. Coker, *Philosophical Magazine*, vol. 20 (1910), p. 740. Numerous articles on photoelasticity and its applications, mostly in *Engineering*, 1911, 1912, 1914, 1921, 1922, 1925, etc.

[10] "Methode purement optique de determination des tensions intérieures, etc.," by H. Favre, *Schweizerische Bauzeitung*, December, 1927.

(*b*) *The Torsion Frame.* This part as a whole is attached to a large tension frame through a set of bolts, as shown in Figs. 2 and 3. The tension frame is built up of angles and plates, with a flat bottom provided with a set of leveling screws. The vertical uprights have holes of $^3/_8$ in. at regular intervals of 2 in., for the adjustment of the torsion frame to different heights. The center of the key is made to coincide approximately with the axis of the optical set, and by the proper adjustment of the lens system, the entire field of the key and the adjacent parts of the shaft and hub illuminated by the parallel rays of the polarized light will be projected on the screen. Torque is produced in the shaft by means of the handwheel *A* working around a screw *B* attached to a spring scale *C*. Any upward pull on the scale is divided equally in the two linkage systems *D* and *F* located at equal distances from the center line of the scale and connected to a horizontal lever *G* at its extremities through $^1/_4$-in. round steel pins. An upward force in *F* corresponds to an equal downward force in *I*,

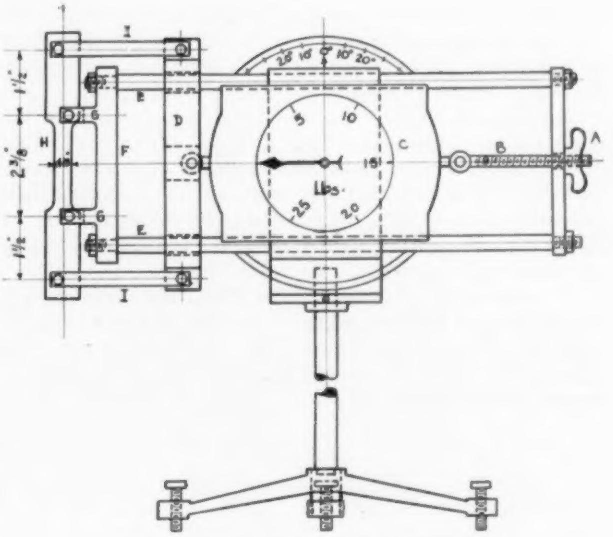

FIG. 4 DIAGRAM OF BENDING FRAME

which in combination with the equal and opposite force in *D*, produces a torque in the shaft around its central axis equal to one-half the scale reading multiplied by the distance between the bars *D* and *I*. The vertical links *D*, *F*, and *I* consist each of two strips of metal $^3/_{32}$ in. by $^1/_2$ in., one on each side of the model. The horizontal bars *G* and *H* are made of $^3/_{16}$-in. steel and are connected to the scale and to the linkage system as shown in the illustration. The model is held rigidly by means of a set of $^1/_8$-in. bolts in between two $^1/_8$-in. brass plates, which in their turn are bolted to the tension frame. The central portion of the brass plates is cut for a diameter of 6 in. in order to expose a corresponding surface of the transparent model to the rays of light for projection purposes.

(*c*) *The Calibration Frame.* The bending frame, a diagram of which is shown in Fig. 4, is used as a calibrating device to obtain a relation between color, order, and stress. It consists of a handwheel *A* working on a screw *B* to produce pull on the spring scale *C* and the bar *D* attached to the latter and sliding freely on the two rods *E*, which at their ends support a bar *F* with the two supports *G*, at a distance of $2^3/_8$ in. apart from center to center. The calibrating beam *H*, made of

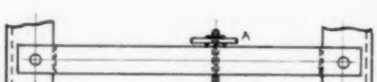

FIG. 2 DIAGRAM OF TORSION FRAME

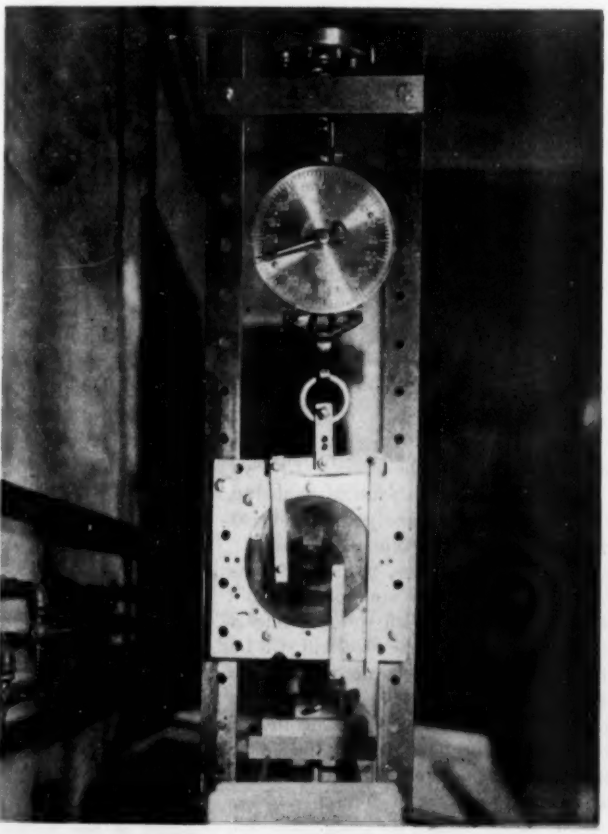

FIG. 3 TORSION FRAME

the same bakelite as that used for the models, is attached to the slots of the supports through $^1/_4$-in. round pins. At the two extremities of the bar *D*, $^1/_{16}$-in. by $^3/_{16}$-in. strips *I* are connected to the ends of the bakelite beam on both sides through similar pins, at distances of $1^1/_2$ in. from the supports outward. The load registered on the scale is equally divided in the strips *I* and acts at the ends of the beam, producing a uniform bending moment for all sections in between the two supports. The bending frame is supported by a $^3/_4$-in. round steel rod fixed on a tripod. It can be moved vertically up and down, as well as turned through any angle in a vertical plane.

(*d*) *Optics of the Apparatus.* The light from the lamp, after passing through the condensing lens, reaches the polarizer and is split into two rays, called ordinary and extraordinary rays. The first is reflected and absorbed by the sides of the prism, while the second passes through the prism and has its vibrations all in a plane perpendicular to the principal plane of the prism. The two large lenses are adjusted to obtain a parallel beam of light and sharp focusing. If the principal plane of the analyzer is set at 90 deg. with that of the polarizer—i.e., when the "Nicols are crossed"—the beam will not pass beyond the analyzer, producing complete extinction of light on the screen. Any rotation of one of the prisms will produce partial illumination. With the two Nicols crossed and the bakelite model not stressed, the field will still be dark. However, when the material is stressed, the rays of the polarized light, while passing through the model, will undergo a relative retardation during the process of the refraction in the material, and as the retardation is proportional to the shear intensity at a given point, color will appear on the screen with a succession of bands of yellow, red, and green of different orders, depending upon the intensity of the stresses in the plate. Should one principal stress at some point in the model be parallel to the plane of polarization of the parallel beam of light, the ray through that point will not be decomposed, and a dark spot will appear on the screen, marring the distinct appearance of the isochromatics. In order to avoid this, the plane-polarized light is changed to circular or elliptic polarized light by introducing two quarter-wave mica plates with their optical axes at 45 deg. with those of the Nicols.

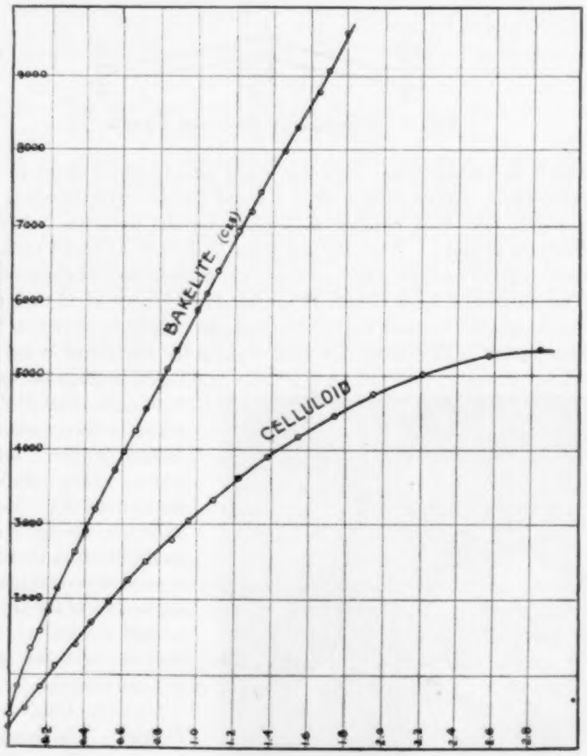

Fig. 5 Stress-Strain Curves

4 Materials

The materials commonly in use for models have been glass, xylonite (celluloid), phenolite, and bakelite. The last one was adopted by the authors on account of its comparatively high optical sensitivity in producing color fringes under moderate stresses. Glass has been used by the early experimenters, such as Brewster, Wertheim, Mesnager, etc. its chief advantages being perfect transparency and no residual stress-optical effect. On the other hand, glass is very difficult and expensive to machine, and must be very thick and highly stressed in order to produce multicolor effects. Xylonite is camphorated nitrocellulose, easily machined, but has considerable initial stresses, especially when not aged. It is not optically very sensitive and undergoes a large permanent set upon removal of stresses even far below its elastic limit, followed by a progressive creep. Phenolite is an organic compound made from phenol and formaline by the catalytic action of ammonia at ordinary pressure. In many respects it has the same qualities as bakelite.

Bakelite is a phenol resinoid, invented by Dr. L. H. Baekeland in 1907. The principal ingredients used in the manufacture of bakelite resinoid are phenol (commonly called carbolic acid) and formaldehyde. The transparent bakelite materials are produced directly from the initial resinoid by processing with heat at a temperature of about 300 deg. fahr. and a pressure of 50 to 100 lb. per sq. in., in a special type of autoclave. The material thus produced is a clear, amber-hued resin-like solid, having a specific gravity of 1.27. It will not melt at any temperature, but begins to char at about 570 deg. fahr It has a stress optical coefficient of about 500 brewsters, as determined by I. Arakawa, at least five times larger than that of celluloid, and therefore for a given thickness a greater number of isochromatics are brought into the field. The chief difficulty in using bakelite has been due to the presence of considerable initial stresses within the material as supplied or after machining it to a complicated shape with holes, corners, etc. After several trials the authors have found it possible to remove these unnecessary stresses by keeping the finished plate in an electrically controlled hot-air furnace at a temperature of 75 to 85 deg. cent. for at least one hour and then gradually cooling down to room temperature within 12 to 14 hours.[11] Under no circumstances should one try a temperature of 110 deg. cent. or above, as the transparency of the material will be affected. Both mechanical and optical tests run in the laboratories of Columbia University have indicated that the properties of bakelite are slightly affected as a result of annealing. Hence, the color-stress relations will no more be the same for the material in its initial and heat-treated conditions. However, by annealing the calibrating beam under the same temperature and time of cooling conditions as that used for the model, any discrepancy of this nature will be eliminated.

Bakelite models have a tendency to warp during the annealing process. It is necessary, therefore, to hold the plates in between two glass plates loaded on top.

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[11] This method of annealing bakelite has been developed by the authors and used since December, 1930. Dr. Max M. Frocht's process of removing the internal stresses is in many respects similar, although developed independently. For details of Dr. Frocht's method, see his paper, "Recent Advances in Photoelasticity," presented at the National Applied Mechanics Meeting (A.S.M.E.), Purdue University, Lafayette, Ind., June, 1931.

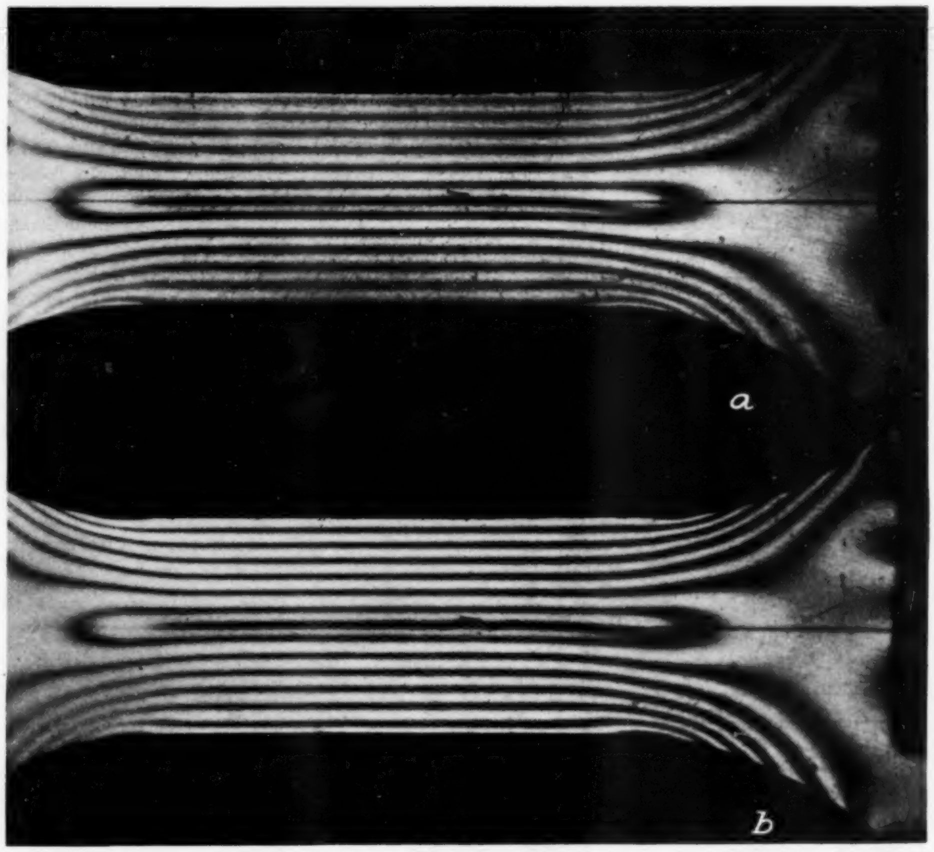

Fig. 7 Calibrating Beam in Flexure

(*a*, isochromatics in black and white obtained with white light; *b*, monochromatics in black and white obtained with mercury light.)

A stress-strain curve of C25 bakelite is given in Fig. 5. A similar curve for celluloid is added for comparison.

Although bakelite has a fairly high apparent elastic limit, yet time-creep tests of specimens of the material, both annealed and non-annealed, show that stresses within about 35 per cent of the maximum strength of the material do not produce permanent set. This corresponds to a working stress of 3200 lb. per sq. in. Bakelite has a considerable hysteresis for higher stresses, and a certain amount of permanent set will occur if the maximum intensity of the stresses is not kept within safe limits.

5 Keys

A key is used commonly to transmit a torque between a hub and a shaft. It will be assumed in this discussion that there is no press fit and that a good sliding fit exists between the hub and the shaft; i.e., that the total torque is transmitted through the key. The key works in shear. Several types of keys are used in machine building, such as the gib-head key, the woodruff key, the snub key, etc., but they differ from each other only in details. Therefore the analysis here is confined to a standard key and with the shearing stresses in and around it.

It will be recognized that with the existing practice of machining keyways, sharp edges bear against the surfaces of the key. This introduces great complexity in the analytical solution of the problem. High stress concentrations exist at certain critical points, far above the elastic limit of the material. Although the design of the key or the keyway is based upon the results of years of experience and experimental study, the frequent failures have shown the necessity of further research by an entirely different method than that used in the past.

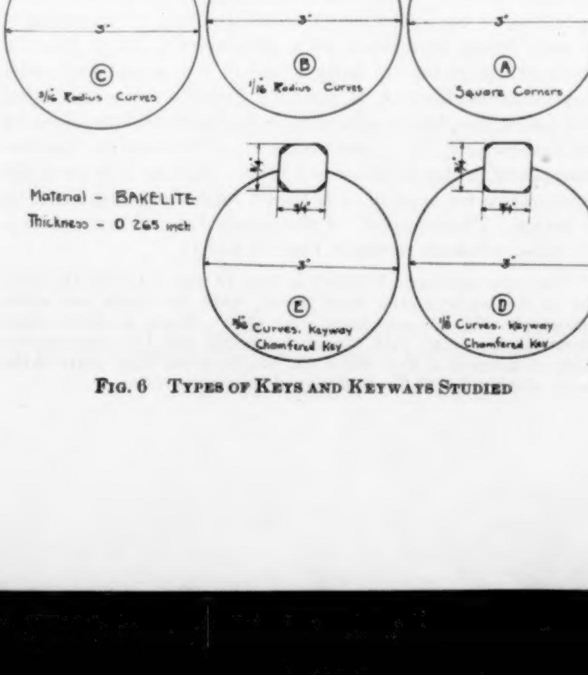

Fig. 6 Types of Keys and Keyways Studied

A $^3/_4$-in. square standard key corresponding to a shaft of 3 in. in diameter was adopted for investigation. Four modifications of the standard design were made by introducing $^1/_{16}$-in. and $^3/_{16}$-in. curves at the corners of the keyways and making the keys to fit these curves or chamfering the corners of the keys, as shown in Fig. 6.

6 Test Procedure

(*a*) *Method of Preparing Models.* The models were machined from bakelite of 0.265 in. thickness and were polished on large rotating disks provided with a layer of felt on their upper surfaces. Levigated alumina dust mixed with water was found to give a very high polish. A careful fitting of the three parts was obtained by a continuous examination of the models in the photoelastic apparatus. Any misfit, and therefore the appearance of local stresses, was easily detected and eliminated. After a satisfactory fit was obtained, the models were annealed as described. Subsequent to annealing, it was found necessary in many cases to fit the three parts again on account of the deformation of the plate as a whole during the process of freeing itself from the internal stresses, while cooling down to room temperature.

(*b*) *The Color-Stress Scale.* The material of the models was calibrated. A color-stress scale was prepared as follows: A beam made from bakelite, using the same material and of the same thickness as the models, was machined to proper shape and dimensions, as shown in Fig. 4. The beam was annealed as described. When stressed in flexure under a given load at the extremities of its cantilever arms, pure bending of a known intensity took place in the middle of the beam, corresponding to a certain extreme fiber stress. The middle portion of the beam was put in the path of the rays of polarized light and projected on the screen. Colored bands symmetrically placed on both sides of the neutral axis and in a direction parallel to the axis of the beam appeared on the screen. Fig. 7*a* is a photograph of the isochromatics in black and white. Fig. 7*b* is that of the monochromatics having replaced the white light used for the isochromatics by the green light of a mercury-vapor lamp.

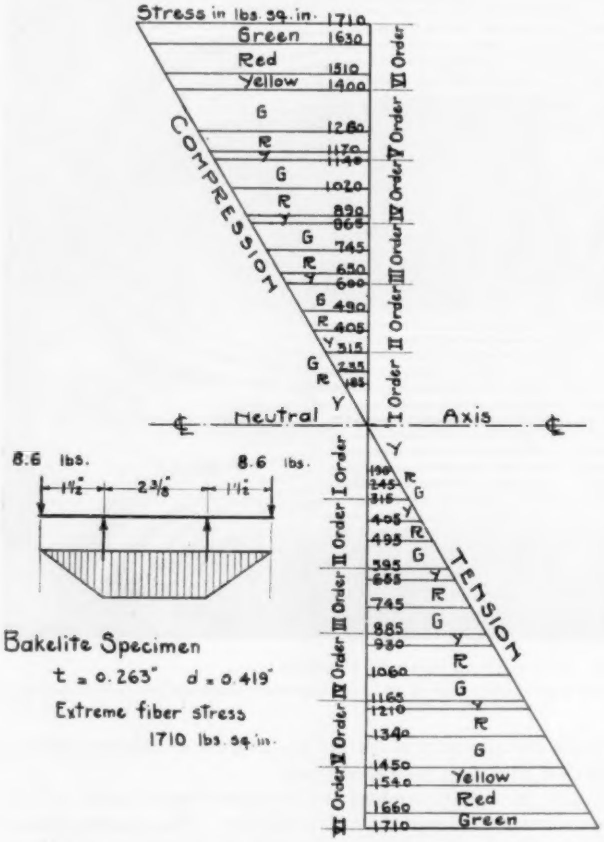

FIG. 8 COLOR-STRESS SCALE FOR C25 BAKELITE

Although the monochromatics give a more distinct picture, the use of isochromatics for the evaluation of stresses is preferred by the authors and is recommended for the following reasons:[12]

(*a*) Black and white photographs of the isochromatics are now obtained as easily as of the monochromatics.

(*b*) Isochromatics when projected or printed are more impressive and enable the observer to follow the changes of intensities of stresses in complicated structures.

(*c*) Black regions corresponding to $p - q = 0$ are easily detected in the isochromatics.

(*d*) With three-color bands in the isochromatics against two-color in the monochromatics, the optical sensitivity of bakelite is increased in the ratio of 3 to 2.

(*e*) The monochromatic fringe method in certain cases requires repeated loading and unloading of the material for the determination of the number and order of a fringe. This process produces permanent set in the material, and hence affects the results. With the isochromatics the first-order colors are easily recognized, and the higher ones are determined by counting from the first-order fringes toward the higher orders. A little practice will enable one to tell the correct order of fringes of orders below the fifth by a mere observation of the hues of the colors or by comparison with a standard color scale.

In order to obtain a color-stress relation for the evaluation of the stresses, the lines of divisions between the several isochromatics as well as the neutral axis were carefully marked. With fresh-cut specimens properly annealed, the neutral axis was in all cases coincident with the center line of the beam. Knowing the intensity of fiber stresses for a given uniform bending moment, and also the cross-section of the beam, a color-stress diagram was prepared, as shown in Fig. 8, where the stress equivalents of the different fringe lines are found from their relative distances from the neutral axis. The stresses given in Fig. 8 are the values of the principal tensile or compressive stresses p, the other principal stress being zero. The maximum shearing stresses are therefore $s = \frac{1}{2}(p - q) = \frac{1}{2}p$. The isochromatics represent also shearing stresses, and the values are given in Table 1.

TABLE 1 ISOCHROMATICS—COLOR-STRESS VALUES FOR BAKELITE C25

Color	Order	Symbol	—($p - q$) Stress, lb. per sq. in.— Tension	Compression	Average	Shear
Yellow	...First	y_1	190	185	187½	94
Red		r_1	245	235	240	120
Green		g_1	315	315	315	158
Yellow	...Second	y_2	405	405	405	203
Red		r_2	495	490	492½	247
Green		g_2	595	600	597½	299
Yellow	...Third	y_3	655	650	652½	326
Red		r_3	745	745	745	372
Green		g_3	885	865	875	438
Yellow	...Fourth	y_4	930	890	910	455
Red		r_4	1060	1020	1040	520
Green		g_4	1165	1140	1152½	576
Yellow	...Fifth	y_5	1210	1170	1190	595
Red		r_5	1340	1280	1310	655
Green		g_5	1450	1400	1425	712
Yellow	...Sixth	y_6	1540	1510	1525	762
Red		r_6	1660	1630	1645	822
Green		g_6				

The stress readings in Fig. 8 show that corresponding values of stress for a given color order are not the same for both sides of the neutral axis. This is due to irregularities in the material and to differences in thickness of the different fibers of the beam on both sides of the axis, inasmuch as tension and compression stresses will produce opposite changes of thickness. The average of the two values was taken in preparing the color-stress list, Table 1.

(*c*) *Magnitudes of Maximum Shearing Stresses.* The bakelite models shown in Fig. 6 were mounted in the loading frame. After the necessary adjustments, a constant torque of 166 in-lb. was applied to the round plate representing the shaft through the leverage mechanism, by introducing a total pull of 70 lb. in the spring scale. This was sufficient in all cases to produce a maximum stress corresponding to the green color of the sixth order. The color bands or isochromatics projected on a piece of white paper fixed to the screen were traced carefully. Photographs of the color bands were taken for a three-color print on Wratten Panchromatic plates, by using filters of red, green, and violet colors (Eastman filters A, B, and C). Later on it was found that black and white photographs of the color bands could be obtained very satisfactorily by a direct exposure of the rays on Eastman Ortho plates, without the use of a filter. Figs. 9*a* to 9*e* show the isochromatics for keys *A* to *E* traced from their projections on the screen. Photographs[13] of the square type of key and keyway under stress are shown in Figs. 10 and 11.

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[12] For a detailed explanation of the reasons, see the discussions by the authors of Dr. Max Frocht's paper on "Recent Advances in Photoelasticity," Applied Mechanics Meeting (A.S.M.E.), Purdue University, Lafayette, Ind., June, 1931.

[13] The color photograph shown in Fig. 10 was taken at the same time as the isochromatics were traced, while the black and white photograph (Fig. 11) was taken long after. Hence it shows slight differences compared with the isochromatics and the color photograph, on account of time effect and reuniting the three parts of the model, which may not be absolutely of identical nature.

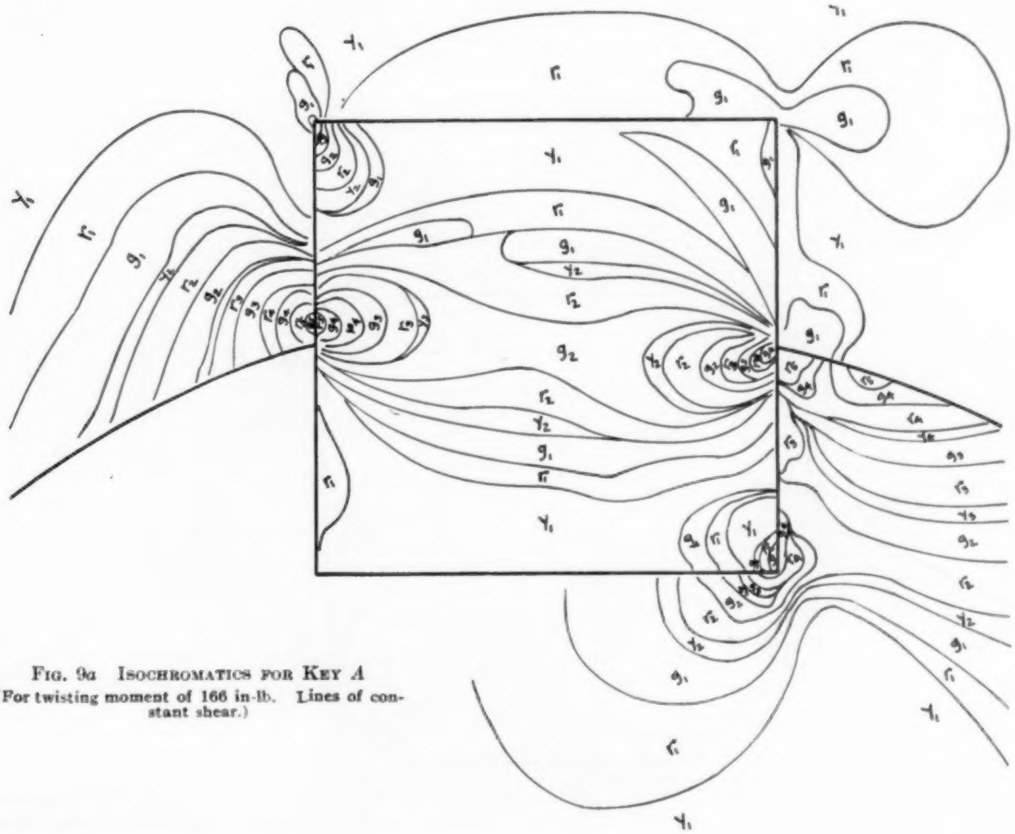
y₁
r₁
g₁
y₂
r₂
g₂
r₃
g₃
g₄
r₄
y₃
y₄

Fig. 9a Isochromatics for Key *A*
(For twisting moment of 166 in-lb. Lines of constant shear.)

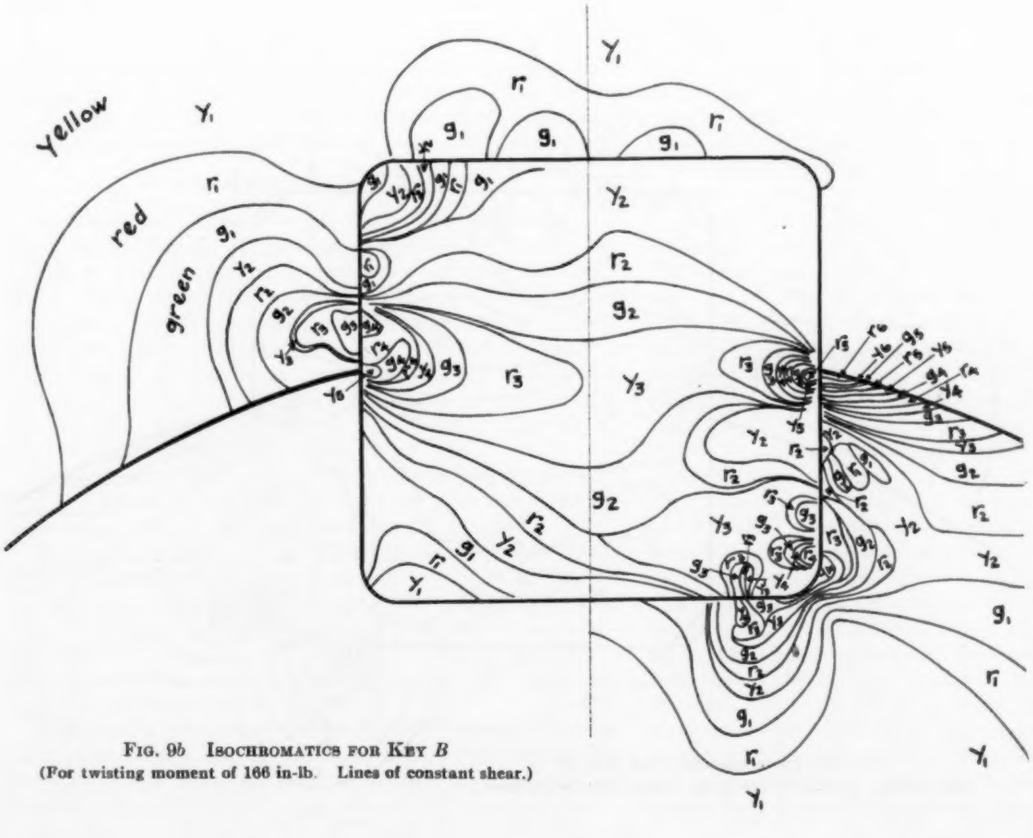
yellow
red
green
y₁
r₁
g₁
y₂
r₂
g₂
y₃
r₃
g₃
y₄
r₄
g₄
y₅
g₅
r₅
y₆
r₆

Fig. 9b Isochromatics for Key *B*
(For twisting moment of 166 in-lb. Lines of constant shear.)

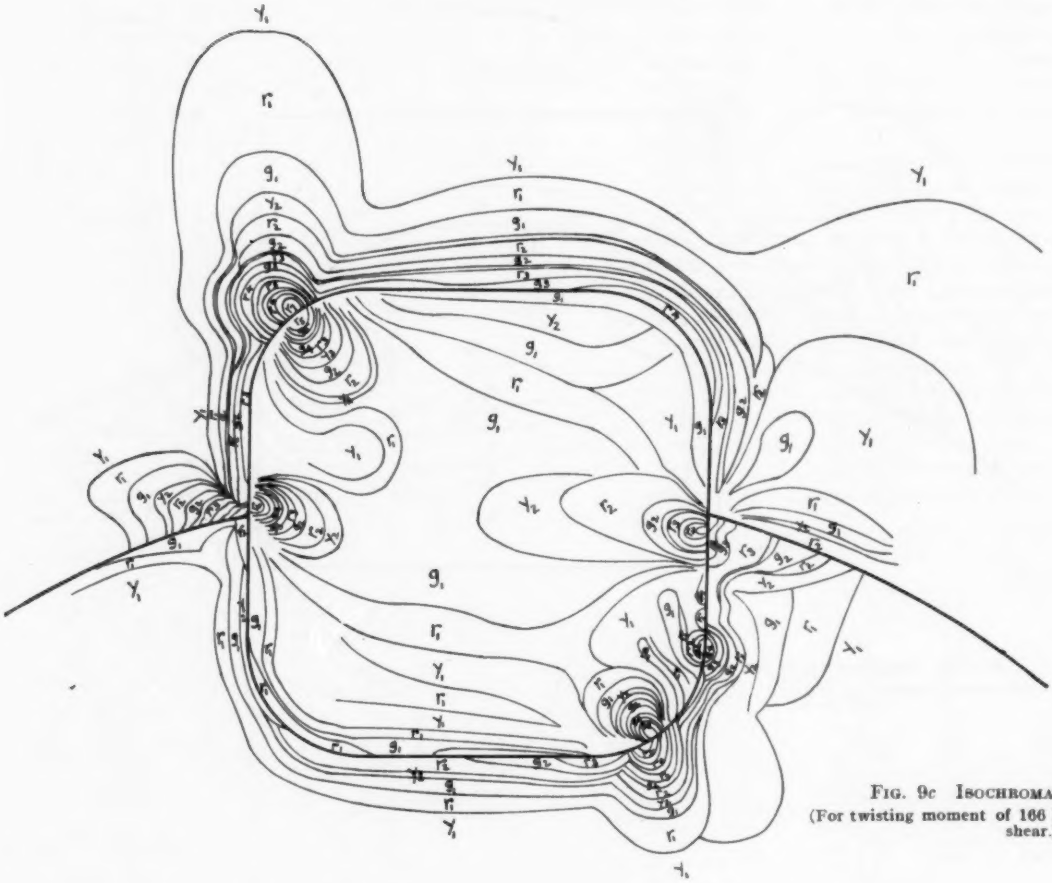
y1
r1
g1
y2
r2
g2
r3
g3
y1
r1
g1
y2
g1
r1
g1
y1
y1
r1
y1
g1
y1
y1
r1
y2
r2
g1
g1
r1
y1
r1
r1
g1
y1
r1
y1
g1
r1
g1
g2
r2
y2
g2
r1
y1
r1
y1

Fig. 9c Isochromatics for Key *C*
(For twisting moment of 166 in-lb. Lines of constant shear.)

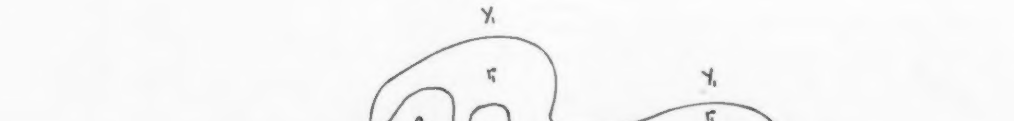
y1
r1
y1
r1
g1
g1
y1
r1
y1
g1
y1
y1
r1
g1
y2
r2
g2
r2
y2
g1
r1
y1
r2
g2
g1
r1
y1
g2
r2
y2
g1
r1
y1
g1
r1
y1

Fig. 9*d* Isochromatics for Key *D*
(For twisting moment of 166 in-lb. Lines of constant shear.)

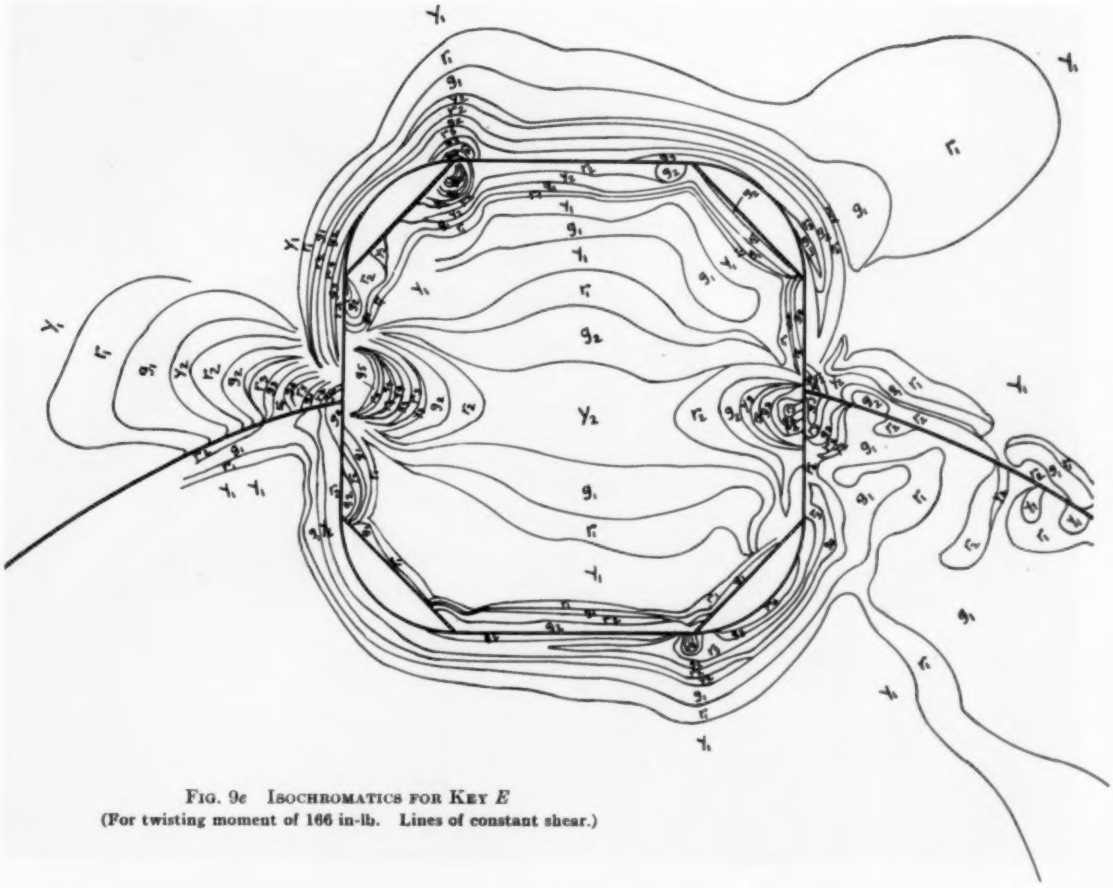

Fig. 9e Isochromatics for Key *E*
(For twisting moment of 166 in-lb. Lines of constant shear.)

It will be observed that in nearly all cases the yellow color beyond the fourth order is not represented in the isochromatics on account of the fact that at high stresses it is very difficult to discriminate it from red and green colors.

7 Observations and Discussions

(*a*) *Shear Distribution in the Keys.* Curves showing the intensities of shearing stresses around the boundary lines and at various sections of the different types of keys are given in Figs. 12 to 16. These are all for a twisting moment of 166 in-lb. The ordinates to the curves representing the magnitudes of the unit shears are taken from the color-shear compilation, Table 1, corresponding to the proper isochromatic line of that point. Similar curves are drawn for the shear stresses in the shaft and hub around the keyway, as shown in Figs. 17 to 21. These are all for a twisting moment of 166 in-lb.

A comparative study of the shear intensities for the different types of keys and keyways allows us to make the following observations, true to all the different types:

(1) The maximum shear intensity on the surfaces of the keys is several times higher (in many cases from two to four) than the average at points where the edges of the shaft and the hub come into contact with the sides of the key. This is due to the pressure concentrations from the sharp corners of the keyway either in the shaft or in the hub, depending as to which one first comes into contact with the sides of the key.

(2) At one or two special corners of the key, depending upon the type of the key and the direction of the torque applied, the intensity of the maximum shear is as high as five times the average.

(3) The total shear carried by the middle horizontal section of the keys (proportional to the areas under the shear curves corresponding to these sections) is larger than that carried by any other section parallel to the middle ones.

(4) The material on the no-load side of the keys, both for the hub and the shaft, is practically free from stress (except in type *C*), which might be expected. Probably in the case of a long key this is true only for that part of the keyway adjacent to the free end of the shaft, while at the other end of the hub the material takes an active part in the transmission of the torque.

(5) Heavy stress concentration exists at the reentrant corners of the keyway. Contrary to expectations, introduction of a large-radius curve at the corners did not show a particular beneficial effect in this respect. Apparently the local stresses depend considerably on the proper fit of the key and on the location of the point where the pressure of the key against the keyway is concentrated.

(6) The effect of chamfering the keys has been found in some cases harmful on account of the introduction of high localized stresses due to sharp chamfered corners of the key. In the case of the key *C*, for example, the maximum shear in the key boundary at a point around the lower right corner is 455 lb. per sq. in. The introduction of a chamfer in the key increased this to 760 lb. per sq. in., without any appreciable reduction in intensity of shear on both sides of the corner of the key in either vertical or horizontal direction. However, the same is not true for the key *D*, where the largest value of the shear was increased only from 655 to 765 lb. per sq. in. at the upper left corner, while that at the lower left corner was decreased to a considerable amount, from 710 to 245 lb. per sq. in. On the other hand, the intensity of the shear on the

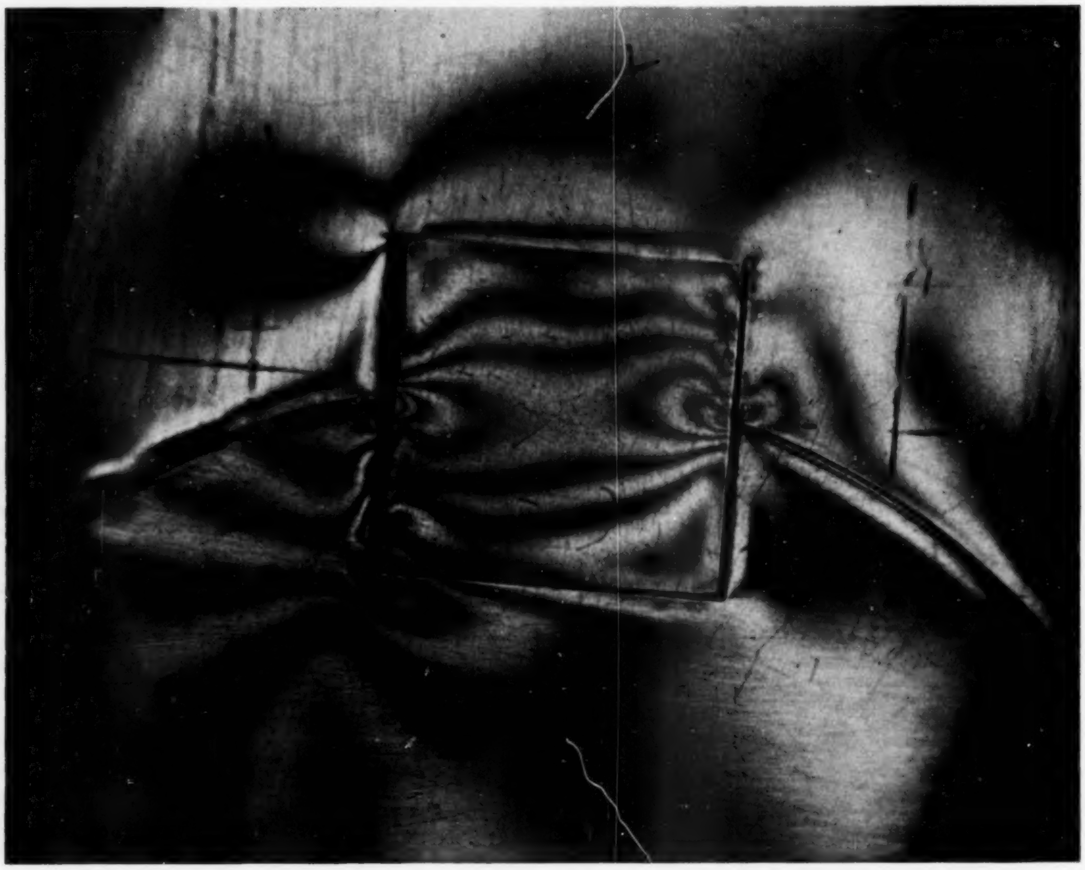

Fig. 11 Isochromatics for Square Key and Keyway—Black-and-White Photograph

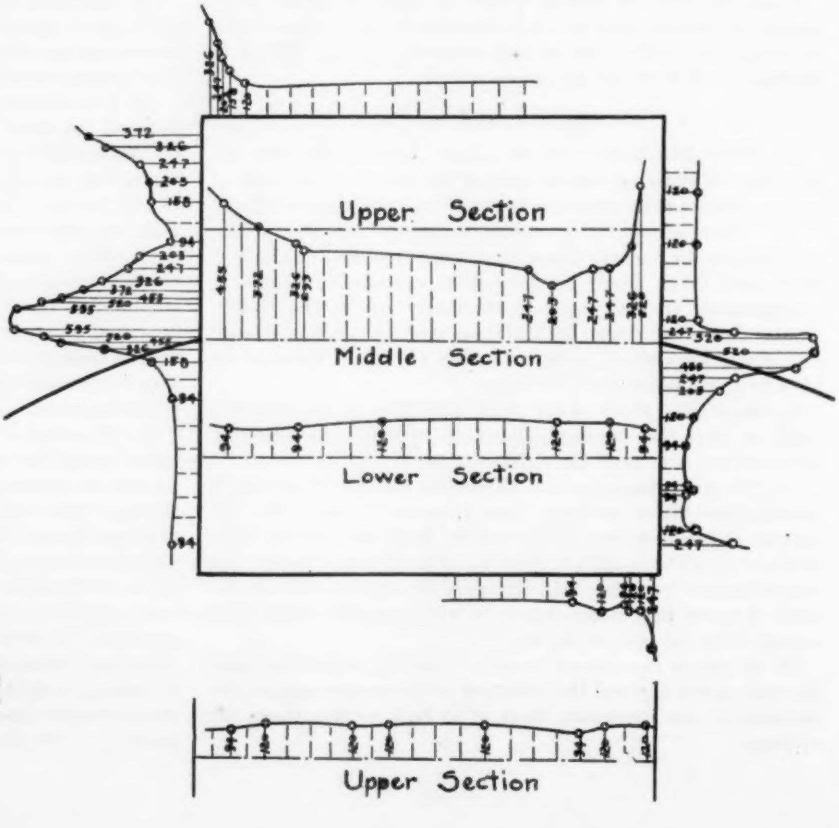

Fig. 12 Shear Distribution in Key *A*

Fig. 10 (Inserted Color-Plate Page at Right) Isochromatics for Square Key and Keyway—Three-Color Photograph

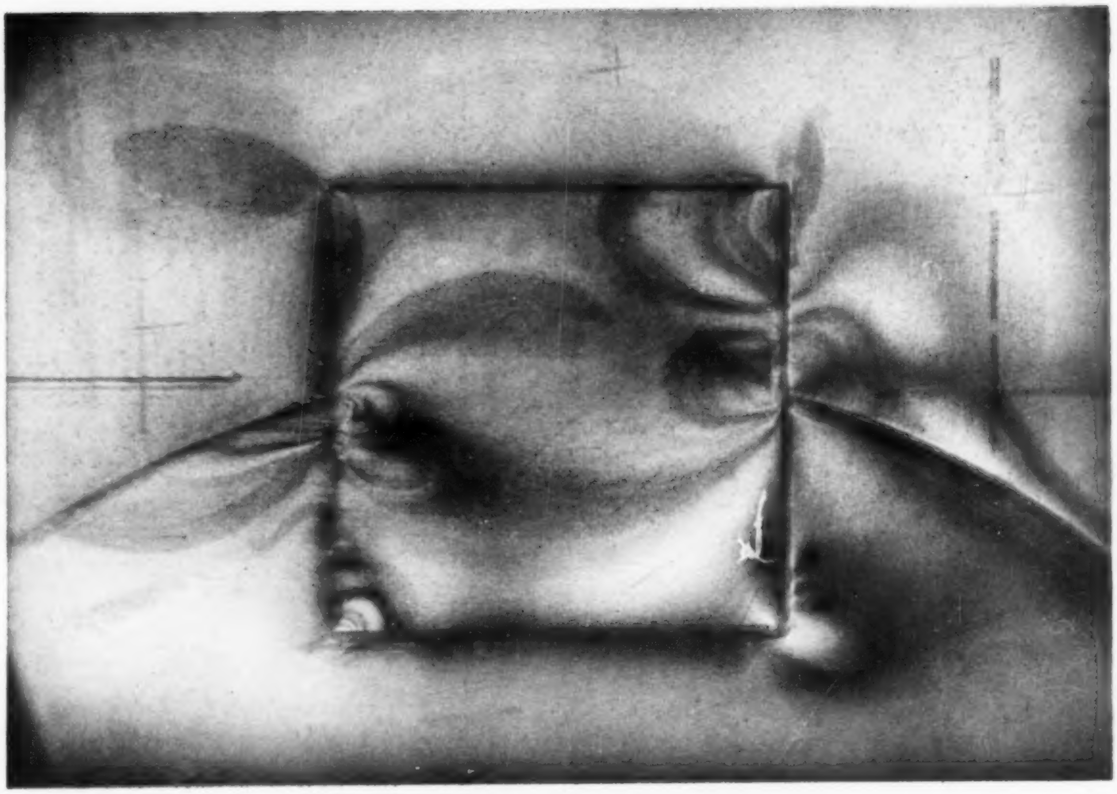

Fig. 10 Isochromatics for Square Key and Keyway

(3-color photograph)

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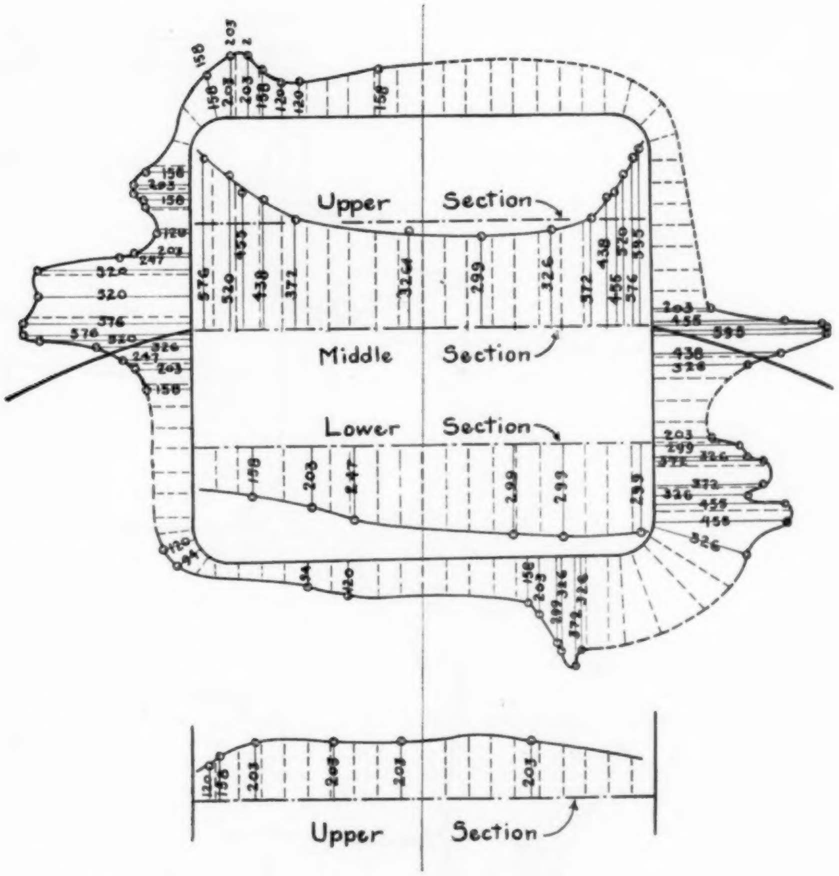

Upper Section
Middle Section
Lower Section
Upper Section
158
203
120
576
520
438
372
326
299
247
595
456

FIG. 13 SHEAR DISTRIBUTION IN KEY *B*

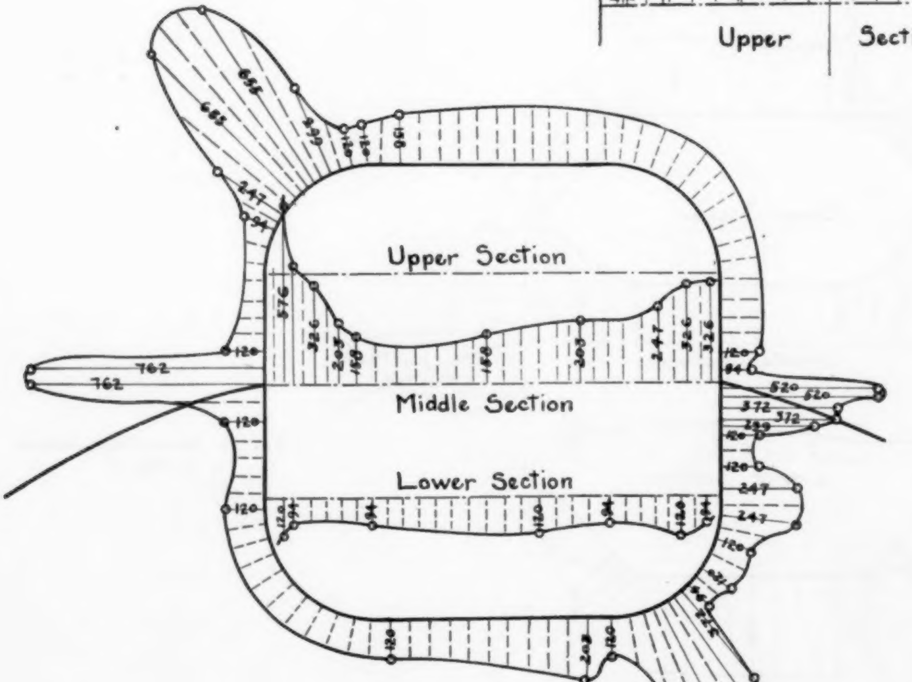

Upper Section
Middle Section
Lower Section
Upper Section
762
685
656
576
326
247
203
190
120
94
520
372
712

FIG. 14 SHEAR DISTRIBUTION IN KEY *C*

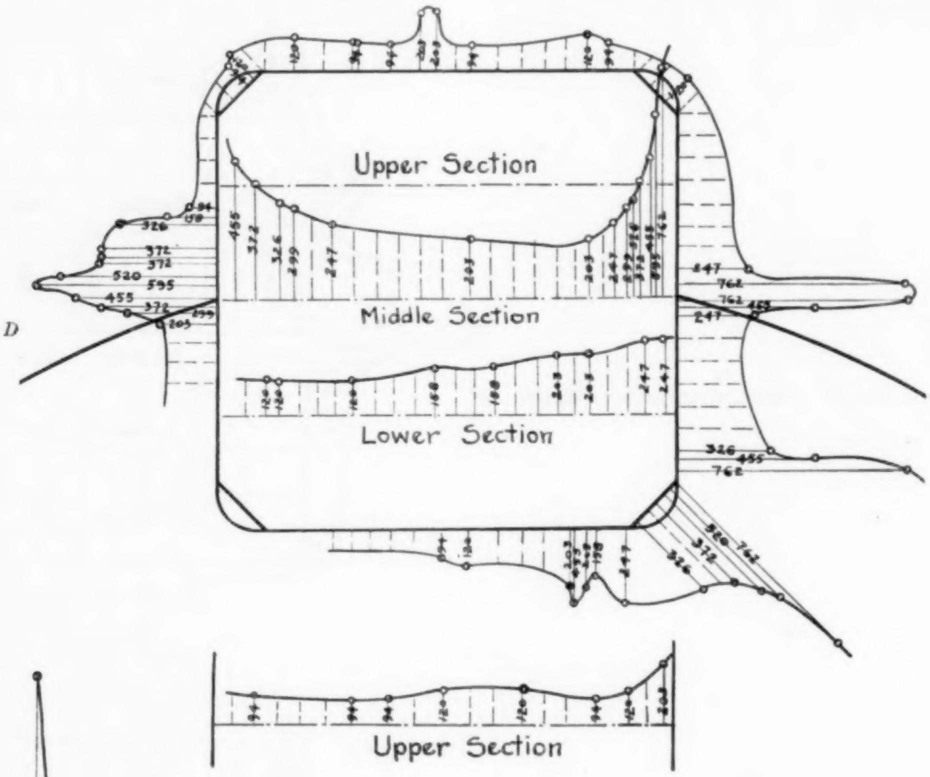

Upper Section
Middle Section
Lower Section
Upper Section

Fig. 15 Shear Distribution in Key *D*

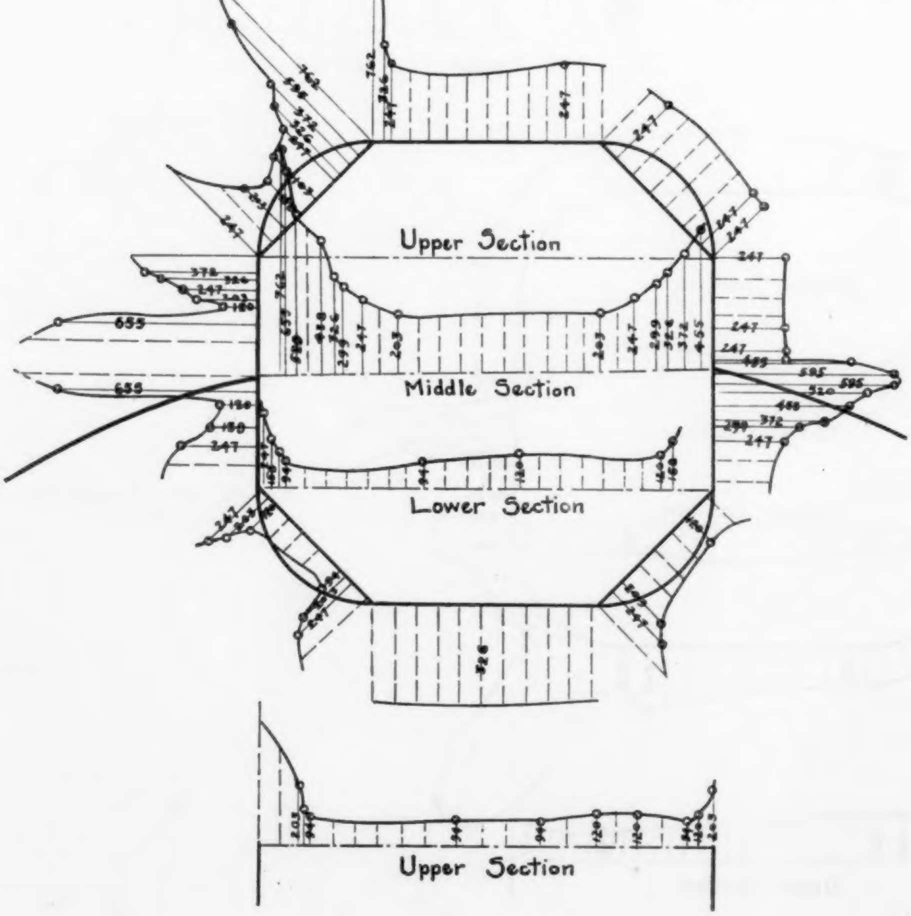

Upper Section
Middle Section
Lower Section
Upper Section

Fig. 16 Shear Distribution in Key *E*

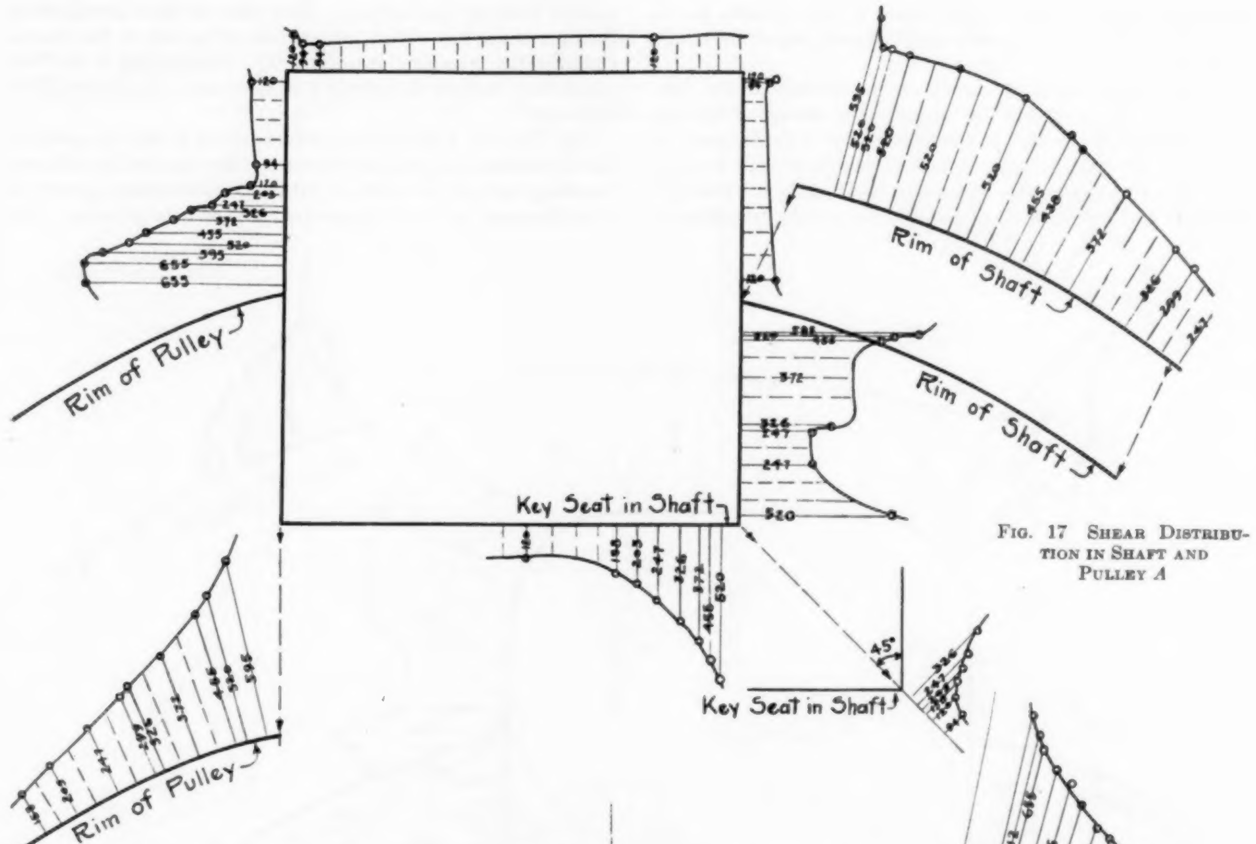

Rim of Pulley
Rim of Shaft
Rim of Shaft
Key Seat in Shaft
Key Seat in Shaft
45°
Rim of Pulley

FIG. 17 SHEAR DISTRIBUTION IN SHAFT AND PULLEY *A*

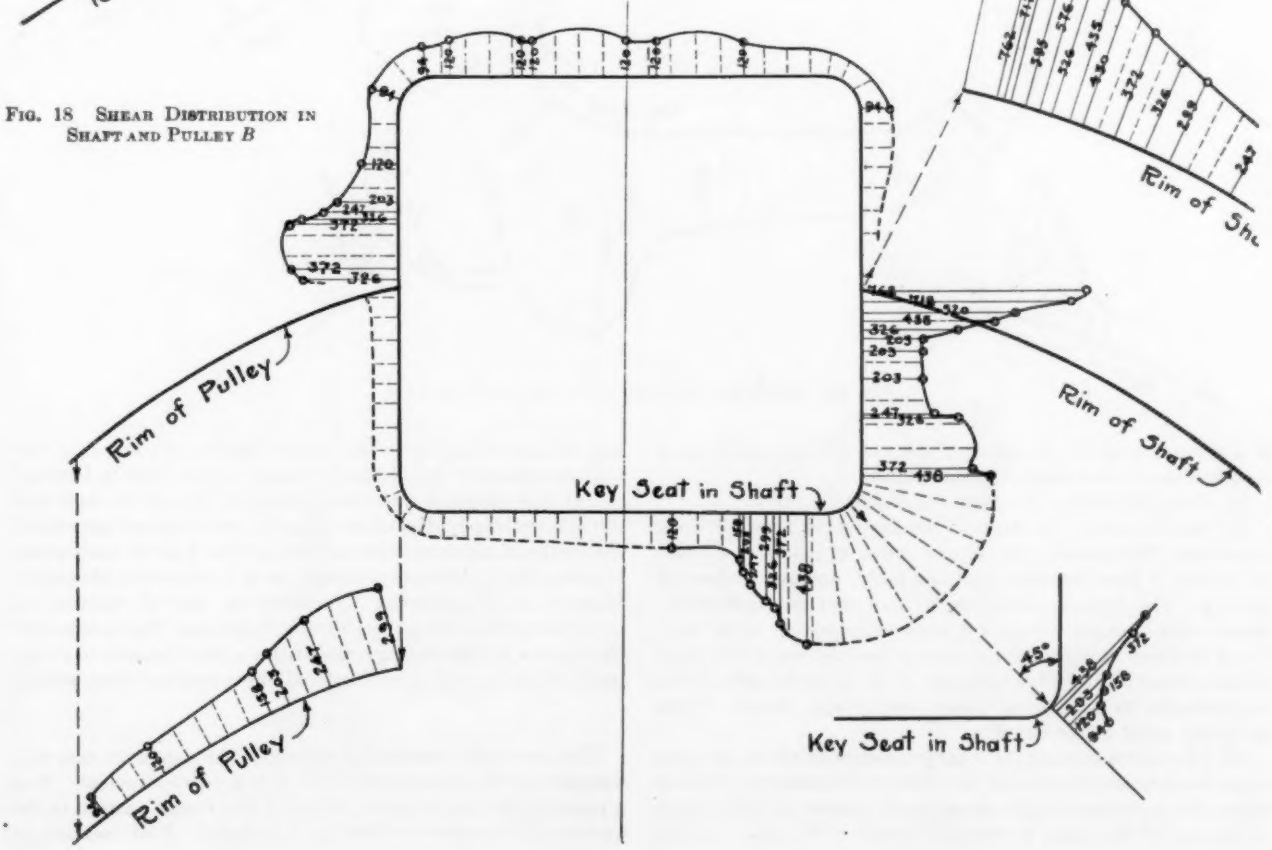

Rim of Shaft
Rim of Pulley
Rim of Shaft
Key Seat in Shaft
Key Seat in Shaft
45°
Rim of Pulley

FIG. 18 SHEAR DISTRIBUTION IN SHAFT AND PULLEY *B*

horizontal boundary faces was increased to some degree, due to the reduction of the bearing areas by the chamfering of the corners of the keys.

(7) The introduction of curves at the corners of the keys throws a certain amount of the torque to the corners of the keyway, especially in the case of the large-radius curved corners of the key *C*. This is largely due to the facility with which a key with larger curves at the corners will adapt itself easily to small displacement and adjustments of position during the transmission of a given torque to its adjacent sides in contact, resulting in compression stresses along diagonal planes.

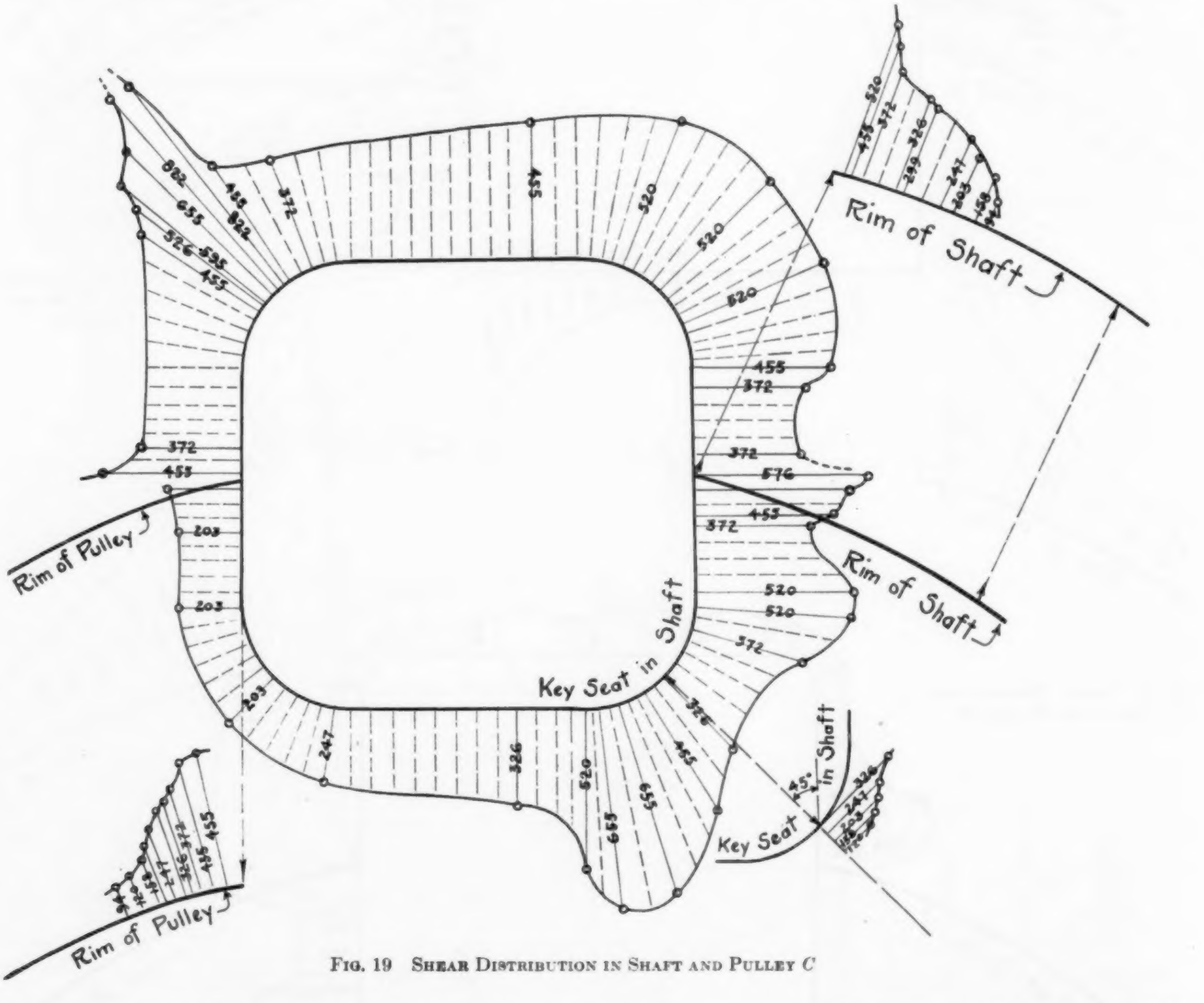


FIG. 19 SHEAR DISTRIBUTION IN SHAFT AND PULLEY *C*

(*b*) *Shear Distribution in Shaft and Pulley.*

(8) Results similar to those of the keys have been obtained concerning the intensity of shearing stress at points of contact of the line of joint between shaft and pulley against the faces of the key. The resulting maximum stresses are from three to five times larger than the averages in most cases, except in the keys *C* and *E*, which have resulted in more or less uniform distribution of shear stress all along the boundary of the keyhole, with a ratio of maximum to average of about two to one, except at one particular point in the key *E*.

(9) The curves showing the shear intensities on 45-deg. sections at the most stressed corners of the keyways illustrate to a striking degree the presence of high concentrated stresses at such points on account of the sharp or rounded corners of the keys, bearing against those of the keyways. This effect is more prominent in the case of the key with $^3/_{16}$-in.-radius curve, due to the reasons stated in the preceding paragraph (7). Chamfering of the keys has greatly reduced the intensity of stresses at the corners of the keyways.

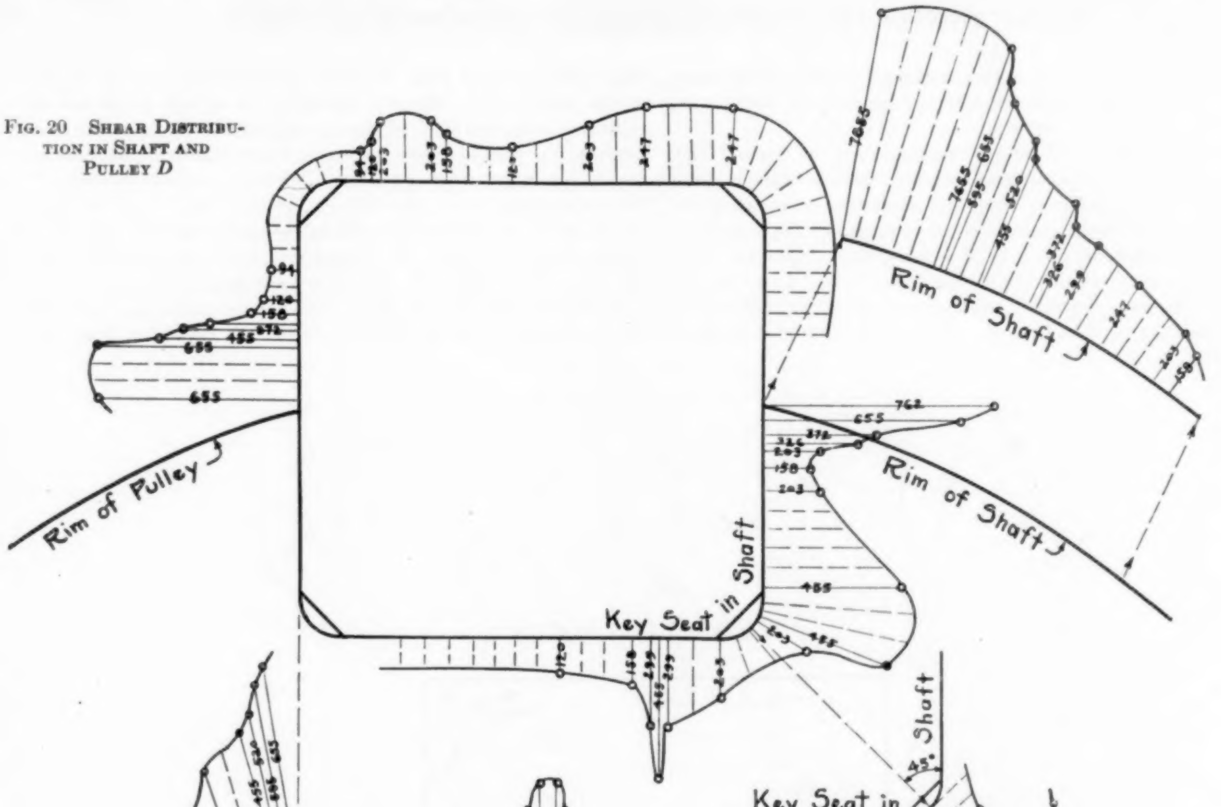
(10) The key *A* with square corners seems to take the effect of the entire torque by the reaction of its sides against the adjacent boundary faces in the keyway, without transmitting a portion of it to the upper or lower horizontal faces on a large scale. This may be accounted for by the greater rigidity of position of this key on account of its relatively longer vertical faces in bearing.

(11) The values of the stresses along the rims of the shaft and the hub are largely dependent upon the existence and amount of the frictional forces at these surfaces, so that a direct comparison of their relative intensities cannot be of a conclusive character. However, it is interesting to observe the way of variation of shear intensities, always considerably larger near the boundary of the keyway and decreasing gradually for a short distance to a very small value, depending on frictional forces acting at these points.

CONCLUSION

The photoelastic method of measuring stresses in the case of a complicated two-dimensional stress state is very convenient. It is a powerful tool for designers, for quick and simple analysis of the stresses in the members they have designed. With bakelite as

FIG. 20 SHEAR DISTRIBUTION IN SHAFT AND PULLEY *D*


Rim of Pulley
Rim of Shaft
Rim of Shaft
Key Seat in Shaft
7685
7685
655
595
520
455
372
326
299
247
203
158
762
655
372
326
203
158
203
455
203
455
120
158
299
299
405
203
94
120
158
372
455
655
655
94
120
203
158
127
203
247
247

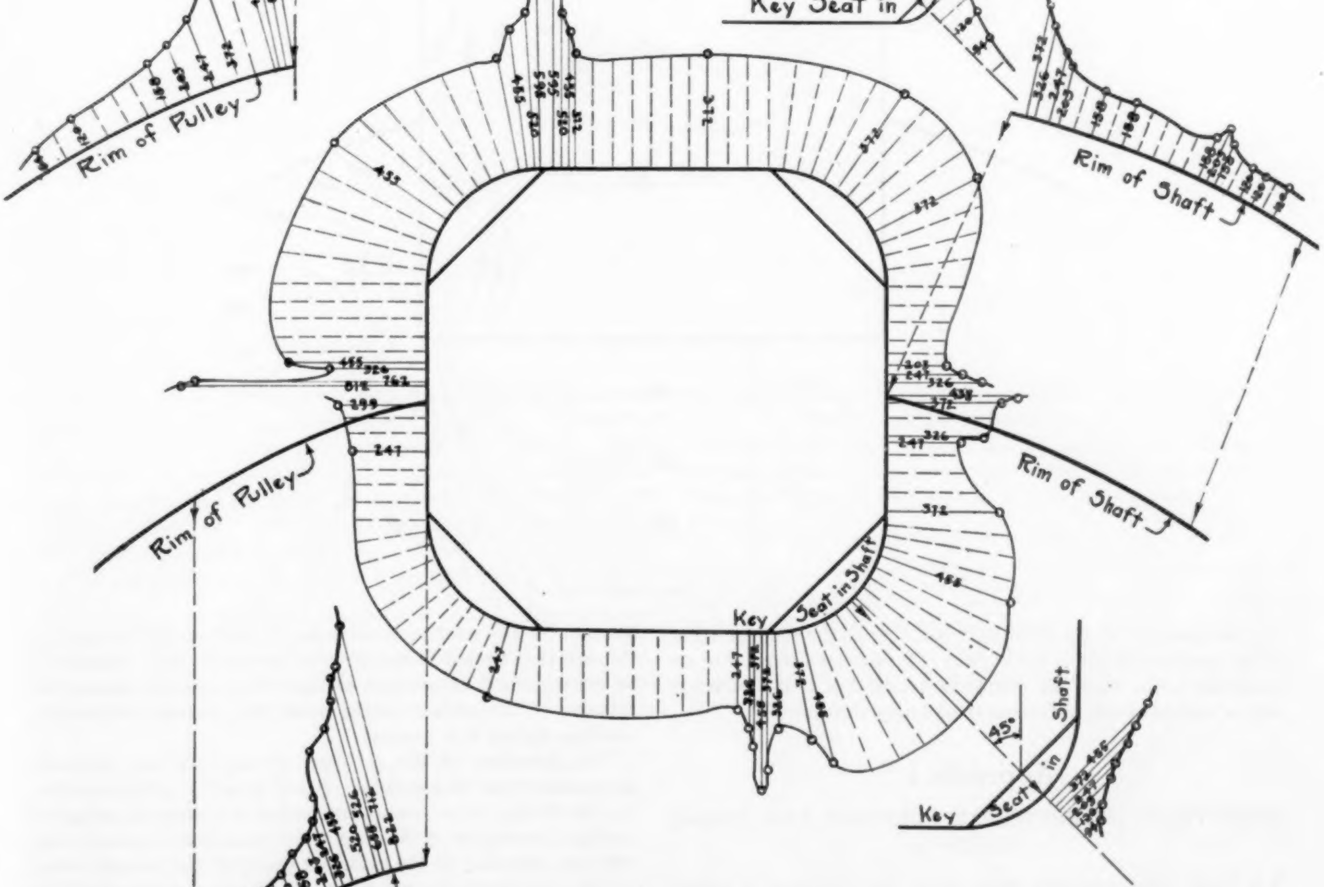

Rim of Pulley
Rim of Shaft
Rim of Pulley
Rim of Shaft
Key Seat in Shaft
45°
Key Seat in Shaft
45°
Key Seat in Shaft
Rim of Pulley
455
372
372
372
455
762
326
299
247
203
247
326
438
372
326
247
372
455
247
326
372
455
520
526
247
120
158
203
247
326
455
520
526
686
716
822

FIG. 21 SHEAR DISTRIBUTION IN SHAFT AND PULLEY *E*

material for the models, the isochromatics give sufficient information as to the stress concentrations and general distribution of stresses in the loaded structure.

Further refinement of measurements, such as finding the directions of the principal stresses and shear lines, is of little practical value for designers. These measurements were therefore not touched upon in the paper, but are discussed in Appendix 1.

As to the construction of keys and keyways, there are very heavy stress concentrations of the order 4 to 1 and 5 to 1 at the corners of the keyways and at points where the edges of the shaft and the hub bear against the key. As a result, cutting in of a key may be expected, and is often observed. As to the concentration at the reentrant angle of the keyway, the increased stress is of no particular harm when the torque is steady, but it is dangerous with a variable load, as fatigue cracks may then occur.

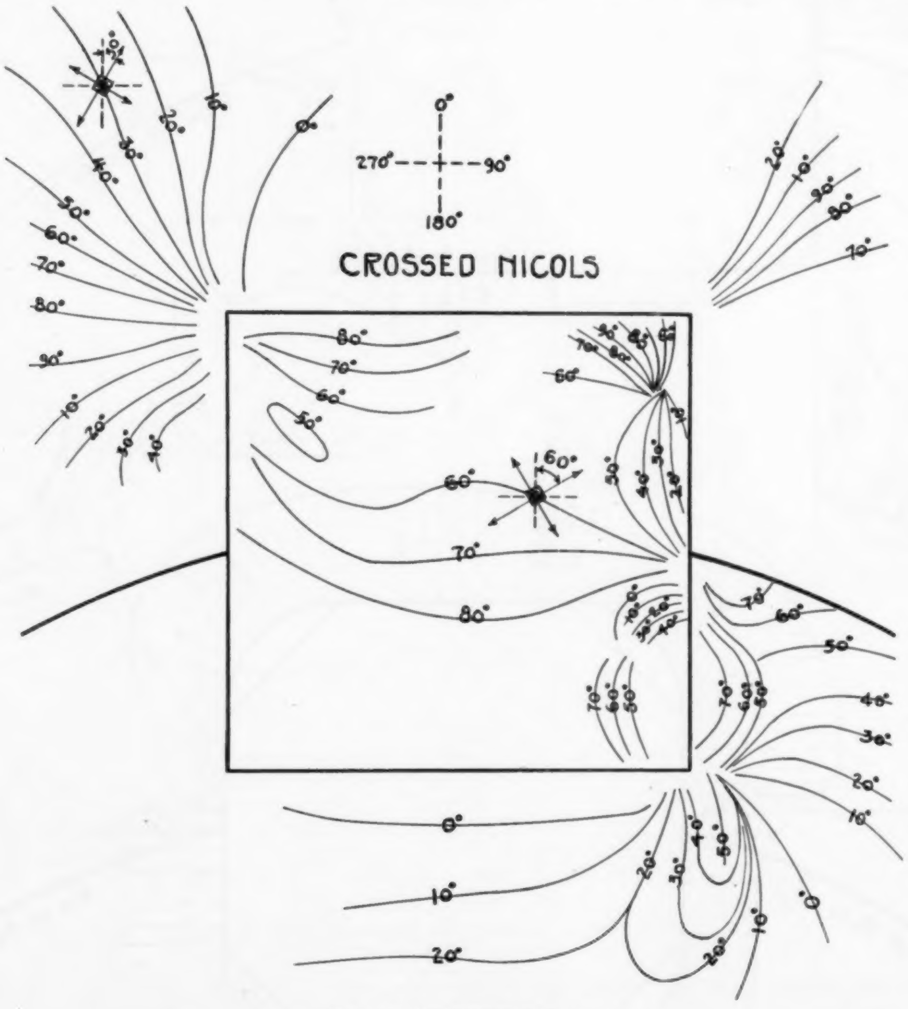


Fig. 22 Isoclinics for *A*

Appendix 1

DIRECTIONS OF PRINCIPAL STRESSES AND SHEAR LINES

With plane polarized light, when the directions of principal stresses at a point happen to be parallel and perpendicular to the direction of the plane of polarization of the analyzer, there will be extinction of light corresponding to this point as the vibrations of the ray emerging out of this point are at right angles to the principal plane of the prism. So there will be a series of black spots on the screen corresponding to similar points on the specimen, forming a set of black bands called "isoclinics." With the crossed nicols set at 0 deg., 10 deg., 20........90 deg., isoclinics of the corresponding numbers are obtained, and their center lines are traced on a paper attached to the screen. Fig. 22 shows the isoclinics for the square key and keyway.

However, there will be other black bands besides the isoclinics corresponding to points where $p - q = 0$, as is evident from the formula for the relative retardation of light during its passage through the stressed model, $\delta = c/\gamma\ (p - q)t$. These bands can be distinguished on account of their fixed position during the rotation of the crossed nicols, while the isoclinics shift their position during this process.

The directions of the principal stresses are now obtained graphically from the isoclinics. Starting with a certain isoclinic, i.e., the 20-deg. curve, crosses are marked at random all along this isoclinic, having one of the arms of the cross at an angle of 20 deg. with the direction of the principal plane of the crossed nicols, and in a direction corresponding to the direction of rotation of the nicols. The same process is repeated with all the other isoclinics. Lines drawn parallel to these crosses and spaced at

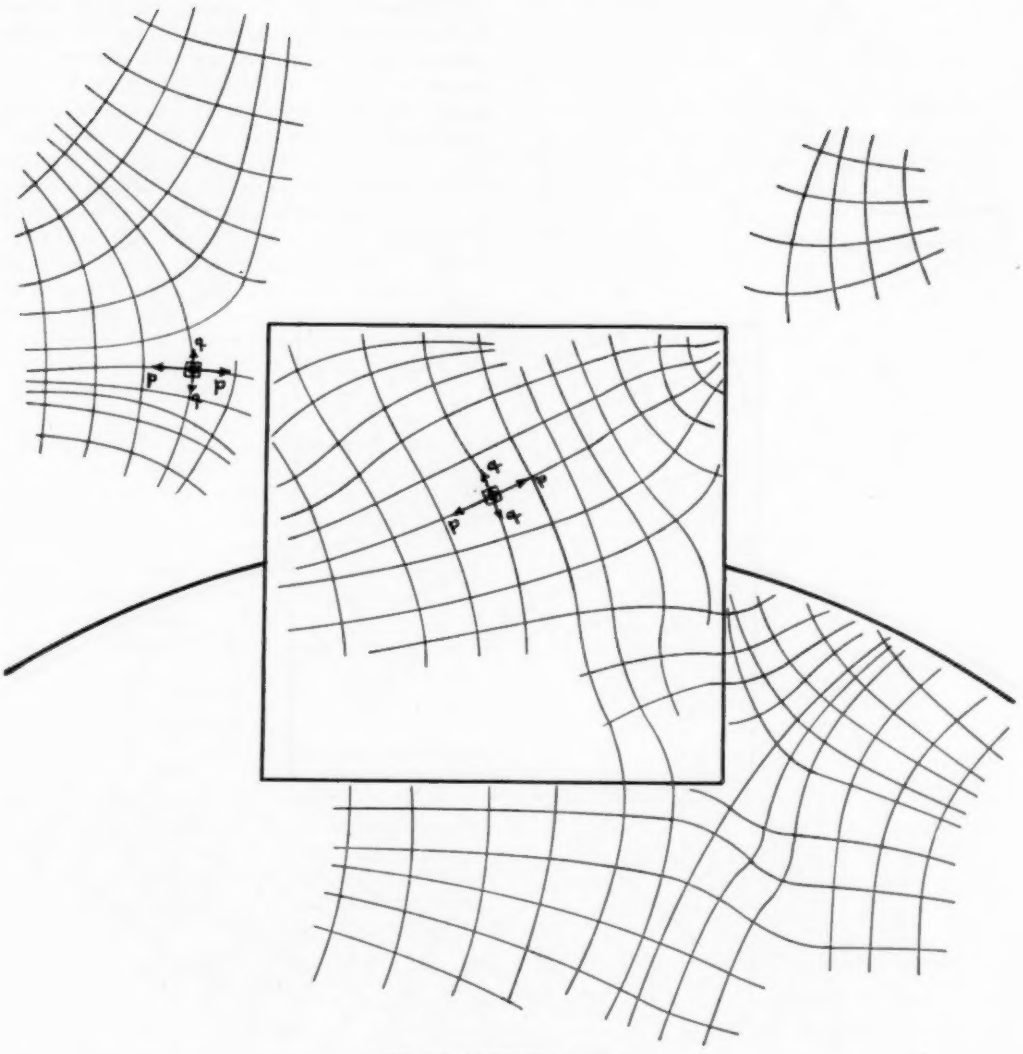


FIG. 23 ISOSTATICS FOR A

regular intervals with regard to a convenient reference line on the specimen are termed "isostatics," and they give the directions of the principal stresses p and q, at right-angles to each other. As to which one will be p or q, the identification will be very easy in many cases by the mere observation of the direction of application of the external load. Isostatics for the square key and keyway are shown in Fig. 23.

The general formula for shear $2\ \tau = (p_1 - q_1) \sin 2\varphi$ is maximum for $\varphi = 45$ deg. and becomes $s = \dfrac{p-q}{2}$, where s is the ultimate value of shear corresponding to a 45-deg. value of φ. Hence the directions of maximum shears are given by curves at 45 deg. with the isostatics, as shown in Fig. 24. As in general most of the failures in machine parts and structural members are due to the effect of maximum shearing stresses, a knowledge of their directions as well as their magnitudes is of great importance to the designer, for a predetermination of the most probable section of failure of the material due to overstressing.

ACKNOWLEDGMENT

The authors are greatly indebted to Dr. G. B. Pegram, head of the Physics Department of Columbia University, for continuous encouragement and helpful suggestions, as well as to Profs. A. H. Beyer, of the Civil Engineering Department, and W. D. Turner, of the Chemical Engineering Department, for their assistance in the work.

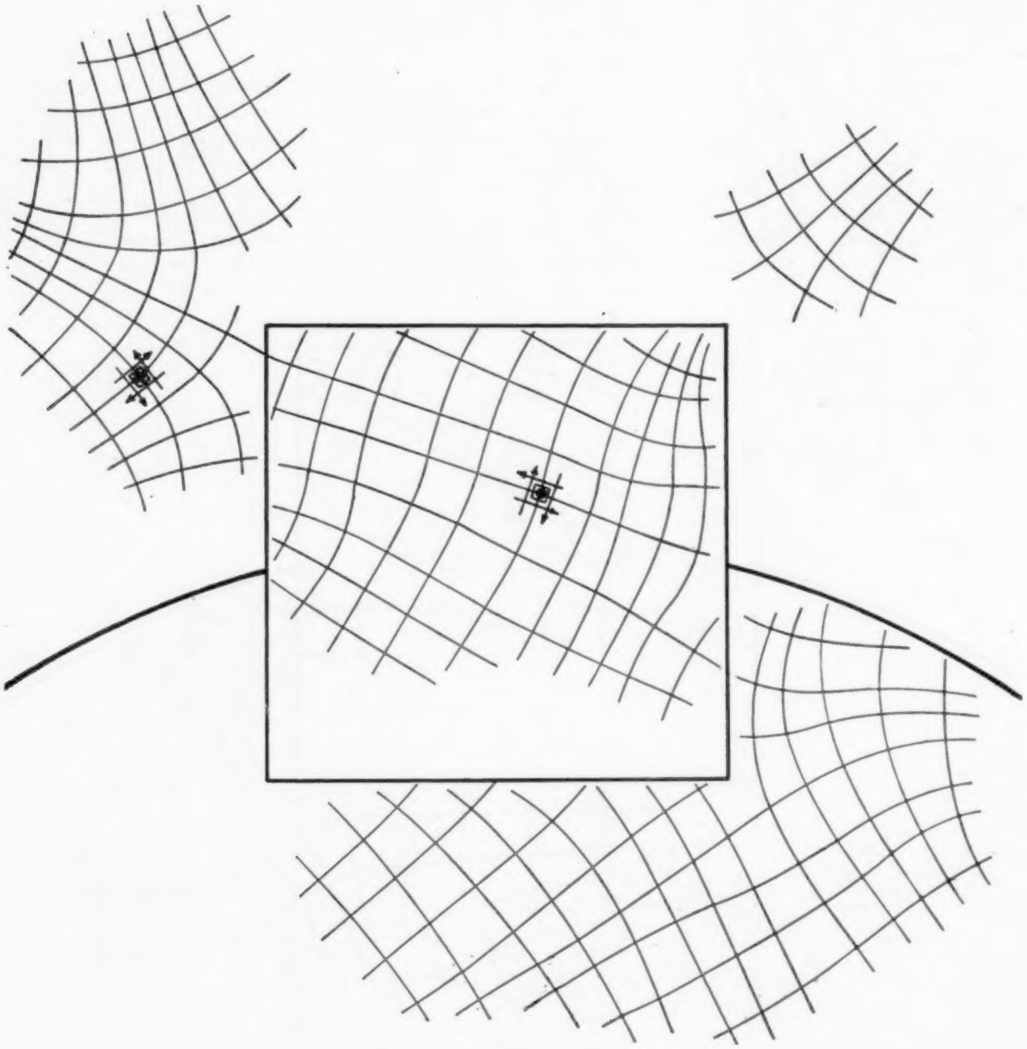


FIG. 24 SHEAR LINES FOR *A*

Discussion

J. P. DenHartog.[14] There is as yet no practical photoelastic method for measuring the stresses in three-dimensional cases. The distinction between two- and three-dimensional stress distributions should be very sharp, but in practical cases it is somewhat vague. In fact, there does not exist any practical stress distribution which can be termed two-dimensional with the exactness of a mathematician. But a rather large number of cases are so close to being two-dimensional that for the engineering accuracy required the photoelastic method is quite sufficient and very useful.

Among the numerous papers on the subject, now and then one appears discussing an application of a definite three-dimensional character, which is implicitly assumed to be two-dimensional. It seems that the present paper is an instance of this, for the following reason. A key has the object of transmitting a torque from a shaft to a disk or rotor of some kind, and it is well known that this transmitted torque is not distributed uniformly along the length of the shaft, but is much concentrated near the edge of the disk. This is illustrated in Fig. 25, where the ordinates indicate the torque transmitted through the key. The exact shape of this curve cannot, of course, be calculated, but an approximate consideration shows it to be something like an exponential curve, which has its steepest slope just at the point of maximum torque transmission. This steepness is of such a nature as to make the problem decidedly three-dimensional, especially at the point of maximum stress, which is of greatest interest.

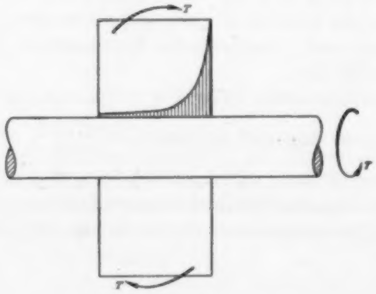


FIG. 25

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[14] Research Laboratories, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Assoc-Mem. A.S.M.E.

Another point in the test procedure which is not entirely satisfactory is the method of applying the torque, which gives a stress distribution quite different from the usual torque stress distribution in a shaft. The effect of this difference on the local stresses near the key due to the local key pressures will probably be small. But on the other hand, the stress in the corner of the keyway, due to the torque, will be very much affected. This may be a possible explanation for the unexpected result that rounding these corners did not improve the stress condition markedly.

The authors state that "most of the failures in machine parts are due to the effect of maximum shearing stresses." This is entirely correct as it relates to a first appearance of slip.

In a restricted area, like a fillet or groove, this is not serious. As the authors state, the real danger is caused by fatigue, and a fatigue failure is determined by the maximum tensile stress rather than by the maximum shear.

Adolph A. Moline.[15] Proper key design is an important factor in large apparatus, especially steel-mill motors, Diesel generator units, etc., in which the torque loads are not accurately known. The authors have contributed a paper which brings to light the somewhat unknown stress distribution in keys.

The authors state that a more favorable stress distribution results if fillets are placed in the corners of the keyways and if the keys are chamfered at the corners. This design, no doubt, reduces the stress concentration at the keyway corners. It is the writer's opinion, however, that the key would have a greater tendency to "roll," and thus fail by compression. Such a condition would also cause very high stresses in the rotor hub.

The writer uses a method of designing keys that may be of interest. It is based on the yield points of the materials involved and on the minimum torque-carrying diameter of the shaft. The shaft is assumed to fail in shear at one-half of its yield point in tension, and the key is assumed to fail in shear at 75 per cent of its yield point in tension. The maximum permissible compression stress in the key and rotor hub is taken as three times the average shear stress in the key. If the yield points are equal, the width of the key is found to be equal to $0.262 \dfrac{d^3}{LD}$, and the key height will be

$$2/3 \times 0.262 \times \frac{d^3}{LD} = 0.175 \times \frac{d^3}{LD}$$

where

d = minimum torque-carrying diameter of the shaft
L = length of the hub bearing against the key
D = diameter of the shaft at the key.

It sometimes is difficult with one key to arrive at a proper key-to-shaft-diameter proportion because of a limited hub length. In such cases, two keys are used, one of them being of the three-piece type in order to insure that both keys are effective.

M. Stone.[16] The discusser is interested in both major aspects of the paper, keys and photoelasticity. The type of investigation that the authors have undertaken—to obtain the stress-distribution and stress-concentration factors at the keyway with load applied through the key—is an advance over the more usual method when these influences are determined for a pure torque application. However, in any such investigation we are always confronted with the predicament of having to interpret the photoelastic findings practically. Photoelastic investigations, to mean anything scientifically, are made within the elastic limit of the model material. The actual concentrations found of 4 or 5 indicate that in the actual keys the yield point is exceeded in several points; i.e., the actual stress condition is part elastic and part plastic. This means that, practically, the plastic flow of the material results in much lower concentrations—which concentrations, even, must not be considered serious for static-stress application.

The discusser has for some time been engaged in the work of rationalizing key design. The limits set are: For a maximum allowable shear stress in the shaft S, the shear stress in the key is set at $1.5\,S$ and the bearing stress at $4.5\,S$. This results in key widths of approximately $^1/_4$ of the diameter or less, and key heights of $^2/_3$ the width—which are practical key dimensions. In application, the bearing stress is held at the yield point of the material used, which means that at several localized areas the stresses exceed the yield point, and without disastrous results.

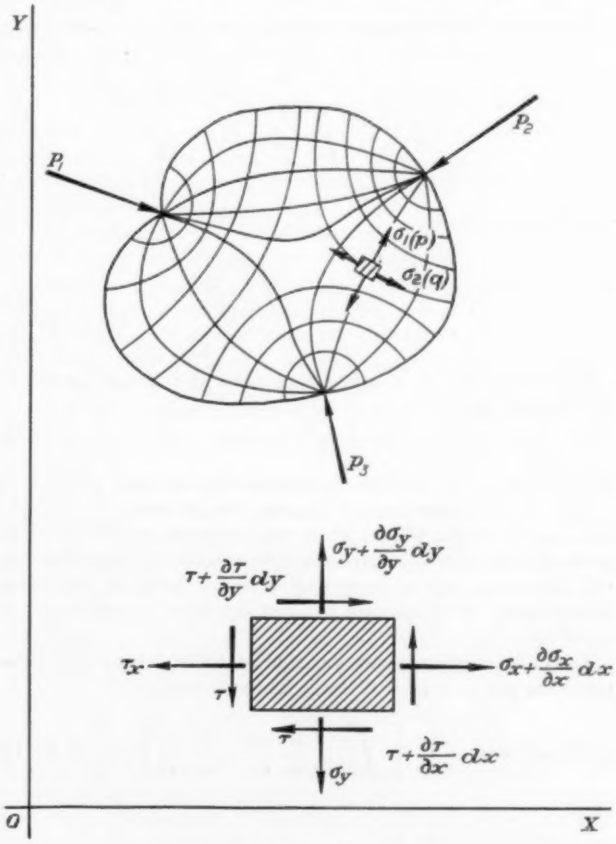
Y
P_1
P_2
$\sigma_1(p)$
$\sigma_2(q)$
P_3
$\tau + \frac{\partial \tau}{\partial y} dy$
$\sigma_y + \frac{\partial \sigma_y}{\partial y} dy$
τ_x
τ
$\sigma_x + \frac{\partial \sigma_x}{\partial x} dx$
τ
$\tau + \frac{\partial \tau}{\partial x} dx$
σ_y
O
X

FIG. 26

Regarding the point the authors make that a chamfered key causes high concentrated stress when the key turns slightly, it must be said that this concentration is highly local, and again is probably of no serious consequence. It is detrimental, however, to have keys with too large chamfers, for by far the most common type of key failure is by rolling out of place.

In the photoelastic section of the paper, the authors have discussed the question of stress separation. Although, in most cases, the photoelastic method's usefulness is to give a qualitative picture of the stress distribution, there are times when the actual distribution of normal stresses is necessary. When lateral extensometers are not available for measuring $(p + q)$ effects (the best method is that due to Favre, using the interferometer), this discusser has resorted to a graphical method, which he feels is simpler than that due to Filon or Mesnager.

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[15] Mechanical Engineer, Power Engineering Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[16] Development Section Engineer, Power Engineering Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Consider the general stress system, as shown in Fig. 26, and the element below it. From considerations of equilibrium, it can be shown that

$$\left.\begin{aligned}\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau}{\partial x} = 0\\ \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} = 0\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots[1]$$

Integrating these equations partially, we get

$$\left.\begin{aligned}\sigma_{y1} = -\int_{y_0}^{y_1}\frac{\partial \tau}{\partial x}\,dy + \sigma_{y0}\\ \sigma_{x1} = -\int_{x_0}^{x_1}\frac{\partial \tau}{\partial y}\,dx + \sigma_{x0}\end{aligned}\right\}\ldots\ldots\ldots\ldots[2]$$

which, added, give

$$\sigma_{y1} + \sigma_{x1} = -\left[\int_{y_0}^{y_1}\frac{\partial \tau}{\partial x}\,dy + \int_{x_0}^{x_1}\frac{\partial \tau}{\partial y}\,dx\right] + \sigma_{y0} + \sigma_{x0} \ldots\ldots\ldots[3]$$

From a knowledge of stress conditions at a point in a plane, it is known that

$$\sigma_y + \sigma_x = \sigma_1 + \sigma_2 \ldots\ldots\ldots\ldots\ldots\ldots[4]$$

where σ_1 and σ_2 are the principal stresses at the same point.

Our method now has for its aim the determination of the $(\sigma_1 + \sigma_2)$ network, which, when superimposed on the $(\sigma_1 - \sigma_2)$ network that comes directly from the photoelastic measurements, will give us σ_1 and σ_2 separated at every point in the stress distribution. We thus wish to construct lines of constant $\sigma_1 + \sigma_2 = K$.

Let us introduce the new variable s into [3], where s is the distance along a $\sigma_1 + \sigma_2 = K$ line. Thus [3] reads

$$\sigma_{y1} + \sigma_{x1} = K = -\left[\int_{s_0}^{s_1}\left(\frac{\partial \tau}{\partial x}\frac{dy}{ds} + \frac{\partial \tau}{\partial y}\frac{dx}{ds}\right)ds\right] + K \ldots[5]$$

or

$$\int_{s_0}^{s_1}\left(\frac{\partial \tau}{\partial x}\frac{dy}{ds} + \frac{\partial \tau}{\partial y}\frac{dx}{ds}\right)ds = 0 \ldots\ldots\ldots\ldots[6]$$

defines a line of constant $\sigma_1 + \sigma_2$. Realizing that

$$\frac{dy}{ds} = \sin\alpha; \qquad \frac{dx}{ds} = \cos\alpha$$

then

$$\int_{s_0}^{s_1}\left(\frac{\partial \tau}{\partial x}\sin\alpha + \frac{\partial \tau}{\partial y}\cos\alpha\right)ds = 0 \ldots\ldots\ldots[7]$$

holds. The only way that [7] can hold is that the integrand itself is zero, and so we have

$$\frac{\partial \tau}{\partial x}\sin\alpha + \frac{\partial \tau}{\partial y}\cos\alpha = 0 \ldots\ldots\ldots\ldots\ldots[8]$$

or

$$\tan\alpha = -\frac{\partial \tau/\partial y}{\partial \tau/\partial x} \ldots\ldots\ldots\ldots\ldots\ldots[9]$$

Thus the slope of the $\sigma_1 + \sigma_2 = K$ line is at every point given by the ratio of the shear-stress gradients, and enables one to proceed from point to point very accurately and quickly. The shear-stress gradients $\partial\tau/\partial y$ and $\partial\tau/\partial x$ are obtained directly from the photoelastic results, so that the $(\sigma_1 + \sigma_2)$ network is as accurately obtained as the $(\sigma_1 \overset{-}{+} \sigma_2 = 2\tau)$ network is known.

R. E. Peterson.[17] In a number of failures of keyed shafting, the writer has noticed that the fatigue fracture occurred at the end of the keyway where a stress concentration additional to that treated by the authors is present. There are two general types of keyway end, as shown in Fig. 27—the profiled keyway, made with an end mill, and the sled-runner keyway, made with a slotting mill. Fatigue cracks originate at the places indicated in Fig. 27.

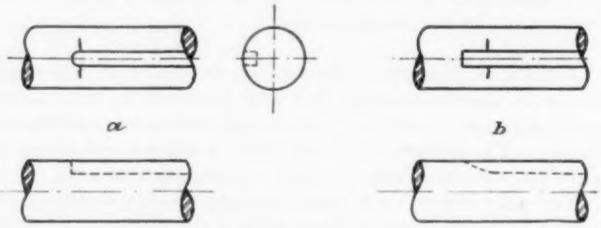

Fig. 27 Failure at Ends of Keyway
(a, profiled keyway; b, sled-runner keyway.)

Bending-fatigue tests now in progress indicate considerable advantage in the sled-runner keyway as compared with the profiled keyway. The point in making this statement is to indicate that the failure of keyed shafting is in general a three-dimensional problem.

Max M. Frocht.[18] Aside from the information which the paper furnishes for the problem of shear stresses in keys and keyways, it raises the fundamental issue of white light versus monochromatic light in photoelasticity. Before discussing this issue, however, the writer wishes to comment on the interpretation of stress patterns, equipment, and annealing process of bakelite.

On the second page the authors say: "If a monochromatic light be used,....the field will be darkened for those points of the plate where the retardation is equal to $^1/_2$, $^3/_2$, $^5/_2$, etc., wave lengths.....Fringes of the first, second, third, etc., orders will correspond to retardations of $^1/_2$, $^3/_2$, $^5/_2$, etc., wave lengths." These, in the writer's opinion, are erroneous statements. The numbers referring to the wave lengths should be replaced by 1, 2, and 3, respectively. The statement in the paper would be correct for the case of light vibrations in one plane in which vibrations of the same amplitude but having phase differences of odd-halves wave lengths annul each other. This is not the case here. The two rays traveling through any point in the stressed model lie in the principal planes—that is, in planes mutually perpendicular and not in one plane. For such conditions dark fringes appear for those points in the model which produce relative retardations or phase differences which in no way disturb the character of the entering ray. If, as is the case in photoelastic investigations, the ray entering the stressed model be circularly polarized, then dark fringes will appear for those stressed points for which the ray emerging from the model is also circularly polarized and of the same sign of rotation. It should be evident that this will be true only for additional phase differences of an integral number of wave lengths. For retardations of odd-halves wave lengths the emerging ray, it is true, remains circularly polarized, but its sign of rotation is reversed relative to the entering ray, and this results in a field of maximum illumination.

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[17] Manager, Mechanics Division, Westinghouse Research Laboratories, East Pittsburgh, Pa.

[18] Associate Professor of Mechanics, Carnegie Institute of Technology, Pittsburgh, Pa.

Quarter-Plates. On the fourth page, second column, the authors state that the function of the circularly polarized light is to remove the isoclinic from the stress pattern. This is undoubtedly an important function of the circular ray. There is, however, another important function which this ray performs and which the authors fail to mentoin. The use of circularly polarized light results in the removal of the directional effect of stress; that is, it is supposed to make the color at a point, or the fringe order, independent of the directions of the principal stresses. The writer will return to this point farther on in the discussion.

Equipment. It seems to the writer that the equipment used by the authors is unnecessarily complicated. This is especially true of the calibration frame, Fig. 4, the only function of which is to apply two couples to a beam. The main frame can be designed to produce bending as well as tension or compression. Nor is there a good reason for the rather intricate design of the calibration beam, shown in Fig. 4, necessitating careful and costly machining, when a simple, straight prismatic bar will suffice.

Annealing. It seems that the two authors do not agree on the exact nature of their claim regarding the annealing process. Thus Mr. Solakian implies essential differences[19] between the writer's process and the one described in the present paper, whereas Mr. Karelitz thinks that the two methods coincide. Mr. Karelitz[19] says: "Our observations regarding the ease of annealing, procedure, and temperature *coincide*[20] with those of the author,[21] which is gratifying." Two points of difference are mentioned by Mr. Solakian: one gives an annealing temperature of 75 to 85 deg. cent., against "about 70 deg. cent." as given by the writer; the second specifies that the models be kept at the annealing temperature for "at least one hour," contrasting it with 15 minutes' heating time given by the writer. In comparing the one hour against the 15 minutes, Mr. Solakian compares time intervals referring to different operations of the annealing process. The 15 minutes refers to the time taken by the writer to bring furnace and model to the required temperature; the time of "at least one hour" specified by the authors refers to the interval for which the models must be kept at the annealing temperature. These intervals represent totally different things. Again, in view of the small temperature gradient of the annealing furnace and the rather high tolerance of the annealing temperature, the writer fails to see the need for the specification of "at least one hour." To be more explicit, if the models be brought to the average annealing temperature of 80 deg. cent. and left alone in the furnace, the temperature at the end of one hour would be approximately 76 deg. cent., the average gradient being about 4 deg. cent.[22] In other words, the furnace will automatically keep the models within the range of the annealing temperature for at least one hour. The specification referred to is thus superfluous and tends only to unnecessarily complicate the process.

With regard to the higher annealing temperature which the authors propose, the writer wishes to observe, first, that their lower limit of 75 deg. cent. is not so very remote from "about 70 deg. cent.," as specified by the writer, and, second, that the scores of models successfully annealed by the writer, and shown photographically in his papers, prove that there is no need for such high temperatures.

Shear Stresses. It seems that this paper contains either too much material or not enough. If, as the authors believe, the principal stresses are of little practical value to the designer, then why go to the trouble of determining isoclinics and stress trajectories? The only practical value of these curves when not used to find the shear values is to facilitate the determination of the principal stresses, which the authors, as a matter of policy, do not determine. The writer cannot subscribe to the over-simplified theory that the principal stresses are of little practical value to the designer. Aside, however, from the importance or unimportance of the principal stresses by themselves, they are necessary in order to verify the correctness of the shear values. A complete solution would contain a static check and a continuity check. A static check is the minimum essential in a problem of stress analysis, and it is regretted that the authors have not made such a check.

Light. Referring to the more fundamental question of white light versus monochromatic light, the writer will discuss the alleged advantages of white light in the order in which they are summarized on the sixth page, the first column.

Paragraph (*a*) states, "Black and white photographs of the isochromatics are now obtained as easily as of the monochromatics." This seems vague and inconsistent with a preceding statement in which the authors concede that "monochromatics give more distinct pictures."

The writer is inclined to agree with the point made in (*b*) that isochromatics when projected or printed are more impressive, but would like to add that they are also less reliable. Compare, for example, Figs. 10 and 11. The two photographs are supposed to be of one and the same stress pattern, with the black fringes of Fig. 11 corresponding to the greens of Fig. 10. A casual inspection, however, reveals numerous differences. The differences between the two pictures are so great that they can hardly be attributed to the effect of creep and explained by the lapse of time between the taking of the pictures. If the effect of creep were pronounced enough to cause such differences, photoelastic research would hardly be possible.

Paragraph (*c*) states that black regions corresponding to $p - q = 0$ are easily detected in the isochromatics. This is true. It is equally true, however, that a little experience enables one to tell these points with relative ease when a mercury-vapor lamp is used. All one needs to do is to remove the color filter, and the dark spots come into relief. Also, due to the stationary character of these points, they are easily recognized with monochromatic light.

Paragraph (*d*) states, "With three-color bands in the isochromatics against two-color in the monochromatics, the optical sensitivity of bakelite is increased in the ratio of 3 to 2." On first thought this sounds plausible. Let us, however, examine this statement more carefully. Suppose we assume that a bakelite model $^{1}/_{4}$ in. thick is subjected to fairly high stresses within the elastic limit—say, stresses of the order of the 15th fringe. Let us further assume that the white-light stress pattern would distinctly reveal all the corresponding 15 greens.[23] To obtain the alleged increase in optical sensitivity all the 45 isochromatic bands of yellow, red, and green would have to be clearly identified. Let us see, however, what actually happens. Quoting as authority the authors of this paper, on the ninth page, the first column, they state: "It will be observed that in nearly all cases the yellow color beyond the fourth order is not represented in the isochromatics on account of the fact that at high stresses it is very difficult to discriminate it from red and green colors." In other words, according to their own experience it would only be possible to recognize 4 yellow bands out of the 15, and we should thus have a maximum of 34 bands[24] instead of 45, whereas with monochromatic light we should obtain 30. The ratio of the sensitivities would thus be 34 to 30 and not 45 to 30, as claimed in paragraph (*d*). The alleged ratio of 3 to 2 would have more weight if we deliberately restricted the stress range to the fourth green. It must be understood, however, that the important thing is not how many points can be obtained within an arbitrarily restricted range of low stress, *but how many points can be obtained for the maximum stress within the elastic limit.*

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[19] See discussion of writer's paper "On Recent Advances in Photoelasticity," Trans. A.S.M.E., paper APM-53-11.

[20] Italics by the present discusser.

[21] The present discusser.

[22] If 80 deg. cent. be the average annealing temperature, 22 deg. cent., the approximate room temperature, and 14 hours, the average time for cooling, we have for the average temperature drop per hour $(80 - 22)/14 = 4.1$ deg. cent., approximately.

[23] This assumption is contrary to fact and it is made solely in order to show that under the most favorable conditions the contention of the authors is unwarranted.

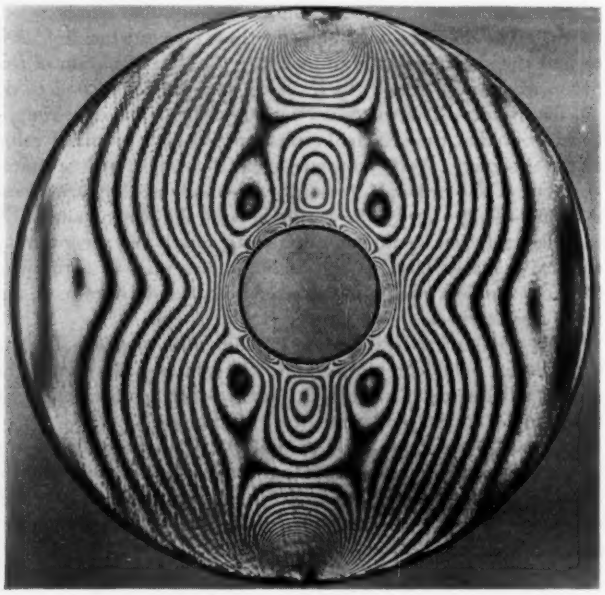

FIG. 28 DISK WITH HOLE UNDER VERTICAL CENTRAL LOADS (Annealed bakelite model. Monochromatic light.)

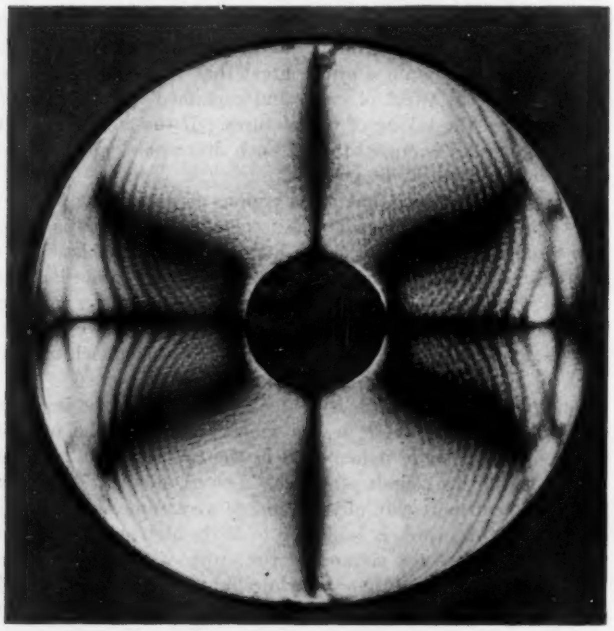

FIG. 29 DISK WITH HOLE UNDER VERTICAL CENTRAL LOADS (Annealed bakelite model. White light. Black curves represent the zero isoclinics. Note vagueness of isochromatics of higher order. Loads slightly greater than in Fig. 28.)

This, however, is not all. The 34 to 30 ratio, as already suggested, has been granted for the sake of discussion only. In reality the sensitivity with monochromatic light is greater than with white light. The authors have undoubtedly observed that isochromatics become narrower and fainter as their order increases. In fact, beyond the fifth green the colors become so crowded and washed out as to be of little value. Assuming that all isochromatics can be traced with accuracy (a task not always easy to perform, especially in the presence of creep) and taking as a practical limit the sixth green, we have a total of 18 isochromatic bands. With the fringe method the patterns are distinct even beyond the fifteenth fringe. We thus have within the elastic limit at least 30 bands with the fringe method, against 18 with white light, or an effect equivalent to an increase in the optical sensitivity of 5 to 3. This increase in sensitivity is clearly shown by Figs. 28 and 29. Attention is called to the absence from Fig. 29 of all the rich detail around the hole and about the vertical diameter of Fig. 28.

Paragraph (*e*) sets forth the imaginary difficulties of determining the fringe order with monochromatic light against the ease and simplicity of white light. Leaving for a moment the question of difficulty in determining the fringe order, the writer wishes to point out that even in the white-light method essentially the same difficulties exist. The process of counting is as necessary with white light as it is with monochromatic light, since the order of a color band must be known before a numerical value can be attached to it. The same is true about the need for slow loading. If the fringe pattern is of the escaping type, so is the color pattern, and to determine the number of escaping isochromatics one has to watch their formation during their slow loading just as in the case of fringes. As a matter of fact the determination of the fringe order presents no difficulties.

Effect of Creep. Mr. Solakian seems also to consider the effect of creep less detrimental with white light than with monochromatic light.[25] The existence of optical creep is, however, a strong point against the white-light method and in favor of the fringe or monochromatic method. It requires only a second or a fraction thereof to take an accurate fringe photograph of any stress pattern regardless of its complexity. The amount of time, however, necessary to sketch isochromatics is vastly larger and varies with the stress pattern. Consider, for example, Fig. 9, *a-e*. It would take at least 15 minutes to sketch any one of these, or approximately 900 seconds. The existence of creep is therefore bound to affect the drawing of the isochromatics at least 900 times more than it does the picture. The sharpness of the fringe photographs obtained by the monochromatic method is evidence of the absence of effect of creep. The fringe photographs should be taken immediately after the load is applied, and the determination of the fringe order should be carried out later. The presence of creep during the determination of the fringe order is of no importance.

Directional Effect. An inherent defect of the white-light method of paramount importance will now be pointed out—the directional effect of stress when white light is used. Consider two elements *A* and *B* in a body subjected to a two-dimensional-stress system, Fig. 30. Let us assume that the principal stresses on these elements are equal in magnitude, but differ in direction. If a ray of plane-polarized light be transmitted through this body, it will be resolved into rectilinear components in directions of the principal stresses whose magnitudes will depend on the direction of the plane of vibration of the impinging ray. If, for concreteness, the impinging plane-polarized ray has vibrations in a vertical plane, the components would be as shown in Fig. 30. In such cases the optical effect of a stress system acting upon an element would depend on the direction of their stresses as well as on their magnitudes. To obviate such a condition the plane-polarized ray is converted into a circularly polarized one, which breaks up into equal components regardless of the directions of the principal stresses. Such a condition is implied in the stress optic law $R = C\ (p - q)$, which does not contain the directions of the stress system. Now, the most effective way to produce circular polarization is by means of a quarter-plate. But a quarter-plate has meaning only when used with monochromatic light. There evidently can be no quarter-plate for white light. If white light be merely the combined effect of the fundamental colors having different wave lengths, then obviously a quarter-plate can be made to fit one but not all the colors simultaneously. Thus, white light gives elliptic polarization and not circular, with a consequent result that there is a directional effect of the stresses at a point. The writer has made some experiments to verify this theory. Strips of celluloid and bakelite were subjected to tension and placed in the path of the circular (?) ray of the polariscope, using white light. The strips were then rotated in the plane normal to the axis of propagation of the ray, and the changes of color were observed. The experiment showed that while the celluloid strip seemed to hold its color, the bakelite strip showed pronounced changes. Starting with a deep green in the vertical position, the color changed and showed a clear tinge of red in the horizontal position, indicating a possible error of approximately 20 per cent. What seemed to be evidence of circular polarization with celluloid thus proved to be evidence of its small optical sensitivity. There is therefore a directional effect when white light is used. Such, however, is not the case with monochromatic light. Here a quarter-plate can be made to produce true circular polarization, thereby eliminating directional effect and increasing the accuracy of the work.

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[24] 4 yellows, 15 reds, and 15 greens.

[25] See paragraph (*a*) of Mr. Solakian's discussion of writer's paper, "On Recent Advances in Photoelasticity," loc. cit.

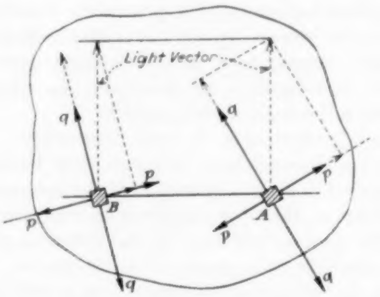

FIG. 30 DIAGRAM SHOWING THAT COMPONENTS OF LIGHT VECTOR DEPEND ON DIRECTIONS OF PRINCIPAL STRESSES

The fringe or monochromatic method has also the advantage that it makes practicable the taking of clear moving pictures and thereby contributes an effective tool for many fields of investigation, especially for the study of impact stresses.

The writer advocates the use of the fringe or monochromatic method because he believes that it has genuine merits and superiorities over the white-light method. This, however, does not mean that he bans white light from his laboratory. There are two occasions on which he uses white light: one is when sketching isoclinics, the other when investigating the sign of stress—two uses of white light which seem to have escaped the attention of the authors.

The paper will undoubtedly stimulate further interest in photoelasticity. It is gratifying that this useful and interesting field of investigation is beginning to receive the attention that it merits.

ALPHONSE A. ADLER.[26] As a member of the Committee on the Standardization of Shafting, Keys, and Keyways, the writer is much interested in the results given in this paper. Requests continually come before this committee to standardize square and rectangular keys for large sizes of shafting. It is the writer's belief that because of the importance of fit of key and keyway and the crudeness of the theory underlying their design, it would be unwise to standardize the larger sizes, since this might convey the impression that such keys represent good machine design.

It is the writer's opinion that the tangent type of key shown in Fig. 1 is much better adapted to heavy work than the rectangular key, since it can be removed and reinstalled with a reasonable degree of certainty of complying with assumptions made in its original design. This is not usually true in the ordinary rectangular key unless it is generous in its proportions.

The tangent type of key is installed in pairs, as shown in Fig. 31, each key having two tapered wedges, which must be driven with an initial compression sufficiently great so that any torque applied in the direction of driving will not release the trailing key of its entire initial compression. Tangent keys are not commonly used, perhaps because their merits for heavy duty are not fully appreciated.

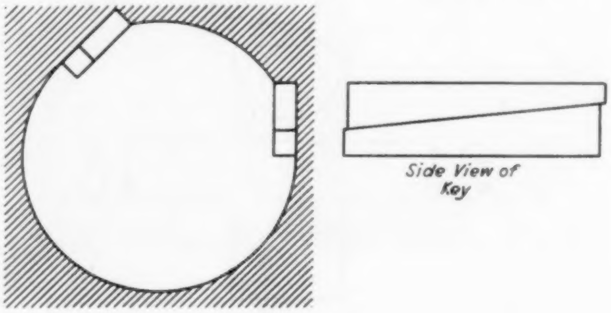

FIG. 31

Moreover, tangent keys might have their taper so adjusted that any reasonable predetermined compression pressure could be obtained along the length of the key to insure proper division of torque along this dimension. This is almost a hopeless task for the ordinary rectangular key, and if it were possible, the dismantling and reassembling would disturb the initial stress distribution.

In view of this, perhaps the authors would like to continue their experiments to test out the merits of the tangent keys. The writer will be pleased to assist by giving the formulas for their design which he used while engaged in teaching machine design.

H. F. MOORE.[27] In this interesting paper reference has been made to tests of the effect of keyways on the strength of shafts made some 23 years ago at the University of Illinois.[28] These tests were all *static* tests and an arbitrarily determined elastic limit[29] was used as a criterion of strength. This gives a fairly reliable index for *static* strength, but should not be relied on as an index of the weakening effect of a keyway under repeated stress.

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[26] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

[27] Research Prof. of Engineering Materials, University of Illinois, Urbana, Ill. Mem. A.S.M.E.

[28] The Illinois tests are reported in Bulletin 42 of the Engineering Experiment Station, not in Bulletin 41 as stated in the A.S.M.E. preprint.

[29] Method II of the 1931 Am. Soc. Testing Materials Tentative Standards, p. 880.

[NOTE: Discussion of this paper jointly with that of Max M. Frocht by Prof. Raymond J. Roark will be found following the latter paper.]

AUTHORS' CLOSURE

The authors are much gratified by the interest the various discussers have taken in the paper.

Mr. Den Hartog's remarks are correct in so far that a key problem is to a certain extent a three-dimensional case. This is also emphasized in Mr. Peterson's discussion. As a matter of fact there are few stress problems of an absolutely two-dimensional nature. However, the general stress distribution in a cross-section of the keyed shaft is probably not very different from that given in the paper, particularly for regions of the key not including the very first tenth division close to the highest-stressed end. Considering the fact that the width of the model used for stress analysis is a very small fraction of the length of the key and assuming that Mr. Den Hartog's stress representation along the length (l) of the key under a torque from both ends is correct quantitatively, the rate of stress variation over a length (δl) equal to the thickness of the model (Fig. 32) is so small as to have a negligible effect on the general stress distribution in the cross-section of the mechanism at that section.

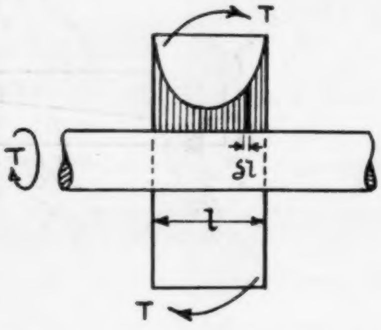

FIG. 32 TORQUE TRANSMITTED THROUGH THE KEY

Several other methods of producing torque in the model were tried, but the one described in the paper was finally adopted because it did not appreciably affect, even with the maximum torque, the stresses developed in the neighborhood of the keyway.

Mr. Stone's proposition of determining a $p + q$ stress network from the $p - q$ stress network by the application of a formula giving the slope of the $p + q$ line in terms of the shear-stress gradient has its definite merits, and seems to be much simpler than the Maxwell-Filon method of graphical integration commonly used for such cases. Mr. Z. Tuzi tried a similar method about five years ago with good results.

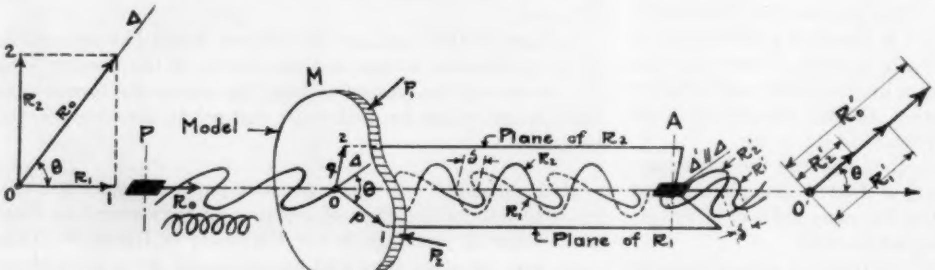

FIG. 33 OPTICS OF POLARIZED LIGHT IN A PHOTOELASTIC APPARATUS

Professor Roark's remarks are to a certain degree true with regard to the limitations of the photoelastic investigations to the range of stresses within the elastic limit of the material. However, the authors believe that it is possible to investigate problems even though the stresses exceed the elastic limit of the bakelite.

Mr. Adler's tangent type of key certainly deserves a special photoelastic investigation to determine its characteristics as compared with the types already tested.

Dr. Frocht's discussion is very interesting. The authors appreciate his thoroughness, although they hardly agree with him with regard to his interpretation of the contents of the paper and especially to the way employed in representing it to the public. The authors will take up the different points about in the same order that they appear in the discussion.

It is not an "erroneous statement" to ascribe the successive appearance of stress fringes to retardations of $^1/_2$, $^3/_2$, $^5/_2$, ... etc. wave lengths of monochromatic light, instead of 1, 2, 3,... etc. The difference between the two, as a matter of fact, is a question of origin of reference. However, it is interesting to note that the discusser himself proves the validity of the authors' statement. According to the discusser's conception, "the statement in the paper would be correct for the case of light vibrations in one plane....... The two rays traveling through any point in the stressed model lie in the principal planes, that is, in planes mutually perpendicular and not in one plane....." Let us refer to Fig. 33, in which R_0 represents (approximately) a ray of circularly polarized monochromatic light emerging from the polarizing prism P and vibrating in any plane Δ at the instant of striking the plate. The ray after its passage through the stressed transparent model M refracts into two rays R_1 and R_2, each vibrating in the planes of the two principal stresses p and q mutually perpendicular to each other. The discusser's observations of light vibrations refer to this section of the optical set, which is a wrong section of reference as the stress fringes are formed due to interference of the two refracted rays taking place after these rays pass through the analyzing prism A, which brings the vibrations of the two rays into *one plane* Δ' parallel to Δ, as shown in the above-mentioned figure—a condition imposed by the discusser for the validity of the authors' statement.

Quarter-Plates. This part is considered under "Directional Effect."

Equipment. The authors do not consider the equipment "unnecessarily complicated." The optical set is the simplest of its kind ever designed, having the smallest number of lenses and other component parts. The calibrating-beam apparatus serves also as a Coker compensator. The suggestion of replacing this with a "simple, prismatic beam" in a research laboratory is a little hard to accept for accurate results.

Annealing. The discusser is much worried about the discrepancies in the methods of annealing of bakelite and the "discovery" of the process. Regarding "the priority" in making public an annealing process for bakelite as claimed by the discusser in his oral discussion, the authors merely wish to remark that their method was informally disclosed to a group of members of the A.S.M.E. Applied Mechanics Division, as part of an exhibit of the photoelastic equipment of Columbia University, on April 21, 1931, about two months before the discusser's paper was presented. The discusser's insistence on lower temperature in annealing and a shorter time of heating than that specified by the authors does not stand a scientific test. In addition, it has not been efficient in certain cases, as is evident from his remark that "in cases of stubborn stresses a second annealing will be found effective."

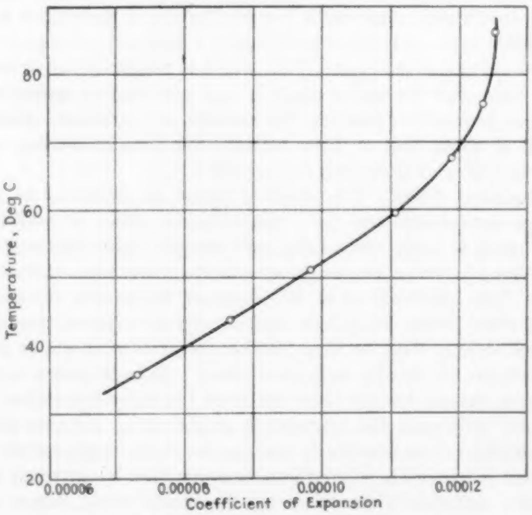

Fig. 34 Relation Between Temperature and Coefficient of Expansion of Bakelite

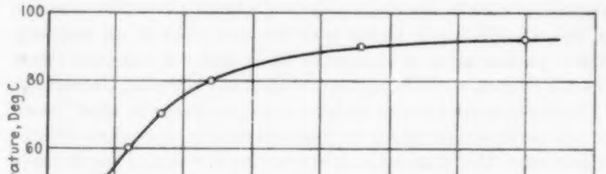

Fig. 35 Relation Between Temperature and Time of Conduction of Bakelite

Fig. 34, showing the relation between temperature and the coefficient of expansion of bakelite, demonstrates that expansion is continuous up to about 80 deg. cent. At this point the material enters a plastic stage, in which cohesive strength is a minimum, so that internal equilibrium is most easily attained. Remembering that bakelite is one of the best-known insulators of heat, one would hardly expect ten minutes' heating to produce a uniform temperature throughout its thickness, even when in the form of a $^{1}/_{4}$-in. plate. This is borne out by Fig. 35, showing temperature differences between interior and exterior plotted against time. It will be seen that after only ten minutes a large difference exists; the interior temperature continues to rise for slightly more than an hour (the minimum limit specified by the authors and for a constant heat value). Without thermal equilibrium, stress equilibrium cannot be expected.

Shear Stress. The authors have already stated that the principal stresses are not necessary to a practical designer, who is mostly interested in the concentration of the stresses in general and the failure phenomenon present in the mechanism. Isoclinics and stress trajectories are given in the Appendix only for one type of key, with a short description, as frequent reference was made in the paper to them. A knowledge of the directions of the maximum shear stresses is an asset to the designer, and the curves representing these could not have been obtained without the help of the isoclinics or stress trajectories. It is a vague statement to say that the principal stresses are necessary in order to verify the correctness of the shear values, as the first can be computed only when the shear values are already known.

Light. The authors are now in a position to state that with the addition of a pair of filters to their optical set, the white light—still keeping the three predominant colors on the screen—gives as distinct a photographic print of the object as that obtained with mercury monochromatic light, as will be seen in Fig. 36, which shows the stress fringes in a Dardelet self-locking bolt and nut connection.

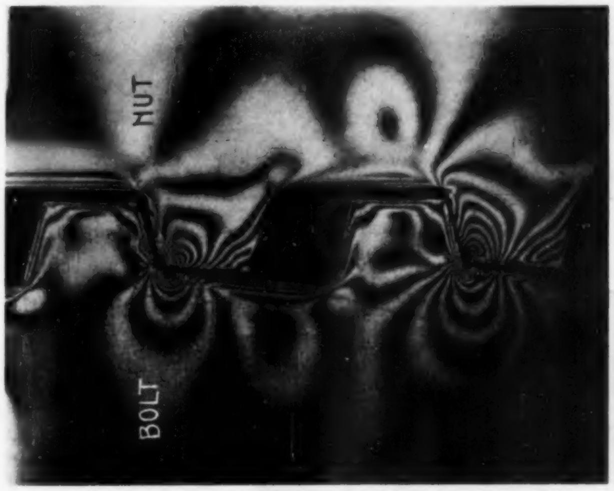

Fig. 36 Stress Fringes in a Dardelet Self-Locking Bolt and Nut

(Taken with white light.)

The authors regret that the discusser compares Figs. 10 and 11 to prove that "the isochromatics are less reliable," as during the presentation of the paper it was mentioned that the color photograph in Fig. 10 was not identical with that given in Fig. 11 for reasons of non-originality in loading conditions for both cases and the difficulties encountered in obtaining a good color print of the image. Besides, footnote No. 13 should have been a sufficient safeguard against comparing the two figures.

The optical sensitivity of bakelite cannot be measured by the total number of fringes produced under a very high stress, and it does not depend upon the quality of light, as it is a characteristic of the material itself, such as is the case in comparing glass, celluloid, phenolite, bakelite, etc. Moreover, it is a prohibitive practice to try to produce a large number of stress fringes, on account of plastic deformation and creep in the material, which is a prevailing phenomenon in nearly all the stress patterns of the discusser. Although attractive to the eye, it renders the pictures worthless from the point of view of stress analysis.

The authors' statement in paragraph (*e*) concerning the ease of determining the order of an isochromatic fringe with white light is found to be "imaginary." Let us refer back to Fig. 36. As will be seen, there are possibly six to eight regions in the picture where the stresses originate or fringes constantly change under a continuously changing tension in the bolt. With monochromatic light there should be about an equal number of observers each counting the number of fringes passing a certain reference mark, or 6 to 8 repeated loadings and unloadings of the model, which being composed of three separate parts (the figure shows only one-half of the model) there will be no originality of stress condition corresponding to each stage, besides the introduction of permanent set in the material as a result of repeated fluctuations of stress. A second case is that shown in Fig. 37, in which the fringes represent the $p-q$ stresses in a portion of a shrink-fit connection, where the diameter of the disk is slightly larger than the hole in the ring. As these fringes are formed during the gradual cooling down of the preheated ring during a long period of time in an annealing oven, the question of counting the fringes during their formation is not practicable. Further cases in which the use of monochromatic light is still more prohibitive have come to the attention of the authors in connection with recent studies of the stresses in reinforced-concrete structures and shrinkage stresses in composite materials, of which a case is that given in Fig. 38, representing the initial stresses in a bakelite specimen developed during the cooling of the material with a bar of metal inserted in the mold. Unless one knows the exact order of one or more of these fringes, the solution of the problem is an impossible one with monochromatic light. As the authors have already stated, for such cases, even in the absence of the neutral line, or one or two fringes, the orders of the other ones can be determined quite easily with white light from the hues of their respective colors of the lower-order isochromatics.

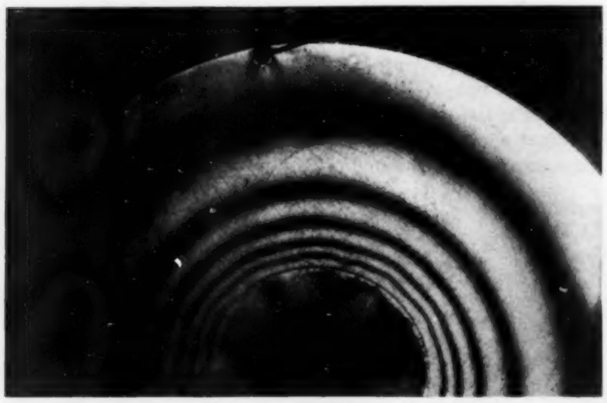

FIG. 37 SHRINK-FIT STRESS FRINGES IN A RING
(Taken with white light.)

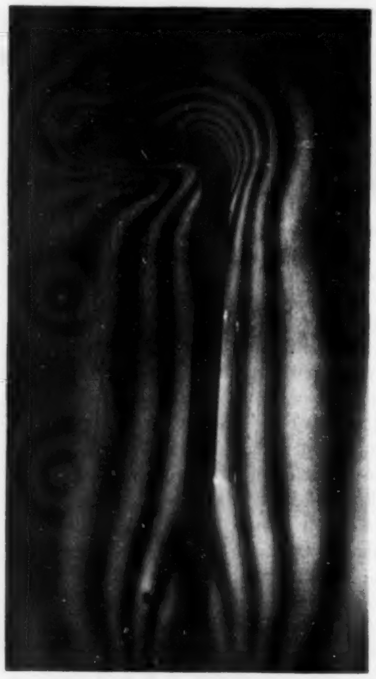

FIG. 38 STRESS FRINGES DUE TO SHRINKAGE AND PULL IN THE ROD
(Taken with white light.)

Effect of Creep. This is practically null for the maximum stress limits adopted by the authors in their tests—about 8 fringe orders. Such an effect can be observed in the experiments of the discusser where more than 25 fringes possibly exist at a point. The sharpness of a fringe photograph is not a sign of absence of creep, on account of the very short time allowed for exposure, amounting to one or two seconds only. But if it does not affect the plate, it certainly has a constantly changing effect on the deformations produced in the model, as a result of which pictures taken at different times after the application of a constant load will differ from each other to a degree which depends upon the rate of change of the creep effect, which is relatively larger for a short time after the load is applied. As a precaution against the slightest creep effect possible, the authors, as a principle, allow a period of about half an hour between the time of loading and photographing or sketching the isoclinics.

Directional Effect. The physical principles involved in this section are correct only for a quarter-wave plate; in addition, they apply to both white light and mercury light—although in less degree to the latter—as there is no absolutely monochromatic light. The observations of the discusser concerning the directional effect of the white light and quarter-wave plate, however, are not strictly true, as there are two quarter-wave plates in a photoelastic set, one for each nicol prism. As both plates are of identical nature, having been cut from the same mica sheet of constant thickness, the retardation produced by the first plate (which may not be necessarily one-quarter wave length for all the different colors of the white light) is neutralized by an equal and opposite retardation produced by the second plate, hence the directional effect of the white light does not exist with a well-designed optical set. An explanation of the change of color obtained by the discusser in his experiments of rotating a piece of stressed bakelite in circularly polarized white light is that possibly the two quarter-wave plates used are not alike in all respects. Such a phenomenon of change of color under a constant stress does not happen with the optical set used at Columbia University.

The authors are pleased to hear from the discusser that "there are two occasions on which he uses white light, one when sketching isoclinics, the other when investigating the sign of the stress—two uses which seem to have escaped the attention of the authors." Unfortunately there is no mention of such a use in either publications of the discusser; on the other hand, one reads the contrary in the expression that: "The sodium lamp (highly monochromatic) which was used for isoclinics was replaced by the mercury-vapor lamp....etc." As the authors' work throughout has been done with white light for isoclinics as well as for the stress fringes, it was not necessary to mention that fact repeatedly. Concerning "the sign of the stress," as stress fringes obtained with either light are lines of constant shear stress or $(p-q)/2$, and not of p or q separately, there is no specific sign for shear stress, whether it is due to tension, or compression, or any possible combination of the two. If the discusser refers to a beam under constant bending moment, the neutral black fringe only serves to separate the tension region from that of compression, but does not tell whether a certain fringe corresponds to tensile stress or compression, unless the conditions of bending are known beforehand. Even then in this case, the distinction is only for a limited field of the beam as the neutral axis is not a continuous line from one end of the beam to the other, and besides, in this specific case, the stresses in the beam are only the p stresses, as the q stresses are zero.

Before concluding, the authors would call attention to another interesting fact in connection with Figs. 28 and 29 which the discusser has inserted to prove the "increased optical sensitivity of bakelite by the use of monochromatic light." At first glance this may seem to be true for one who has not had much experience with photoelasticity. However, it is not very difficult to detect that Fig. 28, corresponding to monochromatic light, is taken with *circular polarized light,* giving very sharp fringes, while Fig. 29, corresponding to white light, is taken with *plane polarized light,* as is evident from the absence of the isoclinics from Fig. 28 and their appearance in Fig. 29; as a result of the presence of these black fringes with a slightly dark hue in the field of the plane polarized light, the color fringes in Fig. 29 do not appear in a way as they should. In addition, the time of exposure for both cases is different, which is why the black isoclinic is much clearer than the color fringe in the print. For such a comparison of the two lights, Fig. 7 in the paper would give the discusser all the information necessary.

The authors thank Professor Moore for his correction of footnote 3.

APM-54-11

The Center of Torsion for Angle and Channel Sections

By W. L. SCHWALBE,[1] URBANA, ILL.

The author presents a method of determining the position of the center of torsion in a channel section when the bar, held at one end, is subjected to a torsional couple. The case of the angle section is first investigated and an expression for the distance from the center line of leg to the center of torsion is determined. The case of the channel section is then analyzed by the same method.

FOR THE determination of longitudinal tensile and compressive stresses that exist in an I-beam or channel when the bar is subjected to a torsional couple only, with one cross-section of the bar remaining plane, that is, free from any torsional warping, it is necessary to know the location of the axis about which the bar twists. For sections such as the I-section with rectangular axes of symmetry, the axis of torsion coincides with the centroidal axis of the bar, but for sections with only one or no axis of symmetry this is not the case.

The terms "shear center"[2] and "center of twist"[3] have been used to denote the point in the plane of the cross-section of a beam through which the plane of the transverse bending loads must pass in order that no torsional effect be present. If the plane of the loads does not pass through this point, then the cross-section rotates with respect to this point and the bar is in combined bending and twist. A. Eggenschwyler[4] has given the formula $x_B = \frac{I_{xy}'h}{I_x}$ for the location of the shear center (Fig. 1); where I_{xy}' is the product of inertia of the half-section with respect to the x and y axes and I_x is the moment of inertia of the whole cross-section with respect to the x-axis. Similar formulas have been obtained by R. Maillart[5] and by C. Weber.[6]

A series of tests was made at the University of Illinois (reported in Bulletin 211 of the Experiment Station) in order to locate the shear center for channel-beam sections ranging in depth from 4 to 15 in. The arrangement of the beam is shown in Fig. 2. The method of locating the shear center is illustrated in Fig. 3.

TABLE 1 VALUES OF x_B

Size of channel	$h\frac{I'_{xy}}{I_x}$ t.d.	m.d.	$\frac{h}{2}\frac{x_c}{y_c}$ t.d.	m.d.	Experimental t.d.	m.d.
4 in.– 5.4 lb.	0.54	0.52	0.46	0.47	..	0.50
6 in.– 8.2 lb.	0.62	0.62	0.54	0.56	..	0.60
6 in.–15.3 lb. (ship)	1.26	1.16	1.14	1.05	..	1.10
6 in.–15.5 lb.	0.49	0.51	0.40	0.42	..	0.35
10 in.–15.3 lb.	0.79	0.81	0.69	0.70	..	0.70
15 in.–33.9 lb.	0.96	0.94	0.81	0.78	..	0.70

t.d., tabular dimensions. Carnegie Steel Handbook (1923).
m.d., measured dimensions.
All dimensions in inches.
Experimental values from Bulletin 211 Experiment Station, Univ. of Ill.

A comparison of values for x_B, as obtained by different methods, is given in Table 1. The method for determining the distance $x_B = \frac{h}{2}\frac{x_c}{y_c}$ (from the web center to the center of torsion) for the channel section when the bar is subjected to torsion only, is given in the following analysis.

ANGLE SECTIONS

The determination of the axis of torsion for channel shapes depends on the location of the axis of torsion for angle shapes, the channel-shaped cross-section being regarded as made up of angle elements. An angle bar, fixed at one end, is subjected at the free end to a torsional couple M (Fig. 4). The moment of the couple M is such as to rotate any cross-section of the bar at distance z from the fixed end, through a small angle θ about a center B (Fig. 5), without appreciably changing the shape of the cross-section. The whole angle section is considered to be made up of two rectangular elements L_1 and L_2, of lengths h and widths w. The rotation of the section as a whole with respect to center B may then be resolved into translations of the legs L_1 and L_2 and rotations of the legs about their respective centroidal centers c_1, c_2 (Fig. 6). In Fig. 6 the center lines only of the sections are shown. Now the translational displacement $\overline{c_1c_1}'$ of the leg L_1 from position I to position I', may be regarded as a bending deflection of the element since one end of the member remains fixed. Similarly, $\overline{c_2c_2}'$ is the bending deflection for L_2. After the bending deflections of the legs L_1 and L_2 have taken place, then the two elements are twisted with respect to centroidal centers c through angles θ until continuity is established at O and the final position of the section, as a whole, results. Longitudinal stresses due to torsion are negligible compared to the bending stresses. This picture of the action furnishes a basis for locating the center of torsion B for the section as a whole in the following way. For a small angle θ the deflection, arc $\widehat{c_1c_1}'$ (see Fig. 7), may be regarded as a straight line, hence the deflection curve of the leg L_1 lies in a plane. This plane may be taken as tangent to the deflection, arc $\widehat{c_1c_1}'$, at the point c_1. The neutral plane N_1 for the bending action of the rectangular section L_1 is perpendicular to the plane of the deflection curve and passes through the centroid c_1. Since the center of rotation B_1 for the arc $\widehat{c_1c}'$ lies on a line which is per-

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Mr. Schwalbe served as graduate research assistant in Mechanics to Prof. A. N. Talbot for two years at the University of Illinois, and received a M.S. degree in Theoretical and Applied Mechanics from the same university. He was engaged in municipal, electric railway, and building construction for a number of years in Milwaukee, Wis. He has been teaching mechanics at the University of Illinois for about ten years.

[2] "The Torsional Effect of Transverse Bending Loads on Channel Beams," Seely, Putnam, and Schwalbe, Bulletin 211, Engineering Experiment Station, University of Illinois (1930).

"Schubmittelpunkt," R. Maillart, *Schweiz. Bauz.*, vol. 76 (1920), p. 266.

[3] "Strength of Materials," S. Timoshenko, vol. 1, p. 194.

[4] *Schweiz. Bauz.*, vol. 83 (1924), p. 259.

[5] *Schweiz. Bauz.*, vol. 77 (1921), p. 196.

[6] *Zeit. für angew. Math. und Mech.*, vol. 4, 1924, p. 347.

Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

pendicular to the tangent of the arc at the point c_1 it follows that the point B lies on the neutral axis N_1. Similar considerations for the leg L_2 lead to the conclusion that B lies on N_2, the neutral axis for L_2. Hence B is determined by the intersection of N_1 and N_2 (Fig. 6).

In order to determine the neutral axis N_1 let the x and y components of the deflection $\overline{c_1c_1}'$ be ξ_1 and η_1, respectively. Then the bending stress in the section L_1 is

$$S_1 = \frac{M_{y1}x_1}{I_{y1}} + \frac{M_{x1}y_1}{I_{x1}} \dots\dots\dots\dots\dots [1]$$

where M_{x1} and M_{y1} are the x and y components of the bending moment in L_1 and I_{x1}, I_{y1} are the moments of inertia of the cross-sectional area with respect to the axes x_1 and y_1, respectively (Fig. 5).

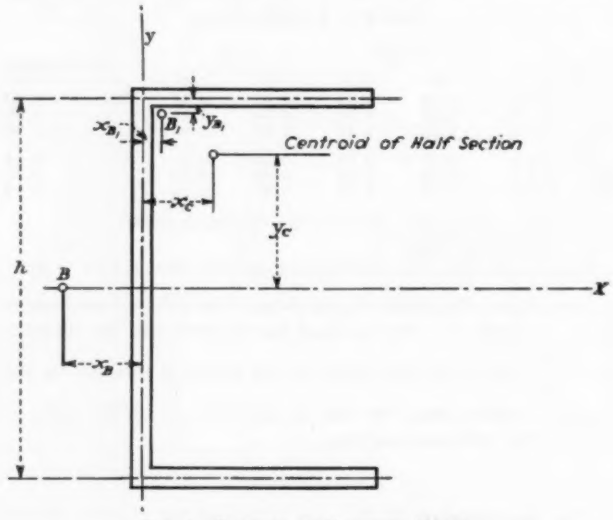
y
x_{B_1}
B_1
y_{B_1}
Centroid of Half Section
x_c
y_c
h
B
X
x_B

FIG. 1 SECTION OF CHANNEL BEAM

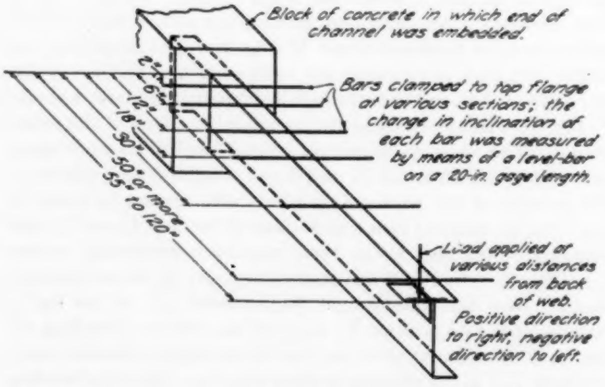
Block of concrete in which end of channel was embedded.
2"
6"
12"
18"
30"
50" or more
55" to 120"
Bars clamped to top flange at various sections; the change in inclination of each bar was measured by means of a level-bar on a 20-in. gage length.
Load applied at various distances from back of web. Positive direction to right, negative direction to left.

FIG. 2 ARRANGEMENT OF CHANNEL BEAM FOR TESTING

For the leg L_2 the bending stress is

$$S_2 = \frac{M_{y2}x_2}{I_{y2}} + \frac{M_{x2}y_2}{I_{x2}} \dots\dots\dots\dots\dots [2]$$

The equations of the neutral axes N_1 and N_2 are obtained from the stress Equations [1] and [2] by putting $S_1 = S_2 = 0$.

$$S_1 = \frac{M_{y1}x_1}{I_{y1}} + \frac{M_{x1}y_1}{I_{x1}} = 0 \dots\dots\dots\dots [3]$$

$$S_2 = \frac{M_{y2}x_2}{I_{y2}} + \frac{M_{x2}y_2}{I_{x2}} = 0 \dots\dots\dots\dots [4]$$

Subjecting Equation [3] to the transformation of coordinates $x_1 = x$, $y_1 = y - h_1/2$, then

$$\frac{M_{x1}}{I_{x1}}\left(y - \frac{h_1}{2}\right) + \frac{M_{y1}}{I_{y1}}\,x = 0 \dots\dots\dots\dots [5]$$

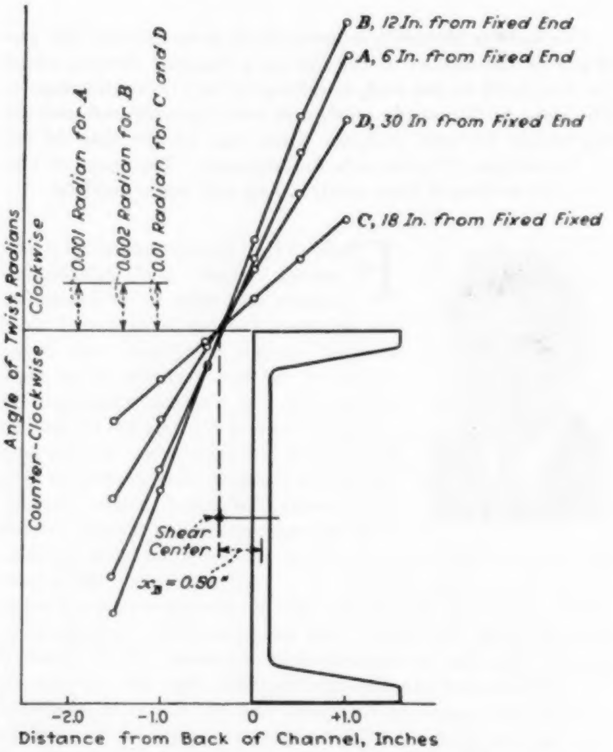

FIG. 3 SHEAR CENTER FOR 4-IN. 5.4-LB. CHANNEL TESTED AS A CANTILEVER WITH LENGTH OF 55 IN. AND LOAD OF 270 LB. AT END

Also, letting $x = x_2 + h_2/2$ and $y = y_2$, Equation [4] becomes, with reference to the origin of coordinates at O,

$$\frac{M_{x2}y}{I_{x2}} + \frac{M_{y2}}{I_{y2}}\left(x - \frac{h_2}{2}\right) = 0 \dots\dots\dots\dots [6]$$

The coordinates of the intersection of Equations [5] and [6] are then given by

$$\left.\begin{aligned} x_B &= \frac{\dfrac{h_1}{2} - \dfrac{h_2}{2}\,\dfrac{I_{x2}M_{y2}}{I_{y2}M_{x2}}}{\dfrac{I_{x1}}{I_{y1}}\,\dfrac{M_{y1}}{M_{x1}} - \dfrac{I_{x2}}{I_{y2}}\,\dfrac{M_{y2}}{M_{x2}}} \\ y_B &= \frac{h_1}{2} - \frac{I_{x1}}{I_{y1}}\,\frac{M_{y1}}{M_{x1}} \cdot x_B \end{aligned}\right\} \dots\dots\dots\dots [7]$$

The ratios of the bending moments in the expressions for x_B and y_B in terms of known quantities may be determined as follows. Since the two legs L_1 and L_2 are continuous at the point O, the bending stress in L_1 at O is equal to the bending stress in L_2 at the point O. This condition may be formulated as $S_{1,o} = S_{2\,o}$ or

$$\frac{M_x h_1}{I_{x1}} - \frac{M_{y2}h_2}{I_{y2}} = 0 \dots\dots\dots\dots\dots [8]$$

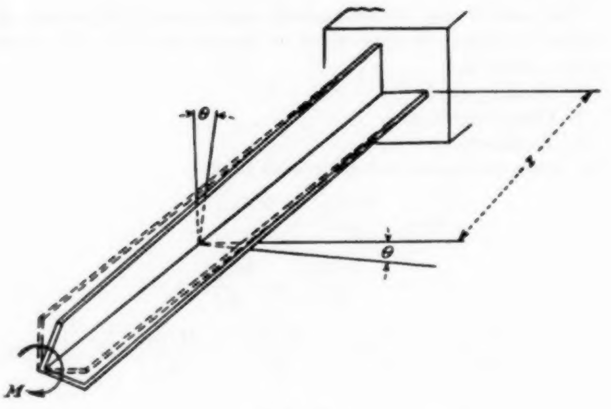

FIG. 4

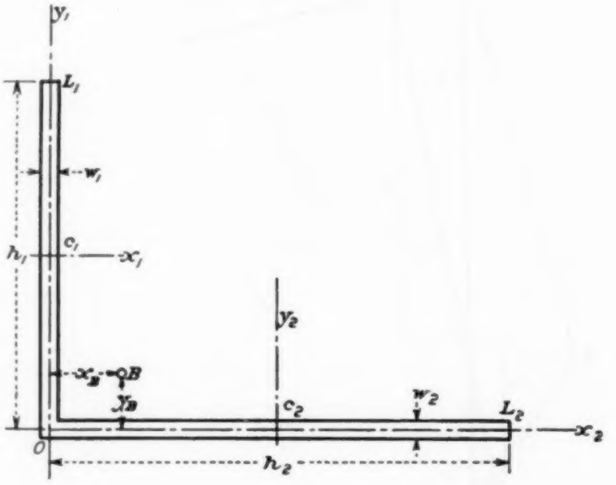
y_1
L_1
w_1
c_1
x_1
h_1
y_2
x_B
B
y_B
c_2
w_2
L_2
x_2
O
h_2

FIG. 5

Again, from the geometric continuity of the section at O, it follows that for a small angle θ (see Fig. 6),

$$\eta_0 = \eta_1 \text{ and } \xi_0 = \xi_2$$

also

$$\xi_1 = \frac{h_1}{2}\theta - \xi_2 \qquad \text{and} \qquad \eta_2 = \frac{h_2}{2}\theta - \eta_0$$

Combining the above expressions and eliminating θ, there results

$$\xi_1 + \xi_2 - \frac{h_1}{h_2}(\eta_1 + \eta_2) = 0 \ldots\ldots\ldots\ldots\ldots [9]$$

Since the deflections ξ and η are continuous functions of the distance z, measured along the axis of the bar, Equation [9] may be differentiated twice with respect to z. Then

$$\frac{d^2\xi_1}{dz^2} + \frac{d^2\xi_2}{dz^2} - \frac{h_1}{h_2}\left(\frac{d^2\eta_1}{dz^2} + \frac{d^2\eta_2}{dz^2}\right) = 0 \ldots\ldots\ldots [10]$$

Multiplying [10] by E, the modulus of elasticity of the material (assuming a homogeneous, isotropic material) and by making use of the bending-moment-curvature relation

$$E\frac{d^2(\;)}{dz^2} = \frac{M}{I}$$

there results

$$\frac{M_{y2}}{I_{y2}} + \frac{M_{y1}}{I_{y1}} - \frac{h_1}{h_2}\left(\frac{M_{y1}}{I_{x1}} + \frac{M_{x2}}{I_{x2}}\right) = 0 \ldots\ldots\ldots [11]$$

Since the whole member is under the action of torsional couples only, it is clear that the summation of the bending moments on legs L_1 and L_2 must equal zero. Hence

$$M_{x1} - M_{x2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots [12]$$

$$M_{y1} - M_{y2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots [13]$$

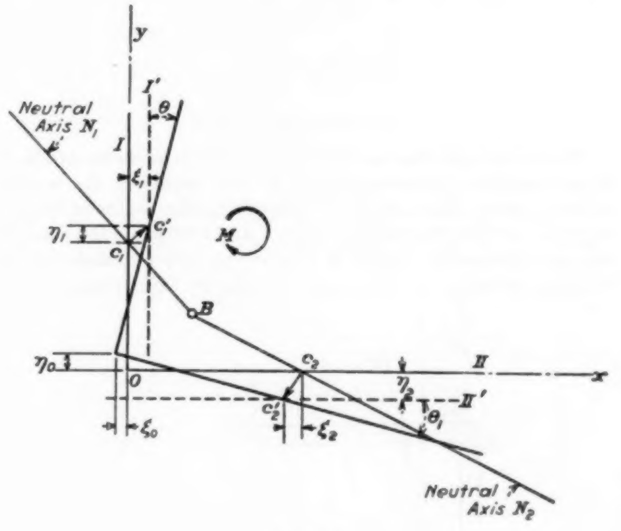
y
I'
θ
Neutral Axis N_1
I
ξ_1
η_1
c_1'
c_1
M
B
η_0
c_2
II
x
O
η_2
c_2'
II'
θ_1
ξ_0
ξ_2
Neutral Axis N_2

FIG. 6

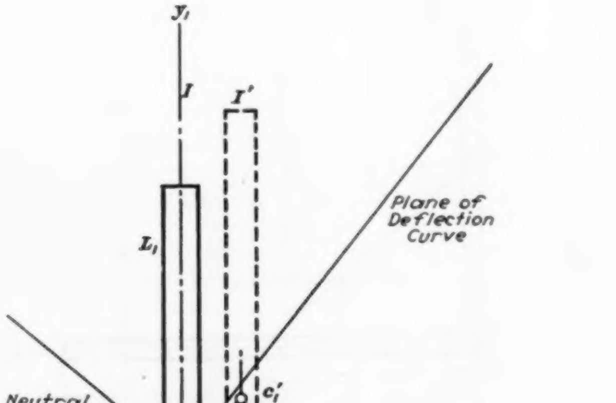
y_1
I
I'
L_1
Plane of Deflection Curve
Neutral Plane N_1
c_1'
c_1
x_1
B

FIG. 7

Equations [8], [11], [12], and [13] are compatible if

$$I_{y2}I_{x2} = I_{y1}I_{x1} \quad\ldots\ldots\ldots\ldots\ldots\ldots [14]$$

From Equations [8], [11], [12], and [13]

$$\frac{M_{y2}}{M_{x2}} = \frac{M_{y1}}{M_{x1}} = \frac{h_1}{h_2}\frac{I_{y2}}{I_{x2}}$$

Substituting in [7] and making use of [14] there results

$$\left.\begin{aligned} x_B &= \frac{h_2}{2}\frac{I_{y1}}{I_{y1}+I_{y2}} \\ y_B &= \frac{h_1}{2}\frac{I_{y1}}{I_{y1}+I_{y2}} \end{aligned}\right\} \quad\ldots\ldots\ldots\ldots\ldots [15]$$

CHANNEL SECTIONS

When a straight channel bar of cross-section as shown in Fig. 8 is subjected to a twisting couple M, one section in the bar remaining plane, the method for determining the center of torsion is similar to that given for the angle. The rotation of the channel section about some center B is resolved into a translation or bending deflection of the angle A_1 (Fig. 9) from position I to position I' and a rotation of this angle about its center of torsion B_1, to the final position; a translation of the angle A_2 from position II to position II′ and a rotation of A_2 about its center of torsion B_2 to the final position, continuity being established at O.

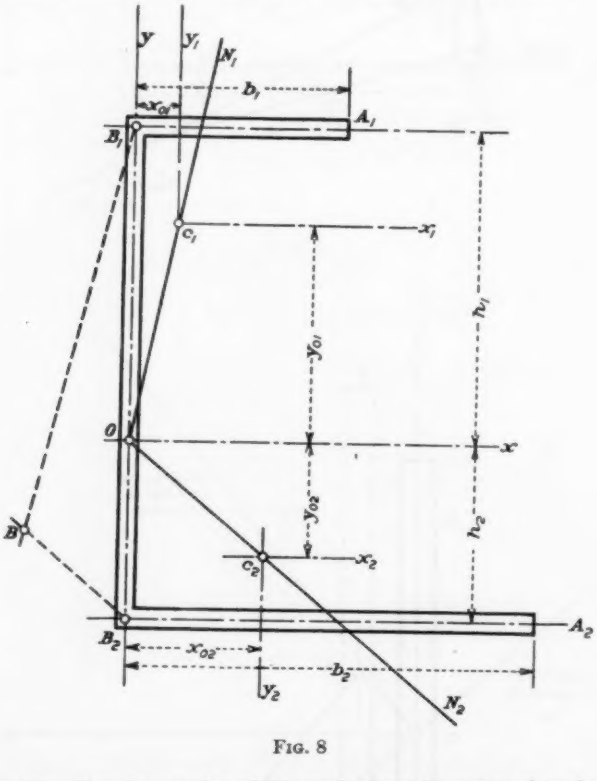

FIG. 8

Since the bending deflections are perpendicular to the neutral bending axes, it is only necessary to determine the intersection of lines drawn through B_1 and B_2, parallel to the neutral axes N_1 and N_2, in order to locate the center B.

The neutral axes N_1 and N_2 are determined if the point O is known, where O is that point on the center line of the web of the channel which has no longitudinal or bending stress; hence it is a point on both neutral axes. If O is known, then the angles A_1 and A_2 with the centroids c_1 and c_2 are determined.

The point O may be determined approximately by satisfying similar conditions as those given in the discussion for the angle alone. That is,

1 Bending stress $S_{1,o} = S_{2,o} = 0$
2 Continuity at O
3 Equilibrium.

The stress conditions may be written thus

$$S_{1.o} = \frac{M_1{}^h x_{o1}}{R_1{}^h} - \frac{M_1{}^v y_{o1}}{R_1{}^v} = 0 \quad\ldots\ldots\ldots\ldots [16]$$

$$S_{2.o} = -\frac{M_2{}^h x_{o2}}{R_2{}^h} + \frac{M_2{}^v y_{o2}}{R_2{}^v} = 0 \quad\ldots\ldots\ldots\ldots [17]$$

$$\frac{M_1{}^h x_{o1}}{R_1{}^h} - \frac{M_1{}^v y_{o1}}{R_1{}^v} + \frac{M_2{}^h x_{o2}}{R_2{}^h} - \frac{M_2{}^v y_{o2}}{R_2{}^v} = 0 \quad\ldots\ldots [18]$$

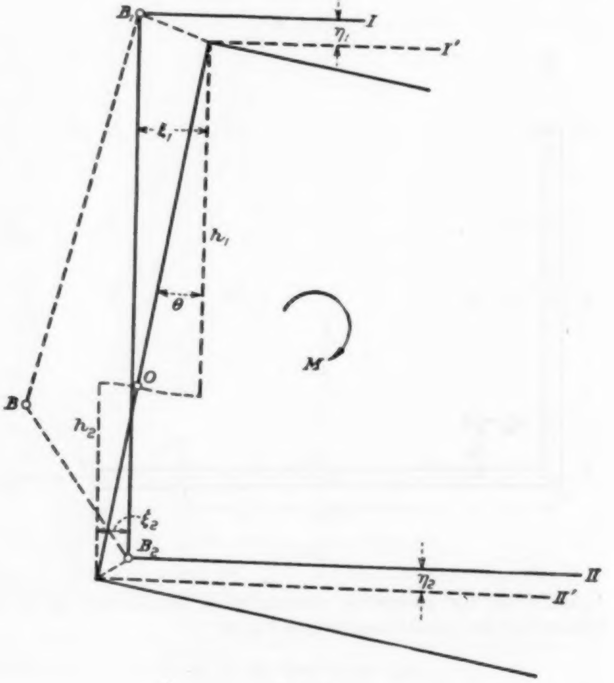

FIG. 9

where M^h is the bending moment for which the horizontal deflection ξ is the effect, and M^v is the bending moment for which the vertical deflection η is the effect.

$$R^h = \sqrt{I_y{}^2 + I_{xy}{}^2} \quad \text{and} \quad R^v = \sqrt{I_x{}^2 + I_{xy}{}^2}$$

where I_x, I_y and I_{xy} are the moments and product of inertia of the angle section with respect to x-y-axes with the origin of coordinates at the centroid c of the angle section. The subscripts 1 and 2 refer to the angles A_1 and A_2 with origins at c_1 and c_2, respectively.

The continuity equations are obtained from the following conditions which are evident from Fig. 9:

$$\eta_1 = \eta_2$$

$$\frac{h_1}{h_2}\xi_2 - \xi_1 = \left(\frac{h_2}{h_1} + 1\right)\xi_o$$

ξ_o is the horizontal displacement of the point O. For a small angle of rotation θ, ξ_o may be neglected, hence

$$\frac{h_1}{h_2}\xi_2 - \xi_1 = 0$$

The equations which express continuity then are

$$\frac{M_1^v}{R_1^v} - \frac{M_2^v}{R_2^v} = 0 \qquad [19]$$

$$-\frac{M_1^h}{R_1^h} + \frac{h_1}{h_2}\frac{M_2^h}{R_2^h} = 0 \qquad [20]$$

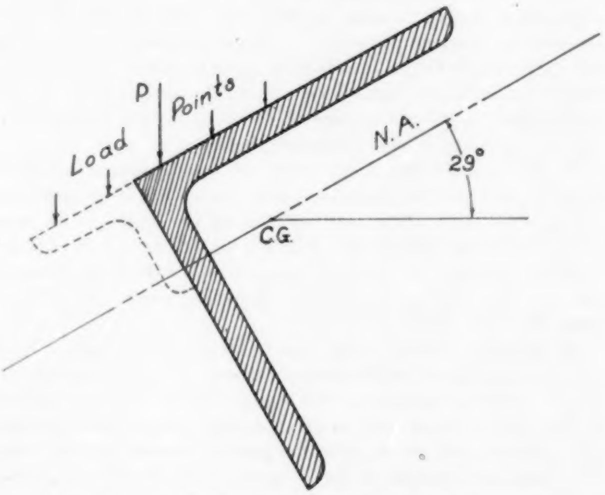

FIG. 10 ARRANGEMENT OF ANGLE BEAM FOR TESTING

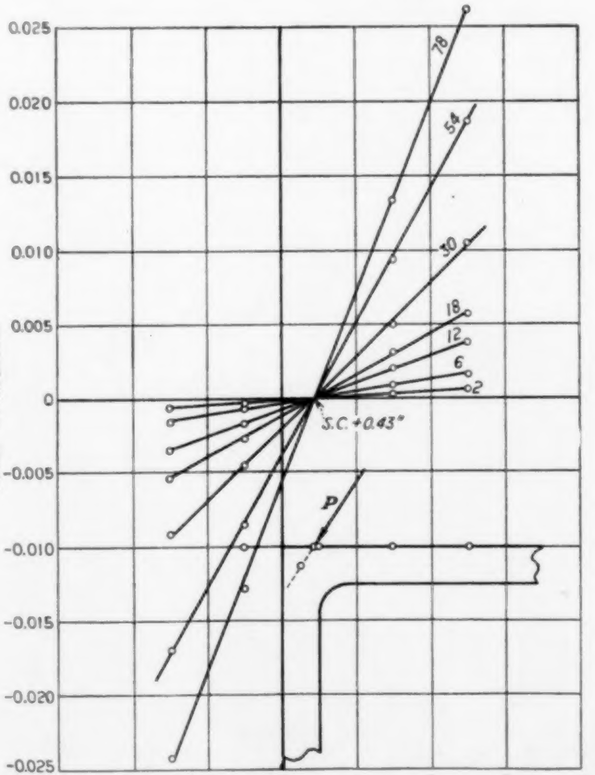

FIG. 11 LOCATION OF SHEAR CENTER OF 6-IN. BY 6-IN. BY $^1/_2$-IN. ANGLE BEAM SET AT 29 DEG.

The condition of equilibrium for the section as a whole is: $M_1 + M_2 = 0$. In terms of the x and y components of M_1 and M_2 the equations of equilibrium become

$$-\frac{I_{(xy)1}}{R_1^h} M_1^h + \frac{I_{x1}}{R_1^v} M_1^v - \frac{I_{(xy)2}}{R_2^h} M_2^h + \frac{I_{x2}}{R_2^v} M_2^v = 0 \qquad [21]$$

$$\frac{I_{y1}}{R_1^h} M_1^h - \frac{I_{(xy)1}}{R_1^v} M_1^v - \frac{I_{y2}}{R_2^h} M_2^h + \frac{I_{(xy)2}}{R_2^v} M_2^v = 0 \qquad [22]$$

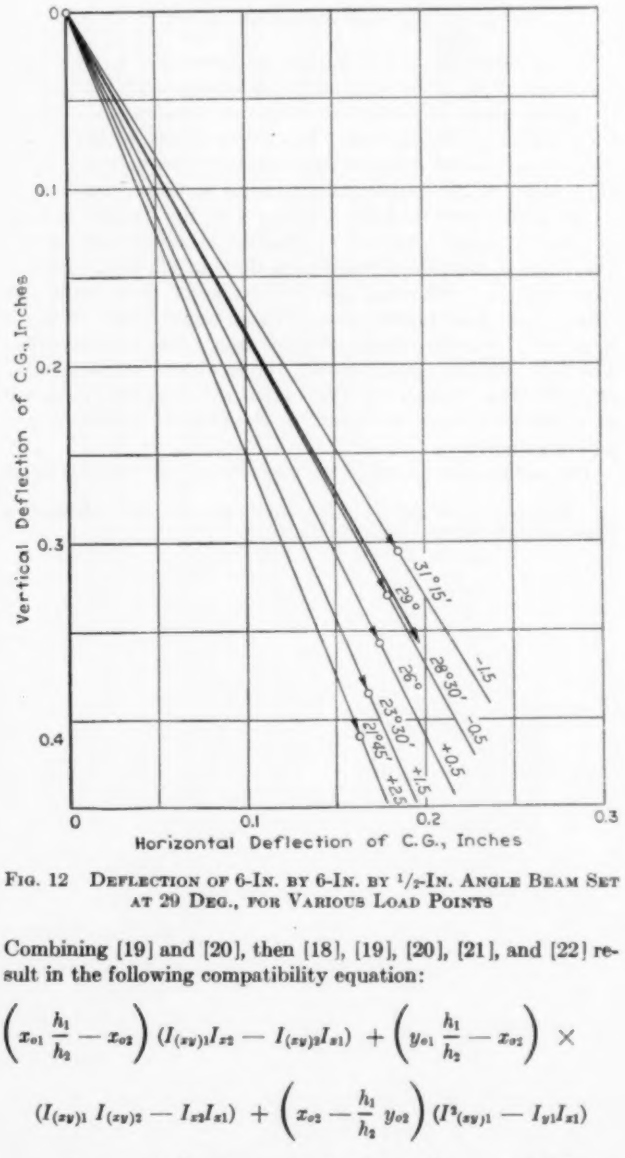

FIG. 12 DEFLECTION OF 6-IN. BY 6-IN. BY $^1/_2$-IN. ANGLE BEAM SET AT 29 DEG., FOR VARIOUS LOAD POINTS

Combining [19] and [20], then [18], [19], [20], [21], and [22] result in the following compatibility equation:

$$\left(x_{o1}\frac{h_1}{h_2} - x_{o2}\right)(I_{(xy)1}I_{x2} - I_{(xy)2}I_{x1}) + \left(y_{o1}\frac{h_1}{h_2} - x_{o2}\right) \times$$

$$(I_{(xy)1}\,I_{(xy)2} - I_{x2}I_{x1}) + \left(x_{o2} - \frac{h_1}{h_2}\,y_{o2}\right)(I^2_{(xy)1} - I_{y1}I_{x1})$$

$$= (y_{o1} - y_{o2})\,(I_{(xy)1}I_{y2} - I_{y1}I_{(xy)2}) - (x_{o1} - y_{o1})\,(I^2_{(xy)2}$$

$$- I_{x2}I_{y2}) - (y_{o1} - y_{o2})\,(I_{x1}I_{y2} - I_{(yx)1}I_{(xy)2}) \qquad [23]$$

Equation [23] is then solved for h_1/h_2 in order to locate O. For any special case the solution is easily found by successive approximations.

Where the cross-section has an axis of symmetry as in the usual rolled-steel channel, Equation [23] results in $h_1/h_2 = 1$. The center of torsion B then lies on the axis of symmetry and at a distance x_B from the web center (Fig. 1). Making use of Equation [15] the value of x_B is found to be

$$x_B = \frac{h}{2}\frac{x_c}{y_c} - \left(x_{B1} + y_{B1}\frac{x_c}{y_c}\right) \qquad [24]$$

For sections of small width w, x_{B_1} and y_{B_1} for the angle section may be put equal to zero, hence

$$x_B = \frac{h}{2}\frac{x_c}{y_c} \qquad [25]$$

Discussion

Wm. J. Putnam.[7] The author has presented a very clear derivation of a usable equation for determining the location of the shear center or center of torsion for unsymmetrical structural shapes of thin section. The writer simply wishes to add some corroborative evidence as to the correctness of the equations given in their application to angle sections.

The author cites in Table 1 values of x_B for channel sections of various weights and sizes, obtained experimentally at the University of Illinois and reported in Bulletin 211 of the Experiment Station. Following the publication of those data, the writer made some further tests using an angle section, 6 in. × 6 in. × $^1/_2$ in., 8 ft. long, embedded in a 12-in. concrete cube. The cube was then held between the head and weighing table of a 200,000-lb. testing machine so that the angle iron projected as a cantilever beam as shown for the channel sections in the author's Fig. 2.

The section was placed in the concrete so that the legs made an angle of 29 deg. to the horizontal and vertical, as shown in Fig. 10, in order that the neutral axis, which is perpendicular to the plane of the deflection, would be parallel to one leg. The load consisted of a weight hung on the free end of the beam in various positions across the top flange or that plane extended.

The angle of twist for each position of the load was measured at seven sections along the beam by means of a level bar on a 20-in. gage length reading to 0.001 in. The results of these readings are shown in Fig. 11, by which the position of the load for no twist is found to be at 0.43 in. from the corner of the angle. Since the angle has equal legs, the shear center will be at the intersection of the action line of the load in this position and the bisector of the angle between the legs.

In this manner the shear center is found experimentally to be at a point 0.27 in. from the back of each leg. Applying the author's Equation [15] and considering the angle to be composed of two rectangles each 5.75 in. by 0.5 in., x_B and y_B are found to be 0.022 in. from the intersection of the center lines of the two legs, which means that the shear center is 0.272 in. from the back of each leg.

Incidentally, while taking the observations on twist, readings were also taken of the horizontal and vertical displacements of the center of gravity of the angle. These data are plotted in Fig. 12 and show that when the load was applied through shear center with the angle in the position stated, the deflection was practically normal to the computed direction of the neutral axis.

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APM-54-12

A New Type of Dynamic-Balancing Machine

By ERNEST L. THEARLE,[1] SCHENECTADY, N. Y.

In this paper is described a novel type of dynamic-balancing machine developed for production balancing. The machine employs an automatic compensating device, making it possible for an unskilled operator to obtain accurate readings of amount and position of unbalance correction with great rapidity. The theory underlying the action of the machine is given and the characteristics of the automatic balancing head are investigated. The method of operating the machine is explained and data on the accuracy of balance obtainable are included.

During the past few years the manufacturers and users of many kinds of machinery have become greatly concerned with the problem of vibration elimination. In general the solution of this problem lies in (1) proportioning the parts of the machine to avoid any conditions of resonance, and (2) reducing to a minimum the forces tending to excite vibration. We are here concerned with the practical accomplishment of this latter task by means of the dynamic balancing of rotating parts.

The modern tendency toward more complete motorization, where a number of electric motors may be mounted on and directly connected to a machine, has led the manufacturer of that machine to pass at least a part of the responsibility for the smooth running of his product to the motor manufacturer. Consequently, the motor manufacturer is demanding dynamic-balancing equipment with which unskilled operators can quickly and accurately balance motor rotors. An effort to satisfy this demand has resulted in the dynamic-balancing machine described in this paper.

The Rotor-Suspension System

It may readily be shown, both analytically and experimentally, that any substantially rigid rotating body can be correctly and completely balanced by the addition (or removal) of two weights, one in each of two arbitrarily chosen transverse planes of the body. In general, two weights are both necessary and sufficient to obtain a correct dynamic balance. The two transverse planes in which the balancing weights are applied (or removed) will be known as the "planes of correction." These planes of correction and the radii at which the balance corrections are applied may be chosen arbitrarily, or to suit the convenience of attachment or removal of weight. The function of the balancing machine is thus to determine four quantities which are the amounts of the two correction weights and their angular positions relative to the rotor to be balanced.

Consider a rotor mounted as shown diagrammatically in Fig. 1, where z is the axis of rotation, S an elastic supporting member such as a spring, and P a pivot permitting angular oscillation of the rotor in a vertical plane about the horizontal axis O. B and C are the chosen balance-correction planes. When rotating about the axis z, such a rotor in a general state of unbalance will oscillate in a vertical plane about the axis O of the pivot.

It will now be possible, by any one of a number of known methods, to determine a balance-correction weight W_1 which, when applied to the rotor in the correction plane C and in the proper angular position, will suppress the previously observed oscillations. When this balance correction is accomplished, the rotor will run smoothly in the support shown in Fig. 1. This complete absence of oscillation indicates that the application of weight W_1 has so altered the distribution of rotating mass that the resultant of all remaining unbalanced forces has no moment about the pivotal axis O and is therefore a single force lying in the transverse plane A containing the pivotal axis.

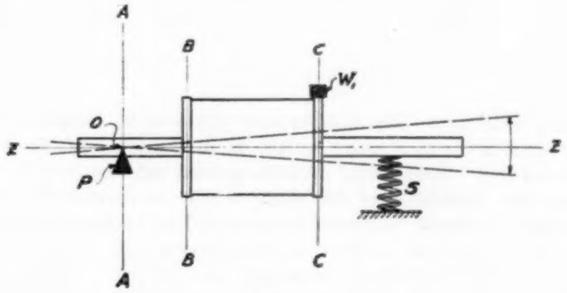

Fig. 1

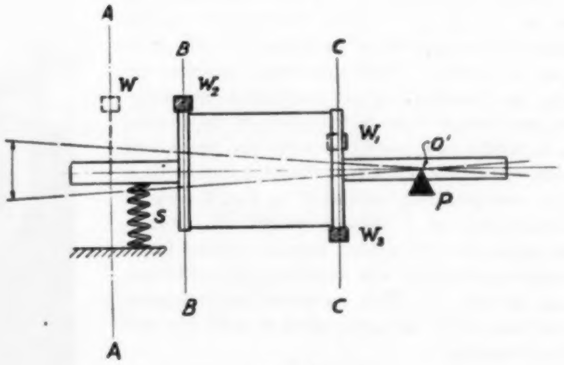

Fig. 2

If the rotor is now mounted as shown in Fig. 2 with the pivotal axis displaced to a new position O', a single weight W can be found which will complete the balance if applied to the rotor at the correct angular position in the first pivotal plane A. Since, as shown, in Figs. 1 and 2, it is impracticable to attach a permanent correction weight to the rotor in the plane A, this hypothetical weight W must be replaced by its equivalent, composed of the two weights W_2 and W_3 in the chosen correction planes B and C, respectively. This method is frequently undesirable, since it requires an application to the rotor of a total of three balancing weights where only two are necessary for a correct balance.

As shown in the process just described, after applying the first balance-correction weight W_1 in the correction plane C, the remaining unbalance is a single force lying in the plane of the

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[1] Research Engineer, General Electric Co. Assoc-Mem. A.S.M.E. Mr. Thearle was graduated from Cornell University, where he served as instructor of applied mechanics from 1920 until 1925. He was professor of mechanical engineering, head of the department, at the University of Arkansas from 1925 until 1928, when he became associated with the Research Laboratory of the General Electric Company.

Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

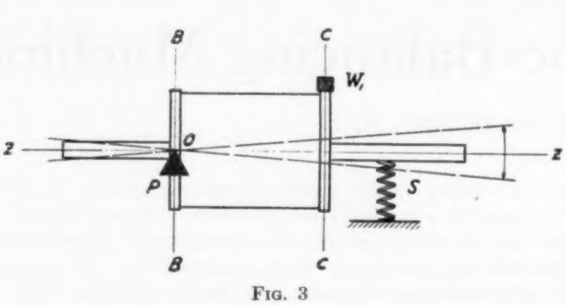

FIG. 3

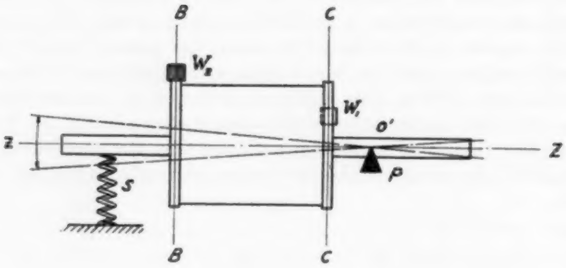

FIG. 4

first pivotal axis. The balance may therefore be completed by the application of a single weight W in the plane of the first pivotal axis. Lawaczeck,[3] in 1908, pointed out that in order to make the application of this single weight convenient, the first pivotal axis should be placed in the second correction plane B.

The rotor to be balanced is now supported as shown in Fig. 3 with the pivot or fulcrum P placed at O, in the second correction plane B. A weight W_1 can be determined which, when applied to the rotor in the plane C and in the correct angular position, will suppress all oscillation of the rotor about the axis O. When this weight has been applied, the resultant of all remaining unbalance must be a single force in the plane of the pivotal axis O, which now coincides with the previously chosen correction plane B. Moving the pivot P to any new position, such as O' in Fig. 4, permits determination of a second weight W_2, which, when applied at the proper angular position in the correction plane B, will suppress all oscillations about the axis O'. This completes the balancing of the rotor with the application of only two correction weights.

The structure supporting the rotor while carrying out the above-described balancing process is shown in Fig. 5. The journals of the rotor rest in half-bearings D mounted in a light but rigid cradle consisting of two longitudinal tubes E clamped in the two cross-members F. The size and spacing of bearings may readily be changed to suit various rotor sizes.

The cradle (with rotor) is carried by four vertical springs and four horizontal springs shown at G and H, respectively, in Fig. 5. Two pairs of flat fulcrum springs, shown at J and K, are clamped to the cradle tubes and project downward between the jaws of fulcrum vises shown at L and M, the latter being rigidly fixed to the frame of the machine. Either pair of fulcrum springs, with their corresponding vises, can be adjusted into any desired transverse plane of the rotor to be balanced. The pivotal axis of the cradle and rotor may be shifted quickly from one transverse plane of the rotor to another by opening the first pair of vises and closing the second pair, thus causing the first pair of fulcrum springs to become inactive and the second pair active. The rotor is driven at constant speed by means of a light and uniform belt from a driving motor.

Consider now the form of restraint imposed upon the cradle and object to be balanced when either pair of fulcrum springs is clamped in its corresponding pair of vises, the other vises being open. Assuming the vises shown at M, Fig. 5, to be open, the fulcrum springs K are inactive and play no part in restraining the cradle. With vises L closed, the lower ends of the fulcrum springs J are rigidly clamped to the frame of the machine. The point O, at the center of the rotor shaft and in the plane of fulcrum springs J, is thus held in a fixed position. Flexure of the fulcrum springs J permits a horizontal oscillation of the cradle and rotor about the vertical axis y and a vertical oscillation about the horizontal axis x. The stiffness of the horizontal cradle springs H is adjusted so that the natural frequency of horizontal oscillation of the cradle and rotor is equal to its natural frequency of vertical oscillation.

For the purpose of analysis, the rotor to be balanced and its supporting cradle, mounted thus, may be replaced by the approximately equivalent system shown in Fig. 6. Since any general state of unbalance can be corrected by the application of two weights, that unbalance may be represented by the same two weights applied in the chosen correction planes, as at u and w in Fig. 6. Here the pivot point O is placed in the plane of un-

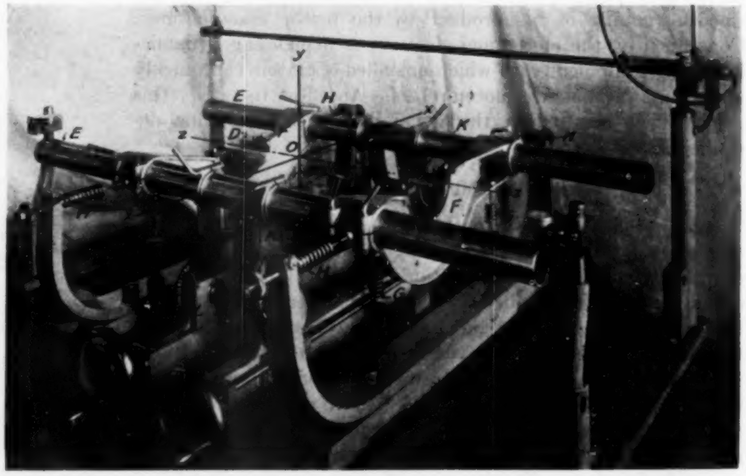

FIG. 5 CRADLE OF 50- TO 250-LB. DYNAMIC-BALANCING MACHINE

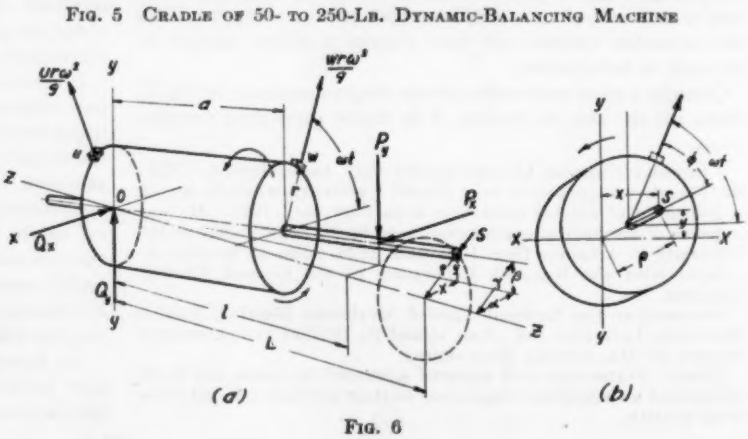

FIG. 6

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[3] See German Patent No. 201,787.

balance u. The unbalance weights u and w are not considered as being a part of the rotor; their respective actions on the rotor are shown, instead, as external forces. The rotor in Fig. 6 is shown in a general position, displaced from its mean or rest position by rotation through an angle α horizontally about the y-axis and an angle β vertically about the x-axis. Its speed of rotation is assumed constant. Our interest lies in the motion of the point S at the center of the shaft, the position of which is described by the coordinates $x = L\alpha$ and $y = L\beta$.

Let

b = a damping factor
g = acceleration due to gravity
I = moment of inertia of rotor and cradle about the axis x or the axis y, assumed to have the same value about any diametral axis through the point O
J = moment of inertia of the rotor about its axis of rotation or shaft center line
k = stiffness factor or spring constant of the equivalent horizontal and vertical cradle-supporting springs, which are assumed adjusted to equal stiffness
u, w = unbalance weights as shown in Fig. 6
ω = rotative speed of rotor, in radians per second

All external forces exerted on the cradle and rotor, except that of gravity, are represented by the vectors appearing in Fig. 6(a). The force of gravity is omitted, as a simplification, since it can be shown to have no influence on the motion of the rotor. Vectors $ur\omega^2/g$ and $wr\omega^2/g$ represent the forces exerted on the rotor by the unbalance weights u and w. This latter expression $wr\omega^2/g$ is approximate; the additional components of this force due to oscillation of the rotor about the point O may be neglected. The reaction at the pivot point O is indicated by vectors Q_x and Q_y. Vectors P_x and P_y represent the resultant of all spring reactions and include all damping or friction forces. The magnitudes of these vectors may therefore be expressed by

$$P_x = kl\alpha + bl\frac{d\alpha}{dt} \qquad [1]$$

and

$$P_y = kl\beta + bl\frac{d\beta}{dt} \qquad [2]$$

Referring now to Fig. 7, the vectors $Id\alpha/dt$, $Id\beta/dt$, and $J\omega$ represent the three components of angular momentum of the rotor and cradle, with respect to their corresponding axes, when the rotative axis of the rotor lies along the line O-n, displaced from the z-axis by the angles α and β. As motion of the rotor continues to any new position such as O-m, with the angular displacement $\alpha + \Delta\alpha$ and $\beta + \Delta\beta$, changes in its angular momentum take place which are represented by the vectors $J\omega\Delta\alpha$, $J\omega\Delta\beta$, $I\Delta(d\alpha/dt)$, and $I\Delta(d\beta/dt)$. The *rates of change* of angular momentum, in the same directions as the *changes*, are shown in Fig. 8 by the vectors $J\omega d\alpha/dt$, $J\omega d\beta/dt$, $Id^2\alpha/dt^2$, and $Id^2\beta/dt^2$.

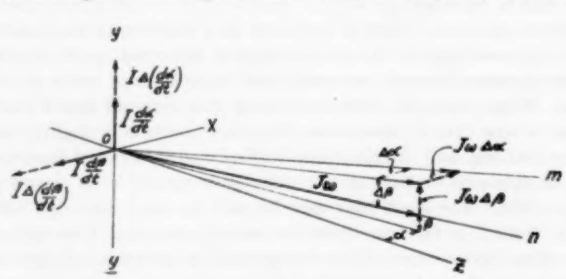

FIG. 7

The sum of the moments of all external forces acting on the rotor, as shown in Fig. 6, about either of the axes, x or y, may be equated to the rate of change of angular momentum with respect to that axis as shown in Fig. 8.[3] It must here be borne in mind that the vectors shown in Fig. 6 are *force* vectors, while those in Fig. 8 are *couple* vectors. Thus

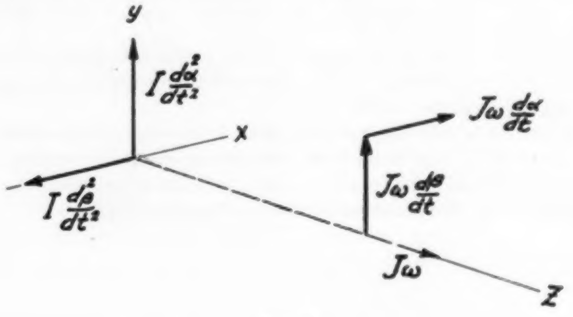
y
$I\frac{d^2\alpha}{dt^2}$
X
$J\omega\frac{d\alpha}{dt}$
$I\frac{d^2\beta}{dt^2}$
$J\omega\frac{d\beta}{dt}$
$J\omega$
Z

FIG. 8

$$I\frac{d^2\beta}{dt^2} - J\omega\frac{d\alpha}{dt} = \frac{awr\omega^2}{g}\sin\omega t - lP_y \qquad [3]$$

and

$$I\frac{d^2\alpha}{dt^2} + J\omega\frac{d\beta}{dt} = \frac{awr\omega^2}{g}\cos\omega t - lP_x \qquad [4]$$

Substituting [1] and [2] for P_x and P_y in [3] and [4] and rearranging,

$$\frac{d^2\beta}{dt^2} + \frac{bl^2}{I}\frac{d\beta}{dt} + \frac{kl^2}{I}\beta - \frac{J\omega}{I}\frac{d\alpha}{dt} = \frac{awr\omega^2}{gI}\sin\omega t \qquad [5]$$

$$\frac{d^2\alpha}{dt^2} + \frac{bl^2}{I}\frac{d\alpha}{dt} + \frac{kl^2}{I}\alpha + \frac{J\omega}{I}\frac{d\beta}{dt} = \frac{awr\omega^2}{gI}\cos\omega t \qquad [6]$$

Considering only forced oscillations of the system, the solution of this pair of simultaneous equations is

$$\alpha = A\cos\omega t + B\sin\omega t \qquad [7]$$

$$\beta = A\sin\omega t - B\cos\omega t \qquad [8]$$

where

$$A = \frac{awr[kl^2 - (I - J)\omega^2]\omega^2}{g([kl^2 - (I - J)\omega^2]^2 + l^4b^2\omega^2)} \qquad [9]$$

and

$$B = \frac{awrl^2b\omega^3}{g([kl^2 - (I - J)\omega^2]^2 + l^4b^2\omega^2)} \qquad [10]$$

The resultant angular displacement γ of the rotor shaft from its rest position, at any instant of time, is given by

$$\gamma = \sqrt{\alpha^2 + \beta^2} = \sqrt{A^2 + B^2} \qquad [11]$$

Substituting [9] and [10] in [11],

$$\gamma = \frac{awr\omega^2}{g\sqrt{[kl^2 - (I - J)\omega^2]^2 + l^4b^2\omega^2}} \qquad [12]$$

Since γ is constant with respect to time, the axis of the rotor shaft describes a right circular cone, and any point at the center of the rotor shaft moves in a circular path. The point S thus moves in a circular path of radius ρ, as shown in Fig. 6, where

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[3] See "Textbook of Mechanics," by E. H. Wood, 1926 ed., sec. 226.

$$\rho = L\gamma \qquad [13]$$

or

$$\rho = \frac{awrL\omega^2}{g\sqrt{[kl^2 - (I - J)\omega^2]^2 + l^4b^2\omega^2}} \qquad [14]$$

This radius ρ will be used here as the amplitude of vibration, since it is proportional to the observed amplitude of vibration of any point on the cradle.

As shown in Fig. 6(*b*), an end view of the rotor, displacement of the point S lags behind the disturbing unbalance weight w by the angle ϕ. Also, $\sin \omega t = \sin \phi$ and $\cos \omega t = \cos \phi$ when γ and $\beta = 0$. Substitution of these in Equation [8] gives

$$\tan \phi = \frac{B}{A} \qquad [15]$$

$$\tan \phi = \frac{l^2b\omega}{[kl^2 - (I - J)\omega^2]} \qquad [16]$$

The resonant speed of the system is here defined as the rotor speed at which the amplitude of cradle vibration is a maximum, with no damping present. When the damping constant b is zero, expression [14] for amplitude obviously becomes a maximum when its denominator vanishes. Imposing this condition gives the resonant speed

$$\omega_r = \sqrt{\frac{kl^2}{(I - J)}} \qquad [17]$$

It may here be pointed out that, since the pivot point O was chosen to lie in the transverse plane containing the unbalance weight u, the above expressions describing the rotor motion are independent of this weight and may therefore serve as a measure of the unbalance weight w alone.

Using data taken from a small balancing machine, carrying a rotor weighing about 34 lb., values of the phase angle ϕ and amplitude ρ were calculated from Equations [16] and [14] and plotted against rotor speed.

The resulting curves, Fig. 9, show the familiar amplitude vs. speed and phase angle vs. speed relations for a system such as this. The amplitude ρ practically reaches its maximum value at the resonant speed of about 1055 r.p.m. and, for the same number of r.p.m. away from this speed, is greater when above the resonant speed than when below.

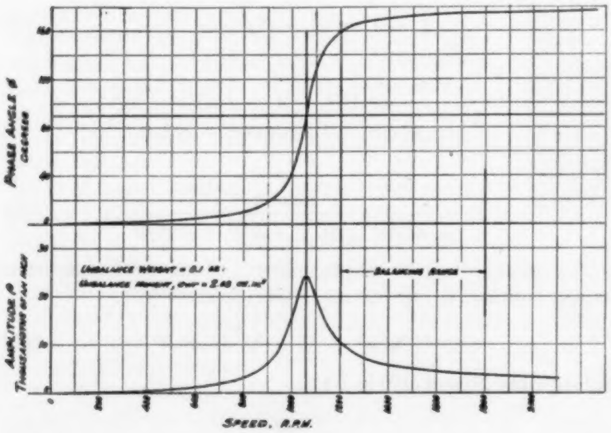

FIG. 9

What is most important to us here is the relation between the phase angle ϕ and speed, as shown by the upper curve of Fig. 9. Since, at constant speed, both the radial displacement ρ of the point S and the phase angle ϕ are constant with respect to time, the motion of the rotor is pure rotation about the axis z. When the speed of rotation of the rotor is somewhat below the resonant speed, the phase angle ϕ is small, hence the displacement of the shaft is in the general direction of the unbalance w. An instantaneous end view of the rotor would appear as shown in Fig. 10(*a*) where its motion is pure rotation about the axis z. The shaft may be said to whirl with its "heavy side" farthest from the axis of rotation.

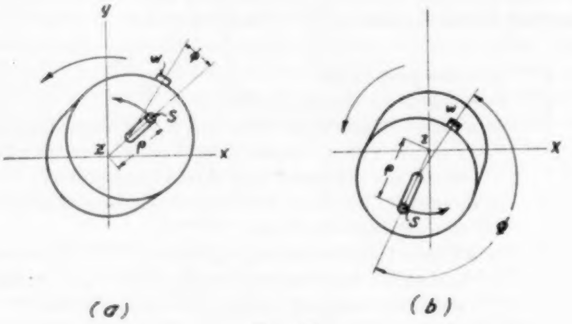

FIG. 10

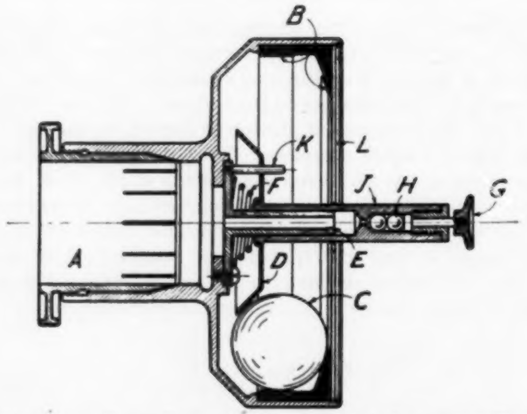

FIG. 11 AXIAL CROSS-SECTION OF ONE TYPE OF AUTOMATIC BALANCING HEAD SUITED TO THE BALANCING OF ELECTRIC-MOTOR ROTORS

When the rotor speed is increased to a value somewhat above the resonant speed, the phase angle ϕ increases quite rapidly when passing through resonance and approaches a value of 180 deg. Thus, when the rotor is running at a constant speed somewhat above that of resonance, the phase angle ϕ is slightly less than 180 deg. and the shaft end is displaced in a radial direction about opposite to that of the unbalance weight w as shown in Fig. 10(*b*). The shaft may now be said to whirl with its "light side" "out," or farthest from the axis of rotation. This motion provides forces which serve to operate an automatic balancing device now to be described.

THE AUTOMATIC BALANCING HEAD

Fig. 11 shows an axial cross-section of one type of automatic balancing head suited to the balancing of electric-motor rotors. This type of head is carried on the end of the rotor shaft, where it is clamped and centered by means of the split-collet chuck A. Pressed into the face of the head is a carefully hardened and ground race B, in which are placed two steel balls C of equal size. These balls are clamped in any position in their race by the conical clutch D, carried by the central staff E, and held against them by the spring F. The clutch is actuated by finger

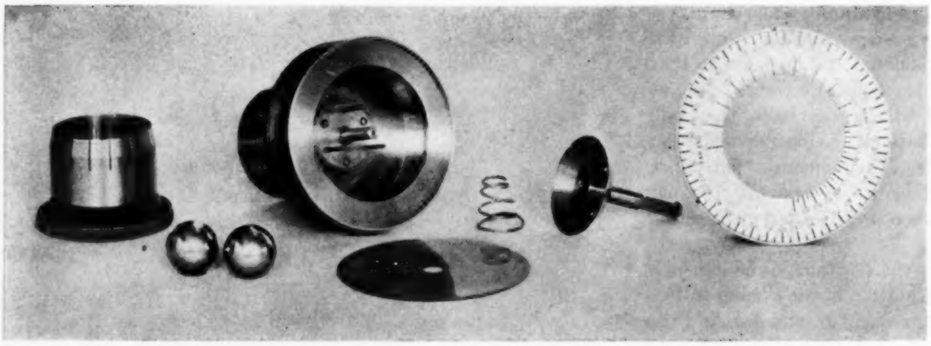

FIG. 12 DISASSEMBLED AUTOMATIC BALANCING HEAD AND SCALE FOR A 1.875-IN. SHAFT

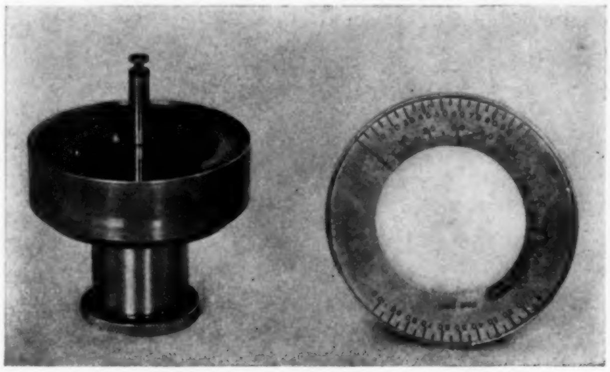

FIG. 13 AUTOMATIC BALANCING HEAD AND SCALE FOR A 1-IN. SHAFT

pressure on the button G, bearing against a ball thrust bearing H in the outer end of clutch sleeve J. Three pins K prevent relative rotation of clutch to ball race. A transparent window L serves to exclude foreign matter.

Fig. 12 shows a balancing head disassembled; a smaller head is shown assembled in Fig. 13. The dials shown, for measuring and bisecting the angle between the balls, are cupped to slip over the outer periphery of the head when needed.

The two steel balls in the balancing head provide the two variables necessary and sufficient to satisfy any condition of unbalance required of the head, within its capacity, when used in conjunction with a pivoted rotor. Thus the head, at any one speed of rotation, may introduce to the rotating system a force, in its plane, in any radial direction, and of any amount varying from zero when the balls are diametrically opposite each other to the full capacity of the head when they are in contact.

Consider this balancing head to be rigidly fixed to the end of the rotor shaft shown in Fig. 6, concentric with the shaft and in the radial plane of the point S. Since we are dealing here with a system pivoted at a point O, the effect of any unbalance weight such as w (Fig. 6) on the system will be proportional to the product of the "unbalance" (wr) and its moment arm (a). In the text following, such a product (wra) will be referred to as a "moment of unbalance."

An axial view of the rotor and head will appear as shown in Fig. 14 where the balls C are shown in any general positions within their race B. The moment of unbalance in the rotor, combined with that introduced by the balls C, is represented by the resultant unbalance vector U. If the speed of rotation of the rotor is somewhat above that of resonance, the phase angle ϕ will be slightly less than 180 deg., and the axis of rotation of the system shown in Fig. 14 will be at Q, with the point S, at the end of the rotor shaft and center of the race, moving in a circle of radius ρ.

Considering its motion and equilibrium to be *relative to the race*, each ball must be treated as existing within a centrifugal force field evidenced by the force $F = Wr'\omega^2/g$ exerted on it in a direction determined by the axis of rotation Q of the system and the center of the ball. At sufficiently high speeds of rotation the influence of gravity becomes negligible.

Assuming the surface of the race to offer negligible rolling resistance, the reaction of the race on a ball will be normal to the race surface and therefore toward its center S, as shown by the vectors N. It is apparent from Fig. 14 that, since the forces F have components tangent to the race, each ball experiences an urge in such a direction e that its displacement in that direction will *decrease* the resultant moment of unbalance of the system. Also, the only condition of rotation of the system under which the balls will be in stable equilibrium relative to the race is that of

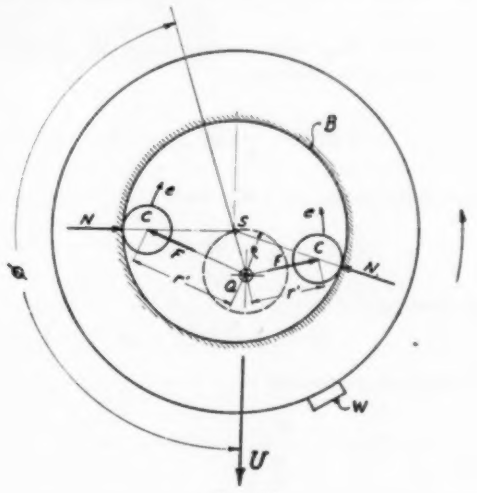

FIG. 14

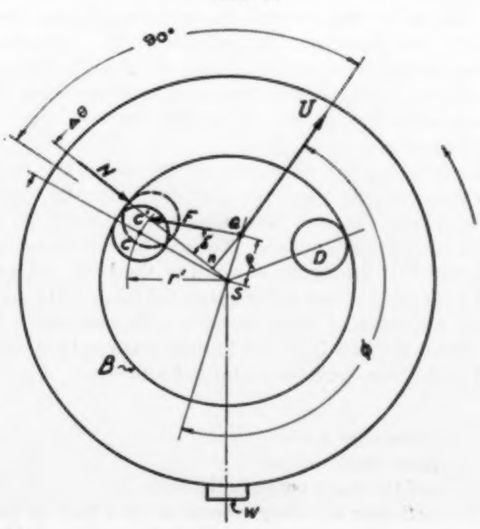

FIG. 15

rotation about the axis S. This condition obtains when the axis Q coincides with axis S, the amplitude ρ is zero, and the moment of unbalance introduced by the balls is equal and opposite to that in the rotor. Thus, when the rotor is running above its resonant speed and the clutch is depressed to free the balls, they will automatically assume the correct positions to suppress oscillation of the rotor and its supporting cradle. The moment of unbalance introduced by the balls must then be equal and opposite to that of the rotor, of which the angle between the balls and its bisecting line serve as a measure.

Let C and D in Fig. 15 represent the positions of equilibrium of the balls relative to their race. Rotation of the system is then about the axis S, and the resultant moment of unbalance of the system is zero. Now assume that the ball C is caused to move through an angle $\Delta\theta$ to a new position C'. This displacement of ball C introduces an unbalance U displaced approximately 90 deg. from position C or C'. The axis of rotation of the system shifts a distance ρ, from S to Q, in a direction determined by the phase angle ϕ. Since equilibrium of the balls is considered as relative to their race, the ball at C', of weight W, must be considered to experience, from the centrifugal force field in which it exists, a force

$$F = \frac{W}{g}(\overline{QC'})\omega^2 \qquad [18]$$

The component of this force tending to return the ball to its equilibrium position is therefore

$$F \sin\delta = \frac{W}{g}(\overline{QC'})\omega^2 \sin\delta \qquad [19]$$

Since, from the geometry of Fig. 15,

$$\sin\delta = \frac{(\overline{Qn})}{(\overline{QC'})} \qquad [20]$$

and, approximately,

$$\overline{Qn} = \rho\cos(180^\circ - \phi) \qquad [21]$$

the restoring force on the ball is

$$F\sin\delta = \frac{W}{g}\rho\omega^2\cos(180^\circ - \phi) \qquad [22]$$

Since comparatively large variations of phase angle ϕ, when near a value of 180 deg., cause only slight variations of the factor $\cos(180^\circ - \phi)$, Equation [22] indicates that a not too great departure of the phase angle ϕ from a value of 180 deg. will have a negligible influence on the stability of the head, and will in no way affect the accuracy of its indications. This has been found to be the case.

In investigating the sensitivity of the balancing head it is necessary to consider the resistance offered by the race to the rolling of a ball. In Fig. 16 is shown a ball C, in its race B, in a state of impending rolling. The axis of rotation of the head is at Q, and that amplitude of whirl of the head just sufficient to start rolling of the ball C is represented by ρ_0. The coefficient of rolling resistance, a linear distance μ, is assumed to be constant. Since the ball is in equilibrium relative to its race, the forces F and N are equal, opposite, and collinear.

Let

d = diameter of a ball
D = diameter of the race
ϵ = half the angle between the balls
μ = coefficient of rolling resistance for a ball on its race
R = ratio of race diameter to ball diameter = D/d

From the geometry of Fig. 16,

$$\sin\delta = \frac{\overline{Sn}}{r'} \qquad [23]$$

also

$$\sin\delta = \frac{2\mu}{d} \qquad [24]$$

hence

$$\frac{\overline{Sn}}{r'} = \frac{2\mu}{d} \qquad [25]$$

But

$$\overline{Sn} = \rho_0 \sin(\epsilon - \delta) \qquad [26]$$

and since ρ_0 is very small compared to the radius r', the angle δ is small compared to the angle ϵ and may be neglected. Thus substituting $\overline{Sn} = \rho_0 \sin\epsilon$ in Equation [25],

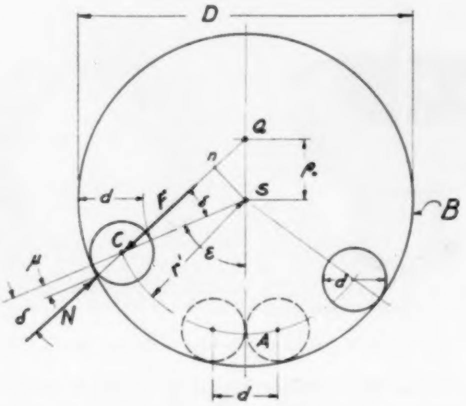

Fig. 16

$$\frac{\rho_0 \sin\epsilon}{r'} = \frac{2\mu}{d} \qquad [27]$$

$$\rho_0 = \frac{2r'\mu}{d\sin\epsilon} \qquad [28]$$

or

$$\rho_0 = \frac{\mu}{\sin\epsilon}\,\frac{D-d}{d} = \frac{\mu}{\sin\epsilon}(R-1) \qquad [29]$$

The sensitivity of the balancing head may be defined as the reciprocal of the amplitude ρ_0 of impending ball motion. Thus

$$\text{Sensitivity} = \frac{1}{\rho_0} \qquad [30]$$

Equation [29] shows that the head sensitivity depends upon the angle 2ϵ between the balls, or upon that fraction of the capacity of the head being utilized. Maximum sensitivity is realized when the head is being used to measure the smallest unbalance moments, the balls taking up positions almost diametrically opposite each other. Then very nearly,

$$\sin\epsilon = 1 \qquad [31]$$

and

$$\text{Maximum head sensitivity} = \frac{1}{\mu(R-1)} \qquad [32]$$

Minimum sensitivity is realized when the head is used at maximum capacity, or when the balls are in contact as at A. Then

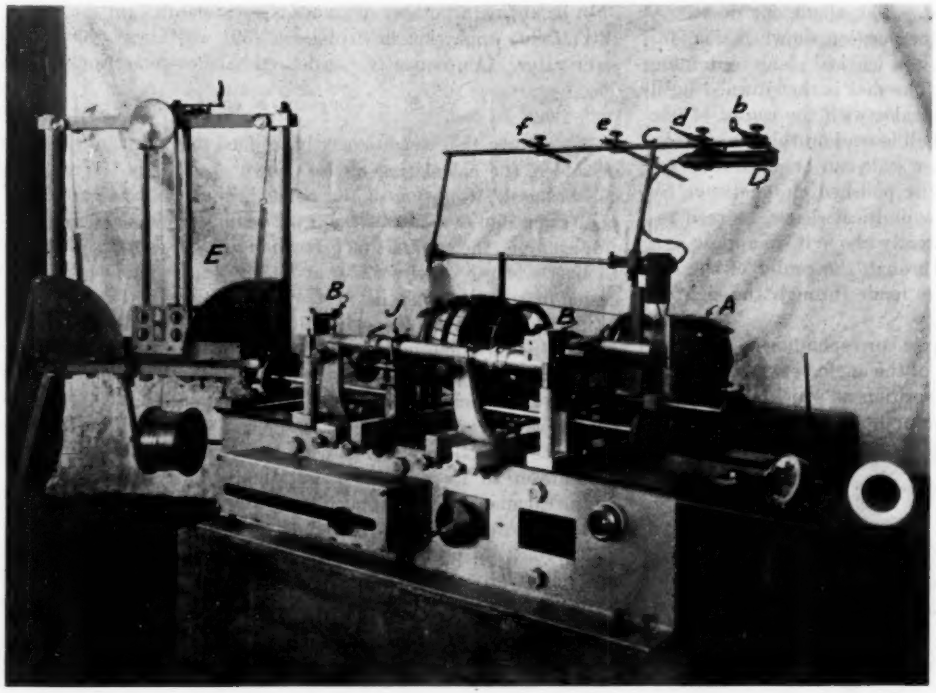

Fig. 17 10- to 50-Lb. Dynamic-Balancing Machine

$$\sin \epsilon = \frac{d}{2r'} = \frac{d}{D-d} = \frac{1}{R-1} \qquad [33]$$

and

$$\text{Minimum head sensitivity} = \frac{1}{\mu(R-1)^2} \qquad [34]$$

Expressions [32] and [34] for head sensitivity indicate that the ratio R of race diameter to ball diameter should be made as small as possible. However, an attempt to carry these proportions too far results in a serious loss in head capacity which becomes zero when $R = 2$. Of course the coefficient of rolling resistance μ should be made as small as possible by careful hardening and finishing of the race and by the use of high-grade steel balls. The actual effective resistance to rolling may be minimized by tapering the apparently cylindrical surface of the race very slightly, so that the balls may be rolled in an axial direction by depressing the clutch.

Operation of the Balancing Machine

Fig. 17 shows a small machine suitable for balancing motor rotors weighing from about 10 lb. to 50 lb. The rotor to be balanced is placed in the machine with its longest shaft extension, upon which the balancing head is clamped, projecting to the right. The fulcrum springs J at the left, which are adjusted to lie in the second correction plane, are clamped; those at the right are left free. The rotor and head are brought to a speed 30 or 40 per cent above that of resonance by means of the driving motor A, controlled by a foot-operated switch. When the clutch of the balancing head is depressed, thus releasing the two balls in their race, they automatically assume such positions as suppress oscillation of the rotor and its supporting cradle. When this condition exists, as shown by the hands of the dial indicators B coming to rest, removing pressure from the clutch button causes the balls to be clamped in their respective positions. The dial indicators B are driven from points at the end of a cradle tube, and show the amplitudes of vibration of these points in thousandths of an inch. These indicators are usually used only to show the absence of vibration, no measurements of amplitude being necessary.

The rotor is now brought to rest and the dial shown in Fig. 13

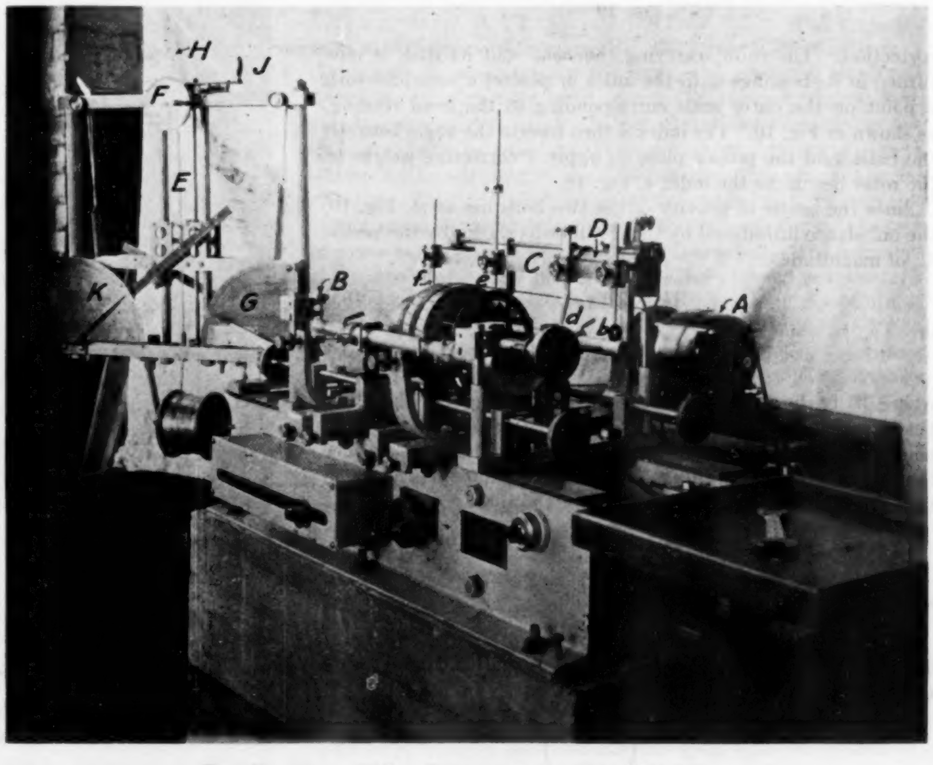

Fig. 18 10- to 50-Lb. Dynamic-Balancing Machine is placed over the face of the head. The alignment device *C*, carrying a lamp *D*, is brought to the position shown in Fig. 18, where its three indices *d*, *e*, and *f* lie in a vertical plane containing the axis of rotation of the rotor. The dial is then turned until the zero line on its inner scale coincides with the center of one ball, and the position of the other ball is read on this inner scale. The setting and reading of the inner scale can be accurately accomplished, since illumination of the polished clutch sleeve by lamp *D* causes the image of this cylindrical sleeve, as seen reflected from the surface of a ball directly above it, to appear as a sharp, bright, radial line passing through the center of the observed ball. These observations are made through the sight *b*, Figs. 17 and 18.

The number read on the inner scale corresponding to the position of the second ball is a measure of the angle between the balls and serves to determine both the position at which a corrective weight should be applied to the rotor and the amount of this correction. The rotor, carrying the head and its dial, is now turned in its bearings until the index or pointer *d* coincides with a point on the outer scale corresponding to the head reading, as shown in Fig. 19. The index *d* then bisects the angle between the balls, and the proper place to apply a corrective weight to the rotor lies under the index *e*, Fig. 18.

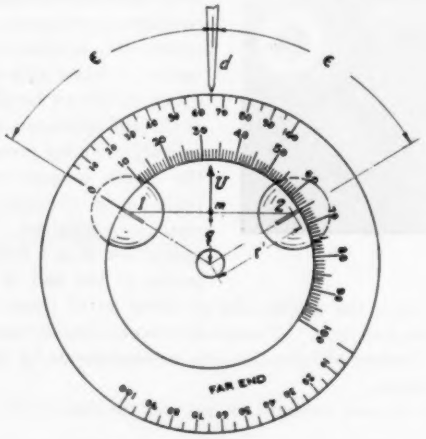

FIG. 19

Since the center of gravity of the two balls lies at *m*, Fig. 19, the unbalance introduced by them will be as shown by the vector *U*, of magnitude

$$U = 2Wq \qquad [35]$$

or

$$U = 2Wr'\cos\epsilon \qquad [36]$$

where *W* is the weight of each ball and ϵ is half of the angle between them. If Fig. 6(*a*) is considered to represent the rotor being balanced, with the center of the head at the point *S*, since the unbalance introduced by the head in its plane has annulled the effect of the unbalance weight *w* of the rotor,

$$\frac{(2Wr'\omega^2\cos\epsilon)}{g}L = \frac{(wr\omega^2)a}{g} \qquad [37]$$

or

$$2Wr'L\cos\epsilon = wra \qquad [38]$$

This determines the unbalance weight *w* equal in magnitude to the correction to be applied. Thus the correction weight

$$w' = w = \left(\frac{2Wr'L}{ra}\right)\cos\epsilon \qquad [39]$$

In balancing a number of rotors of identical design, the factor $(2Wr'L/ra)$ appearing in expression [39] will always have the same value. Consequently, the desired balance-correction weight

$$w' = \text{Constant} \times \cos\epsilon \qquad [40]$$

When correction weights may be applied to a rotor in the form of solder, the calculating device shown at *E*, Figs. 17 and 18, can be used. Rotation of the handwheel *F* (Fig. 18) until the pointer on dial *G* indicates the head reading previously obtained (for example, the reading "65" shown in Fig. 19) causes the device to eject a length of wire solder at *H*, Fig. 18, proportional to the cosine of half the angle between the balancing-head balls. The proper "constant of proportionality" setting for the calculator may be readily determined and fixed by a simple adjustment of the mechanism. If this adjustment is correctly made when setting up the machine, the length of solder ejected will be of the desired weight *w'*. A stroke of the lever *J* cuts off this measured piece of wire solder, and causes the return of the calculator mechanism to its initial configuration. The solder delivered by the calculator is then applied to the rotor at the point indicated by the index *e*, in the first, or right-hand, correction plane.

The fulcrum springs *J*, Fig. 17, at the left end of the rotor are now released by opening their vises. Those at the right, which are adjusted to lie in any convenient transverse plane of the rotor, are clamped by closing their vises. The magnitude and position of the corresponding correction weight to be applied in the second correction plane, at the left end of the rotor, are now determined in a manner similar to that described above. The balancing head remains on the right-hand extremity of the rotor shaft. Since the pivotal point of the system is now *between* the balancing head and the correction plane of the rotor, the outer scale marked "far end" on the dial shown in Figs. 13 and 19 is used in determining the position at which the correction shall be applied. The dial *K* (Fig. 18) of the calculating device, which is subject to a different

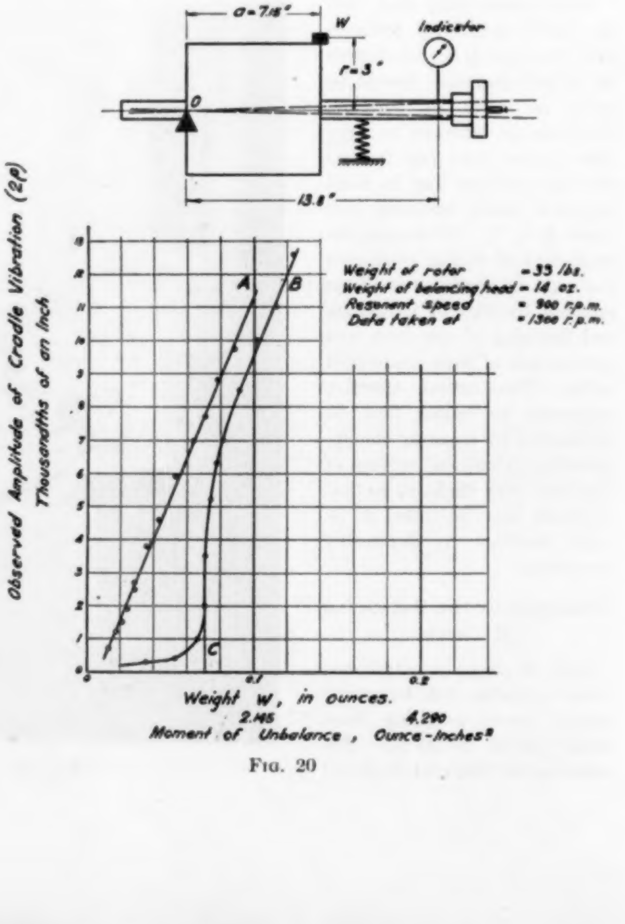

FIG. 20

constant of proportionality, is used in cutting the proper correction weight. The application of this weight, in the second correction plane of the rotor and at a point indicated by the index *f*, completes the balancing process. The balancing head is now removed from the rotor shaft and, if desired, the rotor may be subjected to a check run with both sets of fulcrum springs released.

If the rotor unbalance should in any case happen to be greater than the balancing-head capacity, it will be found impossible to bring the indicator hands to rest by depressing the balancing-head clutch. This indicates that the head capacity is exceeded, and, upon stopping the machine, the two balls in the head will be found in contact. Their positions specify a point on the rotor at which a correction should be applied, of amount at least equivalent to the full capacity of the balancing head. If the application of this correction reduces the rotor unbalance to a value within the head capacity, performance of the machine will again be normal.

Performance of the Balancing Machine

In determining the accuracy of balance obtainable with this machine, a 34-lb. motor rotor carrying a balancing head weighing 14 oz. was placed in the cradle. This rotor, with head, was very accurately balanced. The resonant speed of the system was found to be about 900 r.p.m. Various known weights were then applied to the rotor, and the corresponding amplitudes of cradle vibration were observed at a rotor speed of 1300 r.p.m., about 45 per cent above that of resonance. The values of observed amplitude of cradle vibration, as plotted versus unbalance weight (w) and moment of unbalance (wra) in Fig. 20, correspond closely to *twice* the amplitude ρ used in the above analysis, since they were obtained from observations of the total displacement of the hand of a dial indicator placed approximately in the transverse plane of the balancing head.

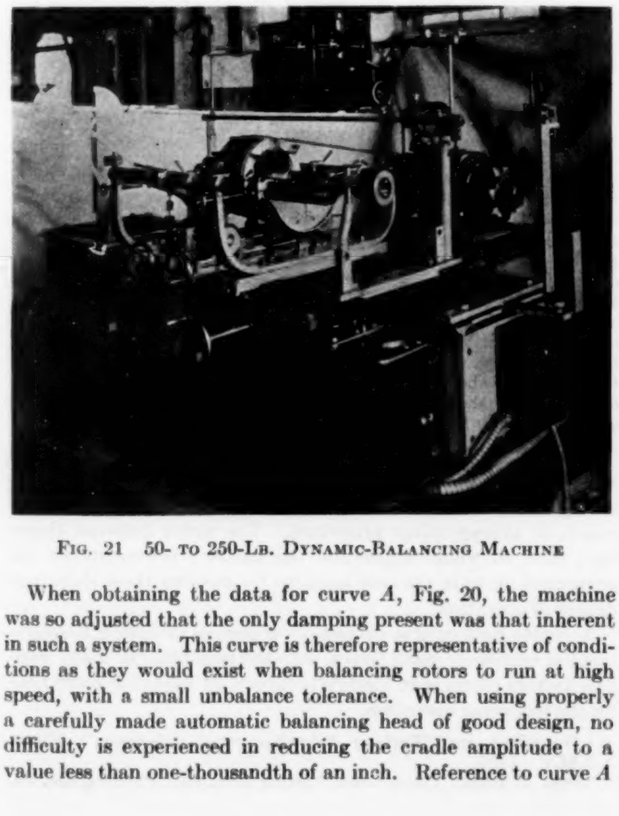

Fig. 21 50- to 250-Lb. Dynamic-Balancing Machine

When obtaining the data for curve *A*, Fig. 20, the machine was so adjusted that the only damping present was that inherent in such a system. This curve is therefore representative of conditions as they would exist when balancing rotors to run at high speed, with a small unbalance tolerance. When using properly a carefully made automatic balancing head of good design, no difficulty is experienced in reducing the cradle amplitude to a value less than one-thousandth of an inch. Reference to curve *A* shows that the balancing head will then indicate the amount and position of a correction to be applied to a rotor of this size within a tolerance of 0.34 oz-in.,[2] which, in this particular case,[4] amounts to less than 0.016 oz.

When the machine is used for the production balancing of standard motor rotors, to be run at a comparatively low speed, the unbalance tolerance may be considerably greater than the machine is capable of meeting. In this case the introduction of some damping, in the form of Coulomb friction resisting motion of the cradle, is found to make the machine more stable and therefore more easily handled by an unskilled operator. The effect of this damping on the behavior of the machine is shown by curve *B*, Fig. 20. As the balls in the head approach their equilibrium positions, causing the moment of unbalance of the system to approach its tolerance, the observed amplitude decreases quite rapidly until, when the tolerance is reached, the damping devices seize. In order that the "cut-off" of the indicators will be absolute, they are driven through devices which may be adjusted to fix an amplitude below which they are inactive. This, in effect, extends the vertical part of curve *B* downward as shown at *C*, and the indicator hands come definitely to rest when the unbalance tolerance is reached. The accuracy of balance obtained is thus less dependent upon the operator's judgment, which may vary from day to day, than it would otherwise be.

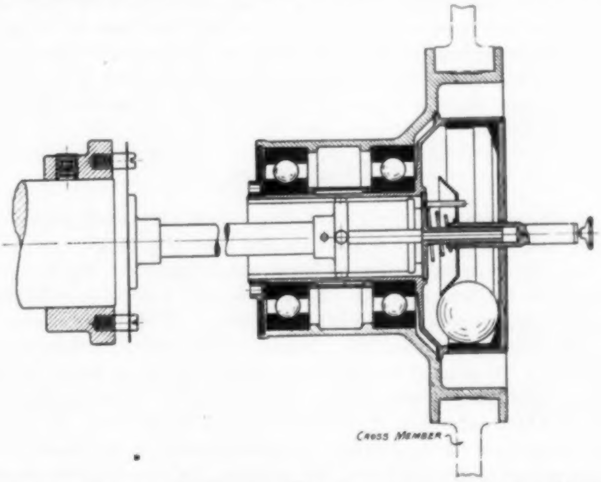

Fig. 22 Automatic Balancing Head Carried in Its Own Ball Bearings, Mounted in a Separate Cradle Cross-Member, and Attachable to a Rotor Shaft Through a Universal-Coupling Device

Since the balancing head functions very quickly the total time required to balance a rotor depends mainly upon that necessary for loading and unloading the machine, stopping and starting the rotor, and applying the corrective weights. As only two runs are necessary, the starting and stopping time is a minimum.

Rotors small enough to be lifted to and from the machine by hand can easily be completely balanced, by an unskilled operator, in less than four minutes. This time includes that required for the application of the corrective weights, and for loading and unloading the machine.

To place this figure on a basis comparable with claims made by some of the more conservative balancing-machine manufacturers, the time required for the application of the corrective weights must not be included. Repeated tests have shown that an operator can easily load the machine, determine and mark the positions of the corrections, cut the corrective weights to size, and unload the machine in a total time of two minutes.

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[4] Wire solder $^1/_8$ in. in diameter weighs about 0.058 oz. per inch of length.

Fig. 21 shows a larger size of machine capable of balancing rotors weighing from 50 to 250 lb.

In many cases, mounting the balancing head directly on the rotor-shaft extension is undesirable. The head is then carried in its own ball bearings within a stationary outer shell which is mounted in a third cradle cross-member, and is driven by a light universal-coupling device which may be attached to either end of the rotor shaft. This arrangement has a number of obvious advantages. Such a balancing head is shown, in section, in Fig. 22.

Discussion

J. Ormondroyd.[5] Several attempts have been made to make automatic balancing machines on the general principles as outlined by the author. One of these was proposed by the French engineer LeBlanc in 1913. A discussion of LeBlanc's attempt will be found in Loewenstein's translation of Stodola's "Dampfturbinen," pages 1134–1135. Hardly a year passes by that some one does not suggest the use of a suitable balancing groove with some solidifying liquids such as solder or babbitt being applied to them which automatically finds its place in the grooves. In spite of all this talk on the subject, the author seems to have made the first practical application of the process. He has done the job so well that one can find nothing about it to criticize, and the writer wishes to express heartiest congratulations for the results which he has obtained.

T. C. Rathbone.[6] The author has described a very clever application of the principle of automatic balancing above the critical speed. Although this principle has been known for some time, the author deserves the greatest credit for putting it into practical use. So far as the writer knows, the attempts to provide automatic balancing by means of mercury or by loose washers on the shaft have never been entirely satisfactory. The author's system employing the two balls which can be fixed in position when they have adjusted themselves to balance, appears to be the practical answer to the problem.

The author describes the necessity for making the natural frequency in the vertical and horizontal direction the same. In the analysis, this is accomplished by making the moment of inertia of rotor and cradle the same for both directions, and at the same time providing equal stiffness in the two directions.

The writer is interested in knowing whether any practical difficulty is encountered in obtaining these conditions when the machine may be called on to handle rotors of various dimensions. It would also be of interest to know whether practical differences in the damping factor b, in the vertical and lateral dimension, would cause appreciable departure from a circular motion. Both the amplitude and the lag angle are affected by the damping, so that theoretically, at least, the motion of the ball-race center could take the form of any ellipse bounded by the rectangle formed by the vertical and lateral amplitude. For an extreme case, the motion could become linear (on a diagonal of the rectangle). If small deviations from the circular motion do occur, particularly where Coulomb damping is deliberately introduced, are the ball positions materially affected?

It appears to be necessary to insure that the center of the ball race coincides precisely with the axis of rotation rather than the axis of the shaft. If the shaft is eccentric at the plane of the ball race, the motivating forces on the balls, as shown in Fig. 16, would be affected in such a way as to cause them to assume false positions. It would be of interest to know if the author has encountered any difficulty on this score.

The writer's experience with vibratory motion on complex structures, where the most curious transitions of ellipses[7] from one shape and inclination to another are produced by changes in speed or in the inertia and elastic characteristics, have perhaps made him unduly concerned over the possibility of compounding to maintain purely circular motion.

T. Y. Olsen.[8] The author has presented in a very clear and interesting manner the mathematical basis for general elements of design in balancing machines. He has also brought into use through the ball-type automatic-balancing head a principle which has long been recognized.

Of historic interest and possible offering a thought for further development is an article discussing this principle on page 184 in the issue of December, 1904, of *Railway Machinery* describing a method using three eccentric disks mounted on a shaft in such a manner that they could adjust themselves and automatically balance the rotor.

The following is an extract from the article, involving tests some time previous to 1904:

> Some years ago William Sellers & Co., Inc., Philadelphia, conducted a series of tests upon an experimental steam turbine, similar in principle to the present Curtis turbine. As the disks rotated at very high speed, some convenient method of balancing was sought, and the plan illustrated [see Fig. 23] was originated. In the figure, *A* is the disk to be balanced, and *a*, *b*, and *c* are three thin disks turned up in the form of eccentrics and mounted on the shaft with the main disk. The bore of the eccentric was a close fit on the shaft, so there will be some frictional resistance between the eccentrics and the shaft; yet they were not so tight that it was necessary to drive them in place. The eccentrics were spaced 120 deg. apart, and as the main disk attained its speed *the eccentrics gradually shifted automatically*, until they reached such position that apparently brought themselves, together with the disk and shaft, into perfect running balance. Just why the vibrations of the shaft were such as to cause this behavior of the eccentrics is a problem for the mathematicians to handle, but such is the fact, and we will leave it for others to explain.

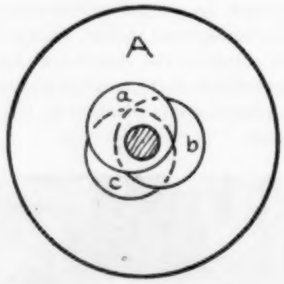

Fig. 23

Balancing is having careful thought wherever rotating parts are manufactured, thereby emphasizing the development of balancing machines for the most convenient and practical application to average plant procedure. The writer believes, therefore, that all would appreciate the author's comments on the provisions made in the new machine for meeting these shop conditions of operation.

Accurate balancing is necessarily a process involving measurement of extremely small forces, with the possibility of many variable factors entering to disturb accuracy, or requiring more time for the balancing operation.

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[5] Experimental Division Engineer, Westinghouse Elec. & Mfg. Co., South Philadelphia Works, Philadelphia, Pa.

[6] Central Station Turbine Division, Westinghouse Elec. & Mfg. Co., South Philadelphia, Pa. Mem. A.S.M.E.

[7] Described in "Turbine Vibration and Balancing," Trans. A.S.M.E., 1929.

[8] Pres. & Treas., Tinius Olsen Testing Mch. Co., Philadelphia, Pa.

The automatic balancing head in Fig. 11 is secured to the end of the shaft by a contracting chuck gripping the shaft near the end in a comparatively restricted area. The ball balancing weights are overhung outside this support area in the chuck. How much looseness is permissible between the shaft and the sleeve *A* without seriously affecting results? How much error is likely to be created by a looseness of, say, 0.001 or 0.002 in. at this point? If a shaft has a tapered or threaded end, can the automatic balancing head be conveniently applied?

In connection with Fig. 20, reference is made to reading unbalance of 0.016 oz. 3 in. from center of shaft or 0.34 oz-in.[2] With this degree of sensitivity of this machine, what is the range or minimum and maximum amount of unbalance which the machine will actually measure to the same degree of accuracy? What is the minimum included angle between the balls, at which the aforestated accuracy can be obtained? In previous text is mentioned that if the amount of unbalance is beyond the capacity of machine, the two balls will come together, and a tentative correction is made, and if necessary repeated until the frame becomes quiet with the balls slightly separated. If unbalance is beyond the capacity of the balls, is there any way of estimating approximately the amount of unbalance for the purpose of making tentative correction?

Are the balls in the balancing head sensitive to the presence of dirt or wear to a degree which might require frequent attention? As an average about how long will the balls run without requiring replacement?

When different weights of rotors are to be balanced, what adjustments are required in the balancing machine?

The armature to be balanced is rotated by a drive belt running around and pulling against its outer surface. What limitations on irregularity of the surface of the part being tested will produce detrimental effects in taking fine readings of unbalance? How would a part to be balanced be driven with the outer surface very irregular or tapered, such as a gasoline-engine crankshaft, steam-turbine rotor, or some electrical armatures? Is it necessary to carefully control variation in the action of the belt which might cause vibration such as to interfere with accurate check on final balance?

Is the motor mounted on the stationary base or on the vibrating unit? Which construction is the better as affecting accurate indications of unbalance?

Has the friction in the indicator dial any noticeable dampening effects when measuring small amounts of unbalance?

The part mentioned to be balanced is rotated at 1300 r.p.m. What benefits are derived from running at this high balancing speed?

Readings for the amount and angular position of unbalance are taken from the automatic balancing head which is located on the end of the shaft or some other position not in the axial plane where corrections are to be made to the part being balanced. If some other material than solder is to be used for making the balancing correction, can the soldering instrument be used to transfer the readings so as to indicate the angular position and ounce-inches correction which is to be made in some other manner?

AUTHOR'S CLOSURE

The automatic balancing device proposed by LeBlanc consists of a hollow drum or cylinder containing a liquid or its equivalent such as mercury, and mounted on a flexibly supported shaft.

When rotating above its critical speed, the shaft is displaced in such a way that the drum containing the liquid is whirling with its "light side" out. The liquid is then supposed to flow to the light side of its container, and thus correct for the initial unbalance.

It is readily seen that this action cannot take place completely, for if the mercury distributes itself in such a way as to balance the drum, then the drum will run true, the mercury will be uniformly distributed, and can introduce no unbalance correction. The LeBlanc automatic balancing device is thus imperative.

The author's experience with devices such as loose rings or disks mounted on the shaft has been that it is very difficult to make them operate. Due to excessive friction, several sharp blows with a hammer were necessary to cause three rings to assume their desired positions even approximately.

Some variation from true circular motion of the cradle, caused either by non-uniform springs or irregular damping, has been found to have no appreciable effect on the action of the balancing head. Eccentricity of the ball race, however, causes the balls to assume false positions. This difficulty is avoided by proper set-up, not mounting a head on a long overhanging shaft, or by using the type of head shown in Fig. 22.

APM-54-13

The Propeller-Type Fan

BY O. G. TIETJENS,[1] EAST PITTSBURGH, PA.

In this paper a method is given showing how it is possible to present the complete characteristics of fans in a very concise manner. This is usually done by giving pressure-volume curves for different sizes and speeds. Here this relationship for all sizes and speeds is shown by three curves.

Theory is developed according to which all data for designing a fan can be determined if the volume per minute and the pressure rise, i.e., the power output of the fan, are given. In addition, it is shown how the efficiency due to air friction can be calculated, and in what way the loss due to the rotational velocity of the air behind the fan is determined by the pressure increase produced by the fan.

The limitation of the theory is then dealt with, and various restrictions regarding the validity of the assumptions made are discussed. Finally a short description of a test equipment for fans is given.

THE propeller-type fan has many advantages compared with the centrifugal type. Generally speaking, we may say that for a given power output the propeller-type fan with its housing is smaller in size and simpler in construction and therefore cheaper than a centrifugal fan. In addition, the speed of rotation is higher for propeller-type fans than for those of the centrifugal type, particularly for fans of large power output, which means a smaller and cheaper motor. Finally, the efficiency of a propeller-type fan, if properly designed, is higher than that of a centrifugal fan, particularly for large volume output, whereas for a comparably small volume and high pressure, the centrifugal type may be considered superior with respect to efficiency.

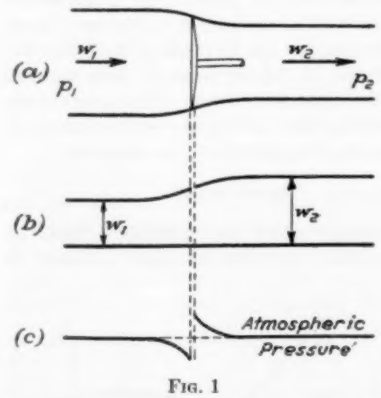

FIG. 1

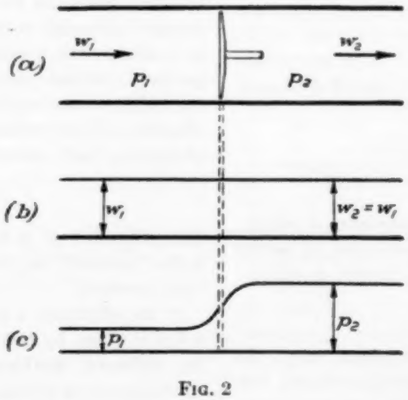

FIG. 2

The highest pressure obtainable is about the same for both types of fans, and for air of atmospheric pressure is approximately 20 in. of water.

For higher pressures a special type of centrifugal blower, the so-called compressor, is developed.

Despite the importance of the propeller-type fan, there is, as contrasted with the airplane propeller, no theory for it, and but few data are available concerning its performance. However, the conditions of operation of an airplane propeller are quite different from those of a propeller-type fan, therefore making it impossible to apply without any modification the well-developed propeller theory to fan design. In brief, this difference consists in the fact that a propeller accelerates a certain amount of air in the axial direction, the reaction force of which represents the thrust, while a fan operating in a housing increases the pressure of the air flowing through the fan area.

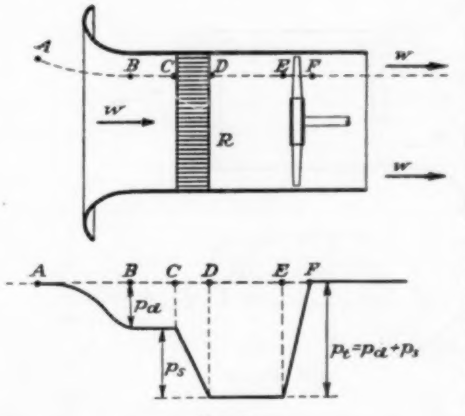

FIG. 3

Figs. 1 and 2 illustrate better the difference between the performances of a propeller and a fan. Instead of considering a propeller moving with a certain velocity through the quiet air, we assume the propeller to be at rest and the air flowing toward it with the same velocity, say, w_1, in the opposite direction. The air acted on by the propeller is accelerated in the axial direction to the velocity w_2, this being done partly ahead of the propeller and partly beyond it, as shown at (*a*) and (*b*), Fig. 1. In the plane of the propeller there is a discontinuity in pressure. As shown in Fig. 1(*c*), the pressure ahead of the propeller decreases from the atmospheric pressure p_1 to a lower pressure corresponding to an increase of velocity at this point. In the propeller plane a sudden increase of pressure takes place above the atmospheric pressure. Beyond the propeller the pressure again decreases, with a corresponding increase in velocity, until the atmospheric pressure is again reached.

The operating conditions under which a fan works are quite different, as shown in Fig. 2. Due to the fact that a fan operates inside a housing, usually having the shape of a pipe, there can be no increase in axial velocity [Fig. 2, (*a*) and (*b*)]. The pressure rise in the plane of the fan must therefore be connected with a certain rotational velocity, which relation will be dealt with later.

I—POWER OUTPUT OF A FAN

Let us consider the performance of a fan in a special case. The task of the fan may be to cool the radiator *R* in Fig. 3 by

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Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

drawing air through from left to right. Considering the pressure variation along a line from A to F we notice first a drop from A to B corresponding to a velocity increase from zero at point A to a certain velocity, say, w, at point B. The pressure drop, usually called dynamic pressure (p_d), is, according to Bernoulli's equation, equal to $^1/_2\rho w^2$, where ρ is the density of the air. From B to C the pressure is practically constant, whereas the static pressure p_s decreases from C to D according to the restriction to the air flow caused by the radiator. From D to E the pressure is again constant. Due to the fact that the static pressure of the air leaving the pipe like a jet is equal to the atmospheric pressure, the fan has to create a total pressure rise equal to the sum of the dynamic and static pressures, or $p_t = p_d + p_s$.

In this case the fan is drawing the air through the radiator. The static pressure varies somewhat differently where the fan is forcing air through the radiator as shown in Fig. 4. In both cases it is seen that the work to be done by the fan is to increase the pressure of a certain volume of air. Therefore, denoting the volume flowing per unit time through the fan area by V, and the total pressure rise by p_t, the product $V \times p_t$ represents the power output of the fan. Considering the physical dimensions of V and p_t,

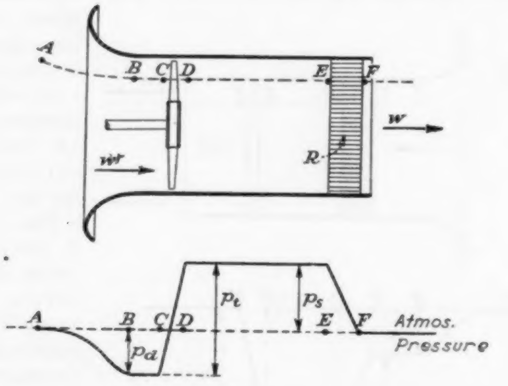

FIG. 4

$$V \times p_t = \frac{\text{length}^3 \times \text{force}}{\text{time} \times \text{length}^2} = \frac{\text{length} \times \text{force}}{\text{time}} = \text{power}$$

we see that the product of V and p_t has the dimension of power. The efficiency η of a fan, being defined as the ratio of power output to power input, is therefore determined by the equation

$$V \times p_t = \eta \times \text{power input.}$$

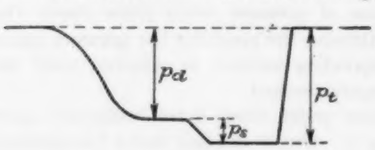

FIG. 5

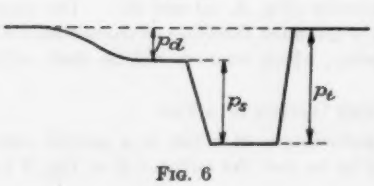

FIG. 6

The conditions under which a fan has to operate vary. In some cases the fan has to deliver a large volume per minute, but has to overcome only a small static-pressure drop; in other cases only a small volume per minute is wanted, but a large pressure rise. In the first case the dynamic pressure forms a large part of the total pressure (as shown schematically in Fig. 5), while in the second case (Fig. 6) the static pressure predominates. The working conditions of a fan are thus characterized by the ratio:

$$\frac{p_d}{p_d + p_s} = \frac{p_d}{p_t}$$

This ratio varies from zero to unity, being close to zero for a compressor and equal to unity for a desk fan.

As an example let this ratio be determined for the particular case of a cooling arrangement on top of a Diesel locomotive as shown in Fig. 7. Assume the volume required to be 16,000 cu. ft. per min., the diameter of the fan 32 in., the diameter of the hub 12 in., and the pressure drop through the radiator 1 in. for the required amount of air. The velocity of the air discharged by the fan is then

$$\frac{16{,}000 \times 144 \times 4}{(32^2 - 12^2)\pi}$$

$$= 3330 \text{ ft. per min.}$$

and the dynamic pressure height therefore 0.68 in. of water.

We then have the ratio

$$\frac{p_d}{p_t} = \frac{p_d}{p_d + p_s} = \frac{0.68}{0.68 + 1} = 0.405$$

By multiplying the numerator and the denominator of p_d/p_t by the volume, it is seen that this ratio shows how much of the total power output is used to impart to the air a certain amount of kinetic energy. In the particular case under consideration about 40 per cent of the total power output of the fan is represented by the kinetic energy of the air discharged by the fan. In some cases, as for instance in the present one, this kinetic energy—although a part of the power output of the fan—must be considered as a loss since no use can be made of it. Only the product (volume per minute) × (static pressure drop through the radiator) can be considered as useful work. The aerodynamic efficiency of the cooling arrangement in Fig. 6—not the efficiency of the fan itself—is therefore determined by the equation

$$V \times p_s = \eta_s \times \text{power input}$$

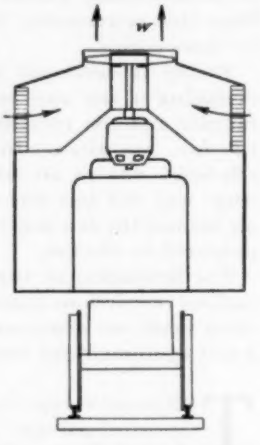

FIG. 7 COOLING ARRANGEMENT FOR DIESEL LOCOMOTIVE

The efficiency η_s is sometimes called the "efficiency based on static pressure" as contrasted with the efficiency η based on total pressure.

If the efficiency η of a fan is given by tests for different working conditions or as a function of p_d/p_t as shown in Fig. 8, it is a very simple matter to calculate the efficiency based on static pressure, all that is necessary being to multiply the efficiency η for a given value of p_d/p_t by the factor $[1 - (p_d/p_t)]$. The broken line represents the limit for the efficiency based on static pressure,

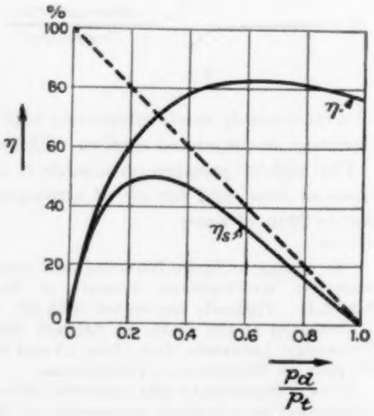

FIG. 8

which line would be reached if the fan itself had an efficiency of 100 per cent. Due to the fact that for $p_d/p_t = 0$ the volume delivered by the fan is zero, the pressure, however, being of finite magnitude, we have no power output for this value of p_d/p_t. Therefore the curve of η as a function of p_d/p_t has to start from zero. This efficiency curve can be obtained by using an experimental set-up such as that shown in Fig. 19. By changing the position of the baffle in this figure the volume of air per unit time and the difference between the pressure inside of the box and the atmospheric pressure can be varied, and therefore also the ratio p_d/p_t.

II—Characteristics of a Fan

The volume delivered by a fan per unit time can be expressed by the equation

$$V = \varphi A u \quad [1]$$

where A represents the fan area and u the peripheral speed. φ being a non-dimensional factor does not depend on the size of the fan or on the speed but only on the design of the fan—as, for instance, on the number of blades—and in addition on the working condition, i.e., on p_d/p_t.

The total pressure produced by the fan can be expressed by the equation

$$p_t = \psi \frac{\rho}{2} u^2 \quad [2]$$

where ρ denotes the density of air, i.e., mass per unit volume. ψ likewise does not depend on the size of fan or on its speed, but only on type of fan, and for a given fan on the ratio p_d/p_t. Fig. 9 shows the typical shape of the φ and ψ curves as functions of p_d/p_t. These curves differ for each design of fan, and may together with the efficiency curve be considered as the characteristics of the fan. As mentioned in the two preceding paragraphs, the curves do not depend on the size of the fan or on its speed.

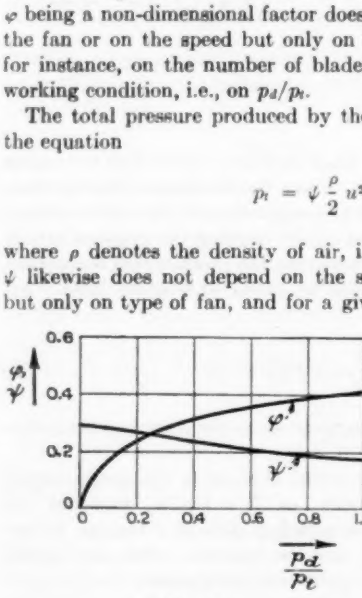

FIG. 9

Another expression for the value p_d/p_t can now be easily developed by using Equations [1] and [2]. Knowing that p_d is equal to $^1/_2\rho w^2$, where w is the axial velocity through the fan area A, we have in connection with [2]

$$\frac{p_d}{p_t} = \frac{\frac{\rho}{2} w^2}{\psi \frac{\rho}{2} u^2} = \frac{1}{\psi}\left(\frac{w}{u}\right)^2$$

Since $V = Aw$, [1] may be written

$$\frac{w}{u} = \varphi$$

and we have

$$\frac{p_d}{p_t} = \frac{\varphi^2}{\psi} \quad [3]$$

In order to obtain the characteristics φ, ψ, and η of a fan with respect to $p_d/p_t = \varphi^2/\psi$, it is necessary to measure the volume delivered per unit time, the difference of static pressure produced by the fan, and the power input to the shaft of the fan. These measurements have to be made for different working conditions or different positions of the baffle in Fig. 19. By measuring the volume we obtain, according to [1], the value φ after having determined the area A and the r.p.m. In addition, the ratio V/A gives the axial velocity at the fan area, and therefore the dynamic pressure $p_d = {}^1/_2\rho w^2$. By measuring the static pressure p_s and adding to this value that of the dynamic pressure just calculated, we obtain the total pressure and can therefore, according to [2], determine ψ. The ratio of the power output Vp_t to the measured power input then gives the efficiency η. In this way we obtain for each position of the baffle a value of φ, of ψ, and of η, and we are thus able to plot these variables as functions of $p_d/p_t = \varphi^2/\psi$.

Fig. 10 shows the φ, ψ, and η curves obtained in this way for a particular fan. It is seen that this fan works with the highest efficiency for high value of the ratio p_d/p_t. Another fan will have a different efficiency curve with its maximum at another value of φ^2/ψ. For each working condition of the fan, i.e., for each value of φ^2/ψ, it is possible, according to [1] and [2], to determine two of the four values p, V, A, and u if the other two are given. The fact that these performance curves for all kinds of fans run from $\varphi^2/\psi = 0$ to $\varphi^2/\psi = 1$ makes it very convenient to compare different fans with each other and to select in a particular case the one giving the best performance.

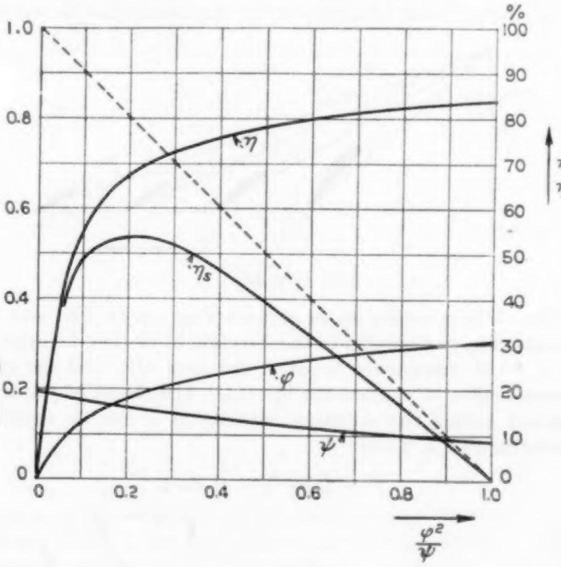

FIG. 10

III—Blade-Element Theory

The power output of a fan, i.e., volume times total pressure, can also be expressed in another way. The volume per unit time being equal to the fan area A times the axial velocity w, we can write

$$Vp_t = Ap_t w = Tw \quad [4]$$

where T is the force acting on the fan area. This force usually is called the thrust. The thrust times the axial velocity is thus equal to the power output of the fan.

If n is the number of blades, a thrust of T/n will be produced by each blade. Dividing the blade in Fig. 11 in elements $b \times dr$, where b is the width of the blade and generally varying with the distance from the hub, the thrust of such an element will

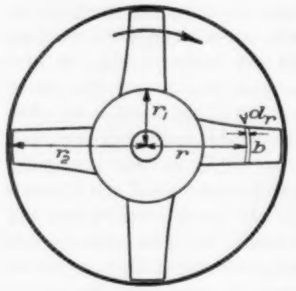

FIG. 11

be dT/n, wherein the general case dT varies with the distance from the hub or with the radius r.

Let us now replace dT by another term. The relative velocity v of a blade element with respect to the air is equal to the geometrical sum of the axial velocity w, which is assumed as uniform over the area A, and the tangential speed $u = \omega r$, where ω is the angular velocity. Dividing the velocities v, w, and $u = \omega r$ by the angular velocity and turning each element through 90 deg., we obtain Fig. 12.

Let us now assume that the performance of a blade element is like that of an element of an airplane wing moving with the speed v through quiet air, and having an angle of attack equal to α. Later it will be seen how far this assumption is justified.

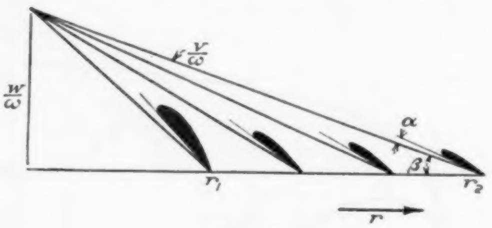

FIG. 12

The air force acting on an airplane wing can be split into two components, one parallel to the direction of motion, i.e., parallel to v, which component is called the drag (D), and the other perpendicular to v called the lift (L). The thrust of the blade element having the direction opposite to w can be expressed according to Fig. 13 by

$$T = L \cos \beta - D \sin \beta \quad [5]$$

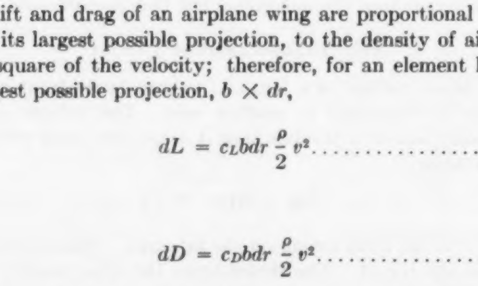

FIG. 13

The lift and drag of an airplane wing are proportional to the area of its largest possible projection, to the density of air, and to the square of the velocity; therefore, for an element having the largest possible projection, $b \times dr$,

$$dL = c_L b dr \frac{\rho}{2} v^2 \quad [6]$$

and

$$dD = c_D b dr \frac{\rho}{2} v^2 \quad [7]$$

The coefficients of lift and drag, c_L and c_D, which depend on the airfoil shape and on the angle of attack, have been obtained by wind-tunnel tests for numerous shapes of airfoils. Fig. 14 shows for a particular airfoil the coefficient of lift as a function of the coefficient of drag, with the angle of attack as parameter.

Due to the fact that the lift is some 15 to 20 times larger than the drag for reasonable angles of attack (α about 3 deg.) and that the angle β is always smaller than 45 deg. (usually around 25 deg.) we may, as a first approximation, neglect the drag entirely. Thus, for an element of an n-bladed fan,

$$\frac{dT}{n} = dL \cos \beta = c_L b dr \frac{\rho}{2} v^2$$

hence the total thrust of a fan having a hub radius equal to r_1, and a fan radius equal to r_2 will be

$$T = n \frac{\rho}{2} \int_{r_1}^{r_2} c_L v b v \cos \beta dr \quad [8]$$

or, since $v \cos \beta = \omega r$,

$$T = n \frac{\rho}{2} \omega \int_{r_1}^{r_2} c_L v b r dr$$

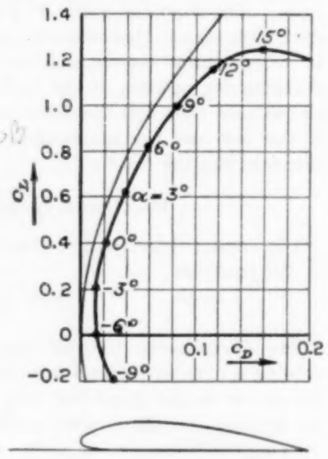

FIG. 14

The relative velocity of the blade element v varies with the radius r. For lightly loaded fans, where the rotational velocity back of the fan can be neglected as compared with the axial velocity, it is desirable to vary b and c_L (by varying the angle of attack along the radius) in such a way that the product $c_L v b$ is constant over the radius. The reason for doing so is given later. Assuming $c_L v b$ = const.,

$$T = \frac{n}{4} \rho \omega c_L v b \, (r_2^2 - r_1^2) \quad [9]$$

This value is somewhat too large since the drag has been neglected.

Due to the fact that the thrust is equal to the power output divided by the axial velocity, or $T = Vp_t/w$, Equation [9] enables us to determine the principal data of a fan, as, for instance, its size and speed, and the number, width, and airfoil section of the blades, if the power output is given.

The tangential speed ωr of the fan being usually several times larger than the axial speed w, we may neglect w^2 as compared to $(\omega r)^2$, writing in [9] ωr instead of v. Further, taking into account the fact that c_L is about 0.5, if we assume a sufficiently large lift/drag ratio to justify neglecting the drag, and that the average density of air is equal to 6.49×10^{-7} lb. min./ft.^{4} we have instead of Equation [9]:

$$\frac{Vp_t}{w} = T = n/4 \times 6.49 \times 10^{-7} \times \omega^2 \times \frac{1}{2} \times b_2 r_2 (r_2^2 - r_1^2)$$

or, with ω = r.p.m. $\times 2\pi$,

$$\frac{Vp_t}{w} = T = 3.2n \left(\frac{\text{r.p.m.}}{1000}\right)^2 b_2 r_2 (r_2^2 - r_1^2) \quad [10]$$

Example. Let V = 22,500 cu. ft. per min. and p_t = 2 in. of water = 10.4 lb. per sq. ft. For a fan radius of 18 in. = $1^1/_2$ ft., and a hub radius of 6 in. = $^1/_2$ ft.,

$$T = Ap_t = (1.5^2 - 0.5^2) \times \pi \times 10.4 = 65.2 \text{ lb.}$$

Assuming 6 blades and a speed of 1750 r.p.m., the width b_2 of the blade at the tip is given by [10] as 0.37 ft. = 4.5 in. The variation of b with the radius is then determined by the condition that $c_L v b$ = const.

The same power output, i.e., the same volume per minute, and the same total pressure, can be obtained also from a fan of a different size and speed and having a different number of blades. What fan, in a particular case, will be the most suitable depends on various factors. Sometimes, in addition to the power output, the r.p.m. is given and the other variables have to be so chosen that Equation [10] will be satisfied; and in the case where the kinetic energy of the air discharged by the fan has to be considered a loss, it is advisable to use a fan of larger diameter, which means a smaller axial velocity for a given volume per minute. Within a certain range all these different factors can be varied for a given volume per minute and for a given static pressure. How far we can go in this direction is governed mainly by the fact that a satisfactory efficiency must be obtained.

Fan losses by which the efficiency η is determined consists of:

(1) Loss due to the fact that each blade element, in producing a lift, has to overcome a certain drag; and

(2) Loss due to the rotational-velocity component of the air discharged by the fan.

The latter loss, which in some cases is the larger, can very often be considerably diminished by using stationary guide vanes back of the fan.

IV—LOSS DUE TO THE DRAG OF THE BLADE ELEMENT

Besides the component dT of the air force which if multiplied by the axial velocity represents the power output of the element, there is another component which acts on the element in the direction opposite to its motions, and which has to be overcome by the motive power. The product of this component and the radius, called the torque, represents, if multiplied by the angular velocity, the power input of the element.

The efficiency of a blade element is therefore

$$\eta = \frac{w dT}{\omega dQ} \quad [11]$$

According to Fig. 13,

$$dQ = r(dL \sin\beta + dD \cos\beta)$$

Thus, in connection with [6] and [7], we have for the efficiency of a blade element at a certain distance r from the axis of the fan

$$\eta_{(r)} = \frac{w}{\omega r} \times \frac{c_L \cos\beta - c_D \sin\beta}{c_L \sin\beta + c_D \cos\beta}$$

or, since $w/\omega r = \tan\beta$,

$$\eta_{(r)} = \frac{\dfrac{c_L}{c_D}\cot\beta - 1}{\dfrac{c_L}{c_D}\cot\beta + \cot^2\beta} = \frac{\dfrac{c_L}{c_D} \times \dfrac{\omega r}{w} - 1}{\dfrac{c_L}{c_D} \times \dfrac{\omega r}{w} + \left(\dfrac{\omega r}{w}\right)^2} \quad [12]$$

In this equation the efficiency of a blade element is expressed as a function of the two variables, c_L/c_D and $\omega r/w$. Fig. 15 shows the efficiency of a blade element as a function of $\omega r/w$ for a number of values of c_L/c_D. It is seen that the efficiency is a maximum for $\omega r/w \approx 1$, and that it decreases at first slowly and then more rapidly as $\omega r/w$ increases. For large values of $\omega r/w$ it is particularly important to use the best possible airfoil section with a large c_L/c_D value.

In order to obtain an expression for the efficiency of the whole fan corresponding to what has just been developed for a blade element, it is necessary to consider the expression

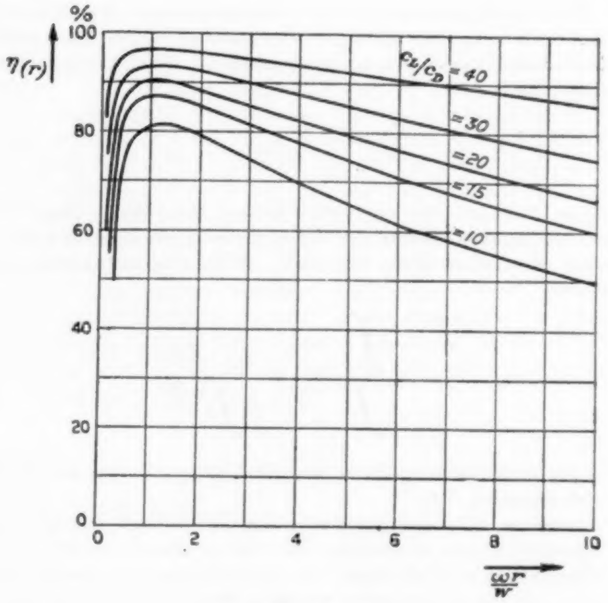

FIG. 15

$$\eta_F = \frac{w \displaystyle\int_{r_1}^{r_2} dT}{\omega \displaystyle\int_{r_1}^{r_2} r dQ}$$

or, since $\omega r dQ = w dT/\eta_{(r)}$

$$\eta_F = \frac{\displaystyle\int_{r_1}^{r_2} dT}{\displaystyle\int_{r_1}^{r_2} \frac{dT}{\eta_{(r)}}} \quad [13]$$

where $\eta_{(r)}$ is the efficiency of the blade element.

If the design of the fan is given, and if a particular working condition is assumed, which means a particular value of $p_d/(p_d + p_s) = p_d/p_t$, the efficiency η_F can be determined, although it is a fairly tedious calculation. For this purpose a certain but arbitrary value of the axial velocity has to be assumed, with which the angle of attack of each blade element is then determined. From these angles of attack and the c_L, c_D curves of the airfoils used at different radii, the thrust of each element can be calculated according to Equation [5] and integrated over the radius from the hub to the tip. If the total thrust calculated in this way is the same as that determined by the equation

$$T = A p_t = A \frac{\rho}{2} w^2 \frac{p_t}{p_d}$$

the axial velocity assumed was the right one; otherwise the calculation has to be repeated, using another value of w. Having in this way determined the axial velocity, the efficiency of each blade element can be calculated by [12], using the same angular velocity as assumed in calculating w. Finally the efficiency η_F can be found by a graphical integration of Equation [13].

More important, however, than the calculation of the efficiency of a fan of given design is the problem designing a fan that will have the highest efficiency possible. According to [13], the efficiency of the fan depends not only on the efficiency of the blade elements, but also on the variation of the thrust along the blades.

The simple assumption of a constant pressure over the fan area leads to the conclusion that the variation of the thrust with the radius of the blade is proportional to r. From the propeller theory we have

$$\frac{dT}{dr} = \rho \Gamma r \omega \qquad [14]$$

where Γ denotes the circulation around the blade. Therefore our assumption relating the thrust distribution involves a constant circulation along the blade. With this assumption we have

$$\eta_F = \frac{\int_{r_1}^{r_2} rdr}{\int_{r_1}^{r_2} \frac{rdr}{\eta_{(r)}}} = \frac{r_2^2 - r_1^2}{2\int_{r_1}^{r_2} \frac{rdr}{\eta_{(r)}}}$$

which can be solved easily by graphical integration in connection with Equation [12].

However, this assumption of a constant-pressure distribution over the fan area necessitates that the rotational velocity components shall be fairly large near the hub, causing a considerable loss according to the kinetic energy of the air because of its high rotational velocity.

Therefore if we do not use stationary guide vanes back of the fan in order to straighten out the air, in this way converting, at least partly, the kinetic energy due to rotational velocity into pressure, it is advisable to distribute the thrust over the blades in such a way that dT/dr will decrease more rapidly with decreasing r than it would if dT/dr were proportional to r. In other words, the circulation does not have to be constant over the blade length, but must decrease in the direction of the hub.

Although the assumptions on which the propeller theory, developed by Prandtl[2] and Betz,[3] is based do not strictly hold for all types of fans, particularly those of high pressure and low axial velocity, i.e., for a low value of φ^2/ψ, a distribution of circulation such as that developed by Prandtl will give better results than a constant circulation along the blade. For a propeller of n blades, the expression of the circulation as a function of the radius is, according to Prandtl,

$$\Gamma = \frac{2\pi r' w}{n} \times \frac{r^2}{r'^2 + r^2}$$

r' being that distance from the axis where $\tan \beta = 1$, or $r' = w/\omega$.

The influence of a finite number of blades, which necessitates a decrease of the circulation near the tips of the propeller blades, can be neglected in the case of a fan, since the fan runs in a housing, and there is but a small clearance space between the blades and the housing.

Assuming such a distribution of circulation, we obtain for Equation [13], making use of [14],

$$\eta_F = \frac{\frac{2\pi r'^1 w}{n}\rho\omega \int_{r_1}^{r_2} \frac{r^2}{r'^2 + r^2} rdr}{\frac{2\pi r' w}{n}\rho\omega \int_{r_1}^{r_2} \frac{r^2}{r'^2 + r^2}\frac{1}{\eta_{(r)}} rdr}$$

$$= \frac{\int_{r_1}^{r_2} \frac{rdr}{\left(\frac{r'}{r}\right)^2 + 1}}{\int_{r_1}^{r_2} \frac{rdr}{\left(\frac{r'}{r}\right)^2 + 1}\frac{1}{\eta_{(r)}}}$$

Substituting the new variable $x = r/r' = r\omega/w$,

$$\eta_F = \frac{1 + x^2 - \log_e(1 + x^2)\Big]_{x_1}^{x_2}}{2\int_{x_1}^{x^2} \frac{Cx^4 + x^5}{(1 + x^2)(Cx - 1)}dx} \qquad [15]$$

where C is equal to c_L/c_D.

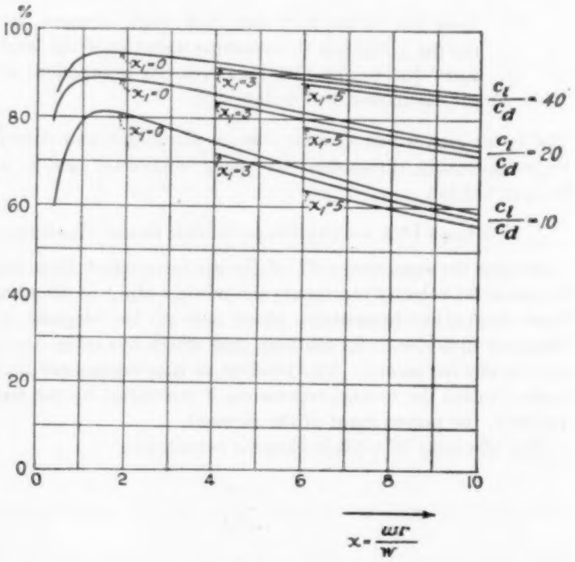

FIG. 16

Fig. 16 gives a solution of [15] for several values of C, which is assumed as constant along the blade, and for values of x_1 equal to 0, 3, and 5; the curve for $x_1 = 1$ is omitted because it practically coincides with the curve for $x_1 = 0$.

V—LOSS DUE TO THE TANGENTIAL VELOCITY COMPONENT OF THE AIR DISCHARGED BY THE FAN

In addition to the loss due to the drag of the blade element, there is still another source of loss which may, in some cases, be even larger. This is due to the rotational velocity of the air beyond the fan, being essentially connected with the pressure rise in the fan area.

In order to explain this, let us consider a two-dimensional flow of air moving with a velocity equal to w from left to right through a row of blades, which latter may move with a velocity u_1 at right angles to w. We assume this flow to be steady, the coordinate system being considered at rest with respect to the blades, and the velocity of flow sufficiently far ahead of the blades is then equal to $v_1 = \sqrt{w_1^2 + u_1^2}$. The pressure at this point may be p_1, and the pressure beyond the blades p_2, according to Fig. 17. Applying the Bernoulli equation to two particles, respectively, ahead of and beyond the blades on the same streamline, we see that a pressure increase must be connected with a velocity decrease:

$$p_1 + \frac{\rho}{2}v_1^2 = p_2 + \frac{\rho}{2}v_2^2$$

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[2] Prandtl, L., "Application of Modern Hydrodynamics to Aeronautics," Rep. No. 116 of National Advisory Committee for Aeronautics.

[3] Betz, A., "Eine Erweiterung der Schraubenstrahltheorie," *Zeit. f. Flugtechnik u. Motorluftschiffahrt*, vol. 11 (1920), p. 105.

or

$$p_2 - p_1 = \frac{\rho}{2}(v_1^2 - v_2^2) = \frac{\rho}{2}[w_1^2 + u_1^2 - (w_2^2 + u_2^2)]$$

The axial velocity, however, being equal on both sides of the blades ($w_1 = w_2$),

$$p_2 - p_1 = \frac{\rho}{2}(u_1^2 - u_2^2) \ldots\ldots\ldots\ldots\ldots [16]$$

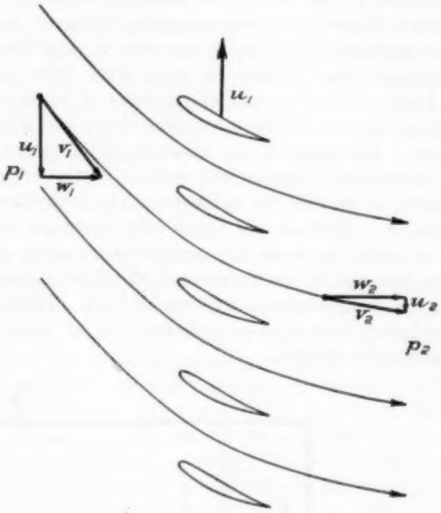

FIG. 17

Returning now to the first coordinate system in which the blades move with a speed equal to u_1 it is seen that the air beyond the blade has a component $u_1 - u_2$ perpendicular to w.

Applying this consideration to the flow through the fan area, it is seen that a pressure rise cannot be obtained without creating a rotational velocity. The kinetic energy due to this velocity must be considered as a loss, provided there are no stationary guide vanes beyond the fan. Assuming that the rotational component is sufficiently small to justify neglecting any centrifugal effects due to it, the loss due to the rotational velocity is

$$\int_{r_1}^{r_2} 2\pi r dr w \rho \frac{(u_1 - u_2)^2}{2} = \pi w \rho \int_{r_1}^{r_2} (\omega r - u_2)^2 r dr \ldots [17]$$

If the variation of the pressure difference on both sides of the fan is given as a function of the radius,

$$p_1 - p_2 = p_t = \frac{\rho}{2}(\omega^2 r^2 - u_2^2) = f(r) \ldots\ldots\ldots [18]$$

Equation [17] can be solved for a certain axial velocity and for a certain fan speed.

It can easily be shown by Equations [17] and [18] that for a constant pressure difference over the fan area corresponding to a constant circulation along the blade, the region near the hub contributes a much larger share of kinetic energy due to rotational velocities than the region near the blade tip. For this reason it is advisable, as mentioned before, to base the design of a fan upon a distribution of circulation which decreases with decreasing distance from the hub.

VI—RESTRICTIONS WITH REGARD TO THE VALIDITY OF THE ASSUMPTIONS PREVIOUSLY MADE

At the very outset it should be emphasized that the considerations regarding the blade-element theory and the formula expressing the efficiency no longer hold when the ratio of dynamic pressure to total pressure, i.e., φ^2/ψ, becomes too small. Particularly is it necessary to exercise care in the case of fans of large power output per unit area. Generally we may say that from about $\varphi^2/\psi = 0.3$ the centrifugal forces acting upon the boundary layer of the blades become more and more important, until finally for values in the neighborhood of $\varphi^2/\psi = 0.1$ the flow around the blades is entirely changed.

Another important question is whether the assumption holds that the performance of an element of a fan blade is the same as it is if considered an element of an air wing. In other words, can we apply the c_L, c_D curves of airfoils, determined by wind-tunnel tests, in designing a fan and calculating its efficiency? Generally speaking, it may be said that this assumption is the more justified the larger the ratio

$$\frac{w}{\omega r_2} = \varphi \ldots\ldots\ldots\ldots\ldots\ldots [19]$$

and the smaller ratio

$$\frac{T}{A\frac{\rho}{2}w^2} \ldots\ldots\ldots\ldots\ldots\ldots [20]$$

These, it may be noted, are the same restrictions made in the propeller theory—with the difference, however, that for a propeller the requirements are usually fulfilled in a high degree due to the large axial velocity w. For a theoretical treatment, therefore, the conditions are more favorable for a propeller than for a fan. Substituting Ap_t for T in [20] and considering that $p_t = \psi \frac{\rho}{2}(\omega r_2)^2$, we have, from [19],

$$\frac{T}{A\frac{\rho}{2}w^2} = \frac{\psi \omega^2 r_2^2}{w^2} = \frac{\psi}{\varphi^2} \ldots\ldots\ldots\ldots [20a]$$

The reciprocal of [20], or [20*a*], represents the ratio of dynamic pressure to total pressure, and it has been previously shown that in a set-up such as that shown in Fig. 3 this value is always less than unity, and the expression [20*a*] is thus greater than unity. However, if there is a diffuser beyond the fan, converting a part of the kinetic energy of the air into pressure, as shown in Fig. 18, φ^2/ψ can become considerably greater than unity, and hence the ratio [20*a*] very small. Following the variation of pressure along a streamline as shown in Fig. 18, and assuming a diffuser which converts 80 per cent of the kinetic energy of the air directly beyond the fan into pressure, we obtain

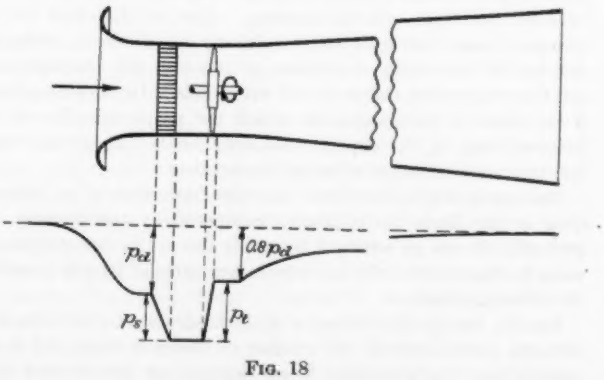

FIG. 18

$$\frac{p_d}{p_t} = \frac{\frac{\rho}{2} w^2}{\frac{\rho}{2} w^2 + p_s - 0.8 \frac{\rho}{2} w^2}$$

If p_s is negligible, we have $p_d/p_t = 5$, or

$$\frac{T}{A \frac{\rho}{2} w^2} = \frac{\psi}{\varphi^2} = \frac{p_t}{p_d} = \frac{1}{5}$$

Tests recently made with different fans, using a test equipment described later, have shown that for a value of $\varphi = 0.2$, the c_L, c_D curves can be used fairly well down to a value of $\psi/\varphi^2 = 0.3$. For larger values of φ this limit probably can be somewhat increased.

There is, however, still another difficulty regarding the application of the c_L, c_D curves. It is well known that for a certain angle of attack the lift and especially the drag of an air wing depend largely on the aspect ratio, i.e., on the ratio of the width of a wing to its length. In addition to the profile drag caused by air friction along the airfoil, there is the so-called induced drag, caused by air flowing around the ends of the wing, due to the pressure drop at these points from the region of higher pressure below the wing to the region of lower pressure above the wing. This flow around the ends of the wing, producing two vortices starting at each end, means an additional drag according to the kinetic energy of these vortices. As shown in the so-called Prandtl wing theory, such vortices are not only produced at the end of the wing but at all points along the trailing edge of the wing where the amount of the circulation varies or, where $d\Gamma/dx \gtrless 0$, if x is parallel to the length of the wing. This induced drag, which is very often the greater portion of the total drag, depends largely on the aspect ratio. Not each arbitrary distribution of the circulation along the wing produces the same induced drag for a given lift and wing area, but it has been proved that a certain distribution can be considered the best one, giving the minimum of drag. In the present case, the circulation has to vary like the ordinates of an ellipse, having its principal axis as the axis of abscissas. For this elliptical distribution of the circulation the induced drag is proportional to the aspect ratio and to the square of the lift.

Applying these considerations to the induced drag of a fan blade, we have to bear in mind that the flow around the blade tip is very restricted due to the small clearance space between the blade and the housing. Also, at the other end of the blade near the hub, the conditions are different, although due to the decreasing circulation at the hub the circumstances are here more like those at the wing ends. In addition, there is no elliptical circulation, for which the proportionality of the induced drag to the aspect ratio only holds. However, these last two arguments are of minor importance.

Bearing in mind, therefore, that the formation of an induced drag at the blade tip is largely hampered by the housing, we probably do not go wrong if we apply the c_L, c_D curve of an air wing having about a 50 per cent larger ratio of length to width than the fan blade.

Finally, the mutual influence of the blades has to be taken into account, particularly if the number of blades is large and if the ratio $w/\omega r_2 = \varphi$ is small. It all depends on the product $d \times w/\omega r_2$, where d is the distance between the blades; the smaller this product the larger the mutual influence of the blades. The effect of this influence is a decreasing lift and an increasing induced drag for a given angle of attack, in accordance with the multiplane wing theory. Therefore the thrust does not increase in exact proportion to the increase of number of blades as assumed in [8], but somewhat less. Here again the influence is more pronounced for a low value of φ^2/ψ than for a high one, making also in this respect a theoretical treatment more difficult for lower values of φ^2/ψ. The results so far obtained, however, do not give a sufficiently clear picture to enable us to see in what manner the blades interfere with each other.

VII—EXPERIMENTAL SET-UP FOR FAN TESTS

A few words may now be said about the test equipment which was built about a year ago at the Research Laboratories of the Westinghouse Electric & Manufacturing Company in order to study systematically the whole question of propeller-type vs. centrifugal-type fans. Numerous tests have been made with different kinds of fans, varying the number of blades from 2 to 32, and using blades of different aspect ratios and different airfoil sections. Furthermore the influence of the clearance space between blades and housing with respect to the efficiency has been studied, as well as the influence of the smoothness of the blade surfaces. All these and many other tests are very useful not only in order to obtain the performance curves of actually built fans, but also in order to check the various assumptions on which the designs of the fans were based. These tests are being continued, and after completion will be described in a later paper on the subject.

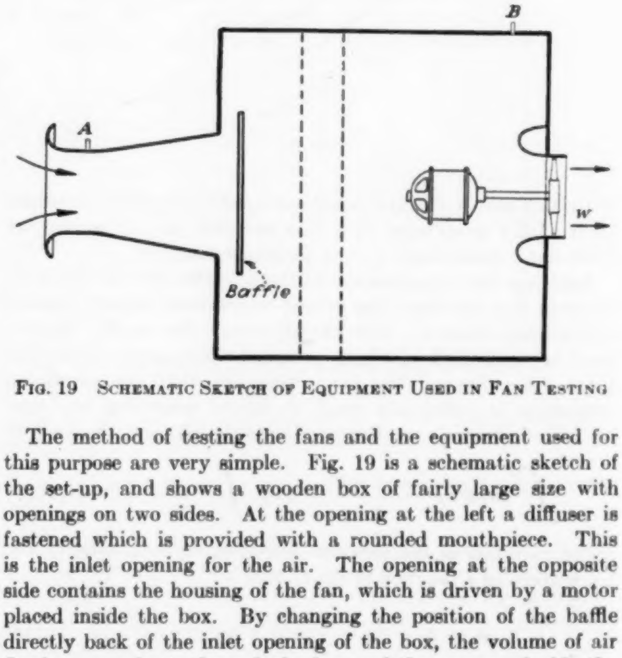

FIG. 19 SCHEMATIC SKETCH OF EQUIPMENT USED IN FAN TESTING

The method of testing the fans and the equipment used for this purpose are very simple. Fig. 19 is a schematic sketch of the set-up, and shows a wooden box of fairly large size with openings on two sides. At the opening at the left a diffuser is fastened which is provided with a rounded mouthpiece. This is the inlet opening for the air. The opening at the opposite side contains the housing of the fan, which is driven by a motor placed inside the box. By changing the position of the baffle directly back of the inlet opening of the box, the volume of air flowing per minute through the box and the pressure inside the box can be varied. A position of the baffle close to the wall represents the condition of a low value of φ^2/ψ, and one far from the wall a high value. In some cases it may be advisable to place screens between the baffle and the fan in order to diminish the kinetic energy of the air flowing at high velocity through the passage between the wall opening and baffle.

The volume of air flowing per unit time through the inlet opening, which volume of course is equal to the volume discharged by the fan, is determined by measuring the pressure difference at point A close to the rounded mouthpiece against atmospheric pressure. The increase of static pressure produced by the fan is measured directly by measuring the pressure at point B against atmospheric pressure. In addition to these two readings, the r.p.m. has to be determined, and also the power input to the shaft of the fan, which latter can be done best by using the motive power as a dynamometer. These readings, carried out for different baffle positions, provide sufficient data with which to calculate the φ, ψ, and η curves, and to obtain thereby a complete knowledge of the performance of the fan being tested.

VIII—Conclusions

In this paper a first attempt has been made to make use of more advanced aerodynamic theory in fan design. While such theory is applied with great success to airplane propellers, it requires considerable modification when applied to fans. This is due chiefly to the fact that the thrust of a propeller is caused by a velocity increase in the axial direction, whereas the pressure rise of a fan is essentially combined with an acceleration of air in a tangential direction. In addition, we may say that, generally speaking, the performance of a propeller is more susceptible to a theoretical treatment than is that of a fan, due to the high axial velocity of the propeller.

However, it may be emphasized that the assumptions on which the theory presented in this paper is based are well founded for that range of applications where the propeller-type fan can be considered superior as compared with the centrifugal type. Only for those working conditions (small volume and large pressure rise) where the propeller-type fan on the whole begins to be less suitable does the theoretical treatment become uncertain. Here further experiments are necessary, showing in what way the assumptions have to be modified for this case.

The propeller-type fan, working under normal conditions, however, can be designed in a rational way according to the outline given in this paper, and the power output as well as the efficiency can be predicted with reasonable accuracy.

Discussion

Th. Troller.[4] The author considers a propeller-type fan of the simplest form; namely, a propeller rotating in a closed tunnel, with no vanes or similar device for straightening the flow. Naturally this propeller has a special significance because of its simplicity and low cost. For many purposes, however, a small improvement of this construction—i.e., the installation of rigid guiding vanes before or behind the propeller—is quite helpful. Such an addition increases but little the cost of construction, but on the other hand makes possible a greater efficiency and pressure rise than obtainable with propeller alone. The calculation of such a fan is similar to the method used by Mr. Tietjens and is even somewhat simpler, for the reason that one assumption of the author may be left out of consideration; namely, the condition of small power loading or small ψ.

The writer wishes to omit the details of the calculation itself, but refers instead to the complete treatment of the question in Heft 10, Aachener Abhandlungen aus dem Aerodynamischen Institut, "Zur Berechnung von Schraubenventilatoren" (The Design of Propeller Fans), by Th. Troller. In this article the writer has discussed fully the principles of the design and calculation of propeller-type fans from an aerodynamical standpoint.

The writer will limit himself to several comments on the physical phenomena in the operation of propeller fans and the special conditions which lead to the consideration of the fans equipped with guiding vanes as an ideal type.

The general problem of the fan is as follows: It is desired to set a certain amount of air in motion against a certain static pressure. The energy required by the fan is equal to the quantity of air times (increase of static pressure plus increase of dynamic pressure of the flow), or more exactly, the integral over the single particles of air. Now, in general, for a given quantity of air and static pressure head this energy is smallest when longitudinal motion of all the fluid particles is constant. That follows from the fact that for a certain sum of the first power of a function (in this case a velocity distribution over the surface, which gives the quantity of air) the sum of the second powers (in this case the dynamic pressures) are smallest for a constant value of the variables. The consideration of the energy imparted to the air without taking into consideration other factors which influence the efficiency of the fan leads, therefore, to the condition that the flow produced by the fan has a uniform axial velocity over the whole cross-section. Calculation of the conditions at the fan shows further that when the author's ψ has greater values than about one, the energy of the air has a greater significance for the efficiency and the construction than all other factors such as friction, etc.

Now it is in general not possible to obtain the necessary condition for higher values of ψ without rigid guiding vanes or a second propeller rotating in the opposite direction. The propeller without vanes sets the air into rotation about the axis. This rotation gives rise to a loss of pressure toward the axis due to the centrifugal force. Since the static pressure against which the air must flow acts uniformly over the whole surface, the air will then flow more slowly or will perhaps even flow backward in this region. In order to prevent this, one must increase the pressure caused by the propeller by giving the propeller a larger working surface toward the axis. From this a new increase of rotation is caused. There results a further increase of under-pressure and further losses through the energy of rotation. This dilemma can be solved only by the use of a guiding vane.

Another point is the operation of the fan under varying conditions: in other words, the taking into account of the entire head-capacity curve instead of a single point on this line. If a fan is to be used under varying operating conditions (for instance, varying static pressures), it is necessary to design the propeller in such a way that the uniform velocity distribution is always obtained. In the aerodynamic construction of the fan there is a possibility of taking this point into consideration, since in the design for a certain operating condition only the product $b \cdot c_1$ is determined. In the Aachen article, the writer has developed the formula which gives the desired expression through which both factors can be determined for the entire propeller. These formulas are of a simple nature and make possible the complete fan design with little labor. Propellers designed on the basis of this formula in this manner have proved themselves always practical.

V. Nekrassoff.[5] The writer would like to take issue with one of the opening statements of the author which says, "Despite the importance of the propeller-type fan, there is, as contrasted with the aeroplane propeller, no theory for it." As far back as 1919 there was published in this country Report 29 of the National Advisory Committee for Aeronautics containing Dr. G. Bothezat's memoir on the general theory of blade screws (including propellers, fans, heliocopter screws, helicoidal pumps, turbo motors).

Even this first memoir, almost exclusively devoted to the problems of aeroplane propellers, has a sufficient amount of data for practical handling of the problems of rational propeller-fan design.

Using this memoir as a basis during research work for the Proctor & Schwartz Company in Philadelphia (1921–1926), the writer made quite a number of tests and theoretical investigations of the disk or propeller type of fans. All these researches finally were condensed in one hexagonal nomogram giving a solution at a glance of all seven fundamental problems of disk fan design. Not long ago the writer was told that this nomogram was still successfully used by the Proctor & Schwartz Company.

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[4] Guggenheim Airship Research Institute, Akron, Ohio.

[5] Bell Telephone Laboratories, New York, N. Y.

In mentioning these facts, the writer would like to emphasize only the following: The theory of the propeller-type fan does exist already; its author, Dr. G. Bothezat, is still living in this country; the United States Navy has adopted as a standard type ventilators (propeller fans) designed by Dr. Bothezat himself; these fans are on the market manufactured by the Bothezat Company, and their performance is the very best one as compared with all other existing propeller fans.

With such purely historical correction, the paper certainly fully preserves all its excellent qualities of usefulness, but hardly can it be accepted as "a first attempt to make use of more advanced aerodynamic theory in fan design," as the author says in the concluding part of his paper.

AUTHOR'S CLOSURE

It may be regarded as a proof of the timeliness of this paper on propeller-type fans that almost at the same time another paper on this subject has been published in Germany by Mr. Th. Troller.

In his paper, the performance of a fan is discussed exclusively in connection with stationary guide vanes in front or behind the fan, and it is pointed out that a constant axial air velocity over the whole cross-section, corresponding to a constant circulation along the blade, will give the highest efficiency. This is particularly true if the total pressure produced by the fan is large. However, if this pressure is comparatively small, the loss of energy due to surface friction of the guide vanes may be larger than the gain of energy due to the conversion of rotational velocity into pressure. In this case, the use of guide vanes will bring practically no increase of the efficiency.

In addition, it happens quite often that there is no space for any guide vanes, as, for instance, for a fan on top of a Diesel locomotive or for a fan mounted on a rotor of a generator. Taking no account of these cases, the author agrees with Mr. Troller that a fan with guide vanes should be preferred if the cost of the fan is not of essential importance.

With respect to the historical remarks of Mr. V. Nekrassoff regarding Dr. G. Bothezat's work on propeller theory, which was outstanding at that time, the modern propeller theory is entirely based on the aerofoil theory as it was developed in 1918 by L. Prandtl. Although Dr. Bothezat realized the importance of the tip vortices of an aerofoil regarding its resistance, he tried in vain to develop a formula for the induced drag. It was shown by Dr. A. Betz in 1914 that the assumption of a constant circulation along the aerofoil and two concentrated tip vortices can be used quite well if one wants to determine the mutual influence of the wing of a biplane or the influence of the neighborhood of the ground. However, this assumption of two concentrated tip vortices, as it was made by Dr. Bothezat, is not sufficiently accurate for a determination of the drag of the aerofoil induced by the "vortex ribbon" behind the aerofoil. This is shown on page 20 of L. Prandtl's Aerofoil Theory.[6] Therefore, the results based on the assumption of only two tip vortices of finite value are in error.

On the other hand, it may be noted that the Report No. 29 of the N.A.C.A. by Dr. Bothezat contains only the first memoir on the propeller, and not the study on fans, although announced in the title. The performance of a propeller at a fixed point, however, cannot be considered equal to the performance of a fan working in a housing with small clearance space at the tip. The distribution of the circulation along the blades (highest efficiency assumed) is quite different in both cases. Dr. Bothezat says on page 166 of his report:

> This first memoir thus contains, besides a general summary of the whole screw problem, a detailed study of the propulsion screw—that is, the propeller—and the different questions in connection with it. A second memoir, directly continuing this one, will contain a special study of screws at a fixed point in their different application principally when used as fans and as helicopters as well as a detailed study of the turbo-motor screws, especiallly as aeroturbines; that is to say, as windmills.

This second memoir, however, containing the study of fans, has not yet been published.

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[6] L. Prandtl, "Tragfluegeltheorie," I. Mitteilung. Nachr. Kgl. Gesellschaft der Wissenschaften, Goettingen, 1918.

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APM-54-14

The Use of Models in Vibration Research

By J. P. DEN HARTOG,[1] EAST PITTSBURGH, PA.

In this paper the theory of dynamic similarity as applied to vibration problems in mechanical engineering is discussed, and a number of specific examples are given.

THE use of models in research of any sort is intimately connected with the science of "dimensional analysis" or "dynamical similarity." At least this has become more and more so since Osborne Reynolds in 1883 published the results of a series of experiments on water flow through pipes in which the pressure drop was expressed not in the velocity of flow V, nor in the diameter of the pipe d, nor the viscosity ν, but in terms of a non-dimensional combination Vd/ν afterward called the Reynolds number.

Reynolds' fundamental work and the subsequent applications and extensions by a great number of scientists, among whom Rayleigh stands foremost, have dealt primarily with hydrodynamic problems. In fact it is no exaggeration to state that most of the great advance made in that science during the last thirty years is due to, or at least has been greatly influenced by, the theory of dynamic similarity.

But the method is by no means restricted to hydrodynamics alone. The whole domain of physics in its broadest sense has been affected by it. In this paper it is intended to discuss some cases in vibration engineering where model tests, mostly in conjunction with some similarity considerations, have led to useful results.

The Theory of Similarity Applied to the Classical Vibration Problem

Consider the simplest case of a single-degree-of-freedom system with viscous damping and a harmonic exciting force (Fig. 1). The equation of motion is usually written as:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = P \sin \omega t \ldots\ldots\ldots\ldots [1]$$

Each term of this equation has the dimension of a force; they are, respectively, the inertia force, damping force, spring force, and exciting force, and are usually measured in pounds. The amplitude x is usually measured in inches here, and in centimeters on the European Continent. This, however, is inconvenient and impractical in more than one respect. The essence of dimensional analysis consists of measuring the various units *not* in inches, pounds, or other standardized units, but rather in units which are of particular use for the problem in hand. For instance, in this case it is better to measure the deflection or amplitude x in terms of the static deflection P/k of the spring under the load P. The quantity $x_{st} = P/k$ is a length intimately tied up with our problem. Then, instead of calculating with x in inches, we calculate with $y = x/x_{st}$, that is, with x expressed in the new unit. It is clear that y is a dimensionless quantity, a pure number.

The damping coefficient c is a force divided by a velocity, and can be expressed in terms of lb/in./sec. But it is more convenient to express it in terms of the "critical" damping coefficient c_c, which is of the same dimension. The critical damping is that damping which is just sufficient to prevent the "free" system from oscillating, but makes it return deadbeat to its equilibrium position from any initial displacement. Therefore, instead of c,

$$\frac{c}{c_c} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{mk}}$$

will be used.

The time t is measured in seconds. But there is a time unit inherent in the phenomenon, namely, the duration of a "free period," $T = 2\pi/\omega_n$ or $T/2\pi = 1/\omega_n$. Expressing the time t in the unit $T/2\pi$ instead of in seconds has many advantages, so that the new variable:

$$\tau = \frac{t}{T/2\pi} = \omega_n t$$

may be introduced.

With these preliminaries, Equation [1] can be attacked. In order to make all terms dimensionless, divide it by P:

$$\frac{m}{P}\frac{d^2x}{dt^2} + \frac{c}{P}\frac{dx}{dt} + \frac{k}{P}x = \sin \omega t$$

or

$$\frac{m}{k}\frac{d^2\left(\frac{x}{P/k}\right)}{dt^2} + 2\frac{c}{2\sqrt{mk}}\sqrt{\frac{m}{k}}\frac{d\left(\frac{x}{P/k}\right)}{dt} + \frac{x}{P/k} = \sin \omega t$$

or

$$\frac{1}{\omega_n^2}\frac{d^2y}{dt^2} + 2\frac{c}{c_c}\frac{1}{\omega_n}\frac{dy}{dt} + y = \sin\frac{\omega}{\omega_n}\omega_n t$$

The last equation is dimensionless except for the time t. Replacing t by its equivalent, τ/ω_n, the equation becomes:

$$\frac{d^2y}{d\tau^2} + 2\frac{c}{c_c}\frac{dy}{d\tau} + y = \sin\frac{\omega}{\omega_n}\tau \ldots\ldots\ldots\ldots [2]$$

In this form it is seen that the dimensionless amplitude y can be considered as a function of the dimensionless time τ, while there are two dimensionless parameters: c/c_c and ω/ω_n.

All the characteristics of the system can be studied from the behavior of a model in which only the damping c/c_c and the forced frequency ω/ω_n can be varied. Giving the model a mass of, say, 1 lb and a spring constant of 20 lb per inch of deflection and determining $x = f(t)$ for a number of values of c/c_c and ω/ω_n experimentally, enables us to predict the motions of any other model of different mass and spring constant.

This solution gives the transient as well as the steady state. If only the steady state is wanted, the dependence on the time

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[1] Westinghouse Research Laboratories, Assoc-Mem. A.S.M.E. Mr. Den Hartog was graduated as an electrical engineer from the University of Delft, Holland, in 1924 and received the degree of Ph.D. in 1929 from the University of Pittsburgh. Since 1925 he has been connected with the Westinghouse Laboratories. He has just been appointed Assistant Professor of Applied Mechanics at Harvard University.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society. Printed in the U. S. A.

r is immaterial and there exists only the relation between y, c/c_c, and ω/ω_n. It is customary to consider y as a function of ω/ω_n and plot the curves for various values of c/c_c, which is the usual family of "resonance curves."

The particular example of a classically damped single-degree-of-freedom system has been tested on models many times. However, it is also amenable to a simple exact calculation, and that fact takes away most of the importance of the model test. In cases where only an approximate solution or no solution at all can be found, a model has to be resorted to. In the following, a number of practical cases which have been solved by this method will be discussed.

LANCHESTER TORSIONAL-VIBRATION DAMPER FOR DIESEL ENGINES

In certain Diesel- or gas-engine installations which are called upon to operate over a wide speed range, it is often advisable to use a vibration damper, for instance, of the Lanchester type.[2]

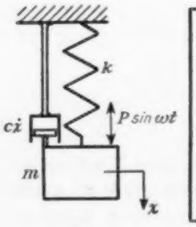

FIG. 1

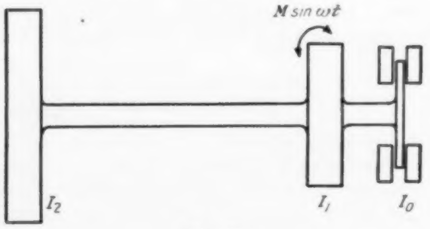

FIG. 2

The calculation of the amplitudes of vibration of such a system is possible only in an approximate manner, so that a model test is useful. Let the simplified system be represented by Fig. 2. A disk of moment of inertia I_2 represents the flywheel or electric generator driven by the engine, and a disk I_1 represents the combined inertia of all cranks, pistons, etc., of the engine. The damper flywheel I_0 can turn freely on the shaft and is restricted from doing so only by the constant friction torque T, which can be given various values by changing the set in the springs provided for that purpose. Let the torque inequalities of the engine be represented by a torque $M \sin \omega t$ acting on I_1. By gradually increasing the frequency ω of this disturbing torque, the angular oscillations α of I_1 will grow larger, until at a certain $\omega_{cr.}$, the "critical speed," the angle α is a maximum. For greater speeds α will decrease again. The maximum of α at the critical speed is naturally of interest, and it is useful to know how this maximum $\alpha_{max.}$ can be affected by changing the damper friction torque T or the damper inertia I_0. In usual constructions I_2 is very large—so large that it can be considered to be "infinitely" large, i.e., I_2 is of no importance for the problem. Then we can write:

$$\alpha_{max.} = f(I_0, I_1, T, M, \omega_{cr.})$$

No other quantities can conceivably affect α. Apply to this equation the usual procedure of dimensional analysis:[3]

$$\alpha_{max.} = I_0^x, I_1^y, T^z, M^u, \omega_{cr.}^v$$

and since $[I_0] = [I_1] = [ml^2]$; $[T] = [M] = [ml^2t^{-2}]$; $[\omega_{cr.}] = [t^{-1}]$ and α is dimensionless, we write:

$$[\alpha_{max.}] = m^{x+y+z+u}\, l^{2x+2y+2z+2u}\, t^{-2z-2u-v}$$

or

$$x + y + z + u = 0$$
$$2z + 2u + v = 0$$

From these equations we solve y and z in terms of the other variables and substitute:

$$\alpha_{max.} = \left(\frac{I_0}{I_1}\right)^x \left(\frac{M}{T}\right)^u \left(\frac{I_1\omega^2_{cr.}}{T}\right)^{\frac{v}{2}} = \left(\frac{I_0}{I_1}\right)^x \left(\frac{T}{M}\right)^{-\frac{v}{2}-u} \left(\frac{M}{I_1\omega_{cr.}^2}\right)^{-\frac{v}{2}}$$

so that the result becomes

$$\alpha_{max.} = f\left(\frac{I_0}{I_1}, \frac{T}{M}, \frac{M}{I_1\omega_{cr.}^2}\right) \qquad [3]$$

More than this the dimensional analysis is not able to predict. Another consideration, however, will lead to the additional fact that $\alpha_{max.}$ is directly proportional to $M/I\omega_{cr.}^2$. This can be seen as follows: If all forces acting on the disk I_1 increase in the same ratio, the motion of I_1 will remain similar to itself. There are four forces (moments) acting, namely: the disturbing moment M, the damper moment T, the spring restoring moment of the shaft, and the inertia moment of the disk. The two latter moments are directly proportional to the angle $\alpha_{max.}$, provided that $\omega_{cr.}$ remains the same. Now assume M and also T to be twice as large as before. In order to get similar motion, it is only necessary to make $\alpha_{max.}$ twice as large so that all four forces increase in the same ratio. In Equation [3] the ratios T/M and I_0/I_1 remain unchanged; $M/I\omega^2_{cr.}$ becomes twice as great and $\alpha_{max.}$ is twice as great. Clearly on account of this, result [3] can be simplified to:

$$\frac{I_1\omega_{cr.}^2\alpha_{max.}}{M} = f\left(\frac{T}{M}, \frac{I_0}{I}\right) \qquad [4]$$

In other words, in a model of the set-up of Fig. 2, we have to measure the various critical $\alpha_{max.}$-values for various values of T and I_0. Then all these results have to be plotted in a diagram with $I_1\,\omega^2_{cr.}\alpha_{max.}/M$ as ordinate and T/M as abscissa. There will be a number of curves, each belonging to a definite value of I_0/I_1.

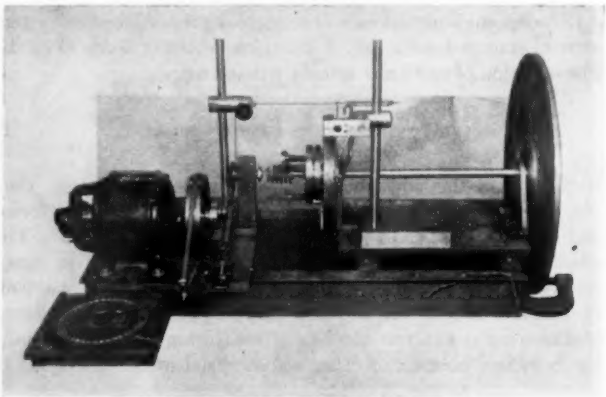

FIG. 3 LANCHESTER DAMPER MODEL

A photograph of the model on which the experiments were actually carried out is shown in Fig. 3, so that it may be said that the problem has been completely solved.[2]

In this example the model analogy has been obtained in the usual manner, without referring to the differential equation as was done with the classical single-degree-of-freedom problem. In the following example, the latter method will again be applied.

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[2] "Torsional Vibration Dampers," Trans. A.S.M.E., vol. 52 (1930), paper APM-52-13.

[3] See, for instance, the excellent article on "Dynamic Similarity," by Levy, in Glazebrook's "Dictionary for Applied Physics," vol. 1.

Running Through a Critical Speed

It is sometimes practically impossible to construct a gas engine that will be free from torsional critical speeds in the entire range from standstill up to the running speed. Many engines of this kind have one or more critical speeds under the running speed. These are not considered dangerous since it is reasoned that the engine only runs through them during starting, and that so quickly that the vibrations have no time to build up. This qualitative statement leads to the problem of investigating the amplitudes of a one-degree system running through its critical speed at a definite constant rate of acceleration. In order to express the problem in mathematical language Equation [1] has to be modified only in its last term, in which the frequency ω is now proportional to the time t, so that:

$$m\ddot{x} + c\dot{x} + kx = P \sin \alpha t^2 \ldots\ldots\ldots\ldots [5]$$

With these changes, the differential Equation [7] becomes

$$N^2 \frac{d^2y}{d\tau_1^2} + 2N \frac{c}{c_c}\frac{dy}{d\tau_1} + y = \sin \frac{1}{\pi N^3} \cdot \tau_1^2 \ldots\ldots \quad [8]$$

from which it is seen that the amplitude y is a function of τ_1, of N, and of c/c_c. We are interested in knowing the maximum amplitude $y_{\text{max.}}$ and not so much the time at which this occurs. Therefore

$$y_{\text{max.}} = f\left(N, \frac{c}{c_c}\right)$$

A model has been built by Mr. J. G. Baker in which the quantity N can be varied by means of a special drive, consisting of a d-c motor with a flywheel and a constant-current regulator. This gives a constant acceleration, the rate of which can be varied from experiment to experiment by a change in the current-regulator circuit. The damping is by means of a d-c magnet, in which the current can be adjusted over a wide range of values.

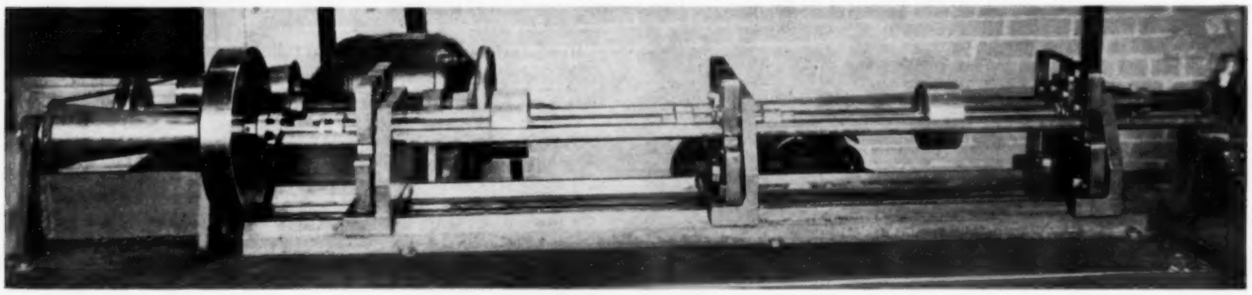

Fig. 4 Model for Testing Critical Speeds

However, to say that αt is the frequency is incorrect. This can be understood from what follows. A cycle of the force occurs when αt^2 increases by 2π. The time T during which this takes place is found from $\alpha(t + T)^2 - \alpha t^2 = 2\pi$, or $2\alpha Tt = 2\pi$, so that $T = \pi/\alpha t$. The frequency thus is:

$$\omega = \frac{2\pi}{T} = 2\alpha t \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

Transform [5] in the same manner that [1] was transformed into [2]:

$$\frac{d^2y}{d\tau^2} + 2\frac{c}{c_c}\frac{dy}{d\tau} + y = \sin\left(\frac{\alpha}{\omega_n^2}\right)\tau^2 \ldots\ldots\ldots [7]$$

The new dimensionless group α/ω_n^2 appears, which is a measure for the rate of acceleration. A more easily imaginable quantity, which is inversely proportional to the acceleration, is the number of "free" cycles N to reach resonance. Since $\omega = 2\alpha t$, in general $\omega_n = 2\alpha t_{\text{res.}}$, where $t_{\text{res.}}$ is the time in which resonance is reached. Since τ is a free period,

$$N = t_{\text{res.}}/\tau = \frac{\omega_n}{2\alpha} \bigg/ \frac{1}{2\pi\omega_n} = \pi\frac{\omega_n^2}{\alpha}$$

Instead of α/ω_n^2, therefore, we write $1/\pi N$.

The various curves reach resonance after N free cycles, that is, at the time $N\tau$. In this manner each rate of acceleration $1/N$ will give a curve with a different resonance point. It is more convenient in this case to have all curves plotted on the same resonance point, which can be done by introducing the new unit of time τ_1:

$$\tau_1 = N\tau$$

The test results are conveniently plotted with y as ordinate against $1/N$ as abscissa, and with c/c_c as the parameter for the various curves.

An analytical solution of this problem has recently been obtained by Lewis.[4]

Models for Lateral-Vibration Phenomena

It may have been noticed that both practical examples discussed thus far deal with the amplitudes of *torsional* vibration. There is a definite reason for this. A torsional model can be built so that all inertia effects are localized in the shaft, flywheels, etc., and that no inertia reaction is transmitted to the bearings. Therefore, in torsional vibrations of this type, flexibility in the bearings does not affect the location of the critical speeds, nor can there be any energy dissipation through the bearings to the foundation.

With lateral vibrations this is not the case, and the loss of energy in the foundation is so difficult to prevent or to predict that no worth-while quantitative model determinations of the amplitude of lateral vibrations have ever been carried out.

The manner in which the critical speeds are affected by gyroscopic action, non-symmetrical shaft sections, bearing flexibilities, etc., can be studied quantitatively. A model for this purpose is shown in Fig. 4, but so far the amplitude determinations made have been only of a qualitative nature.

Lateral vibrations, for instance in whirling shafts, are either due to unbalance and other "forced" effects, or to "self-induction." The phenomena of the latter class are especially interesting, and have become of practical importance in the last few

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[4] F. M. Lewis, "Vibration During Acceleration Through a Critical Speed;" presented at the National Applied Mechanics Meeting of the A.S.M.E., New Haven, Conn., June 23, 1932.

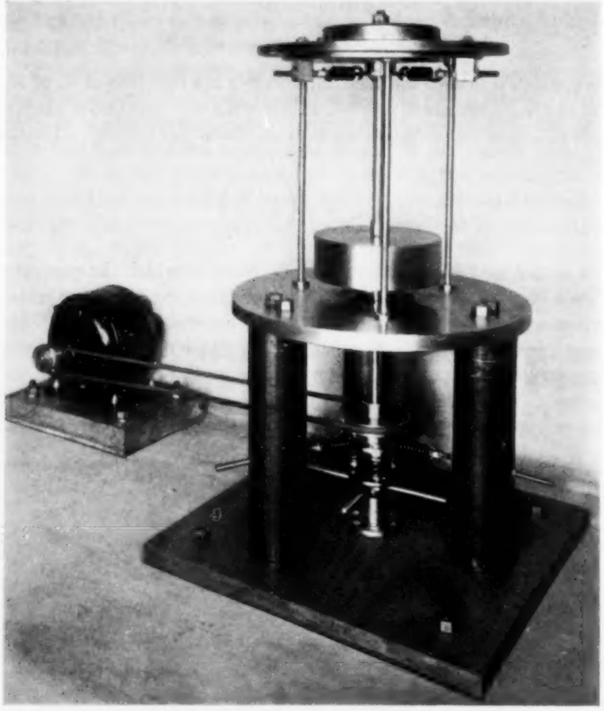

FIG. 5 SELF-INDUCED VIBRATION CAUSED BY THRUST-BEARING FRICTION

years. The first real troubles on machines due to this effect were found in 1924 by Kimball[5] and Newkirk.[6] They explained that the whirling was caused by internal friction in the shaft or by certain lubrication conditions in the bearings, and tested their theories qualitatively on models. Their work has not received the attention it deserves, however; for instance, in Stodola's classical treatise on steam turbines, a phenomenon is described and declared to be inexplicable which is clearly due to internal shaft friction, in the manner that Kimball has explained it.[7]

A different case of self-induced vibration was observed on a vertical-hydraulic-turbine installation, in which the amplitudes grew worse with increasing load on the thrust bearing. A quantitative theory was devised and a model (Fig. 5) built which could be set in violent vibration by the friction on the thrust bearing.[8]

Recently electric transmission lines of which the cables had assumed non-symmetrical sections on account of ice or sleet were found vibrating violently in a transverse wind. The phenomenon was explained as a case of aerodynamic self-induction, and a toy described by Lanchester in 1907 was found to be a model whereby the stability of various ice-coated cable cross-sections could be judged.[9]

In all these cases, however, no reliable amplitude measurements could be made because of the uncertainty of the energy dissipation in the foundation.

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[5] A. L. Kimball, "Internal-Friction Theory of Shaft Whipping," *General Electric Review*, 1924, p. 244.

[6] B. L. Newkirk, "Shaft Whipping," *General Electric Review*, 1924, p. 169.

[7] Stodola, "Steam and Gas Turbines," vol. I, p. 434, New York, 1927; also p. 367 of the 1930 German edition.

[8] J. G. Baker, "Self-Induced Vibrations;" presented at the National Applied Mechanics Meeting of the A.S.M.E., New Haven, Conn., June 23, 1932.

[9] "Transmission-Line Vibration Due to Sleet," A.I.E.E. Summer Convention, Cleveland, June 20, 1932.

APM-54-15

Forced Vibrations With Non-Linear Spring Constants

By J. P. Den Hartog[1] and Stanley J. Mikina,[2] East Pittsburgh, PA.

J. P. Den Hartog

In many technical applications, couplings are used which have springs in them with an initial set. In this paper an exact theory of the forced vibrations of a system with such a coupling is given. Conclusions concerning the shift in critical speed with the amount of initial set are made. The solution obtained is compared with the existing approximate solution. The more general case of a coupling with stops is solved in an appendix.

S. J. Mikina

In the classical theory of vibrations it is assumed that the springs figuring in the system have a linear characteristic, i.e., the spring forces are proportional to the displacements. In many applications, however, the dictates of practical design require the use of spring members whose restoring force cannot be formulated analytically as a simple function of the displacement over the entire range of possible displacements, usually because of discontinuities introduced by clearances, initial spring set, "stops," etc. Three cases of non-linear springs which are encountered quite often in practice are represented schematically in Fig. 1.

In the first there are clearances between the mass and the spring, thus allowing the former to move freely a certain distance before striking the spring. The second case differs from the first in that the springs are built into the apparatus with an initial set, while the mass rests free (but without clearance) between the members that maintain the initial set. Under these conditions, the mass cannot move until a force is applied to it which is at least equal to the force in the springs due to the set.[3] Constructions of this type are frequently used for flexible torsional couplings, such, for instance, as those on the engines of the airship "Graf Zeppelin" before the accident of May, 1929.[4] The third sketch (*c*) shown in Fig. 1 represents a system with elastic "stops." Here the mass is free to execute simple harmonic vibrations until its amplitude of motion reaches a certain limit, at which additional springs or "stops" are struck.

The "natural frequency" of *free* vibrations of systems equipped with springs of any of the three types discussed depends on the amplitude of motion. Methods of determining the relation between amplitude and corresponding natural frequency are known and have been discussed at length in the literature.[5] However, the *forced* vibrations which the systems of Fig. 1 execute under the influence of a harmonic force on the mass have found to date only an approximate solution.[6] The approximation involved consists of assuming sinusoidal motion of the mass and gives crude results, especially for the smaller or slower motions. It is fundamentally possible, however, to obtain an exact solution of this problem by fitting together the linear solutions which hold for different portions of the cycle of motion, as applied before to the problem of vibrations with Coulomb damping.[7] In

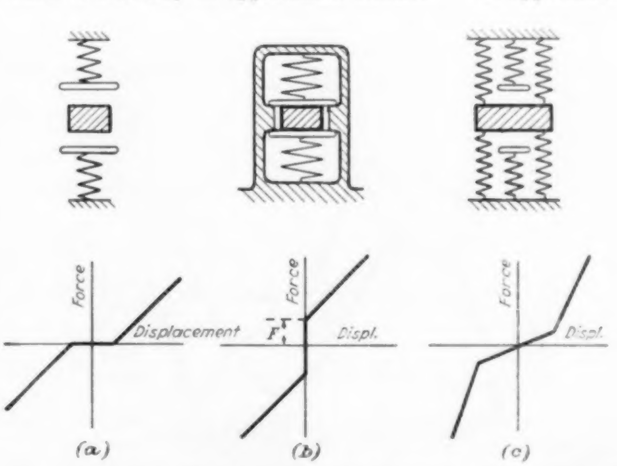

Fig. 1 Three Types of Non-Linear Springs

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[1] Assoc-Mem. A.S.M.E. Mr. Den Hartog was graduated as an electrical engineer from the University of Delft, Holland, in 1924 and received the degree of Ph.D. in 1929 from the University of Pittsburgh. Since 1925 he has been an engineer in the Westinghouse Research Laboratories, with the exception of one year (1930–31) when he was sent to the University of Göttingen for special studies.

[2] Mr. Mikina was born in Dabrova, Poland, in 1907. He was graduated from the University of Michigan in 1930 with a degree in engineering mechanics, and since then has been working on dynamical problems at the Westinghouse Research Laboratories.

[3] This refers to a static loading only. It will be shown later that with periodic forces motion is possible even when the impressed force is numerically less than the force of set.

[4] *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Sept., 1929, p. 465.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] Timoshenko, "Vibration Problems in Engineering," pp. 72–80.

[6] Timoshenko, loc. cit., pp. 81–87. Rüdenberg, "Zeitschrift für Angew. Math. u. Mech.," 1923, p. 463.

[7] J. P. DenHartog, "Forced Vibrations With Coulomb Damping." Presented before the 1930 Applied Mechanics Meeting of the A.S.M.E. at Purdue.

the present paper, case b of Fig. 1 has been worked out completely. The known approximate solution is discussed and compared with the exact results. Cases a and c are more complicated. The general solution for case c (which includes a and b as special cases) has been worked out, however, and is given in Appendix No. 1. Quantitative results for this case may be obtained only by performing numerical calculations similar to those which have been made for the case treated below.

Exact Solution for Springs With Initial Set

The spring force as a function of the displacement as represented in Fig. 1(b) can be written as $-kx \pm F$; where k is the spring constant, x the displacement, and F the force due to set, the + sign holding for $x < 0$ and the — sign for $x > 0$. The differential equation of the motion for $x > 0$ then is:

$$m\ddot{x} + kx + F = P \sin (\omega t + \varphi) \qquad \text{[1]}$$

where φ is a parameter used for the sake of convenience. The following abbreviations may now be introduced:

$$\frac{k}{m} = \omega_n^2 \qquad \frac{F}{k} = x_f \qquad \frac{P}{k} = a \qquad \text{[2]}$$

but it is to be distinctly understood that ω_n is *not* the "natural frequency of free vibration," but is rather some characteristic frequency which is only used for simplifying the notation.

Substituting the expressions [2] into Equation [1],

$$\ddot{x} + \omega_n^2(x + x_f) = a\omega_n^2 \sin (\omega t + \varphi) \qquad \text{[3]}$$

the general solution of which is:

$$x = A \sin \omega_n t + B \cos \omega_n t + aV \sin (\omega t + \varphi) - x_f \qquad \text{[4]}$$

where

$$\beta = \frac{\omega_n}{\omega} \text{ and } V = \frac{\beta^2}{\beta^2 - 1} = \frac{\omega_n^2}{\omega_n^2 - \omega^2}, \text{ the "resonance function"} \qquad \text{[5]}$$

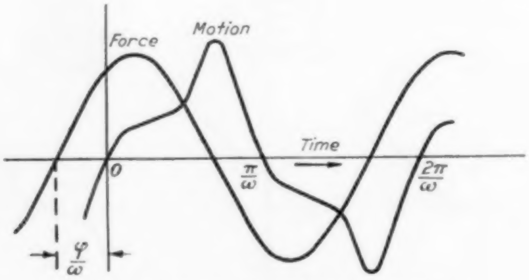

Fig. 2 Force and Motion Curves

Since we desire only the solution representing the steady state of motion and are not interested in transients, the conditions prevailing for forced vibrations are given in Fig. 2. The solution [4] being true for $x > 0$, holds for $0 \leq t \leq \pi/\omega$. Evidently the three boundary conditions which must be satisfied are:

$$\left.\begin{array}{ll} t = 0 & x = 0 \\ t = \dfrac{\pi}{\omega} & x = 0 \\ (\dot{x})_{t=0} = -(\dot{x})_{t=\frac{\pi}{\omega}} & \end{array}\right\} \qquad \text{[6]}$$

or calculated with the aid of [4],

$$\left.\begin{array}{l} B + aV \sin \varphi - x_f = 0 \\ A \sin \beta\pi + B \cos \beta\pi - aV \sin \varphi - x_f = 0 \\ A(1 + \cos \beta\pi) - B \sin \beta\pi = 0 \end{array}\right\} \qquad \text{[7}a, b, c\text{]}$$

These three equations suffice to determine A, B, and the parameter or "phase angle" φ. From [7c],

$$A = B \frac{\sin \beta\pi}{1 + \cos \beta\pi}$$

which, when substituted in [7a, b], gives

$$B + aV \sin \varphi - x_f = 0$$

$$B - aV \sin \varphi - x_f = 0$$

from which

$$A = x_f \frac{\sin \beta\pi}{1 + \cos \beta\pi}; \quad B = x_f; \quad \sin \varphi = 0 \qquad \text{[8]}$$

Substituting these values in [4] gives the solution for the motion:

$$\frac{x}{a} = \frac{x_f}{a}\left(\frac{U}{\beta} \sin \omega_n t + \cos \omega_n t - 1\right) \pm V \sin \omega t \qquad \text{[9]}$$

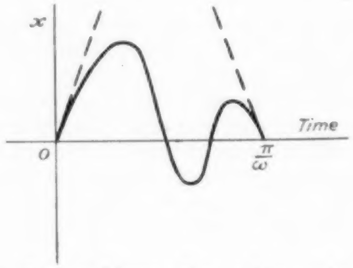

Fig. 3 Type of Motion Ruled Out by Analysis

where

$$U = \frac{\beta \sin \beta\pi}{1 + \cos \beta\pi}$$

the "Coulomb damping function." The $\pm$ sign in [9] follows from the fact that $\sin \varphi$ being zero, φ may be either 0 or 180 deg.

The solution represented by Equation [9] gives only the shape of the motion curve and not its maximum amplitude. The calculation of the latter, however, cannot be effected by the usual analytical methods since these lead to a transcendental in t owing to the simultaneous occurrence of $\sin \omega t$ and $\sin \omega_n t$ which, generally, cannot be expressed in terms of each other. It is therefore necessary to resort in each case to numerical calculation to obtain the maximum amplitude of forced vibrations for a given system. In this connection, it must be remembered that the solution obtained is valid only under the assumptions imposed in Fig. 2: in particular, the motion is not allowed to pass under the zero line in the interval under consideration. In certain cases (for slow forces or large β and small values of P/F), Equation [9] will lead to a motion like that of Fig. 3, which fits all the conditions [6] but passes into the domain of the other differential equation before $t = \pi/\omega$. Such solutions naturally have to be discarded.

Fig. 4 shows a number of calculated motion curves for $\beta = \omega_n/\omega = 1.75$ for various values of $P/F = a/x_f$.[6] It is seen that the curves deviate more and more from a pure sinusoid the smaller P/F becomes. The lowest curve in the figure touches the horizontal axis. Diagrams like that of Fig. 4 have been calculated for a number of values of β. The maximum amplitudes of all the curves in these various diagrams are plotted together in Fig. 5, where the solution appears as a family of curves, each curve giving the amplitude x/a as a function of the frequency for a definite value of P/F. For $P/F = \infty$ the classical resonance curve results, and for smaller values of P/F curves displaced, therefrom. Bearing in mind that the solution is restricted only to values of $x > 0$, an inspection of Equation [9] shows that in the range of $0 < \omega/\omega_n < 1$ these curves must all lie below the classical curve, while in the region above resonance, amplitudes both above and below those corresponding to the classical curve are possible. This is due to the fact that for values of $\beta < 1$, a positive solution may be obtained for φ equal to either 0 or π: $\varphi = \pi$ gives solutions all lying above the classical curve, while $\varphi = 0$ gives solutions lying both above and below it. The solid-line curves in the region where $\omega/\omega_n > 1$ refer to the case in which the phase of the disturbing force is opposite to that of the motion as in the classical curve, while the broken-line curves represent cases where the force and motion are in phase.

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[6] These curves are symmetrical about $\omega t = \pi/2$, as can be shown by substituting $\pi - \omega t$ for ωt in [9]. That this is the case becomes more evident when the solution is obtained in the form of an infinite

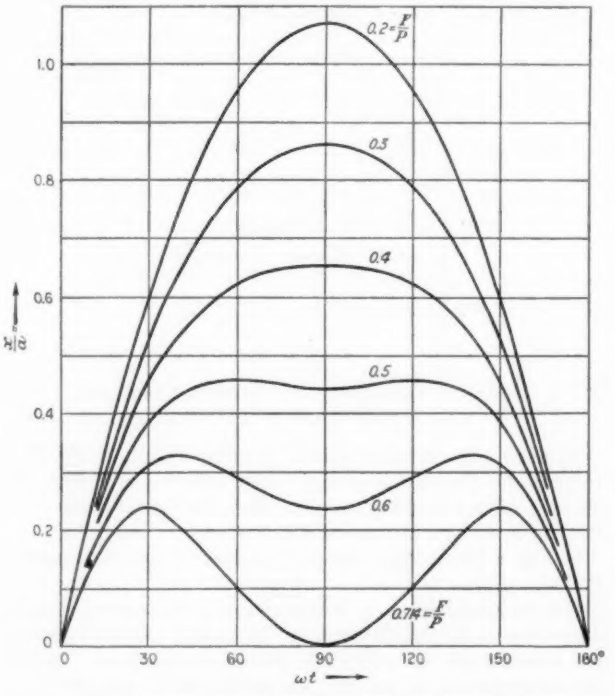

FIG. 4 MOTION CURVES FOR VARIOUS IMPRESSED FORCES

The single solution found below resonance is valid only in a restricted range, the boundary of which is indicated by the thin broken line. Below and to the left of this line no solution of the type of Fig. 2 can exist. It is of interest to know for what values of P/F we obtain no, one, or two solutions. This is shown in Fig. 6 where P/F is plotted against ω/ω_n. Again in the shaded area the analysis does not lead to a result. Outside this area, however, the field is divided into two main regions. Besides the two solutions with real amplitude obviously there must be a trivial solution with zero amplitude for values of P/F less than unity. This is true even in the shaded region where the analytical process breaks down.

The existence of solutions other than $x = 0$ for fractional values of P/F, as indicated in Figs. 5 and 6, seems rather paradoxical at first thought: the impressed force being numerically less than the initial set-up in the springs, we should expect the system to remain quiescent at the origin. It must be remembered, however, that we are dealing here with "forced" vibration, that is to say, with motions which can be maintained by a harmonic force once they have been started, without considering the manner in which the steady state is attained. If, then, we imagine for a moment the existence of an agency here which will displace the system from its position of equilibrium at the origin (as might well be the case in an actual construction as a result of some initial shock or jar due to impact), the resulting motion can be maintained by the existing harmonic force and a solution of the type calculated be realized.

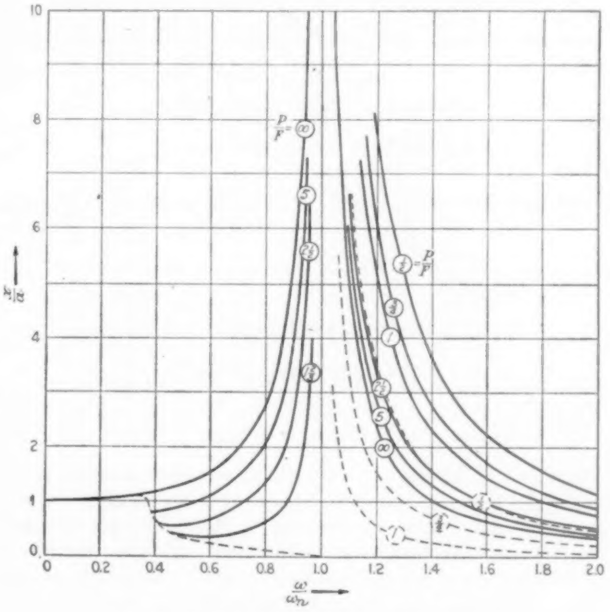

FIG. 5 AMPLITUDE-FREQUENCY DIAGRAM

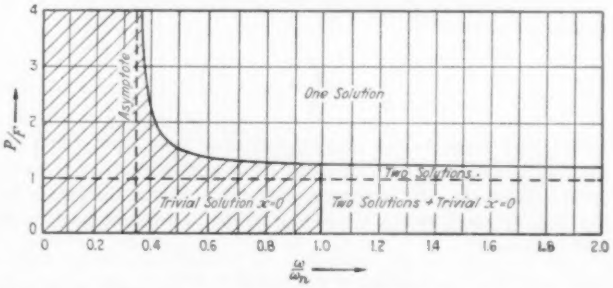

FIG. 6 SHOWING REGIONS OF ONE OR MORE THAN ONE SOLUTION

EXISTING APPROXIMATE SOLUTION[9]

The differential Equation [1] can be written as $m\ddot{x} + f(x) =$

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series by representing the set-up force in the springs by its Fourier development. The motion is then given by

$$\frac{x}{a} = \frac{x_f}{a}\frac{4\beta^2}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin n\omega t}{n(n^2-\beta^2)} \pm V\sin\omega t$$

The analysis involved in this method of solution is analogous to that used in the problem of forced vibrations with Coulomb damping, loc. cit., footnote 7.

[9] Rüdenberg, loc. cit., footnote 6.

$P \sin \omega t$, where $f(x)$ stands for the entire diagram of Fig. 1(*b*). The motion, which in reality is not sinusoidal, is now assumed to have the following property of a sinusoid, namely, that at the point of maximum amplitude, $\ddot{x} = -\omega^2 x$. Further, it is assumed that this maximum coincides with an extreme value of the force. The equilibrium of all forces at the maximum amplitude is then expressed by:

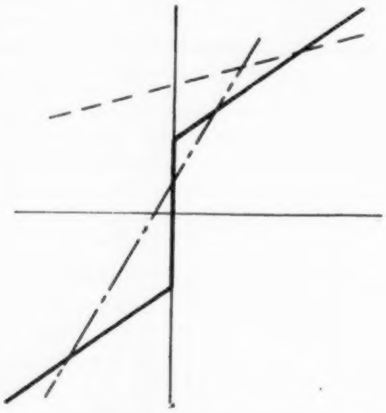

Fig. 7 Illustrating the Approximate Solution

$$-m\omega^2 x + f(x) = \pm P$$

or

$$f(x) = \pm P + m\omega^2 x \quad \text{[11]}$$

This equation admits of a very simple graphical solution, shown in Fig. 7. The right-hand member of [11] is a straight line with slope $m\omega^2$ and ordinate intersection P. Where this line intersects the $f(x)$ curve, a solution for x is obtained. It is seen that besides an intersection at $x = 0$, there may be either one or two intersections, depending on whether P/F is greater or less than unity (whereas the exact solution gives $4/\pi$).

For purposes of calculation, the analytical expression for the points of intersection is:

$$\frac{x}{a} = V\left(\pm 1 - \frac{F}{P}\right) \quad \text{[12]}$$

For very large values of x/a, i.e., near resonance, when the motion is practically a sinusoid, this approximation is fairly good. However, the accompanying table shows that even for motions as great as $2^1/_2$ times statical the error can be as large as 75 per cent.

Values of x/a for $\beta = 1.05$

P/F	Exact solution	Existing approximate solution
1.67	2.46	4.28
2.50	5.21	6.43
5.00	8.96	8.57

This is not very surprising since for small values of the amplitude the motion deviates considerably from a sinusoid, as can be seen, for instance, in Fig. 4. The approximate solution really expresses only a certain relation between the maximum curvature and amplitude of the actual motion curve and the sinusoid replacing it, and does not enter into the wave shape. Therefore rather large errors are to be expected.

Shift of Resonant Speed Due to the Set in Springs

Fig. 5 shows that the resonance curves for all values of P/F go to infinity at $\omega = \sqrt{k/m}$, that is, at the "natural frequency" of the system without set-up in the springs. This at first thought would indicate that the "critical speed" of the system is not affected by a set in the springs, which, however, is contrary to experimental evidence. As mentioned earlier, the failure of the crankshafts of the "Graf Zeppelin" was due to the fact that before the fatal trip a change was made in the set-up of the couplings with a consequent shift in the critical speed.

The results of Fig. 5 are academic in that no damping has been considered. Infinite amplitudes are, after all, unreal quantities. For practical purposes it is better to consider at which frequency the amplitudes become, say, four times the statical. Examining Fig. 5, it is seen very clearly, for instance, in the curve $P/F = 2^1/_2$, that both branches of the (undamped) resonant curve have been shifted to the right.

The influence of damping on the classical case is known to have the effect of rounding the top of the two branches, extending to infinity; the maximum of this top lying approximately midway between the two undamped branches.

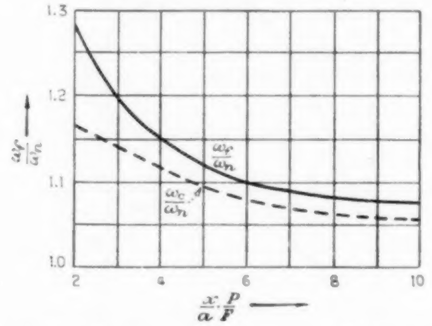

Fig. 8 Resonant Frequency and Natural Frequency Versus Amplitude of Vibration

To draw a practical conclusion from the results of Fig. 5 we define as the "critical frequency" ω_c of a certain P/F curve the frequency lying midway between the two intersections of the P/F curve with the horizontal $x/a = 4$.

In Fig. 8 this critical speed ω_c/ω_n has been plotted against P/F as a broken line.

The "natural frequency of free vibration" is another quantity depending on the amplitude or on P/F, which does not necessarily coincide with our arbitrarily defined critical speed. The free natural frequency ω_f can be easily calculated by computing the time necessary to move one-quarter cycle from the middle position to one extremity. The result of such a calculation is

$$\frac{\omega_f}{\omega_n} = \frac{\pi}{2 \operatorname{arc\,cos}\left(\dfrac{1}{1 + \dfrac{x}{a}\dfrac{P}{F}}\right)} \quad \text{[13]}$$

and is shown graphically as the full line of Fig. 8. It is seen that the natural free frequency and the arbitrary critical frequency do not coincide but follow the same tendency.

An exact solution for the case with viscous damping may be obtained either in a manner similar to that used above for the undamped motion or in terms of a Fourier series, but involves very considerable numerical calculations.

Conclusions

An exact solution for the forced vibrations without damping under a harmonic force of the system of Fig. 1(*b*) has been given, the main results of which appear in Fig. 5.

The existing approximate solution has been discussed and is shown to lead to considerable errors.

The shift in the critical speed, due to the set in the springs, has been discussed in an approximate manner.

In Appendix No. 1 a complete theory of the general case shown in Fig. 1(*c*) is worked out.

Appendix No. 1

SOLUTION OF THE GENERAL CASE

Consider the general case of Fig. 1(*c*), where two different linear flexibilities are involved. The spring force [Fig. 9(*a*)] can be expressed as $-k_1x$ until it reaches the value F; above that, it becomes $-\left\{F + k_2\left(x - \frac{F}{k_1}\right)\right\}$ or using the abbreviation $x_f = F/k_1$ it can be written:

$$-k_2x - (k_1 - k_2)x_f$$

Take as the origin of time the instant when the mass just strikes the stop in its upward motion in Fig. 1(*c*) and call t_1 the (unknown) time at which the mass just leaves the stop in its downward motion [Fig. 9(*b*)]. The differential equations then become:

$$m\ddot{x} + k_2x + (k_1 - k_2)x_f = P \sin(\omega t + \varphi) \ldots\ldots [14]$$

between $0 < t < t_1$ and

$$m\ddot{x} + k_1x = P \sin(\omega t + \varphi) \ldots\ldots\ldots\ldots [15]$$

between $t_1 < t < \pi/\omega$. The general solution of [14] is:

$$x_a = A \sin \omega_2 t + B \cos \omega_2 t + a_2V_2 \sin(\omega t + \varphi) + \left(1 - \frac{k_1}{k_2}\right)x_f \ldots\ldots [16]$$

and of [15] is:

$$x_b = C \sin \omega_1 t + D \cos \omega_1 t + a_1V_1 \sin(\omega t + \varphi) \ldots [17]$$

For the determination of the six unknown A, B, C, D, φ, and t_1, we have the six boundary conditions:

$$\left.\begin{array}{lll} t = 0 & x_a = x_f \ldots\ldots\ldots\ldots & (a) \\ t = t_1 & x_a = x_b = x_f \ldots\ldots & (b, c) \\ t = \pi/\omega & x_b = -x_f \ldots\ldots\ldots & [d] \\ t = t_1 & x_a = x_b \ldots\ldots\ldots\ldots & [e] \\ (\dot{x}_a)_{t=0} & = -(\dot{x}_b)_{t=\pi/\omega} \ldots\ldots & [f] \end{array}\right\} \ldots\ldots\ldots [18]$$

From [18, *a*, *b*, *c*, *d*] we can express the four integration constants in terms of t_1 and φ with the following results:

$$\left.\begin{array}{l} A = \left(\frac{k_1}{k_2}x_f\frac{1 - \cos \omega_2 t_1}{\sin \omega_2 t_1}\right) + \sin \varphi\left(\frac{a_2V_2}{\tan \omega_2 t_1} - \frac{a_2V_2 \cos \omega t_1}{\sin \omega_2 t_1}\right) - \cos \varphi\left(a_2V_2\frac{\sin \omega t_1}{\sin \omega_2 t_1}\right) \\ B = \frac{k_1}{k_2}x_f - \sin \varphi(a_2V_2) \\ C = x_f\frac{\cos \beta_1\pi + \cos \omega_1 t_1}{\sin(\omega_1 t_1 - \beta_1\pi)} + \sin \varphi \cdot a_1V_1 \cdot \left(\frac{\cos \omega t_1 \cos \beta_1\pi + \cos \omega_1 t_1}{\sin(\beta_1\pi - \omega_1 t_1)}\right) + \cos \varphi\left(a_1V_1\frac{\sin \omega t_1 \cos \beta_1\pi}{\sin(\beta_1\pi - \omega_1 t_1)}\right) \\ D = x_f\frac{\sin \beta_1\pi + \sin \omega_1 t_1}{\sin(\beta_1\pi - \omega_1 t_1)} + \sin \varphi \cdot a_1V_1 \cdot \left(\frac{\cos \omega t_1 \sin \beta_1\pi + \sin \omega_1 t_1}{\sin(\omega_1 t_1 - \beta_1\pi)}\right) + \cos \varphi\left(a_1V_1\frac{\sin \omega t_1 \sin \beta_1\pi}{\sin(\omega_1 t_1 - \beta_1\pi)}\right) \end{array}\right\} .[19]$$

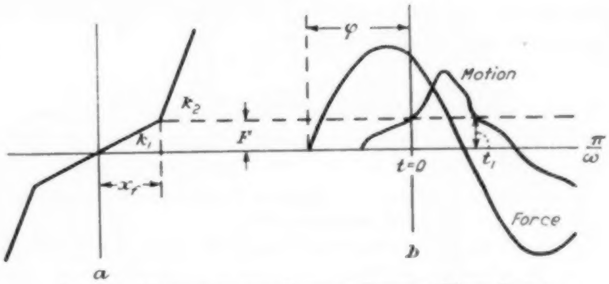

FIG. 9 MOTION IN THE CASE OF SPRINGS WITH STOPS

Substituting these values in [18(*e*)] and [18(*f*)] leads to two equations which can be written in the form:

$$\left.\begin{array}{l} P_1 \cos \varphi + Q_1 \sin \varphi + R_1x_f = 0 \\ P_2 \cos \varphi + Q_2 \sin \varphi + R_2x_f = 0 \end{array}\right\} \ldots\ldots\ldots [20]$$

where P, Q, and R are functions of t_1 and the various frequencies only, but do not contain either φ or x_f, as follows:

$$\left.\begin{array}{l} P_1 = \omega \cos \omega t_1(a_2V_2 - a_1V_1) + a_1\omega_1V_1 \sin \omega t_1 \times \cot(\omega_1 t_1 - \beta_1\pi) - a_2\omega_2V_2 \sin \omega t_1 \cot \omega_2 t_1 \\ Q_1 = \omega \sin \omega t_1(a_1V_1 - a_2V_2) + a_1\omega_1V_1 \cdot \frac{1 + \cos \omega t_1 \cos(\omega_1 t_1 - \beta_1\pi)}{\sin(\omega_1 t_1 - \beta_1\pi)} + a_2\omega_2V_2 \cot \omega_2 t_1 \cdot \frac{\cos \omega_2 t_1 - \cos \omega t_1}{\sin \omega_2 t_1} \\ R_1 = R_2 = \frac{k_1}{k_2}\omega_2\frac{\cos \omega_2 t_1 - 1}{\sin \omega_2 t_1} - \omega_1\frac{1 + \cos(\omega_1 t_1 - \beta_1\pi)}{\sin(\omega_1 t_1 - \beta_1\pi)} \\ P_2 = \omega_1(a_2V_2 - a_1V_1) + a_1\omega_1V_1\frac{\sin \omega t_1}{\sin(\omega_1 t_1 - \beta_1\pi)} + a_2\omega_2V_2\frac{\sin \omega t_1}{\sin \omega_2 t_1} \\ Q_2 = a_1\omega_1V_1\frac{\cos \omega t_1 + \cos(\omega_1 t_1 - \beta_1\pi)}{\sin(\omega_1 t_1 - \beta_1\pi)} + a_2\omega_2V_2\frac{\cos \omega t_1 - \cos \omega_2 t_1}{\sin \omega_2 t_1} \end{array}\right\} .[21]$$

From [20] we can solve for $\sin \varphi$ and $\cos \varphi$ in terms of x_f. Noting that $R_1 = R_2$,

$$\cos \varphi = \frac{R_1(Q_1 - Q_2)x_f}{P_1Q_2 - P_2Q_1}; \quad \sin \varphi = \frac{R_1(P_2 - P_1)}{P_1Q_2 - P_2Q_1}x_f \ldots [22]$$

and since $\sin^2 \varphi + \cos^2 \varphi = 1$, the following condition for x_f is obtained:

$$x_f^2 = \frac{(P_1Q_2 - P_2Q_1)^2}{R_1^2[(P_1 - P_2)^2 + (Q_1 - Q_2)^2]} \ldots\ldots\ldots [23]$$

With these results, the problem is reduced to numerical calculation. At each value of β_1, for a number of values of t_1 (ranging between zero and π/ω), P, Q, R, φ and x_f can be calculated by means of [21], [22], and [23]. With these values substituted in [19] and [16] the shape of the motion of the part between $t = 0$ and $t = t_1$ in Fig. 9(*b*) can be calculated and the maximum amplitude can be found. The results can be plotted as in Fig. 5, one such diagram for each value of the ratio k_1/k_2. The diagram given in Fig. 5 is the one for $k_1/k_2 = \infty$, while the condition of Fig. 1(*a*) is represented by $k_1/k_2 = 0$.

Appendix No. 2

LIST OF NOTATIONS

$a_1 = \frac{P}{k_1}; a_2 = \frac{P}{k_2}$ = static deflection of spring under maximum of impressed force

A, B, C, D = integration constants given by Equation [19]

F = spring force at which stop is hit [Fig. 9]

k_1, k_2 = spring constants in pounds per inch of deflection

P = maximum of impressed force $P \sin \omega t$

$P_{1,2}, Q_{1,2}, R_{1,2}$ = functions given by Equations [21]

t = time

t_1 = time of motion with the stiff springs [Fig. 9]

$U = \frac{\beta \sin \beta\pi}{1 + \cos \beta\pi}$ "Coulomb damping function"

$V_1 = \frac{\beta_1^2}{\beta_1^2 - 1}; \quad V_2 = \frac{\beta_2^2}{\beta_2^2 - 1}$, = "resonance function"

x = displacement

$x_f = F/k_1$ = displacement at which stiff spring is hit

x_a = displacement smaller than x_f, Equation [16]

x_b = displacement larger than x_f, Equation [17]

$\beta_1 = \frac{\omega_1}{\omega}; \quad \beta_2 = \frac{\omega_2}{\omega}$ = frequency ratios

φ = phase angle, Equation [1]

ω = frequency of impressed force: $P \sin \omega t$

$\omega_1^2 = \frac{k_1}{m}; \; \omega_2^2 = \frac{k_2}{m}; \; \omega_n^2 = \frac{k}{m}$ = characteristic frequencies

ω_f = natural frequency of free vibration

ω_c = "critical frequency" as defined in text.

Discussion

J. G. **Baker.**[10] The authors have found periodic motions of the mass of a non-linear single-degree system with which there is equilibrium between the inertia, the restoring force, and the disturbing force of the system. In their analysis they have considered only the possibility that the disturbing-force frequency (the disturbing force was considered simple harmonic) coincides with the fundamental frequency of the motion. It seems equally likely that a forced vibration could exist if the disturbing-force frequency coincided with a higher frequency component of the motion than the fundamental.

It is difficult to speak thus superficially of this possibility in systems with no damping, such as the authors treat, since steady vibrations may be either free or forced. For this reason let us discuss a similar system with a slight amount of linear damping so that any steady vibration must be forced. Suppose this non-linear system is vibrating freely and that there is a large second harmonic in its motion. If we apply a small simple harmonic force at the frequency of and in proper phase relation with this harmonic of the motion, there will be work done by this force on the motion. The energy lost in damping can thus be supplied, and it looks as if the motion could be maintained in this way practically unaltered.

In a linear system, a force at one frequency cannot furnish the energy dissipated by a force at another frequency. If this were true for the non-linear case in question, the energy dissipated by the damping at the fundamental frequency of the motion could not be supplied by the disturbing force we consider, and our proposed condition would be impossible. This objection can be quickly removed, however, by studying for a moment the cases similar to those solved by the authors. The motions are obviously never simple harmonic, so that with damping there will be energy dissipated at other frequencies than that of the disturbing force; yet this energy must come from the disturbing force. It is clear, then, that the theorem mentioned about the impossibility of exchange of energy in linear systems does not apply to non-linear systems.

A series of cases of forced vibrations of non-linear systems with damping can be exactly solved in a very simple manner. These are the cases in which the disturbing force is always equal and opposite to the damping force. If this is the condition, of course the problem may be looked upon as simply the academic case of a free non-linear undamped vibration. The latter case is always solvable. The disturbing force for the foregoing condition would not be simple harmonic, but it would be not far from it, since the motions are usually not far from sinusoidal. In a real case the disturbing force is not often simple harmonic anyway, so that there is no more reason why we should discuss a simple harmonic force than any other arbitrary periodic force. (There is clearly a reason for discussing simple harmonic forces for linear systems since the superposition of solutions is allowable, but this obviously is not the case for non-linear systems.) Further, if only an approximation is desired, it appears that the force could be considered as simple harmonic and of a magnitude large enough to supply all of the energy required by the damping. This would not satisfy the equilibrium requirement, but it would doubtless come very close to it.

To all these remarks and to the paper also can be put the following question: Even if a motion satisfies the equilibrium condition, how do you know that it is stable? Perhaps an infinitesimal change will bring about an entirely different motion. For some cases this question on inspection does not offer difficulty, while for others it appears that there is considerable study required to answer it.

M. **Stone.**[11] The work that the authors have carried out in this paper is a happy continuation of the type of analysis that one of the authors has sponsored in earlier work—i.e., the seeking of complete solutions over restricted ranges of validity and fitting the solutions together at the discontinuities. This type of solution provides the only real basis for adopting any approximate solution that may be analytically simpler.

Under this general subject head, this discusser some time ago had the task of properly designing a spring gear of variable flexibility characteristic with set-up forces that was used for locomotive transmission purposes. The scheme is shown diagrammatically in Fig. 10, together with its flexibility characteristic in Fig. 11. The purpose of such a characteristic is to introduce flexibility variable with displacement, so that no single critical frequency will exist—it will depend on the amplitude of oscillation. The curvature also supplies an automatic stop to the motion; i.e., the force approaches infinity, with little deflection—which is also desirable.

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The differential equation of free oscillatory motion is:

$$m\frac{d^2y}{dt^2} + f(y) = 0 \qquad [24]$$

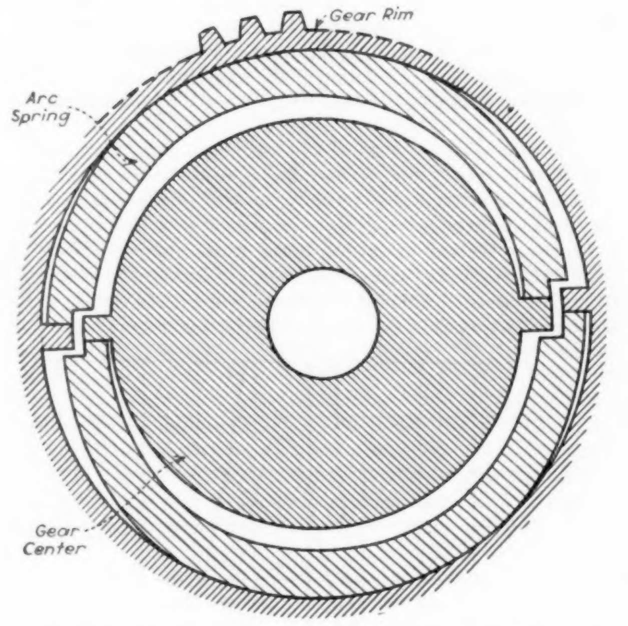

FIG. 10 SPRING GEAR OF VARIABLE FLEXIBILITY CHARACTERISTICS

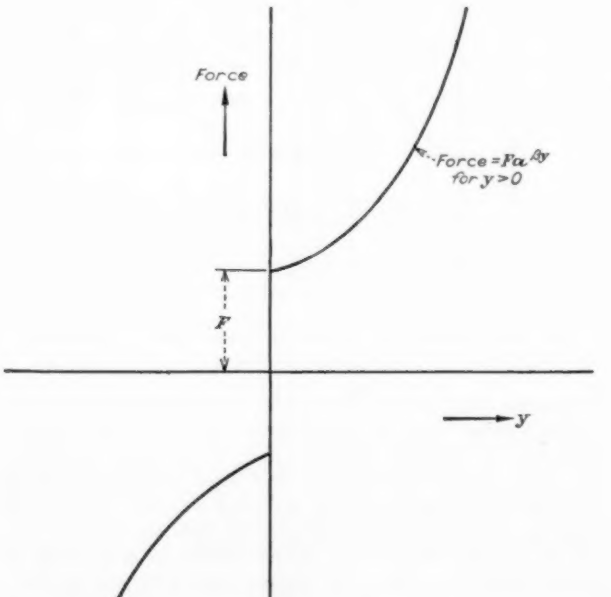

FIG. 11 SPRING GEAR ELASTIC CHARACTERISTICS

By confining attention to the range where $y > 0$, and knowing that the motion is mirrored for $y < 0$, it was found possible to satisfactorily solve this rather difficult case by substituting for $f(y)$:

$$f(y) = Fa^{\beta y} \qquad [25]$$

so that our motion equation becomes

$$m\frac{d^2y}{dt^2} + Fa^{\beta y} = 0 \qquad [26]$$

Here F is the set-up force. It is important to note that if F were zero, the use of this type of function would not be permitted. Continuing the solution:

$$m\left(\frac{dy}{dt}\right)d\left(\frac{dy}{dt}\right) = -Fa^{\beta y}dy$$

or

$$\frac{dy}{dt} = \sqrt{\left[\frac{2F}{m\beta\log_e a}\left(a^{\beta y_0} - a^{\beta y}\right)\right]} \qquad [27]$$

which integrates again into

$$\int_0^t dt = t = \int_{y_0}^0 \frac{dy}{\sqrt{\left[\frac{2F}{m\beta\log_e a}\left(a^{\beta y_0} - a^{\beta y}\right)\right]}}$$

or

$$t = \sqrt{\frac{m}{2F\beta\log_e a}}\left[-\frac{2}{\sqrt{(a^{\beta y_0})}}\tanh^{-1}\sqrt{\frac{a^{\beta y_0} - 1}{a^{\beta y_0}}}\right]. \qquad [28]$$

Now the period of oscillation, for a given amplitude y_0, is 4 × t, as given in [28], and the corresponding natural frequency is defined as

$$w_{cr} = \frac{\pi\sqrt{\frac{F\beta\log_e a\cdot a^{\beta y_0}}{8m}}}{\tanh^{-1}\sqrt{\left(\frac{a^{\beta y_0} - 1}{a^{\beta y_0}}\right)}} \qquad [29]$$

from which it is directly seen that w_{cr} is a function of the amplitude of oscillation y_0, and of the set-up force F as well. This relation is interesting and worth retaining as to its significance. This relation [29] is shown qualitatively in the curve, Fig. 12, and shows a high natural frequency for small amplitudes, a fast-decreasing natural frequency, and then increasing again with large amplitudes.

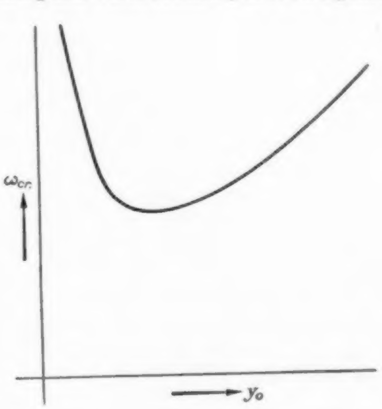

FIG. 12 RELATION SHOWN QUALITATIVELY

The equation of the forced vibration of such a system, or even the free vibration about a steadily deflected position (from 0), does not permit of a complete analytical solution, however. To this end, Rüdenberg's approximate method mentioned in the author's paper gives satisfactory practical results. Where

$$m\frac{d^2y}{dt^2} + Fa^{\beta y} = P\sin wt \qquad [30]$$

it is assumed that at $y = y_{\max}$, $y''_{\max} = -w^2 y_{\max}$, and that $y_{\max}$ occurs where the impressed force is also a maximum. Then

$$m \left. \frac{d^2y}{dt^2} \right|_{\max} + Fa^{\beta y \max} = P \ldots\ldots\ldots\ldots [31]$$

means

$$-mw^2 y_{\max} + Fa^{\beta y \max} = P \ldots\ldots\ldots\ldots [32]$$

from which $y_{\max}$ is obtained graphically.

E. O. WATERS.[12] The authors are dealing with an equation of motion which can be solved easily enough, but which is very clumsy for practical use on account of the ambiguity in the sign of the spring force. However, by an ingenious choice of initial conditions, they have very nearly sidestepped this difficulty, and have been able to present results which give the velocity and displacement of the vibrating body in a form which is no more complicated than that which is encountered in the classical case. But it is only fair to point out that their solution is a very special one; from the standpoint of the pure mathematician, there is not one chance in a million that a mass vibrating between elastic stops, as in Fig. 1(*b*), will behave *precisely* as predicted by their equations. The authors' formulas might be said to give a standard or type form of motion, just as $x/a = V \sin \omega t$ might be considered a standard solution for the classical case.

Actually, the motion of a vibrating body is a composite of that due to the spring force, which gives it one periodicity, and that due to the impressed force, which in general gives it a different periodicity. The result is a vibration with beats. For the classical case, with spring force proportional to displacement from the mean position, this gives an amplitude at the middle of each beat which is far greater than that indicated by the "standard" solution. For example, if $\beta = 1.25$, the "standard" amplitude is 2.778, whereas the actual amplitude at the middle of a beat (assuming initial displacement and velocity of zero, and a phase angle of 90 deg.) is twice this, or 5.555. The same thing occurs with the set-spring case; the "standard" amplitude, for $F = 0.4\ P$, is about 1.33 (see Fig. 5), whereas the maximum amplitude at the middle of a beat, as computed by a step-by-step solution of Equation [4], evaluating the constants and changing the sign of x_f each time that $x = 0$, is about 2.8. It is interesting to note that, for the classical case, the period of the vibrations within any given beat is constant, while with a set spring it increases with the amplitude, so that at the middle of a beat the vibrations take more time than they do at either end.

The exact solution of any problem of this type, with any given initial conditions, is of course possible, but extremely laborious. The authors are to be commended for their analysis of a particular case which is comparatively simple, and which gives an approximate indication of the motion that will generally occur with this kind of spring mounting.

AUTHORS' CLOSURE

Mr. Baker's discussion is a very valuable contribution to the subject, and calls attention to a condition the authors had overlooked. His remarks naturally can be also directly applied to a Coulomb-damped linear vibrating system of which the analysis is very similar. In other words, harmonic forces having frequencies 3, 5, 7...... times higher than the natural frequency of the undamped system are capable of setting up considerable vibrations at the natural frequency due to the presence of Coulomb damping.

Mr. Stone discusses the free vibrations of a spring somewhat different from ours. Comparing his Fig. 11 with our Fig. 1(*b*), it is evident that for small oscillations the same expression for the natural frequency as a function of the amplitude must be found. Indeed, our formula [13] and Mr. Stone's [29] both reduce to

$$\frac{\omega_f}{\omega_n} = \pi \sqrt{\frac{F}{8ky_0}}$$

for small values of y_0.

Professor Waters' discussion bears on the difference between transient and steady-state vibrations. Naturally, any analysis where no viscous friction is considered will lead to results where the transients stay in the system forever, but nevertheless such an analysis is capable of disclosing interesting and useful relations.

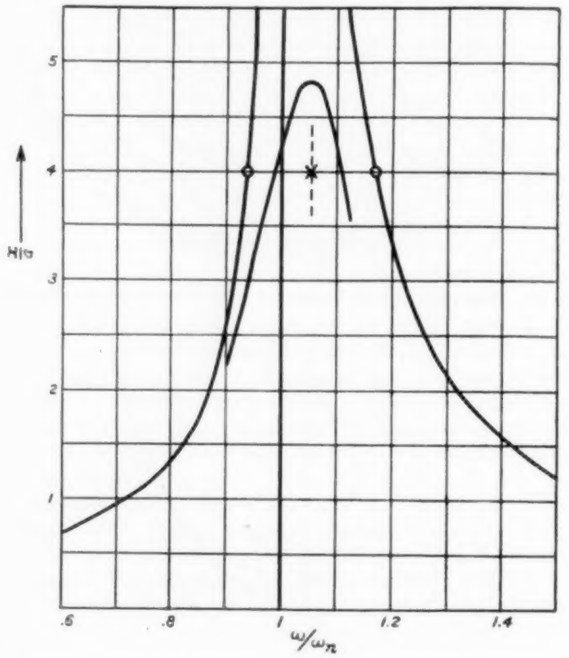

FIG. 13 EXACT CALCULATION INCLUDING VISCOUS DAMPING ($c/c_c = 0.1$; $P/F = 2.5$.)

The authors have not been satisfied with the rather arbitrary definition of "critical speed" on page 4. In order to judge its accuracy they have performed the detail calculations for one case with the exact solution including viscous friction. The analysis follows the lines indicated in footnote 8 and is very involved. The result for a viscous friction of $c/c_{cr} = 0.1$ are shown in Fig. 13. The curve without viscous friction for $P/F = 2.5$ is drawn in also, and the middle between the two intersections with $x/a = 4$ is indicated by a cross. It is seen that the maximum amplitude of the damped curve occurs just above the cross, showing that the arbitrary definition of "critical speed" is justified.

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APM-54-16

Design of Columns of Varying Cross-Section[1]

By DR. A. DINNIK,[2] DNIEPROPETROVSK, U.S.S.R.

Translated by M. Maletz,[3] Chicago, Ill.

In this paper the author develops formulas for special cases of columns of varying cross-section. The uses of these formulas are illustrated by many problems from practice, and tables of constants for every special case facilitate the application of the theory without going into the solution of differential equations.

COMPRESSION members are often given varying cross-sections, the section being increased where an increase in the bending moment is expected; i.e., making the member approximately uniform in strength. So far very little has been done in the development of formulas for the design of such members. In the present paper the author deals with compression members the middle portions of which are of uniform cross-section, and the end portions are tapered according to definite mathematical laws. Such members are quite frequently met in engineering structures, and those with hinged and with free ends will both be considered.

Members With Free Ends

Let us consider a compression member the middle portion of which is of uniform prismatic cross-section, and which has both of its ends tapered as shown in Fig. 1. Let l denote the total length of the member, λ the length of its middle portion, I the constant moment of inertia of the middle part, and i the moment of inertia of the extreme end sections (smallest). Let the moment of inertia of both tapered portions AB and CD vary according to the law:

$$I_x = I\left(\frac{x}{b}\right)^m \qquad [1]$$

where b denotes the distance between section CC and the point of intersection O of the lines CD and C_1D_1. This point O will be taken as the origin of coordinates (not shown in the figure). The solution of the differential equation of the elastic curve for both the mid-portion and the ends shows that the critical load may be expressed in the simple form:[4]

$$P_K = \frac{KEI}{l^2} \qquad [2]$$

which is similar to Euler's well-known formula. Here P_K denotes the critical load, E Young's modulus of elasticity, I the moment of inertia of the middle portion of the strut, l the total length of the member, and K the coefficient of stability, its magnitude being a function of the geometrical form and the method of fastening the strut. Values of this "coefficient of stability" K for various values of i/I and λ/l for the case of a strut with free ends are given in Table 1. The use of this table is illustrated in the following problems.

TABLE 1 VALUES OF THE COEFFICIENT OF STABILITY K FOR STRUTS WITH FREE ENDS

	Values of λ/l					
Values of i/I	0	0.2	0.4	0.6	0.8	m
0.0	5.78	7.04	8.35	9.36	9.80	1
0.0	1.00	1.56	2.78	6.25	9.57	2
0.0	0.0	0.0	0.0	0.0	0.0	3
0.0	0.0	0.0	0.0	0.0	0.0	4
0.01	5.87	7.11	8.40	9.40	9.80	1
0.01	3.45	4.73	6.58	8.62	9.71	2
0.01	2.55	3.65	5.42	7.99	9.63	3
0.01	2.15	3.13	4.84	7.53	9.56	4
0.1	6.48	7.58	8.63	9.46	9.82	1
0.1	5.40	6.67	8.08	9.25	9.79	2
0.1	5.01	6.32	7.84	9.14	9.77	3
0.1	4.81	6.11	7.68	9.08	9.77	4
0.2	7.01	7.99	8.90	9.73	9.82	1
0.2	6.37	7.49	8.61	9.44	9.81	2
0.2	6.14	7.31	8.49	9.39	9.81	3
0.2	6.02	7.20	8.42	9.38	9.80	4
0.4	7.87	8.59	9.19	9.70	9.83	1
0.4	7.61	8.42	9.15	9.63	9.84	2
0.4	7.52	8.38	9.10	9.62	9.84	3
0.4	7.48	8.33	9.10	9.62	9.84	4
0.6	8.61	8.12	9.55	9.76	9.85	1
0.6	8.51	9.04	9.48	9.74	9.85	2
0.6	8.50	9.02	9.46	9.74	9.85	3
0.6	8.47	9.01	9.45	9.74	9.85	4
0.8	9.27	9.53	9.68	9.82	9.86	1
0.8	9.24	9.50	9.70	9.82	9.86	2
0.8	9.23	9.50	9.69	9.81	9.86	3
0.8	9.23	9.49	9.69	9.81	9.86	4

Problem 1. An airplane pine strut, the total length of which is $l = 100$ in., has its middle portion 60 in. long and of constant rectangular cross-section (4 in. × 1 in.). The width of both end portions is tapered according to a straight-line law and is 2.4 in. at the end. The modulus of elasticity is to be taken as $E = 1{,}500{,}000$. It is required to figure this strut for buckling.

Solution. Here is a case where the rigidity EI of the end portions varies according to a straight-line law; i.e., $m = 1$; further, $i/I = 0.6$, and also $\lambda/l = 0.6$. From Table 1 it is accordingly found that $K = 9.76$, and from Equation [2] the critical load is 488 lb. The maximum compressive stress is about 196 lb per sq in., which is considerably less than that allowable.

Problem 2. Same strut as in Problem 1, but with the thickness of the end portions varying according to a straight-line law from 1 in. to 0.59 in., the width being constant (4 in.).

Solution. In this case the moment of inertia varies according to the law of cubes; i.e., $m = 3$; $i/I = 0.2$, and $\lambda/l = 0.6$. From Table 1 it is found that $K = 9.39$, and from [2] the critical load is 470 lb.

Problem 3. The connecting rod of a steam engine is 210 in. long; its middle portion, of a cylindrical cross-section, is 42 in. long. Both ends are conical. The largest diameter of the connecting rod is 8 in., the smallest, 6.36 in. Find the critical load. (These dimensions were taken from a mine hoist in the Don Basin. The influence of the head of the connecting rod is so small that it may be neglected. The diameter of the cylinder is 42 in. and the steam pressure 110 lb; the load on the rod is to be taken as 150,000 lb, i.e., with a factor of safety of 7.)

Solution. Here $\lambda/l = 0.2$, $i = 80.3$ in.^{4}, $I = 201$ in.^{4}, $i/I = 0.4$, and $m = 4$ (the moment of inertia of a cone varying as the law of the fourth power). From Table 1, $K = 8.33$, and from

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[1] A previous paper of the author on the same subject appeared in Trans. A.S.M.E., vol. 51 (1929), no. APM-51-11.

[2] Professor and Head of Department of Applied Mechanics in the Institute of Mines in Dniepropetrovsk.

[3] Designing Engineer, Wisconsin Steel Works, International Harvester Co., South Chicago, Ill. Assoc-Mem. A.S.M.E.; Mem. V.D.I.

[4] The derivation is given in the Appendix.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 2 VALUES OF THE COEFFICIENT OF STABILITY K FOR COLUMNS WITH FIXED ENDS

	Values of λ/l					
Values of i/I	0	0.2	0.4	0.6	0.8	m
0.2	20.36	22.36	23.42	25.44	29.00	1
0.2	18.94	21.25	22.91	24.29	27.67	2
0.2	18.48	20.88	22.64	23.96	27.24	3
0.2	18.23	20.71	22.49	23.80	27.03	4
0.4	26.16	27.80	28.96	30.20	33.08	1
0.4	25.54	27.35	28.52	29.69	32.59	2
0.4	25.32	27.20	28.40	29.52	32.44	3
0.4	25.23	27.13	28.33	29.46	32.35	4
0.6	31.04	32.20	32.92	33.80	35.80	1
0.6	30.79	32.02	32.77	33.63	35.64	2
0.6	30.72	31.96	32.72	33.56	35.60	3
0.6	30.68	31.94	32.69	33.54	35.56	4
0.8	35.40	36.00	36.36	36.84	37.84	1
0.8	35.35	35.97	36.34	36.80	37.81	2
0.8	35.32	35.96	36.32	36.80	37.80	3
0.8	35.33	35.96	36.32	36.78	37.80	4

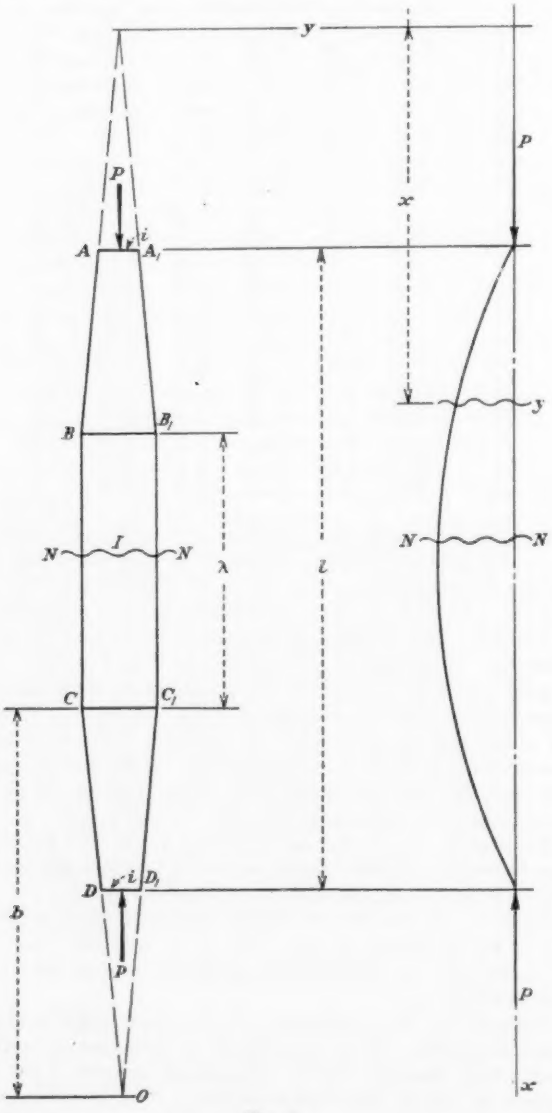

FIG. 1

Equation [2] $P = 1{,}140{,}000$ lb, assuming $E = 30{,}000{,}000$. The critical stress in the smallest section is 36,500 lb per sq in., which does not exceed the elastic limit for steel.

MEMBERS WITH FIXED ENDS

In this case the critical load may be also determined by using Equation [2]. The coefficients of stability are given in Table

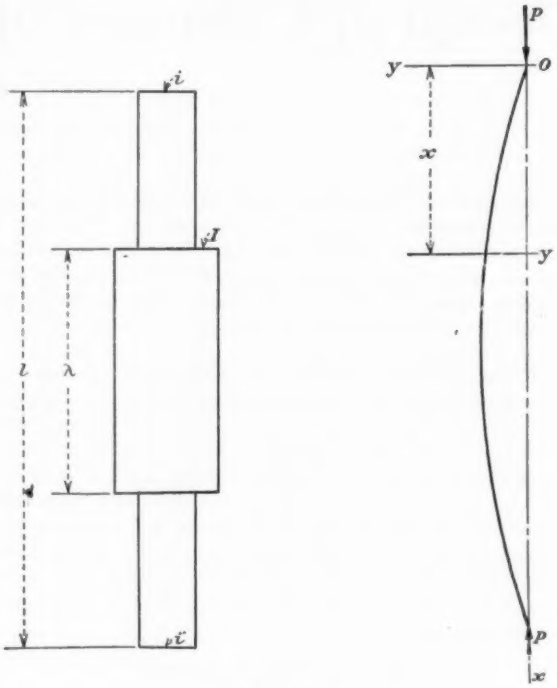

FIG. 2

2, which was computed exactly in the same manner as was Table 1. It is well to bear in mind that for a column of rectangular cross-section with both ends fixed the coefficient of stability is $K = 4\pi^2 = 39.48$.

VARIATION OF RIGIDITY OF COLUMN ACCORDING TO AN EXPONENTIAL LAW

If the moment of inertia varies along the length of the column according to an exponential law such that

$$I_x = Ie^{-\frac{mx}{l}} \qquad [3]$$

then the equation is also integrated in Bessel's functions. The general formula for the critical load is same as Equation [2]. The coefficients of stability for the case of a strut hinged at both ends are given in Table 3.

TABLE 3 VALUES OF THE COEFFICIENT OF STABILITY K FOR STRUTS HINGED AT BOTH ENDS

	Values of λ/l				
Values of i/I	0	0.2	0.4	0.6	0.8
0.01	1.27	1.91	3.14	5.77	9.24
0.1	4.24	5.34	7.22	8.87	9.77
0.2	5.68	6.89	8.24	9.29	9.78
0.4	7.34	8.23	9.07	9.62	9.83
0.6	8.42	8.98	9.43	9.74	9.85
0.8	9.23	9.48	9.64	9.84	9.86
1.0	9.87	....	....	....	....

PROBLEM. An airplane pine strut is 6 ft 8 in. long, its middle portion is 16 in. long, and its cross-section is elliptical. The axes of the (constant) cross-section of the middle portion are $2a = 4$ in. and $2b = 2$ in. The strut is tapered toward both ends according to an exponential law, the axes of the ellipses at the ends being $2a_1 = 3.2$ in. and $2b_1 = 1.6$ in. Assuming an allowable compressive stress of 900 lb per sq in. and taking $E = 1{,}500{,}000$, examine this strut for buckling.

Solution. First compute the moment of inertia of the middle cross-section about its axis of minimum rigidity; i.e. where the danger of buckling is the greatest.

$$I = \frac{\pi ab^3}{4} = \frac{\pi}{2}$$

and for the end sections the moment of inertia is $i = 0.205\pi$; therefore $i/I = 0.4$, and $\lambda/l = 0.2$. From Table 3, $K = 8.23$, whence $P = 3040$ lb. The compressive stress will be only 480 lb per sq in., which is below the assumed allowable stress of 900 lb. Taking a factor of safety of 6, the allowable load on the strut will be a little above 500 lb.

The Stepped Strut

Such a strut consists of parts of different cross-sections as shown in Fig. 2. Let i and I denote the moments of inertia of the end portions and middle portion, respectively, l the total length, and λ the length of the middle portion. The critical load for such a strut is also given by Equation [2]. The coefficients of stability for the case where both ends are hinged are given in Table 4, and where both ends are fixed, in Table 5.

TABLE 4 VALUES OF *K* FOR STEPPED STRUT WITH BOTH ENDS HINGED

	Values of λ/l			
Values of i/I	0.2	0.4	0.6	0.8
0.01	0.15	0.27	0.60	2.26
0.1	1.47	2.40	4.50	8.59
0.2	2.80	4.22	6.69	9.33
0.4	5.09	6.68	8.51	9.67
0.6	6.98	8.19	9.24	9.78
0.8	8.55	9.18	9.63	9.84

TABLE 5 VALUES OF *K* FOR STEPPED STRUT WITH BOTH ENDS FIXED

	Values of λ/l			
Values of $0/l$	0.2	0.4	0.6	0.8
0.01	0.61	1.08	2.39	8.48
0.1	5.87	9.48	15.47	17.13
0.2	11.13	16.26	20.46	21.06
0.4	20.24	24.88	26.31	27.47
0.6	27.71	30.62	31.09	32.46
0.8	34.02	35.31	35.44	36.74

Problem. A column is built up of two 6-in. 8.2-lb channels 20 ft 10 in. long, riveted web to web as shown in Fig. 3. The critical load according to Euler's formula is

$$P_e = \frac{30 \times 10^6 \times 2 \times 1.34}{62{,}500} = 12{,}700 \text{ lb}$$

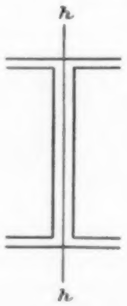

Fig. 3

where 1.34 is the moment of inertia of each channel about the axis *hh*. Assume that it is required to strengthen the mid-portion by riveting on two 4-in. × $^3/_8$ in. cover plates so that the critical load may be increased to 27,300 lb. What should be the length of the stiffened portion?

Solution. The moment of inertia of the two additional plates about *hh* (Fig. 3) is $2 \times 3 \times 64/8 = 4$ in.^{4}. Thus, the moment of inertia of the stiffened portion is $I = 6.68$, the moment of inertia of the end portion, $i = 2.68$, and the ratio $i/I = 0.4$. From [2] the coefficient K is determined as

$$K = \frac{Pl^2}{EI} = \frac{27{,}300 \times 62{,}500}{30 \times 10^6 \times 6.68} = 8.51$$

In Table 4 it is seen that for $K = 8.51$, $i/I = 0.6$, and $\lambda/l = 0.6$; i.e., it is sufficient to strengthen the column along a length $\lambda = 0.6l = 12$ ft 6 in.

Built-Up Columns Stiffened by Lattice Bars

Consider a column built up of four angles and consisting of three parts: the middle part, of constant cross-section, and two end parts of varying cross-section in which the edges follow a curved path (Fig. 4). Let I denote the moment of inertia of the center portion of constant cross-section, and i the moment of inertia of the end section of each of the end portions.

The moment of inertia of any section distant x from the point of intersection of the curved paths is

$$I_x = 4I_c + 4S\eta^2 \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

where I_c denotes the moment of inertia of each angle about its center of gravity, S the sectional area of each angle, and η the distance of the center of gravity of each angle from the neutral plane of the column in bending. Often the first quantity in the second member of Equation [4] may be neglected as being very small in comparison with $4S\eta^2$, and we may put

$$I_x = 4S\eta^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

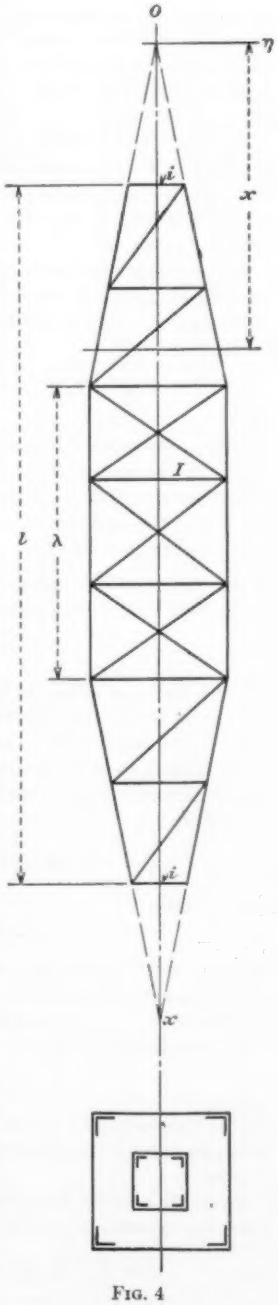

Fig. 4

Let us now consider a special case, when the edge of the column follows a curved path the equation of which is

$$\eta = Ax^n \ldots\ldots\ldots [6]$$

A and n being constants. The moment of inertia will then be

$$I_x = 4SA^2x^{2n} \ldots\ldots [7]$$

i.e., will vary according to an exponential law.

We may now investigate a few special cases. When $n = 0$, the moment of inertia is constant along the whole length of the column—Euler's case.

When $n = {}^1/_2$, then the equation of the edge is

$$\eta^2 = A^2x$$

and the moment of inertia,

$$I_x = 4SA^2x \ldots\ldots [8]$$

i.e., the edge of the column is a parabola, the axis of which coincides with the vertical center line of the column (Fig. 4). The moment of inertia of the end portions varies according to a straight-line law; the coefficients of stability may be found in Tables 1 and 2, for $m = 1$. When $n = 1$, the column is of a pyramidal shape, its moment of inertia varies as the law of squares, and the coefficients of stability are also found in Tables 1 and 2, but for $m = 2$. When $n = 3/2$, the edge of the column is a semi-cubic parabola, the axis of which coincides with the center line of the column. The moment of inertia varies as the law of cubes, and the coefficients of stability are found in Tables 1 and 2 for $m = 3$.

When $n = 2$, the edge of the column is a parabola, the axis of which is perpendicular to the center line of the column and coincides with the ordinate $O\eta$.

$$\eta = Ax^2;\ I_x = 4SA^2x^4 \ldots\ldots\ldots\ldots\ [9]$$

The moment of inertia varies as the law of the fourth power; the coefficients of stability are taken from Tables 1 and 2 for $m = 4$.

If in general the edges follow a curve the equation of which is

$$\eta = Ae^{-\frac{nx}{l}} \ldots\ldots\ldots\ldots\ [10]$$

the moment of inertia varies according to an exponential law, and the coefficients of stability are given in Tables 4 and 5.

The foregoing reasoning is true not only for columns when the edges are curved, but also when they are polygonal, provided the vertices of the polygons lie in the paths of the curves [6] or [10].

Problem. A supporting column in a mine structure 66 ft 8 in. high carries a compressive load of 70,000 lb. The cross-section is conical and the column is built up of four equal-leg angles connected with lattice bars. Select the size of the angles and the distances between them, assuming an allowable compressive stress of 7000 lb per sq in. and a factor of safety of 4.

Solution. Considering the column under simple compression, it is found that the sectional area of each angle must be 2.5 sq in. This corresponds to a $2^3/_4 \times 2^3/_4 \times {}^1/_2$-in. angle,[5] the area of which is 2.53 sq in., and the compressive stress will then be 6900 lb per sq in.

Let us now select the distance between the back of the angles so that the column will not buckle. If the width of the column is taken as 12 in., then the moment of inertia of the section will be—see Equation [4]:

$$I = 4\,(1.62 + 2.53 \times 5.15^2) = 273 \text{ in.}^4$$

where

1.62 = moment of inertia of each angle about the axis through its center of gravity

5.15 = 6 — 0.85, where 0.85 is the distance from the back of the angle to its center of gravity. (All figures taken from a standard steel handbook.)

If the section of the column is uniform along the whole length, the critical load in accordance with Euler's theory will be

$$P = 126{,}000 \text{ lb}$$

i.e., the factor of safety is less than 2.

In order to increase the stability of the column let us make it wider in the middle only. Let us space the angles in the middle of the column 18 in. back to back. The moment of inertia of the new section consisting of the same angles but 18 in. center to center will be

$$I = 4\,(1.62 + 2.53 \times 8.15^2) = 680 \text{ in.}^4$$

and the ratio

$$i/I = 0.4$$

As the moment of inertia varies in this case very closely as the law of squares, the coefficient of stability may be taken from Table 1. For $m = 2$, $\lambda = 0$ and $i/I = 0.4$, $K = 7.61$, and the critical load according to [2] is

$$P_K = 242{,}000 \text{ lb}$$

i.e., by increasing the width of the column in its middle part we have increased its stability nearly twice with a factor of safety of over 3.

Let us try making the width 20 in. at the middle of the column. Then the moment of inertia

$$I = 4\,(1.62 + 2.53 \times 9.15^2) = 855 \text{ in.}^4$$

and $i/I = 0.32$ or about 0.3. The corresponding coefficient of stability from Table 1 is $K = 7.05$, and the critical load

$$P_K = 280{,}000 \text{ lb}$$

i.e., the factor of safety is exactly 4.

But instead of increasing the width from 18 in. to 20 in., we can keep it at 18 in. and make the mid-portion of constant cross-section (prismatic) as shown in Fig. 4. As before let us make the width at the ends 12 in. with a moment of inertia $i = 273$ in.^{4}, and at the middle 18 in. with a moment of inertia $I = 680$ in.^{4}, and let us also make the prismatic mid-portion 13 ft 4 in. long.

Then $l = 800$ in., $\lambda = 160$ in., and from Table 1 for $i/I = 0.4$ and $\lambda/l = 0.2$, $K = 8.42$, and from [2] the critical load

$$P_K = 268{,}000 \text{ lb}$$

i.e., again too low a factory of safety.

Trying another ratio $\lambda/l = 0.4$; i.e., $\lambda = 320$ in. (26 ft 8 in.) and the coefficient of stability $K = 9.15$,

$$P_K = 292{,}000 \text{ lb}$$

This value is again too high. By making $\lambda/l = 0.3$ we find $K = 8.78$ and $P = 280{,}000$ lb, giving exactly the required factor of safety. It is evident that by widening the column in the middle along a certain distance we can increase the stability of the column without increasing the sectional area of the angles, provided that the connections employed are rigid enough to prevent any relative displacement of the angles and the column will act as a whole.

If the angles were bent and given the shape of a parabola with its axis vertical, with the ratios $\lambda/l = 0.3$ and $i/I = 0.4$, the critical load would be 283,000 lb. If the angles were bent and given the shape of a parabola with its axis horizontal, the critical load would be 278,000 lb, or but slightly less.

Remarks

Tables 1, 4, and 5 give coefficients of stability for the case of a column with hinged ends, and of the type shown in Fig. 1. Considering the upper part of the column only, we arrive at the solution of the problem where the lower end is fixed, the upper end free, the lower part is of constant cross-section and the upper part of varying cross-section, varying according to [1] or [3]. The critical load for this case can also be obtained from [2], but the coefficients of stability given in Tables 1 and 3 should be divided by 4. This remark holds good for the coefficients of Table 4 and the cases to which these coefficients are applied.

Conclusion

The author has given formulas for designing columns, the mid-portions of which are of constant cross-section and the ends are of varying cross-section, their moments of inertia varying as the law of exponents or the law of powers. The formula for the critical load in all cases is

$$P_K = K\,\frac{EI}{l^2}$$

The coefficient K is a function of the shape of the column and conditions at the ends. Tables 1–5 give these coefficients for a great variety of cases. Special tables are given for the case where the width of the column varies according to a straight-line law, the thickness however remaining the same; for the case of a column with edges bent to the form of a parabola, the width being kept constant and the thickness varying according to a straight-line law; and further for the cases of a conical column, a built-up column, etc. Finally the case of a stepped column is considered and tables for a column consisting of three parts of different but constant moments of inertia are computed.

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[5] British standard—Translator.

Appendix

Let us consider a column consisting of three parts (Fig. 1), the middle part BC of prismatic and constant cross-section, and two end parts of varying cross-section, tapering toward the ends. Let l denote the total length of the column, λ the length of the middle part, I the constant moment of inertia of the middle part, and i the moment of inertia of the end section. Let the moment of inertia of the end parts AB and CD vary as the exponential law

$$I_x = I\left(\frac{x}{b}\right)^m \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

where m is a positive whole or fractional integer, b is the distance from the section CC to the apex O where the moment of inertia I_x becomes zero. This point is taken as the origin of the coordinates and is not shown in the figure. In view of the symmetry of the column it is sufficient to analyze its upper half only.

The differential equation for flexure for the middle portion of the column is

$$EIy'' + Py = 0 \ldots\ldots\ldots\ldots\ldots\ldots [12]$$

and for the upper portion AB,

$$EI\left(\frac{x}{b}\right)^m y'' + Py = 0 \ldots\ldots\ldots\ldots\ldots [13]$$

Both equations are of the second order, and there are four constants of integration which will have to be determined from the following conditions at the ends:

(1) At the middle of the column in section NN,

$$y' = 0$$

(2), (3) At the section BB, the deflection y and the slope of the deflection curve y' can be found by solving Equation [12] or [13].

(4) At the end AA of the column, $y = 0$.

Remarks. Equation [13] and condition (4) are for the case of hinged ends. In case of fixed ends, we shall have in Equation [13] the reactive moment m at the end section AA, and to condition (4) another condition will be added: namely, at the end of the column $y = 0$ and $y' = 0$.

Equation [13] is generally solved by means of Bessel's functions for any value of m except for $m = 2$. For $m = 2$, however, the integral of [13], as shown in the following paragraphs, can be obtained very simply. This exceptional case is of the greatest practical importance.

CASE WHEN $m = 2$

Equation [13] for this case is

$$EI\left(\frac{x}{b}\right)^2 y'' + Py = 0 \ldots\ldots\ldots\ldots\ldots [14]$$

and its integral

$$y = \sqrt{x}\left\{C\cos(\mu\log_e x) + D\sin(\mu\log_e x)\right\} \ldots\ldots [15]$$

where

$$\mu^2 = \frac{Pb^2}{EI} - \frac{1}{4} \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

The integral of [12] for the middle portion of the column is

$$y = A\cos nx + B\sin nx \ldots\ldots\ldots\ldots [17]$$

where

$$n^2 = \frac{P}{EI}$$

For the case of hinged ends the conditions (1)–(4) at the ends give us the following equation for the determination of the magnitude of 2μ:

$$\sqrt{1+4\mu^2}\tan\left[\sqrt{1+4\mu^2}\,\frac{(1-k^2)\lambda}{2(l-\lambda)}\right] = 1 + 2\mu\cot\left[2\mu\log_e k\right] \ldots\ldots\ldots [18]$$

Substituting the value of 2μ thus found from [18] in [16], the critical load is found to be

$$P = \frac{(1+4\mu^2)\,(1-k^2)^2}{4\left(1-\dfrac{\lambda}{l}\right)^2}\,\frac{EI}{l^2} = K\frac{EI}{l^2} \ldots\ldots\ldots [19]$$

where $k^4 = i/I$.

To find the coefficients of stability K for different ratios i/I and λ/l we first assume in [18] that $i/I = 0, 0.1, 0.2\ldots$ and $\lambda/l = 0, 0.1, 0.2\ldots$ and determine the first root (2μ) of this equation and then substitute the value of 2μ in [19]. The values of the roots of [18], 2μ, are given in Table 6 for various values of i/I and λ/l.

TABLE 6

	Values of λ/l				
Values of i/I	0	0.2	0.4	0.6	0.8
0	0	. .	. .	. .	. .
0.1	3.25	2.85	2.29	1.47	0.40
0.2	4.46	3.83	3.02	1.49	0.53
0.4	7.44	6.24	4.06	3.23	1.38
0.6	12.91	10.62	8.13	5.45	2.60
0.8	28.78	23.33	17.67	11.83	5.87

In the case of a column with fixed ends, Equations [12], [13], and [14] will have an additional constant M_0 (reactive moment), and Equations [15] and [17] will have a constant M_0/P.

From the conditions at the ends the following equation is found for the determination of the magnitude 2μ:

$$\sqrt{1+4\mu^2}\cot\left[\sqrt{1+4\mu^2}\,\frac{(1-k^2)\lambda}{2\,(l-\lambda)}\right] = 1 + 2\mu\cot(2\mu\log_e k) \ldots\ldots\ldots [20]$$

The roots of this equation are given in Table 7, and the coefficients of stability K are given in Table 2 for $m = 2$.

TABLE 7

	Values of λ/l				
Values of i/I	0	0.2	0.4	0.6	0.8
0.2	7.81	6.60	5.10	3.42	1.62
0.4	13.71	11.34	8.66	5.85	2.94
0.6	24.60	20.06	15.21	10.24	5.20
0.8	56.32	45.44	34.25	22.96	11.61

CASE WHEN $m = 4$

In this case the solution is in elementary functions. The differential Equation [13] is now

$$EI\left(\frac{x}{b}\right)^4 y'' + Py = 0 \ldots\ldots\ldots\ldots\ldots [21]$$

or more simply,

$$x^4y'' + U^2y = 0 \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

where

$$U^2 = \frac{Pb^4}{EI} \ldots\ldots\ldots\ldots\ldots\ldots [23]$$

Its integral is

$$y = x\left\{A\cos\frac{U}{x} + B\sin\frac{U}{x}\right\} \ldots\ldots\ldots\ldots [24]$$

From the conditions at the ends in the case of hinged ends, we arrive at the following equation for determining U:

$$\tan\frac{(1-k^2)\,U\lambda}{l-\lambda}+\cot\frac{(1-k^2)\,U}{k^2}=\frac{1}{U}\ldots\ldots\;[25]$$

where

$$k^8=\frac{i}{I}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\;[26]$$

Substituting the roots of the [25] in [23] we find the critical load

$$P=\left\{\frac{(1-k^2)\,U}{1-\dfrac{\lambda}{l}}\right\}^2\frac{EI}{l^2}=K\,\frac{EI}{l^2}\ldots\ldots\ldots\;[27]$$

The roots U of Equation [25] are given in Table 8 for various values of i/I and λ/l, and the coefficients of stability K are given in Table 1 at $m = 4$.

TABLE 8

Values of i/I	Values of λ/l					
	0	0.1	0.2	0.4	0.6	0.8
0.1	..	3.39	2.26	1.90	1.38	0.71
0.2	3.70	3.49	3.24	2.63	1.85	0.95
0.4	6.68	6.18	5.64	4.42	3.03	1.53
0.6	12.14	11.10	10.01	7.69	5.21	2.62
0.8	28.00	25.39	22.71	17.21	11.52	5.79

In the case of fixed ends the conditions at the ends give the following equation for determining U:

$$U\tan\frac{(1-k^2)\,\lambda U}{l-\lambda}=1+\frac{k^2+U\tan\dfrac{1-k^2}{k^2}\,U}{k^2\tan\dfrac{1-k^2}{k^2}\,U-U}\,U\ldots\;[28]$$

The roots of [28] are given in Table 9, and the coefficients of stability K in Table 2 for $m = 4$.

TABLE 9

Values of i/I	Values of λ/l				
	0	0.2	0.4	0.6	0.8
0.2	6.44	5.50	4.30	2.95	1.57
0.4	12.27	10.18	7.80	5.30	2.78
0.6	23.10	21.03	16.61	9.66	4.97
0.8	54.77	44.21	33.32	22.36	11.33

CASE WHEN $m = 1$

In this case Equation [13] is solved by means of Bessel's functions; its integral

$$y=\sqrt{x}\left\{AJ_1\left[\sqrt{\frac{4Pbx}{EI}}\right]+BN_1\left[\sqrt{\frac{4Pbx}{EI}}\right]\right\}\ldots\;[29]$$

where J_1 and N_1 are cylindrical functions.

From the conditions (1)–(4) at the end for columns with hinged ends we arrive at the following equation for determining the auxiliary magnitude U:

$$\tan\frac{(1-k^2)\lambda U}{4\,(l-\lambda)}=\frac{J_1\,(kU)N_0(U)-N_1(kU)J_0(U)}{J_1\,(kU)\,N_1\,(U)-N_1\,(kU)\,J_1(U)}$$

where $k^2 = i/I$ and the coefficient of stability

$$K=\left[\frac{(1-k^2)\,Ul}{l-\lambda}\right]^2\ldots\ldots\ldots\ldots\;[31]$$

U is found from [30], and from [31] and [2] the critical load is determined.

Table 10 contains the roots of Equation [30], and the coefficients of stability are given in Table 1 under $m = 1$.

TABLE 10

Values of i/I	Values of λ/l				
	0	0.2	0.4	0.6	0.8
0	2.41	2.12	1.73	1.22	0.63
0.1	2.83	2.45	1.96	1.37	0.70
0.2	3.31	2.83	2.24	1.56	0.78
0.4	4.68	3.91	3.03	2.08	1.05
0.6	7.33	6.04	4.63	3.12	1.57
0.8	15.22	12.35	9.33	6.27	3.14

For columns with fixed end, instead of Equation [30] we have

$$\tan\frac{(1-k^2)\lambda U}{2\,(l-\lambda)}=\frac{J_0\,(U)\,N_0\,(kU)-N_0\,(U)\,J_0\,(kU)}{J_1\,(U)\,N_0\,(kU)-N_0\,(U)\,J_1\,(kU)}\cdot[32]$$

The roots of this equation are given in Table 11, and the coefficients of stability for this case are found in Table 2 under $m = 1$.

TABLE 11

Values of i/I	Values of λ/l				
	0	0.2	0.4	0.6	0.8
0	2.40	2.12	1.73	1.22	0.63
0.2	5.64	4.73	3.67	2.52	1.35
0.4	8.53	7.03	5.38	3.66	1.92
0.6	13.93	11.35	8.61	5.81	2.99
0.8	29.80	24.0	18.10	12.14	6.15

CASE WHEN $m = 3$

The differential equation for flexure is

$$EI\left(\frac{x}{b}\right)^3y''+Py=0$$

and its integral is

$$y=\sqrt{x}\left\{AJ_1\left[\sqrt{\frac{4\pi b^3}{EIx}}\right]+BN_1\left[\sqrt{\frac{4\pi b^3}{EIx}}\right]\right\}\ldots\;[33]$$

From the conditions at the ends for columns with hinged ends we arrive at the following equation:

$$\tan\frac{(1-k^2)\lambda U}{2\,(l-\lambda)}=\frac{J_1\left(\dfrac{U}{k}\right)N_2\,(U)-N_1\left(\dfrac{U}{k}\right)J_2\,(U)}{J_1\left(\dfrac{U}{k}\right)N_1\,(U)-N_1\left(\dfrac{U}{k}\right)J_1\,(U)}\ldots\;[34]$$

where $k^8 = i/I$

Computing the roots U of Equation [34] for various values of i/I and λ/l and substituting them in [31], we find the coefficients of stability given in Table 1 under $m = 3$. The roots of [34] are given in Table 12.

TABLE 12

Values of i/I	Values of λ/l				
	0	0.2	0.4	0.6	0.8
0.2	5.97	5.21	4.21	2.95	....
0.4	10.42	8.80	6.87	4.70	....
0.6	18.61	....	....	....	....
0.8	42.39	....	....	....	....

For columns with both ends fixed, U is determined from the following equation:

$$\tan\frac{(1-k^2)\lambda U}{2\,(l-\lambda)}=\frac{J_2\,(U)\,N_2\left(\dfrac{U}{k}\right)-N_2\,(U)\,J_2\left(\dfrac{U}{k}\right)}{J_1\,(U)\,N_2\left(\dfrac{U}{k}\right)-N_1\,(U)\,J_2\left(\dfrac{U}{k}\right)}\ldots\;[35]$$

The roots of this equation are given in Table 13 and the coefficients of stability in Table 2 under $m = 3$.

TABLE 13

Values of i/I	Values of λ/l				
	0	0.2	0.4	0.6	0.8
0.2	10.35	8.81	6.87	4.72	2.51
0.4	19.13	15.85	12.15	8.26	4.33
0.6	35.40	28.90	21.9	14.80	7.62
0.8	32.90	66.90	50.5	33.8	17.15

NOTE. Table 12 is not complete because the coefficients of stability K in Table 2 can be figured with sufficient accuracy from comparison of the values under $m = 2$ and $m = 4$ in Table 1.

EXPONENTIAL LAW OF VARIATION OF MOMENT OF INERTIA

Referring to Fig. 1, let the moment of inertia of both end portions CD and AB of the column vary along their lengths according to the following exponential law:

$$I_x = I e^{-\frac{m(b-x)}{b}} \qquad [36]$$

where I is the largest moment of inertia of the portion of varying cross-section and at the same time the moment of inertia of the middle portion, and b is the length of the end portion.

The length of the column is l as before, and λ denotes the length of the middle portion. Let the origin of coordinates be at the center of section AA, and let us denote the smallest moment of inertia, at $x = 0$, by i. From [36]

$$i = Ie^{-m} = k^2 I$$

where

$$k^2 = \frac{i}{I} = e^{-m} \qquad [37]$$

The differential equation for flexure for the middle portion remains as given by Equation [12], and for the upper portion

$$EIe^{-\frac{m(b-x)}{b}} y'' + Py = 0 \qquad [38]$$

This equation is integrated by means of Bessel's function of zero order:

$$y = A\,J_0(u) + B_0\,N(u) \qquad [39]$$

where

$$u = \frac{2}{m}\sqrt{\frac{Pb^2}{EI}}\,e^{-\frac{m(b-x)}{b}} \qquad [40]$$

From the conditions at the ends of a hinged-ended column we arrive at the following equations for determining the critical load:

$$\tan\frac{m\lambda U}{2(l-\lambda)} = \frac{J_0\left(\frac{U}{k}\right)N_1(U) - N_0\left(\frac{U}{k}\right)J_1(U)}{J_0\left(\frac{U}{k}\right)N_0(U) - N_0\left(\frac{U}{k}\right)J_0(U)} \qquad [41]$$

where

$$U = \frac{2}{m}\sqrt{\frac{Pb^2}{EI}}; \qquad k^2 = \frac{i}{I} = e^{-m} \qquad [42]$$

Substituting 0.1, 0.2, etc., for i/I and λ/l in [41], we determine for each value the first root U of [41], and from [42] we then determine the critical load

$$P_{cr} = \left[\frac{mU\lambda}{2(l-\lambda)}\right]^2 \frac{EI}{l^2} = K\frac{EI}{l^2} \qquad [43]$$

Table 14 gives the roots of [41], and the coefficients of stability are to be found in Table 3.

STEPPED COLUMNS

The column consists of three parts of two different cross-sections (Fig. 2). Let i and I denote the moment of inertia of the upper and lower portions and the middle portion, respectively, l the total length of the column, and λ the length of the middle portion.

TABLE 14

	Values of λ/l				
Values of i/I	0	0.2	0.4	0.6	0.8
0.01	0.25	0.24	0.23	0.21	0.13
0.1	0.83	0.82	0.70	0.52	0.27
0.2	3.31	1.31	1.07	0.76	0.39
0.4	4.68	2.51	1.97	1.35	0.68
0.6	7.33	4.69	3.61	2.44	1.23
0.8	15.22	11.04	8.35	5.62	2.82

The differential equations for flexure of the upper or lower and middle portions are:

$$\begin{aligned} E\,i\,y_1'' + P\,y_1 &= 0 \\ E\,I\,y_2'' + P\,y_2 &= 0 \end{aligned} \qquad [44]$$

The conditions at the ends are

$$\begin{aligned} &(1)\quad x = 0 \quad y_1 = 0 \qquad &(3)\quad x = \frac{l-\lambda}{2} \quad y = y_2 \\ &(2)\quad x = \frac{l}{2} \quad y_2' = 0 \qquad &(4)\quad x = \frac{l-\lambda}{2} \quad y_1^1 = y_2^1 \end{aligned} \qquad [45]$$

The integral of [44] is [17]. Taking into account the conditions at the ends when integrating [17] we obtain the following equation for the determination of the critical load:

$$\tan U \tan\frac{k\lambda U}{l-\lambda} = -\frac{1}{k} \qquad [46]$$

$$P_{cr} = \left[\frac{k\lambda U}{l-\lambda}\right]^2 \frac{EI}{l^2} = K\frac{EI}{l^2} \qquad [47]$$

where

$$k^2 = i/I$$

The roots of Equation [46] for various values of i/I and λ/l are given in Table 15, and the coefficients of stability in Table 4.

TABLE 15

	Values of λ/l			
Values of i/I	0.2	0.4	0.6	0.8
0.01	1.50	1.55	1.56	1.57
0.1	0.43	1.34	1.47	1.58
0.2	0.69	1.86	1.38	1.50
0.4	0.49	0.92	1.23	1.43
0.6	0.40	0.79	1.11	1.36
0.8	0.35	0.69	1.02	1.31

For a stepped column with both ends fixed the critical load is found by using Equation [47], but U is found from the following equation:

$$\tan U + k\tan\frac{k\lambda U}{l-\lambda} = 0 \qquad [48]$$

The roots U of this equation for various values of i/I and λ/l are given in Table 16, and the coefficients of stability K in Table 5.

TABLE 16

	Values of 0/l			
Values of i/I	0.2	0.4	0.6	0.8
0.01	0.15	0.28	0.60	2.12
0.1	1.47	2.37	3.87	4.28
0.2	2.78	4.07	5.12	5.26
0.4	5.06	6.22	6.58	6.87
0.6	6.93	7.65	7.77	8.11
0.8	8.51	8.83	8.86	9.09

Compression of Rectangular Blocks, and the Bending of Beams by Non-Linear Distributions of Bending Forces[1]

By J. N. Goodier,[2] Toronto, Ont.

An existing method for the approximate analysis of two-dimensional stress distributions is extended in scope and applied to two examples of some practical interest. The first is a rectangular plate or block compressed by forces applied centrally to two opposite sides, as by knife edges. Tensions of considerable magnitude are developed across the middle plane through the knife edges. These tensions are determined, and illustration given of their dependence on the extent of spreading of the load over the bearing area.

The second is intended as quantitative example of Saint-Venant's principle in the bending of beams. The stresses due to a non-linear distribution of bending forces on the ends are investigated in order to estimate how far from the end one must go to find the linear distribution assumed in the elementary theory of flexure. Conclusions are given in the paper proper and the mathematical analyses from which they are drawn are given in appendices.

Compression of Rectangular Blocks

The thickness (perpendicular to the rectangular section) is either very great (plane strain) or very small (plane stress). Referring to Fig. 1, the forces are confined to small middle portions ab, cd, of opposite edges, and this concentration results in the development of considerable *tensile* stresses across the section $EFGH$, a matter of importance for materials weak in tension, as cast iron and concrete are.

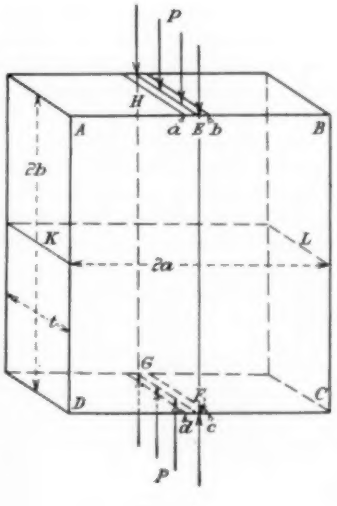

Fig 1

Forces Ideally Concentrated. The tensions are greatest when the forces are ideally concentrated, that is, confined to the lines EH and FG. Numerical results have been worked out for three rectangles, $b/a = 2$, the square $b/a = 1$, and $b/a = {}^1/_2$ (Fig. 2), and are shown in Fig. 3 as curves drawn on ordinates from EF. The greatest tension in each case is given in Table 1, in terms of $P/\pi bt$ (P being the total force), which is the average tension whatever the proportions of the rectangle—and indeed whether the section is rectangular or not (Ap-

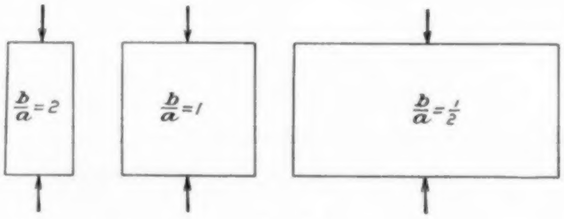

Fig. 2

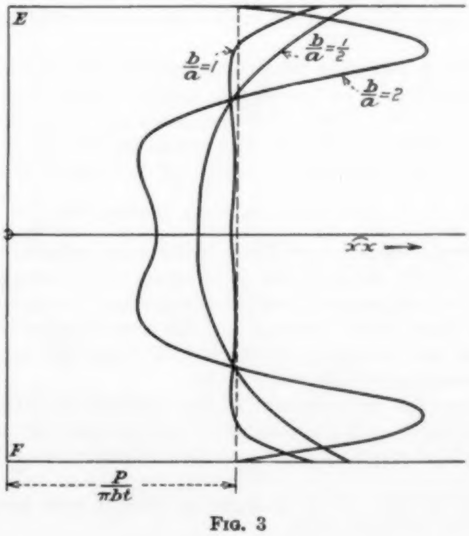

Fig. 3

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[1] Including portions of a dissertation prepared under the direction of Dr. S. Timoshenko for presentation to the faculty of the University of Michigan in partial fulfilment of the requirements for the degree of Doctor of Science.

[2] Ontario Research Foundation. Mr. Goodier was educated in England at Downing College, Cambridge University, where he studied mathematics and mechanical sciences. He took the degree of B.A. through the Mechanical Sciences Tripos, with the Rex Moir Prize for general distinction and the Ricardo Prize for thermodynamics. For two years he worked as a research student under Prof. C. E. Inglis, F.R.S., on problems of elasticity and applied mechanics, stress distributions, bridge vibrations, and the phenomenon of rail corrugation, and, together with H. S. Sayles, B.A., was awarded the John Steddy Winbolt Prize of Cambridge University for a dissertation on "Some Problems of Plane Stress." He was elected a Commonwealth Fellow in 1929. He then went to Ann Arbor to continue work under the supervision of Professor Timoshenko at the University of Michigan, remaining there until August, 1931, when he assumed his present duties.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

pendix No. 2). If the section is circular, for instance, the tensions are constant across the diameter joining the forces and so equal to the average value $P/\pi bt$.

TABLE 1

b/a	2	1	$^1/_2$
Greatest tension	$1.82\ \frac{P}{\pi bt}$	$1.33\ \frac{P}{\pi bt}$	$1.50\ \frac{P}{\pi bt}$

The excessive labor involved in the calculations has limited the number of rectangles investigated to the three already mentioned, but it is possible that rectangles of other proportions $b/a > 2$ might have tensions still higher than that of 1.82 $P/\pi bt$ for $b/a = 2$.

Forces Not Ideally Concentrated. Ideally concentrated forces were considered first for mathematical convenience. They set an upper limit to the tensions, and we have now to show how these tensions are modified when the load is spread over short elements ab and cd, Fig. 1. The correction consists of compressive stresses, intense near the load, and falling off very rapidly as we move away from E or F. But as they begin by being at least as great as the mean applied pressure, they may, even at some distance from E or F, be still great enough to be comparable with $P/\pi bt$. It is shown elsewhere[3] that if we assume the loads uniformly distributed on ab, cd (of length 2α), the local compressive stress, to be superposed on the tensions of Fig. 3, can be found from

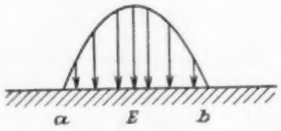

FIG. 4

$$-\frac{P}{\pi\alpha t}\left[\tan^{-1}\frac{\alpha}{y}-\frac{\alpha/y}{1+\alpha^2/y^2}\right]\quad\ldots\ldots\ldots[1]$$

where, confining attention to the end E, y is the distance from E along EF. If the distribution is taken as parabolic (Fig. 4) instead of uniform, the corresponding expression is

$$-\frac{3}{2}\frac{P}{\pi\alpha t}\left[\left(1+3\frac{y^2}{\alpha^2}\right)\tan^{-1}\frac{\alpha}{y}-3\frac{y}{\alpha}\right]\ldots\ldots\ldots[2]$$

This is probably the more practical case.

TABLE 2 LOCAL COMPRESSIVE STRESSES

		y/α							
		0	1	2	4	6	8	10	Factor
Compressive stress	Uniform distribution of load	$\pi/2$	0.2854	0.0636	0.00969	0.00299	0.00128	0.00066	$\frac{P}{\pi bt}\cdot\frac{b}{a}\cdot\frac{a}{\alpha}$
	Parabolic distribution of load	$3/2.\pi/2$	0.2124	0.0412	0.00594	0.00182	0.00077	0.00040	

In order to superpose these compressive stresses on the tension curve from Fig. 3 we choose b/a, and the degree of spreading of the load α/a. Then α/b is fixed, and since the vertical EF in Fig. 3 is equal to $2b$, the distances α, 2α, etc. can be set off along it. For these points we find the compressive stress from Table 2 as a numerical coefficient multiplying $\frac{b}{a}\cdot\frac{a}{\alpha}\cdot\frac{P}{\pi bt}$; b/a, a/α are chosen, and $P/\pi bt$ is the mean ordinate (perpendicular to EF) of the curve of tensions. The ordinate representing the compressive stress can then be drawn on the diagram, and the final curve of stress across EF constructed. The mean of course will no longer be $P/\pi bt$, and the stress close to the loads will be compressive, elsewhere tensile.

The two full curves in each of Figs. 5 and 6 show the effects of distributing each force uniformly and parabolically on $^1/_{10}$th and $^1/_5$th of the loaded sides of the rectangle $b/a = 2$. The broken-line curves, Figs. 5 to 10, are for the concentrated forces. The reduction of maximum tension is much less marked for the parabolic distribution than for the uniform distribution.

Figs. 7 and 9 are for comparison with curve II of Fig. 5. If we imagine the three rectangles to have the same height, as in Fig. 2, and the loads spread uniformly over the same length ab in each case, ab being $^1/_{10}$th of the height $2b$, then curve II in Fig. 5 gives the tensions for the rectangle $b/a = 2$ (that is, the width half the height), Fig. 7 gives the tensions for $b/a = 1$ (a square of the same height), and Fig. 9 the tensions when the width is twice the height ($b/a = {}^1/_2$). These curves show, in short, the effects of adding to the width of the rectangle. It is unlikely, by Saint-Venant's principle, that further addition beyond $b/a = {}^1/_2$ would make much difference.[4]

Figs. 6, 8, and 10 represent the same thing when the loads are distributed parabolically. It appears that, except in the square, both the mode and the extent of distribution of the forces may have an appreciable effect on the maximum tensions.

The distribution of pressure on the middle horizontal section KL (Fig. 1) is drawn for $b/a = 1$ and $b/a = {}^1/_2$ (Fig. 11). It is of course unaffected by the spreading of the load. For $b/a = 2$, the distribution should be uniform.[5] Actually in this solution it departs from the mean value by 5 per cent. This would be accounted for by errors of the same order in the approximate parts of the calculations. The approximations contribute much more to these pressure distributions than to the tension distributions with which we have been concerned, and an error of the same order could only affect the tensions by about 2 per cent.

BENDING OF BEAMS BY NON-LINEAR DISTRIBUTIONS OF BENDING FORCES

According to Saint-Venant's principle, distributions of forces on the ends of a beam which are statically equivalent will produce the same distribution of stress at some distance from the end. The distribution of bending stress to be expected in the interior is of course the familiar linear distribution of the elementary theory. That this distribution may be transmitted throughout the length of the beam without change is shown by the exact stress-function ϕ = constant xy^3, so that if the distribution were linear at the ends it would be the same on all cross-sections.

We consider a non-linear distribution of bending forces on the ends, of intensity proportional to the cube of the distance from the neutral axis (Fig. 12).

If the beam is so short that its span is equal to its depth, the distribution of bending stress is given approximately by Equation [3].

$$\eta^3-2(1-\xi^2)^2\left\{0.08392(5\eta^3-3\eta)+0.004108(21\eta^5-20\eta^3+3\eta)\right\}$$
$$-2\xi^2(1-\xi^2)^2\left\{0.07308(5\eta^3-3\eta)+0.04179(21\eta^5-20\eta^3+3\eta)\right\}$$

where $\xi = x/a$, $\eta = y/b$, as in Fig. 13. At the middle cross-section, $x = 0$, the distribution is not far from linear. It is drawn in Fig. 14. If the span of the beam is twice its depth, i.e., $b/a = {}^1/_2$ the distribution is given approximately by Eq. [4]:

$$\eta^3-2(1-\xi^2)^2\left\{0.09434(5\eta^3-3\eta)-0.001674(21\eta^5-20\eta^3+3\eta)\right\}$$
$$-2\xi^2(1-\xi^2)^2\left\{0.3110(5\eta^3-3\eta)+0.03940(21\eta^5-20\eta^3+3\eta)\right\}$$

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[3] J. N. Goodier and H. S. Sayles in Winbolt Prize Essay, University of Cambridge, 1929.

[4] Comparison of the curve, from Fig. 3, for $b/a = {}^1/_2$, with the corresponding curve for infinite width given by F. Seewald, "Abhandlungen aus dem Aerodyn. Inst. an der Tech. Hochschule Aachen," 1927, shows no agreement, but it may be observed that the mean ordinate of Seewald's curve is not $P/\pi bt$, as it should be according to Appendix No. 2 of this paper.

[5] F. Bleich, *Bauingenieur*, 1923, Heft 9.

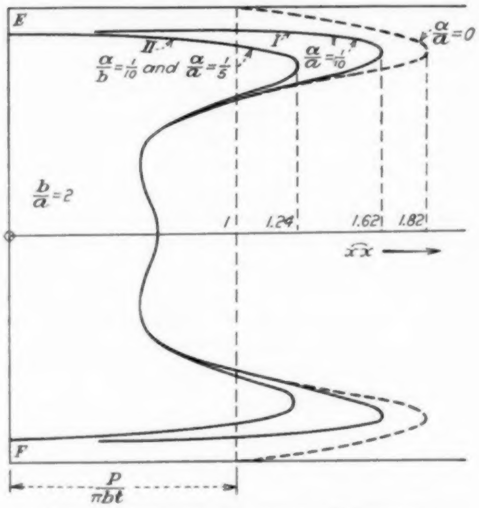

FIG. 5 FORCES DISTRIBUTED UNIFORMLY

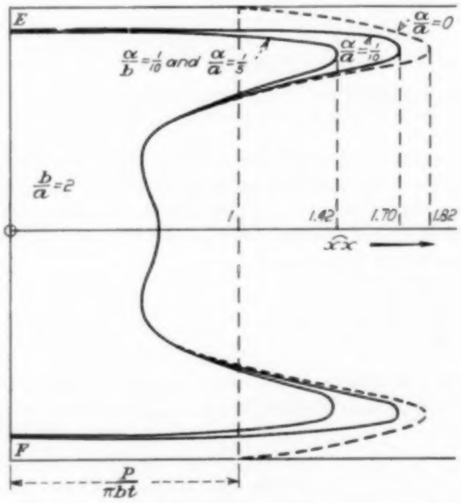

FIG. 6 FORCES DISTRIBUTED PARABOLICALLY

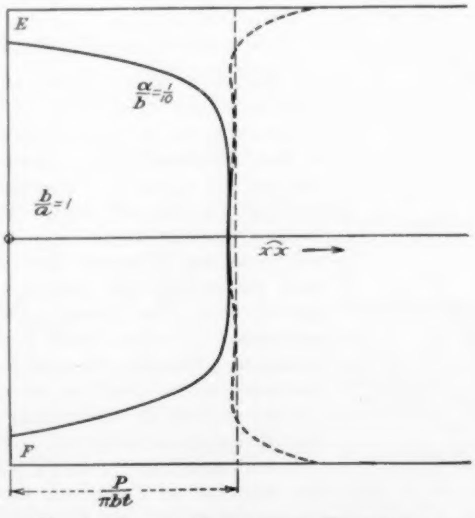

FIG. 7 FORCES DISTRIBUTED UNIFORMLY

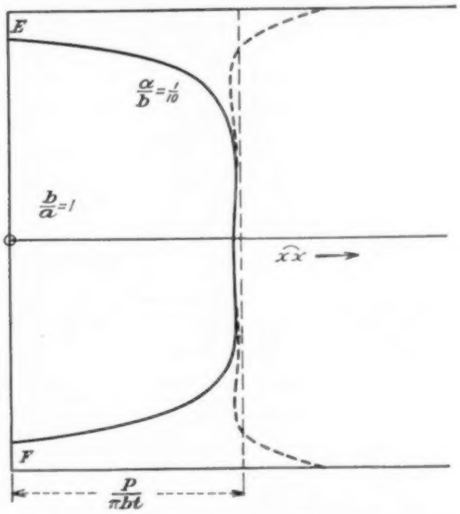

FIG. 8 FORCES DISTRIBUTED PARABOLICALLY

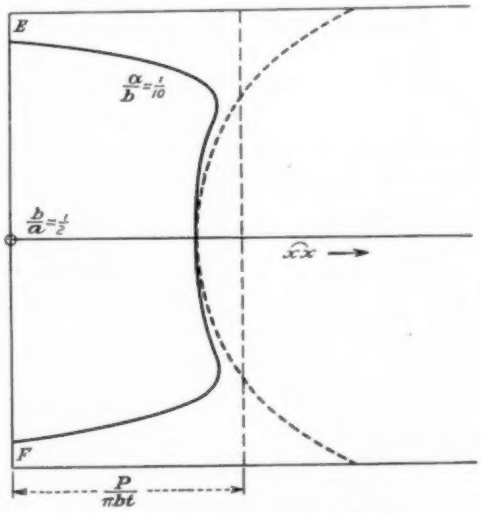

FIG. 9 FORCES DISTRIBUTED UNIFORMLY

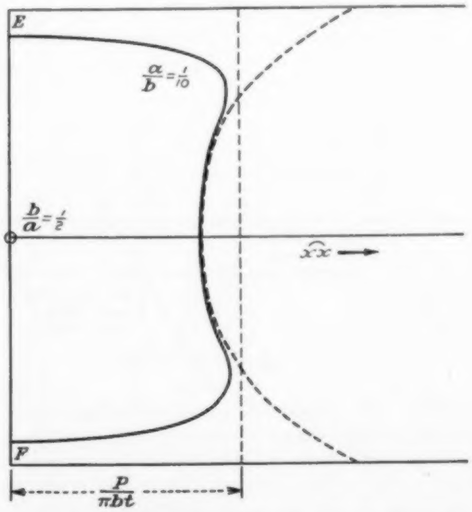

FIG. 10 FORCES DISTRIBUTED PARABOLICALLY

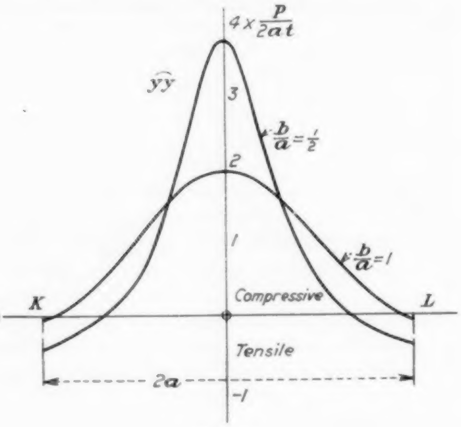

FIG. 11

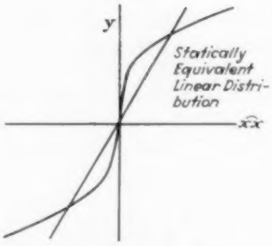

FIG. 12 SQUARE AND RECTANGLE. APPLIED CUBIC DISTRIBUTION ON ENDS

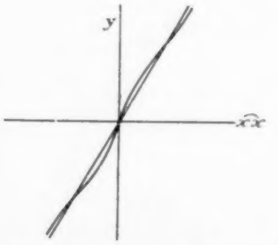

FIG. 14 SQUARE. DISTRIBUTION AT MIDDLE SECTION *AB*, FIG. 13

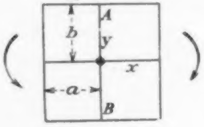

FIG. 13

The distributions over various cross-sections are drawn in Fig. 15. At a distance from the end equal to three-eighths of the depth, linearity is not yet attained. At a distance from the end equal to half the depth, the curve bends slightly the other way, indicating a slightly excessive correction to the cubic distribution represented by the term η^3. At three-fourths of the depth, away from the end, the curve is practically linear, but in the middle of the beam there is again a slight curvature, this time in the same sense as that of the end curve. It appears that the approximate solution does not give an asymptotic approach to the linear distribution, as a true solution would, but, having once attained it, overshoots the mark, then recovers a little too much, and so oscillates slightly about the desired result. This is a defect in the method, arising from the necessity of composing the correction to the end distribution from two curves of the types

$$5\eta^3 - 3\eta \text{ and } 21\eta^5 - 20\eta^3 + 3\eta$$

affected at any cross-section ξ by factors $(1-\xi^2)^2$ and $\xi^2(1-\xi^2)^2$.

We conclude that the linear distribution is probably attained at a distance from the end equal to half the depth. If this is true for the beam of span twice the depth, it will of course hold for longer beams.

Appendix No. 1

THE DETERMINATION OF TWO-DIMENSIONAL STRESS DISTRIBUTIONS IN THE RECTANGLE

1 The solution of two-dimensional problems in elasticity by means of the Airy stress function ϕ, satisfying the biharmonic equation $\Delta^4\phi = 0$ and the prescribed boundary conditions, is equivalent to the determination of the form of ϕ which makes the strain-energy integral—

$$V = \frac{1}{2E}\int\int[\phi_{xx}^2 + \phi_{yy}^2 - 2\sigma\phi_{xx}\phi_{yy} + 2(1+\sigma)\phi_{xy}^2]dxdy \quad [5]$$

taken over the volume of the plate or prism, a minimum. The approximate method applied in this paper consists in assuming a form for ϕ which satisfies the boundary conditions and contains a number of adjustable parameters. These parameters are found from the minimum condition of the integral V.

Such a form may be represented as

$$\phi = \phi_0 + \alpha_1\phi_1 + \alpha_2\phi_2 + \alpha_3\phi_3 + \ldots [6]$$

in which ϕ_0 satisfies the boundary conditions for ϕ, and ϕ_1, ϕ_2, etc. give no contributions to the normal and shear stresses at the boundaries. The adjustable parameters are α_1, α_2, etc. Stresses derived from this function will satisfy the conditions of equilibrium and the boundary conditions, but in general they will only fulfil the compatibility condition for the special values of an infinite number of the parameters α_1, α_2, etc. which make [5] a minimum. The approximation consists in discarding all but the first few terms of [6] and making [5] a minimum with the few remaining parameters α_1, α_2, etc.

If we substitute [6] in [5] we obtain the integrand as a quadratic function of the parameters α. The conditions that V

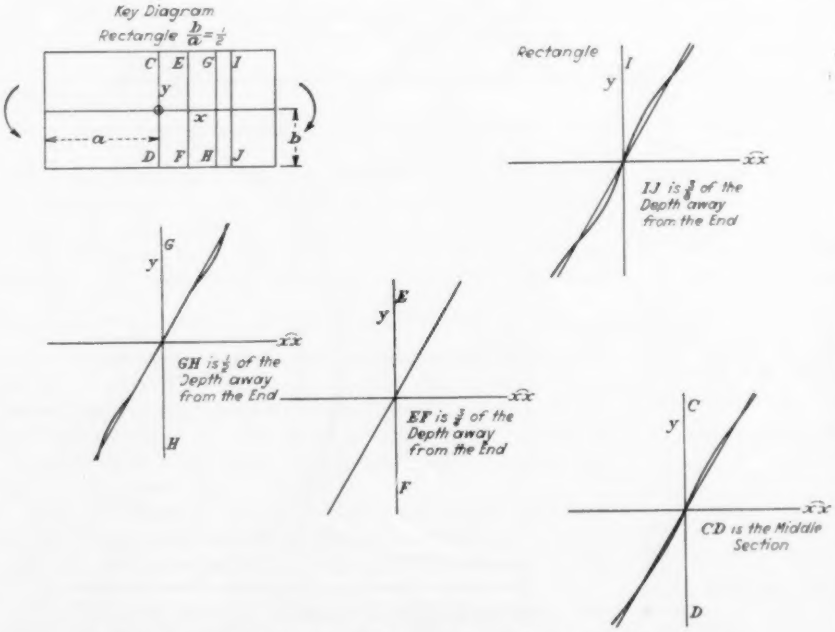

FIG. 15 DISTRIBUTIONS OVER VARIOUS CROSS-SECTIONS OF RECTANGLE

should be a minimum with respect to variations of these parameters, viz.,

$$\frac{\partial V}{\partial \alpha_1} = 0 \qquad \frac{\partial V}{\partial \alpha_2} = 0, \text{ etc.} \ldots\ldots\ldots\ldots\ldots [7]$$

are a set of linear equations for the α's. Writing the stresses derived from [6] as

$$\widehat{xx} = \frac{\partial^2 \phi}{\partial y^2} = \phi_{yy} = \phi_{0yy} + \Sigma \alpha_n \phi_{nyy}$$

$$\widehat{yy} = \frac{\partial^2 \phi}{\partial x^2} = \phi_{xx} = \phi_{0xx} + \Sigma \alpha_n \phi_{nxx}$$

$$\widehat{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} = -\phi_{xy} = -\phi_{0xy} - \Sigma \alpha_n \phi_{nxy}$$

the linear equations for the α's take the typical form

$$\iint \left\{ (\phi_{0xx} + \Sigma \alpha_m \phi_{mxx})\phi_{nxx} + (\phi_{0yy} + \Sigma \alpha_m \phi_{myy})\phi_{nyy} - \sigma(\phi_{0xx} + \Sigma \alpha_m \phi_{mxx})\phi_{nyy} - \sigma(\phi_{0yy} + \Sigma \alpha_m \phi_{myy})\phi_{nxx} + 2(1+\sigma)(\phi_{0xy} + \Sigma \alpha_m \phi_{mxy})\phi_{nxy} \right\} dxdy = 0 \ldots\ldots [8]$$

The first equation is obtained by taking $n = 1$, the second by taking $n = 2$, and so on, the summations in each case comprising all the terms $\alpha_m \phi_m$ from $\alpha_1 \phi_1$ to the end of the series.

When the boundary conditions are prescribed loads and not prescribed displacements, the stress distribution is independent of the elastic constants. We may then take $\sigma = 0$ without affecting the results, and, putting all terms in ϕ_0 on the right, rewrite [8] as

$$\iint \left[\sum_m \alpha_m \phi_{mxx}\phi_{nxx} + \sum_m \alpha_m \phi_{myy}\phi_{nyy} + 2\sum_m \alpha_m \phi_{mxy}\phi_{nxy} \right] dxdy = -\iint \left[\phi_{0xx}\phi_{nxx} + \phi_{0yy}\phi_{nyy} + 2\phi_{0xy}\phi_{nxy} \right] dxdy \ldots [9]$$

For the rectangle bounded by $x = \pm a$, $y = \pm b$, we may conveniently take

$$\phi = \phi_0 + (x^2 - a^2)^2(y^2 - b^2)^2[\alpha_1 + \alpha_2 x^2 + \alpha_3 y^2 + \alpha_4 x^2 y^2 + \ldots \ldots\ldots\ldots [10]$$

provided that the stresses are to be symmetrical with respect to both axes. Retaining only the four parameters written down, [9] gives four equations in which the coefficients of the α's on the left are integrals in terms of the polynomials on the right of [10], and can be integrated at once. The four equations[6] become, putting $b^2/a^2 = k$,

$$\left[\frac{64}{7} + \frac{256}{49}k + \frac{64}{7}k^2\right]\alpha_1 + \left[\frac{64}{77} + \frac{64}{49}k^2\right]\alpha_2 a^2 + \left[\frac{64}{49} + \frac{64}{77}k^2\right]\alpha_3 b^2 + \left[\frac{64}{539} + \frac{64}{539}k^2\right]\alpha_4 a^2 b^2$$
$$= -\frac{9.5.5}{128a^9b^5}\int_0^a\int_0^b \left[\phi_{0xx}\phi_{1xx} + \phi_{0yy}\phi_{1yy} + 2\phi_{0xy}\phi_{1xy}\right] dxdy$$

$$\left[\frac{64}{11} + \frac{64}{7}k^2\right]\alpha_1 + \left[\frac{192}{143} + \frac{256}{77}k + \frac{192}{7}k^2\right]\alpha_2 a^2 + \left[\frac{64}{77} + \frac{64}{77}k^2\right]\alpha_3 b^2 + \left[\frac{192}{1001} + \frac{192}{77}k^2\right]\alpha_4 a^2 b^2$$
$$= -\frac{9.7.5.5}{128a^{11}b^5}\int_0^a\int_0^b \left[\phi_{0xx}\phi_{2xx} + \phi_{0yy}\phi_{2yy} + 2\phi_{0xy}\phi_{2xy}\right] dxdy$$

$$\left[\frac{64}{7} + \frac{64}{11}k^2\right]\alpha_1 + \left[\frac{64}{77} + \frac{64}{77}k^2\right]\alpha_2 a^2 + \left[\frac{192}{7} + \frac{256}{77}k + \frac{192}{143}k^2\right]\alpha_3 b^2 + \left[\frac{192}{77} + \frac{192}{1001}k^2\right]\alpha_4 a^2 b^2$$
$$= -\frac{9.7.5.5}{128a^9b^7}\int_0^a\int_0^b \left[\phi_{0xx}\phi_{3xx} + \phi_{0yy}\phi_{3yy} + 2\phi_{0xy}\phi_{3xy}\right] dxdy$$

$$\left[\frac{64}{11} + \frac{64}{11}k^2\right]\alpha_1 + \left[\frac{192}{143} + \frac{192}{11}k^2\right]\alpha_2 a^2 + \left[\frac{192}{11} + \frac{192}{143}k^2\right]\alpha_3 b^2 + \left[\frac{576}{143} + \frac{256}{121}k + \frac{576}{143}k^2\right]\alpha_4 a^2 b^2$$
$$= -\frac{9.7.7.5.5}{128a^{11}b^7}\int_0^a\int_0^b \left[\phi_{0xx}\phi_{4xx} + \phi_{0yy}\phi_{4yy} + 2\phi_{0xy}\phi_{4xy}\right] dxdy \; [11]$$

2 It remains, before the parameters can be found from these equations, and the function [10] completed, to choose the function ϕ_0 satisfying the symmetrical boundary conditions, and representing the anticipated stress system as closely as possible.

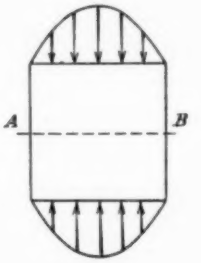

FIG. 16

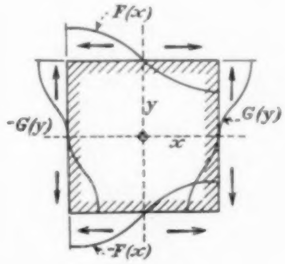

FIG. 17

When two opposite sides of the rectangle are loaded with normal pressures distributed in the same manner (e.g., Fig. 16), say $\widehat{yy} = p(x)$ on the two sides $y = \pm b$, we have to determine ϕ_0 so that

$$\begin{array}{lll} \phi_{0xx} = p(x) & \text{at} & y = \pm b \\ \phi_{0yy} = 0 & \text{at} & x = \pm a \\ \phi_{0xy} = 0 & \text{at} & x = \pm a \text{ and } y = \pm b \end{array}$$

In all the cases occurring subsequently we take simply

$$\phi_0 = \iint p(x)dxdx \ldots\ldots\ldots\ldots\ldots\ldots [12]$$

so that

$$\phi_{0xx} = p(x), \phi_{0yy} = 0, \phi_{0xy} = 0$$

everywhere.

When the applied loads are shearing forces the function ϕ_0 is not so simple. For a stress system symmetrical about both axes, the intensity of the shearing forces on the boundary $y = b$ will be an odd function of x say, $F(x)$, and on $y = -b$ it will be $-F(x)$ (Fig. 17). Similarly we may have a distribution $G(y)$

---

[6] The method of the preceding section of this Appendix is from a paper by Dr. S. Timoshenko, *Phil. Mag.*, vol. 47, June, 1924, p. 1095. Dr. Timoshenko uses three parameters and gives three such equations, which, inter alia, are applied to determine the stresses in a square plate due to parabolic distributions of pressure on opposite sides (Fig. 16). The introduction of the fourth term in [10], viz., $\alpha_4 x^2 y^2 (x^2 - a^2)^2 (y^2 - b^2)^2$, affects the distribution of the pressures across AB by only 2 per cent at most, chiefly at the ends.

on $x = a$ and $-G(y)$ on $x = -a$. The boundary conditions for ϕ_0 are then

$$-\phi_{0xy} = \left\{\begin{array}{ll} F(x) & \text{at } y = b \\ -F(x) & \text{at } y = -b \\ G(y) & \text{at } x = a \\ -G(y) & \text{at } x = -a \end{array}\right. \qquad \ldots\ldots [13]$$

and $\phi_{0xx} = 0$ at $y = \pm b$, $\phi_{0yy} = 0$ at $x = \pm a$

A suitable form for ϕ_0 is

$$\frac{x^2 - a^2}{2a}\int_b^y\left[-G(y) + \frac{y}{2b}G(b)\right]dy$$
$$+ \frac{y^2 - b^2}{2b}\int_a^x\left[-F(x) + \frac{x}{2a}F(a)\right]dx \ldots\ldots [14]$$

It may be verified directly by differentiation that this function fulfils conditions [13]. Its derivation ab initio is given in the next article.

3 Consider a function

$$\psi = (x^2 - a^2)g(y) + (y^2 - b^2)f(x) \ldots\ldots [15]$$

where $g(y)$ is an even function of y only, and $f(x)$ is an even function of x only. Differentiating,

$$\frac{\partial^2\psi}{\partial x^2} = 2g(y) + (y^2 - b^2)f''(x)$$

At $y = \pm b$ this has the constant value $2g(b)$. Similarly[7]

$$\frac{\partial^2\psi}{\partial y^2} = (x^2 - a^2)g''(y) + 2f(x)$$

has the constant value $2f(a)$ at $x = \pm a$.

These constant boundary values can be eliminated without affecting $\partial^2\psi/\partial x\partial y$ by adding to [15] the terms

$$-x^2g(b) - y^2f(a)$$

Further,

$$-\frac{\partial^2\psi}{\partial x\partial y} = -2xg'(y) - 2yf'(x)$$

a function odd in both x and y. If this is to be $\pm F(x)$ at $y = \pm b$

$$-2xg'(b) - 2bf'(x) = F(x) \ldots\ldots [16]$$

and if it is to be $\pm G(y)$ at $x = \pm a$,

$$-2ag'(y) - 2yf'(a) = G(y) \ldots\ldots [17]$$

Equation [16] is to hold for all values of x between $-a$ and $+a$. In particular it will hold for $x = a$, so that

$$2ag'(b) + 2bf'(a) = -F(a) \ldots\ldots [18]$$

Similarly [17] will hold for $y = b$ and

$$2ag'(b) + 2bf'(a) = -G(b) \ldots\ldots [19]$$

Since the shear stresses $F(x)$, $G(y)$, are complementary at $x = a$, $y = b$, $F(a) = G(b)$ and [18] and [19] become identical. They are fulfilled if

$$2bf'(a) = 2ag'(b) = -\frac{1}{2}F(a) = -\frac{1}{2}G(b) \ldots\ldots [20]$$

Taking therefore

$$2f(a) = -\frac{1}{2b}F(a)$$

[17] gives

$$2ag'(y) = -G(y) + \frac{y}{2b}F(a)$$

so that, from this and [20],

$$g(y) = \frac{1}{2a}\int\left[-G(y) + \frac{y}{2b}G(b)\right]dy \ldots\ldots [21]$$

In the same manner, [16] and [20] give

$$2bf'(x) = -F(x) + \frac{x}{2a}G(b)$$

and

$$f(x) = \frac{1}{2b}\int\left[-F(x) + \frac{x}{2a}F(a)\right]dx \ldots\ldots [22]$$

Substituting these results in [15] and adding the terms $-x^2g(b)$ and $-y^2f(a)$ to eliminate the boundary values of $\frac{\partial^2\psi}{\partial x^2}$ and $\frac{\partial^2\psi}{\partial y}$ on $y = \pm b$ and $x = \pm a$, respectively, we obtain the function

$$\phi_0 = \frac{x^2 - a^2}{2a}\int\left[-G(y) + \frac{y}{2b}G(b)\right]dy$$
$$+ \frac{y^2 - b^2}{2b}\int\left[-F(x) + \frac{x}{2a}F(a)\right]dx - x^2g(b) - y^2f(a) \ldots [23]$$

But from [21] and [22]

$$g(b) = \frac{1}{2a}\left\{\int_{y=b}\left[-G(y) + \frac{y}{2b}G(b)\right]dy\right\},\ f(a)$$
$$= \frac{1}{2b}\left\{\int_{x=a}\left[-F(x) + \frac{x}{2a}F(a)\right]dx\right\}$$

Consequently [23] can be expressed as

$$\phi_0 = \frac{x^2 - a^2}{2a}\int_b^y\left[-G(y) + \frac{y}{2b}G(b)\right]dy$$
$$+ \frac{y^2 - b^2}{2b}\int_a^x\left[-F(x) + \frac{x}{2a}F(a)\right]dx \ldots [14]$$

the constants $b^2f(a)$ and $a^2g(b)$ being included for symmetry.

By means of [12], its analogue in terms of y for pressures on the other pair of opposite edges $x = \pm a$, and [14], we can obtain a composite function ϕ_0, satisfying the boundary conditions for ϕ, for the most general case of continuous symmetrical loading of normal pressures and shearing forces on the four sides of a rectangle. After evaluating the integrals in the right-hand members of the Equations [11], α_1, α_2, α_3, α_4, can be found for any value of the ratio b/a and the approximate stress function [10] is completely determined. The results are usually satisfactory if the sides of the rectangle are not of very different magnitudes.

4 For stresses which will be symmetrical about the y-axis, but anti-symmetrical—i.e., the normal stresses represented by odd functions of y—about the x-axis, as in the bending problem of this paper, we take

$$\phi = \phi_0 + (x^2 - a^2)^2(y^2 - b^2)^2[\alpha_1 y + \alpha_2 x^2 y$$
$$+ \alpha_3 y^3 + \alpha_4 x^2 y^3] \ldots [24]$$

Then the Equations [9] for determining the α's become

---

[7] It is necessary that $f''(x)$ and $g''(y)$ should be finite.

$$\left[\frac{64}{7} + \frac{256}{105}k + \frac{64}{55}k^2\right]\alpha_1 b + \left[\frac{64}{77} + \frac{64}{385}k^2\right]\alpha_2 a^2 b$$
$$+ \left[\frac{64}{21} + \frac{512}{1155}k + \frac{192}{715}k^2\right]\alpha_3 b^3 + \left[\frac{64}{231} + \frac{192}{5005}k^2\right]\alpha_4 a^2 b^3$$
$$= -\frac{9\cdot7\cdot5}{128\,a^9b^6}\int_0^a\int_0^b\left[\phi_{0xx}\phi_{1xx} + \phi_{0yy}\phi_{1yy} + 2\phi_{0xy}\phi_{1xy}\right]dxdy$$

$$\left[\frac{64}{7} + \frac{64}{35}k^2\right]\alpha_1 b + \left[\frac{192}{91} + \frac{256}{105}k + \frac{192}{35}k^2\right]\alpha_2 a^2 b$$
$$+ \left[\frac{64}{21} + \frac{192}{455}k^2\right]\alpha_3 b^3 + \left[\frac{192}{273} + \frac{512}{1155}k + \frac{576}{455}k^2\right]\alpha_4 a^2 b^3$$
$$= -\frac{11\cdot9\cdot7\cdot5}{128a^{11}b^6}\int_0^a\int_0^b\left[\phi_{0xx}\phi_{2xx} + \phi_{0yy}\phi_{2yy} + 2\phi_{0xy}\phi_{2xy}\right]dxdy$$

$$\left[\frac{64}{21} + \frac{512}{1155}k + \frac{192}{715}k^2\right]\alpha_1 b + \left[\frac{64}{231} + \frac{192}{5005}k^2\right]\alpha_2 a^2 b$$
$$+ \left[\frac{832}{231} + \frac{4352}{15015}k + \frac{64}{715}k^2\right]\alpha_3 b^3 + \left[\frac{832}{2541} + \frac{192}{15015}k^2\right]\alpha_4 a^2 b^3$$
$$= -\frac{9\cdot7\cdot5}{128a^9b^8}\int_0^a\int_0^b\left[\phi_{0xx}\phi_{3xx} + \phi_{0yy}\phi_{3yy} + 2\phi_{0xy}\phi_{3xy}\right]dxdy$$

$$\left[\frac{64}{21} + \frac{192}{455}k^2\right]\alpha_1 b + \left[\frac{192}{273} + \frac{512}{1155}k + \frac{576}{455}k^2\right]\alpha_2 a^2 b$$
$$+ \left[\frac{832}{231} + \frac{192}{1365}k^2\right]\alpha_3 b^3 + \left[\frac{64}{77} + \frac{4352}{15015}k + \frac{576}{1365}k^2\right]\alpha_4 a^2 b^3$$
$$= -\frac{11\cdot9\cdot7\cdot5}{128a^{11}b^8}\int_0^a\int_0^b\left[\phi_{0xx}\phi_{4xx} + \phi_{0yy}\phi_{4yy} + 2\phi_{0xy}\phi_{4xy}\right]dxdy \qquad \ldots\ldots\ldots\ldots[25]$$

Appendix No. 2

THE RECTANGULAR PLATE COMPRESSED BY CONCENTRATED FORCES

The existence of the tensions across the plane *EFGH* (Fig. 1) may be demonstrated (following a suggestion of Dr. Timoshenko) with the help of the well-known "radial distribution"—the two-dimensional stress distribution induced in the semi-infinite plane by a single concentrated force normal to the boundary (Fig. 18), and given in polar coordinates, as in the figure, by

$$\overset{\frown}{rr} = \frac{2P}{\pi rt}\sin\varphi, \quad \overset{\frown}{r\varphi} = \overset{\frown}{\varphi\varphi} = 0$$

Considering a small semi-cylindrical surface round the line of application of the force, it can be shown that the horizontal resultant of the stresses on each side of the vertical is P/π. Separating the rectangle from the semi-infinite plane section, we obtain the radial system within it maintained by the concentrated force and a set of boundary tractions. Then we may superpose another such system, this time with the force acting on the lower edge, adding another set of boundary tractions. The horizontal components of the complete boundary tractions on one half of the rectangle are balanced by the two horizontal forces P/π (Fig. 19). These boundary tractions are eliminated by applying an equal and opposite set of tractions. It follows that this equal and opposite set will develop tensions across *EF* of total amount $2P/\pi$. The same is true, of course, of a circular disk compressed by forces at the ends of a diameter. It is clear, in fact, that the argument does not depend on the shape of the section except that, if the elements on which the forces bear are not perpendicular to the direction of the forces, the total tension will be $(2P/\pi)\sin\alpha$, where α is the angle which the force makes with the boundary.

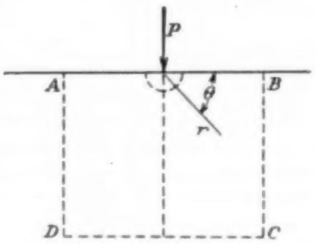

Fig. 18

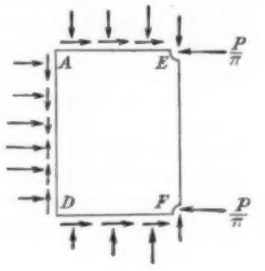

Fig. 19

Taking the origin at the middle of the rectangle, and measuring y upward, the radial distribution due to the upper force P (Fig. 18), in the semi-infinite plane below the horizontal *AB*, is given by

$$\overset{\frown}{xx} = -\frac{2P}{\pi t}\frac{x^2(b-y)}{[x^2+(b-y)^2]^2}, \quad \overset{\frown}{yy} = -\frac{2P}{\pi t}\frac{(b-y)^3}{[x^2+(b-y)^2]^2},$$
$$\overset{\frown}{xy} = \frac{2P}{\pi t}\frac{x(b-y)^2}{[x^2+(b-y)^2]^2}\ldots\ldots\ldots\ldots[26]$$

a positive force P being tensile. This gives tractions on the three sides of the rectangle, *AD*, *DC*, and *CB*. Superposing the radial distribution in the semi-infinite plane above *DC*, due to the lower force P (Fig. 1), i.e.,

$$\overset{\frown}{xx} = -\frac{2P}{\pi t}\frac{x^2(b+y)}{[x^2+(b+y)^2]^2}, \quad \overset{\frown}{yy} = -\frac{2P}{\pi t}\frac{(b+y)^3}{[x^2+(b+y)^2]^2},$$
$$\overset{\frown}{xy} = -\frac{2P}{\pi t}\frac{x(b+y)^2}{[x^2+(b+y)^2]^2}\ldots\ldots\ldots[27]$$

tractions are added on the three sides of the rectangle *DA*, *AB*, and *BC*, so that there are now tractions on all four sides in addition to the required concentrated forces. These boundary tractions are:

(1) A distribution of compressive forces on *AB* of intensity

$$-\frac{P}{\pi bt}\frac{b^4}{a^4}\cdot\frac{16a^4}{(x^2+4b^2)^2}\ldots\ldots\ldots\ldots\ldots\ldots[\text{A}]$$

and the same distribution on the opposite side *DC*.

(2) A distribution of compressive forces on *BC* of intensity

$$-\frac{P}{\pi bt}2\frac{b}{a}a^3\left[\frac{b+y}{[a^2+(b+y)^2]^2} + \frac{b-y}{[a^2+(b-y)^2]^2}\right]\ldots[\text{B}]$$

and the same distribution on the opposite side *AD*.

(3) Shearing forces on all four sides of intensities:

$$\left.\begin{array}{ll}
\text{On } AB(y=b), & -\dfrac{P}{\pi bt}8\dfrac{b^3}{a^3}\dfrac{a^3x}{(x^2+4b^2)^2} = F(x), \text{ say} \\[2ex]
\text{On } DC\ (y=-b), & -F(x) \\[2ex]
\text{On } BC\ (x=a), & \dfrac{P}{\pi bt}2\dfrac{b}{a}a^2\left\{\dfrac{(b-y)^2}{[a^2+(b-y)^2]^2}\right. \\[2ex]
 & \left.\qquad -\dfrac{(b+y)^2}{[a^2+(b+y)^2]^2}\right\} = G(y), \text{ say} \\[2ex]
\text{On } AD\ (x=-a), & -G(y).
\end{array}\right\} \cdot[\text{C}]$$

These three sets of boundary tractions [A], [B], [C], have to be annulled by superposing three stress systems with equal and opposite boundary values. To determine these stress systems we apply the minimum energy method of Appendix No. 1. The first step is to calculate the integrals on the right of Equations [11].

For system [A], we take, in accordance with [12],

$$\phi_{0xx} = \frac{P}{\pi bt}\frac{b^4}{a^4}\frac{16a^4}{(x^2+4b^2)^2}, \quad \phi_{0yy} = \phi_{0xy} = 0$$

The integrations on the right of Equations [11] can now be carried out, and the values of α_1, α_2, α_3, α_4 found for this system. The calculations are too lengthy to record.

In the same manner, system [B] leads to

$$\phi_{0xx} = \phi_{0xy} = 0, \quad \phi_{0yy} = \frac{P}{\pi bt}\,2\,\frac{b}{a}\,a^3\left\{\frac{b+y}{[a^2+(b+y)^2]^2} + \frac{b-y}{[a^2+(b-y)^2]^2}\right\}$$

and the solution for this system can be completed from Equations [11].

System [C] is dealt with by means of the function [14]. The derivatives of the particular form of this function which annuls [C], are

$$\phi_{0xx} = -\frac{P}{\pi bt}\frac{b}{a}\left[\tan^{-1}\frac{2ab}{a^2-b^2+y^2} - \tan^{-1}\frac{2b}{a} - \frac{a(b+y)}{a^2+(b+y)^2} - \frac{a(b-y)}{a^2+(b-y)^2} - 4ab\frac{(3x^2-4b^2)(y^2-b^2)}{(x^2+4b^2)^3} - \frac{4aby^2}{(a^2+4b^2)^2} + \frac{2ab(a^2+6b^2)}{(a^2+4b^2)^2}\right]$$

$$\phi_{0yy} = -\frac{P}{\pi bt}\frac{b}{a}\left[\frac{2a(x^2-a^2)(b+y)[a^2-(b+y)^2]}{[a^2+(b+y)^2]^3} + \frac{2a(x^2-a^2)(b-y)[a^2-(b-y)^2]}{[a^2+(b-y)^2]^3} - \frac{4ab}{x^2+4b^2} - \frac{4abx^2}{(a^2+4b^2)^2} + \frac{8ab(a^2+2b^2)}{(a^2+4b^2)^2}\right]$$

$$-\phi_{0xy} = \frac{P}{\pi bt}\frac{b}{a}\left[\frac{2ax(b+y)^2}{[a^2+(b+y)^2]^2} - \frac{2ax(b-y)^2}{[a^2+(b-y)^2]^2} + \frac{8abxy}{(x^2+4b^2)^2} - \frac{8abxy}{(a^2+4b^2)^2}\right]$$

whence the right sides of Equations [11] can be evaluated, and the approximate stress function for system [C] completed. These somewhat lengthy forms compare unfavorably with the simple expressions we are able to use for the normal boundary tractions. It does not appear, however, that there is any undue complication, if we observe that the expression for $-\phi_{0xy}$, and any "shear stress" derived from [14] gives a linear transition of a value on the boundary to the opposite value at the opposite boundary.

Instead of evaluating α_1, α_2, α_3, and α_4 separately for each of the three systems [A], [B], [C], we may add the right sides of Equations [11] for the three systems and obtain at once the values of the α's corresponding to the complete set of boundary tractions, [A], [B], and [C] together. These total values of the α's are given in Table 3.

The complete stress system for a rectangular block compressed by concentrated forces is now determined as the radial distributions [26] and [27], the contributions from the ϕ_0-functions [A], [B], and [C] and the "correction stresses" derived from

$$(x^2-a^2)^2(y^2-b^2)^2[\alpha_1 + \alpha_2x^2 + \alpha_3y^2 + \alpha_4x^2y^2]$$

as from an ordinary stress function, with the values of α_1, α_2, α_3, and α_4 appropriate to the particular value of b/a in question, as given in Table 3.

If the loads are not ideally concentrated, but distributed on *ab* and *cd*, Fig. 1, the boundary tractions corresponding to [A], [B], and [C] will not be effectively altered if *ab* and *cd* are less than about one-third of the smallest side of the rectangle *ABCD*. The curves of Fig. 3, representing the tensions introduced by superposing equal and opposite boundary tractions, will still be valid, but they must now be combined with the curves of local compressive stresses in the manner already explained in the paper.

TABLE 3

		b/a		
		2	1	$^1/_2$
α_1	=	−0.00297	+0.06294	+0.9426
α_2a^2	=	−0.00088	−0.00132	+0.1617
α_3b^2	=	+0.01874	+0.09054	+0.0546
$\alpha_4a^2b^2$	=	+0.00868	+0.10071	−0.6343

Appendix No. 3

BENDING OF BEAMS BY A NON-LINEAR DISTRIBUTION OF BENDING FORCES

The boundary conditions for the cubic distribution are that on the ends $x = \pm a$, $\widehat{xx} = Ay^3$, the boundaries being otherwise free from load. Clearly the stress function must be even-valued in x and odd-valued in y, so that the form [24] of Appendix No. 1 is appropriate. Also we may take $\phi_0 = \int\int Ay^3dydy$ giving $\phi_{0xx} = \phi_{0xy} = 0$, $\phi_{0yy} = Ay^3$. Then the values of the α's in [24] can be found by solving Equations [25], after evaluating the coefficients.

Discussion

Max M. Frocht.[8] The paper is interesting both from the point of view of method and that of results. The formulation of a suitable stress function for the removal of shear stresses from the boundaries of rectangles represents a valuable contribution to

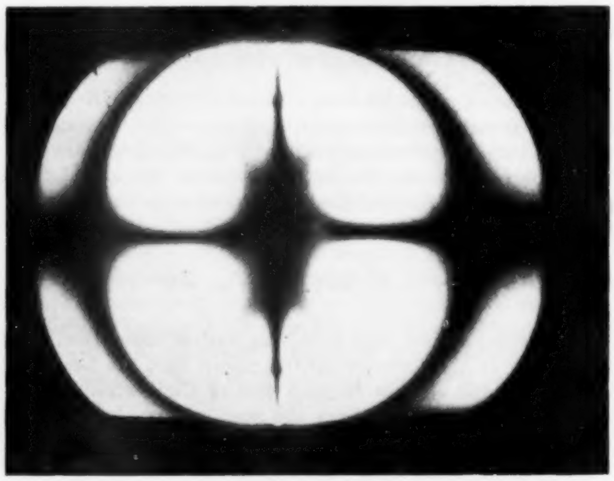

Fig. 20 Glass Model. Cross Represents Zero Isoclinic

the science of stress analysis. Such a function is given by Equation [14], and the shears that are to be removed are shown in Fig. 17.

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[8] Associate Professor of Mechanics, Carnegie Institute of Technology, Pittsburgh, Pa. Participant Applied Mechanics Division, A.S.M.E.

The results regarding the stress distribution in rectangular blocks under concentrated loads are rather significant. Consider, for example, the curves of Fig. 11. It is seen that the maximum compressive stress is about twice the average stress. In other words, an error of approximately 100 per cent is introduced when the compressive stress is computed by the elementary formula $S = P/A$.

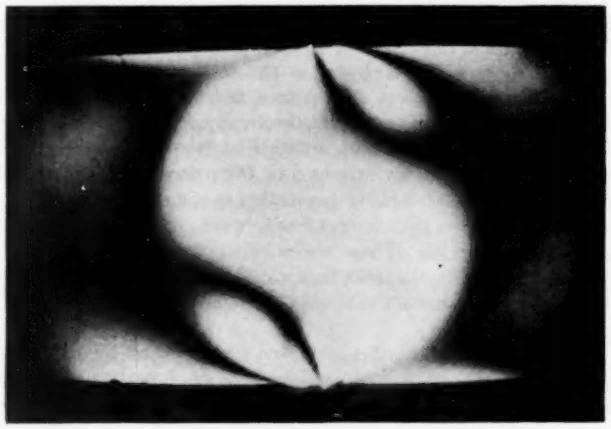

FIG. 21 GLASS MODEL CONTAINING 30-DEG ISOCLINIC
(b/a valve differs from that in Fig. 20.)

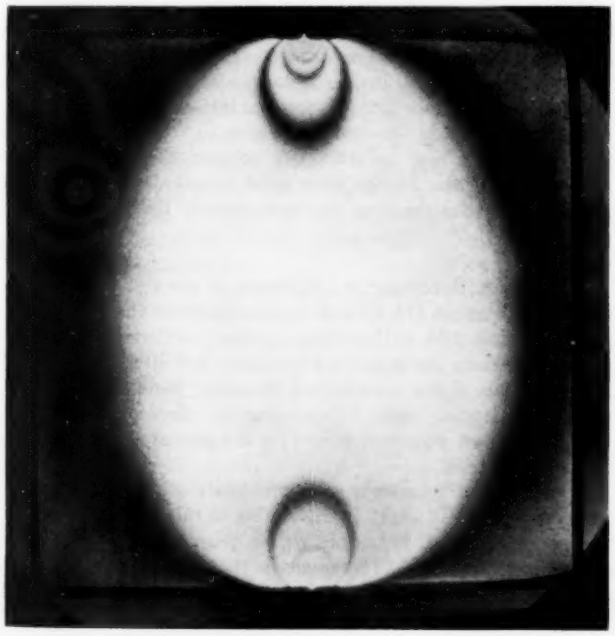

FIG. 22 GLASS MODEL. SQUARE PLATE, ISOCLINICS REMOVED

Average Tensile Stress. The existence of large tensile stresses across the middle plane is no doubt of considerable practical importance, and it is instructive to find that the average stress $P/\pi bt$ is independent of the proportions of the rectangle. It may perhaps be added that the average tension would remain the same as long as the product bt, that is, the cross-sectional area, remains the same, even if b varied. It may further be noted that the total tension is constant for all rectangles and equals $2P/\pi$, regardless of their dimensions.

The generalization, however, that the average tension is always equal to $P/\pi bt$ "whether the section is rectangular or not," is open to question. In Appendix No. 2 the author modifies this statement to take care of concentrated loads inclined to the surfaces upon which they act, and he gives the total tension as $2P \sin \alpha/\pi$, where α is the angle which the force makes with the boundary. The opposing forces may, however, have different inclinations. It would therefore be more correct to state that the total tension is given by (P/π) $(\sin \alpha_1 + \sin \alpha_2)$. But even this correction will not take care of all non-rectangular shapes. Consider, for example, the case of a square block subjected to concentrated diagonal loads. In this the total tension is given by $2P/(\pi + 2)$ and not by the above expression.

Maximum Tension. It is well known that the middle portion of slender bars acted on by axial concentrated loads is subjected to approximately uniformly normal stress and to no lateral stresses. This, together with the previously stated property that the total tension across the central vertical section of a rectangle remains constant, would lead us to believe that the maximum tension increases with the ratio b/a. The results, however, given in Table 1 do not entirely agree with this conclusion. Thus the maximum tension for the square block seems to be less than for the flat block. The discusser is somewhat puzzled by this result.

Need for Further Work. It would be advisable to continue further work on rectangular blocks in order to ascertain that ratio b/a for which the tension becomes a maximum. In view of the laboriousness of the method employed by the author, it would seem more advisable to attack further cases of this problem photoelastically. The shape being rather simple, models could be made of glass and the stress distribution determined with great precision. The discusser has made a start in this direction, and hopes to continue when opportunity permits. Figs. 20, 21, and 22 represent some of the results from this preliminary investigation. These photographs were taken in connection with a study of the isoclinic systems of rectangular blocks. Attention is called to the dark, circle-like curves which persist among all isoclinics and also when the isoclinics are removed, as is the case when a circularly polarized beam of light is used. These curves do not belong to the isoclinics and may thus have one of two meanings: they are either the locus of points of zero stress or of equal principal stresses. Fig. 11 shows two points of zero stress for the square block at approximately the points where the oval cuts the horizontal axis of symmetry. The discusser is wondering whether the author could from his analytical solution throw further light on the meaning of these curves.

C. A. GARABEDIAN.[9] There can be no question that the problems studied by the author are of both theoretical and practical interest. Incidentally, the paper seems to suggest the possibility of further profitable studies in non-linear distributions. And if we may assume that closely similar problems of this sort can be solved by two or more equally ingenious methods applicable to plates and beams, there would still remain the problem of a comparative study to ascertain the relative merits of the methods employed and results obtained. Unfortunately, the value of such a comparative study must be left to conjecture; the individual investigator is sufficiently occupied to solve his special problem by one effective method.

The comment just made is obviously concerned only indirectly with the particular paper under discussion. The author's excellent contribution is but one of many theoretical studies in elasticity which remind us that this field, besides presenting many special problems as yet unsolved, also offers a challenge to those interested in comparison of methods, uniformity of treatment, synthesis of isolated results—in short, in studies which aim at comprehensiveness. So far as the writer is aware, there has been little done in this direction since Saint-Venant.

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Theodor von Kármán.[10] The author states that the curve he obtained for the stress distribution in the case of a rectangular block with the ratio 1/2 between the height $2b$ and the width $2a$ shows no agreement with the corresponding curve for infinite width given by F. Seewald in his paper published in the "Abhandlungen des Aerodynamischen Instituts Aachen," 1927. He also points out that the mean ordinate of Seewald's curve is not $P/\pi bt$, as it should be according to Appendix No. 2 of the paper.

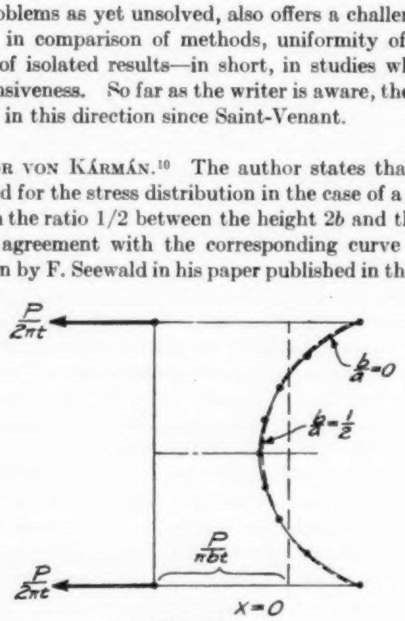

FIG. 23

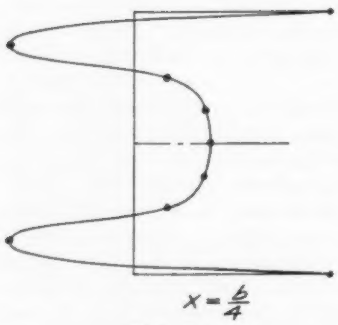

FIG. 24

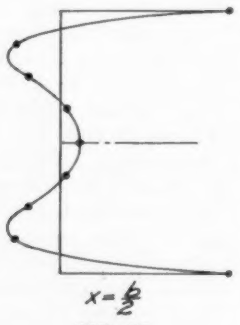

FIG. 25

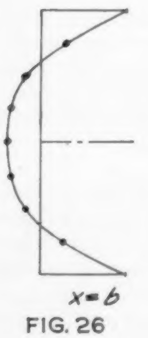

FIG. 26

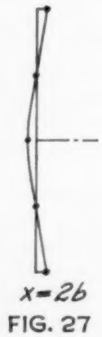

FIG. 27

As Dr. Seewald obtained his *exact* solutions in the case of a block with infinite width, by using a method introduced by the writer in the theory of beams in a previous paper, the latter was interested in this alleged discrepancy and checked the author's and Mr. Seewald's solutions. The result is shown in Fig. 23. Using Seewald's numerical values and diagrams (page 23 of his paper), the writer obtains the solid line, while the author's curve is reproduced by the broken line. The agreement of the two is so close that the author is to be congratulated on the accuracy of the method he used for the approximation.

It seems to the writer that the author's remark is based on the schematic sketch given by Seewald on page 26 of his paper. Unfortunately this figure is somewhat misleading, because it refers to a case wherein the lines of action of the two opposite forces are *nearly* coincident and this does not appear in the drawing; the diagram represents the stress distribution in a section which lies between both forces. For this reason it includes the large compression stresses which appear in the sections near the line of action of the forces, but disappear in the section $x = 0$ in the case of opposite forces acting in the same line, as treated by the author. Therefore the mean ordinate in Seewald's curve should not be $P/\pi bt$ but zero. It seems to the writer that even in the author's case the correct representation of the stress distribution should include the two concentrated forces, as shown in the accompanying Fig. 23. These forces belong to stress distribution; they correspond in the limit to the compression stresses and reduce the mean value of the stress to zero, as it appears in the case of other sections.

L. H. Donnell has kindly calculated for the writer the distribution of the stresses in different sections of a beam of infinite length, the beam being compressed by two opposite forces acting perpendicularly to the length of the beam. Completing Seewald's work, he obtained the curves reproduced in Figs. 24–27. The curves represent the stresses in the sections corresponding to $x = {}^1/_4b$, ${}^1/_2b$, b, and $2b$. It will be noted that the stresses in the section $x = 2b$ are very small; this explains the good agreement between the two curves in Fig. 23. Another interesting point is that the difference between the stresses appearing in the sections $x = 0$ and $x = b$ is nearly constant. The writer recalls that Bleich and the author found a uniform stress distribution across the section $x = 0$ in the case where $a = b$. It is an interesting fact that in this case the difference between the stresses in the two sections of the infinite beam gives apparently a fairly good approximation for the stresses which occur in the block of finite width.

Michael A. Sadowsky.[11] Referring to the test immediately following Equation [6], on page 4, the condition that ϕ_1, ϕ_2, etc. give no contribution to the stresses at the boundaries is true only in the case where the prescribed boundary conditions refer to the stresses. But if these prescribed boundary conditions refer to other magnitudes, such, for instance, as displacements, then ϕ_1, ϕ_2, etc. must give zero values for the prescribed magnitudes at the boundaries.

Appendix No. 2 presents an approximate solution of the proposed problem. It might be interesting to calculate the value of the energy integral, Equation [5], on page 4, for the obtained approximate solution. The reason is that if many approximate solutions of the same problem are found, the values of the energy integral may give a good idea of the achieved degree of approximation.

Author's Closure

Dr. Frocht's remarks on the function [14] raise a question of terminology. This function is not in the strict sense entitled to the description of "stress function." It gives stress formulas which satisfy the boundary conditions [13] and the conditions of equilibrium, but does not in general give compatible strains. It is to correct, as far as possible, this shortcoming that the functions added to ϕ_0 in [10] and [24] are introduced. In the paper I have used the term "approximate stress function" for truncated series, such as [10] and [24], having the form of [6].

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[11] 703 Delaware St., Minneapolis, Minn.

The generalization that the average tensile stress is always equal to $P/\pi bt$, whether the section is rectangular or not, was intended to carry with it all the limitations implied by the argument in Appendix 2 on which it is based. To have enumerated all these and to have considered the analogous cases not included, such as those raised by Dr. Frocht, would have been perhaps to sacrifice conciseness. My own inclination was not to attach very much importance to an estimate of the average tensile stress except as a check on the accuracy of the final results. The maximum tensile stress appears to be the really important quantity.

The considerations brought forward by Dr. Frocht under the heading of *Maximum Tension* imply that as the rectangle is narrowed the tension in the middle portion should steadily decrease until it disappears, the tension near the ends meanwhile increasing so as to preserve the constant mean $P/\pi bt$. This expectation is of course a very plausible one, but nevertheless it is not completely fulfilled by the author's calculations, which indicate that the flat block has a smaller tensile stress in its middle portion than has the narrower square block. I agree with Dr. Frocht that this is a surprising result. It is supported, however, by a consideration of the curves for the infinite strip contributed by Dr. von Kármán. Taking out the middle square from the infinite strip, we should have the boundary tractions of Fig. 26 to annul. It is easily seen that the effect of doing this would almost certainly be to modify the curve in Fig. 23 so as to increase the tension in the middle portion. Certain shears would remain on the edges $x = \pm b$, but there is not much likelihood of their being important.

The question of what shape of rectangle has the greatest tensile stress is one which the author has had in mind throughout the work, although the method of the paper is not sufficiently general to afford a direct answer. The following argument shows that the tension has no upper limit, but may be made to exceed any value by making the rectangle narrow enough. Let us consider in the first place a series of rectangles of given height, but of decreasing widths. When a certain narrowness is attained, the stress distribution in the middle portion will be effectively uniform unidirectional compression. The stress distribution in the upper half of the rectangle is then the same as in the congruent region in the three-boundary problem indicated by Fig. 28—the fourth boundary being at infinity. Any stress formula in this problem must be, for dimensional reasons, of the form

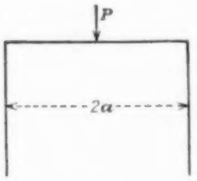

FIG. 28

$$p = \frac{P}{at} f\left(\frac{x}{a}, \frac{y}{a}\right)$$

or, in conformity with the formulas of the paper:

$$p = \frac{P}{\pi bt} \cdot \pi \frac{b}{a} \cdot f\left(\frac{x}{a}, \frac{y}{a}\right)$$

The function $f\left(\frac{x}{a}, \frac{y}{a}\right)$ will have a numerical maximum independent of a. If, therefore, the half-width a is diminished, the maximum tension will increase in the inverse proportion, without limit. In a model the load will be spread on a finite length, and the argument ceases to apply when the width of the rectangle itself becomes comparable with this length. For a given extent of spreading and mode of distribution of the load there *will* be some ratio b/a giving a maximum greatest tensile stress.

There is clearly scope for the photoelastic method in these problems, but optical investigation would bring with it its own peculiar difficulties. The maximum tension is always found close to the applied forces, and it is in such regions that the optical method appears to be least accurate. As the calculations of the paper show, the maximum tension is appreciably affected by both the mode and extent of the distribution of the applied forces, and some degree of spreading of the load is inevitable in a model. These features would detract from the precision of photoelastic results. The theoretical analysis has the advantage of being able to furnish curves, such as those of Fig. 3, setting an upper limit to the tensions. But the full-line curves in Figs. 7 and 9 would appear to be ideal for photoelastic investigation, the uniform distribution having already been realized by Dr. Frocht himself by transmitting the load to a glass model through soft copper.

If the nearly circular curves in the photographs were loci of zero stress, the stress distribution within them would be nearly the same as that in a circular disk. This would be quite at variance with the theoretical results. A simple and ingenious method of discriminating between points of zero stress and points of equal principal stresses is described on page 249 of Coker and Filon's recent "Treatise on Photoelasticity."

Professor von Kármán's remarks, disposing of the apparent discrepancy referred to on page 2, are extremely valuable, and the author is also much indebted to him and to Dr. Donnell for contributing the comparison of the formulas of the paper with certain results of his own exact theory of the infinite strip. He is right in supposing that the author's remark was based on a misconception of Seewald's sketch.

The value of the average *stress*, on a section parallel to the forces, must of course be zero in all cases. In the paper attention was directed to the region where the stress, on $x = 0$, is tensile, and for this reason the terms "tensile stress" or "tension" were used instead of merely "stress." Although the mean *stress* must be zero, the mean *tension*, when the loads are concentrated, must be $P/\pi bt$. The concentrated forces appearing in Dr. von Kármán's Fig. 23 might well have been included in my Fig. 3. (It seems that in Fig. 23 they are given only half the magnitude they should have, if t is the whole thickness.) Their existence, however, is pointed out in Appendix 2.

The first of Dr. Sadowsky's remarks provides a necessary correction to a statement left incomplete through confining attention too closely to the special type of boundary conditions, in terms of stresses, occurring in the paper.

APM-54-18

Thermal Stresses in Spheres and Cylinders Produced by Temperatures Varying With Time

By CLARENCE H. KENT,[1] STATE COLLEGE, PA.

In this paper solutions are given of the problem of stress distribution in spheres and cylinders of elastic material for cases in which the temperature is a function of both the radial coordinate and the time. Two types of temperature variation are included; that due to sudden cooling, as in quenching, and that due to linear surface heating, as in furnace heating. These are applied both to the solid and hollow sphere and cylinder. Examples are included of the temperature and stress distribution in a long solid forging of 30 in. diameter and in a cylindrical drum of 50 in. diameter when each is heated by raising the temperature of its convex surface at a uniform rate.

THE state of stress in spheres and long cylinders due to temperatures which vary with the radial coordinate only is well known. The case of the steady flow of heat in hollow cylinders is covered in several textbooks.[2] When the heat flow is variable instead of steady, the same stress equations may be used provided the temperature is a function of the radius and time only.

Such a temperature distribution occurs in the sudden cooling, as in quenching, of spheres and long cylinders and in the furnace heating of these shapes. Other cases not considered in detail here also produce this type of temperature distribution, as in the radiation of heat to the air from a hot body.

The process of solution is to substitute in the already available stress formulas, expressions for the temperature as a function of radius and time for the given condition of heat flow. For the case of the cooling of a sphere by suddenly changing the temperature of its surface, this expression is a trigonometric series, and for the cylinder a series of Bessel's functions. It is found, however, that these series converge so rapidly that one or two terms prove sufficient for a practical solution. For this reason the labor of substitution in the final result is not great.

Figs. 1 to 8 show the temperature and stress distribution for solid and hollow spheres and cylinders when subjected to sudden cooling and linear heating at assumed rates. Fig. 9 shows the time rate of change of stress on the surface of a solid sphere and long cylinder when the body is cooled by suddenly lowering the surface temperature and thereafter maintaining it constant.

It is noticed that the maximum tensile stresses of both bodies occur at the surface at the beginning of the cooling, and that these values are equal in the two cases and are given by $EaT_1/(1 - \mu)$, where T_1 is the temperature change of the surface, a is the linear coefficient of thermal expansion, and E and μ are the elastic constants. This is seen to be independent of the shape and size of the body and to depend only on the temperature change and the properties of the material. It is equivalent to the stress produced in a thin plate when its temperature is changed T_1 degrees and a change of area is prevented. The very thin layer of the external surface of a body of any shape or size, whose temperature is affected by the sudden change, is prevented from changing its area by the relatively very large mass of the rest of the body. As the time curves show, this stress is only momentary and rapidly drops as the reduction of temperature spreads inward and a contraction of the outside layer is permitted.

If the value of the maximum tension at the surface is sufficient to produce a permanent set in the material, residual stresses will be present when the sphere has cooled. It is obvious that as the result of casting or heat treatment these residual stresses may be very high.

Theoretically, the stress formulas for the cylinder are accurate only when the cylinder considered is of infinite length. They will apply, however, to that portion of a long finite cylinder not near the ends. There is no solution at present which will apply to the end portions. We know that in the case of a thin cylindrical shell with free ends the tangential stress at the ends rises to a maximum some 35 per cent above the value for the rest of the cylinder.[3]

As an example of the formulas developed in this paper, let us take the case of a long cylindrical forging, 30 in. in diameter, such as the rotor of a high-speed alternator. If its operating temperature is 180 F, and if the surface of the rotor is brought up to this value in 30 min from room temperature of 60 F, the average rate of change of surface temperature is 4 deg per min. If we assume this rate of change to be constant, we have the condition which has been termed "linear heating." When this form of heating has continued a sufficient length of time the temperature gradient at any point becomes a constant, that is, all parts of the cylinder are being increased in temperature as the constant time rate of the surface. Stresses in the cylinder have now reached a maximum and continue at this value until the point at which they begin a gradual reduction to zero caused by the leveling off of the temperature curve to a uniform value.

Fig. 6 shows the temperature distribution in the forging 20 min after the heating begins, at which time the steady state

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[2] See Timoshenko, "Strength of Materials."

Contributed by the Applied Mechanics Division and presented at the Second National Applied Mechanics Meeting, Yale University, New Haven, Conn., June 23 to 25, 1932, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] See Trans. A.S.M.E., 1931, paper APM-53-13.

has been reached. The exterior has reached a temperature of 140 deg, and the interior, 70 deg. The temperature curve is a parabola, and retains the same form until the surface temperature reaches its maximum. During this time the stresses remain constant, since thermal stresses depend on the form of the body, its material, and the temperature gradient.

As shown in Fig. 6, the stress curves are also parabolas. On the surface the axial and tangential stresses reach a maximum value in compression of 19,920 lb per sq in. At the center the axial stress reaches the same value in tension, but the tangential stress at the same point attains only one-half this value. The last-named stress is of the most importance, since it will add directly to the stresses produced by rotation.

Fig. 6 represents the stress distribution only for that portion of the cylinder not near the ends. At the ends the axial stress must be zero, and the radial and tangential stresses will be considerably changed by this condition.

Another example is the drum of a steam-turbine rotor. We may take such a drum, with an external diameter of 50 in. and internal of 40 in. Assume the temperature of the convex surface of the drum to be raised from 60 F to 700 F in one hour at a constant time rate. The steady state is quickly reached in the hollow cylinder, and Fig. 8 shows the temperature curve 10 min after the heating begins. The exterior has reached a temperature of 167 F, and the interior, 89 deg. The curve is a parabola to which a logarithmic term has been added.

As in the case of the solid cylinder, the tangential and axial stresses reach a common maximum in compression on the external surface of 14,300 lb per sq in. On the internal surface these stresses reach a value of 8050 lb per sq in. in tension.

It will be noticed that the radial stress does not exceed 600 lb per sq in. at any point. But little error would have been introduced in this problem if it had been treated as a thin cylinder by neglecting the radial stress. In that case both the axial and tangential stresses would have been shifted slightly from the positions of Fig. 8 at all points except on the two surfaces, and would completely coincide with one another. For such a thin cylinder, solutions are available for the end portions.[4]

NOTATION

a = linear coefficient of thermal expansion
A = coefficient of thermal diffusivity = $K/\rho s$
b = internal radius of sphere or cylinder
c = external radius of sphere or cylinder
e_r, e_t, e_z = radial, tangential, and axial unit strains
E = modulus of elasticity
K = coefficient of thermal conductivity
m = constant time rate of change of surface temperature, deg F per sec
ρ = density of material
r = radial coordinate of sphere or cylinder
s = specific heat of material
t = time
T = temperature
u = radial displacement of point in sphere or cylinder
$\sigma_r, \sigma_t, \sigma_z$ = radial, tangential, and axial unit stresses
μ = Poisson's ratio.

SOLID SPHERE

The equations expressing the relation between stresses and a temperature which varies in the radial direction in the solid sphere are:[5]

$$\sigma_r = \frac{2Ea}{1-\mu}\left[\frac{1}{c^3}\int_0^c Tr^2dr - \frac{1}{r^3}\int_0^r Tr^2dr\right] \ldots\ldots [1]$$

$$\sigma_t = \frac{Ea}{1-\mu}\left[\frac{2}{c^3}\int_0^c Tr^2dr + \frac{1}{r^3}\int_0^r Tr^2dr - T\right] \ldots [2]$$

At the point $r = 0$, these equations become:

$$\sigma_r = \sigma_t = \frac{2Ea}{1-\mu}\left[\frac{1}{c^3}\int_0^c Tr^2dr - \frac{1}{3}T\right] \ldots\ldots [3]$$

These equations are developed on the assumption that the sphere is free from stress when all points are at the temperature zero, but since the state of stress is not affected by a uniform temperature change, the symbol T really represents the difference between the temperature of any point and the original uniform condition. We have now to determine the value of T for the given condition of heating or cooling, and substitute in the equations above to determine the stress distribution.

Sudden Cooling. Assume a solid sphere whose initial temperature is uniform throughout and equal to T_1, and whose surface is then kept at zero. The temperature of any point r distant from the center at any time t after cooling begins, is given by the solution of the equation:[6]

$$\frac{\partial}{\partial t}(Tr) = A\frac{\partial^2}{\partial r^2}(Tr)$$

which is:[7]

$$Tr = \frac{2c}{\pi}T_1\left[e^{-pt}\sin\frac{\pi r}{c} - \frac{1}{2}e^{-4pt}\sin\frac{2\pi r}{c} + \frac{1}{3}e^{-9pt}\sin\frac{3\pi r}{c}\right] \ldots\ldots [4]$$

where $p = A\pi^2/c^2$.

EXAMPLE 1. For low-carbon steel the following constants may be used:

$E = 30 \times 10^6$ lb per sq in.
$a = 6.7 \times 10^{-6}$ in. per in. per deg F
$A = 0.027$ sq in. per sec
$\mu = 0.30$

The coefficient A is chosen high because of the low temperature to be assumed. A, a, and E are actually functions of the temperature, but it is sufficiently accurate for practical purposes to choose average values which hold for the temperature range concerned.

Let us assume a solid sphere of 5 in. radius which is heated throughout to a uniform temperature of 100 F and then, by immersion in a liquid, the surface is maintained at zero. Obviously the stress calculation for any temperature difference of 100 deg will be the same regardless of the value of the initial uniform temperature.

Fig. 1 shows the temperature distribution one minute after the cooling has begun. To obtain the temperature at the center, differentiate Equation [4] and place $r = 0$ in a manner similar to that by which Equation [3] was obtained from [1]. The same result may be obtained from [4] by allowing r to approach zero and remembering that:

$$\lim_{r \doteq 0} \frac{\sin\frac{m\pi r}{c}}{\frac{m\pi r}{c}} = 1$$

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[4] See Trans. A.S.M.E., 1931, paper APM-53-13.

[5] See Appendix No. 1.

[6] See Appendix No. 3.

[7] Byerly, "Fourier's Series," p. 116.

We have then, for $r = 0$,

$$T = 2T_1[e^{-pt} - e^{-4pt} + e^{-9pt} \dots]$$

In this case, $pt = 0.64$, and $T = 200\ (0.526 - 0.0772 - 0.0031 - \dots) = 90.4$ F, which illustrates the rapid convergence of the series involved. Other points of the temperature curve are obtained by substitution of the values of r in Equation [4] for $t = 60$.

To arrive at the corresponding stresses we need to evaluate the integrals:

$$\int_0^c Tr^2dr = \frac{2c^3}{\pi^2} T_1 \left[e^{-pt} + \frac{1}{4} e^{-4pt} + \frac{1}{9} e^{-9pt} + \dots \right] = 1385$$

$$\int_0^r Tr^2dr = \frac{2c^2}{\pi^2} T_1 \left[e^{-pt} \left(\frac{c}{\pi} \sin \frac{\pi r}{c} - r \cos \frac{\pi r}{c} \right) - \frac{1}{4} e^{-4pt} \left(\frac{c}{2\pi} \sin \frac{2\pi r}{c} - r \cos \frac{2\pi r}{c} \right) + \dots \right]$$

These series, owing to the introduction of the coefficient $1/n^2$ converge even more rapidly than the temperature equation illustrated above. Substitution of the value of these integrals in Equations [1], [2], and [3] enables the stress curves of Fig. 1 to be easily plotted.

We note from Fig. 1 and also from the symmetry of the body that at the center of the sphere, $\sigma_r = \sigma_t$. Since the tangential stress has the same value in all circumferential directions, the material at the center is subjected to an equal compression in all directions—the equivalent of hydrostatic pressure. If we reverse the temperature conditions, as by plunging a cold sphere into a hot medium, we can produce at this point a condition of equal tension in all directions which will be the maximum tension produced in the sphere.

Under the conditions of Fig. 1, the maximum tension occurs at the surface when $t = 0$. If we place $r = c$ and $t = 0$ in Equation [2], we find:

$$\text{max. } \sigma_t = \frac{EaT_1}{1 - \mu} = 28{,}700 \text{ lb per sq in.}$$

where

$$\pi^2/6 = \left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots\right)$$

This is, of course, a very localized stress existing only at the surface. Fig. 9 shows how rapidly this stress reduces with time as the cooling spreads inward.

The radial stress in this case is always compression and the maximum occurs at the center. The time at which the maximum is reached is found in the usual way by differentiation of Equation [3] with respect to time and equating the result to zero. We obtain:

$$\sum_1^\infty e^{-n^2pt} \left[1 + (-1)^n \frac{n^2\pi^2}{3} \right] = 0$$

$$e^{-pt} = 0.5674; \text{ and } pt = 0.5667$$

$$\text{max. } \sigma_r = 0.385 \frac{EaT_1}{1 - \mu} = 11{,}050 \text{ lb per sq in. compression}$$

The maximum radial and tangential stresses are seen to be independent of the size of the sphere, and the maximum radial is always 38.5 per cent of the tangential. In this example the time at which the maximum radial stress occurs is 53.2 sec after the cooling begins.

Linear Heating.[3] A second important temperature distribution occurs when the sphere is heated or cooled by changing the surface temperature at a known rate. The temperature of any point is found by a solution of the same partial-differential equation as in the preceding case but adapted to the different boundary conditions. Assume the sphere to be heated by increasing the temperature of the surface at a constant rate, producing "linear heating." If the initial temperature of the sphere is zero, the solution becomes:

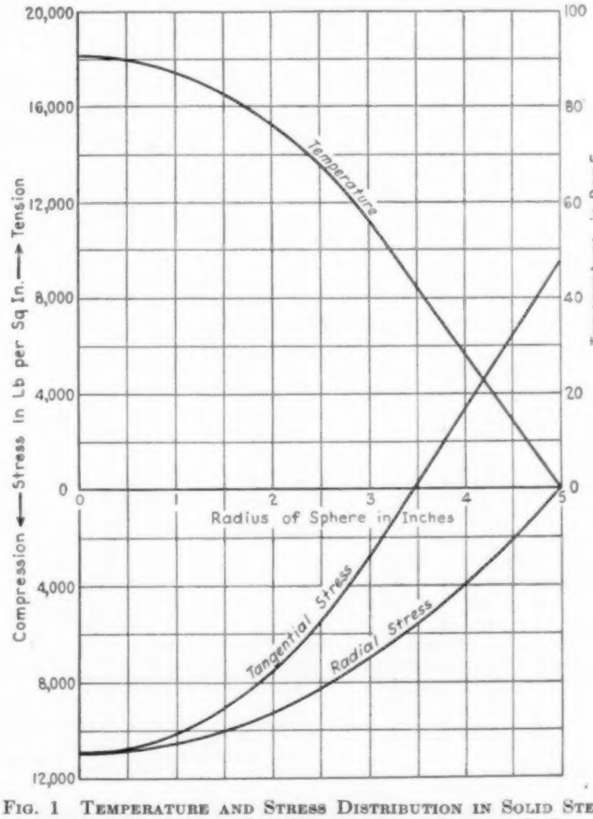

FIG. 1 TEMPERATURE AND STRESS DISTRIBUTION IN SOLID STEEL SPHERE OF 5 IN. RADIUS, ONE MINUTE AFTER SURFACE IS COOLED 100 F BELOW ORIGINAL UNIFORM TEMPERATURE

$$T = mt - \frac{m}{6A}(c^2 - r^2) + \frac{2mc^3}{A\pi^3} \sum_1^\infty \frac{(-1)^{n+1}}{n^3} e^{-n^2pt} \frac{1}{r} \sin \frac{n\pi r}{c} \qquad \dots\dots\dots [5]$$

When this heating has continued long enough to make the product pt, which is $A(\pi^2/c^2)t$, large, the third term of this expression practically vanishes. The temperature gradient has then reached its maximum value and becomes independent of time, as a differentiation of Equation [5] with respect to time will show. In calculating the stresses produced by such heating, we are more interested in the maximum values produced when the steady state has been reached.

They are given by substituting the first two terms of Equation [5] in [1] and [2], from which we obtain:

$$\sigma_r = \frac{Eam}{15A(1 - \mu)}(c^2 - r^2)$$

$$\sigma_t = \frac{Eam}{15A(1 - \mu)}(c^2 - 2r^2)$$

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[3] See Adams and Williamson, *Physical Review*, vol. 14, pp. 99–114.

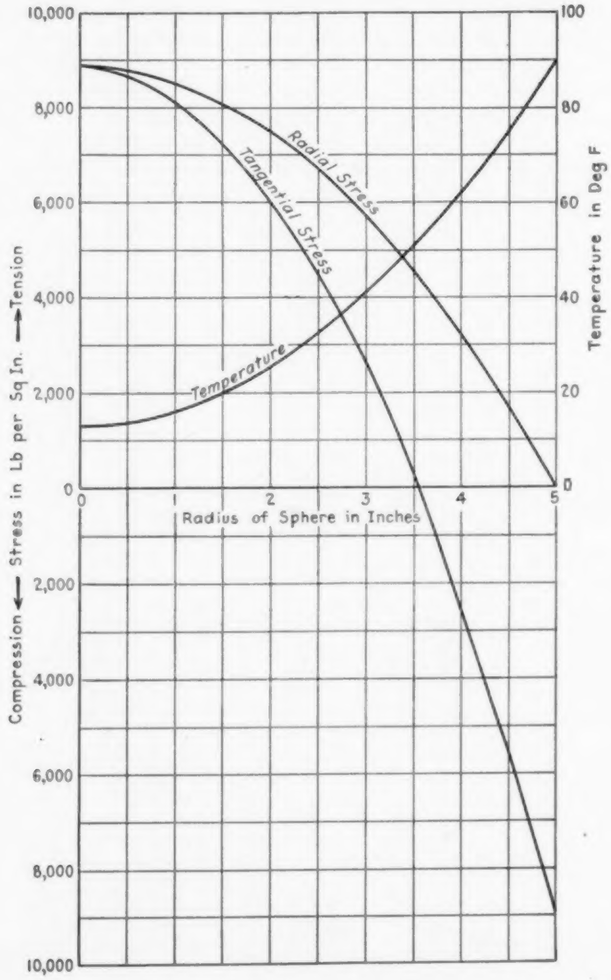

FIG. 2 TEMPERATURE AND STRESS DISTRIBUTION IN SPHERE OF FIG. 1, THREE MINUTES AFTER BEGINNING OF LINEAR SURFACE HEATING AT RATE OF 30 F PER MINUTE

Fig. 2 shows the stresses existing in the sphere of Example 1 after a steady state has been reached, if the temperature of the surface is increased at the rate of 30 deg per min. It is a simple matter, using the equations above, to calculate the rate of heating which will keep the stresses produced below any desired value. The stress distribution is quite similar to that in Fig. 1, except that the curves are parabolic in form. The temperature distribution is shown for a time 3 min after the heating begins at which time the steady state has been reached.

HOLLOW SPHERE

The stress equations for the hollow sphere are:[9]

$$\sigma_r = \frac{2Ea}{1-\mu}\left[\frac{r^3-b^3}{r^3(c^3-b^3)}\int_b^c Tr^2dr - \frac{1}{r^3}\int_b^r Tr^2dr\right]\ldots[6]$$

$$\sigma_t = \frac{Ea}{1-\mu}\left[\frac{2r^3+b^3}{r^3(c^3-b^3)}\int_b^c Tr^2dr + \frac{1}{r^3}\int_b^r Tr^2dr - T\right]\ldots[7]$$

Sudden Cooling. The problem of the temperature of any point in a hollow sphere when it has been heated throughout to a temperature of T_1 degrees and the external surface is then kept at zero, involves the solution of the same differential equation as for the solid sphere but with different boundary conditions. The result is:[10]

$$Tr = \sum_1^n \beta_n e^{-A\alpha_n^2t}\left[\sin \alpha_n(r-b) + \alpha_n b \cos \alpha_n(r-b)\right]$$

EXAMPLE 2. Assume that the sphere of Example 1 is hollow with an internal radius of 2.5 in. It is originally heated to a uniform temperature of 100 F, and then its external surface is kept at zero. Fig. 3 shows the temperature distribution 20 sec after cooling has begun.

As before, we evaluate the following integrals for use in the calculation of stresses:

$$\int_b^c Tr^2dr = \sum_1^n - e^{-A\alpha_n^2t}\frac{\beta_n c}{\alpha_n}\left[1 + (\alpha_n^2b^2)\cos \alpha_n(c-b)\right] = 1847$$

$$\int_b^r Tr^2dr = \sum_1^n e^{-A\alpha_n^2t}\frac{\beta_n}{\alpha_n^2}\Big[[1+\alpha_n^2b(r-b)]\sin \alpha_n (r-b) + [\alpha_n b - \alpha_n(r-b)]\cos \alpha_n(r-b) - \alpha_n b\Big]$$

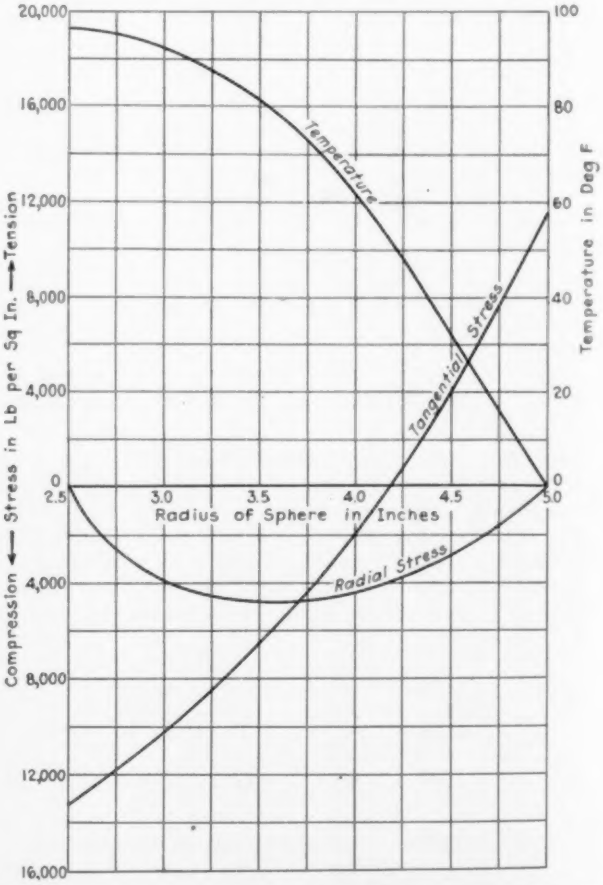

FIG. 3 TEMPERATURE AND STRESS DISTRIBUTION IN HOLLOW STEEL SPHERE OF 5 IN. OUTSIDE AND 2.5 IN. INSIDE RADIUS, 20 SECONDS AFTER EXTERIOR IS COOLED 100 F BELOW ORIGINAL UNIFORM TEMPERATURE

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[9] See Appendix No. 2.

[10] See Appendix No. 3.

As indicated in Appendix No. 3, the results are sufficiently accurate in this case when three terms of the summation are included and β has been calculated on the basis of $n = 3$. The quick convergence of the series is shown by the numerical value of these integrals when $t = 20$:

$$\int_b^c Tr^2dr = 1833 + 13.95 + 0.0996 = 1847$$

For $r = 3.5$,

$$\int_b^r Tr^2dr = 144 - 5.5 - 0.06 = 138$$

Substitution of the value of these integrals in Equations [6] and [7] makes possible the plotting of the stress curves in Fig. 3.

Linear Heating. If the shell is heated by increasing the temperature of the outer surface at a uniform rate, after heating has continued long enough to reach the steady state, the temperature is:

$$T = mt + \frac{m(r^2 - c^2)}{6A} + \frac{mb}{3Arc}(c - r)$$

Substitution of this expression in Equations [6] and [7] gives:

$$\sigma_r = \frac{Eam}{15A(1-\mu)}\left[-r^2 - 5\frac{b^3}{r} + \phi - \psi\right]$$

$$\sigma_t = \frac{Eam}{15A(1-\mu)}\left[-2r^2 - \frac{5b^3}{2r} + \phi + \frac{\psi}{2}\right]$$

where

$$\phi = \frac{c^5 + 5c^2b^3 - 6b^5}{c^3 - b^3}; \qquad \psi = \frac{c^5b^3 - 6c^3b^5 + 5c^2b^6}{r^3(c^3 - b^3)}$$

Fig. 4 shows the stress distribution in the hollow sphere of Example 2 after a steady state has been reached, when it is heated by increasing the temperature of its external surface at the rate of 30 deg per min.

It is interesting to note that when b is small as compared to c and we let $r = b$, the above equation for tangential stress becomes:

$$\sigma_t = \frac{2Eam}{15A(1-\mu)}c^2$$

This is seen to be twice that produced at the center of a solid sphere by the same heating rate. Thus a small cavity in the center of a sphere will have the effect of doubling the maximum tangential stress under linear heating.

LONG SOLID CYLINDER

The stress equations for the solid cylinder when the temperature varies in any manner with the radial coordinate are:[11]

$$\sigma_r = \frac{Ea}{1-\mu}\left[\frac{1}{c^2}\int_0^c Trdr - \frac{1}{r^2}\int_0^r Trdr\right] \ldots\ldots\ldots [8]$$

$$\sigma_t = \frac{Ea}{1-\mu}\left[\frac{1}{c^2}\int_0^c Trdr + \frac{1}{r^2}\int_0^r Trdr - T\right] \ldots [9]$$

$$\sigma_z = \frac{Ea}{1-\mu}\left[\frac{2}{c^2}\int^c Trdr - T\right] \ldots\ldots\ldots\ldots\ldots [10]$$

These equations do not apply to those portions of the cylinder near the ends.

Sudden Cooling. When the cylinder is heated to the uniform temperature T_1 and the convex surface is then held at zero, the temperature of any point r distant from the axis at a time t seconds after the cooling begins is given by the solution of the equation:[12]

$$\frac{\partial T}{\partial t} = A\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right)$$

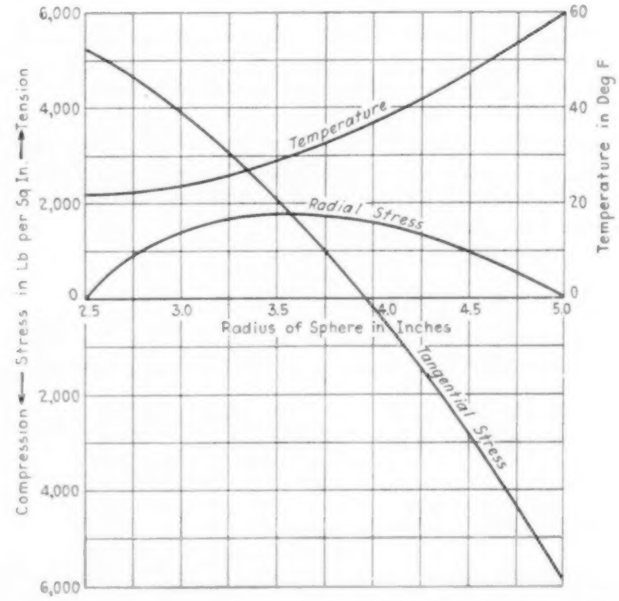

FIG. 4 TEMPERATURE AND STRESS DISTRIBUTION IN HOLLOW SPHERE OF FIG. 3, TWO MINUTES AFTER BEGINNING OF LINEAR SURFACE HEATING AT RATE OF 30 F PER MINUTE

If we let $T = ve^{-A\alpha^2t}$, where v is a function of r only, we have Bessel's equation of order zero:

$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} + \alpha^2v = 0$$

For the boundary conditions of this problem the solution is:

$$T = \frac{2}{c^2}\sum_1^\infty e^{-A\alpha_n^2t}\frac{J_0(\alpha_nr)}{[J_0'(\alpha_nc)]^2}\int_0^c T_1rJ_0(\alpha_nr)dr \ldots [11]$$

where $J_0(x)$ is Bessel's function of order zero of the first kind,

$$J_0'(\alpha c) = \left[\frac{d}{dr}J_0(r)\right]_{r = \alpha c}$$

and α is a real and positive root of the equation $J_0(\alpha c) = 0$, of which there are an infinite number.

The value of $J_0(x)$ is given by:

$$1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \ldots$$

It is a transcendental function which has been found to be a solution of Bessel's equation. It is finite and continuous for all values of x, and it has an infinite number of real roots. Its wave length is approximately 2π, and its amplitude constantly decreases. Tables are available for the determination of its numerical magnitude, and it can be manipulated similarly to other mathematical quantities by the use of known relations. As shown below, the maximum value of $J_0(x)$ occurs where $J_1(x)$ is zero, as is the case with sin x and cos x.

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[11] See Appendix No. 4.

[12] Carslaw, "Conduction of Heat," p. 115.

By use of the formulas:

$$\int_0^c rJ_0(\alpha_n r)dr = \frac{c}{\alpha_n} J_1(\alpha_n c)$$

and

$$J_0'(\alpha_n c) = -J_1(\alpha_n c)$$

we may write Equation [11]:

$$T = \frac{2T_1}{c}\left[\frac{e^{-A\alpha_1^2 t}}{\alpha_1}\frac{J_0(\alpha_1 r)}{J_1(\alpha_1 c)} + \frac{e^{-A\alpha_2^2 t}}{\alpha_2}\frac{J_0(\alpha_2 r)}{J_1(\alpha_2 c)} + \dots\right]$$

Example 3. Assume a solid cylinder of low-carbon steel 20 in. in diameter and of considerable length. It is initially heated to a uniform temperature of 100 F and the convex surface is then kept at zero.

For this value of c the roots of $J_0(\alpha c) = 0$ are:[13]

$$\alpha_1 = 0.2405; \quad \alpha_2 = 0.5520; \quad \alpha_3 = 0.8654$$

At the center of the cylinder 10 min. after cooling has begun the temperature is:

$$T = \frac{2T_1}{c}\left[\frac{e^{-0.938}}{0.2405}\frac{J_0(0)}{J_1(2.405)} + \frac{e^{-4.96}}{0.5520}\frac{J_0(0)}{J_1(5.520)} + \dots\right] = 61.7 \text{ F}$$

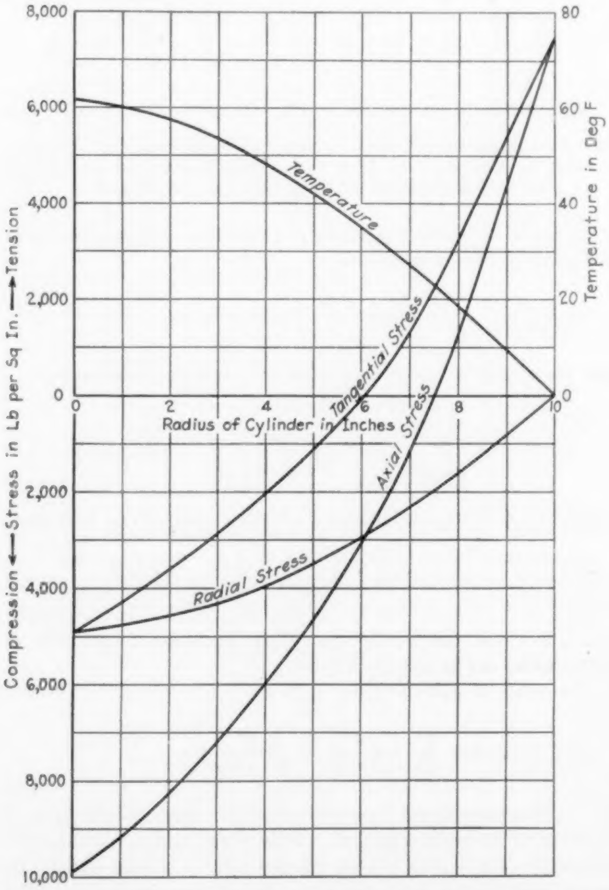

Fig. 5 Temperature and Stress Distribution in Solid Steel Cylinder of 10 In. Radius, 10 Minutes After Surface Is Cooled 100 F Below Original Uniform Temperature

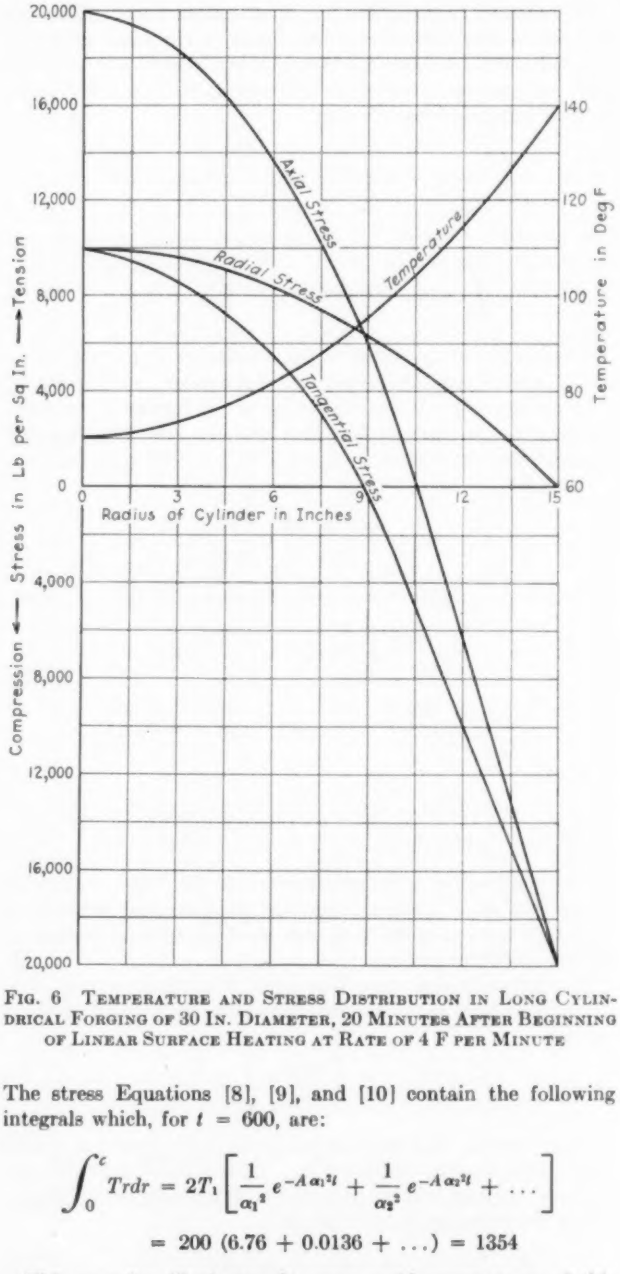

Fig. 6 Temperature and Stress Distribution in Long Cylindrical Forging of 30 In. Diameter, 20 Minutes After Beginning of Linear Surface Heating at Rate of 4 F per Minute

The stress Equations [8], [9], and [10] contain the following integrals which, for $t = 600$, are:

$$\int_0^c Trdr = 2T_1\left[\frac{1}{\alpha_1^2}e^{-A\alpha_1^2 t} + \frac{1}{\alpha_2^2}e^{-A\alpha_2^2 t} + \dots\right]$$

$$= 200\ (6.76 + 0.0136 + \dots) = 1354$$

This equation illustrates the very rapid convergence of this series.

$$\int_0^r Trdr = 2T_1\frac{r}{c}\left[\frac{e^{-A\alpha_1^2 t}}{\alpha_1^2}\frac{J_1(\alpha_1 r)}{J_1(\alpha_1 c)} + \frac{e^{-A\alpha_2^2 t}}{\alpha_2^2}\frac{J_1(\alpha_2 r)}{J_1(\alpha_2 c)} + \dots\right]$$

When these integrals are substituted in the stress equations, the stress curves of Fig. 5 are obtained showing the distribution 10 min after cooling has begun.

The maximum tension occurs at the surface where σ_x and σ_t are equal and at the time, $t = 0$. If we place $r = c$ and $t = 0$ in Equations [9] and [10], we find:

$$\text{max. } \sigma_x = \sigma_t = \frac{Ea T_1}{1-\mu}\sum_1^\infty \frac{4}{c^2}\frac{1}{\alpha_n^2}$$

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[13] Watson, "Theory of Bessel's Functions."

Since we know that the value of the maximum stress at this point is $\frac{EaT_1}{1-\mu} = 28{,}800$, we may conclude that:

$$\sum_{1}^{\infty} \frac{1}{\alpha_n^2} = 25$$

Fig. 9 shows how this maximum value decreases with time so that the tension at the surface has decreased, after four minutes, to about one-half its value at the start.

Linear Heating. When the convex surface of the cylinder is increased in temperature at a constant rate the temperature is again given by a solution of Bessel's equation with the proper boundary conditions. It is:

$$T = mt - \frac{m}{8A}(c^2 - r^2) + \frac{2mc^2}{A} \sum_{1}^{\infty} e^{-\frac{A\alpha_n^2 t}{c^2}} \frac{J_0\left(\frac{\alpha_n r}{c}\right)}{\alpha_n^3 J_1(\alpha_n)}$$

When the heating has continued long enough for a steady state of temperatures to be reached, the last term of the above expression may be dropped. The stress equations for the steady state then become:

$$\sigma_r = \frac{Eam}{1-\mu} \frac{c^2 - r^2}{16A}$$

$$\sigma_t = \frac{Eam}{1-\mu} \frac{c^2 - 3r^2}{16A}$$

$$\sigma_z = \frac{Eam}{1-\mu} \frac{c^2 - 2r^2}{8A}$$

Fig. 6 shows the stress distribution and the temperature curve 20 min after heating begins, at which time the steady state has been reached. The cylinder is a long forging of 30 in. diameter whose surface is being increased in temperature at the rate of 4 deg per min.

Hollow Cylinder

When a long hollow cylinder is heated in such a way that the temperature is a function of the distance from the axis only, the stresses produced in that portion not near the ends, are given by:[14]

$$\sigma_r = \frac{Ea}{1-\mu}\left[\frac{r^2 - b^2}{r^2(c^2 - b^2)} \int_b^c Trdr - \frac{1}{r^2}\int_b^r Trdr\right] \ldots [12]$$

$$\sigma_t = \frac{Ea}{1-\mu}\left[\frac{r^2 + b^2}{r^2(c^2 - b^2)} \int_b^c Trdr + \frac{1}{r_2}\int_b^r Trdr - T\right] \ldots [13]$$

$$\sigma_z = \frac{Ea}{1-\mu}\left[\frac{2}{c^2 - b^2} \int_b^c Trdr - T\right] \ldots\ldots\ldots [14]$$

In the hollow cylinder as well as the solid, the axial stress is the sum of the radial and tangential. The value of the axial deformation, which is a constant, is $e_z = \frac{2}{c^2 - b^2}\int_b^c Trdr$.

Sudden Cooling. Suppose the cylinder to be heated to the uniform temperature T_1, and the inner and outer surfaces then kept at zero, as by immersion in a liquid. The temperature of any point is again given by a solution of Bessel's equation, but since the range of r does not now extend to the origin, Bessel's functions of the second kind are not excluded. The solution is:

$$T = \frac{\pi^2}{2} \sum_{1}^{\infty} \alpha_n^2 \left[\frac{J_0^2(\alpha_n b)}{J_0^2(\alpha_n b) - J_0^2(\alpha_n c)}\right] e^{-A\alpha_n^2 +} U_0(\alpha_n r) \int_b^c T_1 r U_0(\alpha_n r) dr \ldots\ldots [15]$$

where $U_0(\alpha r) = J_0(\alpha r)\, Y_0(\alpha c) - J_0(\alpha c) Y_0(\alpha r)$

α_n represents the real and positive roots of $U_0(\alpha b) = 0$, and

$Y_0(x)$ represents Bessel's function of the second kind,[15] and of order zero. It obeys the same laws in its mathematical treatment as $J_0(x)$, and its numerical values for various values of x are given in the tables of Watson.

The integral in Equation [15] has the following value:

$$\int rU_0(\alpha_n r)dr = \frac{1}{\alpha_n^2}\left(\frac{2}{\pi} + \alpha_n b F_n\right)$$

where $F_n = J_0(\alpha_n c)\, Y_1(\alpha_n b) - Y_0(\alpha_n c) J_1(\alpha_n b)$

Equation [15] may then be rewritten:

$$T = T_1 \frac{\pi^2}{2} \sum_{1}^{\infty} \left[\frac{J_0^2(\alpha_n b)}{J_0^2(\alpha_n b) - J_0^2(\alpha_n c)}\right] e^{-A\alpha_n^2 t} \times \left(\frac{2}{\pi} + \alpha_n b F_n\right) U_0(\alpha_n r)$$

The two variables in this expression are t and r. t appears only in the exponent of e, and r only in two terms of $U_0(\alpha_n r)$. All the other terms are constant and depend only on the form of the cylinder. Thus for all points on the temperature curve after the first it is only necessary to compute values of $U_2(\alpha_n r)$. The summation extends over the positive roots of $U_0(\alpha b) = 0$. This makes $U_0(\alpha b) = U_0(\alpha c) = 0$, which is necessary in order to reduce the expression for T to zero at $r = b$ and $r = c$.

Example 4. Assume a long steel cylinder of 10 in. outside radius and 5 in. inside. Let it be heated to 100 F throughout, and its surfaces, excluding the ends, then kept at zero.

The roots of the equation $U_0(\alpha_n b) = 0$, can be calculated as:[16]

$$\alpha_1 = 0.6264;\ \alpha_2 = 1.2547;\ \alpha_3 = 1.8837.$$

At the point $r = 8$ after the cooling has continued for 30 sec, the temperature is:

$$T = 494\left[\frac{0.0907}{0.0907 - 0.0445}\frac{1}{1.890}\left(\frac{2}{\pi} + 0.6264 \cdot 5 \cdot 0.1441\right)0.0763 + \ldots\right] = 494[0.1624 - 0.0043 + \ldots] = 78.1 \text{ F}$$

The following integrals will be needed in calculating stresses:

$$\int_b^c Trdr = T_1 \frac{\pi^2}{2} \sum_{1}^{\infty} e^{-A\alpha_n^2 t}\, V_n \left(\frac{2}{\pi} + \alpha_n b F_n\right) \times \int_b^c rU_0(\alpha_n r)dr$$

where $V_n = \frac{J_0^2(\alpha_n b)}{J_0^2(\alpha_n b) - J_0^2(\alpha_n c)}$

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[14] Timoshenko, "Strength of Materials," p. 552.

[15] Watson, p. 63.

[16] Gray and Mathews, "Bessel's Functions," p. 242.

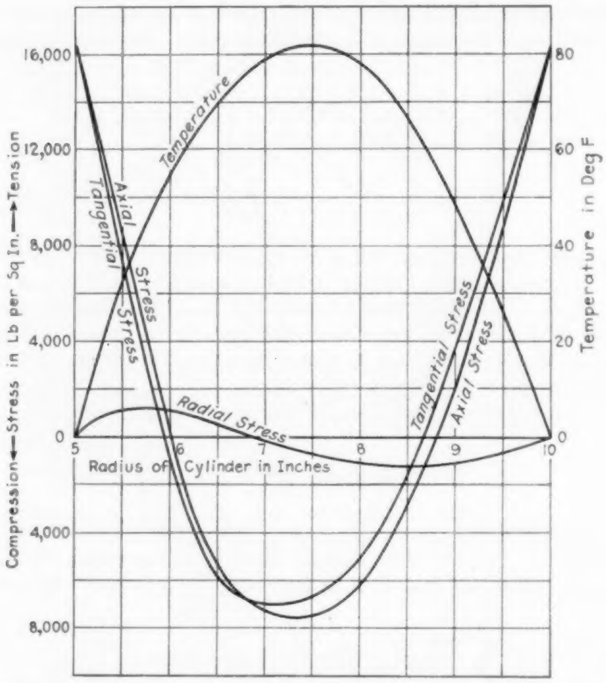

FIG. 7 TEMPERATURE AND STRESS DISTRIBUTION IN LONG HOLLOW CYLINDER OF 10 IN. OUTSIDE AND 5 IN. INSIDE RADIUS, 30 SECONDS AFTER BOTH SURFACES ARE SUDDENLY BROUGHT TO ZERO FROM AN INITIAL UNIFORM TEMPERATURE OF 100 F

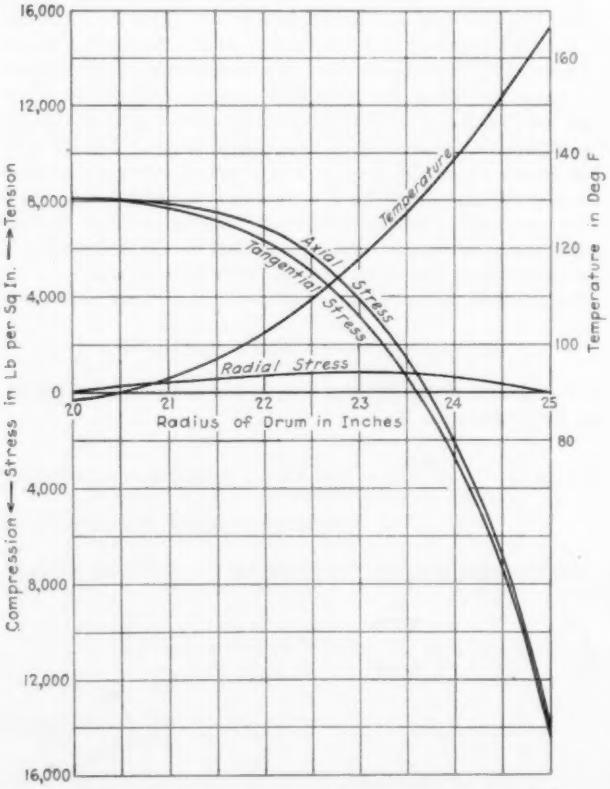

FIG. 8 TEMPERATURE AND STRESS DISTRIBUTION IN LONG STEEL DRUM OF 50 IN. OUTSIDE AND 40 IN. INSIDE DIAMETER, WHEN EXTERIOR SURFACE IS HEATED LINEARLY FROM 60 F TO 700 F IN ONE HOUR: CURVES SHOW CONDITION 10 MINUTES AFTER HEATING BEGINS

Integrating,

$$\int_b^c Trdr = T_1 \frac{\pi^2}{2} \sum_1^\infty \frac{1}{\alpha_n^2} e^{-A\alpha_n^2 t} V_n \left(\frac{2}{\pi} + \alpha_n b F_n\right)^2$$

$$= 494(4.30 + 0.0108 + \ldots) = 2130$$

$$\int_b^r Trdr = T_1 \frac{\pi^2}{2} \sum_1^\infty e^{-A\alpha_n^2 t} V_n \left(\frac{2}{\pi} + \alpha_n b F_n\right)$$

$$\left(\frac{Y_0(\alpha_n c)}{\alpha_n}\left[rJ_1(\alpha_n r) - bJ_1(\alpha_n b)\right]\right.$$

$$\left. - \frac{J_0(\alpha_n c)}{\alpha_n}\left[rY_1(\alpha_n r) - bY_1(\alpha_n b)\right]\right)$$

For $r = 8$, $\int_b^r Trdr = 1338$.

Fig. 7 shows the temperature and stress distribution one-half minute after the cooling has begun. Since the cooling takes place on both surfaces, the radial stress changes sign through the cross-section. Even with a thick cylinder such as has been taken in this example the radial stress is still quite small as compared with the other two.

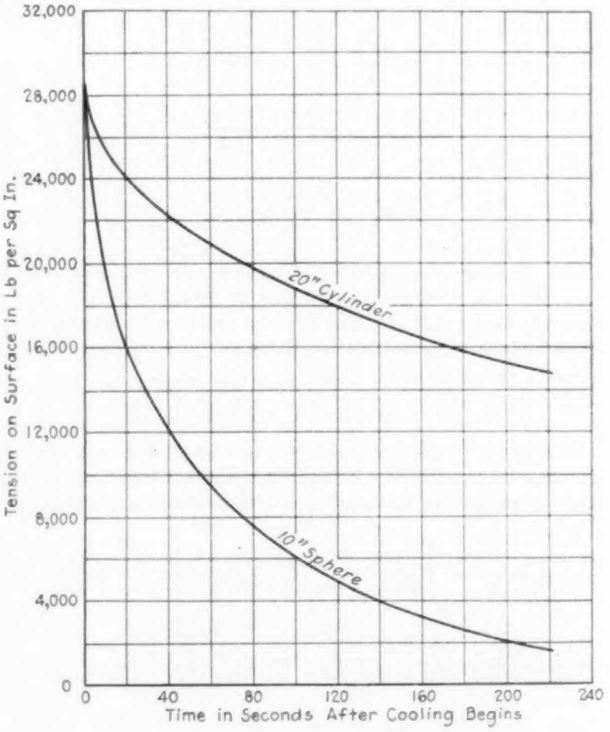

FIG. 9 TENSION ON SURFACE OF SPHERE OF FIG. 1 AND CYLINDER OF FIG. 5 AS TIME INCREASES AFTER SURFACES ARE SUDDENLY LOWERED 100 F IN TEMPERATURE

Linear Heating. For a long cylindrical tube heated linearly on the outside only, the temperature after a steady state has been reached is given by:

$$T = mt - \frac{m}{4A}(c^2 - r^2) + \frac{mb^2}{2A} \log \frac{c}{r}$$

For the same tube heated linearly on both surfaces, the equation becomes:

$$T = mt - \frac{mr^2}{4A} + \frac{m[b^2 \log (r/c) - c^2 \log (r/b)]}{4A \log(c/b)}$$

Substituting the first of these in Equations [12], [13], and [14],

$$\sigma_r = -\frac{Eam}{16A(1-\mu)}\left[r^2 - c^2 + \frac{b^2}{r^2}(c^2 - r^2) - 4\frac{b^2c^2}{r^2}\frac{b^2 - r^2}{c^2 - b^2}\log(c/b) - 4b^2 \log(r/b)\right]$$

$$\sigma_t = \frac{Eam}{16A(1-\mu)}\left[5b^2 - 3r^2 + \frac{c^2}{r^2}(r^2 + b^2) - 4\frac{b^2c^2}{r^2}\frac{b^2 + r^2}{c^2 - b^2}\,\mathrm{lgo}\,(c/b) + 4b^2 \log(r/b)\right]$$

$$\sigma_z = \frac{Eam}{8A(1-\mu)}\left[c^2 + 3b^2 - 2r^2 - 4\frac{b^2c^2}{c^2 - b^2}\log(c/b) + 4b^2 \log(r/b)\right]$$

when $r = c$,

$$\sigma_r = 0;\ \sigma_t = \sigma_z = \frac{Eam}{8A(1-\mu)}\left[3b^2 - c^2 - \frac{4b^4}{c^2 - b^2}\log(c/b)\right]$$

when $r = b$,

$$\sigma_r = 0;\ \sigma_t = \sigma_z = \frac{Eam}{8A(1-\mu)}\left[b^2 + c^2 - \frac{4b^2c^2}{c^2 - b^2}\log(c/b)\right]$$

For a very thick wall when b is small compared to c, these equations become:

$$\sigma_r = \frac{Eam}{16A(1-\mu)}\left(c^2 - r^2 - \frac{c^2b^2}{r^2}\right)$$

$$\sigma_t = \frac{Eam}{16A(1-\mu)}\left(c^2 - 3r^2 + \frac{c^2b^2}{r^2}\right)$$

$$\sigma_z = \frac{Eam}{8A(1-\mu)}(c^2 - 2r^2)$$

As in the case of the sphere, when there is a small cavity in the center of the cylinder the tangential stress at that cavity is twice the corresponding stress at the center of a solid cylinder heated linearly.

Fig. 8 shows the stress and temperature distribution in the drum of a turbine rotor when its exterior surface is heated linearly from 60 F to 700 F in one hour. The temperature curve corresponds to a time 10 min after the heating begins, at which time the steady state is established. The drum has an external diameter of 50 in. and internal of 40 in.

Appendix No. 1

The problem of the state of stress in a sphere of elastic material subjected to a temperature gradient in the direction of the radius was one of those solved by Duhamel[17] in 1838 in the first paper on thermal stresses. We may put this in a more modern form.

Let u be the radial displacement of any point in the sphere. On account of the symmetry of the temperature change there will be no other displacements. Then:

$$e_r = \frac{\partial u}{\partial r};\quad e_t = \frac{u}{r} \quad\ldots\ldots\ldots\ldots\ldots [16]$$

Hooke's law shows that:

$$e_r = \frac{1}{E}(\sigma_r - 2\mu\sigma_t) + aT$$

$$e_t = \frac{\sigma_t}{E}(1-\mu) - \frac{\mu}{E}\sigma_r + aT$$

Solving these for σ_r and σ_t:

$$\sigma_r = \frac{E}{1-2\mu}\left[\frac{1+\mu}{1-\mu}e_r + \frac{2\mu}{1+\mu}e_t - aT\right] \ldots\ldots [17]$$

$$\sigma_t = \frac{E}{1-2\mu}\left[\frac{\mu}{1+\mu}e_r + \frac{1}{1+\mu}e_t - aT\right] \ldots\ldots\ldots [18]$$

There will be but one equilibrium equation obtained by summing forces in the radial direction in Fig. 10. We may use the total differential since σ_r varies only with r, hence

$$\sigma_r - \sigma_t + \frac{r}{2}\frac{d\sigma_r}{dr} = 0 \ldots\ldots [19]$$

Differentiating [17] with respect to r,

$$\frac{d\sigma_r}{dr} = \frac{E}{1-2\mu}\left[\frac{1-\mu}{1+\mu}\frac{de_r}{dr} + \frac{2\mu}{1+\mu}\frac{de_t}{dr} - \frac{d}{dr}(aT)\right] \ldots [20]$$

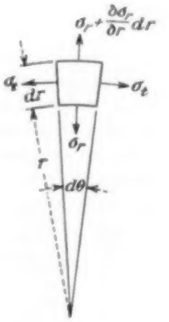

Fig. 10

Substituting [17], [18], and [20] in [19] after replacing e_r and e_t and their derivatives in terms of u from [16], gives

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} - 2\frac{u}{r} = \frac{1+\mu}{1-\mu}\frac{d}{dr}(aT)$$

or we may write:

$$\frac{d}{dr}\left(\frac{du}{dr}\right) + 2\frac{d}{dr}\left(\frac{u}{r}\right) = \frac{1+\mu}{1-\mu}\frac{d}{dr}(aT)$$

Integrating,

$$\frac{du}{dr} + 2\frac{u}{r} = \frac{1+\mu}{1-\mu}aT + 3C_1$$

Multiply by r^2 and write

$$\frac{d}{dr}(ur^2) = \frac{1+\mu}{1-\mu}aTr^2 + 3C_1r^2$$

Integrating

$$u = \frac{1+\mu}{1-\mu}\frac{1}{r^2}\int_0^r aTr^2dr + C_1r + \frac{C_2}{r^2} \ldots\ldots\ldots [21]$$

When $r = 0$, $u = 0$, so $C_2 = 0$.

This value of u is then substituted in the Equations [16], which are then used to replace e_r and e_t in [17] and [18]:

$$\sigma_r = \frac{E}{1-2\mu}\left[C_1 - \frac{1-2\mu}{1-\mu}\frac{2}{r^3}\int_0^r aTr^2dr\right]$$

No external forces act on the surface and the radial stress must be zero here. Thus we write:

$$r = c,\ \sigma_r = 0,\ \text{and } C_1 = \frac{1-2\mu}{1-\mu}\frac{2}{c^3}\int_0^c aTr^2dr$$

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[17] Mémoires divers savants, French Academy, vol. 5, 1838.

Then

$$\sigma_r = \frac{2Ea}{1-\mu}\left[\frac{1}{c^3}\int_0^c Tr^2dr - \frac{1}{r^3}\int_0^r Tr^2dr\right] \ldots\ldots [1]$$

$$\sigma_t = \frac{Ea}{1-\mu}\left[\frac{2}{c^3}\int_0^c Tr^2dr + \frac{1}{r^3}\int_0^r Tr^2dr - T\right] \ldots [2]$$

These equations are indeterminate when $r = 0$. If we multiply them by r^3 and then differentiate with respect to r, we arrive at expressions for σ_r and σ_t when $r = 0$, which are determinate:
For $r = 0$,

$$\sigma_r = \sigma_t = \frac{2Ea}{1-\mu}\left[\frac{1}{c^3}\int_0^c Tr^2dr - \frac{1}{3}T\right] \ldots\ldots\ldots [3]$$

Appendix No. 2

We may easily adapt the equations developed in Appendix No. 1 to the hollow sphere. The development outlined there as far as Equation [21] applies as well to the hollow as to the solid sphere, but the condition which reduces the constant to zero will not now obtain, and the lower limit for the integration will be b instead of zero. The two constants of integration may be evaluated by the fact that $\sigma_r = 0$, both when $r = b$ and when $r = c$. We have:

$$\sigma_r = \frac{E}{1-2\mu}\left[C_1 - \frac{1-2\mu}{1+\mu}\frac{2C_2}{r^3} - \frac{1-2\mu}{1-\mu}\frac{2}{r^3}\int_b^r aTr^2dr\right]$$

The constants are:

$$C_1 = \frac{1-2\mu}{1-\mu}\,\frac{2}{c^3-b^3}\int_b^c aTr^2dr$$

$$C_2 = \frac{1+\mu}{1-\mu}\,\frac{b^3}{c^3-b^3}\int_b^c aTr^2dr$$

so that finally

$$\sigma_r = \frac{2Ea}{1-\mu}\left[\frac{r^3-b^3}{r^3(c^3-b^3)}\int_b^c Tr^2dr - \frac{1}{r^3}\int_b^r Tr^2dr\right] \ldots [6]$$

$$\sigma_t = \frac{Ea}{1-\mu}\left[\frac{2r^3+b^3}{r^3(c^3-b^3)}\int_b^c Tr^2dr + \frac{1}{r^3}\int_b^r Tr^2dr - T\right] [7]$$

Appendix No. 3

As an example of the method of developing the differential equations of heat flow used in this paper, we may select the hollow sphere which is subjected to cooling on the external surface. If the initial temperature is uniform or a function of the radial coordinate, the temperature of any point after cooling begins is a function of its distance from the center and of the time. Thus any imaginary surface which is r distant from the center is an isothermal surface and the lines of heat flow are normal to it.

Fig. 11 shows such a surface of radius r within the sphere. It is a fundamental hypothesis for the theory of heat conduction that the rate at which heat crosses from the inside to the outside of an isothermal surface per unit area per unit time, is given by: $-K(\partial T/\partial n)$, where T is the temperature of the surface, K the coefficient of thermal conductivity of the material, and $\partial/\partial n$ indicates differentiation along the outward-drawn normal to the surface. The negative sign indicates that there must be a decrease in the temperature as the normal increases in order to establish flow in this direction. In this case $\partial/\partial n$ may be replaced by $\partial/\partial r$ since r is the normal to the isothermal surface and the partial derivative $\partial T/\partial r$ is called the temperature gradient. Then the heat per unit time flowing past the surface indicated toward the outside will be:

$$-4\pi r^2K\frac{\partial T}{\partial r}$$

Let us now consider the spherical shell of inner radius r and outer $r + dr$ in Fig. 11. A heat balance on this shell may be stated as follows:

Heat in — Heat out = Heat stored

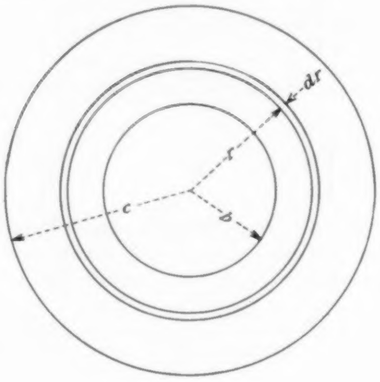

FIG. 11

The temperature of the outer surface of the shell will be:

$$T + \frac{\partial T}{\partial r}dr$$

The second term of this expression is the change in temperature along a distance dr when the rate of change is $\partial T/\partial r$. The heat flowing per unit time across this outer surface toward the outside is:

$$-4\pi(r+dr)^2K\frac{\partial}{\partial r}\left(T + \frac{\partial T}{\partial r}dr\right)$$

The heat stored is the product of four terms: the volume heated; its density; the specific heat of the material; and the change in temperature per unit time. This is given by:

$$4\pi r^2dr \cdot \rho \cdot s \cdot \frac{\partial T}{\partial t}$$

where ρ is the density and s the specific heat of the material.

Terms containing higher powers of dr than the first will vanish and have not been included in the above expression. When $\partial T/\partial t$ is negative, as in this example, the heat stored represents heat given up.

We may now write the heat-balance equation in symbols as:

$$-4\pi Kr^2\frac{\partial T}{\partial r} + 4\pi K(r+dr)^2\frac{\partial}{\partial r}\left(T + \frac{\partial T}{\partial r}dr\right) = 4\pi\rho sr^2\frac{\partial T}{\partial t}dr$$

By performing the indicated operations, discarding terms containing powers of dr higher than the first, and writing $K/\rho s$ as A, the coefficient of thermal diffusivity, we have:

$$A\left(r^2\frac{\partial^2 T}{\partial r^2} + 2\frac{\partial T}{\partial r}\right) = r\frac{\partial T}{\partial t}$$

or, changing the order and making a new variable, Tr, we can write:

$$\frac{\partial}{\partial t}(Tr) = A\,\frac{\partial^2}{\partial r^2}(Tr)\ldots\ldots\ldots\ldots\ldots\ldots[22]$$

This differential equation will now be solved for the special conditions applying to the sudden cooling of the convex surface. Let the sphere be originally at the uniform temperature T_1 and then its exterior is held at zero. The boundary conditions require, first, that there be no heat flow across the inner surface, that is, that the temperature gradient at that point is zero, which assumes a vacuum within the sphere. If there is a gas in this space, unless it is under high pressure, the amount of heat flowing across this surface would be very small. So we may assume that

$$\frac{\partial T}{\partial r}_{(r\,=\,b)} = 0\ldots\ldots\ldots\ldots\ldots\ldots[23]$$

The second condition requires that the outer surface is to be kept at zero, which gives

$$T = 0 \text{ when } r = c\ldots\ldots\ldots\ldots\ldots\ldots[24]$$

The initial condition requires that when $t = 0$, the temperature of the sphere is a constant:

$$T = T_1 \text{ when } t = 0\ldots\ldots\ldots\ldots\ldots\ldots[25]$$

Known solutions of [22] are:

$$Tr = \beta e^{-A\alpha^2 t}\sin\alpha r;\ \text{and}\ Tr = \beta e^{-A\alpha^2 t}\cos\alpha r$$

where α and β are constants to be determined. These solutions may be verified by substitution in Equation [22]. Since these are solutions their sum is also a solution, and we may write:

$$Tr = \beta e^{-A\alpha^2 t}(\sin\alpha r + \cos\alpha r)$$

We may modify this as follows. Let

$$Tr = \beta e^{-A\alpha^2 t}[\sin\alpha(r-b) + \alpha b\cos\alpha(r-b)]\ldots[26a]$$

The change in the angle or multiplication by the constant αb will not affect the solution of [22]. If we solve this for T and take the partial derivative with respect to r, we have:

$$\frac{\partial T}{\partial r} = \beta\left[\frac{\alpha e^{-A\alpha^2 t}}{r}\cos\alpha(r-b) - \frac{1}{r^2}e^{-A\alpha^2 t}\sin\alpha(r-b)\right.$$
$$\left. - \frac{\alpha^2 b}{rb}e^{-A\alpha^2 t}\sin\alpha(r-b) - \frac{\alpha b}{r^2}e^{-A\alpha^2 t}\cos\alpha(r-b)\right]$$

and when $r = b$, $\partial T/\partial r = 0$, so that Equation [23] is satisfied.

To satisfy [24], Equation [26a] must be zero when $r = c$, so that

$$\sin\alpha(c-b) + \alpha b\cos\alpha(c-b) = 0$$

or:

$$\tan\alpha(c-b) = -\alpha b$$

There will be a solution of this equation in both the second and fourth quadrants and the angle $\alpha(c-b)$ will increase approximately by multiples of π. To take care of these multiple solutions we may modify [26a] as follows: Let

$$Tr = \sum_{1}^{n}\beta_n e^{-A\alpha_n^2 t}[\sin\alpha_n(r-b) + \alpha_n b\cos\alpha_n(r-b)]\ldots[26]$$

where α_n is determined by

$$\tan\alpha_n(c-b) = -\alpha_n b\ldots\ldots\ldots\ldots\ldots\ldots[27]$$

and n has all positive integral values from 1 to n. α cannot be determined until we have numerical values for b and c.

The initial condition will determine the constants β_n. If we place $t = 0$, then $T = T_1$ and Equation [26] becomes:

$$T_1 r = \sum_{1}^{n}\beta_n\,[\sin\alpha_n(r-b) + \alpha_n b\cos\alpha_n(r-b)]\ldots[28]$$

Again when we have numerical values for b and c we may choose between them as many values of r as we like and obtain a corresponding number of simultaneous equations, each of which will yield a value of β.

In Example 2, $c = 5$ and $b = 2.5$. To determine α_1, Equation [27] becomes:

$$\tan 2.5\alpha_1 = -2.5\alpha_1$$

By trial we find that if $\alpha_1 = 0.811$, the equation becomes:

$$\tan 2.03 = -2.03$$

which is true. In the same way we obtain:

$$\alpha_2 = 1.965;\ \alpha_3 = 3.18$$

Three values will be found sufficient for ordinary calculations. We may then substitute these in Equation [28], using $r = 2.5$ and $T_1 = 100$. We have:

$$250 = \alpha_1 b\beta_1 + \alpha_2 b\beta_2 + \alpha_3 b\beta_3 + \ldots\ \alpha_n b\beta_n$$
$$= 2.03\beta_1 + 4.91\beta_2 + 7.95\beta_3 + \ldots$$

At the point $r = 3.5$, we have since $r - b = 1$,

$$350 = \beta_1(\sin\alpha_1 + \alpha_1 b\cos\alpha_1) + \beta_2(\sin\alpha_2 + \alpha_2 b\cos\alpha_2) + \ldots$$

In Example 2 the point $r = 4.5$ was also chosen and the corresponding values of β_n are found to be:

$$\beta_1 = 188;\ \beta_2 = -45.4;\ \beta_3 = 11.5$$

The curve represented by the right side of Equation [28] will coincide with rT_1 at the points $r = 2.5$, $r = 3.5$, and $r = 4.5$. A graph of the actual function shows that it approximates the required function, rT_1, for other values of r with sufficient accuracy.

Appendix No. 4

The stress equations applying to the thick cylindrical tube are given in several textbooks. For the solid cylinder we may begin with Equation (*f*) on page 552 of Timoshenko's "Strength of Materials," or

$$ur = \frac{1+\mu}{1-\mu}\int_0^r aTrdr + C_1 r^2 + C_2$$

In a solid cylinder when $r = 0$, $u = 0$, so $C_2 = 0$. From the consideration that $\sigma_r = 0$ when $r = c$, the value of C_1 is found as in the reference cited. Then

$$\sigma_r = \frac{Ea}{1-\mu}\left[\frac{1}{c^2}\int_0^c Trdr - \frac{1}{r^2}\int_0^r Trdr\right]\ldots\ldots[12]$$

$$\sigma_t = \frac{Ea}{1-\mu}\left[\frac{1}{c^2}\int_0^c Trdr + \frac{1}{r^2}\int_0^r Trdr - T\right]\ldots[13]$$

To find the axial stress σ_z we have the Equation (*d*), page 551, of Timoshenko's work, which we may write:

$$\sigma_z = \frac{E}{1+\mu}\left[e_z + \frac{\mu}{1-\mu}\,\frac{1-2\mu}{E}\,(\sigma_z+\sigma_r+\sigma_t) + \frac{3\mu}{1-2\mu}\,aT\right] - \frac{EaT}{1-2\mu}$$

By substituting the values of σ_r and σ_t,

$$\sigma_z = \frac{\mu}{1-\mu}\,\frac{2Ea}{c^2}\int^c Trdr + Ee_z - \frac{EaT}{1-\mu}$$

To evaluate e_z we note that: $\int_0^c \sigma_z dA = 0$, where $dA = 2\pi rdr$.

Substituting the above value of σ_z in this, we have:

$$e_z = \frac{2a}{c^2}\int_0^c Trdr \quad \text{and} \quad \sigma_z = \frac{Ea}{1-\mu}\left(\frac{2}{c^2}\int_0^c Trdr - T\right)$$

The axial stress is seen to be the sum of the radial and tangential stresses. Thus σ_z and σ_t are equal at the surface where the radial stress is zero.

Equations [12] and [13] are indeterminate when $r = 0$. As in the case of the sphere, if we multiply them by r^2 and differentiate with respect to r, we obtain: For $r = 0$:

$$\sigma_r = \sigma_t = \frac{Ea}{1-\mu}\left[\frac{1}{c^2}\int_0^c Trdr - \frac{1}{2}\,T\right] \quad \ldots\ldots [14]$$

APM-54-19

On the Lift Distribution for a Wing of Arbitrary Plan Form in a Circular Wind Tunnel

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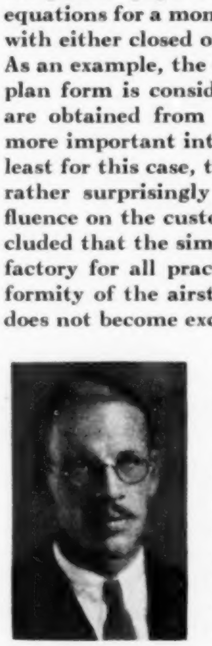
Previous theories dealing with the interference of wind-tunnel boundaries on airfoil characteristics have in general neglected the distortion of the lift distribution along the span caused by the presence of the boundaries. In the present paper this effect is considered and the general equations for a monoplane wing in a circular wind tunnel, with either closed or open working section, are developed. As an example, the case of an untwisted wing of elliptical plan form is considered in detail, and numerical results are obtained from which curves are plotted giving the more important interference effects. It is found that, at least for this case, the distortion of the lift distribution is rather surprisingly small and has an unimportant influence on the customary interference factors. It is concluded that the simple Prandtl type of correction is satisfactory for all practical cases in which the lack of uniformity of the airstream or the size of the airfoil chord does not become excessive.



1 Introduction

THE author's interest in the present problem was aroused by a recent British report[2] describing wind-tunnel tests on a number of airfoils whose span was relatively large in comparison to the tunnel width. In this report it was concluded that when the airfoil span becomes as large as seven-eighths of the wind-tunnel width, the usual theoretical correction factors, which do not take into account distortion of the lift distribution, are no longer valid. This conclusion indicated that a more general investigation of wind-tunnel interference, taking into account the distortion of the lift distribution, might be of interest. In the following a working section of circular cross-section is assumed, both because of simplicity of the analysis and because the wind tunnel at the California Institute of Technology, with which the author has been much concerned, has that form.

2 The Image System and Its Associated Induced Velocities

The customary assumptions of Prandtl's "first-order" wing theory are made, namely, that all induced velocities are small compared to the basic velocity of flow and that the chord length is relatively small so that the wing may be approximated by a "lifting line." The wind-tunnel working section is idealized by assuming the model to be placed in an infinitely long cylindrical tube or jet whose axis is in the direction of the undisturbed flow, and the wing span is assumed to be symmetrically placed with respect to the axis, and the sweep-back to be zero, although otherwise the plan form and distribution of profile of the wing are arbitrary. The type of analysis originally introduced by Trefftz[3] is followed, in which the two-dimensional flow in a "retreating plane" far downstream from the airfoil and perpendicular to the tunnel axis is considered. Then since the trailing vortices are assumed to follow the undisturbed streamlines, they form a plane vortex sheet extending to infinity downstream from the airfoil. If the retreating plane is taken far enough behind the airfoil, then considerations of symmetry indicate that the velocities normal to the tunnel axis have exactly half the values in the plane through the wing which they have at corresponding points in the retreating plane. The symmetry conditions also indicate that the induced velocities parallel to the direction of the basic flow vanish at the wing itself. Since for the present purpose we are primarily interested in the induced velocities at the wing, the problem is reduced to that of determining the velocities in the two-dimensional flow in the retreating plane at the vortex sheet which gives the "wind shadow" of the wing.

This is exactly the same potential-flow problem as is treated in the case of a wing in an unbounded fluid, except that the usual boundary conditions at infinity are here replaced by conditions which must be satisfied at the circular boundary of the wind-tunnel stream. For a closed tunnel with solid walls this boundary condition obviously assumes the form that the circular boundary must be a streamline. For an open tunnel in which the airfoil is in a free jet the condition is that the pressure must be constant over the circular boundary of the jet. Prandtl[4] has shown that this condition is identical with the requirement that the circular boundary be an equipotential curve. The effects of the two types of wind tunnel may therefore be discussed in quite analogical manners through the use of the principle of superposition. It is only necessary to superimpose on the flow corresponding to the airfoil in an unbounded flow an additional potential flow which makes the circular wind-tunnel boundary a streamline for the closed tunnel or an equipotential curve for the open tunnel. In the retreating plane there is a distribution of vorticity along

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[1] Assistant Professor of Aeronautics, Daniel Guggenheim Graduate School of Aeronautics, California Institute of Technology. Professor Millikan was born in Chicago in 1903, and received his Ph.B. from Yale University in 1924, and Ph.D. in Physics and Mathematics from California Institute of Technology in 1928. He was a Teaching Fellow at the California Institute of Technology from 1924 to 1928, and since then has held his present position.

[2] F. B. Bradfield and W. G. A. Perring, "Drag Tests on a Large Model in a Small Wind Tunnel," Aeronautical Research Committee, R. & M. 1371 (1931).

Presented at the Second National Meeting of the Applied Mechanics Division of The American Society of Mechanical Engineers, New Haven, Conn., June 23–25, 1932.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] E. Trefftz, "Prandtlsche Tragflächen und Propellertheorie," *Z. Ang. Math. Mech.*, I, p. 206 (1921); Vortrage aus d. Geb. d. Hydro. u. Aerodynamik (Innsbruck, 1922), Berlin, 1924, p. 41.

[4] L. Prandtl, "Applications of Modern Hydrodynamics to Aeronautics," N.A.C.A. Tech. Rep. 116 (1925), p. 51, 2.

the line determined by the wind shadow of the wing, and the unbounded flow is determined by this distribution of singularities and by the criterion that there be no other singularities in the entire plane. The superimposed flows which are necessary to satisfy our new boundary conditions require the addition of new singularities which must be outside the circular tunnel boundary. It is shown in the standard treatises on hydrodynamics[6] that if for every vortex in the wind shadow a vortex of opposite sign be placed at its image point with reference to the circular boundary, then the boundary will be a streamline of the resultant flow. Hence the superimposed flow for the closed-tunnel case is simply that corresponding to the distribution of image vortices in which each image vortex has a sign opposite to that of the original trailing vortex. Prandtl states that the superimposed flow for the open tunnel corresponds to exactly the same distribution of image vortices but with signs reversed. Since this arrangement of singularities has been less discussed than the previous one, it may be interesting to outline a simple proof of Prandtl's statement.

There is no circulation around the complete wind shadow, so that the distribution of vortices in it may be built up by the superposition of vortex pairs (two vortices with equal strength but opposite sign). Hence it is only necessary to prove that the circle is an equipotential curve for such a vortex pair with its images of the same signs. For a vortex of strength Γ the value of the potential at any point x,y is given by

$$\varphi(x,y) = \frac{\Gamma}{2\pi}\,\theta$$

where θ is the angle between the X-axis and the line joining the vortex to the point x,y. Hence if the circle has a radius R and the vortex pair is given by Γ at x_1, —Γ at x_2, while the images are Γ at ξ_1, —Γ at ξ_2, then for any point $P(x,y)$ on the circle (cf. Fig. 1) the complete potential is given by

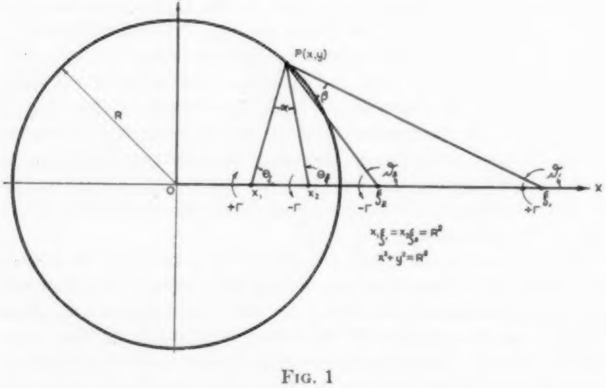

FIG. 1

$$\varphi\,(x,y) = \frac{\Gamma}{2\pi}\,(\theta_1 - \theta_2 + \vartheta_1 - \vartheta_2) = \frac{\Gamma}{2\pi}\,(\beta - \alpha)$$

where

$$x_1\xi_1 = x_2\xi_2 = R^2;\ x^2 + y^2 = R^2;\ \beta = \vartheta_1 - \vartheta_2;\ \alpha = \theta_2 - \theta_1$$

By simple applications of the cosine law to the triangles, x_1x_2P and $\xi_1\xi_2P$, it can immediately be shown that

$$\beta - \alpha = 0 \text{ for } x_1x_2 > 0;\ \beta - \alpha = \pi \text{ for } x_1x_2 < 0$$

Hence for such a distribution of vortices, φ = constant on the circle, and Prandtl's assertion is verified.

The calculations for the two types of wind tunnel may now be carried out simultaneously. In the following, the author will discuss explicitly the closed tunnel. The results can then be immediately applied to the open tunnel by changing the signs of all the image vortices.

Consider a wing span b with its center on the axis of the tunnel whose radius is $R = D/2$, and choose a coordinate system x,y in the plane of the wing or in the Trefftz retreating plane as indicated in Fig. 2. The lift is supposed to be in the direction of the positive Y-axis. In the wing shadow there is a distribution of vorticity whose strength per unit length along the span will be denoted by γ. Then at an element dx at a point x there will be a vortex of strength γdx. The image element will then be at $\xi = \dfrac{R^2}{x}$ and will have a length $d\xi = -\dfrac{R^2}{x^2}\,dx$. Place now at this element an image vortex of strength $\gamma' d\xi = \gamma dx$. Then, since dx and $d\xi$ have opposite signs, the original and image vortices will also have opposite signs, so that this gives the image system for a closed tunnel. For the complete system we have then a distribution of original vortices of strength γ per unit length along the X-axis from $-b/2$ to $+b/2$ and a similar distribution of image vortices of strength γ' per unit length from $x = -\infty$ to $x = -2R^2/b$ and from $x = +2R^2/b$ to $x = +\infty$, where

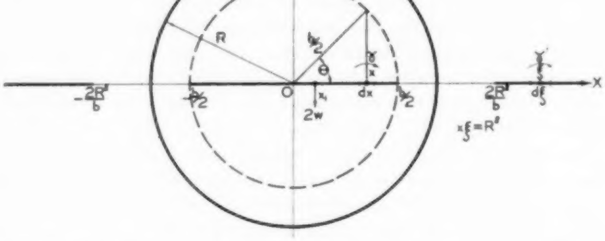

FIG. 2 THE TREFFTZ RETREATING PLANE FAR DOWNSTREAM FROM THE WING

$$\left.\begin{aligned} x\xi &= R^2 \\ \gamma dx &= \gamma' d\xi \end{aligned}\right\} \qquad [1]$$

We are interested in calculating the downwash $w(x_1)$ at an arbitrary point x_1 along the wing span. Then, as has already been mentioned, the downwash induced by the vortex singularities at the corresponding point x_1 in the retreating plane will be just $2w(x_1)$. It should be noted that the downwash is taken as positive in the direction of the negative Y-axis, i.e., opposite to the lift. It will be convenient to split this downwash into two parts, the first denoted by w_0 and giving that portion induced by the trailing vortices in the wing shadow, and the second denoted by w_1, giving the additional downwash induced by the complete system of image vortices. w_0 is exactly the downwash which would be present if the wing were in an unbounded flow, i.e., $R = \infty$, so that w_1 represents the interference velocities caused by the presence of the tunnel boundary. A new variable, θ, is now introduced, giving the position along the span where

$$x = \frac{b}{2}\cos\theta \qquad [2]$$

and the circulation around the wing, $\Gamma(x)$, expanded in a Fourier series of the form

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[6] See, for instance, H. Lamb, "Hydrodynamics," fifth edition, p. 205.

$$\Gamma = 2Vb \sum_{n=1}^{\infty} B_n \sin n\theta \ldots\ldots\ldots\ldots\ldots [3]$$

where V is the flow velocity far upstream from the wing. Then the usual wing theory[6] gives for the "undisturbed" downwash w_0

$$\frac{w_0}{V} = \sum_{n=1}^{\infty} nB_n \frac{\sin n\theta}{\sin \theta} \ldots\ldots\ldots\ldots\ldots [4]$$

The task now is to find the downwash w_1 induced by the system of image vortices. Calling γ, γ' positive for clockwise rotation as indicated in Fig. 2,

$$2w_1(x_1) = -\frac{1}{2\pi}\int_{\frac{2R^2}{b}}^{\infty} \frac{\gamma' d\xi}{\xi - x_1} - \frac{1}{2\pi}\int_{-\infty}^{\frac{-2R^2}{b}} \frac{\gamma' d\xi}{\xi - x_1}$$

or, from [1]

$$2w_1(x_1) = \frac{1}{2\pi}\int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{\gamma dx}{\frac{R_2}{x} - x_1}$$

γ is connected with the circulation around the wing by the relation

$$\gamma = \frac{d\Gamma}{dx} = \frac{-2}{b \sin\theta}\frac{d\Gamma}{d\theta} = -4V \sum_{n=1}^{\infty} nB_n \frac{\cos n\theta}{\sin\theta}$$

so that the expression for w_1 may be transformed into

$$\frac{w_1(x_1)}{V} = -\frac{1}{4\pi}\frac{b^2}{R^2}\sum_{n=1}^{\infty} nB_n \int_0^{\pi} \frac{\cos n\theta \cos\theta}{1 - \frac{bx_1}{2R^2}\cos\theta}\, d\theta$$

Writing, for the moment, $bx_1/2R^2 = \epsilon$,

$$\int_0^{\pi} \frac{\cos n\theta \cos\theta}{1 - \frac{bx_1}{2R^2}\cos\theta}\, d\theta = \frac{1}{2}\int_0^{\pi}\frac{\cos (n-1)\theta}{1 - \epsilon\cos\theta}\, d\theta + \frac{1}{2}\int_0^{\pi}\frac{\cos (n+1)\theta}{1 - \epsilon\cos\theta}\, d\theta$$

A very simple contour integration leads to the result (for $\epsilon < 1$)

$$\int_0^{\pi} \frac{\cos a\theta}{1 - \epsilon\cos\theta}\, d\theta = \frac{\pi}{\sqrt{1-\epsilon^2}}\left(\frac{1-\sqrt{1-\epsilon^2}}{\epsilon}\right)$$

and using this, a little calculation gives

$$\frac{w_1(x_1)}{V} = \frac{-b^2}{4R^2\epsilon\sqrt{1-\epsilon^2}}\sum_{n=1}^{\infty} nB_n \left(\frac{1-\sqrt{1-\epsilon^2}}{\epsilon}\right)^n$$

Now letting θ correspond to x_1 in accordance with [2],

$$\epsilon = \frac{bx_1}{2R^2} = \frac{b^2}{4R^2}\cos\theta = A^2\cos\theta$$

where

$$A = \frac{b}{2R} = \frac{b}{D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

Hence finally

$$\frac{w_1(\theta)}{V} = \frac{-1}{\cos\theta\sqrt{1-A^4\cos^2\theta}}\sum_{n=1}^{\infty} nB_n \times \left(\frac{1-\sqrt{1-A^4\cos^2\theta}}{A^2\cos\theta}\right)^n \ldots\ldots [6]$$

Equation [6] gives the additional downwash induced at all points along the wing span by the presence of the closed-tunnel boundary. For an open wind tunnel [6] is still valid if the sign be changed. In order to determine the circulation distribution, i.e., the B_n's, for a definite plan form and distribution of airfoil profiles this result must be combined with the two-dimensional theory of lift in quite the normal way [cf. footnote 6].

3 The Equations for a Monoplane Wing in a Circular Wind Tunnel

Let the lift per unit length of span be denoted by L', the effective angle of attack at any longitudinal section of the wing by α_e, the geometrical or apparent angle of attack by α, and the chord at any point along the span by t. Angles of attack are measured from the position of zero lift of the wing section under consideration. Then for every point along the span

$$\alpha = \alpha_e + \frac{w}{V} = \alpha_e + \frac{w_0}{V} + \frac{w_1}{V} \ldots\ldots\ldots\ldots [7]$$

The results of the two-dimensional theory of airfoil sections are now introduced by writing

$$L' = 2\pi\eta\alpha_e \frac{\rho}{2} V^2 t$$

where ρ is the fluid density and η is called the efficiency factor and is an empirical constant giving the fraction of the theoretical slope of the lift coefficient vs. α_e curve which is obtained experimentally. For all normal airfoil sections at reasonably high Reynolds numbers, η may be taken as approximately 0.90.

We have also the Kutta-Joukowsky law giving a second expression for L':

$$L' = \rho V \Gamma \ldots\ldots\ldots\ldots\ldots\ldots\ldots [8]$$

Equating these two expressions and using [7],

$$\alpha = \frac{\Gamma}{\pi\eta V t} + \frac{w_0}{V} + \frac{w_1}{V}$$

and introducing Equations [3], [4], and [6],

$$\alpha = \sum_{n=1}^{\infty} B_n \left[\frac{2b}{\pi\eta t}\sin n\theta + \frac{n}{\sin\theta}\sin n\theta - \frac{n}{\cos\theta\sqrt{1-A^4\cos^2\theta}}\left(\frac{1-\sqrt{1-A^4\cos^2\theta}}{A^2\cos\theta}\right)^n\right] \ldots . [9]$$

It is convenient to introduce the chord at the center of the wing, $t_0 = t\left(\theta = \frac{\pi}{2}\right)$, and to write

$$\mu = \frac{2b}{\pi\eta t_0},\ \beta_n = \frac{B_n}{B_1} \ldots\ldots\ldots\ldots\ldots [10]$$

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[6] H. Glauert, "Aerofoil and Airscrew Theory" (1926), p. 139.

Then [9] may be written in the form

$$\alpha \sin \theta = B_1 \sum_{n=1}^{\infty} \beta_n \left(\mu \frac{\sin \theta}{t/t_0} + n \right) \sin n\theta - \sigma(\theta) \quad \text{..[11]}$$

where

$$\sigma(\theta) = \pm \frac{B_1 \sin \theta}{\cos \theta \sqrt{1 - A^4 \cos^2 \theta}} \sum_{n=1}^{\infty} n\beta_n \times \left(\frac{1 - \sqrt{1 - A^4 \cos^2 \theta}}{A^2 \cos \theta} \right)^n \quad \text{.....[12]}$$

Equation [11] serves to determine the coefficients B_n when the plan form of the wing (t/t_0), the distribution of profile (α), and the ratio of span to tunnel diameter (A) are given. The plus sign in [12] corresponds to a closed-working-section tunnel and the minus sign to an open tunnel. It should be noted that in the limit $A = 0$, corresponding to a wind tunnel of infinite diameter, the factor $\sigma(\theta) = 0$ and [11] reduces to the standard equation for a monoplane wing in an unbounded flow.

The most obvious method of proceeding with the determination of the coefficients β_n from [11] is to express $\sigma(\theta)$ as a Fourier series

$$\sigma(\theta) = B_1 \sum_{n=1}^{\infty} \gamma_n \sin n\theta$$

Equations [11] and [12] now assume the form

$$\left.\begin{aligned} \alpha \sin \theta &= B_1 \sum_{n=1}^{\infty} \left[\left(\mu \frac{\sin \theta}{t/t_0} + n \right) \beta_n - \gamma_n \right] \sin n\theta \\ \gamma_n &= \pm \frac{2}{\pi} \sum_{m=1}^{\infty} m\beta_m \int_0^{\pi} \left(\frac{1 - \sqrt{1 - A^4 \cos^2 \varphi}}{A^2 \cos \varphi} \right)^m \\ &\qquad \times \frac{\sin n\varphi \sin \varphi}{\cos \varphi \sqrt{1 - A^4 \cos^2 \varphi}} d\varphi \end{aligned}\right\} \quad [13]$$

where the plus sign corresponds to a closed tunnel and the minus to an open tunnel.

Before discussing the solution of [13] to give the coefficients β_n, it will be convenient to derive the form of the final results in which we are interested. In the present investigation it is desired to determine primarily three wind-tunnel interference effects. The first is the distortion to the lift distribution over the span, the second is the change in total lift coefficient or angle of attack, and the third is change in total drag coefficient. For this purpose a given wing will be considered in two conditions, first in the wind tunnel with a given total lift coefficient (no subscript), and second in a free stream, $A = 0$, with the same total lift coefficient (subscript *f*). The interference factors are then defined by

$$\Delta\alpha = \alpha_F - \alpha, \ \Delta C_D = C_{DF} - C_D, \text{ where } C_{LF} = C_L \quad \text{..[14]}$$

Now, using [3] and [8], we have

$$C_L = \frac{1}{\rho/2\, V^2 S} \int_{-b/2}^{b/2} L' dx = \frac{2b^2}{S} \sum_{n=1}^{\infty} B_n \int_0^{\pi} \sin n\theta \sin \theta d\theta$$

or

$$C_L = \pi M B_1 \quad \text{....................[15]}$$

where

$$S = \text{wing area}; \quad M = \frac{b^2}{S} = \text{aspect ratio} \quad \text{......[16]}$$

Hence B_1 will have the same value for the wing in the wind tunnel and in the free stream, since in our comparison we take $C_{LF} = C_L$. The expression for $\Delta\alpha$ may now be written

$$\Delta\alpha = \alpha(A = 0) - \alpha(A) = \alpha(\gamma_n = 0) - \alpha \quad \text{......[17]}$$

The fundamental expression for total drag coefficient is

$$C_D = \frac{D}{\frac{\rho}{2} V^2 S} \quad \text{where} \quad D = \rho \int_{-b/2}^{b/2} \Gamma w dx$$

But $w = w_0 + w_1$, and from [6] and [12]

$$\frac{w_1}{V} = \frac{-\sigma}{\sin \theta} = -B_1 \sum_{n=1}^{\infty} \gamma_n \frac{\sin n\theta}{\sin \theta}$$

A little calculation now gives

$$C_D = \frac{2b^2}{S} B_1^2 \int_0^{\pi} \left[\sum_{n=1}^{\infty} \beta_n \sin n\theta \sum_{m=1}^{\infty} (m\beta_m - \gamma_m) \sin m\theta \right] d\theta$$

or

$$C_D = \frac{C_L^2}{\pi M} \sum_{n=1}^{\infty} (n\beta_n^2 - \beta_n \gamma_n)$$

Hence

$$\Delta C_D = \frac{C_L^2}{\pi M} \sum_{n=1}^{\infty} \left[n\beta_n^2(A = 0) - (n\beta_n^2 - \beta_n\gamma_n) \right] \quad \text{..[18]}$$

Finally the free stream and actual lift distributions are given by

$$L_F' = 2\rho V^2 b B_1 \sum_{n=1}^{\infty} \beta_n(A = 0) \sin n\theta; \quad L' = 2\rho V^2 b B_1 \sum_{n=1}^{\infty} \beta_n \sin n\theta \quad \text{........[19]}$$

Equations [17], [18], and [19] give all of the interference effects with which we are concerned, in terms of the coefficients β_n and γ_n.

4 OTHER EXPRESSIONS FOR THE COEFFICIENTS γ_n

The evaluation of the integral in the expression for γ_n in [13] appears to be very difficult for arbitrary values of A. We therefore adopt the expedient of expanding the integrand as a power series in A and so obtaining approximate values of γ_n. In carrying out this calculation considerations are restricted to cases in which the lift distribution is symmetrical about the center line of the wing. This covers all practically interesting cases except those in which the effect of aileron displacements is sought for. The restriction mentioned implies that only odd coefficients $\beta_1, \beta_3, \beta_5$, etc., appear, i.e., $\beta_2 = \beta_4 = \beta_6 = \ldots..$etc. $= 0$. Similarly the image vortices will have a symmetrical arrangement so that $\gamma_2 = \gamma_4 = \gamma_6 = \ldots..$etc. $= 0$. Hence in Equations [13] the summation indices n and m may be replaced by indices of the form

$$n, m = 2k - 1 \quad \text{.................[20]}$$

Using this notation the expansion of [12] gives

$$\sigma = A^2 \sin\theta \cdot B_1\left[\frac{1}{2} + \frac{3}{8}(1 + \beta_3)(A^2\cos\theta)^2 + \frac{5}{32}(2 + 3\beta_3 + \beta_5)(A^2\cos\theta)^4 + \frac{7}{128}(5 + 9\beta_3 + 5\beta_5 + \beta_7)(A^2\cos\theta)^6 + \ldots\right]$$

Transforming this to the Fourier-series form given in [13], a little calculation gives for the first four Fourier coefficients:

$$\left.\begin{aligned}
\gamma_1 &= \pm A^2\left[\frac{1}{2} + \frac{3}{32}(1+\beta_3)A^4 + 2\cdot\frac{5}{512}(2+3\beta_3+\beta_5)A^8 + 5\cdot\frac{7}{8192}(5+9\beta_3+5\beta_5+\beta_7)A^{12} + \ldots\ldots\right]\\
\gamma_3 &= \pm A^2\left[\frac{3}{32}(1+\beta_3)A^4 + 3\cdot\frac{5}{512}(2+3\beta_3+\beta_5)A^8 + 9\cdot\frac{7}{8192}(5+9\beta_3+5\beta_5+\beta_7)A^{12} + \ldots\ldots\right]\\
\gamma_5 &= \pm A^2\left[1\cdot\frac{5}{512}(2+3\beta_3+\beta_5)A^8 + 5\cdot\frac{7}{8192}(5+9\beta_3+5\beta_5+\beta_7)A^{12} + \ldots\ldots\right]\\
\gamma_7 &= \pm A^2\left[1\cdot\frac{7}{8192}(5+9\beta_3+5\beta_5+\beta_7)A^{12} + \ldots\ldots\right]
\end{aligned}\right\}\ldots[21]$$

where the plus and minus signs refer as usual to the closed and open tunnel conditions.

The curious symmetry which appears in these expressions renders it easy to write down additional terms after a little experience, but the complexity of the calculations makes it difficult to give a definite formula for any arbitrary term. It may be noted that if A be put equal to 1, then the numerical coefficients of β_1, β_3, β_5, etc., form a matrix symmetrical about the diagonal.

The expansion given by [21] leads to a satisfactory solution of the wind-tunnel interference problem when A does not approach unity too closely. In the limiting case $A = 1$ the integral of [13] can be evaluated directly and expressed as a definite function of m and n. For large values of n the integration becomes a little tedious, but there are no difficulties in principle for arbitrary values of the indices. The same property which was noted at the end of the last paragraph is found to hold also in this case, namely, the coefficients of β_1, β_3, β_5, etc., form a diagonally symmetrical matrix. The numerical results for the first few terms are as follows:

$$\left.\begin{aligned}
\gamma_1 &= \pm(0.727 + 0.367\beta_3 + 0.238\beta_5 + 0.177\beta_7 + 0.139\beta_9 + \ldots\ldots)\\
\gamma_3 &= \pm(0.367 + 0.681\beta_3 + 0.569\beta_5 + 0.461\beta_7 + 0.379\beta_9 + \ldots\ldots)\\
\gamma_5 &= \pm(0.238 + 0.569\beta_3 + 0.652\beta_5 + 0.608\beta_7 + 0.542\beta_9 + \ldots\ldots)\\
\gamma_7 &= \pm(0.177 + 0.461\beta_3 + 0.608\beta_5 + \ldots\ldots)\\
\gamma_9 &= \pm(0.139 + 0.379\beta_3 + 0.542\beta_5 + \ldots\ldots)
\end{aligned}\right\}\ldots[22]$$

5 APPLICATION TO AN UNTWISTED WING OF ELLIPTICAL PLAN FORM

The solution of Equation [13] is particularly simple for an untwisted wing of elliptical plan form, so that this case is chosen for the working out of a numerical example. For such a wing

$$\alpha = \text{constant}, \quad t/t_0 = \sin\theta, \quad \beta_{2k} = \gamma_{2k} = 0$$

Hence [13] gives at once

$$\beta_{2k-1}(\mu + 2k - 1) - \gamma_{2k-1} = \frac{2\alpha}{\pi B_1}\int_0^{\pi} \sin\theta \sin(2k-1)\theta d\theta$$

or

$$\left.\begin{aligned}
\alpha &= B_1(1 + \mu + \gamma_1)\\
\beta_{2k-1} &= \frac{\gamma_{2k-1}}{2k - 1 + \mu} \qquad (k > 1)
\end{aligned}\right\}\ldots\ldots[23]$$

To get free-stream conditions we must put $A = 0$, i.e., $\gamma_{2k-1} = 0$. Hence

$$\alpha_F = B_1(1 + \mu), \quad (\beta_{2k-1})_F = 0 \qquad (k > 1)$$

and [17] and [18] together with [15] give

$$\Delta\alpha = B_1\gamma_1 = \frac{C_L}{\pi M}\gamma_1$$

$$\Delta C_D = \frac{C_L^2}{\pi M}\left[\gamma_1 + \sum_{k=2}^{\infty}\left\{\gamma_{2k-1} - (2k-1)\beta_{2k-1}\right\}\beta_{2k-1}\right]$$

In order to compare our results with those given by Prandtl, which are based on the assumption of an elliptical lift distribution,[4] it will be convenient to rewrite these expressions in a little different form. Denoting the cross-sectional area of the wind tunnel by C so that

$$\pi M = \pi\frac{b^2}{S} = \frac{\pi R^2}{S}4\left(\frac{b}{2R}\right)^2 = 4\frac{C}{S}A^2$$

and writing

$$\Delta\alpha = \delta_\alpha\frac{S}{C}C_L, \qquad \Delta C_D = \delta_D\frac{S}{C}C_L^2 \ldots\ldots\ldots [24]$$

then

$$\delta_\alpha = \frac{\gamma_1}{4A^2},$$

$$\delta_D = \frac{1}{4A^2}\left[\gamma_1 + \sum_{k=2}^{\infty}\left\{\gamma_{2k-1} - (2k-1)\beta_{2k-1}\right\}\beta_{2k-1}\right]\ldots[25]$$

Prandtl's results may be expressed in the form:[4,7]

$$\left.\begin{aligned}
&\delta_\alpha = \delta_D = \delta_P, \quad \beta_{2k-1} = 0 \quad (k > 1)\\
&\text{and}\\
&\delta_P = \frac{1}{8}\left(1 + \frac{3}{16}A^4 + \frac{5}{64}A^8 + \frac{175}{4096}A^{12} + \ldots\ldots\right)
\end{aligned}\right\}\ldots[26]$$

The simultaneous solution of the sets of linear Equations [21] or [22] and [23] may quite easily be carried out to any desired

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[7] L. Prandtl, etc., "Ergebnisse der Aerodynamischen Versuchstanstalt zu Göttingen," 2 Lieferung (1923), p. 17.

approximation. The expansion in powers of A, neglecting terms of degree higher than 14 in A, gives for a closed wind tunnel:

$$\left.\begin{aligned}
\delta_\alpha &= \frac{1}{8}\Bigg[1 + \frac{3}{16}A^4 + \frac{5}{64}A^8 + \frac{175}{4096}A^{12} + \ldots\ldots\ldots \\
&\qquad + \frac{9}{512(3+\mu)}A^{10}\left(1 + \frac{5}{4}A^4 + \ldots\ldots\ldots\right)\Bigg] \\
\delta_D &= \frac{1}{8}\Bigg[1 + \frac{3}{16}A^4 + \frac{5}{64}A^8 + \frac{175}{4096}A^{12} + \ldots\ldots\ldots \\
&\qquad + \frac{9(3+2\mu)}{512(3+\mu)^2}A^{10}\left(1 + \frac{5}{4}A^4 + \ldots\ldots\ldots\right)\Bigg] \\
L' &= 2\rho V^2 bB_1\Bigg[\sin\theta + \frac{3}{32(3+\mu)}A^6\Bigg\{1 + \frac{5}{8}A^4 + \frac{105}{256}A^8 \\
&\qquad + \ldots\ldots + \frac{3}{32(3+\mu)}A^6 + \ldots\ldots\Bigg\}\sin 3\theta \\
&\qquad + \frac{5}{256(5+\mu)}A^{10}\left\{1 + \frac{35}{32}A^4 + \ldots\ldots\right\}\sin 5\theta \\
&\qquad + \frac{35}{8192(7+\mu)}A^{14}\sin 7\theta + \ldots\ldots\Bigg]
\end{aligned}\right\} .[27]$$

It will be observed that δ_α and δ_D differ from Prandtl's value and from one another only in terms of the tenth degree or higher in A, while the distortion of the lift distribution contains terms of the sixth degree. The secondary series in parentheses would appear to diverge for values of A near to unity, but there is a reason to believe that the numerical coefficients of the first few terms only increase and that the coefficients later decrease with increasing degree of A, so as to make the series actually convergent even for $A \longrightarrow 1$. Putting $\mu = 3$, corresponding to an aspect ratio of about 5.5 [27] gives:

$$\left.\begin{aligned}
\delta_\alpha &= \frac{1}{8}\,[1 + 0.188A^4 + 0.078A^8 + 0.043A^{12} + \ldots\ldots\ldots \\
&\qquad + 0.003A^{10}(1 + 1.250A^4 + \ldots\ldots\ldots)] \\
\delta_D &= \frac{1}{8}\,[1 + 0.188A^4 + 0.078A^8 + 0.043A^{12} + \ldots\ldots\ldots \\
&\qquad + 0.004A^{10}(1 + 1.250A^4 + \ldots\ldots\ldots)] \\
\beta_3 &= 0.016A^6\,(1 + 0.625A^4 + 0.410A^8 + \ldots\ldots\ldots \\
&\qquad + 0.016A^6 + \ldots\ldots\ldots) \\
\beta_5 &= 0.0024A^{10}\,(1 + 1.094A^4 + \ldots\ldots\ldots) \\
\beta_7 &= 0.0004A^{14}(1 + \ldots\ldots\ldots).
\end{aligned}\right\} .[28]$$

In the limiting case $A = 1$ the calculations have been carried out for $\mu = 3$, using the approximation indicated in [22]. The result is

$$\left.\begin{aligned}
&\delta_\alpha = 0.192 \qquad\qquad \delta_D = 0.198 \\
&L' = 2\rho V^2 bB_1\,[\sin\theta + 0.076\sin 3\theta + 0.041\sin 5\theta \\
&\qquad + 0.024\sin 7\theta + 0.017\sin 9\theta + \ldots\ldots\ldots]
\end{aligned}\right\} .[29]$$

Decreasing the degree of approximation by one step, i.e., neglecting β_9 and γ_9, caused no noticeable change in the above values of δ_α and δ_D, while neglecting β_7 and γ_7 in addition decreased both about one-half per cent.

The numerical expressions have also been worked out for the open-throat wind-tunnel case. The expansion in powers of A gives

$$\left.\begin{aligned}
\delta_\alpha &= -\frac{1}{8}\,[1 + 0.188A^4 + 0.078A^8 + 0.043A^{12} + \ldots\ldots \\
&\qquad - 0.003A^{10}\,(1 + 1.250A^4 + \ldots\ldots\ldots)] \\
\delta_D &= -\frac{1}{8}\,[1 + 0.188A^4 + 0.078A^8 + 0.043A^{12} + \ldots\ldots \\
&\qquad - 0.004A^{10}\,(1 + 1.250A^4 + \ldots\ldots\ldots)] \\
\beta_3 &= -0.016A^6\,(1 + 0.625A^4 + 0.410A^8 + \ldots\ldots\ldots \\
&\qquad - 0.016A^6 + \ldots\ldots\ldots) \\
\beta_5 &= -0.0024A^{10}\,(1 + 1.094A^4 + \ldots\ldots\ldots) \\
\beta_7 &= -0.0004A^{14}\,(1 + \ldots\ldots\ldots)
\end{aligned}\right\} .[30]$$

The limiting case $A = 1$ leads to the expressions:

$$\left.\begin{aligned}
&\delta_\alpha = -0.175 \qquad\qquad \delta_D = -0.172 \\
&L' = 2\rho V^2 bB_1\,[\sin\theta - 0.052\sin 3\theta - 0.023\sin 5\theta \\
&\qquad - 0.014\sin 7\theta - 0.009\sin 9\theta - \ldots\ldots\ldots]
\end{aligned}\right\} .[31]$$

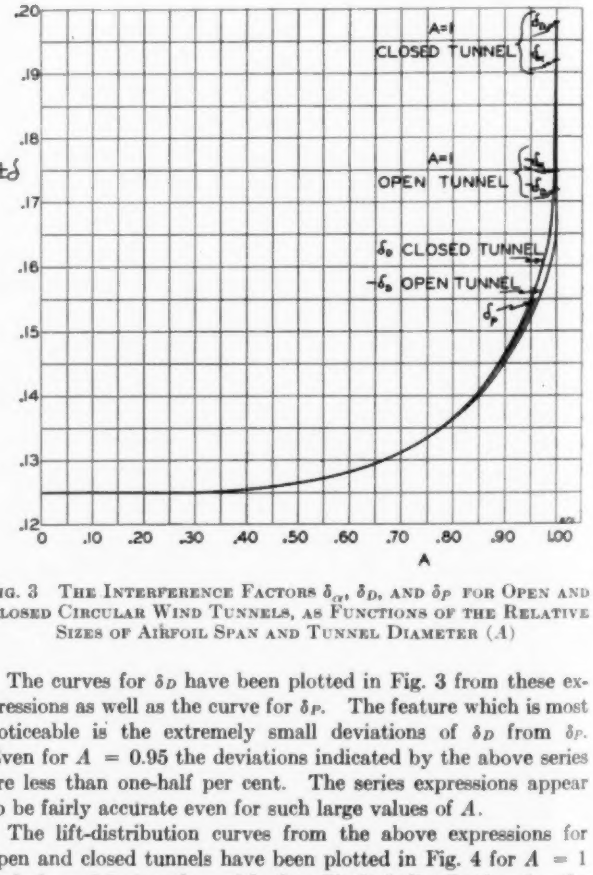

Fig. 3 The Interference Factors δ_α, δ_D, and δ_P for Open and Closed Circular Wind Tunnels, as Functions of the Relative Sizes of Airfoil Span and Tunnel Diameter (A)

The curves for δ_D have been plotted in Fig. 3 from these expressions as well as the curve for δ_P. The feature which is most noticeable is the extremely small deviations of δ_D from δ_P. Even for $A = 0.95$ the deviations indicated by the above series are less than one-half per cent. The series expressions appear to be fairly accurate even for such large values of A.

The lift-distribution curves from the above expressions for open and closed tunnels have been plotted in Fig. 4 for $A = 1$ and $A = 0.9$, together with the elliptical distribution for the airfoil in a free stream $A = 0$. Even in the extreme case $A = 1$ the distortion of the lift-distribution curve is not violent, while for $A = 0.9$ it is so slight as to be of no practical consequence.

6 Discussion of Results

The most interesting result of the foregoing numerical investigation is the surprisingly small distortion in the lift distribution caused by the wind-tunnel interference. In view of this very small effect for elliptical wings, it appears very unlikely that other normal plan forms (e.g., rectangular, tapered) will suffer markedly larger distortions until *A* approaches very near to unity. Also it seems very probable that for wind tunnels with rectangular, elliptical, or other usual cross-sections the distortion will be of the same order as that found here, as long as the wing tips are a reasonable distance from the tunnel boundaries. In closed tunnels there is always a boundary layer which disturbs the uniformity of flow near the walls, and in open tunnels the region near the jet boundary is much disturbed. Hence it is generally unsafe from this point of view to test a model whose span is much over nine-tenths of the wind-tunnel breadth. Hence one may conclude that for tests in circular wind tunnels with a wing plan form similar to an ellipse, if the wing is restricted to the region of uniform velocity, the simple Prandtl correction factors may safely be used and the lift-distribution distortion may be neglected. This conclusion can probably be extended with reasonable certainty to other wind-tunnel cross-sections and wing plan forms. Hence it would seem that the explanation for the discrepancies to which reference is made in the introduction must be sought in some other direction.

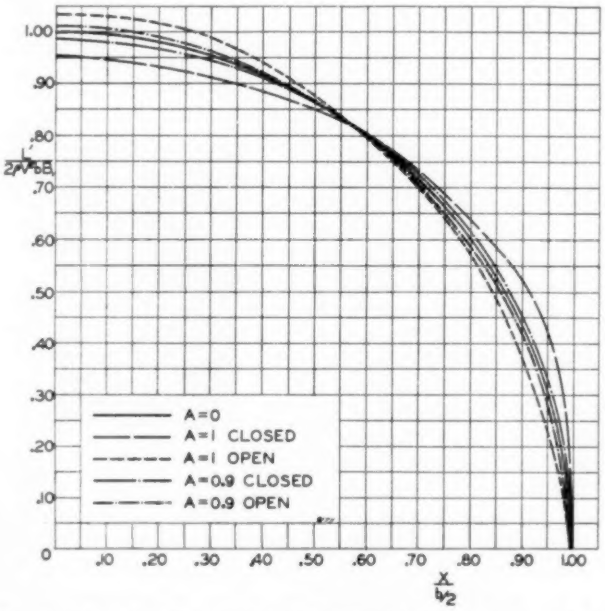

Fig. 4 The Lift Distribution Over the Semi-Span of an Untwisted Elliptical Wing in Open and Closed Circular Wind Tunnels for Various Values of *A*

It should be pointed out explicitly that the effects of streamline curvature due to tunnel walls or of deflection of the jet in the case of open tunnels have not been considered in the present work. Both of these effects vary with the ratio of wing chord to tunnel dimensions, and not with the ratio of span to tunnel dimension.

In conclusion, it may be mentioned that the investigation of other plan forms may be carried out, using the results of the present analysis, by exactly the same methods as have been used for a wing in an unbounded fluid by Glauert[4] or more recently by Lotz.[5]

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[5] I. Lotz, "Berechnung der Auftriebsverteilung beliebig geformter Flügel," *Z.F.M.* (1931), 7, p. 189.

APM-54-20

Design of Spring Gears for Exhaust-Turbine Installations

By J. ORMONDROYD[1] and T. C. KUCHLER,[2] PHILADELPHIA, PA.

J. Ormondroyd

The paper outlines the necessity for flexibility in the coupling linkage of a turbine with a reciprocating engine driving the same shaft. A résumé of the history of the problem is given. The conditions which must be fulfilled for satisfactory operation and the details of the calculation to fulfil these conditions are discussed. For simplicity the design chosen relies on propeller damping to hold the torque oscillations within safe limits. Tests with and without flexibility in the turbine gearing are described. It is shown that the problem depends on the vibration characteristics of the system and that a solution can be made if the drawings of the rotating parts and a few engine indicator cards at different speeds are available.

T. C. Kuchler

THE exhaust turbine for raising the efficiency of existing reciprocating-engined ships is gaining increased notice in marine circles. The combination of the exhaust turbine and the reciprocating engine driving a single propeller shaft gives rise to peculiar mechanical difficulties in the gearing which connects the turbine to the propeller shaft. This paper deals with a solution of this problem.

The steamship *Susan V. Luckenbach* is a cargo vessel of 12,000-ton displacement which was originally equipped with a triple-expansion reciprocating engine which delivered 2600 i.hp. at 77 r.p.m. and having a speed of about 12 knots. Competition required a greater speed for the same operating cost. Having the engine available, this was best accomplished by the installation of an exhaust turbine.

The exhaust-turbine installation consists of a 1000-hp. turbine taking steam from the engine at 19 in. vacuum and exhausting to the condenser at 29 in. vacuum. The power developed by this turbine is transmitted to the propeller shaft through a gear of double-reduction flexible-frame type.

To make the maneuvering easier, the turbine is directly connected to the propeller shaft at all times, and a control is installed to maneuver the turbine automatically with the engine by the use of high-pressure steam.

This turbine, shown in Fig. 1, is designed to utilize either low- or high-pressure steam. The ahead blading consists of four pairs of rows of reaction blading in which either low-pressure exhaust steam or high-pressure steam expanded through a nozzle is used. The astern blading consists of two moving and one stationary row of impulse blades in which only high-pressure steam is used.

The reduction gearing as originally installed is shown in Fig. 2. It was designed to carry the load of 1000 hp. developed by the turbine with a speed reduction from 3600 r.p.m. of the turbine to 84 r.p.m. of the propeller shaft. The gears were made strong enough to carry the full engine torque to which they might be subjected if reversing should be done with the engine alone. The gearing is of the double-reduction flexible-frame type, the pinions and gears being double helical with standard involute teeth. The entire gear train—which consists of a high-speed (or first reduction) pinion, high-speed (or first reduction) gear, with low-speed (or second reduction) pinion mounted on the same shaft, and low-speed (or second reduction) gear—is arranged in a horizontal plane and carried in a single housing. The connection between the turbine and the high-speed pinion is by means of a pin-type coupling and a flexible drive shaft. This drive shaft extends through the pinion and is fastened to it at the end remote from the turbine. The arrangement gives a shaft of considerable length which provides the necessary flexibility to permit the flexible frame to function without restraint. The permissible rocking of the frames insures good tooth alignment, even though the housing is distorted due to weaving of the ship or the main shaft drops due to wear of the engine main bearings.

The result of this installation was to increase the indicated shaft horsepower of the ship from 3000 hp. developed by the engine to 4000 hp. developed by the combination of the engine and turbine with the same fuel consumption. This added horsepower increases the speed of the ship about 2 knots.

The original trials of the installation showed very fine results, with the exception of a violent bumping in the gears. This was at first thought to be due to uneven load distribution in the three engine cylinders. The load distribution was improved without changing the gear bumping. Reducing bearing clearances and backlash had no effect on the gear impacts.

Torsiograph readings were taken during the trials to determine

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[1] Experimental Engineering Division, South Philadelphia Works, Westinghouse Elec. & Mfg. Co. Mr. Ormondroyd received the degree of A.B. from the University of Pennsylvania in 1920. Since then he has been with his present company as engineer. For two years his duties related to electric-heating engineering and from 1924 to 1930 to railway-motor engineering. In 1927 he was made engineer in charge of instruction in the Westinghouse Mechanical Design School. In 1930 he became manager of the company's Experimental Division.

[2] Research Engineer, Engine Division, South Philadelphia Works, Westinghouse Elec. & Mfg. Co. Jun. A.S.M.E. Mr. Kuchler was graduated in 1926 from the Swiss Federal College of Science, as a pupil of Dr. A. Stodola. From 1926 to 1928 he was employed by Sulzer Bros., Winterthur, Switzerland, as a draftsman in the marine Diesel engine department. He was been connected with the Westinghouse Company since 1928, and has been engaged in research and development in connection with high-speed oil engines.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

the variable engine torque. The results from these readings showed quite convincingly that the bumping was caused by this variable torque.

Consideration of the design of a flexible gear drive resulted in the design of a low-speed gear driving the propeller shaft through springs, the variation in torque being taken up by the springs.

This spring-gear wheel is shown in Fig. 3. The gear wheel center, two spiders, and one of the 12 spring assemblies are shown in Fig. 4. The gear wheel is free to rotate on the gear-wheel center, having two bronze bushings at the ends which bear on the journals at the ends of the gear-wheel center. The springs are supported by two internal stops which are used to limit their deflection and hold them in place. There are hardened pins fitted into the ends of the stops which are used as a support point for the spring assembly in the gear wheel and spiders. The spiders are bolted directly to the ends of the gear-wheel center.

The load is transmitted from the gear wheel through the springs to the spider, which in turn drives the gear-wheel shaft. The springs take the load in compression in either direction of rotation as they are always in contact at the semicircular notches in the gear wheel at one end and with the semicircular notches in the spider at the other. Under no load the spring stresses are zero with this arrangement.

Guard plates are attached to the gear wheel to prevent the spring assemblies from coming out in case a spring should be broken. Openings are provided in the gear-housing cover to allow easy and quick spring replacement.

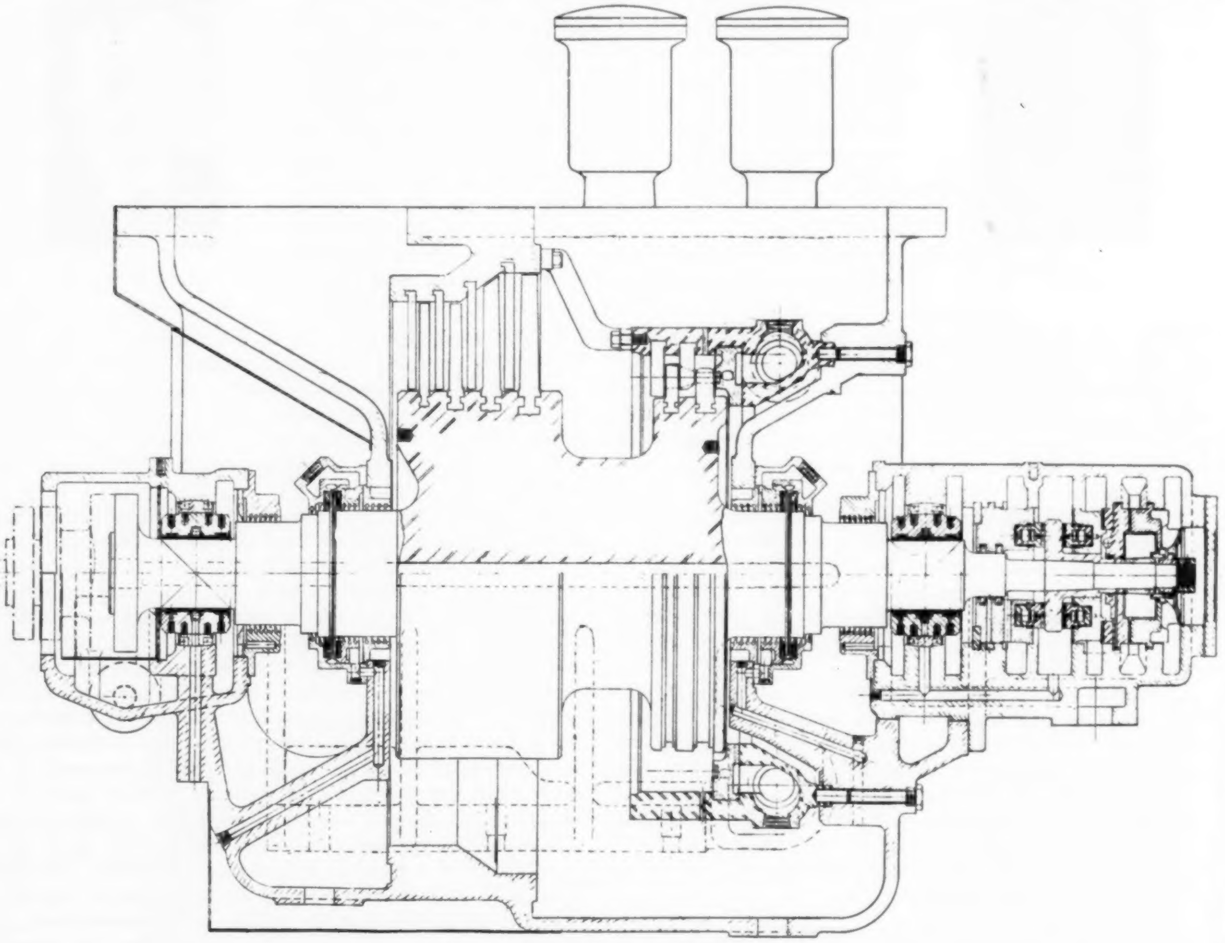

FIG. 1 LOW-PRESSURE TURBINE

The turbine and gear installation in the ship is shown in Fig. 5. The ship has been in service about one year with the spring gear, and the operation is very satisfactory.

HISTORY OF THE SPRING GEAR

As early as 1906 Sir Charles Parsons had taken out patents, not only covering the combination of steam engine and exhaust turbine by means of gearing, but also flexible gears to be used in the unit. He realized that the variable-torque characteristic of the steam engine and the steady-torque characteristic of the turbine necessitated some elastic linkage between the two when used in combination. At full load the steam-engine torque variation may be as much as 60 per cent of the average torque, and this variation may occur several times in one revolution.

This leads to severe bumping in the gears and large impact forces on the gear teeth and housing. Parsons indicated a preference for the flexibility in the high-speed pinion shaft. In 1929 Parsons took out another patent covering the combination of flexibility and friction damping in the first reduction gear wheel. In spite of their early appreciation of the advantages to be gained by the combination of exhaust-turbine and triple-expansion steam engine, the Parsons company equipped its first ship this year.[3] The equipment on the *Kingswood* embodies the flexibility and damping described in the 1929 patent.

The well-known Bauer-Wach engine-exhaust turbine system employs a Föttinger hydraulic coupling between the turbine and the high-speed pinion. The chief purpose of the coupling is to provide a means of disconnecting the turbine from the propeller shaft during maneuvering. Its use involves a loss of 2 to 3 per cent of turbine efficiency in transmission. Incidentally it is supposed to give enough slip to keep the shocks arising from variable torque within safe limits. The main gear in the Bauer-Wach system is connected to the propeller shaft by means of a comparatively flexible quillshaft. The expressed purpose of this shaft is to take care of crankshaft misalignment caused by engine bearing wear.

The Brown Boveri Company has recently designed an exhaust-turbine drive incorporating a flexible, hydraulically damped coupling between the second reduction-gear wheel and the propeller shaft.[4] The slow-speed gear is mounted on the quillshaft, to which the flexible coupling is attached. The coupling is a separate device external to the gear itself. So far the Brown Boveri Company has not made an installation.

In 1929 the Westinghouse Company provided the exhaust turbine and gears, as described, for the *Susan V. Luckenbach.* The second reduction-gear wheel had flexibility introduced into it as shown in Fig. 3. The rim is free to rotate around its center, the connection between the rim and the center being made through tangentially placed coil springs.

Quantitative Design Requirements

While the necessity of flexibility has been recognized extensively, no quantitative discussion of design details ever has been published.

The fundamental requirements demanded of any flexibility introduced into the gear system are:

1 To keep the variation in torque in the gears less than the average torque transmitted by them
2 To keep the variation in torque as small as possible
3 To accomplish the first requirements with safe working stresses in the springs.

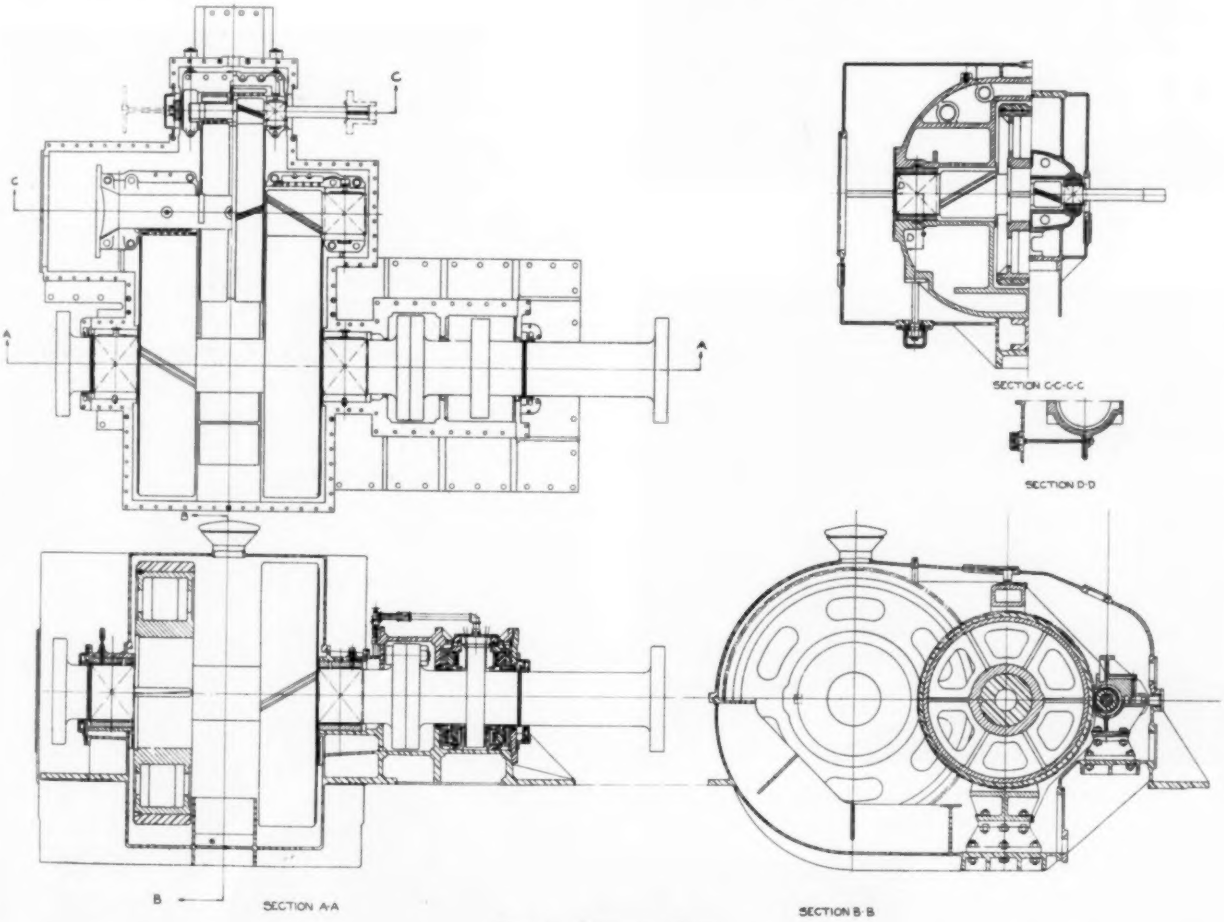

Fig. 2 Original Gear

These conditions can be met by practical flexibilities at the usual running speeds of the engines, but not at very low speed maneuvering. Since, in freight steamers, maneuvering covers such a small percentage of the working time of the engines, the failure to satisfy the fundamental requirements under those conditions is not serious.

The worst variation in torque in the gears will be encountered when the engine-turbine-propeller system is running at a torsional critical speed; therefore, the requirements of the flexibility, stated specifically, are:

4 To keep the variation in torque in the gears less than the average torque transmitted by the gears and as small as possible in the worst torsional critical speed which

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[3] *Engineering*, Feb. 6, 1931, p. 171.

[4] Brown Boveri Review.

may be encountered in or near the usual running range of the engine.

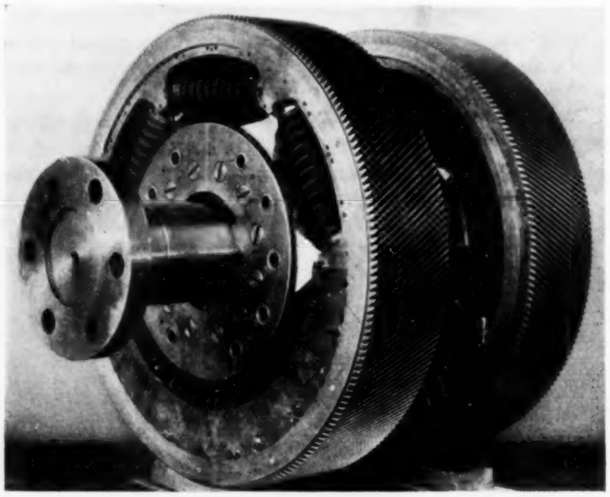

FIG. 3 SECOND REDUCTION SPRING GEAR

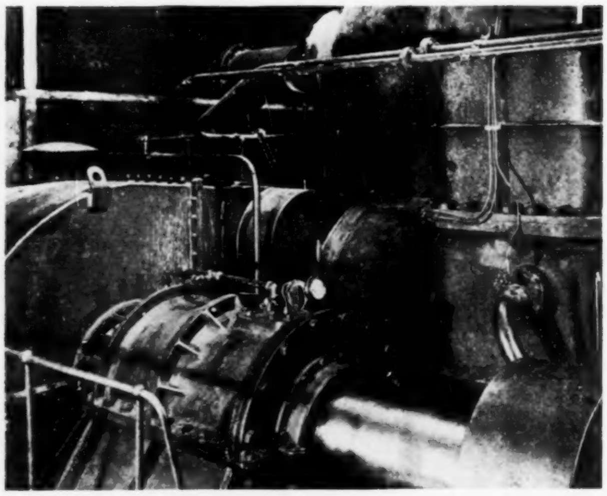

FIG. 5 EXHAUST-TURBINE AND REDUCTION-GEAR INSTALLATION IN SHIP

POSITION OF FLEXIBILITY IN SYSTEM

Before making a study of the torsional critical speeds of the system, it will be well to indicate the reasons for placing the flexibility in the low-speed gear. For practical reasons it was decided to incorporate the flexibility on the gear wheels; the only question to be settled was whether the springs should be in the first reduction gear or the slow-speed gear. With flexibility in the gearing, the turbine mass will not vibrate because of its tremendous inertia and the springs will have deflections corresponding to the vibration of the engine-propeller-shaft system. The conditions which determine the position of the springs are as follows:

1 For a given amplitude of vibration in the crankshaft, the transmitted torque must be the least possible

2 For the same given motion, the stresses in the springs must be the least possible.

Assume that coil springs, tangentially placed, are used at some radius R_2 in the low-speed gear or at R_1 in the first reduction gear. Let the pitch-line radii of the low-speed gear, low-speed pinion, and first reduction gear be r_2, r_3, and r_1, respectively. Let the number of springs used, the diameter of spring wire, mean diameter of spring coil, and number of turns of wire per spring be n, d, D, and N, respectively. Assume that the pro-

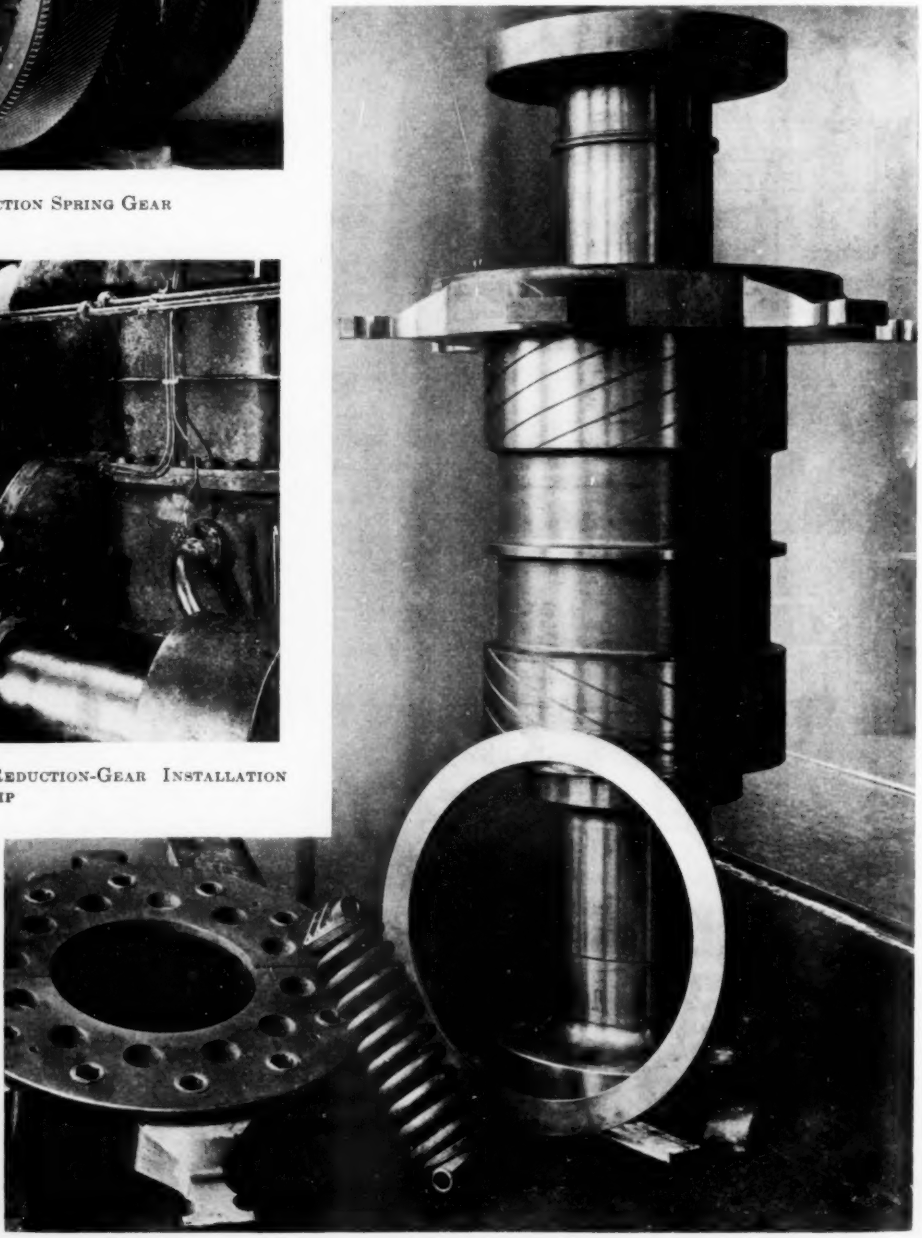

FIG. 4 SPRING-GEAR PARTS

peller shaft moves through an angle α. G is the modulus of elasticity in shear.

For geometrically similar spring systems, the following relations will hold: $n_1 = n_2$; $G_1 = G_2$; $R_1 = \frac{r_1}{r_2} R_2$; $d_1 = \frac{r_1}{r_2} d_2$; $D_1 = \frac{r_1}{r_2} D_2$; $\alpha_1 = \frac{r_2}{r_3} \alpha_2$, where the subscripts 1 and 2 refer to the first and second reduction gears, respectively.

Let T stand for torque transmitted and S stand for the stress arising in the springs for the motion α. Then:

$$T_2 = \frac{n_2 G_2}{8 N_2} \frac{R_2^2 d_2^4}{D_2^3} \alpha_2 \ldots\ldots\ldots\ldots [1]$$

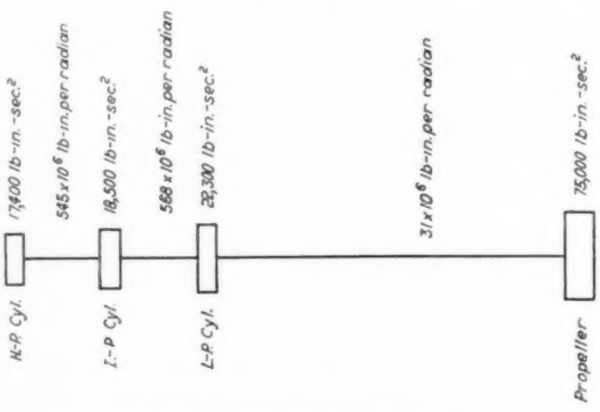
17,400 lb-in.-sec.²
545 x 10⁶ lb-in. per radian
18,500 lb-in.-sec.²
568 x 10⁶ lb-in. per radian
22,300 lb-in.-sec.²
31 x 10⁶ lb-in. per radian
75,000 lb-in.-sec.²
H-P. Cyl.
I-P Cyl.
L-P Cyl.
Propeller

FIG. 6a ORIGINAL SYSTEM AS INSTALLED

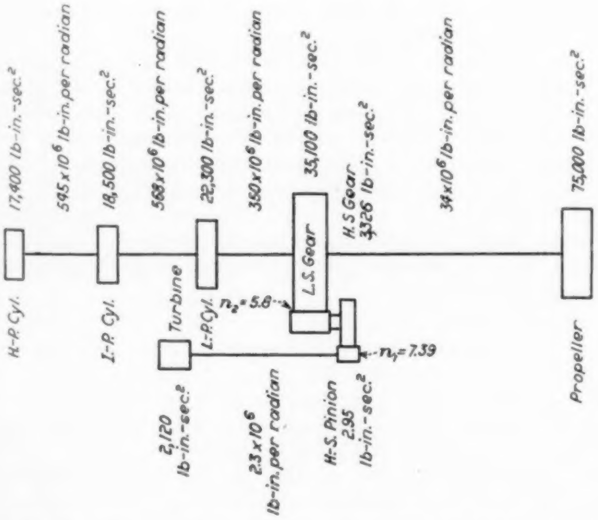
17,400 lb-in.-sec.²
595 x 10⁶ lb-in. per radian
18,500 lb-in.-sec.²
568 x 10⁶ lb-in. per radian
22,300 lb-in.-sec.²
350 x 10⁶ lb-in. per radian
35,100 lb-in.-sec.²
H.S Gear
3,326 lb-in.-sec.²
34 x 10⁶ lb-in. per radian
75,000 lb-in.-sec.²
H-P. Cyl.
I-P. Cyl.
Turbine
L-P. Cyl.
L.S. Gear
n₂ = 5.8
n₁ = 7.39
Propeller
2,120 lb-in.-sec.²
2.3 x 10⁶ lb-in. per radian
H-S. Pinion 2.95 lb-in.-sec.²

FIG. 6b FLYWHEELS AND SPRING CONSTANTS, ACTUAL SYSTEM

$$T_1 = \frac{r_2}{r_3} \left(\frac{r_1}{r_2} \right)^3 \frac{n_2 G_2}{8 N_2} \frac{R_2^2 d_2^4}{D_2^3} \alpha_2 \ldots\ldots\ldots\ldots [2]$$

or

$$T_1 = \frac{r_2}{r_3} \left(\frac{r_1}{r_2} \right)^3 T_2 \ldots\ldots\ldots\ldots [3]$$

$$S_2 = \frac{G_2}{\pi N_2} \frac{R_2 d_2}{D_2^2} \alpha_2 \ldots\ldots\ldots\ldots [4]$$

$$S_1 = \frac{r_2}{r_3} \frac{G_2}{\pi N_2} \frac{R_2 d_2}{D_2^2} \alpha_2 \ldots\ldots\ldots\ldots [5]$$

$$S_1 = \frac{r_2}{r_3} S_2 \ldots\ldots\ldots\ldots [6]$$

In the gears proposed for the ship, $r_2/r_3 = 5.8$ and $r_1/r_2 = 0.679$; therefore, $T_1 = 1.8\ T_2$ and $S_1 = 5.8\ S_2$.

This indicates clearly that the springs should be placed between the center and the rim of the low-speed gear. All calculations, therefore, are based on springs in this position.

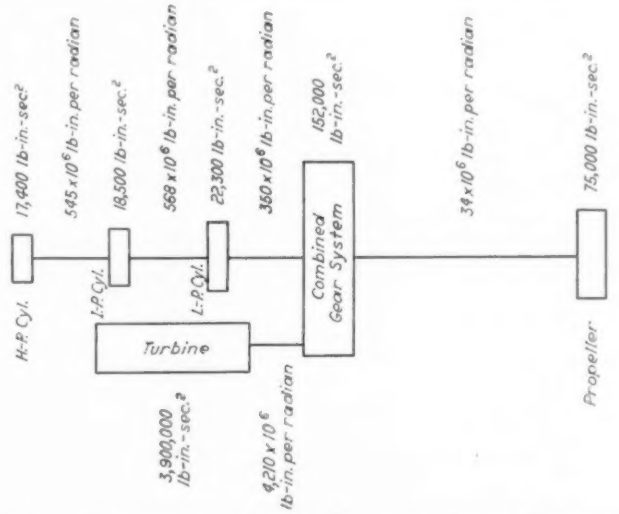
17,400 lb-in.-sec.²
545 x 10⁶ lb-in. per radian
18,500 lb-in.-sec.²
568 x 10⁶ lb-in. per radian
22,300 lb-in.-sec.²
350 x 10⁶ lb-in. per radian
152,000 lb-in.-sec.²
Combined Gear System
34 x 10⁶ lb-in. per radian
75,000 lb-in.-sec.²
H-P. Cyl.
I-P. Cyl.
L-P. Cyl.
Turbine
3,900,000 lb-in.-sec.²
4,210 x 10⁶ lb-in. per radian
Propeller

FIG. 6c FLYWHEELS AND SPRING CONSTANTS, SYSTEM REDUCED TO PROPELLER SPEED

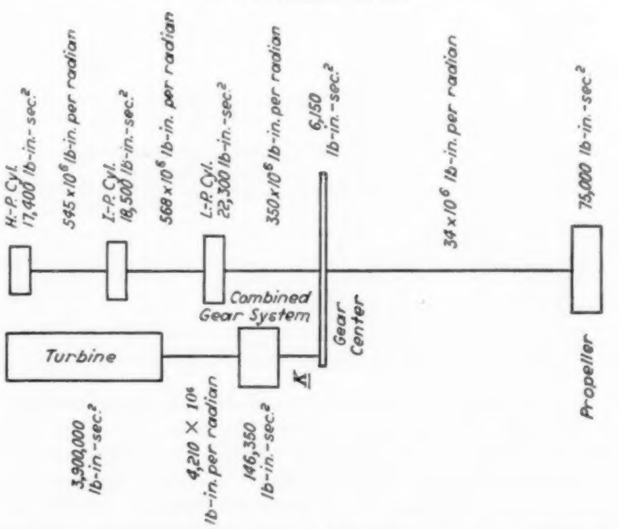
H-P. Cyl. 17,400 lb-in.-sec.²
545 x 10⁶ lb-in. per radian
I-P. Cyl. 18,500 lb-in.-sec.²
568 x 10⁶ lb-in. per radian
L-P. Cyl. 22,300 lb-in.-sec.²
350 x 10⁶ lb-in. per radian
6,150 lb-in.-sec.²
34 x 10⁶ lb-in. per radian
75,000 lb-in.-sec.²
Combined Gear System
Turbine
K
Gear Center
Propeller
3,900,000 lb-in.-sec.²
4,210 × 10⁶ lb-in. per radian
146,350 lb-in.-sec.²

FIG. 6d FLYWHEELS AND SPRING CONSTANTS, SYSTEM REDUCED TO PROPELLER SPEED

TORSIONAL CRITICAL SPEEDS

The method used in calculating the natural frequency of the systems and the relative configuration of the parts of the system at resonance is that of Holzer.[5] The methods of reducing the reciprocating parts and rotating parts including the propeller to equivalent flywheels is well known. These methods also can be found treated in detail.[6]

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[5] Holzer, "Die Berechnung der Drehschwingunger," Springer, 1921.

[6] Lewis, "Torsional Vibration in the Diesel Engine," Proc. Soc. Nav. Arch. and Mar. Eng., 1925. Timoshenko, "Vibration Problems in Engineering," pp. 138–159, D. Van Nostrand Co., New York, 1928. Porter, "The Range and Severity of Torsional Vibration in Diesel Engines," Trans. A.S.M.E., 1928, APM-50-8.

TABLE 1

System and natural frequency	Part	I lb-in.-sec.^{2}	$\frac{I\omega^2}{10^6}$ (lb-in.)	α (radians)	$\frac{I\omega^2}{10^6}\alpha$ (lb-in.)	$\Sigma\frac{I\omega^2}{10^6}\alpha$ (lb-in.)	$\frac{K}{10^6}$ (lb in. per radian)	$\frac{\Sigma I\omega^2\alpha}{K}$ (radian)
Original installation, no exhaust turbine	h.-p. cyl.	17,400	16.1	1.0	16.1	16.1	545	0.03
$\omega_1^2 = 925$	i.-p. cyl.	18,500	17.1	0.97	16.6	32.7	568	0.058
$f_1 = 4.85 \sim$ per sec.	l.-p. cyl.	22,300	20.6	0.912	18.6	51.3	31	1.65
$N_1 = 291 \sim$ per min.	propeller	75,000	69.4	−0.738	−51.1	0.2	...	.....
Engine, exhaust turbine, gears; no flexibility	h.-p. cyl.	17,400	7.83	1.0	7.83	7.83	545	0.0144
$\omega_1^2 = 450$	i.-p. cyl.	18,500	8.33	0.985	8.2	16.03	568	0.0283
$f_1 = 3.38 \sim$ per sec.	l.-p. cyl.	22,300	10.04	0.957	9.62	25.65	350	0.0734
$N_1 = 203 \sim$ per min.	gears	152,500	68.8	0.884	60.7	86.35	...	.....
$\omega_b^2 = 1080$	turbine	3,900,000	1750.0	1.515	2655.0	2741.35	34	80.8
$1 - \frac{\omega_1^2}{\omega_b^2} = 0.583$	propeller	75,000	33.8	−79.9	−2700.0	+41.35	...	.....
Engine, exhaust turbine, gears; no flexibility	h.-p. cyl.	17,400	74.2	1.0	74.2	74.2	545	0.136
$\omega_2^2 = 4260$	i.-p. cyl.	18,500	78.9	0.864	68.1	142.3	568	0.251
$f_2 = 10.4 \sim$ per sec.	l.-p. cyl.	22,300	95.0	0.613	58.3	200.6	350	0.574
$N_2 = 625 \sim$ per min.	gears	152,500	650.0	0.039	25.35	225.95	...	.....
$\omega_b^2 = 1080$	turbine	3,900,000	16,600.0	−0.0132	−219.5	6.55	34	0.193
$1 - \frac{\omega_2^2}{\omega_b^2} = -2.95$	propeller	75,000	320.0	−0.154	−49.3	−43.0	...	.....
Engine, exhaust turbine, gears. $K = 16 \times 10^6$ lb-in. per radian	h.-p. cyl.	17,400	1.755	1.0	1.755	1.78	545	0.00322
$\omega_1^2 = 101$	i.-p. cyl.	18,500	1.87	0.997	1.865	3.62	568	0.00638
$f_1 = 1.6 \sim$ per sec.	l.-p. cyl.	22,300	2.25	0.99	2.23	5.85	350	0.0167
$N_1 = 96 \sim$ per min.	l.-s. gear center	6,150	0.621	0.974	0.61	6.46	...	.....
$\omega_b^2 = 4$	turbine and gears	4,000,000	404.0	−0.04	−16.2	−9.75	34	−0.2865
$1 - \frac{\omega_1^2}{\omega_b^2} = -24.25$	propeller	75,000	7.57	1.26	9.62	−0.13	...	.....
Engine, exhaust turbine, gears. $K = 16 \times 10^6$ lb-in. per radian	h.-p. cyl.	17,400	17.05	1.0	17.05	17.05	545	0.0313
$\omega_2^2 = 980$	i.-p. cyl.	18,500	18.6	0.969	18.0	34.05	568	0.0617
$f_2 = 5 \sim$ per sec.	l.-p. cyl.	22,300	21.9	0.907	19.9	54.95	350	0.157
$N_2 = 300 \sim$ per min.	l.-s. gear center	6,150	6.03	0.75	4.52	59.47	...	.....
$\omega_b^2 = 4$	turbine and gears	4,000,000	3,920.0	−0.0031	−12.17	47.3	34	1.39
$1 - \frac{\omega_2^2}{\omega_b^2} = -241.5$	propeller	75,000	73.5	−0.64	−47.0	0.3	...	.....

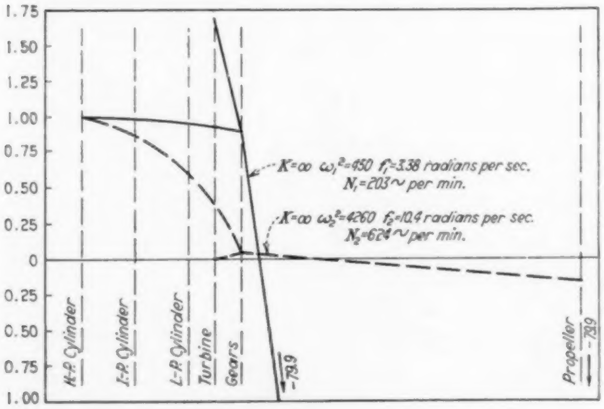

FIG. 7 NORMAL ELASTIC CURVES, NO SPRINGS IN GEARS

Fig. 6*a* shows the equivalent flywheels and spring constants in the original installation without the exhaust turbine, and Fig. 6*b* shows the equivalent flywheels and spring constants of the engine-exhaust turbine-propeller system installed on the *Susan V. Luckenbach*. Fig. 6*c* shows the latter system with all flywheels and springs reduced to equivalent values for rotation at propeller-shaft speed. Fig. 6*d* shows the system with flexibility introduced into the slow-speed gear. All calculations refer to the systems shown in Figs. 6*c* and 6*d*. The first reduction has a gear ratio $n_1 = 7.39$; the second reduction has a gear ratio $n_2 = 5.8$. The overall gear reduction between turbine and gear shaft is $n = 42.8$.

Holzer's numerical method of calculating natural frequencies and normal elastic curves is usually published as applying to systems having all the flywheels on one shaft; while here there is a branched system. The branch contains one flywheel (the turbine rotor), in Fig. 6*c*, and two flywheels, in Fig. 6*d*. However, the system shown in Fig. 6*d* can be considered to have a branch containing one body only when K is small as compared to the torsional stiffness between the turbine and the gearing.

For systems having flywheels in a branch attached to the main-line system by a flexibility, the principle which permits the use of Holzer's method is as follows:

In cases of branched systems the end of the branch spring which is attached to the main system must have the same amplitude as the main system at that point.

For the motion at the point where the branch system is attached to the main system, the amplitude of torsional vibration at resonance is α_4 (using the amplitude of the high-pressure cylinder flywheel as α_1), and the unknown amplitude of the branch flywheel is α_b. The maximum inertia torque of the branch flywheel is $I_b\alpha_b\omega^2$, where ω^2 is the trial value chosen for the particular calculation at hand. The twist in the branch-system spring to balance this inertia torque is $I_b\alpha_b\omega^2/K_b$. Therefore, the following relationship must exist according to the principle enunciated:

$$\alpha_b - \frac{I_b}{K_b}\omega^2\alpha_b = \alpha_4 \quad [7]$$

or

$$\alpha_b = \frac{\alpha_4}{1 - \dfrac{\omega^2}{\omega_b^2}} \quad [8]$$

where

$$\omega_b^2 = \frac{K_b}{I_b} \quad [9]$$

Equation [8] gives α_b its proper magnitude and sign.

Table 1 shows the calculations of the single-noded vibration of the engine and propeller without the exhaust turbine, the single- and two-noded vibrations of the engine-exhaust turbine-propeller system with no flexibility, and the single- and two-noded vibration of the engine-exhaust turbine-propeller system with $K = 16 \times 10^6$. The calculated values of ω^2 are higher than the actual, since damping, neglected flexibilities, and clearances in the system all tend to lower the natural frequencies.

Fig. 7 shows the normal elastic curves for the first and second modes of motion of the system without springs in the gears. Fig. 8 shows the elastic curves for the same system with values of $K = 0$, 16×10^6, 100×10^6, 300×10^6. Only the second mode of motion is shown for all values of K except $K = 16 \times 10^6$, for which both the first and second modes of motion are shown.

Considering the propeller as the chief source of damping in the system, Fig. 7 indicates that the first mode of motion will never appear in an actual installation without springs in the gears. Any violent bumping in the gear teeth must be caused in this case by the second mode of motion, where the propeller damping is very small. Assuming the propeller damping to be proportional to the velocity of the vibration, the energy absorption is proportional to the square of the amplitude. The ratio of energy absorption in the first and second modes is of the order of 300,000 ÷ 1, or even approaching 1,000,000 ÷ 1; for vibration energy inputs of the same order of magnitude. Fig. 8 indicates, on the same basis of reasoning, that the first mode of motion will not be the most troublesome, since the propeller damping in the first mode is about four times as effective as in the second mode of motion when flexibility exists in the gears.

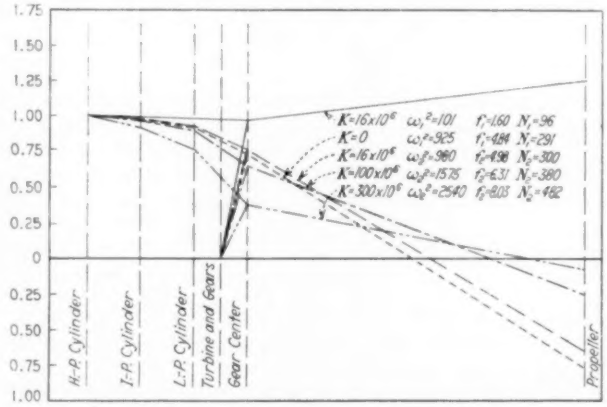

FIG. 8 NORMAL ELASTIC CURVES, SPRINGS IN GEARS

The figures in the fifth row, fourth column, of the last four sections of Table 1 indicate the torque transmitted to the gearing by the vibration per radian amplitude at the high-pressure cylinder. Fig. 9 shows the relationship between the value of the spring constant K and the vibration torque transmitted to the gearing per radian amplitude at the high-pressure cylinder. This curve clearly indicates that the springs should be made as flexible as possible—a conclusion which seems obvious. Fig. 8 leads to the same conclusion, as it shows the propeller amplitude to approach a maximum value as K decreases, indicating an increase in propeller damping as the flexibility increases.

ACTUAL AMPLITUDES AT RESONANCE

The procedure of design based on the conclusions drawn is to make a spring system with nominal load (maximum turbine torque) stresses as high as consistent with known properties of the spring steel used. This gives a first trial spring constant K, which in turn permits the natural frequencies and the normal elastic curve to be calculated. The next step is to calculate approximately the actual amplitudes at the worst critical speed in the running range. This can be done on the assumption that the propeller supplies all the damping. This assumption not only makes the calculation easier, but it is on the safe side, since all the damping arising from hysteresis on the twisted elements and other causes will make the actual amplitude smaller than calculated.

The vibration is caused by the harmonic torques arising from the varying steam pressure in the cylinder and the inertia and weight effects of the reciprocating parts. The harmonic torques caused by weight and inertia can be calculable from the steam-engine dimensions and materials. The harmonic steam torques can be estimated from theoretical indicator cards or, if it is a case of rejuvenating an old ship, from actual engine indicator cards taken before the turbine is applied. The cards will be somewhat different before and after the application of the turbine, but for approximate calculations they will be sufficiently similar.

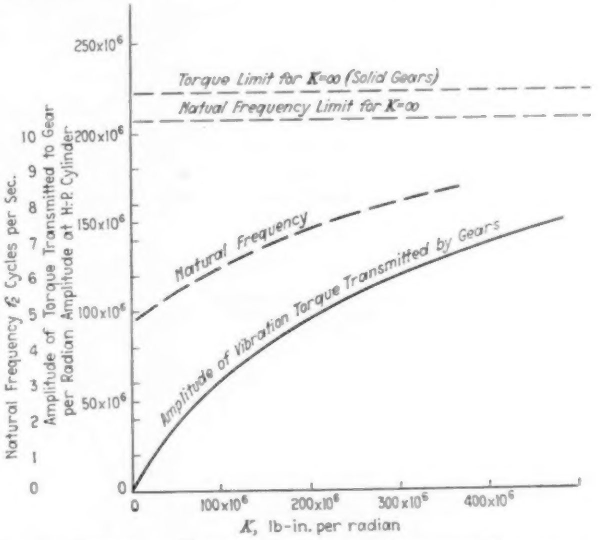

FIG. 9 EFFECT OF VARIOUS SPRINGS ON VIBRATION TORQUE AND NATURAL FREQUENCY

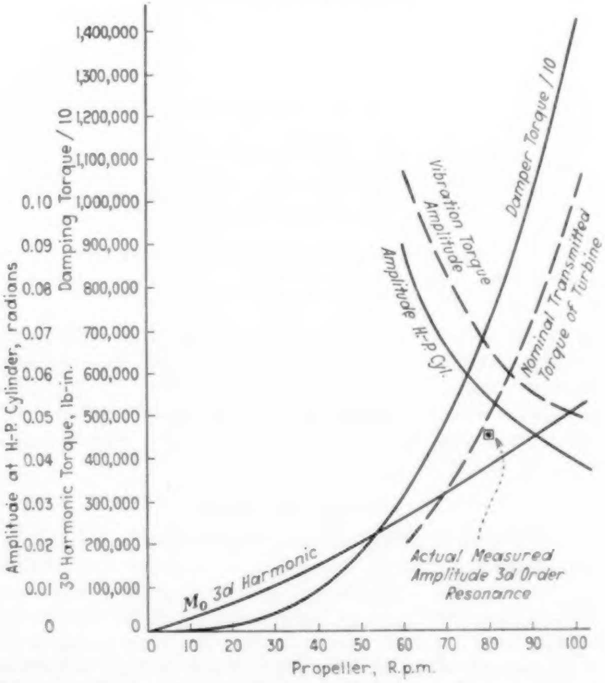

FIG. 10 GRAPHIC CALCULATION OF VIBRATION AMPLITUDE AND TORQUE

For calculation purposes we will use $K = 16 \times 10^6$. From Table 1 the natural frequency of the second mode of motion with $K = 16 \times 10^6$ lb-in. per radian is 5 cycles per sec. or 300 cycles per min. Under this condition the speeds at which resonance may occur are given in Table 2.

TABLE 2

Harmonic	Resonance speed, r.p.m.
1	300
2	150
3	100
4	75
5	60
6	50

In any triple steam engine the first three harmonics will have large amplitudes, while the others will be small. Since the engines of the *Susan V. Luckenbach* operated between 70 and 90 r.p.m., the first two harmonics were not to be feared, even if the calculated critical speed were too high. But the third harmonic might actually occur in the running range. The analysis of several indicator cards taken on the original triple-expansion engine gives the amplitude of the third harmonic as shown in Fig. 10.

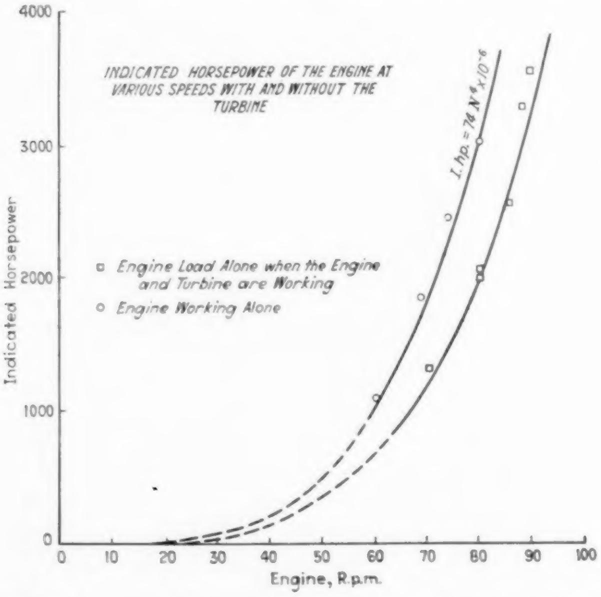

FIG. 11 INDICATED HORSEPOWER VERSUS PROPELLER REVOLUTIONS PER MINUTE
(For steamship *Susan V. Luckenbach*.)

PROPELLER DAMPING

The analysis of several published curves showing the relationship between indicated horsepower and propeller revolutions per minute with three-bladed propellers on battleships and cruisers shows that the i.hp. varies as the r.p.m. raised to powers varying from 3 to 3.75. On a slow-speed freighter, such as the ship equipped, where the lines of the ship are not as fine as those of a cruiser or battleship and a four-bladed propeller is used, the i.hp. probably varies nearly as the fourth power of the r.p.m. Analysis of the curves shown in Fig. 11 shows this assumption to be correct. This ship was known to turn 77 r.p.m. with 2600 i.hp. before the turbines were added.

$$H = KN^4 \quad \text{[10]}$$

$$K = \frac{2600}{(77)^4} = 74 \times 10^{-6} \quad \text{[11]}$$

where

H = indicated horsepower
N = propeller revolutions per minute.

The shaft horsepower H_1 is then

$$H_1 = \eta KN^4 \quad \text{[12]}$$

where η is the mechanical efficiency of the driving engine.

For small vibrations the damping will be a linear function of the vibratory velocity, and the relationship between the damping constant and propeller r.p.m. can be obtained from the H-N curves in Fig. 11.

$$C = \frac{dT}{d\omega} \quad \text{[13]}$$

where

T = shaft torque
ω = angular velocity of propeller
C = damping constant

$$T = \frac{33{,}000 \times 12 \times \eta KN^4}{2\pi N} \quad \text{[14]}$$

$$\omega = \frac{2\pi N}{60} \qquad N = \frac{60}{2\pi}\omega \quad \text{[15]}$$

$$T = \frac{33{,}000 \times 12}{2\pi}\left(\frac{60}{2\pi}\right)^3 \eta K\omega^3 \quad \text{[16]}$$

$$\frac{dT}{d\omega} = 3 \cdot \frac{33{,}000 \times 12}{2\pi}\left(\frac{60}{2\pi}\right)^3 \eta K\omega^2 \quad \text{[17]}$$

$$C = 3 \cdot \frac{33{,}000 \times 12}{2\pi}\left(\frac{60}{2\pi}\right) \eta KN^2 \quad \text{[18]}$$

$$C = 1{,}808{,}000 \ \eta KN^2 \quad \text{[19]}$$

For the combination of the engine and turbine the overall mechanical efficiency is about 81 per cent. Therefore:

$$C = 1.808 \times 10^6 \times 0.81 \times 74 \times 10^{-6} N^2 = 108.5 \ N^2 \ \text{lb-in. per radian per second} \quad \text{[20]}$$

ENERGY METHOD OF CALCULATING AMPLITUDE

The amplitude can now be calculated by equating the energy put into the vibration by the harmonic torque to the energy dissipated by damping.

The energy input per cycle of motion, assuming a motion

$$\alpha = \alpha_1 \sin \omega t \quad \text{[21]}$$

and a harmonic exciting torque

$$M = M_o \sin (\omega t + \varphi) \quad \text{[22]}$$

is

$$W/\sim \ = \pi M_o \alpha_1 \sin \varphi \quad \text{[23]}$$

At resonance $\varphi = 90$ deg., so that the input is:

$$W = \pi \ M_o \alpha_1 \quad \text{[24]}$$

The energy output per cycle of a sinusoidally vibrating body acted upon by a linear damping is

$$W_2 = C \int^{\frac{2\pi}{\omega}} \left(\frac{d\alpha}{dt}\right)^2 dt = \pi C\beta^2\alpha^2{}_1\omega_c \quad \text{[25]}$$

where

α_1 = amplitude at the high-pressure cylinder
α_p = amplitude at the propeller
β = α_p/α_1
ω_c = angular velocity of vibration at resonance.

TABLE 3

	Without springs		With $K = 16 \times 10^6$ lb-in. per radian			
	Calculated	Measured	Calculated at 80	Measured	Calculated at 60	Measured
Order	6	6	3	3	4	4
R.p.m.	104	74	100	80	75	60
Amplitude at high-pressure cylinder radian	...	0.0115	0.053	0.043	0.02	0.008
Amplitude T_v lb-in.	...	2.55×10^6	0.654×10^6	0.523×10^6	0.243×10^6	0.097×10^6
Amplitude T_t lb-in.	...	0.43×10^6	0.512×10^6	0.512×10^6	0.230×10^6	0.230×10^6
Ratio T_v/T_t	...	6	1.26	1.02	1.06	0.42

Equating W_1 and W_2, the amplitude α_1 is:

$$\alpha_1 = \frac{M_o}{\beta^2 C\omega_e} \qquad [26]$$

For resonance with the third harmonic the vibration has a frequency of $3N$ cycles per minute; therefore,

$$\omega_e = 2\pi \frac{3N}{60} = \frac{\pi}{10} N \text{ radians per sec} \qquad [27]$$

Assuming that β remains constant, even when the critical speed varies $\beta = 0.64$ and $\beta^2 = 0.41$, from Table 1, the last figure in the fifth column.

$$\beta^2 C\omega_e = \beta^2 \times 108.5\ N^2 \times \frac{\pi}{10} N = 14\ N^3 \qquad [28]$$

Fig. 10 shows the plotted points of the third harmonic torque taken from analysis of steam pressure, weight, and inertia torques on the triple-expansion engine originally in the ship and the damping torque, both as functions of r.p.m. From Fig. 9 the torque transmitted by the vibration is:

$$T_v = \frac{12.17 \times 10^6\ M_o}{\beta^2 C\omega} \qquad [29]$$

By dividing the corresponding values of the input and output curves, T_v is obtained and plotted. The turbine torque T_t is plotted on Fig. 10, based on the assumption that the turbine puts out one-third of the indicated horsepower at an efficiency of about 70 per cent.

$$T_t = \frac{0.33 \times 0.70 \times 74 \times 10^{-6} N^4 \times 33000 \times 12}{2\pi N} = 1.07\ N^3 \qquad [30]$$

T_v and T_t cross at 83 r.p.m. Actually other damping will make them cross at some lower speed. That is to say, if the third order critical speed occurs at any speed above 80, the first condition for a spring drive will be satisfied.

TEST RESULTS

Torsiograms were taken on the installation in the ship without and with springs in the low-speed gear. The results are given in Table 3.

The calculated T_v/T_t at 100 r.p.m. for the third order vibration was 0.5 and the calculated T_v/T_t at 75 r.p.m. for the fourth order vibration is 0.36.

Without the springs the unit was practically inoperative because of the violent and dangerous impacts in the gear unit. With the springs the unit is safely operative and has made voyages totaling a distance close to 75,000 miles since installation. There is a slight bump at 80 r.p.m., as the measurements indicate there should be; but the ship operates most of the time between 85 and 89 r.p.m., where the gears remain completely in contact in one direction. The improvement made by the spring gear in T_v/T_t is 6 to 1, as shown by measurement. The improvement was even greater than this, since the effects of other harmonics beside the sixth were neglected in computing the vibration torque in the gear without springs.

The unit could be improved in two ways:

1 By decreasing K to some value less than 16×10^6 lb-in. per radian, the slight bump at 80 r.p.m. could be completely removed

2 By adding some definite friction or hydraulic damping to the spring gear.

The cheaper and simpler solution is to depend on propeller damping and to omit artificial damping from the gear wheel, as was done in this installation.

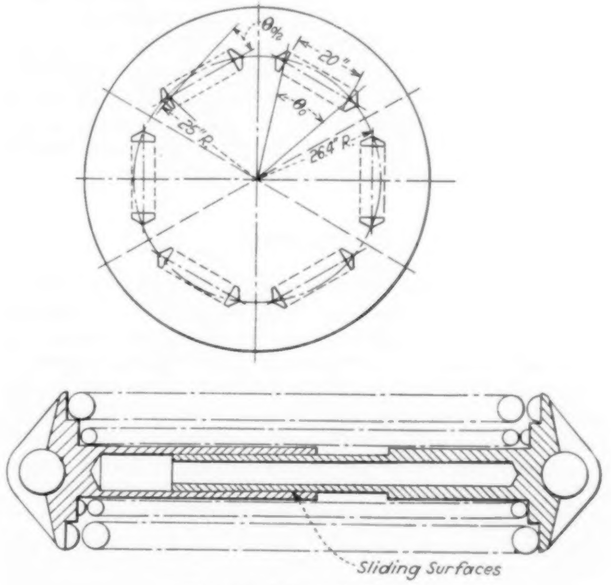

FIG. 12 SPRING DETAILS

	Outer spring coil	Inner spring coil
Diameter of wire, in.	$1^1/_4$	$^{11}/_{16}$
Mean diameter of coil, in.	$5^1/_2$	3
Spring constant, lb. per in.	1790	565
Noml. stress per in. deflection, lb. per sq. in.	12,800	13,250
Actual max. stress per in. defl., lb. per sq. in.	17,500	18,150

SPRING-DESIGN DETAILS

Twelve springs, six on each side of the low-speed gear, were used. The details are shown in Fig. 12. The entire gear was made as a large type C Nuttall coupling, as shown in Fig. 3. The torsional spring constant of the spring system shown in Fig. 12 is:

$$K_1 = nKR^2 \cos^2 \left(\frac{\theta_0 - \theta}{2}\right) \text{ lb-in. per radian} \qquad [31]$$

With $\theta_0 = 44.5$ deg., the average value of K_1 for deflection θ up to 0.1 radian is:

$$K_1 = 0.875\ nKR^2 \qquad [32]$$

where

n = number springs
K = linear spring of each nest of springs
R = radius to center line of spring at the end of the spring.

Since $n = 12$, $K = 2355$ lb-in., and $R = 25$ in., $K_1 = 16 \times 10^6$ lb-in. per radian. At 80 r.p.m. the deflection of the spring gear caused by the turbine torque is 0.035 radian; the amplitude of vibration is also equal to 0.035 radian (at the gear), giving a total maximum deflection of 0.07 radian (1.65 in spring deflection), which corresponds to a maximum stress of about 32,000 lb. per sq. in. in each spring. This stress is the true maximum stress at the outer fiber on the inside of the coil. It is composed of static stress of 16,000 lb. per sq. in. and a variable stress which has an amplitude of 16,000 lb. per sq. in. These stresses are very conservative for the spring steel used.

The spring stops limit the maximum travel to 3 in., with a corresponding maximum stress of 53,500 lb. per sq. in.

CONCLUSION

It has been shown that it is possible to predict the suitability of any chosen gear flexibility if the drawings of the rotating parts for calculating the critical speeds and indicator cards (theoretical or actual) at two or three speeds are available.

The calculation made in designing the springs for the ship installation were in some error because the natural frequencies were calculated too high by some 20 per cent. This can be attributed to lack of experience in calculating such marine installations, and in nowise vitiates the value of the method to those practiced in the art of calculating such things.

Discussion

Hans Bauer.[7] For purposes of discussion this paper may be considered under two headings: first, the application of exhaust turbines in reciprocating-engined steamships, and second, the mathematical theory of oscillations in elastic rotating systems.

Concerning the first classification, the writer is able to offer observations based upon the successful operation of over 200 vessels equipped with exhaust turbines constructed according to the Bauer-Wach system and aggregating approximately 1,000,000 hp. In that system, two single-reduction gears are connected by a Vulcan hydraulic coupling, the latter being interposed between the high-speed gear shaft and the low-speed pinion shaft. The hydraulic coupling serves both to absorb shocks and vibration and as a clutch to disconnect the turbine and high-speed gear from the engine and low-speed gear when maneuvering, or at other times when it is desired to run on the engine alone.

Reversing is best performed when the exhaust turbine is disconnected, since it is practically impossible to synchronize the admission of steam to turbine and reciprocator as required for maneuvering, quite apart from the complicated design that it necessitates. Therefore, if the turbine is kept continually coupled to the engine, the gear teeth and the whole system are repeatedly subjected to reversal of stresses of unknown magnitudes. It may be claimed that this can be overcome by using the turbine alone when maneuvering. To do so, however, restricts the maneuvering power to that available from the turbine, and there are still torque variations due to dragging the engine. The much simpler method is to disconnect the turbine and use the engine for maneuvering.

That the hydraulic coupling provides complete protection of the turbine and gear system against torsional shocks or vibrations originating in the engine and propeller system may be seen from torsiographs which have been published from time to time. This result is considered ample justification for a slippage which, be it noted, constitutes about one-half of 1 per cent of the total power.

To introduce springs into an oscillating system is to invite trouble from vibration. By proper proportioning of one to the other the period may be controlled, but the tendency to oscillate remains and will be felt at some frequency. The authors admit that it is impossible to design a spring coupling to damp out torsional vibrations at all speeds, and they should not be satisfied if they have succeeded in doing this in or near the usual running range of the engine, since, as all marine engineers know, a marine engine may be called upon to run at any speed up to the maximum, according to weather conditions, load, etc., and if torsional vibration occur even for only a short period, it may be detrimental to the gearing. Conditions in an oscillating system like the above linked by a flexible and elastic member will be much more difficult to determine, and almost impossible to control, if the propeller is momentarily lifted out of the water in a seaway.

It is not quite clear what the authors mean by "propeller damping." The degree of propeller damping must necessarily be different under different draft and weather conditions. The propeller cannot prevent resonance as between the turbine system and the engine system. Springs interposed between the two systems obviate the extreme stresses due to short-period oscillations, but are not effective—in fact, they constitute a source of danger when the frequency of the impressed forces approaches the natural frequency of oscillation of the spring-coupled system.

A hydraulic coupling, on the other hand, not being elastic, does not transmit any torsional oscillations.

The authors are to be commended for having gathered and analyzed mathematical researches relating to torsional vibration and the design of spring couplings. To criticize a detail, however, it does not seem advisable to the writer to base designs on stresses as high as 32,000 lb. per sq. in. in spring steel, since the springs may at times be compressed to the maximum travel when, as stated by the authors, the stress would be raised to 52,500 lb. per sq. in. This may occur frequently, as, for example, with a racing propeller in rough weather, or, again, in maneuvering.

R. Eksergian.[8] The authors are to be congratulated on an excellent technical investigation of a complicated problem. An outstanding feature of the presentation is the clearness in picturing the phenomena with skilful approximations.

The normal elastic curves in Fig. 8 show very clearly the advantage of elasticity at the coupling in producing large amplitudes for the propeller damping, which, from Equation [26], shows corresponding reduction in the amplitude of vibration caused by a given disturbing moment.

The writer has been further impressed with the advantage of reducing the elasticities and inertias to a common shaft coordinate. In this way a much clearer picture of the relative inertias and elasticities of the system is obtained. The turbine rotor of only 2120 lb-in.-sec.^{2} polar moment of inertia when reduced to the propeller-shaft coordinate and thereby increased by the square of the products of the gear ratio becomes 3,900,000 lb-in.-sec.^{2} Thus the turbine becomes practically the pivot point of the system. Likewise the turbine spindle, which is fairly flexible in itself, becomes extremely stiff when multiplied by the square of the products of the gear ratio and reduced to the propeller shaft.

The authors' analysis for thus placing the flexibility in the main gear is very interesting. The ratios stated for geometrical spring systems appear obvious except the last one. A little consideration, however, will show that $\alpha_1 = (r_2/r_3)\alpha_2$ is correct if the amplitude at the propeller-shaft junction is to remain the same. We have the two cases shown by Fig. 13.

If the amplitudes α_D at the junction of the propeller shaft remain the same for the two cases, then, with the assumption of the negligible amplitude of the reduced turbine mass due to its enormous value, the authors show that on the basis of a geometrical-similarity comparison, the stresses and torques transmitted are considerably increased when the spring system is placed in the high-speed gear. On the other hand, referring to Fig. 8, the amplitudes at the propeller-shaft junction on the basis of unit amplitude at the engine decrease with increased stiffness of the spring system. But the decreased amplitude at the propeller, however, for the stiffer spring system may result because of the consequent decreased damping at resonance, in possibly larger actual amplitudes for the system under the exerting engine torque. Therefore α_D for the stiffer spring, as in Case 1, may be further augmented as compared with α_D in Case II, which would appear even more to favor the authors' conclusions.

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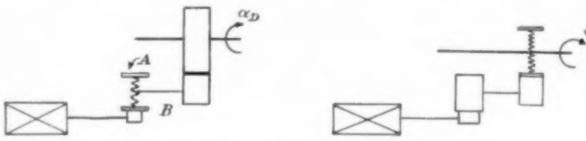

FIG. 13

CASE I

$$\alpha_D = \alpha_c$$
$$\alpha_c r_2 = \alpha_B r_3$$
$$\alpha_B - \alpha_4 = \alpha_1$$
$$\text{If } \alpha_4 = 0,\ \alpha_B = \alpha_1$$
$$\therefore\ \alpha_D = \alpha_1 \frac{r_3}{r_2}$$

CASE II

$$\alpha_D - \alpha_c = \alpha_2$$
$$\alpha_c r_2 = \alpha_B r_3$$
$$\alpha_B = \alpha_4 = 0$$
$$\text{If } \alpha_c = 0,\ \alpha_D = \alpha_2$$
$$\therefore\ \alpha_2 = \alpha_1 \frac{r_3}{r_2}$$

Another point of view in addition to the authors' proof of a condition of inherent high stresses, etc. in the secondary gear is on the basis of the dynamics of the system by a comparison of the elastic constants of the springs for the two cases.

If T is the torque transmitted and ϕ the corresponding relative angular displacement between the spider and gear rim, then the elastic or spring constant is

$$K = \frac{T}{\phi} = \frac{nPR}{\dfrac{8NPD^3}{d^4GR}} = \frac{nR^2d^4G}{8nD^3}$$

Using the authors' geometrical-similarity assumption and noting that the reduced spring constant of the secondary gear is $K_1' = K_1\,(r_2/r_3)^2$, we have the ratio of the elastic constants

$$\frac{K_1'}{K_2} = \frac{n_1R_1^2d_1^4G_1}{8N_1D_1^3}\left(\frac{r_2}{r_3}\right)^2 \Bigg/ \frac{n_2R_2^2d_2^4G_2}{8N_2D_2^3} = \left(\frac{r_1}{r_2}\right)^3\left(\frac{r_2}{r_3}\right)^2$$

The greater the flexibility, the *smaller* the spring constant; therefore, if the ratio C_1'/C_2 is greater than unity, the greatest flexibility is obtained by placing the springs in the primary or low-speed gear. If, on the other hand, the factor $(r_2/r_1)^3 > (r_2/r_3)^2$, the greatest flexibility is obtained in the secondary gear. For this latter condition to be realized, assuming the gear ratio $r_2/r_3 = 5.8$, then $r_1/r_2 < 0.31$, which from constructional limitations alone it is impossible to obtain. With the practical value of $r_1/r_2 = 0.679$, the ratio of the spring constants is $K'_1/K_2 = 10.5$. This shows, when considering the flexibilities of the system, the considerable advantage in placing the springs in the low-speed gear.

The actual torques transmitted for the two cases are: $K_2\,\alpha_{D2}$ with spring in low-speed gear, and $K_1\,(r_2/r_3)\,\alpha_D$ with spring in high-speed gear. Hence the torques transmitted for the two cases are

$$\frac{T_1}{T_2} = \frac{K_1\left(\dfrac{r_2}{r_3}\right)\alpha_{D1}}{K_2\alpha_{D2}} = \frac{\left[K'_1\Big/\left(\dfrac{r_2}{r_3}\right)^2\right]\dfrac{r_2}{r_3}\,\alpha_{D1}}{K_2\alpha_{D2}} = \left(\frac{r_1}{r_2}\right)^3\frac{r_2}{r_3}$$

which agrees with the author's conclusions, provided $\alpha_{D1} = \alpha_{D2}$.

In estimating the modes of oscillation corresponding to the natural frequencies of the system, the authors have shown that the problem is concerned with the second mode of oscillation. The reduced dynamical system shown in Fig. 6 has five degrees of freedom, and by using the Holzer method the two lower frequencies can be accurately estimated by trial solutions.

By further combinations of the inertias and elasticities we can reduce the system to one of two degrees of freedom, which would be in fair agreement with the first mode and approximately accurate for the second mode when compared with the system of five degrees of freedom. Such a resolution, however, may be of interest.

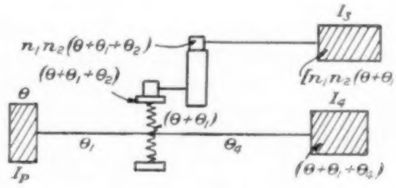
I_3
$n_1n_2(\theta+\theta_1+\theta_2)$
$(\theta+\theta_1+\theta_2)$
$[n_1n_2(\theta+\theta_1+\theta_2)+\phi_3]$
θ
I_4
$(\theta+\theta_1)$
θ_1
θ_4
I_p
$(\theta+\theta_1+\theta_4)$

FIG. 14

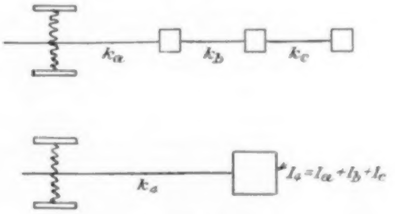

FIG. 15

The approximate two-degree system is shown in Fig. 14. The inertias of the gearing when reduced to the line shaft are small compared with the inertia of the turbine. Therefore they may be neglected or added to the turbine inertia I_3. The inertia of the propeller is I_p, and that of the engine, I_4.

To approximate the elastic constant of the engine shaft to the gearing, if k_a, k_b, and k_c are the elastic constants shown in Fig. 15 between the gearing and engine masses I_a, I_b, and I_c,

$$\frac{1}{k_4} = \frac{\dfrac{I_a}{k_a} + \left(\dfrac{1}{k_a} + \dfrac{1}{k_b}\right)I_b + \left(\dfrac{1}{k_a} + \dfrac{1}{k_b} + \dfrac{1}{k_c}\right)I_c}{I_a + I_b + I_c}$$

Now considering the system shown in Fig. 14, we have for the kinetic and potential energies,

$$T = {}^1/_2\, I_p\theta'^2 + {}^1/_2\, I_3\,(n_1n_2(\theta' + \theta_1' + \theta_2') + \phi_3')^2 + {}^1/_2\, I_4\,(\theta' + \theta_1' + \theta_4')^2$$

$$V = {}^1/_2\, k_1\theta_1^2 + {}^1/_2\, k_2\theta_2^2 + {}^1/_2\, k_3\phi_3^2 + {}^1/_2\, k_4\theta_4^2$$

where θ' and ϕ' are the first derivatives of θ and ϕ with respect to time.

The momentum corresponding to the coordinate θ is a constant, which we may take equal to zero, i.e.,

$$I_p\theta' + I_3\,(n_1n_2\,(\theta' + \theta_1' + \theta'_2) + \phi_3')n_1n_2 + I_4\,(\theta' + \theta'_1 + \theta'_4) = \theta$$

from which we can eliminate the coordinate θ in the expression of the kinetic energy. Or we may differentiate directly, so that,

$$(I_p + (n_1n_2)^2\,I_3 + I_4)\theta'' = -\,n_1n_2I_3(n_1n_2(\theta_1'' + \theta_2'') + \phi_3'') - I_4\,(\theta_1'' + \theta_4'')\ldots\ldots[33]$$

from which we can express the second derivative ϕ'' in terms of θ_1'', θ_2'', θ_4'', and θ_3''.

The equations of motion corresponding to the coordinates θ_2 and θ_3 are,

$$\frac{d}{dt}\left(\frac{\theta T}{\partial \theta'_2}\right) = n_1n_2I_3\,[n_1n_2(\theta'' + \theta_1'' + \theta_2'') + \phi_3''] = -k_2\theta_2 \quad [34]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \phi_3'}\right) = I_3\,[n_1n_2(\theta'' + \theta_1'' + \theta_2'') + \phi_3''] = -k_3\phi_3 \quad [35]$$

which, on combining, give

$$k_2\theta_2 = n_1n_2k_3\phi_3, \qquad \therefore\ \theta_2 = \frac{n_1n_2k_3}{k_2}\phi_3$$

that is, the torque of the elastic gear balances the reduced inertia torque of the turbine shaft. This provides additional elimination of the coordinates.

The two remaining equations are,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \theta_1'}\right) = nn_2I_3\,[n_1n_2(\theta'' + \theta_1'' + \theta_2'') + \phi_3''] + I_4\,(\theta'' + \theta_1'' + \theta_4'') = -k_1\theta_1 \quad [36]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \theta'_4}\right) = I_4\,(\theta'' + \theta_1'' + \theta_4'') = -k_4\theta_4 \quad [37]$$

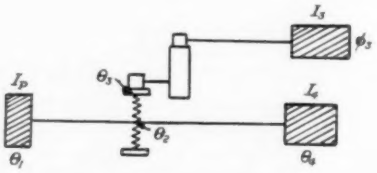

FIG. 16

Combining [34] and [37] with [36], we note that

$$k_1\theta_1 = k_2\theta_2 + k_4\theta_4$$

which we should anticipate immediately from the condition of equilibrium at the coupling. Therefore

$$\theta_1 = \frac{k_2}{k_1}\theta_2 + \frac{k_4}{k_1}\theta_4 = n_1n_2\frac{k_3}{k_1}\phi_3 + \frac{k_4}{k_1}\theta_4$$

On substituting the values of θ_1'' and θ_2'' in terms of ϕ_3'' and θ_4'' in [33], we have

$$\theta'' = -A\phi_3'' - B\theta_4''$$

where

$$A = \frac{n_1n_2\left[\left((n_1n_2)^2\left(\frac{k_3}{k_1} + \frac{k_3}{k_2}\right) + 1\right)I_3 + \frac{k_3}{k_1}I_4\right]}{I_p + (n_1n_2)^2 I_3 + I_4}$$

$$B = \frac{(n_1n_2)^2\frac{k_4}{k_1}I_3 + \left(1 + \frac{k_4}{k_1}\right)I_4}{I_p + (n_1n_2)^2 I_3 + I_4}$$

Substituting the values of θ'', θ_1'', and θ_2'' in terms of ϕ_3'' and θ_4'' in Equations [35] and [37], we obtain

$$\left[1 + (n_1n_2)^2\left(\frac{k_3}{k_2} + \frac{k_3}{k_1}\right) - n_1n_2A\right]I_3\phi_3'' + n_1n_2\left(\frac{k_4}{k_1} - B\right)I_3\theta_4'' + k_3\phi_3 = 0$$

$$\left(n_1n_2\frac{k_3}{k_1} - A\right)I_4\phi_3'' + \left(1 + \frac{k_4}{k_1} - B\right)I_4\theta_4'' + k_4\theta_4 = 0$$

The frequencies are obtained from the solution of the determinate

$$\begin{vmatrix} k_3 - \omega^2\left[1 + (n_1n_2)^2\left(\frac{k_3}{k_2} + \frac{k_3}{k_1}\right) - n_1n_2A\right]I_3, & -\omega^2 n_1n_2\left(\frac{k_4}{k_1} - B\right)I_3, \\ -\omega^2\left(n_1n_2\frac{k_3}{k_1} - A\right)I_4, & k_4 - \omega^2\left(1 + \frac{k_4}{k_1} - B\right)I_4 \end{vmatrix} = 0$$

On substituting for the numerical values given by the authors for the actual installation, a good check is found with the frequency values obtained by them. A closer approximation would be obtained by including the inertias of the gearing with an additional degree of freedom.

The determinate, however, is of use for comparing different spring constants and in approximating the frequencies for similar installations.

In order to calculate the ratio of the amplitudes a different system of coordinates is used as shown in Fig. 16. The equations of motion, with these coordinates are

$$I_4\theta_4'' = k_4(\theta_4 - \theta_2)$$

$$I_3\phi_3'' = -k_3\ (\phi_3 - n_1n_2\theta_3)$$

$$I_p\,\theta_1'' = k_1\ (\theta_2 - \theta_1)$$

and

$$k_2\,(\theta_3 - \theta_2) + k_4\,(\theta_1 - \theta_2) - k_1\,(\theta_2 - \theta_1) = 0$$

$$-k_2(\theta_3 - \theta_2) + n_1n_2k_3\ (\phi_3 - n_1n_2\theta_3) = 0$$

If we assume the amplitude at the engine equal to unity, then

$$\theta_2 = \frac{k_4 - I_4\omega^2}{k_4}, \qquad \theta_1 = \frac{k_1\,(k_4 - I_4\omega^2)}{k_4\,(k_1 - I_p\omega^2)}$$

$$\theta_3 = \left(\frac{k_1 + k_2 + k_4}{k_2}\right)\theta_2 - \frac{k_1}{k_2}\theta_1 - \frac{k_4}{k_2}$$

$$\phi_3 = \left(\frac{k_2 + (n_1n_2)^2 k_3}{n_1n_2k_3}\right)\theta_3 - \frac{k_2}{n_1n_2k_3}\theta_2 = \frac{n_1n_2k_3\theta_3}{k_3 - \omega_2I_3}$$

The last equation could also be used for calculating the frequencies.

HEINRICH HOLZER.[9] To find out the best way of coupling an exhaust turbine of steady-torque characteristic to a reciprocating engine of variable-torque characteristic we may start by considering the fact that vibration in the turbine shafting can arise only from the coupling linkage. Therefore, if the point of the engine shafting at which the turbine branch is attached has no vibratory motion itself, the whole turbine branch will remain without vibrations.

The turbine branch should be attached to the main system at a point where a node of the vibration of the main system occurs. In general the vibration of the propeller system will consist of harmonic components of different periods and phases, and a coupling point without any vibratory motion will not be available. In this case we must of course be content to couple the two systems at one of the nodes of the main harmonic component. At the coupling point the amplitudes of both the main shaft and the turbine branch are not only the same, but also the harmonic coupling torques are equal but of different signs, representing the action—and reaction—torque. By means of these harmonic coupling torques both the main shaft and the turbine branch can be treated as free systems independent of each other. At the turbine branch the harmonic coupling torque is the only exciting torque determining completely the vibratory motion of the turbine-gearing system. Now it is well known that of all vibratory states of a given system, the natural or free vibrations of the system demand the smallest exciting torques down to zero, if there is no damping in the system. The effect of the reciprocating engine on the turbine branch, represented by the harmonic coupling torque, will therefore be eliminated when the turbine-gearing system is tuned to the frequency of the main component vibration of the engine-propeller system. When, for instance, the plant is ordinarily running between 85 and 89 r.p.m. and the main vibration is of third order, we may tune the turbine-gearing system to the free frequency of $3 \times 87 = 261$ cycles per min., if there is not critical speed in or near the working range; we may tune it to $3 \times 80 = 240$ cycles per min. if there is a heavy critical speed of the third order at 80 r.p.m. The flexibility of the springs coupling the low-speed gear to the propeller shaft is therefore of little or no account, though it is never wrong to make the springs as flexible as possible. When running at the speed to which the turbine-gearing system is tuned, there will be no vibration torque whatever in the gearing nor in the springs connecting the low-speed gear to the propeller shaft.

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Of course, the best solution is to tune the turbine branch to the best running conditions and at the same time attach it to the propeller shaft at the point of least amplitude of vibration.

Frank M. Lewis.[10] The paper directs attention to the dynamic problems which arise when reciprocating machinery is used in conjunction with geared transmission. As the authors state, it is necessary that a positive torque be always maintained at the gear faces of such an installation. The irregular torque of the reciprocating machinery can start oscillations in the system which may cause the torque at the gear teeth to pass through negative loops. The pounding thus caused results in rapid deterioration of the gears. It is very difficult to insure this condition of positive torque without the introduction of a considerable amount of elasticity at an appropriate point in the system. In all the installations with which the writer is familiar, elasticity in some form was introduced—hydraulic, electrical, or in the form of mechanical springs. In the installation described springs have been used and the writer believes that the authors have effected a very satisfactory solution of the problem.

Propeller damping coefficients are deduced from the so-called propeller characteristics. The damping coefficient is essentially the slope of the torque–angular-velocity curve. It is this slope which the authors deduce in their Equations [10] to [19]. However, the writer is not quite in agreement with their results. The curve they give is based on an assumption that the ship's speed increases as the revolutions increase, while the slip increases only slightly. The condition of a vibrating propeller differs widely from this, however. Here the ship's speed remains constant while the slip is varying widely as the propeller vibrates. A torque–angular-velocity curve for this condition would have a much steeper slope than the one corresponding to Fig. 11, with a consequent higher damping coefficient. The writer deduced a method of calculating propeller damping coefficients based on these latter assumptions in 1925. While he has no data relative to the propeller dealt with in the paper, the method would probably give a damping coefficient 50 per cent to 75 per cent higher than the one calculated by the authors.

Any check of propeller damping coefficients by direct experiment is a very difficult matter. The best that can be done is to make torsiograph measurements on actual ships where all conditions and exciting forces are known. It is not an easy matter to make tests under such satisfactory conditions that reliable information will result.

In regard to the inertia effect of the water, our knowledge of propellers is also very inadequate, even more so than in regard to the damping effects. It is usual to make a purely arbitrary allowance of some 10 to 25 per cent of the propeller inertia, according to the taste of the calculator. The writer has made many torsional vibration investigations on ships, but has never been able to obtain a sufficiently comprehensive set of observations to permit an accurate calculation of either the inertia or damping effects of the water.

C. Richard Soderberg.[11] On page 8 there is an attempt to determine the damping properties of a ship propeller from the speed-horsepower curve. Now, a speed-horsepower curve such as the one shown in Fig. 11 is based on a steady state of motion, that is, the speed is assumed to be constant for each horsepower reading. This is so important that great care has to be taken in guarding against speed variations during the test. In view of this, it does not appear permissible to draw conclusions of the damping properties from this curve. For the damping phenomenon we are interested in the behavior of the propeller under cyclic variations of the speed; the speed-horsepower curve is entirely unrelated to this phenomenon.

Just what ought to be the correct relation for the damping resistance is difficult to say, but it is probable that it will depend upon the velocity of the oscillation $d\alpha/dt$ as well as on the mean speed of rotation N. The comparatively satisfactory agreement of tested and calculated amplitudes must be regarded as accidental.

O. G. Tietjens.[12] In this paper an attempt has been made to derive the damping constant for small torsional vibrations of a propeller shaft from a curve representing the indicated horsepower as a function of the revolutions per minute of the propeller. Leaving the question open for a moment as to whether the curve shown in Fig. 11 of the paper is the right one or not, it may be of interest to explain in general how the propeller supplies the damping, or in what manner the amount of energy that corresponds to the damping is being dissipated by the propeller.

If we consider the performance of a propeller-blade element having the peripheral speed u and the axial speed w, Fig. 17, we see that the relative speed of the blade element with respect to the water will be $v = \sqrt{(u^2 + w^2)}$. Each blade element has a certain angle of attack α with respect to the relative velocity v. The torsional vibration of the propeller shaft produces a periodic

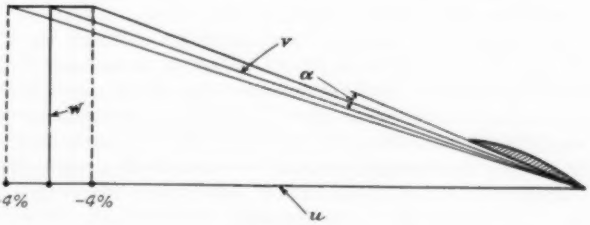

Fig. 17

(The variation of the peripheral speed u of a blade element caused by torsional vibration of the propeller increases and decreases periodically the angle of attack of the blade element.)

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increase and decrease in the peripheral speed u of each blade element. The amplitude of this variation in u is about ±4 per cent in the actual case dealt with in the paper. Therefore, since the axial velocity w remains constant during this periodic variation of the peripheral speed, we see from Fig. 17 that the torsional vibration of a propeller shaft produces a similar variation in the angle of attack of each blade element. These changes of the angle of attack are combined with a small variation of the relative velocity v as shown in Fig. 17. However, the variation of the angle of attack is the much more important factor.

On the other hand, an increase in the angle of attack of a blade element means an increase of the lift of this element, and a decrease in the angle of attack means a decrease of the lift. However, we know[13] that an increase in the lift generates behind the blade element a vortex which rotates in a counterclockwise direction if the blade element moves from the right to the left, and we know that a decrease of the angle of attack causes a vortex which rotates in a clockwise direction. Therefore we see that the torsional vibration of the propeller shaft gives rise to a periodic generation of vortices as shown in Fig. 18. The kinetic energy of these vortices produced per unit time corresponds to the work of damping done by the propeller.[14]

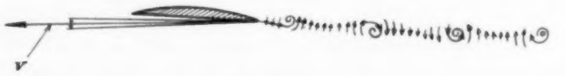

FIG. 18

(The kinetic energy of the vortex path produced per unit time corresponds to the work of damping done by the propeller.)

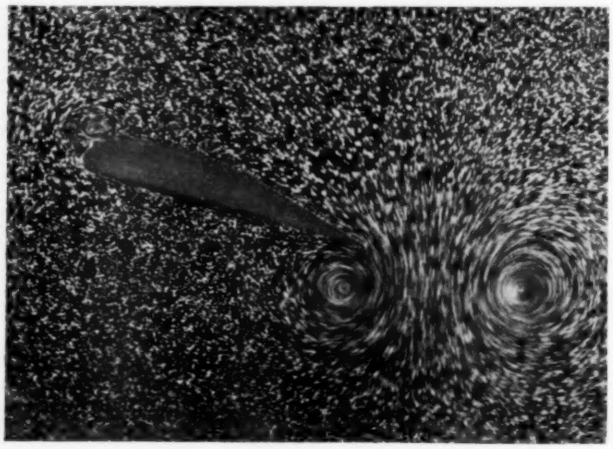

FIG. 19

(Showing two vortices which are generated behind a blade section by a variation in lift of this blade.)

Fig. 19 shows a picture of a wing section with two vortices generated by a variation in lift. The vortex at the right corresponds to an increase of the lift from zero to a certain amount; the vortex close to the trailing edge of the wing section corresponds to a decrease of the lift to zero. Due to the fact, however, that for the propeller-blade elements the variation in lift caused by the torsional vibration is much smaller than that corresponding to Fig. 19, the vortices for the propeller blades will also be much smaller than shown in this figure.

However, it is important to realize that a certain time is required to enable these vortices to reach the size that corresponds to the amount of variation in the angle of attack. If the frequency of the torsional vibration of the shaft, and therefore the frequency of the variation of the angle of attack, becomes too high, say, 100 cycles per second, the time for developing the vortices will be too short and very little or no energy will be dissipated. For very high frequencies, therefore, the propeller will supply no damping at all. In this case the tangent to the curve in Fig. 11 should be a horizontal line and $C = dT/d\omega = 0$. For a smaller frequency of variation the angle of attack C has a certain finite value. Therefore it is seen that the damping constant must be considered an unknown function of the frequency f, or

$$C = \frac{dT}{d\omega} = F(f)$$

However, even if we make the assumption that the frequency of the torsional vibration of the shaft is small enough to guarantee the generation of the vortices to their full size, it is further necessary to take into account a certain relation between the hydrodynamical forces and the frequency. This relation depends on the expression $\pi l/tv$, where l is the width of the blade, v the relative velocity of the blade element, and t the time of one cycle. The smaller the above expression, the less does the lift of a blade element, and therefore also the thrust as well as the torque, depend on the frequency. However, according to a theory developed by W. Birnbaum,[15] even for

$$\frac{\pi l}{tv} = 0.15$$

the amount of the variation in the lift, and therefore also in the torque, is about 20 per cent smaller than that based on the assumption of a steady flow, i.e., for lim $1/t = 0$. Assuming the data given in the paper, i.e., a blade width of about 2 ft., a frequency of 4 cycles per second, and a relative velocity of 4200 ft. per min. or 70 ft. per sec., we obtain approximately

$$\frac{\pi l}{tv} = 0.3$$

which makes it evident that the performance of a propeller with such a variation of its angle of attack will be different from that taking place under steady-flow conditions.

However, even if we have a sufficiently low frequency, or high velocity v, so that the above expression becomes small enough to justify the assumption of steady flow around the blades, the calculation of the damping constant should be based on a curve which gives the indicated horsepower as a function of the propeller revolutions for *constant* ship speed instead of the curve of Fig. 11 of the paper which gives the indicated horsepower as a function of the propeller r.p.m. where the ship speed varies with the r.p.m. As also mentioned by Professor Lewis, this has to be taken into account since the ship speed remains constant while the peripheral speed of each blade element varies due to the torsional vibration of the propeller shaft.

The curve for constant ship speed, however, is steeper and has a larger tangent than the curve of Fig. 11 at points of corresponding values of propeller revolutions. This can easily be seen from Fig. 17 if we realize that with increasing r.p.m. the increase in the angle of attack, and hence the increase in the torque, would be smaller if the speed of the ship, and therefore the axial velocity w increases with increasing propeller revolutions.

If we could make the assumption that the performance of the vibrating blade can be approximated by the performance of the blade in steady flow, we could then obtain the above-mentioned curve of indicated horsepower as a function of propeller revolutions for constant ship speed if the r.p.m. is suddenly increased or decreased and the indicated horsepower is then determined before the ship has had time to change its speed.

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[13] O. G. Tietjens, "Hydro- und Aeromechanik nach Vorlesungen von L. Prandtl," Vol. II, pp. 180–184, Berlin, 1931.

[14] It may be mentioned that these vortices have nothing to do with the so-called Kármán vortex path.

[15] W. Birnbaum, "Das ebene Problem des schlagenden Fluegels," *Zeit. f. Angew. Math. u. Mech.*, 1924, p. 277.

However, since we do not know in what manner the damping depends on the frequency, or more generally, on the term $\pi l/tv$, the derivation of the damping constant from a curve similar to Fig. 11 does not appear to be based on sound physical principles.

These remarks should not be considered as a criticism of this interesting paper as a whole, but as referring only to that part of it which deals with the theoretical derivation of the damping constant.

Authors' Closure

The primary purpose of the Vulcan coupling in the Bauer-Wach exhaust turbine is to permit the disengagement of the low-pressure turbine during maneuvering. Incidentally, it may minimize vibrational effects. All the published studies of this coupling concern themselves with its steady-state operation at constant rotational speed on both sides of the coupling. The Vulcan coupling must have distinct reactions to vibratory motion. There must be a loss of energy associated with oscillatory relative motion between the parts; and there is a torque between the driving and driven members that depends on their relative displacement. This elastic torque would be analogous to the synchronizing torque found in synchronous electrical machinery.

No quantitative values of these damping and elastic torques are known when the coupling is applied. The application may be effective against the vibration or it may not; that depends on the relationship of the vibratory properties of the coupling to the rest of the system.

As a damping device the Vulcan coupling is in its best position on the high-speed shaft. As an elastic device it might as well be torsionally rigid in this position. Without quantitative knowledge of its damping properties, reference must be made to experience. If it is a universally effective protective device against vibration, the gear teeth in all the 200 Bauer-Wach installations will only be worn or wire-edged on the driving side. If it is not, signs of pounding will appear on both sides of the teeth. The authors have been informed that such pounding and wear on both sides of the gear teeth is in evidence in some installations where the Vulcan coupling is used.

The spring gear described in the paper eliminated signs of pounding on both sides of the gear teeth.

It must be remembered that gear-tooth pounding and ultimate breakage are usually fatigue effects. Destructive effects require time and the repeated action of millions of blows. If these blows can be eliminated from the speed range where the ship runs 95 per cent of the time, the number of blows has been reduced to such an extent that other factors become more important in the ultimate life of the equipment. It is also understood that the Bauer-Wach system must be disconnected at lower speeds on account of pounding in the gear teeth.

Propeller damping is a very real thing. It is so effective that the one-noded vibration has practically no amplitude even at resonance. The purpose of a spring coupling is to keep amplitudes of stress harmless even in the event of resonance. Experience shows that it accomplishes this end.

The working spring stresses amount to 16,000 lb. per sq. in. static with an oscillating stress with 16,000 lb. per sq. in. amplitude; these stresses are well within the limit of safety of the material used as one year's continuous service amply proves. The maximum stress of 52,500 lb. per sq. in. occurs at low speeds and is not repeated often. It is also within the limits of safety for the material.

Dr. Eksergian's mathematical treatment reducing the complicated system to one of a few degrees of freedom is very interesting. It can be very useful in calculating the values of the natural frequencies after the numerical data on inertias and spring constants have been obtained.

Mr. Holzer's contention that the whole problem of oscillatory gear stresses could be eliminated by placing the gearing at a node in the elastic system is entirely correct. If the second reduction gear could have been placed at the node of the two-noded mode of motion, the gears would need no elasticity to protect them. Unfortunately insurance regulations and considerations other than torsional vibration determine the masses of the steam engine and propeller and the size of the main propeller shaft. The properties of most ship installations are such as to put the important node somewhere in the propeller-shaft alley far aft of the available engine-room space. These practical reasons are determining factors, especially when the exhaust turbine is installed in a ship originally designed to operate only with a multiple-expansion steam engine.

Dr. Holzer's statement that tuning the turbine and gear system would entirely eliminate any gear-torque reactions at those speeds where the harmonic frequencies were in resonance with the tuned frequency of the branch system, is correct. However, unless this tuned frequency is exactly equal to the 2-noded natural frequency of the system, it does not offer any great advantage since the maximum force in the gear teeth grows very rapidly as the 2-noded frequency recedes in either direction from the tuned frequency of the branch system. Practically, this means that the natural frequency of the branch system would have to be adjustable since it is almost impossible to calculate within limits of 5 to 10 per cent of the actual value of the 2-noded critical speed of one of these installations.

In the case of the *Susan V. Luckenbach* the 2-noded natural frequency is somewhere in the order of 4 cycles per second, which would require a flexibility between the turbine and gearing about 50 times as great as that of the present pinion shaft. Such a flexible coupling would certainly require more space than is used at present between the turbine and the gearing. The spring gear as actually used takes up no more space than an installation which has no flexibility in it. In the engine room of a ship, space is at a great premium.

With adjustable flexibility in the turbine coupling, Dr. Holzer's scheme is not only theoretically correct, but probably would offer actual practical advantages. Without adjustability and with complete dependability on design calculations, we feel that the spring gear used by us in the *Susan V. Luckenbach* probably gives a better practical answer.

The spring gear as used is not only a theoretical cure for a difficult problem, but it works.

Messrs. Lewis, Soderberg, and Tietjens have attacked the authors' method of calculating the propeller damping. The authors cannot defend the method used on physical grounds and can only add their desires to those of the discussors for a good theoretical and experimental method for determining the actual values for propeller damping. They cannot agree with Mr. Soderberg that the method is completely valueless, however.

First of all, the damping constant found by the authors' method agrees in form with those found by Frahm and Lewis:

$$\text{Lewis: } C = \left(1 - S\right)\frac{\dfrac{\partial T}{\partial s}}{T}\,\frac{T}{\omega}$$

$$\text{Frahm: } C = r\,\frac{T}{\omega}$$

$$\text{Ormondroyd and Kuchler: } C = \left(n - 1\right)\frac{T}{\omega}$$

where T = propeller torque (steady rotation)
ω = angular velocity of propeller (steady rotation)
S = slip of propeller
r = Frahm's constant
n = exponent of N (r.p.m.) in expression for propeller torque (steady rotation).

The disagreement between these methods concerns only the value of the constant, which in the authors' method is 3. Lewis, in his paper cited on page 5, gives a method for calculating this constant from propeller-model tests and based on good physical reasoning. Using data published by Porter[16] based on the Lewis method, this constant should be about 3.7 for the propeller in question, although Porter's data refer to three-bladed propellers and the propeller in the authors' case has four blades. Frahm's constant as published by Porter in the same paper was 3.8.

General considerations indicate that the damping constant found by the authors' method will always be too small. This leads to the prediction of larger amplitudes than actually exist, which is on the safe side as far as design prediction is concerned.

The method proposed by Lewis is the best available, although Tietjens indicates that the problem is even more complicated than indicated by Lewis. What is really needed for a large class of ship-machinery vibration problems is an experimental study of propeller damping effects.

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[16] F. P. Porter, "Practical Determination of Torsional Vibration in an Engine Installation," Trans. A.S.M.E., 1929, paper APM-51-52, p. 241.

APM-54-21

Distribution of Pressure Concentrated on a Small Surface

By MICHAEL A. SADOWSKY,[1] MINNEAPOLIS, MINN.

In this paper the author discusses mathematically problems concerning the pressures and deformations that arise when a very rigid die or punch is pressed against a comparatively elastic material, and also in the case where column footings rest on an elastic soil foundation. The author considers (1) the case where the axial section of the die is given and the resulting distribution of pressure along the curved contact surface is required; and (2) where the distribution of pressure over the contact surface is prescribed and the proper curvature of the surface to give such a distribution of pressure is required. Solutions of each of these cases are given for two types of die; namely, when the die is a body of revolution, and when it has a rectangular cross-section the dimension of which perpendicular to the plane of the figure is either very large or very small.

THE present paper deals with the problem of pressures and deformations arising when a very rigid die, or punch, is pressed against a comparatively elastic material. Such a problem arises in the case of various punching machines and also in the case of column footings resting on an elastic soil foundation. In such cases it is reasonable to assume that the die is absolutely rigid in comparison with the body of material with which it comes in contact. This assumption has been made in each of the cases considered.

Two problems of particular interest arise (see Fig. 1):

1 When the axial section of the die is given (curve ABC), and the resulting distribution of pressure along the surface ABC is required.

2 When the distribution of pressure along the curve ABC is prescribed, and the proper shape of the curve to give such a distribution of pressure is required.

The first problem may be easily solved if the curve ABC is a straight line, and the second, if the pressure distribution along the curve is uniform. Only these two special cases of the above problems have been solved in the present paper.

Solutions for each of the above-mentioned problems are given for two types of die, namely:

(a) When the die or punch is a body of revolution (e.g., a cylinder), and,

(b) When it has a rectangular cross-section the dimension of which perpendicular to the plane of the figure is either very large or very small (i.e., a two-dimensional problem).

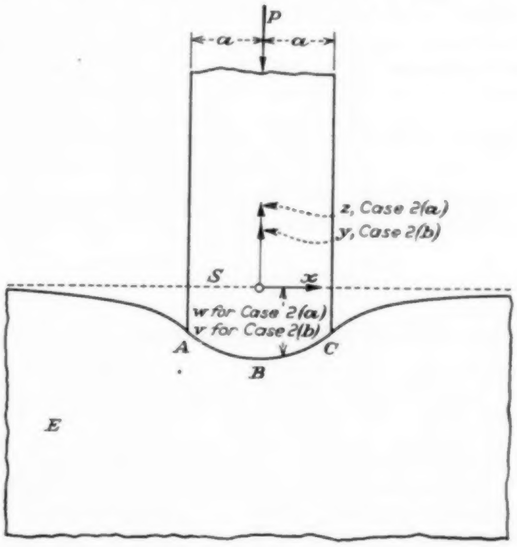

FIG. 1

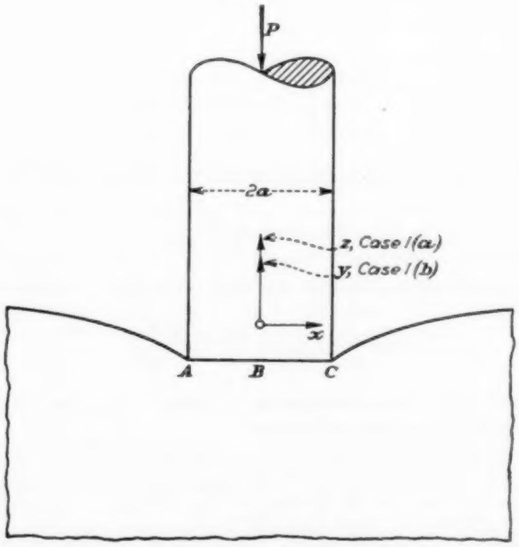

FIG. 2

SUMMARY OF RESULTS

1—DISTRIBUTION OF PRESSURE WHEN ABC IS A STRAIGHT LINE (FIG. 2)

Case 1(a). The die (punch) is a body of revolution, and the

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[1] Dr. Sadowsky was born in Dorpat (Esthonia, formerly Russia) on March 1, 1902. He left Russia before the revolution and lived for six years in Finland. In 1922 he went to Berlin, Germany; in 1926 he was graduated at the Technische Hochschule, Berlin; and in 1927 he took the degree of Dr.-Ing. there. Later he became a Privatdozent for Mathematics of the faculty at the Technische Hochschule. After six years of teaching experience in Berlin, he came to this country, in 1931, going to the University of Minnesota, where he gives general mathematical courses and organizes a mathematical laboratory for teaching practical (applied) mathematics.

Presented at the National Applied Mechanics Meeting, New Haven, Conn., June 23–25, 1932, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

pressing surface *ABC* is plane. Then the pressure (p) depends on the distance (x) from the center B, and is given by the equation:

$$p_x = \frac{P}{2\pi a} \frac{1}{\sqrt{a^2 - x^2}} \quad \ldots\ldots [1]$$

At the boundary ($x = a$) the pressure becomes infinite; at the center ($x = 0$) it is equal to $P/2\pi a^2$.

Case 1(b). Two-dimensional problem.[2] *ABC*, Fig. 2, is a straight line, and the pressure at any point is then given by the equation:

$$p_x = \frac{P}{\pi} \frac{1}{\sqrt{a^2 - x^2}} \quad \ldots\ldots [2]$$

in which P is the load per unit length in the direction perpendicular to the plane of the figure. At the edges $x = \pm a$ the pressure becomes infinite. At the center ($x = 0$) it is equal to $P/\pi a$.

2—SHAPE OF SURFACE *ABC* FOR A UNIFORM DISTRIBUTION OF PRESSURE (p) (FIG. 1)

Case 2(a) (*Fig. 1*). The die (Fig. 1) is a body of revolution. the distribution of pressure is uniform, i.e.,

$$p_x = \frac{P}{\pi a^2} = \text{const.} = p \quad \ldots\ldots [3]$$

The curve *ABC* is then given by the equation:

$$w_{(x,0,0)} = -\frac{4(1-\sigma^2)}{\pi}\frac{ap}{E}\int_0^{\frac{\pi}{2}} \sqrt{1 - \left(\frac{x}{a}\right)^2 \sin^2 \varphi}\, d\varphi \ldots [4]$$

in which σ is Poisson's ratio. The elliptic integral of the second kind occurring here is represented by the notation $E(x/a, \pi/2)$. Its numerical values as a function of x/a may be found in mathematical tables.[3]

The curve *ABC* is like that shown in Fig. 1. Replacing it by a circle through the points A, B, and C, it is found that the radius R of this circle is

$$R = \frac{aE}{4\left(1 - \frac{2}{\pi}\right)(1 - \sigma^2)p} \quad \ldots\ldots [5]$$

The radius of curvature of the curve *ABC* at point B is

$$R_{x=0} = \frac{aE}{(1-\sigma^2)p} \quad \ldots\ldots [6]$$

The difference of ordinates of the curve *ABC* at A and at B is:

$$w_{(a,0,0)} - w_{(0,0,0)} = \frac{2(\pi - 2)(1 - \sigma^2)ap}{\pi E} \quad \ldots\ldots [7]$$

Case 2(b). Two-dimensional problem. Assuming in Fig. 1 a uniform distribution of pressure,

$$p_x = \frac{P}{2a} = \text{const.} = p \quad \ldots\ldots [8]$$

in which P is the load per unit length in the direction perpendicular to the plane of the figure. The necessary shape of the curve *ABC* is then given by the equation

$$v_{(x,0)} - v_{(0,0)} = \frac{paK}{\pi G}\left(\log\left|1 - \frac{x^2}{a^2}\right| + \frac{x}{a}\log\left|\frac{a+x}{a-x}\right|\right) \ldots [9]$$

which can be used for all values of x except $x = \pm a$. For $x = \pm a$,

$$v_{(\pm a,0)} - v_{(0,0)} = \frac{2\log 2paK}{\pi G} \quad \ldots\ldots [10]$$

The constant K which occurs in Equations [9] and [10] has the value

$$K = 1 - \sigma \quad \ldots\ldots [11]$$

in the case of *plane deformation*, i.e., when the dimension of the punch and of the body in the direction perpendicular to the plane of the figure is very large.

In the case of *plane stress distribution*, i.e., when the dimensions of the punch and of the body in the direction perpendicular to the plane of the figure are small, the value of K is given by equation:

$$K = \frac{1}{1 + \sigma} \quad \ldots\ldots [12]$$

Poisson's ratio σ is about 0.3, and the difference between Equations [11] and [12], being a quantity of the second order, is small, and the dimension in the direction perpendicular to the plane of the figure does not substantially affect the shape of the curve *ABC*.

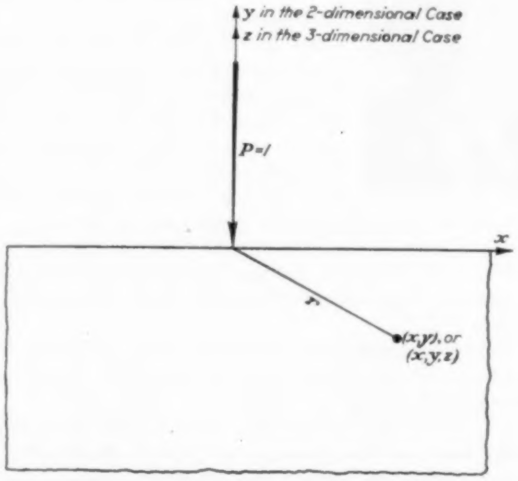

FIG. 3

Fig. 1 shows approximately the shape of the curve *ABC*. The circle passing through points A, B, and C has the radius

$$R = \frac{a\pi G}{4\log 2pK} \quad \ldots\ldots [13]$$

The radius of curvature of *ABC* at B is

$$R_{(x=0)} = \frac{a\pi G}{2pK} \quad \ldots\ldots [14]$$

Replacing the curve *ABC* by a circle the radius of which is equal to the average of the values given by Equations [13] and [14], a distribution of pressure approaching uniform distribution may be obtained.

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[2] The solution is the same for the case of plane deformation and for plane stress.

[3] E. Jahnke and F. Emde, "Funktionentafeln mit Formeln und Kurven," Leipzig-Berlin 1923, pp. 61–64.

Appendix

MATHEMATICAL TREATMENT OF THE PROBLEM AND DERIVATION OF THE RESULTS

THE TWO-DIMENSIONAL PROBLEM

If a single force, the magnitude of which is unity, acts, as shown in Fig. 3, upon an elastic semi-infinite plane, the displacement $v_{(x,y)}$ in the y-direction of a point x,y of this semi-infinite plane is given by the equation

$$v_{(x,y)} = \frac{1}{2\pi G}\left[K \log (x^2 + y^2) - \frac{y^2}{x^2 + y^2}\right] \ldots\ldots [15]$$

where

$$K = \begin{cases} 1 - \sigma, \text{ in the case of plane deformation} \\ \dfrac{1}{1+\sigma}, \text{ in the case of plane stress distribution} \end{cases} \ldots\ldots\ldots [16]$$

Equations [15] and [16] may be found in books on elasticity[4] and their systematic derivation in some special papers.[5]

If a uniform load of the intensity p is distributed on $-a \leqq x \leqq a$, the vertical component of the displacement can be obtained from Equation [15] by integration, or

$$v_{(x,y)} = \frac{p}{2\pi G}\int_{-a}^{a}\left[K \log \left\{ (x-\xi)^2 + y^2 \right\} - \frac{y^2}{(x-\xi)^2 + y^2}\right] d\xi \ldots\ldots [17]$$

where y is negative as seen from Fig. 1.

To determine the displacement $v_{(x,0)}$ on the boundary $y = 0$, we have to determine the limit

$$v_{(x,0)} = \underset{y \longrightarrow 0-0}{\text{limit}}\; v_{(x,y)} = \underset{y \longrightarrow 0-0}{\text{limit}} \text{ of right-hand member of [17]} \ldots [18]$$

Now, as $\log 0 = \infty$, the convergency of the integral [17] is not uniform for the process [18], when $-a \leqq x \leqq a$, and so it is necessary to determine carefully if it is allowable to change the sequence of the ξ-integration and the $y \longrightarrow 0 - 0$ limitation in [18]. This can be done, as is shown by the formula

$$\lim_{c \longrightarrow 0} \int^{b} \log\,(z^2 + c^2)dz = \int_0^b \log\,(z^2)dz \ldots . [19]$$

The proof of Equation [19] is very easy, as it is known that $\int_0^b \log z dz$ is integrable. The result of the limit operation [18] is then

$$v_{(x,0)} = \frac{pK}{\pi G}\int_{-a}^{a} \log \left| x - \xi \right| d\xi \ldots\ldots\ldots [20]$$

[The second term of the right-hand member of [17] when integrated gives $-y \tan^{-1} \dfrac{\xi - x}{y}\Big|_{\xi = -a}^{\xi =}$ the absolute value of which is equal to or less than $\pi|y|$, which $\longrightarrow$ 0 with y.] The integral in Equation [20] can be evaluated, giving

$$v_{(x,0)} - v_{(0,0)} = \frac{paK}{\pi G}\left[\log \left| 1 - \frac{x^2}{a^2} \right| + \frac{x}{a} \log \left| \frac{a+x}{a-x} \right| \right] \ldots [21]$$

This expression can be used for all values of x except $x = \pm a$. For $x = \pm a$ we have:

$$v_{(\pm a,0)} - v_{(0,0)} = \frac{2 \log 2paK}{\pi G} \ldots\ldots\ldots [22]$$

The following table gives numerical values of Equation [21] for various values of the ratio x/a.

x/a	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2	3	10
$v_{(x,0)} - v_{(0,0)} =$	0	0.0100	0.0403	0.0914	0.1646	0.2616	0.3855	0.5409	0.7361	0.9893	1.3863	3.2958	4.16	6.60

By using the table, the shape of the die producing uniform pressure distribution can be drawn. Thus a solution of the problem of case 2(*b*) is obtained. Let us consider now the problem of 1(*b*) and determine the distribution of pressures p when the section *ABC* of the punch is a straight line. In order to solve this problem we have to take Equation [15] and perform an integration over the loaded length *ABC*. The equation from which the distribution of pressures $p(\xi)$ is obtained is:

$$\text{constant} = \int_{-a}^{a} p(\xi) \log \left| x - \xi \right| d\xi \ldots\ldots [23]$$

The right-hand member of this equation represents $v_{(x,0)}$ along *ABC* (see Equation [20]), and has to be constant because *ABC* is a straight line. The absolute value of this constant has no physical meaning, because the solution of an elastic problem always admits an additional displacement of the whole system, and so it cannot be stated that this constant represents the real displacement of the punch. However, this constant is not needed for the determination of the pressures, as will be immediately shown. The pressure $p(x)$ is no longer constant as in Equation [20], but is variable, and so it is seen that [23] is an integral equation, the use of which will determine the unknown function $p(x)$.

To solve this integral equation we can say that [23] represents an equation of the potential theory of two dimensions, the kernel $\log|x - \xi|$ being the logarithmic potential and $p_{(x)}$ the unknown density of charge. As the left-hand member of Equation [23] is a constant, the distance $-a \leqq x \leqq a$ must have a constant potential, and as there is no pressure for $|x| > a$, the density of charge has to be zero for $|x| > a$. The problem can then be solved at once by conformal mapping. Let us take a circle of unit radius in an auxiliary plane of the complex variable ψ, and make it a line of a constant potential. Then we know that the charge distribution along the boundary of this circle is uniform. If we use the conformal mapping, given by

$$\zeta = \frac{a}{2}\left(\psi + \frac{1}{\psi}\right)$$

where the ζ-plane corresponds to the xy-plane of our elastic problem, the circle will be transformed to the loaded distance $-a \leqq x \leqq a$, and the distribution of the charge $p_{(x)}$ will be given by the absolute value of the derivative $d\psi/d\zeta$ determined at the corresponding point, multiplied by the density of the distribution in the auxiliary plane ψ. As this last density is constant, $p_{(x)}$ will be directly proportional to

$$\left|\frac{d\psi}{d\zeta}\right|$$

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[4] E.g., S. P. Timoshenko, "Theory of Elasticity."

[5] See author's papers in *Sitzungsberichte der Berliner Mahtematischen Gesellschaft*, vol. 28, pp. 86–99; and in *Zeitschrift für Angewandte Mathematik und Mechanik*, Berlin, 1927, vol. 7.

This quantity can be evaluated by a simple differentiation, which will be omitted here; the result is

$$p_{(x)} \alpha \frac{1}{\sqrt{a^2 - x^2}}$$

In order to determine the factor of proportionality, use will be made of the relation

$$P = \int_{-a}^{a} p_{(x)} dx$$

which is obvious. Then the final result is:

$$p_{(x)} = \frac{P}{\pi\sqrt{a^2 - x^2}} \dots\dots\dots\dots\dots\dots [24]$$

Equation [24] solves the proposed problem 1(*b*). An extension of the method described here, applicable in the case of other shapes of punches, has been given in a paper by the author.[5]

The Three-Dimensional Problem

Let us consider now the three-dimensional problem. The method of the calculation will be essentially the same as in the two-dimensional case, the only difference being that the function theory and conformal mapping are not applicable for the solution of the three-dimensional potential problems. If a single vertical force, the magnitude of which is unity, acts as shown in Fig. 3 upon an elastic semi-infinite body $z \leqq 0$, the vertical displacement $w_{(x,y,z)}$ of a point (x,y,z) in the elastic body is given by the known formula of Somigliana, namely:

$$w = -\frac{1}{4\pi G}\left[\frac{z^2}{r^3} + 2(1 - \sigma)\frac{1}{r}\right] \dots\dots\dots [25]$$

where

$$r^2 = x^2 + y^2 + z^2$$

If there is a uniformly distributed load, the intensity of which is p, the displacement $w_{(x,y,0)}$ of a point $(x,y,0)$ of the surface $(z = 0)$ can be obtained from Equation [25] by integration and by performing the limiting process $(z \longrightarrow 0 - 0)$. The result is

$$w_{(x,y,0)} = -\frac{p(1 - \sigma)}{2\pi G} \int\int \frac{dF}{r} \dots\dots\dots [26]$$

where

$$r^2 = (x - \xi)^2 + (y - \eta)^2 \text{ and } w_{(x,y,0)} = \lim_{z \longrightarrow 0 - 0} w_{(x,y,z)}$$

The region of the integration is the loaded region, and *dF* denotes its differential.

Referring to the limit process $z \longrightarrow 0 - 0$, see the corresponding portion of the two-dimensional case, Equations [18] and [19].

If the region of integration is a circle of radius a, the integration may be performed by using polar coordinates and the result will be Equation [4].

If the distribution of pressure is non-uniform, but corresponding to a plane circular pressing surface of the punch (*ABC* a straight line), the pressure $p_{(x)}$ for $(y = 0)$ has to be determined from the integral equation

$$\text{constant} = \int\int \frac{p dF}{r} \dots\dots\dots\dots [27]$$

which corresponds to Equation [23] of the two-dimensional case. If we interpret [27] as an equation of the potential theory, $p(x)$ is the charge-distribution function when the circular disk, the radius of which is a, has a constant potential. This problem is solved by the potential theory,[6] and Equation [1] in its solution.

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[6] See, for instance, O. D. Kellogg, "Foundations of Potential Theory," Berlin, Verlag von Julius Springer, 1929, p. 191, Exercise 2.

APM-54-22

The Use of Models in Aerodynamics and Hydrodynamics

By O. G. TIETJENS,[1] EAST PITTSBURGH, PA.

In the present paper the author endeavors to present in a simple manner the various laws of similarity and to indicate the ways in which they may be applied to model-testing practice. After explaining the principles of mechanical similarity, he describes different methods of deriving its laws, following which he discusses the functions of Reynolds', Froude's, and Mach's numbers. The closing section of the paper deals with special applications of the laws set forth to fluid resistance, propellers and fans, cavitation in ship propellers, hydraulic turbines and pumps, flow through pipes and channels, and to fluid problems related to acoustic velocity.

MODEL testing is probably of greater importance in those fields of engineering which are based on aerodynamics and hydrodynamics than in any of the others. This is mainly due to the fact that the applications of the general differential equation in hydrodynamics still offer insurmountable difficulties even in comparatively simple cases.

In considering, for instance, the design of an electrical machine, say, of a turbo-generator, it is known that the theory of electrodynamics can be readily applied for obtaining the electrical data necessary for the design. However, it is out of the question to apply the theory of hydrodynamics in a similar way for calculating the air flow through the machine for purposes of ventilation. The electrodynamic field, the copper losses, the iron losses, and many other details of such a machine can be determined theoretically with a fair degree of exactness, whereas the determination of the pressure drop of the cooling air through the machine and any details of the air flow through the various ducts must be based entirely on experiments.

In carrying out experiments concerning aerodynamical or hydrodynamical problems it is very often of considerable advantage to make them with small-scale models. This is true not only from an economical standpoint, but also because small-scale models are usually simplified to such an extent that the essential features of the problem are brought out more clearly. In addition, it is generally much easier to make systematic changes in models than in full-sized structures, not to mention the considerable saving in time. Therefore, particularly in research work where systematic investigations are carried on, tests with small-scale models will be found most advantageous.

On the contrary, however, in some cases, it is better to use models of a larger scale than the prototype. If, for instance, the flow of steam between small turbine blades is to be investigated, it will be more suitable to use models of turbine blades which are five to ten times larger in size. Or, if the performance of the wing of an insect is to be studied, it can be done advantageously by testing a wing one hundred times larger, and of geometrically similar shape, in thick oil instead of in air.

The purpose of testing models is, generally speaking, to apply the results obtained therefrom to full-scale practice. These results may be of a more or less general character, giving information regarding the kind of flow, the shape of streamlines, etc., or they may consist of numerical data which provide the desired information for the full-scale structure.

In order to insure that the results obtained by model tests can be applied to full-scale practice, certain so-called laws of mechanical similarity must be observed. Conversely, these laws will furnish the rules according to which the desired information for the full-sized structure can be obtained from the model test data.

It is the aim of this paper to present in a simple manner these various laws of mechanical similarity, and to indicate the ways in which they may be applied to model-testing practice. Details of practice in aerodynamical and hydrodynamical experimental work have been purposely avoided, although particulars regarding the more important experimental equipment for model testing such as wind tunnels and water tanks would be of interest, as would also be a description of one of the larger laboratories for aerodynamic and hydrodynamic research work.

I—PRINCIPLES OF MECHANICAL SIMILARITY

Considering, for instance, the flow of a gas or of water around two bodies, the question arises when and under what conditions the flow around one body will be mechanically similar to the flow around the other. Similarity in the mechanical sense implies not only geometrical similarity but also similarity with respect to the forces in question, or dynamical similarity. The flow around two bodies can be similar only if the shapes of the two bodies are geometrically similar. This condition is necessary but by no means sufficient.

Considering a particular case, e.g., the flow around two cylinders of different sizes—which involves the necessary condition of geometrical similarity—it will be found, in general, that the streamlines around these two cylinders are not similar, which means that there is no similarity in the kinematical and dynamical sense.

This is shown in Figs. 1 and 2, which are photographs showing streamlines of water flowing around a thin wire. It is quite obvious from the appearance of the streamlines that there is no mechanical similarity in the two cases, although the shapes of the bodies are similar. The different streamline formations shown are due to the water impinging on the cylinders at different velocities. In Fig. 1 the velocity is extremely low (about 6 in. per min for a wire of about $^1/_{32}$ in. diameter), while in Fig. 2

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[1] Westinghouse Elec. & Mfg. Co. Mem. A.S.M.E. Dr. Tietjens was educated in Germany at the University of Göttingen, where he studied mechanics, hydrodynamics, and aerodynamics under Dr. L. Prandtl. In 1922 he received the degree of Ph.D., after which he was for five years an assistant to Dr. Prandtl. Since January, 1929, he has been connected with the Westinghouse Company, where he is in charge of hydrodynamics and aerodynamics.

Contributed by the Applied Mechanics Division and presented at the Semi-Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, Bigwin Inn, Lake of Bays, Ontario, Canada, June 27–July 1, 1932.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

it is considerably higher (about 1000 ft per min for the same wire).

Similar configurations of streamlines would indicate that the direction of the velocity is the same at corresponding points with respect to the bodies in consideration.

However, the velocity direction at any point of the flow is entirely determined by the *ratio of the forces* acting on a fluid particle at this point. One can say, therefore, that the conditions of mechanical similarity are fulfilled if the ratio of the forces acting on a fluid particle is the same at points similarly situated with respect to the bodies.

For a body moving in air or in water, provided it is completely immersed, the inertia and the viscosity forces are the only ones which have to be taken into account. Therefore mechanical similarity can then be expected if at points similarly located with respect to the bodies the ratios of the inertia forces to the viscosity forces are the same.

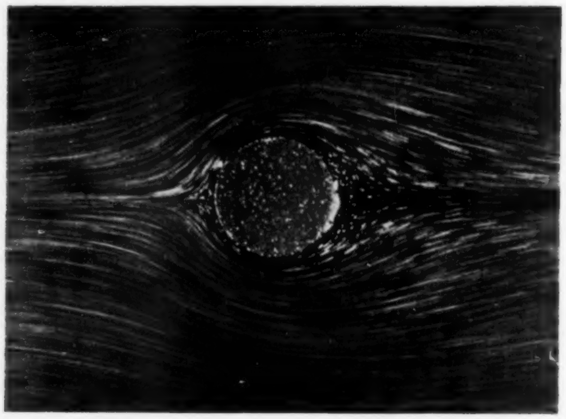

Fig. 1 Streamlines Around a Cylinder of Circular Cross-Section; $R = dV/\nu = 2$

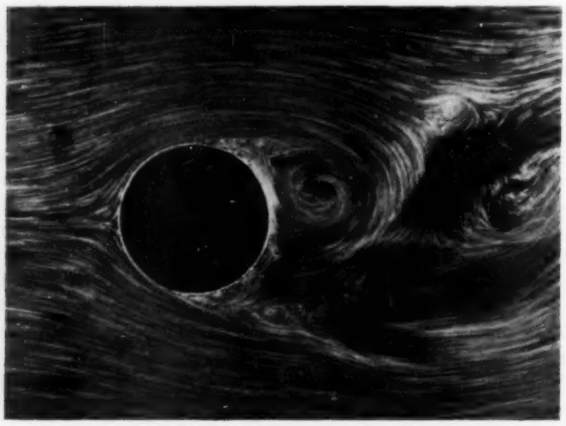

Fig. 2 Streamlines Around a Cylinder of Circular Cross-Section; $R = dV/\nu = 4000$

In considering water waves produced by a ship or by a seaplane hull, those forces which have to be taken into account are the inertia forces and the force of gravity. A model of a ship will therefore produce the same shape of waves as the full-sized ship if the ratio of inertia force to force of gravity is the same at similarly located points with respect to the model and the full-sized ship.

The various laws of mechanical similarity are merely concise expressions of the above-stated fact that mechanical similarity always exists if the ratio of those forces which govern the flow is the same at similarly located points.

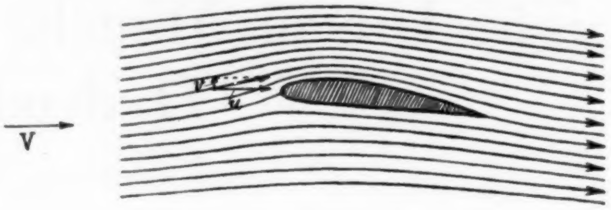

Fig. 3 Flow Around an Airfoil Section

II—DIFFERENT METHODS OF DERIVING THE VARIOUS LAWS OF MECHANICAL SIMILARITY

(a) Ratio of Forces

The manner in which the different forces acting on a single fluid particle will vary if the geometrical dimensions, the unit of time, and the physical constants of the fluid change, may now be investigated.

Consider, as an example, the flow around an airplane wing. The question arises: In what manner do the forces acting on a single air particle vary if the size of the wing changes but in other respects it remains geometrically similar to the original wing? And in what manner do the forces acting on the air particles vary if the speed of the wing changes? Finally, how do these forces vary by a change of the physical constants of the air such as specific weight and viscosity?

The relation between a change in dimensions, a change in the unit of time, and a change in the physical constants on the one hand and the variation of the forces acting on the air particles due to this change on the other, may be derived for the three most important forces, namely:

1 Inertia forces
2 Viscosity forces
3 Gravity forces.

1 The inertia force acting on a single fluid particle is equal to the product of the mass of the particle times its acceleration. If u, v, w are the components of the velocity at any point in the x, y, z directions, respectively, and if ρ denotes the mass density or the mass per unit volume, the x-component of the inertia force per unit volume for a steady flow is (Ref. 17, vol. I)

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) \ldots\ldots\ldots\ldots\ldots [1]$$

From this expression it is seen that the inertia force is proportional to the mass density of the fluid.

Taking into consideration that the acceleration of any fluid particle must be independent of the arbitrary choice of the length and velocity units applied, new units for length and velocity can be introduced in [1] without changing this expression.

Considering again as an example the flow around an airfoil section as shown in Fig. 3, the velocity of the air wing itself, denoted by V, may be introduced as the unit of velocity, which leads to:

$$u = Vu'$$
$$v = Vv'$$
$$w = Vw'$$

where u', v', w' are dimensionless figures.

By taking the width L of the plane for the unit of length, the following equations are obtained:

$$x = Lx'$$
$$y = Ly'$$
$$z = Lz'$$

where x', y', z' are also dimensionless figures.

By introducing u', v', w', and x', y', z' in [1], this expression becomes

$$\rho \frac{V^2}{L}\left(u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} + w'\frac{\partial u'}{\partial z'}\right)\ldots\ldots\ldots\ldots[2]$$

Assuming two similar airfoil sections of different size as shown in Fig. 4, and assuming that the flows around both air wings are similar, which means that the streamlines are geometrically similar, then it is seen from expression [2] that at corresponding points P_1 and P_2 the inertia forces are proportional to $\rho_1 V_1^2/L_1$ and $\rho_2 V_2^2/L_2$, respectively, or generally proportional to $\rho V^2/L$.

2 The force due to the internal friction or viscosity of the fluid[2] is, per unit volume, equal to (Ref. 17, vol. II)

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)\ldots\ldots\ldots\ldots\ldots\ldots[3]$$

if only one component of this force, for instance, the x-component, is considered.

By substituting the above-mentioned dimensionless variables u', x', y', and z' in [3] it becomes:

$$\mu \frac{V}{L^2}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2}\right)\ldots\ldots\ldots\ldots\ldots[4]$$

from which it is seen that the viscosity forces at corresponding points, for instance, P_1 and P_2 in Fig. 4, are proportional to $\mu_1 V_1/L_1^2$ and $\mu_2 V_2/L_2^2$, respectively, or generally proportional to $\mu V/L^2$.

It has been stated before and can be shown by the general differential equation of hydrodynamics that within any liquid, for instance, water, and in any gas, with not too high velocities (lower than about one-fifth of the acoustic velocity), the inertia forces and the viscosity forces are the only ones which have to be taken into account, or which entirely determine the shape of the streamlines. The gravity force drops out on account of the hydrostatic or aerostatic lift. The gravity force, however, has to be taken into consideration in all those cases of fluid flow where the surface of the fluid plays an essential rôle, as, for instance, waves produced by a ship or the flow over a weir. If the water level, however, is not changed or disturbed by the flow as is the case in the flow around a submarine which is sufficiently far enough below the water surface, or in the flow through water turbines and pumps, the inertia and viscosity forces determine entirely the flow, or, in other words, the mechanical similarity. In this case, therefore, it can be said that for mechanically similar flows around geometrically similar bodies the ratio of inertia force to viscosity force will be the same, or, according to [2] and [4],

$$\frac{\text{inertia force}}{\text{viscosity force}} = \frac{\rho_1 \dfrac{V_1^2}{L_1}}{\mu_1 \dfrac{V_1}{L_1^2}} = \frac{\rho_2 \dfrac{V_2^2}{L_2}}{\mu_2 \dfrac{V_2}{L_2^2}}$$

or generally

$$= \frac{\rho V L}{\mu}$$

The statement, therefore, that the expression $\rho VL/\mu$ is the same for the small-scale model as for the full-sized structure indicates that at similarly located points on the model and the prototype the ratio of inertia force to viscosity force is the same, or that the streamlines are similar, and that therefore mechanical similarity is assured.

The expression $\rho VL/\mu$, which is dimensionless because it is the ratio of two forces, is called the Reynolds number. (Ref. 4.) By introducing the so-called kinematic viscosity $\nu = \mu/\rho$, we can also write:

$$\text{Reynolds' number} = \frac{VL}{\nu}\ldots\ldots\ldots\ldots\ldots[5]$$

3 The gravity force acting on a particle having the mass m is equal to mg, and therefore the gravity force per unit volume is ρg.

Considering as an example water waves produced by a ship, the viscosity force can be neglected because it is small compared with the two other forces which govern the flow lines, namely, gravity force and inertia force. It now remains to state that for the condition of mechanical similarity the ratios of these two forces at similarly located points with respect to the model and the full-sized ship must be the same. This leads to the expression

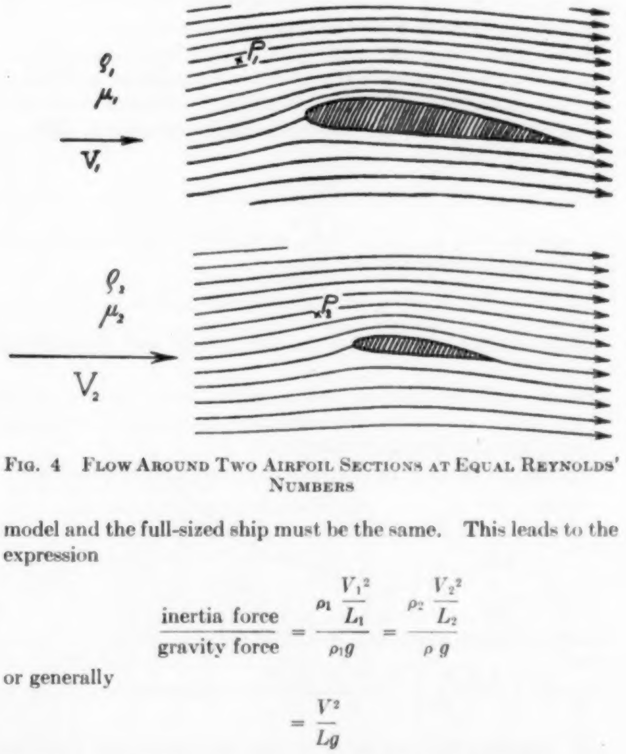

Fig. 4 Flow Around Two Airfoil Sections at Equal Reynolds' Numbers

$$\frac{\text{inertia force}}{\text{gravity force}} = \frac{\rho_1 \dfrac{V_1^2}{L_1}}{\rho_1 g} = \frac{\rho_2 \dfrac{V_2^2}{L_2}}{\rho\, g}$$

or generally

$$= \frac{V^2}{Lg}$$

where V is the velocity and L the length of the model or the ship. In other words, the shapes of the water waves produced by the model ship and the full-sized ship will be similar if the value of V^2/Lg is the same for both.

This dimensionless product, or usually its square root, is called Froude's number (Ref. 3), or

$$\text{Froude's number} = \frac{V}{\sqrt{Lg}}\ldots\ldots\ldots\ldots\ldots[6]$$

In like manner, the other laws of mechanical similarity or model rules can be obtained by establishing the ratio of those forces which are of essential importance for the particular kind of flow in question.

Another method of deriving the model rules of similarity consists of introducing dimensionless variables into the general differential equation of hydrodynamics in a manner analogous to that employed above for the expression of inertia forces and viscosity forces. Since similarity of the flow means identity of the solutions of the differential equation in dimensionless variables, the model rules can be obtained directly in a simple way. (Ref. 17, vol. II.)

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[2] If the velocities in consideration are low compared with the acoustic velocity, say, about one-fifth of the acoustic velocity or less.

(*b*) Derivation of the Model Rules for Similarity by Considering the Dimensions of the Physical Quantities in Question

If neither the differential equation for the flow in consideration nor the expressions representing the forces are known, but if it is understood on what physical quantities, such as velocity, density, viscosity, etc., the movement of the fluid depends, another method for deriving the model rules can be applied. This is by making use of the dimensions of the physical quantities. (Ref. 11.)

It must be possible to express every physical law in a form free from the special applied system of physical units—in other words, in a form where only pure, i.e., dimensionless, figures appear. This raises the question: In what manner is it possible to combine the physical quantities essential for the flow such that this combination will be a dimensionless product?

In the case of air flow[3] or of water without free surfaces where gravity does not have to be considered, the streamline conception depends on the following four quantities: the linear dimension L, the velocity V, the mass density ρ, and the viscosity μ. Therefore in the expression

$$L^\alpha \times V^\beta \times \rho^\gamma \times \mu^\delta$$

the exponents α, β, γ, δ have to be so determined that this expression will be dimensionless; or

$$[L^\alpha \times V^\beta \times \rho^\gamma \times \mu^\delta] = l^0 \times t^0 \times f^0 = 1$$

where l is the unit of length, t the unit of time, and f the unit of force.

Since a dimensionless product remains dimensionless if raised to any power, one of the exponents, for instance, α, is arbitrary and can be assumed equal to unity.

Taking into account the dimensions of the physical quantities

$$[L] = l \qquad [\rho] = \frac{ft^2}{l^4}$$

$$[V] = \frac{l}{t} \qquad [\mu] = \frac{ft}{l^2}$$

the preceding equation may be written as

$$\frac{l \times l^\beta \times f^\gamma\ t^{2\gamma} \times f^\delta\ t^\delta}{t^\beta \times l^{4\gamma} \times l^{2\delta}} = l^0 \times t^0 \times f^0.$$

By equating the exponents of like dimensions the following equations are obtained:

$$\begin{aligned} 1 + \beta - 4\gamma - 2\delta &= 0 \\ -\beta + 2\gamma + \delta &= 0 \\ \beta + \delta &= 0 \end{aligned}$$

from which it follows that

$$\beta = 1, \gamma = 1, \delta = -1$$

or

$$\left[\frac{\rho VL}{\mu}\right] = 1$$

i.e., the Reynolds number represents the only possible dimensionless combination of the physical quantities L, V, ρ, and μ.

In a similar manner, Froude's number could be derived. In this case, however, the essential physical quantities are only the linear dimension L, the velocity V, and gravity g, and it is realized at once that V^2/Lg or $V/\sqrt{Lg}$ is the only possible combination of the physical quantities in question which is a pure number.

If a body moves through still air with a velocity close to or even higher than the acoustic velocity, or if a gas flows through a nozzle with a velocity exceeding the acoustic velocity, it is necessary to take the compressibility of the gas into account. This must be done because of the fact that such high velocities are combined with very large pressure differences, and these latter in turn alter the density, which was hitherto assumed constant throughout the gas. The rate of change in density due to a pressure difference is expressed by the modulus of elasticity.

For a flow of gas where the velocity is close to or exceeds the acoustic velocity but where the viscosity forces can be neglected, the physical quantities that have to be considered as essential[4] are the velocity V, the mass density ρ, and the modulus of elasticity E.

From the equation

$$[V \times \rho^\beta \times E^\gamma] = l^0 \times t^0 \times f^0$$

the relation

$$V\sqrt{\rho/E} \quad \text{or} \quad V/\sqrt{E/\rho}$$

can be obtained. The quantity $\sqrt{E/\rho}$, however, represents the acoustic velocity a.

Two flows of gas close to or exceeding the acoustic velocity are therefore mechanically similar if in both cases the ratio V/a is the same and provided the viscosity forces can be neglected.

III—VARIOUS LAWS OF SIMILARITY

As mentioned earlier, the application of a special law of mechanical similarity depends in a particular case entirely on the forces which determine the flow.

In Fig. 5 an endeavor has been made to present in a simple way the more important laws of mechanical similarity and their relations to the forces in question. In addition to the laws mentioned there are still further ones which involve other forces, e.g., capillary forces and forces encountered in thermodynamics.

However, it must be borne in mind that the conditions for the flow of a gas or of a liquid are not always as simple as has been shown. Quite often the flow depends not only on one of the laws of similarity mentioned, but on two, or even three.

A striking example of this is the fluid resistance of a ship. This resistance consists mainly of frictional resistance due to viscosity forces and of wave resistance due to the gravity force. Hence the ship resistance will depend on the Reynolds number and on Froude's number at the same time. While the Reynolds number involves the necessity of a higher velocity for the model, this velocity should be reduced according to Froude's number. It is therefore impossible to determine the frictional resistance and the wave resistance by testing small-scale ship models.

Theoretically, it is possible to combine both laws of similarity by using two different liquids for the model and ship. In practice, however, this is of no importance since there are no liquids available which have sufficiently different kinematic viscosities.

Similar difficulties arise when the performance of an airplane propeller is to be determined by models in cases where the tip speed of the full-sized propeller approaches or reaches the acoustic velocity. Here the flow around the blades, and therefore the performance of the propeller, depends on the Reynolds number and at the same time on Mach's number. In this case it is rather more important to have the same Mach number for the model and for the full-sized propeller than the same Reynolds number.

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[3] Provided the velocities are low compared with the acoustic velocity.

[4] To be exact, it may be mentioned that for model tests c_p/c_v must be the same for the model as for the full-sized structure.

In regard to the last two examples of fluid flow, some doubt may arise as to the reliability of certain model tests in which but one of the two laws of mechanical similarity involved can be fulfilled, whereas the actual flow may depend on both. In such cases the law on which the flow in question most depends is to be determined, after which it is necessary to investigate by experiment whether any other law which should be taken into consideration may be neglected or not.

In this connection attention may be drawn to a point not hitherto mentioned: Regarding those flows which depend entirely on inertia and friction forces, one can be sure to have mechanical similarity if the Reynolds number is in both cases the same. If not, however, one cannot say that these flows necessary differ widely from each other, for there are quite a number of cases where even for a wide range of Reynolds numbers the flows are practically identical.

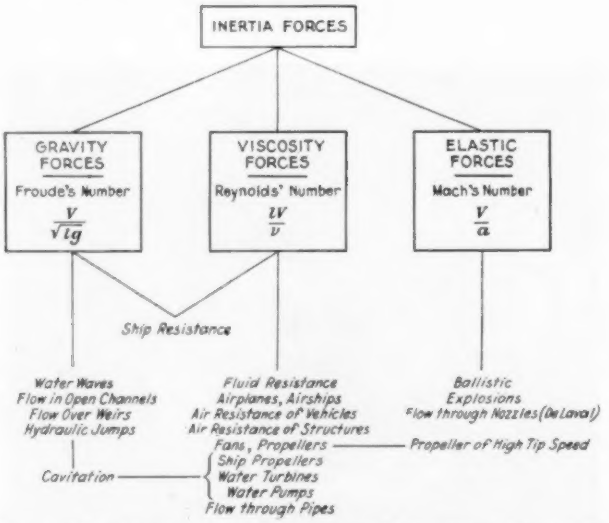
INERTIA FORCES
GRAVITY FORCES
Froude's Number
$\frac{V}{\sqrt{lg}}$
VISCOSITY FORCES
Reynolds' Number
$\frac{lV}{\nu}$
ELASTIC FORCES
Mach's Number
$\frac{V}{a}$
Ship Resistance
Water Waves
Flow in Open Channels
Flow Over Weirs
Hydraulic Jumps
Fluid Resistance
Airplanes, Airships
Air Resistance of Vehicles
Air Resistance of Structures
Fans, Propellers
Ballistic
Explosions
Flow through Nozzles (De Laval)
Propeller of High Tip Speed
Cavitation
Ship Propellers
Water Turbines
Water Pumps
Flow through Pipes

Fig. 5 The More Important Laws of Mechanical Similarity and Their Relations to the Forces Involved

This may be explained more clearly by an example. The resistance of a body moving through still air or of a body at rest in an airstream traveling at a certain velocity, is usually expressed in the following form:

$$\text{Resistance} = cA\,\frac{\rho}{2}\,V^2$$

where A is the projected area of the body, ρ the mass density of the fluid, and V the relative velocity of the fluid with respect to the body. The coefficient of resistance c, which is a dimensionless figure, depends first of all on the shape of the body, as well as on the configuration of the streamlines around the body, i.e., on the Reynolds number. Therefore it can be said that the coefficients of fluid resistance for bodies of geometrically similar shape are equal if the Reynolds number in each case is the same. However, if the Reynolds number is not the same, it cannot be said that in this case the coefficient of fluid resistance would necessarily be different; it can only be said that there is no certainty in the matter, and that it can be decided only by tests.

In order to show this, the relation between the coefficient c and the Reynolds number may be considered for a simple body, say, for a long cylinder of circular cross-section, this cross-section being parallel to the flow. In Fig. 6 the value of c is shown as a function of $R = dV/\nu$, where d is the diameter of the cylinder. It is seen that for a certain range of values of R the coefficient c varies considerably with R, while for other ranges c is almost constant. The range of Reynolds numbers of practical interest is enormously wide, in the present case up to about 8×10^5. In the ordinary manner of plotting, the region of smaller Reynolds numbers (up to about $R = 10{,}000$) is so contracted that no details of the curve can be recognized. In order to circumvent this difficulty it has been found practical to plot the logarithms of the drag coefficients as well as of the Reynolds numbers instead of the quantities themselves. Another way is to plot the drag coefficient and the Reynolds number on logarithmic paper. Fig. 7 shows the curve of Fig. 6 replotted in this manner.

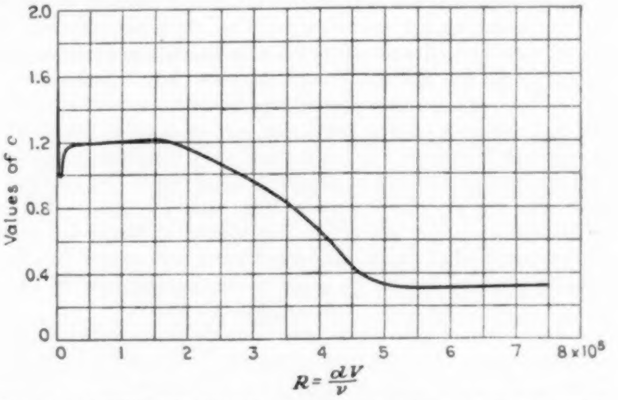

Fig. 6 Coefficient of Air Resistance for a Long Cylinder as a Function of the Reynolds Number

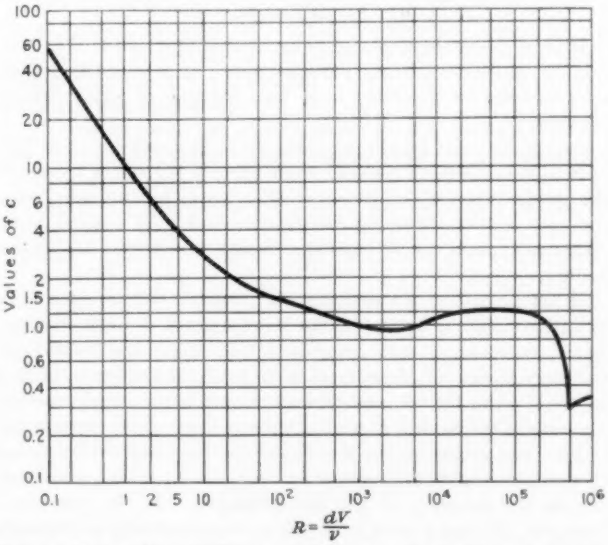

Fig. 7 The Same Curve as in Fig. 6, but Plotted on Logarithmic Paper

From both curves it is seen that, within certain ranges of Reynolds' numbers, model tests made at much smaller Reynolds' numbers can be reliably applied in full-scale practice.

The use of model tests would be considerably restricted were it necessary in every case to observe exactly the laws of similarity. This however, holds only for the Reynolds number. It is therefore often permissible to make model tests at Reynolds numbers which are quite different from those of the full-sized structure, and it is only necessary to make sure that there is no abrupt change in the kind of flow between the Reynolds number of the model test and that of the actual flow.

What is true for the Reynolds number, however, is not to the same extent true for the Froude and Mach numbers. Here it is usually much more important to fulfil exactly the conditions of mechanical similarity.

Reverting to the example of the model of a high-speed airplane propeller, the tip speed of which may be, for instance, 0.8 of the acoustic velocity a, it is necessary to have the same tip speed of 0.8 for the model notwithstanding the fact that the Reynolds number may be much smaller owing to the smaller blade size.

In the opinion of the author, the following statement deserves more attention than it generally receives. It is not sufficient in model-testing practice merely to know on which law of mechanical similarity the flow in consideration depends; but it is also necessary to have at least some general idea as to the manner in which the flow depends on the particular law or laws of similarity. If one were to endeavor to meet exactly the requirements of the various laws of mechanical similarity involved in the problems that come up, but very few model tests could be made. It may be mentioned in this connection that almost all the model tests carried out in wind tunnels and which have been proved so valuable for airplane design, are made at considerable smaller Reynolds numbers than those of the full-sized structure.

A knowledge of the various laws of mechanical similarity is an essential feature of model-testing practice, but experience of many years in the application of these laws is another feature which is certainly not of less importance.

IV—SPECIAL APPLICATIONS

(a) Fluid Resistance

It is of course beyond the scope of this short survey to present the subject with any completeness. It is only thought to refer to some of the more important features of model-testing practice.

One of the most important problems in hydrodynamics is that of fluid resistance. However, since there is no theory by which the fluid resistance, or the drag of a body, can be calculated, the only way of obtaining the data desired is by making tests. Numerous problems of airplane design are solved by model testing.

In the early days of aviation quite a number of tests were made in natural wind or by towing the test specimen through still air.[5] Later, however, it was recognized that there are several advantages in making these tests in an artificial airstream, or in a so-called wind tunnel, and nowadays this is the usual practice. By testing the models in a wind tunnel, the various components of the force acting on the test model can be measured by means of sensitive scales, and plenty of time can be taken, which improves the accuracy of the measurements. In the free wind, however, all results have to be taken from recording instruments which are inherently less accurate. This is due to the fact that the natural wind is always more or less gusty. Another advantage of the wind-tunnel method compared with testing in the natural wind is the possibility of locating all measuring instruments outside the airstream, while the test model itself is held in place therein by means of thin wires or struts. Because of this, the errors arising from the fact that the airstream is affected not only by the test specimen but also by its support, are reduced to a minimum.

If the velocities of the flow under consideration are low compared with the acoustic velocity, permitting the neglect of compressibility, gases can be treated like liquids. In this case, Reynolds' law of similarity has to be taken into account.

It has been found, however, that for certain cases where an especially high accuracy is desired, the results obtained by model tests do not always quite agree with tests made with the full-sized structure. This is due to the fact that usually the model tests are made at a considerable smaller Reynolds number, although it may be taken for granted that they are not carried out in any critical region of the Reynolds number. This departure of the results obtained from model tests from those of full-sized tests is known as the "scale effect."

In order to reduce this scale effect, the general trend in wind-tunnel practice is to obtain larger and larger Reynolds' numbers.

Different methods for obtaining sufficiently large Reynolds numbers have been used, primarily in connection with research work for aviation. One method consists in using water instead of air. Due to the fact that the kinematic viscosity of water is about one-fifteenth of that of air, the Reynolds number is 15 times larger provided the same velocity and linear dimensions are used. A disadvantage of this method, however, is the enormous power required to move a large amount of water with a sufficiently high velocity. Another and better way is to use compressed air. The kinematic viscosity of air under a pressure of, say, 15 atm. is about one-fifteenth of its normal value, so that the Reynolds number becomes 15 times as large. This method naturally involves great structural difficulties since the entire installation has to be able to withstand an internal pressure of 15 atm. In order to avoid these difficulties, wind tunnels are being developed with larger dimensions, which in turn makes possible the testing of larger models, and, therefore, at larger Reynolds' numbers. In the largest wind tunnel, which is installed in Langley Field, Va., the airstream has a cross-sectional area of about 1300 sq ft.

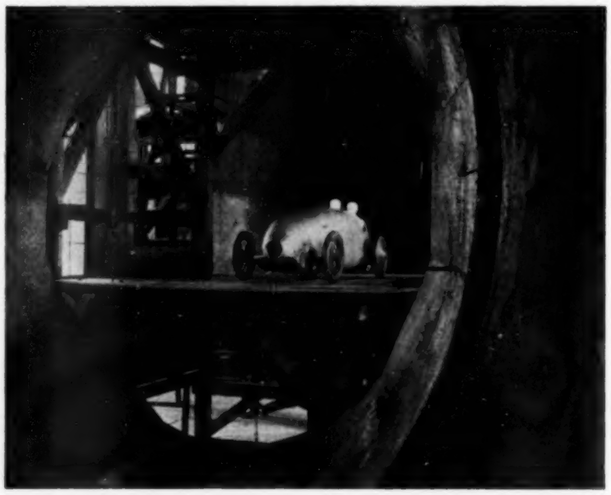

Fig. 8 Arrangement of a Racing-Car Model in a Wind Tunnel

Generally, an attempt should be made to present the data of any experimental work in dimensionless quantities whenever possible. In model-testing practice it is absolutely necessary.

Air forces acting on various bodies, as, for instance, the drag and lift of airplane wings, the drag of airships, as well as the air resistance of vehicles such as trains and automobiles, are usually expressed in the following form:

$$\text{Air forces} = c \times A \frac{\rho}{2} V^2$$

$$c = f\left(\frac{LV}{\nu}\right) = f(R)$$

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[5] For drag measurements in water, primarily in connection with the problem of ship resistance, this method is still used successfully.

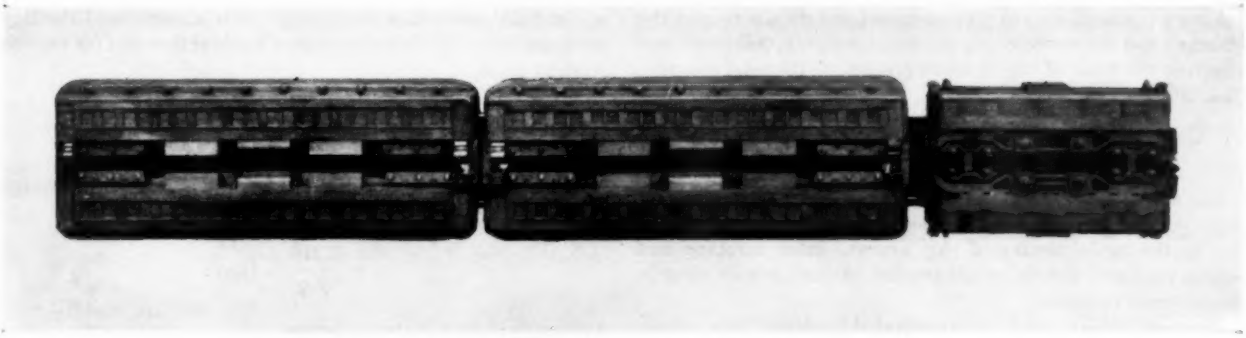

FIG. 9 MODELS OF A PRESENT-DAY TYPE OF TRAIN AS ASSEMBLED FOR TESTS

where A is a characteristic area, ρ the mass density, and V the relative velocity of the air with respect to the body.

Air pressures, or more generally, fluid pressures, are proportional to the dynamic pressure:

$$\text{Air pressure} = k \times \frac{\rho}{2} V^2$$

The dynamic pressure $\rho V^2/2$ is usually taken as the pressure unit, and the various pressures in consideration are expressed in fractions thereof.

Regarding the measurements of air resistance of *vehicles* by model tests in a wind tunnel, some difficulties arise in the satisfactory representation of the ground and track. The ideal method would be to have the ground move below and close to the models with a speed equal to that of the air in the wind tunnel. This could be done by means of an endless belt. However, in general, it is sufficient to have an arrangement which insures that the streamlines below the vehicles are horizontal and parallel to the track. This condition can be approximately obtained by two methods.

In the first method a thin, smooth, and sharp-edged flat plate is rigidly fastened below and fairly close to the wheels of the vehicle, avoiding, of course, any actual contact. Fig. 8 shows such a set-up for testing the air resistance of a racing car. (Ref. 18.)

A second method is the so-called "mirror method." In constructing the model of the vehicle, a duplicate has to be made and assembled, as shown, for example, in Fig. 9 for the model of a train. The air flow around this assembly has a plane of symmetry at the surface where the ground would be for an actual train. This plane of symmetry or of an imaginary ground preserves closely the character of the flow lines occurring with an actual train. From the resistance obtained in testing these assembled duplicates, of course, only half is to be taken for the resistance of a single model. (Ref. 19.)

In testing the *wind pressure on structures* like buildings, bridges, etc., the velocity distribution of the wind in the vertical direction must be taken into account. At the ground the wind velocity is zero, but it increases very rapidly with increasing distance from the ground. For a fairly level ground the wind velocity increases as the $^1/_7$th power of the distance from the ground. The pressure on particular parts of the structure, e.g., on the walls and roof, depends on the shape of the structure as a unit and cannot be determined separately. If other buildings are in the neighborhood of the structure under consideration, their possible influence has to be taken into account. It may be mentioned that suction forces, primarily on roofs of buildings, can exceed considerably the pressure forces.

In model tests to determine the *wind pressure on ships*, the velocity distribution of the wind in the vertical direction must also be considered.

(*b*) PROPELLERS AND FANS

The most important condition to insure mechanical similarity in testing small-scale *airplane and ship propellers* is that of having the same value of $V/D\omega$ for both model and full-scale propeller, where V is the speed of advance of the propeller, ω the angular velocity, and D the diameter of the propeller.

In addition to this the Reynolds number should be the same, and in case the tip speed of the full-scale propeller is near to the acoustic velocity a, the ratio $D\omega/a$ should be the same for both model and prototype.

Provided the tip speed is less than half the acoustic velocity, the compressibility of the air may be neglected and the model tests can be applied to full-scale practice.

In general, it is always possible to obtain the same value of $V/D\omega$ for both the model and full-scale propeller; however, it is usually not very easy to obtain also the same Reynolds number. A possibility for obtaining the same Reynolds number would consist in using a medium of lower kinematic viscosity, for instance, compressed air.

In order to avoid any possible scale effect, a special wind tunnel was built a few years ago at Langley Field, Va., in which full-size propellers can be tested.

The thrust and torque are usually expressed by the following equations:

$$\text{Thrust} = T = c_T \frac{D^2\pi}{4} \frac{\rho}{2} V^2 = k_T \frac{D^2\pi}{4} \frac{\rho}{2} (D\omega)^2$$

or

$$\text{Torque} = Q = c_Q \frac{D^2\pi}{4} \frac{D}{2} \frac{\rho}{2} V^2 = k_Q \frac{D^2\pi}{4} \frac{D}{2} \frac{\rho}{2} (D\omega)^2$$

Therefore the efficiency η of a propeller can be written:

$$\eta = \frac{T \times V}{Q \times \omega} = \frac{c_T}{c_Q} \frac{V}{D\omega} = \frac{k_T}{k_Q} \frac{V}{D\omega}$$

For a *fan*, the conditions regarding mechanical similarity consist mainly of having the same Reynolds number. However, here, as well as in the case of airplane propellers, it is not very likely that any objectionable scale effect will be encountered.

The volume delivered by a fan is given by the equation

$$\text{Volume} = \varphi \times A \times u$$

where A is a characteristic area, for instance, the discharge area of the fan, and u the peripheral speed.

The total pressure, i.e., the sum of static pressure and dynamic pressure, is usually expressed by the equation

$$\text{Total pressure} = P_s + P_d = \psi \times \frac{\rho}{2} u^2$$

A very practical way to present model test data is to plot the efficiency and the coefficients φ and ψ against φ^2/ψ, this latter representing the ratio of the dynamic pressure to the total pressure. (Ref. 20.)

(*c*) CAVITATION IN SHIP PROPELLERS, WATER TURBINES, AND PUMPS

The considerations set forth regarding the model testing of airplane propellers and fans can be employed without modification in the model testing of ship screws, water turbines, and pumps, provided there is no danger that cavitation may occur in the full-sized machine.

However, if this is just to be investigated by model tests, certain model rules have to be taken into account with respect to similarity regarding the occurrence of cavitation. Three methods can be applied:

1 Variation of the pressure heads
2 Equality of the peripheral speed
3 Increase of the water tension by increasing the temperature.

1 If H_a denotes the water head corresponding to the atmospheric pressure minus the water tension, and if H_s is the suction head and H the total head, the expression

$$\frac{H_a - H_s}{H} = \sigma$$

must be the same for both model and prototype. (Ref. 22.) The smaller σ is, the greater the danger of the occurrence of cavitation. Since H is proportional to $u^2/2g$, where u is the peripheral speed of the machine, the above expression can also be written as

$$\frac{H_a - H_s}{u^2/2g} = \sigma'$$

This model rule can be easily derived from Froude's law of similarity, which has to be taken into consideration because of the predominant importance of the formation of free surfaces. (Ref. 21.)

2 Another method for obtaining similarity as regards cavitation is to have the peripheral speed of the model equal to that of the full-sized machine. This, however, involves a very high rpm and necessitates a full-sized construction because of using the same pressure and suction head in the model as in the full-sized machine.

3 A third possibility of obtaining similarity with respect to cavitation consists in increasing the water tension by heating the water. This method, although rather inconvenient in practice, would have the advantage of providing a larger Reynolds number owing to the smaller kinematic viscosity of hot water.

(*d*) FLOW THROUGH PIPES AND CHANNELS

These flows are governed throughout by the Reynolds number, at least to the extent that the velocities are not comparable in magnitude with the acoustic velocity.

The question of interest is, generally the pressure drop along pipe lines or at elbows, or the pressure drop occurring with changes of the cross-sectional area, as for instance, in nozzles and orifices. These matters are of vital importance for the construction and calibration of flowmeter devices.

The pressure drop is generally expressed in the following form:

$$p_2 - p_1 = \lambda \times \frac{l}{d} \times \frac{\rho}{2} V^2$$

where $p_2 - p_1$ is the pressure difference at two points separated by the distance l, d is the diameter of the pipe, V the average velocity of the fluid, and ρ its mass density. λ is a function of the Reynolds number. Within the range of laminar flow and for smooth, straight pipes,

$$\lambda = \frac{64}{R}, \quad \text{where} \quad R = \frac{dV}{\nu} < 1200$$

Within the range of turbulent flow, and for smooth, straight pipes,

$$\lambda = \frac{0.316}{\sqrt[4]{R}} > 1600$$

For rough pipes, λ is always larger.

The coefficient of flow resistance ζ for elbows, throttles, etc., is usually expressed in terms of $\rho V^2/2$ and therefore

$$\text{Pressure loss} = \Delta p = \zeta \times \frac{\rho}{2} V^2$$

There are still two other extensive fields of model-testing practice which really should belong to hydrodynamics: namely, problems concerned with flows in river beds, over weirs, through sluices, etc., and those concerned with ship resistance. However, due to the importance of these problems, they are being specially dealt with in other papers of this symposium.

In closing, the author would again emphasize that it is not sufficient to find out on what various laws of similarity a given flow exactly depends, since it is generally impossible to fulfil the necessary requirements. In practically all cases a compromise has to be made. That law of mechanical similarity which is of essential importance has to be taken into account, while the others may have to be neglected; and only through extended experience can the knowledge be obtained to judge in a particular case which one should be considered and how far it is permissible to neglect the others. This is one of the most difficult sides of experimental practice in aerodynamics and hydrodynamics.

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Air Resistance of High-Speed Trains and Interurban Cars

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In this paper a study is made of the economics possible by reducing air resistance of high-speed passenger trains and interurban cars. A critical survey is given of the train-resistance formulas used hitherto and of the various methods of determining the coefficients of these formulas. It is shown that the general ideas on which the train-resistance formulas are based have to be corrected according to the more modern aerodynamic conceptions on air resistance.

Data on the air resistance of train models of various shapes recently obtained in the Westinghouse wind tunnel are given in a number of curves. The methods employed and the experiment set-up are fully described. It is shown that a considerable reduction of air resistance of the present types of trains and interurban cars can be obtained by streamlining.

The fundamental laws of air resistance are stated, and it is shown on what air resistance mainly depends. The importance of the critical Reynolds number is explained, particularly in connection with the reliability of model tests. A striking example is given showing how, by means of streamlining, the air resistance of a body can be reduced to 1/20 of its original value. A more rational method of determining the total resistance of trains and interurban cars is given, and it is shown that the streamlining of present-type cars for high-speed service is more justifiable from an economical standpoint than the streamlining of the fuselage of an airplane.

O. G. Tietjens

ONE of the important advantages of the railroad train over the bus is its higher speed. Some American trains have schedule speeds of as high as 50 m.p.h. between important cities, and most of the limited trains travel at about 40 m.p.h. On the other hand, the schedule speed of buses in intercity travel reaches nowadays almost 30 m.p.h. or only 25 per cent less than that of the average limited train. This difference in train and bus speeds is illustrated in Fig. 1 by time-scale maps for travel in the United States.

However, after super-highways have been built between the larger and more important cities, there is no reason why the schedule speed of buses cannot be increased to 40 m.p.h. or even higher. This can be done, and without further increase in the power required, by streamlining the shape of the buses. The next important step in the development of bus transportation will likely be in this direction, and it will probably be taken within the next few years. Then the competition of the bus will truly become of vital importance to the railroad.

The speed of almost every means of transportation has been increased immensely during the last two decades. If we think of the automobile of 20 or even 10 years ago, what a change and improvement. The same holds for the airplane; and also the speed of ocean liners is seen to have been increased considerably when we compare the record of 4 days 19 hours for crossing the Atlantic with the time required 30 years ago.

K. C. Ripley

What can be said about the railroad and its endeavor to improve its speed? As early as 1845 a record of 45 miles in 52 minutes was made in England. At one time the train was proceeding at the rate of one mile in 48 seconds, or "at the astonishing velocity of 75 miles an hour." In the same year the *American Railroad Journal* said:

> The other day on the London and Birmingham and on the Great Northwestern Railway a rate of 65 miles an hour was accomplished. The express trains on these lines run at the rate of nearly 50 miles an hour, stoppage included.

This happened more than 80 years ago, and although these records must of course be considered as exceptions, what efforts to increase the speed of trains can be put alongside those made in the cases of ships, automobiles, and airplanes? One remarkable attempt was made a little more than 30 years ago in Zossen, Germany, with the result that with a very heavy and powerful electric locomotive speeds up to 130 m.p.h. were obtained. After having made several runs, however, the rails were spoiled, and the results of the tests offered little encouragement to increase train speed above a certain rate which had been proved to be safe.

Several years ago the matter of designing a new train or coach for high-speed service was again taken up in Germany. This time an endeavor was made to obtain the high speed by decreasing the train resistance rather than by increasing the motive power, as was done in the Zossen tests mentioned. The results of tests recently made with a new type of railway coach, designed by Fr. Kruckenberg, and having a small total resistance, proved that in their method the investigators are on the right track. The fact that this coach is driven by a modified airplane propeller is, with respect to the speed obtained, only of minor importance.

The new coach is light in weight (20.6 tons), low in center of gravity, and streamlined, all of which are departures from conventional railway practice. The speed made on an actual run

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of 173 miles between two large cities was 106 m.p.h. with a top speed of 143 m.p.h., and the run was better by four minutes than the regular passenger-airplane time.

One of the most interesting facts regarding this new German high-speed train is its low energy consumption per mile, chiefly due to its streamlined shape. The coach, carrying 50 passengers, travels 4 miles to the gallon of gasoline. How good an economy this is will be appreciated when it is stated that the power required for coaches of the present type varies almost as the cube of speed; so that an increase of speed from 40 m.p.h to 100 m.p.h. means a more than tenfold increase in the power required for a coach of a given shape and weight.

If the train is to undertake to compete with the airplane in speed, as seems now to be the next logical development, it is important that the railroads give attention to the design considerations that make the high-speed operation of trains possible without appreciable sacrifice in safety and economy. If the railroad companies do seriously consider this matter of high-speed operation of trains, they will need to evolve some new type of streamlined, light-weight train, and probably make even other radical departures from present practice.

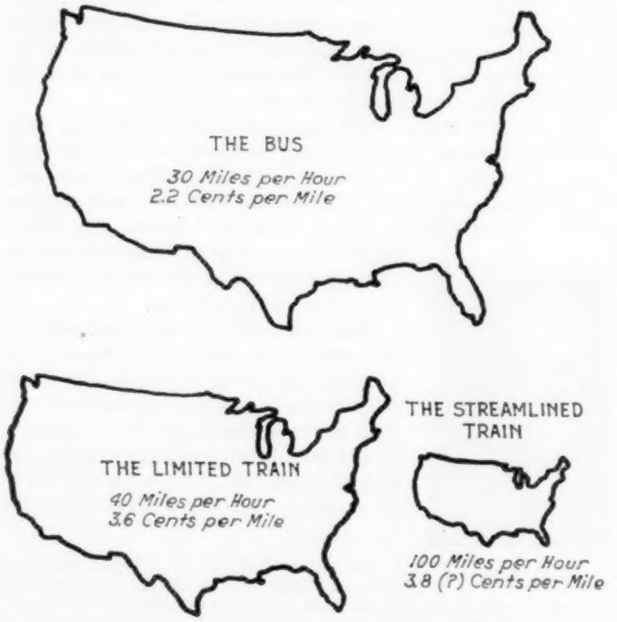

Fig. 1 Time-Scale Maps for Travel in the United States

The effect that an increase in the schedule speed of trains to 100 m.p.h. would have upon the time-scale map for travel in the United States is shown in Fig. 1, and—to anticipate—the cost of this increase in speed to the passenger per mile is also shown in this figure as the difference between the present base rate of 3.6 cents per mile and the new rates, estimated on what is believed to be a conservative basis of distribution of the burden, of 3.8 cents per mile, or less than 6 per cent.

However, even if it be considered inadvisable to increase the speed of trains at the present time, it will be shown later that with existing speeds air resistance is the predominant factor, and that from an economical standpoint streamlining of our fast trains is justified, as is also that of some of our high-speed interurban cars.

For sake of brevity, consideration of the problem of air resistance will be confined here mainly to the case of a train of fixed length—given by that of a locomotive and six coaches—and to that of a single car, as used in frequent schedule service, such as on interurban lines.

I—Train-Resistance Formulas

Those acquainted with train-resistance work are familiar with some of the common train-resistance formulas that agree in showing one part of the resistance as a function of the weight of the cars in the train, and a second part as a function of the shape and size of the train, assuming a straight, level track. The resistance mainly due to weight is commonly called the frictional resistance, and that due to shape and size, the air resistance. The frictional resistance comprises several items, as, for instance, journal, rolling, and track resistance, and wheel-flange friction, which latter, however, does not depend on weight. However, being here interested only in the air resistance and its relative importance to the total resistance, the authors will not consider the separate items of the resistance due to weight, but will include them under the general terms "frictional resistance."

Extensive experimental tests on the resistance of standard types of trains of various weights have been made during the past 30 years. Although the results of these tests differ considerably from each other, they agree that the resistance of a train depends chiefly on its weight and speed. Plotting the resistance for trains of different weight as a function of the speed, it was obvious that the relation obtained could be approximated by a formula such as

$$R = A + BV + CV^2 \dots\dots\dots\dots\dots [1]$$

where A and B are functions of the weight, C a function of the size and the shape of the train, and V the speed of the train.

In what follows, the sum of the first two terms will be called the frictional resistance; the third term, depending on the square of the speed, represents the air resistance.

Without exaggeration, one may say that there are almost as many formulas similar to Equation [1] as there are series of tests. J. A. F. Aspinall, in his famous paper on train resistance,[3] gives a list of 55 train-resistance formulas, including five of his own, and C. O. Mailloux,[4] in his discussion on "Problems of Heavy Electric Traction," estimates the number of train-resistance formulas to be 200 or more. It is not intended to discuss even the more important of these formulas. Mr. Mailloux, in the discussion mentioned, points out the following:

> It may be said of many of the train-resistance formulas that they give results close to actual practice in some specific case; at some particular speed, with a certain kind or length of train, or under other peculiar conditions; but not one of them fits all cases, even approximately, as becomes painfully evident when one sees the way in which the curves wander over the curve sheet when plotted together in the same sheet and according to the same scale.[5]

It has been proposed[6] to develop an average curve by using all experimental data available, or in other words to establish an equation with unknown coefficient and to determine this coefficient by means of all test data available. This, of course, is possible; however, it should be realized that such an "average curve" has only slight value due to the fact that the special features of the train corresponding to this curve—the "average train," so to speak—are not known.

It is therefore obvious that it is necessary to do more than merely introduce a new set of coefficients in Equation [1] in order to accomplish something representing a material advance in our methods of dealing with train resistance in railroad engineering.

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[3] J. A. F. Aspinall before the Institute of Civil Engineers, London, November, 1901.

[4] Discussion of paper on "Problems of Heavy Electric Traction," Trans. A.I.E.E., vol. 23 (1905), p. 723.

[5] E. C. Schmidt and H. H. Dunn, "Passenger Train Resistance," University of Illinois Bulletin No. 110.

[6] E. H. Croft, "Railway Track Resistance," *Electrical Review* (1929), p. 349.

If it is impossible, as will be seen later, to devise a single general formula giving the train resistance with sufficient accuracy, one may rightly be of the opinion that there is not much need for a formula which is so simple that one can calculate in a moment the resistance, but with only ± 50 per cent accuracy. It would be much better to investigate the fundamentals of train resistance. The data obtained by such studies would make it possible to predict with reasonable accuracy the power required. Instead of calculating the total resistance by using one of the many train-resistance formulas, it would be much safer to determine each portion of the total resistance separately, as, for instance, the journal resistance or the air resistance, taking into consideration the particular features of the train.

Considering for a moment the frictional resistance, the first two terms of Equation [1], we may mention the values of A and B as determined by A. H. Armstrong and H. J. Davis. According to Armstrong,

$$A = C_1 \sqrt{W} \quad \text{and} \quad B = C_2W$$

where C_1 and C_2 are constants and W is the weight of the train.

W. J. Davis,[7] in his valuable paper on "The Tractive Resistance of Electric Locomotives and Cars," gives a somewhat more complicated value for A, namely,

$$A = C_1\,(1 + C_2W) \quad \text{and} \quad (B = C_3W)$$

for axle weights in excess of 5 tons, and

$$A = C'\,(1 + C''\sqrt{W}) \quad \text{and} \quad (B = C_3W)$$

for axle weights of less than 5 tons.

These coefficients for the frictional resistance, though of different form, do not give very divergent results. This is mainly due to the fact that for different trains the construction of the bearings, wheels, and tracks is rather similar. The largest uncertainty in train resistance lies in the air resistance.

The coefficient C in the third term of Equation [1], which term represents the air resistance, is usually assumed to be proportional to the cross-sectional area A_1 of the train: or

$$\text{Air resistance} = C' \times A_1V^2$$

Considering the dimensions in this equation, it will be seen that the coefficient C' is proportional to the ratio of the specific weight γ of the air to the specific gravity g, or $C' = c(\gamma/g)$. In other words, the air resistance will vary directly with the density of air. This fact has been very often neglected. In cold weather and with a high barometric pressure, however, the specific weight of the air may be 15 per cent larger than in hot weather and with a low barometric pressure. The coefficient C' determined by air-resistance tests made with trains operating under different weather conditions may therefore vary as much as 15 per cent if the difference of the specific weight of the air has not been taken into account.

Therefore, as is common in aerodynamics, we may write:

$$\text{Air resistance} = c\,\frac{\gamma}{2g}\,A_1V^2 \ldots\ldots\ldots\ldots\ldots [2]$$

This coefficient c by which the air resistance is determined for a train of a given cross-sectional area A_1, moving with a certain speed through quiet air of specific weight, depends largely on the shape of the train. There is no theory by which this coefficient can be calculated, the only way of determining it for trains of different shapes being by tests.

In some train-resistance formulas the resistance of additional coaches is expressed as a percentage of the air resistance of the locomotive and for long trains this can be done without error. The coefficient c in Equation [2] is then split up in a number of coefficients:

$$c = c_0 + c_1 + c_2 + \ldots\ldots + c_n$$

where c_0 represents the coefficient of the locomotive and c_1, c_2, ...c_n the coefficients of the n coaches. It must be understood, however, that the coefficients $c_1 \ldots . c_n$ do not refer to a single isolated coach, but to the coaches considered as parts of a train. For long trains there is little difference between the coefficients of the different coaches, so that we may write

$$c = c_0 + n\,c_1$$

However, it must be emphasized that the manner in which c is split into the two parts c_0 and c_1 holds only for a particular kind of train and therefore cannot be used in a general formula for different trains, even though the trains should accidentally have the same c.

It is very often proposed to divide the total air resistance into one portion due to the front pressure, another due to the rear suction, and a third due to skin friction. This distribution is hardly practicable, especially since the third portion, due to what is called skin friction, actually consists also of a kind of pressure resistance because of the space between adjoining coaches.

In the authors' opinion it is much more advisable, instead of seeking vainly for a few coefficients applicable to all kinds of trains, to investigate the fundamentals of air resistance. Systematic tests with trains of different shapes would show us on what it depends. Such experiments would teach us how we might change the shape of locomotives and coaches without changing the air resistance, and, on the other hand, what changes of the shape would influence the air resistance. The information gained in this way would put us in a much better position to predict the air resistance of a train than that of using train-resistance formulas based on front pressure, etc.

In this respect, much can be learned from aviation engineers. They have no airplane-resistance formulas, but have had an extended experience with air resistance in general. Numerous tests have been made by using wind tunnels, and if the need arises for an air-resistance coefficient of, say, a new fuselage, they will very likely find tests that have been made with a fuselage similar in shape and which will give a coefficient of sufficient accuracy.

If it is agreed that investigations should be made to clear up the still obscure point of air resistance of trains, there is the question of which method is the most suitable. Here again we may ask the aviation engineers for their advice. Generally speaking, there are two methods of testing: one using actual trains moving through quiet air, and the other employing models of trains in wind tunnels. Both methods have their advantages and both were used in earlier days by aviation engineers. However, it was found that the latter method, of using wind tunnels, was not only more convenient but also more exact, and nowadays practically all the tests on airplanes and airships are made with models in wind tunnels.

The main objection to the method of using actual trains is that the total resistance is first determined and the air resistance then figured as the difference between this total resistance and the frictional resistance determined in a different way. Further, it is very seldom possible to have sufficiently quiet air on the day the tests are being made or a wind of constant uniformity. Even a scarcely perceptible wind of 3 m.p.h. will increase the power required to overcome air resistance about 20 per cent, assuming a speed of 50 m.p.h., and will increase the power required about 50 per cent assuming a speed of 20 m.p.h. This can be easily seen if we take into account that the power required to overcome air resistance varies with the third power of the speed:

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[7] *General Electric Review*, vol. 29 (1926), p. 685.

$$\frac{53^3 - 50^3}{50^3} \times 100 = 20\% \text{ and } \frac{23^3 - 20^3}{20^3} \times 100 = 52\%$$

The speed of the air in wind tunnels, on the other hand, can easily be kept constant to within 1 per cent.

II—WIND-TUNNEL TESTS ON TRAINS AND INTERURBAN CARS

When the senior author came to the United States about three years ago in order to organize research work on hydrodynamics and aerodynamics at the Westinghouse Electric & Manufacturing Co., East Pittsburgh, he soon realized the great importance of faster transportation in this country. Finding that concern closely connected with the railway business, the

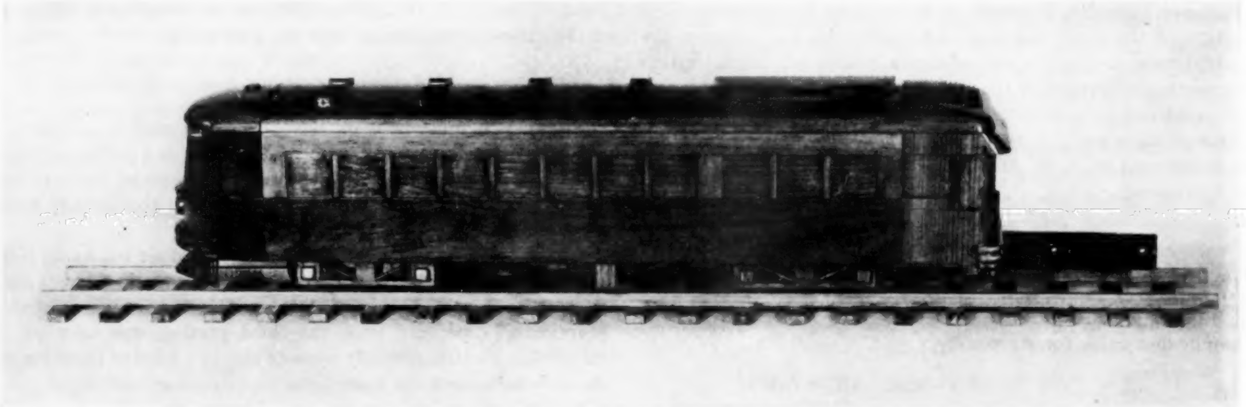

FIG. 2 MODEL OF A PRESENT-TYPE INTERURBAN CAR

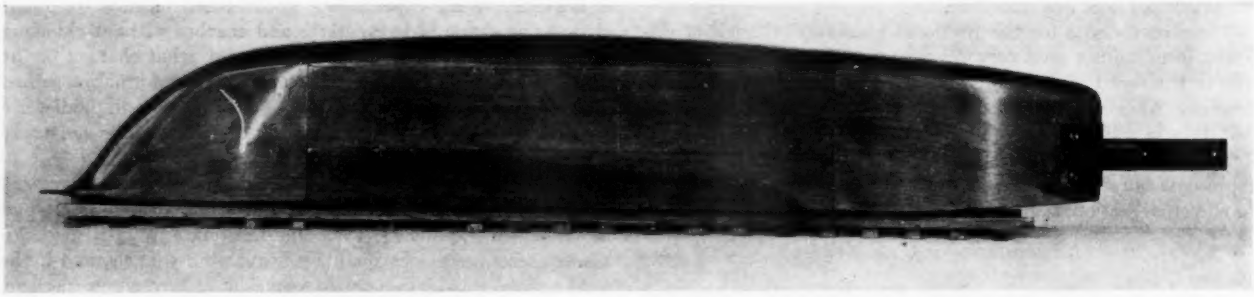

FIG. 3 MODEL OF A STREAMLINED INTERURBAN CAR

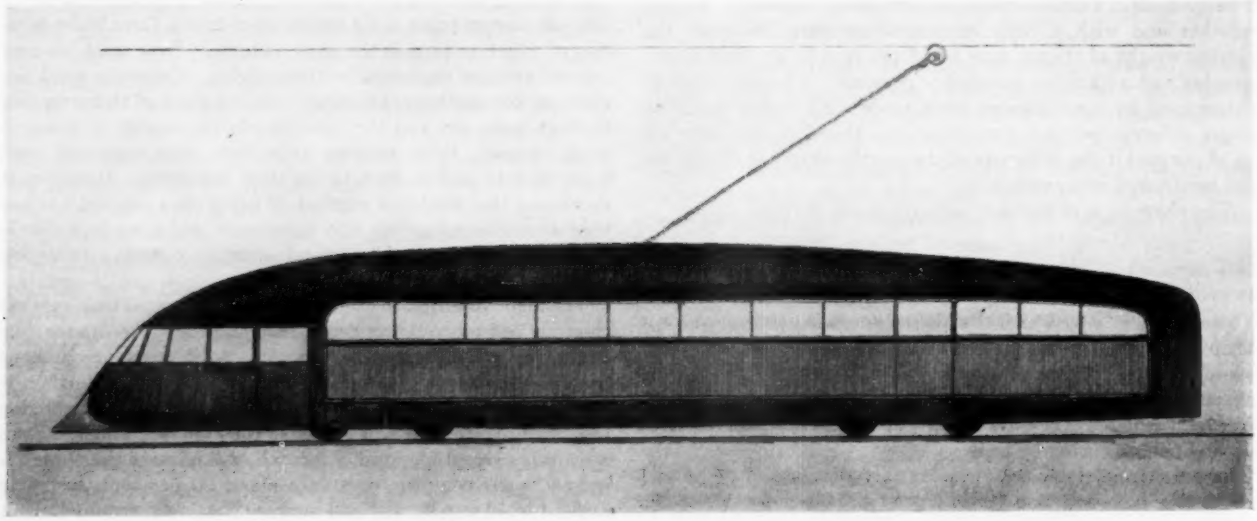

FIG. 4 SKETCH OF A STREAMLINED INTERURBAN CAR

development of new features of design of electric locomotives, coaches, and interurban cars was taken under consideration.

The work was started about two years ago by the building of a special wind tunnel for testing models of present-type trains and interurban cars as well as those of a streamlined shape. Fig. 2 shows as an example the model of a present-type interurban car, which is about 1/25 the size of the actual car. Fig. 3 shows a model of a streamlined interurban car of the same cross-sectional area, but 20 per cent larger in capacity. It will be noted that current-collecting devices were omitted from both types of models, as it is the intention to study the air resistance of these devices separately. Fig. 4 is intended to give an idea of how a streamlined interurban car will look.

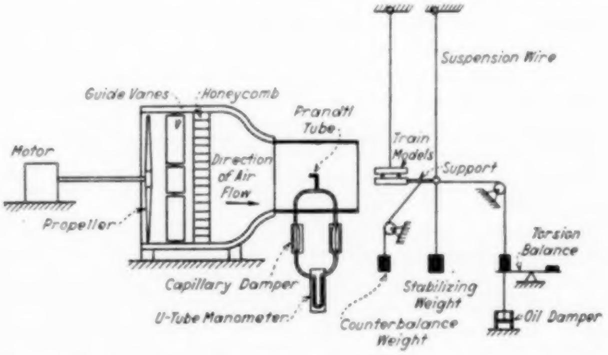

FIG. 5 SCHEMATIC DIAGRAM OF WIND TUNNEL USED IN TESTS

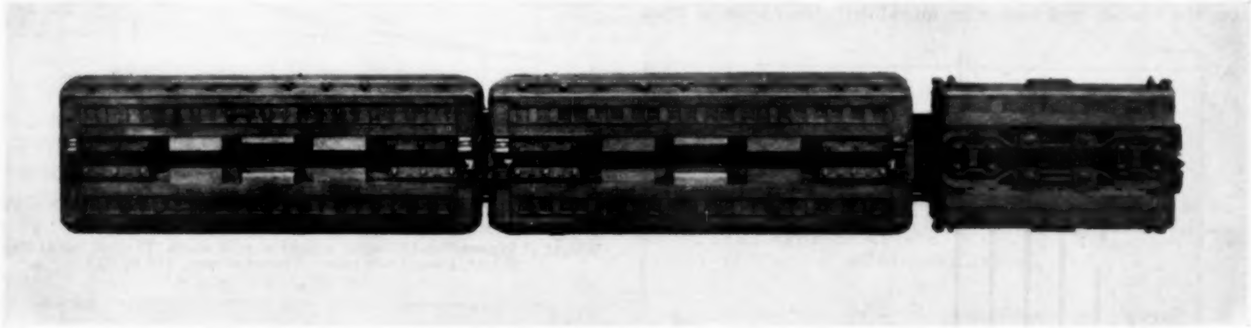

FIG. 6 MODELS OF A PRESENT-TYPE TRAIN AS ASSEMBLED FOR TESTS

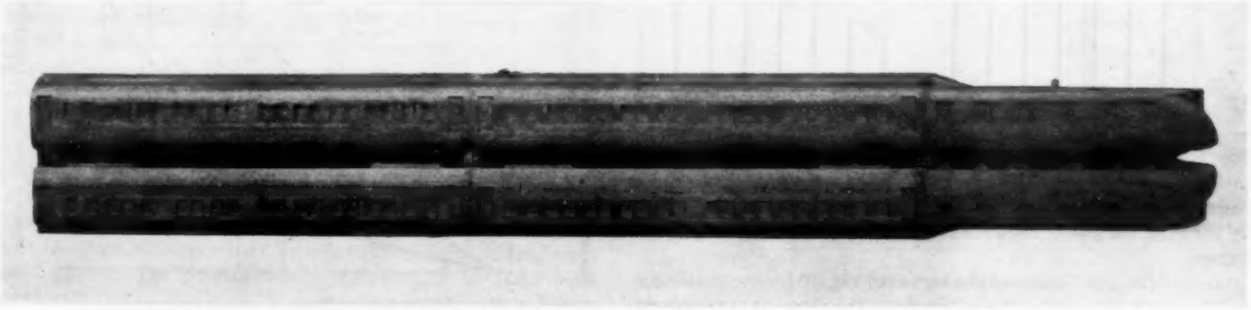

FIG. 7 MODELS OF A STREAMLINED TRAIN AS ASSEMBLED FOR TESTS

The wind tunnel built for these tests is shown in a schematic way in Fig. 5. In tests on small-scale models in a wind tunnel it is better, as will be shown in the succeeding part of the paper, to have a turbulent rather than a uniform air flow and therefore, in order to check the characteristics of this particular wind tunnel against those of certain well-known wind tunnels, a preliminary test was made on a 10-in.-diameter sphere with various air speeds. This and other tests showed the wind tunnel to be satisfactory.

Generally, in constructing the models, a duplicate was made of each train and the two trains assembled as shown in Figs. 6 and 7. This assembly, known as the "mirror method," was used to make the air flow symmetrical about the plane of the imaginary ground, and thus to preserve closely the character of the flow lines occurring with actual trains. From the resistance obtained in testing these assembled duplicates, of course, only half is to be taken for the resistance of a single model. For single cars, such as interurban cars, it was found by experiment that a flat plate parallel to the air flow will give the effect of ground upon a moving train, but with models of long trains the stagnation of air due to a plate stationary with respect to the models would have been for the rear cars objectionably large.

The suspension of the train models in the wind tunnel, and the testing of the air resistance of the models and suspension, and then of the suspension separately to get the air resistance of the models alone, followed conventional practice in these matters. The sensitivity of the test arrangement was sufficient to make the systematic error in the reading of the instruments only about 1 to 2 per cent. Damping devices on the torsion balance and on the manometer permitted mean values being read. The tests were made with a range of air speeds of from about 40 to 70 m.p.h.

III—AIR RESISTANCE OF TRAINS

The results of the tests with trains are summarized in Fig. 8. The tests were made on models of a locomotive alone and with one and two coaches for both types of trains. In Fig. 8 the test results have been extrapolated to trains of six coaches. A straight-line extrapolation was assumed to be permissible owing to the fact that practically no difference could be detected between the air resistance of the first and the second coaches in either of the train models.

The values of coefficient of air resistance shown in Fig. 8 are, if anything, probably somewhat high if used for full-sized trains, and especially so for the streamlined trains. The reason is that in the streamlined models most of the resistance is surface resistance, and owing to the complexity of the present-type models, the models, all of the same smoothness of surface, were not smooth as, with more expense, they might have been made. However, the streamlined locomotive, after having been fitted with a streamlined rear part, was tested before and after being highly waxed and polished, and a reduction of over 40 per cent in the air resistance was obtained simply by this change in smoothness of surface.

Since this paper deals only with tests on air resistance, certain assumptions had to be made regarding the frictional resistance in order to determine the total train resistance. After considering various methods of calculating the frictional resistance, the method of A. H. Armstrong was finally adopted: namely,

$$R_f = 50\sqrt{W} + 0.03\,W \times V$$

where R_f is the frictional resistance in pounds, W the weight of a car in tons, and V the speed of the train in miles per hour.

From this expression for the frictional resistance and from Equation [2], in which the coefficients of air resistance obtained from the wind-tunnel tests were introduced, the curves of train resistance of Fig. 9 were drawn, and from these the curves of Fig. 10 showing the power requirements of the trains were readily obtained.

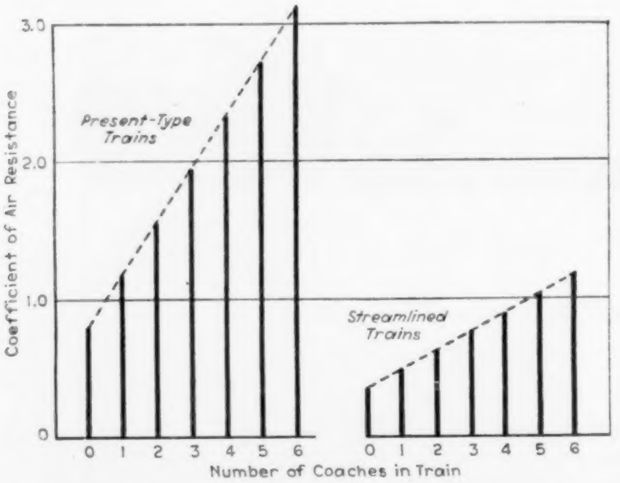

FIG. 8 THE COMPARATIVE AIR RESISTANCE OF VARIOUS TRAINS

To find the significance of the curves of Fig. 10 for the passenger on the one hand and for the railroads on the other, it is necessary to convert them into expressions of dollars and cents' economy.

The passenger is interested in the effect of streamlining and high speed upon the cost of travel per passenger-mile, and the railroad is interested in the comparative earning power of the alternative investments required by streamlined and non-streamlined equipment, everything considered.

It is not possible in an elementary study such as the present one to consider in detail or with great accuracy all of the items that enter into the economy of streamlined trains and high speeds over present-type trains and present speeds, but it is possible to show, by giving approximate values to the more important items, that high-speed, light-weight, streamlined trains have sufficiently great economic advantages over present trains amply to justify, apparently, any railroad in adopting them wherever the existing track, right of way, schedules, etc., permit high speeds of, say, 80 m.p.h.

It will be necessary first to justify the use of high speed for trains by considering the cost of high speed to the passenger per mile, after which it will be significant to inquire into the difference in capitalization required for present-type and streamlined trains operating at the same high speeds.

Such difficultly determined items as the saving in labor overhead due to the higher schedule speed, and the increased cost of track maintenance due to the same cause, will not be considered here in figuring the net cost of high speed to the passenger. The items considered will be those due to changes in power rating of the locomotives with increases in schedule speed. For electric operation these items are the expenses due to increased energy consumption of the train, to increased cost of the locomotive, and to increased cost of the electric transmission system.

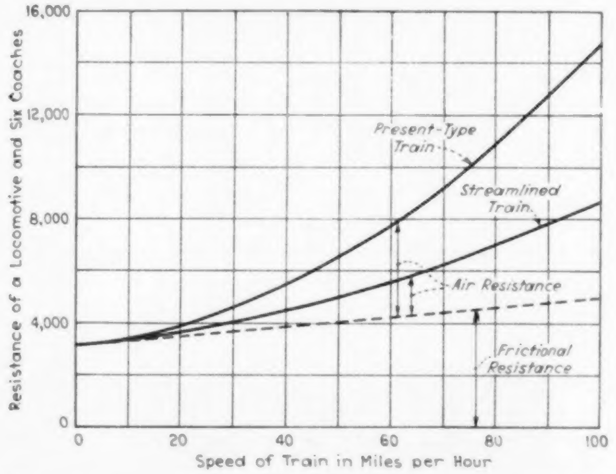

FIG. 9 RESISTANCE IN POUNDS OF FULL-SIZED TRAINS AS CALCULATED FROM THE MODEL TESTS

(Weight of locomotive, 145 tons; weight of each coach, 75 tons; total train weight, around 600 tons. Projected area, 126 sq. ft.)

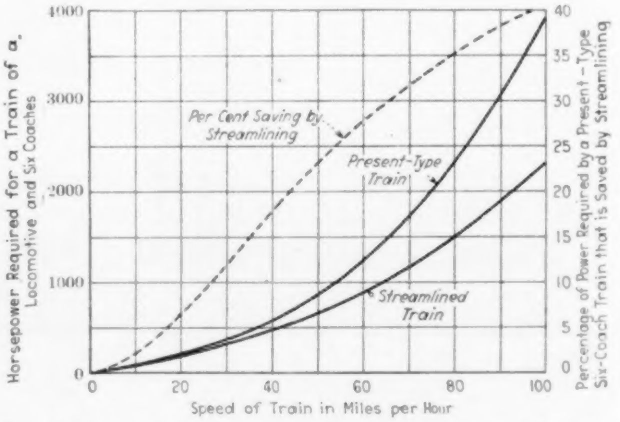

FIG. 10 POWER REQUIREMENTS OF FULL-SIZED TRAINS AS DERIVED FROM FIG. 9

In estimating the expense of increased energy consumption with increased speeds a number of assumptions must be made, for which the following values are offered as reasonable for approximate purposes: Let the cost of electric energy at the car wheels be 1.5 cents per kw-hr. Let the average annual mileage of the train be 100,000 × $\frac{V_s}{40}$, where 100,000 is in round numbers the mileage of present passenger locomotives, and V_s is the schedule speed of the train in miles per hour. Since the average resistance of a train for a distribution of speeds about a schedule speed is for any ordinary distribution equal within a few per cent to the resistance at the schedule speed, let the resistance at the schedule speed be taken as the average resistance of the train. If C_A is the annual cost of electric energy per train and P_s is the horsepower required at the schedule speed by either a streamline or present-type train, as given by Fig. 10, it can be shown that

$$C_A = \$28{,}000 \times \frac{P_s}{1000}$$

To find the annual fixed charge C_B per train due to the cost of the locomotive, assumptions such as the following may be made: Let the weight of a locomotive vary directly as its power rating for a fixed voltage, with a 150-ton locomotive developing 2500 hp. continuous rating, and let the cost of the locomotive vary directly as its weight according to the approximate formula of 50 cents per lb., with or without streamlining. Let the annual straight rate on the locomotive investment to cover interest, taxes, depreciation, insurance, and incidentals be 15 per cent. Let the locomotive have a sufficient power rating to maintain continuously on a straight, level track a train speed of 1.4 times the schedule speed of the train. Then

$$C_B = \$12{,}600 \times \frac{P_s}{1000}$$

If it is assumed that the ratio between the cost of the transmission system for an electrified railroad, including substations and overhead lines, to the cost of the electric locomotives will, for present purposes, be 2 : 3, with an annual rate for figuring fixed charges of 15 per cent, the total annual cost C_{ABC} of the three items that have been considered will, in round numbers, be

$$C_{ABC} = \$50{,}000 \times \frac{P_s}{1000} \quad \ldots\ldots\ldots\ldots\ldots [3]$$

While it is probable that the next developments in increasing the speed of trains will come gradually as in the past, it is of interest to calculate the cost in cents per passenger-mile of an increase in schedule speed to as high as 100 m.p.h. from a schedule speed of 40 m.p.h., for at such a high speed the train would be comparable to the airplane in speed. Assuming, therefore, an increase of schedule speed from 40 to 100 m.p.h., it is found from Fig. 10 that for a streamlined train the additional horsepower required ($P_{s2} - P_{s1}$) is 1800. Assuming that for day-coach travel the average number of passengers per coach is 30, the cost of this additional speed to the passenger per mile can readily be figured from Equation [3] to be 0.2 cent. For sleeping-car passengers, the additional expense would be somewhat greater. This additional expense per passenger-mile is obviously so small compared with the advantages of high-speed travel that the railroads ought to be able to recover it many times over by the increased patronage of the public if the latter are offered this new mode of travel at little or perhaps no increase in fare. The rate for cross-country travel by airplane is at present (1931) about 6.5 cents per passenger-mile on a railway-mileage basis, with a schedule speed on the same railway mileage basis of usually between 100 and 120 m.p.h.

It is interesting, now, to find the capitalized value of the saving effected by streamlined trains over present-type trains for different schedule speeds up to 100 m.p.h. The life of an electric locomotive is variously estimated for depreciation purposes to be from 20 to 30 years. Taking an interest rate of 5 per cent for finding the present worth of an annuity, capitalization factors of 12.5 and 15.4 are, respectively, obtained for annuities running 20 and 30 years. Using these factors to capitalize the annual savings to be found from Equation [3] and Fig. 10, the curves of Fig. 11 are obtained. It should be noted that these are for the savings obtained by streamlining of the kind shown in Fig. 7, and hence do not take into account the savings possible by reducing the projected area of the trains by lowering the height of the trains. Neither do they take into account the savings possible by reducing the weight of the trains.

IV—AIR RESISTANCE OF INTERURBAN CARS

In making the interurban-car models it was desired to provide two, one of a present-type car and one of a perfectly streamlined car, and to have a number of parts on one model that were interchangeable with the corresponding parts on the other. In this way it would be possible to change by gradual steps from a present-type car to a perfectly streamlined car, and by making tests after each of these changes to find the relative importance of the various changes in shape. The two types of models are shown in Figs. 2 and 3, and it will be seen that the streamlined car would have a larger passenger capacity than the present-type car. Eight different tests were made on these models, but for sake of brevity the results of only four will be given here, as shown in Fig. 12.

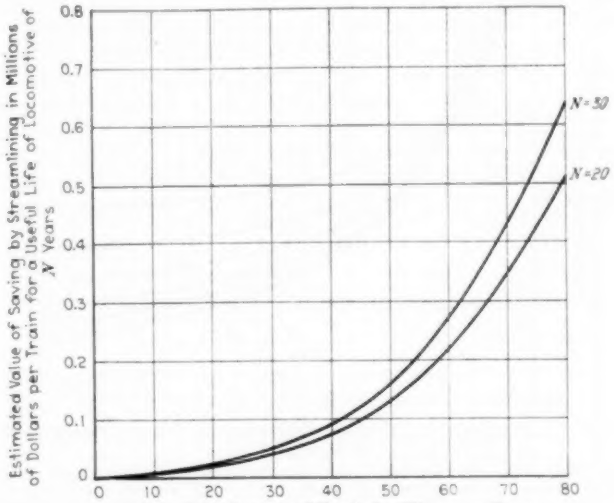

FIG. 11 THE ESTIMATED VALUE OF ADOPTING STREAMLINING FOR HIGH-SPEED TRAINS

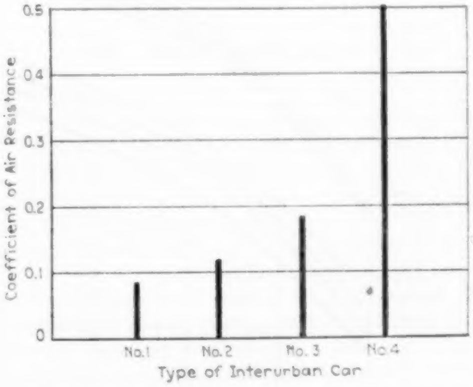

FIG. 12 COEFFICIENTS OF AIR RESISTANCE OF INTERURBAN CARS

Car No. 1 referred to in Fig. 12 is the perfectly streamlined car.

Car No. 2 is car No. 1 made the same in the rear as in the front for two-way operation. Car No. 3 is car No. 1 with the windows and under parts of the present-type car, but with streamlined front, rear, and roof. Car No. 4 is the present-type car. The scale of the models is 1:25. Trolleys were omitted from the models.

The weight of each of the full-sized cars was taken as 25 tons, and the projected area as 83 sq. ft.; so that by using Equation [1], one could, if desired, construct curves of total resistance and of horsepower required by the different full-sized cars as functions of the speed of the cars, assuming no scale effect to exist with the models. If this were done it would be found that for the partially streamlined car, No. 3, the saving by streamlining, in horsepower required to overcome total resistance, would at a speed of 50 m.p.h. amount to about 40 per cent, and at 35 m.p.h., to about 30 per cent. The reason for this large saving effected by streamlining interurban cars as compared with that brought about by streamlining trains is due to the fact that as compared with a train, the air resistance of a car forms a larger part of the total resistance, and this because of the lighter weight of the interurban car in proportion to its projected area.

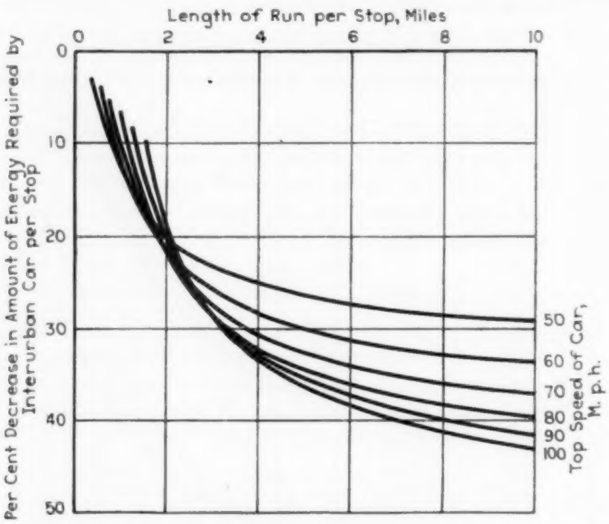

FIG. 13 DECREASE IN ENERGY CONSUMPTION FOR A STREAMLINED INTERURBAN CAR

(Weight of car, 25 tons; projected area, 83 sq. ft.; grade, 0.25 per cent; acceleration, 1.7 m.p.h. per sec.; braking, 2.0 m.p.h. per sec.; reduction of air resistance, 65 per cent; ratio of schedule speeds before and after streamlining, 1.0.)

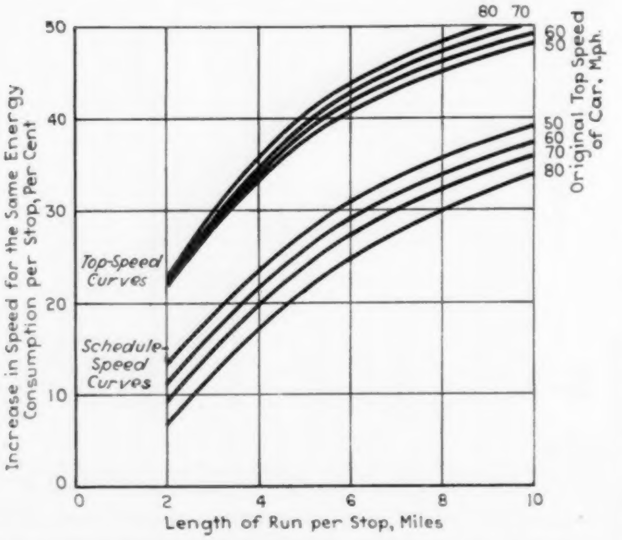

FIG. 14 INCREASE IN SCHEDULE AND TOP SPEEDS FOR A STREAMLINED INTERURBAN CAR

(Length of stop = 5 sec. per mile.)

Such curves of total resistance and of power consumption have a much more limited use for interurban cars than for trains due to the short distance between stops for the former. Much more useful curves for frequent stop service are those showing the effect of streamlining upon the performance of a car as a function of the length of runs in miles per stop. In this way account is taken of the fact that with a frequent-stop service a portion of the operating time is required for accelerating and retarding the car, which means that the average resistance of the car may differ considerably from that existing when the car has reached its normal running speed.

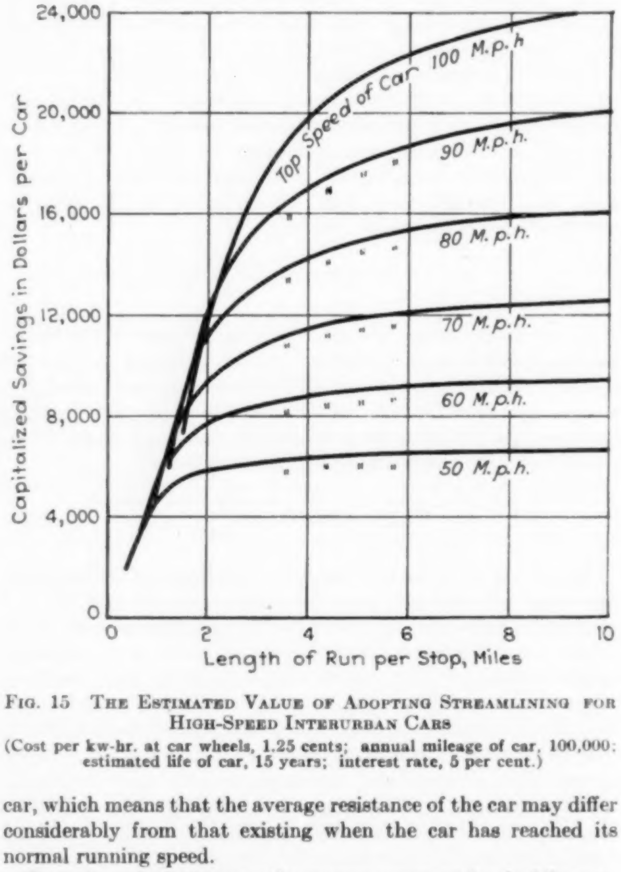

FIG. 15 THE ESTIMATED VALUE OF ADOPTING STREAMLINING FOR HIGH-SPEED INTERURBAN CARS

(Cost per kw-hr. at car wheels, 1.25 cents; annual mileage of car, 100,000; estimated life of car, 15 years; interest rate, 5 per cent.)

To make calculations on the energy consumption in kilowatt-hours per stop for various interurban cars for various lengths of runs per stop and for various normal or "top" speeds, it is necessary to make certain simplifying assumptions. One of the most important of these is as to how the car shall be accelerated—whether at a constant rate by use of oversize motors and special control equipment, or by the common method of a variable acceleration for a large portion of the acceleration period while the "balancing speed" is being reached. For comparatively long lengths of run per stop a constant acceleration up to the normal running speed of the car is not important, but in short lengths of run where it is desired to make a high schedule speed, the matter resolves itself into a question of the comparative economy, everything considered, of a constant acceleration and a given normal speed as compared with the alternative of a decreased acceleration during "balancing," with a necessarily higher balancing speed for the same schedule speed. For short lengths of run and high schedule speeds it is economical to use a constant acceleration up to substantially the top speed of the car, and hence a constant acceleration of 1.7 m.p.h. per sec. up to the top or normal running speed of the car was assumed in calculating the curves of Figs. 13 and 14. The other assumptions upon which the curves were based are given in the captions of the figures. It was assumed for simplicity that, after reaching their normal running speed, the cars would run at a constant speed until braking for making a stop became necessary.

Interurban cars may be streamlined for either of two purposes: to reduce the energy consumption for the same schedule speed, or to increase the speed of the car without increasing its energy consumption. Fig. 13 shows the percentage that the energy consumption of the present-type car can be decreased for the same schedule speed by only partial streamlining, such as in car No. 3. Fig. 14 shows the percentage by which the normal running and the schedule speeds of car No. 4 can be increased without change in energy consumption by partial streamlining, such as in car No. 3.

As in the case of trains it is of interest to gain some idea of the economic importance of streamlining as applied to interurban cars, this time as a function of length of run per stop as well as of speed. With this idea in mind the curves of Fig. 15 have been plotted. These curves include only the savings due to decreased energy consumption for the same schedule speeds and acceleration.

It may be well at this point to call attention to the fact that one large interurban company is now using cars that have a balancing speed of 79 m.p.h.

V—SOME GENERAL ASPECTS OF AIR RESISTANCE

With respect to the reliability of the results of small model tests as applied to actual trains and interurban cars, it may be justified at this point to discuss briefly the question of air resistance in general. The statement previously made, that the coefficient of air resistance is only a function of the shape of a body, has to be corrected to a certain extent. The resistance of a body moving through still air or of a body at rest in an air stream traveling at a certain velocity depends largely on the shape of the streamlines around the body and on the wake in its rear. The form of these streamlines, however, is closely connected with the aerodynamic forces acting on each air particle flowing around the body. There are two sets of forces which have to be taken into account: the inertia forces and the friction forces.

Considering the flow around bodies geometrically similar in shape but differing in size, it can be said that the streamlines are also geometrically similar, and therefore the coefficients of resistance are the same for the bodies in question, if the ratios of inertia forces over friction forces are the same at points similarly situated. This ratio of inertia force to friction force is commonly called the Reynolds number. Therefore it may be stated that the coefficients of air resistance for bodies of geometrically similar shape are the same if the Reynolds number in each case is the same. If, however, the Reynolds number is not the same, it cannot be said that in this case the coefficients of air resistance would necessarily be different; it can only be said that there is no certainty in the matter, and that it can be decided only by tests.

The expression for the Reynolds number is

$$R = \frac{dV}{\nu}$$

where d is a characteristic length of the body—for instance, the diameter of a sphere or the height of an interurban car, V the velocity of the air flowing against the body, and ν the kinematic viscosity.

Applying the Reynolds number to the tests with train models, it is seen at once that the Reynolds numbers of the models are only a small fraction of the Reynolds number of an actual train. The question that arises now is: can the coefficient of air resistance obtained from small-model tests be employed in the calculation of the air resistance of an actual train, notwithstanding this difference in the Reynolds numbers?

In order to answer this question let us consider the relation between the air-resistance coefficient and the Reynolds number for a simple body, say, for a long cylinder of circular cross-section, this cross-section being parallel to the air flow. Fig. 16 shows c as a function of R, and it is seen that for R between 20,000 and 160,000 the coefficient is almost constant and equal to 1.2. When the Reynolds number exceeds 160,000, however, the coefficient gradually decreases to about 0.3 for $R = 500{,}000$, beyond which critical region its value remains constant according to tests so far made.

This change of the coefficient c from 1.2 to about 0.3 is connected with an essential change of the streamlines around the cylinder, which in turn is caused by the fact that the flow in the so-called "boundary layer" changes from a laminar to a turbulent one. This is not the place to go into further details of this interesting and important branch of aerodynamics.[8] Suffice it to state that all those bodies which have a rounded shape, such as a sphere, a streamlined interurban car, etc., have a more or less pronounced critical region of Reynolds' numbers, in which region a considerable decrease of c with increasing Reynolds' number occurs. The better streamlined the body, the smaller is the critical Reynolds number. Bodies with a more or less sharp-edged outline such as a cube or a present-type interurban car do not have this transition of air-resistance coefficients from higher to lower values. For them, c is constant over a wide range and independent of R.

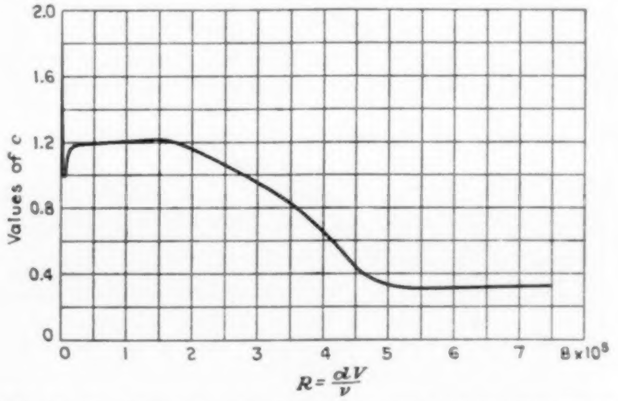

FIG. 16 COEFFICIENT OF AIR RESISTANCE FOR A LONG CYLINDER AS A FUNCTION OF THE REYNOLDS NUMBER

Returning to the question of whether the tests with small models are reliable or not, it may be said that there is no objection to small-model tests if these models have a more or less sharp-edged shape like that of present-type trains. With respect to models of streamlined trains or interurban cars, however, one has to be sure that the tests are made at Reynolds' numbers beyond a certain critical value, which latter in turn depends upon the shape of the model. For bodies like a streamline train the critical Reynolds number is about 100,000. The tests with models of trains and interurban cars described in this paper, however, were made at Reynolds' numbers of about 300,000, or three times the critical value.

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[8] O. G. Tietjens, "Hydro- und Aerodynamik nach Vorlesungen von L. Prandtl," Julius Springer, Berlin, vol. 2 (1931), p. 111.

It may be emphasized, however, that it would be desirable to check the assumption made that the coefficients for streamline bodies will change but very little beyond the critical Reynolds number. This check could be made by testing larger models in a very large wind tunnel, such as the new wind tunnel of the Advisory Committee for Aeronautics at Langley Field, Virginia.

Another very important matter is the question of the streamlining itself. To begin with, it is not sufficient to take a present-type train and round off a few corners or change somewhat the front part of a locomotive. The whole train has to be considered as a unit and the very first layout of the design of locomotive and coaches must be based on experimental data already obtained and on theoretical considerations regarding air resistance. A feeling how a train or an interurban car should look in order to have low air resistance will be of considerable assistance in designing streamline trains. Such a feeling, however, can only come as the result of work for many years on air resistance.

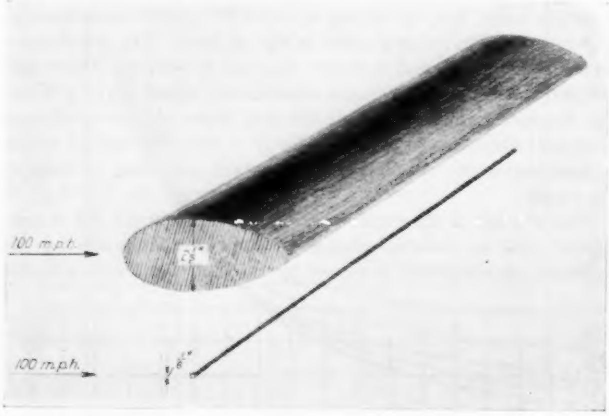

Fig. 17 Strut and Rod Having the Same Air Resistance for a Velocity of 100 M.p.h.

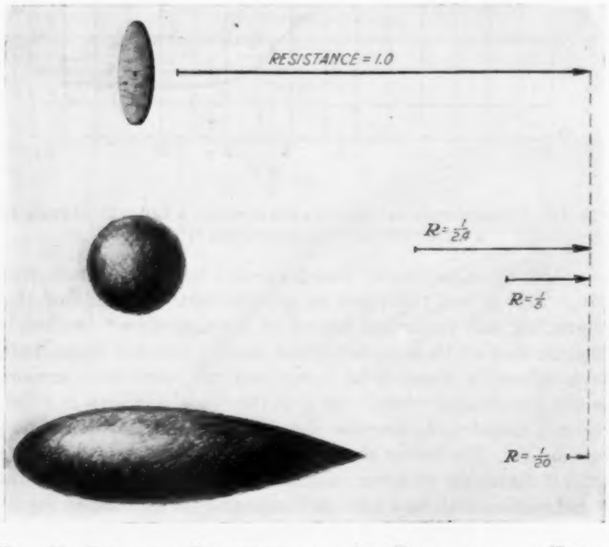

Fig. 18 Relative Difference of Air Resistance of Three Bodies Having the Same Projected Area

Some striking examples may be cited showing that good common sense is not in itself sufficient in rating the excellence of a particular shape of a body as regards its air resistance; but it would lead too far into the theory of air resistance to furnish explanations of the following illustrations.

Fig. 17 shows two bodies of equal air resistance, assuming a speed of 100 m.p.h. The rod has a diameter of $^1/_8$ in. and the strut has a cross-section whose maximum width perpendicular to the direction of air flow is of $2^1/_2$ in.

Fig. 18 shows three bodies having the same projected area. If the air resistance of the disk is denoted by 1, the air resistance of the airship hull is equal to 1/20, and that of the sphere to 1/5 for Reynolds' numbers above the critical and to 1/2.4 for Reynolds' numbers below.

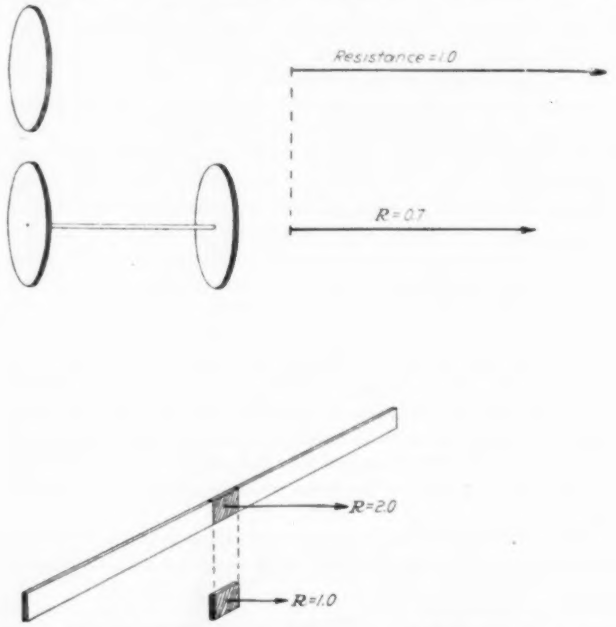

Fig. 19 Air Resistance of Disks and Rectangular Plates

(The air resistance of one disk is less than that of two disks if the two are placed at a certain distance apart. The air resistance of a rectangular plate when isolated is but half what it is when considered as a part of a long plate.)

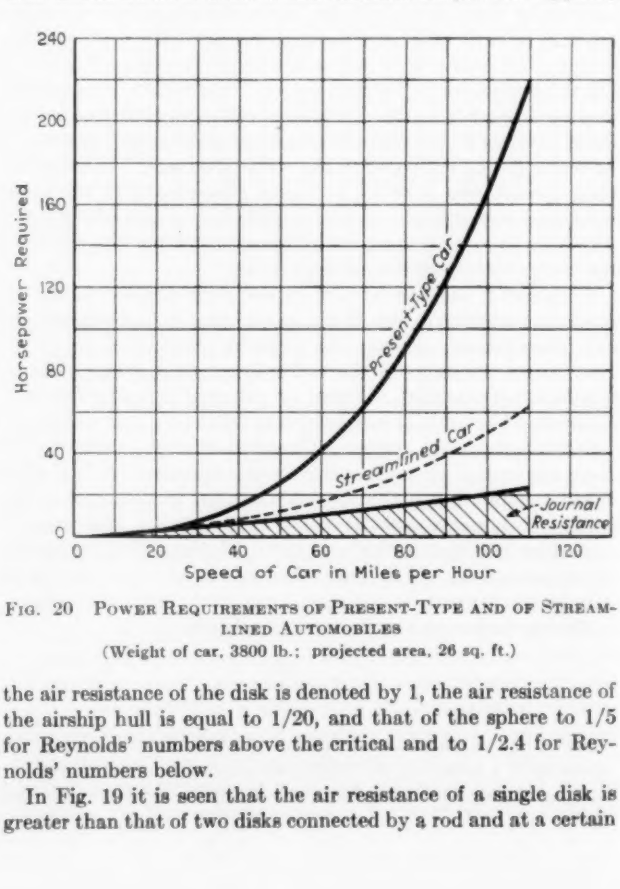

Fig. 20 Power Requirements of Present-Type and of Streamlined Automobiles

(Weight of car, 3800 lb.; projected area, 26 sq. ft.)

In Fig. 19 it is seen that the air resistance of a single disk is greater than that of two disks connected by a rod and at a certain distance from each other. It is also there shown that the air resistance of a square plate is twice as great if considered as a part of a long rectangular plate as it is when isolated.

These few examples will make it obvious that a thorough aerodynamic training is necessary for those who propose to "streamline" modern trains and interurban cars.

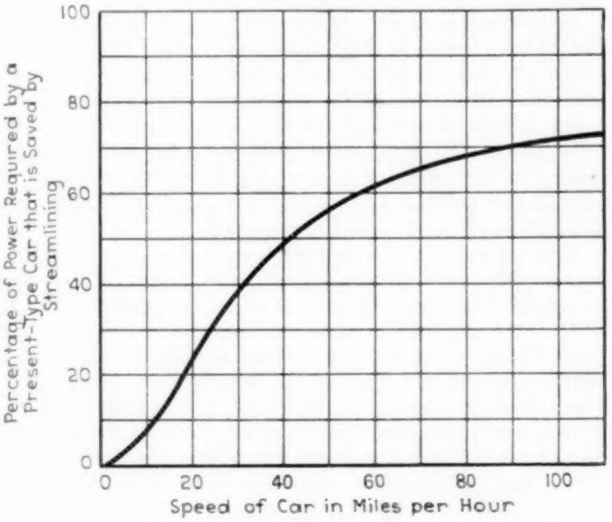

FIG. 21 PERCENTAGE OF THE POWER REQUIRED BY A PRESENT-TYPE CAR SAVED BY STREAMLINING

VI—EFFECT OF STREAMLINING ON AUTOMOBILES

Although the air resistance of automobiles does not strictly come within the scope of the present paper, it nevertheless seems advisable to devote some attention to the important means of transportation it involves. Inasmuch as the automoblie is even nowadays a severe competitor of the railroads, it is highly important for them and particularly for the interurban lines to realize the immense possibilities of the streamline automobile as a means of high-speed transportation especially after super highways have been built.

The air resistance of automobiles was an important factor even years ago when the speed on the open road did not exceed 30 or 40 m.p.h., and is a vital one nowadays when the traveling speed often reaches 60 or 70 m.p.h.

In Fig. 20 the two solid lines show how the power required to overcome air resistance and the power required to overcome journal and rolling resistance depend on the speed. A car weight of 3800 lb. and a projected area of 26 sq. ft. are assumed. These curves are based on numerous tests[9] made in this country and abroad.

According to Fig. 20, a speed of 80 m.p.h. calls for an engine of about 90 hp. of which 14 hp. is required to overcome journal and rolling friction, and the remainder, namely, 76 hp., or 84 per cent of the total power required, is needed to overcome air resistance. Even for the comparatively low speed of 40 m.p.h. the power required to overcome air resistance runs up to 60 per cent of the total power. This comparatively enormous amount of power required to overcome air resistance, however, can be reduced considerably by proper streamlining.

It has been proved by numerous tests made by the senior author in Germany[10] that partial streamlining such as rounding a few corners or sloping the windshield a little more has but little effect on the air resistance. If, however, the body, wheels, and fenders considered as a unit are shaped according to the precepts of modern aerodynamics, a considerable reduction in air resistance can be effected. As a result of the various tests made by the above-mentioned author, a shape was finally developed that had but one-fifth of the air resistance of the present type of car, and yet afforded the same ample space for four or five passengers as did a sedan (Fig. 22).

Assuming, for example, a speed of 80 m.p.h., the streamlined design shown in Fig. 22 requires but 14 + 76/5 = 30 hp. instead of 14 + 76 = 90 hp. The broken line in Fig. 20 refers to a streamlined car such as that shown in Fig. 22, and it is a striking fact that less than 50 hp. is sufficient to maintain a speed of 100 m.p.h., whereas more than 150 hp., i.e., more than three times as much power, is necessary to obtain the same speed of 100 m.p.h. with the present type of car. The Dusenberg car is said to deliver 265 hp. and to attain a speed of 116 m.p.h. or slightly more. The same speed could be maintained with an engine of 70 to 80 hp. if the body of the car were streamlined as shown in Fig. 22.

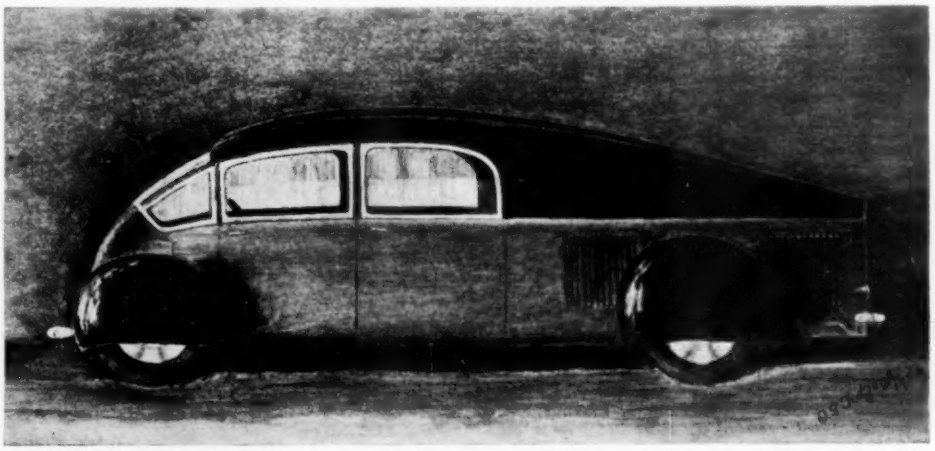

FIG. 22 STREAMLINED AUTOMOBILE WHICH GIVES THE SAVING IN POWER SHOWN IN FIG. 21

In Fig. 21 the saving in total power required is shown for a streamlined car (Fig. 22) in percentage of the total power required for a present-type car. It is seen that even for a speed of 40 m.p.h. a saving of 50 per cent can be effected. This saving increases with the speed, so that at 70 m.p.h. 65 per cent of the power will be saved, and at 90 m.p.h., 70 per cent.

Whereas a present-type car of, say, 80 m.p.h. top speed will travel about 10 miles per gallon of gasoline consumed, 30 miles per gallon could be obtained with a car streamlined according to Fig. 22 and having the same top speed.

These data should be sufficient to convince the reader that within the next few years the bulky-bodied vehicle of today will without doubt be supplanted by the streamlined car.

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[9] Agg, T. R., and N. Wolfard, "Investigation of Air Resistance on Automobiles." Proc. Sixth Annual Meeting, Highway Res. Board, Washington, D. C. (1927). Conrad, L. E., "Wind Resistance of Motor Vehicles." *Public Roads*, vol. 6 (1925), p. 203. Lockwood, E. H., "Air Resistance of Automobiles." *American Highways*, vol. 8 (1929), p. 1.

[10] O. G. Tietjens, "Strömungsforschung" (Research on Aerodynamics), *Technische Rundschau*, March 24, 1926.

VII—Conclusion

The aim of this paper has been to emphasize, first of all, the importance of considering air resistance in its relation to high-speed transportation. It has then been shown in what manner the air resistance can be determined more precisely than by using one of the existing train-resistance formulas. Furthermore it has been pointed out that a considerable saving in the power required can be obtained by the streamlining of high-speed trains and interurban cars, and it has been shown how this saving will affect the economics of the problem of high-speed transportation.

The savings in power consumption made possible by streamlining could be utilized to bring about some of the improvements proposed by Egmont Arens in a recent article entitled "The Train of Tomorrow."[11] Air conditioning, it is believed, will be one of the most desirable of these.

Of course it is beyond any dispute that existing rolling stock cannot be scrapped merely to replace it with streamlined equipment. However, it should be clearly understood that railroad companies anticipating the future development of high-speed trains should start now to build experimental trains of streamlined shape, in order to acquire the experience necessary for the final design of the trains of tomorrow.

As the authors see it, the question is not whether the streamlined shape will be adopted, but when this will happen and who will be the first in this new field of modern transportation.

Discussion

F. W. Pawlowski.[12] Since the authors mention the attempt to greatly increase the speed of electric traction made by the Allgemeine Elektricitaets-Gesellschaft and the Siemens & Halske Company on a German military railroad in 1901, it will be of interest to recall that a similar, although much more thorough, investigation was made in this country on the occasion of the Louisiana Purchase Exposition in 1903, and therefore almost as far back as the other. An "Electric Railway Test Commission" was appointed by the president of the exposition, at the suggestion of Prof. W. E. Goldsborough of Purdue University, to conduct tests with the electric-railway equipment assembled for the exhibit in order "to secure data relating to the train resistance of an interurban car, operated both with and without a trailer, at speeds of from 30 to 70 miles an hour," and also "to obtain similar data on a dynamometer car, specially constructed for the purpose of separately measuring the air resistance . . . with different forms of front and rear vestibules." This dynamometer car, named "Louisiana," was developed and built jointly by several companies, among which the J. G. Brill Car Company, the Pressed Steel Car Company, and the Westinghouse Electric and Manufacturing Company contributed the major parts. One-fourth of the 620-page report of the commission deals with the tests of the car "Louisiana." While the car did not bear any relation to the German car, aerodynamically (as we can see now) it was of a superior and more rational design.

An old engineering principle is that stating, "every engineering problem is a compromise problem," and from that point of view the Kruckenberg car, or the so-called "railway Zeppelin," comes under the category of freaks and its performance into that of stunting. Everybody knows that a car shaped like a cigar or a fish will have much smaller air resistance than a box-type car, and also that a car built like an airplane fuselage (a skeleton of alloy-steel tubing, partially covered with fabric) will be much lighter than the usual type of car; but such a car will deteriorate before it will earn any profit.

The writer's admiration is diverted from the generally lauded automobile engineer to the railway-car engineer. Of the $25,000 which the interurban-car manufacturer receives, he pays out two-thirds to others for the chairs, windows, window sashes, doors, electric power equipment, and various auxiliaries, and then for the remaining one-third he produces a car that will run up to 300,000 miles per year for more than 30 years. In view of this astounding performance, many an engineering feat will certainly fade into the background.

Even an empirical formula should be dimensionally correct and should stand a physical interpretation. While it is possible to ascribe the necessary dimensions to the constants C_1, C_2, or C_3 in the expressions for the functions A and B of the weight of the car, in Equation [1], it is rather hard to find any physical meaning for the first term, as well as any good reason for the presence of the factor V in the second term of this equation.

The well-known chaos in the equations for train resistance should not be surprising at all, considering the date of origin of all these equations. A very similar chaos characterized our notions about the apparently simplest case of experimental aerodynamics, resistance of a flat normal plate.

Splitting the train resistance in two items, one representing the road resistance (or mechanical resistance, as some of the railroad engineers call it) and the other the air resistance, the total train resistance can be expressed accurately enough as:

$$R = K_r W + K_a A V^2$$

where K_r is a pure number and K_a has the dimensions of mass per unit volume and is a product of another pure number and mass density of air at some standard conditions.

By using this formula, together with reasonable values of K_r, as found in literature on railway engineering, and K_a, as determined from modern wind-tunnel tests, an excellent agreement of results can be found with those of the Aspinwall formula.

The motor power in horsepower delivered to the wheels of the car on level road in still air can be therefore written as

$$P = 0.00267\,(K_r W + K_a A V^2)V$$

The coefficient K_r takes care, not only of the work necessary to overcome the car wheels and the bearing friction, but also the deformation work of the whole car (never running on a perfectly even track) and that of the rails and the track, which goes into heat. This last item constitutes an appreciable amount in the whole.

The coefficient K_a can be considered as the specific air resistance of the car, in pounds per square foot per miles per hour, based on cross-sectional area of the car A in square feet and the translational speed V in miles per hour. The relation between this coefficient and the coefficient used by the authors in Equation [2] is:

$$K_a = \left(\frac{5280}{3600}\right)^2 c\,\frac{\gamma}{2g} = \left(\frac{5280}{3600}\right)^2 \cdot \frac{0.0765}{64.4}\,c = 0.00255\,c$$

for standard atmosphere (15 deg. cent. and 76 cm. Hg).

It is of course very important to consider in problems of train resistance the climatic, seasonal, and even hourly variations of air density, but the 15 per cent variation mentioned by the authors is a rather optimistic value. The writer can recall cases, right here in Michigan, when the air density on a crisp winter morning was 30 per cent greater than on a hot summer afternoon.

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[11] *Advertising Arts* (Supplement of *Advertising and Selling*), July, 1931.

[12] Guggenheim Professor of Aeronautics, University of Michigan, Ann Arbor, Mich.

In splitting the coefficient c of Equation [2] into components corresponding to the locomotive and the n cars of a whole train ($c = c_0 + c_1 + c_2 + \quad . \quad . \quad . \quad . \quad + c_n$), it is quite true that "there is little difference between the coefficients of the different coaches, so that . . . " the expression may be written in a simpler form. However, this observation does not apply at all to the last car, so that the simplified expression shall read as follows:

$$c = c_0 + (n - 1)\, c_1 + c_n$$

All of this has been known ever since Prof. W. F. M. Goss, of Purdue University, published in *The Electric Review* of October, 1898, the remarkable results of his wind-tunnel experiments with scale models of railway trains. Wind velocities up to 100 m.p.h. were used in this experiment. While these results might have been rather startling at that time, they are perfectly obvious in view of our present notions concerning aerodynamics.

Undoubtedly an experienced aeronautical engineer is in a position to make offhand suggestions as to the shaping of vehicles with the view of reducing their air resistance; however, in order to determine the quantitative effect of any suggested modification, in each case, an aerodynamic test will be necessary, until the so-called perfectly streamlined shapes will be generally adopted. In the meantime we do not know as yet what the shape of the perfectly streamlined vehicle should be.

Of the two possible methods of aerodynamic tests, the scale-model wind-tunnel method and the full-size road-test method, the former is unquestionably more convenient and less expensive, but it is doubtful whether it is "more exact," and this particularly on account of the very crude reproduction of the ground effect by means of the "reflection method." While the reflection method is the easiest to use, it certainly is not entirely satisfactory.

The laboratory equipment used by the authors, shown in Fig. 5, is interesting, since it is not of the usual type. The wind tunnel seems to be simplicity itself. Unfortunately, however, the dynamometric balance provides for drag measurement only.

It is impossible to guess from the content of the paper why this balance is referred to as "torsion balance;" however, if the authors really make measurements with a systematic error of between 1 to 2 per cent, they should be congratulated for the success in developing such a sensitive and accurate equipment.

In view of the writer's statements regarding the split coefficient for trains composed of several cars, the results for trains as summarized in Fig. 8 of the paper do not appear convincing; besides, extrapolation of results obtained for one and two cars back of the locomotive, up to as many as six cars, is entirely unwarranted.

Professor Goss, who experimented systematically with trains up to 25 cars, found that the resistance of the last car in a train composed of two cars and a locomotive is 3.7 times that of the preceding car. This multiple gradually decreases to a value of 2.5 at 10 cars in the train, and then remains constant up to 25 cars. While the resistance of the intermediate cars in a train varied within $\pm 2^1/_2$ per cent, the resistance of the first car back of the locomotive was always 20 per cent less than that of the intermediate cars. Since Professor Goss used models of nearly the same size as the authors of the paper used ($3^3/_8$ in. wide and $4^1/_2$ in. high) and experimented at higher wind velocities (up to 100 m.p.h.) and therefore was dealing with forces of a magnitude which do not offer difficulties in obtaining accurate measurement, it is impossible to reject his results as erroneous.

If the statement as to a 40 per cent reduction of air resistance of a locomotive model obtained by highly waxing and polishing the model surface is correct, then this will be the most valuable information found in the paper. The original finish of the model was very likely nothing other than the usual finish produced with a fine sandpaper. That waxing and polishing could reduce the air resistance so greatly on a model of the size in question is difficult to understand.

The advantages of streamlining electric trains, from the business point of view, is interesting, for it does not follow the familiar method based usually on performance curves worked out according to the S. B. Cooper method. It seems that "the additional horsepower required" for "an increase of schedule speed from 40 to 100 m.p.h." for a streamlined train, as found from the diagram (Fig. 10), should be 3700 instead of 1800, with the resultant cost per passenger mile, figured from Equation [3], to be 0.41 cent instead of 0.2 cent.

Also it seems that the conclusions as to the advantageous capitalization of the streamlined cars will be considerably reduced if one takes into consideration the longevity and the low cost of the well-built American electric cars of the present (conventional) type and the possibility of a great increase in the cost of cars of some novel design.

A great Eastern electric-railway company for which the writer investigated during the past year 30 car designs, among others one much like the authors' car 1, some of them offering a possibility of cutting the total power at 90 m.p.h. one-half or two-thirds, as compared with the conventional type, has adopted for one line a compromise design with a power reduction of not quite 30 per cent. These people, with years of experience in operating and in buying electric trolley cars, do not deliberately handicap their own interests in adopting an aerodynamically less efficient type of car. In these 30 designs the middle body of the car was left unchanged; the streamlining was limited to the vestibules.

The values of air-resistance coefficients given by the authors (Fig. 12) seem to be remarkably low for all four models.

Among the 30 cars tested, there were at least two corresponding closely to the authors' cars 1 and 4; however, the writer obtained for them much higher coefficients. The models were of practically the same size (scale 1 to 24) and employed wind velocities up to 90 m.p.h. The writer soon will be able to conclude, from the comparison of actual performance of the old-type and the new cars now in operation, how bad or how good are the coefficients obtained.

The writer is unable to agree with the statement that "partial streamlining such as rounding corners or sloping the windshield . . . has but little effect on the air resistance" of vehicles. Experiments made a few years ago with parallelopipeds, approaching roughly the proportions of a sedan body and having 1 sq. ft. cross-sectional area, developed that even a little rounding of edges (with a radius $^1/_{12}$ of the side of the square section) reduces the coefficient of the air resistance 47 per cent at 60 m.p.h. and 77 per cent at 90 m.p.h., as compared with the sharp-edged shape, and that there is a certain radius of rounding the edges at which the coefficient is a minimum for a given wind velocity. Beveling of the flanks of the parallelopiped an amount as observed in some of the modern automobile bodies reduces the coefficient more than 60 per cent at wind speeds between 60 and 90 m.p.h. Even a slight sloping of the windshield reduces appreciably the air resistance.

The proposed design of an automobile, although esthetically pleasing, does not seem to be practical: (1) The protruding tailpiece (on the same wheelbase) will render the already acute parking situation still more difficult; (2) the rear location of the motor will undoubtedly result in a much less reliable car. All possible ways of distributing the vital parts of the car mechanism have been tried, but the real development of automobiles begins with the discovery by Panhard and Levaseur, some 30 years ago, of the present arrangement (engine, clutch, transmission, propeller shaft, rear axle), evidently the most practical one. Any radical change in this arrangement will result in a loss of years of experience responsible for the remarkable reliability of present automobiles.

The authors give an excellent discussion of the so-called scale effect and a presentation of some of the principles of aerodynamics. This section of the paper certainly renders it much clearer to engineers not familiar with this branch of mechanics.

O. C. Cromwell.[13] It will be interesting to bring to attention the early attempt of the Baltimore & Ohio in research on train resistance. In 1890, 40 years ago, Mr. Frederick U. Adams, of Chicago, had assigned to him a train consisting of one baggage car and five coaches at the Mt. Clare shops of the Baltimore & Ohio Railroad, in Baltimore, to be fitted up with what is now called streamline design, to demonstrate the improvement in train air resistance.

The cars were fitted up with uniform contour lines on top of the train, giving an appearance of a torpedo, and the sides were extended below the sills to just above the track, with sufficient clearance to avoid right-of-way obstructions; this extension was to cut out the air resistance of the running gear of the train. Fig. 23 will give an idea of the smooth contour of the exterior of the cars, the photograph having been taken on the viaduct, Relay Station, Baltimore, Md., about 9 miles out of the City of Baltimore.

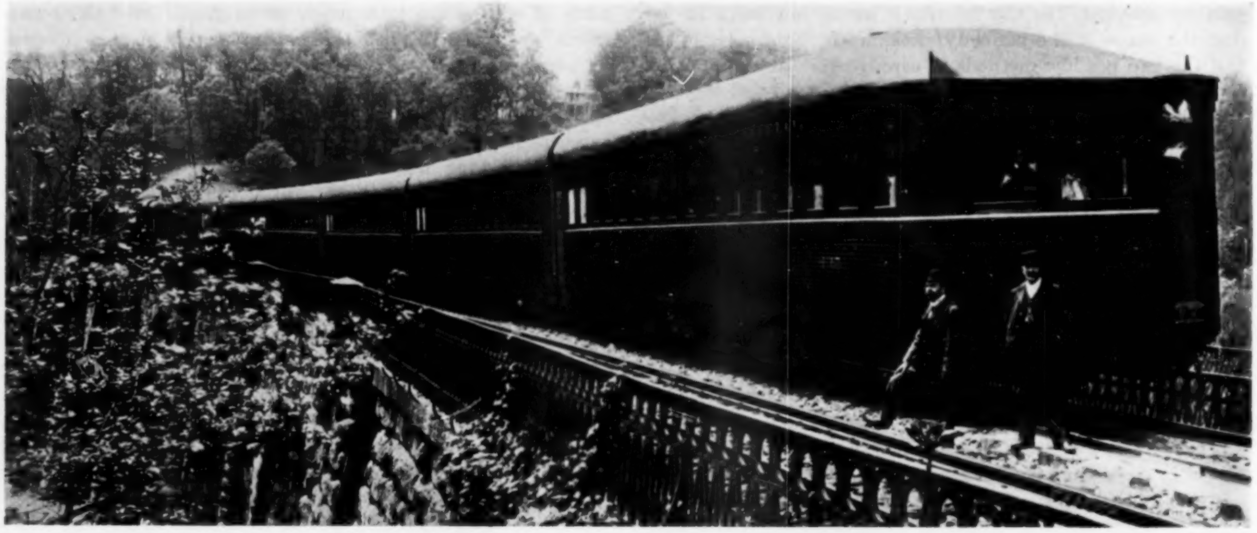
Fig. 23 Baltimore & Ohio "Streamlined" Train Successfully Operated in 1890

After being equipped, this train was popularly termed the "Wind Splitter," and it operated, while being developed, between Baltimore and Washington, a distance of 40 miles, attaining at times a speed of 68 m.p.h., the train being handled by American-type locomotive 822, which was rather light for the service. The drivers were 69 in. in diameter, the weight 94,300 lb., and the tractive power 15,476 lb.

After the development between Baltimore and Washington, the train was operated between Baltimore and Philadelphia, a distance of 96 miles, by American-type locomotive 854, of heavier weight. The drivers were 78 in. in diameter, the weight 114,500 lb., and the tractive power 16,700 lb.

On the return trip, Philadelphia to Baltimore, the train at times attained a speed of 82 m.p.h.

The test demonstrated the values of streamlining, but the experiment was ahead of the times, and after a few minor mishaps with the extension below the sills, the train was removed from service.

William Elmer.[14] It is a little unfortunate that the data in regard to trains is limited to a locomotive and six cars. Most of the heavy through trains are composed of twelve to fourteen cars, and on our railroad we are looking forward to speeds of 90 to 100 m.p.h. in the electrified territory. There is also a lack of information concerning the effects of a quartering wind or a side wind. Our experience with long freight trains on the road indicates that a head wind of, say, 30 m.p.h. against such a train of, say, 100 cars moving at 30 m.p.h. is not so serious as a side wind, and this does not increase the resistance as much as a quartering wind. Experience on a locomotive indicates that under favoring conditions the train resistance may be more than doubled by a quartering wind. This is of course due to the fact that there are one hundred openings between the cars, each of which presents a resistance point.

In the early days of passenger transportation the coaches were of the open-platform type, and while the speeds were of course low, there was a serious amount of air resistance even at speeds of 40 m.p.h. Later the narrow vestibules were introduced, and the air resistance was cut down, and now we have wide vestibules with the corners of the cars and roofs rounded off, but there are still openings between cars amounting to a foot or so from the corners into the diaphragms forming the passageway between platforms.

It would seem to be quite difficult to design steam-railroad cars with smooth surfaces at the ends where the cars couple together, but there is certainly much food for thought in the paper, and undoubtedly the results obtained will justify considerable research on the part of railroads and manufacturers.

Boris A. Bakhmeteff.[15] The paper should be welcomed as a most valuable and timely contribution to the art. Any one who has had to deal with locomotive and train resistance is aware of how scant and insufficient is the actual knowledge in the case. Until lately railroad engineers have been working with notions and figures which date back to a rather antiquated period, and so far no real advantage has been taken of the more recent development of aerodynamics and of the progress in the methods of testing models in wind tunnels.

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[15] Professor, Columbia University, New York, N. Y. Mem. A.S.M.E.

The present state of the art is revealed, for example, by the diversity of basic data regarding the so-called "wind" or head resistance of a locomotive. In American practice[16] the usual formula $W = a \cdot S \cdot V^2$ (W being the resistance in pounds, S the cross-section in square feet, taken to be 120 sq. ft., and V the speed of the wind in miles per hour) is used with a coefficient $a = 0.002$. Translated into the terms of the dimensionless drag coefficient in Equation [2], the coefficient $a = 0.002$ corresponds to $c = 0.784$, or to a so-called disk ratio of about 0.70.

The German practice of evaluating train resistance is based principally on the work of Frank[17] and of Strahl.[18] The latter in discussing the value of the coefficient K to be used in the formula $W = K \cdot S \cdot \left(\frac{V}{10}\right)^2$ (W being the resistance in kilograms, S the cross-section in square meters, and V the speed in kilometers per hour) arrives finally at the figure of $K = 0.6$, his conclusions being drawn from experimental evidence collected by the Prussian railways and based on the premise that the head resistance of a locomotive exceeds by about 10 per cent the resistance of a disk of equivalent section. In other words, Strahl suggests a disk ratio of 1.1, which would make the coefficient C in Equation [2] close to 1.25, or over 50 per cent above the American value.

Whether this discrepancy is to be attributed to the necessarily unexact methods used in experimenting on trains in service or whether the form lines of an American engine are more advantageous than the outline of a Continental locomotive could be answered only by reliable aerodynamic tests.

The authors lay particular stress on the resistance of the train, advocating the streamlining of the train as a whole, and appear to attach less importance to the head resistance of the locomotive as such.

There is no question that in the resistance of the aggregate the part played by the cars usually predominates. Nevertheless, the resistance of the locomotive front should by no means be ignored, particularly with passenger trains operated at high speed.

Garbe, the great German authority, when evaluating the wind resistance caused by a normal passenger train, weighing 600 tons and traveling at 90 km. per hour, figures the locomotive resistance to be nearly 30 per cent of the total.[19]

The American formula when applied with $a = 0.002$ to an engine running at 100 m.p.h., gives a resistance measured at the base of the wheel of near 2400 lb. and a loss in power of over 600 hp. A reduction of head resistance by proper shaping might measurably add to the tractive force, which of course is particularly important at high speeds.

C. E. Barba.[20] The authors' analysis of the helpfulness in considering modification in design of passenger equipment for high-speed and interurban service is most opportune, and indicates the adaptation of the principles long used in ship-hull construction by the use of models to determine the proper waterlines for a minimum of friction at maximum speeds.

The paper presents many interesting results based upon a definite design, but these cannot always be applied. The query therefore arises as to how this data could be utilized without making additional experiments on a complete practical application to a whole train.

The energy generated by a railroad prime mover is utilized in storing up kinetic energy in the train load and overcoming frictional and wind resistances. The authors have shown how the large item of wind resistance can be minimized. Frictional resistance in its relation to the other two factors is a small item and one that already, at speed, is at a more or less irreducible minimum.

Kinetic energy from its nature is directly proportional to mass or weight of train. The use of improved materials of construction will permit of decreasing unit car weight, consequently train load for equal carrying capacity, and with a fixed energy-producing prime mover, increased speed with equal carrying capacity or increased hauling capacity with fixed speed can be attained through reduced light car weight or, as pointed out by the authors, through lessened wind resistance, or by a combination of both.

Energy cannot be destroyed, but must be overcome or dissipated through work done. In normal stops this dissipation is through the work of the brakes. Under abnormal stopping conditions, as through wreck or accident, equal safety with lighter materials of construction and lighter design is obtained in that the energy required to be dissipated in a short period of time takes place in the crumpling up of the vestibules and car platforms and not in damage to the car bodies.

A lighter car structure can be employed, since the kinetic energy at any given speed will be reduced proportionately to the weight of the structure, and therefore the destruction forces to be absorbed in accident will be proportionately reduced for equal carrying capacity.

Since the damage done is therefore proportional to the resistance offered, and as this resistance is based on weight, a lighter structure could therefore be employed, so that the damage it would sustain would be equivalent.

Streamlining would start with the prime mover, and although the front of our present locomotives, either steam or electric, could be modified to produce the necessary effects, the following conditions would bear some investigation as governing determinates in such modified constructions:

(1) The necessity of smokebox inspection at frequent intervals
(2) Relocation of air pumps at front end and the necessary ability to inspect same or to get at them for repacking purposes
(3) The headlight and front marker lamps would have to be relocated
(4) Dummy coupler and uncoupling mechanism could not be covered up
(5) Removal of front cylinder heads for packing-ring renewals would mean removable shields
(6) Steps at front end from running board and handrails would no doubt be eliminated
(7) Passing from one side to the other at bumper beam would not be practicable
(8) Cab vision may be obscured.

The foregoing is based upon the idea of a superimposed body, made of light material, based upon the design of a streamline front end.

Streamlining, theoretically, should also inclose the sides of locomotive, in which the shape of boiler, running board, and space below should blend into the cab. This would present the following difficulties:

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[16] Handbook Amer. Loc. Co., 1917; Kent's M. E. Handbook, etc.

[17] *Z.V.D.I.*, 1907.

[18] *Z.V.D.I.*, 1913.

[19] *Zeitgemässe Heissdampf Lokomotiven*, Berlin, 1924.

[20] Mechanical Engineer, Boston & Maine Railroad, North Billerica, Mass. Mem. A.S.M.E.

1. (1) If half the diameter of the driving wheels is covered over by a shield below the modified running board, the inspection of the spring rigging is not possible, and although such an enclosure is prevalent in European practice, the spring rigging is usually found below the driving box, which is not American practice, although such a design is practicable
2. (2) The cab front would undoubtedly have to be modified, which in some designs would permit the steam turret to be within the cab, as the center portion of the cab front would project along the center line of the boiler
3. (3) The cab and its connection to the tender would undoubtedly have to incorporate a vestibule type of tender; this has been done
4. (4) The height of the tender would require the same contour as in passenger cars, and could be roofed over by having hatches, hinged and overlapping, for coaling purposes. Going up over the tender for taking water should not be considered as good practice where large tenders are involved.

The connection between tender and train or between the units of the train cannot be accomplished as indicated in Fig. 7, as so far no practicable means has yet been discovered wherein two cars do not require the clear space which now exists between vestibule corner posts or end sills to take care of curving, unless the connection made between corner posts is made of a flexible material similar to the diaphragms now in use. This is possible, but hardly practicable, due to the make-up of trains and frequent separations.

Sloping car roofs at ends of cars such as usually prevail are not necessary and neither are clearstories, since any interior ceiling effect can be obtained with a circular or so-called turtle-back roof by the application of falsework attached to the carlins.

In conclusion, it should be remembered that, irrespective of the ideals the present railway fraternity may foster, the final results in design are largely hampered by legislative acts, both state and Federal, that will not permit modernization to the same extent as agencies in other fields of transportation.

The authors are to be congratulated on the presentation of a subject foreign to present engineering practices in railway equipment construction that should assist in stimulating others in performing experiments useful in the development of super-transportation units for high-speed service.

B. S. Cain.[21] This paper indicates very clearly the direction toward which the design of high-speed trains must shortly be directed and gives some idea of the benefits which may result.

It is hardly practicable to divide total air resistance into front pressure, rear suction, and skin friction. However, it seems that for some typical trains of the present day, it should be possible to state approximately what improvements could be made by providing a streamlined locomotive or a streamlined rear-end observation car since such changes may well precede the general replacement of all high-speed Pullman cars; that is, if the authors could show that substantial benefits might be obtained by a partial change while awaiting the day when completely redesigned trains may be available.

It would be interesting to hear the results of any tests with oblique winds on trains, since the effect of a transverse wind may be greater at speed than at rest. Rare as such occurrences may be, there are records of the overturning of trains due to transverse wind.

It also would be interesting to know the effect of streamlining on the head-end stability of a train; that is, the stability of the streamlined locomotive, which would have very little head resistance and might have an appreciable tendency at very high speed to deviate from straight progression.

Referring to the figures of estimated savings, the authors indicate clearly that they have made numerous simplifying assumptions. For example, a 150-ton locomotive developing 2500 hp. is considered, and it is assumed that the locomotive weight will vary with the horsepower. Actually, 150 tons, 2500 hp. corresponds to a recently built locomotive of the Pennsylvania Railroad, which at the same time designed a larger locomotive with 50 per cent more horsepower and only 25 per cent more weight. Actual figures of savings will therefore require to be calculated in greater detail to cover specific cases.

Authors' Closure

By O. G. Tietjens. The discussion by Prof. F. W. Pawlowski is very much appreciated, at least so far as it is of a constructive nature. The principal difference between the tests of the Electric Railway Test Commission mentioned by Professor Pawlowski and the tests of the German companies in 1901 is the different speed applied. The tests with the "Louisiana" car were made at speeds between 30 and 70 m.p.h., which speeds cannot be considered as very high, whereas the Berlin-Zossen tests were made at 130 m.p.h., which is about twice the maximum speed of the "Louisiana" car.

The author agrees absolutely with Professor Pawlowski that even an empirical formula should be dimensionally correct and should stand a physical interpretation. It may therefore be mentioned that in Equation [1] of the paper the quantity A represents that part of the total resistance that does not depend on the velocity; its dimension is of course that of a force, and has the same physical meaning as the first term of Professor Pawlowski's equation. The "good reason" for the presence of the factor V in the second term of Equation [1] is the fact that a certain part of the resistance is approximately proportional to the first power of the speed.

Since the air-resistance coefficient K_a of the second term of the equation suggested in the discussion is not a dimensionless number, but depends on the atmospheric pressure, temperature, and humidity, it is generally conceded to be preferable to use the pure number C instead of K_a or any similar coefficient.

The author cannot quite understand what Professor Pawlowski has in mind when he says that the statement in the paper that 15 per cent variation in air resistance may occur, due to changes in temperature, barometric pressure, and humidity, is rather optimistic. If in some extreme cases, as he observed, the change in air density amounts to 30 per cent, then of course the air resistance also changes 30 per cent, the other conditions remaining unchanged. Just this relation between the air resistance and the atmospheric conditions makes it advisable to use the pure number C in the air-resistance formula instead of K_a, as suggested by Professor Pawlowski.

The author does not agree with Professor Pawlowski that the air-resistance coefficient of the last car will be considerably different from the last-but-one car. The reference to experiments made by Prof. W. F. M. Goss does not mean anything in this connection. The fact that Professor Goss attacked the problem of air resistance of trains in a rational way as early as 1898 must be highly appreciated. However, the results of these tests have no practical value. The objections against the method applied could not be realized at the time Professor Goss carried out his tests. But now, after about 20 years of wind-tunnel practice, it is well known to every one that is at all acquainted with wind-tunnel work that a tube of 60 ft. length and a cross-section of 20 in. by 20 in. will not give results that are free from objections.

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[21] Locomotive Division, Transportation Engineering Department, General Electric Company, Erie, Pa.

Even those wind tunnels of the closed type that have a length of only about twice or three times the diameter, take into consideration the fact that there would be a variation of static pressure in the direction of flow if the diameter of the closed wind tunnel remained constant. The general rule is therefore to increase the diameter of closed wind tunnels slightly in direction of flow. Tests of trains up to 25 cars long in a tube which had a length of more than 20 times its diameter do not represent conditions which can be considered similar to those of the actual train. Furthermore, the method of representing the ground is objectionable and is bound to give wrong results. Although these tests undoubtedly have a certain historical value, and although Professor Pawlowski feels it impossible to reject the results as erroneous, there is no question that they are absolutely out of date on account of the foregoing reasons.

The author agrees that the extrapolation in Fig. 8 of the paper from tests with a locomotive and two coaches to a train of a locomotive and six coaches may be open to discussion. However, it was only intended to give in this paper qualitative figures regarding air resistance of conventional and streamlined trains and to emphasize thereby the importance of streamlining trains and interurban cars as well as automobiles.

With respect to the capitalization of the streamlined cars, it cannot be seen why the cost of cars of some novel design should be necessarily much greater than the cost of the present design, as is the opinion of Professor Pawlowski. The streamlined car may be less expensive due to greater simplicity.

That the smoothness of a very well-streamlined body is of enormous importance regarding its fluid resistance should not be surprising. The statement as to a 40 per cent reduction of air resistance due to polishing the model is made in the paper only for a very well-streamlined body where the total air resistance is almost entirely resistance due to surface friction. The author wishes to refer Professor Pawlowski to tests[22] made at Göttingen, Germany, on air resistance of bodies of various surfaces. The well-known fact may be mentioned, in this connection, that it is very important to have the wetted surface of racing boats as highly polished as possible in order to reduce the frictional resistance to a minimum. The reason for the large influence of the surface condition on the resistance is found in the relation of the breakdown of the laminar boundary layer to the smoothness of the boundary itself.

The extent to which a laminar or a turbulent boundary layer influences the fluid resistance was recently shown in a paper by Dr. C. B. Millikan.[23] The thorough study of this paper may justify Professor Pawlowski's opinion that the statement as to a 40 per cent reduction of air resistance due to a changed surface condition will be for him the most valuable information found in the paper.

It is very interesting for the author to learn from Mr. O. C. Cromwell that the Baltimore & Ohio had built a streamlined train as far back as 40 years ago.

The question of side wind brought up by Mr. W. Elmer and Mr. B. C. Cain is very important. In the near future tests will be carried out at the Research Laboratories of the Westinghouse Electric & Manufacturing Company to determine the influence of side winds on the air resistance of trains of various shapes.

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[22] "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," by L. Prandtl, I. Lieferung, 1921, pp. 120–126.

[23] C. B. Millikan, "The Boundary Layer and Skin Friction for a Figure of Revolution," Trans. A.S.M.E., Jan. 30, 1932, p. 29, paper APM-54-3.

APM-54-24

Vibration During Acceleration Through a Critical Speed

By FRANK M. LEWIS,[1] NEW YORK, N. Y.

This paper gives an exact solution of the problem of running a system having a single degree of freedom and linear damping through its critical speed from rest at a uniform acceleration. An exact expression is found for the envelope in which are located the maximum amplitudes. The results are plotted for various rates of acceleration and for various dampings.

MANY reciprocating engines, turbines, and other types of machinery capable of vibration are normally operated above their critical speed, and must pass through this critical speed in being started, and again in coming to rest. A problem of interest in this connection is the relation between the maximum amplitude of vibration attained, the number of dangerous vibrations, and the rate at which the machine is accelerated through its critical.

Again, in testing such machinery for vibration, continuous records are often taken over a wide range of speeds as the speed of the machine is either increased or decreased. It is known that in such cases the recorded amplitude will be smaller than if the machine were held at constant speed directly on a critical; and further, that an apparent shift in the position of the critical will occur, and that it will appear higher than the true critical when the speed is increasing and lower when it is decreasing.

This paper is a quantitative study of these effects. In particular, the exact solution is obtained for the motion of a single-mass elastic system, with linear damping, under the action of an exciting periodic force, the frequency of which is (*a*) increasing and (*b*) decreasing at a uniform rate. This corresponds to the case of a machine with uniform acceleration.

Passing over for the present the mathematical analysis of the problem, the results, which are shown in dimensionless form in Figs. 1 to 5, inclusive, will be discussed.

Let N = natural frequency of the vibrating system in cycles per second

N' = instantaneous frequency of the exciting periodic force

h = acceleration of the system in cycles per second per second

r = number of undamped free vibrations the system would make from time zero to time t

q = the value of r at which the instantaneous frequency of the exciting force is equal to the natural frequency of the system. q is a measure of the rate at which the system is accelerated.

In the analysis there are obtained the relations

$$q = \frac{N^2}{h} \quad \text{and} \quad N' = N\frac{r}{q}$$

We note that $q = 10$ implies an extremely rapid acceleration rate, probably faster than could be obtained in practice. $q = 500$ implies a rather slow rate; $q = \infty$ implies that the critical is passed through with infinite slowness, and the $q = \infty$ curves are, in consequence, the classical resonance curves for constant frequency.

R, the resonance factor, is the ratio of the force actually in the spring of the system at any instant to the maximum value of the periodic force.

γ, a dimensionless damping constant, is, in the case of constant frequency, the reciprocal of the value of R at resonance.

It is shown in the analysis that a periodic force in which the frequency is increasing at a uniform rate may be specified by

$$P \cos\left(\frac{\pi r^2}{q} + \sigma\right)$$

where σ, a phase-angle constant, may have any value.

The results for the case of zero damping are shown in Fig. 1. Curve A is the periodic-force curve $\cos\left(\frac{\pi r^2}{q} + \sigma\right)$ for $q = 10$ and $\sigma = 0$.

Curve B gives the somewhat irregular motion of the body when initially at rest and then acted upon by the force specified by curve A.

If the body were initially held so that the spring force was equal to the initial value of the periodic force, the resultant motion curve would be of the simpler form given by C.

After the maximum amplitude has been reached the exciting periodic force is able to extract but little of the energy which it has put into the system, and, as there is no damping, the body will continue to vibrate with an amplitude but little short of its maximum. It may be shown that ultimately the body will be vibrating with its natural frequency and with an amplitude $R = \pi\sqrt{q}$.

With a value of σ different from zero, a periodic-force curve such as A and a motion curve such as C are obtained. The variation of σ produces a family of such curves, and the envelope of this family of curves is of particular interest since the maximum amplitudes obtainable are located on it. Further, the envelope is much easier to compute than the curves of actual motion of the body.

R_e denotes this resonance envelope, and the full lines of Fig. 1 give the envelope solution for $q = 10$, 20, 40, and 80, respectively.

The case of a machine with decreasing speed is simulated in the following manner. Assume that at time infinity in the past the body was at rest and was acted on by a periodic force of infinite frequency. At time zero this frequency has decreased to zero. Then if we fix our attention on negative values of time, the body will pass through a critical at some negative value of r. The solution thus obtained for decreasing frequency is very similar to that for increasing frequencies. The shift of the critical is practically the same, the maximum amplitudes now occurring at speeds lower than the constant-frequency critical. The maximum amplitudes are slightly larger for decreasing speed than for increasing.

Since machines entirely without damping cannot be con-

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[1] Professor of Engineering, Webb Institute of Naval Architecture. Mem. A.S.M.E. Professor Lewis has specialized in research relative to the vibration of ships and machinery.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

structed, the results for the case of zero damping are purely of academic interest.

Figs. 2 to 5 show the R_e-curves for various values of the damping constant γ and the acceleration constant q, the upper set of curves being for decreasing frequency and the lower for increasing.

In a qualitative way, at least, many of the features of these curves could be anticipated.

The greater the acceleration rate, the greater the shift of the maximum and the smaller its amplitude.

The greater the damping, the more nearly do the curves approach the resonance curves for constant frequency.

After the maximum is passed, the envelope curves, where the damping is small, are oscillating in character, but are eventually asymptotic to the resonance curves for constant frequency.

It may be noted that unless the acceleration is extremely rapid, or there is little damping, the amplitudes are not a great deal smaller than the constant-frequency amplitude. With normal acceleration rates the shifts of the critical are sufficient to be of importance when making vibration tests.

Attempts to find empirical formulas which will approximate to the amplitude and shift of the critical over the entire range of q and γ have not as yet been wholly successful.

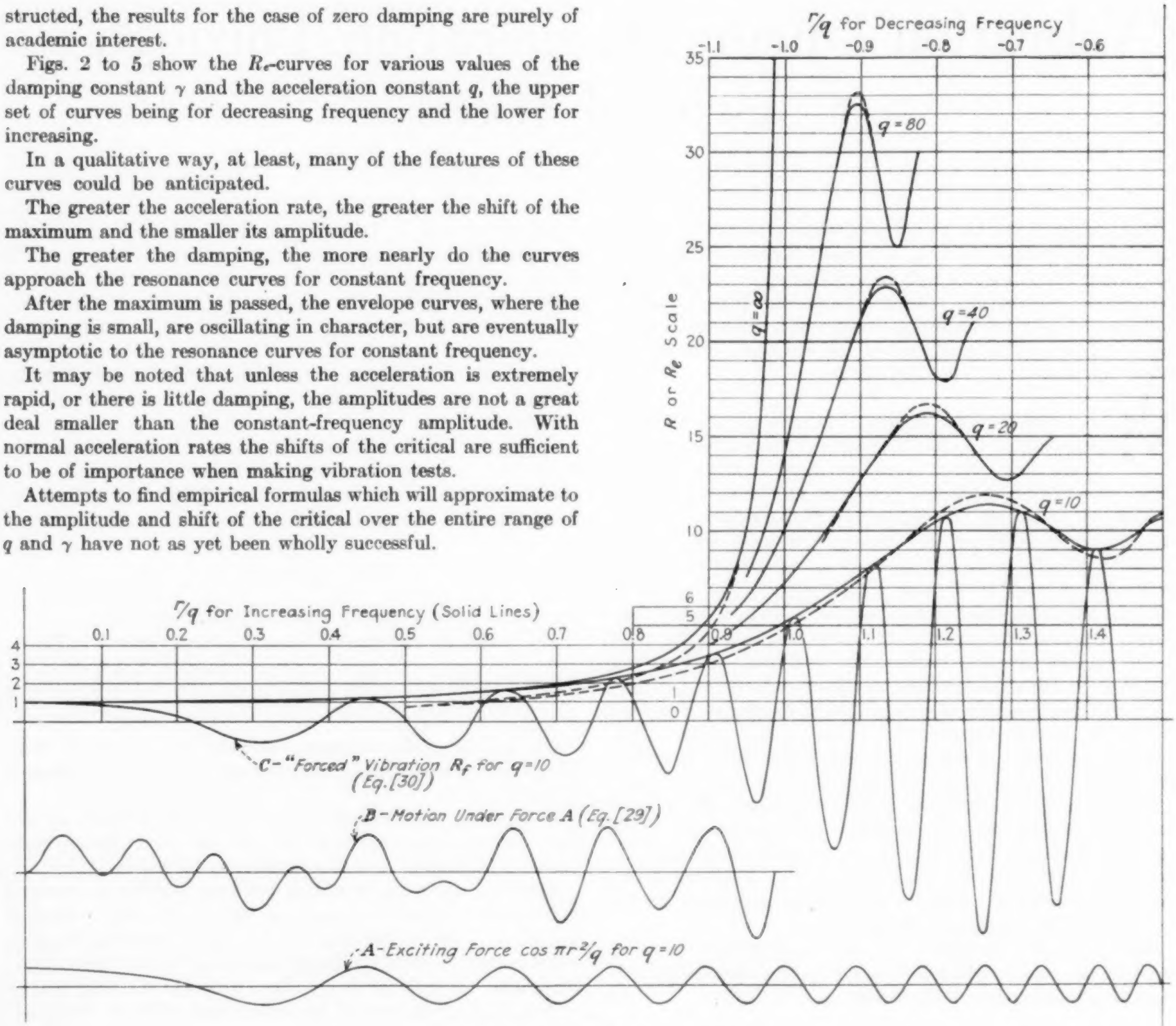

FIG. 1 CASE OF ZERO DAMPING ($\gamma = 0$)

[Solid lines for increasing frequency; broken lines for decreasing frequency. For large values of q, R_e (max.) = $3.67\sqrt{q} \pm 1/4\sqrt{2}$ (— for increasing frequency, + for decreasing). Shift of the maximum from $r/q = 1$ is equal to $0.853/\sqrt{q}$.]

The application of the results is limited for the present to single-mass elastic systems, or systems which can be considered sufficiently close approximations to this type. Further remarks on systems of several degrees of freedom are made in the analysis.

ANALYSIS

Consider the motion of a single-mass elastic system with linear damping under the action of a variable force $f(t)$. Let the weight be W, the damping constant d, and the spring constant k. Then the well-known differential equation of motion of such a system is

$$\frac{W}{g}\ddot{x} + d\,\dot{x} + kx = f(t) \quad \ldots\ldots\ldots\ldots\ldots [1]$$

or

$$\ddot{x} + 2n\dot{x} + p^2x = \frac{g}{W}f(t) \quad \ldots\ldots\ldots\ldots\ldots [2]$$

where

$$\left.\begin{array}{ll} p^2 = \dfrac{kg}{W} & 2n = \dfrac{dg}{W} \\ \text{also let} & \\ \gamma = \dfrac{2n}{p} & p = 2\pi N \end{array}\right\} \quad \ldots\ldots\ldots\ldots [3]$$

It may easily be verified that a solution of [2] subject to the condition $dx/dt = 0$ and $x = 0$ when $t = 0$, is:[2]

$$x = \frac{g}{W(\alpha - \beta)}\left\{e^{\alpha t}\int_0^t f(t)e^{-\alpha t}dt - e^{\beta t}\int_0^t f(t)e^{-\beta t}dt\right. . \quad [4]$$

where

$$\left.\begin{array}{l} \alpha = -n + i(p^2 - n^2)^{1/2} \\ \beta = -n - i(p^2 - n^2)^{1/2} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots [5]$$

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[2] See Jeffrey's "Operational Methods in Mathematical Physics," 1.7 (8).

MUST ASSUME

Equation [4] is the solution for the motion of a body initially at rest and then subjected to any variable force which is a function of the time. If $f(t) = P \cos mt$, this solution reduces to the classical solution for constant frequency, plus a term representing a damped harmonic motion which is due to the initial conditions imposed.

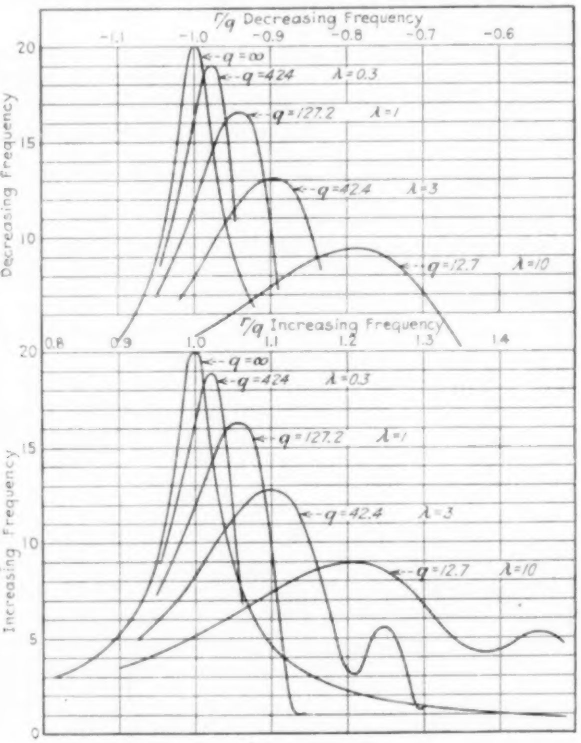

FIG. 2 RESONANCE ENVELOPES ($\gamma = 0.05$)

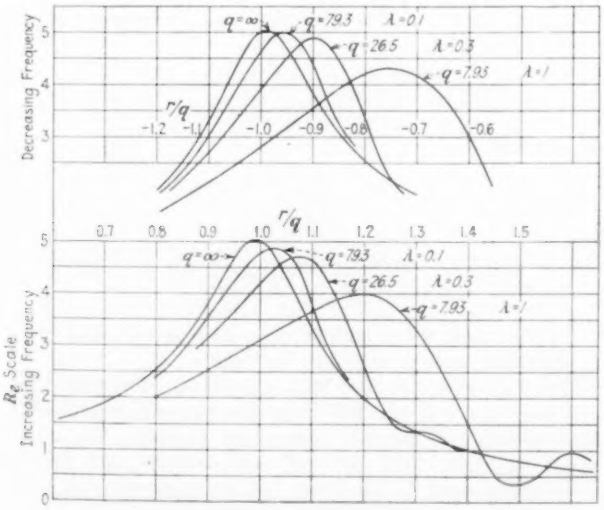

FIG. 4 RESONANCE ENVELOPES ($\gamma = 0.2$)

A periodic force in which the frequency is increasing at a linear rate may be specified by the equation

$$f(t) = P \cos (\pi h t^2 + \sigma) \dots\dots\dots\dots\dots\dots [6]$$

where h and σ are constants, σ being the initial phase angle of the exciting force.

It is perhaps not permissible to speak of "frequency" in connection with a force as specified by [6]. We may, however, define the "instantaneous frequency" of a periodic force $\cos \pi h t^2$ as being the same as the frequency of $\cos mt$, which is equal to $m/2\pi$ when

$$\frac{d}{dt}(\pi h t^2) = \frac{d}{dt}(mt)$$

since this will make the slopes of the two curves equal at the

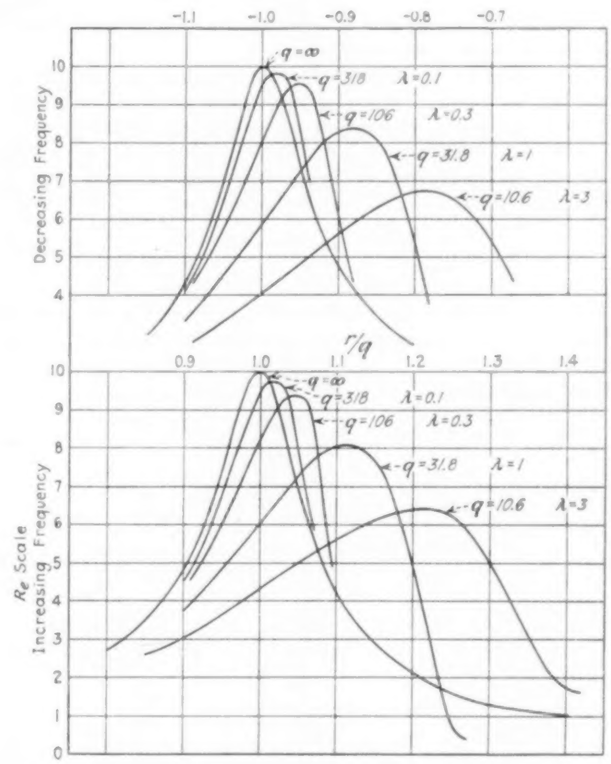

FIG. 3 RESONANCE ENVELOPES ($\gamma = 0.1$)

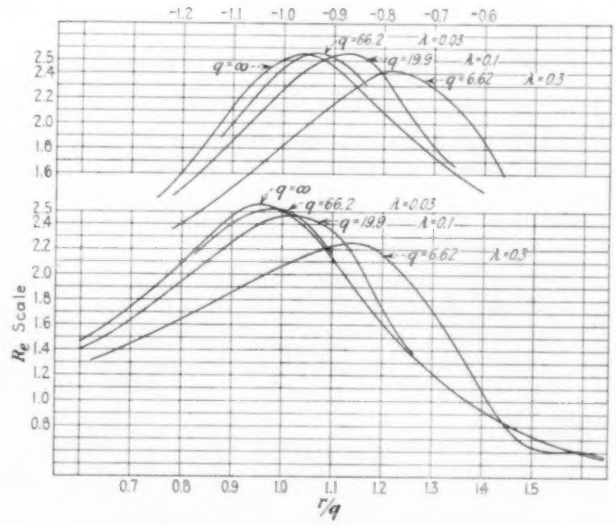

FIG. 5 RESONANCE ENVELOPES ($\gamma = 0.4$)

same angular phase of the motion. From this, if we call N' the instantaneous frequency,

$$N' = ht \quad \text{.........................} [7]$$

from which it follows that h is the acceleration of the system in cycles per second per second.

Substitute [6] in [4] and there is obtained the solution for the case of a body initially at rest and acted on by a periodic force in which the frequency is increasing at a uniform rate.

Before proceeding further it will be convenient to express this solution in dimensionless form. The time variable will be changed from t to r, according to the relation

$$pt = 2\pi r \quad \text{or} \quad Nt = r \quad \text{....................} [8]$$

N being the natural frequency of the system. Let q be the value of r at which the instantaneous frequency of the periodic force is equal to the undamped frequency N of the elastic system. In [7] make $N' = N$, and in [8], $r = q$; then, eliminating t, there is obtained

$$q = \frac{N^2}{h} \quad \text{......................} [9]$$

From [7], [8], and [9] there is obtained

$$N' = N\frac{r}{q} \quad \text{....................} [10]$$

and

$$\pi h t^2 = \frac{\pi r^2}{q} \quad \text{....................} [11]$$

Let

$$R = \frac{kx}{P} \quad \text{........................} [12]$$

and

$$\delta = \left(1 - \frac{\gamma^2}{4}\right)^{1/2} \quad \text{.................} [13]$$

In place of [6], $f(t)$ will be written in the exponential form

$$f(t) = Pe^{i(\pi h t^2 + \sigma)} = Pe^{i(\pi r^2/q + \sigma)} \quad \text{.........} [14]$$

in virtue of the relation $e^{i\theta} = \cos\theta + i\sin\theta$ and with the understanding that the imaginary part of the result will be ultimately discarded.

With these changes of variable, Equations [4] and [5] reduce to the dimensionless form:

$$R = -\frac{i\pi e^{i\sigma}}{\delta}\left\{e^{2\pi i u_1 r}\int_0^r e^{i\pi(r^2/q - 2ru_1)}dr - e^{2\pi i u_2 r}\int_0^r e^{i\pi(r^2/q - 2ru_2)}\,dr\right\} \quad \text{......} [15]$$

where

$$\left.\begin{aligned} u_1 &= \delta + i\gamma/2 \\ u_2 &= -\delta + i\gamma/2 \end{aligned}\right\} \quad \text{................} [16]$$

If reduced to real form, [15] contains four integrals of the type

$$\int_0^r e^{ar}\cos br\cos cr^2\,dr$$

These integrals cannot be reduced to any functions which have been evaluated. We are interested in their value in the vicinity of the critical speed; that is, for large values of r. In cases of engineering interest, involving relatively slow acceleration rates, there may be several hundred nearly equal positive and negative loops in the integrand between time zero and the critical speed. For this reason we consider their evaluation, with the slightest attempt at accuracy, quite impossible by any direct method of numerical integration, or by mechanical or electrical integraphs.

In the case of zero damping, when $a = 0$ the integrals may be reduced to Fresnel's integrals.

The so-called process of contour integration gives a successful method of attack, and even in the case of zero damping furnishes a simpler solution than the one involving Fresnel's integrals. In Equation [15] assume that r may have not only real but complex values. Then $\int_0^r$ is to be interpreted as an integration along the real axis in a complex r-plane.

Consider the first or u_1-term of the integrals in Equation [15] and transform from a complex variable r to a complex variable z_1 according to the relation

$$\begin{aligned} z_1 = x_1 + iy_1 &= -i\pi q\left(\frac{r}{q} - u_1\right)^2 \\ &= -i\pi q\left(\frac{r}{q} - \delta - \frac{i\gamma}{2}\right)^2 \quad \text{......} [17] \end{aligned}$$

then

$$r = \sqrt{\frac{iz_1q}{\pi}} + qu_1 \quad \text{................} [18]$$

In [17] assume that r has real values only, and separate the real and imaginary parts. Then

$$\left.\begin{aligned} x_1 &= \gamma\pi q\left[\frac{r}{q} - \delta\right] \\ y_1 &= -\pi q\left[\left(\frac{r}{q} - \delta\right)^2 - \frac{\gamma^2}{4}\right] \end{aligned}\right\} \quad \text{..........} [19]$$

Eliminating $\left(\frac{r}{q} - \delta\right)$ between these equations there is obtained

$$y_1 = -\frac{x_1^2}{\pi q\gamma^2} + \frac{\pi q\gamma^2}{4}$$

or, writing

$$\pi q\gamma^2 = \frac{1}{\lambda} \quad \text{.....................} [20]$$

$$y_1 = -\lambda x_1^2 + \frac{1}{4\lambda} \quad \text{................} [21]$$

This parabola is the path of integration in the z-plane which is equivalent to the real axis in the r-plane.

Fig. 6 shows a sketch of such a parabola. The point z_1' corresponds to $r = 0$, and the integration proceeds from here in a counter-clockwise direction around the parabola. In order to make Equations [17] and [18] self-consistent, the following conventions must be observed. Of the two values of $\sqrt{i}$ the value $\left(-\frac{1+i}{\sqrt{2}}\right)$ must be used. z_1' must be taken to lie in the (—1) quadrant, that is, arg z_1' lies between $-\pi/2$ and 0. As we proceed around the parabola, z_1 passes through the first, second, and third quadrants, so that ultimately arg z_1 lies between π and $3\pi/2$.

With these conditions and substituting r from [18], the first term of [15] within the braces can be written:

$$e^{2\pi i u_1 r}\int_0^r e^{i\pi(r^2/q\,-\,u_1 r)}\,dr = \frac{1}{2}\sqrt{\frac{iq}{\pi}}\,e^{z_1'}e^{2\pi i u_1 r}\int_0^r \frac{e^{-z_1}dz_1}{\sqrt{z_1}}\ldots\ldots[22]$$

where

$$z_1' = -\,i\pi q u_1^2\ldots\ldots\ldots\ldots\ldots\ldots[22a]$$

Returning to Equation [15], the u_2-term is transformed according to the relation

$$z_2 = x_2 + iy_2 = -\,i\pi q\left(\frac{r}{q} - u_2\right)^2 = -\,i\pi q\left(\frac{r}{q} + \delta - \frac{i\gamma}{2}\right)^2\ldots\ldots[23]$$

Separation into real and imaginary parts will lead to a parabola identical with that given by [21]. The point z_2', corresponding to $r = 0$, lies in the third quadrant, and the part of the parabola along which integration is carried out is located wholly in this quadrant. The u_2-term of [15] reduces to an equation of the same form as [22], subscripts 2 instead of 1 being used.

Consider, in Fig. 6, integration around the path ∞_r, A, B, ∞_r. Around this path by Cauchy's theorem

$$\int_c \frac{e^{-z}\,dz}{\sqrt{z}} = 0$$

since no singularities of the integrand are enclosed within it.

If, however, arg $z > \pi$, integration is around a path such as $\infty_r, A, B, C, D, \infty_r$, and the singularity at the origin is enclosed within the contour.

In the latter case the contour may be deformed into Hankel's integral for the gamma function,[3] and

$$\int_c \frac{e^{-z}\,dz}{\sqrt{z}} = -\,2\sqrt{\pi}$$

In either case there may be written

$$\int_{z_1}^{z_1}\frac{e^{-z}\,dz}{\sqrt{z}} = -\,2\sqrt{\pi}\,K + \int_{z_1'}^{\infty_r}\frac{e^{-z}\,dz}{\sqrt{z}} - \int_{z_1}^{\infty_r}\frac{e^{-z}\,dz}{\sqrt{z}}\ldots\ldots[24]$$

where

$$\left.\begin{array}{l} K = 0 \text{ if } \arg z < \pi \\ K = 1 \text{ if } \arg z > \pi \end{array}\right\}\ldots\ldots\ldots\ldots\ldots[25]$$

The integration in [24] proceeds along lines parallel to the x-axis.

In the first integral write

$$\left.\begin{array}{l} z = z_1' + v \\ z = z_1 + v \end{array}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots[26]$$

and in the second,

where v is purely real. Then

$$e^{2\pi i u_1 r}\int_0^r e^{i\pi(r^2/q\,-\,2ru_1)}\,dr = \frac{1+i}{\sqrt{2}}\sqrt{(q)}e^{2\pi i u_1 r\,+\,z_1'}K - \frac{1+i}{2\sqrt{2}}\sqrt{\frac{q}{\pi}}\,e^{2\pi i u_1 r}\int_0^\infty \frac{e^{-v}dv}{\sqrt{z_1'+v}}\ldots\ldots\ldots[27]$$
$$+ \frac{1+i}{2\sqrt{2}}\sqrt{\frac{q}{\pi}}\,e^{i\pi r^2/q}\int_0^\infty \frac{e^{-v}dv}{\sqrt{z_1+v}}$$

The z_2-integral starts for $r = 0$ in the third quadrant and terminates for $r = \infty$ in the same quadrant. No singularities can be enclosed within the contour, and an expression for this integral similar to [27] with subscripts 2 and with the K-term omitted can be written as follows: Let

$$\phi_1 = \int_0^\infty \frac{e^{-v}dv}{\sqrt{z_1+v}}\ldots\ldots\ldots\ldots\ldots[28]$$

and let ϕ_1', ϕ_2, ϕ_2' designate the corresponding expression when z_1 becomes z_1', z_2, z_2', respectively. Then with this notation, and substituting [27] and its companion equation with subscripts 2 in [15], there is obtained

$$R = \frac{(1-i)e^{i\sigma}\sqrt{q\pi}}{2\sqrt{2}\,\delta}\left\{\left(2\sqrt{\pi}e^{z_1'}K - \phi_1'\right)e^{2\pi i u_1 r} + \phi_2'e^{2\pi i u_2 r} + e^{i\pi r^2/q}\,(\phi_1 - \phi_2)\right\}\ldots\ldots\ldots[29]$$

The terms containing ϕ_1' and ϕ_2' represent a damped harmonic vibration. They arise from the initial conditions $x = 0$ and $dx/dt = 0$ at $t = 0$ which were imposed.

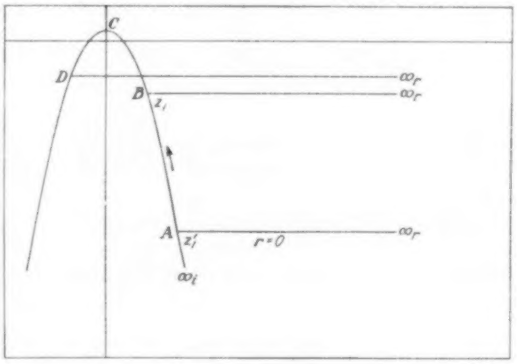

FIG. 6

Where any damping is present in the system, the contribution to R by these terms in the vicinity of the critical will be practically nil. The vibration represented by the remaining terms, in analogy to the case of constant frequency, will be designated as the "forced vibration." For this forced vibration we have

$$R_f = \frac{(1-i)e^{i\sigma}\sqrt{q\pi}}{2\sqrt{2}\,\delta}\,e^{i\pi r^2/q}\left\{2\sqrt{\pi}e^{z_1}K + \phi_1 - \phi_2\right\}\ldots[30]$$

Only the real part of this solution is to be utilized. Assume that the expression within the braces has been evaluated and is of the form $a + bi$, where a and b are real. Then, writing the exponentials in sine and cosine form, and multiplying out, the real part is found to be

$$R_f = \frac{\sqrt{q\pi}}{2\sqrt{2}\,\delta}\left\{\left[(a+b)\cos\pi r^2/q + (a-b)\sin\pi r^2/q\right]\cos\sigma + \left[(a-b)\cos\pi r^2/q - (a+b)\sin\pi r^2/q\right]\sin\sigma\right\}\ldots\ldots[31]$$

This equation represents an oscillating curve. As the phase angle σ is varied we obtain a family of such curves. By the elimination of σ between R_f and $\partial R_f/\partial\sigma = 0$ there is obtained the equation for the envelope of the family of curves, namely,

$$R_e = \frac{\sqrt{q\pi}}{2\delta}\sqrt{a^2+b^2}$$

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[3] See Whittaker and Watson, "Modern Analysis," art. 12.22.

or, in the customary notation of the complex variable,

$$R_e = \frac{\sqrt{q\pi}}{2\delta} \text{ modulus} \left\{ 2\sqrt{\pi}\, e^{z_1}K + \phi_1 - \phi_2 \right\} \ldots . [32]$$

Consider next methods for evaluating the ϕ-integrals of Equation [28]. Writing $v = \tan s$, there is obtained

$$\phi_1 = \int_0^{\frac{\pi}{2}} \frac{e^{-\tan s}\, ds}{\cos^2 s \sqrt{z_1 + \tan s}} \ldots\ldots\ldots\ldots [33]$$

which is more convenient for purposes of direct computation than [28].

The evaluation was carried out as follows:

Referring to Fig. 8, the complex number $z_1 = x_1 + iy_1$ was plotted with reference to the real and imaginary axes, any convenient scale being used and a line $z_1 \infty$ drawn parallel to the real axis, and on this $\tan s$ laid off from z_1 at 5-deg intervals of s.

Writing $z = z_1 + \tan s = \rho e^{i\theta}$

$$z^{-1/2} = \rho^{-1/2} e^{-i\theta/2} = \rho^{-1/2}(\cos \theta/2 - i \sin \theta/2)$$

But $\rho = y_1 \csc \theta$, and by trigonometric reduction,

$$z^{-1/2} = y_1^{-1/2} \left\{ \left(\frac{\sin \theta}{2} + \frac{\sin 2\theta}{4} \right)^{1/2} - i \left(\frac{\sin \theta}{2} - \frac{\sin 2\theta}{4} \right)^{1/2} \right\} \ldots\ldots\ldots [34]$$

the radical terms were computed and plotted as polar curves from a base circle and are the curves marked "real" and "imaginary" in Fig. 8.

The function $e^{-\tan s} \sec^2 s$ was computed for the 5-deg intervals of s.

A straight edge, with scale, is passed through the origin and the successive points on the $z_1 \infty$ line, corresponding to the 5-deg intervals of s. The intercepts of this scale between the base circles and the real and imaginary curves are tabulated. These intercepts are then multiplied by the function $e^{-\tan s} \sec^2 s$ and the products summed by Simpson's rule. Dividing by $y_1^{1/2}$ there are obtained the real and imaginary parts of the ϕ_1-integral.

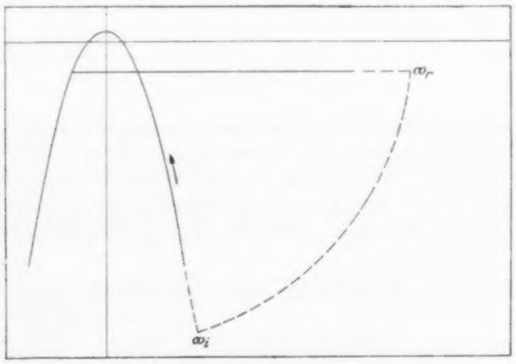

FIG. 7

The same real and imaginary base curves serve for all values of z_1, and it is merely necessary to shift the position of the z-scale to compute for a new value of z_1.

This procedure fails for small values of z_1 and for points on or near the real axis, but for computing such points complex series can be obtained.

The ϕ-function is related to the confluent hypergeometric W-function.[4]

Referring to Whittaker and Watson,[4] write in their notation $k = -{}^1/_4$, $m = {}^1/_4$, and equate the definite integral of Art. 16.12 to the asymptotic series of Art. 16.3. There is thus obtained the following asymptotic expansion, useful for large values of z:

$$\int_0^\infty \frac{e^{-v} dv}{\sqrt{z + v}} = z^{-1/2} \left\{ 1 - \frac{1}{(2z)} + \frac{1 \cdot 3}{(2z)^2} - \frac{1 \cdot 3 \cdot 5}{(2z)^3} \ldots \right\} [35]$$

and it may be shown to hold over the range necessary for our purpose from

$$\frac{3\pi}{2} > \arg z > -\frac{\pi}{2}$$

In like manner, equating the series of Art. 16.41 to the definite integral of Art. 16.12, there is obtained the convergent series

$$\int_0^\infty \frac{e^{-v}\, dv}{\sqrt{z + v}} = (1 - 2K)\sqrt{\pi}\, e^z - 2z^{1/2} \left\{ 1 + \frac{2z}{1 \cdot 3} + \frac{(2z)^2}{1 \cdot 3 \cdot 5} + \ldots \right\} \ldots\ldots [36]$$

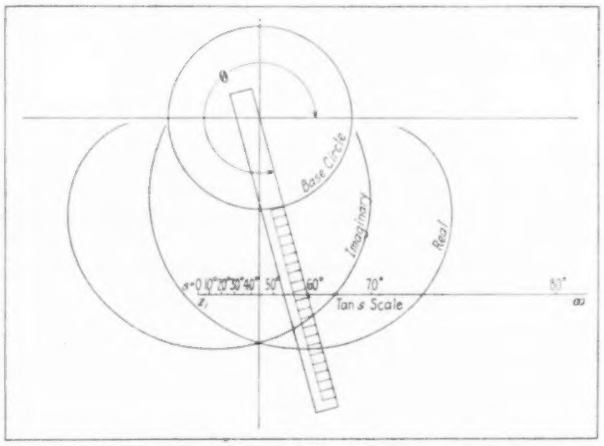

FIG. 8 CONSTRUCTION FOR EVALUATING ϕ-INTEGRALS

Whittaker and Watson do not consider the value of the function for $\arg z > \pi$, but the necessity for the K-term may be independently established.

It will be found that in [30], in the vicinity of the critical speed, the ϕ_2-term contributes only from 1 to 5 per cent of the value of the term within the braces. Further, z_2 will be large. An approximate expression of sufficient accuracy may be developed for ϕ_2. In the asymptotic expansion [35], drop all but the first term. Then

$$\phi_2 = z_2^{-1/2}$$

But

$$z_2 = -i\pi q \left(\frac{r}{q} + \delta - \frac{i\gamma}{2} \right)^2$$

from which it may be shown that z_2^{-12} is approximated by

$$z_2^{-1/2} = \phi_2 = -\sqrt{\frac{\lambda}{2}} \frac{\gamma}{(r/q + \delta)} \left\{ \left[1 - \frac{\gamma}{2(r/q + \delta)} \right] + i \left[1 + \frac{\gamma}{2(r/q + \delta)} \right] \right\} \ldots\ldots [37]$$

While ϕ_2 may be calculated by using the exact expression, the effect of so doing upon the curves plotted in Figs. 1–5 could hardly be detected.

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[4] Cf. Whittaker and Watson, "Modern Analysis," chap. 16.

THE CASE OF DECREASING FREQUENCY

As previously stated, the case of decreasing frequency is simulated by fixing our attention on negative values of r. If we write $\int_{\infty}^{r}$ in place of $\int_{0}^{r}$ Equation [15] will apply to this case.

Referring to Fig. 7, we have the parabola into which the real axis is transformed. The z_1-integral will now start at α_i on the parabola, and the part in which we are interested for $r < 0$ will lie wholly in the (—1) quadrant.

The z_2-integral will likewise start at ∞_i in the (—1) quadrant, but $r = 0$ will lie in the third quadrant.

In either case, the contour is closed by a quadrant at infinity, and it may be proved that the integral around this is zero.

A solution corresponding to [30] may then be written:

$$R_f = \frac{(1 - i)e^{i\sigma}\sqrt{q\pi}}{2\sqrt{2}\,\delta}\, e^{i\pi r^2/q} \left\{ \phi_1 - \phi_2 - 2\sqrt{\pi}\, e^{z_1^2}K \right\} \ldots . \quad [38]$$

where in this case

$$\begin{aligned} K &= 0 \text{ if } \arg z < \pi \\ &= 1 \text{ if } \arg z > \pi \end{aligned}$$

The real and imaginary parts of the expression within the braces are calculated, and [32] applies for the equation of the envelope.

SUMMARY OF THE CALCULATION PROCEDURE

It is desired to calculate the equation of the envelope R_e for specified values of the damping constant γ and the acceleration constant q. If we know the natural frequency of the system in cycles per second and the rate of acceleration in cycles per second per second, q is first determined from [9].

The λ-constant is then determined from [20] and a series of values of z_1 on the parabola of [21] determined. For each such value of z_1, ϕ_1 is calculated by either [33], [34], or [35] as proves convenient. In general, for large values of z, $|z| > 5$, the asymptotic series [35] is used. For small values, $|z| < 0.5$, and for points on the real axis the convergent series [35] is employed, and for all other values, the definite integral [33].

To the resulting real and imaginary parts are added $2\sqrt{\pi} \times e^{z_1^2}K$ and the sums plotted against x_1 as the abscissa.

It may be noted that although K is a discontinuous function, no discontinuity is implied in the motion of the body for there is a discontinuity in the function ϕ_1 which exactly compensates, so that where $\arg z$ passes through π the function $\phi_1 + 2\sqrt{\pi}e^{z_1^2}K$ and all its derivatives are continuous.

In the case of zero damping the parabola in the z-plane degenerates into the negative imaginary axis, considered as looped about the origin, and the sole difference in treatment is that $\sqrt{y_1}$ is used for the abscissa in the plot instead of x_1.

The r/q-values corresponding to the x_1-values are determined from [19].

The real and imaginary parts of ϕ_2 are determined from [37] and added in, and finally a series of points on the envelope are calculated by [32].

It may be noted that the calculation for ϕ_1, which is the difficult part to determine, will furnish data for all the curves over which $\gamma^2 q$ is constant.

It may be noted also that the ϕ_1-calculation for positive values of r is identical with the calculation of ϕ_2 for negative values of r. The calculations for increasing frequency will therefore also detail the curves for decreasing frequency.

SYSTEM OF SEVERAL DEGREES OF FREEDOM

The foregoing methods may, in theory at least, be extended to systems of several degrees of freedom.

Assume a system comprising several masses with elastic connections between, with or without damping, and with a periodic force S acting on one of the bodies.

For each mass of the system a differential equation is written, and in this family of differential equations p is written for d/dt and p^2 for d^2/dt^2.

Solve this family of equations simultaneously for the motion of the particular body desired, treating p as an algebraic constant, and the resulting equation in p can then be expressed in the form

$$x_s = \frac{f(p)}{pF(p)}\, S$$

Determine the roots of the equation $F(p) = 0$. These roots, in the case of zero damping, will be imaginary, but to each pair of roots corresponds one of the natural frequencies of the system, and there are several methods available for determining such frequencies. With damping in the system, the roots are complex and the procedure is limited by the possibility of their determination.

By the rules of operational calculus[5] the foregoing may then be broken up into a number of terms of the form $S/(p - \alpha)$, and each term "interpreted" by the formula

$$\frac{S}{p - \alpha} = e^{\alpha t} \int_{p}^{t} Se^{\alpha t}\, dt$$

There is thus obtained an equation similar to [4], and each term of this equation may be evaluated by the method of contour integration outlined in the paper.

The author is somewhat doubtful whether the importance of the problem would in many cases justify the considerable labor involved in evaluating the solution. It is quite possible, however, that an approximate solution can be obtained, and the exact solution might then serve as a guide for its construction and accuracy.

Discussion

R. G. STURM.[6] In connection with vibration during acceleration through a critical speed, an example of a practical application of this phenomenon might prove of interest. The Stockbridge vibration damper, which is used quite extensively on overhead transmission lines, has two natural frequencies resulting from two degrees of freedom—that is, direct oscillation and rotational motion of the damper weight. The use of this damper has been described by R. A. Monroe and R. L. Templin.[7] The fact that the damper is subjected to varying speeds of vibration which extend over one or both of its critical speeds and at the same time is affected by a considerable damping action which is analogous to a velocity damping, makes an analysis of the action of the damper extremely complicated.

Experimental determination of these natural frequencies reveals the fact that if the damper is vibrated at a speed greater than the critical speeds and then permitted to coast through all speeds to rest, the observed maximum throws occur at speeds less than when the damper is operated at a uniform speed. These observations are qualitatively in agreement with the findings of the author and show quantitative agreement with the theory applied to two degrees of freedom.

It may be stated that theoretical values of maximum throw and critical speeds, as well as the shape of the deformed damper cable, have been checked by experiments.

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[5] Cf. Jeffreys 1.5.

[6] Research Engineer-Physicist, Aluminum Research Laboratories, Aluminum Company of America, New Kensington, Pa.

[7] "Vibration of Overhead Transmission Lines," by R. A. Monroe and R. L. Templin, presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20–24, 1932.

E. A. Tulus.[8] In high-speed Diesel applications it is almost always true that the fundamental mode of vibration is below the operating speed of the installation. In one of the recent cases of Diesel drives brought to the writer's attention, the operating speed was so high and the elastic system was so flexible that in a final design both major orders of vibration of the first and second mode were below the operating speed. Naturally, the question arises in the designer's mind as to how dangerous it is to operate an installation of this sort and as to what the stresses would be in such a drive as the unit passes through the major orders in speeding up to its rated speed.

It is understood that the danger is not serious, since the unit passes through the critical and it takes time to build up the appreciable amplitudes of vibration. But in some cases where the rate of acceleration is very low and the damping in the system is negligible, the amplitudes may reach such values that the operation of the unit is out of question.

The author has solved the problem completely. The writer has been interested in the same problem for some time and has obtained both experimental and theoretical solutions. In obtaining the theoretical solution, certain approximations were made, with the following line of reasoning.

Taking the case of a vibrating body with 1 deg of freedom, the differential equation may be written:

$$m\ddot{x} + c\dot{x} + kx = 0 \quad \ldots\ldots\ldots\ldots [39]$$

Introducing the initial condition of $t = 0$, $\dot{x} = \dot{x}_0$, $x = 0$, the solution of [39] may be written:

$$x = \frac{\dot{x}_0}{w_n} e^{-ct/2m} \sin w_n t \quad \ldots\ldots\ldots\ldots [40]$$

Let us assume now that a force is acting on a vibrating body and that the force varies from m to n as a function of t, as shown on the curve in Fig. 9.

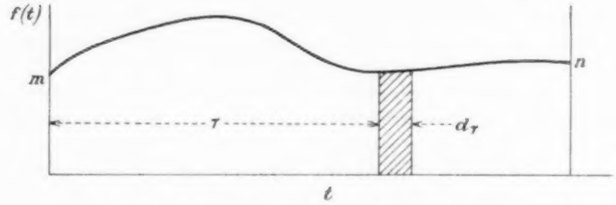

Fig. 9 Force Acting on a Vibrating Body

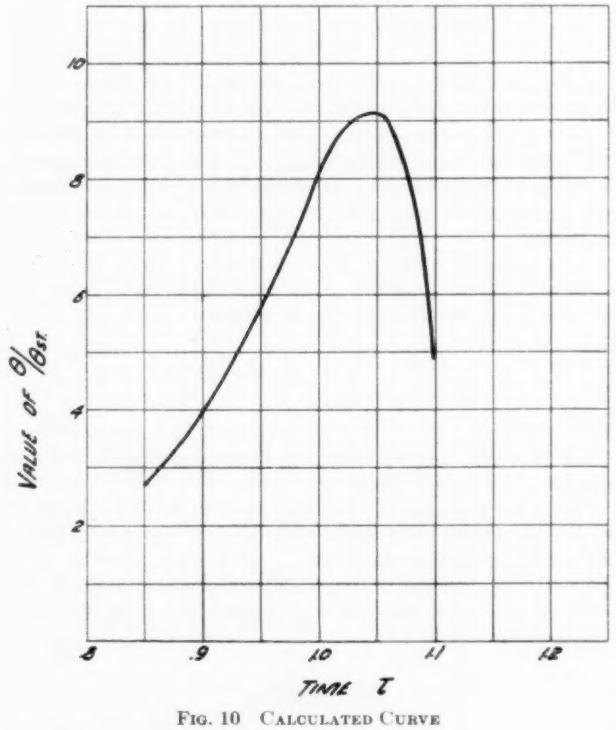

Fig. 10 Calculated Curve
(n = 100; c/cs = 0.05.)

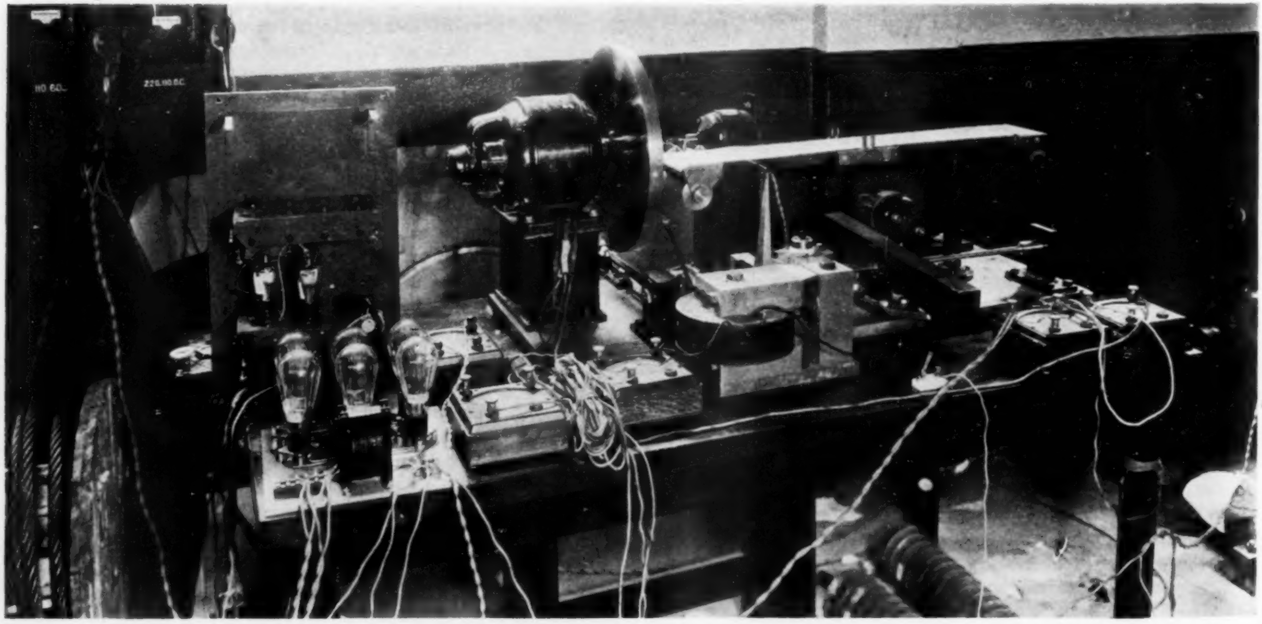

Fig. 11 Model to Test Amplitude of Vibration

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[8] Development Engineer, Power Engineering Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

In order to obtain the displacement of the system at any time τ, we have to divide $f(t)$ into elementary intervals $d\tau$. Due to the action of the force $f(t)$ during this interval, the velocity of the mass m is increased, which can be found from the following equation:

$$f(t)\,d\tau = m\,d\dot{x}_0$$

or

$$d\dot{x}_0 = \frac{f(t)}{m}\,d\tau \qquad [41]$$

From Equation [40] it can be concluded that any addition to the velocity at the time τ will produce at the moment t a displacement:

$$dx = \frac{f(t)\,d\tau}{mw_n} e^{-c/2m\,(t-\tau)} \sin w_n\,(t-\tau)$$

Assuming that $f(t) = P \sin w\tau$ between $\tau = 0$ and $\tau = NT$, where T is the period of any given forced frequency, and taking the summation, we have:

$$X(t) = \frac{P}{mw_n}\int_0^{NT} e^{-c/2m\,(t-\tau)} \sin W_n\,(t-\tau) \sin w\tau d\tau \qquad [42]$$

Integrating [42] from 0 to NT for any desirable number of cycles of forced frequency, it is possible to obtain $X(t)$ for a given damping coefficient and the rate of acceleration. The writer carried calculations for the particular case of $N = 100$ and $c/cz = 0.05$, which is equivalent to the author's notation $\gamma = 0.1$.

The amplitude of vibration as a function of τ is shown in Fig. 10, which checks almost exactly the curve on Fig. 3 in the paper.

It is appreciated that the method is long and requires many calculations, so together with Mr. J. G. Baker,[9] a model was built, as shown in Fig. 11, to obtain similar results experimentally. A magnetic damper was used to introduce damping into the system. Constant rate of acceleration was maintained with a vacuum-tube current regulator. The results of the investigation are shown in Fig. 4 and differ 10 to 15 per cent from the results as obtained by the author. This is within the accuracy of the measuring instruments. As a conclusion, it may be pointed out that the model study presents wonderful possibilities for study of systems of many degrees of freedom where theoretical solution hardly could be approached.

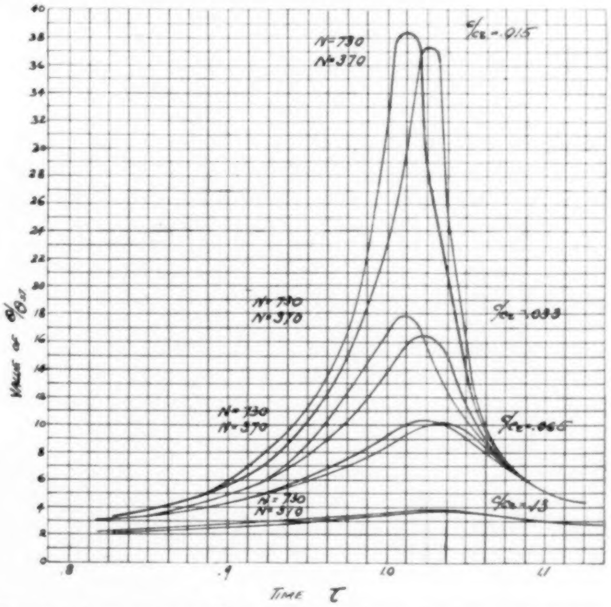

FIG. 12 AMPLITUDE OF VIBRATION DURING ACCELERATION THROUGH CRITICAL SPEED

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[9] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Erratum

On page 116 of the Applied Mechanics section (paper APM-54-10), Equation [9], has been incorrectly derived and should read:

$$\tan \alpha = -\frac{\partial \tau/\partial y - \partial \sigma_y/\partial x}{\partial \tau/\partial x - \partial \sigma_x/\partial y}$$

In view of this fact, the procedure as outlined in the discussion is useless as a practical method of stress determination based on photoelastic readings.

APM-54-25

The Use of Models in Determining the Strength of Thin-Walled Structures

By HAROLD E. SAUNDERS[1] AND DWIGHT F. WINDENBURG,[2] WASHINGTON, D. C.

This paper discusses the status of fundamental theories and explains the circumstances surrounding the use of models to determine the strength of three types of thin-walled structures. The particular types selected represent problems in naval architecture, but their counterparts are frequently encountered in other engineering structures.

Special emphasis is devoted to the necessity for obtaining geometrical similarity between the model and its original and for using identical materials if consistent and reliable data are to be obtained.

The actual methods used in the fabrication of representative models are described in considerable detail, as an aid in their application to other model research. The limits to which riveting and welding can be duplicated are defined, and the use of soldering and spot welding as alternatives is discussed.

In conclusion, the authors point out that the application of intensive thought and study to model research, commensurate with the care devoted to the design of the full-scale structure, is imperative if reliable results are to be expected from the work.

H. E. SAUNDERS

D. F. WINDENBURG

THE keynote of present engineering design is economy. In land structures, weight as a rule is of secondary importance for a given performance, and the primary consideration is cost. However, in ship design, economy of weight is generally predominant, although in merchant work the question of operating cost and profits is again the controlling factor. In the design of airplanes and submarines, weight saving becomes of preeminent importance.

By correct design, much weight can be saved in airplane spars and struts, ship's hulls, ship's bulkheads, machinery and turret foundations, and other ship structures. Correct design, however, involves much thought and requires extended study of many possible combinations of material to find those which are most satisfactory. The selected combinations, in turn, require confirmation to the greatest possible degree before they can be used in building engineering structures. Such confirmation is one of the functions of model tests in engineering.

For example, the design of a submarine hull involves the attainment of the required collapsing pressure with a minimum weight of structure. In detail, this design hinges upon the correct distribution of material in the stiffening frames and shell. Although the problem has been generously treated from the theoretical standpoint, it is imperative that direct experimental tests be made, especially since the demands for military performance and weight saving require the factor of safety to be extraordinarily low. The naval architect is at once confronted with the practicability and reliability of model tests in determining the strength of his submarine-hull design, as it is impossible for him to test the full-scale submarine to destruction.

The use of these models offers the designer a comparatively economical indication of the correctness of his theories, and, in many cases, the only advance proof he can obtain. However, he must recognize the necessity of approaching the model program with the same systematic study as was used in developing the tentative designs of the full-scale structure, and of applying great care and discretion to the conduct of the model tests and to the interpretation of their results.

For example, during the World War, the German Navy (1)[3] prepared an elaborate model-testing program to prove the theories which their scientists had developed in an endeavor to obtain the greatest strength for the least weight in the pressure hulls of their submarines. Their tests were intended to show the influence of the scale of a submarine-hull model upon the collapsing pressure, and also the influence of framing upon the strength of the shell against collapse. They found, however, that in each series of supposedly similar models, the maximum collapsing pressure was more than twice as great as the minimum, the larger models usually possessing the greater strength. Such erratic results could of course give no reliable information upon the influence of framing. Nevertheless the tests indicated that inaccuracies in workmanship were relatively greater in the smaller cylinders and that model-test results could not be applied safely to the design of submarine hulls unless the thickness of plating was at least 0.16 in. In other words, they realized that their models had to be exactly similar before they could expect consistent results. Further experiments were planned, but the work

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[1] Senior Assistant to Officer in Charge, U. S. Experimental Model Basin, Navy Yard, Washington, D. C. Commander Saunders is a graduate of the United States Naval Academy and of the Massachusetts Institute of Technology, Department of Naval Architecture. For three years he was head of the submarine-design section in the Bureau of Construction and Repair, Navy Department, and for five years in charge of the construction of large V-class submarines at the Navy Yard, Portsmouth, N. H. Since October, 1929, he has been in his present position.

[2] Assistant Physicist, U. S. Experimental Model Basin, Navy Yard, Washington, D. C. Mr. Windenburg was graduated from Cornell College, Mt. Vernon, Iowa, with M.A. degree. He had two years of graduate work in physics and mathematics at the University of California. For three years he was head of the mathematics department of the Polytechnic College of Engineering, Oakland, Calif., and for two years head of the mathematics department of the Long Beach Junior College, Long Beach, Calif. Since January, 1929, he has been in his present position.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Numbers in parentheses refer to bibliography at the end of the paper.

could not be carried out because of the general breakdown at the close of the war.

In 1929 the U. S. Experimental Model Basin at Washington undertook an extensive series of model tests to investigate completely the strength of submarine hulls. All the tests to date have been made on small-scale models with a wall thickness about one-third of the minimum indicated by the German experiments of twelve years previous. Additional tests have been made on models of elementary ship structures, using material as thin as 0.03 in. It is the purpose of this paper to show that small-scale model tests on these thin-walled structures are not only possible, but practical, economical, and reliable.

GENERAL SCOPE OF TESTS

Three types of thin-walled structures have been constructed and tested as models. These comprise (*a*) cylindrical shells representing pipes, tanks, submarine hulls, diving bells, and other vessels subjected to external pressure, (*b*) flat, unstiffened bulkheads for submarines, and (*c*) hollow girders representing primary hull structures of surface ships.

These tests have been systematic and extensive, embracing some hundreds of models. The influence of certain features has been investigated experimentally by changing one variable at a time in a series of models. This procedure required the making of many models in each series, but after the technic for the fabrication of one satisfactory model had been developed, the building of the remaining models of that series was generally found to be a simple and economical operation. In later paragraphs of this paper, the fabrication methods for these multiple models are described.

INDIVIDUAL VS. MULTIPLE MODELS

If, for special reasons, a single model is to be built, it should be constructed to a relatively large scale so that the general construction and the details may be faithfully reproduced. An individual model must be made with even greater care than multiple models, as no verification of the test is intended. Hence it is doubtful if there is much saving in time or cost in the construction of a single model. It is certain that definite conclusions can be drawn with greater assurance when they are based upon the closely duplicated results of several identical models rather than upon data from one isolated example.

THE ATTAINMENT OF GEOMETRICAL SIMILARITY

The theory of similitude, upon which the whole art of model testing is based, requires (*a*) geometrical similarity and (*b*) similar or identical materials. Both of these requirements present real, but not insurmountable, problems in thin-walled model construction.

If models are to give results comparable to those expected from the full-size structure, they must be geometrically similar to that structure. For a simple structure, such as a tube of circular cross-section, the attainment of geometrical similarity in a model at first sight presents no great obstacles. However, difficulties soon arise as the scale is reduced.

Suppose it is desired to build and test a model of a long tubular column belonging to a hydraulic ram or elevator. If the original were, say, 10 in. in diameter, we should expect a manufacturing tolerance in diameter and roundness of 0.001 in., or one ten-thousandth part of the diameter. If the model were to be 1 in. in diameter, we should have to specify a tolerance of 0.0001 in., again one ten-thousandth part of the diameter. If the model tube were to be 0.1 in. in diameter, the corresponding tolerance would be 0.00001 in. This is an extremely exacting requirement.

The earliest experimenters with vessels subjected to external pressure soon discovered that their models must be geometrically similar to the original if the test data obtained from them were to be used to predict the performance of the full-sized structure. One of the first of these was Fairbairn (2), whose experiments on the strength of lap-jointed as compared to butt-jointed tubes led him to declare: "These facts are conclusive in showing the necessity of adhering in these constructions to the true cylindrical form."

Later experimenters, in various lines, following further developments of the law of similitude, have had the matter of geometrical similarity brought forcibly to their minds. In spite of repeated warnings, however, tests continue to be made on decidedly inferior models. As Buckingham (3) points out, "it is a familiar fact that in many instances small-scale models do not act at all like their full-sized originals under conditions which seem at first sight to be similar, and that hasty conclusions from model experiments are very unsafe indeed."

A flat plate is one of the simplest geometrical forms, yet it is one of the most difficult to produce accurately to small scale. Rolled sheets are rarely of uniform thickness and they invariably lack homogeneity, even when elaborately fabricated, with the result that they are difficult to form. Black sheet steel, for example, is totally unsuited to the manufacture of external-pressure-vessel models, for it refuses to roll into a smooth, circular cylinder. The high-grade stretcher-leveled steel used in the manufacture of metal furniture and automobile bodies possesses the greatest all-around uniformity in available ferrous material, and is the only material which can at the present time be recommended for the building of models of relatively thin-walled steel structures.

THE USE OF IDENTICAL MATERIALS

If the model and full-sized structure are to give similar results, they must also be made of materials having identical physical properties. Although the variations in the physical properties may be known and approximately allowed for, corrections of this kind usually introduce uncertain factors and hence should be avoided. Fortunately, Young's modulus and Poisson's ratio, which enter as the principal physical constants in most elastic equations, are practically the same for all steels used in engineering construction, and their values can be assumed with reasonable accuracy when reliable tests are not available. On the other hand, both the yield point and the proportional limit are in general poorly defined and they may be found to vary with the direction of rolling.

The following table gives the results of physical tests on many samples of black sheet steel and metal-furniture steel used in constructing models at the Experimental Model Basin:

	Thickness of sheet in inches		Young's Modulus $\times 10^{-6}$		Yield point $\times 10^{-3}$		
Material	Nominal	Tolerance	Low	High	Low	High	Probable value
Black sheet steel.......	0.05	0.001	..	..	22.6	32.6	26.0
Metal - furniture steel..	0.05	0.0005	..	..	30.7	39.3	36.0
Medium-steel plate......	0.25	0.01	29.2	29.7	39.7	40.9	40.0

THE SIMPLE CYLINDRICAL SHELL

Perhaps the simplest model of a thin-walled structure is the circular, cylindrical shell, or common tube. Nevertheless the fabrication of such a model shell or tube which is to withstand external pressure without premature buckling is by no means an easy matter. Seamless tubes, suitable for very thin-walled models, are not made commercially, and it is slow and expensive work to turn down a thick seamless steel tube on a lathe. It would also be unduly expensive to build the necessary apparatus

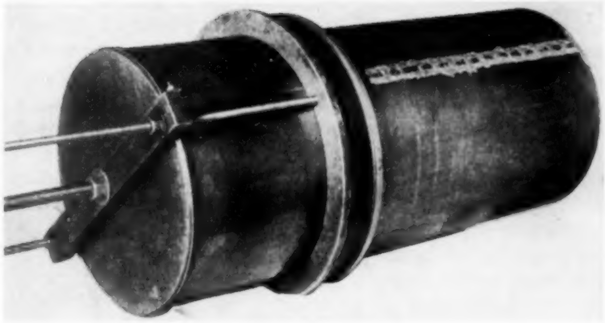

FIG. 1 THIN CYLINDRICAL SHELL DURING FABRICATION, SHOWING CENTER DRAWBOLT FOR PLUNGER AND TWO SIDE DRAWBOLTS FOR RING. SEAM STRAP SECTIONS PARTLY SOLDERED IN PLACE

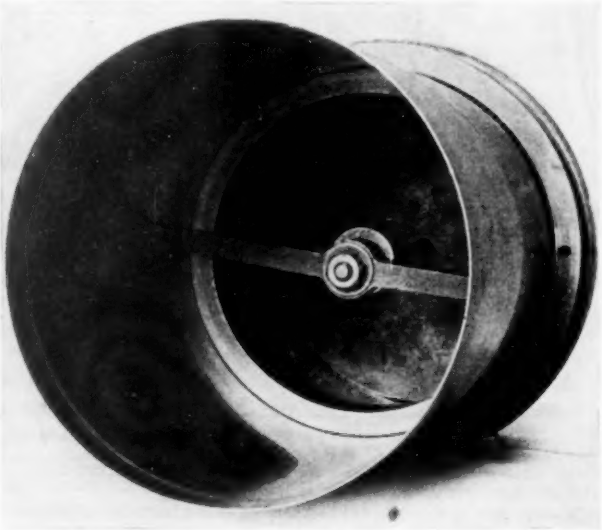

FIG. 2 CYLINDRICAL SHELL WITH INSIDE PLUNGER AND OUTSIDE RING

for forming thin seamless tubes, assuming the walls could be reduced to the dimensions required. Moreover, seamless tubes, machined or drawn, are not representative of large pressure vessels which must be fabricated from steel plates.

For models of such pressure vessels, including submarine hulls, an internal diameter of 16 in. has been found economical and satisfactory. This diameter, for a number of applications, permits the use of sheet steel about 0.05 in. thick for the shell. The edges are butted, and the seam is covered by a narrow longitudinal strap soldered in place.

FABRICATION OF THE SHELL

A thin sheet of steel cannot be rolled uniformly in a cylinder, unless the rolls are heavy and the sheet has first been formed to the proper curvature at the ends adjacent to the butt. To eliminate the latter difficulty, the rolls are first set to the required radius by using a sheet of the same dimensions as will be used for the finished model. The sheet to be fabricated is then cut about a foot too long in the direction of rolling, and is formed in the rolls, with the excess length lapped over. With the two overlapping ends held temporarily in proper position by soft solder, the two butt edges are formed by cutting through the double-thickness laps with a metal saw. The butt edges are then dressed carefully with a file to a previously scribed line.

To maintain the cylindrical form while the butt joint is made, a steel plunger 4 in. long and exactly 16 in. in outside diameter is placed inside the tube and a steel collar or ring about the same length is fitted snugly over the outside of the shell. The plunger and ring are drawn along the tube abreast each other by means of threaded rods as shown in Figs. 1 and 2. They are so rigid and they fit the shell so tightly that they iron out all local irregularities and hold the shell in exact and uniform circular shape while the seam straps are soldered in position. In practice, the plunger and ring are drawn along alternately, about two inches at a time, the ring always remaining over and abreast the plunger. After each movement of the ring, a narrow strip about two inches long and tapered at the edges, as shown in Fig. 3, is soft-soldered over the seam thus exposed. Since the strap is soldered on that portion of the shell which is held rigidly in circular form by the inside plunger, the heat from soldering will cause no warping.

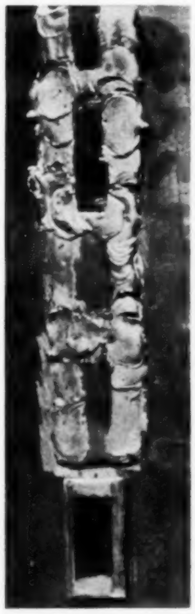

FIG. 3 CLOSE-UP OF SHELL SEAM WITH SHORT SECTION OF STRAP READY FOR SOLDERING IN PLACE NEXT TO OUTSIDE RING

The result is a smooth cylindrical model, without frames or stiffening rings, which is very nearly uniform in diameter and circular in cross-section. However, it is not enough to know that it is a *good* model. The question which must be answered is: *How good?* That is, how closely does the model resemble the full-scale original? The answer to this question requires the building of apparatus and the perfecting of a technic for measuring the contour of the shell of both the model and its prototype.

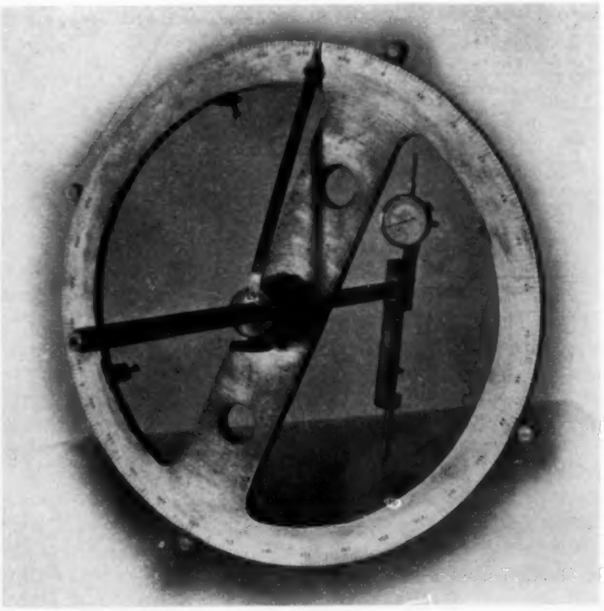

FIG. 4 APPARATUS FOR MEASURING VARIATIONS IN RADIUS OF CYLINDRICAL SHELLS

(The large ring is placed over the open end of the model and secured to it. The cross-arm on the central spindle, carrying the dial micrometer and sliding plunger, may be placed at any desired height in the model. The dial micrometer measures variations in radius at all points around the selected circumference.)

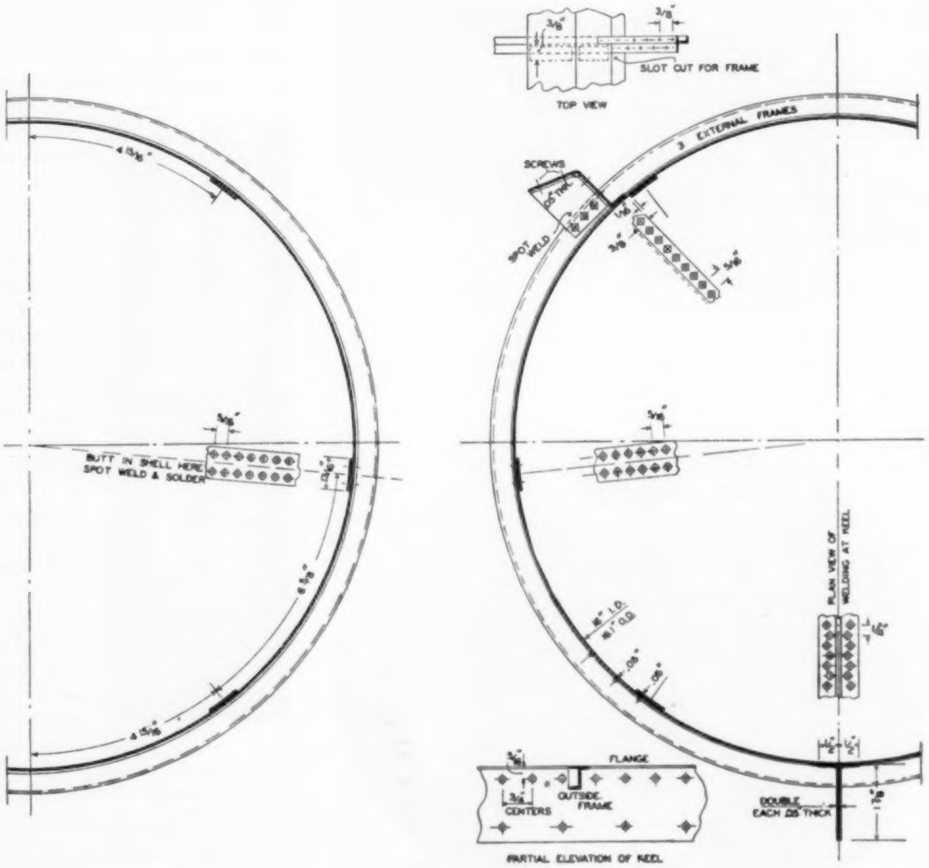

FIG. 6 WORKING DRAWING OF CYLINDRICAL SHELLS STIFFENED BY CIRCUMFERENTIAL EXTERNAL FRAMES AND LONGITUDINAL MEMBERS
(The model on the left has longitudinal straps only; that on the right has keel and upper longitudinals in addition.)

MEASUREMENT OF SHELL CONTOUR

Hitherto, either no measurements of out-of-roundness of vessels subjected to external pressure were made, or else experimenters were satisfied with measuring variations in diameter only (4), (5), (6). Although such measurements give a misleading picture of the true contour of the shell, systematic variations in diameter are very significant in determining the strength of long tubes. However, the problem becomes more complicated for tubes of shorter length, which collapse in a number of lobes greater than two. In this case, as the authors have previously pointed out (7), the variation in circular form within the region of one lobe is the significant feature.

To determine the actual contour of the shell, the measuring apparatus shown in Fig. 4 is used. This consists of a radial arm carrying a sliding plunger and dial micrometer which are swung around inside the shell at any station desired. One end of the shell under test is blanked while the other remains open to view. Dial-micrometer readings for various angles around the circumference are plotted on polar-coordinate paper, giving a graphical picture of the variations from circular form to a greatly magnified scale like that shown in Fig. 5. Similar readings are taken at successive pressure increments during the test and plotted on the same sheet, showing how the deflections increase with increasing load, and illustrating the characteristic lobe formation which always starts at the point of greatest out-of-roundness.

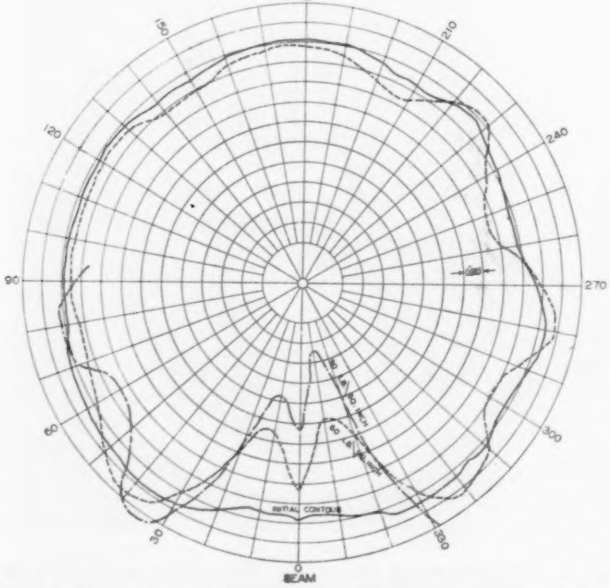

FIG. 5 VARIATIONS IN RADIUS OF CYLINDRICAL SHELL 16 IN. INSIDE DIAMETER AND 0.063 IN. THICK AFTER FABRICATION AND DURING EXTERNAL-PRESSURE TESTS
(Note that maximum variation in radius of the initial contour is only 0.010 in., about one-sixth the shell thickness.)

In using this or similar apparatus, it is essential that the axis of rotation of the radial arm be the geometrical axis of the model. Otherwise, since variations in radius are plotted to a greatly magnified scale, a true circle will appear distorted. Since we cannot be certain that the instrument axis is absolutely centered, especially for long models, the graphical contour thus obtained may show a greater out-of-roundness than actually exists. We are certain, therefore, that no model has a greater out-of-roundness than is indicated by these polar diagrams.

When models are made of identical materials and have about the same degree of out-of-roundness, they collapse at nearly equal pressures. Variations greater than 5 per cent of the collapsing strength can invariably be traced to irregularities in the shell, and premature collapse can usually be predicted by a close examination of the initial contour as shown on the polar diagram of the model.

THE CYLINDRICAL SHELL WITH STIFFENING RINGS AND ATTACHMENTS

The most economical design of a large cylindrical vessel subjected to external pressure requires closely spaced stiffening rings or frames. Moreover the stresses in the shell depend not only upon the spacing of the frames, but upon their rigidity as well. Although the theory underlying the stress distribution in such a structure has been rather carefully worked out (8), (9), we are compelled to rely upon model tests for final confirmation of design.

The problem of duplicating a section of a submarine hull or other large pressure vessel by a model 16 in. in diameter presents few additional difficulties, once the technic of fabricating the thin cylindrical shell has been developed. In actual practice, large pressure vessels are not fabricated from a simple plate in each circumferential belt or course, but from several plates which are generally connected (and reinforced) at the seams by longitudinal straps. A working plan of such a model is shown in Fig. 6.

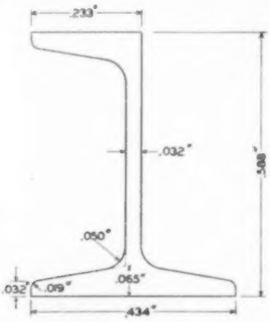

Fig. 7 Profile of Stiffening Frame for Models Shown in Fig. 6

Method of Fabricating Small Stiffening Rings or Frames

The stiffening frames are very small, having a depth of about $^1/_2$ in. with a web thickness of 0.032 in. Actual dimensions of the section are shown in Fig. 7 and the steps required in machining one or more frames from a forged circular ring are shown in Fig. 8. Two separate operations are employed. First, the flanges and the channel side of the frame are completely finished in a lathe and the remainder of the frame roughed out and cut off. Next, the finished side is fitted tightly in a chucking ring while the other side of the frame is finished to size.

The outside and inside diameters of the frame can be measured with micrometers, but the irregular parts must be made to fit templets. These templets are very accurately made on zinc sheets by photographic reductions of large-scale drawings. The correct web thickness is obtained by finishing the frame flush with the chucking ring. Using this method, a skilled mechanic can turn out a very accurate piece of work.

Methods of Attaching Stiffening Rings to Shell

The frames in most large pressure vessels are riveted to the shell, but this construction cannot well be duplicated in small-scale models. In more recent designs the frames are welded to the shell, but welding likewise imposes limitations upon the scale of the model.

Riveted construction can be satisfactorily duplicated with careful work down to about one-eighth scale, or until the rivets become as small as $^1/_8$ in. in diameter. The plates may be as thin as $^1/_{16}$ in., provided suitable material can be obtained and the holes are drilled.

The minimum thickness of plate which will permit fusion welding without objectionable warping is about 0.10 in., but best results are obtained with material at least 0.125 in. thick. The welding itself cannot well be reproduced in beads or fillets less than about $^1/_8$ in. in maximum dimension.

When frames or stiffening rings are internal and require only to be held in place, soft solder is entirely satisfactory, but when the frames are external, something more substantial must be provided to prevent the shell from tearing away from them.

Spot welding is a successful alternative for riveting on thin shells and frames. It can be applied with great facility to the fabrication of many parts of thin-walled models. Although the frame flanges illustrated in Fig. 7 are less than $^1/_4$ in. wide, perfect spot welds are possible by using small electrodes specially shaped to fit the frame. Wood forms and stands made for the purpose are used to hold the model accurately in position at the spot-welding machine so that the operator's whole attention can be devoted to the welding process. The strength of good welds, thus made, is quite comparable to that of riveted joints reduced to scale.

Construction and Testing of Two Similar Models

Two models were constructed according to the plan in Fig. 6. The first model was fitted with six longitudinal seam straps. The straps were made to butt against the ends of the pressure vessel and thus take their full share of the axial load. The second model was made exactly similar to the first, but with the addition of suitable longitudinal members to represent the keel and tank tops of a submarine.

Initial measurements showed a maximum variation from circular form (within the region of one lobe) of about 0.02 in. for each model; this variation was less than half the wall thickness of 0.05 in. In the first model, the greatest variations occurred near the seam, which was reinforced by a longitudinal strap, and near one of the upper longitudinal straps. These variations were undoubtedly caused by excessive heat or electrode pressure in the spot-welding operation. The variation from circular form was on the whole greater than would be expected in a submarine hull, but was likely of the same order of magnitude as would be found in a commercial pressure vessel of large diameter.

The two models, when tested, collapsed at equal external pressures. In the first model, a bulge was formed at 91 per cent of the collapsing pressure. At 98 per cent of the collapsing pressure the second bulge formed, although the model was able to hold that pressure without further damage. With a slight increase of pressure, other bulges followed in quick succession and the pressure fell off. Had the pressure been held, collapse would have been complete. The first bulge appeared near the seam where the variation from circular form was a maximum and the second bulge appeared near the longitudinal strap where the variation was almost as great.

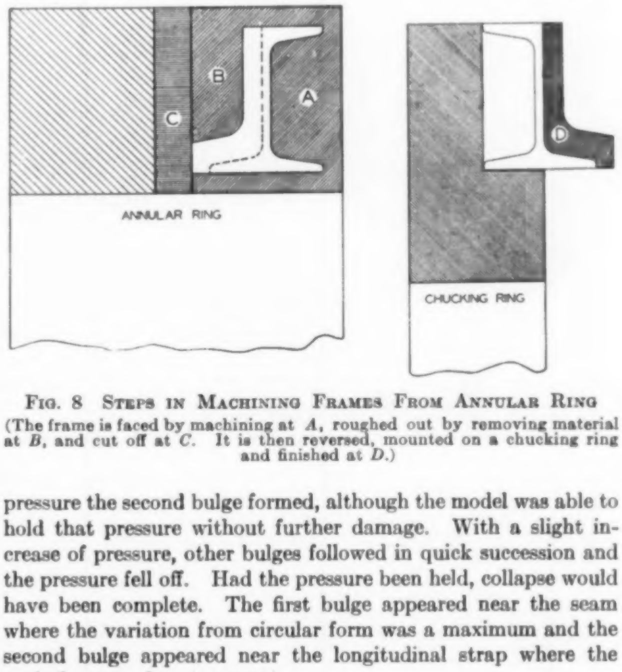

Fig. 8 Steps in Machining Frames From Annular Ring
(The frame is faced by machining at *A*, roughed out by removing material at *B*, and cut off at *C*. It is then reversed, mounted on a chucking ring and finished at *D*.)

In the second model, the greatest variations from circular form occurred near a longitudinal strap and near an upper longitudinal but not near the seam. These variations also were undoubtedly due to the spot welding. This model withstood a pressure of 98 per cent of the collapsing pressure before a bulge appeared. As before, this bulge was at the point of greatest variation from circular form. Four other bulges formed while the pressure was held constant, demonstrating that the element of time, as well as that of load, is directly associated with the failure of structures when stresses in the material reach the yield point. When the pressure was slightly increased the shell tore away from the frame at one of the bulges due to failure of the spot welds. Evidences of rivets pulling through the shell have been noted on similar tests to much larger scale.

THE FLAT, UNSTIFFENED, OR DIAPHRAGM BULKHEAD

A proposal to construct a light-weight compartment bulkhead in a submarine, in which stiffeners were to be partially or entirely omitted and in which reliance was to be placed upon the strength of a circular flat plate loaded as a diaphragm, afforded an excellent opportunity to utilize small-scale models. In this case, the design was too radical and embodied too many unknown features to warrant building a full-sized structure into a ship without thorough investigation, especially as the stresses in the diaphragm plate, in the event of accidental flooding, even at shallow depths, would exceed the yield point of the material. In cases of this kind theoretical considerations of elasticity are of little avail, and the best means of obtaining advance information is by model tests.

The model scale selected for preliminary tests required a flat-plate thickness, in the diaphragm portion of the bulkhead, of only 0.035 in. The diaphragm was to have an integral, substantial connection around its periphery to a relatively heavy ring, representing an elaborate construction in the full-scale design. The dimensions and form of the model bulkhead are shown in Fig. 9.

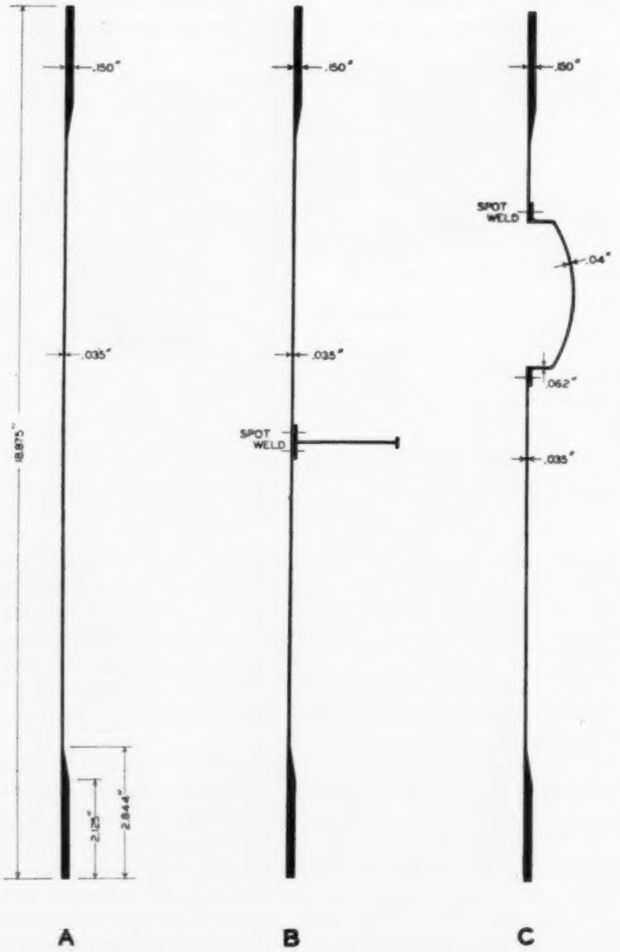

FIG. 9 SECTIONS OF THREE MODELS USED IN TESTING DESIGNS OF CIRCULAR DIAPHRAGM BULKHEADS

(*A* is a plain unstiffened bulkhead, *B* is stiffened by an I-beam across a diameter, and *C* is a bulkhead with circular door.)

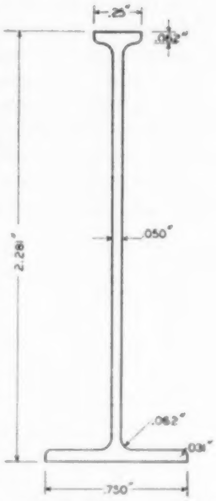

FIG. 10 PROFILE OF I-BEAM FOR MODEL DIAPHRAGM BULKHEAD SHOWN AT *B* IN FIG. 9

Since there was available no suitable magnetic chuck for holding the model while machining, the following method was employed to insure the required uniform thickness in the diaphragm. The bulkhead was first faced off to a plane surface and this face was then soft-soldered to the plane surface of a heavy backing plate. The entire bulkhead was then turned to the exact thickness of the rim. Using this rim surface as a reference plane, the central or diaphragm portion was readily turned to the proper thickness.

The model, when removed from its temporary mounting and calipered with micrometers, was found to have a uniform diaphragm thickness of 0.035 in., plus or minus 0.0005 in. This percentage variation in thickness is no greater than would be expected in the full-scale structure. It is only fair to state, however, that for this as well as for some other types of models such a degree of refinement in a machined model was reached only after building several models of much simpler construction having considerably heavier scantlings.

During hydrostatic tests the diaphragm deflected quite uniformly even when the deflection at the center reached one-sixth of the diameter of the diaphragm. It is of interest to note that the pressure at rupture exceeded the collapsing pressure of models representing the hulls of the same submarine design by nearly 50 per cent.

A modification of the full-scale design required a heavy stiffening member to be worked along a diameter of the bulkhead, breaking up the diaphragm into two semicircular panels, as shown at *B* in Fig. 9. This member was represented in the model by the modified I-section, illustrated in Fig. 10, which was machined from the solid and its wider flange spot-welded to the diaphragm plate.

The machining of this member presented no particular difficulties on account of its comparatively short length (about 14 in.). Both flanges were kept at the same width until the web was finished. When one side of the web was machined to size, the beam was turned over and the web surface supported by parallel bars while the other side was machined.

Another modification required the addition of a structure representing a door frame and a door, eccentrically placed on the diaphragm, as shown at *C* in Fig. 9. The model frame and door were machined as a unit from the solid, to a thickness of 0.04 in. in the door and 0.06 in. in the flanges. The flanges were spot welded to the 0.035-in. bulkhead plate and the latter was then cut out inside the door frame. This bulkhead model, after rupture, is shown in Fig. 11.

Deflections of the model bulkhead across a diameter were measured by an apparatus whose principal parts are shown in Fig. 12. The assembly is of very rigid construction and the bearing surfaces are extraordinarily large, to avoid all deflections in the measuring apparatus and to prevent rotation or rocking of the sliding block when in use. A measuring device of this kind with long or short straight-edge guides, is an inexpensive and useful piece of apparatus for numerous tests of thin-walled-model structures.

THE HOLLOW GIRDER, REPRESENTING AN ELEMENTARY SHIP STRUCTURE

It has long been felt that the present simple theory of determin-

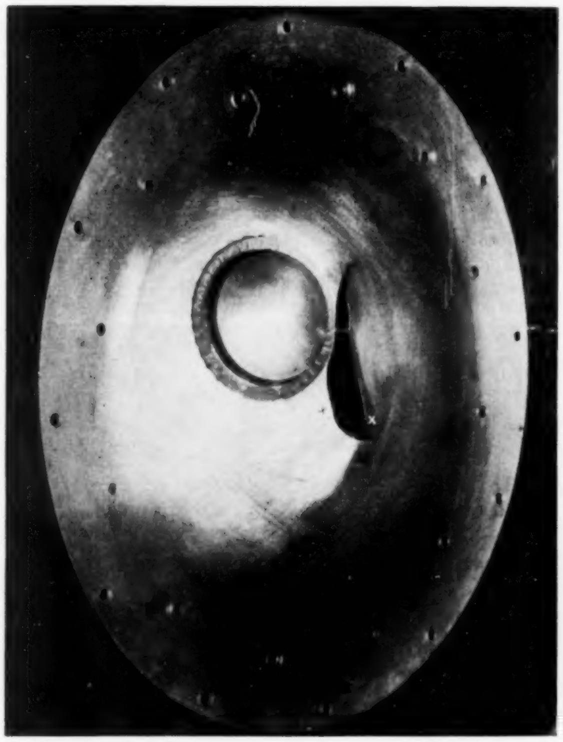

Fig. 11 Photograph After Rupture of Model Diaphragm Bulkhead Shown at *C* in Fig. 9

(Note spot welding around door frame and holes for attachment to test tank.)

ing the longitudinal strength of a ship's hull is far from satisfactory. Although many ships so designed have proved amply strong, they unquestionably possess excess strength in certain places, which of course represents excess weight. Some ships, on the other hand, have been found deficient in strength, owing to abnormally high local stresses or lack of column strength in the compression members, neither of which is indicated by the simple theory.

It is a long, complicated, and expensive job to test a full-sized ship under longitudinal bending load, and only three such tests have ever been made (10), (11). Furthermore, even though full-scale tests are carried to destruction, they show the weakness of the weakest part rather than the strength of the strongest. To be sure, elaborate strain-gage observations are taken, but these data are of no great significance except in tension members, as the additional displacement due to buckling in compression members cannot readily be separated from pure strain. Values of stress computed by ordinary methods, therefore, are erroneous and are certain to be misleading. Moreover the elastic effects are rather obscured by the design of the secondary structures in one ship which may not be duplicated in another.

Again, the naval architect who wishes, for instance, to determine the most economical distribution of material in hull plating and stiffening frames, is confronted with the possibility of obtaining reliable information from model tests. The subsequent paragraphs describe the beginning of a program now in progress at the U. S. Experimental Model Basin in an endeavor to solve this and related problems.

Hollow beams or girders $5^1/_2$ in. deep and 9 in. wide, built of sheet steel 0.031 in. thick, were chosen as suitable and inexpensive models for a general survey of the field. A cross-section of one of these models is shown in Fig. 13. All that has heretofore been said about adherence to true geometrical form in the construction of thin-walled cylindrical shells representing large pressure vessels applies equally to the building of these hollow-girder models; in fact, the problems are essentially the same. High-grade sheet steel is likewise employed to avoid irregularities in the plates which would cause buckling to begin at very low stresses.

The photographs, Figs. 14 and 15, illustrate the principal steps in the fabrication of these models. The sheet steel is first cut to the proper dimensions and bent to U-shape in a brake, following which it is pressed to very nearly exact shape in special metal dies. The male die is removed and a heavy plunger, shown in Fig. 15, analogous to the plunger used in the cylindrical shells, is placed inside the model at one end. A substantial steel plate is bolted over the female die to hold the plunger in position, and removable shims are placed between this plate and the die to regulate the pressure on the plunger as the latter is drawn through from end to end and as the model is worked more and more nearly to final size.

During these drawing operations, if such they may be called, the thin sheet forming the model must of course be held so that it is always in tension. The plunger and sheet are lubricated as for other drawing operations on sheet steel. Free movement of the plunger when the top plate is drawn down tightly on the female die indicates that the thin plate is exactly to shape and size.

The U-shaped sheet is now withdrawn from the die and the two edges for the attachment of the deck plate are formed at right angles to the sides in a brake. The deck or top plate is next applied and the model replaced in the die. A second clamping

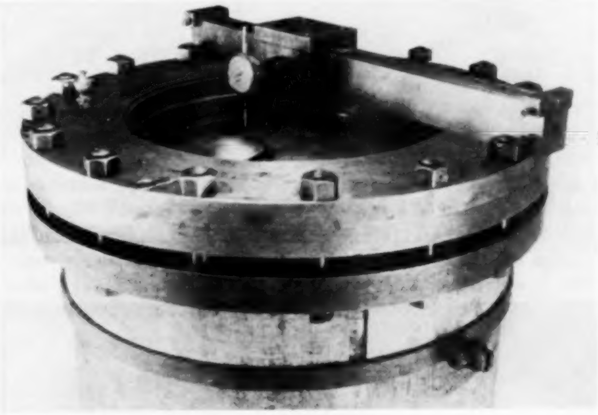

Fig. 12 Straight Edge, Sliding Block, and Dial Micrometer Used in Measuring Deflections on Model Diaphragm Bulkhead Under Test

(Note extremely rigid members employed to limit deflections to the mode only.)

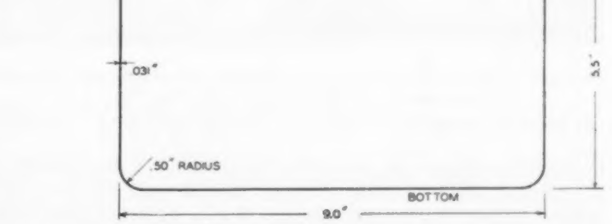

Fig. 13 Cross-Section of Hollow-Girder Model Representing Elementary Ship Structure

plate having notches at intervals along each side to correspond to the slots shown in the female die, holds the two parts of the model securely together while spot welds for tacking purposes are made at the edge of the deck. The plunger may be drawn through the model while these tack welds are being made, keeping the plunger always abreast the electrodes as each weld is made. After removal from the die, the rows of spot welds along the deck edges are completed as indicated in Fig. 13.

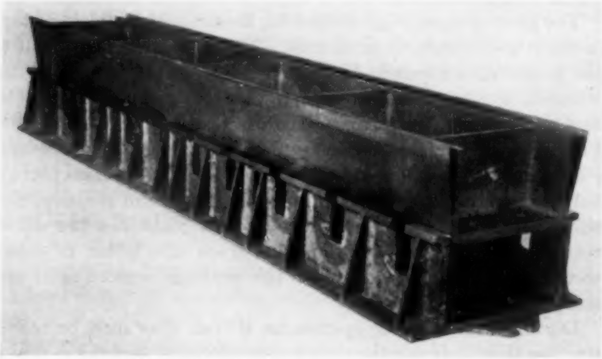

Fig. 14 Special Metal Dies for Forming Bottom and Sides of Model Shown in Fig. 13

(The slots in the lower die are to permit tack welding the deck plate to the sides of the model while the plates are held accurately in place by the forms.)

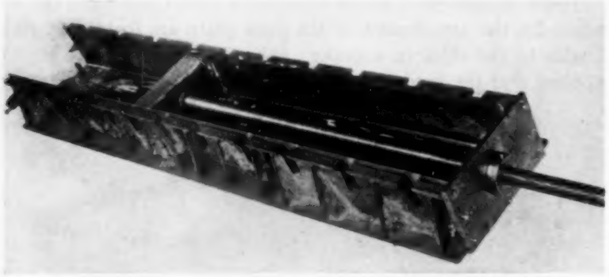

Fig. 15 Plunger and Drawbolt Used to Smooth Out Irregularities in Model While Held in Metal Form

[Note clamps at left for holding sheet in tension while plunger is drawn to right. A heavy clamping plate with clearance grooves cut for the plate edges (not shown in the photograph) holds the plunger in position.]

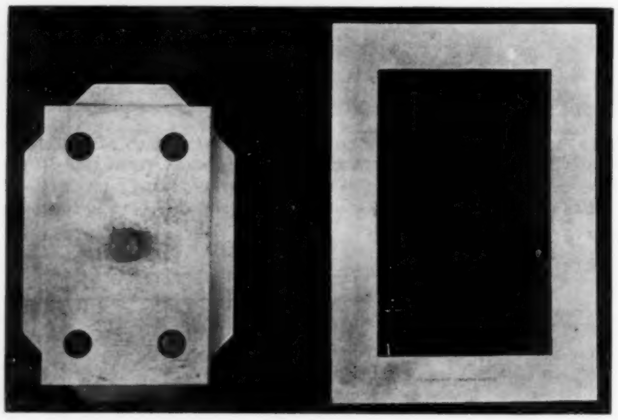

Fig. 16 Die and Punch for Accurately Flanging Bulkheads Fitted in Models Shown in Fig. 13

(At the left a bulkhead plate is secured to the punch preparatory to flanging.)

Bulkheads are placed at intervals inside the model to represent those occurring in all ship structures and to preserve the general form of the section. To prevent distorting the walls of the model when these bulkheads are spot welded in place, the flanges around their edges must be exactly square and the bulkheads must be of exactly the same shape as the model. Flanging all four sides of these bulkheads simultaneously in the simple die shown in Fig. 16 easily satisfies these conditions. Special ends are welded to the model to take the reactions in the testing machines.

The completed model is an extremely thin-walled structure with smooth, flat sides, in which the maximum departure from true shape and flatness is about one-half the plate thickness. Since collapse of these models is invariably caused by buckling of that portion which forms the compression flange of the girder, consistent results can be obtained only with models which possess the same degree of flatness.

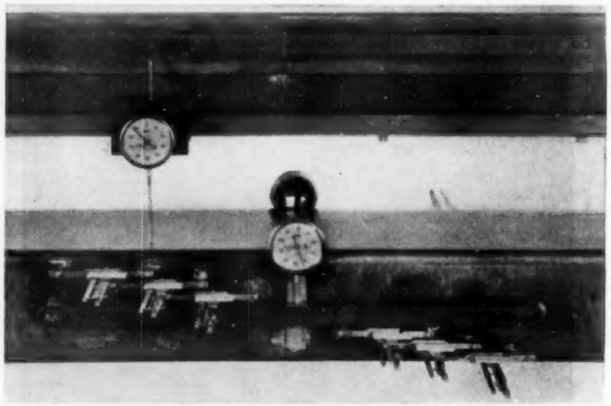

Fig. 17 Hollow-Girder Model Prepared for Longitudinal Bending Test

(The upper dial micrometer is carried by a block sliding on the straight edge and measures the longitudinal profile of the compression member. The two lower dial micrometers, one on each side, measure the deflection at the center bulkhead relative to the adjacent bulkheads. The Huggenberger tensometers measure strains in the side plating.)

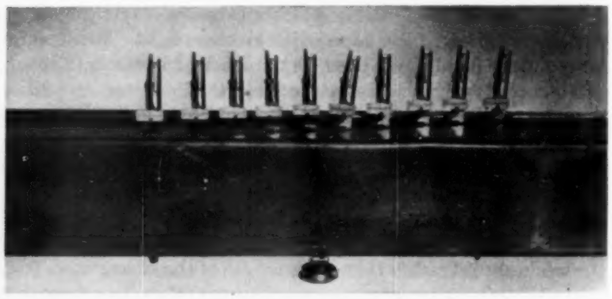

Fig. 18 Hollow-Girder Model After Collapse of the Compression Flange (Bottom)

(Note the three rows of spot welds indicating position of three transverse bulkheads. Holes for hand strain gages are staggered diagonally across the bottom; clamps for Huggenberger tensometers are likewise mounted in staggered positions to permit reading the scales. Note different types of strain-gage mountings shown in Figs. 17 and 18.)

The initial longitudinal profile of the compression flange, as well as the profile during subsequent loadings, is indicated by the sliding block and dial micrometer shown in Fig. 17. The part of the beam under test is the 24-in. midsection which contains three bulkheads spaced 12 in. apart, as shown by the lines of spot welds in Fig. 18. Deflection of the central portion of the beam with reference to the two end bulkheads is given directly by the two lower dial micrometers shown in Fig. 17.

During test, the model is loaded with uniform bending moment, shear being practically eliminated. An important feature to be investigated under these test conditions is the influence of the vertical sides in loading the top and bottom members throughout their entire breadth. Such an investigation requires, of course, a knowledge of the magnitude and distribution of stress over the section.

As previously stated, accurate strain observations are difficult to obtain in the compression members. Hand strain gages have been tried but are no longer used, since even small holes drilled into the thin material form lines of weakness and the pressure required for applying the gage points in the holes causes unnatural bending of the material and hastens collapse due to eccentricity in the compression members. Furthermore, most gages of this type have a base length entirely too great.

Huggenberger tensometers of 1 in. base length have been employed for strain readings. In the future Tuckerman optical strain gages of the same length will also be used. The latter instruments are extremely accurate and were designed at the U. S. Bureau of Standards especially for model-testing work. Both types of gages are sufficiently compact to be placed at close intervals for investigation of stress distribution as shown in Figs. 17 and 18.

Many different types of clamps have been tried for attaching these gages to the thin walls of the model. Although comparatively slight pressure is required to hold the contact points on the smooth steel plate without the necessity of drilling holes, cutting notches, or making special indentations, this pressure may be sufficient with an improperly designed clamp to cause imperceptible buckling of the plate and premature collapse. Some success has been attained by mounting a second strain gage or a dummy gage on the opposite side of the plate, but with a closed or box model this is not always possible.

CONCLUSIONS

The authors have endeavored to show that while the design and fabrication of models for determining the strength of thin-walled structures represent real problems in the attainment of the requisite degree of geometrical similarity and the use of identical materials, these problems are not at all insurmountable.

The authors cannot too strongly recommend the use of multiple models. In fact, it has become their practice to base definite conclusions as to the strength of any given design only upon the closely duplicated test results of not less than three identical models.

Since test results on similar models of thin-walled structures can be closely duplicated, models can be used for making comparative tests among themselves. Once a satisfactory basis for comparison has been established, the influence of systematic variations in scantlings and the effects of auxiliary structures can be evaluated.

Although models are built to a small scale and are usually made elementary in character, it must not be assumed that the care and thought expended upon them can be reduced in the same proportion. In fact, the smaller the scale of the model, the more exacting are the mechanical requirements imposed by the theory of similitude, and the greater the care and thought that must be used in conducting the tests and interpreting the results.

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(9) Viterbo, F., "Sul problema della robustezza di cilindri cavi rinforzati trasversalmente sottoposti da ogni parte a pressione esterna," *L'Ingegnere*, vol. 4, no. 7, 1930, pp. 446–456, and vol. 4, no. 8, 1930, pp. 531–540.

(10) Biles, J. H., "The Strength of Ships With Special Reference to Experiments and Calculations Made Upon H. M. S. Wolf," Trans. Inst. Nav. Arch., vol. 47, 1905, pp. 80–115.

(11) Kell, C. O., "Investigation of Structural Characteristics of Destroyers Preston and Bruce," Trans. Soc. Nav. Arch. and Mar. Engrs., vol. 34, 1931, pp. 35–62.

Discussion

T. McLean Jasper.[4] The writer notes with interest the authors' outline of tests applied to models in determining the strength of thin-walled structures. Models, however, whose dimensions are several times reduced from the actual, must be used with discretion, since in very thin sections representing the model of a thin full-sized vessel, the model may take on some of the characteristics of a thin diaphragm or film. The mechanics of diaphragms or films does not apply to that of plates with considerable stiffness. This brings to mind, therefore, that multiple models may not necessarily give the correct solution when they represent the foregoing general conditions.

Undoubtedly models can be of great help in solving certain problems, and there is no doubt that the authors have used such methods to acquire valuable information in design, and therefore they should be complimented for the untiring efforts that they have applied.

The writer would like to ask what measurements for out-of-roundness were made in the collapse work that they have performed, except the outside- and inside-diameter changes which indicate also the variation of thickness of the tube. Would not the outside diameter variation and the thickness-measurement variation give also an equal degree of knowledge of the test specimens?

Experiments carried out under references 5 and 6 measured both the outside diameters and the thickness of the cylinders along the periphery at many points. Specimens used under reference 4 were made on a mandrel and the turning was carried out with great precision, and it is believed produced specimens of the greatest degree of precision yet used for collapse tests.

These remarks are directed specifically to the statement made by the authors under "Measurements of Shell Contour."

It would be of great interest to learn how it is proposed to use the data contained in Fig. 5, in applying them to test results and also to commercial tolerances in design and construction.

R. G. Sturm.[5] The paper gives a clear demonstration of several cases where exact geometrical similarity between a model and its prototype are really necessary. The effect of small discrepancies, such as deviations from flatness of thin plates, certainly cannot be overlooked in most models and is of extreme importance in models where elastic instability is a factor contributing to failure. There are cases, however, where thin-walled structures may not necessarily behave in the same manner as the prototypes unless a number of similarity conditions in addition to geometric similarity are obtained.

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One of the greatest advantages of model testing is to verify a theory of the behavior of a type of structure when that theory is applied to the model, and to establish confidence in the application of the same theory to the prototype. From theoretical analyses applied to model tests it is often found that failure of the model depends upon some power of the scale ratio, as well as upon a number of other factors, including the physical properties of the model material and details of the structure.

An example of the use of a model for interpreting the behavior of its prototype in which the elastic properties of the model material, as well as the geometrical proportions, enter into the similarity conditions, is that employed in the studies of a model of an 11-ft-diameter pipe line.[6] In this case the stresses in the prototype were determined by a theoretical analysis and checked by actual measurement in the field. Various models were built and a more comprehensive study made of the deflections and stresses. It was indicated by the theory that the stresses themselves would vary as the scale ratio, thereby indicating that the elastic properties of the model material should bear the same ratio to the properties of the material in the prototype as the scale ratio in order to give an exact similitude in behavior. The theoretical analysis, however, furnished the medium for interpreting the test results, even though complete similitude was not obtained.

In many cases it is desirable to magnify the deflections and strains produced by given loads so as to facilitate accurate measurements. In the case of the model of the aforementioned pipe line, the model was made of aluminum sheet particularly for this purpose.

A model of an arch dam[7] has been built of blocks of special rubber compound so as to magnify the deflections and strains of the model, as well as to obtain certain other conditions of similarity, such as discontinuities in the structure.

In the case of the tests of the diaphragm mentioned in the paper, the ductility of the material, as well as the ultimate strength, would influence the ultimate strength of the diaphragm in resisting uniform pressures. The actual stress in the plate at failure will be influenced by the amount of distortion, and the amount of distortion at failure depends upon the ductility of the material; that is, the greater the bulge, the smaller the radius of curvature, and consequently a greater internal pressure is needed for a given ultimate stress in the material.

In the case of the models of ships indicated in the authors' paper for which the loads apparently would be reduced according to the scale ratio, one would expect the measured stresses to be in the same proportion as the scale ratio. Thus the observed stresses in the model would be expected to be only a small part of what would be expected in the prototype. If the structure were to fail by elastic instability at loads scaled down according to the scale ratio and the modulus of elasticity for the model was also reduced according to scale, directly comparable results would be expected. If the structure were to fail by plastic yield of the material, one would expect the model to show a strength many times the strength of its prototype unless the ratio of the strength of the material in the model to the strength of the material in the prototype was the same as the scale ratio. This appears to be another example in which all the physical properties of the model material, including the modulus of elasticity, should be reduced according to the scale ratio in order to maintain true similarity.

The use of models in studying thin-walled structures may be informative and valuable provided the significant factors in similarity are adequately considered. The nature of the particular structure to be tested and the type of loads applied will determine which of all the conditions of similarity are significant.

E. E. Shanor.[8] The work being done at the U. S. Experimental Model Basin, Navy Yard, Washington, D. C., as described in this paper, is very interesting. Engineering, before the development of the mathematical methods of analysis now used, depended largely on model building for a check on designs. Originally the Patent Office required models in connection with a patent application. Engineering then was largely by the cut-and-try method. With the greater use of mathematics by the engineer, it became less needful to confirm new designs by model building and testing, and it is now only in exceptional cases that resort to test by models is made. Such cases generally arise from the difficulty of a mathematical analysis or the uncertainty as to the starting hypothesis. For instance, all bidders on the penstocks required at the Boulder Dam are required to build small-scale models of the Y's and T's and test them to destruction to prove the design on which they are bidding—this because of the difficulty of the mathematical analysis involved.

The models discussed in this paper represent constructions that are hard to analyze mathematically and that involve uncertainties as to the agreement of the actual construction with the assumptions on which the analyses are based. This is especially true of the tube which collapses due to external pressure. Theoretically, the tube should be absolutely round, and the walls should be of uniform thickness and the material of uniform physical qualities. The variation in pressure at which failure will occur and the form of the failure will be determined largely by variations from perfection in form and thickness of wall. It is evident what a task the model builder sets for himself when he attempts to check theoretical formulas for collapse of tubes under external pressure by means of models.

The methods used by the authors in building their models are extremely interesting. One wonders if the soldered seam does not introduce objectionable inequalities; yet a better way is not apparent. Although possibly beyond the scope of the paper, it would have been of immense interest to have included illustrations showing the collapsed tubes and a comparison of actual and theoretical results.

The paper performs a valuable service in showing that accurate and dependable results can be secured through the use of models, provided the requisite accuracy in their building is attained.

W. P. Roop.[9] This admirable exposition of the devices found necessary to obtain compliance with the conditions of similitude in model tests of thin-walled structures well illustrates the specialized character of model work in general. Models have come in for rather wide application recently, and their usefulness is well appreciated. This paper will serve to emphasize the necessity of caution, and it is hoped that it will help to prevent disappointments that might occur from too great reliance on model tests of insufficient scope and detail. The authors' strong recommendation for multiple models is strongly seconded. Possibly there are engineers who have sufficient insight, experience, and skill to assure that information obtained from a single model is complete and exact enough for direct application, but an engineer who could meet this requirement would also be able to design a full-scale structure of novel type without models. For, as the authors well say, a successful model requires no less forethought than is expended on a full-scale structure.

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[6] Discussion of "Design of Large Pipe Lines," by Schorer, by R. L. Templin, and R. G. Sturm, Proc. A.S.C.E., April, 1932, p. 597.

[7] "Building and Testing an Arch Dam Model," by A. V. Karpov and R. L. Templin, *Civil Engineering*, January, 1932.

[8] Vice-President, Petroleum Iron Works Company, Sharon, Pa. Mem. A.S.M.E.

[9] Lt. (CC), U.S.N., Office of Superintending Constructor for U. S. Navy, New York Shipbuilding Company, Camden, N. J.

Many existing structures have given long service which, from the point of view of present-day standards, might be rated unsuccessful due to overweight. It was largely by a process of approximation realized in such full-scale models that the design standards of an older generation were established. The model method is obviously a more economical method of obtaining similar results, but it is an error to press the model engineer too closely in the matter of economy. Models cost less to build, but no less to design, than full-scale structures. The full-scale models have one advantage in that they are capable, in most instances, of at least partial service, but even when due allowance is thus made for what might be called the salvage value of a full-scale model, the balance is all in favor of the small model as a means of progress in design. Where progress is desired, therefore, a generous expenditure for models is in order.

The analysis of model data is hardly less costly and surely not less necessary than forethought in model design; indeed, the two can hardly be separated. Yet it is the writer's belief that the relatively low cost of actual construction of models sometimes leads to an underestimation of the amount of the expenditure on model work, and especially of the study and interpretation of model data, which will lead to greatest overall economy.

The tests under discussion follow two divergent lines, dealing on the one hand with cylinders of circular section under external pressure, and on the other with beams of box section. Of these the first line has been followed much further, and the large number of models tested has resulted in the evolution of an elaborate but highly successful technique. These methods have been in part adapted to use in the box models since the occurrence of failure by instability leads to marked similarities between the cylinders under external pressure and the boxes under bending load. These are, however, important differences; and one would anticipate that with further extension of the box-model program even further divergences in technique would occur.

For example, no strain-gage data were taken on the cylinders. One might infer from what is said that since strain gages are of value only in plating in tension, and since these box girders invariably fail by compression, strain gages are of little value on the boxes. This would, however, be a hasty inference. Distribution of strain in tension may itself serve to indicate imminent occurrence of buckling in compression members. But it would be disappointing to be forced to the conclusion that "buckling in compression members cannot readily be separated from pure strain."

Box models so far tested appeared to have been all of the same bulkhead spacing; with stiffening members more closely spaced, the need of extreme accuracy with respect to flatness would appear to be somewhat diminished, or, in any event, a distinction would have to be made with respect to the extent of plating within which given deviations might occur, whether within the limits of one frame or of several frames. While the thin-walled cylinder under external pressure will always fail in the unstable mode, the thin-walled box girder presents a variety of elastic elements of which the flat plate under compression in its own plane is only one, though under most circumstances of proportion it is doubtless the most significant.

The allusion by the authors to the influence of secondary features of design deserves added emphasis.

It has long been customary to design a structure as a whole (in case of a ship, a tapered box girder with transverse diaphragms), and then attach secondary parts with little consideration of effects other than those of weight. As refinement of the design of the primary structure advances, more consideration must be given to the effect of such incidental features. The function of the present tests is to provide a secure basis for the refinement of the primary structure, and this can be done only by elimination of uncertainties proceeding from the incidental features. Thus while the models do not appear to resemble an actual ship very closely, they do present an idealized representation of the primary structure. When, by use of models, supplemented by full-scale tests, the limits of refinement of the primary structure are approached, a choice will have to be made between two procedures with respect to the incidental features. As a first step in advance, they may be so combined with the primary structure as to produce no interference with its elastic action. By following certain definite practices in design, this result can be rather accurately obtained. A further development, however, would consist in adaptation so as to obtain assistance for the primary structure. This will impose rather definite limitations on design of incidental structure. The safest guide in ascertaining these limitations would be found in tests on models incorporating the incidental structure.

C. O. Rhys.[10] The use of small-scale models undoubtedly offers an excellent method of determining how a full-sized structure will behave. As is well known, the principle has long been successfully employed in the design of ships, models of 12 ft or so in length being towed along in tanks. In this case, the main object is to ascertain resistance to motion through the water, and geometrical similarity is the paramount consideration.

As pointed out in the excellent paper under discussion, when strength considerations are involved, not only geometrical similarity, but identical materials and similarity of loading and fastening become equally important. The authors show in a clear and instructive way how these results are obtained, bringing out the point that, as might be expected, multiple models have great advantages over individual models.

On the fifth page it is stated that spot welding is a successful alternative to riveting for very thin plates. It would be of interest to know whether any tests have been made to compare the joint efficiencies of the two modes of fastening.

In the description of the behavior of two similar models under test, the time interval between the formation of bulges is emphasized. This confirms some recent experience of the writer's with the shell of a heat exchanger which deformed under stresses due to unequal temperatures. Calculation showed that a corrugated effect might be expected. Actually, however, only one corrugation appeared, though dimensionally it agrees with the theoretical result.

William Hovgaard.[11] This paper is of great importance, because it shows that it is possible to study the design of thin-walled structures by means of small models, a problem that hitherto has been unsolved, or solved only in certain cases. It indicates the methods by which such models can be manufactured satisfactorily, and since the information in this respect is the result of many costly experiments and much thought and ingenuity, it is of great value to the profession. In naval architecture as well as in several other branches of engineering the problem of weight saving has come to the front in recent years. Old rule-of-thumb methods and crude empirical formulas must be in many cases replaced by more refined calculations, based on the theory of elasticity, but even the best formulas cannot be safely applied without experiments which enable the designer to determine the physical constants. Full-scale tests can be carried out only in rare cases, and it is necessary to resort to model experiments; but with thin-walled structures and if relatively small models are to be used, an extreme accuracy in the manufacture of the models is required, involving great practical difficulties. As pointed out in the paper, it is necessary to use several identical models in each specific case, so as to obtain a reliable average. All this means high cost, unless special methods of manufacture are developed, and it is here perhaps that this paper has its greatest value by indicating practical and simple methods of manufacture.

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Thus the authors of this paper open up a field of research which hitherto has been cultivated in a somewhat fragmentary and unsatisfactory manner. The systematic and thorough way in which this difficult problem has been approached has yielded results which reveal the great possibilities of the method. It cannot fail to lead to much greater results in the future and a wider application by other investigators.

The Bureau of Construction and Repair of the Navy has here successfully initiated a development of this important line of research, which will be of benefit not only to the Navy, but also to the merchant marine and to several important industries in other fields.

AUTHORS' CLOSURE

It is difficult for the authors to believe that, as Mr. Jasper says, "the mechanics of diaphragms or films does not apply to that of plates with a considerable stiffness," providing the "diaphragms" and "plates" are *geometrically similar*. The "slenderness" of a rod and the "thinness" of a tube or plate depend upon their length, diameter, and breadth, respectively, as well as upon their absolute thickness. A so-called "thin" tube may be 0.05 in. thick or 5 in. thick, depending entirely upon its diameter. So also a "thin" plate or shell may be very thin or it may be very thick, depending upon its other dimensions. Practically, it may be impossible to obtain geometrical similarity in models "whose dimensions are several times reduced from the actual," but this is a purely physical limitation and in no way changes the underlying theory. To be sure, the thickness of model shells cannot be reduced indefinitely, say, to molecular dimensions, but practical limitations prevent an approach to such an extreme.

Concerning measurements of out-of-roundness, the record for each model tested includes a diagram similar to Fig. 5. From such a diagram, it is possible to measure directly not only variations in diameter of the shell, but also variations in radius. Measurements of diameter involve two irregular surfaces, and it is not uncommon to have the effect of irregularities on opposite sides neutralize each other. That is, an outward bulge on one side and an inward bulge on the opposite side may not be indicated by a measurement of the diameter.

Since short, thin tubes, or tubes with closely spaced frames, collapse in a large number of lobes, and since each individual lobe behaves in a manner somewhat analogous to a column or plate under compression, it seems reasonable that the eccentricity in each individual lobe should be significant in determining the resistance to collapse of that particular lobe section. Certainly this eccentricity should be considered independent of any eccentricity that might exist on the opposite side of the model.

Von Mises ("Stodola's Festschrift," Zurich, 1929, p. 429) states: "It is certain that in the case of buckling with a higher number of lobes, even very slight initial irregularities may have a decisive effect." Two tubes, one elliptical and the other containing a dent, may have the same difference in maximum and minimum radii, yet the elliptic shape would have a greater weakening influence than the dent on a tube long enough to collapse in two lobes, while the dent would have a greater weakening influence on a tube short enough to collapse in a large number of lobes. A tube is no stronger than its weakest lobe.

It is for this reason that variations in radii rather than variations in diameters have been employed by the authors in the measurement of out-of-roundness. The necessity for such a distinction does not become evident in dealing with long tubes similar to those mentioned in references 4, 5, and 6, which always collapse in a small number of lobes, but cannot be overlooked in analyzing data on short, thin tubes which collapse in a large number of lobes.

The authors wish to point out that they are always on the alert for evidences of a scale effect which might vitiate the results of their model tests, even where theory indicates that no such effect can be expected. It is for this reason that geometrically similar models of different scales and sizes are invariably built. Their plan has been to use smaller, simpler, and hence cheaper models for a general survey of the field to be covered, and to verify the conclusions thus obtained with larger and more accurately constructed models, eventually confirming results by tests on models of about half-scale. For example, their experiments on submarine strength hull models 6 in. and 16 in. in diameter were confirmed by tests at the Portsmouth Navy Yard on models 8 ft. in diameter, thus precluding the possibility of an unknown scale effect.

Evidently there is some confusion in Mr. Sturm's mind as to the method of applying load to the models of ships described in the paper. In the model arch dam referred to, the height of the dam and the pressure head are reduced according to the scale ratio; since the surface of the dam is reduced by the square of the scale ratio, the total load is reduced by the cube of the ratio. However, the ship models are not tested in water, but are supported by transverse knife edges on the platform of a testing machine. According to the simple theory of beams or columns, equal loads per unit area on the model and prototype, that is, total loads reduced according to the square of the scale ratio, produce equal stresses in the two structures. This is shown in the case of tubes subjected to external pressure, since the model and the full-sized structure collapse at equal hydrostatic pressures.

Concerning the objectionable features of a soldered seam, as pointed out by Mr. Shanor, the authors admit that the seam has proved a source of trouble and has caused considerable uncertainty in interpreting experimental results. The seam usually represents the region of greatest departure from circular form, and, as a rule, collapse begins in a lobe adjacent to the seam.

Various attempts have been made to improve these conditions. Models have been made, using no support whatever at the seam except solder; longitudinal straps from 0.25 in. to a semi-circumference in width have been tried; narrow straps transverse to the seam have also been used. The present method of reinforcing the seam was adopted as possessing the fewest disadvantages.

The authors have at hand many diagrams similar to Fig. 5, and many photographs of collapsed models. One such photograph was published as Fig. 3 in their 1931 paper (7), while Table 1 of the same paper gave a comparison of actual and theoretical collapsing pressures for a number of tube models.

Other photographs and data will be published in a subsequent paper.

It is indeed desirable, as Lieutenant Roop points out, to obtain reliable strain-gage readings on thin-walled structural models while under test. Such readings can be taken by using very sensitive gages of not more than 1 in. base length, clamped to the test member. Preferably, gages should be mounted on both sides of the plate with their points exactly opposite each other. This avoids the introduction of eccentric loading by the gage itself and makes the determination of strain at the middle surface of the plate very simple. When gages with a base length of 1 in. or less are used, the introduction of apparent strain due to buckling does not introduce serious error in stress calculations until the buckling becomes considerable.

The amount of buckling a plate can undergo and still continue to hold its load, as well as the permissible initial eccentricity in a compression member, are functions of the lobe length. Here again the analogy is very close between stiffened plates under edge compression and circular cylindrical shells under external hydrostatic pressure.

The length of the lobes in a plate is determined by the spacing of longitudinal stiffeners, that is, stiffeners which are parallel to the direction of loading. This corresponds to the spacing of transverse circular frames in a tube. Hence the effect of variation from flatness depends more upon the spacing of longitudinal than of transverse stiffeners.

Regarding the use of spot welding instead of riveting, as brought up by Mr. Rhys, the authors possibly should have said that spot welding is the only known alternative for riveting for very thin plates. Comparative tests of riveting, spot welding, and fusion welding on plates about 0.1 in. thick are contemplated in the near future, but it is not likely that any satisfactory comparison can be made on plates 0.05 in. thick or less.

The time interval occurring between the application of load and the formation of bulges in circular cylindrical tubes subjected to external pressure is very pronounced. The same may be expected on all models and structures where the stresses reach the yield point of the material. On some of the large tube models, bulges have appeared after the load had been held constant for several minutes. Likewise, bulges have appeared at intervals of several minutes at constant load, even though the bulges were not adjacent and seemed to have no direct connection with each other. This phenomenon is due no doubt to the gradual redistribution of loads when any small region has been stressed to the yield point and deformation has brought about a corresponding redistribution of stresses.

The experimenter should take care lest the test loads be applied too rapidly and the recorded ultimate collapsing load be too great.

Once the validity of model tests on thin-walled structures has been justified, an unlimited field of application is immediately opened up. Practically the only limitation is the accuracy with which models can be fabricated and the ingenuity with which test results can be interpreted. As previously pointed out, certain features such as riveting cannot be conveniently reproduced to small scale, and the experimenter is confronted with the necessity of providing a suitable alternative. Just how successful a given alternative may be, or how fully the results obtained by its use can be relied upon for full-scale design, are questions to be settled by experience and individual judgment. However, much if not all of the uncertainty is removed when final tests are performed on larger-scale models where the details of construction can be accurately reproduced to scale.

APM-54-26

Stresses in Railroad Track

By S. TIMOSHENKO[1] and B. F. LANGER[2]

S. Timoshenko

With the constant tendency in railroad practice to increase the axle loading and the speed of locomotives, the problem of stresses produced in rails by moving loads becomes more and more important. In a study recently made by engineers of the Westinghouse Electric and Manufacturing Company, principally in connection with the study of the tracking characteristics of electric locomotives, there has been developed a method for experimental determination, not only of vertical but also of lateral forces produced on the rails by a moving locomotive, and it is shown that these lateral forces produce in rails very high stresses. In this paper the authors briefly discuss the theory which has been used as a guide in this experimental research work, and describe some recent experiments in the laboratory and in the field.

B. F. Langer

With the constant tendency in railroad practice to increase the axle loading and the speed of locomotives, the problem of stresses produced in rails by moving loads becomes more and more important. During the last fifteen years an important piece of work in this field has been done by the Special Committee on Stresses in Railroad Track[3] under the direction of Prof. A. N. Talbot. In this work a complete theory for the bending of rails produced by vertical loads has been given, and also an experimental method for determining these loads has been developed.

Further progress in determination of track stresses has been made by engineers of the Westinghouse Electric & Manufacturing Company,[4] principally in connection with the study of the tracking characteristics of electric locomotives. There has been developed in this study a method for experimental determination not only of vertical, but also of lateral forces produced on the rails by a moving locomotive, and it has been shown that these lateral forces produce in rails very high stresses.

In this paper a brief discussion of the theory which has been used as a guide in this experimental research work is given, and some recent experiments in the laboratory and in the field are described. Special attention is given to the local stresses in rails produced by vertical and lateral forces. It seems that this is the first time that these stresses have been studied. The importance of knowledge of these stresses for a logical design of rail profiles cannot be overestimated.

I—RAIL AS A LONG BAR ON ELASTIC FOUNDATION

1 Vertical Bending of Rails

The following theory of vertical bending of rails is based upon the assumption that the rail can be considered as a long bar continuously supported by an elastic foundation.[5] The rigidity of this foundation is defined by the magnitude k of the *modulus of foundation*, which is the load per unit length of the rail necessary to produce a deflection of the foundation equal to unity. The experiments show that the magnitude of the modulus can be always determined in such a manner that the deflection of the rail calculated on the assumption of the continuous foundation is in good agreement with the deflection of the actual rail supported by ties. Using the notations

EI = flexural rigidity of the rail

$$\beta = \sqrt[4]{\frac{k}{4EI}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [1]$$

and applying the equation for the deflection curve

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[2] Mechanics Division, Westinghouse Research Laboratories, East Pittsburgh, Pa. Mr. Langer is a graduate of Stanford University, where he received the degree of B.A. in 1926 and of Engineer in 1928. He has been employed by the Westinghouse Research Laboratories since 1928, where he has been working on mechanical problems relating to railway engineering.

[3] The reports of this committee may be found in the following publications:

1st Progress Report: Proc. A.R.E.A., vol. 19 (1918); Trans. A.S.C.E., vol. 82 (1918).

2nd Progress Report: Proc. A.R.E.A., vol. 21 (1920); Trans. A.S.C.E., vol. 83 (1920); Circular No. S-II-10 of the A.R.A.

3rd Progress Report: Proc. A.R.E.A., vol. 24 (1923); Trans. A.S.C.E., vol. 86 (1923); Circular No. IV-32 of the A.R.A.

4th Progress Report: Proc. A.R.E.A., vol. 26 (1925); Trans. A.S.C.E., vol. 88 (1925).

5th Progress Report: A.R.E.A. Bulletin, vol. 31, no. 319, Sept., 1929.

[4] S. Timoshenko, "Method of Analysis of Statical and Dynamical Stresses in Rail," Proc. 2nd International Congress for Applied Mechanics, Zurich, 1926.

J. P. Shamberger, "A Magnetic Strain Gage," Proc. A.S.T.M., vol. 30 (1930), p. 1041.

B. F. Langer, "An Instrument for Measuring Small Displacements," *Review of Scientific Instruments*, vol. 2, 1931.

B. F. Langer and J. P. Shamberger, "The Measurement of Locomotive Wheel Loading," Proc. 3rd International Congress for Applied Mechanics, Stockholm, 1930.

[5] Such an analysis was made for the first time by Winkler. See his book, "Die Lehre von der Elasticität und Festigkeit," Prag, 1867. See also Dr. H. Zimmermann's book, "Die Berechnung des Eisenbahnoberbaues," 2nd ed., Berlin, 1930.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$$EI\,\frac{d^4y}{dx^4} = -ky \ldots\ldots\ldots \quad [2]$$

we find[6] for the simplest case of a single load P acting in the vertical plane of symmetry on an infinitely long rail (Fig. 1):

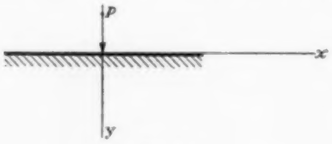

Fig. 1

$$y = \frac{\beta P}{2k} e^{-\beta x} (\cos \beta x + \sin \beta x) \ldots\ldots \quad [3]$$

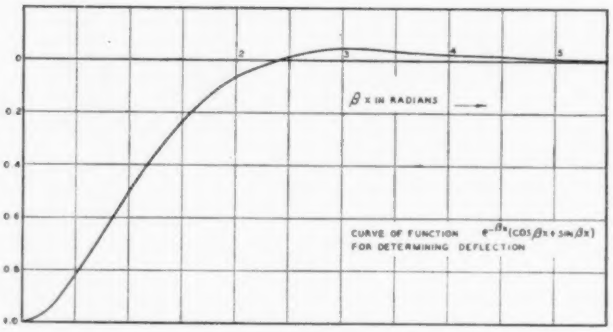

Fig. 2 Deflection Curve

The corresponding deflection curve is shown in Fig. 2. The maximum deflection is under the load P and is obtained from [3] by putting $x = 0$. Then

$$\delta = (y)_{x=0} = \frac{\beta P}{2k} \ldots\ldots\ldots \quad [4]$$

This equation is usually employed in calculating the modulus of foundation k from the deflection of an actual rail. Measuring the deflection of the rail produced by a known vertical load P and using Equation [1], we can easily calculate from [4] the magnitude of k. (See description of the calibration of the track, Section III.)

From the solution [3] it is seen that the deflection curve is a wavy line, the amplitudes of which are damped out with the increase of the distance x from the point of the application of the load P. The length of the wave is

$$L = \frac{2\pi}{\beta} = 2\pi \sqrt[4]{\frac{4EI}{k}} \ldots\ldots\ldots \quad [5]$$

Taking as an example a 130-lb. rail ($I = 72.8$ in.^{4}) and assuming $k = 1500$ lb. per sq. in., we find from Equation [1] that $\beta = 0.020$ in.$^{-1}$ Then from Equation [5] we find that the length of a wave is more than 300 in. If instead of an infinitely long bar a bar of a finite length $2l$ is loaded at the middle, the equation for deflection can be put in the form

$$\delta = \alpha_1 \frac{\beta P}{2k}$$

In which α_1 is a numerical factor depending on the length of the bar. The values of this factor for various values of βl are given in the second line of the following table.[7]

$l\beta =$	1	1.5	2	2.5	3	π
$\alpha_1 =$	1.18	1.09	1.08	1.04	1.02	1.01
$\alpha_2 =$	0.92	1.09	1.05	1.01	0.997	0.996

It is seen that if a bar of a considerable length (say, $l\beta > 2.5$) is taken, the maximum deflection is about the same as in the case of an infinitely long bar.

Having the deflection curve, Equation [3], the bending moment is easily calculated from the equation

$$M = -EI\frac{d^2y}{dx^2} = \frac{P}{4\beta} \epsilon^{-\beta x} (\cos \beta x - \sin \beta x) \ldots\ldots \quad [6]$$

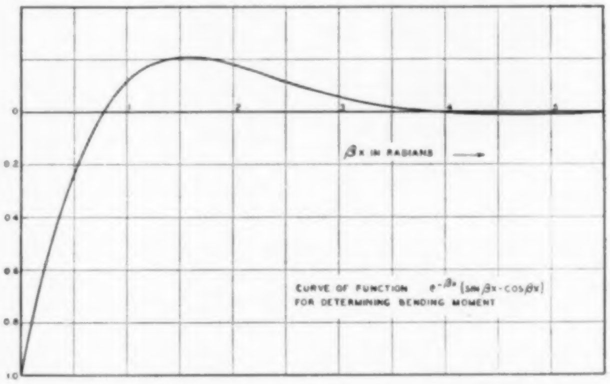

Fig. 3 Bending-Moment Curve

which moment is represented graphically in Fig. 3. The maximum bending moment is under the load. Substituting $x = 0$ in [6],

$$M_{\max} = \frac{P}{4\beta} \ldots\ldots\ldots \quad [7]$$

In the case of a bar of a finite length $2l$ loaded at the middle, the maximum bending moment is

$$M_{\max} = \alpha_2 \frac{P}{4\beta}$$

The numerical values of the factor α_2 for various values of $l\beta$ are given in the table above. It is seen that for $l\beta > 2.5$, the maximum bending moment for a bar of finite length is practically the same as in the case of an infinitely long bar. In our laboratory tests (see Section II) $2l = 240$ in. and $\beta = 1/50$ in.$^{-1}$ Hence the stress conditions near the point of application of the load are practically the same as for an infinitely long bar.

By using Equation [1] the maximum bending moment is

$$M_{\max} = \frac{P}{4} \sqrt[4]{\frac{4EI}{k}} \ldots\ldots\ldots \quad [7a]$$

and using the notation S for the section modulus of the rail, the maximum bending stress is

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{P}{4}\frac{\sqrt[4]{I}}{S}\sqrt[4]{\frac{4E}{k}} \ldots\ldots\ldots \quad [8]$$

Since the right-hand member of [8] remains constant for geometrically similar cross-sections of rails, if E/k remains constant and P varies as the square of the linear dimensions of the cross-section, it can be concluded that $\sigma_{\max}$ remains unchanged if the weight of the rail per unit length increases in the same proportion as the load P. This conclusion gives a certain justification for increasing the weight of the rail in the same proportion as the axle load of the rolling stock that is to run over it.

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[6] For derivation of formulas see S. Timoshenko, "Strength of Materials," vol. II, p. 401. Equation [3] is derived only for $x > 0$.

[7] See H. Zimmermann, "Die Berechnung des Eisenbahnoberbaues," 2nd ed.

The approximate value of the maximum pressure $R_{\max}$ on a tie is obtained by multiplying the maximum deflection, Equation [4], by the tie spacing a and by the modulus of foundation. Then

$$R_{\max} = \frac{\beta P}{2k}\, ak = \frac{P}{2}\sqrt[4]{\frac{ka^4}{4EI}} \quad \text{[9]}$$

It is seen from this that the pressure on the tie depends principally on the tie spacing a. It diminishes with a diminution of a and k, and with an increase in the flexural rigidity EI of the rail.

Having the solution for the vertical deflection produced by a single load P and using the principle of superposition, the deflection curve and the bending-moment diagram for any system of vertical loads can easily be obtained. For the purpose of illustrating the method of calculation, let us consider a 100-lb. rail with $I = 44$ in.^{4} and with $k = 1500$ lb. per sq. in. Then from [1], $\beta = 1/43.3$ in.$^{-1}$ Taking a system of four equal loads (Fig. 4) 66 in. apart, and fixing the origin of coordinates at the point of contact of the first wheel, the values of βx for four wheels are given in the following table.

Wheels	1	2	3	4
βx	0	1.52	3.05	4.57

By using now the curve in Fig. 3 and superposing the effects of all the loads acting on the rail, the bending moment under the first wheel is

$$M_1 = \frac{P}{4\beta}\,(1 - 0.207 + 0.051 + 0.008) = 0.75\,\frac{P}{4\beta}$$

i.e., the bending moment is 25 per cent less than that produced by a single load P. In the same manner, for the point of contact of the second wheel it is found that

$$M_2 = \frac{P}{4\beta}\,(1 - 2 \times 0.207 - 0.051) = 0.535\,\frac{P}{4\beta}$$

It is thus seen that, due to the action of the adjacent wheels, a considerable diminution of the bending moment under the second and the third wheels takes place.

In the same manner the bending moment at any other point can be calculated. For instance, at the point A, midway between the wheels 2 and 3, the bending moment is $-0.22\, P/4\beta$ and the rail is bent convex upward. From this we see that during the motion of such a system of loads as is shown in Fig. 4 the stresses change not only their magnitude but also their sign. Under the action of such cycles of stress, fatigue cracks in rails may occur. Many actual fractures in rails have a typical fatigue-crack appearance. The same kind of fracture occurs also in splice bars (Fig. 5). The cracks usually begin at the points mm, which are the most remote points from the neutral axis nn. Sometimes cracks begin at the holes or at the reentrant corners, where a high stress concentration takes place.

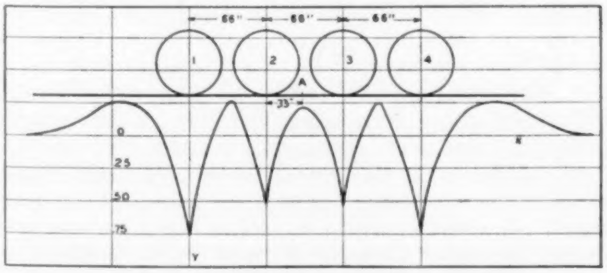

Fig. 4 Bending Moment Produced by Four Equal Wheel Loads
(Moment from one isolated load = unity.)

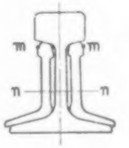

Fig. 5

By using the same method of superposition and the curve given in Fig. 2, the deflection produced by a system of vertical loads also can be easily calculated.

The inverse problem, i.e., of calculating the vertical pressures produced by locomotive wheels on the rail provided either the deflections or the stresses in the rail under the wheels have been determined by experiment, can also be solved without any difficulty. The application of such calculations to our experiments has always shown a good agreement between the sum of the calculated pressures and the actual weight of the locomotive used. This gives a certain justification to our assumption that the rail can be considered as a bar on a continuous elastic foundation.

2 Eccentric Vertical Load

If a vertical load P is applied with an eccentricity e, Fig. 6(a), it can be replaced by the load P centrally applied and by the torque Pe, Fig. 6(b). The central load produces vertical bending of the rail, discussed in the previous article, and the torque produces twisting of the rail.

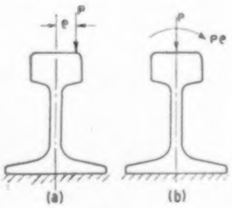

Fig. 6

If a rail with free ends is submitted to the action of two equal and opposite twisting couples M_t, the angle of twist may be roughly calculated from the known approximate equation of Saint-Venant, namely,

$$\phi = \frac{M_t l}{C} \quad \text{[10]}$$

In which l = length of the rail, and

$$C = \frac{A^4 G}{40 I_p}$$

where A = area of the cross-section
G = modulus of rigidity, and
I_p = polar moment of inertia.

To get a more accurate value for the torsional rigidity C, a direct torsion test is necessary. Such tests show that the actual value of the torsional rigidity is somewhat smaller than given by Saint-Venant's formula.

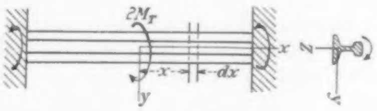

Fig. 7

If the ends of the rail are fixed and a torque $2M_t$ is applied at the middle (Fig. 7), the twist of the rail is accompanied by a bending of the head and base of the rail. At any cross-section at a distance x from the middle, the torque M_t is transmitted partly in the form of simple twist and partly by bending of the head and of the base of the rail. Let M_1 and M_2 represent the first and second parts, respectively, and ϕ denote the angle of twist shown in Fig. 8. Then

$$M_1 = -C\,\frac{d\phi}{dx} \quad \text{[11]}$$

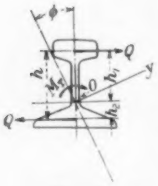

FIG. 8

The part M_2 is represented in Fig. 8 by the couple Qh, in which Q denotes the shearing force due to bending in the head and base of the rail, and h is the distance between the centroids of the head and base sections. Neglecting bending of the web, the position of the center of twist[8] O is determined by the distances

$$h_1 = \frac{hI_2}{I_1 + I_2}, \qquad h_2 = \frac{hI_1}{I_1 + I_2} \ldots\ldots\ldots\ldots [12]$$

in which I_1 and I_2, respectively, denote the moments of inertia of the head and of the base sections with respect to the axis of symmetry of the rail cross-section. The shearing forces Q are then determined from the equation

$$Q = EI_1 \frac{d^3y}{dx^3} = EI_1 h_1 \frac{d^3\phi}{dx^3}$$

which gives

$$M_2 = Qh = Eh^2 \frac{I_1 I_2}{I_1 + I_2} \frac{d^3\phi}{dx^3} \ldots\ldots\ldots\ldots [13]$$

Using the notation

$$\frac{EI_1 I_2}{I_1 + I_2} = D$$

the differential equation of twist becomes

$$M_t = M_1 + M_2 = -C \frac{d\phi}{dx} + Dh^2 \frac{d^3\phi}{dx^3} \ldots\ldots [14]$$

The solution of this equation for a long rail is

$$\frac{d\phi}{dx} = -\frac{M_t}{C}(1 - e^{-\gamma x}) \ldots\ldots\ldots\ldots\ldots [15]$$

in which

$$\gamma = \sqrt{\frac{C}{Dh^2}} \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

It is seen that with an increase in the distance x from the middle cross-section of the rail, the second term in the parenthesis in the right-hand member of Equation [15] diminishes, and this equation approaches Equation [11] for simple torsion. The bending of the head and base of the rail have only a localized effect on the torsion of the rail. The results of the experiments made[9] are in very good agreement with Equation [15].

Imagine now that the rail shown in Fig. 7 is attached along its length to a continuous elastic foundation which resists the rotation of cross-sections of the rail during twist. Assuming that the resisting couple of the foundation per unit length of the rail is proportional at any cross-section to the corresponding value of the angle of twist ϕ and using for the factor of proportionality the symbol k_1, we find by differentiation of Equation [14],

$$\frac{dM_t}{dx} = -k_1\phi = -C\frac{d^2\phi}{dx^2} + Dh^2 \frac{d^4\phi}{dx^4}$$

and the equation for determining the angle of twist is

$$Dh^2 \frac{d^4\phi}{dx^4} - C\frac{d^2\phi}{dx^2} + k_1\phi = 0 \ldots\ldots\ldots\ldots [17]$$

The solution of this equation shows that the twist of a rail affixed to an elastic foundation may be represented by a wavy line, and that the amplitude of the waves decreases with an increase in the distance from the loaded cross-section.

3 LATERAL LOAD

A single lateral force H applied to a rail produces lateral deflection and twist in the rail. Let us assume again that the rail is continuously supported by an elastic foundation which resists the lateral deflection and twist of the rail. Let k_1 be the modulus of foundation with respect to twist (see Art. 2) and k_2 the modulus of foundation with respect to lateral deflection. Denoting by y the lateral deflection of the center of twist of the rail and by ϕ the angle of twist, the deflection at the base of the rail is, from Fig. 9, $y - g\phi$, and the corresponding lateral reaction at the base per unit length is

$$q = k_2(y - g\phi) \ldots\ldots [18]$$

The reactive twisting moment per unit length is

$$m = k_1\phi \ldots\ldots\ldots [19]$$

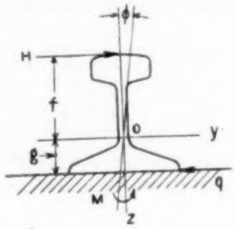

FIG. 9

Now the differential equation of lateral bending of the rail is

$$EI_1 \frac{d^4y}{dx^4} = -k_2(y - g\phi) \ldots\ldots\ldots\ldots\ldots [20]$$

in which EI_1 is the flexural rigidity of the rail in a horizontal plane.

In discussing the twist of the rail we find from statics that the rate of change of torque is given by the equation:

$$\frac{dM_t}{dx} = qg - m$$

Using now the Equations [14], [18], and [19], we find

$$-C\frac{d^2\phi}{dx^2} + Dh^2 \frac{d^4\phi}{dx^4} = k_2(y - g\phi) - k_1\phi \ldots\ldots [21]$$

The two simultaneous Equations [20] and [21] determine the lateral bending and twisting of the rail under the action of the force H. In integrating these equations, the following conditions at the loaded cross-section must be satisfied:

$$\left.\begin{array}{l} \left(\dfrac{dy}{dx}\right)_{x=0} = 0; \quad EI_1\left(\dfrac{d^3y}{dx^3}\right) = \dfrac{1}{2}H; \quad \left(\dfrac{d\phi}{dx}\right)_{x=0} = 0 \\ -C\left(\dfrac{d\phi}{dx}\right)_{x=0} + Dh^2\left(\dfrac{d^3\phi}{dx^3}\right)_{x=0} = \dfrac{1}{2}Hf \end{array}\right\} \ldots [22]$$

The other constants of integration must be chosen so as to make bending and twist of the rail disappear at infinity.

The application of this general theory in particular cases[9] shows that lateral bending and twist are of localized character; therefore if several lateral forces, such as lateral pressures produced by locomotive wheels, are acting on the rail, the maximum stress produced in the head of the rail by one of these forces is not substantially affected by the other forces.

4 EXPERIMENTAL DETERMINATION OF LATERAL FORCES, VERTICAL FORCES, AND THE ECCENTRICITY OF VERTICAL LOADS

In the most general case, the action of a force on the rail can be replaced by a vertical load P with a certain eccentricity e

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[8] Further discussion regarding center of twist can be found in Timoshenko's "Strength of Materials," vol. 1.

[9] See paper by S. Timoshenko in Reports of Zürich Congress, loc. cit., footnote No. 4.

and by a lateral load. In discussing stresses in rails and tracking characteristics of locomotives, it is of practical importance to know all three quantities: P, H, and e. There is no difficulty in finding experimentally the vertical load P. It is clear from Figs. 10(b) and 10(c) that at the two points m and n at the two opposite edges of the base of the rail the stresses produced by a torque M_t or by a lateral load H are numerically equal and of opposite sign. Hence by taking the strain-gage measurements at m and n, the average of these two readings can be used for determining the magnitude of the vertical load. This calculation can be made by using Equation [8] or, with a greater accuracy, by making a preliminary calibration of the track by applying a known vertical load and taking readings at m and n. The effect of the adjacent vertical loads on the readings at m and n can be determined by using the curve in Fig. 3.[10]

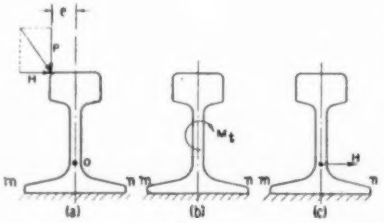

FIG. 10

In our field experiments, a Westinghouse magnetic strain gage[11] was used and the strains produced at m and n by locomotive wheel pressures were recorded to a certain scale on an oscillograph film. Taking readings under all the wheels of the locomotive, the vertical pressures of these wheels on the rail can be calculated as it was explained in Art. 1. In order to obtain a higher accuracy, the curve obtained by calibration was usually applied instead of the theoretical curve, Fig. 3. Figs. 11, 12, and 13 show the strain gages attached to the base of the rail.

To determine the magnitude of lateral force H, Fig. 10(a), the longitudinal strain-gage measurements at the height of the center line of the rail were made as shown in Fig. 12 (strain gage B). The preliminary tests in the laboratory (see Section II) showed that this longitudinal strain depends principally on the magnitude of the lateral force H. The effect of the vertical force P, Fig. 10(a), is small and practically independent of the magnitude of the eccentricity e. Subtracting this small effect from the total reading of the strain gage B, the difference represents to a certain scale the lateral force H. The scale is usually established by making the preliminary calibration tests on the track.

For determining the eccentricity e of the vertical pressure P, the vertical strain gage C (Figs. 11 and 13) was used. This strain gage measures the change in the distance between the head and the base of the rail, which depends principally on the eccentricity e of the vertical load P. Thus having the readings on the strain gages A, B, and C, the quantities P, H, and e, determining completely the pressure of a wheel on the rail, can be calculated provided a preliminary calibration test on the track has been made. Several examples of such calculations will be given in discussing field measurements in Chapter III.

5 DYNAMIC STRESSES IN RAILS

The dynamic deflection of the rail and the dynamic stresses under the action of the moving wheels of a locomotive may become much larger than those calculated on the basis of the static formulas discussed in Art. 1. There are various causes

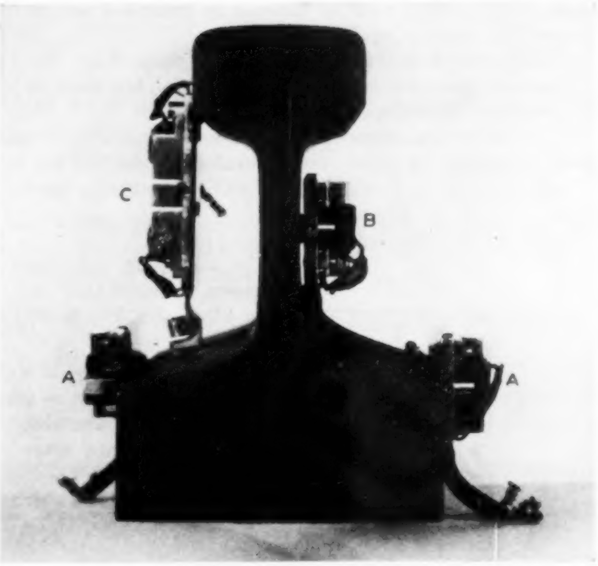

FIG. 11 STRAIN GAGES MOUNTED ON RAIL MODEL. END VIEW

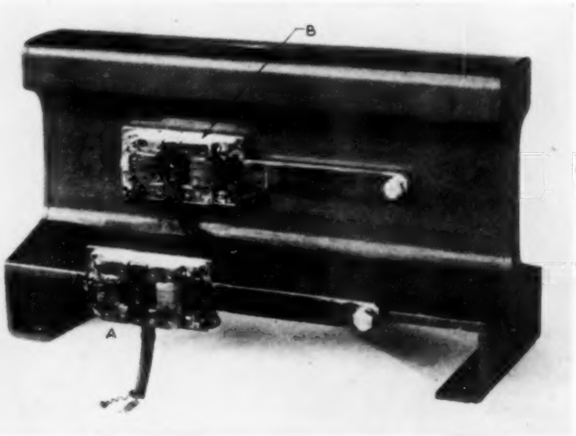

FIG. 12 STRAIN GAGES MOUNTED ON RAIL MODEL. GAGE SIDE OF RAIL

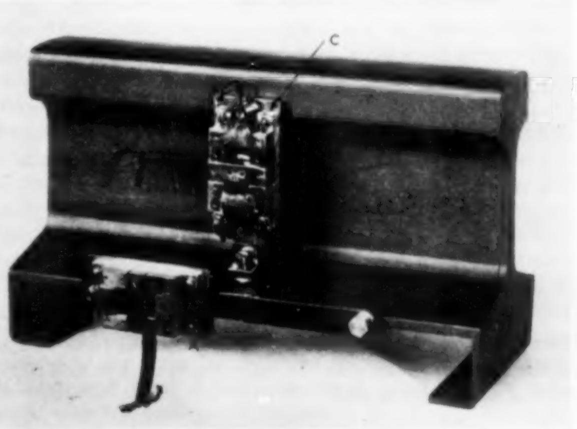

FIG. 13 STRAIN GAGES MOUNTED ON RAIL MODEL. OUTSIDE OF RAIL

which may produce such an increase in deflection and stress, the principal ones being:

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[10] At the suggestion of J. P. Shamberger a method has been developed which will avoid the necessity of making corrections for the stress due to adjacent wheels when calculating vertical loads. Simultaneous measurements will be taken under each wheel of the locomotive.

[11] J. P. Shamberger, Proc. A.S.T.M., vol. 30 (1930), p. 1041.

a Different kinds of irregularities in the shape of the wheel or rail, such as flat spots on the rim of the wheel, low spots on the rail, and discontinuities at the rail joints.

b Variation in the forces acting on the rail caused by variable spring forces on the wheel, the vertical component of the centrifugal force of the counterweights, and the vertical component of the forces in the connecting rods.

c Vibration of the rail under moving loads.

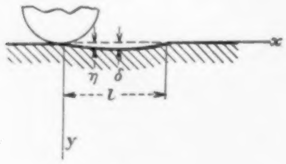

FIG. 14

In discussing the dynamic stresses produced in a rail by the effect of a low spot (Fig. 14), let

u = variable depth of the low spot, a given function of x

W/g = unsprung mass per wheel

$\alpha = 2k/\beta$, the vertical load necessary to produce a deflection equal to unity—see Equation [4]

y = additional deflection of the rail under the wheel due to the dynamic effect of the low spot.

Then the vertical displacement of the wheel due to the dynamic deflection of the rail and to the low spot is $y + u$, and the differential equation of motion of the wheel in the vertical direction is:

$$\frac{W}{g}\frac{d^2(y + u)}{dt^2} + \alpha y = 0 \qquad [23]$$

If v is the speed of the locomotive, then

$$\frac{du}{dt} = \frac{du}{dx}v; \qquad \frac{d^2u}{dt^2} = \frac{d^2u}{dx^2}v^2$$

and Equation [23] can be put in the form

$$\frac{W}{g}\frac{d^2y}{dt^2} + \alpha y = -\frac{W}{g}v^2\frac{d^2u}{dx^2} \qquad [24]$$

If the shape of the low spot and the speed v are known, the right-hand member of [24] can easily be expressed as a function of the time and we arrive at the known equation of forced vibration of a system with one degree of freedom, the solution of which can be easily obtained in each particular case. Take, for instance, the low spot the shape of which is given by the equation

$$u = \frac{\delta}{2}\left(1 - \cos\frac{2\pi x}{l}\right) \qquad [25]$$

in which (see Fig. 14)

l = length of the low spot, and

δ = depth of the spot at the middle of its length.

Then the right-hand member of [24] becomes

$$-\frac{W}{g}v^2\frac{\delta}{2}\frac{4\pi^2}{l^2}\cos\frac{2\pi x}{l}$$

Measuring the time from the moment when the point of contact of the wheel coincides with the beginning of the low spot (Fig. 14), $x = vt$ and [24] becomes

$$\frac{W}{g}\frac{d^2y}{dt^2} + \alpha y = -\frac{W}{g}v^2\frac{\delta}{2}\frac{4\pi^2}{l^2}\cos\frac{2\pi vt}{l} \qquad [26]$$

Using the notations

T = period of vibration of the wheel on the rail[12]

$T_1 = l/v$ = time it takes the wheel to cross the low spot, we obtain a solution of Equation [26] satisfying the initial conditions

$$(y)_{t=0} = \left(\frac{dy}{dt}\right)_{t=0} = 0$$

in the following form:

$$y = \frac{\delta}{2}\frac{1}{1 - \left(\frac{T_1}{T}\right)^2}\left(\cos\frac{2\pi t}{T_1} - \cos\frac{2\pi t}{T}\right) \qquad [27]$$

It is seen that the additional dynamic deflection is proportional to the depth of the spot δ and depends on the magnitude of the ratio T_1/T and on the position of the wheel. The variation of this deflection when the wheel is moving along the spot, i.e., in the interval $0 < t < T_1$ for different values of the ratio T_1/T, is shown in Fig. 15. At the beginning y is zero, and then becomes negative. This means that at the instant the wheel reaches the edge of the low spot the pressure on the rail and the deflection begin to diminish. This happens because the wheel, due to the low spot, begins to accelerate downward. When the wheel advances along the spot, the retardation of the vertical downward movement begins, and with it an increase in pressure and deflection, as seen from the figure. The values of the ratio of the maximum deflection $y_{\max}$ to the depth δ of the low spot, calculated from Equation [27] for different values of the ratio T_1/T, are given in the following table.

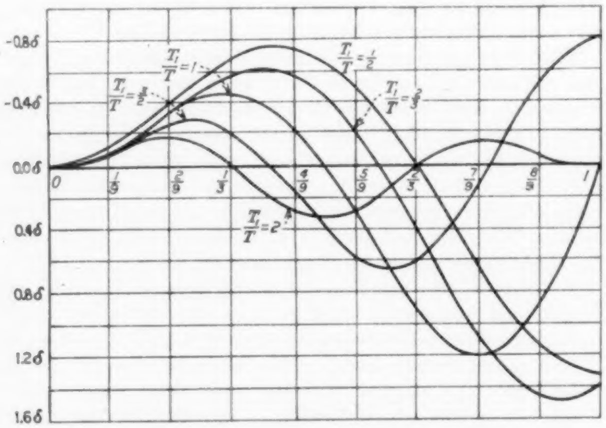

FIG. 15 ADDITIONAL (DYNAMIC) DEFLECTION PRODUCED BY FLAT SPOT ON WHEEL OR LOW SPOT ON RAIL

(δ = depth of flat spot; l = length of flat spot; T = period of free vibrations of wheel on rail; T_1 = time for wheel to cross flat spot.)

T_1/T	2	3/2	1	4/5	2/3	3/5	1/2
$y_{\max}/\delta$	0.33	0.65	1.21	1.41	1.47	1.45	1.33

A maximum additional deflection, equal to 1.47 δ, occurs at the speed corresponding to the ratio $T_1/T = 2/3$, i.e., when the time it takes the wheel to cross the low spot is equal to two-thirds of the period of natural vibration of the wheel on the rail. It can be concluded that the additional (dynamic) pressure due to the low spot is about equal to the load which produces a static deflection equal to 1.5 δ. Thus a comparatively small low spot produces, at certain speeds, a very appreciable dynamic effect, which must be added to the deflection calculated from the static Equation [4].

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[12] The mass of the rail is neglected in this calculation. W/α is the static deflection of the rail under the action of the weight W of the wheel, then the formula for T coincides with the usual formula for the period of free vibration. See S. Timoshenko, "Vibration Problems in Engineering," p. 17.

Similar results can be obtained for low spots of different contours. In the general case, Equation [26] becomes

$$\frac{W}{g}\frac{d^2y}{dt^2} + \alpha y = F(t)$$

and its general solution is[13]

$$y = \frac{g}{W}\frac{T}{2\pi}\int_0^{t_1} F(t)\sin\frac{2\pi(t_1 - t)dt}{T} \quad\ldots\ldots\ldots[28]$$

Calculations made[14] for low spots of several different contours show that the ratio $y_{\max}/\delta$ does not depend substantially on the contour of the low spot provided it is represented by a continuous curve. Hence we can state that the maximum additional dynamic pressure on the rail due to a low spot is about 50 per cent larger than the load which produces static deflection equal to the depth δ of the low spot.

The calculation made above for the case of a low spot can be applied also to the case of a flat spot on the rim of the wheel, because such a flat spot produces exactly the same vertical movements of the wheel as a low spot of the same shape on the rail.

The question of the dynamic effect of clearances between the rail and the tie and the dynamic effect of rail joints can also be treated in a similar manner.

In considering the additional wheel pressure of a locomotive due to the centrifugal force Q of a counterweight, we can use the theory of forced vibration. In calculating this pressure, we take the centrifugal force and multiply it by the magnification factor.[15] Then the additional pressure on the rail is

$$Q\,\frac{1}{1-\left(\frac{T}{T_2}\right)^2}$$

in which T, as before, is the period of vibration of the wheel on the rail and T_2 is the time of one revolution of the wheel.

In this calculation the mass of the rail is neglected, which is justified because the time T_2 is usually large in comparison with the period of natural vibration of the rail.

To determine the period of lowest type of vibration, we note that this type of vibration consists of an oscillation of the rail in a vertical direction as an absolutely rigid body. Let q be the weight of the rail per unit length and k, as before, the modulus of foundation. The static deflection of the rail due to its own weight is q/k, and the period T_3 of the fundamental type of vibration is

$$T_3 = 2\pi\sqrt{\frac{q}{kg}}$$

Taking, for instance, a 130-lb. rail and $k = 1500$ lb. per sq. in.,

$$T_3 = 2\pi\sqrt{\frac{130}{36 \times 386 \times 1500}} = 0.0157 \text{ sec.}$$

This can be always considered as small in comparison with the time T_2 of one revolution of the wheels, and the magnification factor differs but little from unity.

II—LOCAL STRESSES IN RAILS

1 INTRODUCTION

In the previous section the rail was discussed as a beam on an elastic foundation, and it was shown that the usual theory for such bending can be applied to the rail as a whole. However, in the region near the point of application of the load the stresses deviate considerably from those that can be calculated from simple bending theory. In this section these local stresses near the load point will be discussed, and the results of some tests will be given.

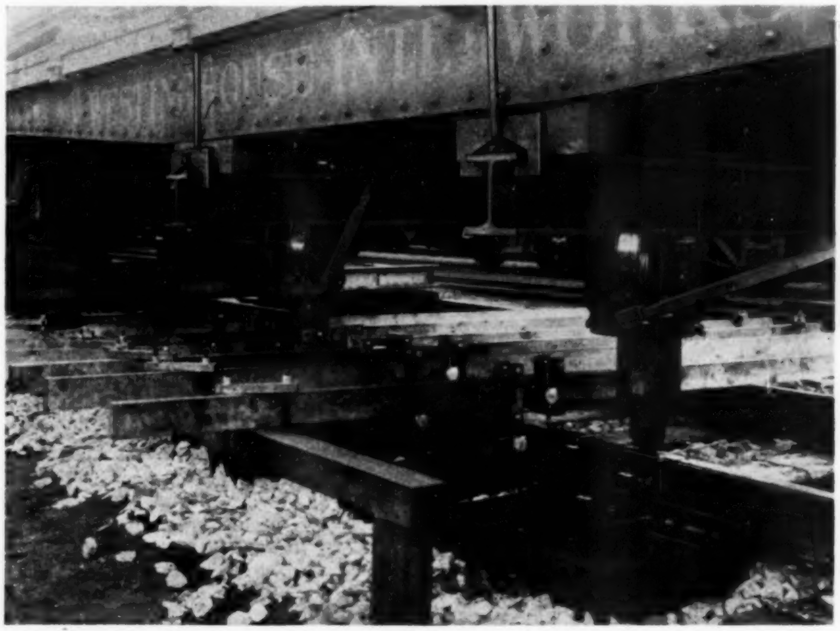

FIG. 16 EQUIPMENT FOR STATIC FIELD TESTS

The tests that will be mentioned were made partly on actual track and partly in the laboratory. The apparatus used for the static tests on actual track is shown in Fig. 16. A loaded flat car was supported from points outside the rail-tie structure. Hydraulic jacks were attached to the under side of the car directly over the rails in order to exert vertical loads. Lateral loads were produced by another hydraulic jack acting in such a way as to spread the rails apart. The magnitudes of the loads were read with hydraulic gages tapped into the high-pressure sides of the oil systems of the jacks. Deflections were measured with dial gages supported from outside the track, and stresses were measured with Huggenberger tensometers.

In the laboratory a 20-ft. length of rail was supported on cantilever springs spaced 21 in. apart to simulate track conditions. Fig. 17 shows the arrangement, and Fig. 18 details of one of the springs. The tests showed that the elasticity of these springs and the clamps holding the rail to them had a vertical modulus of foundation of 1500 lb. per sq. in. and a lateral modulus of 3000 lb. per sq. in. Lateral load was applied after turning the rail, springs, and base plate over on their sides so that the structure rested on the angles which were welded to the base plate.

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[13] See S. Timoshenko, "Vibration Problems in Engineering," 1928, p. 20.

[14] See paper by S. Timoshenko in *Bulletin of Engineers*, St. Petersburg, 1915.

[15] See "Vibration Problems in Engineering," p. 18.

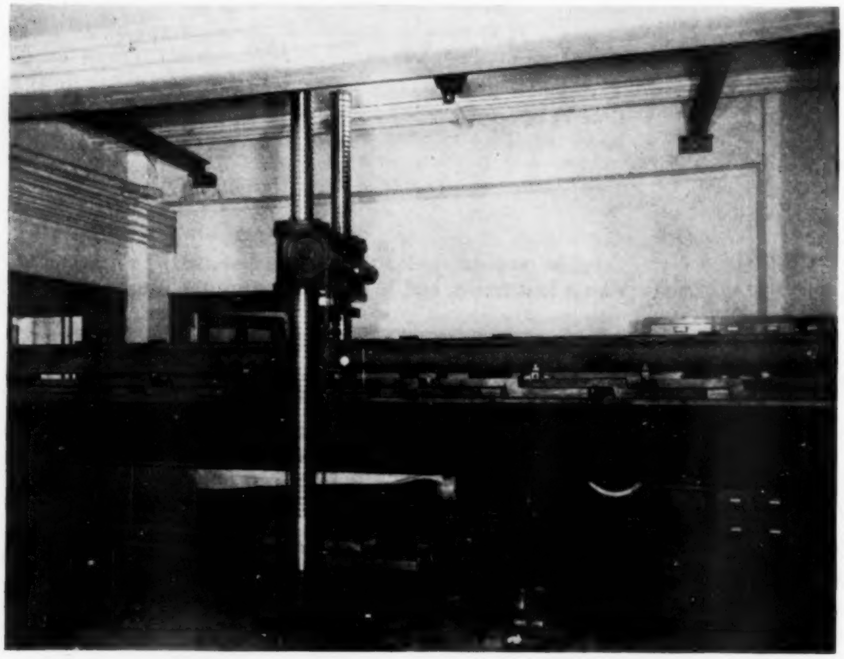

FIG. 17 EQUIPMENT FOR LABORATORY TESTS

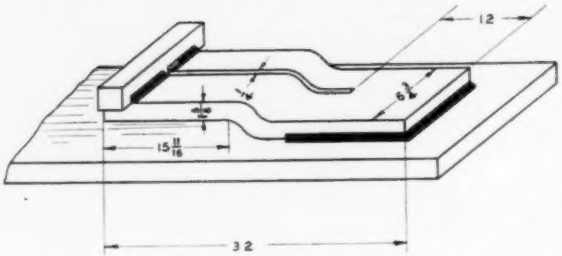

FIG. 18 SPRING USED IN LABORATORY TESTS

2 WEB AND FILLET STRESSES

General. The tendency of American rail design has been toward the increase of vertical rail stiffness, and a thin, high section has resulted. Considering the rail as a beam, it seems justifiable to remove material from the web and place it in the head or base in order to reduce the longitudinal stresses produced by vertical bending of the rail as a whole. However, if the web is made too thin it will compress vertically or else bend, as a narrow plate, away from the longitudinal vertical plane of the rail, and vertical stress in the web will result.

Experience and direct investigation have taught the stress analyst to beware of rapid changes in section. In the cross-section of a rail such rapid changes are to be observed at the points where the web meets the head and where it meets the base. The vertical stresses in the fillets at these points cannot easily be investigated by direct strain-gage measurements on a steel rail. The method used by the authors was that of measuring the stress in the web of an actual rail, after which photoelastic methods[16] were employed to find the relationship between the web stress and the fillet stress.

Concentric Vertical Load. A vertical load applied at the center of the head causes compression in the web. This vertical load, concentrated in a very small area at its point of application, spreads out along the rail as it progresses downward. In the case of a two-dimensional problem the pressure on a semi-cylindrical section of radius *r* and thickness *b* (see Fig. 19) produced by a load *P* is[17]

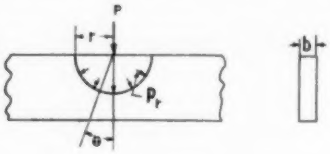

FIG. 19

$$p_r = -\frac{2P\cos\theta}{\pi b r}$$

If we consider only the section of the rail directly under the load, then $\theta = 0$, $\cos\theta = 1$, and the compression in the web varies in inverse proportion to the thickness *b* of the web, and the distance *r* from the load. The applicability of this theory is demonstrated in a qualitative way by the test results shown in Fig. 20. Here, among other data, the vertical web stress produced by a centrally applied vertical load (*A*) is shown for different points along the height of the web. It will be observed that the stress near the bottom of the web is much less than that near the top.

Eccentric Vertical Load. When a vertical load is applied nearer one edge of the head than the other, it produces vertical bending in the web as well as compression. In Fig. 20 one of the curves shows the vertical web stress produced by an eccentric vertical load (*B*). Note that at the top of the web, on the side of the rail at which the load is applied, the stress produced by this type of loading is the highest of any found in the web. Fig. 21 shows the vertical web stress $3^3/_4$ in. above the base of a P. S. 130-lb. rail in actual track produced by centrally applied and eccentrically applied vertical loads. The maximum compression is somewhat higher than that shown in Fig. 20, probably due to the greater stiffness of the foundation (k = 3000 lb. per sq. in.).

Because it caused the most severe condition in the web, the eccentric vertical type of loading was investigated photoelastically. The photoelastic method was applied to thin celluloid models of rail sections.[18] A thin section to which only a single concentrated load is applied does not completely represent a rail because it takes no account of the spreading of the load in the longitudinal direction nor of the torsional rigidity of the head.

This torsional rigidity, in a full rail, opposes the rotation of the head at the section where the load is applied. In the loading arrangement used in the photoelastic tests (see Fig. 22) the force applied to the extension of the head at *A* acted in the same way on the celluloid model. In the tests on steel rails it was found that the rotation of the head was $^1/_7$ of what it would have been if a section only 1 in. thick had been used. Therefore the force at *A* and its moment arm were adjusted so as to reduce by $^6/_7$ the twisting moment on the head produced by the main part of the eccentric load at *B*.

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[16] See "Photoelasticity for Engineers," by E. G. Coker, *General Electric Review*, Nov. and Dec., 1920, and Jan., March, May, 1921.

[17] See Timoshenko, "Strength of Materials," vol. I, p. 147.

[18] The photoelastic tests described here were made by R. V. Baud of the Westinghouse Research Laboratories.

The spreading of the load along the length of the rail was taken into account by correcting the readings in accordance with the amount of spreading found in tests on steel rails. That is, the slope of the curve was adjusted to conform to the slope of curve *B*, Fig. 20.

Fig. 23 shows the results of photoelastic tests on a model representing the P. S. 130-lb. rail. The stress along the boundary of the cross-section was measured. It will be noted that the highest stress occurs in the fillet under the head, and that the stresses near the bottom of the web are comparatively low because of the correction for the spreading of the load along the length of the rail. These results suggested that a more favorable stress distribution could be obtained if material were removed from the bottom of the web and placed nearer the tops. A model of such a rail was made and tested, with the result shown in Fig. 24. The redistribution of material in the web lowered the maximum stress by about 22 per cent.

The photoelastic results were not considered reliable in the

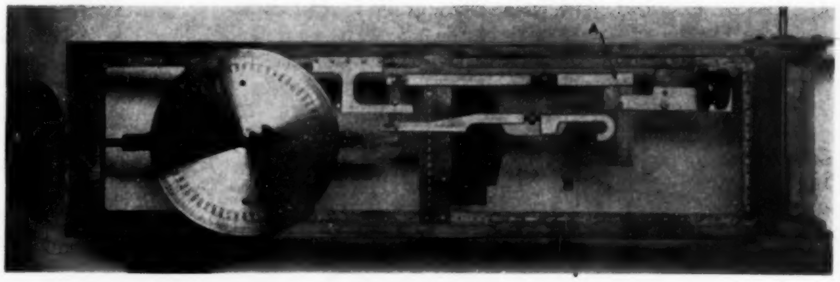

Fig. 22 Loading Arrangement Used in Photoelastic Tests

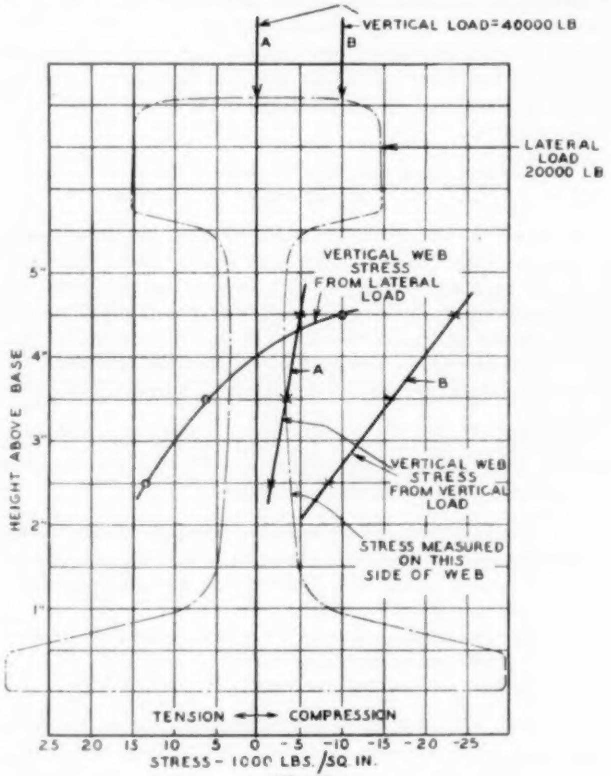

Fig. 20

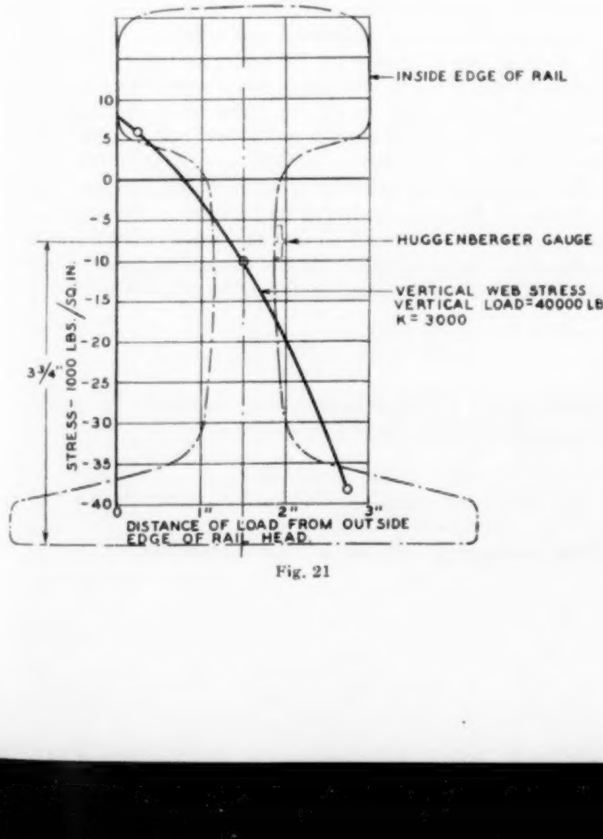

Fig. 21

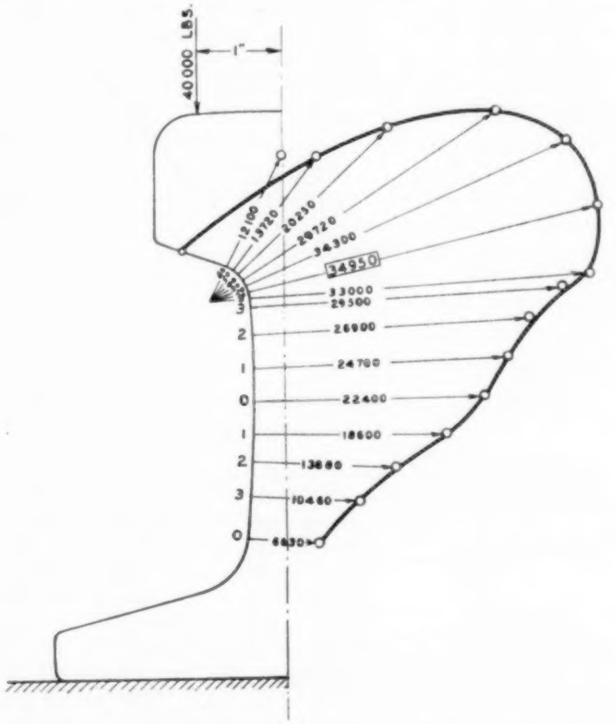

Fig. 23 Results of Photoelastic Test. P. S. 130-Lb. Rail

Fig. 20 (upper left-hand column) Vertical Web Stress Produced by Various Types of Loading. P. S. 131-Lb. Rail in Laboratory

Fig. 21 (lower left-hand column) Vertical Web Stress Produced by Various Locations of Vertical Load. P. S. 130-Lb. Rail in Track region of the base and the fillet at the bottom of the web because the method of attachment of the base of the celluloid model was not similar to that of an actual rail. A few measurements were made of lateral base stress in the base of a P. S. 130-lb. steel rail under the conditions shown in Fig. 25. Strain was measured between points *a* and *b*. The results showed that fairly high lateral stresses in the direction *ab* can be expected in the base of the rail. The conditions represented the most severe possible locations of vertical and lateral load, and were made directly over a tie. The stress thus measured was 28,000 lb. per sq. in. compression.

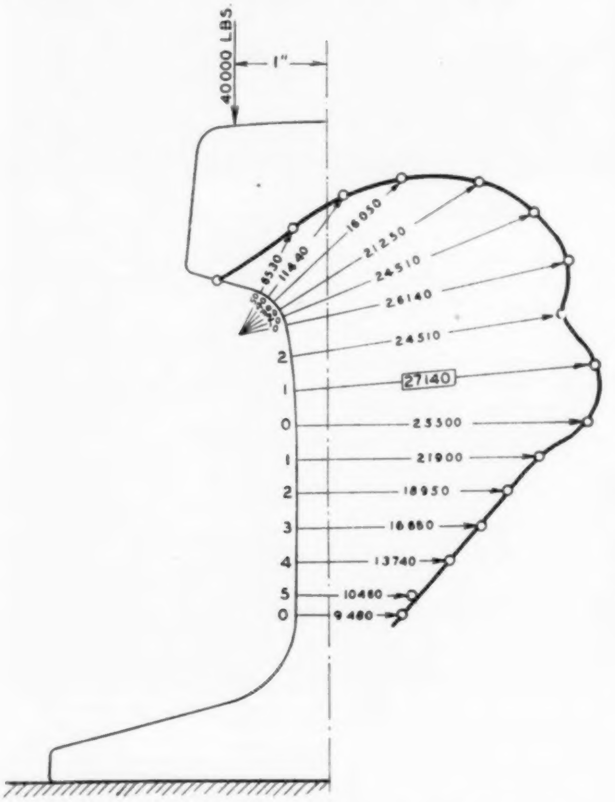

FIG. 24 RESULTS OF PHOTOELASTIC TEST. RAIL WITH UPPER PART OF WEB THICKENED

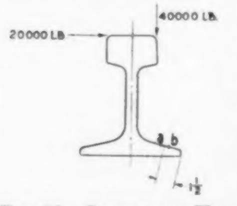

FIG. 25 CONDITIONS UNDER WHICH LATERAL BASE STRESS WAS MEASURED

Lateral Load. When a lateral load is applied to the head of a rail, the web bends, but not in the same manner as it does under the influence of an eccentric vertical load. One of the curves in Fig. 20 shows the distribution of vertical web stress produced by lateral load along the height of the web. The torsional rigidity of the head almost completely prevents rotation of the top of the web, and the curvature is of the type shown in Fig. 26 (*a*) instead of that shown in Fig. 26(*b*). Because of this reverse curvature, the stress in the fillet under the head is compression on the side near the lateral load

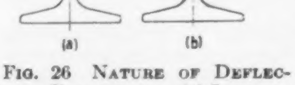

FIG. 26 NATURE OF DEFLECTION PRODUCED BY (*a*) LATERAL AND (*b*) ECCENTRIC VERTICAL LOAD

Summary. For the magnitudes of vertical and lateral load found in practice (see Section III) the highest vertical web stress is produced by a vertical load applied near one edge of the rail head. In most rails this maximum stress is located in the fillet under the head, and the stress in the lower part of the web is comparatively low. The vertical web stress produced by lateral load is usually not excessive. This stress is important because, for the most usual combination of high vertical and lateral load (vertical load applied eccentrically on the gage side of the head and lateral load directed away from the center of the track), the stresses from the two load components (vertical and lateral) are additive, both being compression.

The points brought out by these tests that are of value to the rail designer are that a more uniform stress distribution can be obtained by thickening the upper part of the web at the expense of the lower part, and that bringing the wheel load nearer the center of the rail will lower the stress in the fillet under the head.

3 LONGITUDINAL STRESSES NEAR THE POINT OF LOAD APPLICATION

At points within about 10 in. of the load, the longitudinal stresses are quite different from those that could be calculated from the ordinary bending theory, as may be seen from the experimental data plotted in Figs. 27 and 28. These show the distribution of head and base stress along the length of a P. S. 130-lb. rail in track. The base stress was measured 0.25 in. above the bottom of the rail and the head stress 1.2 in. below the top.

In order to account analytically for this distribution of stress near the load, it is necessary to add to the usual bending stress the effects due to four different phenomena:

(*a*) Torsion (Produced by lateral or eccentric vertical load)
(*b*) Bending of the head alone on the elastic foundation formed by the web (Produced by any type of vertical or lateral load)
(*c*) Reverse bending due to the concentration of the load (Produced by any type of load)
(*d*) Contact stress (Produced mostly by vertical load).

(*a*) *Torsion.* In a rail in pure torsion a cross-section originally plane does not remain so after the twisting moment has been applied. A rail built into a track, however, may be considered as an infinitely long bar with a twisting moment applied at some section along its length. (See Section I, Article 5.) In this case, due to symmetry, the section at which the load is applied cannot deform. But since the cross-sections at a little distance from the load do deform, there must be longitudinal tensions and compressions set up in the rail. A rail twists when a lateral or an eccentric vertical load is applied to the head.

The differential equation for a rail in torsion has been given in Section I, Equation [14], and its solution in Equation [15]. The longitudinal stresses set up near the load are such as would be produced by the forces *Q* in Fig. 8. The lateral deflection of the head is $h_1\phi$. Therefore the bending moment in the head is

$$M_H = EI_1h_1\frac{d^2\phi}{dx^2}$$

Differentiating the expression already obtained for $d\phi/dx$ in Equation [15] with respect to x, we obtain

$$\frac{d^2\phi}{dx^2} = -\gamma\frac{M_t}{C}e^{-\gamma x}\ldots\ldots\ldots\ldots\ldots [29]$$

At the load point, $x = 0$ and $e^{-\gamma x} = 1$, therefore,

$$M_H = -EI_1h_1\gamma\frac{M_t}{C}\qquad\text{[30]}$$

and similarly, the bending moment in the base is

$$M_B = -EI_2h_2\gamma\frac{M_t}{C}\qquad\text{[31]}$$

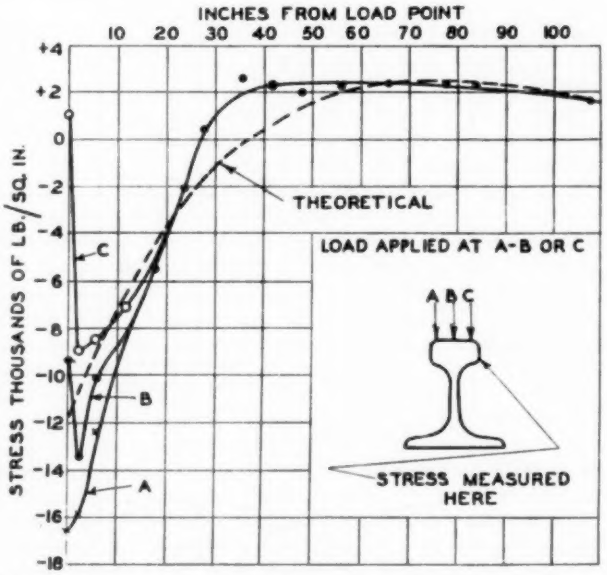

Fig. 27 Longitudinal Head Stress From 30,000 Lb. Vertical Load. P. S. 130-Lb. Rail in Track

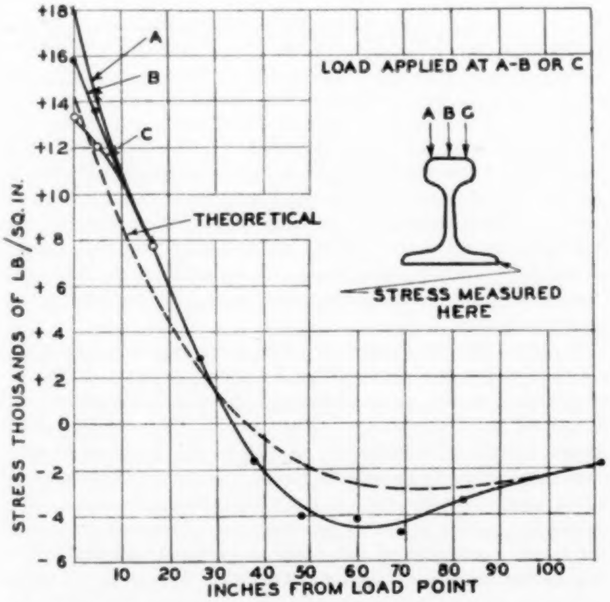

Fig. 28 Longitudinal Base Stress From 30,000 Lb. Vertical Load. P. S. 130-Lb. Rail in Track

From these expressions and the dimensions of the head and base of the rail, the longitudinal stresses set up by the forces Q (Fig. 8) can be calculated.

(*b*) *Bending of the Head Alone.* If the base of a rail were rigidly supported and a vertical or lateral load applied in the usual way, there would still be bending stresses set up in the head. Since the head is much wider than the web, it may be roughly considered as a beam bending on the elastic foundation formed by the web. The theory for this type of bending is similar to that already given for a rail bending on the elastic foundation formed by the ties, ballast, and ground. The only difference is in the calculation of the elasticity of the web foundation. The modulus of foundation, k, is by definition the load per unit length necessary to produce unit deflection at the union between the head and the web.

When the load is vertical, only the compression of the web need be considered. For vertical load, the modulus of foundation of the web is

$$k = \frac{tE}{d}\qquad\text{[32]}$$

where t = average web thickness,
d = web height, and
E = Young's modulus for rail material.

When the load is lateral the bending of the web must be taken into account. In the measurements of vertical web stress described above it was seen that the web bends, not as a cantilever, but in the manner shown in Fig. 26(*a*), without rotation of the top of the web. The deflection at the end of a cantilever, the end of which is prevented from rotating, is one-fourth that of a simple cantilever. Therefore, for lateral load, the modulus of foundation of the web is

$$k = \frac{Et^3}{d^3}\qquad\text{[33]}$$

(*c*) *Reverse Bending Due to Concentration of Load.* Consider a small hemispherical region concentric with the point of load application (see Fig. 29). The pressure on each element of the hemispherical surface has a component in the direction of the length of the rail. These components add together to form the forces F, which cause a bending moment in the rail as well as a uniform tension. The integration of these longitudinal components over the hemispherical area shows that $F = P/\pi$. The uniform tension produced by this effect is F/A, where A is the cross-sectional area of the rail. The bending moment that it produces is F times the distance of the point of application of the load from the neutral axis of the rail. The effect of the forces F should be considered for both vertical and lateral bending.

(*d*) *Contact Stress.* The contact pressure between the rail and the wheel is of importance in the study of the plastic flow of the rail head and the tire. The stress inside the head produced by contact pressure may also be a factor in hastening certain types of rail failure.

The solution for the pressure between the surfaces of two elastic bodies in contact was first given by Hertz, and can be found in most books on elasticity.[19] The solution for the state of stress inside a body produced by a concentrated load on its surface is also well known.[20] However, the calculation of the state of stress inside a body produced by a distributed load on its surface is exceedingly complicated, and is one that must be solved in each particular case of surfaces of contact for a complete, accurate study of the conditions in a rail. Two such solutions have been made independently, and are in perfect agreement. One was made by Prof. N. M. Belajef, Mechanical Laboratory, Institute of Ways of Communication, Leningrad, Russia. The solution best available to American readers is that of H. R. Thomas and V. A. Hoersch.[21] They have put their solution into a very usable form, and have also checked it experimentally by means of the Fry strain-etch method. The problem of the rail and the wheel is considered as that of two crossed cylinders.

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[19] See "Strength of Materials," by S. Timoshenko. For the consideration of the tangential component of contact stress such as can be produced by tractive effort, see Fromm, "Schlupfberechnung beim Rollen Deformierbarer Schieben," *Zeitschrift für Angewandte Mathematik und Mechanik*, 1927, p. 27; also, R. Lorenz, "Schiene und Rad," Z.V.D.I., no. 6, 1928, p. 173.

[20] See A. E. H. Love, "Mathematical Theory of Elasticity," p. 190.

The approximate formula given by Thomas and Hoersch for the maximum shear in two crossed cylinders in contact is

$$\text{Max. shearing stress} = \frac{11{,}750\ P^{1/3}}{(R_1/R_2)^{0.271} R_2^{2/3}} \ldots\ldots [34]$$

where P = load in pounds
R_1 = radius of larger cylinder in inches
R_2 = radius of smaller cylinder in inches
$1 < R_1/R_2 < 8$, and
Poisson's ratio = 0.25

The contact pressure between the two elastic bodies and the maximum shearing stress inside the bodies are functions of the load and the radii of curvature of the bodies. Thus a small wheel with a light load may produce just as much plastic flow in the rail head as a larger wheel with a heavier load. Thomas and Hoersch give the following two numerical examples to illustrate this point.

Wheel diam., in.	Radius of rail head, in.	Load, lb.	Depth to point of max. shear, in.	Area of contact, sq. in.	Max. shearing stress, lb. per sq. in.
33	14	25,000	0.121	0.215	56,500
80	14	60,000	0.179	0.502	59,800

Comparison Between Calculated and Experimental Results. The longitudinal stresses in the head and base produced by eccentric vertical load were calculated for the cross-section directly under the load and compared with experimental results from measurements on an actual track. (See Figs. 27 and 28.) The calculated stresses for concentric load were assumed, perforce, to be equal to the experimental values, since the modulus of foundation of the track could be determined only from the experiments themselves, and there was no purely theoretical method of calculating the stresses from simple bending.

The local stresses produced by eccentric vertical load were calculated according to the theories already outlined. The lateral bending due to twist was calculated from Equations [30] and [31], and modulus of foundation of the web from [32]. The forces F, Fig. 29, were considered to produce uniform tension and bending. The contact stress was calculated according to a theory in which it is assumed that the load is concentrated instead of distributed over a small area. The rail used was P. S. 130-lb.

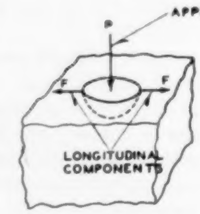

Fig. 29

Fig. 30 presents a comparison between the calculated and the experimental results. The very good experimental confirmation of the calculated stress values shows that the theories outlined above are well founded.

4 Transverse Fissures

Transverse fissures are fatigue cracks which form inside the heads of rails in the transverse plane. They have become increasingly prevalent in America during the past score of years, and are particularly dangerous because they cannot be detected by the usual methods of inspection.

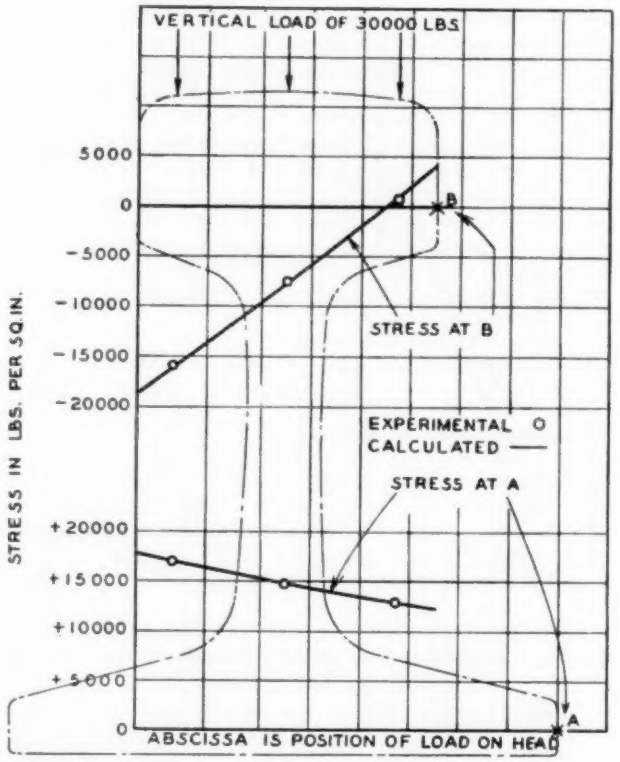

Fig. 30 Comparison of Calculated Longitudinal Stresses With Experimental Results

A long step toward the elimination of accidents from transverse fissures was made when E. A. Sperry introduced his fissure-detector car,[22] by means of which track can be inspected for the presence of such internal flaws. The method is an electrical one. A current is passed through the rail and used to detect changes in resistance such as occur when a transverse fissure is present. Cars of this type have already been used with good success on several railroads.

The detection of fissures in track is at best not a complete solution of the problem. What is really necessary is the determination of the cause of fissures and the development of a means for their prevention. The work of Freeman and others at the Bureau of Standards,[23] as well as the evidence of other observers, points to the conclusion that transverse fissures occur when, and only when, the rail head has internal cracks produced during the cooling period of manufacture. Freeman has studied the tensile properties of rail steels at elevated temperatures as well as the process of cooling of the rail. The study of tensile properties showed that some steels have, around 500 to 650 deg. cent., a ductility less than at normal atmospheric temperature. The study of the cooling process showed that tensile stresses are apt to be set up in the head at the time that the metal on the inside of the head is passing through this brittle range. If, then, we have the combination of a steel subject to brittleness at about 550 deg. cent. and a fairly rapid cooling of rails rolled from that steel, we are apt, according to Freeman, to find internal cracks in the head of the rail which will develop into transverse fissures under the action of wheel loads. There is some question as to whether the amount of the increase in brittleness found by Freeman is sufficient to account for the appearance of cracks during cooling. It may be that the cracks are due to the stresses on the inside of the head being tension in three dimensions (see Fig. 31), which is a very severe stress condition. It appears, then, that a complete solution of the problem may come with the development of an economical method for the slow cooling of rails.

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[21] See "Stresses Due to the Pressure of One Elastic Solid Upon Another," Univ. of Ill., Engineering Experiment Station, Bulletin No. 212.

[22] See "Non-Destructive Detection of Flaws," by E. A. Sperry, *Iron Age*, vol. 122, no. 20, Nov. 15, 1928.

[23] See "Tensile Properties of Rail and Other Steels at Elevated Temperatures," by J. R. Freeman and G. W. Quick. Trans. A.I.M.E., Iron and Steel Division, 1930, p. 225.

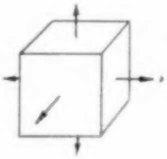


Fig. 31

Attempts have been made to place the entire blame for transverse fissures on the stresses in the rail head produced by bending and contact pressure. There are, undoubtedly, high stresses in the head, and there is no question but that they cause a fissure to grow after it has once started. It seems unlikely, however, that these stresses ever start cracks in sound material, since fissures seldom, if ever, start where the bending and contact stresses are a maximum.

It is seen that transverse fissures are connected with the shape of the rail cross-section only in so far as the ratio between the volume and surface of the head affects the rate of cooling. The evidence now available indicates that the problem is one that can best be solved by the metallurgist and the manufacturer. A suggested line of attack would be the study of residual stresses and the effect upon them of different methods and rates of cooling.

III—FIELD TESTS

1 Methods and Apparatus

The field tests made were primarily for the purpose of studying the action of various types of locomotive running gear on curved and tangent track. The results that were obtained are of major importance in the study of rail stresses because they tell what the forces are which can be exerted on rails in actual operation.

On the basis of the information obtained from the static tests (see Sections I and II), a method was devised whereby strain-gage measurements could be taken under dynamic conditions in such a way that they could be used to determine vertical and lateral wheel loads. This method is described in Section I.

At each test location a short static calibration was made to determine the relationship between vertical load and average base stress, and that between lateral load and web stress, for the particular spot where the tests were being made. The load was also applied at various distances from the gages to determine what correction should be made for the stress due to adjacent wheels. The apparatus used for these static calibrations was similar to that used in the static-stress studies. It consisted of a loaded car to which hydraulic jacks were rigidly attached, so that by moving the car the jacks could quickly be spotted over the point on the track where the calibration was to be made. Lateral loads were applied by means of a jack lying crosswise between the rails.

The calibration car was placed with the hydraulic jacks directly over the magnetic strain gages. Vertical and lateral loads were applied, and readings taken from all the gages. (Figs. 11, 12, and 13 show the location of the instruments.) The car was then moved to two or three locations, 3 to 10 ft. distant from the gages, and the procedure was repeated. At each location the car body was raised off the center pins with jacks in order to isolate the load applied at the center of the car as nearly as possible.

The deduction of wheel loads from stress data can best be explained by means of a simple example. Suppose we have a record of strain-gage measurements from a four-wheeled vehicle which has passed over the test spot, and wish to find the vertical loads. The axle spacing is 60 in. The static calibration has given us the two necessary curves, which are:

Curve No.	Abscissa	Ordinate
(1)	Distance from load	Average base stress from vertical load
(2)	Vertical load	Average base stress at load point

The peak base-stress readings on the north rail are:

	1st axle	2nd axle
Inner base.......	9,600	8,000
Outer base.......	10,800	6,000
Average..........	10,200	7,000

From curve (1) it is found that at 60 in. from the load the rail is in reverse bending and the base stress is —15 per cent of the base stress directly under the load.

Let A = maximum stress due to 1st wheel alone
B = maximum stress due to 2nd wheel alone

Then

$$10{,}200 = A - 0.15B$$

and

$$7000 = B - 0.15A$$

Solving for A and B,

$$A = 11{,}500, \text{ and } B = 8730$$

From curve (2) we can find the wheel loads corresponding to A and B, the corrected base stress values.

The method used for finding lateral loads is similar, with the web stress used in place of the average base stress. Ordinarily, due to the shorter wave length of the stress distribution from lateral load, no correction need be made for adjacent wheels, and curve (2) can be used directly.

For vehicles with more than two axles the calculations are more complicated, but the method is the same. When each stress reading has to be corrected for the effects of three or four adjacent wheel loads, it is best to solve the simultaneous equations by trial and successive approximations rather than by a direct method.

A typical oscillograph record of rail stress is shown in Fig. 32.

2 Tests on the Great Northern Railway

Description of Locomotives and Operating Conditions. Field

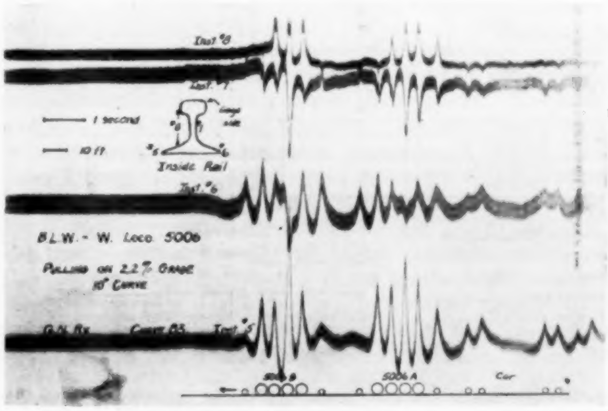
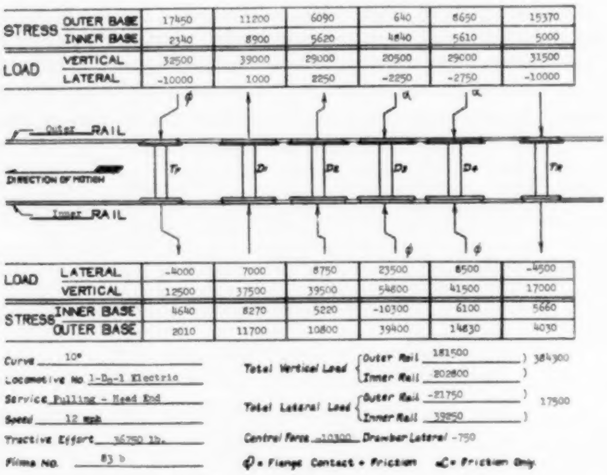


Fig. 32 Typical Oscillograph Record of Rail Strain



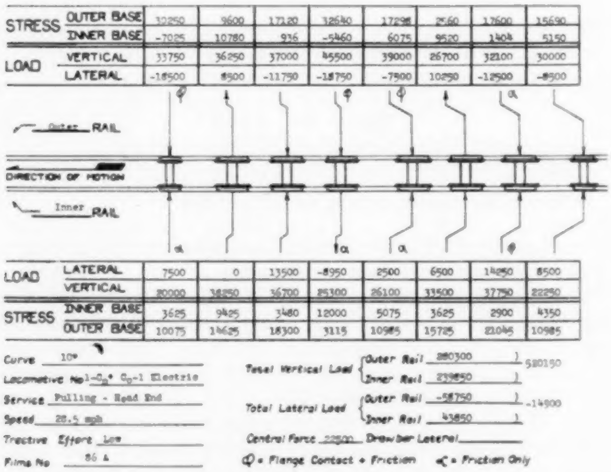
STRESS	OUTER BASE	17450	11200	6090	640	8650	15370
	INNER BASE	2340	8900	5620	4840	5610	5000
LOAD	VERTICAL	32500	39000	29000	20500	29000	31500
	LATERAL	-10000	1000	2250	-2250	-2750	-10000

LOAD	LATERAL	-4000	7000	8750	23500	8500	-4500
	VERTICAL	12500	37500	39500	54800	41500	17000
STRESS	INNER BASE	4640	8270	5220	-10300	6100	5660
	OUTER BASE	2010	11700	10800	39400	14830	4030

Curve 10°
Locomotive No 1-D0-1 Electric
Service Pulling - Head End
Speed 12 mph
Tractive Effort 36750 lb.
Films No. 83 b

Total Vertical Load {Outer Rail 181500) Inner Rail 202800) 384300
Total Lateral Load {Outer Rail -21750) Inner Rail 39250) 17500
Central Force -10300 Drawbar Lateral -750
φ = Flange Contact + Friction α = Friction Only

FIG. 33 WHEEL LOAD DIAGRAM



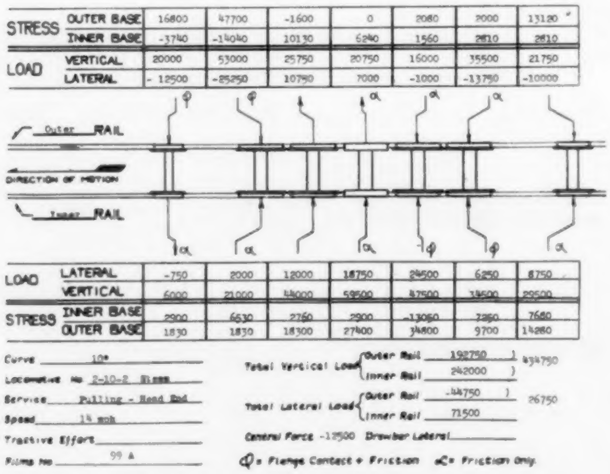
STRESS	OUTER BASE	30250	9600	17120	32640	17298	2560	17600	15690
	INNER BASE	-7025	10780	936	-5460	6075	9520	1404	5150
LOAD	VERTICAL	33750	36250	37000	45500	39000	26700	32100	30000
	LATERAL	-18500	8500	-11750	-18750	-7500	10250	-12500	-8500

LOAD	LATERAL	7500	0	13500	-8950	2500	6500	14250	8500
	VERTICAL	20000	38250	36700	25300	26100	33500	37750	22250
STRESS	INNER BASE	3625	9425	3480	12000	5075	3625	2900	4350
	OUTER BASE	10075	14625	18300	3115	10985	15725	21045	10985

Curve 10°
Locomotive No 1-C0+ C0-1 Electric
Service Pulling - Head End
Speed 28.5 mph
Tractive Effort Low
Films No 86 A

Total Vertical Load {Outer Rail 280300) Inner Rail 239850) 520150
Total Lateral Load {Outer Rail -58750) Inner Rail 43850) -14900
Central Force 22500 Drawbar Lateral
φ = Flange Contact + Friction α = Friction Only

FIG. 34 WHEEL LOAD DIAGRAM



STRESS	OUTER BASE	16800	47700	-1600	0	2080	2000	13120
	INNER BASE	-3740	-14040	10130	6240	1560	2810	2810
LOAD	VERTICAL	20000	53000	25750	20750	16000	35500	21750
	LATERAL	- 12500	-25250	10750	7000	-1000	-13750	-10000

LOAD	LATERAL	-750	2000	12000	18750	24500	6250	8750
	VERTICAL	6000	21000	44000	59500	47500	34500	29500
STRESS	INNER BASE	2900	6630	2760	2900	-13050	7250	7680
	OUTER BASE	1830	1830	18300	27400	34800	9700	14280

Curve 10°
Locomotive No 2-10-2 Steam
Service Pulling - Head End
Speed 14 mph
Tractive Effort
Films No 99 A

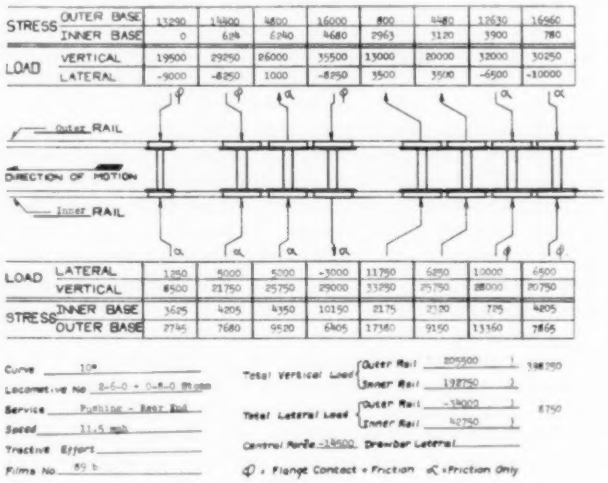
Total Vertical Load {Outer Rail 192750) Inner Rail 242000) 434750
Total Lateral Load {Outer Rail -44750) Inner Rail 71500) 26750
Central Force -12500 Drawbar Lateral
φ = Flange Contact + Friction α = Friction Only

FIG. 35 WHEEL LOAD DIAGRAM

tests were made on the Great Northern Railway during the months of August and September, 1929. Track stresses were measured at three locations between Merritt and Berne on the electrified portion of the Cascade Division, and for all types of locomotives operating over the Division, namely,

Electric, 1-D$_0$-1 (Mikado), G. N. Class Z-1
Electric, 1-C$_0$ + C$_0$-1 (Mallet), G. N. Class Y-1
Steam, 2-8-2 (Mikado), G. N. Class O-4
Steam, 2-10-2 (Santa Fe), G. N. Class Q-1
Steam, 2-6-0 + 0-8-0 (Mallet), G. N. Class M-2

The three test locations included a tangent, a 2-deg. 30-min. curve, and a 10-deg. curve, all on a 2.2 per cent grade. The tangent and the 10-deg. curve had P. S. 130-lb. rail and the 2-deg. 30-min. curve had Great Northern 110-lb. rail. The speeds were mostly low, and never exceeded 30 m.p.h. As a consequence, the only severe conditions were found on the 10-deg. curve.



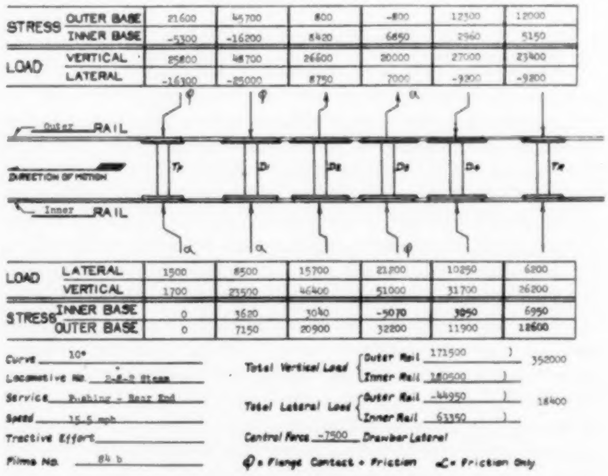
STRESS	OUTER BASE	13290	14400	4800	16000	800	4480	12630	16960
	INNER BASE	0	624	6240	4680	2963	3120	3900	780
LOAD	VERTICAL	19500	29250	26000	35500	13000	20000	32000	30250
	LATERAL	-9000	-8250	1000	-8250	3500	3500	-6500	-10000

LOAD	LATERAL	1250	5000	5000	-3000	11750	6250	10000	6500
	VERTICAL	8500	21750	25750	29000	33250	25750	28000	20750
STRESS	INNER BASE	3625	4205	4350	10150	2175	2320	725	4205
	OUTER BASE	2745	7680	9520	6405	17380	9150	13360	7865

Curve 10°
Locomotive No 2-6-0 + 0-8-0 Steam
Service Pushing - Rear End
Speed 11.5 mph
Tractive Effort
Films No 89 b

Total Vertical Load {Outer Rail 205500) Inner Rail 192750) 398250
Total Lateral Load {Outer Rail -34000) Inner Rail 42750) 8750
Central Force -14500 Drawbar Lateral
φ = Flange Contact + Friction α = Friction Only

FIG. 36 WHEEL LOAD DIAGRAM



STRESS	OUTER BASE	21600	45700	800	-800	12300	12000
	INNER BASE	-5300	-16200	8420	6850	2960	5150
LOAD	VERTICAL	25800	48700	26600	20000	27000	23400
	LATERAL	-16300	-25000	8750	7000	-9200	-9200

LOAD	LATERAL	1500	8500	15700	21200	10250	6200
	VERTICAL	1700	23500	46400	51000	31700	26200
STRESS	INNER BASE	0	3620	3040	-5070	3050	6950
	OUTER BASE	0	7150	20900	32200	11900	18600

Curve 10°
Locomotive No. 2-8-2 Steam
Service Pushing - Rear End
Speed 15.5 mph
Tractive Effort
Films No. 84 b

Total Vertical Load {Outer Rail 171500) Inner Rail 180500) 352000
Total Lateral Load {Outer Rail -44950) Inner Rail 63350) 18400
Central Force -7500 Drawbar Lateral
φ = Flange Contact + Friction α = Friction Only

FIG. 37 WHEEL LOAD DIAGRAM

Results. Figs. 33–37 give the results from one representative test of each of the five locomotives on the 10-deg. curve. The directions of the arrows opposite the wheels indicate the directions of the lateral guiding forces exerted on the wheels, and so help to visualize the manner in which the locomotive is made to follow the curvature of the track against the resistance of friction and inertia. A minus sign preceding the value of lateral load indicates a force on the rail directed away from the center of curvature of the track.

Vertical Loads. Although the sum of all the vertical loads for any one test is always practically equal to the static weight of the locomotive, it will be observed that there is considerable non-uniformity in the loads on separate wheels. This non-uniformity is due to six different phenomena:

(1) Oscillations set up by unevenness in the track
(2) Rotation of counterbalance
(3) Angularity of connecting rod
(4) Longitudinal weight transfer. This is due to the height of the drawbar above the rail. The tractive effort at the level of the rail is resisted by the drawbar force, which produces a moment tending to lift the front of the locomotive into the air when it is pulling
(5) Centrifugal or superelevation force, when the locomotive is not traveling at the speed for which the curve is superelevated
(6) Lateral forces at the rims of the wheels, either flange pressures or friction. The lateral forces at the wheel rims are resisted at the wheel hubs, which causes a turning moment on each axle which can only be resisted by an unbalance in the vertical loads.

In low-speed operation, such as these tests represent, (5) and (6) are by far the most important effects, and they occur only on curves. The tests on the tangent stretch did show some irregularity in the vertical loads, but this may have been due largely to friction in the equalization system not allowing the wheel loads to readjust themselves after the active unbalancing forces had been removed. The effect of lateral force on vertical load distribution can be observed in Figs. 33–37 wherever the lateral forces are particularly high. See, for example, the third driving axle in Fig. 33, or the first driving axle in Fig. 35. On the third driver in Fig. 33 the total lateral force on the axle is 23,500–2250 or 21,250 lb. On a 56-in. driver this produces a turning moment of $28 \times 21{,}250 = 595{,}000$ in-lb. The vertical forces producing the resisting couple are 58 in. apart and so must be 59,500/58 or 10,250 lb. This would produce a difference between the vertical loads of 20,500 lb. Actually, there was a difference of 34,300 lb., the remainder of which must have been due to superelevation and other effects.

Lateral Loads. The tests showed that the lateral forces were negligible on the tangent location and fairly small on the 2-deg. 30-min. curve, as might be expected for low-speed operation. Figs. 33–37 show the lateral forces on the 10-deg. curve.

The importance of lateral wheel loads in the study of rail stress has already been made obvious in Sections I and II. However, when we come to study the action of the locomotive itself, two new factors present themselves which increase still more the importance of the lateral load. One is the sixth of the causes mentioned for non-uniformity in the vertical loads. A lateral load not only produces rail stress by itself, but it increases the accompanying vertical load with a consequent increase in stress. The other factor is that the magnitude of the lateral load, together with the manner in which it is applied, is a measure of flange wear and rail side wear.

The study of the manner of application of the lateral load involves answering two questions: (1) How much of the lateral load is due to friction at the tread and how much to direct flange pressure? and (2) What is the angle between the vertical plane of the wheel and the vertical plane of the rail? (angle of incidence). In order to answer these questions it is necessary to make a detailed study of the action of the locomotive, which is beyond the scope of this paper. However, such a study has been made for a few of the test conditions used, and some of the results will be given. It can be shown that the amount of wear should be roughly proportional to the actual flange pressure and the angle of incidence of the wheel.[24] If we multiply these two values together we get a "wear factor" which can be used for a comparative study of the wear produced by various wheels. Table 1 presents a few typical results.

Table 1 shows that a high lateral load does not necessarily mean excessive flange and rail wear, since the wear depends so much on the actual flange pressure and the angle of incidence. In the first example given in the table the actual lateral load was quite low, but analysis showed that it was made up of a high flange pressure and a large friction force acting in opposite directions, and that the angle of incidence was large. It may be observed that the highest wear factors always occur on leading outer drivers. The last three examples in the table give the highest wear factors found on the locomotives to which they belong. It may be said that in general a high lateral force at the leading outer wheel of a multi-axled wheel base is indicative of excessive flange wear, whereas a high lateral force at one of the rear inner wheels is no cause for alarm.

Stresses. The only stresses which were measured directly were those in a longitudinal direction in the base and in the web of the rail. The longitudinal web stresses were very small and were used only as a means of measuring lateral forces. The highest base stresses measured at the three locations were as follows:

Test location	Position	Stress, lb. per sq. in.
Tangent	Outer base	12,200
2° 30′ curve	Outer base, outer rail	19,400
10° curve	Outer base, inner rail	50,700

These results are typical of the general trend of the stresses found at the three locations and illustrate the importance of the lateral load as a stress producer.

After the forces and eccentricities were determined from the field tests, the head and web stresses were found by using the static-stress measurements described in Section II. The static-load tests showed that the longitudinal base stress is not apt to be the highest stress in the rail. It was found that a lateral flange pressure of 25,000 lb. would produce a stress of about 45,000 lb. per sq. in. compression on the gage side of the head. The maximum base stress found in the dynamic tests produced by a purely vertical load did not exceed 15,000 lb. per sq. in. The point of maximum bending stress on the head, which is the top of the gage side, is very nearly the same distance from the neutral axis as the point where the strain gage was located on the base in the dynamic tests, and so 15,000 lb. per sq. in.

TABLE 1 WEAR FACTORS FOR VARIOUS LOCOMOTIVES

Locomotive	Service	Driver	Lateral load, lb. (exp.)	Wear factor
Electric, 1–D_o–1	1st of 2 cabs pulling	1st outer	1,000	971
Electric, 1–D_o–1	1st of 2 cabs pulling	3rd inner	23,500	72
Electric, 1–C_o + C_o–1	1 cab pulling	3rd outer	21,250	495
Electric, 1–C_o + C_o–1	1 cab pulling	4th outer	5,500	1870
Electric, 1–D_o–1	Regenerating at head end	1st outer	11,750	2540
Electric, 1–C_o + C_o–1	Regenerating in train	4th outer	11,000	2640
Steam, 2–10–2	Pushing at rear end	1st outer	25,250	4800

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[24] Based on the assumption that the wear between two surfaces is proportional to the pressure between the surfaces, the rubbing velocity, and the coefficient of friction.

can be used for the head also. Thus the compression on the gage side of the head due to bending may be said not to have exceeded 60,000 lb. per sq. in.

High stresses may also be expected in a vertical direction in the fillet under the head. The static tests showed that these stresses are produced mostly by vertical load and are affected quite considerably by the location of the vertical load on the head. This position of the vertical load was determined roughly in the dynamic tests by the use of a gage measuring vertical deflection of the head relative to the base. (See Section I, Art. 4.) It was found that this particular deflection was produced by vertical load much more than by lateral load, and was also affected by the position of the vertical load on the head. It could therefore be used as a rough measure of this position. A combination of the data obtained in the static tests, the photoelastic tests, and the dynamic tests led to the conclusion that for the maximum values of vertical and lateral load (60,000 lb. and 25,000 lb., respectively) and the most usual position of the vertical load ($^3/_4$ in. from the gage side of the head), the stress in the fillet under the head attains a value of 60,000 lb. per sq. in.

3 Tests on the Pennsylvania Railroad

Description of Locomotives and Operating Conditions. Field tests were made on the Pennsylvania Railroad during the months of September and October, 1930. The two test locations were both near Claymont, Del., on the Maryland Division.

Stresses were measured for three types of locomotives, which were:

Electric, 2-B_0-2 (4-4-4), P. R. R. Class O-1
Steam, 4-6-2 (Pacific), P. R. R. Class K-4s
Steam, 4-8-2 (Mountain), P. R. R. Class M-1a

The two test locations included a tangent and a 1-deg. curve, with practically no grade. The rail was P. S. 130-lb. Interest was centered chiefly on the high-speed tests, and maximum nominal speeds of 80, 90, and 100 miles per hour were attained with the Mountain, Pacific, and electric locomotives, respectively.

Results. The maximum stresses were found to be produced by dynamic effects at high speed, and were most severe at the tangent location.

It was observed that here, as in the tests on the Great Northern, the sum of the vertical wheel loads for a single test is about the same as the static weight of the locomotive, even though there was considerable non-uniformity in the loads on separate wheels. The non-uniformity was, however, due principally to oscillations and dynamic augments from the counterbalance.

The effect of oscillations on vertical loads is twofold. First of all, lateral oscillations produce lateral wheel loads at the rims which are resisted at the hubs and produce unbalance of the vertical loads as described earlier, in the discussion of the vertical loads found in the Great Northern tests. Secondly, slight unevennesses in the track can, at high speeds, start a rolling motion of the sprung weight of the locomotive, which results in a periodic shifting of the weight from one rail to the other. Studies of the vertical and lateral wheel loads on individual axles showed that the unbalance in the vertical loads could not all be due to the effect of lateral load, and consequently a considerable amount must have been due to rolling.

The tests showed that the highest lateral forces were produced, not by the guiding of the locomotives around the 1-deg. curve, but by lateral oscillations on the tangent.

The general importance of lateral loads has already been sufficiently discussed. From the point of view of rail study these tests serve to point out another cause of lateral wheel loads, that is, locomotive oscillations at high speeds. Flange wear and rail side wear are not of importance in operation on tangent track and low-degree curves because of the small angles of incidence involved.

IV—CONCLUSIONS

1 Properties of Rails

In this paper an attempt has been made to point out some of the functions that a rail is called upon to perform, and to give what information is available on the factors which help it to perform those functions with a maximum of safety and economy. Let us consider for a moment some of the properties that a rail should possess.

Vertical Stiffness. The rigidity of a rail in the vertical plane is a very important factor. On it depends the pressure transmitted to individual ties, and hence it affects the amount of work necessary to maintain the track surface. The impact at the joints also depends partly on the vertical rigidity of the rail, but the problem of rail joints has not been dealt with in this paper.

Another factor which should be kept in mind when considering vertical rigidity is the damage to rolling stock caused by impact. As shown in Section I, Art. 5, the additional (dynamic) pressure due to a low spot on the rail or a flat spot on the wheel increases with the rigidity of the track.

Lateral Stiffness. Lateral stiffness is of importance in the study of locomotive oscillations at high speed as well as the study of operation on sharp curves at low speed. At high speed the problem is that of the lateral oscillations of the locomotive on the spring formed by the elasticity of the track and running gear. The problem is extremely complicated, and nothing definite can be said about the advantages or disadvantages of high lateral rigidity. In low-speed operation on sharp curves, the amount of lateral deflection of the rail will often affect the distribution of the flange pressures that guide the locomotive through the curve.

Soundness of Material. The recent prevalence of transverse fissures shows that there is a great deal of work to be done in improving the initial condition of rail steels. Manufacturers should study the cooling process with particular reference to the formation of residual stresses and internal flaws.

Hardness of Material. It is known that contact stresses produce plastic flow in the rail head, particularly at the joints. The very high axle loads used in America therefore make hard rail material a necessity. A large rail is given less mechanical work during the rolling process than a small one, which results in a coarser grain structure in the large rail. In order to have the same hardness as the small rail, it must have a higher carbon content. This tendency has resulted in carbon contents of 0.75 to 0.80 per cent in rails for main-line use, with consequent increase of brittleness. It is possible that the solution will be found in heat treatment, either of the whole rail or of the wearing surfaces.

Uniform Strength. There are several places in a rail where the stresses may be high. The optimum condition is to make all such places equally strong, and the experiments described in this paper give data which are helpful in designing a rail for uniform strength.

In considering the stress at any point of a rail, due thought should be given to whether the stress is of a cyclical and frequently repeated variety, or only occasional. If of the former the criterion of safety should be based on the fatigue limit of the material. If of the latter, the yield point can be used.

Studies in the fatigue of metals[25] have shown that the conditions of safety for repeated stress are well represented by a curve such as that shown in Fig. 38. In this diagram the abscissa represents "steady" stress and the ordinate, "alternating" stress. Any given stress cycle can be analyzed into a steady and an alternating component, much the same as is a fluctuating direct current in electrical theory. A stress cycle can therefore be represented by a point on the diagram, the coordinates of which are the steady and alternating components of the cycle. The limit of safety is determined by drawing a line from the yield point of the material on the steady-stress axis to the endurance limit on the alternating-stress axis. Points lying between this line and the axes represent safe conditions, and points lying outside the line represent unsafe conditions.[26]

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[25] See Soderberg, C. R., "Factor of Safety and Working Stress," Trans. A.S.M.E., APM-52-2 (1930).

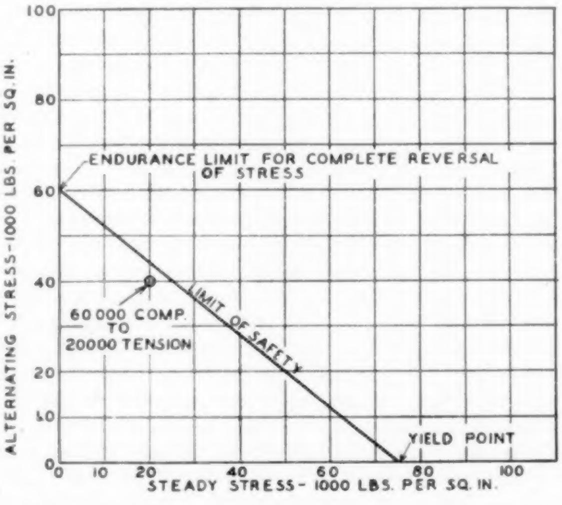

Fig. 38 Diagram Showing Limit of Safety of Stress Cycles

As an example, take a stress range of 60,000 lb. per sq. in. compression to 20,000 lb. per sq. in. tension. The algebraic average of these two is 20,000, which is taken as the steady component. The variation from this point is plus and minus 40,000, which is the alternating component. Thus, for a steel with a yield point of 75,000 lb. per sq. in. and an endurance limit of 60,000 lb. per sq. in., such a stress is safe, but very near the border line, and slight defects in the material can easily cause failure.

Let us discuss some of the stresses mentioned in this paper from the viewpoint of their liability to cause failure. The values given are for 130-lb. rail.

Stresses Produced by Centrally Applied Vertical Load. This type of load produces vertical bending of the rail, compression of the web, and contact stress in the head. It is repeated often enough during the life of the rail to necessitate the consideration of fatigue.[27]

The stress due to vertical bending goes from tension to compression. The range is not apt to be more than from 15,000 lb. per sq. in. in one direction to 5000 lb. per sq. in. in the other.

The compression in the web directly under the load is produced by vertical bending of the head on the elastic foundation formed by the web. Therefore it changes to tension a few inches away from the load, and the maximum tension is 20 per cent of the maximum compression (see Fig. 3). The wave length of this distribution is too short to allow the reversed stress between wheels to be more than 20 per cent of the maximum stress under one wheel. The highest compression that is apt to be found in the web due to centrally applied vertical load is 10,000 lb. per sq. in., so the stress range is from 10,000 lb. per sq. in. compression to 2000 lb. per sq. in. tension.

The contact stress produced by vertical load varies from zero to a maximum and is undoubtedly very high. However, it is also highly localized, and produces mostly plastic flow in the rail head.

Stresses Produced by Eccentrically Applied Vertical Load. This type of load produces all the stresses that are produced by centrally applied vertical load, with the addition of torsion, vertical bending stress in the web (see Section II, Art. 2), and lateral stress in the base (see Fig. 25). Its frequency of application is of the same order of magnitude as that of the centrally applied vertical load.

The torsion produced by this type of load is not severe, due to the shortness of the moment arm of the applied force. The lateral stress in the base is only high when the vertical load is eccentric on the outside of the head and accompanied by a high lateral flange pressure, which is an unusual condition.

The serious condition produced by eccentric vertical load is in the fillet under the head, and is due to bending of the web out of the vertical longitudinal plane. This stress goes from compression to tension, the same as the vertical web stress produced by centrally applied vertical load. Its maximum range is about 50,000 lb. per sq. in. compression to 10,000 lb. per sq. in. tension. When accompanied by a high lateral flange pressure, its maximum increases to about 60,000 lb. per sq. in. compression.

Stresses Produced by Lateral Load. Lateral load produces torsion, vertical stress in the web, lateral stress in the base, and lateral bending, principally in the head. The frequency of application of lateral load only approaches fatigue conditions on curves, where it is applied continuously during the passage of a locomotive. On tangent track the lateral forces are due to locomotive oscillations and they attain high values only occasionally.

The torsion is important only in that it produces lateral bending of the head and base [see Section II, Art. 3(*a*) and so can be considered along with the lateral bending. The vertical stress in the web produced by lateral load is not excessive, but is important because it must usually be added to the stress produced by eccentric vertical load.

The high stress produced by lateral load is that due to lateral bending of the head. This has a maximum of about 45,000 lb. per sq. in. and a reversal of 20 per cent of the maximum. On the gage side of the head, the maximum is compression and must be added to the stress produced by vertical load. Thus the maximum stress range in the head is about 60,000 lb. per sq. in. compression to about 15,000 lb. per sq. in. tension.

The work described in this paper shows that the weak points of present rails are the fillet under the head and the head itself.

2 Acknowledgments

The authors wish to thank the officials of the Westinghouse Electric & Manufacturing Company for having sponsored and guided the development and application of the test methods that were used. The field tests were all made in cooperation with the Westinghouse Railway Engineering Department, represented by Mr. J. P. Shamberger.

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[26] Fatigue tests show that the true limit of safety is a curve and not a straight line. The line shown on the diagram is a convenient simplification of the actual data, and is known to err on the safe side.

[27] An exception must be made here for the augments of vertical load produced by some dynamic effects originating in the locomotive. For example, the augment produced by the rotation of the counterweight of a locomotive will not occur often enough at any one spot in a rail to produce fatigue.

Discussion

R. **Bernard.**[28] Experiments similar to those described in the paper are being carried on in Germany, and a large number of stress measurements on rails have already been performed. For this purpose a carbon-pile instrument, analogous to that first used by the Bureau of Standards in Washington, has been further developed, and by means of it stresses, frequency of vibration, and modulus of foundation have been determined. Some experiments have also been made with magnetic strain gages similar to the one described in the paper.

For German "Reichsbahn" track it has been found that the stress fluctuates from 1200 kg. per cu. cm. to —900 kg. per cu. cm. and the frequencies from 6 to 400 per sec., and that the modulus of foundation is about 10 kg. per cu. cm.

If the different conditions, in America are taken into account, it can be concluded that the foregoing results obtained are in satisfactory agreement with those given in the paper.

C. B. **Bronson.**[29] On page 3, and with reference to Fig. 5, the authors comment on the failure of splice bars, which they call fatigue cracks. Failure originates at the mid-distance of the top fishing or bearing surface and is induced by the notch effect from the sharp angle or edge of the rail head digging into the splice bar. Undercutting or filing of the rail-head bearing, rolling a notch or depression in the bar, and increasing the strength of the bar are all partially effective in overcoming this difficulty. Heat treatment of the splice bars has been very effective in reducing splice-bar failures.

In several places the authors call attention to the high stresses in the fillet joining head and web, and indicate that these are the most severe in the rail section. Rail failures, however, do not so indicate. In the Dudley rail sections, with fairly wide fillets joining head and web, failures of this type are practically unknown. Furthermore, with the greater and almost universal use and installation of canted tie plates, eccentric loading is being very much reduced, with more nearly center-bearing application.

In Section IV, under "Hardness of Material," the authors call attention to the fact that a large rail receives less work in rolling than a small one. While this is in part true, it must be remembered that the rail mills have faced the problem of heavier rail sections by rebuilding and modernizing their plants to keep pace with the changing conditions. As a result of this, much heavier mills are now in use, which produce very effective rolling pressures through the members of the heavy rail sections.

In particular, the New York Central Lines have continued the practice established by the late Dr. P. H. Dudley of using a broad, thin head on all of their rail sections, varying slightly from one weight to another. This has made it unnecessary to step up the carbon and other hardening elements in rolling the heavier rails, which the authors mention as being essential.

B. S. **Cain.**[30] It may be of interest to compare some of the results of this valuable paper with results obtained in somewhat similar work done by the General Electric Company during several years' study of locomotive design and operation.

Equation [27] shows some theoretical basis for the observation made in practice that the vertical forces due to balanced unsprung weight do not increase unduly with increase of speed on good track. In fact, at very high speed the weight does not have time to drop into soft spots. Similar observations are made by the Special Committee on Stresses in Railroad Track.[31] Fig. 15 also shows that the maximum impacts do not occur at the center of the depression or at a joint, but some distance away.

The strain gages used by the General Electric Company were originally of the McCollum-Peters type[32] depending on the compression of carbon piles. Some good results have been obtained with these gages, but experience has led to the preference for an electromagnetic gage of the same general type as that described in this paper. The gage is a development of the electric micrometer,[33] is constructed for a 2-in. gage length, and operates on 2000-cycle current with compact and portable equipment.

When using track-stress measurements to calculate the lateral pressures of locomotive wheels, it has occasionally been found that the rail will slip on the tie plate and may be prevented by vertical load from immediately slipping back. This action must be watched for, as it does not correspond with the theory of lateral bending and, if not detected, may give incorrect values of lateral force. Track-stress measurements at one or two isolated points cannot be relied upon to yield maximum values of stress or load, the records being not even simultaneous on different wheels. With the object of obtaining records over a greater distance, the Pennsylvania Railroad equipped a special track with hardened balls and plates to support the rail and to register maximum forces by indentation of the plates.[34] An instrument which makes continuous records of both vertical and lateral pressures simultaneously on a number of ties has been constructed and used by the General Electric Company[35] and has given many valuable results, especially of vertical pressures on the rail. It has shown particularly the importance of oscillations of certain types of locomotives at high speed in producing high rail stresses, showing that the maximum stresses are indicated by rail-stress gages only if these happen to be located at the exact spots where the maximum stresses occur.

Another attack on the problem of continuously recording wheel pressures has been to mount gages on the locomotives or cars themselves so that the lateral pressures on axles are recorded, either the maximum by means of a hardened ball and plate, or continuously by an electromagnetic strain gage. Records of this sort have recently been obtained. It is to be hoped that these continuous records of forces will ultimately be correlated with the local stresses in the rail section which form the chief subject of this paper.

The authors use in Table 1 a so-called wear factor, based on the assumption that the wear between two surfaces is proportional to the pressure between the surfaces, the rubbing velocity, and the coefficient of friction. Actually, it is also assumed that the coefficient of friction is the same for all cases. The writer does not believe that this factor is necessarily a good indication of wear, because rapid wear is often due to concentration of pressure on a small area, particularly in the familiar cases where incorrect contours are used, leading to cutting. In fact, the term "cutting" indicates a common source of rapid tire and rail wear which is not indicated by the so-called wear factor.

In the particular instance of the electric locomotives of the Great Northern Railway, as given in Table 1, there is a small difference between the maximum wear factors on different locomotives and considerable difference between individual wheels, even after averaging for both directions of motion. On the other hand, data obtained from the railway company covering tire wear on electric locomotives for the last two years, show that an even tire mileage is obtained on both classes of locomotives and "there is no measurable difference between tires wearing, regardless of position." It therefore appears that the "wear factor," while an interesting index of possible high wear in some cases, is not to be relied on as indicating relative performance, even qualitatively.

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[31] Fourth Progress Report A.R.E.A. 1925, pp. 1167–1168.

[32] Proc. A.R.E.A., vol. 31, 1930, pp. 1556–1568. Also Technologic Paper of the Bureau of Standards no. 247.

[33] *Jl. A.I.E.E.*, Sept., 1926, p. 820. Also *Gen. Elec. Rev.*, Nov., 1926, p. 815.

[34] Proc. A.R.E.A., vol. 19, 1918.

[35] *Railway Age*, Aug. 16, 1924, pp. 283–286. Also *Gen. Elec. Rev.*, Feb., 1924, p. 98.

R. Eksergian.[36] This paper is a very important and valuable contribution to the literature of stresses in rails. Very little study up to this time has been given as to the effect of lateral loads in producing stresses in the rail, and the authors' analysis of stress concentration in the rail is extremely interesting.

Some years ago the late Prof. G. Lanza made a critical analysis of the effect of the spacing of the ties as concentrated elastic reactions under vertical wheel loads, and came to the conclusion that, due to the long wave length of the elastic depression of the rail, the assumption of an elastic distributed reaction for the actual concentrated tie reactions was a sufficiently good first approximation. The writer believes that Professor Timoshenko himself had made a similar analysis and come to the same conclusion.

But now in the study of lateral loads on the rail, the authors themselves state that lateral bending and twist are of a localized character, and sufficiently localized so that the lateral pressures of adjacent drivers have no mutual effect. In such a case the writer would like to ask the authors to what degree the assumption of a continuous torsional and lateral elastic reaction given by Equations [17] and [18] is justified when compared with the actual spacing of the tie reactions.

If the vertical loading has no eccentricity it would appear the lateral and vertical loading stresses can be separated, so that, due to the localized condition of stress produced by lateral loadings, stresses can be calibrated for a unit lateral load so that we can quantitatively measure the lateral forces of a wheel base. On the other hand, the authors point out the importance of eccentric vertical loading in producing both torsion and additional bending in the rail. If the flange is bearing up against the head the eccentricity will differ from that when the flange is away from the head. A study of such eccentricities for different configurations of the wheelbase may possibly be of importance in order that we may differentiate experimentally for the obtaining of lateral forces separately. Or can the effect of eccentricity of vertical loading be neglected with reasonably large lateral loads?

The consideration of a distributed elastic torsional reaction at the base of a rail has suggested a torsional elastic track (per pair of rails) reaction in conjunction with unequal loadings for individual wheels per pair of wheels. Such conditions occur wherever lateral forces are exerted, as in the rolling of a locomotive, as seen from an analysis of the phase relations in the dynamic augment of steam locomotives. Any combination of these conditions resolves into a couple exerted on the axle and wheels, together with the total axle loading, which latter divides equally on both rails, and the couple augments the mean axle wheel loading on one side and decreases it on the other. The couple reaction on the track (i.e., per pair of rails) causes a distributed elastic torsion reaction of the roadbed, if we can assume the tie itself as rigid. Actually the flexibility of the tie itself with the assumption of a distributed elastic tie reaction results in a more complicated torsional track elastic reaction. The mutual effect of tie reactions per pair of rails for unequal wheel loadings for the right rail as compared with the left rail, considering the distributed elastic reaction supporting the ties, would appear to be an interesting problem for study in connection with the vertical stresses in the rail.

The writer has made many calculations for the lateral forces exerted by various types of locomotives. A general discussion of the nature of these lateral forces is given in a paper by H. Uebelacker, and in the writer's "Static Adjustment of Trucks on Curves." It is interesting to note that with several pairs of drivers the largest lateral force for a locomotive with several pairs of drivers usually occurs on the next-to-the-rear driver, and the lateral reaction is exerted on the inner rail. Frequently the loadings may range from 20,000 to 30,000 lb. Such high loadings on the basis of the authors' analysis for stresses due to lateral forces must result in excessive rail stresses.

Such large lateral loadings are due to a considerable extent to the lateral guiding reactions of the front and rear trucks, and with swing-link trucks such loadings increase with the curvature. The authors' paper therefore reopens the question from another angle, i.e., that due to lateral rail stress limitations as to the type of truck-guiding apparatus such as a constant-resistance truck in preference to a swing-link truck and the use of an articulated wheelbase versus a completely rigid wheelbase.

In connection with vertical rail stress, the writer, in a paper on "The Balancing and Dynamic Rail Pressure of Locomotives," attempted to place the limitation of the counterbalance problem on the limiting rail stress. In connection with this the various phase angles of the dynamic augments for adjacent wheels were evaluated and the corresponding stresses estimated. The authors' paper has pointed out a further idea along this line, i.e., the estimating of the range of stress for the endurance or fatigue life of a rail. Using the master diagram of the bending moment, it is possible to construct an influence line for the passage of several drivers, and then, considering the phase relations of the dynamic augments of adjacent drivers, the complete variations of stress at a particular section can be estimated.

The relation of flange wear and angle of incidence have been recognized for some time. This relation has usually been stated by the equation, $W = kR\phi^n$, where W is the flange wear, R the lateral force, and ϕ the angle of incidence. It would be very interesting to establish experimentally the exponent n. The idea that flange wear depends essentially on the angle of incidence explains why the large lateral force of the next-to-the-last driver on the inner rail causes no appreciable flange wear.

F. M. Graham.[37] The designer of improved rails for heavy steam or electric service is confronted by a serious lack of available theoretical and experimental knowledge from which to project a rail section of greater dimensions than previously used. He will desire to produce a rail of increased stiffness and with such proportions of dimensions as will insure lateral stability against overturning under maximum combinations of forces which may be exerted in the actual service for which the rail is intended. The section must lend itself to suitable joint connections and conform to necessary manufacturing processes to produce sound steel and also permit heat treatment, if desired. The shape of the head of the rail must conform to the necessary requirements of wheel contacts, and such contacts should be made with the lowest possible unit pressure.

As opposed to the foregoing requirements, the following conditions are known to exist:

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1 Speeds approaching 100 m.p.h. develop very large forces exerted on the rail which cannot be calculated and which have only been observed in very limited tests.

2 Hidden defects in rails raise stresses in service to the point where their frequent repetition results in ultimate failure.

3 The cost of rolling experimental, or trial, sections is prohibitive, and therefore dependence must be placed on experience and the limited theoretical and test data available.

4 Wheel pressures on rails develop unit compressive stresses entirely beyond the elastic capacity of ordinary rail steel, as is shown by the observed cold flow of rail tops throughout their length, and especially at rail ends.

5 The supporting character of subgrade ballast and ties varies continuously from point to point, and especially so as between wet and dry seasons and during the winter season when roadbed and ballast are deeply frozen so that the ties provide supports with widely varying spans and resultant stresses.

The paper under discussion will be helpful in solving some of the above-mentioned difficulties.

The writer disclaims ability to criticize the more difficult mathematical analyses included in the paper, but wishes to call attention to some of the bases of calculations and other practical considerations.

Referring to Section I, wherein the elastic constant k is involved in all longitudinal stress values, it is impractical to determine this value generally on account of the elaborate equipment required, the tedious measurements to be made, and necessary occupation of the track interfering with its use during a single determination. Very low values of k occur at different times and places, resulting in permanent deformation of rails, if actual failure does not occur.

In this section the suggestion is advanced that the weight of the rail should be directly proportioned to the axle loads, disregarding speed, counterweighting, and shape of rail section. In designing a stronger rail section, the height is increased to provide greater stiffness; however, in doing so the size of the head is not increased—it may be decreased. To provide equal stability against lateral forces, it is necessary to increase the width of the base, also its thickness, tending to retain a low position of the center of gravity and thus relatively increasing the distance between the neutral axis and the top of the rail. Calling this distance c, it is suggested that where two different rail sections are assumed to have the same extreme fiber stress, their relative strengths are as $I^{3/4}/c$ for each section.

From the above it can be noted that two rail sections properly designed are not of similar shape.

Referring to Fig. 5 and the accompanying text in regard to joint bars breaking at the center of their length and cracking from the top downward, and stating that this is due to the greater distance from neutral axis to top of bar compared to the distance from the neutral axis to the bottom of the bar, resulting in higher unit tensile stress at top of bar in reverse bending, this explanation seems hardly adequate in view of the relatively low value of the reverse bending moment as shown in Fig. 3. May not this type of failure be brought about by reverse bending stress combined with the large unit pressure of the head of rail on the joint bar at this point, and the resultant severe abrasion and cold working causing considerable distortion or cold movement of the metal?

In the last paragraph of Section II the existence of a couple is assumed which assists in preventing the rotation of the rail from external forces. This assumption is doubtless based on the holding-down capacity of the spikes on the base of the rail. This is an unsafe assumption as spikes are not usually in contact with the top of the base of the rail and a very considerable actual rotation of the base of the rail would be necessary to bring them into action. Even if the spikes should act in this way, their effect would not be of a dependable elastic nature.

Similarly, in the first paragraph of Section III the resistance to lateral movement of the rail is assumed to be of an elastic character. Such movement is actually resisted by friction on the tie plates, which is of course not of an elastic nature, and also by contact of the rail base against spikes or shoulders of the tie plates. Such reactions are irregularly spaced and are of variable strength so that they do not provide a condition which can be properly included in a theoretical determination of rail stresses.

Referring to the tests on the Pennsylvania Railroad, the writer is not aware that simultaneous values of vertical and lateral forces exerted by each wheel have ever been measured. Such measurements have been made as the successive wheels arrive at a point of measurement, and it is probably not correct to assume that these are equal to simultaneous values.

The foregoing comments on some portions of the paper are only offered from a practical standpoint and are not of a constructive character.

The writer wishes to express his appreciation of this very interesting discussion of a difficult subject. It deals with stresses arising from the interaction of such diverse materials as dirt, stone, wood in various stages of decay, and steel when acted upon by modern locomotives at high speeds. The maximum forces often arise from accidental conditions which cannot be foreseen and which cannot be determined theoretically. Safe railroad operation is a question of maximums, not averages.

The writer suggests that the field work outlined by the authors be much extended to include the use of various types of modern locomotives (both steam and electric) at high speeds, using a large number of strain- and vibration-recording instruments so as to cover a considerable length of track instead of a single point. The track conditions should be varied so as to include all maintenance conditions which may be reasonably possible in actual service.

R. Grammel.[38] The authors' investigations provide a splendid example of fruitful cooperation between theory and experiment. The writer ventures to present some additional considerations which may perhaps be of use in furthering these investigations and in their practical application.

When the rail, as is usual, is attached to cross-ties which themselves are bedded on an elastic foundation, there obtains an interrelation between the vertical deflection [3] and the torsion ϕ. This is due to the fact that the loading of the tie near the two ends produces an upward bending of the tie in the middle, and hence a tendency of the ends to incline outward (except in the case when the parts of the tie under the rails themselves rest on some kind of a foundation but not in the middle). For an initial calculation we may assume that the moment of torsion which acts from the tie to the rail is proportional to the vertical deflection z of the rail and appears in Equation [21] as an additional member k_3z on the right-hand side. Here k_3 is a sort of coefficient of bedding, while z has the value given in [3]. A more exact calculation will take into consideration the connection between ϕ and z, and in addition the values of k_3 must be expressed in terms of the coefficient of bedding k. This can be done approximately in the following way. Instead of an infinite number of ties let us assume a single elastically supported "plate" having a coefficient of bedding k, but no bending rigidity in the longitudinal direction of the rail. If we then measure x lengthwise to the rail, y crosswise to the rail horizontally (which means along the tie), and z vertically, we have then to load the "plate" lengthwise to the rail with the unknown forces $Z(x)$ and the unknown moment $M(x)$. In addition to the known loads P (vertical) and H (horizontal) the rails then also carry the reactions $-Z(x)$ and $-M(x)$. In this way we obtain four linear differential equations, namely, for the deflection z' and inclination ϕ' of the "plate," for the vertical deflection z of the rail, for its torsion ϕ, and for its horizontal deflection y. These four equations are mutually interconnected by the magnitudes $Z(x)$ and $M(x)$, which may be omitted by the employment of an auxiliary condition to the effect that z and ϕ are the same as the values of z' and ϕ' at the locus of the rail. The actual calculation is rather complicated, but numerical results can be obtained. It would be of interest to compare them with results of actual measurements.

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Walter M. Haessler.[39] Regarding the lateral rail forces mentioned by the authors, the writer would like to ask how this factor varies according to the height of the center of gravity of the locomotive above the rail. He is cognizant of the fact that these quantities—the lateral rail forces and the height of the center of gravity—are roughly in inverse proportion to each other. Have any experiments been made to indicate in a quantitative manner the relationship existent between these two quantities?

J. J. Karpinski.[40] The tendency of rail design, as stated in the paper, has been toward increasing the vertical stiffness. This was effected by increasing the height of the cross-section and placing more material in the head or base, while making little or no change in the thickness of the web. Thus the 80-lb. Am. Soc. C.E. Standard rail has a web $^{35}/_{64}$ in. thick, while the 85-lb., 90-lb., 95-lb., and 100-lb. rails have the same thickness of web, namely, $^{9}/_{16}$ in. This is an indication that the rail designers have already felt that material in the web was not being properly utilized.

After thorough theoretical and experimental investigations the authors point out that a more favorable stress distribution could be obtained if material were removed from the bottom of the web and placed nearer the top.

It seems to the writer that it would be of interest to extend these investigations to a case of a rail having the web perforated at regular distances near the bottom. In this way it would be possible to form an idea whether it would be possible to treat the rail as a Vierendeel girder. In this case, if necessary, extra thickness could be given to the verticals representing the unperforated part of the web because of the economy of metal attained by punching the web. The stronger verticals would permit increasing the height of the rail cross-section without fear of excessive stresses resulting from an eccentric vertical load, and therefore they would make it possible to diminish the amount of metal in the head, thus facilitating the process of cooling and consequently diminishing the chances of transverse fissures occurring.

C. F. Dendy Marshall.[41] With regard to the question of longitudinal weight transfer when in "motion" which is due to what the writer calls "trim," the authors give as the reason the pull of the drawbar. There are in addition two other factors which they omit to mention, namely, the upward pressure on the slide bars, and the frontal air resistance (with regard to the latter it is true that they were only dealing in that part of their paper with moderate speeds). The slide-bar pressure is a very serious disturbing force. Imagine for a moment that the stroke was increased until it was nearly equal to the diameter of the wheel. Suppose the crankpin to be at the bottom, brakes hard on, and steam admitted in front of the piston. It is obvious that there is a pressure downward on the rail under the driving wheels, and upward on the slide bars, tending to lift the front of the engine.

There is a point in connection with rail joints on which, though not touched on in the paper, the writer would like to lay his opinion before American engineers. In England they are set in line with one another; in the United States, he believes, they are usually alternated. Coming as they do at regular intervals, they are a potential danger if, at any attainable speed, their impacts can coincide, or harmonize, with the natural periods of the vehicles. With rails 45 ft. long, at 61 miles an hour a joint is met every second. The period of pitching, which is induced by parallel joints, is probably far quicker than this. But it has been found in England that the period of rolling (which alternate joints tend to produce) of a heavy tank engine is from 1 to $1^{1}/_{4}$ seconds, and rather less in the case of a tender engine. It would therefore appear that there is some risk about the alternative method, because the impulses, though small, are regular, and the writer is inclined to think that in this lies the explanation of the fact that elaborate and costly spring-compensating arrangements are found necessary in America, although not required in England: the reason being that they distribute the impulses and break up the rhythm.

S. Matsunawa.[42] In Japan, as well as in other countries, the study of stresses in railway track is regarded as of great importance from the economical viewpoint of railway operation, and the writer is very glad to have been afforded the opportunity of reporting some results of the investigations of similar subjects which were carried out by the Japanese Government Railways.

Statistical investigations of rail failures in 1929 covering a total operating track length of some 17,000 km. of the said railways have shown that 65 per cent of the total rail failures occurred within 1.6 meters from rail joints.

In view of this fact we think that investigations of rail stresses should be focused upon the effects of rail joints. In this connection the paper, "On the Strength, Design, and Maintenance of Rail Joints" by Mr. I. Horikoshi,[43] may be worth noting.

Measurement of the stress in railway rails under moving loads is one of the most important tasks imposed upon those who are interested in scientific research on rail stresses, though it is attended with great difficulties. The electric telemeter as devised by Mr. H. Shibata, engineer, of the writer's office, may be numbered among the best apparatus for the purpose, and the attention of experts is called to the descriptions in his paper entitled "Measuring the Stress and Depression of Rails by Means of an Electric Telemeter."[44]

In this apparatus the amount of stress (that is, the strain within a given gage length) and of the depression of rails to be measured is read in terms of electric current which varies proportionally to the amount of stress or depression. The slide-wire method is adopted, and the record is obtained by means of a portable six-element oscillograph designed for the purpose.

A chief advantage of the apparatus is that the stresses at four different points and the depressions at two different points can be recorded at the same time, and that the record can be easily magnified any number of times, as there is no considerable inertia mass of the recording part.

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[41] Guildford, England. M.A., M.I. Loco. E.

[42] Engineer and Chief of the Research Office of the Japanese Government Railways, Tokyo, Japan.

[43] See Bulletin, Research Office, Japanese Government Railways, vol. 19, no. 21, May 25, 1931.

[44] Paper on file at A.S.M.E. Headquarters.

In Fig. 39, AB is a resistance wire, and a current from a battery E is adjusted by the rheostat R. A point C is soldered on AB, and D is a contact point which slides a distance in proportion to the strain or depression of the rail under rolling load. There is some potential difference between C and D according to the amount of current flowing through the resistance wire AB, and to the value of the resistance between C and D. The current in the shunt circuit DVC will therefore vary with the amount of the strain or the depression. Thus, we can measure the strain or depression by recording the variation of this current by means of the oscillograph at any required magnification, which is made possible by adjusting the rheostats R and r.

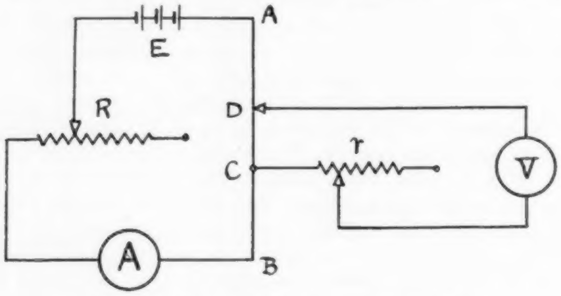

FIG. 39 ELECTRIC TELEMETER CIRCUITS

Thus the longitudinal, lateral, and any other stresses in rail can be measured by applying a proper attachment which reduces different amounts of strain to proper variations of the electric current in the shunt circuit DVC.

H. F. MOORE.[45] The authors have given an interesting discussion of transverse fissures in rails, although their experimental methods are not such as to detect such fissures or to determine the complex stresses beneath the surface of a rail under a wheel load. They refer to Bulletin 212 of the University of Illinois Engineering Experiment Station. In that bulletin a mathematical analysis is developed for internal stresses, and it is found that theoretically there exist very high shearing stresses, which are a maximum beneath the surface of the rail. Whether these stresses are sufficiently great to start a fissure in sound rail steel is as yet a matter of uncertainty. Experiments are now in progress in the Materials Testing Laboratory of the University of Illinois, in connection with the Joint Investigation under the auspices of the American Railway Association, the Technical Committee of the Association of Steel Rail Manufacturers, and the University. Already a transverse fissure has been produced in the laboratory in a shatter-cracked rail which has never been in track service, and tests on specimens of sound rails are now under way. Until further experimental results have been obtained the writer does not believe that it can be safely assumed that it is unlikely that wheel-load stresses can ever start internal fissures in sound rail steel. Further tests may, or may not, show this to be the case.

A. ONO.[46] The calculation of stresses in rails and the experiments in the laboratory and field have so thoroughly and successfully been carried out by the authors that there appears nothing to be added except one thing relating to the rail joint. Some results given in or deduced from a note on the calculation of the joint[47] will be introduced here in the hope that they will prove of interest to engineers interested in the present paper.

The joint is considered to consist of compound beams, and the relative displacement of the rail and fishplates, say, $y_1 - y_2$, is assumed to be represented by an equation such that $\mu = K_1 (y_1 - y_2)$, where μ is the intensity of pressure between the rail and fishplates induced by the action of load, and K_1 is the modulus of joint. In the calculation another constant α may be conveniently taken instead of K_1, viz.,

$$\alpha = \sqrt[4]{\frac{K_1}{4}\left(\frac{1}{E_1J_1} + \frac{1}{E_2J_2}\right)}$$

where E_1J_1 and E_2J_2 are the flexural rigidities of the rail and plates, respectively. As the modulus of elasticity of carbon steel is practically independent of the carbon content, E_1 and E_2 may be taken as equal. It is to be remarked that α plays a similar part to that of the constant β, which depends on the modulus of foundation as well as on the rigidity of rail.

The value of K_1 is unknown, but α may be determined by measuring the deflection of rails jointed together, and comparing this result with that observed on rails without joint. This was done in the laboratory of the Steel Works at Yawata, Japan, by taking specimens supported at both ends and loaded at the middle. Of course the value of α may vary greatly according to the tightness of the joint.

Now the greatest bending moment induced at the joint by a load P may be approximately found by the following equation:

$$M_b = \frac{P}{\beta\,(4 + q)}$$

where

$$q = \frac{4\beta J_1}{\alpha J_2}$$

The numerical value of q for a very tight joint was found to be 0.925 for 37-kg. rails (weight per meter) and 0.725 for a certain 50-kg. rail, used in Japan. Accordingly, the greatest bending moment in fishplates is a little less than that in rails without joints, while the stress in the former is very considerable as compared with that in the latter owing to the relatively small value of the section modulus.

Next the stress in rails may be considered by extending the calculation of the joint. For this purpose we find the positive and negative moments, and then the maximum and minimum values of the bending stress at a certain section of the rail. In this way we obtain the average stress and the alternating stress, as were also mentioned by the authors. Taking these stresses as factors determining the safety of material and assuming, e.g., a straight-line relation between these two for the fatigue limit, an equivalent alternating stress with the average stress zero may be found. Then the breaking section may be determined by finding a section in which the equivalent stress is a maximum.

Such calculation being performed, it can be shown that the fracture of a rail is likely to take place near a joint, so far as the bending stress is concerned, but when the joint is very tight, the straining action is only slightly greater than in long rails without joints. When the joint is so loose that it is quite unable to transmit force and moment, the calculation should be partly modified, as the magnitude of the negative moment is greatest when the load is at an end of the rail. In this case the straining action becomes much higher, say, some 33 per cent over that in long rails, and the distance between the breaking section and an end of a rail is some 76 cm. in 37-kg. rails and 88 cm. in 50-kg. rails. A report made by the research staff of the Steel Works at Yawata states that the transverse fissure of rails is rather localized in a region near an end. This seems to support the result of the calculation, although the writer does not mean to imply that the stress may be conclusively calculated by taking the bending only, as there are many other things to be considered, which were dealt with by the authors of the present paper.

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[47] *Zeitschrift f. Angewandte Mathematik u. Mechanik*, Band 11, 1931, p. 165.

C. Richardson.[48] It is interesting to note the effect of lateral forces on rail stresses. Eccentric loads also tend to increase the unit stresses. In canting one rail it is frequently found that the load is delivered on the outside of the head. In this case the lateral forces plus the vertical forces combine to produce an abnormally high unit stress.

The actual tests cover moderate speeds. It is hoped that further papers will disclose stresses involved at maximum speeds.

Continuation of the research work started should show whether a redesign of the rail is necessary to meet stresses actually incurred. Originally freight-car side frames were designed primarily for vertical stress. Investigation of failures showed that the lateral forces were the actual causes of failure. When the side frames were redesigned for both vertical and lateral forces, the life of side frames was increased to the point where they now outlive the car.

E. Schwerin.[49] This paper extraordinarily enriches our knowledge of the complicated stresses which arise in railroad tracks. The authors show that the probable stresses may be calculated with great accuracy, and that the weak points of the rails now used are the fillet under the head and the head itself; this last result seems to be extremely important for the development of new, more suitable forms of track.

With regard to this research the writer would like to know the opinion of the authors concerning the rise of grooves ("Riffelbildung") on the head of the rail. This phenomenon, which is especially observed at the entrance of stations, and which leads to quick destruction of the head of the rail, was much discussed some years ago. Has the Westinghouse Company already investigated this phenomenon, or are such experiments projected? As far as the writer knows, the cause of this phenomenon has not yet been fully explored, and he would be glad if the authors would express their opinion on this point.

Dr. Ing. Saller.[50] Some months ago the writer submitted an article entitled "Einheitliche Berechnung des Oberbaues" to the *Organ für die Fortschritte des Eisenbahnwesens* for publication, in which he gives simpler equations for the axle sequence than those offered by the authors. This article appeared in No. 2 of that journal for 1932.

Equation [5] of the present paper, as the writer sees it, is incorrect. The value of y in Equation [3] becomes zero when $(\cos \beta x + \sin \beta x)$ equals zero, and this is the case when $\beta x = 3\pi/4$, and hence the total length $2 \times (3\pi/4\beta) = 1.5\ \pi/\beta$.

Referring to Equation [9], the writer would say that it seems to him that here the longitudinal-tie-construction theory is applied to cross-tie construction. The longitudinal-tie construction employs no cross-ties. In the application of the longitudinal-tie theory to cross-tie road construction, the cross-tie and longitudinal-tie widths are often interchanged.

The writer has made a few tests with Thoma's electrical capacity extensometer at the rail base in straight sections, and has found that the influence of the lateral forces is not as great as the authors indicate. This work is still being continued.

A more specific reference to the table in Zimmermann's book (footnote 7) would be appreciated.

A. Spoliansky[51] and J. Lamoen.[52] The mathematical introduction to the paper gives as clear an insight into the stability of rail as may possibly be obtained when one restricts oneself to analytical methods. The theory used (of bending of bars continuously supported by an elastic foundation) has proved to be one of the most useful theories in applied mechanics, especially with reference to foundation problems. The most exhaustive work from this point of view is probably that written by Hayashi.[53] One of the present writers has also treated this question,[54] and has applied the theory to the computation of the horizontal reactions between piers and bridge in a weir consisting of a great number of slender piers supporting a bridge reaching from one abutment to the other.[55] The reactions of the piers on the bridge are in fact concentrated forces, but he has likened the bridge to a bar on an elastic foundation. The results of the approximate computation, assuming continuous support, were 0.5 per cent higher than those obtained by the exact calculation, considering concentrated reactions. This fully corroborates the presumption that the rail of Fig. 17 of the paper is, practically speaking, continuously supported.

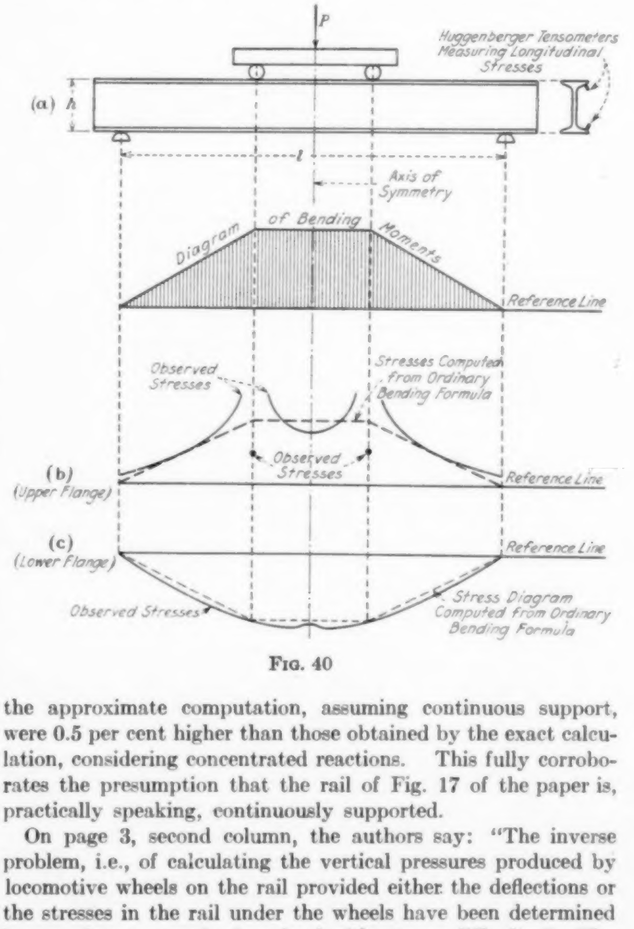
P
Huggenberger Tensometers Measuring Longitudinal Stresses
(a)
h
l
Axis of Symmetry
Diagram of Bending Moments
Reference Line
Observed Stresses
Stresses Computed from Ordinary Bending Formula
Observed Stresses
(b) (Upper Flange)
Reference Line
(c) (Lower Flange)
Reference Line
Observed Stresses
Stress Diagram Computed from Ordinary Bending Formula

Fig. 40

On page 3, second column, the authors say: "The inverse problem, i.e., of calculating the vertical pressures produced by locomotive wheels on the rail provided either the deflections or the stresses in the rail under the wheels have been determined by experiment, can also be solved without any difficulty." The writers quite agree as to the deflections, these being but feebly affected by the contact effects. But to make this method hold for stresses measured under the wheels, it would be necessary that the observed stresses—which are of course the existing stresses—be approximately equal to the stresses computed from the bending formula $M/\sigma = I/v$, M being measured on verticals passing through the wheel centers (see Fig. 4). But as the contact stress intervenes, these calculated stresses will be entirely different from the real stresses. (See Figs. 27 and 28.)

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[49] Prof. Dr.-Ing. E. Schwerin, Berlin-Wilmersdorf, Germany.

[50] Director, German State Railways, Dechbettenerstrasse 32, Regensburg, Germany.

[51] C.E., E.E. Chief Engineer, Constructions Métalliques Enghien-St. Eloi, Enghien, Belgium.

[52] C.E. Laboratory, Institute of Civil Engineering, Liège University, Liège, Belgium.

[53] "Theorie des Trägers auf elastischer Unterlage und ihre Anwendung auf den Tiefbau," by Keiichi Hayashi, Professor at the Kyushu University, Japan. First edition, 1921, Julius Springer, Berlin.

[54] J. Lamoen, "La théorie de la poutre sur fondation élastique et ses applications," *Revue Universelle des Mines*, Nov., 1931.

[55] J. Lamoen, "Over de krachtsverdeeling in een stuw met groot aantal smalle pijlers." *Der Ingenieur*, Sept. 18, 1931.

The writers are much interested in Figs. 27 and 28 showing the effect of contact stress. They have obtained similar results when experimenting with the arrangement shown in Fig. 40 (*a*). Their observations are roughly sketched at (*b*) for the compression in the upper flange, and at (*c*) for the extension in the lower flange. One might think that at a certain distance from the points of contact the computed and the observed stress diagrams would become identical. This is only true, however, for sufficiently great values of l/h, i.e., the transverse dimensions must be sufficiently small as compared with the longitudinal dimensions. When this condition is not satisfied, the ordinary theory of bending is contradictory to the theory of elasticity.

H. R. Thomas.[56] Referring to Fig. 30 and the accompanying text, the relative magnitude of the various local stresses might well have been indicated; and the method of calculating the contact stresses as well as the reason for assuming that the load makes contact at a point instead of over an area would be of interest.

The introduction of a discussion of transverse fissures into Section II—Local Stresses in Rails, does not seem warranted. The kind of repeated stress involved in the statement "Transverse fissures are fatigue cracks which form inside the heads of rails in the transverse plane," is not stated, nor are there any results presented which would indicate the presence of a repeated stress in the interior of the head sufficient in intensity to cause a fatigue crack to form and spread. Practically the whole of this discussion involves matters on which the evidence available (both in this paper and elsewhere) is not conclusive.

As suggested above, there does not seem to be anything in this paper which would warrant the paragraph on Soundness of Material.

Referring to Fig. 38, the use of the expression "Limit of Safety" is unfortunate, since any stress condition approaching this curve is certainly approaching the point of actual destruction.

In dismissing contact stresses as having only the effect of producing "mostly plastic flow in the rail head," the possibility that such cold working of the head may result in high internal residual stresses or may contribute to the start of surface cracks is being disregarded.

In presenting stress measurements and calculations, only locomotive-wheel loads seem to be considered. On account of the much greater frequency of application of car-wheel loads, it would seem that on the whole these loads might be even more important. This is especially the case if their effect in producing plastic flow and wear of the rail head is considered.

Hans Fromm.[57] The investigations which are discussed in the paper give important information regarding the magnitude of the stresses in rails, regarding the causes producing these stresses, and the calculation of the stresses. These investigations showed that it is possible, by using a simple theory, to calculate with sufficient accuracy stresses produced by vertical and lateral forces. It is possible furthermore to obtain from a few suitably chosen strain measurements in the field the system of forces acting on the rail. Combining these two kinds of investigation it is possible to determine the static and dynamic forces acting between the wheels and the rail and to establish in this manner an important foundation (1) for the design of locomotives and cars, and (2) for the design of rail profiles and of railway track.

Especially interesting is the action of lateral forces, which in general consists of the pressure of the flanges and of the lateral friction forces. The lateral forces produce a very considerable increase of stresses in the rails (1) by direct action and (2) by producing changes in distribution of the vertical loads between the wheels.

The wear on the rail depends, in the writer's opinion, principally on the magnitude of the work of friction which is obtained as the product of the friction force and the corresponding magnitude of sliding (*Gleitschlupf*). We have to consider here (1) the wear between the flange and the rail, which is approximately proportional to the acting pressure of the flange, to the coefficient of friction, and to the height of the flange, and (2) the wear of the upper surface of the rail and of the surface of the wheel rim, for which not only lateral but also longitudinal friction forces must be considered. These longitudinal forces appear partially as the reaction against friction between the flange and the rail, but principally they represent the necessary circumferential forces acting on the driving wheels and the forces acting during braking. The work of friction can be calculated[58] in the case of a conical surface of the rim of the wheel as is usually the case in railways. It gives the friction losses due to sliding and also the lever of the rolling friction.

The longitudinal friction forces have only a negligible influence on stresses in the rail at a considerable distance from the point of contact of the wheel and of the rail. But at the surface of contact and near this surface the friction forces can have a considerable effect on the stresses. The calculations[59] in the case of a rim with the usual conical surface and a flat surface of the rail give an increase of the reduced stresses[60] up to 70 per cent. This increase depends on the dimensions, the magnitude of the load, and the coefficient of friction. In the case of a curved contact surface of the rail we have the case of contact of two cylinders with perpendicular axes, for which we have not yet a solution. We must expect here a similar increase in reduced stresses as for the above-mentioned case of a flat contact surface of the rail. Moreover, with continuous wear the contact surface of the rail approaches a flat surface. There exist also some profiles which have the flat surface from the very beginning.

So long as the reduced stress is far below the yield point the wear depends essentially upon the work of friction forces. When the reduced stress approaches the yield point, the hysteresis work becomes noticeable. This is shown by increase of rolling losses above the work of friction forces and on the other hand by the acceleration of wear due to fatigue. If the point of the maximum reduced stress is not on the contact surface but at a certain depth below the rolling surface of the rail, the well-known "pitting" phenomenon may occur.

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[56] Dept. of Theoretical and Applied Mechanics, University of Illinois, Urbana, Ill.

[57] Technische Hochschule, Berlin, Germany.

[58] H. Fromm, "Zulässige Belastung von Reibungsgetrieben mit zylindrischen oder kegligen Rädern," *Z.V.D.I.*, vol. 73, 1929, p. 1031. The influence of the angle of incidence in the case of the angle between the plane of the wheel and the axis of the rail is discussed in *Zeitschrift für technische Physik*, 1928, p. 304.

[59] H. Fromm, *Z.V.D.I.*, vol. 73, 1929, pp. 957, 1029, and vol. 72, 1928, p. 1899.

[60] As reduced stress we take the expression

$$\sigma_{\text{red.}} = \sqrt{2(\tau_1^2 + \tau_2^2 + \tau_3^2)}$$

proposed by von Mises, in which τ_1, τ_2, and τ_3 are the principal shearing stresses.

AUTHORS' CLOSURE

S. TIMOSHENKO. In closing the discussion the present author will consider only questions concerning the introductory theoretical part of the paper and the general arrangement of field measurements. Those concerning field experiments and laboratory tests will be answered by Mr. Langer.

In the theoretical part of the paper the authors begin with the discussion of the action on the rail of vertical loads. A more detailed investigation of this problem has shown[61] that the isolated elastic supports can be replaced with sufficient accuracy by a continuous elastic foundation (see discussions by Dr. R. Eksergian and Dr. Saller). Although the actual support of the rails is very far from this ideal condition, the theory based on such an assumption proved a very useful one, and the authors were guided by this theory in planning their experiments. It was found also that by using this theory vertical loads could be calculated with sufficient accuracy from strain measurements if only the modulus of foundation k were determined by the preliminary calibration tests.

In investigating lateral bending and twisting of the rail, the authors applied the same assumption of continuous elastic support. Due to the simultaneous action of large vertical loads, it can be assumed that all clearances are taken up and that the rotation and lateral deflection of the rail are producing the corresponding elastic deformation of the ties (see discussion by F. M. Graham). The theory developed in this manner gives only a general picture of the deformation of the rail produced by lateral forces and by eccentric application of vertical loads. To obtain the numerical values of lateral forces and of eccentricities from their tests the authors did not use the theoretical equations but based their calculations on the results of the preliminary calibration of the track (see discussion by R. Eksergian) so that the defects of their theory do not affect the calculated values of lateral forces. The arrangement adopted for strain measurements makes it possible to measure the effect of lateral forces separately from the effect of eccentric application of vertical loads (see discussion by R. Eksergian).

The authors made their measurements only in two cross-sections, and in general they cannot be relied upon to yield maximum values of stress or load (see discussion by B. S. Cain and F. M. Graham), but their experiments showed that in the curves and at moderate speeds there is a steady distribution of vertical and lateral forces between the locomotive, wheels, and rails. Under such conditions, their measurements give a true picture of acting forces. In the case of a tangent track, it is necessary to take simultaneous readings in many cross-sections. The possibility of mounting the gages on the locomotive was also considered in this case.

The problem of stress conditions in joints was not investigated by the authors, and they mentioned the cracks in joint bars only because they show that there must be reverse bending stresses which can be explained if the rail be considered as a bar on an elastic foundation. The large local pressures of the head of the rail on the joint bar may also have an effect on the development of the above-mentioned fatigue cracks (see discussions by F. M. Graham and C. B. Bronson). The deformation of a joint depends greatly upon the tightness of the joint, and is affected also by friction. It would be very interesting to determine experimentally with what accuracy the bending moment acting in a rail joint can be calculated (see discussions by S. Matsunawa and A. Ono). In conclusion, the author would like to note that it was not suggested in the paper that the weight of the rail should be directly proportioned to the axle loads (see discussion by F. M. Graham). It is shown only that if the rail profiles are geometrically similar, the maximum bending stress produced by an isolated force remains constant if the "weight of the rail per unit length is proportional to the load."

B. F. LANGER. Mr. Cain criticizes the use of the "wear factor" mentioned in Table 1 because it does not consider high concentrations of pressures due to incorrect contours. The wear factor is not intended to cover the whole subject of flange wear and rail side wear, but merely shows how this wear is affected by the layout of the wheelbase and the distribution of the guiding forces.[62] The fact that it does do this is demonstrated by the side wear of the outer instead of the inner rails on curves and the common practice of interchanging tires between end drivers and inner drivers to equalize the wear over the whole locomotive. A good article on flange wear, with particular reference to the shape of flange and rail contour, may be found in Heumann's "Spurkranz und Schienenkopf.[63]

Dr. Saller's method of finding the wave length of the deflection from Equation [3] is not correct, since he incorrectly assumes that the distance from the point where y is maximum to the point where y is zero is one-half of a wave length. The true wave length is found by putting the variable angle βx equal to 2π. This results in the value given in Equation [5].

Regarding Dr. Saller's criticism of Equation [9], the whole paper considers only cross-tie construction, and not longitudinal-tie construction. Equation [9] is correct for the former, but meaningless for the latter, since it does not consider the stiffness of the tie in bending.

The table referred to in Zimmerman's book (footnote 7) may be found on page 289 of the 2nd edition (1930).

The authors were very much interested in the information given by Messrs. Spoliansky and Lamoen on the effect of localized stress on stress distribution in beams. It was on account of this effect that they avoided the use of stresses in the head of the rail, near the point of contact, in calculating wheel loads. In the base and the web, the localized stresses are present, but do not introduce an error because the wheel loads are calculated from the experimental and not the purely theoretical relationship between stress and load.

Referring to Professor Thomas' discussion, the details of the calculations whose results are shown in Fig. 30 were not considered of sufficient importance to be included. The deformations from contact pressure were calculated from a formula given in A. E. H. Love's "Mathematical Theory of Elasticity," p. 192.

The appropriateness of a discussion of transverse fissures in this paper is apparently a matter of opinion. The authors felt that even though they had not done any original work themselves, it was a subject of such great importance that it could not be completely ignored, and therefore mentioned such work of other investigators which seemed to offer most hope of an ultimate solution. They will look forward with great interest to the completion of Professor Moore's work and its final answer to the question of whether or not fissures can be produced by wheel loads in sound rails. The evidence now available may not be conclusive, but it certainly indicates very strongly that the initial condition of the rail and not the loads imposed on it in service is the factor which determines whether or not it will develop fissures.[64]

In Fig. 38, the expression "Limit of Safety" should not be construed as meaning a safe working stress for design purposes. The well-known "factor of ignorance" should be applied.

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[61] See author's paper in "Bulletins of the Institute of Engineers of Ways of Communication," St. Petersburg, 1915.

[62] See also, "Rail Stresses and Locomotive Tracking Characteristics Found in Tests on the Great Northern Railway," by J. P. Shamberger and B. F. Langer, A.R.E.A. Bulletin, vol. 33, no. 339, September, 1931.

[63] *Organ für die Fortschritte des Eisenbahnwesens*, Dec. 1 and 15, 1931.

[64] In addition to the work referred to in the paper, see also "Effect of Controlled Cooling and Temperature Equalization on Internal Fissures in Rails," by C. P., O. F. A., and N. P. P. Sandberg, *Metals and Alloys*, vol. 3, no. 4, April, 1932.

The authors were very much interested in the strain gage mentioned by Dr. Matsunawa, and hope to be able to obtain a more detailed description of it. The mechanical construction must be quite ingenious to translate such small motions into electrical variations by the simple means of a slide-wire resistance.

Mr. Dendy Marshall mentions two causes of longitudinal weight transfer in addition to the pull of the drawbar. One is frontal air resistance, which was not considered in the paper, but which is negligible at moderate speed. The other he calls "slide-bar pressure," which is the same as the authors referred to as non-uniformity in vertical load due to angularity of the connecting rod. (See section on Vertical Loads, under III—Field Tests.)

Mr. Bronson mentions that the high stresses in the fillet joining the head to the web have been overcome by widening the fillet and canting the rail. This is quite in line with the authors' ideas on correct rail design, although they feel that the canting of the rail, while desirable, should not be relied on entirely to reduce the eccentricity of the vertical load. The fillet should be adequate to withstand the eccentric load in case it does occur.

The determination of the modulus of foundation, k, need not be as difficult as Mr. Graham thinks. If this one value is all that is required, it can be found with sufficient accuracy by measuring the deflections under the wheels of a locomotive of known axle loading by means of a surveyor's level.

Mr. Haessler asks how the height of the center of gravity affects the lateral rail forces. This is a subject on which opinions differ and no experimental work has been done, to the author's knowledge. The commonly accepted idea is that a high center of gravity introduces elasticity between the spring mass and the rail, and hence reduces the lateral forces. This is of importance only at the moment of entrance to a curve or else during severe lateral oscillations. If the motion is steady, the height of the center of gravity can have no effect on the lateral rail forces.

Professor Schwerin asks an opinion regarding the formation of grooves (*Riffelbildung*) across the head of the rail. The authors have not done any work on this subject and can offer no new suggestions as to their cause.

Mr. Graham criticizes the theory of torsion of the rail, since it assumes the existence of a couple which opposes the rotation of the rail. This assumption is not, as Mr. Graham supposes, based on the holding-down capacity of the spikes, but merely on the existence of a vertical load. If there is pressure between the base of the rail and a tie plate, then a resisting moment can take the form of a redistribution of this pressure, without any spikes being used at all.

It is not stated in the paper that simultaneous records of vertical and lateral load under all the wheels of a locomotive have already been made. In footnote 10, it is merely stated that this is contemplated.

The authors heartily welcome Mr. Graham's suggestion that the methods outlined here be extended to the use of more gages and a greater variety of operating conditions. A burning problem at the present time is that of oscillations at high speed. Theories and models can be and are being used, but the final proof of any theory will always depend on full-scale tests. The authors believe that such tests can best be made by developments of the method outlined in this paper. The Otheograph of the General Electric Co. is not satisfactory for the following reasons:

1 The recorded values of lateral load are unreliable since the mounting of the rail is such that purely vertical load can produce lateral deflection and hence record as lateral load. It makes no attempt to record lateral forces directed toward the center of the track.

2 The recorded values of vertical load are in error at high speed due to the inertia of the steel ties supporting the spring mechanism.

APM-54-27

Resistance of Slender Bodies Moving With Supersonic Velocities, With Special Reference to Projectiles

By THEODOR VON KÁRMÁN[1] and NORTON B. MOORE,[2] PASADENA, CALIF.

T. von Karman

The hydrodynamical equations for the steady flow of a compressible fluid with axial symmetry are applied to the case of a slightly disturbed parallel flow with velocity higher than the velocity of sound. The linear equation obtained in this way is integrated by a numerical method, and the solution applied to an approximate calculation of the pressure distribution at the head of a projectile. The question of the drag of projectiles is discussed and the results of the theoretical calculations compared with the values obtained from ballistic experiments. The numerical calculations have been carried out by the junior author, using the methods suggested by the senior author.

N. B. Moore

The present investigation deals with an approximate calculation of the resistance of bodies of revolution, moving through the air with a velocity superior to the velocity of sound. A number of investigations already exist on two-dimensional flow problems involving supersonic velocities, but there have been very few attempts to find solutions of the hydrodynamical equations in the three-dimensional case. On the other hand, most of the experimental data refer to bodies of revolution, in connection with the problem of the resistance of projectiles. This question has aroused the interest of a great number of scientists and engineers, and military institutions have also carried out quite a large number of experiments on the problem. In the present paper very slender bodies of revolution are considered, so that the results give only a first approximation to the case of actual projectiles. However, the pressure distribution on the front part of the bodies seems to be represented in a fairly correct way, and it is hoped that this first approximation will be useful to obtain further and closer approximations.

1—The Equations for Steady Flow With Axial Symmetry in a Compressible Fluid

The authors establish the fundamental equation under the following assumptions:

(*a*) The viscosity of the fluid can be neglected
(*b*) The pressure is a given function of the density
(*c*) The flow is a vortexless motion.

Assumption (*a*) implies that the frictional resistance of the body be neglected; this is of course not quite justified from the practical point of view. However, it seems that the frictional resistance and the "wave resistance" due to the effect of compressibility can be considered separately; in other words, it can be assumed that the shape of the compressibility waves and their contribution to the resistance is not altered very much by viscosity. Assumption (*b*) enables us to consider two limiting cases: the adiabatic and the isothermic compression or dilatation of the fluid elements. The first case corresponds to no heat conduction at all; the second postulates that the heat conduction is so perfect that there is no temperature change whatever in the part of the fluid involved in the flow around the body. The first case is nearer to reality, so the adiabatic hypothesis will be used, putting $p/\rho^k = p_0/\rho_0^k$, where p denotes pressure at an arbitrary point, and p_0 the value corresponding to the undisturbed part of the fluid, the respective densities being ρ and ρ_0; k is the ratio of specific heats and for air has the value $k = 1.41$ approximately.

According to the Helmholtz laws of vortex motion, no vorticity can be created in a perfect fluid if the external forces are conservative and the pressure is a function of the density only. Therefore assumption (*c*) is justified by the two previous assumptions, (*a*) and (*b*).

Introducing cylindrical coordinates x, r, and denoting the velocity components in the x and r directions by v_x, v_r, these velocity components can be expressed by the potential $\Phi\ (x, r)$, putting

$$v_x = \frac{\partial\Phi}{\partial x},\ v_r = \frac{\partial\Phi}{\partial r} \qquad [1]$$

in which case the dynamical equations are identically satisfied if we write

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[1] Director, Guggenheim Aeronautical Laboratory, California Institute of Technology. Mem. A.S.M.E. Dr.-Ing. von Kármán received his M.E. at Budapest in 1902 and Ph.D. at Göttingen in 1908; honorary degree of Doctor of Engineering, University of Berlin, 1929. He was Privat Docent, Göttingen, 1910–1913; Professor of Mechanics and Aerodynamics, Director of the Aerodynamical Institute, University of Aachen, 1913; Member of Gesellschaft der Wissenschaften zu Göttingen, 1925; foreign member of the Royal Academy of Sciences, Turin, 1928; Director of the Graduate School of Aeronautics, California Institute of Technology, 1928.

[2] Assistant in Aeronautics and Teaching Fellow in Mathematics, California Institute of Technology. Mr. Moore is 22 years of age. He took his A.B. in 1929 at the University of California and his M.S. in 1931 at Purdue University, where he was an Assistant in Physics.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$$\frac{1}{2}(v_x^2 + v_r^2) - \frac{1}{2}V^2 = \int_p^{p_0} \frac{dp}{\rho} \ldots\ldots\ldots\ldots [2]$$

and assume that $v_x = V$ and $v_r = 0$ for $x = -\infty$.

The only differential equation to be satisfied is the "continuity equation," written in the vectorial form as

$$\mathrm{div}\,(\rho v) = 0 \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

or in cylindrical coordinates, assuming axial symmetry, as

$$\frac{\partial}{\partial r}(r\rho v_r) + \frac{\partial}{\partial x}(r\rho v_x) = 0 \ldots\ldots\ldots\ldots [3a]$$

Introducing the potential Φ in Equation [3a],

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{\partial^2\Phi}{\partial x^2} + \frac{1}{\rho}\left(\frac{\partial\Phi}{\partial r}\frac{\partial\rho}{\partial r} + \frac{\partial\Phi}{\partial x}\frac{\partial\rho}{\partial x}\right) = 0 \ldots [4]$$

We now calculate $\frac{\partial\rho}{\partial r}$ and $\frac{\partial\rho}{\partial x}$, using Equation [2]. First we write $\int \frac{dp}{\rho} = \int \frac{dp}{d\rho}\frac{d\rho}{\rho}$, where $\frac{dp}{d\rho}$ is a given function of the density. The physical significance of $dp/d\rho$ is well known: it is equal to the square of the "local" velocity of sound c, corresponding to the given density ρ. Differentiating [2],

$$\left.\begin{aligned} \frac{c^2}{\rho}\frac{\partial\rho}{\partial r} &= -\left(v_r\frac{\partial v_r}{\partial_r} + v_x\frac{\partial v_x}{\partial_r}\right) \\ \frac{c^2}{\rho}\frac{\partial\rho}{\partial x} &= -\left(v_x\frac{\partial v_x}{\partial x} + v_r\frac{\partial v_r}{\partial x}\right) \end{aligned}\right\} \ldots\ldots\ldots\ldots [5]$$

Introducing these values into Equation [4],

$$\left(1 - \frac{v_r^2}{c^2}\right)\frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} - \frac{2v_rv_x}{c^2}\frac{\partial^2\Phi}{\partial r\partial x} + \left(1 - \frac{v_x^2}{c^2}\right)\frac{\partial^2\Phi}{\partial x^2} = 0 \ldots\ldots [6]$$

Assuming $p = \frac{p_0}{\rho_0^k}\rho^k$, then $\frac{dp}{d\rho} = k\frac{p_0}{\rho_0^k}\rho^{k-1} = k\frac{p_0}{\rho_0}\left(\frac{\rho}{\rho_0}\right)^{k-1}$

On the other hand, $\int_p^{p_0}\frac{dp}{\rho} = k\frac{p_0}{\rho_0^k}\int_\rho^{\rho_0}\rho^{k-2}\,d\rho = \frac{k}{k-1}\frac{p_0}{\rho_0^k}(\rho_0^{k-1} - \rho^{k-1})$.

Introducing this expression in [2],

$$\frac{1}{2}(v_x^2 + v_r^2 - V^2) = \frac{k}{k-1}\frac{p_0}{\rho_0^k}(\rho_0^{k-1} - \rho^{k-1}) \ldots\ldots [7]$$

or

$$\frac{1}{2}(v_x^2 + v_r^2 - V^2) = \frac{1}{k-1}\left(k\frac{p_0}{\rho_0} - c^2\right) \ldots\ldots\ldots [8]$$

and finally

$$c^2 = k\frac{p_0}{\rho_0} + \frac{k-1}{2}(V^2 - v_x^2 - v_r^2) \ldots\ldots\ldots [8a]$$

If this expression for c be introduced into the differential Equation [6], the latter will contain only the one unknown variable Φ and its derivatives. Hence the problem is reduced to one of finding solutions of [6] which will satisfy certain boundary conditions at the surface of the body.

2—Assumption of Small Perturbations

In order to obtain a first approximation for the solution of Equation [6] it is assumed that v_x differs slightly from V and that v_r is small compared with V. This will be approximately the case if the body is a slender one, i.e., if the inclination of the tangents to the meridian curve with respect to the x-axis is small at every point. According to this assumption we put

$$\left.\begin{aligned} v_x &= V + u = V + \frac{\partial\varphi}{\partial x} \\ v_r &= v = \frac{\partial\varphi}{\partial r} \end{aligned}\right\} \ldots\ldots\ldots\ldots [9]$$

and consider u and v small compared with V. Introducing the expressions [9] into Equation [6] and retaining only the first-order terms in φ, u, v, and their derivatives,

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \left(1 - \frac{V^2}{c^2}\right)\frac{\partial^2\varphi}{\partial x^2} = 0 \ldots\ldots\ldots [10]$$

As for the velocity of sound, it will be seen that according to [8a], c can be considered as a constant with the same degree of approximation, since c is approximately equal to $k(p_0/\rho_0)$.

Equation [10] is a linear equation much employed in the theory of sound waves. In fact, if the time t be introduced, putting $t = x/V$,

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} = \left(\frac{1}{c^2} - \frac{1}{V^2}\right)\frac{\partial^2\varphi}{\partial t^2} \ldots\ldots\ldots\ldots [11]$$

or

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} = \frac{1}{c'^2}\frac{\partial^2\varphi}{\partial t^2} \ldots\ldots\ldots\ldots\ldots [12]$$

where $\frac{1}{c'^2} = \frac{1}{c^2} - \frac{1}{V^2}$.

In case $V > c$, Equation [12] is identical with the equation for a two-dimensional wave motion, with a "source" at the origin $r = 0$; hence for velocities higher than the velocity of sound the steady flow around a body of revolution can be interpreted as a two-dimensional circular wave motion, with a variable source at the origin. Using this analogy it is necessary to assume that the intensity of the source is a function of the time t, depending on the shape of the body. The problem of computing the stationary motion around a body of given shape can be reduced to that of computing the "source distribution" as a function of the time or of the coordinate x.

3—The Solution of the Approximate Differential Equation

The general solution of Equation [10] for the case $V > c$ is given by the following definite integral:[3]

$$\varphi(x, r) = \int_\infty^0 f(x - \alpha r\cosh z)\,dz \ldots\ldots\ldots [13]$$

where $f(\xi)$ is an arbitrary function of $\xi = x - \alpha r \cosh z$ and $\alpha = \sqrt{[(V/c)^2 - 1]}$. It can be shown that [13] is a solution of Equation [10] by differentiation; however, it is preferable to show this by interpretation of the solution. First it is noticed that the value of φ at a given point x, r, depends only on the values of $f(\xi)$ between $\xi = -\infty$ and $\xi = x - \alpha r$. If ξ be measured along the x-axis, the point $\xi = x - \alpha r$ is found by drawing a straight line through the point x, r, with the inclination $\beta = \tan^{-1}(1/\alpha)$, the intersection of this straight line with the x-axis giving the point $x - \alpha r$. β is called Mach's angle, and the cone with the point $\xi = x - \alpha r$ as origin and with the vertex angle 2β, Mach's cone. Obviously, the flow outside of Mach's cone depends only on the values of $f(\xi)$ corresponding to $\xi < x - \alpha r$; in other words, the influence of the values of $f(\xi)$ for $\xi > x - \alpha r$ is restricted to the interior of Mach's cone.

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[3] Cf. H. Lamb, "Hydrodynamics," Cambridge, 1930, p. 278 and p. 498.

Assume now that $f(\xi) = 0$ for $\xi \leqq 0$ and that $f'(0)$ is finite. In this case the integral occurring in Equation [13] can be written as

$$\varphi(x, r) = \int_{\cosh^{-1}\frac{x}{\alpha r}}^{0} f(x - \alpha r \cosh z)\, dz \ldots\ldots [13a]$$

or introducing $\xi = x - \alpha r$ as the variable and substituting $z = \cosh^{-1}\dfrac{x - \xi}{\alpha r}$,

$$dz = \frac{-d\xi}{\sqrt{\left(\frac{x - \xi}{\alpha r}\right)^2 - 1}}\,\frac{1}{\alpha r} = -\frac{d\xi}{\sqrt{(x - \xi)^2 - \alpha^2 r^2}}$$

we obtain

$$\varphi(x, r) = -\int_{0}^{x - \alpha r} f(\xi)\,\frac{d\xi}{\sqrt{(x - \xi)^2 - \alpha^2 r^2}} \ldots [14]$$

The solution [14] corresponds formally to the solution of the potential equation, assuming sources distributed along the x-axis. In fact, putting $\alpha r \sqrt{-1} = q$; then Equation [10] can be written formally as

$$\frac{\partial^2 \varphi}{\partial q^2} + \frac{1}{q}\frac{\partial \varphi}{\partial q} + \frac{\partial^2 \varphi}{\partial x^2} = 0 \ldots\ldots\ldots\ldots [15]$$

which is exactly the potential equation. Furthermore, assume that the sources of the intensity $f(\xi)/4\pi$ are distributed along the x-axis between the points $\xi = 0$ and $\xi = x - \alpha r$. The solution of Equation [15] corresponding to this source distribution is given by

$$\varphi(x, q) = \frac{1}{4\pi}\int_{0}^{x - \alpha r} f(\xi)\,\frac{d\xi}{\sqrt{(x - \xi)^2 + q^2}} \ldots\ldots [16]$$

It follows that the corresponding expression [14] satisfies the original Equation [10].

Corresponding to this analogy $f(\xi)$ is also called the intensity of the source distribution in the present case and the calculation of an example is then undertaken, employing the simplest assumption for $f(\xi)$.

4—THE FLOW ALONG THE SURFACE OF A CONE

Assume that $f'(\xi) = a$, where a is a constant which has the dimension of a velocity. In this case, the integration involved in Equation [13a] is easily carried out, giving

$$\varphi = a\int_{\cosh^{-1}\frac{x}{\alpha r}}^{0} (x - \alpha r \cosh z)\,dz \ldots\ldots\ldots\ldots [17]$$

or

$$\varphi = -ax \cosh^{-1}\frac{x}{\alpha r} + a\sqrt{x^2 - \alpha^2 r^2} \ldots\ldots [17a]$$

The velocity components are

$$\left.\begin{aligned} u &= \frac{\partial \varphi}{\partial x} = -a \cosh^{-1}\frac{x}{\alpha r} \\ v &= \frac{\partial \varphi}{\partial r} = a\alpha\sqrt{\left(\frac{x}{\alpha r}\right)^2 - 1} \end{aligned}\right\}\ldots\ldots\ldots\ldots [18]$$

It is seen that the velocity components are functions of x/r only, so that the velocity is constant along straight lines passing through the origin. Obviously, Equation [18] can be used to describe the flow along the surface of a cone with small vertex angle.

Superposing the velocity components given by [18] on the parallel flow with the velocity V, we obtain

$$\left.\begin{aligned} v_x &= V - a \cosh^{-1}\frac{x}{\alpha r} \\ v_r &= a\alpha\sqrt{\left(\frac{x}{\alpha r}\right)^2 - 1} \end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots [19]$$

In order to obtain the flow along the surface of the cone with the vertex angle 2γ, we write $x/r = \cot\gamma$ and put

$$\tan\gamma = \frac{v_r}{v_x} = \frac{a\alpha\sqrt{\left(\frac{x}{\alpha r}\right)^2 - 1}}{V - a\cosh^{-1}\frac{x}{\alpha r}} = \frac{a\sqrt{\cot^2\gamma - \alpha^2}}{V - a\cosh^{-1}\frac{\cot\gamma}{\alpha}} \ldots [20]$$

or with $\alpha = \cot\beta$ (β = Mach's angle), assuming $\gamma < \beta$,

$$a = \frac{V\tan\gamma}{\sqrt{\cot^2\gamma - \cot^2\beta + \tan\gamma\cosh^{-1}\frac{\cot\gamma}{\cot\beta}}} \ldots\ldots [21]$$

It is interesting to calculate the pressure acting on the surface of the cone. According to Equation [2],

$$\int_{p_0}^{p}\frac{dp}{\rho} = \frac{1}{2}\left[V^2 - (V + u)^2 - v^2\right] = -Vu - \frac{u^2 + v^2}{2} \ldots [22]$$

On the other hand, according to our assumption of an adiabatic process

$$\int_{p_0}^{p}\frac{dp}{\rho} = \frac{k}{k - 1}\,\frac{p_0}{\rho_0}\left[\left(\frac{p}{p_0}\right)^{\frac{k - 1}{k}} - 1\right] \ldots\ldots\ldots [23]$$

Introducing this expression in [22],

$$\frac{p}{p_0} = \left[1 + \frac{k - 1}{k}\,\frac{\rho_0}{p_0}\left(-Vu - \frac{u^2 + v^2}{2}\right)\right]^{\frac{k}{k - 1}} \ldots [24]$$

Developing the right-hand member of the equation,

$$p - p_0 = \rho_0\left(-Vu + \frac{u^2 + v^2}{2}\right)\left[1 + \frac{\rho_0}{2kp_0}\left(-Vu - \frac{u^2 + v^2}{2}\right)\right] \ldots\ldots\ldots [25]$$

and putting $c^2 = kp_0/\rho_0$

$$p - p_0 = \rho_0\left(-Vu - \frac{u^2 + v^2}{2}\right)\left(1 - \frac{Vu + \frac{u^2 + v^2}{2}}{2c^2}\right) \ldots [26]$$

or

$$\frac{p - p_0}{\frac{\rho_0 V^2}{2}} = -\left(\frac{2u}{V} + \frac{u^2 + v^2}{V^2}\right)\left(1 - \frac{2Vu + u^2 + v^2}{4c^2}\right). [27]$$

Fig. 1 represents $(p - p_0)/(\rho_0/2)V^2$ as a function of the ratio V/c for different values of the vertex angle. It is to be remarked that the approximation does not hold for small values of α

(V/c near to 1) and for values of α near to or greater than cot γ. An exact solution for the flow along a cone can be obtained by integrating the ordinary differential equation which results from the partial differential Equation [6], if it be assumed that all the velocity components are constant along straight lines through the origin of the cone. This differential equation has been discussed by Busemann[4] and more recently by G. I. Taylor,[5] but no numerical solutions are available at the present time which can be compared with the approximate solution given in this section. It would be of great value to compare the two solutions, and especially to investigate whether the exact solution involves a discontinuous change in the velocity and pressure (shock wave) even for small angles γ, as Busemann assumes, or whether this phenomenon appears only in the case of cones with larger vertex angles.

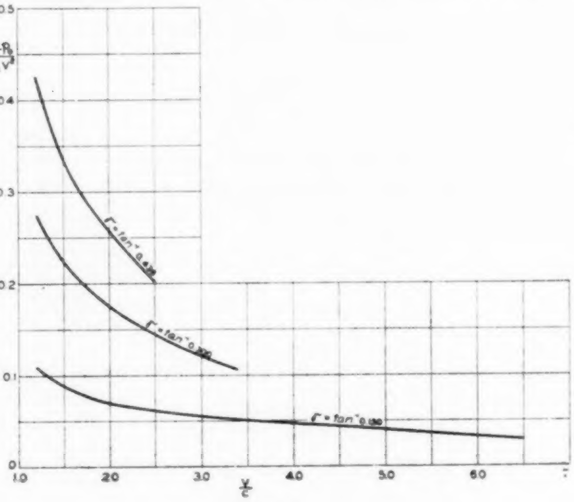

FIG. 1

It is interesting to compare our three-dimensional case with the two-dimensional flow along the two sides of an edge. Assuming small perturbations in the latter case, we have to integrate the equation

$$\frac{\partial^2\varphi}{\partial y^2} = \alpha^2 \frac{\partial^2\varphi}{\partial x^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots [28]$$

The solution for the edge is very simple. We obtain $\varphi = 0$ for $x < \alpha y$ and $\varphi = -a(x - \alpha y)$ for $x > \alpha y$, where a is a constant as above. Hence the additional velocities are

$$u = \frac{\partial\varphi}{\partial x} = -a, \quad v = \frac{\partial\varphi}{\partial y} = a\alpha$$

and the boundary condition gives

$$\tan\gamma = \frac{a\alpha}{V - a}$$

or

$$a = V\frac{\tan\gamma}{\alpha + \tan\gamma} \quad\ldots\ldots\ldots\ldots\ldots\ldots [29]$$

It will be noticed that a discontinuity in the velocities along the line $x = \alpha y$ is always obtained, even for small values of γ.

In the three-dimensional case the discontinuity as far as the approximate equation is concerned is restricted to the point of origin.

Let us consider the velocity distribution along a cylindrical surface r = const. and put $x = \alpha r + x'$. Then, from Equation [18], it is easily seen that

$$\left.\begin{aligned} u &= -a\cosh^{-1}\left(1 + \frac{x'}{\alpha r}\right) = -a\sqrt{\frac{2x'}{\alpha r}}\,(1 + \ldots) \\ v &= a\alpha\sqrt{\left(1 + \frac{x'}{\alpha r}\right)^2 - 1} = a\sqrt{\frac{2\alpha x'}{r} + \frac{x'^2}{r^2}} \end{aligned}\right\} \quad\ldots\ldots [30]$$

It is seen that u and v do not show any discontinuity at the conical surface $x = \alpha r$; however, the derivatives of u and v are infinite at points of this surface. Fig. 2 shows the pressure distribution along the cylindrical surface.

5—THE FLOW ALONG A "SEMI-FINITE BODY" (PROJECTILE)

Proceeding now to the computation of the source distribution for a body of revolution with a sharp point at the origin and cylindrical shape at infinity, we assume that the radius r is a monotonic function of x.

Referring to Fig. 3, the source-distribution curve $f(\xi)$ is approximated by a curve composed of straight segments, so that $f'(\xi)$ is replaced by the "step-shape" function shown.

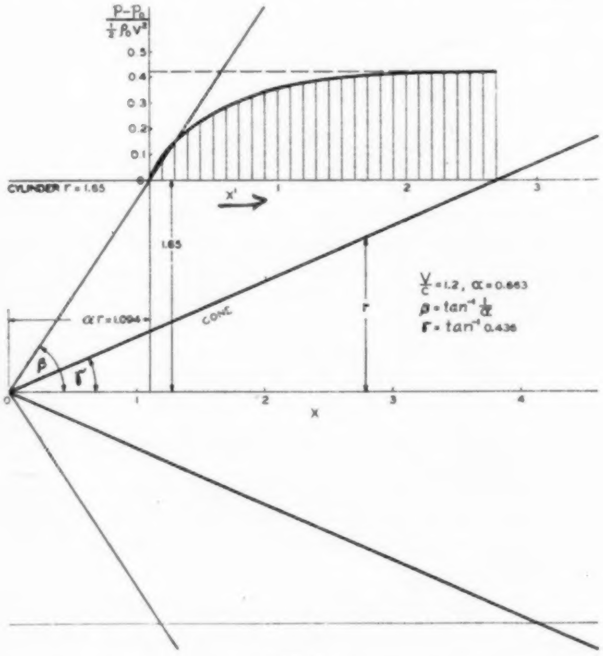

FIG. 2

The expressions for the additional velocities are

$$u = \int_{\cosh^{-1}\frac{x}{\alpha r}}^{0} f'(x - \alpha r\cosh z)\,dz,$$

$$v = -\int_{\cosh^{-1}\frac{x}{\alpha r}}^{0} f'(x - \alpha r\cosh z)\,\alpha\cosh z\,dz \ldots [31]$$

Consider now the points $P_1, P_2, \ldots\ldots P_n$ of the meridian line of the body in question; their coordinates may be designated by $x_1, r_1;\ x_2, r_2, \ldots x_n, r_n$ and the corresponding values of ξ by $\xi_1, \xi_2, \ldots \xi_n$. Denote the constant value of the derivative $f'(\xi)$ between ξ_{i-1} and ξ_i by A_i. Obviously then we can carry out the integrations in [31] and obtain for the velocity components at the point P_n

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[4] Busemann, *Zeit. für angew. Math. u. Mech.*, 1 Bund 9, Heft 6, p. 496 (1929).

[5] Private communication from Mr. Taylor.

$$\left.\begin{aligned} u(x_n, r_n) &= -\sum_{i=1}^{n} A_i \left(\cosh^{-1} \frac{x_n - \xi_i}{\alpha r_n} - \cosh^{-1} \frac{x_n - \xi_{i-1}}{\alpha r_n} \right) \\ v(x_n, r_n) &= \sum_{i=1}^{n} \frac{A_i}{r_n} \left(\sqrt{(x_n - \xi_i)^2 - \alpha^2 r_n^2} \right. \\ &\qquad \left. - \sqrt{(x_n - \xi_{i-1})^2 - \alpha^2 r_n^2} \right) \end{aligned} \right\} \quad \ldots\ldots\ldots [32]$$

The boundary condition for the meridian line can be written

$$\frac{dr}{dx} = \frac{v_r}{v_x} = \frac{v}{V + u} \quad \ldots\ldots\ldots\ldots\ldots\ldots [33]$$

Introducing the expressions [32] in [33] and applying the equation consecutively to the points $P_1, P_2, \ldots$ we obtain linear equations for the constants $A_1, A_2, \ldots$. These equations can be solved consecutively because every one of them introduces a new variable which does not occur in the previous equations.

The source distribution has been calculated for a semi-infinite body with a so-called "ogival" head, i.e., composed of a cylinder and of a head generated by the revolution of a circular arc. The radius of the circular arc is called the "ogival radius," and is usually expressed in "calibers," i.e., as a multiple of the radius of the cylindrical part. In the present example the body has a "six-caliber ogival head." This ratio is chosen because pressure-distribution measurements for a shell of such a ratio are available.

Fig. 3 shows the source distribution for the ratio $V/c = 1.2$.

It will be noticed that the boundary condition [33] could be replaced by $dr/dx = v/V$, because u is considered small in comparison with V. In this case the pressures obtained for the head of the projectile will be somewhat higher, so that our assumption gives a lower limit for the resistance. A closer approximation requires a more exact solution of the non-linear differential equation.

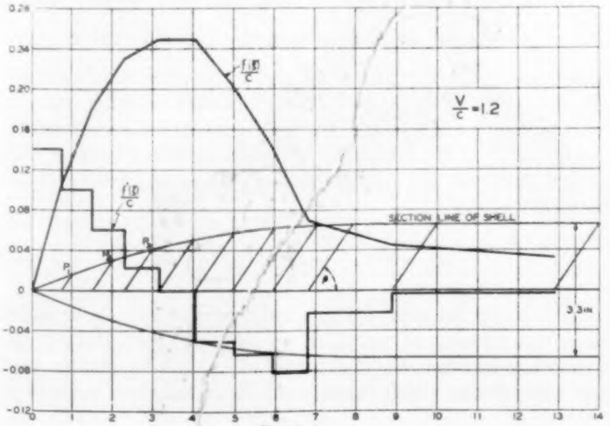

Fig. 3

6—The Expression for the Wave Resistance

If a body moves in a perfect fluid with a velocity less than that of sound, there is no drag so long as the flow follows the surface of the body (i.e., no "wake" is produced). This theorem holds even for a body extended to infinity. The pressure distribution gives no resultant force. Considering the pressure distributions represented in Fig. 4, it is seen that a finite drag results from an integration of the pressures over the surface. We can also deduce this resistance directly by applying the momentum theorem to the system.

A flow of a perfect fluid with velocities below the velocity of sound does not give rise to any resistance for an immersed body, because the velocities induced by the body vanish at infinite distances as $1/R^2$ (R being the distance from the center of the body). This can be shown by applying the momentum law to the portion of the fluid included in a cylinder with the radius R. It is obvious that no finite amount of momentum can be transported through the surface of this cylinder, because the surface increases proportionally with the radius R and the highest term in the expression of transported momentum per unit area is of the order $1/R^2$.

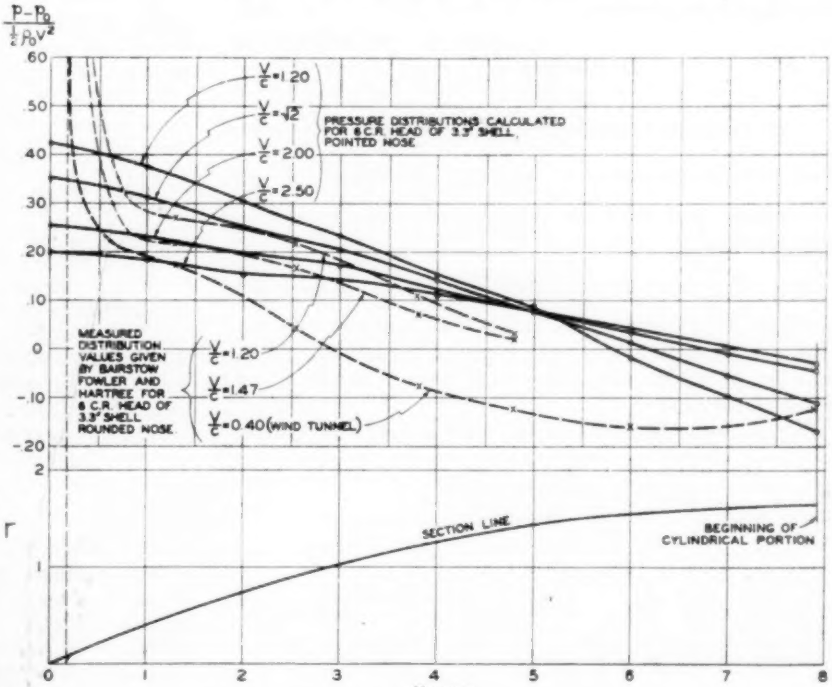

Fig. 4

Considering now the case of flow with velocities higher than the velocity of sound, we find that proceeding to infinity along Mach's lines, the additional velocities vanish as $1/\sqrt{(R)}$, so that the integral which represents the total momentum transported through a cylindrical surface approaches a finite limit different from zero, if the radius of the cylinder increases indefinitely. The drag corresponding to this momentum is called the "wave resistance" of the body in question.

The momentum integral is given by

$$M = \int_{-\infty}^{\infty} 2\pi r \rho v_r v_x dx \ldots\ldots\ldots\ldots\ldots [34]$$

or, introducing $v_x = V + u$, $v_r = v$,

$$M = V\int_{-\infty}^{\infty} 2\pi r\rho v dx + \int_{-\infty}^{\infty} 2\pi r\rho uv dx \quad \ldots\ldots [35]$$

The first term can be written as $V\cdot m$, where m is the mass of fluid passing through the surface of a cylinder with the radius r.

Let us assume that the cylinder includes the body. Obviously the drag on the body is given by the difference between the momentum M and the momentum ρAV^2, where A is the cross-section of the body at infinity. The second term corresponds to the momentum passing through A in unit time and is equal to the term $V\cdot m$ occurring in Equation [35]. Therefore the drag is given by the expression

$$D = -2\pi r\int_{-\infty}^{\infty} \rho uv dx \quad \ldots\ldots\ldots\ldots [36]$$

or, with sufficient accuracy,

$$D = -2\pi r\rho_0\int_{-\infty}^{\infty} uv dx$$

If the body has a finite length (as, for instance, a streamlined body with a sharp point at the rear), the integral $\int \rho v dx$ vanishes, so that we obtain the same expression for the drag as before.

It is easily shown that the expression $2\pi r\int_{-\infty}^{\infty} \rho uv dx$ is independent of r, so that we can consider a cylinder of very small radius close to the x-axis, which carries the sources. In this case we can use the first terms in the development for u and v for small values of r.

We can write the velocity components in the following form:

$$u = -\int_0^{x-\alpha r} f'(\xi)\frac{d\xi}{\sqrt{(x-\xi)^2-\alpha^2 r^2}},$$

$$v = \int_0^{x-\alpha r} f'(\xi)\frac{d\xi\,\frac{x-\xi}{r}}{\sqrt{(x-\xi)^2-\alpha^2 r^2}} \quad \ldots\ldots [37]$$

Proceeding to the limit $r \rightarrow 0$,

$$\left.\begin{aligned} u &\cong -f'(x)\log\frac{x}{\alpha r} + \int_0^x \frac{f'(\xi)-f'(x)}{\xi - x}d\xi \\ v &\cong \int_0^x \frac{f'(\xi)d\xi}{r} = \frac{f(x)}{r} \end{aligned}\right\} \quad \ldots\ldots [38]$$

Hence the drag is equal to

$$D = 2\pi\rho_0\int_0^{\infty} f(x)f'(x)\log\frac{x}{\alpha r}dx - 2\pi\rho_0\int_0^{\infty} f(x)\int_0^x\frac{f'(\xi)-f'(x)}{\xi - x}d\xi dx \quad \ldots\ldots [39]$$

Taking into consideration the fact that $f(x) = 0$ for $x = -\infty$ and $x = \infty$, it is seen immediately that the integral $\int_0^{\infty} f(x)f'(x)dx = 0$, so that we can cancel the term $-\int_0^{\infty} f(x)f'(x)\log(\alpha r)dx$ and obtain

$$D = 2\pi\rho\int_0^{\infty} f(x)f'(x)\log x dx + 2\pi\int_0^{\infty} f(x)\int_0^x \frac{f'(\xi)-f(x)}{\xi - x}d\xi dx \quad \ldots\ldots [40]$$

or by partial integration,

$$D = -2\pi\int_0^{\infty}\frac{[f(x)]^2}{x}dx + 2\pi\int_0^{\infty} f(x)\int_0^x\frac{f'(\xi)-f(x)}{\xi - x}d\xi dx$$

The integral is to be extended over the x-axis, i.e., along the range, in which $f(x)$ and $f'(x)$ differ from zero.

7—THE RESISTANCE OF PROJECTILES

The three components of the resistance of projectiles are:

(*a*) *The head resistance*, i.e., the resultant of the pressure acting on the head of the projectile

(*b*) *The skin friction*, i.e., the resultant of the tangential forces acting along the surface of the body, and

(*c*) *The wake resistance*, which is the contribution of the underpressure (vacuum) at the base.

Let us first try to estimate the contributions (*b*) and (*c*). The skin friction is in the first approximation proportional to the density of the air and to the square of the velocity. Put $D_f = C_f\frac{\rho V^2}{2}S$, where S is the surface area of the projectile. The coefficient C_f is in fact a function of the ratio Vd/ν, called the Reynolds number (d = diameter of the body, ν = kinematic viscosity), and probably of the ratio V/c. For such high values of the Reynolds number as those which correspond to the velocities and sizes of projectiles, the value of C_f can be estimated to be between 0.002 and 0.003. The influence of the ratio V/c is not known, but probably it is not very large. The drag is usually referred to the frontal area A of the projectile. For long projectiles we can assume S/A = 16 to 18, whence, approximately,

$$D_f = (0.03 - 0.05)\frac{\rho V^2}{2}A \quad \ldots\ldots\ldots\ldots [41]$$

a mean value of C_{Df} = 0.04 will be used.

It is somewhat difficult to obtain a fair estimate for the contribution of the underpressure at the base. Fortunately there are some wind-tunnel measurements at low velocities on the resistance of shell models (point A, Fig. 5), and also on the pressure distribution at the head. For a comparatively long shell we can assume that the drag coefficient at low speeds varies between the values 0.2 and 0.24.[6] The resultant of the head pressures has a small negative value, about $-(0.01 \sim 0.015)\frac{\rho V^2}{2}A$. Obviously we shall obtain the probable value of the underpressure at the base if we add to the drag coefficient the absolute value of the negative mean pressure resulting from the head pressures and subtract the coefficient which corresponds to the resultant of the frictions. We shall assume, therefore, that at low velocities the contribution of the underpressure at the base amounts to

$$D_B = C_{DB}\frac{\rho V^2}{2}A \cong 0.2\frac{\rho V^2}{2}A \quad \ldots\ldots\ldots\ldots [42]$$

Considering the case of high velocities, especially those higher than the velocity of sound, it is probable that $D_B/(\rho V^2/A)$ diminishes because the underpressure cannot be greater than one atmosphere, which corresponds to an absolute vacuum at the base Denoting the absolute value of the pressure at the base by p, for low velocities we write

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[6] Cf. Fowler, Gallop, Lock, and Richmond, *Phil. Trans.*, A591, 1920.

$$p_0 - p \cong \lambda \frac{\rho V^2}{2} \qquad [43]$$

where $\lambda = 0.2$. For very high velocities $p_0 - p$ will be constant and equal to one atmosphere. To establish a transition between these limiting cases it appears plausible to write

$$\int_p^{p_0} \frac{dp}{\rho} = \lambda \frac{\rho V^2}{2} \qquad [44]$$

using the same value of λ as at low velocities. For adiabatic dilatation, we obtain

$$\int_p^{p_0} \frac{dp}{\rho} = \frac{k}{k-1} \frac{p_0}{\rho_0}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}\right] \qquad [45]$$

Or, putting $k(p_0/\rho_0) = c^2$,

$$\frac{p}{p_0} = \left[1 - \frac{(k-1)\lambda}{2} \frac{V^2}{c^2}\right]^{\frac{k-1}{k}} \qquad [46]$$

For $V^2 = \frac{2}{(k-1)\lambda} c^2$, $p = 0$; so that the absolute vacuum would be reached at a velocity which is more than four times as great as the velocity of sound.[7] For $V/c < 2.5$ we can use the approximative formula

$$p_0 - p = \lambda \frac{\rho V^2}{2}\left(1 - \frac{V^2}{4c^2}\lambda\right) \qquad [47]$$

Using this formula, the contribution of the underpressure at the base to the total drag will be:

$$D_B = C_{DB} \frac{\rho V^2}{2} A$$
$$= 0.2\left(1 - 0.2 \frac{V^2}{4c^2}\right) \frac{\rho V^2}{2} A \qquad [48]$$

In order to compute the contribution of the head pressures at velocities higher than the velocity of sound, we have to compute the pressure distribution. We assume that for this purpose we can use the pressures corresponding to the flow along the semi-infinite body, which has been considered in section 5. The calculated pressure distribution, using Equation [27], is shown in Fig. 4. With the object of measuring the dynamic pressures acting on the head of an actual shell, very interesting experiments have been carried out in England by Bairstow, Fowler, and Hartree.[8] The method used by these investigators is rather ingenious. They fire shells of the same type under identical conditions, i.e., with same initial velocity and the same elevation, but with holes at different points of the head. Because the time necessary to burn off a fuse with a certain setting depends on the pressure, the shells with different holes will burst at different points of the same trajectory. The relation between the rate of burning and the pressure has been established by laboratory experiments made on shells exposed to different pressures and spinning around a fixed axis, and, assuming this relation is not altered by the translatory movement, the pressure at the different holes can be calculated from observations on the time of burning, or on the distance at which the shells burst. The experimental curve for the pressures obtained in this way (for $V/c = 1.2$ and 1.41) is also shown in Fig. 4. It is to be noticed that the character of the curve is similar, but the experimental values are smaller than the calculated values. However, it seems that the measurements are not extremely reliable; the Messrs. Bairstow, Fowler, and Hartree mention two sources of error which may have considerable influence. First, the zero point of the time of burning was guessed at, and second, the holes on the shells were comparatively large. It is believed that the effective pressures must be superior to the measured values; it will be shown that even the computed higher pressures lead to too small values for the total drag. In addition, the pressure measurements in flight, which have been extended to velocities as low as $0.7c$, can be compared with the pressure distribution obtained by wind-tunnel measurements at a velocity $V = 0.4c$, and given in the same paper. Theoretical and experimental investigations show that the negative values of $(p - p_0)/(\rho V^2/2)$ obtained at $V = 0.7c$ ought to be smaller than the respective values at $V = 0.4c$; however, the measurements in flight give considerably lower values for the higher velocities. It seems, the zero point of the pressure measurements is not exact enough. Of course, the experimental task faced by the British investigators was a very difficult one, but the credit of having done a beautiful piece of experimental research must nevertheless be given, even though the results show a considerable degree of uncertainty.

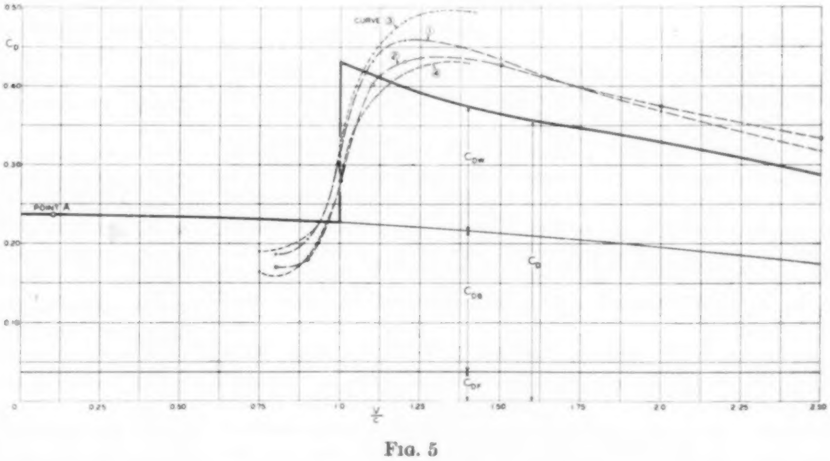

Fig. 5

The pressure of the nose has not been determined by measurement. The British investigators mentioned used Rayleigh's formula for adiabatic compression, assuming a "stagnation point" at the nose. It is believed that this is correct for a shell with a rounded nose; in the case of a shell with sharp point, however, the pressure at the nose is much lower and depends on the vertex angle of the cone.

Fig. 5 shows the total drag as function of the ratio V/c computed as the sum of the friction, the underpressure at the base, and the head resistance resulting from the pressures calculated according to the theory developed in this paper. Unfortunately, no exact data are available for the drag curve of a shell with the shape used in the calculation and in the British experiments. For an approximate comparison, the drag curve for the so-called S-projectile used in the German army is shown in the diagram. This projectile has an "ogival radius" equal to seven calibers; the ogival radius of the shell considered by the authors is equal to six calibers. Curve (1) is deduced from experiments on shells with an ogival radius equal to three calibers.[9] According to an empirical rule given by Heydenreich, a shell with a "six-caliber" ogival radius has about 20 per cent less drag than one with a "three-caliber" ogival radius. Curve (2) shows the values reduced in this way. They are in fairly good accordance with the values obtained for the S-projectile. Finally, Messrs. Bairstow, Fowler, and Hartree give the curves (3) and (4) and state that the total drag of the shell used in their tests lies between the values represented by those two curves.

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[7] See Appendix.

[8] Proc. Roy. Soc., A97, 1920.

[9] See Cranz, "Lehrbuch der Ballistik," Berlin, 1925, p. 60 et seq.

It is to be noticed that the computed values for the total drag represented by the solid line are fairly near to the actual values, and even the variation of the drag with the ratio V/c is given by the theory in a nearly correct way. As was pointed out in section 5, the computed pressure distribution gives a lower limit for the resistance. In addition the drag is increased in actual flight by yaw and conical oscillations of the shell. As far as the transition at the point $V = c$ is concerned, the present approximate theory fails to give any information. It is to be hoped that a further development of the theory, especially in connection with scientifically directed experiments, will lead to a more exact knowledge of the complex phenomena connected with the aerodynamics of supersonic velocities.

Appendix

The magnitude of the depression developed at the base of the projectile has been discussed by several authors. Depreux[10] and Gabeault[11] assume vortex rings in the wake of the projectile, and arrive at the conclusion that for velocities higher than twice the velocity of sound, full vacuum must reign immediately behind the projectile. Furthermore, they conclude that, like the head resistance, the depression at the base is suddenly increased when the velocity of the projectile passes the velocity of sound. They use the assumption that the "circulation" propagates with the velocity of sound; unfortunately they do not give any proof to justify this statement, and it is very difficult to follow their deductions.

It seems to the authors that M. T. Harris[12] gives a more adequate description of the phenomena, in that he discusses the wave fronts shown in photographs obtained by various optical methods. He points out that, corresponding to the depression at the base, the flow immediately behind the base must be convergent.

Using the equations given by L. Prandtl[13] and Th. Meyer[14] and applied recently by P. S. Epstein[15] to a two-dimensional model of a projectile, it is possible to calculate the deflection around the rim of the projectile if the depression is given, or to estimate the depression if the direction of either the deflection or of the wave fronts is known. The authors carried out this calculation for $V = 880$ m per sec, corresponding to the photograph, Fig. 6, which is reproduced from Cranz's "Ballistik."[16] It is easy to show that if full vacuum were to be reached at the base, then the flow behind the base would have to be deflected nearly 90 deg. However, the actual deflection can be estimated from the direction of the waves starting from the rim, and also from the location of the strong compression shock waves which follow the shock between the convergent streams. The authors believe that their estimate of the depression is in fairly good agreement with the conclusions deduced from the photographical records.

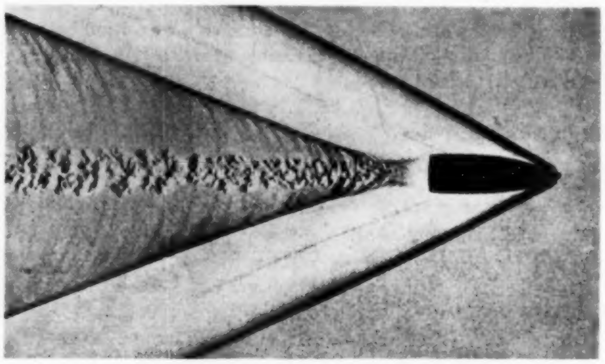

FIG. 6 BULLET MOVING WITH VELOCITY OF ABOUT 880 M PER SEC

Discussion

GEORGE B. PEGRAM.[17] On page 309 and elsewhere in the paper it is stated that behind the projectile an "absolute vacuum" is, according to the mathematical equations, to be reached at a certain velocity. For several obvious reasons the space back of the projectile, even if it is going with several times the velocity of sound, cannot be anything like free of gas molecules. Would it not therefore be better to use a milder term than "absolute vacuum?"

AUTHORS' CLOSURE

The authors agree with Professor Pegram that an absolute vacuum is not possible in one region of a fluid that is in contact with another region of finite pressure. The reason for the discrepancy between the mathematical solution and the reality is due to the fact that deduction of the hydrodynamical equation is based on the assumption that the mean free path of the molecules is small compared with the dimensions of the vessels containing the fluid or of the bodies immersed in it. This is obviously no longer true in the case of the projectile, if the pressure at the base approaches zero. As far as the resistance is concerned, the difference will be quite imperceptible.

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[10] Comptes Rendus, No. 11, p. 439 (1931).

[11] Ibid., No. 25, p. 1630 (1931).

[12] Woolwich Ordnance, Res. Dept. Rep. No. 63 (1925).

[13] *Phys. Zs.*, 5, p. 599 (1904).

[14] *Mitt. des V.D.I.* (Berlin, 1908).

[15] Proc. Nat. Acad. Sci., 17, pp. 532–547 (1931).

[16] Cf. "The Mechanical Properties of Fluids," ch. X (Blackie, 1925).

[17] Professor of Physics, Columbia University, New York, N. Y. Mem. A.S.M.E.

APM-54-28

Strength of Semicircular Plates and Rings Under Uniform External Pressure

By A. M. Wahl,[1] East Pittsburgh, Pa.

The exact solution of the problem of a semicircular ring subjected to uniform pressure involves the solution of a fourth-order partial-differential equation, in conjunction with certain specific boundary conditions. As an exact solution could not be found, the author has developed approximate methods which tests show are sufficiently accurate for practical use in cases where the inner radius of the ring or plate is over 0.4 that of the outer radius. These methods, it is believed, will be found useful by machine designers in the calculation of thrust-bearing plates, split-ring keys, semicircular split diaphragms, and similar mechanical structures.

In connection with analyses of the strength of machine parts of various shapes, the problem of the strength of a semicircular ring or plate with a hole, subjected to uniform pressure on the surface, and simply supported on its outer circumference, was considered. This is the case of a split diaphragm under external pressure. Since problems of this general kind are of considerable interest to machine designers, the methods of solution of this problem will be given in detail, particularly in view of the fact that similar methods may be applied to a wide variety of problems of the same general nature.

The exact solution of the semicircular-plate problem mentioned above involves the solution of a fourth-order partial-differential equation, in conjunction with certain specified boundary conditions. For the case in question, namely, that of the semicircular ring, this exact solution could not be found, and hence resort was had to an approximate method which is discussed below.

In order to show the order of exactitude of the general method, the case of a narrow rectangular plate having one long edge free and the other edges simply supported, was first considered. Two approximate solutions were applied and compared with the known exact solution for this case. A similar approximate method was then applied to the case of a semicircular ring or plate, mentioned before.

It is the author's hope that the methods developed herein may find application in practical machine design to the calculation of similar mechanical structures, such as, for example, thrust-bearing plates, split-ring keys, and semicircular split diaphragms under external pressure. In any case the general method should be applicable to many cases of the same general nature.

I—THE RECTANGULAR PLATE OF CONSTANT THICKNESS SIMPLY SUPPORTED ON THREE EDGES AND THE FOURTH FREE

Exact Solution

This case is represented in Fig. 1, and was chosen because it is, to some extent, similar to the semicircular plate without a hole, and of constant thickness, the plate being simply supported on its circumference and free along its diametral edge. The exact solution for the case shown in Fig. 1 was first obtained by E. Estanave[2] using the general method developed by Levy.[3]

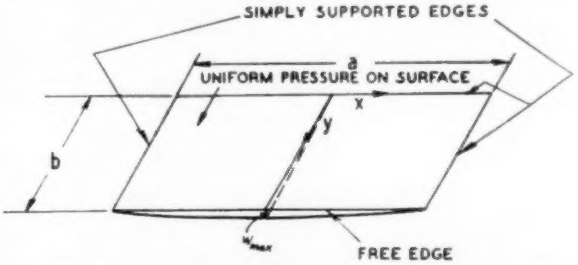

Fig. 1 Rectangular Plate Under Uniform Pressure, Three Edges Simply Supported, the Fourth Free

Essentially this problem[4] consists in obtaining solutions compatible with the given boundary conditions for the biharmonic equation:

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{N} \qquad [1]$$

where w = deflection of the plate at any point having the coordinates x, y (Fig. 1)

p = pressure on surface of plate (assumed uniform in this case)

N = plate rigidity = $Eh^3/12(1 - \nu^2)$

ν = Poisson's ratio

h = thickness of plate (assumed constant), and

E = modulus of elasticity of the plate material.

The function w for this case must satisfy the following boundary conditions (referring to Fig. 1). At $x = \pm a/2$, $w = 0$, and $\partial^2 w/\partial x^2 = 0$, since these edges are supported along straight

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Contributed by the Applied Mechanics Division and presented at the Second National Applied Mechanics Meeting, Yale University, New Haven, Conn., June 23 to 25, 1932, of The American Society of Mechanical Engineers. (This paper comprises portions of a dissertation presented to the University of Pittsburgh in partial fulfilment of the requirements for the Ph.D. degree, June, 1932.)

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] E. Estanave, "Contribution a l'étude de l'équilibre élastique d'une plaque rectangulaire mince," Thesis, Paris, 1900.

[3] *Comptes Rendus*, Oct. 9, 1899, pp. 535–539.

[4] See A. Nádái, "Die Elastische Platten" (Berlin, Julius Springer, 1925), for a more complete discussion of this problem.

edges and no moments act thereon. At $y = 0$, $w = 0$, and $\partial^2 w/\partial y^2 = 0$ likewise. At $y = b$,

$$\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} = 0$$

$$\frac{\partial}{\partial y}\left\{ (2 - \nu)\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right\} = 0$$

The last two equations express the fact that no moments or shearing forces act on the free edge.

It may be shown that the following series will satisfy these conditions:

$$w = \sum_{n=1,3,5}^{\infty} Y_n \cos\frac{n\pi x}{a} \qquad [2]$$

where Y_n is given by Equation [37] in Appendix No. 1.

As a case of particular interest, let us take that where $a = 2b$. The maximum deflection may be shown to occur at $x = 0$, $y = b$, and taking two terms of the series [2] (which is very rapidly convergent), the maximum deflection becomes

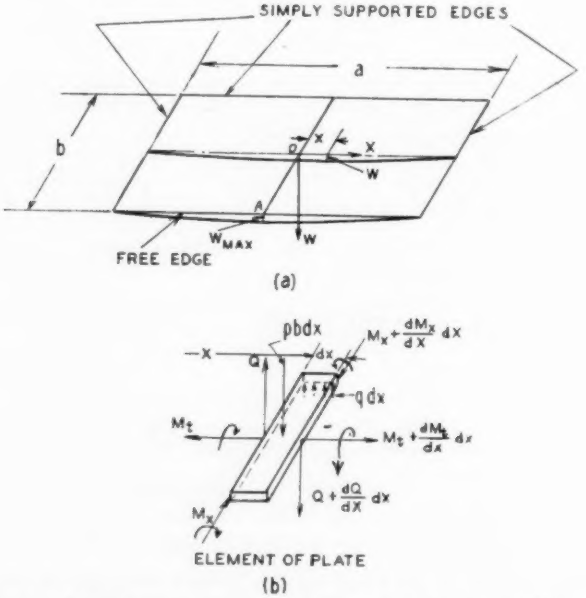

FIG. 2 RECTANGULAR PLATE—APPROXIMATE SOLUTION NO. 1 (BEAM METHOD)

$$w_{\max} = 0.546\,\frac{4pa^4}{\pi^5 N} \qquad [3]$$

(Poisson's ratio being taken as $^1/_4$.)

Using the value $N = Eh^3/12(1 - \nu^2)$, this may be written

$$w_{\max} = 0.0804\,\frac{pa^4}{Eh^3} \qquad [4]$$

APPROXIMATE SOLUTIONS FOR A NARROW RECTANGULAR PLATE

(One long edge free, the other edges simply supported.)

1—Beam Method. This method consists in assuming a narrow rectangular plate with the given conditions of loading and edge support to be a beam, each element of which is acted on by bending moments, twisting moments, loads, and shearing forces, as shown in Fig. 2(*b*). Obviously the solution holds with fair approximation only when b is not over, say, $a/2$, since the assumption that cross-sections of the plate perpendicular to its length deflect without distortion is no longer approximately correct when the width is too great. By applying the conditions of equilibrium of the bending moments, loads, and shearing forces acting on an element of the plate, the following differential equation is obtained:[5]

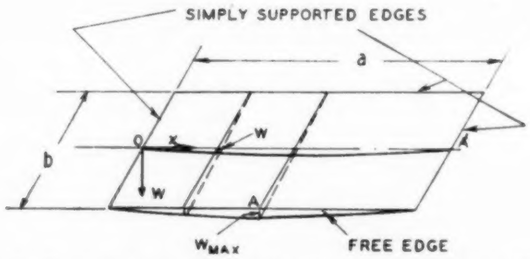

FIG. 3 RECTANGULAR PLATE—APPROXIMATE SOLUTION NO. 2 (ENERGY METHOD)

$$\frac{d^4 w}{dx^4} - \frac{4C}{EIb^2}\frac{d^2 w}{dx^2} = \frac{pb}{EI} \qquad [5]$$

In this w = deflection of center line of the plate at a distance x from the point O, Fig. 2(*a*); C = torsional rigidity of a cross-section perpendicular to the x-axis and may, with sufficient approximation,[6] be taken as

$$C = \frac{bh^3 G}{3}\left(1 - \frac{0.625\,h}{b}\right) \qquad [5a]$$

where h = thickness of plate
b = width of plate perpendicular to free edge
a = length of plate, parallel to free edge
I = moment of inertia of cross-section = $bh^3/12$, and
G = modulus of rigidity of the material.

A solution of [5] which satisfies the boundary conditions $w = 0$ and $d^2w/dx^2 = 0$ for $x = a/2$ and the symmetry conditions at $x = 0$, is:

$$w = \frac{pb^5}{16\,C}\left[\frac{EI}{C}\left(\frac{\cosh\frac{2x}{b}\sqrt{\frac{C}{EI}}}{\cosh\frac{a}{b}\sqrt{\frac{C}{EI}}} - 1\right) + \frac{a^2}{2b^2}\left(1 - \frac{4x^2}{a^2}\right)\right] \qquad [6]$$

The maximum deflection occurs at point A, Fig. 2(*a*), and will be found by putting $x = 0$ in [6]. Thus:

$$w_{\max} = 2(w)_{x=0} = \frac{pb^5}{8C}\left[\frac{EI}{C}\left(\frac{1}{\cosh\frac{a}{b}\sqrt{\frac{C}{EI}}} - 1\right) + \frac{a^2}{2b^2}\right] \qquad [7]$$

Taking $b = 2a$, as before, and $h = b/10$, this becomes[7]

$$(w_{\max})_{a=2b} = 0.0906\,\frac{pa^4}{Eh^3} \qquad [8]$$

Comparing this with the value obtained by the exact solution

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[5] See Appendix No. 2 for derivation of this differential equation.

[6] Timoshenko, S., "Strength of Materials," New York, D. Van Nostrand Co. Todhunter and Pearson, "A History of the Theory of Elasticity," Cambridge University Press.

[7] It should be noted that a considerable change in the ratio h/b would not affect the final result much, since the thickness is assumed small relative to the width.

(Eq. [4]), it is seen that this value is about 12 per cent in error. This error is probably due to the fact that the beam in this case is taken relatively wide, i.e., b not small relative to a, so that the underlying assumptions do not hold with a high degree of accuracy. Nevertheless the degree of approximation appears sufficient for most practical work in the case of plates where the width perpendicular to the free edge is not over half the length.

2—Energy Method. This method[8] consists in assuming the deflection curve of the center line of the plate (OA' in Fig. 3) in the form of a Fourier series and determining the coefficients of this series so as to make the total energy of the system a minimum. (See Appendix No. 2 for a more complete discussion of this method.) Referring to Fig. 3, the deflection w of the center line at a distance x from O is assumed in the form of a series which satisfies the symmetry conditions and those at the ends $x = 0$ or a. Such a series is:

$$w = a_1 \sin\frac{\pi x}{a} + a_3 \sin\frac{3\pi x}{a} + \dots$$

or

$$w = \sum_{n=1,3,5\dots}^{\infty} a_n \sin\frac{n\pi x}{a} \dots\dots\dots\dots [9]$$

Under this assumption, and the further one that cross-sections of the plate perpendicular to the x-axis deflect without distortion, the total energy U stored in the system may be calculated. Then to obtain any coefficient a_n, the equation

$$\frac{\partial U}{\partial a_n} = 0 \dots\dots\dots\dots [10]$$

is used. Thus it is found that

$$a_n = \frac{2pba^2}{\pi^3 n^3 \left[\frac{2C}{b^2} + \frac{EI\pi^2 n^2}{2a^2}\right]} \dots\dots\dots\dots [11]$$

Again, taking $a = 2b$ and $C/ET = 1.48$, and using the first two terms of [9], the maximum deflection at point A, Fig. 3, becomes

$$w_{\max} = 0.0912\,\frac{pa^4}{Eh^3} \dots\dots\dots\dots [12]$$

This solution is quite close to that obtained using the beam method (see Equation [8]), which is reasonable since the underlying assumptions are essentially the same for both methods, and it is about 12 per cent in error compared with results obtained by using the exact method, Equation [4].

II—THE SEMICIRCULAR PLATE OR RING UNDER UNIFORM EXTERNAL PRESSURE

General. The case of a semicircular ring or plate with a hole having the shape shown in Fig. 4(*c*) will now be considered. It is assumed that the inside diameter is not over 0.4 that of the outside diameter, that the thickness is constant, and that a uniform pressure acts on the surface of the plate. (The analysis may, however, be adapted to cases where the thickness is variable in the radial direction and where the pressure is not constant.) The outer circumference of the plate or ring is assumed to be simply supported, and the inner circumference and diametral edges free. Since, as mentioned before, an exact solution involving the integration of the fundamental equation $\nabla^2\nabla^2 w = p/N$ $\left(\text{where } \nabla^2 \text{ is the Laplacian operator } \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ could not be found by exact methods, it was necessary to use an approximate solution, based on the fundamental theory[9] of the bending of an initially curved bar out of its plane of initial curvature.

Approximate Solution for Deflection. Taking w as the deflection of the mean radius def [Fig. 4(*a*)] of the plate, and assuming as before that radial cross-sections deflect without distortion, it is possible to derive the following differential equation for the deflection w (see Appendix No. 3):

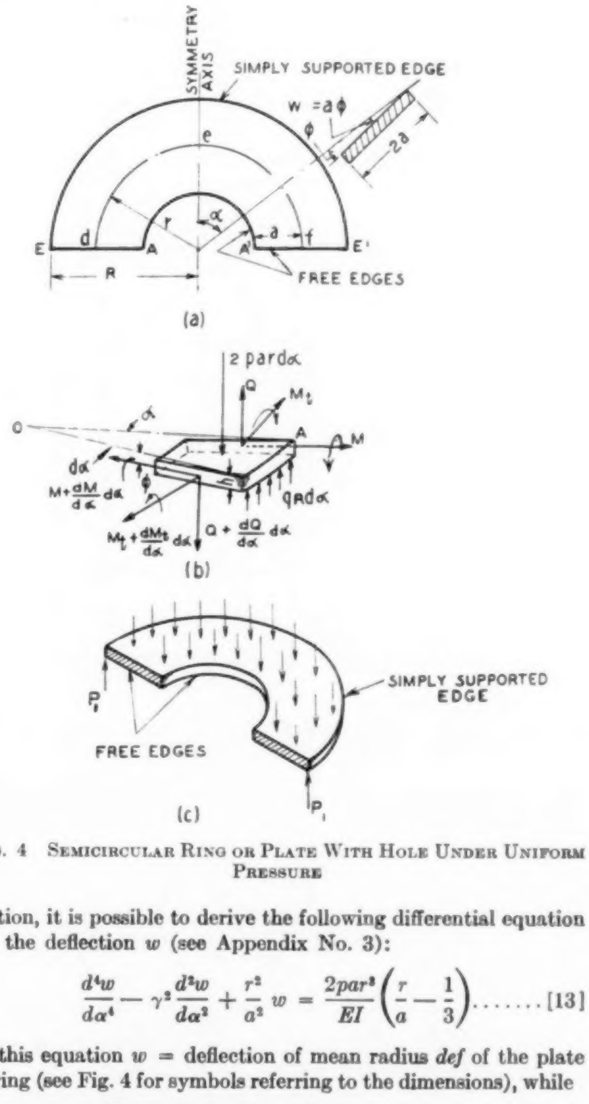

FIG. 4 SEMICIRCULAR RING OR PLATE WITH HOLE UNDER UNIFORM PRESSURE

$$\frac{d^4w}{d\alpha^4} - \gamma^2\frac{d^2w}{d\alpha^2} + \frac{r^2}{a^2}\,w = \frac{2par^3}{EI}\left(\frac{r}{a} - \frac{1}{3}\right) \dots\dots [13]$$

In this equation w = deflection of mean radius def of the plate or ring (see Fig. 4 for symbols referring to the dimensions), while

p = uniform pressure on surface
α = angle from symmetry axis of plate
r = mean radius of plate
a = half-width of plate
I = moment of inertia of a radial cross-section of the plate $= {}^1/_{12} \times 2ah^3$

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[8] Rayleigh, Lord, "Theory of Sound," London, Macmillan Co., 1926, Sec. Ed. Ritz, W., *Crelles Journal*, vol. 135, p. 1, 1908. See footnotes 4 and 6.

[9] This theory was developed by St. Venant, J. H. Mitchell, and A. E. H. Love. See Love, A. E. H., "The Mathematical Theory of Elasticity," Cambridge University Press, 1927, 4th Ed. Also footnote 6, "Strength of Materials," vol. 2, p. 468.

h = thickness of plate (assumed constant), and
C = torsional rigidity of a radial cross-section.

$$C = 2\beta a h^3 G$$

where E = modulus of elasticity of the material
G = modulus of rigidity of the material
β = a factor[10] depending on the ratio $h/2a$ and which may be taken from the curve of Fig. 5. For practical cases where $h/2a < 0.8$, the formula

$$\beta = \frac{1}{3}\left(1 - \frac{0.625h}{2a}\right) \qquad [14]$$

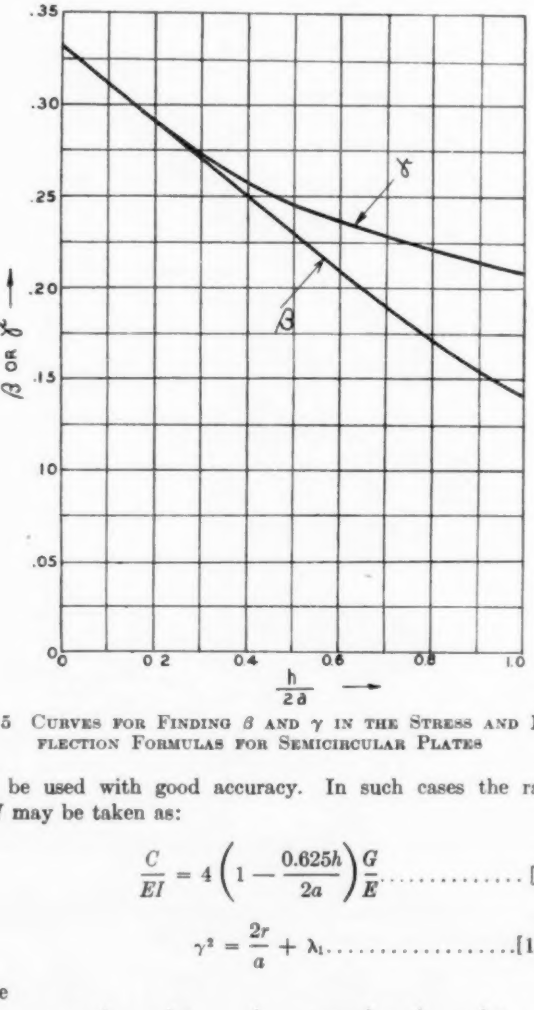

FIG. 5 CURVES FOR FINDING β AND γ IN THE STRESS AND DEFLECTION FORMULAS FOR SEMICIRCULAR PLATES

may be used with good accuracy. In such cases the ratio C/EI may be taken as:

$$\frac{C}{EI} = 4\left(1 - \frac{0.625h}{2a}\right)\frac{G}{E} \qquad [15]$$

$$\gamma^2 = \frac{2r}{a} + \lambda_1 \qquad [15a]$$

where

$$\lambda_1 = \frac{C}{EI}\left(1 + \frac{r}{a}\right)^2 = 4\left(1 - \frac{0.625h}{2a}\right)\frac{G}{E}\left(1 + \frac{r}{a}\right)^2 \qquad [16]$$

A solution of this equation which satisfies the condition of symmetry with respect to the symmetry axis $\alpha = 0$ is:

$$w = C_1' \cosh \gamma_1\alpha + C_2' \cosh \gamma_2\alpha + \frac{2pa^4r}{EI}\left(\frac{r}{a} - \frac{1}{3}\right). \qquad [17]$$

In this equation

$$\gamma_1 = \frac{\gamma}{\sqrt{2}}\sqrt{1 + \sqrt{1 + \frac{4r^2}{a^2\gamma^4}}} \qquad [18]$$

$$\gamma_2 = \frac{\gamma}{\sqrt{2}}\sqrt{1 - \sqrt{1 + \frac{4r^2}{a^2\gamma^4}}} \qquad [19]$$

From the conditions at the end $\alpha = \pi/2$, the two constants C_1' and C_2' may be found, thus giving an expression for w equal to:

$$w = \frac{12\,pa^2r^2}{Eh^3}\left(\frac{r}{a} - \frac{1}{3}\right)\left(C_1 \cosh \gamma_1\alpha + C_2 \cosh \gamma_2\alpha + \frac{a}{r}\right). \qquad [20]$$

where

$$C_1 = \frac{1}{\left(\frac{r}{a} - \gamma_1^2\right)\left(\lambda - 1\right)\cosh \frac{\gamma_1\pi}{2}} \qquad [21]$$

$$C_2 = \frac{1}{\left(\frac{r}{a} - \gamma_2^2\right)\left(\frac{1}{\lambda} - 1\right)\cosh \frac{\gamma_2\pi}{2}} \qquad [22]$$

$$\lambda = \frac{\gamma_1\left(\frac{r}{a} - \gamma_1^2 + \lambda_1\right)\left(\frac{r}{a} - \gamma_2^2\right)\tanh \frac{\gamma_1\pi}{2}}{\gamma_2\left(\frac{r}{a} - \gamma_2^2 + \lambda_1\right)\left(\frac{r}{a} - \gamma_1^2\right)\tanh \frac{\gamma_2\pi}{2}} \qquad [23]$$

In these expressions, γ_1, γ_2, and λ_1 are given by [18], [19], and [16], respectively.

The maximum deflection $w_{\max}$ will occur at points A and A', Fig. 4(a), where $\alpha = \pi/2$ and is found from [20] by substituting $\alpha = \pi/2$ therein.

$$w_{\max} = 2(w)_{\alpha=\pi/2} = \frac{24pa^2r^2}{Eh^3}\left(\frac{r}{a} - \frac{1}{3}\right)\left[C_1 \cosh \frac{\gamma_1\pi}{2} + C_2 \cosh \frac{\gamma_2\pi}{2} + \frac{a}{r}\right] \qquad [24]$$

where C_1 and C_2 are given by [21] and [22].

Calculation of Stresses. In calculating the stresses, the fundamental Equations [63] and [64] of Appendix No. 3 may be used in connection with [20]. The value of bending stress σ' determined in this way will, however, be the *average* bending stress over a radial section, and not the *maximum* bending stress which occurs at the inner circumference of the plate, due to the decreased fiber length. To get an approximation to this maximum stress, the average stress σ' thus obtained must be multiplied by a factor K_1 which may be obtained by taking the ratio of the maximum bending stress in a *full* circular plate having the given ratio of inner to outer diameter as determined from the known circular-plate theory, to the value calculated by considering the circular plate as a beam acted on by external moments which may be calculated from the statical equations of equilibrium. Taking σ' as the average bending stress at any angle α from the axis of symmetry, determined from the beam method, the fundamental Equations [63] and [64] of Appendix No. 3 reduce to:

$$\sigma' = \frac{M}{Z} = \frac{6M}{2ah^2} = \frac{Eh}{2r}\left(\frac{w}{a} - \frac{1}{r}\frac{d^2w}{d\alpha^2}\right) \qquad [25]$$

$Z = 2ah^2/6$ = section modulus of a radial cross-section. The calculated shearing stress τ will likewise be, using [64],

$$\tau = \frac{M_t}{2ah^2\gamma} = \frac{\beta G h}{\gamma r^2}\left(1 + \frac{r}{a}\right)\frac{dw}{d\alpha} \qquad [26]$$

where $2ah^2\gamma$ is the section modulus in torsion of a rectangular section, γ being a function of $h/2a$ and may be taken from the curve of Fig. 5, or calculated with enough accuracy from:[11]

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[10] "Strength of Materials," part 1, p. 76, footnote 6.

$$\gamma = \frac{1}{3 + 1.8\dfrac{h}{2a}} \quad\ldots\ldots\ldots\ldots\ldots\ldots [27]$$

Using the expression for w given by [20], the stresses σ' and τ as functions of α become:

$$\sigma' = \frac{6par}{h^2}\left(\frac{r}{a} - \frac{1}{3}\right)\left[C_1\left(1 - \gamma_1^2\frac{a}{r}\right)\cosh\gamma_1\alpha + C_2\left(1 - \gamma_2^2\frac{a}{r}\right)\cosh\gamma_2\alpha + \frac{a}{r}\right] \quad\ldots\ldots\ldots [28]$$

$$\tau = \frac{12pa^2}{h^2}\left(\frac{G}{E}\right)\left(1 + \frac{0.3h}{a}\right)\left(1 - \frac{0.312h}{a}\right)\left(1 + \frac{r}{a}\right)\left(\frac{r}{a} - \frac{1}{3}\right)\left(C_1\gamma_1\sinh\gamma_1\alpha + C_2\gamma_2\sinh\gamma_2\alpha\right) \quad\ldots\ldots [28a]$$

in which C_1, C_2, γ_1, and γ_2 are given by [21], [22], [18], and [19], respectively.

An examination of these equations shows that, for practical cases, the shearing stress τ is a maximum at $\alpha = \pi/2$ and the bending stress σ' is a maximum at $\alpha = 0$. These maximum values are:

$$\sigma'_{\max} = \frac{6par}{h^2}\left(\frac{r}{a} - \frac{1}{3}\right)\left[C_1\left(1 - \gamma_1^2\frac{a}{r}\right) + C_2\left(1 - \gamma_2^2\frac{a}{r}\right) + \frac{a}{r}\right] \quad\ldots\ldots\ldots [29]$$

$$\tau_{\max} = \frac{12pa^2}{h^2}\frac{G}{E}\left(1 + \frac{0.3h}{a}\right)\left(1 - \frac{0.312h}{a}\right)\left(1 + \frac{r}{a}\right)\left(\frac{r}{a} - \frac{1}{3}\right)\left(C_1\gamma_1\sinh\frac{\gamma_1\pi}{2} + C_2\gamma_2\sinh\frac{\gamma_2\pi}{2}\right) \quad\ldots\ldots\ldots [30]$$

An examination into the sign of the second derivative of the stress shows that at $\alpha = 0$ the bending stress σ' is a maximum.

It should also be noted that this stress $\tau_{\max}$ at $\alpha = \pi/2$ arises mainly because of the action of the concentrated loads P_1, Fig. 4(*c*). It is also clear that near these concentrated loads, local stresses of considerable magnitude may be set up. Hence the stresses τ as given by [28*a*] must be considered as approximations valid only near the mean radius of the plate, and at some distance from the diametral edges. The stress $\tau_{\max}$ given by [30] must also be considered as a very rough approximation.

As mentioned before, to get the true *maximum* bending stress σ at the inner edge of the plate, the *average* stress σ' figured as above, must be multiplied by a correction factor K_1, greater than unity, to take into account the stress increase at the inner edge due to the shorter length of the inner fibers. An estimate of the factor K_1 may be had by taking the ratio of the maximum stress σ_f in a *full* circular plate, having the same ratio of inner to outer radius as the semicircular plate in question, to the average stress σ_f' in a full circular plate figured by the beam method mentioned previously. The stress σ_f may be easily calculated from known circular-plate formulas and is:[12]

$$\sigma_f = K_f\frac{p(r + a)^2}{h^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots [31]$$

where

$$K_f = \frac{3.9\log_e\beta_1 + \dfrac{2.475}{\beta_1^2} - 3 + 0.525\,\beta_1^2}{1/\beta_1^2 - 1} \quad\ldots\ldots [32]$$

and β_1 = ratio $\dfrac{r - a}{r + a} = \dfrac{\text{inner radius}}{\text{outer radius}}$

The average bending stress σ_f' in a full circular plate may be figured as follows: The moments acting on any diametral cross-section are obtained by taking the moments of the edge reactions and subtracting from these the moment due to the pressure load. This gives a moment acting on a diametral section equal to

$$M_r = \frac{p}{3}\left[(r + a)^3 - 3(r + a)(r - a)^2 + 2(r - a)^3\right]$$

and the *average* stress σ_f' thus becomes (since $\sigma_f' = M_r \div$ Section Modulus):

$$\sigma_f' = \frac{p(r + a)^2}{h^2}\left[\frac{1/\beta_1 - 3\beta_1 + 2\beta_1^2}{1/\beta_1 - 1}\right] \quad\ldots\ldots\ldots [33]$$

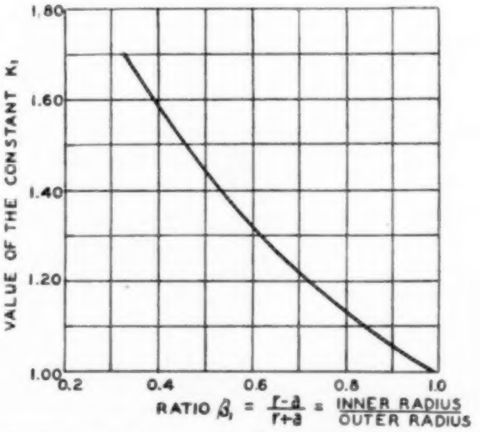

FIG. 6 CURVE FOR FINDING THE CONSTANT K_1 IN THE STRESS FORMULAS FOR SEMICIRCULAR PLATES

The stress multiplication factor then becomes, using [31] and [33]:

$$K_1 = \frac{\sigma_f}{\sigma_f'} = \frac{3.9\log_e\beta_1 + \dfrac{2.475}{\beta_1^2} - 3 + 0.525\beta_1^2}{\left(1 + \dfrac{1}{\beta_1}\right)(1/\beta_1 - 3\beta_1 + 2\beta_1^2)} \quad\ldots\ldots [34]$$

In Fig. 6, for convenience in calculation, values of K_1 are plotted as functions of the ratio $\beta_1 = \dfrac{r - a}{r + a} = \dfrac{\text{inner radius}}{\text{outer radius}}$.

To obtain the true maximum bending stress σ at the *inner* edge of a semicircular plate or ring as a function of the angle α, the value of the average stress σ' given by [28] is multiplied by K_1. Thus the bending stress σ at the inner edge as a function of the angle from the symmetry axis is:

$$\sigma = K_1\sigma' \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots [35]$$

where σ' and K_1 are given by [28] and [34].

The maximum value of bending stress $\sigma_{\max}$ occurs when the angle α is zero. This value is again obtained by multiplying the value of $\sigma_{\max}$ found above by the factor K_1. Thus

---

[11] Cf. "Strength of Materials," footnote 6, p. 76.

[12] Wahl, A. M., and Lobo, G., "Stresses and Deflections in Flat Circular Plates With Central Holes," Trans. A.S.M.E. (1930), APM-52-3.

$$\sigma_{\max} = K_1 \sigma'_{\max} \ldots\ldots\ldots\ldots\ldots\ldots [35a]$$

where $\sigma'_{\max}$ is given by [29].

Tests. In order to check up the stress and deflection formulas given herein, some tests were made on two semicircular plates designated as plates G and H. A special test apparatus, which will be described in a forthcoming paper, was used to carry out these tests. This apparatus consisted essentially of means for applying hydraulic pressure to the surface of a semicircular plate of the shape shown in Fig. 4(a), the plate being simply supported on the outer circumference and free on the inner circumference and diametral edges. The deflections of the plates were measured at various points by dial gages. Also provision was made for attaching Huggenberger extensometers at the inside edge of the plate where the maximum stress occurs. The method of attachment of the extensometers was similar to that described in a previous paper.[12] The plates tested had dimensions as indicated in Table 1.

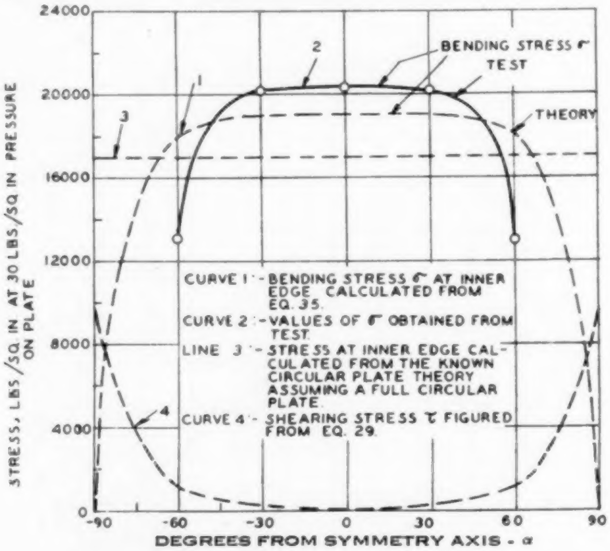

Fig. 7 Stress Distribution, Plate H

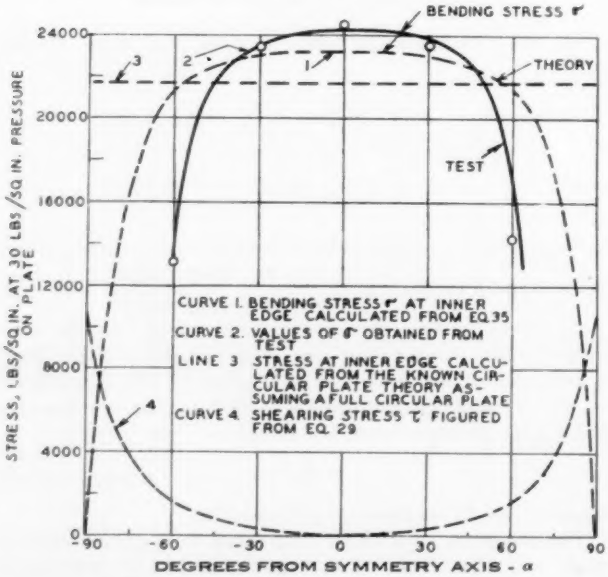

Fig. 8 Stress Distribution, Plate G

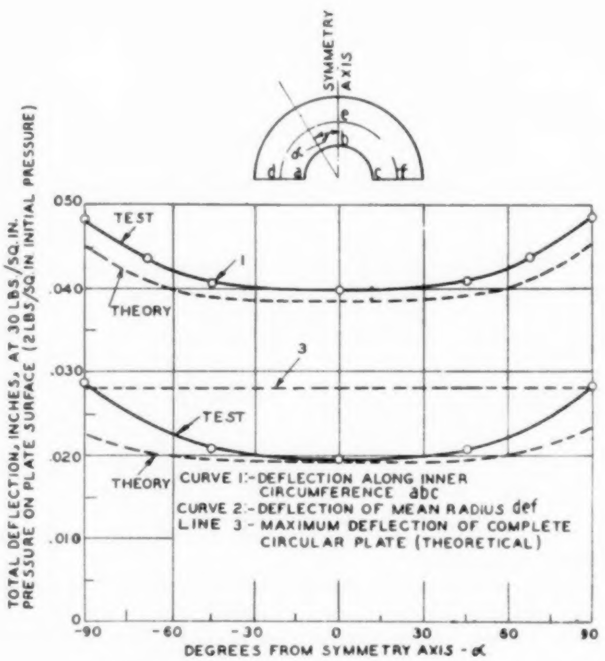

Fig. 9 Deflection Distribution, Plate G

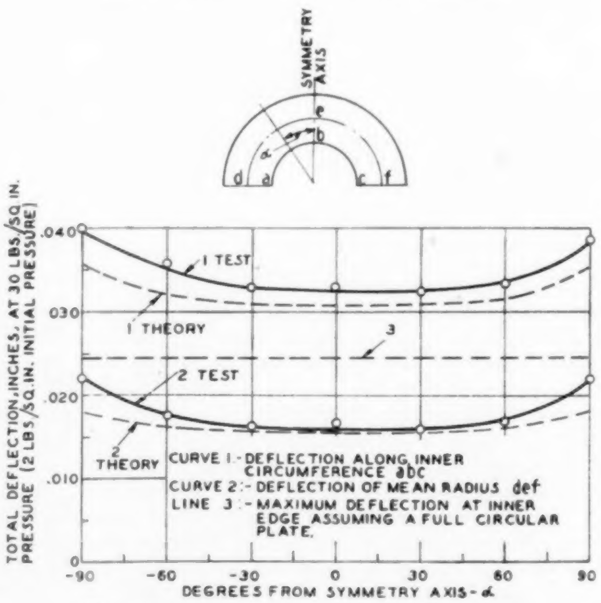

Fig. 10 Deflection Distribution, Plate H

TABLE 1 DIMENSIONS OF SEMICIRCULAR PLATES

(All dimensions refer to Fig. 4)

Plate	R Outer radius, inches	r Mean radius, inches	a Semi-width, inches	h Thickness, inches
G	4.97	3.48	1.48	0.250
H	4.97	3.73	1.23	0.250

In Fig. 7, curve 1 shows the bending stress σ at an external pressure of 30 lb per sq in., calculated from Equation [35], plotted as a function of the angle α from the axis of symmetry for plate H. Curve 2 of this figure shows the test curve representing the bending stresses obtained from the extensometer readings at various angles at the inner edge. It may be seen that as far as the *maximum* stress is concerned the test curve gives results approximating the theoretical curve. At larger angles α, however, the test curve drops off much faster than the theoretical curve. This may be explained by the fact that near the diametral edges $\alpha = \pi/2$, this stress-concentration effect due to the decrease in fiber length is not present to any marked degree, since these portions do not act in a manner similar to those of a complete circular plate. Nevertheless, as far as the maximum stress is concerned, it is seen that the method gives a good approximation. It is also seen that curve 3 of Fig. 7, which represents the stress calculated from the known full-circular-plate theory (Equation [31]) also gives a fair approximation to the maximum bending stress in a semicircular plate having the geometrical shape of plate H.

Curve 4 of Fig. 7 represents the shearing stress τ plotted as a function of the angle α on the basis of Equation [28*a*]. As mentioned above, this shearing stress arises mainly from the presence of the concentrated reactions P_1 at the corners [Fig. 4(*c*)], and hence this curve must be considered as a rough approximation valid only near the center line of the plate and at some distance from the diametral edges.

The curves on Fig. 8 are similar to those on Fig. 7 except that they represent values for plate G which had a ratio of inner to outer radius of about 0.4, as contrasted with a corresponding ratio of 0.5 for plate H. It may be seen that the results obtained for plate H apply in general to plate G. This would indicate that the method given herein applies to the case of rings or plates where the inner radius is as small as 0.4 the outer radius.

In Figs. 9 and 10, the deflections of the mean radius of the plate and those of the inner circumference are plotted as functions of the angle α from the axis of symmetry, assuming the pressure on the plate surface increased from 2 to 30 lb per sq in. The dotted lines represent theoretical curves calculated on the basis of Equation [20], using a modulus of elasticity of 30×10^6 lb per sq in. and a value of G of 11.5×10^6 lb per sq in. This value of E was checked by tension tests on small specimens of the material.[13] The full lines represent test curves. The straight lines marked "3" represent the deflections of the inner edge calculated on the assumption of a complete circular plate having otherwise the same dimensions. It may be seen that the test curves are somewhat higher than those calculated from Equation [20], but this may probably be explained by irregularities in the supporting edges, which tend to flatten out as the load increases, and thus at first make the plate more flexible than calculated. This hypothesis is verified by Fig. 11, which shows pressure plotted against maximum deflection of the plate [at points A, A', Fig. 4(*a*)]. The test curves represented by full lines are slightly concave upward at first, but straighten out later on. The dotted lines show the theoretical curves of maximum deflection of points A and A' calculated from [24]. It is seen that there is good agreement between the slopes of the test and the calculated curves. It should be noted, however, that the maximum deflections of the semicircular plates (or split rings) tested were around 1.6 to 1.7 times the deflections of a complete circular plate or ring having otherwise the same dimensions and loaded with the same external pressure. In contrast with this the maximum bending stresses as measured in these semicircular plates were found to be only about 20 per cent larger than those which would exist in the corresponding complete circular plate under the same external pressure.

III—CONCLUSIONS

The tests thus indicate that the theoretical formulas and approximate methods developed herein for calculating the stress and deflection of semicircular rings or plates with holes, are sufficiently accurate for practical use in cases where the inner radius of the ring or plate is over 0.4 that of the outer radius.

It is hoped that these methods will prove useful in the analysis of mechanical structures similar to the cases herein considered.

ACKNOWLEDGMENT

The author's thanks are due to Dr. A. Nádái and Dr. S. Timoshenko for valuable suggestions and encouragement during the course of this work. Messrs. Fornes and Richmond assisted in the tests and checking of the numerical calculations.

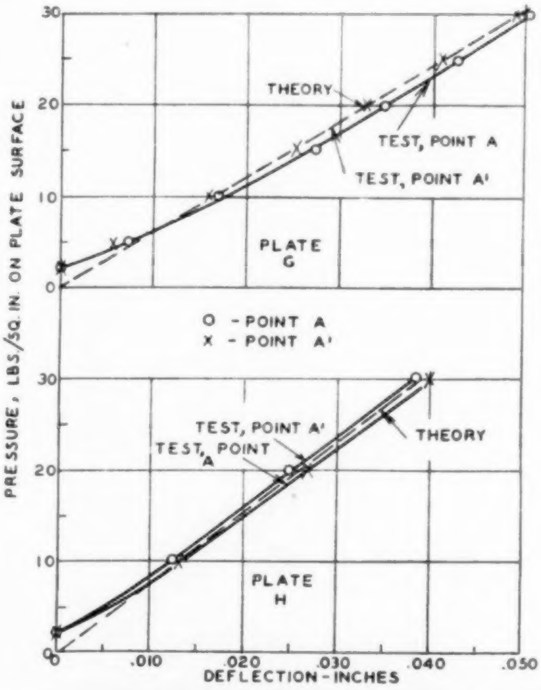

FIG. 11 PRESSURE-DEFLECTION CURVES AT POINTS OF MAXIMUM DEFLECTION OF PLATES G AND H

Appendix No. 1

EXACT SOLUTION FOR RECTANGULAR PLATE HAVING THREE EDGES SIMPLY SUPPORTED AND THE FOURTH FREE

To solve the case of the rectangular plate shown in Fig. 1, the series

$$w = \sum_{n=1,3,5\ldots}^{\infty} Y_n \cos\frac{n\pi x}{a} \ldots\ldots\ldots\ldots\ldots [36]$$

where Y_n is a function of y alone, is assumed. This series obviously satisfies the conditions at $x = \pm a/2$, namely, $w = 0$, $\partial^2 w/\partial x^2 = 0$. In addition for $y = 0$, the conditions $w = 0$, $\partial^2 w/\partial y^2 = 0$ give $Y_n = 0$, $d^2Y_n/dy^2 = 0$ for $y = 0$, while for $y = b$, the conditions

$$\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial}{\partial y}\left[(2-\nu)\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] = 0$$

give

$$\left[\frac{d^2Y_n}{dy^2} - \frac{\nu\pi^2}{a^2}Y_n\right]_{y=b} = 0$$

---

[13] These tension tests were made by T. F. Hengstenberg, of the Mechanics Division, Westinghouse Electric & Manufacturing Co.

and

$$\frac{\partial}{\partial y}\left[-(2-\nu)\frac{\pi^2}{a^2}Y_n+\frac{d^2Y_n}{dy^2}\right]_{y=b}=0$$

Furthermore the value of w given by [36] must satisfy the biharmonic Equation [1]. Substituting [36] in [1], it is found that

$$\sum_{n=1,3,5\ldots}^{\infty}\left(\frac{d^4Y_n}{dx^4}-2\mu^2\frac{d^2Y_n}{dx^2}+\mu^4Y_n\right)\cos\frac{n\pi x}{a}=\frac{p}{N}$$

where $\mu = n\pi/a$.

Multiplying both terms of this equation by $\cos\frac{m\pi x}{a}\,dx$, where m is odd, and integrating between 0 and $a/2$, the condition which Y_n must satisfy becomes:

$$\frac{d^4Y_n}{dx^4}-2\mu^2\frac{d^2Y_n}{dx^2}+\mu^4Y_n=(-1)^{\frac{n-1}{2}}\frac{2p}{n\pi N}$$

A value of Y_n which satisfies this equation when p and N are constants, and also satisfies the above boundary conditions, is:

$$Y_n=\frac{(-1)^{\frac{n-1}{2}}}{n^5}\Big[1-\cosh\mu y+\mu y\sinh\mu y-H\sinh\mu y$$
$$-F\mu y\cosh\mu y\Big]\ldots\ldots[37]$$

where

$$H=\frac{-(3-\nu)}{2(1-\nu)}\tanh\mu b+\mu\frac{b}{2}+F\left(\frac{1+\nu}{1-\nu}-\mu b\tanh\mu b\right)$$

and

$$F=\frac{\frac{-(3-\nu)}{2}\sinh^2\mu b+\nu\cosh\mu b\,(1-\cosh\mu b)}{-(3+\nu)\sinh\mu b\cosh\mu b-(1-\nu)\mu b}$$

Appendix No. 2

APPROXIMATE SOLUTIONS FOR RECTANGULAR PLATE UNDER UNIFORM PRESSURE AND HAVING THREE EDGES SIMPLY SUPPORTED AND THE FOURTH FREE

This is the case shown in Fig. 2. Two different approximate solutions will be developed, namely, (1) the Beam Method and (2) the Energy Method.

Approximate Solution No. 1—Beam Method. This method assumes the rectangular plate to be a beam acted on by bending and twisting moments; it is assumed that the equations for bending and twisting of bars hold in this case. Referring to Fig. 2(a), the x- and w-axes are chosen as shown, the deflections w being taken as those of the longitudinal center line of the plate. Assuming an element of length dx cut out from the plate at a distance x from O, by two neighboring cross-sections perpendicular to the x-axis, the forces and moments acting thereon are shown in Fig. 2(b). These consist of a shearing force Q, a twisting moment M_t, a bending moment M_x, a distributed load $q\,dx$ acting at the edge, and a pressure $pb\,dx$ acting on the surface. The following fundamental formulas hold from the equilibrium of these forces (for meaning of symbols see early part of paper):

$$EI\frac{d^2w}{dx^2}=-M_x\ldots\ldots\ldots\ldots\ldots\ldots[38]$$

$$\frac{2C}{b}\frac{dw}{dx}=M_t\ldots\ldots\ldots\ldots\ldots\ldots[39]$$

$$Q=\frac{dM_x}{dx}\ldots\ldots\ldots\ldots\ldots\ldots[40]$$

$$\frac{dQ}{dx}=q-pb\ldots\ldots\ldots\ldots\ldots\ldots[41]$$

$$\frac{dM_t}{dx}=-\frac{qb}{2}\ldots\ldots\ldots\ldots\ldots\ldots[42]$$

Eliminating the unknown quantities M_x, M_t, Q, and q from these equations by differentiation and resubstitution, the following differential equation is obtained:

$$\frac{d^4w}{dx^4}-\frac{4C}{EIb^2}\frac{d^2w}{dx^2}=\frac{pb}{EI}\ldots\ldots\ldots\ldots\ldots[43]$$

A solution of this equation is

$$w=C_1''\cosh\frac{2x}{b}\sqrt{\frac{C}{EI}}+C_2''\sinh\frac{2x}{b}\sqrt{\frac{C}{EI}}-\frac{pb^3x^2}{8C}$$
$$+C_3''x+C_4''\ldots\ldots[44]$$

From symmetry conditions when the origin is at the center of the plate, the constants C_2'' and C_3'' are zero; hence this becomes:

$$w=C_1''\cosh\frac{2x}{b}\sqrt{\frac{C}{EI}}-\frac{pb^3}{8C}x^2+C_4''\ldots\ldots[45]$$

To determine C_1'' and C_4'', the conditions $M_x = -EI\frac{d^2w}{dx^2} = 0$, $w = 0$ at $x = a/2$, are used. The expression for w thus determined is:

$$w=\frac{pb^5}{16C}\left[\frac{EI}{C}\left(\frac{\cosh\frac{2x}{b}\sqrt{\frac{C}{EI}}}{\cosh\frac{a}{b}\sqrt{\frac{C}{EI}}}-1\right)+\frac{a^2}{2b^2}\left(1-\frac{4x^2}{a^2}\right)\right]\ldots[46]$$

The maximum deflection of the plate at A, Fig. 2(a), will be

$$w_{\max}=2(w)_{x=0}=\frac{pb^5}{8C}\left[\frac{EI}{C}\left(\frac{1}{\cosh\frac{a}{b}\sqrt{\frac{C}{EI}}}-1\right)+\frac{a^2}{2b^2}\right]$$
$$\ldots\ldots[47]$$

In the case where $a = 2b$, taking $C = 0.333\left(1-\frac{0.625\,h}{b}\right)bh^3G$ and $\frac{C}{EI} = 4\left(1-\frac{0.625\,h}{b}\right)\frac{G}{E}$ (see Equation [5a] et seq.) the value of the maximum deflection becomes:

$$(w_{\max})_{a=2b}$$
$$=\frac{3pa^4}{128\left(1-\frac{0.625\,h}{b}\right)h^3G}\left[\frac{\frac{1}{\cosh\frac{a}{b}\sqrt{\frac{C}{EI}}}-1}{4\left(1-\frac{0.625\,h}{b}\right)\frac{G}{E}}+2\right]\ldots[48]$$

Taking as a typical case $t = b/10$ and $G/E = 0.4$, this reduces to

$$(w_{\max})_{a=2b}=\frac{0.0906\,pa^4}{Eh^3}\ldots\ldots\ldots\ldots[49]$$

Approximate Solution No. 2—Energy Method. In this method the coordinates of Fig. 3 are used for convenience, x being taken as the distance from the left edge. As before, the deflections of the center line of the plate parallel to the free edge are taken as w. The method consists essentially of assuming the deflection curve w in the form of a Fourier series[8] of a type which satisfies the boundary and symmetry conditions, and determining the coefficients of this series so that the total energy of the system is a minimum. In addition it is assumed as before that cross-sections perpendicular to the x-axis rotate without distortion, which means that the angular twist ϕ at any point x is $\phi = 2w/b$. The total energy consists of the energy due to bending U_1, the energy due to twist U_2, and the energy lost during the displacement of the load $-U_3$.

$$U_1 = \int_0^a \frac{EI}{2}\left(\frac{d^2w}{dx^2}\right)^2 dx$$

$$U_2 = \int^a \frac{C}{2}\left(\frac{d\phi}{dx}\right)^2 dx = \int_0^a \frac{2C}{b^2}\left(\frac{dw}{dx}\right)^2 dx$$

$$-U_3 = -\int_0^a pbwdx$$

The total energy is thus

$$U = U_1 + U_2 - U_3 \qquad [50]$$

The deflection w is assumed in the following form (which is required from symmetry conditions):

$$w = a_1 \sin\frac{\pi x}{a} + a_3 \sin\frac{3\pi x}{a} + \dots$$

$$w = \sum_{n=1,3,5\dots}^{\infty} a_n \sin\frac{n\pi x}{a} \qquad [51]$$

This form also satisfies the conditions at $x = 0$ or a, i.e., $w = 0$, $d^2w/dx^2 = 0$. Differentiating [51] as required, substituting the result in [50], and integrating, the following expression for the total energy is obtained:

$$U = \frac{C\pi^2}{ab^2}\sum_{n=1,3,5\dots}^{\infty} a_n^2n^2 + \frac{EI\pi^4}{4a^3}\sum_{n=1,3,5\dots}^{\infty} a_n^2n^4 - \frac{2pba}{\pi}\sum_{n=1,3,5\dots}^{\infty} a_n \qquad [52]$$

To determine any coefficient, the energy U is differentiated partially with respect to a_n and the result set equal to zero. Thus:

$$\frac{\partial U}{\partial a_n} = \frac{2C\pi^2}{ab^2}a_nn^2 + \frac{EI\pi^4 a_nn^4}{2a^3} - \frac{2pba}{\pi n} = 0$$

from which

$$a_n = \frac{2pba}{\pi^3n^3\left[\dfrac{2C}{ab^2} + \dfrac{EI\pi^2n^2}{2a^3}\right]} \qquad]53[$$

Each coefficient a_1, a_3, ... may be determined from this equation. Thus

$$a_1 = \frac{12pa^4}{\pi^3Eh^3\left[\dfrac{Ca^2}{EIb^2} + \dfrac{\pi^2}{4}\right]} \qquad [54]$$

$$a_3 = \frac{12pa^4}{27\pi^3Eh^3\left[\dfrac{Ca^2}{EIb^2} + \dfrac{9\pi^2}{4}\right]} \qquad [55]$$

Using $a/b = 2$, $C/EI = 1.48$ (which would be true for $h = {}^1/_8b$), the first two coefficients become

$$a_1 = 0.0461\,\frac{pa^4}{Eh^3} \qquad [56]$$

$$a_3 = 0.0005\,\frac{pa^4}{Eh^3} \qquad [57]$$

The deflection curve for the case where $a = 2b$ is thus

$$w = \frac{pa^4}{Eh^3}\left[0.0461\sin\frac{\pi x}{a} + 0.0005\sin\frac{3\pi x}{a} + \dots\right]$$

It is seen that this is a rapidly converging series. Using the first two terms and taking $x = a/2$, the maximum deflection at A, Fig. 3, becomes:

$$w_{\max} = 2\,(w)_{x=a/2} = 0.0912\,\frac{pa^4}{Eh^3} \qquad [58]$$

Appendix No. 3

DEFLECTION OF SEMICIRCULAR RING OR PLATE WITH A HOLE

Referring to Fig. 4(b), the following forces and moments act on an element cut out by two neighboring radial planes perpendicular to the plane of the plate. Bending moments M and $M + \frac{dM}{d\alpha}d\alpha$, twisting moments M_t and $M_t + \frac{dM_t}{d\alpha}d\alpha$, and external pressure load 2 $pard\alpha$, a force $qRd\alpha$ at the outer edge, and shearing forces Q and $Q + \frac{dQ}{d\alpha}d\alpha$. The equations of equilibrium are derived as follows: Taking moments about the axis OA and considering the fact that the moments M_t and $M_t + \frac{dM_t}{d\alpha}d\alpha$ act at an angle and hence have components in the same direction as M, the following equation results (neglecting terms of higher order):

$$\frac{dM}{rd\alpha} + \frac{M_t}{r} = Q \qquad [59]$$

Likewise, taking moments about an axis perpendicular to OA, the condition of equilibrium gives:

$$-\frac{dM_t}{ad\alpha} + \frac{2}{3}pa^2 + \frac{M}{a} = qR \qquad [60]$$

where q is the load per inch along the edge at an angle α.

By summing up the forces in the vertical direction, the equation

$$\frac{dQ}{d\alpha} + 2apr = qR \qquad [61]$$

results. Assuming that radial cross-sections deflect without distortion, it follows that

$$w = a\phi \qquad [62]$$

where ϕ is the angle which a radial element of the surface makes with the plane of the ring [Fig. 4(b)]. The following equations, fundamental in the theory of bars bent out of their planes of initial curvature, also hold:[9]

$$\frac{M}{EI} = \frac{\phi}{r} - \frac{d^2w}{r^2d\alpha^2} = \frac{w}{ar} - \frac{d^2w}{r^2d\alpha^2} \qquad [63]$$

$$\frac{M_t}{C} = \frac{d\phi}{rd\alpha} + \frac{1}{r^2}\frac{dw}{d\alpha} = \frac{dw}{r^2 d\alpha}\left(1 + \frac{r}{a}\right) \ldots\ldots\ldots [64]$$

Eliminating the unknown quantities M_t, M, q, Q, and ϕ from the six equations, [59], [60], [61], [62], [63], and [64], by successive differentiations and resubstitutions, the fundamental differential Equation [13] is obtained. A solution of this equation, compatible with the symmetry conditions, is:

$$w = C_1' \cosh \gamma_1\alpha + C_2' \cosh \gamma_2\alpha + \frac{2pa^4r}{EI}\left(\frac{r}{a} - \frac{1}{3}\right)$$

To determine C_1' and C_2', the conditions at the end $\alpha = \pi/2$ are used. It is necessary to assume the possibility of a concentrated reaction P_1 at the corner [Fig. 4(c)]. Under this assumption, the boundary conditions become: At $\alpha = \pi/2$, the bending moment $M = 0$, and hence from [63]

$$\left(\frac{w}{a} - \frac{d^2w}{rd\alpha^2}\right)_{\alpha = \pi/2} = 0 \ldots\ldots\ldots\ldots\ldots [65]$$

At $\alpha = \pi/2$, the shearing force $Q = -P_1$ and $M_t = P_1a = -Qa$, from which, using [59], [63], and [64], we obtain the condition:

$$\left[\frac{dw}{d\alpha}\left(\frac{r}{a} + \lambda_1\right) - \frac{d^3w}{d\alpha^3}\right]_{\alpha = \pi/2} = 0 \ldots\ldots\ldots [66]$$

Using the value w given by [17], differentiating, substituting in [65] and [66], and taking $\alpha = \pi/2$, the values of C_1' and C_2' are determined; using these, Equation [20] is obtained.

Discussion

William Hovgaard.[14] The writer believes that Professor Wieghardt's method described in "Drang und Zwang" could be applied to this problem with advantage—at any rate, to the rectangular plates. He also would suggest that perhaps polar coordinates might be applied to the semi-circular plate.

Author's Closure

Regarding Professor Hovgaard's discussion, as the author understands it, Professor Wieghardt's method makes use of the similarity of the differential equation for the stress function corresponding to a state of plane stress or strain and the equation corresponding to the deflection of a flat plate, in order to determine the stress distribution in complicated cases of plane stress or strain by experimental observations on a bent plate, loaded so as to satisfy the required boundary conditions. Thus for a plate loaded only along the boundary, the pressure p is zero, and the plate equation becomes $\nabla^4w = 0$, which is also that satisfied by a plane stress function F. In the present instance, however, we are interested in the stresses and deflections in a plate loaded transversely, and not in the stress distribution in cases where states of plane stress or strain exist. Since the stresses and deflections in plates loaded transversely may usually be measured directly, using apparatus of the kind mentioned in the paper, such tests from experimental solutions of the plate problem, much as Professor Wieghardt's method, form an experimental solution to the problem of plane stress or strain.

Professor Wieghardt's method might be applied to the solution of certain problems involving semi-circular plates where no pressure acts on the surface and all external loads or moments act at the edges. (Such cases were not considered in the paper.) This method would then involve the determination of the stress distribution in the corresponding case of plane stress or strain (by photoelastic or other experimental methods); knowing this distribution, by a process of integration the stress function could be found, and this would enable the determination of the stresses and deflections in the plate to be made. Such a procedure, in the author's opinion, would in general be considerably more complicated than the simple experimental measurement of stresses and deflections on the plate in question. It might, however, be of advantage in a few particular cases where the direct experimental determination of stress and deflection is very difficult.

Regarding the use of polar coordinates, an exact solution of the semi-circular plate problem using these requires that a function for the deflection w be found that will satisfy both the fundamental equation $\nabla^4w = 0$ and the boundary conditions at the edges, these being expressed in polar coordinates. Since the differential expressions for boundary conditions at free edges are rather complicated, no function, to the author's knowledge, has been found that will satisfy the conditions of the case considered in the paper, although solutions have been found for semi-circular plates having all edges either clamped or simply supported (but not free). It is the author's opinion, however, that, where the inside diameter is not too small, the approximate method developed in the paper is sufficiently accurate for most practical needs.

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Slag Tap Furnaces

By A. L. BAKER,[1] CHICAGO, ILL.

The molten-slag tap furnace is followed from its original application to a power-plant boiler and brought up to date. The troubles which have developed in operating large furnaces with this type of ash removal are discussed, with the remedies applied. A comparison is made between similar boilers equipped with a hopper bottom and a slag bottom, and some experimental data are given.

THE extremely rapid development of pulverized fuel in all its applications during the last five years has caused attention to center upon its successful production and combustion under varied conditions. The fact that its use is so widespread testifies to the degree of success attained in these two fundamentals. As is usually the case in any mechanical or chemical development, this success has carried with it a number of unsolved problems of such a minor nature that they have been deliberately set aside pending the practical solution of the major problem. With the major problem solved, however, its large-scale application has transformed the former minor problems into major problems of almost first magnitude. Among these supplementary problems are:

1 Fire hazard
2 Explosions
3 Erosion of all parts of boiler and auxiliary equipment
4 Fouling of boiler, economizer, and air heater
5 Handling of ash in boiler, economizer, superheater, air heater, fans, ducts, breechings, and stacks
6 Handling of ash after removal from boiler
7 Prevention of nuisance caused by ash in stack gases.

Needless to say, most of these difficulties have been solved to a greater or less degree in the various applications according to their local importance. A problem which is always present in any pulverized-fuel installation is the removal of ash from the furnace, and one of the solutions proposed for this is discussed herein.

First Slag Tap Furnace

As often happens, the first application of the principle of removing the ash from a pulverized-fuel furnace in molten form was more or less accidental. The idea is not original, however, as the steel industry has used it with great success for a number of years. They first removed molten slag from blast furnaces into ladles and later broke up the hot slag stream with a water jet located at the slag hole, to form granulated slag. This product was more easily handled and disposed of than the chilled slag or "skull" in the ladle, and occasionally had a commercial value. Other industries also have utilized the same principle prior to its application to the pulverized-fuel furnace.

In the 1926 extension to the Charles R. Huntley Station of the Buffalo General Electric Company at Buffalo, four direct-fired pulverized-fuel boilers were authorized. The first of these, No. 13, was installed and operated at high ratings to determine the practical limitations of capacity of this type of equipment and furnace design. This boiler had an 8-ft.-square well, sunk in the flat furnace floor, into which the pulverized-fuel burners discharged. During the operation of this boiler it was noted that pools of slag accumulated in a molten condition at the bottom of the well. This happened in spite of an attempt to keep the well walls as cool as possible and to prevent slagging of the ash by the installation of bare cast-iron and steel blocks on the well tubes.

With that curious twist of the mind which often distinguishes an engineer, a distinct liability was quickly converted into a real asset. The remaining three furnaces were deliberately and courageously designed for collection of molten slag on their flat bottoms, after experiments with handling the slag and disintegrating it with water had convinced every one that it could be done in a practical way. It was thought that the furnaces could be operated at very high rates of combustion per unit volume because of not having to limit the furnace temperature when burning a low-fusion-ash coal. The only limit was felt to be the effect of temperature and erosion on the walls.

Modern Removal Methods

The most successful method of handling the molten ash is today almost exactly that used on the original experiments at Buffalo. The tap hole leads into a cast-iron spout outside the water walls. This spout is usually equipped with bottom liners to regulate the slag level in the furnace and to permit easy replacement when eroded by the movement of hot slag. The slag runs over the liners when the plug is removed, and in falling into the disintegrating chamber passes through a high velocity jet of water. The jet carries the small particles of hot slag against a break-up plate, and this impact, combined with the sudden cooling action of the water, shatters the red-hot slag stream into very small particles. These are then carried off as desired by a low-velocity jet to a storage pit or sludge pump for final disposition. The slag from Kincaid coal from central Illinois when granulated looks like a mass of very coarse black sand. The individual pieces viewed under a magnifying glass are cellular and show a dull iridescence. The particles have all corners rounded and handle easily as a mass without excessive use of jet water. Erosion of sluice plates seems somewhat less rapid with the granulated slag than with the normal hopper-bottom furnace ash.

Use of Low Fusion Ash Coals

Until this first experiment it was thought that low-fusion-

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

ash coals should be avoided by prudent operators of pulverized-fuel equipment who aspired to relatively high combustion rates and correspondingly high temperatures in the furnaces. The coal burned at Buffalo has about 12 to 14 per cent ash with a fusing point of 1900 to 2000 deg. fahr., and it was successfully burned at rates in excess of 40,000 B.t.u. per cu. ft. per hour with about 18 per cent excess air. Thus the accidental adoption of the molten-slag bottom furnace opened the way for the use of those coals formerly not considered for pulverized fuel, and which were in danger of becoming quite a drug on the market. With all concerned—i.e., the mine owner, mine operator, coal dealer, engineer, and power executive—seeking to lower their fuel cost, and all welcoming this new factor in the burning of the abundant poorer coals, it is to be expected that the adoption of this new bottom should be widespread.

Construction of Early Slag Bottoms

Early liquid or "wet" bottoms were constructed with a flat

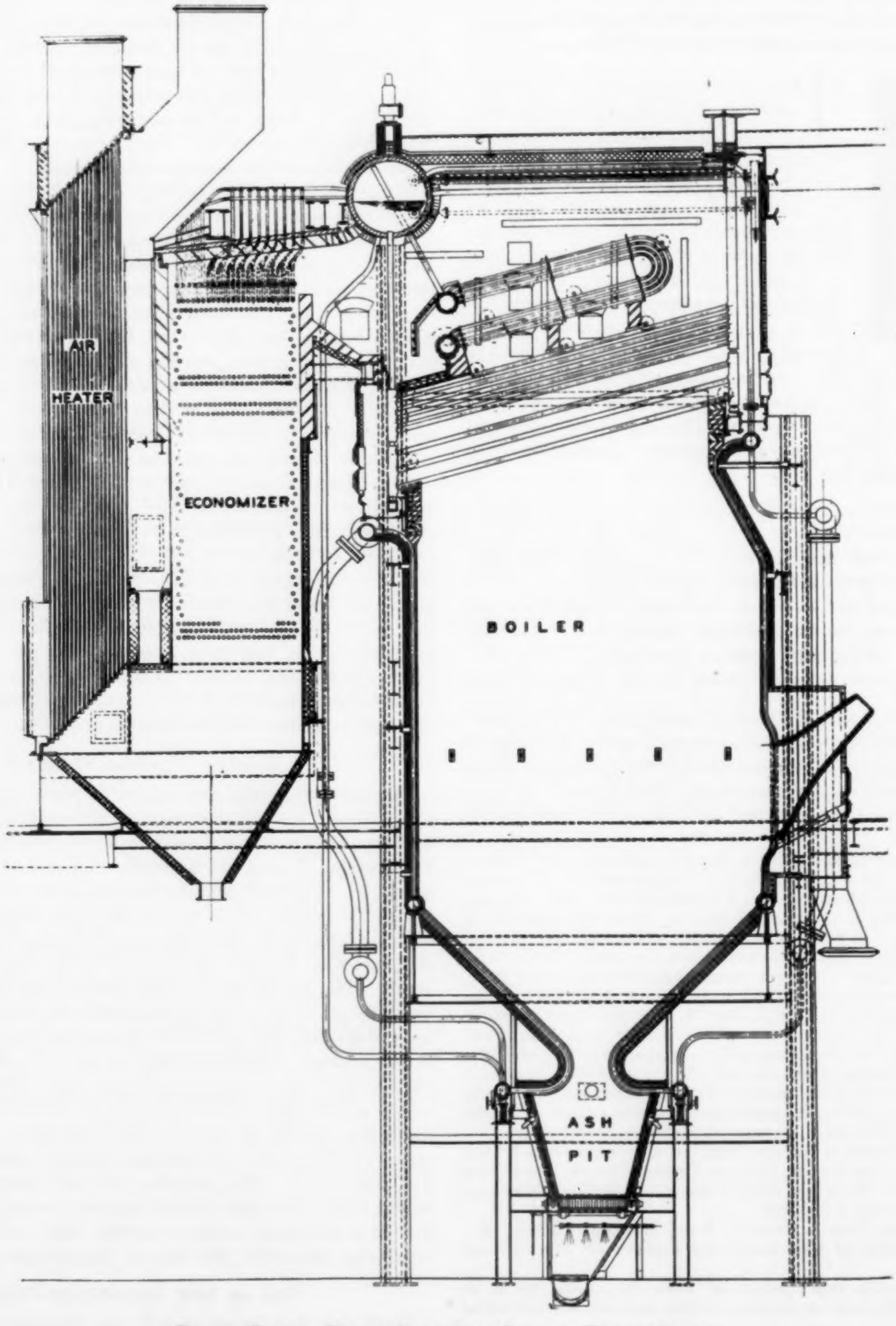

Fig. 1 Boiler, Unit 1, State Line Station, Hopper Bottom

steel bottom plate properly supported on structural-steel members, as illustrated in Fig. 2. This plate was covered with one or more layers of firebrick laid flat and dry. Then 6 in. or more of burned dolomite was added and ramped up to a foot or more in thickness at the corners and along the sides. This use of dolomite, by the way, came to us from the steel industry, which uses it for making open-hearth bottoms and for necessary repairs during a melt. The initial dolomite went into solution or was carried off with the slag soon after melting temperature was reached, but for further shutting down and starting operations of the boiler, the slag itself formed the protective bottom. The slag level was carried according to the convenience of the operators in tapping operations and sometimes attained a depth of $1^1/_2$ ft.

It was soon discovered that slag, when in the molten state, has some of the properties that fuel oil displays on board ship. It can go through cracks apparently invisible to the eye with remarkable facility and volume. Leaks in bottoms were a frequent occurrence, and a shower of molten slag occasionally came through open joints. A firehose quickly applied at the base of the floor was the immediate solution and this point seemed solved.

As the applications grew in size, the problems involved in supporting the slag weights added complications to the steel supporting structures. That old nemesis of many a designer, expansion, began to trouble the operator. As a furnace went into service the dolomite and brick bottom supported the slag. After a period of operation, the dolomite usually went into solution or was eroded by liquid-slag action, and as the temperature and thickness of the slag bed increased, the dolomite and crevices between the bricks were permeated with molten slag. The slag also filled the space between side walls and brick bed, and eventually the bottom became a practically solid mass of molten slag from wall to wall and from bottom plate to top of bed. While the slag remained liquid, no temperature changes in the furnace or bed caused trouble from resulting expansion, but when the boiler was shut down, the liquid slag remaining in the bottom below the slag tap hole quickly froze into a solid black mass. As it cooled, it contracted, and large cracks appeared throughout the bottom. When the furnace was started up, these deep cracks filled with molten slag which froze, and then the whole bottom expanded horizontally. As the mass could not displace within itself, being still a solid, it forced the side walls of the boiler out. This process continued, and the outward pressure on the wall was exerted until the bed was melted down in a continuous line across the furnace. Then the expansion could force slag upward, and the walls were relieved somewhat. However, the pressure was so great during the first part of the starting period that the walls were strained beyond the elastic limit and took a permanent set. This operation proved to be cumulative, and the walls continued to move out slowly but definitely, a movement of an inch taking about six weeks or two months. This condition increased the number of leaks between side walls and bottom of economizer, and added an extra hazard to operation. The speed of outward-wall movement was a function of the frequency of boiler outages.

A further difficulty appeared, in that the movement of hot slag due to flame or air-blast impingement on the bottom in a constant direction caused whirlpools of hot slag. These in turn eroded the bottom material, and eventually caused hot spots in the supporting plate. Apparently the bottom conditions were ideal as a reducing agent for the production of iron from the ash in the coal, and thick chunks of metallic iron were often found in embarrassingly large pieces in front of burner positions. Sulphur compounds were also determined to be present as separate pools within the molten bed. These were of a lower fusion point and less viscous than the slag proper and were very dense. These compounds melted first, therefore, and filled the cracks. On freezing, the typical needle crystals were formed.

Trend of Slag Bottoms in 1928

To summarize the conditions as they existed about 1928, it was thought that the molten bottom furnace would have the following advantages:

1 The first cost of a boiler should be somewhat less.

2 It permitted the burning of low-fusion-ash coals at very high rates of combustion per cubic foot of furnace volume.

3 With the accompanying high temperatures in the vicinity of the liquid bottom, combustion was completed in a minimum of furnace volume—a distinct economy.

4 Satisfactory turbulence could be obtained in a minimum of space with a direct flame impingement on the bottom without damage being caused by erosion to furnace parts.

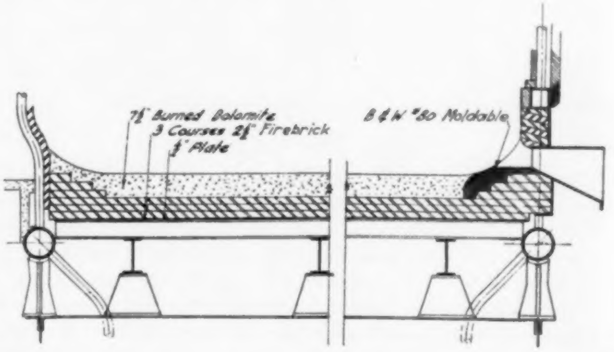

Fig. 2 Boiler 6, Unit 1, State Line Station, Dolomite Bottom

5 Losses due to combustible in the ash could be reduced by burning on the slag surface.

6 A reduction could be made in the height of the boiler-room roof, due to shape of bottom and the reduced furnace volume.

7 Due to completion of combustion in the lower part of the furnace, the gases could be cooled below the fusing point in passing to the generating tubes, and thus prevent slagging of the boiler proper. This assumes that there is provided sufficient cooling surface in the form of tubes or water-cooled walls to absorb the heat.

8 A far simpler system of ash handling could be used and some man power dispensed with.

9 An increase in boiler capacity might be available due to the increased radiation surface of the molten bottom. Absorption of heat by water walls and tubes in the boiler which could "see the fire" should be increased.

10 Reduction of boiler outages due to accumulations of slag not possible to get rid of during normal operation. This of course reduces the spare boiler plant required.

11 Ash from other parts of the boiler setting, such as economizer hoppers and dust-collector hoppers, could be returned to the furnace, melted, and handled with the furnace slag with a minimum of trouble.

12 The production of a "revenue" ash which could be used for fill or railroad grading or in the manufacture of various products requiring a cheap, strong aggregate.

13 The removal of ash is a clean operation, allowing a far cleaner boiler room.

14 It was claimed that a greater percentage of ash recovery was possible due to adhesion of ash particles striking the liquid bottom.

Arrayed against this imposing list of advantages (if true) were the following disadvantages, which were more or less appreciated at the time or which soon developed:

1 It was realized that there were available no materials known to be satisfactory for the make-up of the slag bottom in a permanent form.

2 Expansion of the slag bed when being melted caused wall troubles and leaky tubes.

3 It was difficult to allow for wall expansions parallel to their faces, and at the same time maintain an effective hot-slag seal at the sides and corners.

4 There was a sensible heat loss in the molten slag.

5 High heat radiation from slag bed caused a higher temperature in the furnace below the bottom generating tubes. This caused a somewhat greater slagging up of the boiler tubes.

6 Maintenance of soot-blower units on the lowest generating tubes was high, as these elements were not properly protected against increased temperatures.

The difficulties with slag bottoms in plants of the Chicago district were much greater than in most plants on account of the high percentage of iron and sulphur in the coal, but even so, the troubles encountered were not insurmountable, and there were distinct advantages in continuing the development.

CHICAGO AREA DEVELOPMENT

The first large boiler installation in the Chicago District was made on boiler 6, unit 1, at the State Line Station of the Chicago District Electric Generating Corporation. This boiler was ordered in the summer of 1928 and placed in service in the spring of 1929. The other five boilers of this unit had furnaces as shown in Fig. 1. Except for its flat-brick and dolomite bottom, it was similar in all respects to the other five hopper-bottom boilers of the unit. The boiler was designed to steam at 450,000 lb. of steam per hour normal maximum rating at a working pressure of 700 lb. pressure in the drum and 730 deg. total temperature. The boiler was designed to burn central Illinois "Kincaid" coal[2] at a rating of 1375 per cent and a heat liberation of approximately 34,300 B.t.u. per cu. ft. This boiler was operated as a normal station boiler for the period from July to December, 1929, with the original dolomite bottom constructed as shown in Fig. 2. During this period the following characteristics were noted:

1 Expansion of bed caused a constant outward movement of walls.

2 Slag leaks occurred along walls and at corner seals due to this expansion.

3 Maintenance of brickwork and soot blowers at top of Bailey downtake wall was very high due to high temperatures caused by radiation from slag bed and erosion from molten slag dripping off the generating tubes.

4 Lower generating tubes slagged over more quickly than with the hopper-bottom boilers at the same ratings.

5 Furnace steaming capacity was slightly greater than with the hopper-bottom boilers when boiler was clean, and was reduced after a few hours of steaming, due to difficulty in keeping slag off of the generating tubes.

6 Tapping of slag was simple, though a rather long operation. The time required to tap slag through one spout was about $1^1/_2$ hours for a 24-hour normal day. During this period 425 tons of coal was burned, which contained 64 tons of ash as fired. With an estimated percentage of ash collected in the slag bed of 35 per cent, a total tap per day of 22.4 tons was made. The slag flowed, therefore, at a rate of about 15 tons per hour.

The major trouble experienced in operation was the outward movement of the walls and the subsequent slag leaks. These leaks were dangerous and destructive to the concrete slab beneath the boiler and might have weakened the structural-steel supports of the boiler if not properly protected. It was considered necessary to protect this steelwork by guniting.

After repeated experiments to determine actual wall movements with variation in rating on the boiler, it was finally determined that the walls would be continually moved out unless the boiler were operated at a constant rating without allowing the slag bed to freeze. For temporary protection the bed was trenched all around the walls after eight months of operation, and the walls were jacked back to position. Tie rods and anchors were then installed below the bottom plate, tying the wall headers together with turnbuckles and springs to provide the necessary take-up and expansion. These provisions, though helpful, were not a solution, and it was determined to substitute a new bottom material.

It was demonstrated in this test, however, that the dolomite bottom might be considered a thoroughly practical and economical type for certain applications with less severe conditions and better coal. The trenching operation, which might be required at intervals, is a simple and normal form of maintenance.

SAND BOTTOM

A new bottom was made up, which was placed in service Dec. 17, 1929. Over the bottom plates was laid $2^1/_2$ in. of firebrick. On top of the brick was placed a $2^1/_2$-in. layer of dry sand. This in turn was completely covered with a No. 20 gage plate of special alloy steel containing chromium as a heat-resisting element. On top of this plate was a 10-in. layer of granulated slag for an initial filler. The theory of operation of the bottom was as follows:

After the first slag layer had formed on or in the 10-in. layer of granulated slag, the No. 20 gage plate would form the bottom limit of the molten slag. It was known that the temperature of the molten bed decreased considerably a few inches away from the top and that the fusing temperature of the plate would be a safe amount above the actual temperature at the bottom of bed. The plate would thus prevent penetration of the sand and afford a relatively thin slag bed in which the depth could be controlled. The tap spout was located so as to permit draining the bed to the thin plate level.

This bottom in practice, however, did not corroborate the theory due to the presence of pools of sulphur compounds which attacked the thin plate by erosion and corrosion and permitted penetration of the sand below. The chromium-bearing steel was readily attacked and soon worn through. The presence of the separate sulphur compounds in a molten state was first determined here. They would be in evidence during tapping operations as separate dense streams of very active nature, which would course down the main slag stream like mercury, giving off strong sulphurous fumes which would grow in volume and strength as the bed was tapped nearer the bottom.

This bottom was the first attempt to reduce the side-wall thrust by decreasing the thickness of the bed. It was very simple and relatively inexpensive, and, except for the presence of the sulphur compounds in the Kincaid ash slag, would have been successful.

FIRST COKE-CAN BOTTOM

Experiments were then made on numerous materials,[3] and it was decided to install a bottom made up of round corrugated-steel cans about 8 in. in diameter, filled with compressed coke. The interstices between the round cans were filled with loose coke, iron punchings, coking coal, and other experimental materials. In order to prevent flotation the cans were fastened to the bottom plate with lugs and bolts.

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[2] Analysis of Kincaid coal, as fired: Carbon, 52.05; hydrogen, 3.85; sulphur, 3.75; oxygen, 9.05; nitrogen, 0.80; ash, 15.50; moisture, 15.00; B.t.u. per lb., 10,000; ash-fusing temperature, 1900 to 2100 deg. fahr.

[3] By the Fuller Lehigh Company at Fullerton, Pa.

A 2-in. layer of sand was spread on top of the coke cans to protect the bottom while the initial slag was being formed, and was expected to aid in coking part of the ingredients of the bottom.

It was hoped that the slag would not permeate the coke in the cans, either by absorption or erosion, and that consequently the bottom would remain in a more or less permanent state with its adequate provision to take care of expansion movements. The actual construction is shown in section in Fig. 3.

This new bottom was placed in service on April 20, 1930. It was noted that, as soon as the furnace was heated, gas began to generate from the coke and to seep out along all the walls. The heat of the furnace ignited this gas, and the entire exterior furnace bottom was surrounded for many days with a gas flame of fairly constant volume. It was found that the protective sand layer was blown off the bottom in front of the burners, almost before the boiler was lighted, by the air from the burners, thus exposing the coke to flame impingement from the Calumet burners before the slag could form.

Inspection of the bottom during initial operation showed a number of cans floating in the hot slag. It was at first assumed that these had been insecurely anchored. The wall movement was normal during the first few weeks of operation; that is, the walls returned approximately to cold position each time the boiler was shut down. After this period, however, it was apparent that the slag had begun to permeate the bottom. More cans were observed floating in the bed, the walls started moving out, some major slag leaks developed, and all the old troubles appeared that had occurred on the dolomite bottom. Subsequent examinations of the bed and excavations proved this. Iron was found in metallic form in large chunks weighing as much as 10 tons. No trace of the coke cans could be found in those sections of the bottom most subject to flame impingement and consequently subject to most active slag movement. The photograph of the bottom in section (Fig. 4) is typical of the conditions found in areas subject to slag movement. While theoretically and actually coke is known to resist slag erosion and penetration to a very marked degree, the failure of this bottom is laid to the presence of the sulphurous compounds existing in the molten bed as separate bodies. These compounds are very dense and actively attacked the coke by erosion and chemical combination, aided of course by the high temperature and ideal oxidizing conditions. This bottom is still in service at this writing, however, though it has been necessary to trench the bottom around the walls and to jack the walls in place. The area of coke cans removed in this operation and other excavations has been filled with the original brick, with magnesite and chrome ore (KN) added.

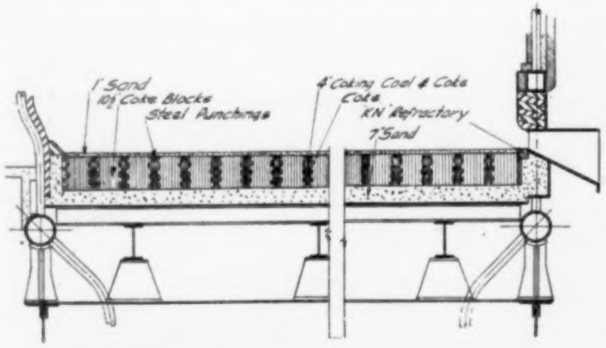

Fig. 3 Boiler 6, Unit 1, State Line Station, Coke-Can Bottom Diagram

First Water-Cooled Bottom

With the rather disappointing performance of this bottom in mind, it was decided to attack the problem of securing a permanent bottom from a new angle. One of the old hopper-bottom boilers at Toronto Station had been converted to a slag tap type by adding a bottom on top of the water tubes of the hopper. It was felt that the successful operation of this bottom was due to the tubes underneath, which carried off heat from the bottom materials and tended to preserve the bottom in its original condition. Probably the wedge action accidentally secured on two sides by placing the bottom on sloping hopper tubes aided considerably. A new water-cooled bottom was designed, therefore, to replace the hopper bottom on the adjacent boiler 5 at State Line Station. This new bottom had a V-shape, though with less slope than the hopper bottom, and was designed to carry a very thin slag bed.

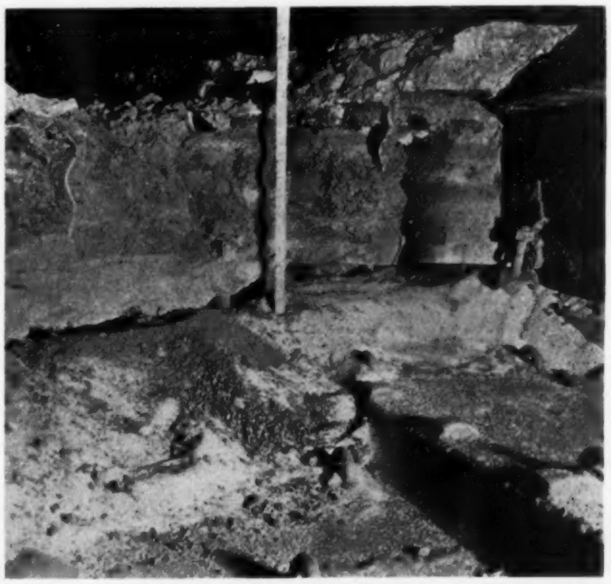

Fig. 4 Boiler 6, Unit 1, State Line Station, Coke-Can Bottom Section of Floor

It was determined to limit the height of the molten bed by tapping more often. With this design, therefore, the slag left in the bottom to freeze would be a very small quantity, and would not present the expansion difficulties of the previous bottoms. The success of the block-faced water side walls in resisting erosion and trouble on the slag line on previous wet bottoms led to the belief that the water-tube cooled bottom, properly faced, would supply the answer to the problem.

Boiler 5 Converted

The bottom installed on boiler 5 (see Fig. 5) consists of $3^1/_4$-in. tubes on 6-in. centers, insulated on the outside. The tubes are completely faced on the furnace side with Bailey bare cast-iron blocks. Over the blocks was placed a layer of dead burned magnesite varying from 3 to 14 in. in thickness. This was in turn surfaced with a 1-in. layer of pyrites. Granulated slag 3 in. deep was then added to control the temperature on the bottom during the first start. This bottom had a total area of about 700 sq. ft., practically the same as boiler 6. Some 1-in. air pipes were installed through the bottom near the throat and connected to the station compressed-air lines. It was thought that oxidation

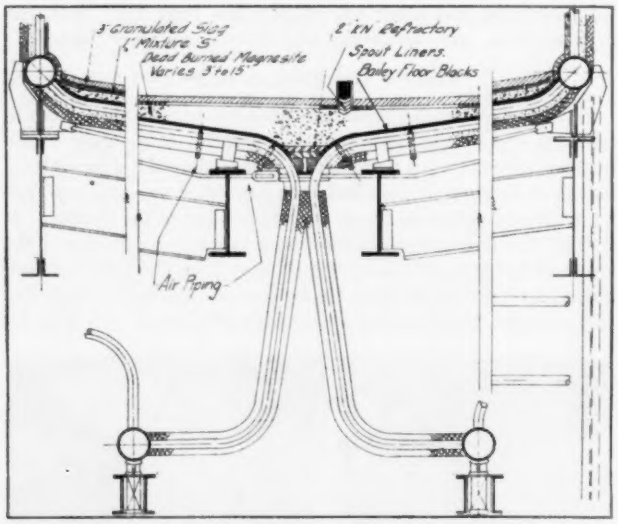

FIG. 5 BOILER 5, UNIT 1, STATE LINE STATION, CONVERTED WATER-COOLED BOTTOM DIAGRAM

of the sulphur compounds could be accelerated in this manner, but in subsequent tests it was found that insufficient penetration of the bed by the air defeated the scheme.

The boiler was first brought up so as to give a rate of increase in temperature at the surface of about 50 to 100 deg. per hour for a period of 36 hours. The pyrites then combined with the magnesite to form a hard crust about 1 in. thick. This crust is an iron-magnesite compound which seems to be solid at 3000 deg. fahr. at surface, and from which the slag can be tapped down to the surface without appreciably affecting the crust by erosion or exposure to furnace-heat conditions.

Figs. 6, 7, and 8 are photographs of the furnace interiors taken at intervals after a considerable period of operation at high ratings. All are taken with furnace down and bottom frozen. Fig. 6 shows a portion of the floor. The pool in the foreground is slag, while along the borders of the bottom can be seen the original magnesite. Fig. 7 shows the floor adjacent to the slag tap hole, and illustrates the cooling cracks which form upon the freezing of the bed. The rough material adjacent to the side wall is the original magnesite. Fig. 8 shows the shrinkage crack between floor and side wall. This crack is continuous along the entire wall, and probably fills first with molten material when the furnace is brought up.

Except for two minor leaks caused by holes being left in the side-wall seals accidentally during construction, this bottom has given no trouble. The wall movement has been practically normal. The lack of a sustained outward movement is caused by the inability of the relatively thin slag bed and bottom to exert a heavy thrust on the walls. That this thin bed can be successfully maintained is undoubtedly due to the cooling effect of the water tubes immediately below.

Cracks occur in this bottom material on shutting down as in the other bottoms, but a system of introducing compressed air into the bottom during the starting period to prevent molten slag getting in them has been tried out with success. This boiler has been in operation and under test for the period from Aug. 28, 1930, to Dec. 11, 1930, and has shown no major troubles or defects. It has been subjected to abnormally hard service conditions during this period. Its obvious disadvantage is in invest-

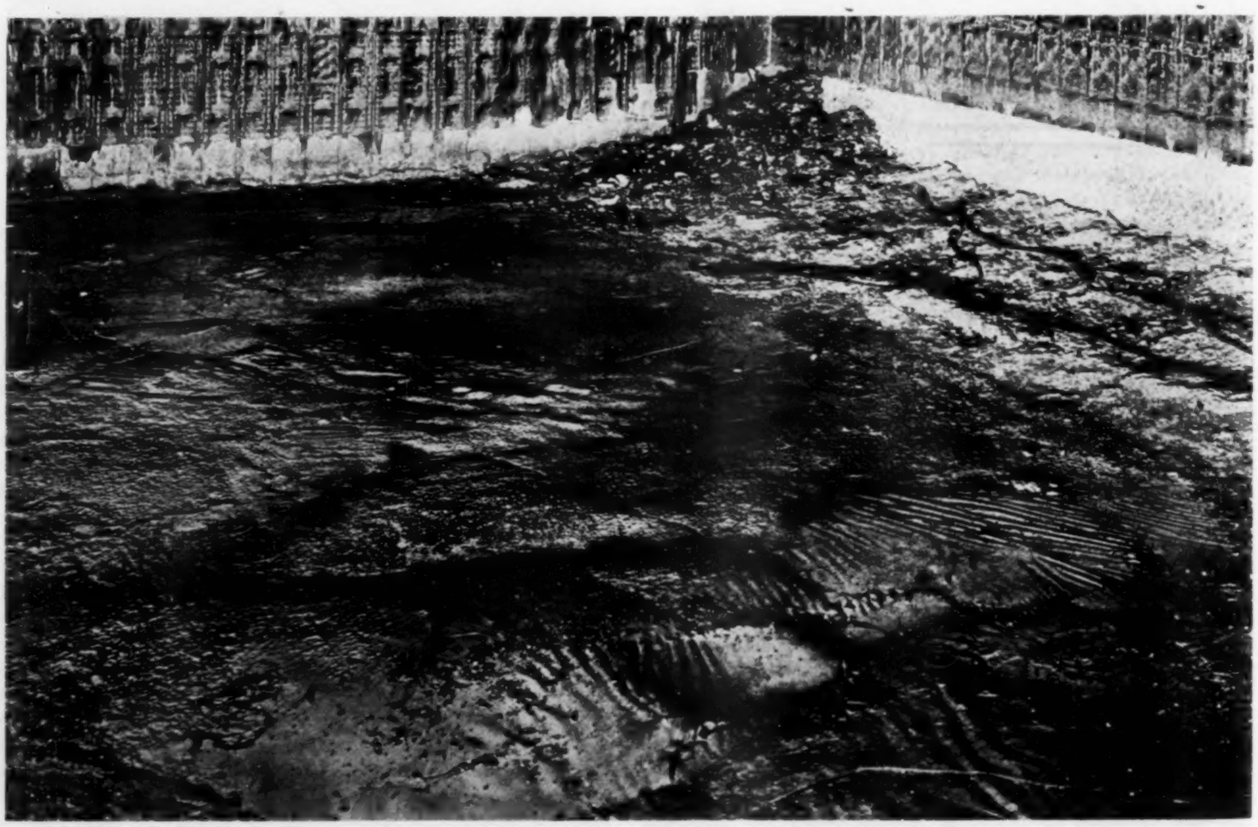

FIG. 6 BOILER 5, UNIT 1, STATE LINE STATION, VIEW OF CORNER

ment cost. It is a much more expensive type than the dolomite bottom, and thus offsets one of the advantages which should be available in this type of bottom.

The boiler was again placed in service on Jan. 1, 1931, after some alterations in the upper furnace for experimental purposes, and additional tests were made during January.

Late Installations

In an effort to design a bottom for handling molten slag which combined the virtues of low first cost, low maintenance, and no wall trouble, the air-cooled bottom was introduced late in the fall of 1930. A typical example of this type of construction, illustrated in Fig. 9, is now installed on boiler 1, at the Michigan City Generating Station of the Northern Indiana Public Service Company.

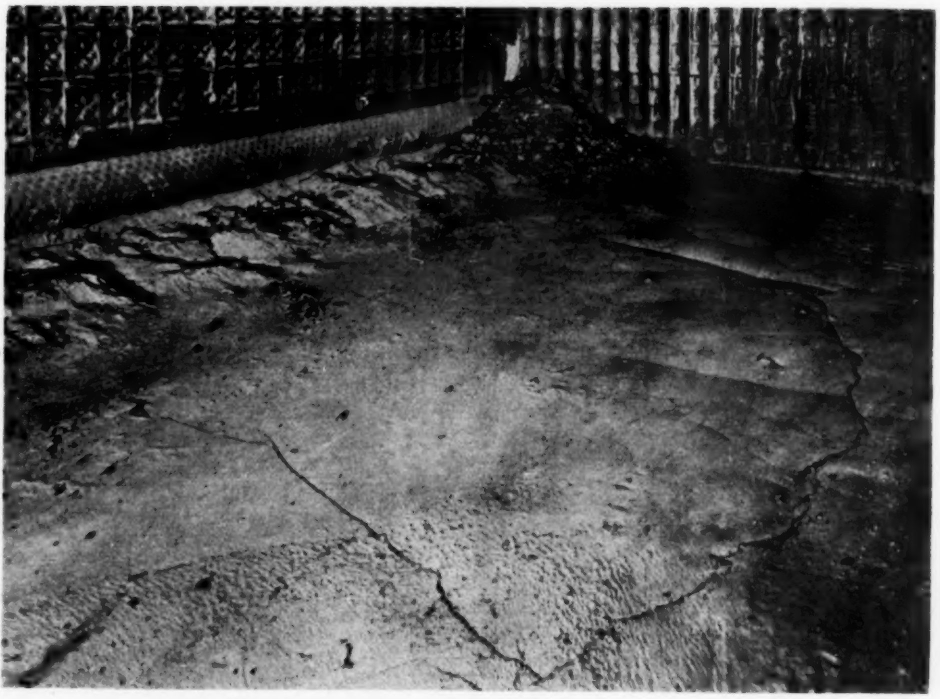

Fig. 7 Boiler 5, Unit 1, State Line Station, View of Floor Cracks

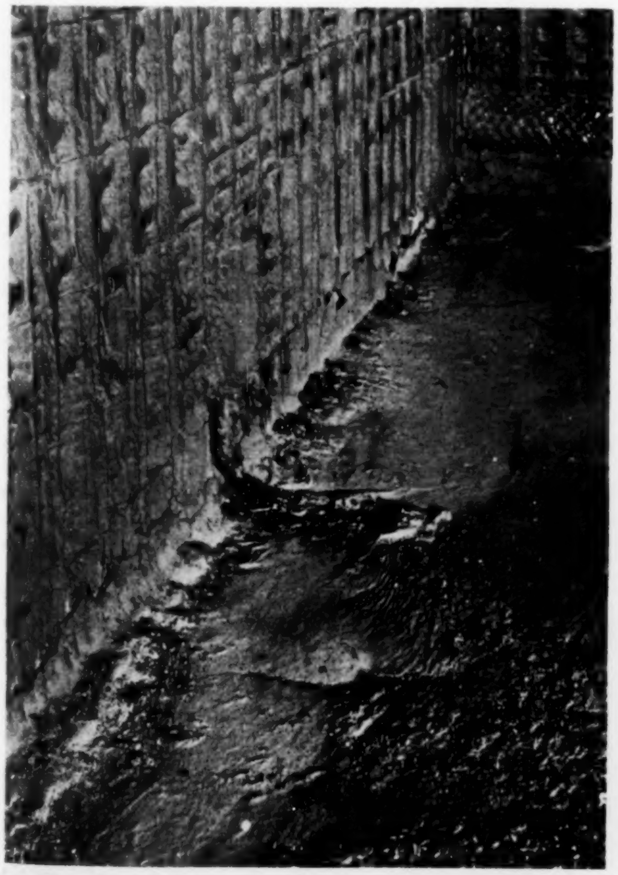

Fig. 8 Boiler 5, Unit 1, State Line Station, View of Slag Tap Hole

A $^1/_4$-in. flat bottom plate is provided, supported by the usual structural members, but unique in that the sides are turned up in a generous curve like a frying pan. This feature was intended to provide a means of taking up expansion within the bottom itself, without carrying thrust into the side walls. Firebrick was laid $12^1/_2$ in. thick on top of the plate, and the brick in turn was covered with 3 in. of magnesite mixed with a special liquid known as "liquid U." A layer of $1^1/_4$-in. split brick covered this mixture, laid with a slope toward the slag tap hole. A mesh screen was placed on the split brick, and 3 in. of granular slag was added for original firing.

At frequent intervals throughout the bottom vertical perforated pipe nipples were installed extending about 6 in. above the bottom plate. These were packed with The Babcock & Wilcox Co.'s No. 80 moldable mix, and dry magnesite was added around the nozzles. These pipes were connected with headers to a motor-driven compressor, which developed about 29 in. of air pressure to insure an even distribution of air to all nozzles. The air is intended to circulate from the nozzles to the split-brick "roof," and then flow to the side walls and through orifices into the furnace. The air thus acted as a cooling medium under the roof, similar to a water-cooled bottom. Metering equipment was also provided to meter all air fed into the furnace through the bottom. This type of bottom is more expensive in first cost than the dolomite type, due mostly to the compressor and piping. It has also the disadvantage of requiring continuous auxiliary power for proper operation. An attempt has been made on this bottom to deflect the heavy side thrust of the expanding slag bed upward by providing special ledge blocks on the wall tubes at the slag line. These blocks extend below the slag bed at a fair angle, the theory being that side expansion of the slag crust will be turned upward and the bed cracked or broken up by the turn.

This boiler is now ready for service (January, 1931), but has not yet carried any appreciable load, pending start of turbine operation.

Flat Water-Tube Bottoms

A development of the water-tube cooled bottom as successfully tried out on boiler 5, at State Line Station, but with a decided saving in first cost, is exemplified in the two other boilers installed at Michigan City. The first of these (boiler 2) is equipped with what may be called a transverse, straight-tube, water-cooled bottom (see Fig. 10). It has the usual flat bottom plate without the frying-pan feature because of the relatively low temperatures expected on it. Above the plate is a layer of loose-laid firebrick which varies from 5 to $7^1/_2$ in. in thickness. Then the $2^1/_2$-in. cooling tubes are laid in, extending from an outside header at the front of the boiler to another header at the tap or downtake wall. These headers are connected to the boiler circulating system with special tubes. The bottom tubes have a slope toward the downtake wall of about $^1/_4$ in. per ft. Above the tubes are laid $^1/_{16}$-in. steel plates overlapped to provide for expansion. On the plate are 4 in. of dry magnesite and then 3 in. of granulated slag, all sloped toward the tap hole. This boiler is also ready for service at this writing, but has not yet carried load.

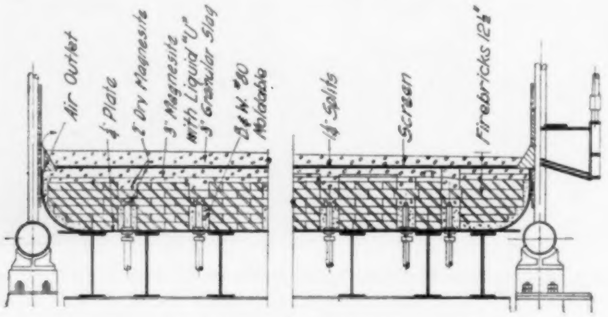

Fig 9 Boiler 1, Michigan City Generating Station, Air-Cooled Bottom

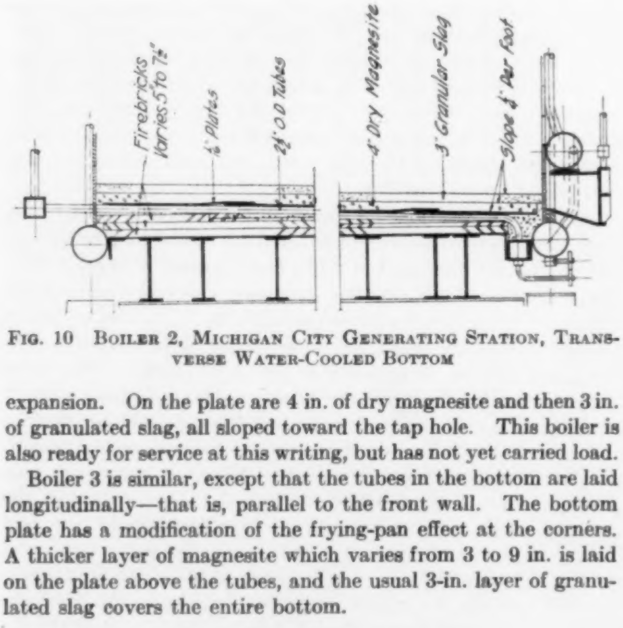

Fig. 10 Boiler 2, Michigan City Generating Station, Transverse Water-Cooled Bottom

Boiler 3 is similar, except that the tubes in the bottom are laid longitudinally—that is, parallel to the front wall. The bottom plate has a modification of the frying-pan effect at the corners. A thicker layer of magnesite which varies from 3 to 9 in. is laid on the plate above the tubes, and the usual 3-in. layer of granulated slag covers the entire bottom.

Other Applications

Other applications in the Chicago district include the following slagging boilers on major power plants ordered or in service:

Waukegan Power Station No. 6 of Public Service Company of Northern Illinois: Unit 4, one water-cooled boiler and two coke-can bottom boilers. Unit 5, three water-cooled boilers.

Powerton Power Station of Super-Power Company of Illinois: Unit 2, boiler 2-1, dolomite bottom; boiler 2-2, hexagonal coke-can bottom; boiler 2-3, corrugated coke-can bottom. Unit 3, boiler 3-1, sloping water-cooled bottom; boilers 3-2 and 3-3, air-cooled bottoms.

Test Data

Some miscellaneous data and curves are given in Tables 1 to 3 to illustrate the physical nature of the molten slag from Kincaid coal.

TABLE 1 TYPICAL ANALYSIS OF SLAG ASH

	Buffalo[a]	State Line	Comparative analysis Slag and flue dust	Flue dust
Carbon	0	0	0	0.98
Silica (SiO_2)	44.52	37.42	43.43	46.36
Aluminum oxide	18.64	16.83	20.98	20.53
Iron and iron oxides	31.25	34.90	27.23	21.52
Titanium oxide	0.76	0.66	0.83	0.85
Phosphorus pentoxide	0.25	not det.	0.18	not det.
Calcium oxide	1.33	7.51	6.28	6.40
Magnesium oxide	1.10	0.99	0.98	not det.
Manganese oxide	not det.	not det.	0.09	not det.
Sulphur trioxide	1.80	0	trace	0.97
Water	0	0	0	1.45
Chlorine	0	0	0	0.94
Undetermined	0.35	1.69	0.01	0
Temperature of molten ash, deg. fahr.	2600–2800	2600–2800		

[a] H. M. Cushing and R. P. Moore, paper presented at First National Meeting of A.S.M.E. Fuels Division, St. Louis, Mo., Oct. 10–13, 1927. Trans. A.S.M.E., paper FSP-50-10, vol. 49–50, 1927–1928.

TABLE 2 SLAG WEIGHTS

Molten, lb. per cu. ft.	185
Cast in block, lb. per cu. ft.	200
Granular, lb. per cu. ft. (dry)	72.5
Dry ash:	
Furnace, lb. per cu. ft.	65
Economizer, lb. per cu. ft.	59[a]
Precipitator, lb. per cu. ft.	55
Fusion point dry ash, deg. fahr.	2250

[a] Variation, 25 to 80 lb. per cu. ft.

TABLE 3 TYPICAL SIEVE ANALYSIS OF FLUE DUST AT BOILER OUTLET

Through 48 mesh	100
Through 100 mesh	99.7
Through 150 mesh	99.3
Through 200 mesh	97.3
Through 325 mesh	91.5

Test data on distribution of ash throughout the boiler setting have been rather unreliable to date. Although many ash balances have been made and reported, there are some factors that are extremely hard to determine accurately, and the unaccounted-for losses are sometimes charged against other partially determined parts of the balance. For instance, it is common practice to assume that all ash fired should be accounted for in slag or ash to the flue. This is incorrect, as the "ash in the coal" determination is commonly made at low temperatures of about 900 deg. fahr., while the burning of the coal occurs in temperatures around 2800 deg. fahr. At these high temperatures it is known that a portion of the ash will be volatilized, and so may not be measured as dust at the stack. This may represent a percentage of the ash as high as 15 per cent.

The distribution of ash, as given in Table 4, varies with boiler rating, the collection in the furnace increasing with rating, as is also the case with a hopper bottom.

TABLE 4 TYPICAL DISTRIBUTION OF ASH

Ash content by analysis (dry basis) of average Kincaid sample, per cent	13.97
Total ash per lb. dry coal, lb.	0.1397
Ash to flue per lb. dry coal, lb.	0.05
Ash collected as slag per lb. dry coal, lb.	0.06
Ash collected on boiler tubes, economizer hoppers, etc., and removed by soot blowing operations, lb.	0.03

TABLE 5 ASH COLLECTION IN FURNACE PLOTTED AGAINST RATING

Boiler rating, lb. per hour	220,000	300,000	375,000
Collection in slag bottom, per cent	25	24	36
Collection in hopper bottom, per cent	23	26	36

A typical curve of collection in the furnace plotted against rating is given in Table 5. These readings must be considered as an approximation only, as there are insufficient test data available for more exact figures.

CONCLUSIONS

1 The slag-bottom boiler is undoubtedly in the process of major development at this writing. It will require considerable time and further experimentation with designs and materials before all the facts required for a final analysis may be established. The problems involved when using many coals other than Kincaid are undoubtedly easier of solution.

2 Sufficient progress has been made, however, with the present installations to establish the following summary of advantages and disadvantages as they may be considered to exist today:

Advantages:

(*a*) A practical molten-slag bottom can be installed at less first cost than a water-cooled hopper bottom.

(*b*) Any low-fusion-ash coal may be burned in pulverized form in an economical furnace using the molten-slag bottom.

(*c*) Any type burner may be used and turbulence of flame assisted by impingement on the molten bottom, if required, without damage to furnace or bottom.

(*d*) A saving in building height and cost may be obtained by using a lower setting. Very little headroom is required for handling the granulated slag.

(*e*) A simple and cleanly system of ash handling can be used.

(*f*) Dry dust collected from other parts of the boiler may be conveniently and automatically melted in the furnace bottom, thus eliminating duplication of ash handling in dry and molten form.

(*g*) Increased radiation absorption from the slag bed develops higher efficiencies from the exposed surfaces.

(*h*) Recent tests in one of the Chicago area stations seem to indicate that fairly coarse coal may be burned readily with an appreciable saving in mill maintenance.

Disadvantages:

(*a*) Expansion movements of the furnace-bottom materials cause wall troubles and slag leaks.

(*b*) A satisfactory and permanent seal between walls and bottom has not yet been developed.

(*c*) The high temperature existing in the vicinity of the lower generating tubes due to radiation from the slag bed causes fouling of these tubes, and consequently the boiler must be blown or hand-lanced more often.

(*d*) Low soot-blower elements near the furnace must be protected from high temperatures.

3 A balancing of the preceding items from the designing engineer's viewpoint would indicate that the slag tap furnace has certainly justified the efforts expended to date. Its development will proceed more rapidly with the present confidence existing in the correctness of its basic principles.

Discussion

P. B. RICE.[4] The Special Research Committee on the Removal of Ash as Molten Slag From Powdered-Coal Furnaces would submit a statement of the work being done under the supervision of the committee by the Pittsburgh Experiment Station of the U. S. Bureau of Mines.

The program which is being followed is a sequence of the limited studies made by the bureau at the Toronto Station of the Pennsylvania-Ohio Power & Light Company, the results of which were presented to the Society[5] in 1929. These studies showed that a coal having an ash with a high fusion temperature did not form a molten bed, so that the ash could not be removed by tapping. It followed that it was necessary to know the possibility of adding materials to flux the ash deposited in the furnace. The bureau undertook to make such a study, the committee supplying funds for all special equipment required and the extra money required for field work, should this be necessary.

The work was started in September, 1929, and it is expected that a full report can be submitted later this year. The program called for the following method of attack:

(1) *Small scale laboratory tests*, by which the fluxing effects of admixtures were to be determined under the ideal conditions of intimate mixture and uniform temperature. It was not proposed to undertake an extended fundamental investigation of the relationship of viscosity to the composition of slags, because years would be required for the complex systems involved; rather it was hoped that a spotting of the field would be sufficient.

(2) *Large scale laboratory tests* in a gas-fired furnace having a slag hearth of $2^1/_2$ by 3 ft. This area would be large enough to check conclusions arrived at from the small-scale tests. It was proposed also to use this furnace to study mass effects, influence of size, and other factors simulating the application of fluxing to a full-size furnace.

(3) *Plant Tests*. The objects of work conducted at plants would be to confirm the laboratory results and to develop the mechanical details for the addition of the flux.

Instead of working with synthetic mixtures, as would be done in a truly scientific investigation, large samples of ashes or slags were obtained from eight power stations using coals which fairly well cover the range of variations in composition of the ash. These ashes had fluid temperatures, determined by the A.S.T.M. method, of from 2470 to 2940 deg. fahr.

Most of the work to date has been on the small-scale laboratory tests, the apparatus consisting of a platinum-wound furnace and platinum crucibles in which to heat the mixtures. The technique consists, in brief, in determining the temperature at which the molten mixture has a standard viscosity as measured by the feel of a platinum stirring rod; the viscosity used as standard corresponds to that of a slag which gave free flow at Toronto. Determinations are not made if the fluid temperature is above 2650 deg. fahr., which was taken as the maximum possible of a slag bed; the lower limit was fixed at 2300 deg. fahr., which is low enough for any application of the slagging method likely to be used.

Materials studied as possible fluxes have included iron oxides, limestone, dolomite, magnesite, fluorspar, saltcake, silica, open-hearth waste slags, low-grade clays, as well as mixtures of some of these. This laboratory work has been, in effect, a study of various component systems, but the complete field has been covered only for a few of the infinite number of combinations possible.

Only a few large-scale laboratory tests have been made, but further work is now under way to study the mass effect and the time required for the flux to penetrate into the bed; also to study the relative action of fluxes which themselves melt at a low temperature, such as saltcake and those which are refractory by themselves, such as limestone.

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[4] General Engineer, Allied Engineers, Inc., New York, N. Y.; Chairman, Special Research Committee on the Removal of Ash as Molten Slag From Powdered-Coal Furnaces. Assoc-Mem. A.S.M.E. (Mr. Rice not being present, the report was read by Mr. P. Nicholls, of that committee.)

[5] Sherman, Nicholls, and Taylor, "Study of Some Factors in Removal of Ash as Molten Slag From Powdered-Coal Furnaces," A.S.M.E. Trans., paper FSP-51-51, pp. 399–413.

The occasion has not arisen as yet to do field work because of a plant being in difficulties due to inability of the slag to run, and the assumption is that coals with refractory ashes have not been used. Studies are, however, being made for one plant which burns oil and in which the deposited slag is not fluid, and recommendations for fluxes have been made.

One of the objects of the committee in presenting this report is to acquaint those using or considering the use of slagging furnaces with the availability of help from the committee or the bureau; also to invite the cooperation of individuals interested in the subject. The benefits possible by the use of fluxes are not limited to continuous slagging of the whole bed. The following conditions are occasions where the use of a flux might be necessary; they are stated in the order of their probability of occurrence:

(*a*) That a flux would be useful, and perhaps sometimes necessary, to make more fluid the portion of the bed around the tap hole; thus the flux would be used only occasionally and in relatively small quantities.

(*b*) That a flux might be used occasionally over the whole bed to make it possible to more thoroughly drain the bed before the boiler was shut down for repairs; the use of a flux would permit of this being done without maintaining an abnormal overload over a long period. The reason for desiring to drain the bed might be one of several, of which one might be the avoidance of strains because of the shrinkage of a large mass of slag.

(*c*) That a flux over the whole bed might be required occasionally because a shipment of coal contained ash with a higher than normal fusibility.

(*d*) That in a large furnace the use of a flux was required continually because the deposited ash was never fluid enough to run.

(*e*) That the slagging-bottom design was applied to smaller furnaces and boilers in which the furnace temperature was materially lower than is customary in larger furnaces; consequently, the range of coals that could be used would be narrower and the need of a flux more probable.

The original reason for starting the investigation was to protect against (*c*) and (*d*), which assume that the deposited ash is not fluid at 2550 to 2600 deg. fahr., or, in other words, a fluxing effect is required to reduce a high-fusion ash to one fluid at 2550 deg. fahr., which is assumed to be the temperature at which the bed can be maintained in large furnaces. The other three conceived possibilities involve the assumption that the deposited ash is fluid at or below 2550 deg. fahr. and that the flux must reduce its fluid temperature still further.

The latter conceptions will explain why the attempt was made in the tests to bring the fluid temperature down to 2300 deg. fahr. or lower, and why studies were made on ashes which had normal fluid temperatures lower than 2550 deg. fahr.

The advisability of extending the investigation to plant application and experimentation will depend on the experience of those operating furnaces equipped with slagging bottoms. Their reactions and recommendations are required as guides.

P. Nicholls.[6] As stated in the report presented by Mr. P. B. Rice, we have been studying some of the factors associated with the slagging-bottom type of furnace as applied to the burning of pulverized coal. Although we have had samples of ash and slag and have obtained data from several operating companies, yet previous to the presentation of Mr. Baker's paper we had not been in touch with the State Line Station. Since then, they have supplied us with small samples, and also with some further records of their own analyses.

The form of construction of the bottom or hearth is outside our investigations except as far as it affects the deposition of ash, the depth of the bed of slag, and the temperature gradient through it, or as the bottom reacts with the slag.

The author attributes the peculiar trouble they have had and the fluidity of the slag to the high percentage of iron. The writer suggests that as far as the ash in the coal is responsible, it is because of its high percentage of calcite, which is much smaller in Eastern coals. Our study of the fluxing phase has developed a fairly complete set of data on the relation between the "flow temperature," or the temperature at which the slag is fluid enough to flow, and the composition of mixtures of coal ash and fluxes; the most interesting set are those including iron and lime. Iron oxides by themselves will not produce a slag fluid at a very low temperature. The "flow temperature" decreases as the percentage of iron oxide increases up to about 22 per cent, when it is about 2640 deg. fahr., with 1 per cent lime; further increase leaves the flow temperature stationary, but somewhat decreases the fluid range. Slag having the equivalent of 42 per cent ferric oxide and 1.4 lime has a flow temperature of 2590 deg. fahr.

The sample of slag from State Line which we received analyzed: SiO_2, 38.6; Al_2O_3, 15.4; Fe_2O_3, 11.7; FeO, 19.3; Fe, 3.1; CaO, 7.6; MgO, 0.8; SO_3, 0.2.

The flow-temperature determination gave 2545 deg. fahr. for the as-received sample, and 2495 deg. fahr. after the iron was oxidized, which values are about 80 deg. lower than they would have been if the lime were around 1 per cent. In addition, the lime increases the fluid range between the plastic and flow state, changing it from a value of 80 deg. to one of 250 deg. The State Line slag differs from the Deepwater in having a 2.5 silica-alumina ratio as against 1.72, which also influences the effectiveness of the 7.6 per cent of lime, which amount has little fluxing effect on the Deepwater.

The author does not emphasize the influence of the flame impinging directly on the bed and the comparatively small travel before it strikes the slag. The suggestion is made, however, that this is the real distinctive feature of the State Line installation and the primary reason for the peculiar properties of the slag as compared, for example, with Deepwater or Toronto. The direct impingement will occur before combustion is complete, and one would expect that it will maintain a highly reducing atmosphere over that part of the bed; it will also produce a high temperature there and, as the author states, will tend to cause a motion at the surface and probably considerable convection in the slag.

Does not this offer a simple explanation for the peculiarities? The particles of iron pyrite will take longer to burn than the coal, and many of them will strike the bed unburnt and, because of their density, with considerable force. There will be a greater probability of their being held by the surface, and, because of the reducing conditions, will have no opportunity to burn. If they are not at once buried in the slag, and even if they melt, their greater density will tend to make them sink and ultimately reach the lowest level permitted by the temperature gradient in the bed.

Speculation on the reason for the large masses of molten iron is questionable, but it is quite possible that the metallic iron is formed in the reducing area at the surface, and that an explanation based on action at the bottom may not be required. The metallic iron would sink lower than the sulphide, and there would be more chance of it remaining there during the tapping. Even so, some of the burning snake appearing on the surface during tapping may be iron burning, though its presence would be hidden by the burning sulphur.

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[6] Supervising Fuel Engineer, U. S. Bureau of Mines, Pittsburgh Experiment Station, Pittsburgh, Pa. This discussion published by permission of the Director, U. S. Bureau of Mines. Not subject to copyright.

Although the author does not so state, it may be assumed that one of the objectives of impinging the flame on the bed was the expectation that it would result in the deposition of an increased quantity of ash. If this be so, how far was it successful? The fact that the percentage of the ash collected in the wet bottom was no greater than that in the dry, does not indicate much success.

A determination of the "flow temperature" of the average ash of the Kincaid coal was made and gave 2580 deg. fahr. in an oxidizing atmosphere, showing that if the whole of the ash of this coal could be deposited on the bed, then it could be tapped successfully.

One would think that the direct impingement of the flame would have tended to cause the retention of a large percentage of the fluxes in the bed, that is, to cause a large selective action. Actually the selective action is less than at Deepwater with its horizontal flame.

	Fe_2O_3 in the coal	Fe_2O_3 (equivalent) in the slag
State Line..........	29.0	37.0
Deepwater.........	25.5	42.0

This selective action in the furnace as fixing the quality and the quantity of ash deposited and of fly ash is fundamental in the slagging operation, and an understanding of it is necessary if maximum slag deposition is desired. The Special Research Committee has approved the suggestion that a study of the selective action shall be the next phase of the investigations the bureau is making on factors connected with the use of fuels in slag tap furnaces.

Design Features and Operating Results of Fairfield Blast-Furnace Power Plant

By F. G. CUTLER,[1] ENSLEY, ALA.

This paper discusses one of the latest blast-furnace power plants, built at the Fairfield Steel Plant of the Tennessee Coal, Iron & Railroad Company, and shows that by the improvement in efficiency in general it was possible to increase the electric power generated from surplus blast-furnace gas per ton of product about 50 per cent over what was average practice nine years ago, this result being due to increased efficiency of boiler plant, as well as to the increased amount of blast-furnace gas available. The operating results for a week's period are given, as well as descriptions of the automatic combustion control employed, the system of utilizing surplus steam from an adjacent plant, the method of handling water systems, and other unusual features.

ONE of the most modern power plants in the steel industry is located at the Fairfield Works of the Tennessee Coal, Iron & R.R. Company, and the operation has been so satisfactory that it is believed that a presentation of some of the unusual features and operating results will be of interest to those engineers who are concerned with blast-furnace power plants.

Descriptions of the plant have been published so that it is not necessary to list the several units composing the station, but reproductions of drawings are included in order to show the principal features.

Fig. 1 is a view showing blast furnaces Nos. 5 and 6 and the power house; Fig. 2 is a general cross-section of the power plant showing the relation of the boilers to the turbo-blowers and power units; Fig. 3 is a cross-section of the boiler setting, showing the relation of the boiler to its auxiliaries, and Fig. 4 a view of the coal-pulverizing and water-treating plants and cooling towers.

No complete test has been made of the entire plant, but as it is equipped with recording meters, gages, and pyrometers, it is possible to check the operating efficiencies continually, and in the following discussion some of the operating results will be presented.

Since it would not be possible within the scope of the paper to furnish complete data, especial attention will be given to answering the questions most frequently asked by those who have visited the plant.

The power plant consists of five boiler units; two 20,000-kw. turbo-generators and three 75,000-c.f.m. turbo-blowers, with auxiliary equipment.

Each boiler is equipped with economizers, air heaters, and superheaters, and the combustion chamber is equipped with air-cooled hollow side walls, a bare-tube front-wall section above the burners, and tubes faced with refractory and cast-iron protection for the back of the combustion-chamber hopper. Four of the five boilers are equipped with powdered-coal burners supplied with coal from storage bins. These bins are filled through a 5-in. pipe line from an adjacent coal-pulverizing plant which has steam-heated rotary-type coal driers, the coal burned being a middle product of the coal washers.

Automatic control of air and gas supply is provided, but there is no automatic control of the powdered-coal feeders as the amount of coal burned at present would not justify its installation.

The turbo-generators and turbo-blowers have surface condensers, and, owing to the shortage of plant cooling water, the condensing water is cooled by natural-draft cooling towers.

There are two steam pressures used at this plant, the higher, of 325 lb. gage and 200 deg. fahr. superheat, supplying the main units and main boiler auxiliaries (fans and two feed pumps driven by geared non-condensing turbines), and the lower, of 200 lb. and about 100 deg. superheat, supplying the condenser auxiliaries, boiler soot blowers, and the blast-furnace plant for operation of bells, mud guns, etc. The line supplying this 200-lb. steam is connected to a battery of waste-heat boilers (located in the adjacent open-hearth plant), this boiler plant supplying steam to a pump and compressor station, and several mills in the Fairfield steel works.

Steam is supplied from the high-pressure system to the low-pressure system through two reducing valves controlled from the 200-lb. system.

The condenser circulating-water pumps of the five main units are each driven by both motor and geared turbine, each of sufficient capacity to drive the pump, and the steam drive is utilized to the extent of the supply when available and to maintain sufficient exhaust-steam supply to the deaerating type of open feedwater heaters.

It is therefore possible to supply steam from the blast-furnace system to the open-hearth system and also to absorb surplus steam from the open-hearth system into the blast-furnace system for operation of auxiliaries when it is available, and maintain a uniform pressure on the mill steam system, which has a variable demand and which is supplied from waste-heat boilers. This connection does away with the necessity of auxiliary gas fuel for the waste-heat boiler.

The condensate is returned to a storage tank above the heaters, and make-up is supplied to an underground sweet well from the water-treating plant which furnishes the open-hearth boilers

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[1] Chief of Bureau of Steam Engineering, Tennessee Coal, Iron & R.R. Co. Mem. A.S.M.E. Mr. Cutler was educated at the University of Kentucky, receiving the degree B.M.E. in 1901 and M.E. in 1904. He was assistant in the steam engineering department of the South Works of the Illinois Steel Company from 1901 to 1906. He then became steam engineer of the Tennessee Coal, Iron & R.R. Co. at its Ensley plant, and later was appointed to the position he now holds. He was a member of the Executive Committee of the Birmingham Section during the organization period in 1915, served as its chairman in 1920–1921, and has presented papers before the Society dealing with steel works problems. He is also a member of the American Iron & Steel Institute, the Birmingham Engineers Society, and the Iron & Steel Electrical Engineers.

Contributed by the Power Division and presented at the Semi-Annual Meeting, Birmingham, Ala., April 20 to 23, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

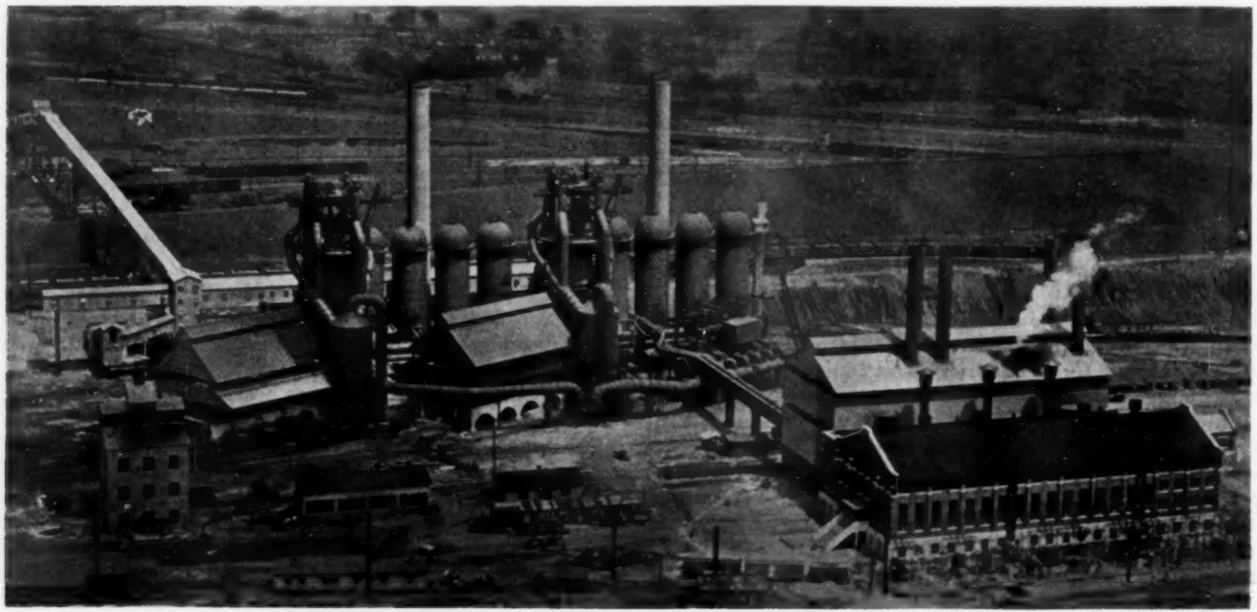

FIG. 1 BLAST FURNACES NOS. 5 AND 6 AND POWER HOUSE, FAIRFIELD WORKS, TENNESSEE COAL, IRON AND RAILROAD COMPANY

with feedwater. Water from the sweet well is admitted to one or more of the blower condensers and pumped to the feedwater tank over the heaters.

When the plant was designed it was expected that there would be a surplus of steam from the open-hearth plant at all times, but due largely to increased demands of steam by the mills there is usually considerable steam delivered from the blast-furnace system to the open-hearth system. As a consequence of this loss of high-pressure steam it is necessary to add about 10 per cent of treated water to the system.

The cooling water required by the power house to make up the loss at the cooling towers and for water supplied to the blast-furnace high-pressure water system, ash-sluicing system, etc., is first treated with lime and iron sulphate and filtered in a separate plant (shown in Fig. 4), and then delivered through the generator air and transformer oil coolers to a sump in the power-house basement. Motor-driven pumps deliver this water to the cooling-tower system and to the turbo-blower jackets so that the total fresh water used by the plant is usually less than 3 per cent of that circulated, and the various cooling surfaces are therefore found to be relatively free from contamination.

The blast-furnace plant is equipped with eleven Cottrell-type electric precipitators, eight of which clean all the gas from the two furnaces going to the boilers and stoves. The stove gas is sent by a separate main from the boiler house through the three remaining precipitators, thus receiving a double cleaning.

The average amount of flue dirt in the blast-furnace gas, as shown by the Brady method and by the continuous dust-recording equipment described by Dr. A. W. Simon of the company in a paper presented at the October, 1930, meeting of the Ameri-

TABLE 1 BLAST-FURNACE AND POWER-PLANT DATA, FAIRFIELD STEEL WORKS

	Week of Sept. 7–13, 1930							7–13 inc. 7-days' average	9–13 inc. 5 wk. days' average
	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.		
Tons pig iron produced in 14 hr.	577	647	542	700	546	632	633	611	611
Lb. coke per ton of product	3,296	2,807	3,483	2,778	3,481	3,213	3,139	3,171	3,219
Lb. coal charged in furnace in 24 hr.	25,536	24,384	25,344	26,112	25,344	27,072	26,494	25,755	26,074
Top temperature, deg. fahr.	625	620	620	625	620	605	630	621	620
Hot-blast temperature, deg. fahr.	1,230	1,260	1,245	1,225	1,240	1,295	1,290	1,255	1,259
Avg. cu. ft. blast per minute to furnace	76,500	74,000	75,200	76,400	75,200	74,700	74,900	75,270	75,200
Avg. blast pressure at blower, lb.	23.2	22.6	23.3	22.9	21.1	21.6	21.2	22.2	22.0
Million B.t.u. per hr. in top gas from coke	721.00	642.55	722.50	685.00	726.50	758.85	738.75	713.59	726.32
Million B.t.u. per hr. in top gas from coal	12.77	12.19	12.67	13.05	12.67	13.53	13.25	12.88	130.03
Million B.t.u. per hr. in top gas, total	733.77	654.74	734.74	698.05	739.17	772.38	752.00	729.18	739.15
Avg. gross boiler hp. generated by boilers	9,832	11,384	14,200	14,176	14,295	14,432	14,519	13,291	14,324
Tons powdered coal burned under boilers in 24 hr.	....	18	21	25	45	35	30	25	31
Million B.t.u. per hr. to steam at 83 per cent efficiency	393.28	455.36	568.00	567.04	571.80	577.28	580.76	530.50	572.98
Million B.t.u. per hr. in powd. coal at 12,000 B.t.u. per lb.	....	18.00	21.00	25.00	45.00	35.00	30.00	24.86	31.20
Million B.t.u. per hr. blast-furnace gas (by diff.)	393.28	437.36	547.00	542.04	526.80	542.28	550.76	505.65	541.78
Per cent of top gas to boilers, calculated	53.50	66.75	74.50	77.75	71.10	70.25	73.10	69.56	73.34
Per cent of top gas to stoves, calculated	22.50	21.20	22.10	21.25	21.90	21.75	22.00	21.81	21.80
Per cent of top gas to bleeders by diff.	24.00	12.05	3.40	1.00	7.00	8.00	4.90	8.62	4.86
Kw-hr. generated in 24 hr. (gross)	271,000	395,000	521,000	537,000	553,000	553,000	554,000	384,428	543,600
Kw-hr. used by power-house auxiliaries	17,400	16,800	18,000	16,900	18,100	18,000	17,300	17,500	17,660
Kw-hr. net delivered to mains	253,600	378,200	503,000	520,100	534,900	535,000	536,700	465,928	525,940
Avg. kw-hr. per hour (gross)	11,300	16,420	21,750	22,390	23,050	23,050	23,100	20,151	22,632
Avg. boiler hp. generated, blast-furnace boilers	9,832	11,384	14,200	14,176	14,295	14,432	14,519	13,261	14,324
Avg. boiler hp. to open-hearth steam system	1,083	805	802	945	700	559	550	778	711
Avg. boiler hp. distributed from furnace plant	8,749	10,579	13,398	13,231	13,595	13,873	13,969	12,483	13,613
Avg. boiler hp. to blast furnaces, excl. of auxiliaries	102	111	120	109	103	106	103	108	108
Avg. boiler hp, turbo-blowers, excl. of auxiliaries	2,408	2,399	2,457	2,459	2,298	2,442	2,282	2,392	2,388
Avg. boiler hp., electric power, excl. of auxiliaries	4,516	6,477	8,644	8,885	9,115	9,438	9,084	8,023	9,032
Avg. boiler hp. auxiliaries and losses (by diff.)	1,723	1,592	2,177	1,778	2,079	1,887	2,500	1,960	2,085
Steam pressure, lb. gage	341	334	323	325	324	324	322	328	324
Superheat, deg. fahr.	183	179	186	191	194	189	187	187	189
Boiler hp. in service	4,383	4,383	4,383	4,383	4,383	4,383	4,383	4,383	4,383
Per cent of rating	224	262	323	323	326	329	331	303	326

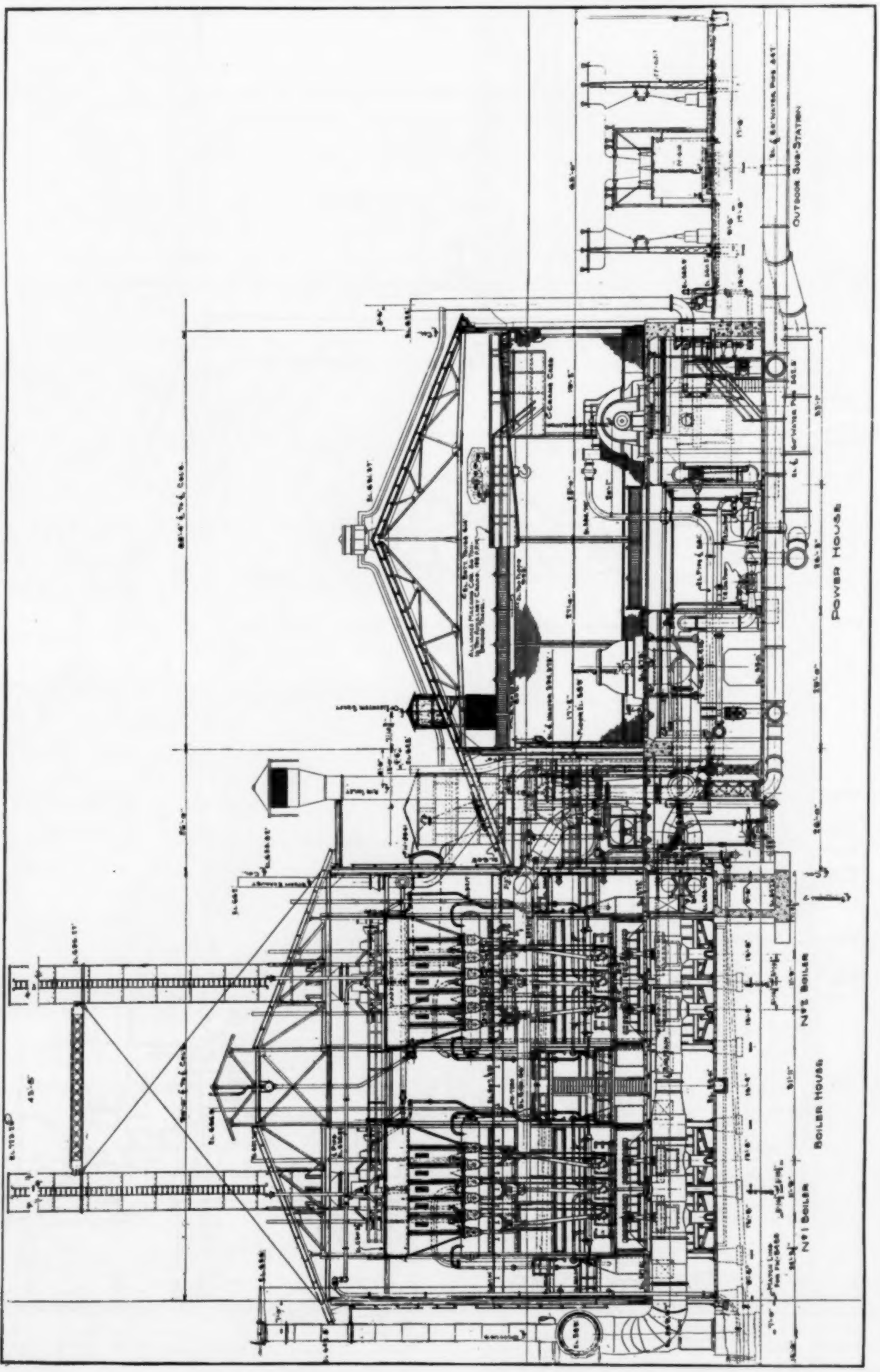
No 1 Boiler
Boiler House
No 2 Boiler
Power House
Outdoor Sub-Station

FIG. 2 CROSS-SECTION OF POWER PLANT

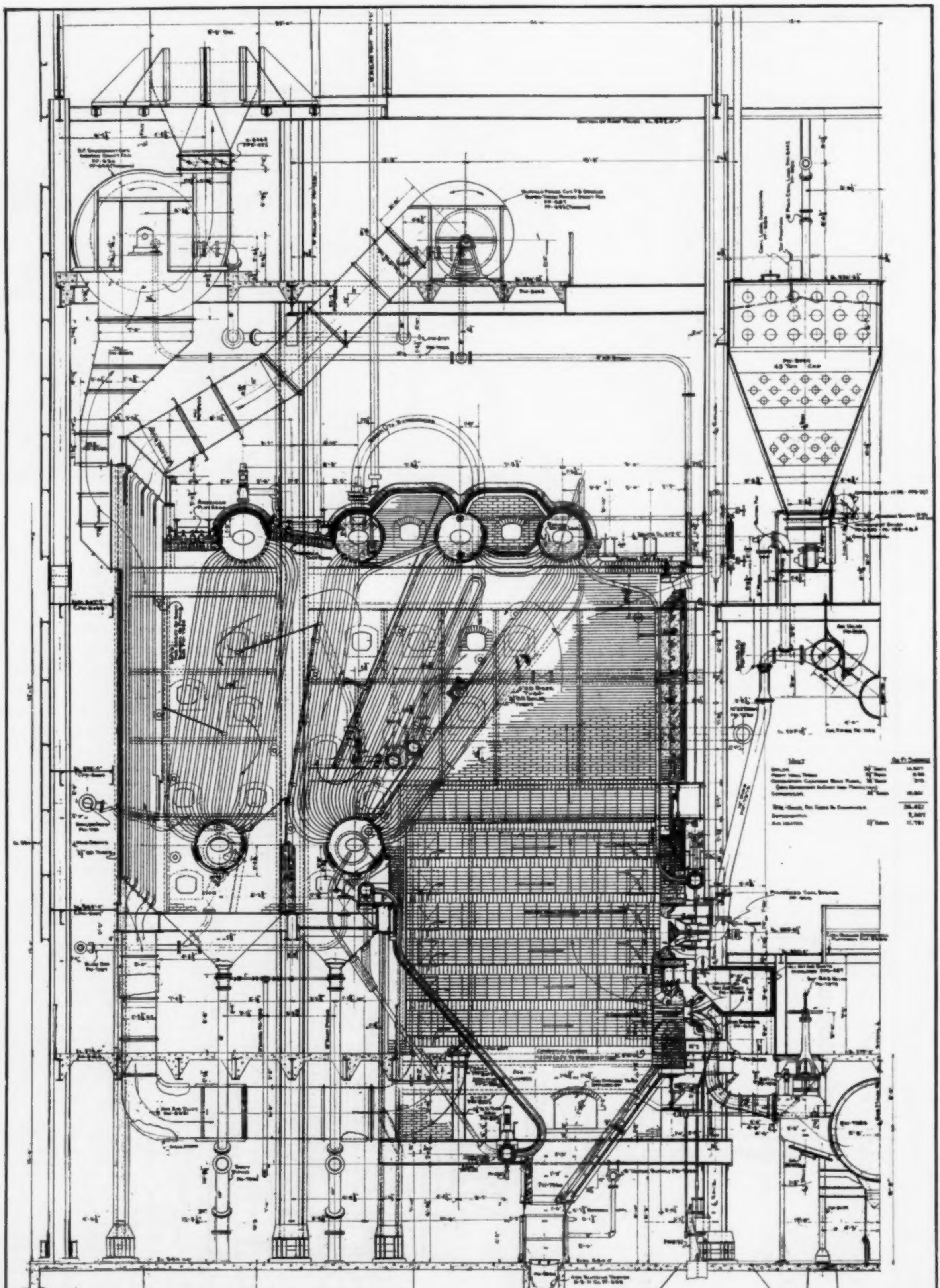

FIG. 3 CROSS-SECTION OF BOILER SETTING

can Iron and Steel Institute, is as follows:

Grains of dust per cubic foot (standard conditions) in:	
Gas leaving dust catcher	3.04
Gas supplied to boilers	0.54
Gas supplied to stoves	0.02

Gas enters the boilers at an average temperature of	470 deg. fahr.
Gas enters the stoves at an average temperature of	280 deg. fahr.
Raw gas enter the precipitator at	542 deg. fahr.

Before starting this power house the electric-power demands of the Tennessee Company were supplied by two power houses at the Ensley works, supplemented by purchase of power from the Alabama Power Company.

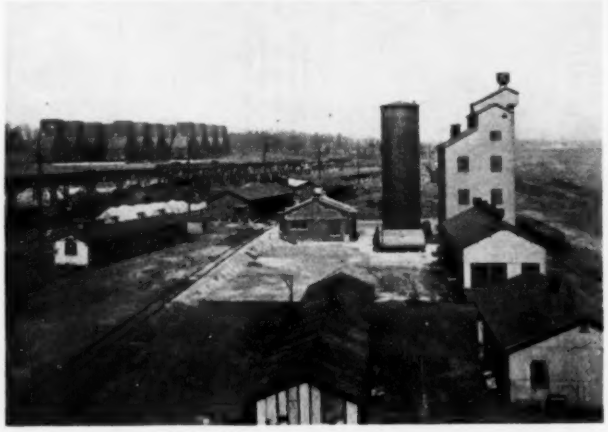

Fig. 4 Coal-Pulverizing and Water-Treating Plants and Cooling Towers

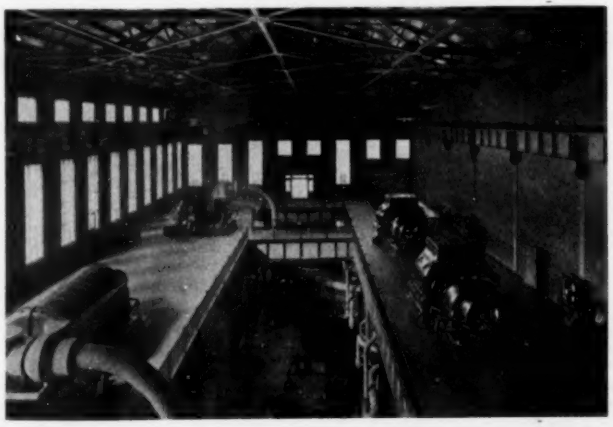

Fig. 5 Interior of Power House Showing Turbo Generators, Turbo-Blowers, and Condenser Auxiliaries

TABLE 2 IMPROVEMENTS EFFECTED IN BOILER AND STOVE EFFICIENCIES

	Assumptions of paper, May, 1922	Results, Sept., 1930
Top temperature, deg. fahr.	400	620
Hot-blast temperature, deg. fahr.	1,250	1,259
Stove efficiency, per cent	60	71
Boiler efficiency, per cent	66.6	83
Heat in top gas per ton of product. B.t.u.	28,006,300	29,189,800
Heat required to heat blast per ton of product, B.t.u.	6,502,380	5,488,400
Heat in surplus gas after blast is heated, B.t.u.	21,503,920	23,701,400
B.hp-hr. per ton of product in surplus	430	592.5
B.hp. per hour per ton per day	17.9	24.7
Avg. b.hp. for tons and coke rate	10,930	15,050
B.hp. required for furnace use	380	108
B.hp. required for blowing furnaces	2,830	2,388
B.hp. required for furnaces and blowing	3,210	2,496
Surplus steam for power, b.hp.	7,720	12,554
Kw. per hour from surplus steam at 2 kw. per b.hp.	15,440	25,108
Kw-hr. per ton of product in surplus	25.3	41.0

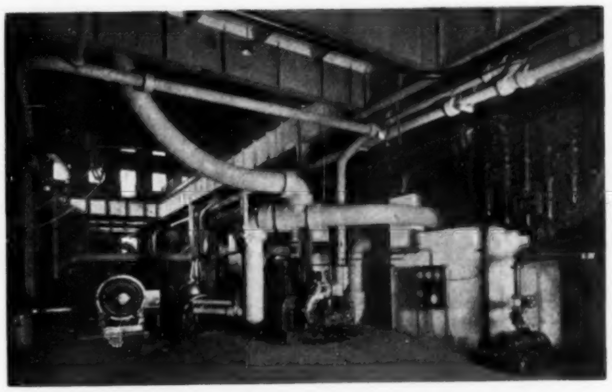

Fig. 6 Condenser Auxiliaries in Power-House Basement

After the plant was started the connection from the power company was maintained for protection and regulation, and an arrangement was made with the power company by which surplus power generated from blast-furnace gas was delivered to the power company's lines whenever the supply of water at their hydroelectric plants was not sufficient for all demands without operation of steam stations.

Owing to the ample supply of water available at the power company's hydroelectric plants there has been a surplus of blast-furnace gas wasted at the bleeders of the blast furnaces

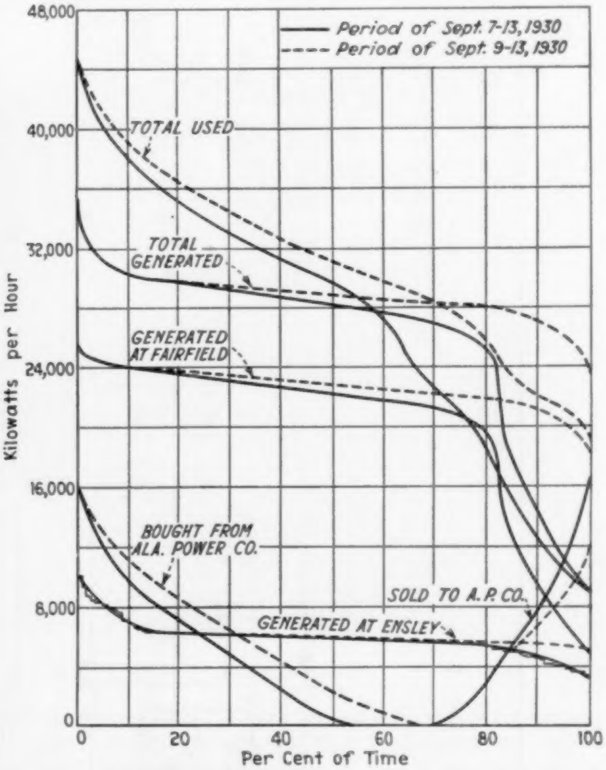

Fig. 7 Total Hourly Power Production at Fairfield and Ensley Works, and Purchase and Sale of Power

	Sept. 7–13	Sept. 9–13
Total kw-hr. generated	4,371,800	3,463,300
Total kw-hr. used	4,597,200	3,760,100
Kw-hr. generated at Fairfield	3,385,000	2,720,000
Kw-hr. generated at Ensley	986,800	743,300
Kw-hr. bought from Alabama Power Co.	534,600	485,400
Kw-hr. sold to Alabama Power Co.	309,200	188,600

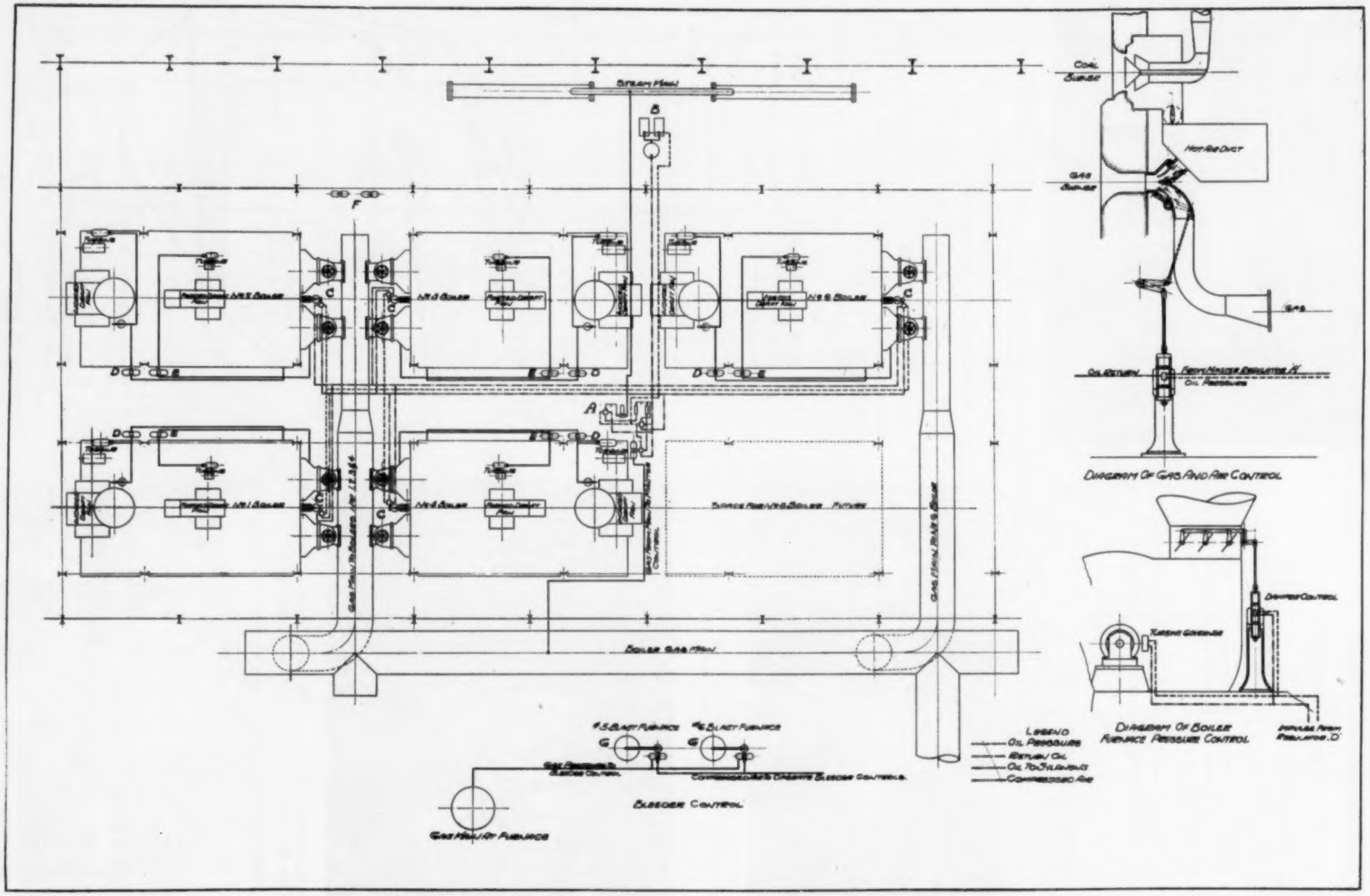
Steam Main
B
A
F
C
D
E
Boiler Gas Main
Coal Burner
Gas Burner
Hot Air Duct
Gas
Oil Return
From Master Regulator 'A'
Oil Pressure
Diagram Of Gas And Air Control
Damper Control
Turbine Governor
Diagram Of Boiler Furnace Pressure Control
Impulse From Regulator 'D'
Legend
Oil Pressure
Return Oil
Oil To Cylinders
Compressed Air
#5 Blast Furnace
#6 Blast Furnace
G
Gas Pressure to Bleeder Control
Compressed Air to Operate Bleeder Controls
Bleeder Control
Gas Main At Furnace

Fig. 8 Diagrammatic Layout of Automatic Combustion-Control System

most of the time, but there have been periods during which the power company took surplus power when practically all of the surplus gas was consumed by the boilers.

One such period, when only No. 6 blast furnace was in service, occurred during September, 1930, and the principal data for a week of this period during which three boilers operated are given in Table 1.

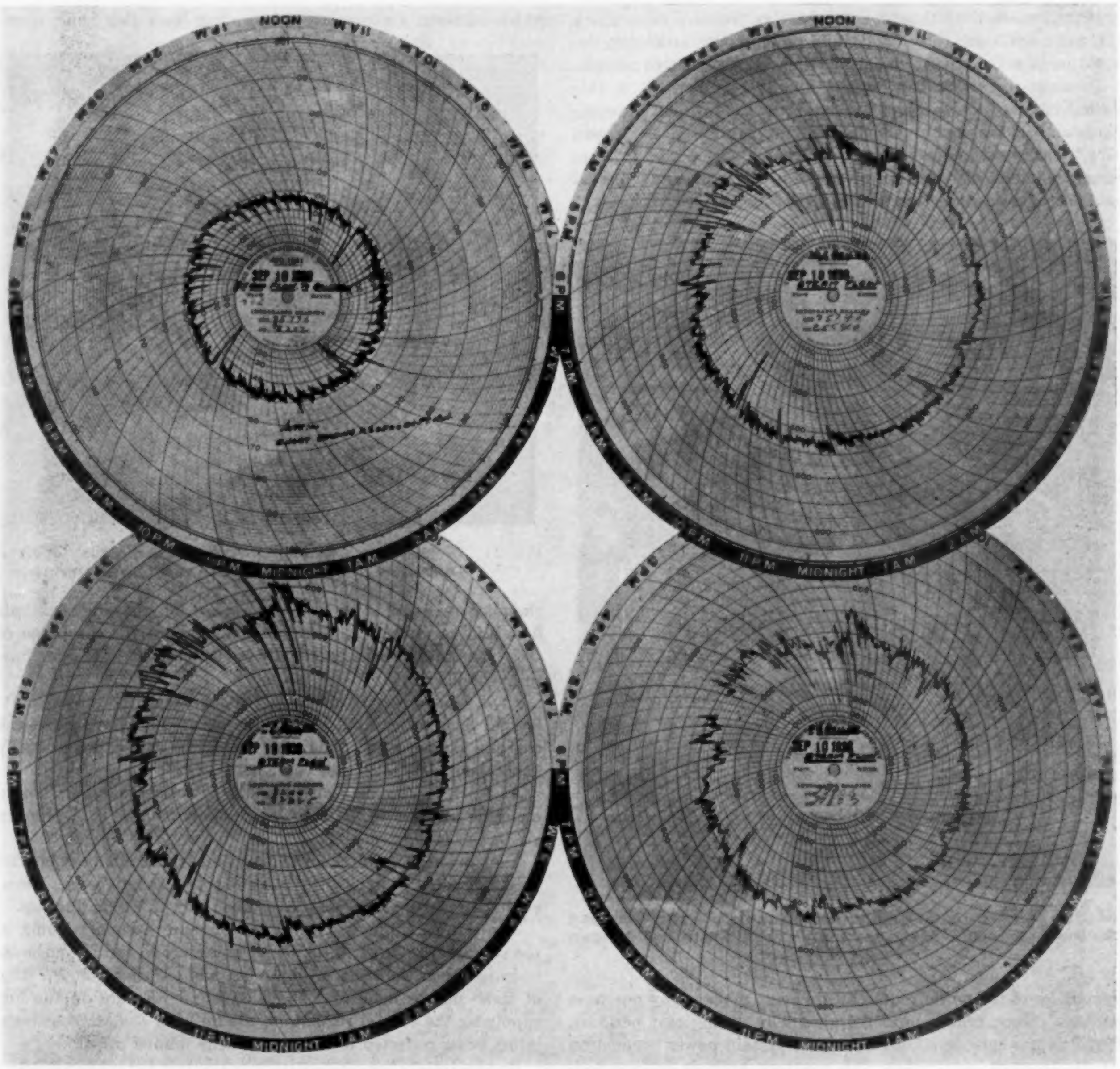

Fig. 9 Charts Showing Gas Flow to Boiler Plant and Steam Flow From Boilers

The B.t.u. in blast-furnace gas from coke as shown in this table are calculated by the method given in a paper presented at the Spring Meeting of the A.S.M.E. in Atlanta, May, 1922, the furnace coke averaging 92 per cent carbon.

The fuel value of the coal charged into the furnace (in order to add tar vapors for better cleaning by the Cottrell precipitators) was included separately at 12,000 B.t.u. per lb. The data for the period from September 9 to 13 (Tuesday to Saturday, inclusive), when there was a market for practically all surplus power, indicate that less than 5 per cent of the top gas was lost at the bleeders, and an average of 22,632 kw. per hour was generated with 5.5 per cent of the boiler fuel being powdered coal. The steam sent to the open-hearth system in this period, if it had been used for power, would have increased the total power generation to about 24,000 kw.

In the paper referred to previously it was shown that the steam available from surplus gas after the blast had been heated was a function of the coke rate, hot-blast temperature, and stove and boiler efficiency, and a formula was given showing the relation of steam from surplus top gas after the blast had been heated for an assumed standard efficiency of 66.6 per cent for boilers and 60 per cent for stoves, these efficiencies being considered good average practice at the time the paper in question was prepared (about nine years ago).

In order to show how boiler and stove efficiencies have been improved in a relatively short time, a comparison is given in Table 2 of the assumed efficiencies of that paper with the actual results at Fairfield for the period of five consecutive days shown in Table 1 during which the power company took surplus power, based on the coke rate of 3219 lb. per ton of product.

A symbolic chart showing total hourly power production at Fairfield and Ensley, and purchase and sale of power, is shown in Fig. 7, the full lines showing conditions for the entire week, and the broken lines those for the five-day period during which the mills operated, this chart being prepared by arranging the total loads in the order of magnitude over the respective periods.

Recording charts taken for one day (September 10) in this period indicate the extent of variation in electric power conditions and in the steam and gas pressure, feedwater, and steam flow from the three boilers in service for this period. From these the effect of reduction of gas production at a cast period is reflected in a reduction of steam and power production, the remainder of the power requirements being made up by purchase or sale. Some coal is burned (principally during cast periods), but it is not considered advisable to maintain power production at the normal rate unless the contract limit is approached. Power is purchased on a kva. contract, and the generators in service are automatically regulated to maintain a uniformly high power factor on the purchased power.

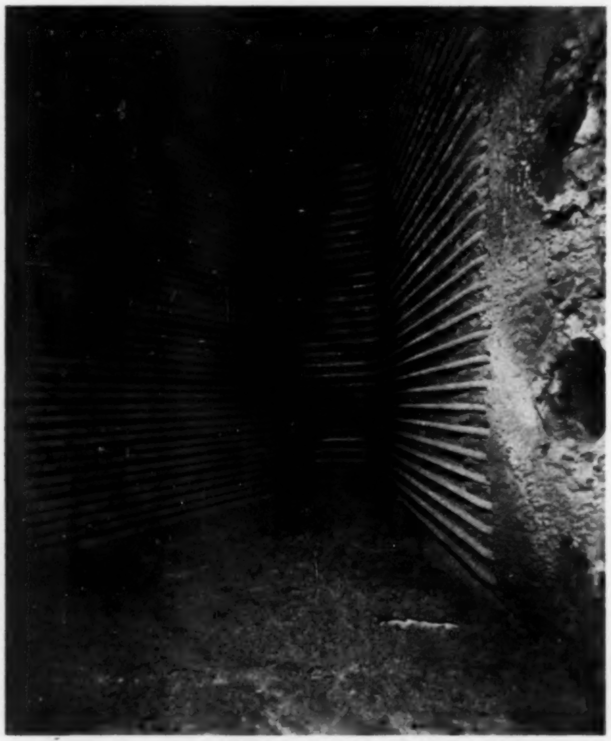

Fig. 10 View Looking Upward in Boiler Combustion Chamber Showing Flue-Dirt Accumulation on Side Walls and Around Front Tubes

The automatic control system is shown diagrammatically in Fig. 8. This system consists of several regulators, each of which has a particular function briefly discussed in the following paragraphs.

At *A* a master regulator operating primarily from steam pressure controls through an oil-pressure system the position of a plunger at the basement level near the gas burners *C*, which plunger operates the gas and interconnected air dampers, admitting gas and preheated air to the combustion chamber, opening these when there is a drop in steam pressure and closing them when the steam pressure on the main rises. An auxiliary regulator also at *A* operating from the blast-furnace-gas main closes the valves at the boilers whenever the gas pressure drops below a set pressure of about 6 in. water column in order to protect the supply to the stoves.

At *D* on the main operating floor of the boiler plant near the water columns are regulators operating from the combustion-chamber pressure to control the speed of the induced-draft fans and position of the louver dampers on the discharge of these fans to maintain practically balanced draft in the combustion chamber.

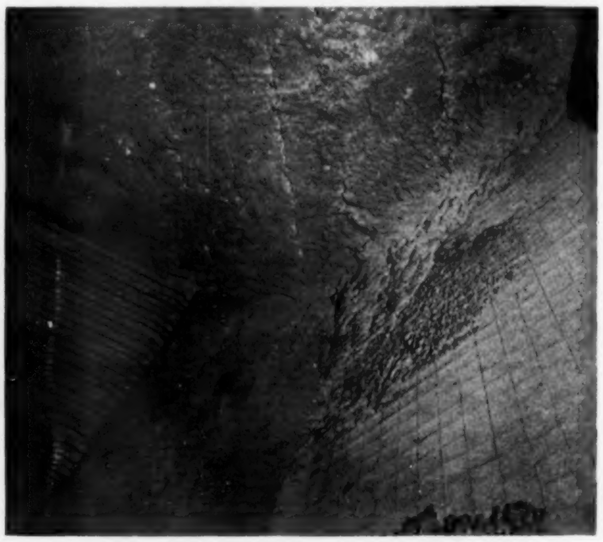

Fig. 11 Interior of Boiler Showing Accumulation of Flue Dirt on Water-Cooled Floor and Side Walls

At *E*, similar regulators control the speed of the forced-draft-fan turbines to maintain constant preheated-air pressure at the coal and gas burners. This pressure may be varied, but is usually kept about $1^{1}/_{2}$ in. water column.

At *F*, regulators control the speed of the primary-air fans to maintain about 6 in. on the blast main supplying cold primary air to the coal burners.

Each of the blast-furnace dust catchers has an auxiliary bleeder with the valve controlled by a regulator *G* to relieve gas whenever the main pressure goes over about 20 in. at the furnace.

The pumps supplying oil to the pressure system running to the several regulators in the boiler house are in the power-house basement near the boiler-feed pumps, and are shown at *B*. One of these pumps maintains about 60 lb. oil pressure on the line supplying the several regulators, the oil after use at these regulators being collected in a tank over the pumps.

By this system the combustion conditions in the boiler are maintained uniformly good, as evidenced by the low stack heats and CO_2 content of the stack gases, the latter usually above 20 per cent. The preheated-air temperatures leaving the side walls are usually within 50 deg. of the stack temperatures and these are recorded by pyrometers on the same chart, and as only about 12 deg. is added to the air temperature in the side walls, the counter-current arrangement of the tubular air heater is effective.

Particular attention is called to the power and boiler charts (Fig. 9) about 1:30 and 2:00 p.m. of September 10, when due to trouble on the power company's system there were two complete interruptions, causing the turbo-generator in service to drop about 22,000 kw. and pick it up again almost immediately.

When the power load was dropped, the automatic regulation system shut off the gas and combustion air supply to the boilers and reduced the steam production to less than one-third of that carried previously very quickly, and when the load was resumed the boiler output was increased almost as quickly, with no serious loss in steam pressure.

It is the opinion of the operators that it would not be possible to successfully operate the plant without the automatic control and boiler instruments provided, and the results, as indicated by the power production, indicate the maintenance of good combustion efficiency.

In order to show the effect of the water-cooling surface in the combustion chamber of the boiler, thermocouples were installed in a number of the blocks fastened on the floor tubes, and tests made indicated that there was a very high rate of steam production in these tubes—as high as 60,000 B.t.u. per sq. ft. per hour, or from 15 to 20 per cent of the total steam production of the boiler; and as a result of employing this heat-absorbing surface and the front-wall bare tubes and air-cooled side walls, practically all the brickwork repairs required to date have been the partial rebuilding of the bridge walls; and the total repair cost, in spite of the relatively high rating as compared with the usual blast-furnace-gas-fired plant, has been very nominal. Photographs taken inside the boiler setting (Figs. 10 and 11) show the condition of the furnace after a usual period of service and the relatively slight deposit of flue dirt on floor wall, side walls, and tubes.

All of the operators were taken from other plants of the Tennessee Company and had had very little, if any, experience with steam pressures over 150 lb., superheaters, turbines, condensers, pulverized coal, or automatic combustion control; however, the operating difficulties encountered have been but nominal.

Discussion

E. J. Billings.[2] The author has described a modern plant in which blast-furnace gas, a by-product, is used as boiler fuel supplemented by pulverized coal. Either fuel, or a combination of the two fuels, may be burned at will in the same furnace with high efficiency in quantities required to meet the steam demands as they may exist. Thus is met one of the first principles in modern steam generation for industry, namely, the efficient and economic use of the cheapest available fuel or fuels.

Modern industry provides many different kinds of by-product boiler fuels besides blast-furnace gas. Only a few need be mentioned to illustrate the range. From the processing of coal there are coke-oven gas, coke breeze, coal tar, and anthracite culm. From the refining of petroleum there are fuel oil, acid sludge, and petroleum coke. From the wood industries there are hogged fuel, tan bark, and dried wood refuse. Natural gas during the season of light domestic demand is sometimes considered a by-product fuel available for boiler use. These are only a few of a long list of by-product fuels, but they illustrate the varying nature of those available.

In almost every case the value of these special fuels for steam-boiler use is compared to that of the cheapest available steaming coal. This is because coal is generally available and therefore less subject to fluctuations in price. Coal is suitable for cheap transportation and storage, whereas the special or by-product fuels must generally be used as and close to where they are produced.

In this connection it is interesting to note that pulverized-coal-fired boiler furnaces may be readily adapted to almost the entire range of by-product fuels and used in the same manner as described by the author. This is a distinct advantage in favor of pulverized-fuel firing because in this age one scarcely knows from one day to the next what by-product fuel may become available at his plant, or conversely, what by-product fuel now available and in use may be suddenly withdrawn because of greater value for other uses.

Another feature evident from the author's paper, which is typical of the modern trend where by-product fuel or process steam is involved, is the reciprocal arrangement whereby power is exchanged with the central-station system. If the author's company owned the central-station system, or vice versa, there would be one design of plant which would most nearly comply with the condition of best overall economics of the situation. In the absence of such condition of ownership it would seem that it would only be necessary to divide the benefits obtained from this efficient overall set-up in some equitable manner between the two companies. Obviously, in this case it requires an equitable contract between the two companies for the exchange of power. It would appear to the writer that the management of both the author's company and the central-station system are to be congratulated on having worked out their mutual problems on the basis of sound overall engineering and economics.

D. S. Jacobus.[3] The author has presented a paper which describes a plant designed for his particular conditions that is a real contribution to the art.

Boiler efficiencies of 66 per cent, as given in his Table 2 for the year 1922, were at one time assumed as typical of boilers serving blast furnaces, and no doubt there are still many boiler plants in operation in which the efficiencies are no higher than this. It is very gratifying to see steel plants such as this one installing boiler units in which the efficiency is 83 per cent and in which the boilers are operated at a higher capacity.

Each of these five units has 14,607 sq. ft. of heating surface in the boiler, 10,601 sq. ft. in the economizer, and 11,781 sq. ft. in the air heater, which compares very favorably with central-station practice.

The daily load curve as represented by the charts is quite uniform and is the type of load that justifies economical units. Table No. 1 indicates that less than 5 per cent of the gas is wasted to the bleeders during five days of the week, and a less efficient plant would have required the burning of more coal.

The results secured through use of automatic control are very interesting and show that load conditions which are uniform on the average, but which are subjected to short sudden swings such as are experienced here, need automatic control as much as, or in some cases more than, loads which vary widely at different times of the day.

The methods of water treatment and cleaning the dust are worked out very completely and reflect credit to the designer.

The use of pulverized coal to make up for any deficiency fits in very nicely where the main supply is a by-product fuel, such as in blast-furnace gas. The furnace design has worked out satisfactorily and is no more complicated than it would be for the use of either fuel alone.

Sanford A. Moss.[4] The author is to be congratulated on the presentation he gives of a blast-furnace plant following power-station practice. For many years power-station practice has steadily improved, and higher and higher pressures and temperatures and various other devices for increasing output for a given amount of fuel have been continuously adopted. Even if a blast-furnace plant makes more than enough gas to operate itself, there is in most cases enough use for surplus power or other uses for the blast-furnace gas to give as much financial gain from improvement in fuel consumption in blast-furnace plants as in power stations. Therefore blast-furnace operators realize that thermodynamic efficiency is just as important for them as for the power stations.

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[2] Vice-President, Fuller Lehigh Company, New York, N. Y. Mem. A.S.M.E.

[3] Advisory Engineer, Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

[4] Mechanical Engineer, Thomson Research Laboratory, General Electric Company, Lynn, Mass. Mem. A.S.M.E.

Of course, the time will come when even the pressures and temperatures cited by the author will have to be exceeded by blast-furnace plants. Power stations have already begun to use very much higher pressures and temperatures. There are, of course, other plants in the United States which have conditions similar to those cited by the author. One of these is an installation in the Youngstown Sheet and Tube Company, where blast-furnace blowers have been operated since the latter part of 1928 on 400 lb. gage pressure and 690 deg. total temperature and 240 deg. superheat, with the same success as is indicated in the paper. These machines also follow modern power-station practice by extracting steam from the main turbine in three places to heat feedwater.

FSP-54-3

Conversion of Coal-Fired Boilers and Furnaces to Gas Firing

By WILLIAM D. EDWARDS,[1] MEMPHIS, TENN.

This paper deals with the use of natural gas in the City of Memphis, the methods and data used in making estimates of gas consumption, the application of the degree-day to this end, and the actual operating data for various types of plants converted to gas fuel.

WHEN, after months of preparation and expectancy, the pipe line finally delivers natural gas to a community, the necessity of building up a load becomes of paramount importance, to which end the installation of gas-burning equipment under existing boilers is a most important factor.

With due regard to all of the advantages obtained by the substitution of gas for other fuels, the prospective user is chiefly interested in its effect upon the operating cost which will result by the combination of fuel expense on one hand and savings to be effected on the other.

The estimate as to the cost of the gas itself can be made with a fair degree of accuracy, remembering, however, that it is a forecast of probable conditions, and as such is subject to modifications.

The other savings in the line of attendance, space, cleanliness, smoke elimination, etc., are questions of the individual plant, upon the conditions of which they may be estimated in the terms of dollars and cents.

To the engineer the question of interest is mainly that of the use of the gas for a given result, embodying its application, volume used, and cost of the same, and it is to this end that the experience covering the last two years of the City of Memphis in the adoption of natural gas for fuel purposes is outlined as a possible guide to other engineers taking up this work.

The natural gas is supplied to Memphis by a 220-mile pipe line, 18 in. in diameter, from the Monroe, La., fields; it is over 90 per cent methane CH_4 and has a heating value of 960 B.t.u.

In selecting the standard by means of which the gas requirements, in relation to the heating load of a given section, can be based, the degree-day is used as the most convenient yardstick upon which are based most of the calculations.

The degree-day as developed by the American Gas Association is a measure of the average daily temperature in its variation from the standard of an average daily temperature of 65 deg.; a day showing an average daily temperature of 35 deg., would be noted as having 30 degree-days. The reports of the U. S. Weather Bureau are recorded each day during the heating season.

In the use of gas, a more direct relation exists between the amount used and the outdoor temperature than is the case with the other classes of fuels. Expressed in terms of degree-days and volume of gas used, it will be found that practically a constant relation exists between the two quantities, which is of great assistance in making estimates of fuel requirements.

The most convenient method of recording the degree-days is to show them in the cumulative form, adding the degree-days of each day to the sum of those of the previous days, from the start of the heating season, showing both the record of the season in question, as well as that for the normal season.

The Memphis section has about 3000 degree-days, starts about October 1, and ends the heating season on the last of April, although some gas is used for heating before and after these dates. While the record is shown for each day, an abstract as given in Table 1 will show the general set-up.

TABLE 1 DEGREE-DAYS FOR SEASON OF 1929–30 AND FOR NORMAL DATE

Date	Degree-days to date	Degree-days, normal	Date	Degree-days to date	Degree-days, normal
Oct. 1..	6	0	Jan. 1..	1238	1162
Oct. 10..	32	0	Jan. 15..	1513	1498
Oct. 20..	32	14	Feb. 1..	2147	1902
Oct. 30..	115	68	Feb. 15..	2379	2210
Nov. 10..	211	173	Mar. 1..	2494	2475
Nov. 20..	358	309	Mar. 15..	2702	2685
Dec. 1..	662	494	Apr. 1..	2922	2858
Dec. 10..	853	668	Apr. 15..	2943	2933
Dec. 20..	977	883	Apr. 30..	2985	2949
				for season	for season

TABLE 2 DEGREE-DAYS PER MONTH FOR SEASON OF 1929–30 AND FOR NORMAL SEASON

Date	Degree-days, 1929–30	Per cent of total	Degree-days, normal	Per cent of season
Oct., inc. Sept.....	115	4.0	76	2.5
Nov..............	514	17.0	400	13.5
Dec..............	607	20.0	662	22.5
Jan..............	891	30.0	741	25.0
Feb..............	350	12.0	579	19.5
Mar..............	437	14.5	392	13.0
Apr..............	71	2.5	99	4.0
Total...........	2985	100	2949	100

The data in the form of Table 1 have proved to be of value in checking up on the operation of installed equipment, and particularly when a question arises over current gas bills.

To the average person the memory of even recently past weather conditions is very vague, and only the Weather Bureau records for the period in question offer a basis for an intelligent analysis of the seemingly high bill.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

In the form given in Table 1, the volume of gas used in an elapsed period, together with the degree-days covered by this same time and the proportion of the latter to the total seasonal degree-days, forms a workable basis to check the cost of gas for the heating season and its relation to the cost estimate given.

As an illustration it will be noted that during the 1929–30 season the degree-days on December 20 had reached a total of 977, this being about one-third of the season's total of 2985, and the amount of gas used up to December 20 would represent one-third of the anticipated season's requirements. Repeated checks show that this works out very closely in actual practice, and if the estimate of probable gas used, as made at the beginning of the season, falls much below the amount indicated as required by the foregoing degree-day proportion, it is time to give this installation an inspection.

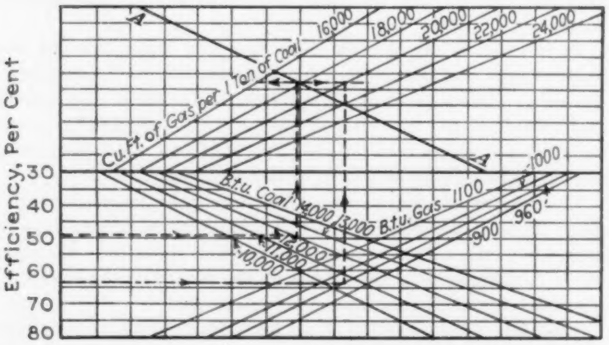

Fig. 1 Gas and Coal Relation, in Terms of Cu. Ft. of Gas and 1 Ton of Coal

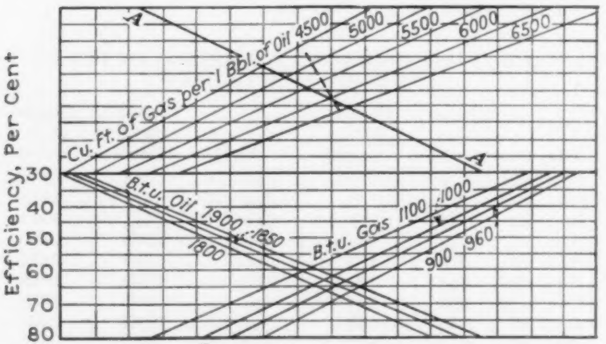

Fig. 2 Gas and Oil Relation, in Terms of Cu. Ft. of Gas and 1 Bbl. of Oil

The question of maintained temperature is of vital importance. The foregoing calculations are based upon an indoor temperature of 70 deg., while in practice the users like to keep a temperature of from 5 to 10 deg. higher. The average outdoor temperature during the heating season, locally, is 50 deg., which makes an average difference, when heating to 70 deg., of 20 deg. rise, a greater difference caused by a higher indoor temperature making a proportional increase in the gas requirements.

The degree-day data tabulated in the monthly form with the percentage of the season's total are of material assistance in determining from the total season volume of gas the amount of gas used per month, this being essential due to the fact that about all rates are based upon a monthly gas consumption. This form is shown in Table 2.

In practically all gas rates an increase in volume used results in a lower unit cost, in accord with which the smaller gas consumption during the fall and spring months costs more, per 1000 cu. ft., than does that of the heavy-loaded winter months; hence the necessity of breaking down the total gas of the season to a monthly basis, and applying the rate for each month, the total of which gives the season's cost, which in connection with the total gas gives the average rate per season.

The degree-day data of Table 2, the estimated volume of gas for the season, and the established rates are the three factors involved in making a forecast of operating cost for a heating season. Two of these can be determined with a fair degree of accuracy, but the third, the volume of gas used per season, is the variable quantity the correct estimate of which depends very much on the good judgment of the engineer in his application of basic data to the plant in question.

The determination of the amount of gas required per season may be based on the total of installed radiation in the building under consideration, or, where it is available, the amount of coal or oil used during prior seasons.

In the former, the data are expressed in terms of cubic feet of gas per season required by 1 sq. ft. of installed radiation. In the latter this is given in the form of the number of thousand feet of gas equivalent to one ton of coal or to one barrel of oil, and here it may be noted that this coal-gas ratio is the football of the game between the coal and gas interests.

On a strictly B.t.u. basis and assuming equal efficiency for both coal and gas firing, one ton of 12,000 B.t.u. coal will be equal to 25,000 cu. ft. of 960 B.t.u. gas, but this does not work out in practice, as 20,000 seems to be the average firing ratio, in the comparison of a good hand-fired coal plant with a good gas-fired one. A poorly operated coal plant may bring this ratio down to 15,000 and an efficient stoker-fired plant will bring this up to the 25,000 of theory. With the efficiency playing the important part that it does, the only basis of comparison is that of effective B.t.u. A ton of 12,000 B.t.u. coal at 45 per cent efficiency gives a little less than 11,000,000 heat units, while a cubic foot of 960 gas at $57^1/_2$ per cent efficiency has an effective value of 550, which compared with the 11,000,000 heat units of coal gives a ratio of 1 to 20,000. The ratio of fuel oil to gas ranges from 5000 to 6000 cu. ft. of gas to one barrel of oil.

Fig. 1 shows a graphic method of determining the coal-to-gas relation, based upon available heat units at the different efficiencies, as assumed for each kind of fuel. In this diagram a horizontal line from the coal efficiency selected is extended to intersect the coal diagonal, from which point a vertical is drawn up to intersect the diagonal line *A-A*, a horizontal line is again drawn through this intersection. Starting back at the efficiency line this process is repeated at the efficiency selected for the gas, the horizontal line intersecting the diagonal shown for gas, and then being carried upward to meet the horizontal line previously drawn through the point on *A-A*, this intersection giving the cubic feet of gas in accordance with its location in reference with the diagonals thus designated.

Fig. 2 shows the oil-and-gas relation in the same manner of operation.

The entire question of determining the relation between coal and gas resolves itself into the efficiencies to be secured by each. In the absence of test data, it is simply a matter of the good judgment of the engineer in his survey. However, it might be remarked that the average coal-fired heating plant probably operates at a very low point of efficiency, and in average conditions the gas job will show from 10 to 20 points higher than will the coal, or 40 per cent for coal may be equal to 50 per cent to 60 per cent for gas.

Gas per Square Foot of Radiation

This second method of estimating gas requirements is subject to variations, and depends upon the character of the burner installation, the care in operation, and the temperatures maintained in the building. It is expressed in terms of cubic feet of gas per season, per square foot of radiation, per degree-day. Locally, with 3000 degree-days a gas consumption of 400 cu. ft. per season would be based on the factor of 0.133 cu. ft. per square foot per degree-day.

The consumption of 400 cu. ft. is about the low limit for steam radiation, and it may range up to nearly twice this amount, 550 to 600 cu. ft. being considered a good average, or a factor of from 0.18 to 0.20 cu. ft. per square foot per degree-day.

In the application of these data to the plants in a colder climate it should be borne in mind that the Memphis conditions cover an unfavorable type of heating service, so far as efficient operation is concerned, with the temperature dropping to the vicinity of zero for short periods; yet the average temperature of the heating season is 50 deg., which means that for normal operation the heating systems are of excess capacity. This may be one of the reasons for the prevalent belief on the part of the majority of home owners that if 70 deg. is good 75 is better, and 80 is best of all. In any event, the combinations of these adverse conditions, which cause extravagant operation, would indicate that in sections having a steady heating load, at a point nearer the plant capacity, the proportionate gas consumption per, degree-day could be reduced from the values given.

This is borne out by a comparison made on coal consumption. A survey of a group of buildings in Memphis as to the relation between coal used per season and steam radiation installed showed an average of 3.3 tons of coal per 100 sq. ft., this for 3000 degree-days, while data from Pittsburgh, with 5000 degree-days showed a use of 3.7 tons per 100 sq. ft. of radiation.

It might be added that a survey of a large group of buildings of various types as to coal used per season in relation to installed radiation forms invaluable data to the engineer interested in gas-conversion work.

Gas Rates

With the determination of the season's gas consumption, so divided as to show the amount of gas approximately used during each month of the heating season, the final factor in calculating the season's cost is that of the gas rates for given volumes used per month.

The convenience in operation, saving in space and labor, cleanliness, and freedom from the smoke nuisance as afforded by the use of gas all have a definite value, but in the end cannot be relied upon too strongly to build up a gas-heating load.

The gas rates must be such that the cost of gas fuel is competitive with that of coal or oil within reasonable limits, with all due credits given its use along the line of economies in attendance and space saved, and which must be so tangible as to be expressed in dollars and cents.

Beyond this, the gas rates must not be such as will exact a too-heavy premium for its use. However, this matter of rates is generally out of the province of the engineer, and is a subject which in the end adjusts itself in accordance to whether or not the gas load of the pipe line meets the expectations.

The gas rates, as published, all seem to be in a more or less complicated form. The figuring of a rate for each individual job involves much time and labor. This was very much simplified in Memphis by the figuring out of the average rate for various gas consumptions per season.

The Memphis rate involves a demand charge based upon the maximum daily demand, which in the absence of a test is assumed to be one-twentieth of the monthly consumption.

In addition there is a charge of 30 cents per 1000 ft. for the first 100,000 cu. ft. and 15 cents per 1000 ft. per thousand for all gas used per month in excess of this figure.

Table 3 illustrates the working out of this rate for a building assumed to have 5000 sq. ft. of radiation and operating on 600 cu. ft. of gas per square foot, requiring 3,000,000 cu. ft. per season.

TABLE 3 OPERATING RESULTS FOR ONE BUILDING

Month	Monthly per cent of seasons	Load, cu. ft. of gas	Demand	Gas at 30 cents	Gas at 15 cents	Total	Rate per 1000 cu. ft., cents
Oct...	4.0	120,000	$ 10.00	$ 30.00	$ 3.00	$ 43.00	36.0
Nov..	13.5	405,000	20.25	30.00	45.75	96.00	23.7
Dec..	22.0	660,000	33.00	30.00	84.00	147.00	22.1
Jan...	23.0	690,000	34.50	30.00	88.50	153.00	22.0
Feb..	19.0	570,000	28.50	30.00	70.00	129.00	22.7
Mar..	13.5	405,000	20.25	30.00	45.75	96.00	23.7
Apr..	5.0	150,000	10.00	30.00	7.50	47.50	31.6
Total...	100	3,000,000	$156.50	$210.00	$345.00	$711.50	

Average rate for the season, 23.72 cents per 1000 cu. ft.

From a seasonal consumption of 500,000 cu. ft., in which practically no 15-cent gas is used, the rate drops from 43 cents per 1000 cu. ft. to 20 cents and lower when a season's consumption reaches 10,000,000 cu. ft. or more. This rate is termed the commercial heating rate.

The industrial rate, with a fairly steady month-to-month load, and with gas consumption figured on a monthly basis instead of a seasonal basis, will range from 27 cents to 17 cents as the monthly gas use increases from 500,000 to 10,000,000 cu. ft.

As a check upon the foregoing data, a table of actual operating results covering one or more season's operation is submitted (Table 4). This is for buildings in the commercial heating classification. The gas used is for heating purposes only. Those buildings in the readings of which water heating was involved have their gas consumption corrected to a heating basis only.

TABLE 4 OPERATING RESULTS OF A GROUP OF BUILDINGS USING NATURAL GAS

(Conversion from coal or oil fuel to gas)

Type	Radiation, sq. ft.	Gas per season, cu. ft.	Gas per sq. ft. of radiation, cu. ft.	Cost per season of radiation	Cost per sq. ft. of radiation, in cents
Steam heating:					
1 Office building..	15,100	6,651,000	445	$1395	9.2
2 Office building..	15,000	8,000,000	530	$1720	11.5
3 Office building..	16,000	9,500,000	600	$1925	12.0
4 Office building..	15,000	10,800,000	700	$2150	14.5
5 Office building..	13,000	9,250,000	710	$1909	14.7
6 Office building..	2,000	1,150,000	575	$ 350	17.5
7 Office building..	625	325,000	520	$ 157	25.0
8 Hotel..........	11,000	5,600,000	510	$1225	11.2
9 Garage.........	4,000	2,647,000	640	$ 658	16.4
10 Factory........	3,750	2,465,000	656	$ 620	16.5
11 Factory........	3,000	2,293,000	760	$ 577	21.0
12 Apartment.....	5,400	4,000,000	740	$ 869	16.0
Total..........	103,875	62,681,000			
Average.........			605		
Hot-water heating:					
13 Office building..	4,200	1,608,000	382	$512	12.2
14 Residence hotel	1,100	433,500	395	$174	15.9
15 Medical clinic..	2,000	900,000	450	$275	13.7
Total..........	7,300	2,941,500			
Average.........			400		

When natural gas was available, the large volume of business secured imposed certain adverse conditions in burner installation work, as far as securing maximum efficiency was concerned. The installation crews had to make a large number of conversions in a short space of time, and in addition many were installed in the summer months when it was not possible to secure satisfactory tests of operation. With such defects as may exist being remedied and with increased experience for the operator in the use of gas, it is probable that the operating costs of the first year will be materially lowered.

Condition of Heating Plants

In the survey of a proposed gas installation job, the engineer will come across heating plants of all sorts and conditions, and his investigations should cover not only the boiler alone, but should embrace the plant in its entirety, with a note of its defects of operation in prior seasons.

He will find boilers of all types and methods of installation, and many of them too small for the work required. Piping systems will have many variations and sizes, with many having no relation to the radiation served, and will probably lack covering in many cases. Radiation will be deficient and poorly vented, and control valves be out of order.

The net result of a series of defects will be unsatisfactory service, and while the troubles may have existed during all previous years, yet with the advent of gas firing they will be blamed to the gas. Overheating three-quarters of a building in order to get heat to the balance is a fruitful source of gas waste.

The defects of a plant should be remedied by calling in the heating contractor; otherwise, a plant with many pronounced shortcomings is a good one for the gas engineer to leave alone.

Conversion Versus Gas-Fired Units

This is another subject which will generally start an argument. In a new installation the lower operating costs of the gas-fired equipment will probably justify its use, the bone of contention being the pros and cons of discarding present boilers for new equipment designed especially for gas firing. The saving in operating of the latter over a conversion job has to be weighed in connection with the additional cost of the new installation, the depreciation of the same, and the interest on the investment.

The records in Memphis show a fuel saving of from 15 to 19 per cent of a gas-fired installation over a conversion type for residential use. On the basis of initial cost alone it would require from 13 to 15 years to cover the difference in cost, to say nothing of depreciation or interest, and this will probably be longer than the life of the boiler.

In view of the foregoing it is probable that when the boiler equipment is found to be in good shape, or readily made so, it is more logical to use it by converting it from an oil or coal burner to gas fuel.

Conversion to Gas Firing

Two major classifications may be made of the installation of gas burners to replace oil or coal in the furnace of a boiler and may be designated as follows:

1 For commercial heating, which embraces the boilers of a heating system of 1000 sq. ft. (or less) of radiation on up to that of the largest buildings, and which includes office buildings, hotels, store buildings, apartment houses, etc.

2 Residential heating, which is, as its name implies, that of installing the smaller class of burners in the steam, hot-water, and hot-air furnaces of the various classes of residences.

Another classification of conversion work may be made, which includes the installation of gas burners, replacing coal, in small boilers for domestic water heating, the smaller type of hot-water heating systems of the Arcola nature, and the conversion of coal-fired stoves or circulating heaters to gas.

In the commercial heating field the atmospheric burner is used in the majority of cases. This burner is of the multiport, rectangular-face type, the mixture of air and gas entering through the several ports, and combustion taking place on the face of the burner; the air supply being under control of the gas valve and so arranged that the air supply is cut off by its dampers when the gas valve closes.

This burner is installed in the ashpit or over the grates as the nature of the furnace would indicate to be the more desirable. When installed in the ashpit, a combustion chamber with its baffle walls is built up with firebrick, and the bottom of this chamber is also insulated. With the burner located over the grates, the latter are left in place, and a firebrick bed is constructed on top of them, together with the necessary baffle walls.

The control of the burner is in accord with such water temperature or steam pressure as it is desired to maintain, using a temperature or pressure regulator of the counterweighted lever type, this lever being connected to the lever of the gas-firing valve, the gas being on or off as the temperature or pressure requirements demand.

The foregoing arrangement is sometimes varied by the use of thermostats, limit control, and motor-actuated gas valves.

The use of automatic gas control, however, is limited to the small and medium-size boilers. With burners having a large gas capacity an element of danger arises in the sudden admission of a large volume of gas to a furnace.

In addition to the atmospheric type of burner, the forced-air burner using a blower, motor, and air-gas mixer is being used to a limited extent, and for certain forms of furnaces it is probably better adapted than the atmospheric. Each type has its place.

Burners and Furnaces

The luminous, semi-luminous, and non-luminous flame burner all have their advocates. The latter burns a blue flame at the burner and produces a sort of light haze in the furnace. The semi-luminous starts with a blue flame, changing to an orange-color tip, while the former runs an orange and red flame, which can be carried to such an extent as to produce a bad soot deposit and a smoky stack. The non-luminous burner has a more intense heat at the burner face and furnace space near it, while the others carry this heat further along into the furnace and are supposed to give off more heat by radiation. Here again it is that the type of furnace and boiler setting may be the deciding factor in the burner selection.

As a general thing too much weight is given to the burner itself and too little to the subject of the location of the burner in respect to the heating surface of the boiler. In addition the question of proper arrangement of baffles directing the flame and heat travel over the heating surfaces of the boiler is one that is of utmost importance. The two foregoing subjects, burner location and baffling, offer a most fruitful field for tests and investigation on the part of combustion engineers.

Illustrating this may be cited a recent installation made in a hot-water heating boiler, of the round vertical type, and handling about 750 sq. ft. of radiation. A gas burner was installed in the usual manner, in the ashpit, which was firebrick lined, insulated, and baffled in the regular manner. Later this burner was removed, was placed in the firedoor, and the flame was deflected direct over the heating surface of the boiler. Under identical conditions as to load and gas consumption the time of raising the water through 100 deg. was reduced from four hours to two hours.

Residential Heating

The burners used for this work are the "on and off" thermostat-controlled type, with limit control of water temperature or pressure. Here the grates are removed, the burner ring being placed in the furnace, around the inner face of which vertical radiants are placed. Two of the standard makes of this type are represented in about all of the installations of the residence heating, and are used for steam, vapor, hot-water, and hot-air furnace work.

Secondary Conversion Installations

The most important factor in this line was the domestic water heating in reference to the conversion of the main heating boiler from coal to gas.

In many buildings, particularly apartment houses, the source of domestic hot water is a cast-iron, coal-fired boiler installed in conjunction with a storage tank, these boilers having a grate varying from 10 to 16 in. in diameter. They presented a problem as to conversion to gas in such a manner as to secure an economy of operation.

At the beginning, there was no satisfactory burner available for this work, and when the recommendation was made to discard this boiler and buy a new gas water heater, the cost of this in combination with the cost of gas burners for the main boiler made a total which generally resulted in "no trade." Blocking the installation of gas for heating, as this frequently did, it was necessary to develop an efficient lower-priced burner for this work. This was done, and the burners were placed in service, and their operation was gradually improved as the results of observation of their work.

The burners are controlled by a water-temperature regulator on the discharge line from heater to tank; this is of the balanced lever "Sylphon" type and cuts the burner on or off in accord with the water-temperature requirements.

In some cases this control has been placed on the return line to the heater, with good results. Burners of this class are shown in Fig. 3.

The development of the efficient and low-priced burner for the small hot-water heating systems and coal-fired hot-air circulators is in progress, the successful completion of which is of considerable importance from the viewpoint of adding to the gas load.

LOOKING AHEAD

The development of equipment for the use of gas fuel is in its infancy. Considering the rapidly increasing number of natural-gas pipe lines and the consequent large amount of this gas distributed one may look for many changes and improvements in the line of both the apparatus for the using of gas and in the methods of control of this use.

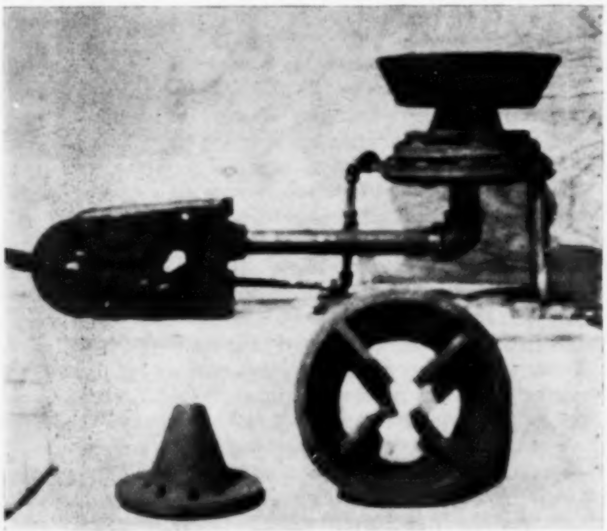

FIG. 3 BURNER FOR DOMESTIC WATER HEATING

With the present practice in mind, one might try to look ahead and forecast some of the places in which this improvement will occur. The following is offered as an individual view, suggesting a line of thought to the combustion engineer interested in this use of gas.

USE AND WASTE OF GAS

The importance of burner location and proper flame direction has been mentioned, and to this may be added the necessity of avoiding the undue consumption of gas which is not required by the heating load or conditions, such as maintaining excessive temperatures, high temperatures during night periods, and when the outdoor temperature is high.

There is a distinct relation between the required temperature of the heating medium (steam, water, or hot air) and the outside temperature, so that it is probable that one may expect decided improvements in the line of control equipment to maintain this relation.

The method of burner operation might be another point of observation; this refers to the "on and off" method of firing, in which the thermostat in the room cuts off the burner with the rise in room temperature, the burner remaining entirely cut off until the cooling of the room starts it again.

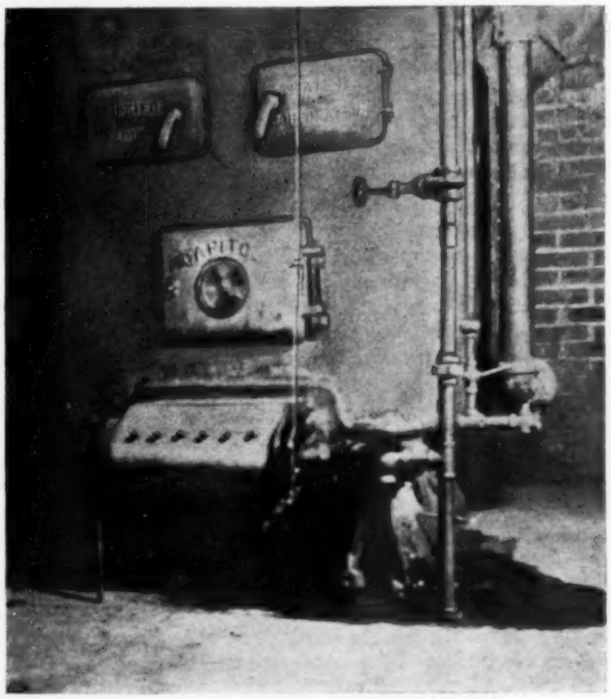

FIG. 4 BURNERS INSTALLED IN THE SLICE DOOR

This intermittent operation imposes a heavy tax upon the burner, as it is always operating against a peak load, calls for a high rate of operation for short periods, and tends to an all-around lowering of efficiency. With a fixed hourly gas consumption it is better to burn it over a long period at a lower rate than over a short period at a high rate. To this end one may look to the "high and low" type of burner, in which the intensity of the flame is under thermostatic or manual control.

Carrying out this theory, a number of burners of this order have been developed and given a season of operation, with excellent results. These burners operate at full, two-thirds, and one-third capacity, with not only the gas being cut off on the burner outlets that are out of commission, but the air to these outlets is also cut off. An installation of this type, with the burner installed in the slice door of the boiler is shown in Fig. 4.

THERMOSTATS

The complete thermostatic-controlled gas-burning installation is near the ideal so far as maintaining an even room temperature is concerned, but even at that, its limitations furnish food for thought.

In spite of many claims as to operating economy on the part of the thermostat enthusiast, it is possible that this maintaining of a constant temperature, sometimes rather high, is at the cost of a higher gas consumption.

Again it is a long step from the day of the coal scoop to the easy chair and the pretty thermostat on the wall, all as shown by the pictures; but perhaps there are many prospective gas users who are desirous of the convenience afforded by gas, but who are willing to regulate the same by hand, if they can thus get the burner installation at a cost within their means. This brings up the question as to whether the use of the thermostat has not been stressed too much and development of an effective manual or semi-automatic control is not desirable.

The advent of natural gas in a community is a more or less disturbing event to the old-established coal industry, and the engineers and would be engineers all start to figure to the end that the use of gas for a fuel is out of the question, the cost being prohibitive, and some remarkable data are evolved by them.

With the passing of time something seems to be the matter with their figures, so they have to revise them. This also happens to the super-enthusiast for gas on the other hand; the high efficiencies he starts out with do not hold up in continuous operation.

This brings up the question as what is the net result in Memphis of about two years of gas operation. The answer is that it has proved to be very successful.

In the commercial heating line practically all of the larger office buildings, stores, hotels, etc., are using gas, and the greater number of the smaller buildings are so equipped.

The successful result of one season's operation of gas in one building justifies other buildings to adopt it.

The costs of operation have proved it to be comparable with coal and materially lower than the average fuel-oil use. In many cases the gas costs have also proved to be lower than the previous season's operation with coal.

In view of the foregoing, the engineer can carry on the work of utilizing natural gas with full confidence that he is contributing to the comfort, convenience, and benefit of the people in general.

FSP-54-4

Recent Developments in By-Products From Bituminous Coal[1]

By A. C. FIELDNER,[2] WASHINGTON, D. C.

After pointing out that the by-product oven has displaced the beehive oven as a producer of metallurgical coke, the author discusses the increasing value of by-products as compared to that of the coke itself. He then takes up successively the subjects of competitive sources of gas for industrial and domestic use; utilization of the hydrogen in coke-oven gas for the production of synthetic ammonia; competition between synthetic and by-product ammonia; utilization of sulphur in coke-oven gas in the production of ammonium sulphate; processes for the recovery of sulphur in connection with gas purification; recovery of phenols from ammonia still wastes; benzol and light oils; production of resins from coal by-products; tar and tar products; and the hydrogenation of coal tar.

TWENTY years ago a favorite theme of the crusader for fuel conservation was the enormous waste of valuable by-products incurred in the manufacture of blast-furnace coke in beehive ovens. Indeed, there was good reason for arousing the national conscience to this needless loss of an irreplaceable mineral resource. In 1910, 83 per cent of the coke production was made in beehive ovens from which surplus gas, tar, light oil, and ammonia, aggregating in value (at 1910 prices) 40 million dollars annually, were discharged into the atmosphere.

In 1929, proportions were reversed and 90 per cent of the production was made in by-product ovens. The average value of by-products obtained in the production of 1 ton of coke in 1929 was $3.60, as compared with $1.16 in 1910, although the value of a ton of coke had increased only from $3.47 to $4.80. As shown in Fig. 1,[3] the production of by-product coke progressed rather slowly until 1915. War needs suddenly expanded the demand for coke and for the by-products benzol and toluol for the manufacture of explosives. Except during the depression of 1921, this accelerating rate of production was maintained up to 1929, largely at the expense of beehive plants. The 1929 production of 60 million tons of coke will probably remain a record-breaking figure for several years. The developments which are now in progress in long-distance transmission of natural gas and in the utilization of volatile petroleum products and refinery gases for augmenting city gas supplies may retard an otherwise accelerating demand for coke and coke-oven gas for domestic and industrial heating.

Trends in Value of By-Products

The increasing value of by-products in comparison to the value of coke is shown in Fig. 2.[3] In 1913 the average value of by-product coke was $3.38, the value of the by-products per ton of coke was $1.41, and the cost of coal per ton of coke was $3.27. In 1929 (not shown in figure), the value of the coke had risen to $4.80 per ton, the value of the by-products to $3.60, and the cost of coal per ton of coke to $5.07. The by-product values in 1929 were 155 per cent higher than in 1913, although the coke value had increased only 25 per cent and the cost of the coal 55 per cent. Actually the increase of the total value of coke plus by-products in this period was very little more than the increase in the cost of the coal, being 55 per cent for coal and 60 per cent for coke plus by-products.

The distribution among the several by-products, of the average gross returns from by-products per ton of coke produced, is shown in Fig. 3.[3] This chart shows important shifts in the trend of income from the different by-products. In 1913 ammonia supplied the greatest income; in 1916 and 1917 war demands for explosives gave benzol and toluol a temporary leadership over ammonia, which, however, was regained by the latter in 1918; and in 1921 gas forged ahead of the other by-products, and ammonia began to decline—a trend which has continued up to the present time. In evaluating the significance of these changes it must be remembered that Fig. 3 represents a composite of all by-product coke plants, most of which are associated with the iron and steel industry. Before 1920 most of the by-product gas was used as a metallurgical fuel and was credited to the oven at very low prices. About 1920 and 1921 the trend toward the use of by-product coke ovens for supplying base-load city gas began to make itself felt in the larger average price returns for this by-product. The decline in returns on ammonia since 1920 is due to the remarkable expansion of synthetic plants which can produce ammonia very cheaply from hydrogen and atmospheric nitrogen. The growing importance of the by-product coke oven as a source of smokeless fuel is shown in Fig. 4[4] by the increasing percentage of total coke sold for domestic use.

The trend in gross value of by-products per ton of coke as shown in Fig. 3 is due to changes in unit prices and in yields of by-products per ton of coke produced. Fig. 5[5] shows the extraordinary rise in price of toluol due to war demands. Between 1919 and 1928 the prize of benzol has been fairly constant, and that of solvent naphtha and toluol has increased slightly. Sulphate of ammonia has declined from 4 cents to less than 2 cents a pound, while tar has risen from 3.2 to 5.3 cents per gallon.

Beginning with the same date, 1919, which may well be regarded as the beginning of the by-product coke-oven era in the coal-carbonization industry, Fig. 6[6] shows a marked upward

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[1] Published by permission of the Director, U. S. Bureau of Mines. (Not subject to copyright.)

[2] Chief Engineer, Experiment Stations Division, U. S. Bureau of Mines. Mr. Fieldner was graduated in chemical engineering from Ohio State University, and subsequently entered the employ of the Denver Gas and Electric Company as apprentice fuel engineer. He then entered the Government service as fuels chemist in the U. S. Geological Survey, and the Bureau of Mines at the Pittsburgh Experiment Station. In 1921 he was appointed superintendent of the station, and in 1927 to his present position, with headquarters in Washington. During this entire period Mr. Fieldner has directed the fuels chemistry research of the Bureau of Mines.

[3] Tryon, F. G., and Bennit, H. L., "Coke and By-Products in 1928." Mineral Resources of the United States, 1928—Part II, Reprint II:30, U. S. Bureau of Mines, 1930, p. 708.

Presented at the National Meeting of the A.S.M.E. Fuels Division, Chicago, Ill., February 10 to 13, 1931.

[4] Tryon, F. G., and Bennit, H. L., op. cit., p. 752.

[5] *Ibid.*, p. 779.

[6] *Ibid.*, p. 777.

trend in the yields of by-products per ton of coke produced. Part of these increases are due to better selection of coals for by-product yields, and others to improvements in oven design and by-product recovery equipment. The marked increase in tar yield is due probably to the use of 100 per cent high-volatile coal at certain large installations of the iron and steel industry. Better oven design has reduced the gas needed for underfiring, and the increased use of producer and blast-furnace gas for heating ovens has released more surplus gas for distribution.

The average yield and value of by-products from a ton of coal for 1929, computed from Bureau of Mines statistics, is given in Table 1. It will be noted that the returns from surplus gas comprise 47 per cent or nearly half of the returns from total by-products. The returns from tar, although higher than 10 years ago, are dominated by the price of fuel oil since approximately half of the total tar production is not sold or refined but is burned as fuel in open-hearth and other metallurgical furnaces.

Present Economic Status of By-Products From Coal

Having briefly reviewed the trends in by-products of the coke-oven industry, let us consider now the present status and future prospects in relation to other sources of these products. Coal as a source of energy must compete with petroleum, natural gas, and water power. The tremendous increase in present and potential production of oil and gas has exceeded all expectations. Natural gas, petroleum, and their products compete with the products of the coal-carbonization plant. Coke for use in blast furnaces and foundries is practically the only major coal product that has no present-day competitor.

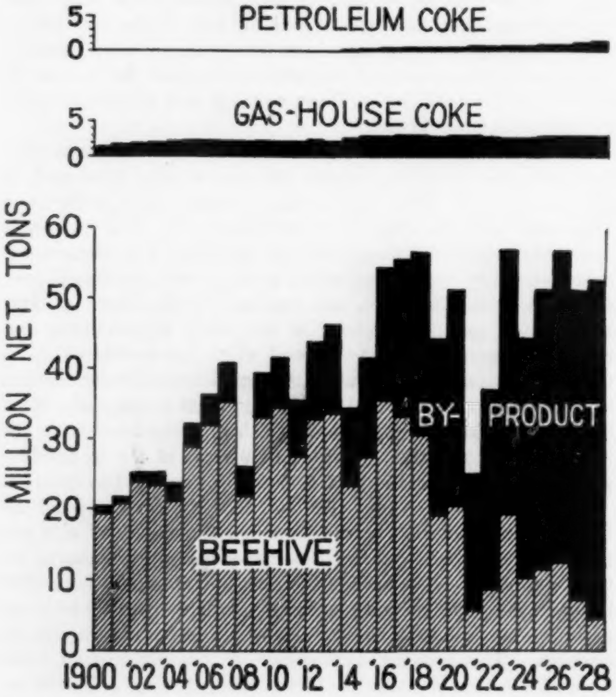

Fig. 1 Production of Coke in the United States, 1900–1928

Competitive Sources of Gas for Industrial and Domestic Use

City gas companies are becoming, to a considerable degree, distributers of purchased and blended gas. They are buying not only the surplus gas of near-by coke-oven plants, but also from oil refineries and natural-gas pipe-line companies. The oil refiners are beginning to appreciate the possibilities of selling their surplus gas, which is increasing greatly in amount due to modern developments[7] in gasoline production. In view of these additional sources of gas, and especially the construction of long-distance (1000 miles and more) pipe lines for transporting natural gas, the economic picture for the immediate future is not so favorable for by-product coke-oven gas as in the past; it will have serious competition.

TABLE 1 AVERAGE YIELD AND VALUE OF BY-PRODUCTS FROM A TON OF COAL IN 1929

	Yield	Value
Ammonium sulphate	22.3 lb.	$0.38
Light oil	2.9 gal.	0.40
Surplus gas	6,600 cu. ft.	1.10
Tar, sold	8.9 gal.	0.22
Tar, used, not sold		0.23
Total of by-products	—	$2.33
Breeze produced	—	$0.14
Coke produced	—	3.34
Total coke	70%	$3.48
Total value coke and by-products	—	$5.84

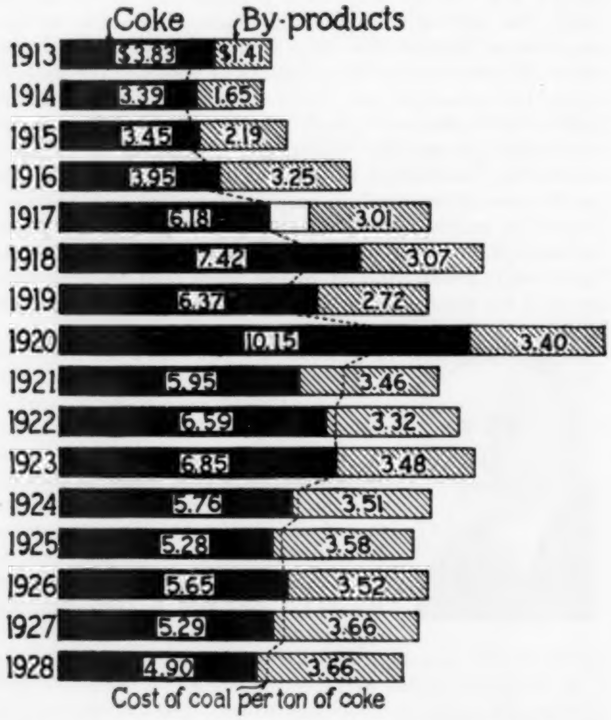

Fig. 2 Average Value of By-Product Coke and Gross Value of By-Products and Cost of Coal per Ton of Coke Produced

(It should be noted that these figures represent the gross values of the products and that there are many other expenses besides the cost of coking coal, including other materials, wages, salaries, purchased electric power, insurance, taxes, and fixed chargs on a large investment. No data on total costs or profits have been collected.)

In the United States most of the by-product coke ovens supplying metallurgical coke are operated in conjunction with steel plants, and the major portion of the gas and the tar is used in the steel plant for heating purposes. However, a few steel-plant ovens make more gas than they can use. Some of these plants sell their surplus to city gas companies, and in recent years they have added to their income by heating the coke ovens with low-B.t.u. blast-furnace gas (100 B.t.u. per cu. ft.), thus releasing all of the much higher-heating-value coke-oven gas (550 B.t.u. per cu. ft.) for sale to the city distributing company.

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[7] Cracking processes produced more than 250 billion cubic feet of cracked gas of approximately 1250 B.t.u. per cu. ft. during the year 1929, according to Egloff, Gustav, and Morrell, J. C., "The Cracking Process in the Gas-Making Industry," *Ind. & Eng. Chem.*, vol. 22 (1920), pp. 1080–1083.

In Europe by-product coke ovens are usually located at the mines, where surplus gas is not so readily used. This condition in the Ruhr district of Germany led to the formation of the Ruhr-Gas A.G., which collects gas from the Ruhr coke ovens and distributes it through a pipe-line system aggregating 590 miles in length and serving an area of 425 square miles, with a population of 10 million people.[8] The present pipe-line capacity of this system is approximately 50 billion cubic feet per annum, and the most distant town served is Hanover. It is proposed to pipe Ruhr coke-oven gas to Berlin and to construct four major long-distance lines. However, the inevitable accumulation of excess coke in order to fulfil contracts for gas during the present industrial depression may put a damper on extending this project. To meet this difficulty some of the companies are heating the ovens with blast-furnace gas, thus providing greater flexibility in the ratio of salable gas to coke produced.

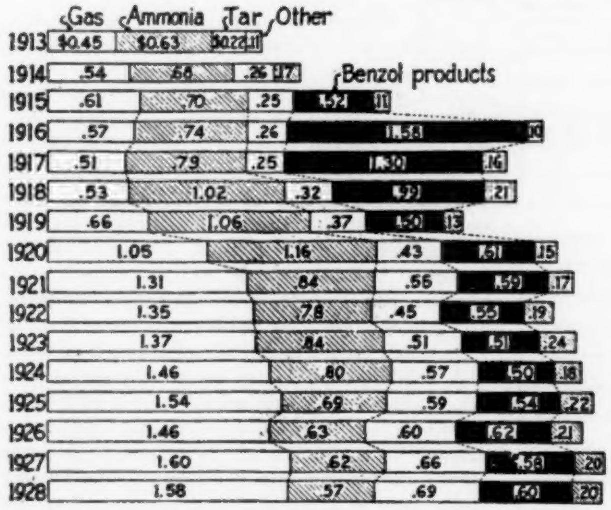

Fig. 3 Gross Value of the Several By-Products per Ton of By-Product Coke Produced, 1913–1928

Chemical Utilization of Coke-Oven Gas

The utilization of the hydrogen in coke-oven gas for the production of synthetic ammonia is another project for utilizing excess gas. The Ruhr-Chemie A.G., a combination of 20 coke-oven operators of the Ruhr district, have under construction and partly completed seven different plants with a total proposed annual capacity of approximately 200,000 metric tons of fixed nitrogen. Some 30,000 tons was produced in 1929, and it was expected that the 1930 production would reach 80,000 tons.[9] However, the present world-wide overproduction of fixed nitrogen may discourage completion of the original plan.

To those who are not familiar with this subject it should be explained that the manufacture of synthetic ammonia from coke-oven gas is quite different from the recovery of the small amounts of by-product ammonia always present in coke-oven gas. The process developed in the Ruhr involves the installation of liquefaction and fractional-distillation plants similar to those used for the production of oxygen from air. Coke-oven gas contains about 50 per cent of hydrogen. This hydrogen is extracted from the gas and chemically combined with nitrogen from the air to form ammonia. The residual gas, after removing the hydrogen, is a valuable fuel gas with a higher heating value per cubic foot than that of the original gas.

Similar plants for producing synthetic ammonia from the hydrogen in coke-oven gas have been installed in Belgium[10] and France.[11] The hydrogen from carbonizing 1 ton of coal yields approximately 110 lb. of fixed nitrogen as ammonia, compared to 5.3 lb. recovered from the gas in the usual by-product plant.

Probably the most complete chemical utilization of coke-oven gas has been made at Bethune, France. Ammonia, methanol, ethyl alcohol, and ethyl ether have been produced from the hydrogen, carbon monoxide, and ethylene in the gas after separating them in a Claude liquefaction and fractional-distillation plant. It is reported that a fourth unit, capable of making 20 tons of ammonia and $1^1/_2$ tons of methanol daily, has been completed recently.

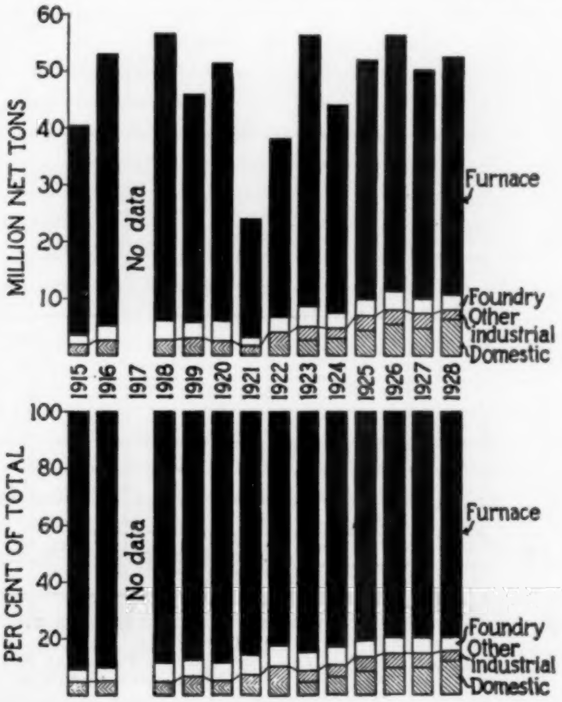

Fig. 4 By-Product and Beehive Coke Sold for Furnace, Foundry, Other Industrial, and Domestic Use

(Figures for furnace coke include all coke used by producer and not sold. The data represent the disposition made of the total production, including exports, except that in 1915 and 1916 the exports are not included.)

Competition Between Synthetic and By-Product Ammonia

As shown in Fig. 5, the decline of ammonium sulphate prices began in 1921, and has continued since that date due to the marked increase in synthetic-ammonia plants. Prior to the World War most of the commercial ammonia was obtained as a by-product from coal distillation. The World War accelerated the commercial development of synthetic processes for the catalytic combination of atmospheric nitrogen and hydrogen. The very rapid increase in world production of synthetic ammonia as compared to by-product ammonia is shown in Table 2.

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[8] Brownlie, David, "Long Distance Gas Transmission in Germany." *Gas Age-Record*, vol. 66 (1930), pp. 553–558.

[9] "German Chemical Developments in 1929." Bureau of Foreign and Domestic Commerce Trade Information Bulletin 690,1930, p. 13.

[10] Pallemaerts, F. A. F., "The Union Chimique Belge Synthetic Ammonia Plant at Ostend." Proceedings International Conference on Bituminous Coal, Carnegie Institute of Technology, 1928, vol. II, pp. 178–197.

[11] Delorme, Jean, "The Processes of Georges Claude for the Separation of Gas by Liquefaction and the Synthesis of Ammonia." Proceedings International Conference on Bituminous Coal, Carnegie Institute of Technology, 1928, vol. II, pp. 223–230.

TABLE 2 WORLD PRODUCTION OF SYNTHETIC AND BY-PRODUCT AMMONIA, IN METRIC TONS OF NITROGEN[12]

Kind of Ammonia	1917	1920	1923	1926	1929
By-product..........	340,000	290,000	325,000	360,000	440,000
Synthetic............	80,000	100,000	220,000	500,000	1,000,000

Tyler[12] estimates the current output (Sept., 1930) of synthetic ammonia for the United States at the rate of 180,000 short tons of ammonia per year, and the total capacity of plants now built, building, and immediately projected as equivalent to 360,000 tons of synthetic ammonia per year. This amount is more than $1^1/_2$ times the 1929 production of by-product ammonia.

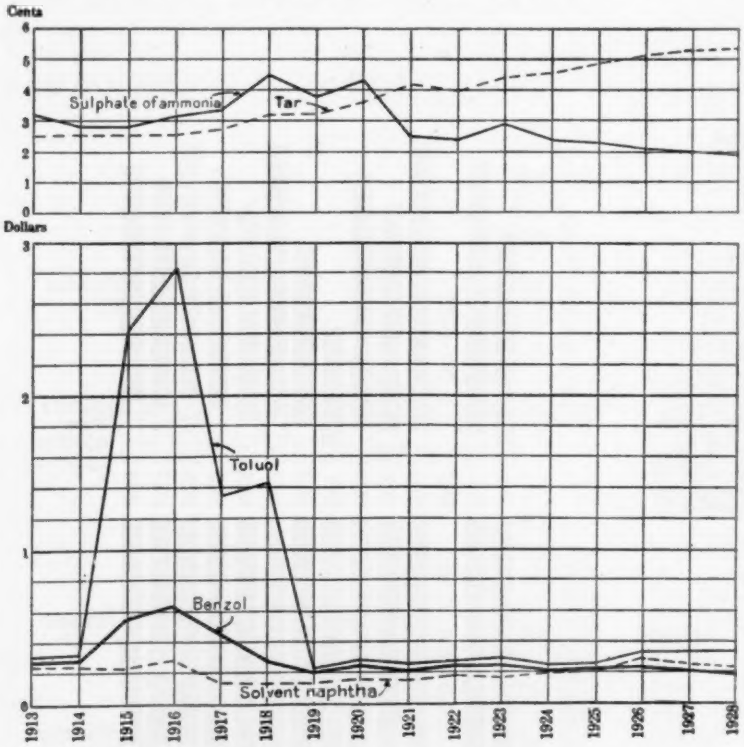

Fig. 5 Average Receipts per Unit for the Principal Refined Light Oils and Other Products, at By-Product Coke Plants, 1913–1928

(The figures for tar and sulphate of ammonia represent cents per gallon and per pound, respectively; the figures for light oils represent dollars per gallon.)

At a recent meeting (Oct. 23, 1930) of the Coke Oven Managers Association in England, J. B. Deakin, the president, stated that:

> This year for the first time we had the depressing sight of ammonia being scrubbed out of coke-oven gas and the resultant ammonia liquor used for coke quenching, and this in a modern up-to-date plant which was particularly well situated for obtaining cheap sulphuric acid."[13]

It is evident that the market price of ammonia is rapidly reaching the point where the returns will scarcely cover the cost of recovery, unless the process is cheapened. For this reason, some consideration is being given to wasting the fixed ammonia by leaving it in the liquor, thus saving steam and lime. Other economies are being introduced in new plants by installing larger saturator units, as, for example, the large new plant at the Prosper Colliery in the Ruhr. All the gas from ovens carbonizing 3750 tons of coal per day is passed through one single saturator producing 45 tons of ammonium sulphate per day.[14]

Utilization of Sulphur in Gas in Production of Ammonium Sulphate

Since sulphuric acid is one of the important elements of cost in the recovery of ammonia by the present sulphate method, investigators have made renewed efforts to utilize the sulphur in coke-oven gas for the production of ammonium sulphate. Two such processes have been developed recently in Germany. These are the Heinrich Koppers "C.A.S." process[15] and the "Tern" process.[16]

C.A.S. Process. In the C.A.S. (Cyanogen-Ammonia-Sulphur) process, cyanogen compounds, ammonia, and hydrogen sulphide are removed from the gas and recovered as ammonium sulphate and free sulphur. It is an improvement on the old Feld process. The essential principle of the C.A.S. process consists of scrubbing the gas with the concentrated ammonia liquor (10–12 per cent NH_3) and a solution of iron thiosulphate and polythionate containing iron hydroxide in suspension. The spent washing liquors containing ammonium thiosulphate, ammonium polythionate, ammonium sulphocyanate, iron sulphide, and free sulphur are then treated in appropriate circuits returning to the scrubbing cycle, with sulphur dioxide and air to regenerate iron thiosulphate, iron polythionate, and iron hydroxide, while free sulphur and ammonium sulphate are recovered in another circuit as end-products, after further treatment with sulphur dioxide and heating under pressure to 180 deg. cent. A part of the free sulphur is burned to generate the sulphur dioxide required in the process. The C.A.S. process has been installed at the Georgeschacht coke plant at Obernkirchen, Germany.

Tern Process. The Tern process, which also eliminates the purchase of sulphuric acid, has been installed by the Thüringer Gas-Gesellschaft at Leipzig, Germany. In this process the tar and ammonia liquor are separated by cooling and condensation in the usual manner. The ammonia is expelled from the liquor by distillation and stored in gaseous form in a suitable container.

The gas is desulphurized in iron oxide boxes by the ordinary dry purification method. The spent oxide is then roasted in a suitable furnace, from which gaseous sulphur dioxide is led off to a storage chamber where it is oxidized to sulphur trioxide (SO_3) in the presence of an electric arc. Gaseous ammonia and sulphur trioxide, in the proper proportions to form ammonium sulphate, are then led from the storage chambers to a reaction chamber which contains sufficient moisture to permit the formation of dry ammonium sulphate crystals. The sulphate is recovered with the aid of Cottrell precipitators.

More industrial experience is required before any opinion can be given on the economics of these new processes. It is largely a question of whether the gas works or the sulphuric acid manufacturer can make the cheapest SO_4 radical.

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[12] Tyler, Chaplin, "Some Aspects of Nitrogen Economics," *Chem. & Met. Eng.*, vol. 37 (1930), pp. 538–539.

[13] *The Gas World* (London), vol. 93, Nov. 1, 1930, p. 17.

[14] Private communication from A. Thau.

[15] *Gas Journal* (London), vol. 188, 1929, pp. 643–644; *The Gas World*, Dec. 7, 1929, pp. 19–20; Thau, A., "Die Ammoniakbindung mit Hilfe des Gasschwefels," *Glückauf*, vol. 65, 1929, pp. 1703–5; "Ammonium Sulphate Made by New Processes," *Chem. & Met. Eng.*, vol. 37, 1930, p. 681.

[16] Westphal, *Gas- und Wasserfach*, vol. 72, 1929, p. 911; *Chem. Zeit.*, 1929, p. 833; Thau, A., "Die Ammoniakbindung mit Hilfe des Gasschwefels," *Glückauf*, vol. 65, 1929, pp. 1703–5; "Ammonium Sulphate Made by New Processes," *Chem. & Met. Eng.*, vol. 37, 1930, p. 681.

Sulphur-Recovery Processes

The recovery of sulphur in marketable quantities in connection with gas purification is one of the notable achievements of recent years. Several plants in the United States have installed such processes. In 1929, the average recovery was 2.6 lb. of sulphur per ton of coal carbonized, and the sales value as ordinary brimstone was approximately $18 a ton. Flotation sulphur, which will be discussed later, has a higher value.

Charcoal Process. Activated charcoal, similar to that used in gas works, is one of the newer solid absorbents used for purifying gases from sulphur compounds. It has the advantage of absorbing organic sulphur compounds as well as hydrogen sulphide, and permits the recovery of free sulphur.[17] The gas, to which has been added sufficient air to oxidize the hydrogen sulphide to free sulphur and a very small amount of ammonia, is passed through cylindrical tanks containing activated charcoal. After a tank becomes saturated with sulphur, it is cut out from the gas circuit and the free sulphur is extracted with ammonium sulphide. After the sulphur is extracted the tank is steamed to remove the last traces of solution and regenerate the charcoal for the next cycle of absorption. The ammonia is then distilled out of the polysulphide solution for another extraction of charcoal, leaving behind the free sulphur.

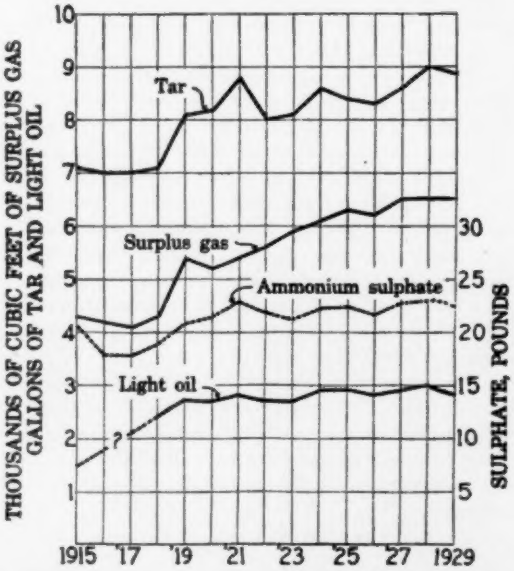

Fig. 6 Average Yield of the Principal By-Products per Net Ton of Coal Carbonized in By-Product Coke Ovens, 1915–1929

(Figures for light oil represent the average at plants recovering light oil.)

This process is used by the German I.G. at the Leuna synthetic-ammonia plant for purifying gases, and was given serious consideration among other processes for purifying the large volumes of coke-oven gases collected by the Ruhr-Gas A.G. for their gas distribution system. However, they finally adopted a similar sulphur-recovery system, using iron oxide in place of charcoal.

Ruhr Iron Oxide Process.[18] The gas is passed through large ferroconcrete boxes, from which the oxide when saturated can be removed through a bottom discharge or from the top with grab buckets. The sulphur is extracted from the oxide with ammonium sulphide in the same manner as in the activated-carbon process. After removing the sulphur the spent oxide is treated in the usual manner for the recovery of cyanogen compounds. It is reported that about 20,000 tons per annum of almost pure sulphur is being recovered by the Ruhr-Gas A.G.

Flotation Sulphur. Ordinary sulphur or brimstone is one of our cheapest commodities, present quotations at the mines being $18 a ton. Consequently the returns from this by-product, while significant, do not pay the expense of purification. However, modern chemical research can aid the by-product coke industry as well as its competitors. The research organizations of American coke-oven builders have tackled the various problems of the industry with courage and intelligent vision. One of their outstanding accomplishments has been the mechanical and chemical improvement of gas purification processes, and especially the development of new by-products from the impurities which have been removed. "Flotation" sulphur, as distinguished from ordinary sulphur, is one of these new products. It is called "flotation" sulphur because it separates out in the purification process in an extremely finely divided, or "colloidal," form and floats to the top of the purification liquid in the form of a froth. Due to its fineness of particle size, this form of sulphur has been found to be especially well adapted to use as a fungicide in spray or dust form.[19] It also has been found to have curative effects on simple forms of skin disease when used as a constituent in soap.

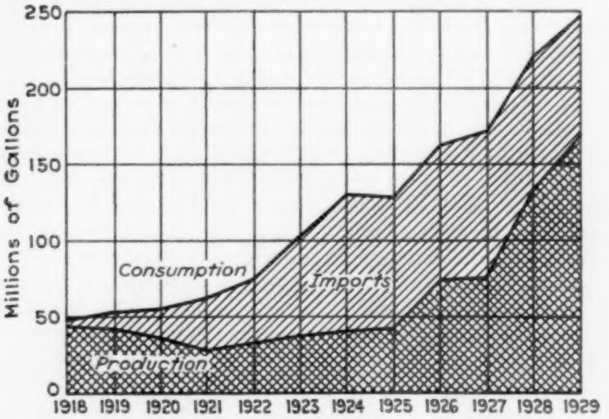

Fig. 7 Factors of United States Consumption of Creosote Oil

There are three modifications of the sulphur-recovery processes:[20] the "Thylox," "Ferrox," and Nickel processes, all using the same apparatus. The common principle of these processes is scrubbing the gas with a liquid containing sodium carbonate and a metal compound (iron or nickel) in solution or suspension. The foul solution then passes to a tank in which fine streams of air are blown up through the liquid, causing the sulphur compounds to be oxidized and the liquid regenerated for further use. The sulphur floats to the top of the liquid and collects in a froth, overflows to a slurry tank, and is filtered off from the water.

The Thylox process uses a clear solution of the purifying reagents and remains clear except for the precipitated sulphur, which consequently is recovered in a pure form. The Ferrox process uses a suspension of an iron compound and therefore yields a sulphur contaminated with iron. However, this should not be objectionable, and perhaps it may be favorable in its use as an agricultural fungicide. The Nickel process uses nickel in place of iron and is used only for oil gas and water gas, as the cyanogen compounds in coal gas consume too much nickel.

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[17] *Gas- und Wasserfach*, vol. 71, 1928, p. 296.

[18] Private communication from A. Thau.

[19] Sanchelli, V., "A New By-Product Sulphur for Agriculture." *Chemical Markets*, vol. 24, 1929, pp. 267–8.

[20] Sperr, F. W., Jr., "Gas Purification in Relation to Coal Sulphur." Proc. International Conference on Bituminous Coal, Carnegie Institute of Technology, 1928, vol. II, p. 42.

Colbert, Frank O., "Thylox Liquid Purification Plant at Racine." *Gas Age-Record*, vol. 65, 1930, p. 783.

"Gas Purification Processes." Pamphlet Published by The Western Gas Construction Co., Fort Wayne, Indiana.

All of these processes remove hydrogen cyanide (HCN) from the gas and about 95 to 98 per cent of the hydrogen sulphide. The remainder is absorbed in an iron oxide catch box.

Thiocyanates. It is a simple matter to introduce a slight modification in the liquid purification process and recover ammonium thiocyanate,[21] which is converted by heat to thiourea. Thiourea can be condensed with formaldehyde to form resins resembling bakelite. This product can be molded or machined into a variety of shapes, either transparent or colored.

The recovery process consists of scrubbing the gas after leaving the secondary washer cooler with weak ammonia liquor, which reacts with part of the hydrogen sulphide in the gas and forms ammonium sulphide. Some finely divided sulphur is then dissolved in the ammonium sulphide to form ammonium polysulphide, with which the gas is scrubbed to form ammonium thiocyanate by reaction with the cyanogen compounds in the gas.

Thus research will probably develop some financial return for another impurity that must be removed from coal gas before it is sent out through the distribution system. Of course, these are minor by-products, but they illustrate the constant thought that is being given to all the details of by-product recovery.

PHENOLS FROM STILL WASTES

Another important example of solving a troublesome waste-disposal problem with at least a partial credit from a salable by-product is the dephenolizing of ammonia-still wastes. Most coke-oven plants use the direct system of ammonia recovery. Such plants produce an average of 30 gal. of still waste per ton of coal carbonized. Plants using the indirect system produce about 90 gal. per ton. Total phenols (including cresols and other phenol derivatives) in ammonia-still wastes from direct-recovery plants average about 2 grams per liter (0.017 lb. per gal.), but may be as high as 4 grams in some cases.[22]

The discharge of such untreated still waste into a public water supply is objectionable because the phenols give rise to a disagreeable taste in the water. Modern water supplies are usually chlorinated. The chlorophenols produced in this process give a medicinal taste to the water even in dilutions as high as 50 or 100 parts per million.

Three different processes have been developed and are now in use at a number of plants for removing the phenols from this waste water. Two of the processes depend on removing the phenols in the vapor phase and the third by extracting the phenols with solvents.

Heffner-Tiddy Process. In this process advantage is taken of the volatility of ammonium phenolate. The ammonia liquor is preheated to 98 deg. cent. or higher, and enters the still countercurrent to the ammonia gas leaving the still. Ammonium phenolate is formed in the vapor state and goes on into the gas, or can be scrubbed out with caustic solution.

Koppers Vapor-Recirculation Process. In the Koppers vapor-recirculation process the hot liquor from the free-ammonia still is sprayed down a tower against an ascending stream of steam and gas or air, which vaporizes the phenols and carries them out of the liquor, which is then returned to the fixed-ammonia still. The phenol-bearing vapors are then scrubbed with caustic soda solution in another part of the cycle, forming sodium phenolate, from which phenol may be recovered by treatment with carbon dioxide or sulphuric acid.

An interesting interutilization of by-products resulting in appreciable savings consists of using the same caustic soda solution first in the dephenolizing tower and then in the benzol washer. Blast-furnace gas containing 22 to 24 per cent of carbon dioxide is then bubbled through the sodium phenolate solution, liberating phenol. After separating the phenol the residual solution containing sodium carbonate and bicarbonate is utilized in the liquid purification plant.

Solvent Extraction Process. In this process the ammonia liquor before entering the still is subjected to a countercurrent extraction with benzol or light oil. The phenols dissolve in the benzol, which is in turn extracted with sodium hydroxide solution, forming sodium phenolate, from which phenol is produced by treatment with acid or carbon dioxide.

Amount of Phenols Recovered. The amount of salable phenols recovered varies somewhat with the type of recovery process used and coke-oven-plant conditions. An average figure for American plants is 0.35 lb. per ton of coal carbonized. Four producers reported a total of 85,000 gal. of crude phenols recovered from ammonia-still waste in the United States in 1929.[23] In 1930, additional plants installed or contracted for have a potential total capacity of 250 million gallons of ammonia liquor, corresponding to 3 million pounds of phenol annually. It is probable that the amount of phenol from this source will increase considerably as new dephenolizing plants are completed.

In the Ruhr district of Germany 12 dephenolizing plants are treating 528 million gallons of ammonia liquor annually and producing 6,600,000 lb. of phenol.

In this connection it is interesting to note that the combined U. S. production of natural phenol from coal tar (not including phenol from ammonia liquor) and synthetic phenol in 1929 was 24 million pounds,[23] as compared with 10 million pounds in 1928. More than 80 per cent of the production was made synthetically from benzene.

BENZOL AND LIGHT OILS

As shown in Fig. 3, benzol products have been a steady source of income to the coke-oven operator during the last 10 years, despite the declining prices of gasoline in recent years. Approximately one-half of the 1929 production of crude light oil was sold as motor benzol at an average unit price of 16 cents per gallon. The higher price of benzol as compared to gasoline is due to its high "anti-knock" properties. It is blended with petroleum gasoline for the production of "premium" motor fuels. There is also an excellent foreign market, 33 million, from a total production of 135 million, gallons of crude and refined benzol being exported[24] in 1929. One-half or more of the benzol exported went to Germany.

As seen from Fig. 6, a permanent increase in average yield of light oil per ton of coal carbonized of from 2.7 to 2.9 gal. took place in 1924. This increase, due largely to the adoption of higher carbonizing temperatures and technical improvements, has been held ever since.

The principal recent improvement in benzol recovery has been in the refining process. Activated carbon and other solid absorbents have been tried as a substitute for oil absorption at certain European plants without practical success.

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[21] Beebee, A. M., "A Recent Development in Gas Purification." *Am. Gas. Assn. Monthly,* vol. 12, 1930, pp. 160–161.

[22] Powell, A. R., Report of Subcommittee on the Disposal of Wastes from Gas Plants. Proc. Am. Gas. Assn., 1929, pp. 928–933.
Hatch, B. F., "Phenol Recovery at By-Product Coke Plants." *Blast Furnace and Steel Plant,* vol. 17, 1929, pp. 1493–6, 1510, 1797–1804; vol. 18, 1930, pp. 618–9, 621.

[23] Census of Dyes and of Other Synthetic Organic Chemicals, 1929. United States Tariff Commission, Tariff Information Series No. 39, pp. 19–20.

[24] U. S. Tariff Commission, op. cit., p. 22.

In washing benzols with sulphuric acid to remove gum-forming constituents, it is recognized that appreciable quantities of light oil are lost due to the drastic action of the strong acid. Progress has been made in Europe in reducing this loss by using a limited amount of less concentrated sulphuric acid. Also, in the United States Jacobson[25] has reported on a new process of benzol washing known as the "Ufer" process, which has been introduced at the plant of the Hamilton Coke and Iron Co., at Hamilton, Ohio. This process avoids the production of the usual "acid sludge," the disposal of which is difficult and from which the spent acid can not be recovered readily. It also reduces light-oil losses. Jacobson states that:

> In the Ufer process, after the benzols have been agitated with sulphuric acid of a suitable strength, a certain amount of water is added to the agitator; agitation is then continued, after which the mixture is allowed to settle. Due to the addition of water, no sludge results, but instead the resins remain dissolved in the light oil, leaving a clear sulphuric acid below, which is suitable for further use in the ammonium sulphate saturator. On distillation of the light oil, the resins remain in the still residue.

Resins From Coal By-Products

Progress has been made in the utilization of the coumarone and indene compounds in light oil for the production of resins. These resins are now on the market and are used in rubber cements, printing inks, varnishes, lacquers, and driers. While this source of resin is at present of minor importance, coal is the parent substance of most of the synthetic resins which have entered into a large variety of uses, such as insulating panels, dials, and other parts of electrical apparatus (the largest use), pens and pencils, ornamental fittings, smokers' articles, novelties, etc. With the cheapening of the raw materials for resin plastics, much larger uses, such as molded furniture, interior trim, and various utensils, are in prospect. Bakelite, the best known of these resins, is made from formaldehyde and phenol, and phenol is produced from coal, both by extraction from the tar and by synthesis from benzol. The marked increase in the production and sales of phenol is largely due to the rapid increase in output of synthetic phenol resins. The total 1929 production of 33 million pounds of synthetic resins of coal-tar origin represents more than a 50 per cent increase over 1928.[26] Also, the cyanogen compounds in coal gas can be converted to thiourea, another source of resin plastics.

Cresol is also used for resins, and much experimental work has been done in recent years on making resins from the complex tar acids which comprise 20 to 30 per cent of low-temperature tar. If commercial success is ultimately achieved in such utilization of low-temperature tar it will help the cause of low-temperature carbonization, since this process yields, per ton of coal, from 20 to 30 gallons of tar, of which nearly a third is tar acids and higher phenols. Without some such special use of low-temperature tar, it is worth no more than its fuel value.

Tar and Tar Products

Next to coke and gas, tar is the most important product of coal carbonization. The average value of tar per ton of coke produced has been gradually increasing since 1913, as shown in Fig. 3. This is due both to increasing yields (Fig. 6) and increasing unit price (Fig. 5).

The expansion of metallurgical coke plants using 100 per cent high-volatile coal and developments in operating practice are responsible for the larger yields. The increased average price is due largely to expansion of demand for creosote oils for wood preservation and tar for protective coatings and road oils.

The total tar production for 1929 was 681 million gallons,[27] of which about 50 per cent was distilled and the remainder used as liquid fuel.

An important development of the last two years is the partial refining of tars at coke-oven plants. In 1928 these plants produced 12 million gallons, and in 1929, nearly 27 million gallons. Many of these stills are of a new continuous type, developed by the Barrett Company.[28] They are internally heated by the sensible heat of hot coke-oven gases. Creosote oil is recovered by fractional condensation.

The rapid increase in consumption and production of creosote oil is shown in Fig. 7. Since 1927, the importation of creosote oil has been declining, and this trend is likely to continue.

The disposal of the large quantities of pitch has always been more or less of a problem to the tar refiner, since the pitch residue is usually more than half of the original tar before distillation. However, uses are expanding. The extensive new pipe-line systems have increased the consumption of pitch and saturated felts in built-up coatings for protection from corrosion. The soft pitch is used extensively in road construction, and pitch coke has been shown to be advantageous for certain metallurgical purposes where a coke fuel free from ash or sulphur is desired.

Hydrogenation of Coal Tar

Inasmuch as 50 per cent of the tar produced in this country is burned as a fuel, its value is set by the market price of the fuel oil with which it must compete.

This situation would also hold true with low-temperature tar if it were produced in commercial quantities. Although much research has been done on finding new uses for low-temperature tar, no real commercial success has yet been achieved. Since a large part of low-temperature tar consists of hydrocarbons similar to those found in petroleum, it is amenable to cracking for the production of motor fuels, and it may also be hydrogenated by the Bergius process in the same way as petroleum. However, the yields of gasoline are less in either case than from petroleum.

It is reported that the I. G. organization in Germany is producing 100,000 tons per annum of synthetic gasoline by hydrogenating lignite tar.

In the United States, the Standard Oil Company of New Jersey, through its Development Company, joined with the I.G. in the further development and commercialization of this method as applied to the conversion of heavy oils and residues to gasoline and the production of high-grade lubricating oil. Extensive research has been conducted and is still in progress in simplifying the equipment and broadening the process, so that now it is described by Haslam and Russell as "a flexible method of treating and refining by the use of hydrogen which should have a wide application in the oil industry."[29]

Several commercial plants have been built in this country for hydrogenating petroleum. There is no question but that low-temperature tar could be treated in similar plants if and when a low-temperature carbonization industry is developed.

Summary

In summarizing the trends and recent developments in the by-product coking of coal, it is most gratifying to note the steady gain in value and yield of by-products during the last 15 years. The design of ovens and by-product-recovery equipment has been constantly improved. Practically every new installation in recent years has had some mechanical or process improvement giving greater capacity, better quality of product, better yields, lower costs, or new by-products.

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[25] Report of Builders' Section, Carbonization Committee Report, Proc. Am. Gas Assn., 1930, p. 67.

[26] U. S. Tariff Commission, op. cit., p. 59.

[27] U. S. Tariff Commission, op. cit., p. 20.

[28] Cooke, F.. "The Direct Recovery of Standard Road Tars and Other Tar Constituents of Coal Distillation Gases by Fractional Condensation." *The Gas World* (London), vol. 92, 1930, Supplement, pp. 13–16.

[29] Haslam, R. T., and Russell, R. P., "Hydrogenation of Petroleum." *Ind. & Eng. Chem.*, vol. 22. 1930, p. 1030.

The by-product oven has not only displaced the beehive oven as a producer of metallurgical coke, but it has in recent years taken an important position in supplying base-load gas for city use in conjunction with coke for the domestic fuel trade. It is probable that further extensive developments in this direction will be arrested to a considerable degree in regions that will be served by long-distance natural-gas pipe lines and possibly by the increasing amounts of refinery gas now becoming available. At present these refinery gases and liquefied gas, such as propane and butane, are supplementing rather than displacing coke-oven gas. The future interrelation of these various types of gases is difficult to predict.

The declining prices of ammonia, due to cheap synthetic ammonia, have made the profitable recovery of this by-product a real problem. European investigators have attempted to avoid the purchase of sulphuric acid for ammonia recovery by utilizing the sulphur in the coke-oven gas. Two new processes for the conjoint recovery of sulphur, ammonia, and cyanogen compounds have been developed in Europe and are being given large-scale tryouts. In America, the sulphur-recovery purification process is making excellent progress and promises to give a new by-product sulphur peculiarly adapted to fungicidal and agricultural purposes. The problem of ammonia-still waste disposal has been solved. Phenol, the troublesome constituent of these wastes, is recovered in salable form, 85,000 gal. of crude phenols being obtained in 1929.

Improvements have been made in light-oil refining which reduce light-oil losses in the washing process, and a larger proportion of coke-oven tars are now being refined, with a resultant increase in the production of creosote oil.

Much work has been done in attempts to develop new uses for low-temperature tar, but it cannot be said that any material commercial outlet, other than utilization as a fuel, has been developed.

FSP-54-5

Grindability of Coal

By R. M. Hardgrove,[1] Fullerton, Pa.

This paper points out the need for a method of determining the grindability of coal and outlines a method by which a prepared sample is partly pulverized and the new surface produced is measured in accordance with Rittinger's Law. Two types of grindability machines, the mortar and pestle and the ball mill, are described, and test data obtained from both types are compared. The relation between pulverizer capacity and grindability is discussed. A representative list of the coals of the United States and Canada is presented showing their grindabilities and the relation between grindability and volatile content. The relation between the grindability of petroleum coke and its method of manufacture and volatile content is discussed.

The testing of power-plant equipment has reached a high stage of development, and the Test Codes of the Society have been helpful in recommending the most accurate methods and standardizing the procedure and calculations, with the result that test data obtained from different plants can be compared quite readily.

The general use of pulverized coal in power plants has added pulverizers to the list of equipment to be tested. Up to the present time pulverizer tests have been quite incomplete and unsatisfactory because there was no method of determining the grinding characteristics of coal. It is similar to testing a boiler without being able to obtain the calorific value of the coal.

Accurate estimates of the performance of new equipment have been difficult to make as there was no way to predict the grindability of the coal that would be used.

In the selection of the best coals to use there are many factors to be considered such as price, freight rates, calorific value, fusing temperature of the ash, and proximate analysis, all of which are quite accurately known or can be determined on a laboratory sample.

If accurate knowledge of the grindability and abrasive qualities of the coal could be determined as easily as the fusing temperature, it would be of considerable help to the coal buyer.

Thus the need for a standard grindability method has been felt by both manufacturers and users of pulverizing equipment, and such a method applies to ores, rocks, cement, and other materials as well as to coal.

The work done in pulverizing certain materials has been investigated by various authorities, and Rittinger's Law, which states that the work done in grinding is proportional to the new surface produced, is generally accepted as being approximately correct. This theory was first proposed by Rittinger in 1867, but was not definitely proved until 1924 and 1925 by Martin.[2] Measurement of the surface of all the particles in a sample has been accomplished on silica by measuring its rate of chemical reaction or the time required to dissolve the sample in hydrofluoric acid. Mechanical screening methods seem to be more practical, however, and if one considers a 1-in. cube split into eight $^1/_2$-in. cubes, each of these small cubes will have three new faces in addition to the three old ones, or the surface of each $^1/_2$-in. cube is just double the original surface. Rittinger's Law can therefore be restated, that the work done in grinding is proportional to the reciprocal of the diameter of the particles.

The method of determining grindability may be outlined as follows:

1 Prepare the coal sample so that all particles are practically the same diameter.

2 Subject this sample to a definite process of pulverizing.

3 Screen this partly pulverized sample and determine the increased surface produced.

4 Based on Rittinger's Law, the final surface of the sample is obtained by applying constants proportional to the reciprocal of the diameters of the particles. The new surface produced is obtained by subtracting the original surface.

5 The relative grindability is obtained by comparing the new surface to that obtained in a similar way on a coal selected as a basis for comparison.

In deciding upon the size of particles in the prepared sample to be tested, it was felt that it should be at least as fine as culm so that tests could be made on that material. The coarser the material the less surface units it possesses, which is an advantage; but more of the sample will be rejected as being too fine, which is a disadvantage. A sample prepared so that all will pass through a 16-mesh screen and all be held on a 30-mesh screen seems to satisfy the requirements the best. All of the work reported in this paper was done on samples sized between these limits.

In preparing the sample it is crushed in a coffee mill, which produces the least amount of fines, until all will pass through a 16-mesh screen, after which the material finer than 30-mesh is screened out and rejected. A definite weight is then selected from this sample.

Table 1 shows the size of screens used in the final screening of the partly pulverized sample. In the second column is given the opening of each mesh in inches. In screening the sample it was assumed that the material between each interval, such as between the 100- and 140-mesh screens, has an average diameter which is the average of the openings of the 100- and 140-mesh screens, or 0.00495, and similarly for the other intervals. The

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Geoffrey Martin, Edgar A. Bowes, and F. Brand Turner, "Connection Between the Surface Area Produced and the Work Done in Tube-Mill Grinding of Quartz Sand," Transactions of the Ceramic Society, 1925–1926, vol. 25, p. 63. Geoffrey Martin, "Some Recent Research in the Science of Fine Grinding Done by the British Portland Cement Research Association Between 1923–1925," Journal of the Society of Chemical Industry, June 4, 1926, vol. 45, p. 2. Edward W. Davis, "Ball Mill Crushing in Closed Circuit With Screens," Bulletin of University of Minnesota, Aug. 27, 1925, vol. 28, no. 42, p. 4.

TABLE 1 SCREENS USED IN GRINDABILITY METHOD

U. S. series screen mesh	Opening, in.	Average opening for material between these sizes, in.	Reciprocals
16	0.046		
		0.0346	29
30	0.0232		
		0.01645	61
60	0.0097		
		0.00775	129
100	0.0058		
		0.00495	202
140	0.0041		
		0.0035	285
200	0.0029		
		0.00265	377
230	0.0024		
		0.00215	465
300	0.0019		525+

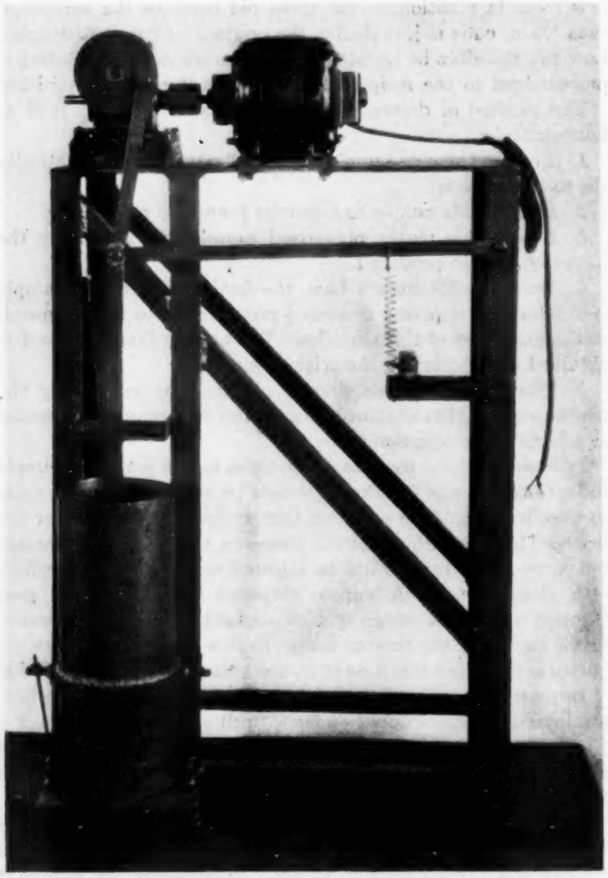

FIG. 1 MORTAR-AND-PESTLE TYPE OF GRINDABILITY MACHINE

fourth column gives the reciprocals of these average openings for the intervals used, which are the constants used in calculating the surface units.

By using a large number of screens of close intervals, a good separation of the final product into various sizes can be obtained. The material through the 300-mesh screen is, however, more difficult to evaluate except by elutriation or other tedious methods which are considered impractical for this purpose. It was felt, however, that much of this material is considerably finer than 300-mesh, and a constant of 1000 is used, which is about double the reciprocal of the opening in a 300-mesh screen. This really assumes that the material through 300-mesh screen will have an average size of 0.001 in., which probably is not far from the truth.

Table 2 gives a sample calculation of surface units on a sample which originally was between the 16- and 30-mesh screens.

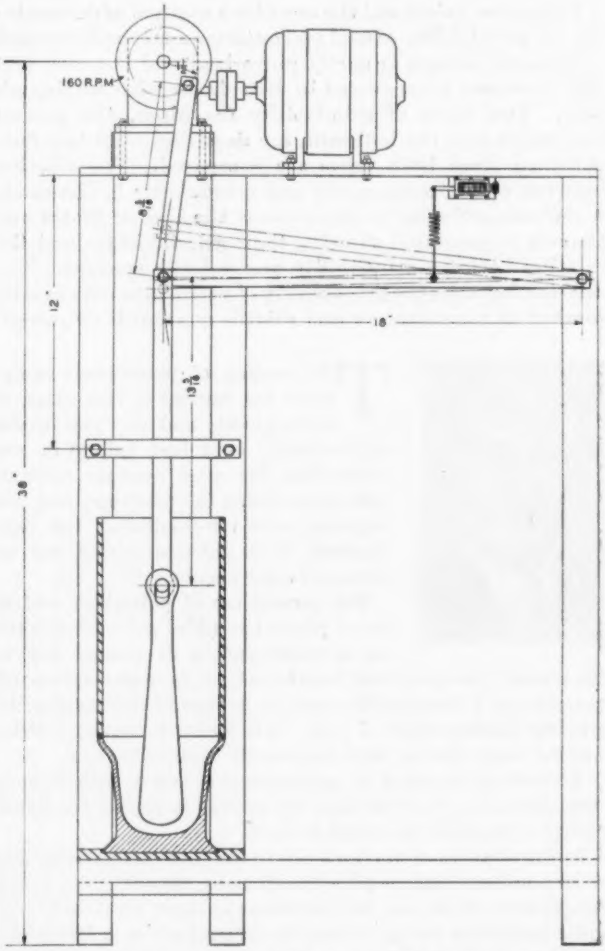

FIG. 2 MORTAR-AND-PESTLE TYPE OF GRINDABILITY MACHINE

Multiplying through by the various factors and adding them up, it is found that 19,740 surface units were produced upon this sample. The original sample being between the 16- and 30-mesh screens, had 2900 surface units before pulverizing, which

TABLE 2 SAMPLE CALCULATION OF SURFACE UNITS

Screen mesh	For Illinois coal run on ball mill, per cent of sample between these two screens		Factor		
16–30	24.4	×	29	=	708
30–60	36.0	×	61	=	2196
60–100	13.8	×	129	=	1780
100–140	7.2	×	202	=	1454
140–200	3.8	×	285	=	1084
200–230	1.6	×	377	=	602
230–300	2.4	×	465	=	1116
Through 300	10.8	×	1000	=	10800
Final surface units					19740
Less original surface units					2900
New surface units					16840

NOTE: If new surface units of standard sample is 29,961, grindability = 16,840 ÷ 29,961 = 56.2 per cent.

must be deducted; thus giving 16,840 new surface units. If the new surface units of a standard sample treated in the same way were 29,961, the grindability is taken as the ratio between these two, or 16,840 divided by 29,961 equals 56.2 per cent.

Description of Apparatus

Mortar and Pestle. Figs. 1 and 2 illustrate the first grindability machine made, which consists of a mortar and pestle with a motor and gearing for raising the pestle and letting it drop on the sample of coal in the mortar at the rate of 160 strokes per minute. The travel of the crank is just a little more than the distance the pestle is raised above the bottom of the mortar, and the speed of operation is such that the fall of the pestle due to gravity is a little slower than the actual travel of the crank, so that a true dropping motion of the pestle is produced. However, it does not travel in a true vertical motion, due to the angularity of the radius arm which tends to give the pestle a swinging motion, permitting it to hit in various positions, thus working all over the bottom of the mortar. The mortar was made sufficiently deep to avoid any appreciable loss of material while grinding.

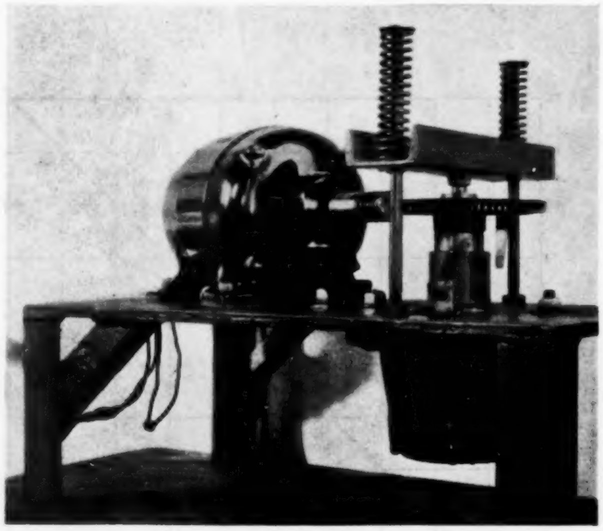

Fig. 3 Ball-Mill Type of Grindability Machine

Ball Mill. Figs. 3 and 4 illustrate the ball mill which was used in the later work. The eight 1-in. balls roll on a stationary ring and are driven from above by a rotating plate, at a speed of 21 r.p.m. A definite downward pressure is exerted on this upper plate by means of the two springs, and the result is a rolling action under pressure very similar to commercial pulverizing mills where large rolls or large balls are rolled against grinding rings or tables, the force being obtained either by centrifugal force or by springs. Dead weights were used during some of the work and gave results equal to those obtained with the springs and are more permanent. Several other minor features are being improved upon.

A small tube mill, using steel balls of a mixture of sizes, was tested in comparison with the first two types. Two other machines were built and tested, but the results from these were not promising enough to warrant inclusion in this paper.

In selecting a coal that would be called 100 grindability, with which all others could be compared, it was found that the semibituminous coals from the New River and Pocahontas fields in West Virginia and from the Upper Kittanning seam in Somerset and Cambria Counties, Pennsylvania, had about the same grindability, and were the easiest-grinding coals to be found in this country. It seemed logical that one of these should be chosen, and the most convenient one was the coal from Somerset County, Pennsylvania. Referring to Table 5, the grindability values given for New River coals average 98 while those for Pocahontas coals average 104.

Characteristics of Grindability Machines

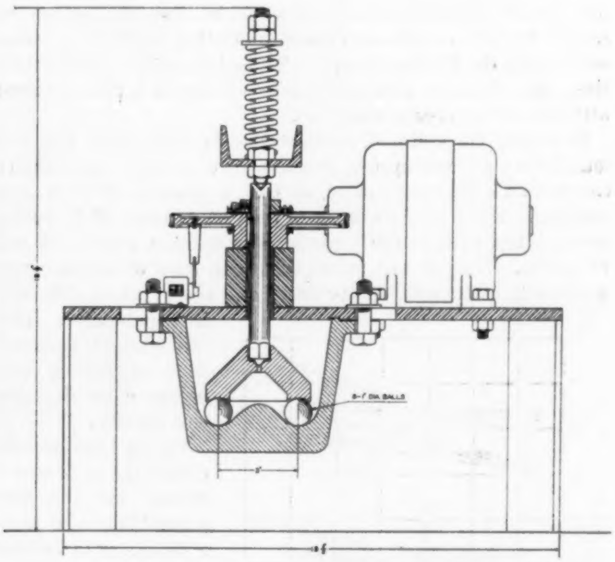

Fig. 4 Ball-Mill Type of Grindability Machine

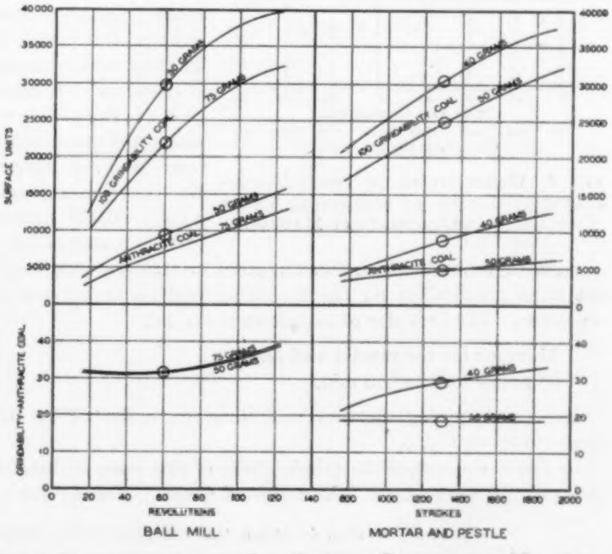

Fig. 5 Characteristics of Two Types of Grindability Machines, Using Various Sizes of Samples

In bringing out the characteristics of the different types of machines, Fig. 5 represents a series of results using this 100-grindability coal and a low-grindability anthracite coal from the Upper Pottsville district near Mahanoy City, Pennsylvania. The results on the mortar and pestle were obtained using a speed of 160 r.p.m., which was found to give the most consistent results. Results for two different sizes of original samples, namely, 40 and 50 grams, are plotted. The surface units on the 100-grindability coal are about 25 per cent higher for the 40-gram than

TABLE 3 CHECK DETERMINATIONS OF SURFACE UNITS ON THREE TYPES OF GRINDABILITY MACHINES

	Ball mill, 60 rev. (Surface units)		Mortar and pestle, 1300 strokes (Surface units)		Tube mill, 1400 rev. (Surface units)	
	100-grindability coal	Anthracite coal	100-grindability coal	Anthracite coal	100-grindability coal	Anthracite coal
	29,448	9,620	30,050	8,250	36,407	12,052
	29,938	9,818	28,975	7,957	36,712	10,080
	29,432	9,848	30,300	8,949	26,403	9,851
	29,852	9,486	31,310	9,340	26,490	9,683
	29,818	9,854	29,202	8,181	26,601	7,621
	30,544	9,946	30,647	7,684	44,306	8,450
	30,232	9,782	30,680	8,748	25,489	10,678
	30,304	9,966	30,895	8,852	29,278	10,295
	29,840	9,548	31,030	8,503	27,704	9,489
	30,198	9,636	31,170	9,055	30,005	8,938
Average	29,961	9,800	30,426	8,552	30,998	9,714
Maximum variation from average	529	186	1,451	788	13,408	2,338
Per cent	1.76	1.9	4.8	9.2	43.0	24.0

they are for 50-gram samples, as would be expected, but on the anthracite they are almost double, indicating too large a charge when using the 50-gram sample. The grindability of the anthracite, using 40 grams, also agrees more closely with that obtained with the other types of machines.

In testing the ball mill, as illustrated in Figs. 3 and 4, it was found that a lower spring pressure gave a flatter grindability curve than a higher pressure, so that a pressure of 30 lb. was adopted. Fig. 5 shows a series of tests made using 30 lb. spring pressure and with two different sizes of samples, namely, 50 and 75 grams. Here it was found that both sizes of samples give practically a flat grindability curve and a very small difference in grindability. This characteristic is important in testing materials of varying specific gravity.

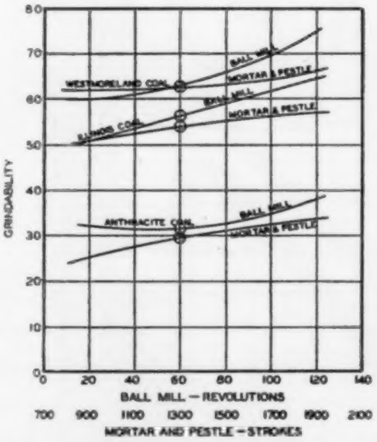

FIG. 6 GRINDABILITY OF THREE REPRESENTATIVE COALS AS OBTAINED, USING TWO TYPES OF GRINDABILITY MACHINES

On all types of machines the surface-unit curves for the 100-grindability coal show a tendency to become horizontal at the higher revolutions as the increased number of fines cushion the grinding. The best working range is therefore at a number of revolutions where the curve has a more uniform slope. By choosing approximately 30,000 surface units as a standard for each, it was found that points can be obtained that give practically the same grindability for the anthracite coal on both types of machines. The best size of sample seems to be:

40 grams for the mortar and pestle
50 grams for the ball mill.

The points giving the best conditions are indicated by the large circles on Fig. 5.

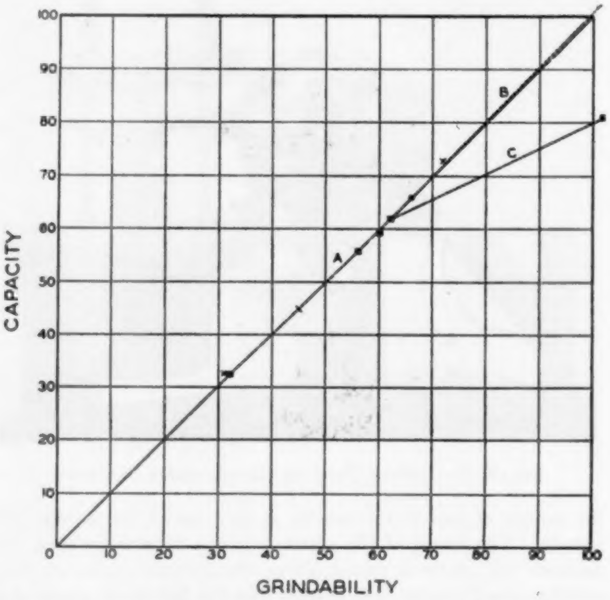

FIG. 7 RELATION BETWEEN MILL CAPACITY AND GRINDABILITY

In Fig. 6 are plotted the grindabilities of this same anthracite coal, Illinois coal, and a Westmoreland County, Pennsylvania, coal. For ease in comparing the two types of machines the abscissas are plotted so that the best number of revolutions coincide.

By properly choosing the working conditions, practically the same grindability of all three coals can be obtained with both types of machines. Preference for either type depends upon its ease of manipulation and the ability to duplicate results rather than on any real difference in the scale of the results obtained.

It is quite evident that in order to establish a standard method various laboratories must use the same size of sample, the same number of revolutions, the same speed, the same method of screening and calculation, and preferably the same type and size of machine.

The tube mill was tested in a similar manner, and the results gave in general about the same values as the other two types, but they were so inconsistent that they have not been included. In Table 3 are listed the surface units for 10 consecutive tests on the 100-grindability coal and the anthracite coal on the mortar-

TABLE 4 MILL CAPACITIES AND GRINDABILITIES OF POINTS PLOTTED IN FIG. 7

	Material	Grindability	Capacity, tons per hour
Curve *A* (×), closed-circuit system, coarse grinding	Coke Chrome ore Limestone	31 45 72	16.2 22.4 36.0
Curve *B* (⊙), tube mill with external classifier and with large recirculating load	Coal Coal	66 102	3.6 5.6
Curve *C* (⊡), air-swept mill with internal classifier	Coal Coal Coal	32 56 60	5.7 10.0 10.6
Smaller mill	Coal	62	4.6
Smaller mill	Coal	102	6.0

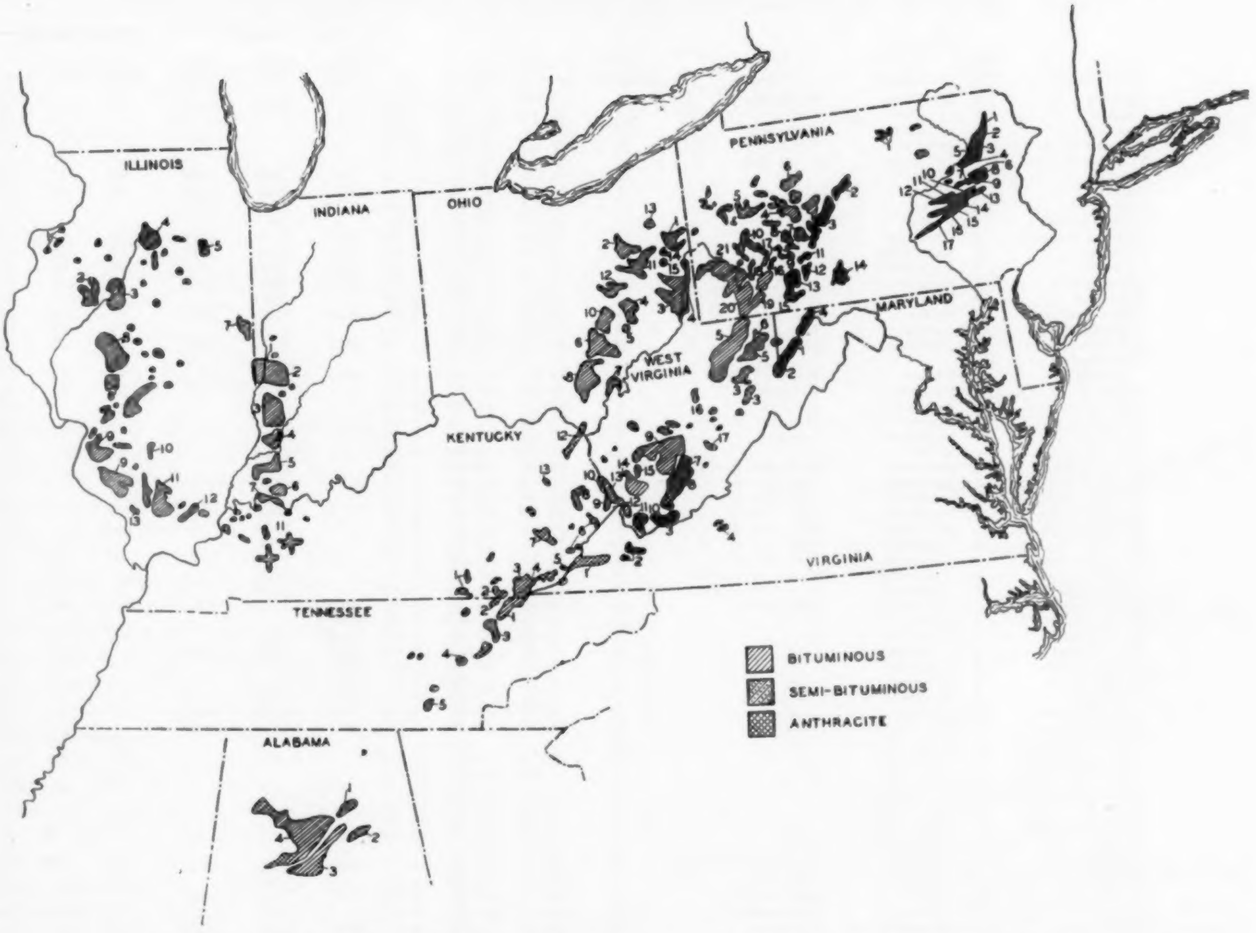
ILLINOIS
INDIANA
OHIO
PENNSYLVANIA
WEST VIRGINIA
KENTUCKY
MARYLAND
VIRGINIA
TENNESSEE
ALABAMA
▨ BITUMINOUS
▨ SEMI-BITUMINOUS
▨ ANTHRACITE

FIG. 8 MAP OF THE ACTIVELY MINED COAL FIELDS OF THE EASTERN PART OF THE UNITED STATES

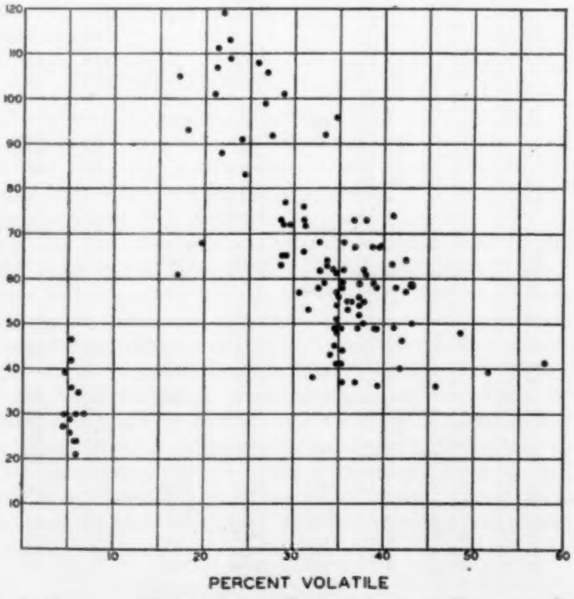

FIG. 9 RELATION BETWEEN THE GRINDABILITY OF COAL AND ITS VOLATILE CONTENT

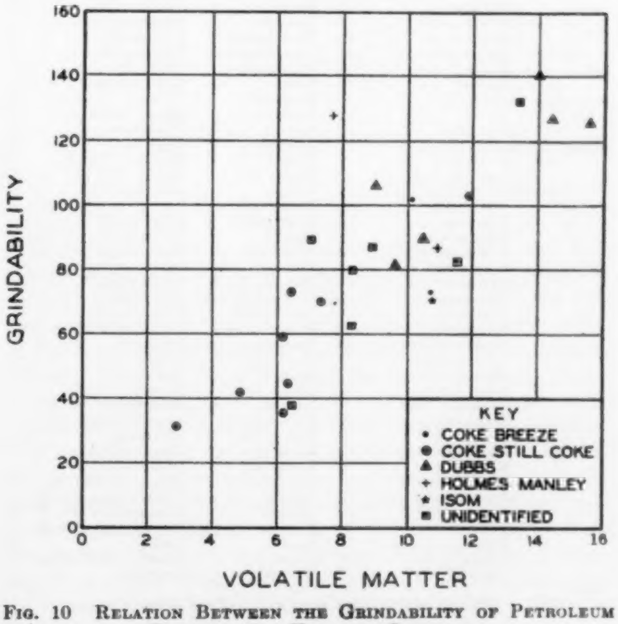

FIG. 10 RELATION BETWEEN THE GRINDABILITY OF PETROLEUM COKE AND ITS VOLATILE CONTENT

TABLE 5 GRINDABILITY OF REPRESENTATIVE COALS OF THE UNITED STATES AND CANADA

State	District	Map No.	County	Seam	No. of samples	Volatile matter	Fixed carbon	Ash	Fusing temperature			Grindability		
									Initial def.	Fusing temp.	Fluid temp.	Max.	Min.	Avg.
Ala.	Cahaba	3	St. Clair	Harkness	1	32.90	57.10	10.00	....	2323	....	...	...	62
	Cahaba	3	St. Clair	Helena	1	35.10	59.10	5.80	....	2340	....	...	...	49
	Cahaba	3	St. Clair	Harkness	1	32.55	54.55	12.90	....	....	....	...	...	58
	Warrior River	4	Jefferson	Pratt	1	29.45	59.60	10.95	....	....	....	...	...	72
	Warrior River	4	Jefferson	Mary Lee	1	28.80	59.00	12.20	....	....	....	...	...	72
Col.	Denver	..	Weld	...	3	51.70	36.00	12.23	2232	2306	2507	41	37	39
	Denver	..	Weld	...	2	48.60	45.80	5.57	2005	2055	2195	49	46	48
	Denver	..	Weld	...	1	57.90	32.85	9.25	2170	2230	2475	...	...	41
Ill.	Fulton County	2	Fulton	No. 5	1	31.10	40.05	18.65	....	1975	....	...	...	66
	Central Illinois	6	Christian	No. 6	8	37.70	48.68	13.80	1925	2050	2100	59	44	55
	Danville	7	Vermilion	No. 6	8	35.46	50.75	13.70	2010	2250	2375	77	60	68
	Danville	7	Vermilion	No. 6	4	34.93	52.5	12.30	2138	2260	2360	59	53	57
	Springfield	8	Sangamon	No. 5	26	37.75	46.69	15.22	1975	2025	2205	71	51	61
	Belleville	9	St. Clair	No. 6	1	34.87	42.02	13.57	....	2095	....	...	...	61
	Saline-Gallatin	12	Saline	No. 5	4	35.00	51.05	8.47	2120	2225	2350	63	53	56
	Murphysboro	13	Jackson	No. 2	2	41.00	51.70	7.30	....	2100	....	74	75	74
Ind.	Sullivan-Linton	3	Sullivan	No. 5	2	89.25	44.00	11.75	2007	2095	2187	58	57	58
	Princeton-Ayrshire	5	Gibson	No. 5	2	38.02	47.20	9.90	2095	2240	2435	73	73	73
Ky.	Elkhorn	6	Letcher	Elkhorn	2	36.45	55.22	8.32	....	2892	....	54	55	55
	Elkhorn	6	Letcher	Hazard	1	35.55	48.05	16.40	....	2690	....	...	...	62
	Hazard	7	Perry	Hazard No. 4	1	34.95	57.55	7.50	....	2720	....	...	...	48
Mich.	Saginaw	..	Saginaw	Saginaw	1	37.65	52.70	9.65	....	2220	....	...	...	50
N. Dak.	...	..	Ward	...	1	61.70	32.90	5.40	....	2360	....	...	...	50
Ohio	Belmont-Pgh	3	Belmont	Pgh. No. 8	1	42.40	50.60	7.00	....	2120	....	...	...	57
	Belmont-Pgh	3	Belmont	Pgh. No. 8	1	42.49	51.02	6.49	2220	2310	2395	...	...	64
	Belmont-Pgh	3	Belmont	Pgh. No. 8	1	34.80	54.05	11.15	....	2120	....	...	...	54
	Belmont-Pgh	3	Belmont	Pgh. No. 8	1	40.80	50.60	8.60	....	2070	....	...	...	63
	Belmont-Pgh	3	Belmont	Pgh. No. 8	1	39.05	51.05	9.90	....	2070	....	...	...	49
	Belmont-Pgh	3	Belmont	Pgh. No. 8	1	37.75	51.25	11.00	....	2100	....	...	...	62
	Belmont-Pgh	3	Jefferson	Pgh. No. 8	1	37.10	58.60	4.90	....	2235	....	...	...	49
	Cambridge	4	Guernsey	Upper Freeport	1	34.55	57.55	7.90	....	2410	....	...	...	49
	Cambridge	4	Guernsey	Upper Freeport	1	32.15	55.05	12.80	....	2240	....	...	...	38
Okla.	McAlester	..	Pittsburgh	Wilburton	10	30.60	55.30	14.00	2075	2115	2295	67	47	57
Pa. Anth.	Wilkes-Barre	4	Luzerne	...	1	5.63	88.40	5.95	3000+	....	....	...	...	24
	Wilkes-Barre	4	Luzerne	...	1	5.95	87.45	6.60	3000+	....	....	...	...	21
	Black Creek	7	Luzerne	...	2	5.35	65.03	29.62	2890	2960	3000+	...	...	36
	Black Creek	7	Luzerne	...	1	5.10	82.40	12.50	3000+	....	....	...	...	26
	Black Creek	7	Luzerne	...	2	5.22	81.85	12.92	3000+	....	....	...	...	29
	Hazleton	8	Luzerne	...	1	5.65	79.60	14.75	2935	3000+	....	...	...	24
	Panther Creek	13	Carbon	...	1	4.70	81.20	14.10	2885	2950	3000+	...	...	30
	Panther Creek	13	Schuylkill	...	1	6.75	83.90	9.35	2875	2950	3000+	...	...	30
	West Schuylkill	15	Schuylkill	...	3	...	...	...	....	....	....	59	40	52
	West Schuylkill	15	Schuylkill	...	4	5.50	79.80	13.20	....	....	....	46	38	42
	West Schuylkill	15	Schuylkill	...	3	4.10	83.10	10.60	....	....	....	31	23	27
	West Schuylkill	15	Schuylkill	...	3	...	...	...	....	....	....	47	39	43
	West Schuylkill	15	Schuylkill	...	1	6.39	77.93	15.68	....	....	....	...	...	35
	West Schuylkill	15	Schuylkill	...	1	5.49	67.25	23.56	....	....	....	...	...	46
	West Schuylkill	15	Schuylkill	...	1	4.90	83.10	10.80	....	....	....	...	...	39
	Lorberry	16	Schuylkill	...	1	...	...	...	....	....	....	...	...	37
Pa. Bit.	Clearfield	3	Clearfield	Upper Kittanning	2	21.87	63.87	14.25	2580	2652	2725	92	83	88
	Low Grade	5	Clarion	Brookville	1	31.55	52.70	15.75	....	2540	....	...	...	53
	Shawmut	6	Elk	Kittanning	1	28.55	61.50	9.95	2140	2460	2585	...	...	73
	Indiana	9	Indiana	Kittanning	21	26.80	63.42	9.85	....	2158	....	119	99	106
	Johnstown	11	Cambria	Freeport	2	22.00	71.30	6.65	2420	2535	2665	123	115	119
	Johnstown	11	Cambria	Kittanning	1	20.65	64.10	7.25	....	2340	....	...	...	101
	Johnstown	11	Cambria	Miller	1	21.40	73.85	4.75	2570	2680	2800	...	...	111
	Somerset[a]	13	Somerset	Upper Kittanning	100[a]	17.45	71.88	9.67	2750	2860	2950	103	98	100
	Youghiogheny	18	Westmoreland	Pittsburgh	1	36.10	56.75	7.75	....	2070	....	...	...	53
	Pittsburgh	21	Allegheny	Pittsburgh	1	37.25	55.14	7.61	....	2380	....	...	...	54
	Pittsburgh	21	Fayette	Pittsburgh	1	33.45	54.85	11.70	....	2585	....	...	...	59
	Pittsburgh	21	Washington	Pittsburgh	1	33.61	57.34	9.05	....	2646	....	...	...	63
	Pittsburgh	21	Westmoreland	Pittsburgh	1	35.34	58.36	6.30	....	2460	....	...	...	58
Texas	Eagle Pass	..	Maverick	Eagle Pass	3	24.10	51.13	26.71	....	2950	....	104	82	91
	Laredo	..	Webb	San Pedro	3	45.84	36.31	17.85	....	2138	....	39	32	36
Tenn.	...	..	Overton	Wiler	1	33.70	53.40	12.90	....	2625	....	...	...	64
Utah	...	..	Carbon	Mesa Verde	1	42.00	51.10	6.90	....	2305	....	...	...	46
	...	..	King	Hiawatha	1	43.05	49.45	7.50	2460	2570	2690	...	...	46
Wash.	...	..	King	McKay	2	41.70	48.5	9.82	....	2580	....	41	39	40
	...	..	Pierce	No. 2	2	32.95	55.5	11.55	....	2752	....	68	67	68
	...	..	Pierce	No. 2	2	29.05	55.30	15.70	....	2662	....	87	67	77
	...	..	Pierce	No. 3	1	21.20	60.30	18.50	....	2775	....	...	...	101
	...	..	Thurston	...	1	19.75	70.65	9.60	....	2425	....	...	...	68
W. Va.	Fairmont	5	Harrison	Pittsburgh	1	39.60	53.45	6.95	....	2200	....	...	...	67
	Fairmont	5	Marion	Pittsburgh	2	39.82	52.85	7.32	2085	2160	2345	67	67	67
	Fairmont	5	Marion	Pittsburgh	1	39.10	56.60	4.30	2330	2425	2540	...	...	49
	Fairmont	5	Marion	Pittsburgh	1	37.25	58.24	4.50	2410	2520	2640	...	...	52
	Fairmont	5	Marion	Pittsburgh	1	35.41	54.99	9.60	2160	2295	2395	...	...	59
	Fairmont	5	Marion	Pittsburgh	1	41.14	53.44	6.40	2030	2210	2360	...	...	49
	Fairmont	5	Monongalia	Pittsburgh	1	37.25	53.25	9.50	....	2320	....	...	...	59
	Fairmont	5	Monongalia	Pittsburgh	1	38.90	53.05	8.05	....	2220	....	...	...	59
	Preston-Taylor	6	Preston	Freeport	1	28.55	60.40	11.05	....	2800	....	...	...	63
	New River	7	Fayette	Sewell	2	27.40	68.30	4.30	....	2817	....	94	90	92
	New River	7	Fayette	Sewell	1	22.65	72.45	5.30	2240	2380	2460	...	...	109
	New River	7	Green Brier	Sewell	1	26.60	69.85	3.55	....	2305	....	...	...	99
	Winding Gulf	8	Raleigh	Dorothy	3	35.25	58.70	6.05	2775	2836	2900	42	40	41
	Kanawha	9	Kanawha	Cedar Grove	2	34.77	52.80	12.42	....	2740	....	42	39	41
	Kanawha	9	Kanawha	Coalburg	2	35.40	52.47	12.12	....	2813	....	45	42	44
	Kanawha	9	Kanawha	Coalburg	2	34.90	54.97	10.15	....	2960	....	47	39	43
	Kanawha	9	Kanawha	Winifrede	2	35.15	55.12	9.72	....	3000+	....	38	36	37
	Pocahontas	10	Mercer	Orista No. 3	1	17.10	72.18	6.34	....	....	....	...	...	105
	Pocahontas	10	Mercer	Pocahontas No. 3	2	22.27	72.17	5.55	2277	2387	2523	119	108	113
	Pocahontas	10	Mercer	Pocahontas No. 3	2	21.25	73.35	5.35	2268	2425	2552	109	104	107
	Pocahontas	10	Mercer	Pocahontas No. 3	2	18.11	71.25	9.62	2620	2740	2817	96	90	93
	Logan	14	Logan	Eagle	2	34.87	61.82	3.27	....	2657	....	52	46	49
	Logan	14	Logan	Island Creek	2	34.85	58.49	6.66	....	2482	....	49	52	51
	Coal River	15	Boone	Dorothy	2	36.75	59.50	3.75	2725	2800	2890	37	36	37

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[a] This is 100-grindability coal, used as a basis of comparison.

TABLE 5 (*Continued*) GRINDABILITY OF REPRESENTATIVE COALS OF THE UNITED STATES AND CANADA

Canada

Province	Town	Seam	No. of samples	Volatile matter	Fixed carbon	Ash	Fusing temperature Initial def.	Fusing temperature Fusing temp.	Fusing temperature Fluid temp.	Grindability Max.	Grindability Min.	Grindability Av.
Alta.	Blairmore	Frank, Blairmore	1	17.00	35.15	57.85	....	2690	....	...	...	61
	Cadomin	Cadomin	1	24.45	60.60	14.95	....	2695	....	...	...	83
B. C.	Coal Creek	No. 2	1	25.75	67.40	6.85	....	2425	....	...	...	108
	Coalmont	Bituminous	1	34.35	52.05	10.50	....	2375	....	...	...	62
	Coalmont	Bituminous	1	36.00	53.10	7.53	....	2285	....	...	...	55
N. B.	North Minto	Grand Lake	1	31.30	47.85	20.85	2020	2070	2215	...	...	72
N. S.	Glace Bay	Emery	1	34.54	47.46	18.00	....	2395	....	...	...	45
	Glace Bay	Emery	1	31.10	48.05	20.85	....	2360	....	...	...	73
	Springhill	....	1	29.41	54.70	11.34	2140	2260	2360	...	...	74
Ont.		Lignite	1	31.15	11.90	5.18	....	2000	....	...	...	76
Vancouver Island	Cassidy	Douglas	1	33.32	41.13	24.42	....	2380	....	...	...	92
	Cassidy	Douglas	1	34.75	47.18	16.90	....	2380	....	...	...	92
	Cumberland	Comox	1	29.25	53.65	17.10	....	2355	....	...	...	65
	Cumberland	Comox	1	28.66	45.70	25.49	....	2580	....	...	...	65
	Nanaimo	Douglas	1	36.65	41.95	21.40	....	2345	....	...	...	73
	Nanaimo	Douglas	1	40.40	46.50	13.10	....	2345	....	...	...	58
	Nanaimo	Newcastle	1	43.15	44.45	12.40	....	2400	....	...	...	50
	Nanaimo	Newcastle	1	37.15	41.00	21.85	....	2350	....	...	...	56
	South Wellington	....	1	38.70	49.60	11.70	....	2410	....	...	...	67
	South Wellington	....	1	36.75	44.40	18.85	....	2305	....	...	...	67

and-pestle, ball-mill, and tube-mill types. The consistency of the results clearly places the ball mill first, the mortar and pestle second, while the tube mill is so erratic that it is eliminated.

The ball mill is quite easy to clean and is entirely closed during grinding. The mortar and pestle is easy to clean, but loses some fine dust during grinding. The tube mill is very difficult to clean as it is necessary to brush off each ball in collecting the sample. The result is that the average operator will lose about 4 per cent of the sample with the tube mill, whereas with the mortar and pestle less than 2 per cent will be lost, while with the ball mill less than 1 per cent will be lost.

In calculating the results it was assumed that any material lost was finer than 300 mesh, and this is probably true for the most part, but the fact that more is lost in the manipulation of the tube mill partly explains why its results are not as consistent.

COLLECTING SAMPLES

As the coal is found in the mine, the different layers have different grindabilities, and it is only natural that in the mining, handling, and shipping operations the softer coal should be broken down into the smaller sizes while the harder coal remains in the larger lumps. It is therefore quite essential that a proper sample be taken which includes the correct proportion of larger lumps and fine sizes. To do this requires just as much care as is necessary in obtaining a sample for calorimeter determinations, and a sample for grindability determinations should be taken in the standard manner of sampling coal, quartered down, and crushed until it is of a size suitable for sending to the laboratory. The effect of taking an improper sample is clearly brought out by a series of grindability determinations made on a coal which had been separated into three sizes at the mines. Each one of these sizes was run separately, and the following results were obtained:

Size	Grindability
On $1\,{}^{1}/_{4}$-in. bar screen................	82
Over ${}^{3}/_{8}$-in. hummer screen..........	89
Through ${}^{3}/_{8}$-in. hummer screen...........	104

TABLE 6 GRINDABILITY OF PETROLEUM COKE

Grindability	Volatile matter	Fixed carbon	Ash	Fusing temperature	Process	Remarks
175	50.10	40.70	0.20	2185	Holmes Manley	Petroleum pitch
151	11.85	85.25	2.90	....		
140	14.00	85.75	0.25	....	Dubbs	
132	13.40	.....	1.00	2395		
128	7.70	92.00	0.30	2150	Holmes Manley or Badger	
127	14.40	83.35	2.25	....	Dubbs	
126	15.55	80.30	4.15	....	Dubbs	
123	12.20	87.75	0.05	....		Cracking still coke
120	15.35	84.05	0.60	2555		Pressure still coke
108	8.00	90.10	1.90	2160		Cracking still coke
107	19.43	77.03	1.53	2240		
106	8.95	90.45	0.60	2095	Dubbs	
103	11.95	87.60	0.45	1975		Coke still coke
102	10.05	89.39	0.60	2000		Coke breeze
99	9.15	90.65	0.20	2060		Hard lump coke
95	17.10	79.20	3.70	2500		
90	10.42	88.78	0.80	1995	Dubbs	
89	6.95	92.95	0.10	2375		
87	8.92	84.01	0.40	2095		
87	10.90	88.55	0.55	1970	Holmes Manley	
83	11.50	72.70	15.80	....		
82	9.60	88.25	2.15	....	Dubbs	
80	8.27	91.18	0.15	....		
76	11.01	81.69	6.80	2520		
76	7.95	91.90	0.15	....		
73	10.70	87.85	1.45	1875		Coke still coke Breeze
73	6.65	93.20	0.15	2030		Coke still coke
72	18.00	80.90	1.10	2425		
71	10.76	81.39	1.71	....	Isom.	Coke still coke
70	7.40	92.30	0.30	....		
63	8.30	91.05	0.65	2055		Coke still coke
59	6.20	93.75	0.05	....		
58	12.10	86.60	1.30	2535		Pressure still coke
57	12.30	86.85	0.85	2160		Stack petroleum coke
47	8.80	90.85	0.35	2090		Coke still breeze
46	13.70	86.20	0.10	2070		Coke still coke
46	9.80	89.35	0.85	2025		Coking mixture of reduced crude
45	6.35	91.35	2.30	2220		Coke still coke
42	4.90	95.45	0.05	....		Coke still coke
38	6.45	93.05	0.50	2295		
37	5.95	89.05	5.00	2025		Coke heel
37	6.80	92.90	0.60	2460		Cracking coil coke
35	6.20	93.75	0.05	....		Coke still coke
33	7.95	91.90	0.15	2950		Stack coke
31	2.95	96.85	0.20	....		Coke still coke
25	13.50	80.10	6.40	....		Caustic tar butts

The establishment of a definite grindability scale that is in general use would be of great value even though the numbers of the scale had no exact relation. However, if this scale can bear some definite relation to mill capacity it will be still more useful. In Fig. 7 are plotted the relative capacities obtained by grinding material of various grindabilities on the same pulverizer. Curve *A* represents three points obtained with different materials in a closed-circuit grinding system with relatively coarse grinding. The material being ground was cushioned very little by fine products and therefore can be said to follow its true grindability quite closely. The points match the theoretical relation as shown by the dot-and-dash line. Points on the line *B* were obtained by grinding coal in a tube mill equipped with an external classifier where a high recirculating load was used, which means that the grinding parts were scavenged quite completely. This also checks the theoretical curve. Curve *C* represents five points obtained from air-swept mills with internal classifiers which shows four points, having grindabilities of 62 and less, checking the theoretical curve quite closely, while the fifth point, having a grindability of 102, shows a lower capacity. This is to be expected, for most of these mills are designed for coals having a grindability around 50 to 70, and the air conditions are designed for that type of material. When one tries to realize the full added capacity by grinding a much softer coal, the scavenging is much less complete and the mill parts are cushioned with fines. This means that the relation between maximum capacity and grindability depends greatly upon the type of mill and the exact conditions of its operation, and that relations anywhere between the curve *C*. or lower, and the theoretical curve can be obtained.

Each coal itself has only one grindability under all conditions, and the design and application of pulverizers should be suited to the capacity and grindability.

It has been suggested that standard capacity ratings of pulverizers should be on the basis of 100-grindability coal. With coals varying in grindability as they do, any standard capacity ratings are misleading, but if such a rating were used it would be much better to rate them at 50 or 60 grindability where the capacities are much less affected by the conditions of operation, as is brought out in Fig. 7.

On the basis of Fig. 7, it seems that the scale established is a very reasonable one, even though it requires individual interpretation with different types of pulverizing mills.

Since a mill usually takes about the same power when it is fully loaded, irrespective of how many tons it is producing, it follows that the power per ton is inversely proportional to the maximum capacity, and therefore inversely proportional to the grindability of the material.

During the four years that this method has been in use over 2000 samples have been tested. In Table 5 are given the grindabilities of representative coals from the various parts of the United States and Canada. In a number of instances several samples from the same mine have been tested. These results have been averaged, and the sixth column gives the number of samples represented, and the maximum, minimum, and average grindabilities are listed. Fig. 8 is a map of the northeastern section of the United States showing the actively mined regions with identifying numbers which match the numbers given in Table 5.

In general there is a relation between the volatile content of the coal and its grindability, as illustrated in Fig. 9. Coals between 16 and 28 per cent volatile are usually of high grindability, and this decreases as the volatile increases, but there are so many exceptions to this trend that determination of volatile is not a definite guide to grindability. The Pennsylvania anthracites show a remarkably uniform variation from very hard in the eastern end of the field to quite soft in the western end, as will be seen by comparing the grindabilities in Table 5 with the map.

Table 6 shows the grindability of a number of petroleum-coke samples with a partial description of the process by which the coke was produced. There seems to be some relation between grindability and volatile as brought out in Fig. 10.

Conclusions

1 There is a definite need for a practical laboratory method of determining the grindability of coal and other materials.

2 Of the three types of machines tested, both the mortar and pestle and the ball mill give results consistent enough for most purposes, but the ball mill gives the best checks on repeated determinations. The tube mill is not consistent enough to warrant its use as a grindability machine.

3 The relation between grindability and pulverizer capacity will have to be determined for each type of pulverizer and the conditions under which it is operated. With very complete scavenging of fines from the grinding parts, the capacity becomes very nearly proportional to the grindability.

4 Thorough and accurate procedure is necessary in obtaining representative field samples that contain the correct percentage of coarse coal and fines, otherwise the laboratory determination of grindability will be misleading.

5 Table 5 lists the grindabilities of representative coals of the United States and Canada that should prove useful to all users of pulverized coal. The agreement between various samples from the same mine is in general fairly close.

6 Petroleum cokes have a much wider variation in grindability than coals, and show a definite trend for the grindability to increase as the volatile increases.

7 The general use of grindability determinations will necessarily require standardization, and it is to be hoped that the A.S.M.E. Test Code Committee will propose a definite method for the determination of the grindability of coal and other materials at an early date.

Discussion

W. W. Pettibone.[3] The paper presents data and information of real value to every one interested in pulverized coal. Efforts to have the equipment and methods described in the paper at least tentatively adopted by the A.S.T.M. Test Code Committee should be encouraged by the A.S.M.E., also by the coal-mining industry.

Certainly, it is reasonable to believe that "grindability" will eventually be included in coal specifications by purchasers of coal for use in pulverizers. At the present time with the manufacturer basing the power consumption and capacity of his equipment upon fuels of more or less known grindability, it is essential for the purchaser of fuels for this equipment to know that the fuels offered are not going to either reduce the equipment capacity or result in materially increased power consumption unless compensation is obtainable in fuel cost.

Fuel specifications now cover other more or less physical items of the fuel such as ash content, moisture, and ash-fusion point, and with the relationship shown of pulverizer capacity and grindability, coal purchasers very likely will eventually include grindability in their specifications where fuels are to be pulverized. If in the case of a 100 grindability coal the power consumption is 13 kw-hr. per ton, then a 75 grindability coal would probably mean about 17.3 kw-hr. per ton, or a 33 per cent increase in power consumption per ton.

The author says that a general relation exists between the volatile content and grindability, but it is questioned whether this general relation is of sufficient value to be of any merit. He qualifies his general statement by reference to the many exceptions to this trend. From Fig. 9 of the paper there is a variation in grindability of 35 per cent volatile coals ranging from 37 to 67, or some 81 per cent. Similarly at about 24 per cent volatile the range of grindability is from about 88 to 120, or about 36.5 per cent. Some of the Arkansas semi-anthracites with a volatile content of about 15 per cent are extremely soft and friable, as is also the case with some of the Colorado sub-bituminous coals, which are quite high in volatile content, so that with so many exceptions and such wide variations in the relationship between volatile matter and grindability we should look for some other relationship if it exists. In this connection the writer is wondering whether determinations of the specific gravities of these various coals might not give a better yardstick by which to correlate the grindabilities of such coals. With the specific gravities varying from 1.10 in the case of some lignites to about 1.35 in the case of Maryland and Pennsylvania bituminous coals and to about 1.50 in the case of anthracites, it appears that some possible relation exists between density and grindability. If so, then this offers a fairly quick, easy, and portable method of approximating a knowledge of this important characteristic of coals for pulverizing. The hardness of coals varies from practically nothing in the case of some lignites to 2.5 to 3 in anthracites, and the specific gravity similarly changes. Hardness and density apparently have some relationship, and certainly hardness and grindability are closely related.

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F. H. Dechant.[4] The paper is extremely interesting from the standpoint of calibration of coal on the basis of grindability factors. It should be of interest and real value in the determination of economics of pulverized-fuel-plant installations. In connection with anthracite coal, in which we in our region are particularly interested, it would be of further interest to know whether the grindability factor of the seam coal, that is, a composite of the coal vein, would show the same factor as the fines.

It has been the writer's belief that certain anthracite-mine operations could be conducted more profitably on the basis of shipping pulverized fuel than on the production of market-sized anthracite, so that the grindability factor immediately becomes important in reaching conclusions along this line of thought.

Ralph A. Sherman.[5] Although it has been understood that pulverized-fuel manufacturers and testing laboratories have been making tests on the grindability of coal for several years, this is the first published paper to come to the writer's attention. The author is to be congratulated on having carried this pioneer work on a difficult problem to the point where it warrants publication. Because of the importance of the problem to the manufacturer, user, and the coal producer, which the author has rightfully emphasized, the paper should bring forth considerable discussion and probably be followed by other papers on the subject.

Rittinger's Law, on which the author based his work, has been proved correct by the work of Martin and his associates, to whom the author refers, but credit should also be given to the work of Gross and Zimmerly[6] in this country, who have done some valuable research on the work done in crushing.

Three major considerations govern the determination of the grindability of coal:

(1) The selection of the sample
(2) The method of applying the work
(3) The measurement of the surface produced.

The author has called attention on the seventh page of the paper to the care necessary in the selection of the sample and has shown the relatively large differences in the grindability of the different sizes of the same sample of coal. The method of preparing the sample by grinding to pass all through 16 mesh and rejecting the part that passes 30 mesh appears to be fundamentally incorrect. Although the coffee-mill type of grinder undoubtedly produces the minimum of fines, the amount will vary with the grindability of the coal, and if any part is rejected, the sample is not representative of the whole lot. Unless data can be presented to prove that this rejection does not affect the results, a basic requirement of an acceptable method must be that no part of the sample be rejected.

Although, because of the wide choice offered, the selection of a method of applying work to the sample may lead to considerable difference of opinion and discussion, this consideration would appear to offer the least difficulty of the three named. It is important that the method selected applies the same work to all samples and that the loss of fines be prevented. The author indicates that the latter feature made the so-called ball-mill machine preferable to the other types used, but it has the added advantage to the Fuller Lehigh Company that it simulates the type of application of work in their type B mill.

The writer has seen no report of an investigation to learn whether coals will stand in the same order of grindability if the crushing is done by impact, by attrition, or by various combinations of these two methods. This question should be investigated, for until this definite information is available each manufacturer will prefer a test machine which simulates the action of his particular type of mill, whereas for standardization it is obviously desirable to have but one type of laboratory test apparatus.

The real problem in the development of a grindability-test method is that of the determination of the surface produced by the work done. The author has calculated the surface from sieve tests down to 300-mesh material with an arbitrary size factor assumed for all coals for the under-300-mesh material. The sieve-test method is doubtless acceptable from the standpoint of accuracy and rapidity down to 200 mesh, but it is less desirable for finer sieves. Difficulty is experienced with even the 200-mesh sieve with certain "sticky" semi-bituminous coals.

The author has calculated the mean size as the arithmetical mean of the two sieve sizes, $(d_1 + d_2)/2$, but as the chief interest is in surface, the mean taken should be the square root of the mean of the squares, $\sqrt{}(d_1^2 + d_2^2)/2$.

The basis for the assumption of an average size of 0.001 in. for the under-300-mesh material is not stated in the paper. Unless it could be definitely proved that this is a correct figure, and it does not appear reasonable that it should be for all coals, such an assumption should not be made. As shown in the sample calculation in Table 2, although only 10.8 per cent by weight of the sample passed 300 mesh, this represented 10,800 of the 19,740 surface units produced, or 54.8 per cent. Therefore, it is extremely important that this surface be determined more accurately.

Elutriation was mentioned by the author as a possible means for the determination of the size, but the writer agrees from experience that this is a tedious and not too accurate method.

The method which has been adopted in the study of the process of the combustion of pulverized fuel at the Battelle Memorial Institute to determine the average size and surface of the under-200-mesh coal is that of microscopic counting. The microscope must be the final standard for the determination of the dimensions of particles of this size, and although tedious, it has been found that much less time is taken for a determination in this way than by air elutriation.

The method is essentially that described by Perrott and Kinney[7] and consists in mounting a sample of the pulverized coal on a slide in Canada balsam, and by comparison of the particles at 100 or 200 magnifications with a calibrated cross-ruled slide placed in the eyepiece, the number of particles of different sizes in a sufficient number of fields is counted to give a good average. The percentage by weight of the various sizes is calculated from $nd^3/\epsilon nd^3$, where n is the average number of particles of a diameter d. The average diameter which is significant as to surface may be calculated from $\epsilon nd^3/\epsilon nd^2$. The specific surface of a spherical or cubical particle is given by $S = 6/\rho d$, where ρ is the density. The product of the fraction by weight of each size by the specific surface gives the total surface of each size in one gram of the sample.

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[5] Fuel Engineer, Battelle Memorial Institute, Columbus, Ohio. Mem. A.S.M.E.

[6] Gross, John, and Zimmerly, S. R., "Crushing and Grinding." (I—Surface Measurement of Quartz Particles. II—Relation of Measured Surface of Crushed Quartz to Sieve Sizes. III—Relation of Work Input to Surface Produced in Crushing Quartz.) Trans. A.I.M.E., Milling Methods, 1930, pp. 7–50.

[7] Perrott, G. St. J., and Kinney, S. P., "The Meaning and Microscopic Measurement of Average Particle Size," *Jour. Amer. Ceram. Soc.*, vol. 6, 1923, pp. 417–439.

Table 7 shows the results of the determination of the surface of two samples of coal ground in a conical ball mill, a semi-bituminous coal from the Sewell bed ground so that 79.2 per cent passed a 200-mesh sieve and a Hocking No. 6 coal of which 79.0 per cent passed a 200-mesh sieve. The specific surface was calculated on the assumption of a density of 1.4 for both coals.

It will be noted that, although the percentage passing through the 200-mesh screen was only 0.2 per cent greater, the surface per gram was 60 per cent greater for the Sewell than for the Hocking coal. For both coals over 95 per cent of the surface and, therefore, by Rittinger's Law, over 95 per cent of the work were represented in the under-200-mesh material and 23 to 25 per cent in the 0.002 mm. particles. The average sizes of the under-200-mesh particles were 0.015 and 0.025 mm. for the Sewell and Hocking coals, respectively. Expressed in inches, these values are 0.0006 and 0.001, as compared with the value of 0.001 inch which the author assumed for the under-300-mesh material.

TABLE 7 CALCULATION OF SURFACE OF PULVERIZED COAL FROM SIEVE TEST AND MICROSCOPIC COUNT

Mesh	Mean size, mm.	Fraction by wt., per cent	Specific surface, cm.²/g.	Total surface per gm. of sample, cm.²	Fraction of total surface, per cent
		Semi-Bituminous Coal—Sewell Bed, W. Va.			
+30	...	0.1	...	..	...
30–50	0.467	0.5	92.6	0.5	...
50–100	0.235	3.2	182.6	5.8	0.3
100–200	0.118	17.0	363.0	61.7	2.7
	0.074	8.2	571.0	46.8	2.0
	0.060	4.8	714.0	34.3	1.5
	0.050	11.2	857.0	96.0	4.2
	0.040	14.0	1072.0	150.0	6.5
	0.030	10.9	1430.0	155.9	6.8
	0.020	14.9	2140.0	318.8	13.9
	0.010	8.6	4290.0	368.9	16.1
	0.005	2.8	8570.0	240.0	10.5
	0.002	3.8	21400.0	813.0	35.5
		100.0		2291.7	100.0
		Bituminous, Hocking No. 6—Middle Kittanning Bed, Ohio			
+30	...	0.2	...	..	...
30–50	0.467	0.4	92.6	0.4	...
50–100	0.235	2.2	182.6	4.0	0.3
100–200	0.118	18.2	363.0	66.1	4.6
	0.074	10.3	571.0	58.9	4.1
	0.060	8.9	714.0	63.5	4.5
	0.050	21.3	857.0	182.5	12.8
	0.040	12.7	1072.0	136.1	9.5
	0.030	10.8	1430.0	154.4	10.8
	0.020	8.9	2140.0	190.5	13.4
	0.010	3.4	4290.0	145.9	10.2
	0.005	1.2	8570.0	102.8	7.2
	0.002	1.5	21400.0	321.0	22.6
		100.0		1426.0	100.0

These data indicate, as would be expected, that one value cannot be arbitrarily taken for the size of the sub-sieve particles. Some method must be developed for the rapid and accurate determination of the size. The microscopic method described is considered more accurate and less time consuming than air elutriation, but has sources of error in that long particles are considered as cubes; also the particles below 0.002 mm., of which there must be a great many, are not counted. Other methods of surface determination are probably possible, and the problem is worthy of extensive research.

The writer does not intend to imply by the criticisms offered that the data presented in the paper are not of value. On the contrary, they are of great value because of the large number of samples reported and the associated data on the coal; the industry owes the author and his company its thanks for making the data available. Further research may prove that the methods used by the author are entirely suited to the purpose, but the writer wishes to emphasize that the research is necessary before a standard test for grindability may be adopted.

Author's Closure

Mr. Pettibone's discussion adds a new angle to the grindability problem by comparing it to the specific gravity. In general, there seems to be a trend in that direction, but it is believed that coals such as lignite which change their grindability considerably with weathering do not change their specific gravity proportionally. The grindability test described in the paper does not take materially longer than a specific-gravity determination, and the same accuracy in collecting and preparing a representative sample is necessary.

In reply to Mr. Dechant's question as to whether the seam coal has the same grindability as the fines, it is believed that the fines would always show a higher grindability than the seam coal as it is the softer portions of the seam that break down into fines

Mr. Sherman has presented a very helpful discussion on the method of measuring the sub-sieve sizes produced in the determination of grindability. The use of microscopic counting is useful in some types of research work, but is quite tedious for a standard routine test like a grindability test.

Due to the small size of the sample used on the slide, it is very difficult to get a representative sample, and duplicate slides often give widely differing results. It is quite noticeable that the specific surface, in square centimeters per gram, for the extremely small sizes, becomes a very large number. As a result, a very small change in the fraction by weight-per cent makes a very big difference in the total surface per gram of sample, and if the microscopic count were carried down to 0.001 mm., a point would be reached where the accuracy would have to be so great that it is prohibitive.

Theoretically, the use of microscopic counting seems correct, but inasmuch as practically all specifications and standards are adopted on a compromise basis between what is wanted and what is available, the screening method which will yield check results on the same sample repeatedly seems to be not only the simplest but the best method.

As stated in the paper, the application of grindability results to actual mill capacities depends on the completeness of scavenging and will require different interpretation on different mills, but the establishment of a definite grindability scale which is reliable is still the working tool needed.

The samples tested by Mr. Sherman, being about 79 through 200 mesh, represent the usual finished product of the pulverizer. In the grindability test, however, the work of grinding is stopped far short of this point, where only about 10 per cent 200 mesh is produced and the sample contains only a few per cent finer than 300 mesh, and the errors in their measurement do not affect the final result as much.

The adoption of the constant of 1000 for the weights finer than 300 mesh was based on obtaining relative results that would check with actual mill performance rather than on actual measurements of the sizes of the particles produced.

FSP-54-6

Burning Oil-Refinery Waste Fuels in a Modern Steam Plant

By H. J. KLOTZ,[1] BOSTON, MASS.

Unsalable by-products in the refining of crude petroleum present a disposal problem. Such undesirable by-products are the acid and alkaline sludges and acid tar, the residues from the purification of lubricating oils and gasoline by the use of sulphuric acid, and the fine coke, or breeze, screened from the salable lump petroleum coke. Refineries were forced to burn the sludges to effect their removal. The chief problem was to get them into condition to be pumped. Louisiana Steam Products, Inc., entered into contract with a large refinery to supply their steam and electric requirements from a plant that would burn non-merchantable refinery waste fuels up to 50 per cent of the total heat content of the fuel required. The equipment used in this installation is described.

THE refining of crude petroleum to secure the materials in commercial demand, principally gasoline, kerosene, and lubricating oils, is accompanied by the production of several by-products. Markets have been developed for most of these, and they are sold in such forms as fuel and gas oils, paraffin, road oils, and coke. In addition to the marketable by-products, there are several that are not only decidedly unsalable, but which have been real problems to the refineries as regards their disposal. Such undesirable by-products are the acid and alkaline sludges and acid tar, which are the residues from the purification of lubricating oils and gasoline by the use of sulphuric acid. Another unsalable residue is the fine coke, or "breeze," screened from the usually salable lump petroleum coke.

The proportions of the unsalable residues vary with the types of processes employed, the grades of crude refined, and the commercial products desired. Improvements in refinery processes are reducing the proportions of unsalable residues so that the oil refineries seem to be headed toward a position comparable to the enviable one of the meat packers, which, as tradition has it, "utilize everything but the squeal." Until this utopia is reached, however, the refineries must continue to struggle with the problem of disposing of these residues. Some of the sludges are, in fact, becoming more difficult to handle even though the proportionate quantities produced are being decreased.

The nature of the acid sludges, being normally of a heavy, pitch-like consistency at atmospheric temperatures, is such that the refineries for many years disposed of them by hauling to dump piles. The material is far from constituting a desirable fill, however, and the refineries were finally forced to burn the sludges as a means of effecting their disposal. The utilization of the heat from their burning was of secondary importance, and consequently high efficiency of burning was not usually tried for. The big problem was to get the sludge in condition to be pumped and burned and to build tanks and pipe lines to withstand the extremely corrosive characteristics.

Great credit is due the men in the various refineries who tackled this problem and worked out schemes which met their particular conditions. The most widely adopted method of burning is by the use of steam atomizing burners, which have been installed under practically all types of boiler. Another method of burning which has been employed to a rather limited extent in some refineries uses a rotary kiln into which the sludge is fed in a semi-solid state by a suitable conveyor. This method avoids the necessity for maintaining the sludges in a state of fairly uniform fluidity, as is required when they are burned in suspension by spraying from any of the usual types of oil burners. The heat liberated by burning the sludges in the rotary kiln is ordinarily made use of by passing the products of combustion over the heating surfaces of a boiler. This method of disposal requires a highly special type of design which cannot be readily adapted to conventional boiler-plant layouts where it is desired to burn sludges in conjunction with other fuels in the same furnaces.

One refinery has developed a sloping grate stoker on which the sludge is burned in a semi-solid state directly in the boiler furnace. This method requires that sludge be burned continuously to protect the stoker. Also, experience indicates that the stoker equipment could not be maintained in a boiler furnace operating at very high ratings and using preheated air for combustion.

Louisiana Steam Products, Inc., was confronted with a different phase of the problem from that presented to the men directly in the refineries. A contract was entered into with a large oil refinery to supply their steam and electric power requirements from a separately owned plant to be built for this purpose adjacent to the refinery. An important condition in the contract was that refinery residuals which were designated as "non-merchantable but burnable refinery waste fuels" must be burned in the new plant up to 50 per cent of the total heat content of the fuel required. This proportion was such as to absorb the entire output of these residuals at the time of making the contract. The residuals are bought on a heating-value basis at a price which, in conjunction with the rates at which steam and electric energy are sold, requires that they be burned as efficiently as natural gas and fuel oil, which are the normal and emergency fuels, respectively, making up the balance of the plant fuel requirements.

In addition to the requirement of obtaining maximum efficiency of burning, it was desirable, in order to keep the cost

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of the plant to the minimum, to obtain the highest possible capacity per burner. The adoption of mechanical-atomizing burners rather than the steam-atomizing type was clearly indicated when to the foregoing considerations was added the desirability of keeping steam losses to the minimum in the plant, which was to operate at high ratings on 80 per cent treated make-up water, containing about 175 parts per million of total solids.

The choice of burners was made against the advice of every one consulted that had had experience in burning sludge. The possible advantages to be gained by using mechanical-atomizing burners appeared to warrant taking the chances involved, and operating results have justified the decision. One marked advantage stood in favor of the project in that a new plant was to be built specifically designed to burn large quantities of sludge where heretofore sludge had to be burned in existing plants.

The character of the acid sludges, which constitute about 85 per cent of the liquid residuals burned, is indicated by the following data on typical samples of the two varieties:

	Lubricating acid sludge	Acid tar
Specific gravity, as received	1.084	1.052
Moisture or weak acid content, per cent	39.68	47.51
Acidity of weak acid, per cent	32.58	35.21
Acidity of sludge, per cent	12.93	16.73
Weight per gallon, pounds	9.03	8.76
Heating value, B.t.u. per pound	10,940	9,525

While these analyses are given as typical, the predominating characteristic of sludges is their extreme variability. The moisture or weak-acid content varies through a range of from about 30 per cent to 50 per cent, and on somewhat rare occasions may go even higher. The corresponding acidity of the sludge may vary through a range of 10 per cent to 20 per cent. Such variations in moisture content cause a wide variation in the consistency of the sludge and require special treatment for the individual batches in order to get them in proper condition for atomization.

Of the remaining 15 per cent of the sludges which are non-acid in character, about one-half are strongly alkaline. The latter so-called "neutralized sludge" is the result of treating the heaviest grades of acid sludge with caustic soda in order to make them fluid enough to be pumped.

The total quantity of all sludges necessary to burn amounts to about 2000 bbl. per day.

Lubricating acid sludge, which is obtained from the purification of lubricating oils and is usually referred to merely as "acid sludge," is of an extremely viscous consistency and must be constantly agitated by open steam jets to prevent solid particles of pitch from settling out. Constant flow through the burner supply piping and recirculation to the supply tank must be maintained for the same purpose. If the sludge is allowed to stand in the piping, it will harden to almost the consistency of coke, requiring its removal by mechanical means. Sludge which has thus hardened can be restored to a burnable condition only by remelting in an agitating tank.

Acid tar, which is obtained from the purification of gasoline, is much lower in viscosity than acid sludge and does not usually have to be agitated with steam jets, but gradual settling out of solid particles will occur if it is allowed to stand too long without being circulated. Although acid tar is more readily handled and atomized than acid sludge, it is the most corrosive of all the liquid residuals. As the two products frequently have practically the same acid content, the difference in corrosive attack is probably due to the inhibiting effect of the lubricating-oil content of the acid sludge.

The alkaline sludges, which are low in viscosity, are readily handled and delivered for atomization through the same steel pumps, piping, and burners provided for fuel oil. The brass lines used for the acid fuels would of course be attacked by the alkali. The only problem encountered in the combustion of the alkaline sludges is that deposits of sodium sulphate are formed which, at times, tend completely to clog the gas passages, especially in the superheater section, and have a very marked fluxing action on refractories when this fuel is burned in conjunction with coke. Extra provisions in soot-blowing equipment are necessary in order to remove these deposits.

Another variety of sludge burned, which there is comparatively little to dispose of, is known as "wax tailings." Aside from the fact that this fuel becomes entirely solidified at atmospheric temperatures, it is pumped and burned as readily as any good fuel oil and is handled similarly in every respect.

Another waste fuel which is obtained in many refineries, but which has not been available for this installation, is known as "Dubbs' residuum." A typical analysis of this fuel is as follows.

A.P.I. gravity, deg. Baumé	7
Flash point, Pensky Martin, deg. fahr.	120
B. S. & W., per cent	30
Viscosity, Saybolt Furol, sec. at 122 deg. fahr.	200
Sulphur, per cent	1.3

This material in its worst form is reported to contain a very considerable amount of carbon in non-colloidal suspension, but experience has shown that it can be handled readily and burned with the equipment used for alkaline sludge. In combustion this fuel also presents about the same problems in fluxing furnace refractories and, to a minor degree, in deposits on boiler and accessory heating surfaces.

FUEL-HANDLING EQUIPMENT

Lead-lined tanks are used for receiving the acid sludge and acid tar from the refinery in batches of 500 to 800 bbl. It is not possible to store these sludges, so they must be received as produced and burned with as little delay as possible. As mentioned, the acid sludge must be constantly agitated by open steam jets in order to keep it of proper consistency for pumping and burning. With all of the sludges, constant flow through the supply piping and recirculation to the supply tank must be maintained in order to prevent stoppage of the lines. When shutting down a line, the piping should be thoroughly blown out with steam, and if an acid-fuel line is to be out of service for an extended period, it is desirable to circulate fuel oil through it in order to effect as complete purging of the corrosive elements as possible.

Cone bottom tanks are used for all the sludges. The general construction of the tanks and the vents provided in the top are shown in Fig. 1. Each tank is provided with a ring of steam jets in the bottom for agitating the acid and alkaline sludges.

Reciprocating pumps of a type which has been in general use on acid sludges for many years have been adopted for handling the liquid fuels. Due to the higher pressures required for mechanical atomization of the sludges, the pumps for this installation are necessarily of more rugged construction. No springs are employed on the pump valves, which are of the plain rectangular block type. The so-called acid-resisting bronze furnished by the manufacturer has been entirely satisfactory in the case of the pumps handling acid sludge. The increased severity of the action of acid tar is clearly demonstrated by the more rapid deterioration of the pumps handling this fuel, and special alloys are being tried in order to obtain liners and valves which will suitably resist its action for longer periods. Up to date one of the phosphor bronzes has proved to be the best material tried for this service.

For piping to handle the acid sludge and acid tar, red brass, which is 85 per cent copper and 15 per cent zinc, has been found entirely satisfactory. The same material, or slight modifications of it, containing 0.5 per cent to 1 per cent lead, has been found satisfactory for valves in the acid-sludge lines. As has been found in the case of the pumps, the combination of corrosion and erosion which is encountered in the flow through valves is apparently much more severe in the acid-tar lines, and valve material which will be more resistant to the action of this fuel is now under investigation. Gate valves have been used exclusively and have been entirely satisfactory on the acid-sludge piping. On the acid-tar lines, however, the gate valves which have failed are to be replaced with cocks, as it is believed that the type of construction embodied in cocks greatly reduces the severity of the erosive action of this fuel.

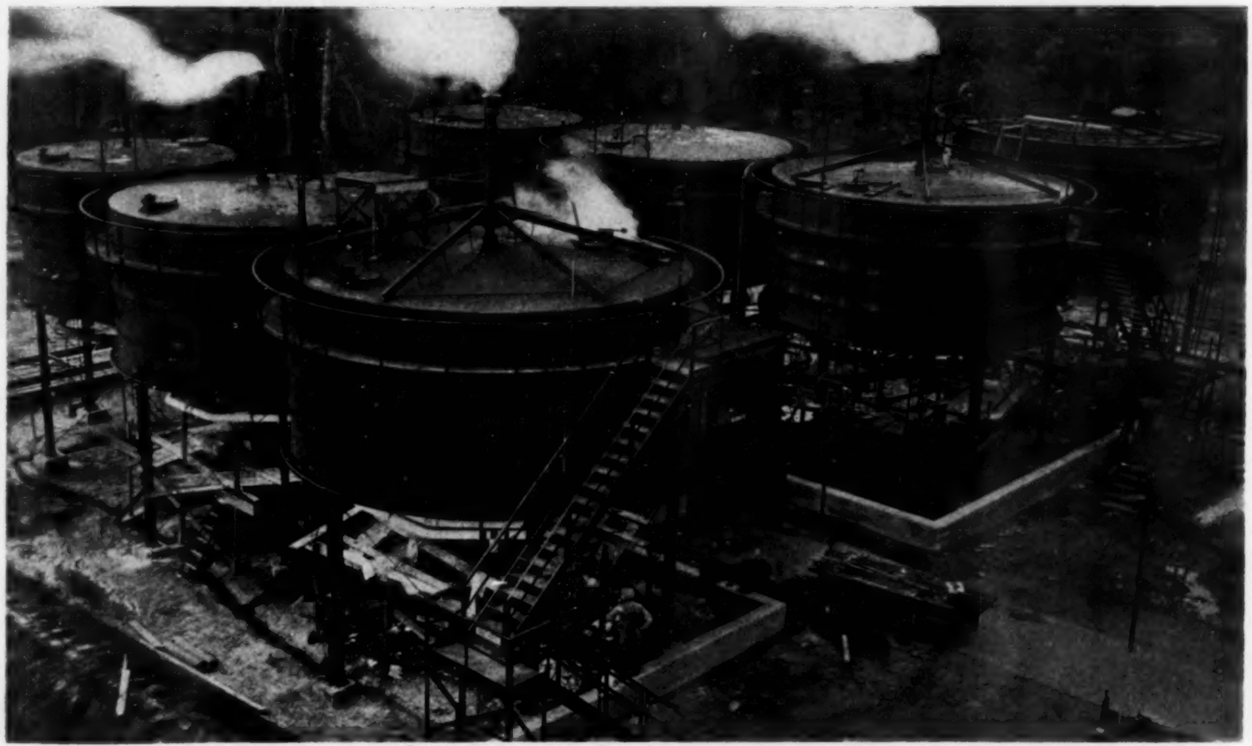

FIG. 1 LIQUID-WASTE-FUEL TANKS

BURNER DESIGN

It is impossible to make viscosity determinations on the sludges by any of the usual methods, because of the tendency for solid particles to settle out. The arrangement and proportions of the mechanical burner tips and the pressures and temperatures necessary for atomization were therefore based entirely on spray tests conducted in the open by spraying the sludges against a vertical steel plate placed about 3 ft. in front of the spray tip. This afforded an excellent method of studying the quality of atomization obtained under various conditions.

As the result of these studies, tips were developed which differ from those generally used in mechanical burners only to the extent that the central opening from which the fuel is sprayed is larger and the slots which feed the central chamber in the type of burner selected are fewer in number and larger in cross-sectional area to avoid as far as possible the marked tendency to clog any small opening which is characteristic of a fuel of this type. The burners have been satisfactory from the standpoint of capacity and quality of atomization secured. Up to 3000 lb. is atomized per burner per hour, which corresponds in heat release per burner to about 1800 lb. of normal fuel oil. The appearance of the furnace when burning sludges compares favorably with that accompanying the burning of a good grade of oil. The flashing into steam of the high moisture content of the sludges is probably responsible for an effect, as the fuel leaves the burner, which is slightly different from that usually observed with mechanical burners, and is probably partly the cause of a slightly longer flame. Occasional solid particles in the fuel stream are evidenced by streamers of flame or "sparklers," which extend beyond the main body of flame, occasionally striking the rear wall. No detrimental effects on the rear walls have been observed from this action.

Acid sludge is supplied to the burners at pressures between 190 and 200 lb. per sq. in. and at temperatures of from 225 to 275 deg. fahr. The temperature required depends upon the characteristics of the particular batch of sludge being burned. For acid tar the corresponding pressure and temperature conditions are 150 to 160 lb. per sq. in. and 165 to 175 deg. fahr. The range in temperatures required with acid tar is not so great, as this fuel is not normally so variable in consistency as acid sludge.

To obtain the high temperatures required to burn acid sludge with mechanical burners it is of course necessary to heat the fuel in a closed-type heater. This heating is being satisfactorily accomplished, so far as thermal transfer is concerned, in a four-pass fuel-oil heater of standard design, with the tubes and tube sheets constructed of a brass of special composition. The material initially selected for the sludge-heater tubes has shown a service life of only about six months. Tests are now under way looking to the selection of material to give a somewhat longer life.

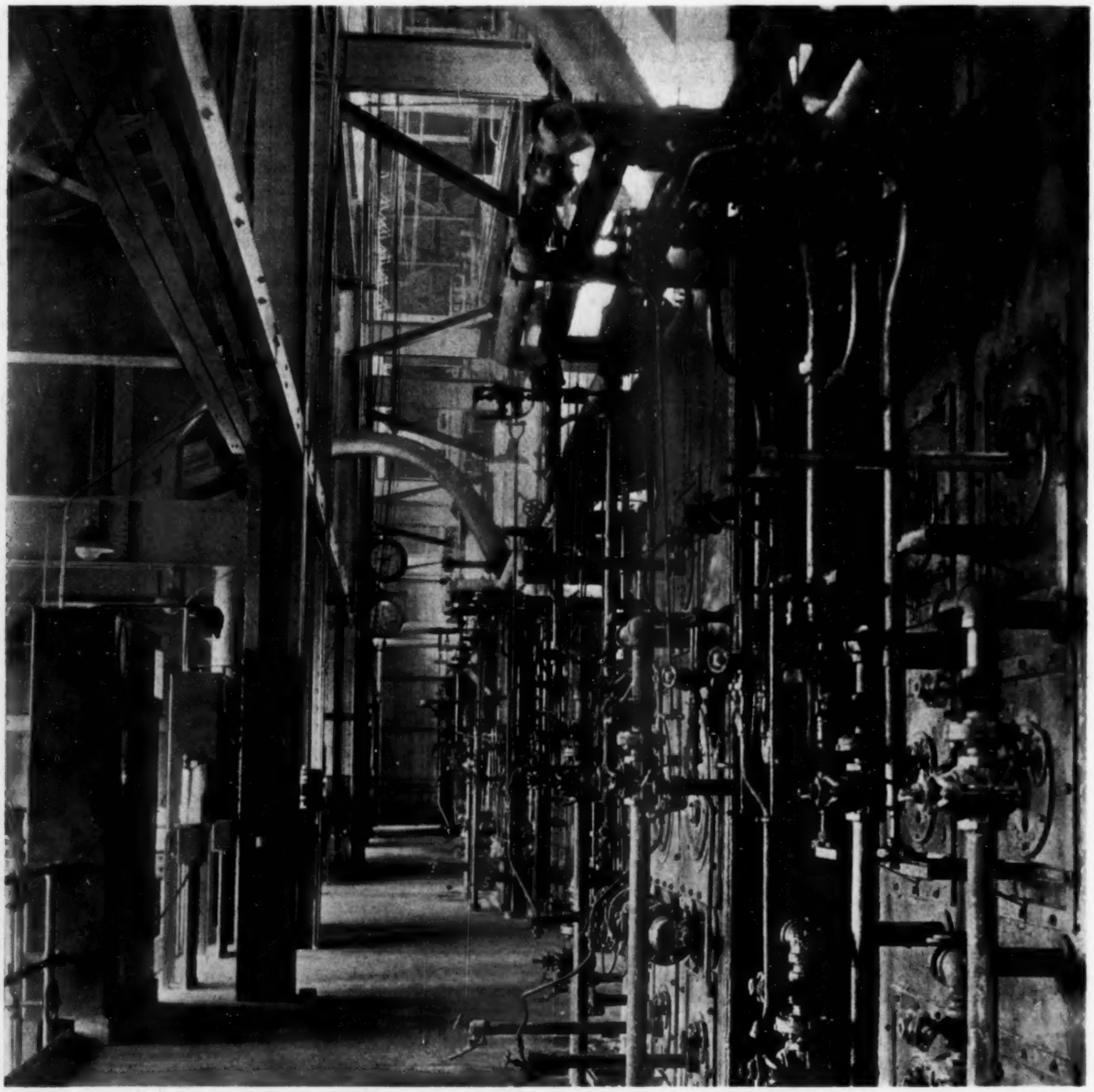

FIG. 2 FIRING AISLE AND BURNER FRONTS AT LOUISIANA STATION

MATERIALS FOR BURNERS

The material used for the burner tips is of course subjected to rather high temperatures and has to withstand a strong erosive action as well as the corrosive attack of the acid fuels. Up to the present aluminum bronze has been the most satisfactory of the materials tried. Investigations are being made with the object of finding some material which will give longer life in this service. Experience so far has included a number of bronzes and some of the best-known ferrous alloys. The majority of the latter have shown extremely short life, some lasting less than five hours on acid tar, but as there are many of the alloys of this class still to be tried, it cannot yet be stated with assurance that non-ferrous materials are the more suitable for this service.

Trials are also to be made of burner tips made of non-metallic materials, such as fused quartz and glass. There seems to be little question of the ability of these materials to withstand the effects of corrosion and erosion, but they may be unsuited to resist the mechanical strains resulting from abrupt temperature changes, as well as those incident to forcing the tips against the burner barrel to make the necessary tight joint.

The extremely accelerated test of materials which is afforded by their use for burner tips is being supplemented by a comparison under less severe conditions. In both the acid-sludge and acid-tar burner supply lines, sleeves are being installed which have been fitted with several successive rings of different materials to be tested. The various specimens are separated by glass rings as an insulating medium to prevent the possibility of gal-

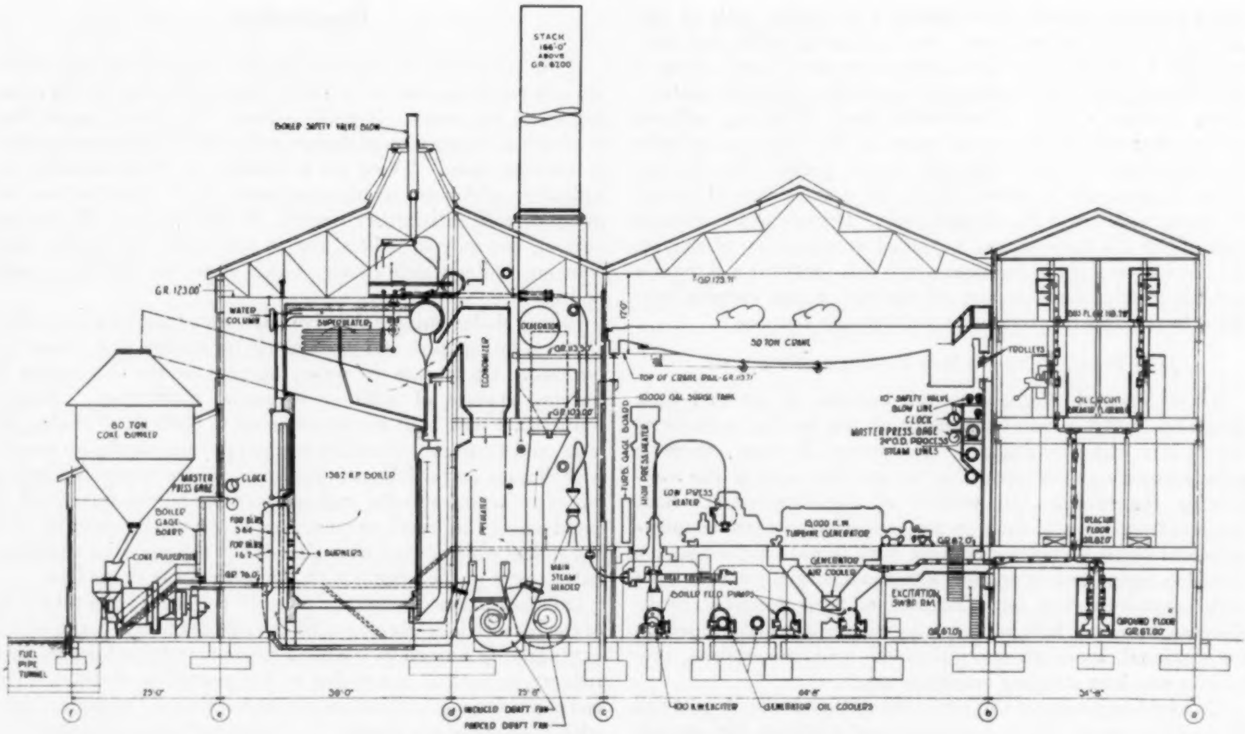
STACK 166'-0" ABOVE GR. 62.00
BOILER SAFETY VALVE BLOW
GR. 123.00'
WATER COLUMN
SUPERHEATER
DEAERATOR
ECONOMIZER
GR. 113.50'
GR. 103.00'
80 TON COKE BUNKER
MASTER PRESS GAGE
CLOCK
BOILER GAGE BOARD
COKE PULVERIZER
1362 H.P. BOILER
6 BURNERS
GR. 76.0
PREHEATER
MAIN STEAM HEADER
INDUCED DRAFT FAN
FORCED DRAFT FAN
FUEL PIPE TUNNEL
25'-0"
30'-0"
25'-8"
GR. 123.71'
17'-0"
50 TON CRANE
TOP OF CRANE RAIL-GR. 113.71'
10,000 GAL. SURGE TANK
TURB. GAGE BOARD
HIGH PRESS. HEATER
LOW PRESS. HEATER
10,000 K.W. TURBINE GENERATOR
GENERATOR AIR COOLER
BOILER FEED PUMPS
EXCITATION SWBD. RM.
400 K.W. EXCITER
GENERATOR OIL COOLERS
GR. 82.0
GR. 61.0
10" SAFETY VALVE
BLOW LINE
CLOCK
MASTER PRESS. GAGE
24" O.D. PROCESS STEAM LINES
TROLLEYS
OIL CIRCUIT BREAKER
REACTOR FLOOR
GROUND FLOOR GR. 61.00'
64'-8"
34'-8"
f
e
d
c
b
a

Fig. 3 Cross-Section of Louisiana Station
(Showing furnace and boiler setting for burning waste fuels.)

vanic action which might occur with the different materials in contact. Comparison of results from the two types of test will enable a choice to be made, under strictly comparable conditions, of the best material for service with these fuels.

Fuel-Burning Arrangement

Combination gas and oil or gas and pulverized-fuel burners are used throughout the installation. Two of the four boilers, which are of 1400 hp. rated capacity, but designed to produce a maximum of 350,000 lb. of 600 lb. pressure, 750 deg. fahr. total temperature steam per hour, have 18 burners each. The other two boilers have 17 burners each, due to the additional space required by the two pulverized-coke burners. All burners on all boilers except the two combination gas and coke burners are arranged for burning gas, oil, or alkaline sludges. Acid sludge and acid tar can be burned on only the two bottom rows of burners on all boilers. The distribution of sludges to the various boilers was arranged to simplify the fuel-supply piping as much as possible. Reference to Fig. 2, which shows a view of the firing aisle and boiler fronts with the burner and piping arrangement, gives some indication of the large amount of piping necessary, even with a comparatively simple arrangement. Of the three headers shown crossing the top of the burner panel of the boiler in the foreground, one is for fuel oil and alkaline fuels, one is for acid fuels, and the third supplies steam for blowing out individual burner connections, which is necessary when changing burners. When an entire waste-fuel line is blown out, steam, air, or fuel oil for this purpose is supplied through the pump suction connections. Compressed air is satisfactory for this purpose on the acid-tar lines only, and is used to avoid the abrupt temperature changes accompanying the use of steam, which frequently causes leaks in the joints on the brass pipe lines. The two bottom rows of burners have connections to both fuel headers, while the top row of burners is connected only to the steel header for supplying alkaline fuels or fuel oil.

The gas header is below the operating floor, with individual connections to each burner, as shown for the middle and top rows. The connections to the bottom gas burners are below the floor, with the operating stems for the gas cocks shown at floor grade. The pulverized-coke supply pipes to the two boilers in the foreground are shown by the two long bends crossing over the firing aisle.

The multiplicity of piping and valves for the various fuels and steaming-out service necessarily introduces rather complex operating features. It has been found, however, that alert and intelligent operators with experience on either oil or gas firing quickly grasp the special features and handle the operation satisfactorily.

The location of the burners in the setting and the furnace proportions are clearly shown in Fig. 3, which is a cross-section of the plant, including a furnace and boiler setting. Three rows of six combination gas and oil burners were used on two of the boilers, while on the other two, three of the combination gas and oil burners were replaced with two combination gas and pulverized-fuel burners for burning petroleum coke.

For the coke burning, one pulverizing mill of the ball-and-tube type and two combination gas-and-coke burners per boiler have been provided for two of the boilers in the plant. Only one of the burners is connected to a pulverizer at present, a second burner being provided for the possible future addition of a second mill for each boiler. Each mill and burner is capable of pulverizing and burning about 3 tons per hour. The arrangement of the coke-pulverizing equipment is shown in Fig. 3.

Operating Results

No difficulty has been experienced in the operation of the coke-pulverizing equipment. It has not been operated long enough to provide any data on wear or maintenance cost. As the coke comes from the stills it is practically pure carbon with an ash content usually less than 1 per cent, but owing to the fact that the coke supply for this installation is coming largely from a yard storage, there are considerable quantities of foreign matter, which produce a very objectionable slag. This slag adheres to the lower row of slag screen tubes in the boiler and attacks the brickwork of the front-wall burner panel. The fluxing action is particularly severe when the coke is burned simultaneously with one of the alkaline fuels. Because of the unusual bulk of the ash there is also a gradual accumulation of slag on the flat bottoms of the furnaces which it is expected will require periodic melting and tapping off through a slag opening provided in the two furnaces arranged for coke burning.

Acid Fuels Have No Bad Effect on Surfaces

No detrimental effects from the burning of the acid fuels have been noted in the furnaces or on the heating surfaces of the boilers and economizers. Apparently, however, there is some corrosive attack upon the chrome-iron alloy of the soot-blowing elements in the vicinity of the superheater. Soot deposits from the acid fuels on surfaces at economizer temperatures and higher, in marked contrast with those from the alkaline fuels, are comparable in amount with those normally encountered with commercial fuel oils. Because of their potential corrosiveness, however, it is advisable to include adequate provision for continual, thorough removal of the acid-fuel deposits, preventing any long-standing accumulation.

To avoid corrosion in the air heaters, it was initially planned to limit the use of acid sludges to rates of operation high enough to insure that the gases would be kept above the dewpoint in passing through the air heaters. In actual operation, however, it was found that deposits adhered to the plates and local corrosion followed, due to the consequent cooling of the coated portions of the plates below the dewpoint. In order to maintain sufficiently high temperatures under all conditions of load and initial air temperatures, arrangements have been made for returning heated air to the preheater inlets, and on one of the preheaters, air connections have been reversed to give parallel flow of gas and air, and an air-tempering steam-coil heater has been installed in the air duct leading to the preheater. The steam heater is normally employed only during starting and light-load periods. The combination of steam heating and recirculation permits constantly maintaining all portions of the plates above a temperature of 250 deg. fahr. if desired. Both experimental investigation and operating experience, however, indicate that 225 deg. fahr. is well above the dewpoint accompanying the highest percentage of sulphuric-acid vapor which is to be expected when burning the largest proportions of acid fuels contemplated in this installation and affords entirely adequate protection from corrosive attack.

Preventing Deposits on Preheater Plates

One of the most important factors in avoidance of preheater corrosion would seem to be the prevention of deposit accumulation on the plates. It has been found necessary to install special blowing elements to insure reasonably thorough cleaning of the air-preheater plates by steam during operation and to supplement the steam blowing by periodic washing with an alkaline solution when the boilers are out of service.

It is considered that the various problems encountered with the unusual fuels burned at the Louisiana Station are no more than to be expected in a plant where such pioneering features were involved. Operating results clearly indicate that the plant is burning these residuals at capacities which are much higher than in general attained and at efficiencies comparable to the best of the normal fuel-oil and gas installations.

Discussion

L. A. Mekler.[2] It is gratifying that refinery waste products are now being used as fuel, not only by the refineries, but by other industrial fuel users. It appears from Mr. Klotz's paper that mechanical atomization of sludges is the only satisfactory method of burning them. There are a number of sludge-burning installations which use steam atomization for sludges that can be pumped, with satisfactory results. Steam-atomized sludges are burned, not only in boilers, but in the heaters of topping and cracking units, which require much closer control than most boilers.

Rotary sludge burners also give satisfactory results with sludges that can be pumped, and particularly for sludges that cannot be pumped. Up to date the rotary burners are the only means of burning sludges of solid or semi-solid consistency. Rotary burners are now built in capacities up to 6000 lb. of sludge per hour, and the improved-design burners give practically no trouble with linings and driving mechanism. Where excessive wearing out of refractories in the wall opposite the burner was found, it could usually be traced to admixture of the earthy material with the sludge which fluxed the wall. When this material was eliminated, most of the trouble with the setting was eliminated.

On installations where only 60 to 70 per cent of the total load of the furnace is taken care of by the sludge burned, it can be advantageously burned in modified steam-atomizing burners with refinery or natural gas acting as the atomizing medium. The use of gas for atomization assists the burning of the sludge, since with this type of atomization the sludge particle is surrounded by an easily ignitible medium which assists the combustion of the sludge. One of the standard burners now on the market is designed to use either steam or refinery gas for atomization. The gas is admitted into the burner near the tip through a tangentially placed opening so that the gas forms a layer between the sludge stream and the body of the burner tip. In this way the erosion and corrosion of the tip by the sludge are greatly minimized.

One operator has complained that the petroleum coke burned in the plant gave most of the trouble. This is rather an extraordinary occurrence. Pulverized petroleum coke is an ideal fuel for power plants. It has a high B.t.u. value, a very low ash content, and as a general rule requires less power for crushing and pulverizing than most of the coals used for the purpose (15.5 to 16 kw-hr. per ton of coke crushed and pulverized).

A public-utility company in the Mid-Continent field purposely built one of its power plants in the neighborhood of refineries which produce large quantities of petroleum coke so as to use this coke as fuel. The writer visited the plant, and the superintendent said he would rather burn petroleum coke than any other fuel he can get. The management is also quite satisfied with the performance of the plant. The boilers produce 135,000 lb. of steam per hour with a fuel consumption of approximately 1 lb. of petroleum coke per 1 kw-hr. The efficiency of the boiler and air preheater is 82 per cent; 15.5 to 16 per cent CO_2 is obtained in the products of combustion under full-load conditions.

Most of the trouble reported by the users of petroleum coke is due to improper handling of the coke before it enters the burner. The trouble due to moisture in the coke will be eliminated if it is kept in storage for at least 24 hours before it is fed to the crushers and pulverizers. A set of grizzlies between the hopper and the crusher with another set of grizzlies between the crusher and the pulverizer will eliminate most of the pitch that is occasionally found in the coke, as well as most of the large pieces of cable and other stray iron. Crushing to about one-quarter to one-half size in the crusher and a magnetic separator will eliminate most of the small iron that is found in the coke entering the pulverizer.

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[2] Combustion Engineer, Universal Oil Products Company, Chicago, Ill. Mem. A.S.M.E.

If the coke was stored for years in a pile and became mixed with dirt and other outside materials before it was used, the burning of the coke would be quite troublesome. Fresh petroleum coke properly handled should not give any trouble.

K. L. Martin.[3] The paper presented by Mr. Klotz is interesting as treating the unusual case when a public utility is located close enough to a refinery so that waste fuel can be piped from one to the other. In the more usual case where the refinery has to dispose of its own wastes, the objection has been made that the use of commercial fuel oil as a flux to produce a liquid which can be burned in a mechanical atomizer is not economical, and preference has been shown for atomizers which have sufficiently large passageways, so that they will burn the sludges as they come.

It has further been observed that the general trend of modern power practice has been toward the use of larger units. Larger turbines and boilers and higher steam pressures, ratings, and superheat are being used. Stokers and pulverized-coal burners have followed the development of water-cooled furnaces and bigger boiler settings so rapidly that what is a record today is commonplace tomorrow.

As meeting both the requirements of a burner suited to viscous fuels carrying large particles of carbon in suspension and designed for the burning of large quantities of oil efficiently, attention is directed to a combination gas and oil burner, capable of evaporating 90,000 lb. of steam from a single burner with interchangeable steam and mechanical atomizers. This means a B.t.u. release of 132 million per hour per burner, resulting from the burning of approximately 900 gal. of oil per hour.

With this unit, it is only necessary to pipe such fuels as may be available to the vicinity of the burners, and connect the fuel to be burned to either a steam or mechanical atomizer mounted in the same register. The heavier fuels would be kept in constant circulation in the usual way past the burner connections, but whatever reached the burner would be atomized and burned. The steam and mechanical atomizers are readily interchangeable, and, one may be substituted for the other in a few seconds. The smallest oil passageway in the steam atomizer is $^5/_{16}$ in. in diameter and in the mechanical $^3/_{32}$ in., and the orifice is $^1/_4$ in., all of which are large enough to pass any material likely to be carried in suspension and will operate for hours without clogging.

Where it is available in refinery work, it has proved very advantageous to use high-pressure still gas for atomizing. In one refinery, gas at 130 lb. pressure has been used in this way. A very rapid flame propagation is obtained, and the high temperature results in complete combustion of solid material in suspension, and a higher furnace temperature is also maintained.

The use of these registers means that about one-third of the usual number of burners are employed. The advantages are stronger furnace-wall construction, ease of regulation and control, and less flame interference.

R. C. Vroom.[4] The paper by Mr. Klotz was most interesting, as the writer was connected with the preliminary experiments leading to the development of the sludge-burning equipment now in use at Louisiana. Furthermore, he recently visited the station and was much impressed with the satisfactory results being obtained with the waste fuels. Furnace conditions were excellent, and in fact as good as could be expected when firing ordinary fuel oil, and the excess air was low. The "sparklers" and longer flame with sludges mentioned in the paper seem not to be due essentially to the fuels being fired, but to the fact that occasionally there are employed tips which have been eaten out by the corrosive action of the acid.

The author has referred to the complexity of the operation, due to the great number of valves and the large amount of piping on the furnace fronts. As he has stated, it seems to be quite easy for the operators to become familiar with the proper procedure. The apparent complexity of the installation is greatly increased by the fact that it was impracticable to run the leads to the individual gas burners inside of the windbox. Had this been done, much of the piping on the furnace fronts would have been eliminated.

While it is true that there is a certain advantage in designing a station initially for waste-fuel firing as compared with using waste fuels in a plant already designed, it is a mistake to lay any particular stress on that fact as it tends to minimize the difficulties overcome at Louisiana. There are a great many things about that installation which are so different from previous practice that it is a distinct achievement to have made such a success of it, even though it was designed initially for firing waste fuels. Burner capacities are materially higher than are ordinarily encountered, water-cooled furnaces are employed, and the maximum capacity (based on boiler heating surface alone) corresponds to 850 per cent of builder's rating and this with about 80 per cent make-up water from the Mississippi River. Furthermore, a very important factor in the operation of this plant is that the fuels are not donated; they have to be purchased at a price per B.t.u. comparable to that paid for the standard fuels employed. In most refineries, the combustion of sludges has been largely for the purpose of getting rid of them.

Mr. Klotz has stressed the importance of circulation, throughout the waste-fuel system. Dead pockets where fuel can settle are extremely undesirable with unstable fuels of the nature employed as they will easily solidify or drop out carbon. It should be noted that when firing the sludges, gas is always fired simultaneously in the same furnace. Sometimes coke is fired also. The reason for using the gaseous fuel together with the sludges is that the sludges provide only about 50 per cent of the total heat input required for the plant. Furnaces are water cooled and partly refractory faced, and there are no ignition arches such as are sometimes employed when firing sludges.

In connection with the question of burner capacity, one of the speakers has raised the point that it is not a difficult trick to spray large quantities of oil. It is perfectly true that gas-oil sprays for use in carburetors are designed for 40 or 50 gal. per min. However, the writer would greatly hesitate to apply for fuel firing in power plants a spray having the coarseness of atomization produced by some of these sprays. It is not a particularly difficult trick to spray a large quantity of fuel, but it is not an easy thing to design an atomizer which will spray it in sufficiently fine form to permit its combustion. While the proper admission of the combustion air to the fuel spray is highly important, still the question of the production of the spray should not be considered as something simple.

In view of the difficulty that has been experienced due to a deposit forming on the heating surfaces of the boiler when firing alkaline fuels, it probably will be wondered why acid and alkaline fuels are not fired simultaneously. Although rather unfortunate results were obtained during preliminary spray tests before the plant was put in operation, when acid and alkaline fuels were mixed by mistake, still both fuels have been fired simultaneously by the same atomizer. Whether this simultaneous combustion will do away with the deposit can be determined only by operating experience. However, the neutralized sludge, which is an alkaline fuel, originally was an acid sludge and was neutralized by the addition of soda. Therefore, except for what excess soda was added, there would appear to be no chemical reaction due to the simultaneous combustion of acid and alkaline fuels, and it is possible that the deposit on the tubes is formed by the neutral salt resulting from the neutralization of the sludge.

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[3] President, Engineer Company, New York, N. Y. Mem. A.S.M.E.

[4] Chief Engineer, Peabody Engineering Corporation, New York, N. Y. Assoc-Mem. A.S.M.E.

Waste fuels are becoming of increasing importance and their efficient combustion must be given due consideration wherever they are produced. Even ordinary fuel oils on the market are generally blends or mixtures, and oils of a given gravity often have characteristics altogether different from those of the same gravity used 10 years ago.

AUTHOR'S CLOSURE

Mr. Mekler states that it would appear from the paper that mechanical atomizing of sludges is the only satisfactory method of burning them. It was not intended to give this impression. As pointed out in the paper, special conditions which apply to the design and operation of Louisiana Station, such as the high rates of operation necessary and the need for a furnace design adapted to the burning of gas and pulverized coke, as well as liquid refinery residuals, necessitated the adoption of a method for burning the sludges in suspension. The mechanical-atomizing type of burner was selected rather than the steam-atomizing type because of the need to limit the make-up water requirements, which were necessarily higher than desirable in this station, and also to limit the amount of water vapor in the exit gases, increase of which would tend to add to the corrosion problem in the preheaters and flue-gas connections.

It is true that the burning of clean petroleum coke in combination with gas or fuel oil should give no difficulty, but freshly produced coke as well as storage coke with a high percentage of impurities, when burned in combination with alkaline sludges, has produced troublesome slag deposits.

The large-capacity burner described by Mr. Martin is very interesting and gives promise of valuable future developmets.

Mr. Vroom in his discussion has emphasized further the point which was brought out in the paper that the residual fuels are purchased at a price which requires that they be burned with the highest possible efficiency to obtain favorable operating costs.

The simultaneous combustion of acid and alkaline sludges has been tried and has been found to give greater difficulty on some occasions than the burning of alkaline sludges separately from the acid fuels. The uncertainty of the results of burning alkaline sludges is due, probably, to the lack of uniformity in the alkaline content of the fuel as delivered by the refinery. As the amount of neutral salts in the products of combustion must depend on the available alkali and sulphate radicals in the fuels burned, it is readily seen that any excess of alkalinity in the alkaline sludges would produce greater deposits of sulphates if such fuel is burned in conjunction with acid sludges.

Studies of methods of processing and burning the residuals are continuing to be made in order that the difficulty due to deposits on the heating surfaces will be minimized.

FSP-54-7

Pulverization and Boiler Performance

By E. H. TENNEY,[1] ST. LOUIS, MO.

While much has been said about the effect of varying degrees of pulverization, this paper gives specific and practical information on the relationship of pulverization of coal to boiler performance. The paper contains a resume of theoretical studies and actual tests that have been made in the past by other investigators and a recapitulation of the factors and theory of effects of fineness of grind on the combustion of pulverized coal and design of furnaces, mills, and boiler arrangements. Results are given of efficiency tests made at Cahokia with various finenesses.

IN THE last six years, during which time the use of pulverized coal in the operation of large boiler furnaces has been greatly increased, the question has been repeatedly raised as to the relationship between the degree of pulverization and boiler performance. In a general way it seems to have been agreed that coarse pulverization results in unsatisfactory and incomplete combustion in the furnace, and that the finer the coal is ground the better the resulting furnace condition. During these years no extensive research has been undertaken for establishing the necessary coordination between theoretical and practical considerations, and work is still being carried on, more or less in the dark, in endeavoring to establish a balance between the economics of milling coal and burning it, without very much being known of the technical details involved.

Inasmuch as this particular problem of pulverized-coal burning has become one which obviously is going to continue indefinitely, it seems appropriate to consider briefly some of the problems involved in the light of these years of experience. If such a discussion will have pointed out some of the questions which eventually must be solved and if it to some small degree stimulates the realization of the need of further study and investigation, the time will have been well spent.

Certainly the degree of pulverization affects a sufficient number of the phases of pulverized-fuel burning to offer a variety of angles from which to study the problem. The combustion of pulverized coal may be considered as a mechanical and chemical problem, and both of these phases are closely concerned with the degree of pulverization. The delivery of coal through piping and burners, its turbulent dispersion through the air supply, and its flotation through the furnace are mechanical functions of the size and weight of the particles. The heating and ignition of the dust cloud, the distillation of volatile content, and the combustion of the solid particles are thermo-chemical reactions hastened or retarded by greater or less fineness.

Fundamental data on the burning characteristics of powdered fuel are so meager, however, as to be of little value in the practical design or operation of boiler furnaces. When complete facts and data have been developed on the various types of fuels and factors relating to furnace performance, one may reasonably expect them to establish the proper sizing of coal particles. Obviously such a result will be brought about only by intensive research into the fundamental characteristics of pulverized coal.

At first glance the determination and collection of this basic information appear to present a great task, for it is known that the fuel from each mining locality or mine seam has certain peculiarities. However, the completion of work on the classification of coals as announced by Dr. A. C. Fieldner at the meeting of the Fuels Division in 1928[2] should greatly simplify the problem, for component chemical and physical properties of the coal and the performance of any single fuel can then be estimated from a knowledge of its composition. Already some research has been completed both in this country and abroad which has demonstrated that burning characteristics can be so studied, and some of the relations between combustion time, for example, and certain chemical constituents of coal have been established.

Theory of Particle Size

Since the size of the coal particles affects nearly every phase of pulverized-fuel firing, the various steps in the process and their relation to particle size must be considered.

The delivery of the fuel particles into and through the furnace is dependent upon the dust-bearing capacity of the air stream. The rate at which particles of various diameter settle out in still air (terminal velocity) is shown in Fig. 1 (from E. G. Bailey).[3] Coarser particles fall more rapidly than fine, and in design the air velocity must be calculated to overcome the settling factor of the coarsest particles in the length of travel under consideration, whether in a burner pipe or in a furnace. For example, if the vertical velocity of gases in a furnace is 10 ft. per sec., then particles over a 50-mesh sieve, or larger than 0.011 in. diameter, will settle out to the furnace bottom. Thus there is at once a definite limitation to the size of particles, for the coal must be maintained in suspension while being burned.

The burning process is effected in two steps: (1) the heating and ignition of the dust cloud, and (2) the subsequent combustion of the dust particles.

Since the velocity of the dust cloud is greatest at the burner outlet, the ignition period may occupy considerable furnace space, and as one of the major problems of pulverized-fuel firing has been the reduction of furnace volume, the speed of ignition is of consequent importance.

The rate at which the dust cloud becomes heated to the ignition point depends upon these factors:

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[1] Chief Engineer of Power Plants, Union Electric Light and Power Company. Mem. A.S.M.E. Mr. Tenney is a graduate of Oberlin College and Ohio State University in Mechanical Engineering. He entered the drafting department of the Union Electric Light and Power Company in 1905, with which firm he has since been connected in various engineering capacities until appointed Assistant Chief Engineer of Power Plants in 1911, which position he held until being appointed Chief Engineer of Power Plants in January, 1919. Collaborated in the design and construction of Cahokia Station, which was one of the first central stations designed to use pulverized coal. During the reconstruction of Ashley Street Station in 1925, he developed the first unit mill and horizontal turbulent type of pulverized-coal firing adopted by central stations, a system which was subsequently applied to 24 boilers in that station, and which in modified form has been used in later sections of Cahokia.

Presented at the Fourth National Fuels Meeting, Chicago, Ill., Feb. 11 to 13, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Trans. A.S.M.E., FSP-50-51 (Sept. to Dec., 1928), p. 1.

[3] "Present Status of Furnace and Burner Design for Pulverized Fuel," Trans. A.S.M.E., FSP-50-72 (Sept. to Dec., 1928), p. 177.

(1) The temperature of ignition source (i.e., the temperature of the combustion chamber) and the position of the furnace heat-reflecting surfaces.

(2) The temperature to which the air and particles are preheated before entering the furnace.

(3) The thermal capacity of the dust cloud—which accounts for greater or lesser temperature rise with addition of a given quantity of heat, and which is increased by excess air.

(4) Rate at which heat is conducted to the interior of the particles, which Dr. Rosin[3] connects with the time lag of ignition.

(5) The opacity of the dust cloud, which increases its rate of radiant-heat absorption.

Of these perhaps only the last two are dependent on fineness. The time taken for the heat to penetrate the coal particle obviously depends upon its diameter. The opacity of the cloud (and its coefficient of radiant-heat absorption) is a function of the fineness, as was shown by Lincoln T. Work,[4] who used a turbidimeter as a measure of fineness. The presence of so-called superfines, which are on the order of 20 microns in diameter, hastens ignition, since these particles become heated to the ignition temperature more quickly than the larger ones, after which they impart heat to the remaining particles by their combustion. A quantitative relation between average fineness and the furnace temperature for a given rate of ignition (not the ignition temperature of the coal) is shown in Fig. 2 for one fuel (from Dr. William E. Gibbs).[5] Obviously the advantages due to high fineness permit, as this curve shows, a lower combustion-chamber temperature for the same rapidity of ignition.

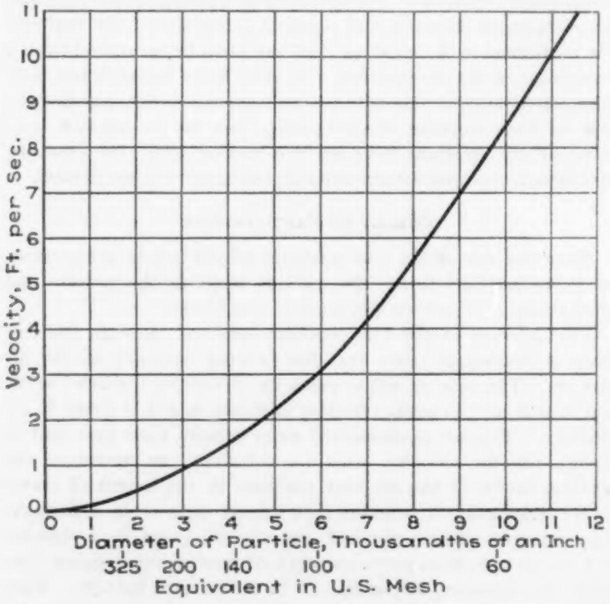

FIG. 1 VELOCITY OF PULVERIZED COAL (SPECIFIC GRAVITY 1.2) FALLING THROUGH STILL AIR AT 70 DEG. FAHR.

Of equal importance to the heating of the dust cloud is that of the ease with which a given coal can be ignited, i.e., the temperature at which ignition takes place. It is known to be lower, the lower the rank of coal, which is commonly attributed to the higher volatile content of the low-rank fuels. Some work has been done, however, which indicates that other characteristics of the coal and its chemical composition are equally important.[6] (See Fig. 4.) In general, however, any factor tending to retard ignition can be offset by greater fineness.

The combustion of the ignited particles requires a considerably longer time period than their ignition, and is therefore relatively more important in seeking high combustion intensity and consequent reduction of furnace size. The rate of combustion may be considered as dependent upon two factors: (1) the combustibility of the coal (or rapidity of its reaction with oxygen) which is generally higher for low-rank fuels of high reactivity,[7] and (2) the rate of contact with oxygen at the interface between the dust particle and the air supply.

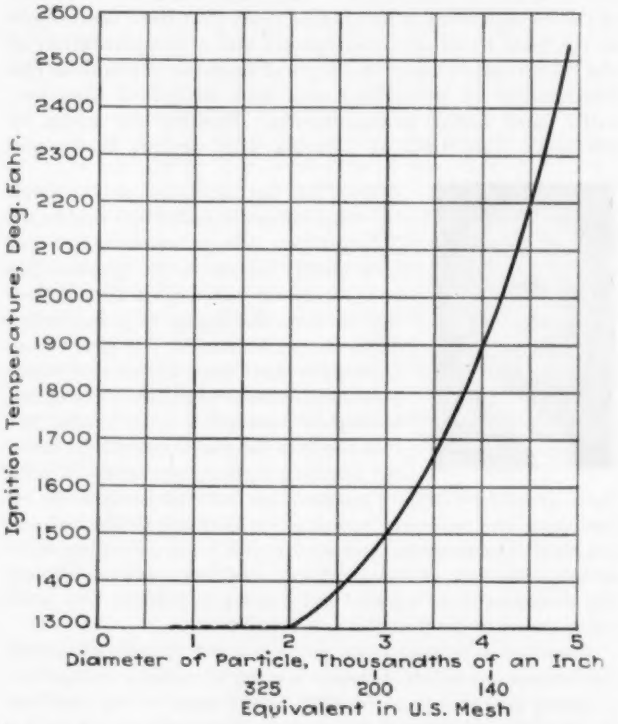

FIG. 2 VARIATION OF THE COMBUSTION-CHAMBER TEMPERATURE FOR IGNITION OF PULVERIZED COAL WITH FINENESS

Photographic studies made of burning fuel particles indicate that the combustion takes place in two stages[6,8]—the release and combustion of the inflammable volatile matter and the combustion of the remaining coke particle. Since the combustion of volatile matter is rapid, the reason for the high combustibility of low-rank, high-volatile fuels is apparent. The combustibility of the remaining coke particle, however, seems to be dependent upon the reactivity of the coal. The reactivity of a fuel follows its rank, somewhat, but the reactivity of the coke particle in the pulverized-fuel furnace is apparently also affected by the furnace temperature, as shown in Fig. 3, based on photographic data collected by Griffin, Adams, and Smith, U.S. Bureau of Mines.[8] The relation between furnace temperature and combustion time is shown for three finenesses. The surprising result is that the higher the combustion chamber temperature the slower the combustion, i.e., the temperature coefficient is negative. The explanation for this relation given by Dr. Fieldner is that coke formed from coal at higher temperatures is known to be less reactive than coke formed at lower temperatures, and in the pulverized-fuel furnace the particles are first formed to coke during the combustion cycle. The higher-temperature furnace consequently results in slower burning. The negative temperature coefficient is exactly opposite to the accepted opinion and should be substantiated, but if and when it is substantiated, then another reason exists, in addition to that of Dr. Rosin,[9] for lower furnace temperatures.

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[4] Trans. A.S.M.E., FSP-50-60 (Sept. to Dec., 1928), p. 87.

[5] "The Dust Hazard in Industry," Dr. William E. Gibbs, published by Ernest Benn, Ltd.

[6] *Fuel*, vol. ix, no. 2, p. 57 (Feb., 1930), A. L. Godbert. *Rev. de Ind. Min.*, Jan. 1, 1924, no. 73, p. 1, E. Audibert.

[7] "Reactivity" is a laboratory determination found by exposing pulverized samples to air in a furnace at 100 deg. cent. for three hours, the reactivity being the weight of oxygen absorbed per 100 grams of sample.

[8] "Rate of Burning of Individual Particles of Pulverized Fuel," *Industrial and Engineering Chemistry*, vol. 21, no. 9, p. 808 (Sept., 1929), Griffin, Adams, and Smith.

The chemical reaction between oxygen and the dust particle is so rapid that the rate of delivering the air supply to the particle is probably more important than the combustibility of the coal. This rate of oxygen reaction is dependent upon these factors:

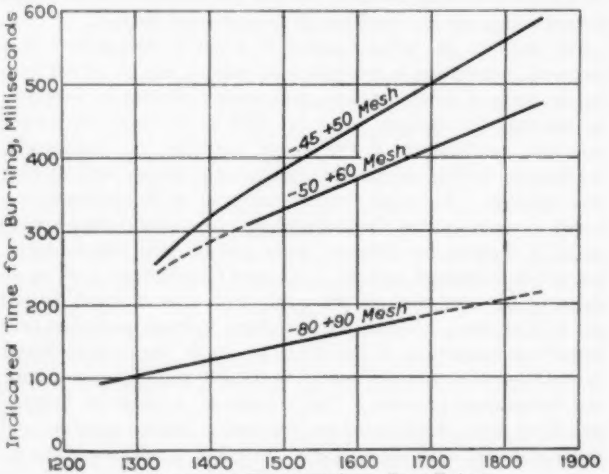

FIG. 3 TIMES FOR BURNING PULVERIZED COAL OF THREE SIZES AT VARIOUS FURNACE TEMPERATURES

(1) The specific surface of the dust (ratio of the area of coal to its mass).

(2) The proportion of air mixed with the dust and the manner in which it is supplied.

(3) The temperature of combustion space, which also determines the rate of chemical reaction between the fuel and air.

(4) The relative motion between the dust and the air.

These points can advisably be considered in some detail, as follows:

(1) Since the combustion reaction between the coal particle and its air supply can take place only at the interface between the two, the advantage of great fineness on this point is apparent. It is probably that the greatest single benefit from fine coal is due to the increase of surface exposed to the air supply.

(2) Each particle of coal dust requires for complete combustion a volume of air 35,000 times greater. The dispersion of the coal particles evenly and quickly into the center of such spheres is important and obviously requires considerable turbulence. The success of turbulent firing in commercial furnaces bears out this point. The delivery of air to the particles is naturally accelerated with excess air, the quantitative effect being shown in Fig. 4. It may be noted that the combustion time in these curves (based on experimental data) is given in fractions of a second, whereas in present commercial practice from two to four seconds are available. This indicates to a certain extent the margin of progress possible, although experimental results under ideal conditions cannot be reached in commercial furnaces. It does suggest, also, that, ignition permitting, there may be a limited advantage in superfines, since the combustion must be timed to the larger particles.

(3) A high temperature of the combustion space has been previously mentioned as retarding combustion, probably by forming a coke particle resistant to oxygen. If so, this effect must be sufficient to offset other combustion factors which are hastened by high temperature, for in general the rate of oxygen reaction increases with temperature, and the oxygen molecules diffuse through the gaseous products of combustion surrounding the coal particles more quickly. The net quantitative relation between furnace temperature and combustion rate has been referred to in Fig. 3, which should probably be accepted with reservations for the present.

(4) Much has been said about the effects of relative motion between the coal particles and the air. Any such relative motion tends to scrub away the products of combustion and facilitates the further delivery of fresh air. The advantages of turbulence in this respect have been advanced, and undoubtedly turbulence does aid this action. It seems more likely, however, that the advantage of turbulence is due more to the mixing of air and dust. In Fig. 1 is shown the velocity at which coal particles fall in still air. This terminal velocity for coal passing through 175 mesh is 1 ft. per sec. or less. As the velocity of the gases through the furnace are more on the order of 10 ft. per sec., the terminal velocity assumes less importance, and the finer the coal the lower the terminal velocity. Further, the relative motion due to turbulence is greatest immediately on leaving the burner in the ignition space, where it is needed least. With very coarse particles, however, say those over 60 mesh, a relative motion between air and particle may be material and may help considerably in reducing the amount of unburned carbon.

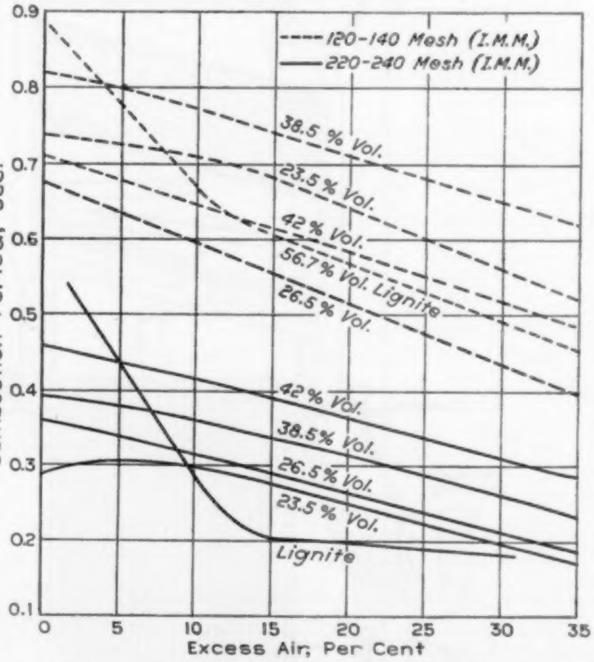

FIG. 4 VARIATION OF COMBUSTION PERIOD WITH EXCESS AIR, RANK, AND FINENESS OF COAL
(*Rev. de l'Ind. Min.*, Jan. 1, 1924, no. 73, p. 1, Audibert.)

One final factor in the subject of fineness of grind assumes considerable importance in practice, i.e., the effects of very coarse particles on slag "birdnesting" in the boiler generating tubes. Apparently results in commercial installations alone can establish the limitations on this point. The fusing of the ash, the path of the gases through the furnace, and the furnace shape all have a bearing on results. The slagging tendency has had much to do with the limit of 1 to 2 per cent over 40 mesh generally established by operators. On the other hand, recent developments in flame directions and in arrangement of boiler tubes to avoid birdnesting have been quite effective. Three methods may be used separately or together: (1) wide spacing of lower generating tubes to prevent slag bridging over and to chill to solidity the heavy particles of molten ash, (2) directing the flame toward the hearth in slag-tap furnaces so as to throw out the heavy particles which are then burned on the floor, and (3) the use of properly placed slag blowers. These devices as applied commercially must be watched for developments.

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[9] Paper before International Conference on Fuels, Dr.-Ing. P. Rosin, abstract in *Power Plant Engineering*, March 15, 1930, p. 326.

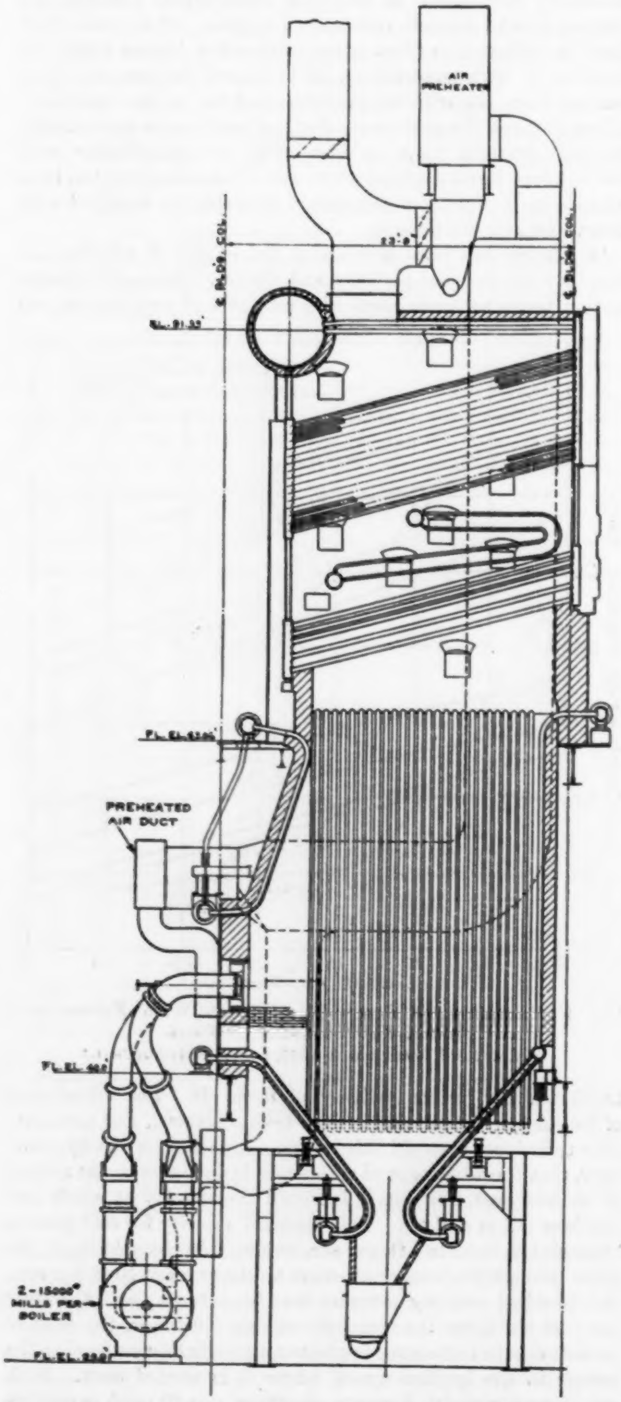

FIG. 5 BOILER NO. 17, ON WHICH FINENESS TESTS WERE MADE

The theoretical and experimental relations between the degree of pulverization and the ignition and combustion of pulverized fuel have been traced. The commercial balance between these factors and the cost of a high degree of pulverization furnish an economic problem—the resultant boiler efficiencies, investment costs, and operating costs all to be considered. The problem for a plant already in operation, the problem as viewed for a plant now being designed, and the problem for improvements toward which one can work are all separate and distinct.

The problem of "what fineness of grind to design for" in a proposed installation is one which at present can be solved only on the basis of what previous experience indicates is desirable. In general, the furnace must be sufficiently large to permit complete combustion of the larger particles, or, conversely, the fineness need be no greater than furnace volume and burning time requires. Although the experimental work previously referred to suggests that these considerations would permit a wide range of fineness for different fuels and furnace temperatures, practice is in general uniform, with most installations working on about 65 per cent through 200 mesh, with over 40 mesh limited to 1 to 2 per cent. Yet with this uniform fineness maximum heat liberations range from 12,000 B.t.u. per cu. ft. per hour to 50,000 in central-station practice and up to 100,000 and higher in marine and locomotive practice. This represents a range in burning time from 3 sec. down to 0.7 sec. for central-station practice, and fineness should vary accordingly. In other words, there are installations where furnaces are either four times as large as theory indicates or coal is being ground to 200 mesh when less than 100 mesh might suffice. The problem quite apparently warrants further attention and study, both by designers and by operators.

CAHOKIA TESTS

The problem as to what fineness to use in a plant already in operation usually becomes one of balancing costs of coal preparation against savings in boiler efficiency. In some circumstances it may be a matter of increasing fineness (and preparation costs) in order to increase boiler capacity, for finer coal giving more rapid combustion may permit higher furnace-heat liberations. The latter possibility, however, is rare, for increased fineness reduces mill capacity, and furnace-heat liberations are usually limited by furnace-wall construction. The first case, i.e., of boiler economy versus preparation costs, was the subject of a number of tests which were made at Cahokia Station.

A series of economy tests were made on No. 17 boiler (Fig. 5),[10] which is an 1801-hp. cold-bottom unit, fired from two 15,000 lb. per hour Simplex unit mills. It was found that at the normal rating of 300 per cent (furnace liberation of 20,000 B.t.u. per cu. ft. per hour) a wide range in fineness could not be fired because the mills, fans, and burner piping were designed for comparatively fine coal and became choked with large quantities of coarse coal. To permit a range of 45 to 75 per cent through 200 mesh, it was therefore necessary to operate at about 200 per cent rating, or 13,000 B.t.u. furnace-heat liberation. To this extent the tests are of more value for study than for practical use in establishing regular plant practice. The wide range of fineness, however, gave an interesting set of conditions to observe.

The tests were with weighed water and coal, of six to eight hours' duration, and sufficient data were taken to calculate boiler efficiency and to record the operating conditions affected by coal size; namely, CO_2, combustible in flue dust, proportion of ash to the stack, stack-gas temperatures, and mill-power consumption. It was realized that proper fineness samples were very important, although difficult to obtain, so each mill outlet was sampled by a vacuum-cleaner sampler at nine different points in a square cross-section of the mill discharge at some distance from the fan. The sampling pipe was held at each of the nine points for 5 min., and the resultant 45-min. composite was automatically corrected for stratification, and an accurate sample was insured. Flue dust was collected in five cyclone-separator samplers spaced across the uptake. Pit ash was weighed and the quantity of stack ash was determined by difference.

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[10] Reproduced from *Mechanical Engineering*, vol. 50, no. 10, p. 772, Oct., 1928.

The test results are shown in Table 1 and are plotted in Figs. 6 and 7. The tests from *A* to *K* are arranged in descending order of fineness through 200 mesh. It may be noted that a high CO_2 of 17 per cent (18 per cent excess air) was held throughout. This point was determined by smoke, the air adjustments being made to avoid unburned hydrocarbons. In this way all the loss due to incomplete combustion was evident as unconsumed carbon in the flue dust and pit ash, which reaches 15.7 per cent and 7.0 per cent, respectively, with the coarsest coal. In regular operation at 300 per cent rating the excess air would be increased to about 25 per cent and the unburned carbon would be about half that shown, but it was believed that the method followed at the lower rating gave a truer indication of relative economy due to coarse coal. In this way it was also possible to maintain a very uniform stack temperature, and altogether the number of observable variables was reduced to the minimum. It is very possible that higher economies could have been found by varying the relation between excess air and flue-dust loss, which indicates how really complicated the whole problem of fineness is and the amount of further investigation necessary to accurately determine good operation.

There has previously been shown (Fig. 1) the relation between particle size and gas velocity necessary for combustion in suspension. At the rating at which these tests were made, between 3 and 4 sec. are available for combustion, and the flame travel is about 40 ft., or 10 to 13 ft. per sec. Particles between 40 and 50 mesh therefore would tend to fall to the furnace bottom.

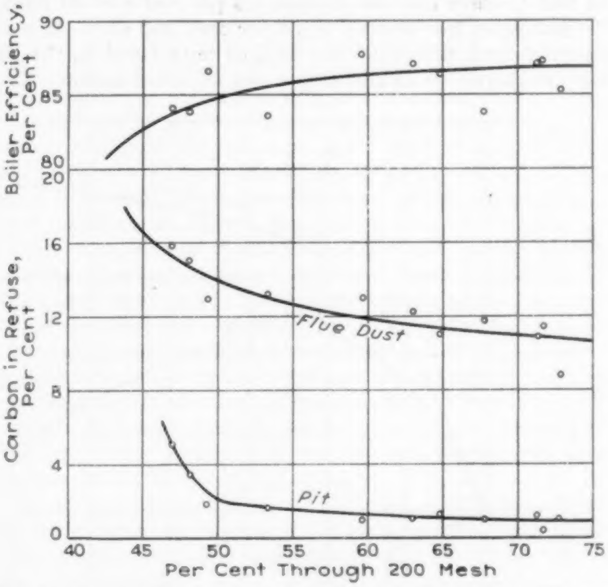

FIG. 6 BOILER EFFICIENCY AND CARBON IN REFUSE VERSUS DEGREE OF PULVERIZATION, BOILER NO. 17, CAHOKIA STATION

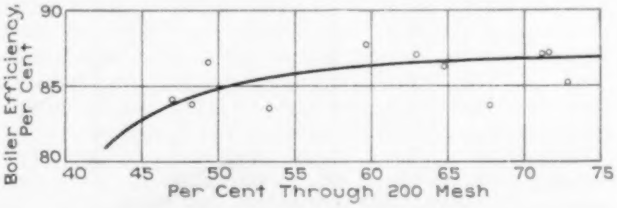

FIG. 7 BOILER EFFICIENCY VERSUS DEGREE OF PULVERIZATION, BOILER NO. 17, CAHOKIA STATION

TABLE 1 BOILER EFFICIENCY VERSUS DIFFERENT DEGREES OF PULVERIZATION, CAHOKIA STATION, BOILER NO. 17

(Data: Initial operation, October, 1927. Heating surface, 18,010 sq. ft., Babcock & Wilcox. Two Simplex 15,000 lb. per hr. unit pulverizers. Four horizontal turbulent burners. Combustion Engineering plate-type air heater, 9920 sq. ft. Furnace volume, 12,000 cu. ft. Side walls, plain tubes on 7-in. centers, recessed into refractory; 465 sq. ft. projected area of tubes. Front arch, plain tubes on 10½-in. centers, recessed into refractory; 93 sq. ft. projected area of tubes. Rear wall, plain tubes on 7-in. centers, recessed into refractory; 238 sq. ft. projected area of tubes. Hopper bottom, bare iron blocks on tubes on 7-in. centers; 479 sq. ft. total area. Total water-wall surface, 1275 sq. ft. Fraction cold, 0.79. Water economizers, none. Coal driers, none.)

Test run	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
Fuel fineness (per cent through A.S.T.M. sieves):											
325 mesh (44 microns)	56.7	60.1	60.6	54.8	51.0	51.9	47.7	43.0	37.3	36.7	34.5
200 mesh (74 microns)	72.8	71.6	71.3	67.7	64.8	63.0	59.7	53.3	49.3	48.2	47.0
140 mesh (105 microns)	83.2	80.1	80.2	75.2	72.9	73.4	70.6	64.6	59.8	58.7	56.1
100 mesh (149 microns)	88.7	87.4	87.3	83.8	82.2	83.4	79.5	74.8	69.8	68.7	65.8
60 mesh (250 microns)	97.3	96.6	96.6	95.2	94.6	94.6	92.8	89.2	86.7	85.3	82.9
40 mesh (419 microns)	99.7	99.7	99.6	99.6	99.0	99.1	98.4	97.0	95.1	92.9	92.6
Proximate analysis:											
Moisture, per cent	8.96	9.50	9.60	9.11	9.70	9.28	8.70	9.90	9.33	10.40	10.21
Volatile, per cent	33.23	34.04	33.17	33.17	33.86	33.66	34.05	33.96	33.09	33.16	31.96
Fixed carbon, per cent	41.51	42.08	42.22	43.08	41.00	43.54	41.36	42.17	40.17	43.80	43.47
Ash, per cent	16.30	18.39	15.01	14.64	15.44	13.52	15.89	13.97	17.41	12.62	14.37
B.t.u. per lb.	10,600	10,623	10,557	10,670	10,490	10,803	10,662	10,730	10,262	10,824	10,541
Moisture in coal fired	2.2	3.1	3.6	2.6	1.9	2.35	2.6	2.6	2.8	3.2	3.7
CO_2 top first pass, per cent	17.2	16.9	17.1	17.2	17.3	17.1	17.1	17.1	17.2	17.0	17.1
Stack-gas temperature, deg. fahr.	341	340	343	336	333	324	323	332	332	326	333
Steam pressure, lb. per sq. in.	328	326	326	324	323	326	318	325	326	322	320
Total steam temperature, deg. fahr.	731	759	755	724	727	735	731	728	745	702	723
Draft, uptake, in. H_2O	0.59	0.59	0.63	0.49	0.49	0.51	0.47	0.50	0.53	0.45	0.50
Draft, furnace, in. H_2O	0.06	0.06	0.06	0.05	0.05	0.06	0.05	0.05	0.05	0.06	0.06
Pounds steam per hr.	103,575	102,276	109,745	101,958	101,349	101,828	99,619	104,756	102,939	101,747	101,196
B.t.u. liberated per cu. ft. furnace volume in 1 hr.	12,814	12,458	13,790	12,686	12,260	12,280	11,280	13,100	12,500	12,700	12,520
Pulverizers (Simplex):											
Average load, lb. per hr. each	7,253	7,037	7,627	7,133	7,011	6,820	6,778	7,330	7,308	7,035	7,126
Per cent of rated capacity	48.3	46.9	50.8	47.5	46.7	45.5	45.1	48.8	48.7	46.8	47.5
Power used, kw-hr. per ton	23.1	23.25	24.7	21.85	21.6	19.8	17.4	16.4	16.5	16.3	14.9
Refuse:											
Per cent to stack	77.7	78.8	72.5	65.2	73.6	72.2	65.3	69.6	65.8	62.6	62.2
Per cent combustible, flue dust	8.8	11.5	10.8	11.9	11.0	12.3	12.9	13.2	12.9	14.9	15.7
Per cent to pit	22.3	21.2	27.5	34.8	26.4	27.8	34.7	30.4	34.2	37.4	37.8
Per cent combustible, pit	1.0	0.4	1.2	1.0	1.3	1.1	1.0	1.6	1.9	3.4	7.0
Boiler efficiency	85.22	87.25	87.12	83.80	86.49	87.17	87.86	83.69	86.58	83.91	84.06

This is borne out by the test data. Tests from *A* to *H* have a quite uniform pit loss with unburned carbon up to 1.6 per cent. At test *I*, where the coal screened 4.9 per cent over 40 mesh, the pit-carbon loss went up to 1.9 per cent, and when the coal above 40 mesh exceeded 7 per cent, on tests *J* and *K*, the pit loss rose sharply to 3.4 and 7.0 per cent unburned carbon. The cause was apparent in the furnace, where a large number of "sparklers" were seen to settle to the pit. The fact that the amount of ash to the pit did not increase in like proportion is due to the fusing tendency of the ash (fusion point 2000 deg. fahr.); with the finer grinds the small particles of ash fuse together in the top of the furnace and can be seen falling down into the pit.

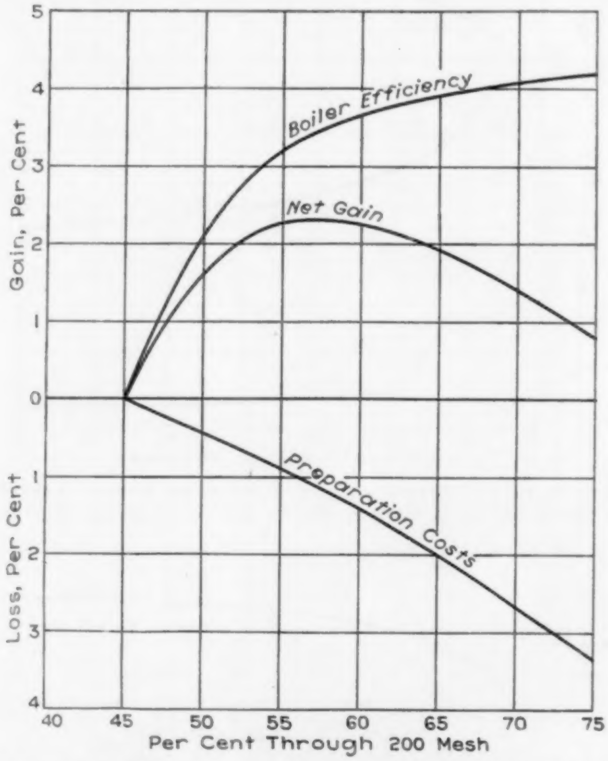

Fig. 8 Per Cent Gain or Loss in Boiler Efficiency and Coal-Preparation Costs Versus Degree of Pulverization, Boiler No. 17, Cahokia Station

(Points boiler efficiency gain from Fig. 7. Loss due to extra coal-preparation costs shown as per cent of cost of coal and equated to equivalent per cent points of boiler efficiency. Lower curve subtracted from upper to give net overall gain expressed in equivalent per cent points of boiler efficiency.)

The southern Illinois coal burned has a volatile content of about 34 per cent, is free-burning, non-caking, and is readily ignited. The dust cloud was observed to be shortened with the finer grinds, but it is doubtful if, at this low rating, any gains in ignition time are of importance. With constant excess air the furnace temperature was uniform, and there was no opportunity to establish any relations between ignition, combustion, and furnace temperature.

The very low amount of excess air (18 per cent), without smoke, indicates the completeness with which the air supply was delivered to the separate coal particles, a result due in great part to the turbulent method of firing. The consistent increase in unburned carbon to the stack as the coal becomes coarser measures the increase in burning time required for the larger particles. In practice there is a partial compensation on this point, for, as the particle becomes larger and its combustion time increases, its tendency to settle out lengthens its stay in the furnace. It may be seen in tests *J* and *K* that only when the size of the particle causes it to fall rapidly does the pit loss become excessive. A tendency also noted on these runs was for slag to adhere to the lower boiler tubes, and it is doubtful if operation could have been continued longer than 24 hours.

It may be noticed that in each test a great portion of the coal that passes through 200 mesh also passes through 325 mesh. Such sizes come under the head of "superfines," which under certain conditions are considered quite necessary for satisfactory ignition and combustion. There is no way of measuring their value in this particular installation. Combustion time is available for much coarser particles, but on the other hand the smaller particles are more easily dispersed into the air supply. This is another important point which must await further investigation.

The boiler efficiency, of course, summarizes all losses, and, allowing for test inconsistencies, the net result is shown by the curve of Fig. 7. It is seen that the curve breaks when the fineness falls below 55 per cent through 200 mesh, and sharply when below 50 per cent. There is no gain when the fineness exceeds 65 per cent.

The desired range for operation is dependent upon the efficiency gained compared with the additional cost of preparation. The latter is made up of power costs and maintenance, which can be assumed as paralleling the power consumption of kilowatt-hours per ton, which can be seen to consistently drop with coarser coal. This cost in cents per ton has been converted into percentage of total coal cost and plotted with 45 per cent through 200-mesh coal as a base in the lower curve of Fig. 8, and is algebraically added to the net gain in boiler efficiency above. (Both percentages are equated as per cent points of boiler efficiency.) The central curve represents the net gain, and the best point of operation is at 57 per cent through 200-mesh.

Although these tests are in no way a complete answer to the problem of degree of coal pulverization, they do suggest the general method by which the problem can be attacked by the operator. There has been discussed the consistent relation between fineness and the possibilities of design, and there have been indicated the complexity and the importance of the problem, for almost every phase of pulverized-fuel firing is closely related to particle size. If the advantages of this form of fuel firing are to be fully realized, then additional research is imperative if a slow trial-and-error method of development is to be avoided.

Discussion

M. K. Drewry.[11] Industry is indebted to the author for the considerable work expended to evaluate optimum coal fineness in actual practice. His concise summary of combustion-rate influences is also valuable.

That higher furnace temperatures actually decrease combustion rapidity is surprising. It has long been felt that benefit derived from designing for higher temperatures was slight, but it was not anticipated that combustion was thereby actually handicapped. The test data showing a halving of combustion rapidity as temperatures are increased from 1200 deg. fahr. to 1800 deg. fahr. are of interest. It supports the contention that combustion troubles in all-metal cooled furnaces at low ratings are entirely due to poor mixing, not lowered furnace temperature.

High admission temperatures of coal and air apparently have a rapid influence on increasing combustion rates. Experience with 650 deg. fahr. preheated air and 350 deg. fahr. supply-coal temperature has shown that low carbon loss is very easily attained with the assistance of this initial preheating. Reaction rates multiply with enormous rapidity, since they double with each 20 deg. fahr. temperature increase at low ranges and 100 deg. fahr. increase at higher ranges.[12] Preheating outside the furnace reduces the period when slowest reactions occur. Large air heaters, heating primary and secondary air to high temperatures, help furnace reactions markedly.

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[11] Assistant Chief Engineer, Power Plants, Milwaukee Elec. Ry. & Light Co., Milwaukee, Wis. Mem. A.S.M.E.

It seems timely that still lower excess air be used than the present average of 25 per cent. Large savings in size of all heat-absorbing equipment can be effected if combustion efficiency is not decreased. More attention to the combustion process, as encouraged by this paper, is desirable.

L. E. Griffith.[13] It costs money to pulverize coal! In practically all the papers on the subject of coal pulverization in the past this fact has been more or less ignored. The author has given a paper which definitely takes this into account and has carefully and thoroughly covered all the factors in the relationship between fineness and efficiency of combustion. He has tabulated results of actual tests showing this relation for a given set of conditions. This tabulation and the curves drawn from the data indicate an increasing efficiency with increase in fineness. It is to be noted, however, that the slope of the fineness-efficiency curve continuously decreases and will tend to ultimately become zero. On the contrary, the slope of the curve showing cost of preparation plotted against fineness would tend to continuously increase.

The law of diminishing returns has been well established and certainly applies to the pulverization and burning of coal. This law is well illustrated by Fig. 8 in the paper. While there is a continuous gain in boiler efficiency by increasing fineness of pulverization, there also is a continuous increase in preparation cost, and the curves indicate that in the case of the unit involved in the author's tests above 57 per cent through 200 mesh the increase in preparation cost is greater than the gain in efficiency and that to pulverize coal to any greater degree would represent an actual loss in the cost of steam production.

As far as the effect of fineness is concerned, Mr. Tenney can make steam in the particular boiler mentioned at the rating operated at the lowest cost per thousand pounds when the coal is pulverized to about 57 per cent through 200 mesh.

Within the last two years there have been papers intending to show the degree and kind of fineness that can be obtained with different types of pulverizing equipment. These papers have given data from which conclusions were drawn that certain types of pulverizers gave a greater degree of fineness or gave a greater percentage of extreme fines. The assumption has been made in these papers that a greater degree of fineness was necessary, and consequently the conclusion was drawn that the particular type of pulverizer which gave the greatest degree of fineness as indicated by the data tabulated in the paper was the most desirable type of pulverizer. The fallacy in such reasoning is clearly demonstrated by the paper.

It has been rather conclusively established that, in the absence of an objectionable percentage of larger particles, high efficiencies can be secured, with proper burner design, if pulverization is maintained at points ranging from 50 to 80 per cent through a 200-mesh screen—depending upon the characteristics of the coal and the furnace involved. The mere process of securing this degree of fineness produces the desirable ratio of so-called "superfines."

Of importance also is the requirement that the pulverizing equipment be capable of maintaining, uniformly, the desired character of pulverization over the necessary range of load demands.

It is to be hoped that the relationships established in the author's paper will encourage further experiments of this character with various types of coal and various forms of furnace installations. It will undoubtedly be found that certain groups of coals for a given set of furnace and operating conditions will have a particular fineness as being the most economical, while other groups will have different degrees of fineness as being the most economical. It is probable that the low-volatile Eastern coals will show the highest degree of fineness for the maximum return—first, because of extreme friability and, second, because of difficulty of obtaining complete combustion of fixed carbon. The second group might be considered as coals which are typical of the Pittsburgh district. It is probable that the desirable degree of fineness in such coals will be found to lie between that of low-volatile Eastern coals and Mid-Western bituminous coals such as that used in Mr. Tenney's plant. This will be due to the fact that such coals do not, in general, possess the same friability or free-burning characteristics encountered with Mid-Western coals.

An accumulation of such data in any given plant should emphasize the importance of giving proper consideration to the effect of the cost of fuel preparations upon the overall cost of steam production.

R. H. Morris.[14] The paper should do much to clarify the situation regarding size specifications for pulverized fuel, and if one can judge by the alacrity with which mill manufacturers attempted to reply to Mr. Work's paper of two years ago, the situation can stand considerable clarification.

From the data in Table 1, apparently it makes little difference whether the fineness be specified as a definite percentage through a 200, 100, 60, or 40 sieve. Replotting Fig. 7 with efficiency against the percentage through any sieve in the series will show that the boiler efficiency always drops below 85 per cent between the *H* and *I* tests, as is shown for 200 mesh. This confirms the opinion that those favoring specifications calling for a minimum percentage through 40 or 48 mesh differ from those calling for a minimum percentage through 200 mesh, only as evaluating an article in dollars differs from evaluating it in cents. The 200 designation dealing with smaller units is perhaps more accurate, but takes more time and care, so that perhaps an intermediate sieve, say 100, would be the most practical one to use. The data given are so consistent that a complete fineness curve could be interpolated or reconstructed from any single given point.

Rosin showed, as early as 1927, that pulverized fuel, when plotted with cumulative percentage against sieve size, gives a characteristic curve, the shape of which depends only upon the absolute fineness of the dust and is practically independent of the kind of pulverizer and kind of coal. This is well substantiated by the author's data for a particular coal and mill, but apparently is in direct conflict with Work's conclusions and the inferences to be drawn from his Figs. 1, 2, 3, and 5.

Unfortunately Work did not mention the kind of coals to which his data apply. There is considerable difference in the characteristics which, as plotted, indicate a distinctive difference in product due to both coals and mills. Careful examination of the data in specific cases, such for instance as *H*-2 for hammer mills, shows that it agrees with certain tube-mill samples over the entire range. With all tests plotted on a single sheet, it is doubtful if samples from several mills could be identified.

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[12] Haslam and Russell.

[13] Vice-President, Riley Stoker Corporation, Worcester, Mass. Assoc. A.S.M.E.

[14] Associate Editor, *Power Plant Engineering*, Chicago, Ill. Assoc-Mem. A.S.M.E.

Undoubtedly the differences shown are directly attributable to the classifier action, which is not and should not be considered as characteristic of the mill except in so far as certain manufacturers have made it so.

Distinct differences in hammer-mill and tube-mill sizes, shown by a distribution of superfines in Work's Fig. 5, are easily explained in the light of Rosin's work. Rosin shows that decreasing the fineness from about 90 per cent through 200 mesh to 80 per cent through 200 mesh changes the distribution of product, expressed as per cent per micron, from a peak of about 4 per cent at 15 microns down to about 2 per cent at 30 microns. A further decrease in fineness to 40 per cent through 200 mesh reduces the peak to about 1 per cent at 60 microns. Work's superfine data covered two samples, one 58 and the other 78 per cent through 200 mesh, more than sufficient to account for the difference in distribution of sizes, even though both samples had been taken from the same mill. It is likely that a careful analysis of Work's data would substantiate Rosin's claims that the characteristic curve is independent of the type of coal and mill.

Tenney's data over the complete range should be correlated with similar data from other installations having different mills and coals. Bureau of Mines investigations show differences of as much as 50 per cent between hand-sieve and machine-sieve methods. Even when the A.S.T.M. specifications are followed, discrepancies in testing make the data useless for absolute values, although relative results are satisfactory for home use. In order to place data from different plants in such shape that the results might be interpreted, it would be advisable to give details of the sampling method and equipment used in sample preparation and the sieve procedure followed.

No one has determined how necessary these superfines really are, although as the author has pointed out, the matter has been investigated theoretically by Rosin, whose conclusions, well backed by theoretical and experimental data, are not always in perfect accord with accepted theories. Apparently within the grinding limits of practice, from 30 to 45 per cent of the coal as fired is below 25 microns. If this 30 to 45 per cent of superfines were screened out, what would be the effect on combustion?

The question is of considerable interest in connection with the power required for grinding—that is, mill efficiency as well as for combustion. In connection with the efficiency of grinding mills, Gross and Zimmerly of the Bureau of Mines (Serial 2952, July, 1929) determined that the efficiency of mills (the ratio of laboratory determinations to actual power consumption of the mills) was 44.8 per cent. This they considered high.

However, after determining that 200 mesh or 74 microns was the proper size, they found that 90.2 per cent of the power expended in grinding was expended uselessly in producing superfines which were not only useless but in some cases a distinct disadvantage. The useful grinding efficiency of the mill was only 4.25 per cent.

If 50 or 25 micron fuel particles are found to be the minimum size necessary to give good results, it is possible that coal-mill useful-work efficiencies are also in the neighborhood of 4 per cent and that work which should require 0.5 to 1 kw. per ton is taking 15 to 25 kw. The possible power saving should be sufficient to warrant experimental work to determine the minimum size necessary, if for no other reason than to find out if this useless power expenditure is an inherent loss like the condenser loss or if it can be eliminated by proper pulverizer and classifier design.

The work of Gross and Zimmerly is not, of course, directly comparable with coal mills. They were working with wet tube mills grinding galena, sphalerite, pyrite, and gangue and in which there was a considerable circulating load of superfines due to the classifier action. The work does, however, indicate the low useful efficiency of grinding mills. Furthermore, they discovered that although the total load of 3000 tons per day was only about 33 per cent gangue, this material absorbed 71.2 per cent of the total useless work, most of which was expended on the superfines.

This brings up the possibility of pulverizing fuel in two stages, with some arrangement of cleaning in between to eliminate a large portion of the ash and inert substances found in the poorer grades of coal. Taken in itself the power saving from eliminating the necessity for grinding the ash and slate and producing superfines might be negligible, but taken with the decreased mill maintenance and improved furnace conditions, the combined benefits of such a practice might make it practical as further refinements in operation are demanded.

M. A. Mayers.[15] The writer notes a few points in connection with the theoretical data presented in this paper.

In the first place it is unlikely that the reactivity has any effect on the burning time in boiler furnaces. Determinations of the reactivity of coke with air show that all cokes have the same reactivity above temperatures in the neighborhood of 1100 deg. fahr. and that the temperature coefficient of reactivity becomes less at this temperature than at lower temperatures. This indicates that some other factor than reactivity governs above this range of temperature.

In partial explanation of the negative coefficient of combustion velocity shown in Fig. 3 of the paper, the laws of the kinetic theory of gases may be invoked. It can be shown that the number of mols of a gas that impinge on a unit of surface in unit time is given by

$$Q = \frac{p}{\sqrt{(2\pi RT)}}$$

where Q = number of mols of gas
p = pressure of gas
R = gas constant
T = absolute temperature

It is evident that this would result in a decreasing frequency of contacts between combustible matter and oxygen as the temperature increased, and hence in a negative coefficient of combustion velocity with temperature.

It is interesting to note from Fig. 4 that the combustion time varies approximately as the diameter of the particle. The writer has made calculations that indicate that this should be the case and is pleased to find experimental justification for this conclusion.

H. G. Lykken.[16] In Fig. 7 the author has plotted boiler efficiency against percentage of 200-mesh material, showing an apparent increase in efficiency with increase of percentage through 200 mesh. In Table 1 are given the data from which the curve is taken.

In this table it will be noted also that as the percentage of 200 mesh increases, there is a decreasing percentage of coarse material in the mixture. In column A, with 72.8 per cent 200 mesh, there is 11.3 per cent on 100 mesh and only 2.7 per cent on 60 mesh; whereas in column K, with 47 per cent through 200 mesh, there is 34.2 per cent on 100 mesh, 17.1 per cent on 60 mesh, and 7.4 per cent on 40 mesh.

In so far as the lower efficiency was accounted for in a large measure by a higher percentage of carbon in the ashpit and particularly in the flue dust with the coarser grinding, should not the curve have been boiler efficiency plotted against the percentage of material on 60 mesh?

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[15] Coal Research Laboratory, Carnegie Institute of Technology, Pittsburgh, Pa. Jun. A.S.M.E.

[16] Chief Engineer, Unipulvo System, Strong-Scott Mfg. Co., Minneapolis, Minn.

Was it not the higher percentage of coarse material, 7.4 per cent on 40 mesh, 17.1 per cent on 60 mesh, and possibly 34.2 per cent on 100 mesh, that caused the loss rather than the lesser percentage of superfines?

A test as to the value of a higher percentage of superfine in reality means nothing unless a series of tests can be run having the identical amount of coarse particles, for the reason that it is patent that it is the coarse material going out unburned that causes most, if not all, of the difference in loss.

By observation, tests, and extensive experience, the writer has failed to get any smaller flame volume, better furnace condition, or better efficiency by increasing the superfine when maintaining an equal degree of elimination of coarse particles—say, everything through 60 mesh. It may be possible that the ideal condition calls for very little superfine. It is the writer's opinion that if all material passed 50 to 60 mesh with the bulk passing 100 mesh, little or nothing can be gained by more than 30 to 40 per cent passing 200 mesh or finer.

It is the coarse material that affects efficiency, not percentage of superfine, and milling specifications should be based upon the degree of fineness at the lower end of the scale, such as all through 50 mesh, 60 mesh, or whatever the requirement may be, letting the percentage of 200 mesh take care of itself. Specifying fineness by a given requirement through 200 mesh does not adequately specify the material, because a given percentage of 200 mesh may mean a much larger percentage of coarse with one type of mill than with some other type.

Ralph A. Sherman.[17] The author discussed the combustion of pulverized fuel from a somewhat different viewpoint from the majority of papers in the past. It is considered a hopeful sign that perhaps we are to get away from the usual paper presenting the relation of burner designs to boiler efficiency or similar subjects and study the more fundamental aspects of the combustion of pulverized fuel.

The writer first wishes to correct the misinterpretation which the author has placed upon the terminal velocities of particles. Particles of coal falling in a viscous medium, such as air, do not continue to accelerate under the influence of gravity, but attain a terminal or equilibrium velocity which depends upon their diameter, their density, and on the kinematic viscosity and the density of the medium. If a particle has a terminal velocity of 10 ft. per sec., gases moving upward at this velocity will maintain the particle in suspension, but it is not true, as the author states, that gases moving with the same horizontal velocity will keep the particle in suspension. For horizontal movement of gases the terminal velocity gives the downward component and is extremely useful in calculating the velocities required in moving coal in ducts or in furnaces. Attention should be called to the fact that in addition to the curve which the author presents in Fig. 1, which is taken from Gibbs, "The Dust Hazard in Industry," Audibert[18] has conducted some experimental studies to determine the terminal velocity and developed an empirical formula for its calculation. Blizard[19] analyzed his data and developed a formula which showed the interesting fact that the terminal velocity should be independent of the temperature of the gas. However, in some recent work by the writer on the sizing of pulverized fuel by rising air currents it appeared that Stoke's law gave better results than the Blizard-Audibert formula.

The author has called attention to the work of Audibert, Godbert, and Griffin, Adams, and Smith on the mechanism of combustion of pulverized coal and has reproduced some of the curves from their work. These investigations are extremely interesting and valuable, but it must be emphasized that, because of the much different conditions from boiler or industrial furnace practice, extreme care must be used in the application of their results to practice. Audibert measured the time of completion of combustion as the time at which the CO_2 content of the gases become constant. He did not sample the dust to determine the carbon content nor did he show how closely the total oxygen content of the gases checked with that originally fired with the coal. His findings that the time of burning of the coal increased with the volatile content is in direct variance with all practical experience.

The work of Griffin, Adams, and Smith was done at temperatures considerably below that in practical furnaces, and the number of particles was so small that they were surrounded by air with the normal content of oxygen in air throughout the period of burning. Dr. S. P. Burke, formerly director of research, Combustion Utilities Company, and now with the West Virginia University, has made a mathematical analysis of the mechanism of combustion which is said to explain the negative temperature coefficient. This is to be given shortly in the publication of the American Chemical Society.

The author has called attention to the wide difference between the burning times of 0.2 to 0.4 sec. found by these various workers for pulverized-coal particles and the time of 2 to 4 sec. which a particle remains in a boiler furnace. There is some question, however, whether the coal remains this long in the furnace. This time is ordinarily calculated from the rate of gas flow in volume per unit time and the total volume of the furnace. It is known that the gases do not fill the entire volume; therefore, the rate of flow is greater and the time in the furnace is less than are normally calculated. The discrepancy between experiment and practice must remain high, and more experimental data are needed to understand all the factors thoroughly.

The data which the author presents on the tests at Cahokia and the conclusions which he draws appear entirely reasonable. Because of the difficulty of close control of all conditions in a boiler installation as is shown by the points which fall off the smooth efficiency curve, a boiler furnace is not well suited to an accurate study of the combustion process, although any results on experimental furnaces must finally be proved there.

Henry Kreisinger.[20] The paper contains useful data on the relation between fineness of pulverized coal and plant efficiency. The data shows that with Illinois coal there is no material gain in the plant efficiency in increasing the fineness of coal beyond 55 per cent through 200-mesh screen, and that there is a decided drop in the plant efficiency when the fineness is increased beyond 60 per cent through 200-mesh screen. What is gained in higher boiler efficiency is more than offset by the increased cost of pulverization. The fineness of 55 per cent through 200-mesh screen for Illinois coal has been recognized as the most economical fineness, and has been used as a standard in the adjustment of the mills. For other kinds of coals the most economical fineness increases in general with the percentage of fixed carbon of the coal, because high-fixed-carbon coals are, as a rule, more difficult to burn than high-volatile coals. It is a common experience that when high-fixed-carbon coals are burned in pulverized form the flue dust contains large percentage of combustible unless the coal is pulverized to a higher degree of fineness.

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[17] Fuel Engineer, Battelle Memorial Institute, Columbus, Ohio. Mem. A.S.M.E.

[18] Audibert, Etienne, "Étude de l'entrainement du poussier par un courant d'air," *Ann. des Mines*, ser. 15, t. 1, 1922, pp. 153–191.

[19] Blizard, John, "The Terminal Velocity of Particles of Powdered Coal Falling in Air or Other Viscous Fluid," *Jour. Franklin Inst.*, vol. 197, 1924, pp. 199–207.

[20] Consulting Engineer, Combustion Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

The foregoing statement of experience seems to be at variance with the results obtained by Audibert and shown in Fig. 4 of the paper. Audibert's results apparently show that more time is required to burn coals containing a high percentage of volatile matter than coals with small volatile-matter content. This is particularly true of the lower group of curves which give the time required for combustion for the coarser coals. In this group of curves the individual curves align themselves according to volatile matter, the coal with the highest volatile matter being at the top and that with the lowest at the bottom of the group.

The upper group of curves is not so consistent. The coal with the lowest volatile matter happens to be the second from the top instead of being at the bottom of the group. This lack of agreement is very likely due to the method of determining the time required for the combustion. In these experiments it has been assumed that the combustion was complete when the percentage of CO_2 in the products of combustion became nearly constant. No determination of the percentage of combustible in the flue dust was made. If Audibert had stated the completeness of combustion in the same manner as the author—that is, by determining the percentage of combustible in the flue dust and ashpit refuse—the curves of Fig. 4 would have arranged themselves in different order. Those who have done work on commercial installations know that the only way to find out how complete the combustion is, is to determine the amount of unburned combustible both in solids and gaseous state.

It is a common fault of many a casual observer of combustion of powdered coal in commercial installations to assume that the combustion is complete at the end of a visible flame. Only the volatile matter burns with visible flame. The combustion of fixed carbon which is left after all the volatile matter has been distilled off and burned continues to burn without any visible flame. The only way to determine how far this combustion of the fixed carbon has progressed at any section of the path of the gases is to sample the solids and determine the percentage of combustible.

Much has been said and written about the reactivity of different forms of carbon or the reactivity of coke from different coals or coke made in different ways. Some of the experiments conducted with the object of determining such reactivity seem to indicate a rather great difference between the reactivity of coke from different coals or coke made in different ways, but when the methods of experimentation are changed the results are also changed. The reactivity of carbon may be defined as the rate at which the carbon combines with oxygen when free oxygen is supplied to the surface of the carbon particles at the furnace temperature, which is about 2000 deg. fahr. or higher. Such definition should at least hold when we speak of reactivity in connection with burning pulverized coal.

One of the methods used for the determination of the reactivity of fixed carbon from different coals is to burn the coal or coke in a fuel bed 4 to 6 in. thick and analyzing the gases collected at different heights in the fuel bed. With some coals free oxygen may be all used up in combustion 3 in. from the grate. With some other coals the free oxygen may disappear 4 to 6 in. from the grate. When the free oxygen has been all combined, the resulting CO_2 is being reduced to CO by the passage through the upper layers of the fuel bed. The reactivity of the carbon is supposedly indicated by the height from the grate at which the oxygen disappears and by the percentage of CO_2 reduced to CO in the upper layers of the fuel bed. In such experiments the sizing of the coal or coke and the shape of the pieces and their arrangements in the fuel bed are very important factors. If the size and the shape of the pieces is such that they form a compact bed and form very irregular and narrow passages for the air and other gases in their upward flow through the fuel bed, the indicated reactivity of the carbon will be high. Thus, when the experiments are conducted with lignite or lignite char, it is found that free oxygen disappears 2 to 3 in. from the grate and practically all the CO_2 is reduced to CO by the time the products of combustion reach the top of a 4-in. or 6-in. fuel bed. It is generally concluded that the reactivity of carbon in lignite is very high, but such conclusion is questionable. The lignite and lignite char disintegrate when heated. Although the fuel when added at the top of the fuel bed may be of definite size, this size changes by the heating of the fuel as the fuel moves down from the surface to the grate. By the time the fuel reaches half-way through the fuel bed, the pieces disintegrate into much smaller pieces and form a compact fuel bed with very narrow and zigzag passages for the air and the gases. This change in the size and arrangement of the pieces causes the free oxygen to disappear much nearer the grate than with fuels that do not disintegrate. For a similar reason the CO_2 is reduced to CO more completely in the upper layers of the fuel bed. If an experiment were made with anthracite coal of a size similar to the pieces of lignite in the lower part of the fuel bed, very likely the anthracite might show as high reactivity as the lignite.

Another factor which may lead to false indication of the reactivity of the carbon is the distribution and the quality of ash. If the ash is uniformly distributed throughout the mass of coal and has such physical properties as will make it to adhere in a layer to the pieces of coal after the carbon in the surface has burned, this layer of ash will tend to prevent direct contact of oxygen with the carbon under this layer of ash. Such fuel will give indication of very low reactivity. However, the cause of this apparent slow burning of the carbon is not a slow combination of oxygen with the carbon, but a lack of sufficient contact of the oxygen with the carbon due to the layer of protective ash. Anthracite coal shows such behavior particularly with low rates of combustion where the velocity of gases passing the surfaces is too low to scrub off the layer of ash. When anthracite is burned in a house-heating furnace, the fire may go out at times. The material left on the grate seems to be nothing but ash. If, however, this ash is taken out and thrown on the ash pile, a good rain will wash the layer of ash and expose to vision good black coal that will burn easily when put back into the fire. Among such coals the Rhode Island graphitic coal shows this behavior to a very marked extent. For a period of half an hour to one hour after a fresh charge has been added to the fire, the coal will burn freely with intense heat. After this period the fire gradually becomes dull and may go out unless the layer of ash on the pieces of coal is scrubbed off by actively raking the surface of the fuel bed. After such raking the fire again becomes bright and hot. An open wood fire behaves in a similar way. After the carbon in the surface layers has burned off, the remaining layer of ash prevents contact of oxygen with the carbon under this layer of ash, and the fire becomes dull and may go out. If the fire is agitated with a poker, most of the ash is shaken off the wood, and the fire will brighten up again.

The apparent difference in the reactivity of different metallurgical cokes may be due to the distribution of ash in the coke. Before coal is charged into the coke oven, the coal is reduced to a small size and the ash is thereby distributed more uniformly through the mass of coal. The ash in some of the coal may be of such a nature that it is not easily scrubbed off the surface by the current of gases, and the combustion of such coke may be slower due to the fact that oxygen does not come in contact with the carbon because of the layer of ash on the surfaces.

Fig. 3 of the paper shows that the process of combustion becomes slower as the temperature of the furnace rises. If we assume that coal burns as fast as oxygen comes in contact with it, we have to account in some way for fewer contacts when the temperature rises. We know that the density of gases decreases as the absolute temperature rises. Therefore, the same volume of gases surrounding a coal particle will contain fewer molecules of free oxygen. However, the oxygen molecules move with higher velocity which is proportional to the square root of the temperature. If it is true that fewer contacts are made, then the effect of reduced density is not quite offset by the increased molecular speed of the oxygen molecules. The effect of temperature on the rapidity of combustion is a factor which deserves a thorough investigation. It is a factor about which there is the greatest divergence of opinion among combustion engineers.

Author's Closure

The information given by Mr. Drewry in reference to more rapid burning when air and pulverized coal are preheated is of interest. It is quite probable that the ignition process is the one most affected by preheating, since, as pointed out in the paper, the ignition temperature is most quickly reached the higher the temperature of the entering air and coal. If, as seems to be the case with some types of firing, the volume of furnace space required for ignition can be greatly reduced by slower coal and air-inlet velocities, better mixing of air with coal, and better radiation of heat into the coal stream, then this particular advantage of preheated air may be of less importance. Where ignition is slow, however, preheated air will undoubtedly be of considerable value. Mr. Drewry is correct in recommending excess air lower than 25 per cent. The boiler described in this test has been successfully operated with 10 per cent excess air. To avoid smoke, however, with rapidly swinging loads, the particular fuel we fire seems to require about 18 per cent in routine operation. Only exceptional conditions can warrant excess air over 25 per cent.

Mr. Griffith brings up the important points that extreme fineness is not always an advantage and that different fuels require different degrees of pulverization. It is becoming apparent that every combination of physical and economical conditions will call for a different mesh analysis.

A very pertinent point is that brought out by Mr. Morris to the effect that, with a given fuel and possibly without reference to the type of mill used, the percentage through one mesh bears a direct relation to the percentages through other meshes.

Mr. Morris's point answers the question of Mr. Lykken as to the preferability of plotting boiler performance against 60-mesh data rather than 200-mesh. Since the percentage through 60 mesh increases with increased percentage through 200 mesh, the validity of deductions from plotted data are independent of the mesh used as a variable. Mr. Lykken is of course quite right in taking the stand that some of the losses caused by low fineness is due to the coarser particles. The percentage through 200 mesh, however, does not come under the classification of superfines, since the latter are ordinarily thought of as 20-micron-diameter particles, whereas 200 mesh is 74 microns. As mentioned in the paper, the tests reported do not permit definite conclusions as to the value of superfines. It would be valuable information if Mr. Lykken would publish his tests where fines were increased while holding the percentage of coarse particles fixed.

Mr. Mayers's mathematical explanation of the negative temperature coefficient adds strong evidence to what laboratory work indicates. The formulas of Burke and Schumann also indicate that combustion speed is reduced by higher temperatures. Mr. Mayers apparently disagrees with Dr. Fieldner in regard to the importance of the reactivity of the coke particle as affecting the combustion time. This is one of the facts which further investigation may definitely determine.

Mr. Sherman is quite correct in pointing out that the terminal velocity of a particle determines the requisite vertical velocity and not horizontal velocity. The text has been so changed. It is quite possible, as he states, that the methods used in research laboratories do not give absolute values comparable with actual practice. The author has made some recent tests of velocity and burning times of pulverized coal during regular plant operation.[21]

It has been found that (allowing for the difficulties of measuring such times in a commercial furnace) the actual burning time is on the order of a half second when air and coal are thoroughly mixed before combustion. This supports the relative accuracy of the work of Audibert and others. The explanation is undoubtedly that suggested by Mr. Sherman, that the apparent burning time as calculated from furnace volume is not a true one, since the whole furnace is not occupied by combustion; in fact, as little as one-eighth may be actively required for combustion.

Mr. Kreisinger gives some very interesting material somewhat in parallel with the studies covered by the paper. It may be pointed out that the reactivity measurements used by Godbert were not determined in a furnace as discussed by Mr. Kreisinger but in a laboratory.

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[21] Third International Conference on Bituminous Coal, Nov. 16, 1931, Carnegie Institute of Technology, Pittsburgh, Pa.

FSP-54-8

Heat Transfer From Condensing Steam to Flowing Water

Accomplished Through Single Metal Tubes Which Are Completely Surrounded by Air-Free Steam

By A. T. BROWN,[1] NEWARK, N. J.

The method developed in this paper correlates in part some of the known experimental data for heat transfer through single tubes at low rates of heat flow with the experiments made by Geo. A. Orrok in 1910. This method may be used at low rates of heat transfer such as occur in condensers to establish an ideal with which to compare the actual performance. Such a comparison indicates how close the performance of the actual condenser approaches the rate of heat transfer which can be secured with a single tube completely surrounded by air-free steam. The formula for the heat transfer through the film on the water side of the tube is taken from a paper presented by Edwin R. Cox at the 1927 Annual Meeting of The American Society of Mechanical Engineers, and this formula is a development of data prepared by Chester W. Rice. The method given in this paper should prove useful in calculating changes in the rate of heat transfer caused by altering the velocity of the water in the tubes in an actual condenser with the inlet water at the ordinary temperatures, say, above 50 deg. fahr.

DURING 1928 the author was given some data on heat-transfer rates prepared by R. J. S. Pigott, now Chief Engineer of the Research Department, Gulf Production and Pipe Line Companies, and saw that a method might be developed whereby the overall coefficient of heat transfer for a single tube could be calculated. The method given herewith is the result of this study which was made with the idea of applying it in the range covered by condenser practice, and it is not intended as a general solution of the problem of heat transfer.

The conclusions given are reached from data all based upon the reports of the following experiments:

Clement and Garland, 1909 Bulletin, Engineering Experiment Station, University of Illinois.

Geo. A. Orrok, Transactions American Society of Mechanical Engineers, 1910.

Webster, Transactions English Shipbuilders, Scotland, 1913.

McAdams and Frost, Massachusetts Institute of Technology, *Journal of Industrial and Engineering Chemistry*, January and February, 1922.

Tabulation by Chester W. Rice of the results of various experiments, and reported in the *Journal of Industrial and Engineering Chemistry*, May, 1924.

Paper by Edwin R. Cox, presented at the 1927 Annual Meeting of The American Society of Mechanical Engineers. (Trans. A.S.M.E., 1927, PET-50-2.)

Morris and Whitman, *Journal of Industrial and Engineering Chemistry*, March, 1928.

D. F. Othmer, University of Michigan, *Journal of Industrial and Engineering Chemistry*, June, 1929.

Albert Eagle and R. M. Ferguson, Report of the British Electrical and Allied Industries Research Association, read as a paper before the Institution of Mechanical Engineering, Nov. 21, 1930, and reported in *Engineering*, Nov. 28, and December 19 and 26, 1930.

In all of the tests made by Clement and Garland and by McAdams and Frost, the steam surrounding the tubes through which the heat transfer was measured varied from 210 deg. fahr. to 330 deg. fahr., as did the tests made by D. F. Othmer and Webster.

The tests reported by Clement and Garland were made on soft steel tubes, while those reported by McAdams and Frost were made on brass tubes.

In the original report of Clement and Garland, 48.4 B.t.u. per sq. ft. per hour per deg. fahr. difference per foot of thickness was used as the conductivity of the metal of the steel tube, and this value closely approximates the conductivity of wrought iron.

The heat conductivity for soft steel as averaged from the values reported by several authorities is $26^1/_3$, and in the calculations given below the values for the resistances on the water side in the Clement and Garland tests have been recalculated, using the lower value of $26^1/_3$.

Morris and Whitman used a conductivity of 35 for the steel tube employed in their test apparatus, but the author has made no change in the data calculated from these tests.

In the ordinary logarithmic form of the formula for heat transfer from a hot fluid of constant temperature to a colder fluid of varying temperature, it is assumed that the coefficient of heat transfer (the B.t.u. per sq. ft. per hour per deg. fahr. mean temperature difference between the hot fluid and the average temperature of the cooling fluid) is constant for each elementary surface of the tube throughout the length of the tube, and this assumption appears now not to be exactly true.

If one attempts to develop a differential equation by the application of the film theory similar to the classic equation from which the logarithmic form of the mean temperature difference is deduced, unwieldy equations seem to develop. Consequently, all of the data given below have been calculated on the basis of the arithmetical mean temperature differences, which are much easier to use.

The following symbols have been used throughout the calculations and formulas given below:

Let

Q = the total heat transferred per hour (B.t.u.) through the surface S in square feet

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[1] Mechanical Engineer, United Engineers and Constructors, Inc. Mem. A.S.M.E. Mr. Brown was born in Fairfield, Conn., on October 13, 1880. He attended Fairfield Academy, Park Avenue Institute, the University School, and the Sheffield Scientific School of Yale University, where he received the degree of Ph.B. in 1902.

Contributed by the Power Division and presented at the Semi-Annual Meeting, Birmingham, Ala., April 20 to 23, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

S = the area of the outer or vapor side of the tube in square feet
d_o = the outside diameter expressed in inches
d_m = the mean diameter of the tube expressed in inches
d_i = inside diameter of the tube expressed in inches
Δt_v = temperature drop from the vapor to the metal of the tube in deg. fahr.
Δt_t = temperature drop through the metal of the tube in deg. fahr.
Δt_w = temperature drop from the inside of the tube to the average temperature of the water in deg. fahr.
T = temperature of the steam outside the tube in deg. fahr.
M.T.D. = ΔT = total overall temperature drop from the vapor to the average temperature of the water = $(\Delta t_v + \Delta t_t + \Delta t_w)$
h_v = total heat transmitted per square foot per hour per deg. fahr. difference between vapor and tube per hour
h_t = same for the metal of the tube
h_w = same for the water film on the water side of the tube
k = conductivity of metal of tube in B.t.u. per square foot per hour per deg. fahr. difference per foot thickness = 63 for Muntz and Admiralty metal tubes = 26.5 average for soft steel
b = thickness of tube in inches
U = coefficient of heat transfer in B.t.u. per square foot per hour per degree of temperature difference overall vapor to water based upon the arithmetical mean-temperature difference

Then

$$Q = SU\Delta T = SU(\Delta t_v + \Delta t_t + \Delta t_w)$$
$$= h_v S\Delta t_v = \frac{12k}{b}\frac{Sd_m}{d_o}\,\Delta t_t$$
$$= \frac{h_w S d_i}{d_o}\,\Delta t_w$$

Let

$\frac{1}{U} = R$ = heat-resistance coefficient overall vapor to the average temperature of the water

$$\frac{1}{U} = \frac{S\Delta T}{Q} = \frac{S}{Q}\,(\Delta t_v + \Delta t_t + \Delta t_w)$$
$$\Delta t_v = \frac{Q}{h_v S}$$
$$\Delta t_t = \frac{Qbd_o}{S\;12k\;d_m}$$
$$\Delta t_w = \frac{Qd_o}{Sh_w d_i}$$
$$\frac{1}{U} = \frac{S}{Q} \times \frac{Q}{S}\left(\frac{1}{h_v} + \frac{bd_o}{12kd_m} + \frac{d_o}{h_w d_i}\right)$$

If we let $r_{v\text{O.D.}}$, $r_{t\text{O.D.}}$, and $r_{w\text{O.D.}}$ represent, respectively, the resistances in the parentheses, we have

$$\frac{1}{U} = (r_{v\text{O.D.}} + r_{t\text{O.D.}} + r_{w\text{O.D.}})\ldots\ldots\ldots\ldots\ldots[1]$$

This is over the whole surface involved.

Note: The resistances must all be expressed on the basis of the outside diameter of the tube.

From the data taken from the various tests and plotted as in Figs. 1 and 2, $r_{v\text{O.D.}}$ varies approximately as some fractional power of Q/S. The actual values can be read from the chart, Fig. 2, which gives the average of the Webster tests and those plotted in Fig. 1.

Then

$$\frac{1}{U} = r_{v\text{O.D.}}\text{ (average of all points from Fig. 2)}$$
$$+ \frac{bd_o}{12kd_m} + \frac{d_o}{d_i h_w}\ldots\ldots[2]$$

It is obvious that there is considerable variation in the data as reported by the various experimenters, but as the values, read from the average line in Fig. 2, together with data (given below) on the resistance on the water side of the tube, check quite closely at low total rates of heat transfer (below 30,000 B.t.u. per sq. ft. per hour) with the tests on overall M.T.D. made by Geo. A. Orrok, the values of $r_{v\text{O.D.}}$ shown in Fig. 2 have been used throughout this paper.

Let

J_w = fluidity (reciprocal of viscosity expressed in centipoises) of the water film at its average temperature $\left(\frac{t_i + t_o + \Delta t_w}{2}\right)$ on the water side of the tube, where
t_i = temperature of inlet water, deg. fahr.
t_o = temperature of outlet water, deg. fahr.

From data in a paper presented by Edwin R. Cox before the A.S.M.E. Annual Meeting, December, 1927. and which was a development of the data presented by Chester W. Rice for the ordinary temperatures involved in condenser work,

$$h_w = \frac{C^{2/3}\,V^{5/6}\,J_w^{1/2}\,G^{5/6}\,C_P^{1/3} \times 10}{K_2 d_i^{1/6}}\ldots\ldots\ldots[3]$$

where C = conductivity of the fluid film in B.t.u. per sq. ft. per hour per deg. fahr. difference per foot thickness = 0.329 for water. (See Table 2, p. 16, Jan., 1922, *Journal of Industrial and Engineering Chemistry*, by W. H. McAdams and T. H. Frost.)

G = specific gravity of film compared with water (assumed for this paper as based upon water at 62 deg. fahr.)
C_P = specific heat of film at constant pressure
K_2 = an experimental constant
V = velocity in feet per second of cooling water over the cross-section of the tube.

or

$$r_{w\text{O.D.}} = \frac{d_o}{h_w d_i} = \frac{d_o K_2}{10 \times C^{2/3}V^{5/6}J_w^{1/2}d_i^{5/6}G^{5/6}C_P^{1/3}}\ldots\ldots[4]$$

Then

$$K_2 = \frac{10r_{w\text{O.D.}}(d_i VG)^{5/6}J_w^{1/2}C^{2/3}C_P^{1/3}}{d_o}\ldots\ldots\ldots\ldots[5]$$

In condenser practice both G and C_P are practically equal to unity, and, consequently,

$$r_{w\text{O.D.}} = \frac{d_o K_2}{10C^{2/3}(d_i V)^{5/6}J_w^{1/2}}\ldots\ldots\ldots\ldots\ldots[6]$$

In order to determine the values of J_w the data plotted in Fig. 4 have been taken, and these values have been used throughout this paper.

For average film temperatures from 35 to 70 deg. fahr., it can be deduced from this chart that (as the curve is practically a straight line between these temperatures)

$$\log\frac{J_w}{0.585} = \log\left(\frac{\text{avg. film temp.}}{35}\right)^{0.81}$$

$$J_w = 0.585\left(\frac{\text{avg. film temp.}}{35}\right)^{0.81}$$

$$\Delta t_w = \frac{Q}{S} r_{w\text{O.D.}} = \frac{Q}{S}\frac{d_oK_2}{3.645(d_iV)^{5/6}\left(\frac{2t+\Delta t_w}{70}\right)^{0.405}}$$

$$\Delta t_w\left(\frac{2t+\Delta t_w}{70}\right)^{0.406} = \frac{Q}{S}\frac{d_oK_2}{3.645(d_iV)^{5/6}} \cdots\cdots\cdots [7]$$

Where t = avg. temp. of cooling water $= \frac{t_i + t_o}{2}$.

$$\Delta t_w(2t + \Delta t_w)^{1/2} = \frac{Q}{S}\frac{d_oK_2}{0.407(d_iV)^{5/6}} \cdots\cdots\cdots [8]$$

Where t = avg. temp. of cooling water $= \frac{t_i + t_o}{2}$.

In the paper by Edwin R. Cox mentioned above, the experimental constant is given as 1/53.5, or 0.0187 in Equation [2] of the paper. The chart of Fig. 11 seems to indicate a value of 0.0175 for a high value of L/d_i for low rates of heat flow.

In the Orrok series of tests made in 1910 on the overall heat transfer through Admiralty metal tubes, the steam was at considerably lower temperatures than in the tests by Clement and Garland, McAdams and Frost, D. F. Othmer, and Webster.

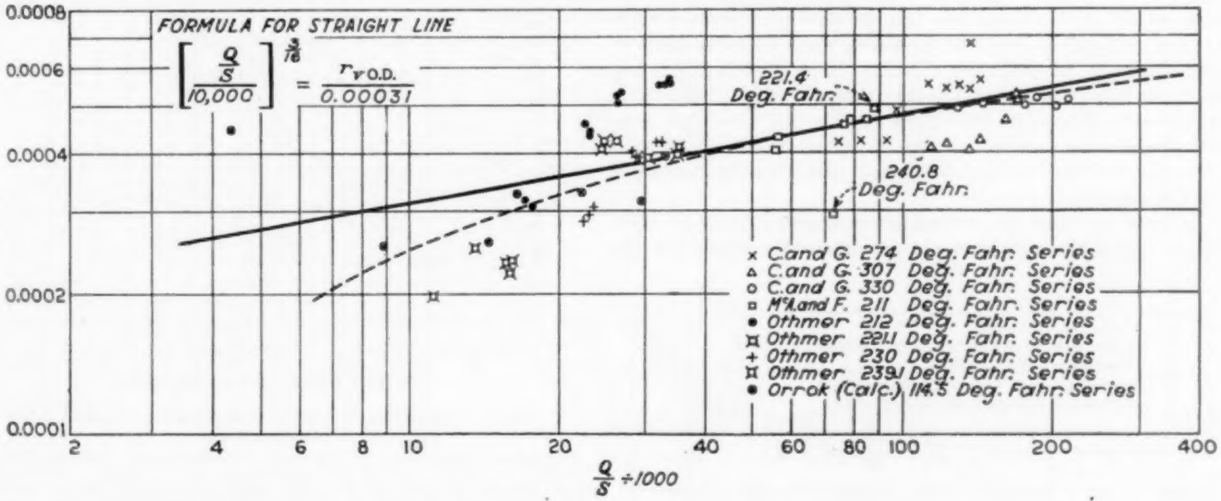

FIG. 1 VARIATION OF $r_{v\text{O.D.}}$ WITH Q/S—VARIOUS TESTS

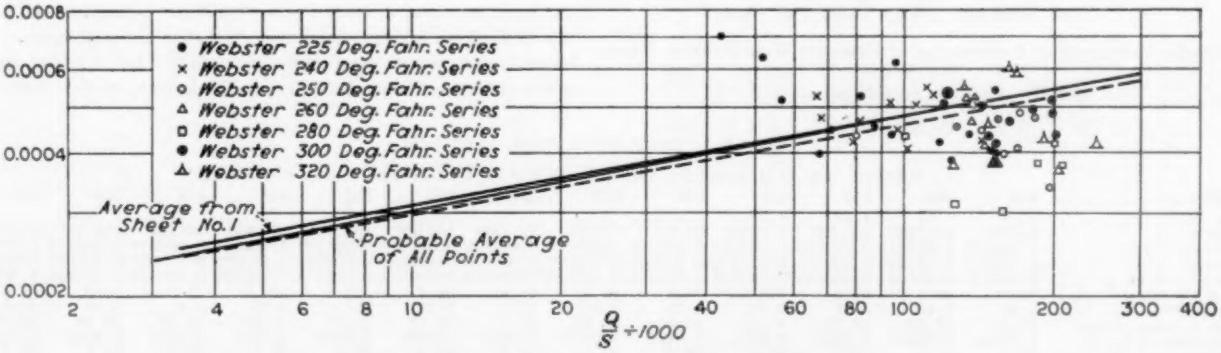

FIG. 2 VARIATION OF $r_{v\text{O.D.}}$ WITH Q/S—WEBSTER TESTS

Also for average film temperatures of 70 to 130 deg. fahr., the chart shows that,

$$\log\frac{J_w}{1.020} = \log\left(\frac{\text{avg. film temp.}}{70}\right)$$

$$J_w = \frac{1.020 \text{ avg. film temp.}}{70}$$

$$\Delta t_w = \frac{Q}{S} r_{w\text{O.D.}} = \frac{Q}{S}\frac{d_oK_2 \quad 11.71}{4.766\ (d_iV)^{5/6}(2t + \Delta t_w)^{1/2}}$$

$$= \frac{Q}{S}\frac{d_oK_2}{0.407(d_iV)^{5/6}(2t + \Delta t_w)^{1/2}}$$

However, as the average resistance of the film on the steam side of the tube seems to be indicated by the dotted line on Fig. 2, we have used these data to calculate the value of the temperature drop Δt_v from the steam to the metal of the tube and from this have obtained values of Δt_w from the observed overall mean-temperature differences reported by Orrok for Admiralty metal tubes.

We have assumed in Table 1 that the exponent of V is $^5/_6$, and have calculated the values of K_2 from Equations [7] and [8], reading Δt_w from the charts, and it is evident that there is nothing consistent in the variation of K_2, assuming that the exponent of V is $^5/_6$ throughout the series of tests, and the only way to secure consistency seems to be the assumption that K_2 is a constant throughout (taking the value as 0.015 determined from the low rate of heat transfer) and then finding the values of the exponent of V in Equations [7] and [8] which will satisfy the Orrok test data. This has been done in Table 2, and the results are plotted on Fig. 3, which would seem to indicate that the exponent of the velocity varies in some way with the rate of heat transfer.

Taking, then, the values of n from the average line on Fig. 3, and reading the values of $r_{wO.D.}$ from the average line on Fig. 2, we are giving in Table 3 a comparison between the actual mean temperature differences as reported by Orrok and the mean temperature differences calculated by means of the method developed in this paper for these tests, which have been picked at random from those reported in the A.S.M.E. Transactions for 1910 by Geo. A. Orrok.

There is a fair agreement even up to mean temperature differences as high as 148 deg. fahr., but the agreement is not always as close as might be desired. However, when we come down to near the range of condenser practice, below 25,000 B.t.u. per sq. ft. per hour, we find that the deviations of the calculated mean temperature differences from those reported by Orrok are generally only by fractions of a degree fahrenheit, and this is in spite of the fact that the values of Δt_w were all read from the charts and are not calculated.

In the worst cases the method seems to give at the very least 85 per cent of the mean temperature differences reported in the Orrok tests, even at rates as high as 216,000 B.t.u. per sq. ft. per hour.

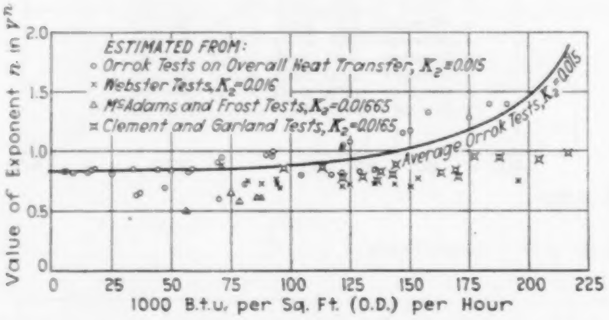

FIG. 3 VARIATION OF EXPONENT n OF VELOCITY WITH RATE OF HEAT TRANSFER
(V = velocity in tube, ft. per sec.)

This seems to establish the fact that for low rates of heat transfer, such as occur in condenser practice, the resistance to the flow of heat on the water side of the tube varies inversely as $V^{5/6}$, or we can say that the value of K_2 assumed (0.015), in combination with Equations [7] and [8] and the resistance of the film on the vapor side and the metal of the tube, can be used to calculate the M.T.D. for a single condenser tube, the same as used in the Orrok tests, when completely surrounded by air-free steam where the rate of heat absorption Q/S is known, or vice versa.

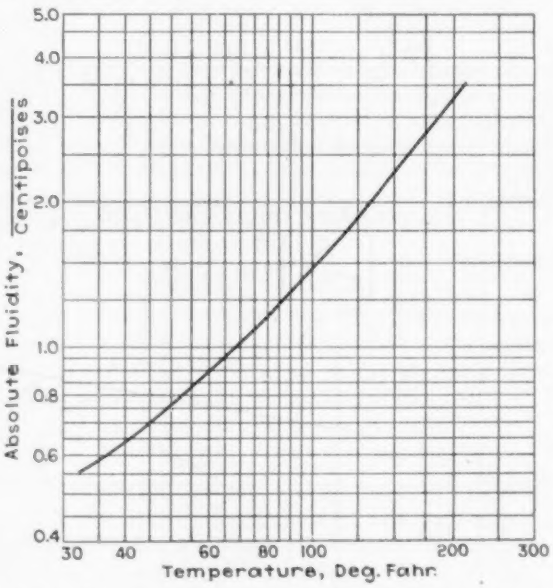

FIG. 4 VARIATION OF ABSOLUTE FLUIDITY OF WATER WITH TEMPERATURE

[Absolute fluidity = 1 ÷ Absolute viscosity. Unit viscosity (metric system) = 1 poise = force of 1 dyne per sq. cm. producing a velocity of 1 cm. per sec. at a distance of 1 cm. Absolute viscosity (metric system), dyne, cm., sec.) = 14.88 × absolute viscosity (English system, poundal, ft., sec.). Data from "Hydraulics of Pipe Lines," by W. F. Durand, p. 262.]

TABLE 1 TEST DATA, GEO. A. ORROK

(b = 0.049 in. 1-in. O.D. Admiralty Tubes, No. 18 B.W.G., I.D. 0.902 in. $r_{tO.D.}$ = 0.000068)

Test no.	348	349	350	351	352	353	354	355	356	357	358	359	360
V	8.59	8.51	8.59	8.59	......	......	......	......	......	......	......	......	......
Q/S	92,700	123,600	158,100	191,200	69,700	92,000	121,500	150,000	183,750	174,600	147,000	124,250	89,250
$r_{wO.D.}$	0.000470	0.000495	0.000520	0.000540	0.000440	0.000470	0.000495	0.000510	0.000530	0.000528	0.000510	0.000495	0.000465
$r_{tO.D.}$	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068
$r_{wO.D.} + r_{tO.D.}$	0.000538	0.000563	0.000588	0.000608	0.000508	0.000538	0.000563	0.000578	0.000598	0.000596	0.000578	0.000563	0.000533
$\Delta t_v + \Delta t_t$	49.8	69.6	93.0	116.2	35.4	49.5	68.4	86.7	109.9	104.0	85.0	70.0	47.6
$(d_iV)^{5/6}$	5.49	5.42	5.49	5.49	5.35	5.4	5.35	5.35	5.4	5.35	5.4	5.46	5.46
Avg. temp. water	45.92	47.03	46.83	49.22	43.55	44.61	45.16	49.05	55.73	55.49	51.58	49.08	46.70
Estimated K_2	0.01046	0.00944	0.0064	0.0044	0.0127	0.0108	0.0098	0.0075	0.0046	0.0058	0.0071	0.0089	0.0112
Estimated Δt_w	38.1	44.9	38.9	32.5	36.3	39.6	46.5	43.0	31.9	38.0	42.0	41.7	39.4
Actual M.T.D.	87.9	114.5	131.9	148.7	71.7	89.1	114.9	129.7	141.8	142.0	127.0	111.7	87.0

Test no.	361	362	600	601	602	605	606	607	608	609	610	620	623
Q/S	70,400	72,100	47,000	34,670	36,050	135,000	129,100	123,000	117,250	104,000	80,250	69,500	57,250
$r_{wO.D.}$	0.000450	0.000470	0.000410	0.000390	0.000375	0.000500	0.000500	0.000495	0.000490	0.000480	0.000455	0.000440	0.000430
$r_{tO.D.}$	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068
$r_{wO.D.} + r_{tO.D.}$	0.000518	0.000538	0.000478	0.000458	0.000443	0.000568	0.000568	0.000563	0.000558	0.000548	0.000523	0.000508	0.000498
$\Delta t_v + \Delta t_t$	36.50	49.6	22.48	15.89	15.96	76.7	73.4	69.25	65.4	57.0	42	35.3	28.5
$(d_iV)^{5/6}$	5.46	5.55	3.4	2.29	......	6.7	6.1	5.45	5.1	4.0	2.8	2.53	5.68
Avg. temp. water	46.57	47.82	54.28	56.83	......	53.27	53.33	53.95	54.32	55.79	57.98	58.38	50.17
Estimated K_2	0.0117	0.0107	0.0183	0.0184	......	0.0142	0.0149	0.015	0.0159	0.0157	0.0173	0.01955	0.01514
Estimated Δt_w	33.2	37.7	49.62	53.71	51.94	57.0	60.1	63.65	68.4	74.0	86.7	92.9	32.4
Actual M.T.D.	69.7	87.3	72.1	69.6	67.9	133.7	133.5	132.9	133.8	131.0	128.7	128.2	60.9

Test no.	643	644	655	656	657	658	659	660	661	662	663	664
Q/S	17,640	8,775	4,430	4,550	16,460	58,400	50,150	44,100	33,400	25,000	15,240	5,050
$r_{wO.D.}$	0.000340	0.000295	0.000257	0.000258	0.000332	0.000430	0.000415	0.000405	0.000385	0.000360	0.000330	0.000264
$r_{tO.D.}$	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068	0.000068
$r_{wO.D.} + r_{tO.D.}$	0.000408	0.000363	0.000325	0.000326	0.000400	0.000498	0.000483	0.000473	0.000453	0.000428	0.000398	0.000332
$\Delta t_v + \Delta t_t$	7.2	3.2	1.44	1.48	6.59	29.1	24.22	20.85	15.15	10.7	6.08	1.68
$(d_iV)^{5/6}$	5.55	5.41	5.6	5.61	5.61	5.61	5.55	5.63	5.58	5.58	5.55	5.5
Avg. temp. water	100.4	110.56	112.13	111.39	101.26	53.08	63.03	72.29	83.27	92.04	102.22	111.36
Estimated K_2	0.0154	0.01355	0.01544	0.0166	0.01455	0.0146	0.0146	0.01455	0.0152	0.0157	0.0152	0.0142
Estimated Δt_w	8.4	3.6	1.86	2.12	7.21	31.6	26.28	21.65	16.65	12.3	7.02	2.22
Actual M.T.D.	15.6	6.8	3.34	3.60	13.8	60.7	50.5	42.5	31.8	23.0	13.1	3.90

This also seems to be confirmed by recent experiments of the British Electrical and Allied Industries Research Association, which has published some data on the transfer of heat from the tube wall to the water in a condenser tube. These tests are reported in the Nov. 28, 1930, issue of *Engineering*. Three tests, all below 20,000 B.t.u. per sq. ft. per hour, are reported, and in later issues some quite complicated mathematical tables and formulas are published, which the author has not had sufficient time to study.

In these tests (Table 4), where the tube was heated electrically, the length and inside diameter and the calculated value of the experimental constant K_2 were very close to the values in the Soenneken tests reported by Chester W. Rice (see Fig. 11).

The values of K_2 plotted in Fig. 11 from the test data in Tables 5, 6, and 7, on the basis that $^5/_6$ is the exponent of the velocity, are all from tests below 50,000 B.t.u. per sq. ft. per hour, and these points seem to indicate that the value of K_2 increases slightly with L/d_i. The full line in Fig. 11 is assumed from the points as shown, omitting from consideration the single point from a single tube taken in an actual condenser, as in these tests (Table 8) on a single tube in the condenser a separate water supply was applied to each tube under test, the Δt_w was calculated from the observed overall mean temperature differences, and as these tubes were not in the front row at the top of the condenser,

TABLE 2 HEAT-TRANSFER TESTS. INVESTIGATION TO DETERMINE IF n, THE EXPONENT OF V, VARIES. DATA TAKEN FROM TESTS BY GEO. A. ORROK ON OVERALL HEAT TRANSFER

(Δt_w = (observed overall M.T.D. — Δt_v — Δt_i). K_2 calculated from tests 656, 644, 655, and 664)

Test no.	351	607	610	657	656	348	349	350	352	353	354	355	356
Q/S	191,200	123,000	80,250	16,460	4,550	92,700	123,600	158,100	69,700	92,000	121,500	150,000	183,750
K_2 (assumed)	0.0150	0.0150	0.0150	0.0150	0.01563	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150
Δt_w (estimated)	32.5	63.65	86.7	7.21	2.12	38.1	44.9	38.9	36.3	39.6	46.5	43.0	31.9
V	8.59	8.52	3.79	8.82	8.82	8.59	8.51	8.51	8.36	8.51	8.36	8.36	8.44
Avg. temp. water	49.22	53.95	57.98	101.26	111.39	45.92	47.03	46.83	43.55	44.61	45.16	49.05	55.73
$V^n = \frac{Q/S \times K_2}{0.3735\,\Delta t_w(2t + \Delta t_w)^{1/2}}$ for average film temperatures of 70 to 130 deg. fahr.:													
$\Delta t_w(2t + \Delta t_w)^{1/2}$ (chart)	.....	835	1230	104	31	.....	.....	.....	.....	.....	.....	510	380
V^n (calculated)	.....	5.91	2.62	6.35	$(V^{5/6})$ (6.14)	.....	.....	.....	.....	.....	.....	11.81	19.49
Log V^n	.....	0.77159	0.41830	0.80277	.....	.....	.....	.....	.....	.....	.....	1.07225	1.28870
Log V	.....	0.93044	0.57864	0.94547	.....	.....	.....	.....	.....	.....	.....	0.92221	0.92634
$n = \frac{\log V^n}{\log V}$	.....	0.821	0.724	0.849	0.833	.....	.....	.....	.....	.....	.....	1.163	1.39
$V^n = \frac{Q/S \times K_2}{3.345\,\Delta t_w\left(\frac{2t + \Delta t_w}{70}\right)^{0.406}}$ for average film temperature at 35 to 70 deg. fahr.:													
$\Delta t_w\left(\frac{2t+\Delta t_w}{70}\right)^{0.406}$ (chart)	42	.....	.....	.....	.....	48.7	58.7	50	45.5	50.5	60.5	.....	.....
V^n (calculated)	20.41	.....	.....	.....	.....	8.54	9.45	14.18	6.87	8.17	9.01	.....	.....
Log V^n	1.30984	.....	.....	.....	.....	0.93146	0.97543	1.15168	0.83696	0.91222	0.95472	.....	.....
Log V	0.93399	.....	.....	.....	.....	0.93399	0.92993	0.92993	0.92221	0.92993	0.92221	.....	.....
$n = \frac{\log V^n}{\log V}$	1.401	.....	.....	.....	.....	0.997	1.049	1.239	0.909	0.981	1.034	.....	.....

Test no.	357	358	359	360	361	362	600	601	602	605	606	620	608
Q/S	174,600	147,000	124,250	89,250	70,400	92,100	47,000	34,670	36,050	135,000	129,100	69,500	117250
K_2 assumed	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150
Δt_w (estimated)	38	42	41.7	39.4	33.2	37.7	49.62	53.71	51.94	57	60.1	92.9	68.4
V	8.36	8.44	8.55	8.55	8.55	8.70	4.84	3.00	3.34	10.95	9.74	3.38	7.82
Avg. temp. water	55.49	51.58	49.08	46.70	46.57	47.82	54.28	56.83	57.17	53.27	53.33	58.38	54.32
$V^n = \frac{Q/S \times K_2}{0.3735\,\Delta t_w(2t + \Delta t_w)^{1/2}}$ for average film temperatures of 70 to 130 deg. fahr.:													
$\Delta t_w(2t + \Delta t_w)^{1/2}$ (chart)	464	505	.....	.....	.....	.....	627	697	668	730	778	1345	910
V^n (calculated)	15.11	11.69	.....	.....	.....	.....	3.009	1.998	2.169	7.43	6.67	2.073	5.18
Log V^n	1.17926	1.06781	.....	.....	.....	.....	0.47842	0.30060	0.33626	0.87099	0.82413	0.31660	0.71433
Log V	0.92221	0.92634	.....	.....	.....	.....	0.68485	0.47712	0.52375	1.03941	0.9885	0.52892	0.89321
$n = \frac{\log V^n}{\log V}$	1.278	1.153	.....	.....	.....	.....	0.699	0.63	0.642	0.838	0.833	0.598	0.8
$V^n = \frac{Q/S \times K_2}{3.345\,\Delta t_w\left(\frac{2t + \Delta t_w}{70}\right)^{0.406}}$ for average film temperatures of 35 to 70 deg. fahr.:													
$\Delta t_w\left(\frac{2t+\Delta t_w}{70}\right)^{0.406}$ (chart)	...	.....	55.4	50.8	42	48.9	.....	.....	.....	.....	.....	.....	.....
V^n (calculated)	.....	.....	10.06	7.88	7.52	8.44	.....	.....	.....	.....	.....	.....	.....
Log V^n	.....	.....	1.00260	0.89653	0.87622	0.92684	.....	.....	.....	.....	.....	.....	.....
Log V	.....	.....	0.93197	0.93197	0.93197	0.93952	.....	.....	.....	.....	.....	.....	.....
$n = \frac{\log V^n}{\log V}$	.....	.....	1.075	0.963	0.94	0.988	.....	.....	.....	.....	.....	.....	.....

Test no.	609	623	366	643	644	655	658	659	660	661	662	663	664
Q/S	104,000	57,250	216,000	17,640	8,775	4,430	58,400	50,150	44,100	33,400	25,000	15,240	5,050
K_2 assumed	0.0150	0.0150	0.0150	0.0150	0.01346	0.01445	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.01471
Δt_w (estimated)	74	32.4	15.8	8.4	3.6	1.86	31.6	26.28	21.65	16.65	12.3	7.02	2.22
V	5.89	8.91	8.7	8.72	8.52	8.79	8.82	8.72	8.86	8.75	8.75	8.72	8.63
Avg. temp. water	55.79	50.17	54.32	100.4	110.56	112.13	53.08	63.03	72.29	83.27	92.04	102.22	111.36
$V^n = \frac{Q/S \times K_2}{0.3735\,\Delta t_w(2t + \Delta t_w)^{1/2}}$ for avg. film temperature of 70 to 130 deg. fahr.:													
$\Delta t_w(2t + \Delta t_w)^{1/2}$ (chart)	1005	.....	.....	110	53 $(V^{5/6})$	28 $(V^{5/6})$	.....	325	279	222	171	100	33 $(V^{5/6})$
V^n (calculated)	4.155	.....	.....	6.44	(5.97)	(6.12)	.....	6.195	6.35	6.042	5.87	6.12	(6.03)
Log V^n	0.61857	.....	.....	0.80889	.....	.....	.....	0.79204	0.80277	0.78118	0.76864	0.78675	.....
Log V	0.77016	.....	.....	0.94052	.....	.....	.....	0.94052	0.94743	0.94201	0.94201	0.94052	.....
$n = \frac{\log V^n}{\log V}$	0.803	.....	.....	0.859	0.833	0.833	.....	0.842	0.848	0.829	0.816	0.836	0.833
$V^n = \frac{Q/S \times K_2}{3.345\,\Delta t_w\left(\frac{2t + \Delta t_w}{70}\right)^{0.406}}$ for average film temperatures of 35 to 70 deg. fahr.:													
$\Delta t_w\left(\frac{2t+\Delta t_w}{70}\right)^{0.406}$ (chart)	...	42	19.9	.....	.....	.....	41.5	.....	.....	.....	.....	.....	.....
V^n (calculated)	.....	6.115	48.65	.....	.....	.....	6.31	.....	.....	.....	.....	.....	.....
Log V^n	.....	0.78640	1.68708	.....	.....	.....	0.80003	.....	.....	.....	.....	.....	.....
Log V	.....	0.94988	0.93952	.....	.....	.....	0.94547	.....	.....	.....	.....	.....	.....
$n = \frac{\log V^n}{\log V}$	.....	0.829	1.796	.....	.....	.....	0.846	.....	.....	.....	.....	.....	.....

TABLE 3 COMPARISON BETWEEN THE M.T.D. OBSERVED BY GEO. A. ORROK AND REPORTED IN 1910 TRANSACTIONS A.S.M.E. AND THE M.T.D. CALCULATED ON THE BASIS OF THIS PAPER

(Tubes 1 in. O.D. × No. 18 B.W.G., or 0.902 in. I.D.)

Test no.	337	338	341	342	347	351	355	359	366	374	376	384	548	712
Q/S	119,700	68,900	136,200	146,000	71,100	191,100	149,900	124,250	216,000	91,500	123,700	64,850	72,200	8,730
$r_{wO.D.}$ (average from chart) × 1000	0.472	0.426	0.485	0.490	0.430	0.518	0.495	0.475	0.525	0.450	0.485	0.421	0.430	0.290
$r_{tO.D.}$ (average from chart) × 1000	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068
$(r_{wO.D.} + r_{tO.D.}) \times 1000$	0.540	0.494	0.553	0.558	0.498	0.586	0.563	0.543	0.593	0.518	0.553	0.489	0.498	0.358
Avg. temp. water	45.07	44.47	46.47	48.25	45.83	49.22	49.05	49.08	54.32	51.35	53.33	51.35	54.08	101.53
V	8.60	8.8	8.66	8.51	8.77	8.59	8.36	8.55	8.7	8.29	8.4	9.03	11.2	3.53
K_2 (chart)	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
n (chart)	0.92	0.85	0.97	1.0	0.85	1.34	1.02	0.93	1.84	0.88	0.93	0.85	0.85	0.833
$Q/S \dfrac{K_2}{3.345 \times V^n}$ (35–70)	....	48.9	....	....	50.6	47.85	....	....	....	....	....	44.9	41.4	....
V^n (chart)	7.2	6.31	8.15	8.51	6.3	17.9	8.7	7.35	53	6.35	7.2	6.48	7.82	2.88
$Q/S \dfrac{K_2}{0.3735 \times V^n}$ (70–130)	668	....	671	689	....	....	692	679	163.7	579	690	....	....	121.6
Δt_w (chart)	54.4	38.4	54.7	56.0	39.3	36.7	55.5	54.8	14.8	47.5	54.2	34.2	31.2	8.3
$(\Delta t_v + \Delta t_t)$	64.7	34.1	75.4	81.5	35.4	112.0	84.4	67.4	128.0	47.4	68.4	31.7	36.0	3.13
Calculated M.T.D.	119.1	72.5	130.1	137.5	72.7	148.7	139.9	122.2	142.8	94.9	122.6	65.9	67.2	11.43
Actual M.T.D. reported	116.4	81.5	132.6	149.6	69.7	148.7	129.7	111.7	143.8	110.9	145.2	62.7	72.2	11.8
Difference, deg. fahr.	+2.7	—9.0	—2.5	—12.1	+3.0	....	+10.2	+10.5	—1.0	—16.0	—22.6	+3.2	—5.0	—0.37
Difference in per cent of actual M.T.D.	+2.3	—11.04	—1.9	—8.1	+4.3	....	+7.9	+9.4	—0.7	—14.4	—15.6	+5.1	—6.9	—3.1

Test no.	713	655	656	657	642	643	644	682	645	646	647	700	701	702	703
Q/S	4,140	4,430	4,545	16,460	25,700	17,640	8,780	4,465	15,240	23,750	31,650	2,987	9,560	15,175	22,340
$r_{wO.D.}$ (average from chart) × 1000	0.250	0.255	0.255	0.325	0.351	0.330	0.290	0.255	0.320	0.350	0.365	0.239	0.295	0.320	0.342
$r_{tO.D.}$ (average from chart) × 1000	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068
$(r_{wO.D.} + r_{tO.D.}) \times 1000$	0.318	0.323	0.323	0.393	0.419	0.398	0.358	0.323	0.388	0.418	0.433	0.307	0.363	0.388	0.410
Average temp. water	110.33	112.13	111.39	101.26	91.59	100.4	107.67	112.2	100.59	92.20	84.42	110.74	100.37	93.56	82.6
V	3.57	8.79	8.82	8.82	8.72	8.72	8.52	8.68	8.72	8.72	8.63	3.61	3.68	3.38	3.76
K_2 (chart)	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
n (chart)	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833
$Q/S \dfrac{K_2}{3.345 \times V^n}$ (35–70) °F	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
V^n (chart)	2.9	6.13	6.15	6.15	6.05	6.05	6.0	6.0	6.05	6.05	6.02	2.91	2.94	2.75	3.0
$Q/S \dfrac{K_2}{0.3735 \times V^n}$ (70–130) °F	57.3	29.04	29.7	107.5	170.5	117	58.7	29.9	101.1	157.7	211.1	41.2	130.5	221.7	299
Δt_w (chart)	3.8	2.0	2.0	7.4	12.20	8.10	3.90	2.0	7.00	11.30	15.5	2.7	9.0	15.6	21.6
$(\Delta t_v + \Delta t_t)$	1.32	1.43	1.47	6.47	11.77	7.04	3.14	1.44	5.93	9.94	13.7	0.92	3.47	5.89	9.16
Calculated M.T.D.	5.12	3.43	3.47	13.87	23.97	15.14	7.04	3.44	12.93	21.24	29.2	3.62	12.47	21.49	30.76
Actual M.T.D. reported	5.10	3.30	3.60	13.8	22.80	15.60	6.80	3.2	13.30	21.7	30.0	3.80	12.9	20.7	30.9
Difference, deg. fahr.	+0.02	+0.13	—0.13	+0.07	+1.17	—0.46	+0.24	+0.24	—0.37	—0.46	—0.8	—0.18	—0.43	+0.79	—0.14
Difference in per cent of actual M.T.D.	+3.9	+3.9	—3.6	+0.5	+5.1	—3.0	+3.5	+7.5	—2.8	—2.1	—2.7	—4.7	—3.3	+3.8	—0.5

TABLE 4

(British Electrical and Allied Industries Research Association, *Engineering* (London), Nov. 28, 1930)

(d_o = 0.75 in. d_i = 0.653 in. L = 75 in. L/d_i = 114.8)

Q/S O.D.	3,470	10,510	17,500
Δt_w	2.81	8.27	13.46
V	8	8	8
$(d_iV)^{5/6}$	3.96	3.96	3.96
Avg. temp. water	38.70	39.67	40.63
K_2 (base $V^{5/6}$)	0.0166	0.0167	0.0166

as air was undoubtedly present, and the condition of fouling was unknown, the calculated value of K_2 should be and is higher than for a single tube completely surrounded by air-free steam.

As a matter of interest, if we assume that the value of K_2 for the test apparatus used by the following experimenters can be taken from this curve (Fig. 11), we can calculate how the exponent n of V may vary in order to satisfy these test results:

	L/d_i	K_2 chart
Clement and Garland	80.5	0.01650
Webster	60.0	0.01600
McAdams and Frost	97.5	0.01665

By this method the agreement is not good (see Tables 9, 10, and 11 and Fig. 3) between the exponent n of V calculated from the Orrok, the Webster, and the McAdams and Frost tests.

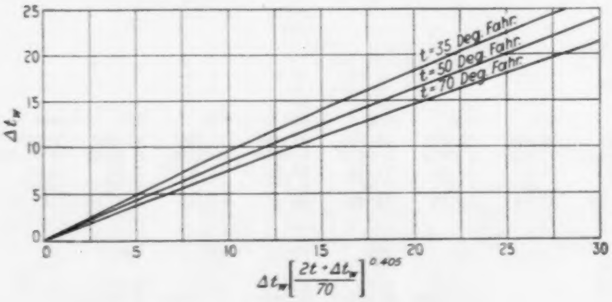

Fig. 5

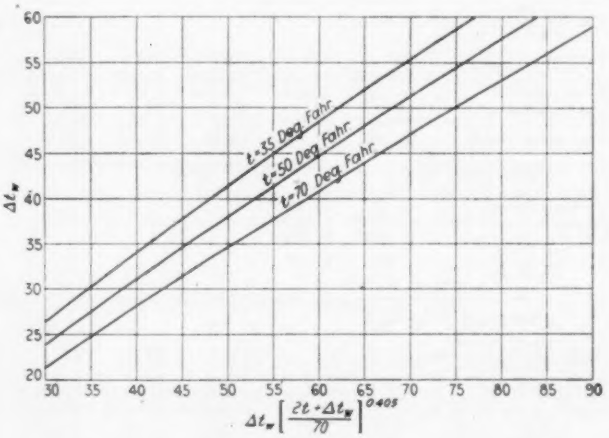

Fig. 6

FIGS. 5 AND 6 HEAT TRANSFER THROUGH WATER FILM INSIDE TUBE (Average film temperature, 35 to 70 deg. fahr.)

$$\Delta t_w\left[\frac{2t+\Delta t_w}{70}\right]^{0.405} = \frac{Q}{S} \times \frac{d_oK_2}{3.645(d_iV)^{5/6}}$$

where Δt_w = drop in deg. fahr. through water film inside tube to average temperature t of cooling water
d_i, d_o = inside and outside diameters of tube, in.
V = average velocity of cooling water over cross-section of tube, ft. per sec.
K_2 = experimental constant
Q/S = B.t.u. per hr. per sq. ft. of external surface of tube.

The Clement and Garland tests appear to be consistent with the calculated results for the Orrok tests at low heat rates, but the exponent n of V appears to rise less rapidly with increasing heat flows than is estimated from the Orrok tests. The same thing appears from the Webster tests.

In view of the considerable variation in the resistance on the

TABLE 5 SOENNEKEN TESTS, REPORTED BY C. W. RICE

(*Journal Industrial & Engineering Chemistry*, May, 1924)

Drawn brass tube 75.6 in. long, $d_i = 0.669$ in., $L/d_i = 113$

$$K_2 = \frac{r_{w\,\mathrm{I.D.}}\ 10(VG)^{5/6}\ J_w^{1/2}\ C^{2/3}\ Cp^{1/3}}{d_i^{1/6}} \qquad d_i^{1/6} = 0.935$$

(Q/S for all of these tests is below 40,000)

Test no.	$r_{w\mathrm{I.D.}}$	Calc. avg. film temp.	V, ft. per sec.	G	$(VG)^{5/6}$	$J_w^{1/2}$	$C^{2/3}$	$Cp^{1/3}$	Est. K_2
1	0.001005	61.35	4.335	1.0	3.4	0.949	0.4766	1	0.01654
2	0.000835	85.5	4.335	0.9965	3.39	1.106	0.4766	1	0.01596
3	0.000742	114.9	4.335	0.991	3.35	1.30	0.4766	1	0.01647
4	0.000635	160.6	4.335	0.978	3.31	1.58	0.4766	1	0.01693
5	0.001234	67.6	3.28	0.999	2.69	1.0	0.4766	1	0.01692
6	0.001046	90.6	3.28	0.996	2.68	1.135	0.4766	1	0.01623
7	0.000941	118.5	3.28	0.99	2.66	1.322	0.4766	1	0.01686
8	0.000888	143.5	3.28	0.984	2.64	1.482	0.4766	1	0.01772
9	0.00199	64.4	2.022	1.0	1.79	0.975	0.4766	1	0.01769
10	0.0014	125.1	2.022	0.989	1.77	1.36	0.4766	1	0.01720
11	0.00126	155.2	2.022	0.98	1.76	1.549	0.4766	1	0.01750
								Avg.	0.01691

TABLE 6 TESTS REPORTED BY C. W. RICE

(*Journal Industrial & Engineering Chemistry*, May, 1924)

Stanton (III) drawn copper tube: $d_i = 1.39$ cm.; I.D.: $L = 47$ cm.

$$\frac{L}{d_i} = \frac{47}{1.39} = 33.8;\ K_2 = \frac{r_{w\mathrm{I.D.}}\ 10\ (VG)^{5/6}\ J_w^{1/2}\ C^{2/3}\ Cp^{1/3}}{d_i^{1/6}} \qquad d_i^{1/6} = 0.9045 \text{ in.}$$

Test no.	$r_{w\mathrm{I.D.}}$	Calc. avg. film temp.	V, ft. per sec.	G	$(VG)^{5/6}$	$J_w^{1/2}$	$C^{2/3}$	$Cp^{1/3}$	Est. K_2
1	0.000672	68.2	4.04	0.999	3.20	1	0.4766	1	0.01134
2	0.000811	68.3	3.22	0.999	2.65	1	0.4766	1	0.01133
3	0.000944	69.8	2.74	0.999	2.55	1.009	0.4766	1	0.0128
4	0.001094	68.5	2.26	0.999	1.97	1	0.4766	1	0.01135
5	0.001266	69.2	1.905	0.999	1.7102	1.005	0.4766	1	0.01149
6	0.001608	69.4	1.43	0.999	1.346	1.005	0.4766	1	0.01146
7	0.002363	69.9	0.941	0.999	0.954	1.009	0.4766	1	0.012
								Avg.	0.01168

Stanton (I) copper tube: d_i 0.736 cm.; $L = 46$ cm.; $L/d_i = 62.5$; $d_i = 0.2895$ in.
$d_i^{1/6} = 0.813$ in.

1	0.0003755	71.7	9.71	0.999	6.61	1.024	0.4766	1	0.01491
2	0.000342	97.6	9.71	0.999	6.60	1.182	0.4766	1	0.01565
3	0.0003185	114.2	9.71	0.999	6.55	1.28	0.4766	1	0.01566
								Avg.	0.01541

Stanton (II) copper tube: $d_i = 1.07$ cm.; $L = 44.5$ cm.; $L/d_i = 41.6$; $d_i = 0.4215$ in.; $d_i^{1/6} = 0.866$ in.

1	0.001046	84.7	3.005	0.997	2.5	1.104	0.4766	1	0.01589
2	0.000843	134.5	3.005	0.986	2.49	1.42	0.4766	1	0.0164
								Avg.	0.01615

TABLE 7 TESTS BY MORRIS AND WHITMAN

($^1/_2$-in. std. steel pipe: $d_o = 0.84$ in.; $d_i = 0.622$ in.; $d_o/d_i = 1.35$; $L/d_i = 121.5/0.622 = 195.5$)

("Heating Water With Steam," *Journal Industrial & Engineering Chemistry*, vol. 20, no. 3, March, 1928. Estimated data on water film inside tube)

(Q/S for all of these tests is below 45,000)

Test no.	$r_{w\mathrm{O.D.}}$	Avg. film temp.	V, ft. per sec.	G	$(d_iVG)^{5/6}$	$J_w^{1/2}$	$C^{2/3}$	$Cp^{1/3}$	Est. K_2
A-1	0.00253	159.75	0.955	0.979	0.645	1.575	0.4766	1	0.0146
A-2	0.00252	159.8	0.985	0.979	0.653	1.575	0.4766	1	0.0147
A-3	0.00209	161.15	1.35	0.978	0.84	1.578	0.4766	1	0.0157
A-4	0.001605	159.1	1.875	0.979	1.11	1.565	0.4766	1	0.0157
A-5	0.001712	158.95	1.92	0.979	1.13	1.565	0.4766	1	0.0172
A-6	0.001455	158.1	2.36	0.979	1.34	1.555	0.4766	1	0.0172
A-7	0.001323	157.15	2.735	0.980	1.503	1.552	0.4766	1	0.0175
A-8	0.001184	156.45	2.78	0.98	1.553	1.548	0.4766	1	0.01615
A-9	0.001164	155.4	3.25	0.98	1.76	1.545	0.4766	1	0.01795
A-10	0.001022	153.2	3.42	0.981	1.85	1.53	0.4766	1	0.01644
A-11	0.001115	155.95	3.51	0.98	1.88	1.55	0.4766	1	0.01845
A-12	0.000866	151.75	4.02	0.982	2.11	1.52	0.4766	1	0.0158
								Avg.	0.01645

TABLE 8 TESTS ON TWO SINGLE NEW ADMIRALTY-METAL TUBES

(Both located in the top of an actual two-pass condenser)

$s_o = 1$ in.; $d_i = 0.902$ in.; $b = 0.049$; L (active length $= 17$ ft. $^1/_4$ in.;
$L/d_i = 226.5$; $r_{t\mathrm{O.D.}} = 0.000068$

Tube no.	1	1	2	2
Q/S	18900	32100	16750	29850
Observed M.T.D. overall	36.25	39.25	37	39.7
Avg. temp. water in tube = t	49.75	50.35	47.0	44.3
V	4.22	9.65	4.22	9.7
Assumed n	0.84	0.84	0.84	0.84
V^n	3.35	6.7	3.35	6.75
$r_{w\mathrm{O.D.}} \times 1000$	0.333	0.369	0.325	0.362
$r_{t\mathrm{O.D.}} \times 1000$	0.068	0.068	0.068	0.068
$(r_{w\mathrm{O.D.}} + r_{t\mathrm{O.D.}}) \times 1000$	0.401	0.437	0.393	0.430
$(\Delta t_w + \Delta t_t)$	7.6	14	6.6	12.8
Est. Δt_w	28.65	25.25	30.4	26.9
$\Delta t_w \left(\frac{2t + \Delta t_w}{70}\right)^{0.405}$ (see chart)	36.6	31.8	38	33
$K_2 = \dfrac{3.345 \times V^n \times \Delta t_w \left(\frac{2t + \Delta t_w}{70}\right)^{0.405}}{Q/S}$	0.0217	0.0222	0.0254	0.0249
				Avg. 0.0236

vapor side as shown in Figs. 1 and 2, probably no exact consistency should be expected in the values of the exponent n of velocity V for the various tests.

So far the method given above seems to present a satisfactory basis for the calculation of the heat transfer from saturated air-free steam through a single tube which is completely surrounded by the steam, at the ordinary temperatures and rates of heat absorption encountered in condenser work, and the data for higher rates of heat flow for single tubes shown in this paper are presented for the use of any one interested in following this matter further and are to be used with due caution.

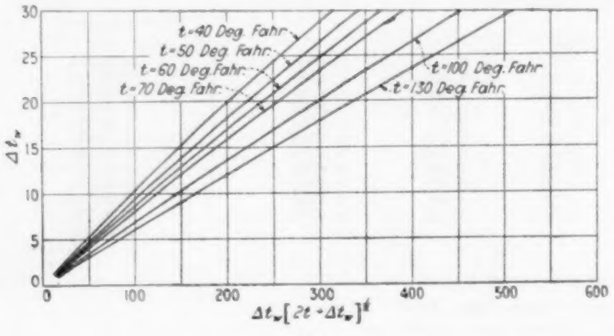

Fig. 7

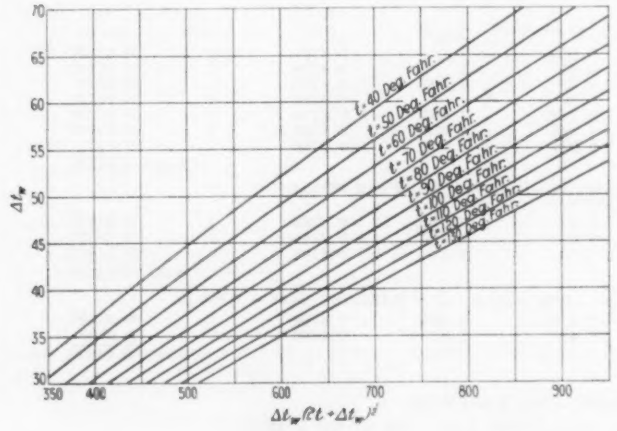

Fig. 8

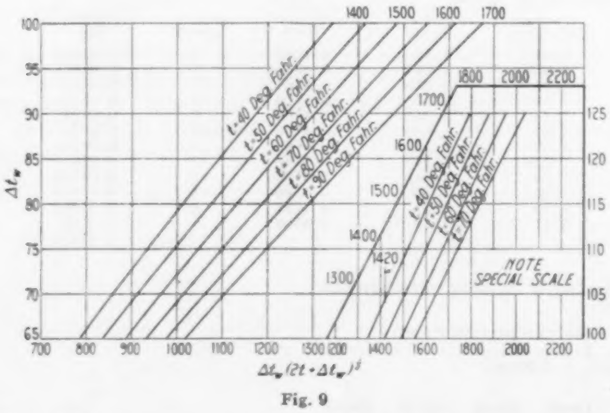

Fig. 9

Figs. 7–9 Heat Transfer Through Water Film Inside Tube
(Average film temperature, 70 to 130 deg. fahr.)

$$\Delta t_w[2t + \Delta t_w]^{1/3} = \frac{Q}{S} + \frac{d_o K_2}{0.407(d_i V)^{5/6}}$$

In a heater, where the tubes are spaced rather close together, or in a practical condenser, all of the surface does not work uniformly, and there are varying amounts of air present in the steam and of scale or dirt on the tubes. Each tube is not completely surrounded by steam, and consequently the total resistance to the flow of heat from the steam to the condensing water will be widely different from the performance of the ideal single tube. If we call $r_{sasO.D.}$, the resistance of the condenser or heater, due to scale, air, and arrangement of surface, we can write,

$$\frac{1}{U} = (r_{vO.D.} + r_{tO.D.} + r_{wO.D.}) + r_{sasO.D.} \ldots\ldots\ldots [9]$$

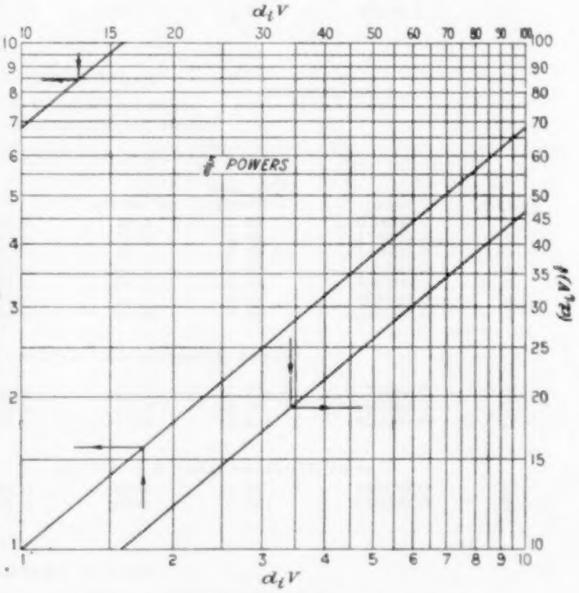

Fig. 10 Chart Giving ⁵/₆th Powers of $d_i V$

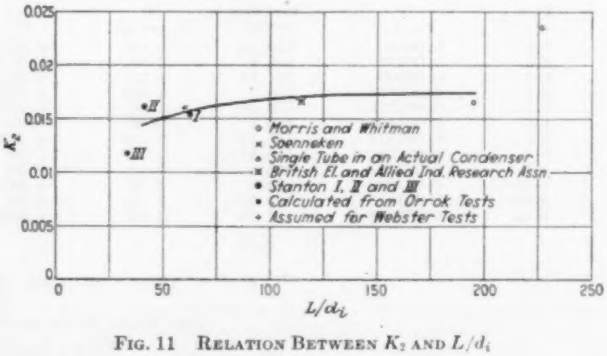

Fig. 11 Relation Between K_2 and L/d_i

The expression in the parentheses covers the value of the theoretical heat transfer of a single tube determined from the method given above, and $r_{sasO.D.}$ is the difference between the actual performance obtained and the theoretical performance which would be obtained if the condenser consisted of a single tube.

In condensers where the tubes are packed closely together, the values of $r_{sasO.D.}$ are usually of the order of 40 to 75 per cent of the total resistance $1/U$.

At present we have no method of calculating the value of $r_{sasO.D.}$ from the design of a particular condenser, and little seems to be known as to how it varies in actual condensers with changing temperatures and velocities of circulating water, but a study should be made to determine the best methods to follow in re-

TABLE 9 HEAT TRANSFER—CLEMENT AND GARLAND TESTS

(Steel tube 1.253 in. O.D. × 0.985 in. I.D.)

Test no.	B-1	B-2	B-3	B-4	B-5	B-6	B-8	C-1	C-2	C-3	C-4	D-1	D-2	D-3	D-4
Q/S	137,500	135,800	142,900	129,500	121,400	113,200	96,800	170,400	169,000	163,100	143,800	216,700	203,800	187,600	176,800
B.t.u. per sq. ft. per hr., I.D.	175,000	172,750	181,750	164,750	154,500	144,000	123,000	216,800	215,000	207,500	183,000	275,500	259,000	238,800	225,000
Δt_w est.	50.35	69.68	59.05	72.48	78.82	82.11	95.46	63.25	61.90	73.20	95.35	48.7	62.5	70.01	80.30
V	16.78	12.37	14.89	10.15	7.89	5.88	3.89	17.64	15.50	11.43	5.33	17.13	14.05	10.39	8.06
Avg. temp. water	65.95	68.22	66.80	69.42	71.93	75.49	85.89	76.3	77.4	80.4	83.2	69.1	70.8	74.6	78.0
K_2, assumed	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165	0.0165
For avg. water-film temperatures (70–130 deg. fahr.)															
$V^n = \dfrac{3.12 \times Q/S \times K_2}{\Delta t_w(2t + \Delta t_w)^{1/2}}$	10.25	6.96	8.99	6.3	5.29	4.64	3.195	9.49	9.55	7.5	4.8	16.72	11.67	9.29	7.36
$\Delta t_w(2t + \Delta t_w)^{1/2}$ (see chart)	691	1004	818	1056	1180	1255	1560	925	910	1120	1540	667	898	1040	1236
n, calculated	0.831	0.764	0.813	0.794	0.806	0.867	0.855	0.785	0.823	0.826	0.89	0.991	0.93	0.951	0.956

TABLE 10 WEBSTER TESTS

(L = 30 in.; d_o = 0.825 in.; d_i = 0.5 in.; L/d_i = 60)

Test no	58	59	61	72	76	62	81	34
Q/S O.D	82,000	88,200	153,000	96,300	135,000	94,000	150,500	71,400
Δt_w estimated	112.1	115.5	99.0	102.8	103.5	121	102.8	116.6
V	3.87	4.1	10.73	6.42	9.45	3.85	12.5	2.46
Avg. temp. water	65.5	65.0	57.4	59.2	58.3	70.1	56.8	73.4
K_2 assumed from chart	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016
$\Delta t_w(2t + \Delta t_w)^{1/2}$	1750	1810	1448	1532	1540	1955	1516	1892
For avg. film temps. (70–130 deg. fahr.)								
$V^n = \dfrac{Q/S \times K_2}{0.277\,\Delta t_w(2t + \Delta t_w)^{1/2}}$	2.71	2.815	6.11	3.63	5.06	2.775	5.73	2.18
n, calculated	0.737	0.733	0.763	0.693	0.722	0.757	0.69	0.866

Test no	35	36	37	50	61	62	63	64
Q/S O.D	94,200	121,800	152,500	124,200	153,000	94,000	144,000	196,800
Δt_w estimated	113.9	95.4	81.2	113.3	99.0	12.1	114.6	11.3
V	4.67	9.83	16.13	7.26	10.74	3.85	8.57	12.25
Avg. temp. water	64.1	57.6	54.3	59.5	57.4	70.1	60.4	58.9
K_2 assumed from chart	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016
$\Delta t_w(2t + \Delta t_w)^{1/2}$	1772	1384	1120	1725	1448	1956	1756	1720
For avg. film temps. (70–130 deg. fahr.)								
$V^n = \dfrac{Q/S \times K_2}{0.277\,\Delta t_w(2t + \Delta t_w)^{1/2}}$	3.07	5.08	7.86	4.16	6.1	2.775	4.73	6.61
n, calculated	0.728	0.711	0.742	0.719	0.762	0.758	0.724	0.754

ducing this factor to a minimum as it has a large effect upon the amount of surface required for a given service.

It would seem that the various manufacturers of condensers should undertake such a study, as only in recent years have they begun to arrange condenser tubes so that the steam can reach the condenser surfaces effectively.

In the tests reported by Orrok in 1910, it was found that the overall resistances varied approximately as the reciprocals of the square root of the velocities for a single tube completely surrounded by steam. This is the sum of the three terms in the parentheses, Equation [9]. This rate of variation has been assumed to hold for an actual condenser and has been used in the past in calculating changes in vacua, due to changes in velocity of the water in the tubes for an actual condenser. A consideration of Equation [9] shows that if the term $r_{sasO.D.}$ is as large as the sum of the terms in the parentheses, such a method would result in corrections for changes in velocity, which would be twice the actual changes that can be secured by velocity change alone. This has actually happened in the experience of the author, where the actual benefits secured by increasing the velocities were about one-half of the expected benefits.

Fig. 12 shows the results obtained by calculating from actual test data the values of $r_{sasO.D.}$ for a two-pass 50,000-sq. ft. condenser, model of 1923; and for the same inlet-water temperature and the same flow of steam there seems to be no change in the value of $r_{sasO.D.}$ with changes in velocity, and, consequently, if the value of $r_{sasO.D.}$ is known for one rate, we should be able to use this method to determine the change in the coefficient of heat transfer due to changes in the water velocity alone, provided there is no change in the amount of air present in the steam and the tubes have the same degree of cleanliness in both cases.

However, this condition does not seem to hold quite so well for a single-pass condenser (see Table 12) where $r_{sasO.D.}$ seems to vary with the velocity and temperature of the inlet water, the greatest change occurring at low temperature, and the variation in $r_{sasO.D.}$ seems much less at higher temperatures of inlet water than at low.

TABLE 11 VALUES OF n OF V^n CALCULATED FROM TESTS BY McADAMS AND FROST

(Brass tube 0.675 in. O.D. × 0.49 in. I.D. × 47.75 in. long. L/d_i = 97.5)

Test no.	M-1	M-2	M-3	M-4	M-6	M-7
Q/S O.D.	76300	78500	87800	85400	56100	55500
B.t.u. per sq. ft. per hour I.D.	105000	108000	121000	117600	77300	76500
K_2	0.01665	0.01665	0.01665	0.01665	0.01665	0.01665
Δt_w	82.1	83.3	85.0	80.2	96.0	97.1
V	5.39	6.90	7.16	7.45	3.31	3.20
Avg. temp. water	83.1	80.0	81.0	80.8	83.4	83.3
Avg. water-film temp.	124.2	121.7	123.5	120.9	131.4	131.9
$\Delta t_w(2t + \Delta t_w)^{1/2}$	1296	1308	1388	1250	1558	1576
For avg. water-film temps. (70–130 deg. fahr.)						
$V^n = \dfrac{3.005 \times (Q/S) \times K_2}{\Delta t_w(2t + \Delta t_w)^{1/2}}$	2.945	3	3.285	3.42	1.804	1.76
Log V^n	0.46909	0.47712	0.51654	0.53403	0.25624	0.24551
Log V	0.73159	0.83885	0.85491	0.87216	0.51983	0.50515
$n = \dfrac{\log V^n}{\log V}$	0.642	0.569	0.605	0.612	0.493	0.486

It is evident that further study of test data on condensers must be made to learn how $r_{sasO.D.}$ varies for different designs, if very accurate results are to be secured in calculating changes in vacua due to varying velocities and at all temperatures of inlet water.

In Table 13 a sample calculation for a two-pass condenser is shown.

Using the values of $r_{sasO.D.}$ obtained with one pump operation, it is shown in Table 13 that the overall coefficient of heat transfer in this condenser can be calculated quite accurately for the condition of two-pump operation for the same temperature of inlet water and the same quantity of steam condensed per hour.

In conclusion the data submitted in this paper seem to transfer the problem in condensers from the consideration of the behavior of a single tube to the arrangement of the surface and the

TABLE 12 42,000-SQ. FT. SINGLE-PASS CONDENSERS

(7/8-in. O.D. × No. 18 B.W.G. tubes. d_i = 0.777 in.)

	Condenser "A"						Condenser "B"					
No. circ. pumps in service	1	2	1	2	1	2	1	2	1	2	1	2
Lb. steam per hr.	135,391	136,673	356,405	356,926	294,156	298,783	274,500	273,300	320,200	318,100	355,100	353200
B.t.u. per lb.	945	947	919	922	916	915	917[a]	919[a]	916[a]	917[a]	921[a]	917[a]
Q/S	2990	3030	7500	7580	6210	6320	6000	5980	7000	6940	7780	7720
Abs. pressure, in Hg	0.355	0.335	0.663	0.553	0.557	0.485	1.373	1.22	1.52	1.298	1.584	1.298
Temp. vacuum, deg. fahr.	49.44	47.90	66.83	61.63	61.84	57.93	89.0	85.28	92.25	87.21	93.57	87.21
Temp. inlet	34.42	34.25	34.37	33.71	34.31	33.51	71.86	72.96	72.12	73.05	71.38	71.25
Temp. rise	5.15	2.98	13.49	7.86	10.98	6.53	9.76	5.63	11.72	6.57	12.90	7.41
Avg. temp. water	37	35.74	41.12	37.64	39.8	36.78	76.74	75.78	77.98	76.34	77.83	74.94
Observed M.T.D.	12.44	12.16	25.71	23.99	22.04	21.15	12.26	9.50	14.27	10.87	15.74	12.25
V	4.05	7.0	3.95	6.75	4.0	6.75	4.15	7.25	4.02	7.25	4.07	7.13
$(d_iV)^{5/6}$	2.6	4.09	2.55	3.95	2.58	3.97	2.65	4.2	2.59	4.2	2.61	4.15
K_2	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175	0.0175
$\frac{Q}{S}\frac{d_o\ K_2}{3.645(d_iV)^{5/6}}$	4.825	3.111	12.36	8.06	10.1	6.68	85.25[b]	53.5[b]	101.6[b]	62.2[b]	112[b]	70[b]
Δt_w estimated	4.6	3.0	11.2	7.6	9.2	6.3	6.6	4.3	7.9	5.0	8.7	5.6
1000 × $r_{sO.D.}$, estimated	0.238	0.239	0.281	0.285	0.273	0.275	0.270	0.270	0.278	0.278	0.281	0.281
1000 × $r_{tO.D.}$, estimated	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069
1000 × $r_{wO.D.}$, estimated	1.538	0.990	1.494	1.002	1.481	0.996	1.100	0.719	1.129	0.721	1.117	0.726
1000 × ($r_{sO.D.}$ + $r_{tO.D.}$ + $r_{wO.D.}$)	1.845	1.298	1.844	1.356	1.823	1.340	1.439	1.058	1.476	1.068	1.467	1.076
Observed 1000 × $(1/U)$	4.160	4.015	3.431	3.166	3.550	3.348	2.045	1.588	2.040	1.568	2.023	1.588
Calc. value of 1000 × $r_{sasO.D.}$	2.315	2.717	1.587	1.810	1.727	2.008	0.606	0.530	0.564	0.500	0.556	0.512

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[a] B.t.u. per lb. removed.

[b] $\frac{Q}{S}\frac{d_o K_2}{0.407(d_iV)^{5/6}}$

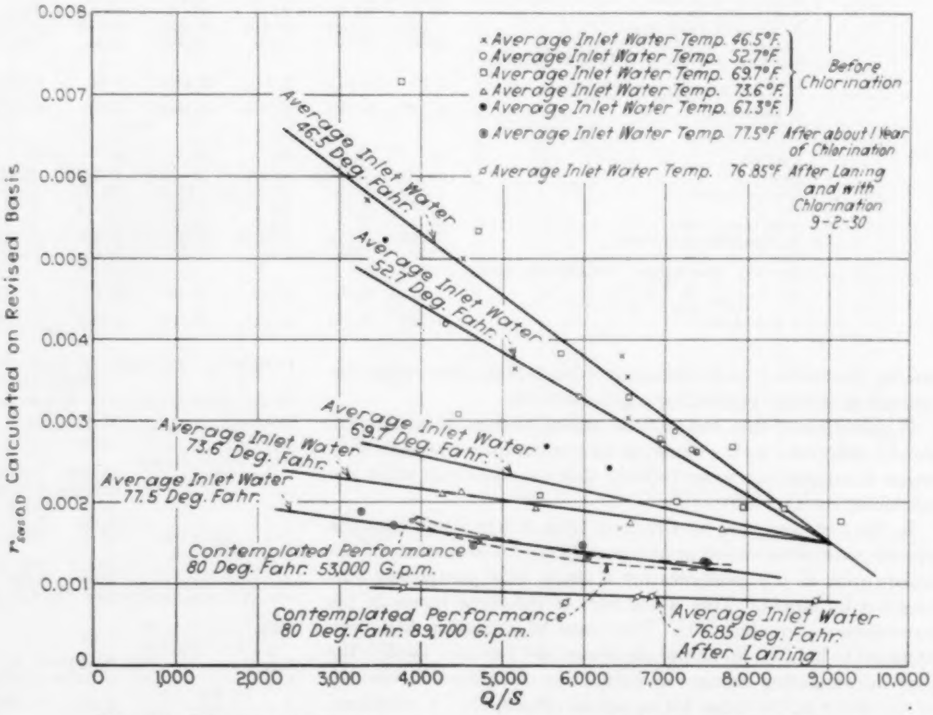

FIG. 12 RELATION BETWEEN $r_{sasO.D.}$ AND Q/S FOR AN ACTUAL CONDENSER

elimination, so far as possible, of scale, dirt, and air, and probably considerable work will be necessary before the variation of the term $r_{sasO.D.}$ is thoroughly understood.

It would seem, however, that there must be sufficient test data in existence which, if compared properly on this basis, would yield clews as to how this factor varies with different conditions, and it is to be hoped that some one will undertake the task for the benefit of the condenser industry.

If we accept the Orrok tests as the criterion in judging condenser performance, the method developed in this paper apparently can be used to calculate quite accurately the mean temperature difference for a single tube completely surrounded by air-free steam for rates of heat flow below 30,000 B.t.u. sq. ft. per hour, and with a lesser degree of accuracy for rates as high as 216,000 B.t.u. per sq. ft. per hour, but its application to an actual heater or condenser cannot be expected to give accurate mean temperature differences until more is known about the behavior of the resistance ($r_{sasO.D.}$) due to the arrangement of the tubes, etc.

TABLE 13 50,000 SQ. FT. TWO-PASS CONDENSER

(Admiralty Metal tubes 1 in. O.D. × 0.87 in. I.D.)

Run no.	H	A	F	E
Inlet water deg. fahr.	76.04	79.85	76.76	77.41
Vacuum (30 in. bar.), in. Hg	27.85	27.99	28.315	28.51
Temp. vacuum, deg. fahr.	103.5	101.25	95.6	91.5
Condensate, lb. per hr.	400310	397930	245270	244100
Avg. tube velocity, ft. per sec.	5.0	8.8	4.9	8.7
Temp. rise of circulating water	16.05	9.17	10.33	5.74
Avg. temp. water	84.07	84.44	81.93	80.28
B.t.u. per sq. ft. per hr.	7530	7500	4700	4650
Actual arith. M.T.D., deg. fahr.	19.43	16.81	13.67	11.22
K_2	0.0175	0.0175	0.0175	0.0175
$(d_iV)^{5/6}$	3.4	5.4	3.35	5.38
$\frac{Q}{S}\frac{d_o\ K_2}{0.407(d_iV)^{5/6}}$	95.1	59.7	60.3	37.15
Δt_w est. deg. fahr.	7.2	4.6	4.4	2.9
1000 × $r_{sO.D.}$	0.280	0.280	0.258	0.258
1000 × $r_{tO.D.}$	0.092	0.092	0.092	0.092
1000 × $r_{wO.D.}$	0.956	0.613	0.936	0.624
($r_{sO.D.}$ + $r_{tO.D.}$ + $r_{wO.D.}$) × 1000	1.328	0.985	1.286	0.974
Overall resistance $\frac{\text{M.T.D.}}{Q/S}$ × 1000	2.580	2.241	2.910	2.413
Est. $r_{sasO.D.}$ × 1000	1.252	1.256	1.624	1.439[a]
M.T.D., run A calc. on basis of H	....	16.78° F.	....	....
M.T.D., run E calc. on basis of F	....	....	....	12.08°F.

---

[a] Less air leakage than run F.

In its present state this method appears to be useful in correcting the mean temperature difference for changes in velocity of circulating water in the tubes in condenser practice for inlet-water temperature of the order of, say, 50 to 90 deg. fahr. approximately, provided the value of $r_{sasO.D.}$ is known for one state, and then there is no change in any of the conditions except velocity in the tubes.

The results appear to be not so accurate below 50 deg. fahr. inlet. Probably the term $r_{sasO.D.}$ varies more with changing velocity at these low temperatures.

It is to be noted that the changes in the velocities, Tables 12 and 13, are very large (of the order of 75 per cent) and cover the change from one-pump to two-pump operation. If the change in velocity to be covered is, say, only 25 per cent, the difference would obviously be less.

The condenser industry should find this information useful, as it calls attention to the fact that the design of the condenser has as much, if not more, effect upon the M.T.D. than the other resistances ($r_{sO.D.} + r_{tO.D.} + r_{wO.D.}$).

The value of this method must rest largely upon the painstaking work of the various experimenters and the others mentioned, who have developed the formulas and made the experiments on which this method is based, and the author wishes to take this opportunity of extending to them his appreciation of their work.

The data on which Table 12 is based were supplied through the courtesy of the Worthington Pump and Machinery Corporation. The author would appreciate having any errors called to his attention.

Discussion

N. Artsay.[2] A few years ago the idea of comparing the average condenser performance with the possible performance of a single clean tube gained ground, and a great many papers appeared here and abroad dealing with the single-tube results. The comparison has shown such a low efficiency of the condenser-tube banks that several dormant improvements were dug out and incorporated in the designs. So in less than a decade the cooling surface went down from 2.0 to 0.6 sq. ft. per kw.

It should be pointed out that the analysis of separate resistances to heat transfer in single tubes will be of doubtful use for the tube banks, because the controlling resistance for a total condenser bank is on the vapor side and consists of several new resistances or their equivalents which are not sufficiently distinct in a single tube. The list would run approximately as follows:

(1) Decreased vapor temperature toward the inner part of the bank due to the increased partial pressure of air and to the reduction of total pressure of the mixture by hydrodynamic resistance.

(2) Reduction of velocity of steam past the tubes, and consequently the aggravated effect of air due to lack of sweeping effect.

(3) Increased amount of dripping condensate which reduces the exposed metal surface.

(4) The undercooling of exhaust steam, which cannot be detected by the usual thermometers.

(5) Accumulating dirt.

The water side of the cooling surface also presents new problems not encountered in single tubes: dirt, fouling, uneven distribution of flow among the tubes (to the extent of backflow in some tubes of two-pass condensers), liberation of air in top tubes of siphon flow condensers when placed too high, etc.

Taking all in all the mentioned factors encountered in the tube banks far outweigh those determining the performance of single tubes, and before attempting to predict the performance of given tube banks at various water velocities and rates of condensation a great deal more of experimental and theoretical investigation should be made.

Orrok, in his tests, has found that the overall heat transfer is proportional to the temperature difference in power $^7/_8$ which indicates the effect of condensate on the tube surface. In "Technische Mechanik und Thermodynamik," Feb., 1930, E. Schmidt, W. Schurig, and W. Sellschopp report on heat transfer by condensing steam when settling in the form of droplets and in films. High rates of heat transfer were aimed at in these experiments; a 6-in. copper disk placed vertically was cooled on one side by a central jet of water, and saturated steam was on the other side. Total transfers as high as 400,000 B.t.u. per sq. ft. per hour were reached.

The overall heat transfer coefficients B.t.u. per hour per sq. ft. per deg. fahr. obtained were as follows:

	Water velocity, ft. per sec.	Overall heat-transfer coefficient
Polished plate, droplets	16.4	2160
Polished plate, droplets	35.0	3060
Etched plate, 100 per cent film	16.1	945
Etched plate, 100 per cent film	34.5	1050
Etched plate, 30 per cent film	34.5	2750
Chrome-plated disk coated with kerosene, droplets	17.0	2600
Chrome-plated disk coated with kerosene, droplets	33.0	3430
Chrome-plated disk coated with kerosene, droplets	36.0	3550

With the film condensation the limit for overall heat transfer was in the neighborhood of 1100 B.t.u. per hour per sq. ft. per deg. fahr. attained at about 20 ft. per sec. water velocity; any further increases of velocity did not improve the heat transfer. However, with the condensate settling in the form of droplets, the curve of heat transfer versus water velocity was still rising after 35 ft. per sec.

These experiments demonstrate clearly the importance of the vapor side and call for a cautious interpretation of single-tube results. The figures given indicate that in a vertical row of tubes placed in saturated steam the heat-transfer coefficient in the third from the top and in the lower tubes will be practically independent of water velocity due to the dripping of condensate.

Somebody should extend the single-tube experiment providing a determined shower of condensate over the tube and measure the additional resistance.

Nusselt's theory of heat transfer during condensation takes the resistance of dripping condensate as the major factor and checks well with the experimental figures for banks. The splendid performance of Ginabat (Delas) and of the newest Escher Wyss condensers where measures were taken to divert the condensate testifies to the importance of this factor.

AUTHOR'S CLOSURE

Mr. Artsay points out the importance of the resistance on the vapor side of the tube. This is in accordance with the findings of this paper, as the term $r_{ssO.D.}$ in Equation [9] for an actual condenser with perfectly clean tubes seems to cover resistances external to the tube proper—i.e., in the arrangement of the surface and in the amount of air present in the steam, etc. This conclusion is based upon the fact that calculated changes in the term $r_{wO.D.}$, Equation [9], due to changing velocity of water inside the tubes, seem to cover satisfactorily the changes in heat transfer in an actual condenser, provided the flow of steam, the amount of air present, and the temperature of the inlet water are not altered at the same time that the velocity of the circulating water is changed.

It has been known for a long time that the rule of the square root of the velocity for calculating changes in heat transfer in an actual condenser was not exactly correct, and Equation [9] seems to explain why this is so.

At least one manufacturer and one user of condensers are now experimenting with this method, and interesting conclusions may be expected during the coming year.

---

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FSP-54-9

Development of Pulverized-Coal Firing and Study of Combustion

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This paper reviews briefly the development of burning pulverized coal under steam boilers. It points out the difficulties experienced in the early installations and how some of these difficulties were overcome by changes in the design of furnaces and burners and also by the adoption of more advantageous methods of firing. The paper discusses the present practice in rates of combustion in furnaces of various designs and the factors that set the limitations. A large part of the paper is devoted to a detailed study of combustion of pulverized coal.

THE early installations of powdered coal under steam boilers had furnaces with solid refractory walls. Many of these installations were boilers changed over to powdered coal from stoker firing, and the same design of furnace walls had been used as was then the practice with stoker furnaces. Even the new installations were set with refractory furnaces, although the larger units had the air-cooled hollow-wall construction with the refractory tied to the steel frame for stability.

Some of the first installations were short lived. The furnaces were small, and the refractory walls were rapidly destroyed by erosion of molten ash and by flame impingement. It was this rapid destruction of the furnace walls that held back the application of powdered-coal firing to steam boilers. It became evident that the furnaces must be made large in order to reduce the impingement of flames and to avoid the rapid wasting away of the furnace walls. Large furnaces really meant the removal of the refractory walls from the flame, and giving the flame more elbow room.

The desirability of avoiding impingement of flame against furnace walls made it necessary to employ methods of firing with limited turbulence in the furnace and to admit air in sheets between the flame and the furnace walls. The fact that a large part of the air was introduced into the furnace along the walls resulted in slow mixing and long flame. When higher ratings were required, it was necessary to admit into the furnace a greater excess of air to lower the furnace temperature and thereby reduce the slagging of the ash and the washing away of the furnace-wall refractories.

The removal of the refuse deposited at the bottom of the furnace was another problem in the early installations. The refuse was either fused or became sticky and could not be removed without shutting down the boiler and cooling the furnace. Resort was generally made to large excess air to lower the furnace temperature and to avoid the fusion of the refuse and make its removal easier.

Water-Cooled Furnaces

Greater excess air increased the stack losses and lowered the overall efficiency of the steam generating unit. Therefore, efforts were exerted to reduce the furnace temperature by placing water-cooled surfaces in the furnace. The water-cooled surfaces were placed first over the bottom of the furnace, then over the rear wall, and then by several successive steps over the side walls and the front wall.

With water-cooled furnaces it was not necessary to introduce excess air into the furnace beyond that amount needed for nearly complete combustion, in order to lower the furnace temperature and to avoid slagging of the ash. The lowering of temperature was much more usefully accomplished by allowing the water-cooled walls to absorb part of the heat liberated by combustion. At present practically all large steam-generating units are built with water-cooled furnaces.

The water-cooled walls either replace refractory walls or supply protection to the refractories. Thus, the modern water-cooled furnace makes possible the realization of higher rates of combustion without large excess air, thereby maintaining high efficiency.

The development of water-cooled furnaces removed the limitations to the turbulence in the furnace, and brought about the application of methods of firing with very intensive mixing and more rapid combustion. Impingement of the burning mixture against the furnace walls ceased to be destructive because there were either no refractories to be washed away or because what refractories were used were protected by water-cooled tubes.

Intensive mixing or turbulence fills the furnace space more completely with the burning mixture, so that the furnace volume is more fully utilized in the combustion of coal, which results in higher average rate of heat liberation.

Methods of Firing

Three general methods of firing pulverized coal have been developed; namely, the vertical, the horizontal, and the tangential. The vertical firing was the first to meet with a definite commercial success. It was followed shortly by the horizontal

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

firing. Various modifications and combinations of the two methods were devised and put in service. The Gilbert Station of the New Jersey Power and Light Company is an example of a modified horizontal firing. In this installation rows of horizontal burners are placed in opposite walls of the furnaces so that the streams of burning mixture impinge on each other in the middle of the furnace, producing intensive mixing and quick combustion. This modification of the horizontal firing was given the name of opposed firing.

A noted combination of the two methods of firing is used at the East River Plant of the New York Edison Company. Vertical burners firing downward are placed, as usual, in the arches. A set of horizontal burners is placed in the front walls close to the arches. The streams of the mixture of air and coal from the horizontal burners impinge against the streams of air and coal from the vertical burners. The impingement takes place close to the burners and results in quick ignition and intensive mixing. This method of firing has been adopted because at this plant low-volatile Eastern coal is burned which is slow to ignite by straight vertical or horizontal firing. Such combination of firing should be especially useful for burning anthracite coal and coke, which fuels have low volatile matter content and are difficult to ignite when burned in pulverized form. However, on account of the high temperature close to the arch produced by the quick ignition, the arch and front wall should be water cooled to prevent their destruction.

In the tangential method of firing the principle of producing intensive mixing by impingement is carried to the limit. Four sets of burners are placed in nearly the same horizontal plane, one set in each corner of the furnace, the streams of coal and air from each set being directed into the furnace tangent to a circle having a diameter $^1/_4$ to $^1/_2$ of the sides of the furnace. Generally 20 to 50 per cent of the air needed for combustion is supplied with the coal as primary air; the remainder of the air is supplied through the burner around the coal nozzle as secondary air and is forced into the furnace in the same direction as the mixture of primary air and coal.

The streams of mixture of coal and air from the burners in adjacent corners meet at right-angles, and the jets from the diagonal corners move nearly in opposite direction. The impingement of the streams against one another produces a very intensive mixing or turbulence. Due to the fact that the jets are tangent to a circle, the burning mixture is given a rotative motion on its way through the furnace and completely fills the combustion space. Thus, all combustion space is utilized in the burning of the coal, and high average rates of heat liberation are possible.

However, on account of the impingement of the burning mixture against the furnace walls, the tangential method of firing is applicable only to a completely water-cooled furnace. A refractory furnace would not long stand the impingement and the maintenance and outage of the furnace would be high.

There are now several large installations using the tangential method of firing. Among these are two units at the Kips Bay Station of the New York Steam Corporation, having a capacity of 450,000 and 800,000 lb. of steam; three units at the Hell Gate Station of the New York Edison Company, with a capacity of 400,000 lb. each, and two 1400-lb. pressure units at the Fordson Plant with a maximum capacity of 700,000 lb. There are also a number of smaller installations with this method of firing.

Although the corner location of the burners makes it a comparatively easy mechanical problem to fit the burners to a completely water-cooled furnace, the coal-pipe system is somewhat complicated. This is especially true with the direct-firing method where the coal from the mill must be divided into two to four streams which must be piped to different corners of the furnace. To simplify the coal piping and still retain most of the mixing advantages of the tangential firing, a few installations were made with one large burner placed in the bottom of the furnace. The mixture of coal and air is given rotary motion as it is forced up into the furnace from the burner. Although these installations give in general satisfactory results, the combustion is not as rapid as it is with the tangential method of firing. The rotative motion of the burning mixture, completely filling the furnace, is there, but the impingement of a number of streams against one another giving intensive mixing is lacking. When streams of secondary air are admitted through the corners and made to impinge on the rotating mixture from the burner, the results obtained are very near those obtained with the tangential method of firing.

Rates of Heat Liberation

The past experience has demonstrated the best practice for the rate of heat liberation for any furnace design. A few years ago there was a hope for high rates of heat liberation per cubic foot of available combustion space. Predictions were made that the rates of heat liberation would go to or even exceed 100,000 B.t.u. But this hope has not been realized, and it looks as though we are farther than ever from such realization.

At present a good engineering practice is to design refractory furnaces for rates of heat liberation not exceeding 20,000 B.t.u. for continuous operation, and if the ash is fusible, the rates generally do not exceed 15,000 B.t.u. With water-cooled furnaces the rates of heat liberation are limited to 30,000 B.t.u. for continuous operation, with a peak rate of 35,000 B.t.u. Higher rates are the exceptions and require extra good coal and fine pulverization.

The limit to the rate of heat liberation in refractory furnaces is the excessive erosion of the refractories by molten ash resulting in high furnace maintenance and difficulty in removing the refuse from the furnace. In water-cooled furnaces the limit is generally the deposition of slag on boiler tubes and the clogging of the gas passages through the boiler. Losses from incomplete combustion in many cases are not a serious limitation. With intensive mixing coal could be burned at considerably higher rates before the economic limits due to incomplete combustion would be reached.

Fineness of Coal

Finely pulverized coal burns quicker than coarse coal. Also high-volatile coal ignites and burns quicker than high fixed-carbon coal. There is no need pulverizing coal to a fineness that will give nearly complete combustion at a rate of heat liberation of 40,000 B.t.u. when the construction of the furnace and the fusion of the ash permit a rate of only 15,000 B.t.u. There is no need pulverizing Illinois coal or Texas lignite as fine as the high fixed-carbon Eastern bituminous coals or anthracite. A large part of the combustible of the Illinois coal and Texas lignite burns as gas, which can be burned quickly, whereas most of the combustible in the Eastern coal and anthracite burns as coke or fixed carbon, which burns slowly.

In addition to the relative contents of volatile matter and fixed carbon the various coals behave differently when heated. The particles of anthracite retain their shape and do not stick together when they collide. The Eastern bituminous coals fuse and become sticky. While passing through the stage of fusion, they change their shape, and when two or more particles collide, they may fuse into large pieces. The Illinois coal does not generally change shape and does not become sticky. The individual particles stay separate until completely burned. The lignite and the Western sub-bituminous coals crack and disintegrate. The particles break up into a larger number of smaller pieces, which burn more quickly than the larger particles. When sub-bituminous and lignite coals are burned in pulverized form, it is comparatively easy to reduce the combustible in flue dust to 1 or 2 per cent. On the other hand, it is considered good combustion when the flue dust from burning Eastern high fixed carbon bituminous coal contains 20 per cent combustible. The flue dust from anthracite coal-burning installation often contains 50 per cent combustible.

Therefore the fineness of pulverization should to a large extent be determined by the nature of the coal. Table 1 gives the fineness for the various coals for the best economic results, considering the cost of pulverization as well as the losses from incomplete combustion.

TABLE 1 ECONOMIC FINENESS FOR VARIOUS COALS

	200 mesh	100 mesh	50 mesh
Anthracite	85	97	100
Eastern low volatile	75	92	98
Eastern high volatile	65	85	97
Illinois	55	80	92
Sub-bituminous	45	75	90
Lignites	35	65	88

PROCESS OF COMBUSTION OF PULVERIZED COAL

The early experiments with powdered coal have shown that intensive mixing produced quicker and more complete combustion. Since then designers of pulverized-coal equipment strove to devise methods of firing and to design burners that would produce intensive mixing in the furnace and quick combustion. A closer study of the process of combustion shows the advantage of intensive mixing.

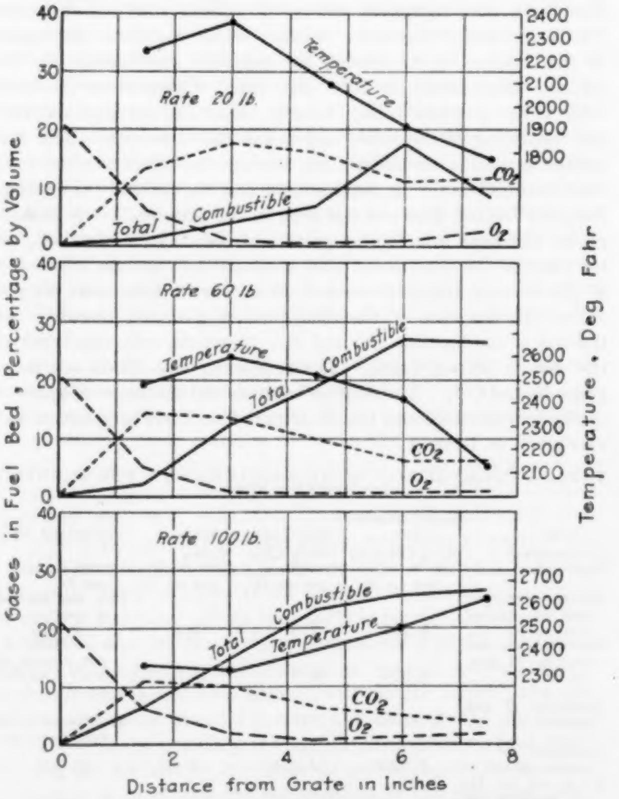

FIG. 1 COMBUSTION OF ANTHRACITE IN FUEL BED

(Composition and temperature of gases in 6-in. fuel bed of anthracite coal. Rates of combustion 20, 60, and 100 lb. per sq. ft. of grate per hour. Oxygen combines with carbon as fast as it is brought in contact with it. Practically all free oxygen combines with carbon in the first 3 in. of the fuel bed.)

Pulverized coal is supplied to the furnace in a mixture with air. The coal particles are heated largely by radiation from the flame of the already burning coal. The heating drives off the volatile matter as gas, and when this gas has been heated to a temperature of about 1100 deg. fahr., it bursts into flame and burns with the oxygen immediately surrounding it. The heat liberated by this combustion further raises the temperature of the coal particles and speeds up the distillation of volatile matter.

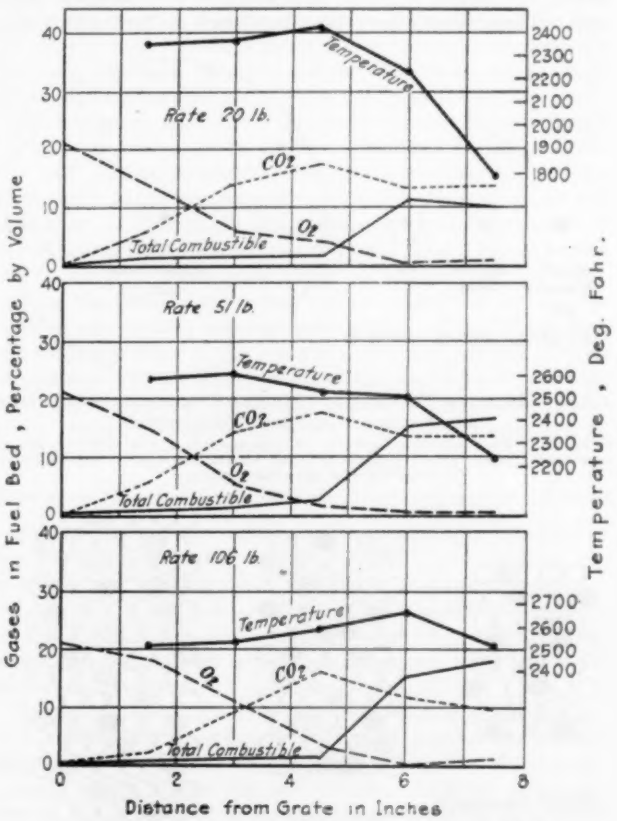

FIG. 2 COMBUSTION OF COKE IN FUEL BED

(Composition and temperature of gases in 6-in. fuel bed of coke from Pittsburgh coal. Rates of combustion 20, 51, and 106 lb. per sq. ft. of grate per hour. Oxygen combines with coke as fast as it is brought in contact with it.)

The distilled volatile matter ignites and burns first; when most of the volatile matter has been distilled and burned, the fixed carbon begins to burn.

Combustion of fuels in boiler furnaces is a chain of two consecutive processes. The first link of this chain is the coming of oxygen and combustible in contact. The second process or link in the chain is the chemical combination of the oxygen with the combustible when they have made contact. The speed of combustion is always controlled by the slower of the two links of the chain. The process of bringing oxygen and combustible in contact is by far the slower of the two links. Time is required for the oxygen and combustible to make contact. Once brought in contact the chemical combination at the ordinary furnace temperature is virtually instantaneous. In other words, the combustible can be burned only as fast as oxygen is brought in contact with it.

This principle is fully utilized in controlling the rate of combustion on grates. The more air that is forced through the fuel bed, the faster the oxygen makes contact with the combustible and the higher is the rate of combustion. During its passage through the spaces between the pieces of fuel, the oxygen comes in contact with the combustible and combines with it as fast as the contacts are made. The limit is reached only when the flow of air through the fuel bed becomes so great that the pieces of fuel are lifted and blown away and holes or craters are formed in the fuel bed. With some fuel it is possible for the rate of combustion or gasification to greatly exceed 100 pounds of fuel per hour per square foot of grate before this limit is reached.

Several years ago the Bureau of Mines made a series of experiments, the results of which demonstrated the foregoing principle. These results have been published in Technical Paper No. 137. Figs. 1 and 2 are reproduced from this publication. They show the composition of the gases at various distances from the grate and how quickly oxygen combines with combustible.[2]

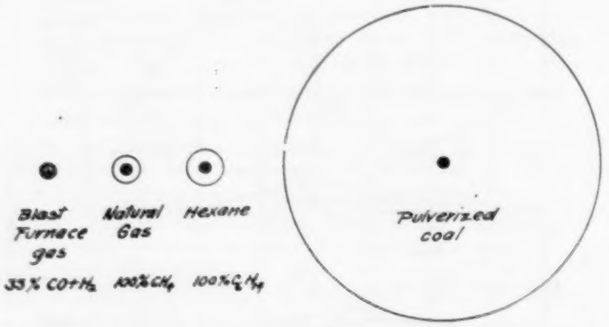

FIG. 3 RELATIVE VOLUMES OF VARIOUS COMBUSTIBLES AND THE AIR NEEDED FOR THEIR COMBUSTION

(The black dots represent spheres of combustible. The outside circles represent spheres whose volume is equal to the air needed for complete combustion of the combustible.)

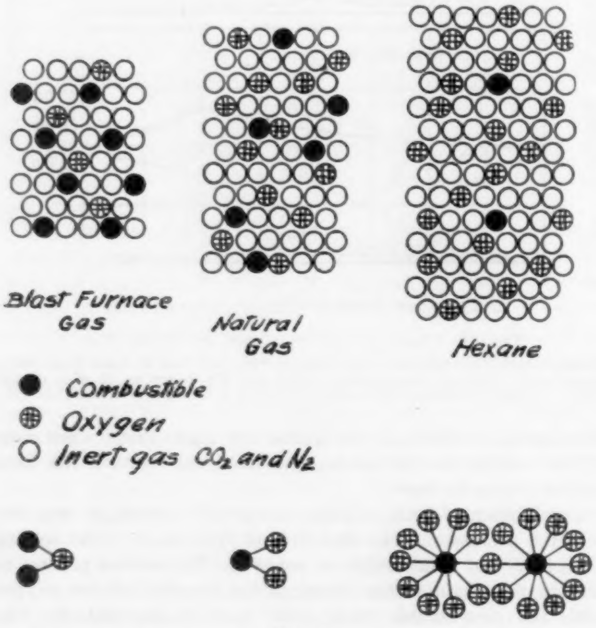

FIG. 4

(At the top are shown cross-sections through three gaseous fuels in a homogeneous mixture with all the air needed for their complete combustion. At the bottom are shown the number of molecules that are to combine with each molecule of the three fuels.)

When fuel is burned in pulverized form, the combustible burns only when it comes in contact with free oxygen, and therefore burns only as fast as it makes contact with oxygen. The distilled volatile combustible being in gaseous state diffuses into the atmosphere immediately surrounding the coal particles, meets the free oxygen, and burns. In other words, it goes halfway to meet the free oxygen and therefore makes contact with it quickly. The fixed carbon on the other hand stays in the coke residue in solid state and must wait until free oxygen works its way to it and makes contact with it.

A particle of pulverized coal is highly concentrated combustible which requires for its complete combustion a large volume of air many times as great as its own volume. Even if the mixture were perfectly uniform—that is, if the particles of coal were so spaced that all the air needed for the complete combustion surrounded each particle in a sphere—a large part of the oxygen molecules would be an appreciable distance from the coal particles, and time would be required to make contact with their surface. In case of gaseous fuels a mixture with air can be obtained in which all the oxygen molecules needed for complete combustion are comparatively near the combustible molecules and contacts between them can be made more quickly than in the case of pulverized coal.

This great concentration of combustible in a particle of pulverized coal and the large volume of air needed for its combustion compared to some of the gaseous fuels is illustrated in Fig. 3. The black dots represent spheres of various combustibles, and the large circles represent spheres whose volumes are equal to the volume of air needed for complete combustion at the furnace temperature of 2000 deg. fahr. Comparison is made with three gaseous fuels; namely, blast-furnace gas, natural gas consisting of methane, and a gas called hexane. The gas hexane is used in the illustration to show the effect of more concentrated fuel on the air requirements and the necessity of mixing. Actually hexane does not last long in the furnace. It is broken up by the heat into lighter gases and soot. It is assumed for the sake of the illustration that methane and hexane are stable at the furnace temperature and do not break into more simple gases. In the case of the blast-furnace gas only one-third of the gas is combustible CO and H_2; therefore, only one-third of the dot is shown black. The remaining two-thirds are inert gases N_2 and CO_2. The relative volumes and diameters of spheres of the combustibles and the air required for their combustion are also given in Table 2.

TABLE 2 COMPARISON OF AIR REQUIREMENTS FOR BURNING DIFFERENT FUELS

Fuel	Blast-furnace gas	Natural gas	Hexane	Pulverized coal
Composition	33% CO + H_2	100% CH_4	C_6H_{14}	....
Heat value	100 B.t.u. per cu. ft.	1000 B.t.u. per cu. ft.	4500 B.t.u. per cu. ft.	13,500 B.t.u. per lb.
Volume of oxygen per cu. ft. of gas, cu. ft.	0.165	2.0	9.5	(Air per pound of coal) 9.8 lb.
Volume of air per cu. ft. of gas, cu. ft.	0.825	10.0	45.6	Air at 2000° F. per pound of coal) 46,000 cu. ft.
Diameter of gas sphere	*D*	*D*	*D*	*D* (diameter of coal particle)
Diameter of sphere of air	1.22*D*	2.225*D*	3.6*D*	35.8*D*
B.t.u. per cu. ft. of mixture at 2000° F.	11.8	19.6	20.8	21.2

It is apparent that in the case of the gaseous fuels, in a nearly homogeneous mixture with all the air needed for combustion, the free oxygen is close to the combustible with which it is to make contact. However, with pulverized coal a large part of the oxygen is a considerable distance from the coal particle and more time is required for all the oxygen to make contact with the combustible in the coal particle than in the case of the gaseous fuels. Furthermore, the gaseous fuels can be easily subdivided into very small volumes which in the limit may become individual molecules, in which case the mixture may become completely homogeneous. In such a mixture the oxygen molecules would be close to the molecules of combustible, and contacts between the two would occur rapidly although not instantaneously. On the other hand, there is a limit to the fineness to which coal can be pulverized. The limits are both physical and economical. There are no available means to tear coal into individual molecules. Even within the limits of practical pulverization, very fine particles tend to cling together to form clusters, and thus the benefit of finer pulverization is partly lost. Besides, fine pulverization is costly. The average-sized particle of pulverized coal is very large compared to the size of a molecule of combustible gas, and much more time is required for all the free oxygen needed for complete combustion to come in contact with the combustible of the coal particle.

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[2] A more detailed study of this subject is given in a paper, "Burning Characteristics of Different Coals," by the present author and B. J. Cross in Trans. A.S.M.E., paper FSP-50-52 (Sept.-Dec., 1928).

Although contacts between oxygen and gaseous combustible in a mixture occur comparatively easily, they do not take place instantaneously. Even if the mixture is perfectly homogeneous an appreciable length of time is required for all combustible to make contact with the oxygen. The necessity for the time factor is illustrated in Fig. 4, which shows a section through the mixtures and the probable arrangements of the molecules of the three gaseous fuels in nearly homogeneous mixture with all the air needed for combustion. The shaded circles represent the molecules of the combustible, the cross-sectioned circle represents oxygen molecules, and the white circles the inert gases, nitrogen and, in the case of blast-furnace gas, also carbon dioxide. At the furnace temperature the molecules are in rapid motion and collide frequently with one another. In all three mixtures all the oxygen molecules must make contact with the combustible molecules. Some of the oxygen molecules are close to the combustible molecules, and contacts may occur quickly. Other oxygen molecules are farther away, and must work their way around and between the inert molecules toward the combustible molecules, and more time elapses before the contacts are made. As the combustible molecules combine with the oxygen, there are fewer active molecules left, and the contacts become less frequent.

In the heavier combustible gases such as methane and hexane, a number of oxygen molecules must make a contact with each combustible molecule, and many of the molecules must come a comparatively long distance to make the contact. This is especially true of hexane. But even with this heavy gas the number of molecules is small and the distance they have to travel short as compared with the combustion of pulverized coal. So large is the number of combustible molecules in a small particle of coal, and so great is the number of oxygen molecules that must make contact with the coal particle that it is impossible to draw a diagram. If the coal particle consists mostly of fixed carbon, all the oxygen molecules must come to the particle of coal to make contact, and the rate of contact making is slow. If the particle is coal with high volatile matter content, the volatile matter leaves the coal particle and diffuses into the air and the rate of contact making is high. It is well known how slowly burning are anthracite and coke in pulverized form, and how difficult it is to reduce the combustible in the flue dust below even 30 per cent.

MIXING IN FURNACE INCREASES RATE OF CONTACT MAKING

Mixing in the furnace greatly increases the rate of contact making. It brings the unused free oxygen molecules next to the surface of the coal particle. It also carries away the products of combustion and the distilled volatile matter from the coal particle, thus equalizing the distribution or concentration of free oxygen and the unburned combustible in the furnace.

It is the mixing in the furnace that brings combustible and free oxygen nearer together so that they can make contact and combine. Mixing outside of the furnace is ineffective in producing rapid combustion. All the devices that were designed for mixing outside of the furnace failed in producing rapid combustion unless the mixing was carried into the furnace. A uniform mixture of pulverized coal and air is very difficult to obtain, and if it is ever obtained it can not be maintained. The coal is much heavier than air and tends to settle out of the mixture. It is kept in suspension by the movement of the mixture. Even during the flow of the mixture the concentration of the coal particles in various parts of the section of the stream is constantly changing. A slight change in the direction of the stream tends to concentrate the coal at the outside curve of the stream.

With intensive mixing, the coal particle moves faster or slower than the air, or moves in different direction. In other words, the coal particle constantly changes its position with respect to the air, moving out of one location where it made contact and used up all the free oxygen near it, into a new location where there is still uncombined oxygen. At the same time it leaves behind the products of combustion either as CO or CO_2, and H_2O. During the distillation process the particles leave behind also most of the distilled gaseous combustible. A careful observer can easily see some of the larger coal particles moving through the furnace with a comet-like tail of burning volatile matter behind them. Mixing continually equalizes the concentration of free oxygen and combustible in all parts of a cross-section of the stream of burning mixture.

EFFECT OF CONCENTRATION ON THE RATE OF CONTACT MAKING

Even with intensive mixing and the resulting equalization of the concentration of the free oxygen and the combustible, the rate of contact making rapidly decreases as the process of combustion goes on. There are fewer oxygen molecules left; some of the very small particles have burned completely, and the larger ones have smaller surface for the oxygen to make contact with.

In case of a gaseous fuel in a homogeneous mixture with air, each combustible molecule within a given unit volume of this mixture has a chance to make a contact with any of the molecules of oxygen within that unit volume. In other words, the rate at which contacts occur is proportional to the product of the concentrations of the two kinds of molecules. If this concentration is expressed in percentage by volume, then the rate of the contact making is proportional to the products of the percentage of oxygen and the percentage of combustible gas. Thus, if C is the percentage of combustible gas and O the percentage of oxygen, then the rate of contact making $= K(C \times O)$. This is true at any time during the process of combustion as long as the mixture is homogeneous.

If a chemical combination takes place every time a contact between two active molecules occurs, which can be assumed with reasonable certainty, every contact removes one molecule of oxygen and at least part of the molecule of combustible from active participation in the contact making, and also in the process of combustion. The rate of contact making during the process of combustion is then expressed by the equation:

$$-\frac{dC}{dt} = K(C \times O) \dots\dots\dots\dots\dots\dots [1]$$

The oxygen in a furnace consists of two parts; namely, that needed for complete combustion of the combustible, and the excess oxygen that is left after all the combustible has been burned. The first part of oxygen is always proportional to the combustible, and the second part remains constant. We can then write

$$O = aC + b$$

where a and b are constant; b stands for the excess oxygen and a for the number of oxygen molecules required to burn a molecule of combustible. Equation [1] then can be written

$$-\frac{dC}{dt} = KC\ (aC + b)\ \dots\dots\dots\dots\dots\dots [2]$$

On integration and the determination of the constant of integration the equation becomes

$$Kt = \frac{1}{b}\log_e\left(\frac{aC+b}{C}\right) - \frac{1}{b}\log_e\left(\frac{aC_1+b}{C_1}\right)\dots\dots [3]$$

where C_1 = initial concentration of combustible and t = period of time since process of combustion has started.

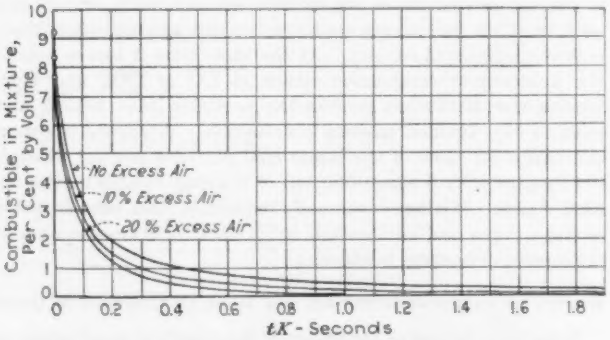

Fig. 5 Process of Combustion of Natural Gas

(This consists of 100 per cent of methane, in a homogeneous mixture with three excesses of air 0, 10, and 20 per cent.)

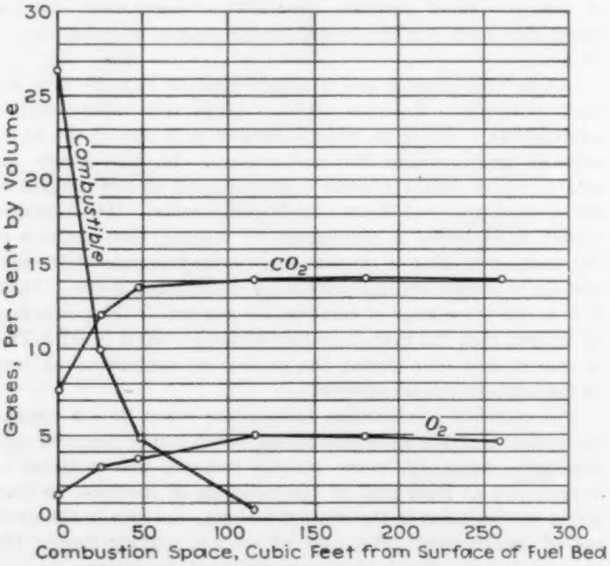

Fig. 6 Process of Combustion of Gaseous Combustible Rising From a Fuel Bed of a Stoker-Fired Fuel Bed

(Air for combustion was supplied through various openings above the fuel bed.)

With no excess air Equation [3] becomes

$$Kt = \frac{1}{ac} - \frac{1}{ac_1}\dots\dots\dots\dots\dots\dots\dots [4]$$

For natural gas containing 100 per cent of methane, and burning with 0, 10, and 20 per cent of excess air, the foregoing equations give the three curves shown in Fig. 5. The curves are plotted with the unburned combustible as ordinates and the time as abscissas.

The curves show two interesting features which are frequently observed in the actual operation of furnaces. The combustible burns at first at high rates, but this rate rapidly decreases as the mixture moves away from the burner, and the rate becomes very low when the combustible has been reduced to 1 per cent. In other words, the first 6 or 7 per cent of the combustible burns much more quickly than the last 1 per cent. It is the burning of the last 1 per cent of combustible that requires time and furnace volume. Within a practical furnace volume the combustible cannot be burned completely, it can be only reduced to economically insignificant quantity. When gaseous fuels are burned it is often difficult to determine the last small traces of combustible in the furnace gases. We know that with pulverized coal we can never reduce the combustible in flue dust to zero.

The rate of heat liberation during the combustion of the first 6 or 7 per cent of combustible is several times as great as the average rate for the entire furnace.

The excess air does not affect materially the initial rate of combustion. It is very useful, however, in burning the last 1 per cent of combustible. The first 10 per cent of excess air is more effective in burning the last 1 per cent of combustible than subsequent additions to the excess air.

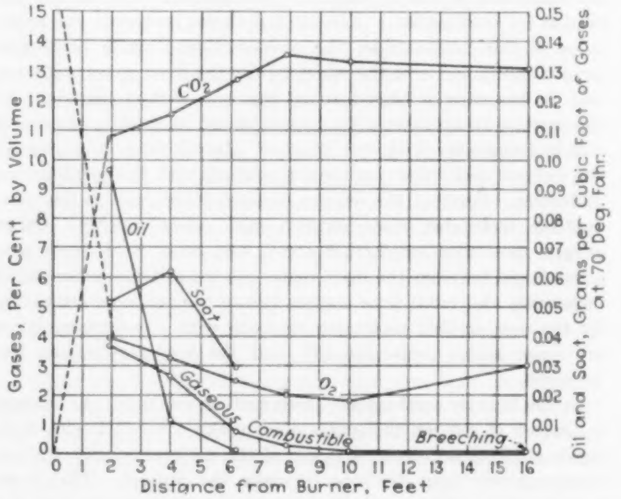

Fig. 7 Process of Combustion of Fuel Oil in a Marine Boiler Furnace

(The atomized oil upon entering the furnace partly burns and partly breaks up into soot and gaseous combustible. About 75 per cent of the combustion occurs within the first 2 ft. distance from the burner.)

Equations [3] and [4] and the curves are applicable to gaseous fuels with which it is possible to express the concentration of combustible in percentage by volume. In oil burning and the burning of pulverized fuel, other factors would have to be substituted for the concentration of combustible. Such factors might be partly proportional to the number of globules of oil or particles of coal and their surfaces per unit volume; that is, the factor should include the fineness of atomization of oil or of pulverization of coal. Most of the oil volatilizes in the furnace and behaves as heavy gas. A large part of high-volatile coal also volatilizes and burns as gas. The curves representing the process of combustion of these two fuels when all the air is supplied through or closely around the burner would be very similar to the curves shown in Fig. 5. With anthracite and other low-volatile fuels the combustion would be slower and the first part of the curves would not be so steep.

For experimental studies in which the air supply could be accurately measured, the sampling of the burning mixture at various distances from the burner and the analytical work could be simplified, if the decrease of the free oxygen were used as the indication of the progress of combustion instead of the unburned combustible. Equation [1] would then become

$$-\frac{dO}{dt} = KC(O + b)$$

where O is the oxygen necessary to burn completely the combustible C, and b the excess of oxygen. The combustible C is then always proportional to O and can be expressed by the equation $C = aO$

Equation [1] in its simplest form then becomes

$$-\frac{dO}{dt} = KaO(O + b)$$

The curves obtained by this equation would be similar to those of Fig. 5, but would be asymptotic to horizontal lines representing the constant excess of oxygen b. Comparison of these curves with those obtained from experimental results would show to what extent the equation would have to be modified.

Experimental data on the process of combustion in furnaces are meager. Obtaining such data requires special apparatus and much careful work. Figs. 6 and 7 show some experimental results from the work of the Bureau of Mines. Fig. 6 shows the progress of combustion of gaseous combustible rising from the fuel bed of a stoker-fired furnace. The oxygen was supplied by air admitted through various openings over the fuel bed and was not therefore mixed with the combustible gases at the point when they were leaving the fuel bed.

Fig. 7 shows experimental data on the progress of combustion in an oil-fired marine-boiler furnace. This figure indicates how difficult it is to determine the total unburned combustible because part of it is in gaseous state, part in liquid, and part in solid state. It is necessary to use two scales to show its amount. The drop in CO_2 and the rise in O_2 in the breeching is due to leakage into the setting.

The curves of these two figures show a strong resemblance to the curves of Fig. 5. Both of these figures indicate very rapid combustion at the beginning of the process. This is especially true of the oil-burning furnace. Two feet from the mouth of the burner a large part of the combustible was burned, as is indicated by 10.7 per cent of CO_2. In the next 6 ft. of the distance the CO_2 increased less than 3 per cent.

CONCLUSIONS

Intensive mixing in the furnace is the most important factor within the control of the designer and operator for producing rapid combustion.

Mixing outside of the furnace is ineffective in producing rapid combustion because a uniform mixture of air and coal cannot be maintained. Without mixing in the furnace, volatile matter distilled from the coal would hang around the coal particles in spheres with only the outside of the surface of these spheres exposed to the contact of the oxygen molecules.

Mixing in the furnace tends to keep the coal particles uniformly distributed across any section of the stream and makes the atmosphere surrounding the coal particles a homogeneous mixture of gases; in other words, mixing in the furnace constantly equalizes the concentration of free oxygen and unburned combustible over any section of the moving stream of burning mixture.

Combustion constantly destroys equalized concentration; mixing reestablishes it. Beyond such constant equalization of concentration, mixing can have no further effect on the rapidity of combustion. With equalized concentration, combustion proceeds at a definite rate depending on the concentration of the combining substances.

Discussion

Ralph A. Sherman.[3] The author has attempted to discuss the problem of the combustion of fuels, particularly of pulverized coal, on a more fundamental basis than has normally been done in the multitude of papers on the subject. He has assumed as his major premise that the rate of combustion is governed only by the rate of contact of combustible and oxygen because this process is much slower than the rate of chemical reaction. This conclusion was reached, apparently, from the results of the experimental work on the combustion of coal in fuel beds which the author conducted when with the Bureau of Mines, where it was definitely shown that the rate of diffusion of oxygen into the film of gas surrounding the particle of fuel governed the rate of formation of CO_2.

Therefore the author sets up the basic differential equation:

$$-\frac{dC}{dt} = K(C \times O) \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

Even for fuel beds, however, this equation does not hold strictly true, for with a given size of a given fuel C is constant, and therefore the rate of combustion is proportional only to the concentration of the oxygen. With a different size of fuel the rate of combustion will also be proportional only to the concentration of the oxygen as before, but will be different from the first by a factor dependent on the relative surfaces of the particles.

Other of the combustion reactions, as

$$\begin{aligned} CO_2 + C &\longrightarrow 2CO \\ 2CO + O_2 &\longrightarrow 2CO_2 \\ 2H_2 + O_2 &\longrightarrow 2H_2O \\ C + H_2O &\longrightarrow CO + H_2 \end{aligned}$$

are relatively slow compared to that of $C + O_2 \longrightarrow CO_2$, and their velocities depend on the specific rate of reaction rather than on the rate of diffusion or of contact making. This assumes, of course, in the combustion of gases good mixing such as would occur in a well-designed burner. With imperfect mixing, such as that of the combustible gases arising from a fuel bed with secondary air, the rate of combustion may well be expected to depend upon the rate of contact making. As the degree of homogeneity is unknown, it can scarcely be hoped to reduce the rate of contact making to a fundamental mathematical expression.

Considering the combustion of coal in pulverized form in its simplest state, that of particles of carbon of uniform size, the rate of combustion at any time should depend only on the concentration of the oxygen, but as the size of the particles decreases, although their number remains constant, a factor for the change in surface should be introduced into the differential equation. When one further considers the combustion of an actual coal in pulverized form where the particles are initially of widely varying size, where they may, as the author points out, coalesce or disintegrate to particles of different size, where the number of particles change as small ones are completely burned, where the evolution of volatile matter gives as intermediate products

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[3] Fuel Engineer, Battelle Memorial Institute, Columbus, Ohio. Mem. A.S.M.E.

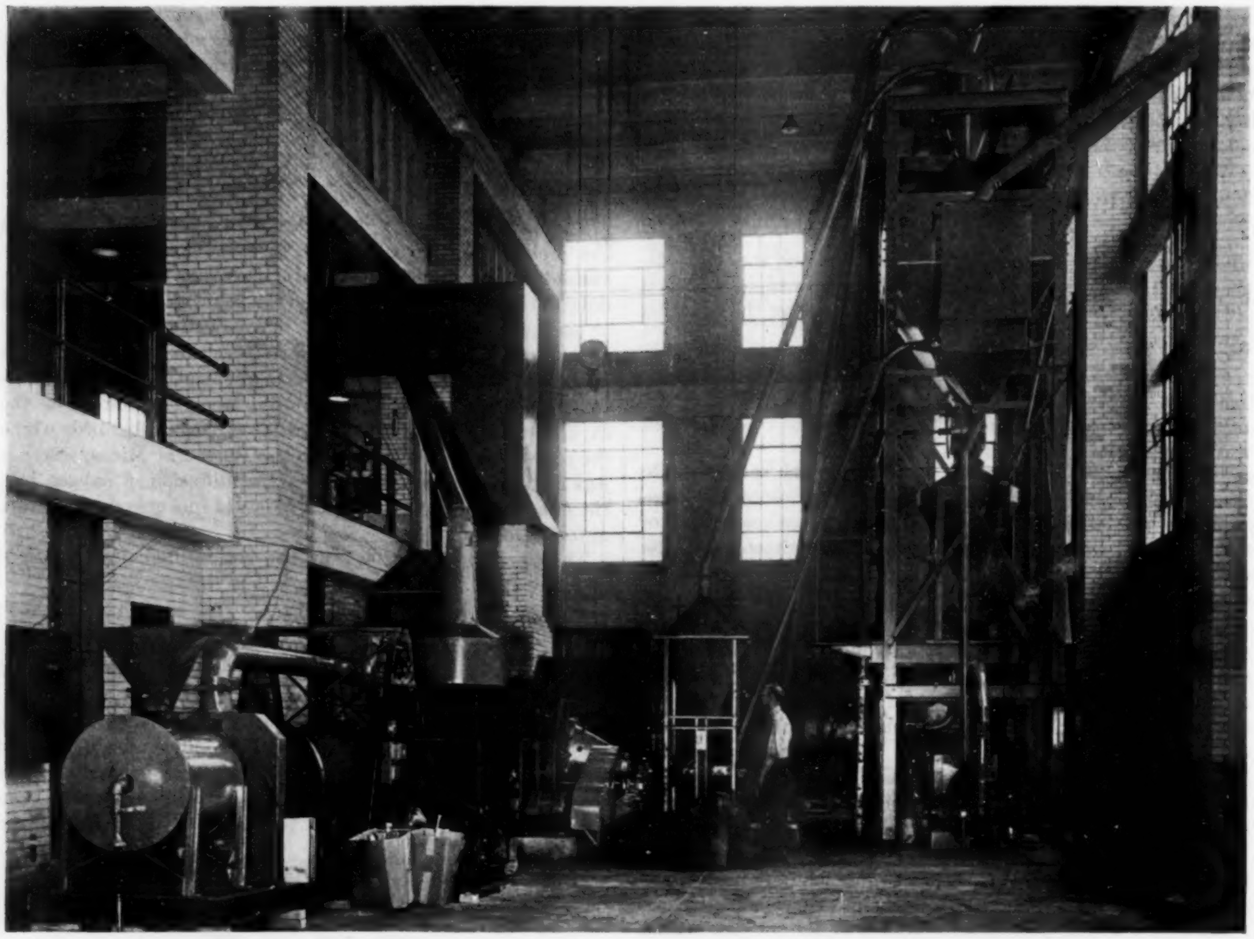

FIG. 8 EQUIPMENT FOR THE STUDY OF COMBUSTION OF PULVERIZED COAL AT BATTELLE MEMORIAL INSTITUTE

gaseous hydrocarbons, hydrogen, and soot, and where the reactions $CO_2 + C \longrightarrow 2CO$ and $H_2O + C \longrightarrow CO + H_2$ may and do occur, the possibility of expressing the rate of reaction in as simple a form as the author suggests is remote.

Experimental data on the process of combustion of pulverized fuels are extremely meager. Audibert[4] made a small-scale study of the process, using a number of fuels, two different sizes, and different amounts of excess air. At the Bureau of Mines two studies have been made by Griffin, Adams, and Smith[5] and Smith and Gudmundsen[6] on the rate of burning of individual particles. These studies are valuable, but are not directly applicable to the present problem.

Realizing the lack of data, the Battelle Memorial Institute felt that as great a contribution as it could make to the advance of the science of fuel utilization would be the undertaking on its own endowment of a study of the process of the combustion of pulverized fuel. Fig. 8 shows the equipment which has been installed for the study. In the left foreground is a rotary drier, from which the coal is handled to the hopper at the rear through a constant-weight feeder into a ball mill. The coal is discharged through the conical superfine classifier, by adjustment of which the size may be controlled, and is collected in the cyclone collector at the top of the 2-ton storage bin. The coal is discharged into a 500-lb. weighing bin mounted with the screw feeder on Toledo scales, so that the rate of feeding at any time is accurately shown. The coal is picked up by the entire amount of air, which is measured by an orifice, and fired through a simple burner into a horizontal, refractory-lined furnace, $3^1/_2$ ft. in diameter and 18 ft. long. The steel shell is welded or cemented at joints to insure against infiltration. A series of sampling holes and observation doors on the sides of the furnace are provided for the sampling of gases and solids and measurement of the temperature. The temperature of the wall is determined by a series of thermocouples embedded in the refractories at four points along the furnace.

The present program includes the study of the process of combustion with different degrees of fineness, with different amounts of excess air, and with different coals. The effect of air temperature, wall temperature, and burner design will be studied later.

Although, as the author states, the quantitative sampling of solids from the gases is difficult, yet we are attempting to determine the completeness of combustion in that way and have developed a method which is not too difficult. The installation has only recently been completed, and some details of the apparatus are not yet working smoothly, but some results have been obtained which may supplement the author's paper.

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[4] Audibert, Etienne, "Experimental Study of the Combustion of Pulverized Coal," *Rev. Ind. Min.*, vol. 4, 1924, pp. 1–32.

[5] Griffin, H. K., Adams, J. R., Smith, D. F., "Rate of Burning of Individual Particles of Solid Fuel," *Ind. and Eng. Chem.*, vol. 21, 1929, pp. 808–829.

[6] Smith, D. F., and Gudmundsen, A., "Mechanism of Combustion of Individual Particles of Solid Fuels," *Ind. and Eng. Chem.*, vol. 23, 1931, p. 277.

Fig. 9 shows typical results, gas composition, and gas temperature as determined by a shielded velocity-type thermocouple, wall-surface temperature, percentage combustible in dust, and the carbon in the dust as a percentage of the total carbon in the gas, plotted against the distance from the burner. These results were obtained when burning at a rate of 200 lb. per hr., with approximately 30 per cent excess air, a semi-bituminous coal from the Sewell bed, Fayette County, West Virginia, with 0.7 per cent moisture and 3.6 per cent ash. The screen size of the coal was:

Screen size of coal	Per cent
On 20 mesh	0.0
Through 20 mesh, on 50 mesh	0.2
Through 50 mesh, on 100 mesh	1.0
Through 100 mesh, on 200 mesh	11.0
Through 200 mesh	87.8
	100.0

The distance from the burner is not a good scale for the abscissas, as the gases, because of temperature change and expansion from the burner mouth, do not move at uniform velocity. From readings of the velocity with a water-cooled pitot tube the time to each sampling point has been approximately determined and used in the following calculations and figures.

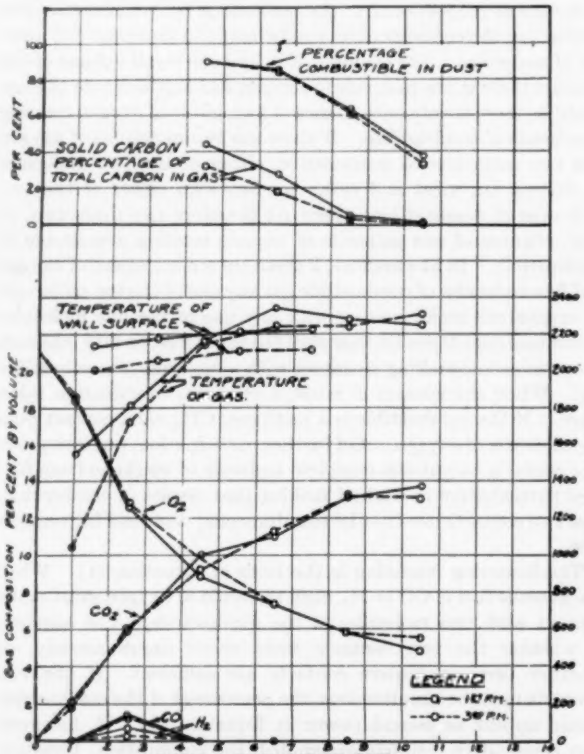

Fig. 9 Typical Data From Experimental Furnace Burning Pulverized Coal

Fig. 10 shows the concentration of O_2 remaining in the gases as calculated from the gas composition shown in Fig. 9 and the air-combustible ratio known from the firing conditions. The type of the curves is similar to those shown in the author's paper with the oxygen concentration becoming asymptotic to the constant excess air lines.

Fig. 11 shows the percentage of the total combustible unburned (1) as calculated from the oxygen remaining and (2) as determined by the quantitative sampling of the solids. The first method is in error, as the calculation is based on the oxygen necessary for the entire combustible, whereas in the later part of the burning time only carbon is being burned. The two methods of calculation do not check, and further careful work is necessary to determine whether the difficult sampling of the solids can be abandoned for the simpler sampling of the gases only.

To determine how closely the rate of combustion might be governed by the rate of contact making as suggested by the author or by the concentration of the oxygen, only values of K have been calculated from the expression

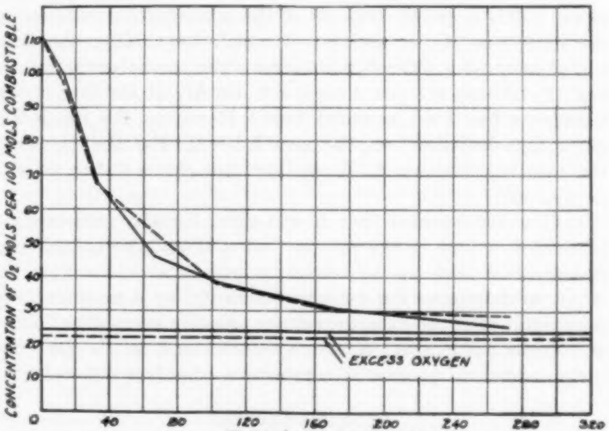

Fig. 10 Rate of Change of Concentration of Oxygen in Experimental Furnace Burning Pulverized Coal

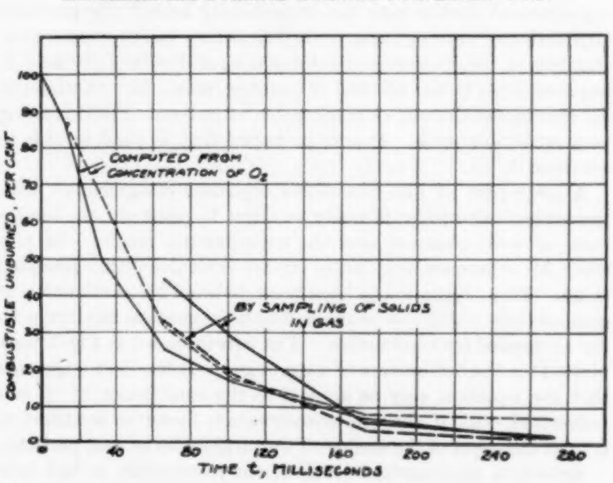

Fig. 11 Percentage of Combustible Unburned in Experimental Furnace Burning Pulverized Coal

$$K_1 = \frac{1}{t} \cdot \frac{1}{ab} \log_e \frac{(O + b)\,O_1}{(O_1 + b)\,O}$$

which is the integrated form of the final equation given on the last page of the paper, and from

$$K_2 = \frac{1}{t} \log_e \frac{O_1 + b}{O + b}$$

which is the integrated form of the differential equation $-\frac{dO}{dt} = K(O + b)$; t is the time in milliseconds, a the initial ratio of oxygen to combustible, b the constant excess of combustible,

O the concentration of oxygen at time t, and O_1 is the initial concentration of the oxygen. Following are the results:

	t, milliseconds					
	13	33	66	103	173	283
$K_1 \times 10^{-6}$..........	1.3	2.9	2.5	3.4	3.7	6.4
$K_2 \times 10^{-3}$..........	9.0	10.1	12.5	10.4	7.6	5.0

Neither of the "constants" is, as was to be expected, a constant. This is partly because of the admitted inaccuracies in the data, one of the greatest of which is probably the time t, (1) because it is difficult to determine the velocities accurately, and (2) because the zero time is not directly at the face of the burner as has been assumed here. However, the maximum percentage deviation from the mean is less for K_2; that is, where the rate is calculated to depend on the concentration of the oxygen only.

It is to be expected that K will differ for the various complicated reactions in the process, but a thorough experimental investigation such as we intend to pursue should enable us, if not to determine the velocity constants, by a mathematical interpretation of the data, to evaluate properly some of the factors and throw some light on this successful and much used but little understood process of combustion of pulverized coal.

Author's Closure

Mr. Sherman's discussion is a valuable contribution to the study of combustion of fuels. Although such theoretical and experimental studies may not immediately benefit the practical application of burning fuels, nevertheless they increase our understanding of the processes of combustion, and eventually lead to improvements in our method of burning fuels. Mr. Sherman in his experimental study of combustion of pulverized coal is doing very creditable work. It is to be hoped that he shall be able to continue it.

A large part of Mr. Sherman's apparent disagreement with the author's deduction is really an effort to point out the limitations of such equation and the experimental results. In this effort Mr. Sherman went really farther than the author intended to go. Thus, Equation [1] has been deduced for combustion of gaseous fuels which can be made into homogeneous mixture with the air needed for combustion. The curves shown in Fig. 5 were plotted for the combustion of natural gas. It has been suggested that the equation may be applied to the combustion of oil and pulverized coal, if the fuel-concentration factor is modified to include the effect of the surface of the oil globules or coal particles.

Results of experimental study of the combustion in fuel beds were used to show that combustion takes place as fast as oxygen can be brought in contact with the combustible. The rate of combustion can be increased tenfold by increasing the weight of air passing through the fuel bed tenfold. This can be done within certain practical limits regardless of the extent of surface. We can bring 10 lb. of air in contact with 1 sq. ft. of surface and burn a definite amount of coal, or we can bring 100 lb. of air in contact with the same foot of surface and burn 10 times the amount of coal. In the same manner we can take 2, 3, or more square feet of surface and bring 10 and 100 lb. of air in contact with the surface. The amounts of coal burned are the same as when the air comes in contact with only 1 sq. ft. It is a matter of the amount of oxygen coming in contact with the surface.

The practical limit to increasing the rate of combustion by increasing the air flow through the fuel bed is reached when the velocity of the air becomes so high that it lifts the coal and holes are formed in the fuel bed. The air then passes through these holes without making contact with the coal. The fusion of the ash into clinker may also affect the flow of air and its contact with the surface of coal. If the size of coal is too large, the spaces between the lumps may be too wide, so that the air may flow through the large passages without making contact with the surface of the coal.

After establishing with reasonable certainty the fact that combustion takes place as fast as oxygen can be brought in contact with combustible, the next move was to find how fast contacts between oxygen and combustible occur in the burning of various fuels. The simplest case is the combustion of gaseous fuels such as natural gas, because the fuel is as finely subdivided as the oxygen in air, and the combustible mixture can be made completely homogeneous. A homogeneous mixture is one having the same concentration of free oxygen, as well as combustible, in every part of the volume of the mixture. Applying this to a stream of burning mixture, if we take a sample of the gases from every square inch of the entire cross-section of the stream of mixture and find the percentage of free oxygen the same, and also the percentage of combustible the same in all the samples, the mixture could be called homogeneous. In such a mixture the rate of contacts of oxygen with combustible would be in proportion to the number of molecules of free oxygen times the number of molecules of the combustible gas. Inasmuch as the number of molecules is proportional to the percentage by volume, this percentage, or the concentration, can be made to represent the number of molecules. For example, if in any one small volume of the mixture there is one molecule of oxygen and one molecule of combustible, there is only once chance of a molecule of oxygen meeting a molecule of combustible. If there are two molecules of oxygen and two molecules of combustible, either of the two molecules of oxygen can meet and make contact with either of the two molecules of combustible; there are therefore two times two, or four, chances of one molecule of oxygen meeting a molecule of combustible. In like manner, if there are ten molecules of oxygen and five molecules of combustible gas any one of the ten molecules of oxygen can make contact with any one of the five molecules of combustible; there are therefore ten times five, or fifty, chances of one molecule making a contact with a molecule of combustible gas. When the contact is made, a chemical combination takes place. If the combustible is a methane, CH_4, each contact puts one molecule of oxygen out of further participation, but it requires two contacts to put one complete molecule of methane from further participation in contact making that results in combustion. The two contacts need not be simultaneous; they can be consecutive.

The foregoing reasoning is the basis of Equation [1]. When the gaseous fuel is CO or H_2, each molecule of oxygen must make contact with two molecules of the combustible. The question is whether the two contacts must occur simultaneously, or whether two consecutive contacts are sufficient. If the two contacts must be simultaneous, the percentage of the combustible would appear as second power in Equation [1]. If, however, two consecutive contacts accomplish the combustion, Equation [1] remains as it stands.

It has been suggested in the paper that the equation can be made to apply to the combustion of pulverized coal if the factor representing the concentration of combustible is modified to take care of the variation of the surface of the coal particles. This surface varies with the fineness and also with the progress of the combustion process. We know that very finely pulverized coal burns quicker than coarsely pulverized coal. Also for any uniform fineness the surface varies approximately as $^2/_3$ power of the volume or mass of the coal particle. This volume, however, does not approach zero volume as the combustible burns, but the volume of the ash residue. In cases of coal with very uniform distribution of ash, it may not be possible to burn out the combustible in the center of the larger coal particles because the ash residue may form a protective layer on the surface and prevent contacts of oxygen with the combustible inside of the ash shell. This and other similar factors make it very difficult if not impossible to formulate an exact mathematical law applicable to all coals and all degrees of fineness. Attempts to formulate such law may indicate the direction of the influence of these factors, which is about all the practical value we can hope to deduce from such formulas. Nevertheless a study of these factors is not devoid of benefit; it increases our understanding of the process of combustion of pulverized coal.

Mr. Sherman has done considerable work in his attempt to test Equation [1] with the results of his experiments. The curves given in his Figs. 10 and 11 show a remarkable similarity to the curves of Fig. 5. That his constants *K* did not come out as constants is not at all surprising. Equation [1] applies to a homogeneous mixture of gaseous fuel with oxygen. With pulverized coal a homogeneous mixture cannot be made. The mixture can be only uniform; that is, in the limit a stream of burning mixture could be made to contain the same number of the same size, evenly spaced coal particles with the same percentage of free oxygen in every square inch of the entire cross-section of the stream of the burning mixture. This condition was far from fulfilled in Mr. Sherman's experiments. At a cross-section 2 ft. from the burner, the percentage of CO_2 varied from 8 to 16 per cent. The percentage of oxygen and the concentration of combustible probably varied in similar manner. The velocities varied from about 70 ft. per sec. in the forward direction to 5 ft. in the rearward direction. The weighted averages of the concentrations of combustible and oxygen when substituted in the equation give results greatly different from the average of the individual results that would be obtained if the concentration values of each small area of the cross-section of the furnace having nearly uniform concentration were substituted in the equation. The errors introduced in such computations are far too great to prove or disprove the validity of any equation. Such errors have the tendency to flatten the first part of the curve and make it less steep.

Lack of uniformity of concentration is a hindrance to rapid combustion. Some parts of the section of the stream of burning mixture contain too much oxygen and too little combustible. Other parts, again, contain too much combustible and too little oxygen. In either case the combustion is slower than it would be if the concentration were equalized over the entire section of the stream of burning mixture. It is the intensive mixing that constantly tends to equalize the concentration and thereby causes rapid combustion. A furnace in which the mixing has been carried to the limit and has continuously produced equal concentration has not as yet been put in operation.

FSP-54-10

Improvements in Missouri-Kansas Coals and Their Burning Equipment

By E. L. McDONALD,[1] KANSAS CITY, MO.

In the producing of Missouri-Kansas steam coals, there always have been certain undesirable qualities, the reduction of which would materially improve the quality of the coal; namely, high ash, high sulphur, low fusion temperature, excessive moisture, and inconsistent quality. While these undesirable qualities exist in most coals, the character of the local coals and the mining method (mostly strip pitting) make them more evident in these fields. The effort put forth in recent years to improve the quality and the results obtained are briefly covered in this paper. During the past 25 years improvements have been made to reduce various operating problems and to increase boiler-room efficiency. The major improvements made in this field and the actual results obtained are also covered.

IN ORDER to discuss the improvements in Missouri-Kansas coals and their burning equipment, particularly the latter, it is necessary to go back over a number of years, as these improvements were not made in a single step, but rather in a series of steps extending over a long period of time.

A period of 25 years has been adopted as it includes a period with which most people are generally familiar and it includes equipment of which both extremes still exist in some of the plants of today.

It is the intention of this paper to bring out the operating problems existing during this period, the changes or improvements made to overcome these problems, and the results of these improvements.

It is realized that many of the operating problems existing in this field are those common to other fields and have been solved in the same or in some equally efficient manner.

It also is realized that some of the improvements in operating conditions and efficiencies have been derived by the installation of the same equipment that has been generally adopted in other fields, and in order to link together the various steps that have gradually brought the boiler-room operation and efficiency in this field up to its present standard, it will be necessary to include, at least briefly, some facts that are already known. However, generalities will be avoided and the paper will be confined to showing how these developments have improved conditions in a specific field. The data shown are those obtained from equipment actually installed and from tests actually run on this equipment.

The paper has been confined to chain-grate stoker equipment, as the marked improvement made in underfeed stokers and pulverized-coal equipment has been ably covered by two previous papers.[2]

The object in combining the improvements in Missouri-Kansas coals with improvements in their burning equipment is twofold; first, to show that all of the improved operating conditions in this field cannot be credited to improvements in equipment and, second, to show what has been accomplished in improvement of coal by more careful mining.

Improvements in Coal

Of all the boiler-room operating problems, that created by slag and clinkers predominated. For years high percentages of ash or impurities were considered inherent in the coals. Means were devised to combat their effects, but little was done to reduce their existence.

The extent to which impurities can exist in these coals is more evident when it is realized that practically all are from strip-pit mines, which necessitates the use of large steam or electric shovels for the removal of 15 to 40 ft. of overburden before the thin seam (18 to 36 in.) can be reached. Unlike the deep shaft seam, this seldom has a shale cover or floor which permits the easier separation of the coal and impurities.

The incentive for better mining methods, better drainage of pits, and more careful picking of coal, which have created the improvements in Missouri-Kansas coals, has been the selling of coal on a B.t.u. basis, an idea instilled by the consumer, whereby the coal operator is rewarded for better coal and penalized for poorer coal.

With coal purchased in this manner, delivered at the station, the coal operator pays the freight on all of the moisture and impurities and receives nothing in return.

There is of course a limit to which a reduction of ash is desirable in chain-grate stokers, as a very low ash offers little protection to the rear of the stoker, resulting in increased maintenance. There is a physical limit to which moisture and impurities can be removed, and also there is, what is perhaps of most importance to the shipper, a commercial limit, governed by the increased cost of production and the increased sale value of the coal. However, the fact that it is economically practical to reduce moisture and impurities is evident from the gradual increase in the heat value caused by these reductions during the past several years.

Fig. 1 shows the yearly composite average B.t.u. of coal "as received" from several of the larger shippers during the past several years and indicates clearly the gradual improvement in quality, in some cases as much as 1000 B.t.u. per lb.

The various curves represent different yearly tonnages, the minimum being about 25,000 tons and the maximum about 125,000 tons a year. Each curve represents coal from a specific shipping point.

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[1] Efficiency Engineer, Kansas City Power and Light Company. Mem. A.S.M.E. From 1904 to 1919 Mr. McDonald was with the Commonwealth Edison Company, in plant operation and betterment work. Received technical education during this period at Armour Institute. Wrote "Burning of Mid-West Coals" for April, 1927, Fuels Meeting and several other papers on power-plant subjects. Has conducted about 400 8- to 24-hour evaporation tests on Illinois, Kansas, Missouri, Oklahoma, and Arkansas coals.

Presented at the Fourth National Fuels Meeting, Chicago, Ill., Feb. 11 to 13, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "Development and Recent Design of Stoker-Fired Equipment for Steam," by F. G. Worker and F. S. Bennett. "Progress in Central-Station Use of Pulverized Coal," by E. H. Tenney.

The reliability of the curves will be better appreciated when it is stated that each car of coal is sampled by an automatic sampler and analysis is made on composite samples of not over ten cars and usually less; hence, it is evident the points represent not a few, but hundreds of samples and analyses.

A knowledge of the mine improvements made and a check on the moisture, ash, and ashless B.t.u. show that the increased heat value is not due to a change in the inherent qualities of the coal, but rather to better mining methods, reducing impurities, and better drainage of strip pits, reducing surface moisture.

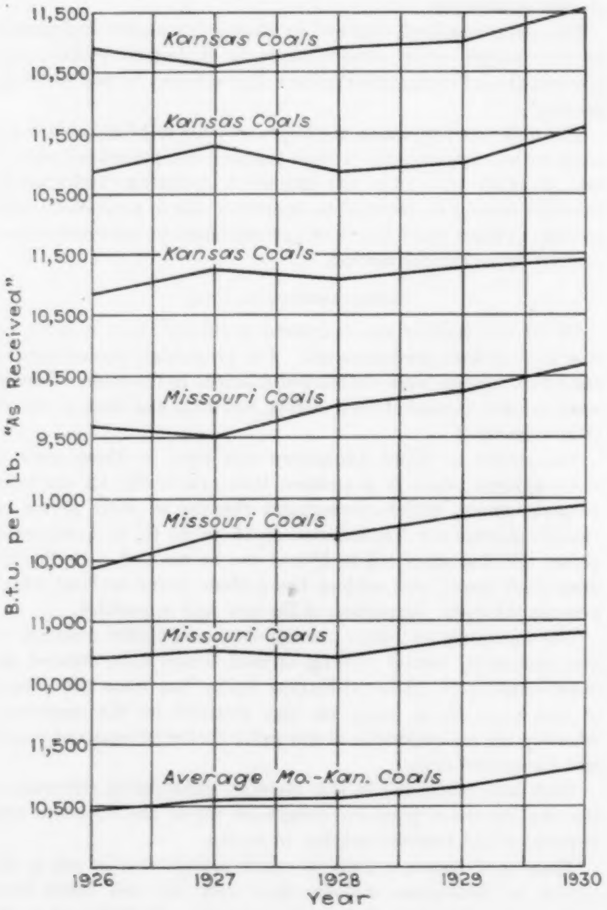

Fig. 1 Improvement in Quality of Missouri-Kansas Coals Due to More Careful Mining

This has gradually resulted in not only a better, but a more consistent quality of coal. Moistures of 20 to 22 per cent have been practically eliminated. An observation of the fuel bed seldom discloses the partly burned islands of coal and impurities found so frequently in the past. Ignition is better, lower ashpit losses exist, and more dependable ratings are available.

It should not be construed from the foregoing that the quality of this coal is without fault or that all Missouri-Kansas coals have so improved, for such is not the case; however, where an improvement has not taken place it usually is due to the lack of an incentive such as selling on a B.t.u. basis. The possibility of improvement in the quality of any of these coals should be evident from the data submitted, and the benefits derived from such must be given consideration when comparing the present-day operation with the past, for surely some of the improvements covered in this paper can be credited to the improved quality of the coal and not all to the improvements in equipment.

Improvements in Equipment

The most predominant boiler-room equipment in the modern central station of 25 years ago in this field was the natural-draft chain-grate stoker, set under a 300 to 500 hp. boiler, underfeed stokers coming into more general use a little later.

While a cross-section of such a furnace (Fig. 2, 1905) may help to visualize the operating problems of that period, they can only be fully appreciated by those who have burned the low B.t.u. Middle West coals with their high percentage of moisture, sulphur, and low-fusion ash.

The effort of the old loosely constructed chain-grate stoker, with about 20 per cent air space, through which 6 to 8 per cent of the coal sifted, trying to liberate 45,000 to 70,000 B.t.u. per cu. ft. of furnace volume with 8 to 10 ft. of flame travel, was met with slag, clinkers, smoke, high maintenance, low efficiencies, and short periods of operation—simple words, but by no means simple problems. It is the gradual improvements made to reduce these specific problems and the results accomplished that form the basis for the remainder of this paper.

First Improvement

The first real improvement came from the realization that these problems were greatly the result of overworked furnace capacities and short flame travel, and the answer, obviously, was higher set boilers.

In accomplishing this, two thoughts occurred; first, the increasing of boiler height only and, second, the reversal of the boiler and gas flow (Fig. 2, 1908-*A*). The first increased the furnace volume and lengthened the flame vertically; the second, more horizontally, resulting in longer flame travel and the utilization for more space directly below the boiler. This became the more generally adopted header-type boiler for some years to come.

The first installations of this type were built with a bridgewall extending 4 to 5 ft. above the grate, in back of which was a so-called soot hopper, which in actual operation developed into a slag hopper, collecting an almost molten mass of slag that fused into a solid body, which took hours to cool and more hours to remove.

The operating difficulties attending this form of construction were somewhat reduced by the next improvement that soon followed, which consisted of a bridgewall sloping from a short distance above the water box to a short distance below the rear boiler header, as shown by Fig. 2, 1908-*B*. This reclining bridgewall reflected heat to the passing gases. It also permitted some of the molten slag to drip back to the rear of the stoker and hence into the ashpit, although furnace temperatures permitting this were usually undesirable from a maintenance standpoint.

While this form of construction improved conditions a little, it still possessed many deficiencies, one in particular being the accumulation of slag on the low sloping wall, which gradually built up, cutting down the furnace volume and obstructing the first pass opening. No means of removal while in service was available. Hand-lancing of the boiler tubes permitted holding the boiler on the line about 20 days at gradually reduced ratings. The work of removing the slag deposit from the bridgewall was not only hot and dirty, but also hazardous.

Surprising as it now seems, this type of boiler unit remained for over 10 years, with little effort being made to overcome its many deficiencies. Slag and clinkers were accepted as operating problems, and while means were devised to facilitate their removal, little was done to prevent their formation, and it is questionable how long this unit would have remained had not the increase in turbine capacities demanded an increase in boiler capacities.

Boiler Size Doubled and Economizers Added

This demand came rather abruptly, and boilers more than doubled in size in almost a single jump, resulting in higher settings, longer flame travel, steeper bridgewalls, and of course larger combustion chambers. The advantages that might have been gained from the latter in slag reduction were lost, however, in the tendencies for higher boiler ratings, and normal operation found this unit running at an even higher B.t.u. liberation per cubic foot of furnace volume than the boiler of 10 years previous. However, there remained two points gained, longer flame travel and a steeper bridgewall, both of which permitted higher ratings for longer periods of time.

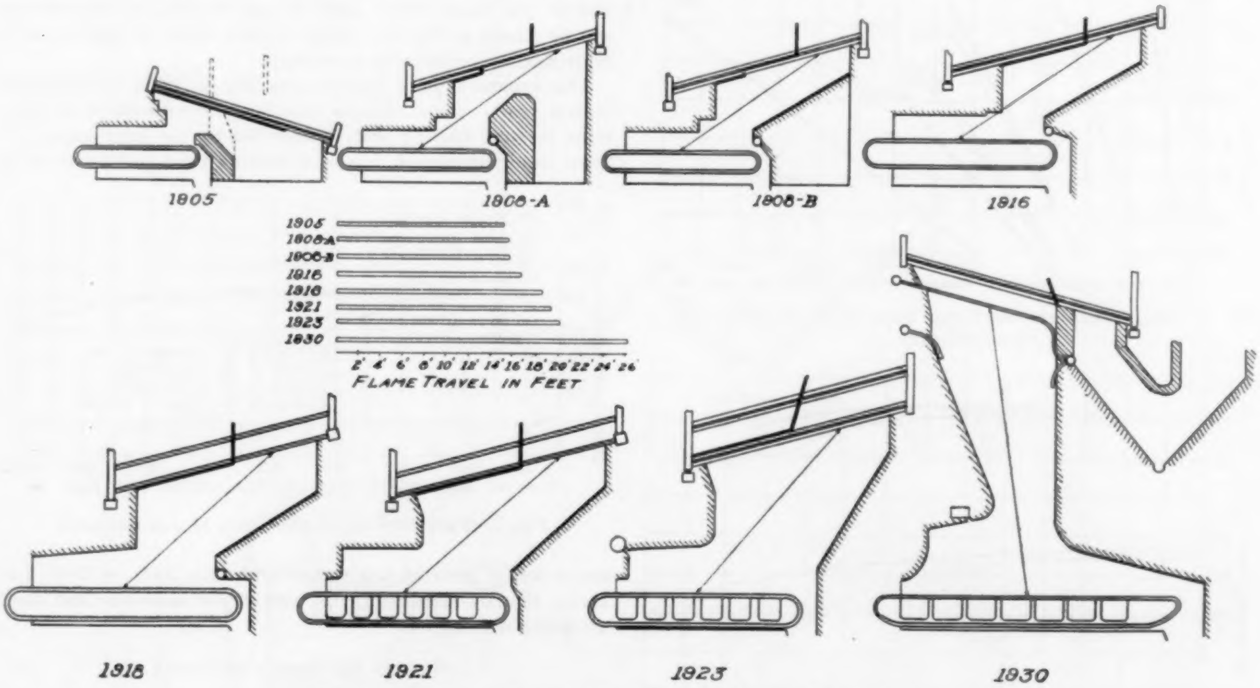

Fig. 2 Changes in Furnace Design for Missouri-Kansas Coals

This larger boiler unit (1200 to 1400 hp.) brought with it the installation of two stokers under a single boiler, and, more important, the installation of an economizer, which was the first real step in improving boiler-room economy.

The old cast-iron economizer, operating under full boiler-feed pressure, reduced the stack-gas temperatures, increased the efficiencies, and likewise increased the boiler-room maintenance, which accounts for its replacements in subsequent installations by the steel tube economizer. Incidentally, the oxygen problems encountered with the first steel-tube economizers threatened for a while to parallel the maintenance of the cast-iron economizer.

Forced-Draft Stokers

The next and perhaps the most important improvement in the actual burning equipment for this grade of coal was the introduction of the forced-draft chain-grate stoker. This not only permitted quicker pick-up, greater capacities, and higher economies, but it also permitted the burning of a greater variety of coals, ranging from the low grade of strip-pit Missouri to the semi-anthracite and anthracite of Oklahoma and Arkansas. It also brought an added step in the improvement of rear-wall construction. By eliminating the old water box, a steeper bridgewall was possible, less slag accumulated, and again longer periods of operation resulted. It is safe to say that the elimination of the old water box caused no grief among the maintenance men, particularly in plants with poor feedwater.

Air Preheaters

The continued advance of turbine bleeding for feedwater heating and the economies gained soon made evident the lack of place for large boiler and economizer surface. The reduction in economizer area in turn made room for the next step or improvement in fuel-burning equipment, namely, the air preheater.

The first large air preheater purchased for central station use in this country was installed in the Northeast Station at Kansas City in 1923. It was a plate-type heater containing 9600 sq. ft. of surface and was installed in conjunction with 12,700 sq. ft. of boiler and 8100 sq. ft. of economizer surface.

On this installation, air was heated from room temperature up to about 175 deg. Tests on the unit showed the overall gain to be in excess of that calculated, due to heat absorbed from the exit gases, and was accounted for by more rapid ignition, lower stack loss (less excess air), and lower combustible losses.

The quicker ignition created by preheated air permitted the utilization of a greater portion of the grate for fuel burning, resulting in higher combustion rates and higher capacities. This was particularly evident with the low-grade Missouri coals containing a high percentage of moisture and ash.

Contrary to the original belief that the air preheaters would be of short life, due to the high sulphur content of Missouri-Kansas coals, this first installation is still in daily use after seven years of service, and many air-heater installations have since followed, some preheating air up to 600 deg. with 350 to 400 deg. as a safe operating limit.

Along with the improvements due to the air-preheater installation came the improvements due to wider tube spacing in the first few rows of boiler tubes, and once again flame travel and furnace volumes were increased through higher-set boilers, all of which were steps in the right direction to help reduce the still-existing slag problem.

OVERFIRE AIR THE NEXT IMPROVEMENT

For several years considerable experimenting had been carried on (particularly by the Commonwealth Edison Company) with overfired air as an aid to the better combustion of Middle Western coals and as a means of reducing smoke. This can possibly be inserted here as the next step in the progress of improvements in Missouri-Kansas fuel-burning equipment, as anything that assists in reducing smoke surely deserves a place in the line of progress.

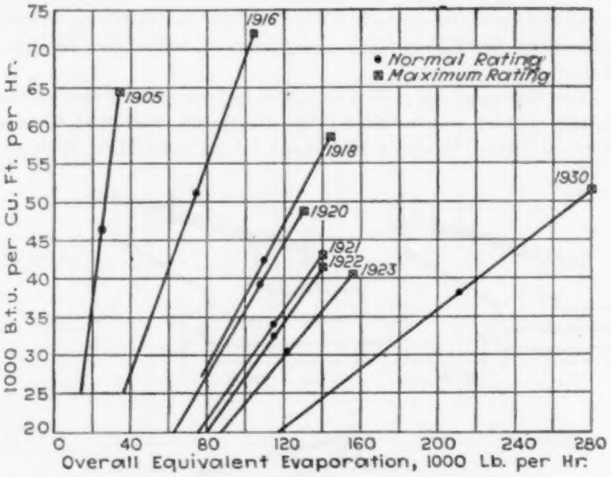

FIG. 3 B.T.U. LIBERATED PER CUBIC FOOT OF FURNACE VOLUME ON VARIOUS UNITS

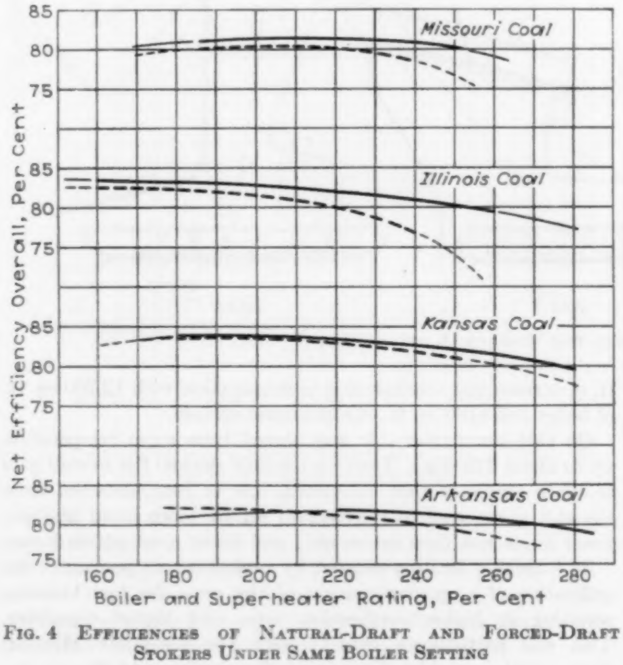

FIG. 4 EFFICIENCIES OF NATURAL-DRAFT AND FORCED-DRAFT STOKERS UNDER SAME BOILER SETTING

Application was first made in this section principally as a means of smoke reduction, and its value when burning some of the long-flamed, high-volatile Missouri coals was particularly evident; however, subsequent tests showed a gain in excess of that anticipated.

WATER-TUBE SIDEWALLS

Throughout these years various means had been tried to reduce two of the most annoying of operating problems, sidewall clinkers and sidewall maintenance, perhaps more serious in the Middle Western field than in others because of the high percentage of low-fusion ash in these coals.

Numerous kinds of refractory materials, air-cooled tile, and waterwall boxes were tried with various degrees of success, but it remained for the water-cooled sidewall to make the greatest improvement in this direction. This form of construction, together with the radiant-heat superheater, which made its appearance shortly afterward, has done much toward the reduction of furnace maintenance, and the latter has played its part in improving the plant economy of many of the older plants; and so the water-cooled sidewall and the radiant superheater can be placed as the next major improvement, at least in Missouri-Kansas fuel-burning equipment.

This general type of construction (Fig. 2, 1923) remained for several years. Some changes were made in subsequent installations in tube spacing and furnace height, the most important being the improvement in stoker design, which permitted using one stoker in place of two stokers under the 20-ft. to 24-ft. furnaces, thereby eliminating the two center sidewalls and their attending deficiencies.

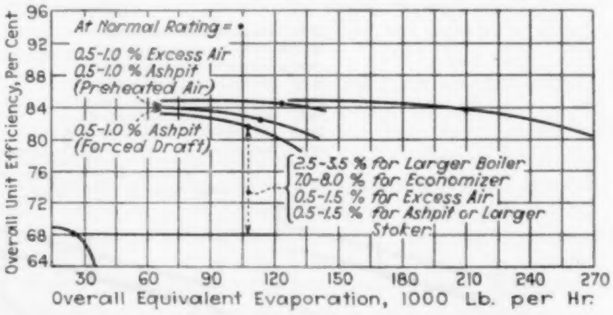

FIG. 5 TEST EFFICIENCIES ON FIVE MAJOR CHANGES

MODERN EQUIPMENT OF TODAY

With the adoption of forced-draft stokers, economizers, air preheaters, waterwalls, etc., there appeared to be little room for increased efficiency in the present modern unit.

Steam rates had increased, but furnace volume increased in greater proportion, resulting in a gradual reduction of B.t.u. liberated per cu. ft. This, together with the gradually increasing flame travel and better tube spacing, permitted longer periods of operation, but not long enough, as slag and maintenance still controlled.

The answer was obviously to again increase flame travel, and this time a very material increase was made in the height of the boiler setting (Fig. 2, 1930).

In this unit, improvements in stoker construction permitted increasing the preheated-air temperature to 350 or 400 deg. for normal operation, and the water-wall surface has been extended to include the front and rear walls. No longer is the refractory surface of the arch exposed to the full blast of furnace temperature or flame, the brunt of the force being taken by the water tubes placed in front and in some places entirely replacing the refractory surface. But the value of this water wall extends beyond its ability to prolong the life of firebrick. Its high rate of heat absorption has permitted an upward trend in the B.t.u. liberated per cu. ft. of furnace volume, and the normal rate of today is close to the maximum rate of the unit previously shown.

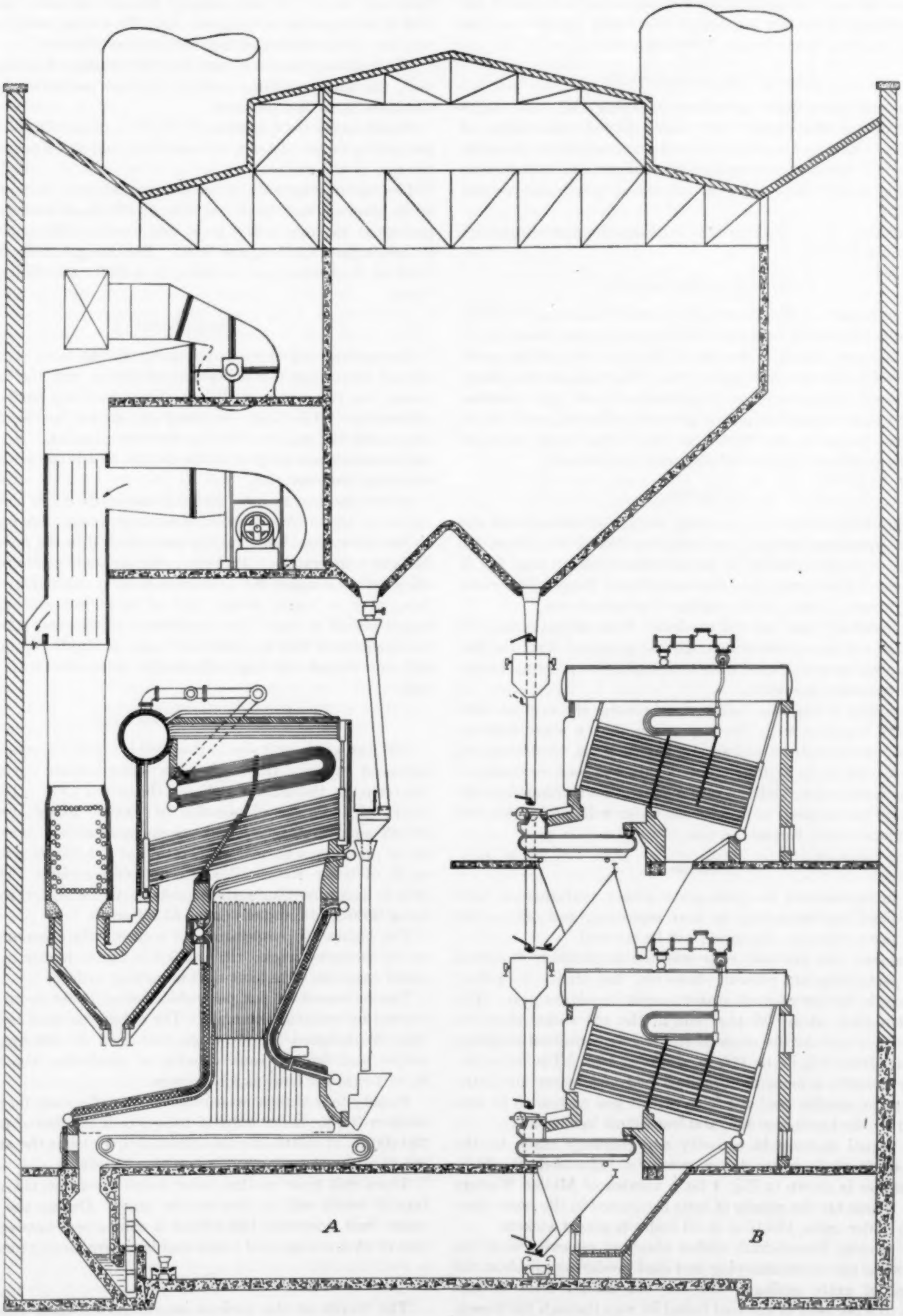

FIG. 6 EQUIPMENT OF 1930 (*A*) AND OF 1905 (*B*) IN SAME STATION
(Grand Avenue Station, Kansas City Power & Light Co.)

It has permitted increasing boiler capacities far in excess of the same amount of surface applied to the boiler proper and has aided in securing much longer operating periods.

RESULTS OF IMPROVEMENTS

Time and space have permitted covering only the major changes during this period. The more rugged construction of equipment, the many improvements in mechanical features, the value of soot-removal equipment, boiler-room instruments, etc., all of which have aided in improving conditions, remain untouched.

A summary of the results obtained from the more important changes will be made.

EVOLUTION OF BRIDGEWALL

The evolution of the rear wall or bridgewall is of interest. The gradual change in contour of this wall is evident from Fig. 2. During the past 25 years, the rear of the original wall has gradually described an arc, until today it has turned almost completely over. Each change was an improvement over the previous wall, and each change brought a gradual reduction, and finally an almost complete elimination of two of the most vexing of operating problems, bridgewall slag and maintenance.

FURNACES

The gradual tendency to prolong the life of refractories and increase operating periods by reducing the B.t.u. liberation previous to the installation of water walls is evident from Fig. 3, which shows the average and maximum B.t.u. liberated per cubic foot of furnace volume in the various furnaces shown.

As typical of most curves produced from actual data, the points are not as consistent as might be produced from the frequently used manufactured data or imaginative powers; nevertheless, the trend is evident.

The radical increase in boiler size between the unit of 1905 and 1926, together with the impression that a short increase in flame travel alone would permit higher B.t.u. liberation, accounts for the jump at that point. This was soon corrected in the next installation, and the rate of liberation continued downward until the modern unit, with its water walls and arches and longer flame travel, turned the tide.

STOKERS

The improvements in chain-grate stoker performance have been voiced too frequently to bear repeating, and only a few points, not commonly discussed, will be covered.

Of course, the greatest step was the application of forced draft, permitting air control; however, the ability to reduce air spaces in the forced-draft stoker was an important gain. The reduction from about 20 per cent in the old stoker down to about 9 per cent in the modern stoker has permitted reducing coal sizes from $1^1/_2$ in. or $1^3/_4$ in. down to 1 in. The latter obviously permits a more uniform fuel bed and better air distribution; the smaller coal particles permit less carbon to be embedded by the low-fusion ash, and less ashpit loss results.

The actual increase in capacity and economy made by the forced-draft chain-grate stoker over the induced-draft chain-grate stoker is shown in Fig. 4 for a number of Middle Western fuels. These are the results of tests conducted in the same plant on two boiler units, identical in all respects except stokers.

The modern forced-draft stoker also has removed from the boiler room one more annoying and dust-producing problem, the disposal of grate siftings. On the old natural-draft stokers from 6 to 8 per cent of the coal found its way through the loosely constructed grates in the form of fine dry siftings, the dust and disposal of which were usually a problem. Containing 90 to 95 per cent of the heat value of the coal, economy demanded that it be returned to the grate, and the mixing with fresh coal was not always accomplished with full satisfaction.

While siftings have not been entirely eliminated in the present unit, the small quantity and the modern methods of disposal eliminate them as a problem.

Stoker speeds have increased from 20 to 65 and 70 ft. per hour, permitting faster pick-up, thinner fires, and more protection to the rear of the stoker.

The fuel-burning rate of the old natural-draft unit was about 40 lb. average and 55 lb. maximum. Without forced draft or preheated air, this was a little high, and on the unit of 1916 it was dropped to 35 lb. and 50 lb. This has gradually increased until on the modern unit of today it is 48 lb. and 65 lb., respectively.

EFFICIENCIES

Throughout all these years many things have contributed toward increasing the boiler-unit efficiency, but the major increase has come from additional heat-absorbing surface. The efficiencies of the units discussed are shown in Fig. 5, which also shows the major causes for the various gains. No attempt will be made here to go into the details, which are evident from what has been covered.

It is of interest to note that the test efficiency of the modern unit is no higher than the unit of seven years ago, due principally to the commerical limit having been reached in the reduction of flue-gas temperature. However, the present unit obtains its efficiency at a higher B.t.u. liberation, at a much higher steam flow, over a longer range, and of most importance, over a longer period of time. In considering efficiencies, it must also be remembered that the inherent losses in Middle Western coals will not permit the high efficiencies obtainable with Eastern coal.

INCREASED CAPACITIES

The increase in unit size in this field is not as impressive as the increased capacity from the same water-heating surface, best illustrated by the units of 1923 and the unit of 1930. The former produced a normal evaporation of 100,000 lb. of steam with 20,843 sq. ft. of boiler and economizer surface, whereas the latter produces a normal evaporation of 175,000 lb. with 20,900 sq. ft. of boiler, waterwall, and economizer surface. The difference in heat-absorbing surface exists in the air heater, the former being 9600 sq. ft. and the latter 23,750 sq. ft.

The higher air temperature, of course, only accounts for part of the increased work; the balance is due to higher m.t.d. and larger amounts of radiant-heat absorbing surface.

The increased output per cubic feet of boiler room is made evident by an actual example. The old double-deck boiler room (Fig. 6) designed 28 years ago contained 40 500-hp., 185-lb. stoker-fired boilers, each capable of producing about 30,000 lb. of steam per hour as a maximum.

Twenty-four of these were removed to make room for six of the modern 700-lb. boiler units of today, each capable of producing 250,000 lb. of steam as a maximum, or over twice the amount of steam under the same roof as previously existed.

There still exist in this boiler room 16 of the old units the fate of which will in time be the same. During the last few years their operation has offered a comparison between operation of 25 years ago and today that is not frequently available.

SUMMARY

The merits of the various improvements have been shown throughout the paper. The increased capacities and efficiencies are of course quite evident; however, the most impressive comparison and perhaps the most logical proof of the vast improvements in operating conditions is the ability to keep the modern unit in continuous operation for long periods of time.

Operating periods of 20 to 25 days, considered good in the old units, have been lengthened to over 100 days.

Slag, clinkers, smoke, high maintenance, and low efficiencies, representing problems in the old boilers, have gradually been reduced, until today, while by no means eliminated, they are but a shadow of their former selves.

The greatest single stride in all of this progress has come with the modern unit of today, where flame travel was increased from 20 to 26 ft. at a single step and water-cooled front and rear arches were installed. This water-cooled rear arch has not only aided in reducing the ashpit loss, but also in protecting the rear of the stoker from the intense furnace heat.

Such protection, together with the rugged construction of this modern stoker with its heavy framework and drive and its water-cooled rear shaft, has brought the maintenance cost down and the reliability of the stoker up to a place previously unknown.

The handling of ash has been so simplified by the modern installation that only three men now are required to handle ash from the same coal tonnage that formerly required eighteen men. Ash can be removed at any time without admitting excess air. High maintenance, dirt, dust, and the hazards of handling hot ash are no longer problems.

It has not been intended for this paper to convey the idea that the improvements made have eliminated all of the operating problems and that the boiler room of today is a playhouse, for such is not the case.

Many of these improvements have brought with them new problems, but the prolonged periods of operation make evident that they also have reduced other more serious problems, and only those who have operated both extremes can fully appreciate the value of improvements in Missouri-Kansas coals and their burning equipment.

Discussion

J. G. Worker.[3] The author's experiences certainly should interest users of chain-grate stokers, particularly for Missouri and Kansas coals.

A recent survey of the coal consumed by public-utility plants during 1928 shows that out of a total consumption of 42,000,000 tons of coal, there was about 20,000,000 tons burned on underfeed stokers, 12,000,000 tons on chain-grate stokers, 7,000,000 tons as pulverized coal, and 2,000,000 tons on overfeed stokers and by hand firing. This report also showed that there was a little over 1,000,000 tons of Missouri and Kansas coal burned by public utilities during 1928; so that if the author has shown a way to save even 5 per cent of this by better application of stokers and boilers it seems that this work has been worth while.

As to the author's curve, shown in Fig. 3, indicating a gradual lowering of the heat-release rate in chain-grate furnaces, it is pleasing to see that the author in his 1930 installation has shown a higher heat-release rate than the 1923 installation. Would it not be possible in the next installation to even increase this heat-release rate up to 70,000 B.t.u. per hour? It seems that 25 ft. from the grate to the tubes is a somewhat expensive design. Have we not gone too far in the building of large combustion furnaces when we consider the effect that modern furnace elements such as water walls, slag drips, and preheated air have in overcoming the difficulties that directed us toward decreasing the heat-release rate and building these larger furnaces? Would it not be possible to use the 1923 application with all of the present modern furnace elements of construction and still obtain as good results as the 1930 installation?

A few years ago a prominent engineer in the Middle West declared that the limit of chain-grate development was 125,000 lb. of steam per hour in a boiler. It is noted that the author has shown that there will be chain-grate installations of 250,000 lb. of steam per hour. This surely shows a marked development in coal-burning capacity of chain-grate stokers. The writer would like to ask the author what is the possibility of developing chain-grate stokers for 500,000 lb. of steam per hour. This would probably mean a burning of 80 lb. of coal per square foot per hour.

The author did not bring out any of the developments in the design of the stoker structure itself. It would be interesting if the author would mention some of the major designs in modern forced-draft chain-grate stokers compared with the natural-draft chain-grate stokers of 20 years ago. There must have been many important improvements in chain-grate link design or air-control compartments and arch construction. Surely some of the stoker structure designs have helped to bring about the results that the author has so ably mentioned.

F. H. Rosencrants.[4] In view of Mr. Worker's remarks concerning the limitations in the size of traveling-grate stokers, it will perhaps be of interest to known that in England a traveling-grate stoker 32 ft. wide by 22 ft. long is now in successful operation at the Central Electric Station in Liverpool. It is believed that this is the largest traveling-grate stoker thus far built.

John Van Brunt.[5] The author has contributed a concise and valuable summary of the development of the chain-grate stoker and furnaces for Mid-Western fuels. This development has been paralleled in other plants, but this is the first attempt to knowledge to bring the data together and present them in such excellent form.

Mr. Worker referred to Fig. 3 and to the form of curve drawn through the maximum and normal points of the lines showing the relation between overall evaporation or B.t.u. released per cubic feet per hour. This curve shows very clearly the influence of the water-cooled furnaces on the possible maximum combustion rates, but it is not probable that the maximum combustion rates reached in 1930 will be materially increased. The limit is the rate at which the slag accumulation in the boiler tubes will prevent continuous operation of boilers at the designed ratings.

Mr. Worker's question as to why stokers of this type could not be built in larger size than those referred to in the paper can be easily answered. At the present time standard designs are available for widths up to 24 ft. to 25 ft. Wider furnaces can be equipped by installing two stokers in a furnace, and with other designs it would be possible to extend the widths to 28 ft. or 30 ft. The cost of designing special machines of this type is high, and it is largely a question of dollars and cents to the buyer. Manufacturers can design and build these extremely large stokers, but unless the purchaser is willing to pay for the cost of design, it is not likely that the manufacturers will urge larger sizes.

Author's Closure

Mr. Worker suggests the possibility of 70,000 B.t.u. heat release in future stoker furnaces and also suggests that 25 ft. from the grate to the lower row of tubes may be expensive furnace construction.

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[3] General Sales Manager, American Engineering Company, Philadelphia, Pa. Mem. A.S.M.E.

[4] International Combustion Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

[5] Vice-President for Engineering, Combustion Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

The author believes that with some modification of the present furnace design, a figure of 70,000 B.t.u. could be reached as a peak. However, higher combustion rates and lower settings mean more maintenance and more slag problems, and it is simply a matter of what conditions one is content to operate under. Each succeeding step in longer flame travel has brought a reduction in slag problems, and the present length is not deemed excessive. Also, when it is considered that the 1930 unit in question occupies the same space formerly occupied by four smaller units and produces over twice the capacity, the higher setting will not appear so out of proportion.

Mr. Worker is quite correct in his statement that it would be of interest to know some of the detailed mechanical improvements in the design of the modern stoker, but this is a subject by itself, on which a lengthy paper could easily be written, and it was deemed best not to include it.

Mr. Van Brunt has covered the inquiry regarding the possibilities of a stoker for 500,000 lb. evaporation per hour.

FSP-54-11

Comparative Performance of a Large Boiler Using Oil and Natural-Gas Fuels

By F. G. PHILO,[1] LOS ANGELES, CALIF.

Tests were made with utmost care by well-trained and experienced observers, and the data presented are comparable in accuracy with carefully conducted tests burning coal. The boiler tested was a Babcock & Wilcox cross-drum, straight-tube type with interdeck superheater. The furnace was completely inclosed by Bailey-type water-cooled walls. The Babcock & Wilcox air heater was of tubular construction. Peabody combination oil and gas burners were used. The highest rate of output at which the boiler was tested was 450,751 lb. of steam per hour, which represented operation of the 3416-hp. boiler at 413.2 per cent of rating. Efficiency-capacity characteristics are straight lines between 100 per cent and 400 per cent rating. Efficiency with oil fuel is approximately 4 per cent higher than with gas due to higher "hydrogen losses" with gas fuel. Superheated-steam temperatures are considerably higher when burning gas than when burning oil fuel, due jointly to lower water-wall heat absorption and larger flue-gas volume while burning gas fuel. Maximum heat liberation was 27,300 B.t.u. per cu. ft. of furnace volume, although it is felt heat-liberation rates as high as 60,000 B.t.u. per cu. ft. might be satisfactorily maintained in this furnace.

PERFORMANCE data showing the operating characteristics of a large boiler burning oil and natural-gas fuels were obtained by a series of 14 tests made between June and November of 1929. Boiler 50 was tested. This is one of the three boilers serving unit 10 in the Long Beach Steam Station of the Southern California Edison Company, Ltd. These tests were made with utmost care by experienced men under the direction of the efficiency engineer of the Long Beach Steam Station. The A.S.M.E. Boiler Test Code was followed and results were computed in accordance with the code.

Because of widespread extension of natural-gas transmission pipe lines and increasing consumption of oil and natural gas for central-station and industrial power, it is felt that the data presented will be of value and timely interest. Considerable related data on forced- and induced-draft fan, water-wall, superheater, and air-heater performance were obtained, but are largely omitted as irrelevant to the purpose of this paper. For the sake of brevity actual test readings and calculations are omitted.

It is felt that, although a calibrated venturi meter was used for measuring the boiler feedwater in these tests, the data obtained are comparable in accuracy with carefully conducted tests burning coal. While using oil or gas fuel such difficulties as obtaining representative fuel samples, indeterminate quantities of unburned fuel in solids discharged from the stack, and unburned fuel in the ashpit refuse are not encountered. Once steady temperature conditions are obtained throughout the boiler and furnace, boiler tests, using liquid and gaseous fuels, can be obtained with a high degree of accuracy within periods of possibly not over two to three hours. In this series of tests trial runs were of four hours' duration at constant steam flow and excess air. Observations were taken every half hour during the test run. Approximately one hour before each test run, the boiler was

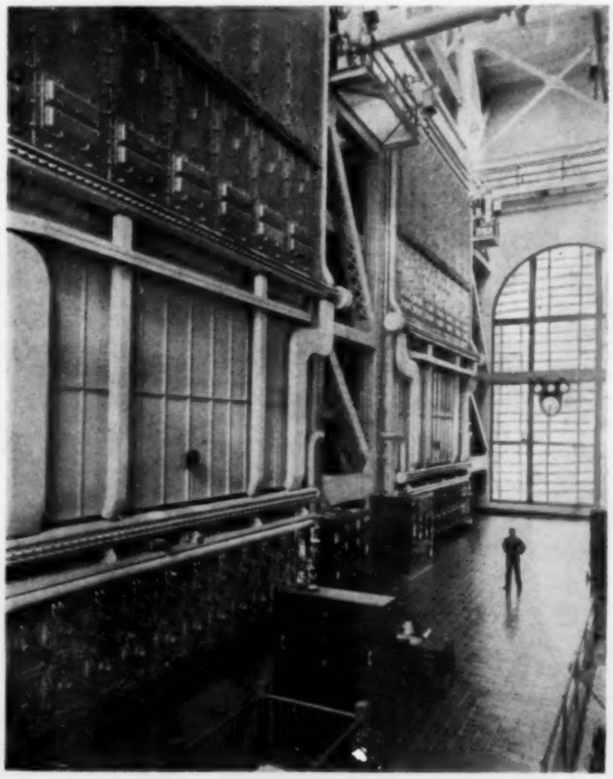

FIG. 1 BOILER-ROOM, UNIT 10 BOILERS, LONG BEACH STEAM STATION

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[1] Superintendent of Steam Generation, Southern California Edison Company, Ltd. Mem. A.S.M.E. Mr. Philo was educated at Ohio State University and in 1911 participated in the Delray tests of large boilers of the Detroit Edison Co. From 1912 to 1914 he was boiler-room engineer for the Corn Products Refining Co., Edgewater, N. J., and later was efficiency engineer for the Harwood and Hauto power stations of the Lehigh Navigation Electric Co. During 1918–1919 he placed in operation the Burling Slip Station "A" of the New York Steam Corporation, and was appointed chief operating engineer of its downtown New York district. From 1919 to 1924, Mr. Philo was connected with Stone & Webster, Inc., as engineer in the power-station betterment division. Most of his time was spent in power-station betterment and initial operation of new power stations. In 1924, after placing the Long Beach steam plant of the Southern California Edison Co., Ltd., in operation for Stone & Webster, Inc., he was appointed to his present position.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

brought to the desired steaming rate and steady operation ascertained before readings were taken.

The highest rate of output at which the boiler was tested was 450,751 lb. of steam per hour. This output represented operation of the boiler at 413.2 per cent of the boiler rating of 3416 boiler hp. The relations at various boiler ratings of the more important characteristics of the equipment under test are shown in Figs. 4 and 5. The efficiencies obtained with oil fuel parallel closely the efficiencies obtained with gas, but are about 4 per cent higher. The ratings given are the heat output of the boiler, superheater, and water walls expressed as percentages based on the 34,162 sq. ft. of heating surface in the boiler.

Comparative Efficiencies With Oil and Gas Fuels

Within close limits of accuracy the efficiency-capacity characteristics of the combination of furnace, boiler, superheater, and air heater burning both fuels may be assumed to be straight lines between 100 per cent and 400 per cent of rating. These are shown in Fig. 6. It is interesting to note that the unaccounted-for losses in both oil- and gas-fuel tests are approximately 1 per cent in each case. No surprise at this should be expected, however, as such indeterminate losses as carbon in stack gases and ash refuse are not a factor in the burning of liquid and gaseous fuels.

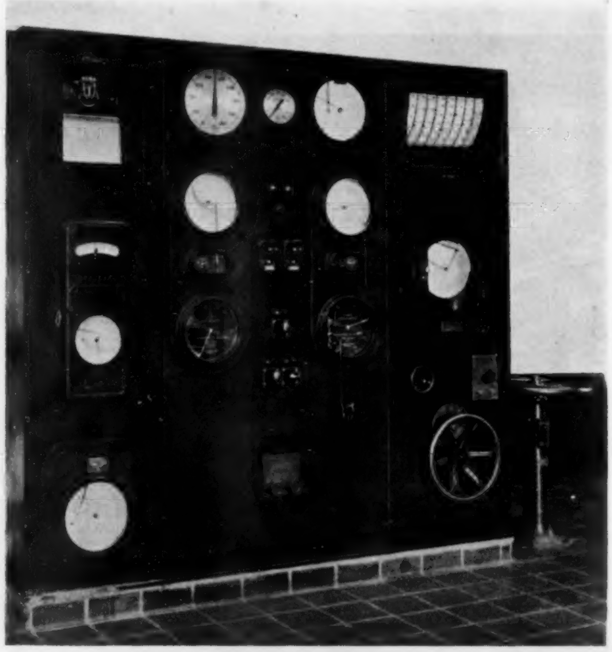


Fig. 2 Boiler Instrument and Combustion Control Board

In the series of tests using gas fuel the heat-balance calculations assisted very materially in establishing what is considered the correct shape and slope of the efficiency-capacity curve. For some undetermined reason the efficiency as shown in tests using gas fuel at the two lowest ratings did not fall in line with the efficiencies shown in the tests at higher capacities. This deviation, however, was only approximately 1 per cent, and was probably caused by the usual difficulties in gas measurement at the lower rates of gas flow. The corresponding heat-balance efficiencies indicate this error plainly.

The difference in heat loss due to the amounts of water formed by combustion of the hydrogen in the two fuels accounts for the difference in efficiencies obtained with oil and gas fuels. The "hydrogen losses" at 400 per cent boiler rating are theoretically 10.3 per cent for gas fuel and 6.1 per cent for oil fuel.

This theoretical difference, 4.2 per cent, checks closely with the difference of 4.15 per cent as shown in the tests. It is interesting to note that, for each pound of gas burned, approximately 2 lb. of water are formed in combustion, and for each pound of oil burned 1 lb. of water is formed. This water of combustion passes up the stack as superheated steam. On a cold, still day this steam is observed as a cloud of vapor above the stacks.

Combustion is normally controlled by complete Bailey automatic control, but during test periods control was by push button, with the fuel control set for constant flow and only such changes in fan speed as were necessary to maintain constant excess air. Peabody combination oil and gas burners were used. The oil burners are of the Peabody return type, but the return feature was not used during the tests.

All instruments were calibrated carefully before and after each series of tests. Water to the boiler was metered through a $3^1/_4$-in. by 8-in. Builders Iron Foundry venturi tube in the boiler-feed lines, the flow being recorded and integrated on a standard recorder which was calibrated by means of a water column against builders specifications for the tube. Previous checks on similar equipment against weighed quantities indicate an accuracy by this method within 0.3 of 1 per cent at ranges above 20 per cent of full-meter capacity.

Gas fuel was metered by a standard orifice in general use for this purpose, the orifice being located in a specially constructed 68-ft. straight section of 12-in. pipe. Pressure and differential connections were taken at four equally spaced points around the circumference of the pipe. The differential drop across the orifice was measured by a U-tube manometer at the higher rates of flow and with an Ellison inclined-tube manometer at the lower rates of gas flow. Orifice-meter calculations include determinations of the gravity, temperature, and absolute pressure of the gas taken with standard equipment, and the proper coefficients applied. With a given orifice the accuracy of this method of metering is greatest at the higher flow differentials, but the maximum error at all points taken should not exceed 0.5 of 1 per cent. The heating value of the gas was determined continuously by a Thomas recording gas calorimeter which was checked against hydrogen. This record was also checked by periodic determinations with a Junkers calorimeter. The specific gravity of the gas was determined with a standard Edwards density balance. The oil was weighed cold and fed to the boiler through heaters maintaining by thermostatic control approximately 250 deg. fahr. temperature at the oil-burner supply header. The heating value and specific gravity of the oil were determined independently by Smith-Emery Company of Los Angeles and in the station laboratory. Chemical analyses were made of a sample of oil taken from Oil Test No. 6 and a sample of gas taken from Gas Test No. 1. Since all oil used was from the same supply tank and all gas was received from the same sources, these were taken as representative for all tests.

The temperature of the superheated steam leaving the boiler, the feedwater entering the boiler, and flue gases leaving the induced-draft fan were taken with calibrated industrial glass thermometers. Other important measurements were made with a potentiometer-type Leeds and Northrup recorder. Flue-gas analyses were made with two sets of Orsat apparatus, and samples of gas were taken alternately from the east and west sides of the boiler. Pressures of the air and flue gases were measured with water in glass manometers.

The maximum overall pressure drop in the air and gas system was 16.2 in. water with oil fuel, and the boiler operating at 413.2 per cent of rating. The resistance of the oil-burner diffuser plates increases the pressure drop between the forced-draft fans and the furnace when burning oil. The greater volume of flue gases makes the drop from furnace to stack greatest when burning gas. The maximum heat liberation per cubic foot of furnace volume during these tests was approximately 27,300 B.t.u. per cu. ft. at 413.2 per cent rating. The relation of heat liberation to boiler rating is shown in Fig. 7. Based on observation of the combustion characteristics of the fuels used in these tests it is felt that heat-liberation rates of at least 60,000 B.t.u. per cu. ft. of furnace volume might be satisfactorily maintained with oil or gas fuel with this furnace and the combustion equipment used.

Hydrogen and Dry-Gas Losses Compared

It is interesting to note the relative magnitudes of the hydrogen and dry-gas losses. At 300 per cent rating, burning oil fuel, the hydrogen loss was 1.44 times the dry-gas loss. At the same rating, burning gas fuel, the hydrogen loss was 2.47 times that of the dry-gas loss. It is interesting also to note that the total boiler losses with gas fuel are 1.35 times those of oil fuel at 300 per cent boiler rating. The dry-gas losses of both fuels decrease uniformly with the rating, while the losses due to hydrogen are approximately uniform within the range of capacities shown. These relations are shown in Figs. 8 and 9.

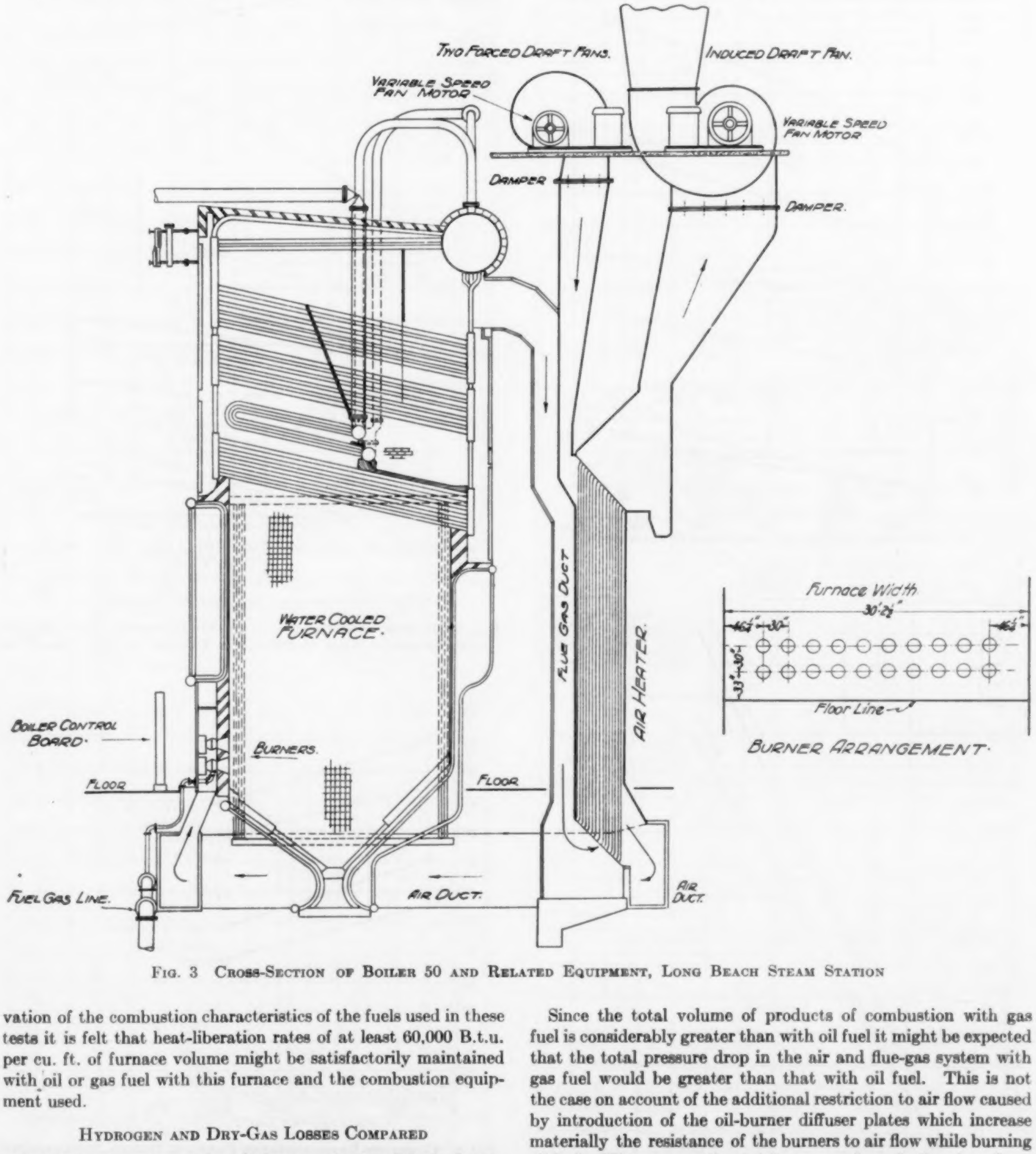

Two Forced Draft Fans.
Induced Draft Fan.
Variable Speed Fan Motor
Variable Speed Fan Motor
Damper
Damper.
Water Cooled Furnace.
Flue Gas Duct
Air Heater.
Boiler Control Board.
Burners.
Floor
Floor
Fuel Gas Line.
Air Duct.
Air Duct.
Furnace Width
30'-2½"
46½"
30"
46½"
30"
33"
Floor Line
Burner Arrangement.

Fig. 3 Cross-Section of Boiler 50 and Related Equipment, Long Beach Steam Station

Since the total volume of products of combustion with gas fuel is considerably greater than with oil fuel it might be expected that the total pressure drop in the air and flue-gas system with gas fuel would be greater than that with oil fuel. This is not the case on account of the additional restriction to air flow caused by introduction of the oil-burner diffuser plates which increase materially the resistance of the burners to air flow while burning oil fuel. The comparative total pressure drops in the air and gas system using both fuels and the effect of damper regulation are shown in Fig. 10.

When segregated, as shown in Fig. 11, it will be noticed that

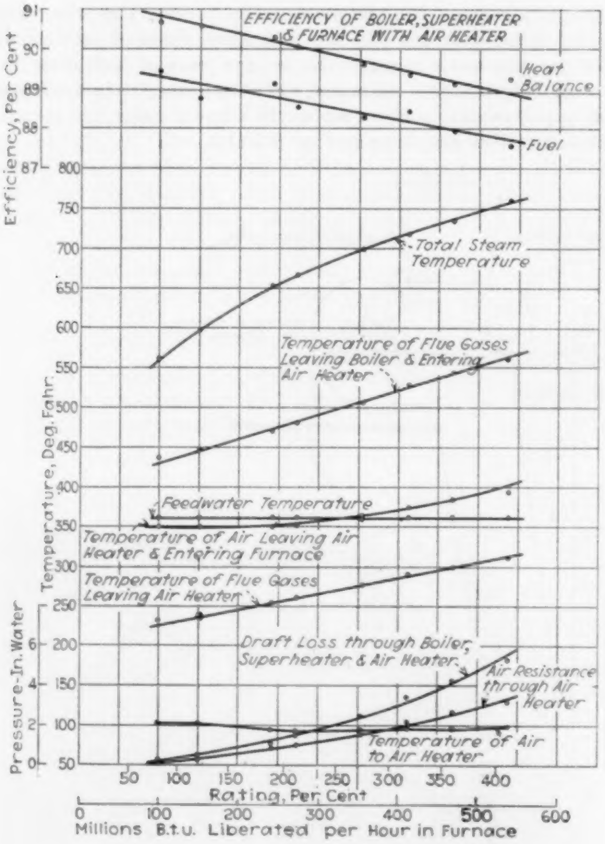


Fig. 4 (Upper Left-Hand Figure) Boiler, Air Heater, and Superheater Performance Characteristics With 15 Per Cent Excess Air and Oil Fuel

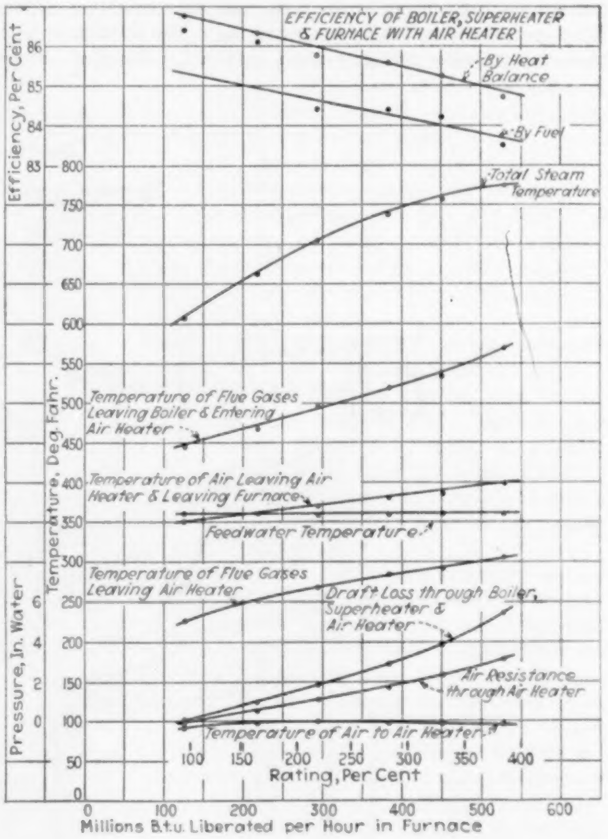
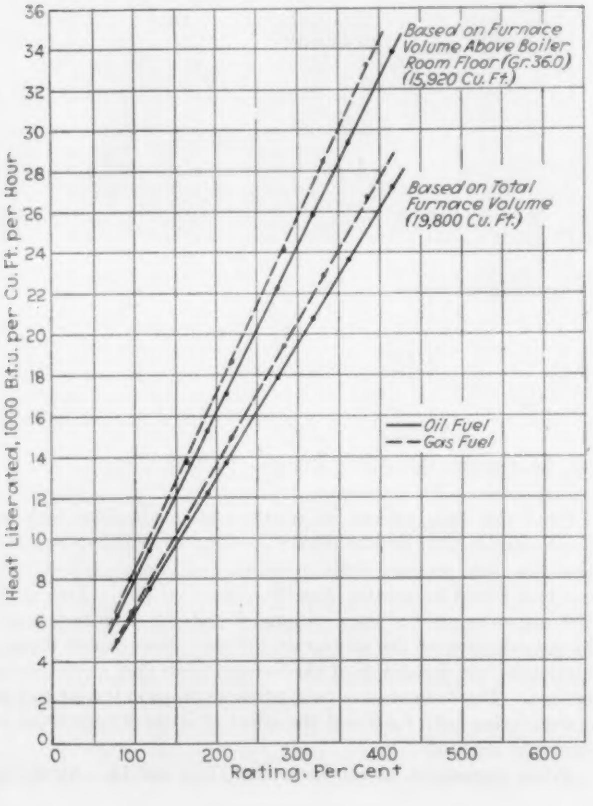


Fig. 5 Boiler, Air Heater, and Superheater Performance Characteristics With 15 Per Cent Excess Air and Gas Fuel



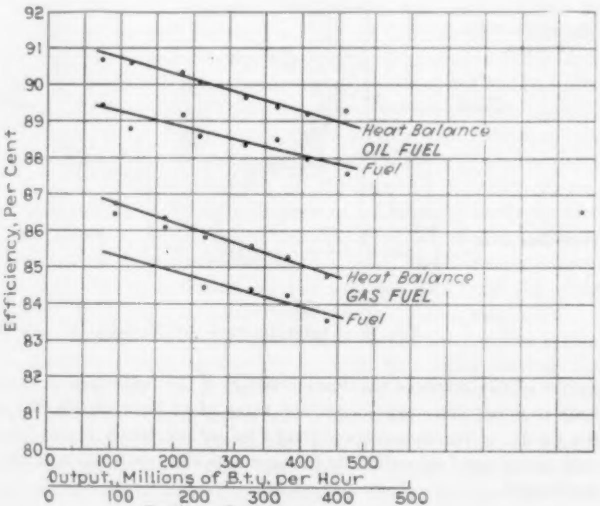


Fig. 6 Combined Efficiencies of Furnace, Boiler, Superheater, and Air Heater

Fig. 7 (Lower Left-Hand Figure) Heat Liberation per Cubic Foot of Furnace Volume

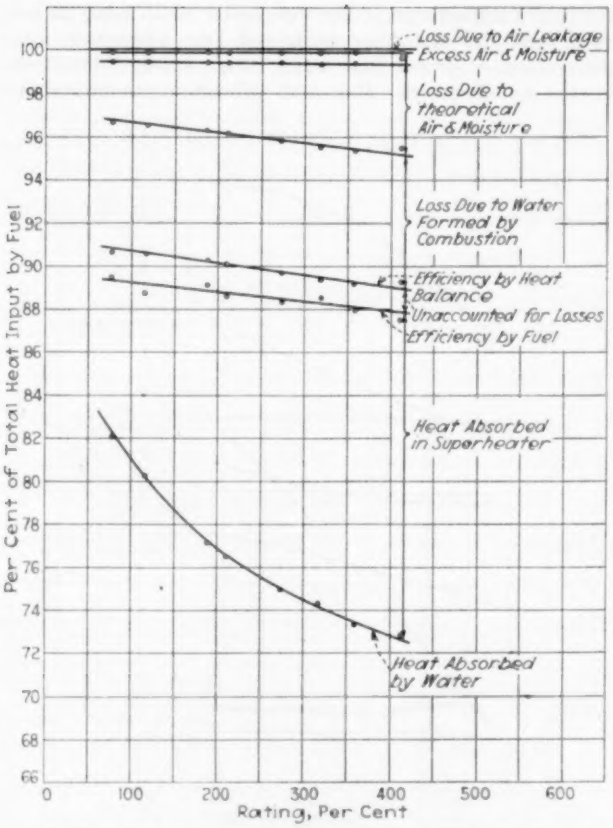


FIG. 8 HEAT BALANCE VS. BOILER RATING WITH OIL FUEL

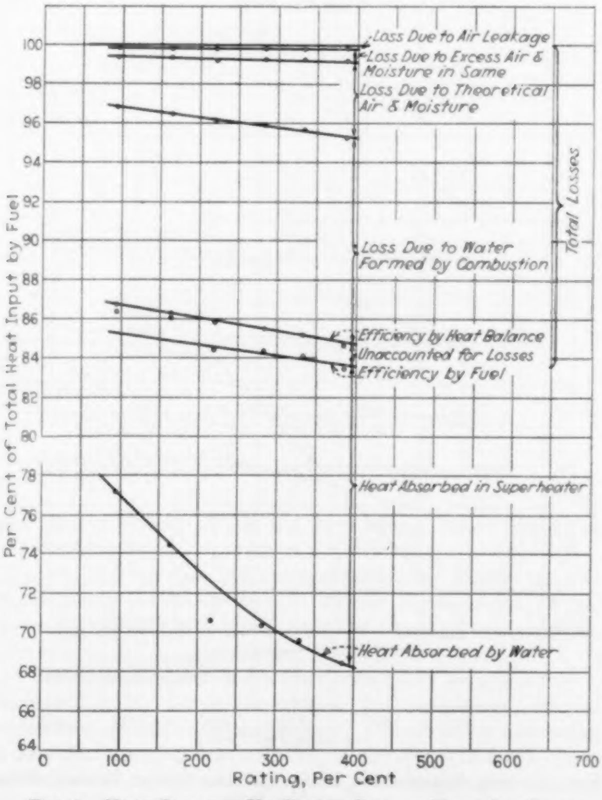


FIG. 9 HEAT BALANCE VS. BOILER RATING WITH GAS FUEL

the oil-fuel air pressures are considerably higher than those for gas, but the induced-draft requirements are greater for gas than those for oil fuel, due principally to the decreased boiler efficiency while using gas fuel and increased volume of combustion products per pound of gas fuel, caused mainly by the larger volume of water vapor formed in the combustion of the natural gas.

Curves of the more important temperatures observed are shown in Figs. 12 and 13. These curves agree very closely with the exception of the temperature of the superheated steam. Fig. 14 illustrates to a larger scale the superheater characteristics. As might be expected the superheated-steam temperatures are considerably higher when burning gas than when burning oil fuel due, jointly, to lower water-wall heat absorption with gas fuel and larger flue-gas volumes while burning gas fuel.

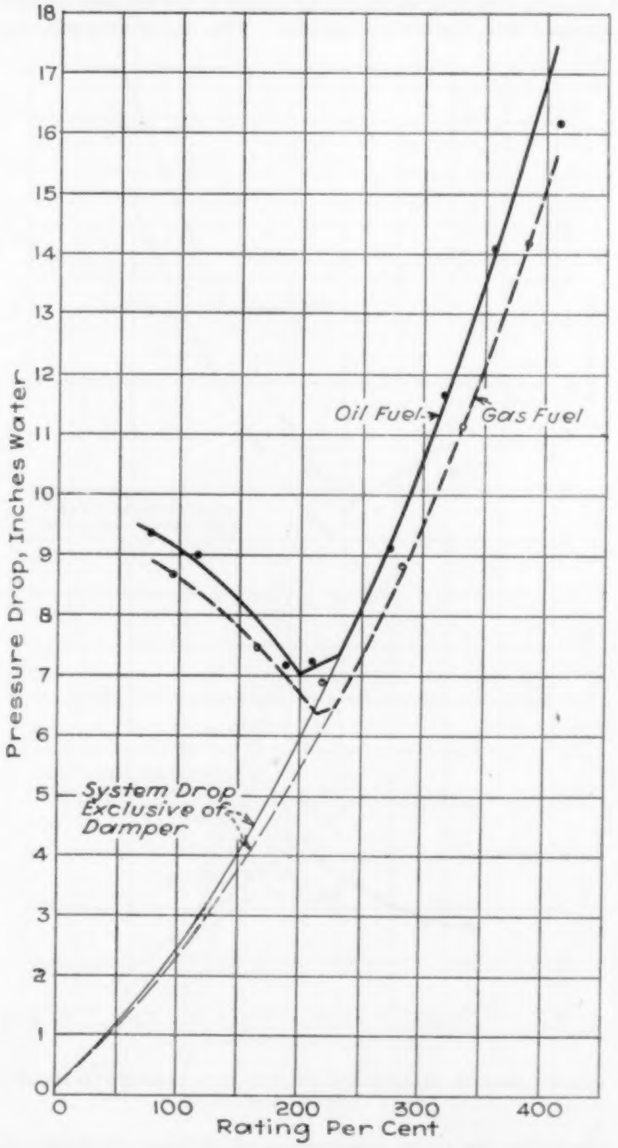


FIG. 10 TOTAL PRESSURE DROP IN FORCED AND INDUCED-DRAFT SYSTEMS

SUPERHEATED-STEAM TEMPERATURE HIGHER WITH GAS THAN WITH OIL

In the installation reported superheated-steam temperatures in all cases fell below the manufacturer's guarantee, especially at the lower capacities. It is felt that the reason for this is largely chargeable to abnormally high heat absorption by the water-cooled furnace walls, since no slag or other accumulations occur on these walls as is the case when burning coal. By burning oil and gas fuel with the equipment provided, it is possible to run with 15 per cent excess air within all ordinary limits of capacity. The characteristics of most coal-burning installations have been such that the excess air supply to the furnaces usually increased with the lower capacities. This had a compensating effect upon the steam temperatures which tended to flatten the steam-temperature curve. With low exit-gas temperatures and low values of excess air it has been found, by practice, that the overall plant efficiency might be raised in some instances by deliberately admitting increased amounts of air to the fires, especially at the lower capacities, which cause higher steam temperatures with a beneficial effect upon the turbine greater than that caused by the loss of the boiler efficiency.

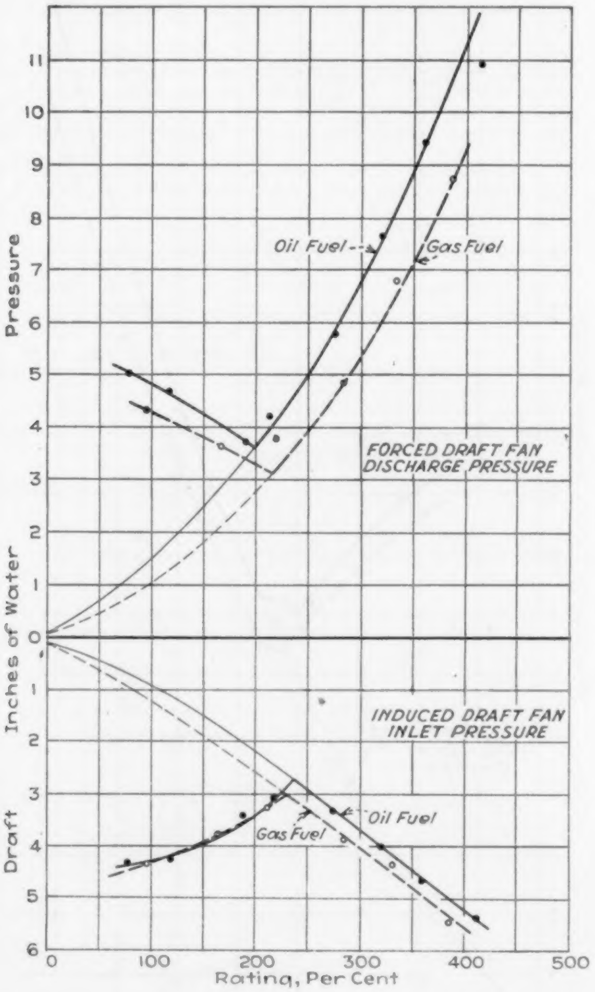


Fig. 11 Boiler Rating Vs. Forced and Induced-Draft Fan Pressures

From the observed performance of the water-cooled furnace walls with both fuels, shown in Figs. 19 and 20, it will be noted that the total heat absorption of the water-cooled furnace walls at 400 per cent rating is approximately 35 per cent greater with oil fuel than with gas fuel. This difference largely accounts for the poorer performance of the superheater while using oil fuel. Such variations in furnace water-wall heat adsorption make the problem of proper superheater design a very difficult one for the manufacturers. It is very difficult to obtain accurate

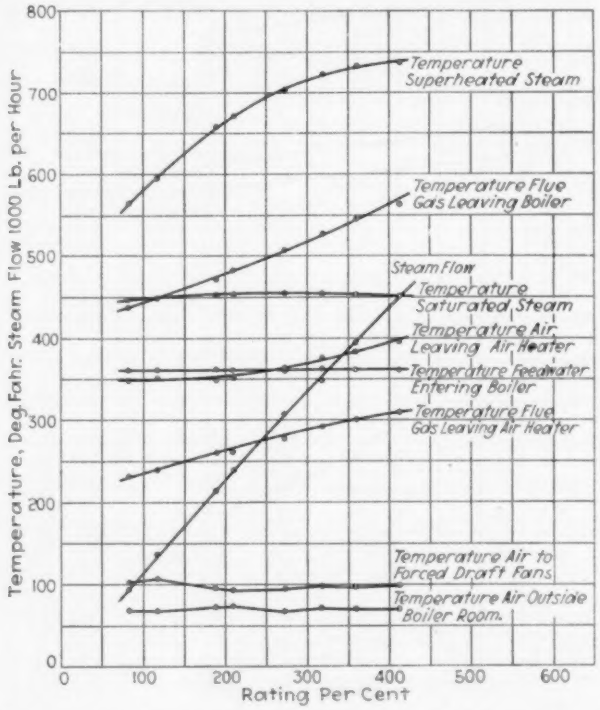


Fig. 12 Air, Feedwater, Flue Gas and Steam Temperatures With Oil Fuel

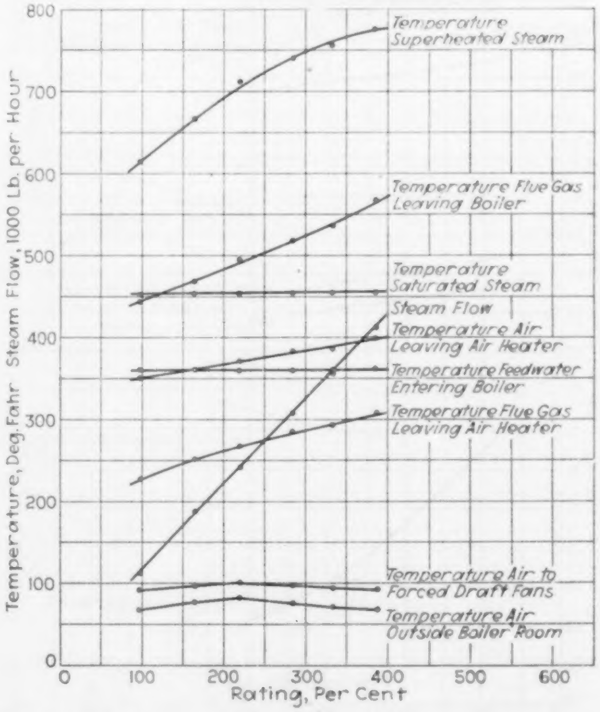


Fig. 13 Air, Feedwater, Flue Gas and Steam Temperatures With Gas Fuel

data on water-cooled furnace-wall absorption by using accepted methods; however, the data shown, it is felt, indicates fairly closely the heat absorption of the water-cooled furnace walls. It is interesting to note that in the case with oil fuel the water-cooled furnace walls with but 3.4 per cent of the total heating surface apparently absorbed approximately 50 per cent of the total heat absorbed at 250 per cent rating and 67 per cent of the total heat absorbed at 100 per cent rating. This relation is shown in Fig. 20.

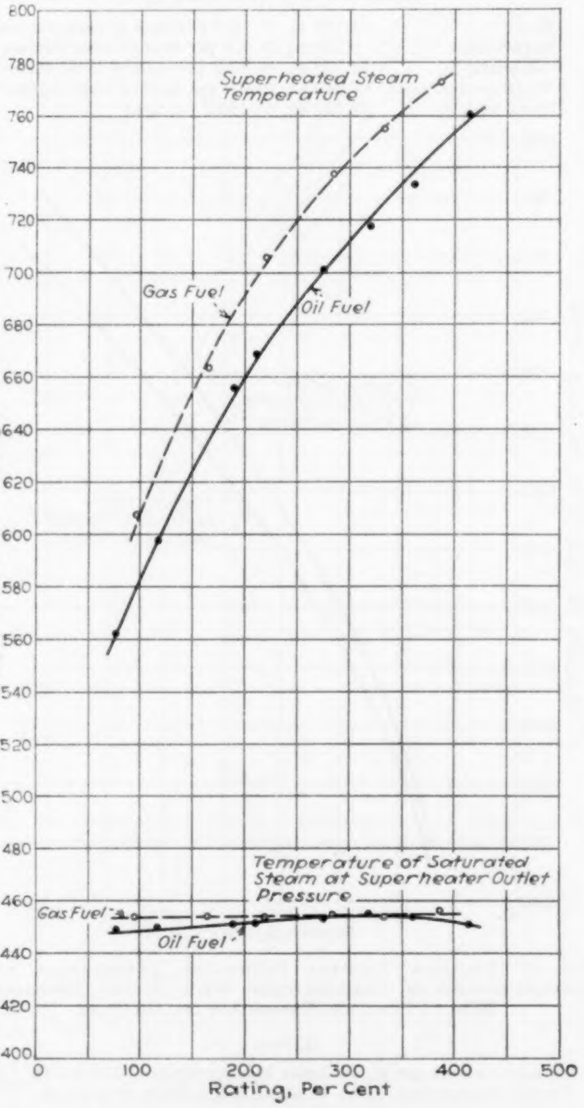


Fig. 14 Superheated-Steam Temperatures

Curves showing observed temperatures, heat absorption, and specific heats are shown in Figs. 22 and 31, inclusive. These curves are selected from the original test report as being of sufficient interest for inclusion in this paper.

A heat-balance diagram for this boiler while using gas fuel is shown in Fig. 21. Again the ratio of "hydrogen losses" to total stack losses appears quite prominently. Also it is very interesting to note that the total heat delivered in the steam from the boiler is 109.5 of the heat input of the fuel. This apparent enigma is due almost entirely to the amount of heat in the feedwater returned from extraction headers being greater than the total boiler losses.

Appendix 1

Boiler Data

Water tube, cross-drum type manufactured by Babcock & Wilcox Co. Heating surface 34,162 sq. ft. Drum 60 in. dia. Tubes 4 in. O.D. Length 24 ft. 0 in. No. 6 B.w.g. thickness. Number of tubes 1326, including three rows of circulating tubes. Arrangement: 51 tubes wide, 23 tubes high in three decks. Lower deck 5 tubes high. Two upper decks are 9 tubes high each.

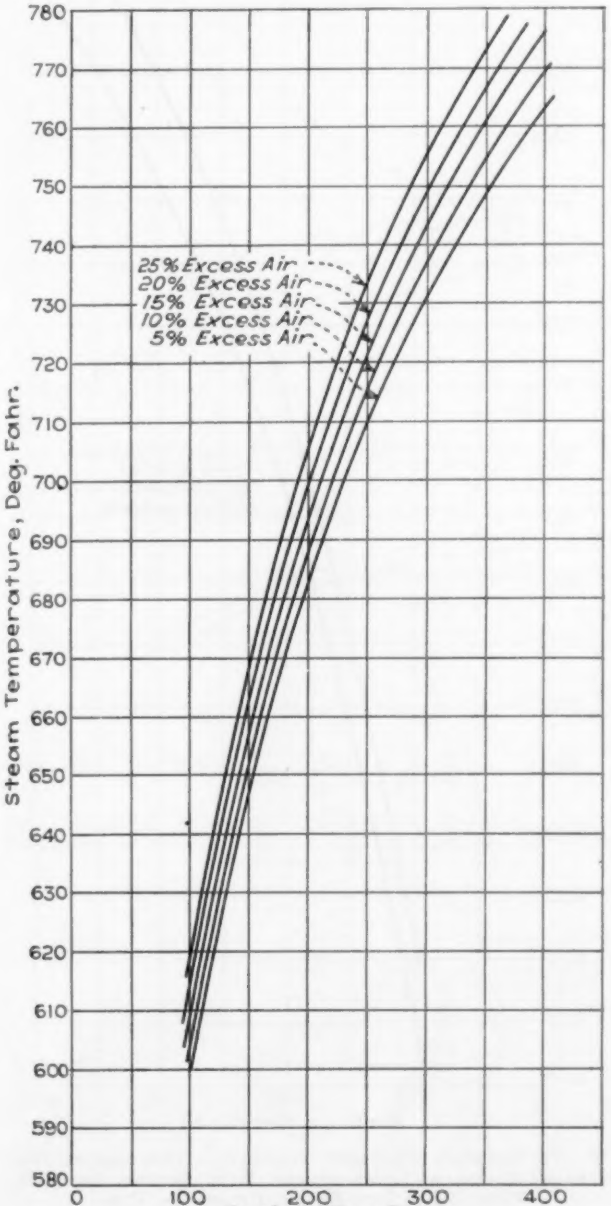


Fig. 15 Superheated-Steam Temperatures as Affected by Variations in Amount of Excess Air With Gas Fuel

Superheater Data

Convection type, quadruple deck, three pass, manufactured by Babcock & Wilcox Co. Superheater located interdeck, immediately above lower deck of boiler tubes. Heating surface 6553 sq. ft. Total number of tubes 462. Passes have equal number of tubes and equal surface. Tubes 2 in. O.D., No. 10 B.w.g. thickness.

AIR-HEATER DATA

Tubular type manufactured by Babcock & Wilcox Co. Heating surface air-touched 51,232 sq. ft. Internal gas-touched surface 46,350 sq. ft. Air-passage area 54.5 sq. ft. Gas-passage area 60.6 sq. ft. Number of tubes 2176. Tubes $2^1/_2$ in O.D., No. 11 B.w.g. thickness. Length 36 ft.

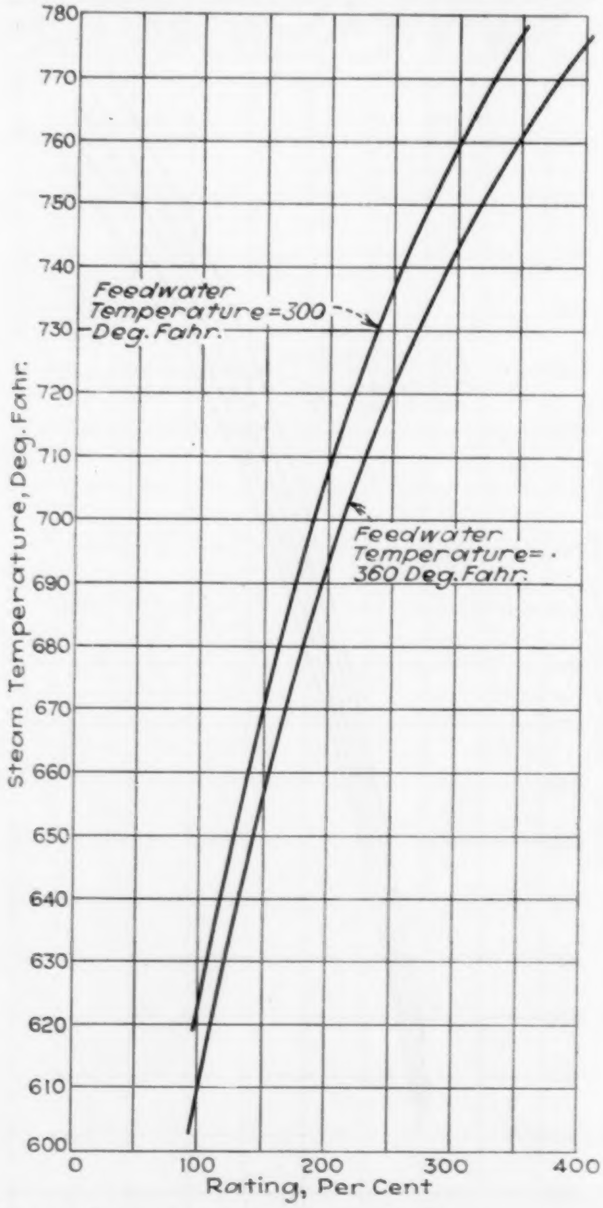


FIG. 16 RELATION BETWEEN FEEDWATER TEMPERATURE AND SUPERHEATED-STEAM TEMPERATURES WITH BOILER OPERATING WITH 15 PER CENT EXCESS AIR AND GAS FUEL

WATER-COOLED FURNACE WALLS

Bailey, refractory-faced iron-block type. Manufactured by Fuller-Lehigh Co. Heating surface 3240 sq. ft. The side walls have 42 water-cooling tubes each. The front wall, front-ashpit wall and rear wall have 60 cooling tubes each. Tubes $3^1/_4$ in. O.D. No. 6 B.w.g. thickness. Wall blocks type BAG with $^3/_4$-in. thick "carbosite" refractory face. Block is cast around molded refractory material. Corners are made with type WAC and WAJ blocks with $1^1/_4$-in. and $2^1/_2$-in. thick No. 80 B. & W. fire-brick refractory cemented in.

Furnace width 30 ft. $2^1/_2$ in. Depth 20 ft. $10^3/_4$ in.

Furnace Volume:

Above firing aisle floor	15,920 cu. ft.
Below firing aisle floor	3,880 cu. ft.
Total volume	19,800 cu. ft.

SUMMARY OF HEATING SURFACES

Boiler	34,162 sq. ft.	35.9 per cent of total surface
Superheater	6,553 sq. ft.	6.9 per cent of total surface
Air heater	51,232 sq. ft.	53.8 per cent of total surface
Water-cooled walls	3,240 sq. ft.	3.4 per cent of total surface
Total surface	95,187 sq. ft.	100.0 per cent.

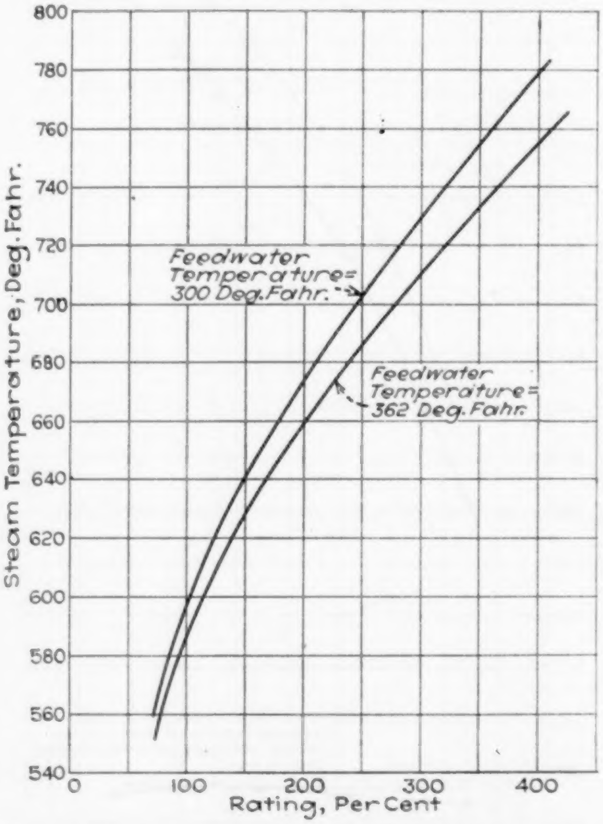


FIG. 17 RELATION BETWEEN FEEDWATER TEMPERATURE AND SUPERHEATED-STEAM TEMPERATURES WITH BOILER OPERATING WITH 15 PER CENT EXCESS AIR AND OIL FUEL

RATIOS

Furnace volume per sq. ft. boiler heating surface, 0.58 cu. ft.
Furnace volume per sq. ft. total heating surface, 0.45 cu. ft.

BURNERS

Combination oil and gas burners, Peabody Engineering Corp.

FORCED AND INDUCED DRAFT FANS

Two forced draft fans per boiler. No. 56, RB-C-2, radial flow, double inlet, double width. Each fan driven by a type BTS, 140-hp. variable-speed motor. 6 poles. 2200 volts. 3 phase. 50 cycles. Speed range 1210–600 r.p.m.

One induced-draft fan. No. $10^1/_2$, RR-B-X, radial flow, double inlet, double width. Driven by a type BTS, 300-hp. variable-speed motor. 10 poles. 2200 volts. 3 phase. 50 cycles. Speed range 630–360 r.p.m.

Fans manufactured by Green Fuel Economizer Co.

Motors manufactured by General Electric Co.

Motors have wound rotor with commutator, rotor transformers and brush-shifting mechanism actuated by the Bailey Automatic Combustion control.

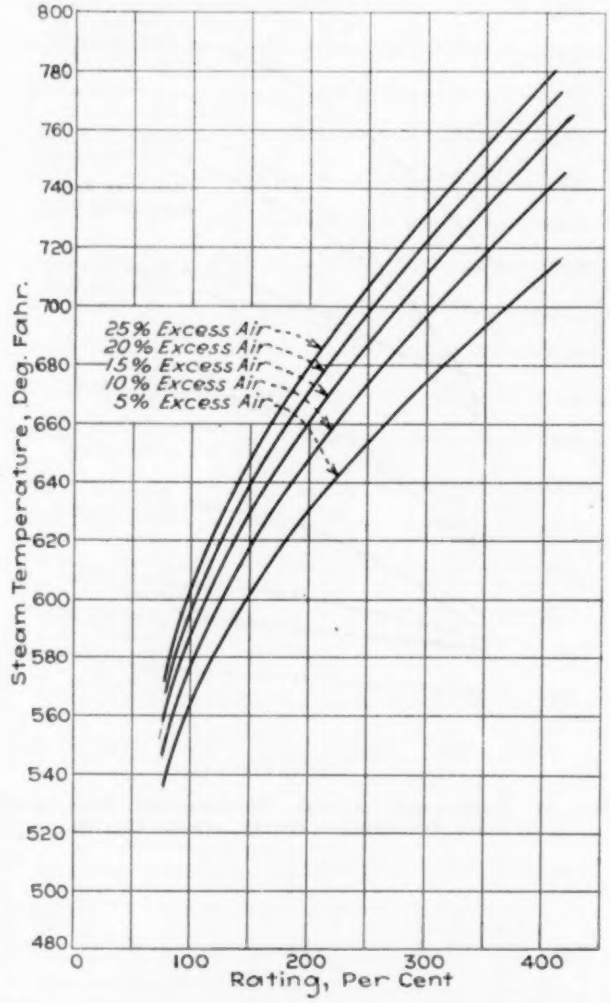


FIG. 18 SUPERHEATED-STEAM TEMPERATURES AS AFFECTED BY VARIATIONS IN AMOUNT OF EXCESS AIR WITH OIL FUEL

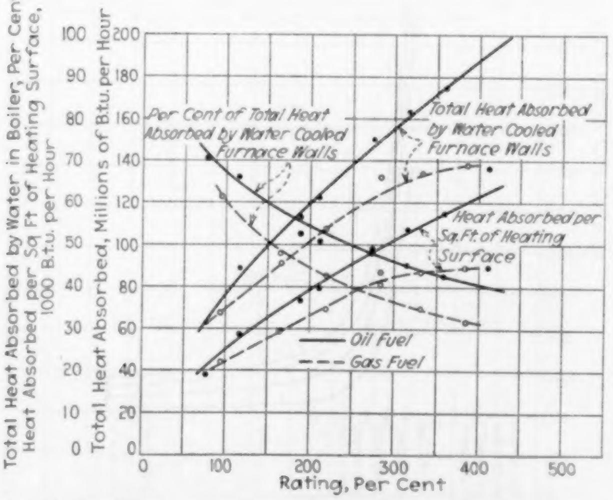


FIG. 19 PERFORMANCE OF BAILEY WATER-COOLED FURNACE WALLS

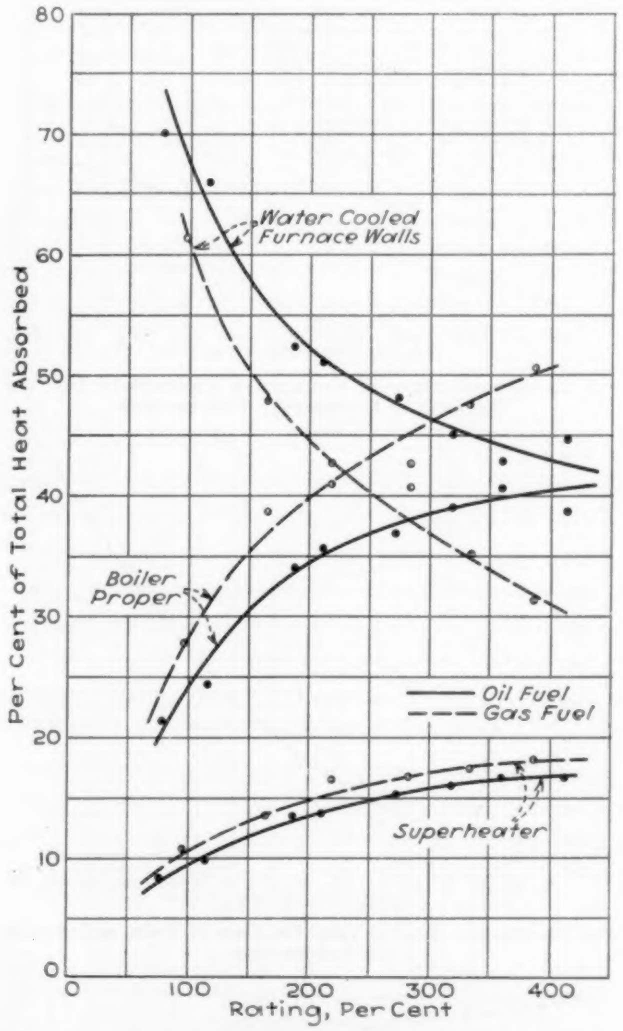


FIG. 20 HEAT ABSORPTION OF BOILER HEATING SURFACE EXPRESSED AS PER CENT OF TOTAL HEAT ABSORBED BY WATER AND STEAM IN BOILER

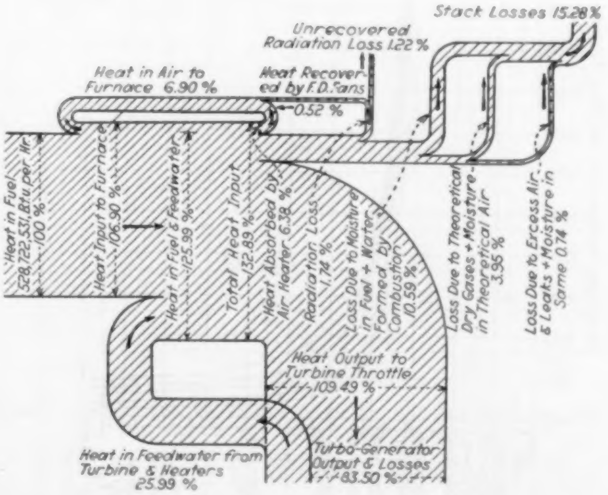

Stack Losses 15.28%
Unrecovered Radiation Loss 1.22%
Heat in Air to Furnace 6.90%
Heat Recovered by F.D. Fans
0.52%
Heat in Fuel 528,722,531 Btu per Hr. 100%
Heat Input to Furnace 106.90%
Heat in Fuel & Feedwater 125.99%
Total Heat Input 132.89%
Heat Absorbed by Air Heater 6.38%
Radiation Loss 1.74%
Loss Due to Moisture in Fuel + Water Formed by Combustion 10.59%
Loss Due to Theoretical Dry Gases + Moisture in Theoretical Air 3.95%
Loss Due to Excess Air & Leaks + Moisture in Same 0.74%
Heat Output to Turbine Throttle 109.49%
Heat in Feedwater from Turbine & Heaters 25.99%
Turbo-Generator Output & Losses 83.50%

FIG. 21 HEAT BALANCE DIAGRAM GAS FUEL

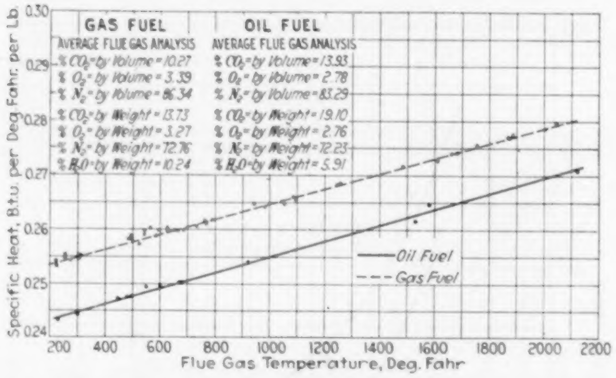


Fig. 22 Specific Heat of Combustion Products of Oil and Gas Fuel as Functions of Temperature

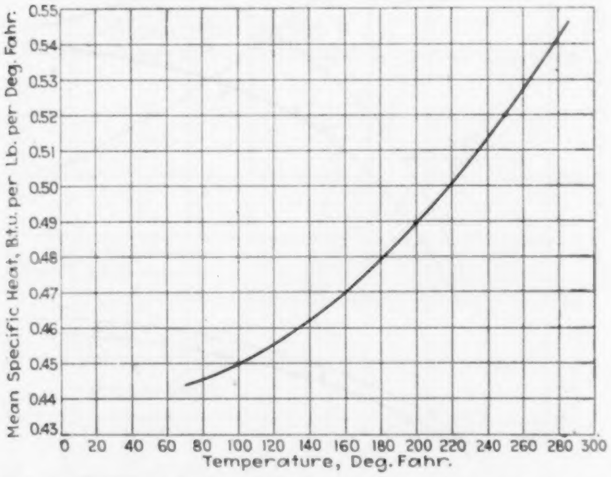


Fig. 23 Specific Heat of Fuel Oil Used in Tests as Function of Temperature

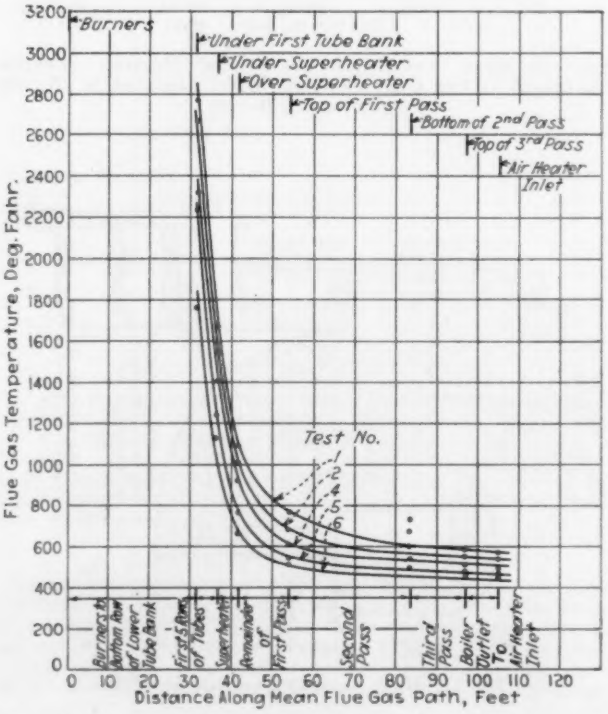


Fig. 26 Gas Temperature Survey Through Boiler and Superheater With Gas Fuel

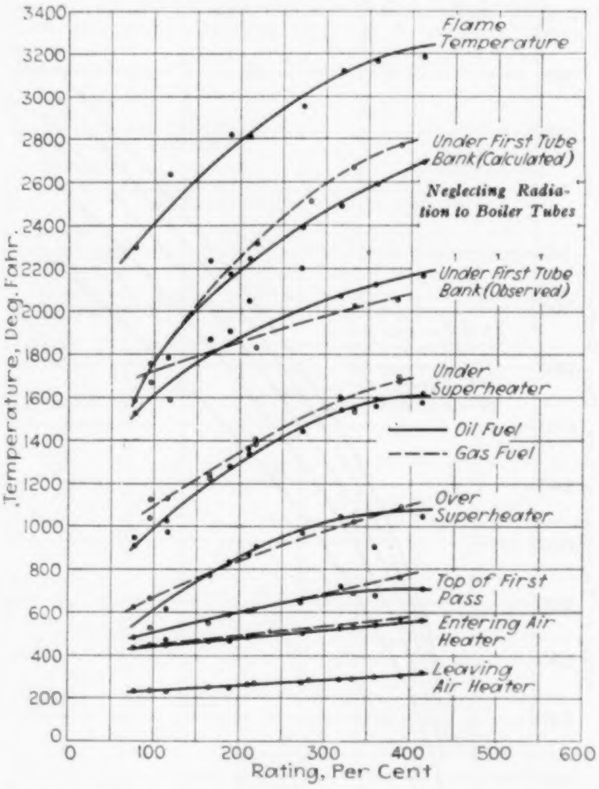


Fig. 24 Distribution of Gas Temperatures Throughout Furnace, Superheater, Boiler, and Air Heater

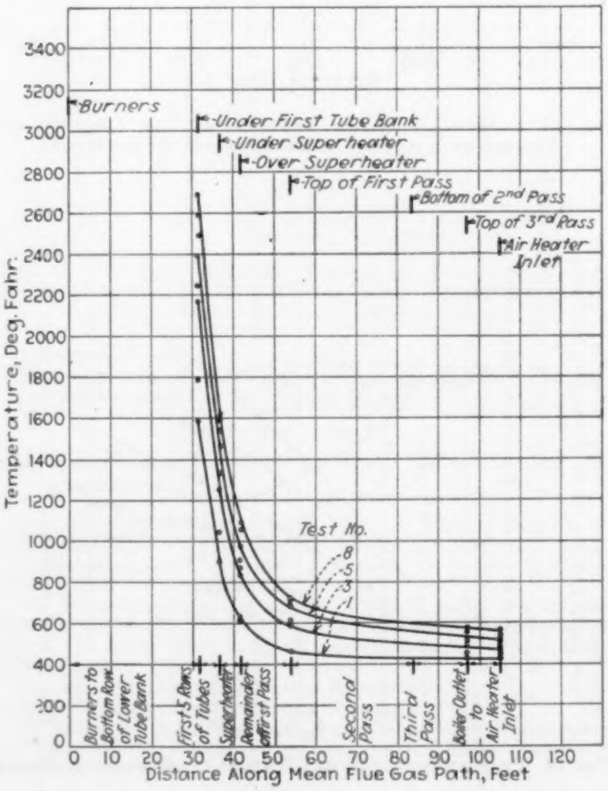


Fig. 25 Gas Temperature Survey Through Boiler and Superheater With Oil Fuel

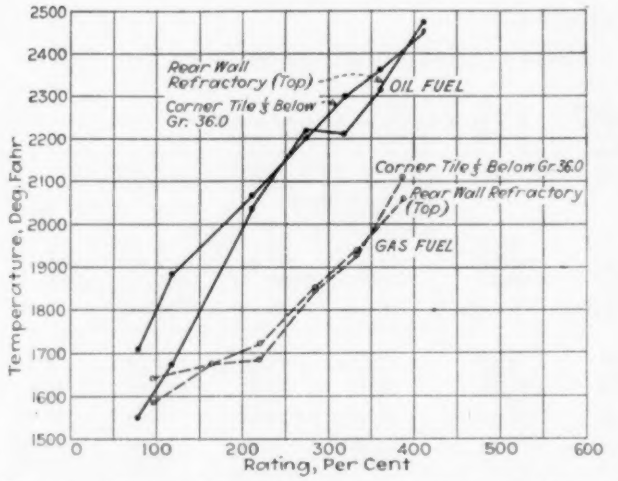


Fig. 27 Furnace Temperature as Indicated by Optical Pyrometer

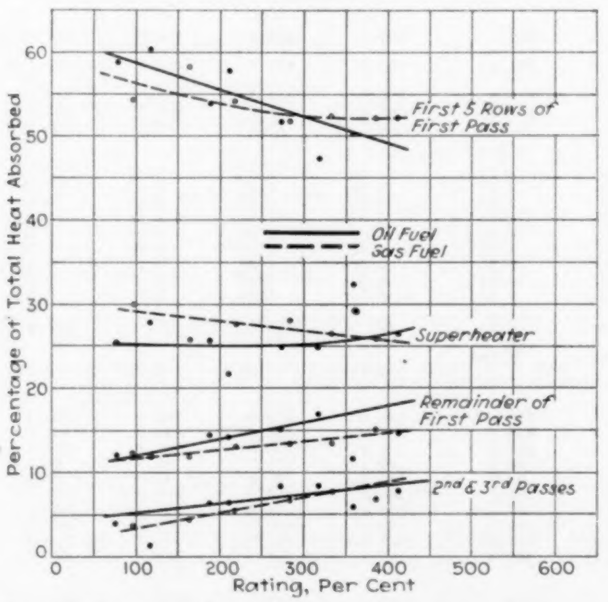
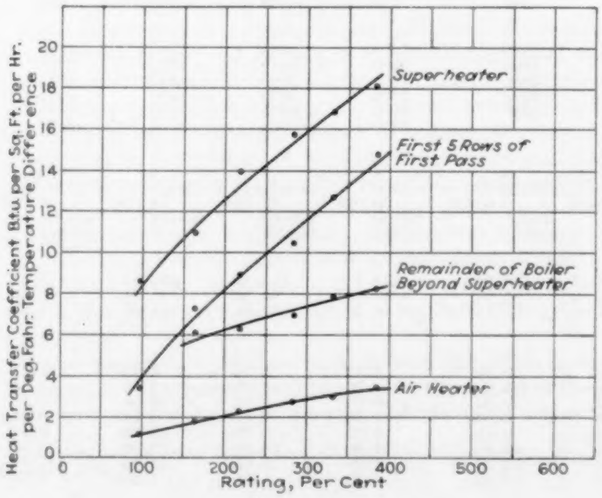


Fig. 28 Relative Heat Absorption of Superheater and Sections of Boiler Heating Surface



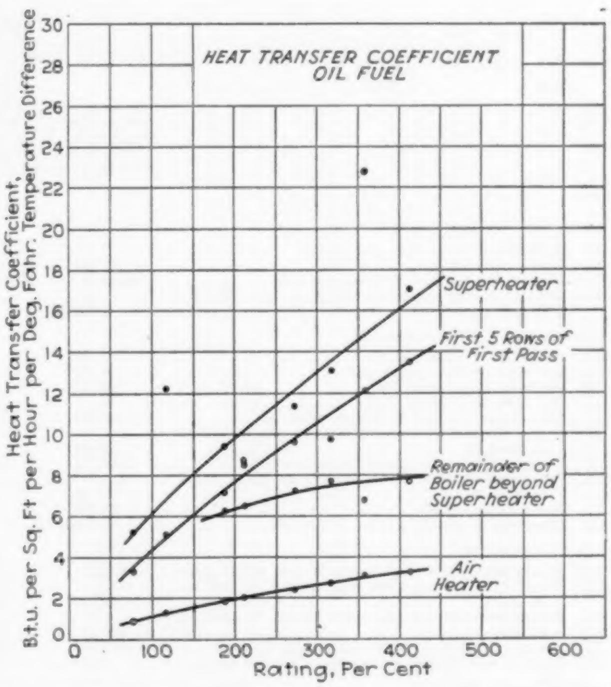


Fig. 29 Heat Transfer Coefficients of the Air Heater, Superheater, and Portions of the Boiler With Oil Fuel

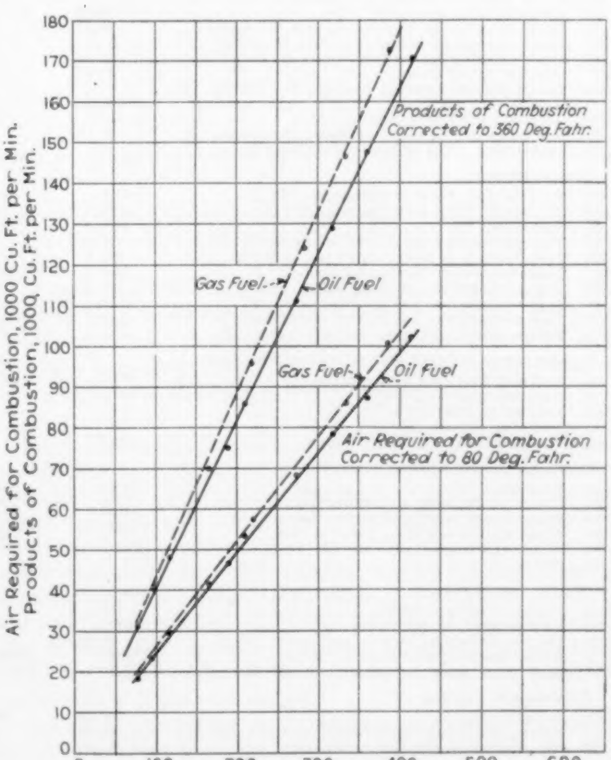


Fig. 31 Air Requirements and Products of Combustion of Oil and Gas Fuel Used During Tests

Fig. 30 (Lower Left-Hand Figure) Heat Transfer Coefficients of the Air Heater, Superheater, and Portions of the Boiler With Gas Fuel

Appendix

TEST OF BOILER 50, LONG BEACH STEAM STATION, SOUTHERN CALIFORNIA EDISON COMPANY, LTD.

Fuel:[a]	Item	Gas fuel			Oil fuel			
1		Methane CH_4, % by volume		74.7	Carbon C, % by weight			86.5
2		Ethane C_2H_6, % by volume		23.7	Hydrogen H_2, % by weight			11.0
3		Oxygen O_2, % by volume		0.1	Nitrogen N_2, % by weight			0.29
4		Nitrogen N_2, % by volume		1.4	Sulphur S_2, % by weight			0.94
5		Carbon dioxide CO_2, % by volume		0.1	Oxygen O_2, % by weight			1.02
6					Moisture, % by weight			0.20
7					Ash, % by weight			0.05
8					Flash point, °F			198
9					Specific gravity at 60° F., deg. A.P.I.			17.9
10	Test No	6	3	1	1	4	7	8
11	Rating, per cent	96.48	284.16	386.34	78.3	211.13	359.96	413.17
12	Heat value per pound of fuel as fired[b]	22760.1	22667.7	22486.7	18600	18600	18600	18600
13	Density of fuel at 14.9 lb. per sq. in. abs., 60° F., lb. per cu. ft	0.05457	0.05457	0.05423	59.039	58.958	58.958	58.921
	Flue gas at air-heater outlet:							
14	Per cent by volume CO_2	9.86	10.24	10.23	13.6	13.6	13.8	14.8
15	Per cent by volume O_2	4.02	3.51	3.44	3.0	3.0	2.8	1.6
16	Per cent by volume N_2	86.12	86.25	86.33	83.4	83.4	83.4	83.6
17	Per cent by volume CO	0	0	0	0	0	0	0
18	Dry gas per pound of fuel as fired, lb	18.84	18.16	18.18	16.15	16.15	15.93	14.90
19	Water formed by combustion per pound of fuel as fired, lb	2.03	2.03	2.03	0.99	0.99	0.99	0.99
20	Air supplied per pound of fuel as fired, lb	19.87	19.20	19.21	16.14	16.14	15.92	14.89
21	Air supplied per pound of fuel as fired, theoretical, lb	16.44	16.44	16.44	13.76	13.76	13.76	13.76
22	Per cent excess air	20.86	16.73	16.85	17.30	17.30	15.70	8.21
23	Moisture in air, lb. per lb. of dry air	0.0101	0.0121	0.0108	0.0083	0.0110	0.0118	0.0121
	Steam pressures:							
24	Steam pressure, boiler drum, lb. per sq. in. gage	422.5	446.2	464	402.6	426.9	456.8	461.4
25	Steam pressure entering superheater, lb. per sq. in. gage	422.4	444.0	461.8	401.6	424.7	451.4	453.5
26	Steam pressure leaving superheater, lb. per sq. in. gage	420.2	425.8	433.5	399.9	414.1	421.2	413.8
27	Steam pressure at header, lb. per sq. in. gage	417.6	419.4	421.3	398.9	408.7	408.5	397.5
28	Total drop boiler drum to header, lb. per sq. in.	4.9	26.8	42.7	3.7	18.2	48.3	63.9
	Air pressures and drafts:							
29	Pressure at forced-draft fan discharge, in. water	4.31	4.95	8.77	5.02	3.93	9.42	10.91
30	Pressure after forced-draft fan damper, in. water	0.41	4.45	8.87	0.25	3.22	9.53	11.16
31	Rear windbox pressure, in. water	...	2.71	5.97	0.10	2.18	6.83	8.0
32	Front windbox pressure, in. water	...	2.03	4.75	0.05	1.78	5.73	6.77
33	Furnace draft, in. water	0.12	0.25	0.09	0.07	0.07	0.07	0.10
34	Draft entering air heater, gas side, in. water	0.34	2.30	3.73	0.21	1.06	3.10	3.77
35	Draft leaving air heater, gas side, in. water	...	3.16	5.35	0.27	1.50	4.4	5.37
36	Draft entering induced draft fan, in. water	4.36	3.86	5.42	4.30	3.24	4.67	5.27
37	Draft entering stack	0.69	1.06	1.38	0.70	1.04	1.74	2.05
38	Fuel pressure entering burners, lb. per sq. in. gage	0.15	1.05	2.52	27	48.5	139	185.8
39	Number of burners used	20	20	20	20	20	20	20
40	Gas burners: 72 holes, diameter per burner, in.	0.189	0.189	0.189	...	...	...	...
41	Oil burners: Size of burner tip, bore, in	...	...	...	0.11	0.147	0.147	0.147
	Temperatures:							
42	Superheated steam temperature, °F	614	739.7	775	566.0	672.5	734.8	736.3
43	Saturated steam temperature, °F	453.1	452.1	456.1	448.4	451.6	453.4	451.6
44	Superheat, °F	160.9	287.6	318.9	117.6	220.9	281.4	284.7
45	Temperature air outside boiler room, °F	68.8	74.8	66.7	69.4	71.7	69.7	69.9
46	Temperature at forced-draft fan inlet, °F	90.1	98.6	92.7	103.6	92.6	95.9	99.3
47	Temperature air leaving air heater, °F	350.8	382.6	399.2	349.8	352.2	384.8	394
48	Temperature fuel entering boiler, °F	68.5	73.3	67.1	236.4	252.8	257.2	255
49	Temperature flue gas leaving boiler, °F	446.6	518.8	568	436.4	480.6	545.2	562.3
50	Temperature flue gas leaving air heater, °F	260.7	284.8	307.8	233.6	260.9	301.7	310.7
51	Temperature feedwater entering boiler, °F	362	358.5	360.4	362	362	362.8	360.3
	Temperatures in boiler setting by thermocouple:							
52	Entering first pass (calculated), °F	1762	2510	2774	1588	2244	2590	2692
53	Entering superheaters, °F	1088	...	1683	927	1350	1582	1603
54	Leaving superheaters, °F	665	...	1094	630	867	919	1046
55	Top of first pass, °F	522	...	766	486	618	680	705
56	Top of third pass, °F	467	...	582	452	520	552	574
	Temperatures by optical pyrometer:							
57	Rear wall, °F	1585	1855	2060	1550	2035	2315	2470
58	Bailey block front wall above burners, east, °F.	1775	1800	2160	2100	2440	2515	2730
59	Bailey block front wall above burners, west, °F.	1785	1770	2105	2000	2460	2340	2825
60	Front wall around top burners, east, °F	2270	2255	2385	2590	3030	3250	3220
61	Front wall around top burners, west, °F	2250	2250	2380	2690	2930	3150	3150
62	Flame, °F	...	...	...	2300	2820	3170	3180
	Hourly quantities:							
63	Duration of test, hours	4	4	4	4	4	4	2
64	Fuel as fired, lb. per hr	5610	16990.7	23512.5	5356	14581.8	25025.5	28856.5
65	Density of fuel at 14.9 lb. per sq. in., 60° F.	0.05457	0.05457	0.05423	59.039	58.958	58.958	58.921
66	Actual water evaporated, lb. per hr	112827	309307	413497	94060	238846	394268	450751
67	Equivalent water evaporated, lb. per hr	113730	334980	454971	92273	248878	424252	486811
	Efficiency and rating:							
68	Boiler rating, %	96.48	284.16	386.34	78.3	211.13	359.96	413.17
69	Efficiency of boiler, superheater, and air heater, %	86.42	84.38	83.50	89.44	88.55	87.95	87.56
70	Efficiency of boiler and superheater, %	80.90	78.53	77.12	84.28	83.01	81.84	81.75
	Heat balance, per cent of heat input:							
71	Heat loss due to water in fuel and water formed by combustion, %	10.10	10.35	10.59	5.97	6.02	6.13	6.15
72	Heat loss due to theoretical dry gases, %	2.50	3.34	3.86	2.79	3.21	3.96	4.10
73	Heat loss due to moisture in theoretical air, %.	0.06	0.09	0.09	0.05	0.07	0.09	0.10
74	Heat loss due to excess air with moisture, %	0.50	0.49	0.63	0.43	0.55	0.61	0.28
75	Heat loss due to air leakage with moisture, %.	0.09	0.15	0.11	0.09	0.06	0.06	0.08
76	Heat absorbed by water in boiler and walls, %.	77.19	70.36	68.40	82.05	76.50	73.33	72.82
77	Heat absorbed by superheater, %	9.23	14.02	15.10	7.39	12.05	14.62	14.74
78	Unaccounted-for, %	0.33	1.20	1.22	1.23	1.54	1.20	1.73

Appendix (*Continued*)

TEST OF BOILER 50, LONG BEACH STEAM STATION, SOUTHERN CALIFORNIA EDISON COMPANY, LTD.

		Gas fuel			Oil fuel			
	Test No.	6	3	1	1	4	7	8
	Air heater:							
79	Heat available to air heater, B.t.u. per lb. of fuel	1697.9	1932.2	2405.6	1292.9	1529.4	1758.8	1688.6
80	Heat absorbed by air heater, B.t.u. per lb. of fuel	1254.9	1325.9	1435.4	965.2	1035.2	1142.3	1086.3
81	Air-heater efficiency, %	73.9	68.6	59.7	74.7	67.7	64.9	64.3
82	Heat input per cubic feet of furnace volume, B.t.u. per hr	6448	19452	26703	5056	13772	23640	27257
	Heat absorbed per square foot of heating surface, B.t.u. per hour:							
83	Bottom five rows of boiler tubes, B.t.u. per sq. ft.	3611	15882	25285	2565	11009	18958	21992
84	Superheater, B.t.u. per sq. ft	1954	8432	12189	1130	5015	10445	12140
85	Remainder of first pass, B.t.u. per sq. ft	423	2112	3786	270	1402	2225	3181
86	Second and third passes, B.t.u. per sq. ft	99	891	1365	74	503	907	1236
87	Water-cooled walls, B.t.u. per sq. ft	22063	43190	44882	19044	39908	57151	68551
	Per cent of total heat absorbed by various heating surfaces:							
88	Heat absorbed by bottom five rows of boiler tubes, %	21.58	30.58	35.74	16.95	26.28	30.36	27.29
89	Heat absorbed by superheater, %	10.68	16.62	18.09	8.26	13.60	16.63	16.84
90	Heat absorbed by remainder of first pass, %	4.88	7.84	10.31	3.44	6.45	6.86	7.60
91	Heat absorbed by second and third passes, %	1.42	4.12	4.63	1.18	2.89	3.49	3.69
92	Heat absorbed by water-cooled walls, front, %	9.35	5.93	3.30	9.42	6.40	5.73	5.75
93	Heat absorbed by water-cooled walls, sides, %	21.57	18.09	13.32	28.54	21.97	17.41	18.08
94	Heat absorbed by water-cooled walls, rear, %	11.33	6.37	5.96	13.90	9.33	6.95	8.10
95	Heat absorbed by water-cooled walls, floor, %	19.19	10.45	8.65	18.31	13.08	12.57	12.65
	Heat-transfer coefficients, B.t.u. per square foot per degree Fahrenheit temperature difference per hour:							
96	Bottom five rows of boiler tubes	3.43	10.52	14.8	3.36	8.52	12.08	13.53
97	Superheater	8.60	15.78	18.1	5.25	8.77	22.76	17.05
98	Boiler surface beyond superheater	6.94	6.94	8.28	...	6.47	6.78	7.66
99	Air heater	1.18	2.75	3.43	0.95	2.0	3.06	3.25
	Forced-draft fans, two fans per boiler:							
100	Inlet-air temperature, °F	90.1	98.63	92.7	103.6	92.6	95.9	99.3
101	Static head on fan, in. water (actual)	4.31	4.95	8.77	5.02	3.93	9.42	10.91
102	Air flow per fan, cu. ft. per min. (actual)	12822	36357	52278	10101	27670	46402	50006
103	Revolutions per minute	659.9	748.2	1000.9	706.6	641.7	1006.7	1089.7
104	Power to fan motor, kw. (actual)	24.5	46.9	100.0	25.3	31.3	98.3	122.6
	Induced-draft fan, one fan per boiler:							
105	Inlet-flue-gas temperature, °F	227.8	284.8	307.8	233.6	260.9	301.7	310.7
106	Static head on fan, in. water (actual)	3.67	2.65	4.04	3.62	2.20	2.95	3.22
107	Gas flow, cu. ft. per min. (actual)	35809	114273	163960	26786	76774	137694	151202
108	Revolutions per minute	393.8	422.7	531.9	381.3	372.2	477.9	506.4
109	Power to fan motor, kw. (actual)	43.3	80.4	191.2	31.6	43.7	133.9	174.2
	Damper opening:							
110	Forced-draft fan damper opening, deg	16.2	63.0	79.2	...	53	78	...
111	Induced-draft fan damper opening, deg	11.7	63.9	84.5	...	40	78	...

[a] Fuel analysis from one representative sample, applied to all tests.

[b] The heat value of fuel gas was determined for each test with a Thomas calorimeter. The heat value of fuel oil was determined once, from the supply that was used on all tests.

Discussion

D. S. Jacobus.[2] This is a most valuable paper. The accuracy of the results is shown by cross-checking and is also evidenced through the pains employed in securing exact experimental data. The paper should serve as a model for the making of tests of the sort.

The comparative results for the oil and gas fuel are of great interest, as well as those showing the amounts of heat absorbed by the boiler and furnace wall cooling tubes.

Fig. 7 shows the relation between heat liberated in the furnace and the boiler rating, and if drawn to different scales would also show the relation between the heat absorbed and the heat available for absorption. This relationship is shown to be a straight line passing through the origin for both oil and gas fuel, the only difference being in the slope of the lines, that for the gas being a little steeper than the other. This is a fundamental relationship and one not always reported. In the present case it is especially interesting to find that the lines pass through zero.

In view of the fact that tests of such accuracy on equipment of such a large size are very costly, the results which are herein given to the public are rare and should be appreciated in proportion thereto.

It was the writer's privilege to work with Mr. Philo in the Delray tests. It is indeed gratifying to note the success of those with whom one has striven in the past, and the writer wishes to extend his congratulations for the masterly way in which the author has conducted the tests and in which he has presented the results.

It would be well to add the data, which are referred to in the paper as being largely omitted, on forced- and induced-draft fans, water walls, etc. It is seldom that one finds as accurate test results as those given in the paper, and any supplementary matter of the sort would still further increase its value.

E. B. Powell.[3] The author presents much valuable data, which have been greatly needed, on the comparative characteristics of oil and gas as steam-boiler fuels. We have had impressions of performance and theoretical deductions upon which we have based expectations, but the author, by eliminating all elements of difference except those of the fuel consumed and by securing complete and equally efficient combustion of each, definitely measures the differences inherent in the two fuels. The care and thoroughness with which the observations have been taken are clearly evidenced by the exceptionally high degree of consistency shown by his results.

The data of the paper are, however, of even greater value in their contribution to the analysis of heat transfer in the steam-generating unit, particularly in the comparison which they afford of radiation from luminous and from non-luminous flame. These latter data will be of material assistance in furthering steam-boiler and furnace design. It will be noted that, for the same net heat liberation, for the arrangement and proportions of heat-absorbing surface of the steam-generating unit here represented, the two fuels show practically identical efficiencies. Data for the boiler and superheater, considered independently from the air heater, suggest very slightly higher efficiency for the oil-fuel condition. Doubtless a unit having a considerably smaller proportion of convection surface would show an appreciable loss in absorptive efficiency when burning gas fuel as compared with that obtainable with oil, but it is most interesting to note that, in this instance, the absorptive capacity of the convection surface handling the somewhat greater mass of combustion products was sufficient to compensate for a very marked deficiency in absorption on the part of the radiant surface.

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E. H. Peabody.[4] This paper does not at first sight give an intimation of its really stupendous importance in the field of modern steam-engineering practice. Of its dozen pages (half of which constitute the appendix), scarcely more than two are devoted to written statements. These few pages describe in simple language the basic facts concerning the charts and tables which record the results of weeks and months of careful and laborious observation, but which form a veritable treasure box of up-to-the-minute fact and information.

It is a commentary on the spirit of service which permeates our great public-utility companies and leading engineering organizations that such tests as these reported by the author can be carried out with such accuracy and the results published to the world. Information of this kind can be obtained only under exceptionable conditions; it is accessible to few, and its compilation requires a lavish expenditure of time, enthusiasm, and money. The writer welcomes the opportunity to express his feeling of indebtedness for the gift of actual operating data of such great helpfulness. It must be a satisfaction also, to every one concerned with the design, to find in this report confirmation of expectations entertained before the plant was built.

The comparative performances of the various elements making up the steam-generating unit when burning oil and natural gas are of particular interest to operating engineers and equipment designers, because of the increasing availability of natural gas in new territories throughout the country, and because of its use under highly forced conditions. By segregating the component parts of the unit and thoroughly analyzing the data applying to each part, the author has presented a clear picture of the function of each piece of equipment and its operating characteristics under varied conditions of fuel and load. He also has demonstrated that natural gas, a comparatively new fuel to many engineers, can be successfully applied to steam generation under most exacting conditions. It should be remembered that but a year or two has elapsed since it was generally considered the height of folly to burn gas other than by means of natural draft. This fact alone demonstrates how rapid has been the progress.

The writer's attention has been particularly attracted to one point in the paper. In Fig. 19 the curves on heat absorbed per square foot of heating surface indicate that there is apparently a limit to the absorption rate in the furnace walls when burning gas, while with oil the rate continues to increase. This is also shown in Fig. 20, in which the percentage of total heat absorbed in the furnace walls and boiler section closely approach each other when burning oil and tend to maintain equality, while corresponding curves on gas attain equality at approximately 230 per cent of rating and become widely separated at higher capacities. Combustion gas temperatures and furnace temperatures do not indicate the reason for this apparent limit to the heat-absorption rate in the radiant section when burning gas. It is possible that the rate of combustion per unit of furnace volume determines the point at which "saturation" of the radiant heating surface occurs. This brings up the question of proper furnace shape and proportioning of radiant surface to convection surface in connection with gas-fired furnaces, and it is hoped that the author will find time to further discuss this point.

It is interesting to note that the water-cooled furnace walls absorbed approximately 35 per cent more heat with oil fuel than with gas fuel at 400 per cent of boiler rating. This has of late been a somewhat controversial matter. It will be very helpful if similar data are obtained on other types of furnaces, such as bare tubes and tubes covered with bare cast-iron blocks.

In spite of the radically different distribution of absorbed heat by boiler and furnace as shown by Fig. 20, the temperatures of the gases leaving the boiler are practically the same as indicated by Figs. 4 and 5.

One assumption made by the author—namely, that the capacity-efficiency characteristics fall on a straight line between 100 and 400 per cent of rating—jars a bit on the sensibilities of the writer. We know that it must be a curve even if a very slight one within these limits, and so why not favor the appearance of the true conditions rather than use the "simplified spelling" of a straight edge. Anyhow, the latest trend of theory is that there are no such things as straight lines.

The writer must take exception also to the author's use of the word "rating" in the sense of "capacity." It is a somewhat lamentable thought that the usage he is adopting is a very common one with the younger generation, but why should a perfectly good word like "capacity" be thrown into the discard and a word substituted which already has a very definite and a very different meaning. In doing this we are weakening the language. It is perhaps a small point to bring up in connection with a paper of such outstanding importance, and the writer does so only by way of emphasizing the fact that a bad practice is rapidly springing up among us.

It is the writer's impression that the Society has looked askance on tests of only four hours' duration. This is an academic attitude which this paper should go far to prove to be antiquated and ridiculous. A continuation of any of these tests longer than four hours would have served no useful purpose whatever and resulted only in the necessity of increased vigilance to prevent observational errors.

E. G. Bailey.[5] The paper is of very timely interest to many engineers, even though today they are burning neither oil nor gas. The furnaces used in conducting the tests by Mr. Philo were designed for the use of pulverized coal should that later be found desirable.

With the extension of high-pressure gas lines to the North and East from the Texas and Louisiana gas fields, it seems likely that more power stations now burning pulverized coal may be interested in burning a certain amount of gas as a surplus fuel. It is believed that when the time comes to do this it can be quite readily applied to existing pulverized-coal-fired furnaces, at which time the very valuable data collected by Mr. Philo will be of much benefit.

It is interesting to note that this type of furnace-wall construction gave an average absorption rate in excess of 60,000 B.t.u. per sq. ft. per hour through furnace walls using a refractory-faced cast-iron block lining, even at the relatively low heat liberation in the furnace of approximately 27,000 B.t.u. per cu. ft. per hour. This high rate of absorption may be accounted for by several factors, among which are extremely high furnace temperatures with low excess air used in burning oil and gas and the relatively clean condition of the heat-absorbing surface.

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[5] President, Fuller Lehigh Co., New York, N. Y. Mem. A.S.M.E.

The radiant-heat-absorbing surfaces in the boiler and furnace walls are not subject to accumulations of ash and slag on the surface and are therefore in a much better position to absorb radiant heat than they would be in a coal-fired furnace.

In Fig. 24 the author shows the difference between calculated and observed temperatures taken under the first tube bank. This differential is apparently due to the radiation absorption to the boiler tubes, which evidently has not been included in the author's calculations.

Considering the difference in temperature between the calculated and observed results for gas firing, and assuming that the total effective boiler-tube surface exposed to radiant heat is 140 per cent of the total area of the first row of boiler tubes, the rate of heat absorption to these tubes would be 60,000 B.t.u. per sq. ft. per hour. Using the same absorption rate per square foot of boiler-tube surface with oil firing, a gas temperature entering the front bank of boiler tubes of 1900 deg. fahr. is obtained. The difference between this temperature and the actual observed temperature may perhaps be accounted for by the effect of direct radiation to the thermocouple with oil firing.

The type of construction installed on these furnaces lends itself to determining rates of heat absorption to furnace walls very readily. In order to make such determinations, certain of the refractory-faced cast-iron blocks are provided with thermocouples at definite points in the cast-iron backing of the block so that the temperature gradient across this iron can be measured, from which the corresponding rate of heat transfer may be calculated. These blocks are made up very accurately with welded junction chromel-alumel thermocouples staked into the cast iron of the block at locations which are very accurately maintained. The calibration constants for determining the rate of heat transfer from any given temperature gradient were accurately determined from a laboratory furnace set up using full-size blocks and tubes and a furnace of sufficient size to give results comparable to those obtained in actual practice.

In making these calibration tests a large number of readings were taken determining the gradient curves corresponding to certain conditions, and the rate of heat transfer was determined through individual blocks by accurately weighing the water evaporated. With the precautions taken to insure accurate results it is felt that the calibration constants used are of a high degree of accuracy and that the rates of heat absorption obtained from the thermocouple blocks in actual practice are accurate within the limits of practical test work.

The accuracy of this method is even greater in connection with oil and gas firing than with coal firing, since there are no ash and slag deposits on certain portions of the walls which might give unequal rates of transfer on adjacent portions of the wall surface.

It is very interesting to note that the radiation and unaccounted-for losses for this series of tests are less than 2 per cent for all tests and are in the neighborhood of 1 per cent for quite a few of the tests. This would indicate very low loss of unburned gaseous fuel. By calculations from the gradient curves used for calculating the absorptions to the furnace walls it appears that the surface temperature of the blocks will range from 1800 to 2500 deg. fahr. This is borne out by wall temperatures of 2000 to 2300 deg. fahr. given in Fig. 27. It is evident that under such conditions there is much less chance of unburned fuel escaping from the furnace than in the case of water-cooled walls having a surface temperature of the order of 500 deg. fahr. This is an important consideration as the usual control of furnace operation does not give an indication of the possible losses due to unburned fuel, so that unless complete tests such as those reported by the author are made such a loss may exist without being suspected.

The author points out that while these furnaces are operated at a maximum rate of heat liberation of only 27,000 B.t.u. per cu. ft. per hour, rates as high as 60,000 B.t.u per cu. ft. per hour could probably be maintained satisfactorily. Other installations with similar furnace and wall construction have exceeded 50,000 B.t.u. per cu. ft. The relatively low heat release of the present furnaces is in line with current practice when these furnaces were designed for possible conversion to burn pulverized coal.

Ralph A. Sherman.[6] The paper will be used often and long as a source of experimental data by designing engineers. Because of this, it is felt that certain of the data given in the curves should have more explanation than is given in the test. The authority for the specific heats of the gases as given in Fig. 22 and of the oil as given in Fig. 23 should be quoted.

It would also be of interest and value to know how the gas temperatures were measured. The writer has been personally guilty of making many measurements of temperatures of furnace gases with an exposed thermocouple. Most of these were made in coal-fired furnaces where the presence of suspended ash made the use of any other type impractical. However, the author had an excellent opportunity to use the shielded velocity type of thermocouple with his ashless fuels, and if this type was used his data are of much greater value than otherwise. It is doubtful that this type was used, however, from the discrepancy shown in Fig. 24 between the calculated and observed temperatures below the first tube bank. The observed temperatures were lower than the calculated temperatures and the observed temperatures when burning gas were lower than those when burning oil, which is not as it should be because of the demonstrated greater rate of transfer of heat from the oil flame in the furnace.

Author's Closure

The author is grateful to those who prepared written discussions of this paper and is pleased by the remarks, which are interpreted as indicating the rapidly spreading interest in liquid and gaseous fuels, particularly for central-station use.

Dr. Jacobus suggests that data on related equipment referred to as being omitted from the paper be included. It was felt that the inclusion of too much of the data accumulated in the tests would make the paper unnecessarily bulky. As an appendix, data obtained in four of the eight oil-fuel tests and three of the six gas-fuel tests are included. The tests selected to report are at low, intermediate, and high ratings, thereby covering the full range investigated, without making the appendix burdensome.

The primary purpose in presenting this paper, as defined by the Power Committee, was a comparison of the characteristics of the two fuels as burned in one boiler. It is felt that inclusion of data on fans and other auxiliary boiler equipment is irrelevant to this purpose.

As Mr. Peabody states, natural gas is now easily burned under modern steam-generating units with standard equipment. The use of forced and induced draft causes no difficulty in the use of gas as fuel, and the ease with which it can be handled and its cleanliness make gas a highly desirable fuel.

The variations in heat absorption for the two fuels, as noted by Mr. Peabody, and shown in Fig. 19 and Fig. 20, have been a source of considerable discussion. Perhaps the result of this variation is most noticeable in the curves showing superheated-steam temperatures, Fig. 14. The author's experience has shown that superheated steam temperatures are usually higher with gas fuel than when oil fuel is used. The variations encountered depend largely on the type of furnace, refractory settings giving smaller variations than water walls, the variation increasing as the water-wall surface and water-wall absorption rate is increased.

Oil fires have higher temperatures than gas fires and are highly luminous. Combustion with oil fuel is delayed to a certain extent, so that a considerable proportion of the furnace is filled with luminous flame, varying of course with the rating, amount of excess air, temperature of the air and oil temperature, and pressure. These conditions tend to increase the heat absorption of the furnace walls and tubes facing the furnace, when oil fuel is used, but the resultant furnace temperature is still higher than it is with gas. With oil fuel and at a given rate of steam flow, the temperature, volume, and specific heat of flue gases entering the superheater are less than with gas fuel. These factors predominantly influence the final steam temperature. With gas fuel, combustion is extremely rapid. There is no visual evidence of luminous flame, and in all probability the combustion "area" is much reduced.

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With the special test blocks installed in the water-cooled walls, reliable comparative measurements of heat transfer are secured, and these results were used as the basis for the water-wall data given in Figs. 19 and 20.

Perhaps Mr. Peabody is correct in his statement that the efficiency graphs should, theoretically, be curved lines. At extremely low ratings there would undoubtedly be a break in the curve, since increased efficiency, with decreased rating, will only hold to a point where combustion efficiency falls off and heat losses from the boiler become proportionally large enough to be a governing factor. In the case of the tests reported, it was thought that the straight lines shown more nearly indicated the trend of results of the tests than any curve which could have been drawn through the test points.

Mr. Bailey has pointed out the discrepancy between the temperatures observed under the first tube bank shown in Fig. 24 and the curve of calculated temperatures at this point. The designation of the curves for "calculated" temperature is unfortunate, since no deduction was made for radiant heat absorbed by the lower rows of boiler tubes. Notation of this is made in Fig. 24 for final printing. The difficulty of correctly measuring flue-gas temperatures in this location, which has been quite thoroughly discussed by Kreisinger, Barkley, and others, is well recognized. In the tests presented, it was found that these temperatures were by no means uniform, and after some rather unsuccessful attempts to secure more uniform results with shielded couples, the tests were run with exposed couples. Some couples used were made of No. 22 chromel-alumel wire and others of No. 16 iron-constantan wire, the results being reported as showing "approximate" rather than true temperatures. If it were possible to calculate accurately the average gas temperature at this point, it would undoubtedly be more nearly in agreement with the observed values, which, admittedly, are probably too low.

The comments on measurement of temperatures below the first tube bank given in reply to Mr. Bailey's discussion will be of interest to Mr. Sherman.

The data for the mean specific heats of the flue gases given in Fig. 22 of the paper were taken from "Principles of Combustion in the Steam Boiler Furnace," by Arthur D. Pratt, published in 1920 by the Babcock & Wilcox Co. Values for each constituent of the flue gases were taken from Fig. 1, page 32, of this text and combined in the proportions indicated by the average flue-gas analysis as given, to secure the composite curve presented. The curve of specific heat of fuel oil was derived from experimental data obtained from the Union Oil Company of California.

FSP-54-12

Load-Deflection Relations for Large Plain, Corrugated, and Creased Pipe Bends

By E. T. COPE[1] and E. A. WERT,[2] DETROIT, MICH.

E. T. Cope

This paper presents the results of tests made on 10-in. and 14-in. seamless steel pipe loaded in the same manner as the simple cases shown on pages 526 to 529, "Piping Handbook,"[3] by Walker and Crocker. Comparison has been made between calculated deflections and the results of tests in the six cases for a plain pipe bend. The relation between the results of tests on corrugated and creased pipe bends and the calculated results for plain pipe bends of the same dimensions are shown. Such calculations are included as are necessary to show the method of analysis by which the load-deflection relations of a plain quarter-bend and its tangents are determined.

E. A. Wert

INTRODUCTION

THE limitations placed on the design of main steam piping for unit No. 6, Marysville Power Plant, made desirable the use of piping made up of elements having much more than customary flexibility. The installation of a 50,000-kw. turbo-generator in the space originally provided for a 30,000-kw. unit called for main steam piping of greater diameter. Large pipe in close quarters required unusual means to obtain sufficient flexibility and it was decided to resort to corrugated[4] pipe. Accordingly nine pieces in all corrugated quarter-bends and tangents were secured.

There did not appear to be available test data on the relative flexibility of corrugated bends and plain bends of the same dimensions. The desirability of accurate test data showing load-deflection relations for these bends was most evident. These data and the conclusions drawn from them should be especially valuable for future design. At the same time it was thought worth while to compare the load-deflection relations of a large plain pipe bend, as secured by actual test and as calculated. The investigation was further extended to include the creased[5] pipe bend and a bend made up of a short-radius forged elbow having a tangent welded to each end.

This paper presents the results of the tests made in checking the flexibility formulas for loadings of a 10-in. quarter-bend made of 400-lb. A.S.A. seamless steel pipe having a ratio of bend radius to pipe diameter of 6 to 1. The loadings were similar to those illustrated in Cases V to X, inclusive, of "Piping Handbook."[3] The results are shown for similar loadings on each of two 14-in. corrugated bends made of 400-lb. A.S.A. seamless steel pipe, two 14-in. creased bends made of 400-lb. A.S.A. seamless steel pipe, and one 10-in. bend made up of a forged seamless steel elbow having a tangent corresponding to dimensions of 250-lb. A.S.A. pipe welded to each end. The results of tests on a 14-in. corrugated expansion U-bend and on a 10-in. corrugated U-bend borrowed from the 10,000-kw. British Thomson-Houston Company high-temperature turbine installed in Delray P. H. 3 are also included. Load-deflection relations for pipe bends made of plain pipe having the same dimensions as the corrugated and creased bend tested have been calculated and are compared with the results of the tests.

A 14-in. corrugated tangent made of 400-lb. A.S.A. seamless steel pipe was also loaded as in Cases II, III, and IV. The flexibility of this tangent is compared with that of a plain tangent of the same dimensions. Fig. 1 shows the schemes of loading. The drawings of the pipe bends and the tangent tested are shown in Figs. 2 to 8, inclusive.

I—TEST PROCEDURE APPLIED TO 90-DEG. PIPE BENDS

Loadings of each 90-deg. bend according to the schedule shown in Fig. 1 were carried out by bolting one flange of the bend to the top of the turbine casing of a partly dismantled vertical shaft turbo-generator. The generator had been removed. This heavy casting proved to be a satisfactory bending table. The turbine top, the framework of the building, adjacent abandoned

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[1] Research Department of The Detroit Edison Company. Mem. A.S.M.E. Mr. Cope was graduated in 1907 with the degree of B.S. in M.E. from the Towne Scientific School, University of Pennsylvania, and was granted the degree of Mech. Eng. in 1910 by the same university. After graduation he spent three years with the Westinghouse Machine Company of Pittsburgh, and three years as instructor in the Engineering College at the University of Michigan. Since 1913 he has been connected with The Detroit Edison Company, and since 1919 in the Research Department. He is co-author with W. A. Carter of a paper on "The Rational Design of Covering for Pipe Carrying Steam Up to 800 Deg. Fahr.," presented at the Spring Meeting of the A.S.M.E. in 1925.

[2] Piping Division, Drafting and Surveying Bureau, The Detroit Edison Company. Mem. A.S.M.E. Mr. Wert was educated at Carnegie Institute of Technology and the University of California. He served for two years (1917–1919) in the U. S. Air Service as instructor and in engineering duties. For five years he was employed as a designer with The Detroit City Gas Company. Since 1925 he has been engaged in piping design with The Detroit Edison Company.

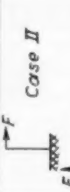
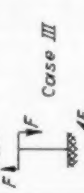
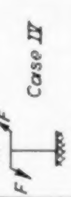
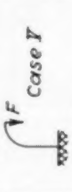
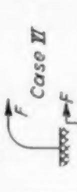
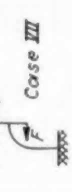
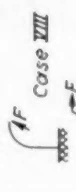
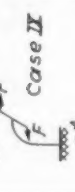
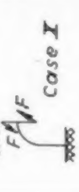
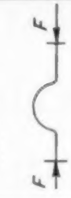
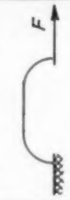
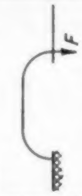
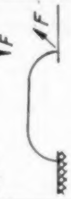
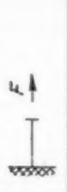
[3] All "cases" referred to are given on pages 526 to 529, inclusive, of "Piping Handbook," second edition (1931), by J. H. Walker and Sabin Crocker, McGraw-Hill Book Co., Inc. Attention is called to the revision of Cases VIII, IX, and X in the second edition. The revised cases are also given in the closure of a paper on "Frictional Resistance and Flexibility of Seamless-Tube Fittings Used in Pipe Welding," by Sabin Crocker and Arthur McCutchan, presented at the December, 1930, meeting of the A.S.M.E.

[4] P. Koffoski, of Poland, patentee.

Contributed by the Power Division and presented at the Hartford, Conn., Meeting, June 1–3, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] Alfred Schmidt, of Düsseldorf, Germany, patentee.

Kind of Loading and Case No. based on "Piping Handbook" pp. 526-529	 Case II	 Case III	 Case IV	 Case V	 Case VI	 Case VII	 Case VIII	 Case IX	 Case X					
10-in. Plain Tangent Item 1, p. 2	X	–	–	–	–	–	–	–	–	–	–	–	–	–
10-in. Plain Bend Item 2, p. 2	–	–	–	X	X	X	X	X	X	–	–	–	–	–
14-in. Corrugated Tangent Item 3, p. 2	X	X	X	–	–	–	–	–	–	–	–	–	–	X
14-in. Corrugated Bend No. 1 Item 4, p. 2	–	–	–	X	X	X	X	X	X	–	–	–	–	–
14-in. Corrugated Bend No. 2 Item 5, p. 2	–	–	–	X	X	X	X	X	X	–	–	–	–	–
14-in. Corrugated Expansion U-Bend Item 6, p. 2	–	–	–	–	–	–	–	–	–	X	–	–	–	–
10-in. Corrugated U-Bend Item 7, p. 2	–	–	–	–	–	–	–	–	–	–	X	X	X	–
14-in. Creased Bend No. 1 Item 8, p. 3	–	–	–	X	X	X	X	X	X	–	–	–	–	–
14-in. Creased Bend No. 2 Item 9, p. 3	–	–	–	X	X	X	X	X	X	–	–	–	–	–
10-in. Short-Radius Bend Item 10, p. 3	–	–	–	X	X	X	X	X	X	–	–	–	–	–

X Indicates Test Made
– Indicates no Test Made

FIG. 1 SCHEDULE OF TESTS MADE ON THE VARIOUS SPECIMENS OF PIPE

steam turbines, and the traveling crane hoist were used as points of attachment for chains or cables used in applying load. Load was applied by the use of chain falls or turnbuckles or both. In the cases where a single pull was applied to the end of the pipe, the chain fall was satisfactory. In the cases where a moment was applied to the end of the pipe, a chain fall and a turnbuckle were used in series. Load was measured by Chatillon traction dynamometers which were calibrated in the Universal testing machine before and at intervals during the tests. When a moment was applied, the two dynamometers were calibrated, one against the other. This proved to be a worth-while added precaution.

Measurements of deformation were made between fixed points on the pipe and fixed points attached either to the bending table or the building. A micrometer having, when necessary, a filler piece of fixed but unknown length was used in making the readings.

Contact points for the micrometer, on the pipe and on the fixed points, consisted of short brass pins driven tightly into small drill holes in the pipe and the fixed points. The outer ends of the brass pins were filed round.

The movement of the free end of the bend was measured in three directions at each load. Measurement of movement of the pipe a few inches above the fixed end was also made. This measurement had no direct bearing on the problem, but immediately revealed unexpected displacement of the fixed end during the progress of the test.

Experience in calibrating helical springs has shown that the spring must be loaded heavily two or three times before consistent load-deflection readings can be secured. This same treatment was found to be necessary with new pipe bends. The load on the pipe was rapidly increased. At each step in the loading, readings of the major deflection were obtained and a load-deflection curve plotted. When the proportional limit was slightly exceeded, the load was removed and the no-load readings were repeated. The load necessary to stress the pipe to the proportional limit was reapplied and released two or three times until the no-load readings before and after successive loadings became consistent. Then the formal loading of the pipe was carried out. In most instances this procedure made possible the securing of successive deformation readings that agreed within a few thousandths of an inch. The test assemblies of the various kinds of 90-deg. bends are shown in Figs. 9 to 15, inclusive.

II—MATHEMATICAL ANALYSIS OF A 10-IN. PLAIN PIPE BEND

The mathematical determination of deflections of pipe bends lying in one plane is based on the grapho-analytical method, using area-moment diagrams and the principles of superposition.

The condition in which only a single force or moment acts at the free end is assumed. These solutions are not of direct application to the problem of determining flexibility of piping subjected to thermal expansion. When piping is subjected to thermal expansion, both ends are considered as anchored and the remaining forces and restraining moments are considered as acting at hypothetical "free" ends. The principles of superposition are used, whereby the load-deflection characteristics

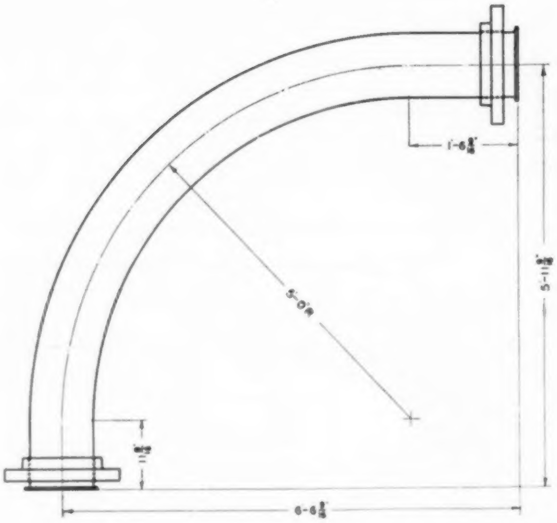

1-6 5/8"
5'-0"R
5-11 3/8"
11 3/8"
6-6 3/8"

Fig. 2 10-In. Quarter-Bend With Tangents
(Mean outside diameter, 10.69 in.; mean thickness (nominal), 0.39 in.; mean inside diameter, 9.90 in.)

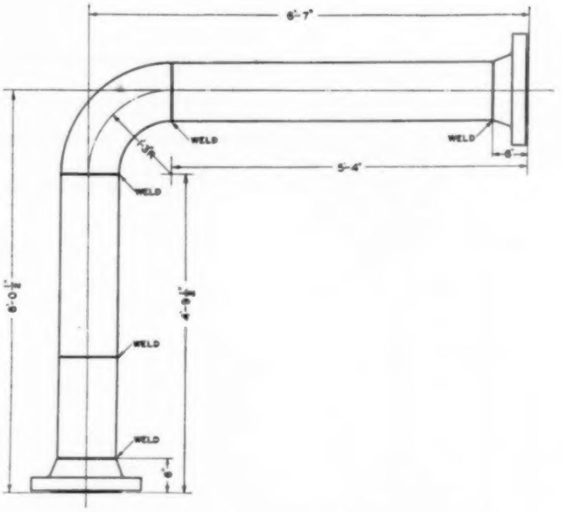

6'-7"
1'-3"R
WELD
WELD
6"
5-4"
WELD
6'-0 1/2"
4'-9 1/2"
WELD
WELD
6"

Fig. 3 10-In. Tube Turn With Tangents
(Mean Dimensions. Tangents—Outside diameter, 10.774 in.; thickness of wall, 0.31 in.; inside diameter, 10.15 in. Bend—Outside diameter, 10.84 in.; thickness of wall, 0.35 in.; inside diameter, 10.14 in.)

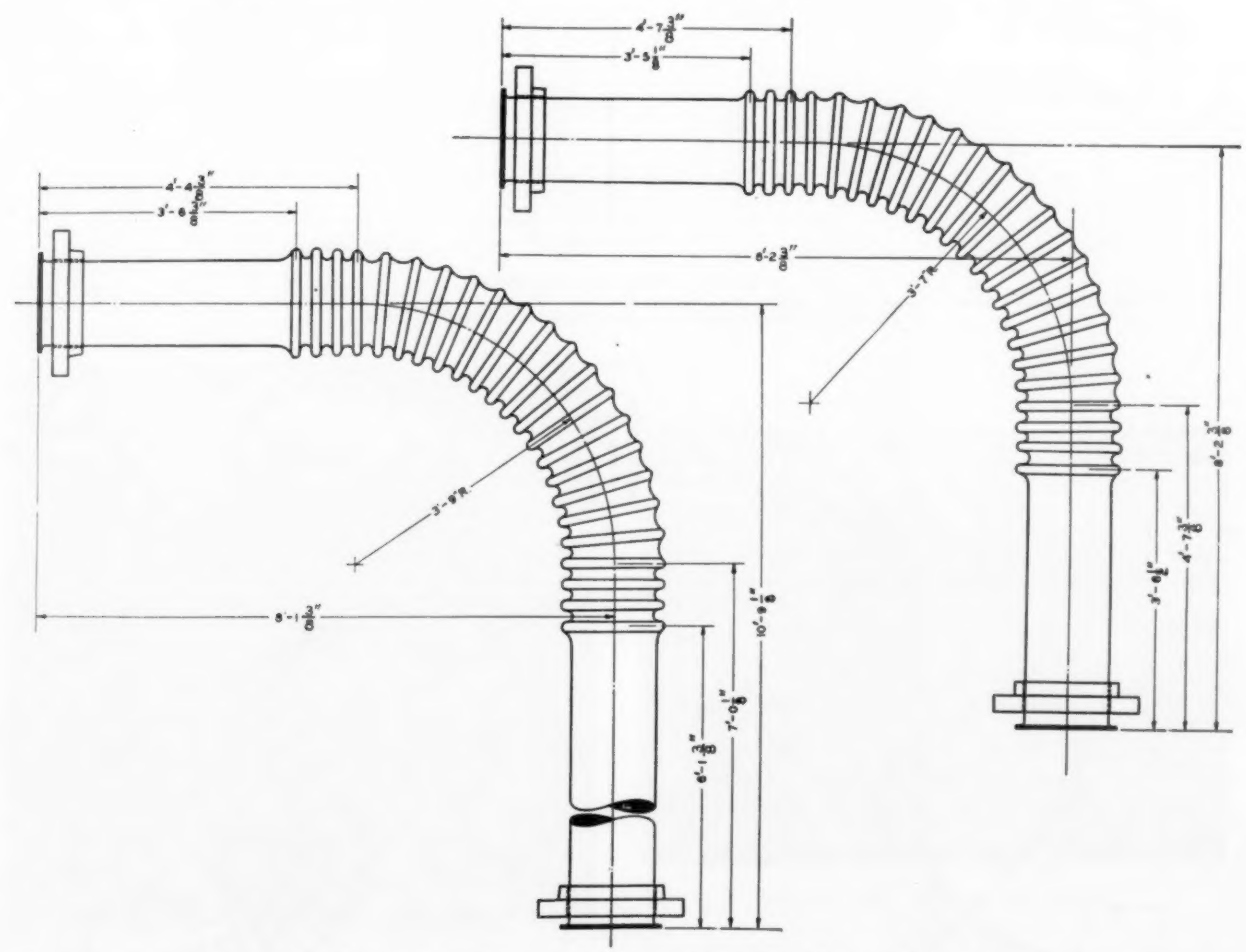

4'-7 3/8"
3'-5 1/8"
8'-2 3/8"
3'-7"R
8'-2 3/8"
4'-7 3/8"
3'-6 1/2"
4'-4 3/8"
3'-6 1/8"
3'-9"R
8'-1 3/8"
10'-9 1/8"
7'-0 1/8"
6'-1 3/8"

Fig. 4 14-In. Corrugated Quarter-Bends With Tangents
(Mean Dimensions. Bend No. 1 (right): Tangents—Outside diameter, 14.06 in.; nominal thickness of wall, 0.50 in.; inside diameter, 13.06 in. Corrugations—Outside diameter of crests, 17.06 in.; outside diameter of troughs, 14.13 in. Bend No. 2 (left): Tangents—Outside diameter, 14.06 in.; nominal thickness of wall, 0.50 in.; inside diameter, 13.06 in. Corrugations—Outside diameter of crests, 17.00 in.; outside diameter of troughs, 14.06 in.)

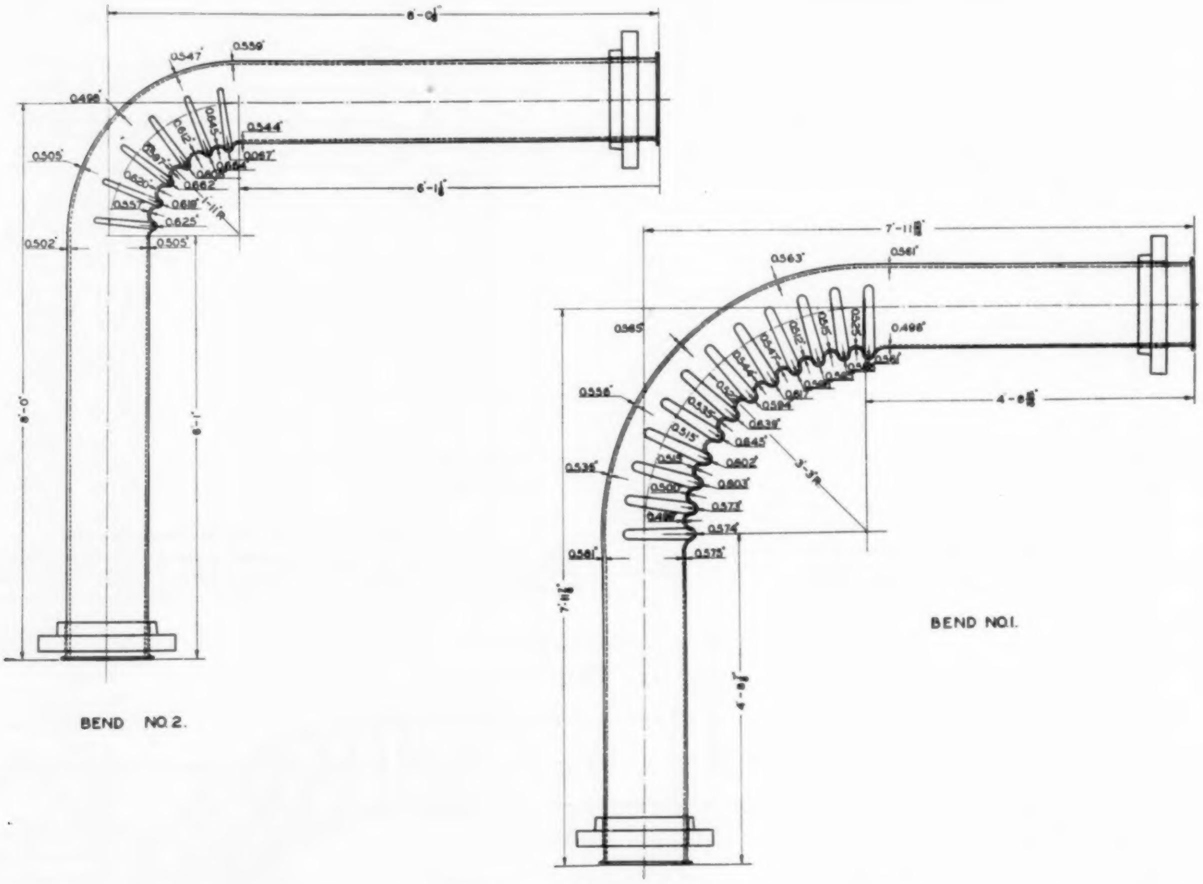

0.547"
0.559"
0.498"
0.505"
0.612"
0.645"
0.544"
0.567"
0.667"
0.654"
0.620"
0.662"
0.557"
0.618"
1'-11"R
0.625"
0.502"
0.505"
6'-1¼"
8'-0"
6'-1"
BEND NO. 2.
7'-11⅝"
0.563"
0.561"
0.565"
0.498"
0.547"
0.612"
0.615"
0.561"
0.556"
0.527"
0.594"
0.535"
0.639"
0.515"
0.645"
0.602"
0.535"
0.500"
0.603"
0.573"
0.574"
0.581"
0.575"
3'-3"R
4'-8⅝"
7'-11⅝"
4'-8⅝"
BEND NO. 1.

FIG. 5 14-IN. CREASED BENDS
(MEAN DIMENSIONS OF TANGENTS. Bend No. 1: Outside diameter, 14.04 in.; wall thickness, 0.50 in.; inside diameter, 13.04 in. Bend No. 2: Outside diameter, 14.02 in.; wall thickness, 0.53 in.; inside diameter, 12.96 in.)

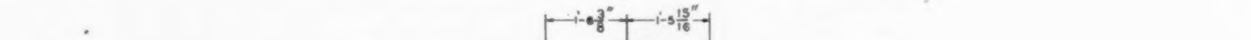

1'-6$\frac{3}{8}$"
1'-5$\frac{15}{16}$"
3'-5"R
3'-6"R
3'-2$\frac{7}{16}$"
10'-1$\frac{7}{16}$"
10'-1$\frac{11}{16}$"
3'-1⅝"
3'-6"R
3'-6"R
1'-6$\frac{3}{8}$"
1'-6"
¼"
9'-11⅝"
9'-11$\frac{15}{16}$"

FIG. 6 14-IN. CORRUGATED EXPANSION U-BEND
(MEAN DIMENSIONS AT ENDS. Outside diameter, 14.06 in.; wall thickness, 0.50 in.; inside diameter, 13.06 in.)

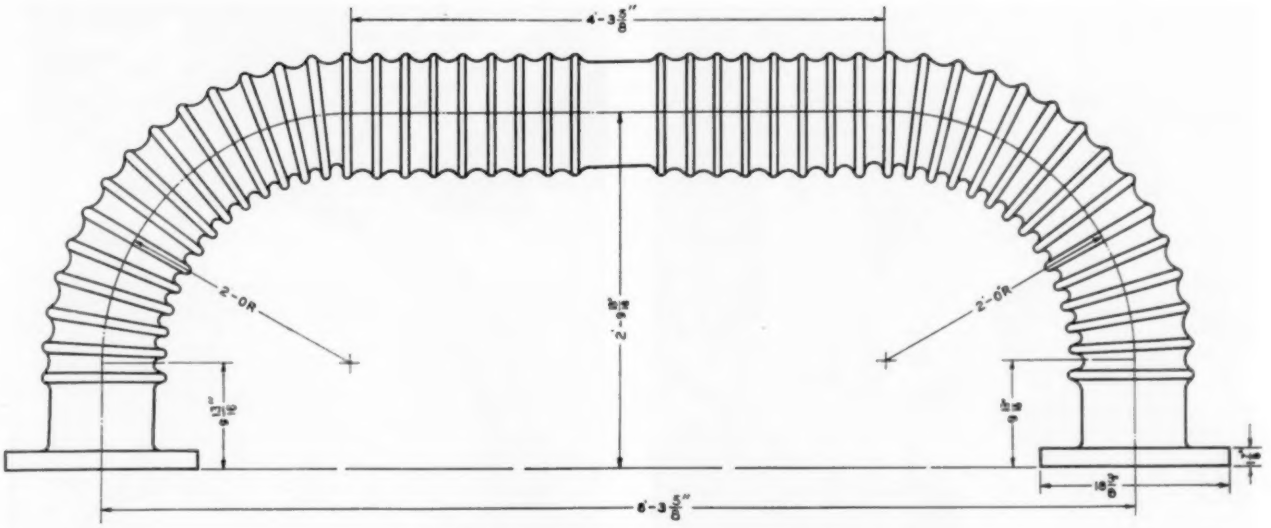

4'-3 5/8"
2'-0"R
2'-0"R
6'-3 5/8"

FIG. 7 10-IN. CORRUGATED EXPANSION U-BEND
(MEAN DIMENSIONS AT ENDS. Outside diameter, 10.36 in.; wall thickness, 0.22 in.; inside diameter, 9.92 in. Corrugations: Outside diameter of crests, 12.11 in.; outside diameter of troughs, 10.25 in.)

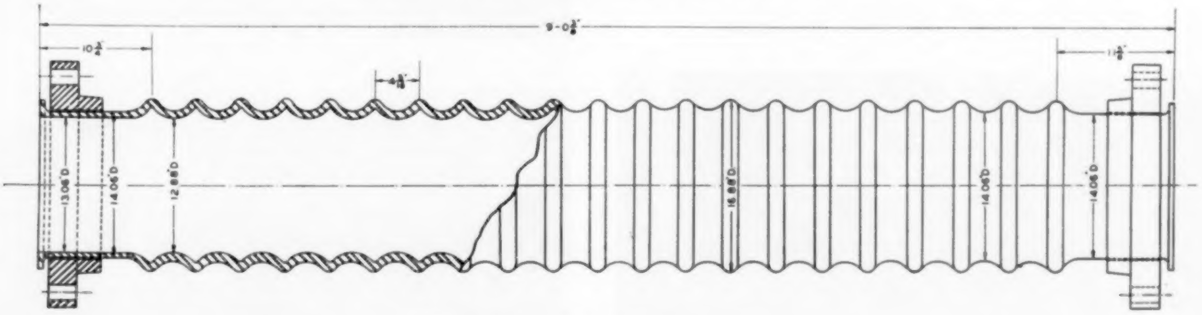

13.06"D
14.06"D
12.86"D
14.06"D
14.06"D

FIG. 8 14-IN. CORRUGATED TANGENT

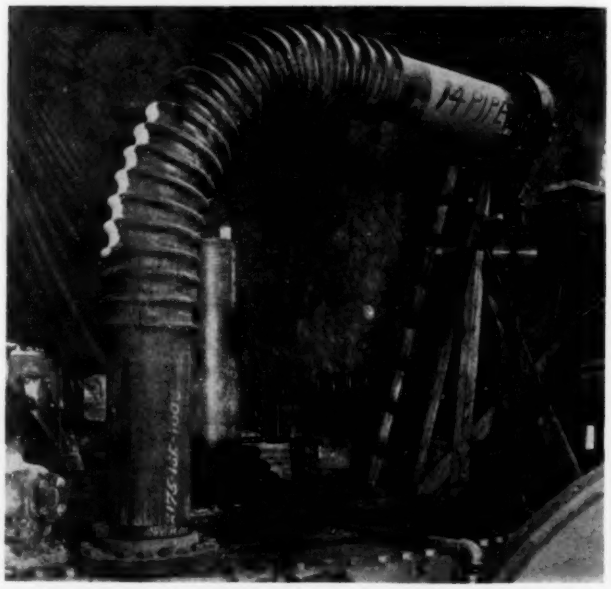


FIG. 9 14-IN. CORRUGATED PIPE BEND

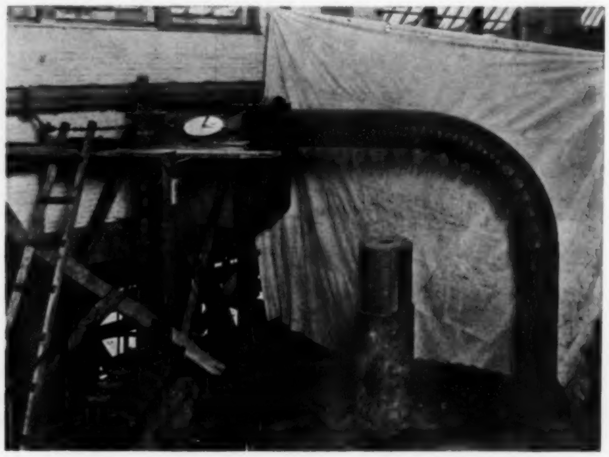


FIG. 10 14-IN. CREASED PIPE BEND

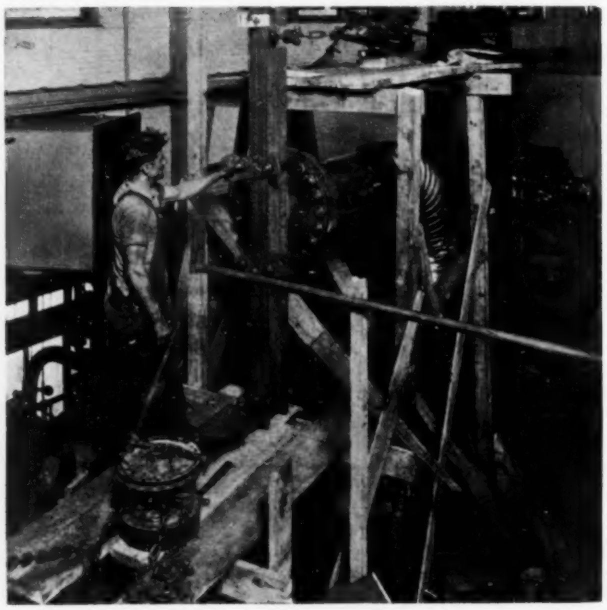


FIG. 11 14-IN. CORRUGATED PIPE BEND

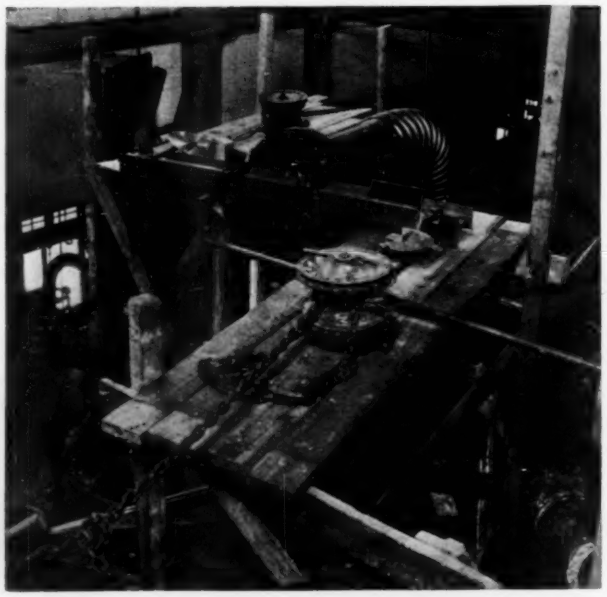


FIG. 13 14-IN. CORRUGATED PIPE BEND

of the pipe bend are solved by first transposing the thrusts and bending moments which exist at the anchor points, applying them to each component part of the pipe bend, and then making an algebraic summation of, first, the deflections of the parts due to such transposed forces and moments and, second, the deflection of the parts due to the cumulative rotations of the parts between each component part and the anchored end of the bend.

Ten elementary cases are required to determine completely the relations between the possible forces and moments which may act on these elements and the resulting rotations and deflections.

These ten cases are treated in "Piping Handbook," pages 526 to 529, inclusive. The applications of the elementary cases to specific pipe bends are taken up one at a time.

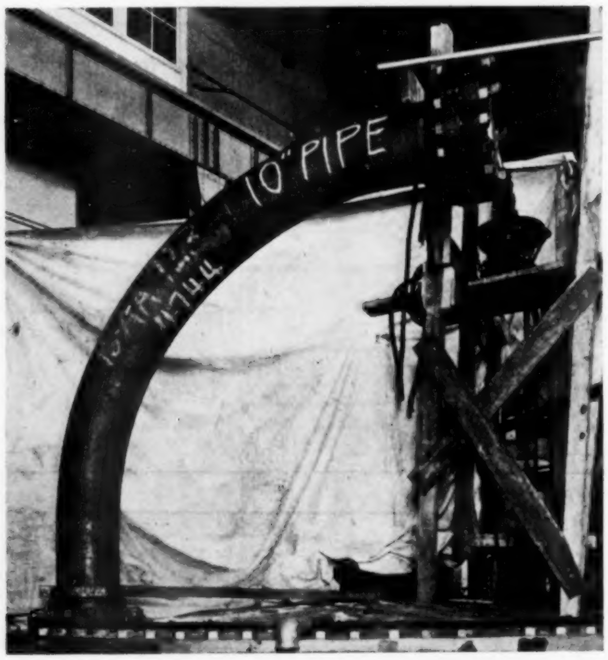


FIG. 12 10-IN. PLAIN PIPE BEND

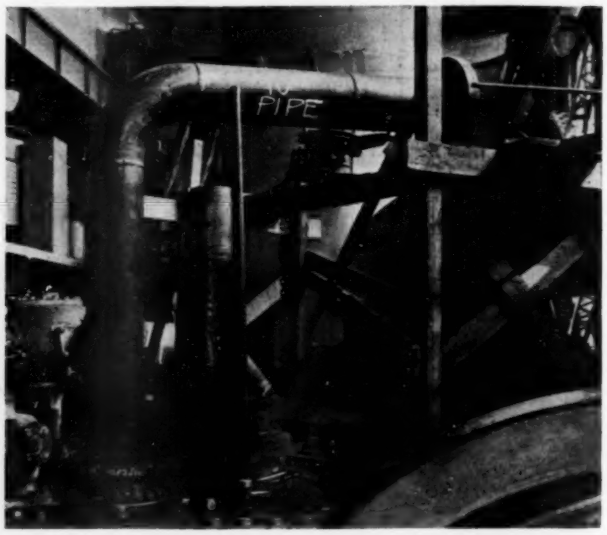


FIG. 14 10-IN. TUBE TURN WITH TANGENTS

PIPE BEND LOADED WITH A TRANSVERSE FORCE ACTING AT THE FREE END IN THE PLANE OF THE BEND

The manner in which forces and moments are transposed and deflections combined for a 10-in. plain pipe bend, Fig. 2, having a transverse force F acting at its free end in the plane of the bend, is explained as follows [Fig. 16 (*a*) shows the bend as a line drawing]: The component parts of the bend are shown with the transposed forces and moments in Fig. 16 (*b*). The component part *ce* is a straight cantilever with a transverse force F applied at its free end as in Case II. The abscissa of the moment diagram is the true length of this part. *bc* is a quarter-bend having a radial force F (called F_r) and a moment Fl, both acting in the plane of the bend, applied at its free end. The value of l in the moment Fl is the length of *ce*. These two loadings are, respectively, Case V*a* and Case VII*a*. The abscissa of the moment diagram is the developed length of this quarter bend, $\pi R/2K$ or $\pi \times 60/2K$, where K is the rigidity multiplication factor.[6] This is called the virtual length of the bend and equals 188.5 in. *ab* is a cantilever having the moment $Fl + F_rR$ which equals $13.81F + 60F$ or $73.81F$ acting at its free end *b*. The abscissa of its moment diagram is its true length. This loading is Case III. The force F is transmitted axially through *ab* to the anchor point.

In addition to the deflections at the free ends of these cantilevers, due to bending within themselves, rotations of the component parts between their hypothetical fixed ends and the anchored end of the pipe bend, represented by ψ_b and ψ_c, produce deflections at the ends of the cantilevers. These are added algebraically to the individual deflections of the parts.

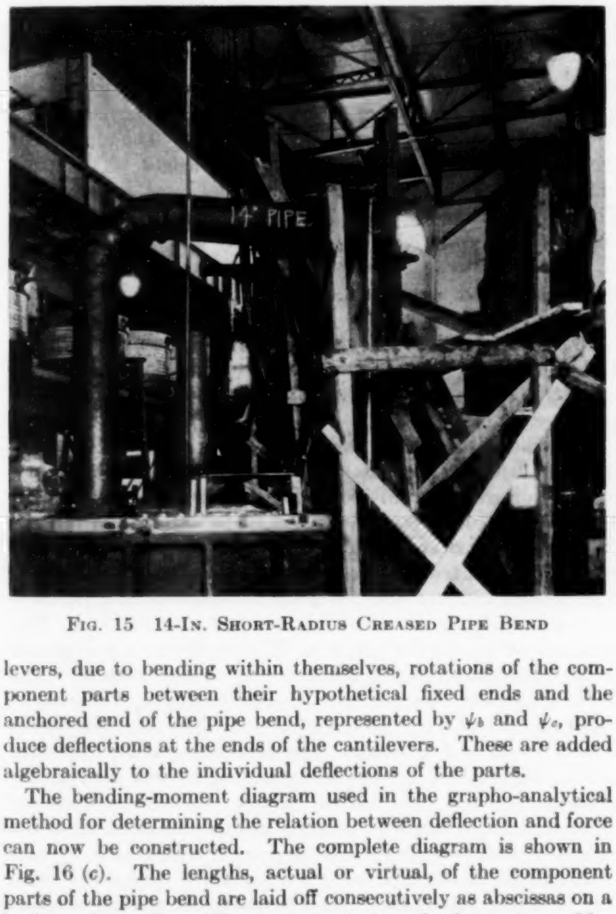


FIG. 15 14-IN. SHORT-RADIUS CREASED PIPE BEND

The bending-moment diagram used in the grapho-analytical method for determining the relation between deflection and force can now be constructed. The complete diagram is shown in Fig. 16 (*c*). The lengths, actual or virtual, of the component parts of the pipe bend are laid off consecutively as abscissas on a horizontal base line. The moment at any point is represented by the ordinate at that point. The moment diagram for the part *ce* is a triangle, the value of whose ordinate is zero at *e* and Fl, which equals $13.81F$, at point *c*. The moment Fl carries through the section of piping to *b*, hence a horizontal line is drawn from the top of the ordinate at *c* toward the left and ends at the ordinate at *b*. The quarter bend *bc* is subjected to an additional bending moment due to the transposed radial force F. This diagram takes the form of a half-segment of a parabola. The value of its ordinate is zero at *c* and FR, represented by and equal to $60F$, at *b*. The moment at *b* is the sum of the ordinates at this point which equals $13.81F + 60F$ or $73.81F$. This moment is carried through to the left and ends at the ordinate at *a*. The difference in the flexural rigidity of the component part *bc* and the other component parts, caused by flattening of the cross-section due to flexure in the plane of the bend, is taken into account by the use of the virtual length of the part *bc* for the abscissa in the bending-moment diagram. The bending moment acting on each component part corresponds to the transposed forces and moments as shown in Fig. 16 (*b*). In the computations, all rotations above the abscissa have been considered as clockwise, those below as counterclockwise. The direction of rotation of the resulting moments is then apparent from an inspection of the free-body diagram in Fig. 16 (*b*).

From the bending-moment diagrams described above, deflections at the free end of each component part can now be obtained. The deflection at the free end of each component part is the algebraic sum of all the deflections which may be due to all or part of the following:

a Rotation caused by deflection of preceding component parts
b Force acting at free end of component parts
c Moment acting at free end of component parts.

Deflections due to rotations are equal to the length of a component part multiplied by the algebraic sum of all the areas of the bending-moment diagram preceding the fixed end of the part, divided by EI.

Deflection due to forces and moments is the result of successive rotations of the elementary length comprising the component part. The total deflection at any point equals the algebraic sum of all the deflections preceding the part in question.

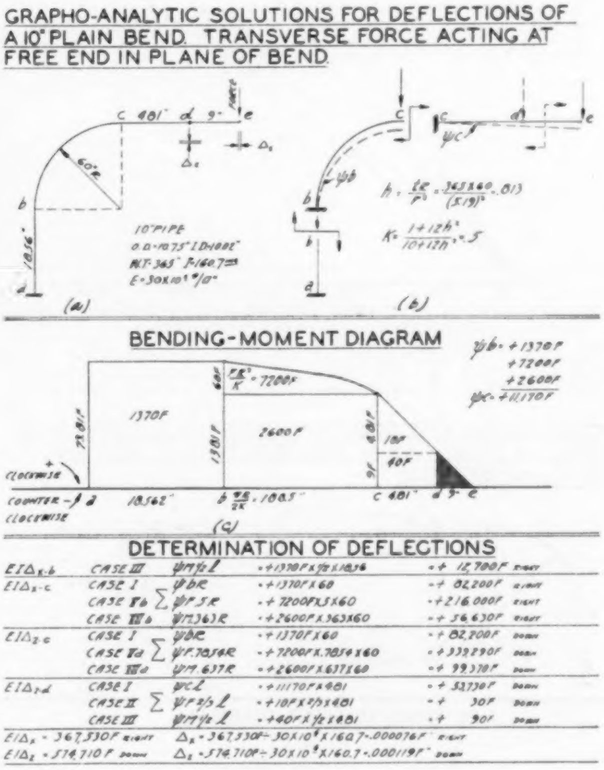

GRAPHO-ANALYTIC SOLUTIONS FOR DEFLECTIONS OF A 10" PLAIN BEND. TRANSVERSE FORCE ACTING AT FREE END IN PLANE OF BEND
c 481" d 9" e
FORCE
Δy
Δx
60"R
b
a
108.56"
10" PIPE
O.D.=10.75" I.D.=10.02"
WT.=.365" I=160.7 in⁴
E=30×10⁶ #/□"
(a)
ψb
ψc
h = tR/r² = .365×60/(5.19)² = .813
K = (1+12h²)/(10+12h²) = .5
(b)
BENDING-MOMENT DIAGRAM
ψb = +1370F
+7200F
+2600F
ψc = +11,170F
FR²/K = 7200F
1370F
2600F
10F
40F
73.81F
60F
13.81F
4.81F
9F
+ CLOCKWISE
COUNTER- CLOCKWISE
a 108.562" b πR/2K = 188.5" c 4.81" d 9" e
(c)
DETERMINATION OF DEFLECTIONS
EIΔx-b CASE III ψm½l = +1370F×½×108.56 = +12,700F RIGHT
EIΔx-c CASE I ψbR = +1370F×60 = +82,200F RIGHT
CASE Vb Σ ψF.5R = +7200F×.5×60 = +216,000F RIGHT
CASE VIIb ψm.363R = +2600F×.363×60 = +56,630F RIGHT
EIΔy-c CASE I ψbR = +1370F×60 = +82,200F DOWN
CASE Va Σ ψF.7854R = +7200F×.7854×60 = +339,290F DOWN
CASE VIIa ψm.637R = +2600F×.637×60 = +99,370F DOWN
EIΔy-d CASE I ψcl = +11170F×4.81 = +53,730F DOWN
CASE II Σ ψF⅔l = +10F×⅔×4.81 = +30F DOWN
CASE III ψm½l = +40F×½×4.81 = +90F DOWN
EIΔx = 367,530F RIGHT Δx = 367,530F÷30×10⁶×160.7 = .000076F" RIGHT
EIΔy = 574,710F DOWN Δy = 574,710F÷30×10⁶×160.7 = .000119F" DOWN

FIG. 16

The total downward deflection at the point *d* (the point at which downward deflections were measured in the tests on this pipe bend), Fig. 16 (*a*), is obtained by the summation of all the increment downward deflections from the fixed end of the pipe bend up to the point *d*. The total deflection to the right at the point *c* of the pipe bend is the summation of all the increment deflections to the right.

The values of deflection obtained by the above method of calculation and from tests made on this bend are shown both tabulated and graphically in Fig. 17. No allowance has been

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[6] Reference 1, p. 552.

TABULATED RESULTS-10" PLAIN BEND

FORCE	Δ_x THEORY	Δ_x TEST	% DIFF.	Δ_z THEORY	Δ_z TEST	% DIFF.
1000#	0.076"	0.075"	1.3%	0.119"	0.120"	.8%
2000#	0.152"	0.161"	5.5%	0.238"	0.269"	11.5%
3050#	0.232"	0.261"	11.1%	0.362"	0.421"	14%
4000#	0.304"	0.347"	12.3%	0.476"	0.568"	16.1%
5000#	0.380"	0.434"	12.4%	0.595"	0.717"	17%

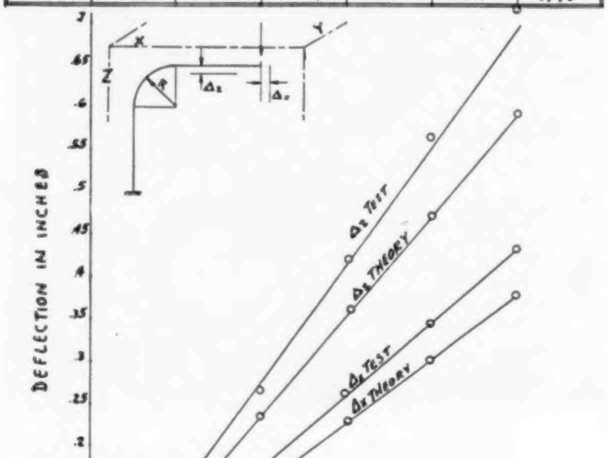
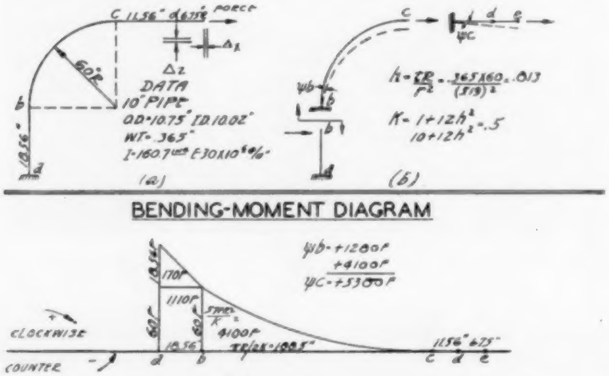


FIG. 17

GRAPHO-ANALYTIC SOLUTIONS FOR DEFLECTIONS OF A 10" PLAIN BEND. AXIAL FORCE ACTING AT FREE END IN PLANE OF BEND



DETERMINATION OF DEFLECTIONS

$EI\Delta_{x-b}$ CASE II $\Sigma\ \psi r\ 2/3\,l = +170F \times 2/3 \times 18.56 = 2100\,F$ right
CASE III $\psi M\ 1/2\,l = +1110F \times 1/2 \times 18.56 = 10300\,F$ right

$EI\Delta_{x-c}$ CASE I $\Sigma\ \psi bR = +1280F \times 60 = 76800F$ right
CASE VIIa $\psi r\,.624R = +4100F \times .624 \times 60 = 153{,}500F$ right

$EI\Delta_{z-c}$ CASE I $\Sigma\ \psi bR = +1280F \times 60 = 76{,}800\,F$ down
CASE VIIb $\psi r\,.876R = +4100F \times .876 \times 60 = 215{,}500F$ down

$EI\Delta_{z-d}$ CASE I $\Sigma\ \psi c\,l = +5380F \times 11.56 = 62{,}190\,F$ down

$EI\Delta_x$ 242700F RIGHT $\Delta_x = 242{,}700F \div 30 \times 10^6 \times 160.7 = .0000503F''$ right
$EI\Delta_z$ 354490F DOWN $\Delta_z = 354{,}490F \div 30 \times 10^6 \times 160.7 = .0000735F''$ down

FIG. 18

made for changes in the value of moment of inertia because of flattening of the quarter-bend or other parts of the pipe during fabrication. The value of I for the cross-section at the middle of the quarter-bend was computed, using actual measurements and using nominal dimensions. In each case the nominal thickness of metal was assumed. The results are:

Using measured dimensions, $I = 160.4$ in.⁴
Using nominal dimensions, $I = 160.7$ in.⁴

PIPE BEND LOADED WITH AN AXIAL FORCE ACTING AT THE FREE END IN THE PLANE OF THE BEND

The solution of the 10-in. pipe bend already described, when subjected to an axial load at its free end, is shown in Fig. 18. The component parts of the pipe bend are shown with transposed forces and moments in Fig. 18 (*b*). The component part *ce* is a straight cantilever acted on by an axial force. The abscissa of the bending-moment diagram is the true length of the part. *bc* is a quarter-bend acted on by the axial force *F*, called F_a. The abscissa of the bending-moment diagram is the developed length of the bend divided by the rigidity multiplication factor *K*. The length used as abscissa is known as the virtual length of the quarter-bend and equals $\pi R/2K$ or 188.5 in. This loading of the quarter-bend is Case VI. *ab* is a cantilever acted on by the force *F* and the moment F_rR in the plane of the bend. F_rR equals 60*F*. The abscissa of the bending-moment diagram is the true length of this cantilever. These loadings are Case II and Case III, respectively

Rotations of the elements are added algebraically to the individual deflections of the component parts. The bending-moment diagram is constructed in the following manner [Fig. 18(*c*)]: Component part *ce* is acted on by an axial force, consequently there is no bending. The ordinate of the diagram is zero. The quarter-bend *bc* is acted on by an axial force *F*. The bending-moment diagram is the half-segment of a parabola having an ordinate of value zero at *c* and *FR* (60*F*) at *b*. The ordinate 60*F* carries through to *a*. The cantilever *ab* is subjected to an additional bending moment due to the force *F*. The bending-moment diagram is a triangle having an ordinate of value zero at *b* and *F* × *ab* or 18.56*F* at *a*. The total ordinate at *a* is *RF* + *ab* × *F* or 78.56*F*. The summations of the respective deflections obtained by the same method as was used in the previous problem gives values for the relations between the downward displacement at points *d* and the displacement to the right at *e*, and the force *F*. *d* is the point at which the downward displacement of the pipe bend was measured during the tests. The values of deflections obtained by the above method of calculation and from actual tests are shown tabulated and graphically in Fig. 19.

PIPE BEND LOADED WITH A MOMENT ACTING AT THE FREE END IN THE PLANE OF THE BEND

The relation of force to deflection of the 10-in. plain pipe bend under discussion when subjected to a bending moment at its free end in the plane of the bend is next determined. Fig. 20 shows the plan of the bend and the analysis of this loading. The component parts of the bend with transposed moments are shown in Fig. 20 (*b*). The component part *ce* is a straight cantilever having a moment 72*F*, acting at its free end. This is Case III. The abscissa of the bending moment diagram is the true length of *ce*. The quarter-bend *bc* is acted on by a moment in the plane of the bend as in Case VII. The abscissa of the bending-moment diagram is the virtual length of the developed quarter bend and equals 188.5 in. *ab* is a straight cantilever acted on by a moment at its free end as in Case III. The abscissa of its bending-moment diagram is its true length. Rotations of the parts are added algebraically to the individual deflections as previously described. The complete bending-moment diagram is shown in Fig. 20 (*c*). The bending moment *M*, which equals 72*F*, carries through the entire length of the pipe bend. The summation of increment deflections to the right at point *e* and downward at point *d* per unit value of *F* are shown. Fig. 21 shows the values of deflections for different values of *F* both in a table and graphically.

TABULATED RESULTS-10"PLAINBEND

FORCE	Δ_x THEORY	Δ_x TEST	% DIFF.	Δ_z THEORY	Δ_z TEST	% DIFF.
1000"	0.050"	0.073"	31.5%	0.073"	0.093"	21.5%
2000"	0.101"	0.132"	23.4%	0.147"	0.169"	13.0%
3000"	0.151"	0.186"	18.8%	0.220"	0.244"	9.8%
4000"	0.201"	0.248"	18.9%	0.294"	0.327"	10.0%

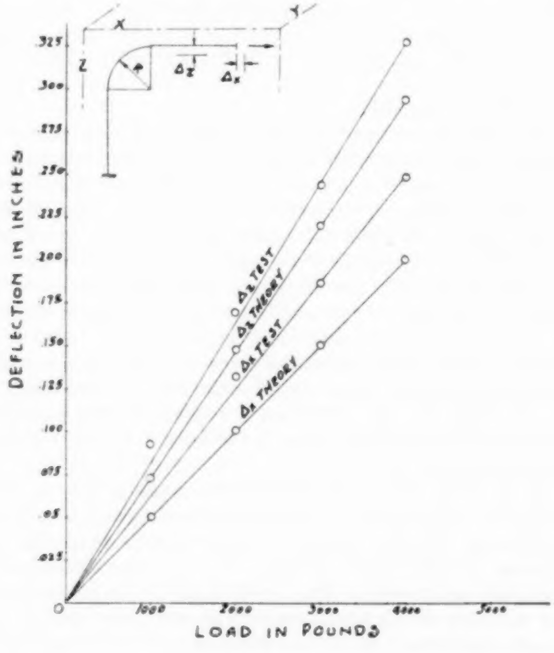


FIG. 19

PIPE BEND LOADED WITH A FORCE ACTING AT FREE END IN PLANE PERPENDICULAR TO PLANE OF BEND

The force applied in this manner causes rotation of the elements of the section of the piping in two planes, namely, the *XY* and *YZ* planes. This is shown in Fig. 22 (*a*). The resultant deflection along the *Y*-axis is determined as follows: The displacement is determined for each component part, caused by the action of the force *F* in the *XY*-plane. To this displacement is added the displacement of each component part caused by the action of the force *F* in the *YZ*-plane.

The manner in which forces and moments are transposed and deflections combined in the *XY*-plane is explained as follows: The component parts of the pipe bend with the transposed forces and moments acting upon them are shown in Fig. 22 (*b*). The component part *ce* is a straight cantilever with a transverse force *F* acting at its free end (Case II). *bc* is a quarter-bend acted upon by a moment equal to *F* × *ce* or 13.93*F* and a force *F*, both of which are in a plane perpendicular to the plane of bend and tangent to its free end. (Cases VIII*b* and IX, respectively.) *ab* is a straight cantilever acted upon by a torsional moment caused by the force acting in the *XY*-plane. This tor-

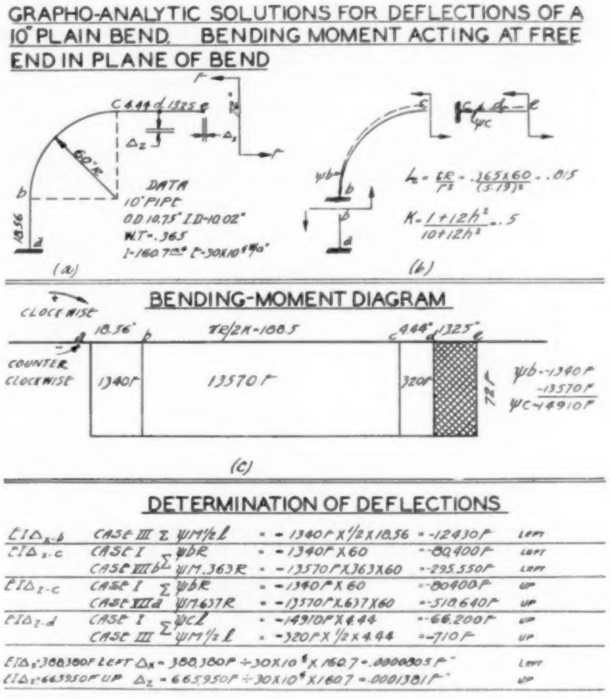

GRAPHO-ANALYTIC SOLUTIONS FOR DEFLECTIONS OF A 10" PLAIN BEND. BENDING MOMENT ACTING AT FREE END IN PLANE OF BEND
F
c 4.44 d 13.25 e
Δ_z
Δ_x
60"R
b
18.56
a
DATA
10" PIPE
O.D 10.75" I.D-10.02"
W.T-.365
I-160.7 in.4 E-30x10^6 #/□"
(a)
ψb
ψc
$h = \frac{tR}{r^2} = \frac{.365 \times 60}{(5.19)^2} = .815$
$K = \frac{1+12h^2}{10+12h^2} = .5$
(b)
BENDING-MOMENT DIAGRAM
CLOCKWISE
COUNTER CLOCKWISE
18.56" πR/2R-100.5 4.44" 13.25"
1340F
13570F
320F
72F
ψb-1340F
-13570F
ψc-14910F
(c)
DETERMINATION OF DEFLECTIONS
$EI\Delta_{x\text{-}b}$ CASE III Σ ψM½ℓ = -1340F × ½ × 18.56 = -12430F LEFT
$EI\Delta_{x\text{-}c}$ CASE I Σ ψbR = -1340F × 60 = -80,400F LEFT
CASE VIIIb ψM.363R = -13570F × .363 × 60 = -295,550F LEFT
$EI\Delta_{z\text{-}c}$ CASE I Σ ψbR = -1340F × 60 = -80400F UP
CASE VIIIa ψM.637R = -13570F × .637 × 60 = -518,640F UP
$EI\Delta_{z\text{-}d}$ CASE I Σ ψcℓ = -14910F × 4.44 = -66,200F UP
CASE III ψM½ℓ = -320F × ½ × 4.44 = -710F UP
$EI\Delta_x$ = 388,380F LEFT Δ_x = 388,380F ÷ 30×10^6 × 160.7 = .0000805F" LEFT
$EI\Delta_z$ = 665,950F UP Δ_z = 665,950F ÷ 30×10^6 × 160.7 = .0001381F" UP

FIG. 20

TABULATED RESULTS - 10" PLAIN BEND

FORCE	Δ_x THEORY	Δ_x TEST	% DIFF.	Δ_z THEORY	Δ_z TEST	% DIFF.
1000"	.080	.083"	3.6%	0.138"	0.155"	11%
2000"	.160"	.179"	10.6%	0.276"	0.308"	10.3%
3000"	.240"	.282"	14.8%	0.414"	0.475"	12.8%

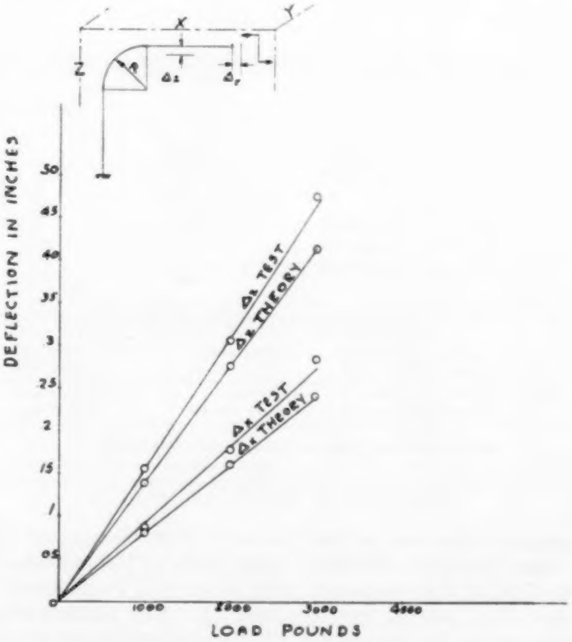


FIG. 21

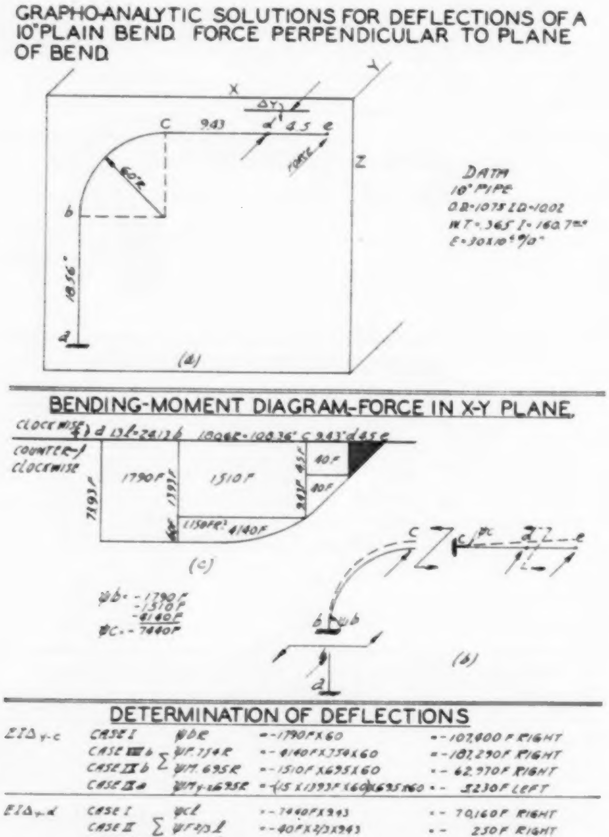

GRAPHO-ANALYTIC SOLUTIONS FOR DEFLECTIONS OF A 10" PLAIN BEND FORCE PERPENDICULAR TO PLANE OF BEND
X
Y
Z
ΔY
C
943
d
4.5
e
60R
FORCE
b
18.56"
a
(a)
DATA
10" PIPE
O.D.=10.75 I.D.=10.02
W.T.=.365 I=160.7
E=30x10⁶ #/□"
BENDING-MOMENT DIAGRAM-FORCE IN X-Y PLANE
CLOCKWISE
COUNTER-CLOCKWISE
1790F
1510F
40F
4140F
73.93F
(c)
(b)
ψb=-1790F
-1510F
-4140F
ψc=-7440F
DETERMINATION OF DEFLECTIONS
EIΔy-c CASE I ψbR =-1790F×60 =-107,400F RIGHT
CASE VIII b Σ ψF.754R =-4140F×.754×60 =-187,290F RIGHT
CASE IX b ψM.695R =-1510F×.695×60 =-62,970F RIGHT
CASE IX a ψMy₂.695R =-(.5×13.93F×60)×.695×60 =-5230F LEFT
EIΔy-d CASE I ψcl =-7440F×9.43 =-70,160F RIGHT
CASE II Σ ψF2/3l =-40F×2/3×9.43 =-250F RIGHT
CASE III ψM½l =-40F×½×9.43 =-190F RIGHT

FIG. 22

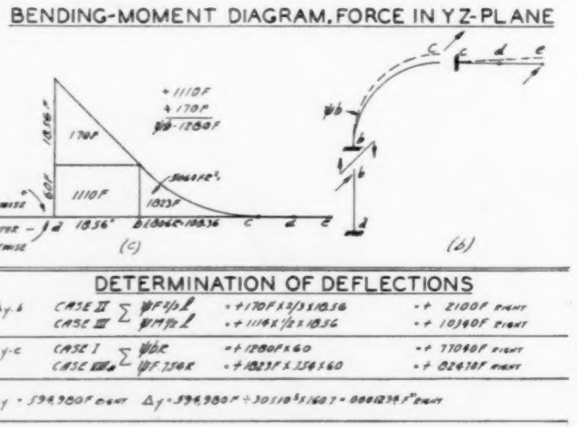

BENDING-MOMENT DIAGRAM, FORCE IN YZ-PLANE
+1110F
+170F
ψb=1280F
170F
1110F
1823F
60F
18.56"
CLOCKWISE
COUNTER-CLOCKWISE
a
b
c
d
e
(c)
(b)
DETERMINATION OF DEFLECTIONS
EIΔy-b CASE II Σ ψF2/3l =+170F×2/3×18.56 =+2100F RIGHT
CASE III ψM½l =+1110F×½×18.56 =+10,300F RIGHT
EIΔy-c CASE I Σ ψbR =+1280F×60 =+77,040F RIGHT
CASE VIII a ψF.754R =+1823F×.754×60 =+82,470F RIGHT
EIΔy =594,980F RIGHT Δy=594,980F÷30×10⁶×160.7=.000123F" RIGHT

FIG. 23

sional moment is equal to $F \times ce + FR$ = ($13.93F + 60F$ or $73.93F$). In addition to the deflections at the free ends of these component parts due to bending within themselves, rotations of the elements between the hypothetical fixed end and free end of each component part are represented by ψ_b and ψ_c which produce deflections at the free ends of the component parts. These deflections are added algebraically to the individual deflections of the component parts (Case I). To determine the sign of ψ_b and ψ_c, see Appendix II. The bending-moment diagram for the force in the XY-plane is constructed as follows [Fig. 22(*c*)]: The lengths, either actual or virtual, of the component parts of the pipe are laid off consecutively as abscissas on a horizontal base line, and the moment at any point is represented by the ordinate at that point. Part *ce* is in bending and is represented by its actual length, 13.93 in. Quarter-bend *bc* is both in bending and torsion. Its abscissa is represented by a virtual length of $1.806R$, which equals 108.36 in. (Cases VIII, IX, and X). *ab* is in torsion and the abscissa is represented by[7] $1.3 \times l$ which equals 24.13 in. The moment diagram for part *ce* is a triangle, the ordinate of which is zero at point *e* and Fl (represented by $13.93F$) at point *c*. This moment Fl carries through the section of the piping to point *b*, hence a horizontal line is drawn to end at *b*. The quarter-bend is subjected to an additional bending moment due to the transposed force. This diagram is represented by the half-segment of a parabola, the value of the ordinate of which is zero at point *c*, and $60F$ at point *b*. The total moment at *b* equals $13.93F + 60F$ or $73.93F$. The moment at point *b* is carried through as a torque moment to point *a* and hence a horizontal line is drawn from *b* to *a*. The force as applied causes all the parts to be rotated in a counterclockwise direction, with respect to the XY-plane. Thus all of the bending-moment diagrams shown in Fig. 22 (*c*) are drawn below the horizontal base line.

The deflection at the free end of each component part is obtained by the same method as previously explained. In addition to these deflections, the moment acting at the free end of quarter-bend *bc* also produces a rotation of the part *bc* in the YZ-plane (Case IX*a*). The total rotation ψM_{ys} of this part multiplied by $0.695R$ gives the value of the additional deflection caused by the moment acting in the XY-plane. This deflection in the YZ-plane is at all times of opposite sign to the deflection in the XY-plane (this refers only to the deflections on part *bc* caused by the moment acting at the free end of this part). For the determination of the direction of the rotation in the YZ-plane, see Appendix No. 2. Values of the deflections of each component part along the Y-axis, due to the actions of the force F in the XY-plane, are now determined.

As previously stated this force F also causes a bending in the YZ-plane. The manner in which forces and moments are transposed in the YZ-plane and the deflection of each component part obtained, is explained as follows: The component parts of the pipe bend are shown in Fig. 23 (*b*), with the transposed forces and moments acting upon them. The component part *bc* is a straight cantilever with a force F acting at its free end. *bc* is a quarter-bend acted on by a force F in a plane perpendicular to plane of the bend and perpendicular to the free end of the pipe. *ab* is a straight cantilever acted upon by a moment which equals $60F$ and a transverse force F. Deflections due to rotations of the elements are added algebraically to the individual deflections of the component parts. The only rotation to cause a lateral displacement is represented by ψ_b.

The same method as previously stated is used to construct the bending-moment diagram. Part *ce* is not laterally displaced by the action of the force in the YZ-plane, and thus its length may be neglected. Quarter-bend *bc* is both in bending and in torsion and its abscissa is represented by $1.806R$ which equals 108.36 in., a virtual length (Case VIII). *ab* is a straight cantilever acted on by a moment and a force at its free end, hence its abscissa is represented by its true length, 18.56 in.

The bending-moment diagram for the entire pipe bend, due to the action of the force in the YZ-plane, is drawn as follows [Fig. 23(*c*)]: The ordinate of the moment diagram for part *ce* is zero throughout. The moment diagram for quarter-bend *bc* is in the form of the half-segment of a parabola, the value of the ordinate is zero at point *c* and $60F$ at point *b*. This moment, $60F$, carries through the section of the piping to the point *a*, hence a horizontal line is drawn to end at *a*. The straight cantilever *ab* is subjected to an additional bending moment due to the transposed force *F*. This diagram is represented by a triangle, the ordinate of which is zero at point *b*, and *Fl*, represented by 18.56*F*, at point *a*. The force as applied causes all the parts to be rotated in a clockwise direction with respect to the *YZ*-plane. Thus all of the bending-moment diagrams are drawn above the horizontal base line. By applying these areas to the elementary cases to which they correspond, the deflection at the free end of each component part is calculated.

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[7] Reference 1, p. 529.

The total deflection along the *Y*-axis is equal to the summation of the deflections of the component parts due to the actions of the force *F* in both the *XY*- and *YZ*-planes. This total summation for the point *d* is shown in Fig. 23 and is represented by $EI\,\Delta y = 594.980F$ *right;* $\Delta y = 0.0001234\ F$ in., *right.*

The tables and curves shown in Fig. 24 show the deflections determined by theory and by tests.

PIPE BEND LOADED WITH A BENDING MOMENT ACTING AT ITS FREE END IN A PLANE PERPENDICULAR TO THE PLANE OF THE BEND AND PERPENDICULAR TO ITS FREE END

Fig. 25 (*a*) shows this pipe bend with the load applied. The component parts of the pipe bend with the transposed moments acting on them are shown in Fig. 25 (*b*). *ce* is a cantilever acted on by the bending moment, which equals 72*F*, at its free end (Case III). *bc* is a quarter-bend acted on by same moment in a plane perpendicular to the plane of the bend and tangent to its free end (Case IX). This moment is transposed to the cantilever *ab* as a torque moment and carries through to the anchor point *a*. Rotations of the component parts are represented by ψ_b and ψ_c.

The same procedure as in the other loadings is used in the construction of the bending-moment diagram which is shown in Fig. 25(*c*). The abscissa of the bending moment diagram *ce* is its actual length 13.93 in. The abscissa of the bending-moment diagram of the quarter-bend is a virtual length which equals 1.806 × *R* or 108.36 in. (Case IX). *ab* is a straight cantilever in torsion. The abscissa of its bending-moment diagram is represented by a virtual length equal to 1.3 times its actual length. This is 24.13 in. The bending moment which is 72*F* at *e* carries through from point *e* to point *a*, hence a horizontal line is drawn from *e* to end at *a*. The couple as applied causes all the parts to be rotated in a counterclockwise direction with respect to the *XY*-plane, hence all of the bending-moment diagrams are drawn below the horizontal base line.

The computation of deflections at the free end of each component part is carried out in the same manner as previously described. The additional deflections of the quarter-bend *bc* caused by the moment in the *XY*-plane producing a rotation in the *YZ*-plane are determined in the same way as discussed in the eighth preceding paragraph. The summation of all the increment deflections preceding any point determines the total deflection at that point. Fig. 25 shows the calculated deflections of the component parts and the total deflection of pipe bend along the *Y*-axis at point *d*, which is the point at which deflections were measured in the test. The table and curves of Fig. 26 show the calculated deflection for different loads as compared with the deflections obtained by test at the same loads.

PIPE BEND LOADED WITH A TORQUE MOMENT ACTING AT ITS FREE END IN A PLANE PERPENDICULAR TO ITS FREE END

The pipe bend with load applied is shown as a line drawing in Fig. 27 (*a*). The component parts of the bend with transposed moments are shown in Fig. 27 (*b*). Part *ce* is a straight cantilever having a torque moment 72*F* at its free end. *bc* is a quarter-bend acted on by the same torque moment at its free end. Part *ab* is a straight cantilever having a bending moment acting at its

TABULATED RESULTS-10" PLAIN BEND

FORCE	ΔY THEORY	ΔY TEST	% DIFF.			
1000#	0.123"	0.193"	36.2%			
2000#	0.247"	0.346"	28.5%			
3000#	0.370"	0.520"	29.0%			
4000#	0.494"	0.695"	28.5%			
5000#	0.617"	0.874"	28.5%			

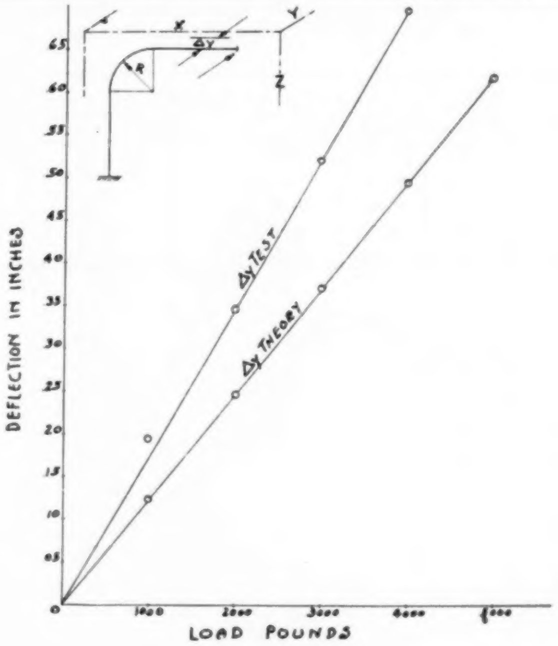
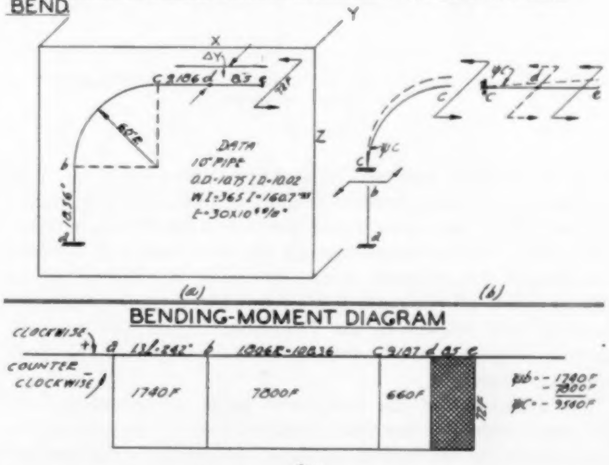


FIG. 24

GRAPHO-ANALYTIC SOLUTIONS FOR DEFLECTIONS OF A 10" PLAIN BEND, MOMENT IN PLANE PERPENDICULAR TO PLANE OF BEND AND TANGENT TO FREE END OF BEND



DETERMINATION OF DEFLECTIONS

EIΔy-c CASE I ψbR = −1740F×60 = −104400F RIGHT
CASE IX b ∑ψM.695R = −7800F×.695×60 = −325260F RIGHT
CASE IX a ψMy = .695R = −.15×72F×60×.695×60 = −27020F LEFT

EIΔy-d CASE I ∑ψcl = −9540F×9.107 = −87.690F RIGHT
CASE III ∑ψM½l = −660F×½×9.107 = −3030F RIGHT

EIΔy = 493,310F RIGHT Δy = 493,310F ÷ 30×10⁶×160.7 = .0001023F" RIGHT

FIG. 25

free end. Rotations of the component parts are represented by ψ_b and ψ_c.

The bending-moment diagram, Fig. 27 (*c*), is constructed as follows: The moment acting on part *ce* produces no lateral displacement. This is Case IV. The abscissa of the bending-moment diagram is a virtual length of the part *ce* equal to 1.3 times its length. This is represented by 16.55 in. The quarter-bend *bc* is subjected to both bending and torsion, consequently the abscissa of its bending-moment diagram is a virtual length equal to 1.806R, which equals 108.36 in. (Case X). *ab* is acted on by the bending moment 72F at its free end (Case III). The abscissa of its bending-moment diagram is equal to its true length. The ordinate of the moment diagram of *ce* is the torsion moment 72F. This ordinate carries through the bend from *e* to *a*, hence a horizontal line is drawn from *e* to end at *a*. The couple as applied causes all the parts to be rotated in a counterclockwise direction with respect to the YZ-plane. All of the bending-moment diagrams are, in consequence, drawn below the horizontal base line.

The deflections of the component parts are determined by the same method as has been described before. In addition to these deflections, the moment acting at the free end of the quarter-bend *bc* also produces a rotation of *bc* in the XY-plane (Case X*b*). The total rotation, ψM_{xy} of the part *bc* multiplied by 0.306R gives an additional deflection along the Y-axis caused by the moment acting as shown in Fig. 27 (*b*). Due to the bending of the quarter-bend *bc* in the XY-plane, there is a deflection produced on the component part *ce* at *d*. This deflection at *d* is equal to the rotation of the bend *bc* in the XY-plane multiplied by the distance *cd*. This is called $\psi c_{xy}l$. For the value of this

TABULATED RESULTS 10" PLAIN BEND

FORCE	ΔY THEORY	ΔY TEST	% DIFF			
1000#	0.102"	0.125"	18.4%			
2000#	0.204"	0.246"	17 %			
3000#	0.306"	0.383"	20 %			
4000#	0.408"	0.512"	20.3%			
5000#	0.510"	0.644"	21%			

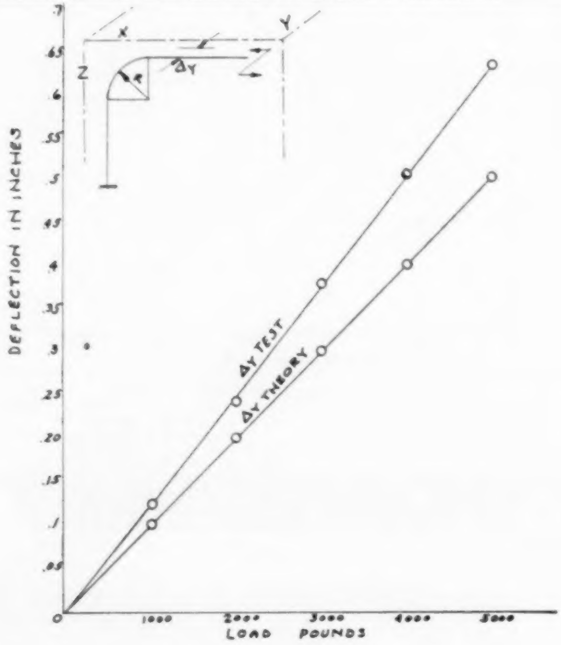
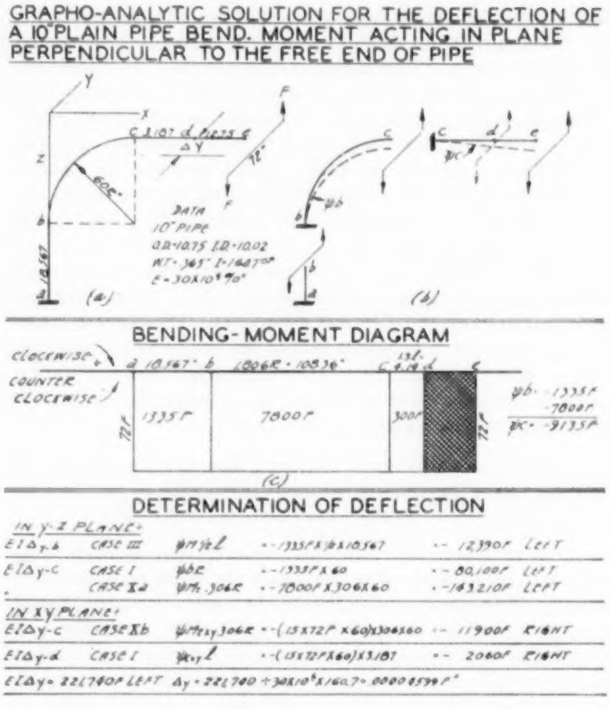


FIG. 26



GRAPHO-ANALYTIC SOLUTION FOR THE DEFLECTION OF A 10" PLAIN PIPE BEND. MOMENT ACTING IN PLANE PERPENDICULAR TO THE FREE END OF PIPE
Y
X
Z
c
d
e
ΔY
F
72"
60R
a
b
18.567
DATA
10" PIPE
O.D.=10.75 I.D.=10.02
W.T.=.365" I=160.7
E=30X10⁶
(a)
ψb
ψc
(b)
BENDING-MOMENT DIAGRAM
CLOCKWISE
COUNTER CLOCKWISE
a 18.567" b 1.806R=108.36" c d e
72F
1335F
7800F
300F
ψb=-1335F
-7800F
ψc=-9135F
(c)
DETERMINATION OF DEFLECTION
IN Y-Z PLANE:
EIΔy-b CASE III ψM½l =-1335F×½×18.567 =-12390F LEFT
EIΔy-c CASE I ψbR =-1335F×60 =-80,100F LEFT
CASE Xa ψMx.306R =-7800F×.306×60 =-143210F LEFT
IN XY PLANE:
EIΔy-c CASE Xb ψMxy.306R =-(1.5×72F×60)×.306×60 =-11900F RIGHT
EIΔy-d CASE I ψcxyl =-(1.5×72F×60)×3.187 =-2060F RIGHT
EIΔy=221740F LEFT Δy=221740÷30×10⁶×160.7=.0000459F"

FIG. 27

TABULATED RESULTS 10" PLAIN BEND

FORCE	ΔY THEORY	ΔY TEST	% DIFF			
1000#	0.046"	0.098"	53%			
2000#	0.092"	0.196"	53%			
3000#	0.138"	0.311"	56%			
4000#	0.184"	0.419"	56%			
5000#	0.230"	0.520"	56%			

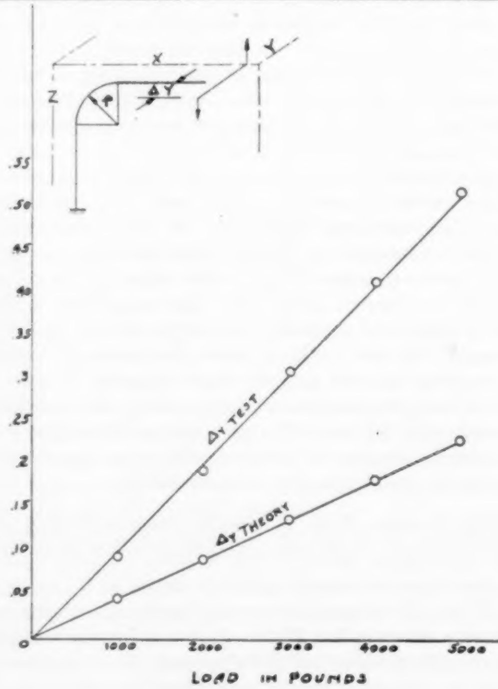


FIG. 28

TABULATED RESULTS 10" TUBE TURN

FORCE	Δ_x THEORY	Δ_x TEST	% DIFF.	Δ_z THEORY	Δ_z TEST	% DIFF.
1000#	0.067"	0.075"	11%	0.198"	0.254"	22%
1500#	0.100"	0.099"	1%	0.297"	0.357"	17%
2000#	0.133"	0.129"	3%	0.396"	0.465"	12.7%
2500#	0.167"	0.161"	3.6%	0.495"	0.573"	12.7%
3000#	0.200"	0.194"	3%	0.595"	0.687"	13.4%

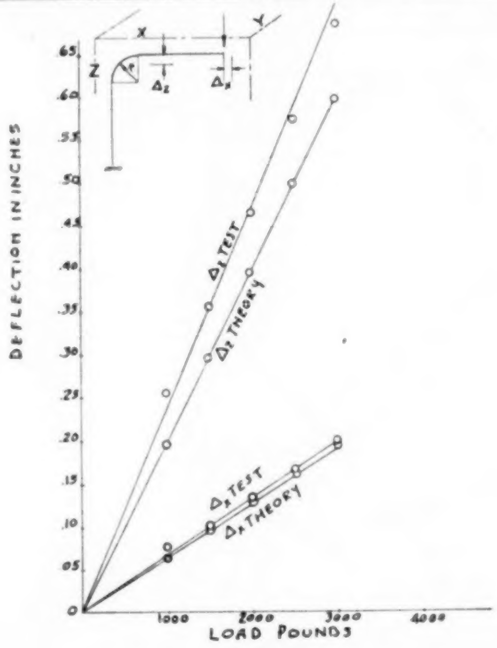


FIG. 29

TABULATED RESULTS 10" TUBE TURN

FORCE	Δ_x THEORY	Δ_x TEST	% DIFF.	Δ_z THEORY	Δ_z TEST	% DIFF.
1000#	0.066"	0.102"	35%	0.212"	0.237"	10.5 %
1400#	0.093"	0.132"	29.5%	0.296"	0.310"	4.5 %
1800#	0.119"	0.165"	27.9%	0.361"	0.394"	3.3 %
2200#	0.145"	0.194"	25 %	0.466"	0.469"	.6 %

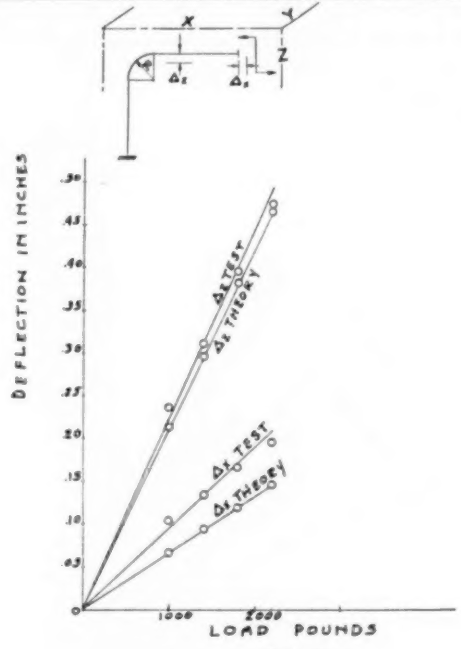


FIG. 31

TABULATED RESULTS 10" TUBE TURN

FORCE	Δ_x THEORY	Δ_x TEST	% DIFF.	Δ_z THEORY	Δ_z TEST	% DIFF.
1000#	0.041"	0.050"	18%	0.052"	0.060"	13.3%
1500#	0.062"	0.073"	15%	0.078"	0.089"	12.3%
2000#	0.082"	0.091"	9.8%	0.104"	0.118"	11.8%
2500#	0.103"	0.116"	11.2%	0.130"	0.146"	10.9%
3000#	0.124"	0.137"	9.4%	0.157"	0.182"	13.7%

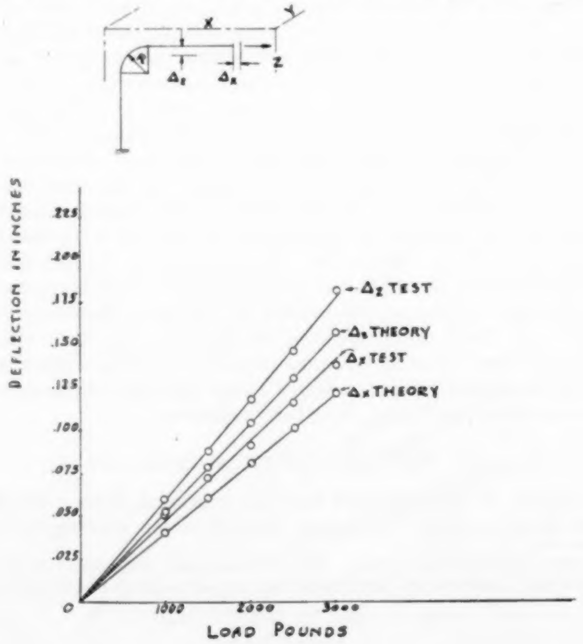


FIG. 30

TABULATED RESULTS 10" TUBE TURN

FORCE	Δ_Y THEORY	Δ_Y TEST	% DIFF.			
1000#	0.185"	0.227"	18.5%			
2000#	0.370"	0.453"	18.5%			
3000#	0.555"	0.686"	19.%			

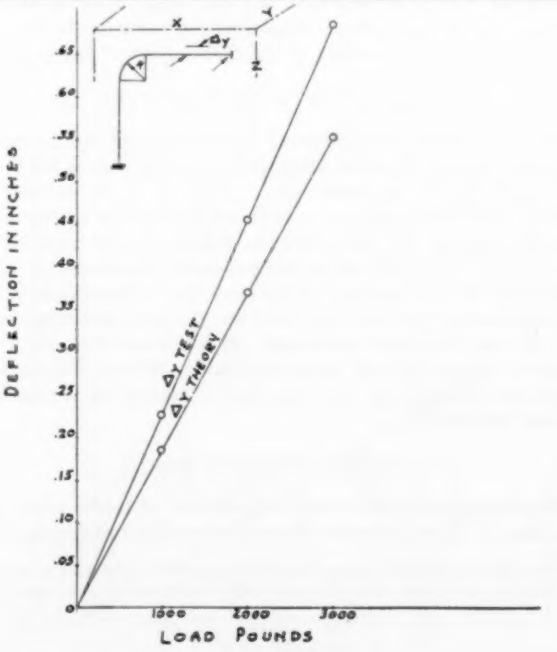


FIG. 32

TABULATED RESULTS 10" TUBE TURN						
FORCE	ΔY THEORY	ΔY TEST	% DIFF.			
1000#	0.136"	0.218"	37.5%			
1400#	0.190"	0.285"	33.3%			
1800#	0.245"	0.356"	31.2%			
2200#	0.299"	0.437"	31.5%			
2600#	0.354"	0.508"	30.4%			

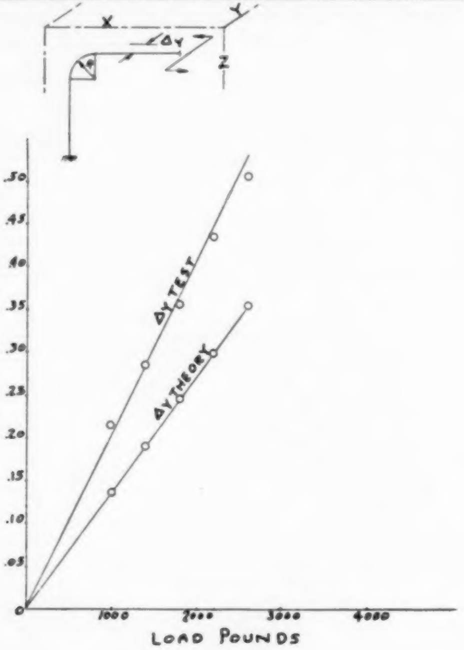


FIG. 33

TABULATED RESULTS 10" TUBE TURN						
FORCE	ΔY THEORY	ΔY TEST	% DIFF			
1000#	0.052"	0.110"	52.6%			
2000#	0.104"	0.219"	52.7%			
3000#	0.156"	0.326"	52.0%			

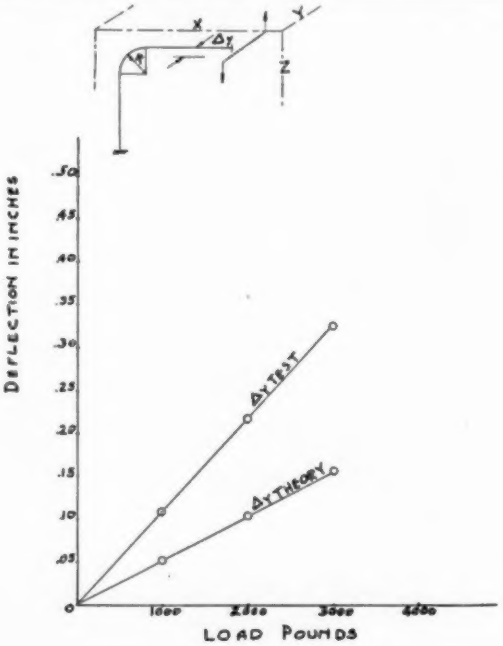


FIG. 34

rotation see Case X. To determine its direction in the XY-plane see Appendix No. 2. All increment deflections preceding point d are combined to determine the total deflection along the Y-axis. The determination of the calculated deflection of each component part and the summation of the deflections are shown in Fig. 27. The tables and curves in Fig. 28 show the relation between the calculated deflection and the deflection obtained by the tests. In the calculations, no allowance is made for the reduction in I because of flattening of the cross-section of the pipe.

III—TUBE TURN WITH TANGENTS

The "tube turn" with tangents was tested and the grapho-analytical solution of the six loadings was carried out, as has just been explained for the plain bend. Figs. 29 to 34, inclusive, show the load-deflection curves both calculated and by test for these six loadings. In calculating the deflections, the value for I was determined by the use of the measured thicknesses of the walls of the quarter-bend and of the tangents. These thicknesses were obtainable when this pipe bend was cut open and measured after the test had been completed. It was found that the use of correct values for wall thickness made a difference of about 10 per cent as compared with the results obtained by the use of nominal thicknesses.

IV—CORRUGATED PIPE BENDS

The grapho-analytical method of solution of bends made of plain pipe of the same major dimensions[8] as the two corrugated bends tested have been carried out for all the six cases of loading. Figs. 35 to 46, inclusive, show the load-deflection relations obtained by tests of the corrugated bends and by calculation for plain pipe bends having the same major dimensions.

V—CREASED BENDS

Figs. 47 to 58, inclusive, show the load-deflection curves obtained by tests of the two creased bends and the calculated load-deflection curves for plain bends of the same major dimensions.[8]

In the study of corrugated and creased pipe bends, no effort was made to arrive at a factor by which I and S of a plain pipe bend might be multiplied to satisfy the load-deflection relations of a corrugated or creased pipe bend of the same dimensions. The possibility of arriving at such a factor was discussed with Prof. S. Timoshenko, but he advised that at present such a factor could not be obtained by calculations as there were too many variables involved. He further called attention to the fact that such a factor obtained for one corrugated bend would not necessarily apply to another bend unless the physical proportions of pitch and depth of corrugations, thickness of pipe wall, etc., were identical. In this study the relations of load to deflection of the pipe bends in question and of a plain pipe bend of the same major dimensions,[8] alone, have been considered.

VI—14-IN. CORRUGATED EXPANSION U-BEND

Because of its shape and size this expansion U-bend could not be conveniently erected on the turbine-top bending table

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[8] By "plain pipe of the same major dimensions" is meant a smooth pipe having the same diameter and wall thickness as that used in making the bend under test. The center-to-face dimensions of the plain pipe bend and the bend under test are assumed to be the same.

TABULATED RESULTS 43"R CORRUGATED BEND

FORCE	Δ_x PLAIN THEORY	Δ_x TEST	$R=\frac{\Delta_x \text{TEST}}{\Delta_x \text{THEORY}}$	Δ_z PLAIN THEORY	Δ_z TEST	$R=\frac{\Delta_z \text{TEST}}{\Delta_z \text{THEORY}}$
600+	0.031″	0.079″	2.5	0.081″	0.209″	2.6
1200+	0.061″	0.139″	2.3	0.162″	0.382″	2.4
1800+	0.092″	0.207″	2.3	0.243″	0.563″	2.3
2400+	0.122″	0.269″	2.2	0.324″	0.754″	2.3
3000+	0.153″	0.334″	2.2	0.405″	0.936″	2.3

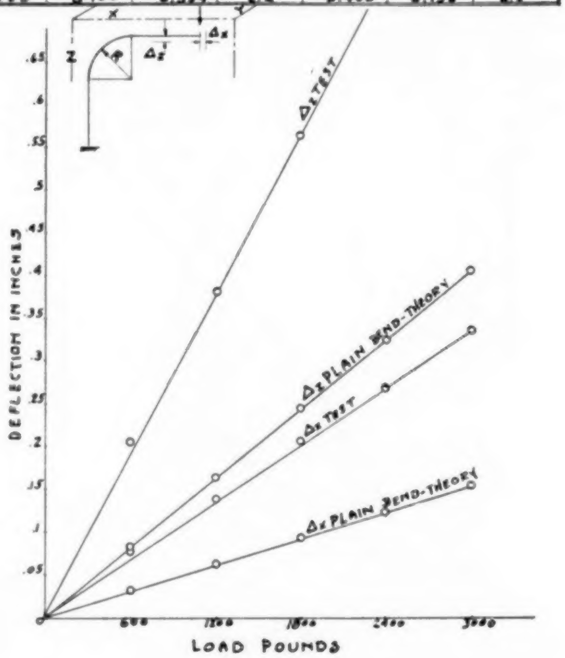


Fig. 35

TABULATED RESULTS 43"R CORRUGATED BEND

FORCE	Δ_x PLAIN THEORY	Δ_x TEST	$R=\frac{\Delta_x \text{TEST}}{\Delta_x \text{THEORY}}$	Δ_z PLAIN THEORY	Δ_z TEST	$R=\frac{\Delta_z \text{TEST}}{\Delta_z \text{THEORY}}$
600+	0.016″	0.032″	2	0.026″	0.064″	2.5
1200+	0.032″	0.062″	2	0.052″	0.117″	2.3
1800+	0.049″	0.090″	1.8	0.077″	0.168″	2.2
2400+	0.065″	0.124″	1.9	0.103″	0.229″	2.2
3000+	0.081″	0.152″	1.9	0.129″	0.277″	2.1
3600+	0.097″	0.180″	1.9	0.155″	0.329″	2.1

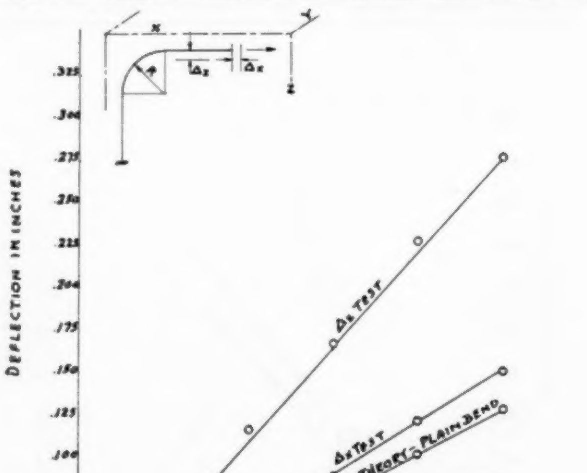


Fig. 36

TABULATED RESULTS 43"R CORRUGATED BEND

FORCE	Δ_x PLAIN THEORY	Δ_x TEST	$R=\frac{\Delta_x \text{TEST}}{\Delta_x \text{THEORY}}$	Δ_z PLAIN THEORY	Δ_z TEST	$R=\frac{\Delta_z \text{TEST}}{\Delta_z \text{THEORY}}$
1000#	0.037″	0.094″	2.5	0.110″	0.259″	2.4
2000#	0.074″	0.180″	2.5	0.220″	0.484″	2.2
3000#	0.111″	0.273″	2.5	0.330″	0.715″	2.2
4000#	0.148″	0.364″	2.5	0.440″	0.937″	2.1

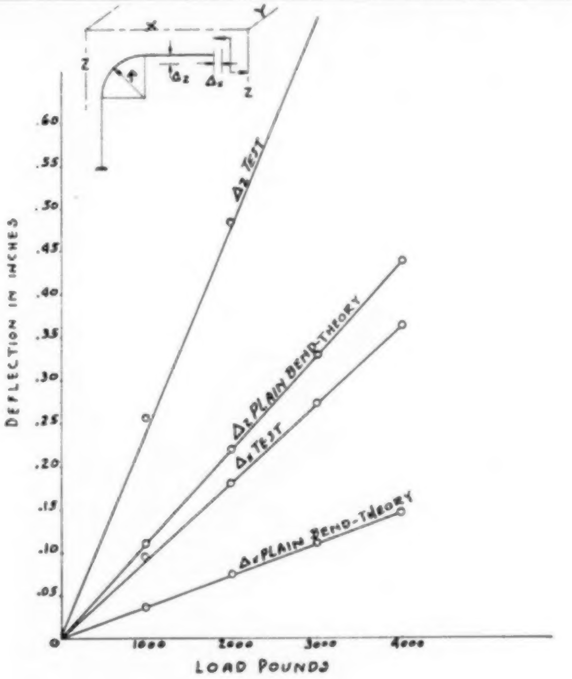


Fig. 37

TABULATED RESULTS 43"R CORRUGATED BEND

FORCE	Δ_y PLAIN THEORY	Δ_y TEST	$R=\frac{\Delta_y \text{TEST}}{\Delta_y \text{THEORY}}$			
600 *	0.059″	0.163″	2.8			
1200 *	0.120″	0.280″	2.3			
1800 *	0.180″	0.421″	2.3			
2400 *	0.240″	0.566″	2.3			
3000 *	0.300″	0.715″	2.4			

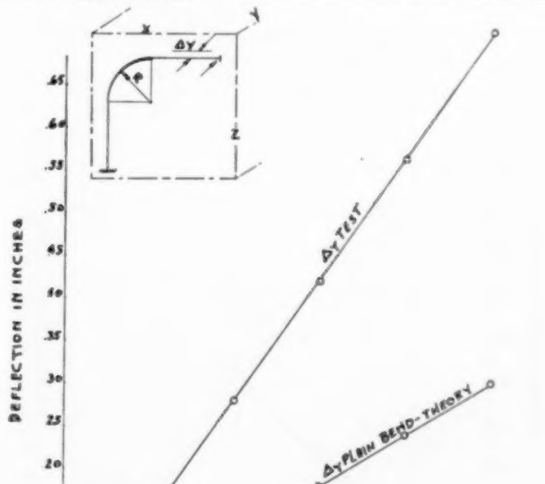


Fig. 38

TABULATED RESULTS 43°R CORRUGATED BEND

FORCE	Δ_Y PLAIN THEORY	Δ_Y TEST	$R = \frac{\Delta_Y \text{ TEST}}{\Delta_Y \text{ THEORY}}$			
1000^{+}	0.056"	0.130"	2.3			
2000^{+}	0.113"	0.242"	2.2			
3000^{+}	0.169"	0.383"	2.3			
4000^{+}	0.226"	0.518"	2.3			

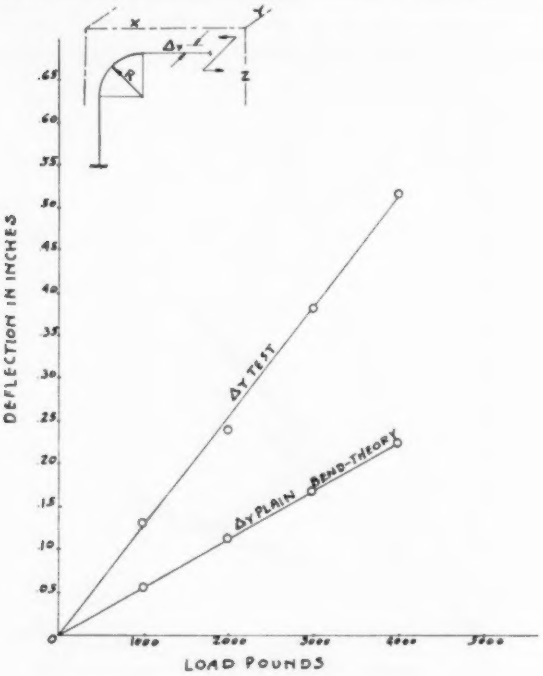


FIG. 39

TABULATED RESULTS 43°R CORRUGATED BEND

FORCE	Δ_Y PLAIN THEORY	Δ_Y TEST	$R = \frac{\Delta_Y \text{ TEST}}{\Delta_Y \text{ THEORY}}$			
1000^{+}	0.023"	0.106"	4.6			
2000^{+}	0.046"	0.211"	4.6			
3000^{+}	0.069"	0.328"	4.7			

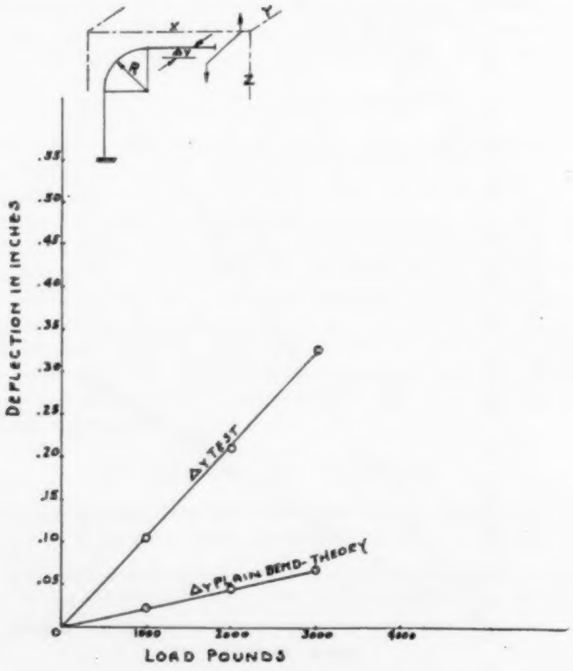


FIG. 40

TABULATED RESULTS 45°R CORRUGATED BEND

FORCE	Δ_X PLAIN THEORY	Δ_X TEST	$R = \frac{\Delta_X \text{ TEST}}{\Delta_X \text{ THEORY}}$	Δ_Z PLAIN THEORY	Δ_Z TEST	$R = \frac{\Delta_Z \text{ TEST}}{\Delta_Z \text{ THEORY}}$
1000^{+}	0.075"	0.135"	1.8	0.145"	0.348"	2.4
2000^{+}	0.150"	0.258"	1.7	0.290"	0.658"	2.3
3000^{+}	0.225"	0.380"	1.7	0.435"	0.982"	2.3
4000^{+}	0.300"	0.502"	1.7	0.580"	1.311"	2.2
5000^{+}	0.375"	0.618"	1.6	0.725"	1.648"	2.2

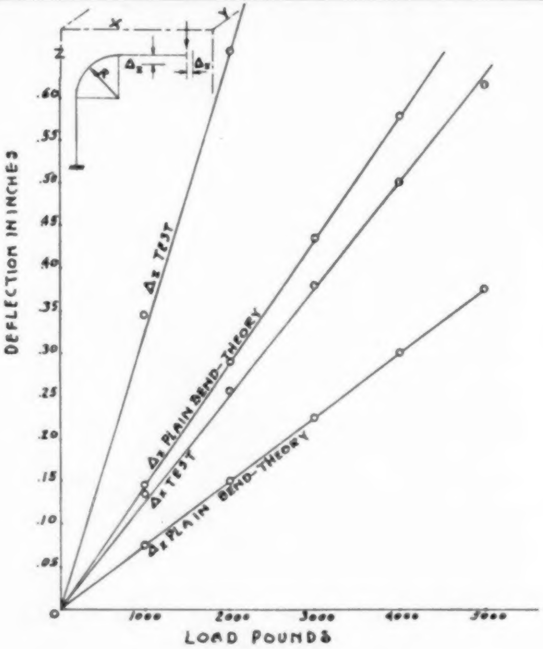


FIG. 41

TABULATED RESULTS 45°R CORRUGATED BEND

FORCE	Δ_X PLAIN THEORY	Δ_X TEST	$R = \frac{\Delta_X \text{ TEST}}{\Delta_X \text{ THEORY}}$	Δ_Z PLAIN THEORY	Δ_Z TEST	$R = \frac{\Delta_Z \text{ TEST}}{\Delta_Z \text{ THEORY}}$
1000^{+}	0.055"	0.102"	1.9	0.063"	0.113"	1.8
2000^{+}	0.110"	0.202"	1.9	0.126"	0.221"	1.8
3000^{+}	0.165"	0.298"	1.8	0.189"	0.326"	1.7
4000^{+}	0.220"	0.398"	1.8	0.252"	0.429"	1.7
5000^{+}	0.275"	0.489"	1.8	0.315"	0.546"	1.7

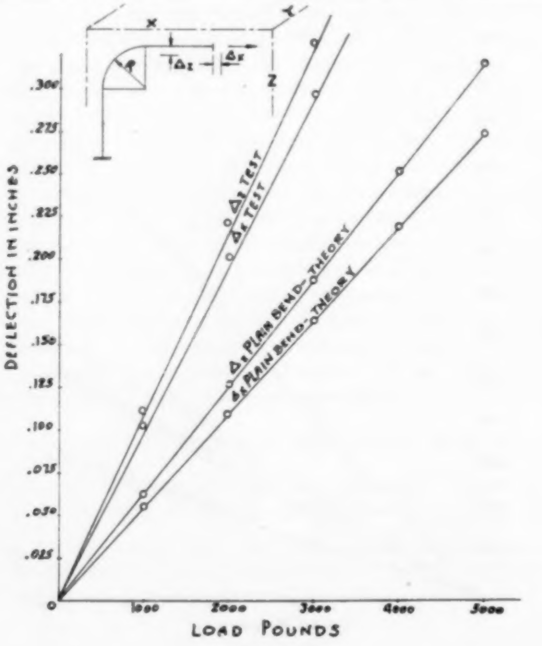


FIG. 42

TABULATED RESULTS 45"R CORRUGATED BEND

FORCE	Δ_X PLAIN THEORY	Δ_X TEST	$R = \frac{\Delta_X TEST}{\Delta_X THEORY}$	Δ_Z PLAIN THEORY	Δ_Z TEST	$R = \frac{\Delta_Z TEST}{\Delta_Z THEORY}$
1000#	0.055"	0.070"	1.3	0.118"	0.219"	1.9
2000#	0.110"	0.132"	1.2	0.236"	0.430"	1.8
3000#	0.165"	0.212"	1.3	0.356"	0.681"	1.9
4000#	0.220"	0.303"	1.4	0.472"	0.899"	1.9
5000#	0.275"	0.384"	1.4	0.590"	1.129"	1.9

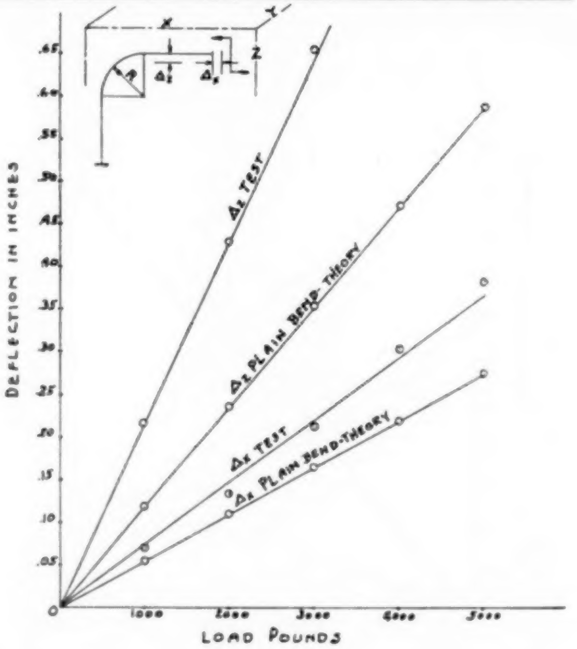


FIG. 43

TABULATED RESULTS 45"R CORRUGATED BEND

FORCE	Δ_Y PLAIN THEORY	Δ_Y TEST	$R = \frac{\Delta_Y TEST}{\Delta_Y THEORY}$			
1000#	0.138"	0.263"	1.9			
2000#	0.276"	0.528"	1.9			
3000#	0.414"	0.815"	1.9			
4000#	0.552"	1.083"	1.9			

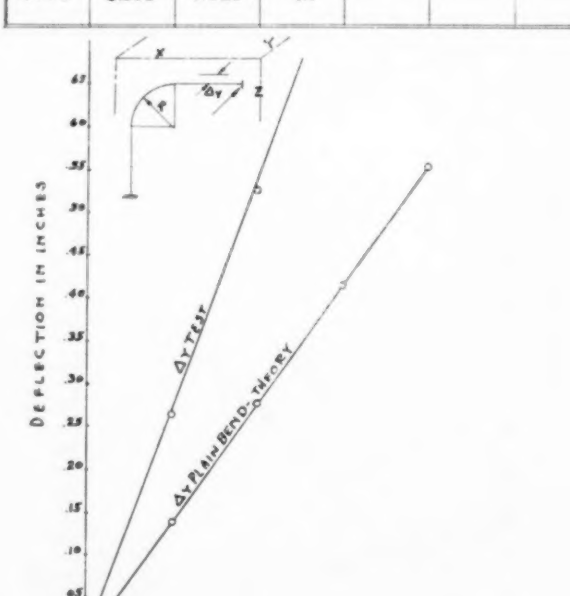


FIG. 44

TABULATED RESULTS 45"R CORRUGATED BEND

FORCE	Δ_Y PLAIN THEORY	Δ_Y TEST	$R = \frac{\Delta_Y TEST}{\Delta_Y THEORY}$			
1000#	0.068"	0.141"	2.1			
2000#	0.136"	0.276"	2.0			
3000#	0.204"	0.414"	2.0			
4000#	0.272"	0.556"	2.0			

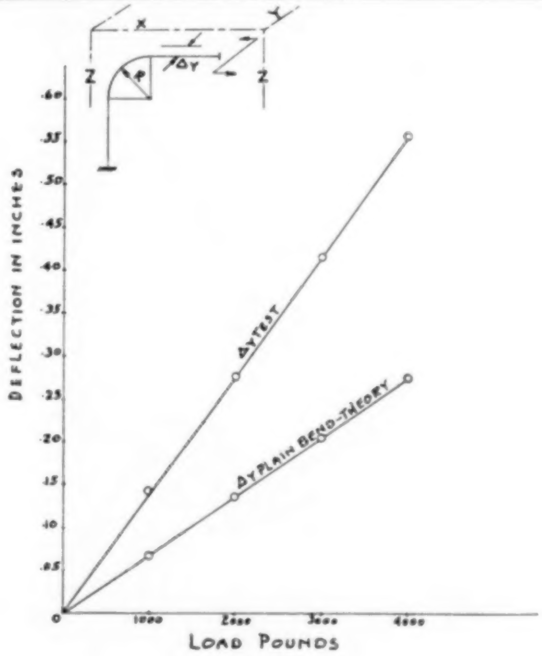


FIG. 45

TABULATED RESULTS 45"R CORRUGATED BEND

FORCE	Δ_Y PLAIN THEORY	Δ_Y TEST	$R = \frac{\Delta_Y TEST}{\Delta_Y THEORY}$			
1000#	0.040"	0.148"	3.7			
2000#	0.080"	0.275"	3.5			
3000#	0.120"	0.395"	3.3			
4000#	0.160"	0.520"	3.2			

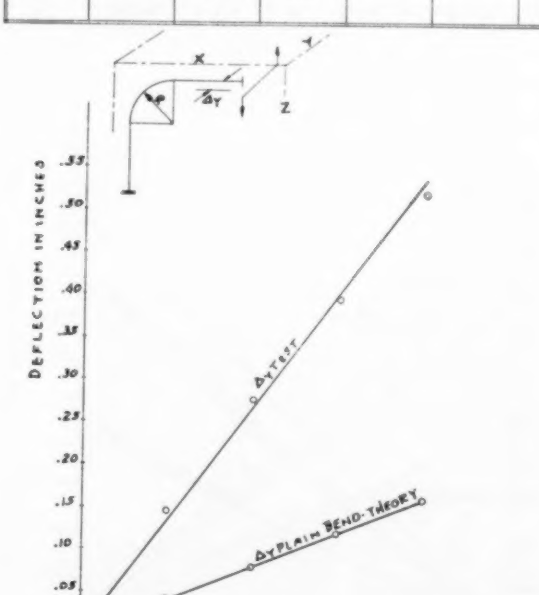


FIG. 46

TABULATED RESULTS 39"R CREASED BEND

FORCE	Δ_x PLAIN THEORY	Δ_x TEST	$R = \frac{\Delta_x \text{TEST}}{\Delta_x \text{THEORY}}$	Δ_z PLAIN THEORY	Δ_z TEST	$R = \frac{\Delta_z \text{TEST}}{\Delta_z \text{THEORY}}$
1000+	0.047"	0.068"	1.4	0.139"	0.172"	1.2
2000+	0.094"	0.123"	1.3	0.278"	0.321"	1.2
3000+	0.141"	0.173"	1.2	0.417"	0.477"	1.1
4000+	0.188"	0.227"	1.2	0.556"	0.634"	1.1
5000+	0.235"	0.279"	1.2	0.695"	0.789"	1.1

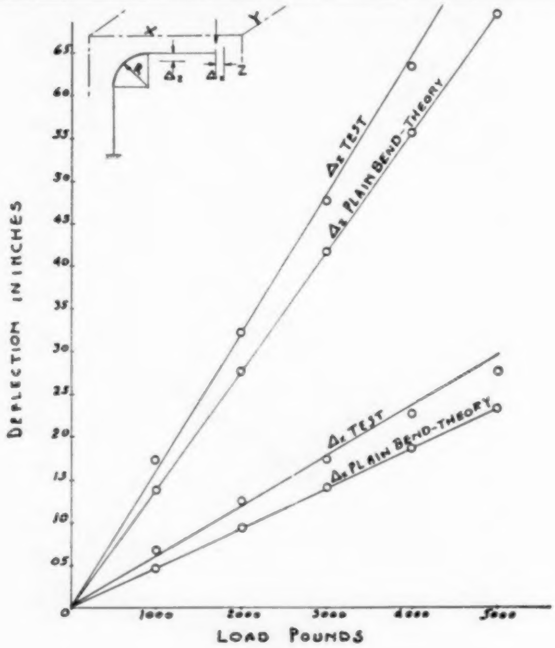


FIG. 47

TABULATED RESULTS 39"R CREASED BEND

FORCE	Δ_x PLAIN THEORY	Δ_x TEST	$R = \frac{\Delta_x \text{TEST}}{\Delta_x \text{THEORY}}$	Δ_z PLAIN THEORY	Δ_z TEST	$R = \frac{\Delta_z \text{TEST}}{\Delta_z \text{THEORY}}$
1000+	0.024"	0.041"	1.7	0.042"	0.055"	1.3
2000+	0.048"	0.077"	1.6	0.084"	0.100"	1.2
3000+	0.072"	0.120"	1.7	0.126"	0.150"	1.2
4000+	0.096"	0.160"	1.7	0.168"	0.198"	1.2
5000+	0.120"	0.197"	1.6	0.210"	0.244"	1.2

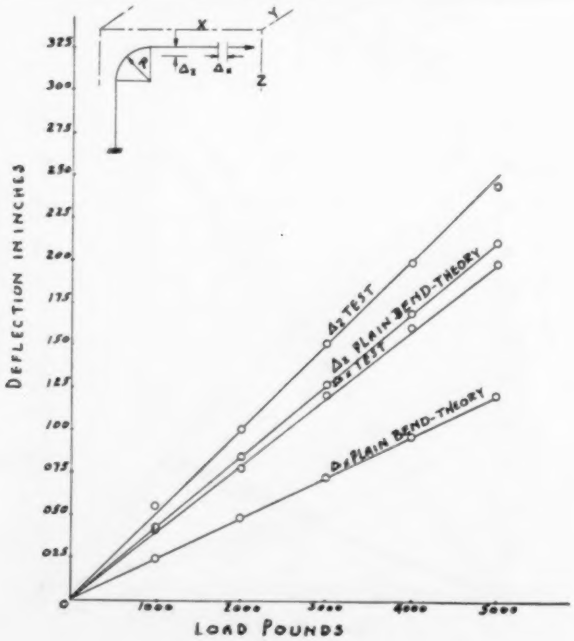


FIG. 48

TABULATED RESULTS 39"R CREASED BEND

FORCE	Δ_x PLAIN THEORY	Δ_x TEST	$R = \frac{\Delta_x \text{TEST}}{\Delta_x \text{THEORY}}$	Δ_z PLAIN THEORY	Δ_z TEST	$R = \frac{\Delta_z \text{TEST}}{\Delta_z \text{THEORY}}$
1000+	0.035"	0.038"	1.1	0.109"	0.121"	1.1
2000+	0.070"	0.080"	1.1	0.218"	0.231"	1.1
3000+	0.105"	0.125"	1.2	0.327"	0.353"	1.1
4000+	0.140"	0.168"	1.2	0.436"	0.466"	1.1
5000+	0.175"	0.215"	1.2	0.545"	0.575"	1.1

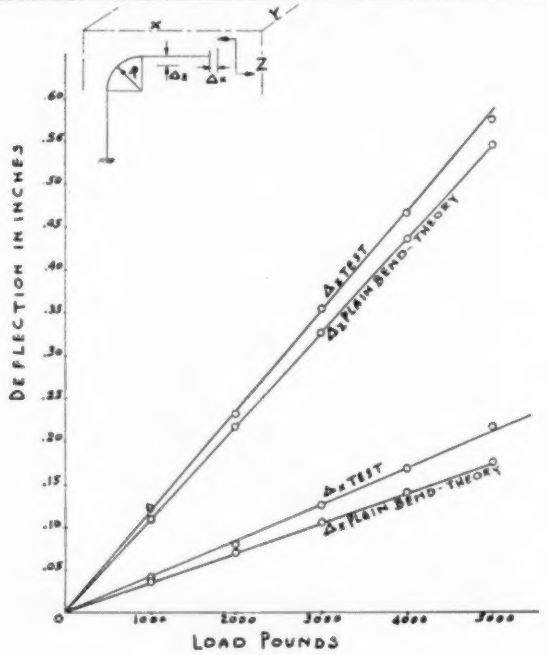


FIG. 49

TABULATED RESULTS 39"R CREASED BEND

FORCE	Δ_Y PLAIN THEORY	Δ_Y TEST	$R = \frac{\Delta_Y \text{TEST}}{\Delta_Y \text{THEORY}}$			
1000+	0.095"	0.140"	1.5			
2000+	0.190"	0.265"	1.4			
3000+	0.285"	0.400"	1.4			
4000+	0.380"	0.521"	1.4			
5000+	0.475"	0.649"	1.4			

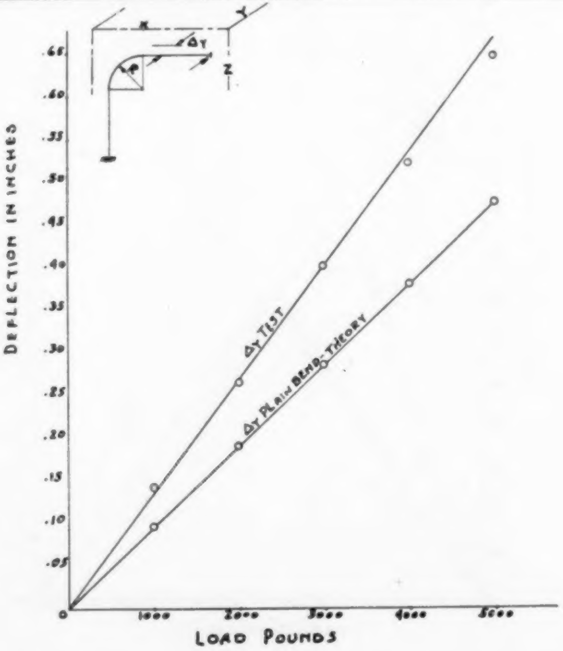


FIG. 50

TABULATED RESULTS 39"R CREASED BEND

FORCE	Δ_Y PLAIN THEORY	Δ_Y TEST	$R = \frac{\Delta_Y \text{TEST}}{\Delta_Y \text{THEORY}}$			
1000#	0.064"	0.070"	1.1			
2000#	0.128"	0.138"	1.1			
3000#	0.192"	0.204"	1.1			
4000#	0.256"	0.272"	1.1			
5000#	0.320"	0.341"	1.1			

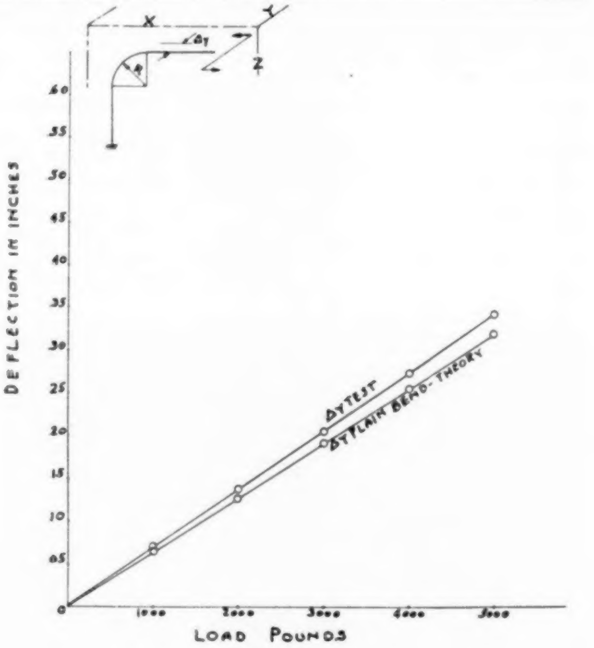


FIG. 51

TABULATED RESULTS 39"R CREASED BEND

FORCE	Δ_Y PLAIN THEORY	Δ_Y TEST	$R = \frac{\Delta_Y \text{TEST}}{\Delta_Y \text{THEORY}}$			
1000#	0.021"	0.054"	2.6			
2000#	0.042"	0.106"	2.5			
3000#	0.063"	0.155"	2.6			
4000#	0.084"	0.201"	2.4			
5000#	0.105"	0.235"	2.2			

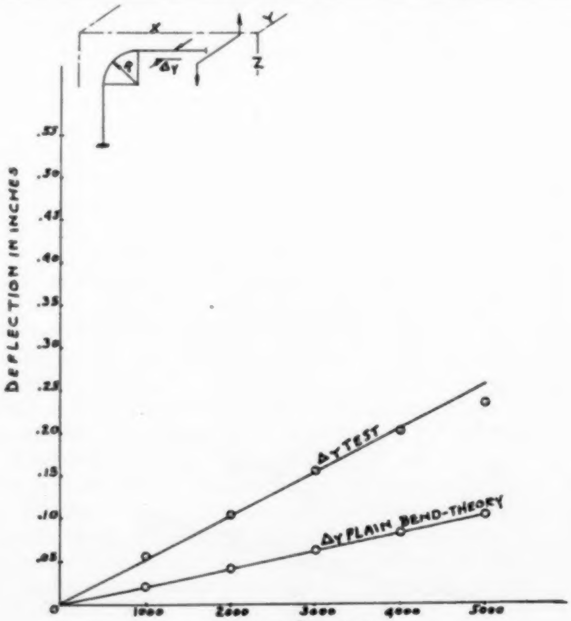


FIG. 52

TABULATED RESULTS 23"R CREASED BEND

FORCE	Δ_X PLAIN THEORY	Δ_X TEST	$R = \frac{\Delta_X \text{TEST}}{\Delta_X \text{THEORY}}$	Δ_Z PLAIN THEORY	Δ_Z TEST	$R = \frac{\Delta_Z \text{TEST}}{\Delta_Z \text{THEORY}}$
1000#	0.041"	0.048"	1.2	0.157"	0.167"	1.1
2000#	0.082"	0.101"	1.2	0.314"	0.337"	1.1
3000#	0.123"	0.155"	1.3	0.471"	0.505"	1.1
4000#	0.164"	0.206"	1.3	0.628"	0.694"	1.1
5000#	0.205"	0.251"	1.2	0.785"	0.865"	1.1

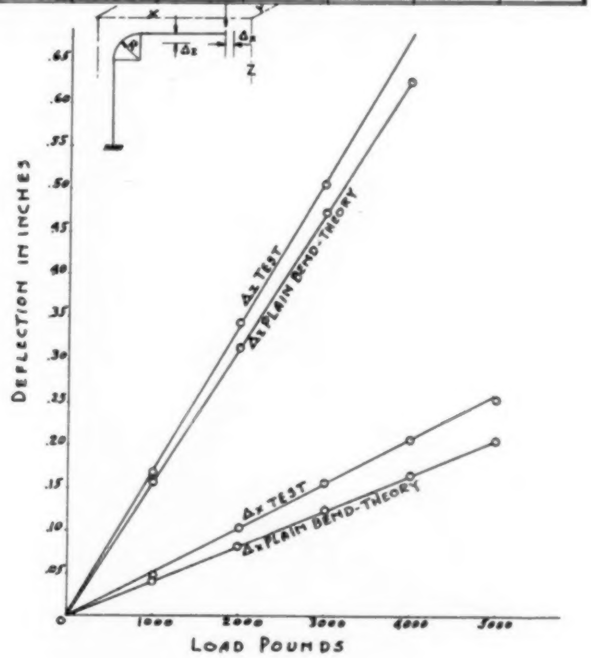


FIG. 53

TABULATED RESULTS 23"R CREASED BEND

FORCE	Δ_X PLAIN THEORY	Δ_X TEST	$R = \frac{\Delta_X \text{TEST}}{\Delta_X \text{THEORY}}$	Δ_Z PLAIN THEORY	Δ_Z TEST	$R = \frac{\Delta_Z \text{TEST}}{\Delta_Z \text{THEORY}}$
1000#	0.022"	0.038"	1.7	0.037"	0.043"	1.2
2000#	0.044"	0.072"	1.6	0.074"	0.085"	1.1
3000#	0.066"	0.106"	1.6	0.111"	0.120"	1.1
4000#	0.088"	0.141"	1.6	0.148"	0.158"	1.1
5000#	0.110"	0.180"	1.6	0.185"	0.199"	1.1

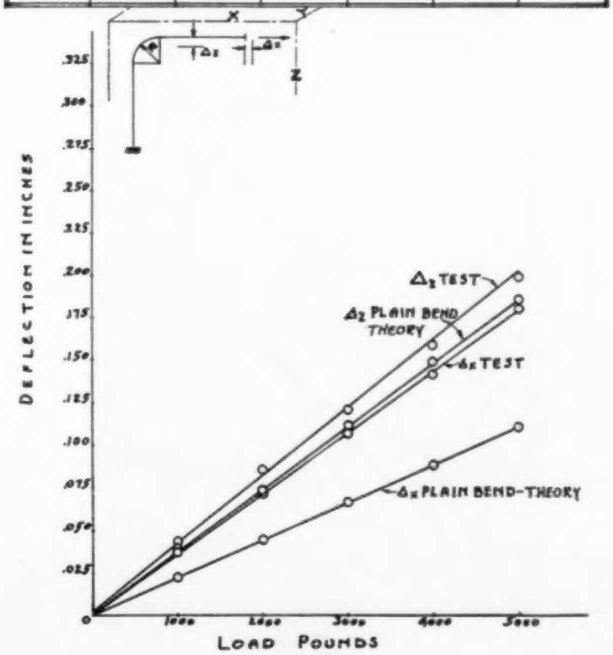


FIG. 54

TABULATED RESULTS 23" R CREASED BEND

FORCE	$\Delta_x \frac{\text{PLAIN}}{\text{THEORY}}$	Δ_x TEST	$R = \frac{\Delta_x \text{TEST}}{\Delta_x \text{THEORY}}$	$\Delta_z \frac{\text{PLAIN}}{\text{THEORY}}$	Δ_z TEST	$R = \frac{\Delta_z \text{TEST}}{\Delta_z \text{THEORY}}$
1000#	0.031"	0.023"	.75	0.143"	0.135"	.94
2000#	0.062"	0.040"	.64	0.286"	0.276"	.96
3000#	0.093"	0.060"	.64	0.429"	0.410"	.95
4000#						
5000#						

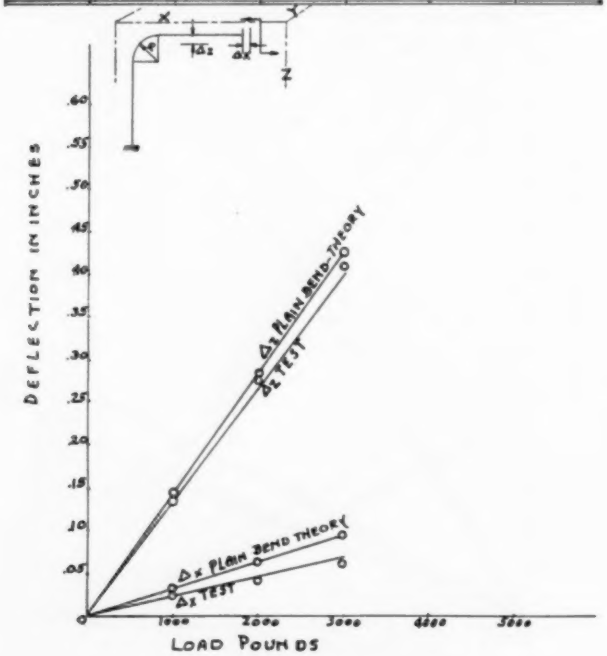


FIG. 55

TABULATED RESULTS 23" R CREASED BEND

FORCE	$\Delta_Y \frac{\text{PLAIN}}{\text{THEORY}}$	Δ_Y TEST	$R = \frac{\Delta_Y \text{TEST}}{\Delta_Y \text{THEORY}}$			
1000#	0.098"	0.162"	1.7			
2000#	0.196"	0.309"	1.6			
3000#	0.294"	0.457"	1.6			
4000#	0.392"	0.612"	1.6			
5000#	0.490"	0.763"	1.6			

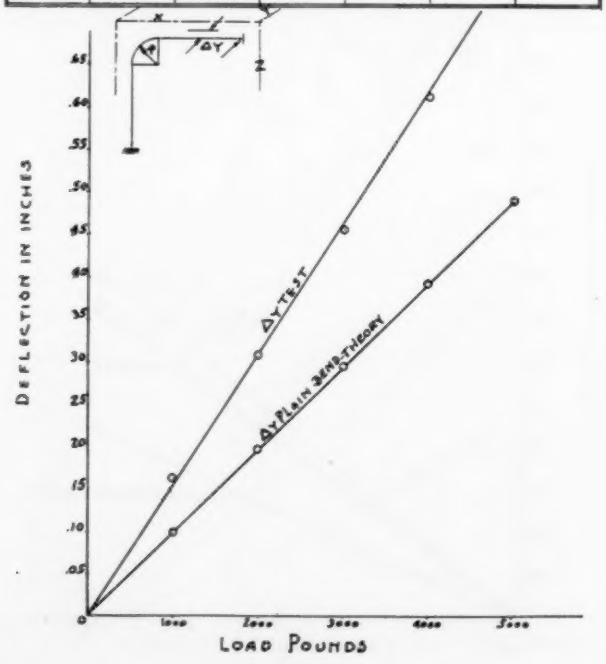


FIG. 56

TABULATED RESULTS 23" R CREASED BEND

FORCE	$\Delta_Y \frac{\text{PLAIN}}{\text{THEORY}}$	Δ_Y TEST	$R = \frac{\Delta_Y \text{TEST}}{\Delta_Y \text{THEORY}}$			
1000#	0.066"	0.083"	1.3			
2000#	0.132"	0.167"	1.3			
3000#	0.198"	0.245"	1.2			

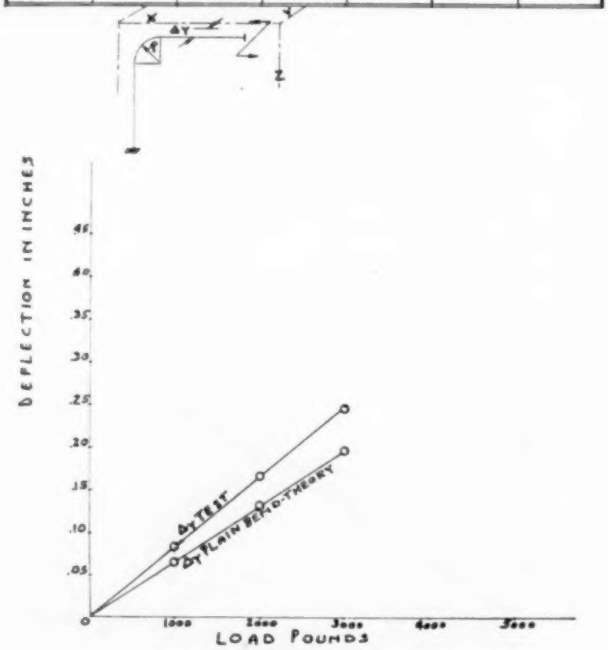


FIG. 57

TABULATED RESULTS 23" R CREASED BEND

FORCE	$\Delta_Y \frac{\text{PLAIN}}{\text{THEORY}}$	Δ_Y TEST	$R = \frac{\Delta_Y \text{TEST}}{\Delta_Y \text{THEORY}}$			
1000#	0.022"	0.057"	2.6			
2000#	0.044"	0.114"	2.6			
3000#	0.066"	0.167"	2.5			
4000#	0.088"	0.229"	2.6			

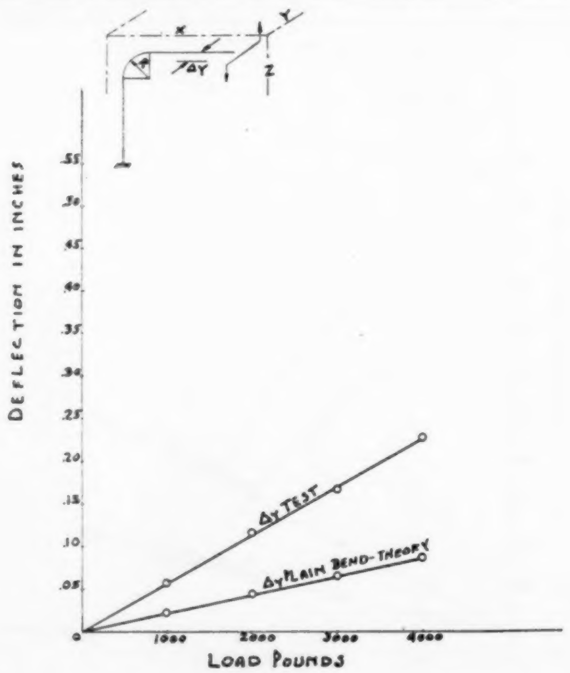


FIG. 58

used in testing all the other pipe specimens. It was suspended by rope slings from the frame of the building. Fig. 6 shows the drawing of this bend. Fig. 59 shows this bend prepared for compression loading, which was the only loading applied. Load was applied by a turnbuckle attached by cables to the top and the bottom of a yoke at each end of the bend. This is clearly shown in Fig. 59. The two traction dynamometers, one of which was in series with each turnbuckle, had been calibrated just preceding the test. It was planned that readings, from which deflections were to be obtained, were to be made between short iron posts welded to the pipe at each end of the bend, both on the top and the bottom of the pipe. Change of distance between these posts with change in load were measured with a micrometer having an iron-pipe filler piece of fixed length. The deflections shown by the top and bottom readings were in such close agreement that it was evident that the pipe was deflecting as perfectly as could be desired. This, together with the fact that the amount of deflection probable, when the pipe was loaded to the proportional limit, would exceed the adjustable length of the micrometer, led to the substitution of a steel tape for the micrometer and a chisel mark at the top of each end flange for a reference point. This arrangement provided ample flexibility and was quite satisfactory.

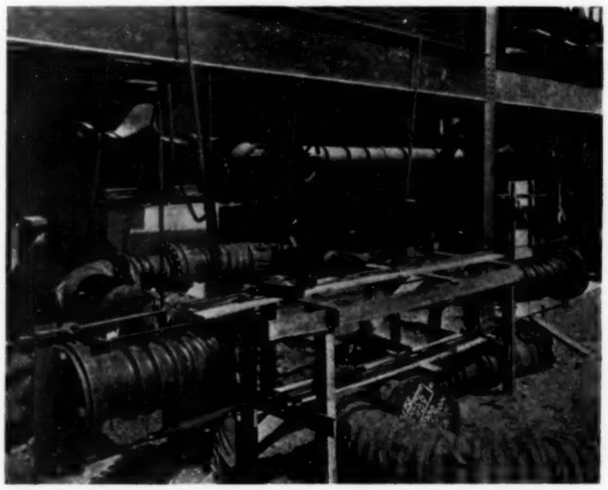
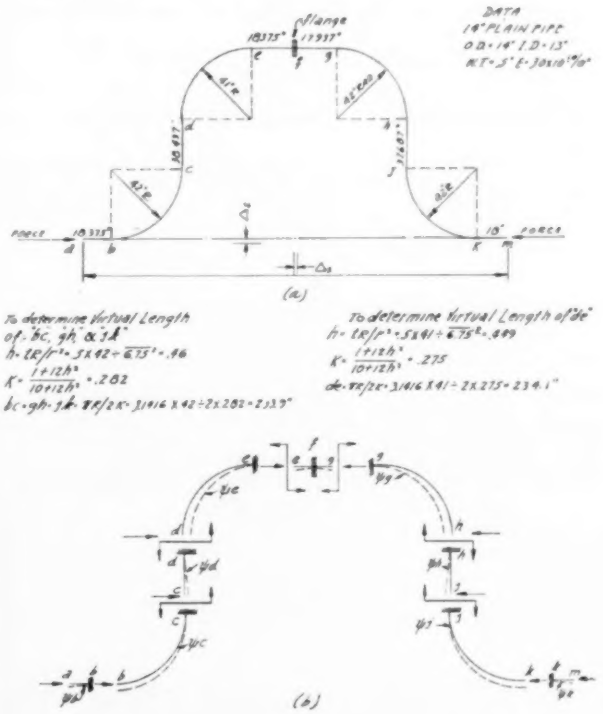


FIG. 59 14-IN. CORRUGATED EXPANSION U-BEND

The grapho-analytical solution by which the load-deflection relations of a plain pipe having major[8] dimensions identical with those of the corrugated pipe is as follows: The expansion U-bend [Fig. 60 (*a*)] is divided into two S-bends, each of which is considered anchored at the flange *f*. In Fig. 60 (*b*), the component parts of the expansion U-bend are shown with the transposed forces and moments acting upon them. Rotations of the elements, as discussed in the previous analyses, are also indicated. The bending-moment diagrams corresponding to these forces and moments for S-bend *a–f* are shown in Fig. 61 (*c*). Those for S-bend *f–m* are shown in Fig. 62 (*c*). The total deflections at point *a* of S-bend *a–f* and point *m* of S-bend *f–m* are determined as follows: From Fig. 61 (*c*) and Fig. 62 (*c*) are obtained the rotations and "moments of areas" due to the force and moment on each component part. The deflections on each component part are the result of the actions of these rotations, forces, and moments; hence the application of these to the elementary cases to which they correspond determines the deflections of each part. The summation of these increment deflections gives the total deflections at points *a* and *m*. The addition of the Δ*x* deflection, Fig. 60 (*a*) at points *a* and *m*, determines the total Δ*x* deflection. The Δ*z* deflections, Fig. 60 (*a*) at points *a* and *m*, are equal to each other and represent the total Δ*z* deflection.

GRAPHO-ANALYTIC SOLUTION FOR DEFLECTIONS OF A PLAIN EXPANSION "U" BEND HAVING THE SAME MAJOR DIMENSIONS AS A CORRUGATED EXPANSION "U" BEND. AXIAL FORCE ACTING AT THE FREE ENDS.

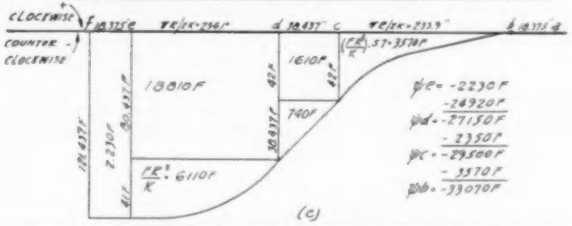


To determine Virtual Length of "bc", "gh" & "jk"
$h = tR/r^2 = .5 \times 42 \div \overline{6.75}^2 = .46$
$K = \frac{1+12h^2}{10+12h^2} = .282$
$bc = gh = jk = \pi R/2K = 3.1416 \times 42 \div 2 \times .282 = 233.9"$

To determine Virtual Length of "de"
$h = tR/r^2 = .5 \times 41 \div \overline{6.75}^2 = .449$
$K = \frac{1+12h^2}{10+12h^2} = .275$
$de = \pi R/2K = 3.1416 \times 41 \div 2 \times .275 = 234.1"$

FIG. 60

BENDING-MOMENT DIAGRAM FOR PART (af)



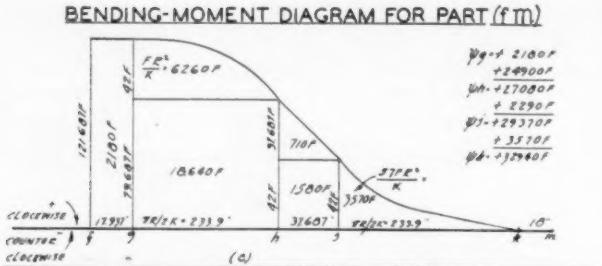
DETERMINATION OF DEFLECTIONS

$EI\Delta_{z\text{-}e}$	CASE III	$\psi M \tfrac{1}{2} l$	$= -2230F \times \tfrac{1}{2} \times 18.375$	$= -20{,}490F$ DOWN
$EI\Delta_{x\text{-}d}$	CASE I	ψeR	$= -2230F \times 41$	$= -91{,}430F$ RIGHT
	CASE Va	$\sum \psi F .7854R$	$= -6110F \times .7854 \times 41$	$= -196{,}750F$ RIGHT
	CASE IIIa	$\psi M .637R$	$= -18810F \times .637 \times 41$	$= -491{,}260F$ RIGHT
$EI\Delta_{z\text{-}d}$	CASE I	ψeR	$= -2230F \times 41$	$= -91{,}430F$ DOWN
	CASE Vb	$\sum \psi F .5R$	$= -6110F \times .5 \times 41$	$= -125{,}250F$ DOWN
	CASE IIIb	$\psi M .363R$	$= -18810F \times .363 \times 41$	$= -279{,}950F$ DOWN
$EI\Delta_{x\text{-}c}$	CASE I	ψdl	$= -27150F \times 38.437$	$= -1{,}043{,}560F$ RIGHT
	CASE II	$\sum \psi F \tfrac{2}{3} l$	$= -740F \times \tfrac{2}{3} \times 30.437$	$= -10{,}960F$ RIGHT
	CASE III	$\psi M \tfrac{1}{2} l$	$= -1610F \times \tfrac{1}{2} \times 38.437$	$= -30{,}940F$ RIGHT
$EI\Delta_{x\text{-}b}$	CASE I	$\sum \psi cR$	$= -29500F \times 42$	$= -1{,}239{,}000F$ RIGHT
	CASE IIa	$\psi F .624R$	$= -3570F \times .624 \times 42$	$93{,}560F$ RIGHT
$EI\Delta_{z\text{-}b}$	CASE I	$\sum \psi cR$	$= -29500F \times 42$	$= -1{,}239{,}000F$ DOWN
	CASE VIb	$\psi F .876R$	$= -3570F \times .876 \times 42$	$= -131{,}350F$ DOWN
$EI\Delta_{z\text{-}a}$	CASE I	ψbl	$= -33070F \times 18.375$	$= -607{,}660F$ DOWN

$EI\Delta_x = 3{,}205{,}460F$ RIGHT $\Delta_x = 3{,}205{,}460F \div 30 \times 10^6 \times 484 = .0002207F"$ RIGHT

$EI\Delta_z = 2{,}495{,}130F$ DOWN $\Delta_z = 2{,}495{,}130F \div 30 \times 10^6 \times 484 = .0001718F"$ DOWN

FIG. 61



DETERMINATION OF DEFLECTIONS

EIΔz-g	CASE III	ψM½ℓ	= +2180F × ½ × 17.937	= + 19,550F DOWN
EIΔx-h	CASE I	Σ ψgR	= +2180F × 42	= + 91,560F LEFT
	CASE Ia	ψF.7854R	= +6260F × .7854 × 42	= + 206,500F LEFT
	CASE IIIa	ψM.637R	= +18640F × .637 × 42	= + 498,690F LEFT
EIΔz-h	CASE I	Σ ψgR	= +2180F × 42	= + 91,560F DOWN
	CASE Ib	ψF.5R	= +6260F × .5 × 42	= + 131,460F DOWN
	CASE IIIb	ψM.363R	= +18640F × .363 × 42	= + 284,190F DOWN
EIΔx-j	CASE I	Σ ψhℓ	= +27080F × 37.687	= +1,020,360F LEFT
	CASE II	ψF⅔ℓ	= + 710F × ⅔ × 37.687	= + 17,840F LEFT
	CASE III	ψM½ℓ	= +1580F × ½ × 37.687	= + 29,770F LEFT
EIΔx-k	CASE I	Σ ψjR	= +29370F × 42	= +1,233,540F LEFT
	CASE IIa	ψF.624R	= + 3570F × .624 × 42	= + 93,560F LEFT
EIΔz-k	CASE I	Σ ψjR	= +29370F × 42	= +1,233,540F DOWN
	CASE IIb	ψF.876R	= + 3570F × .876 × 42	= + 131,350F DOWN
EIΔz-m	CASE I	ψkℓ	= +32940F × 18	= +592,920F DOWN

EIΔx = 3,192,020F LEFT   Δx = 3,192,020F ÷ 30 × 10⁶ × 484 = .0002198F″ LEFT

EIΔz = 2,484,570F DOWN   Δz = 2,484,570F ÷ 30 × 10⁶ × 484 = .0001711F″ DOWN

TOTAL DEFLECTIONS

Δx = .0002207F″ + .0002198F″ = .0004405F″

Δz = .0001711F″

Fig. 62

TABULATED RESULTS 14″ EXPANSION U-BEND

FORCE	Δx PLAIN THEORY	Δx TEST	R = Δx TEST / Δx THEORY	Δz PLAIN THEORY	Δz TEST	R = Δz TEST / Δz THEORY
1200+	0.528″	1.3438″	2.5	.205″	0.60″	2.9
2400+	1.056″	3.0313″	2.9	.410″	1.14″	2.8
3600+	1.584″	4.5313″	2.9	.615″	1.70″	2.6
4400+	1.936″	5.4375″	2.8	.752″	2.00″	2.6
5200+	2.288″	6.5938″	2.9	.889″	2.40″	2.7
6000+	2.640″	7.6875″	2.9	1.026″	2.80″	2.7
6800+	2.992″	8.7813″	2.9	1.162″	3.18″	2.7
7600+	3.344″	9.6250″	2.9	1.299″	3.45″	2.7
8000+	3.520″	10.2187″	2.9	1.368″	3.68″	2.7
8400+	3.696″	10.7813″	2.9	1.436″	3.90″	2.7
8800+	3.872″	11.7187″	3.0	1.504″	4.22″	2.8
9200+	4.048″	12.6563″	3.1	1.573″	4.54″	2.9

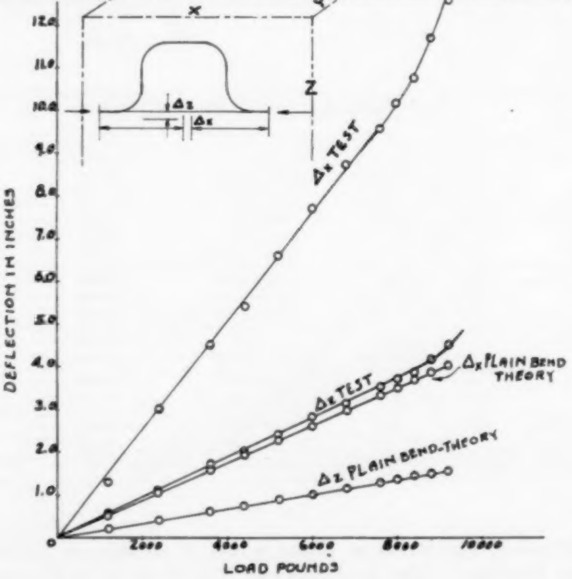


Fig. 63

The calculations of these deflections are shown in Fig. 60 for S-bend *a–f* and Fig. 61 for S-bend *f–m*.

The table and curves of Fig. 63 show the load-deflection relations for the corrugated expansion U-bend as found by the test and the load-deflection relations for a plain expansion U-bend of identical major dimensions.[8] The ratio between the deflections of the two pipes at any load is shown by the value of *R*.

VII—10-IN. CORRUGATED U-BEND

The high-temperature British Thomson-Houston turbo-generator, which, at the time of the tests, was being assembled in Delray P. H. 3, has two corrugated U-bends between the high-pressure and low-pressure cylinders. One of these bends was borrowed for test. The drawing of this bend is shown in Fig. 7 and the bend in one test assembly in Fig. 64. In carrying out the three loadings, one flange of this bend was bolted to the turbine-top bending table, the other flange extended clear of the table. The bend was so flexible that it was decided to load it by the use of standard 50-lb. weights. 600 lb. was the maximum load. Before the bend was erected on the bending table, the faces of the flanges were tested to see how nearly they lay in the same plane. They were found to be so nearly in the same plane that no correction was thought necessary. When the pipe was bolted to the bending table, the free end deflected so much at no load that an adjustable attachment was made to this end. By this attachment an upward pull was applied just sufficient to bring the face of the free flange into the same plane as the face of the fixed flange. Throughout each loading this upward pull was maintained constant. In two of the three loadings of this bend, a horizontal load was applied. This was accomplished by attaching a light rope to the free flange, passing it over a pulley properly placed in relation to the bend, and attaching the weights at its free end. During readings the pulley was tapped lightly. Displacement of the free end of the bend was measured in three directions, using a micrometer.

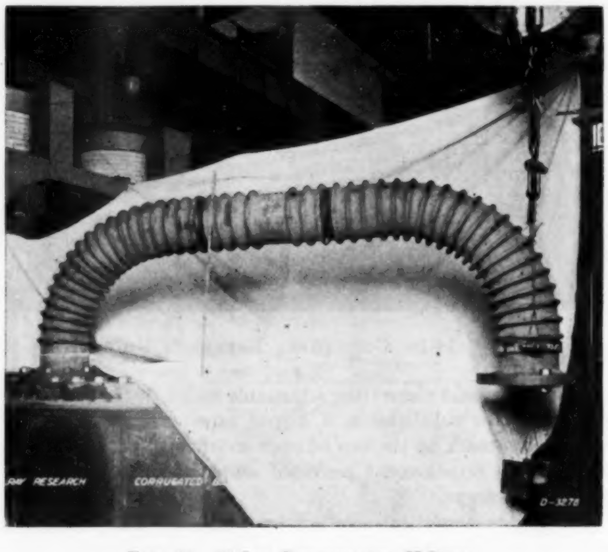


Fig. 64 10-In. Corrugated U-Bend

Grapho-analytical solutions for the determination of deflections of a 10-in. plain U-bend having the same dimensions[8] as the 10-in. corrugated U-bend were carried out. The loadings were identical with those used in the tests on the corrugated bend. From the results of the tests and of the mathematical solutions, comparisons of flexibility were obtained. Figs. 65, 68, and 71

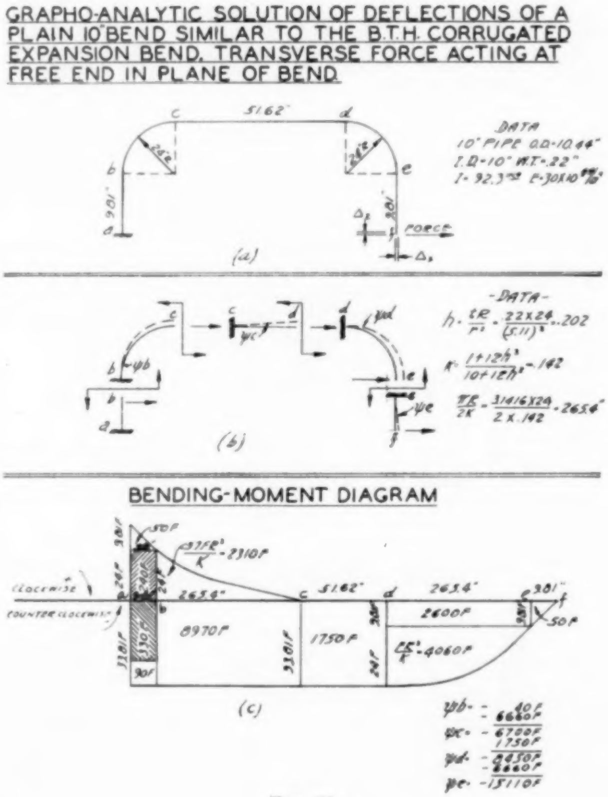

GRAPHO-ANALYTIC SOLUTION OF DEFLECTIONS OF A PLAIN 10" BEND SIMILAR TO THE B.T.H. CORRUGATED EXPANSION BEND. TRANSVERSE FORCE ACTING AT FREE END IN PLANE OF BEND
c
51.62"
d
b
e
a
9.01"
24R
FORCE
Δz
Δx
(a)
DATA
10" PIPE O.D.=10.44"
I.D.=10" WT.=.22"
I=92.3 E=30×10⁶
(b)
-DATA-
h = tR/r² = .22×24/(5.11)² = .202
K = (1+12h²)/(10+12h²) = .142
πR/2K = 3.1416×24/(2×.142) = 265.4"
ψb
ψc
ψd
ψe
BENDING-MOMENT DIAGRAM
50F
.57FR²/K = 2310F
CLOCKWISE
COUNTER CLOCKWISE
265.4"
51.62"
265.4"
9.01"
8970F
1750F
2600F
CE²/K = 4060F
90F
240F
330F
33.01F
24F
9.01F
50F
(c)
ψb = −40F −6660F
ψc = −6700F 1750F
ψd = −8450F −6660F
ψe = −15110F

Fig. 65

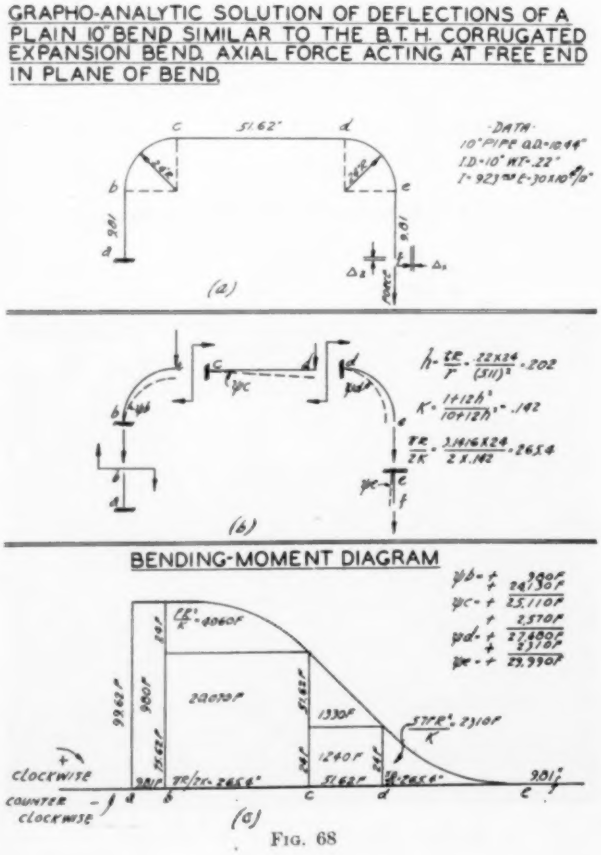
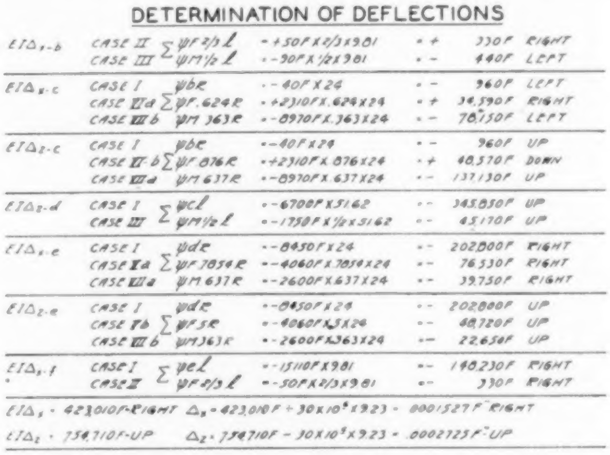

GRAPHO-ANALYTIC SOLUTION OF DEFLECTIONS OF A PLAIN 10" BEND SIMILAR TO THE B.T.H. CORRUGATED EXPANSION BEND. AXIAL FORCE ACTING AT FREE END IN PLANE OF BEND
c
51.62"
d
b
e
a
9.01"
24R
Δz
Δx
FORCE
(a)
DATA
10" PIPE O.D.=10.44"
I.D.=10" WT.=.22"
I=92.3 E=30×10⁶
h = tR/r² = .22×24/(5.11)² = .202
K = (1+12h²)/(10+12h²) = .142
πR/2K = 3.1416×24/(2×.142) = 265.4
ψb
ψc
ψd
ψe
(b)
BENDING-MOMENT DIAGRAM
ψb = + 990F + 24,130F
ψc = + 25,110F + 2,570F
ψd = + 27,680F + 2,310F
ψe = + 29,990F
CE²/K = 4060F
99.62F
90.0F
75.62F
24F
24,070F
51.62F
1330F
1240F
.57FR²/K = 2310F
+ CLOCKWISE
COUNTER CLOCKWISE −
9.01F
πR/2K = 265.4"
51.62F
πR/2K = 265.4"
9.01"
a
b
c
d
e
f
(c)

Fig. 68

show the steps necessary for the construction of the bending-moment diagrams. These steps are identical with those applied in the study of the loadings of a plain bend. In (*b*) of each of these figures the pipe bend is shown broken up into its component parts, each having the transposed forces and moments applied



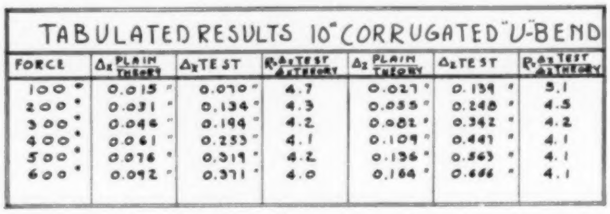
DETERMINATION OF DEFLECTIONS

$EI\Delta_{x-b}$	CASE II	$\Sigma\, \psi F\, 2/3\, l$	$= +50F \times 2/3 \times 9.01$	= +	330F	RIGHT
	CASE III	$\psi M\, 1/2\, l$	$= -90F \times 1/2 \times 9.01$	= −	440F	LEFT
$EI\Delta_{x-c}$	CASE I	$\psi b R$	$= -40F \times 24$	= −	960F	LEFT
	CASE VIa	$\Sigma\, \psi F\, .624R$	$= +2310F \times .624 \times 24$	= +	34,590F	RIGHT
	CASE VIII b	$\psi M\, .363R$	$= -8970F \times .363 \times 24$	= −	78,150F	LEFT
$EI\Delta_{z-c}$	CASE I	$\psi b R$	$= -40F \times 24$	= −	960F	UP
	CASE VI-b	$\Sigma\, \psi F\, .876R$	$= +2310F \times .876 \times 24$	= +	48,570F	DOWN
	CASE VIIIa	$\psi M\, .637R$	$= -8970F \times .637 \times 24$	= −	137,130F	UP
$EI\Delta_{z-d}$	CASE I	$\Sigma\, \psi c\, l$	$= -6700F \times 51.62$	= −	345,850F	UP
	CASE III	$\psi M\, 1/2\, l$	$= -1750F \times 1/2 \times 51.62$	= −	45,170F	UP
$EI\Delta_{x-e}$	CASE I	$\psi d R$	$= -8450F \times 24$	= −	202,800F	RIGHT
	CASE Xa	$\Sigma\, \psi F\, .7854R$	$= -4060F \times .7854 \times 24$	= −	76,530F	RIGHT
	CASE VIIIa	$\psi M\, .637R$	$= -2600F \times .637 \times 24$	= −	39,750F	RIGHT
$EI\Delta_{z-e}$	CASE I	$\psi d R$	$= -8450F \times 24$	= −	202,800F	UP
	CASE Xb	$\Sigma\, \psi F\, .5R$	$= -4060F \times .5 \times 24$	= −	48,720F	UP
	CASE VIII b	$\psi M\, .363R$	$= -2600F \times .363 \times 24$	= −	22,650F	UP
$EI\Delta_{x-f}$	CASE I	$\Sigma\, \psi e\, l$	$= -15110F \times 9.01$	= −	140,230F	RIGHT
	CASE II	$\psi F\, 2/3\, l$	$= -50F \times 2/3 \times 9.01$	= −	330F	RIGHT

$EI\Delta_x = 423{,}010F$-RIGHT $\Delta_x = 423{,}010F \div 30 \times 10^6 \times 9.23 = .0001527F''$ RIGHT

$EI\Delta_z = 754{,}710F$-UP $\Delta_z = 754{,}710F - 30 \times 10^6 \times 9.23 = .0002725F''$ UP

Fig. 66



TABULATED RESULTS 10" CORRUGATED "U"-BEND

FORCE	Δ_x PLAIN THEORY	Δ_x TEST	$R = \frac{\Delta_x \text{ TEST}}{\Delta_x \text{ THEORY}}$	Δ_z PLAIN THEORY	Δ_z TEST	$R = \frac{\Delta_z \text{ TEST}}{\Delta_z \text{ THEORY}}$
100#	0.015"	0.070"	4.7	0.027"	0.139"	5.1
200#	0.031"	0.134"	4.3	0.055"	0.248"	4.5
300#	0.046"	0.194"	4.2	0.082"	0.342"	4.2
400#	0.061"	0.253"	4.1	0.109"	0.441"	4.1
500#	0.076"	0.319"	4.2	0.136"	0.563"	4.1
600#	0.092"	0.371"	4.0	0.164"	0.666"	4.1

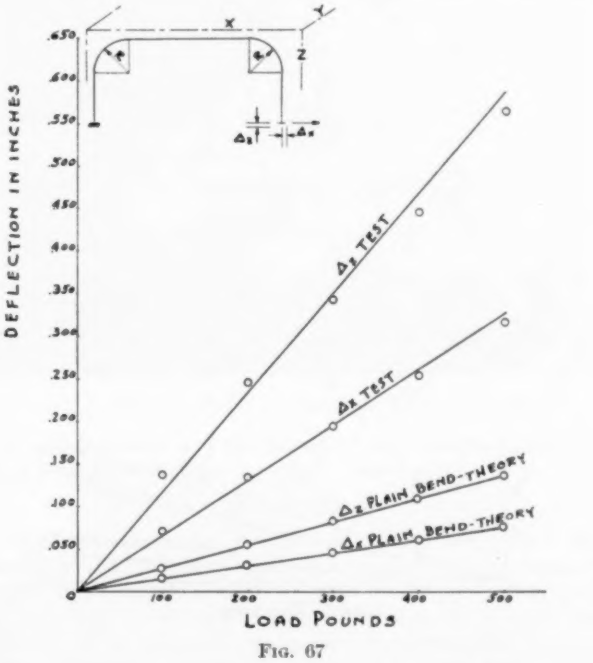


Fig. 67

to the hypothetical free end. (*c*) of each figure is the bending-moment diagram for moments and forces in the plane of the bend. Fig. 71(*c*-1) is the bending-moment diagram for moments and forces in the plane perpendicular to the plane of the bend. The abscissa of each of these bending-moment diagrams is laid out in exactly the same manner as is described in the study of the plain bend. The directions of rotation of the component parts are determined in the manner already described. The determination of deflection, with the calculation of each increment shown in detail, is shown in Figs. 66, 69, and 72. These are shown in sufficient detail that further explanation is thought unnecessary.

DETERMINATION OF DEFLECTIONS

$EI\Delta_{x-b}$	CASE III	Σ ψM½l	= +980F × ½ × 981	= + 4810F	RIGHT
$EI\Delta_{z-c}$	CASE I	ψbR	= +980F × 24	= + 23,520F	DOWN
	CASE V-a	Σ ψF.7854R	= +4060F × .7854 × 24	= + 76,530F	DOWN
	CASE VII a	ψM.637R	= +20,070F × .637 × 24	= + 306,830F	DOWN
$EI\Delta_{x-c}$	CASE I	ψbR	= +980F × 24	= + 23,520F	RIGHT
	CASE V b	Σ ψF.5R	= +4060F × .5 × 24	= + 48,720F	RIGHT
	CASE VII-b	ψM.363R	= +20,070F × .363 × 24	= + 174,850F	RIGHT
$EI\Delta_{z-d}$	CASE I	ψcl	= +25110F × 5162	= + 1,296,180F	DOWN
	CASE II	Σ ψF⅔l	= +1330F × ⅔ × 5162	= + 45,770F	DOWN
	CASE III	ψM½l	= +1240F × ½ × 5162	= + 32,000F	DOWN
$EI\Delta_{z-e}$	CASE I	Σ ψdR	= +27680F × 24	= + 664,320F	DOWN
	CASE VI a	ψF.624R	= +2310F × .624 × 24	= + 34,590F	DOWN
$EI\Delta_{x-e}$	CASE I	Σ ψdR	= +27680F × 24	= + 664,320F	LEFT
	CASE VI-b	ψF.876R	= +2310F × .876 × 24	= + 48,570F	LEFT
$EI\Delta_{x-f}$	CASE I	Σ ψel	= +29990F × 981	= + 294,200F	LEFT

$EI\Delta_x$ = 755,190F LEFT Δ_x = 755,190F ÷ 30×10⁶×92.3 = .0002727F" LEFT

$EI\Delta_z$ = 2,479,740F DOWN Δ_z = 2,479,740F ÷ 30×10⁶×92.3 = .0008955F" DOWN

Fig. 69

TABULATED RESULTS 10" CORRUGATED U-BEND

FORCE	Δ_x PLAIN THEORY	Δ_x TEST	$R = \frac{\Delta_x \text{TEST}}{\Delta_x \text{THEORY}}$	Δ_z PLAIN THEORY	Δ_z TEST	$R = \frac{\Delta_z \text{TEST}}{\Delta_z \text{THEORY}}$
100#	0.027"	0.135"	5.0	0.090"	0.464"	5.1
200#	0.054"	0.262"	4.8	0.180"	0.924"	5.1
300#	0.082"	0.392"	4.8	0.270"	1.376"	5.1
400#	0.109"	0.511"	4.7	0.360"	1.835"	5.1
500#	0.136"	0.631"	4.6	0.450"	2.285"	5.1
600#	0.163"	0.742"	4.5	0.540"	2.748"	5.1

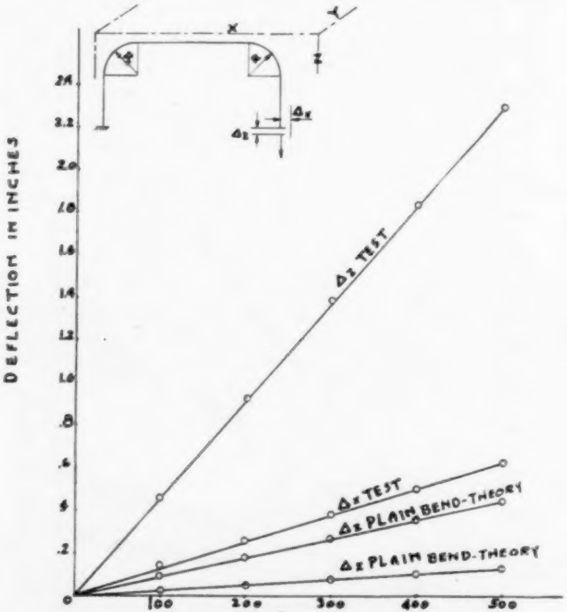


Fig. 70

The load-deflection values obtained by tests on the corrugated pipe and by calculation for a plain U-bend of the same major dimensions[8] are shown in the tables and curves of Figs. 67, 70, and 73. The ratio of the actual to the calculated flexibility for each loading is shown by the value of R in the tables.

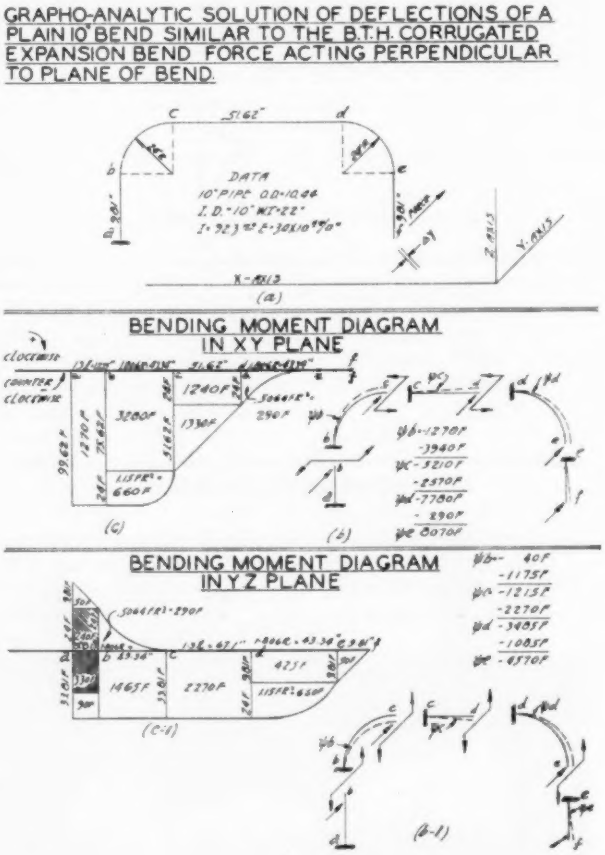

GRAPHO-ANALYTIC SOLUTION OF DEFLECTIONS OF A PLAIN 10" BEND SIMILAR TO THE B.T.H. CORRUGATED EXPANSION BEND FORCE ACTING PERPENDICULAR TO PLANE OF BEND.
c
51.62"
d
b
e
DATA
10" PIPE O.D.=10.44
I.D.=10" WT=22"
I=92.3 E=30×10⁶
X-AXIS
Y-AXIS
Z-AXIS
FORCE
(a)
BENDING MOMENT DIAGRAM IN XY PLANE
+ CLOCKWISE
COUNTER CLOCKWISE
1240F
3200F
1330F
1270F
99.62"
51.62"
.5064FR²=290F
1.15FR²=660F
(c)
(b)
ψb=-1270F
-3940F
ψc-3210F
-2570F
ψd-7780F
- 290F
ψe 8070F
BENDING MOMENT DIAGRAM IN YZ PLANE
ψb- 40F
-1175F
ψc -1215F
-2270F
ψd -3405F
-1085F
ψe -4570F
.5064FR²=290F
1465F
2270F
425F
330F
(c-1)
(b-1)

Fig. 71

DETERMINATION OF DEFLECTIONS

(a) IN XY PLANE:-

$EI\Delta_{y-c}$	CASE I	ψbR	-1270F × 24	= - 30,480F	RIGHT
	CASE VIII b	Σ ψF.754R	- 660F × .754 × 24	= - 11,940F	RIGHT
	CASE IX b	ψM.695R	-3280F × .695 × 24	= - 54,710F	RIGHT
	CASE IX a	ψM$_{yz}$.695R	-(15 × 51.62F × 24) × .695 × 24	= - 4,540F	LEFT
$EI\Delta_{y-d}$	CASE I	ψcl	= -5210F × 5162	= -268,940F	RIGHT
	CASE II	Σ ψF⅔l	= -1330F × ⅔ × 5162	= - 45,770F	RIGHT
	CASE III	ψM½l	= -1240F × ½ × 5162	= - 32,000F	RIGHT
$EI\Delta_{y-e}$	CASE I	Σ ψd$_{yz}$R	= -(15 × 51.62F × 24) × 24	= - 6,530F	RIGHT
	CASE I	ψdR	= -7780F × 24	= -186,720F	RIGHT
	CASE VIII a	ψF.754R	= -290F × .754 × 24	= - 5,250F	RIGHT

(b) IN YZ PLANE:-

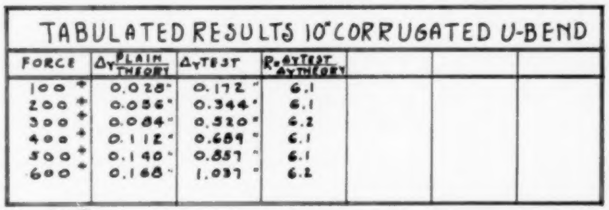
$EI\Delta_{y-b}$	CASE II	Σ ψF⅔l	= +50F × ⅔ × 981	= + 330F	RIGHT
	CASE III	ψM½l	= -90F × ½ × 981	= - 440F	LEFT
$EI\Delta_{y-c}$	CASE I	ψbR	= -40F × 24	= - 960F	LEFT
	CASE VIII a	Σ ψF.754R	= +290F × .754 × 24	= + 5250F	RIGHT
	CASE X a	ψM.306R	= -1465F × .306 × 24	= - 10,760F	LEFT
	CASE X b	ψM$_{xy}$.306R	= -(15 × 33.81F × 24) × .306 × 24	= - 890F	RIGHT
$EI\Delta_{y-d}$	CASE I	ψc$_{xy}$l	= -(15 × 33.81F × 24) × 5162	= - 6280F	RIGHT
$EI\Delta_{y-e}$	CASE I	ψc$_{xy}$R	= -(15 × 33.81F × 24) × 24	= - 2920F	RIGHT
	CASE I	ψdR	= -3485F × 24	= - 83,640F	RIGHT
	CASE VIII b	Σ ψF.754R	= -660F × .754 × 24	= - 11,940F	RIGHT
	CASE IX b	ψM.695R	= -425F × .695 × 24	= - 7090F	RIGHT
	CASE IX a	ψM$_{xy}$.695R	= +(15 × 9.81F × 24) × .695 × 24	= + 590F	LEFT
$EI\Delta_{y-f}$	CASE I	Σ ψe$_{yz}$l	= -(15 × 33.81F × 24) × 9.81	= - 1190F	RIGHT
	CASE I	ψel	= -4570F × 9.81	= - 44,830F	RIGHT
	CASE II	ψF⅔l	= -50F × ⅔ × 981	= - 330F	RIGHT

$EI\Delta_y$ = 709,740F RIGHT Δ_y = 709,740F ÷ 30×10⁶×92.3 = .0002852F" RIGHT

Fig. 72

VIII—CORRUGATED TANGENT

Four loadings were applied to the 14-in. corrugated tangent made of 400-lb. A.S.A. seamless pipe. These were Cases II, III, and IV, and an axial load. The same method of loading was used and the same precautions taken as with the pipe bends. Fig. 8 is a drawing of the tangent. Figs. 74, 75, and 76 show the test assemblies for the three numbered cases. Figs. 77, 78, and 79 show the load-deflection relations obtained by tests on the corrugated tangent and the calculated load-deflection relations for a plain tangent of the same major dimensions.[8]



TABULATED RESULTS 10" CORRUGATED U-BEND

FORCE	ΔY PLAIN / THEORY	ΔY TEST	R = ΔY TEST / ΔY THEORY			
100 #	0.028"	0.172"	6.1			
200 #	0.056"	0.344"	6.1			
300 #	0.084"	0.520"	6.2			
400 #	0.112"	0.689"	6.1			
500 #	0.140"	0.857"	6.1			
600 #	0.168"	1.037"	6.2			

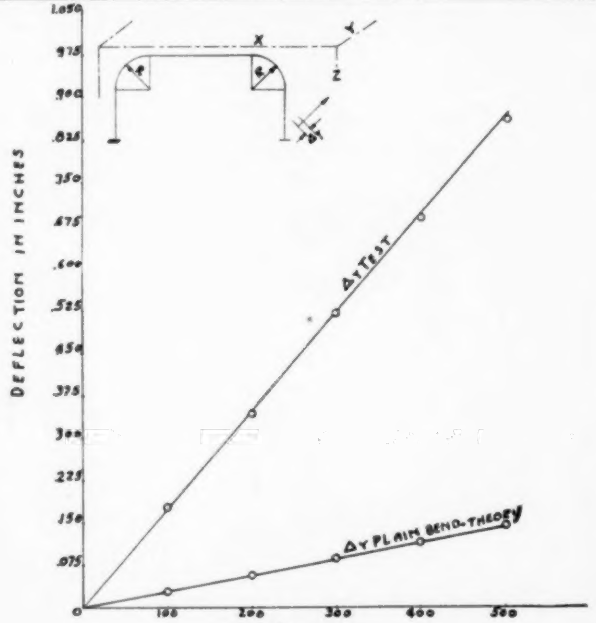


FIG. 73

FIG. 74 14-IN. CORRUGATED TANGENT

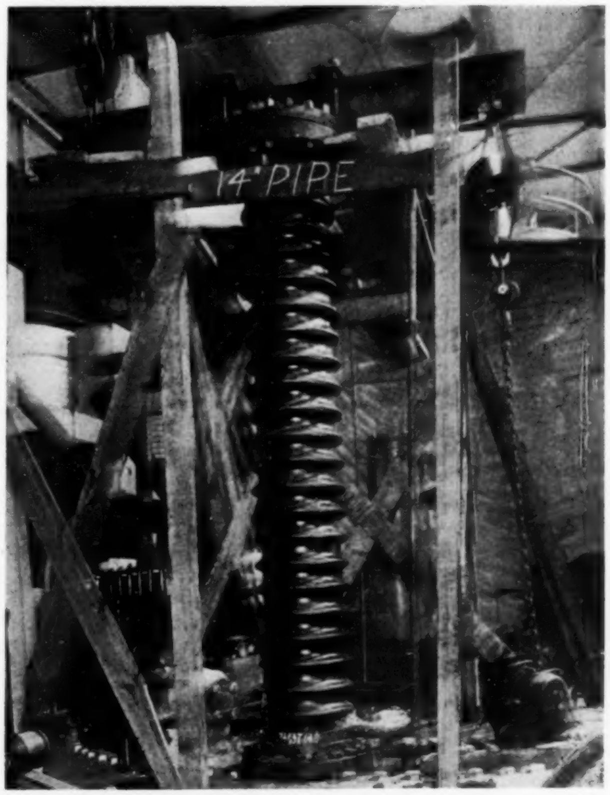


FIG. 75 14-IN. CORRUGATED TANGENT

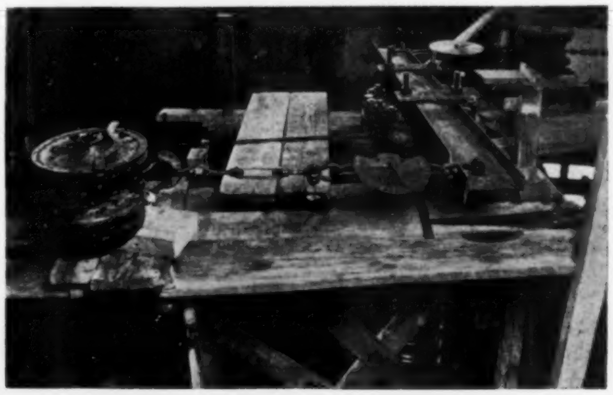


FIG. 76 14-IN. CORRUGATED TANGENT

CONCLUSIONS

This paper shows the results of tests made to compare the flexibility of a plain 10-in. pipe bend having a ratio of radius of bend to pipe diameter of 6:1 with the results obtained by calculation in all the elementary forms of loading. In all cases the flexibility of the pipe as shown by tests was greater than the calculated flexibility. Use was not made of the amended formula for the determination of K, the rigidity multiplication factor, as proposed by Mr. H. E. Jenks in his discussion of A.S.M.E. FSP-51-52.[9] The plain pipe bend made up of a forged quarter-bend having a tangent welded to each end showed the same characteristics as the bend of larger radius. While the ratios of flexibility of these two bends obtained by test and by calculation did not agree in all cases, they were, nevertheless, of the same order. Both pipe bends showed excessive flexibility when subjected to a torsional moment at the free end. The authors have not been able to account for the lack of agreement between tests and theory in this case. The study shows that the formulas apply equally well to a pipe bend having a large ratio of bend radius to pipe diameter, fabricated in the usual way, and to a pipe bend having a small ratio of bend radius to pipe diameter made up of a forged elbow with tangents welded to it.

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[9] "Design of Steam Piping to Care for Expansion," by W. H. Shipman, Trans. A.S.M.E., 1929, FSP-51-52.

TABULATED RESULTS 14" CORRUGATED TANGENT

FORCE	Δ PLAIN THEORY	Δ CORRUGATED TEST	R. = $\frac{\Delta \text{TEST}}{\Delta \text{THEORY}}$			
600 #	0.016"	0.087"	5.4			
1200 #	0.033"	0.164"	5.0			
1800 #	0.049"	0.249"	5.1			
2400 #	0.066"	0.332"	5.0			
3000 #	0.082"	0.418"	5.1			
3600 #	0.098"	0.502"	5.1			

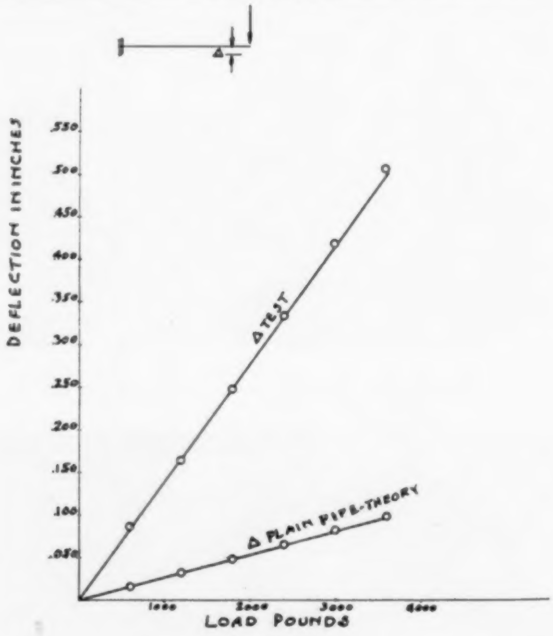


FIG. 77

This paper further shows the results of comparisons between corrugated and creased pipe bends and a corrugated tangent and plain pipe of the same major dimensions[8] as each corrugated and creased pipe. It was thought desirable to express in the form of a ratio the relation between flexibility obtained by test of each corrugated and creased pipe and that obtained by calculation of a plain pipe of the same major dimensions. The approximate average ratio for each kind of pipe is shown in the following table:

Kind of Pipe	Ratio
14-in., 90-deg. corrugated bend	2.2
14-in., 90-deg. corrugated bend	1.8
14-in., 90-deg. creased bend	1.3
14-in., 90-deg. creased bend, short radius	1.3
14-in. corrugated expansion U-bend	2.8
10-in. corrugated U-bend	4.5
14-in. corrugated tangent	5.1
14-in. corrugated tangent in torsion	0.9

As was to be expected, the corrugated tangent was less flexible in torsion than a plain pipe tangent of identical dimensions.

The authors would call attention to the fact that the tests were performed on a very small number of pipe bends of each kind. The results obtained and the conclusions drawn must be treated not as generalizations but rather as applying only to the particular pipe specimens tested. They fully realize that this work is fragmentary. It is believed, however, that it has been done in such a manner that others who may test pipe of the same general characteristics will be able to compare the results of their work with the results here reported.

ACKNOWLEDGMENT

The authors wish to acknowledge with thanks the assistance which certain of their associates in The Detroit Edison Company rendered in making the tests and computing the results. Without their assistance this work would not have been possible. Mr. J. H. Walker, engineer assistant to the general manager, Mr. Sabin Crocker, of the Engineering Division, and Mr. Frank Thorp, piping engineer, by their endorsement, made the investigation possible. Mr. W. F. Carter, plant engineer, and Mr. Frank Nadolni, assistant plant engineer, made available for use such equipment in Delray P. H. 1 as was necessary for carrying out the tests. The use of the partly dismantled equipment made unnecessary the construction of a bending table. This greatly reduced the cost of carrying out the tests. Mr. Samuel Smith, of the Drafting and Surveying Bureau, checked the computations and the sheets of curves. He also developed Appendix No. 2, which the authors believe is a unique solution for the determination of the sign of the rotation. Messrs. Harold Cartier, Fred Shapoe, Charles Moore, and Robert Gill, student apprentices

TABULATED RESULTS 14" CORRUGATED TANGENT

FORCE	Δ PLAIN THEORY	Δ CORRUGATED TEST	R. = $\frac{\Delta \text{TEST}}{\Delta \text{THEORY}}$			
1000 #	0.026"	0.150"	5.8			
2000 #	0.052"	0.269"	5.2			
3000 #	0.079"	0.411"	5.2			
4000 #	0.103"	0.539"	5.2			

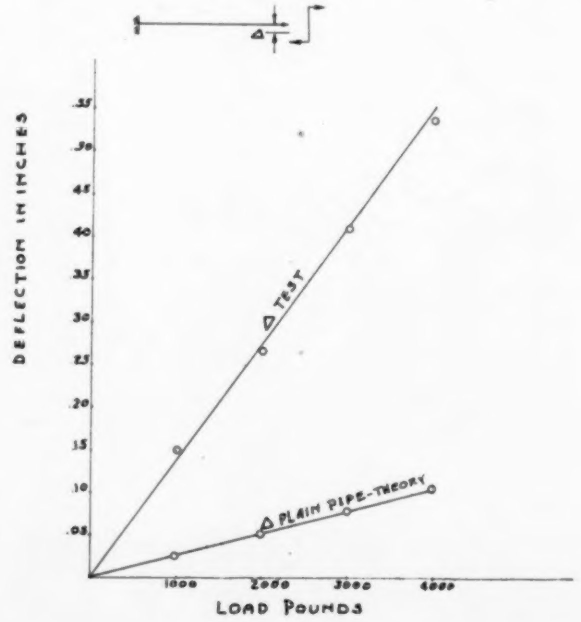
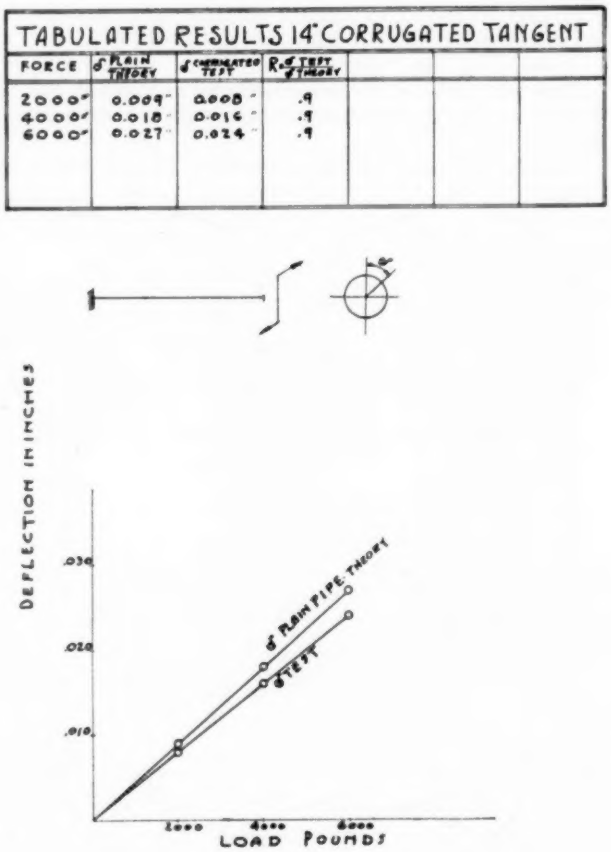


FIG. 78



TABULATED RESULTS 14" CORRUGATED TANGENT

FORCE	δ PLAIN THEORY	δ CORRUGATED TEST	R = δ TEST / δ THEORY			
2000#	0.009"	0.008"	.9			
4000#	0.018"	0.016"	.9			
6000#	0.027"	0.024"	.9			

FIG. 79

in the Research Department, carried most of the actual work of setting up the pipes for test and carrying out the loadings.

The authors further wish to thank the management of the Pittsburgh Piping and Equipment Company and that of Tube Turns, Incorporated, for the samples of their respective pipe bends made up especially for the tests.

Appendix No. 1

LIST OF SYMBOLS

The symbols used in this paper are defined below:

Δ = deflection in direction indicated, inches

Δ_X = deflection along the X-axis, inches

Δ_Y = deflection along the Y-axis, inches

Δ_Z = deflection along the Z-axis, inches

F = generalized force acting as shown, lb.

l = length of cantilever or section of pipe, inches

E = modulus of elasticity of pipe material at the working temperature of the piping, lb. per sq. in.

S = Section modulus

G = modulus of shearing elasticity = $\dfrac{E}{2(1 + \lambda)}$ lb. per sq. in.

λ = Poisson's ratio

I = moment of inertia of pipe cross-section, inches to the fourth power (in.^4)

J = polar moment of inertia = $2I$, inches to the fourth power (in.^4)

K = rigidity multiplication factor = $\dfrac{1 + 12h}{10 + 12h}$

h = $\dfrac{tR}{r^2}$ = pipe-bend ratio

t = thickness of pipe wall, inches

R = mean radius of bend, inches

r = mean radius of pipe cross-section, inches

ψ_M = rotation due to a moment at the end of a cantilever = area of rectangular bending-moment diagram, divided by EI

ψ_F = rotation due to a transverse force at the end of a cantilever = area of triangular bending-moment diagram, divided by EI

ψ_b, ψ_c, etc. = algebraic sum of the rotations of the elements between the point designated by the subscript and the fixed end of the section of piping = algebraic sum of the areas of the bending-moment diagrams to the left of the point designated by the subscript, divided by EI

Appendix No. 2

MATHEMATICAL DETERMINATION OF THE DIRECTIONS OF ROTATION IN PLANES AT RIGHT ANGLES TO THE PLANE OF THE QUARTER-BEND WHEN LOADED SIMILARLY TO CASES IX AND X[10]

It is seen from Cases IX and X that a couple acting at the free end of a quarter-bend and perpendicular to the plane of the bend produces rotations in both the planes at right angles to the plane of bend. In the derivation of formulas[11] for these rotations, there was not brought out the relationship between the direction (clockwise or counterclockwise) of these two rotations that exist for the different ways in which a quarter-bend can be anchored. Taking these formulas as a basis, the manner in which the relationship between the direction of the two rotations to account for the way in which the quarter-bend is anchored, is arrived at in the following manner. The different ways in which a quarter-bend can be anchored are shown in Fig. 80.

Consider the quarter-bends as shown in Fig. 81 with parts a, b, e, and f, having a couple at the free end in the plane perpendicular to the plane of the bend. Assume the couple acting on X-axis.

Let:

ψx_b = rotation along X-axis due to bending

ψx_t = rotation along X-axis due to twist

ψx = rotation along X-axis due to combined bending and twist

ψy_b = rotation along Y-axis due to bending

ψy_t = rotation along Y-axis due to twist

ψy = rotation along Y-axis due to combined bending and twist

$$GI_p = 2EI/2(1 + 0.3) = EI/1.3$$

$$\psi x_b =$$

$$EI d\psi a_b = M a ds = MaRd\theta$$

$$= -M \cos \theta R d\theta$$

---

[10] Developed by Samuel Smith, Drafting and Surveying Bureau, The Detroit Edison Company.

[11] Formulas developed by Prof. S. Timoshenko are reproduced as an appendix to the closure of the paper on "Frictional Resistance and Flexibility of Seamless Tube Fittings Used in Pipe Welding," by Sabin Crocker and Arthur McCutchan, presented at the December, 1930, meeting of the A.S.M.E.

$$EId\psi x_b = —EId\psi a_b \cos\theta$$

$$EId\psi x_b = M\cos^2\theta Rd\theta$$

$$\psi x_b = \frac{MR}{EI}\int_0^{\pi/2}\cos^2\theta d\theta$$

$$= \frac{MR}{EI}\left[\frac{\theta}{2}+\frac{1}{4}\sin 2\theta\right]_0^{\pi/2}$$

$$\therefore \psi x_b = \frac{MR}{EI}\left(\frac{\pi}{4}+0—0—0\right) = \frac{\pi MR}{4EI}$$

$$\psi x_t =$$

$$GJd\psi c_t = Mcds = McRd\theta$$

$$= M\sin\theta Rd\theta$$

$$GJd\psi x_t = GJd\psi c_t \sin\theta$$

$$\frac{EI}{1.3}d\psi x_t = M\sin^2\theta Rd\theta$$

$$\psi x_t = \frac{1.3MR}{EI}\int_0^{\pi/2}\sin^2\theta d\theta$$

$$= \frac{1.3MR}{EI}\left[\frac{\theta}{2}—\frac{1}{4}\sin 2\theta\right]_0^{\pi/2}$$

$$\therefore \psi x_t = \frac{1.3MR}{EI}\left(\frac{\pi}{4}—0\right) = \frac{1.3MR}{4\ EI}$$

$$\therefore \psi x = \psi x_b + \psi x_t = \text{positive value}$$

$$\psi y_b =$$

$$EId\psi y_b = EI_d\psi a_b \sin\theta$$

$$EId\psi a_b = —M\cos\theta Rd\theta$$

$$EId\psi y_b = —M\sin\theta\cos\theta Rd\theta$$

$$\psi y_b = —\frac{MR}{EI}\int_0^{\pi/2}\sin\theta\cos\theta d\theta$$

$$= \frac{MR}{EI}\left[\frac{\sin^2\theta}{2}\right]_0^{\pi/2}$$

$$= —\frac{MR}{2EI}\left(\frac{1}{2}—\frac{1}{2}\cos\theta\right)_0^{\pi/2}$$

$$= —\frac{MR}{2EI}\left(\frac{1}{2}+\frac{1}{2}—\frac{1}{2}+\frac{1}{2}\right)$$

$$\therefore \psi y_b = —\frac{MR}{2EI}$$

$$\psi y_t =$$

$$GJd\psi y_t = GJd\psi c_t \cos\theta$$

$$GJd\psi c_t = M\sin\theta Rd\theta$$

$$\frac{EI}{1.3}d\psi y_t = M\sin\theta\cos\theta Rd\theta$$

$$\psi y_t = \frac{1.3MR}{EI}\int_0^{\pi/2}\sin\theta\cos\theta d\theta$$

$$= \frac{1.3MR}{EI}\left(\frac{\sin^2\theta}{2}\right)_0^{\pi/2}$$

$$\therefore \psi y_t = \frac{1.3MR}{2EI} = \frac{0.65MR}{EI}$$

$$\therefore \psi_y = \psi y_b + \psi y_t = —\frac{MR}{2EI}+\frac{0.65MR}{EI} = \text{positive value.}$$

Thus when the quarter-bend is anchored as shown in Fig. 80, parts *a*, *b*, *e*, and *f*, the two rotations are both in the same direction.

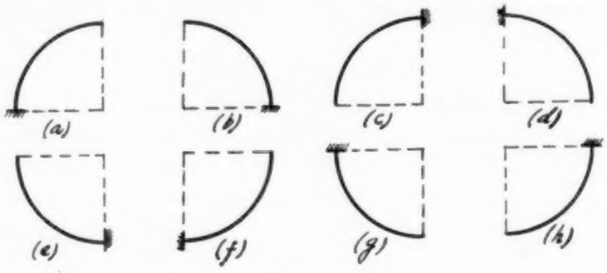


Fig. 80

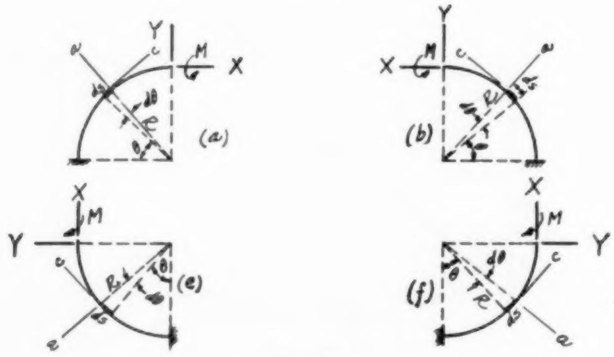


Fig. 81
(M_a = —$M \cos\theta$; M_c = $M\sin\theta$; d_s = $Rd\theta$.)

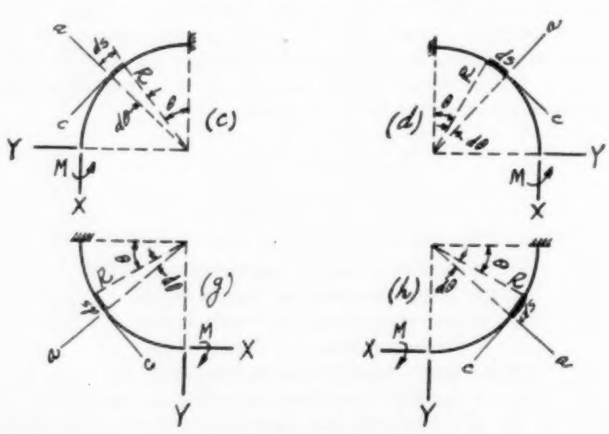


Fig. 82
(M_a = $M\cos\theta$; M_c = —$M\sin\theta$.)

Consider the quarter-bends as shown in Fig. 82, with parts *c*, *d*, *g*, and *h*, having a couple at the free end in the plane perpendicular to the plane of the bend. Assume the couple acting on the *X*-axis.

$$M_a = M\cos\theta$$

$$M_c = —M\sin\theta$$

$$\psi x_b =$$

$$EId\psi a_b = Mads = MaRd\theta = M\cos\theta Rd\theta$$

$$EId\psi x_b = EId\psi a_b \cos\theta$$

$$EId\psi x_b = M \cos^2 \theta R d\theta$$

$$\psi x_b = \frac{MR}{EI}\int_0^{\pi/2} \cos^2 \theta d\theta$$

$$\psi x_b = \frac{\pi MR}{4EI}$$

$$\psi x_t =$$

$$GJd\psi c_t = McRd\theta = -M \sin \theta R d\theta$$

$$GJd\psi x_t = -GJd\psi c_t \sin \theta$$

$$\frac{EI}{1.3} d\psi x_t = MR \sin^2 \theta d\theta$$

$$\psi x_t = \frac{1.3MR}{EI}\int_0^{\pi/2} \sin^2 \theta d\theta$$

$$\psi x_t = \frac{1.3\pi MR}{4EI}$$

$$\psi x = \psi x_b + \psi x_t = \text{positive value}$$

$$\psi y_b =$$

$$EId\psi y_b = EId\psi a_b \sin \theta$$

$$EId\psi a_b = M \cos \theta R d\theta$$

$$EId\psi y_b = M \sin \theta \cos \theta R d\theta$$

$$\psi y_b = \frac{MR}{EI}\int_0^{\pi/2} \sin \theta \cos \theta d\theta$$

$$\psi y_b = \frac{MR}{2EI}$$

$$\psi y_t =$$

$$GJd\psi y_t = GJd\psi c_t \cos \theta$$

$$GJd\psi c_t = -M \sin \theta R d\theta$$

$$\frac{EI}{1.3} d\psi y_t = -M \sin \theta \cos \theta R d\theta$$

$$\psi y_t = -\frac{1.3MR}{EI}\int_0^{\pi/2} \sin \theta \cos \theta d\theta$$

$$\psi y_t = -\frac{0.65MR}{EI}$$

$$\therefore \psi y = \psi y_b + \psi y_t = \frac{MR}{2EI} - \frac{0.65MR}{EI} = \text{negative value.}$$

Thus when the quarter-bend is anchored as shown in Fig. 80, parts *c*, *d*, *g*, and *h*, the two rotations are in opposite directions.

From the previous data the relationship between the direction of the rotations not in the plane of bend can readily be ascertained from the following diagram. Draw two lines at right angles to each other and let the intersection of these lines be considered as the anchored end of a quarter-bend. Each quadrant is given a number.

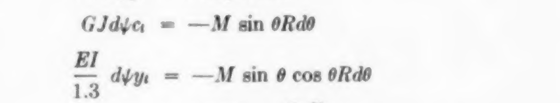


FIG. 83

From the previous data and Fig. 83 the following conclusions are noted: If the end of a quarter-bend is anchored so that the bend lies in quadrants I or II, the rotations in each plane at right angles to the plane of the bend are both in the same direction; but if the quarter-bend lies in quadrants III and IV, the rotations in each plane at right angles to the plane of the bend are in opposite directions.

Discussion

W. P. CURLEY.[12] This paper is of value, not only because it provides design data for corrugated and creased bends, but also because it adds another to the growing number of tests which are proving the present bending theory, as applied to plain pipe, to be substantially correct.

Tests made to check Case V in the "Piping Handbook" for the 10-in. plain bend and the 10-in. "tube-turn" with tangents prove the statement made in the paper "Frictional Resistance and Flexibility of Seamless-Tube Fittings Used in Pipe Welding," presented by Messrs. Crocker and McCutchan at the December, 1930, meeting of the A.S.M.E., that less reacting force is set up by a given deflection in the case of a short-radius fitting with tangents than in the case of a plain pipe bend.

No measurements are reported of the fiber stresses set up in the pipe walls due to bending. The use of strain gages in these tests would not have been practical. Perhaps there is a simpler method of estimating fiber stress, or of arriving at a safe working stress due to bending which may be used for design purposes.

It is evident, in most of the cases, that these stresses did not approach the elastic limit of the pipe material. In Fig. 63, however, the Δ_x and Δ_z test curves turn upward slightly when the applied force is between 8400 and 8800 lb. Since the relation, force to deflection, is a straight-line function, this indicates that the elastic limit of the material at some point in the pipe wall has been reached.

Current bending theory tells us that the maximum longitudinal bending stress S should occur at points e and g [Fig. 60(a)] and may be calculated from the formula

$$S = \frac{\beta Mr}{I} \text{ .... } [1]$$

where $\beta = \frac{2}{3K}\sqrt{\frac{6h^2 + 5}{18}}$ and M = bending moment. (Other symbols as given in Appendix No. 1, p. 27, of the paper.)

The moment arm at e or g is 121.69 in., so that $M = 121.69F$ in.-lb. β is found to be 1.39, r = 6.75 in., and I = 484 in. to the fourth power. Substituting these values in [1], $S = 2.36F$ lb. per sq. in.

For loads of 8400 and 8800 lb. the maximum longitudinal bending stresses are calculated to be approximately 19,800 and 20,800 lb. per sq. in., respectively.

If this one test were taken as representative for corrugated bends, it follows that such bends could be designed from [1] to operate at a stress slightly less than 19,800 lb. per sq. in. without exceeding the elastic limit of the material.

A procedure such as this should provide a simplified method for obtaining design data on any type of bend under any form of loading.

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[12] Tube-Turns, Inc., Louisville, Ky.

The Δ_s test curve for the 10-in. plain bend (Fig. 17) also turns upward, at an applied force of 5000 lb. The maximum longitudinal fiber stress occurs at b [Fig. 16(a)] and is calculated from [1] to be 11,200 lb. per sq. in. for a load of 5000 lb. ($M = 73.81F$, $\beta = 0.94$, $r = 5.19$, $I = 160.7$.) Thus, at a calculated stress of only 11,200 lb. per sq. in. the elastic limit at some point in this bend has been exceeded. This is probably because nominal dimensions are used in making the calculations, no account being taken of the thinning of the outer wall or of the shearing which takes place during fabrication of the bend.

Although this one test cannot be taken as representative, it indicates that it is unwise to design a plain bend to operate at more than 11,200 lb. per sq. in. fiber stress if the elastic limit of the bend is not to be exceeded.

The other units were apparently not stressed beyond their elastic limits. The stress in the tube-turn unit at an applied force of 3000 lb. was only approximately 18,900 lb. per sq. in. from [1]. Professor Hovgaard, in experiments at the Massachusetts Institute of Technology,[13] found the elastic limit of this type of fitting to be 21,000 lb. per sq. in.

It does not seem accurate to assign a ratio of flexibility as high as 1.3 to the 90-deg. creased bends, since this value was reached or exceeded in only a few of the possible cases. If average values are considered, the ratio for both the plain bend and the tube-turn with tangents should also be approximately 1.3 because both of these units show excessive flexibility under certain loadings.

Apparently, for equal center-to-face dimensions and equal pipe size, the 90-deg. tube-turn with tangents and the 90-deg. creased bend are equally flexible. The advantage falls to one type under certain conditions of loading and to the other type under other conditions.

A. M. HOUSER[14] AND S. HIRSCHBERG.[15] The present trend of higher pressures and temperatures in the steam power plant emphasizes the importance of this paper as the problem of providing adequate flexibility becomes increasingly more intricate as the temperatures and pressures increase. The paper demonstrates the uniqueness of the theoretical application of the grapho-analytical method by the use of area-moment diagrams and the principles of superposition.

In their conclusions, the authors express in the form of a ratio of 1.3 the relation between flexibility obtained by tests of creased pipe bends and that obtained by calculation of plain pipe bends of the same major dimensions. Particular attention is called to the results obtained for Case VI (axial force acting at the free end in the plane of the bend) in comparing the test results of the creased bends and the theory of the plain bends of the same dimensions as illustrated in Figs. 48 and 54 of the paper. The writers conducted a series of tests on creased and plain pipe bends of the double-offset expansion U-bend type having the same major dimensions to compare their relative flexibility. The results of these tests did not indicate a flexibility relation of 1.3 of the creased bend over the plain bend, but indicated that the plain pipe bend is more flexible than the creased pipe bend.

Two sets of 3-in. and one set of 6-in. double-offset expansion U-bends were tested. The movement of the free end of the bend was measured in the direction of the axial force applied in the plane of the bend. The dimensions of the bends tested are shown by single line drawings in Figs. 87–89; the dimensions are to center lines. The test procedure applied is briefly described as follows:

The bends were mounted vertically with the run horizontal by bolting the fixed end to a cast-steel tee which in turn was securely bolted to an I-beam. A 3-ft. straight length of 3-in. extra strong steel pipe was bolted to the free end of the bend by means of a flange. The movement of the free end was restricted to the axial direction by guiding the straight length of pipe with two sets of guide rollers above and below the pipe. The test procedure for the 6-in. bends differed from that described above for the 3-in. bends, in that straight lengths of 6-in. extra strong steel pipe were bolted to both ends of the bend having guiding rollers located 10 pipe diameters (5 ft.) from the flanges on the bend.

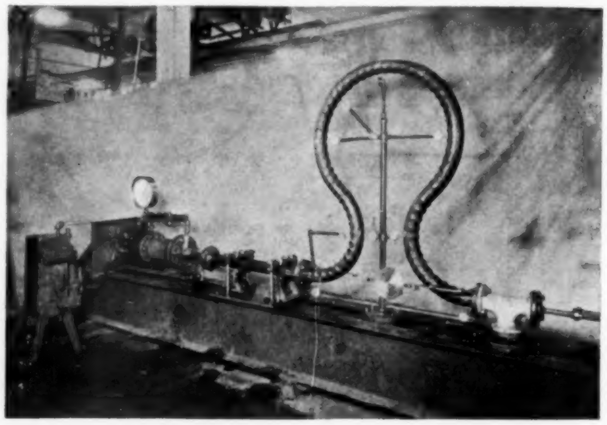

FIG. 84 TEST ARRANGEMENT FOR THE 3-IN. PIPE BENDS

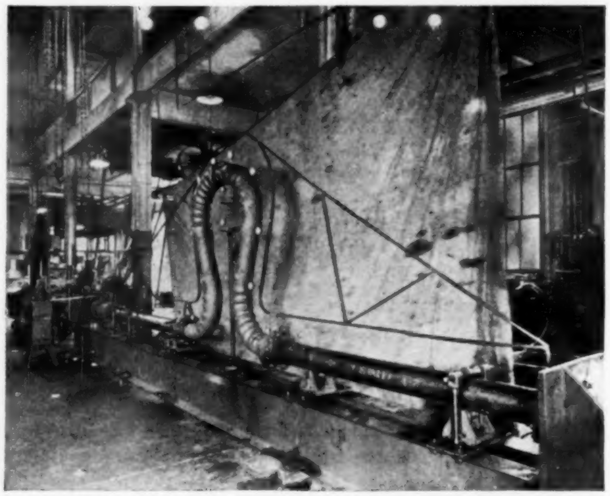

FIG. 85 TEST ARRANGEMENT FOR THE 6-IN. PIPE BENDS

The fixed end was anchored 2 ft. beyond the guiding roller; another roller was located 2 ft. from the first roller on the free end. The load or force was applied to the free end by a hydraulic pump and jack combination, having an area in compression of 25.2 sq. in. and in tension of 26.2 sq. in. The test assemblies of the various bends are shown in Figs. 84 and 85.

To duplicate as nearly as possible the action of pipe bends when installed in a steam line, 150 lb. steam pressure was permitted to flow through the bend during the tests. The steam was admitted at the tee connection near the fixed end, and exhausted on the free end at the extreme of the straight length of pipe. A dynamometer was attached across the flanged ends of the bends to note the amount of force exerted due to the temperature increase. All deflection readings were read on a fixed steel

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[13] Int. Congress of Applied Mechanics, Stockholm, 1930, "Bending of Curved Pipes."

[14] Product Engineer, Crane Co., Chicago, Ill. Mem. A.S.M.E.

[15] Crane Co., Chicago, Ill.

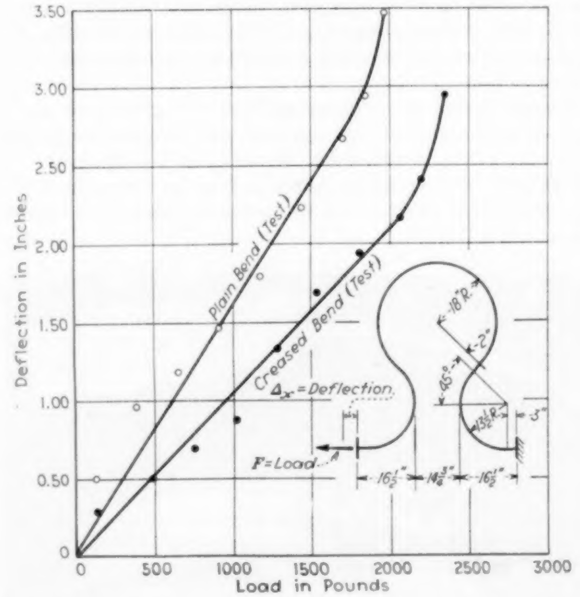

FIG. 86 LOAD-DEFLECTION RELATIONS—PLAIN VS. CREASED BENDS IN TENSION

(Bends made from 3.5-in. O.D. seamless steel tubing; wall thickness, 0.218 in.)

Force	Δ_x, plain bend	Δ_x, creased bend	$R = \frac{\Delta_x \text{ creased}}{\Delta_x \text{ plain}}$
500	0.80	0.50	0.625
1000	1.60	1.05	0.65
1500	2.40	1.57	0.65

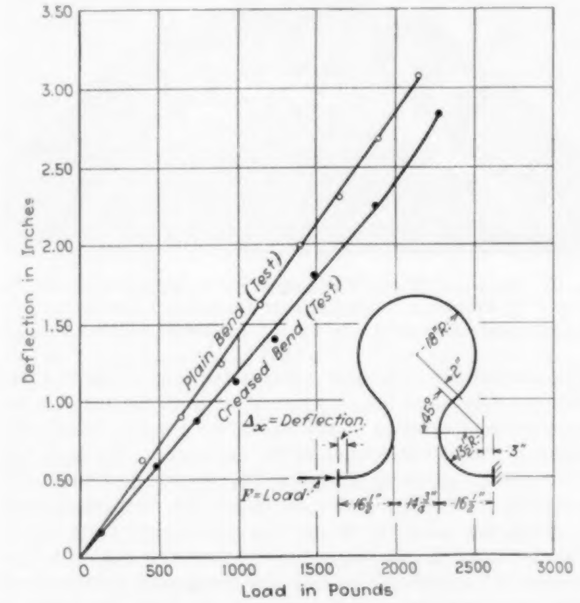

FIG. 87 LOAD-DEFLECTION RELATIONS—PLAIN VS. CREASED BENDS IN COMPRESSION

(Bends made from 3.5-in. O.D. seamless steel tubing; wall thickness, 0.218 in.)

Force	Δ_x, plain bend	Δ_x, creased bend	$R = \frac{\Delta_x \text{ creased}}{\Delta_x \text{ plain}}$
500	0.72	0.60	0.83
000	1.44	1.20	0.83
500	2.15	1.80	0.83
000	2.86	2.42	0.84

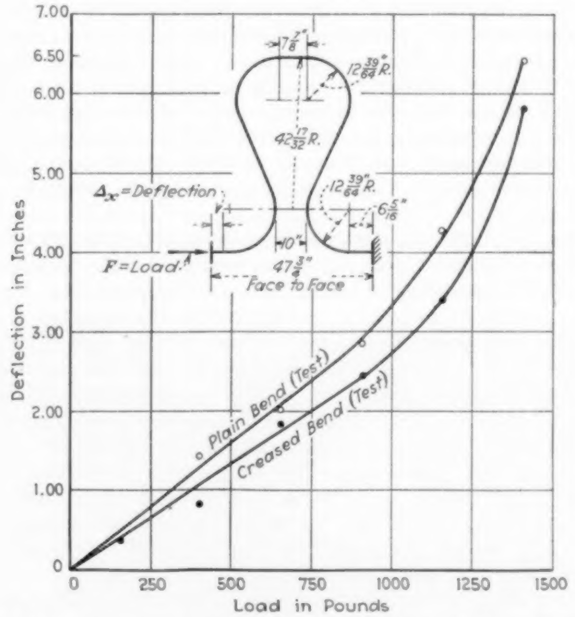

FIG. 88 LOAD-DEFLECTION RELATIONS—PLAIN VS. CREASED BENDS IN COMPRESSION

(Bends made from 3.5-in. O.D. seamless steel tubing; wall thickness, 0.218 in.)

Force	Δ_x, plain bend	Δ_x, creased bend	$R = \frac{\Delta_x \text{ creased}}{\Delta_x \text{ plain}}$
151	0.531	0.375	0.706
403	1.437	0.843	0.586
655	2.031	1.853	0.910
907	2.875	2.468	0.916

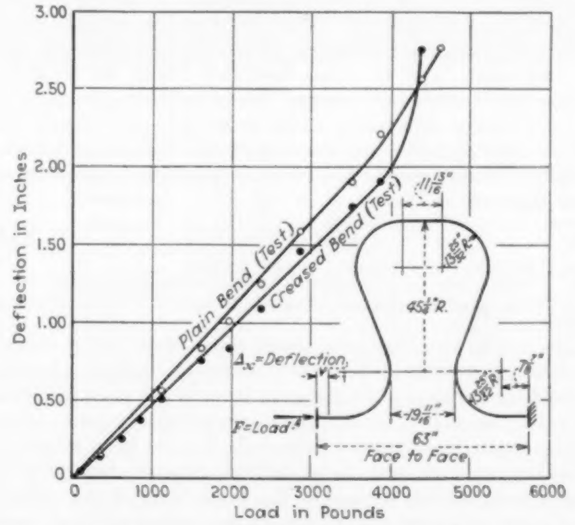

FIG. 89 LOAD-DEFLECTION RELATIONS—PLAIN VS. CREASED BENDS IN COMPRESSION

(Bends made from 6.625-in. O.D. seamless steel tubing; wall thickness, 0.280 in.)

Force	Δ_x, plain bend	Δ_x, creased bend	$R = \frac{\Delta_x \text{ creased}}{\Delta_x \text{ plain}}$
100	0.031	0.031	1.00
352	0.136	0.125	0.92
604	0.25	0.25	1.00
856	0.375	0.375	1.00
1108	0.561	0.520	0.93
1612	0.843	0.755	0.89
1990	1.03	0.843	0.82
2368	1.25	1.093	0.88
2872	1.59	1.46	0.92
3502	1.91	1.74	0.91
3880	2.22	1.90	0.86

scale at the free end of the bend. A needle-point indicator was attached on the free end of the bend, the movement of which could be easily read on the fixed scale. Ames indicating dials were placed at strategic points along the periphery of the bends to note the action of the bends under different conditions. Measurement of movement at the fixed end was also made, which revealed unexpected displacement of the fixed end during the progress of the test. This seems to be in accord with the authors' findings.

The values of loads and deflections obtained from actual tests are shown graphically in Fig. 86 for both the 3-in. creased and 3-in. plain pipe bends when in tension. Values for the same bends are shown graphically in Fig. 87 when the bends are subjected to compression.

Fig. 88 shows the deflections for different values of force, both graphically and tabulated for the 3-in. bends in compression. Fig. 88 illustrates the difference in design of this 3-in. bend as compared to the 3-in. bend shown in Fig. 87. Furthermore, it should be noted from Fig. 88 that the 3-in. bend with the long tangent is decidedly more flexible than the 3-in. bend shown in Fig. 87, although the overall dimensions are approximately equal. However, the proportional limit appears to be reached sooner with the long-tangent bend.

The tabulated results and curves of Fig. 89 show the deflections for different loads obtained from tests on the 6-in. creased bend and the 6-in. plain bend made to the same major dimensions.

The readings on the indicating dials proved that the bends move symmetrically with the vertical axis, although the vertical axis shifts as the deflection is changed, due to fixing of one end of the bend. When a bend is installed in a steam line, both ends are hypothetically "free" ends, the bend absorbing the expansion of pipe due to temperature between the anchors at both ends.

It may be interesting to know that the creased bends illustrated in Figs. 88 and 89 were fabricated in Crane Co.'s Corwith Works in Chicago, under the personal supervision of Mr. Alfred Schmidt, of Düsseldorf, Germany, the inventor of this type of bend.

Conclusions. The foregoing discussion shows the results of comparisons between creased pipe bends and plain pipe bends of the same major dimensions based on actual tests. Below is a table indicating in the form of average ratios the relation between flexibility of each creased pipe bend and plain pipe bend obtained from these tests:

Size	Kind of Bend	Ratio
3 in.	Double offset expansion U-bend, creased	0.83
3 in.	Double offset expansion U-bend, creased (long tangent)	0.77
6 in.	Double offset expansion U-bend, creased	0.92

The results obtained from actual tests conducted on creased and plain pipe bends made to the same major dimensions indicate a decided advantage of flexibility in favor of the plain pipe bend over the creased pipe bend. The results obtained by the authors when comparing actual test data of the creased bend with theoretical figures of the plain bend, indicate different characteristics. The writers believe, because of the importance of this subject, that more comparative test data on the various kinds of bends are necessary to establish a sufficient foundation for theoretical application.

B. R. Leapingwell.[16] As manufacturers of the corrugated pipes tested by the authors, we do not think that a corrugated bend should be compared with a plain bend of the same radius, because in the case of all the corrugated bends tested it would be manifestly impossible to manufacture plain bends to such small radii without thinning the tube excessively. It is of course recognized that uniform wall thickness can be obtained on plain bends of small radii by using tube turns, and welding tangents on. Such a method, however, would not be considered suitable in this country for the high pressures and temperatures which are now in use.

To keep the stress-multiplication factor down, the bends either have to be of a much larger radius or the tube has to be unnecessarily thick to allow for thinning during bending. An example of such thinning is shown in Fig. 90, which shows the reduction of wall thickness on the outside of the bend to be over 25 per cent.

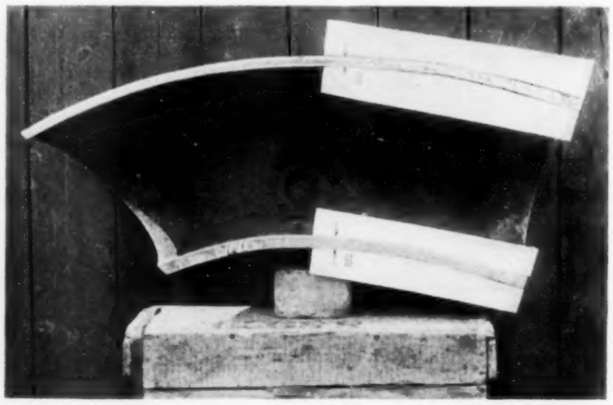

FIG. 90 SHOWING REDUCTION OF WALL THICKNESS ON OUTSIDE OF BEND

(Bore, 10 in.; thickness of straight tube, $^{23}/_{32}$ in.; mean radius of bend, 2 ft., 6 in.)

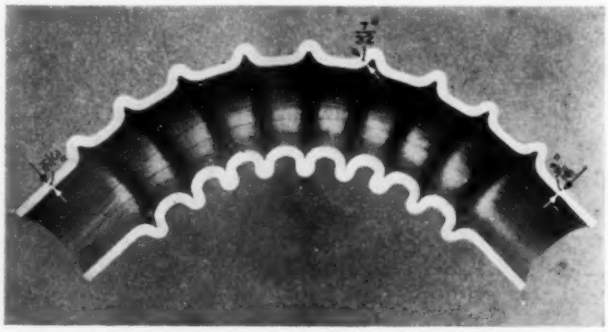

FIG. 91 SHOWING WALL THICKNESS OF CORRUGATIONS TO BE GREATER THAN THAT OF ORIGINAL TUBE

(Actual outside diameter of tube, $2^{3}/_{8}$ in.; mean radius of bend, $6^{3}/_{8}$ in.)

It is our practice to compare a corrugated bend or pipe to a pipe of similar dimensions but in which there is no flattening of the cross-section or thinning of the walls, thus leaving the effective moment of inertia the same as for the plain tube. We agree that for the reasons given by Professor Timoshenko no formula can be derived to obtain a factor of rigidity for corrugated pipes. We obtain this value for our pipes by actual test; however, the figures obtained are not very consistent, probably due to varying thickness of tube around the bore and consequent unevenness in height of corrugations; but generally speaking, the factor of rigidity rises as t/r increases in value. The results of the tests carried out by the authors are very interesting, particularly with regard to the flexibility under torsion for both corrugated tangents and bends; one would have expected the corrugated pipes to be less flexible under torsion. In a series of tests carried out on 2-in. nominal-bore corrugated pipes, owing to the large radius of the bend the rigidity factor for plain tube approached unity, and a less favorable flexibility ratio would be obtained for a corrugated pipe if compared to a plain pipe bend to a standard radius.

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[16] Director, Aiton & Co., Ltd., Derby, England.

As The Detroit Edison was not able to obtain data as to the wall thickness of the corrugated pipes, owing to their being required for service at the Marysville Power Plant, the photograph, Fig. 91, may be of interest, as it shows that the wall thickness of the corrugations is appreciably more than that of the tube from which the corrugated pipe is made. There is a limiting height of corrugation at which the sides thereof touch; the correct height is from three to four times the thickness of the tube.

The results of the tests carried out by Detroit Edison show the great advantage that there is in corrugating a straight pipe, as the flexibility ratio is so much greater in the case of tangents than it is on bends. It is our practice to utilize corrugated tangents in piping layouts, as they both increase the flexibility and reduce the cost considerably.

Benjamin Mills, Jr.[17] The authors have made a very valuable contribution toward establishing a rational method of stress analysis for piping systems by the tests and calculations reported in their paper. As a number of their tests were made on creased bends it is felt that some further data relating to them will prove of interest.

Our company, The Pittsburgh Piping and Equipment Company, entered into the manufacture of this product at the instance of one of the large engineering organizations of this country and with the full knowledge that we were pioneering. Our investigations have convinced us that these bends have great merit when correctly made and properly used, that they are a decided advantage in close quarters, and that their continued manufacture is fully warranted.

Some of the difficulties encountered in the production of these bends have been enormous, as data for their proper manufacture and formulas governing their action in service when made of the thicknesses of pipe used in this country were sadly lacking in the information supplied us by the patentee. Bends as made in accordance with instructions received from that source did not prove to have the flexibility expected or desired. Extensive experiments were undertaken to determine the most suitable spacing and dimensions of creases in relation to the diameter and wall thickness of the pipe for various bending radii. We found it necessary to develop entirely different methods of manufacture from those used on the light-walled pipes in general use in Germany. Our tests have shown conclusively that in order to obtain the best results with creased bends, correct design and proper manufacture are of paramount importance.

One of our engineers during the past year visited the two plants in Germany where creased bends are manufactured and reports that one of these firms also makes corrugated pipe and bends. The corrugated type has been made for some years, but the creased bend is a recent development and is being used to a far greater extent than the corrugated. Two of the large builders of steam turbines, Siemens-Schuckert A. G. and the Brown Boveri A. G., are now using creased bends exclusively where they had previously used corrugated. Corrugated bends are now used only in cases where extreme flexibility is the determining factor and drop in pressure is of small consequence or at least of secondary importance. Comparative tests made in a state coal mine in Holland showed the pressure drop in corrugated bends to be $2^1/_2$ times greater than in creased bends of the same dimensions.

The latest power plants in Germany have made use of large numbers of creased bends not only to care for expansion but as a precautionary measure, as these bends are considered much safer than plain ones, due to the practically uniform wall thickness throughout.

The German works of a large American boiler manufacturer has made use of creased bends in its boiler construction in certain instances where it was found impossible to keep joints tight with plain bends.

These bends are not made only for the purpose of taking care of expansion but can be used to advantage in many cases. Large-diameter pipes of comparatively thin wall can be bent on short radii, which makes them particularly adaptable to exhaust and bleeder lines, reheat piping, and gas lines. A decided advantage is gained by their use on shipboard due to the cramped quarters, and it obviates the necessity of employing bends made up of welded sections, which latter gave serious trouble on one German ship and were replaced with creased bends.

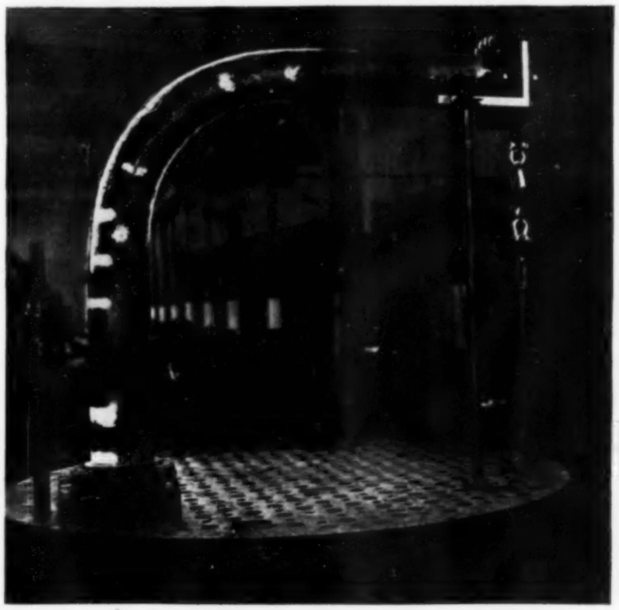

Fig. 92 Set-Up for Testing Plain Pipe Bend

In order to compare more directly the relative merits of creased and plain bends than has been done in the paper, we made up a plain square bend of 14-in. O.D. tubing $^1/_2$ in. thick on a 39-in. radius, duplicating the creased bend No. 1 (Fig. 5) used by the authors in their tests.

One of our bending tables was used in making the test set-up, a special steel plate $4^1/_2$ in. thick being used to make the connection between the end flange of the bend and the cast-iron plate of the table. A general view of the test set-up is shown in Fig. 92. A steel carpenter's square was used for making deflection measurements in both directions from a turned steel pin tapped into the pipe wall as shown in the figure, which shows also the traction dynamometer used to determine the loads imposed.

Gradually increasing loads were imposed on the bend until 9000 lb. was applied, and deflection measurements were taken at each 1000 lb. and at some 500 lb. intervals. No permanent set occurred, and the structure was loaded to 9000 lb. several times. The deflections have been plotted and are shown in Fig. 93 together with the deflections obtained by the authors in their test as shown in Fig. 47 and also the theoretical values. A very interesting feature is the close agreement of our test with the theory when the extreme shortness of this radius for plain bending of 14-in. pipe is considered. It gives us much pleasure to make this contribution toward substantiating the method of calculation used.

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[17] Boston, Mass. Discussion prepared by Messrs. D. G. Jones, G. S. Larsen (Mem. A.S.M.E.), E. Vollmer (Jun. A.S.M.E.), and G. W. Petrie, Jr., members of the engineering staff of the Pittsburgh Piping and Equipment Company, Pittsburgh, Pa.

The plain-bend theory shows that the flexural rigidity of a bend decreases as the radius of the bend is shortened. The correctness of this theory has been substantiated by a great number of tests. The tests made by the authors and also by ourselves show that short-radius creased bends have even less flexural rigidity than would be indicated by the theory for a plain bend of the same radius. As the creased bends are made on much shorter radii than possible for plain bends, it is evident that considerable increase in flexibility may be obtained by substituting a short-radius creased bend for a standard-radius plain bend, all other conditions being equal. To illustrate, let us consider an expansion bend made of 10-in. 400-lb. pipe. If this bend is made up of four 90-deg. bends as shown on page 533 of the "Piping Handbook" with R = 60 in., the force required to deflect the bend 1 in. will be 3770 lb.; if the bend is made up of four 90-deg. bends of 30 in. radius with straight pipe between so that the total overall dimensions are the same as in the first case, the force required to deflect the bend 1 in. will be only 2100 lb.

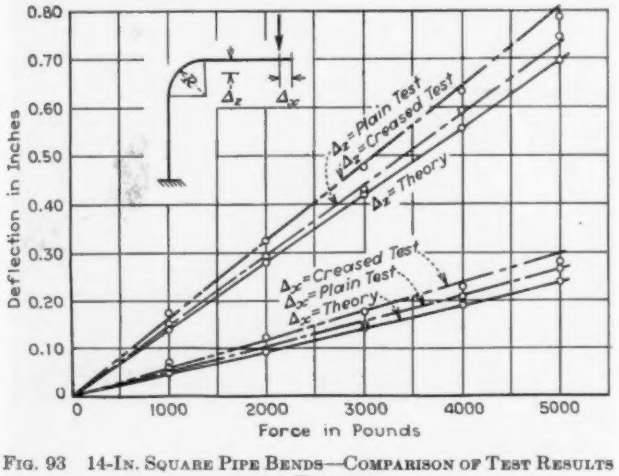

FIG. 93 14-IN. SQUARE PIPE BENDS—COMPARISON OF TEST RESULTS AND THEORY

Force, lb.	Deflections, in. Δ_x Theory	Δ_x Plain test	Δ_x Creased test	Δ_z Theory	Δ_z Plain test	Δ_z Creased test
1000	0.047	0.059	0.068	0.139	0.150	0.172
2000	0.094	0.121	0.123	0.278	0.280	0.321
3000	0.141	0.145	0.173	0.417	0.430	0.477
4000	0.188	0.207	0.227	0.556	0.593	0.634
5000	0.235	0.262	0.279	0.695	0.745	0.789

Creased bends have been found to give much greater deflection under tension than plain bends. By referring to the paper, it will be seen that the authors' tests indicate this to be as much as 64.2 per cent in Fig. 48 and 63.6 per cent in Fig. 54. By combining the decidedly increased values under tension with the somewhat greater values under compression, it can be readily seen that creased bends will freely absorb much greater movement than plain bends even of the same radii. Our practice in designing these bends is to base our calculations on the rigidity factor of plain bends for the same radii, ignoring the additional flexibility imparted by creasing and thus obtaining a greatly increased margin of safety in the installation.

A point which would appear to be worthy of considerable investigation is the probably excessive creep in piping sections where corrugated piping is used. The greatly increased surface exposed to pressure at high temperatures will surely express itself, and we would like to ask how this has been taken care of in the installations made, and whether any measurements have been taken or are considered on this phase of the subject.

While making no extravagant claims for creased bends, we nevertheless desire to repeat that by their use the gain in flexibility of short-radius bends can be utilized to the full with safety, and that when installed to take full advantage of their peculiar characteristics, anchor forces and flange stresses can be reduced.

Harold E. Jenks.[18] The use of higher temperatures with consequent need for more flexible piping to care for added expansion has recently brought about the development of creased and corrugated pipe bends. In connection with the use of such bends, two main factors are to be considered:

1 Relative flexibility as compared with equivalent plain bends
2 Stresses obtained, as compared with those in equivalent plain bends carrying the same total expansion.

It is also possible that the resistance to flow or turbulence of flow inside the pipe caused by creases or corrugations may in some cases affect the design.

By "equivalent" plain bends is meant plain bends having the same center-line dimensions, effective diameter, and wall thickness as the creased or corrugated pipe.

The paper under discussion presents the results of experiments made to determine relative flexibility only. In the writer's opinion these results, although a step in the right direction, are somewhat faulty due to the attempt to keep one end of the pipe bends "fixed." An absolutely fixed end is difficult to obtain, and calculations indicate that it has not been obtained in these tests. A slight variation from the assumed fixed condition results in errors so large as to seriously impair the test data as an effective check on the theory.

Let us examine the first case (the 10-in. plain bend under vertical loading) presented on pages 7 and 8. Solution by the analytic method contributed by the writer to Mr. W. H. Shipman's paper in 1929 gives the following equations for Δ_z and Δ_x in terms of the flexibility constant K:

$$\Delta_z = P(10)^{-8}(1969 + 2829K)$$
$$\Delta_x = P(10)^{-8}(1845 + 5039K)$$

The constant K is here the reciprocal of the value used by the authors. Using the tabulated test values of Δ_z and Δ_x, we obtain K from the foregoing equations as follows:

P	1000	2000	3050	4000	5000
K(from Δ_z)	1.955	2.15	2.33	2.37	2.37
K(from Δ_x)	2.01	2.30	2.37	2.45	2.48

It would appear that the gradual increase in K with load is here caused by a movement (rotation or translation or both) of the "fixed" end, roughly proportional to the load.

Similar calculations for the second case presented on pages 8 and 9 (the same bend under horizontal loading) give the following values of K:

P	1000	2000	3000	4000
K(from Δ_z)	3.42	2.98	2.73	2.73
K(from Δ_x)	2.70	2.39	2.27	2.29

In this case the rapid decrease of K with load would indicate a rather abrupt change in end conditions. The values of K are in no case even close to the Karman theoretical value of 2.01.

It would certainly be impossible to make a close determination of K from either of the above sets of figures, and the lack of consistency and large variation from the theoretical value is undoubtedly due principally to change of fixity at the assumed fixed end.

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[18] Atmospheric Nitrogen Corporation, Hopewell, Va.

It will be noted that the flexibility ratios given in the paper for the creased and corrugated bends are obtained by dividing the actual test results for the creased or corrugated bend by the theoretical results for an equivalent plain bend. Under the conditions of these experiments the theoretical results for plain bends are in all cases lower than actual results due to lack of perfect fixity as just explained, hence it is evident that the tabulated flexibility ratios are too high. For example, it is not at all impossible that the ratio for the creased bends is actually less than unity instead of 1.3 as tabulated, at least for loading in the plane of the bends. This possibility is strengthened by the fact that a ratio less than unity was actually found for one case and that this case represents the type of loading under which the 10-in. plain bend most nearly checked the theory. In connection with creased bends, the writer has before him data recently taken by a large pipe-manufacturing company. This company tested creased bends of a given size and shape against equivalent plain bends and found that the plain bends were in all cases more flexible than the creased bends. Loading was applied in the plane of the bend only. Unfortunately a good check on the theory was not obtainable for the reason that these experiments were attempted with both ends of the bends fixed, a condition that was not fully realized. However, it would seem that such a direct method of comparison would give more accurate flexibility ratios than would the indirect method of comparison with theoretical values for a hypothetical fixed-end bend, the method used in the present instance.

So far as actual flexibility results up to the present time are concerned, the writer must conclude that the creased bend is still a doubtful quantity, possibly slightly less flexible than an equivalent plain bend; and that the corrugated bend, while considerably more flexible than the plain bend, is probably somewhat less flexible than the tabulated ratios in this paper would indicate.

Experiments in which bends with hinged ends are used are believed to be the surest means toward obtaining reliable and consistent results. A few experiments made in this way under the simpler types of loading should give much valuable data. As pointed out in this paper, the flexibility ratios so obtained would not in general apply to a bend of any other size and shape than the one tested.

In many cases, notably those where high pressures as well as high temperatures are employed, the question of pipe stress becomes of importance. Professor Hovgaard has shown that in a plain pipe subject to expansion stresses only, the tangential, or flattening, stresses will at some points exceed the elastic limit long before the pipe as a whole shows a permanent set. That is, portions of the pipe may be in a plastic state and may fail when pressure stresses are added, or at least may corrode much more rapidly than they would otherwise do. Similar conditions may obtain in creased and corrugated bends. A corrugated bend of a given type under a given load may be found to deflect twice as much as an equivalent plain bend, but so far as we now know, it may at the same time be subject to three times the stress that the plain bend sustains.

It therefore seems desirable that strain measurements be made on creased and corrugated bends under load, in order that some idea of the stresses can be obtained. Stress and not flexibility is the basis of rational design and until we know something of stresses in these bends we shall be unable to properly design them or to determine their applicability to high-pressure, high-temperature service.

Alfred Schmidt.[19] With regard to the different types of bend considered, it would seem that bends made by welding pipe ends to a forged steel elbow will scarcely suffice for the highest-pressure work where large expansions have to be cared for, as the welds are subjected to bending stresses in taking up such expansions. It is the writer's belief that expansion bends as far as possible should be one-piece constructions.

According to the tests described in the paper, corrugated bends show a greater capacity for expansion as compared with creased bends, and at first sight would seem to be preferable to them. However, the tests on corrugated bends were not made on an equal basis as regards the creased bends, since in the former the corrugations extend into the tangent portions of the bends. To compare the two, it would be necessary to extend the creasing similarly in the tangent portions of the creased bends. However, the writer does not believe this to be desirable, since the idea involved in the creased-pipe patent is that of producing a flexible pipe with the lowest possible pressure drop. The creased pipe, at its outer wall where the steam impinges owing to its centrifugal force, is entirely smooth. The creases are so disposed that no more are used than are absolutely necessary. From tests made with corrugated and creased pipes having the same number of corrugations and creases, it has been found that the pressure drop in the former is about $2^1/_2$ times that in the latter. For the pipes tested by the authors it would be at least from 4 to 5 times. For this reason, the writer would advise taking up expansion only by creased pipes, and he believes that corrugated bends with corrugations in their straight parts should be used only in very difficult cases, where a very great expansion is to be cared for. Comparing now corrugated and creased bends having the same tangent lengths, the tests show only 10 per cent greater elasticity for the corrugated pipe, while, as already mentioned, the pressure drop is $2^1/_2$ times as much. Further, the method used in manufacturing corrugated bends is fundamentally different from that employed in making creased bends and results in much higher stresses in the material.

Of great interest are the tests comparing the flexibility of creased and plain bends in which a ratio of 1.3 has been determined. From numerous tests made in Düsseldorf a ratio of 1.5 has been obtained. It is possible that the difference may be due to the different materials used, and perhaps also to different methods of forming the creases.

It is very important to know that in the case of creased pipes the flexibility increases greatly with decreasing radius, without any thinning of the walls. It has also been observed during the tests that the flexibility of plain bends also increases with decreasing radius. These tests differ in this respect from those made by the authors, but the writer believes that further ones will help to clear up the discrepancy. Practically, however, it is impossible to use smaller radii for plain bends, because the thinning of the walls increases with decreasing radii. In practice plain bends should not be made with a smaller radius than 5 × pipe diameter D, or at the very lowest, $4D$. If we now compare a creased pipe bend with a radius of $2D$ and a plain pipe bend with a radius of $4D$ to $5D$, we get the following:

$$\frac{\text{Creased bend (radius } = 2D)}{\text{Plain bend (radius } = 4D)} = 4.17$$

$$\frac{\text{Creased bend (radius } = 2D)}{\text{Plain bend (radius } = 5D)} = 6.25$$

and for double-offset expansion U-bends,

$$\frac{\text{Creased double-offset expansion U-bend}}{\text{Plain double-offset expansion U-bend}} = 1.9 - 2.4.$$

These flexibility comparisons show clearly that the use of creased pipes offers many important advantages.

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[19] Engineer, Düsseldorf, Germany.

A. E. R. de Jonge.[20] The writer thanks The Detroit Edison Company for allowing the publication of this information and the authors for having undertaken the arduous task of securing data on load-deflection relations in pipe bends.

Such experiments are always useful in two directions—first, because they give first-hand information about the actual behavior of the tested elements under loads approaching working conditions and, second, because they allow a check on the theories involved in the calculation of such members.

There are very few data on expansion by heat available for plain pipe bends and apparently none for corrugated and creased pipe bends, so that these tests fill a gap badly felt by designers of pipe lines. Thus a critical review of the tests and their results should benefit the profession as a whole.

Early during the year the writer's cooperation was asked in solving the mystery of the rather large discrepancies between theory and test results. But while he made at the time various suggestions, there still remain some points in connection with the test set-up and the execution of these tests about which he is not quite clear, and he would request the authors to kindly supply this information, as it is otherwise not possible to check the results.

1 How high above the ground, i.e., its foundations, does the turbine casing stand to which the pipe bends were fixed? It seems to the writer that with the bend standing above this casing the total height must have been some 15 ft. at least, and with the forces used there must have been some bending of the casing which may easily account for part of the deviations between the test results and those derived from theory. It is the more necessary to check the distortion of this casing very carefully, as the authors state that the fixed points were attached to the bending table or to the building. They have neglected, however, to state definitely in each instance where the fixed point was located. It would also be of interest to know the construction of the old turbine casing, its wall thickness, cross-sections, etc., so that a check regarding its deflection could be applied, if this has not already been allowed for.

2 Where exactly were the brass plug measuring points located on the pipes? To measure a deflection accurately so that it can be compared with that derived from theory it is necessary to either measure at the center line of the pipe or at two points at an equal distance from the center line on opposite sides of it. Was this done?

3 If points on the wall of the pipe were chosen, has due allowance been made for rotations of these points in Cases VIII, IX, and X?

4 The fixed end, according to (1), must have moved under the influence of the forces and moments applied. Apparently the motions of some pins near this end were observed, but relative to which fixed system—to the bending table or to the building? The writer had suggested that index arms be secured to the fixed end of a length of several feet so that any movement might easily be observed. It seems a pity that this was not done.

The writer regrets that on account of the lack of this information he has not been able to check the results obtained nor to account for the discrepancies found in the various cases. But there are other data lacking, which, if supplied, might have been helpful in tracing the source of the discrepancies.

The pipe bends are never exactly circular in cross-section, as the outer fibers have been stretched during the process of hot bending, while the inner ones have been compressed. Thus the outer wall of the bend is thinner and the inner wall thicker than the mean thickness of the bend. Besides, an unavoidable flattening takes place during this process. It seems regrettable that the actual dimensions of the cross-section have not been ascertained, and not only at one particular place but all along the center line in steps of, say, 12 in. Further, the deformation of the sections under load should have been obtained with reference to the dimensions of the sections at no load. A shift in the neutral axis influences in Cases IX and X the rotational component due to torsion and thus the deflection at the free end of the bend. It is true that Mr. Cope has given the writer, during the correspondence prior to the reading of this paper, in his letter two additional data with regard to the cross-section, but they are quite insufficient for a definite check.

The writer further regrets deeply that the test value of the plain tangent in Case II, as indicated in the table on the second page, has not been given in the paper. This might have given some valuable clue with regard to how far the test results differ from theory in this simplest case. It would have been still more valuable if this plain tangent had been tested under the influence of a torsion moment, as in Case IV, which was done for the corrugated tangent.

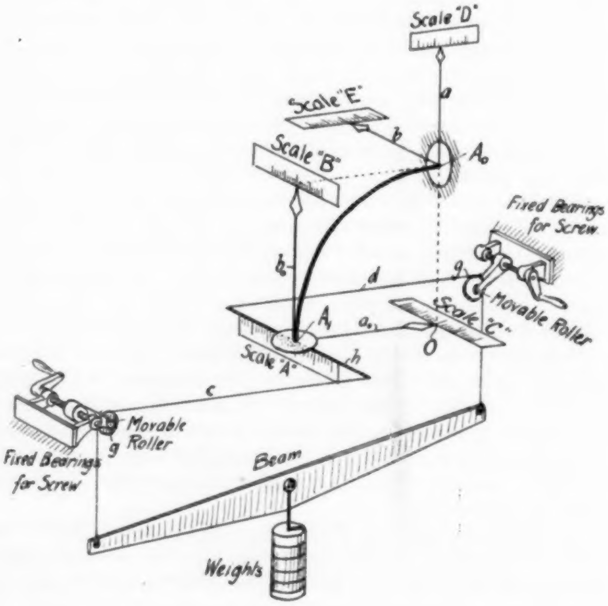

Fig. 94

It seems useless to endeavor to account for the cause of the discrepancy without having the means at hand to check the results, but it seems that the large discrepancies in Cases VIII, IX, and especially X, should have received the authors' most serious consideration with a view to finding their real cause.

With regard to the corrugated bend, there seems to have been some systematic error for the deflections under a load of 1000 lb., as practically all the test points fall widely off the curve. A careful investigation into the source of this error might have been indicated and might have revealed the source of the discrepancies.

About the test values obtained in the case of corrugated and creased bends there can be little said, as no direct comparison with theory can be effected at the present time. Attention may only be drawn to Fig. 79, in which the ordinates are given as "deflections." Should these not be given as "rotations," and if given in inches, should not the radius be stated on which these were measured?

Thus the experiments, as recorded, satisfy the first condition stated at the beginning, in that they give information about the behavior of large-size bends under working loads. They do not give information, however, with regard to the angular deviations or rotations in these cases, since the latter were apparently not determined, and thus have only a limited value with regard to satisfying the second condition, that of checking the theory.

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[20] Dipl.-Ing., New York, N. Y. Mem. A.S.M.E.

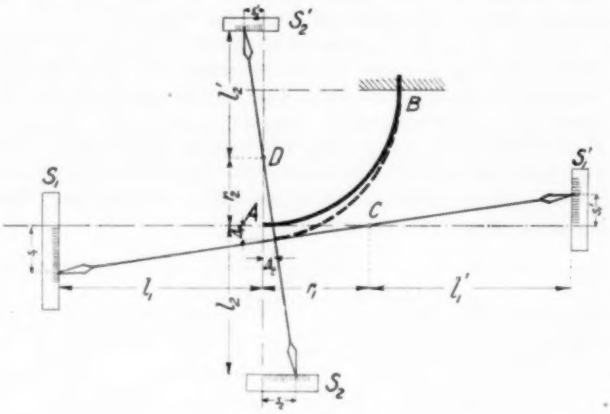


FIG. 95

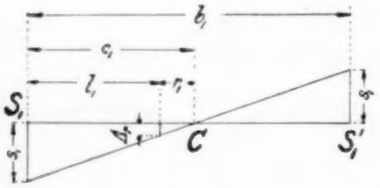


FIG. 96

It may be mentioned that the writer has made experiments with model pipes to test the theory. He has used a much more elaborate set-up than have the authors, which he was forced to adopt owing to certain errors occurring in his initial test results made with test set-ups similar to those of the authors. By gradually refining his methods he was able to get, even in the most sensitive Case X (see Fig. 94), fairly good agreement between test results and theory, the deviation in this case being less than 5 per cent, so that he may state that he feels that the formulas as given in the second edition of the "Piping Handbook" and in the writer's discussion to Messrs. Crocker and McCutchan's paper on "Frictional Resistance and Flexibility of Seamless-Tube Fittings Used in Pipe Welding," Trans. A.S.M.E., paper FSP-53-17, are fairly accurate in giving deflections and rotations, which latter the writer had included in his experiments, and may thus be used with confidence in the case of small pipes.

It is obviously impossible to give here all the results of these experiments. One example, however, may be given, as it will illustrate the way in which the difficulties of measuring exactly were overcome. This example refers to Case V, as listed in Walker and Crocker's "Piping Handbook."

All loads and measurements were taken in the metric system, and in the following description of the test and of the data regarding the pipe dimensions are thus given in kilograms and millimeters.

The pipe used was a copper tube of 6 mm. external and 5 mm. internal diameter. The pipe was annealed at a dull red heat for three hours. The modulus of elasticity was determined after the annealed tube had been allowed to rest at room temperature for one week and was found to be 11,050 kg. per sq. mm., or approximately 11,000 kg. per sq. mm., which figure was used in the calculations and is equivalent to about 15,645,000 lb. per sq. in.

The pipe was then lightly treated with acid and filled with tin solder, which was binding at the tube wall and thus formed, together with the tube, a solid rod, parts of which after cooling were bent to various diameters. The tube bend, the test results of which follow, had a radius of centerline of $R = 48$ mm. (radius at the inside of the bend 45 mm.) and consisted of a quarter bend. After the tube was bent cold to form, it was again heated in order to remove the solder, and was wiped clean of solder on the inside, so that a very fine coating only remained (which compensated for the oxide removed).

No noticeable flattening of the tube had occurred during these operations; at least no such flattening could be detected by means of a micrometer.

The tube was then provided with two steel index wires, as shown diagrammatically in Fig. 95, which extended along the tangent and the radius of the free end of the tube center line in both directions so as to allow the deviations to be read on four metric scales S_1, S_1', S_2, S_2'. The reason for this arrangement was that the deflections of the bend at the applied loads were so small that without magnification the readings would not have been accurate enough, as their order of magnitude was found to be in tenths of millimeters.

Besides, a very sharp and sensitive criterion for the accuracy of the measurements was thus obtained in the location of the points denoted by C and D and which according to the theory should have a location so that $AC = r_1 = \pi R/4$ and $AD = r_2 = \frac{1}{2} R$, where R is the radius of the tube-bend center line. Further, it was possible thereby to also determine the angular deviation of the tangent to the free end A of the bend as well as the deflections Δ_r in the direction of the radius and Δ_t in the direction of the tangent.

The set-up was as shown in Fig. 95, the pipe end B being fixed vertically downward, while the free end A of the bend was horizontal. The loads were applied by means of a knife edge at A at right-angles to the plane of the bend and by means of rings and thin but very strong silk cords in such a way that the indicator arm AS_2 was free to move between the cords, yet leaving the weights suspended always absolutely vertical below the end point A.

The length of the indicator arms was adjusted so that the distance of the scales from the points C or D was 10 times the respective distance AC or AD, or since $R = 48$ mm. and $AC = r_1 = \pi . 48/4 = 37.7$ mm., therefore $AS_1 = l_1 = 9r_1 = 339.3$ mm. and $CS_1' = l_1' = 10r_1 = 377$ mm., and likewise $AD = r_2 = \frac{1}{2} 48 = 24$ mm., thus $AS_2 = l_2 = 9r_2 = 216$ mm., and $DS_2' = l_2' = 10r_2 = 240$ mm.

If, therefore, the theory is correct, there must be $s_1' = s_1$, $s_2' = s_2$ and $\Delta_r = \dfrac{1}{10} s_1$, $\Delta_t = \dfrac{1}{10} s_2$.

If, however, it is found by observation that $s_1' \neq s_1$, $s_2' \neq s_2$, then there is, with reference to Figs. 95 and 96:

$$b_1 = l_1 + r_1 + l_1' = 754 \text{ mm.}$$

and likewise:

$$b_2 = l_2 + r_2 + l_2' = 480 \text{ mm.}$$

and if, as in Fig. 95, C' and D' are the respective points of intersection of the tangent and normal with the lines joining the ends of s_1 with s_1' and s_2 with s_2', respectively, then there is (see Fig. 96):

$$c_1 = \frac{s_1}{s_1 + s_1'} b_1$$

and

$$c_2 = \frac{s_2}{s_2 + s_2'} b_2$$

from which:

$$\Delta_r = s_1 - \frac{l_1}{b_1}(s_1 + s_1')$$

an

$$\Delta_t = s_2 - \frac{l_2}{b_2}(s_2 + s_2')$$

The values for Δ_s and Δ_t determined from these equations check well with those found from the theory, as shown from the following table:

Load	Observed						From theory	
F (kg.)	s_1 (mm.)	s_1 (mm.)	s_1' (mm.)	s_2' (mm.)	Δ_r (mm.)	Δ_t (mm.)	Δ_r (mm.)	Δ_t (mm.)
0.1.....	0.25	0.25	0.15	0.15	0.025	0.015	0.026	0.016
0.2.....	0.50	0.50	0.34	0.35	0.05	0.030	0.051	0.033
0.4.....	1.0	1.0	0.70	0.70	0.10	0.07	0.103	0.065
0.6.....	1.5	1.5	1.0	1.0	0.15	0.10	0.154	0.098
0.8.....	2.0	2.0	1.3	1.3	0.20	0.13	0.206	0.131
1.0.....	2.6	2.6	1.7	1.7	0.26	0.17	0.257	0.164
1.2.....	3.2	3.2	2.0	2.0	0.32	0.20	0.309	0.196
1.4.....	3.75	3.8	2.3	2.3	0.35	0.23	0.360	0.229
1.6.....	4.25	4.3	2.7	2.75	0.40	0.25	0.412	0.262
1.8.....	4.7	4.75	3.0	3.0	0.46	0.30	0.463	0.295

It may be seen from the table that the values agree fairly well, considering the very small quantities involved.

The bend was tested for all other Cases VI to X of the "Piping Handbook" with good agreement. Other bends were tested, too, having a different ratio of R/D, where D is the outside diameter of the pipe. Some tests were performed on solid rods, which also gave good agreement.

With regard to the larger sizes, the writer would reserve his decision until a more careful investigation has produced the cause of the deviations determined by the authors.

The writer may be allowed to mention that these experiments were carried out entirely on his own initiative and at his own expense in the interest of the profession.

There remains one point in the mathematical analysis of the so-called grapho-analytic method about which the writer would like to ask the authors. On the seventh page, in the tenth line from the bottom of the first column, the authors mention a "half-segment of a parabola." Should this not be a sinusoidal curve instead of a parabola?

Authors' Closure

Mr. Curley states that "a less reactive force is set up by a given deflection in the case of a short-radius fitting than in the case of one having a long radius." We are not able to find this conclusion stated in the Crocker-McCutchan paper (FSP-53-17). Attention is called to the results in a specific case of loading, and the reader is cautioned against sweeping conclusions. Mr. Curley's statement holds in Case V, also in Cases VIII and IX. In Case VI the reverse is true. Cases VII and X show mixed performance. Figs. 19 and 30 show the results of this kind of loading on two smooth pipe bends, one of normal radius of curvature and the other having a short radius. The dimensions of these bends are shown in Figs. 2 and 3, respectively. The values of I are as follows:

$$\text{Longer-radius bend:}\quad I = 160.7 \text{ in.}^4$$
$$\text{Short-radius bend:}\quad I = 159.9 \text{ in.}^4$$

In the tests the flange at the end of the longer tangent was bolted to the bending table. In neither case was there any allowance made for flattening of the tube during fabrication of the bend. The short-radius bend was stiffer, that is, showed less deflection for a given load than did the bend of longer radius. A further demonstration of the correctness of this conclusion is shown in Fig. 97, in which the values of the deflection in the direction of the force and perpendicular to it are shown in terms of the applied force. As the radius of the bend is decreased from seven pipe diameters the value of the deflections in both directions decreases. It is of interest to note that in the case in which the bend radius was eight pipe diameters, the bend was not as flexible as when the radius was seven pipe diameters.

In the case of the corrugated expansion U-bend, Fig. 6, to which attention is called by the discusser, the load-deflection relations of which are shown in Fig. 63, the pipe bend was intentionally loaded sufficiently to cause permanent set. This was done merely to find out at what load such set would begin. The pipe bend was actually 1 in. shorter after the loading than before it was applied. The portion of the wall in which plastic flow occurred was not determined.

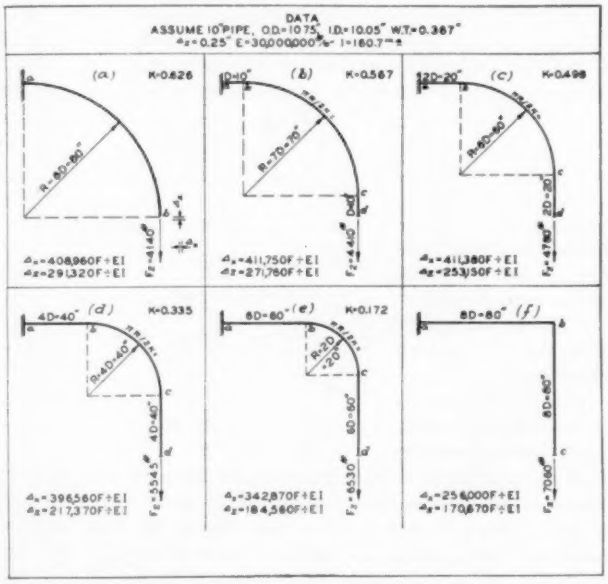


FIG. 97

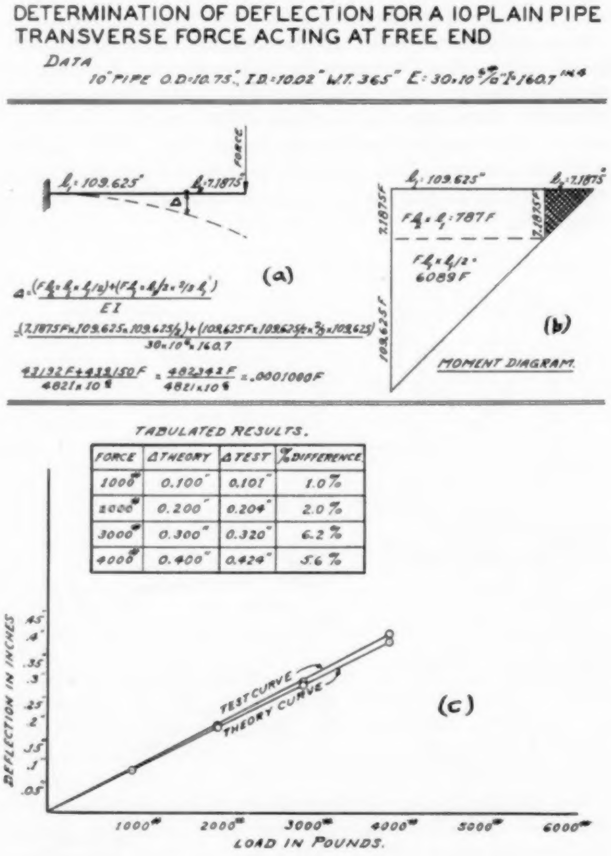
In setting down in the conclusions a table of ratios intended to show the approximate average values of the flexibility of creased and corrugated pipe bends in terms of the flexibility of smooth pipe bends of the same major dimensions, the authors intended only to show an approximate average value for each kind of pipe. The value of the ratio of flexibility in any case of loading should be taken from the load-deflection curve belonging to that case of loading. These values are obviously the results of dividing the deflections obtained by tests on creased or corrugated pipes by those obtained by calculation of an ideal smooth pipe bend of the same major dimensions.

The tests carried out by Messrs. Houser and Hirschberg are interesting, and the authors are indebted to them for contributing data obtained from tests on pipe of smaller size. The authors' experience has been confined to large sizes of pipe only. These tests represent loading in one plane and the combination of only two cases of loading.

The authors agree with Mr. Leapingwell that short-radius bends are not available generally; they are, however, procurable in the United States. There are at least two manufacturers who are prepared to furnish short-radius bends of acceptable uniformity of wall thickness. The question of the radius of smooth pipe bend to be used in the mathematical analysis for comparison of deflection of corrugated bends and smooth-pipe-bend equivalents was discussed by the authors, both between themselves and with other members of the engineering staff of their company. The opinion prevailed that for the purpose of comparison, the smooth pipe bend should be assumed to be of the same major dimensions as the corrugated bend under test. This opinion was supported by the fact (mentioned above) that short-radius bends are being made in America.

The following are values of measurements of wall thickness made on the quarter-bend portion of Fig. 3. These readings were made at intervals on the inside and the outside of the bend, and at the ends of diameters perpendicular to the plane of the bend. The values in inches of measurements made, with their position on the bend, are as follows:

In plane of bend Inside of bend		Outside of bend		Perpendicular to plane of bend			
a	0.355	*d*	0.350	*g*	0.350	*k*	0.358
b	0.350	*e*	0.361	*h*	0.353	*l*	0.342
c	0.365	*f*	0.362	*j*	0.355	*m*	0.348



FORCE	ΔTHEORY	ΔTEST	% DIFFERENCE
1000#	0.100"	0.101"	1.0%
2000#	0.200"	0.204"	2.0%
3000#	0.300"	0.320"	6.2%
4000#	0.400"	0.424"	5.6%

Fig. 98

These measurements were so made that *a*, *d*, *g*, and *k* were in the same plane perpendicular to the axis of the bend. This is also true of the readings *b*, *e*, *h*, and *l*, also *c*, *f*, *j*, and *m*. The average of these readings is 0.354 in. The maximum variation above and below the average are: above 3.1 per cent; below, 3.4 per cent.

The authors are indebted to Mr. Mills for publication of the results of tests carried out on a smooth pipe bend of the same major dimensions as creased bend No. 1, Fig. 5. The results of this one test are a confirmation of the test methods used by the authors, which were essentially the same as those used by Mr. Mills, and of the mathematical solution as applied to smooth pipe bends in this method of loading.

Up to the present time, no measurements have been made to determine whether or not permanent deformation has occurred in the walls of any of the corrugated bends. As soon as opportunity to measure one or more of these corrugated bends arrives, measurements to determine permanent deformation will be made. The rigid fastening of the fixed end of each pipe tested was a matter of as much concern to the authors during the setting up of the pipes for test as it was to Mr. Jenks in attempting to explain the divergence between the results of tests and of calculations. Each pipe tested was fitted with A.S.A. standard flanges. The nuts on the bolts which held the fixed end to the bending table were drawn up using a standard-length wrench and a sledge hammer, as is the usual practice in making up high-pressure steam-pipe joints. Fig. 98 shows the results of a previously unpublished test carried out by the authors before the pipe bends were tested. The method of loading is shown at (*a*); (*c*) shows a comparison between the results of the test and the results of calculations of load-deflection values for an ideal pipe of the same major dimensions. It is to be noted that the test results are within about 5 per cent of the calculated ones. There could scarcely have been a serious deviation from "fixed-end" conditions during this test. In all of the tests in which "fixed ends" were established the procedure was identical with that described.

The authors regret that it was not possible to obtain strain measurements on the corrugated and creased pipe bends during the tests.

Mr. de Jonge has called attention to certain features of the tests which he feels make difficult, if not impossible, a definite comparison of the deflections obtained by test with those obtained by calculation. The authors wish first to restate the fundamental purpose of the tests. This has been done in the second paragraph of the paper. They were particularly interested in comparing the behavior of pipe bends bought for use in a definite steam line and even now performing their intended function in that steam line, with the calculated performance of ideal pipe bends of the same major dimensions.

The height, diameter, etc., of the old turbine, the top of which was used as a bending table, were not measured. The turbine top was a single casting approximately 12 ft. in diameter and 4 in. thick. It was fastened to the turbine casing or shell by ninety-six $1^1/_4$-in. bolts. The whole structure was very rigid. In most cases the measurements were made between a station on the pipe and one attached to the turbine top. In such cases a possible deflection of the turbine casing could have no effect on the value of the measurement. The actual deflection of the turbine and base is impossible of exact solution because of the construction of the base. If one takes such dimensions as are available and makes a rough calculation of the probable deflection, the value obtained is so small that it could have only a negligible effect on the measurements made. The following calculation based on approximate dimensions shows the correctness of this conclusion:

Outside diameter of turbine casing....................	11 ft.
Inside diameter of turbine casing.....................	10 ft. 7 in.
Height of turbine casing above foundation............	14 ft.
Height of point of application of load above foundation of casing..	20 ft.
Value of *E* for material of turbine casing..............	15×10^6
Load applied to free end of pipe as shown in Fig. 10 was..	5000 lb.

The deflection at the top of the turbine casing may then be calculated by substituting in the usual formula for the deflection of a cantilever beam.

Substituting the values in the equation for deflection, Δ = 0.0004 in. The lower section of the turbine base was not circular, but consisted of a heavy rectangular box which constituted the shell of the condenser. The walls were approximately 2 in. thick and were tied together by cross-braces, making a complex structure of great rigidity.

Measurement at the center line of the pipe could have been carried out, but would have entailed a greater expenditure of time than the authors were able to give. The load was applied to the blind flange, which was tightly bolted to the flange on the pipe, except in the case of the pipe shown in Fig. 3, which had flanges welded to the tangents. In no case was the load applied to the pipe section. All flanges were of standard size. There were found, after a diligent search, no mathematical formulas which could be applied to this form of loading, by which deformation or flattening of the pipe could be obtained. The authors would be grateful to Mr. de Jonge if he could suggest a means of determining the amount of this deformation. If the flanges were not tightly bolted together, the load would have been carried on the top of the pipe in Case V loading. Any deformation of the pipe would have decreased the vertical diameter, thus showing less deflection than would have been shown if the pipe section were not deformed. In the various test set-ups the bolts holding the flanges together were drawn up tightly so that the Van Stone end of the pipe was tightly clamped between the flanges. The test shown in Fig. 98 shows no serious effect of flattening of the pipe.

Rotation of the reference points on the pipe in cases 8, 9, and 10 was not measured because the measuring tool, as described in the second paragraph, page 2, was in most instances as long as 5 ft. The error introduced was negligible, since the displacement of the point at which readings were made in Case IX (shown in Fig. 25) was 0.020 in. along the X-axis and 0.003 in. along the Z-axis at 5000 lb. load. The departure of the value of the reading made from the movement parallel to the Y-axis was in this case the difference between 1.000, the cosine of the angle of 0 deg., and the cosine of the angle the sine of which is 0.0202 (the combined movement along the X- and Z-axes) divided by 60. The angle whose natural sine or tangent is 0.0202 divided by 60 is 0 deg., 1 min. The natural cosine of this angle is given as 1.0000.

In Case X loading, under a torque moment of 360,000 in-lb., the displacement of the point at which readings were made was 0.014 in. along the X-axis and 0.048 in. along the Z-axis. The combined displacement was 0.050 in. The angle, the sine or tangent of which is 0.50 divided by 60, is 0 deg., 2 min., 44 sec. The natural cosine of this angle is given as 1.0000.

The effect of deflection of the turbine top on the apparent deflection of the bend is small. Calculations have been made using the equation published on page 208, Vol. 1, of the 1920 edition of "Drang and Zwang," by A. and L. Föppl. The assumptions made were:

Diameter of top	12 ft.
Thickness of top	4 in.
Load of 5000 lb. applied as in Fig. 16 on the 10-in. bend shown in Fig. 12:	
E for cast iron	10 × 10^6 lb. per sq. in.

For the sake of simplifying the problem, the turbine top was assumed to be a flat plate of uniform thickness held rigidly at the circumference. By the solution of the equation noted above the deflection of the plate at the outside circumference of the Van Stone flange was obtained. This deflection produced a rotation of the pipe bend as a whole. Transferred to the point d, Fig. 16, the downward deflection due to this rotation was equal to 0.015 in. The deflection of the pipe at d recorded as shown in Fig. 17 was 0.717 in. The error chargeable to this source would be of the order of 2 per cent.

Measurements were also made as indicated in the fourth paragraph under "Test Procedure Applied to 90-Deg. Pipe Bends," found on page 2. The arm used in making these measurements is clearly shown near the fixed end of the bends in Figs. 9, 12, and 14. The arm used was clamped to the pipe. This arm was 4 ft. $10^1/_4$ in. long from center of pipe to the reference point. The following data are taken from the original report of these tests:

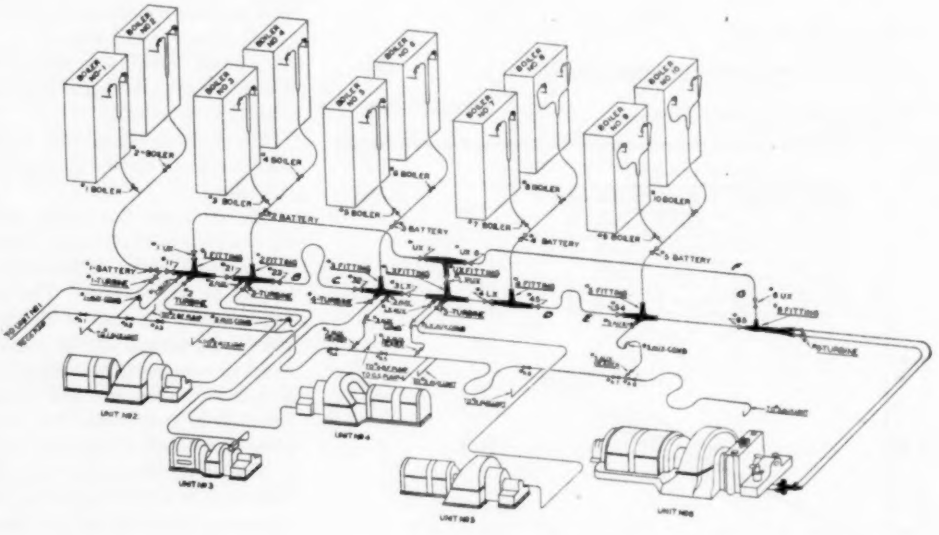


FIG. 99

Loading 10-in. pipe, Fig. 2	Load in-lb.	Deflection at 4 ft. $10^1/_4$ in. from center of pipe, in.
Case V	1000	0.003
	2000	0.010
	3000	0.017
	4000	0.023
	4975	0.030
Case VI	1000	0.012
	2000	0.021
	3000	0.031
	4000	0.043

The arm was located $6^3/_8$ in. above the surface of the bending table.

The results of the test on a smooth pipe tangent are shown in Fig. 98. This has been introduced in the answer to Mr. Jenks's discussion. The results of this test have direct bearing on the answer to Mr. de Jonge's fear that the deflection of the turbine top would vitiate the accuracy of the tests.

Torsional deformation of the pipe may be expressed either as angular displacement or as the tangential displacement of an arm of fixed and known length, when the deformation is small. In the comparison of actual and ideal pipes the length of the arm was taken the same. Perhaps it would have been better in this paper to have used angular rather than tangential displacement in plotting the load-deflection curves.

Mr. de Jonge is correct in stating that the curved boundaries of the bending moment diagrams are sinusoidal rather than parabolic curves. Since the plotted shapes of the boundaries were not used in determining the moments of areas, this error had no effect on the calculations of deflections.

The authors congratulate Mr. de Jonge on being able to produce a quarter-bend which reacted so splendidly under the various cases of loading. They are at somewhat of a loss to see just how to make application of torque loading, as shown in Fig. 94 where the torsional moment is 400,000 in-lb. or even more.

A satisfactory explanation of the apparent departure of the test results from those calculated, in Case X, has not been secured. The authors are still attempting to answer this question. Below are tabulated ratios obtained by dividing the deflection obtained by test by those obtained by calculation for seven pipe bends. Attention is particularly called to the fact that the ratio for Case X loading is uniformly approximately twice that for the cases in which the load is applied in the plane of the bend.

	Approximate ratio of test deflection to calculated deflection			
Pipe specimen	Case V	Case VI	Case VII	Case X
10-in. smooth bend, Fig. 2	1.15	1.19	1.14	2.21
10-in. tube turn with tangents, Fig. 3	1.08	1.15	1.23	2.12
14-in. corrugated bend, Fig. 14, right	2.30	2.07	2.30	4.60
14-in. corrugated bend, Fig. 14, left	2.00	1.80	1.60	3.40
14-in. creased bend, Fig. 15, No. 1	1.15	1.40	1.15	2.45
14-in. creased bend, Fig. 15, No. 2	1.18	1.35	0.82	2.60
12-in. creased bend O. D. 12.695, I. D. 11.850, Ctr. to face 7 ft., 0 min. and 5 ft., 0 min	1.36	1.91	1.46	2.70

The authors agree with Mr. Schmidt that at present the welded-pipe-bend member is not an accepted substitute for the one-piece bend in high-pressure piping. Mr. Schmidt's objection to a direct comparison of the particular creased and corrugated bends tested is a valid one. The authors recognize that the corrugations on the tangents of the corrugated bends introduce a source of extra flexibility. However, they reported the bends as they were tested. It should be again noted that no generalized conclusions were drawn. The fact that so few of each kind of bend were tested made any generalized statement unwise.

The pressure drop through the corrugated pipe bend has been the subject of considerable speculation. At the Marysville Plant, tests were conducted to measure the pressure drop between points *B* and *C*, points *D* and *E*, and points *F* and *G*, Fig. 99. The section of pipe between *B* and *C* is an expansion U-bend made of smooth pipe, 14 in. outside diameter. The section between points *D* and *E* is the corrugated expansion U-bend shown in Fig. 6. That between *F* and *G* is a corrugated 90-deg. bend similar to the bends shown in Fig. 4 except that the radius of the bend was 3 ft. 0 in. The tests were made at night when the load on the plant was small enough so that Unit No. 6 could carry it alone. The pressure drops between points *B* and *C* and between points *D* and *E* were first measured. Steam was taken from boilers No. 1 and No. 2 only, and delivered to Unit No. 6 through the steam header. The east crossover between valve No. 1UX and valve No. UX1 was cut off by closing valve No. UX1. The west crossover between valve No. UX6 and valve No. 6UX was cut off by closing valve No. 6UX. In consequence, all of the steam taken by Unit No. 6 passed through both of the expansion U-bends. Pressure drop was measured by a differential mercury manometer built for use on high-pressure work. The results of these tests are tabulated as follows:

Steam flow, 1000 lb. per hour	Steam pressure, lb. per sq. in. gage	Steam temp., deg. fahr.	Pressure drop, lb. per sq. in., Bend *B-C*, smooth pipe	Bend *D-E*, corrugated pipe
177	278	625	0.92	1.71
231	274	624	1.45	2.62
280	274	660	2.27	4.55
315	272	650	2.87	5.50
378	264	655	3.84	7.68

At the maximum steam flow the velocity of the steam was of the order of 13,700 ft. per min.

Fixed anchors are located at No. 1 fitting, No. LX fitting, and No. 6 fitting. The length of pipe between No. 1 and No. LX fittings is 85 ft. 9 in. Between No. LX fitting and No. 6 fitting there are 115 ft. 6 in. of pipe. There is, therefore, 34 per cent greater expansion to provide for at bend *D-E* than at bend *B-C*. Bend *B-C* is 45 ft. high and 25 ft. 4 in. between flanges. Bend *D-E* is 10 ft. high and 20 ft. between flanges.

The test of pressure drop through the corrugated bend between points *F* and *G* was carried out by bypassing the steam from the main header to the west crossover between No. LX fitting and No. 6 fitting. This was accomplished by opening valve No. 6UX (closed during the previous test) and closing valve No. 65 (open during the previous test). Pressure drop between points *F* and *G* was measured in the same manner as described above. The values obtained from this test are as follows:

Steam flow, 1000 lb. per hr.	Steam pressure lb. per sq. in. gage	Steam temp., deg. fahr.	Pressure drop, lb. per sq. in.
176	279	640	0.58
219	276	648	0.94
266	273	655	1.38
292	272	670	1.92
348	266	666	2.54

The velocity at maximum steam flow was approximately 12,600 ft. per min. through the corrugated bend.

In order to show the effect of the bending moment and stress in the pipe, as calculated for three combinations of bends and tangents in the simple case shown in Fig. 100, the necessary calculations were carried out. The two ends of the pipe were considered as rigidly fixed. This is the condition assumed in the analysis of any pipe line subject to thrusts and moments due to thermal expansion. The corrugated tangent was assumed to be five times as flexible as a smooth tangent of the same material and the same major dimensions. This relationship was taken from Figs. 77 and 78. The creased bends employed were assumed to be equal in flexibility to smooth bends of the same material and dimensions. If corrugated bends had been employed, they would have shown greater flexibility than the creased bends used. For a description of the method of analysis used in analyzing the moments and stresses in these three cases, the reader is referred to Chapter VII of the second edition of "Piping Handbook," by J. H. Walker and Sabin Crocker.

The pipe was assumed to have been installed when its temperature was 60 deg. fahr., and to be operated at 700 deg. fahr. The other data, pipe diameter, wall thickness, etc., are shown on Fig. 100. It is to be noted that the forces and moments acting at the right-hand end of the pipe are much reduced when a corrugated tangent is substituted for a smooth pipe. The force perpendicular to the axis is reduced most in the (*c*) combination, Fig. 100. It is, however, to be noted that the moment decreases in that section of the pipe in which the corrugated tangent has been substituted for a plain tangent. This is best illustrated by

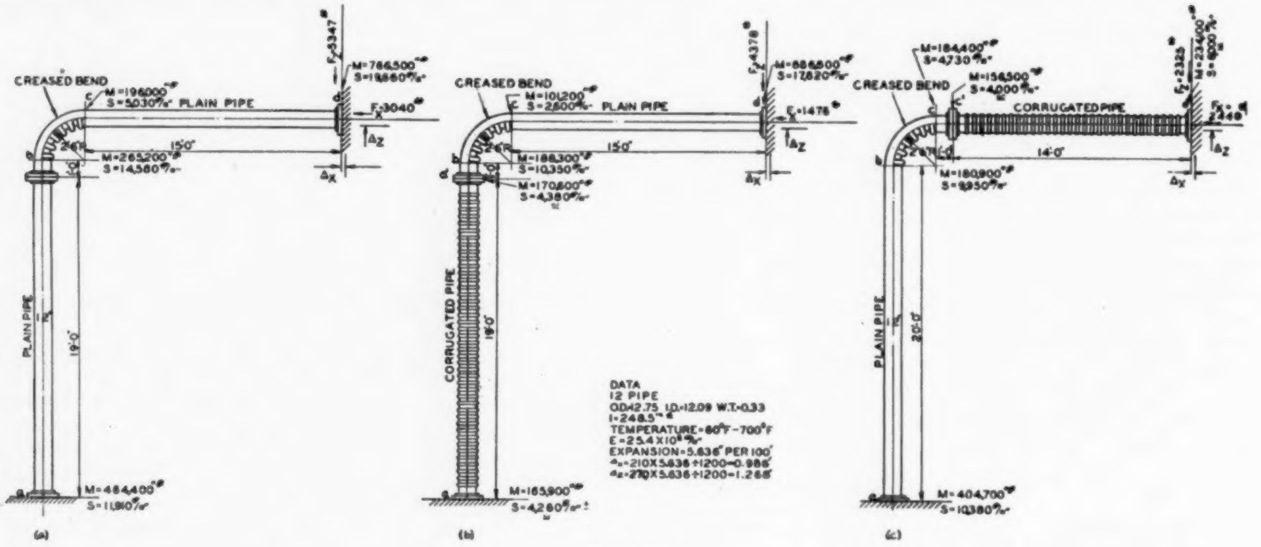

CREASED BEND
M=196000″#
S=5030#/□″
PLAIN PIPE
F_z=3347#
M=766500″#
S=19660#/□″
F_x=3040#
Δz
Δx
15′0″
2½R
M=265200″#
S=14580#/□″
PLAIN PIPE
19′0″
M=464400″#
S=11910#/□″
(a)
CREASED BEND
M=101200″#
S=2600#/□″
PLAIN PIPE
F_z=4378#
M=686800″#
S=17620#/□″
F_x=1476#
15′0″
M=188300″#
S=10350#/□″
M=170800″#
S=4380#/□″
CORRUGATED PIPE
19′0″
DATA
12 PIPE
O.D.12.75 I.D.=12.09 W.T.=0.33
I=248.5″⁴
TEMPERATURE=60°F−700°F
E=25.4×10⁶ #/□″
EXPANSION=5.636″ PER 100′
Δx=210×5.636÷1200=0.986″
Δz=270×5.636÷1200=1.268″
M=165900″#
S=4260#/□″
(b)
M=184400″#
S=4730#/□″
CREASED BEND
M=156500″#
S=4000#/□″
CORRUGATED PIPE
F_z=2325#
M=234000″#
S=6000#/□″
F_x=2440#
14′0″
M=180900″#
S=9950#/□″
PLAIN PIPE
20′0″
M=404700″#
S=10380#/□″
(c)

FIG. 100

comparing the moment at point *a* Fig. 100 (*a*) and (*b*), and point *d*, Fig. 100 (*a*) and (*c*).

At point *a*, Fig. 100 (*a*), moment = 464,400 in-lb.
At point *a*, Fig. 100 (*b*), moment = 165,900 in-lb.
At point *d*, Fig. 100 (*a*), moment = 766,500 in-lb.
At point *d*, Fig. 100 (*c*), moment = 234,100 in-lb.

There are at present no published data by which the stresses in a creased or corrugated pipe may be calculated. The authors are undertaking to analyze additional data which they have collected in the tests of other creased pipe bends and hope to be able to separate out the flexibility of the creased quarter-bend from that of a creased quarter-bend and its tangents. Considerable progress has been made to date. If the material shows promise of sufficiently consistent results on the six creased bends tested, they hope to publish the results in the near future

In Appendix No. 3 of this closure are shown the values of rotation and deflection for smooth pipe bends of 22¹/₂ deg., 30 deg., 45 deg., and 60 deg. These coefficients are based on work submitted by Mr. A. E. R. de Jonge and published in the closure of the paper on "Frictional Resistance and Flexibility of Seamless Tube Fittings Used in Pipe Welding," by Sabin Crocker and Arthur McCutchan, presented at the December, 1930, meeting of the A.S.M.E.

There is also included Appendix No. 4. This presents the analysis of loadings, both forces and moments applied to a smooth-pipe cantilever when the pipe is in an inclined position. This work was carried out by Mr. Samuel Smith of the Drafting and Surveying Bureau, The Detroit Edison Company.

The inclusion of the last appendix furnishes the piping designer with the remaining, at present unpublished, data for analyzing piping design under all ordinary conditions of direction, curvature, loading, etc.

Appendix No. 3

ARRANGEMENT IN TABULATED FORM OF THE MOMENT, ROTATION AND DEFLECTION OF 60-, 45-, 30-, AND 22¹/₂-DEG. ARCS OF BENDS[21]

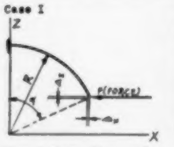
In a piping layout it is at times convenient to use arcs of bends other than 90 deg. The 60-, 45-, 30-, and 22¹/₂-deg. arcs of bends are more commonly used, and it is therefore thought advisable, so as to simplify their application in problems dealing with the flexibility of pipe, to arrange in tabulated form the moment, rotation, and deflection of these bends for the more usual cases necessary to use in determining the flexibility of a pipe line by the grapho-analytic method. The results shown in these tables are derived from the general formulas[22] for the arcs of bends acted upon in various ways.

Appendix No. 4

MATHEMATICAL FORMULAS FOR THE ROTATION AND DEFLECTION OF STRAIGHT CANTILEVER PIPES INCLINED AT AN ANGLE TO THE HORIZONTAL PLANE[23]

Straight cantilever pipes inclined at an angle to the horizontal plane are very often inserted in pipe lines, and it therefore is advisable to be able to determine their effect on the flexibility of these lines. In order to apply these diagonals in problems

Case I


Z
R
α
Δz
F(FORCE)
Δx
X

Angle	Moment at Fixed End	ψ_F Rotation	Δx-Deflection	Δz-Deflection
60°	0.500FR	$0.342FR^2/KEI$	ψ_F 0.397R	ψ_F 0.501R
45°	0.293FR	$0.152FR^2/KEI$	ψ_F 0.233R	ψ_F 0.424R
30°	0.134FR	$0.047FR^2/KEI$	ψ_F 0.106R	ψ_F 0.304R
22½°	0.076FR	$0.200FR^2/KEI$	ψ_F 0.048R	ψ_F 0.260R

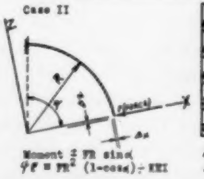
Moment = FR (1−cos α)

$\psi_F = FR^2 (\sin\alpha - \alpha\cos\alpha) \div KEI$

$\Delta x = FR^3 [\alpha(0.5 + \cos^2\alpha) - 1.5 \sin\alpha\cos\alpha] \div KEI$

$\Delta z = FR^3 (0.5 \sin^2\alpha - \alpha\sin\alpha\cos\alpha + \cos\alpha - \cos^2\alpha) \div KEI$

Case II


Z
R
α
Δz
F(FORCE)
X
Δx

Angle	Moment at Fixed end	ψ_F Rotation	Δx-Deflection	Δz-Deflection
60°	0.866FR	$0.500FR^2/KEI$	ψ_F 0.614R	ψ_F 0.250R
45°	0.707FR	$0.293FR^2/KEI$	ψ_F 0.467R	ψ_F 0.147R
30°	0.500FR	$0.134FR^2/KEI$	ψ_F 0.336R	ψ_F 0.067R
22½°	0.383FR	$0.077FR^2/KEI$	ψ_F 0.254R	ψ_F 0.040R

Moment = FR sin α

$\psi_F = FR^2 (1-\cos\alpha) \div KEI$

$\Delta x = FR^3 (0.5\alpha - 0.5 \sin\alpha\cos\alpha) \div KEI$

$\Delta z = FR^3 (1 - \cos\alpha - 0.5 \sin^2\alpha) \div KEI$

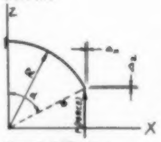
---

[21] Developed by Samuel Smith, Drafting and Surveying Bureau, The Detroit Edison Company.

[22] Formulas developed by A. E. R. de Jonge are reproduced as an appendix to the closure of the paper on "Frictional Resistance and Flexibility of Seamless-Tube Fittings Used in Pipe Welding," by Sabin Crocker and Arthur McCutchan, presented at the December, 1930, meeting of the A.S.M.E.

[23] Developed by Samuel Smith, Drafting and Surveying Bureau, The Detroit Edison Company.

Case III



Angle	Moment at Fixed End	ψ_F = Rotation	Δx-Deflection	Δz-Deflection
60°	0.866FR	$0.407FR^2/KEI$	ψ_F 0.364R	ψ_F 0.558R
45°	0.707FR	$0.262FR^2/KEI$	ψ_F 0.244R	ψ_F 0.463R
30°	0.500FR	$0.128FR^2/KEI$	ψ_F 0.117R	ψ_F 0.330R
22½°	0.383FR	$0.074FR^2/KEI$	ψ_F 0.068R	ψ_F 0.253R

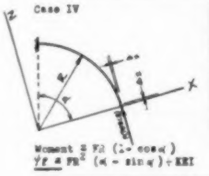
Moment = FR sin α

$\psi_F = FR^2 (\alpha \sin\alpha + \cos\alpha - 1) \div KEI$

$\Delta_x = FR^3 (1.5 \sin^2\alpha - \alpha \sin\alpha \cos\alpha + \cos\alpha - 1) \div KEI$

$\Delta_z = FR^3 [\alpha (0.5 + \sin^2\alpha) + 1.5 \sin\alpha\cos\alpha - 2 \sin\alpha] \div KEI$

Case IV



Angle	Moment at Fixed End	ψ_F = Rotation	Δ x-Deflection	Δz-Deflection
60°	0.500FR	$0.181FR^2/KEI$	ψ_F 0.691R	ψ_F 0.306R
45°	0.293FR	$0.078FR^2/KEI$	ψ_F 0.551R	ψ_F 0.179R
30°	0.134FR	$0.024FR^2/KEI$	ψ_F 0.378R	ψ_F 0.079R
22½°	0.077FR	$0.010FR^2/KEI$	ψ_F 0.304R	ψ_F 0.057R

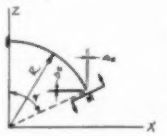
Moment = FR (1 − cos α)

$\psi_F = FR^2 (\alpha - \sin\alpha) \div KEI$

$\Delta_x = FR^3 (1 - \cos\alpha - 0.5 \sin^2\alpha) \div KEI$

$\Delta_z = FR^3 (1.5\alpha - 2 \sin\alpha + 0.5 \sin\alpha\cos\alpha) \div KEI$

Case V



Angle	ψ_m = Rotation	Δx-Deflection	Δz-Deflection
60°	1.047MR/KEI	ψ_m 0.327R	ψ_m 0.389R
45°	0.785MR/KEI	ψ_m 0.193R	ψ_m 0.334R
30°	0.524MR/KEI	ψ_m 0.068R	ψ_m 0.244R
22½°	0.393MR/KEI	ψ_m 0.051R	ψ_m 0.186R

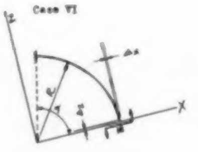
Moment is constant

$\psi_m = \alpha MR \div KEI$

$\Delta_x = MR^2 (\sin\alpha - \alpha\cos\alpha) \div KEI$

$\Delta_z = MR^2 (\alpha \sin\alpha + \cos\alpha - 1) \div KEI$

Case VI



Angle	ψ_m = Rotation	Δx - Deflection	Δz-Deflection
60°	1.047MR/KEI	ψ_m 0.477R	ψ_m 0.173R
45°	0.785MR/KEI	ψ_m 0.373R	ψ_m 0.100R
30°	0.524MR/KEI	ψ_m 0.286R	ψ_m 0.048R
22½°	0.393MR/KEI	ψ_m 0.194R	ψ_m 0.026R

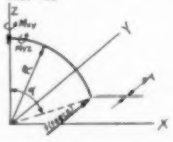
Moment is constant

$\psi_m = \alpha MR \div KEI$

$\Delta_x = MR^2 (1 - \cos\alpha) \div KEI$

$\Delta_z = MR^2 (\alpha - \sin\alpha) \div KEI$

Case VII



$M_{xy} = FR \sin\alpha$

$M_{yz} = FR (1 - \cos\alpha)$

$\lambda = 0.3$

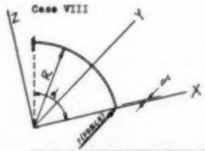
$\psi_{Fyz} = FR^2 [(1 + 0.5\lambda)(\sin\alpha - \alpha\cos\alpha)] \div EI$

$\psi_{Fxy} = FR^2 [(1 + 0.5\lambda)\alpha \sin\alpha - (1+\lambda)(1 - \cos\alpha)] \div EI$

$\Delta_y = FR^3 [(2 + 1.5\lambda)\alpha + 0.5 \sin\alpha\cos\alpha - 2(1+\lambda)\sin\alpha] \div EI$

Angle	Mxy	Myz	ψ_{Fxy}-Rotation	ψ_{Fyz}-Rotation	Δ_{Yxy}-Deflection	Δ_{Yyz}-Deflection
60°	0.866FR	0.500FR	$0.393FR^2/EI$	$0.304FR^2/EI$	ψ_{Fxy} 0.461R	ψ_{Fyz} 0.461R
45°	0.707FR	0.293FR	$0.258FR^2/EI$	$0.172FR^2/EI$	ψ_{Fxy} 0.372R	ψ_{Fyz} 0.372R
30°	0.500FR	0.134FR	$0.127FR^2/EI$	$0.064FR^2/EI$	ψ_{Fxy} 0.264R	ψ_{Fyz} 0.264R
22½°	0.383FR	0.076FR	$0.074FR^2/EI$	$0.023FR^2/EI$	ψ_{Fxy} 0.210R	ψ_{Fyz} 0.210R

Case VIII



$M_{xy} = FR \sin\alpha$

$M_{yz} = FR (1 - \cos\alpha)$

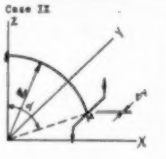
$\psi_{Fxy} = FR^2 [(1 + 0.5\lambda)\alpha - (1+\lambda)\sin\alpha + 0.5\lambda(\sin\alpha\cos\alpha)] \div EI$

$\psi_{Fyz} = FR^2 [(1+\lambda)(1 - \cos\alpha) - 0.5\lambda \sin^2\alpha] \div EI$

$\Delta_y = FR^3 [(2 + 1.5\lambda)\alpha + 0.5 \sin\alpha\cos\alpha - 2(1+\lambda)\sin\alpha] \div EI$

Angle	Mxy	Myz	ψ_{Fxy}-Rotation	ψ_{Fyz}-Rotation	Δ_{Yxy}-Deflection	Δ_{Yyz}-Deflection
60°	0.866FR	0.500FR	$0.143FR^2/EI$	$0.537FR^2/EI$	ψ_{Fxy} 0.557R	ψ_{Fyz} 0.557R
45°	0.707FR	0.293FR	$0.056FR^2/EI$	$0.306FR^2/EI$	ψ_{Fxy} 0.442R	ψ_{Fyz} 0.442R
30°	0.500FR	0.134FR	$0.017FR^2/EI$	$0.157FR^2/EI$	ψ_{Fxy} 0.310R	ψ_{Fyz} 0.310R
22½°	0.383FR	0.077FR	$0.009FR^2/EI$	$0.077FR^2/EI$	ψ_{Fxy} 0.236R	ψ_{Fyz} 0.236R

Case IX



Angle	ψ_{Mxy}-Rotation	ψ_{Myz}-Rotation	Δ_{Mxy}-Deflect.	Δ_{Myz}-Deflect.
60°	−0.113MR/EI	1.269MR/EI	ψ_{Mxy} 0.341R	ψ_{Myz} 0.341R
45°	−0.076MR/EI	0.978MR/EI	ψ_{Mxy} 0.193R	ψ_{Myz} 0.193R
30°	−0.038MR/EI	0.667MR/EI	ψ_{Mxy} 0.068R	ψ_{Myz} 0.068R
22½°	−0.022MR/EI	0.505MR/EI	ψ_{Mxy} 0.047R	ψ_{Myz} 0.047R

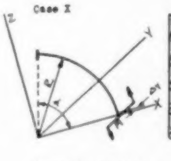
Moment is constant

$\psi_{Mxy} = -MR\,\lambda \sin^2\alpha \div 2EI$

$\psi_{Myz} = MR [(1 + 0.5\lambda)\alpha + 0.5\lambda \sin\alpha\cos\alpha] \div EI$

$\Delta_y = MR^2 [(1 + 0.5\lambda)(\sin\alpha - \alpha\cos\alpha)] \div EI$

Case X



Angle	$-\psi_{Mxy}$-Rotation	ψ_{Myz}-Rotation	Δ_{Mxy}-Deflect.	Δ_{Myz}-Deflection
60°	−0.113MR/EI	1.236MR/EI	ψ_{Mxy} 0.477R	ψ_{Myz} 0.477R
45°	−0.078MR/EI	0.826MR/EI	ψ_{Mxy} 0.406R	ψ_{Myz} 0.406R
30°	−0.038MR/EI	0.537MR/EI	ψ_{Mxy} 0.274R	ψ_{Myz} 0.274R
22½°	−0.022MR/EI	0.399MR/EI	ψ_{Mxy} 0.204R	ψ_{Myz} 0.204R

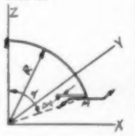
Moment is constant

$\psi_{Mxy} = -MR(0.5\lambda \sin^2\alpha) \div EI$

$\psi_{Myz} = MR [(1 + 0.5\lambda)\alpha - 0.5\lambda \sin\alpha\cos\alpha] \div EI$

$\Delta_y = MR^2 [(1+\lambda)(1 - \cos\alpha) - 0.5\lambda \sin^2\alpha] \div EI$

Case XI



Angle	ψ_{Mxy}-Rotation	ψ_{Myz}-Rotation	Δ_{Mxy}-Deflection	Δ_{Myz}-Deflection
60°	1.139MR/EI	−0.113MR/EI	ψ_{Mxy} 0.383R	ψ_{Myz} 0.383R
45°	0.828MR/EI	−0.076MR/EI	ψ_{Mxy} 0.342R	ψ_{Myz} 0.342R
30°	0.537MR/EI	−0.038MR/EI	ψ_{Mxy} 0.254R	ψ_{Myz} 0.254R
22½°	0.398MR/EI	−0.022MR/EI	ψ_{Mxy} 0.196R	ψ_{Myz} 0.196R

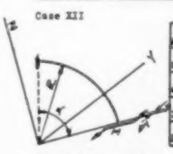
Moment is constant

$\psi_{Mxy} = MR [(1 + 0.5\lambda)\alpha - 0.5\lambda \sin\alpha\cos\alpha] \div EI$

$\psi_{Myz} = -MR(\lambda \sin^2\alpha) \div 2EI$

$\Delta_Y = MR^2 [(1 + 0.5\lambda)\alpha \sin\alpha - (1+\lambda)(1 - \cos\alpha)] \div EI$

Case XII



Angle	ψ_{Mxy}-Rotation	ψ_{Myz}-Rotation	Δ_{Mxy}-Deflection	Δ_{Myz}-Deflection
60°	1.269MR/EI	−0.113MR/EI	ψ_{Mxy} 0.124R	ψ_{Myz} 0.124R
45°	0.978MR/EI	−0.076MR/EI	ψ_{Mxy} 0.068R	ψ_{Myz} 0.068R
30°	0.667MR/EI	−0.037MR/EI	ψ_{Mxy} 0.027R	ψ_{Myz} 0.027R
22½°	0.505MR/EI	−0.027MR/EI	ψ_{Mxy} 0.015R	ψ_{Myz} 0.015R

Moment is constant

$\psi_{Mxy} = MR [(1 + 0.5\lambda)\alpha + 0.5\lambda \sin\alpha\cos\alpha] \div EI$

$\psi_{Myz} = -MR(\lambda \sin^2\alpha) \div 2EI$

$\Delta_y = [(1 + 0.5\lambda)(\alpha - \sin\alpha) - 0.5\lambda \sin\alpha(1 - \cos\alpha)] \div EI$

dealing with the flexibility of pipe it is necessary to know the rotation and deflection of these diagonals when acted upon at the free end either by a force or moment. Formulas for these are available if the force or moment acting at the free end is in the plane of the diagonal, but the authors are unaware of any if the force or moment is not in the plane of the diagonal. Therefore this appendix is confined only to developing formulas for the rotation and deflection of straight cantilever diagonals when acted upon at the free end either by a force or moment not in the plane of the diagonal. These formulas can readily be applied to the grapho-analytic method for determining the flexibility of a pipe layout.

CASE A—Straight cantilever inclined at an angle to horizontal plane and acted upon at the free end by a moment in vertical plane perpendicular to plane of pipe. (Fig. 101.)

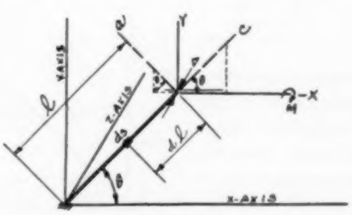


FIG. 101

$$Ma = M \sin\theta$$

$$Mc = M \cos\theta$$

ψx_b = rotation along x-axis due to bending

ψx_t = rotation along x-axis due to twist

ψ_w = rotation along x-axis due to combined bending and twist

$$EI d\psi a_b = Mads = M \sin\theta\, ds$$

$$d\psi x_b = d\psi a_b \sin\theta$$

$$EI d\psi x_b = (M \sin\theta\, ds)\sin\theta = M \sin^2\theta\, ds$$

$$\psi x_b = M\frac{\sin^2\theta}{EI}\int_0^l ds = \frac{Ml \sin^2\theta}{EI}$$

$$GJ d\psi c_t = Mcds = M\cos\theta\, ds$$

$$d\psi x_t = d\psi c_t \cos\theta$$

$$GJ d\psi x_t = (M\cos\theta\, ds)\cos\theta = M\cos^2\theta\, ds$$

$$\psi x_t = \frac{M\cos^2\theta}{GJ}\int_0^l ds = \frac{Ml\cos^2\theta}{GJ} = \frac{1.3\, Ml\cos^2\theta}{EI}$$

$$\psi x = \psi x_b + \psi x_t$$

$$\psi x = \frac{Ml\sin^2\theta}{EI} + \frac{1.3\, Ml\cos^2\theta}{EI}$$

$$= \frac{Ml}{EI}(\sin^2\theta + 1.3\cos^2\theta)$$

$$= \frac{Ml}{EI}\left[\tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta + 1.3(\tfrac{1}{2} + \cos 2\theta)\right]$$

$$= \frac{Ml}{EI}\left(\tfrac{1}{2} - \frac{\cos 2\theta}{2} + 0.65 + 0.65\cos 2\theta\right)$$

$$= \underline{\underline{\frac{Ml}{EI}\left(1.15 + 0.15\cos 2\theta\right)}}$$

$$d\Delta = d\psi a_b dl = M\sin\theta\, dsdl$$

$$\Delta = \int_0^l\int_0^l M\sin\theta\, dsdl$$

$$= \int_0^l M\sin\theta\,(s)_0^l\, dx = \frac{Ml\sin\theta\, dl}{EI} = \frac{Ml\sin\theta}{EI}\left(\frac{l^2}{2}\right)_0^l$$

$$= \frac{Ml^2}{2EI}\sin\theta$$

Deflection due to $d\psi c_t = 0$

$$\Delta = \frac{Ml^2}{2EI}\sin\theta$$

$$\Delta = \frac{Ml^2\sin\theta}{2EI} = \frac{Ml(1.15 + 0.15\cos 2\theta)}{EI}x$$

$$x = \frac{l\sin\theta}{2.3 + 0.3\cos 2\theta}$$

$$\therefore\ \Delta = \underline{\underline{\psi x\,\frac{2.3 + 0.3\cos 2\theta}{l\sin\theta}}}$$

Case B—Straight cantilever inclined at an angle to horizontal plane and acted upon at the free end by a moment in horizontal plane. (Fig. 102.)

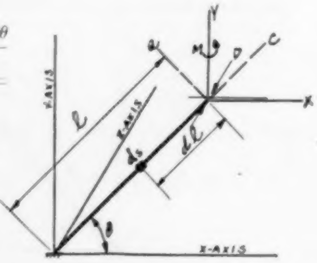


Fig. 102

$$Ma = M\cos\theta$$

$$Mc = M\sin\theta$$

ψy_b = rotation along y-axis due to bending
ψy_t = rotation along y-axis due to twist
ψy = rotation along y-axis due to combined bending and twist

$$EI d\psi a_b = Mads = M\cos\theta\, ds$$

$$d\psi y_b = d\psi a_b \cos\theta$$

$$EI d\psi y_b = M\cos\theta\, ds\cos\theta$$

$$\psi y_b = \int_0^l \frac{M\cos^2\theta\, ds}{EI} = \frac{Ml\cos^2\theta}{EI}$$

$$GJ d\psi c_t = Mcds = M\sin\theta\, ds$$

$$d\psi y_t = d\psi c_t \sin\theta$$

$$\frac{EI}{1.3}d\psi y_t = M\sin^2\theta\, ds$$

$$\psi y_t = \frac{1.3\, M\sin^2\theta}{EI}\int_0^l ds = \frac{1.3\, Ml\sin^2\theta}{EI}$$

$$\psi y = \psi y_b + \psi y_t = \frac{Ml\cos^2\theta}{EI} + \frac{1.3\, Ml\sin^2\theta}{EI}$$

$$= \frac{Ml}{EI}(\cos^2\theta + 1.3\sin^2\theta)$$

$$= \frac{Ml}{EI}\left(\tfrac{1}{2} + \tfrac{1}{2}\cos^2\theta + \frac{1.3}{2} - \frac{1.3\cos^2\theta}{2}\right)$$

$$= \underline{\underline{\frac{Ml}{EI}(1.15 - 0.15\cos 2\theta)}}$$

$$d\Delta = d\psi a_b dl = \frac{M\cos\theta\, ds\, dl}{EI}$$

$$\Delta = \int_0^l\int_0^l \frac{M\cos\theta\, dsdl}{EI} = \int_0^l \frac{M\cos\theta\,(s)_0^l\, dl}{EI}$$

$$= \int_0^l \frac{Ml\cos\theta\, dl}{EI} = \frac{Ml^2\cos\theta}{2EI}$$

$$= \frac{Ml^2\cos\theta}{2\,EI} = \frac{Ml(1.15 - 0.15\cos 2\theta)}{EI}x$$

$$x = \frac{l\cos\theta}{2.3 - 0.3\cos 2\theta}$$

$$\therefore\ \Delta = \underline{\underline{\psi_y\,\frac{l\cos\theta}{2.3 - 0.3\cos 2\theta}}}$$

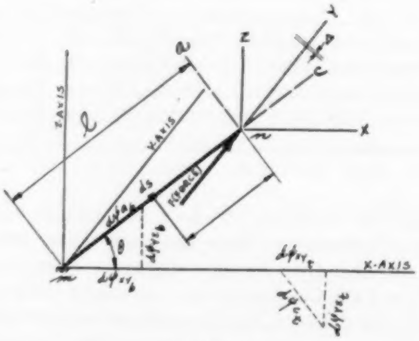


Fig. 103

Case C—Straight cantilever inclined at an angle to horizontal plane and acted upon at free end by a force perpendicular to the plane of pipe. (Fig. 103.)

$$Mxy = Fdl\cos\theta$$

$$Myz = Fdl\sin\theta$$

$$\psi xy = \psi xy_b + \psi xy_t$$

$$\psi yz = \psi yz_b + \psi yz_t$$

To determine ψxy (rotation in xy-plane)

$$Mxy = Fdl \cos\theta$$

$$Ma = Mxy \cos\theta \qquad Mc = Mxy \sin\theta$$

$$EId\psi a_b = Mads = Mxy \cos\theta\, ds = Fdl \cos^2\theta\, ds$$

$$d\psi xy_b = d\psi a_b \cos\theta$$

$$EIdxy_b = Fdl \cos^2\theta\, ds \cos\theta = Fdlds \cos^3\theta$$

$$\psi xy_b = \frac{E \cos^3\theta}{EI} \int_0^l \int_0^l dsdl = \frac{Fl^2 \cos^3\theta}{2EI}$$

$$GJd\psi c_t = Mcds = Mxy \sin\theta\, ds = Fdl \cos\theta \sin\theta\, ds$$

$$d\psi xy_t = d\psi c_t \sin\theta$$

$$GJ = EI/1.3$$

$$d\psi xy_t = \frac{1.3\, Fdl \cos\theta \sin\theta\, ds \sin\theta}{EI}$$

$$\psi xy_t = \int_0^l \int_0^l \frac{1.3\, F \sin^2\theta \cos\theta\, dsdl}{EI} = \frac{1.3\, fl^2 \sin^2\theta \cos\theta}{2\,EI}$$

$$\psi xy = \psi xy_b + \psi xy_t$$

$$\psi xy = \frac{Fl^2 \cos^3\theta}{2\,EI} + \frac{1.3\, Fl^2 \sin^2\theta \cos\theta}{2\,EI}$$

$$= \frac{Fl^2 \cos\theta}{2\,EI} (\cos^2\theta + 1.3 \sin^2\theta)$$

$$= \frac{Fl^2 \cos\theta}{2\,EI} \left[\tfrac{1}{2} + \tfrac{1}{2} \cos 2\theta + 1.3\, (\tfrac{1}{2} - \tfrac{1}{2} \cos 2\theta) \right]$$

$$\underline{\underline{\psi xy = \frac{Fl^2 \cos\theta}{2\,EI} (1.15 - 0.15 \cos 2\theta)}}$$

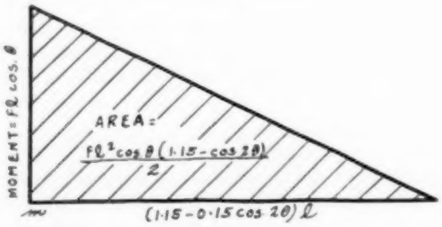


FIG. 104 MOMENT DIAGRAM. FORCE ACTING IN xy-PLANE

To determine ψyz (rotation in yz-plane)

$$Myz = Fdl \sin\theta$$

$$Ma = Myz \sin\theta \qquad Mc = Myz \cos\theta$$

$$EId\psi a_b = Mads = Myz \sin\theta\, ds = Fdl \sin^2\theta\, ds$$

$$d\psi yz_b = d\psi a_b \sin\theta$$

$$d\psi yz_b = \frac{Fdl \sin^2\theta\, ds \sin\theta}{EI} = \frac{F \sin^3\theta\, dsdl}{EI}$$

$$\psi yz_b = \frac{F \sin^3\theta}{EI} \int_0^l \int_0^l dsdl = \frac{Fl^2 \sin^3\theta}{2\,EI}$$

$$GJd\psi c_t = Mcds = Myz \cos\theta\, ds = Fdl \sin\theta \cos\theta\, ds$$

$$d\psi yz_t = d\psi c_t \cos\theta$$

$$GJD\psi yz_t = Fdl \sin\theta \cos\theta\, ds \cos\theta$$

$$d\psi yz_t = \frac{Fdl \sin\theta \cos^2\theta\, ds}{GJ} = \frac{1.3\, F \sin\theta \cos^2\theta\, dlds}{EI}$$

$$\psi yz_t = \frac{1.3\, F \sin\theta \cos^2\theta}{EI} \int_0^l \int_0^l dsdl$$

$$\therefore \psi yz_t = \frac{1.3\, Fl^2 \sin\theta \cos^2\theta}{2\,EI}$$

$$\psi yz = \psi yz_b + \psi yz_t$$

$$\psi yz = \frac{Fl^2 \sin^3\theta}{2\,EI} + \frac{1.3\, Fl^2 \sin\theta \cos^2\theta}{2\,EI}$$

$$= \frac{Fl^2 \sin\theta}{2\,EI} (\sin^2\theta + 1.3 \cos^2\theta)$$

$$= \frac{Fl^2 \sin\theta}{2\,EI} \left[\tfrac{1}{2} - \tfrac{1}{2} \cos 2\theta + 1.3\, (\tfrac{1}{2} + \tfrac{1}{2} \cos 2\theta) \right]$$

$$\underline{\underline{\psi yz = \frac{Fl^2 \sin\theta}{2\,EI} (1.15 + 0.15 \cos 2\theta)}}$$

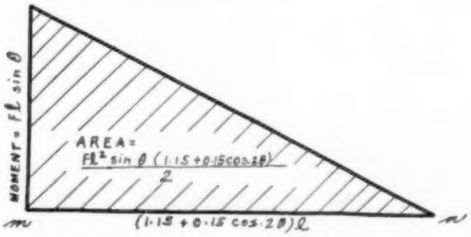


FIG. 105 MOMENT DIAGRAM. FORCE ACTING IN yz-PLANE

To determine deflection (Δ):

$$d\Delta = d\Delta xy + d\Delta yz$$

$$d\Delta xy = d\psi a_b dl + d\psi c_t(0)$$

$$= \frac{Fdl \cos^2\theta\, dsdl + 0}{EI}$$

$$d\Delta yz = d\psi a_b dl + d\psi c_t = \frac{Fdl \sin^2\theta\, dsdl + 0}{EI}$$

$$d\Delta = \frac{Fdl \cos^2\theta\, dsdl + Fdl \sin^2\theta\, dsdl}{EI}$$

$$= \frac{Fdldsdl(\cos^2\theta + \sin^2\theta)}{EI} = \frac{Fdldsdl}{EI}$$

$$\delta = F \int_0^l \int_0^l \int_0^l dldsdl = F \int_0^l \int_0^l dl(s)^l{}_0 dl$$

$$= F \int_0^l \int^l ldldl = F \int_0^l \frac{1^2}{2}\, dl = \frac{Fl^3}{3EI}$$

$$\Delta = \Delta xy + \Delta yz = \frac{Fl^3}{3EI}$$

$$\frac{Fl^3}{3EI} = (\psi xy + \psi yz) =$$

$$\left[\frac{Fl^2 \cos\theta}{2\,EI} (1.15 - 0.15 \cos 2\theta) + \frac{Fl^2 \sin\theta}{2\,EI} (1.15 + 0.15 \cos 2\theta) \right] x$$

$$x = \frac{2}{3} l \left[\frac{1}{\cos\theta(1.15 - 0.15 \cos 2\theta) + \sin\theta(1.15 + 0.15 \cos 2\theta)} \right]$$

$$\therefore\ \Delta xy = \underline{\underline{\psi xy \frac{2}{3} l \left[\frac{1}{\cos\theta(1.15 - 0.15 \cos 2\theta) + \sin\theta(1.15 + 0.15 \cos 2\theta)} \right]}}$$

and

$$\Delta yz = \underline{\underline{\psi yz \frac{2}{3} l \left[\frac{1}{\cos\theta(1.15 - 0.15 \cos 2\theta) + \sin\theta(1.15 + 0.15 \cos 2\theta)} \right]}}$$

High-Pressure Boiler and Turbine Operation at Northeast Station

By J. A. Keeth,[1] Kansas City, Mo.

This paper describes the installation and major problems that came up during $2^1/_2$ years' operation of the high-pressure equipment at the Northeast Station of the Kansas City Power & Light Company. Reheater equipment is discussed, together with the experimental work involved in raising the steam temperature to the desired point, and incidentally obtaining a type of construction that would withstand the high furnace temperatures and give reasonable life. The effect of the reheater troubles on the use obtained from the equipment is also covered.

A general operating history of the turbine is given, dealing in considerable detail with the turbine-blade deposits. Discussion of this phase of the troubles brings out the various changes that have been made in boiler feedwater in an effort to find a relationship between boiler-water concentration and turbine-scale formation. The method used for turbine-scale removal is given.

Only minor troubles were experienced with boilers, high-pressure superheater, and auxiliaries. The effect of continued operation on boiler efficiency has been covered graphically, and the operating performance of the high-pressure equipment as well as the whole station for a typical month has been given showing the heat consumption of the two pressure turbines, the load distribution, and the net result for the station. Considerable detail is given concerning the utility factor of the entire installation and the effect the various individual parts have had on this factor.

In a paper of this nature, it is necessary to discuss the faults more in detail than those parts which have functioned perfectly from the start. It is safe to assume that the Northeast high-pressure installation has been no different from others in that some one part or parts have given more or less trouble. It must be remembered that, when this installation was made, equipment of the nature needed was not entirely past the experimental stage. The manufacturers offered equipment the fundamental design of which was based on experience obtained on other installations of lower pressure where they believed operating conditions were similar. In the installation described, present experience has shown that, due to one cause or another, conditions obtained on this job were not entirely what was expected by the manufacturer, and some of the troubles covered in this paper have been the result. Where troubles of this nature occur in what might be considered pioneering installations, criticisms of manufacturers or designing engineers should not be of a destructive nature.

While this paper must of necessity stress certain parts of the equipment more than others, it is not the intent or desire of the author to cast reflections on any manufacturer or his product.

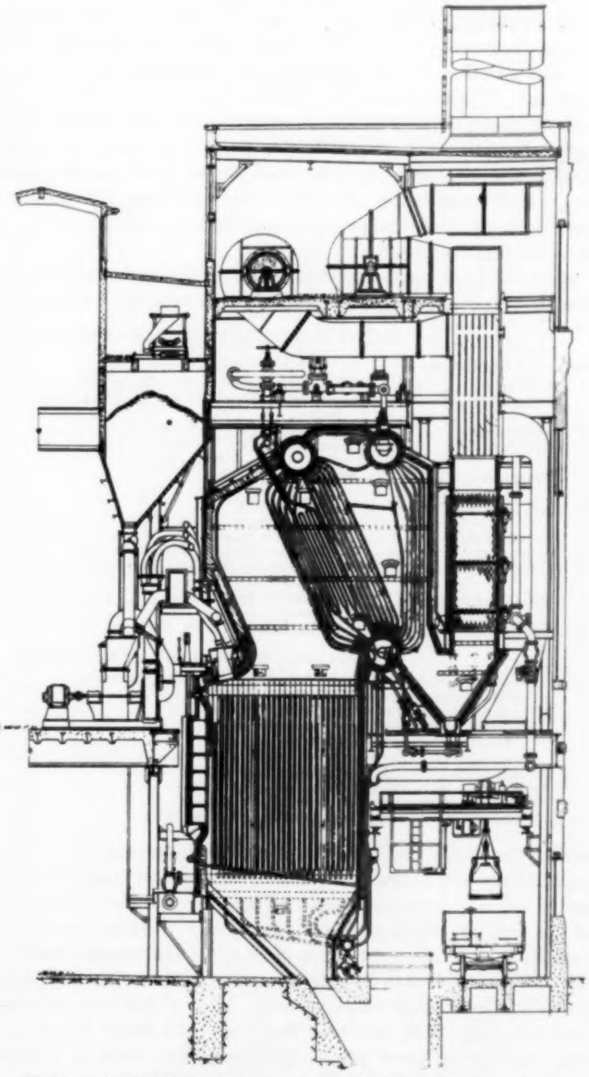


Fig. 1 Cross-Section Through 1400-Lb. Boiler Unit

While errors have occurred, they were in most instances from causes which were beyond both the manufacturers' and the engineers' scope of knowledge or control at the time the design was made. The most whole-hearted cooperation has been experienced in every case between the manufacturers and the purchaser's engineers. The solving of the problems has undoubtedly broadened the industry knowledge, and it is through the working

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Presented at the Kansas City Meeting, Kansas City, Mo., Sept. 7 to 9, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

out of difficulties of this nature that the art of engineering advances.

Many descriptions have appeared in various engineering publications of high-pressure boiler and turbine installations made in this country. Among these have been some brief comments on the high-pressure equipment at the Northeast Station of the Kansas City Power & Light Company. Most of these articles have been written from the design and construction point of view and have touched on the predicted performance only as it was necessary to justify the investment. While many of the high-pressure units have been in operation for several years, few figures covering the actual performance of the equipment have appeared. It can be safely assumed that the installations have been successful, otherwise the use of the higher pressures would not have come into such wide use.

In February, 1929, a paper was presented at the Midwest Power Conference in Chicago which showed the economic basis for the selection of the high-pressure equipment at Northeast Station.[1] The boilers and turbine had been put in service only a short time previous to the presentation of that paper, hence any comments made therein had to be in the nature of predictions. More than two years have now elapsed and it should, therefore, be of considerable interest to see how near the estimates have been met in actual practice, and also to know something of the operating experiences.

Since the equipment was first put in service in January, 1929, a considerable amount of experimental work with many changes resulting therefrom, had to be done on the radiant reheaters. These experiments and changes extended over a period of nearly a year and a half and were the primary cause for the low utility factor of the equipment as a whole. It usually required one to two months to determine the effect of many of the changes made, as they frequently had to do with the ability of materials to stand up under continued high temperatures. Most of this work was completed in July, 1930, and from that time until about February of this year, operation of the equipment settled down to a regular routine which, it was believed, would be indicative of operation during the normal life of the equipment. However, in February some trouble began to develop which showed that certain parts of the reheaters would not stand up against the temperatures obtaining and give a reasonable life. Further changes were decided upon, the materials for which could not be obtained until June, thus leaving the answer for its entire success to some date in the future.

MAJOR EQUIPMENT OF NORTHEAST STATION

No.	Equipment	Manufacturer and Type	Size
2	Boilers	Combustion Engr. Corp., Type VX	16,950 sq. ft.
	Water walls	Combustion Engr. Corp., Bare and finned tube	3,050 sq. ft. per boiler
2	Superheaters	Superheater Co., Convection	2,650 sq. ft. per boiler
2	Reheaters	Superheater Co., Shielded Radiant	
6	Pulverizers	Raymond Imp. No. 90	15,000 lb. per hr. each, 3 to each boiler
36	Coal burners	Lopulco	9—8 in. to each boiler 9—4 in. to each boiler
2	Economizers	Combustion Engr. Corp., Finned tube	10,891 sq. ft. each
2	Air heaters	Combustion Engr. Corp., Plate type	39,270 sq. ft. each
2	Boiler-feed pumps	A. S. Cameron, 6-stage centrifugal, motor driven	900 g.p.m.
1	Boiler-feed pump	DeLaval Steam Turbine Co., motor driven	900 g.p.m.
1	Turbo-generator	Westinghouse E. & M. Co.	12,500 kva.

GENERAL OPERATING CONDITIONS

Maximum pressure for which boiler is designed, lb. gage	1,400
Normal operating drum pressure, lb. gage	1,350
Maximum evaporation per boiler, lb. per hr	225,000
Normal evaporation per boiler, lb. per hr	185,000
Pressure at turbine throttle, lb. gage	1,250
Temperature at turbine throttle, deg. fahr	725
Back pressure at turbine exhaust, lb. gage	315
Superheat in exhaust at full load, deg. fahr	22
Pressure of steam out of reheater, lb. gage	290
Temperature of steam out of reheater, deg. fahr	700

Equipment and Operating Conditions

The equipment consists of two 1700-hp., 1400-lb. boilers and one 10,000-kw., 1200-lb. turbine. The boilers are arranged for pulverized-coal firing, using the unit system. Furnaces are completely water walled, with radiant reheaters, originally shielded by water-wall tubes, located in the side walls. The superheater is of the convection type, located in the first pass of the boiler. Fig. 1 shows a cross-section of the boiler unit.

Reheaters

The early months of the operation of this installation involved frequent interruptions. However, after the first few weeks practically all of the outages were due to reheaters. It was apparent from the start that the reheaters had insufficient surface, under the furnace conditions that prevailed, to raise the temperature of the exhaust steam to a point even approximating that desired. The many changes made in the reheaters and burners, and also air adjust-

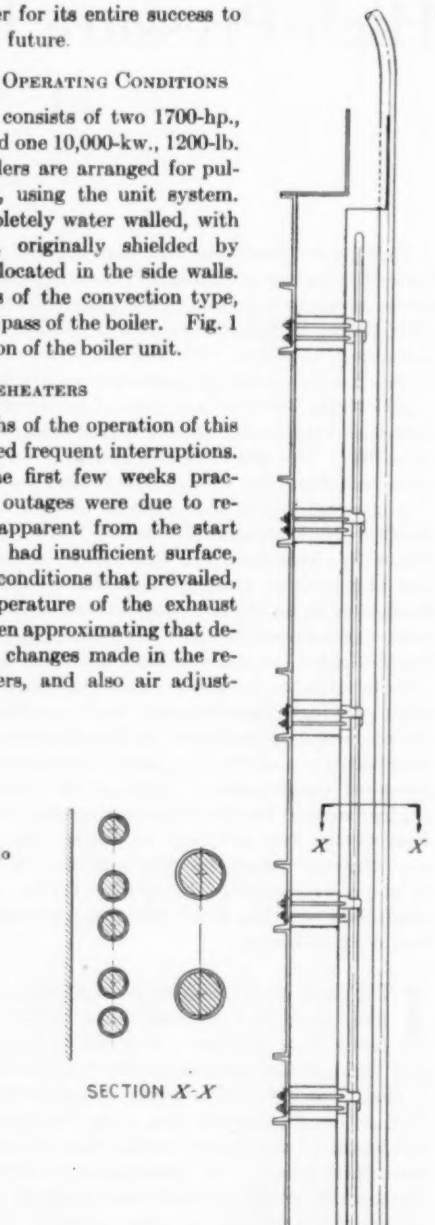

Fig. 2 Original Reheater Arrangement

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[1] "High-Pressure Turbine Installation of Kansas City Power & Light Company," by Edwin Jowett, Trans. A.S.M.E., vol. 51, no. 1.

ments, involved much "cutting and trying." This resulted in one or the other of the boilers being out of service almost continually until the changes completed in July, 1930, had been made. Many of the changes made on these reheaters required anywhere from a week to six weeks to complete and required also the manufacture and shipment of new tubes and other miscellaneous parts, all of which involved the customary delays in obtaining material from the mill and in shipment. After the July change was made, very reliable service was obtained from the equipment until February, 1931. At that time, leaks began to develop in the reheater tubes which, examination showed, were due to overheating of the tube walls. It was possible, however, to continue operation with occasional shutdowns until June, 1931, after which a further change was made in tube materials with the view of obtaining better life from the reheaters.

While the problem of increasing the reheat temperature would appear to involve nothing more than an increase in surface, this very solution became particularly difficult to accomplish due to lack of space and excessive temperature resulting from the scrubbing action of the flame where space could be found available. Fig. 2 shows the original reheater arrangement in each side wall of each boiler. The reheater was designed for a rise of 250 deg., whereas only 125 deg. was obtained. The lack of temperature rise was due, at least in part, to the fact that the path of flame travel in the furnace exposed only a portion of the reheater surface to radiant heat. When changes were made in the firing conditions to eliminate this trouble, large quantities of slag accumulated on the reheater and water-wall tubes which offset any gain made. Various arrangements of slag blowers were tried without effective results.

The next effort to increase reheat temperature was made by removing every other water-wall tube, thus reducing the shielding effect on the reheater tubes. While this change had some effect, it was far from adequate to meet the needs. Additional reheater elements were next attached to the furnace side of the water-wall tubes as shown in Fig. 3. These gave practically the desired temperature, but the life

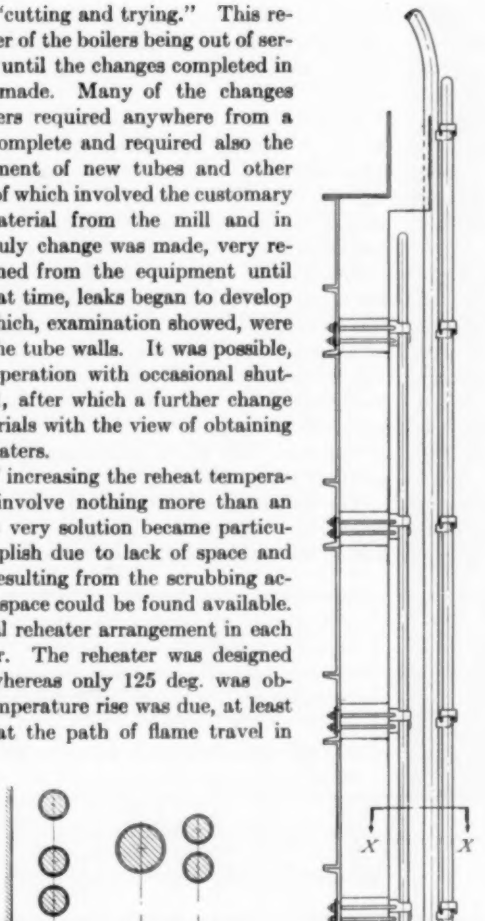

Fig. 3 First Major Change in Reheater

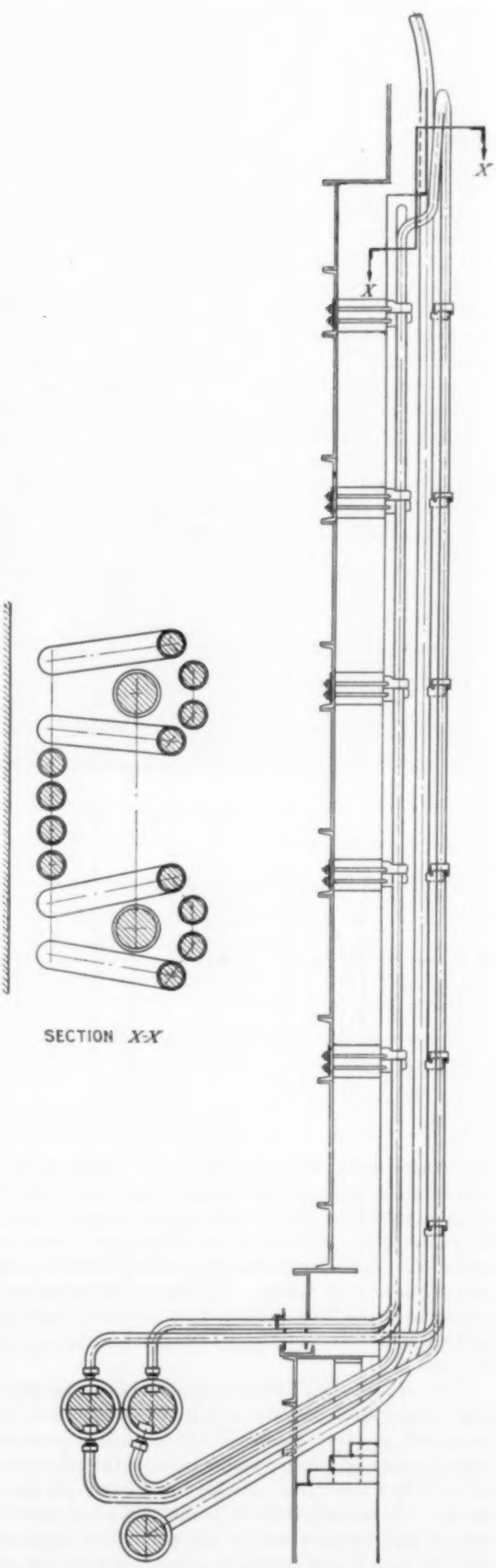

Fig. 4 Second Major Change in Reheater

of the tubes in certain locations of the side walls proved to be only a few days. Thermal couples embedded in the tube walls showed temperatures on the hot or outlet leg as high as 1200 deg. These temperatures were taken at point *A*, Fig. 3, so that tube-metal temperatures considerably in excess of this existed within the furnace. The additional elements were, therefore, removed from these hotter parts of the walls, and only those were left in where subjected to less severe conditions. The final result was a temperature rise but little better than that originally obtained.

It was noted, however, when the change shown in Fig. 3 was made that only the outlet tube or hot leg of the element attached to the furnace side of the water-wall tube failed or showed any distress due to excessive heat. It was, therefore, believed that elements could be made with inlet legs on the outside or furnace side of the water-wall tubes and outlet legs behind and shielded by the water-wall tubes. Accordingly, this arrangement was made up and installed as shown in Fig. 4. All of the old elements behind the water-wall tubes were left in place except those that had to be removed to make room for the hot leg of the new elements. This change was completed in July, 1930, and appeared to be the permanent solution to the problem.

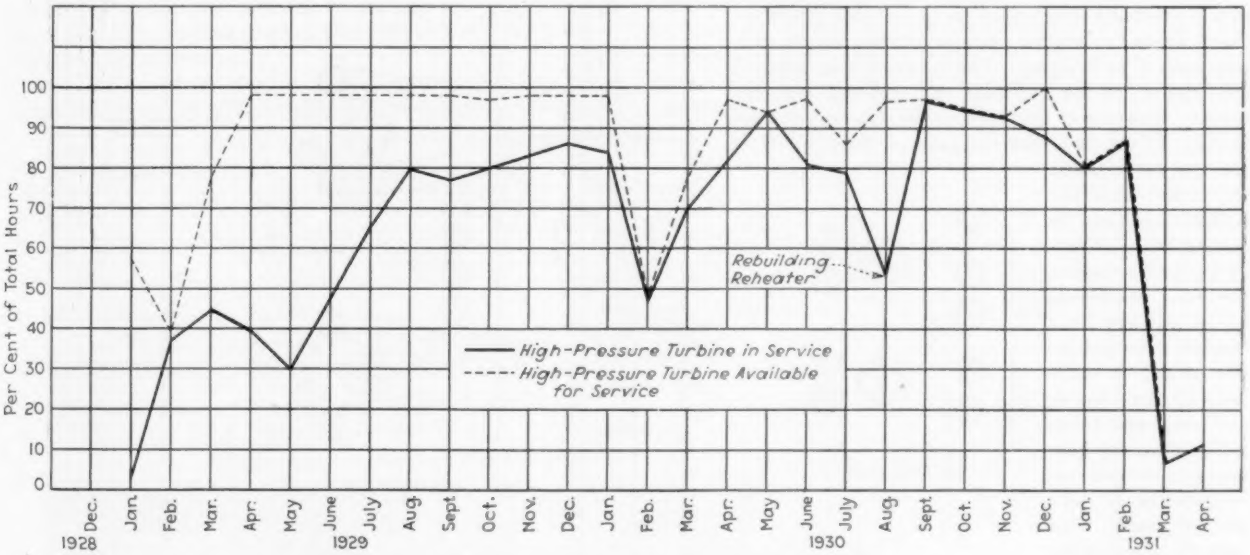

Fig. 5 Turbine Service and Availability Record

The proper reheat was being obtained. However, as previously mentioned, occasional leaks began to develop in the furnace side reheater tube near the top after seven months of practically continuous service. Laboratory examinations of these tube failures showed that metal temperatures of around 1100 deg. had obtained near the points of failure. Sections of the tubes removed from points near the failure showed no alarming signs of overheat externally, but showed a heavy internal coating of magnetic oxide of iron.

Up to this time all reheater tubes had been manufactured from low carbon steel meeting A.S.T.M. specification No. A83-27. Inasmuch as there appeared to be, on this particular job, no other location for reheaters more suitable, the only solution seemed to be to find a material suitable for the tubes in the existing location. An investigation of Enduro or KA-2 showed it unsuitable at high temperature for use where fuels contained sulphur, and also that KA-2S would be unsuitable where the sulphur content of the fuel exceeded 2.5 per cent. Inasmuch as most of the fuel used at the station contains at least 3 per cent sulphur, these two alloys, representing the best of their class, were ruled out. Experience at the plant on booster superheaters shows the chrome-vanadium alloys to be little or no better than low-carbon steel.

A year's use of the experimental high-temperature superheater at Detroit had shown calorized low-carbon-steel tubes quite effective in resisting high temperatures both outside and inside the tube. While the year's use seemed to show some thinning of the outer calorized coating, it was believed to be in such condition that a very reasonable life could be expected from the calorized units. As these metals just mentioned seemed to cover the better known high-temperature resisting metals for tubes, it was decided to try the calorizing. Accordingly, two complete sets of elements were made up to the same form as those shown in Fig. 4, except that the leg attached on the furnace side of the water-wall tube was calorized both inside and out. The two reheaters so equipped were put in service the early part of August of this year, and will, it is confidently believed, prove the final solution to the reheater trouble.

The preceding outlines only the major changes made to the reheaters. Many minor changes were made on the first and second installations such as the use of orifices for changing steam distribution, alteration of burners, and finally the addition of auxiliary burners near the bottom of the side walls. All of these changes, particularly the latter, have contributed their share in the solution of the problem. The last-mentioned item, namely, the auxiliary burners, has been particularly effective in improving furnace conditions and reducing slag. While all of the changes were more or less of a "cut-and try" nature, they were all based on careful experiment and observation which took much time to accomplish.

Particular reference has been made to this reheater trouble because of its bearing on the utility factor of the equipment. During the first six weeks' operation, there were, of course, many other difficulties which caused some outages. These were, however, of a nature more or less expected with the starting of new equipment and were soon overcome.

High-Pressure Turbine

The dependability or service availability of the turbine has been high. Some trouble was experienced during the first three months with balance of the generator rotor and turbine alignment. However, after correcting these troubles, its service availability was nearly 100 per cent until its first inspection after 13 months' use. With the exception of a few minor items, the turbine was found to be in perfect condition. During the second year the turbine continued to give almost equally good service from the mechanical standpoint. However, trouble experienced during the first year from scale deposit on the blades continued throughout this second year, necessitating frequent washing. This trouble evidenced itself by loss of load and a gradual building up of the pressure on the reaction blading. More detailed comment is made later concerning turbine-scale troubles.

About the first of March of this year, the high-pressure turbine was taken out of service for its second annual inspection. The unit had apparently been operating well up to this time, but it had been noted that the last two washings had not had the usual effect in increasing its load-carrying capacity or reducing the pressure on the reaction blading. It was presumed that the blades were badly scaled up and would require scraping. On opening the turbine, it was found that many of the wedges holding the revolving impulse blading had come out, and some had passed through the reaction blading, bending it so badly that the area through the blading was greatly reduced. It was this condition, and not the scale formation, which had caused the last load reduction. The manufacturer felt that the wedges had been loosened by the sudden temperature changes while washing the turbine and recommended returning the spindle to the factory to have a different type of wedge installed which would be less subject to this objection.

This was accordingly done, and in addition some field changes were made to the wedges for the nozzle and impulse blades. Some defective material used in the high-pressure and dummy packing had also failed, causing considerable damage to the remainder of the packing. New packing was installed on the high-pressure end of the unit. There seems to be little doubt

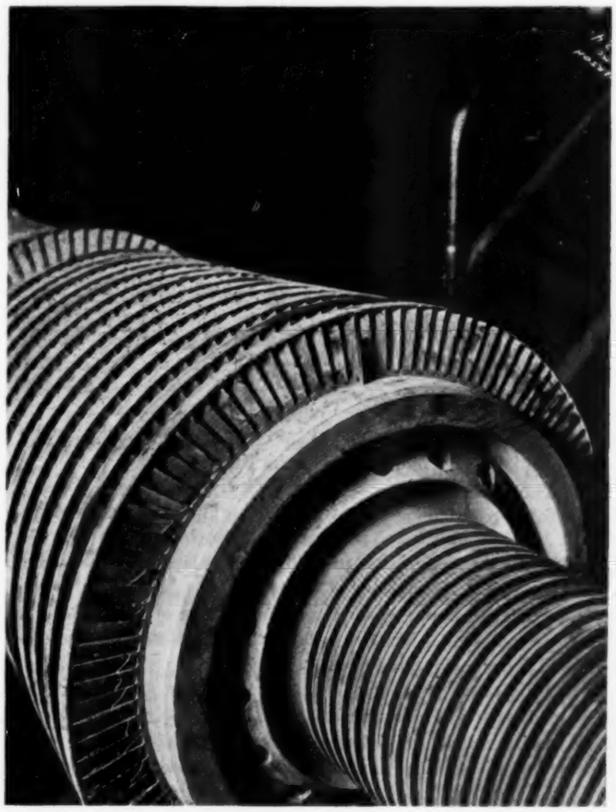

FIG. 6 TURBINE SCALE DEPOSIT

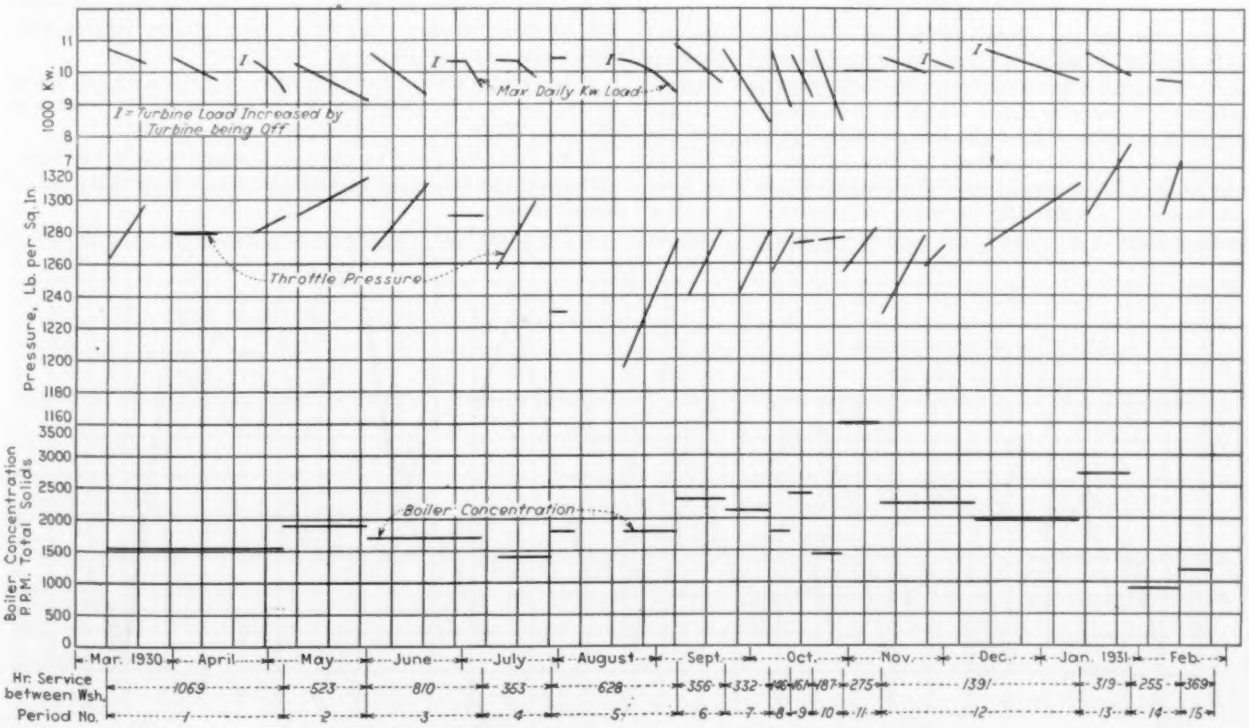

FIG. 7 CURVES SHOWING EFFECT OF SCALE ON TURBINE BLADES

that the troubles which appeared at this inspection have been permanently corrected. There was at no point in the unit any evidence whatever of water cutting or erosion, both the stainless-steel impulse and nickel-reaction blading being in perfect condition. There had been no leakage in the casing joint. The trouble with the wedges was probably due to lack of knowledge on the part of the manufacturer of the conditions that the unit would have to meet in being washed, and in the case of the packing, to defective material. Neither case of trouble could be attributed to the high operating pressure.

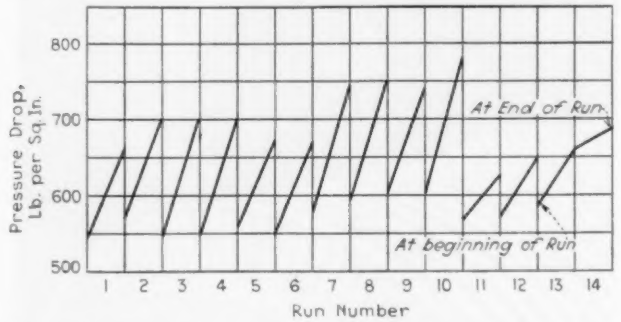

FIG. 8 CURVES SHOWING EFFECT OF WASHING ON PRESSURE DROP THROUGH REACTION BLADING

During the early months of this year some trouble was experienced with the oil-balanced throttle valve. Several changes were made which have apparently corrected the troubles. These troubles with the throttle valve accounted for the decrease in the availability of the turbine just prior to the March overhaul.

Curves, Fig. 5, show the amount of time the turbine has actually been in service and also the amount of time the turbine has been available for service.

TURBINE-BLADE SCALE

The trouble with scale deposit on the turbine blades has been, and still is, sufficiently serious to warrant somewhat detailed comment.

No scale deposit has ever been formed in either the primary superheater or reheater tubes. It seems to form only on the last three rows of blades of the high-pressure turbine and again in the lower stages of the low-pressure turbines operating off the exhaust of the high-pressure unit. The analysis of the scale from both locations is essentially the same and shows it to consist largely of silicates of iron and aluminum which are quite insoluble in water. Fig. 6 shows the scale deposit found on the last row of high-pressure turbine blades at the time of the first annual inspection of the turbine.

In general, the scale has been quite effectively removed by washing the unit with saturated steam when operating at less than 1000 r.p.m., and exhausting to atmosphere. Due to the insoluble character of the scale, it is believed that the washing effects a cracking of the scale due to temperature change, and also an actual scrubbing action due to the entrained water. In some cases, allowing the unit to stand idle for sufficient time to become cold has proved quite effective in scale removal. Here the action must be purely a weathering or cracking.

Curves, Fig. 7, show the operation of the turbine from March, 1930, to March, 1931, divided into periods between washings. On these curves are plotted the maximum possible daily kilowatt load, throttle pressure, and boiler-water concentration. The letter *I* opposite the maximum daily loads indicates the increase in load due to the unit having been out of service at least long enough to cool off. The failure to increase the load by washing as shown the latter part of January and middle of February was explained by the bent condition of the turbine blades as found on inspection and as previously described. Fig. 8 shows the effect on the pressure drop through the reaction blading by each washing.

An enormous amount of experimental work has been done in an effort to eliminate the scale formation on the turbine blades. While it may seem that a solution should have been reached by this time, it must be remembered that the collecting of data for a study of this kind requires much time. This same problem of turbine scale is confronting some of the largest plants in the country and is commanding the study of many research engineers, including Dr. Straub of the University of Illinois. He estimates that his investigation program will take nearly two years to complete. In general, the experimental work done here and the conclusions drawn therefrom are enumerated below:

1 Numerous tests were made to determine the carry-over with various boiler-water concentrations. Results showed no apparent difference in carry-over with concentrations varying from 3600 p.p.m. down to 100 p.p.m.

2 Tests were made to note the effect of water level and rating on carry-over. No difference was observed within reasonable operating limits.

3 All make-up going to the high-pressure boilers was eliminated to reduce concentration and impurities. Results showed no improvement.

4 The phosphate feed was reduced and the PO_4 in the boiler

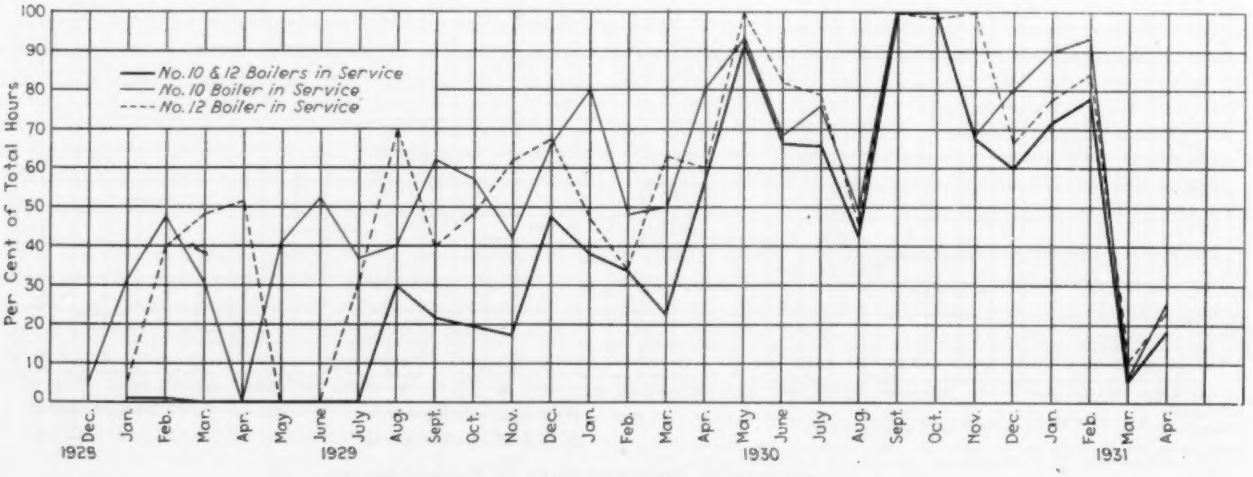

FIG. 9 BOILER SERVICE AND AVAILABILITY RECORD

water was cut from 60 p.p.m. to 3 p.p.m. This reduction has showed no benefits.

5 The concentration was reduced again to about 200 p.p.m. to see if the carry-over tests in item 1 were correct. Scale formation was apparently just as rapid.

A definite plan for further investigation of the turbine-scale trouble along other lines has been laid out and is in progress, and it is hoped that a solution will soon be found.

Boilers and Auxiliaries

The boilers, water walls, superheater, and economizer have developed no defects and give no indications that maintenance will be any higher than on the 300-lb. pressure boiler units in the plant. There have been three boiler-tube failures—the first due to a defective tube, the second damaged due to the first failure, and the third due to almost complete stoppage of circulation in the tube by mud resulting from a serious condenser leak. The tubes are absolutely free of scale and have never been turbined. Careful and correct control of the oxygen content of the feedwater has successfully prevented any pitting or corrosion either in the boiler, water walls, or economizer.

Boiler auxiliaries which include pulverizers, forced- and induced-draft fans, and boiler-feed pumps have proved quite as dependable as the low-pressure service and, therefore, require no particular comment. The same can be said of feedwater regulators, valves, pipe joints, and piping.

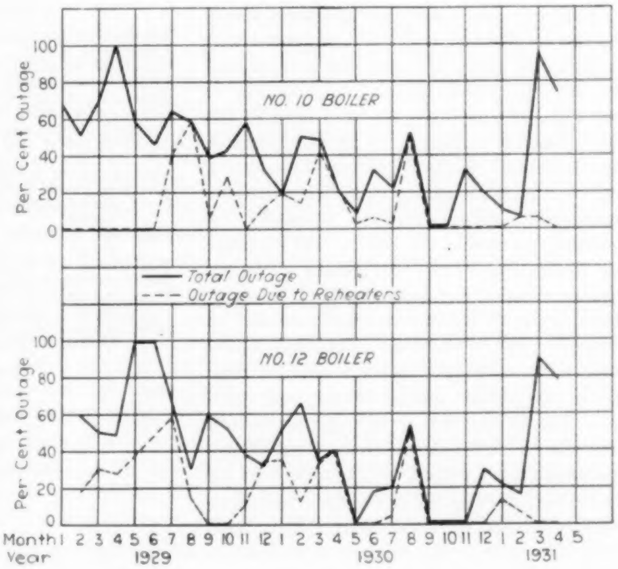

Fig. 10 Boiler Outage Due to Reheaters

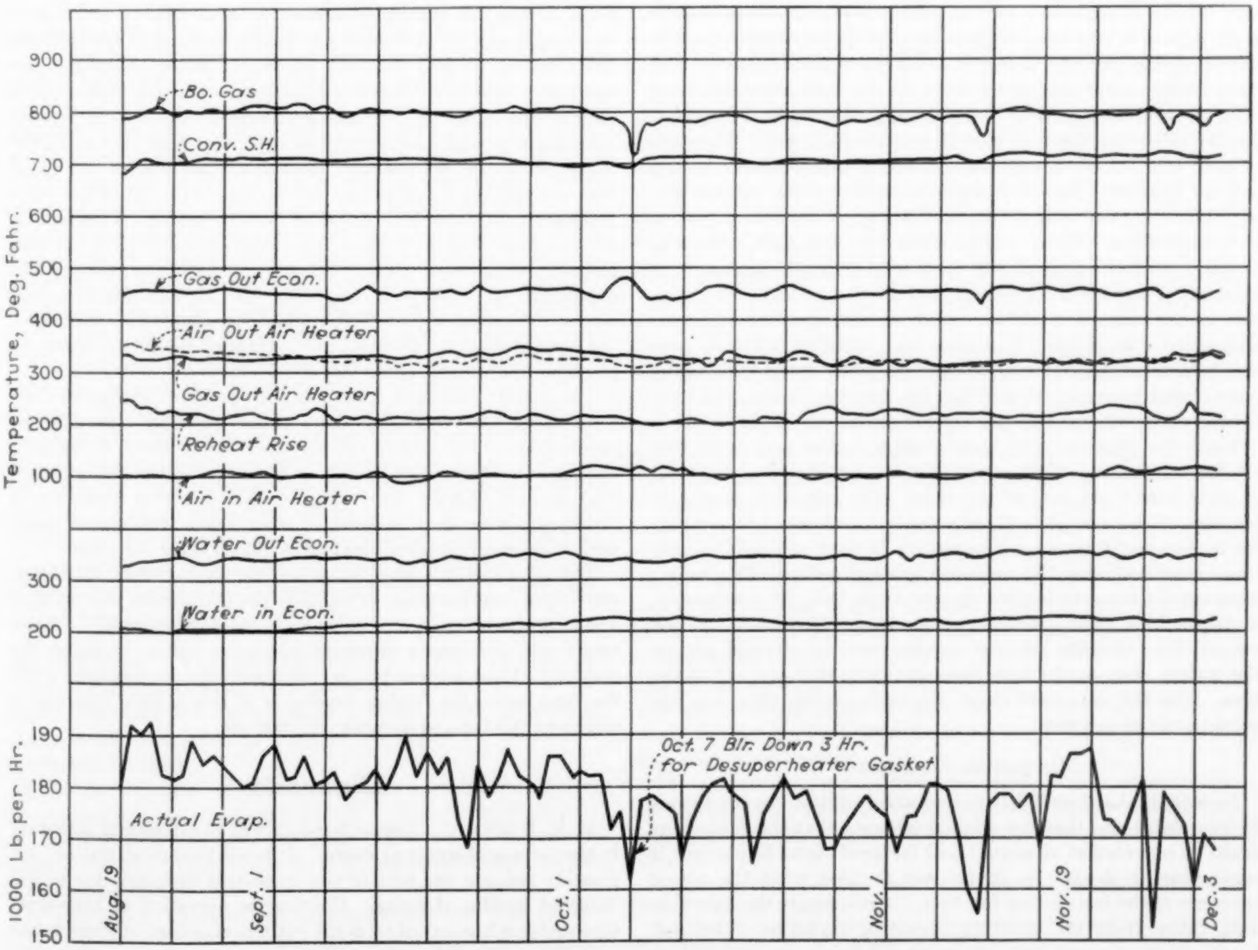

Fig. 11 One Hundred and Six Days' Run of 1400-Lb. Boiler Unit

NORTHEAST STATION PERFORMANCE, SEPTEMBER, 1930

Total power generated, kw-hr.	39,407,000
Net output, kw-hr.	36,705,700
Generated on high-pressure turbine, kw-hr.	6,561,000
Generated on 300-lb. turbines on exhaust from high-pressure unit, kw-hr.	22,150,000
Generated on 1200-lb.–300-lb. combination, kw-hr.	28,711,000
Generated on 300-lb. units with steam from 300-lb. boilers, kw-hr.	10,696,000
Auxiliary power used by high-pressure boilers, kw-hr.	1,334,600
Auxiliary power used by 300-lb. units operating on exhaust from high-pressure unit, kw-hr.	735,000
Total auxiliary power used by 1200-lb.–300-lb. combination, kw-hr.	2,069,600
Auxiliary power used by 300-lb. units with 300-lb. boilers, kw-hr.	632,400
Heat rate of high-pressure unit based on 82.5 per cent boiler efficiency, B.t.u. per kw-hr.	4,400
Heat rate of 300-lb. units operating on exhaust from high-pressure unit, B.t.u. per kw-hr.	15,955
Heat rate of 300-lb. units operating with 300-lb. boilers,[a] B.t.u. per kw-hr.	17,206
Heat consumption high-pressure unit (6,561,000 × 4400), million B.t.u.	28,869
Heat consumption 300-lb. units on exhaust of high-pressure unit (22,150,000 × 15,955), million B.t.u.	353,390
Heat consumption of 300-lb. units on steam from 300-lb. boilers (10,696,000 × 17,206) million B.t.u.	184,030
Total consumption, million B.t.u.	566, 289

[a] Actual 300-lb. boiler efficiency 76.5 per cent due to large amount of banking.

$$\text{B.t.u. per kw-hr. generated} = \frac{566{,}289{,}000{,}000}{39{,}407{,}000} = 14{,}370$$

$$\text{B.t.u. per kw-hr. output} = \frac{566{,}289{,}000{,}000}{36{,}705{,}700} = 15{,}414$$

Reference has been made in several places to the service factor of the equipment and a curve is given to show the service and availability of the turbine. Curves, Fig. 9, show the service obtained from each of the high-pressure boilers. As graphic proof of the statement made regarding reheaters, a curve, Fig. 10, is shown to indicate the percentage of outage due to reheaters. A gradual increase in the service will be noted up to the time of the completion of the reheater rebuilding in June, 1930, after which time, and up to the turbine overhaul in March, 1931, a condition obtained which marked a very good service record. The period of outage during March and April was due to turbine repairs as previously described. After the reheater troubles are permanently repaired, continuous runs for the boilers of from 100 to 125 days will probably mark the practical limit. After such intervals, some repair work or replacements are required on the induced-draft fans. A general cleaning of the boiler and air heater has proved advisable and usually some minor refractory repairs must be made after such a period.

There is very little change in the operating efficiency of the boiler units over continuous runs of 100 days or more. This is very clearly indicated by Fig. 11, which shows all essential operating pressures and temperatures for each day throughout a 106-day operating period. A study of this curve sheet indicates that considerable credit should be given to the methods which have been devised and utilized for removing slag while in service, particularly when the type of coal used is considered. The coals contain more than 15 per cent low-fusing ash and have a high sulphur content. Space limitations restricted the furnace size such that a normal combustion rate of nearly 20,000 B.t.u. per cu. ft. is maintained without the use of turbulent burners. The slag problem at first appeared the most serious factor against even reasonably long operating periods and its solution has been the result of untiring effort toward better combustion and improved slag-removal facilities. The nine 4-in. auxiliary burners near the bottom of the furnace have done much toward obtaining more complete combustion within the furnace proper, and have thus greatly reduced the slag deposit within the boiler itself.

From the time the two high-pressure boilers first came into service up to May, 1931, a period of 28 months, they were out of service 42.6 per cent of the time. The reheaters caused 40 per cent of this outage. If the reheaters had given no trouble, the boilers could have been in service 74.4 per cent of the total time, even including the earliest operating period. The boilers were out of service during March and April, 1931, for overhauling of the turbine, or 6.5 per cent of the 28-month period. If this outage along with the reheater trouble were not charged against the boilers, they would have been available 80.9 per cent of the time. For the year 1930 alone, the boiler availability was approximately 85 per cent.

Operating Economies

Now that it has been fairly well established that the equipment is dependable and that the original estimate that the equipment could be operated at an annual load factor of about 80 per cent is reasonable, it should be of interest to note what the actual economy of the installation has been. Accordingly the following data taken from the monthly operating report of Northeast Station for September, 1930, shows what is being accomplished by the operation of this unit. It should be borne in mind that the efficiency of this cycle was somewhat impaired during this month and also during several previous months by the reduction in output of the high-pressure unit by the scale deposit on the blades. When this trouble is corrected, results should be somewhat better.

Had there been no high-pressure equipment operated at the plant during this month, the 300-lb. pressure boilers would have operated at an overall efficiency including banking of about 79 per cent instead of 76.5 per cent as shown, lower banking losses and better boiler load factor accounting for the difference. This difference in boiler efficiency along with the normal auxiliary power consumption would have resulted in a plant heat rate of 17,600 B.t.u. per kw-hr. net output. This is a reduction for the plant heat rate of 2186 B.t.u. per net kw-hr. due to the use of the high-pressure equipment during this particular month. This is a reduction somewhat in excess of that originally calculated. It was, however, obtained under more favorable conditions; namely, the proportion of total load carried on the 1200-lb.–300-lb. combination was higher than that originally anticipated. Failure of the system load to increase, due primarily to the business depression, has brought about this condition.

The results obtained from the high-pressure equipment has been such, however, that there is no question regarding its justification. The cost of the installation was about $1,650,000, and based on the actual operating economy of the equipment a load factor of 36.5 per cent would result in a saving which would justify the increase in investment over 300-lb. equipment using coal at the original calculated cost of 67,000 B.t.u. for 1 cent.

The situation can be summarized by saying that, all things considered, this company is satisfied the investment was justified and that operation at this pressure is entirely practical. Unless newer and unforeseen developments prove better, or unless the cost of fuel has become too low to justify the investment, when the time comes for further expansion of this station, the use of additional 1200-lb. equipment will be made.

Discussion

M. K. Drewry.[3] The author does well to call special attention to the turbine clogging problem. It is felt that most stations are possibly losing a per cent or two in station economy due to undetected turbine clogging. Continuous operation of high-pressure turbines has served to point out the existence of these losses that escape detection in usual operation of condensing machines. The percentage capacity loss in the former case is greatly more, and is readily detected therefore.

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[3] Assistant Chief Engineer, Power Plants, Milwaukee Elec. Ry. & Light Co., Milwaukee, Wis. Jun. A.S.M.E.

Low boiler-water concentration is the only known positive corrective of turbine clogging at the Lakeside Station. Long-time experience with four high-pressure turbines proves the method thoroughly efficient, but learning the technique of correct boiler-water treatment with high make-up or the elements of proper boiler design for negligible entrainment is certainly desirable for less favorable conditions.

Reliable means of measuring boiler-water carry-over are needed to intelligently study this subject of turbine clogging. When dealing with boiler-water concentrations of 100 p.p.m. and carry-over of 0.1 per cent, only $^1/_{10}$ of 1 p.p.m. of boiler water appears in the condensed steam. Obviously, extremely sensitive measurements are necessary. Electrical conductivity measurements are affected by dissolved gases to the extent that several hundred per cent error may occur under some conditions. Sensitive phosphate tests after concentration by evaporation appear to offer practical help. Priming to the extent of 3 per cent moisture carry-over has been noticed twice after adding 100 p.p.m. of sodium phosphate to high-pressure boilers. Chemistry of the boiler water is of major importance in this problem.

The freedom from corrosion mentioned by the author shows commendable operation. Absence of feed-pump troubles is also of special interest. Outlet flue-gas temperature is well maintained at about 320 deg. fahr. for 100-day periods. A 73 per cent station generation by 1200-lb. equipment reduced the low-pressure station heat consumption by $12^1/_2$ per cent, indicating that the higher pressure is 17 per cent more efficient.

Bert Houghton.[4] It has been a pleasure to read the frank discussion of the difficulties in developing the Northeast Station's high-pressure installation. Definite progress can be made when basic facts are clearly disclosed, and the Kansas City Power and Light Company is to be congratulated on its initiative in putting in one of the first high-pressure installations and then in coming forward with detailed experiences.

Now that this boiler, reheater, and turbine have apparently been brought to a point where any major change in design is not to be expected, it would be interesting to have the benefit of a careful and detailed test on the operating characteristics of each element. The operating performance integrated over a period of time, as given in the paper, is of course valuable information and useful in justifying high-pressure installations; but it is also most interesting and necessary to have the efficiency characteristics over the range of loading, and for this installation it would complete the story which already has been so freely given. It would be also of interest to have the design heat rates included so that the three items could be compared—i.e., design heat rate, test heat rate, and operating heat rate.

John Hunter.[5] The paper has much of interest, particularly the last paragraph in which it is stated that the investment was justified and that the operation of 1200-lb. steam pressure is entirely satisfactory, and for further expansion of the station 1200-lb. will be used.

In the development of this high-pressure equipment, it has taken a great amount of courage, as well as engineering study and skill, and the men who have put forth this effort are to be congratulated for obtaining the results as shown by this paper on the Northeast installation.

The author lays stress on the trouble encountered with the reheater which no doubt was very aggravating, but engineers have found when placing tubes in the furnace exposed to radiant heat, carrying a gas only, that they have a serious condition with which to contend. This was demonstrated when the first superheater tubes were placed in a furnace exposed to the radiant heat. Many different designs and metals were used, causing interruptions in serviceability of the boiler units. However, the final difficulties were overcome by the manufacturers and operating engineers. It is gratifying to note by the paper that the troubles with the reheaters, like the superheaters, have been overcome.

John J. Grebe.[6] In view of the difficulties encountered by scale deposits in the turbines at the low-temperature end, it might be asked if any experimental work has been done to determine whether such scaling can be reduced by carrying out the first part of the superheating of the steam in tubes which have a carefully limited metal temperature. With a low steam velocity and low metal temperature, the solids may have a chance to precipitate in this section of the superheater, from which they might be removed with less trouble. The reason for this remark is that we expect in our design of a combination steam and diphenyloxide boiler to reduce the carry-over of the solids to a minimum by drying and preheating the steam with diphenyloxide-heated surfaces at approximately 700 deg. fahr.

John Van Brunt.[7] Referring to the auxiliary burners installed along the bottom of the side wall, the writer would confirm the statement by the author that the installation of these burners resulted in the shifting of the hot-flame area further forward in the furnace, giving a more even distribution of heat to the side-wall reheaters. The writer would commend to the attention of operators in general the very excellent results over a period of 106 days as indicated in Fig. 11. This chart evidences the fact that efficiency was well sustained for the entire period, that slag accumulation in the furnace or boiler was well under control, and that the operation in general reflects great credit upon the operating force.

Author's Closure

As to the effect of concentration on turbine-blade deposit, as brought forth in the paper, the amount of deposit on turbine blades did not appear to be any less when operating boilers with low concentration than with high concentration. Both tests operating at low concentration were unfortunately of rather short duration due to other causes necessitating removal of the turbine from service. The fact that the rate of decrease in load during these two short periods of low-concentration operation was practically as rapid as with high-concentration prompted the statement that in this case concentration did not appear to be the determining factor. Certain tests conducted by C. W. Foulk[8] seem to substantiate the author's belief that it is possible to have greater carry-over with low concentration.

Since presentation of the paper, operation of the boilers has been continued for about 90 days with a total concentration of about 100 p.p.m., with practically no improvement in the turbine-blade-scale trouble.

The author does not believe that turbine-blade-scale trouble can be remedied by the use of a low-steam-velocity superheater, as suggested by Mr. Grebe. The velocity of the steam does not seem to have any direct bearing on the dropping out of solids. The velocity of the steam through the lower stages of the turbine is many times that in the superheater, yet the deposit occurs in the turbine and not in the superheater.

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[4] Operating Superintendent, Brooklyn Edison Company, Brooklyn, N. Y. Mem. A.S.M.E.

[5] Consulting Engineer, St. Louis, Mo. Mem. A.S.M.E.

[6] Director Physical Research, Dow Chemical Company, Midland, Mich. Assoc-Mem. A.S.M.E.

[7] Vice-President, Engineering, Combustion Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

[8] Paper on "Foaming and Priming of Boiler Water," presented at A.S.M.E. Annual Meeting, 1931.

Concerning the operating efficiency over the entire range of load, the installation described by the author was designed for base-load operation at its most economical capacity, namely, full load, and has been operated most of the time at as near that point as the condition of equipment would permit. The turbine operates either at 2500-kw. load or approximately 10,000-kw. load, depending upon whether one or two boilers are in service. The gain in economy with only one boiler in service, or when operating at 2500 kw., is low due to the high water rate of the single-valve turbine at light load. In this particular case, it is felt that the two boilers can be kept in service a sufficient amount of time to justify the full-load design.

FSP-54-14

Decomposition of Sodium Carbonate in Steam Boilers[1]

By REINHOLD F. LARSON,[2] URBANA, ILL.

This paper deals quantitatively with the extent of decomposition of sodium carbonate, which is present in many boiler waters and has an important bearing on the inhibition of caustic embrittlement and corrosion. Data are presented in the form of curves obtained from tests on a steaming laboratory boiler at various high pressures and with various feedwaters. The extent of decomposition occurring is found to be dependent upon several factors, the principal one being the amount of sodium carbonate in the feedwater. An attempt is made to correlate laboratory data with those obtained from operating plant boilers.

SODIUM CARBONATE as a dissolved salt in the water of a steaming boiler is not stable. The carbonic acid formed by hydrolysis cannot exist at the higher temperatures, but loses its carbon dioxide to the vapor phase. The continuous evolution of steam carries with it the carbon dioxide liberated.

As early as 1891, Paul[3] noticed the increase in causticity in boilers in which carbonate of soda was present in the feedwater, but apparently did not detect the carbon dioxide in the evolved steam. He described the reaction as follows:

$$Na_2CO_3 + H_2O \longrightarrow HCO_2Na + NaOH + {}^1/_2O_2 \ldots [1]$$

Hall, Kartch, and Robb,[4] who made an extended study of this decomposition reaction, paying particular attention to the noncondensible gases in the steam evolved from a small laboratory boiler at various low pressures, pointed out that the reaction is best expressed by:

$$Na_2CO_3 + 2H_2O \longrightarrow 2NaOH + H_2O + CO_2 \ldots\ldots [2]$$

They detected an enormous increase in the carbon-dioxide contents of the steam taken off when sodium carbonate was present in the boiler water. The oxygen content remained about the same as before introducing carbonate. The boiler water increased steadily in hydroxyl alkalinity. They found no evidence for believing that reaction [1] took place. Various other investigators have shown that reaction [2] occurs, but at temperatures below 50 deg. cent. is too slow to be measured.

The reaction is simply one in which one of the products is continuously carried away, the other remaining to increase the concentration of soluble salts in the boiler water. In an operating commercial boiler a continuous supply of reacting carbonate is brought into the boiler with the feedwater. Undoubtedly the amount decomposed is influenced by the partial pressure of the carbon dioxide in the vapor phase that is maintained. Temperature is known to accelerate the reaction. It has therefore been stated that the rate of decomposition of sodium carbonate is a function of the operating pressure and the percentage rating of the boiler.

Investigations concerning the quantitative determination of this decomposition reaction have been confined principally to laboratory apparatus that did not duplicate boiler-plant operation. Pressures and rates of boiling have been considerably lower than those used in modern boiler plants. Conditions of continuous feed and constant water level have in general not been maintained; consequently, the information obtained has had little application in a practical way.

An attempt to obtain information from an operating boiler was reported by C. E. Joos.[5] Some interesting information was obtained. He also reported a survey made of a large number of power boilers operated at various pressures and ratings and gave complete boiler-water data. The amounts of sodium hydroxide and sodium carbonate in a boiler water, if expressed as sodium carbonate and arranged in the ratio

$$\frac{NaOH \text{ as } Na_2CO_3}{(NaOH \text{ plus } Na_2CO_3) \text{ as } Na_2CO_3}$$

express the degree to which decomposition has proceeded, if all of the sodium hydroxide may be assumed to have originated from the carbonate. This ratio may be expressed as per cent decomposition. In these data it varies from 18.8 per cent to 88.9 per cent. There appears to be little correlation between per cent decomposition and either pressure or rating. No conclusions can be drawn, therefore, from these data other than that much of the sodium carbonate in a boiler is converted to sodium hydroxide, and the proportions of these that may exist in an operating boiler cover a wide range.

IMPORTANCE OF THE REACTION

It is desirable to know more about this decomposition and the extent to which it may occur in any particular boiler and in boilers in general. Many natural waters used as boiler feed contain sodium carbonate in large quantities. The pretreatment of feedwater to remove scale-forming calcium and magnesium com-

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[1] Part of research originally suggested by F. G. Straub, Special Research Assistant Professor of Chemical Engineering in the Engineering Experiment Station, and conducted in the Graduate School of the University of Illinois under the general supervision of D. B. Keyes, Professor of Industrial Chemistry.

[2] Department of Mechanical Engineering, University of Illinois. Jun. A.S.M.E. Mr. Larson was graduated from the University of Illinois in 1923 with the degree B.S. in M.E. He worked for four years principally in a sales-engineering capacity for several companies and returned to the University of Illinois in 1927, where he has since taught courses in power-plant engineering and has supervised associated laboratory work. Summers have been spent at various central-station power plants.

[3] Paul, J. H., "Corrosion in Steam Boilers," Trans. Soc. Engrs. (1891), pp. 147–186.

[4] Hall, Kartch, and Robb, "A Physico-Chemical Study of Scale Formation and Boiler-Water Conditioning," Bulletin 24, Mining and Metallurgical Investigations, Carnegie Institute of Technology.

Contributed by the Power Division and presented at the Hartford, Conn., Meeting, June 1–3, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] Joos, C. E., "Conversion of Sodium Carbonate to Caustic Soda Within Steam Boilers," *Power*, Nov. 12, 1929, pp. 762–764.

pounds often leaves considerable sodium carbonate in the treated water. Soda ash is a chief constituent of many boiler compounds.

It may be well to observe at this time that this decomposition phenomenon is common to all carbonates found in boiler feedwaters. However, many of these are only slightly soluble, and their contribution to the carbon-dioxide content of the steam and the increasing causticity of the boiler water is so slight in comparison to that contributed by sodium carbonate alone that for most purposes they can be neglected. The presence of magnesium chloride is known to accelerate the decomposition of calcium carbonate somewhat by precipitating out the highly insoluble magnesium hydroxide, thus preventing the accumulation of the hydroxyl ion. The use of sodium phosphate as a secondary treatment in the boiler drum effects a base exchange reaction precipitating out the insoluble calcium phosphate, thus building up the sodium-carbonate content, which in turn decomposes with a resultant increase in causticity. Where soluble phosphate is maintained in the boiler water, calcium carbonate may be considered potential sodium carbonate.

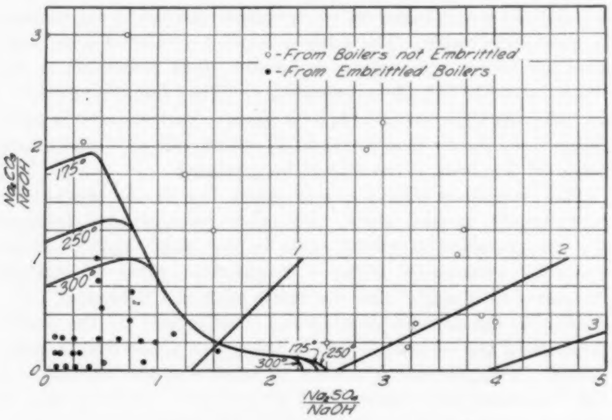

FIG. 1 RATIOS OF $\frac{Na_2CO_3}{NaOH}$ AND $\frac{Na_2SO_4}{NaOH}$ NECESSARY TO PREVENT EMBRITTLEMENT

The presence of hydroxyl alkalinity to a limited extent has been shown to inhibit corrosion. The maintenance of a hydroxyl-ion concentration of about 100 parts per million and a pH value of from 9 to 10.5 have been recommended. Soda ash is often used in small quantities to build up hydroxyl alkalinity and at the same time increase the pH value.

The presence of sodium hydroxide and the absence of sufficient quantities of certain inhibitors has been established as the chief contributing cause of caustic embrittlement, an intercrystalline cracking of the boiler plate which occurs in the seams and highly stressed areas of the boiler. Parr and Straub have definitely shown that the concentration of this caustic is possible in the seams and capillary spaces of the boiler, and to an extent sufficient to attack the grain boundaries of the adjacent highly stressed metal. Among the inhibitors are sodium carbonate and sodium sulphate. The presence of the latter in sufficient quantities is known to effectively prevent this type of cracking, and is recommended in the A.S.M.E. Code. In calculating the quantity of sodium sulphate necessary, the carbonate has been assumed as potential hydroxide, and the ratios for various pressures based on total alkalinity. The maintenance of these ratios has put a heavy burden on the boiler-plant operator who is also striving to keep concentration down by means of blowdown. It naturally results in excessive blowdown.

Later developments in the study of embrittlement have shown that the 1 to 1 ratio may be a trifle low, the other ratios amply safe, and that the sodium carbonate present may also act as an inhibitor. Fig. 1 is a chart taken from Bulletin No. 216 of the University of Illinois Engineering Experiment Station, "Embrittlement in Boilers," by Professor F. G. Straub. It is drawn from data on the solubility relations of sodium carbonate and sodium sulphate in a solution of 100 grams per liter of sodium hydroxide, a concentration of caustic found necessary to embrittle specimens in the laboratory. Wherever this concentration could be reached with sodium carbonate and sodium sulphate still in solution, embrittlement has occurred; wherever the ratios were such that these salts would precipitate out before this dangerous concentration was reached in the boiler seams, boilers were not embrittled. This chart shows clearly the extent to which carbonate is an inhibitor, and explains why certain boilers have operated for years with a high carbonate content in the boiler and with little sulphate, and have experienced no embrittlement. It offers the plant operator a chance to use the inhibiting power of carbonate in calculating the amount of sulphate necessary to insure safety from this type of cracking.

LABORATORY TEST WORK

Object. This series of tests is an attempt to study the decomposition reaction in various synthetic carbonate waters at various operating pressures and boiler ratings, and to seek an explanation for cases of extremely high and extremely low decomposition found in some operating boilers.

Test Apparatus. The apparatus used is pictured in Figs. 2 and 3. It consisted of a 10-in. extra-heavy seamless tube 30 in. long and heavily flanged at the ends so as to make up a cylindrical boiler suitable for high pressures. All connections to the boiler were made through the flanges, as shown in Fig. 3. The boiler was then suspended from a steel frame, and the furnace chamber was bricked in as shown. At a later date Transite baffles were added to protect the studs from the action of excessive heat.

The feedwater inlet was at the left or front end. A special high-pressure gage glass was procured and installed as shown. A blowdown line with valve was installed, as was also a blowdown condenser permitting volumetric measurement of blowdown at room temperature. A steel thermocouple well was installed as shown and bent so as to be completely immersed. An iron-constantan couple was made and inserted, with leads properly insulated with asbestos. A Leeds & Northrup recording potentiometer was used to obtain a temperature check on the reading of a pressure gage installed on the opposite end of the boiler.

In the rear flange was fitted a steam outlet with a needle-valve control. This led to a steam condenser from which condensate could be drawn into 5-gal. bottles and analyzed for alkalinity and carbon dioxide. Samples of boiler water were drawn off through a carbon filter and capillary tube into a steel bomb of about 300 cc. capacity. This bomb could then be disconnected and cooled before releasing the pressure.

Gas was used as a fuel, air being supplied by a fan. Separate valves for gas and air enabled considerable adjustment of gas-air ratios. The furnace was baffled so as to obtain a fairly long gas travel, the boiler being heated on all sides.

Feedwater was supplied by means of a displacement pump having an adjustable stroke, and driven by means of an induction motor through belt and speed-reducing gears. The source of water for building up the synthetic feeds used was condensed steam collected from radiators used to heat the building, and was stored in a steel barrel. This condensed steam averaged about 170 deg. fahr. and was practically free from dissolved carbon dioxide. The feed line was projected to the bottom of the barrel and kept the barrel full and at a maximum temperature at all times.

Synthetic feedwater was made up by weight in 100-lb. and

40-lb. lots, the required salts being weighed accurately in the anhydrous state on an analytical balance.

Operation. In most tests the boiler was filled about half full of the solution to be used as feed, and the boiler was fired. When the pressure had risen to the correct value, the steam take-off was begun and a sample of boiler water was drawn. The time taken to build up pressure varied for the various tests from $^1/_2$ hour to $1^1/_2$ hours, the latter for the higher pressures. The feed pump was then started and the steam valve manipulated so as to maintain a constant predetermined water level. It generally required an hour to secure the intended steaming rate. The pump was then allowed to run continuously, and being induction-motor driven, ran at approximately constant speed. Steam pressure was maintained by gas-burner adjustment. In general, the water level did not vary more than $^1/_4$ in. from normal when samples were drawn and pressure not more than 5 per cent. This condition was attained through constant attention of the operator in the early stages of a test.

A careful check was made on the total water fed to the boiler. All tests, unless specified, were run at the same setting of the pump stroke, but due to the variation in the volumetric efficiency of the pump and the regulation of the motor, the actual rates varied from about 95 lb. per hour at 150 lb. gage pressure to about 82 lb. per hour at a steam pressure of 1500 lb.

Water Analysis. The amount of boiler water drawn off for each sample varied from 0.4 to 0.6 lb., depending on how well the boiler pressure would fill the bomb. No difference in results was obtained by evacuating the bomb other than obtaining a full bomb each time, so that this procedure was given up. The sampling operation generally took several minutes, and no noticeable disturbance took place in the boiler. It is felt that a representative sample was obtained, as blowdown samples were identical with those taken from the bomb at the same time. The samples obtained were generally quite clear, although at times an iron-oxide suspension was noticeable. This settled within several hours and did not interfere with analysis. Samples were kept stoppered in flasks awaiting analysis.

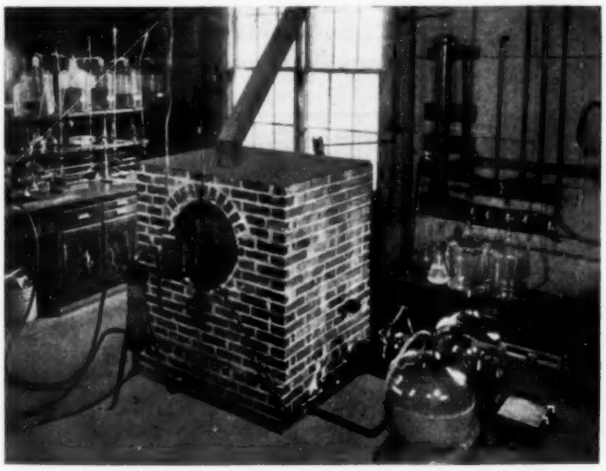

FIG. 2 APPARATUS USED IN TESTS OF DECOMPOSITION REACTION IN VARIOUS SYNTHETIC CARBONATE WATERS

Approximately $^1/_{10}$ normal hydrochloric acid was used in the ordinary double titration with phenolphthalein and methyl orange as indicators. The modified Winkler barium-chloride method was used as a check. Equal amounts were measured out in a pipette and transferred to separate flasks. The amounts used were such that at least 20 cc. of acid was used to obtain an end-point. For strongly concentrated waters a stronger standardized acid was used, as extreme dilution by the acid makes the methyl-orange end-point difficult to check. To the second sample was added 5 to 10 cc. of very slightly alkaline barium chloride. A 10 per cent solution was used, made alkaline with sodium hydroxide to set any dissolved carbon dioxide, and then neutralized to the end-point of phenolphthalein. The sample was then well shaken and allowed to stand about 5 minutes. It was then titrated with acid, being continuously agitated to avoid acid concentration, to the phenolphthalein end-point. The cubic centimeters used were recorded as hydroxide. The checks obtained by the two methods were very good when only carbonate and hydroxide were present in the boiler water.

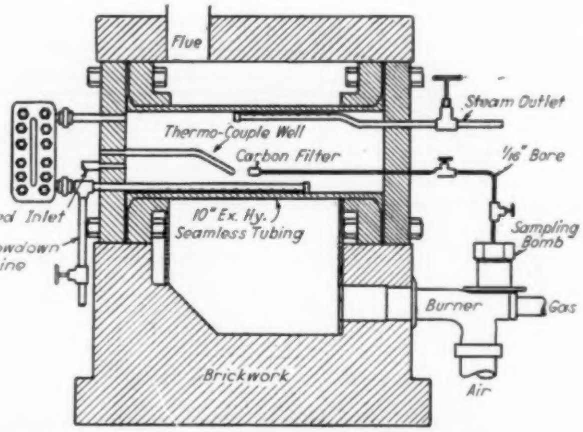

FIG. 3 SECTION THROUGH BOILER OF TEST APPARATUS

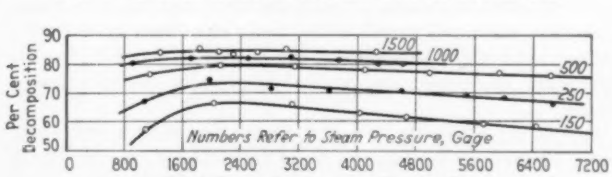

FIG. 4 400 P.P.M. NA_2CO_3 FEED

With the double-titration method there is a slight tendency to stop short of the true phenolphthalein end-point, giving a slightly higher carbonate content. Where only hydroxide is present, as in the case of the second titration, the end-point is sharply defined. In all of the foregoing titrations care was taken to keep all flasks stoppered and titrations were made as rapidly as possible in order to avoid contamination with carbon dioxide from air and breath. Such contamination increases the carbonate and decreases the hydroxide.

DISCUSSION OF RESULTS

Expressing the ratio:

$$\frac{\text{sodium hydroxide expressed as sodium carbonate}}{\text{total alkalinity expressed as sodium carbonate}}$$

in terms of per cent decomposition, and plotting these values against time and against total alkalinity, well-defined smooth curves are obtained. Fig. 4 is the series obtained from the tests using 400 parts per million sodium carbonate in the feedwater. It is apparent that a balance or equilibrium is soon established between the decomposition rate and the incoming sodium carbonate. The maximum per cent decomposition was obtained about two hours after steaming was started, thereafter decreasing steadily with increase in boiler-water concentration. It is noticeable that the higher pressures have proportionately less effect than the lower pressures. In all of these tests slightly over two concentrations occurred per hour.

Fig. 5 was obtained from tests using 150 parts per million sodium carbonate, and shows a similar balance after a somewhat longer time. Fig. 6 was obtained from a 50 parts per million feed. The extremely slow approach to equilibrium is noticeable especially so at the higher pressures. The irregular building up is probably due to failure to exactly duplicate the steaming rates during the first hour or so. Had these tests been run for a longer time, the maximums would undoubtedly have been more clearly defined. As might be expected, the decomposition is higher for this purer feedwater, and operating pressure becomes less a factor.

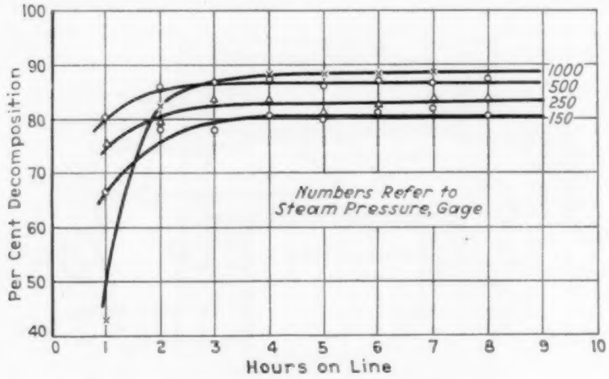

FIG. 5 150 P.P.M. Na_2CO_3 FEED

It was not advisable to run a similar test with a still purer water because of the time required to build up an appreciable concentration. Therefore the boiler was charged with a more concentrated solution at the start, and a 10 parts per million feed of sodium carbonate begun. Two such tests were run, the initial charges being 500 parts per million and 4000 parts per million sodium carbonate. The maximum per cents obtained were 91.5 and 90.4, respectively, with an operating pressure of 250 lb. A change in pressure to 500 lb. caused an increase of 1.2 per cent in the first test. Untreated water was then fed for five hours in the latter test, the decomposition rising to 94.2 per cent as an average for the last three hours.

In most tests a slight loss in boiler-water alkalinity was noticeable. Condensed steam from the boiler showed from 10 to 20 parts per million methyl-orange alkalinity expressed as sodium bicarbonate, indicating some carry-over. This did not change appreciably with boiler-water concentration. The presence of a few small leaks in connections below the water level at high pressures, and the possibility of baking of the salt on the surface above the water level, may account for the fact that total alkalinity actually decreased when 10 parts per million sodium carbonate was fed.

It was noticed in several tests that a sample drawn while the water level was falling would show a higher decomposition, and the opposite for a rising water level, coming back to normal when the water level was kept constant. This would indicate an equilibrium controlled by the partial pressure of the carbon dioxide in the steam space. A test was therefore run with sodium bicarbonate only in the feed, and in quantity sufficient to give the same sodium concentration, but twice the amount of available carbon dioxide. Fig. 7 gives the results obtained and indicates that the amount of available carbon dioxide in the feedwater and consequently the partial pressure of carbon dioxide in the steam space is a controlling factor.

A steady decrease in decomposition with increase in total alkalinity of the boiler water is especially noticeable in the case of 400 parts per million feedwater. Later tests revealed the same dropping-off tendency in the case of 150 parts per million sodium carbonate in the feedwater, but to lesser extent. This would indicate that total alkalinity or possibly sodium-hydroxide alkalinity alone was a factor. Several tests were run at various concentrations, keeping the total alkalinity constant with various amounts of blowdown ranging from 4 to 30 per cent. The fact was noticed that while the total alkalinity was kept constant, the decomposition also became constant, and consequently the sodium-hydroxide concentration was kept constant. A heavy blowdown as was necessary at a low boiler-water concentration was accompanied by a decrease in decomposition. This was to be expected, as the blowdown also carries out carbonate undecomposed, and if increased to the limiting case, or 100 per cent, would result in a decomposition approaching zero. A 30 per cent blowdown with a 150 part per million feed dropped the decomposition about 6 per cent, and a $7^1/_2$ per cent blowdown dropped the decomposition about 1.2 per cent, both at 250 lb. operating pressure.

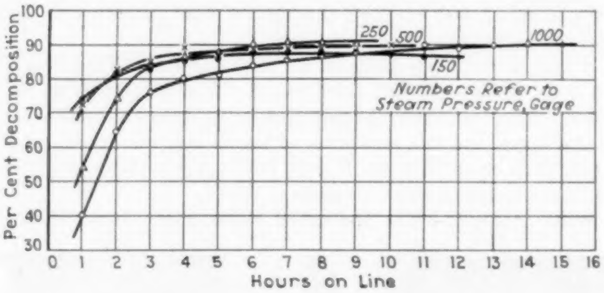

FIG. 6 50 P.P.M. Na_2CO_3 FEED

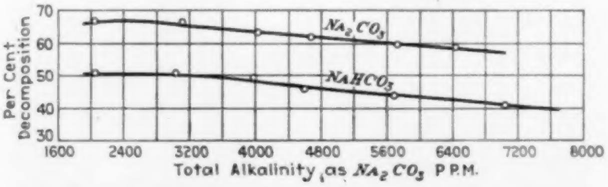

FIG. 7 EFFECT OF PARTIAL PRESSURE OF CO_2 ON DECOMPOSITION OF Na_2CO_3 AT 150 LB. STEAM PRESSURE, 400 P.P.M. FEED

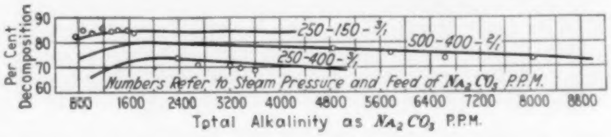

FIG. 8 DECOMPOSITION OF Na_2CO_3 IN THE PRESENCE OF LARGE AMOUNTS OF Na_2SO_4

(Curves represent decomposition obtained with Na_2SO_4 absent. Approximate ratios Na_2SO_4 to alkalinity as Na_2CO_3 are indicated.)

To check whether total ionic strength, through its effect on activity coefficients, might be a factor, several tests were run with large quantities of sodium sulphate added to the feedwater. Fig. 8 shows the plotted results of such tests. The curves were drawn from previous data obtained using sodium carbonate alone. The fair agreement indicates that total ionic strength has but slight effect over the ranges run in these tests. The maintenance of the A.S.M.E. ratios of sulphate to alkalinity should not appreciably affect the decomposition that can be expected.

It was discovered that with sodium sulphate present in the boiler water the Winkler barium-chloride method of hydroxide determination is unreliable. The phenolphthalein end-point comes too soon, indicating too little hydroxide. It was thought that the large quantity of barium sulphate brought down was masking the true end-point. Excessive dilution of the sample and subsequent titration gave the same result. The loss in hydroxide seems to increase with the amount of barium chloride used. Solubility data on barium hydroxide do not explain this loss of hydroxyl ion from the solution. Even small quantities of sulphate radical in the boiler water cause a similar loss in hydroxide when this method is used.

The calculation of decomposition in these later tests with sulphate present was based entirely on double titration, using phenolphthalein and methyl-orange indicators. This method of determination gave consistent results, however; the color change of methyl orange is less marked in the presence of sodium sulphate, and quite broad in dilute solutions. It is better to use a stronger acid and a larger sample in titrating highly concentrated boiler waters.

Early in the schedule of testing several runs were made with different rates of steaming. Fig. 9 gives the results obtained and indicates that boiler rating alone has little effect on decomposition and is not a factor in these accelerated tests. An additional high-steaming-rate test was run with 400 parts per million feed, and showed fair agreement with a previous low-rate test.

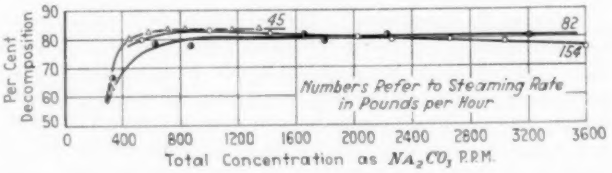

FIG. 9 VARIABLE STEAMING RATE 150 LB. PER SQ. IN., 150 P.P.M. Na_2CO_3

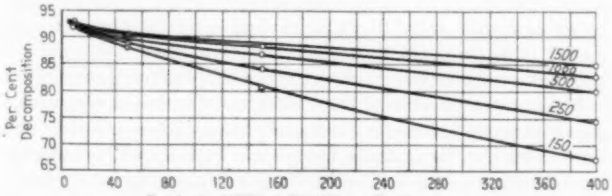

FIG. 10 MAXIMUM DECOMPOSITION OF Na_2CO_3 FOR VARIOUS PRESSURES AND COMPOSITION OF FEEDWATER
(Total boiler-water alkalinity 2400 p.p.m. as Na_2CO_3.)

An attempt was made to correlate decomposition data for various pressures and composition of feedwater, for some particular boiler-water concentration that might be maintained by means of blowdown. The curves in Fig. 10 were drawn from selected data on decomposition at about 2400 parts per million total boiler-water alkalinity expressed as sodium carbonate. The curves are extrapolated somewhat on the assumption that decomposition would approach 100 per cent for a very pure feedwater. A similar family of curves might be drawn for a higher boiler-water concentration or alkalinity, and also for a lower total alkalinity, differing somewhat at higher feedwater concentrations of carbonate, but very little for feedwaters containing very little sodium carbonate.

Such a chart as Fig. 10 might be useful in estimating decomposition and checking boiler-water analyses, unless there exist other factors of equal importance that have not been investigated. The curves are based on attaining the equilibrium previously referred to, a condition which may not always occur in the operating boiler at all times. Quick or sudden changes in steaming rate of great magnitude and intermittent blowdown are probably factors.

Fig. 11 was obtained from a calculation of partial pressure of carbon dioxide in the steam based on decomposition data at a total alkalinity of 2400 parts per million. The curves are drawn as straight lines that converge at a point, indicating about 92 per cent decomposition at zero partial pressure. Theoretically, the decomposition should proceed to 100 per cent completion on reducing the partial pressure of the carbon dioxide to zero and allowing sufficient time. However, removing the carbon dioxide is a dilution process and very slow in the last stages. It appears that a small amount of carbon dioxide has at this point tremendous effect on the completion of this reaction, and practically the limit of decomposition from a feedwater containing but very little potential carbon dioxide seems to be from 92 to 94 per cent.

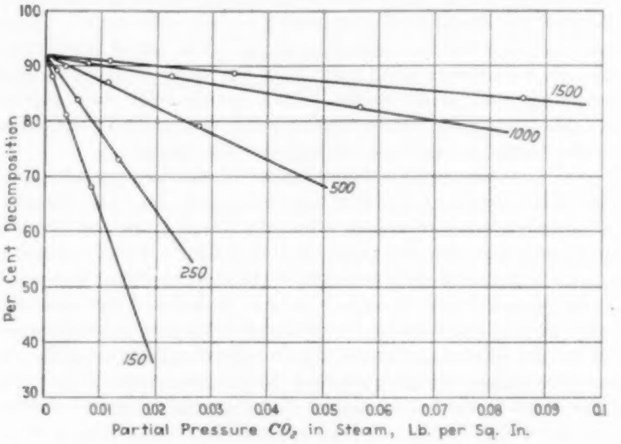

FIG. 11 EFFECT OF CO_2 PARTIAL PRESSURE ON DECOMPOSITION OF Na_2CO_3
(Total boiler-water alkalinity 2400 p.p.m. as Na_2CO_3.)

To check this, two tests were run, using for a feedwater the purest water available—namely, that used in making up these various synthetic solutions. This water showed a methyl-orange alkalinity equivalent to 10 parts per million sodium carbonate, but an accurate carbon-dioxide evolution test accounted for only 5 parts per million expressed as sodium carbonate. Starting with a maximum decomposition of 91.5 per cent obtained with a 10 part per million feed at 250 lb. pressure, the purer water was fed to the boiler for two hours. The final sample drawn showed a decomposition of 93.7 per cent at a total boiler-water alkalinity of 321 parts per million. A second similar test was started with a decomposition of 90.4 per cent similarly obtained, and regular steaming was continued for five hours. The last three samples were almost the same and averaged 94.2 per cent at a boiler-water alkalinity of 3500 parts per million.

No chemical reaction is absolutely instantaneous, but requires some time. Under some conditions the time required may approach zero. Usually in the early stages the speed is very rapid, but as completion is approached and the amount of reacting substance becomes small, the speed decreases considerably. Even under favorable conditions it may require considerable time for completion. Judging from this line of reasoning, it may be possible in the rapidly steaming modern high-pressure boiler, where as many as 15 concentrations may occur in an hour, that the decomposition rate for a very pure water may be exceeded and sodium carbonate accumulate in the boiler faster than it can decompose to maintain an equilibrium dependent upon partial pressure of the carbon dioxide. Such a boiler would show a lower decomposition, and this may be the reason for the low decomposition occurring in some boilers. In the test boiler a change of from one concentration per hour to almost four per hour did not produce any appreciable drop in decomposition.

Most of the boiler-water data that the author has examined show per cents of decomposition lower than that predicted by the curves on Fig. 10, or what might be predicted at some other boiler-water alkalinity corresponding to the case at hand. There are various ways that this may occur. The use of insufficient lime in a lime-soda plant may leave the water bicarbonate and be interpreted as carbonate. Some plants treat with carbon dioxide after passing the softening plant to prevent precipitation of the normal carbonate in the feed lines and heater.

The methods used for determination of carbonate and hydroxide in the average boiler plant are open to error. The presence of certain organic matter in a boiler water is known to displace the methyl-orange end-point and to indicate high carbonate. In the presence of sodium sulphate the methyl-orange end-point is not so easily defined, is likely to be exceeded, and indicates high carbonate. The Winkler barium-chloride method in the presence of sodium sulphate gives too low an indication of hydroxide. In all, the ordinary methods are likely to indicate low decomposition. The direct method of breaking down the carbonates and absorbing the carbon dioxide evolved is tedious and hardly adapted to boiler-plant routine.

Some plants have reported decomposition higher than 95 per cent. Such might occur in a central station using a small evaporated make-up. However, overtreatment with lime in a lime-soda softening plant would result in such a boiler water, high in hydroxyl ion and indicating a higher decomposition than actually exists. A recent test with sodium hydroxide also in the water gave a decomposition several per cent higher than what could be predicted from the sodium carbonate alone.

Summary and Conclusions

The principal controlling factor in establishment of the extent of decomposition of sodium carbonate in a boiler water is the partial pressure of the carbon dioxide in the vapor phase. This in turn depends on the amount and availability of the potential carbon dioxide contained in the feedwater.

The effect of boiler operating pressure decreases with a decrease in carbonate content of the feedwater.

For the range of steaming rates covered in these tests, boiler rating does not appear to be a factor governing the extent of decomposition.

The total alkalinity existing in a boiler water is a factor, the extent of decomposition decreasing with increase in boiler-water alkalinity.

Discussion

W. L. Abbott.[6] For several years past the Engineering Experiment Station of the University of Illinois has been investigating the phenomenon of caustic embrittlement of boilers due to the decomposition of sodium carbonate contained in the boiler water. This study, which has been carried on by the author of the paper and Professor Straub, at first under the direction of Dr. Parr and later under Dr. Keyes, is tardily being recognized as a great achievement in increasing the safety of steam boilers.

In the light of these discoveries the cause of many mysterious boiler explosions is made plain and the means for foreseeing and preventing such disasters are recognized and understood by such authorities as boiler manufacturers, boiler insurance companies, and the Boiler Code Committee of the A.S.M.E.

Naturally such a radical doctrine as the theory of caustic embrittlement, which ran counter to time-honored theories, which appeared to be at variance with experience in numerous instances, and which jostled business interests, had to prove its merit before being generally accepted; however, few there are now who deny the reality and the danger of the phenomenon of caustic embrittlement, although some would call it by some other name.

Fortunate, indeed, are boiler makers and boiler users that this discovery of the nature and cause of caustic embrittlement came as early as it did and preceded the higher boiler pressures and the higher boiler ratings now in general use, as it is in high temperatures and in high concentrations of solubles in the boiler water that the embrittling action is most pronounced.

Dr. Samuel W. Parr will long be remembered for his many and valuable contributions to science and industry in the field of applied chemistry, but none of those discoveries better illustrate his keen and practical research instinct than is shown in his discovery, isolation, and treatment of the bacillus of embrittlement and none of them are of a greater and more practical value to industry.

Everett P. Partridge.[7] The subject of carbonate decomposition in steam boilers has always been obscured by inadequate data. The extensive investigation by Mr. Larson is the first important contribution in this field, but it also suffers from this same defect, since carbonate concentrations were determined by standard methods which give high values, and the figures for decomposition are therefore somewhat lower than the true values. This is the case particularly in the more dilute solutions. Thus when an apparent decomposition of 90.4 per cent was obtained with a feed containing 10 p.p.m. sodium carbonate, and this increased to an apparent value of 94.2 per cent after feeding untreated water, it is probable that practically complete decomposition had really been obtained before the last three samples were taken.

In laboratory studies of the hydrolysis of carbonates at atmospheric pressure the writer has found that a dilute carbonate solution which had been almost completely hydrolyzed, as indicated by determination of its carbon dioxide content by an evolution process, showed an apparent residual carbonate of as high as 20 per cent of the total alkalinity when titrated with phenolphthalein and methyl orange.

In a paper soon to be published by Schroeder and Partridge it is shown that both the standard phenolphthalein-methyl orange and the Winkler barium chloride titrations give inaccurate high values for carbonate in solutions containing small concentrations of carbonate and relatively high concentrations of hydroxide. Among other things this work checks the observations of Mr. Larson on the unreliability of the Winkler method in solutions containing sulphates. It should be noted that checks between the phenolphthalein-methyl orange and barium chloride methods do not necessarily indicate accuracy in the determination of carbonate.

The fact that the analytical methods employed by the author tend to give high values for carbonate means that the lines in all of his figures showing per cent decomposition should actually lie somewhat higher than they do. In Figs. 10 and 11 the lines should actually reach 100 when the abscissas are zero.

The author has referred at several points to a condition of "equilibrium" in his experimental boiler. It would seem highly desirable in this work of a chemical nature to refer to this condition as a "steady state" and to reserve the term "equilibrium" for use in its precise sense. Thus, if the author had used a closed system in which the steam produced was condensed and returned to the boiler with its content of carbon dioxide, the eventual balance between hydroxide and carbonate in solution and carbon dioxide in the steam might have approached true equilibrium at low ratings. If the author had closed his boiler tightly and held it at constant temperature for a sufficient period of time, true equilibrium would probably have been attained. Incidentally, it is to be regretted that this was not done, since a determination of the actual equilibrium at each boiler pressure would have made it possible to measure the deviation from equilibrium of any steady state during operation, and would have made it possible to verify experimentally the statement that the partial pressure of carbon dioxide in the steam is the factor which principally controls the decomposition reaction. It is also unfortunate that carry-over of boiler water prevented accurate determination of the carbon dioxide in the steam.

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[6] Chief Operating Engineer, Commonwealth Edison Company, Chicago, Ill. Mem. A.S.M.E.

[7] Supervising Engineer, Non-Metallic Minerals Experiment Station, U. S. Bureau of Mines, New Brunswick, N. J.

As the author has recognized, the process of converting carbonate into hydroxide with the evolution of carbon dioxide in the steam may be regarded as occurring in two steps, the first step being the hydrolysis of carbonate to hydroxide with the production of dissolved carbon dioxide, and the second being the transfer of this carbon dioxide from the solution to the vapor phase. The ionic reactions of the first step probably proceed more rapidly than the essentially physical process of the second step. The partial pressure of carbon dioxide in the steam will then control the rate of decomposition only in so far as equilibrium exists between it and the dissolved carbon dioxide in the solution.

It is the belief of the writer that the partial pressure of carbon dioxide in the steam should be regarded more as a result of conditions in the boiler water than as a principal factor in controlling these conditions. The greater degree of decomposition obtained with falling water level and the smaller degree of decomposition with rising water level do not necessarily indicate an equilibrium controlled by the partial pressure of carbon dioxide in the steam space. These effects may rather be interpreted as due to change in the rate of introduction of carbonate relative to the more or less constant rate of removal of carbon dioxide in the steam. The results from the use of $NaHCO_3$ in place of Na_2CO_3 likewise do not demonstrate that the partial pressure of carbon dioxide in the steam controls the decomposition, although the amount of carbon dioxide which must be transferred from the water to the steam does undoubtedly determine the degree of decomposition.

Bearing in mind the fact that the author's figures for carbonate decomposition are probably consistently low, it is apparent from his results that the actual amount of carbonate present in any boiler operating under ordinary conditions is so low as to be almost negligible from the standpoint of regarding sodium carbonate as an inhibitor of caustic embrittlement.

Leo F. Collins.[8] During 1929 a set of experiments, similar to those conducted by the author, were run at the Congress Street Heating Plant of The Detroit Edison Company. These experiments were instigated because of the fact that when using sodium carbonate as a boiler-water conditioner it was difficult to maintain the proper ratios of sulphate to carbonate to prevent scale and also to meet the A.S.M.E. recommendations to thwart intercrystalline cracking of the boiler metal. The rapid decomposition of sodium carbonate was the controlling factor. Accordingly it was decided to determine, if possible, what those factors are which govern the decomposition of sodium carbonate as criteria for improving the method of treatment. Since the conclusions drawn were substantially the same as those drawn by the author, it is hoped that the presentation of some of the data will be appropriate.

The first attempt made was to determine the rate of decomposition in an operating boiler. The boiler, a W-type Stirling (operating at 125 lb. gage and having a rated capacity of 36,600 lb. of steam per hour), was filled with fresh water containing no sodium alkalinity and was put into operation at its rated capacity.

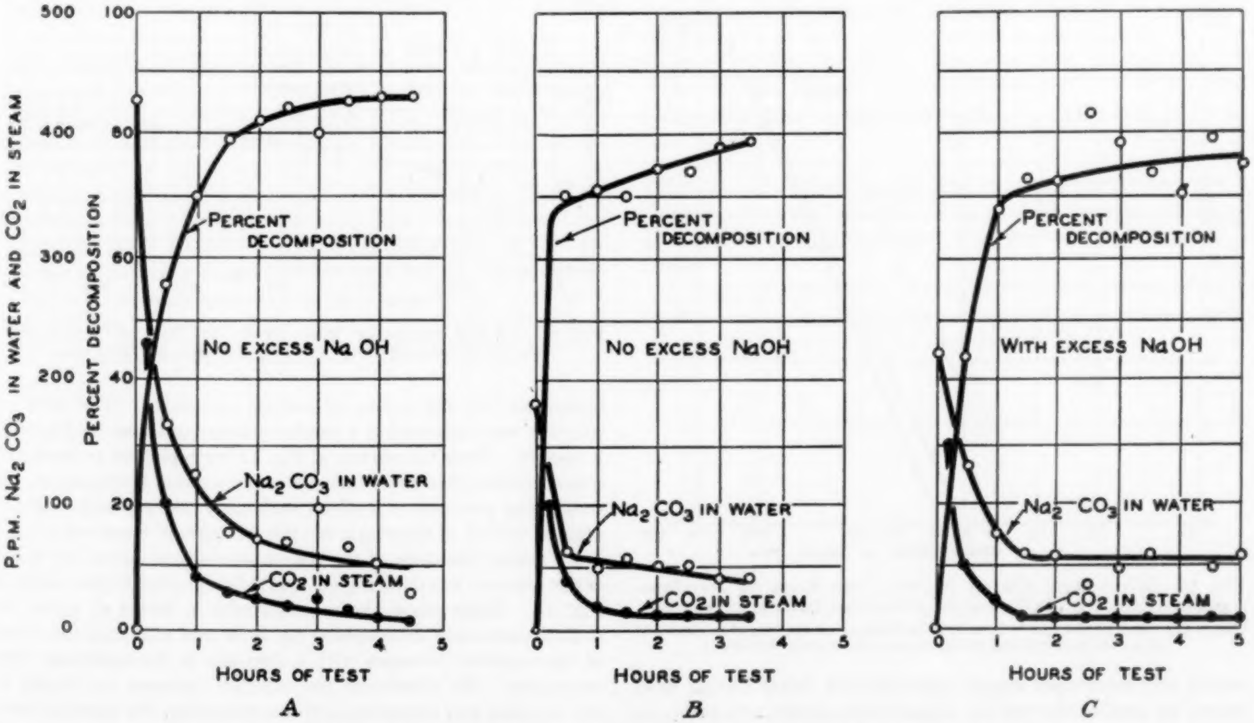

Fig. 12 Curves Showing the Progress of Decomposition of Sodium Carbonate in a Boiler

(With and without an excess of sodium hydroxide present. Boiler operating at 125 lb. pressure and at 100 per cent of rating.)

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[8] Chemist, The Detroit Edison Company, Detroit, Mich.

A weighed amount of sodium carbonate, 430 p.p.m., was then introduced directly into the boiler through the auxiliary feed line. At half-hour intervals samples of feedwater, blowoff water, and condensed steam were collected (after having been cooled by passing through the copper cooling coils) and analyzed. In this manner the data for test *A*, Fig. 12, were collected. In a similar manner the data for test *B* were collected, but the initial dosage was only 188 p.p.m. of sodium carbonate. It will be noted that in these tests complete decomposition was not attained. This, it is believed, was caused by the presence of approximately 10 p.p.m. of CO_2 in the feedwater, which retarded the decomposition.

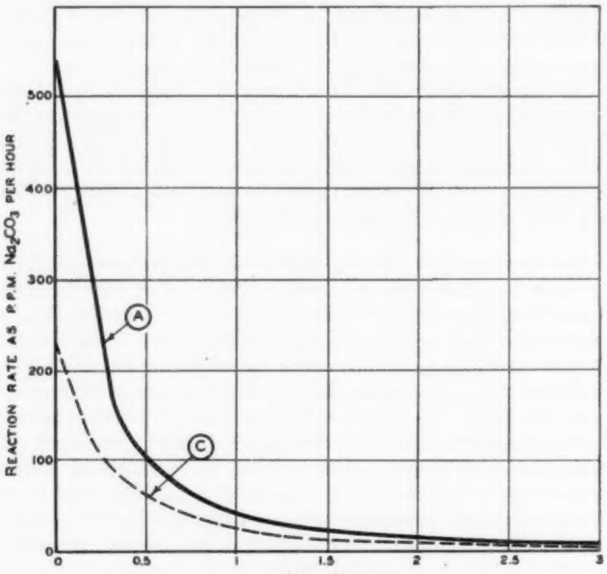

FIG. 13 CURVES SHOWING REACTION RATE $\Delta C/\Delta T$ DURING DECOMPOSITION OF Na_2CO_3

(As shown by tests *A* and *C* of Fig. 12. ΔC = rate of decomposition, p.p.m. Na_2CO_3 per hour. ΔT = time in hours. Curve *A* = without excess NaOH and with high concentration Na_2CO_3. Curve *C* = with excess NaOH and with small concentration Na_2CO_3.)

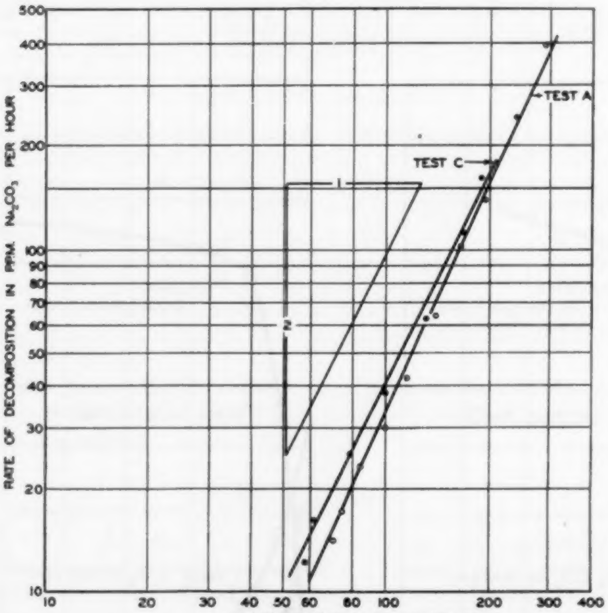

FIG. 14 LOGARITHMIC GRAPH SHOWING THE RATE OF DECOMPOSITION OF Na_2CO_3 AS A FUNCTION OF THE Na_2CO_3 CONCENTRATION

(Since the slope of the line is 2 to 1, it indicates that the reaction proceeds at a rate proportional to the square of the concentration.)

In both of these tests it will be noted that there was initially present no sodium hydroxide. It was desired to know what the effect of this alkali was on the rate of decomposition of sodium carbonate. Accordingly when it was found, in test *B*, that the reaction had practically ceased, although there was still present about 40 p.p.m. of sodium carbonate, the boiler was given another dosage of 188 p.p.m. of the salt. Hence at the beginning of test *C*, the boiler water contained about 111 p.p.m. of sodium hydroxide and 228 p.p.m. of sodium carbonate. The data for this test were collected in a similar manner to those used in tests *A* and *B*. Thus the curves of Fig. 12 were plotted to show the concentration of sodium carbonate and sodium hydroxide in the water, the per cent of sodium carbonate decomposed, and the amount of CO_2 in the steam for the duration of these tests.

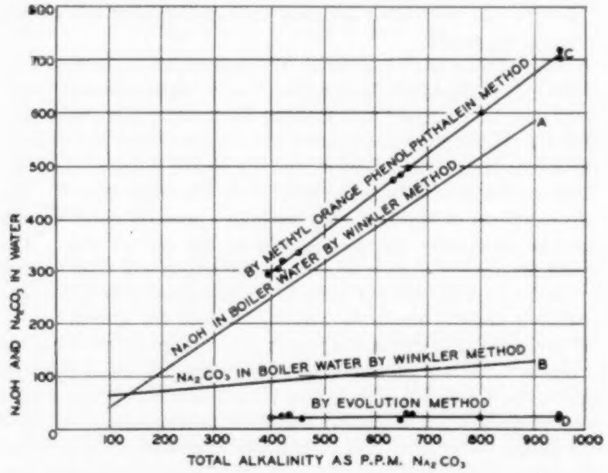

FIG. 15 CURVES SHOWING INCREASE IN Na_2CO_3 CAUSED BY INCREASING NaOH CONCENTRATION IN BOILER WATER

(The CO_2 in steam remained practically constant at about 15 p.p.m.)

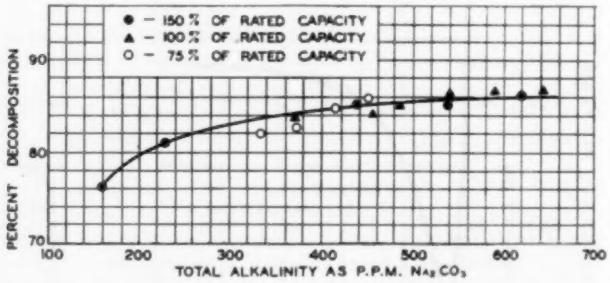

FIG. 16 CURVE SHOWING PER CENT OF BOILER ALKALINITY THAT IS NaOH FOR DIFFERENT BOILER RATINGS

By taking the slope of the sodium-carbonate curve, the speed of the reaction was determined, and this is plotted in the curves of Fig. 13. These show the reaction rate in terms of p.p.m. of sodium carbonate decomposed per hour and also that the speed of the reaction decreases with a decrease in the carbonate concentration. To determine the relation between the speed of the reaction and the carbonate concentration, the reaction rates from the curves of Fig. 13 and the concentration of sodium carbonate present in the water, from the appropriate curves of Fig.

12 (tests *A* and *B*) for the same time interval, were plotted on a log scale.

These curves, Fig. 14, have a slope of approximately 2 to 1. The slope of the curve for test *C* is less than for test *A*, which indicates that the presence of sodium hydroxide decreased the rate of the reaction. Both of the curves, however, indicate that for the conditions encountered the rate of decomposition is approximately proportional to the square of the concentration of sodium carbonate present.

With these data as criteria it was decided to determine just how high the concentration of sodium carbonate could be raised in this boiler by increasing the sodium-hydroxide concentration. The concentration of sodium hydroxide was increased as shown by curve *A*, Fig. 15, and it was found that the sodium-carbonate concentration increased as shown by curve *B*. The data for these curves were collected with the boiler operating at 75, 100, and 150 per cent of rated capacity. The effect of differences in steaming rates could not be noted. The curve of Fig. 16 demonstrates this fact. The points as plotted in Fig. 16 are average values for many analyses and were obtained when a constant amount of sodium carbonate was being added to the feedwater.

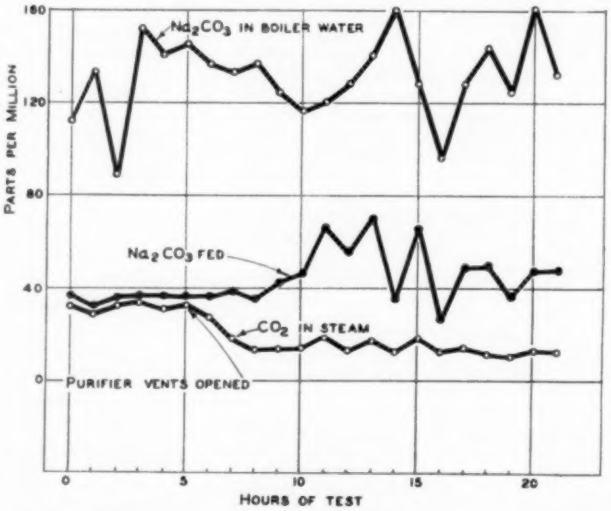

Fig. 17 Curves Showing Interrelation of Sodium Carbonate in Boiler Water, Sodium Carbonate Fed, and CO_2 in Steam Leaving Boiler

(The purifier vents were closed for the first five hours of the test, and then were opened.)

In the foregoing tests the hydroxide and carbonate were determined by the Winkler method. In collecting the data for Fig. 12 there was present no appreciable quantity of sodium sulphate, and hence it is believed that the data are quantitative. However, the data for curves *A* and *B* of Fig. 15 are not believed to be quantitative because of the inaccuracy of the Winkler method when sodium sulphate is present. On the same plot are shown the hydroxide and carbonate concentrations determined by the more accurate methods, and these figures are believed quantitative.

The agreement between the two methods is poor, but each indicates that the sodium carbonate increases with increase in the hydroxide concentration of the water, and this was the object of the test. The relation holds true, however, when the CO_2 and sodium carbonate entering the boiler are constant quantities. Fluctuations in the concentration of either will noticeably affect the carbonate concentration in the boiler water, as is shown in Fig. 17. Blowing down a boiler has the same effect.

In conclusion it appears that for the conditions encountered in these tests the decomposition of sodium carbonate is governed by the following:

(*a*) The partial pressure of CO_2 in the vapor phase

(*b*) The concentration of sodium carbonate present in the boiler water

(*c*) The concentration of sodium hydroxide in the boiler water.

These conclusions, drawn from an operating boiler, are substantially the same as those drawn by Mr. Larson in the laboratory.

Author's Closure

The author has on hand data that will substantiate the accuracy of the methods of analysis used in these series of experiments. No blanks were deducted from the titration values, but inasmuch as the amounts of acid used were for the most part 20 cc. or more, the error introduced should not exceed a few per cent. Particular care was taken to keep this source of error small. If the amount of sample and the concentration are such that only a few cubic centimeters of acid are used, this error is proportionately large and may account for the apparent high carbonate observed by Mr. Partridge. The author has checked the A.P.-H.A. and the Winkler barium-chloride methods with a carefully conducted CO_2 evolution on the same sample of pure solution where the per cent decomposition was approximately 90 per cent and the total carbonate content about 200 p.p.m. The determination of carbonate content for 100 p.p.m. or less by CO_2 evolution calls for excessively large samples in order that errors in the analytical balance and those due to contamination be small.

A series of experiments with the true chemical equilibrium of this reaction at high pressures is under way, the data from which are to be published at an early date.

DEPOSITORIES FOR A.S.M.E. TRANSACTIONS IN THE UNITED STATES

Alabama	Auburn	Engineering Library, Alabama Polytechnic Institute
	Birmingham	Public Library
Arkansas	Fayetteville	Engineering Library, Arkansas University
California	Berkeley	Library, University of California
	Long Beach	Public Library
	Los Angeles	Public Library University of Southern California
	Oakland	Oakland City Library Teachers' Professional Library
	Pasadena	Library, California Institute of Technology
	Santa Clara	Library, University of Santa Clara
	San Diego	Public Library
	San Francisco	Public Library (Civic Center) Engineers Club of San Francisco Mechanics Institute
	Stanford University	Library, Stanford University
Colorado	Boulder	Library, University of Colorado
	Denver	Public Library
	Fort Collins	Colorado State Agricultural College
Connecticut	Bridgeport	Public Library
	Hartford	Public Library
	New Haven	Public Library Library, Yale University
	Waterbury	Silas Bronson Library
Delaware	Newark	University of Delaware
	Wilmington	Wilmington Free Institute
District of Columbia	Washington	Scientific Library, United States Patent Office Library of Congress Bureau of Standards Library George Washington University Catholic University
Florida	Gainesville	University of Florida
	Jacksonville	Free Public Library
	Miami	Public Library
	Tampa	Public Library
Georgia	Atlanta	Carnegie Public Library Georgia School of Technology
	Savannah	Public Library
Idaho	Moscow	University of Idaho
Illinois	Chicago	John Crerar Library Western Society of Engineers Library, Armour Institute of Technology Public Library of Chicago
	Moline	Public Library
	Urbana	University of Illinois
Indiana	Evansville	Public Library
	Fort Wayne	Public Library
	Indianapolis	Public Library Indiana State Library
	Notre Dame	Library, University of Notre Dame
	Terre Haute	Rose Polytechnic Institute
	West Lafayette	Library, Purdue University
Iowa	Ames	Iowa State College
	Des Moines	Public Library
	Iowa City	State University of Iowa
Kansas	Kansas City	Public Library, Huron Park
	Lawrence	Library, University of Kansas
	Manhattan	Kansas State Agricultural College
	Wichita	Wichita City Library
Kentucky	Lexington	University of Kentucky
	Louisville	Speed Scientific School University of Louisville
Louisiana	Baton Rouge	Louisiana State University
	New Orleans	Howard Memorial Library Louisiana Engineering Society Public Library
Maine	Orono	University of Maine
Maryland	Annapolis	United States Naval Academy
	Baltimore	Johns Hopkins University Engineers Club of Baltimore Public Library
Massachusetts	Boston	Northeastern University Boston Public Library
	Cambridge	Harvard University (Engineering Library) Massachusetts Institute of Technology
	Fall River	Public Library
	Lowell	Lowell Textile Institute
	Lynn	Free Public Library
	New Bedford	Free Public Library
	Springfield	Springfield City Library
	Tufts College	Tufts College
	Worcester	Worcester Polytechnic Institute Free Public Library
Michigan	Ann Arbor	University of Michigan
	Detroit	Public Library Cass Technical High School Highland Park Public Library University of Detroit
	East Lansing	Michigan State College
	Flint	Public Library
	Grand Rapids	Public Library
	Houghton	Michigan College of Mining & Technology
	Jackson	Public Library
Minnesota	Duluth	Public Library
	Minneapolis	University of Minnesota Minneapolis Public Library (Engineering and Circulating Libraries)
	St. Paul	James Jerome Hill Reference Library
Mississippi	Columbus	Mississippi Agri. & Mech. College
Missouri	Columbia	University of Missouri
	Kansas City	Public Library
	Rolla	Missouri School of Mines and Metallurgy
	St. Louis	Engineers Club of St. Louis Public Library Washington University Mercantile Library
Montana	Bozeman	Montana State College
Nebraska	Lincoln	University of Nebraska
	Omaha	Public Library

Nevada

Reno	University of Nevada Library

New Hampshire

Durham	University of New Hampshire

New Jersey

Bayonne	Free Public Library
Camden	Free Public Library
Elizabeth	Free Public Library
Hoboken	Stevens Institute of Technology
Jersey City	Free Public Library
Newark	Newark College of Engineering
	Free Public Library
New Brunswick	Rutgers University
Paterson	Free Public Library
Princeton	Princeton University
Trenton	Free Public Library

New York

Albany	New York State Library
Brooklyn	Polytechnic Institute
	Pratt Institute
	Brooklyn Public Library
Buffalo	The Grosvenor Library
	Engineering Society of Buffalo
	Buffalo Public Library
Ithaca	Cornell University
Jamaica, L. I.	Queens Borough Public Library
New York	Engineering Societies Library
	Public Library
	College of the City of New York
	Cooper Union
	Columbia University
	N. Y. University Library
Potsdam	Clarkson College of Technology
Rochester	Rochester Engineering Society
Schenectady	Union College
Syracuse	Syracuse University
	Public Library
Troy	Rensselaer Polytechnic Institute
Utica	Public Library
Yonkers	Public Library

North Carolina

Chapel Hill	University of North Carolina (Engineering Library)
Raleigh	North Carolina State College

North Dakota

Fargo	North Dakota State College
Grand Forks	University of North Dakota

Ohio

Ada	Ohio Northern University
Akron	Public Library
	University of Akron
Canton	Public Library
Cincinnati	University of Cincinnati
	Public Library
	Engineers Club of Cincinnati
Cleveland	Public Library
	Case School of Applied Science
	Cleveland Engineering Society
Columbus	State of Ohio Library
	Public Library
	Ohio State University
Dayton	Engineers Club of Dayton
Toledo	Public Library
	University of Toledo
Youngstown	Public Library

Oklahoma

Norman	Oklahoma University
Oklahoma City	Public Library
Stillwater	Oklahoma Agricultural and Mechanical College
Tulsa	Public Library

Oregon

Corvallis	Oregon State Agricultural College
Portland	Portland Library Association

Pennsylvania

Allentown	Free Library
Bethlehem	Lehigh University
Easton	Public Library
	Lafayette College
Erie	Public Library
Lewisburg	Bucknell University
Philadelphia	Engineers Club
	Drexel Institute
	University of Pennsylvania
	Franklin Institute
Pittsburgh	University of Pittsburgh
	Engineers' Society of Western Pennsylvania
	Carnegie Institute of Technology
	Carnegie Library (Schenley Park)
	Carnegie Free Library of Allegheny
Reading	Public Library
Scranton	Public Library
State College	Pennsylvania State College
Swarthmore	Swarthmore College
Villanova	Villanova College
Wilkes-Barre	Public Library

Rhode Island

Kingston	Rhode Island State College
Providence	Brown University
	Providence Engineering Society
	Public Library

South Carolina

Clemson College	Library, Clemson College

Tennessee

Kingsport	Public Library
Knoxville	University of Tennessee
Memphis	Goodwin Institute
Nashville	Vanderbilt University

Texas

Austin	University of Texas
College Station	Agricultural & Mechanical College of Texas
Dallas	Public Library
El Paso	Public Library
Fort Worth	Carnegie Public Library
Houston	Rice Institute
	Public Library
Lubbock	Texas Technological College (School of Engineering)
San Antonio	Carnegie Library

Utah

Salt Lake City	University of Utah
	Public Library

Vermont

Burlington	University of Vermont

Virginia

Blacksburg	Virginia Polytechnic Institute
Charlottesville	University of Virginia
Norfolk	Carnegie Public Library
Richmond	Virginia State Library

Washington

Pullman	State College of Washington
Seattle	Public Library
	Engineers Club
	University of Washington
Spokane	Public Library
Tacoma	Public Library

West Virginia

Morgantown	West Virginia University

Wisconsin

Madison	Library, University of Wisconsin
Milwaukee	Public Library
	Board of Industrial Education, Vocational School Library
	Marquette University

Wyoming

Laramie	Wyoming University

FSP-54-14

Radiant-Superheater Developments

By M. K. DREWRY,[1] MILWAUKEE, WIS.

Sustained high steam temperatures at all operating ratings, not peak readings at one narrow load range, are being sought in the industry's quest for higher operating efficiencies. Uniform average temperatures, not maximum values, determine commercial economy and even safety of higher steam temperatures. Since premiums are paid for high-temperature equipment on the basis of maximum temperatures, it is important that operating temperatures be uniformly high. Decreasing trends of boiler duty and increasing tendencies of superheater and reheater duty, plus multi-valving of turbines, emphasize the inadequacy of purely convection surfaces for maintaining uniform temperatures. A 200-deg. fahr. drop in reheated-steam temperature from full load to quarter load when using only convection surfaces was diminished to 20-deg. fahr. decrease in the planning of a recent large 1200-lb., 850-deg. fahr., single-turbine, single-boiler installation, simply by prescribing two-thirds radiant-superheating equipment. Objectives of this paper are:

1 To tell of considerable experiences with radiant superheaters in a station operating many since 1923, showing that their design and operation are based upon sound fundamental principles that have been proved by successful service.

2 To give operating data evaluating the desirable "rising" radiant characteristic at lower loads and to state basic differences of radiant and convection superheaters.

3 To treat the proper applications and to list the incidental merits of radiant superheaters.

4 To treat desirable application of radiant superheaters and reheaters in future large high-temperature, high-pressure, single-boiler, single-turbine installations, illustrating with calculated data based upon experience that essentially constant temperature can be obtained at all ratings.

Experiences in only one station are cited, yet developments of the industry prompting use of all designs of radiant superheaters are treated.

PART I

Operating Experiences

TWENTY boiler-units at the Lakeside Station have been equipped since 1923 with various types of radiant superheaters as the art advanced, and thoroughly satisfactory installations (shown in Figs. 4, 5, and 6) were developed. Original installations employing the special heavy forged-steel elements proved that massiveness of element structure was unnecessary, and in fact undesirable, for 300-lb. steam-pressure service.

Figs. 1 to 3 show these early superheaters. Fig. 7 contrasts their heavy cross-section with the simple light-weight elements later found so desirable. Table 1 gives major information and data of all installations, treating them chronologically.

Since the simple, light-weight round-tube design first used in 1927 is the basis of present designs, and since information gathered during its development is still of constructive value, this paper will deal with it almost wholly. Fig. 4 pictures the first installation, and data are given in the "Installation F" column of Table 1.

The Starting Problem

How to get the light 2-in. tubes past the starting period, when no steam was available to cool them, without overheating, was a problem difficult of practical solution. Unlike the case of their predecessors, scarcely no storage of heat in the metal of the elements could be depended upon to prevent overheating, for the new elements weighed only one-tenth as much per equivalent heat-absorption area. Calculations showed exceedingly rapid heating, and served to warn of the starting dangers.

Cooling with steam from other boilers, or with its own after exceeding atmospheric pressure, was found not only inadequate, but impractical. Calculations showed that opening reasonable-sized bleeders would give only a false sense of security. Since the radiant elements were in series with convection ones, high excess-air variations would be of no value in reducing steam temperatures because of the opposing temperature characteristics of the two sections.

Furnace temperature taken previously in a similar furnace suggested that if the coal-feed rate was set at a certain particular value immediately upon lighting the burners, and maintained positively uniform until header pressure was reached and steam flow created, then starting would be reasonably rapid, and maximum flame temperature at the superheaters would not exceed 1000 deg. fahr. Obviously, the tubes could not exceed the flame temperature however long they were exposed. If the installation could be brought into service within a reasonable length of time, and if the operating personnel could be acquainted with the new method and would use it consistently, then the first step toward success of the new elements was assured.

Previous starting practice consisted of gradually increasing coal feed until it became quite high upon opening of the non-return valves, and consumed three hours when starting with cold boiler water. The special steady-coal-feed method of starting required four hours, but was considered practical in view of benefits likewise derived by other portions of the boiler unit.

Element-metal temperatures, furnace temperatures, radiant-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 SUMMARIZED RADIANT-SUPERHEATER EXPERIENCE AT THE LAKESIDE STATION

Installation order	A	B	C	D	E	F	G	H	I		K
General:											
1 Year	1923	1924	1925	1926	1927	1927	1927	1927	1928	1929	1930
2 Boiler unit no.	7	9–16	1–8	17	17	5	1, 2, 4, 6, 7, 8	3	9–16	19	18, 20
3 Nature	Trial	Orig.	Orig.	Orig.	Orig.	Replace.	Replace.	Replace.	Replace.	Orig.	Orig.
4 Service	Suph.	Suph.	Booster	Suph.	Reheat	Booster	Booster	Booster	Suph.	Suph.	Suph.
Design:											
5 Form of element	Rect.	Rect.	Rect.	Rect.	Rect.	Round	Round	Round	Round	Round	Round
6 Distance from wall	Flush	Flush	Flush	Flush	Flush	10 in.	10 in.	10 in.	10 in.	Flush	Flush
7 Steel used	Carbon	0.45 C	0.45 C	0.45 C	0.45 C	0.10 C	Chr. Va	Enduro	Chr. Va	Chr. Va	Chr. Va
8 Area, sq. ft.	161.5	335	109	910	690	96	144	144	418	902	902
9 Weight per sq. ft. of area	.....	80	80	90	80	7.6	7.6	7.6	7.6	13.3	13.3
10 Pressure stress, lb. per sq. in.	.....	.....	.....	.....	.....	1,200	1,200	1,200	1,200	3,000	3,000
11 Normal steam velocity, ft. per sec.	90	100	70	50	130	180	120	120	160	50	50
12 Mass flow rate, lb. per hr., per sq. ft.	150,000	162,000	115,000	400,000	214,000	292,000	195,000	195,000	262,000	440,000	440,000
Operating Data:											
13 Normal steam flow, lb. per hr.	32,500	105,000	65,000	250,000	250,000	65,000	65,000	65,000	105,000	250,000	250,000
14 Steam pressure, lb. gage	300	300	300	1,300	300	300	300	300	300	1,300	1,300
15 Steam temperature, deg. fahr.	700	710	710	740	700	700	720	720	710	750	750
16 Normal transfer rate, B.t.u. per sq. ft. per hr.	35,000	53,000	44,000	45,000	53,500	57,000	37,500	37,500	44,000	45,000	45,000
17 Estimated maximum metal temp., deg. fahr.	.....	1,050	1,050	950	1,050	950	950	950	950	900	900

(Elements changed from special rectangular cross-section to simple round tubes. Weights per square foot of exposed area decreased from 80 lb. to 8 lb. Steam velocities increased from 90 ft. per sec. to 160 ft. per sec. for better metal cooling. The 1300-lb. pressure superheaters are the most recent, and though their steam is 40 deg. fahr. higher and its velocity is lower, their metal temperature is 50 deg. fahr. lower than that of the 300-lb. elements. Experience with 37 radiant-superheater installations is summarized.)

NOTE: "Rect." signifies rectangular shape shown in cross-section, Fig. 7. Installation A superheated half of boiler output. Areas of installations J and K are based upon projected tube area, as are transfer rates.

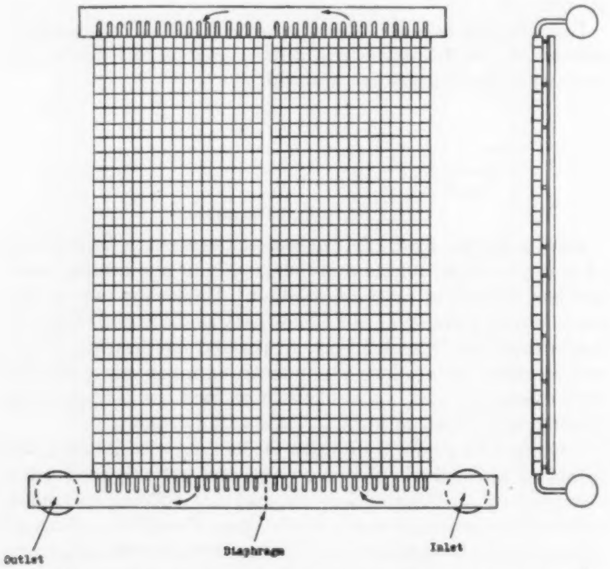

FIG. 1 INSTALLATION A; TRIAL RADIANT SUPERHEATER, NO. 7 BOILER

(Half of a convection superheater was removed to try this installation composed of cast-iron blocks shrunk on to 2-in. steel tubes. Heat-transfer rates and other characteristics were obtained, but the cast-iron element construction was proved impractical, and steel elements were later substituted. This installation was made in 1923, and upon it was based the design of completely radiant superheaters for eight 150,000-lb.-per-hr. boiler units, installation B.)

heat transfer rates, and all significant information were taken continuously during the starting and first days of operation. Conscientious, practical opinion had conceded only a few hours' life of the new elements, and reliance upon theory, only, was not reassuring enough to eliminate doubts of their safety after passing the starting period. One had only to look at the small tubes projecting into the almost white-hot flames to doubt all theory and suspect rapid failure.

Contrary to almost general expectations, the simple carbon-steel tubes lasted the day, the week, the month, and finally the year, without a single failure. Half of them are still in service after three and one-half years' operation. There are 800 other similar elements of a slightly better metal that have been in service three years with only two failures.

TABLE 2 METAL ANALYSES AND CONDUCTIVITIES

Metal	Carbon steel	Chrome-vanadium	Enduro A
Used in boilers	No. 5	All except No. 5 and No. 3	No. 3
Analysis:			
Carbon	0.10	0.20	0.10
Chromium	.....	1.0	17.0
Silicon	0.05	0.15	1.0
Manganese	0.41	0.75	0.40
Phosphorus	0.012	0.01	0.02
Sulphur	0.49	0.04	0.15
Nickel	.....	..	0.20
Vanadium	.....	0.20	.....
Thermal conductivity at 600 deg. fahr., B.t.u. per sq. ft., per inch thick, per deg. fahr., per hr	310	..	170
Relative values, per cent	100	..	55

(The chrome-vanadium steel is essentially only a refined carbon steel and is used extensively in the automobile industry—S.A.E. No. 6120. Its cost in tube form is about 15 cents per pound, and that of Enduro A is in excess of 50 cents per pound. Tensile-test properties are given in Fig. 14, and creep characteristics in Fig. 15.)

Application of exact physical laws, not approximate judgment, had suggested the way to an important development. Further replacements of the special heavy rectangular elements were made with tubes of a slightly better steel (see Table 2), and shortly 16 boilers were so equipped.

RADIANT-HEAT TRANSFER

That the new elements effectively absorbed heat throughout their periphery was proved immediately by the fact that only 60 per cent as many as of the heavy elements were required initially to get full steam temperature. Reflection from the brickwork backing wall not only helped maintenance of the elements by maintaining uniform metal temperatures front and rear, but increased heat-absorption efficiency. In radiant-superheater design it is desirable whenever steam scrubs a surface, and thus causes pressure drop, that useful-heat absorption occurs at that point. The superseded elements were not efficient from this standpoint, for only one face was heated, and connecting nipples caused much of the pressure loss. Study of the cross-sections in Fig. 7 will convey a comparison of the old and new elements.

METAL TEMPERATURES 950 DEG. FAHR.

Obviously, metal temperature, not steam temperature, is the vital determinant of radiant-superheater maintenance. Consideration of the reliability of high steam temperatures too often omits study of the temperature difference between heated steam and heating metal.

At the time when the first round-tube elements were installed, little information on the heat-transfer rate from metal tubes to

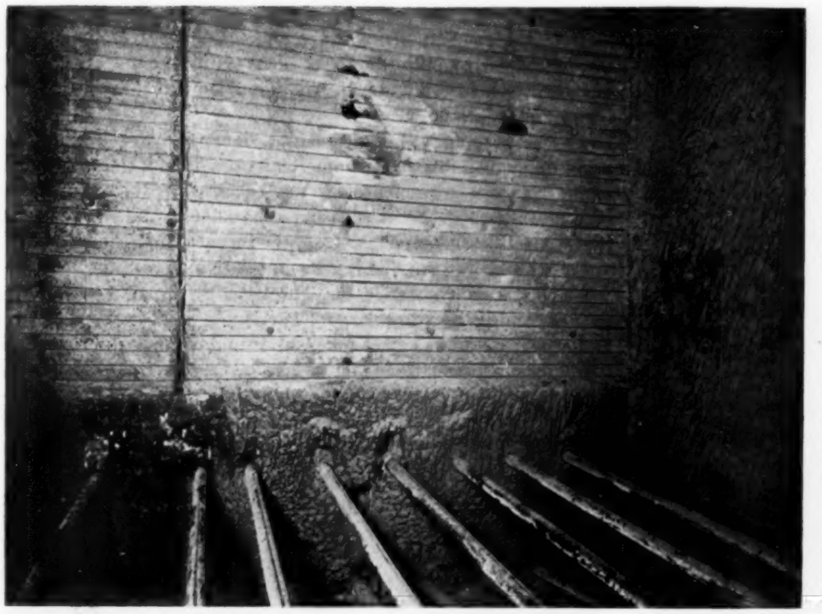

Fig. 2 Installation B; First Application

(A complete new boiler room with 700 deg. fahr. radiant superheaters was a distinct, progressive step in 1924. Installation C, made in the older boiler room to boost the 600 deg. fahr. temperatures also to 700 deg. fahr., was a somewhat similar design applied on another group of eight boilers. A 1927 extension utilized a similar design, as shown in Fig. 3.)

superheated steam was readily available. To supply the want, metal temperatures were measured, and heat-transfer rates to a water-cooled tube nearby, plus furnace temperatures, were observed as well. Chromel-alumel thermocouples, well peened into the tube metal and heat-insulated for a few inches from the tube with porcelains, were employed for the vital measurements. In connection with the second installation of round elements in boiler units 9 to 16, collection of this data was repeated, and the results of both as compared with those calculated from Nusselt's formula are also shown in Fig. 9. Fair agreement is apparent. Internal smoothness of the particular tubes used may affect the calculated values appreciably, and accurate metal-temperature measurement under the difficult furnace conditions is not easily accomplished.

Because of agreement of measured and calculated metal temperatures, it is quite safe to assume that the round superheater elements operate at a temperature not in excess of 950 deg. fahr. It is possible that this temperature may be exceeded sometimes when starting, but the fact that the observed creep was found influenced by steam-flow turbulence (see Fig. 10) shows quite positively that most creep occurs during operation with metal temperatures not exceeding 950 deg. fahr.

Creep

It was realized early that the new-type elements would probably be subject to creep, even though their operating temperature was not over 950 deg. fahr. and the tensile stress due to 300-lb. steam pressure was only 1200 lb. per sq. in. Accordingly, circumference measurements were taken early and frequently, and data shown in the illustrations (Figs. 10 to 13) were obtained.

Chief points and conclusions from these creep measurements were:

1 Rate of circumference increase was found directly proportional to operating hours, indicating that creep was responsible, not infrequent overheating due to improper operation (see Fig. 11).

2 Greatest increase in circumference occurred at the middle of the element, where minimum turbulence from the bends occurred. This also indicated that starting or irregular steam circulation was not responsible (see Fig. 10) for the expansion.

3 Since the first element reached 29 per cent enlargement before failure, it is reasonable to set 20 per cent maximum creep as practical for most superheater installations, providing oxidation does not cause premature failure.

4 Since actual operating values of metal temperature and stress lie considerably under the expected creep curve for low carbon steels (see Fig. 15), it is expected that stresses due to temperature gradient through the tube wall, fluctuation of metal temperatures due to changes in furnace conditions, and slight oxidation of the metal surface are important in correlating laboratory and service results.

5 Creep of Enduro A tubes is appreciable compared with the standard of laboratory results. Fig. 15 shows a wider divergence than with carbon-steel elements. Fig. 16 shows Enduro creep data.

6 Dangers of creep are greatly lessened by the fact that time of ultimate failure can be prophesied early with accuracy. The probable life of elements can be estimated

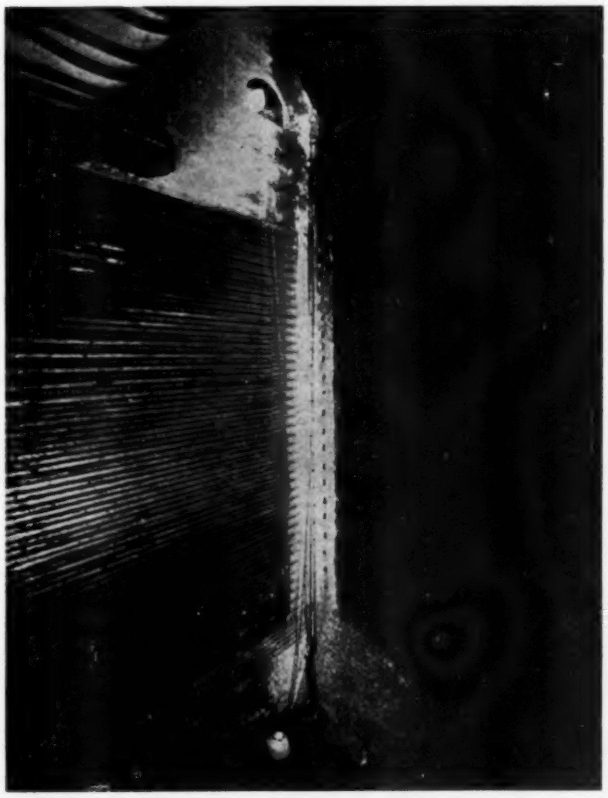

Fig. 3 1300-Lb. Application

(Both superheater and reheater were made radiant in the first 1300-lb. boiler unit. Due to the desirable heat-transfer characteristics of high-pressure steam, no maintenance has been required of the 1300-lb. superheater during its 4 years' service. The superheater is on the left, and the reheater is on the right.)

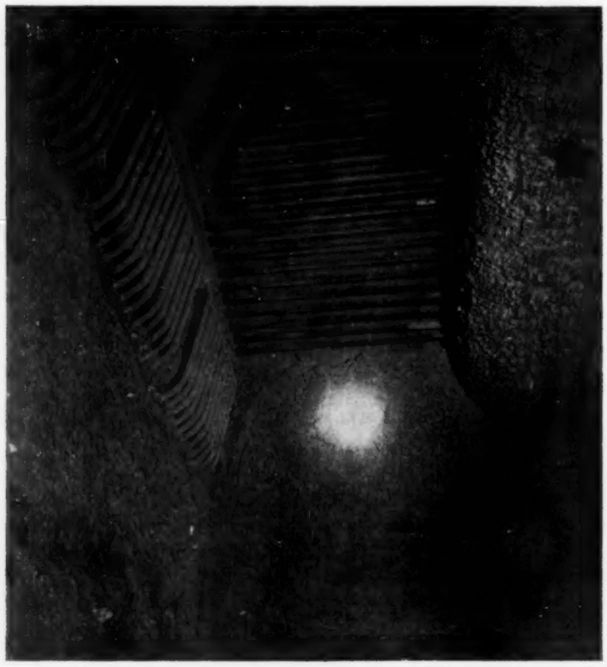

FIG. 4 SIMPLE REPLACEMENT ELEMENTS, INSTALLATION F

(Two years' life of the apparently rugged but actually weak rectangular elements prompted development of less expensive replacement elements. Above, at left, are simple low-carbon steel tubes, and though predicted to serve only a few hours before failure, are partly still in service after 3 years.)

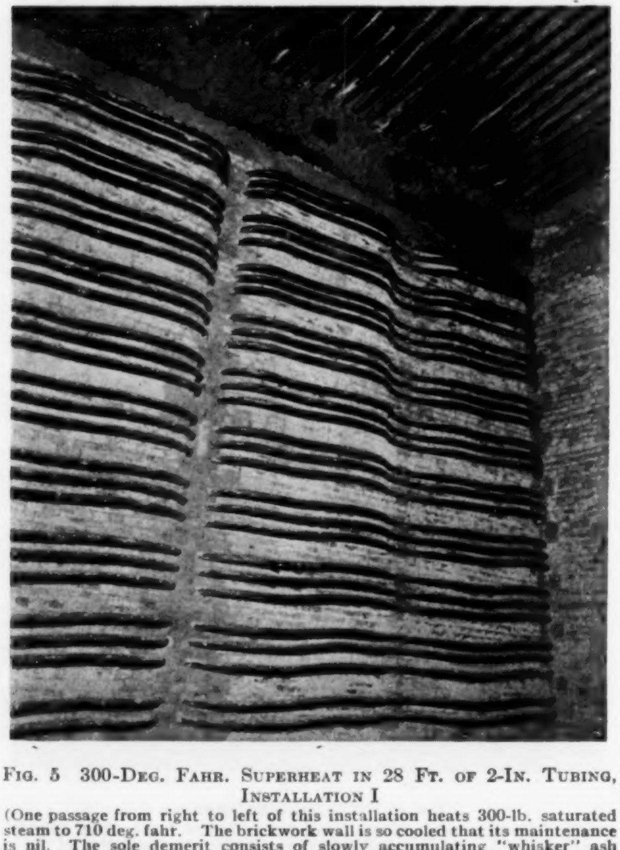

FIG. 5 300-DEG. FAHR. SUPERHEAT IN 28 FT. OF 2-IN. TUBING, INSTALLATION I

(One passage from right to left of this installation heats 300-lb. saturated steam to 710 deg. fahr. The brickwork wall is so cooled that its maintenance is nil. The sole demerit consists of slowly accumulating "whisker" ash deposits on the wall, requiring hand removal weekly or twice monthly. Intermediate supports are required to keep metal stresses very low, for creep occurs easily at 950 deg. fahr.)

within the first few months or years of operation if accurate micrometer measurements are made. Reliance can be placed upon trustworthy performance of tubes subject to measured rates of creep, and forced outages will not occur until estimated life is reached.

7 Use of standard, light-weight, inexpensive tubes having fairly high creep rates are commercially practical if removal every several years is convenient. So few tubes are required to obtain high steam temperatures with radiant surfaces, and renewal as a rule is so easily accomplished from the furnace, that the old standard of 20-year life may be less desirable than 5-year life.

8 Oxidation resistance may prove a principal merit of the high-chromium-nickel steels. Experience with the one superheater using stainless-steel tubes, and similar in all other respects to the other installations, shows higher creep relative to the inexpensive tubes than study of laboratory data would indicate. Reference to Table 3 shows a creep rate of 1.5 per cent per 10,000 hours for stainless steel and 1 per cent for the low-chrome-vanadium material. Published thermal conductivities of stainless steels show them to average only 55 per cent the conductivity of boiler-tube material. Thus, the temperature drop through the metal for same transfer rates and thicknesses is almost double, and temperature stresses are therefore much greater. This actual operating experience suggests that high-chrome-nickel metals are of lesser benefit than laboratory tests indicate them to be, though more experience is desirable to confirm the point.

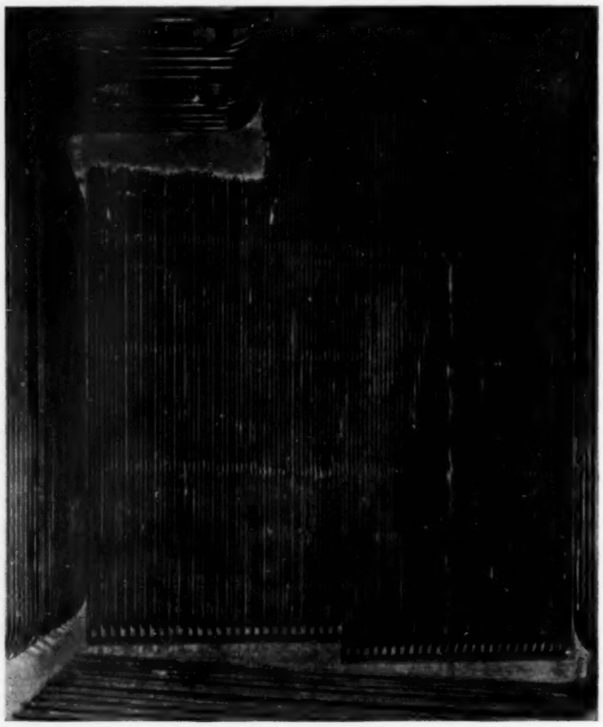

FIG. 6 MOST RECENT INSTALLATION, ONE OF THE THREE 1300-LB. SUPERHEATERS, INSTALLATIONS J AND K

(High pressure assists radiant superheater design markedly, and these superheaters are completely satisfactory after $1^1/_2$ years' service. They have the cleanliness characteristics of fin-tube walls, and have not required a penny's maintenance. They effectively cover both side walls, which is difficult with water walls, and absorb heat with essentially the same effectiveness as bare water tubes.)

9 More experience with creep of metals in service, operating under various conditions of heat transfer and metal temperature, is highly desirable. This particular experience indicates that

TABLE 3 LONG-TIME OPERATING CREEP DATA

Boiler-unit designation	Operating hours	Years of service	Per cent creep: Carbon steel	Per cent creep: Enduro A	Per cent creep: Chrome-vanadium
3	13,840	3.3	...	2.6	..
4	14,366	3.3	...	..	2.2
5	16,020	3.6	7.4	..	..
6	14,728	3.4	..	..	1.25
7	15,834	3.3	...	..	1.0
8	15,400	3.2	...	..	1.8
10	9,249	2.4	...	..	0.6
13	12,163	2.6	...	..	1.2
Summary:					
Creep per 10,000 operating hours			4.6	1.5[a]	1.0
Creep per year			2.0	0.7	0.5
Relative creep rate			4.5	1.5	1.0

(The 400 elements in operation three years were measured to obtain the data. Percentage increase in circumference is reported as "per cent creep." The chrome-vanadium tubing is not much more expensive than high-grade carbon-steel tubing, for it contains insufficient chromium, 1.5 per cent, to class it as a stainless steel and the vanadium is only a fraction of 1 per cent. It appears that the Enduro metal is handicapped by its lesser thermal conductivity, increasing the important temperature stresses. Its external appearance and smoothness, however, is considerably better than the chrome-vanadium elements, indicating possibly longer life because of better resistance to oxidation. Paying 5 cents more per pound for the chrome-vanadium elements than for the carbon ones is certainly desirable, but 50 cents per pound for the Enduro metal is unquestionably unwarranted, even though the weight involved for the entire superheater is small.)

[a] Averaged from Fig. 16.

laboratory stress values for given creep are many times higher than occur in practice unless other than pressure stresses are assumed.

Radiant-superheater experience at the Lakeside Station is best summarized by the following points:

1 Special heavy rectangular elements have been displaced by lighter round elements because (*a*) slow steam velocities and large temperature differences in the front and rear of the rectangular elements caused their failure; (*b*) reasonably high mass-flow rates and uniform circumferential heat absorption of the round elements caused low metal temperature and elimination of thermal stresses.

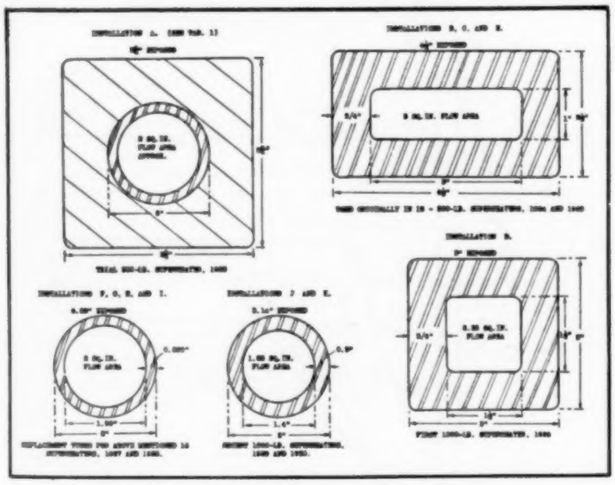

FIG. 7 DEVELOPMENTS SHOWN BY CROSS-SECTIONS

(The simple round-tube sections are greatly superior to the special, heavy elements. The thinnest tube section, lower left, absorbs 20 per cent more heat than the second heavy element, upper right, though it weighs only 10 per cent as much. Its flow area is one-third less, causing desirably higher steam velocities, and effective surface exposed to the cooling steam is about 50 per cent more. Most important of all, enormous and destructive temperature stresses were reduced to almost negligible items by the new design. The square 1300-lb. section, lower right, is successful, but the round section, lower middle, used in subsequent installations is less expensive.)

2 Flush-with-wall position of rectangular elements was discarded for the projecting-tube design of the round elements, until favorable transfer conditions with 1300-lb. pressure steam allowed reapplication of the initial mounting.

3 A refined-steel tube material, containing small amounts of chromium and vanadium, has been found satisfactory for 950 deg. fahr. maximum metal temperature.

4 Round elements set out from a brick wall and with several diameters' spacing absorb heat throughout their periphery at rates closely equal to equivalent flat surfaces in the plane of the wall. Reflection of radiant heat from the brickwork explains this phenomenon.

5 Round-tube elements replacing the unsuccessful rectangular ones weighed only 10 per cent as much, cost less per pound, and did not require expensive steel backing.

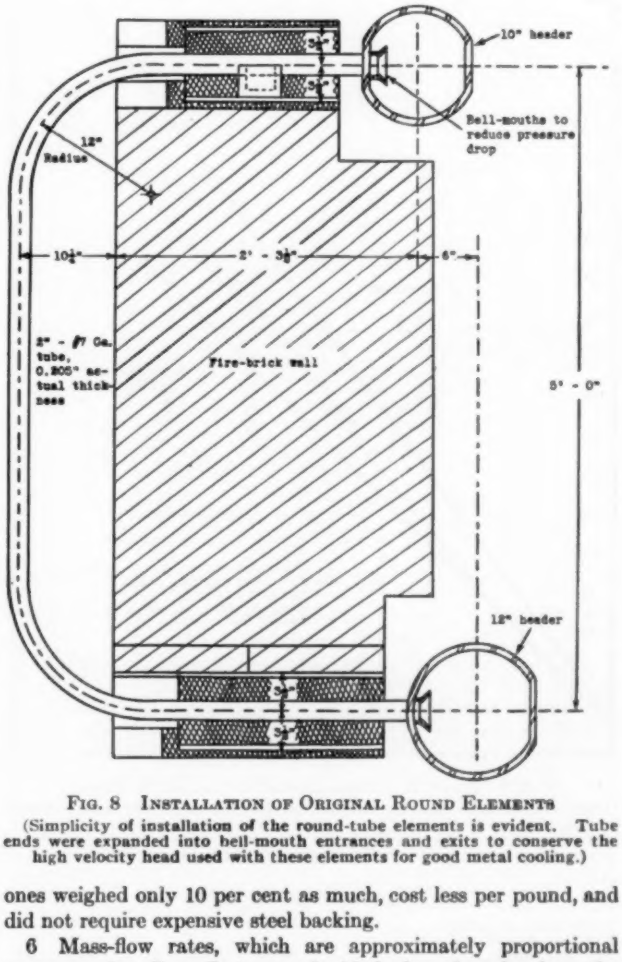
10" header
Bell-mouths to reduce pressure drop
12" Radius
10½"
2' - 3⅛"
6"
2" - #7 Ga. tube, 0.205" actual thickness
Fire-brick wall
5' - 0"
12" header

FIG. 8 INSTALLATION OF ORIGINAL ROUND ELEMENTS

(Simplicity of installation of the round-tube elements is evident. Tube ends were expanded into bell-mouth entrances and exits to conserve the high velocity head used with these elements for good metal cooling.)

6 Mass-flow rates, which are approximately proportional measures of cooling effect, were doubled when changing from the rectangular to round elements. Pressure drop was held down by elimination of connecting nipples and use of bell mouths to diminish entrance and exit losses.

7 Projecting-element design is of maximum efficiency and subjects the tubes to minimum thermal stresses, but has the sole objection that the brickwork backing must be cleaned periodically of ash deposits.

8 Sixteen sets of rectangular-element superheaters were replaced after three years' average life with elements that cost less than 10 per cent as much and that will last at least twice as long.

PART II

RADIANT-SUPERHEATER CHARACTERISTICS

The slope of any superheated-steam temperature versus rating curve is dependent upon so many factors that each particular installation has an individual characteristic, whether the superheater is convection or radiant. Fig. 17 shows the curve for a 300-lb. radiant superheater, Fig. 20 a 1300-lb. one, and the latter shows how the combination of a radiant superheater, a constant-exhaust pressure 1200-lb. turbine, and a convection reheater cause a uniform reheater-outlet temperature.

Much can be learned of radiant-heat-absorption rates in existing installations with water-cooled tubes of known area by metering heat input at various locations and ratings. Metal temperature has little effect upon heat absorption, and the influence can be evaluated accurately.

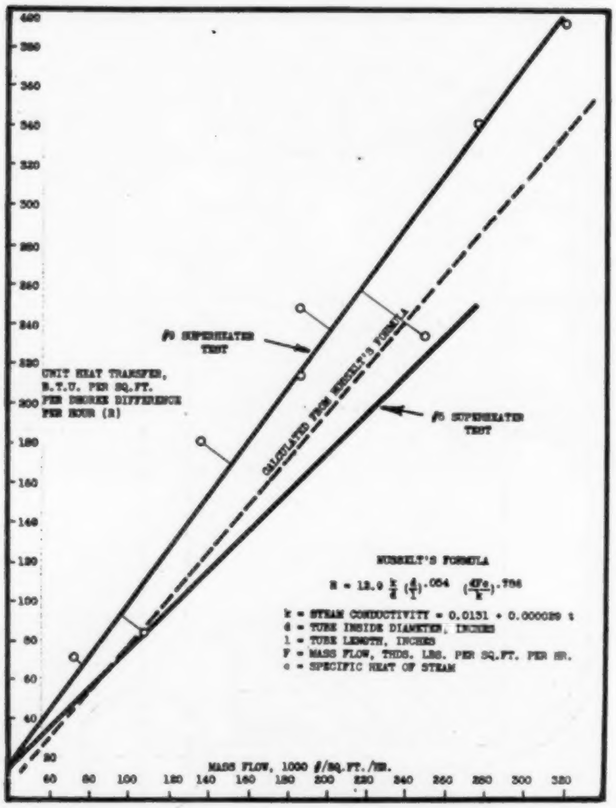

Fig. 9 Heat-Transfer Inside Superheater Tubes

(Metal-temperature measurement by thermocouples peened into elements in operation allowed calculation of the above unit heat-transfer data. Since fair agreement with Nusselt's expression was obtained, maximum operating metal temperatures were concluded not over 950 deg. fahr.)

TABLE 4 METAL TEMPERATURES

1	Pressure, lb. gage....	300			1300		
2	Steam temperature, deg. fahr..........	700			750		
3	Tube thickness, in...	0.2			0.3		
4	Assumed mass-flow rate, lb. per sq. ft., per hr.............	250,000			500,000		
5	Heat input rate, B.t.u. per sq. ft., per hr..	30,000	50,000	70,000	30,000	50,000	70,000
6	Temperature drop through metal:[a]						
	a Carbon steel....	20	33	47	30	50	70
	b High chromium steel.........	35	59	83	53	88	123
7	Temperature drop, steel to steam[a]	112	187	262	50	83	116
8	External tube temperature:						
	a Carbon-steel tube.........	832	920	1009	830	888	936
	b High-chromium-steel tube....	847	946	1045	853	921	989

(With 1300-lb. pressure, metal temperature averages approximately 100 deg. fahr. nearer the steam temperature than with 300-lb. steam—see line No. 8. High chromium steel nearly doubles temperature drop through the metal—compare lines No. 6*a* and No. 6*b*—and thus greatly increases the important temperature stresses. Temperature drop through 0.3-in. chromium tubes slightly exceeds the temperature drop from metal to steam. Tube temperatures range between 830 deg. fahr. and 1050 deg. fahr., depending upon combinations of conditions.)

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[a] Following bases: Thermal conductivity of carbon steel, 310 B.t.u. per sq. ft., per hour, per 1 in., per deg. fahr.; of high chromium steel, 170 B.t.u. per sq. ft., per hour, per 1 in., per deg. fahr. Nusselt's formula, shown on Fig. 9, used for calculating values in line No. 7.

With sufficient experimental background, performance of radiant surfaces can be calculated quite accurately for various ratings and furnace types. A method applied with practical exactness is treated in the Bureau of Mines bulletin, "An Investigation of Powdered Coal as Fuel for Power-Plant Boilers," by Messrs. Kreisinger, Blizard, Augustine, and Cross. Simultaneous temperature and heat-transfer measurements have indicated quite consistently a constant of 1300 for the radiation formula:

$$\text{B.t.u. per hour per sq. ft.} = 1300\left[\left(\frac{T_1}{1000}\right)^4 - \left(\frac{T_2}{1000}\right)^4\right]$$

The distinction between radiant and convection superheaters consists principally of location. Fig. 21 shows 75 per cent radiant-heat absorption at the entrance of a convection reheater, and convection superheaters are in service having higher local rates of heat absorption than some radiant installations.

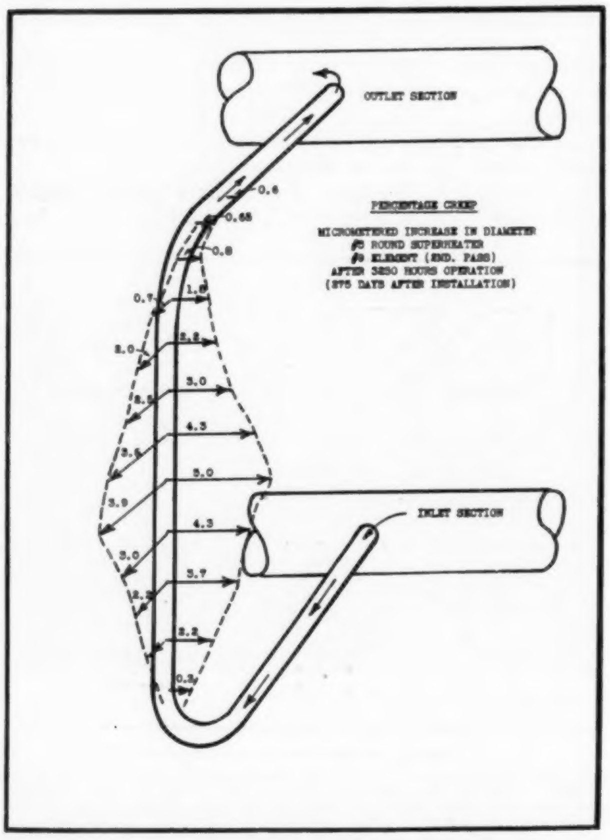

Fig. 10 Turbulence Versus Creep

(That expansion was not due to overheating when starting, but occurred very slowly during operation, is the most important deduction from the above isometric sketch. Turbulence at the bends maintained lower metal temperature than in the middle, and creep values, shown graphed in direction of diameter measured, were correspondingly lower. Higher pressure drop and greater turbulence cause better cooling and lesser creep.)

Fundamental Heat-Transfer Differences, Convection and Radiant Superheaters

For given metal temperatures, pressure drop must increase as rates of heat transfer become larger. The length of steam path decreases, but velocities must increase more rapidly than does heat input rate, and pressure drop varies with the square of velocity. Temperature drop through the metal becomes an important consideration at furnace-heat-transfer rates.

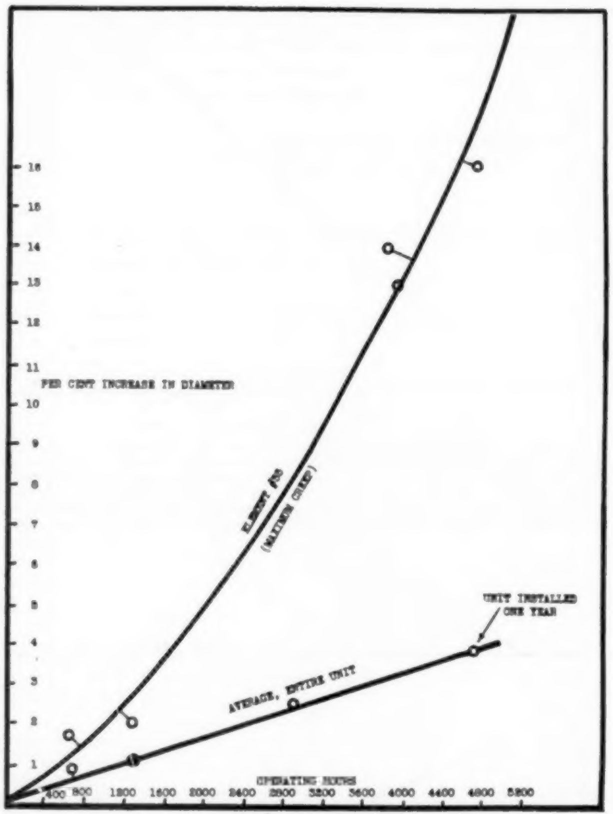

Fig. 11 Creep Rates of Experimental Installation

(That creep is a linear function of operating hours is the most important conclusion from the lower curve. When expansion is so large that the tube walls thin greatly and diameter enlarges appreciably, thus increasing metal-temperature and unit stress due to pressure by several times, the creep curve probably bends upward, as shown by the upper curve. Half of the elements have been removed at convenient times before failure, and half are still in service after 3½ years. They now average 7.4 per cent creep.)

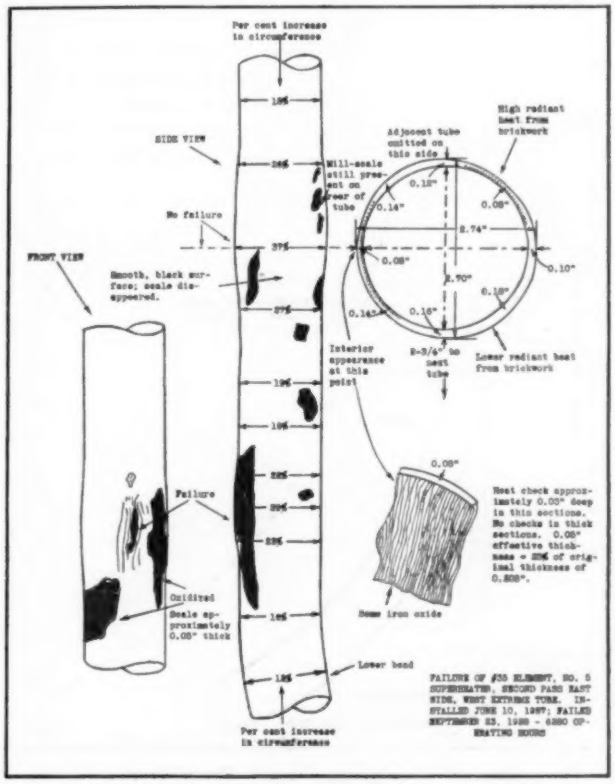
Per cent increase in circumference
SIDE VIEW
FRONT VIEW
18%
28%
Mill-scale still present on rear of tube
No failure
37%
Smooth, black surface; scale disappeared.
27%
19%
19%
22%
20%
23%
16%
13%
Failure
Oxidized
Scale approximately 0.08" thick
Lower bend
Per cent increase in circumference
Adjacent tube omitted on this side
High radiant heat from brickwork
0.12"
0.14"
0.08"
2.74"
0.08"
0.10"
2.70"
0.18"
0.14"
0.16"
Interior appearance at this point
3-3/4" to next tube
Lower radiant heat from brickwork
0.08"
Heat check approximately 0.03" deep in thin sections. No checks in thick sections. 0.08" effective thickness = 25% of original thickness of 0.203".
Some iron oxide
FAILURE OF #35 ELEMENT, NO. 5 SUPERHEATER, SECOND PASS EAST SIDE, WEST EXTREME TUBE. INSTALLED JUNE 10, 1937; FAILED SEPTEMBER 23, 1938 - 6880 OPERATING HOURS

Fig. 12 Creep Failure

(A 37 per cent increase in circumference without failure occurred near the top of the tube, even though expansion around the periphery was not uniform. The original 2-in. tube section could readily be passed through the section shown. Since the effective thickness was only one-fourth of the original, and the inside diameter 60 per cent more, pressure stresses were 6.4 times original stresses. Metal temperature was undoubtedly higher, because of lesser steam velocity. However, temperature stresses were greatly reduced, and the thin tube continued to bear its load. Temperature stresses appear very important, and thin tubes desirable.)

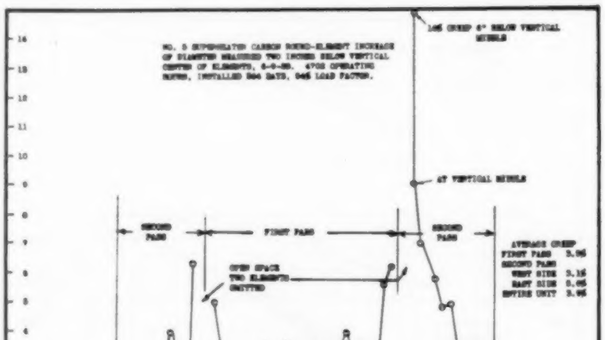

Fig. 13 Creep Variations in an Actual Installation

(Occasional omission of elements, causing higher radiant-heat reflection from the brickwork backing wall, has a marked influence on creep rates. Use of lesser elements in the hotter pass maintained metal temperature about the same as in the first pass. Low-carbon-steel elements increased about 4 per cent in diameter the first year of operation.)

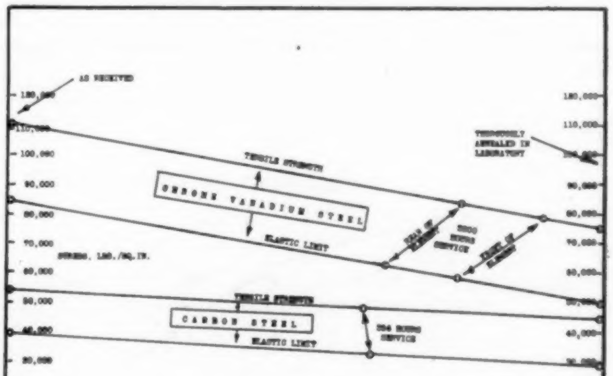

Fig. 14 Annealing Effect of Service at 950 Deg. Fahr.

(That heating to the critical temperature is not necessary for annealing is indicated by the above data. Long-time maintenance at 950 deg. fahr. accomplishes the same results with both kinds of steels. In laboratory testing of metals for creep, it is important that specimens be first thoroughly annealed to simulate ultimate service conditions.)

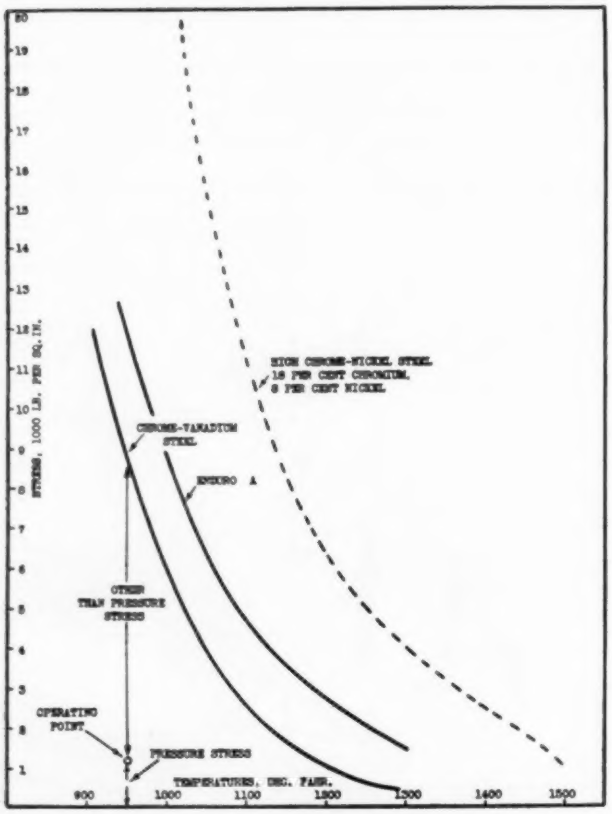

FIG. 15 LABORATORY VERSUS FIELD RESULTS

(Pressure stress is insignificant in producing creep, as study of these curves and operating point will show. The curves are laboratory data, from F. H. Norton, for 1 per cent creep per 10,000 hours. The operating point applies for all 300-lb. steam-pressure round superheater elements. Field creep of the chrome-vanadium elements coincides accurately with that for which the curves were drawn, i.e., 1 per cent per 10,000 hours. Carbon elements increased 4.6 per cent and Enduro A elements 1.5 per cent per 10,000 hours. Additional strength of the Enduro A elements was ineffective because of higher temperature stresses due to lesser metal thermal conductivity. Addition of nickel extends the creep stress considerably, as the dotted curve shows, but thermal conductivity is still lower than that of Enduro A. Obviously, other than pressure stresses are predominant in superheater design.)

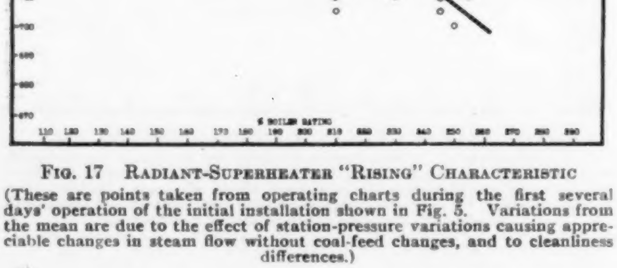

FIG. 17 RADIANT-SUPERHEATER "RISING" CHARACTERISTIC

(These are points taken from operating charts during the first several days' operation of the initial installation shown in Fig. 5. Variations from the mean are due to the effect of station-pressure variations causing appreciable changes in steam flow without coal-feed changes, and to cleanliness differences.)

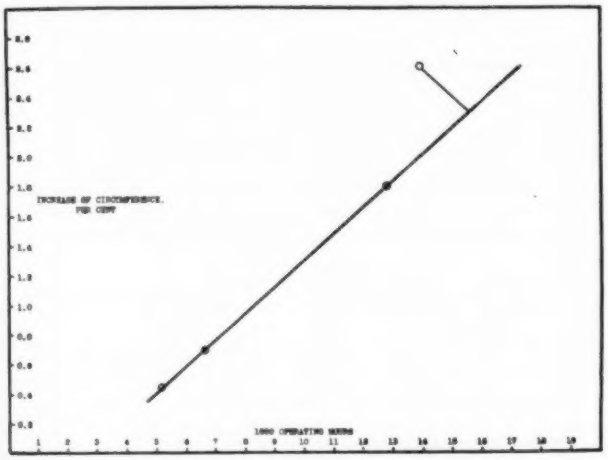

FIG. 16 CREEP OF ENDURO A TUBING

(Four sets of circumference measurements of all 40 Enduro A elements in No. 3 boiler plot against operating hours as shown above. Load factor the last thousand hours has been low, and unreported starting hours high, which possibly explains why the last point is above the curve. Several scattering Enduro elements showed 1.5 per cent creep, while occasionally others with 5 per cent maximums caused the recent average of 2.6 per cent.)

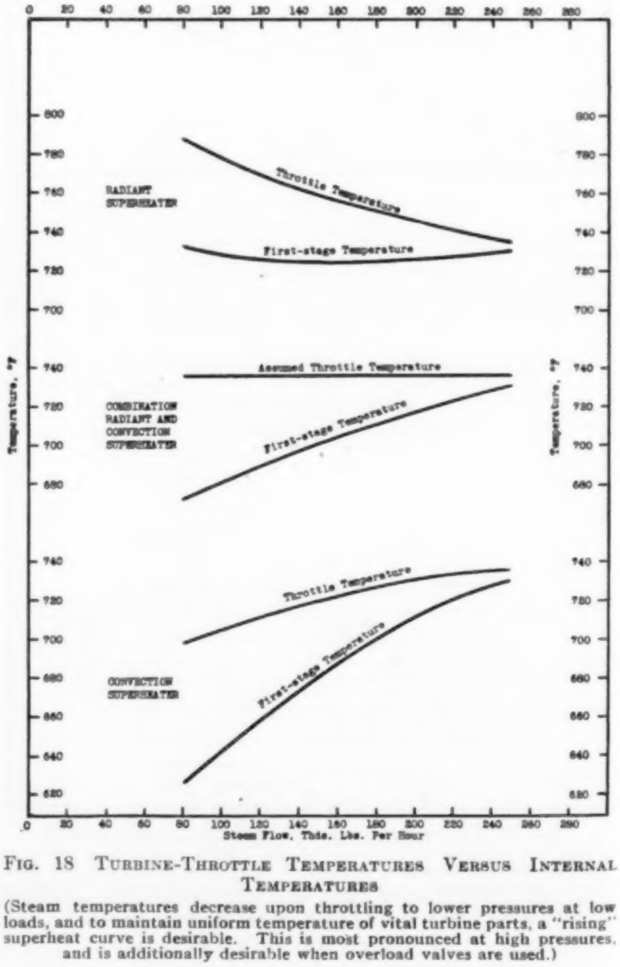

FIG. 18 TURBINE-THROTTLE TEMPERATURES VERSUS INTERNAL TEMPERATURES

(Steam temperatures decrease upon throttling to lower pressures at low loads, and to maintain uniform temperature of vital turbine parts, a "rising" superheat curve is desirable. This is most pronounced at high pressures, and is additionally desirable when overload valves are used.)

The following two major expressions show that cooling of tube metal is not quite proportional to the steam velocity (varies with 0.8 power) and that pressure drop varies with square of velocity. Table 4 solves for metal temperatures under a variety of conditions to illustrate the point that high heat-transfer rates impose a handicap on superheating surfaces.

Heat-Transfer Expression (*Nusselt*):

$$R = 12.9\,\frac{K}{d}\left(\frac{d}{l}\right)^{0.054}\left(\frac{dFc}{K}\right)^{0.786}$$

where

K = conductivity of steam = 0.0131 + 0.000029 t (deg. fahr.)
d = inside tube diameter, in.
l = length, in.
F = mass flow, 1000 lb. per sq. ft. per hour
c = specific heat of steam

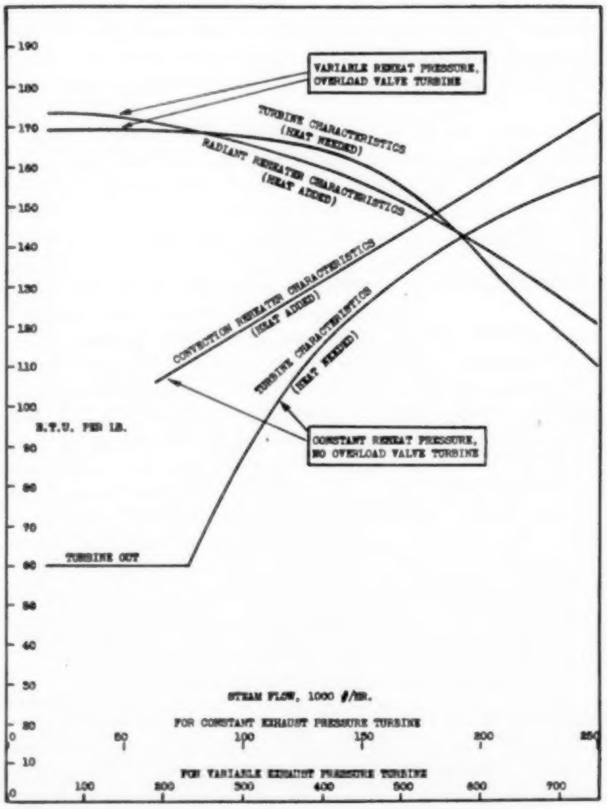

Fig. 19 Contrasts in Reheater Requirements

(Directly opposite reheater characteristics are required for two actual installations A convection reheater suits one well: a radiant reheater fits the other equally well. Turbine type causes the marked difference. Coincidence of the reheater and turbine characteristic curves results in constant reheater-outlet temperature Agreement of slope, at least, is very important.)

The first exponential term $(d/l)^{0.054}$ may be omitted, since for usual length tubes it is very nearly unity, giving

$$R = 12.9\,\frac{K}{d}\left(\frac{dFc}{K}\right)^{0.786}$$

Therefore, the heat-transfer rate varies with 0.786 power of mass flow.

Pressure-Drop Expression (*Fritzsche*):[2]

$$p = \frac{12.8\,v^2Lc}{Vd}$$

where

v = mean velocity, ft. per sec.
L = length of pipe, ft.
c = coefficient, varied by temperature, etc.
V = specific volume, cu. ft. per lb.
d = inside pipe diameter, in.

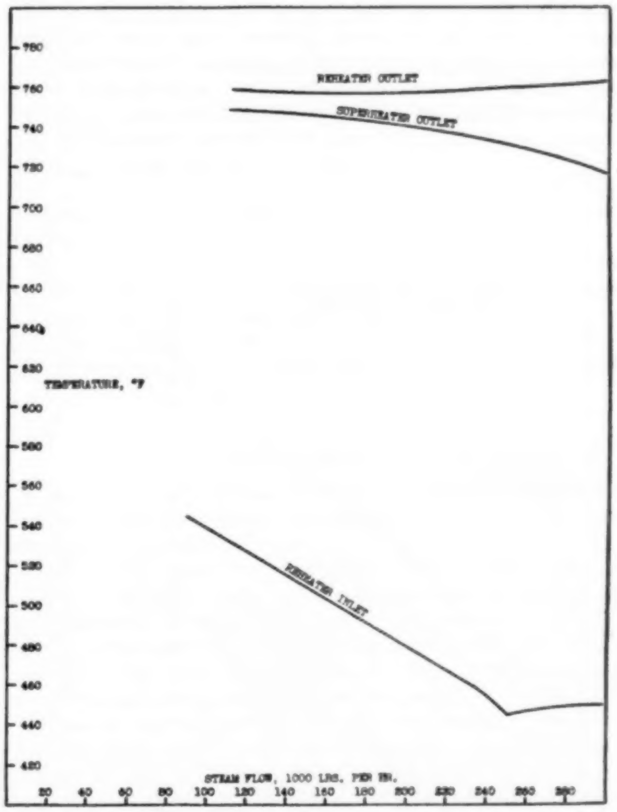

Fig. 20 Where a Convection Reheater Fits Well

(Rapidly increasing exhaust temperatures of a constant back-pressure 1200 lb. turbine as it carries lower loads requires convection-reheater characteristics to result in the flat reheater-outlet temperature curve shown. The rising characteristic of the 1300-lb. radiant superheater is shown. These are actual test data, and were taken with constant excess air in the furnace.)

Thus, from these two expressions,

(1) $R = cF^{0.786}$
(2) $p = c_1F^2L$

Consider two superheaters transferring the same amount of heat, but located in different gas-temperature zones:

$$R = c_2\frac{F}{L} \qquad \text{or} \qquad L = c_3\frac{F}{R}$$

From (1), $F = c_4R^{\frac{1}{0.786}} = c_4R^{1.27}$

Substituting these values in (2),

$$p = c_5\,(R^{1.27})^2\,\frac{R^{1.27}}{R}$$

or, finally, $p = CR^{2.81}$

---

[2] See Marks's Mechanical Engineers' Handbook.

Therefore, pressure loss in superheaters having the same duty and similar design increases rapidly if the same metal temperature is maintained. Doubling of heat-absorption rate requires $2^{2.81}$ or seven times as much pressure drop for the same metal temperature.

Since the time of their initial inception, radiant superheaters have been less likely to cause unscheduled forced outages than many convection installations, merely because uniformity of cooling all tubes has been fortunately dictated by radiant design. Low average steam velocities in the tube banks of many convection superheaters and high header velocities caused velocity heads in the latter to exceed pressure drops across the tubes. Severe tube failures without warning resulted. Later, use of long, continuous tubes between the inlet and outlet headers caused full superheater-pressure drop across each element, and velocity heads in the headers then had little influence in distorting flows through any elements. This principle is naturally dictated in radiant-superheater design, for only a relatively few short tubes can be used, and the pressure drop through each is considerable.

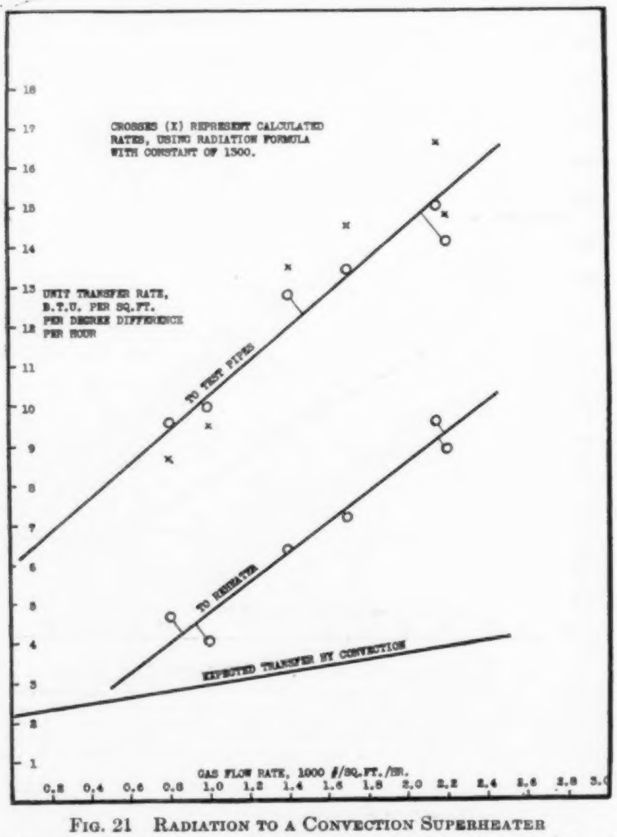

Fig. 21 Radiation to a Convection Superheater

(Test values show 60 per cent average heat absorption by radiation and 40 per cent by convection at high loads. At the entrance of the reheater, where gas temperatures of 1700 deg. fahr. occurred at maximum loads, 75 per cent occurred by radiation and 25 per cent by convection. Single convection superheaters operating in high gas temperatures can have quite desirably flat temperature curves, but as higher steam temperatures and reheat are required, outlet-gas temperatures become so low at low loads that badly "drooping" temperature curves are inevitable.)

Pressure Drop

Uniform heat input throughout the circumference of radiant elements and elimination of all pressure losses not conducive of lower metal temperatures allow minimum pressure drop for given metal temperatures. Round sections are most efficient hydraulically, and use of cores is inefficient. Wherever steam scrubs a tube surface and thus loses pressure, desirable metal cooling should occur. Uniform metal temperature represents maximum efficiency.

The projecting round-tube elements used in the 300-lb. pressure installations are most efficient from the standpoint of uniform peripheral-heat absorption. The flush-with-wall 1300-lb. elements (Fig. 6) are most practical at the higher pressure because of the more favorable heat-transfer characteristics.

Entrance and exit losses, of appreciable percentage with short-length tubes, were diminished by using rounded entrances and discharges in the 300-lb. installations. (See Fig. 8.) A velocity head is conserved, equaling 1 lb. per sq. in.

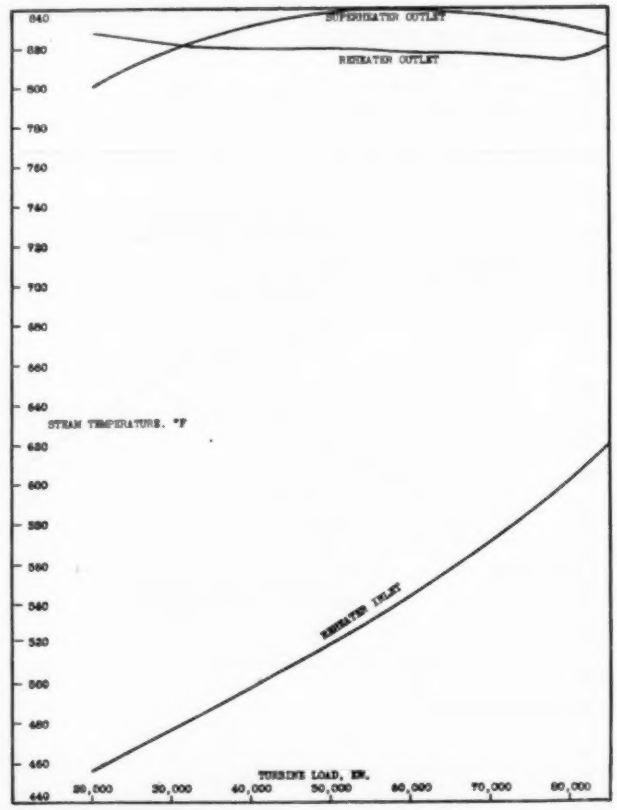

Fig. 22 Uniform High Steam Temperature

(Throttle and reheat-steam temperatures of 825 deg. fahr. throughout a load range of one to four are difficult to average in the case of a single-turbine, single-boiler installation. The above curves, based upon test data, show only 40 deg. fahr. superheat variation and 20 deg. fahr. reheat-temperature variation for this four to one range accomplished by major use of radiant surfaces. Notice the rapidly rising reheater-inlet temperature curve, due to variable reheat pressure and use of two turbine overload valves, which would accentuate the undesirable slope of a convection superheat curve. With simply convection surfaces, the above 20 deg. fahr. reheat-temperature variation would change to 200 deg. fahr.—630 deg. fahr. at 20,000 kw. and 830 deg. fahr. at 80,000 kw. Equipment to give the above results has been purchased for a new station.)

Careful pressure-drop measurements across the initial 1300-lb. round-tube radiant superheater gave surprisingly low results. The usually used Babcock formula gave results nearly twice too large, and the tests were accordingly checked on several occasions. Agreement with Fritzsche's data[3] was found later. Internal tube smoothness probably caused the disparity. Since roughness would probably increase the stationary-film thickness and thus not improve heat transfer, it is important that radiant-superheater tubes be smooth for maximum economy.

Test data and results of application of Fritzsche's formula to the aforementioned superheater follow:

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[3] As published in Marks's Mechanical Engineers' Handbook, 1916, p. 358.

Data: 212 2-in. chrome-vanadium tubes, 0.30 in. thick, 1128 sq. ft. wall area. Four passes, average length of tube per pass = 31.3 ft. Two bends per tube, 9 in. radius. Headers 7 in. inside diameter.

Steam flow, 1000 lb. per hour	250
Superheater-outlet-steam temperature, deg. fahr	740
Superheater-inlet-steam temperature, deg. fahr	576
Superheater-inlet pressure, lb. gage	1278
Superheater-outlet pressure, lb. gage	1258
Actual pressure drop, lb. per sq. in	20
Calculated pressure drop, Babcock formula	35
Calculated pressure drop, Fritzsche formula	20

CLEANLINESS

Experience with all installations indicates almost 10 per cent decrease of heat transfer the first few days of operation that is never regained. Further temperature loss due to ash accumulations is readily recovered by permanently located soot blowers. The initial surface condition of the unused black elements has apparently a higher radiant-heat absorption coefficient than the slightly roughened brown tubes after service. The Enduro A tubes have been observed to retain their original smoothness longer than the carbon-steel elements and to require less cleaning of ash.

The projecting tube elements installed in the 300-lb. boiler units have the sole minor demerit of necessitating ash removal from the brick backing wall. The 10-in. space fills slowly with fragile "whisker" ash, which requires removal weekly or twice monthly. Because the high-pressure boilers are operated continuously for months without outage, and because this type of hand cleaning would not be convenient, the flush-with-wall installation was used in the later equipment.

The vertical round-tube superheater, using 2-in. tubes on $2^{1}/_{2}$ in. centers and backed with insulating tile, retains its original cleanliness as well as a fin-tube furnace wall. Low-grade coal of 2050 deg. fahr. fusion-temperature ash and 4 per cent sulphur has been burned over long periods with scarcely any soot blowing necessary. Ash particles do not collect on the brick backing, being cooled sufficiently between the tubes to fall downward as dry dust. Radiant-superheater soot-blower steam averages 1/40th of 1 per cent of the high-pressure boiler output.

Internal cleanliness has never been a problem. Outside of finding some water-soluble sodium sulphate in the headers and nipples of the first high-pressure superheater when starting feed-water treatment, no deposits ever have been detected. Steam velocities are high, and bends so liberal that suspended matter cannot collect.

Consideration of the usual radiant-heat transmission expression, which states that heat transferred varies directly with difference of the fourth powers of the absolute temperature between the heating and heated media, shows that metal-temperature differences have practically no influence in reducing heat absorption. A 1000-deg. fahr. tube, compared with a 900-deg. fahr. one operating at a heat-transfer rate of 50,000 B.t.u. per sq. ft. per hour, absorbs only $2^{3}/_{4}$ per cent less heat, and only $11^{1}/_{2}$ per cent less than a 70-deg. fahr. tube.

INSTANTANEOUS STEAM-TEMPERATURE REGULATION

Changing from the heavy rectangular elements to the light round tubes, weighing only 10 per cent as much, distinctly modified the 300-lb. steam-temperature curve because of lesser heat-storage capacity of the latter elements. Rapid and frequent variations, as much as 30 deg. fahr. in a few minutes, occurred due to station-pressure variations. Large boiler-water content ($1^{1}/_{2}$ hours' normal output) caused high or low steam flows without coal-feed variations as pressure changed; and without the great weight of metal of the usual convection superheaters to stabilize temperature, steam-outlet temperatures fluctuated. Good pressure regulation is the only remedy. Instantaneous temperature regulation in the high-pressure boilers is much better due to lesser water content (20 minutes' operation), lesser water-temperature variation with the same percentage pressure variation, and better actual pressure regulation.

When using the heavy rectangular elements, steam temperatures were about 25 deg. fahr. low for the first hour of operation each morning. Storage of heat in metals is important and readily calculated.

INFLUENCE OF STEAM PRESSURE UPON RADIANT-SUPERHEATER DESIGN

Three conditions facilitate radiant design at higher steam pressures. They are:

1 Economic mass-flow rates increase directly with pressure.
2 Specific heat increases.
3 Conductivity of steam increases.

Of course, pressure stress increases, but experiences treated previously indicate that other stresses than that from pressure appear of most importance in causing creep.

At 1300-lb. steam pressure, mass-flow rates equivalent to those in a boiler tube with 3 ft. per sec. water velocity can be obtained economically. The transfer rates may not be as favorable as in the boiler tube due to difference in film conditions, but it is of interest to know that the weight flows can be made equivalent.

PART III

PROPER APPLICATIONS

Quick or unsteady starting and very high rates of furnace-heat transfer render the radiant superheater, in its present state of development, undesirable.

Cooling with steam from extraneous sources, when starting rapidly, is generally futile. Available quantities are so limited that practically no benefit is derived. Calculations, solving for metal temperatures by employing the heat-transfer data given previously, will show that impractically large steam flows are needed to limit metal temperatures within reason.

Minimum starting periods practical with particular installations can be determined fairly accurately by calculation, and checked by test. Maximum water cooling in the furnace walls and minimum water storage in the boiler drums allow quickest starting without overheating uncooled radiant-superheating elements, and in such cases considerations other than superheater maintenance determine the rate of starting.

Heat-transfer rates in excess of 60,000 B.t.u. per sq. ft. per hour have not been averaged in practice. Metal temperatures and temperature stresses increase with higher heat-transfer rates, and a practical limit exists, probably in the region of 75,000 B.t.u. per sq. ft. per hour, depending upon the particular conditions.

Experience with furnace types is needed to prescribe proper radiant-superheater area. Modification of area is not simply accomplished as with convection surfaces. If of the combination radiant and convection type, adjustments can be made in the convection section. If simply radiant, brickwork or water-wall surfaces must be modified if adjustment cannot be made conveniently by flame position or dirtiness. When starting a new radiant installation, low temperatures can be readily insured by partial application of a plastic refractory covering.

INCIDENTAL MERITS OF THE RADIANT SUPERHEATER

Greater furnace-heat absorption and elimination of troublesome furnace refractories have constituted a positive trend of the industry for several years. Higher pressures and temperatures, plus reheat, cause an important trend toward smaller water-cooled surface and larger superheating surface. That some of the furnace cooling surface be for superheating is almost necessary to prevent distortion of designs in the new installations.

Radiant superheaters located on the furnace side walls eliminate many difficult water-wall connections. It is believed that rear-wall and front-wall connections can be made much more conveniently in most installations than can side-wall connections. Greatly fewer outside, inactive tubes are needed, and connections are more direct. Radiant superheaters in the wide walls solve these problems at minimum cost.

Crediting costs of displaced water walls toward the expense of radiant superheaters has an element of fairness. As a rule, additional boiler surface is not expensive after drum water-storage capacity is set, and convection superheaters have thus but a slight credit in reducing boiler cost. Overall costs may be lowest when using radiant superheaters.

Thermally, radiant superheaters are most efficient. Higher steam temperatures reduce radiant-heat transfer but slightly, but they diminish convection-heat transfer greatly, especially at lower loads and in cases where much superheating is done. Heat transfer by radiation is accomplished without draft loss.

Greater excess air (lower CO_2) reduces superheated-steam temperatures from a radiant superheater, but increases them from a convection heater. Exigencies of operation sometimes require an increase of excess air. It increases the safety of an installation to have the radiant characteristics apply.

Temperatures from radiant superheaters can be adjusted by flame position. If they are located in the side walls, changing end-feeder speeds cause a rapid influence at any boiler rating. Radiant-transfer rates vary inversely with the square of distance between the hot and cold bodies. Changing primary-air pressure on all burners, or secondary-air distribution, modifies rear-wall heat absorption.

Uniform turbine-casing temperatures are obtained by the rising radiant-superheater characteristics, as shown by Fig. 18. Even a flat temperature characteristic is less desirable in this respect. Use of overload valves exaggerates the need of higher throttle temperatures at low loads so that more uniform casing temperatures may occur. High temperatures are safest at low loads, since heat-transfer rates through superheater tubes are less, and therefore the important temperature stresses are reduced. Metal temperatures of radiant superheaters decrease slightly with rating in spite of higher steam temperatures. High excess air (low CO_2) need not be used at low ratings to maintain desirable superheat temperatures as in the case of many convection superheaters.

PART IV

Application in Reheat Installations

The following factors show the radiant-superheating surface desirable for reheat installations:

1 Characteristics of turbines arranged for reheat with variable exhaust pressure, and especially those having more than one overload valve, are such that lesser reheat is needed as larger loads are carried. Reference to Fig. 19 shows this condition, contrasting it with that caused by a constant-reheat-pressure type of installation.

2 Use of a separate reheat boiler to obtain uniform reheat temperatures greatly increases boiler-room investment costs and complicates operation. Single-boiler, single-turbine installations are better able to be taken out of service every night, an ultimate requirement of all equipment. Radiant equipment assists single-turbine, single-boiler design.

Use of a separate reheat boiler generally requires two standard boilers. In the case of a recent 80,000-kw. extension, one boiler unit was chosen instead of two because of simpler, equally reliable operation, and of an investment saving in boiler plant of $4 per kilowatt.

3 Convection-superheater characteristics "droop" more at low loads as more superheating surface is installed in regions of lower gas temperature. With a primary convection superheater occupying the standard position, characteristics of a convection reheater following it are of exaggerated undesirability. This is especially true as higher steam temperatures are used, higher feed-water temperatures occur, and steam pressure is high. Heat absorbed in evaporating water so lessens, and superheating duty so increases, that usual practice is greatly distorted. In an installation under construction, 690 B.t.u. per lb. will be added in the boiler and 385 B.t.u. per lb. in the superheaters (56 per cent as much in the latter as in the former). Superheating duty is becoming so high that extensive furnace cooling with water walls cannot be practiced without causing a low temperature head for convection-superheating surfaces.

4 When re-superheating is accomplished with facility, use of two stages of it to obtain greater economies, not simply to reduce moisture in lower turbine stages, may occur. Use of radiant equipment will assist attainment of these additional gains.

5 Regulation by inherent characteristics, not by special control, is a merit of considerable desirability when applied to high steam temperatures. Excessive temperatures for short periods or rapid changes up or down may not cause permanent harm in the boiler room, but great trouble in the turbine room. Relieving the operating force of all temperature-control attention and hazard is certainly desirable.

CONCLUSIONS

1 Developments of the industry toward more superheating emphasize the need of radiant superheaters.

2 Important saving in plant investment costs can be effected by reheating in single boilers serving single turbines. Radiant-superheating surfaces permit this design by insuring practically uniform steam temperatures throughout wide load ranges without any attention.

3 Sufficient experience with simple round-tube superheaters has been obtained to accurately predict their behavior under most furnace conditions. Creep rates have been measured, and metal temperatures are known. For present steam temperatures, they are thoroughly satisfactory, and have been purchased for 850 deg. fahr. steam temperature.

4 High pressures are particularly adapted for radiant superheaters, for pressure stresses appear a minor factor in causing creep. Higher allowable mass-flow rates, higher specific heat, and better steam conductivity more than compensate for higher metal stress due to pressure.

5 Radiant superheaters have incidental merits of facilitating furnace cooling by covering side walls that are not as easily water cooled as front and rear walls, of reducing total heating surface costs because of better transfer efficiency, and of allowing lower CO_2 without excessive steam temperatures when burning unusually poor coals. Temperature adjustment by flame position and more uniform internal turbine temperatures are other merits.

6 Very high heat-transfer rates and quick starting preclude their usage.

7 More and better operating results of metals at high temperatures are needed to extend present upper steam-temperature limits. The experience treated herein suggests that temperature stresses are very important determinants of tube maintenance, and they should be evaluated with exactness. High-chromium steel appears considerably handicapped by its lower thermal conductivity, which greatly enhances the important temperature stresses. Oxidation resistance is needed by low-carbon steels.

8 Frequent and accurate creep and metal-temperature mea-

surements of tubes in service will supply a further development need that the laboratories cannot furnish. Much excellent laboratory creep work has been done, but applying its results with large safety factors is undesirable. The best justification of this paper would develop if it serves as an effective request for more complete operating information.

Discussion

T. McLean Jasper.[4] The general subject of superheating ties up intimately with the strength of metals at elevated temperatures. The most significant thing bearing on the service of materials brought out by this paper is the fact that Enduro steel, operating at only 30 to 50 deg. fahr. higher temperature than the low-carbon steel, due to metal heat-transfer rate, shows a creep rate materially higher than that of carbon steel. The manufacturers of Enduro steels would, the writer feels sure, say that this was not according to their best information based on laboratory tests. This indicates another example in which laboratory tests are not in agreement with carefully made practical tests.

Any one dealing with the practical problem will very properly follow the dictates of the service test.

In Figs. 5 and 6 are shown some superheater elements in place. The writer is wondering if the coefficient of expansion is not largely responsible for the condition of increased creep in the Enduro tubes. Their ends are fairly well fixed, and they will therefore assume a much greater longitudinal strain with increased temperature than will the plain-carbon or chrome-vanadium tubes.

Another significant observation is the confirmation of the value of low-alloy-content chrome-vanadium steel for elevated temperature work. This steel has greater resistance to oxidation than low-carbon steel, and also has a good heat-transfer rate and without any material increase in coefficient of expansion.

N. Artsay.[5] It is a long time since such a good paper on a burning question has been presented before the Society. This story of development in the radiant superheaters is an illuminating lesson.

It is a pity, indeed, that thermal stresses are so little known and understood by the majority of engineers. Every housewife knows the danger of pouring hot water into a thick glass vessel and every foundryman knows the danger of rash cooling, but the mechanical engineers, knowing for a long time the existence of thermal stresses in pipes, rails, and in other obvious cases, and knowing vaguely about such stresses in Scotch marine and Lancashire boilers, had very seldom thought of such in boiler tubes.

The question of stresses in tubes subject to a certain rate of heat transfer has been dealt with theoretically by Lorentz in Germany, as far back as 1907 (V.D.I., 1907, p. 743), later on by Mellanby and Kerr in England (Proc. I.M.E., 1927), by Orrok in the United States ("High Pressure Boilers," Trans. A.S.M.E., 1928), by Munzinger in Germany (Kraftwerk, April, 1930), and by Doucherty (London *Engineering*, April 25, 1930).

From all these sources we may draw a general formula:

$$S = a\eta \frac{Htd_o}{Kd_i}$$

where

S = thermal stress, lb. per sq. in.
a = a coefficient
η = ratio of Young's moduli at actual metal temperature and at 60 deg. fahr.
H = heat transfer, B.t.u. per hour × sq. ft.
K = conductivity, B.t.u. per hour × sq. ft. × deg. fahr. per inch thickness
t = thickness of tube metal, in.
d_o = outside tube diameter
d_i = inside tube diameter

According to Orrok, $a = 130$.

Let us compare thermal stresses in the tubes reported by Drewry.

η = 0.675 and 0.83 at 950 deg. fahr. for open-hearth and alloy steels, respectively
K = 310 and 170, respectively
H = 50,000 B.t.u. per sq. ft. × hour
t = 0.205 in.
d_o = 2 in.
d_i = 1.59 in.

The temperature difference is θ between outer and inner tube surfaces.

$$\theta_1 = \frac{50{,}000 \times 0.205 \times 2}{310 \times 1.59} = 41.6^\circ\text{F., open-hearth steel}$$

$$\theta_2 = \frac{50{,}000 \times 0.205 \times 2}{170 \times 1.59} = 76^\circ\text{F., Enduro metal}$$

Thermal stresses:

$$S_1 = 130 \times 0.675 \times 41.6 = 3650 \text{ lb. per sq. in.}$$
$$S_2 = 130 \times 0.83 \times 76 = 8200 \text{ lb. per sq. in.}$$

Pressure stress is 1200 lb. per sq. in. (original).

Respective total tension stresses in inner fibers are:

4850 lb. per sq. in., plain low-carbon steel
9400 lb. per sq. in., alloy steel.

Thinning down to one-half thickness will increase the pressure stress to 2400 lb. and reduce the thermal stress to 1900 and 4100 lb. per sq. in., respectively, so that the total tension stresses at halving of local thickness are 4300 and 6500 lb. per sq. in., respectively.

These figures indicate that the rate of creep should slow down as time goes on. The author's curve of creep on Fig. 16 for the Enduro tubes indicates, if extended down, almost no creep for the first 3000 hours. For the plain-carbon steel (Fig. 11) the actual points tend to show 0.8 per cent initial creep from the beginning and the slowing down of the creep rate as years of operation go by. Thus the alloy steel shows a peculiar behavior. An assumption could be advanced that while occasional overheatings during starting up periods did not harm the plain-carbon steel, they did change something in the alloy steel, and so it began to creep. The mentioning of irregular creep behavior of Enduro tubing (caption under Fig. 16) seems to support the assumption. Moreover, the annealing effect (Fig. 14) at 950 deg. fahr. is only 17 per cent for plain-carbon steel and 33 per cent for the chrome-vanadium steel; for the Enduro tubes data are not given, but it is reasonable to expect an even greater effect on the higher alloy steel. The author's observation of the difference in the field and the laboratory behavior of steels is very fortunate indeed, and his suggestion of more study and records of this difference should be heartily supported.

European practice with very high steam temperatures (Witkowitz, 932 deg. fahr.) indicates a way for a more economical use of materials. The total steam temperature is achieved in three stages: Saturated steam is admitted into a radiant superheater made of plain-carbon-steel tubes, then into the second (cooler) convection zone, also made of low-carbon tubes, and at last into the final alloy-steel superheater placed in the hot convection zone. In the latter part the molybdenum steels were most successful.

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[4] A. O. Smith Corporation, Milwaukee, Wis. Mem. A.S.M.E.

[5] Principal Assistant Engineer of Geo. A. Orrok, New York, N. Y. Mem. A.S.M.E.

The laboratory study of creep should be supplemented by the determination at various temperatures of Young's modulus and of heat conductance for steels under consideration. At present there is felt a woeful lack of this information, while it is as important as the creep curves themselves. Any intelligent use of creep data is impossible without calculating thermal stresses, and so very little has been done to verify the conductance for instance. It seems certain that thermal conductance goes down as metal temperature rises, which aggravates the situation.

Let it be hoped that this excellent work of Drewry will bring more correlation in the present haphazard efforts to analyze the creep. The writer trusts that the necessity of a joint meeting on creep to fix a general program of investigations will soon be realized.

H. J. KERR.[6] The author is to be congratulated on his paper, which combines the application of fundamental principles with a description of scientific operation developed on the basis of the theories used. The design of radiant-heat superheaters is a complicated one if for no other reason than the fact that furnace characteristics vary over a tremendous range. To obtain the full value of the paper, which is certainly a valuable one, the exact conditions which the author started out to meet and which he apparently has well met should be considered, and it should not be assumed that the answers which he has found are applicable to all conditions.

In general the conditions with which the author has dealt are:

1 Low-duty furnaces in which the heat of combustion per cubic foot of furnace volume is not as high as is often called for
2 Coal having a high-fusion-temperature ash
3 Load conditions permitting of careful combustion control, as represented by a starting time of 4 hours
4 An operating personnel that has cooperated with him in securing the best results
5 A maximum steam temperature of 750 deg. fahr.

For these conditions he has apparently developed round-tube radiant-heat superheaters which are meeting the required service conditions, though one might question whether the replacement of tubes at certain intervals as a maintenance problem is entirely satisfactory.

In considering the results, however, from the broader angle of using various kinds of furnaces, some points brought out in the paper may be enlarged upon.

If one takes the radiation formula as given and assumes a transfer rate of 50,000 B.t.u., a furnace temperature of the order of 2100 deg. fahr. is obtained. It would be interesting if the author would say a few words on this, explaining just how this temperature is obtained or just what this temperature actually is. In the majority of furnaces throughout the country it will not be safe to assume either this temperature or this transfer rate as being the maximum to be expected. It is evident, then, that a radiant-heat superheater must be placed in a carefully picked location in the furnace, and where the requirements call for installing the superheater on all walls the problem of meeting higher transfer rates which will surely be obtained locally, at least, must be faced.

Fig. 15, showing the relationship between creep values and the actual pressure stress, is extremely interesting. The author apparently assumes that the major part of the difference between the pressure stress and the creep stress represents temperature stress. Our results do not confirm this conclusion. The apparent discrepancies here would appear to be the result of obtaining on the tubes at some period in the operation higher temperatures than those which are credited to them. In general, our investigation of this matter indicates that we can come very much closer to the creep curve with our pressure stress without obtaining measurable creep over a period of two years.

In comparing square or rectangular tubes with round tubes, it should be borne in mind that the comparison in the paper is on round tubes 0.3 in. thick versus square tubes or rectangular tubes $^3/_4$ in. thick. If this comparison had been made between round tubes and small square tubes of approximately the same thickness, there would not have been the divergence between the tube temperatures obtained. The small square tube, furnishing a flush wall, has advantages, particularly at low ratings where the mass flow is low. In general, the most critical zone for tube temperature is at the very low ratings, particularly where the rising-curve characteristic exists (the rising curve is not characteristic of all radiant-heat superheaters). This is an important factor, particularly where boilers must be placed in service in a shorter time than that permissible at Milwaukee or where firing conditions will not permit of starting up in such a way as to shield the superheater.

With round tubes placed out in the furnace away from the walls, we know from experience that, when dealing with a slagging coal, slag accumulates behind the tubes and around them. It is possible under certain conditions to operate with water tubes located in this manner, but the variation in heat absorption is too great for the satisfactory operation of superheater installations of this type except where the temperature conditions are sufficiently low to insure an absolutely dry ash which will not adhere. If from this type of construction you proceed to the round tube close together, so as to eliminate the formation of slag, then you are back to the condition where only one-half of each tube receives radiation from the furnace, and if the heat head is high enough, the front side of the tube becomes considerably hotter than the rear side, and the tubes tend to bow away from the furnace. In the case of round tubes this bowing will unfortunately not be perpendicular to the wall in all cases, and the tendency is for these tubes to warp out of shape, necessitating a complete wall outside the superheater and a considerable variation in the heat application to the different tubes. In this respect the small square tube, if given an initial bow to the furnace and held, will remain in its proper position for long periods of time. Thus the square tube eliminates the necessity of an outside brick wall.

Fig. 13 indicates clearly how close these tubes are to the limit of overheating when the removal of one tube along the wall will increase the rate of creep on the tube adjoining from about 6 per cent to 15 per cent.

Fig. 17 shows one of the disadvantages of radiant-heat superheaters in general, indicating as it does a variation in steam temperature of some 50 deg. at a given rating during the first several days of operation. Undoubtedly a method of operation has been developed at Milwaukee to eliminate this condition at the present time, but it is nevertheless one of the disadvantages. In our experiments with radiant-heat superheaters we have found that the steam temperature is affected very materially by steam flow, feedwater, and pressure fluctuations.

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[6] Chief Engineer, Service Department, Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

This is very noticeable on marine installations when maneuvering the ship.

Dealing now with the question of materials, there is no doubt that some of the low-alloy materials available for superheater use have superior creep values up to approximately 1000 deg. fahr., but in general there is very little improvement above this temperature. Again, as the author points out, an important factor in determining the material to be used is the oxidation rate. In general, at tube temperatures of 1000 deg. fahr. and above, there is little if any gain in using low-alloy-steel tubes in place of carbon-steel tubes. Work by Porter, Fellows, and Rummel shows that when plain steel exceeds a temperature of 950 deg., dissociation of the steam takes place and the hydrogen formation increases fairly rapidly. Therefore a material to be satisfactory for this purpose should not only give an increased stress for a given creep rate at 1000 deg., but should also show a distinct improvement in oxidation rate on the inside of the tubes.

Investigation of and experience with Nirosta KA2 steel with steam temperatures of the order of 1000 deg. and tube temperatures over 1200 deg. fahr. indicates clearly that this material meets this requirement at temperature levels well above those permissible with low-alloy steels.

While it is true that the conductivity of Nirosta KA2 is materially less than that of plain steel, the use of this material should certainly lead to a big improvement over plain or low-alloy steels at heat transfers far above anything contemplated for radiant-heat superheaters, even with steam temperatures as low as 700 deg. fahr. Nirosta-tube superheaters at our Bayonne Works have operated satisfactorily under conditions where plain steel immediately showed signs of failure. One of these Nirosta superheaters has been maintained in operation for endurance tests and has operated for more than a year without any indication of failure, with steam temperatures from 750 to 850 deg. fahr. and metal temperatures as high as 1200 deg. fahr. The higher stress permissible with this material permits of reduction of thickness, which compensates for the lower conductivity.

In general, operators are very much opposed to controlling combustion conditions to meet any other requirements than those of capacity and efficiency, and in general radiant-heat superheaters do not permit this simplicity. The experience at Milwaukee indicates that the control of the combustion can be worked out to quite a satisfactory standard, but it should be borne in mind that this is a necessary requirement of radiant-heat superheaters. How serious a problem this control may be is somewhat affected by type of furnace and again by the mental attitude of those who are to operate it. This last statement might also hold for the regulation of steam temperature by desuperheating. The author feels that such a method is not the most desirable method, whereas it appears to the writer that control of steam temperature by automatically desuperheating the steam at an intermediate position of the superheater represents the most foolproof method for obtaining constant superheat which has been developed.

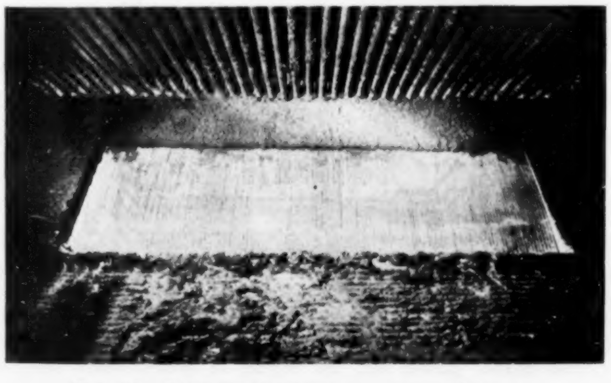

Fig. 23 Calumet Superheater After Service

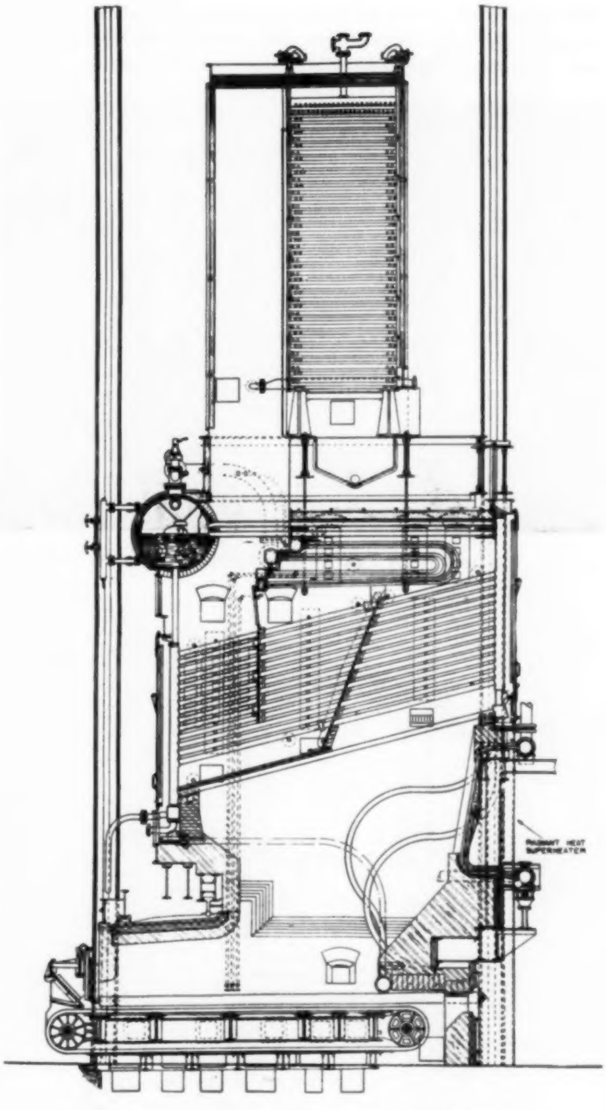

Fig. 24 Boiler and Furnace at Calumet
(Showing location of radiant-heat superheater.)

To show the range of requirements for radiant superheaters, Fig. 23 is a photograph of a small square-tube element superheater installed at the Commonwealth Edison Company's Calumet Station, Chicago. After two years of service with outlet steam temperatures up to 740 deg. fahr., heat inputs up to 58,000 B.t.u. per hour per sq. ft. of wall area, and a pressure drop of 7 lb., with a mass flow of 300,000, this superheater gives every evidence of having an indefinite life. This superheater is set over a forced-blast chain-grate stoker in a furnace having no other wall cooling and a maximum heat release of 40,000 B.t.u. per hour per cu. ft. Fig. 24 is a drawing of the boiler and furnace showing the location of the radiant-heat superheater.

As another example, Fig. 25 shows a photograph of a radiant-heat superheater installed at the Trenton Channel Plant of the

Detroit Edison Company which has been in operation for some two years with final steam temperatures of 1000 to 1100 deg. This superheater has plain-steel tubes at the low-steam-temperature end and Nirosta KA2 at the high-temperature end, the tubes supporting brick liners so as to reduce the heat-transfer rate and give a metal-tube temperature of approximately 100 deg. above max mum steam temperature with a mass flow of the order of 120,000 lb. The writer is expecting that Mr. Van Duzer, of the Detroit Edison Company, will report as to the operating conditions in detail and the present condition of this superheater.

Fig. 25 Radiant-Heat Test Superheater at Trenton Channel Plant

As still another example, there is shown in Fig. 26 a print of the superheater now being installed at the Delray Plant of the same company to heat 90,000 lb. of steam an hour to a maximum temperature of 1100 deg. fahr., with a pressure drop of approximately 25 lb. through the unit and to meet certain efficiency specifications. The flow through this unit is first through a steel-tube superheater, from there through a KA2 superheater, which is parallel flow, and finally through the radiant-heat superheater in the furnace, which is of the same construction as that shown in Fig. 24.

Steam temperatures in the various sections will be, roughly, as follows:

	Deg. fahr.
Inlet	700
Outlet of plain steel section	800
Outlet of KA2 parallel flow section	1000
Outlet of radiant heat section	1100

It might be added that designs are now being made for boilers with superheaters to give a final temperature of 1000 deg. fahr., and such superheaters will be located in a position where they will receive both convection and radiant heat.

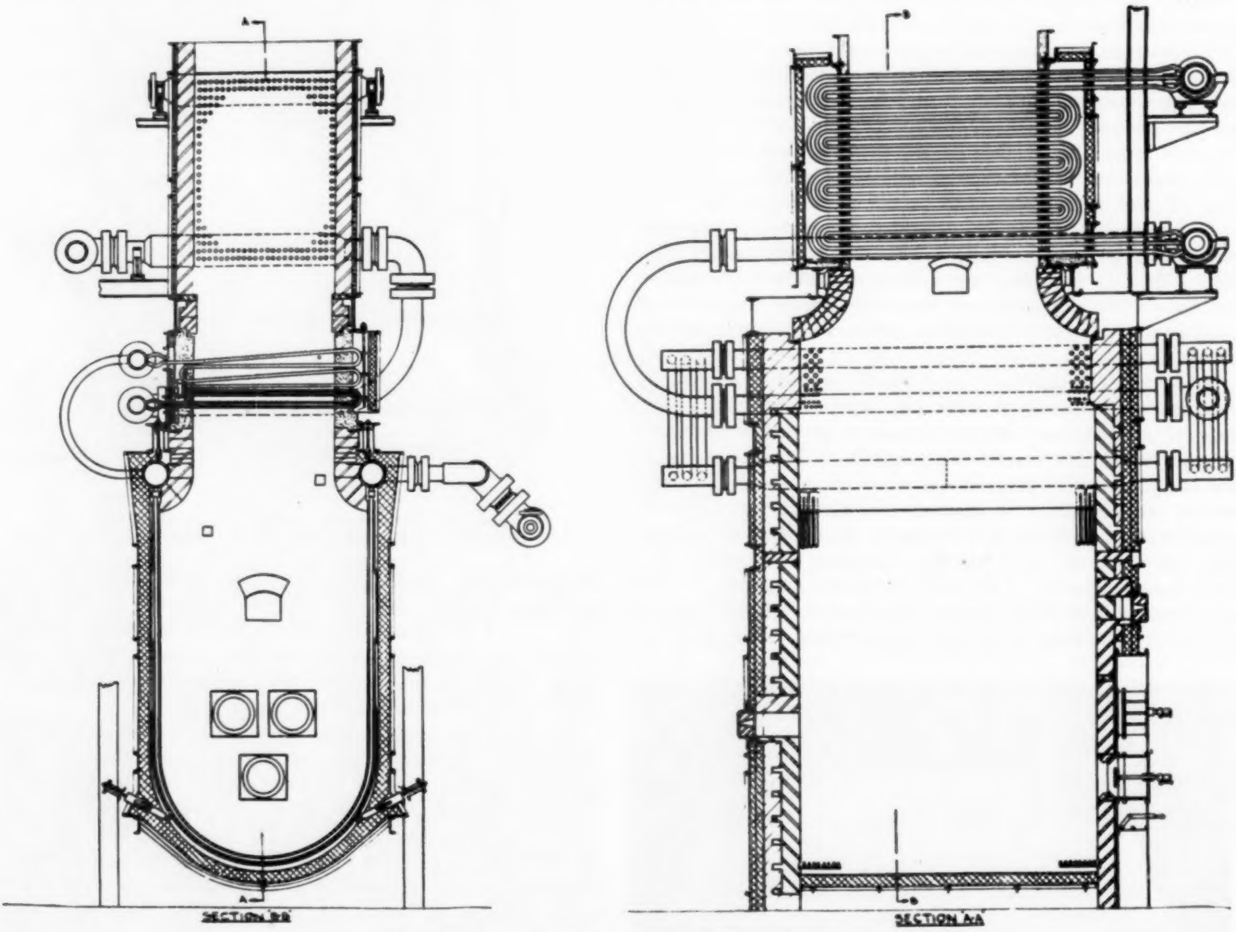

Fig. 26 Superheater for the Delray Plant

The author points out the benefit in heat-transfer rate of increasing pressure up to 1400 lb. The writer is glad to be able to add to this the fact that this benefit continues at higher pressure. We have operated our high-pressure radiant-heat superheater up to 3500 lb. pressure, and at 2500 lb. pressure, with a heat input of 40,000 B.t.u. per hour per sq. ft., we are obtaining a temperature difference between the steam and the outside of tube of only 60 deg. fahr.

R. M. Van Duzer, Jr.[7] The Detroit Edison Company has had in service, since the early part of 1929, an experimental separately fired superheater capable of raising the temperature of 6000 lb. of steam per hour from 700 deg. fahr. to 1100 deg. fahr. The tubes in this superheater, which was supplied by the Babcock & Wilcox Company, were made from a 0.08-C steel and from Enduro KA2. The carbon-steel tubes were installed in the rear wall and the KA2 tubes in the side walls, both of which were covered with protecting tile. Fig. 27 shows the details of this equipment. Steam enters the lower header and flows in four parallel paths to the outlet fitting.

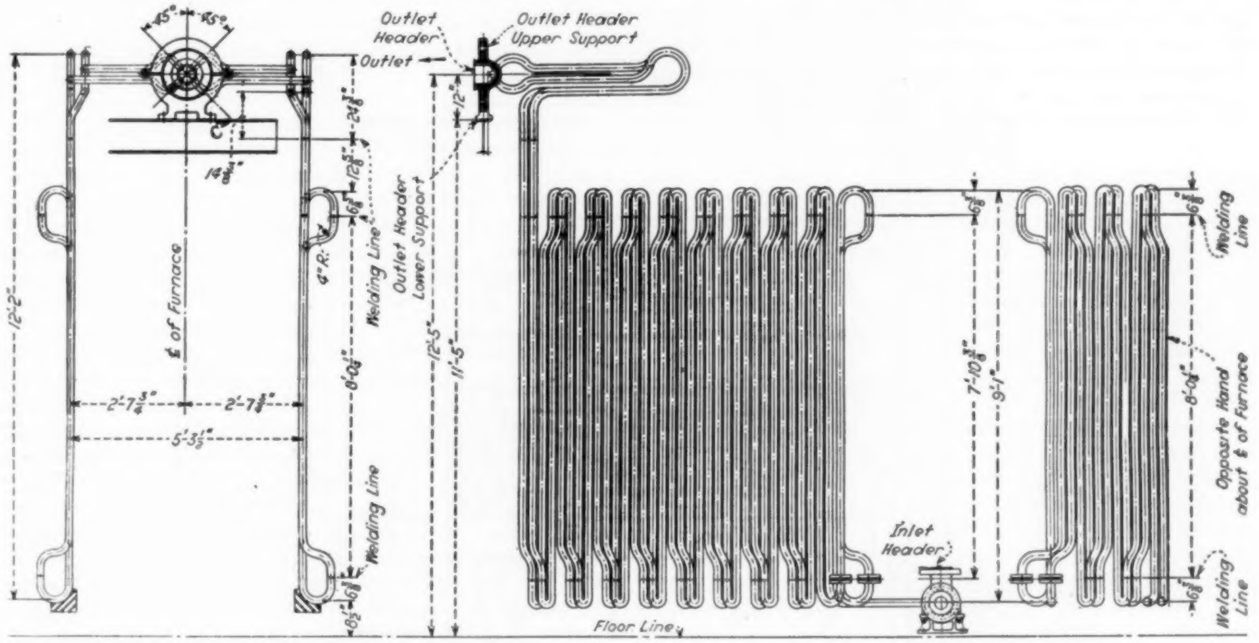

Fig. 27

Table 5 gives a summary of the hours in service to May 25, 1931. No trouble has been experienced with this superheater since its installation.

Table 5

Steam temperature	Hours
Below 900 deg. fahr.	325
900 deg. fahr.	1,458
1000 deg. fahr.	2,398
1050 deg. fahr.	438
1100 deg. fahr.	6,221
Total	10,840

After the superheater, as supplied by the Babcock & Wilcox Company, had been in operation over a year, a section of calorized tubes was added to the outlet, where it was subjected to the hottest steam. This installation was made to determine the effectiveness of the aluminum coating against scaling, and also to find a material that could be substituted for the expensive 18-8 alloy steel. This addition consisted, essentially, of eight $1^3/_4$ in. outside diameter, $^3/_8$-in. wall, 8 ft. long, 0.15-C calorized-steel tube sections, which were placed in the combustion chamber with the lower ends subject to direct flame impingement. No refractory protection was placed on these tubes.

It was realized at the time that the service was rather severe and that use of this material would not be warranted under like conditions in regular operating equipment. The location selected, however, was the best available for securing the information in a short time.

The tubes were removed after 3821 hours of service because of severe growth of two of the forged return bends, located in the flame travel. This growth amounted to nearly 20 per cent. The tubes, themselves, were badly scaled and had increased in diameter from 6 to 8 per cent in the hottest portions. This corresponds to a creep rate of from 16 to 21 per cent in 10,000 hours. The tube metal temperatures varied from 1000 deg. fahr. to 1200 deg. fahr. over the operating range, and the maximum combined stress due to internal pressure and heat flow, from Orrok's equation, was 3100 lb. per sq. in. These creep rates are not to be taken as absolute values, as the measuring points provided on the tubes were affected by the scale formations.

The success of the 18-8 material in this experimental superheater served as the basis of design for the Babcock & Wilcox separately fired superheater which will be placed in service at Delray No. 3 the latter part of this month, supplying steam to the British Thomson-Houston high-temperature turbine-generator. A cross-section of this unit is shown in Fig. 28.

Steam at 700 deg. fahr. is led into the top convection section, which is composed of low-carbon steel. From there, it passes to the middle combined convection and radiant section made of alloy steel; and then on to the lower alloy radiant section. The unit contains approximately 4000 sq. ft. of surface and is designed to raise the temperature of 90,000 lb. of steam per hour from 700 deg. fahr. to 1100 deg. fahr.

The steel used for the two lower sections differs from the material used in the experimental installation in the silicon

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[7] Detroit Edison Company, Detroit, Mich.

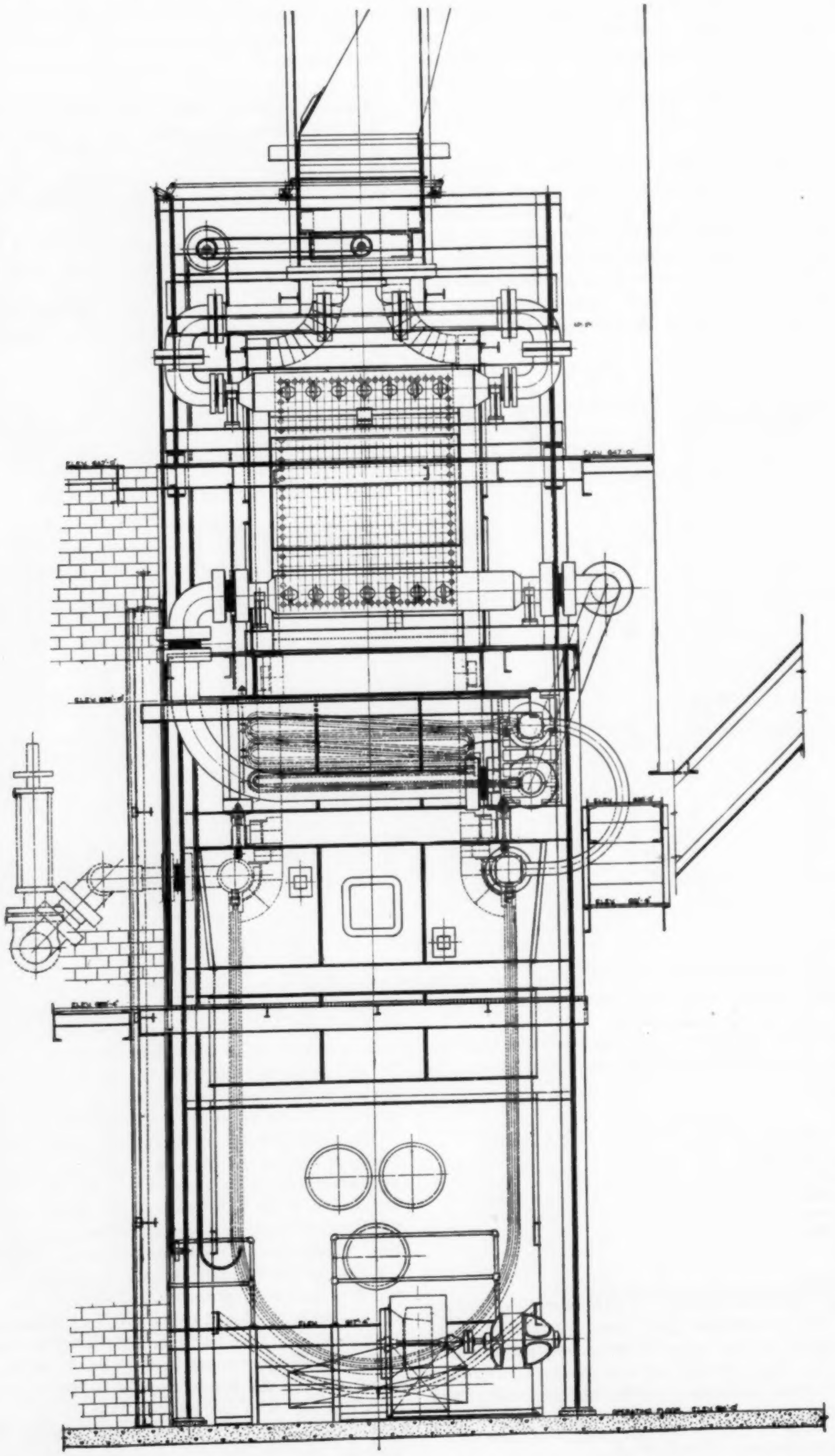

Fig. 28

content. To increase the resistance to sulphur corrosion from the flue gases, 2 to $2^1/_2$ per cent of silicon was added.

A group of high-carbon tubes have been installed in the middle section of the superheater for comparison with adjacent KA2 tubes. Part of this group is calorized, while the remainder has a protective coating of KA2 steel which was applied with a Schoop metal spray gun.

The creep data presented by the author are valuable and reassuring to those who are interested in the development of superheating equipment for higher steam temperatures. It is hoped that additional material will be published in the future on the same subject.

Three other items should receive consideration, together with creep values, in the selection of materials for high-temperature superheater tubing. These are the rate of scale formation, the effect of dissociated steam, and the change in crystal structure indicating a tendency toward embrittlement. Information of this nature concerning the three steels reported in the paper would be of value.

W. E. Caldwell.[8] The paper furnishes a wealth of information on operating experience with radiant superheaters as well as on the operating technique which has been developed in their use and maintenance. The simplicity of the radiant superheater is so attractive that it requires little else to commend it where conditions will permit its use. However, it requires more careful coordination of various related apparatus than does the more conventional type.

Radiant superheaters appear better suited to pulverized-fuel firing than most other methods, since their successful use depends largely on control of furnace conditions and a large furnace which is also desirable for most types of pulverized-fuel firing. The shape of the furnace and the arrangement of the burners are also important considerations, and the latitude in this respect is somewhat limited. The protection of radiant superheaters against unexpected loss of load is a simple problem with pulverized fuel and adequate water cooling in the furnace, since the fire may be quickly extinguished and superheater safety valves depended upon for the final cooling effect.

Paralleling the development of the radiant-type superheater, the conventional type has been making increased use of radiant heat for controlling the shape of the curve. Some years ago a superheater separated from the furnace by only six banks of boiler tubes was considered hazardous, while today a number of successful installations are in use protected by only four banks of tubes and having excellent temperature characteristics. In a revamping undertaking in Hell Gate Station recently a top-bank convection superheater on a stoker-fired boiler was replaced by an interdeck superheater having maximum pressure drop of 15 lb. and located but four tubes above the furnace. The steam-temperature variation normally does not exceed 20 deg. between 80,000 lb. per hour evaporation and 260,000 lb. per hour, the average temperature being 770 deg. With an interdeck superheater on a pulverized-fuel boiler in the same station, the temperature varies from 690 deg. to 710 deg. for a rating of 150,000 lb. to 400,000 lb. per hour and with a pressure drop through the superheater of 19 lb. at the maximum point.

The maintenance problem with interdeck superheaters is largely a question of hanger material and arrangement, but this problem appears to have been satisfactorily solved in recent designs. With the bent-tube type of boiler, the hanger problem practically disappears, since the elements hang from the headers. In the bent-tube type of boiler there is also considerable latitude in the location of the elements with respect to the radiant heat, and the elements receive the protection afforded by the adjacent boiler tubes.

A serious fault in many superheater installations, both radiant and conventional types, is the excessive pressure drop considered necessary for protection. This pressure drop is especially pronounced at high loads, but it influences station performance at all loads, since it establishes the superheater safety-valve setting which, in turn, affects the steam pressure throughout the station. The magnitude of this loss in station performance is not generally appreciated, and in many instances the cost of this superheater protection is far greater than the value of an occasional element which might be required in a less conservative design. While all may not agree that the radiant superheater is susceptible to general application, it appears obvious that the design reported in the paper could never be criticized for being overconservative.

In considering the cost of occasional replacements on the comparatively short life of the elements suggested by the author, it should be recalled that this cost may be offset by a substantial credit on capital savings and on fuel saving from pressure drop alone as contrasted with some other more durable type of superheater.

From the data reported it appears that the pressure loss is less than with most conventional superheaters. In any event if the creep takes place in normal operation rather than during the starting periods it would not seem possible to obtain any further reduction in pressure drop without a corresponding reduction in life of the elements. While in general we are prone to consider occasional replacement as excessive maintenance, maintenance in itself is not always an important measure of the adequacy of apparatus. Maintenance is quite often a manifestation of the far more important consideration, availability, which, in turn, adversely influences capital investment.

Maintenance may be roughly divided into two classes: controllable maintenance, which is subject to scheduling, and unforeseen maintenance, which comes unexpectedly and involves an embarrassing loss of capacity. With the method outlined by the author of predicting superheater-tube life, superheater maintenance may be scheduled so as not to conflict with periods of maximum system demand. Thus one handicap often associated with maintenance is removed.

The substitution of radiant-superheating surface for water walls lessens the quantity of water in the boiler-circulating system. This permits a smaller boiler drum for the same degree of water-level stability.

On the other hand, the radiant superheater and reheater absorbs only about 30 per cent of the total heat absorbed by the complete unit, as contrasted with 50 per cent absorption by some water-wall furnaces reported in recent papers. Thus, for equal furnace size the radiant-superheater installation would result in higher gas temperatures entering the boiler accompanied by increased slagging tendencies in the tube bank. This disadvantage might be offset by increasing the furnace size or by increased depth of slag tubes below the bottom boiler bank.

The reduction in draft loss by the radiant superheater is not clear gain, since it necessitates, by its replacement of water walls, boiler-heating surface elsewhere which may correspondingly increase the total draft loss.

The substitution of radiant superheaters for water walls and consequent reduction of water in the boiler has merit in that it shortens the time required for starting from cold, as indicated by the author. The author refers to re-superheating in two steps to obtain greater economies, not simply to reduce moisture in the lower turbine stages.

There are different schools of thought as to the value of reheat and its purpose, and after all, dry steam results in rather substantial economies besides fuel alone. Turbine maintenance and availability due to water erosion of the lower stage blades is a factor of importance, and there is much room for improvement in this respect. While admitting the complications of certain reheat arrangements, whether radiant or otherwise, the effect of reheat on turbine maintenance should not be lost sight of, quite apart from thermal benefits. Turbine maintenance over the past ten years has been a more serious factor than many realize. To combat this problem more expensive materials are being used in blade construction, and yet wet steam sometimes results in excessive maintenance to other parts of the turbine than moving blades.

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It is well known that most turbine erosion is due to moisture, and it is also recognized that corrosion cannot take place in the absence of moisture, even though corrosive gases may be present in the steam. With the use of sufficient reheat stages to maintain dry steam all the way to the exhaust, cheaper turbine materials and workmanship may be employed, which when credited to reheat might give added attractiveness to reheat cycles in new plant designs. Turbine manufacturers in recent years have made great strides in developing blade material to resist the destructive influences of moisture in the lower stages of the machine. The high cost of modern blade materials, together with the greater cost of machining, is not generally appreciated, and the improved resistance to blade erosion in more recent machines is adding more to the purchase price than may users realize. Blade maintenance may be eliminated or reduced either by the use of more expensive blade materials or extensive use of reheat, and it is barely possible that in some instances double reheat cycles may be cheaper in the long run when all things are considered.

In Fig. 20 the curve of superheater and reheater performance for a constant back-pressure turbine seems to indicate that a convection superheater and radiant reheater would have been more desirable, excluding the consideration of temperature strains in the high-pressure cylinder with changing load. That is, the break in the reheater inlet temperature at the 250,000-lb. point would indicate saturated steam from the exhaust of the high-pressure unit. The presence of this moisture might result in ultimate blade erosion at the exhaust end of the high-pressure unit or the accumulation of deposits from the steam which would not adhere in the presence of superheat.

In Fig. 22 reheat is accomplished at a comparatively high point in the expansion range. As a consequence the available range for reheat at the higher loads is comparatively limited. No doubt this arrangement was dictated by careful evaluation of all contributing influences, but it would appear that full advantage had not been taken of the superheat in the initial expansion.

The desirable shape of the superheat or reheat curve is influenced largely by the choice of the control arrangement of the main turbine, which in most instances is dictated by nature of load, cost of capital, fuel and labor, as well as limitations of manufacture.

A superheat curve which might be attractive for multi-nozzle control might be less desirable for throttle control or bypass control, and the radiant superheater for use alone offers little opportunity for change in the natural shape of this curve. It would seem that for the initial superheating certain types of interdeck superheaters lend themselves more readily to obtaining the desired shape of curve than the radiant type alone. It would also appear that for the initial superheating a single interdeck design might be more economical than a combination of radiant and convection superheaters used together, due to the duplication of piping and headers. For reheating the natural characteristics of the radiant reheater are very attractive.

Radiant superheaters under some conditions are also subjected to convectional influences within the furnace, and it is possible that in time the arrangement of apparatus and control of the furnace may be so governed as to widen the control of the temperature range even for the radiant superheater.

In 1924 six pulverized-fuel boilers were installed in Sherman Creek Generating Station to provide additional capacity. The boilers were comparatively small (650 hp.) and were provided with water-cooled side walls and radiant superheaters on the rear walls. These boilers, by the way, were the first pulverized-fuel boilers ever equipped with water walls. The radiant superheaters are of the round-tube return-bend type and are held against the rear wall by clamps made of a high-heat-resisting material. The six boilers are practically identical, but each furnace is fired by a different combustion system or different burner arrangement. Four of the boilers are fired by the bin and feeder system, while two are fired by unit mills. Several different burner combinations have been used on some of the boilers, and a great mass of data and experience obtained.

The experience with creep of the superheater elements has been much the same as that reported by the author, except at an accelerated rate with some of the units. One of the problems with these superheaters was the anchorage to the wall and also the impinging effect of the flame in certain furnaces, due largely to burner arrangement. The pressure drop through these superheaters is about 15 lb. at a maximum flow of 60,000 lb. per hour, and at the average rating the pressure drop is about 8 lb. The temperature range is between 500 and 600 deg., which varies for the different units. The shape of the curve of some of these radiant superheaters practically parallels the usual convection type, due probably to convection heat or flame impingement with increasing rating. With the burning away of the supports some of the elements would lean toward the furnace and result in much shorter life than the average. Visual inspection through observation openings indicated when attention was necessary, and in the early days of this installation micrometer measurements would have been superfluous.

The creep took place not only circumferentially but longitudinally as well, causing bowing between supports which, unless corrected, subjects the units to very severe conditions. On one furnace in which impingement was especially severe a protecting water screen was installed in front of the superheater, which gave complete protection. A curious effect of this water screen was the change in the shape of the superheat curve. Previous to the installation of the screen the curve exhibited the characteristics of a convection superheater, while after the installation the shape resembled more closely what would be expected from a pure radiant superheater, due perhaps to the water screen intercepting the convection effect of the turbulent furnace gases.

From time to time the superheater elements unprotected by a water screen are removed, straightened out, replaced, and used in this manner until they are unfit for further use, due to excessive reduction in the wall thickness.

In the early days of this installation the appearance of the superheaters indicated the path of the gases and temperature distribution almost as clearly as could be determined by more precise investigations. The straightening of these tubes was accomplished by a device made from a large valve yoke and stem clamped to a building column, so that by means of the hand wheel, pressure could be applied through the stem to force the tube into shape.

The failure of a superheater tube in service is much less destructive than many would imagine and less serious usually than the failure of a boiler tube or water-wall tube. The opening in a failed superheater increases the steam flow and cooling effect, which protects it against further distress. In a pulverized-fuel furnace a major failure of a superheater element usually extinguishes the fire and is not extremely hazardous if adequate safeguards are provided on the combustion system. The unexpected failure of a superheater element might be embarrassing to boiler capacity, but otherwise it is not ordinarily serious.

A problem with some conventional superheaters is pressure-drop limitations established by the boiler width or space available for the installation. In recent installations this has been overcome by the use of elements in parallel, vertical as well as horizontal, thus making up the deficiency in width by an increase in height. While there are a number of conventional superheaters which come within the 20-lb. pressure drop referred to in the paper for the radiant superheater, the majority of installations greatly exceeds this pressure drop and as a result would suffer by comparison with this figure. From a consideration of adiabatic heat drop, 1 deg. of superheat is equal in value to about 3 lb. pressure, and in evaluating superheater performance both temperature and pressure loss are major considerations in striking a balance with the many other factors.

Various types of superheaters are sensitive to different influences which are difficult to predict in the design stage. Besides the influence of the ratio of gas to fuel, some superheaters are sensitive to change in the ratio of products of combustion to steam resulting from changes in the feedwater temperature and similar influences. What is needed is a low-cost superheater or reheater, the temperature from which may be adjusted for sufficient range to take care of changing conditions which frequently arise. The design reported on by the author seems to meet some of these conditions in an admirable manner.

Some plants are so situated that slight changes in market conditions may render any one of the three fuels, coal, fuel oil, or gas, available for economical use. This range of fuel renders the choice of a superheater design very difficult and becomes an important consideration at the time of purchase.

Among the lessons which may be drawn from the paper might be mentioned the indicated durability of ordinary carbon steel used for steam piping. From the observations of creep and temperature presented by the author, it appears probable that the steel piping in use today would probably be satisfactory with steam temperatures up to 850 deg., with a creep rate so low as to cause no particular concern.

As our knowledge and experience with the limitations of various materials advance we will be able to design and operate apparatus much more economically than was possible with some of the earlier "rule of thumb" standards which had been established. One of the difficult problems of the present day in evaluation procedure is to strike a rational balance between physical life and obsolescence. The author's paper deals in a fine way with a design of apparatus which but a few years ago would have been considered radical, but which on close analysis appears more rational than some of our earlier efforts with conservative designs.

H. DAHLSTRAND.[9] Only too seldom is there given complete, detail information concerning the behavior of machinery in actual operation. In a paper such as this, where all the details have been closely studied, the information will be of the greatest value to designers of boilers and superheaters and also of other machinery operating at high temperatures. Experience has shown that data gathered during the actual operation are of great value. Coordinating these data with laboratory tests is of particular value to the designer of machinery in working up detailed designs. This is particularly important when making designs for the high temperatures now used in power plants.

One of the interesting sections of this paper is that covered by complete creep data on the material used in the superheat tubes. Very little has been published concerning the creep under actual operating conditions, and information on this subject will enable us to make up designs with greater confidence. From the results it is shown that the creep is appreciably more than that shown by standard laboratory methods. This, of course, can be expected, as it is impossible to duplicate the operating conditions in the laboratory work. In exposing material to changing temperatures, while these changes may not be great, they will be sufficient to materially increase the stresses in certain parts of the sections, and when the temperature changes are reversed, other parts of the sections may be stressed higher than that calculated in the design. This obviously should increase the rate of creep over that shown by laboratory tests.

One of the important facts for steam-turbine designers brought out by the paper is the importance of maintaining, as far as possible, uniform temperature at various ratings. Naturally it is of importance when starting any unit to bring up the temperature slowly so that no dangerous stresses are set up at that time. Sudden changes in temperature as far as steam turbines are concerned may cause dangerous distortions.

The writer thoroughly agrees that complete operating information on equipment operating at high temperatures is of the greatest importance and should be emphasized in all high-temperature installations.

GEO. A. ORROK.[10] Papers dealing with operating experiences in the development of power are among the most valuable which the Society publishes, and when one of these papers, covering years of operating experience, has included in its scope, scientific discussions along research lines, which show the connection between theoretical and practical viewpoints, the members of the Society are to be congratulated.

The paper deals with over seven years of experience with radiant-heat superheaters where the entire problem has been worked out to a successful solution under severe operating conditions of pressure and temperature.

Perhaps the outstanding statement in the paper is that the pressure stresses in a radiant-heat superheater are so small as to be of only minor importance. The really important thing to consider is the heat stresses, and therefore the thinnest section of metal which will stand the punishment is the best and most economical thickness to use. In the writer's own experience he had recommended as the safest and best solution that thickness of tube which makes the pressure stresses and the heat stresses equal in numerical value. The author's observations on creep and on the values due to creep but duplicate those of other experiences where similar apparatus has been used. Increases of the outer diameter of the tube amounting to from 15 to 20 per cent are common in 18 months of service. The inner diameters of the tube may in the same time increase as much as 45 per cent.

The most serious condition occurs when there is tuyere action from a stoker where the point of the flame actually impinges on the tube surface or where the oil flame or powdered-coal flame impinges in a similar way. Here the tube oxidizes very rapidly, creep and bulges occur at a very much faster rate, and failures similar to those shown in Fig. 12 almost invariably happen. When the flames do not impinge on the tubes, the useful life of course is much longer, but creep does take place, and the life of a superheater tube under these conditions when made of open-hearth steel may be upward of three years.

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The heat conductivity of open-hearth steel with approximately 0.12 carbon is nearly twice that of high-carbon steels and perhaps $2^1/_2$ times that of alloy steels. The conductivity of wrought iron is about the same as of high-carbon steel. Why these variations occur is not known, and this would appear to be a fruitful field for research, since apparently the best metal for heat conductivity and therefore for the lowest outside temperature and lowest heat strains is the standard open-hearth tube steel which has been so largely used. High steam velocities through the tube also reduce the outer surface temperatures, and the operator should be willing to take quite a drop through his superheater in order to take advantage of the increased heat transfer and lower outer surface temperature.

The author's statement of the advantages of the combination of the convection and radiant-heat superheaters is most striking, and in general this paper merits the closest attention.

AUTHOR'S CLOSURE

Mr. Orrok's discussion of tube thicknesses suggests the point that thin tubes have an important investment credit when metals of the 18–8 class, costing 50 to 60 cents per pound, are employed. Mr. Orrok is to be commended on his early advocation of reasonable tube-wall thicknesses.

As both Mr. Orrok and Mr. Caldwell point out, radiant-superheater failures occur as small, unharmful steam leaks. Ability to continue operation until convenient maintenance occurs is a vital merit. Forced outages and scheduled maintenance are widely separated procedures.

Mr. Kerr's liberal discussion is a valued addition to the subject. The considerable interest and work of his company in radiant superheaters appears another confirmation of the point that these superheaters warrant serious attention of the industry.

The furnace temperature of which Mr. Kerr asks is obtained at the point of heat-absorption measurement. It is the equilibrium temperature which a thermocouple assumes, and which when applied in the radiation formula indicates a constant of 1300. Tests have shown the validity of the method throughout a heat transfer range of 10,000 B.t.u. per sq. ft. per hour to 125,000 B.t.u. per sq. ft. per hour.

Other points suggested by Mr. Kerr's discussion are:

1 Low ash-fusion temperature does not preclude use of radiant superheaters. A 4 per cent sulphur coal of 2050 deg. fahr. ash-fusion temperature was burned in Lakeside boilers for several months last fall without a suggestion of dirtiness trouble from the flush-type round-tube superheaters. Reasonable-temperature, bare-tube furnaces are not subject to the wide temperature and transfer variations that Mr. Kerr mentions.

2 As the paper mentions, side-wall locations are preferable for radiant superheaters to facilitate boiler design. They would not be advisable for most front walls, for heat-absorption rates exceed the prescribed 60,000 B.t.u. per sq. ft. per hour and are not as nearly proportional to steam-flow rates.

3 Regarding temperature stresses: The paper gives considerable experimental information indicating their probable importance. Good reasons are advanced showing that the tubes were not occasionally overheated. Moreover, the relatively stronger Enduro tubes actually expanded more because of their poorer thermal conductivity and therefore higher temperature stresses. It seems prudent to concede the existence of these temperature stresses.

4 As the paper infers, radiant superheaters are impractical for hydro-plant or transmission-line standby when cold boilers must be put into service in a few minutes. Very heavy elements with great heat-storage capacity would most nearly meet this requirement. Reasonably good reliability of other power-plant equipment and proper load scheduling can easily tolerate four-hour starting from cold, and the plant in general will benefit thereby.

5 The paper states that metal temperature is lowest at low ratings, assuming of course that coal feed and steam flow are in reasonable equilibrium. Both test data and calculations confirm this point.

6 Mr. Kerr referred to the initial and only experimental installation of carbon-steel tubes when he stressed the point of a tendency of certain elements to expand unduly.

7 Operators will agree that low CO_2 is occasionally necessary during exigencies. Convection superheaters then give high temperatures, but radiant superheaters afford low, safe temperatures during the emergency. No other regulation by combustion changes is necessary in a properly designed installation.

8 Superheating to high temperatures and then continuously desuperheating at and above normal rating, so that good economy and reliability can be obtained occasionally at low ratings, does not seem efficient nor reliable.

It is pleasing to know that Mr. Kerr's company has had favorable experience with 18–8 material at metal temperatures as high as 1200 deg. fahr. That experience insures boiler-room success with 1000 deg. fahr. steam, and the industry will be very glad to get the report. The Babcock & Wilcox Company is doing an excellent thing in obtaining experience with new equipment in its own plant rather than trying it on a large scale for the first time in the customer's station. Mr. Kerr is commended for his valued statements of his company's experiences.

Mr. Caldwell suggests that reversal of the types of superheater and reheater treated in Fig. 20 might be advisable for prevention of moisture in the turbine exhaust. The text is incomplete in not indicating that partial bypassing of the high-pressure turbine occurs above 250,000 lb. per hour to obtain higher steam flow from the high-pressure boilers. The exhaust is superheated at all times.

The 20 lb. pressure drop of the 1300 lb. pressure superheater is equivalent to only 5 lb. pressure drop at 300 lb. steam pressure, which better illustrates the low pressure drop of an actual radiant-superheater installation. Mr. Caldwell's discussion in all aspects of superheater economics merits much application.

Mr. Jasper's remarks of the greater thermal expansion of high chromium alloys are important. In installations where only a portion of the tube periphery is exposed to the furnace, greater stresses tending to cause bowing will occur than in the case of carbon-steel tubes. It is understood that expansion of chrome steel is 60 per cent greater than that of carbon steel.

Mr. Artsay's discussion is valued for its calculations of temperature stresses and correlation of existing expressions of these stresses. He is right in stating that accurate laboratory creep knowledge is only a small portion of a designer's needs. His theoretical contribution will be used in attempts to still further improve installations.

Mr. Dahlstrand's statements properly suggest that turbine design for higher temperatures will be more difficult than boiler-room and piping design, and that therefore turbine equipment should be given the benefit of as uniform temperatures as possible. Whereas 1 per cent creep per 10,000 hours is permissible in the boiler room, less than 0.01 per cent creep per 10,000 hours may be permissible in the turbine room. This latter rate corresponds to 0.002 in. elongation in 20-in. casing bolts per 10,000 hours. The bolts should still exert compressive force on the casing joint, and possibly may need tightening yearly to compensate for creep.

When 1000 deg. fahr. superheat and reheat are used in high-pressure installations, then more heat will be added in the superheating equipment than in the boiler for evaporation. Higher feedwater temperatures and higher boiler pressures are decreasing boiler duty at the same time that superheating duty is increasing.

These changes result in an important distortion of present designs.

Water-cooled furnace walls are a present trend of the industry, but the future high-temperature installation cannot afford them since they absorb too much heat and would leave insufficient heat for superheating. If retrogression to refractory walled furnaces is assumed, heat capacity of flue gases after passing a few rows of boiler tubes is also insufficient for efficient superheating, especially at low ratings. Detroit is leading the way toward temperatures of 1000 deg. fahr., and the time that 1000 deg. fahr. reheat will also be used is probably not far away. It seems appropriate that the industry receive general acquaintance with radiant superheaters at this time so that liberal experience may be had with them before they are positively necessary.

Summarizing, it is felt that the discussions far excel the paper in value, representing, as they do, widespread experience instead of limited application. Appreciation is expressed to the discussors on behalf of the Society and the power industry.

FSP-54-15

Performance of Modern Steam-Generating Units

By C. F. HIRSHFELD[1] and G. U. MORAN,[2] DETROIT, MICH.

C. F. Hirshfeld

This paper is based on data collected upon the operation of 244 large steam boilers over a period of three years. A paper was presented at the 1929 Annual Meeting of the A.S.M.E. covering one year's records, but it was thought advisable to continue for at least two years in order to obtain more conclusive results. The primary object was to study boiler outages with special attention given to causes, frequency, and duration, and the effect of such factors as severity of use, design, fuels, and methods of firing on boiler reliability. This paper gives a general summary of the data and a short discussion of results. It will be followed by a more complete analysis of the individual records in the near future.

G. U. Moran

At THE request of the Power Division, the authors presented a paper under this title at the 1929 Annual Meeting of the A.S.M.E. It was based on boiler-operating data obtained from steam-power stations well distributed throughout the United States. These data covered experience with 186 boiler units. The primary purpose was to evaluate the reliability of modern steam-generating equipment, particularly as affected by the severity of use, the type of design, and other nameable variables.

In that paper only tentative conclusions were drawn, because it was evident to the authors that for many reasons the numerical values could not be regarded as inclusive or conclusive. The number of boilers studied was comparatively small, and the time period of one year did not serve to give a true picture of the performance of any single unit. The personal factor also served to distort the findings to an undesirable extent, because different individuals had obviously placed different interpretations upon the items of performance recorded.

The results reported did, however, appear to be of such value as to justify the Power Division in asking that the study be continued on a larger scale and for a longer period of time and that the results thereof be reported in a second paper. It was thought that the experience gained in the first year would increase the reliability of data collected during subsequent years, both through more exact definition of the data desired and through greater skill in their interpretation.

It was decided to continue the study for two more years and to add as many boiler units as possible so as to obtain more representative results. The present paper is based on the records of 244 boiler units for periods varying from one to three years. The data in hand fall within the period July 1, 1928, and July 1, 1931, except for variation of a few weeks in the case of a small number of units.

Printed log sheets were furnished to each operator reporting. The logs were so arranged that each day's operating data were reported for each boiler. Every factor that might have a bearing on the unit's performance was included in the log. Complete instructions and definitions were furnished so as to make the records as nearly as possible directly comparable. The log sheet used is shown in Fig. 1. Design data necessary for a proper interpretation and coordination of the results were obtained by means of a separate questionnaire of conventional type.

As in the first study, information was gathered only on boilers operating at pressures above 200 lb. gage and containing over 10,000 sq. ft. of saturated heating surface, except that a few units with smaller surface but operating at very high ratings were included.

The following definitions were given the operators for guidance in filling out the log sheets and were used by the authors in working up the data:

1 A "steaming" unit is one which is passing steam to the line.

2 A "banked" unit is one in which a pressure approximately equal to line pressure is maintained, but from which there is no flow of steam to the line.

3 A "reserve" unit is one which is cold and in which the pressure is at or near atmospheric, but which may be put on the line in the time that it normally takes to lay and light the fire and bring the steam pressure up to that in the line.

4 A "crippled" unit is one which is in such condition that it is incapable of being put on the line within the time that is normally required to bring a boiler from cold to steaming.

5 The "availability factor" is the ratio between the time during which the unit is not crippled and the total time reported.

6 The "use factor" is the ratio between the time during which the unit is not crippled or in reserve and the total time reported.

The figures submitted by the different operators reporting indicate that in most cases they have interpreted the definitions

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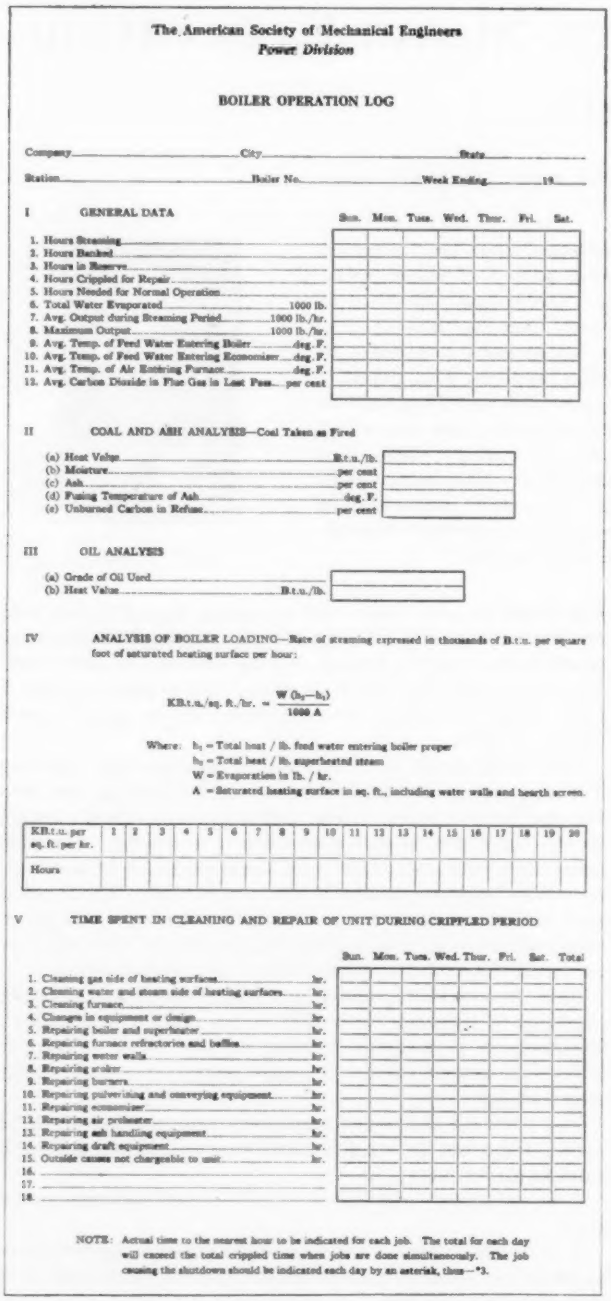
Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.



The American Society of Mechanical Engineers
Power Division

BOILER OPERATION LOG

Company__________ City__________ State__________

Station__________ Boiler No.__________ Week Ending__________ 19____

I GENERAL DATA

	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
1. Hours Steaming							
2. Hours Banked							
3. Hours in Reserve							
4. Hours Crippled for Repair							
5. Hours Needed for Normal Operation							
6. Total Water Evaporated ... 1000 lb.							
7. Avg. Output during Steaming Period ... 1000 lb./hr.							
8. Maximum Output ... 1000 lb./hr.							
9. Avg. Temp. of Feed Water Entering Boiler ... deg. F.							
10. Avg. Temp. of Feed Water Entering Economizer ... deg. F.							
11. Avg. Temp. of Air Entering Furnace ... deg. F.							
12. Avg. Carbon Dioxide in Flue Gas in Last Pass ... per cent							

II COAL AND ASH ANALYSIS—Coal Taken as Fired

(a) Heat Value__________B.t.u./lb.
(b) Moisture__________per cent
(c) Ash__________per cent
(d) Fusing Temperature of Ash__________deg. F.
(e) Unburned Carbon in Refuse__________per cent

III OIL ANALYSIS

(a) Grade of Oil Used__________
(b) Heat Value__________B.t.u./lb.

IV ANALYSIS OF BOILER LOADING—Rate of steaming expressed in thousands of B.t.u. per square foot of saturated heating surface per hour:

$$\text{KB.t.u./sq. ft./hr.} = \frac{W\,(h_2 - h_1)}{1000\,A}$$

Where: h_1 = Total heat / lb. feed water entering boiler proper
h_2 = Total heat / lb. superheated steam
W = Evaporation in lb. / hr.
A = Saturated heating surface in sq. ft., including water walls and hearth screen.

KB.t.u. per sq. ft. per hr.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Hours																				

V TIME SPENT IN CLEANING AND REPAIR OF UNIT DURING CRIPPLED PERIOD

	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total
1. Cleaning gas side of heating surfaces ... hr.								
2. Cleaning water and steam side of heating surfaces ... hr.								
3. Cleaning furnace ... hr.								
4. Changes in equipment or design ... hr.								
5. Repairing boiler and superheater ... hr.								
6. Repairing furnace refractories and baffles ... hr.								
7. Repairing water walls ... hr.								
8. Repairing stoker ... hr.								
9. Repairing burners ... hr.								
10. Repairing pulverizing and conveying equipment ... hr.								
11. Repairing economizer ... hr.								
12. Repairing air preheater ... hr.								
13. Repairing ash handling equipment ... hr.								
14. Repairing draft equipment ... hr.								
15. Outside causes not chargeable to unit ... hr.								
16.								
17.								
18.								

NOTE: Actual time to the nearest hour to be indicated for each job. The total for each day will exceed the total crippled time when jobs are done simultaneously. The job causing the shutdown should be indicated each day by an asterisk, thus—*3.

Fig. 1 Log Used in Collecting Boiler-Operating Data

in the way in which they were intended. In a few cases there appears to have been a different interpretation of the meaning of "crippled" from that intended. In these cases the operators appear to have reported the boilers in "reserve" condition where the majority would have reported them as crippled. Where such interpretations appear to the authors to affect the average results reported herein, the fact has been noted and the necessary corrections have been indicated.

Due to the large mass of data collected and the short time available between the receipt of the last records and the date of preparation of the paper, it has been impossible to complete all the studies contemplated. This paper is therefore only a preliminary report. It gives certain high-spot values which may be compared with those published about two years ago. Study of the data received will be continued, and the authors hope to present a comprehensive treatment within the next few months. The presentation of the completed study will end this endeavor, as it is not believed that any worth-while purpose would be served by continuing the collection of information on this subject.

A summary of the data collected during the three-year period is given in Table 1. The units are grouped according to steam pressures and firing methods, as was done in the first paper. In addition, each class is now subdivided according to the length of time represented by the operating records reported.

In some cases boilers were reported as "crippled" for long periods of time from causes not directly chargeable to the units themselves. We have here a question of definition or interpretation which is troublesome. For example, if a reheat boiler is in condition to operate but cannot be operated because the turbine to which it is connected is shut down, it is certainly fair to regard the boiler as not crippled. On the other hand, if a boiler which is in condition to operate is shut down while the stack to which it is connected is being painted, one might logically take the stand that while the boiler itself was in operative condition the complete steam-generating equipment of which it formed a part was crippled. The authors intend to make a detailed study of the causes of boiler outage as reported and to work up complete records with respect to such causes. Pending the acquisition of the necessary information and until they have had time to digest the data now in hand and supplementary information now being acquired, they have been forced to adopt the interpretation of "crippled" that was used by each operator reporting.

In the present paper they have therefore assumed that when an operator reported a unit as crippled for causes outside the unit, this represents the fact. They have, in such cases, subtracted these hours from the total number of hours reported as crippled and from the total hours covered by the report. This is recognized as a makeshift, but is the best that can be done at present.

The figures given in Table 1 under the heading "Group performances" correspond to those given in the previous paper as "Averages." They were obtained in the same way in both cases; the designation has been changed because experience has shown that the use of the word average for these values is confusing to some readers.

The method of deriving the figures is best explained by reference to the table. The first two units reported are stoker-fired and operate at pressures between 500 and 700 lb. There were 14 for which there were records extending over three years and 10 for which there were records for two years—24 boilers in all.

The figures given in columns 4, 5, 6, and 7, opposite the group of 14, were obtained by totaling the total hours reported for these boilers, totaling the hours reported in each of the four conditions indicated by column headings, and expressing the latter totals as percentages of the total hours reported for the group of 14 boilers. The same thing was done for the group of 10 boilers.

The figures given under "Group performances" were then obtained by totaling the hours reported for the group of 24 boilers, totaling the hours reported under each heading (i.e., steaming, banked, etc.), and then expressing the latter as percentages of the total hours.

The figures given for the class, as for example those indicated as "Performance of stoker-fired class," are not the averages of those given above them in the table. They are obtained by totaling hours for all boilers in the class and handling, as described, for single groups.

TABLE 1 SUMMARY OF OPERATING RECORDS

Period covered in years	Number of units in group	Pressure class, lb. per sq. in.	Percentage of total period				Use factor	Availability factors			Group performances[a]					
			Steaming	Banked	Reserve	Crippled		Individual High	Individual Low	Group	Percentage of total period: Steaming	Banked	Reserve	Crippled	Use factor	Availability factor
							Stoker-Fired Units									
3.0	14	500–700	57.3	18.1	5.9	18.7	0.754	0.846	0.774	0.813						
2.0	10	500–700	74.5	3.8	5.5	16.2	0.783	0.852	0.802	0.838	62.8	13.5	5.8	17.9	0.763	0.821
3.0	1	400–499	69.3	6.9	2.6	21.2	0.762	0.788	0.788	0.788						
2.5	3	400–499	60.4	5.0	0.6	34.0	0.654	0.713	0.622	0.660						
2.0	4	400–499	48.4	25.3	2.2	24.1	0.737	0.788	0.730	0.759						
1.5	4	400–499	83.0	7.0	0.6	9.4(?)	0.900	0.913	0.898	0.906	63.1	12.4	1.3	23.2	0.755	0.768
3.0	24	300–399	70.6	9.0	4.9	15.5	0.796	0.936	0.708	0.845						
2.0	13	300–399	58.7	9.3	15.1	16.9	0.680	0.964	0.707	0.831						
1.5	4	300–399	72.5	8.0	2.1	17.4	0.805	0.904	0.760	0.826						
1.0	13	300–399	70.9	9.6	11.9	7.6(?)	0.805	1.000	0.933	0.924	68.0	9.1	7.8	15.1	0.771	0.849
3.0	12	200–299	55.8	14.8	11.5	17.9	0.706	0.936	0.705	0.821						
2.5	3	200–299	24.8	48.8	15.1	11.3	0.736	0.899	0.875	0.887						
2.0	6	200–299	62.6	12.8	8.4	16.2	0.754	0.905	0.761	0.838						
1.5	4	200–299	72.8	3.2	6.3	17.7	0.760	0.922	0.660	0.823						
1.0	7	200–299	56.7	23.5	3.2	16.6	0.820	0.980	0.650	0.834	55.2	18.0	10.0	16.8	0.732	0.832
Performance of stoker-fired class											63.2	12.6	7.3	16.9	0.758	0.831
							Pulverized-Fuel-Fired Units									
3.0	1	500–700	64.3	2.1	14.3	19.3	0.664	0.807	0.807	0.807	64.3	2.1	14.3	19.3	0.664	0.807
3.0	8	400–499	66.4	12.2	10.3	11.1	0.786	0.949	0.788	0.889						
2.0	6	400–499	54.7	7.3	23.8	14.2	0.620	0.972	0.710	0.858						
1.0	15	400–499	71.0	3.9	19.3	5.8(?)	0.749	1.000	0.732	0.942	65.8	8.0	16.5	9.7	0.738	0.903
3.0	12	300–399	52.1	12.7	18.2	17.0	0.648	0.954	0.664	0.830						
2.0	8	300–399	68.0	8.6	3.4	20.0	0.766	0.843	0.738	0.800						
1.0	19	300–399	64.5	8.5	16.3	10.7	0.730	0.954	0.711	0.893	59.0	10.6	14.4	16.0	0.696	0.840
3.0	2	200–299	88.9	2.9	0.1	8.1	0.918	0.925	0.914	0.919						
2.5	2	200–299	41.6	28.4	11.4	18.6	0.700	0.825	0.807	0.814						
2.0	3	200–299	86.0	5.5	....	8.5	0.915	0.939	0.884	0.915						
1.0	6	200–299	71.0	3.5	11.3	14.2	0.745	0.905	0.602	0.858	73.2	9.3	5.4	12.0	0.825	0.879
Performance of pulverized-fuel-fired class											63.8	9.3	13.8	13.1	0.731	0.869
							Oil- and Gas-Fired Units									
3.0	6	....	54.5	26.7	11.6	7.2	0.812	0.969	0.897	0.928						
2.0	4	....	52.8	14.7	18.9	13.6	0.675	0.908	0.779	0.864						
1.5	3	....	47.0	2.3	12.5	38.2	0.493	0.706	0.489	0.618						
1.0	11	....	71.7	5.6	8.9	13.8	0.773	0.977	0.758	0.862	57.9	16.1	12.4	13.6	0.740	0.864
							Combined Fuels									
3.0	3	....	39.0	36.6	19.2	5.2	0.756	0.972	0.936	0.948						
2.5	3	....	86.2	3.7	0.2	9.9	0.899	0.929	0.868	0.901	60.5	21.7	10.5	7.3	0.822	0.927

[a] In the paper presented in December, 1929, the figures here designated as Group Performances were referred to as Group Averages. In both cases they were obtained in the same way as is indicated in the texts of both papers.

TABLE 2 EFFECT OF LENGTH OF STUDY ON RESULTS OBTAINED—PERFORMANCE FIGURES BY CLASSES

1929 findings, all boilers						1931 findings, all boilers						1931 findings, after elimination of questionable units[a]					
Period covered in years	Number of units in group	Use factor	Availability factors, Individual High	Low	Group	Period covered in years	Number of units in group	Use factor	Availability factors, Individual High	Low	Group	Period covered in years	Number of units in group	Use factor	Availability factors, Individual High	Low	Group
								Stoker-Fired Units									
1	101	0.769	1.000	0.430	0.839	3	122	0.758	1.000	0.622	0.831	3	111	0.752	0.980	0.622	0.825
								Pulverized-Fuel-Fired Units									
1	62	0.723	0.952	0.494	0.877	3	82	0.731	1.000	0.602	0.869	3	73	0.730	0.972	0.602	0.855
								Oil- and Gas-Fired Units									
1	17	0.753	0.949	0.581	0.897	3	24	0.740	0.977	0.489	0.864	3	24	0.740	0.977	0.489	0.864

[a] Certain numbers given in Table 1 are followed by a question mark. Inspection will show that they represent extraordinarily low values for crippled hours. It is believed that the low values either result from an unintended interpretation of definitions or that the age of the units is not sufficient to have brought about a representative average performance.

The performance figures obtained in the present study are entered, with those reported in 1929, in Table 2. It is interesting to note the similarity between the results of the two studies. The stoker-fired, pulverized-fuel-fired, and oil- and gas-fired units maintain approximately the same relative positions in both cases, but the availability has decreased slightly in all cases in the study extending over the longer period. This seems to indicate that some outages are not annual occurrences and are not necessarily encountered in so short a period as one year. It might be interpreted as indicating a decreasing availability with age, but while this would be a natural expectation, it is not believed to have had much influence in this case because of the fact that boilers of radically different ages were considered in both the short-time and long-time studies.

The 400-to-499-lb. stoker-fired class continues to show the lowest availability. Certain facts known to the authors lead them to believe that while the figures reported represent the facts, at least for the first three entries in the group, they should not be regarded as truly representative on a national basis. There is really no reason why this group should be poorer than the groups above and below, it if equally well designed, constructed, and operated.

The fact that the availability of this group is so much lower than that of any of the other stoker-fired groups naturally leads one to suspect that the lower availability for the stoker-fired class in comparison with that of other classes may result from the inclusion of the low-availability group in obtaining the figure for the class. The inclusion of this group does have an effect upon the class figure, but not so great an effect as one would expect. It happens that the total number of boiler years represented by the low group is comparatively small. The omission of the group in obtaining the class figures would raise the availability factor from 0.831 to 0.837, an almost negligible variation.

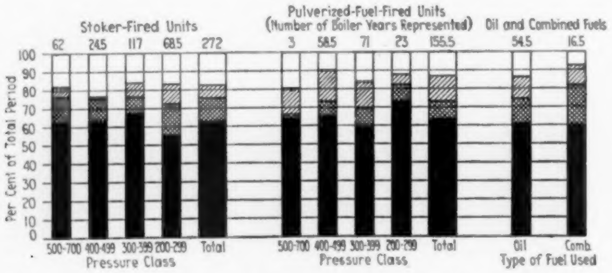


FIG. 2 PERFORMANCE OF STEAM-GENERATING UNITS BY GROUPS
(Legend: White space, crippled; single-hatching, reserve; double-hatching, banked; black, steaming.)

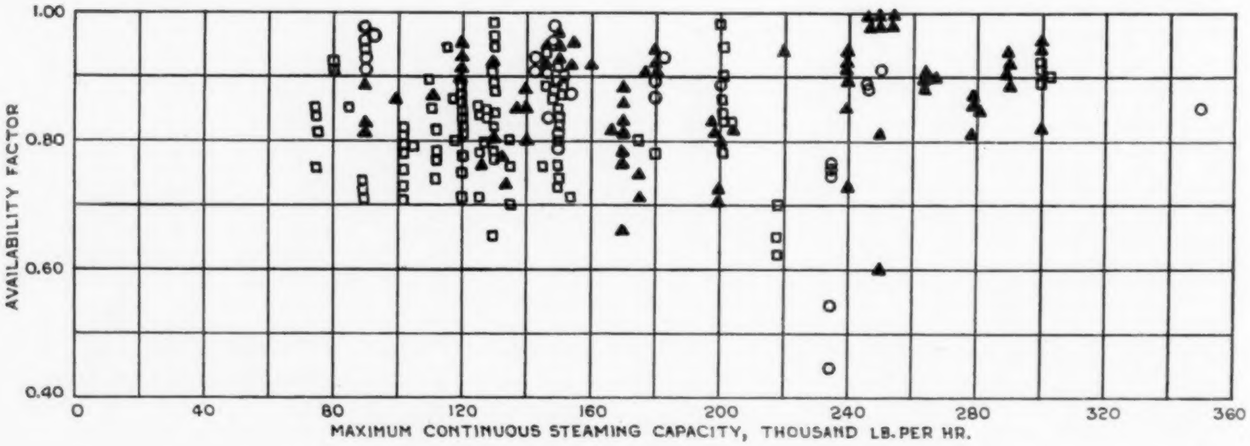


FIG. 3 EFFECT OF UNIT SIZE ON AVAILABILITY
(Legend: Squares, stokers; triangles, pulverized fuel; circles, oil.)

Comparison of either figure with the value 0.869 obtained for the pulverized-fuel-fired class might be assumed to indicate a slightly higher degree of reliability for pulverized-fuel-fired equipment. To draw such a conclusion one must be willing to concede that the number of boilers studied, the length of the period covered, and the methods of collecting and compiling data are all such as to justify the drawing of such fine distinctions. The authors are not prepared to say that this is the case. They are, however, making more detailed studies of the causes of outages in the two classes for the purpose of determining whether any real difference directly chargeable to the firing method can be found.

An attempt to determine the effect of size upon the availability factor of boilers resulted in the production of Fig. 3. Availability factors were plotted against the maximum continuous steaming capacities as given by the operators. Steaming capacities were used in an effort to eliminate the effects of different distribution of heating surface between boiler, economizer, and preheater, as well as the effects of radical differences in boiler design.

It is reasonable to suppose that larger units can more often be given greater reliability within economic limits than can smaller units. The figure appears to show some justification for such an assumption. The smaller units appear to average availability factors in the neighborhood of 0.83 to 0.85, whereas the larger units appear to give values averaging more nearly 0.90. However, it should be noted that some of the smaller units give performances substantially equal to the best of the larger units. It is only the greater spread in the case of the smaller units that results in the figures just given. One is driven to the conclusion that skill in design, construction, and operation, or the economic possibilities of individual cases, are in fact more controlling than the size of the unit.

An interesting study was made possible by the fact that the man-hours used on crippled boilers was reported for 43 units. The number of boilers is rather small, but it is thought worth while to present the results obtained. This is done in Fig. 4 and in Table 3. In the table the units are classified only as to method of firing, and they are arranged in the order of availability factors. For convenience, work done during crippled periods has been expressed in terms of maintenance man-hours per million pounds of water evaporated during the total period studied. The column headed "Regular overhaul" refers to all repair work done during those periods which are set aside for regular inspection and overhaul. The following columns include all work done during crippled periods outside of those regular periods of overhaul.

If we ignore a few points in the figure which are probably abnormal, the trend is probably in favor of the large units as shown by the dotted line. The large capacity unit thus appears to possess advantages with respect to repair and maintenance costs as well as with respect to space requirements and operating personnel. It is to be noted, however, that the greatest part of the possible gain appears to be obtained in going to capacities of the order of about 120,000 to 140,000 lb. of steam per hour; the gain obtained by doubling such sizes is negligibly small.

In the 1929 paper data were given with respect to each unit reported on so that the reader could note variation between units and appraise their significance for himself. It is planned to do the same with the data now in hand for inclusion in a later and greatly enlarged paper. At that time the authors expect to be able to present the results of a number of different types, such as the effects of rate of driving, the effects of different features of design, etc. They regret that it has been impossible to do all

TABLE 3 MAINTENANCE MAN HOURS DURING CRIPPLED PERIOD PER MILLION POUNDS OF WATER EVAPORATED

Availability factor	Total maintenance	Regular overhaul	Change in equipment	Boiler	Furnace	Fuel-burning equipment	Economizer preheater	Ash-handling equipment	Draft equipment	Instruments
					Stoker-Fired Units					
0.943	1.8	0.70	.....	0.39	0.37	0.33	....	....	....	....
0.900	1.8	0.47	.....	0.21	0.67	0.33	0.09	....	0.06	....
0.882	2.9	1.08	0.12	0.71	0.57	0.34	0.05	....	....	....
0.873	3.1	1.05	.....	0.28	1.62	0.04	0.12	....	....	....
0.869	3.5	1.51	.....	0.29	1.22	....	0.42	....	0.05	....
0.865	2.2	1.39	.....	0.29	0.27	0.14	0.03	....	0.03	....
0.862	3.6	1.20	0.28	0.98	0.61	0.37	0.14	....	....	....
0.861	1.3	0.52	0.14	0.11	0.29	0.22	0.05	....	....	....
0.856	3.3	0.74	.....	0.88	1.51	0.04	0.16	....	....	....
0.853	1.3	0.56	0.18	0.13	0.28	0.13	0.05	....	0.01	....
0.851	3.5	1.19	0.07	0.58	1.40	0.16	0.08	....	....	....
0.842	3.1	0.86	.....	1.29	0.71	0.26	0.02	....	....	....
0.837	4.6	1.73	0.08	0.76	1.56	0.04	0.10	....	0.31	....
0.833	2.6	1.22	.....	0.35	0.89	0.13	....	....	....	....
0.827	4.3	1.48	.....	0.43	1.66	0.25	0.40	....	0.07	....
0.822	3.0	1.28	.....	0.70	0.75	0.21	....	....	0.03	....
0.819	4.5	0.88	0.13	0.47	1.60	1.04	0.23	0.03	0.09	0.07
0.818	1.8	0.54	0.27	0.17	0.43	0.29	0.08	....	0.04	....
0.818	3.1	1.95	.....	0.53	1.35	....	0.25	....	....	....
0.803	1.7	0.50	0.47	0.20	0.33	0.14	0.02	....	....	....
0.791	7.5	2.01	3.15	0.19	0.47	1.47	0.15	....	0.03	....
0.786	6.7	1.45	.....	0.50	2.06	1.61	0.71	....	0.36	....
0.785	7.7	2.33	0.14	0.50	0.77	2.59	0.23	0.89	0.27	....
0.782	4.2	0.61	.....	0.93	2.36	0.25	0.04	....	....	....
0.777	4.6	1.13	0.60	0.80	1.58	0.30	0.15	....	....	....
0.758	21.1	4.37	10.30	2.01	1.15	2.90	0.28	....	0.06	....
0.741	6.0	1.07	.....	0.35	1.88	1.66	0.48	0.03	0.59	....
0.730	10.7	1.90	4.57	0.67	0.83	1.72	0.22	0.78	0.05	....
0.706	6 5	1.55	.....	0.48	2.44	1.44	0.30	0.18	0.06	0.05
					Pulverized-Fuel-Fired Units					
0.889	7.6	4.45	0.01	0.29	0.40	0.38	2.02	....	....	....
0.871	2.4	0.51	0.02	0.24	1.41	0.03	0.12	0.05	0.05	....
0.870	2.4	0.39	.....	0.18	1.57	0.04	0.09	0.08	0.06	....
0.865	6.0	1.55	0.01	0.28	1.75	0.30	2.01	0.07	....	....
0.857	1.3	0.56	.....	0.13	0.53	....	0.04	0.06	....	....
0.828	6.5	0.96	1.02	0.29	1.74	0.55	1.92	....	....	....
0.811	6.9	2.24	0.17	0.36	0.73	0.09	3.13	0.15	0.01	....
0.810	2.5	0.49	0.06	0.43	1.16	0.08	0.15	0.03	0.06	....
					Oil-Fired Units					
0.776	2.7	0.02	.....	1.80	0.86	....	....	....	0.02	....
0.764	5.3	1.83	.....	2.53	0.90	....	....	....	0.04	....
0.759	4.1	0.24	.....	3.30	....	....	0.24	....	0.29	....
0.746	4.2	0.15	0.01	2.66	1.30	....	....	....	0.03	....
0.542	22.7	4.50	.....	16.20	1.93	....	....	....	0.04	....
0.444	16.4	1.32	0.15	10.00	4.86	....	....	....	0.07	....

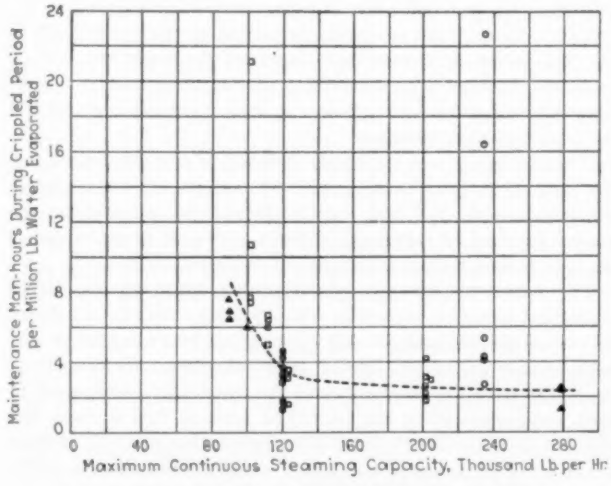


Fig. 4 Effect of Unit Size on Maintenance Labor

(Legend: Squares, stokers; triangles, pulverized fuel; circles, oil.)

this work in time for the presentation of a more complete and comprehensive paper.

As a matter of interest, Table 4 has been presented to show the availability and use factors of steam turbines as reported by the Turbines Subcommittee, Prime Movers Committee of the N.E.L.A. These data were presented in the subcommittee's report of August, 1931.

In concluding, the authors desire to express their thanks to the operators who furnished the performance records upon which this paper is based. Without their kind cooperation the work would have been much more arduous and of far less value to all concerned.

TABLE 4 TURBO-GENERATOR AVAILABILITY

(N.E.L.A. Prime Movers Committee Turbines Report, August, 1931)

Year of operation	Number of turbines	Use factor	Availability factor
1	16	0.7873	0.9494
2	26	0.8274	0.9105
3	21	0.8164	0.9157
4	30	0.7287	0.9118
5	17	0.8096	0.9131
6	56	0.6315	0.9198
7	33	0.5474	0.8912
8	15	0.5488	0.9213
9	13	0.4493	0.8822
10	20	0.5950	0.8886

Discussion

A. G. Christie.[3] This paper presents facts regarding the availability of boilers that have heretofore been lacking. In studying the data presented, one finds that both use factors and availability factors presented in the 1931 figures are slightly less than the 1929 figures. The use factor may have been influenced by the lesser loads and greater reserve capacity in the stations during the last two years. The availability factor is modified to some extent by the age of the units. There has been little new construction during the last two years, and it is probable that the average age of the units covered in the 1931 figures exceeds that of the 1929 figures. Age also may have had something to do with the lower figures for the 400-to-499-lb. class of boilers.

Fig. 2 indicates that the stoker-fired units tend to be crippled to a greater extent than pulverized-coal-fired units. This is also shown by the figures in Table 1. It would be interesting to know the reason for this and whether stokers of recent designs have shown any improvement in this respect over older types.

The data presented in Fig. 3 show a trend toward higher availability factors for units with the larger steaming capacities. One deduces from this that boilers operated at these higher outputs are more reliable than smaller units. This is a highly significant result, for it indicates that there is no detriment on the score of reliability to the construction of properly designed boiler units of even greater continuous steaming capacities than at present. Since boiler and turbine availability factors are not greatly different, one can therefore plan on future power stations with one boiler per turbine, even with turbines of larger capacity than the average size now used.

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[3] Professor of Mechanical Engineering, Johns Hopkins University, Baltimore, Md. Mem. A.S.M.E.

Encouragement toward the use of larger boilers may also be derived from Fig. 4, which indicates the trend toward lower maintenance costs with steaming capacity of boilers. Hence the use of large boiler capacities not only reduces the unit cost of the station, but at the same time lowers the operating cost.

A comparison of Tables 1 and 4 indicates that the availability factors of turbines still exceed those of boilers. If the unit boiler-turbine system is to become standard practice, further study and care must be given to the boiler plant to bring its availability factor on a par with that of the turbine plant.

D. W. Flowers.[4] The writer is struck with the care used in preparing the data and with the need for such data by operators and designers in discovering whether the investment dollar is being used in the boiler plant to the best advantage.

There is a considerable difference in availability factor between the best and the worst boiler, and it is hoped that the further analysis planned by the authors will give a review of the causes for the various units being "crippled," in such a way that designers in the future may avoid the troublesome construction.

The authors note that the availability of these boilers has slightly decreased over the figures for 1929. The writer has noticed the same thing in our plant (Comal Station, New Braunfels, Texas) in 1931 as compared with 1930. The cause of this decrease is the reduced demand for electric generating capacity. In 1930 when we could use all of the higher efficiency boilers to good advantage all the time, routine inspections and necessary repairs were carried on with as large a crew as could be worked efficiently, and were carried on at night with overtime labor at increased rates. In 1931, when at least one boiler is idle all the time, we are saving total expense by doing maintenance work with reduced crews and all on a straight time basis, but this practice is slightly reducing the availability factor of the boilers. Perhaps the business depression has set similar conditions to work in the stations under investigation and inspection in this paper.

Ollison Craig.[5] The authors are again giving figures of availability of units classified as to methods of firing. Table 1 shows the average availability of pulverized-fuel-fired units to be greater than that of stoker-fired units.

The figures given in Table 3 offer a clue to the difference in availability. The total maintenance in terms of man-hours per million pounds of water is somewhat greater for stoker-fired units than for pulverized-fuel-fired units, as an average. Of the actual maintenance expended in each case, change of equipment accounts for 16 per cent in the case of stoker firing, but only 4 per cent in case of pulverized fuel firing. Maintenance of fuel-burning equipment accounts for 14 per cent of the total in the case of stoker firing, but only for 4.7 per cent in the case of pulverized-fuel firing. Furnace maintenance percentage is about the same for each.

It would be interesting to know why fuel-burning equipment maintenance for stoker firing is about three times that for pulverized-fuel firing. The log shown in Fig. 1 provides means for determining temperature of preheated air in cases when air preheaters are used. Data are apparently available for determining the relation between fuel-burning equipment maintenance and temperature of air preheat. This would probably offer the clue to the higher maintenance for stokers and indicate an economic limit of air-preheat temperature applied to stokers.

In the case of stoker firing it would be interesting to know how availability is affected by the average rating carried on the unit.

Vern E. Alden.[6] The immediate objective of the work which the Power Division requested the authors of this paper to undertake four years ago is the centering of the attention of operating and designing engineers on the importance of keeping the outage or "crippled" hours for large steam generating units down to a minimum. The reduction of outage hours is, however, only a means to an end, and that end is the limiting of the power-station investment for carrying the system load to the lowest possible figure from year to year, as the load grows. This broader conception of the problem introduces additional factors. Outage or "crippled" hours fall into two classes:

(1) Those which can be foreseen and, to a considerable extent, scheduled so that they will not cause embarrassment in the handling of the system peak load or appreciably affect the overall fuel consumption for the system.

(2) Those outages which come unexpectedly and will conceivably take the boiler unit out of service at the time when its capacity is needed most. It is believed that considerable progress has been made by the operating and designing engineers in reducing the percentage of total outage hours which cannot be scheduled in advance. It would be most interesting if the authors, in their later paper, would present such figures as are available, indicating the progress which is being made in this direction.

A reduction in the maximum available steaming capacity is only less serious in degree than outage of the boiler unit. It is probable that many steam-generating stations are more handicapped by reductions in steaming capacity of the boiler units than they are by actual outages.

It would have been extremely difficult, if not impossible, for the authors to have made a critical analysis of this factor in connection with 244 boiler units distributed in, perhaps, 80 generating stations. It is not difficult, however, to make such a study in connection with a single generating station.

The trend toward the use of boiler units having larger individual steaming capacities which has its origin in the desire to reduce the investment and the cost for operating labor and maintenance per unit of station send-out, makes it necessary to study the record of availability of the boiler-plant equipment in each steam-generating station. The kind of fuel burned, the water conditions, and the type of equipment installed will present a new set of problems in connection with each installation. The objective in all cases will, however, be substantially the same—the insurance that the maximum amount of steaming capacity expressed in thousands of pounds of steam per hour will be available at the time of maximum system needs. The accomplishment of this end result is in part the work of the designing engineer and in part that of the operating engineer. The designing engineer can, if he has as a background a complete understanding of operating problems, accomplish much by the careful working out of the details of design. The operating engineer can make his contribution through the careful planning and scheduling of regular inspections and preventive maintenance work and through the working out of the most improved methods of operation.

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Some of the major causes for the outages of boiler units are the following:

(1) The formation of slag on the boiler tubes at the inlet to the first pass.

(2) Inability to maintain a uniform fuel bed and the formation of clinkers in the fuel bed.

(3) The failure of refractories.

(4) The formation of scale or the occurrence of corrosion inside the tubes of the boilers, water walls, and economizers, all resulting from inadequate feedwater treatment.

(5) Improper feeding of the fuel and improper mixing of the fuel with the air, coming as the result of shortcomings or the deterioration of the coal feeders or of the burners, either in the case of pulverized coal or oil firing.

(6) Failure of certain of the component parts of the stoker to function properly.

(7) Difficulties incident to the removal of ash and clinkers from a stoker installation or ash and slag from a pulverized-fuel-burning installation.

(8) Troubles with superheaters which may be either leaks in the rolled joints or at the handhole caps, failures of the individual elements incident to improper steam distribution, or failure of the supports due to overheating.

(9) Direct or indirect troubles with soot blowers.

(10) Troubles with economizers.

(11) Troubles incident to improper water circulation in the water-cooled furnace walls.

(12) Troubles incident to air-preheater corrosion.

(13) Troubles incident to the erosion of induced-draft-fan impellers.

It would be of considerable value to operating and designing engineers if the authors would include as a part of the paper they have promised to present in the near future, a more complete list of the various causes of boiler outages which their studies have disclosed. In general, it has proved to be necessary only that we clearly define a problem, in order that we very definitely insure its solution by the steam-station operating and designing engineers of this country. The real progress which has been made over a period of the last ten years in reducing the outages of turbine-generators is the proof of this statement. The authors certainly deserve the thanks of the Power Division and of the Society for the work which they have done in defining this problem and in stimulating further study of the individual situations by the responsible designing and operating engineers.

R. C. Vroom.[7] The unusually low availability factors for the last two oil-fired units of Table 3 raise a question as to exactly what caused the large amount of outage. As both the boiler and furnace maintenance are high, it would appear that these units are operating regularly at high rates of heat liberation in the furnace and at high rates of evaporation per square foot of heating surface. Pertinent data in regard to the operating conditions would be of interest.

A. J. German.[8] Nearly all of those that have commented on this paper have suggested that the authors should include the various causes of boiler outages, thereby possibly locating the weakest link. The writer is of the opinion that this is too much to expect of the authors. Had the writer been asked by the authors to answer their questionnaire for a few years back, he would have had to answer that the first year our stokers were by far the principal cause of boiler outages, and undoubtedly would have given reasons for it. If he had answered the questionnaire the following year, he would have answered something as follows: "We are sorry that our answer to the question last year must now be changed; namely, we have installed new stokers and they have not been the cause of any interruption in boiler performance, but due to the fact that with these stokers we now could operate the boilers at a higher rating, and do the same work with one less boiler, we are now experiencing refractory trouble; hence that has now become the weakest link and the principal cause of boiler outages."

Again, if he had answered the third year's questionnaire, he would be compelled to again change the answer; namely, he would have to inform them that our furnaces had now been redesigned, air cooled, etc., and since then we have been able to again increase the boiler rating, and tests have proved that it could be done with a better over-all efficiency, but it required us to operate the forced-draft fans at a capacity which had never been necessary up until then. This is now the weakest link.

The writer is of the opinion that nearly all plants that have been built for some years and designed for a considerably lower capacity than was later attempted to get out of them, have had somewhat similar experiences, for the reason that all of us want to keep up with the latest improvements.

If the writer is correct in the foregoing statement, then it seems that this would be proof that we could not expect to have anyone collect such data and have it of any use.

Authors' Closure

The authors are highly gratified at the interest aroused by this preliminary report. They are particularly indebted to the discussors for their many suggestions relative to the further and more detailed presentation of the data at hand. The final paper is being prepared with these suggestions in mind, so it is thought unnecessary to go into more detail at the present time.

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[8] Chief Engineer, Scovill Manufacturing Company, Waterbury, Conn. Mem. A.S.M.E.

FSP-54-16

Influence of Inlet Boxes on the Performance of Induced-Draft Fans

By LIONEL S. MARKS,[1] CAMBRIDGE, MASS., and E. A. WINZENBURGER,[2] WALTHAM, MASS.

L. S. Marks

This paper presents experimental results of tests to ascertain the effect of inlet boxes on the performance of induced-draft fans. The tests showed the magnitude of the changes in performance that can be expected and emphasizes the need for testing fans with their inlet boxes attached. The conclusion drawn from the tests is that it is not sufficient for the fan user to know the characteristics of a selected fan with free inlet, but he also should know the characteristics when the inlet box is attached.

E. A. Winzenburger

IT IS well known that the performance of a centrifugal fan may be influenced considerably by the direction and velocity of approach of the air to the inlet. In induced-draft fans the direction and velocity of approach of the gases are determined by the form and dimensions of (1) the inlet box that is furnished with the fan and (2) the inlet ducts leading the gases to the inlet box. The performance of an induced-draft fan with open inlet will in general be different from its performance with inlet box attached. The performance on the test block of an induced-draft fan with inlet boxes attached may be quite different from the performance of the same fan when installed, if the inlet ducts are not well designed.

The object of the tests described herein is to ascertain the effect of the inlet boxes on performance. Certain types of inlet box in fairly common use were selected for investigation. The actual dimensions and forms of these boxes are not standardized, so that the results obtained must be recognized as applying only to the particular conditions of the tests. The tests were intended to bring out the magnitude of the changes in performance which might reasonably be expected and to emphasize the necessity of testing fans with their inlet boxes attached. If the particular ducts to be furnished in the actual installation give an inlet flow different from the ideal, straight, symmetrical flow of standard test practice, the fan performance will be affected, and in such cases testing should be carried out, not only with inlet boxes attached, but also with inlet ducts in position.

The tests were carried out at the Gordon McKay Laboratory of the Harvard Engineering School with the assistance of W. A. Abrahamson and H. D. Watson. The fan used was a double-inlet, Turbovane, induced-draft fan supplied by the B. F. Sturtevant Company. The wheel, Fig. 1, is double-inlet, 38 in. diameter, $23^5/_8$ in. wide, and has radial tips. It is provided with curved aluminum inlet rings of the form shown in Fig. 2.

The boxes used are shown in Figs. 3, 4, and 5. Fig. 3 is a simple rectangular box. Fig. 4 is a rectangular box modified by sloping part of the top and bottom sheets as indicated. Fig. 5 is a scroll-inlet box provided with vanes for guiding the air into the fan wheel. The rectangular and modified rectangular boxes were tested also with a vertical guide sheet about 6 ft. long in the center of each duct and extending up to the locations shown in Figs. 3 and 4. It was thought that some guidance of the air stream might improve the fan performance. The scroll-inlet boxes were tested also with the guide vanes removed.

This, it was recognized, would impair the performance of the fan, but would make evident the importance of proper design of the boxes.

The Standard Test Code for Centrifugal Fans of the National Association of Fan Manufacturers and the American Society of Heating and Ventilating Engineers was followed closely except in one minor particular. The inlet ducts were about 4 diameters in length instead of the 6 diameters specified, but they were provided with rounded inlets as indicated in Fig. 6. The inlet-air measurements were made at the specified location of $1^1/_2$ diameters from the inlet boxes. This modification gives a more uniform flow in the inlet ducts by avoiding the *vena contracta* that is inevitably formed at the entrance to the standard type of duct. The influence of this modification on the fan performance is not measurable.

The power input was determined by an electric dynamometer on the fan shaft. In order to eliminate insensitiveness due to frictional resistance in the weighing system of the dynamometer, the weighing levers were removed and the dynamometer was balanced by weights hung through a single knife edge from the short arm projecting from the dynamometer shell. The only frictional resistance remaining is that of the ball bearings

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[1] Professor of Mechanical Engineering, Harvard University. Mem. A.S.M.E. Professor Marks was born in Birmingham, England, on September 6, 1871. He received his degree of B.Sc. from the University of London in 1892 and M.M.E. from Cornell University in 1894. He was with the Ames Iron Works, Oswego, N. Y., in 1894 and then went to Harvard University as instructor in mechanical engineering, in 1900 being made assistant professor, and in 1909 being advanced to his present position. He is the author of "Steam Tables and Diagrams," "Gas and Oil Engines," "Mechanical Engineers' Handbook," "The Airplane Engine," and of numerous articles in the engineering press.

[2] Research Engineer, Waltham Watch Company, Waltham, Mass. Jun. A.S.M.E. Mr. Winzenburger was born on October 19, 1903, in Stettin, Germany. He received a M.E. degree from the Polytechnic Institute of Brooklyn in 1929. During the next two years he was an assistant in mechanical engineering at the Harvard Engineering School, at the same time studying for his master's degree, which he received in 1931.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

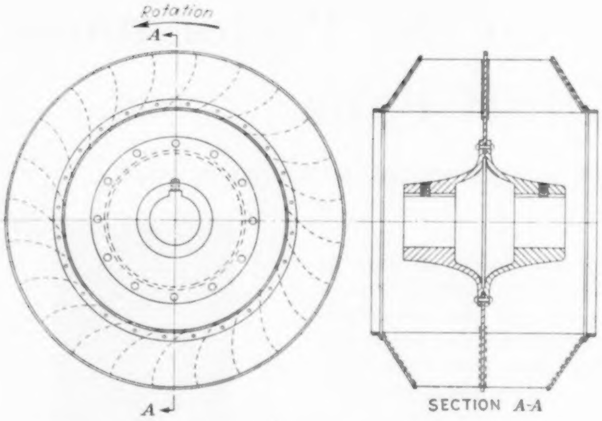
Rotation
A
A
SECTION A-A

Fig. 1 Double-Inlet 28-In. Fan With Radial Tips

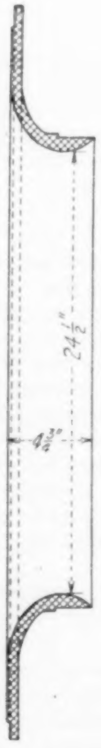

Fig. 2 Curved Aluminum Inlet Ring

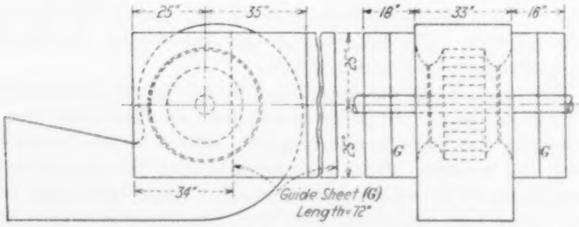
25"
35"
18"
33"
16"
25"
25"
34"
G
G
Guide Sheet (G)
Length=72"

Fig. 3 Rectangular Inlet Boxes

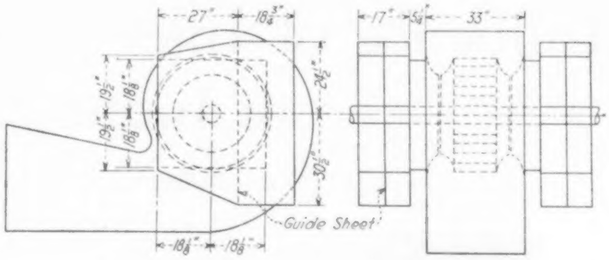
27"
18 3/4"
17"
5 1/4"
33"
19 1/2"
19 1/2"
18 1/8"
18 1/8"
24 1/2"
30 1/2"
Guide Sheet
18 1/8"
18 1/8"

Fig. 4 Modified Rectangular Inlet Boxes

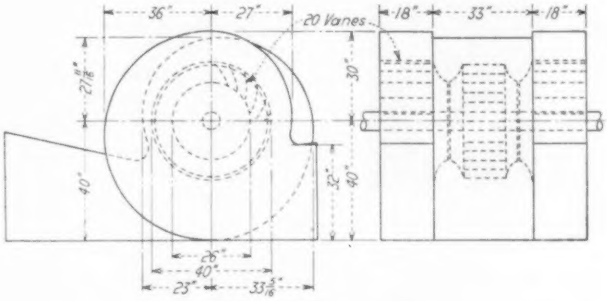
36"
27"
20 Vanes
18"
33"
18"
27 11/16"
30"
40"
32"
40"
26"
40"
23"
33 5/16"

Fig. 5 Scroll Inlet Boxes

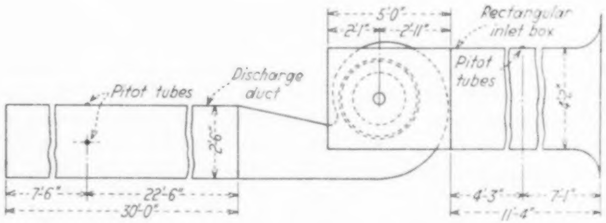
5'0"
2'1"
2'11"
Rectangular inlet box
Pitot tubes
Discharge duct
Pitot tubes
2'6"
4'2"
7'6"
22'-6"
30'-0"
4'3"
7'-1"
11'-4"

Fig. 6 Side Elevation of Fan, Showing Inlet and Discharge Ducts

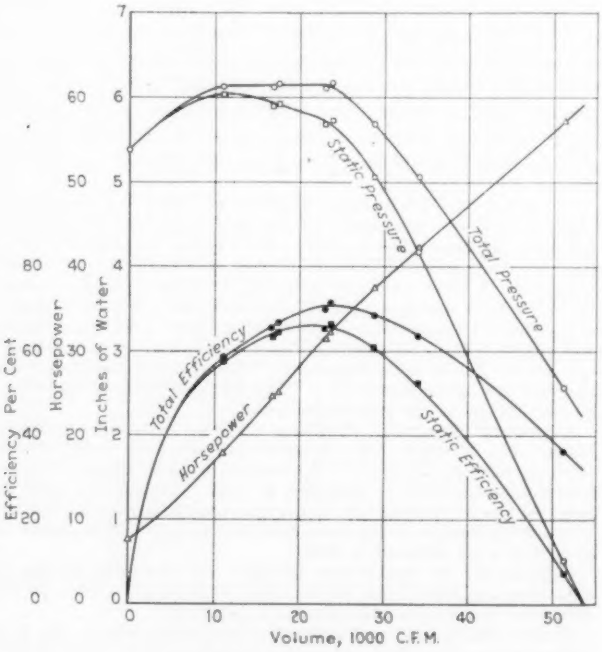

Fig. 7 Free Inlet

supporting the shell. Both the dynamometer arm and the dead weights were easily measurable within an accuracy of 0.1 per cent. The power input was determined also by measurement of the electrical input to the dynamometer corrected for electrical losses. With carefully calibrated instruments and with daily measurements of stray power (following the method detailed by C. L. Dawes, "Electrical Engineering," Second Edition, Vol. I, pp. 423–426) and armature resistance, the power developed by the dynamometer is determinable with an error not exceeding a small fraction of 1 per cent.

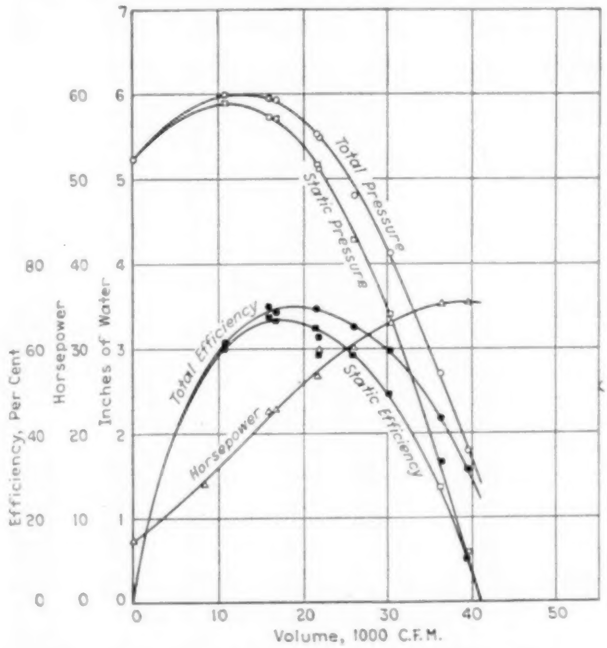

FIG. 8 RECTANGULAR INLET BOXES WITHOUT GUIDE SHEETS

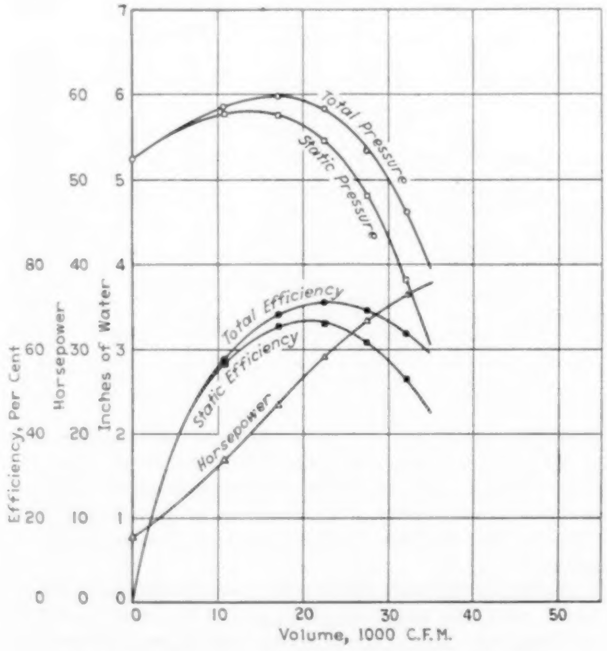

FIG. 9 RECTANGULAR INLET BOXES WITH GUIDE SHEETS

It was found that the agreement between the electrical and mechanical measurements of power was not as close as the apparent individual accuracies of the two methods would suggest. The electrical measurement was from 1 to 2 per cent greater than the mechanical measurement. The frictional resistance of the ball bearings may be partly responsible for this difference. The electrical measurements have been used in the tabulations and curves of this report, both because they are

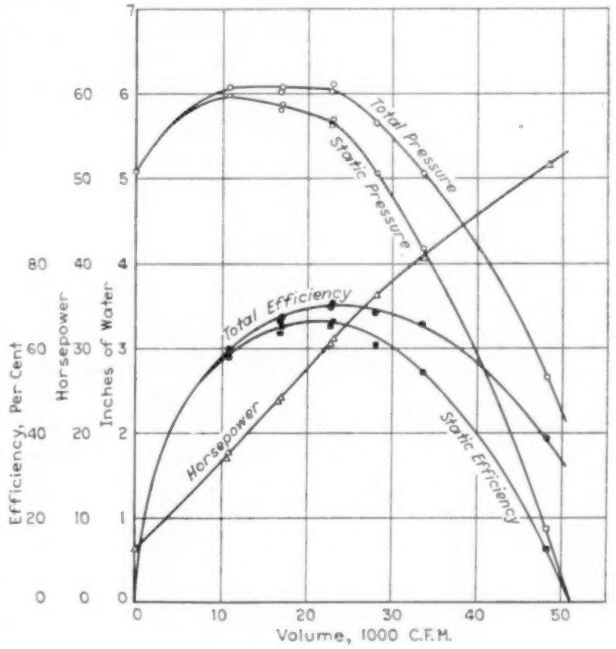

FIG. 10 MODIFIED RECTANGULAR INLET BOXES WITHOUT GUIDE SHEETS

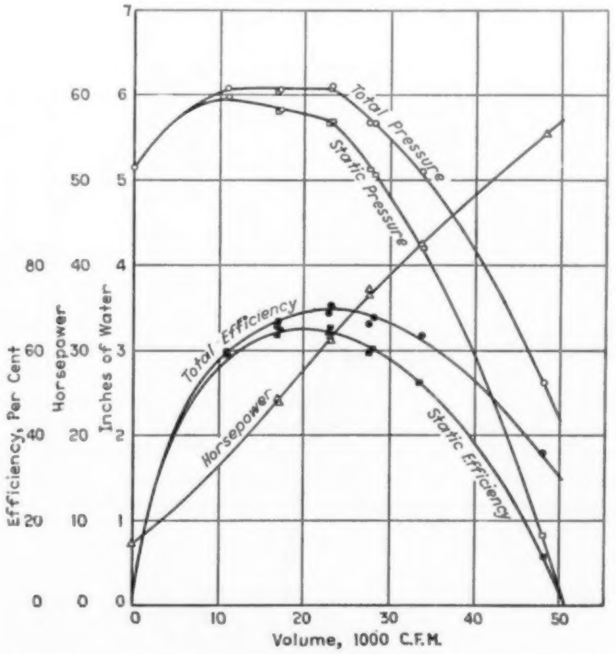

FIG. 11 MODIFIED RECTANGULAR INLET BOXES WITH GUIDE SHEETS

the higher values and also because it is thought that they have a greater accuracy. It was found that horsepower and efficiency plotted on the basis of the electrical measurements lie more closely on a smooth curve. For accurate testing, the direct readings alone of an electrical dynamometer cannot be considered satisfactory and should always be checked by electrical readings.

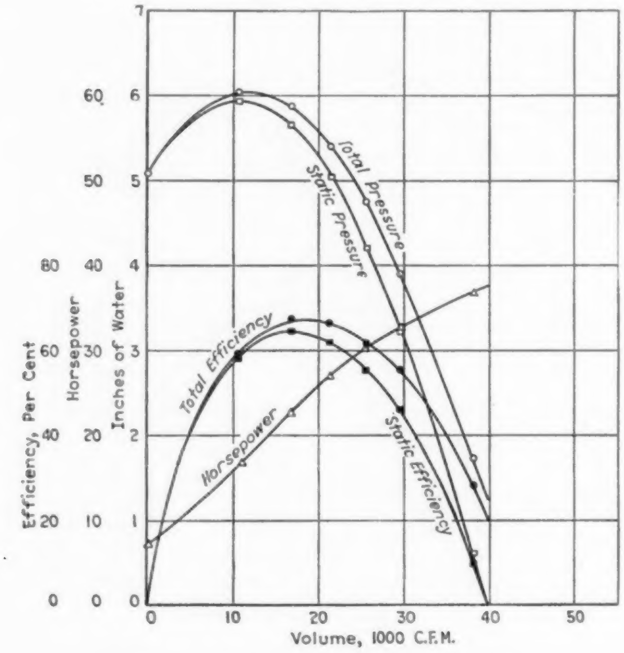

FIG. 12 SCROLL INLET BOXES WITH VANES

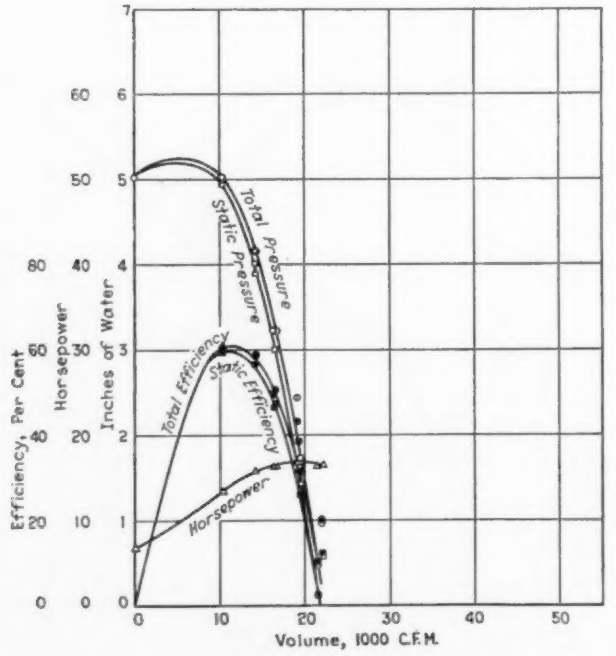

FIG. 13 SCROLL INLET BOXES WITHOUT VANES

TABLE 1 RESULTS OF FAN TESTS
(Speed, 900 r.p.m. Air density, 0.0750)

		Pressure		Efficiency		
Date	Volume c.f.m.	Static, in. water	Total, in. water	Static, per cent	Total, per cent	Horsepower
Free Inlet						
3/20/31	0	5.384	5.384	0	0	7.66
	10980	6.045	6.135	58.00	58.90	18.02
	17450	5.930	6.164	64.45	67.00	25.23
	23750	5.745	6.180	66.55	71.55	32.30
	28730	5.060	5.695	60.92	68.60	37.58
	33950	4.178	5.065	52.61	63.82	42.42
	51380	0.524	2.570	7.40	36.30	57.25
3/30/31	16850	5.901	6.120	63.10	65.40	24.83
	22960	5.694	6.111	65.20	70.00	31.55
Scroll Inlet Boxes Without Vanes						
4/6/31	14120	3.898	4.152	56.08	58.20	15.84
	16340	3.010	3.216	46.56	49.74	16.62
	19400	1.438	1.730	26.07	31.35	16.86
	21520	0.131	0.491	2.70	10.12	16.42
4/8/31	0	5.015	5.015	0	0	6.75
	10310	4.945	5.027	59.78	60.75	13.44
	14150	4.010	4.164	56.55	58.70	15.81
	16510	3.006	3.216	47.52	50.82	16.44
	19100	2.160	2.442	38.30	43.30	16.98
	21950	0.587	0.961	12.27	20.10	16.54
Scroll Inlet Boxes With Vanes						
4/10/31	0	5.090	5.090	0	0	7.08
	10560	5.942	6.027	58.30	59.12	16.96
	16720	5.659	5.873	64.80	67.28	22.97
	21270	5.045	5.391	62.10	66.35	27.17
	25560	4.204	4.704	55.28	61.82	30.62
	29530	3.235	3.904	46.10	55.63	32.64
	38150	0.618	1.744	9.99	28.18	36.93
Rectangular Inlet Boxes Without Guide Sheets						
4/24/31	0	5.235	5.235	0	0	7.36
	10780	5.915	6.004	60.35	61.26	16.60
	15900	5.735	5.955	67.40	69.95	22.61
	21800	5.120	5.486	58.65	62.85	29.95
4/27/31	25900	4.291	4.806	58.02	65.00	30.10
	30300	3.420	4.128	49.20	59.40	33.10
	36350	1.677	2.702	27.10	43.68	35.40
	39570	0.585	1.797	10.27	31.57	35.50
4/29/31	16730	5.725	5.940	66.05	68.55	22.85
	21600	5.155	5.514	65.05	69.58	26.93
Rectangular Inlet Boxes With Guide Sheets						
5/1/31	0	5.240	5.240	0	0	7.73
	10660	5.765	5.852	56.85	57.70	17.02
	17000	5.757	5.980	65.60	68.10	23.47
	22530	5.450	5.841	66.35	71.10	29.13
	27530	4.763	5.346	61.80	69.35	33.40
	32140	3.824	4.620	53.00	64.00	36.50
Modified Rectangular Inlet Boxes Without Guide Sheets						
5/6/31	17030	5.865	6.088	65.00	67.40	24.20
	22930	5.700	6.103	66.30	70.90	31.08
	33580	4.191	5.061	54.45	65.78	40.66
5/8/31	0	5.082	5.082	0	0	6.36
	10850	5.985	6.075	58.08	58.95	17.60
	27900	5.060	5.657	60.98	68.12	36.40
	48150	0.865	2.664	12.69	39.08	51.62
5/15/31	10750	5.980	6.069	58.90	59.80	17.36
	16820	5.805	6.022	63.95	66.37	24.00
	22700	5.632	6.028	65.30	69.85	30.80
Modified Rectangular Inlet Boxes With Guide Sheets						
5/19/31	10820	5.985	6.075	59.15	60.00	17.25
	16880	5.812	6.031	63.40	65.75	24.37
	22810	5.672	6.072	64.35	68.85	31.64
	27940	5.069	5.670	60.60	67.80	36.73
5/20/31	0	5.150	5.150	0	0	7.38
	17050	5.835	6.059	65.00	67.50	24.06
	22930	5.695	6.100	65.67	70.35	31.30
	27550	5.108	5.688	59.70	66.50	37.10
	33630	4.233	5.105	52.70	63.55	42.50
	48050	0.845	2.638	11.65	36.00	55.40

Pitot tubes were used in the standard method and locations for the determination of static and total pressures. Since it is known that, with some fans, this standard method does not yield accurate results, a Moss nozzle was attached to the end of the discharge duct during one of the tests, and readings were taken of the total pressure at the nozzle in addition to the usual pitot-tube readings. The discharge volumes determined by the two methods agreed closely at three speeds of operation and gave evidence that for this fan the pitot-tube measurements were accurate.

The tests were made at 800 r.p.m., except with the scroll-inlet boxes, when the speed was 900 r.p.m. The conditions during each test were steady, except for the rectangular air boxes without guide sheets. For this condition, with volumes of 22,000 c.f.m. and over, the performance was unstable. The fan would develop periodically a roaring noise which coincided with a power increase of 10 to 15 per cent, but without noteworthy change in the discharge volume. The addition of guide sheets in the inlet ducts improved this condition notably.

The usual procedure in a series of tests was to start with blocked outlet and then to test with 20, 30, 40, 50, 60, 80, and 100 per cent openings. Tests were then repeated at some of the smaller openings as a check.

The principal results of these tests, reduced to 900 r.p.m. and an air density of 0.075 lb. per cu. ft., are presented in Table 1. Characteristic curves for the seven sets of tests are given in Figs. 7 to 13. A comparison of the static pressures for various volumes discharged is shown in Fig. 14. Comparisons of horsepowers and of static efficiencies are shown in Figs. 15 and 16, respectively.

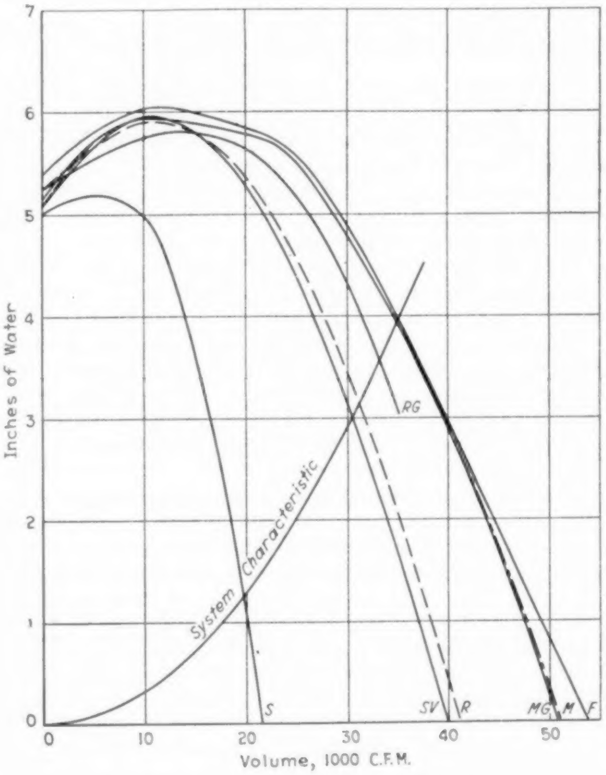

FIG. 14 COMPARISON OF STATIC PRESSURES

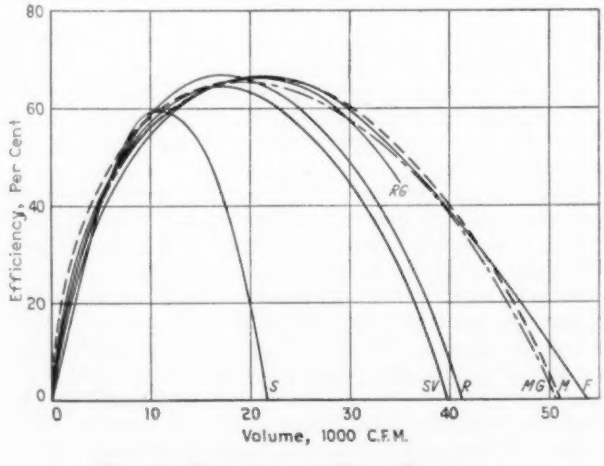

FIG. 16 COMPARISON OF STATIC EFFICIENCY

It is immediately evident that the modified rectangular boxes, both without guide sheets *M* and with guide sheets *MG*, give performances practically identical with that of the free-inlet fan *F*.

A comparison of performance may be on the basis of static pressure at a given volume, volume at a given pressure, volume at a given efficiency, or efficiency at a given volume. Judged in any of these ways, the order of merit for the remaining boxes is (1) rectangular boxes with guide sheets *RG*, (2) rectangular boxes without guide sheets *R*, (3) scroll-inlet boxes with vanes *SV*, and (4), far below all the others, scroll-inlet boxes without vanes *S*. As indicated previously, this last condition is not one which would be used by any intelligent fan manufacturer and is consequently left out of consideration in the discussion that follows.

A quantitative appraisal of the inlet boxes may be made at some selected capacity and static pressure by comparing the necessary speeds and the power consumptions with those of a free-inlet fan. With a discharge volume of 35,000 c.f.m. and a static pressure of 3.96 in., the speed of the free-inlet fan is 900 r.p.m. and the power consumption is 43 hp. Table 2 shows the percentage change from these figures for the other boxes.

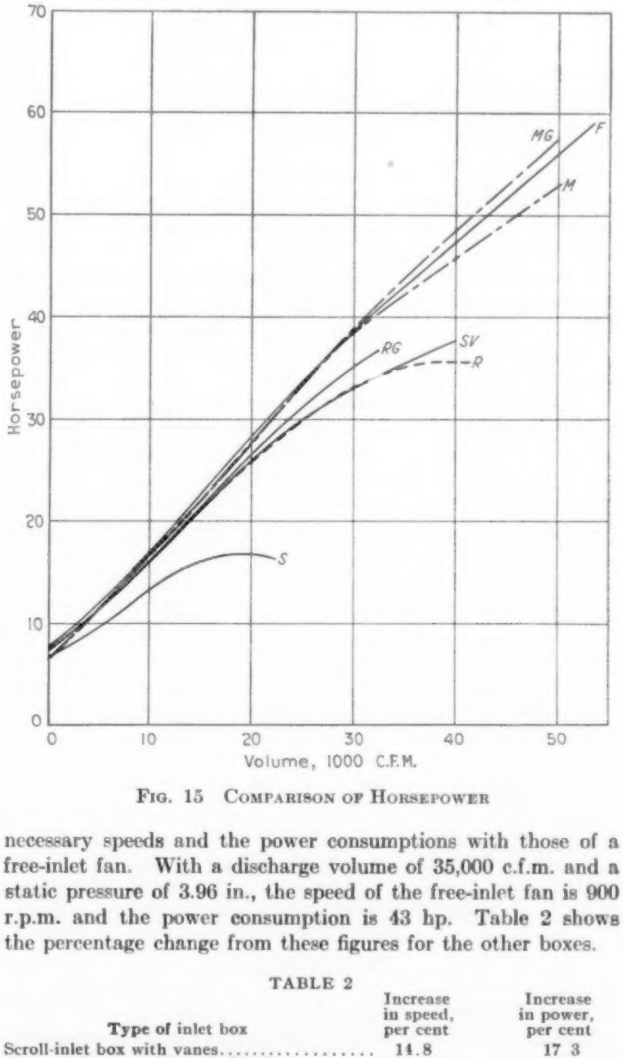

FIG. 15 COMPARISON OF HORSEPOWER

TABLE 2

Type of inlet box	Increase in speed, per cent	Increase in power, per cent
Scroll-inlet box with vanes	14.8	17.3
Rectangular box without guide sheets	12.2	10.5
Rectangular box with guide sheets	5.3	0.7
Modified rectangular box without guide sheets	0.3	−1.2
Modified rectangular box with guide sheets	0.3	3.0

It should be recognized that the increased power consumptions of the scroll-inlet and rectangular boxes result from the fact that at the selected capacities the efficiencies are lower than for the free-inlet fan. Fig. 16 shows that the maximum efficiencies of all the fans (excluding *S*) are substantially the same. If a larger fan is used with the scroll-inlet boxes and rectangular boxes, the operating point for a given capacity will be higher up on the efficiency curve, and the power consumption consequently will be decreased.

A comparison on this basis has been made for a static pressure of 5 in., a volume of 29,250 c.f.m., and a power consumption of 37.5 hp. This performance, which is that obtained from a 38-in. wheel with free inlet at 900 r.p.m., corresponds to a static efficiency of slightly over 60 per cent. From Fig. 16 the capacities with the various inlet boxes at the same efficiency can be measured, and from Fig. 14 the corresponding static pressures can be ascertained. To find the size and speed of fans to give the same performance as the free-inlet fan, use is made of the relations existing between geometrically similar fans: namely, (1) the volume of discharge is proportional to the square of the linear dimensions and to the first power of the tip speed, and (2) static pressures are proportional to the square of the tip speed. Table 3 shows the fan sizes and r.p.m. necessary for each type of inlet box to give the same performance as the free-inlet fan at the selected point.

TABLE 3

Type of inlet box	Wheel diameter, in.	Speed, r.p.m.
Scroll-inlet box with vanes	42.8	812
Rectangular box without guide sheets	40.0	900
Rectangular box with guide sheets	37.7	944
Modified rectangular box without guide sheets	36.9	952
Modified rectangular box with guide sheets	38.65	884

The addition of guide sheets in the inlet ducts results in a marked improvement in the performance with rectangular boxes. With the modified rectangular boxes, no improvement is found or is to be expected, in view of the excellent results obtained without the guide sheets.

The outstanding conclusion to be drawn from these tests is that it is not sufficient for the fan user to know the characteristics of a selected fan with free inlet. He needs to know also the characteristics when the inlet box is attached. With a rectangular inlet box, which is probably the most common form in use, in order to obtain the same performance which the fan gives with free inlet, it is shown that it is necessary to speed up the fan 12.2 per cent and the power input is increased 10.5 per cent. If the user wishes to obtain the free-inlet pressure at the rated volume without the increase in power input, he would have to select a 40-in. fan instead of a 38-in. fan. The instability which characterized the performance with rectangular boxes may be regarded as an indication of the unsuitability of these particular boxes.

It is noteworthy that the comparatively minor difference between the modified box and the straight rectangular box resulted in a considerable change in performance. The mere provision of sufficient cross-section for the approaching air is not sufficient to ensure good performance; it also is necessary to consider the direction of flow in the vicinity of the inlet and to form the boxes in such way as to provide appropriate guidance.

Discussion

W. A. Rowe.[3] This paper brings out forcibly the reasons which prompted the revisions in the code in respect to exhausting fans with inlet boxes. The exception cited by the authors to this standard-test-code set-up, while contemplated at the time of the carrying out of these experiments, is not true with the code as finally revised and adopted. The code does not now provide an inlet duct having a length of 6 mean diameters, as it was found that a short duct with an entrance nozzle produced a more uniform velocity gradient and more accurate reading. The form as adopted is as follows:

> Centrifugal fans with inlet boxes shall be tested with inlet boxes in place. A duct (or a duct for a double-inlet fan) of the same size and form as the inlet-box opening shall be connected thereto as shown on plate G. The inlet test duct shall be not less in length than the mean of its two sides. The open end shall be provided with a nozzle inlet formed of cylindrical sides curved to a radius of not less than one-half of the short dimension of the duct and not less than a 75-deg. arc. Readings shall be taken in said ducts at a point one-half of a mean side from the open end, exclusive of the length of the nozzle. The readings are to be taken in accordance with plate G and calculations made in accordance with the calculation sheet for centrifugal fans exhausting; see plates A, B, and C.
>
> Ratings based on such tests shall be clearly stated as applying to centrifugal fans with inlet boxes and include the loss in the boxes.

As a result of this action, fan rating tables are now available showing the operation of the fans with the boxes in place. However, these rating tables are based on the particular proportioned box provided with the fan, and the authors very properly point out the reason for this. The results they have obtained apply not only to the particular conditions of the test made by them, but to the particular fan or type of fan used in the test.

While the results of this test show the effect on the fan performance in the case of rectangular inlet boxes of certain dimensions as affecting the aspect ratio and area of the inlet to the box, it will be found that with other proportions and areas the effect may be greater or lesser. It is possible by trying out various combinations to determine a box of such proportions as will provide a minimum change in the characteristics of the free fan without excessive width overall, which on induced-draft fans determines the span between bearing centers. Sometimes an apparently minor change makes a very material difference in the performance of the fan, as mentioned in the last paragraph of the paper, and it is necessary to cover wide ranges of proportion and design before arriving at the best combination of all factors.

C. S. Messler.[4] These tests confirm in general the results that we have obtained in investigations along the same lines.

In the case of rectangular boxes the authors have used a box with a ratio of height to width of 2.78 to 1. We think this ratio is too large for best results with this type of inlet box. We have obtained much better results using a plain rectangular box having a ratio of 2 to 1. These tests were made using a fan of the same type as that used by the authors, but with a wheel 52 in. in diameter. At 75,000 c.f.m. and a static pressure of 9.8 in., corresponding to an orifice 65 per cent of wide open, the speed of the free-inlet fan was 1000 r.p.m. and the increased speed for the rectangular boxes is 1.5 per cent.

We have made no tests on a box that would be comparable to the modified rectangular boxes used by the authors, but this design seems to have some decided advantages over the plain rectangular type.

In the case of the scroll inlet boxes, our investigations have proved conclusively that without vanes or baffles the air reaches the fan inlet with such a whirling motion as to form a vortex at the center of the inlet, so that the effective area of the inlet is reduced by about one-half. Under some conditions we found air flowing into the inlet around the outside area and out from the inlet at the center. The volume handled by the fan under these conditions was less than one-half that handled by the fan with free inlet. This confirms results obtained by the authors. It is therefore imperative when using scroll inlet boxes to provide vanes or baffles to direct the flow of air into the fan inlet. The fan performance can be varied over quite a wide range, depending on the design of vanes or baffles used.

A. A. Criqui.[5] The paper states that the standard method of fan testing does not in some cases yield accurate results, although the Moss nozzle test results checked very closely in this case with the standard fan-testing-code results. It would be interesting to know what criticism the authors make on the standard method.

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[3] Chief Engineer, American Blower Corporation, Detroit, Mich.

[4] Chief Engineer, Green Fuel Economizer Company, Beacon, N. Y. Mem. A.S.M.E.

[5] Buffalo Forge Company Buffalo, N. Y.

It is stated that when tested with rectangular inlet boxes without guide sheets the fan developed a condition of instability with periodic recurrence of a roaring noise and an increase in power, and without corresponding change in the volume of air delivery. The addition of guide sheets in the inlet ducts improved this condition. This condition no doubt was caused by a combination of the fan characteristics and the inlet box, and can easily be corrected in the box design.

The tests conducted by the authors show that the modified rectangular inlet box was the most efficient of those tested.

Results of the writer's tests agree with those of the authors in that rectangular inlet boxes should be modified for best results.

It is not thought that the authors wish to claim that the figures given in their conclusions may be regarded as general. Obviously the results apply only to the particular proportions of boxes tested. Rectangular boxes could be made of sufficient area so that results would be practically the same as with free inlet. Such large boxes would not be economical, however, and it therefore becomes a problem in which it is necessary to make the box design as efficient as possible in as small a space as possible to keep both the power consumption and the cost of the unit within economical limits. This can be accomplished by the use of rectangular boxes with proper modifications.

H. F. Hagen.[6] The opening paragraph of the paper might well have been emphasized by italics. Fans, particularly induced-draft fans, more often than not, are selected and bought only after the power house has been designed and the necessary space has been allotted to all the other equipment. The fans and the required flue work have to go into the space that is left; and it sometimes seems to the fan engineer that his most difficult problem is dodging beams. Many power-plant designers believe that directions of flow do not matter, so long as the flue areas are large enough to give a low velocity. While it is true that disappointment in boiler output is sometimes due to inaccurate fan guarantees, yet in still more cases reduced output is due solely to beam-dodging flues. The authors' tests demonstrate authoritatively the large variations in performance that result from careless inlet-box design.

Consider a fan bought from a standard test-code performance with free inlets, as in Fig. 7, and then being provided with boxes, as in Fig. 3, with an unexpected reduction in capacity, as shown in Fig. 8. The cross-sectional area of the boxes is nearly double the fan inlet area, but in spite of this the performance has been materially reduced. Suppose the fan had been bought for 29,250 c.f.m. at 5 in. static pressure, from the performance of Fig. 7. When installed according to Fig. 8, at 29,250 c.f.m., the static pressure would be only 3.6 in. This reduction is in proportion to the differences between expected and actual conditions that are found in so many modern power plants.

A fan is, as shown by these tests, decidedly susceptible to changes in inlet flow, and this change in performance occurs whether the change in inlet flow is caused by different inlet boxes or different duct connections.

An apparently minor modification of the box of Fig. 3 to that of Fig. 4 brought the performance back to that of Fig. 10, which is substantially equal to the original.

However, it must be remembered that in these tests the inlet duct gave a straight flow to all the boxes. And, as the authors state in their first paragraph, the performance may be again affected "if the inlet ducts are not well designed." It was for this very reason that the design of Fig. 5, a spiral box with vanes, was introduced. This is an expensive construction, and, because of smaller capacity, requires a larger fan to do the same work efficiently, again increasing the cost. Table 3 shows that to secure the same volume at the same efficiency a wheel 42.8 in. in diameter is required as against a 36.9-in. wheel for the simpler modified box.

However, if the effect of the spiral box is allowed for and the fan is sold from the curve of Fig. 12, the design might reasonably be expected to give more consistent performance, less affected by poor flue design than any of the others. Its higher cost would then be justified by insuring exactness of performance in actual service. The design has been successful in that it has greatly decreased complaints and has in general given the consistent performance expected of it.

The careful testing and checking of results, as mentioned in the paper, show an attitude worthy of general imitation. The standard test code was followed, but with extra precautions. The code provides that the horsepower shall be determined by an electric dynamometer or a calibrated electric motor. The authors used both methods and secured satisfactory checks, incidentally finding that more than usual precautions were necessary in using a dynamometer. Volume measurements were made with the pitot-tube traverse, the method preferred in the code; but a nozzle determination, also a well-recognized method and described in the code, was secured as a check.

The authors are to be congratulated on the completeness of their presentation of results and for their use of the readily intelligible, customary forms of plotting.

From more general considerations the writer believes that the fan industry and fan users should feel encouraged at this evidence of interest in fan problems displayed by a competent laboratory. A complete series of comparative tests, such as that described in the paper, is difficult to prosecute in the habitually harassed commercial laboratory, even when the technical skill may be available. And unless the particular work planned is followed through to completion, one reaches no conclusions. The results of the tests will be of direct help to the writer and will serve as valuable guides to other designers. They will tend to promote confidence between user and manufacturer. There are many problems before the industry at the present time whose solution and demonstration will necessitate just such painstaking and accurate investigation, and preferably by an independent laboratory. It is hoped that this paper indicates an interest in fans on the part of Professor Marks and his colleagues that will continue.

L. Menkin.[7] It has long been recognized by ventilation engineers that the performance of a fan, when installed, will be quite different from that obtained on the test floor of the manufacturer. The actual conditions of the air flow, prior to entering the fan and after leaving it, are of primary importance in determining the fan characteristics. The velocities of approach of the air flow to a fan inlet are such as to give it a condition of turbulence, with a high Reynolds number. The degree of that turbulence, as the air flow changes its direction to enter the fan and acquires a rotational motion, is bound to have a strong influence on the fan characteristics.

In comparing the merits of the various inlet boxes tested by the authors, it is rather unfortunate that no conclusions can be reached as to the exact factors responsible for the various differences in fan performance. For instance, in the tests of the rectangular and the modified rectangular inlet boxes, the better performances of the latter may be attributed to any one of the following factors or to a combination of them:

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[6] B. F. Sturtevant Company, Boston, Mass.

[7] Ventilation Designer, Board of Transportation, New York, N. Y. Assoc-Mem. A.S.M.E.

1 The sloping of the top and bottom sheets
2 The fact that there is a collar box between the fan housing and the inlet box
3 The inlet box is offset from the center of the fan
4 The entrance area of the modified inlet box is larger than that of the regular rectangular box.

For proper comparison, it would be interesting to see the performance of the same fan with a simple rectangular box, but which would be provided with a collar box of the same size as was provided for the modified rectangular box, and which would be offset from the center of the fan the same amount as the modified box, and would have the same width and height at entrance as the modified box. Under such conditions it would be an easy matter to determine the effect of sloping the top and bottom sheets of the box. Similarly, by varying any one of the other factors, one at a time, their comparative values could be determined. It would be especially interesting to get valuable data for an accurate comparison of the merits of a scroll inlet box with vanes with those of the modified rectangular box. The scroll inlet box has been advocated by some manufacturers, but if it could be conclusively demonstrated that the modified rectangular box is better and that a saving of 10 per cent on the wheel size can be effected for the same results, a considerable economy would have been attained by reducing the size of fans, and thereby the size of the buildings housing them. This would be of especial importance in very large ventilation installations, such as the exhaust system of a vehicular tunnel.

Joseph M. Barnes.[8] The writer would ask the authors if they made any tests as to the effect of installing fixed baffles horizontally in the inlet boxes, at right angles to the direction of gas flow, in an attempt to more accurately distribute the flow over the entire cross-sectional area of the box, and if so, whether this detail had any marked effect on fan efficiency.

Authors' Closure

The authors are glad to know that the recently revised, but as yet unpublished, standard test code prescribes the practice which they are advocating, of testing induced-draft fans with inlet boxes in place. This practice will force manufacturers to investigate more carefully the influence of the inlet boxes and will result in better fan performance. Such information as that supplied by Mr. Messler on the desirable proportions of rectangular inlet boxes is a case in point.

In reply to Mr. Criqui, the standard test method may give results which are considerably in error when the flow is pulsating. This condition occurs particularly in certain fans with a small number of blades. The pitot-tube reading under the circumstances will inevitably indicate a volume which is too large. The error may be considerable.

Mr. Hagen justifies the use of spiral inlet boxes by the statement that they give more consistent performance independently of the inlet-duct design. This belief seems reasonable to the authors and is now under investigation by them.

The further investigation suggested by Mr. Menkin would be interesting and valuable. The investigations now recorded are preliminary and exploratory. The authors hope to carry out a more systematic investigation at some future time.

In reply to Mr. Barnes, the only attempt at distributing the flow of inlet air was by the use of vertical guide sheets.

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[8] Chief Draftsman, Philadelphia Electric Company, Philadelphia, Pa. Mem. A.S.M.E.

FSP-54-17

Solubility of Calcium Salts in Boiler Water[1]

By FREDERICK G. STRAUB,[2] URBANA, ILL.

The purpose of this investigation has been to determine the behavior at elevated temperatures of the various chemical compounds found in boiler waters. This includes a study of the solubility of the various compounds under conditions simulating those found in high-pressure boilers. Such a study involves the determination of the solubility of the more insoluble compounds such as calcium carbonate and calcium sulphate separately and combined, as well as in the presence of the more soluble salts such as sodium carbonate, sodium sulphate, and sodium hydroxide. These results should aid materially in determining the proper water treatment for high-pressure-boiler operation.

IN THE operation of high-pressure steam boilers the prevention of scale receives first consideration since it is realized that the deposition of scale on the heating surface of high-pressure boilers operating at high ratings is very undesirable. The question of concentration of soluble salts also receives consideration since it influences scale deposition, carry-over, turbine-blade deposits, etc. There has been a tendency to predict scale in higher-pressure boilers unless phosphate treatment is resorted to above 200 lb. Thus, Hall[3] states:

> The formation of adherent anhydrite scale on the evaporating surfaces of a boiler operating at 185 deg. cent. (150 lb. gage pressure) is prevented by using soda ash to maintain the following conditions in the boiler water:
>
> $$CO_2 \text{ p.p.m.} > 0.0883\ SO_4 \text{ p.p.m.}$$
>
> It is evident from the expression
>
> $$CO_3 > K \text{ (at operating pressure } p) \times SO_4$$
>
> that the maintenance of scale-free conditions depends upon keeping a definite supply of CO_3 in the boiler water. In case conditions arise such that the necessary CO_3 concentrations cannot be maintained, then it is no longer feasible to use soda ash as the conditioning chemical.... Thus higher operating pressures are characterized by more rapid decomposition of CO_3 and at the same time by the necessity for maintaining higher CO_3 concentration, as the value of K is greater at higher than at lower operating pressures....
>
> A satisfactory and practical answer to the complete conditioning of water for high-pressure operation may be given as follows: If the make-up water is evaporated, and the only impure water entering the boiler consists of the condenser leakage, then conditioning the boiler with phosphate is the only chemical control necessary. If the make-up water is not evaporated, and is of some magnitude, then economy may demand a pretreatment (with lime and soda ash for any water, or by a base exchange process if the water does not contain temporary hardness) followed by the final conditioning of the boiler water with phosphate.... In any event, however, the final control of the boiler water must be based on the use of a stable substance such as phosphate.

If the A.S.M.E. Boiler Code recommendation for the prevention of embrittlement is followed, the amount of sulphate becomes too high for the carbonate and scale would be predicted by the older theory. However, in many instances ratios of over 3 are being maintained in plants at higher pressures, calcium is entering the feedwater in various amounts, phosphate is not being added, the amount of CO_3 is decidedly low, and yet scale is not a problem.

TABLE 1 ANALYSES OF BOILER WATERS FROM PLANTS NOT HAVING SCALE DIFFICULTIES

Plant	A	B	C	D	E
Pressure, lb. gage	700	700	400	225	160
NaOH	103	79	144	90	200
Na_2CO_3	8	5	42	29	26
Total alkalinity	144	109	233	148	291
Na_2SO_4	1130	415	3350	5030	3100
NaCl	190	43	1240	692	125
$\frac{Na_2SO_4}{\text{Total alkalinity}}$	7.8	3.8	14.4	34.0	10.6
$\frac{CO_3}{SO_4}$	0.0059	0.01	0.011	0.0047	0.007

The composition of typical boiler waters from several such plants is given in Table 1. The feedwater conditions at these plants are as follows:

Plant A. 98 per cent condensate returns; 2 per cent zeolite-treated water make-up; condenser water about 12 grains hardness; condenser leakage low.

Plant B. 98 per cent condensate returns; 2 per cent evaporator make-up water lime-soda softened prior to evaporation; condenser water about 12 grains hardness; condenser leakage low.

Plant C. 80 per cent condensate (process) returns; 20 per cent make-up; hot-process softener used for treatment; untreated water about 16 grains total hardness with about 7 grains sulphate hardness.

Plant D. 80 per cent condensate returns; 20 per cent make-up water about 17 grains total hardness with 9 grains sulphate hardness; internal treatment, sodium carbonate and organic.

Plant E. 80 per cent condensate returns; 10 per cent make-up water. The make-up water is treated by a cold-process lime softener and has about 3 grains residual hardness, with 3 grains excess soda ash leaving the softener.

According to existing theories, all of these plants should form scale. The carbonate-to-sulphate ratio according to Hall should be greater than 0.0883 in all of these plants. However, in them, some operating at 700 lb. pressure with calcium present in varying amount, the operators are not bothered with scale troubles.

These discrepancies between theory and operation may be readily explained on the basis that we do not have sufficient data available to formulate theories which hold at the higher pressures. Thus, Hall's figures were calculated using the solubility product, and not having exact values for the solubility of calcium carbonate

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[1] Part of research being conducted at the Engineering Experiment Station, University of Illinois, and financed by the Utilities Research Commission of Chicago. Released by permission of Dean M. S. Ketchum, Director of the Engineering Experiment Station.

[2] Special Research Assistant Professor in Chemical Engineering, University of Illinois. Mr. Straub was graduated from the University of Illinois in 1920. He became a research fellow at Mellon Institute, and later entered the employ of the Semet-Solvay Company, Syracuse, N. Y. In 1921 he became associated with Pennsylvania State College, where he received his M.S. degree in metallurgical engineering in 1923, and then was connected with the Guggenheim Brothers Research Laboratory until he returned to the University of Illinois. He has been conducting the special research for the Utilities Research Commission on boiler-feedwater treatment for the last seven years at the University of Illinois.

[3] Bulletin 24, Carnegie Institute of Technology, 1927.

Contributed by the Power Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

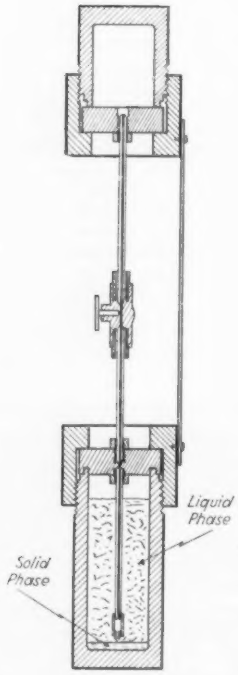

Fig. 1 Section of Bomb Used in Solubility Test

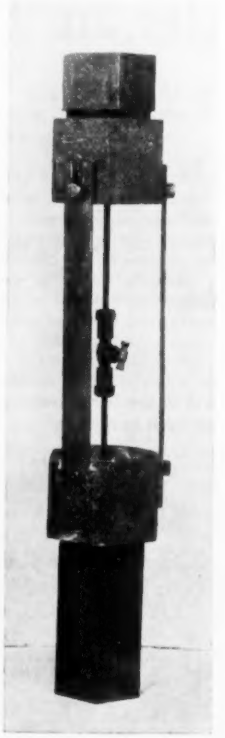

Fig. 2 Bomb Shown in Section in Fig. 1

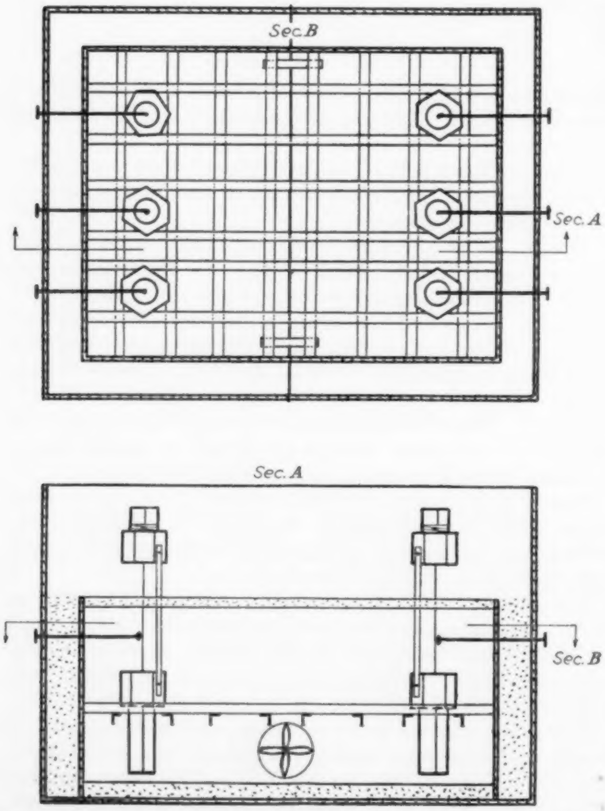

Fig. 3 Constant-Temperature Box, Showing Position of Bombs

his determination of the solubility of calcium sulphate was only made up to about 225 lb. pressure. Partridge[4] has checked Hall's figures on the solubility of calcium sulphate, but he also stopped at about the same pressure. Neither Partridge nor Hall determined the influence of additional salts on the solubility of calcium sulphate. In order to obtain further data in respect to the solubility of the various calcium compounds occurring in boiler waters and to study the influence of soluble salts, research has been conducted along these lines. This work has not been completed; however, sufficient data have been collected up to 1500 lb. pressure to show the general trend of the results.

In order to obtain the desired data an extremely large number of solubility tests had to be run. The procedure in the majority of the previous investigations conducted along this line at lower pressures has involved the operation of a small boiler. Naturally the results obtained from such tests were few in number since so much time had to be spent in obtaining equilibrium conditions. The present investigation had departed from this method of testing in that a large number of small steel bombs were used for the solubility determinations. Four constant-temperature boxes were constructed to hold these bombs. With each box at a different temperature and with six bombs in each box, twenty-four separate solubility tests were run simultaneously, thus obtaining results much faster than possible by the older procedure.

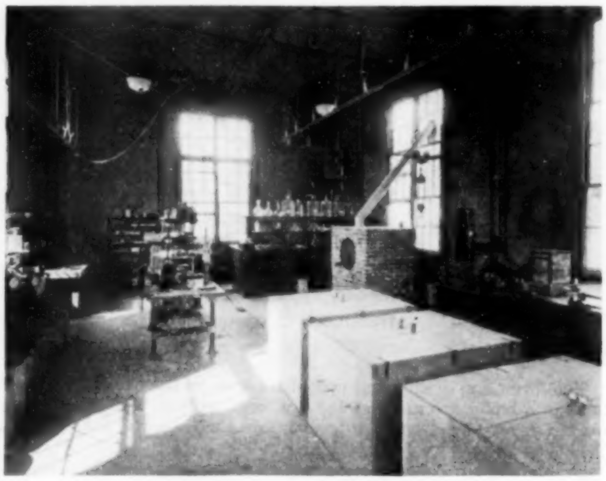

Fig. 4 View of Laboratory Showing Three of the Heating Boxes in the Foreground

The details of construction of the steel bombs are given in Fig. 1. Fig. 2 shows a photograph of the testing bomb completely assembled. Fig. 3 shows how the bombs are assembled in the heating box, while Fig. 4 shows three of the heating boxes in the foreground. The boxes are electrically heated by means of resistance wire. Constant temperature was maintained by means of automatic potentiometer temperature regulators. The accuracy of the temperature control was ± 3 deg. fahr.

The procedure involved in these studies was to put the desired solid phase in the larger or bottom bomb. The desired solution was then added boiling hot and the bomb closed by screwing the top on. The smaller or top bomb, previously cleaned and closed, was then attached by means of the connections on the capillary copper-lined steel tubing to the lower bomb. The valve between the two bombs was closed. The assembled bomb was put in the heating boxes and held at the desired temperature until equilibrium was obtained. In these tests the time in the furnace

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[4] "Formation and Properties of Boiler Scale," E. P. Partridge, Engineering Research Bulletin No. 15, University of Michigan, 1930.

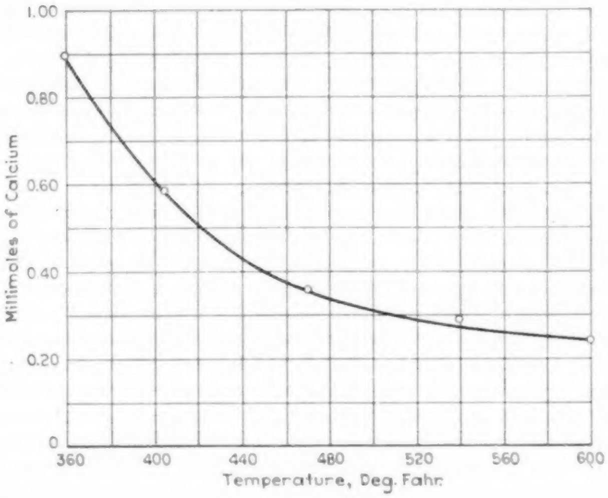

Fig. 5 Solubility of Calcium: Solid Phase Added, Calcium Sulphate
(Millimoles × 40 = parts per million of calcium.)

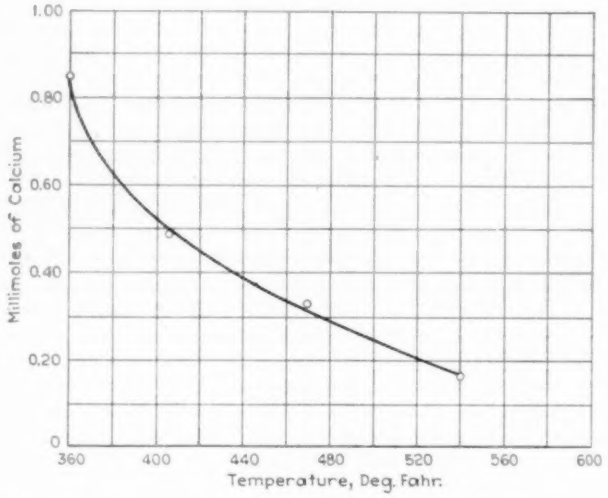

Fig. 7 Solubility of Calcium: Solid Phase Added, Calcium Sulphate and Calcium Carbonate
(Millimoles × 40 = parts per million of calcium.)

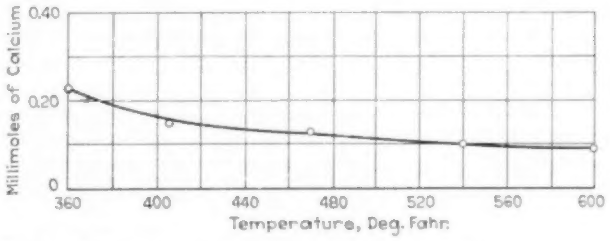

Fig. 6 Solubility of Calcium: Solid Phase Added, Calcium Carbonate
(Millimoles × 40 = parts per million of calcium.)

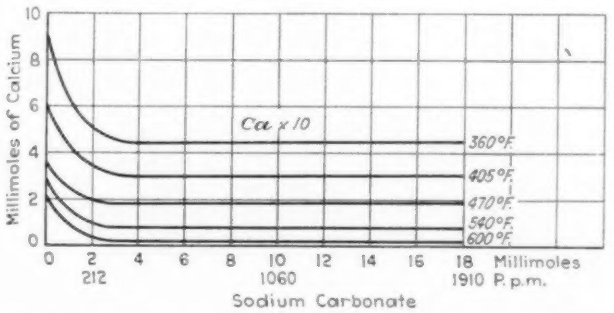

Fig. 8 Solubility of Calcium: Solid Phase Added, Calcium Sulphate; Salt Added, Sodium Carbonate
(Millimoles × 40 = parts per million of calcium.)

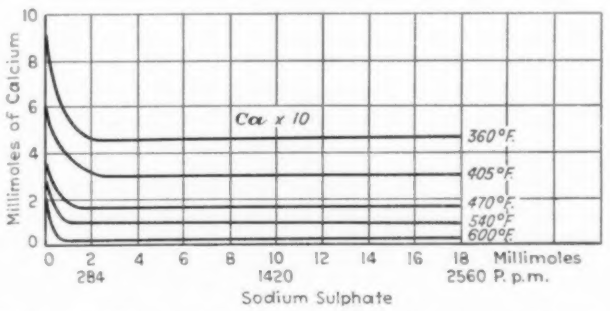

Fig. 9 Solubility of Calcium: Solid Phase Added, Calcium Sulphate; Salt Added, Sodium Sulphate
(Millimoles × 40 = parts per million of calcium.)

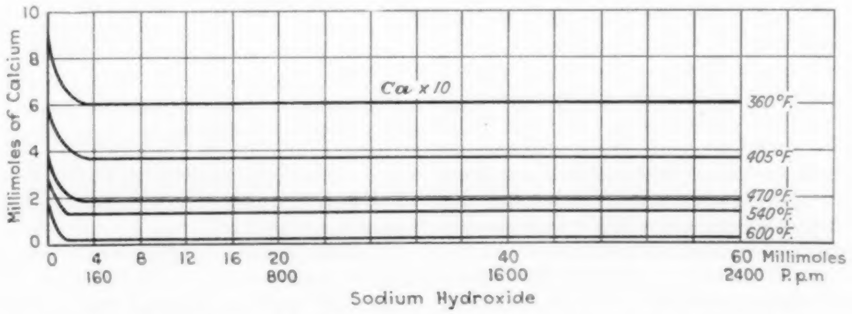

Fig. 10 Solubility of Calcium: Solid Phase Added, Calcium Sulphate; Salt Added, Sodium Hydroxide
(Millimoles × 40 = parts per million of calcium.)

to reach equilibrium was found to be rather short—about 6 to 10 hours. However, the furnaces were run about 72 hours. Consequently, plenty of time was given for equilibrium conditions to be established.

When it was desired to sample the bombs, the valve was opened by means of the handle extending through to the outside of the heating boxes. The steam pressure in the lower bomb forced the solution through the filter and the capillary tube into the upper bomb. After the top bomb was filled with the solution, the valve was closed and the complete bomb was removed from the heating boxes. When cool, the top bomb was removed, and the volume of solution accurately measured and completely analyzed.

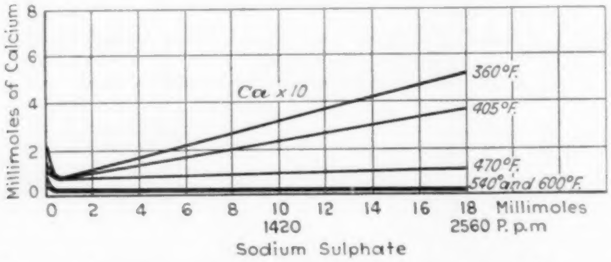

FIG. 11 SOLUBILITY OF CALCIUM: SOLID PHASE ADDED, CALCIUM CARBONATE; SALT ADDED, SODIUM SULPHATE

The solubility tests reported were run at 360, 405, 470, 540, and 600 deg. fahr., corresponding approximately to 150, 250, 500, 1000, and 1500 lb. gage. The solubilities of calcium sulphate and calcium carbonate were determined in distilled water separately and combined. The influence of soluble salts such as sodium carbonate, sodium hydroxide, and sodium sulphate upon each of these calcium compounds was also studied. The results of these tests are given in graph form in Figs. 5 to 14, inclusive.

The solubility of calcium sulphate at 360 deg. fahr. is approximately the same as that determined by Hall and Partridge. As the temperature increases the solubility decreases. A much larger decrease in solubility takes place between 360 and 470 deg. than between 470 and 600 deg. Thus, for 100 deg. fahr. increase in temperature above 360 deg. there is a decrease of 0.54 millimole, while 130 deg. increase above 470 deg. causes a decrease of only 0.12 millimole.

The solubility of calcium carbonate is rather difficult to determine since it is largely influenced by the partial pressure of carbon dioxide over the solution. Thus, at room temperatures as the partial pressure of the carbon dioxide increases the solubility of the calcium increases. The influences of the partial pressure of carbon dioxide on the solubility of calcium carbonate at elevated temperatures has not been determined. However, some work is being done in the author's laboratory along this line. In determining the solubility of calcium carbonate, water free from carbon dioxide and pure Iceland spar crystals were used. The partial pressure of carbon dioxide over these solutions would be a function of the decomposition of the calcium carbonate and would depend on the temperature. Under these conditions the solubility of calcium carbonate was found to decrease with increase in temperature. This is in accord with the work of Frear and Johnston,[5] who show that the calcium carbonate decreases in solubility with increase in temperature.

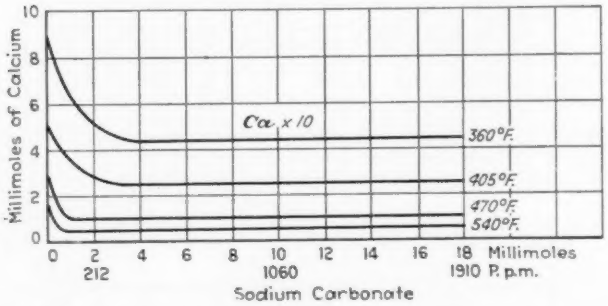

FIG. 12 SOLUBILITY OF CALCIUM: SOLID PHASE ADDED, CALCIUM CARBONATE AND CALCIUM SULPHATE; SALT ADDED, SODIUM CARBONATE

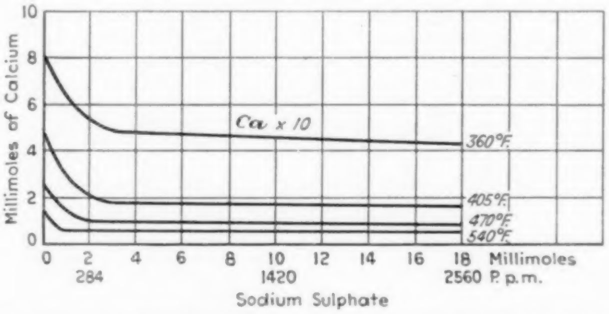

FIG. 13 SOLUBILITY OF CALCIUM: SOLID PHASE ADDED, CALCIUM CARBONATE AND CALCIUM SULPHATE; SALT ADDED, SODIUM SULPHATE

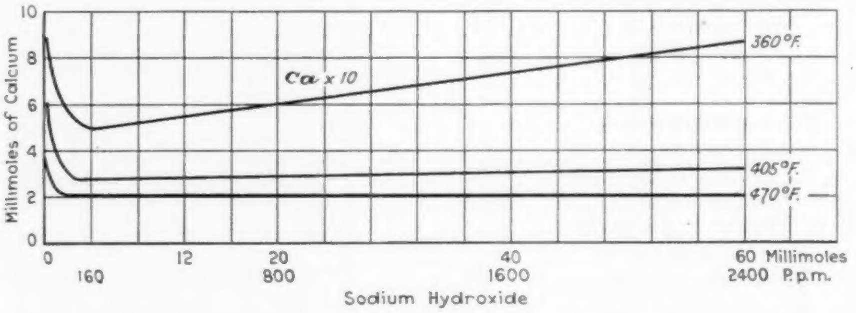

FIG. 14 SOLUBILITY OF CALCIUM: SOLID PHASE ADDED, CALCIUM CARBONATE AND CALCIUM SULPHATE; SALT ADDED, SODIUM HYDROXIDE

However, if these determinations which were made below 212 deg. are used to calculate the solubility at the higher temperatures, the results so obtained will be lower than those actually found. Hall has based his calculations on the assumption that calcium carbonate increases in solubility with increase in temperature.

If calcium sulphate and calcium carbonate are added as a solid phase, the solubility of the calcium, as shown in Fig. 7, is very similar to that of calcium sulphate alone, being a trifle lower at practically all temperatures reported.

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[5] *Jl. Am. Chem. Soc.*, vol. 51 (1929), p. 2082.

When sodium carbonate is added to calcium sulphate, the solubility of the calcium drops rather rapidly until about 300 parts per million of sodium carbonate have been added. The addition of sodium carbonate up to 2000 parts per million does not have any additional effect on the calcium solubility. In these tests there was always an excess of calcium sulphate present. When sodium sulphate was in solution with calcium sulphate added as a solid phase, the results were almost the same as when sodium carbonate was added. This may readily be seen by comparison of the curves in Figs. 8 and 9.

The addition of sodium hydroxide with calcium sulphate as the solid phase also gave results much similar to those obtained when adding sodium carbonate or sulphate. After a small amount of hydrate has been added, putting in more has no further influence on the calcium content of the solution.

The results obtained from adding sodium carbonate and sodium hydroxide solutions to calcium carbonate as the solid phase have not been plotted. This is due to the fact that upon the addition of as little as 0.25 millimole of either salt the solubility of calcium dropped to zero and stayed at that point as more was added. Thus the solubility of calcium was zero in the presence of sodium carbonate or sodium hydroxide when calcium carbonate was the solid phase added.

When calcium carbonate was the solid phase added the addition of a small amount of sodium sulphate decreased the calcium solubility, but as more was added the calcium content increased at 360 and 405 deg., but remained constant at 470 deg. and higher temperatures. This is shown in Fig. 11.

When both calcium carbonate and calcium sulphate were present in the solid phase, the addition of Na_2CO_3 gave results similar to those obtained when calcium carbonate was not added. In these tests there was always an excess of sulphate. The addition of sodium sulphate or hydrate to this mixed solid phase gave similar results, as shown in Figs. 12 and 13.

In all these tests there was an excess of solid phase added. Tests were run starting with a solid phase of calcium sulphate and adding an excess of sodium carbonate. At the end the calcium solubility was equal to zero at all temperatures, which was the same as calcium carbonate with sodium carbonate present.

In attempting to determine the CO_2/SO_4 ratio for the conversion of calcium sulphate to calcium carbonate, the sulphate content of the solution was determined. It was assumed that if 100 per cent of the carbonate reacted with the sulphate the calculated sulphate would be equal, mole for mole, to the carbonate added.

The results obtained are presented in Table 2 and show that practically 100 per cent of the carbonate reacts with the sulphate at 360 deg., and even at 600 deg. only a small percentage of carbonate is not accounted for as sulphate.

In actual boiler water there will nearly always be free hydrate available. Therefore, the foregoing tests were repeated with hydroxide present. The results of these tests are given in Table 3. The calculated sulphate is obtained as before. The tabulated differences show that the determined sulphate agrees with the theoretical within the experimental error. Thus, when an excess of calcium sulphate is present in a solution containing hydrate alkalinity, practically all of the sodium carbonate reacts with the sulphate to form sodium sulphate and calcium carbonate. The retarding influence of the sodium sulphate, even up to concentrations of 3000 parts per million or 170 grains per gallon and at 1500 lb. pressure, can practically be neglected, and the presence of a measurable amount of sodium carbonate should convert the calcium sulphate to calcium carbonate. When sodium carbonate exists in the solution, the solubility of the calcium drops to extremely low values due to the fact that the calcium sulphate has been completely converted to calcium carbonate.

TABLE 2 RELATION BETWEEN CALCULATED SULPHATE AND DETERMINED SULPHATE WHEN SODIUM CARBONATE SOLUTION WAS ADDED TO $CaSO_4$ AS SOLID PHASE

(Results in millimoles per liter)

Temperature, deg. fahr.	Na_2CO_3 added	SO_4 at end	Difference
360	3.75	3.70	—0.05
360	12.5	12.40	—0.10
360	18.7	18.6	—0.10
405	3.75	3.70	—0.05
405	12.5	12.00	—0.50
405	18.7	17.9	—0.80
470	3.75	2.90	—0.85
470	12.5	11.80	—0.70
470	18.7	17.5	—1.2
540	3.75	2.85	—0.90
540	12.5	11.2	—1.3
540	18.7	17.4	—1.3
600	4.25	3.0	—1.25
600	14.15	12.6	—1.55
600	21.2	19.1	—2.1

TABLE 3 RELATION BETWEEN CALCULATED AND DETERMINED SULPHATE WHEN SODIUM CARBONATE AND HYDROXIDE SOLUTION WERE ADDED TO CALCIUM SULPHATE AS SOLID PHASE

(Results in millimoles per liter)

Temperature, deg. fahr.	Calculated SO_4	Determined SO_4	Difference
360	4.50	4.50	0.0
360	13.25	13.20	—0.05
360	19.45	19.20	—0.25
405	4.50	4.60	+0.10
405	13.25	13.30	+0.05
405	19.45	20.00	+0.55
470	4.50	4.70	+0.22
470	13.25	13.50	+0.25
470	19.45	19.20	—0.25
540	4.50	4.30	—0.20
540	13.25	13.50	+0.25
540	19.45	20.00	+0.55

Discussion of Results

The results which have obtained are still incomplete, and the effect of other salts such as chlorides, silicates, and phosphates and the influence of higher temperatures are now being studied. However, they should help in determining the desirable water conditions for higher-pressure boilers.

The solubility of calcium carbonate, being practically zero at the temperatures studied when sodium carbonate or hydrate is present, illustrates why calcium carbonate scale does not form in boilers operating with an appreciable alkalinity. The calcium is all thrown out of solution as insoluble calcium carbonate, and forms the finely divided solid or sludge.

The solubility of calcium sulphate decreases rather rapidly up to 500 lb. pressure, and somewhat slowly above this pressure. The amount of soda ash necessary to prevent the reconversion of calcium carbonate to calcium sulphate in solutions having free hydrate present is decidedly low, even at the highest temperature studied and in the presence of large amounts of sodium sulphate. This shows that with measurable amounts of sodium carbonate and hydrate remaining in solution the calcium sulphate is entirely converted to calcium carbonate, which in turn has an extremely low solubility under these conditions. Thus, if equilibrium is maintained, calcium sulphate cannot exist as a solid phase with sodium carbonate and hydrate in the liquid phase. The influence of the sodium sulphate in solution on the formation of calcium carbonate is practically negligible at the concentrations studied.

The interpretation of boiler-water conditions in terms of these data must be done somewhat conservatively. The data are equilibrium data, and it is entirely possible that such conditions are not reached in all parts of the boiler. At the same time the rate of steam generation at the heating surface may have more to do with scale formation than the question of relative solubility. However, these results should indicate with a fair degree of accuracy whether or not scale should form. They will explain why scale is not a factor in the boiler waters referred to in Table 1.

All of these waters have sufficient carbonate and hydrate to cause the precipitation of calcium carbonate in the boiler water and prevent calcium sulphate from forming.

Research is in progress which will attempt to correlate the solubility equilibrium data with scale prevention.

The conclusions which may be made at this time are as follows:

1 The solubility of calcium carbonate decreases with increase in temperature

2 The solubility of calcium carbonate in the presence of sodium carbonate of hydrate is practically zero at the temperatures studied

3 The solubility of calcium sulphate is not directly proportional to the sulphate in solution

4 Calcium sulphate as a solid phase cannot exist in equilibrium with solutions containing sodium carbonate, even at pressures up to 1500 lb.

5 The old calculated ratios of CO_2/SO_4 to prevent calcium sulphate formation as a solid phase are extremely high.

Discussion

Cyrus William Rice.[6] The writer would like to offer some comments, as we find this solubility to be influenced by other constituents in the formation of structural deposits of calcium under operating conditions.

Our experience has clearly indicated that the increasing concentrations of the sulphate ion through the addition of sodium sulphate to the boiler concentrations have a decided influence on the deposition of calcium as calcium sulphate where calcium is the predominating hardness constituent in the feedwater; that sodium sulphate under similar conditions causes deposits of calcium silicate where silica is contained in the feedwater in a form unknown to us. These effects have all followed the addition of sulphate for the purpose of increasing the sulphate to carbonate ratio.

In cases of zeolite operations this addition of sulphate has often proved so troublesome that the practice had to be discontinued.

Under either of these circumstances the quality of the feedwater and the operating pressures remained the same, and the only change that brought about the troublesome deposits was the increasing concentrations of the sulphate radical. These same deposits of calcium sulphate and calcium silicate have also followed the excessive use of sodium hydroxide.

The solubility of calcium in the boiler concentrates is also influenced by organic matter and some of the silicates. This has been such in a number of cases as to cause sulphate deposits with soda concentrations in excess of 1000 p.p.m. and in a few instances objectionable silica deposits have occurred with soda concentrations approximating 2500 p.p.m. $CaCO_3$.

A review of many complete laboratory analyses of boiler concentrates compared with similar analyses of deposits from fire-exposed boiler tubes has also indicated that deposits of calcium sulphate do not take place within boilers in most of the treatments with sodium carbonate, where the product obtained by dividing all the SO_4 into the CO_3 in the boiler concentrates exceeds 0.0883.

While this is the case, there are exceptions to this, as we know that ratios as low as 0.01 do not cause scale deposits and that a ratio several times the 0.0883 has been accompanied by very troublesome formations on the evaporating surfaces.

Because of this, rules of this character have no definite value in controlling the chemical conditioning of boiler feedwaters.

In referring to the various examples given by the author where the ratio of 0.0883 did not apply, the writer can see why Plant A, because of its zeolite supply, did not scale. Plant B may also fall in this class because of the small amount of make-up and possibly too low calcium content in the feedwater and boiler concentrates to cause deposits.

In either Plants A and B the leakage of calcium into the condensate from the cooling water would also have to be small in amount. Plants C, D, and E, because of the low percentage and character of the make-up, are different, and according to our experience under similar conditions should cause structural deposits of calcium.

In that the amount of dissolved calcium in the boiler concentrates has an important influence in causing deposits, a complete analysis of these, including the calcium and magnesium, should prove interesting reading in all the cases cited.

Getting down to the basic facts in treating feedwaters, it is the manner in which dissolved calcium is removed from the boiler concentrates and the extent to which this takes place that is really the important issue in the conditioning of feedwater.

With calcium to the possible minimum, magnesium is no consideration, although complete analyses of the boiler concentrates invariably show more dissolved magnesium than calcium in a water that has proved itself non-scale-forming with soda treatments.

As stated, the manner in which calcium is removed is most important. This is not mentioned in any part of the paper, and the charts this gives may prove misleading to those who do not have a clear understanding of the subject.

For instance, the charts show about the same amount of calcium with either sodium carbonate or sodium sulphate in use, yet the addition of the carbonate in the treatment of a feedwater produces an insoluble sludgy material, and treatment with sodium sulphate causes hard, adherent structural deposits. In either case, the amount of calcium remaining, as it is indicated by the charts at the various temperatures, is enough to cause objectionable deposits under similar conditions within boilers.

To prevent structural deposits, calcium must be practically eliminated from the boiler concentrates. This can be effected only through the treatment of such concentrates with chemicals that will precipitate calcium as an insoluble non-adherent material.

The index of the calcium concentration in the boilers is therefore one of the best guides for controlling structural deposits, and the writer believes that experiments for determining its solubility with different treating chemicals under different concentrations of the treating agents and in the presence of the many different salts common to feedwater will prove very interesting. Experiments for the removal of calcium and silicate from waters with soda treatments of all kinds, including phosphates and in the presence of hydroxides, sulphates, chlorides, and carbonate of soda, will also be valuable and interesting, because of the great number of industrial waters in which silica has proved itself so troublesome.

Joseph D. Yoder.[7] The method the author describes for sampling water is ingenious and should be satisfactory for the purpose if large enough to give as large samples as required for accuracy in determinations of substances with low solubilities.

He shows a great similarity of chemical reactions at 1500 lb. to those prevailing at 150 lb. pressure. This knowledge should be reassuring to designing and operating engineers as evidence that the prevention of scale offers no obstacle to the installation of high-pressure boilers, regardless of whether the percentage of make-up is large or small.

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[6] President, Cyrus William Rice & Co., Inc., Pittsburgh, Pa. Mem. A.S.M.E.

[7] Manager, Water Purification Department, Cochrane Corporation, Philadelphia, Pa. Mem. A.S.M.E.

That the investigation of chemical equilibria at the high pressures is to be extended to include other salts, particularly silicates and phosphates, is of interest and promises to be of further value.

In the consideration of these data, it is necessary to keep in mind that the prevention of adhering deposit is not solely one of relative solubilities. Consideration must be given to the fact that some precipitates possess less tendency to adhere to surfaces than others, and furthermore that with different conditions the same precipitate offers different tendencies to adhere. As an illustration, if calcium carbonate in the absence of excess CO_3 radicals as sodium carbonate crystallizes from solution, the tendency will be for the calcium carbonate to adhere to the evaporating surfaces, whereas sodium carbonate in excess causes the calcium carbonate to precipitate as a non-adhering sludge. Within reasonable limits the higher the excess of CO_3 radical the less tendency will there be for the deposits to adhere to the evaporating surfaces. As there are practical objections to the high concentrations of sodium carbonate or other soluble solids in the boiler, the goal of feedwater treatment is to prevent adhering deposits and to let remain the smallest quantity of soluble salts.

The author reports that the solubility of calcium carbonates decreases with increase in temperature and that at the temperatures investigated, 360 to 540 deg. fahr., the solubility of calcium was zero with as little as 0.25 millimol or 13 p.p.m. sodium carbonate in solution.

That calcium should have zero solubility under these conditions is surprising and does not seem consistent with other observed facts. Samples of treated water withdrawn from boilers at 250 lb. pressure, 405 deg. fahr., invariably show the presence of some calcium carbonate in solution and generally as much as 8 to 13 p.p.m., with appreciable quantities of sodium carbonate and sodium hydrate. This may be partially accounted for in that some of the calcium carbonate in suspension dissolves at the lower temperature before filtration of the sample preliminary to making the analysis.

There are other indications that the solubility of calcium carbonate is not zero at boiler pressure. An illustration of this is that if a boiler contains some calcium-carbonate deposits when the feeding of sodium phosphate is begun, the phosphate reacts with the calcium-carbonate precipitate to form calcium phosphate, which signifies that the calcium phosphate has a solubility somewhat less than the solubility of calcium carbonate.

Mr. Straub establishes that the ratio of sulphate to carbonate may be very much higher than Hall believed and that a ratio of sodium sulphate to sodium carbonate of 3 to 1 can be carried at the higher boiler pressures without the formation of calcium-sulphate scale.

In the treatment of feedwater, the writer has observed but little tendency to form calcium-sulphate scale with the 3-to-1 ratio. He does, however, advocate the use of phosphate to supplement lime and soda or zeolite treatment of water for boiler pressures higher than 250 lb., and for certain water supplies finds it desirable for pressures as low as 100 lb. The demonstrated value of the phosphate treatment is not so much to prevent a calcium-sulphate scale as to prevent a calcium-silicate-carbonate scale which may contain a small percentage of calcium sulphate and also to make possible carrying lower alkalinity and therefore less sodium sulphate in the boiler water.

Owing to the greater dissociation of sodium carbonate to sodium hydrate at the higher boiler pressures, it obviously becomes necessary to carry higher sodium alkalinity at higher pressures. When it becomes necessary to maintain three times as much sodium sulphate in the boiler water as sodium alkalinity, the result is higher solids in the boiler than is desirable with the natural inclination of the operators to reduce the alkalinity below that required to prevent the calcium-silicate-carbonate deposit. This same tendency to reduce the alkalinity may lead to the formation of calcium-sulphate scale, particularly if the chemical treatment is not always uniform.

The feeding of phosphate makes it possible to keep a very low alkalinity in the boiler and has the further advantage that the calcium-phosphate precipitate offers less tendency to adhere to the evaporating surfaces than calcium carbonate. From the practical standpoint, the feeding of phosphate insures a cleaner surface of the boiler metal, for which reason it is fed regardless of what may be the exact theory involved. The writer's opinion is that its effectiveness is primarily due to the very low solubility of calcium phosphate, the stability of the PO_4 radical, and the physical structure of the precipitate, which is not favorable to crystallization.

Hall[8] and Partridge[9] have advanced the theory that calcium carbonate does not form scale, because its solubility increases at higher temperatures and therefore it becomes insoluble in the main body of the stream of water and not at the surface of the metal, whereas calcium sulphate is less soluble at the higher temperatures and therefore it becomes insoluble at the evaporating surfaces and forms scale. This theory never seemed tenable, and on a previous occasion the writer cited a number of reasons why it did not give an adequate explanation of scale formation. It is pertinent that Straub now finds that the calcium carbonate becomes less soluble with increase in temperature, which completely undermines the theory of scale formation advocated by Hall and Partridge in that he demonstrates their basic premise to be false.

It will be interesting to have determined the solubility of calcium phosphate at increasing temperatures, for it also does not produce scale in the boiler if the PO_4 radical is in excess. There are some indications that, like calcium sulphate and calcium carbonate, it also becomes less soluble with increase in temperature, although the writer has no knowledge of its solubility having been determined at temperatures corresponding to boiler pressures.

R. E. HALL.[10] The paper furnishes data on solubilities of some calcium salts at the temperatures characteristic of present-day boiler waters that are of great interest and that are certain to be of great usefulness. Up to a dividing line of the boiling temperature of a solution under normal atmospheric pressure, a wealth of exact solubility data exists; at pressures above that of the dividing line, there is a great dearth of data of any accuracy commensurate with modern methods of determination. For instance, the solubility of the common gases, oxygen and carbon dioxide, at these higher pressures (temperatures) is not known. Dependable determinations on the solubility of sodium carbonate above 212 deg. fahr. are of recent date. The data on sodium sulphate at elevated temperatures ought to be reviewed.

The varying phases and the solubility of calcium sulphate up to the temperature of 430 deg. fahr. have been given careful consideration by Boyer-Guillon, Melcher, Hall, Robb, and Coleman, and Partridge and White. The work of Professor Straub extends the solubility data to 600 deg. fahr. and brings out the important fact that anhydrite is the stable phase at these temperatures. Thus no complications from phase relationships are involved in considering the problem of calcium-sulphate removal from boiler waters to be used at high operating pressures.

While the presentation of solubility data in the form of curves only, best meets the needs of the present paper, they are of such importance that it is hoped that eventually the values of all of the individual determinations will be presented.

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[8] Hall, R. E., Carnegie Institute of Technology Bulletin No. 24, 1927.

[9] Partridge, E. P., University of Michigan Engineering Research Bulletin No. 15, 1930.

[10] Director, Hall Laboratories, Inc., Pittsburgh, Pa. Mem. A.S.M.E.

In a boiler water the solubility of any one salt is largely influenced by the other salts that are present. The author considers a number of cases, among others the effect of sodium sulphate upon the solubility of calcium sulphate, and finds that up to a concentration of 2560 p.p.m. of sodium sulphate the solubility of the calcium sulphate is considerably decreased by the presence of the sodium sulphate. Our own work on this effect of sodium sulphate on the solubility of calcium sulphate, carried out at a temperature of 360 deg. fahr., was reported in Bulletin 24 of the Carnegie Institute of Technology and the U.S. Bureau of Mines. When sodium sulphate was not present, the solubility of calcium sulphate was found to be 120 p.p.m.; when 1370 p.p.m. sodium sulphate was present, the solubility was lowered to 81 p.p.m., but began increasing as more sodium sulphate was added, so that there were 83 p.p.m. of calcium sulphate when the sodium-sulphate concentration was 4250 p.p.m., 86 p.p.m. when sodium sulphate was 7550 p.p.m., and 103 p.p.m. when sodium sulphate was 14,900 p.p.m. Bulletin 24 states:

> The decrease followed by an increase in solubility of the calcium sulphate, with increasing concentration of the sodium sulphate, points to an increase in the concentration of complex ions and molecules, and an increasing disturbance of the simple equilibria existing in the pure solution. A limit would be reached when equilibrium becomes established between the solution and Glauberite, $Na_2Ca(SO_4)_2$, as solid phase.

The author has not considered the influence of sodium chloride on the solubility of calcium sulphate or calcium carbonate. The effect is very striking. Thus, at 360 deg. fahr., at which temperature the solubility of pure calcium sulphate is 125 p.p.m., when 1000 p.p.m. of sodium chloride are in the solution the solubility of the calcium sulphate is approximately 250 p.p.m., or twice as much; for 2000 p.p.m. of sodium chloride is 300 p.p.m., and for 5000 p.p.m. of sodium chloride has increased to 410 p.p.m. These data are found on page 64 of Bulletin 24. Doubtless before the author completes this work he will determine the effect of the addition of sodium chloride on the calcium-carbonate solubility relationships.

Basing their calculations upon the data of Johnston and Frear, which became available in 1929, both Mr. E. P. Partridge and Professor Straub find by extrapolation of these data that the solubility of calcium carbonate decreases with increase in the temperature of the solution. Professor Straub's experimental data also point to this conclusion. This is contrary to the data of Kendall, whose values were the most accurate available prior to publication of the work of Johnston and Frear. The fact that the solubility curve of calcium carbonate has a negative slope with increasing temperature explains what apparently has been an anomaly in boiler-scale formation. A boiler with any considerable percentage of make-up, and treated with sodium carbonate, usually becomes covered with a thin coating of true scale, which ordinarily keeps flaking or peeling off, and which moving by gravity to the mud drum or the blowdown header is largely removed by blowdown at that point, but of which some considerable mass is usually left at the time when the boiler is opened. The fact that the temperature-solubility curve of calcium carbonate has a negative coefficient places it in the class of the truly scale-forming substances, such as calcium sulphate or calcium hydroxide, and thus explains this formation of scale. The slope is slight, and hence the tendency for scale formation is not so marked as with a greater negative slope, but is none the less real.

The data now available therefore point to a negative solubility-temperature coefficient of the calcium-carbonate curve. As brought out by the equilibrium calculations of Partridge and as indicated in the experiments of Straub, in which static equilibrium is derived in experimentation in bombs, the ratio of carbonate to sulphate in the water necessary to repress formation of calcium sulphate is considerably less than the values we arrived at some years ago on the basis of the solubility data of Kendall. It will therefore be interesting to review the results when ratios of this type are applied to the conditions of dynamic equilibrium as they exist in the boiler waters.

In our earliest experimental work (1922), we ran a 42-day test on a boiler operating at 150 lb. pressure and using practically complete make-up of South Pittsburgh filtered water without treatment. We analyzed everything going to and coming from the boiler, and on this basis arrived at the fact that 89 per cent of all calcium sulphate going to the boiler was deposited there as scale.

At that time the fundamental theorem of scale prevention—namely, suitable control of the chemical equilibria at the evaporative surfaces—was unknown, and hence any maintenance of carbonate-sulphate ratios for the elimination of scale formation was not considered. Our next move, therefore, was to maintain a definite slight (5 to 15 g.p.g.) alkalinity in the boiler water. Under those conditions considerable sludge developed in the boiler water, but on the surfaces of the tubes calcium-sulphate scale was deposited. It contained very little carbonate and very little silica.

In the fall of 1923 and winter of 1924, on the basis of data then available, the carbonate-sulphate equilibrium ratios were worked out. They demonstrated that both sulphate concentration and operating pressures must be considered in any decision on the amount of chemical required for conditioning the water.

Treatment was then set up at the plant in question on the basis of these ratios, and has continued without interruption on that basis to the present time.

One of the first difficulties we met in application of these ratios was that on boiler waters colored by organic contamination we would find a sufficiency of carbonate to meet the carbonate-sulphate ratio, determinations being made by the titrations with phenolphthalein and methyl orange in the customary way, and yet on the surfaces of these boilers calcium-sulphate scale would be deposited. It was at this juncture that we developed the modified Winkler method of titration on boiler waters (Bulletin 24), since it avoided completely any effect of organic matter on the titration. We have used this method of determining the alkalinity relations in boiler waters from that time on.

Because of the decomposition of sodium carbonate into caustic in the boiler water and because elimination of scale by the use of sodium carbonate and its necessarily concomitant causticity has, so far as our experience goes, always been relatively uncertain, whether slight or high causticity and carbonate concentrations have been maintained, we were led in the end to prefer in all cases the use of phosphate as the final conditioning radical in the boiler water, since it is stable and stays in the water ready for use. The difference between carbonate and phosphate is the difference between a backfield defense man whose tackling is uncertain and one whose tackle is a death grip.

And then, too, it is our conception of a boiler water that the alkalinity therein should be the minimum compatible with freedom from corrosion and cleanliness of surfaces.

Just recently a plant operating at 400 lb. pressure for certain reasons changed temporarily from the use of phosphate to maintenance of a recommended alkalinity in the boiler water. Under control by phosphate and with from 30 to 50 per cent make-up, the evaporative surfaces had remained clean to the metal for many months, and there had been no tube loss. With return to caustic and carbonate as final conditioning chemicals, and with the maintenance consistently of a small concentration of alkalinity in the boiler water, tubes burned out within six weeks' time, and the scale that caused the burn-outs was calcium-sulphate scale.

Apparently the experimentation with static equilibrium in bombs is not in agreement with operation and the dynamic equilibria of boiler waters. We evidently must know more in order to bridge this hiatus.

Everett P. Partridge[11] **and W. C. Schroeder.**[12] The results outlined in the paper are of particular interest in connection with the study of the SO_4–CO_3 ratio for the prevention of sulphate scale made by the writers.[13] The author is to be congratulated on the opportunity afforded him to investigate solubility relations in complex solutions at elevated temperatures and on the exceptional facilities at his disposal. His method of attack does not allow him to obtain successive check samples from the same solution after varying periods of time, but if a sufficient number of duplicate determinations are made in each case, it should be possible to establish the equilibrium concentrations quite accurately, in spite of the analytical difficulties introduced by the low solubility of the different calcium salts at the higher temperatures. In this connection the writers hope that the author will eventually publish his original data in as complete a form as possible, since this will allow more intelligent interpretation of the results than is possible with the smooth curves given in the paper. In this type of work the data frequently show perverse inconsistencies, but these inconsistencies may lead to subsequent important conclusions.

The manner in which the author has presented his results may lead to misinterpretation in several cases, since he has indicated the solid and liquid phases added to his bombs, rather than those actually present at the time of sampling. In any case where Na_2CO_3 solution was added to an initial solid phase of $CaSO_4$, the equilibrium solid phase would contain $CaCO_3$, while the solution should contain chiefly Na_2SO_4, as he has himself noted in his Tables 2 and 3. The initial phases added in different runs are compared in Table 4 with those that must actually have been present if equilibrium were attained.

TABLE 4

Figure number	—Initial substances— Solid phase	Liquid phase	—Substances at equilibrium— Solid phase	Liquid phase
8	$CaSO_4$	Na_2CO_3	$CaSO_4$ $CaCO_3$	Na_2SO_4 Na_2CO_3
10	$CaSO_4$	NaOH	$CaSO_4$ $Ca(OH)_2$[a]	NaOH Na_2SO_4
11	$CaCO_3$	Na_2SO_4	$CaCO_3$ $CaSO_4$[a]	Na_2SO_4 Na_2CO_3
12	$CaCO_3$ $CaSO_4$	Na_2CO_3	Ca_2CO_3 $CaSO_4$	Na_2SO_4 Na_2CO_2
13	$CaCO_3$ $CaSO_4$	Na_2SO_4	$CaCO_3$ $CaSO_4$	Na_2SO_4 Na_2CO_3
14	$CaCO_3$ $CaSO_4$	NaOH	$CaCO_3$ $CaSO_4$ $Ca(OH)_2$[a]	NaOH Na_2SO_4

[a] Above certain concentrations as subsequently noted.

In the case of these figures where new substances should have appeared in the solid phase, interpretation of the results would depend upon complete analysis of the final solution and petrographic identification of the substances in the residual solid material.

The method of attack employed affords several opportunities for cross-checking results. For example, Figs. 8, 12, and 13 represent results obtained with systems which should have been almost identical if equilibrium had been attained and experimental error had been eliminated. In each case the final solid phase must have been $CaSO_4 + CaCO_3$, and the solution must have been chiefly Na_2SO_4, with a relatively low content of Na_2CO_3. Barring the possible effects of crystal form and particle size upon solubility, the results obtained when $CaCO_3$ was produced in the solid phase by precipitation should not differ from those obtained when $CaCO_3$ was initially present. In Figs. 8 and 12 the equilibrium was being approached from one direction, in Fig. 13 from the opposite direction, and the agreement is generally within the probable limits of experimental error. Examination of the original data might indicate even closer agreement than is shown in the three figures.

It would be expected that the addition of solid $CaCO_3$ to a system saturated with respect to $CaSO_4$ would have little effect on the equilibrium concentration of Ca, which would tend to be slightly higher, while the SO_4 would tend to be slightly lower than when the solid $CaCO_3$ was not present. Substantial agreement would therefore be expected between Figs. 5 and 7. This agreement is realized at the three lower temperatures. The writers believe that the point at 540 deg. fahr. in Fig. 7 is low and that further experiments will yield results closely in agreement with Fig. 5. Similarly, the close agreement between Fig. 9 and Fig. 8 would be expected, since the final conditions in the two cases differed only by the presence of $CaCO_3$ in the solid phase and of a low concentration of Na_2CO_3 in the liquid phase in Fig. 8. The original data upon which Figs. 8, 9, 12, and 13 are based might therefore be profitably considered together. The maximum discrepancy between values for corresponding molecular concentrations and temperatures in these four figures is only slightly greater than 0.1 millimols per liter, or 4 p.p.m., a range of variation which is quite possible and even probable in this type of work.

The initial drop in Ca concentration with increase in Na_2SO_4 concentration shown in Figs. 8, 9, 12, and 13 is quite in line with modern theories of solution, while the practically constant values for Ca at higher concentrations of added salt may also be interpreted satisfactorily by these theories. The drop in Ca concentration with increase in NaOH concentration shown by the curves in Fig. 10 is, however, difficult to understand until it is realized that at a concentration of about 2 millimols per liter of NaOH at 360 deg. fahr., and at progressively lower concentrations at higher temperatures, $Ca(OH)_2$ would appear in the solid phase. Further addition of NaOH would then merely produce $Ca(OH)_2$ in the solid phase and Na_2SO_4 in solution. The increasing concentration of Na_2SO_4 would tend to depress the solubility of $CaSO_4$, while the presence of an appreciable concentration of NaOH would tend to depress the solubility of $Ca(OH)_2$, resulting in Ca concentrations lower than those corresponding to the solubilities of the respective substances in pure water. Incidentally, $CaCO_3$ was undoubtedly present in addition to $Ca(OH)_2$ and $CaSO_4$ in the solid phase in Fig. 10 as well as in Fig. 14, unless the NaOH solution used had been specially prepared and protected to prevent absorption of CO_2 from the air.

Very little difference would be anticipated between Figs. 10 and 14, even if no $CaCO_3$ had been present in the solid phase in the former case. The apparent discrepancy between the curves for 360 deg. fahr. in the two figures may be attributed to experimental error, although it is impossible to guess which more nearly represents the truth.

In the course of our own work on solubility equilibria in boiler waters we have found a decided tendency for $CaCO_3$ to give high values in the absence of appreciable amounts of other dissolved salts. Due to the hydrolysis which $CaCO_3$ undergoes in contact with water, interpretation of results is only possible when a complete analysis is available. The values given in Fig. 6 are valuable as approximations, but are not significant as equilibrium data until the corresponding concentrations of OH, CO_3, and any other ions present in the solution as impurities and the concentrations of CO_2 in the vapor phase are known. The results of the author's equilibrium experiments will be awaited with great interest.

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[11] Non-metallic Minerals Experiment Station, U. S. Bureau of Mines, New Brunswick, N. J.

[12] Department of Engineering Research, University of Michigan, Ann Arbor, Mich.

[13] Paper on "The SO_4–CO_3 Ratio for the Prevention of Sulphate Boiler Scale," presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

In two of his groups of experiments, Figs. 8 and 12, the author produced $CaCO_3$ in the solid phase by the action of Na_2CO_3 solution upon solid $CaSO_4$. In Fig. 11 the reverse reaction was taking place, which accounts satisfactorily for the increase in Ca concentration with increasing concentration of Na_2SO_4 used as starting solution. As the Na_2SO_4 concentration was increased, the system would tend to approach saturation with respect to $CaSO_4$ as well as $CaCO_3$. An approximate calculation on the basis of some of our measurements indicates that at 360 deg. fahr. $CaSO_4$ probably first appeared in the solid phase when the initial solution contained about 12 millimols per liter Na_2SO_4. At the higher temperatures, saturation with respect to $CaSO_4$ would be reached at progressively lower initial concentrations of Na_2SO_4. The minima in the curves of Fig. 11 at low concentrations probably are not real, since the values in the absence of any added salt are likely to be high as a result of supersaturation.

This paper has presented briefly a great amount of data in generalized form. The writers feel that in attempting to simplify the presentation the author has perhaps offered too great an opportunity for misinterpretation. In considering boiler-water equilibria, particularly in previously unexplored regions, it is much safer and much more conclusive to measure what is taken out than what is put in. Since the paper states that the solution samples were completely analyzed, it should be possible, when the investigation is completed, to present the results on the basis of direct measurement. In representing solubility equilibria, particularly when the solution is complex, it is very convenient to plot the logarithms of stoichiometric solubility products for the saturating substance against the square root of the total ionic strength of the solution, as described in the paper by the writers. The writers have a strong predilection for petrographic determination of solid-phase constituents and would urge that this convenient and valuable tool be employed in cases where formation of a new solid is suspected.

With slight reservations, the writers agree with the conclusions drawn by the author. The second conclusion, that the solubility of calcium carbonate in the presence of sodium carbonate or hydrate is practically zero, may, however, be misinterpreted to mean that feeding soda ash equivalent to the Ca concentration of a feedwater will prevent scale. Tests of scale formation in an experimental boiler described in the writers' paper show that this is not so. In two tests when soda ash slightly more than equivalent to the Ca content of the feedwater was fed continuously, thin deposits of anhydrite scale were formed, although the Ca concentration in the boiler water was extremely low. Since definitely measurable, although small, concentrations of CO_3 were present at all times in the boiler water, this result necessitates modification of the author's fourth conclusion.

The solubility of $CaCO_3$ in the presence of a low concentration of Na_2CO_3 is not actually zero, but it is so low that accurate measurement is difficult. Our own work has shown that at 365 deg. fahr., corresponding to 150 lb. per sq. in. gage pressure, the safe operating SO_4–CO_3 ratio for the prevention of sulphate scale is at least 100 and may be appreciably greater, so that the maintenance of 10 p.p.m. of CO_3 in a boiler water should protect it against sulphate scale if the SO_4 concentration is below 1000. This CO_3 must be actually present, however. A great deal of unjustified criticism has been directed at internal conditioning with soda ash simply because careless sampling and unreliable titration methods for CO_3 indicated relatively high concentrations of this ion when it actually was present in very low concentrations. An accurate method for combined carbon dioxide such as that worked out by the writers may allow the use of soda ash as a conditioning chemical for boilers well above the pressure limit of 200 lb. per sq. in. set by Dr. Hall. The limiting factor will be the rate of hydrolysis of Na_2CO_3 at the higher temperatures.

One caution seems advisable in the utilization of equilibrium data for boiler-water control. In any actual boiler depositing material from solution under operating conditions, there must be a constant tendency toward supersaturation, which may result in an approximately steady state appreciably different from the equilibrium attainable in a closed system. Although equilibrium data are indispensable as the first step, the definition of practical operating limits in boiler-water control should be made on the basis of steaming tests in an experimental boiler, preferably an actual industrial boiler operated under competent control.

AUTHOR'S CLOSURE

The author appreciates very much the suggestions made by those who have discussed his paper. They all emphasize the fact that we still have a large amount of work to do before we can correlate theories of scale formation with the question of solubility of the calcium salts. The data which the author has collected will be published from time to time in detailed form, as suggested by Dr. Partridge. In the present paper the material had to be presented in a very condensed form.

The author would again like to call attention to the fact that the direct relation between solubility of the calcium salts and scale formation on the heating surfaces of steam boilers has not been established, and we may find that relative solubility is not a major factor in scale formation.

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Long Beach......Public Library
Los Angeles......Public Library
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Oakland.........Oakland City Library
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Pasadena........Library, California Institute of Technology
Santa Clara......Library, University of Santa Clara
San Diego........Public Library
San Francisco....Public Library (Civic Center)
Engineers Club of San Francisco
Mechanics Institute
Stanford University.........Library, Stanford University

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Denver..........Public Library
Fort Collins......Colorado State Agricultural College

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Bridgeport.......Public Library
Hartford.........Public Library
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Wilmington......Wilmington Free Institute

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Moscow..........University of Idaho

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Fort Wayne......Public Library
Indianapolis......Public Library
Indiana State Library
Notre Dame......Library, University of Notre Dame
Terre Haute......Rose Polytechnic Institute
West Lafayette...Library, Purdue University

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Des Moines......Public Library
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Lawrence........Library, University of Kansas
Manhattan.......Kansas State Agricultural College
Wichita..........Wichita City Library

Kentucky

Lexington........University of Kentucky
Louisville........Speed Scientific School
University of Louisville

Louisiana

Baton Rouge.....Louisiana State University
New Orleans.....Howard Memorial Library
Louisiana Engineering Society
Public Library

Maine

Orono............University of Maine

Maryland

Annapolis........United States Naval Academy
Baltimore........Johns Hopkins University
Engineers Club of Baltimore
Public Library

Massachusetts

Boston...........Northeastern University
Boston Public Library
Cambridge.......Harvard University (Engineering Library)
Massachusetts Institute of Technology
Fall River........Public Library
Lowell...........Lowell Textile Institute
Lynn............Free Public Library
New Bedford.....Free Public Library
Springfield.......Springfield City Library
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Worcester........Worcester Polytechnic Institute
Free Public Library

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Detroit..........Public Library
Cass Technical High School
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East Lansing.....Michigan State College
Flint............Public Library
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Minneapolis......University of Minnesota
Minneapolis Public Library (Engineering and Circulating Libraries)
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Columbia........University of Missouri
Kansas City......Public Library
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St. Louis.........Engineers Club of St. Louis
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Camden	Free Public Library
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Newark	Newark College of Engineering Free Public Library
New Brunswick	Rutgers University
Paterson	Free Public Library
Princeton	Princeton University
Trenton	Free Public Library

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Brooklyn	Polytechnic Institute Pratt Institute Brooklyn Public Library
Buffalo	The Grosvenor Library Engineering Society of Buffalo Buffalo Public Library
Ithaca	Cornell University
Jamaica, L. I.	Queens Borough Public Library
New York	Engineering Societies Library Public Library College of the City of New York Cooper Union Columbia University N. Y. University Library
Potsdam	Clarkson College of Technology
Rochester	Rochester Engineering Society
Schenectady	Union College
Syracuse	Syracuse University Public Library
Troy	Rensselaer Polytechnic Institute
Utica	Public Library
Yonkers	Public Library

North Carolina

Chapel Hill	University of North Carolina (Engineering Library)
Raleigh	North Carolina State College

North Dakota

Fargo	North Dakota State College
Grand Forks	University of North Dakota

Ohio

Ada	Ohio Northern University
Akron	Public Library University of Akron
Canton	Public Library
Cincinnati	University of Cincinnati Public Library Engineers Club of Cincinnati
Cleveland	Public Library Case School of Applied Science Cleveland Engineering Society
Columbus	State of Ohio Library Public Library Ohio State University
Dayton	Engineers Club of Dayton
Toledo	Public Library University of Toledo
Youngstown	Public Library

Oklahoma

Norman	Oklahoma University
Oklahoma City	Public Library
Stillwater	Oklahoma Agricultural and Mechanical College
Tulsa	Public Library

Oregon

Corvallis	Oregon State Agricultural College
Portland	Portland Library Association

Pennsylvania

Allentown	Free Library
Bethlehem	Lehigh University
Easton	Public Library Lafayette College
Erie	Public Library
Lewisburg	Bucknell University
Philadelphia	Engineers Club Drexel Institute University of Pennsylvania Franklin Institute
Pittsburgh	University of Pittsburgh Engineers' Society of Western Pennsylvania Carnegie Institute of Technology Carnegie Library (Schenley Park) Carnegie Free Library of Allegheny
Reading	Public Library
Scranton	Public Library
State College	Pennsylvania State College
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Villanova	Villanova College
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Clemson College	Library, Clemson College

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Kingsport	Public Library
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Memphis	Goodwin Institute
Nashville	Vanderbilt University

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Austin	University of Texas
College Station	Agricultural & Mechanical College of Texas
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Virginia

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Piezometer Investigation

By CHARLES M. ALLEN[1] and LESLIE J. HOOPER,[2] WORCESTER, MASS.

C. M. Allen

This investigation had for its object the determination of a standard piezometer for commercial use and the study of the hydraulic effect of various conditions of operation upon different forms of piezometers. Piezometers were compared in operation with suitable reference orifices, the differentials being measured with hook gages. A number of factors concerning the performance of piezometer orifices were analyzed. The paper gives results covering a variety of forms and conditions and should assist in establishing a standard form of piezometer.

L. J. Hooper

THE object of this investigation was (1) to determine a stable type of piezometer suitable for use as a standard in commercial work, and (2) to determine those factors which have the greatest bearing in obtaining correct results.

The experimental work was conducted at the Alden Hydraulic Laboratory of Worcester Polytechnic Institute. The general layout of the apparatus may be seen in Fig. 1. A 12-in. pipe line was used for this work, as suitable water velocities, ease of regulation, and uniformity of flow and pressure conditions were readily obtained. This line is connected to the main 40-in. penstock above a 36-in. by 16-in. venturi meter and below the Johnson differential surge tank. The 40-in. penstock, which is about 400 ft. long, takes its water from the second or middle pond and supplies a 100-hp. water wheel under 30 ft. head in the main laboratory.

From Fig. 1 it is seen that the 12-in. line branches off from the main penstock and runs horizontally about 33 ft. across the laboratory.

Then it rises 4.2 ft. and again runs horizontally 34 ft. at right angles to its former course to a 12-in. by 6-in. venturi meter. At the end of the venturi meter is a regulating valve, and from there the water discharges either into a waste way or, for calibration purposes, into a weighing tank.

At the entrance to the 12-in. line two $^3/_8$-in. mesh screens were placed to prevent leaves and sticks from entering the pipe and causing trouble with the test piezometers. As the direction of the current of water in the main penstock was across these screens, they cleaned themselves every time the 12-in. line was shut down.

For this work smooth water-flow conditions are essential. In Fig. 1 it is seen that there are two right-angle bends in the pipe above the test section. These bends were in planes normal to each other. This condition would tend to give a whirl to the water that might seriously affect the accuracy of the measurements. Therefore a water-flow straightener was placed in the pipe just below the second elbow. This was a sheet-metal vane 3 ft. long that divided the pipe into quadrants. This effectively checked any tendency of the water to whirl. To further quiet the water, stilling screens, which consisted of two $^1/_4$-in. mesh screens close together with the wires at 45 deg. to each other were placed in the pipe at the flange joint upstream from the test section.

The test section was located about 6 ft. upstream from the venturi meter and 0.4 ft. upstream from the end of the 18-ft. section of pipe. In Fig. 1 it is seen that the water straightener was 29.6 ft. and the stilling screens 17.9 ft. upstream from the piezometer section. In order to have the pipe wall and hence the surface flow as smooth as possible, the pipe was scraped for a distance of 7 ft. upstream from the test section and brushed with a stiff wire brush for 3 ft. upstream.

At the test section 12 holes were drilled radially at equal intervals around the pipe and tapped for a $^3/_8$-in. pipe thread. The orifices were made up in brass plugs which were screwed into these holes. After being put in place the inside surface of the plug was filed and scraped flush with the wall of the pipe. (See Fig. 2.)

Since there is no independent method of measuring pressure head in a pipe other than piezometers, it was necessary to choose arbitrarily comparison or reference orifices. Four of these reference orifices were installed on two diameters at right angles to each other. These orifices had $^1/_8$-in. holes, square edges, and surfaces flush with the penstock wall. Other piezometers to be compared were installed in the intermediate holes, as shown in Table 1.

The elevation of the pressure head at the test section was several feet above the roof of the building. The small differences in the readings of the various piezometer orifices made the use of sensitive hook gages imperative. Therefore a differential-pres-

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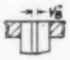
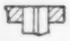
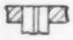
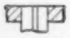
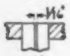
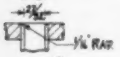
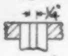
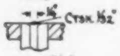
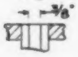
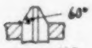
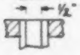
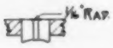
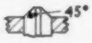
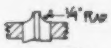
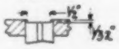
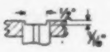
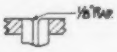
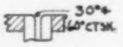
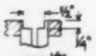
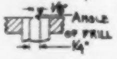
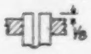
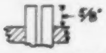
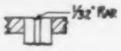
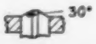
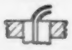
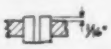
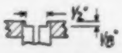
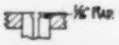
[1] Professor of Hydraulic Engineering, Worcester Polytechnic Institute. Mem. A.S.M.E. Professor Allen was born at Walpole, Mass., on Dec. 12, 1871. He received his B.S. in Mechanical Engineering from Worcester Polytechnic Institute in 1894, his M.S. in 1899, and his D.Eng.Hon. in 1929. He has been on the teaching staff of the institute in part-time capacity since his graduation, for the past 20 years serving as Professor of Hydraulic Engineering. Professor Allen has been engaged for 30 years in consulting and testing-engineering practice in paper-mill and water-power lines, making power and efficiency tests throughout the United States and Canada. He has done considerable research work in connection with the friction of gears, rating of all forms of water-measuring apparatus, and, in recent years, hydraulic model testing. He is a Vice-President of the Society.

[2] With Prof. C. M. Allen, Worcester Polytechnic Institute. Jun. A.S.M.E. Mr. Hooper was born in Essex, Mass., on Feb. 15, 1903. He received his B.S. degree in Mechanical Engineering at Worcester Polytechnic Institute in 1924 and M.E. degree from the same institution in 1929. The first three years after graduation he was engaged in hydraulic test work in Brazil, and for the past four years has been engaged in hydraulic research and commercial work at the Alden Hydraulic Laboratory of the Worcester Polytechnic Institute, and has just been appointed a part-time Instructor at the institute.

Contributed by the Hydraulic Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 PIEZOMETERS TESTED

Piez No.		Description
1		Square edged 1/8" orifice used as a reference.
2		Square edged 1/8" orifice used as a reference.
3		Square edged 1/8 orifice used as a reference.
4		Square edged 1/8 orifice used as a reference.
5		Square edged 1/16" orifice.
5A		Piezometer with 27/32" orifice and 1/16" radius of rounding.
6		Square edged 1/4" orifice
6A		Burr removed by countersinking with large drill point.
7		Square edged 3/8" orifice.
7A		Conical piezometer of 60 degrees.
8		Square edged 1/2" orifice.
10		Piezometer with 1/8" orifice and 1/16" radius of rounding.
10A		Conical piezometer of 45°. 1
10B		Modification of conical piezometer.
10C		Piezometer recessed 1/32"
10D		Piezometer recessed 1/16"
11		Piezometer with 1/8" orifice and 1/8" radius of rounding.
11A	See Fig. 14	Inserted tube piezometer. Similar to a pitot tube but lacking the impact orifice. Tube 5" long and 1/2" o.d. Four 1/8" square edged orifices 3" from point.
11B		Burr removed by two light countersinking operations.
11C		Piezometer recessed 1/4"
11D		Piezometer counterbored from inside
12		Piezometer plug projecting 1/8"
12A		Piezometer plug projecting 5/8"
12B		Piezometer with 1/8" orifice and 1/32" radius of rounding
12C		Conical piezometer of 30 degrees.
12D		Similar to trailing Pitometer orifice.
13		Piezometer plug projecting 1/16"
13A	See Fig. 14	Teaball piezometer. One inch sphere on end of a 1/8" pipe. No. 60 holes drilled all over the ball at regular intervals.
13B		Piezometer plug recessed 1/8"
13C		Cup shaped recess in piezometer tube.

Note- All piezometers have 1/8" orifice except where noted.

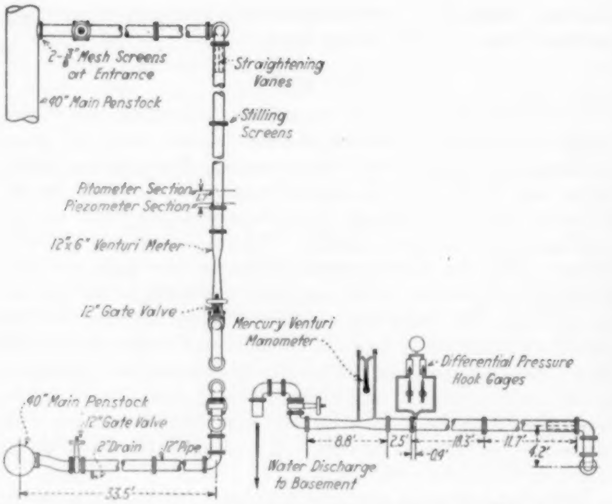

2-⅛" Mesh Screens at Entrance
40" Main Penstock
Straightening Vanes
Stilling Screens
Pitometer Section
1.7'
Piezometer Section
12"x 6" Venturi Meter
12" Gate Valve
Mercury Venturi Manometer
Differential Pressure Hook Gages
40" Main Penstock
12" Gate Valve
2" Drain
12" Pipe
33.5'
Water Discharge to Basement
8.8'
2.5'
0.9'
18.3'
11.7'
4.2'

Fig. 1 Layout of Apparatus in Comparison of Piezometer Orifices

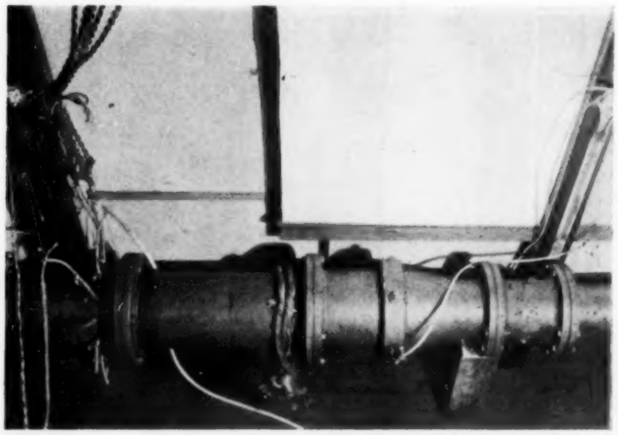


Fig. 2 Piezometer Section and 12-In. by 6-In. Venturi Meter

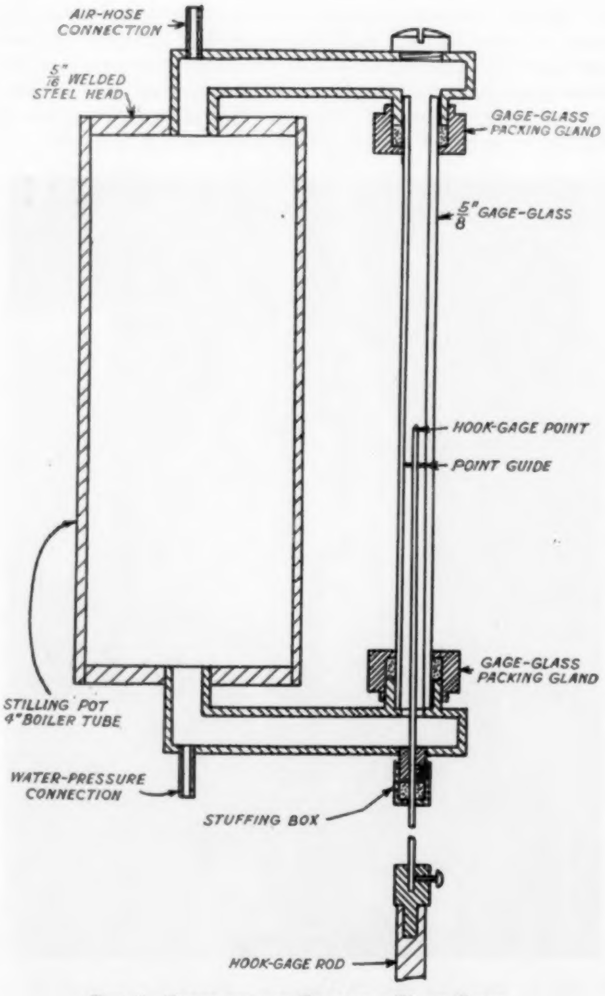

AIR-HOSE CONNECTION
5/16" WELDED STEEL HEAD
GAGE-GLASS PACKING GLAND
5/8" GAGE-GLASS
HOOK-GAGE POINT
POINT GUIDE
GAGE-GLASS PACKING GLAND
STILLING POT 4" BOILER TUBE
WATER-PRESSURE CONNECTION
STUFFING BOX
HOOK-GAGE ROD

Fig. 3 Differential-Pressure Hook Gage

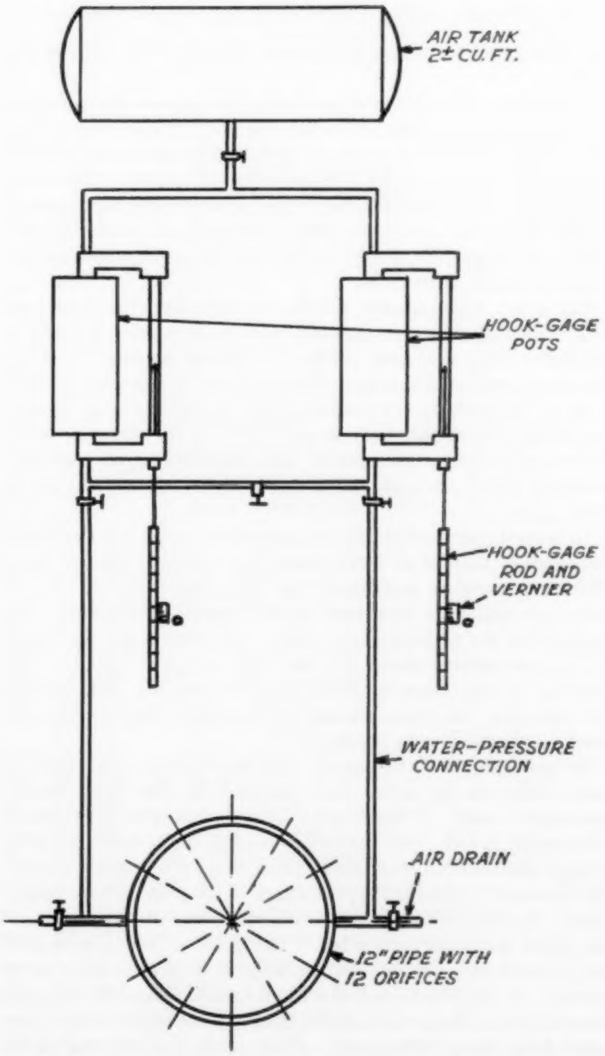

AIR TANK 2± CU. FT.
HOOK-GAGE POTS
HOOK-GAGE ROD AND VERNIER
WATER-PRESSURE CONNECTION
AIR DRAIN
12" PIPE WITH 12 ORIFICES

Fig. 4 Connections for the Differential-Pressure Hook Gages

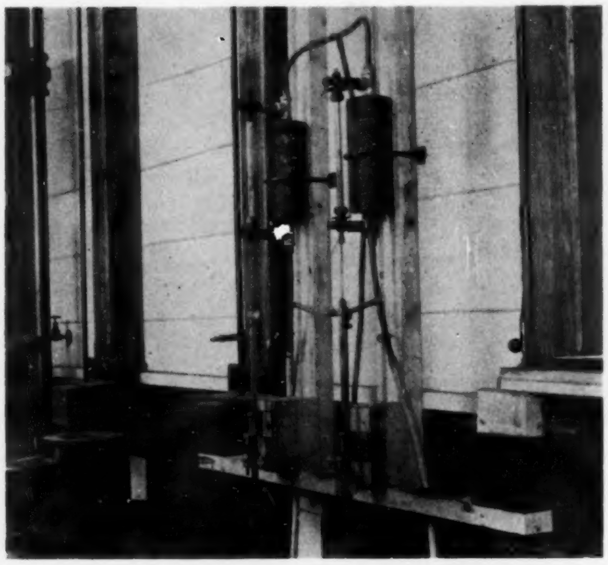


FIG. 5 DIFFERENTIAL-PRESSURE HOOK-GAGE MANOMETER

sure manometer with two hook gages was made up for this test work.

A sectional view of one of the head gage pots is shown in Fig. 3. The stilling pot was made up of a piece of 4-in. boiler tube with two heads welded in, $^1/_2$-in. pipe fittings being used to connect regular water-column fittings to the stilling pot. The lower water-glass fitting was equipped with a small stuffing box, through which passed the hook-gage point. About 1 in. down from the point a small guide was soldered so that the point was kept in a central position in the tube.

Fig. 4 is a diagrammatic sketch of connections for these gage pots. Two of these instruments were mounted side by side on a suitable base, as shown in Fig. 5. The air connections at the top were joined, and a common line went into an air tank of about 2 cu. ft. capacity. An ordinary tire pump was used to raise the pressure in the air tank so as to balance the water pressure. The water-pressure connections were first tied by an equalizer, and then passed through valves to the orifice connections on the 12-in. pipe.

In normal operation all the air passages were open, so that equal air pressure existed in both hook-gage pots and the air tank. The water-pressure equalizing tube was closed and the water-pressure tubes were connected to the desired piezometers. Between runs the water-pressure tubes were closed and the water-pressure equalizing connection was open so as to obtain a zero reading. From time to time the zero reading was checked by removing the gage glasses and laying a machinist's spirit level on the hook-gage points.

Before trying out the gages it was feared that there might be some difficulty in seeing the "pimple" on the water surface through the glass. It was found that not only was the point clearly observable in the usual manner, but that there were two other equally accurate ways of taking the reading of the water surface. By observation from below, the image of the point is very plainly seen. As the point approaches the water surface the image and the point approach each other. The point at which they meet can be seen very clearly, and the water level is accurately determined. In the third method a small frosted lamp bulb was held directly behind the water column, throwing the point and the meniscus into strong silhouette. Here again the meeting of the point with the water surface is very clearly seen. Fig. 6 shows the meniscus and the point at time of reading according to the third method. All three of these methods were compared for accuracy and there was no choice among them.

PROCEDURE

In preparation for a run the air was first blown out of the venturi-meter connections and the zero reading was checked on the mercury U-tube of the venturi meter. Then the regulating valve was opened until the desired flow was obtained in the 12-in. pipe. In the meantime the water-equalizing valve on the hook-gage pots had been opened so that a zero reading could be taken. Then the water-pressure hose from each gage was connected to the desired orifices, and care was taken to remove the air all along the water-pressure line. Before opening the line valves wide, the air pressure in the tank was raised until it balanced the water pressure. After the line valves were opened to insure stable hydraulic conditions, 10 to 15 min. elapsed before taking any readings. Then two check readings were taken with about a 2-min. interval between. A reading of the venturi meter was recorded at the same time.

With the readings of an orifice completed, No. 2 hook gage was connected to the next orifice to be read, while No. 1 was left connected to the main reference orifice throughout the tests. As each orifice was connected the air was blown out of the tubing before starting to take a reading.

Following a run, the flange bolts were removed and that section of pipe containing the piezometer section was revolved 180 deg., and another run was made at the same velocity and using the same reference orifice. Since the water-flow straighteners and stilling screens were not moved, it seems safe to assume that the water came down the pipe in much the same way for both runs. By averaging the results of the two similar runs the effect of oblique or turbulent flow on the piezometers was removed.

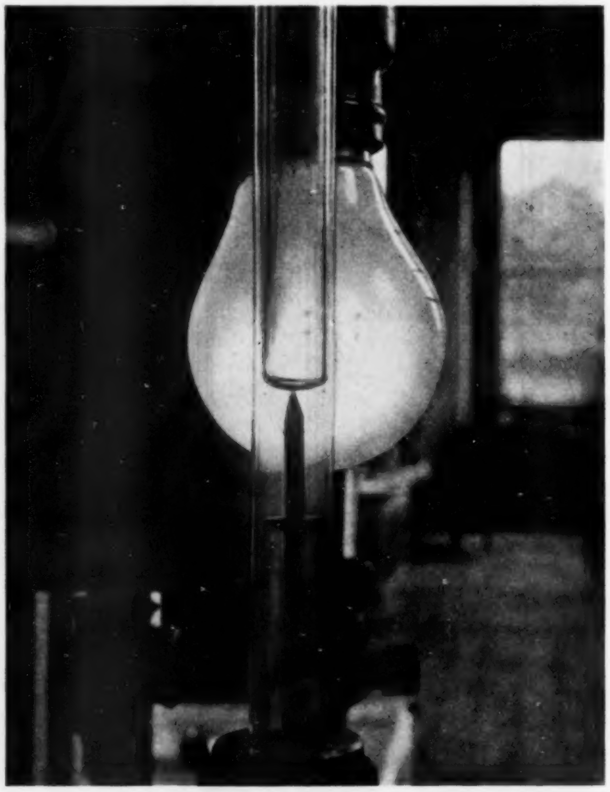


FIG. 6 READING THE DIFFERENTIAL-PRESSURE HOOK-GAGE MANOMETER

At the start, the reference orifices did not agree as well as might be expected. During the course of tests the four reference piezometers were improved until at the end of 19 runs they all read practically the same when they were located at the same point with respect to the flow. After six more runs had been made, the procedure in regard to revolving the pipe was abandoned, inasmuch as the same answer was obtained by averaging the reading of the reference piezometers.

DATA AND COMPUTATIONS

Three sample data sheets, which contain the data for one complete set of runs, are shown in Tables 3, 4, and 5. The computations were made as follows, the first set of readings in Table 3 being used as an example:

Difference in level = reference reading — test reading — zero correction
= 1.480 — 1.379 — 0.108 = —0.007

Mean velocity head = computed from venturi deflection
= $V^2/2g$
= $Q^2/A_1^2 2g$
= (venturi constant)^2 (deflection)/(A_1^2 $2g$)
= (5.66)^2 (0.95)/(0.6168) (64.4)
= 0.81 ft. of water.

Deviation = difference in level/velocity head
= —0.007/0.81
= —0.9 per cent of the mean velocity head

RESULTS

Table 2 presents a summary of all the results that were obtained in this piezometer study. The more obvious conclusions indicated by this study are described in detail.

Care in Making Connections. The utmost care is necessary in removing air and leaks from all pressure connections. In starting the tests, the zero correction by water level did not check that

TABLE 2 RESULTS OF PIEZOMETER STUDY

Run No	Mean Pipe Velocity ft/sec	Piezometer Error in Percent of Mean Velocity Head												Remarks
		#1	#2	#3	#4	#5	#6	#7	#8	#10	#11	#12	#13	
4-5	8.5	REF	-0.7	-0.8	-1.8	0.0	0.2	0.2	-0.3	-0.5	3.0	-9.6	-7.5	Pipe scraped 7 ft and brushed 3 ft upstream Standards (Nos 1,2,3 and 4) Painted
6-8	7.3		0.6	0.6	1.7	0.2	1.4	1.0	1.2	1.6	3.1	-10.9	-7.9	Sandpapered paint from standards and cleaned edges with a match
7-9	4.0		-0.2	0.0	-2.2	-1.4	0.3	0.2	-1.8	-0.3	1.3	-9.4	-7.5	
10-11	7.0		-0.3	-0.3	-2.1	0.2	0.7	-0.3	-0.1	-0.8	2.7	-10.5	-7.3	Stilling screens installed 18 ft upstream
12-14	7.2		0.1	0.1	-1.1	-0.3	0.4	0.0	0.9	-0.1	+2.2	-10.0	-6.4	Piez. Nos. 1,2,3 and 4 squared and polished under microscope. Nos. 5,6,7 and 8 squared No 10 smoothed and rescraped
13-15	4.0		-0.9	-0.5	-1.0	-0.4	0.2	0.0	-0.3	0.4	2.0	-9.1	-6.9	
16-18	7.3		0.1	0.1	-0.3	-0.2	0.2	-0.2	-0.2	-0.3	2.0	-11.0	-6.9	No. 4 filed down flush and squared
17-19	4.0		0.4	0.1	-0.3		-0.2	0.0	0.0	0.0	1.4	-9.0	-7.9	
22-24	7.2		-0.1	4.0	3.5	3.8	5.6	5.0	5.4	5.3	8.3	-4.0	-0.5	Tubercles placed 1¼" on ℄ in front of all orifices except No 1 and No 2. Tubercles ¼" high and 1¼" in diameter.
23-25	4.0	REF	0.6	3.0	4.2	1.8	6.4	5.6	4.9	5.9	8.2	-3.1	-0.2	
26	4.0	-0.8	REF	0.8	0.9	-0.4	2.0	1.2	1.2	0.0	2.0	-9.2	-7.2	As above
27	7.2	-0.6		0.2	0.1	2.1	1.7	1.5	1.0	0.2	2.3	-9.8	-7.2	Tubercles: 5/8" OD x 1/8" high
28	4.0	-0.2		1.0	0.8	1.0	1.6	1.2	1.2	0.4	2.5	-8.0	-5.8	Second row of tubercles added at 2¼" ℄ Size ¾" OD x 3/16" high
29	7.2	-0.1		0.4	0.6	0.5	1.7	1.1	1.1	0.5	3.1	-8.5	-5.7	
30	7.2	0.2		-0.4	0.4	1.7	1.6	0.5	0.2	-0.3	2.1	-8.6	-5.8	Two more rows of tubercles added at ℄ distances of 3¾" and 5¼" — Total 4 rows Size 1" x ¼"
31	4.0	0.0		-0.3	0.8	1.6	1.7	0.7	0.1	-0.2	2.2	-8.8	-6.4	
32	7.2	0.1		-0.9	0.1	1.5	2.3	0.6	0.2	-0.5	3.1	-8.5	-5.9	Added two more rows at ℄ distances of 5¾" and 7¼". Total of 6 rows Size - 1" x ¼"
33	4.0	-0.4		-0.8	0.2	0.9	1.9	0.6	-0.4	-0.4	2.6	-8.0	-4.8	
34	7.2	-0.6		-1.2	-0.8	1.6	1.4	0.5	-0.8	-1.5	-1.1	-9.8	-7.7	Small tubercles in first row removed from in front of orifice Nos 3,4,8,10,11,12 + 13
35	4.0	0.0	REF	-0.8	-0.4	-0.5	2.0	1.8	-0.3	-0.9	0.9	-9.2	-8.8	
36	7.2	2.7	2.9	-0.6	REF	2.4	3.0	2.5	0.5	-0.5	1.7	-9.0	0.1	Six rows of tubercles arranged as above but orifice Nos 3,4,8,10,11 + 12 left free
37	4.0	3.2	2.4	-0.4	REF	1.6	3.0	3.1	0.8	0.0	1.2	-8.8	0.5	
38	7.2	-0.4	REF	-1.1	-2.3	0.9	0.4	-0.5	-2.4	-2.3	-0.3	-3.5	-0.7	Strips of ¼" mesh screen over all orifices except Nos 1 + 2
39	4.0	0.0		-0.8	-2.4	-1.6	0.6	0.0	-1.6	-2.0	0.4	-3.2	-0.4	
40	7.2	-3.5		0.0	-3.5	-2.6	-0.6	0.7	4.0	1.6	0.0	-11.8	-13.7	Whirling water. Vanes 4" x ¾" fastened on pipe at an angle of about 40° Orifice Nos 1,2,5,6 + 13 left free
41	4.0	-3.2		0.0	-2.4	-3.7	-0.6	1.2	3.2	2.0	-0.4	-9.6	-15.4	
42	7.0	0.0		-1.0	-0.8					-43.7	0.3	-34.4	-86.0	Smooth water. No. 10 - 45° cone No. 11 - inserted tube. No 12 - 5/8" plug projecting 1/8" No. 13 - teaball
43	7.0	-1.2		0.1	-3.2					-42.9	-0.6			
44	7.0	0.4	REF	-0.9	-0.3					-32.8	-1.0	-1.2		No. 10B No. 11B No. 12B No. 13A
45	7.0	-5.0	15.0	13.5	9.8	13.7	14.6	15.6	11.9	-22.9	7.4	6.0	REF	Whirling water - 4" x 1" plates attached to pipe wall at an angle of 35° to axis. End of vane 4" upstream from piezometers.
46	3.95	-4.9	11.1	12.2	17.9	10.3	8.7	11.1	9.6	-14.7	13.9	18.0		
47	4.0	18.0	20.8	17.2	16.8	20.8	18.4	18.8	20.8	-44.0	20.0	17.6		Six bladed spinner placed in pipe 1.7 ft upstream from piezometers.
48	7.0	12.1	17.5	12.3	13.2	15.8	13.5	14.0	16.2	-52.2	13.0	13.6	REF	
49	7.05	-30.4	REF	-18.5	-0.6	-2.9	0.5	-4.4	-18.8	-50.0	-1.6	-3.5		Butterfly valve 0.5 ft upstream on ℄.
50	7.0	6.0		7.9	-0.5	-0.6	0.8	-3.2	0.2	-25.9	-2.1	-2.8		Butterfly valve 1.0 ft upstream on ℄
51	7.0	1.2		-6.8	-11.4	4.8	2.2	-1.8	-1.4	-49.0	-12.7	-15.8	1.9 ℄	Butterfly valve 1.5 ft upstream on ℄.
52	7.0	-1.8		-0.9	-1.0		0.4			3.3				No. 6A. No. 10C No. 11 backed out
53	7.0	0.0		-1.9	-1.5	0.4	0.5	-41.0	-1.7	-1.8	-3.3	-34.2	-3.0	No 5A. No 7A. No 10D No 11C No 12C No 13B
54	7.0	0.1	REF	-0.1	-0.7						0.6	-13.6	-2.5	No 13C No 12D No 11D

Note - Ref is the reference or comparison orifice for the indicated series of tests.

TABLE 3 SAMPLE DATA TEST SHEET NO. 9

(Piezometer Tests, March 6, 1928)

Zero Readings:	No. 1	1.583	1.584	1.531
	No. 2	1.475	1.476	1.422
	Diff.	0.108	0.108	0.109

Piezometer No.	Reference reading	Test reading	Venturi deflection	Difference of level	Mean velocity, H	Deviation, per cent	Remarks
4	1.480	1.379	0.95	—0.007	0.81		[a]
	80	80		—0.008		—0.9	
3	1.478	1.377	0.94	—0.007	0.80		[b]
	77	77		—0.008		—0.9	
2	1.529	1.423	0.96	—0.003	0.82		
	32	27		—0.004		—0.4	
5	1.524	1.414	0.95	0.001	0.81		
	23	13		0.001		0.1	
6	1.535	1.425	0.96	0.001	0.82		
	33	23		0.001		0.1	
7	1.521	1.419	0.97	—0.007	0.83		
	20	18		—0.007		—0.8	
8	1.514	1.412	0.96	—0.007	0.82		
	11	09		—0.007		—0.8	
10	1.504	1.406	0.96	—0.011	0.82		
	02	05		—0.012		—1.4	
11	1.492	1.375	0.96	0.008	0.82		
	91	73		0.009		1.0	
13	1.456	1.415	0.96	—0.068	0.82		
	54	13		—0.068		—8.1	
12	1.424	1.405	0.96	—0.090	0.82		
	24	05		—0.090		—11.0	
4	1.481	1.373	0.30	—0.001	0.25		[c]
	66	59		—0.002		—0.6	
3	1.4465	1.340	0.30	—0.0025			
	435	37		—0.0025		—1.0	
2	1.426	1.317	0.30	0.000			
	24	15		0.000		0.0	
5	1.374	1.280	0.30	—0.015			
	74	80		—0.015		—6.0	
7	1.352	1.244	0.30	—0.001			
	51	43		—0.001		—0.4	
6	1.328	1.220	0.30	—0.001			
	26	18		—0.001		—0.4	
8	1.356	1.249	0.292	—0.002			
	54	46		—0.001			
	52	45		—0.002		—0.8	
10	1.450	1.343	0.30	—0.002			
	485	42		—0.0025		—0.9	
11	1.434	1.323	0.30	0.002			
	34	23		0.002		0.8	
13	1.4355	1.347	0.30	—0.021			
	365	48		—0.021		—8.4	
12	1.456	1.370	0.30	—0.023			
	58	72		—0.023		—9.2	

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[a] No. 1 up. Run No. 16.

[b] No. 4 leveled and carefully squared. Found surface projected 0.005 in.

[c] No. 1 up. Run No. 17. Conditions as for No. 16.

found by the machinist's level by 0.002 ft. A bubble of air about the size of a pea was worked out of one of the pipes, and thereafter the readings were checked. In another case the readings of one test on piezometer No. 5 with an opening of $^1/_{16}$ in. were lower by 0.005 ft. than those of a previous test under identical conditions. The trouble was found to be a leak in a pressure-tube connection, and the quantity of water escaping was only one drop per second.

Expression of Error. An examination of the results, Table 2, indicates that the error of any piezometer remained constant when expressed as a per cent of the velocity head. Runs were made at two velocities, and for like conditions the percentage errors remained constant within the limits of accuracy of measurement. As might be expected, the error of a piezometer is caused by the local water velocity passing by the orifice, and the error therefore is correctly expressed as a per cent of the local velocity head at the piezometer. A partial traverse made with a teaball piezometer proved this to be a fact. But due to the difficulty

TABLE 4 SAMPLE DATA SHEET NO. 10

(Piezometer Tests, March 6, 1928)

Zero Readings:	No. 1	1.469	1.470	1.470
	No. 2	1.361	1.361	1.361
	Diff.	0.108	0.109	0.109

Piezometer No.	Reference reading	Test reading	Venturi deflection	Difference of level	Mean velocity, H	Deviation per cent	Remarks
4	1.543	1.432	0.97	0.002	0.83		[a]
	42	31		0.002		0.2	
3	1.531	1.413	0.97	0.009			
	28	09		0.010		1.1	
2	1.512	1.398	0.97	0.005			
	10	96		0.005		0.6	
5	1.490	1.384	0.97	—0.003			
	89	83		—0.003		—0.4	
6	1.474	1.361	0.97	0.004			
	70	58		0.003		0.4	
7	1.452	1.3385	0.97	0.0045			
	50	37		0.004		0.5	
8	1.434	1.321	0.97	0.004			
	32	19		0.004		0.5	
10	1.416	1.300	0.97	0.007			
	15	1.299		0.007		0.9	
11	1.406	1.272	0.97	0.025			
	04	70		0.025		3.0	
13	1.373	1.317	0.97	—0.047			
	72	16		—0.047		—5.7	
12	1.341	1.313	0.97	—0.091			
	39	11		—0.091	0.83	—11.0	
2	1.455	1.344	0.30	0.002	0.25		[b]
	54	43		0.002		0.8	
3	1.455	1.343	0.30	0.003			
	55	43		0.003		1.2	
4	1.430	1.321	0.30	0.000			
	27	18		0.000		0.0	
5	1.451	1.339	0.30	0.003			
	51	40		0.002		1.0	
6	1.468	1.359	0.30	0.000			
	67	58		0.000		0.0	
7	1.468	1.3585	0.30	0.001			
	69	59		0.001		0.4	
8	1.472	1.362	0.30	0.001			
	71	61		0.001		0.4	
10	1.473	1.362	0.30	0.002			
	73	62		0.002		0.8	
11	1.477	1.363	0.30	0.005			
	77	63		0.005		2.0	
13	1.439	1.348	0.30	—0.018			
	38	47		—0.018		—7.3	
12	1.412	1.325	0.30	—0.022			
	12	25		—0.022		—8.7	

---

[a] No. 3 up. Run No. 18. Conditions as in No. 16.

[b] No. 3 up. Run No. 19. Conditions as in No. 16.

TABLE 5 SAMPLE DATA SHEET NO. 11

(Piezometer Tests, April 7, 1928)

Zero Readings:	No. 1	1.584	1.570
	No. 2	1.478	1.464
	Diff.	0.106	0.106

1	1.433	1.327	0.93	0.000	0.79		[a]
	33	27		0.000		0.0	
3	1.433	1.336	0.93	—0.009	0.79		
	33	36		—.0009		—1.0	
4	1.455	1.355	0.94	—0.006	0.80		
	56	56		—0.006		—0.8	
10	1.337	1.579	0.93	—0.345	0.79		[b]
	37	79		—0.345		—43.7	
12	1.373	1.546	0.95	—0.279	0.81		
	72	45		—0.279		—34.4	
11	1.545	1.437	0.95	0.002	0.81		[c]
	45	37		0.002		0.2	
13	1.133	1.696	0.91	—0.671	0.78		[d]
	33	96		—0.671		—86.0	

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[a] No. 1 up. Run No. 42. Conditions as in No. 16.

[b] $^1/_8$-in. hole, but with 45 beveled sides. Plug projecting $^5/_8$ in.

[c] Pitot tube rated at center of pipe.

[d] Tea-ball piezometer at center of pipe.

of determining the actual velocity of the water immediately adjacent to the pipe wall, except where noted the piezometer errors in this report are expressed in terms of the mean velocity head existing at the test section. (See Figs. 7 and 8.)

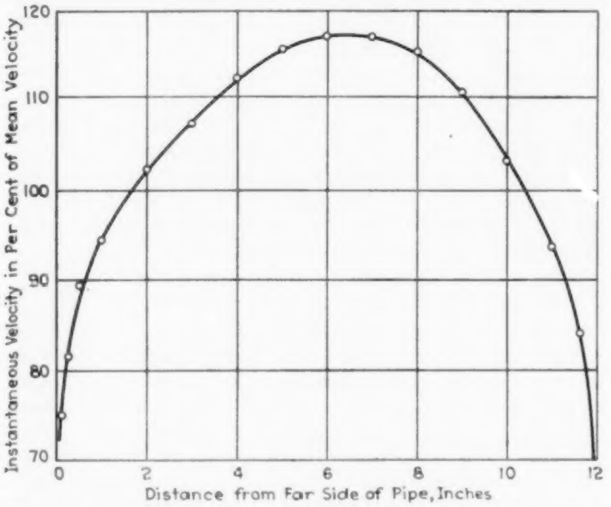

FIG. 7 MEAN OF FOUR PITOMETER TRAVERSES OF THE 12-IN. PIPE LINE
(Pipe factor, —0.85.)

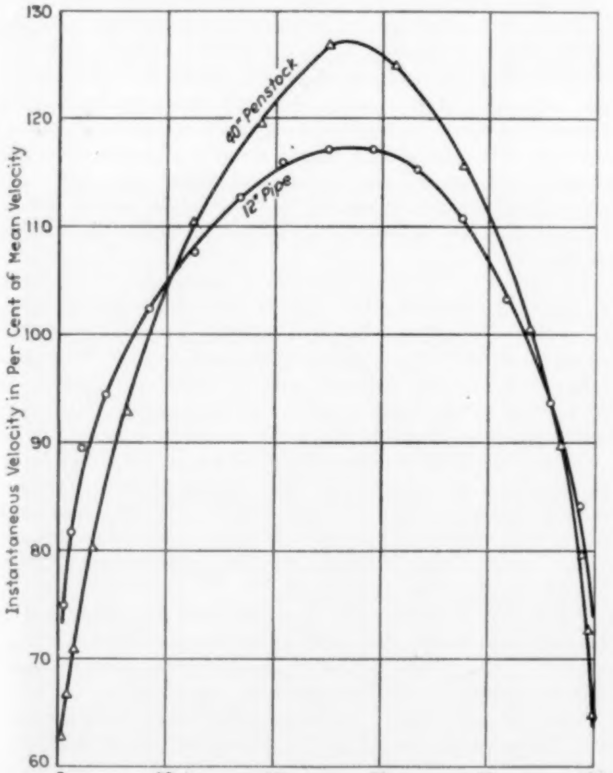

FIG. 8 INSTANTANEOUS VELOCITY CURVES IN STEEL PIPES
(Dotted triangles, 40-in. tuberculated steel penstock; pipe factor, —0.79. Dotted circles, 12-in. smooth steel pipe; pipe factor, —0.85.)

Agreement of Reference Piezometers. As first installed, there were variations as high as 4 per cent among the four reference piezometers. In removing these discrepancies a great deal was learned about the action of square-edged orifices.

Burr. From a study of the results of runs Nos. 10 to 19, inclusive, Table 2, it is seen that orifices Nos. 2 and 3 actually changed 0.4 per cent due practically entirely to the removal of the burr, since the surfaces of these piezometers were little changed. This burr was not on the surface of the piezometer plug, i.e., projecting into the flow of the water, but overhung the hole of the orifice about 0.001 in., and was caused by the dragging effect of the file when the orifice plug was being dressed down to the pipe surface.

The presence of this burr was expected, but following the filing process a piece of hardwood that fitted very snugly was forced and twisted into the hole, and it was thought that the burr was removed by this treatment. To the unassisted eye the edge was smooth and square. The true state of the orifice edge was seen only under a microscope and was remedied with a fine rat-tail file.

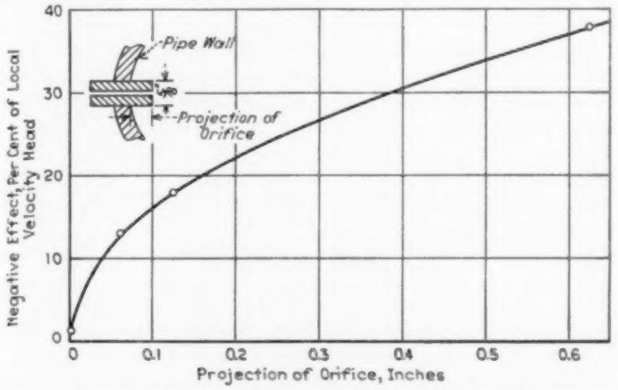

FIG. 9 EFFECT OF PROJECTION OF ORIFICE PLUG UPON PRESSURE READINGS

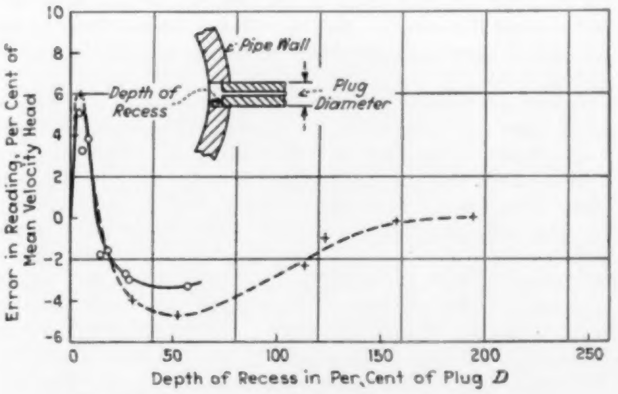

FIG. 10 EFFECT OF RECESSED PIEZOMETER PLUG

Surface Finish of Plug. In examining the reference orifices under a binocular microscope it was noted that the surface left on the piezometer plug by a smooth file was relatively rough. Therefore while the burr was removed from all square-edged orifices, the surfaces of piezometers Nos. 1, 2, and 4 were polished with crocus cloth and the surface of No. 3 was left untouched. In subsequent tests it was noted that No. 1 and No. 3 gave the same reading when in the same place. Since one was polished and the other was not, it would indicate that a piezometer plug is not sensitive to minute abrasions on its surface.

Projection of Plug. Probably the greatest source of error was that caused by the orifice plug projecting beyond the surface of the pipe. Orifices Nos. 12 and 13 were purposely projected $^1/_8$ in. and $^1/_{16}$ in., respectively, and the average error was found to be about —10.0 per cent and —7.4 per cent. In the first few runs orifice No. 4 had a —2.2 per cent error. This was later reduced to —1.1 per cent, but the reason for the improvement was not guessed at the time. Finally the true reason for the discrepancy was surmised. It was found that the center of the orifice plug projected 0.005 in. beyond the pipe surface. This surface was leveled as near as possible, and the error was decreased to —0.3 per cent. In Fig. 9 the negative error, expressed as a percentage of the local velocity head, is plotted against the projection in inches of the piezometer plugs. It will be noted that the curve at first rises rapidly from the origin. Then, in Fig. 10, the effect of allowing the piezometer plug to be recessed in the pipe wall is noted. Here again the curve rises very abruptly from the origin. This should emphasize the great importance of getting a piezometer plug or plate absolutely flush with the penstock wall.

Final Agreement of Reference Piezometers. When the known sources of error were removed, the four $^1/_8$-in. square-edged orifices checked each other to well within 1 per cent. Further, the discrepancies remaining are largely due to oblique or turbulent flow of the water since piezometers No. 1 and No. 3 check each other to 0.1 per cent when in the same place, i.e., when the pipe was rolled over, but there is a variation of 0.5 per cent between them on opposite sides of the pipe.

Size of Orifice. From a comparison of performance of piezometers Nos. 1 to 8 and including No. 5*A*, Table 2, it is seen that within the limits tested the error of a piezometer is not a function of the size of the orifice. All these piezometers mutually check within 0.7 per cent. The variation in size between the piezometer orifices was from $^1/_{16}$ in. to $^{27}/_{32}$ in. It may be noted, however, that a very small orifice, such as No. 5, which had a $^1/_{16}$-in. hole, may be overdamped and sluggish in action, leading to erroneous readings, since the operator assumes that the reading is correct because it is steady. Also the effect of a minute leak in the piping may cause a considerable error in the reading in the smaller piezometers.

Length of Straight Piezometer Tube. Referring to Fig. 10, another factor governing the performance of piezometers is brought out. In addition to illustrating the fact that the piezometer plug must be flush, this curve shows that the piezometer orifice that pierces this plug should be continued for over two diameters into the plug before its shape is changed. This fact is of great importance when piezometers are constructed in thin wall pipe.

Piezometers With Rounded Edges. Orifices No. 10 and No. 11 were made with $^1/_8$-in. holes and a radius of rounding on the edge of $^1/_{16}$ in. for No. 10 and $^1/_8$ in. for No. 11. It is seen that No. 10 checks the standard readings very consistently and accurately throughout. No. 11 has a positive error of 2.1 per cent. Piezometer No. 10 was rescraped because it looked rough under the microscope, yet the performance of the orifice was changed very slightly. The relatively large radius of rounding on the edge of piezometer No. 11 apparently allowed the water to eddy into the opening to give a small positive error. A rounding of $^1/_{16}$ in. and of $^1/_{32}$ in. on a $^1/_8$-in. hole gave the same results as the standard orifices. From these results, for a commercial standard, a piezometer having a $^1/_8$-in. hole and a $^1/_{32}$-in. radius of rounding is recommended for small pipes, while for larger pipes a piezometer having a $^1/_4$-in. hole and a $^1/_{16}$-in. rounding is recommended.

Effect of Pipe-Wall Roughness on Piezometers. In runs Nos. 22 to 39, inclusive the effect of various combinations of wall roughness on the piezometers was studied. In runs Nos. 22 to 25 inclusive a single tubercle made of tar, $1^1/_4$ in. in diameter and $^1/_4$ in. high, was placed $1^1/_4$ in. on center line in front of each of the test orifices. It will be noted that the effect of such a large tubercle immediately upstream from the piezometer was to cause a positive error of about 5 per cent. Various other arrangements of tubercles were tried, as will be noted in Table 2. However, the effect of adding rows of tubercles upstream from the piezometers, as shown in Fig. 11, is to reduce the error gradually until it becomes slightly negative. Leaving 3 in. of clean and smooth pipe wall between the orifice and any roughness or tubercles was sufficient in these tests to insure correct readings. In commercial practice, therefore, it would seem that if a strip of pipe wall about 6 in. wide and 12 in. upstream from the orifice was scraped smooth, correct results would be assured.

In runs Nos. 38 and 39 strips of $^1/_4$-in.-mesh wire screen were laid over all test orifices, as shown in Fig. 12, and the effect for the most part was a very slightly increased negative reading. This checked the action of a large number of tubercles in the pipe immediately upstream from the orifice.

Effect of Whirling Water. In runs Nos. 40 and 41 and Nos. 45 to 48, inclusive, the effect of whirling water upon the piezometers was studied. (See Fig. 13.) The reference orifice in this case was the so-called tea-ball piezometer, which was rated and held at the center of the pipe for these tests. The only conclusion that can be drawn is that under such conditions large positive errors can be obtained. Referring to Fig. 18, the effect of whirling water was tried upon the 12-in. by 6-in. venturi meter. In this case it will be seen that the coefficient of the meter was increased at low discharges and decreased at the high discharges.

Pressure Distribution Below a Butterfly Valve. In runs Nos. 49 to 51, inclusive, the effect upon the piezometers of a butterfly valve above the test section was studied. The valve was 12 in. in diameter and $2^1/_2$ in. thick, and modeled after the butterfly disks which are in common commercial use. The center line of the butterfly valve was located 0.5 ft., 1.0 ft., and 1.5 ft. above the center line of the piezometers. The pressure distribution found at the piezometer section during these tests was very different from that existing with smooth water-flow conditions. The errors are erratic and sometimes relatively large. The data available do not indicate any remedy for this condition except to take a greater distance between the butterfly valve and the piezometer section.

Inserted Tube. The inserted tube, which is shown in Fig. 14, was rated by placing it at the center of the pipe with its axis parallel to that of the pipe. The error was found to be 0.3 per cent, which is an exact check with the reference orifices. The error became —7.0 per cent, however, when the tube was rotated about 10 deg. out of its proper position. It is therefore evident that such a piezometer must be used with the greatest care.

A traverse was made across the pipe with this tube. The deviation was from —1.3 per cent to +1.3 per cent and directly proportional to the true position. This deviation is probably accounted for by the fact that the water flow was not absolutely smooth and axial. Quantitatively, the traverse indicated that the pressure head is constant all across the pipe.

Tea-Ball Piezometer. This type of piezometer, illustrated in Fig. 14, was rated at the center of the pipe and showed a large negative error. While it does not read the true pressure head, it is not sensitive to the direction of the water flow.

Conical Piezometers. As the name implies, this type of piezometer, illustrated in Figs. 15 and 16, consists of a cone which projects into the water flow, with the orifice located at the apex of the cone. Various angles of the side of the cone with the flow of the water from 0 deg. to 90 deg. were tried, both at the wall and mounted on a rod in various parts of the pipe. Very large negative effects can be obtained in this manner up to as much as 15 times the local velocity head acting at the apex of the cone. However, wherever one of these conical piezometers is used, it must be rated, as the amount of the negative effect cannot be

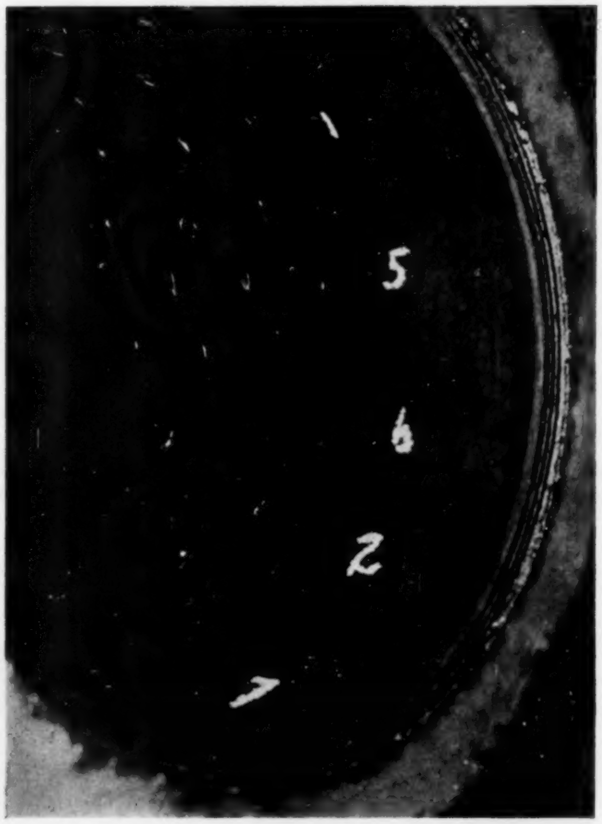

FIG. 11 TUBERCLES OF TAR ATTACHED TO PIPE WALL

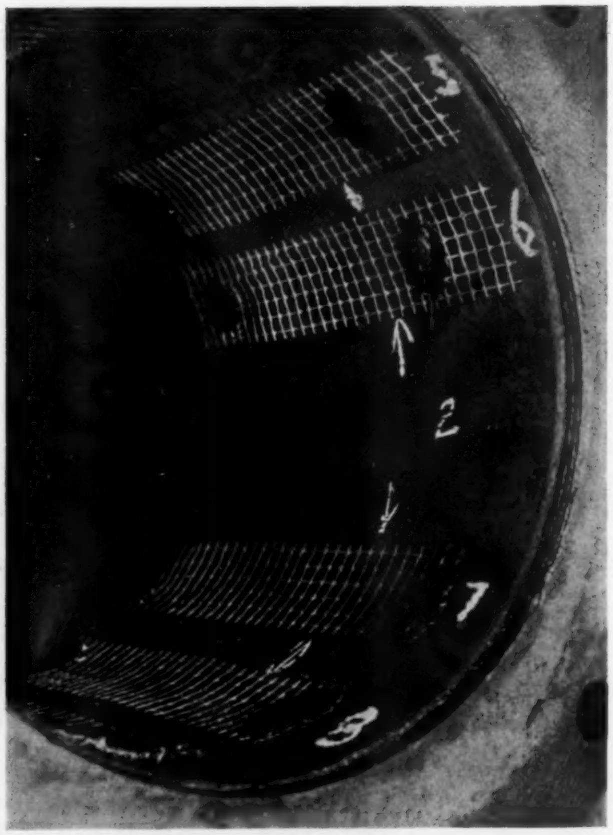

FIG. 12 WIRE SCREEN LAID OVER PIEZOMETERS TO SIMULATE PIPE ROUGHNESS

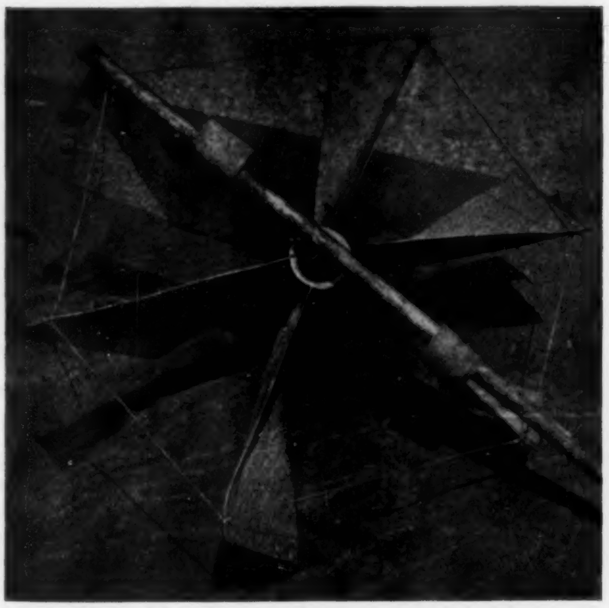

FIG. 13 SPINNER USED TO GIVE WHIRLING WATER FOR PIEZOMETER AND VENTURI TESTS

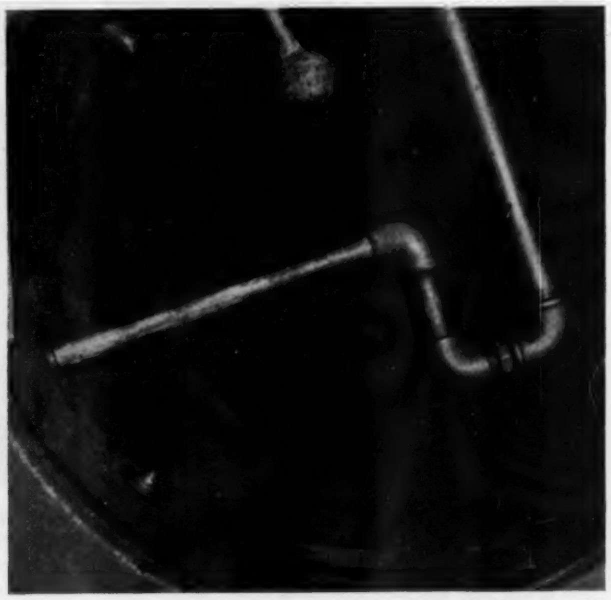

FIG. 14 INSERTED-TUBE AND TEA-BALL PIEZOMETERS

predicted. This is due to the fact that the flow past the tip is the resultant of the primary flow in the pipe and a secondary flow along the tip. This secondary flow is caused by the difference in velocities along the pipe traverse. The dynamic pressure of the water striking the side of the tip is directly proportional to the velocity of the water at that point. Inasmuch as the velocity is constantly changing in the traverse there is a difference in pressure from the bottom of the cone and the apex which is dependent upon the position of the cone in the pipe. Where there is a difference in pressure in a free fluid, there must be a flow which is the secondary flow referred to. Therefore it is seen that a conical piezometer will have a varying rating according to position in the pipe and also according to the distribution of velocities in the pipe.

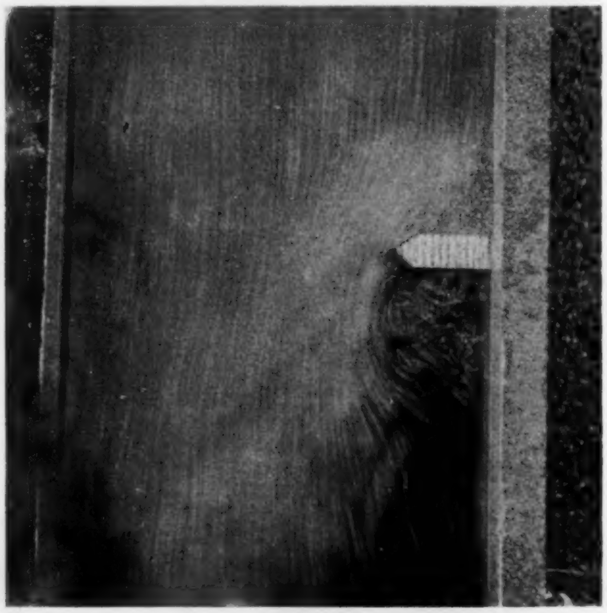

Fig. 15 Flow Past a Piezometer Mounted on a Rod

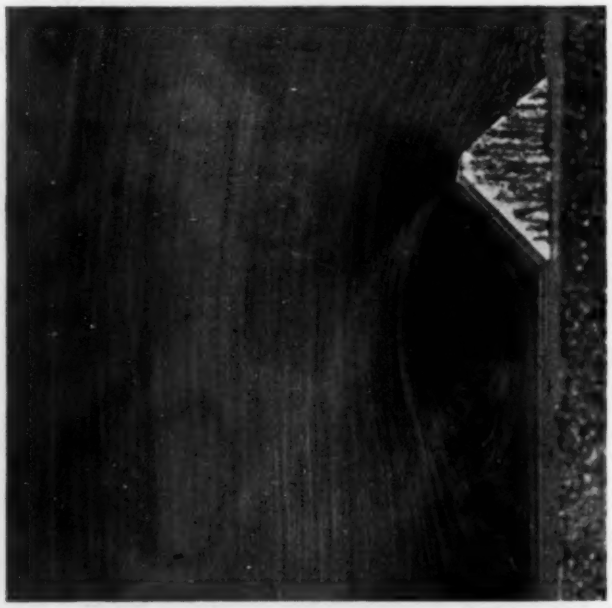

Fig. 16 Flow Past a 45-Deg. Conical Piezometer on the Wall of the Pipe

From the foregoing discussion it is seen that conical piezometers can be used to increase greatly the deflection in a water-gaging or venturi section and for other similar uses. However, due to its sensitivity to more than the ordinary number of factors, it should be used with great care and thoroughly rated.

TABLE 6 RECESSED PLUG

(Tests on Sept. 15, 1931)

Depth of recess, per cent of plug D	Deflection, per cent of mean H_v	Remarks
5.0	+6.0	[a]
30.0	—4.0	
51.5	—4.7	
113.0	—2.3	
123.0	—1.0	
158.0	—0.2	
194.0	0.0	
4.4	+5.1	[b]
8.8	+3.8	
13.3	—0.1	
17.8	—1.6	
26.7	—2.8	
6.2	+3.3	[c]
14.2	—1.8	[d]
57.2	—3.3	[e]
28.6	—3.0	[f]

[a] These tests were made with a $^{13}/_{16}$-in. plug containing a $^{1}/_{8}$-in. standard piezometer. The plug could be moved inside a sleeve screwed into the pipe, which in itself was a standard piezometer. Thus the amount of recess was readily varied.

[b] Piezometer No. 11 in run No. 52 was backed out on its thread.

[c] Piezometer No. 10, run No. 52.

[d] Piezometer No. 10, run No. 53.

[e] Piezometer No. 11, run No. 53.

[f] Piezometer No. 13, run No. 53.

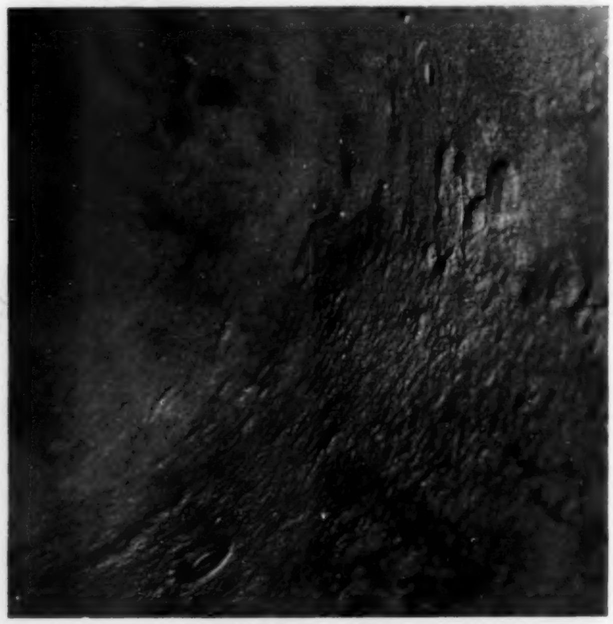

Fig. 17 Inlet Piezometers of Venturi Meter After Tubercles Were Removed

Venturi-Meter Study. During the piezometer investigation, the effect of wall roughness, whirling water, and improvement of meter-inlet piezometers upon the operation of the 12-in. by 6-in. venturi meter was studied. Fig. 17 shows the condition of two inlet piezometers after the tubercles had been removed and before the piezometer plugs were filed down smooth and flush. The results of these tests are shown in Figs. 18 and 19.

CONCLUSIONS

The following results were indicated by these tests on piezometer openings:

1 The error of a piezometer is a constant percentage of the local velocity head existing at the orifice.

2 The size of the opening has no effect within the limits tested, which were from $^1/_{16}$-in. to $^{27}/_{32}$-in. opening and mean velocities up to 7.3 ft. per sec. in a 12-in. pipe.

3 A true square-edged orifice is accurate, but very sensitive to slight changes in construction and position.

4 A standard piezometer plug or plate must be absolutely flush with the conduit wall.

5 A piezometer that projects beyond the pipe-wall surface has a large negative error.

6 The burr must be carefully removed from a square-edged orifice.

7 A piezometer plate or plug is not sensitive to minute abrasions on its surface.

8 A piezometer orifice must continue with parallel sides for over two diameters into the plug before its shape is changed.

9 The inside penstock wall should be scraped clean for at least 1 ft. upstream from a piezometer.

10 The piezometers to determine true pressure head should not be installed where there is any chance of the water whirling in the pipe or immediately downstream from an obstruction, such as a butterfly valve, a rivet head, or a plate joint.

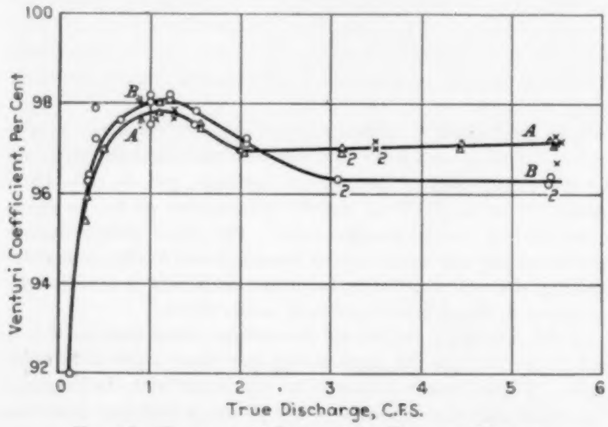

FIG. 18 TESTS OF 12-IN. BY 6-IN. VENTURI METER
(Curve *A*, with and without butterfly valve; curve *B*, with spinner. Circles, six-vane spinner in pipe, 5.3 ft. upstream; crosses, butterfly valve in pipe, 5.3 ft. upstream; triangles, calibration under normal circumstances.)

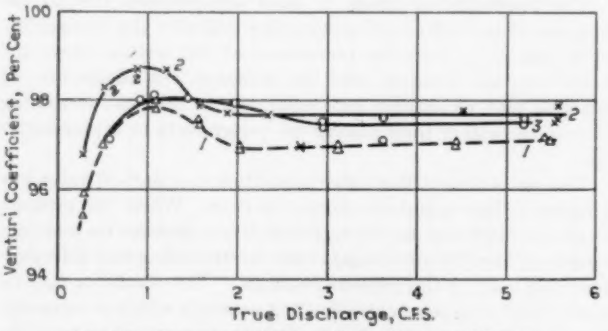

FIG. 19 TESTS OF 12-IN. BY 6-IN. VENTURI METER
(Order of tests: Curve 1, as used in piezometer study; curve 2, with tubercles removed; curve 3, with inlet piezometers improved.)

11 An inserted tube pointed on the upstream end and with piezometers on the side gives a correct pressure-head reading, but it is very sensitive to angularity.

12 A tea-ball piezometer has a large negative error, but it is not sensitive to the direction of flow of the water.

13 A conical piezometer has a very large negative effect, but must be used with extreme care and calibrated.

14 An orifice with a large radius of rounding has a small positive error.

15 An orifice with $^1/_4$-in. hole and $^1/_{16}$-in. radius of rounding is recommended for standard practice.

Discussion

ARTHUR L. COLLINS.[3] The work of the authors on piezometers has been made public at an opportune time, and the information should be a valuable guide to engineers who are interested in measurements where they must make use of pressures obtained through the piezometer.

The authors have stated that the object of this investigation was (1) to determine a stable type of piezometer suitable for use as a standard in commercial work and (2) to determine those factors which have the greatest bearing in obtaining correct results. It does not appear that the first object has been satisfactorily accomplished, because the data do not indicate how a piezometer can be constructed so that it will provide a means for taking a pressure measurement without the measurement being affected by the velocity in the pipe. It will be a matter of coincidence when such a piezometer is discovered. This is apparent since the reference orifices when first installed had variations as great as 4 per cent between them. Even then there was no way to prove that all four piezometers were not in error.

In some classes of measurements where the piezometer is depended upon, such as the pitot tube, orifice plate, and venturi, and where the low differential pressures are required in pipe-line tests for coefficient of roughness, these errors are very important.

Difficulty in obtaining an ideal piezometer may not be the fault of the design of the piezometer so much as it is the rolling action of the water at the pipe wall. Under a normal flow, where the velocity distribution is unaffected by various factors, the slope of the distribution curve has a tendency to follow the parabola. Under this condition the liquid probably strikes the wall in a direction oblique to the axis of the pipe.

The failure of the piezometer to function accurately has already had its effect upon many hydraulic problems. For example, it has prohibited to a large extent the use of the pitot tube, which should be one of the simplest and most accurate instruments that can be used for flow measurements in pipes. Also, the piezometer is bringing into disrepute in many cases measurements made by the orifice plate where it is used on the end of a pipe as a nozzle or between the flanges, and the taps for the piezometer are made by tapping into the pipe. In the orifice plate the way to minimize the effect of the piezometer on differential pressure is to have a high ratio of pipe diameter to orifice diameter. Of course, the objection to the high ratio is the loss of head and dissipation of energy through the restricted area.

The manufacturers of the deep-well turbine pump in California are the most extensive users of the orifice plate for field tests. It was introduced to them about 1920. The end orifice plate is generally used with a low ratio of pipe diameter to orifice diameter, in order not to build up a false head on the pump being tested. The piezometer is drilled in the field, and of course may create readings which are far from correct. Where the differential head is only a few inches and the initial pipe velocity is quite high, the measurement cannot be very reliable. However, innumerable acceptance tests are made with the orifice plate, and the results are not questioned, because some reliable company is credited with having calibrated the orifice plate under laboratory conditions and found it to be correct. Such tests mislead both the manufacturer and the buyer, and no doubt will be abandoned when better methods of water measurement are introduced.

A few examples of how the piezometer has affected the general information on pitot tubes will be called to attention. The A.S.

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[3] Consulting Engineer, San Francisco, Calif.

M.E. Fluid Meter Bulletin, Part 1, 1924, pages 39, 40, and 42, illustrates by Figs. 17A, 17B, 18, 19, 21, and 23, types of tubes the reliability of which can now be judged by these piezometer studies. The A.S.M.E. Power Test Code of 1926 particularly recommends 17B. The coefficient for this tube is given as 0.98. No doubt the 0.02 correction is due to the piezometer, and the value may change depending upon various conditions. It is noted that in the revised Third Edition, Part 1, 1931, the 17B design is now shown on page 42, Fig. 17, and the coefficient has not been mentioned, but multiple piezometers are recommended.

The next example is the Testing Code, approved by the Machinery Builders Society, dated Oct. 17, 1917. Under Pitot Tubes it reads: "The static pressure over the cross-section shall be measured by from four to eight carefully constructed piezometers." . . ."and condition of piezometer orifices shall be such that no piezometer shall vary its readings by more than 10 per cent of the velocity head from the average of all piezometers." Then the code further states that after the measurement has been conducted the velocity is multiplied by 0.976 to correct for oblique or sinuous flow.

Inasmuch as an error in the piezometer due to the velocity means an error in the measurement, one is now also able to judge to a certain extent the risk taken in a pitot-tube measurement when the piezometer is used. It is quite noticeable that the pitot-tube coefficients are less than 1.00, which naturally favors the waterwheel manufacturer, inasmuch as a low measurement is in their favor. However, the pump manufacturer in a performance test is on the other side of the fence and favors the high-capacity readings, and should demand a tube coefficient of 1.00 plus, to be on the safe side.

In the writer's field experience with the pitot tube he has proved that the type of tube using the piezometer in the pipe wall will give readings which may be in error as much as 5 per cent. This error is not permissible, and since the method is specified in the Power Code, it practically prohibits the use of the system. In order to overcome the handicap offered by the piezometer, it is necessary to use a type of pitot tube where the piezometer feature has been eliminated.

There is another point worth mentioning along this same line. The pitot-tube traverses shown in the paper apparently were taken with the overhung type of tube which crowds the water to the opposite side. On one side of the pipe the velocity will be much higher than the velocity on the other side of the pipe, due to the insertion of the tube. For that reason the curve of velocity distribution does not indicate the true velocity in the pipe. As a matter of general information the writer would like to ask for a description of the pitot tube and its coefficients. Is the coefficient the same for the 12-in. pipe and the 40-in. pipe?

F. G. Switzer.[4] The measurement of pressure certainly seems a very simple thing at first, but it needed further investigation to bring out some of the difficulties. The authors deserve credit for a difficult job well done.

In the connections from the pressure tap to the pressure-measuring device such as a water or mercury column, it is customary to insert a valve, which is partly closed, to reduce pulsations. It should be noted that such practice may introduce errors which mount rapidly with increasing tendency to pulsation. Pulsations are usually not entirely eliminated, but merely reduced to a point where accurate observations of column height may be made. Hence there is flow back and forth through the throttling valve. Loss of head through this valve is proportional to nearly the square of the velocity. Velocities above and below the average will therefore be damped by different percentages. The average pressure on the manometer side will not be quite the same as the average pressure in the pipe under observation. While this difference may not be great, its presence may be noticeable in work of a high degree of accuracy. Some method of viscous damping is much to be preferred. One method of accomplishing this is to insert a bundle of fine wire into a short section of the connecting tubing. This makes the removal of air bubbles very difficult, no matter how the piping is arranged with air vents. It is not to be recommended unless extreme care is used in installation and observation. It is suggested that a more practical method of securing viscous damping be devised.

For measuring air pressures, an entirely different device has been used which apparently can be made to measure pressures in liquids also. No reference to this device can be cited. It consists of a tube with a large number of small holes drilled through the walls from all directions. Around that portion of the tube there is wound a closely coiled helix of fuzzy material such as twine used for tying up packages. This arrangement has been used in experimental work at Cornell University with great success. Its chief advantage is that it can be used anywhere in a flowing stream and is not directional, no matter in what direction the tube is placed with respect to the flow lines. This makes it possible to measure pressure distribution in any cross-section.

E. B. Strowger.[5] Although according to the writer's experience it has been customary to insist on rather rigid specifications for the installation of piezometer openings, yet to date there seemed to be available no definite information as to the errors introduced by various modifications. The paper quite definitely establishes the effects of various modifications in the piezometer opening, some of which may be either artificially or accidentally produced in the ordinary course of construction.

Of the conclusions stated by the authors, those numbered 8, 9, and 15 are perhaps the outstanding ones from a practical viewpoint. These three conclusions are concerned with the length of the orifice wall, the condition of the penstock wall just upstream from a piezometer, and the rounding of the orifice edge to prevent burrs. The results found will serve to preclude controversies on these points.

The possibility of errors in head measurement due to the vagaries of the individual piezometers indicates the advisability of having more than one piezometer at the section where the head is to be measured, and also indicates the advisability of bringing each piezometer pipe separately to the head-measuring instrument rather than joining the piezometers to a piezometer ring.

The use of a throttling valve in addition to a shut-off valve in a piezometer line is perhaps worthy of note. Where the pressure to be measured may have a variation in one direction for a certain length of time, the use of a gate valve for throttling may introduce a serious error in the pressure readings. This condition may be exemplified by a piezometer line to a penstock which is connected to a surge tank, a gate valve for throttling purposes being in the piezometer line. If the valve disk is nearly closed, it may become locked closed by the action of the changing pressure tending to deflect the disk over against the seat and thus cut off more or less completely the opening to the penstock. One precaution is to drill a hole through the center of the disk to insure a positive opening under all conditions of throttling.

It should be pointed out that the function of piezometers for use in connection with taking Gibson diagrams is somewhat different in principle from their function in measuring absolute pressure head. In the former case, the piezometer is used for transferring the supernormal pressures existing in the penstock by virtue of the water hammer produced by destroying the momentum of the water. Any slight modification of the piezometer opening while changing the absolute head reading for "steady state" conditions, in the case of the diagram, only results in altering the position of the "running line," the impulse area recorded being unaltered. This has been established by many experiments, using combinations of various piezometers located around the periphery of the pipe on the piezometer sections.

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John W. Brasko.[6] Due to the care which must be exercised with piezometers, the writer is curious to learn what results, in the opinion of the authors, may be expected from the use of a piezometer consisting of a $^1/_2$-in. H pipe nipple screwed into a $^1/_2$-in. H coupling, which is welded over a $^3/_8$ drilled hole in the wall of standard thickness pipe.

We have adopted this method of pressure-tapping pipes as a much more satisfactory mechanical job than that of screwing a nipple into a tapped hole in the pipe. We feel, also, that the elimination of the problem of making the piezometer flush with the pipe wall and the consequent reduction of burr formations are strong points in favor of this method.

If such a piezometer gives inaccurate readings, is it reasonable to assume, in cases of flow measurement by orifice plate where the pressure differential across the orifice is to be measured, that the error in the one piezometer is cancelled by the same error in the other piezometer by reason of a manometer arrangement.

John. B. Drisko[7] **and Hunter Rouse.**[7] A short series of investigations similar to those performed by the authors, dealing with only five types of piezometer openings, was carried out in 1905 in Dresden, Germany, by Dr.-Ing. Paul Schuster.

Schuster's work[8] was essentially an investigation of flow through a Francis turbine. Preliminary tests of various pitot tubes and piezometer openings were carried out in the towing tank of the Uebigau Shipyard. The piezometer openings investigated were five in number:

(1) Square-edged hole with diameter of 1.75 mm. (0.069 in.).

(2) Hole with diameter of 1.75 mm., corners removed with 90-deg. countersink to a maximum diameter of 1.8 times the hole diameter

(3) Square-edged hole with diameter of 2.7 mm. (0.106 in.).

(4) Hole with diameter of 2.7 mm., corners removed with 90-deg. countersink to a maximum diameter of 1.5 times the hole diameter

(5) Hole with diameter of 2.7 mm., corners rounded on a radius of 0.4 times the hole diameter.

The five piezometer openings were mounted at intervals of 25 mm. in a vertical row on a brass plate 4 mm. thick. The plate, the leading edge of which was sharpened with the bevel on the back side, was mounted on the towing carriage exactly parallel to the direction of travel and was towed in still water.

The velocities of the carriage were recorded chronographically. Glass tubes of 6.5 mm. inside diameter containing the piezometer water columns were mounted on a board faced with millimeter cross-section paper, by means of which the column heights were determined. Test data are as follows:

Velocity		Piezometer error due to dynamic pressure head, in mm.				
m. per sec.	ft. per sec.	(1)	(2)	(3)	(4)	(5)
0.76	2.49	—1	—1	—2	—1	—1
0.84	2.76	—1	—1	1	0	0
1.02	3.34	—2	—2	0	—1	1
2.05	6.72	0	—2	0	—1	0
2.57	8.42	—10	—8	—10	—8	—4

Schuster concludes that the suction effect of moving water is best avoided and hence the pressure is indicated most accurately by the use of a well-rounded opening. He gives the following approximate values for the error of piezometer column heights due to the velocity effect:

Square-edged opening, —3 per cent of velocity head
Countersunk opening, —2.5 per cent of velocity head
Well rounded opening, —1 per cent of velocity head

Similar errors as found by Allen and Hooper are:

Square-edged orifice, 0 per cent of velocity head
Round-edged orifice No. 11, 2.1 per cent of velocity head

Allen and Hooper arbitrarily chose square-edged orfices as references; hence rounded orifices have a positive error. Schuster found the rounded openings more nearly correct, which would give the square-edged openings a negative error. A comparison of the foregoing tables indicates that in both cases the well-rounded orifice gave higher readings.

The values of "piezometer error in per cent of mean velocity head" from Allen and Hooper are probably smaller than if based upon actual velocity, inasmuch as they were figured for mean pipe velocity and not for the lesser velocity actually existing near the piezometer openings in the pipe wall. The corresponding Schuster values were figured for a velocity which actually existed close to the piezometer openings and are hence somewhat larger than the former values.

The fact that Schuster's piezometer openings were set into a flat plate and that those of Allen and Hooper were set into a cylindrical surface may account for some of the discrepancy noted.

It is barely possible that some of Schuster's work was done in water quiet enough to give laminar flow conditions. In general, it is found that the first one or two runs made in a towing tank after the water has been undisturbed for some time, as for instance overnight, are unsatisfactory because the water has become quiescent and conditions correspond to those of laminar flow. Once having been disturbed, the water in the tank may be said to be turbulent, and gives corresponding results. Since the flow with which Allen and Hooper were working was undoubtedly turbulent, the possible dissimilarity of flow conditions under which the two series of investigations were done may be given as a remote source of difference in the results.

It is to be regretted that the wide range of piezometer inlets discussed in the paper did not include various forms of openings leading to tubes set at angles differing from 90 deg. with the pipe wall. The fact has long been known that a tube pointing slightly against the direction of flow, however flush it may be with the pipe surface, will give values above the actual pressure head, because of the additional effect of a component of the velocity head—similar conditions causing a decreased reading for a tube pointing somewhat downstream. In addition to having quantitative data on the influence of such angularity, it would also be interesting to know whether the proposed rounding of the opening would tend to reduce this effect.

The danger of inaccuracies due to this cause, especially in cases where openings must be drilled by hand with no means of procuring absolutely normal holes, may be fully understood only through such careful tests as are contained in this paper. One of the writers recently conducted a series of experiments involving 75 hand-drilled piezometer connections on the side of an open flume for measuring the effect of curving flow; as the pressures were often below the atmospheric and the velocities correspondingly high, many of the tubes were rendered useless by their imperfect construction, despite great care in finishing the openings at the wetted face.

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[7] River Hydraulic Laboratory of Massachusetts Institute of Technology, Cambridge, Mass.

[8] Published in 1910 by the Verein deutscher Ingenieure, "Mitteilungen über Forschungsarbeiten," Heft 82.

Allen and Hooper's ingenious method of arranging the gage box brings to mind a number of gage forms which have proved satisfactory in various laboratories for very delicate setting. It is claimed of the gage point lowered from above that small particles of dust or drops of water will cause the springing of the meniscus before the metal point is actually in contact with the water surface. This difficulty is overcome by the hook gage immersed in the water, but the latter is often sharply pointed and the contact noted from above when the surface is punctured; needless to say, this instant of contact cannot be observed with great accuracy from above the surface, while the sharpness of the point makes it equally difficult from below.

The apparatus used at the Worcester laboratory apparently includes a gage whose point is not so acute as to involve the latter difficulty. A slightly arched chisel edge has been used at the Walchensee experiment station of the Forschungsinstitut of Bavaria, with which one may measure without great difficulty to within 0.02 mm. (0.0008 in.). A small lamp illuminates the hook, whose reflection is sharply visible in the under side of the water surface; the hook is raised until it apparently just touches its reflection above, but without quite puncturing the surface.

Many experimenters believe that the glass gage pipe should be from $1^{1}/_{2}$ to 2 in. in diameter, not only to insure a flat portion of the water surface between the meniscus curves at the sides of the glass, but also to overcome the lag due to capillarity, and the variable capillary rise if the pipe varies slightly in diameter. This usually makes the tube large enough to serve both as stilling box and gage glass, and hence lessens the effect of changing volume as the gage is raised or lowered.

H. S. Bean.[9] The paper is the most comprehensive study of pressure measurements with a piezometer that has come to attention. Since K. Büchner[10] published the results of his observations, and possibly before, there have been many references to more or less causal observations on the use of piezometers, but in all cases these observations have been incidental to the attainment of some other objective.

The effects of the various designs and flow conditions that were tried and the conclusions drawn are very much what would be expected. It is gratifying to see that the piezometers used in the Chicago tests, by the A.G.A. Committee on the Measurement of Large Volumes of Gas,[11] conformed exactly to the recommendation of the authors.

The authors do not specifically state whether they used other sizes of lines than those shown in Fig. 1. Presumably they did not. Also, they recommend the use of a piezometer with a $^{1}/_{8}$-in. hole and a $^{1}/_{32}$-in. radius of rounding for small pipes, while for larger pipes a piezometer with a $^{1}/_{4}$-in. hole and a $^{1}/_{16}$-in. rounding is recommended. However, they have not suggested a dividing line between "small" and "larger" pipes. It will be very helpful to know what their recommendations on this point would be.

John R. Freeman.[12] The paper interests me, because about 55 years ago I had a hand in a much more fundamental and more precise research on piezometers. The principal results of this research were published in the *Transactions* of the American Academy of Arts and Sciences in Boston in the year 1878, pages 28 to 53, and illustrated by four plates of drawings and diagrams.

These researches were carried on by Mr. Hiram F. Mills, Chief Engineer of the Essex Water Power Company at Lawrence, Mass., who was one of the most eminent hydraulic experimenters in America in the epoch of 50 years ago. They were made as a preliminary to a long series of other experiments on the flow of water in pipes, only a few of which have ever been published.

These researches on piezometers by Mr. Mills, unfortunately, have remained almost unknown to the engineering profession, because presented to a society which at that time contained relatively few engineers in its membership.

The writer had the rare good fortune to be one of Mr. Mills' young assistant engineers at the time these experiments were made, and soon afterward became his principal assistant, and continued in that capacity for about 10 years in a great variety of hydraulic work, including much of an experimental nature.

These experiments by Mr. Mills were much more fundamental than those of Allen and Hooper because they compared the piezometric level with that of a free water surface flowing in an open channel, whereas Allen and Hooper's experiments simply compare one piezometer with another, and have all their piezometric orifices arranged around the same cross-section of a closed pipe flowing full of water under pressure. Thus they simply show the errors which may be introduced in such measurements by the careless projection of a burr on the inside of the piezometric orifice or the lowering of the water column caused by carelessly permitting the end of a connecting pipe to protrude inside of the channel, which errors are, however, often permitted by those who fail to realize the truths brought out by Professor Allen's experiments.

The precision of measurement in the Mills experiments was also greater inasmuch as the micrometer gages used in the comparisons were all carefully read to about 0.0001 in., under excellent conditions for measuring head in the piezometer chamber, whereas the limit of measurement in the Allen-Hooper experiments was 0.01 in. For most practical purposes this difference of precision is unimportant.

Mr. Mills' experiments aimed to accurately determine once and for all the truth about a moot point in physics which had been raised by Darcy in a statement in his monumental work "Recherches Hydraulique," published about 75 years ago, which (translated) reads: "Piezometers do not indicate the entire head of a conduit at the point where they are attached, but the head diminished by a certain amount, the diminution being due to the velocity at the base of the piezometers, where the water by its cohesion acts on the manometric column, the height of which it lowers."

Darcy also stated that in a piezometer attached to a reservoir of large diameter, feeding a pipe in which water was flowing, the lowering of the piezometric column by suction would be less than in a piezometric column attached to the pipe.

The idea at the bottom of this piezometer question in physics is that water has a definite cohesive strength (which with proper apparatus may be measured as a tensile strength amounting, as I remember it, to about 75 lb. per sq. in.). And it appeared not unreasonable to Darcy and others to suppose that this cohesion would lower the piezometric column when water was flowing rapidly past the orifices.

Dubuat, ablest of the hydraulic experimenters of 120 years ago, had erroneously concluded that water would press against the side of an open conduit in which water was flowing, with a pressure less than that due to its depth by the amount of pressure that would produce its mean velocity. Mr. Mills stated that although Navier in 1819 had controverted this idea, Dubuat's conclusion had been commonly presented in treatises on hydraulics up to the time of Darcy.

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[10] *Z.V.D.I.*, vol. 48, II, 1904, p. 1097 (especially p. 1100).

[11] Bureau of Standards Research Paper No. 335, "Experiments on Metering Large Volumes of Air," *Journal of Research*, no. 7, July, 1931.

[12] Consulting Engineer, Providence, R. I. Mem. A.S.M.E.

Mr. Mills therefore sought the origin of this wrong idea of Dubuat, that the piezometric height would be less than the true height, by connecting certain of his piezometers to a pipe with an open end, which could be projected through the wall of the conduit; and he found, just as Allen and Hooper have now found, that the water column in the piezometer chamber was thereby lowered proportionately to the velocity of the water passing the end of the projecting pipe, because of the deflection of the current by this projection.

The apparatus designed by Mr. Mills for the purpose of definitely settling this question of whether or not there was a lowering of the piezometric column by suction, when connected to an orifice truly in the plane of a conduit wall, comprised in brief an open trough, nearly 30 ft. in length, about 1 ft. in depth, and about 4 in. in width of channel, constructed of clear, well-seasoned pine wood with greatest care by an uncommonly expert mechanic, within the walls of which about 20 orifices of different forms were cut, on opposite sides of the trough, of various sizes and shapes and having various inclinations.

We are now chiefly concerned with the results of the three series of experiments:

(A) With orifice in the plane of the wall of the flume and with the connecting passage at right-angles to that wall
(B) With orifice truly in the plane of wall of the flume, but with passage leading from the orifice to piezometer inclined at an angle to wall
(C) With orifice in end of pipes which projected into the conduit by various amounts.

Nearly 6000 separate observations were made, covering velocities all the way from about $^1/_2$ ft. per sec. to about 9 ft. per sec. The flume was frequently emptied and inspected for possible changes in shapes due to swelling of the wood, by means of steel straight-edges, some short and some long, under conditions of lighting so arranged that deviations of 0.001 in. in or out from the plane of the orifice could readily be perceived. Some of the orifices were in smooth brass plates carefully inserted in the conduit wall.

The piezometric water columns were measured in small reservoirs averaging perhaps 6 in. in diameter, constructed of tin boxes, attached to the side of the flume, with great precautions to prevent leakage.

The principal cause for possible errors in measurement of comparative heights lay in the "wrinkles" in the surface of the water in the open conduit when flowing at high velocities. Notwithstanding the care taken to maintain a constant level at the entrance of the flume, this surface at higher velocities became somewhat "wrinkled" along its course by small diagonal standing waves.

Every practicable precaution was taken to obtain accuracy. To eliminate the personal equation, the observers at the several stations along the trough, all of whom were skilled and experienced in such experiments, frequently shifted their location.

As a result of this elaborate series of experiments, Mr. Mills concluded that the height of the water shown by piezometers of class A was the same as the height of water at the middle of the stream flowing in the trough, as nearly as it was practicable to measure, and that there was no lowering of the piezometric column by suction or cohesion of the water flowing past the piezometric orifice.

For class B, by experiments on four orifices he found, notwithstanding the edges of the orifice coincided as nearly as practicable with the plane of the conduit wall, that when the connecting passage was inclined 45 deg. facing upstream, the effect was to slightly raise the piezometric column, and when inclined downstream to slightly lower it, probably because of some slight sinuosity in the current past the orifice.

For class C, with orifices in the end of a pipe, Mr. Mills found, as Allen and Hooper have found, a definite lowering proportional to the velocity head of the water flowing past the end of the pipe.

As the final result of nearly 6000 observations on about 20 different orifices, Mr. Mills concluded his paper with the words: "From these results, it is evident that it is entirely within the practicable limits of construction to make piezometers that will indicate the true height of the stream, within the practicable limits of observation."

During the 10 years or more following these experiments, in my work for Mr. Mills in measuring water used by the factories at Lawrence, Mass., and Manchester, N. H., in various turbine tests, and in a great variety of experiments on the flow of water in conduits, great care was always taken when setting up piezometers for water-measuring apparatus to make sure that the piezometric orifice was truly in the plane of the surface of the pipe, and in order that there could not be any possibility of a burr or projection, the edges of the orifice were always very slightly rounded. Great care was taken when a pipe was screwed into a conduit wall that by no possibility should it project inside the surface of this wall.

I am prompted to make a few further comments relative to the precautions necessary to obtaining the greatest practicable accuracy in piezometric observations.

First, as to the size of the glass tubes for observing the height of water, I assisted Mr. Mills in a long series of experiments on this matter, using glass tubes of many sizes up to 2 in. in diameter, with water of various temperatures, with the water surface in the reservoir rising in some cases, falling in others, and in experiments in which the actual mean level in the reservoir did not change, but was simply agitated, as by small waves. Unfortunately, no description of these experiments was ever published.

My recollection is that we found it necessary to have our glass tubes at least 1 in. in internal diameter, in order to fully avoid the effects of surface tension, or capillarity, from the inner walls of the glass tubes.

We found this degree of surface tension and error which existed in the small tubes of, say, $^1/_2$ in. internal diameter, varied with both temperature and with the degree of cleanliness of the inner wall of the tube.

Subsequent to these experiments, in all of our precise work, care was taken to have tubes not less than about 1 in. in diameter. With these and proper apparatus of loop-vernier index, proper lighting, and accurate scale, the height can quickly be measured with certainty to within about 0.001 in.

Another matter which commonly does not receive sufficient attention, in precise work, and which may have led to a lack of precision in Professor Allen's experiments, is that the conduit channels between the piezometric chamber and the pipe should be such that water should flow just as easily in one direction as in another; otherwise a sort of check-valve action occurs which, if there are pulsations in the water of the conduit, may materially affect the accuracy of the piezometric measurement.

In the Mills' experiments previously discussed, great care was taken to have the orifices in the walls of the conduit and the connections to the piezometric chamber so shaped that the coefficient of contraction at all edges of the connecting channel should be the same for inward flow as for outward flow.

Another matter which Professor Allen has touched on is the importance of avoiding even the faintest leakage in piezometric connections. I have had frequent occasion to observe that the importance of this precaution is often not appreciated. It is of highest importance for precise experimentation that there be no leakage whatever. Possibly the fact that water possesses slight velocity may be at the bottom of this possible source of error.

Long tubes and tubes of small diameter in many cases present opportunities for error in piezometric observations by reason of air bubbles released from solution which collect in these tubes.

On one occasion, perhaps 25 or 30 years ago, in some experiments in the large hydraulic flume and measuring weir in the hydraulic laboratory of Cornell University, I found that the hook gage in the still-box measurements of depth over the weir, supposed to be "absolutely correct," was about 1 in. in error, as determined by holding an ordinary leveling rod, first on the crest of the weir and then at the water level of the weir box, using an engineer's Y-level under such conditions as to insure accuracy.

This test was made with a foot or more in depth flowing over the weir in order that any effect from settlement under the weight imposed by operating conditions should be discovered. The experienced engineer at that time in charge of Cornell's hydraulic flume could hardly believe his eyes when this amazing error was revealed. Probably it was due to an air trap or an accumulation of small bubbles in the connections.

In our many water experiments and measurements at Lawrence 50 years ago, under Mr. Mills, we learned to take the utmost care to arrange our piezometric connections so that they could be flushed out by a swift current, immediately before beginning observations, also at frequent intervals, and, moreover, found it necessary to carefully avoid slopes in connecting pipes which might interfere with the free rising of bubbles, and also found it necessary at intervals of half an hour more or less, in many cases, to hammer our rubber-tube connections in a way to produce violent pulsations of the water column, for the purpose of loosening air bubbles disengaged from solution in the water, while it stood in the connection.

Precise experimentation in hydraulics involves an everlasting setting of traps to guard against unconscious error of many kinds.

AUTHORS' CLOSURE

Arthur L. Collins: In answer to the statement which Mr. Collins makes that "it does not appear that the first object of the investigation has been satisfactorily accomplished," may we call attention to the work of Mr. Hiram Mills referred to in Mr. John R. Freeman's discussion. Results of his work, which was done in an open flume where the actual elevation of the free-water surface was checked against the piezometer heads, check the work described in this paper very closely. In addition, the four reference piezometers, when properly installed, check within 1 per cent of the mean velocity head of each other, and later four other piezometers, Nos. 5A, 6A, 11B, and 12B, were installed according to the specifications developed in the paper, and they checked the reference piezometers within 1 per cent of the mean-velocity head.

In regard to the use of pitot tubes in this investigation, the static piezometers of the tube referred to as No. 11A and shown in Fig. 14 were compared as to performance with the reference piezometers, and in this case checked very well. However, a pitometer was used in making the traverses in the 12-in. and 40-in. pipes, the curves of which were printed in the paper, and the indicated velocities were corrected for the change in area occasioned by the presence of the rod. Pitometers have been carefully calibrated recently in the 12-in. line and in the 16-in. throat of a 36-in. by 16-in. venturi meter against weighing-tank measurements, and the value of the coefficient was the same in both cases. A pitot tube should not be in error as much as 5 per cent if it is carefully calibrated under hydraulic conditions similar to those in which it is to be used.

F. G. Switzer and E. B. Strowger: The damping devices referred to in the discussions of Professor Switzer and Mr. Strowger are probably satisfactory, but at the Alden Hydraulic Laboratory it is customary to pinch a rubber hose to give the desired degree of damping. This system, we believe, has merit on account of its simplicity, flexibility, and availability. In the first place, it is very easy to obtain full discharge in the pressure line for the elimination of dirt or trapped air, or to change the water in the line because of temperature conditions. Secondly, the channel of approach to the throttling section is just the same on both sides. Then, if the hose is pinched so that the water column or pressure gage is just alive, it is probable that the flow through the throttling section is viscous. And finally the throttling is very readily changed to meet the pressure conditions.

John W. Brasko: Fig. 10 of the paper shows that a $^3/_8$-in. hole must have parallel sides for $^3/_4$ in. before any change of diameter of the piezometer tube occurs. Therefore, for a large number of cases where the pipe-wall thickness may be considerably less than $^3/_4$ in., this form of piezometer will undoubtedly be in error. Also the importance of removing the burr from the inside edge of the piezometer opening cannot be overemphasized.

John B. Drisko and Hunter Rouse: With reference to the tests of Dr. Ing. Paul Schuster in 1905, it is evident that the hydraulic conditions of the two investigations are so dissimilar that comparisons are probably of little value.

H. S. Bean: If reference be made to the results of tests on piezometers Nos. 5A and 12B, it will be seen that they give the same results. Hence the choice of the size of the orifice depends upon other considerations than accuracy.

John R. Freeman: With reference to Mr. Freeman's statement of relative accuracy of Mr. Mills' and this work, it should be understood that the error given in this paper was an over-all figure (about 1 per cent), whereas the figure given for Mr. Mills' work really applies only to the measurement of the elevation of the water surface in the piezometer stilling pots. Referring to Plates Nos. 3 and 4 of Mr. Mills' paper, it will be found that there are a number of points which lie more than 2.0 per cent of the mean velocity head away from the smooth curve of all the points, and each of these points is the average of ten readings. The determination of the free-water surface, a very difficult thing to measure accurately, introduced errors which makes the two papers comparable as to accuracy.

It is to be regretted that Mr. Mills' paper on piezometers was so thoroughly hidden that the material therein was practically unknown to the engineering profession for so long. The authors were unable to obtain a copy of his paper until after the present paper was written, but find, fortunately, that his work and the results of this paper practically check.

HYD-54-2

Hydraulic Butterfly Valves

By ROSS L. MAHON,[1] SAN FRANCISCO, CALIF.

This paper describes some of the special features of design found in the modern butterfly valve and which have been adopted within recent years, during which this type of equipment has been developed to meet some of the unusual requirements existing in many modern hydroelectric projects. Means for making this type of valve more effective in limiting leakage are discussed in some detail and attention is given to general considerations affecting the design of disks and bodies. Some interesting types of operating mechanisms are outlined, and various kinds of protective and auxiliary equipment are described. Recent applications of butterfly valves for free-discharge service have broadened their useful field, and the suggestion is made that valves of this type might well find future use for other purposes than those existing in the hydraulic-power industry.

The butterfly type of valve as developed in the hydraulic-power industry during recent years has such a wide field of application and finds such general use in modern hydroelectric installations that information in regard to it will be of interest. As the size of hydraulic-power units has increased, more attention has been given to the design of auxiliary equipment for the modern hydroelectric plant, such as governors and valves. It is only by making a careful study of modern operating requirements and by cooperating with the engineers of the large public utilities that the turbine designers have been successful in meeting the needs.

Much has been published during recent years on hydraulic turbines, but little has appeared regarding the valves used in the pipe lines or penstocks to control the large quantities of water delivered to the units. During those earlier years when the unit capacities were limited and the penstocks and turbines were small, the penstock valve was usually of a standard type, such as was built by valve-manufacturing concerns for ordinary waterworks service, and little attention was given to developing special designs. As the turbine units increased in size and higher heads were utilized, more attention was directed by the designers to those factors that make for efficiency and safety, and the development of suitable penstock valves began to receive the same careful attention as the turbine units themselves.

It was during this period that the butterfly valve was recognized as being admirably adapted for hydraulic service. Its simplicity, its low cost, and its dependability brought it into general use in hydroelectric work. Although the butterfly valve covers a wide range of engineering conditions and is suited to many types of installation, it should not be understood that it is used to the exclusion of other types of valve. For example, under extraordinarily high heads, special types of gate valves are used. For installations having special requirements, and particularly where valves of very large diameter are needed, balanced needle valves with hydraulically operated internal plungers find their application. The Larner-Johnson valve is a fine example of this type of equipment.

The modern butterfly valve, in its essentials, shows little change from the original conception. A cylindrical or tapering valve body is provided with flanges or riveting necks for attaching to the pipe line and carries two large bearings placed diametrically opposite each other. These support a transverse shaft on which is mounted the circular disk. An external operating mechanism rotates the disk about 90 deg. to move it from full-closed to full-open position. The support of the disk on a central axis results in a hydraulically balanced condition at either the closed or open positions; hence makes possible the use of a comparatively simple operating mechanism.

The first butterfly valves were used with small turbine units and the heads were low. Even though they were equipped with no special means for reducing leakage, the comparatively small size and the low spouting velocity encountered did not make water tightness of great moment. Such water as did pass through the valve when closed was usually taken care of by a drain located in the penstock or turbine casing downstream from the valve.

As butterfly valves of larger size came into common use and as they were employed under higher heads, the problem of properly handling the leakage became a difficult one, and those engaged in the manufacture of this equipment began to give attention to some means for reducing the clearance between the closed disk and the body. The castings which form the body of a large butterfly valve may be machined to an accurate fit with the disk in the shop of the valve manufacturer but, after being installed in a concrete foundation and subjected to the stresses that are set up, they can deform to such an extent that a close fit with the circular disk cannot be maintained. Furthermore, even if a proper fit between disk and body can be assured when such a valve is installed, ordinary water wear may cause an enlargement of the clearances, with a resulting increase in leakage.

It thus became evident that some sort of adjustable device was needed, not only to permit proper setting of the clearances between disk and body when the valve was installed, but to permit periodic readjustment. As a result a number of ingenious devices have been developed by the different valve manufacturers, some of which are described.

As the valves became larger in diameter and the heads under which they were used were increased, it became necessary to develop more powerful operating mechanisms. In many cases purchasers' specifications required that these operating mechanisms be sufficiently powerful to close the valve while normal full water flow was passing through it. For some installations it was even specified that the valve be capable of closure under full spouting velocity, such as might occur in the event of a rupture in the turbine casing or penstock below the valve, and today it is quite

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usual to require that the operating mechanism for a large valve be arranged to close it automatically whenever the water velocity exceeds a predetermined maximum. A description of the different types of operating mechanisms now in common use will therefore be of interest.

The butterfly valve finds its application not only at the lower end of the penstock adjacent to the turbine, but at the upper end of the pipe line where it emerges from the surge tank, tunnel, or forebay; and large butterfly valves, because of their low first cost and ease of operation, are being frequently used in diversion dams on large power projects to serve as head gates.

So much has been done to improve butterfly-valve design that it is believed the field of application is much broader than has been realized. This is true in connection with condenser systems and in pumping plants and pipe lines, particularly where the valves are of large diameter.

Design of Disks and Bodies

Hydraulic-turbine design has been developed to a point where unusually high efficiencies are obtained, and the valve manufacturer must build his equipment so as to minimize hydraulic losses within the valves themselves. Every effort is made in designing a modern butterfly valve to reduce friction and velocity losses, as well as those resulting from unnecessary turbulence or irregular flow conditions.

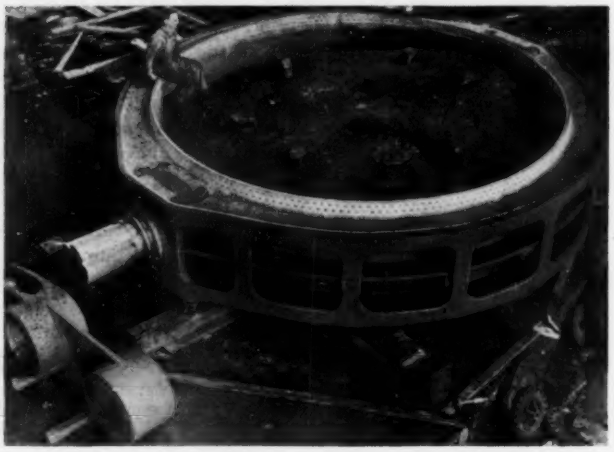

Fig. 1 Butterfly Valve 17 Ft. in Diameter for 180-Ft. Head
(Parting plane between halves passing through center line of valve shaft.)

It is therefore important that a smooth velocity curve be maintained as the water passes through the valve body and around the disk. If the velocity must increase, as is often the case when the valve is located adjacent to the inlet of a turbine casing, the water should be accelerated as gradually as possible. Proper determination of the water passage areas within the valve itself, as well as in the adjacent portions of the penstock and connecting pipes, will accomplish this result.

It is now the generally accepted practice to make the internal surface of the valve body and its connecting pipes as smooth as possible. Rivets with countersunk heads on the inside are used in fabricating joints, and sudden and abrupt changes in the area of the water passage are avoided.

The disk of one of these valves must be designed for heavy bending moments as, when it is closed against full penstock pressure, it will act as a rigid beam supported at its central point. The necessary strength is obtained by introducing heavy ribbing into the disk casting. In the modern valve these ribs are placed within the disk, which of course is hollow, and its outer surfaces are made as smooth as possible in order to present the least resistance to water flow. A characteristic streamline cross-section is found in almost all of the larger modern valves.

When the size of a valve body or disk is such as to make it impossible or inconvenient to transport it on standard railway equipment or to handle it easily with the derrick or crane facilities at the point of installation, recourse is had to sectionalization. The body can be made in two or more sections, with radial, flanged, bolted joints. In certain cases shrink keys have been used, as shown in Fig. 1. Disks of unusually large diameter are likewise made in two or more sections. Those comprising two sections may have the parting plane between the halves either at right angles to the axis of rotation, as shown in Fig. 2, or passing through the center line of the valve shaft, as shown in Fig. 1.

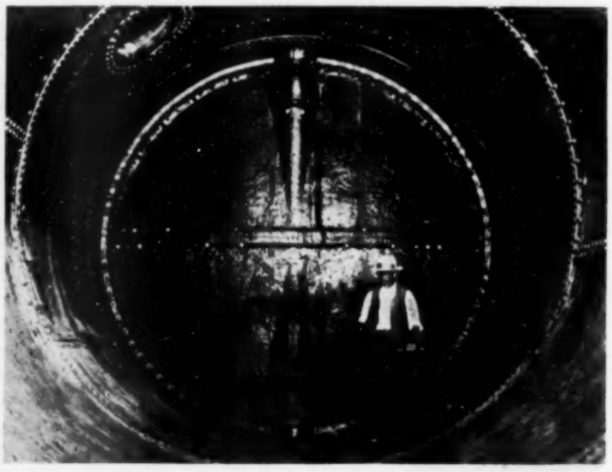

Fig. 2 Butterfly Valve 14 Ft. in Diameter for 40-Ft. Head
(Parting plane between halves at right angles to axis of rotation.)

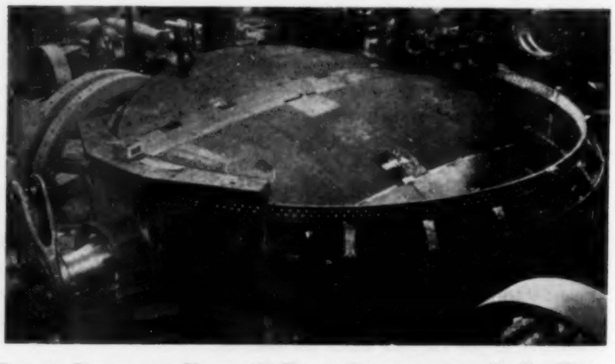

Fig. 3 Butterfly Valve 27 Ft. in Diameter for 89-Ft. Head
(Three-piece disk, with the two parting planes parallel to disk axis.)

When a three-piece disk is used, the two parting planes are preferably located parallel to the disk axis, as shown in Fig. 3, and the center section is made to carry the principal bending moments. Bolted joints are used in assembling the sections and may be located either internally or externally to suit the design. The internal joint has the advantage of leaving the outer surface of the disk free from irregularities, although the same result can be obtained by covering an externally bolted joint with a steel plate, welded or otherwise firmly attached to the face of the disk.

The bearings, which are located in the valve body and which carry the main valve shaft, are of conservative design, with large areas to transmit properly the enormous forces from the disk to the body structure without causing excessive bearing pressures.

It is quite common practice to design these bearings so that they can be removed from the body without disturbing the disk or shaft. Renewable bronze bushings are usually fitted to the bearings, and means are provided for ample lubrication, usually grease, forced into the bearings under high pressure.

Means for Reducing Leakage

Probably the most common method of insuring a practically watertight joint between the closed disk and the body of a butterfly valve is the use of metal seal rings. These are usually made of bronze or some similar non-ferrous metal and can be attached either to the body or the disk, or to both. Their design involves a means for adjusting the seal-ring position on the body or disk in order to make it match accurately the surface against which it seats.

When the ring is attached to the disk, it is usually made of comparatively thin cross-section and often sectionalized into several pieces with overlapping joints. It may be provided with a series of jackscrews or adjustable wedges, located in the disk as shown in *A*, Fig. 4, and by means of which it can be moved out radially in order to bring it into contact with the internal bore of the valve body or body-seat ring. Suitable clamping bolts make it possible to hold it firmly in its proper position after the adjustment is made. If both the adjustable seat ring on the disk and the seating surface in the body are made with a tapering or inclined contact surface, it is possible to have the disk at right angles to the axis of the valve body when it is in its tight closed position, and this is oftentimes desirable. The seating surface in the body may be machined on the body casting itself, although it is frequently desirable to attach a stationary bronze seat ring to the internal bore of the body, thus making it possible to renew the ring should excessive wear occur.

Another type of adjustable disk-seat ring is similar to that described, except that it fits into a slot in the edge of the disk and is so arranged that when the disk is moved into the closed position, pressure water in the penstock from the upstream side of the disk is used to force the seat ring firmly against the seating surface in the body. This construction is shown in *B*, Fig. 4.

In still another type where the closed position of the disk is at an angle of several degrees with the axis of the valve body, as shown in *C*, Fig. 4, the seat ring is loosely held by screws in a slot in the edge of the disk. As the disk comes to the closed position, the seat ring is brought into contact with the cylindrical machined surface in the valve body and is forced back into the disk against a special packing ring placed in the bottom of its groove, and in such a way as to serve not only as a flexible backing for the seat ring, but likewise as a gasket to prevent leakage.

Another very effective arrangement, shown in *D*, Fig. 4, consists of an adjustable bronze ring attached to the internal bore of the valve body. Radial movement of this ring is effected by means of jackscrews and bolts, all of which are adjusted from the outside of the valve body. A renewable ring may or may not be attached to the outside periphery of the disk itself.

One of the most ingenious, simple, and effective sealing devices is the rubber-tube seal ring, shown in *E*, Fig. 4. This is particularly well adapted for use on very large butterfly valves, and in its usual form consists of a heavy rubber tube set in a groove or slot in the valve body at the point where the disk seats. This tube is connected with a pressure system capable of supplying water at a pressure somewhat in excess of that which exists within the valve and penstock. As the disk of the valve is moved into the closed position, this tube is deflated, and after it reaches the closed position, pressure water is admitted to the tube, causing it to expand and to make a tight contact with the machined periphery of the disk adjacent to it. This arrangement has given very satisfactory service, and can be varied by placing the rubber tube in the outer edge of the disk instead of on the valve body.

The fact that these methods of obtaining a tight joint between disk and body are effective in reducing leakage is evidenced by data obtained from the various butterfly valve manufacturers and indicating the leakage that took place under actual test for valves of different diameters under a wide range of test pressures. All of these valves were fitted with special sealing devices to reduce leakage. (See Table 1.)

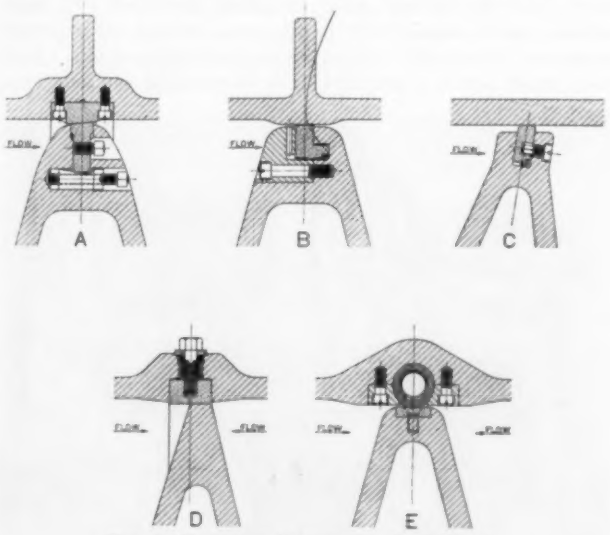

Fig. 4 Types of Adjustable Seal Rings

TABLE 1 LEAKAGE TEST DATA FOR BUTTERFLY VALVES

Diameter of valve, in.	Leakage test pressure, lb. per sq. in.	Leakage, c.f.s.
36	80	0.002
36	130	0.003
48	140	0.004
51	375	0.015
60	155	0.016
66	200	0.033
66	210	0.013
72	110	0.002
72	260	0.025
72	403	0.007
78	65	0.001
78	67	0.008
78	120	0.002
84	48	0.001
84	145	0.014
84	310	0.011
90	103	0.003
90	109	0.004
114	125	0.018
150	56	0.011
168	143	0.045
168	286	0.095
204	78	0.022

Valve-Operating Mechanisms

No attempt will be made to describe all of the different types of valve-operating mechanisms that have been devised for the operation of the many kinds and sizes of butterfly valves in use. There are, however, a limited number of typical arrangements that find general application and during the last few years there have been a few unusual installations that merit some mention.

For the smaller-sized butterfly valves, operating under low and medium penstock pressures, a mechanical operating stand consisting of gearing (which may be of the spur, worm, or bevel type) is well suited. Such a mechanism is ordinarily actuated by means of an electric motor arranged for push-button control from some convenient point adjacent to the valve, and the gearing operates a threaded nut that engages with a spindle or stem connected to a lever on one end of the main butterfly-valve shaft. An operating stand of this type is normally placed above the valve, and the electric motor is usually provided with limit switches and a solenoid brake to prevent overtravel and to permit centering the butterfly disk in its exact closed position. Auxiliary hand control is provided for by a handwheel and some type of clutch device that will permit throwing either the electric motor or the handwheel into mesh with the gearing. A mechanism of this kind has the advantage of being self-locking in any position and is comparatively simple to design, where the forces involved in the operation of the valve disk are not excessive. Figs. 5 and 6 are typical of this class of equipment. A threaded spindle attached at its lower end to a lever on the main valve disk engages with an oscillating nut in the operating stand. Fig. 7 shows a somewhat different type of operating rig in which a vertical threaded lead screw raises or lowers a trunnion nut connected to the main valve lever by means of links.

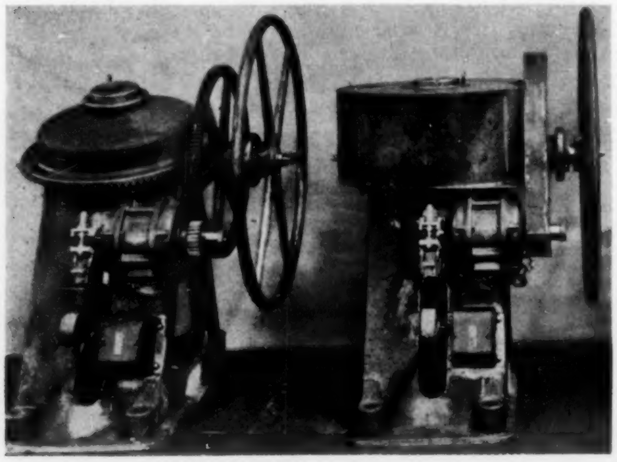

FIG. 5 ELECTRIC-MOTOR-DRIVEN OPERATING STANDS

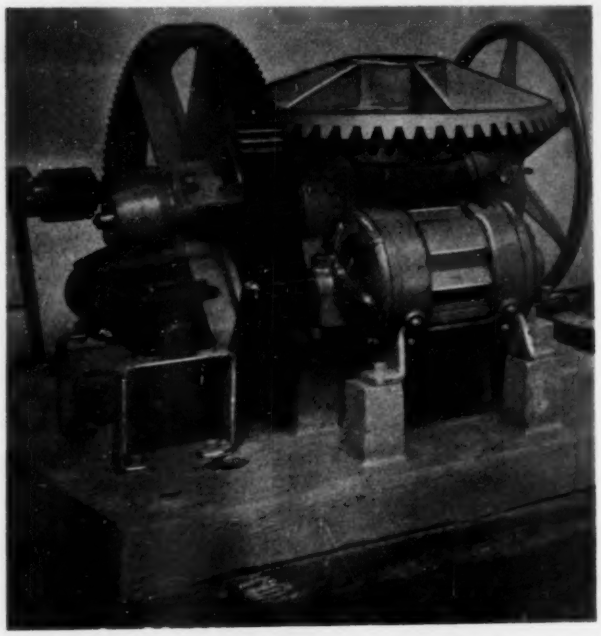

FIG. 6 OPERATING MECHANISMS WITH ELECTRIC-MOTOR DRIVE

There is a definite limit, however, to the amount of power that can be generated by this type of operating gear without involving the use of excessively large and cumbersome machinery. This limit is often exceeded with some of the equipment in use today, and for the larger butterfly valves, as well as those that operate under high heads, some form of hydraulic operating mechanism can be employed to advantage.

Where the penstock pressure is relatively high, a hydraulic cylinder receiving pressure water directly from the pipe line provides an effective means for operating the butterfly-valve disk. Water is admitted to either end of the cylinder, which may be of the vertical or the horizontal type, through a small control valve, which can be actuated by an electric motor or a solenoid arranged for remote control.

As penstock valves are not ordinarily required to remain in other than the full-closed or full-open positions, there is no necessity for having the mechanism self-locking at intermediate points. However, if this self-locking feature is desired, a simple restoring mechanism may be added to the operating cylinder that will hold the valve disk in any mid-position desired. A mechanical safety lock is sometimes provided at each end of the piston stroke so that the disk may be securely held in either the closed or the open position. The operating cylinder used with this type of mechanism may be of the oscillating type, mounted on trunnions, or of the stationary type, with a separate connecting rod and crosshead; it may also be of the trunk-piston type.

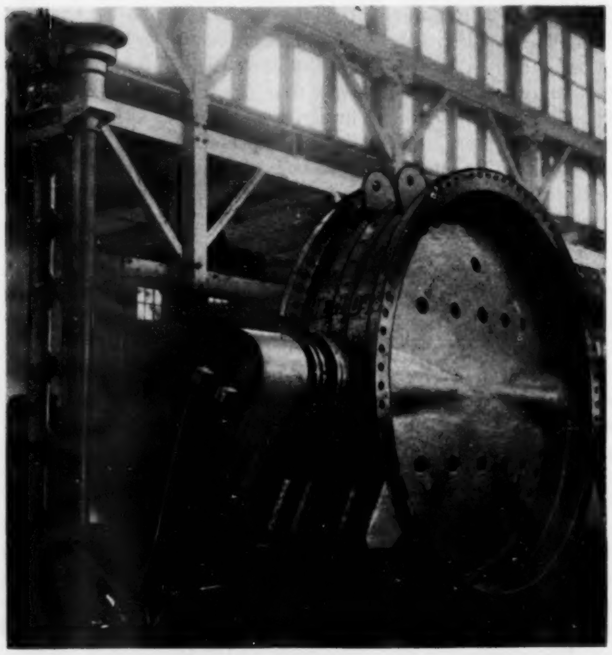

FIG. 7 BUTTERFLY VALVE 10 FT. 9 IN. IN DIAMETER WITH OPERATING MECHANISM

For the operation of very large butterfly-valve disks under low penstock pressures, an operating cylinder, using water from the pipe line, becomes excessively large and unwieldy. In such a case it is customary to use an operating cylinder of comparatively small dimensions, which is supplied with high-pressure oil from an accumulator or pressure tank, charged by a pump driven by an electric motor. An arrangement of this kind is particularly advantageous where several large butterfly valves are located adjacent to each other and where pressure oil can be delivered to all of them from a single source of pressure-oil supply.

An interesting new type of hydraulic mechanism that has been used on large butterfly valves built within the past few months is illustrated in Figs. 8 and 9. The hydraulic cylinder utilizes oil at about 1500 lb. pressure, supplied to it by means of a special high-speed multi-cylinder reversible displacement pump, mounted directly on top of the cylinder itself and direct-connected to an electric motor. No pressure tank or accumulator is needed, as the pump is connected directly to the cylinder and simply transfers oil from one side of the piston to the other. Instead of the usual limit switches, special flow motors are used at each end of the piston stroke to break automatically the electric-motor circuit and to bring the mechanism to rest. At the end of its stroke in either direction the piston comes in contact with a rigid mechanical stop, the oil pressure behind the piston builds up to a point in excess of its normal operating pressure, and it is then discharged through an adjustable pressure relief valve into the flow motor, closing an electric contact and shutting down the electric motor driving the pump. To prevent accident when work is being done within the valve or penstock, special shut-off valves with safety locks are located in the high-pressure oil pipes at either end of the cylinder. A handwheel that can be thrown into mesh with the pump provides a means for auxiliary hand operation. The upper side of the operating cylinder is cast to form a small tank, which provides a source of auxiliary oil supply to make up any leakage that may occur through the piston-rod glands of the operating cylinder.

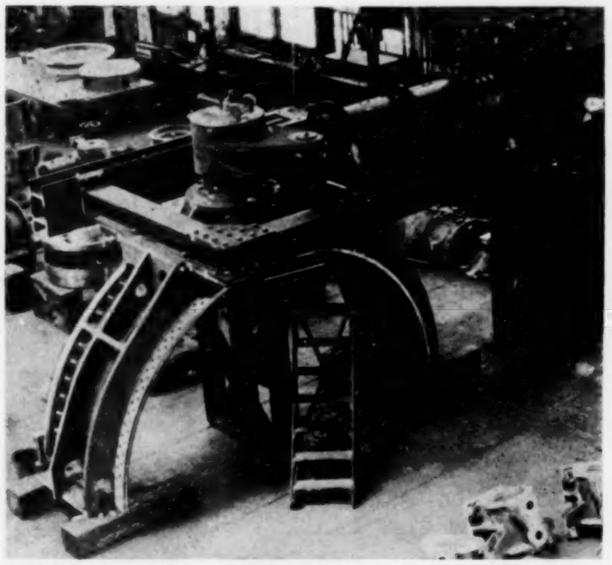

Fig. 8 Upper Half Body of 14 Ft. Diameter Valve With Operating Mechanism

Automatic Features and Bypasses

When a butterfly valve is located at the upper end of a penstock serving a modern hydroelectric plant, it is often required that it be so arranged as to close automatically should the velocity of the water column exceed normal—as, for example, in the case of a rupture in the pipe line below the valve. Such provision will insure prompt action in reducing the flow of escaping water down the penstock grade and will protect the power house and penstock supporting structures in the event of a break.

One device that has been thoroughly effective in accomplishing this automatic closure consists of two pitot tubes inserted within the penstock, adjacent to the butterfly valve, and connected by means of brass or copper tubing to a special diaphragm type of control valve that is extremely sensitive to pressure changes. The control valve can be connected up to actuate the operating mechanism of the butterfly valve, whether it be of the mechanical type driven by an electric motor or of the hydraulic type. If the butterfly valve or the penstock adjacent to it is designed to provide some venturi effect, piezometers can be used as effectively as the pitot tubes.

Another device often used and one that is less expensive than the mechanism just described consists of a vane or paddle located within the penstock and adjacent to the valve, and mounted on a lever that carries an adjustable balancing weight at its outer end. The weight can be set in such a way as to make the vane sensitive to any desired water velocity within the penstock, and by means of a suitable tripping device cause the operating mechanism on the butterfly valve to move its disk to the closed position when excess velocity occurs.

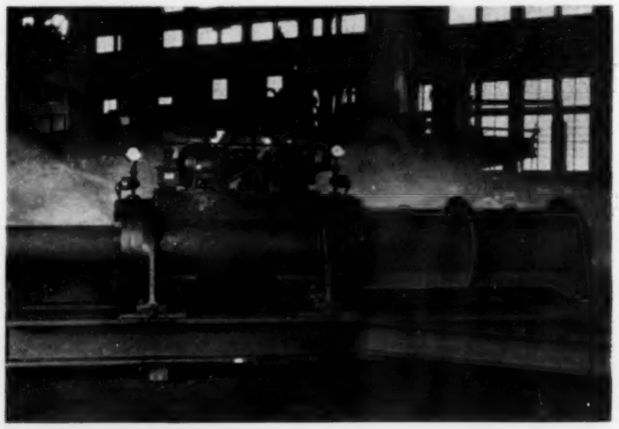

Fig. 9 High-Pressure Operating Cylinder With Pumping Unit

There are many cases on record where similar devices to cause automatic closure have been used with butterfly valves located at the lower ends of penstocks and adjacent to hydraulic-power units and for the purpose of protecting the power-hour structure and equipment in case of a ruptured turbine casing.

Another safety device that has been applied to many valves now in actual service makes it impossible to open the main valve until the pressure on both sides of the closed disk has been equalized. This is of use in preventing disastrous penstock pressure surges that can easily result from the sudden opening of a large valve, permitting the entry of a large volume of water under high pressure into an empty pipe line or turbine casing. Very little expense is involved for the addition of this automatic protection to practically any kind of valve-operating mechanism, and its use definitely eliminates the human factor in making certain that bypass equipment, almost always installed with high-head butterfly valves, will be used in the proper manner.

Bypass valves are of many types, depending on the conditions under which they are installed, and they may be connected directly to openings provided in butterfly-valve bodies, as shown in Figs. 10 and 11, or placed in piping that connects to the penstock above and below the butterfly-valve body. Under high heads and because the bypass is inherently a free discharge valve, the present-day trend is toward the special needle type, as a bypass valve of this construction is more certain to be free from vibration and to insure ease of operation than is an ordinary gate valve. This same needle type of bypass is also well suited for quick automatic opening.

Butterfly Valves for Free Discharge

A description of the modern butterfly valve would not be complete without mention of its comparatively recent use for free-discharge service.

For many of the sizable dams constructed during recent years it has been necessary to provide for the discharge of comparatively large quantities of water that must be bypassed at certain seasons of the year to supply the needs of power plants, irrigationists, and owners of riparian rights located below the dams. Due to the fact that the water level in a storage reservoir usually varies within wide limits, it is necessary to locate the discharge valves in the dam well below the high-water elevation, and the valves will be required at times to operate under relatively high heads. Daily or monthly variation in the quantity of the discharged water, coupled with the fact that the head on the valves changes with the water level, has made it necessary to develop a special type of valve known as the discharge regulator. This is so designed that its orifice can be readily and quickly adjusted for any required discharge up to its maximum and that no excessive wear or cavitation will result, although the valve be operated at partial opening for long periods.

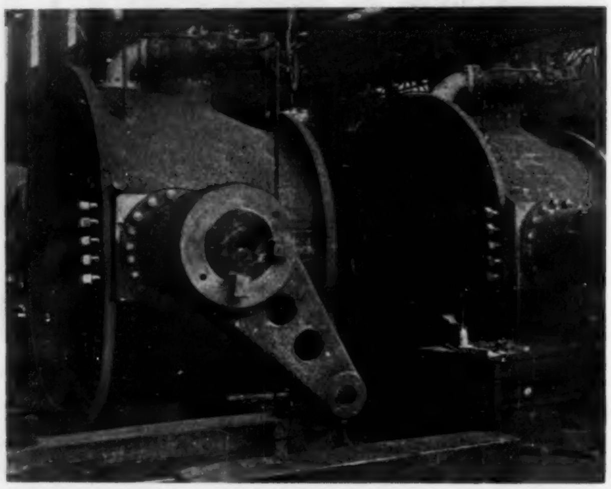

Fig. 10 Two Butterfly Valves Each 9 Ft. in Diameter for 836-Ft. Head

Early free-discharge valves used under very low heads were often ordinary slide gates, with circular or rectangular orifices, but as higher dams and greater discharge pressures were encountered, they proved unsatisfactory for continuous service, although still used in some installations as emergency sluices. In order to eliminate the difficulties experienced and as pressures were continually increasing, new forms of discharge regulators made their appearance, and the needle type of valve soon demonstrated its superiority over the slide gate.

However, with the increase in size and pressure, the forces required to move the needles assumed large proportions, and required the use of powerful operating mechanisms. This led to the development of the modern large needle type of free discharge regulator known as the Larner-Johnson valve, and in which the needle forms part of a hydraulic plunger, moving within a cylinder located inside the body of the valve, and actuated by pressure water taken from the conduit. Many of these valves have been operating for years, under conditions that would make impossible the use of any of the older types of equipment. Water wear and vibration have been eliminated, and mechanisms that are both simple and reliable are used to control the hydraulic forces that actuate the plungers or needles.

When the total water quantity that must be discharged through a dam is no more than can be passed through a single valve, the Larner-Johnson regulator fulfils all requirements, and will operate just as effectively at positions of partial opening as at full capacity. Of recent years, however, dams of such size are being constructed, and the required discharge is in many cases of such magnitude, that a single free-discharge regulator will not suffice, and a battery of these units becomes necessary. If these are all of approximately the same size and have about the same capacity, only one of them need be a regulator, capable of operating at fractional openings, and the remaining valves may all be of a type that is required to function only in the wide-open position, and hence may be of much simpler and less expensive construction than the regulator.

It is for this type of auxiliary free-discharge service that the butterfly valve has already proved itself well adapted and thoroughly dependable, even under fairly high heads. Actual tests, performed both in the laboratory and in the field, have enabled the designer to determine accurately the forces imposed on the disk as it is moved between open and closed positions in the discharging water column, and to design an operating mechanism that will provide plenty of power and insure freedom from vibration.

The shape of free-discharge valve bodies and disks now in use differs very little from those of butterfly valves in penstock service, although research work now in progress may indicate the desirability of adopting special features permitting the discharging jet to break clear of the valve structure at a sharp edge and admit air at certain points to prevent the formation of a vacuum, and the resulting tendency toward cavitation.

At the present time there are no less than ten butterfly valves, up to and including 84 in. in diameter, rendering eminently satisfactory service under free-discharge conditions; some of these operate under heads well in excess of 200 ft. Even under the most severe conditions, vibration is entirely absent and no difficulties have been experienced. Some of these have actually seen use as discharge regulators and have been operated for considerable periods in a position of fractional opening, even though not originally intended for this kind of service. Unless their continued use under these conditions causes cavitation and wear not yet apparent, it seems reasonable to predict that the butterfly type of valve may well fulfil all the requirements for an economical and dependable free-discharge regulator.

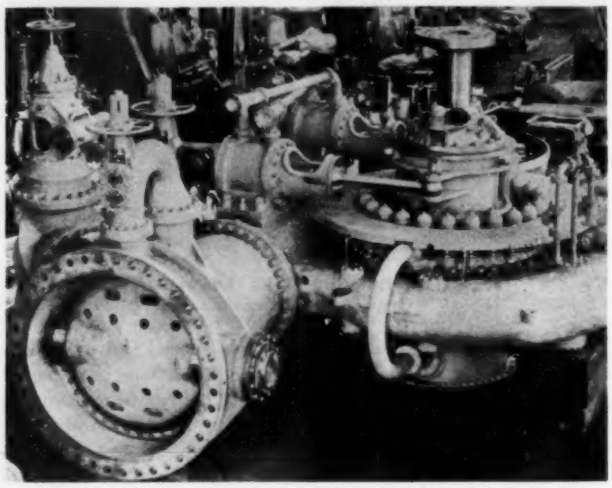

Fig. 11 Butterfly Valve 79 In. in Diameter for 940-Ft. Head (Located at inlet of large turbine casing.)

Butterfly Valves for High Heads

Among the butterfly valves now installed and operating under extremely high heads are those built for the Pigeon River Plant of the Carolina Power & Light Company, each 9 ft. in diameter. Two of them are shown in Fig. 10. The normal static head under which they are installed is 836 ft., but in order to take care of possible pressure rise in the penstock, it was considered necessary to design them for a maximum head of approximately 1215 ft. They are located at the lower end of their penstocks, adjacent to turbine casings, and the operating mechanism for each of them, not shown in the illustration, is of the hydraulic type, consisting of a cylinder and piston actuated by penstock-water pressure that is admitted to the cylinder through an electric-motor-operated control valve. Each butterfly valve is provided with a bypass valve by means of which the turbine casing below can be primed or filled with pressure water before opening the disk of the butterfly.

Provision is also made in the design of the control valve so that the main butterfly-valve disk will close automatically in case the pipe line or turbine casing below it is broken and the velocity through the butterfly valve increases beyond normal. In this case each of the valve bodies was made of tapering construction in order to eliminate the use of a separate taper section. Each of the disks is equipped with a special bronze seal ring in order to reduce the leakage to a minimum.

In the Oak Grove Plant of the Portland General Electric Company on the Clackamas River near Portland, Ore., there are two 79 in. diameter butterfly valves installed under an operating head of about 940 ft., and both serving as main inlet valves for large reaction turbine units. One of these valves is shown in Fig. 11 and is operated either by means of an electric motor or by an impulse water motor taking its supply directly from the penstock. The other is operated by means of a hydraulic cylinder with control valve and likewise takes its water supply from the penstock. Both valves have their disks equipped with bronze seal rings to eliminate leakage, and renewable bronze rings are likewise placed in the valve bodies at the points where the disks seat when in the closed position. Bypass valves are included. Both valves were subjected to a hydrostatic test at the manufacturer's works under a pressure of 1400 ft., or about 615 lb. per sq. in. Each of them has a tapering body, which is more or less characteristic of the high-head type of butterfly valve.

Butterfly Valves of Large Diameter

Probably the largest diameter butterfly valves ever built are those installed at the Conowingo Development of the Susquehanna Power Company, located near Philadelphia. These are each 27 ft. inside diameter and work under a head of 89 ft. Figs. 3 and 12 give a conception of their general construction and show the manner in which, because of their large size, it was necessary to sectionalize each body and disk. In this case each disk has a vertical axis of rotation and is operated by means of a hydraulic cylinder, using pressure oil supplied from a central oil-pressure system with accumulator tank. Aside from their large diameter, these valves are of particular interest because of the rubber-tube seal rings located in the valve bodies.

At the Calderwood Development of the Knoxville Power Company, where the head is 213 ft. there are three 16 ft. diameter butterfly valves that also are equipped with rubber sealing tubes, but in this case the tubes are placed in the periphery of the disk.

In the Diablo Plant of the Seattle City Lighting Department on the Skagit River near Seattle, two 14 ft. diameter valves are being installed under a head of 330 ft. In this case each of the disks rotates on a vertical axis, and both the bodies and the disks are sectionalized. Each disk is equipped with an adjustable bronze seal ring, seating against a stationary bronze seat ring set into the valve body. Operation of the disk is effected by means of a high-pressure hydraulic cylinder, shown in Figs. 8 and 9, and using oil supplied from a pump driven by an electric motor. Provision is made for automatic closure in case of excess velocity.

Fig. 2 shows a 14 ft. diameter butterfly valve with vertical spindle and adjustable bronze seal ring installed in the diversion dam at the upper end of the tunnel that supplies water to the Chelan Power House of the Washington Water Power Company. This valve is actuated by means of an electric motor and gearing located directly above it.

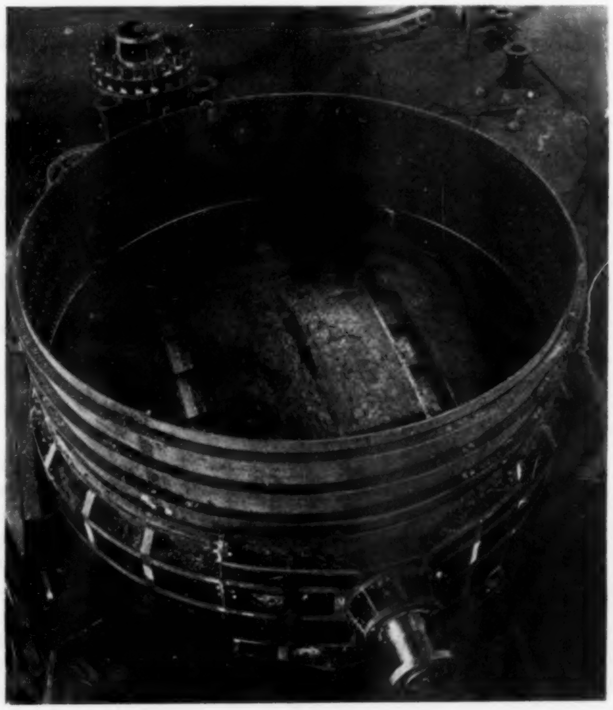

Fig. 12 Butterfly Valve and Connecting Pipe 27 Ft. in Diameter for 89-Ft. Head

At the Ariel Power House of the Northwestern Electric Company on the Lewis River near Portland, Ore., a 17 ft. diameter butterfly valve with a horizontal spindle is located at the inlet to the turbine casing where the head is 180 ft. This valve is shown in Fig. 1, and is at the lower end of a large penstock which passes directly through the lower part of the dam, the power house being located adjacent to the dam itself. An adjustable bronze seal ring is attached to the periphery of the disk. Operation is by means of a high-pressure hydraulic cylinder with an oil pump driven by an electric motor, of the type shown in Fig. 9, and excess velocity causes automatic closure.

Conclusion

Certain of the more important features of modern butterfly valves have been described in order to give a conception of the progress that has taken place during recent years. Difficulties accompanying the use of these valves under unusually high pressures have been successfully overcome, effective means have been devised practically to eliminate leakage, and by adopting special types of construction it has been possible to build them of very large size. New types of operating mechanisms have been developed capable of applying tremendous forces to the valve disks. Automatic operation has been brought to a state of dependability and perfection considered impossible several years ago, and the large number of these valves that are today in successful operation bears witness to the ingenuity and skill of the engineers responsible for their development.

The widespread use of butterfly valves for controlling water flow in large pipe lines proves their fitness and adaptability to this type of service.

The outstanding development in butterfly-valve design in the past four years is its adaptation to free-discharge service, with water velocities that are 10 to 20 times those produced in the average penstock.

There is reason to believe that this type of valve, because of its simplicity of design, its reasonable cost, and its inherent flexibility, has by no means reached the limit of its possibilities.

Acknowledgment

In the preparation of this paper and in the collection of descriptive matter and engineering data the author has received valuable assistance from the following organizations, in addition to his own, and to whom he acknowledges his thanks: Allis-Chalmers Manufacturing Co., Milwaukee, Wis.; Joshua Hendy Iron Works, San Francisco, Calif.; I. P. Morris & De La Vergne, Inc., Philadelphia, Pa.; Newport News Shipbuilding & Drydock Co., New York City.

Discussion

D. J. McCormack.[2] The paper tends to widen the scope of use of butterfly valves. There is a distinct field for installing butterfly valves in pipe lines under normal heads where the velocities are relatively low—that is, at the upper ends of pipe lines or even at the lower ends before the penstock is reduced in size, but far enough away from the turbines so that the disturbance set up by the valve is not carried into the turbine. Many recent tests have shown that such a disturbance carried into the wheel results in a much greater loss than the actual head loss as measured from end to end of the valve. The valve should be set sufficiently away from the turbine to permit the flow to be straightened out before reaching the turbine casing.

Under high heads, say over 600 ft., the disk becomes so thick at the center, to accommodate the spindle and to obtain sufficient strength of all parts, that the velocity through the valve is unduly increased, and the diameter of the valve should be correspondingly increased and made larger in diameter than the pipe. However, this is seldom done. Instead, the butterfly valve is generally installed at the lower end of the pipe line, just where the pipe is reduced to a smaller diameter, and the reduction in area is effected in the valve by necking it down very decidedly. The velocity in the smaller pipe between the valve and turbine, of 25 to 35 ft. per sec., necessarily sweeps the disturbance caused by the valve into the turbine before it can be smoothed out. For this purpose, a valve providing a flush passage through it is the proper thing to use. It is more than justified, since there is practically no head loss through it and no disturbance set up to affect the results of the turbine.

Due to its design characteristics, a butterfly valve is not suitable for free discharge work. In order to eliminate vibration, the disk has to be held rigidly in any position, which is almost impossible with a butterfly valve, because the forces are transmitted from the disk, through a stem and lever, to the operating mechanism. The deflection and lost play in the disk, stem, lever, and operating mechanism do not permit of this. Experience in trying to use a 9-ft. butterfly valve for free discharge under 200-ft. head has proved that some other form of valve should be used for this work. The vibration is terrific.

In 1913 the writer was granted a patent on a flexible sealing ring for a butterfly valve and thought at that time that it was a wonderful thing to make such a valve tight. However, since then, experience has materially changed his ideas. Seal rings in the periphery of the disk, unless they are rigidly and positively held tight without any possibility of loosening up, are a dangerous thing. It is much better to use a babbitted or flexible seat rigidly attached to the housing. There is very little, if any, leakage for the greater part of the periphery of the disk at top and bottom. Most of the leakage occurs at the corners where the stem passes through the housing. A detachable seal ring is of no benefit in reducing this leakage.

When discussing butterfly valves for a wider scope than ordinarily used, as brought out in the paper, it is proper to mention the Dow type of valve, which has some advantages for this work not inherent in a butterfly design of valve. Due to the inclination of the disk in the valve housing, one-half of the disk will be pressed up tightly to its seat by the water pressure on the upstream side. The other half of the disk in a butterfly valve is free to deflect away from its seat. However, in the Dow valve a connecting rod attached to the outer edge near the middle of the disk and designed with a toggle-joint action, takes up any such deflection and provides an absolutely tight periphery on that half of the disk. In the wide-open position, the disk is rigidly held in the fixed position without any lost motion occurring through the stem, lever, etc., which would be the case with a butterfly valve. Around the trunnions special adjustable glands are provided to seal off the ordinarily large amount of leakage between the disk and the housing and at the four corners where the stem passes through. The thickness of the disk does not have to be so great, because the stem need only be designed to take care of bending. Therefore, the head loss through the Dow valve is appreciably less than through a butterfly valve. The Dow valve does not need a bypass, but can easily be cracked open slightly to fill the penstock or turbine casing below. It has been used extensively for free discharge work and stands up remarkably well, with practically no vibration.

Author's Closure

The discussion brings out a few points which have often been discussed by hydraulic engineers and in regard to which some further information will undoubtedly be of interest.

Under high heads, in order to get sufficient bearing area to support the spindle or shaft of a butterfly valve, the bearings and shaft must be of comparatively large diameter, thus requiring a fairly thick disk. In practically every case, however, the butterfly-valve shaft, when designed of proper diameter to carry the load on the disk when it is in the closed position, is more than sufficient to give the necessary torsional rigidity for outside operation by means of a lever instead of using any type of connecting rod attached directly to the disk itself and inside the pipe. Properly proportioned water passages can be obtained through these high-head butterfly valves by using a valve body diameter somewhat larger than the connecting pipes on both sides, adopting a proper curvature for the inside of the butterfly-valve body, and using suitable designed taper sections at one or both ends of the valve body to connect it to the line.

A large number of these high-head butterfly valves are in successful use today, and as far as is known, they have not shown any vibration whatever, even at the partially closed position with full velocity flowing through them. In fact, as stated in the paper, some of these butterfly valves are operating under unusually high heads. Two free-discharge valves are installed under a head of

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[2] Sales Manager, S. Morgan Smith Company, York, Pa. Mem. A.S.M.E.

280 ft., and there seems to be no reason for not going to even higher heads and obtaining successful operation from the butterfly type of valve under free-discharge conditions.

Many years ago hydraulic turbines were equipped with wicket gates with "inside gate operating mechanism." In other words, the links and pins which connected the turbine gates to the gate-shifting ring were placed inside the water passageway, and therefore subject to cavitation, oxidation, water wear and erosion from grit, etc. As improvements were made in turbine design, it became standard practice on the part of hydraulic engineers to remove all possible mechanical connections from the water passages, and the present form of turbine guide vanes, with spindles extended on one or both ends through stuffing boxes and operated by means of levers on the outside, became the accepted method of accomplishing the desired results. This change resulted in no trouble from vibration, even though the turbine guide vanes closely resembled free-discharge valves in their function and the velocity of the water passing through them is in many cases very high.

The butterfly type of valve, as described in the paper, is sometimes called a wicket valve or pivot valve, and in principle of operation is exactly like the modern form of turbine guide vane or wicket gate which has been accepted and standardized by all of the large hydraulic turbine manufacturers without exception. It seems reasonable, therefore, to expect the butterfly type of valve to perform successfully as a free-discharge valve, even though the water passing through the valve and over the disk may be traveling at a comparatively high velocity.

In selecting the proper type of valve, either for pipe line or free-discharge hydraulic service, the engineer should consider which of two general types of design is better suited to his needs. In the true butterfly type of valve he has the outside operating mechanism with a streamline form for the valve disk, together with tapering body, etc. to insure undisturbed flow and practically constant velocity with the least possible obstruction from the disk. All of the operating mechanism is outside of the valve, and therefore is subject to inspection at all times and easily arranged for thorough grease lubrication.

On the other hand, he will have available a type of valve similar to the butterfly, but having the disk operated by means of a connecting rod which attaches to the disk itself near the periphery on the downstream side. This connecting rod must in turn connect to a crosshead located some distance downstream from the center line of the main butterfly-valve spindle or shaft. This, of course, results in a longer body for this type of valve than for the true butterfly. With the disk of this type of valve in the open position, the connecting rod forms an obstruction to the water flow and is at right angles to the shaft of the butterfly-valve disk. In order to prevent unnecessary water wear on the pins and connections between this connecting rod and the disk, a housing of some form is usually built around it, which still further increases its cross-sectional area, and therefore reduces the size of the water passage. In some cases this additional obstruction will more than offset the saving which could be effected by making the disk any thinner than is necessary for the true butterfly type of valve.

It may also be interesting to note that adjustable seal rings, both of the metal and rubber types, have been successfully used for many years, and if properly designed, will not loosen up, nor are they dangerous in any sense of the word. Within the author's experience, there has been no case reported of properly designed seal rings causing any trouble whatever through becoming loose, and the adjustable feature of these seal rings will always permit of their being accurately placed in such a way as to prevent leakage due to the distortion of the disk from pressure on its upstream side, from settlement of the concrete foundation, and from distortion of the valve body or any other part of the mechanism.

It is quite common practice in modern hydroelectric work to use a large butterfly valve, sometimes under a high head, without a bypass. An instance of this is the case of two large 14-ft.-diameter true butterfly valves with outside operating mechanisms, located at the main inlet of two turbines operating under a head of about 330 ft. Filling of the turbine casing is accomplished by means of cracking open the main butterfly-valve disk.

It is believed that engineers engaged in the design and operation of waterworks systems, particularly where they involve piping of comparatively large diameter, can well afford to investigate the advantages of the butterfly type of valve. Due to the inherent low cost of this type, it should find an application in large city networks where it can be used as a quick closing, sectionalizing valve to close automatically in the case of a break in the water main, which without such protection could easily do a great amount of damage to large buildings and their foundations, subways, etc. Automatic closure of these butterfly valves can be accomplished in any one of a number of different ways, such as sudden drop in pressure, sudden increase in velocity, etc. The butterfly type of valve, being inexpensive, would seem to be quite applicable to service as a check valve in waterworks lines where space is limited and cost is a primary consideration. It is believed that a study of modern butterfly-valve design on the part of the waterworks engineer will unquestionably develop some interesting facts.

HYD-54-3

Research Institute for Hydraulic Engineering and Water Power

Description of the Latest Project of the Kaiser Wilhelm Society for the Promotion of Science at Obernach, Near Munich, the World's Largest Open-Air Hydraulic Experiment Station

By HUNTER ROUSE,[1] CAMBRIDGE, MASS.

HISTORY

TO Oskar von Miller, founder of the renowned Deutsches Museum at Munich and responsible for the development and perfection of the Bavarian power system, is given first credit for the movement that has led to the creation of the world's largest open-air hydraulic experiment station. In an address before the Kaiser Wilhelm Society for the Promotion of Science early in 1924, Dr. von Miller proposed the establishment of an institute of such size that hydraulic models could be studied at practically full scale. Engineers and investigators, for decades the sponsors of experiments on an exceedingly small scale, had begun to realize the limitations and possibility of error in model study requiring complete dependence upon the theory of similitude, and turned hopefully to a means of vindicating theory and practice in a more conclusive manner.

In April, 1926, there was organized the Research Institute for Hydraulic Engineering and Water Power, latest of the many branches of the Kaiser Wilhelm Society. Contrary to the usual custom of placing the laboratory management under the staff of a technical university, the new institute was made an integral organization, financed by such bodies as the National Government and the State of Bavaria, the City of Munich, and a number of private power and industrial concerns, in return for the privilege of experimental investigation. Though the Kaiser Wilhelm Society is concerned principally with purely scientific research, the services of the institute were to be available to all who applied for them, the only charges being the actual cost of labor, material, and time.

Generally speaking, the institute was planned to serve the twofold purpose of the supervision of a permanent outdoor laboratory with equipment to carry on large-scale and full-scale model experiments and the training of a staff of engineers who could conduct investigations in various parts of the country at the actual sites in question. Prior to the completion of the station, the institute carried on experiments on the Ampelsbach near the Achensee in Tyrol, at two points on the Isar River north and south of Munich, and finally assisted in a series of flow measurements on one penstock of the Walchensee Power Plant in which four standard methods were used simultaneously.

The search for a desirable location for the new station was governed by several factors: constant water supply in sufficient quantity to provide for all future needs; large unobstructed land area to permit favorable layout of the long canals and river channels; and proximity to both the Munich headquarters and as many of the larger hydraulic works as possible. The region just to the south of the Walchensee, a lake some 50 miles from Munich at the foot of the Bavarian and Tyrolean Alps, was finally selected as that fulfilling these requirements to the greatest degree. Actual construction was begun in 1928, and by the spring of 1930 the installation of the basic equipment had been completed and preliminary rating experiments were under way.

ORGANIZATION, PURPOSES, AND METHODS

At Loristrasse 2, Munich, are the executive headquarters of the research institute, composed of an office for the leaders, with ample space for group consultations, a smaller secretarial room, and large quarters for six or eight men who do the drafting and computation of results.

Two former assistants of Dr. Thoma's laboratory in the Munich technical university are at present responsible for the leadership of the institute: Dr.-Ing. Otto Kirschmer (now Professor of Hydraulics at the Dresden Technische Hochschule) as director, and Dipl.-Ing. B. Esterer as supervisor of experimentation. On the permanent staff are one diplom-ingenieur and five young graduates of technical trade schools for mechanical engineering. During the time of experimentation all are on constant duty at the station, several returning afterward to the Munich office for the work of calculation; additional assistants are employed when occasion demands.

Labor for construction work is easily secured at the nearby villages, and as there are new projects in building throughout the summer, there is usually a force of from 10 to 20 men at work.

In a laboratory where the models are of very small scale, an assistant is generally able to take complete charge of a single experiment and without help carry to completion both the actual handling of the model once it is built and the resulting computation. Here at Walchensee the work is on so great a scale that it requires the combined efforts of from three to ten experienced men, more than one series of experiments seldom being in progress at a given time. Since accuracy is dependent upon weather conditions, once these are favorable the experiment is carried through to completion without stop, often requiring uninterrupted work in shifts for 20 or 30 hours.

While each assistant makes himself familiar with all the apparatus and experimental methods, several of them have perfected themselves in certain systems of flow measurement and represent the institute at construction sites or power plants where their services are requested. In close cooperation with the research institute is the laboratory of Dr. Thoma, where most of the staff have at some time or other supplemented their practice with small-scale study. Throughout the winter, when outdoor work is practically impossible, projects are continued indoors at the Munich technical university, and there is a hearty feeling of collaboration between the two. It is hoped that the other laboratories in the country will sense the value of this attitude, those at Dresden and Berlin having already made steps toward

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[1] Massachusetts Institute of Technology Traveling Fellow in Hydraulics, 1929–1931; now member of staff, Massachusetts Institute of Technology, River Hydraulic Laboratory.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

substantiating their small-scale results by further study through the institute.

An all-too-prevalent belief is that the Walchensee laboratory is attempting to establish the falsity of the theory of similitude. The error of this assumption cannot be too highly emphasized. Its work is on the contrary the proof that the theory of similitude is really a dependable tool, once its factors of conversion have been experimentally determined, and the general study of these conversion relations. In the past there has been little opportunity to vindicate the blind reliance upon similitude in the accurate study of water flow where the scale is reduced often a hundredfold and more, and until this is clearly illustrated by experiments upon the same structures with models varying from very small to practically full scale, many engineers will continue to mistrust small-model study.

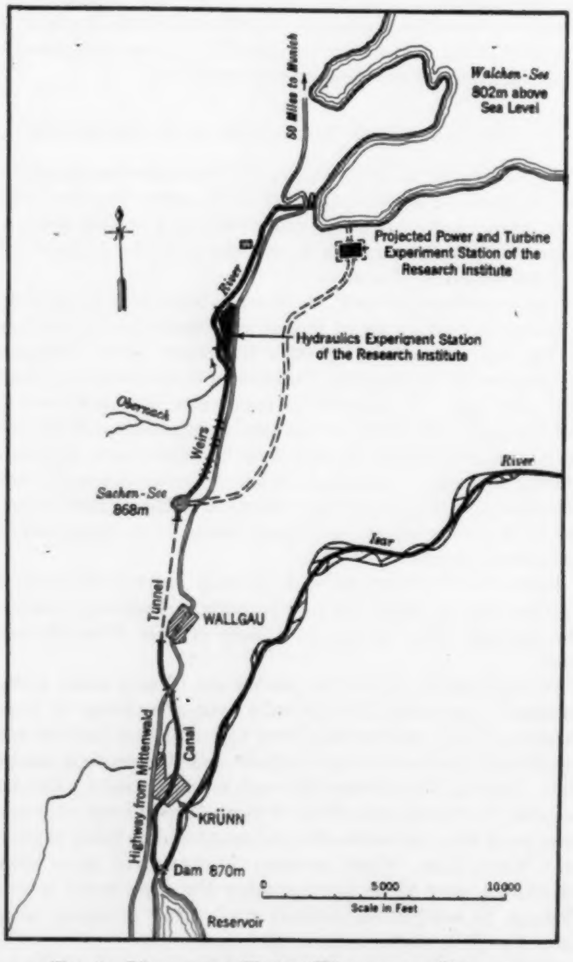

Fig. 1 Map of the Krünn-Walchensee Region

(Showing Isar diversion canal, location of the experiment station, and the projected power plant and turbine research station. Distance above sea level in meters.)

Sponsors of the latter type of research hold quite justifiably that such methods allow greater accuracy in measurement and permit thoroughness of study in a short time and with an economy that is impossible on larger structures. The research institute is not endeavoring to displace the small laboratory, but to give it new value in the eyes of the engineering world; for its directors realize as well as any that small-scale study holds a place in the experimental field which no other method can usurp. On the other hand, there are demands which only a station of the Walchensee type can meet—perfection of flow-measurement methods for commercial use, tests of canal forms and linings, siphon experiments where the atmospheric conditions cannot easily be reproduced to scale, and other experiments of a like nature; and though other experimenters hold that small-scale indoor equipment provides the greatest accuracy, it is claimed by the leaders of the institute that increasing the size as they do involves little sacrifice of exactitude in measurement and that this is offset by greater proximity to actual conditions. In brief, outside of time and financial considerations, the relative advantages and disadvantages of both systems practically compensate each other. Where they have been proved dependable, small-scale experiments are the more desirable, except under circumstances requiring actual or nearly actual proportions.

THE EXPERIMENT STATION

From its source in the Alps of Tyrol near Innsbruck, the Isar River flows over the German border and takes a northerly course toward Munich. Just before the village of Krünn, somewhat more than 5 miles above the Walchensee, the Isar valley forks, and the river takes the deeper route to the right so that it passes completely around the eastern side of the Walchensee. This lake, roughly triangular in shape, $3^1/_2$ miles long and 640 ft. in depth, has a mean surface level 660 ft. higher than that of the Kochelsee, little more than a mile to the north. The Walchensee Power Plant takes complete advantage of this excellent site, and not only draws the entire discharge from the Walchensee, but also diverts the normal Isar discharge of from 15 to 25 cu. m. per sec. (530 to 880 sec-ft.); in winter a minimum of 5 cu. m. per sec. (175 sec-ft.) is reached.

This borrowing of water from the Isar is accomplished as follows: At the valley fork near Krünn, as is shown in Fig. 1, a small regulation reservoir has been constructed, so that in dry seasons the total flow of the river may be diverted through a concrete canal and tunnel system into a small pond, the Sachensee, $2^1/_2$ miles from the Walchensee, and thence a mile farther through an open-cut channel to the point where the Obernach stream enters the valley. Here the channel ends, and its discharge joins that of the Obernach and flows through the latter bed to its outlet at the Walchensee. Just below this juncture, the Obernach bends away from the highway, circling back parallel to it again at a point $^1/_2$ mile farther downstream. This bend, with the highway on the other side, encloses a flat section of land of about 10 hectameters (25 acres), which was chosen by the institute as the most nearly ideal station site in the region. Here is ample space for development of large-scale layouts and a constant water supply large enough for all purposes.

Due to a steady fall in the valley, amounting to 66 m. (217 ft.) from the Sachensee to the Walchensee, future expansion of the Bavarian power system calls for a new plant at the southern end of the Walchensee making use of this head by means of still another diversion canal, with a branch of the institute station at this lower point for turbine and other high-head experiments. In this way the laboratory will have grounds, equipment, and water-supply facilities that surpass as a whole any existing in the world.

Station Layout

At a point some 60 ft. above the weir where the Isar River diversion channel (with a maximum flow of 25 cu. m. per sec.) discharges into the Obernach waters is the intake canal for the entire laboratory supply, as is shown in Fig. 2. The flow is regulated by two hand-driven or electrically driven vertical gates 3.5 m. (11.5 ft.) in width and rising to a maximum height of 2 m. (6.56 ft.), admitting from 8 to 12 cu. m. per sec. (280 to 425

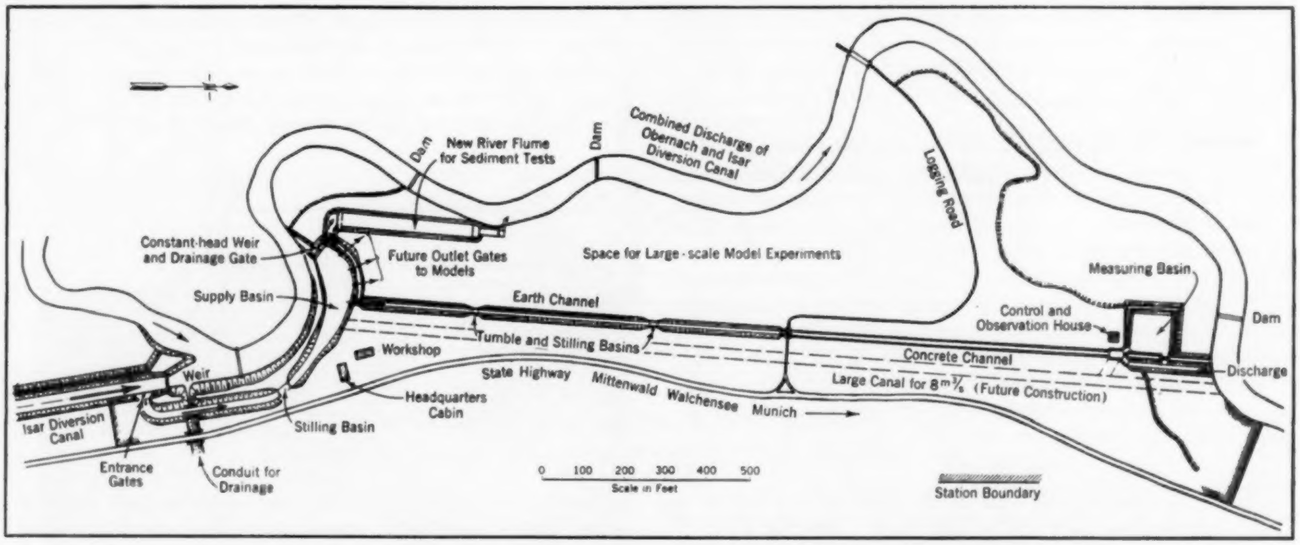
Combined Discharge of Obernach and Isar Diversion Canal
Dam
New River Flume for Sediment Tests
Dam
Logging Road
Constant-head Weir and Drainage Gate
Future Outlet Gates to Models
Space for Large-scale Model Experiments
Measuring Basin
Supply Basin
Earth Channel
Control and Observation House
Dam
Tumble and Stilling Basins
Workshop
Concrete Channel
Weir
State Highway
Mittenwald
Walchensee
Munich
Large Canal for 8 m³/s (Future Construction)
Discharge
Headquarters Cabin
Isar Diversion Canal
Stilling Basin
Entrance Gates
Conduit for Drainage
0
100
200
300
400
500
Scale in Feet
Station Boundary

FIG. 2 GENERAL LAYOUT OF THE EXPERIMENT STATION AT END OF SUMMER OF 1931

sec-ft.) depending upon the Isar discharge. This water is carried about 350 ft. through the concrete canal, parallel to the highway, and is then discharged freely into the concrete stilling basin at the upper end of the supply basin. (See Fig. 3.) Since this basin may serve as many as four or five future side-channels to separate experiments, it has been given a capacity of 2500 cu. m. (88,250 cu. ft.), with a 10-m. (32.8-ft.) constant-head weir and a 1-m. drainage gate at the west end, wasting into the Obernach. This basin is unlined, with a side slope of 1 to $2^1/_2$, the coarse gravel and clay in the soil withstanding the erosion to a fair degree. A single gage pit, of type A, described later, is used for observation of the water level.

In the original construction a single experimental canal was constructed, for a maximum flow of 4 cu. m. per sec. (141 sec-ft.). Plans have been made for a similar channel for double that discharge to be built alongside at a later date, but this is considered at present as unnecessary. This existing main canal runs almost due north from the mid-point of the outer bank of the supply basin for a distance of 575 m. (1889 ft.); inflow is regulated at the upper end by a single vertical gate 3 m. (9.85 ft.) in width, giving an opening 2 m. (6.56 ft.) high. This gate has been calibrated for discharge versus gate opening. As the canal is unlined at this end, the entrance is protected for a distance of 15 ft. by coarse wire screening over stone riprap on sides and bottom. To allow experimentation on various conditions of channel flow, and at the same time to avoid following directly the pronounced downward slope of the valley, this canal has been divided into four sections of different lengths, with a temporary wooden spill

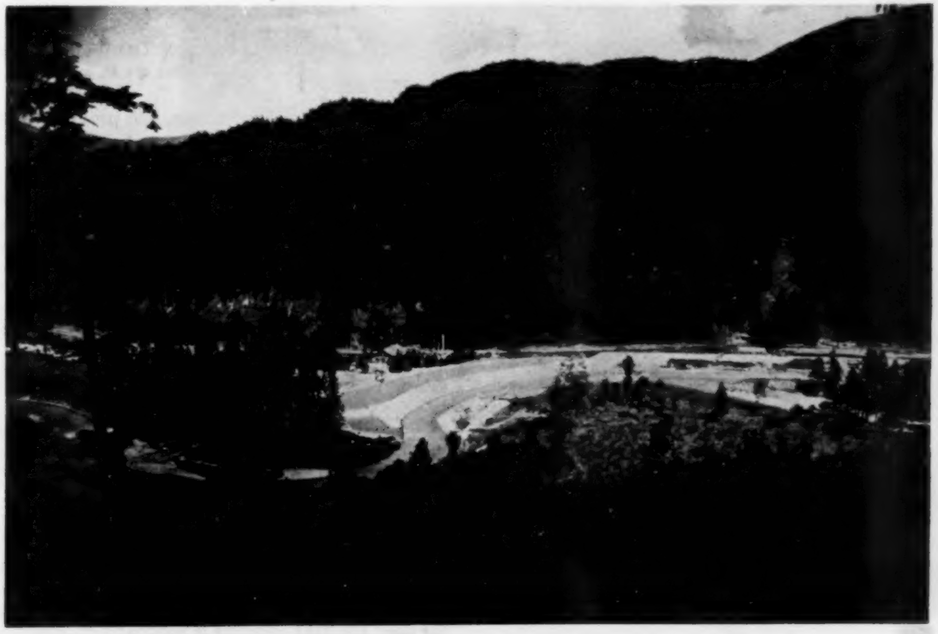

FIG. 3 VIEW OF SOUTHERN END OF STATION
(Supply basin and constant-head weir clearly visible, with main canal intake and headquarters cabin in background; Obernach bed in foreground.)

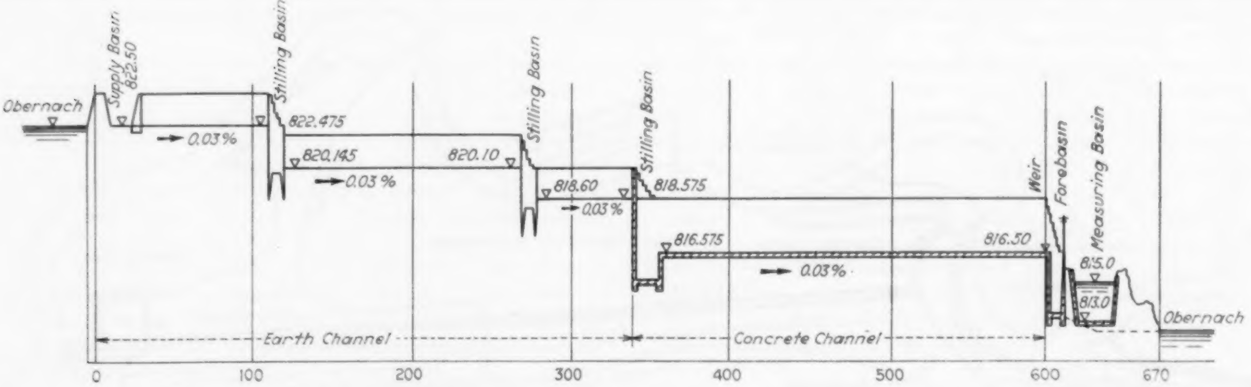

FIG. 4 FOUR SECTIONS OF MAIN EXPERIMENTAL CANAL IN PROFILE
(Scale is not exact. Elevations are mean height above sea level in meters.)

and stilling basin at the ends of the first two. Each of the individual sections has a 0.03 per cent slope. (See Fig. 4.)

The first three sections, approximately 280, 475, and 230 ft. in length, respectively, have the same trapezoidal cross-section, 9 m. wide at top, 3 m. at bottom, 1.5 m. deep. (See Fig. 5.) At first all three were simple earth canals, but the first and second have since been given a lining of tar-bound gravel of grades varying at intervals.[2] The first two spill sections have been made of coarse timbers to allow for future modification and a height of weir to vary with the discharge. The stilling basin at the end of the third earth canal has been made of concrete, extra large to give sufficient energy reduction at the beginning of the fourth section.

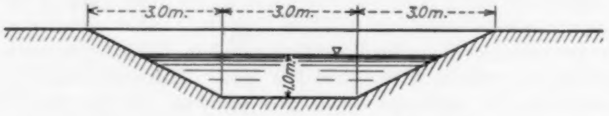

FIG. 5 TYPICAL CROSS-SECTION OF EARTH CANALS
(Velocity, 0.80 m. per sec.; wetted section, 5.0 sq. m.; flow, 4.0 cu. m. per sec.)

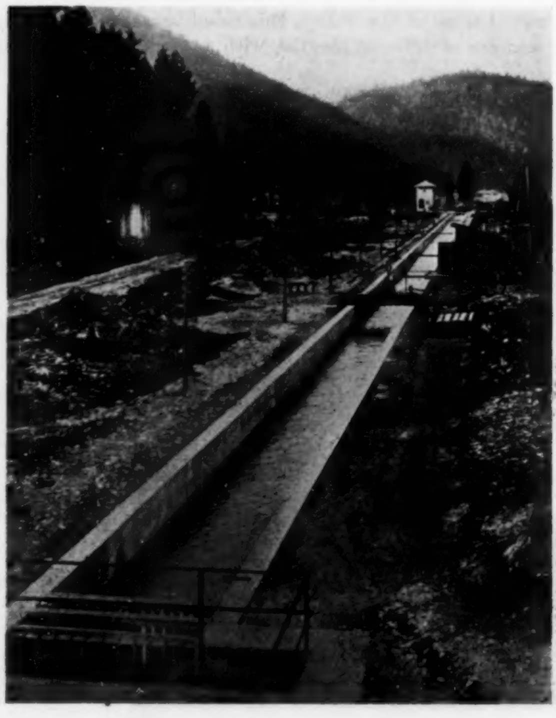

FIG. 6 FOURTH (CONCRETE) SECTION OF EXPERIMENTAL CANAL
(View from titration scaffold looking north; salt-velocity shack at right, control house in distance.)

It is in this fourth section, illustrated in Fig. 6, that the principal flow research of the institute has been conducted. Sides and bottom are of concrete (see Fig. 7), unfinished except for the stripping of the wooden forms, and provided at frequent intervals with boarded-in stop-log grooves for temporary bulkheads in the seepage tests, weir insets, and stilling screens. The entire length is provided with numerous standpipes embedded in the concrete, and gage pits just outside the channel, for the two different systems of water-level observation. Outflow at the lower end is

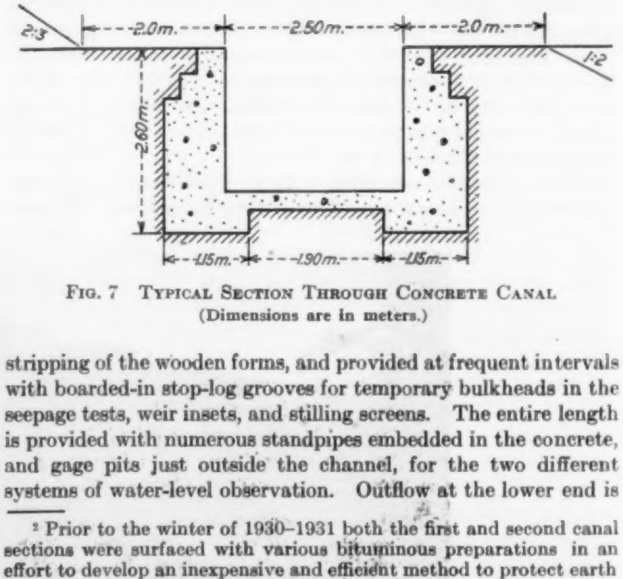

FIG. 7 TYPICAL SECTION THROUGH CONCRETE CANAL
(Dimensions are in meters.)

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[2] Prior to the winter of 1930–1931 both the first and second canal sections were surfaced with various bituminous preparations in an effort to develop an inexpensive and efficient method to protect earth channels from erosive action. This investigation included not only a study of proper materials, but also of a practical means of application. Twelve different kinds of lining were tried in consecutive sections, consisting of hot- and cold-water emulsions, of tar, and of asphalt. These were applied by spraying to a thickness of 0.5 to 1.0 mm., by soaking to a thickness of 2.0 to 5.0 mm., and finally by a poured paving operation similar to that used for city streets. Further experiments have been conducted with what is called "asphalt-beton," crushed stone with a bituminous binder. All twelve kinds of lining were submitted for nine months to constant discharge through the canal, with the accompanying frost action. It was found that the light covering was soon disrupted by the frost, but the heavier forms were uninjured. The institute has succeeded in producing mechanically an economical and practical surfacing now ready for use; the light covering is applicable to temporary channels, while the thicker lining is suitable for all other conditions. The costs are lower than those for concrete, while the asphalt has the further advantages of being elastic, watertight, and easily repaired.

measured by a sharp-crested suppressed weir adjustable to various elevations. Here the flow discharges into a forebasin with a capacity of 96 cu. m. (3390 cu. ft.) below the outlet sills, plus 72 cu. m. (2543 cu. ft.) with outlets closed. There are two outlets to the forebasin, symmetrically arranged and controlled by two Tainter gates to turn the discharge into either the waste channel leading back into the Obernach or into the large measuring basin. The gates are of steel, 3.5 m. (11.5 ft.) wide and opening 1.0 m. (3.28 ft.), swinging on a 1.80-m. (5.91-ft.) radius. The measuring basin has a usable capacity of 1500 cu. m. (52,900 cu. ft.), is of finished concrete, and has recently been given a $^1/_4$-in. to $^3/_8$-in. coat of bituminous waterproofing to eliminate all seepage. The outlet from this basin is into the waste channel through a single vertical gate.

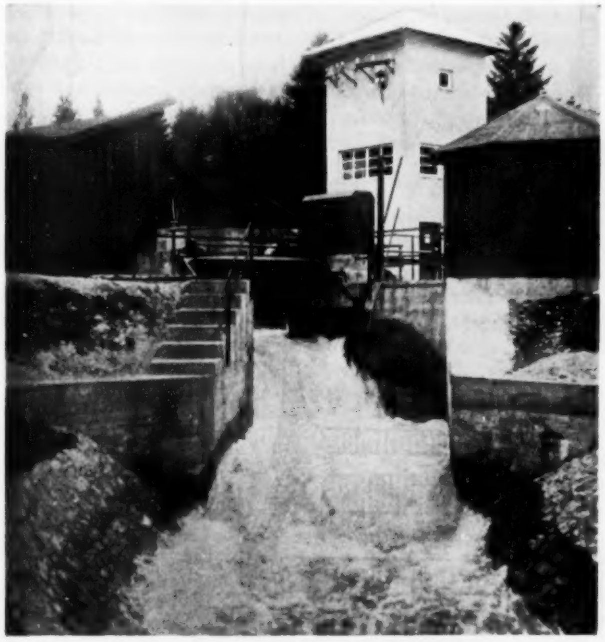

FIG. 8 DISCHARGE FROM FOREBASIN INTO WASTE CHANNEL
(Scale shelter at left, control house center, and gage house right.)

There are at present three permanent buildings on the station property: the headquarters cabin and the workshop between the highway and earth canal at the supply-basin end and the control house at the lower end of the concrete canal. There also are small shelters for the salt-velocity pump near the beginning of the concrete canal, for weighing scales at the forebasin, and for the gages at the measuring basin.

In the headquarters cabin there are rooms designed for executive purposes and for drafting and for chemical analysis, a photographic darkroom, and a kitchen and sleeping quarters for several assistants. The workshop is completely equipped with forge and metal-working machinery and carpentry tools, so that the station can build practically all its apparatus on the site of experimentation.

The control house is of concrete, with windows on three sides commanding a complete view of the canal and measuring basin. In the basement are transformers reducing the potential of the main supply from 6000 v. to either 380 v. three-phase or 220 v. single-phase, convertors to direct current, and storage batteries. In the first floor are further electrical connections, consisting of a switchboard for direct current varying in potential from 2 v. to 40 v., an electric clock sending impulses every second to all parts of the station, and a telephone. Here are also the controls for the gates leading from the forebasin and measuring basin, time of movements being recorded chronographically. Instruments for recording current-meter turns and the salt-velocity curves may be operated here as well. (See Fig. 8.)

The entire plant is connected by telephone so that all points are in communication throughout experimentation, and electric lines the length of the major canal provide current for both lighting and electrical measuring methods. Since this original canal parallels the main highway, there is ready access to all points by wagon or truck; bicycles and motorcycles owned by the assistants allow rapid travel to and from the more distant points without loss of time.

Though still only partially cleared of timber, the remainder of the land in the institute's possession offers possibilities of expansion through many years to come. (See Fig. 9.) Aside from the downward slope of the valley, the section is comparatively flat and admirably suited for large-scale model development; due to its elongated shape, discharge into the Obernach is possible at practically every point with only short waste channels.[3]

FIG. 9 VIEW OF NORTHERN END OF STATION
(Measuring basin on extreme left, with Obernach bed in foreground; practically entire tract owned by the institute is visible.)

VOLUMETRIC MEASURING BASIN, APPARATUS, AND TECHNIQUE

Claims by the institute directors of an experimental accuracy equal to that of any of the small-scale laboratories have their

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[3] The adaptability of the main concrete canal for model experiments was illustrated in the study of a round-crested spillway in the fall of 1930 at a scale of 1 to 4.375 for the Preussische Versuchsanstalt für Wasserbau und Schiffbau. The latter had already tested models

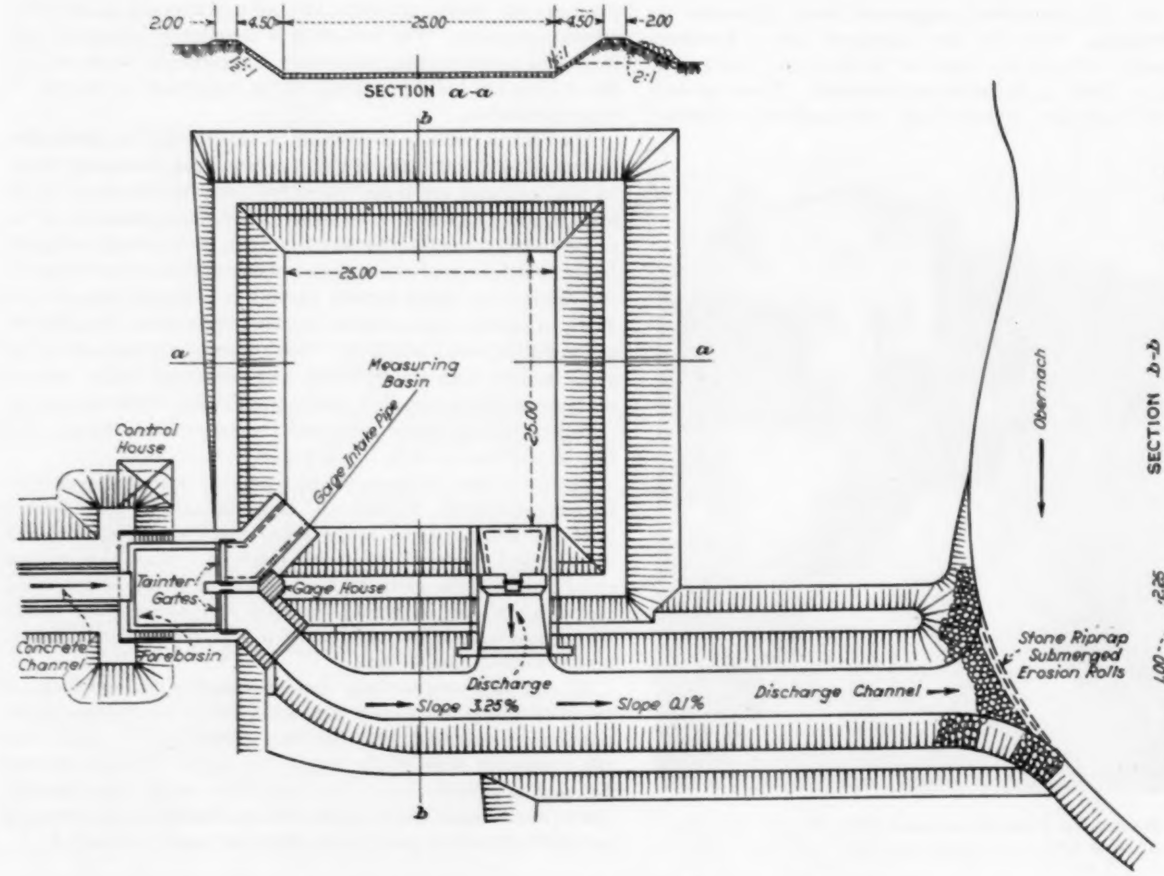

FIG. 10 DESIGN OF FOREBASIN, MEASURING BASIN, AND WASTE CHANNEL
(Dimensions are in meters.)

foundation in a thorough, painstaking planning of each experiment before the run is started; the use of instruments that are probably among the finest that have ever been developed for this purpose; and a patient, capable analysis of and compensation for all unavoidable error. Care and foresight in the arrangement of all detail of layout and procedure have produced an exactitude that is extraordinary in such large-scale work.

Measuring Basin. The principal factor in the accuracy of the results, that upon which all other computations are based, is the actual volumetric record of discharge given by the large measuring basin at the lower end of the main canal. (See Fig. 10.) Whereas most other laboratories depend for their standard upon the mean value of a number of comparative methods of measurement, the research institute measures volumetrically the water that passes through its canals during active experimentation, with apparatus that is even more accurate than conditions require—a calculated and proved error of ±0.1 per cent is the maximum that is present.

Before the measuring basin was used in the first series of experiments, two comparative rating curves were plotted—one from careful measurement and geometrical computation of its capacity with relation to depth of water, and the other by actual tabulation of gage readings as quantities of water, measured by weight, were successively introduced. The concrete canal was boarded up at the lower end and allowed to fill, the forebasin also being filled level with the sills of the Tainter gates. Water was then pumped from the canal into the balance tank, weighed, and discharged into the forebasin. Gage readings were made for each different water stand in the forebasin, and when the latter was filled to capacity, its contents were discharged into the measuring basin, and the process was repeated until the measuring basin had been rated to depths varying from 0 to 2 m. (6.56 ft.). The balance used has a capacity of 3000 kg. (6620 lb.), sensitive to 0.0013 per cent.

For experiments in the concrete canal with built-in models requiring only a small flow, the forebasin alone is used as the measuring tank. A temporary steel chute carries the discharge from the canal over the sill of the Tainter gate leading to the waste channel, the closing of the one gate being sufficient to direct the water into the basin during time of filling. Under these circumstances the basin may be filled only to a point somewhat below the sill level, so that the basin had to be rerated to cover this lower range of 1.25 m. (4.1 ft.).

Error in recording the actual time of filling the measuring basin has been practically removed by the absolutely symmetrical construction of the forebasin and the Tainter gates. (See Fig. 11.) The two gates are coupled so that both begin or cease to move at the same instant, one closing at the same rate that the

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at scales varying from 1 to 75 to 1 to 8.75, but a discrepancy between performance in nature and in a small-scale model necessitated further experimentation at greater scale. The model was built into the main canal with cement mortar and a rubble bed, the canal floor being raised about 2 ft. for a length of perhaps 300 ft. by a temporary wood planking bolted in place. Temporary stilling screens were built in, and measurements were made with the usual gages along the canal wall and the forebasin at the end converted into a smaller measuring basin. A needle weir at the end of the canal regulated the tail water.

other opens, and vice versa. Electrical contact is made when each gate reaches or leaves its lowest position and is recorded automatically in ink on a paper ribbon that contains a simultaneous double chronographic record. (See Fig. 12.) Hence for a single filling of the basin, four points in time will be indicated on the ribbon: t_1 marking the instant the basin gate A begins to open, and t_2 about 3 seconds later indicating the instant the waste gate B from the forebasin is entirely closed; t_3 some 3 or 4 minutes afterward gives the instant B once more begins to open, this interval depending upon the rate of flow in the canal, and t_4 the instant A is again completely shut, after which no more water flows into the basin. Either the time t_1 to t_3 or t_2 to t_4 is used as the time of filling.

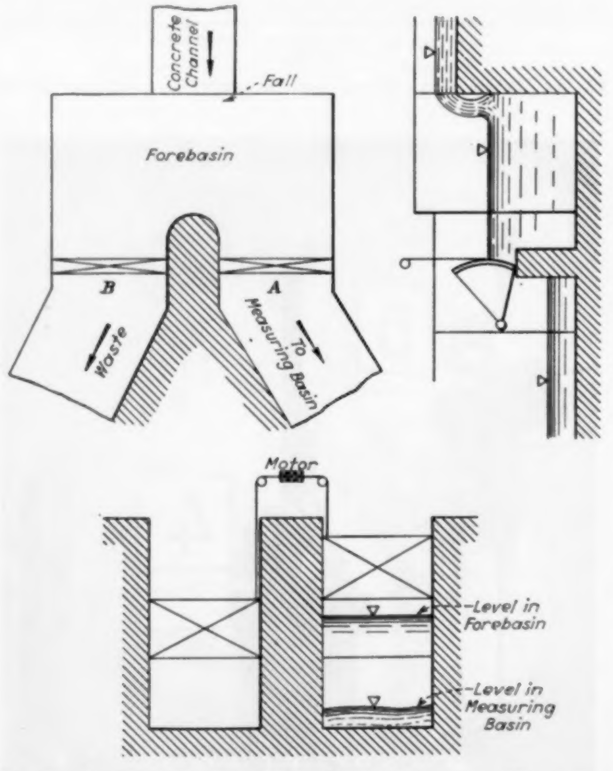


FIG. 11 SECTIONS SHOWING SYMMETRICAL DESIGN OF FOREBASIN AND OUTLET GATES

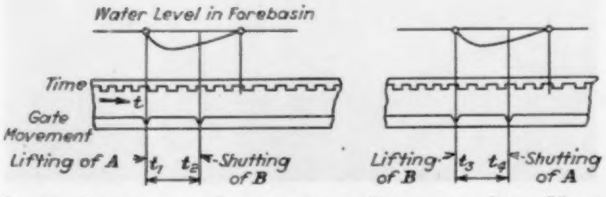
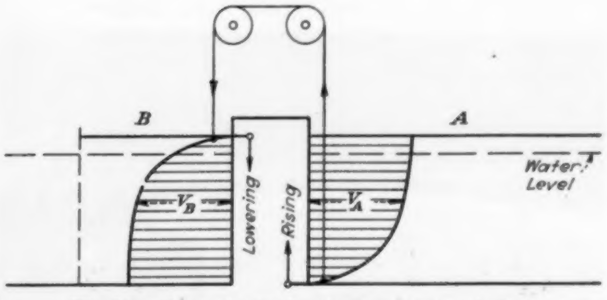


FIG. 12 SPECIMEN OF CHRONOGRAPHIC RECORD OF GATE MOVEMENT



(Fig. 13a Curve of gate elevation versus gate velocity.)

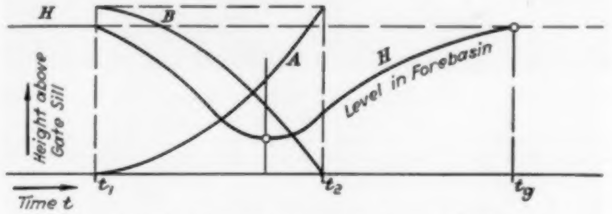
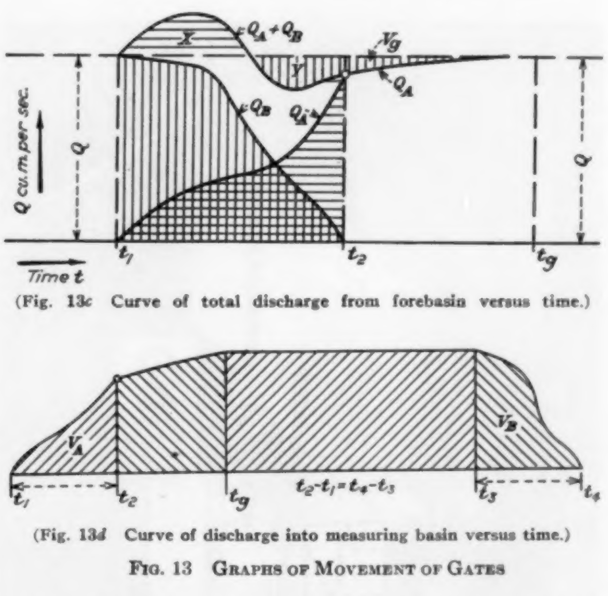
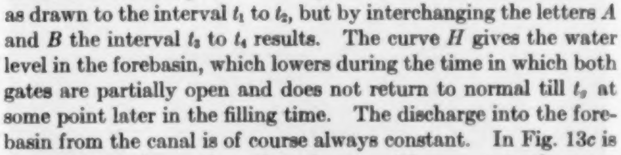


Fig. 13b Curve of gate opening versus time.)



(Fig. 13c Curve of total discharge from forebasin versus time.)



(Fig. 13d Curve of discharge into measuring basin versus time.)

FIG. 13 GRAPHS OF MOVEMENT OF GATES

A proof of the mathematical accuracy of this method is as follows:[4] The gates in every case open well above the water surface, so that there is free discharge under them even under maximum conditions. In Fig. 13a is shown the elevation of the bottom edge of gates A and B with respect to rate of motion. (It is to be borne in mind that the curves are only general and that this in no way affects the accuracy of the proof.) Curves A and B in Fig. 13b show the respective gate openings during the interval of motion, plotted now against time. These curves refer as drawn to the interval t_1 to t_2, but by interchanging the letters A and B the interval t_3 to t_4 results. The curve H gives the water level in the forebasin, which lowers during the time in which both gates are partially open and does not return to normal till t_g at some point later in the filling time. The discharge into the forebasin from the canal is of course always constant. In Fig. 13c is shown the discharge through each gate with respect to the same time axis in curves Q_A and Q_B, and the summation of the flow through both gates in $Q_A + Q_B$; the latter varies in accord with the curve H in Fig. 13b.

Between the times t_1 and t_g the total flow into the forebasin is exactly equal to its total discharge; that is, the total volume of water passing through the forebasin is equal to the area enclosed between the basic time axis in Fig. 13c and the horizontal line of normal flow Q; but since the actual summation of discharge through both gates is the area between the time axis and the

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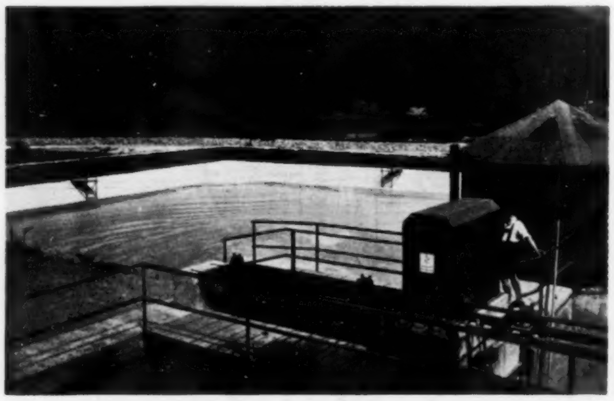

FIG. 14 VIEW OVER FOREBASIN TOWARD MEASURING BASIN DURING FILLING

(Gate mechanism visible at right; vertical rods are hand controls to divert seepage from under gates into discharge; gage house right background.)

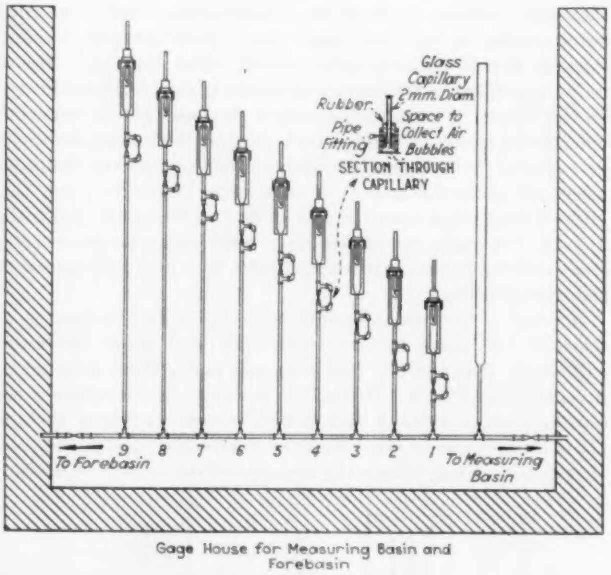
Glass Capillary 2mm. Diam
Rubber
Space to Collect Air Bubbles
Pipe Fitting
SECTION THROUGH CAPILLARY
9
8
7
6
5
4
3
2
1
To Forebasin
To Measuring Basin
Gage House for Measuring Basin and Forebasin

FIG. 15 DIAGRAM OF GAGES AND CONNECTIONS IN GAGE HOUSE

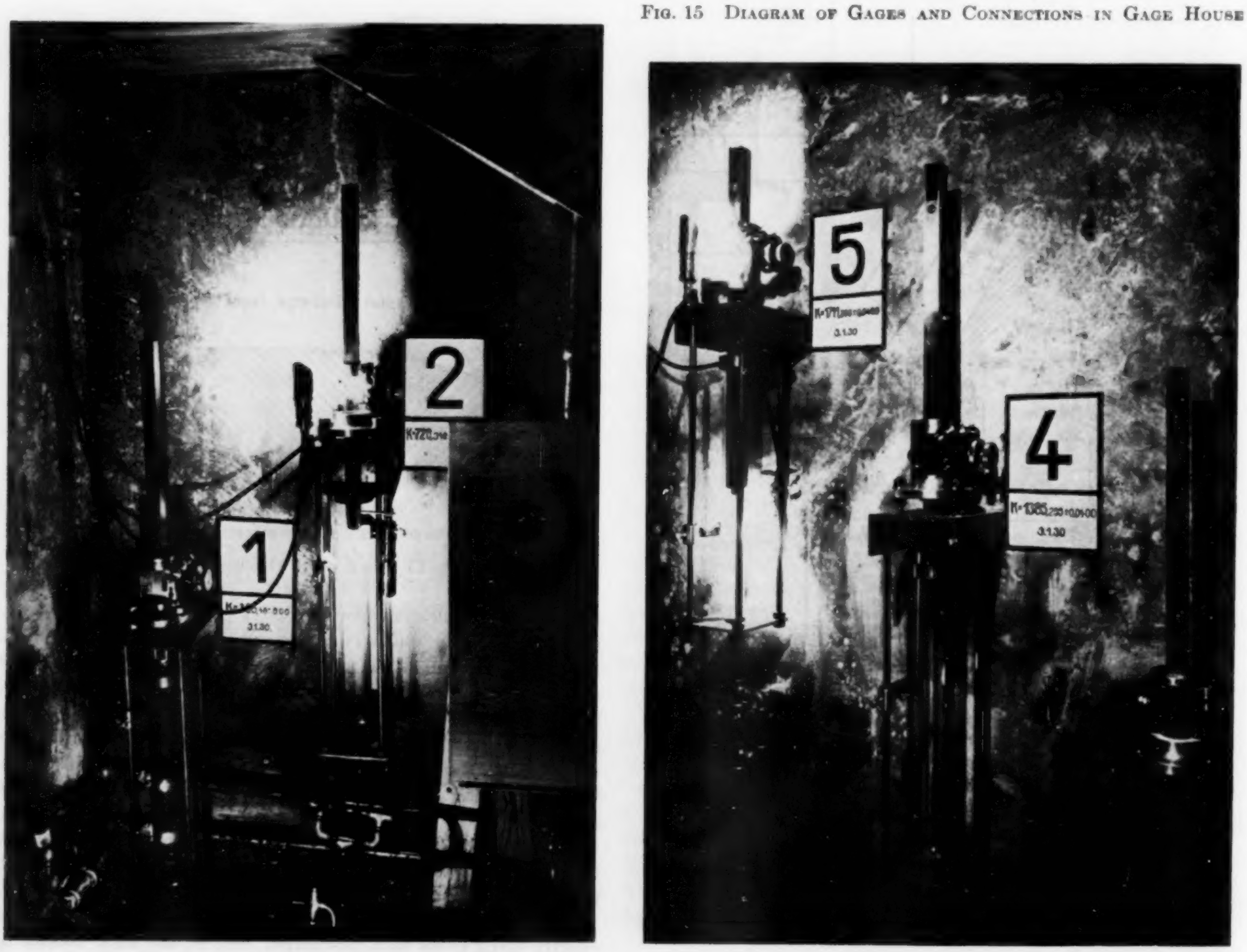

Fig. 16a

Fig. 16b

FIG. 16 TYPE A GAGES IN GAGE HOUSE NEXT TO MEASURING BASIN

(Lighting facilities, small reading mirrors, capillaries, and connections are visible.)

curve $Q_A + Q_B$, then the positive area X must equal the sum of the negative areas Y and V_g. Expressed mathematically:

$$\int_{t_1}^{t_g} (Q_A - Q_B)dt = Q(t_g - t_1)$$

From the diagram it is clear that

$$V_A + V_B = Q(t_2 - t_1) + V_g$$

and

$$V_g = Q(t_g - t_2) - \int_{t_1}^{t_g} Q_A dt$$

From these relations there may now be constructed the diagram of discharge into the measuring basin as shown in Fig. 13*d;* the V_B in the last interval of time is obviously the same as the V_B of the first interval, except that it now designates flow through gate A during its period of closing. In other words, the level and discharge curves at the second gate action are identical with those at the first change, except that the letters A and B must be interchanged and t_3, t_4, and t_g' substituted for t_1, t_2, and t_g.

The flow into the measuring basin during each interval may be tabulated as follows:

Time interval	Inflow
t_1 to t_2	V_A
t_2 to t_g	$Q(t_g - t_2) - V_g$
t_g to t_3	$Q(t_3 - t_g)$
t_3 to t_4	V_B

The total flow given by the addition of these factors is:

$$V = V_A + V_B + Q(t_g - t_2) - V_g + Q(t_3 - t_g)$$

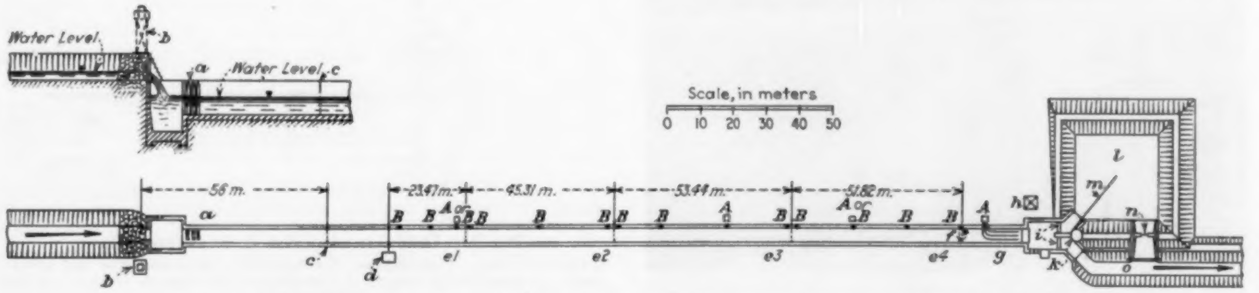

FIG. 18 PLAN OF CONCRETE CANAL, GIVING LOCATION OF MEASURING INSTRUMENTS AND APPARATUS

(Legend: *a*, stilling racks; *b*, titration injection apparatus; *c*, sampling pump for titration; *d*, salt-velocity injection pumps; *e*, salt-velocity electrodes; *f*, current-meter bridge; *g*, measuring weir; *h*, control and observation house; *i*, Tainter gates; *k*, gage house; *l*, measuring basin; *m*, gage inlet pipe; *n*, outlet gate; *o*, discharge canal; *A*, pit gage, type A; *B*, standpipe gage, type B, electrical.)

Through substitution of the equivalent value already found for $V_A + V_B$ this becomes:

$$\begin{aligned} V &= Q(t_2 - t_1) + Q(t_g - t_2) + Q(t_3 - t_g) \\ &= Q(t_3 - t_1) \end{aligned}$$

Hence the method itself is entirely correct, and whatever inaccuracy is involved can come only through unsymmetrical motion of the water in the forebasin, leakage through the gates and basin structure, unfavorable weather conditions, and the personal error involved in reading the gages.

The influence of possible unsymmetrical flow is practically negligible. The interval during which the gates are in motion is constant for a given run, but may be varied for different runs, with a minimum length of 2 sec. at the beginning or end of filling. A very small error during only these 2 sec., as compared to the total filling time of at least 180 sec., thus becomes actually $^1/_{90}$th as large.

Under each Tainter gate in its closed position seeps a small quantity of water; a depression has been left in the top edge of each sill to collect this discharge of perhaps 0.2 liter per sec., so that the seepage under the closed gate may be conducted through a short length of pipe to join the discharge through the open gate. Valves in these pipes are regulated by hand during the interval of gate movement (see Fig. 14) so that the seepage is turned into the measuring basin during time of filling and thereafter once again into the waste channel. Seepage under the outlet gate from the measuring basin is collected in the same way and run into a concrete container, from which a small propeller pump, running continuously, returns it to the basin.

Errors resulting from weather conditions and the reading of the gages will be discussed in the following section.

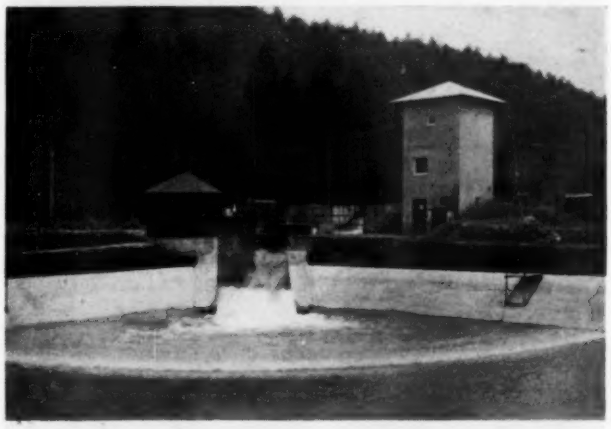

FIG. 17 DISTURBANCE IN MEASURING BASIN AT START OF FILLING

Hook and Point Gages. In the gage house for the forebasin and measuring basin is a remarkably fine set of instruments, arranged to allow the utmost convenience in setting and reading, and so exact in their results that they give an unavoidable error of less than 0.1 mm. (0.00033 ft.). These gages are nine in number, three giving readings for the forebasin and covering a range of 600 mm. (1.97 ft.) and six for the measuring basin with a range of 400 mm. (1.31 ft.), overlapping some 3 cm. on the readings of that gage above and that below. (See Fig. 15.) All are arranged around a narrow spiral stairway leading to the main control valves at the bottom of the concrete pit, and may be set and read from a sitting position on the nearest step. A large glass standpipe with floating pointer covers the entire range of readings, from which the gage to be used is determined.

From the screened inlet at the center of the basin floor, a 3-in. pipe runs across the floor and through the walls into the bottom of the gage pit. Here, following the shut-off valve, are the connections for the pipes leading to the six vernier gages and the standpipe. (See Figs. 16*a* and 16*b*.) Connections to the three forebasin gages are identical. All nine instruments are similar, consisting of a glass standpipe 4 cm. (1.58 in.) in diameter, easily removable for cleaning, with a main inlet tube that can be closed after the water has reached its approximate level, pressure then being transmitted through a glass capillary tube 2 mm. (0.079 in.) in diameter, to avoid momentary fluctuations. The tube extends past the rubber fittings above and below so that both ends will remain free of air bubbles and sediment, as is shown in Fig. 15.

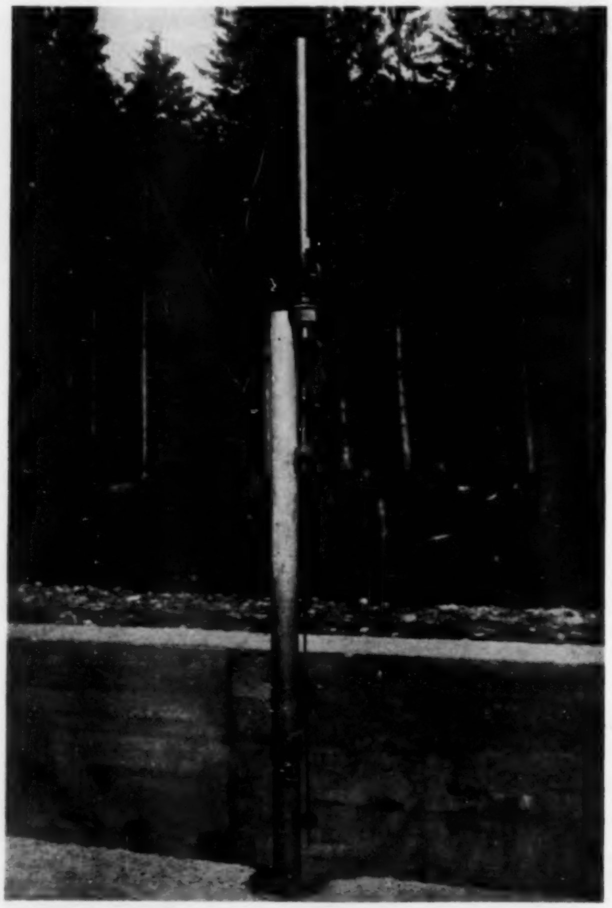

Fig. 19 Standpipe Gage, Type B

(With pointed extension rod hanging outside; note insulator balls and headphone connections.)

Surmounting the standpipe is a centimeter scale with vernier attachment reading directly to tenths of a millimeter (0.0039 in.); three of the instruments read to fiftieths of a millimeter—an accuracy greater than is necessary. The finer adjustment is made by means of a worm gear, with a cone release to permit rapid preliminary setting. The point, of the general hook type, has a fairly broad, arched knife edge to give further accuracy. A movable mirror at the side, set just below the level of the water, gives a simultaneous picture of both the knife edge and its reflection in the water surface above, permitting a very easy and delicate adjustment. Two small electric bulbs illuminate the vernier and the reflected image of the hook below. Both the setting of the instrument and the reading of the results can be accomplished rapidly and without undue change of position.

As is the custom of the institute, results are directly plotted against time of day on graph paper, wherever possible, rather than tabulated in notebooks. This serves to eliminate practically all gross errors, and in the case of the measuring-basin readings, the instrument man can see from his plotting the effect of weather and the resulting length of time necessary for a record from which a satisfactory average may be drawn. Before filling, the basin is never quite empty, a zero reading being made with enough water left in from the last run to require the use of either gage 1 or gage 2. Readings are made at the rate of one every minute, for at least 15 minutes, or in case of wind disturbances until the plot gives a definite average line. Since the sides of the basin are not perpendicular, wind has a pronounced effect upon the readings, giving results that are slightly low. For this reason a wait of almost an hour is necessary after filling, so that the surging due to inflow may quiet down before final readings are begun. (See Fig. 17.) The latter are plotted on the same sheet as the zero curve, but often last as long as one or two hours in severe cases. So delicate is this method that rain or undue seepage makes the records worthless, and the run must be repeated.

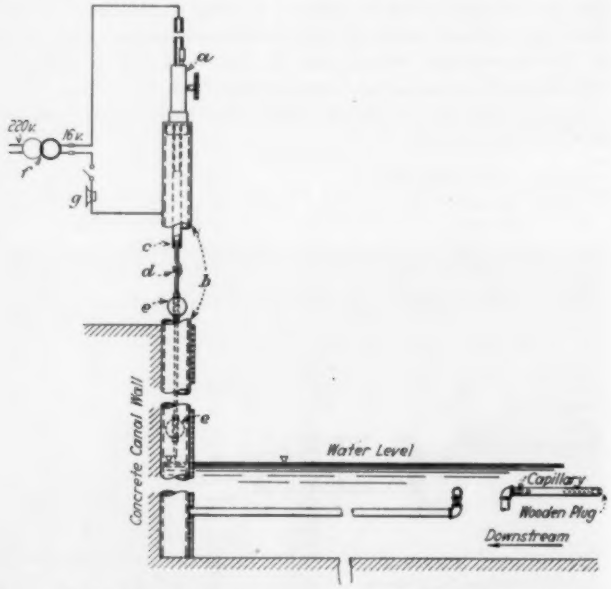

Fig. 20 Diagram of Type B Standpipe Gage

(Legend: *a*, scale; *b*, standpipe; *c*, conductor; *d*, extension rod and contact point; *e*, wood-ball insulator; *f*, transformer, 220 volts to 16 volts; *g*, telephone receiver.)

Fig. 21 Reading Water Elevation Electrically With Type B Gage

(Electrode 3 visible in foreground.)

At various points along the concrete canal (marked *A* in Fig. 18) are hook gages identical with those at the measuring basin, but located in individual pits beside the canal. The last gage downstream gives readings at the right and left of the weir alternately by a simple valve action. Pressure is transmitted to all gages through small pipes leading from the center of the channel. The inlets are somewhat above the canal floor, the pipes pointed upstream, plugged at the end, and perforated with small holes on the sides. Within each pipe is a capillary tube 3 mm. (0.118 in.) in diameter to eliminate the effect of waves in the water.

The type B gage has the same inlet connections, but differs in form and accessibility. Set in the west canal wall are iron standpipes 2 in. in diameter rising about 4 ft. above the ground. Pointed metal rods of known length are inserted in these pipes, insulated from them by wooden balls fitted tight on the rods, but sliding easily through the pipes. Each rod is hooked to a metal scale reading by vernier to 0.1 mm. and lowered till it touches the water surface in the standpipe. Contact is given electrically through a pair of headphones, current coming from the line along the canal and passing through the circuit formed by the rod and pipe as terminals and the water between as conducting medium; a sharp buzz is heard the instant the point touches the water. (See Figs. 19, 20, and 21.) Readings are made to 0.1 mm., but the water level is so variable due to flow disturbances that constant reading is necessary to give a satisfactory average record. Often four or five gages are read in turn by one assistant, who carries a single scale and headphone from one standpipe to the next.

The gage pits and standpipes were originally located to serve the needs of the first series of experiments, but are at the same time so evenly distributed along the main channel that they are in favorable positions for practically all future needs. For general purposes the standpipe system gives all necessary accuracy, with the pit gages at the several points for use where the flow is stable enough to allow more care in observation. Additional pits and standpipes are built in as occasion demands. A recent improvement has been the enclosing of the usual standpipe with a cylinder of steel 20 cm. (7.9 in.) in diameter, so that the increased volume of water thus contained will further dampen momentary fluctuations.

In two lines, 10 m. on either side of the channel axis, are pipes for the determination of the groundwater level to be used in connection with the correction for seepage through the floor and sides of the canal and measuring basin. Readings are made either electrically or by inserting a rod and then measuring to the wetted point. At intervals during experiments with the concrete canal, the latter is divided into five sections by watertight stoplogs, the channel is filled to capacity, and the lowering of the water surface due to seepage is read from the gages over a period of from 36 to 72 hours. These data, together with those from the groundwater readings, furnish the correction for each section in question to be applied to the flow measurements; the same correction was made for the measuring basin prior to its waterproofing.

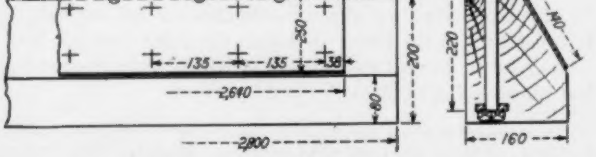

FIG. 22 DESIGN OF MEASURING WEIR AT END OF CONCRETE CANAL
(Dimensions in millimeters.)

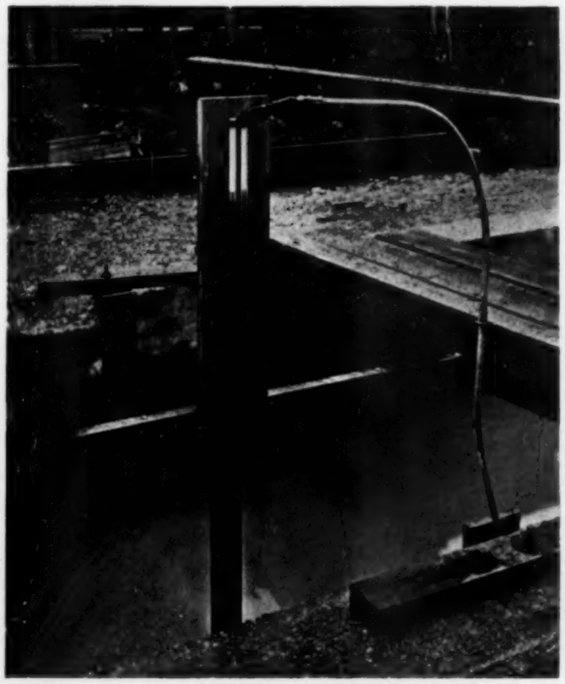

FIG. 23 DISCHARGE OVER MEASURING WEIR
(Showing piezometer for measuring degree of ventilation of sheet.)

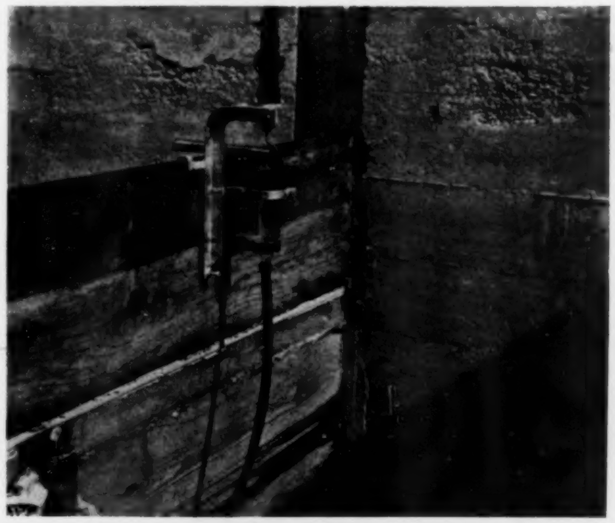

FIG. 24 GAGE AND RIDER USED FOR ACCURATE SETTING OF WEIR CREST
(Hook gage set to crest elevation by spirit level as shown, and then lowered into water in glass standpipe; rubber tube leads to type A gage in nearest pit.)

INITIAL MAJOR EXPERIMENTS AT THE STATION

Simultaneous measurements of flow through the concrete canal by means of the four leading European and American methods composed the first series of studies by the research institute. The sharp-crested suppressed weir, the Allen salt-velocity, the chemical or titration, and the current-meter methods of calculation of discharge were tested for five different rates of flow, all based upon the volumetric record taken at the measuring basin several times during each run. This used to fullest advantage the original installation on the station grounds and at the same time investigated basic practice with an accuracy that never before had been attained.

Sharp-Crested Weir

A steel weir crest of the standard form (see Fig. 22) was bolted to a wooden beam so that it might be raised or lowered in the stop-log groove at the lower end of the concrete canal, the space below being boarded in and calked to watertightness. Thin boards set into the grooves above the crest preserved the smoothness of the walls and at the same time gave free passage of air to the steel ventilation sheet at the downstream side of the crest. An open U-piezometer served as a check upon the completeness of ventilation. (See Fig. 23.)

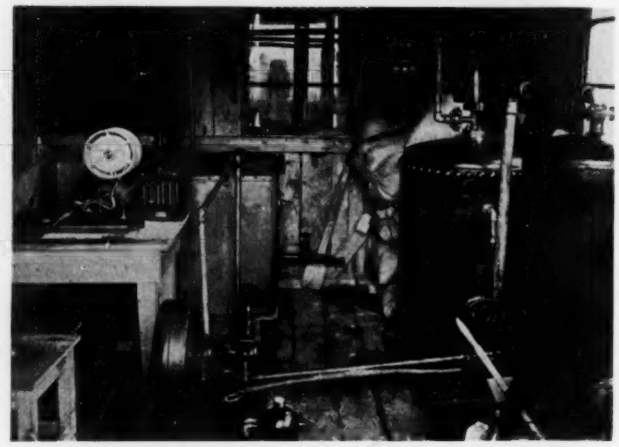

Fig. 25 Pump and Pressure Tanks for Salt-Velocity Injection
(Chronograph at left records current-meter turns during titration sampling.)

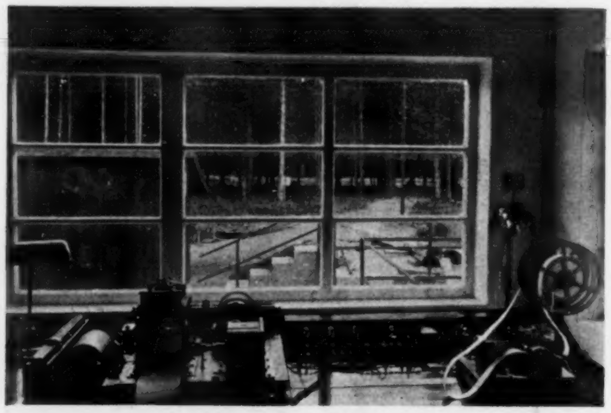

Fig. 26 Chronographs in Control House
(At left, the spark recorder for salt-velocity curves; at right, the six-pen instrument for current meters and forebasin gate movements; electric switchboard between.)

The weir crest was first set roughly by surveyor's level, and then accurately by means of a rider and standpipe hook gage connected by a rubber tube to the gage in the last pit. Since the latter gage read to a precisely known elevation, the water coming to the same level in both standpipes allowed accurate calculation of the crest level till the desired point was reached, the hook on the rider gage being checked to the crest by means of the usual spirit-level method. (See Fig. 24.)

Throughout the run the heads on the weir at the right and the left sides were read at the pit gage alternately at 1-min. intervals, the results being plotted directly against time of day. The gage intakes were located about 3 m. (9.85 ft.) upstream from the weir, 0.4 m. (1.31 ft.) from the side walls.

Allen Salt-Velocity Method

Apparatus for the injection of the salt solution was located under shelter about 60 m. from the head end of the canal. This consisted of a 640-liter (22.6-cu. ft.) tank in which a saturated solution was mixed, a 1.2-hp. gasoline pump and compressed-air tank, and a third container in which the solution was held under a 10-atmosphere pressure prior to injection. (See Fig. 25.) Injection was made through spring valves on the upstream side of the pipe system.

Electrodes, each consisting of six galvanized iron pipes $^3/_8$ in. in diameter, were set in sockets in the canal bed at four points below the injection apparatus (see Fig. 21); each pipe was about 42 cm. (1.4 ft.) from the next, so connected electrically as to give opposing terminals alternately across the section. The electrical recording was made on a chronograph in the control house near the measuring basin, by which the time curve was reproduced in ink on a paper ribbon and the graph of current intensity burnt into the paper by a spark gap to insure certain action. (See Fig. 26.) The record of each succeeding electrode was intensified to give approximately similar ordinates for all curves. Unfortunately with the larger discharges the third electrode had to be omitted to prevent overlapping of curves, only one recording instrument being available.[5]

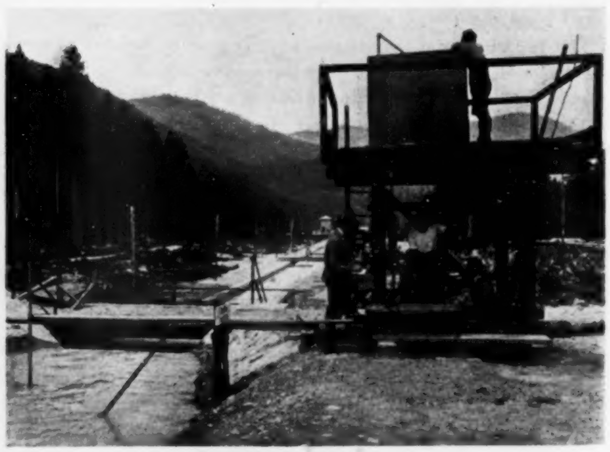

Fig. 27 Titration Injection Apparatus
(Mixing tank on scaffold, constant-head system just below, and calibrated rating tank on ground.)

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[5] Considerable effort was made to determine the effect of length of injection, location and number of injection valves, injection pressure, and location and number of electrodes in the cross-section upon the resulting curves. The injection was varied between 0.5 sec. and 5.0 sec. under a pressure both increased and diminished 100 per cent, through a single valve at one side of the canal, through two valves symmetrically arranged, and finally through six. The electrodes were varied from several in number to an entire network of points over the cross-section. All these changes showed no influence whatever on the resulting measured discharge, the salt clouds in the canal spreading visibly over the cross-section within 10 to 15 m. (33 to 50 ft.) from the injection valves, depending upon the velocity of the water. The injection decided upon was from 42 to 45 liters (quarts) of solution over an interval of 1.5 sec., through two valves; this was later enlarged to a six-valve system.

For the purpose of determining the center of gravity of the areas enclosed by the curves, the latter are enlarged and reproduced to scale on sheets of cardboard, cut out, and the centers found by free suspension in the air; this was found to give more rapid results than mathematical methods, without loss of accuracy.

It is interesting to note that the Allen method was practically unused in Germany until its study by the research institute in 1928.

Chemical Salt-Dilution, or Titration Method

The principle of flow measurement, through chemical analysis of discharge before and after the introduction of a salt or similar solution, has in the past taken a number of various forms,[6] all apparently highly complicated and requiring considerable time and trouble and the assistance of a trained chemist. The experience of the institute is that, of all four methods tried, the titration gives the most accurate results in the shortest time, although handled entirely by a mechanical engineer, and is well adapted to measurements where all other methods are practically impossible.

For the injection of the salt solution at the station, the scaffold and system of tanks shown in Fig. 27 were constructed just above the end of the last earth channel. A saturated solution is mixed in the 2-cu. m. (70.6-cu. ft.) tank on the upper platform; below the outlet pipe from this tank is a constant-head apparatus, the excess solution flowing over the inner circular crest A (Fig. 28) into the narrow space B between the inner and outer cylinders. Discharge from this outer tank is governed by a small orifice giving practically constant flow yet maintaining a head in the outer tank. The ratio between the head h' on the crest A and the waste head h is arbitrary; in these experiments it was approximately 1 to 300. Consequently, a small variation in the discharge head H causing a change in overflow at A is greatly magnified by the variation in h. Hence by a slight regulation of the valve on the mixing tank to hold h fairly constant as shown by the gage, the variation in h' is thus rendered inappreciable.[7]

A swinging funnel below the orifice allows the discharge to be diverted into either the rating tank or the pipe leading to the canal. At the beginning of the run, the flow is regulated until the conditions become constant; the discharge, 0.5 liter (quart) per sec., is then directed into the calibrated tank for about 3 min., next into the injection pipe throughout the run (20 to 25 min.), and at the end once again into the calibrated tank. In other words, not only is a practically constant head maintained, but the flow is measured volumetrically twice during the run.

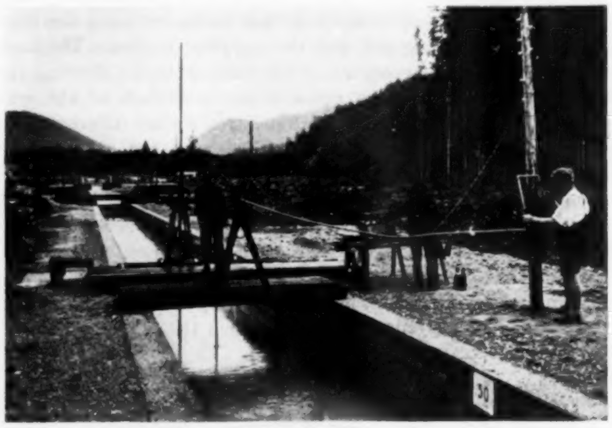

Fig. 29 Pump System for Withdrawing Stream Samples for Titration

(Current-meter carriage is seen just upstream from sampling bridge.)

The injection takes place just upstream from the large concrete tumble-bay, which gives a good mixture of the solution with the canal water. Just below the tumble-bay are three wooden stilling screens at the beginning of the concrete canal. Samples are taken at a point about 56 m. (184 ft.) downstream from the injection point by means of a hand pump and a long pipe whose intake can be placed at any point in the channel cross-section. (See Fig. 29.) Samples were taken for these runs at 15 points

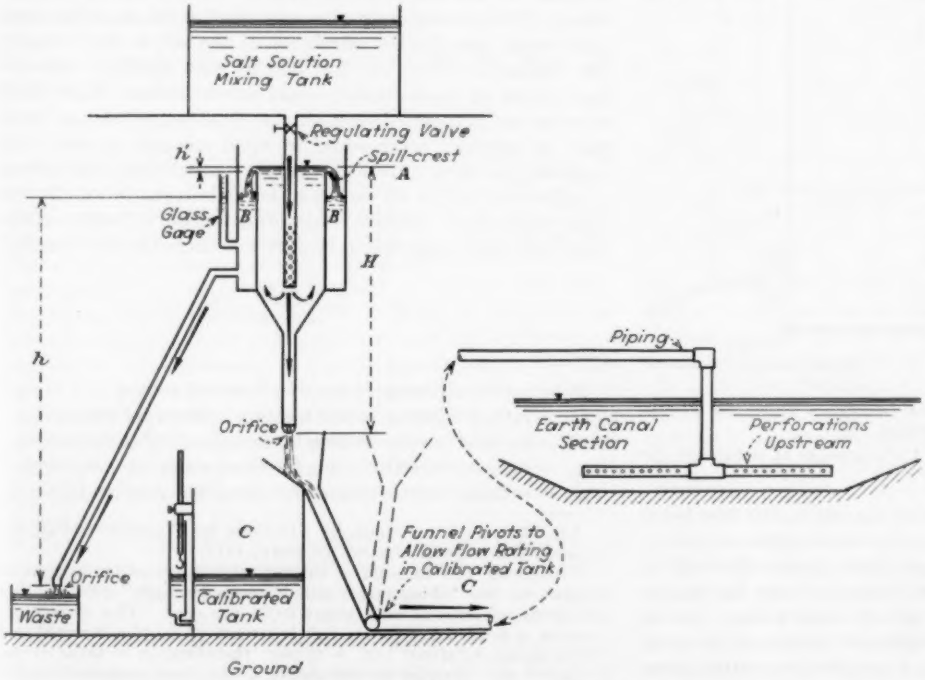

Fig. 28 Diagram of Titration Injection Apparatus

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[6] "Chemi-Hydrometry," by B. F. Groat, A.S.C.E. paper No. 1366, 1916. "Saline Method of Water-Flow Measurement," by W. D. Peasley, *General Electric Review*, Feb., 1916, p. 132.

[7] A mathematical proof of the principle (printed by permission of the Forschungsinstitut für Wasserbau und Wasserkraft) is as follows:

The discharge of excess solution over crest A is $q = a \cdot h'^{3/2}$, where a is a constant. The same discharge expressed as flow through the waste tube is $q = b \cdot h^{1/2}$, where b is also a constant.

Differentiating and equating:

$$dq = \frac{3}{2} a \cdot h'^{1/2}\, dh' = \frac{1}{2} b \cdot h^{-1/2}\, dh$$

$$\frac{dh'}{dh} = \frac{b}{3a \cdot h'^{1/2} \cdot h^{1/2}}$$

Since $a = q/h'^{3/2}$ and $b = q/h^{1/2}$, then $dh'/dh = h'/3h$. Hence, as h during the experiments at the station was 500 mm. and h' was 5 mm., the ratio dh' to dh reduces to 1 to 300. A variation in h of 20 mm. gives a change in h' or H of 0.07 mm., or a matter of ±0.00175 per cent error when H is one meter.

(5 points each in 3 horizontals), during which electrically recorded current-meter readings were made 2 ft. upstream at each point simultaneously. The following samples were taken for each run in 1-liter capped bottles, first washed carefully in the solution to be sampled: 3 bottles of canal water before injection; 1 bottle of concentrated solution at beginning and 1 bottle at end of run; 15 bottles of canal water plus injected solution.

For the purpose of gaging small streams, a portable constant-head discharge apparatus has been devised. (See Fig. 30.) The tank is of light steel construction and may be carried easily on a wheelbarrow, the only other materials necessary being the bags of bulk salt, a mixing pail, and the sampling bottles. The tank is air-tight, with the exception of the inlet at the top through the narrow tube leading to a point about two-thirds of the way down, and the outlet at the bottom fitted for six different-sized orifices. The tank is completely filled with the concentrated solution, the inlet pipe is screwed in place, and the orifice is opened at the bottom. Within about 10 sec. the air inside has reached a negative pressure equivalent to the head of water above the lower end of the inlet, and air from outside flows in through the pipe at exactly the rate that the solution is discharging through the orifice. Since the inlet pipe is always thereafter filled with air, the only head acting upon the orifice is that between the lower end of the inlet and the orifice, the flow hence remaining constant till the water level in the tank has dropped to the end of the inlet. The smallest orifice gives a discharge of about 27 cc. (1.65 cu. in.) per sec. for nearly an hour; the largest, about 300 cc. (18.35 cu. in.) per sec. for some 5 min. All six orifices have been carefully rated with both water and the saturated solution. The jet from the tank is made to enter the stream preferably at a fall or rapids to ensure good mixing. Samples are taken by hand at a distance downstream depending upon the degree of turbulence.

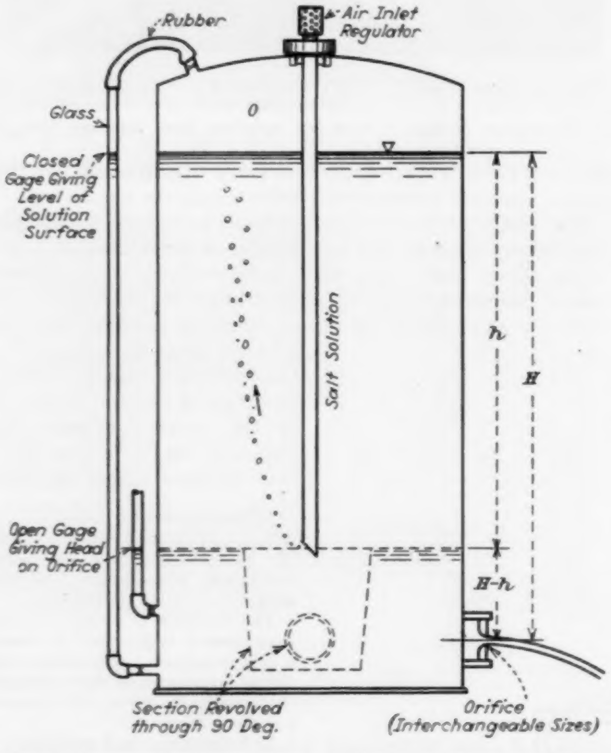
Rubber
Air Inlet Regulator
Glass
O
Closed Gage Giving Level of Solution Surface
Salt Solution
h
H
Open Gage Giving Head on Orifice
H-h
Section Revolved through 90 Deg.
Orifice (Interchangeable Sizes)

Fig. 30 Section Through Portable Injection Tank for Titration Method

($H - h$ = constant head on orifice. Under-pressure at O balances the variable head h.)

This portable apparatus makes the method highly suited to gaging in mountainous country, where current meters are of little value. Several small streams in the Bavarian highlands were rated by the institute to try the method, with such success that two of the assistants spent three months in the fall of 1930 rating streams in the Austrian Alps for the Tauernwerk power project of the A.E.G. (Allgemeine Elektrizitäts Gesellschaft) by this method.

Fig. 31 Portable Injection Tank

(Measuring the flow from a melting glacier.)

Fig. 31 illustrates stream-rating with the portable injection tank for the titration method. There is a discharge of 250 liters (quarts) per second from the Karlinger glacier. The measurements are for the projected Tauernwerk in the Grossglockner territory, Austria.

Mellet's system of chemical analysis[8] has been adopted by the institute for the determination of flow values from the samples taken. This system deals with only relative values rather than with actual quantitative calculation of the salt in each sample. The various solutions are first diluted with distilled water so that all are of approximately equal concentration. Equivalent volumes are evaporated over an electric stove, potassium chromate as indicator being added in equal amounts to each concentrate, and silver nitrate of constant concentration added from a calibrated burette till each solution is brought to exactly the same reddish hue. Burette readings are reduced to 1-liter equivalents, and these numerical values are substituted in the formula:[9]

$$Q_2 = Q_1 \frac{n_1}{n_2 - n} - Q_1$$

in which:

Q_2 = rate of discharge of canal in liters per second
Q_1 = rate of injection of salt solution in liters per second
n_1 = reduced burette reading for sample of injected solution
n_2 = reduced burette reading for canal water after injection
n = reduced burette reading for canal water before injection.

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[8] R. Mellet, Mitteilungen der Abteilung für Wasserwirtschaft des Schweiz. Dept. des Innern no. 10, Bern, 1917.

[9] Although this formula is theoretically incorrect, it is that in general use (see "Wasserkraft und Wasserwirtschaft," 1928, s. 101) and gives an error of only about 0.001 per cent. The correct expression is derived as follows: Salt in stream ($Q_2 \cdot n$) plus injected salt ($Q_1 \cdot n_1$) equals total salt $[(Q_2 + Q_1)n_2]$; therefore, $Q_2 = Q_1(n_1 - n_2) \div (n_2 - n)$. (Printed by permission of the Forschungsinstitut für Wasserbau und Wasserkraft.)

The steps in analysis may be summarized as follows:

1 Accurate volumetric dilution and measurement of samples
2 Evaporation almost to dryness and addition of equal volumes of potassium chromate
3 Titration with silver nitrate, always to the identical slight reddish tinge of the first sample
4 Reduction of all burette readings to 1-liter equivalents
5 Substitution in the formula.

Accuracy depends upon absolute cleanliness both in the securing of samples and throughout the analysis; the use of the same pipette, burette, and reagents, of approximately similar volumes of samples, and of equal-sized evaporation dishes for each individual run; and the judging of the color similarity and the reading of the burette by the same person under the same light conditions.

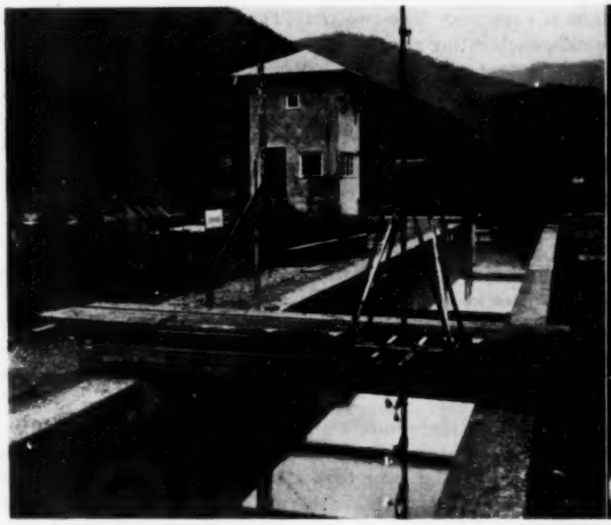

Fig. 32 Bridge and Carriage for Current-Meter Measurements

(Gage pit type A for weir readings is seen just behind and to left of carriage.)

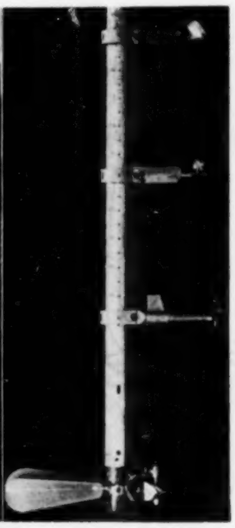

Fig. 33 Near View of Rod Supporting Current Meters

Current Meter Method

A small wooden bridge with wheel grooves was built across the concrete canal 21 m. (68.9 ft.) upstream from the end weir, to carry the steel meter carriage. The meter is bolted to a streamline shaft adjustable vertically by crank and windlass, so that the meter may be moved to any point in the canal section. The carriage is of such weight as to be easily movable by one person, yet stable enough to remain in the set position. The different meters were always used in succession in each run. Three could also be used simultaneously, so that by a single shift of the shaft either five or six horizontals were covered. The meters were in such positions, depending upon the discharge, that points as near as possible to the surface and bottom were reached, the others lying at even intervals in between. The verticals used were approximately 0.1, 0.3, 0.7, and 1.25 m. from either wall of the 2.50-m. canal. (See Fig. 32.)

Electrical leads ran to the control house, where a six-pen Ott chronographic recorder registered automatically the count of turns. The top and bottom pens marked the seconds directly from the clock, three kept track of the meter count, and the last was connected to the gate contacts for taking the filling time of the measuring basin. A spring mechanism moved the paper ribbon forward at the rate of 1 cm. per sec. An assistant on duty at the recorder marked on the paper ribbon the date, time, and location of meter at each change, signaling by buzzer to the carriage operator for movement of carriage or shaft as soon as the instrument had registered successfully a sufficient number of turns.

A similar bridge exists on the concrete channel leading from the Isar diversion canal to the supply basin. The apparatus for the meter readings accompanying the taking of samples for titration is identical, except that the recording is done in the salt-velocity pump house.

In these first experiments, efforts were made to compare the action of several different types of meter, a number of vane and propeller Ott instruments, 12 and 18 cm. (4.73 and 7.09 in.) in diameter, and an old-model Price (Gurley, 1903), each receiving identical tests. (See Fig. 33.) As the Price meter was 27 years old, only the results of the other meters were published. When the current meter was used for the determination of velocity distribution for other methods, that is, for titration and weir, three meters of the same type were used together on the same shaft.

Procedure of Experimentation and Computation

Five different rates of discharge, varying from 0.5 to 4.0 cu. m. (17.7 to 141 cu. ft.) per sec., were selected for study, the head gate on the canal being set at the beginning of each run to give as nearly as possible the discharge determined upon. A period of two hours was allowed for the flow to become constant, after which readings on the various gages along the length of the canal and at the measuring weir were begun and continued throughout the run. The current meters were first used, the measuring basin being filled during this time and the salt-velocity method immediately following the filling of the basin; the titration completed the sequence. This procedure was carried through three times for each run.

In the event of rain, the run was stopped, and then continued as soon as weather permitted. At the termination of each run, the results of the weir discharge and the measuring basin records were rapidly computed, so that in case of appreciable error the run might be immediately repeated under the same flow conditions. Approximately a month was required for the five runs.

All data were then taken to the Munich offices, and the calculation of results was carried out by a staff of six assistants in a period of about six weeks. A high degree of accuracy was sought throughout the period of experimentation and computation, often far more than would be thought necessary. This was done for a number of reasons: to reduce all error to a minimum, to make records of each step so complete as to obviate reproducing the same step at some time in the future, and in some cases to prove that certain influences had no appreciable effect upon the gross results—that is, canal seepage losses, rate of fall in water level of the measuring basin during gage reading extended as correction over filling time, etc.

The measuring-basin record was drawn from the large rating graph, and carefully marked as to time of day of filling. Seepage records for each section were also graphed, as seepage versus depth of flow. At the end of the calculations for each individual experiment, the results were corrected by use of the following routine chart, using for example the titration method:

1 Date
2 Titration number
3 Water depth at each seepage section
4 Seepage in each section from graph
5 Total seepage of all sections
6 Titration results: (*a*) by mixture of equal volume from each sample; (*b*) by weighting each sample for velocity
7 Mean head at weir h_t during titration
8 Mean head at weir h_m during filling of basin
9 $\Delta h = h_t - h_m$
10 $\tan \alpha$ (from basin rating curve)
11 $\Delta Q = \tan\alpha \cdot \Delta h$
12 Q from basin measurement •
13 Q from titration $= Q_t \pm \Delta Q$
14 Departure from basin results.

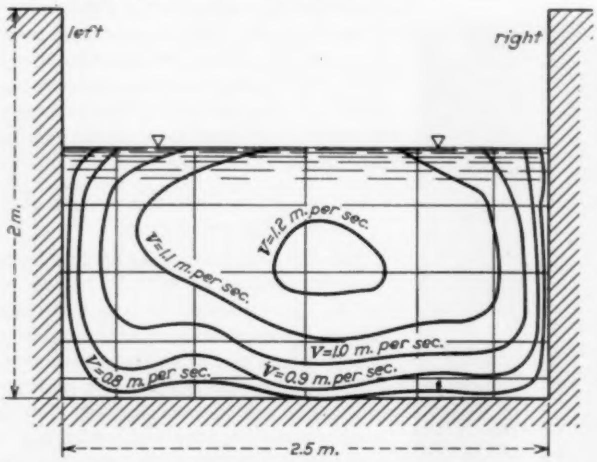

FIG. 34 VELOCITY DISTRIBUTION 20 M. (65.6 FT.) ABOVE WEIR DURING EXPERIMENT NO. 26
(Q = 3.3405 cu. m. per sec. (118 sec.-ft.); depth of water = 1.28 m. (4.2 ft.).)

RESULTS OF INITIAL EXPERIMENTS

Accuracy of weir measurements depends upon two interrelated qualities: the existence of uniform flow conditions with a weir of standard dimensions in good condition and the use of a reliable formula for discharge computation. Since the weir used was constructed as carefully as possible and flow conditions were as uniform as could be made (Fig. 34), the assumption that the first qualification was met leaves only a comparison of the resulting discharge values with those given by the accepted formulas.

In Fig. 35 are plotted curves of Bazin, Frese, S.I.A. (Swiss Society of Engineers and Architects), and Rehbock, together with the institute discharge results as given by the volumetric data of the measuring basin. The formulas[10] are as follows:

$$\text{Bazin (1888)}: \mu_0 = \left(0.6075 + \frac{0.0045}{h_0}\right)\left[1 + 0.55\left(\frac{h_0}{p + h_0}\right)^2\right]$$

$$\text{Frese (1890)}: \mu_0 = \left(0.615 + \frac{0.0021}{h_0}\right)\left[1 + 0.55\left(\frac{h_0}{p + h_0}\right)^2\right]$$

$$\text{S.I.A. (1924)}: \mu_0 = \left(1 + \frac{1}{1000h_0 + 1.6}\right)\left[1 + 0.5\left(\frac{h_0}{p + h_0}\right)^2\right]$$

(where $Q = \frac{2}{3}\sqrt{2g}\cdot\mu_0\cdot l\cdot h_0^{3/2}$ and p = crest height above canal floor, all dimensions being in meters)

$$\text{Rehbock (1929)}: \mu_0 = \left(0.6035 + 0.0813\,\frac{h_0}{p} + \frac{0.00009}{p}\right)\left(1 + \frac{0.0011}{h_0}\right)^{3/2}$$

(from $Q = \left(1.782 + 0.24\,\frac{h_e}{p}\right)\cdot l.h_e^{3/2}$ where $h_e = h_0 + 0.0011$ m.)

The crest height p of the plots in Fig. 35 is 523 mm. (1.716 ft.); experiments were also made with a height p of 850 mm. (2.795 ft.) with similar results.

Both were tested previously as models at a scale of approximately 1 to 7, giving results that checked almost perfectly with the Rehbock 1929 formula.

Since the four accepted formulas were derived from experiments conducted in laboratories to scale, the outcome of the institute tests seems to substantiate the doubt that exists with regard to the use of the sharp-crested weir formulas for large discharges, calling obviously for considerable research with the weir at full scale under greater heads than have so far been investigated.

In Table 1 are given the results of the three other methods, with the percentage departure from the measuring-basin data. The mean velocity varied from 0.3 m. (0.985 ft.) per sec. to 1.2 m. (3.94 ft.) per sec. The two Ott "Texas" meters gave excellent results, each rating slightly under actual discharge. The Price meter average error of 1.845 per cent greater than actual discharge was at first attributed to the comparatively small cross-section of the canal, but a larger Ott meter showed an average error of only —0.378 per cent. It is of course pointless to base a comparison upon the action of a meter 27 years old, in however good a condition it may be. The negative errors of 0.3 per cent and 0.6 per cent are a proof of the accuracy of the Ott instruments, but it depends nevertheless upon the rating of the meter prior to usage, since only after careful and dependable rating may a meter be so reliable.

Table 2 gives the relation of error in the salt-velocity method to the location and distance apart of the electrodes used, the results improving as the distance from the point of injection was increased.

Nearly all of the values are lower than the actual discharge, and a great number of individual tests is necessary before the average reduces the error to a satisfactory point. As may be seen from Table 1, the error of this method (— 1.2 per cent) is the greatest of all three. A further drawback to its practical use is the impossibility of determining beforehand the most favorable location of the electrodes.

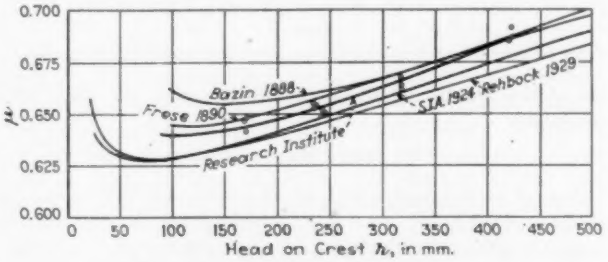

FIG. 35 GRAPH OF WEIR RESULTS
(Coefficient μ versus head on crest, plotted with curves from formulas by Bazin, Frese, S.I.A. 1924, and Rehbock 1929. Weir height, 523 mm., or 1.716 ft.)

Experience in penstock tests at the Walchensee Power Plant led to the same conclusion.

Most consistently accurate of all was the titration method, the departure being almost equally distributed above and below the measuring-basin data, and giving average errors of only + 0.03 per cent and + 0.06 per cent, respectively, for a mean of all points in the canal cross-section and a weighting of each sample to the existing velocity at that point. Since the current meter is suitable for only uniform flow conditions and the titration method depends upon turbulence for satisfactory mixing of the concentrated salt solution with the discharge, the two methods cover well the field of stream gaging.

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[10] Theo. Rehbock, "Wassermessung mit scharfkantigen Überallwehren," *Z.V.D.I.*, bd. 73, no. 24 (1929).

TABLE 1 COMPARATIVE RESULTS BY CURRENT METER, SALT-VELOCITY, AND TITRATION METHODS OF DISCHARGE MEASUREMENT

	Ott meter, form V "Texas," vane propeller			Ott meter, form V "Texas," screw propeller			Allen salt-velocity method					Titration or chemical method				
Experiment No.	Q in m³/s, measuring basin	Q in m³/s, current meter	Departure in % from Q_{mb}	Q in m³/s, measuring basin	Q in m³/s, current meter	Departure in % from Q_{mb}	Q in m³/s, measuring basin	Q in m³/s, electrodes e_2–e_3	Departure in % from Q_{mb}	Q in m³/s, electrodes e_2–e_4	Departure in % from Q_{mb}	Q in m³/s, measuring basin	Q_m in m³/s, mean over section	Departure in % from Q_{mb}	Q_v in m³/s, weighted to velocity	Departure in % from Q_{mb}
1	0.5522 0.5525	0.5508 0.5455	—0.25 —1.27	....	....	....	0.5531	0.5488	—0.78	0.5406	—2.26	0.5547	0.5534	—0.23	0.5521	—0.48
2	....	....	....	0.5541 0.5544	0.5505 0.5590	—0.65 +0.83	0.5542	0.5457	—1.53	0.5457	—1.53	0.5551	0.5526	—0.45	0.5562	+0.20
3	....	....	....	....	....	....	0.5532	0.5462	—1.27	0.5474	—1.05	0.5529	0.5502	—0.49	0.5498	—0.56
4	0.5694 0.5705	0.5673 0.5699	—0.36 —0.10	....	....	....	0.5701	0.5571	—2.28	0.5589	—1.96	0.5705	0.5688	—0.29	0.5672	—0.56
5	....	....	....	....	....	....	0.5774	0.5617	—2.72	0.5606	—2.90	0.5785	0.5743	—0.72	0.5739	—0.79
6	....	....	....	0.5796 0.5794	0.5775 0.5798	—0.36 +0.06	0.5796	0.5590	—3.56	0.5717	—1.37	0.5800	0.5763	—0.63	0.5733	—1.15
7	0.8807 0.8828	0.8735 0.8780	—0.82 —0.55	....	....	....	0.8833	....	....	0.8601	—2.62	0.8833	0.8867	+0.39	0.8867	+0.39
8	....	....	....	0.8792 0.8804	0.8770 0.8860	—0.24 +0.63	0.8796	....	....	0.8683	—1.28	0.8809	0.8786	—0.26	0.8802	—0.07
9	....	....	....	....	....	....	0.8796	....	....	0.8682	—1.30	0.8864	0.8861	—0.03	0.8867	+0.03
10	1.3437 1.3426	1.3410 1.3400	—0.20 —0.20	....	....	....	1.3416	1.3286	—0.97	1.3311	—0.78	....	....	....	....	....
11	....	....	....	1.3457 1.3432	1.3560 1.3560	+0.76 +0.96	1.3442	1.3507	+0.49	1.3331	—0.83	....	....	....	....	....
12	1.3813 1.3850	1.3920 1.3890	+0.77 +0.29	....	....	....	1.3806	1.4204	+2.88	1.3829	+0.17	1.3855	1.3888	+0.24	1.3920	+0.47
13	....	....	....	....	....	....	1.3901	1.4193	+2.10	1.3852	—0.35	1.3939	1.4018	+0.57	1.4057	+0.85
14	....	....	....	1.3938 1.3962	1.3900 1.3930	—0.27 —0.23	1.3940	....	....	1.3967	+0.19	1.3963	1.4092	+0.93	1.4165	+1.45
15	1.8015 1.8002	1.7820 1.7880	—1.08 —0.68	....	....	....	1.7999	....	....	1.7803	—1.09	1.8002	1.8166	+0.91	1.8140	+0.77
16	....	....	....	1.8023 1.8010	1.7950 1.8000	—0.40 —0.05	1.8002	....	....	1.7950	—0.29	1.7999	1.8187	+1.05	1.8165	+0.92
17	2.1581 2.1645	2.1360 2.1430	—1.02 —1.00	....	....	....	2.1604	2.0870	—3.40	2.1157	—2.07	2.1645	2.1688	+0.20	2.1720	+0.34
18	2.2835 2.2852	2.2804 2.2124	—0.13 —3.18	....	....	....	2.2846	....	....	2.2569	—1.21	2.2875	2.3203	+1.43	2.3100	+0.98
19	....	....	....	2.2898 2.2870	2.2660 2.2500	—1.04 —1.62	2.2873	....	....	2.2785	—0.38	2.2856	2.2714	—0.62	2.2753	—0.45
20	....	....	....	....	....	....	2.2825	....	....	2.2643	—0.79	2.2838	2.2571	—1.17	2.2676	—0.71
21	....	....	....	2.4919 2.4910	2.4700 2.4690	—0.88 —0.88	2.4914	....	....	2.4718	—0.78	2.4904	2.4982	+0.32	2.5034	+0.52
22	....	....	....	....	....	....	2.4953	....	....	2.4840	—0.45	2.4954	2.4912	—0.17	2.4931	—0.09
23	3.3287 3.3287	3.3010 3.2880	—0.83 —1.22	3.3314 3.3327	3.3150 3.3220	—0.49 —0.32	....	....	....	....	....	3.3350	3.3399	+0.15	3.3318	—0.10
24	....	....	....	....	....	....	3.3297	....	....	3.3095	—0.61	3.3318	3.2349	—2.91	3.2753	—1.70
25	3.3314 3.3346	3.3400 3.3410	+0.26 +0.19	....	....	....	3.3320	....	....	3.2713	—1.82	3.3328	3.3246	—0.25	3.3312	—0.05
26	....	....	....	3.3379 3.3405	3.3310 3.3500	—0.21 +0.28	3.3361	....	....	3.3100	—0.78	3.3405	3.3501	+0.29	3.3373	—0.09
27	3.9796 3.9790	3.9770 3.9480	—0.07 —0.78	....	....	....	3.9790	....	....	3.8795	—2.50	3.9790	3.9869	+0.20	3.9778	—0.03
28	....	....	....	3.9799 3.9801	3.9396 3.9199	—1.01 —1.51	3.9792	....	....	3.9181	—1.54	3.9768	4.0680	+2.29	4.0369	+1.51
Average in per cent			—0.556			—0.302			—1.003		—1.192			+0.029		+0.062

NOTE: e_2, e_3, e_4 = electrodes; Q_m = average value of all points in cross-section; Q_v = average value of all points after each has been weighted to velocity.

TABLE 2 SALT-VELOCITY METHOD

(Variation in accuracy of results between different pairs of electrodes.)

Electrode combination[a]	Number of measurements	Average departure from measuring-basin results in per cent
0–e_1	109	—5.94
0–e_2	97	—5.09
0–e_3	42	—3.37
0–e_4	71	—2.85
e_1–e_2	103	—4.69
e_1–e_3	43	—2.54
e_1–e_4	78	—2.37
e_2–e_3	27	—1.00
e_2–e_4	79	—1.19

[a] 0 = point of injection; e_1, e_2, e_3, e_4 = electrodes.

ENGELS RIVER EXPERIMENTS

In the fall of 1930 work was started on a large, shallow flume of trapezoidal cross-section in which Geheimrat H. Engels, former director of the River Hydraulic Laboratory at the Technische Hochschule of Dresden, planned to conduct a series of river studies. These were to correlate his previous small-scale experiments on behavior of river sediment, performed indoors at the Dresden laboratory and based upon actual conditions of the Elbe River. (See "Hydraulic Laboratory Practice," pp. 95, 105.)

The flume was located some distance to the west of the upper end of the main canal (see Fig. 2), so that only a short intake canal was necessary from the western end of the supply basin. The flume was approximately 10 m. (33 ft.) wide and 100 m. (330 ft.) long and was lined with packed stone and coarse gravel from the Obernach bed. Discharge into the upper end of the flume was regulated by a vertical gate similar to that on the main canal, while at the lower end a short concrete canal section identical with the main concrete canal was constructed, with the same measuring weir that had been rated in the initial major experiments already described. The water discharged directly into the Obernach from the tumble bay below the weir.

Experimentation was started after the completion of the measuring apparatus early in the summer of 1931. Unfortunately it is impossible at this writing to describe in detail the exact methods and results of this research, as these have not yet been made public. A report will, however, appear in the near future in one of the American engineering papers.

The plans for procedure as proposed in July, 1931, are shown in Fig. 36. In all, five different channel and levee layouts were to be studied, each under the three discharges, giving, respectively:

(*a*) Low-water stage, with a water depth over channel sediment bed of 5 cm. (1.97 in.)

(*b*) Medium high-water stage, water depth 10 cm. (3.93 in.), just topping the channel banks (*c*) Maximum high-water stage, water depth 20 cm. (7.87 in.), in which the whole flume or river bed between main levees would be flooded.

These three separate stages were to be supplemented by rising and falling stages, for the purpose of studying the influence upon bar formation of varying discharge.

Preliminary experiments with different grades of sediment from the Obernach were first conducted in a straight temporary wooden flume of the same width as the model channel, to determine the most satisfactory slope and grain diameter. Sediment of a grain diameter varying from 0 to 5 mm. (0 to 0.2 in.) and a slope of 0.1 per cent or 0.05 per cent were finally chosen.

This sediment was leveled in the model at a depth of 10 cm. below the low-water banks before each run commenced; thereafter additional sediment was fed into the upper end of the channel in measured quantities, and collected at the lower end in a catch basin. Water depths were taken at intervals over the channel length by means of piezometer connections leading from the channel bed to gages of type A outside the flume. A trussed timber traveler had been built across the flume, moving longitudinally along tracks on each bank, to facilitate measuring the contours of the bars formed in the channel during the various flow stages.

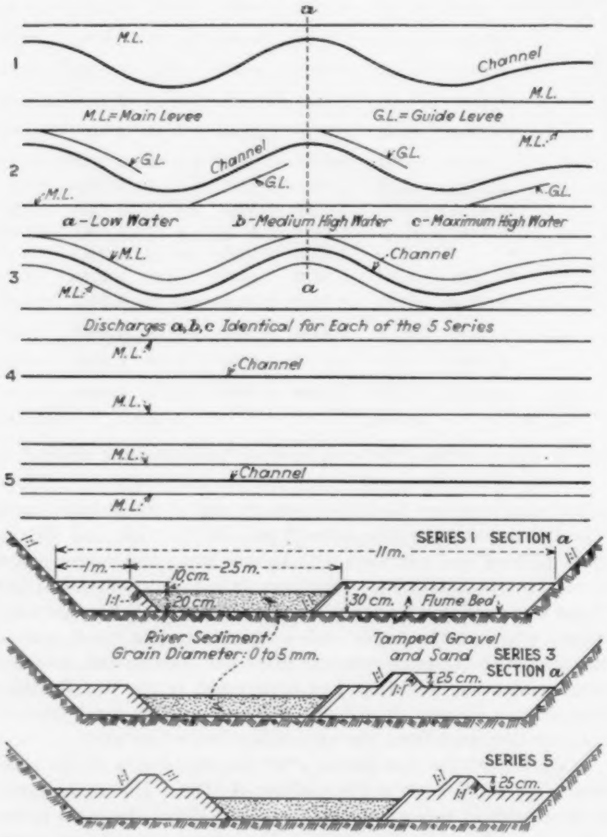

Fig. 36 Engels River Flume

Fig. 37 is the Engels river model at flood stage, without dykes. Though the water covers the bed of the flume, the serpentine outline of the submerged banks is clearly visible.

Fig. 38 is a view of the Engels river flume, looking downstream. The bed is dry, showing the construction of the continuous dykes. The tracks at right and left carry the observation and contour platform visible in the distance.

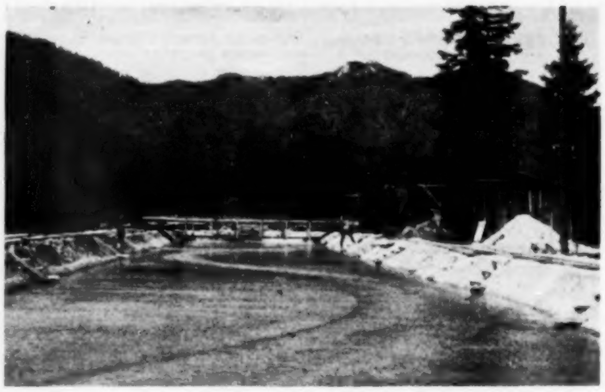

Fig. 37 Flood-Water Conditions Without Dykes

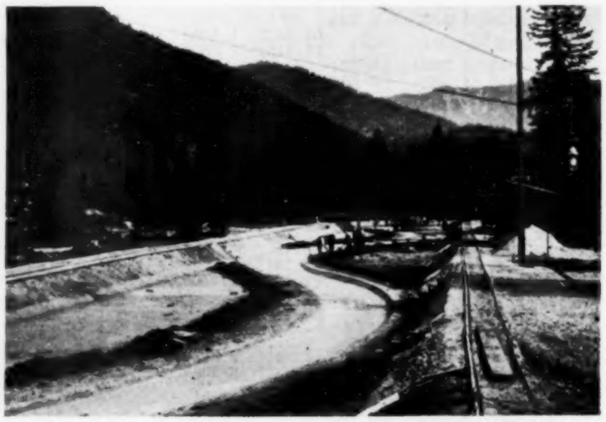

Fig. 38 Dry Model With Continuous Dykes

HYDRAULIC INVESTIGATIONS AFIELD

Soon after the founding of the research institute, but prior to the completion of the experiment station, its staff began a series of studies on canal flow, using actual channels in the vicinity to the south of Munich for their research—namely, the Isar diversion canal below Krünn and the Ampelsbach canal near the Achensee in Tyrol. Results of the experiments have not yet been published, since it is intended to carry on the investigations several years longer with further observations in other localities. At the same time, a study over a period of years is being conducted by the institute dealing with the deposition of sediment by the Isar River at its entrance into the small reservoir at Krünn just before its diversion.

The first model study completed by the institute staff was carried on in the laboratory of the Munich Hochschule, for the design of a spillway crest at one of the Mittleren Isar power plants northeast of Munich. Results were made known in the first publication of the institute in 1928; they may also be found in "Hydraulic Laboratory Practice" under the chapter about the Munich laboratory, in a brief discussion by Dr. Kirschmer.

Flow Measurements at the Walchensee Power Plant

For the purpose of comparing the leading European and American methods of measurement of penstock discharge for the determination of turbine efficiency, a series of tests was made at the Walchensee Power Plant by several English and German firms in collaboration with the research institute. The power plant permitted the use of one penstock and an 18,000-hp. Pelton turbine. Salt-velocity and pitometer measurements were conducted by the British Pitometer Company, Ltd., of London, commissioned by the Canadian and General Finance Company, London; current-meter tests were supervised by J. M. Voith, Heidenheim-am-Brenz; a Gibsonmeter was installed and operated by Dipl.-Ing. P. Volkhardt, Munich; electrical measurements of power output were in charge of the staff of the Walchensee Power Plant; and the research institute contributed a portion of the salt-velocity apparatus and supervised the measurement of the penstock for head, diameter, and location of apparatus, and published the description of the measurements and a tabulation of results in the *Z.V.D.I.*[11]

The six steel penstocks of the power plant lead from a concrete surge tank down an approximately 40-deg. slope to the plant 183 m. (600 ft.) below, giving an over-all head of 192 m. (See Fig. 39.) They vary in diameter from 2.250 m. (7.39 ft.) at the top to 1.850 m. (6.07 ft.) at the lower end, and are anchored at six points, two of these being on the slope between the surge tank and the plant; at the upper anchor on the slope, the penstocks make a small vertical bend. Penstock No. 5, selected for the tests, supplies two Pelton turbines, one being shut down entirely during the runs.

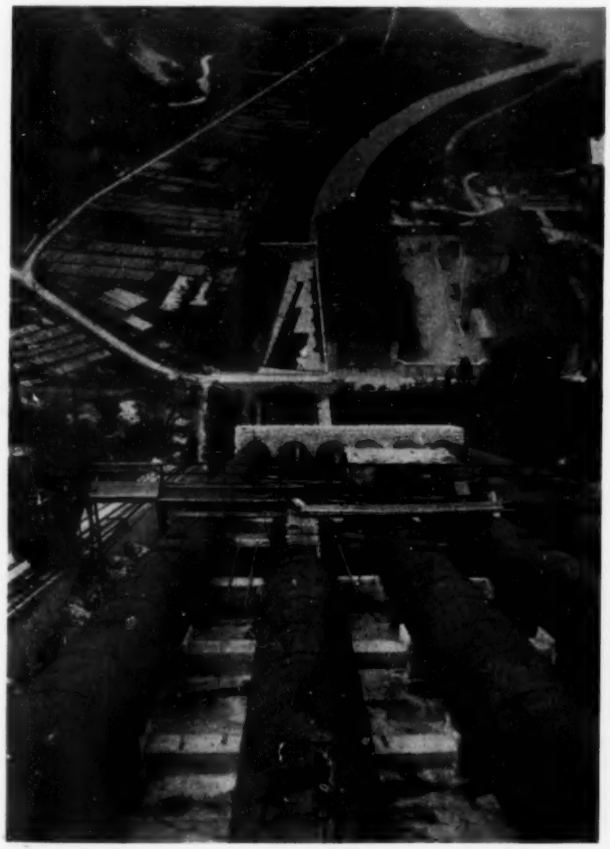

FIG. 39 VIEW FROM SURGE TANK DOWNWARD UPON PENSTOCKS AND PLANT OF THE WALCHENSEE KRAFTWERKE

(Platform for recording instruments is visible at site of current meter and pitot-tube measurements.)

Injection apparatus for the salt solution was located just below the surge tank, operating under a pressure of 7.8 atmospheres to overcome the head on the penstock at this point of 20 m. (65.60 ft.) and the excessively high friction loss in the valves and joints of the apparatus. A single injection valve, furnished by the British Pitometer Company, was set at the midpoint of the penstock cross-section, in such a way that it could be withdrawn for inspection without shutting down the flow. A single injection, 25 to 35 liters (quarts) of solution, lasted from 1 to 2 sec. Four electrodes were distributed over nearly the length of the penstock, in order to offset the difficulty of determining in advance the most advantageous location. They consisted of pairs of short copper strips held rigidly 3.2 mm. (0.126 in.) apart, each strip 52 by 19 by 1.6 mm. (2.05 by 0.75 by 0.063 in.) in size. They were of necessity set at the wall of the penstock, rather than extended over the entire diameter, to avoid complicated construction. It was believed that this would not influence the results materially, since they were four in number and set at various points around the circumference.

Some 18 diameters below the surge tank, the supporting rods for both the pitometer and the current meters were built into the penstock wall in the same cross-section at an angle of 90 deg. from each other, so that when the pitometer was in use, the other was drawn as far as possible out of the way against the penstock wall; the pitometer could be completely withdrawn. Two Ott form-V "Texas" electrically recording current meters were fastened upon their shaft at a distance of 1.038 m. (3.41 ft.) from each other, so that the shaft could be moved to cover the entire diameter in 28 points, coming to within 5.3 cm. (2.09 in.) of the walls; readings for each run were completed in less than half an hour. At the end of the experiments the meters were again rated on the same shaft.

The pitometer measurements were made simultaneously with two instruments, working from opposite ends of the penstock diameter. Due to the presence of the current shaft, the pitot tubes could be inserted only to within 25.5 cm. (10 in.) of the penstock axis. Readings were made from a wood scale behind a glass U-tube filled with carbon tetrachloride, the specific gravity being determined before each experiment. Measurements were made at 16 points on the diameter, the process lasting about twice as long as the current-meter readings.

TABLE 3 MEASURED DISCHARGES AND LOAD ON TURBINE

Experiment no.	Load N, in metric horse-power	Method of measurement							
		Allen		Current meter		Pitot tube		Gibson	
		Q m³/s	Eff. η %	Q m³/s	Eff. η %	Q m³/s	Eff. η %	Q m³/s	Eff. η %
1	8,234.24	...	...	3.848	83.58	3.852	83.54	...	...
2	12,835.26	...	...	5.970	83.99	5.945	84.35	...	...
3	12,744.11	...	...	5.923	84.05	5.903	84.36	...	...
4	5,414.00	...	...	2.598	81.39	2.695	78.49	...	...
5	17,069.66	...	...	8.221	81.10	8.304	80.32	...	...
6	......	...	...	4.004	...	4.054	...	...	...
7	5,110.21	2.470	80.76	2.476	80.63	2.548	78.35	...	...
8	8,664.58	4.010	83.40	3.951	85.66	4.071	83.13	...	...
9	12,867.27	5.969	84.19	5.967	84.23	5.941	84.61	...	...
10	17,275.85	8.143	82.94	8.157	82.74	7.996	84.41	...	...
11	5,715.28	...	...	...	...	...	...	2.709	82.41
12	5,674.37	...	...	...	...	...	...	2.686	82.52

NOTE: All loads and discharges converted to a head of 192 m.

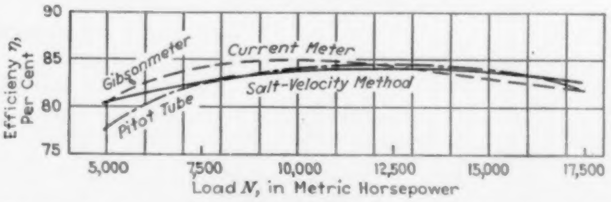

FIG. 40 GRAPH OF EFFICIENCY VERSUS LOAD AS MEASURED BY CURRENT METER, PITOT TUBE, GIBSONMETER, AND SALT-VELOCITY METHOD

Depending upon the intensity of back pressure on the shutting down of a turbine to give indirectly the previous velocity in the penstock, the Gibsonmeter was designed to record photographically the intensity record, in such a way that the internal friction in the instrument is reduced to almost nothing. Unfortunately, incorrect regulation of a spring in the mechanism was not discovered till the films were developed, so that only two experiments gave usable results. The fact that the turbine required a double valve action to close down gave further difficulty under high loads.

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[11] Vergleichs-Wassermessungen am Walchenseewerk," by Dr.-Ing. O. Kirschmer, *Z.V.D.I.*, 1930, bd. 74, no. 17.

Results of the experiments (see Table 3 and Fig. 40) show clearly that, with the conditions under which the tests at the Walchensee plant were made, the current meters gave by far the most dependable results, judging by mean values and a comparison with ratings made at the plant four years previously. The pitometer readings required some four times as long a period, and the unstable level in the U-tube proved particularly bothersome under low discharges. The salt-velocity method showed excessive departure from its own average of readings except on the stretches from the injection point to the third and fourth electrodes, and as these were in use on only four out of twelve runs, the other values were discarded. The cause of the poor results is laid to the very short electrodes used and the difficulty in determining the center of gravity of the graphed areas. These computations were made independently by both the British Pitometer Company and Professor Allen. The two successful Gibsonmeter results are within 0.5 per cent of the current-meter readings.

Although these experiments were conducted by recognized authorities in the field of practice, they demonstrate only too well the great differences in the results of even the best measurement methods known today and the necessity of some absolute standard with which each method can be compared. Comparative tests among several such methods show, at best, simply comparative values, from which the true results can only be assumed. It is to this end—the actual volumetric checking during measurement—that the research institute is striving. When the performance of instruments under actual flow conditions is established, discharge measurement will be far more accurate.

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Engineers Club of San Francisco
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Stanford University.........Library, Stanford University

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Bridgeport.......Public Library
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Cincinnati	University of Cincinnati
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Cleveland	Public Library
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Columbus	State of Ohio Library
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	Ohio State University
Dayton	Engineers Club of Dayton
Toledo	Public Library
	University of Toledo
Youngstown	Public Library

Oklahoma

Norman	Oklahoma University
Oklahoma City	Public Library
Stillwater	Oklahoma Agricultural and Mechanical College
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Pittsburgh	University of Pittsburgh
	Engineers' Society of Western Pennsylvania
	Carnegie Institute of Technology
	Carnegie Library (Schenley Park)
	Carnegie Free Library of Allegheny
Reading	Public Library
Scranton	Public Library
State College	Pennsylvania State College
Swarthmore	Swarthmore College
Villanova	Villanova College
Wilkes-Barre	Public Library

Rhode Island

Kingston	Rhode Island State College
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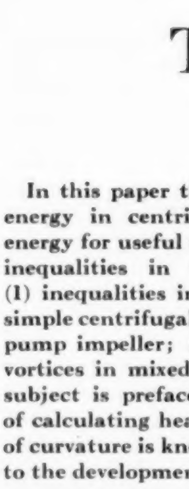
HYD-54-4

Turbulence in Centrifugal Pumps

By J. W. MACMEEKEN,[1] BERKELEY, CALIF.

In this paper the author discusses whirlpool or vortex energy in centrifugal pumps, the harnessing of this energy for useful work, and the effect of different kinds of inequalities in vortices. He deals respectively with (1) inequalities in the forced vortex of an impeller in a simple centrifugal pump; (2) waste energy in a centrifugal pump impeller; and (3) the synchronization of parallel vortices in mixed-flow impellers. The treatment of the subject is prefaced by a brief description of a method of calculating head losses in river bends when the radius of curvature is known, the results obtained with which led to the development of the pump study presented.



THE theory of centrifugal or turbine pumps has been elaborated in great detail during the last fifteen years as a result of the broad advance in the science of applied fluid dynamics. Since the modern centrifugal pump first came into use with Young's invention in 1851, its application in modern improved forms has placed it in the first rank among other important power machines in which efficiency of operation and economy of construction have reached a highly perfected stage of development. The development of the theory of kinematic and dynamic similarity has made possible the wide application of successive improvements obtained by research combined with analytical development of theory. During a period of years, a wide industrial field of application for centrifugal pumps has offered a stimulus for gradual improvement, which justifies very careful study as a means to ultimate economy of power and material.

The following discussion is not intended to introduce any new theory or contradiction of recognized theory of centrifugal-pump performance and design. It has for its object rather the introduction of some aspects of centrifugal-pump problems from the point of view of our knowledge of the observed characteristics of the vortex. A centrifugal pump is essentially a hydraulic machine which allows the generation of hydraulic pressure by means of a forced vortex created by a rotating impeller, combined with a free vortex at its outlet, so constructed as to make all the static and kinetic energy generated available as far as possible in the discharge pipe. This combination of forced spiral vortex and free spiral vortex is capable of certain variations and imperfections, some of which have an adverse effect, and some of which merely result in variations in pump characteristics without impairing the efficiency of the machine or the continuity of the characteristic curve.

The author had occasion several years ago to make a detailed study of hydraulic conditions in a section of the St. Lawrence River from the point of view of head available for hydroelectric power. It was noted that sections of the river and of large power canals, in which flow occurred in curved directions (partial free vortex), showed marked variations in surface levels and in loss of head, which could not be explained by the usual formulas for flow in rivers, such as Bazin's 1897 formula combined when necessary for change in cross-sections of channel with a modified Bernoulli equation. It was necessary to find a means of calculating the water-surface levels in curved channels by some method based on mathematical principles known to be correct to within a small margin of error, and to check these calculations against more than one actual surveyed condition of the river. After this had been done satisfactorily, a means was established for calculating the effect on water levels and available power head of construction work on the river to alter its flow in modified and partially artificial channels in order to gain increased available power head. Correction factors or coefficients of local distribution as a function of the channel curvature were found by both field observation and by calculation. These factors are remarkably similar in value to those used in the case of centrifugal pumps to correct mean velocities to the mean energy values, so that it appears that the flow of water in the curved river channels and in the curved path through a centrifugal pump may be treated both in the same way and calculations may be made based on geometrical considerations only. In short, the flow of rivers in curves may be calculated on the same basis as the flow in straight canals by using correction factors which are a function of the curvature and may be calculated, and similarly the flow of water in a centrifugal pump and its effect on generated head may be calculated by the use of the same correction factors as a function of channel curvature.

In addition to foregoing considerations, this discussion deals with a method of obtaining synchronization of effect of vortex action in mixed-flow impellers where the flow may be regarded as a series of parallel spiral forced vortices in the impeller, modified to some extent by included local free vortex action.

NOTATION

ω = angular velocity, or angular displacement
R = radial distance from center of rotation
g = acceleration due to gravity
U = peripheral velocity of impeller vane
W = absolute velocity
V = relative velocity
V' = value of V when used for mean total-energy calculations
H = head
H_t = theoretical head
C = peripheral component of W. C is also used as a coefficient in the formula $V = C\sqrt{(rs)}$ and is the value for uniform motion in a straight line
V_r = radial component of W or V
$\tan \beta_2$ = tangent of actual exit angle of impeller
$\tan \beta_2'$ = tangent of apparent exit angle
L = length of channel
r = the hydraulic mean radius

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$Z_{1,2}$ = fall in water surface in an open channel between two points

$(1 + \rho)$ = calculated from the curvature of the streamline boundaries of a channel and the position of the first and second centroids with reference to the local center of curvature

η_H = hydraulic efficiency

γ = Bazin's coefficient in the formula $C = 157.8/[1 + (\gamma/\sqrt{r})]$

V_R = radial velocity

METHOD OF CALCULATING HEAD LOSSES IN RIVER BENDS WHERE THE RADIUS OF CURVATURE IS KNOWN

When the loss of head in a large curved river channel as shown in Fig. 1 was computed by Bazin's 1897 formula where $C = 157.8/[1 + (\gamma/\sqrt{r})]$, in the equation, head lost by friction $= V^2L/C^2r$, it was found that the computed loss of head was very much less than the actual loss even when the channel had been accurately surveyed and carefully measured. When the value of γ was determined for a similar straight channel, it was found that the head lost in a carefully surveyed river bend could only be explained by the following equation: Head lost between two sections of channel a given distance apart =

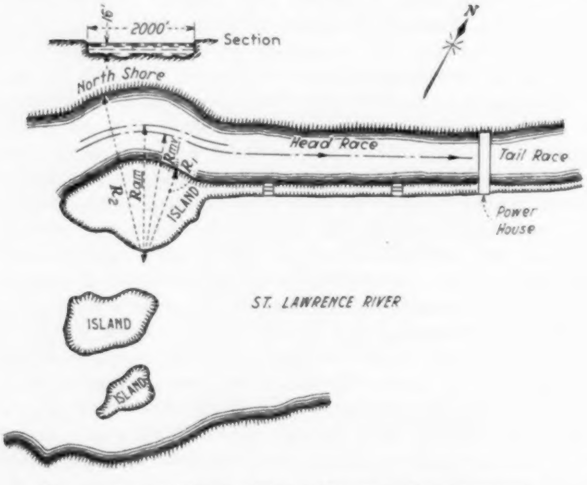


FIG. 1 LARGE CURVED RIVER CHANNEL INVESTIGATED
(Additional coefficient to use with Bazin's formula to correct for curved channels: $(1 + \rho) = {}^1/_2(R_1^2 + R_2^2)/[(R_1 + R_2)/2]^2$.)

$$Z_{1,2} = \frac{(1 + \rho_1)V_1^2 - (1 + \rho_2)V_2^2}{2g} + \int_{L=0}^{L=1-2} \frac{V^2(1 + \rho)dL}{C^2r}$$

$(1 + \rho)$ is a coefficient of local distribution, and its value was obtained in two ways: First, by gaging the stream at sections 1 and 2 a short distance apart, recording all the values of the velocity at a uniformly distributed number of points, and finding both the average velocity and ΣV^2/number of observations; then

$$(1 + \rho) = \text{ratio } \frac{\Sigma V^2/N}{(\text{average } V)^2}$$

The value of $(1 + \rho)$ was next computed by the formula

$$(1 + \rho) = \frac{\dfrac{R_1^2 + R_2^2}{2}}{\left(\dfrac{R_1 + R_2}{2}\right)^2}$$

and the same values resulted.

When values of $(1 + \rho)$ were inserted in the equation for head lost $Z_{1,2}$ between a consecutive series of points located in mid-channel about 300 ft. apart in a channel two miles long, the solving of the equation to explain the existing hydraulic gradient for known river discharge and known channel cross-sections resulted in approximately the same values of C and of Bazin's γ as were found for straight sections of the same kind of channel. The reason apparently is that all losses are proportional to V^2. This is not correct, but the error in assuming it to be so is very small. Therefore the mean value of V^2 should be used rather than the square of V as capacity/area of the channel section.

The fact that the results by both methods were the same showed that the variation was caused by curvature and could be calculated. Before calculating this coefficient, the direction of flow was traced by a series of floats whose bearings were referred to a base line on the shore, so that the curvature of the streamlines of flow as well as of both banks of the stream was known.

The consistent reliability of the results obtained by both methods gave the author great confidence in the reliability of the formula. The value of the coefficient $(1 + \rho)$ varied from 1.20 to 1.70 in curved channels down to 1.06 in straight channels adjoining bends in the stream.

The fact that these values were within the same range as similar coefficients obtained from the analysis of a large number of tests of centrifugal pumps, prompted a study of the significance of this effect, particularly as the variation in value of the coefficient with variation in the ratio of values of R_2/R_1 showed marked functional similarity in both pump and river channels. The purpose of this "coefficient of local distribution," which might be called a "coefficient of turbulence," is to convert "mean velocity" into its equivalent value for "mean-energy" calculations.

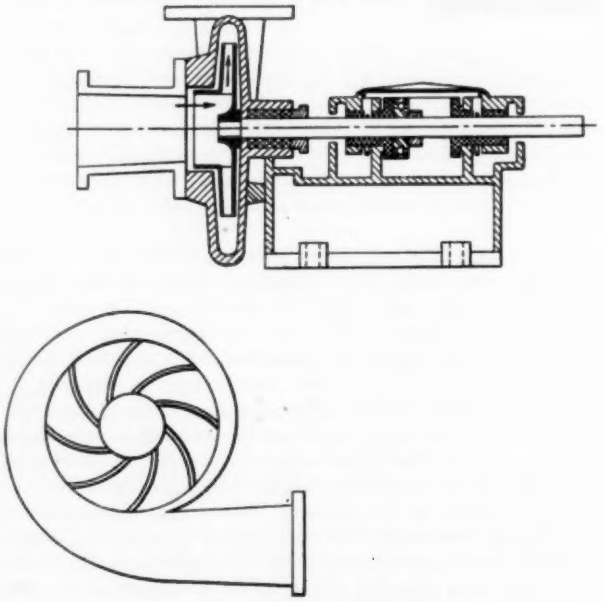


FIG. 2 SIMPLE CENTRIFUGAL PUMP

I—INEQUALITIES IN THE FORCED VORTEX OF AN IMPELLER IN A SIMPLE CENTRIFUGAL PUMP

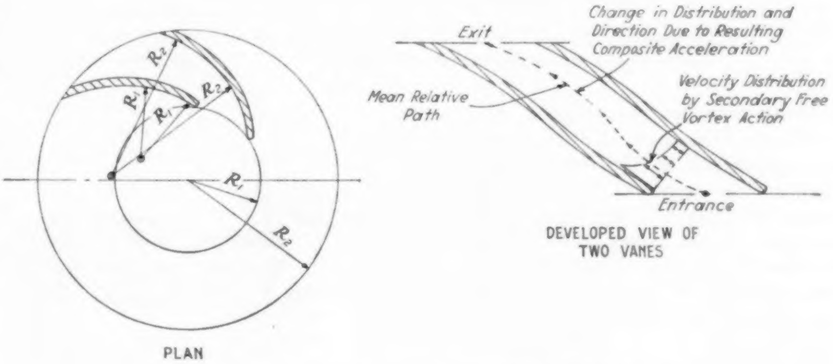
A simple centrifugal pump is shown in Fig. 2. This consists of an impeller and an outlet chamber. Water flows through the impeller theoretically as a forced spiral vortex and through the outlet chamber into the pipe as a free vortex, or rather as a free spiral vortex. This simple centrifugal pump is narrow for its diameter, and the equations for centrifugal force and total energy, assuming a perfect forced vortex in which all particles have the same angular velocity, are generally given as follows:

$$\text{Total energy per pound at periphery} = \frac{\omega^2 R^2}{g} = \frac{U^2}{g} \quad \text{. . . . [1]}$$

when there is no flow

$$\text{Total energy when flow takes place} = \frac{W^2}{2g} + \frac{U^2 - V^2}{2g} \quad \text{. . [2]}$$

or more correctly, the difference between these values for the outlet and inlet radii of the impeller. The element $W^2/2g$ represents the kinetic energy discharged into the free vortex of the outlet where the value of W becomes greatly reduced under the law of constant moment of momentum, viz., $W_1R_1 = W_2R_2$, etc. per unit mass.



R2
R1
R2
R1
R1
R2
PLAN
Exit
Change in Distribution and Direction Due to Resulting Composite Acceleration
Mean Relative Path
Velocity Distribution by Secondary Free Vortex Action
Entrance
DEVELOPED VIEW OF TWO VANES

FIG. 4 TYPICAL IMPELLER WITH MODIFIED INVOLUTE VANES

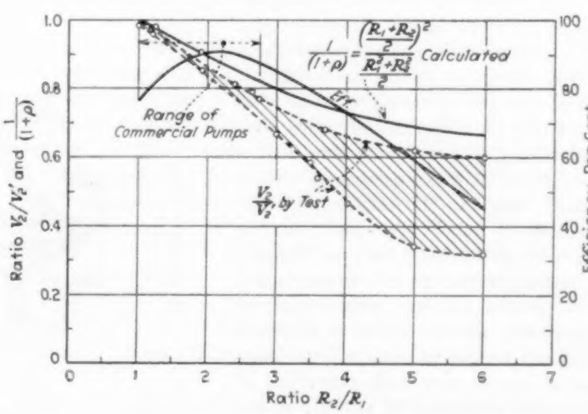


FIG. 5 CALCULATED CORRECTION FACTORS COMPARED WITH TEST RESULTS

The element $(U^2 - V^2)/2g$, which represents the static head developed in the impeller, being a function of V^2, V^2 should not be calculated as (average V)^2, but rather as the mean value of V^2, V not being constant across the channels of the impeller. The mean value of V^2 may be calculated from the test of a pump by measuring all purely mechanical losses and obtaining the hydraulic efficiency

$$\eta_h = \frac{\text{Water hp.}}{\text{B.hp-losses}}$$

and theoretical head

$$H_t = \frac{\text{Actual head}}{\text{Hydraulic efficiency}}$$

An equivalent form of Equation [2] for total energy is

$$H_t = \frac{U_2C_2 - U_1C_1}{g} = \frac{U_2C_2(1-e)}{g} \quad \text{. . . . . [3]}$$

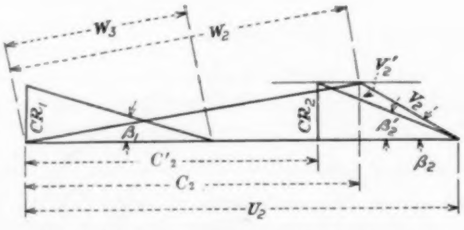


FIG. 3 DISCHARGE-VELOCITY DIAGRAM

$$H = \frac{U_2C_2 - U_1C_1}{g} = \frac{U_2^2}{2g} + \frac{W_2^2}{2g} - \frac{V_2^2}{2g}$$

By solving this equation for C_2, other values being known, the discharge-velocity diagram (see Fig. 3) may be constructed and a new or apparent value of V_2 may be found, which we may call V_2'. This value is always greater than the average value of V_2 measured as Q/A. The reason obviously is that this new value V_2' represents in the energy equation the square root of the mean value of V^2, because the static pressure in the impeller is a function of V^2, and the mean value of $V^2 = \Sigma V^2$/number of observations should be used rather than (average V)^2, because V is not uniform across the impeller channel. All impeller vanes in pumps of this simple type are either involute curves or approximations to such curves (see Fig. 4), and each water channel is bounded by two curves, one of which has a radius approximately equal to R_2, and the other a radius approximately equal to R_1, so that there is a secondary variation in velocity in each impeller channel bounded by two vanes in accordance with the laws of a free vortex, because water is not an elastic medium, and the influence of the vanes for creating a perfect forced vortex is only local. This variation of V between two vanes should be a function of the bounding radii R_1 and R_2 as in the flow in a curved river channel, and the

$$\frac{\text{Mean value of } V^2}{(\text{Average } V)^2} = \frac{\dfrac{R_1^2 + R_2^2}{2}}{\left(\dfrac{R_1 + R_2}{2}\right)^2} = (1 + \rho) \qquad [4]$$

which latter is a factor of local distribution in terms of curvature. This may be expressed for convenience in the form of a change in tan β_2. Correction factors calculated in this way are shown plotted in Fig. 5 compared with actual test values. In Fig. 5, the similarity of variation of V_2/V_2' according to test values, to the variation of the factor $1/(1 + \rho)$ is clear, $(1 + \rho)$ being a coefficient of local distribution and a function of channel curvature. The lower values of V_2/V_2' require to be multiplied by an additional coefficient $(1 + \alpha)$ having a value of from 1.04 to 1.06 in the range of commercial pumps. The coefficient $(1 + \alpha)$ is a coefficient of local distribution, expressing inequalities due to change of cross-section only, and when the impeller

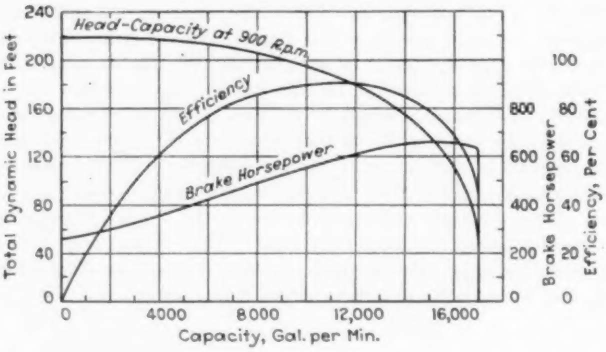


FIG. 6 CHARACTERISTIC CURVE OF AN 18-IN. DOUBLE-SUCTION CENTRIFUGAL PUMP AT CONSTANT SPEED

channels are so proportioned geometrically that there is little change in cross-section in the relative path through the impeller, its value approaches unity. This explains the variation in the values of V_2/V_2' due to effects other than curvature, which are incidental to the geometrical construction of the impeller. Thus the coefficient $(1 + \alpha)$, which is a coefficient of local distribution for a channel of varying cross-section, and $(1 + \rho)$, which is a coefficient to convert (mean velocity)^2 into mean energy on a basis of the mathematical mean value and is a function of channel curvature, afford a means of calculating the head generated by a centrifugal pump, supplementary to the fundamental Equation [2] or [3], which assumes a perfect forced vortex in the impeller. These coefficients make allowance for the effect of partial free vortex distribution of velocity in the space between vanes.

In general, it has been found by experience that the values of these coefficients are independent of the number of vanes, vary with curvature of vanes and with the unit velocity in the impeller, but most of all are dependent on the ratio $R_2:R_1$. This explanation of the apparent reduced value of tan β_2 as due to merely local free vortex action makes it clear why this effect causes no measurable falling off in hydraulic efficiency. Such suggested causes as "component of whirl at exit," "tractive effort of the vane," "impedance of the casing," etc., would be more likely to cause a reduction in efficiency. If the effect is caused merely by free vortex effect in the curved channels of the impeller mixed with the general forced vortex effect of the impeller, the underlying cause is the only condition under which water could flow through the impeller in accordance with the laws of fluid dynamics, and therefore no loss in efficiency should be expected.

It may be argued that free vortex effect on secondary distribution of velocity between the vanes would result in expected distribution of flow opposite to that which has been observed in experimental research. Consider, therefore, the effect of this secondary distribution on the composite action of acceleration commonly referred to as "Coriolian effect." As soon as this secondary circulation begins to appear in the initial stages of the impeller passage where composite acceleration is small, it creates local differences of pressure (rapidly increasing toward the end of the channel), which result in the generally accepted distribution of flow at the exit end of the impeller passage. This explains why impellers having considerable prerotation have a more equal distribution of velocity at the exit. However, once inequalities in the relative velocity have been produced, they are likely to persist, necessitating some cross-flow at the exit of the impeller. The viscosity of the fluid then resists the necessary

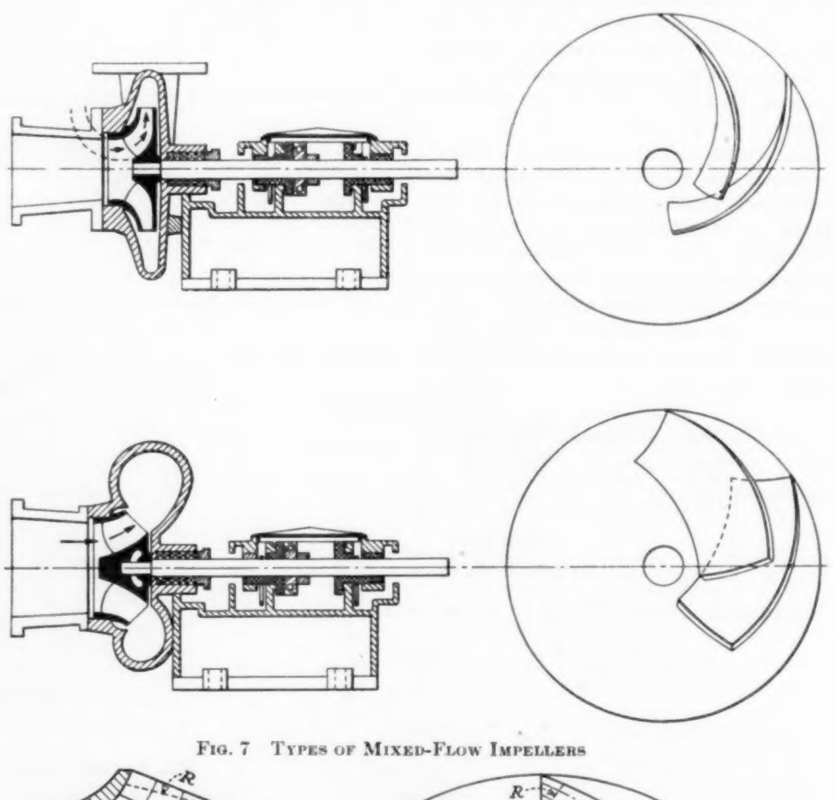


FIG. 7 TYPES OF MIXED-FLOW IMPELLERS

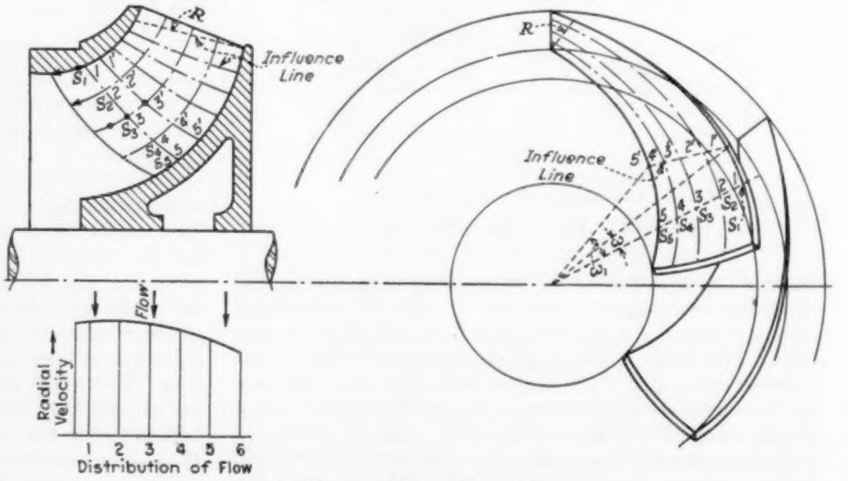


FIG. 8 TYPICAL MIXED-FLOW IMPELLER

interchange of particles at the exit of the impeller to some extent, and therefore the actual head generated is influenced slightly by the viscosity of the fluid pumped. It should be noted that variation in flow at the exit of the impeller due to the effect of composite acceleration is the direct result of initial variations due to free vortex effect between vanes, and therefore the free vortex effect due to curvature of channel is a measure of the exit variations. (See Fig. 4.)

Referring to Fig. 5, note that the calculated coefficient of distribution $(1 + \rho)$ is very closely in agreement with test results throughout the range of R_2/R_1 values for commercial impellers. The visualization of these effects is only of practical value as a guide in research work in obtaining the variations of vane angularity and channel proportions which are best for hydraulic efficiency and least conducive to fluid turbulence.

Fig. 6 shows a characteristic curve plotted from an official test of a modern centrifugal pump in operation in the water-supply system of a large city. The continuity of characteristics and the high overall efficiency are an example of the application of carefully studied design, embodying all recent improvements, in which the various considerations involving turbulence in vortex action have been reduced to a minimum.

II—WASTE ENERGY IN A CENTRIFUGAL-PUMP IMPELLER

A forced vortex is stable when all its particles revolve with the same angular velocity. It is also stable if the angular velocity of the inner particles is less than that of particles at a greater radius. However, there are faults in design which may result in higher angular velocity of the inner particles. Some of these are: too much prerotation, proportions involving poor continuity so that part of the impeller does not run full of fluid, local vapor pockets, either permanent or periodic in formation, and a profile outline of passage which, when combined with the space between the vanes, results in lack of continuity of area and direction. All these considerations may create an instability of vortex and result in local eddies and whirlpools, the inherent energy of which is supplied by the motive power driving the pump without creating useful head. These effects may be combined with flow of dissolved gases from the fluid and rapid deterioration of metal. An example of this is the case of a centrifugal pump accidentally run under reversed rotation when one month's operation with pure water caused deep holes to be cut in the casing, but no wear whatsoever in the impeller. The reason for this is that under these conditions the vortex in the impeller was stable but that in the case was unstable.

Other possibilities of waste energy arise in faulty impellers of the mixed-flow type where more head is generated in one part of the impeller than in an adjacent element, so that local back flow may occur at certain capacities. This effect is discussed in the following paragraphs.

III—SYNCHRONIZATION OF PARALLEL VORTICES

In the case of mixed-flow impellers such as are illustrated in pump types shown in Fig. 7, a typical impeller being shown in Fig. 8, conditions of flow become more complex. The drawing of this impeller shows that it is bounded by two walls which are of approximately parabolic cross-section. The impeller may be divided into any number of parts or units of equal volume as shown by lines 1, 2, 3, 4, 5. Each of these lines is the boundary of a unit of volume. A flow section represented by unit volume of the impeller is therefore an annulus bounded by two surfaces of revolution of approximately parabolic profile. The ring area of a unit volume may be constant from R_1 to R_2, or it may vary continuously in value. Each unit of volume is acted on by the impeller vane in such a way that it becomes a forced vortex with a steady gain in centrifugal force, the amount of which is controlled by the angularity of the vane. The vane may either be so proportioned that all units of volume are acted on in the same way or so that they are all acted on differently. That is, the pumping may be done uniformly or it may be done more on one side. The designer therefore has a wide choice in porportioning the degree to which power is applied to each unit of the total impeller volume. An impeller of this kind may function as a series of parallel vortices, the characteristics of which are determined by the action of the vane on each particle of the fluid, and each particle of the fluid is influenced in addition by the integral effect of all other particles. Obviously, an infinite number of designs are possible, each of which may generate the same capacity and head for a given speed, but each should have a different characteristic performance curve through a range of capacity, and a variation in efficiency would be expected for different designs.

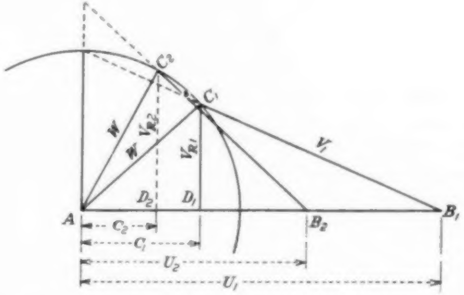


FIG. 9 VELOCITY DIAGRAM OF IMPELLER SHOWN IN FIG. 8

Two requirements must be met in this impeller for stability of characteristic and highest possible efficiency: (*a*) The head generated in each unit of volume should be the same for uniform capacity and the work done on each particle of fluid should be the same. (*b*) As flow occurs in three-dimensional space instead of in a single-plane, the forced vortex of each unit volume must have stability in a lateral as well as a radial direction, as mentioned in Section II. For simplicity, let us make certain assumptions of flow as a basis on which to determine the requirements necessary for condition (*a*). As a second step, these assumptions of flow may be so chosen as to either satisfy condition (*b*) also, or they may be transposed into dynamically equivalent conditions to meet the requirements of (*b*) in addition to those of (*a*).

Referring to the impeller shown in Fig. 8, let us assume that the fluid approaches the revolving vane with a uniform absolute velocity W_1 in all units of volume; also that it leaves the impeller with a uniform absolute velocity W_2 in all units of volume. These are justifiable assumptions because parallel units of volume in the discharge case should have the same velocity in order that the total energy there should be constant in successive layers measured axially parallel to the shaft. In other words, there is a reason both at the entrance and exit for the absolute velocity's being equal in all units of volume. A third assumption that should be made is that the same amount of centrifugal force should be imparted to each particle of the fluid after each unit of time throughout its course through the impeller, provided each particle takes the same total time to pass through the impeller.

Referring to the impeller shown in Fig. 8 and the velocity diagram in Fig. 9, let 1, 2, 3, 4, 5 be a series of points on the vane reached by fluid particles moving along the flow line after the same unit of time; these points are then all points of equal angular displacement from the edge of the vane, and the distances S_1, S_2, etc. represent the velocities in a radial direction to some particular scale. The centrifugal force imparted to a single particle independently at any radius R is equal to C^2/R for unit mass, where R is the radial distance of the particle considered as unit mass from the center of rotation. Hence, for constant centrifugal force applied to each particle along the line 1–5, $C_1^2/R_1 = C_2^2/R_2$, etc., and the radial velocities at each of these five points may be computed as follows:

$V_{r1}^2 = W^2 - C_1^2$; $V_{r2}^2 = W^2 - C_2^2$, etc. (W being constant) Hence

$$V_R = \sqrt{W^2 - KR} \qquad [5]$$

K and W^2 being constant.

The mean value of V_R is:

$$\bar{V}_R = \frac{\int_{R_5}^{R_1} V_R dR}{R_1 - R_5}$$

$$= \frac{\int_{R_5}^{R_1} (W^2 - KR)^{1/2} dR}{R_1 - R_5}$$

$$= \frac{\frac{2}{3k}\left[(W^2 - kR_1)^{3/2} - (W^2 - kR_5)^{3/2}\right]}{R_1 - R_5}$$

$$= \frac{\frac{2}{3k}\left[V_{R_1}^3 - V_{R_5}^3\right]}{R_1 - R_5} \qquad [6]$$

This equation for the mean value of V_R enables us to calculate also the mean radius for purposes of design.

Fig. 10 shows the relative variation of V_R plotted to a scale, and since the points 1–5 represent equal angular displacements, the spaces S_1, S_2, etc. are proportional to V_{R1}, V_{R2}, etc. and represent the relative pitch of the vane. In other words,

$S_1 = V_{R1} \times$ unit time (see Fig. 8)
$S_2 = V_{R2} \times$ unit time (to some scale)

and S_1, S_2, S_3, etc. are velocity-time products representing constantly applied centrifugal force; i.e., lines 1–5 and 1′–5′ are *lines of equal influence*. If the velocities thus obtained are not correct for the requirements of condition (*b*) we may reduce the velocity on any flow line, provided we increase the corresponding time element. This is the same thing as increasing the angular displacement of a point on any flow line, or again, as increasing the length of vane without changing the position of the influencing line. The sum of the spaces S_1–S_5 to S_1'–S_5', etc. measured along each flow line, determines the relative length of vane in radial profile. If the time taken for each particle to pass through the impeller is the same, each flow line will have the same constant angular displacement. However, since $S = V_R \times t = V_R \times \omega$, the radial velocity may be altered to suit condition (*b*), provided the angular displacement ω is altered in the inverse ratio.

Condition (*b*) requires that each point on a transverse section of the vane at the same angular displacement from the end of the vane, shall have a vane angle whose tangent is the ratio of the velocity at maximum capacity to the peripheral velocity of the point. The fact that the total-energy Equation [3] is a linear function at all capacities establishes the stability of the forced vortex at all capacities provided the variation of angle at all points of equal angular displacement is such that the tangent is proportional to $V_{\max}/R$, where $V_{\max}$ is constant and R variable. This is fully explained by Joseph Lichtenstein in Trans. A.S.M.E., paper HYD-50-3, 1928, pages 2 and 3.

The vane angles and the relative variation of V_R having been determined to meet condition (*b*), the relative angular displacement of points 1–5 on each influence line can be calculated, and the influence lines combined with the value of ω may be used as construction lines for the designing of the vane by a method of graphical integration. But now the influence lines are no longer meridional sections and the assumptions made have been modified to the dynamic equivalent by altering the values of time t or angular displacement ω inversely to the ratio: Value of V_R according to (*b*) to its value under the original assumptions.

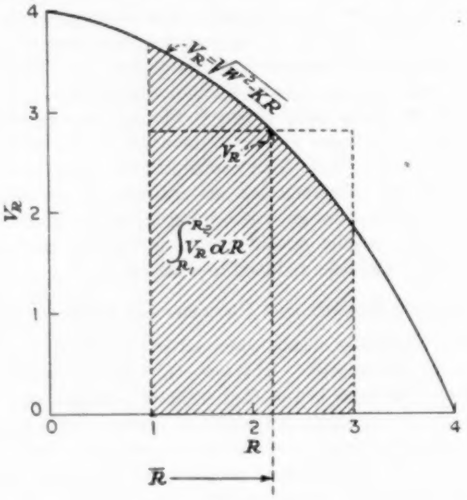


FIG. 10

The significance of Equation [5] can be realized best by manipulation of one or two particular designs. In plotting influence lines on the profile view of the impeller, the space between them will vary similarly to the ordinates in Fig. 10 between the range of values of R occurring on the influence line.

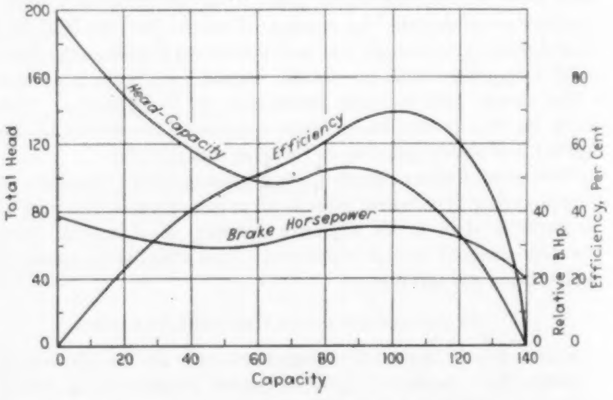


FIG. 11 CHARACTERISTIC SHOWING FAULTY DESIGN

Note that if for any point in an influence line the calculated value of KR becomes equal to W^2, V_R will become equal to 0, and there will be no flow. Also if KR is greater than W^2, V_R will be a function of $\sqrt{-1}$, which is an imaginary quantity. Manipulation of Equation [5] will show that imaginary values of V_R can be reached at one point in an influence line provided the average radial velocity is very low compared to the value of W and K, or provided the value of K is high for the ratio of values of R along the same influence line.

When zero or imaginary values of V_R are reached by calculation, the design should be changed to employ higher average values of V_R or lower values of C, at least in the early stages of the vane. Calculations with these new values will establish C_R as positive and real at some new average value of C_R and K. For this reason Equation [5] is valuable in the designing of new types because its manipulation provides a means of detecting such faulty design as too low radial velocity, too high acceleration, or too much variation in the range of R values along one influence line. The characteristic shown in Fig. 11 is an example of faulty design of this kind where zero and imaginary values of V_R follow from Equation [5]. Curves shown in Figs. 6 and 12 are examples of the absence of such faults.

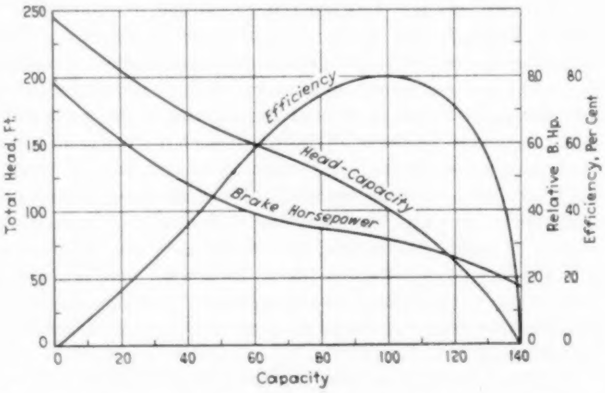


FIG. 12 CHARACTERISTIC SHOWING ABSENCE OF FAULTS FOUND IN FIG. 11

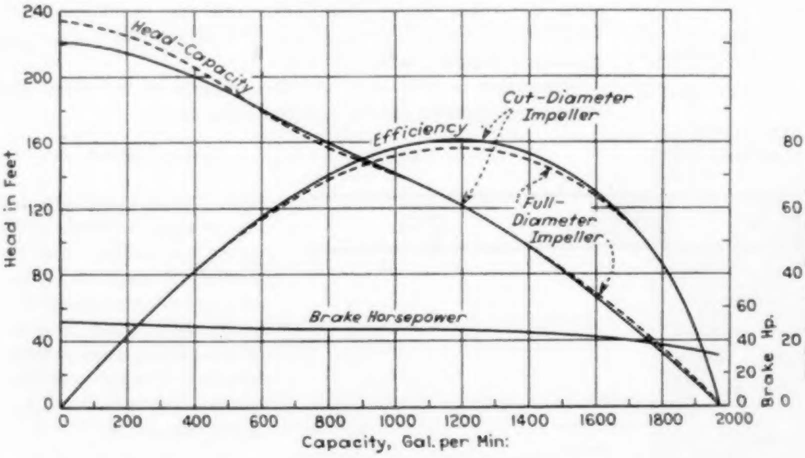


FIG. 13 SHOWING PRACTICAL USEFULNESS OF INFLUENCE LINES PLOTTED FROM EQUATION [5]

The curve shown in Fig. 13 provides an interesting example of the practical usefulness of influence lines plotted from Equation [5]. An impeller such as that as shown in Fig. 8 was tested with a certain discharge edge. Part of this discharge edge was then cut off, as shown at R, Fig. 8, so as to conform with the length of vane indicated by the influence lines. The new discharge edge, although resulting in a smaller average impeller diameter, gave exactly the same head capacity under test, but with a higher efficiency, thus pointing to the value of these influence lines in determining the relative lengths of vane for all flow lines so that all parallel units of impeller volume shall pump uniformly. For this purpose the correct values of ω for each point on an influence line must be determined in order to satisfy condition (*b*), as previously pointed out. The use of such a routine in pump design may appear rather laborious, but it is well worth while in the development of new types because the laws of dynamic similitude make each new type available for wide, general application. A designer can develop a fixed rule for tabulating values according to Equation [5] step by step with the graphical method required for constructing the vane layout. This involves little extra labor and tends toward more reliable design. It is worth recalling that impellers in which unit volumes are separated by internal metal partitions have been most inefficient. The proper balance of vane angularity and length of vane is the better method for efficiency and stability of characteristic. The influence of vane length is such that if ideal variation of angularity in the vane results in less head being developed by one unit of volume than in another, then increased length of vane will enable the fluid to be acted on for a greater length of time in accordance with the influence lines, calculated to fulfil condition (*a*) and values of ω calculated to fulfil condition (*b*).

CONCLUSION

The general discussion of stability and equalization of vortex in Section III includes no considerations which have not been applied repeatedly in actual practice. The careful application of these principles, when made part of the general routine of design according to an established system of procedure, has resulted in a certain reliability of attainment. The variations referred to in Section I are of course included also in the mixed-flow impellers discussed in Section III, where the object in view was regarded as the equalization of vortex effects in parallel units of volume for a uniform integral effect.

With reference to the distribution of flow in curved channels, the coefficient of distribution for mean energy value, viz.,

$$\frac{\dfrac{R_1^2 + R_2^2}{2}}{\left(\dfrac{R_1 + R_2}{2}\right)^2} = (1 + \rho)$$

is somewhat different from that which would result from Isaachen's formula for velocity distribution in elbows, but apparently it has the advantage of agreeing better with field observations of bends in large rivers. If, as Isaachen claims, $V \times R$ = a constant = C, then the mean value of V^2 is

$$\int_{R_1}^{R_2} \frac{V^2 dR}{R_2 - R_1} = \frac{\dfrac{C^2\,(R_2 - R_1)}{R_2 R_1}}{R_2 - R_1} = \frac{C^2}{R_2 R_1}$$

and the mean value of V is $= \int_{R_1}^{R_2} \frac{V dR}{R_2 - R_1}$

$$\frac{\int_{R_1}^{2} V dR}{R_2 - R_1} = \frac{\int_{R_1}^{R_2} C \times \frac{1}{R}\, dR}{R_2 - R_1} = \frac{C\,[\log R_2 - \log R_1]}{R_2 - R_1}$$

$$= \frac{C \log \dfrac{R_2}{R_1}}{R_2 - R_1}$$

Therefore the coefficient of distribution for mean energy value would be:

$$\frac{\dfrac{\left(C \log \dfrac{R_2}{R_1}\right)^2}{(R_2 - R_1)^2}}{\dfrac{C_2}{R_2 \times R_1}} = \frac{R_2R_1\left(C \log \dfrac{R_2}{R_1}\right)^2}{C^2\,(R_2 - R_1)^2}$$

This gives slightly lower computed values of the coefficient.

Discussion

B. M. Woods[2] and M. P. O'Brien.[3] The paper serves as a reminder of the fact that the elementary theory of centrifugal pumps and reaction turbines, developed by Euler for an infinite number of blades, leads to an absurdity when applied in the usual manner to a runner having a finite number of blades. The pressure is assumed to be uniform over areas concentric with the axis of rotation, and under this condition it is evidently impossible that a torque can be exerted on the fluid or that any energy can be transmitted to it. Even in the hypothetical case of frictionless flow, the elementary theory, as explained in most texts, is mechanically unsound and needs to be corrected in such a manner as to give a pressure difference between the two sides of the blades. The torque equation, $T = \left(\frac{Q\gamma}{g}\, r_2C_2 - r_1C_1\right)$ and the head equation, $H = \frac{1}{g}\,(U_2C_2 - U_1C_1)$, are applicable, provided that a means of predicting the actual mean value of C can be devised. In other words, the flow does not follow the blade angle at discharge, and the necessity for such a deviation can be shown from elementary considerations of torque and power, but this discrepancy in fundamental conceptions has not been given much consideration in the American literature on the subject. As the German hydraulic engineers have made some notable contributions to the solution of this problem, it may be of interest to present a brief outline of their method of attack.

The lift of an airfoil presents approximately the same difficulty as the torque of a centrifugal pump, in that the theoretical pressure distribution, obtained from Bernoulli's equation on the assumption of purely translatory motion, gives no resultant thrust in any direction and consequently no lift. As the lift was not to be denied, it was necessary that a theory be developed which would agree with observation, and Professor Prandtl, of Göttingen, brought forward the conception of a circulatory motion superimposed upon that of pure translation. This circulation is assumed to be in such a direction that the velocities due to the two component motions are additive above the airfoil and subtractive below it, thus giving a pressure difference sufficient to account for the lift. However, such a circulation could not arise in a frictionless liquid originally at rest, and the theory was expanded to include the effect of viscous forces, which are negligible everywhere in the fluid except in the immediate vicinity of solid boundaries. The elaboration and application of this theory by von Kármán, Betz, Joukowsky, and others has resulted in an agreement between theory and experiment in aerodynamics which is absent in most of the other branches of fluid mechanics.

In 1918 Kucharski[4] applied the circulation theory to the frictionless flow in a pump impeller having a finite number of radial blades extending to the axis of rotation, and his success in solving this idealized problem mathematically paved the way for a more elaborate treatment by Spannhake[5] and the development by Pfleiderer[6] of a simple method of predicting the effect of a finite number of runner blades.

Circulation is defined as the line integral of the tangential velocity component around a closed path, or

$$\Gamma = \int W \cos \alpha \, ds$$

where Γ is the circulation and α is the angle between the direction of motion and the element of path ds. Considering two-dimensional flow symmetrical about a point, and computing the circulation around a circular path concentric with this point of symmetry, we obtain

$$W \cos \alpha = C = \text{constant}$$

and

$$\int C ds = 2\pi r C = \Gamma$$

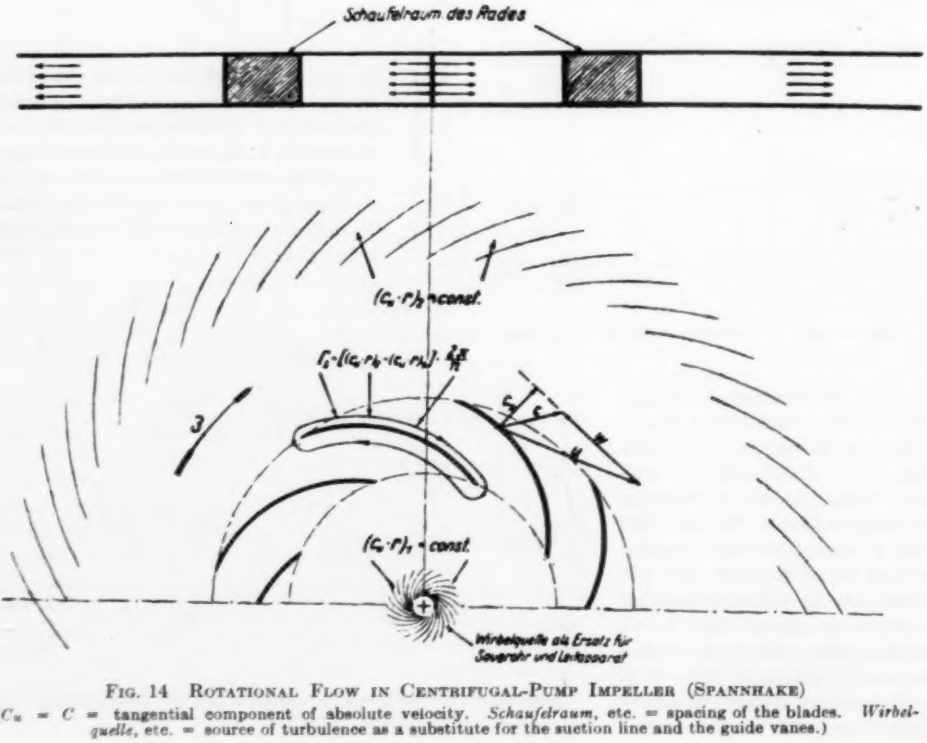

Schaufelraum des Rades
(c_u·r)_2 = const.
Γ_s = [(c_u·r)_2 − (c_u·r)_1]·2π/z
3
c_u
c
w
u
(c_u·r)_1 = const.
Wirbelquelle als Ersatz für Saugrohr und Leitapparat

Fig. 14 Rotational Flow in Centrifugal-Pump Impeller (Spannhake)

(C_u = C = tangential component of absolute velocity. *Schaufelraum*, etc. = spacing of the blades. *Wirbelquelle*, etc. = source of turbulence as a substitute for the suction line and the guide vanes.)

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[3] Associate Professor of Mechanical Engineering, University of California, Berkeley, Cal. Assoc-Mem. A.S.M.E.

[4] Kucharski, "Stromungen in rotierenden Kanälen," *Zeit. fur gesamte Turbinenwesen*, p. 201, 1917; "Stromungen einer reibungsfreien Flussigkeit bei Rotation fester Korper," Munich and Berlin, 1918.

[5] W. Spannhake, "Hydraulische Probleme," *Z.V.D.I.*, 1926.

[6] C. Pfleiderer, "Die Kreiselpumpen," Julius Springer, 1924.

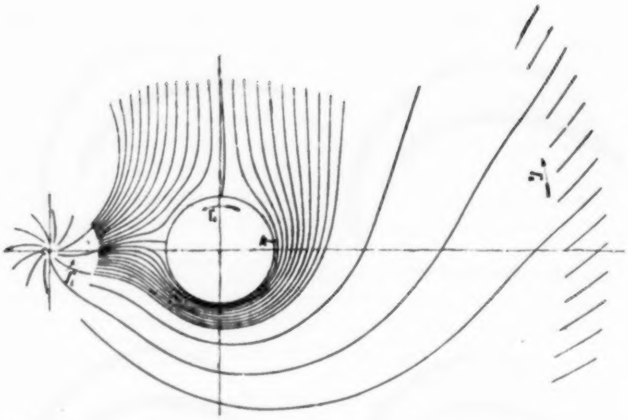

FIG. 15 RADIAL AND ROTATIONAL FLOW AROUND A CYLINDER (AFTER SPANNHAKE)

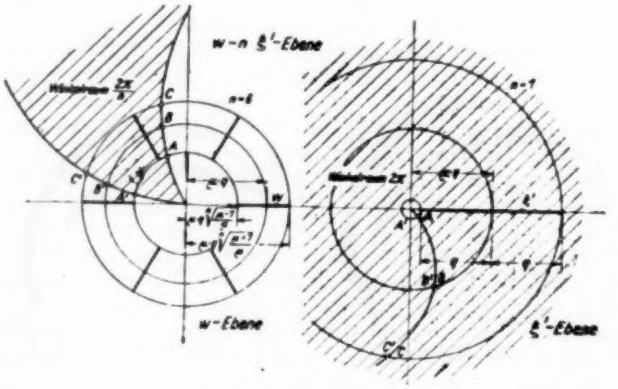

FIG. 16 TRANSFORMATION OF FLOW IN FIG. 15 INTO *n* SEGMENTS (SPANNHAKE)

(*Winkelraum* = angular area; *Ebene* = plane.)

Using this relation, the torque equation becomes

$$T = \frac{Q\gamma}{2\pi g}\,[\Gamma_i - \Gamma_a]$$

Referring to Fig. 14, these circulations are composed of a number of elements contributed by the circulation around the individual blades, and since the circulation is to be computed in the same sense for each blade, the components along the radial lines will cancel, giving

$$\eta\Gamma_s = \Gamma_i - \Gamma_a$$

where η is the number of blades and Γ_s is the circulation about each blade. The circulation around the blades is therefore related to the torque, head, and power. The theory really involves no new principles, since we have merely replaced $(r_2u_2 - r_1u_1)$ by the term $\frac{\eta\Gamma_s}{2\pi}$, but the changed conception is important, since it provides a connection with the aerodynamical theory. Fig. 15 shows a combination of radial and circulatory flow from a central source past a cylinder, about which the circulation is Γ_s. The circulation along any closed path surrounding the source alone is Γ_i, while that along a path including both source and cylinder is Γ_a, and the change Γ_s is a measure of the torque exerted by the cylinder on the fluid.

From the equations describing the flow in Fig. 15, Spannhake obtained the flow pattern around the radial blades in Fig. 16 by conformal transformation which made the sector $\therefore \frac{2\pi}{\eta}$ represent the entire plane of Fig. 15. By comparing the product (ηC) obtained by this method for η blades with the same quantity for an infinite number of blades, the correction for the elementary theory is obtained. In view of the high degree of turbulence in centrifugal pumps, this rather complicated treatment may appear unwarranted, but it should be remembered that a torque *is* developed and must be explained. The corrections obtained are in the right direction and of about the proper magnitude, so that the method appears to be worthy of consideration.

From a less rigorous analysis, but with practically the same assumptions, Pfleiderer has developed a method of correcting for the effect of a finite number of runner blades. His equation is

$$H_w = H\left(1 + \frac{\psi}{\eta}\,\frac{r_2^2}{S}\right)$$

where H_w is the head for the same quantity and speed computed by Equation [2] (Equation [3], page 3, original paper), H the actual head developed, r_2 the discharge radius, η the number of blades, S the statical moment of the center line of the runner passage about the axis of rotation, and ψ a coefficient depending on the blade angle at discharge. Pfleiderer gives the value of ψ as lying between 0.8 and 1.0 for pumps with diffuser vanes. Using this equation and correcting H for friction and impact loss, the head-capacity curve for a small modern centrifugal pump was constructed and the results were compared with data obtained in the laboratory at the University of California. The two curves intersected near the point of best efficiency, and the divergence was small through the entire operating range. Pfleiderer's method is an example of the practical results which often follow rigorous mathematical treatment.

A notable contribution to the experimental data on centrifugal-pump performance has been made recently by the Technische Hochschule at Munich, where the flow in a pump has been

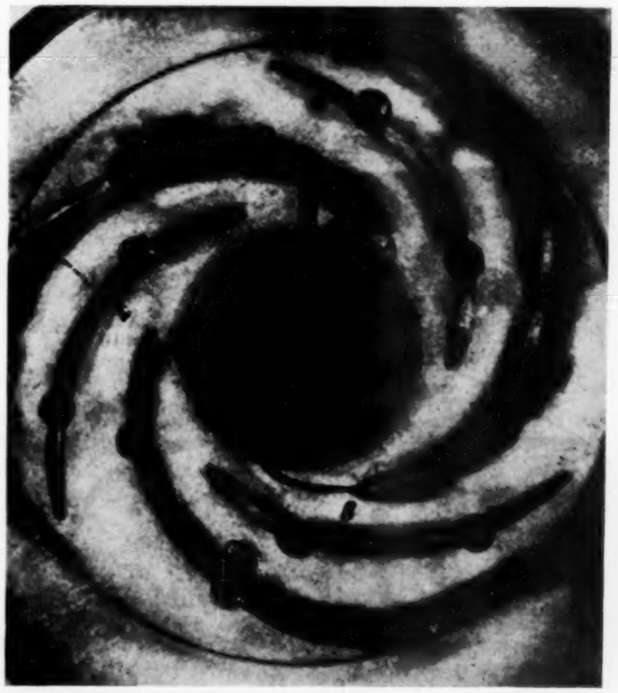

FIG. 17 FLOW IN CENTRIFUGAL-PUMP IMPELLER (AFTER K. FISCHER)

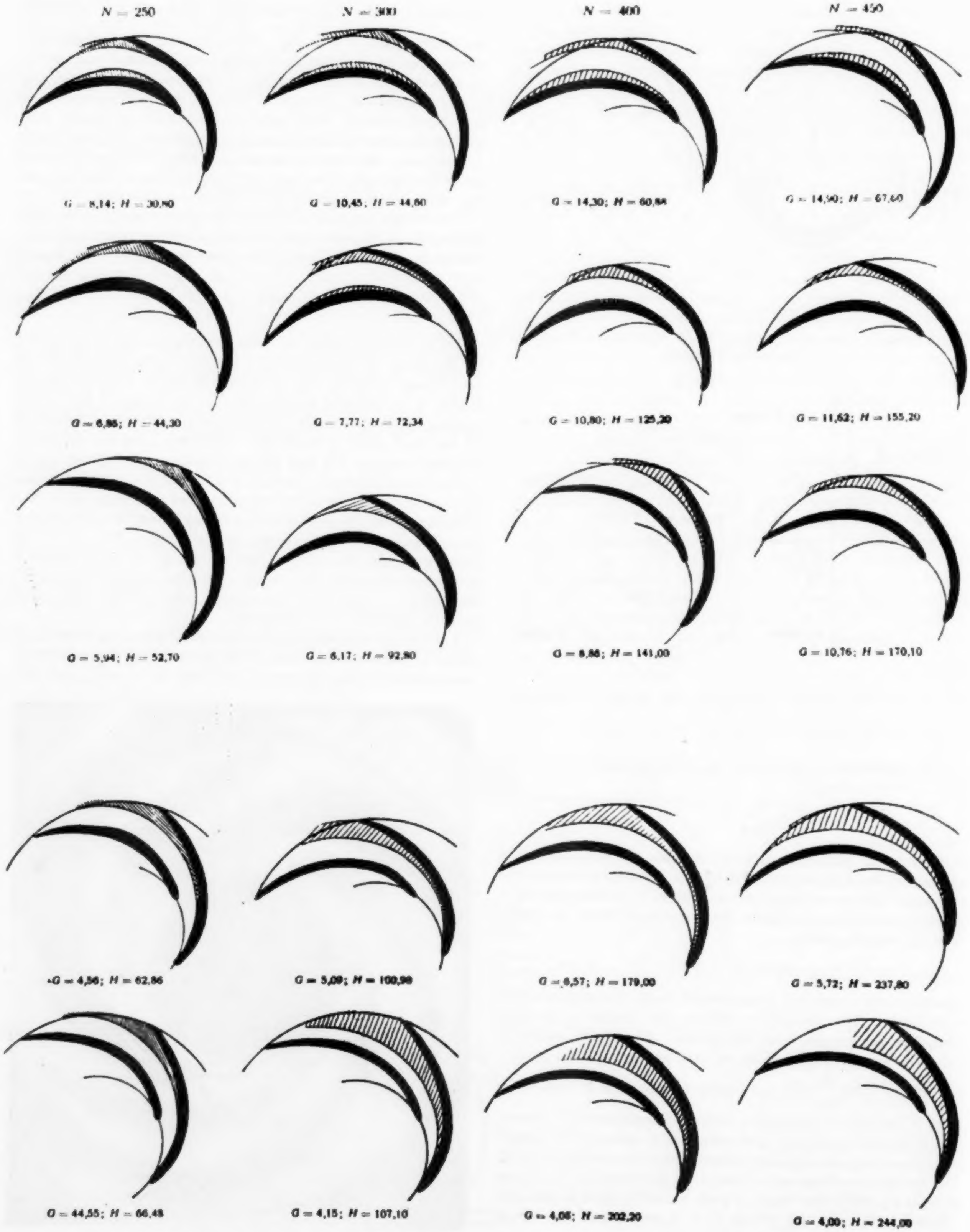
N = 250
N = 300
N = 400
N = 450
G = 8,14; H = 30,80
G = 10,45; H = 44,60
G = 14,30; H = 60,88
G = 14,90; H = 67,60
G = 6,88; H = 44,30
G = 7,77; H = 72,34
G = 10,80; H = 125,20
G = 11,62; H = 155,20
G = 5,94; H = 52,70
G = 6,17; H = 92,80
G = 8,86; H = 141,00
G = 10,76; H = 170,10
•G = 4,56; H = 62,86
G = 5,09; H = 100,98
G = 6,57; H = 179,00
G = 5,72; H = 237,80
G = 44,55; H = 66,48
G = 4,15; H = 107,10
G = 4,06; H = 202,20
G = 4,00; H = 244,00

Fig. 18 Dead-Water Spaces in Centrifugal-Pump Impellers (After K. Fischer)

observed under approximately operating conditions.[7] The impeller was fitted with a glass side and the seal was made at the outer periphery, so that the location of dead-water spaces could be discovered by injecting colored liquid. Direct observations were made with a rotoscope, while photographs were obtained with a camera revolving with the pump. Fig. 17 shows one of the photographs obtained, while Fig. 18 shows a series of diagrams of the dead-water spaces. What makes these experiments of considerable interest is that the impeller passages were never found to be completely filled with active fluid.

We are indebted to the author for the interesting questions raised concerning the circulation theory and for illuminating comments. It is somewhat to be regretted that his treatment is not more definite in certain particulars.

Boris A. Bakhmeteff.[8] The author touches on many subjects of great interest. This discussion deals with only one point, which appears to be the most important in the paper—namely, the taking into account of the unequal distribution of velocities when calculating the mean energy of flow, and of applying, "when converting mean velocity into its equivalent of mean energy," a special correcting coefficient which the author calls the "coefficient of local distribution."

The question is of basic character in hydraulics, and the profession should be thankful for bringing it forward, especially as it seems that considerable misunderstanding prevails regarding the very fundamentals of the case. It should be understood, therefore, that this discussion is offered solely with the purpose of elucidating the matter.

The case brought forward by the author, and which so often has been neglected, dates back to a celebrated "Memoire" by Coriolis of nearly a hundred years ago,[9] in which the great savant, of "acceleration" fame, first laid down the reasoning regarding energy in fluid motion, which since has been the cornerstone of hydraulics and has passed from textbook to textbook in establishing the so-called Bernoulli equation.

Among other features, Coriolis pointed out that the mean kinetic energy carried by a stream (Fig. 19), in which the individual velocities v differ, is greater than the kinetic energy corresponding to the mean velocity v_m obtained by dividing the discharge Q by the cross-sectional area a. Coriolis in this connection suggested introducing a certain correcting coefficient α_k into the relation:

$$\text{Mean kinetic energy head} = \alpha_k \frac{v_m^2}{2g} \dots\dots\dots [7]$$

which coefficient α_k fulfils a purpose analogous to that of the author's "coefficient of local distribution."

To determine the value of α_k, first assume that the whole liquid in Fig. 19 flows with the mean velocity v_m. In this case the total energy carried by the fluid passing through the cross-section a per unit of time, would be

$$\gamma Q \frac{v_m^2}{2g} = \frac{\gamma}{g} a \frac{v_m^3}{2} \dots\dots\dots\dots\dots\dots [8]$$

where γ is the weight of the liquid per unit volume, while g is the acceleration of gravity.

In actual flow one will have to calculate the kinetic energy carried by each separate filament, and then obtain the total energy carried by the stream, by summing up the energies of the individual streamlines. In figuring the kinetic energy carried by a filament passing through an element of the cross-section da (shaded area in Fig. 19), the element of flux across da will be $dq = da\, v$, where v is the particular velocity corresponding to the cross-section point, and the elementary kinetic energy is

$$\frac{\gamma}{g} dq \frac{v^2}{2} = \frac{\gamma}{g} da \frac{v^3}{2} \dots\dots\dots\dots\dots\dots [9]$$

The total kinetic energy carried by the stream per unit of time is obviously

$$\frac{\gamma}{g} \int_a \frac{v^3}{2} da \dots\dots\dots\dots\dots\dots\dots [10]$$

where the value of the integral is taken over the whole cross-sectional area.

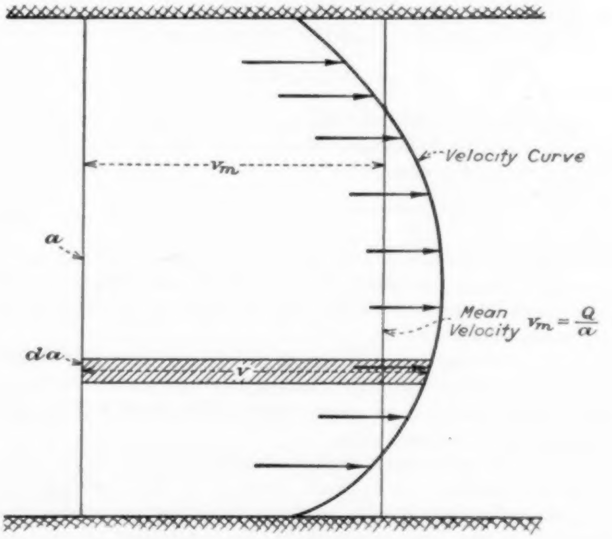

Fig. 19

To evaluate the correction coefficient, which is to express the total energy in terms of the energy head corresponding to the mean velocity v_m, we put

$$\alpha_k \frac{\gamma}{g} a \frac{v_m^3}{2} = \frac{\gamma}{g} \int_a \frac{v^3}{2} da \dots\dots\dots\dots\dots [11]$$

from which

$$\alpha_k = \frac{\int_a v^3 da}{v_m^3 a} \dots\dots\dots\dots\dots\dots\dots [12]$$

In other words, the correction coefficient α_k is the ratio of the mean value of the *cube* of the individual velocities to the *cube* of the average velocity, and in the author's notations should be expressed as

$$\frac{\Sigma(v^3/N)}{(\text{average } v)^3}$$

instead of being the ratio of the mean *square*, as is suggested in the paper. The coefficient α_k depends on the mean *cube* and not on the mean *square*, for the reason that kinetic energy is the product of one-half of the velocity square by the mass flux, and mass flux in its turn is proportional to the velocity. It should, however, be mentioned in this connection that the misapprehension indicated above and which figures in the paper, is quite widespread and is to be found at times in textbooks of recognized standing.

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[7] K. Fischer, "Untersuchung der Strömung in einer Zentrifugal-Pumpe," Mitteilungen des Hyd. Inst. der Technische Hochschule, Munich, 1931.

[8] Professor, Columbia University, New York, N. Y. Mem. A.S.M.E.

[9] *Annales des Ponts et Chaussées*, 1836.

As shown by Coriolis, the coefficient

$$\alpha_m = \frac{\int_a v^2 da}{v_m{}^2 a}$$

obtained from calculating the mean square of the velocities, and which is the coefficient used by the author,

$$(1 + \rho) = \frac{\Sigma(v^2/N)}{(\text{average } v)^2} \dots\dots\dots\dots [13]$$

is a correction coefficient to be used when evaluating the actual momentum of flow

$$\alpha_m \frac{\gamma}{g} Qv_m = \alpha_m \frac{\gamma}{g} av_m{}^2 \dots\dots\dots\dots [14]$$

in terms of the momentum corresponding to the mean velocity.

Both α_m and α_k are > 1, so that one may put

$$\alpha_k = 1 + \delta, \text{ and } \alpha_m = 1 + \rho \dots\dots\dots [15]$$

Coriolis further showed that, in many practical cases, and in general when the individual velocities v do not deviate substantially from v_m, the relation between δ and ρ in [15] is close to

$$\delta = 3\rho \dots\dots\dots\dots\dots\dots [16]$$

meaning that the size of δ in the real correction coefficient $\alpha_k = 1 + \delta$, which is to be used in figuring out the mean kinetic energy head, is about three times the value of ρ in $(1 + \rho)$, obtained by the mean-square formula.

The author on many occasions points out that the results obtained from the use of the mean-square formula are far below the values derived from observations. The above will explain the source of the discrepancy.

As a matter of record it might be interesting to recall that the Coriolis treatment, as outlined, is more or less customary in French hydraulic manuals, particularly of the more advanced class.[10] In fact, one often finds the line expressing the average energy head in the so-called Bernoulli equation presented in the form

$$z + \frac{p}{\gamma} + \alpha_k \frac{v_m{}^2}{2g}$$

The value of α_k naturally depends on the circumstances of the case. As an average characteristic for uniform flow in open channels one often encounters the figure $\alpha_k = 1.1$, which was obtained on the basis of Bazin's experiments on the flow of water in canals.

Another point, to which the writer wishes to draw attention, is a matter of terminology. The term "coefficient of local distribution," suggested by the author, is most appropriate. On the other hand, equally unfortunate is the term "coefficient of turbulence." In fact, the word "turbulence," since the days of Osborne Reynolds, has been used to differentiate the so-called *laminar* or steady streamline motion, from the sinuous and eddying molecular movement which characterizes the unsteady flow of viscous fluid above the so-called critical velocity. Within the last decade or so the phenomenon of turbulence has been the object of repeated and preferred studies, and the meaning of the word in the Reynolds sense has been universally and internationally adopted. It would seem, therefore, that to apply the term in any other sense would be misleading.

The problem of expressing mean kinetic energy in terms of mean velocity by proper correction coefficients is of substantial importance. So the writer would like to further clarify the matter by some concrete examples.

1 We shall consider first the hypothetical case of linear distribution of velocities (Fig. 20), the velocity curve in a cross-section of the width d being a straight line ac. For simplicity, we shall further assume the movement to be two-dimensional, with a depth of flow in the direction perpendicular to the plane of the drawing equal to unity. We shall designate the relation of the minimum velocity v_c to the maximum v_a by

$$v_c/v_a = \beta \dots\dots\dots\dots\dots\dots [17]$$

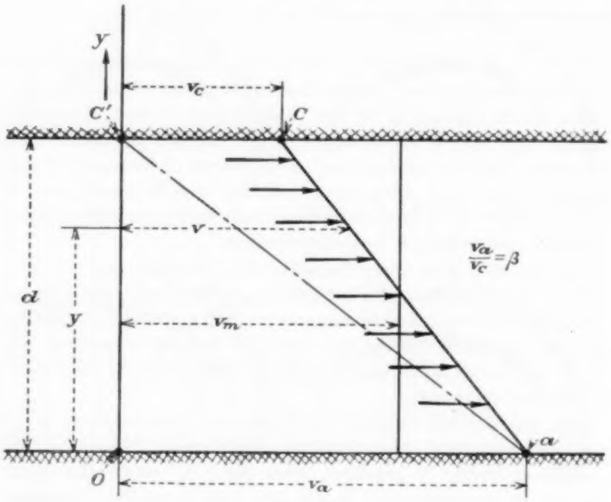

FIG. 20

With this the average velocity is

$$v_m = \frac{v_a + v_c}{2} = \frac{v_a}{2}(1 + \beta)$$

The velocity v at the distance y from o

$$v = v_a\left(1 - \frac{y}{d}(1 - \beta)\right)$$

and the correction coefficient

$$\alpha_k = \frac{\int_0^d v_a{}^3\left[1 - \frac{y}{d}(1 - \beta)\right]^3 dy}{v_m{}^3 d}$$

Carrying out the elementary integration, we obtain

$$\alpha_k = 2\frac{(1 + \beta + \beta^2 + \beta^3)}{(1 + \beta)^3} \dots\dots\dots\dots [18]$$

and in a similar manner

$$\alpha_m = \frac{\int_0^d v^2 dy}{v_m{}^2 d} = \frac{4}{3} \cdot \frac{1 + \beta + \beta^2}{(1 + \beta)^2} \dots\dots\dots [19]$$

The following table shows the values of α_k and α_m depending on β:

β	$\alpha_k = 1 + \delta$	$\alpha_m = 1 + \rho$
0	2.00	1.333
0.5	1.10	1.035
1.0	1.00	1.000

---

[10] Bresse, Boussinesq, Flamant, etc. See also Forchheimer's "Hydraulik."

The practical value of this rather hypothetical case is in the following. Consider flow with a velocity curve, as given in Fig. 21, with v_a and v_c the maximum and the minimum velocities in the center and at the borders. Obviously, no matter what is the form of curve, the triangle *cac′* will give a higher value of the correction coefficients α_k and α_m than the actual curve passing through the same points. Therefore, for any velocity curve lying outside the triangle, the values of α_k and α_m derived from [18] and [19] will be in excess of the actual value. Under such circumstances relations [18] and [19] can be used advantageously for control.

This is particularly important when one notices from the foregoing table that, even for rather extreme conditions, characterized by $\beta = 0.5$, the correction coefficient is relatively small. In other words, the difference between the mean cube and the cube of the mean velocity is oftentimes not very substantial, and the mistake incurred in ignoring the correction to be made may not be very great. The greatest value of α_k corresponding to $\beta = 0$, and featured in Fig. 20 by the broken line *ac′* is $\alpha_k = 2$.

2 The other practical case will be that of flow in concentric circular paths (Fig. 22), under the assumption that the distribution of velocities follows the law

$$vR = \text{const.} \qquad [20]$$

The author refers to it as the Isaachen formula for velocity distribution in elbows. As a matter of fact, relation [20] is one of the most general laws of fluid mechanics, featuring the so-called potential or irrotational movement of ideal fluid in concentric paths.

One often meets with the tendency on the part of the engineer to discard any notions suggested by hydrodynamics of ideal fluids. And it is very true that in so many cases the movement of frictionless liquids has little in common with the flow of actual viscous fluids. On the other hand, as amply demonstrated by the recent developments of aerodynamics and those of the more advanced analysis of hydraulic machinery, there are many cases where the teachings of so-called classical hydrodynamics may be of considerable assistance; particularly when, as in the present case, one bears in mind the concrete physical premises which underlie the term of irrotational or potential flow. Most important for the engineer is the fact that the fundamental physical premise of potential or irrotational flow is movement with an equal distribution of energy throughout the moving fluid as a whole.

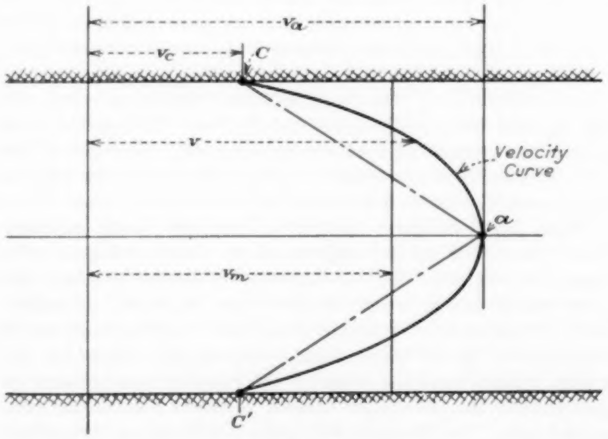

FIG. 21

To illustrate, assume (Fig. 22) that fluid is flowing out of a large container *A*, where it was previously at rest. Now each particle in the container obviously is possessed by the same quantity of energy. Since $v = 0$, this energy, measured per unit weight, is the potential energy head $z + \frac{P}{\gamma} = H$. By disregarding the insignificant losses which occur over the short stretch between the container and the elbow, it is correct to assume that in the movement through the elbow the individual particles will continue to carry approximately the same amount of energy.

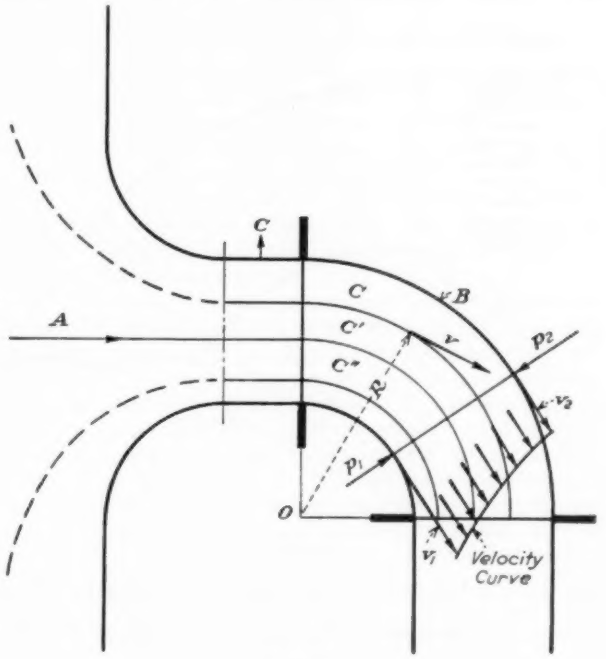

FIG. 22

In other words, the line of the Bernoulli equation

$$z + \frac{P}{\gamma} + \frac{v^2}{2g} = \text{const.}$$

will apply to each and every particle, so that for any particle *c*, *c′*, or *c″*, the relation between velocity and pressure will be

$$z + \frac{P}{\gamma} + \frac{v^2}{2g} = H \qquad [21]$$

As shown in hydrodynamics, equal distribution of energy (Equation [21]) in concentric flow translates itself into the kinematic condition $vR = \text{const.}$

Obviously [20] may be used in conjunction with [21] to establish a complete picture of the distribution of velocities and pressures over a cross-section. To make things still more concrete, may it also be recalled that the reduction of velocity toward the outer points in accordance with Equation [20] and the corresponding increase of pressure from P_1 to P_2 counterbalance the effect of the centrifugal forces acting normally to the concentric paths.

To find now the values of α_k and α_m, assume once more, as in the preceding, the flow to be two-dimensional. A discharge q flows through the rectilinear part *C* (Fig. 23) with a uniform velocity $v_m = \frac{Q}{d}$.

The curvilinear part has the outer and inner radii R_2 and R_1 and the width $d = R_2 - R_1$.

The geometric configuration is fully characterized by the mean radius $R_0 = \dfrac{R_1 + R_2}{2}$ and the relative width of the channel

$$\frac{d}{R_0} = 2\,\frac{R_2 - R_1}{R_2 + R_1} = \beta \ldots\ldots\ldots\ldots\ldots [22]$$

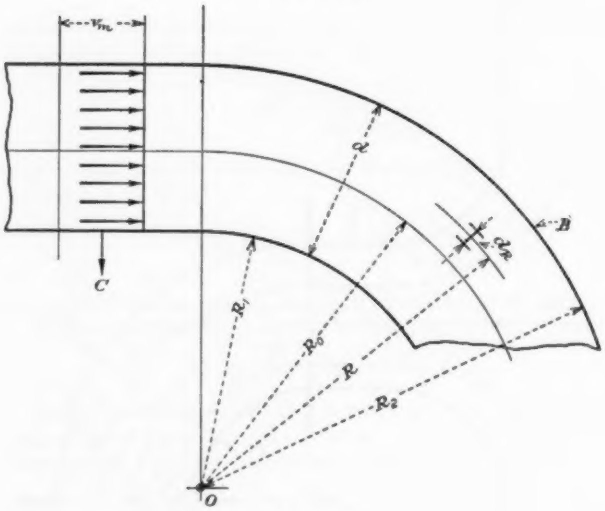

FIG. 23

Designating in Equation [20] the value of the constant by C, so that $v = \dfrac{C}{R}$, the elementary discharge across an element dR, with a radius R, is

$$dq = v\,dR = C\,\frac{dR}{R}$$

Hence the total discharge

$$Q = v_m(R_2 - R_1) = \int_{R_1}^{R_2} v dR = C\int_{R_1}^{R_2}\frac{dR}{R} = C\log_e\left(\frac{R_2}{R_1}\right)$$

and the value of the constant C in terms of the other elements of flow

$$C = \frac{v_m(R_2 - R_1)}{\log_e(R_2/R_1)} = \frac{Q}{\log_e(R_2/R_1)} \ldots\ldots\ldots\ldots [23]$$

To find α_k we have

$$\alpha_k = \frac{\int_{R_1}^{R_2} v^3 dR}{v_m{}^3(R_2 - R_1)} = \frac{C^3\int_{R_1}^{R_2}\frac{dR}{R^3}}{\left[\frac{C^3(\log_e(R_2/R_1))^3}{(R_2 - R_1)^3}\right]}$$
$$= \frac{(R_2 - R_1)^3(R_2 + R_1)}{2(R_1\,R_2)^2(\log_e(R_2/R_1))^3}$$

And by similar procedure

$$\alpha_m = \frac{\int_{R_1}^{R_2} v^3 dR}{v_m{}^2(R_2 - R_1)} = \frac{(R_2 - R_1)^2}{R_1R_2\left(\log_e\left(\frac{R_2}{R_1}\right)\right)^2}$$

Introducing

$$\beta = d/R_0 \text{ and } R_0 = \frac{R_1 + R_2}{2}$$

we obtain finally

$$\left.\begin{aligned} \alpha_k &= \frac{\beta^3}{\left(1 - \frac{\beta^2}{4}\right)^2\left(\log_e\left(\frac{2+\beta}{2-\beta}\right)\right)^3} \\ \alpha_m &= \frac{\beta^2}{\left(1 - \frac{\beta^2}{4}\right)\left(\log_e\left(\frac{2+\beta}{2-\beta}\right)\right)^2} \end{aligned}\right\} \ldots\ldots\ldots [24]$$

As is seen, the correction coefficients depend solely on the value of the relative width $\beta = d/R$.

The following table gives some figures permitting a concrete appreciation of the effect of curvature on the values of the correction coefficients:

β		α_k	α_m	$\left(1 + \frac{\beta^2}{4}\right)$
1.00	....	1.34	1.105	1.250
0.50	....	1.06	1.020	1.060
0.10	....	1.005	.....	1.002

Again it is clear that the correction coefficient assumes substantial values only when the curvature is extreme. The figures in the last column correspond to the author's coefficient

$$(1 + \rho) = \frac{\left(\frac{R_1{}^2 + R_2{}^2}{2}\right)}{\left(\frac{R_1 + R_2}{2}\right)^2} \ldots\ldots\ldots\ldots\ldots [25]$$

which, as he suggests, is derived from empirical observations. It is easily seen that the value of his coefficient is

$$\frac{1}{2}\,\frac{R_1{}^2 + R_2{}^2}{\left(\frac{R_1 + R_2}{2}\right)^2} = \frac{1}{2}\,\frac{R_0{}^2(1 + \beta/2)^2 + R_0{}^2(1 - \beta/2)^2}{R_0{}^2} = \left(1 + \frac{\beta^2}{4}\right)$$

It is seen further that the actual correction coefficients α_k resulting from figuring the mean cube are somewhat in excess of [25], although for medium values of β they practically coincide.

F. G. SWITZER.[11] The paper is full of interesting points. It has already been pointed out by Professor Bakhmeteff that kinetic energy must be obtained by integrating the cubes of the velocity instead of the squares. This point should be emphasized again, because it is a mistake quite frequently made.

While pump designers have for a long time made empirical corrections to the velocity diagram for exit from the impeller of a pump, we find here a method suggested for determining how this correction should be made. At best, this can be only an indication. It assumes, as we always have, that all of the water leaves the impeller at the same angle. Experiments made by the writer indicate that the angle of the relative velocity may be greater than the vane angle, instead of less as has usually been agreed upon. Furthermore, this angle is different at the shrouds and at the center of the impeller.

In Fig. 4 of this paper are shown radii of the vane and radii of the impeller using the same symbols. Does this mean that the vane radii and runner radii are the same? This hardly agrees with usual practice.

Non-uniform flow between runner blades has been noted in pumps of much lower specific speed than the mixed-flow runner of Fig. 8. In fact, there may be such differences as to cause water to flow out of the eye of the impeller, even in a pump of better than average efficiency. In such a case, changes in the suction passages may prove beneficial. It is not always the impeller that is to blame for this. In some pumps it has been found that the flow through any one passage between vanes may vary as the impeller rotates, being greater than average during part of one revolution and less during another part of the revolution. At first glance this may appear to be due to unequal pressures in the volute. However, tests show that radical changes in the volute which cause radical changes in pressure distribution do not show corresponding changes in this pulsation of flow. More can be done to improve this difficulty by changes in the suction passages. While measurements have not been made, it seems probable that variations in pressure around the eye of the impeller are of more significance than variations of pressure around the volute.

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[11] Professor of Hydraulic Engineering at Cornell University, Ithaca, N. Y., and Consulting Engineer for Goulds Pumps, Inc.

O. G. Tietjens.[12] Generally speaking, one may say that the theoretical treatment of centrifugal pumps and blowers is based on the assumption that the medium (it may be water or air) is sufficiently guided by the impeller blades, since these blades usually are closely spaced. The well-known formulas [2] and [3] of the paper are based on this assumption also. The general idea is that the direction of the flowing water between the single blades is entirely determined by the shape of the blades, and it is assumed that the flow lines within the blades of a certain impeller would be the same if the number of blades were indefinitely increased with decreasing thickness of the blades. The pump with an infinitely large number of blades of zero thickness is the conception on which the simple flow-line theory of centrifugal pumps is based. The viscosity of the water, of course, is neglected in this theory, as it is also done in Equations [2] and [3] of the paper.

Fortunately, this comparatively simple method can be used as a good approximation even for pumps of a rather small number of blades, say eight blades. Although in this case the flow lines within the impeller are by no means similar to each other, the performance of the pump in general is fairly well described by the flow-line theory.

However, if we want detail information—if we want to know, for instance, in what manner the performance of a pump depends on the number of the blades or on the ratio of the inlet and outlet radii of the impeller, or if we want to know the actual shape of the flow lines within the impeller, etc.—then the simple flow-line theory proves to be insufficient. In this case another theory can be used which, however, is considerably more complicated. This theory was developed first in connection with the design of airplane propellers and later applied to Kaplan pumps, and recently[13] it has been applied also to centrifugal pumps.

The author does not follow the simple flow-line method in those parts of his paper where he leaves the ground of well-known data, and he also does not follow the more modern hydrodynamic theory mentioned. This is by no means a fault. However, the author introduces such a large number of new conceptions without defining them that it is very often impossible to follow him. For example, these ambiguities raised the following questions in the writer's mind: What is the difference between a forced vortex and a free vortex? What is meant by a stable vortex, and what is meant by an unstable vortex? What are the laws of a free vortex mentioned at the top of the third page? What is meant on the same page by saying that the influence of the vanes for creating a perfect vortex is only local? What is the significance of the following sentence at the top of the fifth page: "An impeller of this kind may function as a series of parallel vortices, the characteristics of which are determined by the action of the vane on each particle of the fluid, and each particle of the fluid is influenced in addition by the integral effect of all other particles." What, specifically, are the characteristics of parallel vortices? The well-known treatises on "Vortex Motion" by W. Thomson or by Helmholtz, for instance, do not help one to understand the behavior and the laws of the vortices dealt with in the paper. In developing a new theory, it is absolutely necessary to have a clear picture of the physical meaning of newly introduced conceptions, and it is also necessary to define these conceptions and so make it possible for the reader to comprehend their significance.

The writer is sorry that this lack of clearness makes it impossible for him to discuss the paper in detail. The pumps may be well built, though the theory given in the paper does not lead to increased confidence in their design.

It may be of interest to draw attention to one point mentioned on the fifth page, under II. The author deals here with the waste energy in a centrifugal-pump impeller, and he points out that this may be caused by the flow of dissolved gases from the water. His statement, however, that deep holes were cut in the casing of a pump which accidentally ran under reversed rotation makes it evident that this erosion was caused by cavitation and not merely by dissolved gases. It is well known that under certain conditions in centrifugal pumps and also in propeller-type pumps the pressure may decrease at certain points below the water tension due to local extremely high velocities, and that the water then vaporizes. If these vapor bubbles move on their way into parts of higher pressure, they collapse, causing high local forces by which the erosion effect is produced. During recent years several investigations have been made, mostly by using high-speed moving pictures, in order to explain the mechanism of this mysterious cavitation. This work has been quite successful, and now we know under what conditions and in what manner cavitation occurs. However, considerable work must still be done to find out what the shape of the runner and of the housing must be in order to avoid local high velocities which may give rise to cavitation.

A. J. German.[14] Messrs. Stepanoff and Macmeeken are certainly to be congratulated in having gone into such extensive details on the subject of centrifugal pumps, for they have brought out in their papers the vast amount of research work carried on, resulting in a great improvement in higher efficiency in operation and economy of construction.

The discussion, so far, has been in the nature of tests, designs, and formulas. The writer is not in a position to elaborate on these questions, but would like to comment instead on personal observations made of a great number of centrifugal pumps where used to supply the circulating water for surface condensers, both in the marine and stationary field, having particular reference to where the cooling agent is sea water, brackish or badly contaminated, causing the condenser tubes to deteriorate rapidly.

Investigations reveal that the circulating pump is one of the contributory causes of short tube life, and in some cases the principal one.

Centrifugal pumps, some years ago, were of larger dimensions than they are now, and were engine driven; hence of slow speed, unquestionably causing very much less turbulence than the modern high-speed centrifugal pump. It is now generally agreed that the greater the turbulence, the more rapid the tube deterioration. Undoubtedly, the centrifugal-pump manufacturers were forced into this much higher pump speed so that the same work could be performed with a lower-priced pump; hence the statement by one of the papers, "greater economy of construction."

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[12] Research Laboratories, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

[13] See, for instance: A. Busemann, "Das Foerderhoehenverhaeltnis radialer Kreiselpumpen mit logarithmisch-spiraligen Schaufeln," *Zs. f. angew. Math. u. Mech.*, no. 8 (1928), p. 372.

[14] Chief Engineer, Scovill Mfg. Co., Waterbury, Conn. Mem. A.S.M.E.

While this greater economy of construction is unquestionably a fact when limited to the pump, it seems with the overall efficiency, including tube life, this economy of first cost of pump disappears.

Nearly duplicate installations in some plants have come to attention where the quantity of steam to be condensed was identical, turbine and condenser the same size, but one of the condensers received its cooling water from a large, slow-speed centrifugal pump, whereas the other condenser, of a later installation, was supplied with cooling water from a smaller but higher-speed pump. It was found that the tubes of the second installation began to deteriorate before any tube replacements were necessary on the first installation, undoubtedly caused by the fact that the water of the second condenser entered the tubes in a considerably more turbulent condition than that of the other. It seems that a better overall efficiency would be had by sacrificing, if found necessary, somewhat on first cost of pump and pump efficiency if this would bring about considerably less turbulent circulating water.

Occasionally we are told that some years ago condenser tubes were of superior quality than they are today, for they used to last for years, and that in some modern installations they give shorter life. One of the causes is that circulating water is now passing through the tubes in increased quantity and turbulence. While condenser-tube material has not reached a point of perfection, there is no doubt that, like other advancements, the art of making condenser tubes has greatly improved.

Author's Closure

The discussion of Professors B. M. Woods and M. P. O'Brien is of value to engineers in industry, because we are indebted to the universities for bringing the latest hydrodynamic theories closer to applicability to commercial work. We have found that certain velocity distributions give excellent results in practical application.

After a general picture with the circulation theory is made possible with simplified assumptions, modifications should be introduced leading to the same velocity distribution that has been proved by actual results, and it is to be hoped that this evolution of the theory will take place at an early date.

The equations involving the modern concept of Prandtl, called "circulation," were realized as important by pump designers several years ago, because the form of the function suggests that some components of velocity should be kept constant as far as possible in the course through the impeller. The use of coefficient of distribution is also of assistance in attaining practical corrections to this end. Previously published articles in the German press showing the calculated effect of the number of vanes on a basis of calculation resulting from the "circulation" theory, are of little value in practical pump design, because the margin of error is substantial and because the best shape of vane is different for each total number of vanes. However, the conception of "circulation" is important for realizing the minimum conversion of energy for head generation.

The method of Pfleiderer, in the experience of the author, may be used reliably for calculating the performance of the impeller, provided the value of the coefficients is assumed, but the assumption of change of direction in the flow of the fluid at the impeller exit by his theory does not agree with the conditions required for minimum shock in the discharge guide vanes. This is the reason for the author's stating his method of calculation by means of two coefficients of distribution, $(1 + \rho)$ for curvature and $(1 + \alpha)$ for change of cross-section. This method explains the condition of best efficiency in the guide vanes as well as the performance of the impeller. This is more apparent still when the same impeller is used in guide-vane casings of different angularity and characteristics. The significance of this is more clearly shown in the mathematical analysis of these effects given by Professor Bakhmeteff in his discussion when he points out that these coefficients of distribution should have different values for kinetic-energy effects compared with their values for momentum effects.

The author notes the reference by Professors Woods and O'Brien to the photographic study of eddies, disturbances, and distribution of flow, and undoubtedly much could be gained by these methods. At the present time the introduction of a new feature into a design must be made so as not to interfere with established features, and the effect noted in test results. Certain new features based on theory have in this way shown marked advantage under test. Photographic study of these would show their effect on flow-line distribution and on eddy formation. The efficiency of a pump is largely a matter of energy distribution in various components, for the best results, and this is best studied by mathematical analysis and may or may not be visible by photographic means. A thorough study of pump design by photography would require the development of a new technique, but may have great possibilities for the future. However, the hydraulic efficiencies already obtained in a wide range of the most prevalent types are so incredibly high that any further improvement possible may be too small to be really appreciable. The use of the camera might make the causes of the superiority of one design more intelligible to the student, particularly in the development of types free from cavitation. Photographic study might extend the range of types for which a similar statement would be true. Photographic study has great possibilities for improving our actual knowledge of conditions for partial and over capacities.

The discussion by Professor Bakhmeteff is of very great interest and importance as a further development of the use of coefficients of distribution in the application of elementary principles of dynamics to pump design. The author notes the important step taken by Professor Bakhmeteff in his Equations [8] and [10], in which he uses the total energy per unit of time as a function of v^3, whereas the energy per unit of mass is a function of v^2. This is an important step usually lost sight of in textbooks and is of importance also in calculating the axial thrust of a propeller.

Commenting on the difference in value between Professor Bakhmeteff's coefficient compared with the corresponding value derived from the author's Equation [4], the author found it necessary to use a coefficient $(1 + \alpha)$ additional to $(1 + \rho)$ due to curvature.

It is necessary to use this additional coefficient of distribution $(1 + \alpha)$ when the area of cross-section of the channel is changing. Professor Bakhmeteff's coefficient would require a smaller value of $(1 + \alpha)$, but it is noted that for the usual range of $R_2:R_1$ the author's coefficient [4] gives very nearly the same calculated value.

The author expects to find Professor Bakhmeteff's discussion, in all its fullness, of the greatest value in the future for pump design. It furnishes an exact means of calculating what would otherwise be empirical constants. Its use by the designer will result in a more accurate evaluation of primary quantities such as coefficients of distribution both for kinetic energy and for changes of momentum, and a resulting accuracy in secondary effects, the values of which are derived from these primary considerations. His clear distinction between the equations for use with kinetic energy and those for change in momentum effects should give a better understanding of the design of volute cases and guide vanes.

Referring to Professor Bakhmeteff's criticism of the use of the work "turbulence," and the accepted application of this term since the time of Osborne Reynolds, the author has used the word (perhaps wrongly) in the sense of its derivative meaning, a "spinning top" or "whirlpool" condition, which is the essential condition of vortex flow for head generation in a pump, there being variations and inequalities in that vortex; that is, all particles do not follow coincident paths. The subject is not a common one, and the author may be excused for using his own terminology in some aspects of the discussion.

Professor Switzer's interesting comments *re* the angle at which the water leaves the impeller, are worth noting because different shapes of impellers and casings show striking dissimilarity in effects as measured by the pitot tube, in both direction and magnitude. Velocities may be made much more uniform by profile and vane shapes of a nature to tend toward uniformity of flow resulting from uniformity of dynamic effect and absence of interfering surfaces. The direction of flow of the water varies, of course, depending on whether the capacity is the peak-efficiency condition of the pump or otherwise. The effect of shrouds is in some designs very local and influences a small percentage of the total volume of fluid.

His question concerning the radii of the vane and the radii of the impeller suggests that if the vane is of involute shape the scribing radii are generally proportional to the impeller radii, so that the ratio is the same. At any rate, the performance of any impeller has been found to be not appreciably changed by a vane construction resulting from two radii r_1 and r_2, which are equal to the inlet and exit radii of the impeller, as long as continuity in all components of velocity is carefully preserved.

In full agreement with Professor Switzer, the author realizes the importance of suction design, but has found that in the case of faulty characteristic curves such as Fig. 11 (original paper), changes in the suction may magnify or minimize the defect more or less, but only changes in the impeller and discharge guide can completely remedy the defect, as tests indicate. The design of the suction approach is, of course, very important and is now well known, but the author assumed that successful suction types would always be used. The author has found that the effect of forward or backward vanes in the suction approach is generally applied to the whole characteristic curve and does not correct local disturbances, as shown in his curve, Fig. 11. It is also his experience that one suction design is better than others, and the best design should always be used.

Mr. Tietjens raises some questions about the definition of terms. Owing to the nature of the subject, dealing with considerations and viewpoints that are not common in engineering textbooks, the author has used his own notation, such terms as are not defined in pages (1) and (2) being taken in their usual denotation. The terms "forced vortex" and "free vortex" are found in the older textbooks, and may have been used loosely. In a forced vortex we have flow with a rotary component with the addition of work. In a free vortex we have flow with a rotating component without interference from solid surfaces. The term "parallel vortices" is used to denote a series of vortices side by side and discharging in parallel rather than in series. Parallel vortices may be either separated by partitions, or, if in perfect equilibrium, imaginary partitions are assumed for purpose of analysis.

Regarding Mr. Tietjens' remarks on cavitation, an interesting fact is that in pumps handling a saturated saline fluid, destruction of metal occasionally takes place only in local areas where there is no flow. At other times it takes place where velocities are highest. Some chemists and metallurgists ascribe this to differences of polarity in the fluid.

Mr. German's instructive discussion about deterioration of condenser tubes resulting from circulating pumps suggests the following thoughts. A circulating pump of the propeller type, when operating submerged in a tank, has been observed to discharge water with a large number of entrained vapor bubbles, at a total pumping head of, say, 10 ft. When this same pump was operated at a higher speed and delivering against 25 ft. head, the quantity of air bubbles was greatly reduced but still appreciable. Low- and moderate-specific-speed types of pumps under similar conditions show no such delivery of air bubbles. This air is, of course, rich in oxygen and hydrogen and is the probable cause of tube deterioration. It would be desirable to limit the specific speed of circulating pumps, particularly when sea water is used for circulating water. The reason for this is that in the higher specific speeds, even of the most efficient design, local low pressure areas (especially at partial capacities) subject to vapor formation are likely to occur.

HYD-54-5

Leakage Loss and Axial Thrust in Centrifugal Pumps

By ALEXEY J. STEPANOFF,[1] BERKELEY, CALIF.

The paper reports on experiments performed by the author at the Hydraulic Laboratory of the University of California with the equipment furnished by the Byron Jackson Company, Berkeley, Calif. Factors governing leakage loss and means to reduce leakage loss were studied. With a special pump, leakage was measured under actual operating conditions, using wearing rings of various types, width, and clearances. Tests were performed at four speeds and different heads. The results are presented in the form of curves, the coefficient of discharge through the clearances being plotted as ordinates against pressures at the wearing ring as abscissas. To facilitate the calculation of the pressure at the wearing ring the pressure distribution in the pump case was studied and an approximate formula was developed. The axial thrust in single- and multi-stage pumps was measured with a specially designed thrust bearing which indicated the thrust directly in pounds under actual operating conditions. New sources of thrust were established—those caused by unequal pressures on the impeller shrouds from the inside due to various causes, the most important of which are an uneven radial velocity distribution and cavitation. In multi-stage pumps, with impellers mounted back to back, generally considered balanced on account of symmetry, it was discovered that there is considerable axial thrust produced by unequal pressures in adjacent stages acting upon the shaft shoulders and impeller hubs extending through the walls of the adjacent stages. Means to eliminate this thrust were found.

INTRODUCTION

IN THE design of centrifugal pumps a few years ago, very little attention was paid to leakage loss, as efficiencies then were comparatively low and this loss constituted only a small portion of the total loss. With modern efficiencies well over 90 per cent, leakage loss takes on an important aspect. To measure leakage loss under actual operating conditions, a commercial centrifugal pump was modified slightly in design. This pump permitted the use of various types and sizes of wearing rings, including those of the labyrinth, stepped, spiral-grooved, circular-grooved, and plain types, as well as plain rings of various widths. The various types were tested with constant clearances, the plain ring being tested with variable clearances. The effect of width on leakage also was observed by testing a plain ring with a constant clearance, but varying the width in steps of $^{3}/_{16}$ in. All tests were performed at speeds of 1400, 1700, 2000, and 2500 r.p.m., with pressures varying from 10 to 160 ft. The results are presented in the form of curves, with the coefficient of discharge calculated from the leakage flow through the clearances plotted as ordinates and the pressure as abscissas.

In order to determine the leakage loss in a new design, it would be necessary to calculate the difference in pressure across the wearing ring. The pressure distribution in the pump case was studied by inserting pipe taps at various points and actually measuring the variation in pressure, including a point adjacent to the wearing ring. This revealed that the pressure at the discharge nozzle is greater than in the volute case for capacities less than the capacity at maximum efficiency. For capacities greater than the capacity at maximum efficiency the pressure at the discharge nozzle is less than in the volute case. At the point of maximum efficiency, the pressure distribution in the volute case was found to be uniform. When operating at capacities other than that of maximum efficiency, the variable pressure distribution throughout the pump case produces a side thrust upon the rotating element. An approximate formula was developed for calculating the pressure at the wearing ring which agreed very closely with the observed values.

In centrifugal-pump design, multi-stage pumps with impellers arranged in groups, mounted back to back, were considered balanced hydraulically due to their symmetry. Investigation has disclosed that this is not the case and that a considerable axial thrust results. This axial thrust is produced by unequal pressures in adjacent stages acting upon shaft shoulders and impeller hubs extending through the walls of the pump casing, the magnitude of the thrust depending upon the operating head, dimensions of the shoulders and hubs, and the arrangement of the stages. By proper arrangement of the stages and by providing special collars on the runner hubs of one group of runners, this thrust was reduced to a very small value or entirely eliminated. The analytical conclusions were confirmed by actual measurement. Due to large oil companies abandoning transportation of oil by mobile tanks and resorting to pipe lines with high-speed booster centrifugal pumps, the elimination of axial thrust is of extreme importance, since ball thrust bearings have their thrust capacity greatly reduced at the higher speeds. These high-speed oil-line pumps operate at 3600 and 3000 r.p.m., at 60 and 50 cycles, respectively.

For measurement of axial thrust, a special thrust bearing was designed which enabled the reading of thrust directly in pounds. A special L-shaped arm hinged at the apex converted the horizontal thrust into a vertical thrust, permitting the use of a platform scale. The thrust was measured at speeds of 1400, 1700,

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[1] Design Engineer, Byron Jackson Company. Assoc-Mem. A.S.M.E. Dr. Stepanoff was born in 1895 in Russia. He was graduated in mechanical engineering in 1918 in Russia; 1920–1923, was superintendent of the city pumping station, City of Harbin, China; 1923–1925, with American Brake Shoe and Foundry Co., San Francisco, Calif. He received B.S. degree in 1926 and M.S. degree in 1927 from the University of California. Since May, 1927, he has been designing engineer for Byron Jackson Company, Berkeley, Calif. He received Ph.D. degree in mechanical engineering in 1930 from the University of California.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

2000, 2500, and 3000 r.p.m. for two different diameter impellers, viz., $8^1/_2$ in. and $10^1/_8$ in. Various balancing devices in the form of radial ribs and low-pressure chambers on the back side of a single suction impeller were tested at the various speeds and the two different diameter impellers. From these results it was impossible to explain the shape of the thrust-capacity curves by the pressure distribution in the case. A new source of thrust was established—that due to unequal pressures on the impeller shrouds from inside, owing to various causes, the most important of which is an uneven radial velocity distribution and cavitation.

NOTATION

Q_t = quantity of liquid discharged by the runner
Q = quantity of liquid discharged by the pump, or measured pump capacity
Q_L = amount of leakage
H_t = theoretical total head
H = actual or useful total head
H_L = pressure difference on two sides of wearing rings
WHP = pump water horsepower, pump output
BHP = motor brake horsepower, pump input
e_v = volumetric efficiency
e_m = mechanical efficiency
e_h = hydraulic efficiency
e = pump efficiency
A = area of the clearance between wearing rings
C = coefficient of discharge
g = gravitational constant
s = radial clearance between the runner and case wearing rings
D = mean diameter of clearance
u_2 = runner peripheral velocity at discharge
u_1 = runner peripheral velocity at entrance
c_{2m} = radial velocity of flow through the runner
c_{2u} = tangential component of the absolute velocity at runner discharge
V_e = velocity through the runner eye
β_2 = vane angle at discharge
α_2 = angle between the absolute velocity of discharge from runner and u_2.

PART I LEAKAGE LOSS

1 Leakage Loss—Definitions and Descriptions

LEAKAGE LOSS in centrifugal pumps is understood to be the loss of high-pressure liquid through the clearances between the rotating element and the stationary pump case. A portion of the liquid pumped is "short circuited," or returned back to the suction. Sometimes this loss is referred to as "short circuit loss" or "clearance loss." The pump capacity measured is less than the amount of liquid passing through the runner by the amount of leakage. The ratio of the quantity of liquid discharged by the pump to that passed through the runner is the volumetric efficiency of the pump.

$$\frac{Q}{Q_t} = \frac{Q}{Q + Q_L} = e_v \qquad [1]$$

The volumetric efficiency is related to the pump efficiency in the following way:

$$e = e_m\, e_h\, e_v = \frac{\text{Pump output}}{\text{Pump input}} = \frac{WHP}{BHP} \qquad [2]$$

Water horsepower (WHP) is equal to $\frac{Q\gamma H}{550}$, where Q is measured in cubic feet per second, γ in pounds per cubic foot, and H in feet.

$$e_m = \frac{BHP - \text{Mechanical losses}}{BHP} \qquad [3]$$

The mechanical losses include power lost in bearings, stuffing-box friction, and the disk friction of the runner.

e_h is hydraulic efficiency, equal to the ratio of the head measured at pump discharge, or total useful head, to the theoretical head, or

$$e_h = \frac{H_t - \text{Hydraulic losses}}{H_t} \qquad [4]$$

The hydraulic losses are the following: Shock loss at the entrance to the runner, friction loss in the runner, shock loss at the discharge from the runner into the pump case, friction loss in the volute case, and all losses due to turbulence, whatever its causes may be.

H_t is the theoretical head, or head attainable with a viscousless fluid and a runner with an infinite number of vanes. It can be calculated using formulas developed by the theory.[2]

From the foregoing it is seen that the mechanical losses are losses of power, the hydraulic losses are losses of head, and the leakage loss is a loss of capacity. This should be kept in mind when investigating the performance of a pump. Thus, if the pump efficiency is better or worse than anticipated, each of the three sources of losses should be studied independently.

The clearance between the rotating element and the pump case may be axial or radial. Figs. 1 and 2 show examples of the axial clearances used in the early pumps. This type of clearance presents two major disadvantages: First, the exact machining and assembly required does not allow any end play; and, second, a high-velocity jet is projected across the flow in the runner eye. This jet becomes considerable where clearances are worn, and impairs the pump efficiency. The axial clearances are used in modern pumps only in connection with the balancing devices shown in Figs. 71 and 72 and described in Section 14.

Close radial clearances in centrifugal pumps prevent the return of high-pressure water from the pump case to the runner suction. In the early pumps close clearances were provided at the entrance to the runner eye as well as at the discharge, as

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[2] Dr. C. Pfleiderer, in his book "Die Kreiselpumpen," Julius Springer, Berlin, p. 48, Equation [7], and p. 69, gives the following expressions for the pump efficiency:

$$e = \frac{QH}{(Q + Q_L)\, H_t + \text{Mechanical losses}}$$

$$= \frac{1}{\left(1 + \frac{Q_L}{Q}\right)\frac{H_t}{H} + \frac{\text{Mechanical losses}}{QH}}$$

It will be shown that these are the same as Equation [2] of this paper. Substituting for Mechanical losses, Input × $(1 - e_m)$, we get:

$$e = \frac{1}{\left(\frac{Q + Q_L}{Q}\right)\frac{H_t}{H} + \frac{\text{Input}\ (1 - e_m)}{QH}}$$

$$= e = \frac{1}{\frac{1}{e_v}\cdot\frac{1}{e_h} + \frac{1 - e_m}{e}} = \frac{e_v e_h e}{e + (1 - e_m)\, e_v e_h}$$

or

$$1 = \frac{e_v e_h}{e + (1 - e_m)\, e_v e_h}$$

$$e + e_v e_h - e_m e_v e_h = e_v e_h$$

$$e = e_m e_v e_h$$

shown in Figs. 1, 2, and 3. In modern pumps close clearances are made at the runner eye only, as shown in Figs. 30 and 75. Close clearances at the runner discharge were brought into pump practice from hydraulic turbines, where the flow is in the opposite direction, the intake being at the outer runner diameter.

Clearances at the outer runner diameter have been discontinued in centrifugal pumps for these reasons: (*a*) The clearance area at the outer diameter is much greater than that at the runner eye with the same radial clearance. It is easier to make close running fit at a smaller diameter. (*b*) It is difficult to make a double fit equally accurate. (*c*) Usually one pump is designed to cover a certain range of heads and capacities, which is accomplished by cutting the runner diameter. A close clearance at the outer diameter would mean cutting the runner vanes only, leaving the runner shrouds of full diameter and resulting in some unnecessary disk friction and a lower efficiency. (*d*) With close clearances in modern pumps it is always possible, due to inaccuracy in machining or wear of bearings, that the runner may rub against the case wearing rings. If rubbing takes place at this large diameter, the torque required to overcome friction will be much greater. (*e*) The cost of machining of a close clearance at the runner outside diameter is greater, with no gain in efficiency; therefore this method of sealing has no advantage.

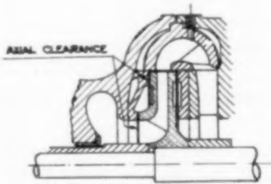

Fig 1 Axial Clearance

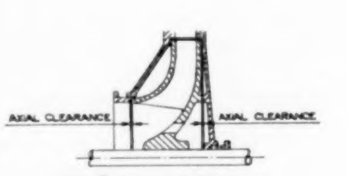

Fig. 2 Axial Clearance

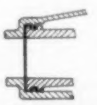

Fig. 3 Close Clearance at Runner Discharge

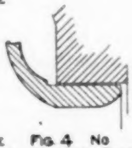

Fig. 4 No Wearing Rings

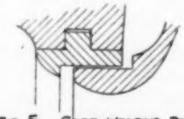

Fig. 5 Case Wearing Ring & No Runner Wearing Ring

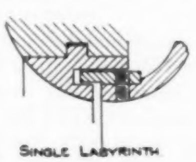

Fig. 6 Single Labyrinth

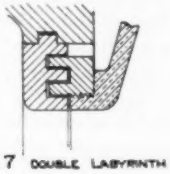

Fig. 7 Double Labyrinth

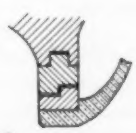

Fig 8 Stepped Wearing Rings

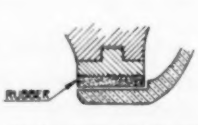

Fig 9 Case Wearing Ring Lined with Rubber.

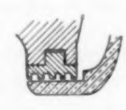

Fig 10 Case Wearing Ring with Circular Grooves

Figs. 4 to 10 show several designs of wearing rings. The pump case and runner in Fig. 4 have no wearing rings. This construction is used for small, cheap pumps. Figs. 5, 9, and 10 show case wearing rings and no runner rings. The rest of the designs provide case and runner wearing rings which can be replaced when clearances become too large. Thus the original clearances are reestablished. In Figs. 6 and 7, respectively, are shown single- and double-labyrinth type of wearing rings. Fig. 9 shows a rubber-lined wearing ring. It is maintained that this type of ring lasts longer with cold, gritty water and that smaller clearances may be used. The disadvantage of this type of wearing ring is that the pump cannot run dry (started without priming or lose priming), even for a short time, without "freezing."

It is evident that in double-suction pumps the leakage loss occurs on both sides of the runner (Fig. 11), while in single-suction pumps leakage takes place only on one side. But for the sake of hydraulic balance single-suction pumps are often provided with a chamber on the back side of the runner of the same diameter as the runner wearing ring and separated from the high-pressure case by wearing rings similar to those used at the runner eye, and connected in some way to the pump suction. Fig. 30 shows one of these pumps, a detailed description of which is given in Section 8. It is seen that, for the sake of hydraulic balance, some portion of the pump capacity is sacrificed in this way.

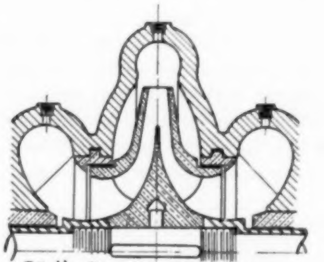

Fig 11 Double Suction Pump

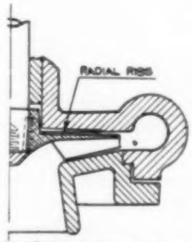

Fig. 12 Open Runner Pump

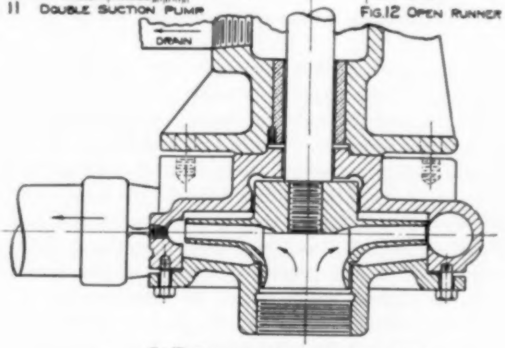

Fig 13 Close Clearance Bushing Used Instead of a Stuffing Box

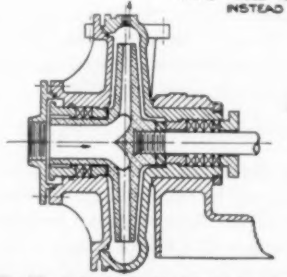

Fig 14 A Stuffing Box Used Instead of a Close Clearance Wearing Ring

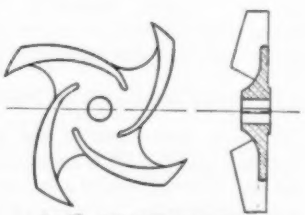

Fig 15 A Runner without Shrouds

In multi-stage pumps, in addition to the loss of the high-pressure water at the runner eye, there is a loss of capacity due to leakage from stage to stage through the clearances between the shaft and the case wall separating the adjacent stages. Depending on the arrangement used, the pressure between two adjacent stages may be that of one stage or more (Fig. 81). If there is one stage pressure across the clearances between the case and shaft, the per cent of liquid lost in this way is that of one stage only, as the same liquid may be regarded as short-circuited from stage to stage.

There is another example where a portion of the pump capacity is sacrificed and additional leakage is introduced for the sake of simplicity of construction. Fig. 13 shows a section of a vertical pump with a close-fitted bushing instead of the stuffing box.

Any leakage through the clearance is drained to the pump intake. With a close clearance this bushing works as a water-lubricated bearing. Special bronzes with a large percentage of lead are especially suited for this purpose.[3]

There are two types of "open runner" pumps. In one type the runner vanes are shrouded on one side only, while in the other the runner vanes are not shrouded, but are cast, with suitable reenforcement, integral with the hub (Figs. 12 and 15). In these pumps running clearance is provided between the runner vanes and the pump cover. The determination of the leakage loss for these pumps is difficult because the clearance is uncertain and it is impossible to separate the short-circuited water from the main flow. These pumps are used for special services, such as pumping milk, pulp, etc., where easy access to the runner vanes is required for cleaning.

Propeller or screw pumps belong to the open-runner type. Due to the very low pressure and large capacity of these pumps, the amount of leakage is a small percentage of the total pump capacity.

In addition to the leakage through the clearances between the pump case and the rotating element, a very small part of the pump capacity may be used for sealing the stuffing boxes or cooling the pump bearings. In the first case, if the stuffing box is under the suction pressure, there is always danger that air may be drawn through the stuffing box and the pump lose priming. To prevent this a cage ring is provided in the middle of the stuffing box (Fig. 30), and high-pressure water is fed in from the pump case. In high-pressure pumps a similar cage ring is inserted in the stuffing box to relieve the pressure on it, and the leakage through the first section of the packing is drained to the pump suction. This leakage is also a net loss. The amount of water used for sealing the stuffing boxes and for cooling the bearings is usually negligible. The amount of leakage from a high-pressure stuffing box may be considerable, depending upon the pressure and the method of packing the stuffing box.

2 LEAKAGE LOSS AND DESIGN OF CENTRIFUGAL PUMPS

A study of the leakage loss is important from several points of view. First of all, a knowledge of the amount of the leakage and the factors governing it and of its effect on pump performance may indicate the ways to reduce this loss and thus to improve the pump efficiency. There is no information about the amount of leakage in the literature on centrifugal pumps. R. L. Daugherty[4] estimates the leakage loss as 1.5 per cent in one case and 13.3 per cent in another case. C. Pfleiderer[5] states that the leakage loss may be from 3 to 10 per cent. L. C. Loewenstein[6] assumes the loss to be 6 per cent. F. Neumann[7] estimates this loss as from 4 to 6 per cent. Many treatises do not mention the leakage loss at all.

While the theory of the leakage loss is considered in a later section, it is convenient, for the purpose of the study of factors governing the leakage loss, to represent the amount of leakage by the following formula:

$$Q_L = AC\sqrt{(2gH_L)}$$

where Q_L is the quantity of water short-circuited through the clearance between the runner and case wearing rings; H_L is the pressure difference across the clearance; A is the net area of the clearance equal to $D\pi s$, where D is the mean diameter of the clearance space and s is the radial clearance. Owing to the small value of s compared with D, the diameter of the wearing ring can be used instead of the mean diameter of the clearance without appreciable error. C is a coefficient of discharge, including the coefficient of contraction and the velocity coefficient, and accounting also for the resistance of the ring due to friction or change in direction or section of the flow. Then

$$Q_L = \pi DsC\sqrt{(2gH_L)}$$

It is evident that the smaller the factors on the right-hand side of this equation are, the smaller will be the leakage.

Each factor is to be considered separately, as follows:

(*a*) The pressure on the clearance H_L varies with the pressure developed by the pump, being smaller for a low-pressure pump and higher for a high-pressure pump. For the same pump this pressure increases as the pump pressure increases and is a maximum at shut-off or no discharge. The ratio of the pressure at the wearing ring to the pressure in the volute case, or the pump total head, depends upon the ratio of the wearing-ring diameter to the mean diameter of the volute case. It decreases as this ratio decreases. This is due to the centrifugal effect produced by the water set in rotation in the space between the runner and the case walls.[8] This effect is greater for smaller clearances between the runner and the pump case. Still greater reduction in pressure at the wearing rings is effected by providing radial ribs on the runner back.

(*b*) It is evident that the smaller the radial clearance between the runner and the case wearing rings, the smaller will be the leakage. But a very small clearance requires exact machining of all parts of the pump, because there is always danger of rubbing in of the wearing rings due to inaccurate bearing centering or too liberal running clearances in the bearings. In this case the saving in the leakage is more than counterbalanced by the power lost in overcoming friction in the wearing rings, which takes place at a comparatively large diameter. Again, it should be kept in mind that pump shafts are not perfectly rigid, but have a certain deflection due to the runners and their own weight. For the wearing-ring clearances the following values represent good practice: For rings of 6 in. diameter and smaller, the clearance is made 0.010 in. on the diameter[9] with 0.001 in. additional for every inch of the wearing-ring diameter over 6 in., or:

$$s = 0.010 + (D - 6) \times 0.001 \text{ in.}$$

where s and D are in inches. This gives for rings of

6 in. diameter, 0.010 in. clearance
12 in. diameter, 0.016 in. clearance
18 in. diameter, 0.022 in. clearance
24 in. diameter, 0.028 in. clearance
36 in. diameter, 0.040 in. clearance.

The wearing ring should be made sufficiently strong to permit chucking for machining without distortion. The final cut of the runner wearing ring should be taken after it is pressed on the runner. The pump shafts should be handled with care as they are easily bent or sprung during machining. Usually the shafts, made of steel, which requires heat treatment, are distorted after that operation and should be trued in the lathe very carefully.

There have been attempts to use a stuffing box instead of a close clearance between the wearing rings to eliminate the leakage loss, as shown in Fig. 14.[10] They were not successful, because the friction in the stuffing box more than counterbalanced the saving in the leakage. Also, a long revolving tube in front of the runner vanes is detrimental to the runner performance.

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[3] A bronze much used for similar services has a composition of copper, 70 per cent; lead, 26 per cent; and tin, 4 per cent.

[4] R. L. Daugherty, "Centrifugal Pumps," p. 116, McGraw-Hill Book Company.

[5] C. Pfleiderer, "Die Kreiselpumpen," p. 57, Julius Springer.

[6] L. C. Loewenstein and C. P. Crissey, "Centrifugal Pumps," p. 140, Van Nostrand Company.

[7] Fritz Neumann, "Die Zentrifugalpumpen," p. 31, Julius Springer.

[8] It has been found by the author in tests described later that for the same pump case the ratio of the pressure at the wearing ring to the pump total head depends very little upon the runner diameter, the ratio being fixed by the case size and the ring diameter.

[9] For small wearing rings on a pump with very exact machining and ball bearings, the minimum clearance may be reduced to 0.008 in.

The decrease of pump efficiency with time is mainly due to an increase of leakage resulting from the wear of the wearing rings. This wear is caused by the presence of abrasive matter in the water, which passes through the clearance at high velocity. Also, due to bearing wear, the rotating element drops, and the runner wearing rings start to rub against the case wearing rings. It is good practice, therefore, to provide both the pump case and the runner with wearing rings and to make each ring of a different material. If both are of bronze, bronzes of different hardnesses should be used. The following compositions may be recommended: For the runners or runner wearing rings, Cu, 88 per cent; Sn, 10 per cent; Zn, 2 per cent. For the case wearing rings, Cu, 85 per cent; Sn, 5 per cent; Pb, 5 per cent; and Zn, 5 per cent.

(*c*) The leakage decreases also with a decrease in the clearance diameter. A reduction of the diameter of the clearance for a given runner-eye area may be obtained by a reduction of the runner hub and shaft diameters. Fig. 75 shows an example of the hub entirely eliminated from the suction side of the runner and the shaft diameter reduced in the runner eye and shaped to conform to the direction of the flow. Here, again, is a danger of weakening the shaft so that the static deflection will grow above the permissible maximum. The strength of the pump shaft for a maximum deflection is not always a determining factor for the size of the pump shaft; it also should be strong enough to transmit the required torque and it should be checked for critical speeds.

The small runner-eye diameter is advantageous also from a hydraulic point of view, because (1) with a small runner-eye diameter the inevitable shock losses at entrance take place at a smaller diameter and absorb less power; (2) for the same runner diameter, the head produced is higher for a smaller runner-eye diameter, or for the same head a runner of smaller diameter may be used. The disk friction will be reduced in this way and the pump efficiency be improved. (3) For the same radial velocity a higher vane entrance angle and a lower relative velocity may be used.

(*d*) The decrease of the coefficient of discharge in the formula $Q_L = AC\sqrt{(2gH_L)}$ may be effected by an increase of the resistance to the flow through the clearance by a change in direction of the flow, by a sudden change of the section, by using labyrinth rings, etc., as illustrated in Figs. 6, 7, 8, and 10. To find the coefficients of discharge for different rings, and also to investigate the relative advantages of various rings, is a part of this work.

The consideration of the leakage affects the design of the pump case, as will be seen from the following. Since water is discharged from the runner with a high velocity, the leakage water does not separate from the main flow at that point, but from a space farther in the volute where the outward radial component is destroyed and the flow is reversed, as shown in Fig. 16. If the pump case is designed with small clearance between the case wall and the runner, the leakage flow will interfere with the main flow, thus impairing the efficiency. Fig. 17 shows an example of a tight case design. Another disadvantage of a case of this shape is that if the runner is displaced from its central position because of inaccurate assembly, poor casting, or excessive bearing end play, the stream from the runner may strike the corners of the volute. In multi-stage pumps with long cases the conditions are even worse. Due to an improper allowance for shrinkage, the center lines of the volutes may differ from those of the runners on the shaft, and rubbing of the runners against the case will result. If a pump with the volute shaped as in Fig. 17 is run with a runner cut in diameter, the stream will hit the corners of the volute and produce disturbance, since the flow from the runner is divergent. The modern pumps are designed with ample clearance between the runner and case wall, as shown in Figs. 75 and 30. These clearances are made approximately $^1/_2$ in. per ft. of the runner diameter, and never less than $^3/_8$ in., the exact value being determined by the runner profile.

Another disadvantage of the tight pump case is that, due to slight non-central runner location in the case, it is possible even with a double-suction runner, inherently balanced, to have an axial thrust which may be variable, thus producing shaft vibration, as explained in Section 17.

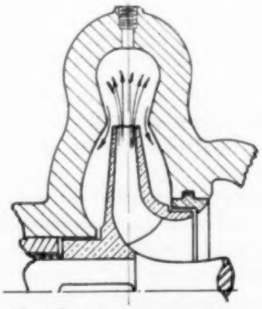

FIG. 16 VOLUTE CASE WITH AMPLE CLEARANCE BETWEEN THE RUNNER SHROUDS AND PUMP CASE WALL

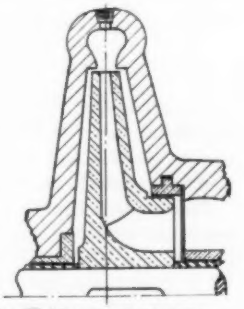

FIG. 17 VOLUTE CASE WITH SMALL CLEARANCE BETWEEN THE RUNNER AND CASE.

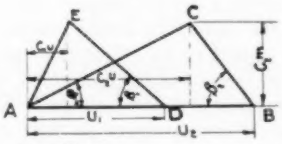

FIG. 18

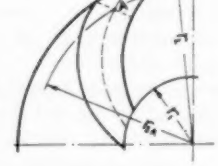

FIG. 19

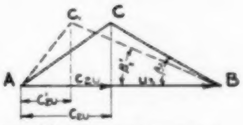

FIG. 20 DISCHARGE TRIANGLE

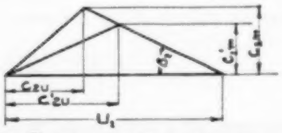

FIG. 21 DISCHARGE TRIANGLES FOR TWO RADIAL VELOCITIES C_{2m} AND C'_{2m}

From a hydraulic point of view the volute case with an ample clearance between the runner and pump case shows a decided advantage over the tight case, the difference in efficiency being from 3 to 10 per cent at high capacities. In a tight case the high jets discharged by the runner come in contact with the stationary walls. The shock and friction losses are higher than when the runner discharges into a body of revolving water.

It should be noted that one of the main reasons for the low efficiency of small pumps is an excessive leakage loss. This is due to the fact that the clearances cannot be reduced below a certain minimum. The wearing-ring diameter is comparatively large, the overlap of the wearing rings is small, and due to small dimensions no labyrinth or stepped rings can be used. Again, since the small pumps are usually of a low-specific-speed type or of small capacity and high head, for the same head the amount of short-circuited liquid constitutes a higher percentage for a small pump. The other reasons for the low efficiency of small pumps are: (1) The low-specific-speed type of pump is inherently less efficient than the medium-specific-speed type of pump. The high percentage of leakage loss is only one reason for this. There is no demand for small pumps of the high-specific-speed type, as pumps of small capacity and low head have little application (the deep-well turbine pump being an exception, as the total head in this pump is split between several stages). (2) Lower mechanical efficiency is inherent in all small machines and mechanisms. (3) Simplicity of design and manufacture must be maintained on small pumps, as the price is the main selling factor, the efficiency being of secondary importance.

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[10] *Engineering*, Jan. 28, 1921.

3 Leakage Loss and Theory of Centrifugal Pumps

In the elementary theory of centrifugal pumps it is shown that the theoretical head produced by a pump is equal to

$$H_t = \frac{u_2c_{2u} - u_1c_{1u}}{g} \qquad [5]$$

where H_t is the theoretical head produced, u_2 and u_1 are the peripheral speed of the runner at the outlet and inlet, respectively, and c_{2u} and c_{1u} are the tangential components of the absolute velocity, as shown in Fig. 18.

Though this equation has been generally accepted for many years, the actual values of head produced never have approached this theoretical head even under the most favorable conditions. Very often the value of the hydraulic efficiency H/H_t, where H is the actual head, is found to be lower than the over-all efficiency of the pump, which is absurd. The general conclusion is that the theoretical head, as given by the foregoing formula, is too high. One of the reasons for this discrepancy is to be seen in the fact that this theory admits an infinite number of vanes or that all particles of liquid move in a similar manner and leave the blade with an angle β_2 of the vane. In actual practice the number of vanes is limited to from 2 to 16. This conclusion is made logical by the fact that by increasing the number of vanes the head produced may be increased a certain amount. Naturally, attempts have been directed toward improving this formula by introducing some corrective factors, but up to the present time the problem has not been solved. The difficulty lies in the very simplicity of the formula,

$$H_t = \frac{u_2c_{2u}}{g} \qquad [6]$$

the term u_1c_{1u} being equal to zero for the radial inlet.

It was proposed to substitute for u_2, not the peripheral velocity corresponding to r_2 radius of the runner, but a value of peripheral velocity of the point A, Fig. 19, in the middle of the line drawn from the tip of the vane normal to the stream lines. It was supposed that from this point the vane ceased to act upon the water streams in the runner. It is evident that the value of u_{2A} for the point A will be smaller for a smaller number of vanes. But this is true only if the angle β_2 is far from 90 deg. With the increase of β_2, the difference between the u_2 and u_{2A} becomes smaller and disappears when β_2 equals 90 deg.

The second method of correction of the fundamental formula consists in correction of c_{2u} so as to take into account the number of vanes of the runner. But as in the triangle at the outlet u_2 and c_{2m} are determined from the actual test or are assumed, the only way to make c_{2u} smaller is to conclude that water leaves the runner with an angle β_2' smaller than the actual vane angle β_2.

Dr. Pfleiderer[11] developed a corrective factor for c_{2u} and gave also an expression for the angle of the deviation of the stream as compared with the actual vane angle. Thus a discharge triangle ABC' is substituted instead of the ABC given by the elementary theory (Fig. 20). It seems, however, that all attempts to correct the elementary formula by introducing some terms to account for various factors affecting the head produced will be futile until it is known just what the theoretical head should be. The only information available about the theoretical head is that the ratio of the actual to the theoretical head or hydraulic efficiency should be less than unity and greater than the pump efficiency $e < e_h < 1$, and every designer or investigator has to use his judgment as to what "reasonable" value for the hydraulic efficiency should be used for different types of pumps.

But if the leakage loss is known, the hydraulic efficiency and theoretical head can be determined with great accuracy from the pump efficiency:

$$e = e_m\, e_v\, e_h,$$

$$e_h = e/e_m\, e_v \qquad [7]$$

where e_v is the volumetric efficiency obtained from

$$e_v = Q/(Q + Q_L) \qquad [8]$$

Q_L being the amount of leakage.

e_m is the mechanical efficiency.

$$e_m = \frac{\text{Power input} - \text{Mechanical losses}}{\text{Power input}}$$

The mechanical losses include friction in bearings and disk friction of the runner. The first can be easily measured by driving the shaft, without the runner, in the pump case filled with water under pressure. A considerable amount of the experimental data on disk friction are available, which enables the designer to calculate this loss with great accuracy. The description and the results of these tests are found in the following articles and treatises: Hesse, Bulletin No. 2, University of California, 1887; J. N. LeConte, *Journal of Electricity, Power,*

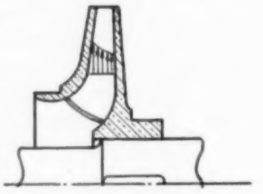

Fig. 22 Radial velocity distribution for small capacity (theoretical)

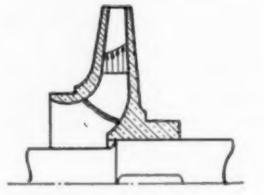

Fig. 23 Radial velocity distribution for large capacity

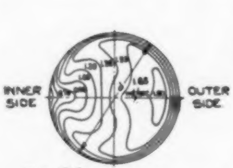

Fig. 24 Velocity distribution in an elbow

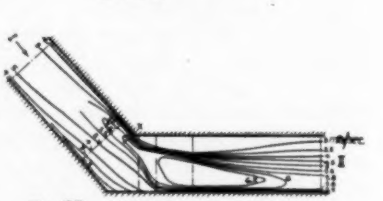

Fig. 25 Velocity distribution in a knee.

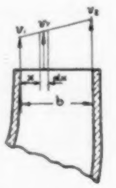

Fig. 26-a.

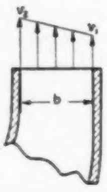

Fig. 26-b

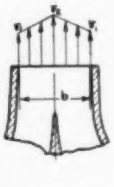

Fig. 26-c

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[11] C. Pfleiderer, "Die Kreiselpumpen," p. 86.

and Gas, Sec. 3, 1910; J. N. LeConte, *Hydraulics;* A. H. Gibson, "Hydraulics and Its Applications;" F. Zur Nedden, Trans. A.S.M.E., 1915.

From the hydraulic efficiency e_h obtained from Equation [7], the theoretical head H_t is found:

$$H_t = \frac{H}{e_h}$$

where H should be obtained by test.

4 THEORETICAL HEAD WITH NON-UNIFORM RADIAL VELOCITY

Equations [5] and [6], Section 3, for the head developed by the runner, assume that all water particles are moving along similar paths with uniform velocities, although experience shows that the flow with uniform velocities does not exist, being approached in the most favorable conditions only. Since in every runner particles of water are deflected 90 deg. before they are acted upon by the vanes, the velocity is no longer uniform. It is usually supposed that for this case the formula for the head developed still holds if the mean values of velocities are used. It will be shown that for this case the theoretical head for an infinite number of vanes, with no losses, is smaller, the difference being greater the more pronounced is the difference between the maximum and minimum radial velocities.

With an ideal fluid and the flow along a curved path, the velocities change according to the law vr = constant, where r is the radius of curvature of the path. This gives low velocities at the outer side and high at the inner side of the curved path. With actual liquids and high mean velocities this velocity distribution holds only for a portion of the curved path; then the inertia forces (centrifugal) come into play, and the liquid leaves the inner wall and the velocity distribution is reversed—i.e., the high velocities are at the outer side and the low at the inner side. The latest and very extensive tests by H. Nippert[12] and R. Bambach[13] show that such distribution of velocities is found in 90-deg. elbows. With small mean velocities of flow and large values of the ratio r_m/d (where r_m is the radius of curvature of the mean stream lines and d is the diameter of the elbow), the theoretical velocity distribution vr = const. was confirmed by these tests except for layers of water near the walls which were affected by friction. The pressures are higher at the outer wall of the turn. With high mean velocities of flow and small values of the ratio r_m/d, or sharp turns after a short portion of the turn, the higher velocities are shifted from the inner wall to the outer. Water leaves the inner side due to inertia, and forms a "dead water" space filled with eddies at the inner wall. Figs. 24 and 25, taken from the Nippert and Bambach papers, illustrate this very clearly. This velocity distribution is maintained for a long distance in the straight portion of pipe and is brought to the normal distribution very slowly by wall and internal friction. The maximum pressures are near the outer wall in this case also. The values of the velocities producing the first or the second form of the velocity distribution, and the relation of the critical velocity to the r_m/d, were not established by these tests.

Assuming that the radial velocity varies uniformly from v_1 at the front shroud to v_2 at the back shroud and at some point located at a distance x from the front shroud, Fig. 26(*a*), the radial velocity is $v_r = v_1 + ax$, where $a = \frac{v_2 - v_1}{b}$, b being the runner width at the discharge. The head produced at this point is:

$$H_t = \frac{u_2 c_{2u}}{g} = \frac{u_2}{g}\left(u_2 - \frac{v_r}{\tan \beta_2}\right) = \frac{u_2^2}{g} - \frac{u_2 v_r}{g \tan \beta_2} \ldots . [9]$$

The volume of water discharged by an element of the runner dx wide is equal to

$$dQ = v_r \pi D_2 dx \ldots\ldots\ldots\ldots\ldots\ldots [10]$$

The power output of this element is:

$$dP = \gamma H_t dQ = \gamma H_t v_r \pi D_2 dx \ldots\ldots\ldots\ldots [11]$$

Substituting for H_t its value from [9], we get:

$$dP = \left(\frac{u_2^2}{g} - \frac{u_2 v_r}{g \tan \beta_2}\right) \gamma \pi D_2 v_r dx$$

For brevity denote:

$$\frac{u_2^2}{g} = A; \quad \frac{u_2}{g \tan \beta_2} = B; \quad \gamma \pi D_2 = C$$

The formula then takes the form:

$$dP = (A - Bv_r)\, Cv_r dx = C(Av_r - Bv_r^2)dx$$

Substituting for v_r its value $v_r = v_1 + ax$,

$$dP = C[A(v_1 + ax) - B(v_1 + ax)^2]dx$$

Integrating between the limits 0 and b, we obtain the power output of the runner:

$$P = \int_0^b CA(v_1 + ax)dx - \int_0^b CB\,(v_1 + ax)^2 dx$$

$$= CA\left[\frac{(v_1 + ax)^2}{2a}\right]_0^b - BC\left[\frac{(v_1 + ax)^3}{3a}\right]_0^b$$

$$= CA\left[\frac{(v_1 + ab)^2}{2a} - \frac{v_1^2}{2a}\right] - BC\left[\frac{(v_1 + ab)^3}{3a} - \frac{v_1^3}{3a}\right]$$

But $v_1 + ab = v_2$; then

$$P = CA\,\frac{(v_2^2 - v_1^2)}{2a} - CB\,\frac{(v_2^3 - v_1^3)}{3a}$$

Substituting for a its value $a = \frac{v_2 - v_1}{b}$ and denoting $\frac{v_2 + v_1}{2}$ the mean radial velocity by v_m, the expression for the power output takes the form:

$$P = CA\,\frac{(v_2 - v_1)}{a}\,\frac{(v_2 + v_1)}{2} - CB\,\frac{(v_2 - v_1)}{a}\,\frac{(v_2^2 + v_2 v_1 + v_1^2)}{3}$$

$$= CAbv_m - CBb\,\frac{(v_2^2 + v_2 v_1 + v_1^2)}{3}$$

The average head produced by the runner is obtained by dividing the runner power output by the weight of water discharged W.

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[12] H. Nippert, "Über den Strömungverlust in gekrümmten Kanälen," *Forschungsarbeiten*, heft 320, V.D.I. Verlag, 1929. In addition to the experimental results and graphs of velocity distribution in 90-deg. elbows of circular and rectangular section, an extensive bibliography on this subject is given.

[13] R. Bambach, "Plötzliche Umlenkung von Wasser in geschlossenen unter Druck durchströmten Kanälen," *Forschungsarbeiten*, heft 327, V.D.I. Verlag, 1930.

$$W = \gamma \int_0^b dQ = \gamma \int_0^b v_r \pi D_2 dx$$

$$= \gamma \int_0^b \pi D_2 (v_1 + ax) dx = \gamma \pi D_2 \left(v_1 b + \frac{ab^2}{2} \right)$$

$$= \gamma \pi\, D_2 \left[v_1 b + \frac{(v_2 - v_1)\, b^2)}{2b} \right] = \gamma \pi D_2 b \, \frac{v_2 + v_1}{2}$$

$$W = \gamma \pi D_2 b v_m = C b v_m \ldots\ldots\ldots\ldots\ldots [12]$$

which could be written at once, since the velocity involved is a linear function of x.

$$H_t = \frac{P}{W} = A - \frac{B}{v_m} \left(\frac{(v_2^2 + v_2 v_1 + v_1^2)}{3} \right)$$

$$= A - \frac{B}{v_m} \left[\frac{(v_2 + v_1)^2 - v_2 v_1}{3} \right]$$

$$= A - \frac{B}{v_m} \left[\frac{3\,(v_2 + v_1)^2 + (v_2 + v_1)^2 - 4 v_2 v_1}{12} \right] \ldots\ldots [13]$$

$$= A - \frac{B}{v_m} \left[\left(\frac{v_2 + v_1}{2} \right)^2 + \frac{(v_2 - v_1)^2}{12} \right]$$

$$= A - \frac{B}{v_m} \left[v_m^2 + \frac{(v_2 - v_1)^2}{12} \right]$$

$$= A - B v_m \left[1 + \frac{(v_2 - v_1)^2}{12\, v_m^2} \right]$$

$$H_t = \frac{u_2^2}{g} - \frac{u_2 v_m}{g \tan \beta_2} \left[1 + \frac{(v_2 - v_1)^2}{12\, v_m^2} \right] \ldots\ldots\ldots [14]$$

Comparing this formula with [9], it is seen that the average theoretical head produced by a runner with radial velocity varying from v_1 to v_2 is smaller than that for the mean radial velocity v_m. The difference in the head is greater for the greater difference between v_2 and v_1, this difference disappearing when v_2 is equal to v_1. The effect of the variable radial velocity on the head produced will more clearly be understood if it is kept in mind that the elementary formula [9] gives the head per pound of liquid pumped and that more pounds per second of the low-pressure water is discharged by the runner where the radial velocity is greater, since the head developed is smaller for a higher radial velocity.

From the formula [14] it is seen that for the same ratio of v_2 to v_1 the correction factor is more effective for the runners, for which the second term $u_2 v_r / g \tan \beta_2$ is greater in comparison with the first u_2^2/g. In the normal designs the second term is greater for runners of high specific speed; hence the correction affects more the theoretical head for these runners. In addition, the factor $\left(1 + \frac{(v_2 - v_1)^2}{12\, v_m^2} \right)$ is greater for wide runners, since the velocity distortion is greater and the length of the water path is smaller for runners of high specific speed. This explains the reason for a greater discrepancy between the theoretical and the actual heads for runners of higher specific speed.

It is evident that the formula [14] holds for the velocity distributions shown in Figs. 26(*b*) and 26(*c*), with a linear relation between the maximum and minimum velocity. For any other relation between the minimum and maximum radial velocities the form of the expression for the correction factor will change, but the significance of the expression will remain the same.

5 Other Theoretical Considerations Involving Leakage Loss

Sometimes in the analysis of the pump performance the power at shut-off or no discharge is taken as that used to overcome the mechanical losses and to pump the leakage.[14] This may lead to very erroneous conclusions, as these losses are only a portion of the power at shut-off, the rest being dissipated in hydraulic friction and in turbulence caused by the circulation of water inside the runner. These latter losses are comparatively small in the low-specific-speed runners—i.e., narrow runners with many long vanes—but they increase for medium-specific-speed runners. The horsepower curve for this type of pump is a horizontal line, while for the high-specific-speed runners of the propeller type the horsepower is a maximum at shut-off and decreases as the pump output increases. It is evident that deductions made on the assumption that the power at shut-off is used to overcome the mechanical losses and to pump leakage are absurd for pumps with this shape of the horsepower curve.

Knowledge of the leakage loss is of importance from another point of view. The present design of centrifugal pumps is based on the application of the laws of similarity of flow for homogeneous pumps, which laws when applied to pumps state that runners of the same type or geometrically similar give heads proportional to the squares of their diameters and capacities proportional to the cubes of their diameters. These, combined with the so-called "affinity laws,"[15] form the foundation of the pump design. But both of these laws hold for the quantities discharged by the runner, which is different from the measured pump capacity by the amount of leakage. This is one of the reasons why these laws are not accurate for large multiplications, as the leakage loss is not proportional to the pump size. The other reasons for the discrepancy are that for large multiplications the shaft sizes and runner hubs cannot always be made strictly to factor and that the roughness of castings is not proportional to the pump size.

The excessive leakage, being a loss by itself, impairs the pump efficiency by destroying the favorable velocity ratios through the runner. Thus the drop in efficiency with time is mainly due both to loss of capacity by leakage through the worn wearing ring and to decrease of hydraulic efficiency on account of upset velocity ratios. The same takes place in multi-stage pumps if there is a great difference in pressure between the adjacent stages.

The leakage loss affects the shape of the pump head-capacity curve in distinction from that of the runner, since the runner capacity is equal to that of the pump plus the leakage through the clearances. Thus the zero discharge for the pump does not mean that for the runner. In addition to that, there is a difference in the pressures in the volute and the discharge nozzle not accounted for by the usual form of friction or shock losses, which may be termed as a loss due to "cross-flows."[16] It is important to note that the theory, elementary and improved, applies only to the runner and does not consider the pump case at all.

The question of leakage is again encountered when viscous fluids like heavy oils are to be pumped. It was found by many experimenters that, as a general rule, when pumping viscous oils, both the head and the capacity decrease as the viscosity increases. From the study of a large number of tests the author has found that for viscosities not over 5000 S.s.u., the decrease of the head and capacity is such that the specific speed N_s =

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[14] A. F. Sherzer, "Centrifugal-Pump Economics," Trans. A.S.M.E., 1929, paper FSP-51-15.

[15] Affinity laws are the following relations between pump head, capacity, and speed: If a pump is run at different speeds, the head varies directly with the square of the speed and the capacity varies directly with the speed.

[16] Detailed discussion of this is found in Section 9.

$NQ^{1/2}/H^{3/4}$ remains constant. The exceptions to this rule were two small pumps (one $1^1/_2$ in. and one 2 in.), in which the capacity increased with an increase in viscosity, while the head decreased quite normally with the viscosity. The probable explanation of this is that in small pumps pumping water the leakage loss is excessive, while with increased viscosity the leakage is reduced so that the measured pump capacity is increased.

6 PREVIOUS RESEARCH ON LEAKAGE

In the early stage of centrifugal-pump design very little attention was paid to the leakage loss because the efficiencies were so low that the leakage loss was a small part of the total loss. The attention of the designers was directed toward the perfection of hydraulic design and mechanical construction. But as time went on pump efficiencies were improved, so that today pumps are built with efficiencies over 90 per cent. Now, the leakage loss, which has been estimated to be from 3 to 5 per cent, has become an important item and deserves more consideration than formerly.

While the theoretical calculation of the leakage loss was a comparatively simple matter, the experimental determination of the necessary coefficients offered certain difficulties. Of these the most important are (1) the impossibility of separation of leakage water from the main flow and (2) the great variety of sealing appliances in use. The latter is greatly simplified at the present time, as a small number of wearing-ring types have been worked out and adopted generally by all manufacturers.

One of the earliest investigators in this field was Dr. Becker,[17] who in 1907 published the results of his tests with the apparatus shown in Fig. 29. This apparatus consisted of a hollow cylinder with a concentric piston having small radial clearance. The dimensions shown in the figure are in millimeters. Water under pressure was applied to one end, and the leakage was measured at the opposite end. The primary aim of this test was to check Poiseuille's law for viscous flow through an annular clearance and to find Reynolds' number for the critical speed. To eliminate the effect of the entrance and discharge losses the edges of the piston were rounded off. Dr. Becker developed a formula for the calculation of the flow:

$$V = \frac{D_m \pi s^2 (P_1 - P_2)}{12\, l \eta}$$

where D_m is average diameter of the clearance space, s is radial clearance, l is length, $(P_1 - P_2)$ is a differential pressure, and η is viscosity of liquid; V is volume of discharge per second. The clearances tested were 0.062 in. and 0.042 in. The critical velocity was found to be at Reynolds' numbers 1346 and 1348.[18]

While these tests were of great interest, they did not furnish any information for calculation of the leakage loss in pumps because the arrangement used was far from that found in pumps. There we have the flow through a radial clearance formed by two concentric cylindrical surfaces, one stationary and one rotating. The axial length of the channel formed in this way is very short, and the resistance to the flow is mainly at the entrance and the discharge. Besides that, all means are used to increase the resistance to the flow by sudden change of direction of flow (as is found in labyrinth or stepped rings), or by sudden change of the cross-section of the clearance by means of grooves, etc.

In Fig. 28 is shown another apparatus tested by Dr. Becker. This consisted of a drum with four steps revolving in a container shaped to suit the drum, forming four close clearances and four changes in direction of the flow. This arrangement was a step forward in reproducing the actual conditions existing in pumps. But the sealing rings of this form are not used at the present time.

Professor Susuki of the Tokio University ran a test to determine the leakage loss through both the axial and the radial clearances.[19] His apparatus for measuring leakage through the radial clearances consisted of a cylinder (Fig. 27) with three nozzles. Water under pressure was admitted through the middle nozzle and passed through the small annular clearance between the cylinder walls and two pistons, and was discharged through the nozzles on both ends of the cylinder. This arrangement did not give any axial thrust. The clearances tested were 0.010 in., 0.018 in., 0.025 in., and 0.050 in. The pistons were driven at 800, 1000, 1600, and 2000 r.p.m. Pressures varied from 0 to 185 ft. for the smallest clearance and from 0 to 36 ft. for the largest. The results were presented in the form of velocity versus pressure curves. From the study of these results the following conclusions can be drawn: (1) For the same clearance and the same pressure, velocities of flow decreased slightly with the speed of rotation of the piston. Expressing the amount of leakage by a formula:

$$Q = AC\sqrt{(2gH_L)} \qquad [15]$$

where A is the area of the clearance, C is a coefficient of discharge, H_L is the head across the clearance, and Q is the amount of leakage. Comparing the coefficients of discharge for the same

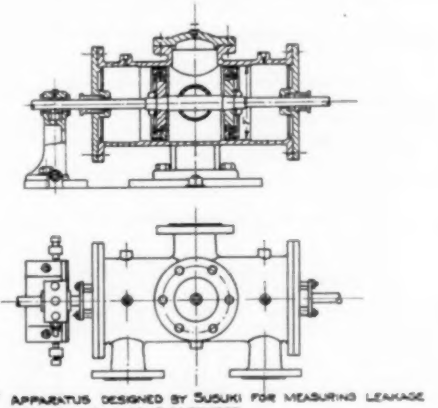

FIG. 27 APPARATUS DESIGNED BY SUSUKI FOR MEASURING LEAKAGE THROUGH ANNULAR CLEARANCES

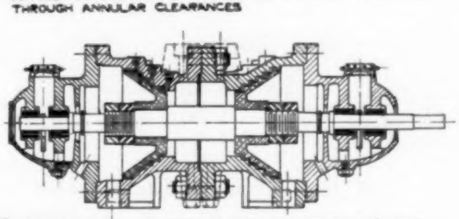

FIG. 28 APPARATUS WITH STEPPED CLEARANCE TESTED BY BECKER

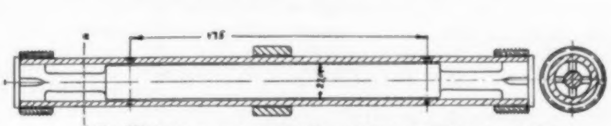

FIG. 29 APPARATUS USED BY BECKER FOR INVESTIGATION OF FLOW THROUGH THE ANNULAR CLEARANCE

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[17] Becker, "Strömungsvorgänge in ringsformigen Spalten," *Z.D.V.I.*, 1907, s. 1133.

[18] Dr. Becker used the expression $R = 2Sv\gamma/\eta$ for the Reynolds number, which corresponds to that for the pipe flow $R = Dv\gamma/\eta$, the mean hydraulic radius in the first case being $s/2$ and in the latter case $D/4$.

[19] Journal of the Faculty of Engineering, Tokio Imperial University, vol. 18, no. 2, 1929 (in English).

clearance and speed, it is seen from these results that C is increasing with the increase of pressure and C is increasing for larger clearances and the same pressure and speed.

The latest tests on the leakage through small annular clearances are those of Dr. Masanao Yendo published in June, 1930.[20] He experimented with an apparatus shown in Fig. 80. Two disks were tested, 10 and 20 mm. wide (0.394 and 0.788 in.), with the clearances 0.25 and 0.50 mm. (0.010 and 0.020 in.) on the radius. The maximum pressure used was 5.0 m. (16.4 ft.). The following tabulation gives the values of the coefficient of discharge obtained, using formula [15]:

Values of coefficient of discharge C in the formula

$$Q_L = AC\sqrt{(2gH_L)}$$

where Q_L = amount of leakage
A = area of the clearance
D = diameter of the disk
b = radial clearance, inches
H_L = difference of pressure on two sides of the disk, feet of water
U = disk peripheral velocity, feet.

	b = 0.010 in.					
	Width = 0.788 in.			Width = 0.394 in.		
H_L/U	62.0	41.3	20.65	62.0	41.3	20.65
1.64	0.196	0.239	0.36	0.313	0.365	0.455
3.28	0.271	0.324	0.417	0.405	0.465	0.531
6.56	0.358	0.420	0.527	0.492	0.551	0.593
9.84	0.413	0.474	0.570	0.543	0.594	0.632
13.12	0.451	0.512	0.597	0.576	0.617	0.642
16.4	0.485	0.543	0.615	0.601	0.639	0.660

	b = 0.020 in.					
	Width = 0.788 in.			Width = 0.394 in.		
H_L/U	62.0	41.3	20.65	62.0	41.3	20.65
1.64	0.293	0.372	0.463	0.473	0.520	0.594
3.28	0.383	0.440	0.515	0.544	0.594	0.638
6.56	0.465	0.506	0.558	0.614	0.653	0.679
9.84	0.508	0.540	0.580	0.648	0.680	0.698
13.12	0.534	0.565	0.595	0.672	0.698	0.713
16.4	...	...	...	...	...	0.720

7 CALCULATION OF LEAKAGE LOSS

Dr. Neumann in his book on centrifugal pumps[21] develops the following formula for calculation of the amount of leakage through the clearance between the runner and case wearing rings:

$$Q = fC\sqrt{(2g\eta H_n - w_2^2 - \varphi^2u_2^2)} \qquad \text{..........[16]}$$

where (Neumann's notation)

Q = amount of leakage per second
f = area of clearance space = $D\pi i$
D = diameter of wearing rings
i = radial clearance
H_n = actual head produced by pump
$1/\eta$ = hydraulic efficiency
w_2 = absolute velocity of water leaving the runner
u_2 = peripheral velocity of the runner at discharge
φu_2 = peripheral velocity of water in space between the pump case and runner shrouds
C = coefficient of discharge through the clearance between the wearing rings.

The expression under the radical is the difference of pressure on both sides of the clearance space. The term $\varphi^2u_2^2$ represents the decrease in pressure in the space between the pump case and the runner shrouds on account of rotation of water caused by skin friction of the runner shrouds. This formula includes three coefficients C, η, and φ which are not known and for which no method of estimating has been offered. It has been pointed out that the value of hydraulic efficiency $1/\eta$ obtained by calculation from actual tests is considerably lower than the pump overall efficiency. This is absurd.[22]

Dr. Pfleiderer[23] gives the following formula for calculating the amount of leakage:

$$V_{sp} = M_iD_i\pi b_i\sqrt{(2gH_{pi})} \text{ (Pfleiderer's notation)} \qquad \text{....[17]}$$

where $D_i\pi b_i$ represents the area of the clearance space and M_i a coefficient of discharge. For this Pfleiderer gives a formula:

$$M_i = \frac{1}{\sqrt{(0.04L_i/b_i - 1.5)}} \qquad \text{..............[18]}$$

where L_i is the width of the wearing ring or length of the water path and b_i is the radial clearance. The first term in the denominator takes into account the friction loss, while the second accounts for the velocity head, increased 1.5 times to provide for the contraction at the entrance. The same formula may be presented in the form:

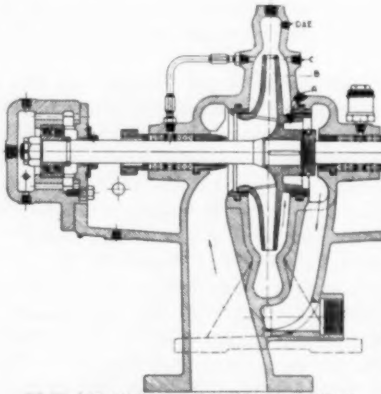

FIG. 30 3-INCH SINGLE SUCTION PUMP WITH ARRANGEMENT FOR MEASURING LEAKAGE THROUGH WEARING RINGS

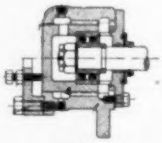

FIG. 30-a

$$H_{pi} = 1.5v^2/2g + \lambda L\frac{U}{4F}v^2/2g \qquad \text{...........[19]}$$

where U/F is the mean hydraulic radius equal to $D\pi b_i/2D = b_i/2$ and λ is a friction coefficient. For this Pfleiderer takes the mean value obtained by Becker[24] with stationary clearance walls equal to 0.08.

The pressure across the clearance H_{pi} is calculated from:

$$H_{pi} = H_p - \frac{(u_2/2)^2 - (u_1/2)^2}{2g} = H_p - \frac{1}{4}\frac{u_2^2 - u_1^2}{2g} \qquad \text{..[20]}$$

where H_p is the pressure in the pump case and u_2 and u_1 are the peripheral velocities of the runner at the outside diameter and the wearing ring, respectively.

The subtractive term in the last formula provides for reduction of pressure near the wearing ring due to rotation of water between the pump case and runner shrouds. The assumption is made that water rotates with half the angular velocity of the runner. This formula applies only to the simplest case of the plain wearing ring; even in this case the result is not accurate enough, because the coefficient of friction varies with pressure and clearance, and the value of the volute pressure has to be calculated

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[20] Reports of the Yokohama Technological College, no. 1, p. 22, June, 1930 (in English).

[21] F. Neumann, "Die Zentrifugalpumpen," Verlag von Julius Springer, s. 16.

[22] The calculation of the amount of leakage as given by Neumann is presented by L. Loewenstein and C. Crissey in the book "Centrifugal Pumps," p. 17, Van Nostrand.

[23] C. Pfleiderer, "Die Kreiselpumpen," Julius Springer, 1924, s. 163.

[24] See footnote 17.

LEAKAGE LOSS FOR DIFFERENT WEARING RINGS IN PER CENT OF NORMAL CAPACITY

3" SINGLE SUCTION PUMP WITH A BALANCING CHAMBER

RUNNER DIAMETER 10⅛". WEARING RING DIAMETER 4⅛" NOMINAL FIG. 31

	SPEED R.P.M.			
	1400	1700	2000	2500
CAPACITY AT MAXIMUM EFFICIENCY (NORMAL) G.P.M.	270	325	375	475
TOTAL HEAD AT MAXIMUM EFFICIENCY (NORMAL) FEET	56	88	117.5	184
HEAD ON WEARING RING H_L IN FEET	35	55.5	74	116
HEAD ON WEARING RING PER CENT OF TOTAL HEAD	62.5	63.0	63.0	63.0

$$Q_L = 2CA\sqrt{2gH_L}$$

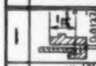
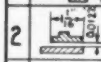
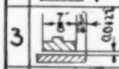
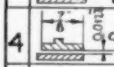
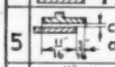
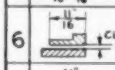
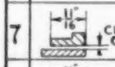
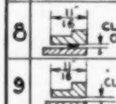
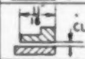
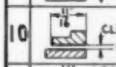
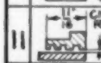
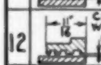
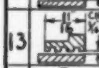
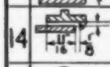
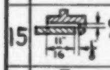
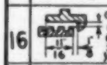
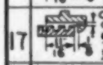
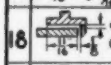
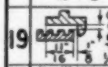
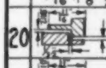
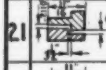
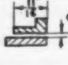
WHERE:

Q_L IS LEAKAGE ON BOTH SIDES OF RUNNER

C = COEFFICIENT OF DISCHARGE

A = CLEARANCE AREA

H_L = HEAD ON WEARING RING OVER SUCTION LEVEL

N°	WEARING RINGS	LEAKAGE LOSS PER CENT AT SPEEDS R.P.M.				COEFFICIENT OF DISCHARGE AT SPEEDS R.P.M.				REMARKS
		1400	1700	2000	2500	1400	1700	2000	2500	
1	 0.0122 CLEARANCE ON DIAMETER	1.52	1.80	2.00	2.18	0.176	0.200	0.220	0.244	LEAKAGE LOSS LESS FROM 41 TO 46.6% AGAINST WEARING RING #2
2	 0.0122 CLEARANCE ON DIAMETER	2.85	3.32	3.52	3.70	0.330	0.368	0.388	0.414	LEAKAGE LOSS GREATER FOR SMALLER OVERLAP. THE PROJECTING OF RUNNER WEARING RING AGAINST CASE WEARING RING AND VICE VERSA, HAS VERY LITTLE IF ANY, EFFECT ON COEFFICIENT OF DISCHARGE.
3	 0.0122 CLEARANCE ON DIA.	3.08	3.60	3.86	4.17	0.356	0.400	0.424	0.468	
4	 0.0122 CLEARANCE ON DIAMETER	3.16	3.62	3.87	4.07	0.366	0.402	0.426	0.456	
5	 CLEARANCE 0.0122 ON DIAMETER	3.68	4.17	4.38	4.50	0.426	0.462	0.482	0.504	
6	 CLEARANCE 0.0122 ON DIA	3.52	4.03	4.33	4.50	0.408	0.448	0.476	0.502	LEAKAGE INCREASES WITH LARGER CLEARANCE, DUE TO BOTH THE INCREASE OF CLEARANCE AREA AND INCREASE OF COEFFICIENT OF DISCHARGE
7	 CLEARANCE 0.017" ON DIAMETER	6.06	6.65	6.70	6.70	0.504	0.528	0.528	0.538	
8	 CLEARANCE .0205" ON DIAMETER	7.92	8.62	8.66	8.60	0.542	0.574	0.573	0.578	
9	 CLEARANCE 0.0294" ON DIAMETER	13.20	13.90	14.05	14.07	0.634	0.640	0.641	0.654	
10	 CLEARANCE 0.039" ON DIAMETER	18.72	19.65	19.85	20.00	0.678	0.682	0.684	0.700	
11	 CIRCULAR GROOVES ROUGH EDGES CLEARANCE .017" ON DIAMETER	3.91	4.28	4.31	4.48	0.325	0.340	0.340	0.360	LEAKAGE REDUCED FROM 32.2 TO 35.5 PER CENT AGAINST WEARING RING #7 MAXIMUM GAIN AT LOWER SPEED
12	 CIRCULAR GROOVES WITH SMOOTH EDGES CLEARANCE .017" ON DIAMETER	4.83	5.38	5.58	5.52	0.402	0.428	0.440	0.443	LEAKAGE REDUCED FOM 16.7 TO 20% AGAINST RING N° 7.
13	 CIRCULAR GROOVES 1/16 WIDE, 3/64 DEEP, 1/8 APART. CLEARANCE .0294 ON DIA.	12.70	13.55	13.75	13.60	0.610	0.622	0.628	0.632	LEAKAGE DECREASED FROM 2.2 TO 4 PER CENT AGAINST RING #9
14	 CLEARANCE .013" ON DIA.	4.42	4.88	5.04	5.05	0.481	0.508	0.521	0.521	
15	 CLEARANCE .015" ON DIA.	3.18	3.68	3.94	4.08	0.390	0.432	0.460	0.485	CALCULATED
16	 CLEARANCE .0115" ON DIAMETER 6 R.H. TH'DS P. INCH SINGLE THREAD 1/16 WIDE 3/64 DEEP	2.52	2.88	2.92	2.98	0.309	0.338	0.342	0.354	LEAKAGE REDUCED FROM 20.8 TO 27 PER CENT AGAINST RING N° 15 MAX. GAIN AT HIGHER SPEED
17	 CLEARANCE .0115" ON DIAMETER 6 R.H. TH'DS P. INCH TRIPLE THREAD 1/16 WIDE 3/64 DEEP	2.13	2.40	2.53	2.58	0.262	0.282	0.296	0.306	LEAKAGE REDUCED FROM 33 TO 36.8 PER CENT AGAINST RING #15 MAX. GAIN AT HIGHER SPEED
18	 CLEARANCE .0215 ON DIAMETER	8.53	9.04	9.15	9.19	0.560	0.568	0.572	0.584	CALCULATED
19	 CLEARANCE .0215 ON DIAMETER 6 R.H. TH'DS P. I. TRIPLE THREAD 1/16 WIDE 3/64 DEEP	6.24	6.68	6.89	6.82	0.410	0.420	0.430	0.432	LEAKAGE REDUCED FROM 24.7 TO 26.8 PER CENT AGAINST RING #18
20	 CLEARANCE .010" ON DIAMETER	1.94	2.18	2.27	2.40	0.274	0.295	0.305	0.328	LEAKAGE LESS THAN WITH RING N° 22 FROM 24 TO 33 PER CENT
21	 CLEARANCE 0.010" ON DIAMETER	2.07	2.34	2.45	2.52	0.292	0.316	0.328	0.344	
22	 CLEARANCE .010" ON DIAMETER	2.55	3.03	3.28	3.44	0.360	0.410	0.440	0.470	CALCULATED

from the theoretical head, which is subject to a considerable error. Dr. Pfleiderer himself in his illustrative design (p. 98) did not use these formulas. He roughly estimated the leakage loss as 5 per cent of the pump capacity.

8 Description of the Apparatus Used for Measuring Leakage

The author's original intention was to measure the leakage through clearances between the case and the runner wearing rings by using a double-suction pump with the runner discharge sealed by a band and driving the shaft at various speeds while applying water under pressure from an outside source to the pump case through the discharge nozzle, and to measure the leakage through the clearance between the rings on the suction nozzle. So far as leakage is concerned, this method might have been quite satisfactory, but it would have left the question as to the pressure distribution in the pump case in doubt, as this might be affected by the discharge from the runner. Again, in every pump under actual operating conditions, there are observed very small vibrations of the shaft which may affect the flow through the small clearances, while if running with no discharge, these vibrations may be absent or considerably reduced. Therefore this idea was discarded, and a new scheme was devised in which the leakage was measured under actual operating conditions, pressures being produced by the same pump. However, to compare the two methods a sealing band for the runner was provided, and the leakage was measured with pressure from an outside source.

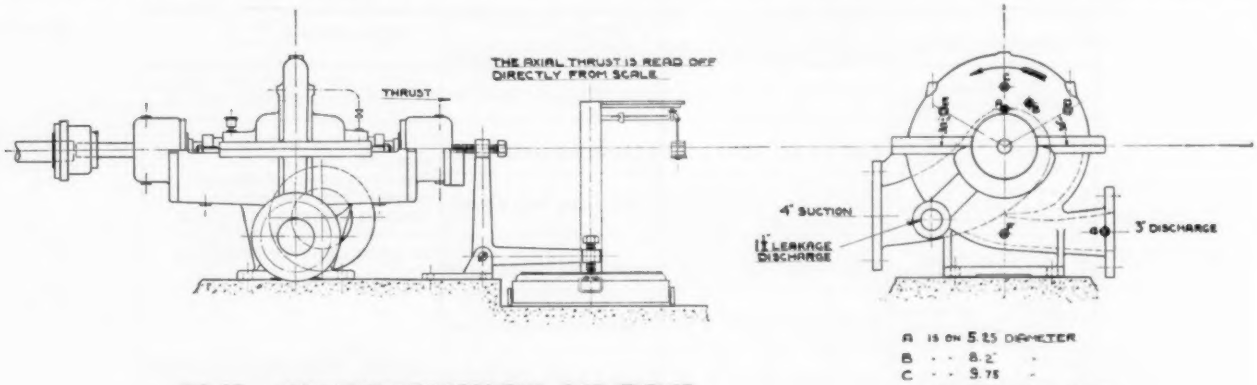


FIG. 32 ARRANGEMENT FOR MEASURING AXIAL THRUST

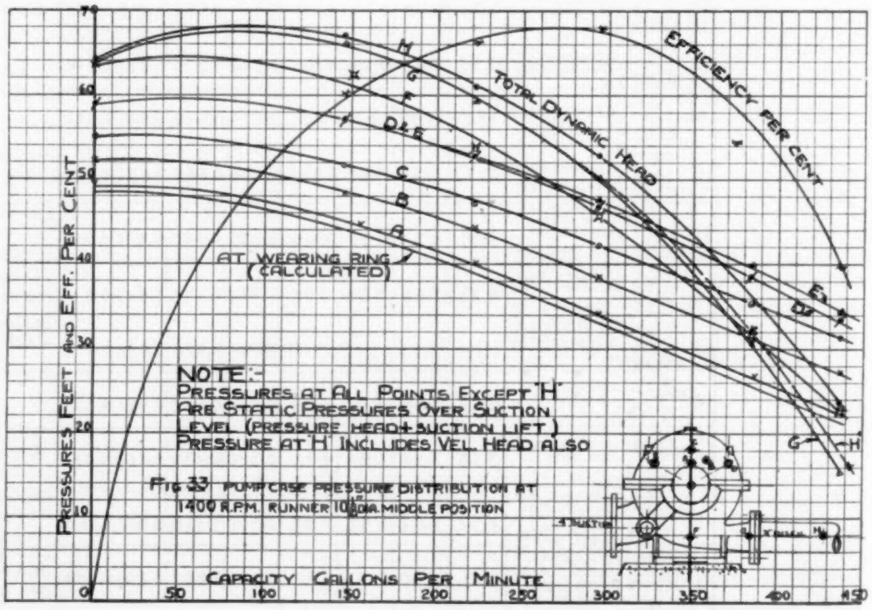


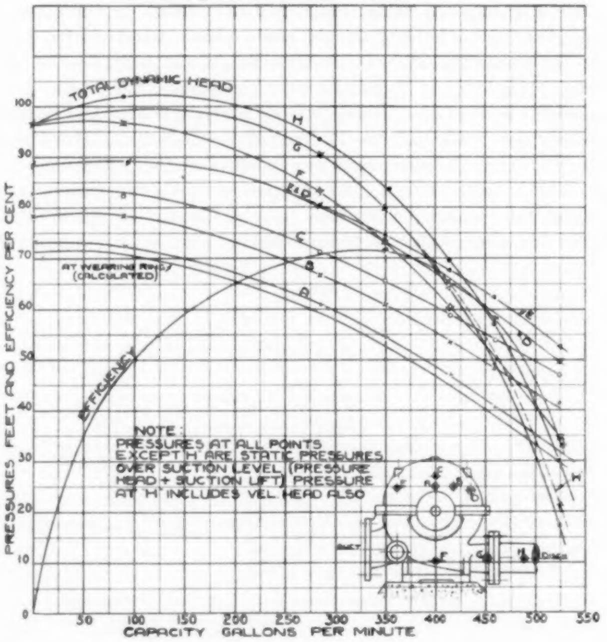
FIG 33 PUMP CASE PRESSURE DISTRIBUTION AT 1400 R.P.M. RUNNER 10 3/16" DIA. MIDDLE POSITION

To measure leakage through the wearing-ring clearances, a 3-in. single-suction pump was used, as shown in Fig. 30. To separate the leakage water from the main flow, a chamber was provided on the back side of the runner. This was separated from the pump case under discharge pressure by a close clearance. The diameter of the runner wearing ring on this side was the same as that on the suction side ($4^1/_8$ in. nominal). Leakage through the clearance between the wearing rings on the runner back was discharged through a $1^1/_2$-in. nozzle and weighed. It should be noted that the pressure difference across the wearing rings on the suction side of the runner is greater than that on the back of the runner because the pressure in the runner eye is smaller than that of atmosphere by the amount of the suction lift and velocity head. However, the results were plotted against pressure. For any suction lift and capacity the true leakage could be obtained from these curves, as explained later.

The different wearing rings tested, with the percentage of leakage, are tabulated in Fig. 31. To compare the effects of a labyrinth and the width of the wearing ring on leakage through the same clearance, different sizes were obtained by cutting the same ring by $^3/_{16}$-in. steps. To find the leakage through different clearances, five rings were made of the same width and with clearances from 0.010 to 0.040 in. nominal (the actual dimensions are given in Fig. 31, wearing rings Nos. 6 to 10). To study the effect of circular grooves on the coefficient of discharge, the grooves were cut on the rings previously tested, in order to assure the same radial clearance. The effect that the spiral grooves on the runner wearing rings had on the resistance to the flow was tested by using two rings, one with single thread and another with triple thread (higher pitch). Both were tested with two radial clearances. The direction of the spiral grooves was such that, when revolving, they produced a pumping effect from the leakage chamber into the pump case.

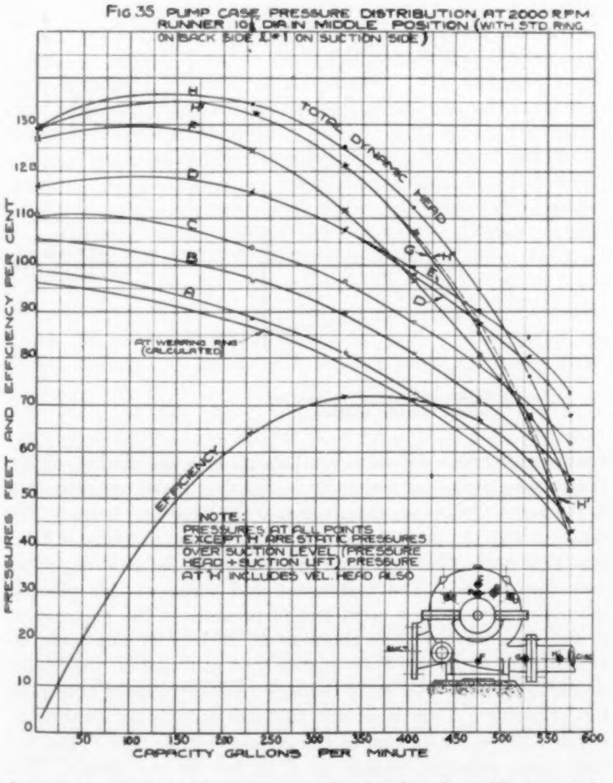
To study the pressure distribution in the pump case, eight taps, *A* to *H* inclusive, were made in the pump-case walls, as shown in Figs. 30 and 32. Tap *A* was drilled as close as possible to the runner wearing ring. Taps *B* and *C* were drilled on 8.2 in. and 9.75 in. diameters. Taps *D, E,* and *F* were drilled on the volute case 120 deg. apart and opposite the widest part in the volute section, which is approximately on the same diameter as the center of gravity of the section. Tap *F* was drilled near

FIG. 34 PUMP CASE PRESSURE DISTRIBUTION AT 1700 R.P.M. RUNNER 10⅛ DIA IN MIDDLE POSITION



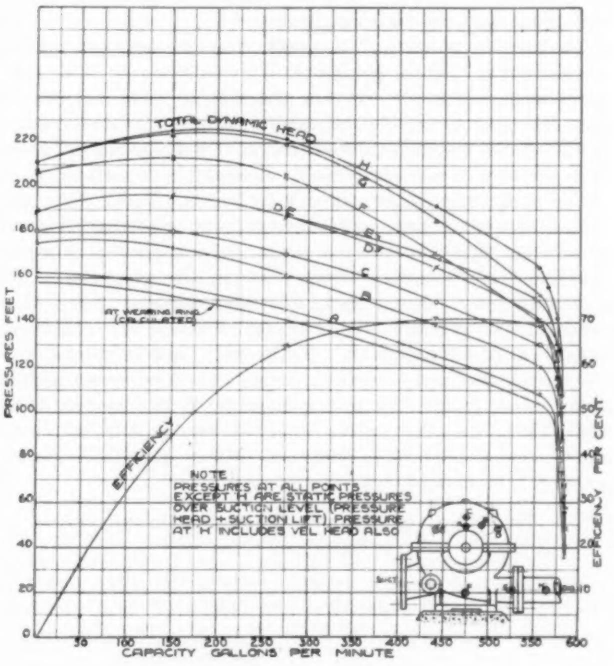
NOTE: PRESSURES AT ALL POINTS EXCEPT 'H' ARE STATIC PRESSURES OVER SUCTION LEVEL (PRESSURE HEAD + SUCTION LIFT). PRESSURE AT 'H' INCLUDES VEL HEAD ALSO

FIG. 35 PUMP CASE PRESSURE DISTRIBUTION AT 2000 R.P.M. RUNNER 10⅛ DIA IN MIDDLE POSITION (WITH STD RING ON BACK SIDE & #1 ON SUCTION SIDE)



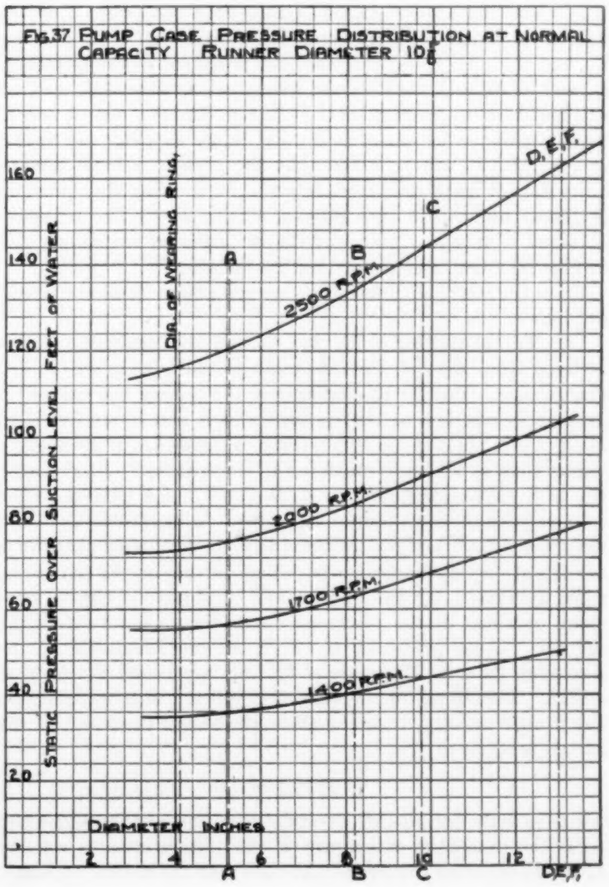
NOTE: PRESSURES AT ALL POINTS EXCEPT 'H' ARE STATIC PRESSURES OVER SUCTION LEVEL (PRESSURE HEAD + SUCTION LIFT) PRESSURE AT 'H' INCLUDES VEL HEAD ALSO

FIG. 36 PUMP CASE PRESSURE DISTRIBUTION AT 2500 R.P.M. RUNNER 10⅛ DIA IN MIDDLE POSITION



NOTE: PRESSURES AT ALL POINTS EXCEPT 'H' ARE STATIC PRESSURES OVER SUCTION LEVEL (PRESSURE HEAD + SUCTION LIFT) PRESSURE AT 'H' INCLUDES VEL HEAD ALSO

FIG. 37 PUMP CASE PRESSURE DISTRIBUTION AT NORMAL CAPACITY RUNNER DIAMETER 10⅛



the beginning of the nozzle, or volute throat. Tap *G* was located at the discharge flange. Tap *H* was drilled on the discharge pipe 18 in. from the discharge flange.

The primary purpose of the study of the pressure distribution was to obtain, from the pressure in the volute case or total pump head, some means of estimating the pressure near the wearing rings and to check the theoretical formulas given for this purpose. However, the results were rather unexpected and proved to be of importance in other respects, as discussed in Section 9.

The pressures were measured with mercury manometers and calibrated pressure gages. All eight taps for measuring pressures at different points in the pump case were connected into one manifold with necessary cocks allowing the pressures at all points to be measured with the same pressure gage. The capacity was measured with a 90-deg. V-notch weir calibrated against a measuring tank. A General Electric-Sprague dynamometer was used for driving the pump and for measuring the pump power input. All tests were made at four speeds, 1400, 1700, 2000, and 2500 r.p.m., with the runner $10^1/_8$ in in diameter, and at 1700, 2000, 2500, and 3000 r.p.m., with the runner cut to $8^1/_2$ in. in diameter.

9 Discussion of Results—Pressure Distribution in the Pump Case

The results of pressure measurements in the pump case with the $10^1/_8$ in. runner diameter and at 1400, 1700, 2000, and 2500 r.p.m. speeds are shown in Figs. 33 to 36. The pressures at points *A* to *G*, inclusive, are static pressures above the pump-suction level or pressures in the case plus the suction lift. The curves for *H* include the velocity head and are the total or kinetic head of the pump. The nearest point to the wearing ring where the pressure could be measured was point *A* on the 5.25 in. diameter, the diameter of the wearing ring being 4.125 in. To find the pressure at the ring, the pressures at the points *A*, *B*, and *C* were plotted on the diameters as abscissas, and by extrapolation the pressure on the 4.125 in. diameter was obtained. Such curves for pressures at normal pump capacity are shown in Fig. 37. It will be noted that the pressure curves become quite flat for small diameters below 5 in., showing that pressure is almost uniform in that portion of the case. The pressures obtained in this way for the 4.125 in. diameter are shown also in Figs. 33 to 36, and are marked "calculated."

From the study of these curves several points should be noted, as follows:

(*a*) For capacities below the normal the pressures are increasing from *A* to *H*, the maximum pressure being on the discharge pipe. But for capacities over the normal the curves rearrange themselves, so that pressures at *G* and *H* are less than at any other point in the volute case. Pressures at *H* are slightly higher than at *G* (static pressures at *H* are shown by the dotted line H^1). Pressure curves for *A*, *B*, *C*, *D*, and *E* have the same relative position as at partial capacities. This result was rather unexpected, as it is a general belief that the maximum pressure is always at the discharge flange. This picture shows that pressures generated by the runner are considerably higher than those obtained at the discharge nozzle for capacities over the normal. These pressures, however, are not available at the discharge flange. The reasons for this are found in the case design. Assuming, for the sake of simplicity, that water enters the runner radially, then the theoretical head produced by the runner is equal to

$$H_t = u_2c_{2u}/g \qquad [21]$$

where u_2 is the peripheral velocity of the runner at the outlet and c_{2u} is the tangential component of the absolute water velocity at discharge from the runner, as shown in the velocity triangle at the discharge (Fig. 38). β_2 is the vane angle, w_2 the relative velocity of water, and α_2 is the angle between absolute and peripheral velocities. The volute case is designed so that the volute angle conforms to the absolute velocity angle α_2 at the normal capacity. At capacities over the normal the angle α_2 increases with the capacity, and the velocity triangle takes the form *ABC'*, the angle α_2 being changed to α_2'. While at the normal capacity the discharge from the runner immediately in front of the lip is approximately in the direction of the main flow in the volute (Fig. 39), at capacities over the normal the angle α_2 is increased so that the discharge from that portion of the runner is directed across the flow in the volute (Fig. 40). The effect of this is that (1) the nozzle is cut off from the pump case by the high-velocity stream and the pressure from the pump case cannot be transmitted through it, and (2) a dead space filled with turbulent water is formed behind the lip in the pump nozzle extending a considerable distance into the discharge pipe. This makes the actual velocities higher than those corresponding to the areas, and consequently makes the pressures lower. This explains why the pressure at the pump discharge flange is lower than that in the discharge pipe 18 in. from the flange at capacities over the normal. The turbulence and shock at this place in the pump case incur a great loss of pressure. The pressures at *G* and *H* are lower than those at *E*, *D*, and *F*, but since the velocities at *E*, *D*, and *F* are higher than at *G* and *H*, the reverse should be the case, as some pressure recovery should take place in the nozzle. Thus it is seen that not only is no pressure recovered in the nozzle against the pressures *E*, *D*, and *F*, but also that some pressure present at *F* is lost in the nozzle. This makes the shape of the head-capacity curve drop rapidly after the maximum efficiency point.

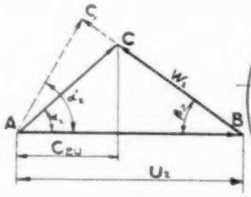


Fig. 38 Increase of Discharge Angle with Capacity.

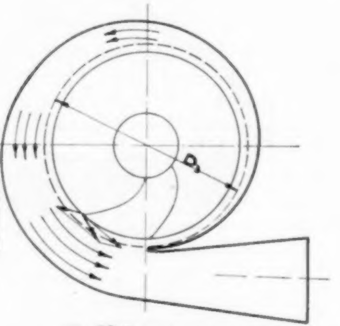


Fig. 39 Flow at Nozzle Throat at Normal Capacity.

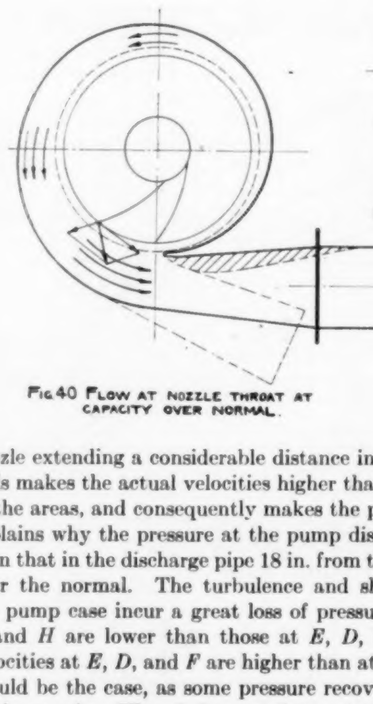


Fig. 40 Flow at Nozzle Throat at Capacity over Normal.

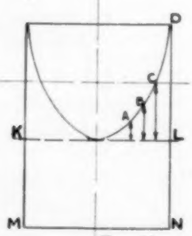


Fig. 41 Decrease in Pressure Due to Rotation.

It seems that conditions can be improved if the nozzle is turned through an angle as shown by the dotted line of Fig. 40. Higher heads can be obtained at the discharge flange, and efficiencies can be improved for capacities over the normal. Another way to make the head generated by the runner available at the discharge flange is to bring the main flow in the volute further out from the runner—i.e., to put the volute sections on a larger base circle. In this connection it should be noted that with a circular case the head capacity curve is considerably flatter than with a volute case.[25] The efficiency curve is also flatter.

(*b*) Refer to the pressures at D, E, and F, which are 120 deg. apart. F is near the nozzle throat (Fig. 32). The taps at these points are approximately opposite the center of gravity of the volute sections. It will be noticed that pressures at D and E are the same for capacities smaller than the normal, and that the pressure at E is greater than that at D for capacities larger than the normal. This difference may be explained by the fact that E is on a larger diameter than D, which makes the average peripheral velocity of water particles and the head produced higher for E than for D. At partial capacities not all the water in the volute case is discharged through the nozzle; part of it is circulated in the pump case, which equalizes the difference in pressure at different points in the volute. The pressure at F is higher than at D and E at partial capacities and lower at capacities over the normal. The pressure at F is lower than that at G and H at partial capacities and higher at capacities considerably greater than the normal. This means that at partial capacities there is a pressure recovery in the nozzle, but at capacities over the normal no pressure recovery takes place in the nozzle. There is, rather, a loss of pressure available at F.

(*c*) The important point to note, now, is that the pressure lines D, E, and F intersect practically at one point and that this point is the point of maximum efficiency. This indicates that at this point there is a constant pressure in the volute case and that the runner is discharging against the same pressure at all points. Hence, there is no unbalanced reaction from the volute case (side pressure) on the runner. The aim of a good design is to proportion the volute sections so as to get a uniform velocity and a constant pressure in the volute case. However, for capacities under and over the normal this balance is destroyed; the pressure varies along the volute case. The velocities vary, also, and there is a side thrust from the volute case on the runner. If the shaft is not strong enough, the deflection may become more than the running clearance between the wearing rings, and the runner will rub against the case wearing rings. Again, the difference in the volute pressure affects the runner performance so that the runner discharge varies as it passes the points of different pressure in the volute, causing pulsating flow and a lowering of efficiency.

(*d*) By inspecting the pressure curves for points A, B, C, D, and E, it will be noticed that they differ from each other by an amount approximately constant for the whole head-capacity range, so that the pressures at A, B, and C can be expressed in terms of the pressure in the pump case by a formula

$$H_a = H_v - \varphi^2 \frac{u_2^2 - u_a^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots [22]$$

where H_v is the volute pressure, H_a is the pressure at A, u_2 is the peripheral velocity of the runner at discharge, u_a is the peripheral velocity corresponding to a diameter at A, and φ is a coefficient to account for the difference in angular velocity of the runner and of the water in the space between the runner and the pump case. This equation is obtained in the following manner: If water in an open cylindrical vessel is set in rotation with an angular velocity constant for all points, we have a forced vortex. The water surface will take the form of a paraboloid of revolution (Fig. 41). Water at point D will be higher than at point O by a distance $u_d^2/2g$, where u_d is the peripheral velocity of water at D, equal to ωr_d, ω being the angular velocity of rotation and r_d the radius at D. Similarly, point C on the surface is $u_c^2/2g$ over point O. If water is rotated with the same angular velocity in a closed cylindrical vessel (*KLMN*, Fig. 41), no paraboloid of revolution can be formed, but pressures will be produced equal to $u_d^2/2g$, $u_c^2/2g$, etc., at the points with radii r_d, r_c, etc. The pressure difference between points D and C will be $(u_d^2 - u_c^2)/2g$. In a pump, assuming that the pressure at the runner discharge is the same as in the volute H_v, the pressure difference between the point D in the volute (or at the runner periphery) and the point C will be $(u_2^2 - u_c^2)/2g$. Since the pressure at the runner discharge is H_v, that at C is equal to $H_v - (u_2^2 - u_c^2)/2g = H_c$. Since the angular velocity of water in the space between the runner and the case walls is lower than that of the runner, a coefficient φ is introduced, so that if ω is the runner angular velocity, that of water outside the runner will be $\varphi\omega$, and the formula for the pressure at the point A becomes

$$H_a = H_v - \varphi^2(u_2^2 - u_a^2)/2g$$

Calculating φ for the point A at 1400, 1700, 2000, and 2500 r.p.m., the following values, respectively, were obtained:

$$\varphi_a = 0.548,\ 0.559,\ 0.552, \text{ and } 0.557$$

The same for the point B are:

$$\varphi_b = 0.648,\ 0.681,\ 0.665, \text{ and } 0.667$$

Those for the point C are:

$$\varphi_c = 1.09,\ 1.205,\ 1.18, \text{ and } 1.17$$

It is seen that the value of φ is practically the same for each point at different speeds, but varies considerably from point to point, and for the point C it is greater than unity, which is impossible. The cause of this lies in the inaccuracy of the formula, which presupposes that the pressure in the volute case is the same as that at the runner discharge (substituting for u_a the value of u_2, the pressure H_a becomes equal to H_v). The latter is really lower, because the velocity in the volute case is considerably lower than the absolute velocity of discharge from the runner, and a recovery of pressure takes place after water leaves the runner. Again, the head produced by a runner depends to a certain extent on the dimensions of the volute case (mean radius). It is higher if the same volute sections are put on a larger base circle, since a forced vortex is formed of a larger diameter and the head produced is higher.

The values of φ can be brought to a closer agreement if instead of using u_2^2, which corresponds to the runner diameter, a value is substituted which corresponds to the diameter of a circle tangent to the volute-case lip (D_3, Fig. 39). For the pump tested, this diameter is equal to $11^3/_4$ in. On substituting for u_2 the value of the peripheral velocity u_3 (corresponding to D_3), the values of φ vary from 0.456 to 0.488 for all speeds and all points, the average value being 0.467.

The pressure distribution in the pump case with the runner cut to $8^1/_2$ in. is of the same character as that for full diameter, as is seen from Fig. 42, which is for 2500 r.p.m. Similar results were obtained for 1700, 2000, and 3000 r.p.m. The values of φ obtained from these tests with $8^1/_2$ in. diameter runner are as follows: For point A, 0.418; for B, 0.394; for C, 0.372. These values of φ are constant for all speeds.

Figs. 43, 44, and 45 show the pressure distribution in the pump case for a $10^1/_8$-in. runner with six radial ribs on the runner back $^3/_{16}$ in. high and 8 in. from tip to tip. The general arrangement of the pressure curves with respect to each other is similar to those discussed. The pressure drop between the points *A*, *B*, *C*, and *D* is greater than in the previous tests. This is to be expected, as the angular velocity of rotation of water between the runner and the case walls is greater. The average value of the coefficient φ for the point *A* is 0.632, for *B* it is 0.568, and for *C* it is 0.512. These values of φ are constant for all speeds. The variation of the coefficient φ shows that water is revolving with different angular velocities at different radii, which is probably due to the fact that the radial ribs do not extend to the runner outside diameter.

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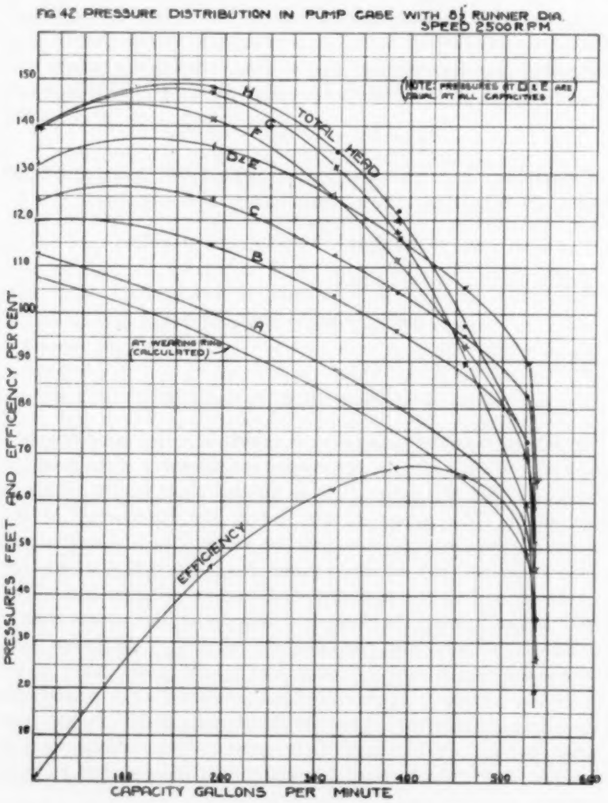
[25] A. F. Scherzer, Proc. A.S.C.E., October, 1927.

10 PRESSURE AT THE WEARING RING

The purpose of the study of the pressure distribution in these tests is to find the means of calculating or estimating the pressure near the wearing ring from the pump total head or volute pressure, knowing that it will be possible to calculate the leakage through the clearance between the wearing rings if the coefficients of discharge for various types of wearing rings are available. It is hardly possible to find a relation between the pressure in the volute case, or pump total head, and the pressure at the wearing ring for the whole head-capacity curve, as these vary for different capacities in different ways. Fortunately for the normal capacity, the pressures at different points in the volute become equal. This point is of most interest to the designer as it is the point of maximum efficiency.

FIG 42 PRESSURE DISTRIBUTION IN PUMP CASE WITH 8½" RUNNER DIA.
SPEED 2500 R P M



(*a*) *Pressure at the Wearing Ring in Terms of Total Head.* From these experiments it was found that the pressure at the wearing rings at normal capacity bears a certain relation to the total head which is constant for all speeds and is not affected by the runner cuts. This ratio for the pump tested is 0.63. Knowledge of such a factor, which will be denoted by K_r, provides the simplest way of determining the pressure at the rings. It probably will vary slightly with the diameter of the volute base circle, and to a smaller degree with the diameter of the wearing rings.

(*b*) *Pressure at the Wearing Ring in Terms of Volute Pressure.* The volute pressure can be expressed in terms of the theoretical head and the hydraulic efficiency:

$$H_v = H_t e_h - \frac{(c_2^2 - c_1^2)}{2g} \qquad [23]$$

where H_t is the theoretical head, e_h is the hydraulic efficiency, and c_2 and c_1 are, respectively, the absolute velocity of discharge from and entrance to the runner. The disadvantage of this method is that it involves the hydraulic efficiency, which is not known. The only information available about it is that it should be less than unity and greater than the overall pump efficiency. Also, the true value of c_2 is not known. The value of c_2 as given by the theoretical triangle of velocities at discharge is considerably higher than the actual value, as has been shown by many investigators.

It seems that a more convenient and more accurate way to determine the volute pressure is from the actual pump total head which is known, by assuming the nozzle efficiency. This will enable us to find the pressure at the nozzle throat, which is equal to the volute pressure at normal capacity. The pressure at the nozzle throat is calculated from

$$H_d - H_v = K_n \times (V_n^2 - V_d^2)/2g \qquad [24]$$

where H_d is the pressure at the discharge flange, H_v is the pressure at the nozzle throat and volute, V_d is velocity at the discharge flange, V_v is the velocity at the nozzle throat, and K_n is the nozzle efficiency. The average value of the nozzle efficiency calculated from these tests is 0.60 for a nozzle length of $8^1/_2$ in. and a ratio of areas of 2.15. The advantage of this method is that it contains only one constant, which can be easily found, or very closely estimated, for any pump.

Using this expression for the volute pressure, the formula for the pressure at the wearing rings becomes:

$$H_L = H_d - K_n(V_v^2 - V_d^2)/2g - \varphi^2(u_2^2 - u_r^2)/2g \qquad [25]$$

where H_L is the pressure at the wearing ring and u_r is the peripheral velocity of the runner wearing ring.

For calculation of the leakage through the clearance between the wearing rings into the runner suction, the pressure H_L should be corrected for the velocity head in the runner eye, since the pressure in the runner eye is lower than the suction lift by the velocity head.

(*c*) *H_L as Function of Peripheral Velocities u_2 and u_1.* For calculation of the pressure at the wearing ring for normal capacity a simple approximate relation can be deduced which gives results with sufficient accuracy for all practical purposes. The fundamental formula for the head developed by the runner:

$$H = \frac{u_2 c_{u2} - u_1 c_{u1}}{g} \qquad [26]$$

can be transformed by using the trigonometric relations between the component velocities from the discharge and entrance triangles into

$$H_t = \frac{u_2^2 - u_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} + \frac{c_2^2 - c_1^2}{2g} \qquad [27]$$

where the factors designated by *u* are the peripheral velocities, *w* the relative velocities and *c* the absolute velocities; the subscript ($_1$) refers to the entrance and subscript ($_2$) to the discharge.

In this formula the first two terms represent the pressure head

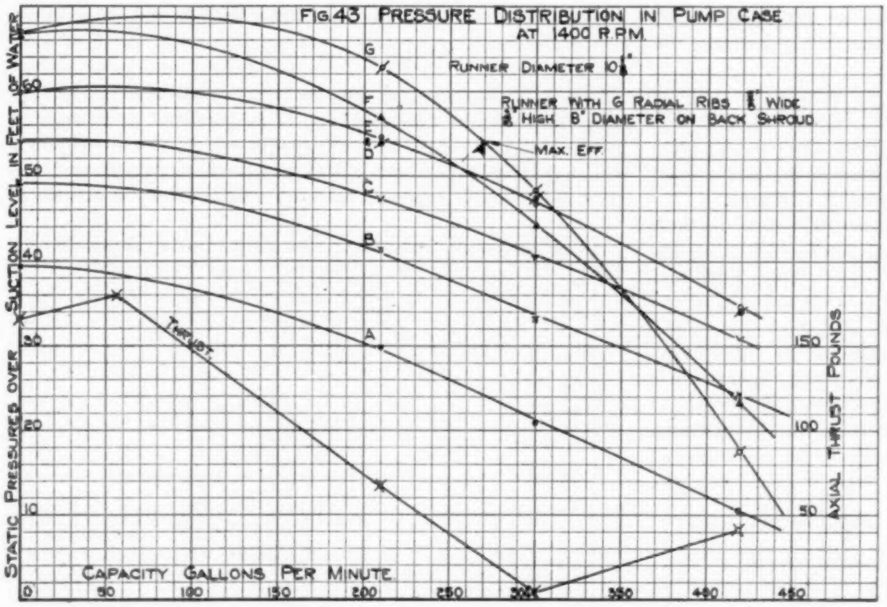
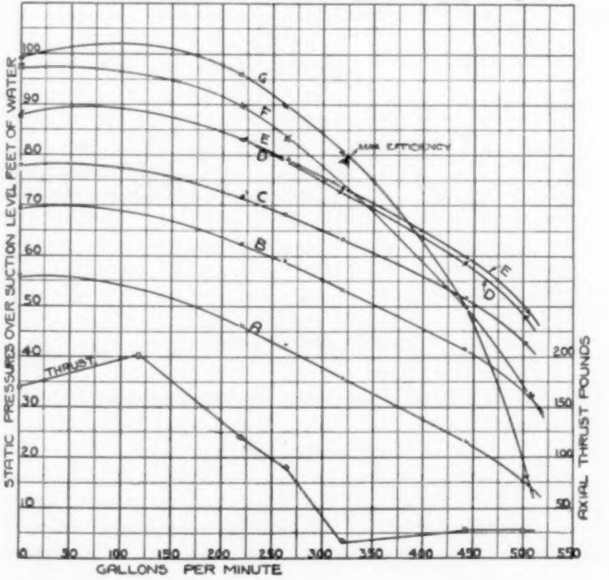


FIG. 43 PRESSURE DISTRIBUTION IN PUMP CASE AT 1400 R.P.M.

FIG. 44 PRESSURE DISTRIBUTION IN PUMP CASE AND AXIAL THRUST AT 1700 R.P.M. RUNNER WITH 6 RADIAL RIBS ⅝" WIDE 3/16 HIGH 8" DIAMETER ON THE BACK SHROUD. RUNNER DIA 10⅛"



and the last term the velocity head. Since the pressure at the wearing ring depends upon the pressure at the runner discharge, the first two terms need be considered only. Due to rotation of the water particles in space between the runner and case walls this pressure is reduced by an amount

$$\frac{\varphi^2(u_2^2 - u_r^2)}{2g} \ldots\ldots\ldots\ldots\ldots\ldots [28]$$

where u_r is the peripheral velocity of the runner wearing ring.[26] Then for the pressure at the ring we get:

$$H_L = \frac{u_2^2 - u_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} - \frac{\varphi^2(u_2^2 - u_r^2)}{2g}$$

This expression can be simplified, as in a modern pump of good efficiency w_1 is equal or slightly greater than w_2, and the term $(w_1^2 - w_2^2)/2g$ is always positive and small enough to be omitted. The dropping of this term will partly account for the hydraulic losses in the runner passages. An assumed value of 0.50 for φ is very close to the value observed in these tests. Another approximation will be made by taking u_r, the peripheral velocity of the runner wearing ring, equal to u_1, the peripheral velocity of the runner eye. The formula takes a very simple form:

$$H_L = \frac{u_2^2 - u_1^2}{2g} - \frac{1}{4}\,\frac{(u_2^2 - u_1^2)}{2g}$$

$$= \frac{3}{4}\,\frac{(u_2^2 - u_1^2)}{2g} \ldots [29]$$

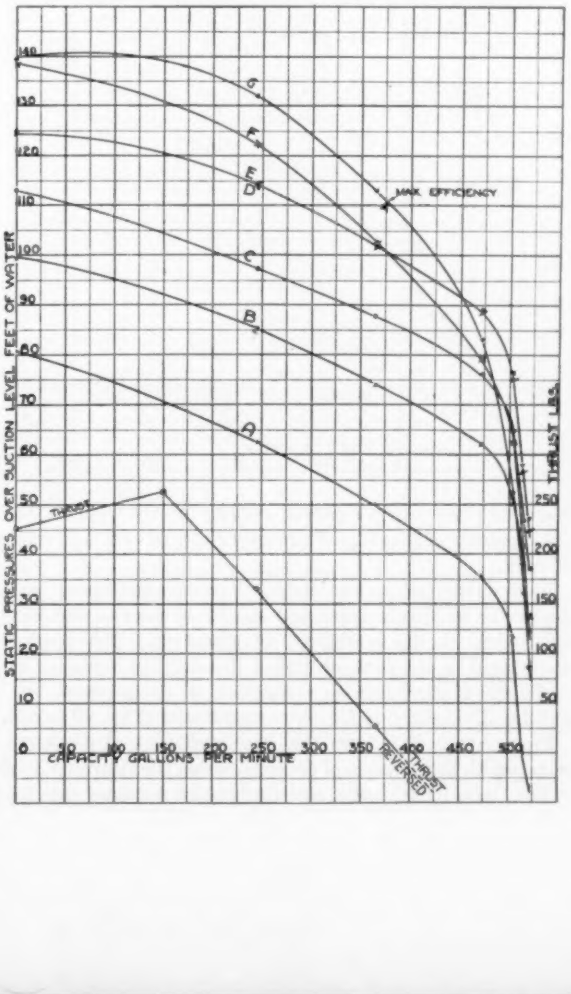
The values of H_L as calculated by this formula and actually measured are given in the following table:

Runner diameter	$10\frac{1}{8}$ in.				$8\frac{1}{2}$ in.	
Speed, r.p.m.	1400	1700	2000	2500	2000	2500
H_L, calculated feet	36.8	54.7	75.8	119.0	49.0	76.2
H_L, measured feet	36.0	55.0	76.0	120.0	49.0	73.0

11 Leakage Loss

For the sake of comparison of the relative advantages of

FIG. 45 PRESSURE DISTRIBUTION IN PUMP CASE AT 2000 RPM RUNNER 10⅛ D WITH 6 RADIAL RIBS ⅝" WIDE 3/16" HIGH AND 8" DIA ON BACK SHROUD



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[26] Because the substitution of u_3 for u_2 has only small effect on the value of φ at small diameters, u_2^2 is used here for convenience.

different rings and the use of results of the test for calculation of the leakage for any wearing ring diameter and clearances, the following formula has been adopted:

$$Q_L = CD_r s\pi \sqrt{(2gH_L)}$$

where Q_L is the amount of leakage per second, D_r is the diameter of the wearing ring, s is the radial clearance, H_L is the head across the clearance, and C is the coefficient of discharge.

From a measured leakage and pressure difference on both sides of the wearing rings, for each ring the coefficient of discharge C was calculated for four speeds, 1400, 1700, 2000, and 2500 r.p.m., which correspond to peripheral velocities u_r of the runner wearing ring equal to 25.2, 30.6, 36.0, and 45.0 ft. per sec., respectively. With a very small value of the radial clearance, in comparison to the wearing ring diameter, the curvature of the ring should have very little, if any, effect on the coefficient of discharge. Thus the results of these tests can be extended for any ring diameter or speed, the choice of the coefficient of discharge depending only upon the type of the ring, the clearance, and the peripheral velocity of the runner ring u_r. The diameter of the ring affects only the area and the peripheral velocity.

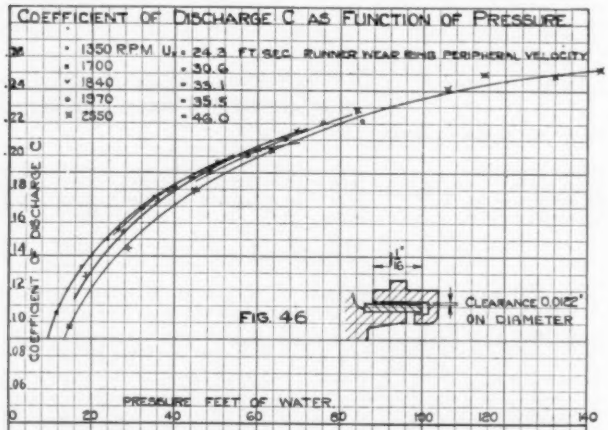

FIG. 46

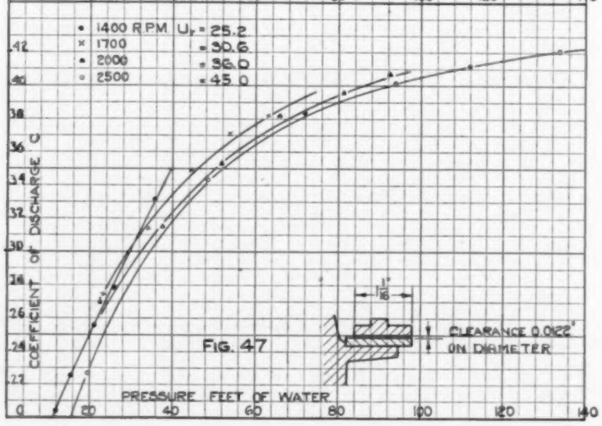

FIG. 47

In the rings where two radial clearances are found (such as stepped or labyrinth rings), the smaller of the two was used for calculation of the coefficient of discharge. The results are represented in the form of curves showing the variation of the coefficient of discharge as a function of the pressure for four peripheral velocities u_r for every ring tested (Figs. 46 to 65).

From curves it is seen that the coefficient of discharge increases with the increase of pressure across the clearance, rapidly at first, then more slowly, and almost at a constant rate for the higher pressures. The reason for this is the fact that the formula:

$$Q_L = CA \sqrt{(2gH_L)}$$

where $A = \pi D_r s$, assumes that the resistance to the flow, or drop of pressure across the clearance, is proportional to the square of the velocity; while, as in the case of the pipe flow, the power is less than 2, approximating 2 as a limit for very rough surfaces. The difference in exponent is compensated for by a change in the coefficient C, as is seen from the following modification of the formula:

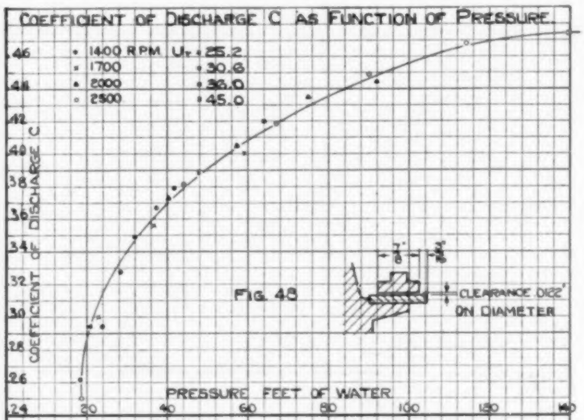

FIG. 48

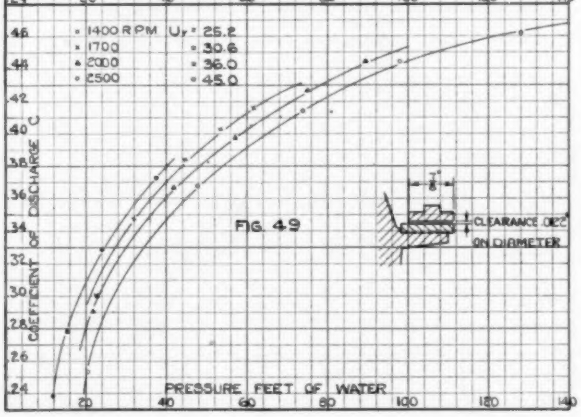

FIG. 49

$$Q_L = CA\sqrt{(2gH_L)}$$

or, dividing by A:

$$V = C \sqrt{(2gH_L)}$$

where V is the mean axial component of the velocity through the clearance space.

Then:

$$V^2/2g = C^2 H_L$$

or

$$H_L = 1/C^2 \; V^2/2g$$

The true fractional exponent of the velocity is not convenient for practical use, since it depends on surface conditions. It is customary to use the second power and a variable coefficient. Thus, it is seen that, as the pump pressure increases, the leakage increases faster than the square root of the pressure across the clearance.

The increase of the coefficient of discharge with pressure may seem contradictory to the general results obtained with sharp-edged orifices, where the coefficient of discharge always decreases with an increase in pressure.[27] The discrepancy is, however, apparent only, and can be explained by the difference in conditions in both cases. In the case of a sharp-edged orifice the coefficient of discharge is composed of the two parts $C = C_cC_v$. One, C_c, is a coefficient of contraction (the contraction occurs on all sides in this case), and C_v is a velocity coefficient which accounts for the resistance to the flow presented by the orifice. The coefficient of contraction decreases with an increase in pressure. The velocity coefficient increases with an increase in pressure from 0.96 to 0.994 according to Weisbach, but not enough to counterbalance the decrease in the coefficient of contraction; hence the coefficient of discharge decreases with an increase in pressure. The increase of the coefficient of velocity in the case of a sharp-edged orifice is due to the fact that the formula assumes that the pressure drop is proportional to the second power of the velocity, whereas the true exponent is slightly less than 2. Thus the decrease of the coefficient of discharge is due to a predominance of the coefficient of the contraction over the velocity coefficient. In the case of the flow through clearances between the wearing rings, the reverse is true. The coefficient of contraction is of less importance, since the contraction is incomplete. Due to projection of the runner wearing ring beyond the case wearing ring at entrance, the contraction is on one side only. At the same time the resistance to the flow is considerably greater than in the case of a sharp-edged orifice; hence the coefficient of discharge for the flow through the clearance between the wearing rings increases with higher pressures.

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[27] A. H. Gibson, "Hydraulics," 1925.

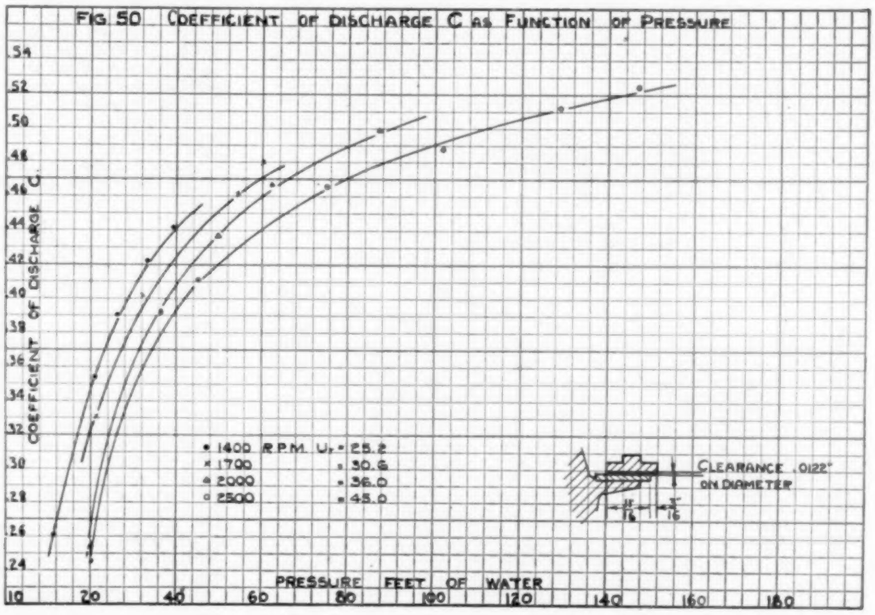


FIG. 50 COEFFICIENT OF DISCHARGE C AS FUNCTION OF PRESSURE

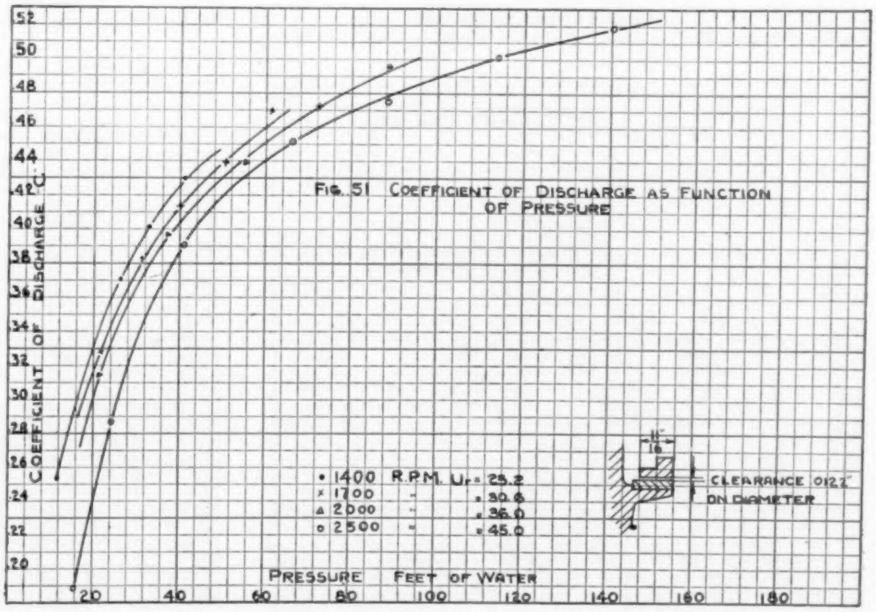


FIG. 51 COEFFICIENT OF DISCHARGE AS FUNCTION OF PRESSURE

Since the pressure in the chamber on the back side of the runner (where the leakage was measured) was equal to that of the atmosphere, and the pressure in the runner eye was below atmospheric pressure by the amount of the suction lift and velocity head in the runner eye, the leakage measured was smaller than the leakage on the suction side of the runner. From measurements of suction lift and pressure at the wearing rings for each capacity, which have been plotted above the suction pressure of the pump in Figs. 46 to 65, the leakage on the suction side of the runner could easily be calculated. These calculations were made for all rings tested at four speeds and the pump normal capacity, and the results are tabulated in Fig. 31, where the leakage is expressed as percentage. From this table it is seen that the leakage loss varied from 1.52 to 20.0 per cent.

From the curves in Figs. 46 to 65 it is seen that for the same pressure the coefficient of discharge decreased slightly as the peripheral velocity of the runner ring increased (increase in r.p.m.). This was probably due to the fact that the mean absolute velocity of the flow through the clearance was greater for greater relative velocities, causing the resistance to the flow to become greater and the coefficient of discharge to become smaller. The effect of the relative velocity on the coefficient of discharge varied slightly for different wearing rings; the general trend was, however, the same for all of them—that is, toward a higher coefficient of discharge for lower speeds.

To study the effect on the coefficient of discharge of the width of the wearing ring, different radial clearances and various resistances in the form of grooves, turns, sudden enlargement of area, etc., it is convenient to choose the points of comparison with the same pressure across the clearance and the same speed. The points at normal head and capacity have been chosen for this purpose and the leakage in per cent of the normal capacity has been calculated and is given in Fig. 31, with the corresponding coefficients of discharge. The upper lines of this table

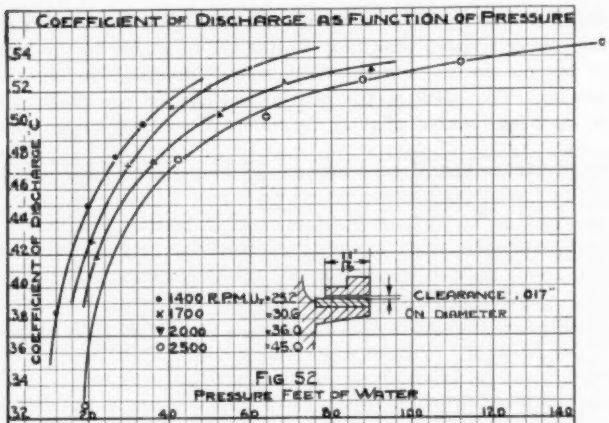


FIG 52

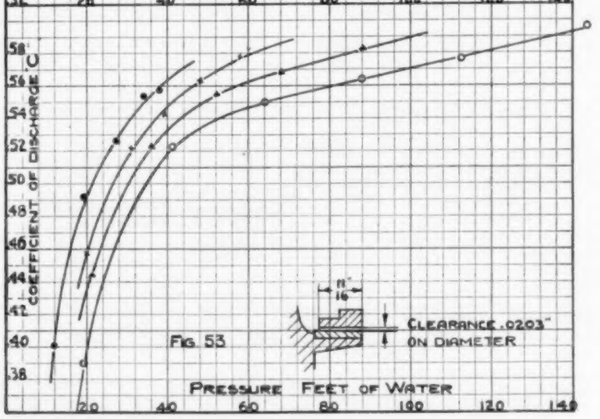


FIG. 53

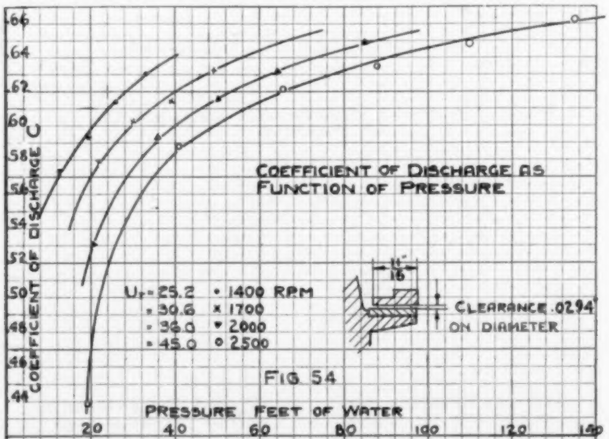


FIG 54

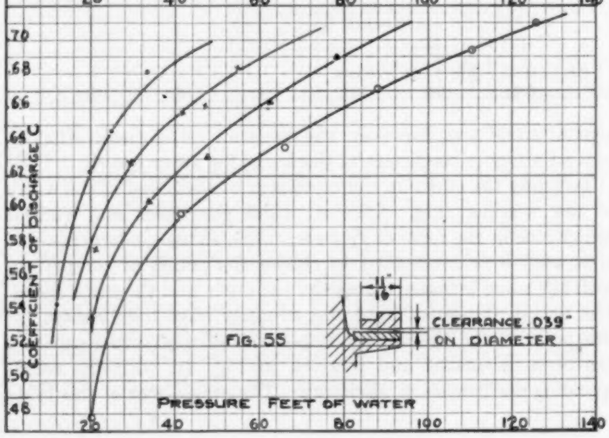


FIG. 55

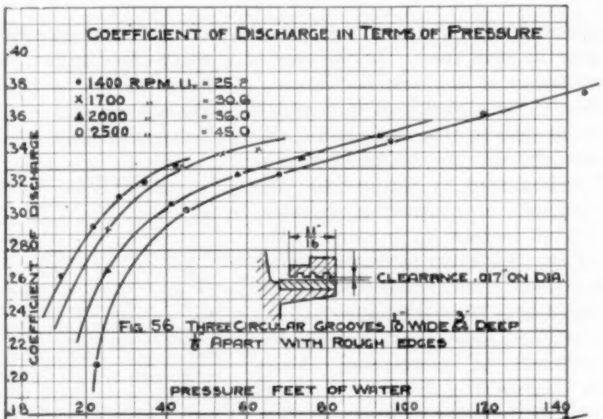


FIG. 56 THREE CIRCULAR GROOVES 1/16" WIDE 3/64" DEEP 1/8 APART WITH ROUGH EDGES

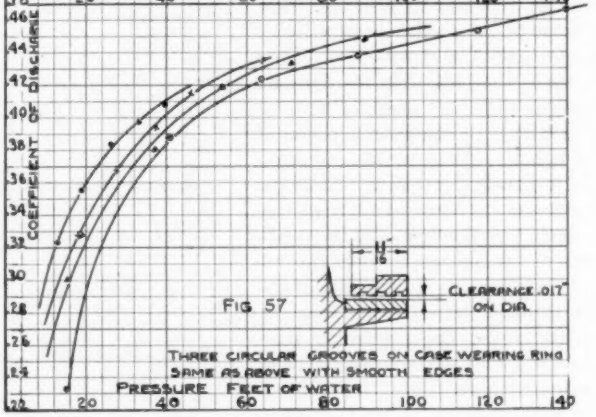
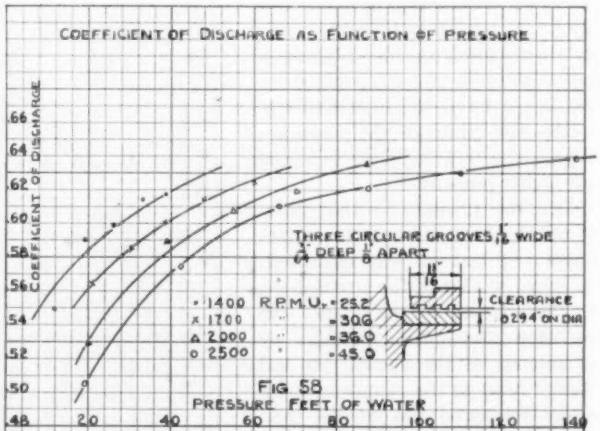


FIG. 57

THREE CIRCULAR GROOVES ON CASE WEARING RING SAME AS ABOVE WITH SMOOTH EDGES



THREE CIRCULAR GROOVES 1/16" WIDE 3/64" DEEP 1/8" APART

FIG 58

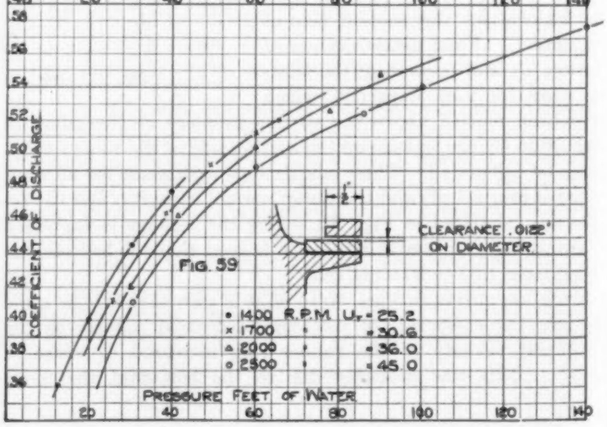
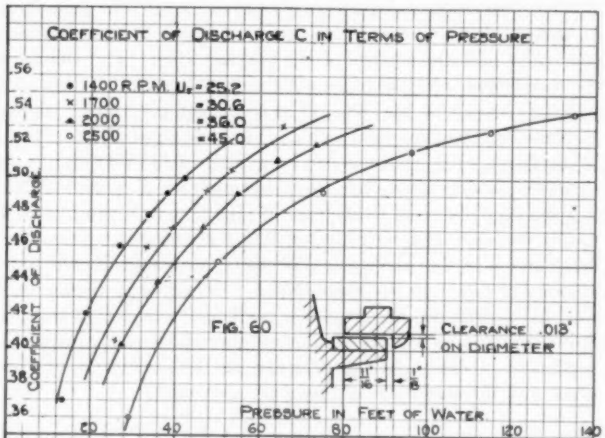


FIG. 59



COEFFICIENT OF DISCHARGE C IN TERMS OF PRESSURE

FIG. 60

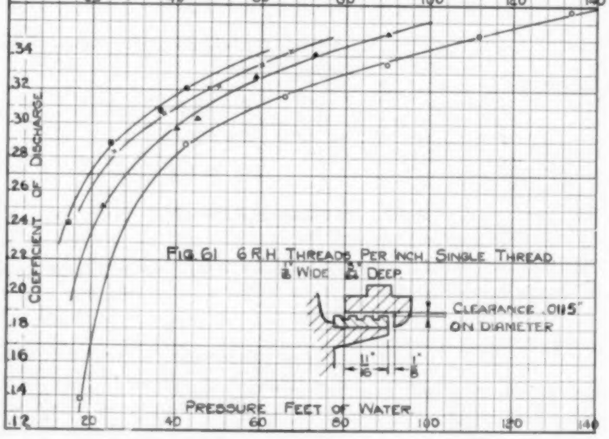
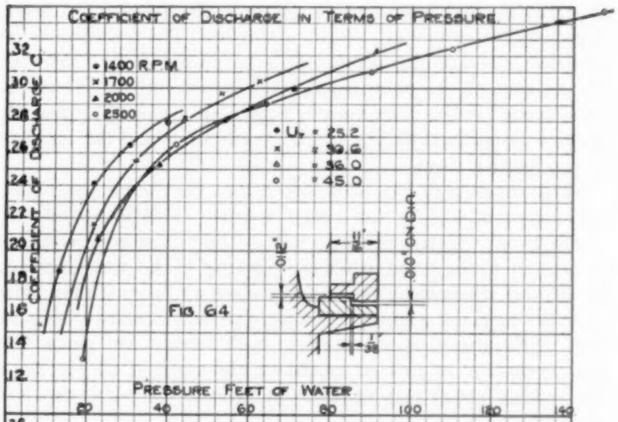


FIG. 61 6 R.H. THREADS PER INCH SINGLE THREAD 1/8 WIDE 1/32 DEEP



COEFFICIENT OF DISCHARGE IN TERMS OF PRESSURE

FIG. 64

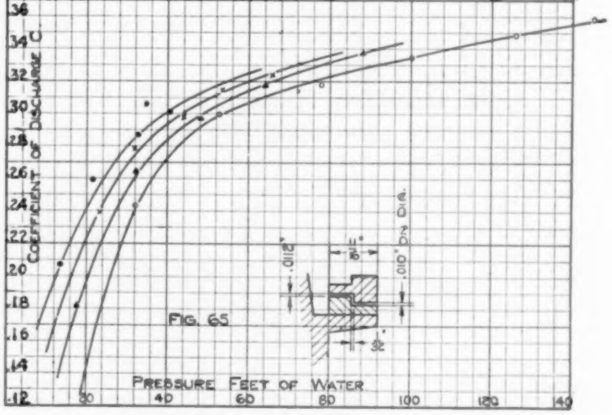
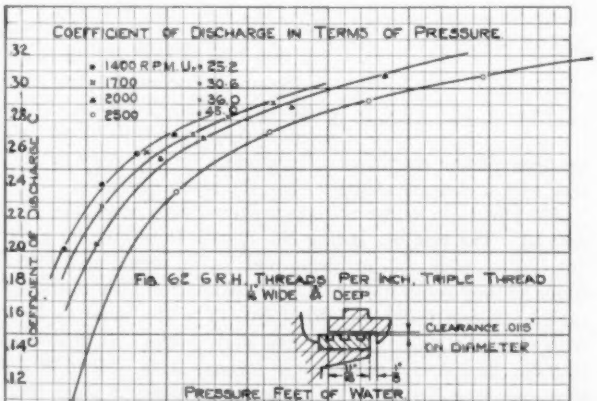


FIG. 65



COEFFICIENT OF DISCHARGE IN TERMS OF PRESSURE

FIG. 62 6 R.H. THREADS PER INCH, TRIPLE THREAD 1/8 WIDE 1/32 DEEP

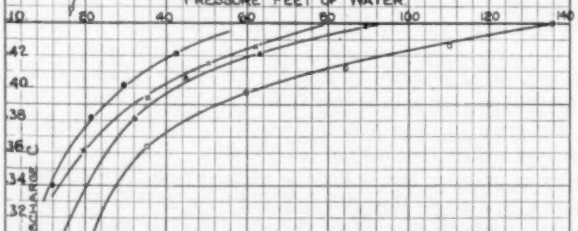


FIG. 63 6 R.H. THREADS PER INCH, TRIPLE THREAD 1/8 WIDE 1/32 DEEP

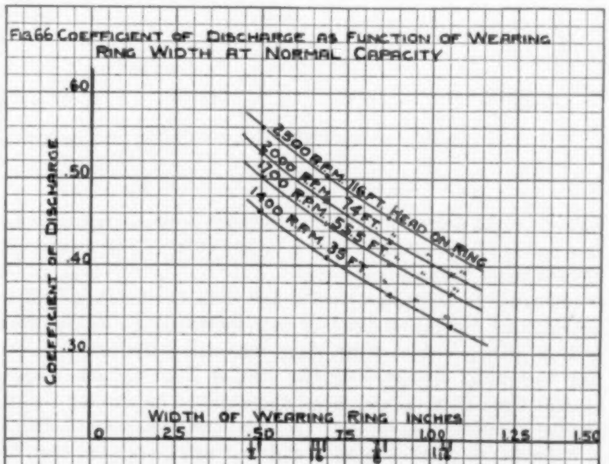


FIG. 66 COEFFICIENT OF DISCHARGE AS FUNCTION OF WEARING RING WIDTH AT NORMAL CAPACITY

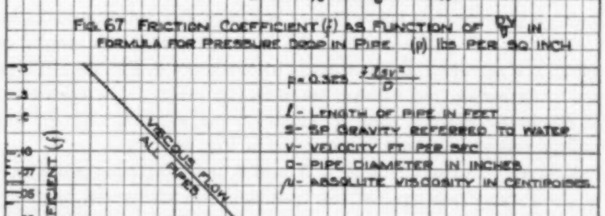


FIG. 67 FRICTION COEFFICIENT (f) AS FUNCTION OF $\frac{DV}{\mu}$ IN FORMULA FOR PRESSURE DROP IN PIPE (p) lbs PER SQ. INCH

$$p = 0.323\ \frac{f\,l\,s\,v^2}{D}$$

l - LENGTH OF PIPE IN FEET
s - SP. GRAVITY REFERRED TO WATER
v - VELOCITY FT. PER SEC.
D - PIPE DIAMETER IN INCHES
μ - ABSOLUTE VISCOSITY IN CENTIPOISES

give the normal head and capacity of the pump at different speeds with a runner of $10^1/_8$ in. diameter. The pressures across the clearance are also given. It is to be noted that these are 63 per cent of the pump total head for all speeds. The same ratio holds for the diameter cut to $8^1/_2$ in., indicating that this ratio is determined by the volute mean radius and the diameter of the wearing rings.

From the data in Fig. 31 it is seen that the percentage of leakage increases slightly for higher speeds. This is because the pump head and the pressure across the clearance increase as the square of the speed, while the capacity of the pump increases directly as the speed. Now, if the discharge through the clearance between the wearing rings were proportional to the square root of the pressure difference on both sides of the rings, then the percentage of leakage would be the same at all speeds, as the leakage would vary in the same ratio as the pump capacity. But since the flow through the clearance is proportional to the $1/n$ power of the head, where n is less than 2, the leakage increases faster than the capacity.

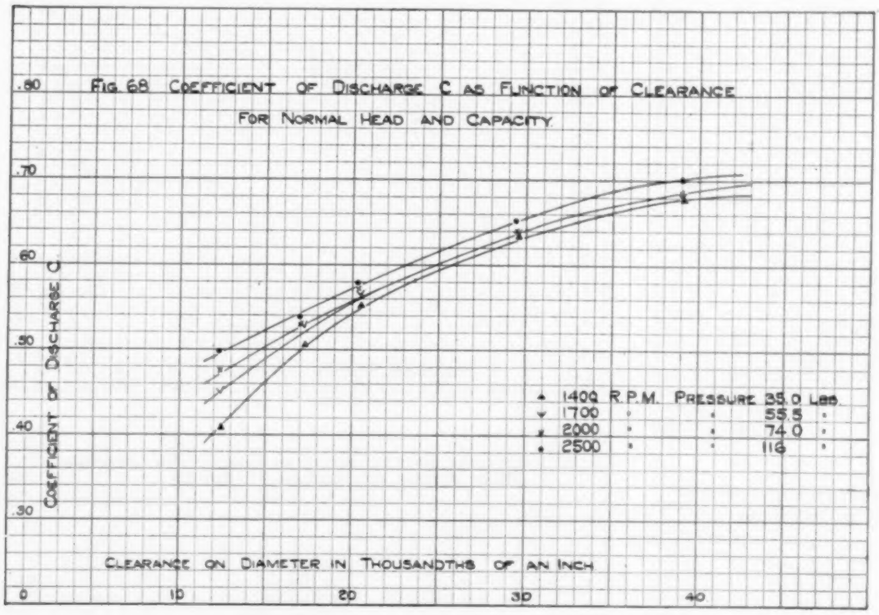


Fig. 68 Coefficient of Discharge C as Function of Clearance for Normal Head and Capacity

Referring to the rings Nos. 1 and 2 in the table, it will be noticed that cutting off the labyrinth increased the leakage from 41.0 to 46.6 per cent of its value with the labyrinth. In this ring the radial clearance on the smaller diameter was slightly greater than that on the large diameter. The overlap on the smaller diameter was $^3/_8$ in. The axial clearance was $^1/_{16}$ in.

Referring to rings Nos. 2, 4, and 6, it will be noticed that the leakage increased as the width of the ring was cut from $1^1/_{16}$ in. to $^{11}/_{16}$ in. A further cut to $^1/_2$ in. was tested, the results of which are not given in the table, but are given in Fig. 59. These are in agreement with those listed in the table. The coefficient of discharge plotted against width of the wearing ring for the points under consideration is given in Fig. 66.

Referring to rings Nos. 3, 4, 5, and 6, it is seen that projection of the runner ring against the case ring and vice versa has very little, if any, effect on the coefficient of discharge. However, in the case of the runner ring projecting outside the case wearing ring, the effect of the different runner speeds disappears (see Fig. 48). This is probably due to a slight pumping action of the projecting portion of the runner wearing ring.

By comparing the leakage for the rings Nos. 6, 7, 8, 9, and 10, the effect of the radial clearance can be seen. The percentage of leakage loss increased due to an increase of the discharge coefficient and an increase of the leakage area. The change of the coefficient of discharge with the clearance for the same points is shown in Fig. 68. It is seen that the rate of change increased for smaller clearances. The change of the coefficient of discharge with clearance can be explained as follows: As the clearance is decreased, the Reynolds number $R = dv/\nu$ (where d is the pipe diameter in the case of pipe flow or mean hydraulic radius for any other form of section, v is the mean velocity of flow, and ν is the kinematic viscosity) decreases for the same velocity of flow and the same liquid. Referring to the curve in Fig. 67, it will be seen that the coefficient of friction for pipe flow increases as the Reynolds number decreases; hence the coefficient of discharge will decrease, as the coefficient of resistance and the coefficient of discharge are related:

$$1 + f = 1/C^2$$

where f is coefficient of friction. This follows from

$$Q = CA\sqrt{(2gH_L)}$$

or

$$V = C\sqrt{(2gH_L)}$$

or drop in pressure

$$H_L = \frac{V^2}{C_2{}^2 g} = fV^2/2g + V^2/2g$$
$$= (1 + f)V^2/2g$$

It is interesting to note that with the rings set eccentrically (due to inaccuracy of machining or shaft deflection), although the clearance area remains the same, the leakage is smaller. This was observed several times during these tests. The reason for this fact is that the increase in coefficient of discharge in the portion with larger than the mean clearance is more than counterbalanced by the decrease of the coefficient of discharge for the portion of the ring with a clearance smaller than the mean clearance.

The effect of circular grooves is shown with rings Nos. 11, 12, and 13. The first one is ring No. 7, with three circular grooves cut $^1/_{16}$ in. wide, $^3/_{64}$ in. deep, and $^1/_8$ in. apart, and with the edges as they come from the lathe, bearing minute burrs, measuring not over 0.001 in. Ring No. 12 is the same as No. 11 with the burrs taken off but with sharp edges. It is seen that ring No. 11 shows a reduction in leakage from 32.2 to 35.5 per cent over ring No. 7. This reduction dropped to from 16.7 to 20.0 per cent after the burrs were taken off.

The advantage of circular grooves almost disappears for large clearances, as is shown by ring No. 13. With a radial clearance of 0.0294 in., the gain is only from 2.2 to 4.0 per cent.

Referring to rings Nos. 15, 16, and 17, it is seen that the spiral grooves reduce the leakage considerably. Compared to a ring without grooves, the leakage is reduced from 20.8 to 27.0 per cent with a single threaded ring and from 33.0 to 36.8 per cent with a triple (higher pitch) thread. The fact that the gain is considerably greater with the triple thread suggests that the reduction in leakage is not solely due to an increase in the resistance to the flow, but that actual pumping of the leakage water back to the case takes place. The effect of the spiral grooves is reduced with large clearances, but not to such an extent as in

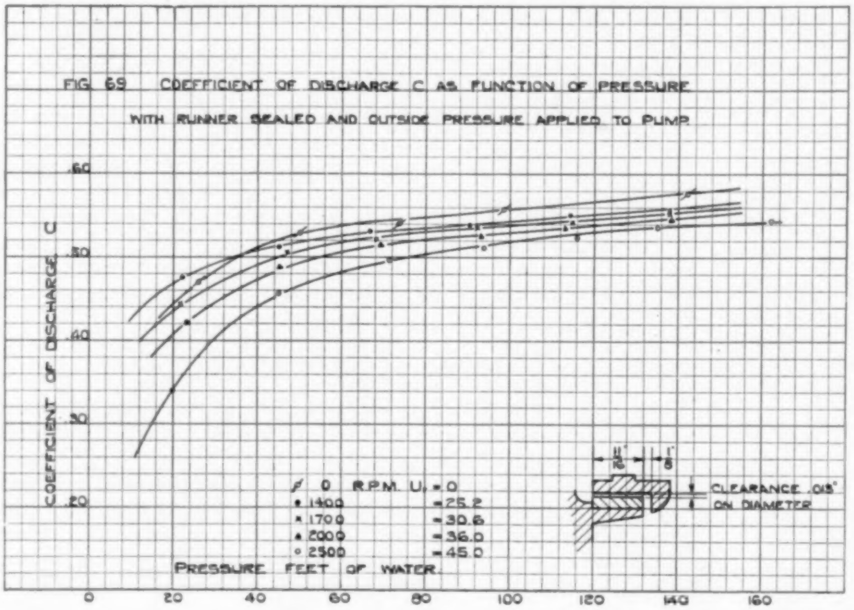


FIG. 69 COEFFICIENT OF DISCHARGE C AS FUNCTION OF PRESSURE WITH RUNNER SEALED AND OUTSIDE PRESSURE APPLIED TO PUMP

the case of the plain circular grooves. This is seen from the results with rings Nos. 18 and 19, the clearances being 0.0215 against a clearance of 0.0115 in. for rings previously referred to.

Although the spiral grooves show a decided advantage over plain circular grooves, the net gain is probably less than that given by the figures in the table, since, naturally, some power is required to pump the leakage by spiral grooves.

The figures for rings Nos. 15, 18, and 22 in the table are marked "calculated." By this is meant that no test has been made with the rings using the clearances specified for these rings in the table; the figures given were obtained for the sake of comparison by interpolating between the five clearances tested. However, these figures may be considered as accurate as the original.

The effect of a step in the wearing ring diameter may be seen from figures given for rings Nos. 20, 21, and 22. The reduction in leakage with a stepped ring against a plain one of the same width and clearance is from 24 to 33 per cent. Rings Nos. 20 and 21 differ in that in the first the runner wearing ring at the entrance projects $^3/_{16}$ in. against the case wearing ring. This ring shows a leakage loss slightly less than with the runner ring, the edges of which were flush with the edge of the case wearing ring.

A series of tests was made with outside pressure applied to the pump case, the runner outlet being sealed by a band. The runner was driven at 1400, 1700, 2000, and 2500 r.p.m. The coefficients of discharge were calculated and plotted in Fig. 69. Comparing these with the results given for the same ring with the pump's own pressure, it is seen that they are almost identical. These curves show that the direction of the curves for the coefficient of discharge at low speeds and high pressure, which could not be obtained with the pump's own pressure, is the same as that for 2500 r.p.m. The power required to drive the rotating element was independent of the pressure in the pump case and was lower than that at pump shut-off. The curves in Fig. 70 show the power required to drive the rotating element with the runner sealed, the power to pump the leakage at shut-off, and the sum of the two with the power at pump shut-off.

It will be noticed that the power at shut-off is considerably greater than that necessary to drive the rotating element with the runner sealed plus the power to pump the leakage. The difference is wasted by internal circulation and eddies in the pump case near the runner periphery. It should be noted that the power to drive the rotating element with no discharge varies as a certain power of the speed, the exponent for higher speeds being approximately 2.8 and for lower speeds about 2.25. This is due to the fact that at high speeds the disk friction, which varies as the cube of the speed (theoretically, the exponent found by experiment is slightly lower), is the main loss; while at lower speeds the friction in the stuffing boxes and bearings, which varies as the square of the speed, constitutes a greater portion of the total power than at the high speeds.

12 PROCEDURE FOR CALCULATION OF LEAKAGE LOSS

The following procedure is recommended for calculating the leakage loss. Usually pumps are designed and sold to operate at the normal head and capacity or at the maximum efficiency point. It is in the attainment of this end that the designer is mostly interested and for which he makes the analysis of losses due to various causes. Therefore, the procedure for calculation of the leakage at this point will be outlined:

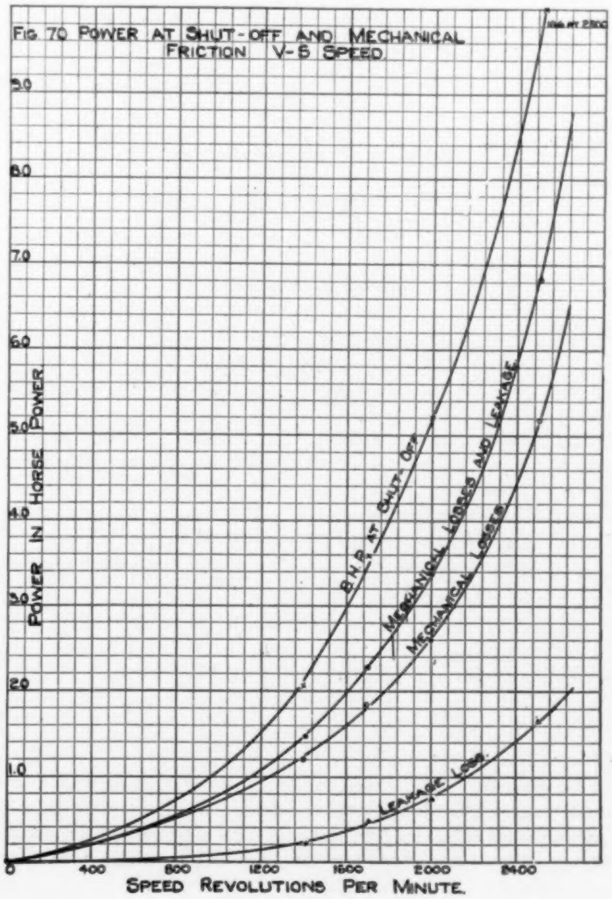


FIG. 70 POWER AT SHUT-OFF AND MECHANICAL FRICTION V-S SPEED

(1) Calculate the volute pressure from a given total head, estimating the efficiency of discharge nozzle, which in the case of the tested pump was 60 per cent. The volute pressure is calculated from

$$H_d = K_n(V_v^2/2g - V_d^2/2g) + H_v$$

where

H_d = the pressure at discharge flange, equal to $H_d = H - V_d^2/2g$
H = pump total head
V_v = velocity in the volute case
V_d = nozzle velocity at the discharge flange
K_n = efficiency of the nozzle.

(2) For the volute pressure thus obtained, calculate the pressure at the wearing ring from

$$H_L = H_v - \varphi^2(u_3^2 - u_r^2)/2g$$

For the tested pump φ was found to vary from 0.456 to 0.488.

If u_2 is used instead of u_3, the value 0.5 can be used for the coefficient φ.

As an alternative the pressure at the wearing ring may be calculated directly from the total head by estimating the factor K_r in the formula

$$H_L = K_rH$$

For the pump tested, this factor was 0.63 and did not depend upon the small variation of the runner diameter. For the analysis of the performance of existing pumps, the volute pressure and pressure at the wearing ring can be obtained by measurement, and the factors K_n and K_r can be calculated.

Probably the quickest way to calculate the pressure at the wearing ring is by using formula [29], Section 10:

$$H_L = \frac{3}{4}\,\frac{(u_2^2 - u_1^2)}{2g}$$

(3) Correct the pressure at wearing ring H_L for velocity head at the runner eye

$$H_L' = H_L + \frac{V_e^2}{2g}$$

where V_e is velocity through the runner eye.

(4) From a given speed and wearing ring diameter, calculate the runner wearing ring peripheral velocity u_r.

(5) For calculated u_r and a given type of the ring, choose the coefficient of discharge for the pressure across the wearing ring as found in (3).

If the design of the wearing ring is different from those for which the test data are available, use the data of the type nearest to it and increase or reduce the coefficient of discharge to correspond to the difference in design. For values of u_r not given in the data available, interpolate between the given values.

(6) With the coefficient of discharge and the pressure across the clearance found, calculate the leakage from

$$Q_L = CA\sqrt{(2gH'_L)}$$

(7) Calculate the volumetric efficiency

$$e_v = \frac{Q}{Q + Q_L}$$

Allowing in these calculations an error not greater than 5 per cent, and in a modern pump a leakage loss of not more than 5 per cent, the total error will be only 0.25 per cent of the pump's input. This is smaller than the error in measuring the input and the output of the pump.

PART II AXIAL THRUST

13 AXIAL THRUST IN CENTRIFUGAL PUMPS

WHILE in the double-suction pumps, due to symmetry of construction, the runner is balanced for the axial thrust, in the single-suction runners some means should be provided to take up the runner axial thrust. This thrust is due to the fact that the pressure in the pump case acts on a larger area on the back side of the runner than on the front side. This thrust is partly balanced by the force to the right, in Fig. 30, due to the change in momentum of the incoming water which results from the change in direction of the flow from the axial to radial.

$$\text{Force to right} = WV_e/g = AV_eV_e\gamma/g = 2A\gamma V_e^2/2g$$

where A is the runner-eye area, V_e is the velocity in the runner eye, γ is the density of water, and W is the weight of water per unit time.

Axial thrust may be taken up by a thrust bearing or it may be reduced or entirely eliminated by hydraulic means. The most widely used method of balancing axial thrust in the single-suction, single-stage pumps consists in providing another wearing ring on the runner back. In the chamber thus formed the suction pressure is admitted. The latter is obtained either by drilling holes through the runner web into the suction (Fig. 2) or by connecting this chamber by a pipe or a channel cast on the pump case to the suction nozzle. Thus, to balance the axial thrust, a portion of the pump capacity is sacrificed by allowing water to leak through an additional clearance. Concerning the amount of axial thrust, there is available no information checked by actual measurements. The difficulty in calculating the axial thrust lies in the fact that the pressure on the portion of the runner shroud opposite the suction is considerably smaller than that of the pump discharge or even the volute pressure, due to rotation of water in the space between the runner and the case walls. Sometimes runners are provided with the radial ribs on the back side of the runner to increase this rotation (see Fig. 12). To study the pressure distribution in the pump case, to measure the axial thrust of a single-suction pump, and also to investigate to what extent the axial thrust may be balanced by different means are the objects of this investigation.

The pressure distribution in the space between the runner and the pump case depends on the clearance between the runner and the stationary walls. The pressure is smaller for smaller clearances.[28] It is therefore important with small clearances to have the runner exactly in the middle position in the pump case, because otherwise, even in the case of a double-suction pump, there will be an axial thrust due to a difference in the angular velocity of rotation of water on the two sides of the runner. This was checked experimentally in the laboratory of the University of California by J. N. LeConte.[29] In modern pumps there is usually provided an ample clearance on both sides of the runner so that slight displacement of the runner from the middle position does not produce an axial thrust.

It should be noted that when holes are drilled through the runner hub to reduce the pressure in the balancing chamber on the back of the runner, the runner performance may be impaired by the flow through these holes directed against the main flow in the runner. The outside connections are to be preferred.

The pressure in the balancing chamber will always be slightly higher than in the runner eye, since some pressure is necessary to produce the velocity and to overcome the resistance through the balancing holes. Furthermore, since the flow into the runner is directed against the holes, the pressure in the chamber will be higher than that in the runner eye by the amount of the kinetic head of the flow impinging against the holes ($V_s^2/2g$); and if the diameter of the wearing ring on the back of the runner is the same as that on the front, the total force due to this head is equal to $AV_s^2/2g$, or equal to one-half of the force in the opposite direction, due to change in momentum of the flow into the runner. Hence, to obtain a perfect balance by means of the balancing chamber on the back of the runner, the diameter of the chamber should be larger than the wearing ring diameter on the suction side.

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[28] J. H. Polhemus and J. Healy, "Dredge-Pump Pressures and Thrust Loads," Trans. A.S.M.E., 1929, paper HYD-51-4.

[29] J. N. LeConte, "Hydraulics," McGraw-Hill Book Company, 1926.

In the case of the open runners (Figs. 12 and 15), the axial thrust is still greater because the pressure on the suction side of the runner varies from suction pressure to discharge pressure, while on the opposite side there is a full volute pressure reduced only by rotation. To reduce the thrust with the open runners, a portion of the back shroud may be cut out (Fig. 15) without seriously affecting the pump performance.

In the vertical pumps the weight of the rotating element adds to the axial hydraulic thrust. This weight may become quite large in deep-well turbine pumps. It is possible to reduce the load on the thrust bearing by making the hydraulic thrust work against the weight of the rotating element, in which case the runners should be turned with the suction side upward.

In addition to what may be termed external sources of the axial thrust, there are internal causes. To these belong the following:

(*a*) The water reaction on the vanes set at an angle different from 90 deg. to the plane of rotation, as is found in the propeller of screw pumps and to a smaller degree in any mixed-flow, single-suction runner. This thrust is of the same nature as that on a ship propeller and is considerable for the large propeller pumps.

(*b*) The axial thrust caused by uneven pressure on the two runner shrouds inside the runner. The causes of the difference in pressure are the difference in the head produced along two stream lines near the shrouds due to a difference in the outside diameter, vane angularity (in mixed-flow runners), the length of the stream lines, and the difference in the radial velocities near the shrouds. Of these the uneven radial velocity is the most important, since in almost all pumps, except the pure-axial-flow pumps, water is turned through 90 deg. just near the points where it comes under the influence of the vane. The effect of the turn is twofold: (1) The pressure near the outer wall of the turn becomes higher than that near the inner wall, thus producing the thrust directed from the suction. This thrust is the same as that due to change of momentum of the incoming flow described before, and equal to WV_s/g. (2) The uneven velocity distribution. Now, since the diameter and the vane angle are the same for the two stream lines, the head produced by the vane is higher where the radial velocity is smaller, as is seen from the velocity triangle (Fig. 21). For a radial inlet the head produced is

$$H_t = \frac{u_2 c_{2u}}{g}$$

For a smaller radial velocity c_{2m}, the value of c_{2u} is greater; hence the head produced is larger. The thrust due to this cause is variable, depending on the pump capacity, since the velocity distribution in a 90-deg. turn depends upon the mean velocity of the flow and the radius of curvature. Under certain conditions this may be reversed, as discussed in detail in Section 4.

Because double-suction runners are symmetrical in construction, the thrust due to this last cause is absent. However, if for some reason the symmetry of the flow in a double-suction pump is destroyed, considerable axial thrust will result caused from inside of the runner. The most common cause for non-symmetrical flow in the runner is a non-symmetrical approach such as is found when an elbow is put next to the suction flange in the plane different from the plane of symmetry of the pump, as is usually the case when two pumps are operated in series.

(*c*) The cavitation, or local formation of vacuous spaces inside the runner, results in a heavy axial thrust. Depending on the runner profile and working capacity, these vacuous spaces may form at the vane entrance tips near the front or back shrouds; then the unbalanced pressures on the opposite shroud inside the runner produce axial forces. Owing to the character of the cavitation, the axial thrust caused by it varies considerably in magnitude and also may vary in direction.

It should be noted that in some instances a moderate axial thrust is desired to keep the rotating element from floating or whipping. Accordingly, it has been found that in the deep-well pumps the bearings wear less if the column shaft is running under tension due to thrust. Of course, this thrust must be provided for when selecting the thrust bearing.

14 AXIAL THRUST IN MULTI-STAGE PUMPS

In the case of multi-stage pumps the end thrust is of more importance on account of the higher pressures involved and the summation of the thrust of several stages. If the number of stages is even, the pump can be made inherently balanced by arranging runners in groups, with suctions in opposite directions. Due to symmetry of construction the pump is hydraulically balanced. But this scheme requires expensive pattern work. If the runners are arranged with suctions in the same direction, the stages may be cast separately and a pump with any required number of stages assembled, or the pattern may be made in sections and a one-piece casting may be made with the required number of stages. In both cases the thrust in all stages is in one direction, and special means should be provided to take care of this thrust.

The balancing of every runner by means of a chamber on the back of the runner and drilled holes through the runner web is secured in multi-stage pumps also. It takes much room in the axial direction, however; it increases the span between the bearings; it is expensive; and it tends to lower the pump efficiency.

The old and very effective methods of balancing a considerable thrust are the use of a balancing disk and a drum. Both methods sacrifice a portion of pump capacity to eliminate the thrust.

The balancing disk operates as follows: In Fig. 72 the thrust is to the right. A balancing chamber is provided on the back of the last stage runner, separated from the pump case by a disk keyed to the shaft. The disk rotates with the shaft, with a small clearance from the case wearing ring. The balancing chamber is connected to the pump suction or is vented to atmosphere. The pressure in the chamber can be regulated by a valve. If the pressure on the runner backs exceeds that on the disk, the disk is moved to the right, the clearance between the disk and the case wearing ring becomes smaller, and the pressure drops in the balance chamber until the pressure on the disk moves it to the left, making the clearance larger. Now the pressure builds up in the balance chamber, and the disk is moved to the right until an equilibrium is reached, when forces to the right balance those to the left.

Fig. 71 shows a balance disk with two active throttling surfaces floating between two fixed seatings with very slight clearance. If the pressure on the high-pressure side of the disk exceeds the axial thrust to the left, the piston moves to the right, increasing the clearance between the large wearing ring and the piston and

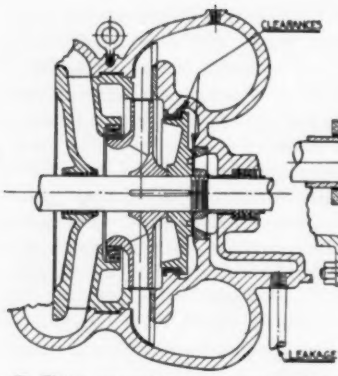

FIG. 71 BALANCING DISK WITH TWO THROTTLING SURFACES

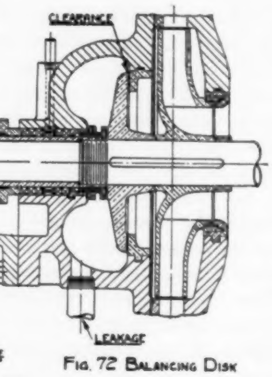

FIG. 72 BALANCING DISK

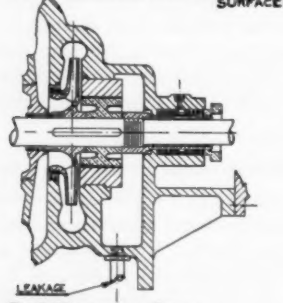

FIG. 73 BALANCING DRUM

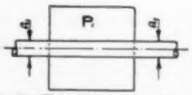

FIG. 74a END THRUST = 0

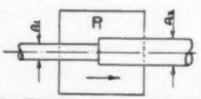

FIG. 74b END THRUST = $P_1(A_2-A_1)$

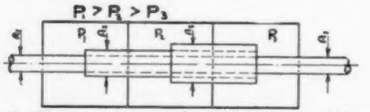

FIG. 74c END THRUST = $(P_1-P_3)(A_2-A_1)+(P_2-P_3)(A_3-A_2)$.

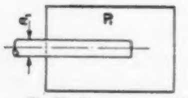

FIG. 74d END THRUST = P_1A_1

SIMPLE CASES OF THE END THRUST DUE TO SHOULDERS ON SHAFTS

decreasing the clearance between the small wearing ring and the piston. The pressure in the balance chamber builds up more quickly than in the case of a single throttling surface, and the rotating element is brought to a balance with a considerably smaller leakage than with a single-faced disk.

The disadvantages of this method of balancing are that (1) a portion of the pump capacity is wasted, (2) some power is consumed on the disk friction of the balancing disk, and (3) it requires a certain amount of axial play for the rotating element. The advantages are (1) the complete automatic balance (no thrust bearing is necessary), and (2) the elimination of the high-pressure stuffing box. The calculation of the amount of leakage in these balancing methods is complicated by the variation of clearances and pressures. However, the leakage can easily be measured. This kind of leakage will not be considered in this investigation. The balancing drum is used to reduce or eliminate small end thrust when the thrust bearing employed does not allow any axial play. It consists of a drum keyed to the shaft (Fig. 73) fitted with a small clearance in the case wall. The last stage pressure acts on one side of the drum, while the other side is under suction pressure. Thus a force to the right is obtained which will partly or completely balance the pump hydraulic thrust to the left. The pressure on the low-pressure side can be regulated. The disadvantages of this method are that (*a*) it is not automatic, (*b*) the pump may be perfectly balanced for one capacity and unbalanced for another, (*c*) the balance is destroyed after the clearance is increased on account of wear, (*d*) the greater span between bearings which results increases the tendency toward deflection and vibration, necessitating the use of a larger shaft.[30]

While multi-stage pumps with runners arranged in two equal groups with suctions in opposite directions are considered to be hydraulically balanced, this is true only for pumps with two stuffing boxes and with shafts of a constant diameter through the whole pump and runner hubs not extending through the walls separating the two adjacent stages. Usually the span between the bearings in multi-stage pumps is large and shafts are made heavier at the middle portion for strength. The shoulders on the shaft and the runner hubs produce axial thrust which may be considerable. It is probable that neglecting this fact has caused the failure of many high-speed pumps. The thrust capacity of ball bearings decreases rapidly with the speed of rotation. Multi-stage pumps running at 3600 r.p.m. are coming into general use now. Such pumps, to operate safely, must have axial pressures balanced as closely as possible in order to minimize end thrust.

To show how the end thrust for a pump with runners arranged back to back can be calculated, a few elementary examples will be considered which will illustrate clearly the principle involved. Fig. 74 shows several closed vessels filled with liquid under pressures P_1, P_2, and P_3 above atmosphere, and shafts pass through the walls of these vessels of diameters D_1, D_2, and D_3 and areas A_1, A_2, and A_3, respectively. The end thrusts are calculated, the direction being shown by arrows. Now consider a four-stage pump with runners arranged as shown in Fig. 75. Let

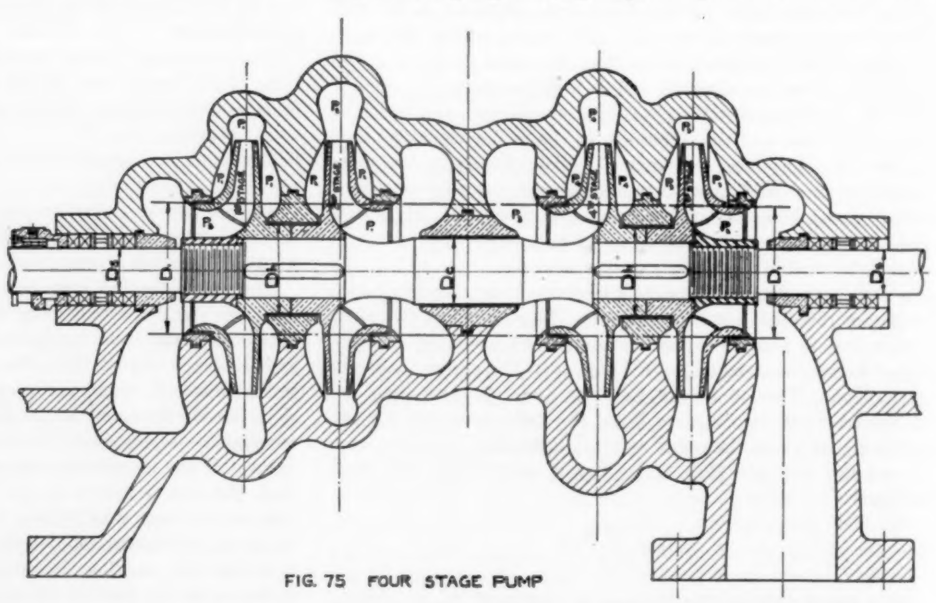

FIG. 75 FOUR STAGE PUMP

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[30] For description of numerous modifications of the balancing devices, see C. Pfleiderer, "Die Kreiselpumpen," pp. 237–259.

P_s = suction pressure

P_1 = volute pressure in the first stage = $P_s + P$, where P is the pressure produced by one stage

P_2 = volute pressure in the second stage = $2P + P_s$

P_3 = volute pressure in the third stage = $3P + P_s$

P_4 = volute pressure in the fourth stage = $4P + P_s$

D_s = diameter of shaft or shaft sleeve at the stuffing box, equal on both sides

D_h = runner hub diameter

D_c = shaft diameter at the center

D_1 = runner wearing ring diameter

A_1, A_s, A_h, A_c = areas of the circles corresponding to diameters D_1, D_s, D_h, and D_c.

Assume that the pressures in the clearances between the pump case and the runner shrouds are uniform and equal to that of the corresponding volute pressure. Assume also that the suction pressure of the second stage is the same as volute pressure of the first stage and that the suction pressure of the third stage is the same as the discharge pressure of the second, etc. These assumptions do not affect the final result, since there is always a one-stage pressure difference P between two similar points in two successive stages.

It is evident that the pressures on both sides of the runner shrouds from the diameter D_2 to diameter D_1 are equal and balanced. Consider now the pressures on each runner after exclusion of areas between D_2 and D_1. Call pressures to the left positive, those to the right negative. Then the forces on the rotating element are:

$$\begin{aligned}
(A_1 - A_h)P_1 - (A_1 - A_s)P_s &= \text{first stage}\\
(A_1 - A_c)P_1 - (A_1 - A_h)P_2 &= \text{second stage}\\
(A_1 - A_h)P_4 - (A_1 - A_c)P_3 &= \text{fourth stage}\\
(A_1 - A_s)P_2 - (A_1 - A_h)P_3 &= \text{third stage}
\end{aligned}$$

The axial thrust is equal to the sum of these forces. Adding and rearranging:

$$\begin{aligned}
T &= A_1(2P_1 + P_3 + P_4 - P_s - P_2 - 2P_3)\\
&\quad + A_h(P_2 + P_3 - P_4 - P_1) + A_c(P_3 - P_1)\\
&\quad + A_s(P_s - P_2)
\end{aligned}$$

Substituting for P_1, P_2, P_3, and P_4 their values $P + P_s$, etc., and reducing:

$$T = 2P(A_c - A_s) = 2P(D_c^2 - D_s^2)\,0.785 \ldots\ldots [30]$$

It is seen that, due to symmetry, D_1 and D_h do not appear in the final result. The whole system can be represented diagrammatically as equivalent to a simple case shown in Fig. 76. This result holds for the arrangement of stages as shown, 1, 2, 4, 3, which gives a minimum thrust. By reasoning similar to the above, the thrust for the arrangement of stages 1, 2, 3, 4 is found to be

$$T = 3P(A_c - A_s) + 2P(A_h - A_c) \ldots\ldots\ldots [31]$$

This is identical with a scheme shown in Fig. 77. The end thrust for this diagram is equal to

$$\begin{aligned}
T &= (A_c - A_s)(P_4 - P_1) + (A_h - A_c)(P_2 - P_1)\\
&\quad + (A_h - A_c)(P_4 - P_3)\\
&= 3P(A_c - A_s) + 2P(A_h - A_c)
\end{aligned}$$

To obtain an idea of the magnitude of the thrust produced in this way, take a numerical example. A 4-in. four-stage pump with an arrangement of stages as shown in Fig. 75 and 100 lb. pressure per square inch per stage has the dimensions, D_s = $1^3/_4$ in., D_h = $3^1/_4$ in., and D_c = $2^5/_8$ in. Substituting these values into formula [30], we get:

$$T = 2 \times 100 \times 0.785(2.625^2 - 1.75^2) = 597 \text{ lb.}$$

For the arrangement of stages 1, 2, 3, 4, the thrust is:

$$T = 100 \times 0.785(2.625^2 + 2 \times 3.25^2 - 3 \times 1.75^2) = 1475 \text{ lb.}$$

The runner dimensions (except the hub diameter) do not affect the result because of the symmetry of construction. In the first case the pressures on the shoulders formed by the runner hubs are opposite in direction and cancel each other, while in the second case these terms are of the same sign and add to the thrust due to the increase in the shaft diameter in the middle. From Equation [31] it is seen that with the arrangement of stages 1, 2, 3, 4 (not symmetrical) an axial thrust is obtained even with the shaft of constant diameter, due to runner hubs, making $A_c = A_s$. Formula [31] becomes

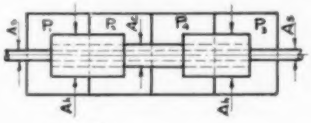

FIG. 76

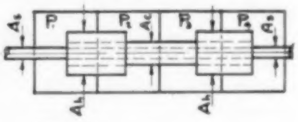

FIG. 77

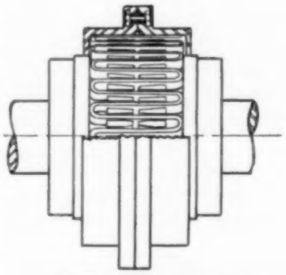

FIG 78 FLEXIBLE COUPLING WITH A SPRING

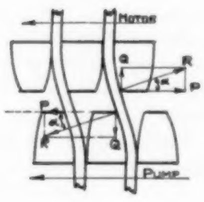

FIG. 79 AXIAL AND TANGENTIAL COMPONENTS TRANSMITTED BY A COUPLING

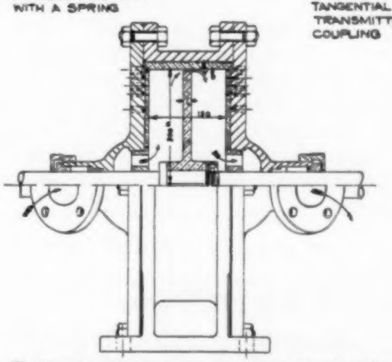

FIG. 80 DR YENDO'S APPARATUS FOR MEASURING LEAKAGE THROUGH THE RADIAL CLEARANCES

$$T = 2P(A_h - A_s) \ldots\ldots\ldots\ldots\ldots\ldots [32]$$

Take for another example a six-stage pump, Fig. 81. By the procedure outlined, the thrust is calculated for the arrangement of stages 1, 2, 5, 6, 4, 3 to be

$$T = 2P(A_c - A_s) + P(A_h - A_c) \ldots\ldots\ldots [33]$$

For the arrangement 1, 2, 6, 5, 4, 3, the thrust is equal to

$$T = 2P(A_c - A_s) - P(A_h - A_c) \ldots\ldots\ldots\ldots [34]$$

For the same values for D_s, D_h, and D_c, as in the previous example, and for a stage pressure of 100 lb. per sq. in., the first arrangement gives an axial thrust of 890 lb., while the second arrangement gives only 310 lb.

In a multi-stage pump with one stuffing box and one internal bearing there is a thrust due to the pressure acting on the enclosed shaft end, which should be added to the thrust due to shoulders on the shaft and runner hubs. Suppose in the pump shown in Fig. 75 the right-hand stuffing box is replaced by a closed bearing. The pressure on this end of the shaft is $2P$, the area is A_s, and the additional thrust is:

$$T' = P_2 \times A_s = A_s(2P + P_s) \ldots\ldots\ldots\ldots [35]$$

For P_s equal to the atmospheric pressure this gives

$$T' = 200 \times 0.785 \times (1.75)^2 = 480 \text{ lb.}$$

From the expression [35] it is seen that the thrust due to an internal bearing depends upon the suction pressure on the first stage. The thrust increases as the suction pressure increases, and it may reach a great value if the pump is operating under considerable positive head (if, for instance, two pumps are working in series). Since the main object of making pumps with one stuffing box is to eliminate the high-pressure stuffing box, the thrust due to this cause is always toward the first-stage suction, or in the same direction as the thrust due to shoulders on the shaft.

The thrust on the enclosed shaft end may be reduced or eliminated by using an uneven number of stages and grouping them so that the thrust due to one stage will balance that due to the enclosed bearing.

The thrust due to shoulders on the shaft and runner hubs can be balanced by making the runner hubs of different diameters in such a way that the pressure on the shoulder formed is opposing the end thrust. Thus, if the runner hubs on stages 3 and 4 are made larger than those on stages 1 and 2 (Fig. 75) (D_{h1} denoting diameter of the runner hubs for stages 1 and 2, D_{h2} the diameter of the runner hubs for stages 3 and 4, and A_{h1} and A_{h2} denoting the corresponding areas) and the calculations are repeated with these changes, the following expression is obtained for the axial thrust:

$$T = 2P(A_c - A_s) - P(A_{h2} - A_{h1}) \ldots\ldots\ldots [36]$$

By equating the thrust to zero we get:

$$2(A_c - A_s) = (A_{h2} - A_{h1}) \ldots\ldots\ldots\ldots\ldots [37]$$

For the same values of A_c and A_s and $D_{h1} = 3^1/_4$ in., A_{h2} is equal to 14.3 sq. in., or $D_{h2} = 4.27 \sim 4^1/_4$ in.

In a similar manner the balance can be obtained by making the runner wearing rings of different diameters. It is evident that in the case of a two-stage pump this is the only way to eliminate the axial thrust due to runner hubs, as there is only one group of runners. This balancing method can be used for reducing or eliminating moderate thrust, whatever its cause, by underbalancing or overbalancing the thrust due to shoulders on the shaft and runner hubs.

In addition to the hydraulic thrust from the pump, the thrust-bearing capacity should be ample to take from the coupling the thrust caused by misalignment of the rotating element of the pump and driver even if a flexible coupling is used. The ideal coupling should transmit the torque from the driver to the pump and allow a certain amount of end play for the driven shaft. Pure torque can be transmitted only if axes of rotation of both elements coincide; otherwise the coupling will produce a radial and axial load on the bearings of both pump and driver. The amount of thrust is proportional to the degree of misalignment. For high-speed pumps the choice of the coupling is of importance. Certain types of couplings, even with perfect alignment, produce axial thrust in both directions. Figs. 78 and 79 show a type of coupling with a tape put between the teeth cut on the coupling flanges as a flexible member. Since the pressure between the tape and the teeth is at an angle to the plane normal to the axis of rotation, components P-P transmit the torque, while components Q-Q produce the axial thrust.

15 Unbalanced Axial Forces on Pump Casing

When considering the axial thrust in centrifugal pumps, the attention is usually confined to the unbalanced axial forces on the rotating element, while in all cases there is an unbalanced and opposing force on the pump case, which is almost self-evident. However, to illustrate the point involved a simplified case will be considered. In Fig. 85 a body with two cylindrical portions of diameter D_1 and D_2 and a shaft of diameter d is enclosed in a casing under pressure P_2, while that outside of the casing is P_1. It is evident that an unbalanced force $(P_2 - P_1)\pi/4$ $(D_1^2 - d^2)$ will act on the body directed downward and at the same time an equal and opposite force will be exerted on the casing wall directed upward. In pumps the case is complicated by the fact that the pressure in the pump case is not uniform and the runner is revolving; however, in all cases there is a force on the pump case exactly equal to the axial thrust on the rotating element due to external pressure. Normally this force on the pump case is of no consequence as the case is rigid enough to stand this force without deformation. In the case of deep-well pump (Fig. 86) this force should be considered if the number of stages is great (say, 25). In this case the force on the pump case becomes of the order of several thousand pounds, and the pump casing (outer column) is not rigid enough to stand it without deflection. This deflection, together with the shaft elongation due to hydraulic thrust which is in the opposite direction, may be more than the running clearance in the stages, and the rubbing of runners against the pump case (bowls) will result. These conditions are more pronounced at the starting of the pump. Suppose that the rotating element was adjusted so that the runners are in the middle position in the pump case. When the pump is not running, the shaft is under tension due to the weight of the runners and shaft, and the outer column is under its own weight and that of the bowls. When the pump is started, the head is very nearly zero. Under these conditions the axial thrust may be considerably higher than at normal operating conditions, and the shaft will be subjected to an additional tension equal to the thrust of all stages. With a very great length of the shaft (several hundred feet) this extension may be of the order of $^1/_4$ to $^1/_2$ in., while the pump cases will be subjected to an equal lifting force. Due to a greater strength of the casing (outer and inner column) the deflection of the casing will be a small portion of the shaft extension. When the outer column is filled with water, the head on the pump is built up and the conditions approach normal. The thrust will be reduced as will the reaction on the pump case, and the weight of the water column in the casing will more than counterbalance the lifting force due to reaction. The problem of the column and shaft extension in deep-well pumps with a great number of stages is a very serious one. The normal clearances between the pump case and the runners may not be sufficient, due to shaft extension under hydraulic load. The rotating element should be adjusted so that, when stationary, the runners are near the top case wall and when the pump is running will be placed in the middle position in the case. The lifting of the rotating element due to shaft whipping is hardly possible under a load of several thousand pounds and close bearing spacing.

16 Description of the Apparatus for Measuring Axial Thrust

For measuring the axial thrust in a single-stage pump a special thrust ball bearing was designed, shown in Fig. 30-*a*. The axial thrust from the runner was transmitted through the shaft and ball bearing to a piston which was free to move axially in the bearing housing. To prevent the oil from leaking from the housing through the clearance between the sliding piston and the housing, an oil guard was provided and filled with oil to the same level as the oil in the bearing housing. The axial motion of the piston could be limited to a predetermined value by setting a stop bolt with a lock nut; otherwise the piston could move out more than would be allowed by the axial clearances between the rotating element and the stationary parts inside of the pump. The pressure due to axial thrust was transmitted by means of a bell crank with a 1 to 1 ratio of the arms directly on to the platform scales as shown in Fig. 32. The platform scales had a reduction of 40 to 1, and the scale beam had about 1 in. travel; the maximum travel for the piston was about $^{1}/_{40}$ in. The pump construction allowed the rotating element to move not less than $^{1}/_{8}$ in. toward the suction and over $^{1}/_{4}$ in. in the opposite direction. In the middle position the piston face was flush with the finished face of the bearing housing. The slightest displacement of the piston from this position could be seen by eye. In this way it was always possible to see if the rotating element was free to move, and in which direction it was thrusting.

An important item in the equipment for measuring the axial thrust was the flexible coupling. Such a coupling should allow the free axial motion of the pump rotating element and should transmit the torque without producing any radial or axial thrust load on the pump bearings. This was accomplished very satisfactorily by a coupling with leather disks as a flexible member floating on pins screwed into both coupling halves. For the axial thrust of from 200 to 400 lb., 5 lb. was sufficient to bring the system out of balance.

It should be noted that near the shut-off the axial thrust was very unsteady, the jumps being of the order of plus or minus 10 per cent. The values recorded are the average estimated values. To take up the lost motion and deflection of the scales, beams, and rods, adjusting screws were provided in both arms of the bell crank.

The axial thrust was measured with the following arrangements:

1 Runner diameter $10^{1}/_{8}$ in.:
   (*a*) No balancing means, with the case wearing ring on the back of the runner removed
   (*b*) With six radial ribs on the back of the runner, 8 in. from tip to tip, $^{3}/_{16}$ in. high and $^{5}/_{8}$ in. wide
   (*c*) With the balancing chamber connected to the suction nozzle by a 1-in.-diameter hole drilled through the casing wall opposite the $1^{1}/_{2}$-in. leakage nozzle, with a wearing ring on the back of the runner and no radial ribs
   (*d*) Same as (*a*), with the runner displaced $^{3}/_{16}$ in. from the middle position away from the suction.

2 Runner diameter $8^{1}/_{2}$ in.:
   (*a*) No balancing means
   (*b*) Six radial ribs on the back of the runner 7 in. from tip to tip
   (*c*) With balancing chamber connected to the suction nozzle by a 1-in.-diameter hole
   (*d*) With nine $^{3}/_{8}$-in. holes drilled through the runner web to connect the balancing chamber to the runner suction
   (*e*) Same as (*a*), with the runner displaced from the middle position toward the balancing chamber.

The runner could be displaced from its middle position by replacing the adjusting collar on the shaft in the thrust bearing by one $^{3}/_{16}$ in. wider, the rest of the assembly not being affected by this change.

A series of tests on axial thrust in multi-stage pumps were made at the testing department of Byron Jackson Company, Berkeley, Calif. The object of these tests was to measure the axial thrust in multi-stage pumps with runners arranged in two opposing groups and thus to check the statements arrived at analytically. The pumps tested belong to the high-speed (3600 r.p.m.) oil-pipe-line type built in sizes 3, 4, and 6 in. The capacities ranged up to 1500 g.p.m., the pressures developed were up to 1000 lb. per sq. in., with maximum power of 400 hp., special equipment being built for these tests.

Fig. 81 shows a 4-in. six-stage pump with runners arranged in pairs set back to back. The arrangement of stages is 1, 2, 6, 5, 4, 3. In Section 14 it was pointed out that this arrangement gives minimum thrust due to shoulders on the shaft and runner hubs extending through the walls separating two adjacent stages. The runner hubs of the third and fourth stages are of a larger diameter to eliminate the thrust due to this cause.

Fig. 81*a* shows a 4-in. four-stage pump with stage arrangement 1, 2, 4, 3. To balance the axial thrust caused by the shoulders on the shaft, balancing collars are provided on the runner hubs of the third and fourth stages.[31] The tests on these pumps included the measurements of the axial thrust without any balancing means and runner hubs of the same diameter. After that the pumps were balanced either by increasing the runner hub diameters of one group or by providing collars on the runner hubs. The tests were repeated with a positive head on the suction of the first stage. All tests were run at 3600 r.p.m.

The bearing for measuring the axial thrust in multi-stage pumps is essentially of the same design as that shown in Fig. 30*a*, only of a larger size and modified to take thrust in both directions. The bell crank also has been changed to transmit the thrust in both directions directly on scales. It is mounted on a special stand independent of the pump base. The height of the supporting cross-member can be adjusted to suit the pump. The picture in Fig. 82 shows the set-up for measuring the thrust outside (to the right). For thrust in opposite direction a tension link is substituted for the compression rod between the bearing and the bell-crank arm.

17 Discussion of Results of Axial Thrust in Single-Stage Pump

From what has been stated in Section 13, it is evident that, unless the pump shown in Fig. 30 has some balancing means, it will thrust to the left, and the piston holding the thrust bearing in Fig. 30*a* will move outside. Figs. 87, 88, and 89 show the axial thrust actually measured with a runner $10^{1}/_{8}$ in. in diameter, plotted against pump capacity. The head-capacity and efficiency curves are also shown for reference. Figs. 90, 91, and 92 show the same with the runner cut to $8^{1}/_{2}$ in. An inspection of these curves shows that the axial thrust varies for different capacities, being minimum at about 1.35 of the normal capacity. From this point the thrust increases both ways, and near shut-off it decreases again. The sharp-pointed minima for the thrust curves in Figs. 89 and 92 are due to an abrupt drop of the head-capacity curves at high speeds. If run with a positive head on suction, the head-capacity curve would take its normal shape and the thrust curve would be similar to those in Figs. 87, 88, 90, and 91. The shape of the axial thrust curves cannot be explained from the shape of pressure curves in Figs. 33 to 36 and 42 for points *A*, *B*, *C*, and points on the diameter of the wearing ring. These show a continuous drop of pressure from

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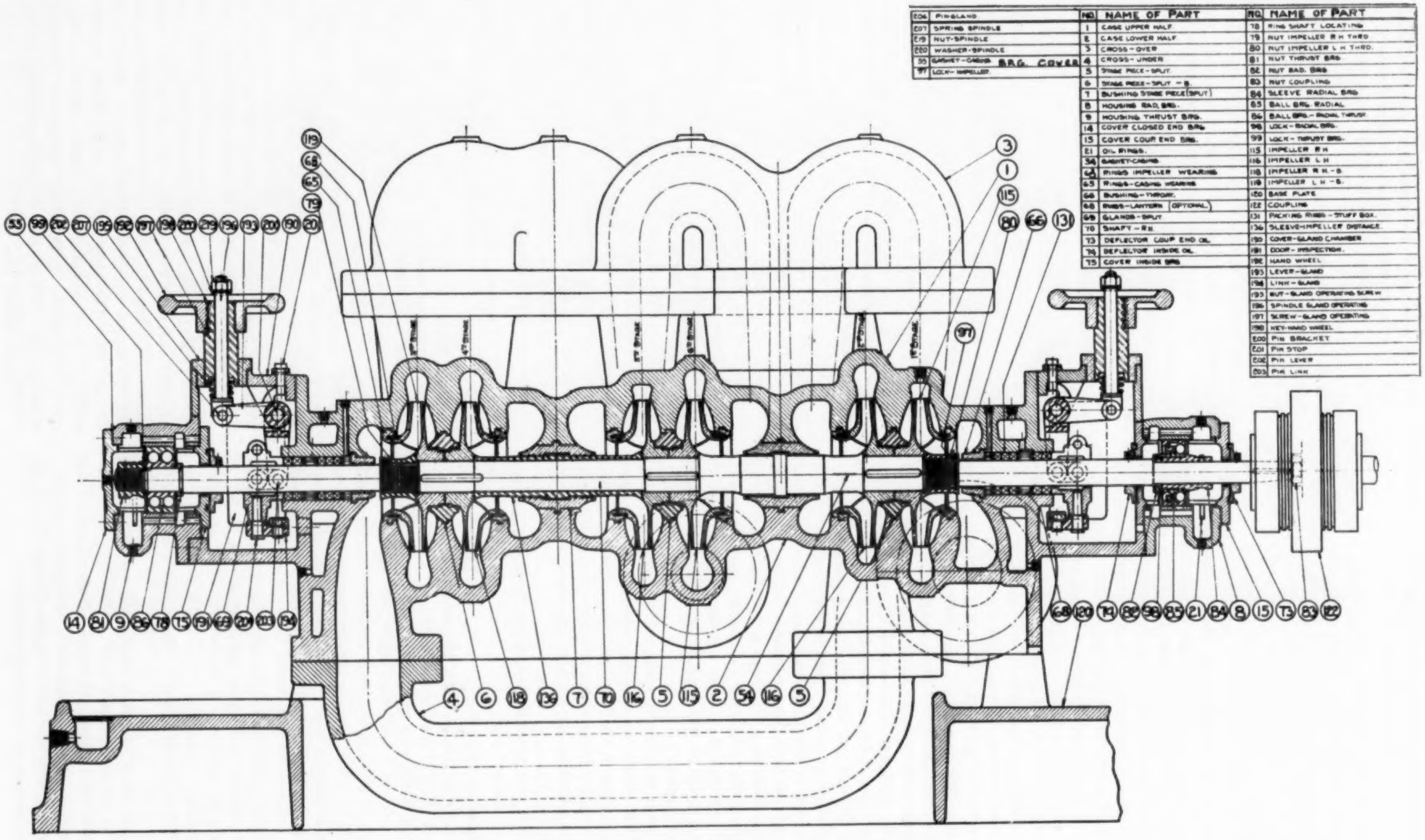
[31] Patent applied for.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68
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131
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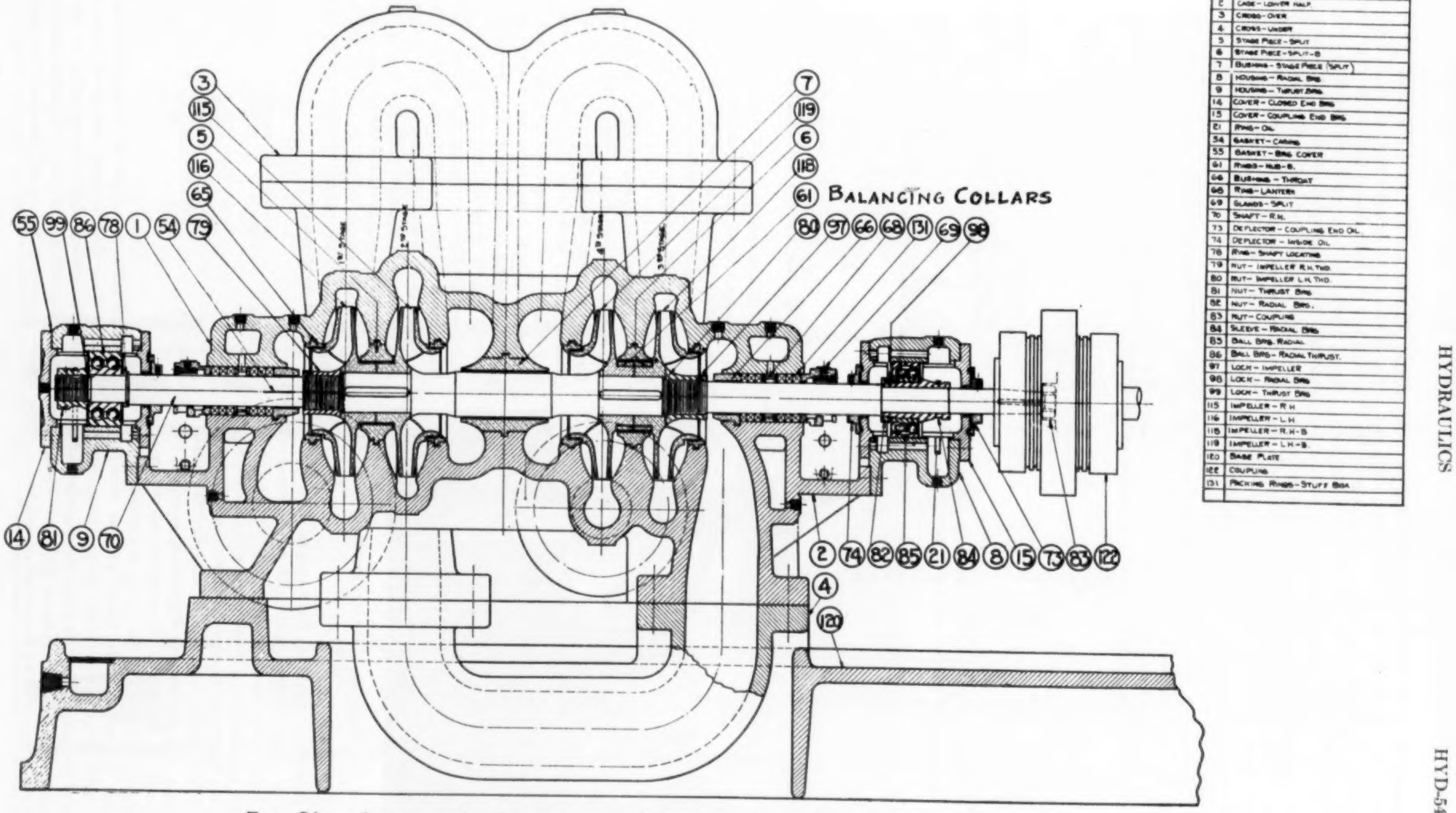
116
5
115
2
54
116
5
68
120
74
82
98
85
21
84
8
15
73
83
122

204	PIN-GLAND
207	SPRING SPINDLE
219	NUT-SPINDLE
220	WASHER-SPINDLE
55	GASKET-CASING BRG. COVER
97	LOCK-IMPELLER

NO.	NAME OF PART	NO.	NAME OF PART
1	CASE UPPER HALF	78	RING SHAFT LOCATING
2	CASE LOWER HALF	79	NUT IMPELLER R H THRD
3	CROSS-OVER	80	NUT IMPELLER L H THRD.
4	CROSS-UNDER	81	NUT THRUST BRG
5	STAGE PIECE-SPLIT.	82	NUT RAD. BRG
6	STAGE PIECE-SPLIT-B.	83	NUT COUPLING
7	BUSHING STAGE PIECE (SPLIT)	84	SLEEVE RADIAL BRG
8	HOUSING RAD. BRG.	85	BALL BRG. RADIAL
9	HOUSING THRUST BRG.	86	BALL BRG.-RADIAL THRUST
14	COVER CLOSED END BRG.	98	LOCK-RADIAL BRG.
15	COVER COUP. END BRG.	99	LOCK-THRUST BRG.
21	OIL RINGS.	115	IMPELLER R H
54	GASKET-CASING	116	IMPELLER L H
64	RINGS IMPELLER WEARING	118	IMPELLER R H-B
65	RINGS-CASING WEARING	119	IMPELLER L H-B.
66	BUSHING-THROAT.	120	BASE PLATE
68	RINGS-LANTERN (OPTIONAL)	122	COUPLING
69	GLANDS-SPLIT	131	PACKING RINGS-STUFF BOX.
70	SHAFT-RH	136	SLEEVE-IMPELLER DISTANCE.
73	DEFLECTOR COUP END OIL	190	COVER-GLAND CHAMBER
74	DEFLECTOR INSIDE OIL	191	DOOR-INSPECTION.
75	COVER INSIDE BRG.	192	HAND WHEEL
		193	LEVER-GLAND
		194	LINK-GLAND
		195	NUT-GLAND OPERATING SCREW
		196	SPINDLE GLAND OPERATING
		197	SCREW-GLAND OPERATING
		198	KEY-HAND WHEEL
		200	PIN BRACKET
		201	PIN STOP
		202	PIN LEVER
		203	PIN LINK

FIG. 81 SECTIONAL ELEVATION 6-STAGE MULTIPLEX-PUMP


55
99
86
78
1
54
79
3
115
5
116
65
7
119
6
118
61 BALANCING COLLARS
80
97
66
68
131
69
98
1st STAGE
2nd STAGE
3rd STAGE
4th STAGE
14
81
9
70
2
74
82
85
21
84
8
15
73
83
122
4
120

NO	NAME OF PART
1	CASE – UPPER HALF
2	CASE – LOWER HALF
3	CROSS – OVER
4	CROSS – UNDER
5	STAGE PIECE – SPLIT
6	STAGE PIECE – SPLIT – B
7	BUSHING – STAGE PIECE (SPLIT)
8	HOUSING – RADIAL BRG
9	HOUSING – THRUST BRG
14	COVER – CLOSED END BRG
15	COVER – COUPLING END BRG
21	RING – OIL
54	GASKET – CASING
55	GASKET – BRG COVER
61	RINGS – HUB – B.
66	BUSHING – THROAT
68	RING – LANTERN
69	GLANDS – SPLIT
70	SHAFT – R.H.
73	DEFLECTOR – COUPLING END OIL
74	DEFLECTOR – INSIDE OIL
78	RING – SHAFT LOCATING
79	NUT – IMPELLER R.H. THD
80	NUT – IMPELLER L.H. THD.
81	NUT – THRUST BRG
82	NUT – RADIAL BRG.
83	NUT – COUPLING
84	SLEEVE – RADIAL BRG
85	BALL BRG. RADIAL
86	BALL BRG – RADIAL THRUST.
97	LOCK – IMPELLER
98	LOCK – RADIAL BRG
99	LOCK – THRUST BRG
115	IMPELLER – R H
116	IMPELLER – L H
118	IMPELLER – R.H – B
119	IMPELLER – L H – B.
120	BASE PLATE
122	COUPLING
131	PACKING RINGS – STUFF BOX

FIG. 81a SECTIONAL ELEVATION 4-STAGE O. L. MULTIPLEX-PUMP

the shut-off point to the point at which the discharge valve is wide open. There is a small increase in the thrust with the increase of capacity. This is due to a decrease in pressure in the runner eye on account of the increase of the velocity head through the runner eye $V_e^2/2g$. This, however, is more than counterbalanced by the increase in the component of thrust which is due to a change of momentum in the axial direction of the incoming flow and which is directed opposite the first. This is equal to WV_e/g or $A\gamma V_e^2/g$, which is twice the thrust due to velocity head. Thus the net effect of the two is a decrease of the thrust to the left, with an increase of capacity equal to $V_e^2/2g$ per unit area of the runner eye. To give an idea of the magnitude of the forces corresponding to WV_e/g and $V_e^2/2g$, these are calculated for the following normal capacities and speeds:

Speed, r.p.m.	Capacity, g.p.m.	V_e, ft-sec.	$V_e^2/2g$, ft.	WV_e/g, lb.	Measured thrust, lb.
1400	270	10.35	1.66	12.0	190
1700	325	12.5	2.4	17.0	265
2000	375	14.4	3.22	23.3	420
2500	475	18.2	5.1	37.3	...

From these considerations it is seen that the shape of the thrust curves cannot be explained by the change of pressure outside the runner. However, considering the causes of the axial thrust inside the runner, a satisfactory explanation can be found. It is the aim of a good pump design to shape the vanes and runner profiles so that an equal head is produced along all the stream lines so that the losses due to cross-flows and back-flow will be eliminated. For a runner with a plain vane and the length of the stream lines near the shrouds approximately equal, this is possible if the radial velocity is the same for all stream lines. With a 90-deg. turn in front of the vanes, neglecting the effect of friction near the runner walls, this is possible only for one value of the radial velocity, and hence for one pump capacity. On both sides of this point, since the radial velocities are different near the runner shrouds, the pressures produced along these stream lines are not equal. This causes unbalanced forces on the runner shrouds inside. Since the axial thrust is to the left (Fig. 30) for small capacities near shut-off, there must be higher pressure on the back shroud than on the front shroud, the difference between the two producing the drop of axial thrust near shut-off. For large capacities the reverse is true. The pressures on the front shroud are higher than those on the back shroud; the difference between the two produces the increase of the axial thrust for high capacities regardless of the rapid decrease of pressures in the pump case. The velocity distribution necessary to produce this effect is as follows: At small capacities, higher radial velocities are near the inner wall of the turn (at front shroud) and low velocities are at the outer side of the turn (Fig. 22). Then the head produced at the back shroud will be higher than that at the front shroud, as explained in Section 13. At high radial velocities (large capacities) the picture is reversed. Higher velocities are at the back shroud and low at the front (Fig. 23).

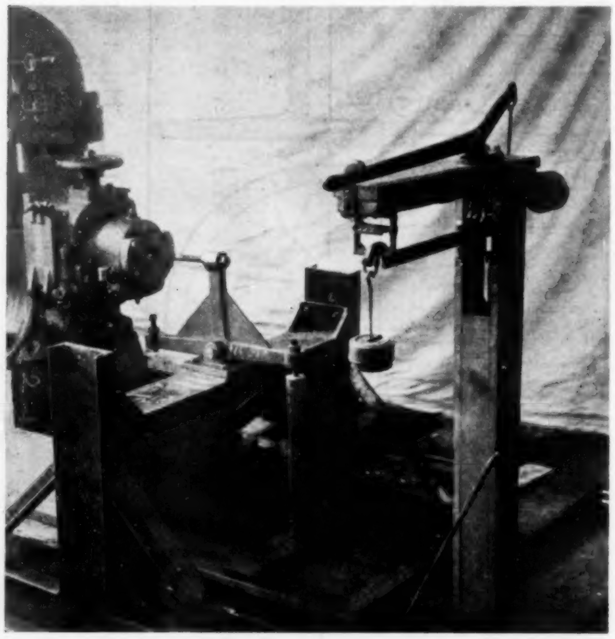


FIG. 82 SET-UP FOR MEASURING AXIAL THRUST OF A SIX-STAGE PUMP

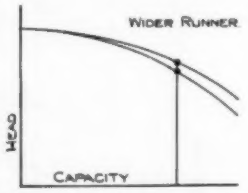


FIG. 83 DIFFERENCE IN HEAD DUE TO A DIFFERENCE IN RUNNER WIDTH

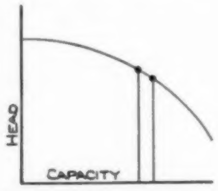


FIG. 84 DIFFERENCE IN HEAD DUE TO A DIFFERENCE IN LEAKAGE

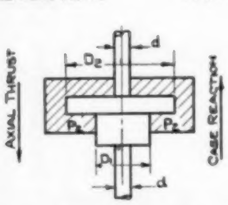


FIG 85 CASE REACTION AND AXIAL THRUST $T = .785(D_1^2 - d^2)(P_2 - P_1)$

Irregularities in the form of sudden breaks and bumps in the shape of the head-capacity curves which are often observed with the wide runners (high specific speed) are usually explained by cross-flows and back-flows at the runner discharge, as a result of a difference in the head produced along the stream lines near the runner shrouds. But it has never been mentioned that these conditions cause an axial thrust from the inside of the runner.

From Figs. 87 to 92 it is seen that at shut-off, when the mean radial velocity through the runner is negligible (only the leakage water being circulated through the runner), the axial thrust is due only to the external pressure on the runner. Taking the pressure on the back side of the runner over the area opposite the runner eye equal to that at the wearing ring, which is approximately true (Figs. 33, 34, 35, 36, and 42), and multiplying it by the net area of the runner eye (12.0 sq. in.) gives very closely the thrust as measured:

Speed, r.p.m.	Runner diameter, in.	Pressure at ring, ft.	Thrust calculated, lb.	Thrust measured, lb.
1400	$10\frac{1}{8}$	48.5	252	255
1700	$10\frac{1}{8}$	71.0	370	360
2000	$10\frac{1}{8}$	96.0	500	500
1700	$8\frac{1}{2}$	...	...	280
2000	$8\frac{1}{2}$	69.0	360	375
2500	$8\frac{1}{2}$	107.5	560	560

For all other capacities the thrust calculated in this way is smaller than that actually measured, approximating it for capacities between those corresponding to the maximum efficiency and minimum thrust. With the valve wide open (maximum capacity), the thrust calculated is considerably smaller

Nº	NAME OF PART
1	Strainer
2	Suction Pipe
3	Suction Flange
4	Bottom Case Bearing Bushing
5	Bottom Case
6	Sand Cap
7	Series Case Std
9	Keys For Series Case Runners
9a	Key For Top Case Runner
10	Top Case With Stuffing Box
11	Case Ring
12	Top Case Runner
13	Series Case Runner
14	Stuffing Box Ring
15	Key Ring
16	Pump Shaft
17	Top Case Flange
18	Leather Cup
19	Packing
20	Case Wear Rings
21	Top Case Bearing
22	Shaft Coupling Intermediate
23	Plain Bearing
24	Outer Column Spider
24a	Rubber For Spider
25	Outer Column Coupling For T.& B. Column
27	Column Shaft
27a	6 Ft Section of Shaft
28	Adjusting Flange
29	Adjusting Flange Nut
30	T-Bearing Top Section Lower
31	T-Bearing Top Section Upper
32	T-Bearing Bottom Section
33	Head Shaft Coupling
34	Pump Shaft Coupling
35	Inner Column Std 6 Ft Sections
35a	Inner Column Through Head
35b	Inner Column Top Section [Pipe Col. Only]
35c	Inner Column Bottom Section
35d	Inner Column Bottom Section To Pump
36	Outer Column Intermediate
36a	Outer Column Top Section
36b	Outer Column Bottom Section
36c	6 Ft Section Of Outer Column
37	Discharge Elbow
38	Pipe Plug
39	Adjusting Nut Collar
39a	Rubber Sealing Ring
40	Adjusting Nut Lock
42	Dust Cover
43	Oil Retainer Lower Bearing
43K	Oil Retainer Kingsbury Bearing
44	Sleeve Nut Lower
45	Ball Bearing Lower
46	Head Shaft
47	Air Deflector Lower
48	Hollow Sleeve
49	Oil Cover
50	Rotor
51	Stator
52	Motor Cover
53	Oil Retainer Upper Bearing
54	Air Deflector Upper
55	Oil Splasher
56	Head Shaft Keys
56K	Shaft Key For Kingsbury Bearing
57	Ball Bearing Upper
58	Sleeve Nut Upper
60	Head Shaft Nut
61a	Top Cap
64	Oil Cup Upper & Lower Bearing
65	Sight Feed Oil Cup
66	Motor Barrel D.C.
67	Motor Half Coupling
68K	Pump Half Coupling
69	Coupling Pins
69a	Leather Links
73	Oil Pipe Bracket
74	Thrust Pot B.I. Head
75	Thrust Pot D.C. Head
76	Thrust Pot Cover D.C. Head
78	Kingsbury Thrust Bearing Runner
79	Kingsbury Thrust Bearing Shoe
80	Kingsbury Thrust Brg. Leveling Plate
81	Kingsbury Thrust Brg. Base Ring
82	Distance Piece For Kings. Brg. B.I.
82a	Distance Piece For Kings. Brg. D.C.
83	Water Cooling Connection

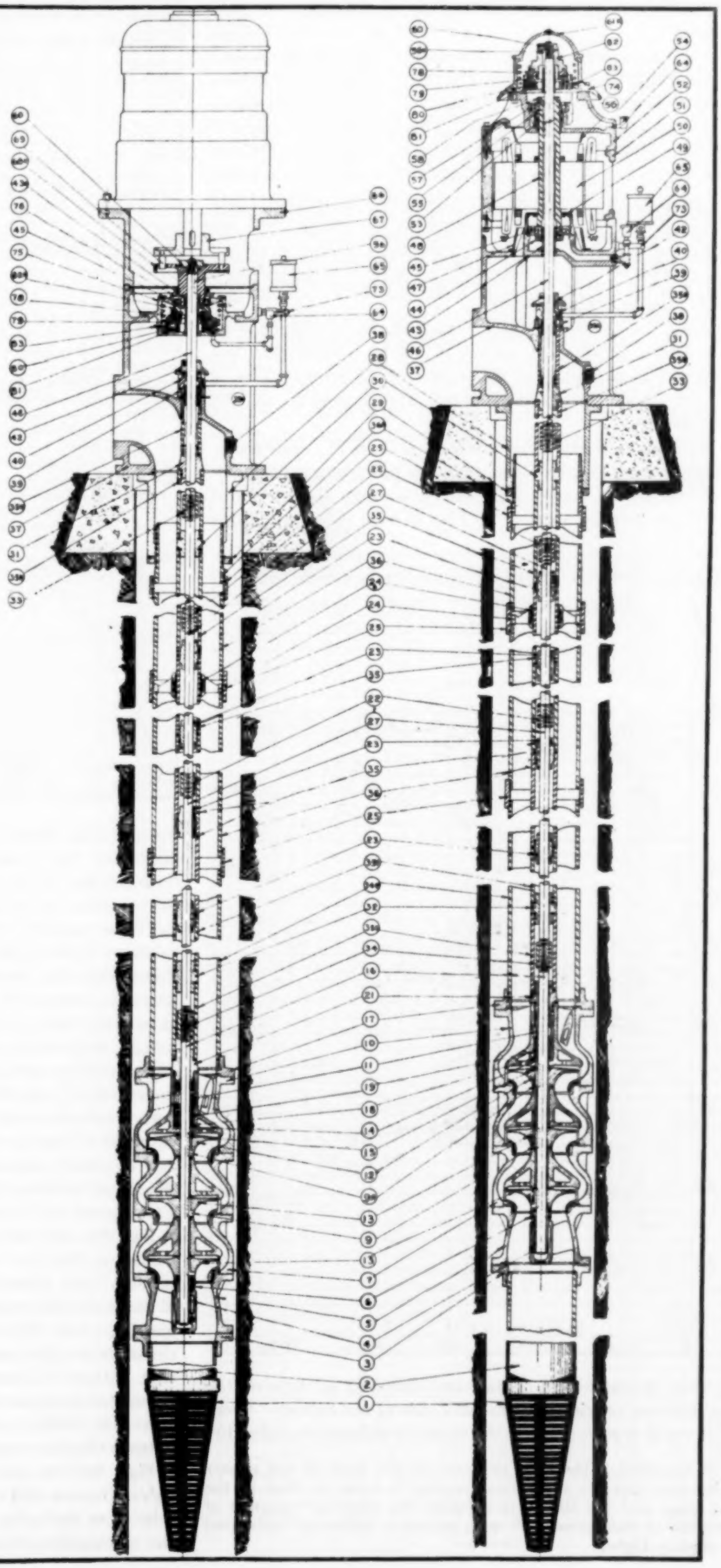


Fig. 86

than that obtained by measurement,[32] as is seen from the following table:

Speed, r.p.m.	Runner diameter, in.	Capacity, g.p.m.	Pressure at ring, ft.	Calculated thrust, lb.	Measured thrust, lb.
1400	$10^1/_8$	450	22.5	117	180
1700	$10^1/_8$	525	30.0	155	260
2000	$10^1/_8$	575	42.5	221	325
1700	$8^1/_2$	450	....	...	250
2000	$8^1/_2$	475	31.0	160	305
2500	$8^1/_2$	570	40–45	220	400

It is believed that the difference between the thrust calculated and the thrust measured is due to the uneven pressure on the runner shrouds inside. This is caused by the difference in the head produced as a result of the uneven radial velocities along the stream lines near the shrouds.

Comparing the thrust at shut-off and maximum efficiency of the same runner at different speeds, it will be noticed that the thrust varies approximately as the square of the speed. This is because the pressures produced vary approximately as the square of the speed.

Figs. 96, 97, and 98 show the axial thrust for a $10^1/_8$-in. runner with the balancing chamber connected to the pump suction by means of one 1-in.-diameter hole drilled through the case wall opposite to the leakage nozzle. In Fig. 30 a set of wearing rings with a close clearance and of the same diameter as those on the suction side was provided on the back side of the runner. From these curves it is seen (1) that the thrust is reduced considerably but not eliminated, (2) that the shape of the thrust curves is similar to the shape of those for the pump with no balancing means (Figs. 87, 88, and 89), (3) that the minimum thrust with no balancing is reduced to about one-half and takes place at the same capacity, (4) that the thrust at the shut-off is reduced to about 0.25 of that with no balancing means. The reason for this is that the increase of pressure in the pump case with the decrease in capacity does not affect the pressure in the balancing chamber, while with no balancing means the pressure on this portion of the runner shroud increases toward the shut-off point.

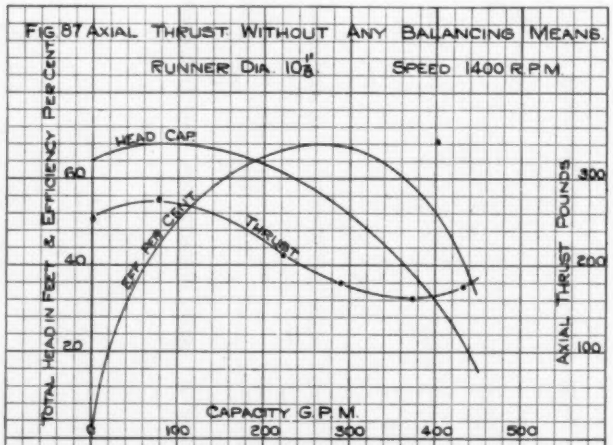


FIG 87 AXIAL THRUST WITHOUT ANY BALANCING MEANS. RUNNER DIA. 10 1/8" SPEED 1400 R.P.M.

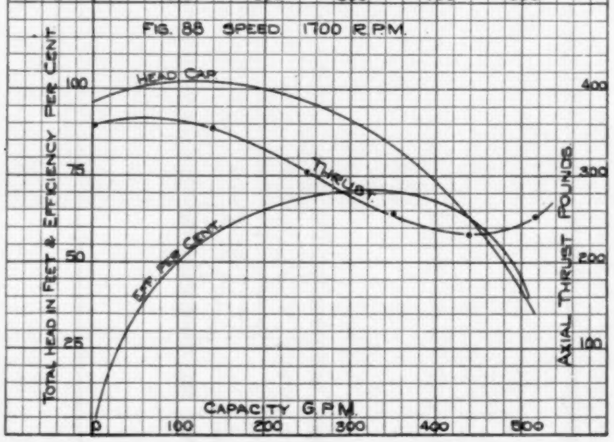


FIG. 88 SPEED 1700 R.P.M.

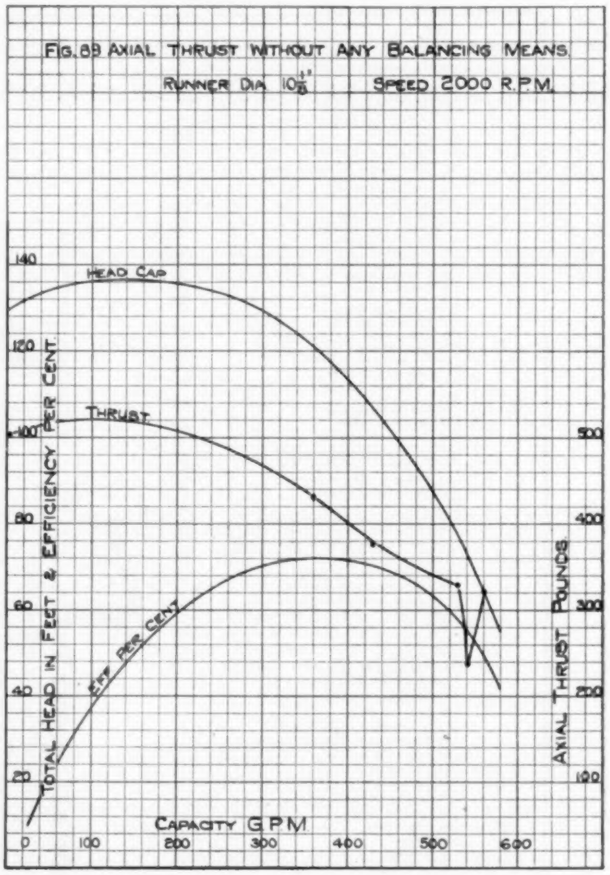


FIG. 89 AXIAL THRUST WITHOUT ANY BALANCING MEANS. RUNNER DIA. 10 1/8" SPEED 2000 R.P.M.

It should be noted that the thrust at shut-off is lower than the minimum at capacities over the normal. In Section 13 it has been shown that the pressure in the balancing chamber cannot be equal to that in the runner eye. The difference in pressure due to stated causes is, however, not sufficient to account for the thrust obtained with this method of balancing. Apparently a portion of the thrust originates inside of the runner.

Figs. 99, 100, 101, and 102 show the thrust measured with the balancing chamber connected to the runner suction by means of nine $^3/_8$-in. holes drilled through the runner web. The shape of these curves is similar to the shape of those obtained with one 1-in. hole drilled to the pump suction; but the minimum thrust is smaller with nine holes, being approximately 0.25 of that without balance. This is because the total area of nine $^3/_8$-in. holes (equal to 0.99 sq. in.) is greater than that of one 1-in. hole (which is 0.785 sq. in.), which makes pressure in the balance chamber nearer that in the runner eye.

Figs. 93, 94, and 95 show the axial thrust curves with an $8^1/_2$-in. runner and six radial ribs $^5/_8$ in. wide, $^3/_{16}$ in. thick, and 7 in. from tip to tip. From these tests it is seen that, as far as the elimination of the axial thrust is concerned, this method is

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[32] The assumption that the pressure on the back of the runner over the area opposite the runner suction is equal to that at the wearing ring, and the failure to consider the effect of variation of the pressure at the runner eye with capacity, make the calculated thrust slightly higher.

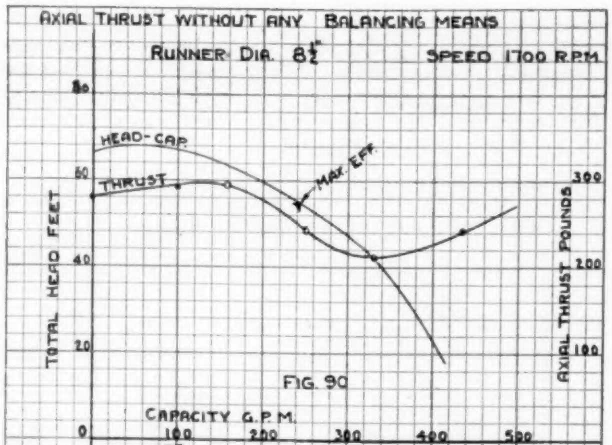


FIG. 90

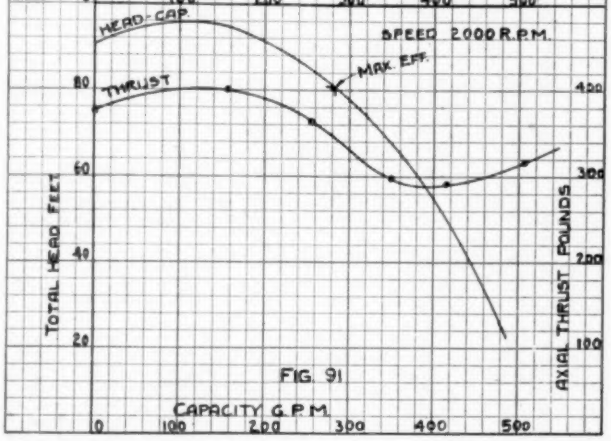


FIG. 91

most efficient. The thrust is not only eliminated but reversed in many cases. The effect of the radial ribs on the back side of the runner is to increase the angular velocity of rotation of water in the space between the runner and case walls. This reduces the pressure on this side of the runner so that the difference between the pressures on the two runner shrouds from the outside reduces or balances the thrust caused by unbalanced pressure on the back side of the runner over the area opposite to the runner suction. In some cases this difference produces a thrust in the opposite direction. The thrust occasioned by the pressure on the back side of the runner over the area opposite the runner suction is considerably lower with the radial ribs on the runner than without them because the pressure at the wearing ring is reduced approximately in the ratio of 1 to 0.62. The mean angular velocity of water in the space between the runner and case without ribs is calculated to be 0.467 ω, where ω is the runner angular velocity. With radial ribs the angular velocity increased to 0.632 ω for *A*, 0.568 ω for *B*, and 0.512 ω for *C*.

The shape of thrust curves for runners with radial ribs is similar to those for runners without any balancing means, the thrust being reduced approximately by the same amount for all capacities. The amount of reduction of thrust caused by ribs on the runner backs varies as the square of the speed.

It should be mentioned that for small capacities approaching the shut-off the thrust is very unstable, especially with the radial ribs on the back of the runner. The probable reason for this is that this portion of the head-capacity curve is unstable.

Figs. 43, 44, and 45 show the axial thrust with six radial ribs $^5/_8$ in. wide, $^3/_{16}$ in. thick, and 8 in. from tip to tip for the runner $10^1/_8$ in. in diameter. The shape of the curves is similar to the shape of those discussed, and what has been said about the first applies to the latter.

A series of tests with the runner displaced from the middle position $^3/_{16}$ in. away from the suction does not show any appreciable difference in the axial thrust, the difference being within the error of observation. The explanation of this is probably in the fact that, even with the runner displaced $^3/_{16}$ in., there is still a clearance between the runner and the case of about $^1/_4$ in., and according to Newton's assumption that if the space between the two surfaces, one stationary and one moving, is filled with liquid, the liquid velocities vary linearly from zero at the stationary surface to a maximum equal to the velocity of the moving surface. Thus the mean velocity in both cases is the same, equal to one-half of that of the moving surface. It is interesting to note that with the runner displaced from the middle position $^3/_{16}$ in., the rotating element has a tendency toward axial vibration, which can be seen by eye or felt by hand at the movable bearing of the pump. This is due to jet reaction of the runner discharging against the case wall on one side.

18 Discussion of Tests on Axial Thrust in Multi-Stage Pumps

The calculations of the axial thrust in multi-stage pumps with runners arranged back to back, as advanced in Section 14, are based on the assumption that every runner produces an equal portion of the total head. While this is aimed at in the normal design, in practice, however, the heads produced in various stages may differ, due to several causes, the most common of which are as follows:

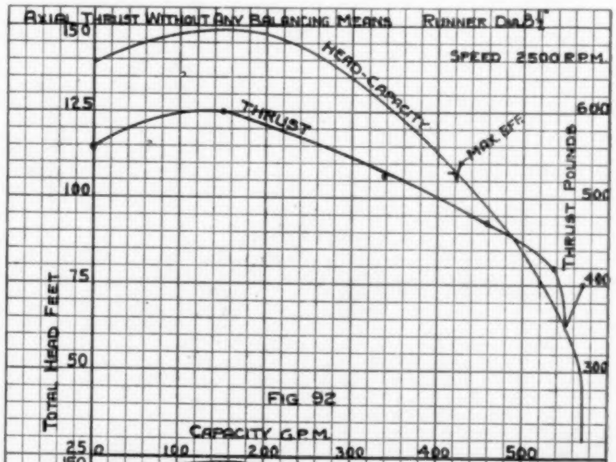


FIG. 92

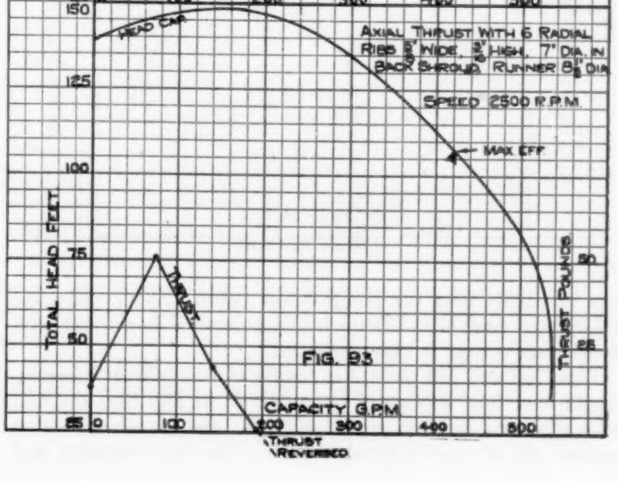


FIG. 93

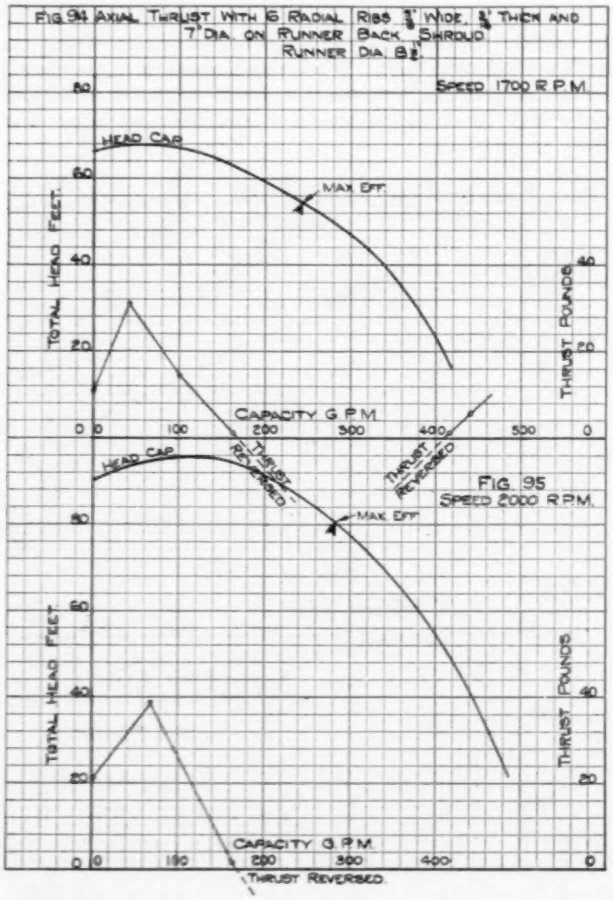
(1) A difference in the runner width due to inaccuracy of castings. With runners of different widths having different head-capacity curves and working at the same capacity, the wider runner will work at a higher head, as shown in Fig. 83.

(2) With the identical runners, if the leakage from stage to stage or through the stuffing boxes or leak-offs is different, some runners will work at a higher capacity and hence a lower head, as is seen from Fig. 89.

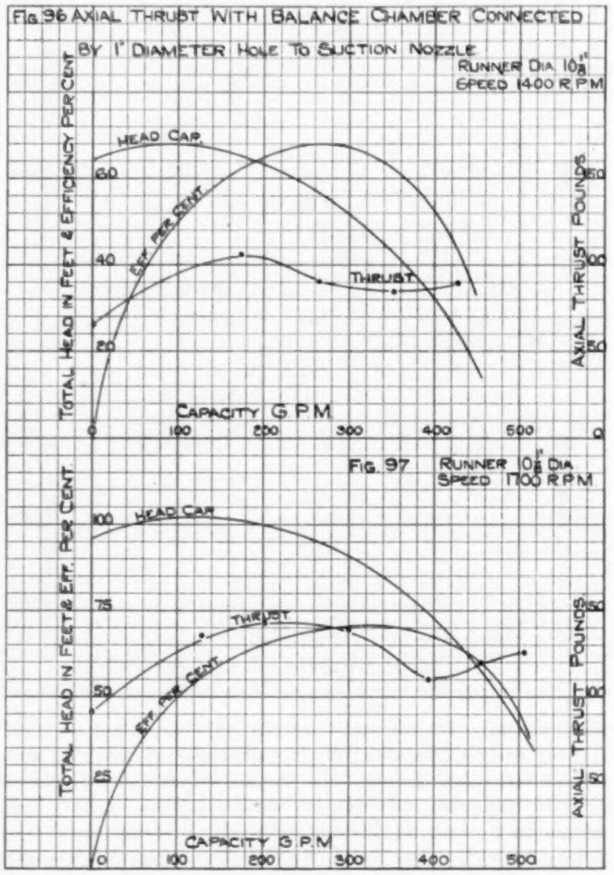
(3) If the pump is working under suction lift, the head-capacity curve for the first stage may be different from that for the rest of the runners, depending on the amount of the suction lift. This difference is greater for larger capacities and may be quite considerable near the pump cut-off, as the first stage may cut off earlier than the rest of the stages.

(4) For mechanical reasons in multi-stage pumps the form of the suction nozzle for different stages is very often made differently. Since the position of the stop and the shape of the channel of approach to the runner entrance affect the amount of pre-rotation of incoming water, the head produced will vary, and unbalanced axial forces will result.

The variation in the head produced by different stages due to all the foregoing causes is more pronounced at high capacities, and especially near the pump cut-off point. This explains the unexpected axial thrust frequently encountered in practice with multi-stage pumps at high capacities.



Figs. 103 and 104 show the results of axial thrust measurements of a 4-in. four-stage pump shown in Fig. 81*a*, the first with a positive head on suction of 60 lb., and the second with a suction lift of approximately 10 ft. The head-capacity and efficiency curves are also given for reference. The results represented by these curves are typical for the whole line of pumps of this kind. From inspection of these curves it is seen that the variation of the thrust for different capacities is mainly due to causes from inside the runner, but to these are added several new, peculiar to multi-stage pumps.



From these tests it is seen that without balancing rings the axial thrust for capacities near the normal is over 600 lb. with a suction lift and over 700 lb. with a positive head on suction, the direction of thrust being toward the first stage suction in both cases, as anticipated. With the balancing collars the thrust is brought practically to zero in both cases. The reduction of thrust is approximately the same for all capacities. The difference in the thrust with suction lift and positive head on suction is probably due to the fact that with the suction lift the first stage produced a head slightly lower than the rest of the stages; hence the pressure on the back of the runner of the second stage is not quite balanced by that of the first stage, causing the total thrust toward the first stage suction to decrease. This becomes more pronounced at capacities over 800 g.p.m., and at capacities near cut-off the thrust is reversed. The measurements of pressure between the stages show that near cut-off the head produced by the first stage drops considerably faster than in the rest of the stages. The unbalanced thrust produced by the second stage becomes more than enough to balance the thrust caused by the shoulders on the shaft and runner hubs. The pressure measurements between the stages for the points other than near cut-off did not allow the making of any definite conclusions because the readings were taken on the curved interstage piping with different velocities.

The increase of axial thrust at capacities over 1100 g.p.m. with a positive head on suction may be caused by the local vacuous spaces (cavitation) inside the runner near the back shroud at the vane entrance tips. This local cavitation is not followed by a sudden drop in the head produced by the first stage; otherwise, the pressure on the back of the runner of this stage would be lowered and the thrust toward the first stage suction would be smaller, as takes place with the suction lift.

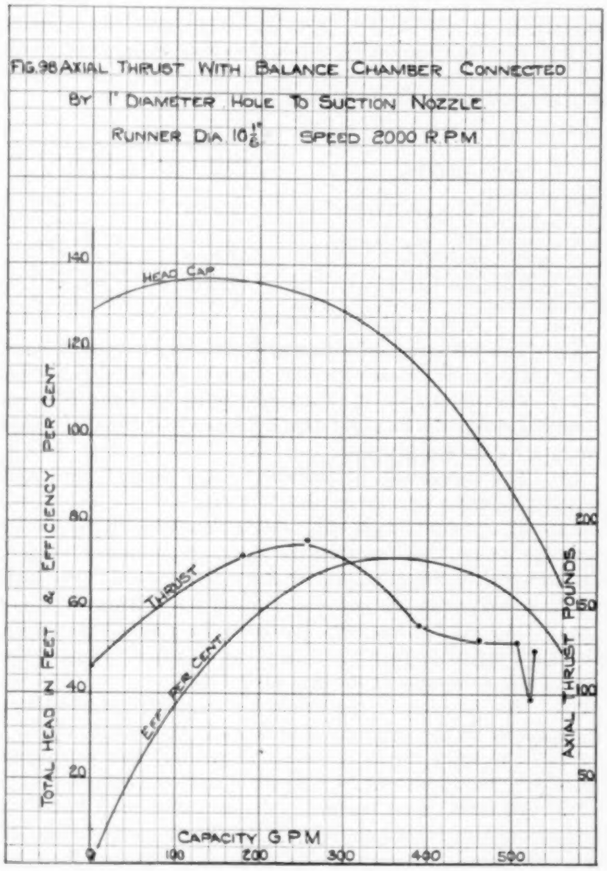


FIG. 98 AXIAL THRUST WITH BALANCE CHAMBER CONNECTED BY 1" DIAMETER HOLE TO SUCTION NOZZLE RUNNER DIA $10\frac{1}{8}$" SPEED 2000 R.P.M.

The cause of the cavitation at the vane tips at the entrance lies in the fact that at high capacities water enters the vane with a high velocity and at a higher angle than that of the vane, producing dead vacuous spaces on the front side of the vane. Air and vapor bubbles appear and collapse in these spots, producing noise and causing pitting of the vane. Depending on the runner profile and form of the approach to the runner, the cavitation may take place either near the front or the back runner shrouds. The axial thrust caused by the cavitation varies between the wide limits, sometimes changing the direction, setting the whole pump in vibration. It is quite probable that the increase of the thrust at high capacities in the case of the single-stage pump, as discussed in Section 17, is also due to cavitation.

It should be noted that the pump efficiency with the suction lift is one point higher than with a positive head on suction for capacities near that at which the cavitation starts (800–900 g.p.m.). This was confirmed by repeated tests and was observed on several single-stage double-suction pumps. This is in agreement with water turbine practice as described in "Hydraulische Probleme."[33] The explanation is found in the fact that, at conditions of approaching cavitation, water leaves the runner walls, and the friction and shock losses are thus reduced and the efficiency is improved.

PART III CONCLUSIONS

WHILE some of the conclusions based on these tests are quite general and hold for all types of centrifugal pumps, the others are limited in their application to the conditions duplicating or approaching those under which these tests were made. Briefly the results of this study can be summarized as follows:

(1) In a centrifugal pump with a volute case, the maximum pressure is at the discharge flange for capacities lower than the normal. For capacities greater than the normal, the pressure, in the volute case, is higher than that at the discharge flange. The reason for this lies in the case design.

(2) The pressures at different points of the volute are equal at the point of maximum efficiency. There is no side thrust on the runner from the volute case under these conditions. On both sides of this point pressures vary along the volute.

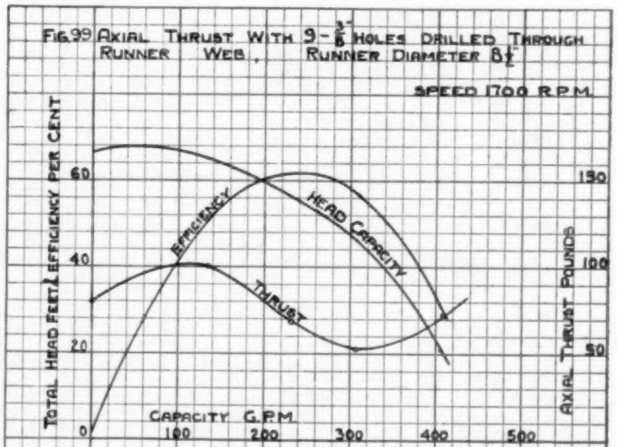


FIG. 99 AXIAL THRUST WITH 9 - $\frac{3}{8}$" HOLES DRILLED THROUGH RUNNER WEB. RUNNER DIAMETER $8\frac{1}{2}$" SPEED 1700 R.P.M.

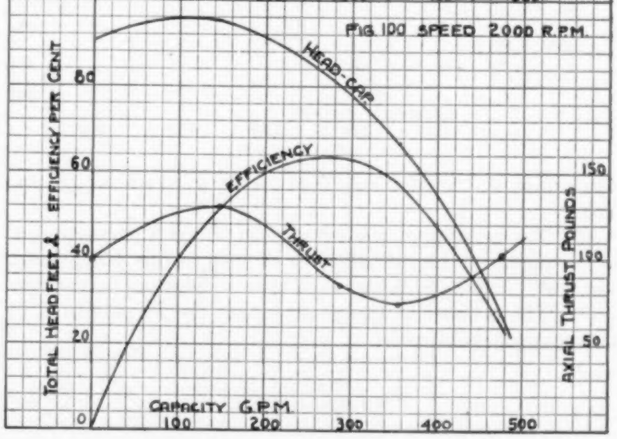


FIG. 100 SPEED 2000 R.P.M.

(3) The volute pressure at the point of maximum efficiency can be determined with a sufficient degree of accuracy from the pump discharge pressure by assuming the nozzle efficiency. For the pump tested the nozzle efficiency was found to be 60 per cent.

(4) The pressures at different radii in the space between the runner and case walls can be expressed by the formula:

$$H_a = H_v - \varphi^2 \frac{(u_2{}^2 - u_a{}^2)}{2g}$$

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[33] V.D.I. Verlag, 1926, Berlin.

For notation see Section 9. The coefficient φ was found to be 0.467 and is constant for all speeds, diameters, and small runner cuts.

(5) With radial ribs on the back of the runner, the angular velocity of rotation of water in the space between the runner and the case is increased considerably. This reduces the pressure on this side of the runner according to the preceding formula.

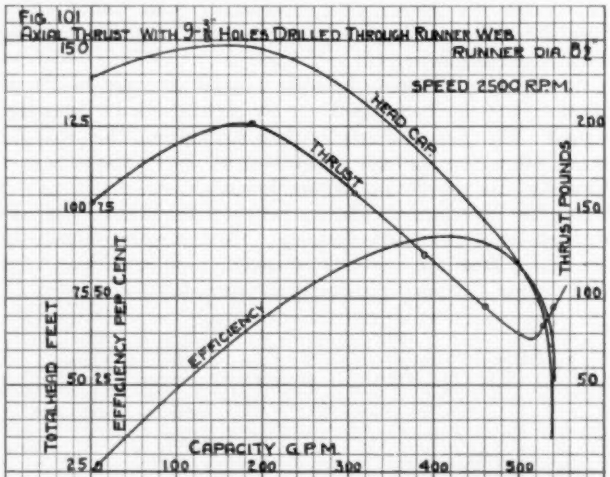
FIG. 101

AXIAL THRUST WITH 9-⅜" HOLES DRILLED THROUGH RUNNER WEB

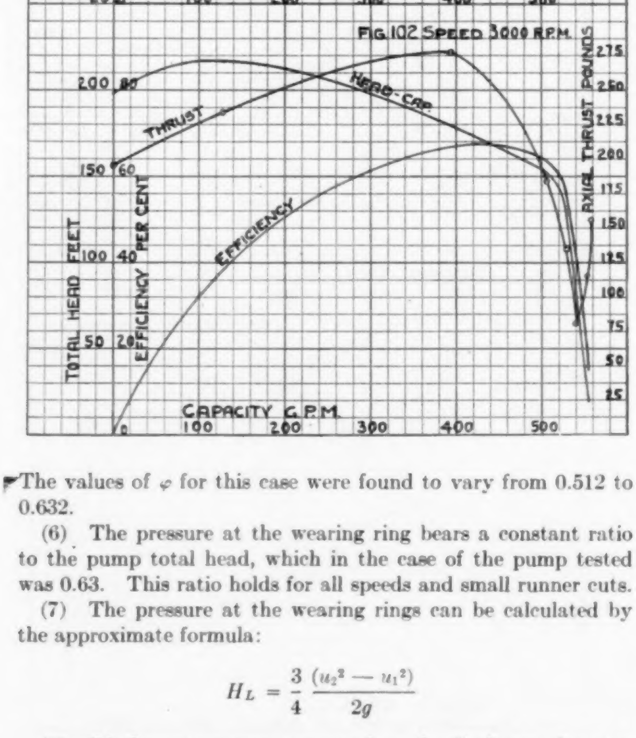
FIG. 102 SPEED 3000 R.P.M.

The values of φ for this case were found to vary from 0.512 to 0.632.

(6) The pressure at the wearing ring bears a constant ratio to the pump total head, which in the case of the pump tested was 0.63. This ratio holds for all speeds and small runner cuts.

(7) The pressure at the wearing rings can be calculated by the approximate formula:

$$H_L = \frac{3}{4}\,\frac{(u_2^2 - u_1^2)}{2g}$$

(8) Discharge pressures measured on the discharge pipe at a distance of from 5 to 10 pipe diameters from the discharge flange are higher than those at the discharge nozzle for capacities over the normal. The reason for this is that the actual velocity at the flange is higher than that corresponding to the discharge area, a portion of the section being occupied by dead turbulent water.

(9) If the amount of leakage is represented by the formula $Q_L = CA\sqrt{(2gH_L)}$, the coefficient of discharge varies as follows: (*a*) For the same speed the coefficient C increases with the pressure H_L. (*b*) For the same pressure it decreases slightly with the increase of speed. (*c*) For the same clearance, speed, and pressure the coefficient C increases as the width of the wearing ring decreases (Fig. 66). (*d*) For the same width, speed, and pressure the coefficient C decreases as the radial clearance is decreased (Fig. 68).

(10) The following table shows the advantages of the different types of wearing rings tested over a plain ring of the same width and radial clearance, for operation at the point of maximum efficiency and for speeds from 1400 to 2500 r.p.m.:

Type of ring	Radial clearance on diameter, in.	Saving in leakage in per cent of that for a plain ring
Single labyrinth	0.0122	41.0 to 46.6
Circular grooves with rough edges	0.0170	32.2 to 35.5
Same with smooth edges	0.0170	16.7 to 20.0
Circular grooves with smooth edges	0.0294	2.2 to 4.0
Spiral grooves, single thread	0.0115	20.8 to 27.0
Spiral grooves, triple thread	0.0115	33.0 to 36.8
Spiral grooves, triple thread	0.0215	24.7 to 26.8
Stepped ring	0.0100	24.0 to 33.0

(11) The advantage of a ring with circular or spiral grooves over a plain one decreases with an increase of the radial clearance.

(12) The spiral grooves of a higher pitch are more effective in cutting down the leakage; this indicates that there is actual pumping by the spiral groove.

(13) The percentage of leakage for the normal head and capacity increases with an increase of speed.

(14) The power to drive the rotating element with the runner discharge sealed, together with the power to pump the leakage, is considerably lower than that at the pump shut-off. For high specific speed pumps the power at shut-off is a maximum.

(15) The axial thrust with no balancing means varies with the pump capacity, showing a minimum at capacities over the normal and a decrease at the shut-off point.

(16) The increase in the thrust for large capacities cannot be explained by the variation of pressure in the pump case.

(17) The axial thrust measured is greater than calculated considering the forces due to pressure outside the runner.

(18) In addition to the thrust due to unbalanced forces on the runner shrouds from the outside, there is an axial thrust due to a difference in pressure on the inside of the runner shrouds, which results from a difference in the head produced along the two stream lines near the runner shrouds and cavitation. Of the several causes of this, the uneven radial velocity distribution is most important.

(19) The balancing of the axial thrust by means of a chamber on the back of the runner of the same diameter as the runner suction does not eliminate the axial thrust, but reduces it to about 25 per cent for the useful part of the head-capacity curve.

(20) With an ample clearance between the runner and the pump case, a small runner displacement from the middle position does not affect the axial thrust.

(21) Multi-stage pumps with runners arranged in two groups back to back, and usually considered hydraulically balanced, have an axial thrust due to a variation in the shaft diameter and the runner hubs extending through the walls separating two adjacent stages. The amount of thrust due to this cause depends on the arrangement of stages.

(22) The axial thrust due to the shoulders on the shaft and runner hubs can be balanced by making the runner hubs of different diameters.

(23) In multi-stage pumps with runners arranged in two opposing groups, the variation in the head produced by different stages causes axial thrust, as symmetry is destroyed under these conditions.

(24) The cavitation or formation and collapsing of vapor-filled vacuous spaces inside the runner, manifested by noise, vibration, and runner pitting, is always followed by a consider-

able change in the axial thrust. The direction of the thrust due to this cause depends on the location of vacuous spaces, which in turn depends upon the runner profile and form of the approach to the runner. Causes of cavitation are high suction lift, high velocities (overspeeding), and difference between the direction of the incoming water and the vane angle at the entrance.

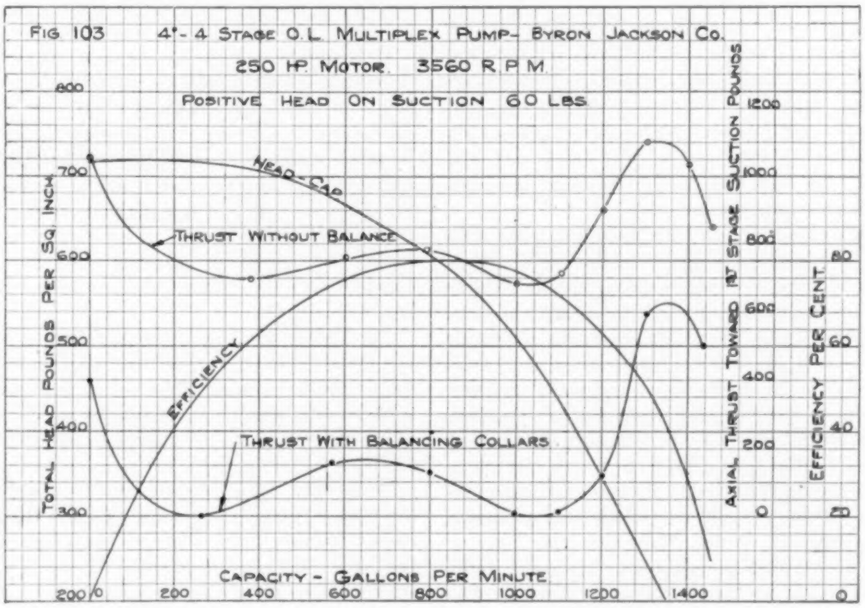
Fig 103

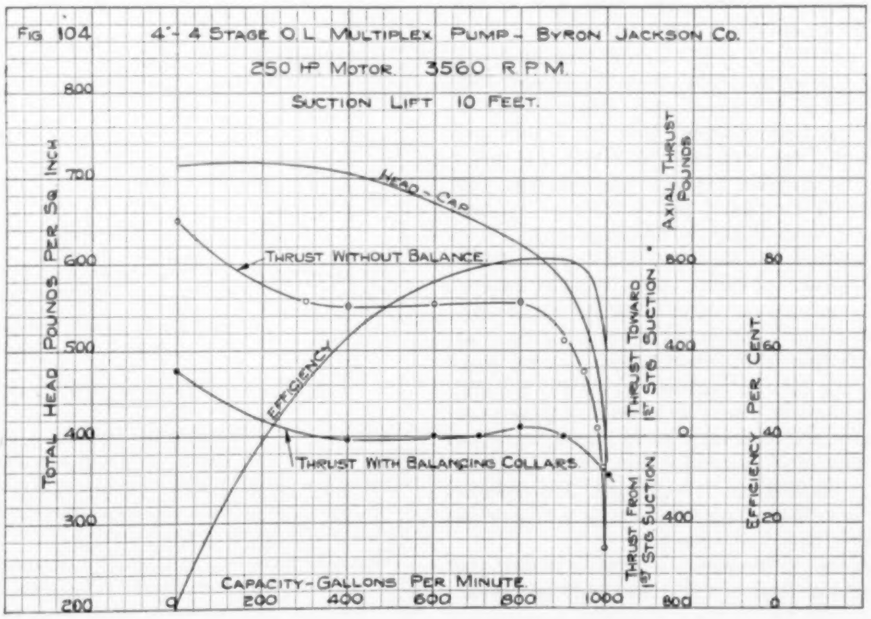
Fig 104

(25) Pumps with one stuffing box and one internal bearing have additional thrust due to unbalanced pressure on the enclosed shaft end. The magnitude of this thrust depends on the suction pressure of the pump.

(26) The theoretical head produced by a runner with an uneven radial velocity distribution is smaller than that given by the elementary theory.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to all those who, by suggestions and criticism, helped the completion of this work. Particularly he is indebted to Prof. M. P. O'Brien and Mr. E. D. Howe, of the Department of Mechanics of the University of California, for their assistance and cooperation in the actual tests involving special "set-ups" and equipment without interfering with the regular laboratory procedure.

The author is also indebted to Byron Jackson Company for the loan of equipment and to Messrs. R. Schurman, Vice-President, and A. Hollander, Chief Engineer, of the Byron Jackson Co., whose interest and support in this study made this work possible.

Discussion

F. G. Switzer.[34] Such research work as this is exceedingly interesting. It is through this kind of thing that the centrifugal pump has reached its present importance. As we learn more about the details of its characteristics we will find that this type of pump will be gradually still further improved, giving it an ever-widening field of application. Many commercial-pump users are hesitant in adopting the centrifugal pump until they can be assured of the high efficiencies which other types of pump are capable of developing, particularly in the range of high head and low capacity. But when multi-stage centrifugals can show 80 per cent efficiency at heads as high as 1200 ft. and capacities as low as 1000 g.p.m., the lower cost and simplicity of construction and operation will make the centrifugal pump of increasing importance.

For some time past the writer has been spending what time he could find available in work of a nature similar to Mr. Stepanoff's. With the cooperation of Goulds Pumps, Inc., there have been made available pumps, pump parts of special design, and complete testing equipment. These were installed in the hydraulic laboratory at the School of Mechanical Engineering at Cornell University. The work done there has been under the direction of the writer. This work has not duplicated any of the work of the author, but the trails have crossed. Pressure distribution in the volute has been looked into a little, not with respect to leakage, but for the purpose of trying to learn something more about volute design. Had the author taken measurements at more than three points (*D*, *E*, and *F*) in the main body of the volute, it is not believed that he would have made the statement he does in the second of his conclusions. Considerable change can often be made in the volute proportions, changing therefore the velocities and hence the pressures, without any great change in efficiency. Pressure variations around the volute may be quite large, and occur in quite a short section of the volute. This is particularly true near the beginning and again near the end of the volute. The effect of increased flow which is illustrated in Fig. 40 can be still further explained. The writer has rather conclusive evidence that flow in a certain portion of the small end of the volute may be reversed in direction under these conditions of high capacity, and this alone will go much further toward explaining the conditions observed than any comparatively small change in discharge angle. It is too bad that efficiencies are not indicated, in addition to leakage. The writer is of the opinion that certain of the types of wearing ring used which showed marked reduction in leakage would show little if any gain in efficiency. This would have been discovered if similar changes had been made in the front wearing ring.

The use of radial ribs for the purpose of reducing leakage, and hence wear in clearances, seems to be well established in the dredge-pump field, while practically never used in any other types. The writer has often wondered why they were not more generally used, particularly in high-head, low-capacity pumps. Here more is to be gained by reducing leakage than in other combinations of head and capacity.

Another explanation for the decrease in coefficient of discharge with increasing peripheral speed of the runner may be offered. It has been found that the flow in these clearance spaces is not axial; it is spiral. The drag of the runner causes tangential motion. Hence the length of path may change somewhat. Put in another way, we may reason that for ring No. 10, say, increasing pressure at constant peripheral speed causes an increased axial component of velocity with about the same tangential component. Hence the spiral of flow becomes steeper and the length of path shortens. The shorter path means increased total flow and is shown by the increasing coefficient of discharge. Also, with fixed pressure and varying peripheral speed, there is a nearly constant axial component of velocity and an increasing tangential component with higher speeds. At higher speeds the path is then longer and the discharge coefficient should go down, which it does.

It would have been exceedingly interesting and informative if tests had been made on narrower rings. Fig. 66 shows the trend of discharge coefficient with width of ring. If these curves could be projected backward to zero ring width, it would be much easier to separate the effects of friction from entrance and discharge losses. Also it might be expected that the curves for different speeds would converge there to a single value independent of speed. This same idea may be expressed by saying that increasing the ring clearance as shown in Fig. 68 should cause the curves to converge at greater clearances, as they actually do.

Fig. 70 shows some interesting data, too. In trying to separate the losses of pumps at or near maximum efficiency, the writer has found that the so-called mechanical losses often appear to be considerably greater than can be accounted for by the sum of bearing and gland frictions and disk friction. No explanation has been found for this.

While the actual measurement of end thrust is not new, the work reported here is more inclusive than any other published records known to the writer. For some years, Goulds Pumps, Inc., has been measuring end thrust. As a result of these studies, very simple means have been found for reducing end thrust. In Fig. 105 is shown a test curve of end thrust of such a pump. It is seen that the arrangement used is effective in reducing end thrust in a new pump and keeping it at a reasonable value with an old pump which is worn beyond the point where the wearing rings should be renewed.

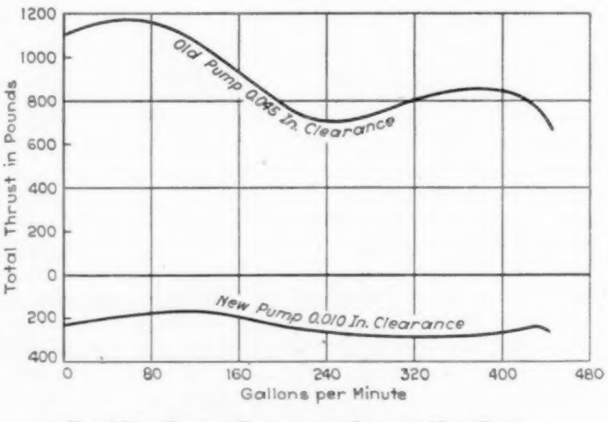

Fig. 105 Thrust Curves for Old and New Pump

That there can be any marked differences of pressure on the front and back shrouds inside an impeller as narrow as the one tested seems unlikely. Any tendency to such differences would be the cause for cross-flow within the impeller and reduced efficiency. Had the author measured pressures in the clearance spaces at the front of the impeller, he would no doubt have found sufficient differences there to account for considerable unbalance. He apparently assumes the same pressures on the outside of both shrouds. Tests have shown that this may not be true. Differences as much as 30 or 40 lb. per sq. in. have been observed between front and back clearance spaces.

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[34] Professor of Hydraulic Engineering, Cornell University, Ithaca, N. Y., and Consulting Engineer for Goulds Pumps, Inc. Mem. A.S.M.E.

Michael D. Aisenstein.[35] The data collected on measuring the thrust of Byron Jackson pumps and a companion of the different types of wearing rings are interesting. It is, however, regrettable that in no place does the author refer to the original experiments on leakage by Prof. A. F. Sherzer,[36] whose experiments on leakage were along the same line and whose methods were followed.

The opening statement in the introduction, that with modern efficiencies of 90 per cent, leakage losses take an important aspect, is not correct.

It is because of the tendency on the part of some designers to improve the efficiency of centrifugal pumps by reducing the clearance joint that some pump manufacturers have resorted to advertisements such as to "beware of close clearances" or advertised that the efficiencies as obtained by them were not due to close clearances. This situation has made many prospective users of pumps very suspicious when any improvement in efficiency is claimed, as immediately close clearances, thin shafts through the stuffing boxes, etc., are thought of as the reasons for this improvement.

The high efficiencies that are obtained by some of the leading manufacturers is the result of reduction of the hydraulic losses owing to a better understanding of the phenomena of hydrodynamics, due to the selection of specific speeds for which the disk friction becomes a minimum for the same head produced by the impeller. Take for example a highly efficient double-suction pump of medium specific speed, a capacity of 9000 g.p.m., pumping fresh water at 75 lb. per sq. in. pressure difference across the clearance. The pump efficiency is 90 per cent. The diameter of the ring is 12 in. and the clearance 0.016 in. on the diameter, using the table on the fourth page. Then from the equation the leakage in gallons per minute for two suctions will be:

$$GPM_L = 2\pi DSC\sqrt{2gH_L}$$

We have, assuming $C = 0.55$,

$$GPM_L = \frac{2\pi \times 12 \times (0.016/2) \times 0.55\sqrt{64.4 \times 2.31 \times 75} \times 448}{144}$$

$$GPM_L = 109$$

and volumetric efficiency

$$l_v = \frac{9000}{9109} = 98.9 \text{ per cent}$$

and if the whole leakage should be eliminated, the limit of increase of pump efficiency due to the absence of leakage loss would be

$$l = \frac{90.0}{98.9} = 91 \text{ per cent}$$

Since the accuracy of any commercial test is not greater than 1 per cent, it would be difficult to determine the gain in efficiency if the entire leakage loss should be eliminated.

On the tenth page the author criticizes Dr. Pfleiderer's Equations [17], [18], [19], and [20] as not being practical; particularly, Equation [20] is claimed as not accurate enough. Mr. Stepanoff's Equation [29] is the same as Equation [17], with the exception that the volute pressure H_p is called $\frac{u_2^2 - u_1^2}{2g}$, and it so happens that in this form it agrees closer with the particular test.

As to the computation of the head H_p, with the present knowledge of centrifugal pump theory it can be calculated very accurately without much difficulty.

The author's criticism that most of the textbooks in illustrative design roughly estimate the leakage losses rather than calculate them is without foundation. It is well known to every designer, as well as to the criticized authors of the textbooks, that to start a new design certain factors are to be assumed. The total volume of liquid, which includes the leakage that should pass through the suction pipe and impeller, should be assumed, since on this capacity the whole design is based and dimensions and angles of the impeller and size of the volute are determined. For instance, if a single-suction pump should deliver 340 g.p.m., assuming 5 per cent leakage, the delivered volume through the impeller will be

$$GPM = 340 \times 1.05 = 357$$

Assuming a 4-in.-inside-diameter suction pipe, the velocity through the suction pipe will be

$$c_s = \frac{357 \times 0.409}{4^2} = 9.1 \text{ ft. per sec.}$$

If the velocity in the impeller eye is taken as 12 ft. per sec. and the hub of the impeller from the horsepower requirements is determined to be $1^3/_8$ in., from equation,

$$C_e = \frac{GPM \times 0.321}{\pi/4\,(d_e^2 - d_h^2)}$$

or

$$d_e = \sqrt{\frac{357 \times 0.321}{12 \times 0.785} + 1^3/_8{}^2} = 3.75 \text{ in.}$$

After d_e is determined and the proper thickness, say $^3/_8$ in., is allowed for metal, the different formulas as given in the textbooks could be used to check the leakage loss, using the ring of diameter $3^3/_4 + {}^3/_4$ in. and whatever clearance it is decided to use. The statement on the fifth page that the main reasons for the low efficiency of small pumps is an excessive leakage loss is incorrect. As shown by the author himself, in considering the small pump as tested by the author, on the eleventh page, it is found for a clearance of 0.012 in. for plain wearing rings in case [6] or [4] that the leakage loss varies from 3.16 to 4.5 per cent, which, considering the pump efficiency of 68 to 72 per cent, is a relatively small loss, and very little improvement should be expected in this direction.

The main reason for low efficiency of high-pressure (low-specific-speed) small-volume, small pumps is the disk friction and unfavorable hydraulic design due to too large impeller hub for relatively small impeller diameter because of required size of the shaft, narrowness of the port width, etc. So far as small, low-head pumps are concerned, the mechanical losses, such as stuffing-box tightness, may overshadow all other losses and account for an efficiency drop of 20 per cent.

With reference to the derivation of equations for theoretical head with non-uniform radial velocity (see seventh page) the whole integration is incorrect. As long as velocity $v_r = v_1 + ax$ is variable, the angle β_2, or rather the corrected true angle β'_2, which should have been used (see Fig. 20), is also variable, and the term $\frac{u_2}{g \tan \beta_2} = B$ could not be treated as a constant; consequently the whole discussion has no theoretical or practical meaning.

So far as Equation [36] is concerned, the assumption that when the runner hubs D_{h2} and D_{h1} are of different size, the thrust pressure per stage P is the same in each case is incorrect, and therefore the resulting Equation [37] is in error.

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[35] Chief Engineer, United Iron Works, Oakland, Calif. Jun. A.S.M.E.

[36] *Engineering News-Record*, vol. 92.

Another disadvantage of having hubs of different diameter is that the peripheral velocity will be greater for larger hubs and still greater unbalancing will take place due to wear, increasing rapidly with time. A much better way to balance a four-stage pump would be to make the shaft through the stuffing box of the same size, that is, $D_s = D_c$, which would also increase the rigidity of the shaft in the packing box, which is a desirable feature as it would increase the life of the packing.

On the ninth page the author discusses characteristics of a centrifugal pump when pumping viscous oils without reference to the article on "Characteristics of Centrifugal Oil Pumps," by the writer, published in Trans. A.S.C.E., Vol. 94, 1930, page 437, in which the same phenomena are discussed and the same explanations are offered.

To determine what actually takes place so far as different flows are concerned, velocities should have been measured at many points and checked against velocity diagrams as obtained from theory. The method of measuring static pressure in different places in the casing, without recording the velocities, as followed by the author, is very unsatisfactory, and the deductions based on such methods are unreliable so far as conditions of flow are concerned in passages of such shape as exist in a centrifugal pump, and should be taken with a grain of salt.

A point which the writer wishes to stress is that however well a multi-stage pump is balanced hydraulically when new, it will become unbalanced in time due to wear; further, heavy thrust due to momentary obstructions or faulty suction conditions may always take place; and for these reasons an adequate thrust bearing of ample capacity should be provided.

It is a well-known fact that the thrust capacity of ball bearings is reduced with speed. According to reputable ball-bearing manufacturers, at 3600 r.p.m. the largest double-row ball bearing that could be safely used would be a No. 409, with a thrust capacity of about 1740 lb. at 3450 r.p.m., for periods of 5000 to 25,000 hours. This is not enough to take care of any contingencies which may occur in an oil pipe line of any size, say, 4 or 5 in. discharge diameter. For this service bearings of the Kingsbury type with thrust capacities increasing with speed should be used.

Of course, for relatively small pumps or slow-speed pumps, ball-bearing-fitted pumps are satisfactory, and as they are much cheaper than the combination of sleeve and Kingsbury-type bearings, there is some advantage in the use of the former type of bearing.

Most of the oil companies resent the use of ball bearings in high-speed pipe-line pumps. They are cheaper so far as initial cost is concerned, but they are expensive to maintain, and expenditures due to shutdowns prove to be many times the additional cost of the pump when fitted with an adequate bearing.

For this reason the writer disagrees with that portion of the paper which claims (on the twenty-sixth page) that the pipeline pumps as shown on the thirtieth and thirty-first pages can be safely operated if the axial pressures were balanced when the pumps were new.

In conclusion, the writer wishes to warn any designer of centrifugal pumps not to get on the dangerous ground of improving efficiency by decreasing the running clearances to improve the volumetric efficiency (0.008 or 0.010 in. for rings of 6 in. in diameter as recommended on the fourth page is dangerous), to thin out the shaft through the stuffing boxes so as to reduce the friction losses, and use the ball bearings where they should not be used to reduce bearing losses and be able to keep smaller clearances at the running joints.

A. PETERSON.[37] This paper should prove of considerable value to manufacturers of centrifugal pumps who have not made or who do not have facilities to conduct such painstaking experiments.

The table, Fig. 31, giving the leakage loss for different kinds of wearing rings, is extremely interesting and valuable. The author's tests confirm similar tests carried out about 25 years ago by the company with which the writer is associated.

Contrary to the author's assertion that years ago very little attention was paid to leakage losses in centrifugal pumps, it may be of interest to bring out that as early as 1905 very complete tests were made by the De Laval Steam Turbine Company to determine leakage through wearing rings, both of the plain type and the labyrinth type, and the apparatus used was very similar to the one shown in Fig. 27, as used by Professor Susuki of the Tokyo University in 1929.

As a result of these tests, the company about a year later, or in 1906, adopted as standard the double-labyrinth ring shown in Fig. 7, and it has been used ever since practically without any modifications.

The writer does not believe that many pump builders or users appreciate the tremendous advantage of labyrinth rings over plain rings, because if they did, labyrinth rings would be more extensively used.

The author's tests indicate that for the same clearance the single-labyrinth ring, as shown in Fig. 6, gives from 41 to 46.6 per cent less leakage than the plain ring. The results of our tests were that the double-labyrinth ring, as shown in Fig. 7, gives about one-third the leakage of the plain ring.

This is of considerable importance in centrifugal-pump design, because not only does it result in higher initial efficiencies, but also in better sustained efficiencies. In view of the one-third lower velocity of the water through the labyrinth ring, there naturally will be less wear of this type of ring than of the flat ring, and this stands out particularly in the case of pumps handling gritty water.

Practically the only reason why the efficiency of a centrifugal pump drops off by use is wear in the rings, and therefore anything that can be done to reduce the amount of wear is of considerable advantage.

Leakage losses are something that cannot be seen, and unless the pump is tested at regular intervals, it may be operated for a considerable time with worn rings and at very much reduced efficiency.

As a rule labyrinth rings are not used so much for the purpose of obtaining a higher initial efficiency as for the purpose of permitting larger clearances without a sacrifice in efficiency. This is also important, as it is not then necessary to resort to the means outlined at the bottom of the fourth page—that is, to machine the revolving ring after it is put on the impeller. This is objectionable, because it is not always convenient for a customer, in case he orders a replacement ring, to take the pump rotor to a machine shop to have a finishing cut taken on the impeller ring.

Another advantage that stands out in connection with large-clearance labyrinth rings is that for a certain fixed amount of wear the leakage increases a smaller percentage than for the same amount of wear in the small-clearance plain rings.

Assuming that a plain ring with 0.012 in. clearance on the diameter has a leakage of 5 per cent and that the same amount of leakage is obtained with a labyrinth ring with 0.030 in. clearance on the diameter and that both rings have worn 0.012 in. on the diameter, we have a loss in efficiency in the case of the flat ring of 5 per cent and with the labyrinth ring a loss of only 2 per cent, assuming no change in coefficient of discharge. This example clearly demonstrates the great advantage of the labyrinth ring.

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[37] Chief Engineer Pump and Compressor Department, De Laval Steam Turbine Company, Trenton, N. J. Mem. A.S.M.E.

The pressure readings taken by the author at various points in the volute are of considerable interest, and they also agree with similar tests conducted by the De Laval Company.

On the fifteenth page there is brought out the importance of designing the volute so as to obtain uniform pressure in it, and the author also shows that the pressure can only be uniform at one point, that is, at maximum efficiency. For other conditions the pressure distribution is unequal, and this throws additional load on the shaft that may result in breakage, if the shaft is not strong enough to withstand such load. The author refers to this load as side thrust or side pressure. It seems to me more logical to consider it as a radial load.

On the same page in the first column reference is made to pressure at E being higher than at D due to E being on a larger diameter than D, making the average peripheral velocity of the water particles higher. Is not the peripheral velocity of the water at E lower than at D, and is not this the reason for the pressure at E being higher than at D?

In the second column of the sixteenth page the statement is made that the pressure H_L should be corrected for velocity head in the runner eye, since the pressure in the runner eye is lower than the suction lift by the velocity head. It is not believed that this is quite correct, because the pressure at the runner eye is as a rule equal to the suction lift less friction in the suction passages up to that point. If the runner eye should happen to be of smaller area than the area where the suction-lift reading is taken, correction should be made for only difference in velocity heads at the two points and not for the entire velocity head at the runner eye.

In his calculations of pressure at the wearing ring in terms of volute pressure, the author, on the seventeenth page, states that the relative velocity w_1 at the inlet is equal to or slightly greater than the relative velocity w_2 at the outlet. In most centrifugal pumps, particularly those operating at comparatively high r.p.m., there is a considerable difference between these velocities, the relative velocity at the inlet often being more than twice as great as the velocity at the outlet. Judging by the proportions of the pump as shown on the tenth page, a ratio of about 2 to 1 exists, and even higher ratios are not uncommon. This means that the term $\frac{w_1^2 - w_2^2}{2g}$ may be quite an item, and dropping it may in some cases affect the results materially. For the pump shown on the tenth page the term $\frac{{}_1{}^2w - w_2^2}{2g}$ is at least 21 for 2500 r.p.m., and if this figure were used in the formula at the bottom of the seventeenth page the calculated pressure at the wearing ring would be 140 instead of 119 ft. Therefore, in order to have the calculated results agree with test results, a constant other than 0.50 for φ would have to be used.

Without doubt there will be conditions where dropping of the term $\frac{w_1^2 - w_2^2}{2g}$ will affect the results even more than in this particular case.

The writer would like to ask if, when conducting the leakage tests, any pressures were taken in the chamber on the back side of the runner. Judging from statement on the nineteenth page, the pressure was assumed to be equal to that of the atmosphere. This assumption, however, may lead to some errors, particularly where the leakage through the rings was great, as in the case of ring 10 with a leakage of 10 per cent, or 47.5 g.p.m. It would require a velocity head of almost 4 ft. to discharge this amount of water through the $1^1/_2$-in. orifice, and possibly the back pressure due to other restrictions may have been even higher; and for this reason the actual pressures in the balancing chamber should have been measured in order to obtain accurate figures for the coefficient of discharge.

In the second part of the paper, on axial thrust, reference is made to hydraulically balanced single-suction multi-stage pumps. Such pumps are not built, as indicated on the twenty-fifth page; that is, with a valve in the balancing pipe for regulating the pressure. The balancing arrangement is always made automatic, either similar to Fig. 71 or with a leak-off through a small annular clearance around the shaft. Although these balancing arrangements may have their disadvantages, they are, however, superior to practically any on the market today, simply because they work automatically and no unknown factors enter into the design.

As to the disadvantage pointed out by the author, that a portion of the pump capacity is wasted, this applies just as much to the pump with impellers arranged back to back.

Take, for instance, the six-stage pump shown in Fig. 81, if this pump were designed for a total pressure of 900 lb. there would be a drop in pressure in the clearance space between the sixth and second stage of 600 lb., which would result in an appreciable leakage and loss of capacity if the bushing and shaft should become even slightly worn.

One advantage of the automatic hydraulic-balancing arrangement is that means can be provided in the balancing pipe for continuous measurement of the leakage, and adjustments and replacement of parts can therefore be made as soon as the leakage is found to be excessive.

With internal-tightening joints subjected to considerable pressure drops, such as in the back-to-back impeller pump, there is no way to determine the leakage except by either taking the pump apart and measuring the clearance or by running a complete test, which is not always convenient.

With the high pressures as nowadays encountered in connection with boiler-feed pumps and high-pressure oil pumps, the item of leakage through balancing rings and through bushings separating stages on back-to-back impeller pumps may be of considerable magnitude, and anything that can be done to check this leakage should be of value in maintaining the original efficiency of the pump, either by adjustment or replacement of parts.

The power requirements of modern high-pressure boiler-feed pumps are such that even a comparatively small percentage loss in efficiency becomes an item of great importance.

At the bottom of the first column of the thirty-seventh page reference is made to increase in efficiency of a high-pressure pump with suction lift as compared with pressure on the suction. This is something that the writer has not found in his experience with centrifugal pumps, and he would be interested to know if there are any other centrifugal-pump manufacturers who have found a condition similar to that which the author has obtained in his tests.

It is rather difficult to reconcile the two head-capacity curves in Figs. 103 and 104. Plotting the two curves on the same sheet, it is found that between 560 g.p.m. and 960 g.p.m. the curve with suction lift is higher than the one with pressure on the suction, the maximum difference being around 20 lb., which seems to require an explanation other than that given at the bottom of the thirty-seventh page. It is doubtful if the condition approaching cavitation with the water leaving the runner walls accounts for the difference in head and efficiency. It is difficult to conceive how the pump could be operating at conditions approaching cavitation over such a great range of capacity as from 560 g.p.m. to 960 g.p.m. There must be some other explanation for the increase in efficiency and head when the pump is operating at 10 ft. suction lift.

The writer would like to ask why only a 250-hp. motor was used to drive this pump. The horsepower required by the pump is such that the motor would be overloaded almost 50 per cent if the pump were handling water, and this overload seems high for the present-day electric motor.

In going back to the second column of the twenty-second page, it is stated that, with the rings set eccentrically, the leakage is smaller for the same clearance area.

The writer has not had an opportunity to verify this by tests, but he happened to run across a reference to such tests in *Zeitschrift des Vereines deutscher Ingenieure* in the April 12, 1930, issue, according to which the leakage through rings set eccentrically is greater than with concentric rings, and with full eccentricity the leakage under some conditions was $2^1/_2$ times as great as with concentric rings. The writer would like to know if the author can offer some explanation for this discrepancy.

John Mann.[38] Some of the discussers seem to think that pump builders are going to a lot of unnecessary trouble in balancing for end thrust when Kingsbury bearings could be built to carry any end thrust which might occur in an unbalanced pump.

Their remarks would possibly apply to the propeller type of pump, but in the radial-vane type of pump there is no loss in efficiency due to balancing, and practically no extra cost.

The pumps under discussion were mainly 3600 r.p.m. and up to eight stages, developing pressures of approximately 100 lb. per sq. in. per stage. These pumps, with impellers back to back and having their areas balanced, can have end-thrust loads as high as 5000 lb. This end thrust can be balanced by the Goulds method of making suction impellers slightly smaller in diameter than the discharge impellers, or by the Byron Jackson method of changing impeller-hub diameters. Pumps so built and balanced have a light end-thrust load. Ball bearings can then be used to locate the rotating element endwise. These bearings over a wide range of speeds are reliable, require very little attention, and are of comparatively low cost, whereas if no effort is made to balance the pumps for axial thrust, and Kingsbury bearings are used, these bearings would be costly and require a lot of attention at the higher speeds to keep them running.

The author has contributed to the improvement of centrifugal pumps by the publication of his comprehensive tests on wearing-ring leakage and axial thrust.

Referring to his statement that there is no available information on axial thrust checked by actual measurements, Goulds has been making such measurements continuously since 1927 and has collected a large amount of data on axial thrust on many types of pumps, such as: Double-suction, single-stage; single-suction, single-stage (as in Fig. 30), with wearing-ring clearances varied; single-suction, single-stage, open impellers; single-suction, multi-stage, runners grouped one way; single-suction, multi-stage, runners in two groups, opposed. These tests have not been published. (As a result of these tests, end thrust in Goulds pumps is known and provided for in the design.)

The machine used to measure thrust consists of a cylinder and plunger, the end thrust being balanced by hydraulic pressure on the plunger, the pressures being read directly from a pressure gage.

Various tests made since 1927 would seem to indicate that the main cause of axial thrust other than unbalanced areas is due primarily to a difference in pressure on the two sides of a runner. It is believed that if the author, in his tests on axial thrust, had obtained pressures on both sides of the runner, he would have come to quite different conclusions in regard to the unknown causes of thrust other than unbalanced areas. This difference in pressure is caused by a difference in leakage. For example, if there is excessive end thrust on a double-suction pump, this thrust would be due to the clearance space on the wearing ring on one side of the runner having worn more than the wearing ring on the opposite side of the runner. While the other causes mentioned by the author may be present, they are so small that they can be neglected.

A two-stage pump with runners back to back will have a leakage from the second-stage chamber to the first-stage chamber. This will cause a slight drop in the average pressure on the back of the second-stage runner and a slight increase in pressure on the back of the first-stage runner. Although this pressure change is low, the area on which the pressure acts is great. The thrust in this two-stage pump, and in the multi-stage pumps with runners grouped back to back, is balanced by making the suction runners smaller in diameter than the discharge runners.

The difference in head developed by the runners is then an unbalanced pressure on the suction-eye areas of the runners in a direction from suction to discharge.

Pumps similar to Fig. 30 have a greater leakage from the front shroud of the runner to the suction chamber than from the back shroud to the discharge chamber. This difference in leakage causes a difference in pressure on the two shrouds of the runner and a thrust toward the suction.

To balance this thrust, the discharge wearing ring is made larger in diameter than the suction wearing ring. The pressure on the discharge shroud of the runner is then on a smaller area than the pressure on the suction-side shroud. In the first paragraph on the twenty-fifth page, the author seems to have neglected the effect of this large thrust load in the opposite direction when increasing the wearing-ring diameter to balance for impact. The shape of the thrust curve and the amount of balancing to be done on this pump depend on the means taken to transfer the leakage from the discharge chamber to the suction side of the pump.

Referring to the conclusions on axial thrust, conclusion (16) can be explained by the change in pressure on the full shroud diameter due to leakage.

As to conclusion (17), the major portion of axial thrust should be calculated on pressure change due to leakage loss.

As to conclusion (18), it is very doubtful that uneven radial velocity distribution in the runner is a cause of axial thrust in the pumps under discussion in the paper.

As to conclusion (23), pumps with runners arranged in two opposing groups have an unbalanced end thrust as per Equation [30]. This is the same as pumps with runners arranged in pairs back to back, as in Fig. 75.

Symmetry is no more a factor in axial thrust in the group arrangement than in the pair arrangement of Fig. 75. That this is so is shown by the thrust curves without balance, as in Figs. 103 and 104.

[Discussion of this paper jointly with that of J. W. Macmeeken by A. J. German will be found following the latter paper.]

AUTHOR'S CLOSURE

The author wishes to thank the discussers for their contributions to this paper. He feels flattered that Professor Switzer and Messrs. Peterson and Mann have gone through the paper with such extreme care. Their points of view, different from the author's in some cases, are appreciated.

Referring to Professor Switzer's valuable discussion, the author thinks that any decrease in leakage is a net saving, and that the pump efficiency will improve with improved volumetric efficiency. Such minute increase in power as that caused by the increased skin friction (steps or grooves on the rings and decrease in clearance) or power used to pump the leakage back by spiral grooves, could not be detected by these tests.[39] The author agrees with Professor Switzer that the radial ribs on the impeller shrouds are the most effective means to reduce the leakage, or even to eliminate it at high speeds, as was observed in these tests. The danger, however, is that the radial ribs will throw the pump out of balance for axial thrust. The general belief that radial ribs take much power seems to be greatly exaggerated. Investigation in this direction would be of value. The author uses radial ribs occasionally for balancing axial thrust. Professor Switzer's explanation of the decrease of the coefficient of discharge through the wearing rings with increased peripheral speed is essentially the same as the author's, only differently worded.

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[38] Chief Engineer, Goulds Pumps, Inc., Seneca Falls, N. Y.

Messrs. Switzer and Mann are inclined to think that the major cause of axial thrust other than the unbalanced areas is the difference of pressure on the two sides of the impeller from outside rather than from inside. The author would agree with that if these discussers would show what causes this difference in pressure on the two sides of the impeller with such ample clearances around the impeller as are found in modern pumps. On page 35, the author has mentioned that he conducted a series of tests (40 runs) with the impeller displaced from its central position in the casing $^3/_{16}$ in., thus making a difference in the clearance space of $^3/_8$ in., and did not find any appreciable change in the axial thrust. Thus this cause for the difference in pressure on two shrouds of the impeller is hardly possible with large clearance spaces. To what extent may the difference in leakage affect the pressures in the clearance spaces? Not more than the difference in the velocity head in these spaces, the leakage only flowing through—which is really negligible. This does not apply to the pressure in the balancing chamber on the back of the impeller, which depends greatly upon the amount of throttling in the wear ring, and as this is decreased with worn rings, the pressure in this chamber increases, resulting in increased axial thrust as shown in Fig. 105. The form of the thrust-capacity curves (increase of thrust at high capacities) is substantially the same with no balancing chamber on the back of the impeller, as seen from Figs. 87–94. It has been shown in the paper that the amount of balancing required depends upon the way the balancing chamber is connected to the pump suction. No explanation accounts for the increase of thrust at low pressures (high capacities) except the supposition of different internal pressures at the two shrouds. The variation in thrust at conditions approaching cavitation, when local low-pressure spots are formed inside the impeller, can hardly be questioned.

The case of a double-suction pump with different clearances in wearing rings quoted by Mr. Mann may serve very well to prove the author's point of view, because with different amounts of leakage on the two sides of the pump, the impeller halves work at different capacities, and hence at different heads, which causes axial thrust from inside the impeller. The author is very anxious to know some details about the pump with a difference in pressure on both sides of the impeller of 30 or 40 lb. as mentioned by Professor Switzer. This must be some unusual case. In the case of a dredge pump tested by J. H. Polhemus and J. Healy (see footnote 28) with an impeller 70 in. in diameter, a head of 92 lb., and a clearance of $^1/_{32}$ in. at the front shroud and of $^8/_{32}$ in. on the back, the maximum difference in pressure observed was 24.5 lb.; but when the clearances were changed to $^9/_{32}$ in. on the front shroud and $^1/_{32}$ in. on the back, the difference decreased to 4 lb. Note the relatively small clearances and the size of the impeller used in these tests.

Professor Switzer will notice that author's readings taken at various points in the volute agree with similar tests by the DeLaval Company, as stated in Mr. Peterson's discussion. As to the possibility of cross-flows inside the impeller caused by the difference in pressure near the two shrouds, it should be noted that there are many examples in hydraulics illustrating that the pressure alone does not determine the direction of the flow—the flow through the elbow being one. At points removed from the point of the peak efficiency, these cross-flows no doubt exist, and account for the drop in efficiency on both sides of the peak.

The author shares Mr. Peterson's enthusiasm about the labyrinth rings, which are standard with the Byron Jackson pumps except on very small, cheap ones, where other means to reduce leakage are used. As to the radial load on the pump shaft caused by uneven pressure distribution in the volute casing, the author prefers the term "side thrust" to distinguish it from the radial load due to weight of the rotating element and to suggest the hydraulic nature of this load. Mr. Peterson's explanation of the difference in pressure at points *D* and *E* in the volute case seems very probable, depending, however, on the volute proportions.

If the suction lift is measured with a monometer on the suction nozzle, then this reading should be corrected for the difference in the velocity heads at the point where the reading is taken and at the impeller eye to obtain the difference in pressure across the clearance, as pointed out by Mr. Peterson. H_L in this paper is the static head above the pump-suction level as clearly defined in the first paragraph of sec. 9, page 14, and it should be corrected for the full velocity head at the impeller eye, the loss of the suction pipe being neglected.

The ratio of the relative velocities at the inlet and outlet $w_1/w_2 \geqq 2$ seems unusually high. Probably the discrepancy is explained by the difference in location of points to which these velocities refer. The author calculates these velocities at the vane tips where the vane overlapping starts at entrance and stops at discharge (dotted line through *A*, Fig. 19) and not from the theoretical velocity triangles. R. Defeld[40] gives for this ratio values ranging from 1.0 to 1.5, which agree with the author's observations. Furthermore, [27] refers to the theoretical head; the actual head is lower by the amount of hydraulic losses, but dropping of the term $(w_1^2 - w_2^2)/2g$ will partly account for these hydraulic losses, as mentioned in the first paragraph of page 17. The size of the special nozzle through which the leakage was discharged was not $1^1/_8$ in., as appeared by mistake in preprints of the paper at the bottom of the first column, page 12, but $1^1/_2$ in. as shown clearly in Fig. 32 on the same page. In all cases, except those with the two largest clearances, this nozzle was not running full, thus assuring atmospheric pressure in the chamber on the back of the impeller.

It seems that the automatic feature of the balancing disk was sufficiently emphasized in the paper. As to the leakage loss in pumps with impellers arranged in two opposing groups as compared with those using balancing disks, the first have the advantage that the clearances are on a small diameter and the throttling surfaces are wide and can be provided with steps and grooves to reduce the leakage.

As to the possibility that pumps approaching cavitation conditions show slightly better efficiency than when operating with a positive head on suction, the author never gave this any thought until he had learned that it was an established fact in hydraulic-turbine practice as reported by J. Ackeret referring to Thoma and

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[39] Professor Howe and Mr. Ponomareff of the University of California are experimenting now with a very delicate apparatus which will enable them to throw some light on the questions of the effect of the clearance, eccentricity, and viscosity upon the power consumption for close-running fits.

[40] Raymond Defeld, "A Practical Treatise on Single and Multistage Centrifugal Pumps," Chapman & Hall, 1930, p. 45.

Schilhansl's observations, in "Hydraulische Probleme," V.D.I. Verlag 1926, page 105. H. Föttinger confirmed the same fact on page 110 of the same work. After that the author had the occasion to observe the same phenomenon on three different pumps tested at different times.

A 250-hp. motor was used in these tests because the pumps were built for light gasoline and mounted on a base to take a motor of this size. These motors stand an overload of 50 per cent for an hour without excessive heating.

As to the discrepancy between the author's experience with the eccentric clearances and those of Dr. Schneckenberg briefly reported in the April 12, 1930, issue of *Zeitschrift des Vereines deutscher Ingenieure*, called to the author's attention by Mr. Peterson, the following comments will be made. After a study of the Dr. Schneckenberg article, it seems that the discrepancy is caused by a difference in the test conditions and methods of observation and may be apparent only, as will be seen from the following:

(1) Both throttling surfaces of the Dr. Schneckenberg apparatus were stationary, while in the author's set-up one surface was revolving.

(2) The maximum difference in the leakage between the concentric and eccentric clearances was observed by Dr. Scheckenberg with the smallest clearances 0.01 mm. (0.0004 in.) and 0.1 mm. (0.004 in.), referring to the viscous flow, this difference almost disappearing with 0.4 mm. (0.016 in.) and 1 mm. (0.040 in.), the flow being turbulent in this case. Only the last two clearances are comparable with those used by the author. The rotation of one of the throttling surfaces accompanied by the microscopic vibration tends to set up turbulent flow much earlier than with both surfaces stationary.

(3) In the author's arrangement the resistance to the flow or pressure drop on the two sides of the wearing rings included the entrance and exit losses and was uniform around the wearing ring, while Dr. Schneckenberg's throttling surfaces were 181.5 mm. long (7.25 in.) and pressure readings were taken a certain distance from the ends, thus excluding the entrance and discharge losses. This difference in the arrangement is important because the leakage (or coefficient of discharge) was plotted against the pressure in both cases.

(4) In the case of eccentric clearance in Dr. Schneckenberg's tests, the pressure readings were taken in the middle between the maximum and minimum clearances, where the clearance was practically equal to that with concentric location. The leakage measured was the total for the whole circumference and was plotted against the pressure difference thus obtained. No doubt the pressure drop varied for different points on the circumference, and so did the velocities. The pressures measured as explained above could hardly be used as a basis for comparison of leakage with concentric clearances and a uniform pressure for all the points on the circumference.

The author's experience with eccentric clearances was limited to a few cases where rubbing in the wearing rings was observed and inconsistent results were obtained. The trouble was removed in all cases after the concentricity of the clearances was restored. No doubt, in all cases when no rubbing in the rings was observed, a certain amount of eccentricity was present, affecting adversely the accuracy of the results.

The author is glad to find an able supporter in Mr. Mann on the question of the use of ball bearings in high-speed multi-stage pumps. The Byron Jackson Company in a period of less than two years has installed over one hundred 3600-r.p.m. multi-stage pumps in sizes up to 500 hp. Some of these pumps have been in continuous service for twenty months. During this time only one trouble with ball bearings has been reported. On inspection of this bearing, a broken thermometer was found in the bearing housing, partially ground to fine dust which was deposited on the balls. After cleaning, this bearing was reinstalled and is running to entire satisfaction. The wearing rings and stage pieces were replaced in some cases where the pipe line was not initially free from sand and scale. The cheapness of the ball bearings is a secondary consideration. The deciding factors in their favor are their efficiency, reliability, minimum attention required, and reduction of the shaft span between the bearings.

In conclusion (23) the expression "runners arranged in two opposing groups" was intended to cover both the runners set in pairs back to back and set in two opposing groups with runners of each group facing in one direction. Naturally, as far as the axial thrust is concerned, the two schemes are identical.

The reference by Messrs. Peterson and Mann to the experimental work performed by their respective companies, the results of which have not been published, leaves the author's statement that there is little or no information available on leakage loss and axial thrust still in force. After the submission of this paper for publication the author learned about two works on leakage loss, one by A. F. Sherzer, briefly reported in *Engineering News-Record*, Vol. 92, pages 1056–58, and another one by M. Hannocq, of Liége University, reported in *Revue Universalle des Mines*, Vol. 6, and mentioned in Defeld's book previously referred to. Both of these investigators experimented with pumps with impeller suctions sealed and measured the leakage at the suction nozzle with an outside pressure applied to the volute case. Professor Sherzer tested plain rings with different clearances, and his results are in close agreement with those presented in this paper. The author was unable to obtain a copy of the publication containing the results of Mr. Hannocq's tests.

Referring to Mr. Aisenstein's criticism—it would seem that this discusser did not take into consideration the simple theoretical propositions advanced in sections 4 and 14, the first one dealing with the theoretical head developed by an impeller with uneven radial velocity distribution and the latter about the axial thrust caused by the shaft shoulders and impeller hubs in multi-stage pumps. The material contained in this paper is essentially the thesis submitted by the author to the University of California, which brought to him a doctor's degree and resulted in his advancement to his present position. To this discusser the integration seemed to be incorrect because the term $u_2/(g \tan \beta_2) = B$ on page 7 could not be treated as a constant on account of variation of the angle β_2. It would seem that he overlooked the author's statement in the first paragraph of section 4 that the deduction is intended for "the theoretical head for an infinite number of vanes," and that thus any variation of the angle β_2 is excluded, which is perfectly consistent with the radial velocity's not being the same for different water particles. With this in mind the whole development stands the most searching scrutiny.

The best proof of the usefulness and correctness of the principles involved in the calculation of the axial thrust due to the shaft shoulders and impeller hubs is on the testing stand. Numerous tests, of which Figs. 103 and 104 are illustrations, proved not only the existence of the axial thrust due to this cause, but that its value can be calculated accurately for the useful range of the head-capacity curve. A slight correction may be necessary to account for the difference in the head produced by different stages, as discussed on page 35.

It may be a matter of opinion whether an improvement of 1 per cent in efficiency of a 90-per cent pump is worth while or not, but certainly the endeavor to attain it should not be stopped by fear of inaccuracy in testing, as this is no worse for a pump of 91 per cent efficiency than it is for one of 90 per cent.

The author is not in favor of advertising injunctions such as "beware of close clearances," which is just another talking point similar to the stereotyped "extra long stuffing box," the latter being applied now to every pump in which no one expects to find more than five packing rings. But for a stuffing box of a hot-oil pump working at 1500 lb. and a temperature of 600 deg. fahr., the term "double extra long" would by no means be strong enough. The small clearances consistent with a perfect mechanical operation free from seizing or rubbing are entirely justified, but they do require skilled labor and cost money. Mr. Peterson pointed out, and rightly, that the reduced leakage not only improves the initial efficiency but also the sustained efficiency on account of greatly reduced wear.

Dr. Schneckenberg, in the article previously referred to, states that in modern high-pressure boiler-feed pumps the clearances between the throttling surfaces are made very small, in fact, smaller than those in the bearings. This of course means that these surfaces themselves serve as bearings. Apparently, the German engineers are not afraid of close clearances.

It would seem that the object of the tests on leakage has escaped Mr. Aisenstein's attention. It was not the author's intention to develop some new formula to calculate the leakage through the clearances in the wearing rings. He used the oldest formula of hydraulics, $v = \sqrt{2gh}$, dating back to Torricelli's time (1643). To find the experimental coefficients for this formula for various types of rings and the ways to estimate the pressure across the clearance was the object in view. The author also appreciates very well the necessity of making certain assumptions and estimates in any kind of engineering calculations, and lengthy arithmetical figuring was hardly necessary to prove that. To guide the design in making these assumptions and estimates was the thought when these data were offered for publication. Dr. Pfleiderer's classical work on centrifugal pumps and textbooks previously published does not contain this information.

It is to be regretted that Mr. Aisenstein did not give particulars regarding the easy method of calculating the volute pressure H_p mentioned in his discussion.

In this discusser's statement of the reasons for the low efficiency of small pumps, the author did not find anything not contained in the last paragraph on page 5, where it is stated in the first item that the "low-specific-speed type of pump is inherently less efficient."

It was hardly necessary to give references on all the questions touched in the introductory and descriptive part of the paper, but if the author were to mention any reference on pumping oils, it would be R. L. Daugherty's reports contained in the Goulds Bulletins 126 and 130.

The experience of several leading pump manufacturers has fully demonstrated that 3600-r.p.m. ball-bearing pumps intelligently designed are just as reliable as 1750-r.p.m. pumps with Kingsbury bearings were in the past, and Mr. Aisenstein's warning in the last paragraph of his discussion might have had weight fifteen years ago, but not now.

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HYD-54-6

Hydraulic Jump in Sloping and Horizontal Flumes

Investigation of a Rational Solution Offering Formulas for the Tail-Water Depth

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The formulas presented in this paper were developed on a rational basis by the author in an attempt to solve for the tail-water depth of the hydraulic jump when it is produced in either sloping or horizontal flumes. A recent study of the mathematical analysis of this phenomenon has brought to light certain interesting relationships which are discussed in this paper. The conclusion drawn from this investigation is that Prof. W. C. Unwin's momentum formula is really a special case of a more general formula which can be used to include jumps produced in both sloping as well as horizontal flumes. Data from a paper by the author entitled, "Computation of the Tail-Water Depth of the Hydraulic Jump in Sloping Flumes"[2] are used to substantiate the theoretical conclusions reached herein.

ONE of the most interesting phenomena included in the study of hydraulics is the passage of water moving at a high velocity and below the critical depth into a stream of water moving at a considerably reduced velocity above the critical depth. This condition of flow has generally been termed the "hydraulic jump," and for over one hundred years there have been many experiments conducted upon it to determine its characteristics. In 1819 Bidone made experiments with a laboratory model in an attempt to discover, if possible, the relation between the depth of water before the jump to the depth of water after the jump. His work, like that of his followers, Gibson, Ferriday, and Lane, was done on small-scale models with jumps produced in horizontal flumes. While their work was very important, it was difficult to tell much about the nature of the action of the jump from their experiments. Prof. W. C. Unwin was the first to correctly analyze the mathematics of jumps produced in horizontal flumes on a rational basis, and his work is recorded in the Encyclopedia Britannica.

With the increasing demand for water power in industry, the necessity of large dams and levees to prevent destruction of property from river floods, and the new application of the jump to the process of water purification, it became imperative to determine more about the laws governing the production of the jump so that its occurrence could be predicted with certainty and the limits of its action more easily controlled. Kennison and Hinds followed in this work with observations on large jumps produced in a horizontal position only, but it was not until 1917 that Riegel and Beebe, of the Miami Conservancy District, extended the study to include jumps produced in sloping flumes. These two latter men were interested in properly controlling the flood run-off waters leaving large retarding basins placed beside the Miami River. In 1920 J. W. Ellms and A. G. Levy conducted a series of experiments on jumps produced in sloping and expanding flumes. Their work was done on a full-sized model flume in which a jump was produced prior to the installation of three such flumes in the Baldwin Filtration Plant at Cleveland, Ohio. In 1927 the author and J. R. Fleming conducted experiments on jumps produced in a sloping and expanding flume capable of carrying 1.5 cfs of water and the slope of which could be varied from approximately 9 to 17 deg.

The results of all the work done on jumps produced in sloping flumes proved conclusively that it was incorrect to apply Professor Unwin's classical momentum formula to this condition of expanding flow. It appeared necessary to develop another formula along rational lines which would have included in it a factor to take care of any variation in the slope of the flume. Several empirical formulas were developed by the author, and the work was described in a paper.[2] Since its publication a further study of the mathematics of the jump has been made and a plausible rational solution developed. Before going directly to the mathematical solution for the jump produced in sloping flumes, a history of the mathematics for jumps produced in horizontal flumes will be briefly reviewed.

History of the Mathematics of the Jump for Horizontal Flumes

During the past century many interesting attempts have been made to obtain the proper mathematical formula which would express the conditions of flow observed. As mentioned before, Bidone's experiments were made in small laboratory model flumes, and it was therefore very difficult to measure accurately the depths of water before and after the jump. This inaccuracy led to the application of false principles and the development of formulas from non-identities.[3]

Belanger attempted to compute the depth of tail water of the hydraulic jump in a horizontal flume by solving the equation

$$d_1 + \frac{v_1^2}{2g} = d_2 + \frac{v_2^2}{2g}, \text{ for } d_2$$

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[2] Robert W. Ellms, "Computation of the Tail-Water Depth of the Hydraulic Jump in Sloping Flumes," Trans. A.S.M.E., paper HYD-50-5, 1927–1928.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, andnot those of the Society.

[3] Throughout this paper all flumes will be considered to have unit width and rectangular section. Here, also, g will equal 32.2 fps and w equal 62.5 lb.

In Fig. 1, and the foregoing formula:

d_1 = depth of water before jump
d_2 = depth of water after jump
v_1 = velocity of water before jump
v_2 = velocity of water after jump
g = acceleration due to gravity, fps.

From this false equation he easily derived the formula

$$d_2 = \frac{v_1^2}{4g} \pm \sqrt{\frac{v_1^2}{2g}\left(d_1 + \frac{v_1^2}{8g}\right)} \quad \ldots\ldots\ldots\ldots [1]$$

According to this, energy losses occurring from impact or sudden expansion were considered by him to be negligible, but it is now known that the term $d_2 + \frac{v_2^2}{2g}$ should have had added to it the h_e losses to make the equation a true identity. Here the h_e represents the loss of head due to impact.

Professor Merriman considered that "the lost velocity head is $\frac{(v_1^2 - v_2^2)}{2g}$, and that this is lost in two ways: first, by impact $h' = \frac{(v_1 - v_2)^2}{2g}$ and, second, by the uplifting of the whole quantity of water through the height $1/2(d_2 - d_1)$." From this he found that

$$d_2 = \pm\sqrt{2d_1 \frac{v_1^2}{2g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

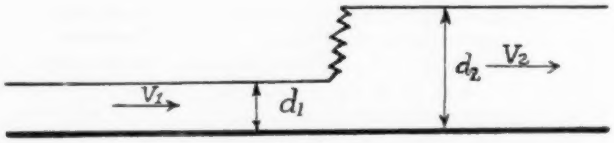

FIG. 1

It has been proved that the loss in head by "sudden expansion" is not equal to $\frac{(v_1 - v_2)^2}{2g}$, but is probably this quantity multiplied by a constant. The lifting of the mass of water through the distance stated is incorrectly applied in this equation.

Mr. Andrews mentioned that possibly "the difference between the velocity heads should be equated to the difference between the height of the center of gravity of the water masses in the two phases of flow, rather than the difference between the elevation of the two water surfaces." Thus

$$\frac{v_1^2 - v_2^2}{2g} = \frac{d_2 - d_1}{2}$$

The resulting equation solved for d_2 is

$$d_2 = \frac{v_1^2}{2g} \pm \sqrt{\frac{v_1^2}{g}d_1 + \left[\frac{v_1^2}{2g}\right]^2} \quad \ldots\ldots\ldots\ldots [A]$$

Considering the center of mass as a measure of the potential energy of water has caused more than one such error in hydraulics.

Professor Unwin seems to have properly analyzed the forces acting in this phenomenon when he suggested that it might be regarded as a case of Newton's second law of motion. He therefore equated the static force acting on the water to the loss of momentum that a unit mass of water would experience in changing its velocity from V_1 to V_2 in unit time. As Newton expresses this law, "The rate of change of momentum is proportional to the applied force and takes place in the direction in which the force acts." Stated in algebraic terms $F \propto \frac{M \cdot v}{t}$, or $Ft = M(V_1 - V_2)$, in which F is the dynamic force that causes the change of momentum from MV_1 to MV_2 in the time t. Professor Unwin considered the dynamic force in this problem as being equal to the difference between the pressure exerted by two columns of water equal in height to d_1 and d_2, respectively.

Thus from Fig. 1 the total force in favor of the flow is $w\int_0^{d_1} d_1$ and the total force of the water opposing the flow is $w\int_0^{d_2} d_2$.

Integrating these forms between their limits and taking their differences will give $\frac{wd_1^2}{2} - \frac{wd_2^2}{2}$, or $w\left(\frac{d_2^2 - d_1^2}{2}\right)$, the latter expression giving a positive sign to the form. Equating this to the change of momentum $\frac{wQ}{g}(V_1 - V_2)$, where $M = \frac{wQ}{g}$, and placing $Q = d_1V_1$, will give

$$\frac{w(d_2^2 - d_1^2)}{2} = \frac{wd_1V_1}{g}(V_1 - V_2) \quad \ldots\ldots\ldots\ldots [3]$$

Substituting $V_2 = \frac{d_1V_1}{d_2}$ and solving for d_2, gives the three roots

$$d_2 = \pm\sqrt{\frac{2d_1V_1^2}{g} + \frac{d_1^2}{4}} - \frac{d_1}{2} \quad \ldots\ldots\ldots\ldots [4]$$

$$d_2 = d_1$$

Equation [3] is a cubic with two positive roots and one negative root.

Table 1 shows how the four equations just developed compare with Bidone's original measurements.

TABLE 1

d_1	V_1	Observed d_2 (Bidone)	Belanger	Merriman	Andrews' Equation [A]	Unwin
0.149	4.59	0.423	0.424	0.439	0.781	0.373
0.155	4.47	0.423	0.438	0.437	0.749	0.367
0.208	5.59	0.613	0.642	0.636	1.148	0.537
0.242	6.39	0.764	0.811	0.777	1.436	0.663
			No energy loss	Sudden expansion	Center of mass	Momentum

It is interesting to note that Bidone conducted his experiments in a flume which had a slope of approximately one-tenth of a foot per foot, and as a result all his measurements are higher than the momentum formula gives. Without discussing the discrepancies in Table 1, it will suffice to mention that even rationally developed formulas are subject to serious error if the problem is not properly analyzed and that to attempt to fit a formula to poorly taken data will prove just as disastrous.

The truth of Professor Unwin's formula has been proved beyond any question or doubt. Many experiments covering a large variation in the value of d_1 and V_1 have been made and checked with this formula, so that its application to a jump produced in a horizontal flume may be made without fear of using it incorrectly. The 1929 edition of Professor King's "Handbook of Hydraulics" has a special chapter devoted to the hydraulic jump that covers the subject so completely that any one may readily apply the principles stated therein to his particular problem.

MATHEMATICS OF THE JUMP IN SLOPING FLUMES

With the extended application of the jump it became desirable to more easily control its action. This was accomplished by causing it to occur in a sloping flume. By so doing the mathematics required to compute the resulting tail-water depth became greatly involved. Introducing this additional variable caused a complete failure of the classical momentum formula when applied to this flow condition. Empirical formulas now developed, however, give accurate results for the range of all present data on sloping flumes.[4]

The real difficulty encountered in the development of a rational formula was to find a satisfactory explanation of the action taking place within the jump and to determine just how this action should be expressed mathematically. An attempt is here made to apply the principles which underlie the conservation of linear momentum and to correlate flow conditions for jumps produced in both horizontal and sloping flumes.

Analysis From Newton's Second Law of Motion

The first step in the solution of this problem is to determine the value and direction of the applied forces acting on the water.

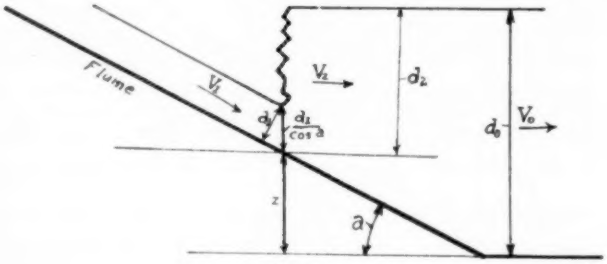

FIG. 2

If we consider Professor Unwin's case of a jump in the horizontal position, it will be observed that the dynamic forces acting on the water may be correctly translated into equivalent static forces. Fig. 2 is a diagram showing the profile of a jump produced in a sloping flume of rectangular cross-section and unit width.

The depth of the stream before impact is d_1 measured normal to the bottom of the flume, and its corrected vertical depth is $\frac{d_1}{\cos a}$, where a is the angle of inclination of the flume with the horizontal. The depth of water after impact is d_2. While the water does not rise immediately to this value as drawn in Fig. 2, still it is this depth which is effective in opposing the flow. The hydrostatic force in favor of the flow is $w \cdot \int_0^{\frac{d_1}{\cos a}} \frac{d_1}{\cos a}$, and the force opposing the flow is $w \cdot \int_0^{d_2} d_2$.

Integrating these functions, subtracting their limits, and taking the difference between them will give $\frac{wd_2^2}{2} - \frac{wd_1^2}{2 \cos a}$. Simplifying this, we have a statement of the force which causes the change of momentum from before to after impact. It is $w \frac{d_2^2 - \frac{d_1^2}{2 \cos a}}{2}$ in magnitude, and its direction is in a horizontal plane. **This assumes cos a to have constant values.**

The effective value of the momentum of the water before impact is the product of its mass times its velocity times the cosine of the angle at which it strikes. This is $Mv(\cos a)$, and for our case it is $\frac{wQV_1 \cos a}{g}$, where a is the angle from normal at which the water before the jump would impinge on a supposedly vertical tail-water wall. Fig. 3 shows this condition.

The velocity vector ($V_1 \cos a$) is co-linear with the tail-water velocity vector (V_2). This is exactly the condition that occurs during the production of the hydraulic jump in a sloping flume.

The instant impact occurs, the water flows practically horizontal. Hence we may therefore state that the value of the momentum after impact is simply equal to wQV_2/g, where V_2 equals the velocity of the tail water which corresponds to the height d_2 to which the water jumps. We are not concerned with the indeterminate flow conditions in the turbulent part of the jump or further down the flume, but only with the forces acting and the resulting theoretical change of momentum which takes place the instant impact occurs.

From the foregoing steps the final basic equation for jumps produced in either a sloping or horizontal flume may be developed, as follows. Starting with the original momentum equation $Ft = MV_1 - MV_2$ and making the proper substitutions will give $w \cdot \frac{d_2^2 - \frac{d_1^2}{\cos^2 a}}{2} \cdot t$ as the magnitude of the impulse and $\frac{wQV_1}{g} \cos a - \frac{wQV_2}{g}$ as the change of momentum.

Equating these two expressions, substituting $Q = d_1V_1$, and t to be unity, will give

$$w \cdot \frac{d_2^2 - \frac{d_1^2}{\cos^2 a}}{2} = \frac{wd_1V_1}{g} \cdot (V_1 \cos a - V_2) \ldots\ldots [5]$$

This is the final equation for a jump produced in either a sloping or horizontal flume. If the flume has zero slope, the cosine of the angle becomes unity and this formula reduces to Professor Unwin's familiar formula, Equation [3] of this paper. We are chiefly interested, however, in its treatment for jumps produced in sloping flumes. In order to more clearly understand the nature of this mathematical problem, it is best that Equation [5] be reduced to a cubic equation which can be readily plotted. This is done in the following steps.

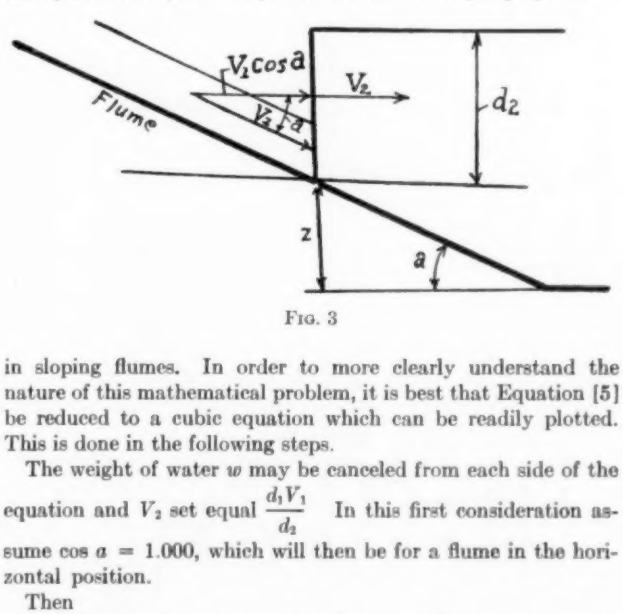

FIG. 3

The weight of water w may be canceled from each side of the equation and V_2 set equal $\frac{d_1V_1}{d_2}$ In this first consideration assume $\cos a = 1.000$, which will then be for a flume in the horizontal position.

Then

$$\frac{d_2^2 - d_1^2}{2} = \frac{d_1V_1}{g}\left(V_1 - \frac{d_1V_1}{d_2}\right) \ldots\ldots\ldots\ldots [6]$$

$$d_2^3 - d_2d_1^2 = \frac{2d_1V_1^2d_2}{g} - \frac{2d_1^2V_1^2}{g}$$

$$d_2^3 - \left[\frac{2d_1V_1^2}{g} + d_1^2\right] d_2 + \frac{2d_1^2V_1^2}{g} = f(d_2) \ldots\ldots [7]$$

The independent variables in this function are d_1 and V_1, while d_2 is the only dependent variable. If d_1 is set equal to unity and

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[4] **The statement and discussion of these formulas are given in the author's previous paper.**

V_1 assumed to have various positive values above zero, Equation [7] may be reduced to any of the following forms:

Let $V_1 = 4.02$, then Roots for $f(d_2) = 0$
$d_2^3 - 2d_2 + 1 = f(d_2)$...... $+1, +0.6, -1.6$....[8]

Let $V_1 = 5.67$, then
$d_2^3 - 3d_2 + 2 = f(d_2)$...... $+1, +1.0, -2.0$....[9]

Let $V_1 = 8.03$, then
$d_2^3 - 5d_2 + 4 = f(d_2)$...... $+1, +1.6, -2.55$..[10]

Let $V_1 = 11.35$, then
$d_2^3 - 9d_2 + 8 = f(d_2)$...... $+1, +2.4, -3.4$...[11]

For the conditions chosen in [8] no jump will occur, because V_1 is less than the critical velocity.

The conditions for [9] are for critical flow, and no jump can be produced.

Equations [10] and [11] are for conditions favorable to produce a jump.

These formulas are plotted on the graph, Fig. 4. It will be noted that since d_1 was chosen equal to unity, all the curves pass through $d_2 = +1$, and that when the velocity V_1 reaches the critical velocity the curve $f(d_2)$ has just become tangent to the d_2 axis.

From this it will be seen that it is necessary to select the proper value of d_2 when solving Equation [5], or Equation [3] which is derived from [5], for the particular flow conditions being studied.

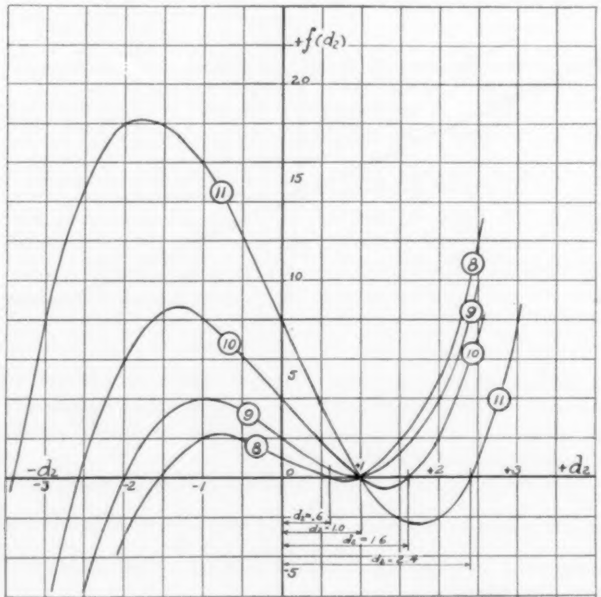

FIG. 4 HORIZONTAL FLUME DEPTH GIVEN BY POSITIVE ROOT OF $f(d_2)$, EQUATION [5]

$$d_2 = +\sqrt{\frac{2d_1V_1^2}{g} + \frac{d_1^2}{4}} - \frac{d_1}{2}$$

$$d_1 = +1.0, \cos a = 1.000$$

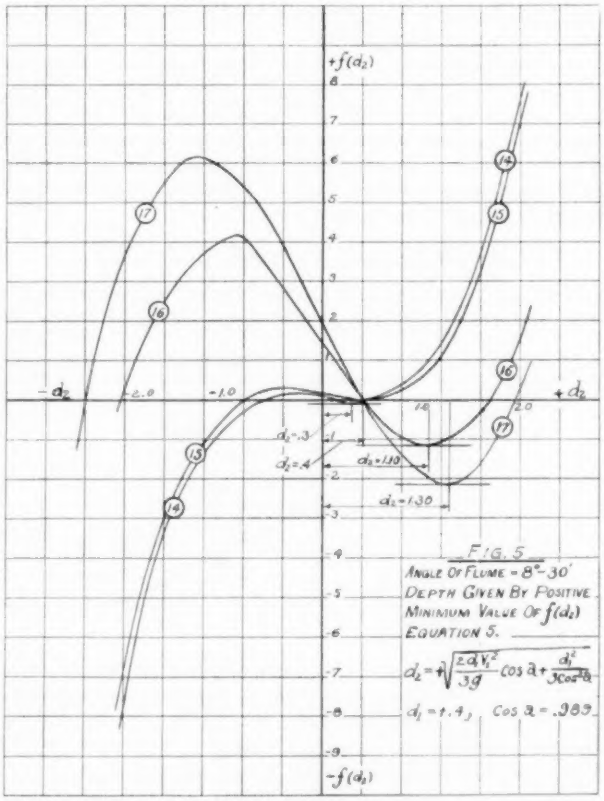

FIG. 5 ANGLE OF FLUME = 8° 30′. DEPTH GIVEN BY POSITIVE MINIMUM VALUE OF $f(d_2)$, EQUATION [5]

$$d_2 = +\sqrt{\frac{2d_1V_1^2}{3g}\cos a + \frac{d_1^2}{3\cos^2 a}}$$

$$d_1 = +0.4, \cos a = 0.989$$

It is now of interest to see what takes place when the jump is produced in a sloping flume, or in other words where a is not equal to zero. Canceling w from Equation [5], as before, and substituting $V_2 = \frac{d_1V_1}{d_2}$, we have

$$\frac{d_2^2 - \frac{d_1^2}{\cos^2 a}}{2} = \frac{d_1V_1}{g}\left(V_1 \cos a - \frac{d_1V_1}{d_2}\right) \ldots\ldots [12]$$

which may be reduced to

$$d_2^3 - \left[\frac{2d_1V_1^2\cos a}{g} + \frac{d_1^2}{\cos^2 a}\right]d_2 + \frac{2d_1^2V_1^2}{g} = f(d_2) \ldots [13]$$

Assume $\cos a = 0.989$, or $a = 8° 30'$. Also assume $d_1 = 0.4$ ft.

Let $V_1 = 2.00$, then Roots for $f(d_2) = 0$
$d_2^3 - 0.262d_2 + 0.040 = f(d_2)$.... $+0.4, +0.18, -0.55$..[14]

Let $V_1 = 3.58$, then
$d_2^3 - 0.478d_2 + 0.127 = f(d_2)$.... $+0.4, +0.4, -0.80$..[15]

Let $V_1 = 12.09$, then
$d_2^3 - 3.58d_2 + 1.45 = f(d_2)$.... $+0.4, +1.7, -2.05$..[16]

Let $V_1 = 14.10$, then
$d_2^3 - 4.87d_2 + 1.98 = f(d_2)$.... $+0.4, +1.95, -2.40$..[17]

For the conditions chosen in [14] no jump can be produced, because V_1 is less than the critical velocity.

The conditions for [15] are for critical flow, and no jump will occur.

Equations [16] and [17] are for conditions favorable to produce a jump.

These formulas are plotted on Fig. 5. Since d_1 was chosen equal to 0.4 ft., all the curves pass through $d_2 = +0.4$ ft. Here again the curve $f(d_2)$ is tangent to the d_2 axis just when V_1 equals the critical velocity.

Since Equation [5] has been developed along rational lines, it is only logical to believe that the larger positive root and value of d_2 would be the height to which the water would rise for jumps produced in a sloping flume. An attempt to use this value for data already obtained on jumps produced in sloping flumes failed completely and resulted in a calculated value approximately twice as large as the measured one. By substituting the measured value of d_2 obtained from previous experimental data in Equation [5], it was observed that the function of d_2 was very close to its positive minimum value. This immediately suggested the thought that the value for d_2 which would satisfy present experimental data was such a one that the value of the momentum equation would be a minimum. Thus in Equation [16] the $f(d_2)$ is a minimum when $d_2 = 1.12$ ft. Actual measurements for flow under the conditions of this formula give a measured height of 1.10 ft. Similarly, for Equation [17] the function is a minimum when $d_2 = 1.30$ ft., while the measured height of the jump was found to be 1.43 ft.

It is to be understood that Equation [5] has been developed entirely along rational lines and that experimental data have been used only to bring out and fortify the reasoning and conclusions herein disclosed. Since we have selected the positive minimum value of Equation [5] as that value of d_2 which accurately states the height to which the water will rise for jumps produced in sloping flumes, it becomes a simple matter to derive by calculus a proper formula which will accurately state this value. It is only necessary to take the first derivative of Equation [5], set it equal to zero, and solve for d_2. For convenience we may use Equation [13], which was derived directly from Equation [5], and write:

$$d_2^3 - \left[\frac{2d_1V_1^2}{g}\cos a + \frac{d_1^2}{\cos^2 a}\right] d_2 + \frac{2d_1^2V_1^2}{g} = f(d_2)$$

$$\frac{d(f \cdot d_2)}{dd_2} = 3d_2^2 - \frac{d_1^2}{\cos^2 a} - \frac{2d_1V_1^2}{g}\cos a \,.\,. [18]$$

Setting this function equal to zero and solving for d_2 gives:

$$3d_2^2 = \frac{2d_1V_1^2}{g}\cos a + \frac{d_1^2}{\cos^2 a}$$

and the two roots of this quadratic equation are:

$$d_2 = \pm\sqrt{\frac{2d_1V_1^2}{3g}\cdot\cos a + \frac{d_1^2}{3\cos^2 a}}\,.\, [19]$$

This formula is not a root of Equation [5]. It simply gives the value of d_2 which will make Equation [5] have a minimum positive value.

TABLE 2 COMPUTED AND OBSERVED TAIL-WATER DEPTHS FOR SLOPING FLUMES (EQUATION 19)

Run no.	Q cfs	cos a	Z	d_1	v_1	d_2 comp.	d_2 observed	$d_0 = d_2 + Z$	d_0 observed
1			3.00	0.468	10.00	1.000	0.910	4.000	3.910
2			2.40	0.340	11.31	0.948	0.710	3.348	3.110*
3	1.5	0.954	1.80	0.267	12.21	0.900	0.700	2.700	2.500
4			1.20	0.219	12.92	0.866	0.671	2.066	1.871
5			0.60	0.190	13.16	0.812	0.694	1.410	1.294
6			3.00	0.355	9.23	0.800	0.745	3.800	3.745
7			2.40	0.250	10.75	0.770	0.425	3.170	2.824*
8	1.0	0.954	1.80	0.187	11.62	0.705	0.595	2.505	2.395
9			1.20	0.158	11.94	0.672	0.565	1.872	1.765
10			0.60	0.138	12.08	0.629	0.430	1.229	1.030*
11			1.76	0.368	10.45	0.925	0.871	2.685	2.631
12			1.54	0.325	10.85	0.900	0.808	2.440	2.348
13	1.5	0.976	1.32	0.292	11.17	0.875	0.775	2.195	2.095
14			1.10	0.264	11.48	0.853	0.725	1.953	1.825
15			0.88	0.242	11.69	0.829	0.690	1.709	1.570
16			0.66	0.223	11.89	0.807	0.555	1.467	1.215*
17			1.76	0.265	9.68	0.725	0.635	2.485	2.395
18			1.54	0.234	10.01	0.702	0.660	2.242	2.200
19	1.0	0.976	1.32	0.210	10.35	0.685	0.610	2.005	1.930
20			1.10	0.192	10.53	0.666	0.563	1.766	1.663
21			0.88	0.176	10.71	0.648	0.576	1.528	1.456
22			0.66	0.164	10.84	0.630	0.575	1.290	1.235
23			3.46	0.21	11.63	0.771	0.91	4.23	4.37
24			3.79	0.22	11.86	0.806	0.88	4.60	4.67
25	20	0.989	4.12	0.24	11.41	0.765	0.86	4.89	4.98
26			4.43	0.26	11.17	0.828	0.89	5.26	5.32
27			4.77	0.27	11.51	0.870	0.86	5.54	5.63
28			5.03	0.36	9.54	0.845	0.95	5.88	5.98
29			5.48	0.43	8.57	0.841	0.94	6.32	6.42
30			3.55	0.28	13.46	1.031	1.16	4.58	4.71
31			3.79	0.30	13.06	1.035	1.14	4.83	4.93
32			4.06	0.32	12.98	1.068	1.15	5.13	5.21
33	31	0.989	4.37	0.38	11.77	1.060	1.19	5.43	5.56
34			4.77	0.40	12.09	1.120	1.10	5.89	5.87
35			4.95	0.49	10.35	1.075	1.20	6.03	6.15
36			5.40	0.64	8.91	1.082	1.26	6.48	6.66
37			5.55	0.81	7.39	1.060	1.41	6.61	6.96*
38			3.77	0.37	14.53	1.288	1.37	5.06	5.14
39			3.86	0.37	14.59	1.285	1.43	5.15	5.29
40			4.07	0.40	14.10	1.300	1.43	5.37	5.50
41	42	0.989	4.37	0.49	12.21	1.220	1.41	5.59	5.78
42			4.62	0.47	13.32	1.335	1.39	5.96	6.01
43			4.82	0.57	11.70	1.310	1.41	6.13	6.33
44			5.40	0.80	9.57	1.310	1.46	6.71	6.86
45			5.55	1.03	7.79	1.280	1.63	6.83	7.18*

* Starred values in measured error as per Fig. 6.

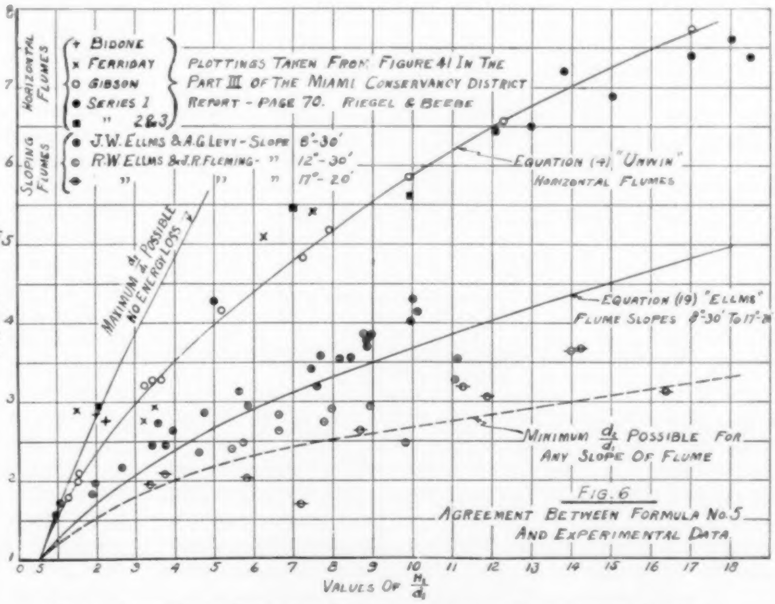

FIG. 6 AGREEMENT BETWEEN EQUATION [5] AND EXPERIMENTAL DATA

This formula has been used to compute all the data available on jumps produced in sloping flumes, and the extraordinary agreement between computed and observed values for d_2 are presented in Table 2. Runs Nos. 1 to 22, inclusive, were made in a small-capacity flume the angle of which was set at 17° 20′ for one set of tests and then changed to 12° 40′ for another set of tests. These readings made by the author are nearly all lower than the calculated value. The remainder of the runs listed in the table were made by J. W. Ellms and are nearly all a little higher than the calculated value. The flume used in these tests was non-adjustable and of high capacity. Its slope was 8° 30′ with the horizontal. Columns 6 and 7 give the computed and observed tail-water depths for the conditions of flow listed, while the last two columns give the depth of tail-water corrected for station elevation.

A further agreement between experimental data and Equation

[19] is shown in Fig. 6. Here are plotted the values of $J = d_2/d_1$ against the value of $\frac{H_1}{d_1} = \frac{V_1^2}{2gd_1}$. This method of comparing data with computed values is given by Messrs. Riegel and Beebe in a report entitled "The Hydraulic Jump as a Means of Dissipating Energy," which is a Technical Report of the Miami Conservancy District, Part 3, issued 1917. The plotted values of Bidone, Ferriday, Gibson, and Series 1, 2, and 3 are picked directly from Fig. 41, p. 70, of this report and are spotted along the theoretical "momentum curve" stated by Unwin for horizontal flumes merely to readily compare on the same graph the data now available for sloping flumes.

The curve farthest to the left represents "no energy loss," and therefore gives exceptionally high values for J. It will be observed that some early experiments of Ferriday, Bidone, and Series 2 and 3 fell to the left of the "no energy loss" curve, indicating a gain in energy which is impossible. The curve marked "Equation [19]" is really three separate lines representing three flume slopes. These lines are so close together that only one is drawn here. The dotted line is a boundary line for the minimum d_2/d_1 possible for any slope of flume and any value of d_1 or V_1. The points which are below this line are obviously in error. Likewise the points which fall above the Equation [4] line, and incidentally on the "no energy loss" curve, are impossible data. Regardless of the points in experimental error, these data taken in two widely different flumes by different experimenters remarkably substantiate the validity of Equation [19]. It may also be noted that all curves intersect where $(d_2/d_1) = 1$ and $(H_1/d_1) = 0.5$; that is, at the condition for critical flow.

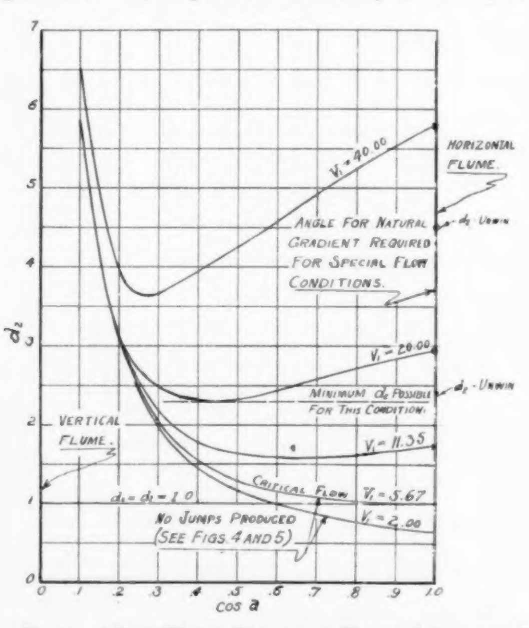

FIG. 7 GRAPH OF TAIL-WATER DEPTH AND FLUME ANGLE FOR $d_1 = 1.0$, EQUATION [19]

Fig. 7 illustrates the variation in the tail-water depth (d_2) for constant d_1 and V_1 conditions when only the angle of the flume is altered. Thus when $\cos a = 1.0$ the depth, d_2 is calculated by Unwin's formula. This calculation will hold true until the slope of the flume is increased just a little greater than the angle of natural gradient required to suit the special flow conditions in the flume. This position is represented by the vertical dotted line just to the left of $\cos a = 1.0$. As the angle of the flume increases, the depth d_2 falls off to some minimum value at which time $d_1/\cos a$ increases faster than d_2 was decreasing. This causes a rapid increase in d_2 for the smaller angles. At the condition $V_1 = 5.67$ for critical flow, no jump will occur, and d_2 only increases as the $d_1/\cos a$ causes it to increase. The curve for $V_1 = 2.00$ is only theoretically possible, and no jump will be produced.

Thus it may be seen that for $d_1 = 1.0$ and $V_1 = 20.00$, d_2 will equal 4.5 for a flume in the horizontal position. If the angle of the flume is tilted just a little steeper than the natural gradient required for the particular flow condition, d_2 will drop to 2.94 ft. This position is also shown on Fig. 6 at $(H_1/d_1) = 6.23$ and $(d_2/d_1) = 2.94$. If the angle of the flume is increased, d_2 will constantly drop to a minimum of 2.33, also shown on Fig. 6 at $(H_1/d_1) = 6.23$ and $(d_2/d_1) = 2.33$ on the dotted line. A further decrease of the flume angle will cause an increase in d_2 to infinity, where $a = 0.0$ for a flume in the vertical position. This is clearly shown on Fig. 7.

The physical reason for using the minimum positive value of Equation [5] for the tail-water depth (d_2) will now be discussed.

It is necessary to show purely from a physical standpoint why the jump produced in a sloping flume behaves according to the mathematics in this article. Referring to Fig. 2, it will be seen that it is impossible under the conditions shown for the depth of water d_1 above the jump to continue indefinitely on down the flume. It would be possible for this condition to occur in jumps produced in horizontal flumes. For the conditions of impact shown in Fig. 2, d_2 can be any value from $d_2 = (d_1/\cos a)$ to the depth stated by the momentum formula, Equation [4]. However, it cannot equal the latter depth, because there is an additional loss of energy due to the angular impact of the water. The rise in the surface of the water must therefore be a compromise between these two depths, and must necessarily be where the function of d_2 in Equation [5] is a minimum. The transition between the horizontal and sloping flume is therefore at the natural gradient for the particular flow condition. This angle is affected by flume friction, depth and velocity of water, shape of flume, and the like. No attempt has been made to correct any of the present formulas for the natural slope.

Some one might raise the question as to whether the water can be considered to flow horizontally immediately after impact. It is quite evident from an inspection of the turbulent impact taking place during the production of the jump that its flow is far from unidirectional. The only logic which may be applied here is that the momentum formula commonly used for jumps produced in a horizontal flume applies to that condition of turbulent flow as well as does Equation [19] now apply for jumps produced in sloping flumes. In either case the path of the center of mass of the moving water is very unsteady. Again, for both cases it cannot be said that the time of impact t is unity. It is quite evident from bubbles of entrained air in the turbulent part of the jump that internal impact and eddy losses occur a considerable distance down the flume.

A study of Fig. 6 or the data in Table 2 will show that the computed and measured values of d_2 do not agree exactly and that in general the runs made by the author are lower than their computed value, while the runs made by J. W. Ellms are higher than given by Equation [19]. It is very possible that different observers would read consistently high or consistently low tail-water depths. It is also possible that the flare of the flume and the effect of the floor of the flume (horizontal section) would cause these discrepancies. It is also quite likely that considerable velocity head was regained in the large expanding flume used by J. W. Ellms while the narrow flume with a steep descent used by the author caused a loss of velocity head.

CONCLUSIONS

The principal development in this paper is a comprehensive and rational mathematical solution of the jump for both sloping and horizontal flumes as expressed in Equation [5]. This formula states the relationship which exists between linear and angular impact. From it are derived the convenient forms [4] for horizontal flumes and [19] for sloping flumes.

The natural gradient determines the critical slope which indicates the correct form of Equation [5] to use.

Increasing the angle of the flume decreases the height of jump except for low flows and exceedingly steep flume slopes. This means that more energy is dissipated in jumps produced in sloping flumes than for those produced in horizontal flumes.

Essentially the same conditions of turbulent flow, time of impact, flume friction, and other related phenomena occur in jumps produced in either flumes. The factor of gained or lost velocity head after impact for jumps in sloping flumes has not been studied.

Data on jumps produced in sloping flumes materially substantiate the validity of Equation [5]. While these data do not prove the mathematical theories advanced in this paper, still both the mathematics and the physics of this phenomenon appear logical. The foregoing article is an attempt to stimulate constructive thinking about the jump so as to broaden its application and modify it to suit special flow conditions where impact occurs.

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Discussion

R. W. Angus.[5] The author has given a brief review of certain formulas for the jump and has shown his reasons for concluding that in horizontal rectangular flumes Professor Unwin's is the most accurate. The writer has found it convenient to put Equation [3] in the form $\frac{d^2}{2} + \frac{Q^2}{gd} = \text{const.}$, on both sides of the jump, and then to draw the curve corresponding to this equation for a given Q, plotting against d. This method is illustrated by Bakhmeteff in his "Hydraulics of Open Channels" and is helpful in studying this problem.

Referring to the author's Table 2, the last two columns lead one to think that greater accuracy has been obtained by formula [19] than is actually the case, for the relatively large values of Z mask the value of d. The formula is used to calculate d_2, and the writer has selected at random every fifth run so as to shorten his discussion. With these values, the results to two decimal places are:

Run No.	5	10	15	20	25	30	35	40	45
$\frac{d_2 \text{ computed}}{d_2 \text{ observed}} =$	1.17	1.46	1.20	1.18	0.89	0.89	0.90	0.91	0.79

The minimum difference is 9 per cent, but the errors vary from this value to 46 per cent and are both plus and minus, so that there is some uncertainty as to the true value. The points plotted in Fig. 6 also suggest the same uncertainty in the result, and it would be impossible to draw any one curve corresponding, even roughly, to the great variety of points shown. The points on Fig. 6 show that the curve, if such is possible for the 8° 30′ slope, is much steeper than the one for the 17° 20′ slope, and that the one for the 12° 30′ slope lies between them; in fact, there are evidently three distinct experimental curves, while the paper shows that for the theory presented the three computed curves nearly coincide.

The writer is not altogether clear about some parts of Table 2. The columns headed d_1 and v_1 give information for finding Q, since $Q = v_1 d_1$, but this does not agree with the discharges given in column 2, and further, the three lower values of Q appear to be in error.

The author is to be commended for his table of experimental results and his constructive attack on a very troublesome problem.

Carroll F. Merriam.[6] The basis for the Unwin formula can perhaps best be visualized by the analogy illustrated in Fig. 8. Suppose that a stream of water at high velocity is deflected into a tank mounted on wheels as shown; the impulse of the striking water would cause a definite thrust against the car. Suppose at the same time an equal quantity of water were allowed to discharge horizontally from the tank at low velocity in the same direction as the entering stream; it is evident that there would be a reaction against the tank in a direction opposing the thrust of the impulse of the entering stream. Furthermore, since the quantities are the same in both cases, the net thrust would be determined by the flow and the difference in velocity in and out of the tank.

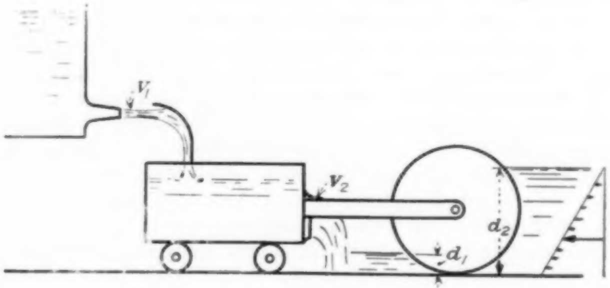

Fig. 8

Now assume that the car on which the tank is mounted is connected to a roller dam which is free to move on a horizontal plane, and that the force exerted would just balance the hydraulic thrust when the water is dammed to a height equal to that of the backwater of the hydraulic jump.

It has been demonstrated that although the action in the jump is exceedingly complex and consists of the pelting of a myriad of particles against a wall of water with a considerable absorption of kinetic energy, nevertheless this phenomenon is mathematically equivalent to this example.

The author deserves credit for demonstrating wherein several investigators have tried to fit theory to erroneous observations, and that finally when observations were carefully made by Gibson, the Unwin equation was found to satisfy the conditions exactly.

It is not clear, however, that the author has satisfactorily explained the marked lack of continuity in Fig. 7, where passage through the critical slope at a cosine slightly less than unity produces an abrupt change in d_2. Is it not possible that the jump, since the location where the author assumes the reaction to take place is not definitely located with respect to the actual phenomenon, may take place as though at some other point farther down the slope where the corresponding values of d_1, d_2, V_1, and V_2 would satisfy the momentum equation and be continuous with the solution for slopes less than the critical?

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Furthermore, the data represented on Fig. 6 indicate that there is considerable effect on the ratio d_2/d_1 caused by the angle of slope of the flume, whereas the author's Equation [19] shows so little effect that the curves for three different angles are drawn as one. This suggests that these differences may be accounted for by lack of definiteness as to the point where the jump can be considered to be concentrated, and that inconsistencies in the results may arise from the fact that d_2 is measured by reference to the height above the bottom of the flume at this point.

AUTHOR'S CLOSURE

The author wishes to thank those who have contributed to the discussion of this paper and hopes to clear up some phases of this research which may have been misleading. Recent mathematical investigations of the hydraulic jump have thrown new light on this problem, and their results will explain apparent discrepancies.

Before giving direct answer to the points mentioned in the foregoing discussions it will be best to briefly summarize what the author believes is one of the most important published papers on the hydraulic jump when it is produced in sloping flumes. The work referred to is a paper entitled "Supporting Force and Its Application to the Computation of Backwater Curves," by Dr. Ing. Th. Musterle, of Karlsruhe. This paper was approved by the structural engineering department of the Technische Hochschule at Karlsruhe as a dissertation, and has recently been translated by Herbert N. Eaton, Acting Director of the Bureau of Standards, Washington, D. C.

To summarize, Musterle explains that an independent method has now been developed for the computation of backwater curves on the basis of the law of supporting force as stated in the principle of the conservation of linear momentum. According to Newton, the applied force is equal to mass times the acceleration, or

$$F = m\frac{dv}{dt} \qquad \text{[A]}$$

To apply this principle to a moving stream, Musterle shows that this law can be expressed in the form:

$$P_1\Delta F_1 - P_2\Delta F_2 + \Delta G \sin a = m\frac{dv}{dt} \qquad \text{[B]}$$

where $P_1\Delta F_1$ and $P_2\Delta F_2$ represent the normal fluid pressures at any two given stations denoted respectively by the subscripts 1 and 2, *and "$\Delta G \sin a$" represents the resultant of the force of gravity in the direction of flow as given by the flume angle "a."* He shows in his paper that Equation [B] can be expressed in the form:

$$S_1 - S_2 + G \tan a - W = 0 \qquad \text{[C]}$$

in which

$S = b\Phi\left[\frac{d^2}{2} + \frac{dv^2}{g\cos^2 a}\right]$, which is an expression for the hydrostatic force and impulsive force at any vertical section of the stream. Here b = flume width, Φ = fluid density, d = depth of stream (vertical section), v = velocity of water, a = flume angle, and g = acceleration due to gravity. $G \tan a = b \times \left[\frac{d_1 + d_2}{2} l.\Phi \tan a\right]$ where l is the length of reach between the vertical sections 1 and 2.

W represents the frictional force opposing the flow and is therefore negative in this expression. Musterle states, "The actual magnitude of the frictional forces can only be determined in Nature. We have a measure of the influence of friction and slope in the well-known Chézy formula:"

$$v = c\sqrt{RI} \qquad \text{[D]}$$

in which v = mean velocity over the cross-section
c = velocity coefficient
R = area of stream cross-section divided by the hydraulic radius, and
I = flume slope.

Through several substitutions it was shown that the frictional force could be stated as:

$$W = \frac{d^2v^2(b + d_1 + d_2)l \cdot \Phi}{c^2\left[\frac{d_1 + d_2}{2}\right]^2} \qquad \text{[E]}$$

With the density of water, Φ, equal to unity, Musterle's final equation took the form:

$$b\left[\frac{d_1^2}{2} + \frac{d_1v_1^2}{g\cos^2 a}\right] - b\left[\frac{d_2^2}{2} + \frac{d_2v_2^2}{g\cos^2 a}\right] + b\left[\frac{d_1 + d_2}{2} l \cdot \tan a\right] - \frac{(d_1v_1)^2(b + d_1 + d_2)l}{c^2\left[\frac{d_1 + d_2}{2}\right]^2} = 0 \qquad \text{[F]}$$

According to Musterle, "The solution of this equation for d_1 or d_2, when the other quantities are known, is very difficult. It is best accomplished by cut-and-try methods." A study of this equation will reveal that the values of d_1 or d_2 are dependent upon the values of c. For smooth channels, c is large, and when this value is squared the last term of this equation has relatively very little value. Equation [F] is developed on the basis that the frictional forces on the side walls and floor of the flume, throughout the length of reach l, are sufficient, or insufficient, to produce a jump.

The author wishes to make clear that Equation [F] can only be used when the value of the velocity coefficient c can be determined. To attempt to solve any of the data given in this paper by the use of Equation [F] will show that these hydraulic jumps were produced on the flume slopes by the *artificial interposition of a baffle* downstream from where the jump was made to take place, and that they were not produced by flume side-wall and floor friction for which his equation was developed. As a result there is no way to predict the value of c which should be used in Equation [F] to give the proper tail-water depths. Equation [19], however, will give the correct tail-water depth for the type of jumps discussed in this paper. For the flume used by A. G. Levy and J. W. Ellms, as well as the one used by the author, the water was backed up on the slope so as to cause a jump to form there. The position of the jump on the slope was regulated by the height of weir downstream from the jump. For filtration-plant work it was necessary to regulate the tail-water level between certain elevations in order to insure the occurrence of the jump within the range of the slope. No jump would have occurred naturally in these steep flumes from only the frictional forces in the length of reach the jump was made to take place. This is why baffles were employed.

It will now be of interest to show how the author's Equation [19] can be derived from Musterle's Equation [F] after the latter is corrected for angular impact, and thus to emphasize the definite mathematical relationship between these two equations.

Since it has been shown by Musterle's equation that the values of d_1 or d_2 are dependent upon the value of c, it will be of interest to learn what effect the limiting values of friction produce on the backwater levels. To increase c to infinite proportions will decrease the frictional resistance W and cause the height of jump, d_2, to approach the value stated by Professor Unwin for horizontal flumes, corrected only by the flume slope. To decrease the value of c will increase the resistance to the flow. The limiting value of c is zero, for which case W will be infinite. Here we may consider that the length of reach l is infinite, which means that the *total* resistance to the flow of the water after impact must necessarily be infinite. For the type of jumps described in this paper the water before the jump plunged into a pool of slowly receding tail water, the rate of discharge being controlled by submerged baffles. This condition approximated an infinite resistance to the flow for which l could be considered infinite, or the tail water considered to have infinite mass. If $l = \infty$, then the third term, as well as the fourth term, in Musterle's equation equals infinity, while d_2 approaches an infinite depth, and v_2 approaches zero. This makes Equation [F] indeterminate. This indeterminate flow condition proved a stumbling block to an orderly study of this phenomenon many times before. For $c = 0$ and $b = 1$, Equation [F] can now be written as

$$\frac{d_1^2}{2} + \frac{d_1 v_1^2}{g\cos^2 a} - \frac{d_2^2}{2} - \frac{d_2 v_2^2}{g\cos^2 a} + \infty - \infty = f(d_2)\ldots \text{[G]}$$

In this equation, and for the type of jump under investigation, it would seem natural that the proper value of d_2 would be such a one as to make the change of momentum have a minimum value, a conclusion previously reached. To put Equation [G] into the same terms used in developing Equation [19], it is necessary to have

$$\begin{aligned} d_1 \text{ (Musterle)} &= \frac{d_1}{\cos a} \text{ (Equation [19])} \\ d_2 \text{ (Musterle)} &= d_2 \text{ (Equation [19])} \\ v_1 \text{ (Musterle)} &= V_1 \cos a \text{ (Equation [19])} \\ v_2 \text{ (Musterle)} &= V_2 \cos a \text{ (Equation [19])} \end{aligned}$$

Then

$$\frac{d_1^2}{2\cos^2 a} + \frac{d_1 V_1^2}{g\cos a} - \frac{d_2^2}{2} - \frac{d_2 V_2^2}{g} = f(d_2)\ldots\ldots\ldots \text{[H]}$$

Since Musterle's equations are developed on the basis of collinear velocities throughout the length of reach considered, and since it has previously been pointed out that this is not the case for the type of jumps studied by the author, it is necessary to correct V_1 again for angular impact. Thus in Equation [H], V_1 should be multiplied by $\cos a$, which indicates that the V_1 and V_2 velocity vectors are not in the same straight line, or even parallel, but differ by the amount of the flume angle a. Correcting V_1 in this manner yields

$$\frac{d_1^2}{2\cos^2 a} + \frac{d_1 V_1^2 (\cos a)}{g} - \frac{d_2^2}{2} - \frac{d_2 V_2^2}{g} = f(d_2)\ldots\ldots \text{[I]}$$

The first derivative of this function with respect to d_2, set equal to zero and solved for d_2, gives

$$d_2 = \pm\sqrt{\frac{2d_1 V_1^2(\cos a)}{3g} + \frac{d_1^2}{3\cos^2 a}}$$

which is the author's Equation [19] and which expresses in the positive value the height of jump d_2 necessary to produce a minimum change of momentum.

It must be borne in mind that frictional disturbances in the vicinity of the jump will influence the height of water directly after impact, and also that Equation [19] is developed on the theory that the length of slope is infinite. For all practical purposes this condition can nearly be realized by the use of baffles, which should be placed as far down the slope as possible, or even better on the horizontal floor of the discharge flume. Equation [19] is only strictly true for jumps produced in flumes of constant width. For flumes with flaring sides, d_2 is slightly reduced by the value $\sqrt{(b_1/b_2)}$, in which case a small finite length of reach must be used. Care should be used in applying this correction factor since no correction for flare has been introduced in the velocity vectors used in the development of Equation [19].

The author is indebted to Dr. Musterle for having developed the laws governing the type of jump produced by flume friction for sloping flumes. Heretofore a great deal of misunderstanding existed so that it was confusing even for those men who were thoroughly familiar with the action of the hydraulic jump and who perhaps felt that different flow conditions could be imposed on a moving stream to produce a jump.

It is important at this time to distinguish between the two types of jumps because it will answer the question raised by Mr. Merriam as to the "lack of continuity in Fig. 7, where passage through the critical slope at a cosine slightly less than unity produces an abrupt change in d_2." The explanation is that when the value of c in Equation [F] is such that it overcomes the effect of the slope, the jump will be produced according to Musterle's discussion and Equation [19] is invalid. In Fig. 7, therefore, for flumes of slight slope or high flume friction, d_2 should not be read from this graph but calculated by Equation [F].

In answer to Professor Angus' observation as to the accuracy of the data in Table 2, it will suffice to mention that measurements on turbulent waters are very difficult, especially when small laboratory experimental models are used. For this reason data taken on the large experimental flume used by Levy and Ellms were considered to have more weight than those taken by the author.

As for the values of Q stated in Table 2, the author only inserted them to give the reader some idea of the magnitude of the tests. Since flumes with flaring sides were used the value of the flume width was different for each of the 45 runs. The first four values of Q are for a small flume whose slope could be changed. The last three values of Q are for a large experimental flume which had a fixed slope. This large flume had a throat width of approximately five feet and a width at the downstream end of approximately 10 feet. The maximum flow which this flume could carry was a little over 40 cfs. The flume used by the author could only carry 1.5 cfs, and had a throat width of 0.25 ft and a downstream width of 0.67 ft.

From a consideration of the variation in flume sizes, the data are in good agreement. The author is positive that local frictional effect and the difficulty of making accurate water measurements on the small flume are the causes for observed discrepancies. It might also be mentioned that in Professor Angus' table of the ratio between computed and observed d_2, Run No. 10, which is off 46 per cent, will be noted as a starred run, for which the footnote of Table 2 explains as being in measured error. Aside from this run, the error in the data from Professor Angus' table is a plus 20 per cent and a minus 9 per cent.

Naturally many more data will have to be taken to confirm this work. It would be best to establish standard methods of measuring the jump so that results can be easily compared. With the help of Equation [19], and Musterle's Equation [F], much can be learned about the jump. The author believes that the theories advanced in these two papers can be further applied to other types of hydraulic flow.

Hydraulic Practice in Europe

By ROBERT W. ANGUS,[1] TORONTO, ONT., CANADA

The title describes in general the nature of this paper, but the author has confined the discussion to such features of European practice as are different from those in America. Various types of reaction turbines and draft tubes are illustrated, and also some two-nozzle Pelton wheels and one vertical four-nozzle machine. Types of valves, gates, and weirs peculiar to the Continent are discussed, and also some features of pumped-storage plants, with the special design of pump that is peculiar to them.

AS WOULD be expected, hydraulic practice in Europe is in many respects very much the same as in America, the interchange of engineers between the continents providing a means of unifying the methods of treatment of the various problems. But certain features have always been retained in Europe, and the author will endeavor to bring these to the attention of engineers in America. Reference will first be made to hydraulic power plants and their accessories; the large number of plants recently built in France, Germany, Italy, Austria, Switzerland, Scandinavia, etc., indicates that there has been great activity in Europe along this line. The power generated is used for operating the railroads, many miles of which have been electrified in Central Europe, and a great amount of power has been distributed to the municipalities and to industries.

REACTION TURBINES

Water powers, with both low heads and high heads, are very common; a discussion of the former will first be given. The low-head plants invariably contain reaction turbines, either of the Francis, propeller, or Kaplan type, of which the latter has until very recently been a purely European development. From the time of its invention by Viktor Kaplan, of Brünn, Czechoslovakia, the Kaplan turbine was put into usable shape by Swiss and German firms, and a number of machines were installed. One of the earlier plants was at Donaustetten, on the Danube near Ulm, Germany, and the latest, or at least the most prominent new one, is at Ryburg-Schworstadt, the first unit of this plant having been put into service in 1930, but the complete plant was not available till 1931. In the earlier developments the servomotor for the runner blades was on top of the machine and was stationary, with a piston revolving inside it. In some of the older units also the runner blades were moved by a forked lever, which engaged with lugs projecting from an inside rod through the hollow mainshaft. The turbines are now built entirely with the servomotor contained in the shaft just above the runner.

It is almost general practice in Europe to machine the runner blades of Kaplan and propeller turbines, this being accomplished by means of a vertical boring machine with the tool guided by a small-sized model. The machines are well worked out and produce a good copy of the model, and after the blades have been finished in this way they are ground. This method not only produces a very accurate blade giving a high efficiency, but as the blades are all alike, the runner is well balanced.

A section of the Donaustetten Power House is shown in Fig. 1, and there are four Kaplan units, two by Escher, Wyss & Co. and the others by Voith. The head is 25 ft, and each turbine produces 1740 hp at a speed of 214 rpm, the specific speed being 161. As the runner diameter is 6.85 ft, this corresponds to a rim speed 1.91 times the theoretical water velocity. The form of draft tube used is clearly shown and is quite typical of those with this kind of turbine, being made with a quarter-turn and a horizontal dividing wall down the center of the elbow and a vertical dividing wall under the outer end. The leading dimensions for the Voith unit are marked on the figure and show that the outlet of the draft tube is 39.4 ft from the turbine axis, which makes a long tube. One of the units showed efficiencies, as reported by Professor Braun, of Stuttgart, above 90 per cent for outputs between 760 and 1600 hp, while the maximum was 93 per cent at 1100 hp; the head during the test was below normal.

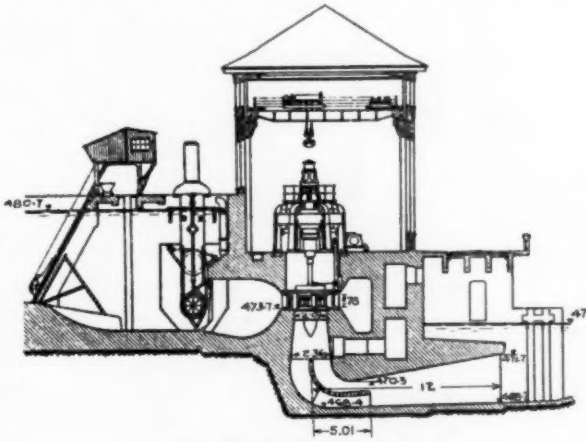

FIG. 1 DONAUSTETTEN POWER HOUSE
(Dimensions are in meters.)

A section through the Ryburg-Schworstadt Power House and turbine is shown in Fig. 2, taken from an Escher-Wyss publication. This plant, as is well known, is on the Rhine River, near the Swiss town of Ryburg (and Möhlin), and the works extend across the river from the Swiss to the German side, the spillway being in Switzerland and the power house in Germany. There are four Kaplan turbines, all exactly alike, and the designs are the results

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[1] Professor of Mechanical Engineering, University of Toronto. Mem. A.S.M.E. Professor Angus was born in St. Thomas, Canada, on January 24, 1873. His public- and secondary-school education was received in London, Canada, and in 1897 he was graduated from the University of Toronto with the degree of Bachelor of Applied Science in Mechanical and Electrical Engineering. His first practical engineering experience was obtained from E. Leonard & Sons in London and in the drafting room of the Standard Oil Company in Cleveland, Ohio. After his graduation he returned to Cleveland to work on a special gas-engine design, and later he had valuable experience with the Shoenberger Steel Company (the U. S. Steel and Wire Company), Pittsburgh. When the separate department of mechanical engineering was established at the University of Toronto, Professor Angus was appointed its first head. He was made Professor in 1906, which position he still holds.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of conferences between the engineers of the firms of J. M. Voith, Escher, Wyss & Co., and Ateliers des Charmilles. The first firm built two of the turbines, and each of the others built one. Each turbine has five runner blades and is built for a normal output of 35,000 hp at 36 ft head and 75 rpm, corresponding to a specific speed of 159, which is extremely high on such a large unit. The runner diameter is 23 ft, for which φ is 1.875, and some of the leading dimensions are marked. The figure shows the same type of draft tube as before, with both horizontal and vertical division walls, and a length of 115 ft from the axis of the turbine to the discharge end of the draft tube. The position of the runner-blade servomotor is shown in the mainshaft and also the general disposition of the parts is indicated. The first unit was put in service on November 3, 1930.

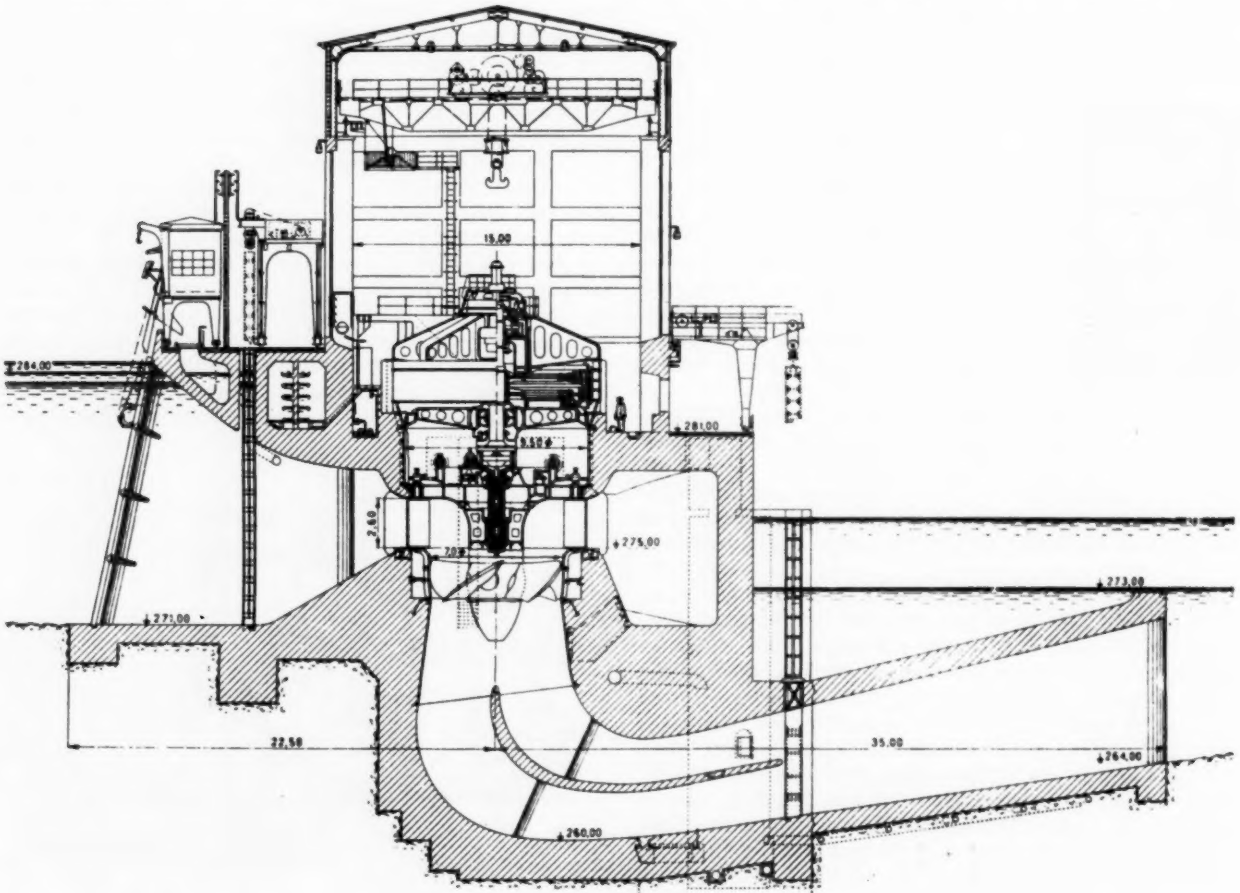


FIG. 2 RYBURG-SCHWORSTADT POWER PLANT
(Dimensions are in meters.)

By agreement, all units were to be accepted on the basis of test results on the first unit operated, which was the Escher-Wyss one, and therefore unusual care was exercised in making the test. Water measurements were made with a set of 15 current meters, and a timber-work guide was built in front to direct the water in straight lines. The results of the tests were given in *Power*, January 19, 1932, and indicate a maximum efficiency of 92.7 per cent at an output of 23,750 hp and about 83 per cent at 40,000 hp, while the efficiency exceeded 90 per cent between 10,000 and 35,000 hp. As the hub has a maximum diameter of 9.84 ft, the velocity head of the water after leaving the runner is close to 50 per cent of the entire head, so that the draft tube has an important service to perform. Very careful model tests were made on a wheel about 27.6 in. in diameter, with the object of finding out how to lessen cavitation, and it will be noted that the normal tail-water level is approximately at the center of the runner, the unit being set very low.

The type of draft tube shown in these two plants is also often used with Francis turbines, but instances of higher head installations are the developments at Eglisau, Mühleberg, and Chancy-Pougny in Switzerland. The first plant is on that part of the Rhine which is entirely in Swiss territory and is nearly north of Zurich, the power house being about 2 miles west of the Swiss town of Eglisau. The works cross the entire river, and they raise the water so much at the town that a new bridge had to be built as part of the project. There are seven vertical turbines, four having been built by Escher, Wyss & Co., Zurich, and three by Theodore Bell & Co., Lucerne, each guaranteed to deliver 6000 hp at 36 ft head and 83.4 rpm, these figures corresponding to a specific speed of 73, which would be a suitable speed for an ordinary high-speed Francis turbine.

An examination of Fig. 3 shows, however, a somewhat unusual shape of a runner, having a diameter of 6.56 ft over the blades at the crown and an exit diameter of about 13 ft, the distributor height being 3.3 ft. A photograph of this type of runner is given in Fig. 4*a* and shows the very large exit diameter relative to that at inlet. The draft tube is also of interesting design, being quarter-turn and curved throughout its length. Its top diameter is 14.1 ft, and the outlet end is rectangular, 16.4 ft by 25 ft, with well-rounded corners. Under a head of 32.8 ft, tests showed an output of 6210 hp with 79.6 per cent efficiency, and the latter rose to 85.9 per cent at 4676 hp, in both cases exceeding the expected results based on model tests.

A similar type of runner was used at a recent plant of the Bernischen Kraftwerke A.-G. located at Mühleberg on the River Aar, about 20 miles down from Berne. Here there are eight vertical machines, which may be run at either 133.3 rpm or 166.6 rpm, depending on the frequency desired, and at the former speed these produce 8100 hp each when the head is 65 ft, corresponding to a specific speed of 65. These turbines and draft tubes are almost identical with the Eglisau units and gave similar efficiencies.

Probably the most interesting examples of these unusual units are in the station at Chancy-Pougny, which stretches across the Rhone River a few miles below Geneva, the generating station being located on the Swiss side, while most of the overflow dam and transformer station are in France. Discussion will be confined to the five vertical generating units, three built by Ateliers des Charmilles and two by Escher, Wyss & Co.; there is also a 350-hp auxiliary unit. The main units were guaranteed for 8700 hp at a head of 29.1 ft and speed of 83.3 rpm, giving a specific speed of 115, so that the ordinary high-speed Francis unit would not be satisfactory. Both builders designed runners similar to those just described, the Escher-Wyss units being almost exactly the same as at Eglisau, while the Ateliers des Charmilles units have a slightly different form, as shown in Fig. 4*b*. A section of the plant is given in Fig. 5, and some figures of the elevations, etc., are marked on; the drawing shows a section through the draft tube and turbine. The Escher-Wyss units are spaced 46 ft on centers, while the others are 49.3 ft on centers.

Considerable interest centered in the turbines used on this plant because of the high specific speed, and careful tests were

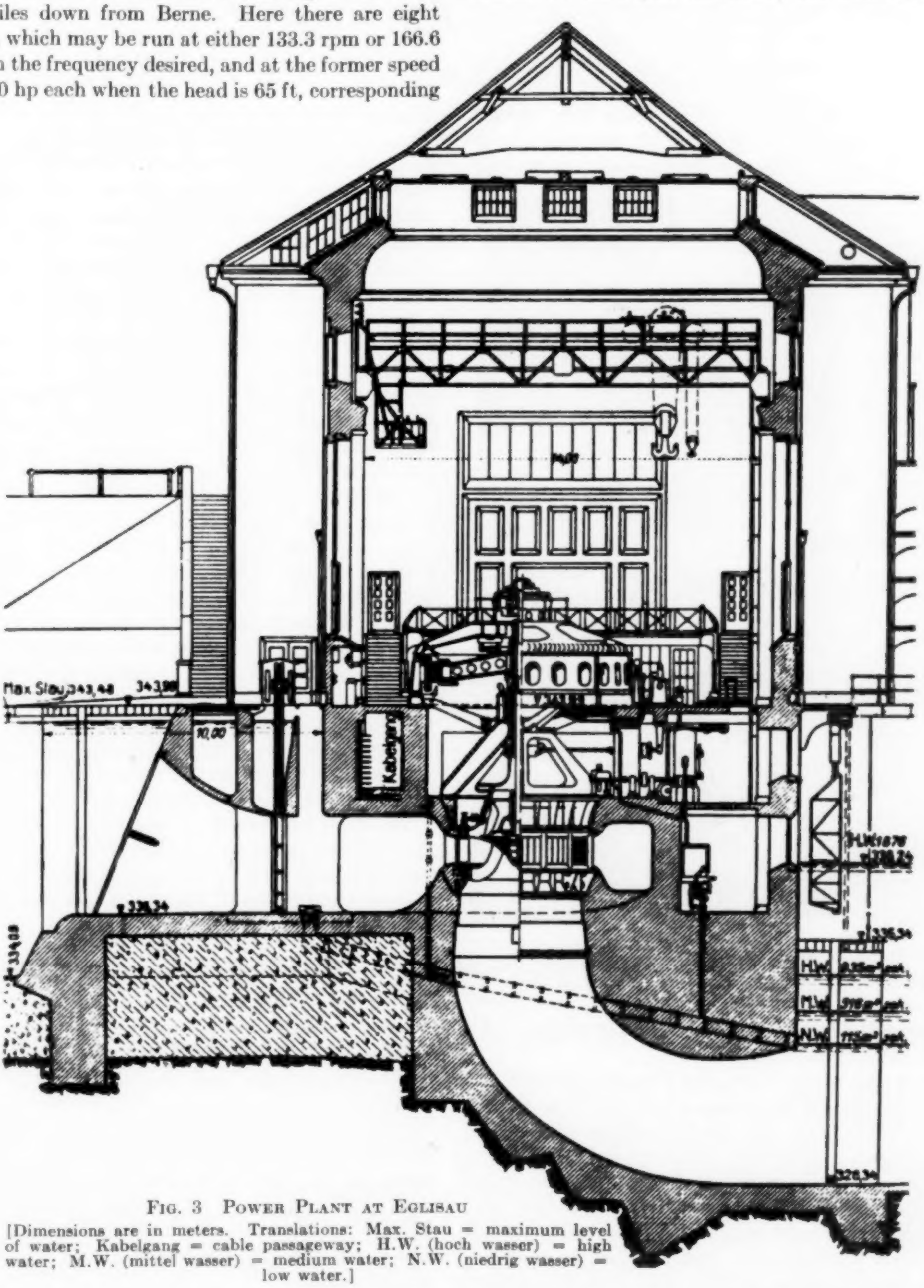


Fig. 3 Power Plant at Eglisau

[Dimensions are in meters. Translations: Max. Stau = maximum level of water; Kabelgang = cable passageway; H.W. (hoch wasser) = high water; M.W. (mittel wasser) = medium water; N.W. (niedrig wasser) = low water.]

carried out on one unit from each builder, the piers at the front of the power house being extended by timbers to give parallel flow toward the screens, as shown in Fig. 6. A set of nine current meters was mounted on a frame, as shown, and these were used to measure the water supplied, while the head and power produced were determined in the usual manner. One of the units gave a maximum efficiency of 88.9 per cent with an output of 9150 hp and a maximum output of 9450 hp with 85 per cent efficiency, while the other builder's unit gave its maximum efficiency of 78.7 per cent at its maximum output of 10,300 hp, all tests being at 29.1 ft head.

At the maximum output of power these units have specific speeds of 120 and 125, respectively, which is high for this type of wheel. Naturally, it is difficult to cast such a runner on account of shrinkage stresses, and they are often made in sections with the outside band shrunk on.

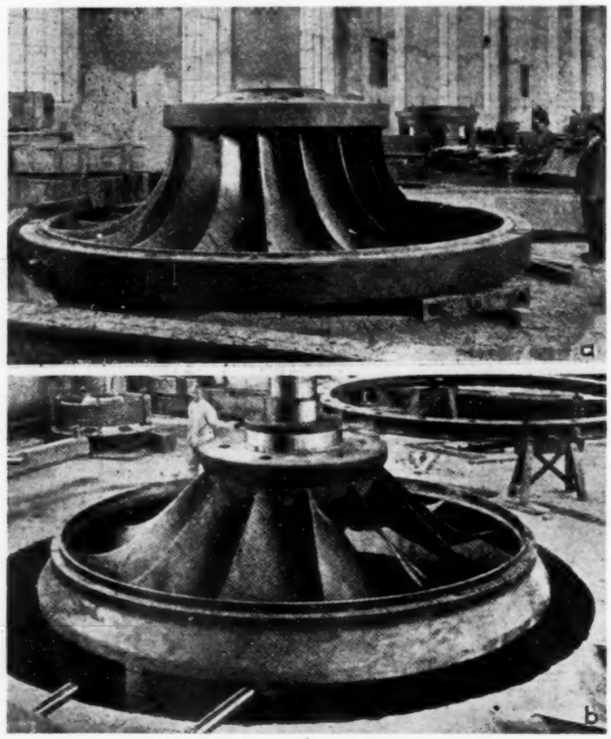


FIG. 4 TURBINE RUNNERS USED AT CHANCY-POUGNY
(*a*, Escher-Wyss runner; *b*, Ateliers des Charmilles runner. Picture from *Schweizer Bauzeitung*.)

Another unusual type is that built with a conical distributor, a design which enables the maximum capacity to be installed on an old setting without changing the center-to-center distances of the unit; in other words, the diameter of the unit for a given power is relatively small. One of the most interesting stations equipped with these units is that at Chevres, near Geneva, where they were placed by Ateliers des Charmilles. There are several types of turbines in this power house, which is quite old, but the units now under discussion displaced some vertical units with four runners on one shaft and which developed 1200 hp at 122 rpm and at 23 ft head.

These old turbines did not produce enough power and were replaced by new units with conical distributors, as shown in the photograph, Fig. 7, and the sectional drawing, Fig. 8, both from the builders' circular. The new units, placed in the same space as the older ones, gave 1850 hp at 21.3 ft head and 130 rpm, but the space was so cramped that the efficiency at this output was only 81.8 per cent. Results exceeding 90 per cent have been obtained with this design in other plants with more favorable settings. The distributor guides are moved either by a large bevel gear or else by the ordinary link mechanism with ball-and-socket joints. The turbines have been used with heads as low as 6.3 ft, with propeller and high-speed Francis runners, and they are made by several turbine builders.

The Lawaczeck turbine is best known through its adoption for a unit at the Lilla Edet Plant in Sweden. Careful tests were made to compare its merits with those of the Kaplan turbine in the same plant. The Lawaczeck turbine is not unlike the Chancy-Pougny turbines of Ateliers des Charmilles, but has not the outer band, and the machine at Lilla Edet has a runner diameter of 17.7 ft. In the comparative tests this turbine showed efficiency characteristics similar to propeller runners. It has a specific speed of 136 based on a head of 21.3 ft and an output of 10,000 hp at 62.5 rpm; these figures may be compared with those of the Kaplan wheel in the same plant, and which is 19 ft in diameter, 11,200 hp, and runs at 62.4 rpm, which corresponds to a specific speed of 140.

The very interesting two-bladed runner developed by Theodore Bell & Co. is known in America, being also investigated by the S. Morgan Smith Company, York, Pa. The runner has all the appearance of an auger and is the result of earlier research in the Bell laboratory and a careful investigation made by Dr. F. Prasil of Zurich. One of its most prominent early applications was in the Matte Hydraulic Power Station of the City of Berne.

The head on this plant is only 9.84 ft, and the plant had been equipped with three vertical Jonval turbines of 130 hp each at 28 rpm, which gave a specific speed of 18.3. As the demands for power increased it was desired to increase the output by the installation of new turbines, but as the space was limited, the power company desired the new turbines to be placed on the same foundations as the old ones, and the old draft tubes were to be used as far as possible. For this purpose Bell & Co. installed three of their new type of turbine, but had to reconstruct the draft tubes to a large extent. The new turbines developed 320 hp each at 9.84 ft head and 214 rpm, these figures corresponding to the very high specific speed of 218, and the units gave an efficiency of 85 per cent; these results are extremely good.

The draft tubes used are quarter-turn without division plate, the diameter at the top end being 5.57 ft, and at outlet the tubes vary a little in size, but all are oval, and one of them is 17.4 ft wide and 6.22 ft high. The tube slopes upward on the discharge part.

HIGH-HEAD TURBINES AND PELTON WHEELS

In impulse wheels the difference between the European and American designs is much more striking than with the reaction turbines. While the usual practice in America is to make single-nozzle runners and to obtain large powers by using two overhung wheels with the generator between them, it has been very common practice in Europe to use two nozzles per runner, and frequently two or three of the latter are mounted on the same shaft, but they are always placed close together and on the same side of the generator. A number of vertical turbines of the Pelton type have also been constructed, some with four nozzles, and as many as five have been used. It is possible to give only a few illustrations.

The machine shown in Fig. 9 was built by Costruzioni Meccaniche Riva, Milan, Italy, for the Mese Power Plant. This unit is of very large size, having an output of 35,000 hp under a head of 2430 ft and at a speed of either 500 rpm or 420 rpm, for frequencies of 50 and 42 cycles, respectively. There is but one runner with two jets, each having a maximum diameter of 6.30 in., the outside diameter of the runner being 8.81 ft. The positions of the jet deflectors and the mechanism for operat-

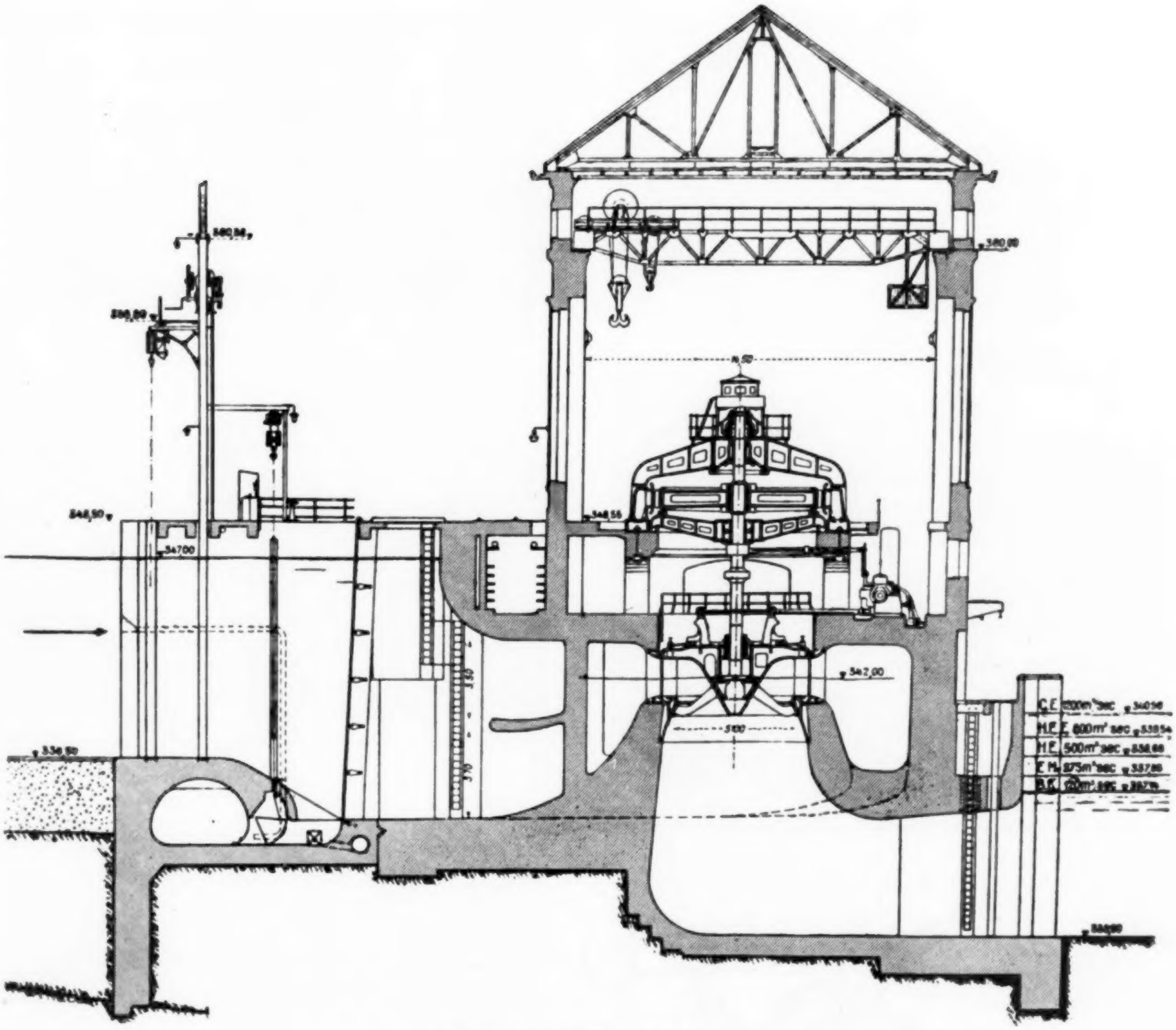


FIG. 5 CHANCY-POUGNY PLANT

[Dimensions are in meters. Translations: C.E. (élévation culminare) = highest possible elevation; H.E.E. (haut élévation extrême) = extreme high elevation; H.E. (haut élévation) = high elevation; E.M. (élévation moyenne) = medium elevation; B.E. (basse élévation) = low elevation. Picture from *Schweizer Bauzeitung.*]

ing them are clearly shown. There are 20 buckets, each of axial width 18.7 in. inside and each secured to the disk by two bolts. The center distance between the main bearings is 9.9 ft, the center of the runner being 4.84 ft from the center of the nearest one, and the shaft is 20.5 in. in diameter at the runner and 15.8 in. and 17.7 in. in the bearings.

Another turbine of large size is that at Schwarzenbach, a pumped-storage scheme belonging to the Badische Landes-Elektrizitäts-Versorgungs A.-G., of Baden, Germany. This plant contains both Francis and Pelton turbines, and those of the latter have an output of 27,500 hp at 500 rpm under a head of 1170 ft, which corresponds to a specific speed of 12.1 for the unit. One of the turbines in the plant, as made by J. M. Voith, Heidenheim, is shown in Fig. 10, and the shaft with the three runners mounted on it in Fig. 11, the buckets being made in pairs and each pair being held to the main disk by three bolts. Fig. 10 shows the arrangement of the distribution piping, also the mounting of the six nozzles, two per runner, and the mechanism for operating the needles. The pitch diameter of the wheel is 53.2 in., and the construction used requires the main bearings to be quite far apart.

Another large unit, built for a plant in Korea, with a head of 2180 ft, has an output of 45,300 hp at 360 rpm. In this case a single runner is used, and it is supplied by two jets about 90 deg apart, each being arranged with a deflector and moving needle.

The power plant at Jenbach, near Innsbruck, Austria, uses the Aachensee as a storage basin, and the head available for the turbines is 1250 ft, so that Pelton wheels have been used. There are eight units, these being of varying size, but the newest and largest one is for 31,000 hp at 500 rpm, and this machine uses 249 cfs. In this case there are two runners mounted on the same side of the generator, each with two jets, or four jets for the unit, and the specific speed of the unit is 11.83, or 5.91 per nozzle. In these units, as in many others, braking is effected by a small jet which may be made to strike on the backs of the buckets.

Many examples could be given if space permitted, for European practice has quite frequently adopted the two-nozzle runner and in some unusual proportions. In one instance, that of the

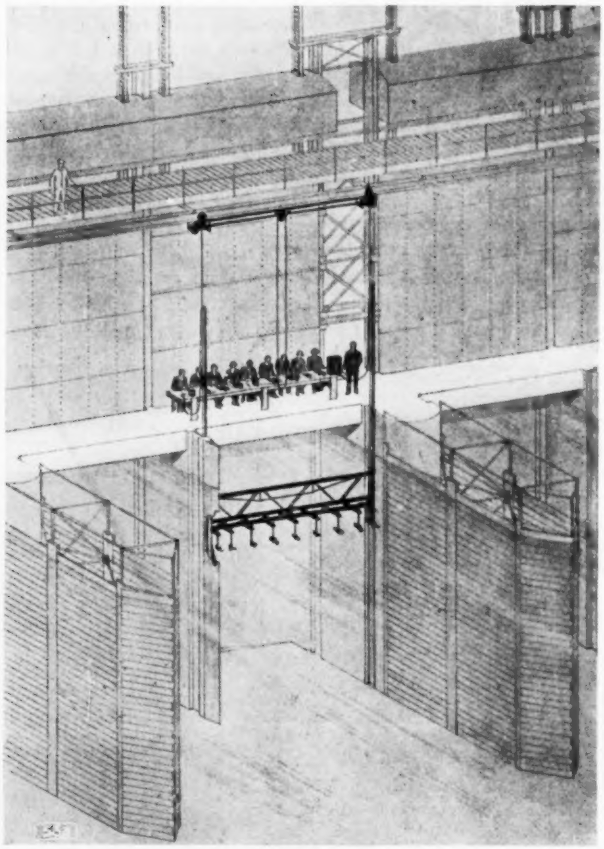


FIG. 6 MEASURING METHOD USED AT CHANCY-POUGNY

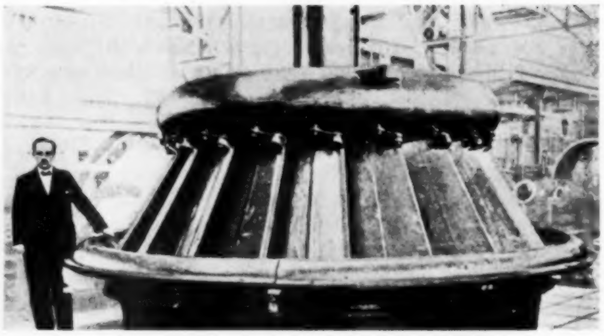


FIG. 7 TURBINE WITH CONICAL DISTRIBUTOR

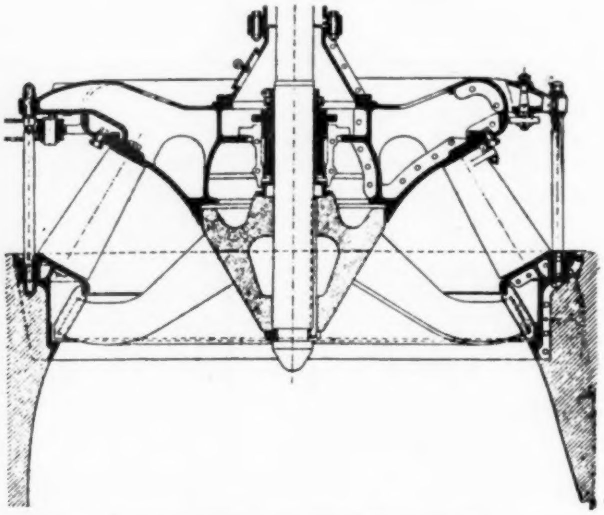


FIG. 8 SECTION OF TURBINE WITH CONICAL DISTRIBUTOR

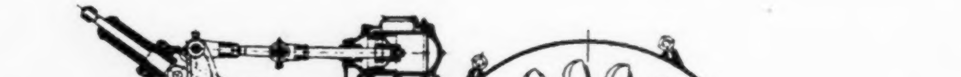


FIG. 9 MESE POWER PLANT 35,000-HP TURBINE
(Dimensions are in millimeters.)

Burglauenen Power Plant of the Jungfrau Railway, Switzerland, where the water used is sandy and unsuitable for a Francis unit, although the head is only 525 ft, an impulse turbine of 2900 hp was built by Escher, Wyss & Co., its speed being 500 rpm, so that the specific speed is 10.7. Just one runner with two jets was employed, giving the unusually high specific speed of 7.55 per nozzle, and since the maximum jet diameter is 5.57 in. and the pitch diameter of the runner is 37.4 in., the ratio is 1 to 6.7, a large value, which results of course from the high specific speed. This machine has a flywheel 7.2 ft in diameter and 11.25 in. face, the main bearings are 5.56 ft on centers, and the shaft is 8.65 in. in diameter at the bearings and 11 in. at the runner. This turbine gave a maximum efficiency of approximately 90 per cent at 800 hp, and the efficiency gradually fell as the load increased, being 85.5 per cent at 2000 hp and 76.5 per cent at 2900 hp.

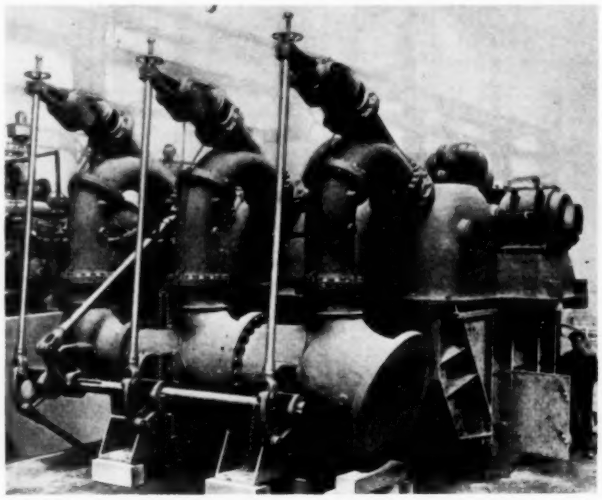


FIG. 10 SCHWARZENBACH TURBINE
(This has 27,500 hp and 1170-ft head, built by J. M. Voith.)

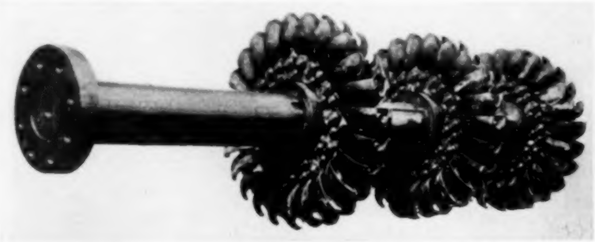


FIG. 11 RUNNERS FOR SCHWARZENBACH TURBINE

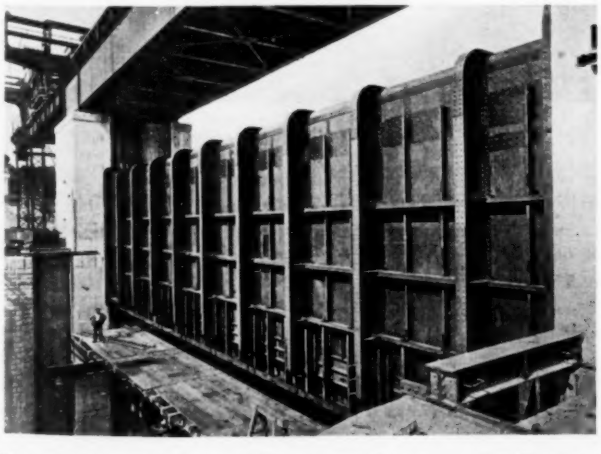


FIG. 13 UPPER PART OF ONE OF THE GATES AT THE RYBURG-SCHWORSTADT PLANT

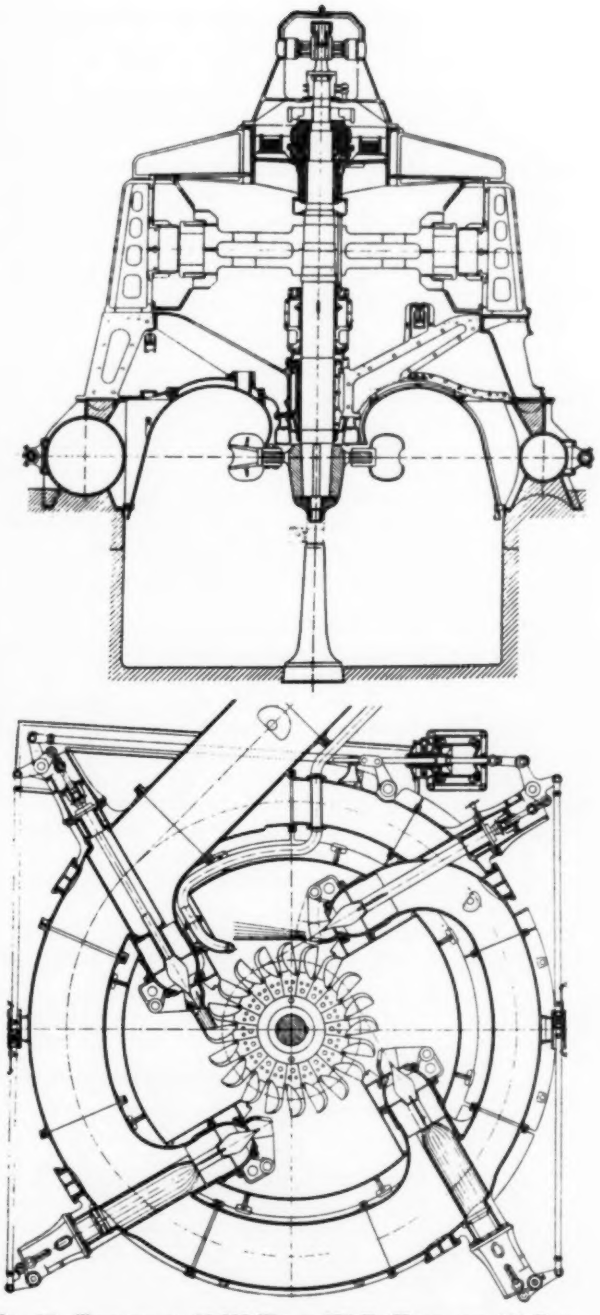


FIG. 12 TURBINE OF 18,600 HP AT 670-FT HEAD AND 250 RPM

But one further instance of Pelton units will be given, this referring to the vertical-shaft type more or less frequently made. These usually have four or more nozzles per runner, one built by Franco Tosi in Legnano in 1928 having five jets. The disposition of the parts in such a construction is shown in Fig. 12, which gives two sections of an 18,600-hp unit, running at 250 rpm under a head of 670 ft, the specific speed being 10, but as four nozzles are used the specific speed of each is only 5. The wheel was built by Escher, Wyss & Co. for the Whitehall Securities Corporation, London, which installed it in Chile. There is a spiral distributor pipe somewhat similar to the Francis turbine, and each nozzle has a needle, as well as a deflector, both of which are made of steel, the construction permitting of easy removal of the runner. This unit probably cost more than a Francis turbine would have done, and of course the Francis unit would have been at higher speed, but the water is gritty and it was thought that this type of wheel would have relatively little wear. The rotating parts were tested at 100 per cent overspeed.

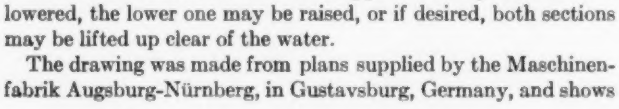
The horizontal section through the distributor shows how the needles are connected and gives some idea of the large external diameter of the casing necessary with such a set-up.

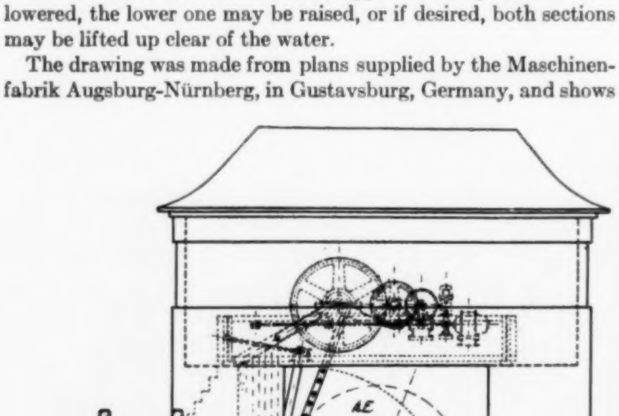
Control Gates

In America control gates are largely of the Stoney type, and these have been used to a limited extent in Europe. A gate that finds very common application there is also a lift gate, but it is divided into two parts, and its construction is such that it may be adapted to very large openings. For example, the weir or spillway at the Ryburg-Schworstadt Plant has four openings each 78.5 ft clear width, there being a pier 16.4 ft wide between each pair of openings, and each of the latter is controlled by a lift gate of the type mentioned. The upper part of one pair is shown in Fig. 13.

The drawing, Fig. 14, gives in detail the construction of one of the gates for the Austrian plant at Pernegg on the Mur River, not far from Graz. This plant has three vertical Francis turbines of 9100 hp each, at 59.5-ft head and 150 rpm, and the water for these turbines is diverted from the river and carried to the power house by a trapezoidal canal over a mile long. The dam extends across the River Mur and has in it three openings each 49.2 ft clear width, with piers 13.95 ft wide between the openings. The normal depth of the river at the dam is 38 ft, and the gates are made in two sections so that the upper one may be raised or lowered, the lower one may be raised, or if desired, both sections may be lifted up clear of the water.

The drawing was made from plans supplied by the Maschinenfabrik Augsburg-Nürnberg, in Gustavsburg, Germany, and shows the sections arranged for the maximum height of water, the dotted position above showing the whole structure lifted out. The motor-operated lifting gear above serves to draw up the sections, which have a weight of 114 tons for each double gate, and the total maximum lift is 40.7 ft. The upper section is 10.8 ft and the lower one 27.6 ft high, the former being mounted on four wheels 19.7 in. in diameter, while the lower section has eight wheels 29.5 in. in diameter and 10 in. wide, both sets of wheels traveling on very heavy rails. The detail drawings of Fig. 14*a* show at (*a*) the method of sealing the sides of the upper section and at (*b*) that used on the lower section, while (*c*) shows the method of sealing the bottom and top of the lower section. Leakage is prevented by contact of a block of wood with the





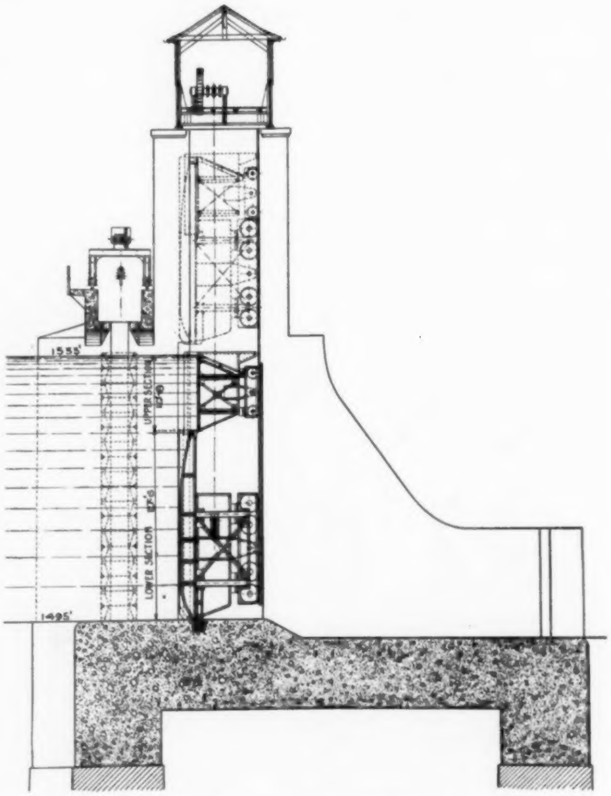


Fig. 14 Double Gates at Pernegg Power Plant, Austria
(Dimensions are in feet.)

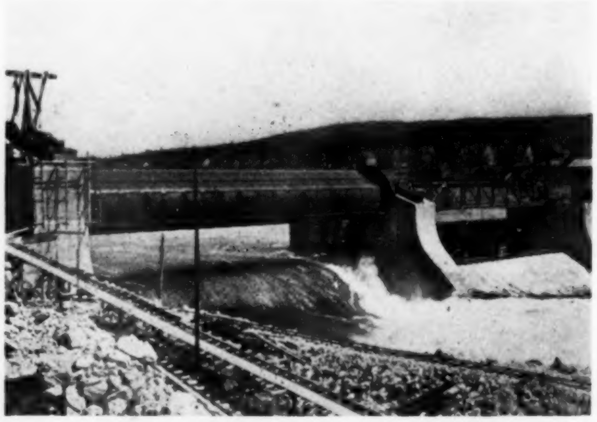
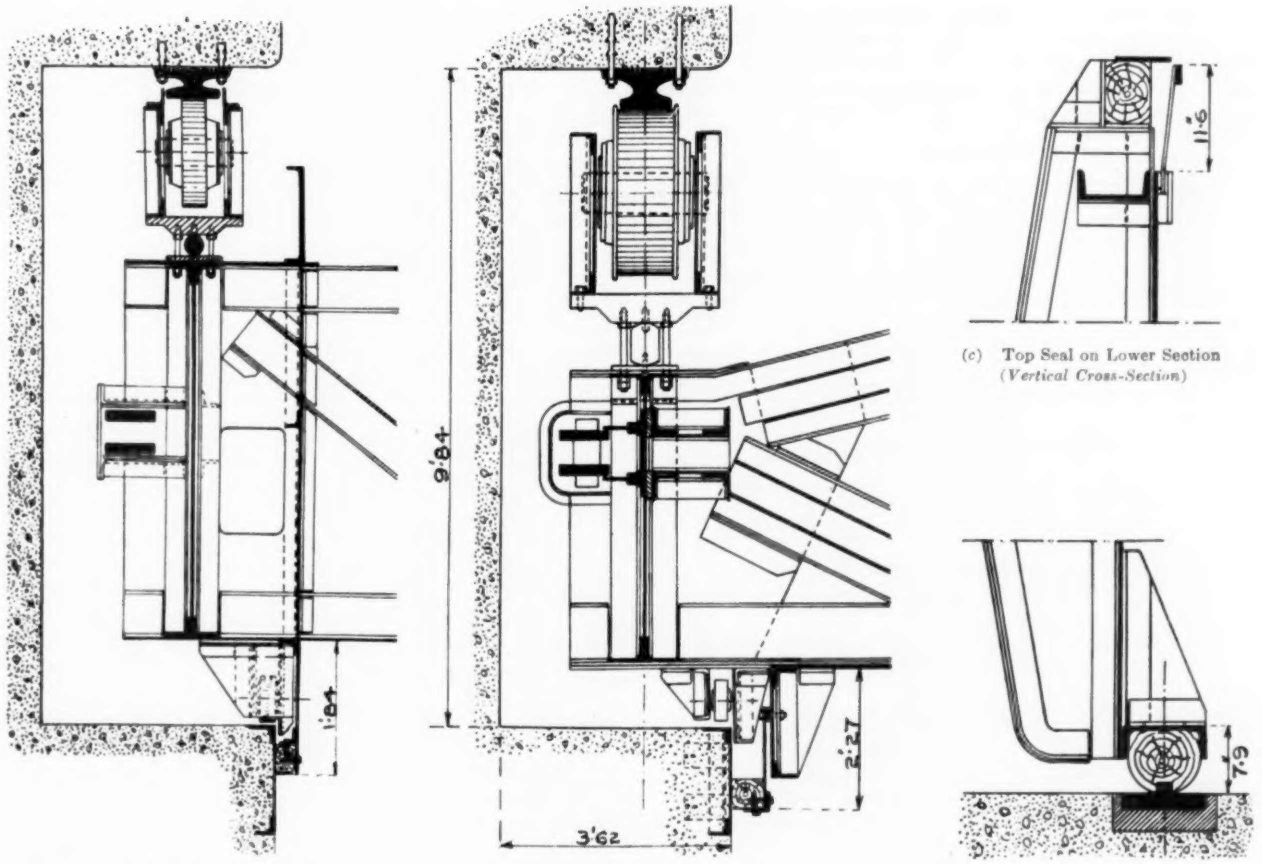


Fig. 16 Roller Weir at Raanaasfos, Norway

Fig. 15 Design of Roller Weir



(*a*) Side Seal on Upper Section (*Horizontal Section*)

(*b*) Side Seal on Lower Section (*Horizontal Cross-Section*)

(*c*) Top Seal on Lower Section (*Vertical Cross-Section*)

(*c*) Bottom Seal on Lower Section (*Vertical Cross-Section*)

FIG. 14*a* METHOD OF SEALING GATES SHOWN IN FIG. 14
(Dimensions are in feet and inches.)

wall in the pier, this wood being fastened to the gate by a piece of steel 0.236 in. thick and 22 in. wide on the upper section and 0.315 in. thick on the lower section.

An interesting apparatus is the roller weir, one form of which is shown in Figs. 15 and 16. (Fig. 15 is from "Der Bauingenieur," and Fig. 16 was supplied by M.A.N., Germany.) The basis of the device is a stiff steel-plate cylinder, to which is attached eccentrically a circular sector, the face of which acts as a dam. Attached to the ends of the roller there are spur gears which engage with racks on the piers, and the hoisting apparatus at the top, acting through the heavy chain, causes the gear to roll up the racks, carrying the cylinder with it, and this allows water to flow underneath, or if desired the roller may be lifted clear of the water, in a nearly horizontal position.

These roller weirs (Walzenwehre) are made in various shapes, with the sector placed in several positions on the roller, sometimes extending only downward, so that water passes over the upper surface of the roller. Water-tightness at the bottom is obtained by a wooden block connected to the weir by a steel plate. The rollers are often made without the sectors at all, and they have been used up to about 100 ft in length; that is, the clear width between the piers is about 100 ft. A few of these weirs are in use in America.

While this type of weir is largely used in dams, it is also applied to the control of the water entering turbines, as in the case of the Donaustetten (see Fig. 1) and other plants. The lifting chain is usually replaced by a cylinder and piston acting directly on the

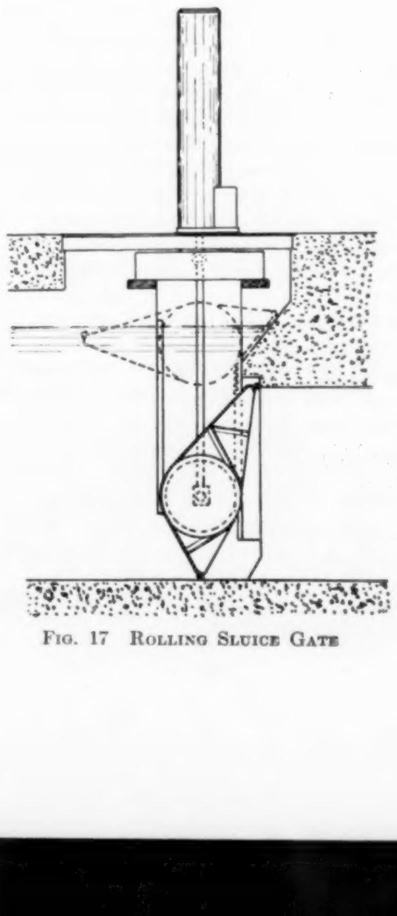


FIG. 17 ROLLING SLUICE GATE

axis of the roller. A diagram of such a roller gate is shown in Fig. 17, in closed and open position. The chains, racks, etc., are let into recesses in the piers so that the moving water does not come in contact with them. These gates are frequently subjected to severe ice conditions.

VALVES

Various types of valves of large and small size are in use in European practice, and of course the ordinary gate valves and disk valves are in quite common use, and some firms build the Johnson valve. For pumped-water-storage plants the design of the valves is of much importance, because the pressures are high and the lines fairly long.

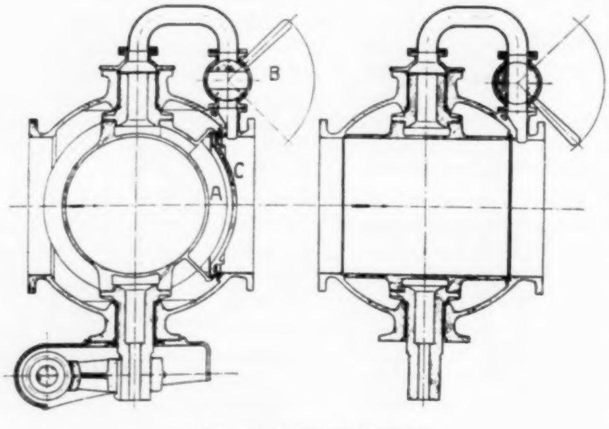


FIG. 18 ROTARY VALVE

The greatest attention is paid to the form of check valve to prevent undue water hammer in case the source of power should fail when the pump is delivering water to the upper reservoir. The ordinary check valve is not effective in this connection, on account of the suddenness of its closing.

One form of valve has had fairly broad application and has been developed by a number of the larger builders on the Continent, so that it should be mentioned. This is the rotary valve, the one shown (see Fig. 18) being made by Escher, Wyss & Co. and differing very little in its hydraulic features from those of Theodore Bell and others. In principle this valve is similar to the plug cock, but the turning piece is cylindrical instead of conical, and when it is wide open it offers as clear a passage for the water as the same length of smooth pipe. Fig. 18 shows a valve operated by worm and wheel, but in the larger sizes the central piece is often rotated by a servomotor, either directly or through a rack and pinion.

The left-hand figure shows the valve shut and the right-hand one gives the open position, the construction being evident from the drawings. The body of the valve is spherical, with a seat at the right side, and there is a corresponding seat on the side of the rotating piece, these seats coming in contact only after the rotating piece has come to its closed position. This is accomplished by closing the cock *B*, when water gradually leaks into the space *A*, pushing the disk *C*, with its finished seat, up against the seat on the valve body and ensuring perfect tightness. To open the valve, cock *B* is opened, balancing the pressure on the disk and allowing the valve to be easily moved.

Other makes of valves have a somewhat different method of accomplishing the same result.

PUMPED-STORAGE PLANTS

A very interesting hydraulic feature, occurring particularly in Germany and Switzerland, is the storage of power for the purpose of balancing out peak loads, the storage being in the form of potential water power. As the matter is already fairly familiar here, little need be said, except to discuss the plants in a general way. Such plants are not economical for low heads, 100 ft being perhaps the minimum, since the storage of any amount of power at very low head requires very large equipment and reservoirs.

TABLE 1 PUMPED-STORAGE PLANTS

Plant	Present power, kva	Ultimate power, kva	Reservoir capacity Upper, million cu ft	Reservoir capacity Lower, million cu ft	Mean head, ft	Present Turbine, hp	Present Pump, hp	Type of unit
Schwarzenbach.....	46,000	70,000	507.0	12.8	1148	62,500	25,100	Horiz.
Niederwartha......	107,500	215,000	71.4	71.4	469	120,000	100,000	Horiz.
Herdecke..........	140,000	140,000	57.0	56.0	508	194,000	102,600	Horiz.
Bringhausen.......	144,000	144,000	27.0	27.1	954	164,000	116,000	Horiz.

Among the largest of these plants are those given in Table 1, all having been put in operation since 1926. In addition to these there are a number of other very large ones, some with the power units having vertical shafts; but only brief reference will be made to these. The power plant in these installations usually has an electrical machine which may be run either as a motor or generator and is generally coupled to the water turbine, so that these always run together. The other side of the electrical machine is connected by friction, hydraulic, or magnetic coupling to the pumps, this coupling being so designed that the pump may be thrown on while the motor is up to full speed. Since the power required to drive some of the pumps is from 20,000 to 30,000 hp and it is desired to bring them up to speed in about 10 sec. the clutches themselves offer no small problems.

In some plants, as at Wäggital, Switzerland, the pumps are driven by an entirely separate motor, but this is not usual. Since the turbine is running all the time that the electrical machine is in operation, including the time when the pump is delivering water, some method must be devised for keeping the water from the runner when it is running idle. This is usually done by some special form of valve, such as the roller sluice, such a device being watertight, quick in action, and offering no resistance when wide open. The water is frequently driven from the turbine casing and draft tube by means of compressed air.

It is usual to employ the same pipe line for both power and pumping, and these lines are sometimes very long, as in the case of the Niederwartha scheme, in which there are now two pipes, each with a surge tank 55.7 ft in diameter, the pipe lines being 6200 ft long in each case. Later, there will be four pipe lines, these varying from 10.5 ft to 8.2 ft in diameter. In this plant all of the water used by the turbines is first pumped to the upper reservoir, but in some other plants only part of the water is pumped, the natural flow being appreciable.

The heads are generally fairly high, probably the largest being that at Tremorgio in the Alps, near the St. Gothard, where the nine-stage pump delivers 14.5 cfs against a head of 3020 ft, the speed being 1000 rpm.

PUMPS

The large pumped-storage schemes have given rise to some interesting pump designs. At Herdecke, near Essen, the designed head is 525 ft, at which the desired delivery is 468 cfs and the driving speed is 300 rpm, conditions that made it necessary to use a two-stage pump. The builders decided to divide the first stage between two single-entry impellers, placed back to back, these two delivering water to the second-stage double-entry impeller, this design largely eliminating end thrust on the shaft and pressure on the stuffing boxes. The first stage impellers are 8.18 ft in diameter, with an exit width of 0.558 ft, while the second-stage impeller diameter is 8.39 ft and its exit width is 1.116 ft. The guides in the ring on the second stage move with change of speed. Since the requirements were so unusual, it was thought desirable to investigate by the use of models, and two models were made with scales of 1 to 8.72 and of 1 to 4.8, the speed of the latter being 1440 rpm.

In the models, as in the full-size machines, the builders divided the vanes on the second stage into two parts, the outer of which is fixed and the inner movable, this arrangement being probably first used by Escher, Wyss & Co. The results of the tests on the models of the Herdecke pump are given by Dr. Hahn in the *Zeitschrift des Vereines deutscher Ingenieure* of June 21, 1930, and it is from this article that the curves of Figs. 19 and 20 have been prepared. The two sets of curves in Fig. 19 (*a* and *b*), were obtained from the small model running at 1495 rpm, while Fig. 20 is a comparison between the results on the large and small models when reduced to the same basis. In Fig. 19-*a* curves 1 and 5 give, respectively, the head and power on a discharge base for the two-stage pump, and these may be compared with curves 2, 4, and 6 for the single-stage pump with spiral casing, 4 being the efficiency, and with curves 3 and 7 for the single-stage pump with guide ring. To avoid complication the efficiency curve for the two-stage pump is omitted, but it is similar to curve 4, the efficiency reaching a maximum of about 82 per cent at about 1.10 cfs and having 75 per cent at 1.25 cfs. The effect of the guide ring is evident.

In Fig. 19-*b* sets of curves are given for the two-stage pumps, and these show the effect of moving the vanes in this ring, these guides being shown in Fig. 20 in the open, partly closed, and closed positions, only the inner section being turned about the center. The curves in Fig. 19-*b* are for different values of the opening *e* in Fig. 20, these curves being numbered, 1, 2, 3, 4, and 5 for values of *e* equal to 0.945 in., 0.606 in., 0.37 in., 0.244 in., and 0.126 in., respectively. The power curves were so close together that only three of them are shown at the bottom of the figure, the head curves being at the top and parts of the efficiency curves in the center. Clearly, the positions of the guides have a marked effect on the capacity of the pump and also on its efficiency, and in addition the power required at starting may be much reduced by their use.

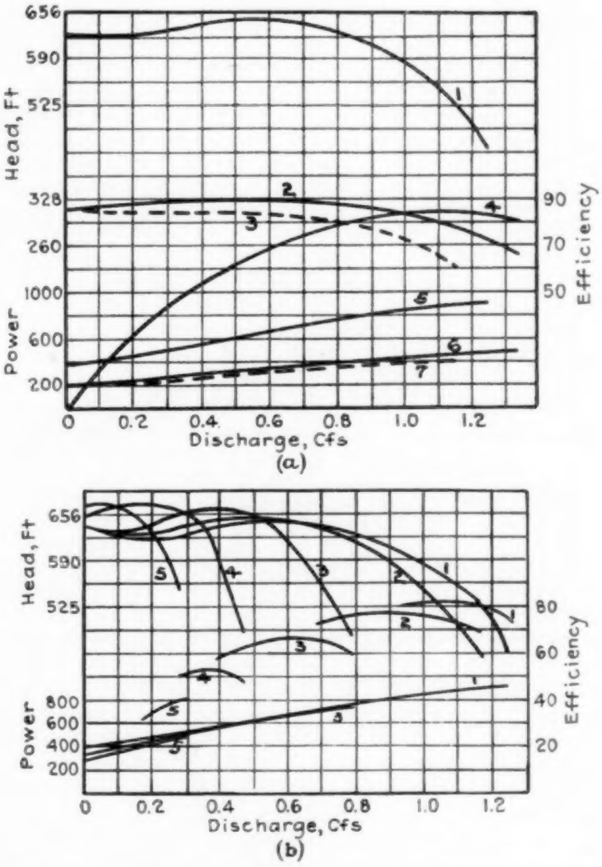


FIG. 19 TEST CURVES ON SMALL MODEL OF HERDECKE PUMP

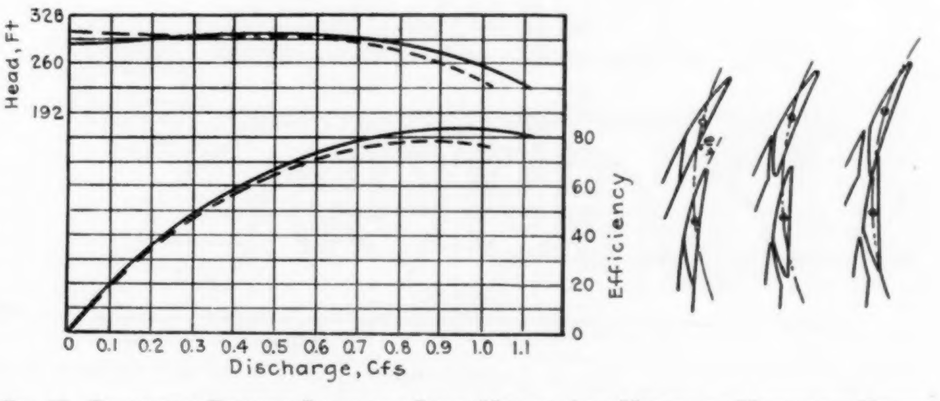


FIG. 20 COMPARISON BETWEEN LARGE AND SMALL MODELS, ALSO METHOD OF MOVING THE GUIDES
(Full lines show the large model; broken lines, the small one.)

The experimental pump and also the full-size machine were made by Gebrüder Sulzer, Winterthur, and J. M. Voith, Heidenheim. A diagram of the pump is shown in Fig. 21. There is a Föttinger coupling between the pump and motor, so that the pump may be started up when the motor is running at full speed. The governor for regulating the guides is similar to a turbine governor.

The Schluchsee Power Storage Station, built in 1929, is located at the plant at Hausern, Germany, a small town on the Rhine between Schluchsee and Waldshut and almost directly north of Zurich. In this case the unit is vertical, with the electrical machine on top, the Francis turbine below and rigidly coupled to it, and then the friction clutch with the pump below it. Much interest has been aroused also in regard to the Schwarzenbach plant of the Badische Landes-Electrizitäts Versorgungs A.-G. The turbine, shown in Figs. 10 and 11, is of 27,500 hp at 1170-ft head and 500 rpm, and is coupled to the electrical machine, the other side of which drives the pumps through gears which bring the pumps up to 1000 rpm. There are two pumps on the unit, with their shafts parallel, and driven from the same gear on the electric motor, the drive being through a magnetic clutch capable of bringing the pumps up to full speed in 10 sec if the motor is already up to speed. Each pump has an output of nearly 78 cfs at a head of 755 ft and requires 8320 hp. These pumps are similar to the Herdecke machines, being two-stage, with the first stage composed of two single-entry impellers discharging into one double-entry second-stage impeller. The cross-section of the Escher-Wyss pump is given in Fig. 22, where it is seen that the guide vanes from all three impellers are movable.

These pumps have characteristics similar to those already described.

The pumped-storage plant that has attracted most attention recently is the one completed within the last two years at Niederwartha, close to the city of Dresden, Germany. In this case there are four horizontal units, two by Escher, Wyss & Co. and two by Voith and Sulzer. Each unit consists of a 23,000-hp to 30,000-hp Francis turbine permanently coupled to an electrical machine delivering 21,500 to 26,875 kva as a generator and being capable of 29,200 hp as a motor. The electrical machine is connected to the pump by a friction coupling designed to be thrown in when the motor is up to speed. The pumps are for 330 to 415 cfs at heads from 506 to 453 ft, the speed being 375 rpm; the shaft horsepower required by the pumps is 23,800 to 27,000 hp.

The pumps are set as low as possible, the water in the lower pond varying in level from that of the pump shaft to 14.3 ft above it. The pump builders differed in their designs, Sulzer using a single-stage double-entry pump with adjustable guide vanes, while the firm of Escher-Wyss decided on two-stage pumps, the first stage being divided into two single-entry low-lift impellers which deliver water to a single double-entry second-stage impeller. A photograph of the station is shown in Fig. 23. Experience with the plant up to the present would indicate that the single-stage pumps are rather better suited to their work and have run with perfect satisfaction.

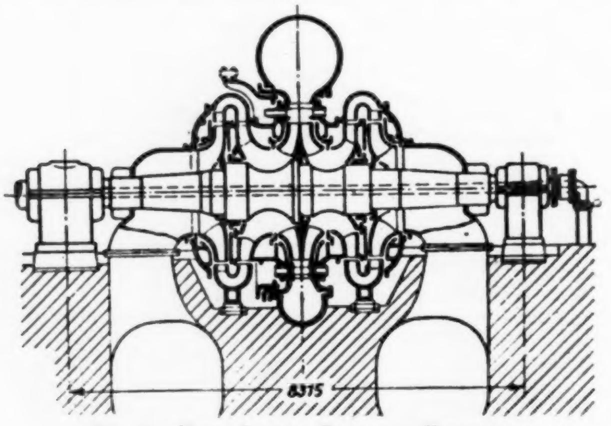


Fig. 21 Voith-Sulzer Pump at Herdecke

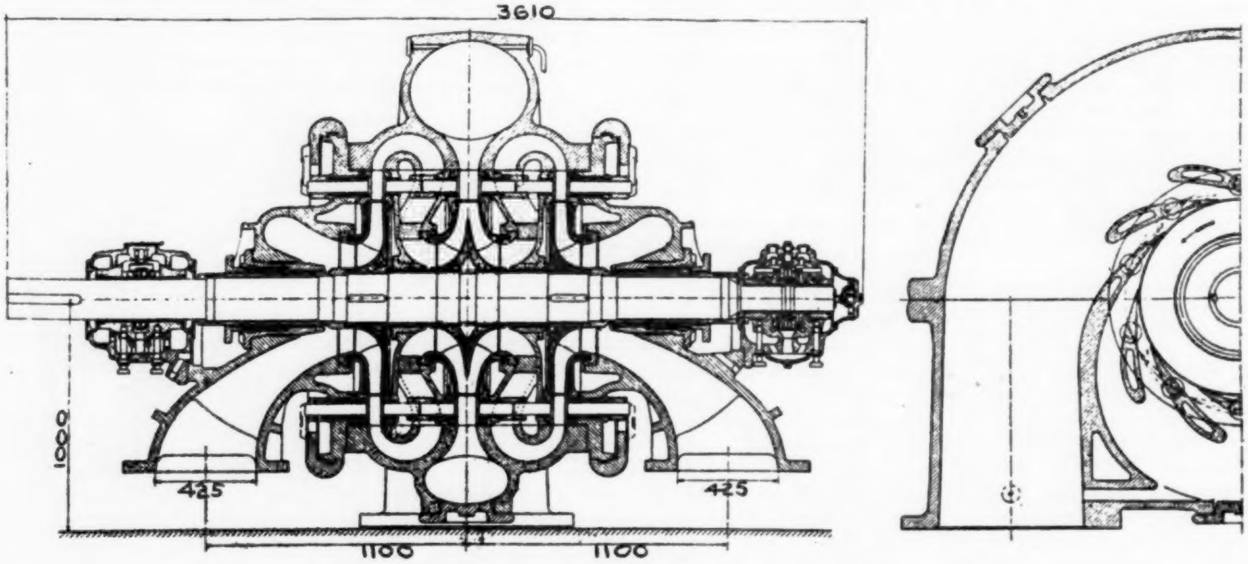


Fig. 22 Cross-Section of a Pump for the Schwarzenbach Works

Pumps for Ordinary Service

It is almost general custom on the Continent, and in England too, to use a solid casing on high-head pumps instead of the horizontally split American type, and while the practice in America is to omit the guide ring as far as possible, almost all pumps in Europe have the rings, particularly if they are more than one stage. In very high-lift pumps the cylindrical casing is cast in one piece, with ports of the water passages in it, and the guide rings and impellers are slipped in axially; in this way the maximum pressure causing leakage is only due to a single stage. Some companies make the pump up in sections, each of which is a unit of impeller with guide ring and casting with water passages, so that any number of stages may easily be built up by adding sections, these sections being held together by long bolts passing through the length of the machine.

End thrust is very largely balanced hydraulically, although very rarely the impellers are placed back to back, and some builders make two- and three-stage pumps with double-entry impellers which are largely in balance, but this arrangement has decided limitations. Considerable attention has been given to bore-hole pumps and to sinking pumps and the author has just seen a drawing for such a pump of five stages, for 212 cfm against a head of 1250 ft when running at 1480 rpm. Much ingenuity also has been shown in the design of low-head, high-speed pumps in which the flow is largely axial. Many larger boiler-feed and other high-pressure pumps also have been built, some of which are steam turbine driven at very high speeds.

The Escher-Wyss publication recently contained a description and section of a pump built by that firm for boiler feed, the drive being a direct-connected steam turbine. The pump is for a pressure of 1705 lb and delivers 4000 lb of water per minute at 3800 rpm. which requires 905 hp. The pump is 10-stage and the impellers are in two sets, the sets being placed back to back, the impellers being single entry. The passages are so arranged that the second stage is at the opposite end of the pump to the first stage and the third to the second, etc., so that the water travels back and forth along the pump and finally leaves at the center. This scheme provides nearly perfect end-thrust balance and avoids high pressures on the stuffing boxes, in addition to which it is relatively easy to draw off water between the sixth and seventh stages, send it through the heater, and back again to the seventh stage. The casing is cast steel, in one piece, and a thrust bearing is provided to take up the small unbalanced end thrust.

Some of the Sulzer bore-hole pumps are also very interesting in construction, as these have to deliver large volumes of water and the space they occupy is small. Sulzer Brothers estimate an average flow of 17,000 to 22,000 gal per hr from a 12-in. bore hole and 40,000 to 55,000 gal per hr from an 18-in. hole. Some of their pumps are arranged with two double-suction impellers in parallel in one case, working in series with other similar units; naturally the water passages are very restricted, but the efficiencies are by no means low.

Research Work

Hydraulic research is proceeding on a very large scale in Europe, and only very brief reference need be made to it. Excellent laboratories exist in the workshops of Theodore Bell & Co., Lucerne; J. M. Voith, Heidenheim, Germany; Escher, Wyss & Co., Zurich; Sulzer Brothers, Winterthur; Costruzioni Meccaniche, Milan; Verkstaden, Kristinehamn, Sweden, and other places, and these are responsible for the mechanical developments taking place, and they have supplied data for building the present turbines and pumps with their accessories.

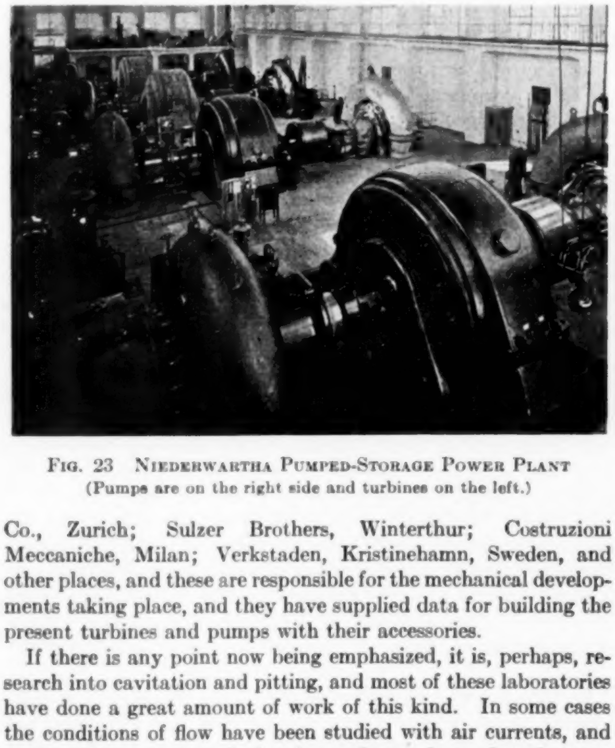


FIG. 23 NIEDERWARTHA PUMPED-STORAGE POWER PLANT
(Pumps are on the right side and turbines on the left.)

If there is any point now being emphasized, it is, perhaps, research into cavitation and pitting, and most of these laboratories have done a great amount of work of this kind. In some cases the conditions of flow have been studied with air currents, and most of them use glass draft tubes and some form of stroboscope in which the action may be studied. Many experiments also have been made on turbine nozzles, turbines, pumps (some of which have been outlined), etc.

The engineering colleges have shown a marked interest in hydraulic research, as is evidenced by the additions to the buildings in Brunswick, Hanover, Dresden, Zurich, Milan, etc., stress being laid on stream flow and open-channel experiments, although investigations on turbines, pumps, and other machinery are under way. There is great research activity among all of the European colleges, and much useful work is being published, one case being that of the important work done by Dr. Thoma, Munich, on the flow in centrifugal pumps.

Only passing reference may be made to the Forschungsinstitut fur Wasserbau und Wasserkraft, near Munich, as this has already been described in various journals. It is a most commendable laboratory, mostly outdoors. It has a possibe flow of 280 cfs from the Obernach River, and it fortunately enters the experimental flume without any great amount of agitation. The experimental field is nearly 2000 ft long and is equipped with measuring reservoirs, weirs, open channel about 8 ft wide and 7 ft deep, and there is a large area available for constructing river models. This institute publishes papers of its researches, which may be had on application.

It will be interesting to members of the Hydraulic Division to know that the reports of many of the researches conducted in Germany are frequently available in printed form and may be had by communicating with the particular person under whom the work has been done. This is at least true of the laboratories under the control of Dr. Thoma at Munich, Professors Rehbock and Spannhake at Karlsruhe, and Professor Oesterlin at Hanover, and some of the other laboratories as well.

Translations of some of these reports are now being undertaken and will be available to the members of the Society very soon. These are a further evidence of the debt that hydraulic engineers owe to Mr. John R. Freeman, through whose generous assistance, in cooperation with the Society, the work is being done.

Discussion

A. J. R. Houston.[2] The paper covers the field so thoroughly that only the following other features of European water-power practice occur to the writer:

(1) Cleaning racks by reversing flow. In some low-head plants where the turbine racks are divided into two or more parts by vertical walls, the racks may be cleaned by reversing the flow in one set at a time. This of course involves reducing the output of the unit during the cleaning operation. The system is not in general use, and rack-cleaning machines are perhaps as common there as here.

(2) In connection with low-head plants, the vertical-shaft geared units are interesting. The generator shaft is vertical as well as that of the turbine, and destroyer-type herringbone gears are used. The unit becomes very much more complicated mechanically, as in addition to the gearing there must be two thrust bearings. However, the speed of the generator is very high, and it becomes correspondingly small in size. Several such plants have been built.

(3) Sand-extraction schemes. In some plants an attempt is made to eliminate some of the material carried in suspension in the water. These devices are more frequently used in connection with high-head than with low-head plants. They are mostly of the settling-basin nature, usually with provision for continuous cleaning. One of the best known is the Dufour.

(4) Roller gates. It might be mentioned that with roller weirs which would call for a shield, the roller is sometimes replaced by trusses. The gate is raised by rolling the gear wheels up racks exactly as with the original roller type. The pure roller gates are often made sinkable so as to pass ice or other rubbish with a minimum loss of storage. The writer does not share the extreme enthusiasm for the roller weir of some American river-flow engineers, who seem to feel that it sometimes offers the only possible solution. While the roller dam is a highly satisfactory device, there are many other good forms of gate available, and it should in general be subjected to competition with other types.

(5) Automatic gates. Mention might also be made of the various gates for automatically maintaining the storage level. These are made in a wide variety of designs. Some are equipped with counterweights; others not. Often they are of the floating-leaf type, and also many are similar to the bear-trap gate so well known in this country.

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[2] Harza Engineering Company, Chicago, Ill. Jun. A.S.M.E.

(6) Hydraulic clutches. In connection with pumped-storage plants it would probably be fair to say that the hydraulic clutch is now being favored over the friction and magnetic devices for coupling the pump to the electrical machine. The hydraulic device seems to take hold more smoothly.

S. Logan Kerr.[3] Several years ago the writer visited most of the plants described, and had the opportunity as well to visit the shops and laboratories of many of the European water-wheel builders. The hydraulic design of many of these plants has been the subject of exhaustive test and research work conducted in laboratories, in advance of construction. In many instances, considerable savings have been effected, but in many others it appears that the turbine installation has been studied in so far as the runner and guide vanes are concerned, and many of the vital portions of the plant, such as the intake and draft tube, have not been the subject of as much research as would be desirable.

This tendency, however, is now disappearing, and the entire structure is considered as a unit, and the principle that has been employed in American research work for a number of years is being more fully appreciated—namely, the interdependence of the various elements of any hydraulic-turbine installation.

The development work on impulse wheels for high-head applications in Europe has been most interesting, and is still considerably at variance with many elements of American practice. The multi-jet wheel appears to be in satisfactory use in many plants in Europe, and when conservative velocities are used with jets free from turbulence, good results should be obtained.

The question of efficiency in European turbine plants, as compared with American performance, has always been a moot question, as the test methods for water measurement have differed so radically. With the wide-spread use of current-meter testing in European plants, there has undoubtedly developed a high order of skill in the use of these meters, in many instances equaling the accuracy obtainable with the Gibson and Allen methods in America. The performance of the high-speed propeller and Kaplan-type units, in both Europe and America, has been virtually the same with respect to the values of efficiency attained. There have been many installations in America where accurate tests have demonstrated that the performance is in excess of 93 per cent, comparing very favorably with European practice.

With the impulse units, however, some reasonable doubt may be in order in certain of the installations, particularly where a single current meter has been used, located in the conduit, and its calibration assumed from laboratory testing. It would be of great interest if one of these impulse-wheel installations could be tested with the Allen salt velocity method and compared with the current-meter or other form of measurement used in most of the European efficiency tests. Performance of many American installations of impulse wheels has undergone considerable revision when the flow has been measured precisely by the Allen method instead of by pitot tubes or current meters. The difference between the actual performance and the estimated performance is so great as to result in a complete revision in design methods.

It would have been of great interest if the author could have described the type of power-plant auxiliaries, such as governors, oil-pressure systems, etc., utilized in the various European hydroelectric plants. It was the writer's impression, gained from visits to most of the plants described, that the turbine equipment itself compared favorably with American practice. There were many details of design of the operating mechanism, bearings, etc. that could be profitably employed by American designers. On the other hand, the ruggedness of the average American turbine was lacking in most of the European installations, and great stress was laid on savings effected in weight and materials, as compared with American practice, which is directed principally at savings in labor.

With regard to the governors and the oil-pressure systems, it was a distinct disappointment to see the complicated mechanisms used in even the most modern installations in Europe. The ease and convenience of control available with any of the modern American governors seem to be sadly lacking with the European designs. The use of a load limit and other simple adjustments, including the automatic change-over from hand control to governor control, were lacking in most cases, and the starting up and shutting down of units appeared to be a tremendous task, as compared with the simplicity of operation in most American installations.

The governor drives were almost invariably gear or a combination of gear and belt, driven from the main shaft of the turbine, and not a single installation of motor-driven flyballs could be found, even though they had become accepted standard practice in American installations.

In this respect the writer feels that the turbine manufacturers in United States and Canada have progressed much farther than the European designers. In the last few years considerable work has been done in Europe on the use of inertia-type governors for hydroelectric plants, but for ordinary turbine regulation it does not seem, at present, to be of sufficient advantage to warrant the expenditures necessary for development and engineering work of this type of governor.

The continued interchange of information and experience between European engineers and the engineers in this country and Canada cannot help but benefit both parties, as each has developed distinct advantages in many different features of their equipment. It is hoped that this international form of cooperation in engineering will continue to the benefit of all concerned.

Eric C. Molke.[4] The paper, especially the chapter on control gates, was most interesting, as the writer has just returned from Europe, where he saw several of the plants described. The spans on European hydraulic gates are generally much longer than those used on this continent, and the time will come when, on certain large rivers on this side, even under more serious climatic conditions, large-span control gates will be installed with advantage. The details showing how to take care of the big reactions at the gate guides are of great interest to engineers. The roller weir is especially very suitable for such spans, and from what was observed in Europe, has stood up well, even under very unfavorable conditions. The writer had an article on "Types of Gates Used on Dams in Europe," in *Power*, June 7, 1932.

W. Spannhake.[5] The design of turbines, especially the fast-running types for low heads, neither here nor elsewhere, has reached satisfactory development with respect to the danger of cavitation. In the future possibly some details in the general layout will be slightly altered in order to reduce cavitation. In my laboratory at the technical university a great many cavitation tests on models of turbines for the power station at Ryburg-Schworstadt have been made for the Atéliers des Charmilles. In addition to these, the writer has tested a new draft-tube device intended to give improvement, as compared with the usual construction, in the direction of less depth and length without decrease of efficiency, in spite of the existence of a whirl in the draft tube in the positive sense of the rotation of the runner, this whirl allowing the runner blades to be designed for lower relative velocities and in consequence for reduced possibility of cavitation.

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[3] Research Engineer, Baldwin-Southwark Corporation, I. P. Morris Division, Philadelphia, Pa. Assoc-Mem. A.S.M.E.

[4] Montreal, Quebec, Canada.

[5] Professor, Technische Hochschule, Karlsruhe, Germany.

Paul E. Gisiger.[6] There is a striking difference in layout and arrangement of spillway equipment of European and American plants. While in the United States and Canada 50 ft is about the maximum so far used for the width of a gate opening, and as a rule gates are operated with one or (for a large number of gates) two traveling gantry cranes, in Europe widths of 80 ft for vertical-lift gates and still wider openings for roller-type gates have been used, and almost always individual hoists and operating mechanisms are provided for each gate. At first sight it would appear that European engineers are unduly conservative in duplicating hoisting facilities, but this feature, as well as very wide gates, is a result of geological and hydraulic conditions. Most European rivers, and especially those originating in the Alps, run through valleys covered with heavy glacial and alluvial deposits, and foundations usually have to be carried down to rock by the pneumatic-caisson method. Gate sills are usually not much higher than the original river bottom, so that the piers, and especially their foundations, are the major items of cost of the spillway. It is economical, therefore, to have wide openings and as few piers as possible. On the other hand, Alpine rivers have much more regular characteristics of flow than most American rivers, with the maximum flows amounting to only from four to six times the 180-day flow. Therefore, a very much smaller spillway capacity is usually required on a European river than on an American river having a comparable average or 180-day flow. For instance, while the Ryburg-Schworstadt development has only four gates at a site with a 180-day flow of about 35,000 cfs, the Safe Harbor development has 32 gates at a site with a 26,000-cfs 180-day flow. The number of gates is of course no measure of total discharge for the reasons stated, but it illustrates the wide difference of conditions that influence the choice of hoisting equipment. The number of gates usually required on a large low-head development in America puts separate hoisting equipment for each gate out of question, but with the four-gate to 10-gate installation most frequently met with in Europe the economics are largely in favor of individual hoists.

Carroll F. Merriam.[7] Much can be said on the subject of difference in engineering practices between Europe and the United States and Canada, and in many cases European designs are open to the criticism, when viewed by our engineers, that their construction is light, complicated, and involves considerable expenditure of labor, and that efficiency may be overemphasized. Consideration must be given in the first place to the fact that whereas we live in a land of plenty where materials are cheap, they have been forced by lack of natural resources to conserve materials and resources most carefully, while conversely, through overcrowded population, labor is correspondingly cheap. We have only to look at the illustrations in the author's paper to realize the force of this statement and appreciate that although our construction may be more rugged and generous in materials, it would be as equally unsuited to their conditions as their designs would be to ours.

Nevertheless, there are many lessons that we may profitably learn from European practice, and the author is to be congratulated for his continued interest in bringing to this country the results of his experience and contact with European engineers.

In recent American construction we have profited greatly by European contacts and have installed the most powerful Kaplan turbines in the world. Many ideas have been incorporated in the design of the dam and power house that have been taken from European practice.

In this connection there should be mentioned the double-leaf gates, very similar to those shown in Fig. 14, except that the upper leaf is practically equal in size to the lower and is controlled by an independent hoist remotely operated from the power-plant control room. These gates have proved very successful in assisting to maintain the desired constant pond elevation.

The design of Kaplan units of such large size has brought about many problems, particularly that of cavitation, and we have instituted important studies in conjunction with foreign authorities for the purpose of investigating this problem and advancing our knowledge of its control.

The plants for pumped storage common in Europe bring some most interesting economical problems, and although it may at first seem doubtful that steam-generated energy can profitably be turned into stored water and again reconverted to electricity with an overall efficiency of only slightly over 60 per cent, nevertheless careful analysis has shown that many factors that are not immediately apparent must be taken into consideration.

In regard to current-meter testing, the method illustrated in Fig. 6 is simply a forerunner to subsequent tests in which the corps of observers is replaced by a chronograph. This method is being used for important tests in this country, but with several modifications. The fundamental idea has been taken from foreign practice and applied to American conditions, with improvements. This may be considered as simply one example of what should result as we become more familiar with European ideas and come to a better understanding with engineers from abroad.

T. H. Hogg.[8] The paper brings together a great amount of information regarding modern European hydraulic practice, which is greatly amplified by the illustrations selected to accompany the text. The writer will comment only on five items in connection with plants and equipment described.

Changes in draft-tube design in Europe during the past 15 years have proceeded in general along the same lines as those followed in America. The quarter-turn of the draft tube is given a smaller radius of curvature than formerly, but the most striking change is seen in the great increase in the horizontal section beyond the quarter-turn. This is seen in the cross-sections of the Donaustetten, Ryburg-Schworstadt, and to a lesser degree in the Chancy-Pougny plants. The same feature is noticeable in many recently built plants in this country, for example, those at Chats Falls on the Ottawa River and Alexander on the Nipigon River.

One feature of draft-tube design in which European and American practice differ materially is the addition, in European draft tubes, of horizontal and vertical division walls. Such division walls were used formerly in a few plants in America, but with unsatisfactory results in some instances, as for example in the St. Timothee plant on the St. Lawrence River. High efficiency is evidently obtained with these in the European examples described by the author.

The concentric type of draft tube has never been favorably received in Europe. In our experience it has effected an improvement in efficiency and steadiness of operation at part gate in many instances. In some cases it has been found necessary to reduce the height of the central cone, in order to obtain the best results as to capacity and high-load efficiency.

It is interesting to note the simplification in the matter of water control in such large plants as that at Ryburg-Schworstadt, where steel gates of the stop-log type are substituted for headgates at entrance to the turbine settings.

The very considerable problem that is presented in the design of couplings between motors and pumps in pumped-storage plants suggests that the pump runner be permanently connected to the motor and that valves be provided to keep water out of the pump casing at times when the turbine is operating. This would seem to be reasonable, as a similar arrangement is used with respect to the turbine runner.

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[6] Safe Harbor Water Power Corporation, Baltimore, Md.

[7] Pennsylvania Water and Power Company, Baltimore, Md. Mem. A.S.M.E.

[8] Chief Hydraulic Engineer, Hydro-Electric Power Commission of Ontario, Toronto, Ontario, Canada.

It is interesting from the standpoint of economics to note that the Niederwartha plant near Dresden is in a locality where numerous steam-electric stations operate using brown coal for fuel, and off-peak power from these steam plants is utilized to drive the pumps in this station. The pumping head is the same as the head used on the turbines, so that the energy returned to the system from the pumped-storage plant is less than that used to run the pumps by the hydraulic and mechanical losses in pumps, penstocks, and turbines. Conditions are materially different from some cases investigated by the writer in which the pumping head was 10 to 15 per cent of the head on the turbines, and the output from the pumped-storage plant therefore much greater than the input. In one such instance the energy made available from the pumped-storage plant was only slightly less expensive than energy from other available sources. This is due to the seasonal variation in load demand, causing the output from the pumped-storage plant to have only surplus-power value during certain months of the year.

It is noticeable that there is less uniformity in design of control gates in Europe than is seen in American plants. The stony sluice has been superseded here in many cases by gates with fixed rollers. The rugged design of the gates at Ryburg-Schworstadt is worthy of note, and the division of the gate into upper and lower sections with its quite evident advantages is commendable. Roller gates of large size similar to those at Raanaasfos are in use in America, one installation recently completed having a span of 110 ft.

In Europe as in America there has been progress in the application of propeller and Kaplan-type runners to higher and higher heads. The installations at La Gabelle on the St. Maurice River, where a head of 60 ft is used, and at the Seven Sisters Development on the Winnipeg River, where the designed head is 66 ft, show a great increase in the head used for propeller units as compared with that used 10 years ago. The writer understands that heads approaching 100 ft are in use on turbines of the Kaplan type in Europe.

Albert J. Luchinger.[9] The author has given a complete description of the special types of hydraulic machinery peculiar to Europe. The paper shows that hydraulic machinery on the European Continent does not differ fundamentally from American practice. Having worked both in America and Europe, one fundamental difference in hydraulic practice, however, has occurred to the writer, and that is the difference in relations between the mechanical and the construction part of the developments. The chief engineer on hydro developments in America is normally more a mechanical one; in Europe, he is more a civil engineer.

The American developments are built around the turbines and gates; the mechanical part is the one that is first ordered and normally governs the design. In Europe the construction part dictates to a certain extent the machinery. Both methods have their advantages and drawbacks, and both methods given their special note to the developments. The difference is not a difference in technical quality, but in predominance of certain otherwise similar features. An interchange of engineers and ideas between the two continents can only help both of them, but need not level off their characteristic viewpoints.

It might be of interest to give a brief description of some unusual power-plant details now in construction in Europe.

The Wettingen plant, on the Limmat, in Switzerland, is one of the rare medium-head plants of that country. The main feature of the design was the selection of a spillway structure that would result in minimum scour to the river bed. A public contest was opened for the best design. The peculiar feature of this structure is the use of two types of discharge openings and three types of gates in the same cross-section.

The total spillway length is divided into four discharge sections, each 33 ft wide. The four crest gates are automatic tilting gates and take care of the flow up to normal high water. The four sluiceways are opened in times of high flood; they serve further for slicing out the accumulated gravel or exceptionally lowering the pond level. Each sluice opening has two gates—a tainter gate for easy and quick operation and a sliding gate in front of it. This second gate serves as a tight closure in time of low stream flow and during repairs on the tainter gate.

The maximum possible combined discharge of crest and sluice openings is 450 sec-ft per 1 ft of spillway length. Extended model tests in the hydraulic laboratory in Zurich have shown the great advantage of operating both crest and sluice gate together in times of flood. The depth of scour below the dam for separate part discharge over the crest or separate part discharge through the sluiceway was just as great as for combined total flow through both discharge openings. The impact of the two flows of different directions destroys a big amount of their energy.

The Kembs power house, on the Canal d'Alsace, near the Rhine, just below the city of Basel, now under construction, is the first of a number of power stations that are projected on a large power and ship canal parallel to the Rhine between Basel and Strasbourg. The interesting feature of this development is the spillway openings in the power house between the turbines and the generators. These openings are provided with automatic gates, which allow the water to pass the power station in case the turbine gates are closed. This provision also guarantees a continuous flow in the canal below the station in case of any shutdown of its turbines. This spillway within the power house saves the cost of separate spillway structures and also allows an easy discharge of the surface ice from the pond. There are six Kaplan turbines of 33,000 hp at 40 ft head.

As a most interesting hydraulic feature, the author discusses a number of the most recent pumped-storage plants. In the last few years there has been great activity in Europe along this line. These storage plants are particularly valuable in connection with large low-head plants like those on the Rhine. The power station at Kembs will be connected to a large pumped-storage plant in the Vosges mountains, the development of Lac Blanc and Lac Noir.

Two natural lakes of about 350 ft difference in elevation and 2500 ft horizontal distance are connected by a gallery and penstocks. Practically all the water used by the turbines is first pumped to the upper lake, the natural flow being very small. The pump, turbine, and electrical generator of each of the four units are mounted on one vertical shaft with fixed couplings. Both turbine and pump are running all the time the electrical machine is in operation; the one running idle is kept free from water by means of compressed air. Each of the four Francis turbines rates 28,000 hp, with 330 ft head; each of the four pumps has an output of 460 cfs and requires 24,000 hp, with 330 ft head and 272 rpm.

These three developments have been carried out by the Swiss firm of Locher & Co., Zurich, which firm is also consulting on the construction of the dam at Kembs and the development of Lac Blanc and Lac Noir. The turbines and pumps of this last development are built by Escher, Wyss & Co., Zurich.

Author's Closure

The author is grateful to those who have so kindly discussed and amplified his paper. The discussion has been constructive throughout and little need be said in regard to it.

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[9] Engineer, Locher & Co., Zurich, Switzerland.

Referring to the point raised by Mr. Houston, the author was struck with the very general use of rack-cleaning machines, and these seemed to have a real duty to perform, judging by the amount of material they removed; some of the machines are illustrated in the paper.

Mr. Kerr points out the discrepancies that may arise in comparing the efficiencies of American and European plants due to the differences in the methods of water measurement. Undoubtedly there is need of more uniformity in test methods, but an effort has recently been made by German engineers to compare several methods of discharge measurement, and this comparison is still in progress. For many of the low-head European plants the current-meter method is almost essential, and the results of tests being made on a very large American plant in this way are being eagerly awaited.

The author did not gain the same impression that Mr. Kerr did with regard to the lightness of the parts of European turbines, and the complication of governor mechanisms. Europeans are more economical, in general, than Americans, but the proportions of their machines are usually so good that one ordinarily does not notice a suggestion of weakness. The inertia governor seems to be in favor in Europe, but it need not be a complicated device, for if the flyballs are displaced from the plane passing through the axis of rotation of the spindle in such a way that they move in planes parallel to the spindle, then the governor shows the inertia effect.

The author agrees with Mr. Kerr in regard to the need of continual cooperation between engineers everywhere, and that is one of the reasons the paper was prepared.

Cavitation, as Professor Spannhake points out, is still a serious problem everywhere, and the author mentions the stress that is being laid upon its study in Europe. The work being done by Professor Spannhake on this subject at the Massachusetts Institute of Technology is particularly valuable, and will bring the best experience of two continents together.

The comments of Messrs. Gisiger and Merriam are important, in view of many points of similarity between the Safe Harbor plant and some of the latest European developments, particularly that at Ryburg-Schworstadt. The author has found comparisons most interesting. In a personal letter, Mr. Merriam has pointed out to the author that in the original report on the Ryburg-Schworstadt plant an error had been made in the generator efficiencies, which were taken from 0.8 to 1.3 per cent too high, and that the actual turbine efficiencies were about one per cent lower than the paper states.

The difficulty of designing couplings for pumped-storage plants is mentioned by Mr. Hogg, but the author was impressed with the remarkably ingenious and perfect devices used for this purpose, enabling starting and operation to be effected without shock of any kind.

The Wettingen plant mentioned by Mr. Luchinger contains three 10,000-hp Kaplan turbines running at 214 rpm under a head of 75.5 ft, the highest head yet used on this type of turbine, but the specific speed has the very low value of 97. The center of the spiral casing is at minimum tail-water level, and about 6 ft below maximum tail-water level, and the draft tube is of the general design shown in Figs. 1 and 2. The author appreciates the reference to the Kembs plant, which was omitted on account of lack of space. The rapid development of the Rhine water powers is of great interest to engineers everywhere.

Information has been received from Bell and Company of a rotary valve 7.55 ft in diameter for a pressure of 95.5 lb per sq in., and they have also sent a drawing of a turbine with a conical distributer where the guides are almost radial. This turbine has 117 hp at 8.53 ft head and 130 rpm, which corresponds to a specific speed of 96.

HYD-54-8

Investigation of the Flow Conditions in a Centrifugal Pump[1]

By K. FISCHER[2] and D. THOMA,[3] MUNICH, GERMANY

This paper gives an account of an investigation recently carried out in the laboratory of the Technical University of Munich, into the flow conditions in the impeller of a centrifugal pump, the results of which may be summarized briefly as follows: (1) Practically all flow conditions for an actual fluid are fundamentally different from those theoretically derived for an ideal frictionless fluid. (2) The dead-water zones which form on the low-pressure sides of the blades undoubtedly cause a reduction in the angle at which the fluid leaves the blades from the actual blade angle. (3) The actual velocity distribution along circles concentric with the axis of rotation of the impeller is not the same as the supposed distribution computed on the basis that the fluid passages are completely filled with active flow. (4) The relative velocity along the high-pressure side of the blade should theoretically approach zero for relatively small reductions in the discharge below normal and with the fluid passages completely filled with active flow. Actually this velocity is greater than the theoretical value, and reverse flow along the pressure side of the blade does not take place even at small discharges. (5) In addition to (4), it was definitely proved that the flow at very small discharges is no longer stable. (6) The entrance flow may be assumed to be radial for all practical operating conditions. (7) The increase in the meridional velocity over the ordinary computed value alters the shape of the exit triangle considerably, which is of special importance in designing the guide-vane entrance angle.

I—INTRODUCTION

THE power actually required by the impeller of a centrifugal pump is considerably less than the theoretical value computed for an impeller containing an infinite number of blades. This is due to the variations in pressure and velocity, along circles concentric with the axis of the pump, created by the relative vortex in the fluid passages. The present methods of computation consider both these eddies and the accompanying reduction of the angle of exit from the impeller blades and give, by the use of correction factors, a result which is in sufficient agreement with the actual facts. It has been impossible, however, to definitely fix the value of the angle of exit for the actual delivery heads, and an exact knowledge of the flow conditions, particularly at exit from the impeller, is in many cases especially desirable. The assumptions which must be made when designing an impeller (e.g., that the fluid passages are always completely filled with actively flowing fluid) have often led to erroneous conceptions of the true flow conditions, especially in the case of small discharges. This is clearly shown by the research herein described. It is true that, since thorough investigation of the formation of "dead" zones on airplane wings, no further doubt has existed about the error in assuming the fluid passages to be completely filled with active flow. This phenomenon, however, was supposed to have but little influence on the power required by the impeller.

Theoretical investigation of the effect of the formation of dead zones was very difficult to conduct because no information was available concerning the extent of the zones and the conditions responsible for their formation. These phenomena have been clarified by the investigations described here in which an optical instrument called a "rotoscope" was employed which was designed by Dr. Thoma[4] several years ago for investigating the flow conditions in hydraulic turbines.

II—DESCRIPTION OF THE RESEARCH APPARATUS

The Pump

As shown by Fig. 1, the method of introducing water to the impeller differed but little from that employed in actual practice; however, the special purpose of the apparatus was given consideration in the design of the bearings and driving mechanism. The single impeller bearing was fitted to the outside of the stationary horizontal suction pipe so that the visibility would not be impaired by an outside bearing. The most essential fact is that the entire impeller and the guide vanes were clearly visible through a single plate-glass wall which took the place of the outside pump case and formed the front shroud of the impeller.

The Impeller

The impeller consisted of six blades cast integral with the

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[1] On reviewing Dr. A. J. Stepanoff's excellent paper entitled "Leakage Loss and Axial Thrust in Centrifugal Pumps," Dr. Thoma feels that he has made a valuable contribution to our knowledge of the subject. Some recent tests made, under Dr. Thoma's direction, in the laboratory of the Hydraulic Institute of the Technical University of Munich have cleared up a number of the uncertain flow phenomena in pump impellers which Dr. Stepanoff has discussed. These tests and the accompanying report, made by Mr. Karl Fischer, will, it is hoped, throw new light on some of the hitherto highly speculative flow conditions. The English text of this report was prepared by Mr. C. P. Kittredge, B.S., Massachusetts Institute of Technology, and holder of one of the John R. Freeman Traveling Fellowships.

[2] Diploma Ingenieur and Doktor Ingenieur, Technische Hochschule, Munich, and Regierungs-Baumeister.

[3] Professor of Hydraulics and Director of the Hydraulic Institute of the Technical University of Munich. Mem. A.S.M.E. Dr. Thoma was graduated from the Technical University of Munich in 1906 as a Mechanical Engineer. From 1906 to 1908 he was employed as designer by the firm of Briegleb, Hansen & Co., of Gotha, manufacturers of water turbines, governors, etc. For the following two years he was associated with Prof. A. Föppl as assistant in technical mechanics. During this time he remained in touch with Briegleb, Hansen & Co., and in 1910 became head of its governor department. He was chief engineer of its turbine-testing stations, as well as head of the governor department, after 1913. A new testing laboratory was built according to his plans in 1913–1914. After 1915 he was also superintendent of all the offices of the firm for the designing of all kinds of water-power machinery and water-power plants. In 1920 he was appointed Professor of Technical Mechanics of the Technical University at Munich. In 1921 he assumed his present position.

Contributed by the Hydraulic Division and presented at the Semi-Annual Meeting of The American Society of Mechanical Engineers, Bigwin Inn, Lake of Bays, Ontario, Canada, June 27–July 1, 1932.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] D. Thoma, "Die Versuchsanstalt für Wasserturbinen in Gotha," published by Engelhardt-Reyher, Gotha, 1918.

back shroud. The front shroud was a detachable circular glass plate. The blades were designed according to standard practice. Both the blades and the back shroud had painted surfaces in contact with the water. The impeller was designed to deliver 8.3 liters per sec (132 gpm) against a head of 1.65 m (5.41 ft) at 400 rpm, and 9.40 liters per sec (149 gpm) against a head of 1.885 m (6.18 ft) at 450 rpm.

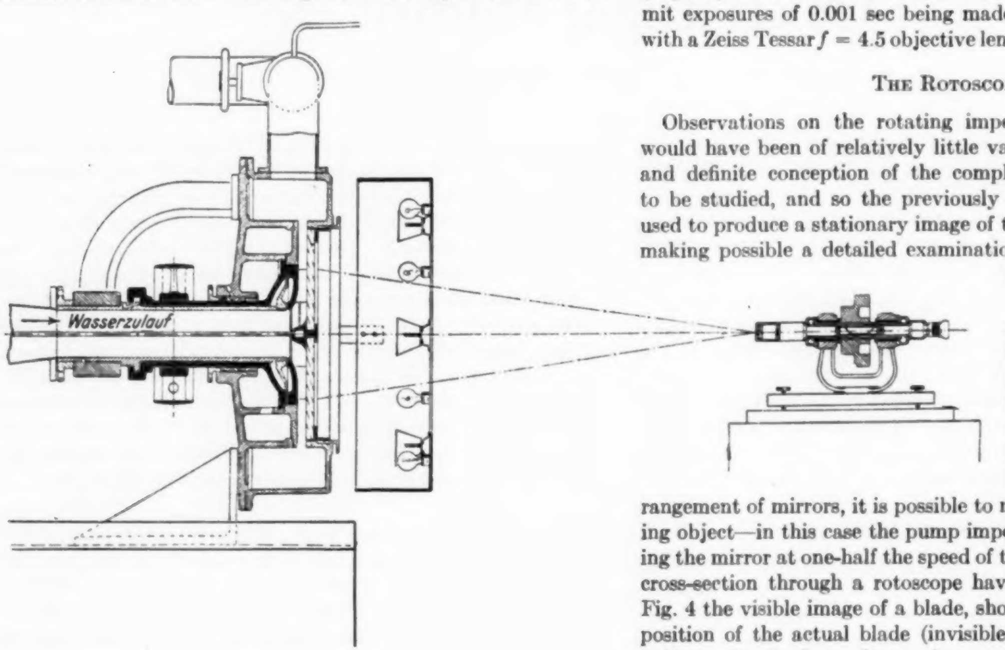

Fig. 1

Impeller Data

Clear distance between blades at entrance b_1 = 22.5 mm (0.886 in.)
Clear distance between blades at exit b_2 = 20.0 mm (0.787 in.)
Entrance diameter d_1 = 140 mm (5.51 in.)
Exit diameter d_2 = 280 mm (11.02 in.)
Suction-pipe diameter d_s = 100 mm (3.94 in.)
Number of blades z = 6
Blade angle at entrance β_1 = 17°
Blade angle at exit β_2 = 28°

As shown in Figs. 1 and 2, the illumination consisted of six concentrically arranged 100-watt lamps for ordinary use and six additional 1000-cp carbon arc lights for use when photographing the flow. The total illumination was sufficient to permit exposures of 0.001 sec being made with a camera equipped with a Zeiss Tessar f = 4.5 objective lens.

The Rotoscope

Observations on the rotating impeller with the naked eye would have been of relatively little value in obtaining a correct and definite conception of the complicated conditions of flow to be studied, and so the previously mentioned rotoscope was used to produce a stationary image of the rotating parts, thereby making possible a detailed examination of the flow phenomena with respect to the blades.

The principle of the rotoscope is that the image of a stationary object when viewed in a rotating inclined mirror revolves at half the speed of the mirror. Therefore, by means of a suitable arrangement of mirrors, it is possible to make the image of a rotating object—in this case the pump impeller—stationary by rotating the mirror at one-half the speed of the object. Fig. 3 shows a cross-section through a rotoscope having three mirrors, and in Fig. 4 the visible image of a blade, shown by solid lines, and the position of the actual blade (invisible), shown by dotted lines, indicate clearly how the stationary image is produced. The illustrations, *a*, *b*, *c*, and *d* show the displacement of the image relative to the blade when the blade is stationary and the mirror system revolves, and *a'*, *b'*, *c'*, and *d'* the displacement of the blade relative to its image when the impeller is revolving at twice the speed of the rotoscope. In each case the diameter of the circle shown dotted represents the apparent plane of reflection. It will be seen that in the latter case the image remains stationary. In actual practice the rotoscope is constructed as shown in Fig. 5 with an inverting prism replacing the three mirrors and a lens system provided to increase the field of view of the instrument.

General Arrangement of Apparatus

The water was supplied to the pump under a positive head of

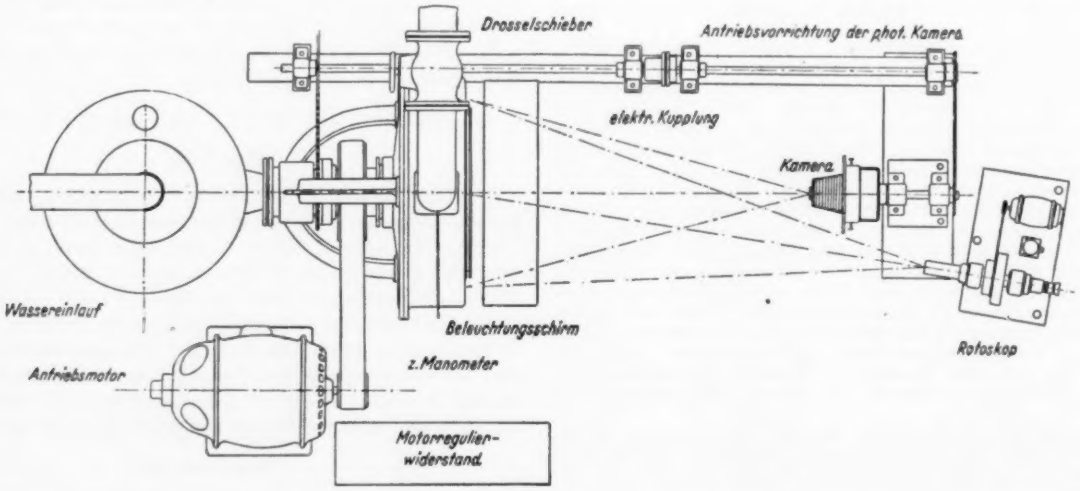

Fig. 2

(*Wassereinlauf* = water inlet; *Antriebsmotor* = driving motor; *Motorregulier-widerstand* = motor regulating resistance; *z Manometer* = to manometer; *Beleuchtungsschirm* = illuminating arrangement; *Drosselschieber* = throttle valve; *elektr. Kupplung* = electromagnetic coupling; *Antriebsvorrichtung* = driving arrangement.)

about 1.2 ft from a small constant-head tank, which was in turn fed by the laboratory supply system. Figs. 1, 2, 6, and 7 show the arrangement of all parts of the equipment. The discharge water was throttled through a gate valve to vary the head and discharge, and then piped to a weighing tank on the floor below where it was accurately measured. The pump was belt-driven by an electric motor the speed of which was regulated by means of a variable-resistance unit. The total static head was measured by a mercury manometer.

In order to keep the effect of viscosity, which owing to the relatively small dimensions of the pump became more important than under most practical conditions, from influencing the final results, the rotating speed was made as high as the strength of the plate-glass wall would permit (about 450 rpm). The Reynolds numbers, based on the hydraulic radii at entrance to and exit from the water passages of the impeller were 164,000 and 108,000, respectively. These are frequently exceeded in practice, but they are already so high that no fundamentally different flow conditions are to be expected in a larger impeller.

METHOD OF OBSERVATION

As already stated, the rotoscope was used for making a continuous observation of the flow conditions to be investigated. It was placed at a distance of from four to five feet from the pump impeller, and, as shown in Fig. 8, was driven by a small belt-connected electric motor equipped with a variable resistance for coarse speed adjustment. Fine speed adjustment was obtained by braking the flywheel of the rotoscope gently with the finger tips, so that an absolutely stationary image of the blade could be easily produced. In order to secure a permanent record of the flow conditions a camera, rotatable about its optical axis, was used in place of the rotoscope and driven by a two-piece countershaft from the pump driving mechanism as shown in Figs. 6 and 7. The two parts of the countershaft could be connected at will by the electromagnetic coupling shown in Fig. 9. A chain drive was used to avoid any speed difference between the pump and camera.

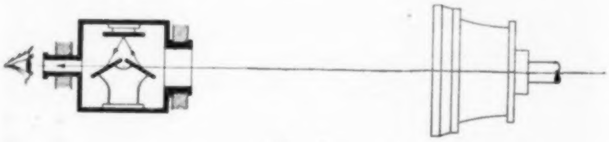

FIG. 3

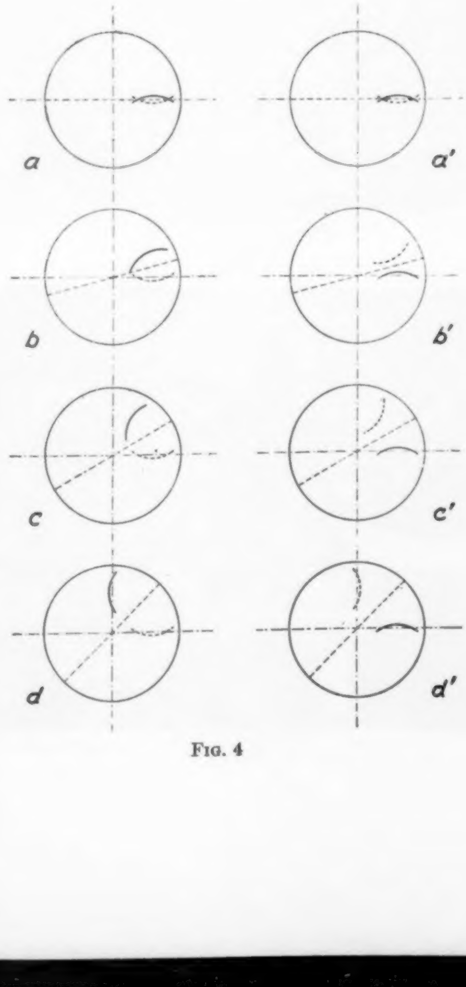

FIG. 4

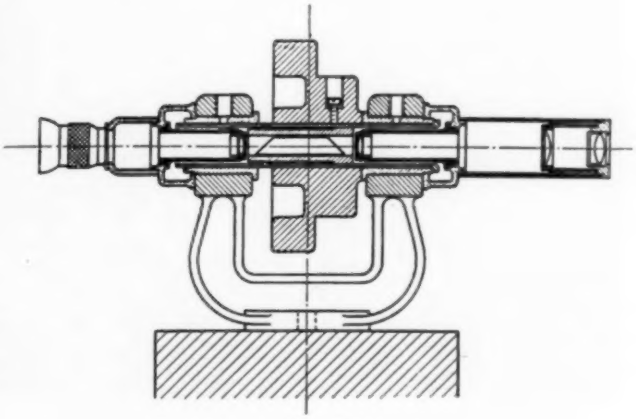

FIG. 5

Theoretically both the camera and the rotoscope should have their optical axes coincide with the prolonged axis of rotation of the impeller, but, when necessary, the position of the rotoscope may be altered slightly, as shown in Fig. 7, without excessive blurring of the image taking place. In this way the impeller could be observed through the rotoscope while the pump discharge was being adjusted. When the proper flow conditions were obtained the camera was set in motion and the shutter tripped at the critical instant to record the particular phenomenon under observation.

The flow conditions in the impeller were made visible by injecting a special dark-colored dye into the water at several points. The dye was brought into the impeller through a brass pipe, 5 mm in diameter, as far as the glass plate. From there two branch pipes, each 2 mm in diameter, carried the dye in one case to the inner nose of one of the blades, as shown at *B* in Fig. 10, and in the other case to about the middle of the high-pressure side of another blade, as shown at *C* in Fig. 18. In order to make sure that these branch pipes did not cause unexpected disturbances in the flow, the inner nose of one of the blades was equipped with three 0.5-mm (0.02-in.) holes, shown at *A* in Fig. 10, which were connected to the nearest attachment bolt for the glass shroud by a canal cut in the interior of the blade. This particular attachment bolt was made hollow and the dye brought to it through a rubber tube from the central supply pipe, outside the glass shroud. A further precaution was taken while injecting the coloring material to apply only enough pressure to get the dye into the water so that disturbances in the flow conditions would not be caused by too high an injection velocity.

III—RESULTS OF THE EXPERIMENTS

According to the researches made by Oertli,[5] the flow in a centrifugal impeller is not exactly two-dimensional, but the flow along the blades at mid-depth will be sufficiently characteristic of the total flow as long as the operating conditions do not vary

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[5] Oertli, Heinrich, "Untersuchung der Wasserströmung durch rotierende Zellenkreiselrad," in the Eidgen. Hochschule Laboratory, Zurich.

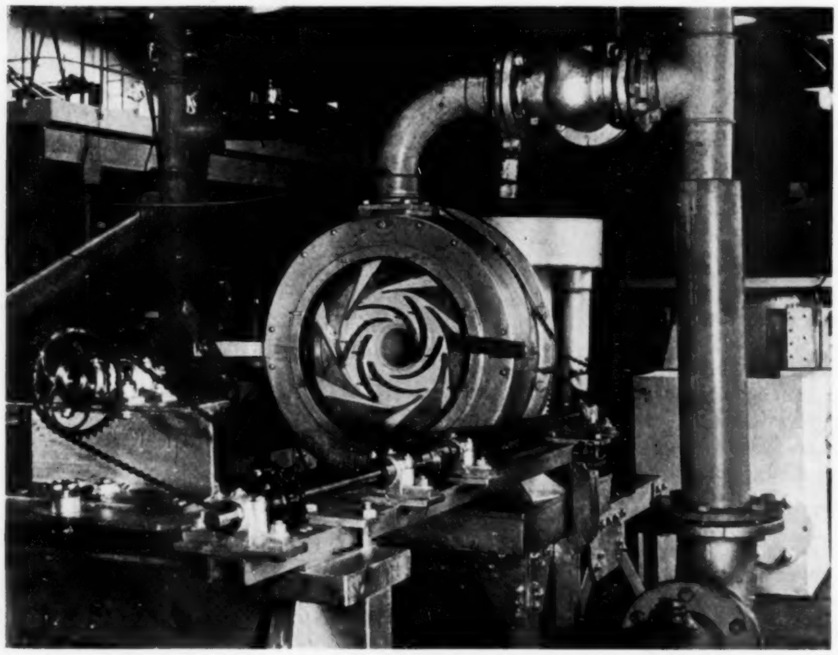

Fig. 6

too widely from their normal value. Accordingly, the points of injection of the coloring material were located at the mid-depth of the respective blades.

Flow Conditions at Entrance to Impeller

It was necessary in this case to consider the absolute flow conditions in order to determine the error in assuming radial entrance flow in impeller design. Neither the rotoscope nor the rotating camera was necessary for this purpose, and direct observation was more desirable. Figs. 11–15 show five different entrance-flow conditions, the data for which are listed in Table 1. It was impossible to detect any difference in the flow phenomena above a speed of 350 rpm.

It will be seen in these photographs that the variations from radial entrance flow are confined within narrow limits. Prerotation of the water is noticeable only at very small discharges (Figs. 14 and 15). The chief cause of this is the reverse flow from the water passages of the impeller, and to a lesser extent the increase in the relative frictional effects at small discharges. It will be shown later that below a certain critical discharge a definite reverse flow takes place in the water passages of the im-

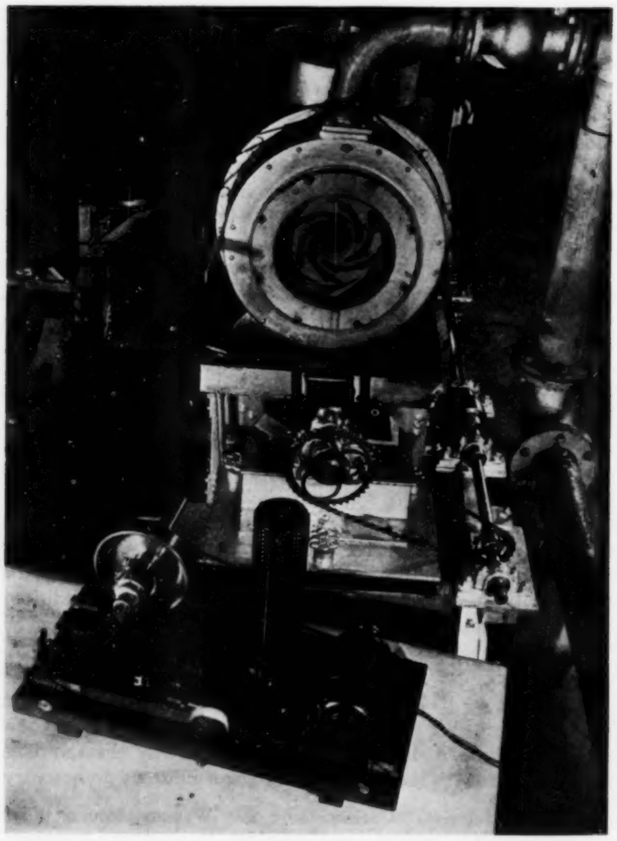

Fig. 7

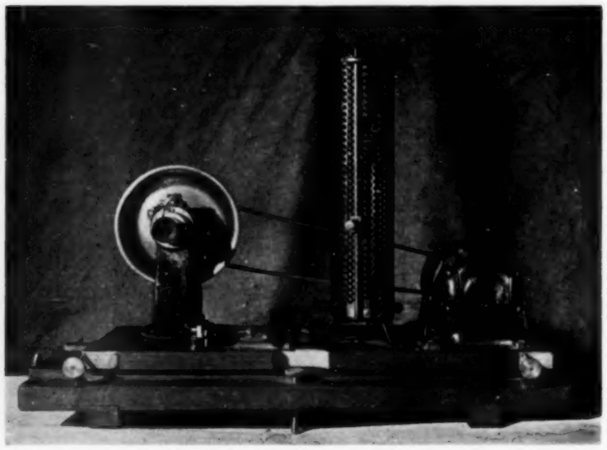

Fig. 8

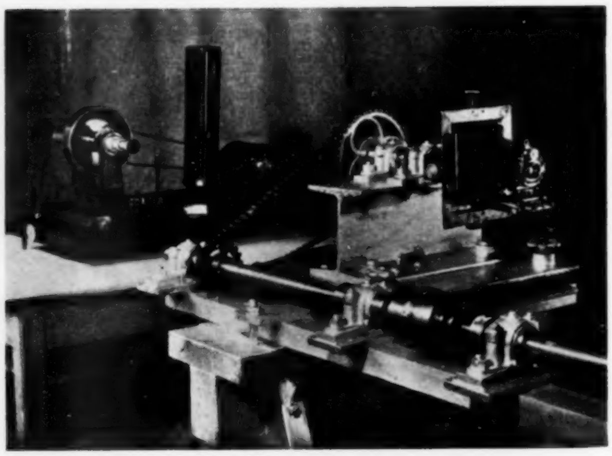

Fig. 9

peller (Fig. 21). It is sufficient here to state that below a critical discharge, reverse flow in some of the water passages takes place, that this flow is not stationary with respect to the impeller blades, and that its location in the impeller changes periodically. The water which flows inward from the impeller also has considerable energy of rotation, and as it mixes with the incoming water from the suction pipe, part of this rotation energy is given up, causing prerotation of the latter to take place. This mixing phenomenon, together with the variations in the reverse flow, causes a very uneven velocity distribution, as is clearly shown by the broken and complicated form of the dye streams in Figs. 14 and 15.

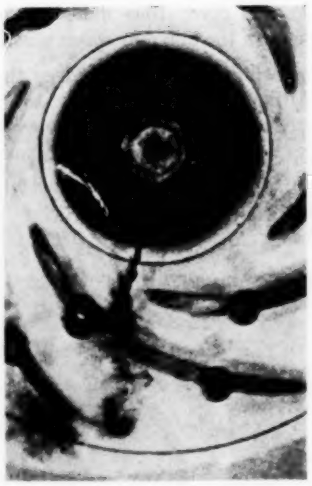
Fig. 11

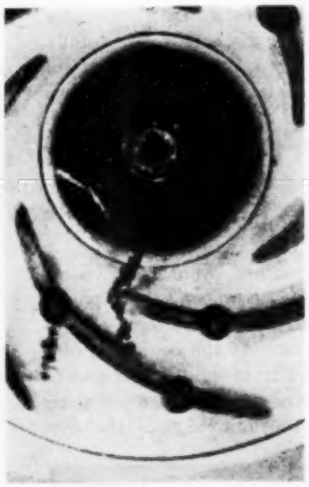
Fig. 12

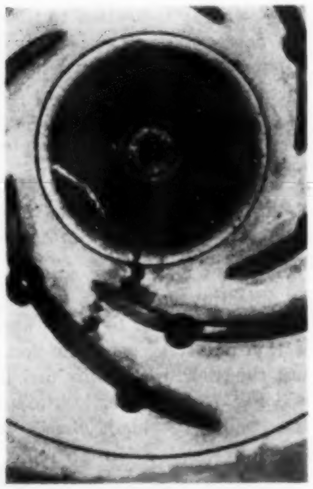
Fig. 13

TABLE 1 DATA ON VARIOUS ENTRANCE-FLOW CONDITIONS

Fig. No.	Speed, rpm	Q Liters per sec	Q Gal per min	$\frac{Q}{Q_{\text{normal}}}$	H, meters	H, feet	Remarks
11	450	14.81	235	1.58	0.752	2.47	Q above normal
12	450	9.40	149	1.00	1.885	6.18	Q normal
13	450	5.74	91.0	0.61	2.351	7.72	Q below normal
14	450	3.20	50.8	0.34	2.568	8.42	Q below normal
15	450	1.85	29.3	0.20	2.589	8.50	Q below normal

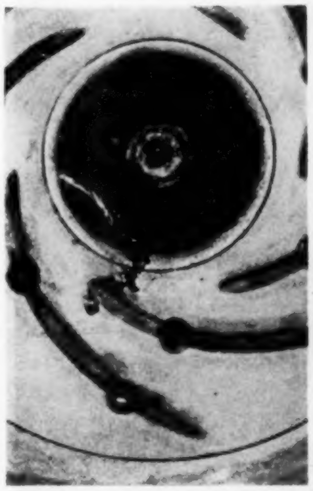
Fig. 14

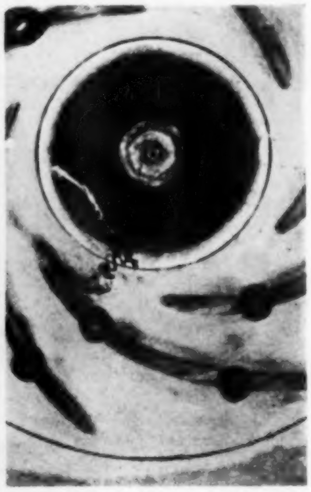
Fig. 15

Flow Conditions in the Water Passages of the Impeller

A great many theoretical investigations have been made in an attempt to solve the problem of the relative-flow conditions in rotating fluid passages. In analyzing the problem, the relative flow has generally been divided into two components, one a nonrotational component representing the flow through a stationary canal, and the other a constant rotational flow having the same angular velocity as the impeller. The resultant of the two components is supposed to give the actual flow. The advantage of this method is that the flow conditions may be easily computed for every speed and discharge. Kucharski[6] has made a strict mathematical derivation of the flow conditions in a rotating impeller having a finite number of straight blades which extend a finite distance radially outward from the axis of rotation, with the space between filled with an ideal fluid and neglecting friction. The fluid passages, formed by curved blades of a finite length and cut off by entrance and exit circles, as found in actual practice,

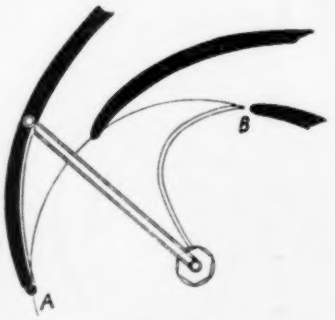

Fig. 10

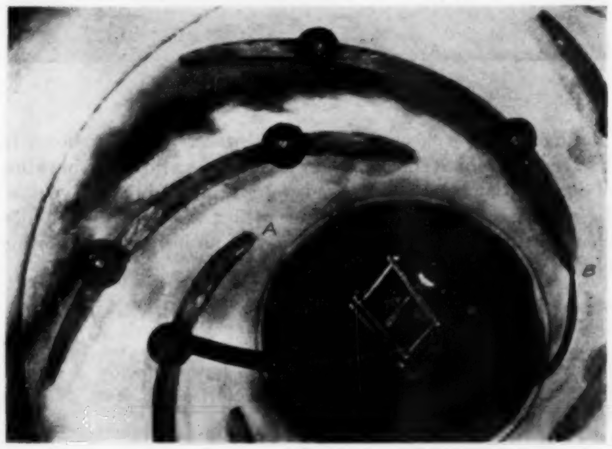

Fig. 16

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[6] Kucharski, "Strömungen einer reibungsfreien Flüssigkeit bei Rotation fester Körper," Munich and Berlin, 1918.

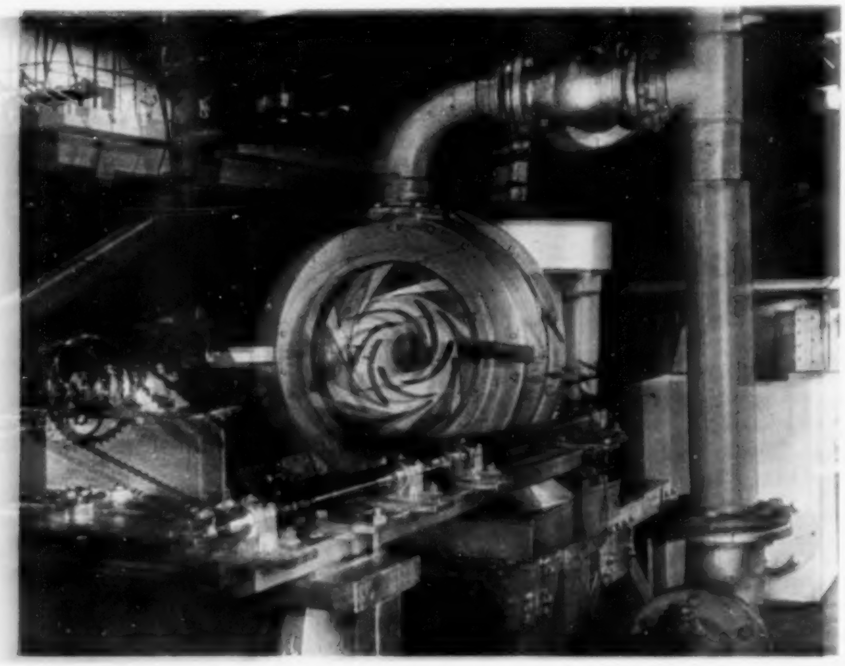

Fig. 6

too widely from their normal value. Accordingly, the points of injection of the coloring material were located at the mid-depth of the respective blades.

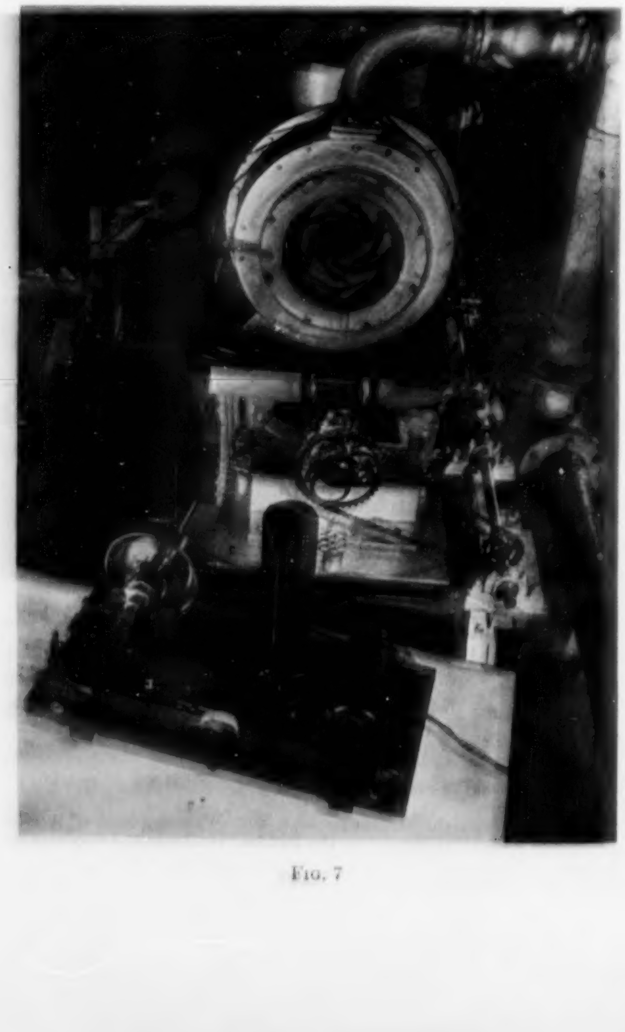

Fig. 7

Flow Conditions at Entrance to Impeller

It was necessary in this case to consider the absolute flow conditions in order to determine the error in assuming radial entrance flow in impeller design. Neither the rotoscope nor the rotating camera was necessary for this purpose, and direct observation was more desirable. Figs. 11–15 show five different entrance-flow conditions, the data for which are listed in Table 1. It was impossible to detect any difference in the flow phenomena above a speed of 350 rpm.

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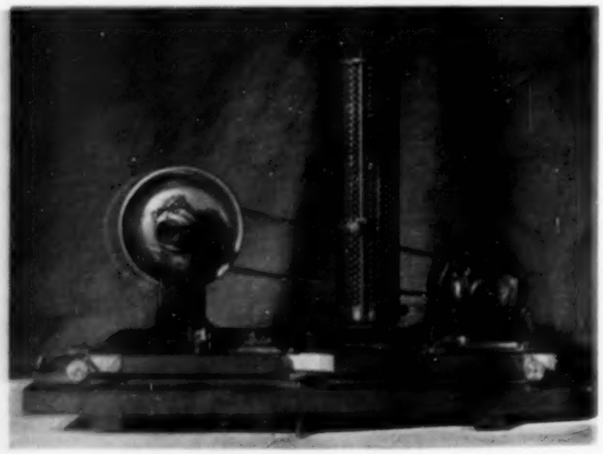

Fig. 8

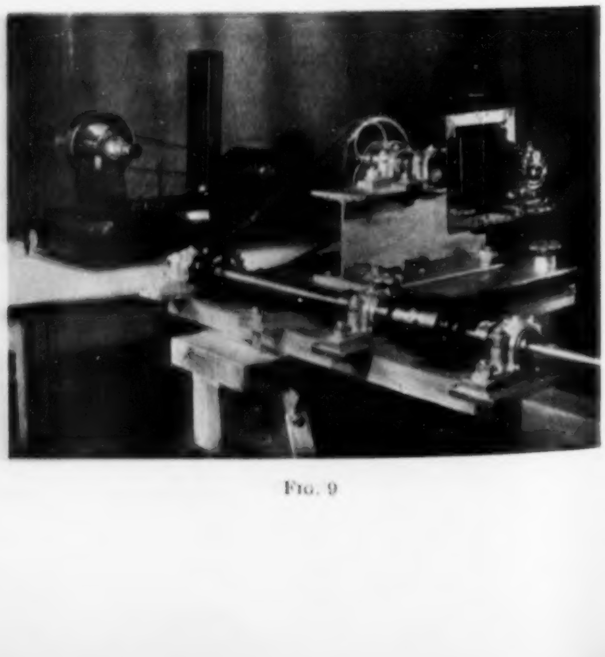

Fig. 9

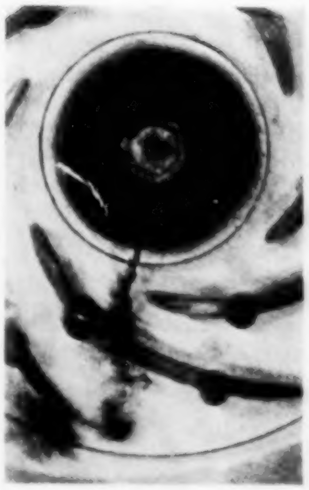

Fig. 11

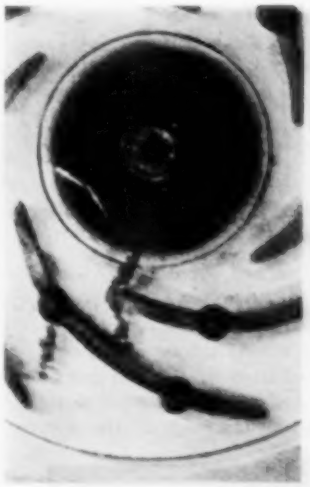

Fig. 2

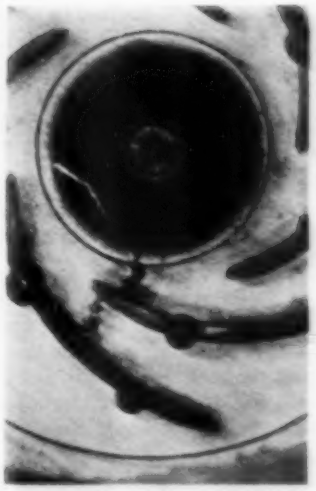

Fig. 13

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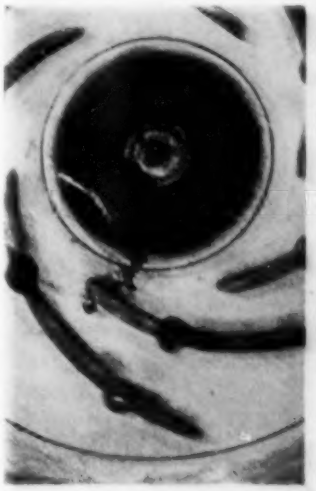

Fig. 14

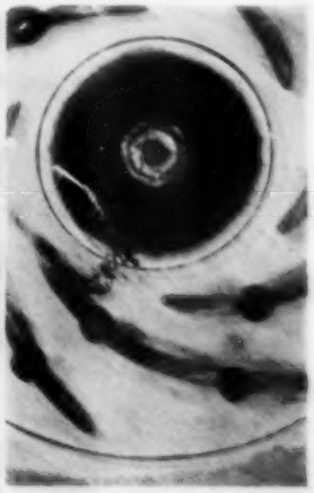

Fig. 15

Flow Conditions in the Water Passages of the Impeller

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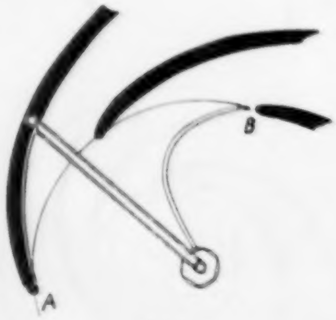

Fig. 10

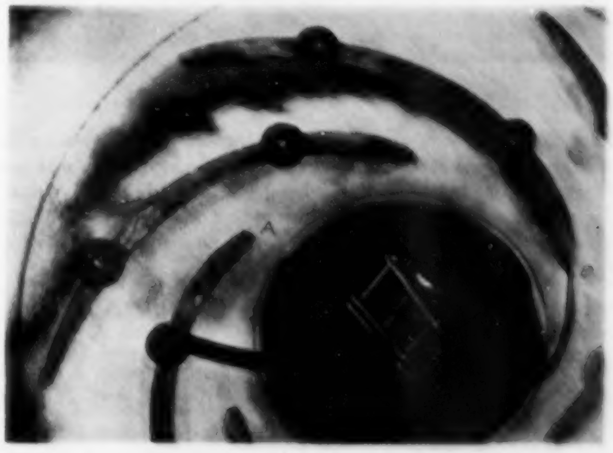

Fig. 16

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[6] Kucharski, "Strömungen einer reibungsfreien Flüssigkeit bei Rotation fester Körper," Munich and Berlin, 1918.

present many difficulties to theoretical investigation.[7] Actually the flow conditions, especially at small discharges, are totally different from those theoretically deduced, due to the effects of friction and instability and the fact that the active flow breaks away from the faces of the blades. At no time in the research herein described were the water passages of the impeller completely filled with active flow.

Establishing the Border Between the Dead-Water Zones and the Active Flow

The flow conditions, as already described, were made visible by injecting a coloring material into the water at the points labeled *A*, *B*, and *C* in the accompanying illustrations, the injection point *A* giving the best results. At the beginning of a test the color was usually first injected from the tubes *B* and *C*, and Fig. 16 shows the location of one of the color streams along the inside of a blade shortly after the admission of the dye.

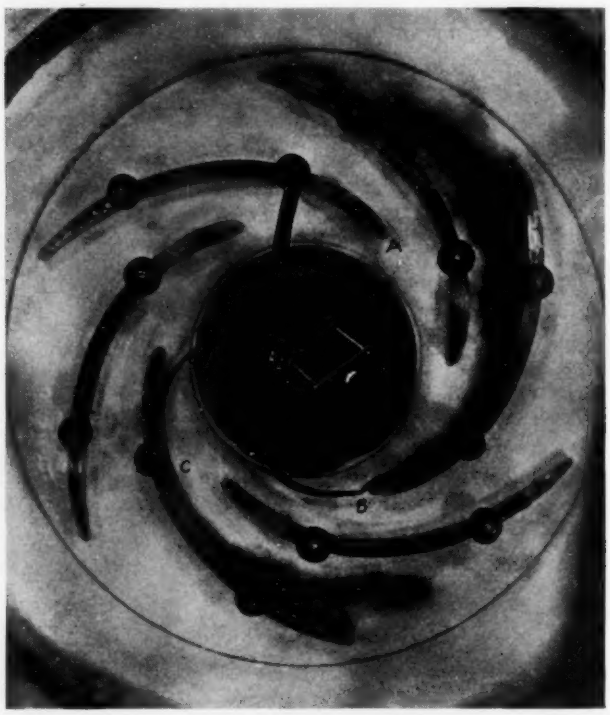

Fig. 18

The dye slowly collected in the dead-water zones as shown in Fig. 18. Except at small discharges there was a rather definite line of demarcation between the dead-water zones and the active flow. In any case, however, the water which may have been discharged through the dead zones was but an insignificant portion of the total flow.

Flow for Different Values of the Ratio Q/Q_{normal}

(*a*) ***Flow at Discharges Below Normal*** ($Q/Q_{\text{normal}} < 1$). If the value of the ratio (Q/Q_{normal}) was lowered below a certain limit the flow conditions were characterized by two different criteria:

1 The outward flow was no longer stable and varying conditions of reverse flow appeared

2 At any given time different flow conditions were to be found in the different water passages.

The following examples show how the change from stable flow to pulsating flow takes place:

Fig. 17 shows the absolute flow conditions in one of the water passages as determined by observations on the relative flow conditions with the rotoscope for

n = 252 rpm; Q = 2.46 liters per sec (39.0 gpm);
Q/Q_{normal} = 0.47; H = 0.75 m (2.46 ft)

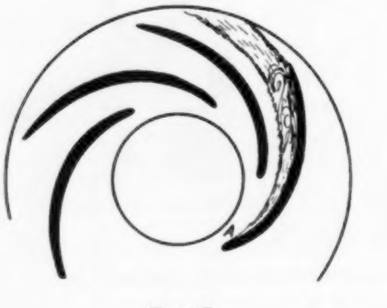

Fig. 17

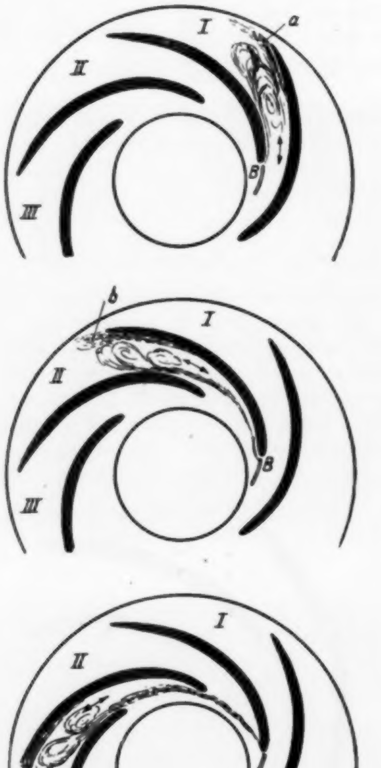

Fig. 19

No difference between the flow conditions in the various water passages could be determined for this speed and discharge. More than half the water passage is, however, taken up by the dead-water zones composed of whirls and eddies, while the active flow occupies only a narrow strip along the high-pressure side of the blade. For the photograph, Fig. 16, the operating conditions were:

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[7] Spannhake, "Hydraulische Probleme," Mitteilungen des Instituts für Strömungsmaschinen der Technischen Hochschule Karlsruhe, Heft 1, 1930.

$n = 250$ rpm; $Q = 2.79$ liters per sec (44.2 gpm);
$Q/Q_{\text{normal}} = 0.53$; $H = 0.77$ m (2.52 ft)

which corresponds closely to the case just mentioned.

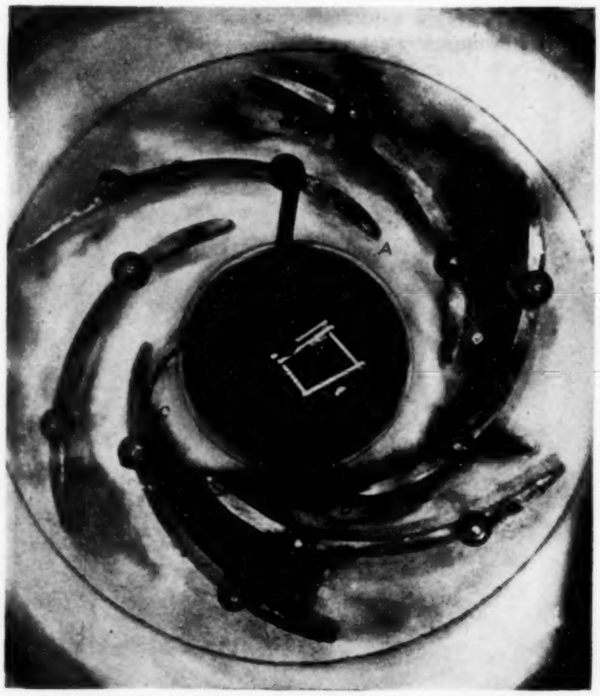


Fig. 20

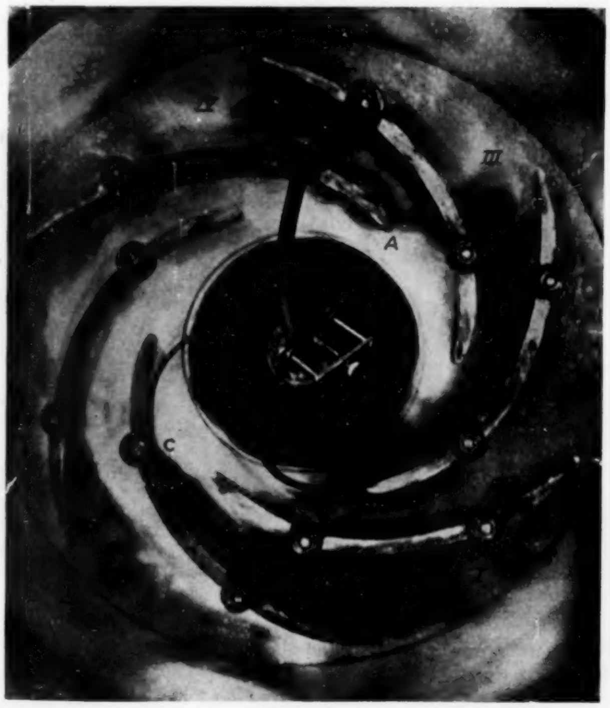


Fig. 21

If the discharge is now decreased considerably a rather uniform pulsating flow appears which has a definitely timed cycle, and the previous uniform flow conditions in the different water passages disappear. This phenomenon is undoubtedly brought about by the impeller blades themselves. The water passages are now almost entirely filled with dead water which surges back and forth parallel to the blades with a periodic motion. These conditions are always such that outward flow is taking place in some passages while there is inward flow in others. Some of the inward flow crosses the entrance circle, and instead of returning to the same passage it enters the second or third passage beyond. In all probability the water delivered by the impeller is forced out across the exit circle by this periodic swinging motion taking place in the space between the blades.

An example of the phenomena taking place during one period is illustrated in Fig. 19 for

$n = 240$ rpm; $Q = 1.038$ liters per sec (16.46 gpm);
$Q/Q_{\text{normal}} = 0.206$; $H = 0.79$ m (2.59 ft)

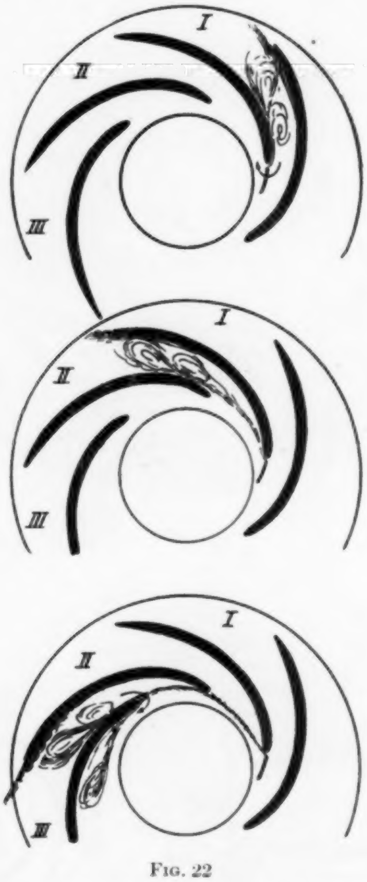


Fig. 22

At the beginning of a period the color was injected from the tube B and entered water passage I (Fig. 19, Condition 1). It flowed closely along the pressure side of the blade and then gradually broke away to form eddies which filled the entire breadth of the passage. This required only a small fraction of a second, but the eddies remained surging back and forth in the passage for a relatively long period of time, and at the point a it was noted that the color was always slowly breaking away from the blade. Meanwhile an outward flow took place in passage II about a third to a quarter of a second after the beginning of the period, and the dye stream entering this passage revealed eddies nearly filling the space between the blades as shown in Fig. 19, Condition 2. As before, the eddies surged back and forth, with the color leaving the blade at the point b. The end of the outward flow in passage II was indicated by the sudden entrance of the dye stream in III, where exactly the same phenomena as in passages I and II took place (Fig. 19, Cond. 3). This marked the end of one period, the duration of which was from 5 to 7 revolutions of the impeller or about 1.5 sec.

Figs. 20 and 21 require no further explanation. The reverse flow in passages I and IV and the outward flow in passage III are clearly visible. Fig. 22 was constructed for

$$n = 243 \text{ rpm};\ Q = 0.133 \text{ liter per sec } (2.12 \text{ gpm});$$
$$Q/Q_{\text{normal}} = 0.025;\ H = 0.812 \text{ m } (2.66 \text{ ft})$$

The irregularities in the flow became clearer as the violence of the back-and-forth surges of the eddies increased.

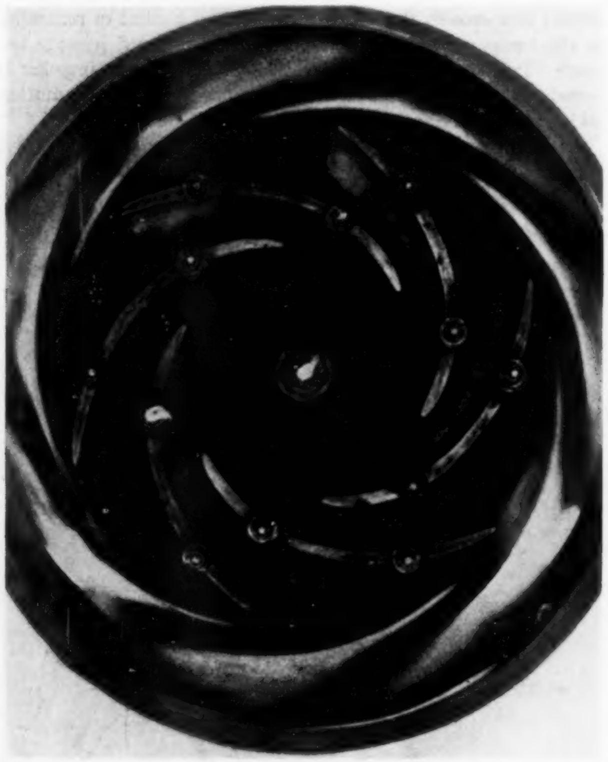


Fig. 23

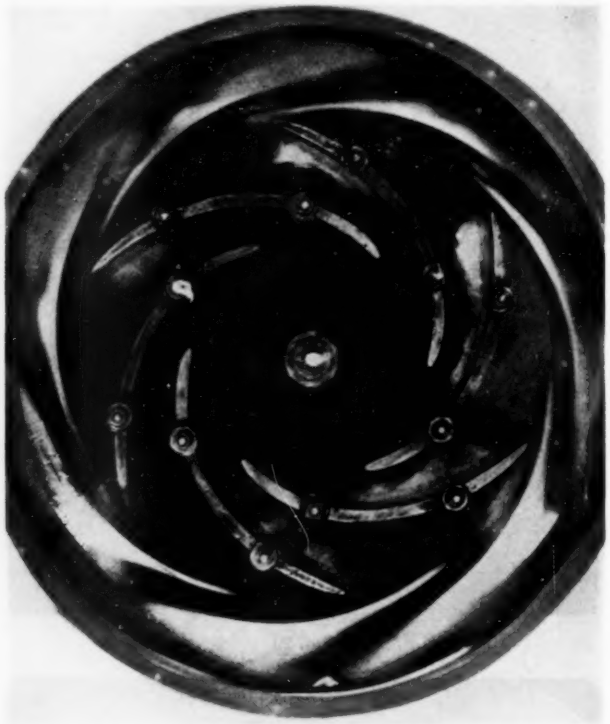


Fig. 24

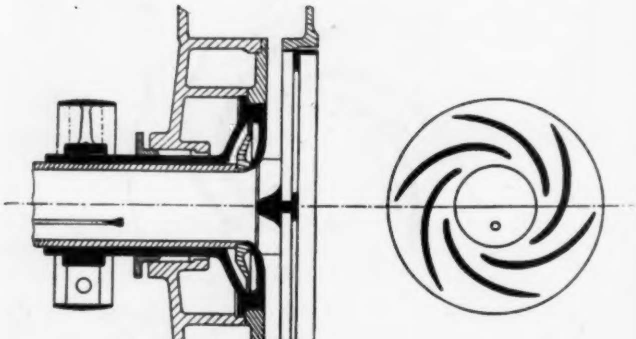


Fig. 25

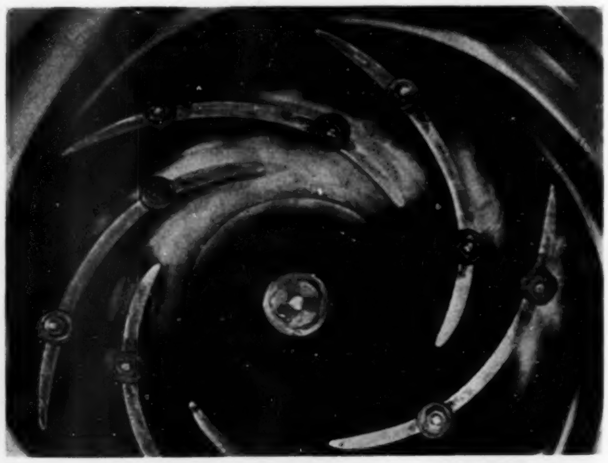


Fig. 26

In connection with the description of the flow conditions by different discharges below the normal value (9.40 liters per sec = 149 gpm), some additional photographs have been included for which a different coloring system was used to show the flow phenomena. To obtain these photographs the pump was started with clear water and then the dye was suddenly forced into the suction pipe through a quick-opening valve. In this way it was possible to completely color a slug of water in the suction pipe and photograph it as it flowed through the impeller. In Figs. 23 and 24 the active flow is shown by the black portion, while the dead-water zones on the low-pressure sides of the blades remain filled with clear water.

This method could not be used with discharges as large or larger than half the normal value because the color spread too rapidly due to increased turbulence in the flow. To prevent this a horizontal partition was placed in the suction pipe and the dye inlet located in the lower half as shown in Fig. 25. With this arrangement only half of the water was colored, so that the blades passed alternately from the colored to the clear water. The dye remained in the dead-water zones, which appear black in the upper half of Fig. 26 ($Q/Q_{\text{normal}} = 0.47$), and the active flow is shown by the white areas.

(*b*) ***Flow at Normal Discharge*** ($Q/Q_{\text{normal}} \cong 1$). Fig. 27 shows the flow conditions for

$$n = 400 \text{ rpm};\ Q = 8.86 \text{ liters per sec } (140.5 \text{ gpm});$$
$$Q/Q_{\text{normal}} = 1.06;\ H = 1.41 \text{ m } (4.63 \text{ ft})$$

Here, for the first time, a favorable flow condition with negligible entrance shock appears, as shown by the dye streams injected at points *A* and *B*. (The mouth of the tube *B* was in the center line of the blade produced.) At exit from the impeller the conditions are still somewhat unsatisfactory since the dead-water zones on the low-pressure side of the blades (shown black at *Z-Z*) fills about one-third of the width of the water passage and consequently causes almost a 50 per cent increase in the velocity of the active flow. These dead-water zones reduce the actual exit angle from 28 deg (blade angle) to about 20.5 deg. Scarcely any separation of the flow from the high-pressure faces of the blades could be detected under these conditions.

(*c*) ***Flow at Discharges Above Normal*** ($Q/Q_{\text{normal}} > 1$). In Fig. 28 the conditions are

$$n = 400 \text{ rpm};\ Q = 14.30 \text{ liters per sec. } (227 \text{ gpm});$$
$$Q/Q_{\text{normal}} = 1.71;\ H = 0.61 \text{ m } (2.00 \text{ ft})$$

The water enters the impeller at an angle greater than the blade entrance angle as shown by the path of the dye stream injected at *B*. The dead-water zone on the low-pressure side of the blade is somewhat smaller than under normal conditions (about $^1/_4$ the width of the passage at *Z-Z* as against $^1/_3$ for normal conditions), while on the high-pressure side of the blade a breaking away of the active flow and resultant dead-water formation are clearly visible.

RELATION BETWEEN THE DEAD-WATER ZONES ON THE LOW- AND HIGH-PRESSURE SIDES OF THE BLADES FOR DIFFERENT DISCHARGES

The results of the investigations show that dead-water zones form on the low-pressure sides of the blades for all discharge conditions. These zones increase in size as the discharge is decreased from above normal to very subnormal conditions, and at the same time the points at which the water leaves the blades move toward the entrance edges.

On the high-pressure sides of the blades the reverse phenomena take place, viz., the dead-water zones are larger and the points of separation of the water from the blades nearer the entrance edges under above normal than they are under normal, and below normal conditions when the dead-water zones (at normal speed) disappear entirely and the water follows the blades throughout their length.

In order to give a clearer idea of the relative extent of the dead-water zones and their location in the water passages, Fig. 29 has been constructed with data obtained from photographs of the flow conditions for different values of head, speed, and discharge. It should be borne in mind that it was very difficult to locate an absolutely definite boundary between the active and dead-water zones for most of the flow conditions investigated, the mixing between the two zones being much more noticeable at small discharges. Fig. 29 also shows that for the four pump speeds investigated the width of the zone of active flow (except at very small discharges) always remained practically constant from entrance to exit. When the dead-water zone disappeared from the pressure side of the blade, the opposite dead-water boundary remained almost parallel to the blade's curvature.

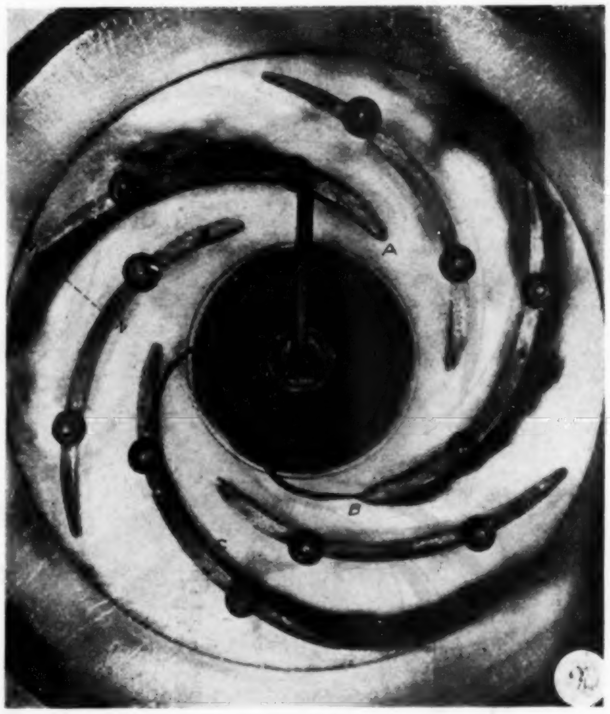


FIG. 27

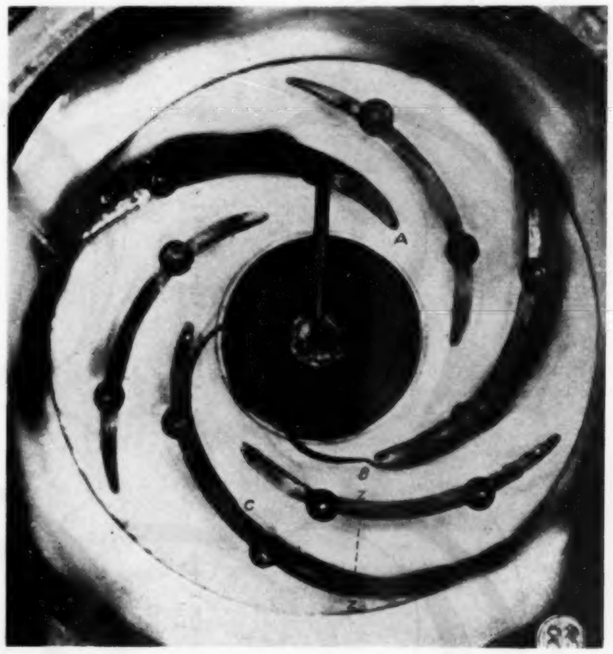
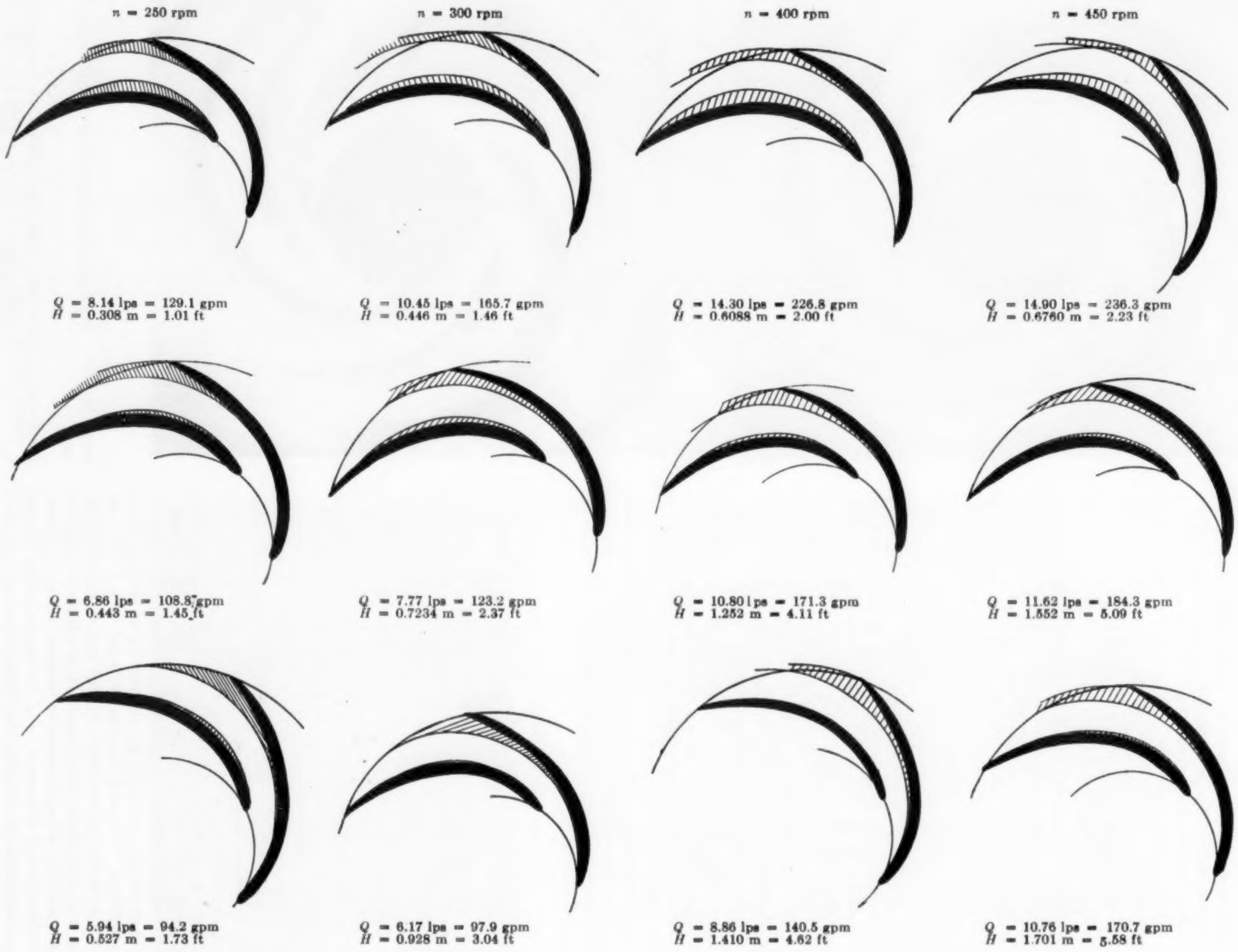
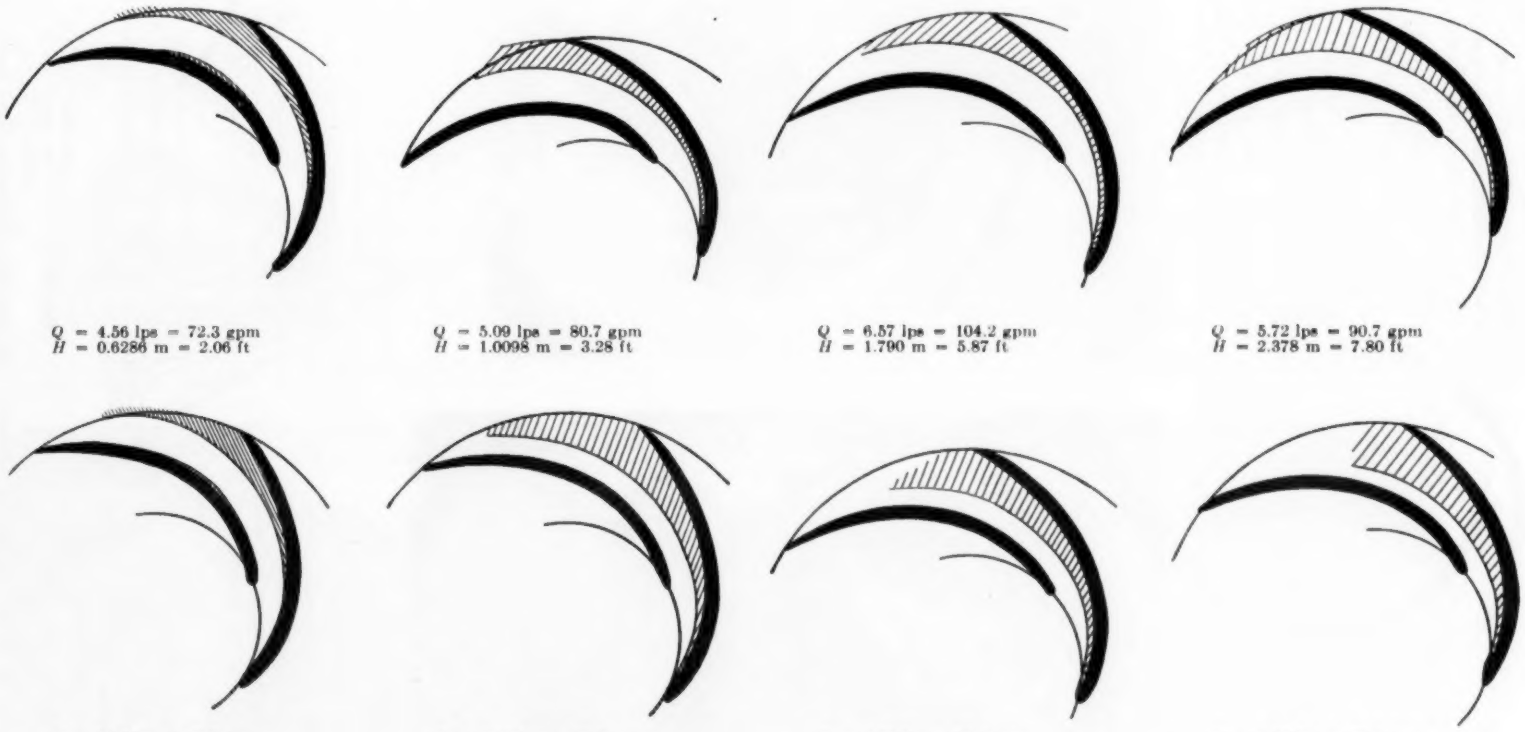


FIG. 28

IV—COMPARISON BETWEEN THE FLOW OF A FRICTIONLESS FLUID AND THE ACTUAL FLOW OF WATER BETWEEN THE BLADES

If the flow of a frictionless fluid be considered as the resultant of two separate and independent components of (1) non-rotational


n = 250 rpm
n = 300 rpm
n = 400 rpm
n = 450 rpm
Q = 8.14 lps = 129.1 gpm
H = 0.308 m = 1.01 ft
Q = 10.45 lps = 165.7 gpm
H = 0.446 m = 1.46 ft
Q = 14.30 lps = 226.8 gpm
H = 0.6088 m = 2.00 ft
Q = 14.90 lps = 236.3 gpm
H = 0.6760 m = 2.23 ft
Q = 6.86 lps = 108.8 gpm
H = 0.443 m = 1.45 ft
Q = 7.77 lps = 123.2 gpm
H = 0.7234 m = 2.37 ft
Q = 10.80 lps = 171.3 gpm
H = 1.252 m = 4.11 ft
Q = 11.62 lps = 184.3 gpm
H = 1.552 m = 5.09 ft
Q = 5.94 lps = 94.2 gpm
H = 0.527 m = 1.73 ft
Q = 6.17 lps = 97.9 gpm
H = 0.928 m = 3.04 ft
Q = 8.86 lps = 140.5 gpm
H = 1.410 m = 4.62 ft
Q = 10.76 lps = 170.7 gpm
H = 1.701 m = 5.58 ft



Q = 4.56 lps = 72.3 gpm
H = 0.6286 m = 2.06 ft

Q = 5.09 lps = 80.7 gpm
H = 1.0098 m = 3.28 ft

Q = 6.57 lps = 104.2 gpm
H = 1.790 m = 5.87 ft

Q = 5.72 lps = 90.7 gpm
H = 2.378 m = 7.80 ft

Q = 4.55 lps = 72.2 gpm
H = 0.6648 m = 2.18 ft

Q = 4.15 lps = 65.8 gpm
H = 1.071 m = 3.51 ft

Q = 4.06 lps = 64.4 gpm
H = 2.022 m = 6.63 ft

Q = 4.00 lps = 63.4 gpm
H = 2.440 m = 8.00 ft

FIG. 29 (lps = liters per second)

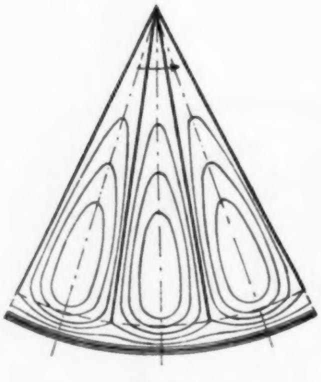


FIG. 30

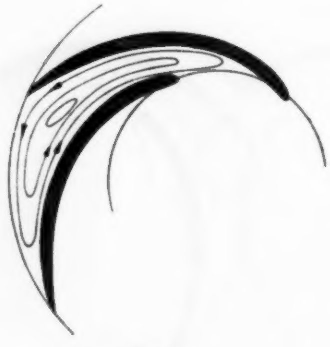


FIG. 31

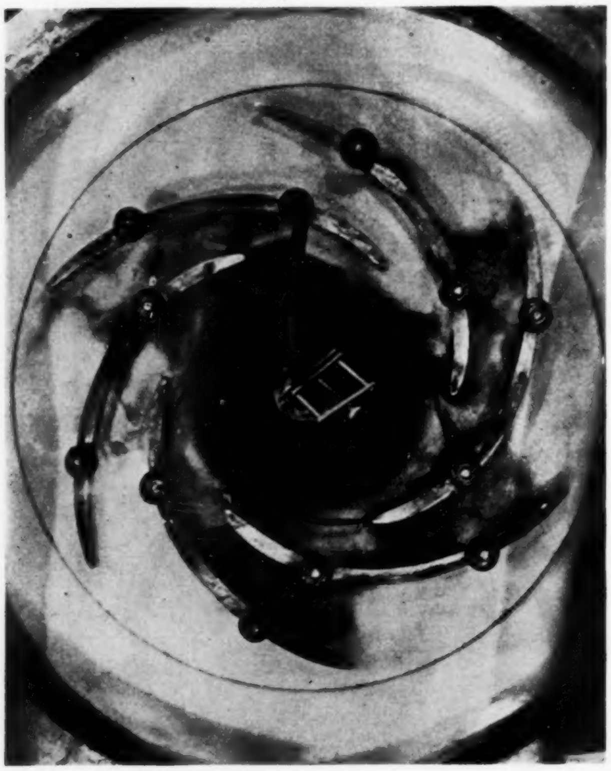


FIG. 32

flow through the fluid passages and (2) pure rotational flow with the angular velocity of the canal eddy = $-\omega$, the general character of the resultant flow will be governed at small discharges almost entirely by component (2), with component (1) gaining influence as the discharge is increased.

PURE ROTATIONAL FLOW ($Q \cong 0$)

For pure rotational flow ($Q = 0$) between straight blades of a finite length extending radially outward from the axis of rotation and neglecting friction, Kucharski derived theoretically the flow conditions shown in Fig. 30. If these conditions are applied to a curved passage the result is as shown in Fig. 31, wherein the fluid flows in a closed path around a stationary nucleus. A comparison of this condition with the actual phenomena where friction exists shows a fundamental change in the flow conditions. As seen in Fig. 32, the main negative eddy is broken up into a series of continually changing smaller eddies having no relation to the original. The results of these investigations show, as was expected, that the friction effects predominate if the pump discharge is zero, whereby the flow through the impeller is only enough to supply the leakage losses.

RADIAL-FLOW CONDITIONS AT SMALL DISCHARGES

By making the same assumptions as for pure rotational flow, Kucharski constructed the diagram shown in Fig. 34 which, transcribed for a curved passage, appears in Fig. 35. For small values of Q/Q_{normal} the rotational eddy overcomes the effect of radial flow to such an extent that reverse flow may be expected to take place along the high-pressure side of the blade. There is no apparent reason why the flow of a frictionless fluid should show continually changing conditions in the different fluid passages; it would more probably show the same constant flow conditions in all passages as long as the working conditions of the pump remained unchanged.

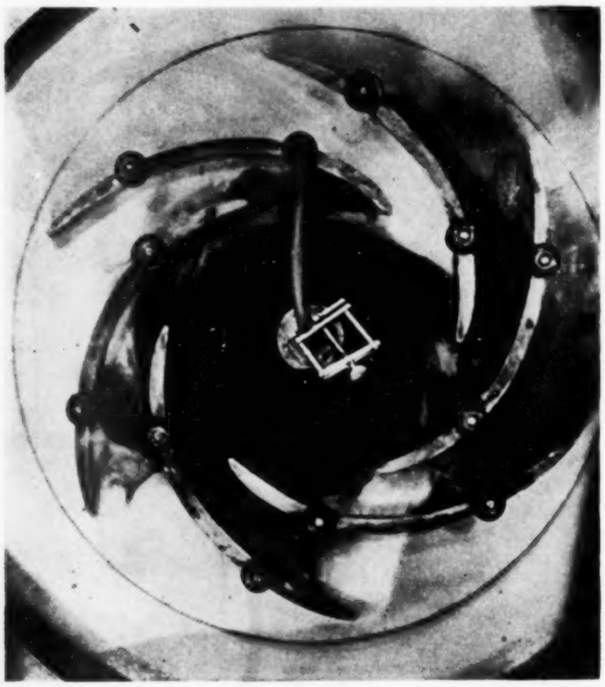


FIG. 33

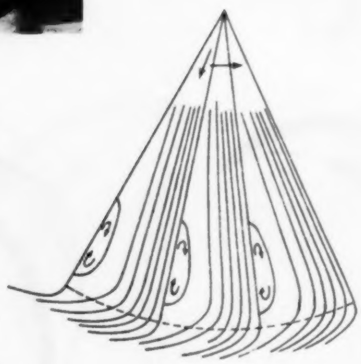


FIG. 34

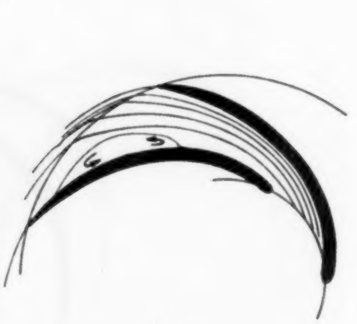


FIG. 35

The actual phenomenon is shown in Fig. 17; the discharge was reduced to a point where reverse flow would be expected with a frictionless fluid, but is still large enough to insure steady flow conditions in all passages. The figure shows that (1) no reverse flow exists, and (2) that a considerable dead-water zone builds up on the low-pressure side of the blade.

V—EFFECT OF THE DEAD-WATER ZONES ON THE ACTUAL TOTAL HEAD

The phenomena already described offer the following explanation for the reduction in the power required by the impeller.

The equation for the theoretical head delivered by an impeller having an infinite number of blades and radial flow at entrance [Fig. 36(*a*)] is

$$H_{\text{theoretical}} = (c_{2u} \times u_2)/g$$

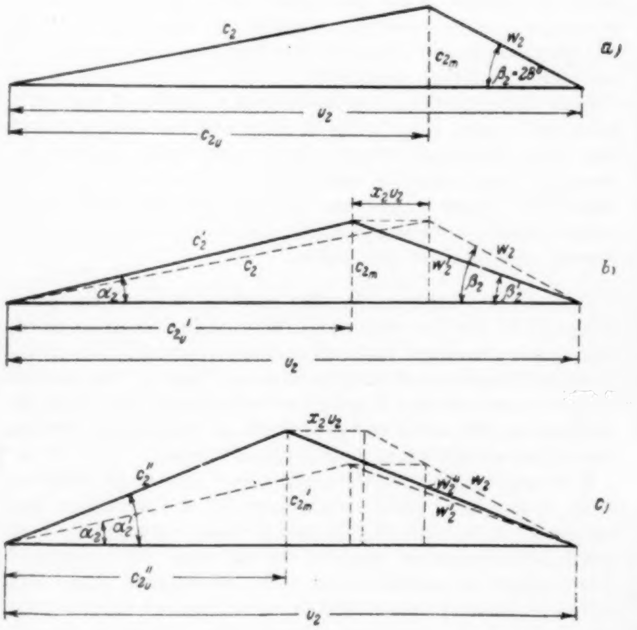


Fig. 36

Fig. 36(*b*) shows the average exit-velocity triangle for a finite number of blades and considering that the space between them is completely filled with active flow, as given by Pfleiderer.[3] The diagram is qualitatively correct as shown by the actual entrance- and exit-velocity distributions in Fig. 37, where the dye streams of passages *A* and *B* have been redrawn in passage *C*. In the same period of time the particles *P* and *Q* move the distances *P-P″* and *Q-Q″*, respectively. Since *P-P″* is much longer than *Q-Q″*, the higher velocity must be on the low-pressure side of the blade. Consequently the average relative velocity at exit is increased and the actual exit angle decreased, both of which cause a reduction in the power required by the impeller.

The fact that the water passages are almost never completely filled with active flow causes a still further increase in the average relative velocity at exit and consequent reduction of the actual total head as shown by Fig. 36 (*c*).

VI—INFLUENCE OF THE INCREASED MERIDIONAL VELOCITY c'_{2m} ON THE SIZE OF THE ENTRANCE ANGLE OF THE GUIDE VANES

The theoretical path of any water particle leaving the impeller is a logarithmic spiral, provided the width of the water passages remains constant. The difference between the angles α and α' is shown in Fig. 36; the tangent of α', which is the actual angle at which the water enters the guide vanes, is given by

$$\tan \alpha' = c'_{2m}/c_{2u}$$

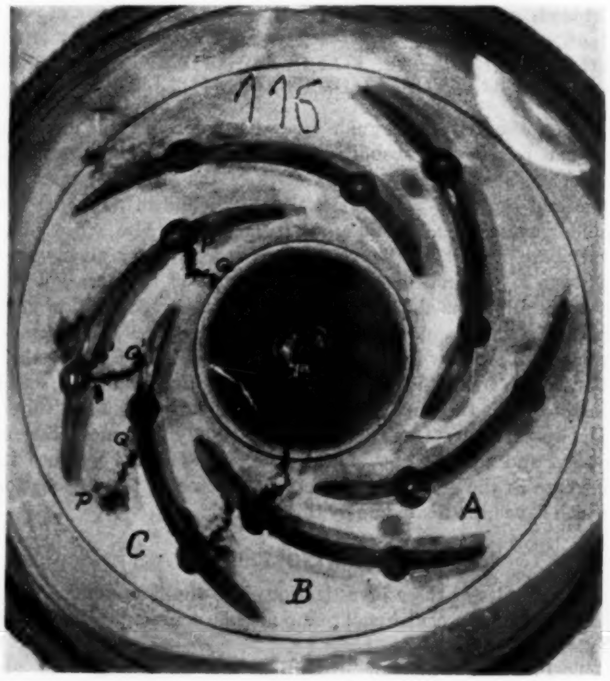


Fig. 37

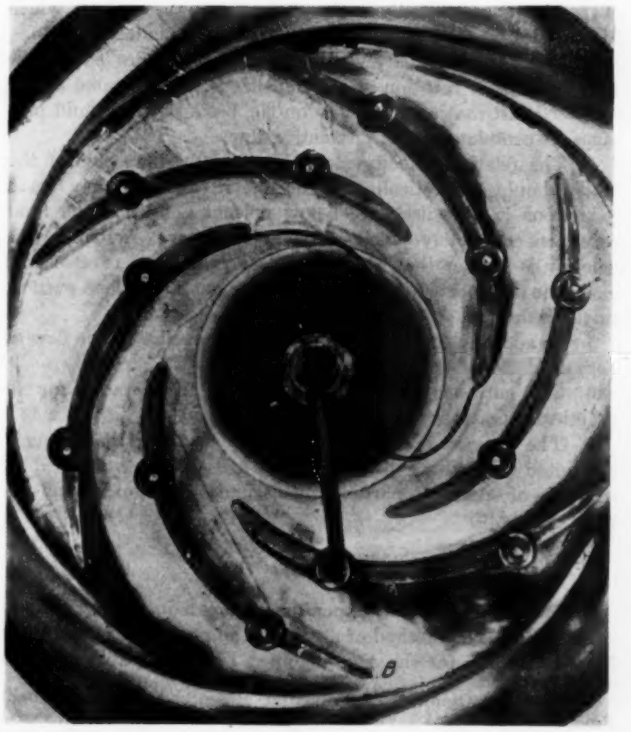


Fig. 38

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[3] C. Pfleiderer, "Die Kreiselpumpen," Berlin, 1924, p. 89 et seq.

The presence of dead-water zones along the blades causes an increase of c'_{2m} over c_{2m}, and consequently an increase in α', so that if the vanes are constructed with the entrance angle equal to α there will be a shock loss in the active flow at entrance. A redistribution of the velocities must take place in the space between the impeller blades and the guide vanes if this loss is to be avoided.

VII—INFLUENCE OF FIXED GUIDE VANES ON THE RELATIVE FLOW IN THE IMPELLER

The presence of fixed guide vanes has a considerable influence on the relative-flow conditions in the impeller and causes a pulsating flow in the water passages of the latter. Neglecting this and the dead-water zones usually results in designing the guide vanes with too small an entrance angle. Also, when guide vanes are present, the flow conditions at exit from a given water passage in the impeller at a given instant of time are not the same at all points along the exit circle. In Fig. 38 the flow at *A* is restricted to a narrow passage, which produces shock losses, while at *B* the flow is practically unhindered. All the investigations made with guide vanes showed an entirely different set of conditions in the impeller from those made without the vanes. The research by Omori[9] showed that only the most careful design and construction of the guide vanes would produce an increase in the efficiency of the pump, some of the reasons for which have been cleared up by this research.

VIII—CONCLUSIONS

The results of these investigations into the flow conditions in the impeller of a centrifugal pump may be summarized briefly as follows:

1 Practically all flow conditions for an actual fluid are fundamentally different from those theoretically derived for an ideal frictionless fluid.

2 The dead-water zones which form on the low-pressure sides of the blades undoubtedly cause a reduction in the angle at which the fluid leaves the blades (β'_2) from the actual blade angle (β_2).

3 The actual velocity distribution along circles concentric with the axis of rotation of the impeller is not the same as the supposed distribution computed on the basis that the fluid passages are completely filled with active flow.

4 The relative velocity along the high-pressure side of the blade should theoretically approach zero for relatively small reductions in the discharge below normal and with the fluid passages completely filled with active flow. Actually this velocity is greater than the theoretical value, and reverse flow along the pressure side of the blade does not take place even at small discharges.

5 In addition to (4), it was definitely proved that the flow at very small discharges is no longer stable.

6 The entrance flow may be assumed to be radial for all practical operating conditions.

7 The increase in the meridional velocity over the ordinary computed value alters the shape of the exit triangle considerably, which is of special importance in designing the guide-vane entrance angle.

Discussion

R. W. Angus.[10] The paper describes an ingenious method of research. The method of studying the problem of pump flow is most helpful, for the experimenters have been able to make any experiments desired and then photograph the results by a relatively stationary camera just at the most desirable instant.

The experiments show that the impeller is rarely filled with water, and that as the discharge varies, the water area decreases instead of the relative velocity. There is also included the statement that the angle of flow of the water at exit is different from the blade angle, which has frequently been assumed in pump theory. The problem still left to be solved is how to find the exact depth of water in the impeller beforehand for given head and discharge. Is there any way of predicting what will happen before the pump is built?

The authors' experiments were made at several speeds, and taking those shown at Fig. 29, the pump would appear to have a rather unusual shape of head-discharge curve, and the maximum head was always low, the probable limitation in this respect being the strength of the glass-sided pump. Is it possible that in a larger pump, or even in a pump of this same size operating at higher speeds and pressures, the flow conditions would seriously differ from those described?

Some years ago the writer carried out a number of tests on a pump with radial guide vanes at entry and was unable to find that these had much effect, which agrees with the authors' conclusion that entry is radial under all ordinary operating conditions. There is, however, evidence that the water in the suction pipes of some pumps whirls, and this must be true at the impeller entry in these same pumps.

Carroll F. Merriam.[11] The authors' conclusions point distinctly to the fact that water does not behave as assumed, and we may conclude that pump designers should now be encouraged to make use of the facts as demonstrated. The question may be asked whether it would be advantageous to fill in the dead spaces with metal and thus break up the parasitic circulation of flow which tends to result in loss of efficiency.

It is unfortunate that the authors were limited to comparatively low speeds, since certain parts of the apparatus had necessarily to be made of glass, which would not stand the high speeds of a commercial pump of the size used. It is suggested that it might be possible to use such a commercial pump with bull's-eye windows which might be viewed through the rotoscope supplemented by some form of stroboscope. Such an investigation might throw light as to whether the conditions in a high-speed pump are correctly similated in the low-speed laboratory model used by the authors. It would certainly be desirable to know whether the dead spaces shown to exist in the laboratory are still present in pumps of commercial design.

Authors' Closure

The views advanced in the discussion must be appreciated as valuable contributions to the problem. No claim is made by the authors that the investigations described in the paper present a perfect and final knowledge of the flow in centrifugal pumps, which has been shown to be very complex.

As Professor Angus has pointed out, a problem still to be solved is that of finding the exact depth of the actual flow through the impeller, and of predicting, before the pump has been constructed, what is likely to occur. However, the authors hope that their experiments may be of service to designers in so far as they give a hint as to the changes that can be made in the shape of the runners to improve the efficiency of centrifugal pumps.

The change suggested by Mr. Merriam is one which deserves close consideration. The actual effect of such a change will of course have to be determined by tests in which the power input can be measured. The relatively large influence of bearing friction rendered this measurement impossible with the experimental pump.

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[9] Tokusaka Omori, "Experimental Researches on Centrifugal Pumps."

[10] Professor of Mechanical Engineering, University of Toronto, Toronto, Ont., Canada. Mem. A.S.M.E.

[11] Pennsylvania Water and Power Company, Baltimore, Md. Mem. A.S.M.E.

The question as to whether the flow, in the case of a larger pump or one of the same size operating at higher speeds, or, again, of a commercial pump, would not seriously differ from that described, certainly deserves investigation by tests on a suitable pump, which could be designed in accordance with the suggestions made by Mr. Merriam. In the opinion of the authors it is probable that the flow, in these pumps also, has the peculiarities revealed by the small experimental pump, but of course only up to the limit of cavitation, which, as is well known, seriously affects the flow.

The interesting fact that, as observed by Professor Angus, the water in the suction pipes of certain pumps is liable to whirl, may be explained by the fact that the reverse flow, which takes place in some of the water passages of the impeller, mixes with the incoming stream in the suction pipe. This effect has been observed in the experimental pump and has been described on page 5 of the paper.—D. THOMA.

Lignum-Vitae Bearings for Roll Necks of Medium-Sized Rolling Mills

By K. W. Atwater,[1] Chicago, Ill.

The performance of metal roll-neck bearings is frequently unsatisfactory, particularly with regard to rapidity of wear and excessive power loss in friction. Roller bearings, although free from these disadvantages, are comparatively expensive, and are limited in applicability because of their space requirements. These circumstances provide a definite field of usefulness for bearings made of very hard wood. Tests of various species of lignum-vitae and of different methods of cutting bearings from the log have indicated specifications under which satisfactory lignum-vitae bearings can be produced. Bearings produced according to these specifications are used successfully in a great many rolling mills and can be lubricated adequately by means of water sprays. With such lubrication, the rate of wear and the power loss in friction are considerably lower than those of metal bearings lubricated with oil or grease. In some rolling mills the use of this type of bearing has effected power saving as high as 25 per cent in comparison with metal bearings. Tests have also shown that their life in certain cases is 15 times as long as that of metal bearings. Experience has indicated that the best results with such bearings can be obtained by polishing the roll necks before installation in the mill.

THE selection of suitable roll-neck bearings for small- and medium-sized rolling mills, such as rod, merchant, skelp, strip, hoop, and sheet-bar mills, has been a rather formidable problem. Difficulties have frequently been encountered in the use of metal roll-neck bearings, for the reason that such bearings are not easily lubricated. Lubricants which lack a natural affinity for metals are washed off the bearing surface by the water which is applied to cool the neck and bearing. The use of lubricants containing fatty acids, which have a natural affinity for metals, or of systems of forced or pressure lubrication, might be expected to lubricate metal bearings adequately, but the constant presence of foreign matter remains as an obstacle to efficient bearing operation. Under these circumstances it has become almost a tradition that the performance of metal roll-neck bearings is marked by rapid wear, short life, and excessive power losses in friction.

In some cases these difficulties can be overcome by the use of anti-friction bearings, which must always be adequately lubricated, and which are always so constructed as to retain the lubricant and to exclude water and foreign matter. Because of their space requirements, however, anti-friction bearings cannot easily be applied to small- and medium-sized rolling mills. Moreover, since the satisfactory performance of anti-friction bearings depends upon careful selection of materials, painstaking care in manufacture, and thorough inspection, such bearings are often rather expensive. These circumstances provided a definite field of usefulness for roll-neck bearings made of very hard wood.

Wooden roll-neck bearings were first introduced in Germany during the World War, as a result of the fact that wood, unlike metals, was not seized in transit as a contraband of war. Experiments with oak bearings indicated that oak was by no means hard enough for this purpose. Lignum-vitae bearings, because of the resin content, great density of grain, and extreme hardness of this wood, were found to be fairly satisfactory.

The successful use of lignum-vitae roll-neck bearings was not immediate, however. It has been found that the numerous species of lignum-vitae, which differ principally in density of grain, resin content, and specific gravity, are not all equally suitable for use in roll-neck bearings. Experience has also shown that the method of cutting bearings from lignum-vitae logs has a considerable effect upon bearing performance. By means of extensive investigation, the species of lignum-vitae having the greatest specific gravity and certain definite methods of cutting bearings from the log have been selected to provide the most satisfactory bearing performance. Lignum-vitae bearings which are constructed according to these specifications are called "millwright" bearings.

A characteristic feature of these bearings is the grain edge on each wearing surface. This feature is desirable in the respect that it provides the maximum resistance to wear and is secured by an oblique method of cutting. The surface which receives end thrust, as well as the principal bearing surface, is provided with edge grain. In some cases the lignum-vitae log is not large enough to permit the construction of a large bearing in a single piece. In such cases, the bearing is made up of small pieces. The separate parts are then doweled together to form the completed bearing.

Although the selected species of lignum-vitae approaches uniformity more nearly than other varieties of wood, it is nevertheless subject to variations in hardness, grain density, and specific gravity. For this reason, experiments are being made with a modified method of construction which is intended to produce bearings of uniform performance in spite of the non-uniformity of the material. The essential feature of this method is the construction of each bearing from several pieces of lignum-vitae, each of which has been cut from a different log, with edge grain running in a different direction. Although this modification is still in the experimental stage, it appears to hold considerable promise of success.

Roll-neck bearings of lignum-vitae are not subject to the disadvantages of metal bearings. They can be lubricated adequately by means of water sprays, which serve simultaneously to cool the bearing and the roll neck. With water lubrication, the rate of wear and the power losses in friction are considerably

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Contributed by the Iron and Steel Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

lower than those of metal bearings lubricated with oil or grease. The operating efficiency, including consideration of initial cost, life, tonnage rolled, cost of maintenance, and continuity of operation, is very much higher than that of metal bearings.

These statements are based upon the results which have been obtained through the use of such bearings in a great many rolling mills. The following specific data may be of interest in this connection.

The average life of metal roll-neck bearings on an 18-in. continuous-strip mill was about 80 hr. running time. Lignum-vitae bearings on the same mill, measured after 672 hr. running time, showed $^3/_8$ in. of wear on the bearing surface. On certain finishing stands of a mill in the Chicago district, where metal bearings had to be replaced every three or four weeks, lignum-vitae bearings served for 13 months without replacement. Power savings as high as 23 per cent, in comparison with metal bearings, are not infrequent. It has been reported that in the hot-strip mill of the Riverdale works of the Acme Steel Company the use of these bearings effected a power saving of 40 per cent in comparison with metal bearings. The operating efficiency, as previously described, exceeds that of metal bearings; frequently in the ratio of 10 to 1, and in one particular installation in the ratio of 17 to 1.

In the finishing stands of a continuous rod mill in the Pittsburgh district, it was observed that rather rapid wear of both the lignum-vitae bearings and the roll necks occurred immediately after installation, and continued until the roll necks had become highly polished, removing about $^1/_8$ in. of the material of the bearing. This condition having been attributed to the initial roughness of the roll-neck surface, the practice was adopted of buffing the roll necks before installation in the mill. The results of this practice were gratifying. The starting torque of the mill was reduced considerably, and the life of the bearings was lengthened by 50 per cent, which was 31 times the previous average life of metal bearings.

Present knowledge seems to indicate that the coefficient of friction and the accompanying power loss in friction approach closely to those of anti-friction bearings. It is certain that in many respects these bearings are more suitable than metal bearings for application to roll necks. It is therefore evident that they merit consideration in the selection of roll-neck bearings for small- and medium-sized rolling mills.

Discussion

C. G. Williams.[2] The author has made out a good account in savings for wood against metal bearings, but the writer would like to ask how he will rationalize his statements with experience derived from the actual use of lignum-vitae bearings in paper mills where an operating savings in power alone of 6 per cent has been shown by a change from lignum-vitae bearings to ball bearings and of 11 per cent by a change to tapered roller bearings, and a life of six years, with an apparent life of double that time. This is a business where, admittedly, machinery and bearings are on the whole a hundred years behind present practice and where pressures per square inch are but normal compared to pressures on rolling-mill machinery.

There is another item in the use of lignum-vitae bearings and that is lubrication or lack of lubrication. (The author states that lubrication is to be a spray of water.) Experiments with these bearings in woodworking machinery have shown that one can expect short life if the commonly used lubricants are put on these bearings. In a heavy-duty bandsaw (re-saw) lignum-vitae bearings lasted but three months, with oil as a lubricant. Babbitt bearings when substituted lasted nine months. Lignum-vitae bearings, again installed and lubricated with a stream of running water, lasted two years. The roller bearings have lasted eight years thus far.

In paper-mill machinery, lignum-vitae has been in the past a standard bearing. In one case where a bearing was renewed after a few months' wear, the firm stated that it used a common lubricating oil, which in this case was the cheapest that could be purchased. With this oil the bearing lasted but $2^1/_2$ months. The bearing was 26 in. long, and the shaft was 9 in. in diameter. The load was approximately 48,000 lb. The bearing had been designed for circulation of water inside the bearing and a flood of water over the entire surface of the bearing. The circulation holes had been plugged up and the bearing oil-lubricated. When refitted with lignum-vitae and the original plans of water lubrication carried out, the bearing lasted 18 months. Another lignum vitae bearing, 24 in. long, on an 8-in. shaft, water-lubricated, had a life of 27 months under a pressure of 37,000 lb., but the wear was about $^9/_{16}$ in. Can this amount of wear be tolerated in a bearing for a roll neck. The writer would consider it doubtful, from experience with this type of machinery. A substitution of roller bearings has lasted eight years, with a saving of 7 per cent in power.

There is another item of cost to the installation and use of lignum-vitae bearings that has been but slightly touched—that is, the cost of raw material and the waste that is encountered in cutting the log. Actual experience has shown that it costs double for material and four times for labor to install lignum-vitae over installing babbitt, and anti-friction bearings having the same load-carrying capacity can be installed in less space, though costing double to install, if length of service life is not considered.

While lignum-vitae bearings gave good service in cane crushers at a Louisiana sugar mill, their life was but two seasons. A rejuvenating of this machinery by using roller bearings gave a saving of 31 per cent in operating cost, and after 10 years there is promise of not less than 10 years' more wear.

There is a lowered cost in installation and in renewal of the lignum-vitae bearing, but what about the actual from day to day operating cost and life as against the life and operating cost of anti-friction bearings?

After years of experience in the use of lignum-vitae bearings in various types of machinery, these statements would seem to be too sanguine to convince operators.

(*Discussion by J. R. Adams and E. S. Glauch will be found following the paper by S. M. Weckstein.*)

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IS-54-2

Water-Lubricated Soft-Rubber Bearings

By W. F. BUSSE[1] and W. H. DENTON,[2] AKRON, OHIO

W. F. Busse

W. H. Denton

Rubber bearings lubricated with water are found to have remarkably low coefficients of friction and to give excellent service under many conditions. The softness of the rubber makes these bearings stand up in the presence of sand and grit, and it also makes the laws of lubrication governing these bearings differ from the classical laws worked out by Reynolds and others for cylindrical metal bearings.

This paper gives a comparison of the frictions of rubber and metal bearings lubricated with water at various loads and speeds. Metal bearings have the lower static coefficient of friction, but the coefficient of running friction is usually lower for rubber bearings. The difference in favor of rubber bearings increases as the load increases, provided the shaft is smooth and the load is applied after the shaft gets up to speed. Rubber bearings have been tested at loads up to 850 lb. per sq. in.

Rubber bearings are particularly suitable for use on shafts running at high speeds not only because the friction is very low but because the softness of the rubber allows the shaft to turn on its center of gyration even though this differs slightly from the geometrical center, thus reducing the dynamic load in the bearing and the vibration in the machine.

INTRODUCTION

Bearings of soft vulcanized rubber have been used for many years in marine service under conditions necessitating the use of water as the lubricant, where sand and grit are often present. The toughness, resiliency, and the low coefficient of running friction of rubber make it a superior material for this service, and it has been found to outwear harder materials such as metals and lignum vitae by as much as ten times their service life.

The softness and resilience of rubber which permit it to be deformed enough to allow sand grains to travel through the bearing without scoring it or the shaft, also have an important effect upon the performance of the bearings when lubricated with water free from grit. The resilient rubber can accommodate itself to slight irregularities present in a shaft, and hence a thinner film of lubricant will serve than would be required by a hard bearing.

A fundamental study of the friction of different types of soft-rubber and of metal bearings under various conditions has shown that the classical laws of bearing lubrication worked out by Towers, Reynolds, etc. for cylindrical metal bearings do not apply to the commercial type of fluted rubber bearings. Because of the difference in the fundamental laws underlying their performance, rubber bearings are superior to metal bearings under many conditions of industrial as well as marine service. This paper compares some of the relations which have been found between the coefficient of friction and the load, speed, shaft roughness, etc. of soft rubber and of metal bearings lubricated by water. It also gives data which should help engineers in using these bearings to greatest advantage. There seems to be no upper limit to the speeds at which water-lubricated rubber bearings can be used if sufficient lubrication is provided. Rubber bearings have yielded good service at shaft speeds of 12,000 r.p.m. or 4330 ft. per min. The upper limit of loads these bearings can carry depends very much on the operating conditions, but bearings have been run when loaded as high as 850 lb. per sq. in.

BEARING THEORY

A simple application of the theory of dimensions[3] shows that if the friction of a cylindrical bearing is a function of v, the bearing speed in r.p.m., the viscosity of the lubricant η, the unit load P, and certain dimensionless ratios such as diameter to length, clearance to length, etc. then the coefficient of friction f should be a function of the combination $(\eta v/P)$, or, for geometrically similar bearings, $f = \phi(\eta v/P)$, a form to which the more detailed equations of Reynolds, Harrison, etc. can be reduced.

The assumptions made in these derivations are not generally true, though they should not be very far wrong for smooth metal bearings of small clearance in the range of viscous-film lubrication. A large number of experiments have been made to see how accurately the formula fits observed facts, but in relatively few cases have the conditions necessary for a rigorous

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[2] Junior Physicist, B. F. Goodrich Company. Mr. Denton was born in Toledo, Ohio, in 1906. He received the degree of Bachelor of Engineering Physics from Ohio State University in 1929, since which year he has been on the staff of the Physical Research Laboratory of the B. F. Goodrich Company.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] M. D. Hersey, Jr., *Jl. Wash. Acad. Sci.*, vol. 4 (1914), p. 542; Trans. A.S.M.E., vol. 137 (1915).

test been satisfied. No journal or bearing is perfectly rigid, so it will vibrate at high speeds and be deflected by high loads, and changes in these factors will also vary the temperature and rate of shear of the lubricating film, thus changing the viscosity. Since it is almost impossible to measure the temperature of the lubricant in the film, the actual viscosity may be quite different from the value taken for the calculations.

From our physical picture of the action of a bearing, and also from Harrison's[4] mathematical analysis, we should expect that in the range where the eccentricity of the bearing is small, the coefficient of friction will be a linear function of the factor $(\eta v/P)$. Hersey[3] found experimentally that the function ϕ was of the form

$$f = \phi\left(\frac{\eta v}{P}\right) = a + b\left(\frac{\eta v}{P}\right)$$

where a and b were essentially constant over a considerable range of values of $(\eta v/P)$. In some cases it has been possible to express the results by a function of the form

$$f = K\left(\frac{\eta v}{P}\right)^b$$

where K is a constant and b is a constant somewhat less than unity. Illmer's summary[5] of bearing-friction research showed that very often the results were given by an equation of the type

$$f = K\frac{\eta^r v^m}{P^n}$$

where K is a bearing constant, and the exponents r, m, and n are determined by bearing conditions. The values of the exponents usually ranged from $^1/_3$ to $^3/_4$. Such variables as the character of oil, rate of oil feed, and journal clearance seemed to have a considerable effect on the values of the exponents of the equation. The difficulties mentioned above probably account for the fact that the values of r, m, and n are not identical as demanded by the theory.

It should be noted that actual friction measurements on bearings as well as the mathematical theory of Reynolds,[6] Harrison,[4] etc. all show that over a limited range the coefficient of friction decreases as the viscosity of the lubricant decreases. However, since these results apply only under conditions of perfect film lubrication, the viscosity cannot be reduced indefinitely to produce lower and lower frictions in the bearing. Under a given set of bearing conditions, as the viscosity of the lubricant is reduced the lubricating film will finally become thinner than the height of irregularities in the metal surfaces and the moving metal parts will come in contact, thus increasing the friction and rate of wear of the bearing.

Because rubber is flexible, it can accommodate itself to slight irregularities of a shaft. Consequently a thinner film of lubricant will suffice to keep a rubber and a metal surface separated than would be necessary to keep two similarly rough metal surfaces apart. This probably accounts for the fact that a low-viscosity liquid such as water is a suitable lubricant for rubber bearings run at moderate speeds under moderate loads, while water can be used satisfactorily with metal bearings only at high speeds with light loads.

Another factor may also help to account for the satisfactory service of rubber bearings. A detailed study of the hydrodynamics of bearing lubrication by Sommerfeld,[7] Mitchell,[8] etc. led to the introduction of the tilting bearing pad found in the present Mitchell and Kingsbury thrust bearings. By allowing the wedging angle to change with the speed and the load these bearings operate at a much higher efficiency, and can carry much greater pressures than the conventional thrust bearings. Fluted rubber bearings allow the shaft to rest on a series of longitudinal strips of rubber running the length of the bearing (see Fig. 1). These strips can be deformed slightly, and so the angle of approach between the bearing surface and the shaft will change as the load is changed, producing the same effect as the pivoted bearing pads do in a modern thrust bearing. This probably accounts for the fact that over a considerable range of pressures the coefficient of friction of a rubber bearing is unaffected by changes in load. The grooves also allow sufficient cooling water to flow through the bearing, to wash out sand and grit that might otherwise get into it. Smooth cylindrical rubber bearings do not give a coefficient of friction which is independent of load, though the change of friction with load is less than for cylindrical metal bearings.

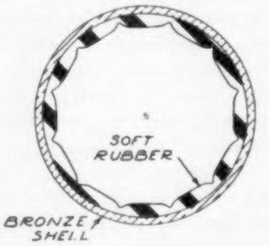

Fig. 1 Cross-Section of a Fluted Rubber Bearing

It should be understood that the statements about rubber bearings in this paper apply only to bearings of soft rubber. Hard-rubber bearings, even those containing graphite, give frictions about the same as are found with metal bearings.

EXPERIMENTS

Effect of Shaft Speed and Roughness

The static friction of soft rubber against dry metal varies from about 0.3 to 0.8, depending on the smoothness and cleanliness of the surfaces. The friction is even greater when the surfaces are wet. The static friction of a rubber bearing should therefore be high, but it should decrease with speed as a lubricating film is built up between the shaft and the bearing. The rate of decrease should depend very much on the smoothness of the metal surface. Preliminary experiments were carried out to show how the friction of wet rubber against metal depended on the speed and smoothness of the shaft.

By means of a lever acting through a roller bearing, a flat strip of rubber 3 in. wide and $^3/_{32}$ in. thick fastened to a metal backing was pressed against a cast-iron pulley 6 in. in diameter and having a 3-in. face, and the tangential force exerted on the rubber was measured for various pulley speeds and conditions of the pulley surface. Lubrication was obtained by allowing the bottom of the pulley to dip in a pan of water. To study the effect of surface roughness the face of the pulley was first knurled to a depth of 0.009 in. to give a surface which would make viscous-film lubrication almost impossible. After this surface was tested, the knurling was removed and right- and left-hand threads 0.002 in. deep and 48 to the inch were cut over the face of the pulley, to give a surface simulating a badly scored shaft. When tests of this surface were completed the face of the pulley was cut as smooth as possible in a lathe, and retested. Then the wheel was polished with crocus cloth to a smoother finish, and was finally given a light coating of wax and repolished to a glossy finish, the tests being repeated on each surface.

The effects of speed and surface roughness on the coefficient of friction are shown in Fig. 2. The friction of the rubber on the knurled surface remains at the high value of the static friction up to a surface speed of 1400 ft. per min. The friction on a badly scored surface decreases somewhat with speed, but remains much too high for a good bearing. Only the smoothest surfaces give a reasonably low coefficient of friction down to peripheral speeds of 100 to 200 ft. per min. It may be mentioned that roughness of the surface of the rubber is nearly, but not quite, as influential as roughness of the metal surface for increasing the friction.

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[4] Glazebrook, "Dictionary of Applied Physics," London, 1922, vol. 1, p. 376.

[5] *S.A.E. Journal*, vol. 26, p. 67, 1930.

[6] Phil. Trans., Part I, p. 194, 1886.

[7] *Zeit. f. Math. u. Phys.*, vol. 5 (1904), p. 50.

[8] *Ibid.*, 1905.

In other tests the width and thickness of the rubber were varied; but, with a reasonably smooth pulley surface, these changes had relatively little effect on the friction over the range from 0.5 in. to 3 in. in width and $^3/_{16}$ in. to $^3/_8$ in. in thickness. Tests were run at several loads, but, except at the lower speeds, the load had only a small effect on the coefficient of friction over the range of pressures from 5 to 260 lb. per square inch of contact area. Raising the temperature of the water increased the friction, probably because of the lower viscosity of the water at the higher temperatures.

Comparison of Water-Lubricated Rubber and Metal Bearings

This work was done primarily to ascertain how the friction of rubber and metal bearings compared under conditions commonly encountered in marine service. However, water has some distinct advantage as a lubricant in studying bearing theory, for it gives a convenient way of investigating the effects of the factor $\eta v/P$ over the lower ranges which are usually ignored in fundamental studies of bearing lubrication. The results of the investigations showed that in the region of low values of $\eta v/P$, where most water-lubricated bearings operate, the fundamental laws governing the relation between friction, load, and speed are very different for soft-rubber and for metal bearings, and indicated that rubber bearings might find uses in industry under conditions where water-lubricated metal bearings could not be employed.

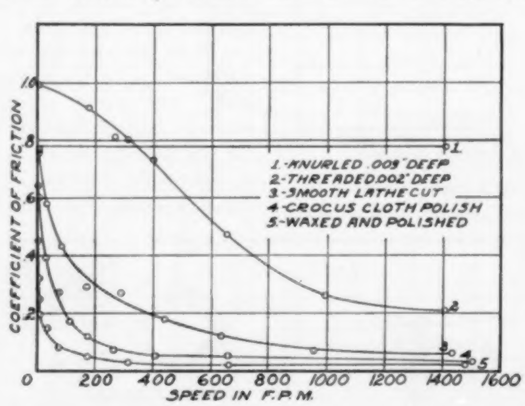
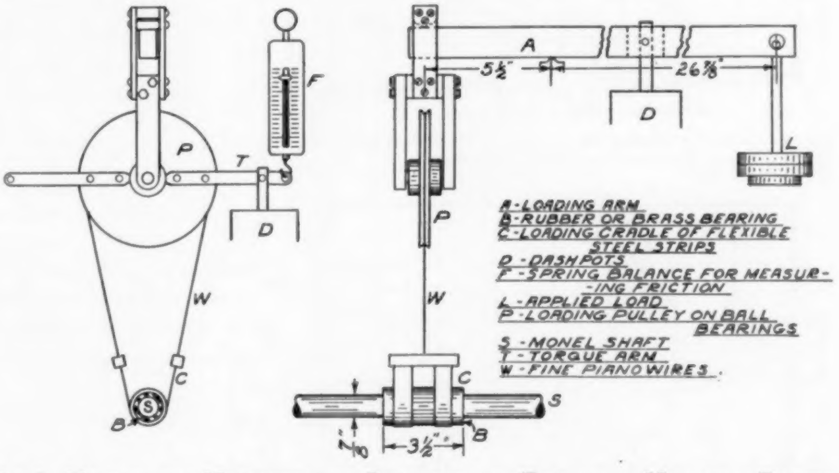


Fig. 2 Effect of Speed and Roughness of Metal Surface on the Coefficient of Friction Against Wet Rubber

The difference in the fundamental laws governing metal and fluted soft-rubber bearings is shown very strikingly by the friction-load-speed curves obtained with $^7/_8 \times 3^1/_2$-in. rubber and brass bearings lubricated with water. The apparatus for testing these bearings is shown in Fig. 3. The bearings B were run on a monel metal shaft S submerged in water. Above the rubber bearing was a 6-in. pulley P mounted on ball bearings, around which was a flexible steel wire W for transmitting the load to the bearing from the loading arm A. The load was applied directly to the bearing by a cradle C of two thin flexible steel straps. The connection between the wire and the cradle was made flexible so that the load would distribute itself equally between the two straps. The torque set up by the bearing was transmitted through the cradle and wire to the upper pulley, and the friction was measured by using a spring balance at the end of the torque arm on the upper pulley. The effect of vibration was reduced by dashpots D on the loading and torque arms.


F
P
T
D
W
C
B
S
A
5½"
26⅞"
D
L
P
W
C
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B
3½"
A-LOADING ARM
B-RUBBER OR BRASS BEARING
C-LOADING CRADLE OF FLEXIBLE STEEL STRIPS
D-DASHPOTS
F-SPRING BALANCE FOR MEASURING FRICTION
L-APPLIED LOAD
P-LOADING PULLEY ON BALL BEARINGS
S-MONEL SHAFT
T-TORQUE ARM
W-FINE PIANO WIRES.

Fig. 3 Machine for Measuring the Coefficient of Friction of Metal and Rubber Bearings

For the lowest friction values it was found essential to use a very flexible wire to transmit the load to the cradle. Several strands of fine piano wire twisted together were found to be most satisfactory, though for frictions below 0.002 the stiffness of even this wire had an appreciable effect on the readings. The sensitivity of the apparatus was increased in the lower friction range by snubbing a cord around the bearing and measuring the friction force directly instead of using the torque arm. Tests showed that, in most cases, the additional force applied to the bearing by the friction-measuring device was negligible compared with the total load; but where necessary, the proper corrections were made. The difficulties of finding a flexible material strong enough to carry large loads set the upper limits of loading and the lower limits of the frictions that can be tested by this method.

The change of friction with load for a water-lubricated brass bearing is shown in Fig. 4, and part of the data are shown on an enlarged scale in Fig. 5. The curves are what one would expect from the theory, the coefficient of friction at a given speed decreasing with load in the range of viscous lubrication, reaching a minimum, and then increasing asymptotically to the value for the static friction at high loads, where the metal surfaces are no longer separated by a lubricating film. Fig. 5 shows that the maximum loads which this brass bearing will carry under conditions of viscous-film lubrication are about 0.4, 1.0, 3.5, and 7.0 lb. per sq. in. for speeds of 66, 183, 366, and 733 ft. per min., respectively. These low loads explain why water is used as a lubricant for metal bearings only because of necessity.

If the torque is plotted against load, a rather interesting series of curves is obtained, as shown in Fig. 6. Although the points are rather scattered, it appears that at high loads—i.e., when there is metallic contact—the frictional force F in the bearing seems to follow the equation

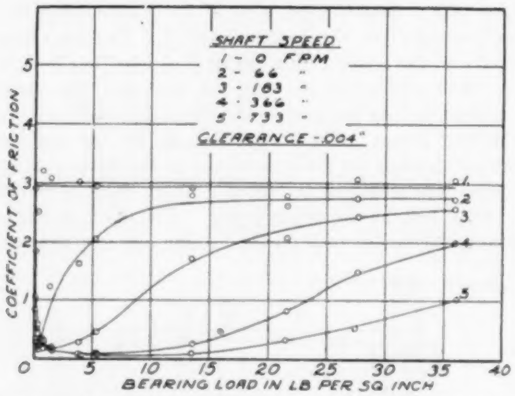

Fig. 4 Effect of Load on Coefficient of Friction of a Cylindrical Brass Bearing

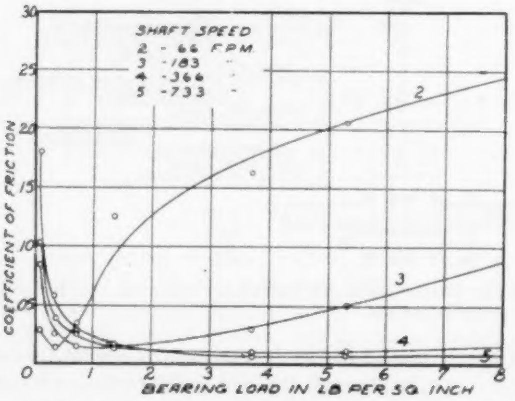

Fig. 5 Effect of Load on Coefficient of Friction of a Cylindrical Brass Bearing for Light Loads

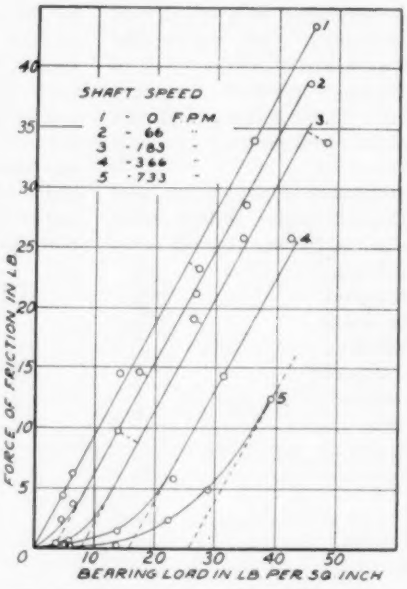

Fig. 6 Friction of a Brass Bearing

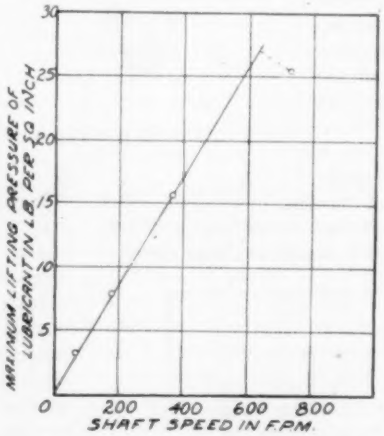

Fig. 7 Change of Lifting Pressure in Lubricant With Speed

$$F = C\,(L - x)$$

where L is the load applied, C is the static coefficient of friction, and x is the intercept of the extended straight portion of the curve on the load axis. x may be interpreted as being the supporting force exerted by the lubricant after there is full metallic contact in the bearing. The value of x in this test proves to be proportional to the speed of the shaft as shown in Fig. 7. The equation may therefore be written

$$F = C\,(L - kv)$$

where v is the speed of the shaft and k is a constant determined by the viscosity, clearance, etc. This equation applies of course only when the loads are great enough to make the film thickness small compared with the irregularities in the bearing surfaces. It is probably also necessary that the bearing projections be of approximately a uniform height, for when a badly scored bearing was tested, the friction-load curves did not become straight lines until very much higher loads were reached. The intermediate region of mixed viscous and solid friction was much larger under these conditions. Further tests on bearings of different sizes and clearances are required to determine if this law is generally valid. A bronze bearing (88-10-2) was tested in the same way, and gave about the same results as the brass bearing, except that it did not wear away so rapidly.

When a $^7/_8 \times 3^1/_2$-in. fluted rubber bearing was tested on the same shaft under the same conditions, very different results were obtained. To make sure that the conditions were the same in testing the two bearings, the rubber and brass bearings were tested alternately at each speed, and each bearing was tested at the same place on the shaft. Changes in shaft roughness or other experimental conditions, therefore, could not account for the differences which were found. The results are plotted in Fig. 8.

It is seen that when the rubber bearing is run at a given speed the coefficient of friction is almost independent of the load, over the range from 2 to 45 lb. per sq. in. bearing pressure. This implies that the frictional force is directly proportional to the load. This relationship does not resemble that of a plane bearing[9] where we should expect the coefficient of friction to be proportional to $\sqrt{1/P}$. Instead it more nearly resembles the change of static friction with load, though the running frictions are much lower than the static friction. It is probable that these results are due to the change with the load in the angle between the rubber and the shaft at each flute. This was confirmed by testing a smooth cylindrical rubber bearing without flutes, having a clearance of 0.008 in. and a $^1/_8$-in. rubber wall. The curves of coefficient of friction against load for this bearing, shown in Fig. 9, are similar to those of the metal bearing, but the fric-

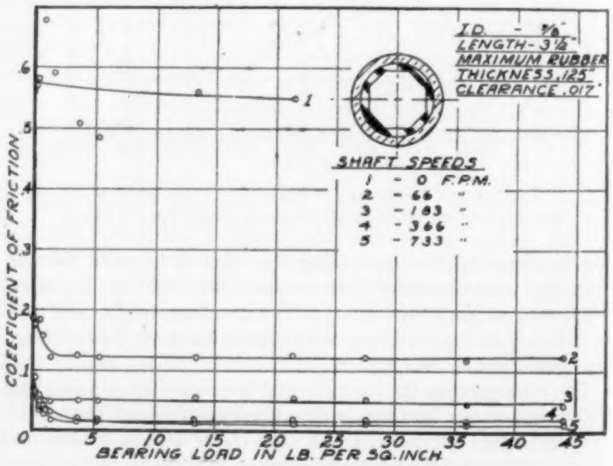

Fig. 8 Effect of Load on Coefficient of Friction of a Fluted Rubber Bearing

tions are lower. They are not far from the friction of the fluted rubber bearing. The coefficients of friction of the rubber bearings at the higher speeds ranged from about 0.01 to 0.005, which is the order of friction found in good ball bearings.

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[9] Archbutt and Deeley, "Lubrication and Lubricants," London, 1927, p. 100.

Even in the cylindrical rubber bearing the rubber is flexible enough to allow some deformation and change in the angle of approach as the load changes. Hence the friction of the cylindrical rubber bearing does not increase as rapidly with load as it does in a metal bearing, in spite of the fact that a rubber bearing has a higher static friction than a metal bearing.

A comparison of Figs. 4 and 8 shows that at very slow speeds and light loads the metal bearing has a lower friction than the fluted rubber bearing, but at higher speeds, and especially at higher loads, the friction of a rubber bearing is much less than that of a metal bearing. Figs. 10 and 11 give a comparison of the coefficient of friction and the friction loss in the rubber and the metal bearing when run at a speed of 800 r.p.m. or a peripheral speed of 183 ft. per min.

The upper limit of the speed at which rubber bearings may be used has not yet been determined. However, the fact that Kay Don used a rubber bearing on *Miss England II* when he broke the world's record showed that the limit for this type of bearing is above a shaft speed of 12,000 r.p.m. or a peripheral speed of 4330 ft. per min. At these higher speeds the rubber bearings are particularly effective for they allow a shaft to turn on its center of gyration even if this does not correspond exactly to the geometrical center, thus reducing the dynamic stresses in the bearing and the vibration in the whole structure.

Tests of Rubber Bearings at High Loads

The use of rubber for severe service, as in the roll necks of steel mills, demands a knowledge of how these bearings will behave under high loads. The apparatus used for the tests already described could not be used to secure complete friction-load-speed curves of large bearings at high loads, but tests previously made by Mr. Annis of the B. F. Goodrich Company's engineering department showed that under certain conditions rubber bearings may be used at loads up to 850 lb. per sq. in.

Bearings 2 in. I.D. × 5 in. long were mounted on a shaft and loaded by a lever arm. The bearing was surrounded by a tank of $3^1/_2$ gal. capacity and water flowed through this tank at the rate of 1 gal. per min. It was found that friction at the packing glands through which the shaft passed caused negligible heating. The rise in temperature of the water on flowing through the tank therefore gave a measure of the energy consumed in the bearing, and this was used to calculate the coefficient of friction. This method leaves much to be desired from the standpoint of precision, but at least it gives the order of magnitude of the frictions at these loads.

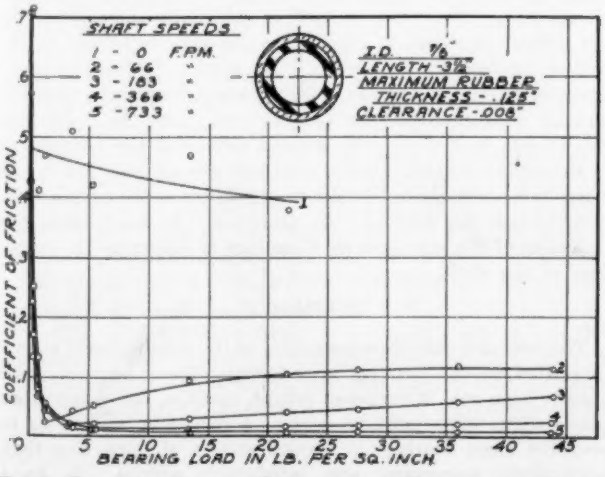

Fig. 9 Effect of Load on Coefficient of Friction of a Cylindrical Rubber Bearing

In making the tests a load was applied to the bearing and it was run for 20 min. to reach equilibrium conditions, after which the temperature of the water leaving the tank was taken. The load was then increased 50 lb. per sq. in., and the process repeated.

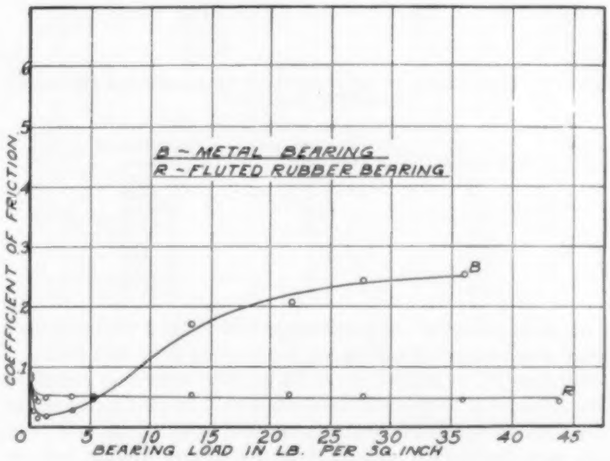

Fig. 10 Comparison of Coefficient of Friction of Water-Lubricated Metal and Fluted Rubber Bearings

(Speed. 800 r.p.m. or 183 ft. per min.; bearing size, $^7/_8$ in. × $3^1/_2$ in.)

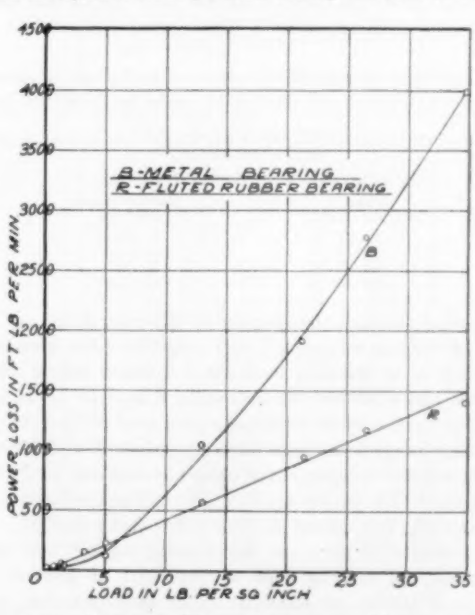

Fig. 11 Power Losses in Water-Lubricated Metal and Fluted Rubber Bearings

(Speed, 800 r.p.m. or 183 ft. per min.; bearing size, $^7/_8$ in. × $3^1/_2$ in.)

A typical test on a 2.025-in. I.D. × 3-in. O.D. × 5-in. long spiral-grooved rubber bearing is shown in Table 1. After this test the bearing was worn only slightly, with no evidence of scoring. The shaft was very smooth as it had been coated with a graphite soap before the test. The maximum temperature of the water was 42 deg. cent., a rise in temperature of 9 deg. The bearing temperature went up to 47 deg. cent. A plain cylindrical bearing of $2^1/_{16}$ in. I.D. was run on the 2-in. shaft

TABLE 1 COEFFICIENTS OF FRICTION OF A 2.025 × 3 × 5-IN. SPIRAL RUBBER BEARING

(Shaft speed, 500 r.p.m. or 262 ft. per min. Clearance, 0.025 in.)

Load, lb. per sq. in.	Coefficient of friction
150	0.038
300	0.026
500	0.018
600	0.015
700	0.014
800	0.012
850	0.012

TABLE 2 COEFFICIENT OF FRICTION OF A 2.062 × 5-IN. PLAIN RUBBER BEARING

(Speed, 700 r.p.m., or 366 ft. per min.)

Load, lb. per sq. in.	Temperature rise, deg. fahr.	Coefficient of friction
55	1.8	0.0580
135	1.8	0.0236
184	2.7	0.0260
334	4.5	0.0240
384	4.5	0.0208
484	4.5	0.0164
590	5.4	0.0164
640	5.4	0.0148

in the same way, and the results shown in Table 2 were obtained. There was no sign of scoring on the bearing after this test.

Bearings up to 6 in. I.D. × 24 in. long were also tested by Mr. Annis, but they could be loaded only to 70 lb. per sq. in. The shaft on which the bearings ran was mounted in ball bearings, and the energy consumed in the rubber bearing was determined from the power used by the driving motor. Again this is not an entirely satisfactory experimental arrangement, but it should give the upper limits of the friction, and the data are consistent with the results which were obtained by other methods.

A typical test on a 5.933 × 24-in. fluted bearing similar to that shown in Fig. 1, but having 10 faces, is shown in Table 3.

TABLE 3 COEFFICIENTS OF FRICTION OF 10-FACE 5.933 × 24-IN. BEARING

(Speed, 420 r.p.m., or 660 ft. per min.)

Load, lb. per sq. in.	Coefficient of friction
34.77	0.0253
41.74	0.0236
56.61	0.0197
69.50	0.0158

Water flowed through the bearing at the rate of 0.3 cu. ft. per min. The bearing was coated with graphite when installed and it was broken in without load for 1.5 hours before the test. In another test a 20-face fluted bearing 6 in. I.D. × 24 in. long was run under the same conditions at a load of 72.5 lb. per sq. in. and was found to have a friction coefficient of 0.00844.

A 12-face fluted bearing of the same size was run for 70.5 hours with a load of 72.5 lb. per sq. in. The initial coefficient of friction was 0.0202, but during the test it dropped to 0.0037. There was no measurable wear on the bearing during this test. A plain cylindrical bearing had a coefficient of friction which ranged from 0.0021 to 0.00075. This indicates that a plain bearing yields the lowest friction values; but, as a matter of safety, it may be necessary to use flutes in the bearings to allow more cooling water to flow through.

While the foregoing figures are not as precise as might be desired, they at least show the upper limits of the frictions which may be expected from soft-rubber bearings. It must be emphasized that these results can be obtained only when the bearings are used on smooth shafts with the load applied after the shaft reaches its running speed, and with sufficient cooling water supplied to the bearing. As the temperature of the water entering the bearing is raised the amount of water must be increased or the maximum load capacity of the bearing will be markedly decreased. The reason for this is that nearly all the heat generated in the bearing must be removed by the water, and the temperature of the lubricating water in the thin film between the rubber and the shaft must be kept below the boiling point. Much higher loads can of course be carried by rubber bearings if the water is admitted to the bearing surface under pressure.

Deflections in Rubber Bearings

The deflections of a rubber bearing vary greatly with the hardness of the rubber, the design of the bearing, etc. The curves of Fig. 12 show the deflections of thin-wall-type fluted rubber bearings up to 5 in. in diameter, and of a thick-wall-type bearing 19.68 in. in diameter made for marine service. The dimensions of these bearings are given in Table 4.

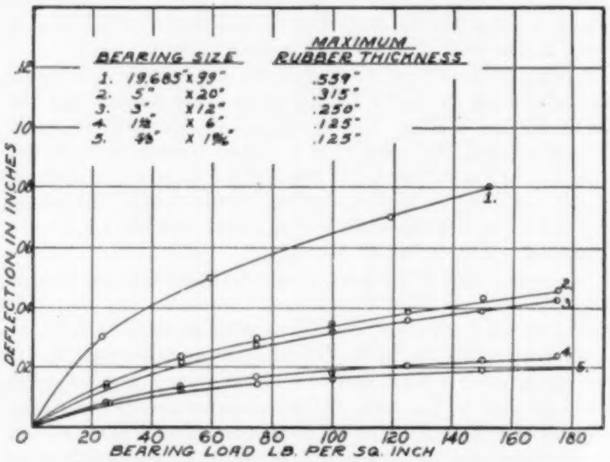

FIG. 12 LOAD-DEFLECTION CURVES FOR MARINE TYPE FLUTED RUBBER BEARINGS

TABLE 4 DIMENSIONS OF BEARINGS USED IN LOAD-DEFLECTION TESTS

Bearing no.	Internal diameter, in.	Outside diameter, in.	Length, in.	Maximum thickness of rubber, in.
1	19.68	21.65	99.0	0.559
2	5.0	6.125	20	0.315
3	3.0	3.75	12	0.250
4	1.5	2.0	6	0.125
5	0.625	1.0	$1^{9}/_{16}$	0.125

In every case but No. 1 the deflection tests were made on complete bearings. The results for this bearing were obtained by measuring the deflection of a single flute and calculating the deflection of the shaft in a complete bearing.

It is of course possible to make rubber bearings which give much smaller deflections than the standard marine bearings shown here. Since rubber itself is essentially incompressible, the deflections shown in Fig. 12 are due entirely to the distortion of the flutes in the bearing. By making the bearing cylindrical instead of fluted the deflections will be greatly reduced, and by using graphited hard-rubber end plates the deflections can be still further reduced without sacrificing the advantages of the rubber bearing. When sand and grit are plentiful in the water it is advisable to increase the thickness of the rubber wall, even though this increases the distortion. In many cases the deflection of the bearing is an advantage in taking up the vibration of the shaft.

SUMMARY

The foregoing data show something of the possibilities of water-lubricated rubber bearings in industry. These bearings have already been used in deep-well pumps, turbines, and many other places where water must be used as a lubricant, as well as in propeller shaft bearings in various classes of ships, and they have given economical and satisfactory service. In sand washers and other places where conditions are equally severe, they have yielded service many times that of metal bearings. Under proper conditions, the cofficient of friction of soft rubber bearings compares favorably with that of the best roller bearings.

Rubber bearings will carry loads of 600–800 lb. per sq. in., provided the shaft is very smooth and that the load is applied only after the shaft gets up to a peripheral speed of about 500 ft. per min. It is also necessary that sufficient cooling water flow through the bearing to keep the temperature of the lubricating film below the boiling point. Still higher loads can be carried if the water is forced into the lubricating film under pressure, or if the shaft revolves at higher speeds.

Soft-rubber bearings are particularly suitable for use on high-speed shafts, not only because a rubber bearing can withstand high loads at these speeds, but because the bearing acts as a cushion which reduces the vibration and decreases the magnitude of the dynamic loads set up by the revolving shaft.

Discussion

R. E. B. SHARP.[10] The statement is made in the paper: "The static friction of soft rubber against dry metal varies from about 0.3 to 0.8, depending on the smoothness and cleanliness of the surfaces. The friction is even greater when the surfaces are wet." From Fig. 2 the coefficient with the metal surface waxed and polished is 0.45. An examination of Figs. 8 and 9 indicates this static coefficient of friction to be 0.57 and 0.48, respectively, for the fluted type of bearing and the cylindrical type. It is assumed that these tests were made with the rubber and metal surfaces in a wet condition.

If it is true that static friction with wet surfaces is greater than with dry surfaces, it is difficult to understand why horizontal-shaft rubber bearings, wherein the shafts are started from rest under load with the surfaces wet, stand up against wear to the extent that they are known to. Actual tests showing the difference in static coefficient of friction for wet and dry surfaces would be of interest.

It is noted that the minimum coefficient of friction from Fig. 2 with the metal surface waxed and polished is about 0.02 from 400 f.p.m. upward, whereas in Fig. 9, which pertains to the cylindrical rubber bearing, the coefficient is about 0.01 for varying loads and for a speed of 733 f.p.m. It would be interesting to know whether the authors have any comments to offer on the reason for this difference in coefficient.

Referring to Fig. 12, the writer has just succeeded in obtaining readings of actual deflection for two different loads in the case of a fluted type of rubber bearing with an internal diameter of 16.5 in. and a length of 25 in. Fig. 13 shows the authors' curves 1, 2, 3, 4, and 5 reproduced to a larger scale, with the points obtained by the writer plotted thereon. For bearing load of 13.3 lb. per sq. in. (5000 lb.) the deflection was 0.005 in., and for a deflection of 26.6 lb. per sq. in. (11,000 lb.) the deflection was 0.0105 in.

Fig. 14 shows the set-up for these tests. The deflections were obtained with micrometers. The maximum rubber thickness was 0.435 in. The bearing contained a total of 24 separate rubber strips vulcanized to metal backs.

It will be noted that these points through which curve 6 passes show less than half the deflection indicated by the authors' curve 1, which is a calculated curve for a 19.68-in. bearing. In view of the fact that the 16.5-in. bearing was designed for use in a vertical hydraulic turbine, which condition requires that the oscillation of the shaft be kept to a minimum, it is gratifying to note that the comparatively heavy load against the bearing produces so small a deflection.

The rubber in this bearing is designated by the Goodrich Rubber Company as their stock No. 8230. The increasing use of rubber bearings for marine, industrial, and hydroelectric application makes it desirable to have a standard method of determining hardness of the rubber used. Such a gage would correspond with the Brinell method of determining the hardness of metal.

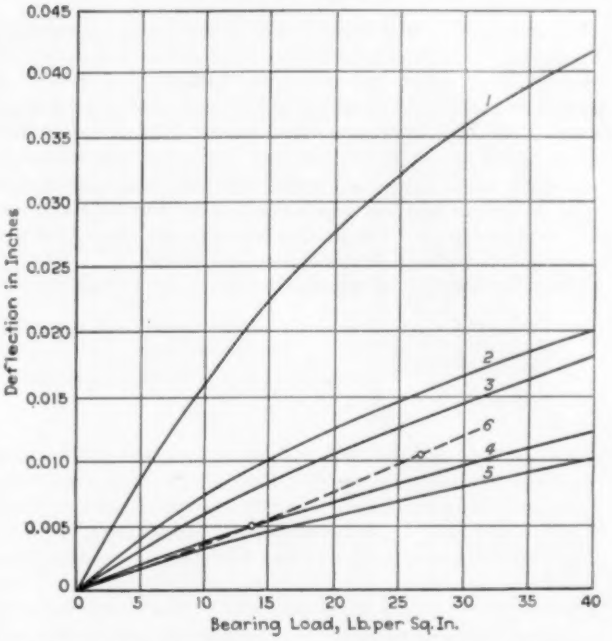

FIG. 13 ACTUAL DEFLECTION FOR TWO DIFFERENT LOADS

(Curves 1, 2, 3, 4, and 5 from authors' Fig. 12; curve 6, results of test of 16.50-in. by 25-in. fluted rubber bearing, maximum rubber thickness 0.435 in., 24 strips per bearing.)

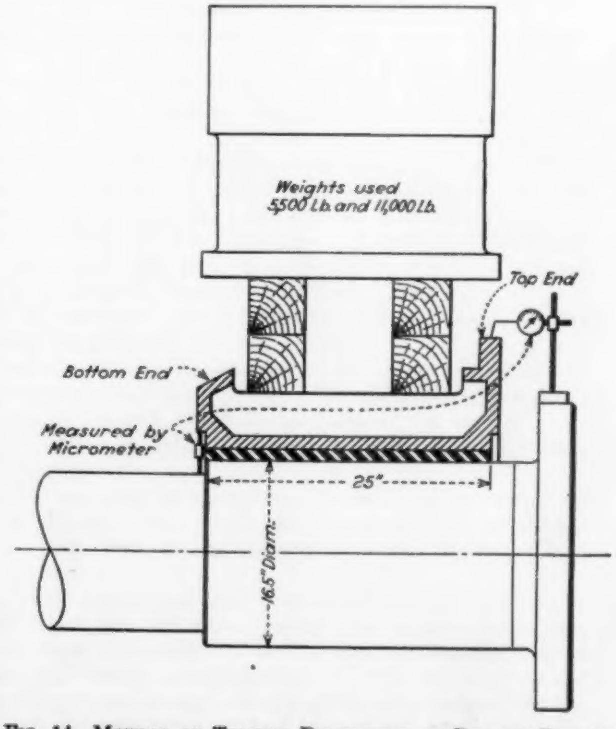

FIG. 14 METHOD OF TESTING DEFLECTION OF RUBBER BEARING

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(*Discussion by J. R. Adams and E. S. Glauch will be found following the paper by S. M. Weckstein.*)

AUTHORS' CLOSURE

It should not be understood, as was done in one discussion, that the static coefficient of friction of wet rubber is always over 0.8. We have found that the static coefficient of friction of dry rubber varies from about 0.3 to 0.8, while that of wet rubber is somewhat higher, ranging from around 0.33 to 0.9 or 1.0, depending on surface conditions, etc. However, this relatively high static friction does not mean that industrial and marine types of rubber bearings which start under full load will not give good service, for the friction immediately drops to lower values as soon as the shaft moves over the surface.

The difference in the minimum frictions shown in Figs. 2 and 9 is probably due to differences in the contact areas and the condition of the surfaces in the two tests, and also, to some extent, to inaccuracies in the machine used to get the data of Fig. 2. This machine was made to show only the relative effect of surface roughness and speed on the friction, and all later data were taken on the more refined machine shown in Fig. 3.

The figures Mr. Sharp presents on the deflection of a 16.5-in. × 25-in. turbine bearing at various loads are a good example of what can be done through proper designing to reduce the shaft deflections where necessary. The deflection data given in Fig. 12 show that even marine bearings which are designed primarily to resist sand abrasion give deflections much smaller than most people would expect. Bearings can be designed for roll necks which would have deflections much lower than those of turbine bearings.

IS-54-3

Straight Copper-Lead Alloys Versus Leaded Solid-Solution Bronzes for Heavy-Duty Bearings

F. R. Hensel

L. M. Tichvinsky

By F. R. HENSEL[1] and L. M. TICHVINSKY,[2] EAST PITTSBURGH, PA.

This paper describes the metallurgical investigation of various bronze bearing metals and deals with a comparative study of the bearing performance of those metals. The technological possibilities of producing heavy-duty bearings were investigated. Experimental installations of bearings with severe requirements are under trial.

Introduction

Bearing bronzes containing a high percentage of lead have been used recently very extensively, and it appears to be the general belief that their main advantage over ordinary babbitt bearings is the increased tensile strength. The strength and hardness of the bronzes result from the fact that they are solid-solution alloys. There are, however, a number of factors related to the actual bearing performance which seem to have been somewhat overlooked. This has led the authors to a study of straight copper-lead alloys which show practically no solid solubility, and therefore only half the strength of bronzes, but which nevertheless show a number of very interesting features regarding their performance as bearings. The lower strength of these alloys was compensated by joining the bearings inseparably to steel shells. The methods of obtaining an inseparable joint were studied in considerable detail.

II Metallurgical Study of Bronze Bearing Metals

(*a*) *General.* The production of lead-bearing copper alloys is attended with considerable difficulty, because lead and copper are practically insoluble in each other, as seen from the copper-lead diagram in Fig. 1. The amount of lead used in these alloys is in the neighborhood of 25 per cent. The melting point of an alloy of that composition is approximately 1000 deg. cent. Crystals of pure copper form along the line *AB*. At a constant temperature of 953 deg. cent., copper crystallizes until the melt has the composition *C*. Along the line *CE* pure copper continues to crystallize until the eutectic point is reached at a temperature about 1 deg. cent. below the melting point of pure lead. The eutectic contains only 0.06 per cent of copper. This description shows that nearly all the copper freezes out, almost pure, before the lead becomes solid. The danger of liquation of lead in such a case is very pronounced. It depends to a large extent upon the velocity of crystallization. By increasing the speed of solidification and therewith the number of copper nuclei, it is possible to trap the lead within the growing copper crystals. The addition of certain alloying elements also influences greatly the crystallization characteristics of a copper-lead matrix.

(*b*) *Casting Characteristics of Straight Copper-Lead Alloys.* The casting experiments that were made to study this question are recorded in Tables 1 and 2. The main variables were casting temperature, chemical composition, and type of casting (sand or chill casting).

The alloys were prepared in gas-fired clay graphite crucibles. Pure scrap copper was melted down and deoxidized with 0.25 per cent P, added in the form of 15 per cent phosphor-copper, before the addition of lead was made. The material was heated to above 1200 deg. cent., then taken out of the furnace and cooled to the desired casting temperature. The casting temperature was measured with a chromel-alumel thermocouple. Before casting the alloy was thoroughly stirred.

For sand castings a pattern was used which permitted two pieces to be cast at the same time. The machined test pieces had a 2-in. gage length and 0.505 in. diameter test section. For chill castings a cast-iron mold $2^{1}/_{2}$ in. in diameter and 18 in. long was employed.

The lowest casting temperature for sand castings was found to be 1050 deg. cent. Below that temperature the metal froze in the gate. Sand castings showed only a small amount of lead on the surface.

All chill castings exhibited a lead coating from $^{1}/_{16}$ to $^{1}/_{8}$ in. thick which extended approximately 4 in. up from the bottom of the ingot. The surface of the chill castings was blowy, and a slight pipe formed during solidification. The use of split molds for chill castings increased the danger of losing lead, as it leaked through the joint when the lead content and casting temperature were high. The series 51 corresponds to an intended composition of 80 per cent Cu and 20 per cent Pb, the series 52 to 75 per cent Cu and 25 per cent Pb, the series 53 to 70 per cent Cu and 30 per cent Pb. (See Table 1.)

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Contributed by the Iron and Steel Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The series 52-x was prepared in order to get a comparison with a solid-solution bronze. In this case the additions were made so that the sum of Pb + Sn + Zn amounted to 25 per cent, the remainder being copper. (See Table 2.)

TABLE 1

Mark	Type of casting	Location of test piece	Casting temperature, deg. cent.	Intended composition, per cent Cu	Intended composition, per cent Pb	Actual composition, per cent Cu	Pb	Sn	Al	Zn	P	Lead retained, per cent
51-1	Sand	..	1100	80	20	87.49	11.68	0.46	0.09	..	0.025	59
51-2	Chill	T[a]	1100	80	20	86.65	12.66	0.16	0.09	..	0.027	63
51-3	Chill	B	1100	80	20	86.75	12.39	0.14	0.12	..	0.048	62
51-4	Chill	T	1050	80	20	85.09	13.78	0.31	0.10	..	0.03	69
51-5	Chill	B	1050	80	20	85.57	13.42	0.31	0.04	..	0.027	67
52-1	Sand	..	1150	75	25	88.52	11.06	0.16	0.05	..	...	44
52-2	Sand	..	1100	75	25	80.35	19.26	0.07	0.05	..	...	77
52-3	Sand	..	1050	75	25	73.86	25.53	0.02	0.05	..	...	100
52-4	Chill	T	1150	75	25	87.53	12.15	0.05	0.03	0.12	...	49
52-5	Chill	B	1150	75	25	87.93	11.86	0.05	0.05	0.10	...	47
52-6	Chill	T	1100	75	25	77.90	21.72	0.02	0.08	0.11	...	87
52-7	Chill	B	1100	75	25	78.46	20.94	0.07	0.13	0.15	...	84
52-8	Chill	T	1050	75	25	71.49	28.06	0.05	0.05	0.13	...	112
52-9	Chill	B	1050	75	25	72.88	26.76	0.04	0.07	0.10	...	104
53-1	Sand	..	1150	70	30	73.24	26.33	0.24	0.10	..	0.064	88
53-2	Sand	..	1100	70	30	73.59	25.95	0.24	0.10	..	0.067	87
53-3	Sand	..	1050	70	30	73.46	26.13	0.24	0.10	..	0.063	87
53-4	Chill	T	1150	70	30	75.87	23.24	0.13	0.07	..	0.087	77
53-5	Chill	B	1150	70	30	75.04	24.40	0.12	0.11	..	0.078	81
53-6	Chill	T	1100	70	30	75.87	23.34	0.12	0.09	..	0.076	78
53-7	Chill	B	1100	70	30	75.61	23.65	0.08	0.08	..	0.084	79

[a] T—top of ingot; B—bottom of ingot.

TABLE 2

Mark	Type of casting	Location of test piece	Casting temperature, deg. cent.	Composition, per cent Cu	Pb	Sn	Al	Zn	P
52-x-1	Sand	..	1150	75.93	17.08	3.65	0.12	3.19	0.075
52-x-2	Sand	..	1100	74.91	17.82	3.66	0.11	3.38	0.067
52-x-3	Sand	..	1050	75.06	17.94	3.62	0.10	3.31	0.074
52-x-4	Chill	T[a]	1150	75.36	17.48	3.43	0.08	3.55	0.088
52-x-5	Chill	B	1150	75.36	17.51	3.40	0.10	3.45	0.086
52-x-6	Chill	T	1100	75.32	17.42	3.39	0.11	3.31	0.086
52-x-7	Chill	B	1100	75.27	17.50	3.52	0.07	3.43	0.078
52-x-8	Chill	T	1050	75.66	17.18	3.50	0.09	3.46	0.093
52-x-9	Chill	B	1050	75.48	17.45	3.30	0.09	3.49	0.091

[a] T—top of ingot; B—bottom of ingot.

The series 51 and 53 of straight copper-lead alloys show a fairly constant amount of lead retained. The retained lead amounts to 60 to 70 per cent of that added in series 51 (20 per cent Pb intended), and to about 85 per cent in the series 53 (30 per cent Pb intended). Both series contain certain amounts of tin and phosphorus, which are taken up into solid solution by copper. The amount of tin averages 0.2 to 0.3 per cent and that of phosphorus ranges between 0.03 and 0.08 per cent. The casting temperature apparently has no great effect on the amount of lead retained if 1150 deg. cent. is not exceeded.

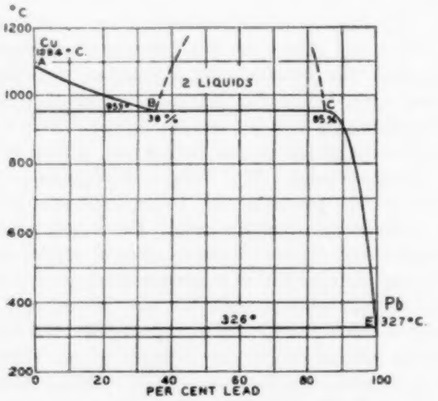

Fig. 1 Copper–Lead System

Conditions are entirely different in the series 52 (25 per cent Pb intended), which represents the purest copper-lead series, as there is only 0.05 Sn present and no phosphorus. Here the amount of lead retained varies greatly with the casting temperature, both in sand and chill castings. Although at a casting temperature of 1150 deg. cent. nearly 50 per cent of lead is lost; practically none is lost when the casting temperature is lowered to 1050 deg. cent. This large difference, however, is due to another cause. The 1150 deg. cent. castings were poured first and the 1050 deg. cent. castings last, from the same crucible. Although in every case the metal was stirred thoroughly before pouring, some segregation must have occurred in the crucible, as otherwise a lead content equivalent to 110 per cent of the intended composition could not have been retained in some of the samples. (See Table 1.) Fig. 2, in which the lead content as a function of the casting temperature is shown, therefore displays the combined effect of two factors—namely, the casting temperature and the crucible segregation on the amount of lead retained in the casting.

In the case of series 52-x, the lead content was constant and independent of the casting temperature or type of casting—i.e., chill or sand casting. Tin and phosphorus apparently keep the lead better distributed within the molten copper.

(*c*) *Tensile Properties.* The physical properties of the chill and sand castings are recorded in Tables 3 and 4. Some of the results are plotted in Fig. 3 and Fig. 4. From each chill casting two test pieces were cut, the location of which can be seen from Fig. 5. The chips for chemical analysis were machined from the tensile specimens after testing.

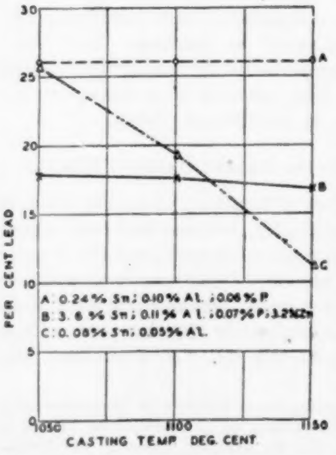

Fig. 2 Effect of Casting Temperature

(On amount of lead retained in Cu-Pb alloys as a function of their composition.)

The physical properties of straight copper-lead alloys are nearly independent of the casting temperature when temperatures from 1050 to 1150 deg. cent. are employed. The lead content seems to have relatively little influence upon the tensile properties such as ultimate strength, reduction of area, and elongation, although there is an unmistakable drop in most of the properties with increasing lead content. The Brinell hardness is a function of the lead content. While an alloy with 13 per cent Pb has a hardness of about 40 Brinell, the hardness of an alloy containing 26 per cent Pb is but 18. There is further a decided difference in Brinell values for sand and chill castings. The values for sand castings are considerably below those of chill castings. This holds true for leaded bronzes (52-x) also.

In the case of the copper-lead bronze (52-x), the highest casting temperature (1150 deg. cent.) yielded the highest values for ultimate strength, elongation, and reduction of area.

A comparison of the tensile properties of straight Cu-Pb alloys and babbitts is given in Table 5.

The tensile properties of the two types of bearings are not

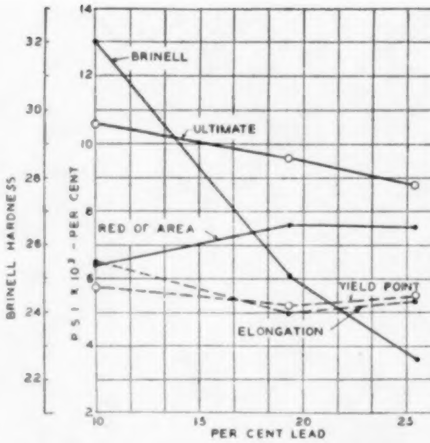

Fig. 3 Effect of Lead on the Physical Properties of Cu-Pb Alloys

TABLE 3

Mark	Type of casting	Casting temperature, deg. cent.	Composition, per cent Cu	Composition, per cent Pb	Yield point per sq. in.	Ultimate strength per sq. in.	Elongation, per cent	Reduction, per cent	Brinell
51-1	Sand	1100	87.49	11.68	6750	11550	4.8	6.1	35.3
51-2	Chill	1100	86.65	12.66	7000	12350	4.5	4.9	38.1
51-3	Chill	1100	86.75	12.39	8000	14350	5.0	5.7	40.2
51-4	Chill	1050	85.09	13.78	6500	12350	5.1	5.3	38.1
51-5	Chill	1050	85.57	13.42	7000	12600	4.6	4.9	35.3
52-1	Sand	1150	88.52	11.06	5500	10300	5.8	6.5	31.2
52-1	Sand	1150	88.52	11.06	6000	11000	7.1	6.1	33.6
52-2	Sand	1100	80.35	19.26	5000	9250	4.7	7.6	24.8
52-2	Sand	1100	80.35	19.26	5250	9950	5.6	7.6	25.4
52-3	Sand	1050	73.86	25.53	5000	8950	5.7	8.8	22.8
52-3	Sand	1050	73.86	25.53	6000	8700	5.4	6.5	22.3
52-4	Chill	1150	87.53	12.15	5250	12250	8.0	8.8	28.6
52-5	Chill	1150	87.93	11.86	5000	11650	6.3	7.2	29.8
52-6	Chill	1100	77.90	21.72	4500	10300	6.5	4.9	24.8
52-7	Chill	1100	78.46	20.94	5000	10400	6.1	7.6	25.9
52-8	Chill	1050	71.49	28.06	4600	8850	4.6	7.6	24.8
52-9	Chill	1050	72.88	26.76	4750	11250	7.5	9.1	25.9
53-1	Sand	1150	73.24	26.33	8500	9650	1.8	5.3	22.8
53-1	Sand	1150	73.24	26.33	5750	8550	3.8	3.8	20.5
53-2	Sand	1100	73.59	25.95	6250	9000	4.5	5.7	19.3
53-3	Sand	1050	73.40	26.13	6250	8400	1.9	7.2	18.9
53-3	Sand	1050	73.40	26.13	6000	8450	3.6	4.9	18.6
53-4	Chill	1150	75.87	23.24	5250	10750	7.2	8.0	27.1
53-5	Chill	1150	75.04	24.40	5250	10250	6.3	6.9	27.1
53-6	Chill	1100	75.87	23.34	5000	7500	2.5	1.4	27.8
53-7	Chill	1100	75.61	23.65	5250	10500	7.3	8.0	28.6

TABLE 4

Mark	Type of casting	Casting temperature, deg. cent.	Composition, per cent Cu	Composition, per cent Pb	Composition, per cent Sn	Composition, per cent Zn	Yield point per sq. in.	Ultimate strength, per sq. in.	Elongation, per cent	Reduction, per cent	Brinell
52-x-1	Sand	1150	75.93	17.08	3.65	3.19	11500	22800	17.0	21.9	43.6
52-x-1	Sand	1150	75.93	17.08	3.65	3.19	11750	22750	16.2	21.5	43.6
52-x-2	Sand	1100	74.91	17.82	3.66	3.38	13250	21200	11.7	11.0	43.6
52-x-2	Sand	1100	74.91	17.82	3.66	3.38	12250	23100	13.7	15.5	42.4
52-x-3	Sand	1050	75.06	17.94	3.62	3.31	12750	22150	11.7	12.5	44.8
52-x-3	Sand	1050	75.06	17.94	3.62	3.31	13200	22000	12.0	14.0	44.8
52-x-4	Chill	1150	75.36	17.84	3.43	3.55	13500	23000	10.0	11.8	50.3
52-x-5	Chill	1150	75.36	17.51	3.40	3.45	13250	25500	14.3	14.4	51.8
52-x-6	Chill	1100	75.32	17.42	3.39	3.31	12500	21250	8.5	10.3	50.3
52-x-7	Chill	1100	75.27	17.50	3.52	3.43	13250	22750	8.8	12.5	51.8
52-x-8	Chill	1050	75.66	17.18	3.50	3.46	12750	19350	5.4	9.5	51.8
52-x-9	Chill	1050	75.48	17.45	3.30	3.49	12500	17900	5.0	6.5	51.8

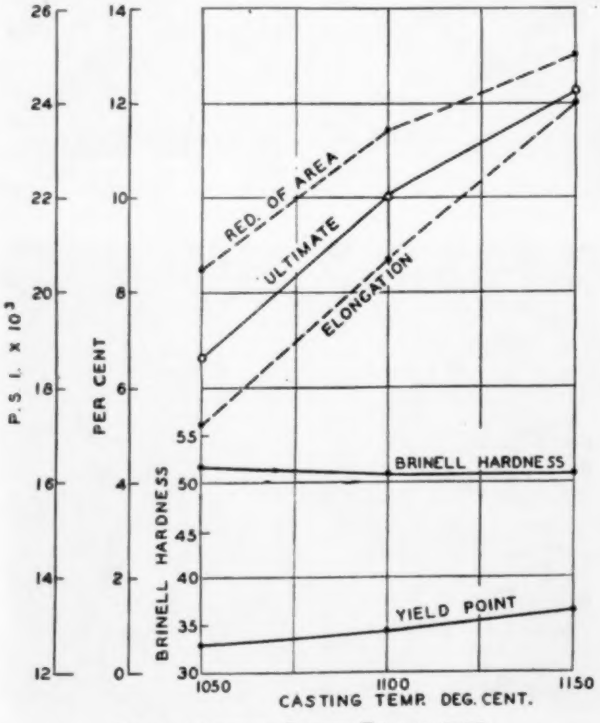

Fig. 4 Effect of Casting Temperature
(On the physical properties of an alloy containing percentages of, approximately, 75 Cu, 17 Pb, 4 Sn, and 4 Zn.)

TABLE 5

Alloy	Composition, per cent	Tensile strength per sq. in.	Elongation, per cent	Reduction tion of area, per cent	Brinell
Babbitt	85 Sn, 10 Pb, 5 Cu	11900	5.5	11.0	27–30
Babbitt	8 Sn, 78 Pb, 14 Sb	10700	8.0	7.8	24–25
Copper-lead	75 Cu, 25 Pb	10500	7.0	8.0	27

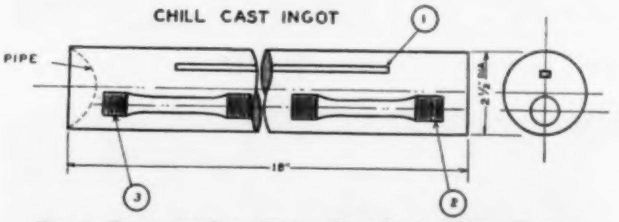

Fig. 5 Positions From Which Test Pieces Were Taken
(1, electrical-conductivity test piece, $^5/_{16}$ in. square by 12 in. long; 2, tensile-test piece from bottom of ingot, 0.505 in. in diameter, 2 in. gage length; 3, same as 2, except taken from top of ingot.)

greatly different. The advantage of the copper-lead alloys, however, becomes evident at elevated working temperatures. While babbitts lose practically all tensile strength above 200 deg. cent. on account of their low melting points (ranging from 225 to 240 deg. cent.) the straight copper-lead alloys still show more than 5000 lb. per sq. in. at temperatures just above 200 deg. cent.[3]

(d) *Electrical Conductivity.* The electrical conductivity of straight copper-lead alloys was determined, as it enables an approximate comparison of the thermal conductivity to be made. High thermal conductivity seems to be one of the main requirements of a good bearing because it assists in obtaining quick dissipation of heat and eliminates concentration of heat on certain spots, thus reducing the likelihood of burnt-out bearings.

The test pieces for conductivity measurements were machined from the chill castings. Their location can be seen from Fig. 5. The test pieces were machined to 12 in. length, $^5/_{16}$ in. square. The drop length was 8 in. The results on series 51, 52, 53, and 52-x are given in Table 6.

The electrical resistivity is a function of the chemical composition. In a mechanical mixture of two constituents, the resistivity of eutectiferous alloys can be calculated by means of the rule of mixtures from the resistances of the two pure metals.

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[3] According to results obtained by Mr. Pike, of the Kalif Corporation.

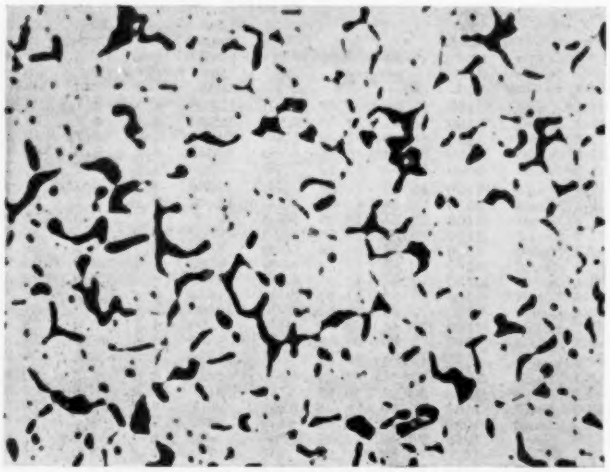

Fig. 6 Sample 51-1, Sand Cast From 1100°C. (11.68% Pb) ×100

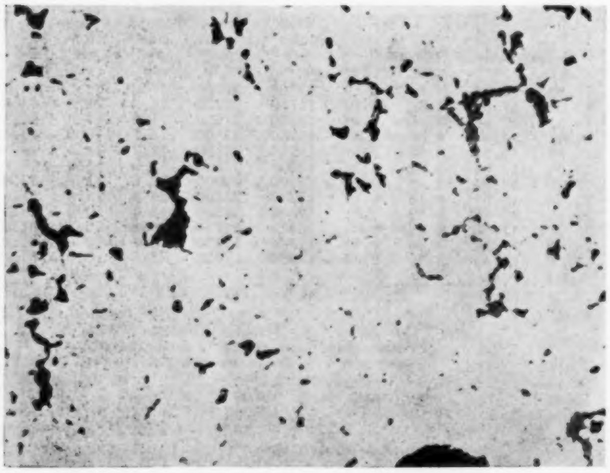

Fig. 7 Sample 51-2, Chill Cast From 1100°C. (12.66% Pb) ×100

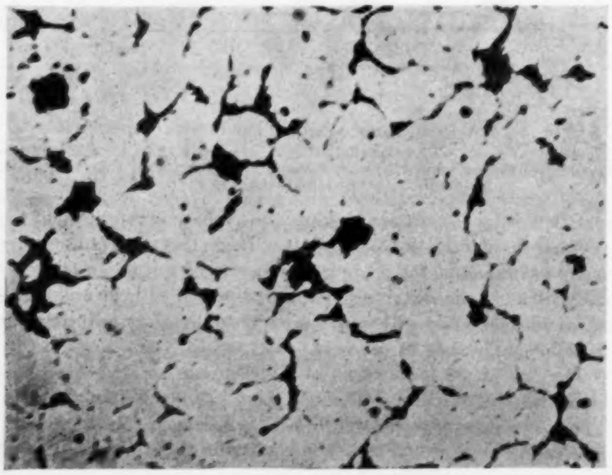

Fig. 8 Sample 52-1, Sand Cast From 1150°C. (11.06% Pb) ×100

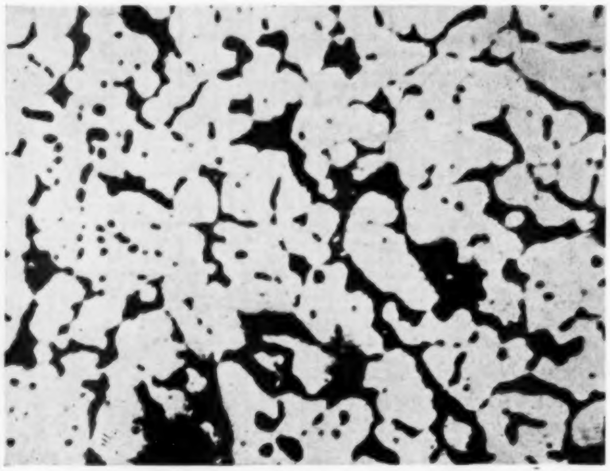

Fig. 9 Sample 52-3, Sand Cast From 1050°C. (25.5% Pb) ×100

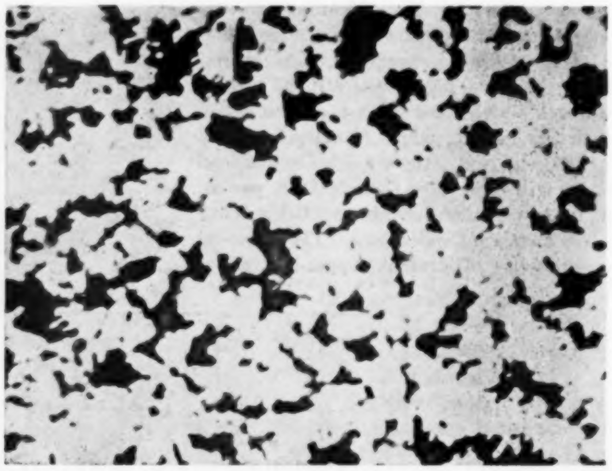

Fig. 10 Sample 52-3, Chill Cast From 1050°C. (28.06% Pb) ×100

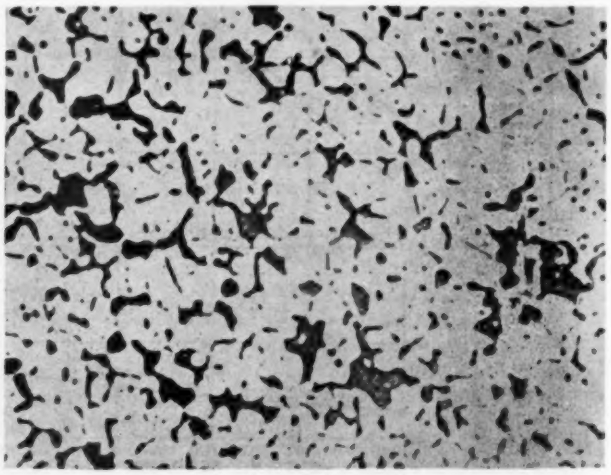

Fig. 11 Sample 53-1, Sand Cast From 115°C., Average Area (26.33% Pb) ×100

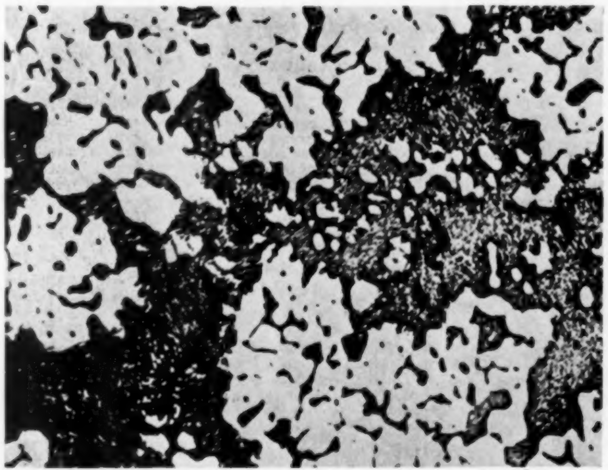

FIG. 12 SAMPLE 53-1, SAND CAST FROM 1150°C., SEGREGATED AREA ×100

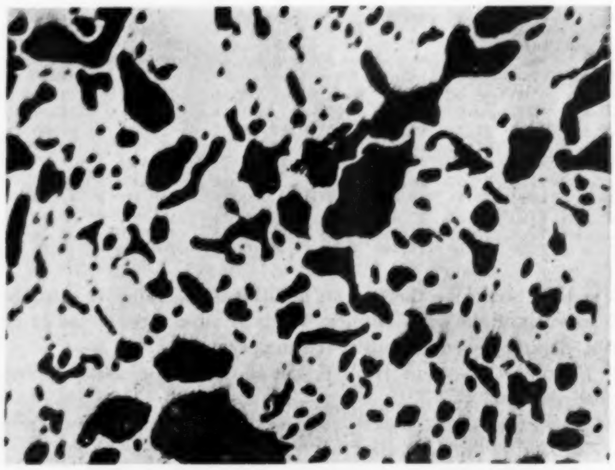

FIG. 15 STRUCTURE OF DW ALLOY (BRONZE) ×100

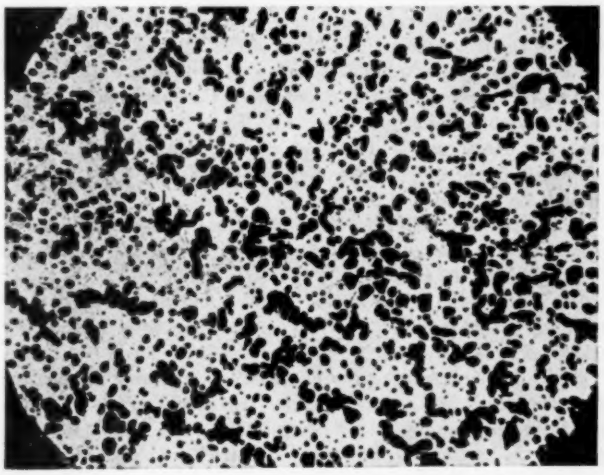

FIG. 13 STRUCTURE OF WE ALLOY (BRONZE) ×100

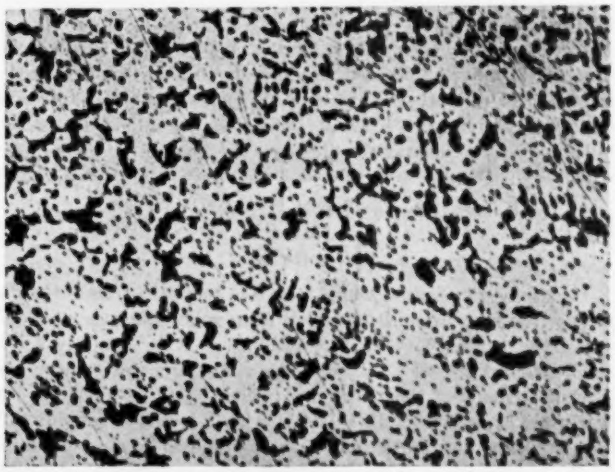

FIG. 16 SAMPLE WE-3-A, POURED AT 1040°C. IN CHILL MOLD ×50

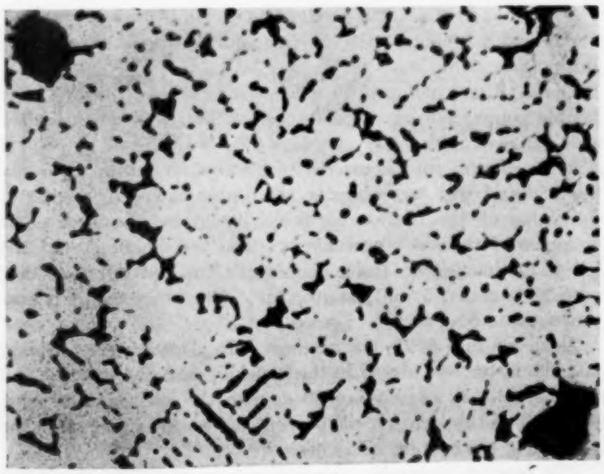

FIG. 14 STRUCTURE OF WE-0 ALLOY (STRAIGHT CU-PB) ×100

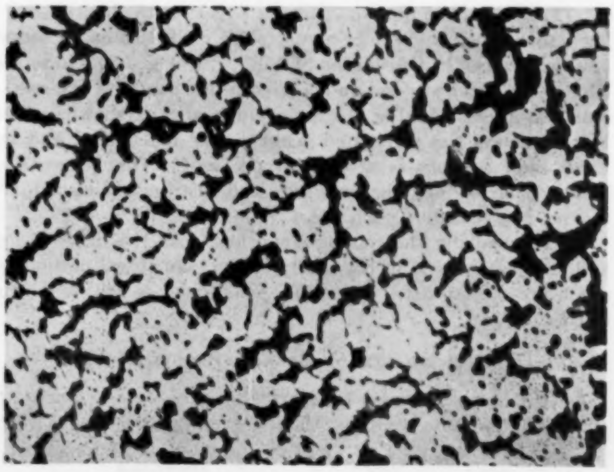

FIG. 17 SAMPLE WE-3-C, POURED AT 1040°C. IN CHILL MOLD ×50

TABLE 6

Mark	Casting temperature, deg. cent.	Composition, per cent Cu	Pb	Sn	Zn	P	Resistivity, microhms, per cu. cm.	Conductivity, per cent
51-2	1100	86.65	12.66	0.16	..	0.04	5.358	33.07
51-4	1050	85.09	13.78	0.31	..	0.03	5.120	34.60
52-4	1150	87.53	12.15	0.05	0.12	..	2.580	69.60
52-6	1100	77.90	21.72	0.04	0.13	..	2.480	72.90
52-8	1050	71.60	27.50	0.05	0.12	..	3.150	57.00
53-4	1150	75.87	23.24	0.13	..	0.09	5.448	32.70
53-6	1100	75.81	23.34	0.12	..	0.08	5.560	32.08
52-x-4	1150	75.36	17.48	3.43	3.55	0.09	10.800	16.46
52-x-6	1100	75.32	17.42	3.39	3.31	0.09	10.600	16.78
52-x-8	1050	75.66	17.18	3.50	3.46	0.09	10.900	16.31

For solid solutions there is an additional resistance due to the formation of the solid solution which in most cases seems to be but slightly dependent upon the temperature.

From the results in Table 6, the alloys can be classified into three groups:

(1) Pure Cu-Pb alloys with a conductivity ranging from 57 to 73 per cent according to Pb content.

(2) Cu-Pb alloys with small amounts of impurities (Sn, P), with a fairly constant conductivity near 33 per cent that is independent of the lead content.

(3) Solid solution Pb-Cu bronzes with a conductivity value of 16.5 per cent.

With regard to the effect of the impurities Sn, Zn, and P, it must be borne in mind that phosphorus and tin lower the conductivity of copper much more than zinc. For small amounts of P, Sn, and Zn, respectively, there must be added about the following relative amounts to decrease the conductivity of copper equally:

1 part P
5 parts Sn
60 parts Zn

It is therefore of utmost importance to keep the straight Cu-Pb alloys free from impurities which form solid solutions and decrease the conductivity, as do P and Sn.

(*c*) *Microstructure.* The microexamination was made on Brinell coupons of the tensile-test pieces of the straight Cu-Pb series 51, 52, and 53. In series 51 with a low lead content (12 per cent), the distribution of lead was found to be more uniform in the sand castings than in the chill castings. (See Figs. 6 and 7.) In the sand cast, series 52, the increasing lead content with lower casting temperature is easily noticeable. (See Figs. 8 and 9.) The size of the primary grains is largest at the highest casting temperature (1150 deg. cent.). The chill castings of series 52 showed a better distribution of lead. (See Fig. 10.) In series 53 excessive lead segregation occurred in the center of the sand-cast test piece. In Fig. 11 an average area is shown, while Fig. 12 shows the segregated center. The amount of lead in those sand castings is around 26 per cent. This percentage seems to be so high that uniform lead distribution cannot be secured in a straight Cu-Pb alloy.

By so changing the casting conditions that the distributing action is carried out mechanically, as by centrifugal casting, a much better lead distribution can be obtained, as will be shown farther on.

III PERFORMANCE OF BEARING-BRONZE METALS

(*a*) *General.* A series of tests was made in order to obtain some comparative results regarding the following bearing alloys:

Sample	Mark	Description
52-x	DW	Solid solution bronze
52-x	WE	Solid solution bronze
52	WE-0	Straight Cu-Pb alloy
52	WE-1	Straight Cu-Pb alloy
52	WE-2	Straight Cu-Pb alloy
52	WE-3	Straight Cu-Pb alloy

Tables 7 and 8 give some data on the properties of the bearings subjected to the tests described.

TABLE 7

Sample mark	Cu	Pb	Sn	Brinell hardness	Electrical conductivity, per cent of pure copper	Heat conductivity, watt-cm. system	Casting temperature, deg. cent.	Microstructure, figure
DW	71	27	2	33	15	0.72	..	15
WE	69	26	4	60	10	0.48	1050	13
WE-0	82	18	..	23	60	2.90	1050	14

TABLE 8

Sample mark	Pb	Cu	Sn	Fe	Zn	Ni	Brinell	Casting temperature, deg. cent.	Microstructure figure
WE-1	15.0	84.8	0.07	0.01	0.037	0.005	28	1200[a]	18
WE-2	20.3	79.4	0.07	0.01	0.033	0.008	26	1100[a]	19
WE-3	20.5	79.3	0.07	0.01	0.032	0.009	30	1040[b]	16, 17

[a] Sand mold. [b] Chill mold.

The microstructures of the tested materials can be seen from Figs. 13 to 19, inclusive. The bearing alloys WE-1, WE-2, and WE-3 were prepared with the object of obtaining some information on the influence of the microstructure on the bearing performance. The material was supplied for test in shape of ingots, from each of which two bearings were machined—one from the top and the other from the bottom of the ingot. In the case of the ingots poured at 1200 deg. cent. and 1100 deg. cent. in sand molds, no difference in the structure of the machined bearings from the same ingot was observed. It is, however, noticeable that the ingot cast at 1200-deg. cent. showed a lead content of only 15 per cent, while the one cast at 1100 deg. cent. from the same ladle showed 20.3 per cent.

In the case of ingot WE-3 cast at 1040 deg. cent. in a chill mold, a considerable difference in the structures of WE-3-A and WE-3-C is seen in Figs. 16 and 17.

(*b*) *Test Set-Up.* The tests were made on the bearing-testing machine shown in Fig. 20.[4] The test bearings were mounted by means of steel bushings in a very rigid housing made in two halves bolted together. The load was applied by tightening the loading nut, the load being registered by a Chatillon spring dynamometer, the maximum load being 5000 lb. A flexible arrangement of the loading gear prevented eccentricity of load application.

In the tests there was used a standard shaft material having the following composition: C, 0.3 to 0.4; Ni, 1.0 to 1.5; Cr, 0.45 to 0.75; Mn, 0.50 to 0.80; $P_{max.}$, 0.040; $S_{max.}$, 0.045.

A sleeve-type shaft was designed as shown in Figs. 21 and 22. The interchangeable sleeves were machined to the same grade of finish.

The shaft was mounted on roller bearings and connected by a flexible coupling to a variable-speed motor, which provided a wide range of speeds by means of the Ward Leonard control system. The motor was carefully calibrated, so that by recording the field and armature currents, the torque at any moment and the value of the friction coefficient could be obtained. The diameter of the shaft was 2.5 in. The test bearings had the following dimensions: Inside diameter, 2.5 in. (+0.001, —0.000); outside diameter, 3.5 in.; length, 2 in. Thus the projected area was equal to 5 sq. in.

As shown in Fig. 23, the bearings were placed in the bushing, and the latter was placed in the housing, with the bushing and bearing split at an angle of 30 deg. to the horizontal housing joint. The key between the bearing and bushing prevented the rotation of the bearing with the shaft.

The bearing lubrication was supplied by an oil cup through a hole registering with a duct in the housing and was regulated by a needle valve. The oil cup was calibrated, and the oil feed could therefore be conveniently regulated. The oil used (M-2268) was of the same grade throughout the tests, its viscosity being 45 centipoises at 100 deg. fahr. and 5 centipoises at 210 deg. fahr.

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[4] Described in A.S.M.E. Transactions, vol. 52, no. 7, 1930.

The temperature of the bearing in degrees centigrade was recorded by means of a thermocouple inserted in a small hole drilled about 1 in. deep in the axial direction and $^1/_8$ in. below the bearing surface of the lower half of the split bearing.

In the same manner, the temperature of the steel bushing was recorded in order to observe the heat gradient between the bearing and the bushing.

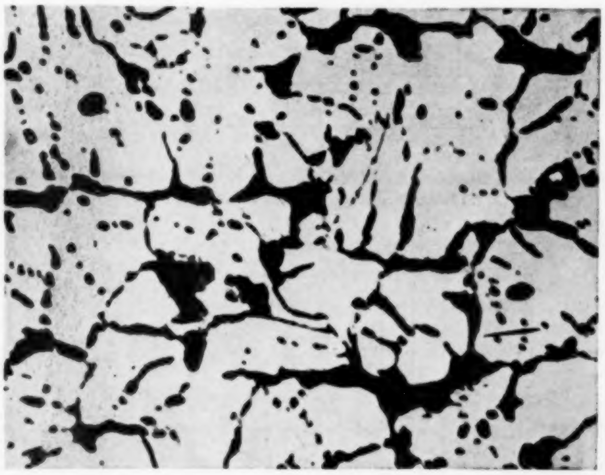

Fig. 18 Sample WE-1, Poured at 1200°C. in Sand Mold ×50

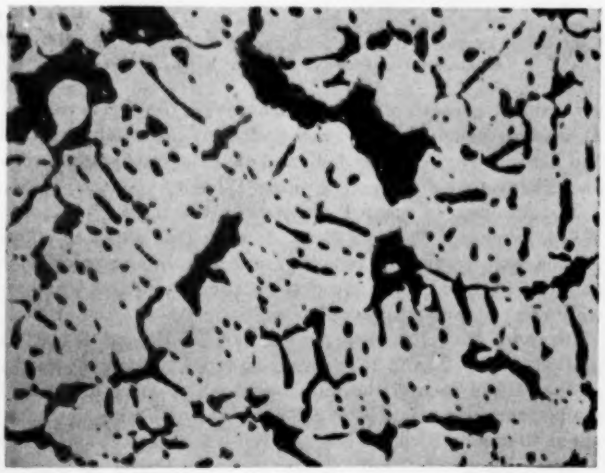

Fig. 19 Sample WE-2, Poured at 1100°C. in Sand Mold ×50

(c) *Method of Test.* All of the bearings mentioned were tested in separate groups, during which the bearings were subjected to certain definite conditions identical for each group; namely, load, speed, lubrication conditions, and, when possible, the time length of the test. These groups were:

1 Test of the bearings with continuous oil supply.

2 Test without continuous oil supply, the bearing being lubricated by the oil remaining between bearing and shaft.

3 Test without oil; the bearing assembly was washed in gasoline and dried.

This division of tests into three main groups permits a convenient comparison of the test results. The first group represents the usual operating bearing conditions, when the amount of

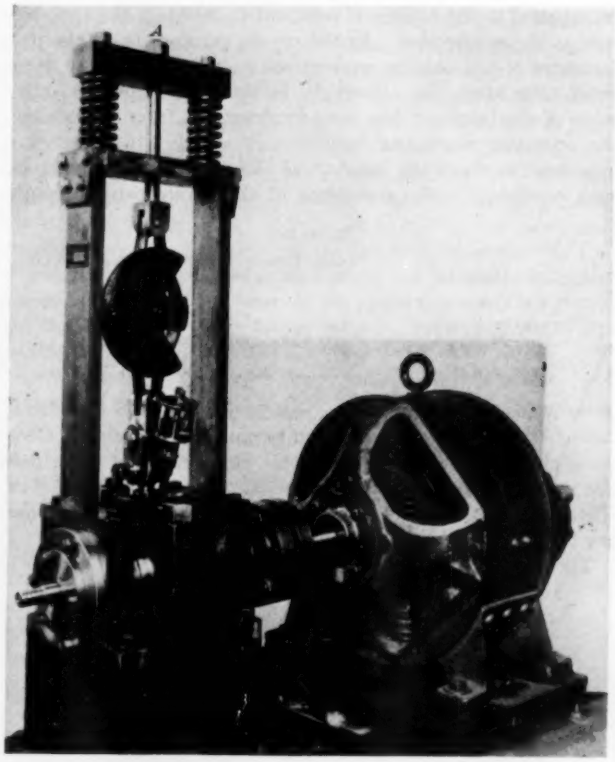

Fig. 20 Machine for Testing Bearings

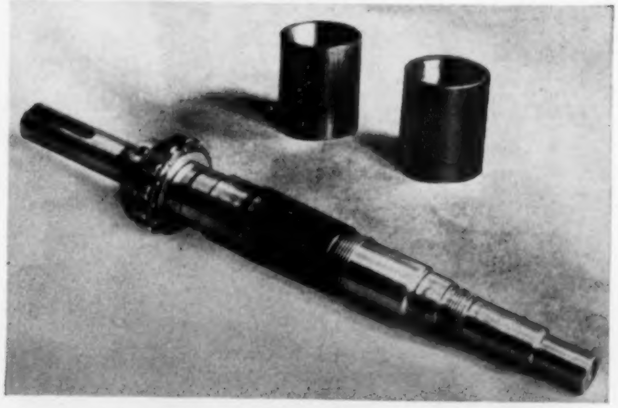

Fig. 21 Sleeve-Type Shaft Used in Bearings Tests

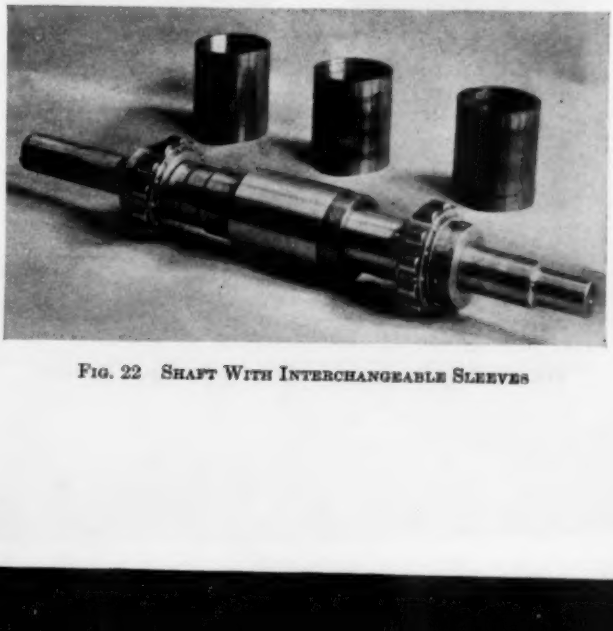

Fig. 22 Shaft With Interchangeable Sleeves

oil supplied to the bearing is sufficient to maintain the temperature at the rated value. In the second group of tests, the performance of the bearing remains the same as in group 1 for a short time after the oil supply is shut off. However, after some of the lubricant has been squeezed out from the bearing, the operating conditions become very severe. It is therefore important to know the behavior of the bearing materials under such conditions. The conditions of this group can frequently occur in practice when for some reason the oil supply is suddenly cut off—for instance, when an oil pump or oil line fails. Complete absence of lubricant, as in the third group of tests, does not generally occur in practice with these bearing materials. These tests were undertaken in order to obtain some data on bearing bronzes as to heat conductivity, wear, etc.

TABLE 9

Group	Material	Load, lb.	Pressure per sq. in.	Speed, r.p.m.	Time, min.	Curves in figs. nos.
1	WE-1, WE-2, WE-3	500	100	500 1000	60 20	26–29
2(*a*)	DW, WE, WE-0	250	50	400	80	24–25
2(*b*)	WE-1, WE-2, WE-3	500	100	500 1000	60 20	26–29
3	DW, WE-0	250	50	400	80	30–31

The tests of the bearings with continuous oil supply (group 1) were run for a length of time sufficient for the bearing to wear in. A constant value of the friction coefficient (constant armature and field-current readings) indicated that the bearing was worn in. The test conditions are given in Table 9.

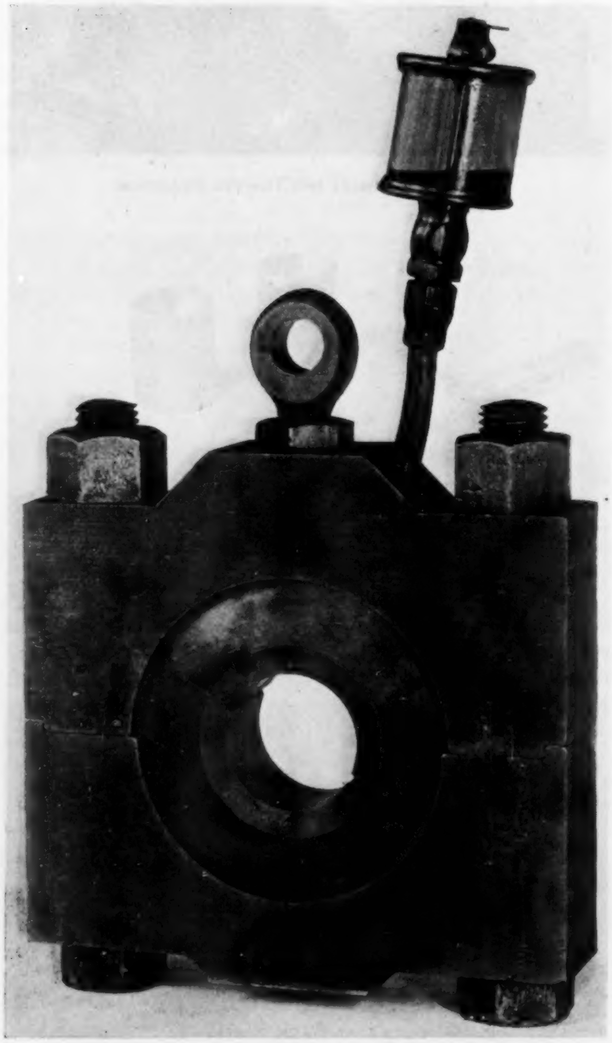

FIG. 23 TEST BEARINGS AND BUSHING PLACED IN HOUSING

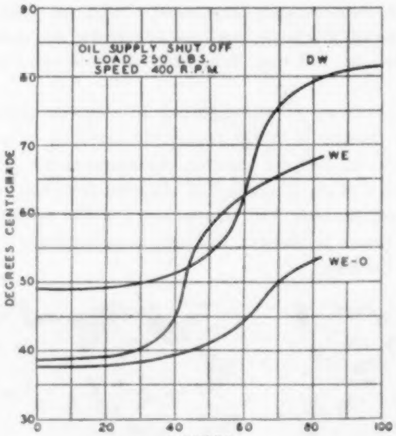

FIG. 24 TEMPERATURE-TIME RELATION OF BRONZE BEARINGS DURING TEST WITHOUT OIL SUPPLY

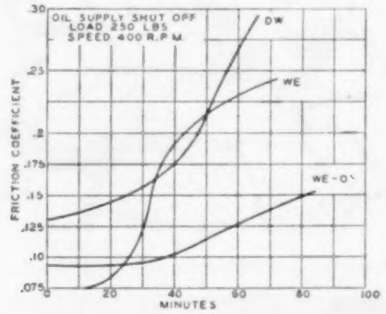

FIG. 25 FRICTION COEFFICIENT-TIME RELATION OF BRONZE BEARINGS DURING TEST WITHOUT OIL SUPPLY

Test Results. In group 1, the tests on the WE-1, WE-2, and WE-3, as plotted in Figs. 26 to 29, show that all three alloys can be well applied in practice, since the friction coefficient and the temperatures derived from their operations are well within the allowable working limits.

In group 2(*a*), the tests on the DW, WE, and WE-0, as plotted in Figs. 24 and 25, show, as expected, a temperature rise in the case of each of the three bearings. A sharp increase in temperature occurs in the WE bearing after 30 min., and after about 40 min. in both the DW and WE-0 bearings.

The friction coefficient changes correspondingly, as seen in the curve in Fig. 25. The values of the temperature and the friction coefficient do not change greatly within a time range of approximately 30 min., indicating that a safe performance of the bearings can be expected within that period. The DW bearing seized the shaft after 60 min., and the WE bearing after 45 min.

The micrographs, Figs. 13, 14, and 15, show the lead distribution of the three materials tested. The lead particles are very coarse in the DW (solid-solution sand casting) metal. A more even distribution of lead is shown in the WE-0 alloy (straight Cu-Pb, chill casting), and a very uniform structure is characteristic for the WE alloy (solid-solution, chill casting).

In group 2(*b*), as plotted in Figs. 26 to 29, quite different results were obtained during the tests of the straight copper-lead bearings WE-1, WE-2, and WE-3 when subjected to more severe

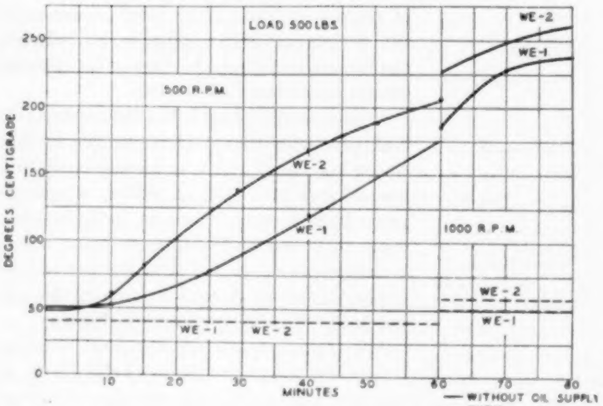

FIG. 26 BEHAVIOR OF TWO CU-PB BEARINGS DURING TEST WITH AND WITHOUT OIL SUPPLY

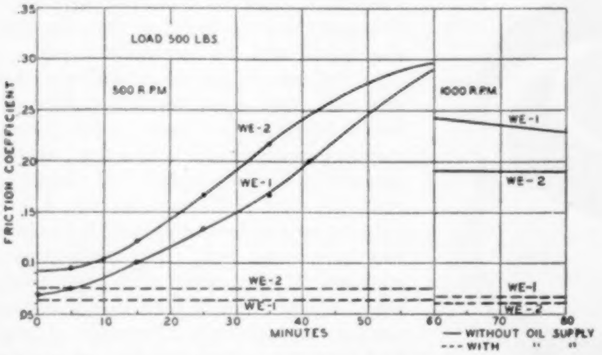

FIG. 27 BEHAVIOR OF TWO CU-PB BEARINGS DURING TEST WITH AND WITHOUT OIL SUPPLY

testing conditions under a load of 500 lb. at 500 and at 1000 r.p.m.

Since the microstructure of the bearings from each of the ingots 1 and 2 was the same, the average values of the data obtained are given on the curves in Figs. 26 and 27. In the case of the tests on the two bearings machined from ingot 3, where different structures were noticed, in the curves in Figs. 28 and 29 two values are plotted corresponding to the bearings machined from the upper part *A* and the lower part *C* of the ingot. Somewhat lower values of temperature and friction coefficient were observed in the case of the material with the finer structure.

The duration of the safe performance of these bearings can be seen from the curves in Figs. 26 and 27 (run at 100 lb. per sq. in. pressure and 500 r.p.m.) to be not more than 10 min. for the WE-1 and 5 min. for the WE-2.

Group 2 tests show that the bearing material will work satisfactorily for a short while after the oil supply is stopped while some oil remains in the clearance. The curves in Figs. 24 and 25 show that not only does the straight copper-lead alloy WE-0 give a lower temperature of operation than the solid-solution bronzes DW and WE, but also a lower friction coefficient and a more satisfactory period of operation during the conditions of this group of tests, when the bearings were subjected to 50 lb. per sq. in. at 400 r.p.m. The curves in Figs. 26 to 29 show that, if the load and speed are increased, the bearing will fail after a short time, the duration of the safe performance being only 4 to 5 min. The results obtained during the test of the bearings WE-3-A and WE-3-C confirm the general belief that fine structure is necessary for a good high-lead-bearing bronze. The effect of the microstructure was, however, less pronounced than originally expected.

In group 3, as plotted in Figs. 30 and 31, two bearing materials—namely, the solid-solution bronze DW and the straight Cu-Pb alloy WE-0, were subjected to dry test under a pressure of 50 lb. per sq. in. at 400 r.p.m. The first temperature reading at the start of the test was 24 deg. cent. in the DW and 27 deg. cent. in the WE-0. From the beginning of the test, the temperature in each case increased rapidly, especially in the DW bearing, where at any time the temperature was approximately twice that in the WE-0 bearing. The values of the friction coefficient are also higher for the DW than for the WE-0 bearing. It was in this group of tests that the wear of the bronze metals was first observed by a decrease of the load, the load being immediately increased to the initial value by the tightening of the loading nut.

Group 3 tests show that the heat gradient from the shaft to the outer frame surface was greater in the DW bearing than in the WE-0. This statement may be verified by comparing the values of the heat conductivity of the two alloys, which comparison shows that the straight copper-lead alloy has a heat conductivity four times that of the DW metal; in other words, the rate of heat dissipation from the WE-0 alloy is four times greater than from DW metal.

Summary. The results obtained during the tests of groups 1, 2, and 3 show that the straight Cu-Pb alloys are superior to leaded solid-solution bronzes regarding their general bearing performance.

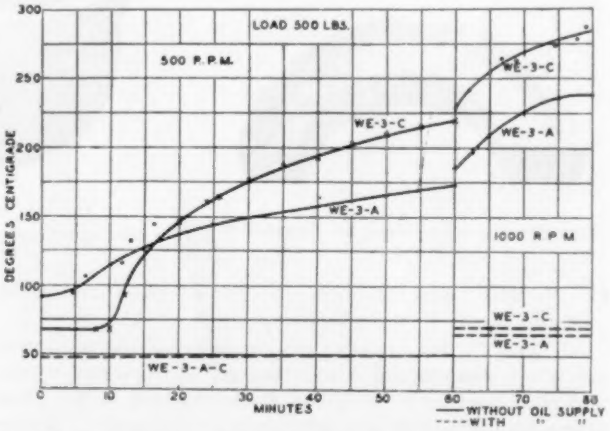

FIG. 28 INFLUENCE OF MICROSTRUCTURE OF THE BEARING MATERIAL UPON THE TEMPERATURE-TIME RELATION DURING THE TEST WITH AND WITHOUT OIL SUPPLY

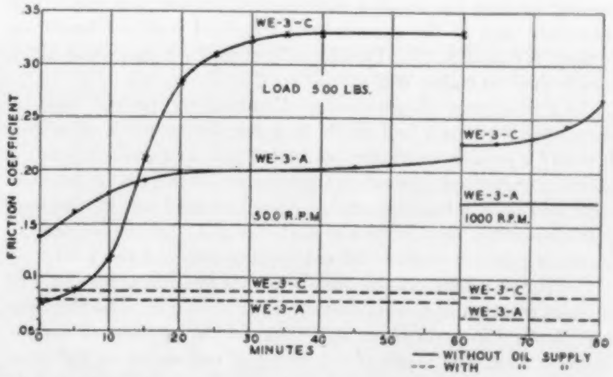

FIG. 29 INFLUENCE OF MICROSTRUCTURE OF THE BEARING MATERIAL UPON THE FRICTION COEFFICIENT-TIME RELATION DURING THE TEST WITH AND WITHOUT OIL SUPPLY

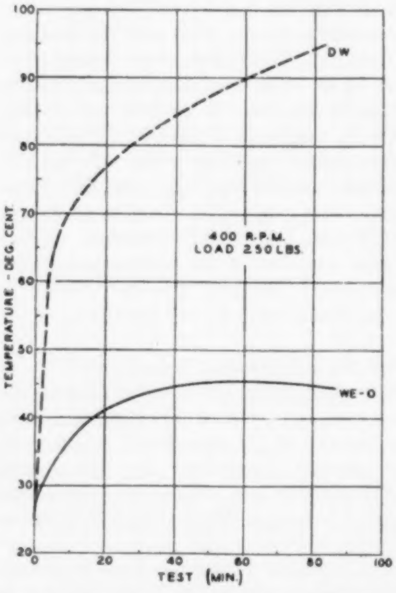

FIG. 30 TEST WITHOUT OIL (DRY BEARINGS)

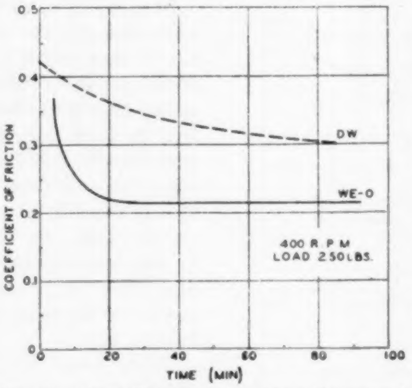

FIG. 31 TEST WITHOUT OIL (DRY BEARINGS)

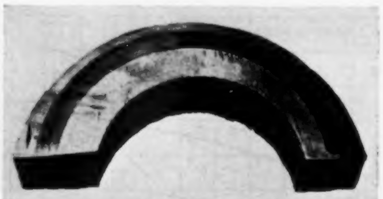

FIG. 32 BEARING MADE BY IMMERSION PROCESS

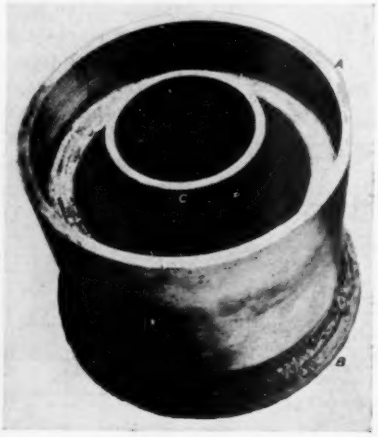

FIG. 33 STEEL SHELL
(For producing bearings according to immersion process.)

IV TECHNOLOGICAL ASPECTS OF PRODUCING HEAVY-DUTY BEARINGS

(*a*) *General.* The object of this investigation was to provide means for strengthening leaded bronzes and straight copper-lead alloy bearings by means of a steel backing shell. There were a number of difficulties encountered in obtaining a perfect and inseparable joint between the leaded bronze and the steel without affecting the lead distribution of the bronze. After some experiments with straight casting methods, efforts were directed to the investigation of the suitability of a centrifugal casting process for making steel-backed bronze bearings. The materials used in the experiments described were solid-solution bronze WE and a 25%-Pb-75%-Cu straight copper lead alloy, which shall be called WE-x.

(*b*) *Preliminary Experiments.* Experiments proved that in immersing two steel half shells in a graphite crucible of molten bronze for periods varying from 2 to 10 min. an excellent bonding action was secured, indicating that a true alloying action between steel and bronze had occurred. After the steel was at the temperature of the molten bronze and "wetted" by the bronze, an increasing length of time did not seem to improve the joint.

Copper at a temperature just below its melting point can hold in solid solution approximately 3 per cent of iron. At the same temperature iron dissolves approximately 8 per cent of copper. In both cases the range of solubility of one metal in the other decreases with the temperature. Most alloys of the two metals, therefore, will consist of a mixture of the two constituents, both of which are solid solutions—one of iron in copper and the other of copper in iron. In order to minimize the formation of an alloy layer between the bronze and steel, the time of hot dipping should be as short as possible.

It may be mentioned that the methods for making copper-clad steel are similar in that a red-hot steel billet is momentarily immersed in a hot bath of molten copper so as to form a very thin and uniform alloy layer over the entire surface, after which molten copper is cast around the hot billets.

Fig. 32 shows a section through an unmachined bearing prepared in this way.

For making a few experimental bearings it was decided to use the steel shell both as crucible and mold, and therefore forgings were made as shown in Fig. 33. On one end of the steel sleeve *A*, a bottom plate *B* was welded, in the center of which a standard iron pipe *C* was fastened, so that the pipe was coaxial with the bore of the sleeve.

In one case the molten bronze was cast in a cold steel shell, which was then reheated to the melting point of the bronze, held at temperature for 1 to 3 min., stirred properly, and slowly cooled. In the other case, the steel shells were preheated in a reducing atmosphere, and the bronze was cast into the hot shell. Both methods were successful to the extent that good joints were obtained, but in each case a certain tendency to blowing and formation of large lead particles was noticeable. By using the first method and holding the metal molten in the shell for an excessively long time (10 min.), the lead coalesced to form very large particles, as shown in Fig. 34, as compared with the very small particle size characteristic of chill castings of the same bronze, as shown in Fig. 13. This hot-casting or "immersion" method was used for the preparation of four large rolling-mill bearings for a special experimental set-up. The dimensions of this bearing are given in Fig. 35, and the good appearance of the bronze is evident from Figs. 36 and 37. The

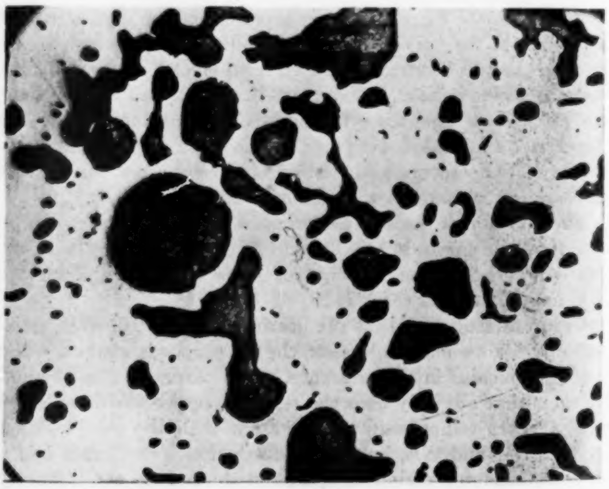

FIG. 34 LARGE LEAD PARTICLES IN BEARING MADE ACCORDING TO IMMERSION PROCESS

joint between the bronze and the bearing shell was perfect.

The hot-casting method was not applied to the straight copper-lead alloys, as their tendency to segregation is too strong.

Centrifugal Casting of Bearing Bronzes

The experiments were first performed with the bearing bronze WE. A small gas furnace was used for melting and to preheat the ladles and the steel shells. The temperature of the molten metal was measured with a chromel-alumel thermocouple and kept at 1050 to 1080 deg. cent. The steel shells were preheated by dipping them in the molten metal for about 2 min. They were then placed quickly in the centrifugal casting machine. As soon as the machine was up to speed, the bronze was poured into the mold through a hole in the front end of the machine. Immediately after pouring, a blast of cold air was turned on the outside of the steel shell to cool the bronze as rapidly as possible and thus prevent any large lead segregation.

In a preliminary experiment with the WE bronze the alloy was centrifugally cast into a cold steel shell. The joints so produced were poor; the lead distribution, however, was extremely fine, as shown in Fig. 38.

The object of the tests with preheated shells was to determine the influence of the rotational speed upon the qualities of the joint and the lead distribution.

The centrifugal force is a function of the inside diameters of the steel shells. Only small shells with $2^{1}/_{8}$ in. and $1^{7}/_{16}$ in. inside diameters were used.

The speeds employed were 1000, 1200, 1500, and 1750 r.p.m. The resultant joints are shown in Figs. 39 to 41. All of them are perfect and strong. The difference in lead segregation is not very noticeable up to 1500 r.p.m. At 1750 r.p.m., however, there are large lead globules near the joint. The large amount of lead did not weaken the joint when it was tested, but if the bearing should be run at elevated temperatures, as may happen in service, the excessive amount of lead may weaken the joint.

The strength of the welded joint was tested by a simple bending test, as shown in Figs. 42 and 43. For this test rings $^{3}/_{8}$ in. wide were cut out of the bearings. These rings were then bent flat in an Amsler tensile-testing machine. If the joint was poor, due to an oxidized surface, the bronze broke away easily, while a good joint could be bent flat without showing any signs of parting between bronze and steel. The joints tended to be poor at the extreme ends of the bearings due to the rapid cooling of the ends of the shell prior to casting, caused by contact with the cold metal parts of the casting machine. The joints in the central portion of the bearings were always strong.

The same series of experiments as with the WE bronze was carried out with the straight Cu-Pb alloys WE-x. Unfortunately, the small furnace available was unable to produce temperatures as high as was desirable. The maximum temperatures obtainable were 1050 to 1080 deg. cent., while a temperature of about 1150 to 1200 deg. cent. would have been required for satisfactory working. Another source of trouble with the WE-x alloy was the large amount of slag which formed very rapidly on the surface of the metal. Thus, in dipping the metal out of the crucible, some slag was always brought along, and part of it was poured into the bearing. The slag, however, was light, and most of it stayed in the inside of the bearing.

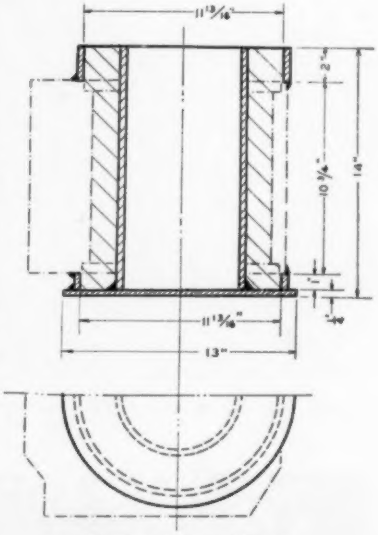

Fig. 35 Steel Shell
(For producing large rolling-mill bearings.

It is quite apparent from the photographs, Figs. 44–46, that the

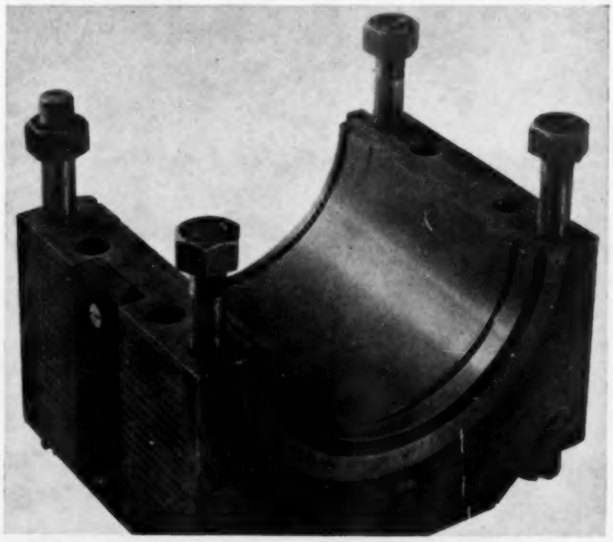

Fig. 36 Bearing Cast by Immersion Process

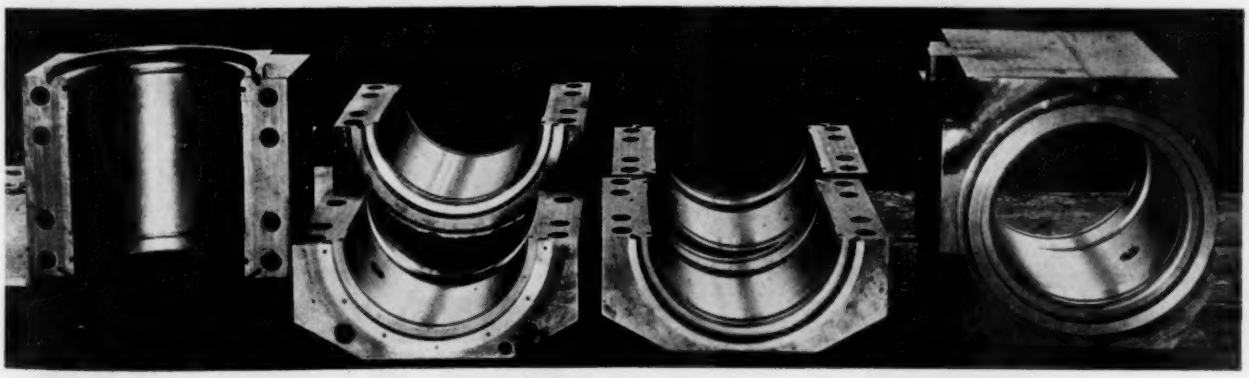

Fig. 37 Four Rolling-Mill Bearings Cast by Immersion Method for Special Test

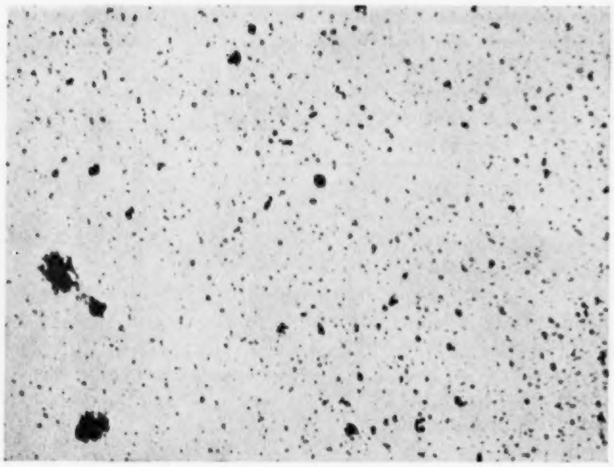

FIG. 38 1000-R.P.M. ALLOY WE CAST INTO COLD SHELL AT 1030°C. ×100

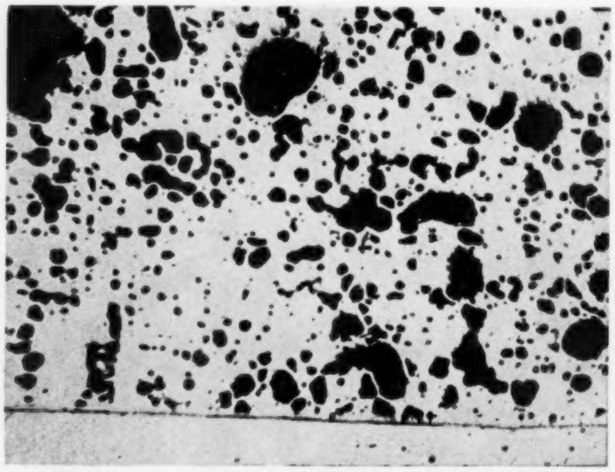

FIG. 39 1500-R.P.M. ALLOY WE CAST INTO PREHEATED SHELL AT 1030°C. ×100
(Inside diameter of shell, $2^1/_8$ in.)

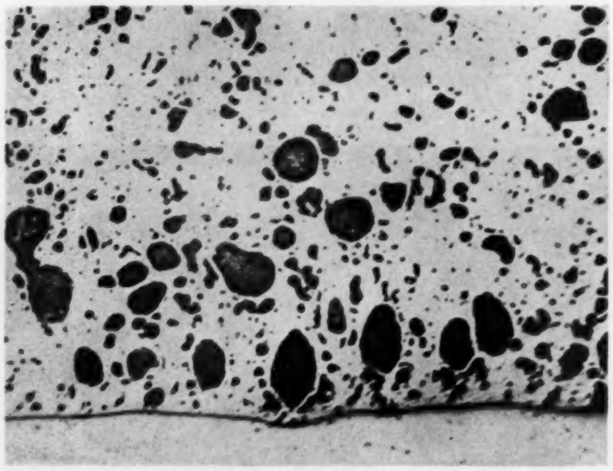

FIG. 40 1750-R.P.M. ALLOY WE CAST INTO PREHEATED SHELL AT 1030°C. ×100
(Inside diameter of shell, $2^1/_8$ in.)

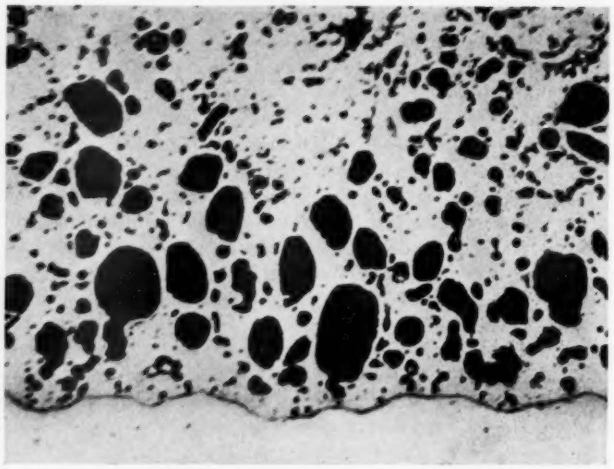

FIG. 41 1750-R.P.M. ALLOY WE CAST INTO PREHEATED SHELL ×100
(Inside diameter of shell, $1^7/_{16}$ in.)

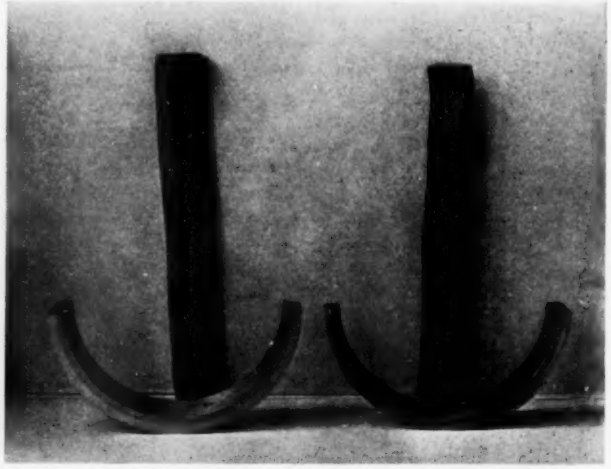

FIG. 42 BEND TEST OF CENTRIFUGALLY CAST BEARINGS
(Left, straight Cu-Pb alloy; right, WE alloy.)

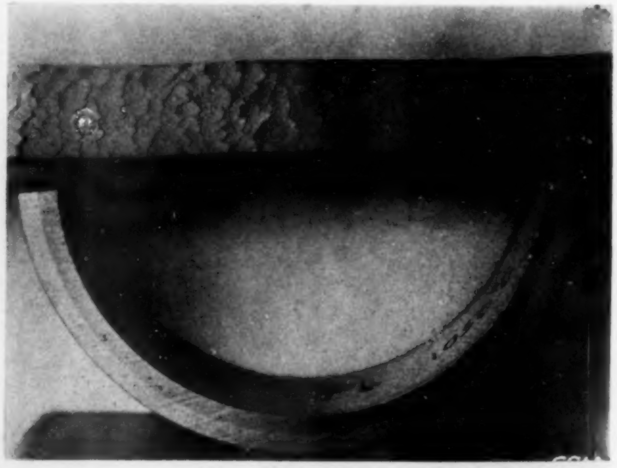

FIG. 43 BEND TEST OF CENTRIFUGALLY CAST BEARING
(WE-x alloy, a straight Cu-Pb alloy.)

lead segregation was not greatly influenced by the speed, which was varied from 600 r.p.m. to 2000 r.p.m. A good bond between the steel and the bearing alloy was secured in every case. The lead is uniformly and finely distributed in a dendritic pattern. There is a sort of columnar arrangement of the lead streaks, while the bearings cast with the WE bronze showed the lead in a globular form.

At a speed of 1700 r.p.m. the WE bronze exhibited a decided lead segregation near the joint, while the bearing cast with the Cu-Pb alloy at the same speed shows hardly any lead segregation. The effect of the speed on the condition of the inside surface of the WE-x alloy was such that a slow speed (600 r.p.m.) produced a rough surface, while a speed of 2000 r.p.m. produced a smooth, glossy surface.

Conclusions

(1) The lead content and its distribution in straight Cu-Pb alloys depends primarily on the casting temperature, in that with lower casting temperatures, quicker solidification occurs and a larger amount of lead is retained in suspension.

(2) Pure Cu-Pb alloys (especially sand castings) have a tendency to pronounced lead segregation if the lead content exceeds 25 per cent.

(3) Small amounts of impurities have a notable effect in retaining a good suspension of the lead in the copper. They reduce the influence of the casting temperature on the lead content and upon the distribution of lead in the casting. A further study of alloying elements acting in this direction should enable improvements to be made in the properties of Cu-Pb alloys.

(4) The physical properties of straight Cu-Pb alloys are nearly independent of the casting temperature within a temperature range of 1050 to 1150 deg. cent. They depend but slightly on the lead content within the limits of 12 to 25 per cent Pb. The Brinell hardness is the only factor that is decreased with increasing Pb content. The physical properties of straight Cu-Pb alloys have values approximately one-half of those of solid-solution bronzes of the same lead content.

(5) The electrical conductivity of Cu-Pb alloys is about six times as high as that of solid-solution bronzes of the same lead content or of babbitts. The conductivity is lowered considerably by small amounts of impurities, especially by P and Sn.

(6) The performance tests of straight copper-lead alloys showed that they are very suitable as bearing materials, on account of low coefficient of friction and low working temperatures.

(7) The influence of the microstructure is such that, with a finer lead distribution, somewhat better bearing performance is obtained.

(8) The bearing temperatures during the performance tests were considerably lower with the straight copper-lead bearings than with the solid solution bronzes because of the higher thermal conductivity of the copper-lead alloys.

(9) It is practicable to produce steel-backed copper-lead bronze or leaded bronze bearings by a centrifugal casting process.

(10) A strong joint between the steel and bearing metal can be obtained if the steel shell is at or preferably above the melting point of the bronze when it is poured into the shell.

(11) The rotational speed for a 2 to 3 in. diameter bearing should be about 1500 r.p.m. for solid-solution bronzes. Any speed between 1000 and 2000 r.p.m. may be used for straight Cu-Pb alloys.

(12) It is suggested that a bath of tin bronze or pure copper without lead be used for preheating the steel shells to avoid the poisonous fumes from vaporized lead. The preheating temperature of the shells in the case of solid-solution bronzes should be 1100 to 1150 deg. cent.; in the case of straight Cu-Pb alloys, it should be around 1150 to 1200 deg. cent.

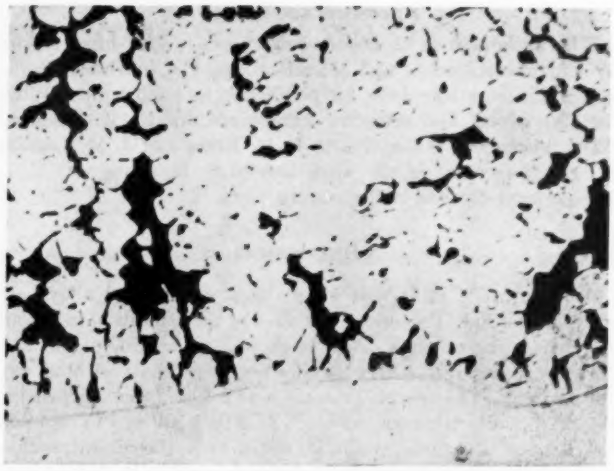

Fig. 44 600-R.P.M. Cu-Pb Alloy Cast Into Preheated Steel Shell ×100
(Inside diameter of shell, $2^1/_8$ in.)

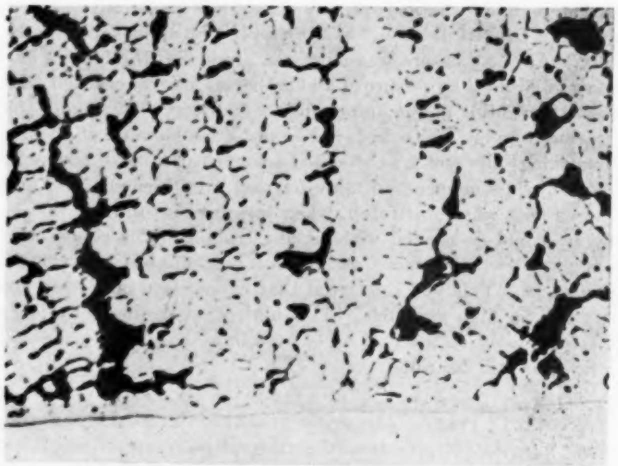

Fig. 45 1500-R.P.M. Cu-Pb Alloy Cast Into Preheated Steel Shell ×100
(Inside diameter of shell, $2^1/_8$ in.)

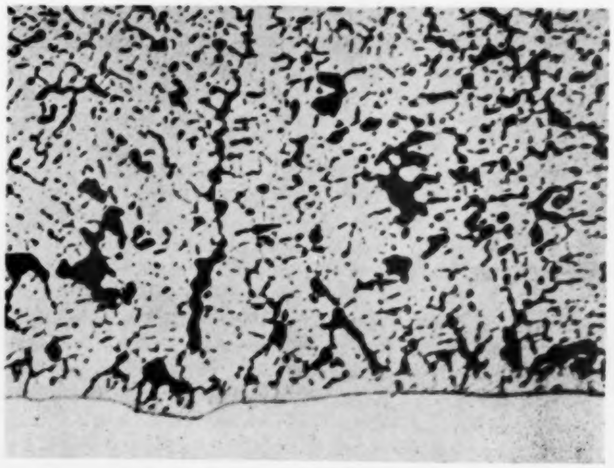

Fig. 46 1700-R.P.M. Cu-Pb Alloy Cast Into Preheated Steel Shell ×100
(Inside diameter of shell, $2^1/_8$ in.)

ACKNOWLEDGMENTS

The authors wish to acknowledge their indebtedness to the Westinghouse Electric and Manufacturing Co., in whose laboratories this work was done, for permission to publish the results. Helpful advice and criticism were given by G. B. Karelitz. Acknowledgment is also due to P. H. Brace and J. M. Lessells for encouragement of the work and to S. E. Amos and E. I. Larsen for assistance in the various tests.

Discussion

W. TRINKS.[5] It is well known that copper-lead alloys are excellent for light pressures. At most of the exhibits of mechanical engineering we see a salesman clamping a bearing of this material on or against a very rapidly revolving shaft without any lubrication. The bearing pressure is so selected that the bearing material finally becomes red hot. It is then allowed to cool and taken apart, and the salesman impresses upon the spectators that no harm has been done either to the bearing or to the shaft.

For heavy pressure the story is quite different. In 1923 it was the writer's goodfortune to make tests under conditions approaching those of a roll-neck bearing. A pressure was used of 3000 lb. per sq. in. and a rubbing speed of almost 800 ft. per min. A few grades of babbitt did well, but the copper-lead alloys failed markedly. As the pressure was increased gradually, the bearing behaved well until a pressure of 900 lb. per sq. in. was reached, and then the reading of the ampere meter suddenly shot up and the bearing became quite hot. In order to eliminate accidental happenings the test was repeated several times, with the same results. The copper-lead alloy bearing was then removed and the babbitt was substituted, when pressures up to 3000 lb. per sq. in. could be carried without any trouble. The copper-lead alloy was tried once more, and again it began to smoke at 900 lb. per sq. in. This test convinced that copper-lead alloys are not suitable for high pressures. The lubrication engineer of a large steel company states that he is not allowed to use copper-lead alloys for mill bearings.

ROBERT D. PIKE.[6] The writer would be interested in receiving some information on intended versus actual composition of centrifugally cast straight copper-lead alloys particularly as applying to Figs. 44, 45, and 46. As a result of several years' experience in making similar microstructures by centrifugal casting, the writer concluded that in order to get a centrifugally chill-cast straight copper-lead bushing averaging 25 per cent of lead, one must start with about 27.5 per cent in the pot; and if the melt were centrifugally cast into hot backs, invariably the actual analysis of the copper-lead liner showed 20 per cent of lead or less. The authors show a considerable reduction of lead intended versus lead introduced in the sand and chill castings, but give no parallel information in the case of centrifugal castings, where such information would, it is thought, be of great interest.

It has been the writer's experience that in centrifugal casting the lead is thrown out bodily, and nothing discernible in the microstructure would lead one to suspect that lead had been lost. Analysis alone will show this.

There also appears to be some discrepancy between the statement that all centrifugally cast joints were weak on the ends and that satisfactory working conditions were not obtained with the straight copper-lead alloys and the statement that all the bonds between straight copper-lead and steel were good. It need scarcely be pointed out that any process for making a satisfactory bearing in the commercial sense must produce a bond practically 100 per cent perfect over the entire bearing in all bearings. Short of this the bearings will fail disastrously in heavy-duty service.

(Discussion by J. R. Adams and E. S. Glauch will be found following the paper by S. M. Weckstein.)

AUTHORS' CLOSURE

With regard to the remarks of Prof. W. Trinks, it is our opinion that the alloys which he used in his tests in 1923 were copper-lead bronzes—in other words, solid-solution alloys. We believe that with our straight copper-lead alloys, especially if centrifugally cast, better bearing performance under high pressure could be obtained on account of the higher heat conductivity, which as we have shown is about six times that of a solid-solution bronze.

Unfortunately we are unable at the present time to furnish the information requested by Mr. R. D. Pike, since no chemical analysis of the centrifugally cast alloys was made. We agree, however, with Mr. Pike's assumption that in centrifugally casting, a certain per cent of lead is lost. With regard to his statement in the last paragraph we wish to point out that the weakness of the joint on the ends of the bearings was entirely due to unsatisfactory experimental conditions and can be easily overcome.

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[5] Professor of Mechanical Engineering, Carnegie Institute of Technology, Pittsburgh, Pa. Mem. A.S.M.E.

[6] Vice-President and General Manager, Kalif Corporation, Emeryville, Calif. Mem. A.S.M.E.

IS-54-4

Application of Impregnated Fabric Bearings to Roll Necks

By ARTHUR J. SCHMITT,[1] CLEVELAND, OHIO

The suitability of celoron for roll-neck bearings has led to widespread use of the product in that field. Its resilience allows bearings to absorb severe impact loads without permanent deformation. Celoron bearings are most commonly lubricated with water, in which case the rate of wear and the power consumed in friction are much less than those of metal bearings. The application of a superior lubricant such as oil or grease improves their performance appreciably. Celoron bearings serve most efficiently on highly polished roll necks, and they have produced substantial power savings in comparison with metal bearings, and their life in numerous installations has been found to be many times that of metal bearings.

BEARINGS are made from several varieties of phenolic materials, which differ somewhat with respect to their method of manufacture, as well as in their properties. The statements which follow may be applicable in general to any phenolic material, but they should be considered as applying specifically only to the particular phenolic substance which is called "celoron."

Manufacture and Properties of Celoron. Celoron is manufactured by impregnating sheets of woven fabric, such as linen or cotton duck, with resin, superimposing one sheet upon another to provide the requisite thickness; then subjecting the laminated mass to accurately controlled heat and pressure in massive hydraulic presses. The simultaneous application of heat and pressure, called curing, converts the resin and fabric into a dense, hard, tough substance which is suitable for a wide variety of uses.

The hardness of celoron is about 40 Brinell. The material is considerably stronger in compression than in tension, having a compressive strength of about 35,000 lb. per sq. in. as compared to its tensile strength of 17,000 lb. per sq. in. It is resilient as well as being hard and tough, and is often molded to final shape, although it can be machined easily. Because of its laminated structure, celoron has definite directional properties; that is, its properties at right angles to the plane of laminations are not precisely the same as those parallel to that plane. The strength, hardness, toughness, and resilience, if necessary, can be modified within certain limits by means of minor variations in the characteristics of the resin and fabric and in the curing operation.

One of the earliest practical applications of celoron was in the construction of gears, piping, and agitators for use in chemical plants. The material is particularly well suited to this type of service because of its extremely great resistance to the corrosive effect of acids and other liquid chemicals, as well as its ability to withstand considerable mechanical abuse. Its suitability for use as a bearing material first became apparent when a chemist, eager to find a bearing material which would not be subject to rapid corrosion, constructed and installed some bearings made of celoron. This experiment resulted in performance so gratifying that the construction of this type of bearings continued and was rapidly carried into other industries.

Following this development, a comprehensive investigation was undertaken for the purpose of definitely adapting celoron to bearing use. This investigation demonstrated the capability of such a bearing to render satisfactory service under adverse conditions. It was found, also, that this characteristic could be improved for certain purposes by incorporating a lubricant in the material at the time of manufacture. Celoron is now often produced with an admixture of graphite, which is compounded with the resin, so that its distribution is uniform throughout the material. About 5 per cent of graphite is the maximum proportion which can be included in this way without impairing the strength of the material. The marked suitability of those bearings for service under adverse conditions suggested their application to roll-neck bearings; and their worth in this field, as in many others, has been proved by abundant experience.

Characteristics of Celoron Roll-Neck Bearings. The conditions under which roll-neck bearings operate are, without doubt, more severe than those of most other bearing applications. Roll-neck bearings are traditionally heavily loaded and poorly lubricated, and in some cases they are subjected repeatedly to severe impact loads. The constant presence of scale and other injurious substances constitutes an additional detrimental circumstance. Moreover, excessive wear of roll-neck bearings is extremely objectionable, because it may cause the rolled product to be off gage, and because it involves frequent delays for adjustment or replacement of bearings.

The degree of performance which may be expected from celoron roll-neck bearings is indicated by the following description of their characteristics:

Load-Carrying Capacity. In numerous installations, celoron bearings carry loads ranging from 1000 to 2500 lb. per sq. in. of projected bearing area. In one particular installation, where the space is severely limited, a load of 5000 lb. per sq. in. is carried. These figures compare favorably with the load-carrying capacities of other types of bearings.

Absorption of Impact Loads. Celoron bearings are resilient and are thereby able to absorb and distribute impact loads over the entire bearing area, so that excessive wear due to impact or vibration is avoided. As an illustration of this characteristic consider a piece of celoron which rests upon an anvil and which is struck with a hammer. The resilience of the material will allow it to distribute the force of such an impact so that the surface will not be injured, whereas an identical piece of bronze subjected to an identical impact would become distinctly dented or deformed.

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[1] District Manager, Continental-Diamond Fibre Company. Mr. Schmitt received his education in the Chicago public schools and worked for several automobile manufacturers. He organized and was president of the Walnart Electric Manufacturing Company, of Chicago, from 1923 to 1928, when it was merged with the Continental-Diamond Fibre Company.

Contributed by the Special Research Committee on Heavy-Duty Anti-Friction Bearings and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

A considerable part of the long life of celoron bearings is attributed to this characteristic.

Coefficient of Friction. The coefficient of friction of celoron bearings, like that of other types of bearings, is quite variable and depends upon several factors. Its value under certain conditions have been observed in laboratory tests. The average coefficient of friction of plain celoron as determined in laboratory tests (shaft speed about 225 ft. per min.) is as follows:

Lubricant	Load (lb. per sq. in. of projected bearing area)		
	1500	2500	3500
Oil	0.040[a]	0.060	0.070
Water	0.042	0.063	0.076[a]
None (dry)	...	0.370	...

---

[a] Estimated.

These data apply only to plain celoron which has not been impregnated with graphite. The coefficient of friction of bronze bearings, observed in similar tests, has been found to be either slightly less or considerably greater than that of celoron, depending upon the type of bronze. The values listed were observed under conditions which were not comparable to those under which roll-neck bearings operate. Consequently, the coefficient of friction of celoron roll-neck bearings for given conditions of load and lubrication may differ somewhat from the corresponding value given.

In general, the significant facts are, first, that celoron bearings carry capacity loads with very moderate friction losses when water only is used as a lubricant; second, that the friction losses under capacity loads are minimized when oil is used as a lubricant; third, that with no lubrication whatever, capacity loads can be sustained for limited periods of time without destruction of the bearings.

Statements of power saving through the substitution of one type of bearing for another have little direct significance without accompanying statements of the circumstances. Although very large power savings have been made in certain cases by the use of celoron bearings, the circumstances in such cases were perhaps somewhat exceptional and would be of little general interest. Records show many actual cases, covering a range of more nearly average conditions, in which substitution of celoron for metal boarings has effected power savings considerably in excess of 10 per cent. It may be safe, therefore, to predict a minimum power saving of 10 per cent through such substitution, provided that the conditions, whatever they may be, remain unchanged.

Rate of Wear. The rate of wear of celoron, like that of other roll-neck bearings, is affected by many factors, such as load, lubrication, temperature, presence of foreign matter, and surface condition of the roll neck. Laboratory tests have indicated that the rate of wear increases slightly with increasing loads, as would be expected, and that the substitution of oil for water as a lubricant, other conditions remaining the same, results in a very marked decrease in the rate of wear. Comparative results obtained in practice show clearly that the condition of the roll-neck surface has an extremely great influence upon the rate of wear. Rough roll necks, being harder than celoron, have a filing effect upon the bearings, and although the roughness may ultimately be worn off the roll necks, a considerable portion of the bearings is likely to be sacrificed during the process. This leads to the interesting corollary that, if the roll necks carried in celoron bearings are initially smooth, they remain smooth and become highly polished during use, being subjected to almost negligible wear. The mirror-like surface, which celoron bearings impart to roll necks, not only reduces the wear of both neck and bearing, but is also partially responsible for the low coefficient of friction of these bearings.

The rate of wear is much less than that of most metal bearings operating under identical conditions. This comparison can probably be expressed most conveniently in terms of the tonnage rolled during the lift of the bearings. Experience has indicated that the tonnage ratio, with rolling and lubrication conditions unchanged, is 3 to 1 in many instances, and frequently even greater, in favor of celoron as compared to metal bearings.

In addition to the rate of wear, there is one salient difference between celoron and metal bearings from the standpoint of wear. Metal bearings, as is well known, wear more rapidly at the edge nearer to the roll body due to the deflection or spring of the rolls. This, together with the rigidity of the bearings, frequently causes the contact between the bearing and roll neck to be restricted to the bearing edge. The result is that the portion of the bearing nearer to the roll body gradually assumes a conical shape, while the outer portion of the bearing remains nearly cylindrical, so that however much or little the roll may deflect, the load must always be carried on a limited portion of the bearing area. When celoron bearings are used, however, the resilience of the material allows the bearing to follow the deflection of the roll neck, at least to some extent, so that the load, although slightly limited in distribution, can be supported on a larger bearing area than in the case of metal bearings. Consequently, the wear is more evenly distributed than with metal bearings.

Lubrication and Maintenance. It has already been mentioned that celoron bearings serve satisfactorily when water only is used as a lubricant. In this case, although the application of large quantities of water is not essential to successful operation, it doubtless serves to prolong bearing life by removing or preventing the entrance to foreign matter. It is known also that the increased cost of lubrication with oil or grease, instead of water, is more than compensated for by reduced friction losses and extended bearing life. Oil and grease have no destructive effect upon celoron, and they may be used as freely as desired. It has been found in practice that these bearings can be operated for short periods of time with no lubrication whatever. Operation without lubrication is not recommended, of course, but there is a distinct advantage in using bearings which can withstand such an emergency.

No particular care is required in the storage and maintenance of celoron bearings. They differ in this respect from wood bearings, which must be handled carefully and kept somewhat moist in order to prevent splitting. Celoron is unaffected by moisture or dryness, and can be stored indefinitely with depreciation. The life of such bearings can probably be extended by inspection and cleaning of the bearing surfaces during roll changes, although this precaution is not essential.

Cost. In general, the first cost of molded celoron bearings is comparable with that of bronze of lignum-vitae bearings of the same size. The first cost of a machined bearing may be somewhat higher, because of the additional material and labor which are involved in cutting a bearing out of a solid block. The cost of molds is not excessive; hence, molded bearings can be produced economically whenever about 200 bearings of the same size are required annually. The dimensions of molded bearings are held to a tolerance of 0.030 in. or less, depending upon the size of the bearing, so that no machining whatever is required.

The first cost is certainly not prohibitive. The fact should be emphasized, in addition, that the ultimate cost of roll-neck bearings involves not only the first cost, but also the length of life, cost of lubrication, cost of the power consumed in friction, continuity of operation, and maintenance of uniform gage in the rolled product. Comparison has already been made between celoron and metal bearings with respect to length of life and power losses in friction. With regard to the maintenance of uniform gage in the rolled product the following statement will be of interest: A producer of automotive axle stock in the Buffalo district, after installing celoron bearings, reported that off-gage rejections had been reduced from 7 per cent of total production to practically zero. Inasmuch as fewer shutdowns are required for the adjustment or replacement of bearings, the continuity of operation, in comparison to metal bearings, is greater. For these reasons, the ultimate cost of bearings is likely to be appreciably less than that of any other type of bearing which is in common use at the present time.

Design. The principles of design of celoron roll-neck bearings are quite simple. In addition to the elementary specifications (length and diameter of roll neck, radius of fillet at roll neck, and dimensions of bearing chock), only two precautions need to be observed: First, the thickness of the bearing section should not be more than 1 in. for neck diameters up to 20 in., to insure accuracy of rolling; second, the plane of the laminations of the material should be perpendicular to the axis of the roll neck, to provide the maximum resistance to wear.

The absorption of end thrust by bronze roll-neck bearings often calls for some special feature, such as overhanging section or a separable thrust collar. With the use of celoron bearings, no special provision is required unless the end thrust forces are unusually great, in which case provision can be made as readily as with bronze bearings. In any case, the portion of the roll-neck fillet or roll body which transmits end thrust should be smoothly polished to minimize the wear of the thrust surface.

Applications. Celoron roll-neck bearings have been applied successfully to a great many rolling mills of different types, such as bar, billet, sheet, strip, skelp, and rod mills. The experience which has been gained, by means of these applications, has shown conclusively that these bearings can be used advantageously under a wide variety of rolling conditions.

Conclusions. Celoron bearings, like other types of bearings, perform most efficiently under ideal conditions. Unlike many other types of bearings, however, these bearings perform quite well under adverse conditions. Consequently the advantages of their use are most evident when inadequate lubrication or excessive scale and grit make long-continued bearing operation very difficult, as in most rolling mills. This, being the chief distinction of celoron bearings, merits emphasis. It is regretted that the wide variability of the factors which enter into rolling-mill operation precludes a more extensive use of numerical data. It is hoped, however, that even with meager numerical data, the general statements which have been given will be of some assistance in the judicious selection of roll-neck bearings.

(Discussion by J. R. Adams and E. S. Glauch will be found following the paper by S. M. Weckstein.)

IS-54-5

Application of Tapered Roller Bearings in Merchant-Bar Mills

By S. M. WECKSTEIN,[1] CANTON, OHIO

Tapered roller bearings have played an important part in the numerous developments in the art of rolling metals. Their use has made possible a considerable decrease in the consumption of power by the elimination of friction, a considerable decrease in the cost of the electrical equipment required, a decrease in the cost of lubrication and general maintenance, an increase in the speeds of operation and consequently the tonnage produced, and a marked increase in the accuracy of the product. In this paper the actual experience, over a period of five years, in the proper design, mounting, and operation of bearings in merchant-bar mills is discussed. Extracts are given from the results of a series of power tests made on the same mills alternately equipped with plain and tapered roller bearings. Various means of lubrication of rolling mills, pinion stands, and gear drives are cited. The design of suitable bearing closures is also taken up and shown in detail. The illustrations shown are of equipment in actual operation in various mills.

THE art of rolling metals has seen numerous developments during the past few years. The general trend in industry to bring down manufacturing costs by a reduction in the number of operations has resulted in a tightening of specification on the material to be fabricated. This demand for higher standards and for a greater variety of special sections held to close tolerances has largely dictated the adoption of new equipment and new rolling practices.

It is natural that in rolling-mill equipment the bearings should come in for considerable investigation. Upon them falls the responsibility of decreasing the power consumption by the elimination of friction, decreasing the cost of the electrical equipment required, decreasing the cost of lubrication and general maintenance, increasing the speeds of operation and consequently the tonnage produced, and increasing the accuracy of the product. The importance of these points is very clearly illustrated in the number of cluster, four-high, and two-high mills for the rolling of strip and sheets and three-high and two-high merchant-bar mills for the rolling of ferrous and non-ferrous materials that have been built with anti-friction bearings in the last few years and that are now under contemplation. A study of these various installations would be very interesting. Because of the extent of the subject this paper will be limited to the use of anti-friction bearings in merchant-bar mills.

Tapered roller bearings have been successfully applied in merchant-bar mills and have the following advantages. With them it is possible to obtain perfect alignment and accurate setting of the rolls and to maintain this setting indefinitely. This is due to the fact that the wear in the bearings is practically negligible. It is possible to make provision on the mill for adjusting devices so that the setting of the rolls can be changed by any required increment in a minimum of time. With this accurate control of the mill, a product held to close tolerances is assured.

The bearing can be set with the required amount of radial clearance to suit the individual mill. In other words, by varying the lateral looseness of the bearing the desired amount of radial clearance can be obtained to suit either the roughing, intermediate, or finishing stand. The product can therefore be controlled all along the line, and proper tolerances can be maintained. Also because of the ability of the bearing to carry thrust as well as radial loads, accessory bearings or thrust washers are eliminated. The simplicity of the mounting and the ease of roll changes and roll setting are illustrated in Figs. 1 to 3, which feature an application on an 18-in. three-high bar mill.

Fig. 1 shows a general arrangement of the mill. The same size of bearing is used on all the roll necks, and the mounting is the same with the exception of the floating end of the middle roll. Each bearing is assembled in a bearing box, forming a complete unit that can readily be slipped on and off the roll neck. The bottom roll and bearing boxes are fitted directly into the mill housing. The middle-roll bearing boxes are made so that the top-roll bearing boxes can be fitted into them. In this way it is possible to locate positively the middle box on the ledge in the housing by means of wedges under the housing cap. The bottom roll is carried on the bottom screw, while the top roll is balanced from the top lifting rods. The right-hand half of the end view shows the fixed end of the mill, while the left-hand side shows the floating side of the mill.

Figs. 2 and 3 show the detail construction. Fig. 2 shows on the right-hand side the construction used for the fixed end of the top and bottom rolls. Each bearing is mounted as a unit in the bearing box, and the entire assembly, together with inside and outside filler rings, is slid on the neck and clamped by means of the nut shown. To reduce the cost of the rolls and eliminate threading them, a groove is cut in the neck and a two-piece threaded ring is fitted and doweled in the groove. The nut is bolted to the keyed filler ring after the bearing has been pulled up tight on the neck and against the inside filler ring. To obtain a very fine adjustment increment on the locking arrangement the following scheme is used. The keyed outer filler ring has a proper number of equally spaced tapped holes. The nut has an odd number (one more or one less) of holes drilled in it to receive the locking cap screw. The bearing box is provided with a special flange used for the alignment of the passes. With the bearing fixed to the neck, as described, the entire roll can be moved in or

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

out the desired amount merely by adjusting the push-and-pull nuts in contact with the bearing-box flange.

The construction used on the floating end of the top and bottom roll is shown in the left-hand side of Fig. 2. It will be noted that the bearing mounting is the same as for the fixed end. The bearing box itself is made without a flange and is not located endwise in the mill housing. In this way it can float in the mill housing and properly position itself when pass alignment is made or when roll expansion occurs.

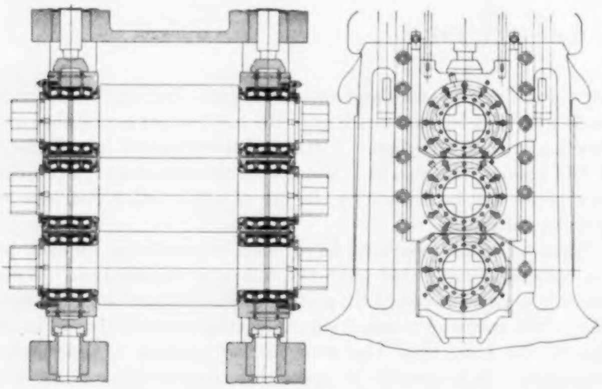

FIG. 1 GENERAL ARRANGEMENT OF THREE-HIGH 18-IN. BAR MILL
(In side view, left side of center line shows floating end and right side shows fixed end.)

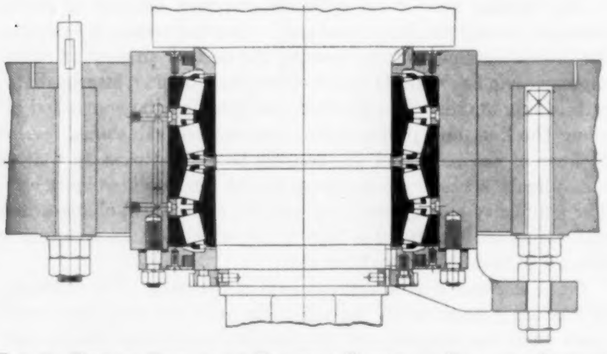

FIG. 2 DETAIL DESIGN OF BEARING BOXES ON TOP AND BOTTOM ROLLS
(Left side of center line shows floating end of top or bottom roll and right side shows fixed end of top or bottom roll.)

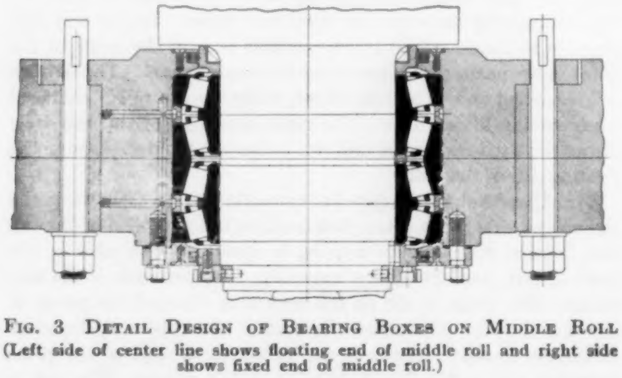

FIG. 3 DETAIL DESIGN OF BEARING BOXES ON MIDDLE ROLL
(Left side of center line shows floating end of middle roll and right side shows fixed end of middle roll.)

The mounting of the bearings on the middle-roll necks is slightly different at each end of the roll, as shown in Fig. 3. The two bearing chocks are made exactly the same and are rigidly clamped down on the ledge in the housing. They are also bolted to the housings through the flanges, as shown. Provision for expansion must therefore be made by the floating of the cones on one of the necks of the middle roll, as illustrated in the left-hand side of Fig. 3. It is to be understood that the roll necks are exactly the same at both ends. The provisions for floating are obtained by a slight change in the design of the inner and outer filler rings. The right-hand half of the illustration shows the fixed end of the roll, which is the same as the mounting used for the bearings of the top and bottom rolls.

The bearings shown in these illustrations are provided with cup and cone spacers. These spacers are ground to exact size so as to give the predetermined running clearance in the bearing and are supplied as part of the bearing. The inner and outer filler rings as well as the cone, or inner race, spacer ring are slotted to permit lubricant to reach the contacting surfaces and also to get down under the races, which are fitted with a given looseness on the roll necks. The grease for lubricating the bearings is supplied from two distinct inlets which register with the annular groove and holes drilled in the cup, or outer race spacers. This assures a uniform supply of lubricant to all parts of the bearings. It also has been found necessary to vent all the bearing boxes to eliminate air pockets, which in many cases caused lack of lubrication, with consequent bearing failures. The closures for retaining the lubricant and excluding water and scale are of special design. The details of the closures will be discussed farther on.

In view of the fact that the bearings are mounted loosely on the necks, it has been necessary to establish the proper looseness for various size necks. It has been found that the clamping arrangement used very definitely retards the amount of neck wear due to the loose fit. It has further been definitely established that on two- and three-high merchant mills in sizes from 28-in. to 16-in. roll diameter, the rolls are worn out long before the neck wear becomes a factor in the operation of the mill.

On smaller mills operating at higher speeds, the more rapid roll-neck wear has dictated a design calling for races fitted tight on the necks, as shown in Fig. 4. This layout features the use of a two-row bearing, but the same principle of mounting has worked out in service in a high-speed mill using four-row bearings. The mounting of the bearings on all the necks is the same. The inner races are fitted with a press fit of from 0.0005 to 0.002 in. tight and in addition are clamped up tight by the same locking arrangement as used on the slower speed mills. The design of the bearing box is altered so as to provide for the easy removal of the bearings. The boxes are made so that the inner closure plate is a separate piece. Also the inner filler ring is provided with a shoulder. When the inner closure is bolted to the bearing

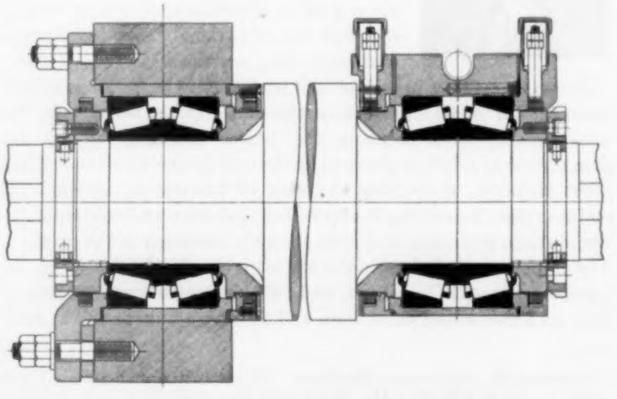

FIG. 4 DETAIL OF BEARING-BOX DESIGN ON ROLL NECKS OF HIGH-SPEED MILLS
(Upper side of center line shows horizontal section through fixed chock and lower side through adjustable chock.)

box, a special split plate from $^{1}/_{8}$ to $^{3}/_{16}$ in. thick is inserted between them. The bearing box, with inner filler ring, inner closure, separating plate, bearings, and outer closure, is assembled as a unit, heated in oil at about 300 deg. fahr., and easily slipped on the roll neck. The outer filler ring and locking nut are then added, and the assembly is complete. To remove the bearings, the split plate is removed, and the inner closure is drawn up tight to the bearing box. The shoulder on the filler ring then contacts with the inner closure, to which pressure can be applied by means of a press or properly designed puller. Very little difficulty is encountered in removing the bearings.

It is interesting in this connection to relate an experience in the removal of bearings mounted in this way on a two-high 12-in. mill operating at a little over 400 r.p.m. The bearings used were of the four-row type and of 6 in. bore. The fit of the bearing on one neck was found to be approximately 0.003 in. loose, and to remove the bearings, it required approximately 43,000 lb. Another bearing was found to be 0.0003 in. loose, and it required 35,000 lb. to remove the bearing. The third bearing was found to be 0.0002 in. tight, and 28,000 lb. was required to remove the bearing. The fourth bearing was 0.0007 in. tight, and only 15,000 lb. was required. This evidently shows that with the loose fit and the high speed the inner races creep excessively on the necks, roughing them up. There also is evidence of corrosion or oxidation between the neck and the races, making it hard to remove them. In other words, with the fit held somewhere between the limits of 0.0005 to 0.002 in. tight, the least amount of force is required to remove the bearings.

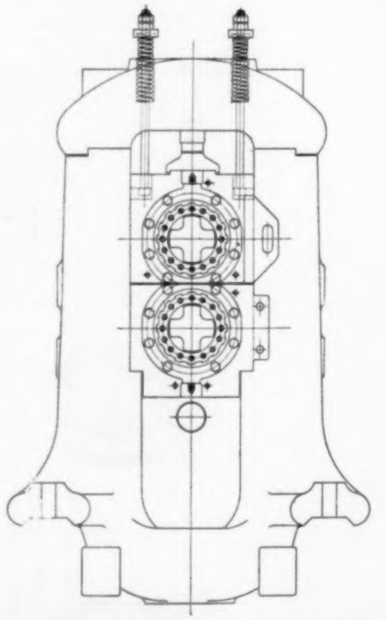

Fig. 5 General Arrangement of Two-High Mill

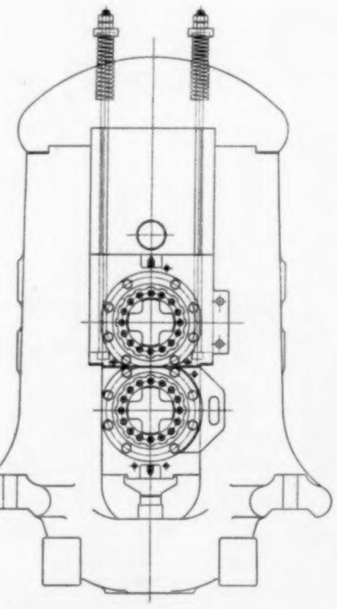

Fig. 6 General Arrangement of Two-High Mill

Referring again to Fig. 4 it will be seen that in a two-high mill one roll can be fixed and the other be made adjustable for pass alignment. On both rolls the boxes at one end are permitted to float to take care of expansion and ease of pass alignment, as previously described. The upper left-hand section shows the method of attaching the fixed roll to the housing. The lower left-hand section shows the scheme used to obtain pass alignment. The bearing box is made with a special flange, between which and the housing an adjustable wedge is inserted. By loosening the nuts holding the flange, the wedge can be adjusted in or out to give the desired setting of the passes in the rolls. A more detailed view of the construction of the closures is shown in the upper right-hand section. The lubrication is the same as for the lower-speed mills.

Figs. 5 and 6 show end views of the general arrangement of the bearing boxes in the mill housings. Fig. 5 shows the arrangement of the middle and top roll with the through spindle at the bottom. Fig. 6 shows the spindle at the top. In both cases the middle roll is fixed, while the top or the bottom roll is adjustable.

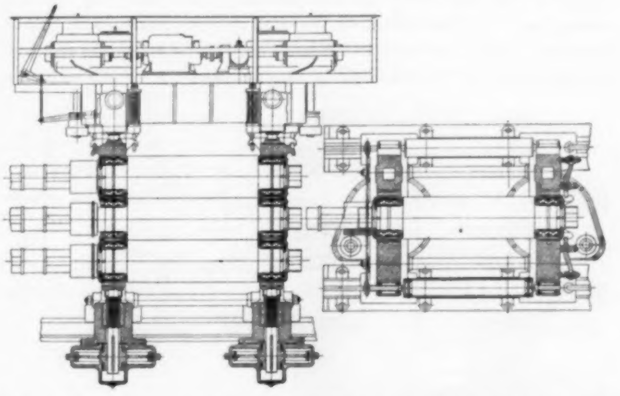

Fig. 7 Method of Aligning Roll Passes on 28-In. Three-High Mill

Before leaving the subject of the mills themselves and proceeding with pinion stands and drives, it will be interesting to study a few other methods, very successfully used on other mills, for the aligning of the passes on the rolls. Fig. 7 shows the design of the 28-in. three-high bar mill used in the plant of the Timken Steel and Tube Company. The general construction is the same as discussed for the other mills. The pass-alignment feature is different, as can be seen in the right-hand section. The bearing box is provided with lugs in which slots are machined. Special clamps fitting into the slots are bolted to the mill housing. By the use of ball seats and special washer, proper functioning of the clamps can be obtained from the push-and-pull nuts at the extreme ends. The holes in the clamps are elongated so that the clamps can be slid out of the way when a roll change is made. This design has worked out satisfactorily on this mill with one exception—the slots in the lugs pounded out. This was remedied by remachining them and inserting hardened steel liners or wedges.

Vertical mills, and especially vertical finishing mills, require suitable means for the alignment of the roll passes. The design shown in Fig. 8 shows the general application of the bearings on an 11-in. vertical finishing mill which has been in service for about two years. The mounting of the bearings is practically the same as for the horizontal mills. The closures must be more positive, and consequently flingers forming labyrinths have been utilized. The rolls are hung from the top bearing boxes. The bearings, because of their thrust carrying capacity, easily support the weights of the rolls and boxes and the thrust loads from rolling. The bottom boxes are permitted to float so as to take care of roll expansion and not to interfere with pass alignment, which is obtained from the top box, as shown in Fig. 9. The top bearing closure cap is designed with an extension having six threads per inch. A special bracket is bolted to the mill housing and attached to the threaded extension of the bearing box cap by means of two jam nuts and a specially designed adjusting plate. The bracket has 23 equally spaced holes and the adjusting plates 24 equally spaced holes. By loosening the jam nuts and by tightening or loosening the adjusting plate an increment of vertical rise and fall of the roll of 0.0003 in. can be obtained. When the desired set-up is reached the adjusting plate is locked to the bracket by means of a pin inserted in the two mating holes. The jam nuts are brought up tight to hold the set-up rigidly.

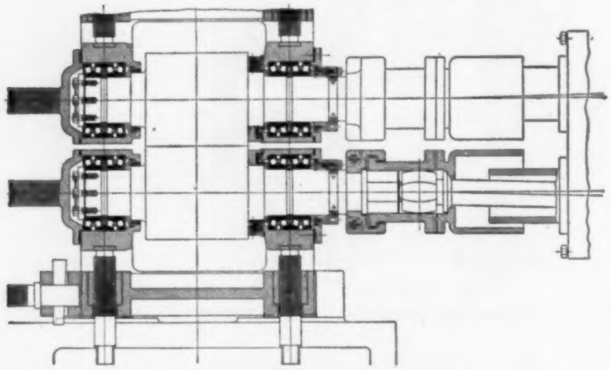

Fig. 8 General Arrangement of 11-In. Vertical Mill

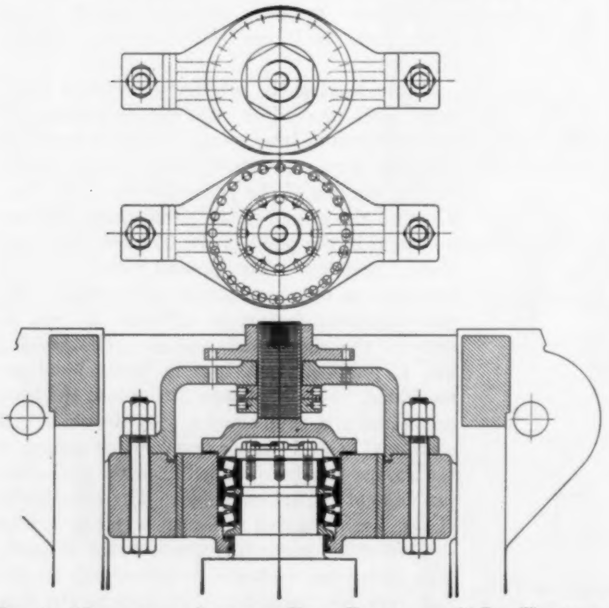

Fig. 9 Method of Aligning Roll Passes on 11-In. Vertical Mill

Pinion Stands

The application of tapered roller bearings on pinion stands has kept in step with their use on the roll necks themselves. In view of the lighter loads, less capacity has been required, and as a general rule one-half of the roll-neck bearing is used on the pinion necks; in other words, where a four-row bearing is used on the roll neck, a two-row bearing is used on the pinion neck. Originally the bearings were mounted loose on the pinion necks without clamping in order to allow for the float of the double-helical pinions. This type of mounting is gradually being abandoned in favor of the design shown in Figs. 10 to 12, which show the general arrangement of three-high and two-high pinion stands and a detail of the bearing mounting on the neck. It will be noted from these figures, and especially by reference to Fig. 12, that each bearing box is provided with a flange which is bolted to the pinion housing, thus positively locating the bearing boxes. The inner races are mounted on necks with a tight press fit. The outer races are given a loose fit in the bearing boxes, permitting them to float. The amount of float is controlled by the pilots on the closure plates. The outer closure plates are machined so as to permit about $^1/_8$-in. float. The inner closure plate pilot is cut off short, since its only function is to properly pilot the closure. The bearings, in addition to being press-fitted, are clamped on the necks by means of the usual locking arrangement. The inner and outer filler rings act as additional closures to retain the lubricant in the bearing boxes and to keep out the gear lubricant. To remove the bearings from the necks it is necessary only to take the assembled pinion and bearing boxes out of the stand, remove the locking nut on the neck, and

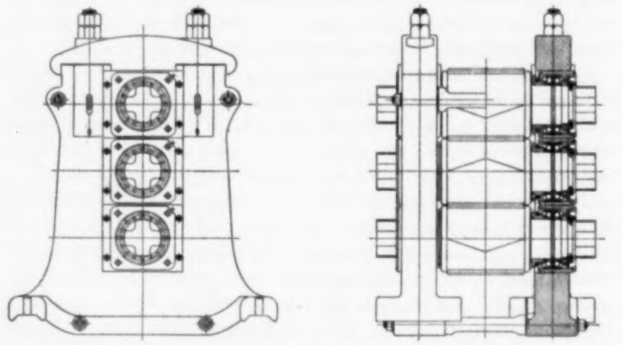

Fig. 10 General Arrangement of Three-High Pinion Stand

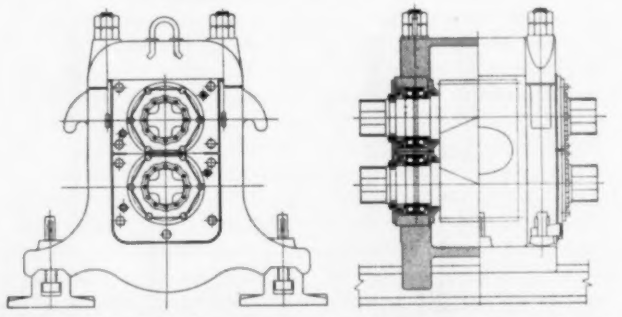

Fig. 11 General Arrangement of Two-High Pinion Stand

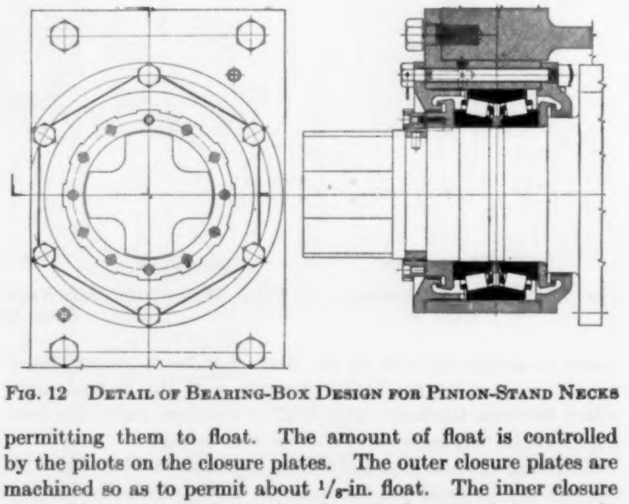

Fig. 12 Detail of Bearing-Box Design for Pinion-Stand Necks

apply pressure to the inner closure plate, which will automatically come in contact with the inner filler ring and push directly on the inner races. The bearing box, bearings, and closures can at all times remain as an assembled unit.

For the lubrication of the pinion-stand bearings, either grease or oil may be used and introduced to the bearings as shown in Fig. 12. It is important that the bearing box be properly vented.

Gear Drives

With the general increase in the speed of rolling and with the further development of the continuous and semi-continuous merchant-bar mills, gear drives have been called upon to operate at much higher speeds than ever before. To make this practical, anti-friction bearings were installed. Tapered roller bearings have been especially desirable because of their ability to carry the thrust loads from the mills and gears. A number of various designs have been worked and have proved satisfactory in service over a period of more than five years.

Fig. 13 shows a typical single-helical-gear reduction unit. A double bearing is fitted tight on each end of the shaft and properly enclosed in a bearing box. The bearing box at one end is machined with an annular projection which fits into a groove in the gear housing and cap. This box rigidly locates the shaft against endwise movement and carries the axial loads. The opposite box is made similar except that the annular projection is machined off, thus permitting the box to float and take care of variations in machining and expansion. Each box is fitted with a round dowel pin extending down from the cap to prevent possible rotation of the box in its seat. This dowel pin is drilled so that the lubrication pipe can be threaded into the bearing box. The design, as shown in Fig. 13, is developed for separate lubrication of bearings and gears. The bearings are completely enclosed and isolated from the gears. The bearing boxes may be packed with grease or a level of oil may be maintained. This design is suitable for relatively small bearings operating at low speeds.

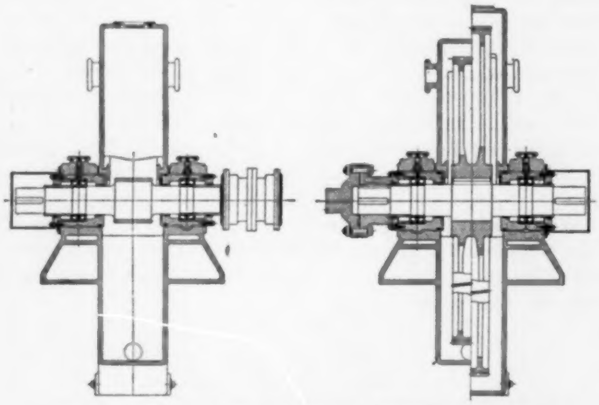
FIG. 13 SINGLE-HELICAL-GEAR REDUCTION UNIT

In quite a number of gear units using larger bearings and operating at higher speeds, it has been found necessary to incorporate more complete lubrication systems in order to keep the temperatures down. This has been accomplished in two ways. In one case the bearings have been completely isolated from the gears and a complete circulating oil system has been used to lubricate them. The oil is introduced between the bearings, and the returns from the bearing boxes are located so as to maintain an oil level around the bearings. The amount of oil to be circulated through the bearings is best determined by a careful check during the operation of the unit. In a number of other cases it has been found possible to lubricate the bearings satisfactorily from the splash of the gears. In these instances the inner closures on the bearing boxes are omitted, and the designs are made so that a level can be maintained around the bearings, with the excess oil overflowing into the gear case. To assure positive lubrication of the bearings it has been found best to locate baffles on the inside walls of the gear case. The splash from the gears is caught in these baffles, and is drained from these by means of pipes direct to the bearings. It is necessary to drill and tap the sides of the case at the baffles to receive the pipes which should carry the oil to the proper point in the bearing boxes. In this system it is important that the quality of the oil be such as to properly lubricate both gears and bearings.

Another satisfactory method of bearing mounting in gear reduction units is illustrated in Fig. 14. In this design the bearings are lubricated from the splash of the gears. The cost is considerably reduced by the elimination of the bearing cartridges featured in Fig. 13 and the mounting of the bearing direct in the gear case. For this purpose two-row bearings having unit outer races are used. The design shown is worked out for double-helical gears. The low speed or take-off shaft is fixed against endwise movement by means of the center bearings, thus taking care of the thrust from the mill and from the gears. All the other shafts are allowed to float so as to properly mesh with the mating gears. Another feature of this design is that the bearings shown are set at the bearing factory by means of spacers between the inner races. This completely eliminates the necessity of bearing adjustment during the assembly of the unit.

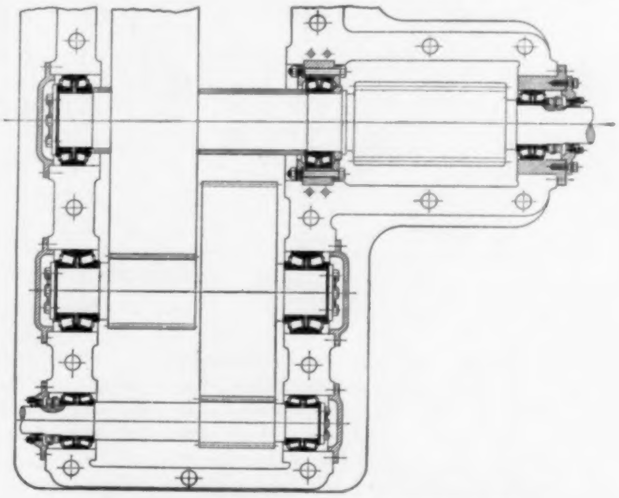
FIG. 14 DOUBLE-HELICAL-GEAR REDUCTION UNIT

Closures

In the preceding illustrations there have been shown various closures that have been used on the mills, pinion stands, and drives. The mill bearings, due to the presence of water and scale, naturally require a more positive closure than do the other applications. After considerable testing had been done, a closure was developed that has proved efficient under bad water conditions, and a small model was built. (See Fig. 15.) The bearing mounting is the same as that used on the actual mills. The closure consists of a strip of metallic packing properly jointed at the ends with a flexible joint. This packing rides on the outside diameter of the inner and outer filler rings. Around this packing is used a thin steel band, the two ends of which are drilled with a small hole to permit of connecting them with the specially designed threaded adjusting bolt which fits into a collar. This collar is round in the body, but upset at the bottom to keep

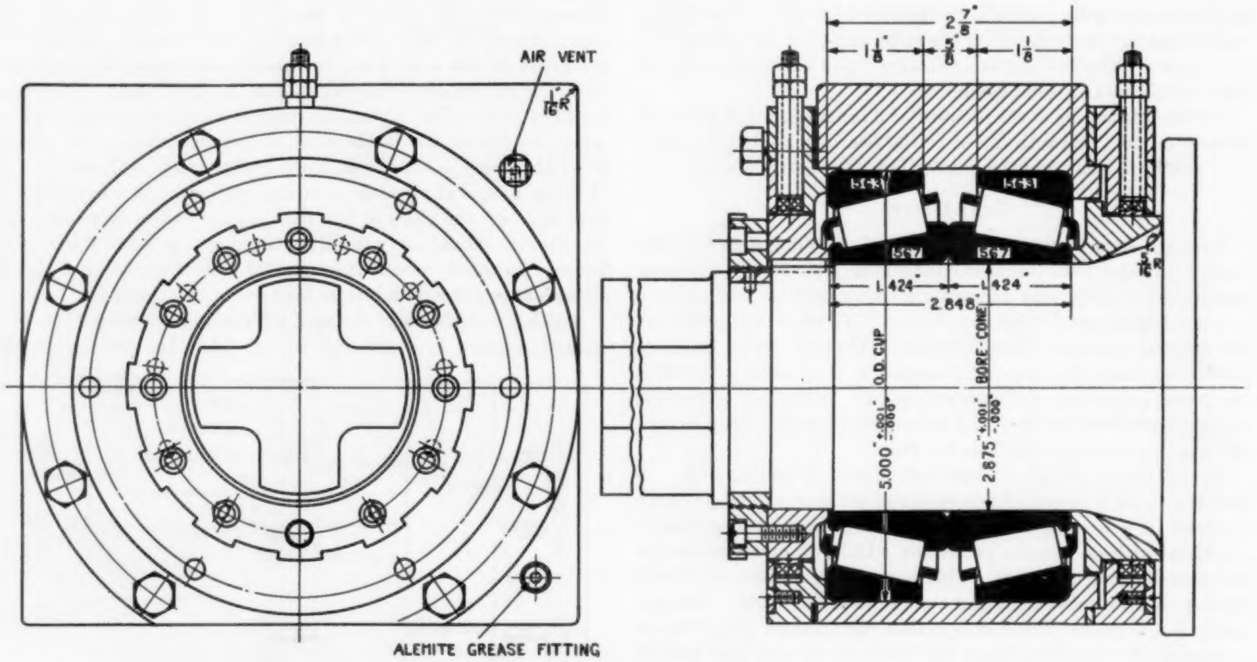

Fig. 15 Details of Clock-Spring Closure

it from turning when inserted in the closure plates. By tightening the two nuts on the adjusting bolt, the required fit of the packing on the filler ring is obtained. In mills where other closures were used ineffectively to keep water and scale out of the bearings and to stop grease leakage, this type of closure has practically eliminated both conditions. A slight improvement over this design has been made by the addition of threaded pipe caps to protect the adjusting nuts, as shown in Fig. 4. Another improvement is being contemplated in that tests are now being made on the possibility of making this closure automatically adjustable by the addition of suitable springs.

It will be of interest to cite some savings in power and maintenance that are being obtained by the use of anti-friction bearings. In a series of power tests made by the General Electric Company at the plant of the Timken Steel and Tube Company it was possible to make comparative tests on two stands of 22-in. three-high mills equipped with plain and with Timken bearings. The tests were made while rolling 8-in. by 9-in. blooms down to 2$^{11}/_{16}$ in. square in 15 passes, 7 passes being taken in the first stand and 8 passes in the second stand. The steel rolled was carbon 0.45, manganese 0.67, and nickel 3.50. Table 1 shows the results obtained.

From these results and from other tests and study of merchant mills equipped with anti-friction bearings a power saving is indicated of from 25 to 50 per cent, with an average saving of about 35 per cent. This means a large saving, in building a new mill, on the cost of the electrical equipment.

TABLE 1

Plain bearings		Timken bearings		Per cent
Roll speed	Hp-hr.	Roll speed	Hp-hr.	saving
83.5	30.653	82.5	15.800	48.5
71.0	25.001	75.2	14.888	40.5
61.0	25.874	63.2	13.362	48.3
52.0	22.806	52.7	13.546	40.6

The saving possible on the cost of lubrication is also an important item. In Fig. 7 is shown a 28-in. three-high mill. This stand is one of a complete unit consisting of two stands of 28-in. three-high mills and one three-high pinion stand. The mills are equipped with bearings of 16 in. bore, 24 in. outside diameter, and 12$^1/_2$ in. wide. The pinion stand is equipped with two bearings, each of 16 in. bore, 24 in. outside diameter, and 3$^5/_8$ in. wide, separated by a spacer so as to make the bearing box practically as wide as the mill bearing box. These 12 mill bearing boxes and 6 pinion-stand boxes consumed 350 lb. of grease in one week's operation of 24 hours per day. This is equal to 58.3 lb. of grease per day, or 3.25 lb. of grease per day per box.

With the further improvement being made in the bearing design and mounting and the improvement in closures and general operating condition, greater savings in power and maintenance may be expected.

Discussion

A. G. Witting.[2] There of course cannot be any question about the desirability of having the bearing-friction losses in a rolling mill reduced to a minimum nor that the anti-friction bearing of the roller type may be particularly suitable. Such bearings may, besides, offer special advantages for cold-rolling mills and sheet mills or in general for mills where the gage tolerances are extremely small.

There may, however, be two sides to the question when this type of bearing is urged for mills rolling ordinary merchant-bar products. It has been brought out in the paper and the discussion that the greater difference in roll and roll-neck diameters may require more expensive material for the rolls; also, much more accurate machining, including polishing of the roll necks, will be necessary, and the question may be raised if this additional cost brings corresponding benefits in cases where the maintenance of an exact gage is not of paramount importance.

There is another factor that may have to be considered because it may affect the rolling costs more than the expense for rolls and the bearings themselves; this is the ease with which a roll change can be made. In a mill where such changes occur frequently, perhaps several times a day, it is out of the question to have bearings that must be pressed off the rolls, whether it takes 80 or only 10 tons. It is also impractical and expensive to have a great number of spare sets all made up beforehand.

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[2] Chief Engineer, Illinois Steel Company, Gary, Ind.

To make the application of roller-type bearings practical for an ordinary mill, even provided that the saving in power and longer length of the bearings compensate for the higher first cost, it is necessary that the bearing is assembled in one piece and that the roll neck and the inner race of the bearing are made tapered so that the bearing can be quickly slipped on the roll neck and located in its proper place by a whack of a sledge hammer and similarly loosened, ready to be lifted off, by a hammer blow

(The two following discussions apply to all five papers in the symposium on bearings.)

J. R. ADAMS.[3] The question of modernizing older rolling-mill installations is dependent very often on the space available between the window housings of the mill and the roll necks. In many cases these mills were originally designed for rolls made of cast iron. Later these mills may have been changed over to make use of forged-steel rolls, but maintaining the same roll-neck diameter. The fact the necks of forged and heat-treated steel rolls are very much stronger than the necks of the cast-iron rolls originally used had meanwhile been lost sight of.

There are probably many cases where the neck diameter of forged and heat-treated steel rolls may be reduced without danger of failure, thereby enlarging the space between the neck and the window housings a sufficient amount to permit the use of anti-friction bearings.

E. S. GLAUCH.[4] Papers have been presented on various types of bearings, and one might obtain the idea that the bronze and babbitt types of bearings for steel-mill service are obsolete. However, the writer is of the opinion that there are still quite a number of places in steel-mill service, especially on rolling mills, where the bronze and babbitt-lined bearings are giving good service and meet the requirements imposed upon them.

Cases have been reported where the various bearings showed a lower friction loss than had been obtained with metal bearings, prior to the installation of some of the bearings mentioned.

The writer is quite sure that in many cases these large savings would not have been possible had the bronze or babbitt-lined bearings been well lubricated, for he has made a great many tests in the past few years on rolling mills and has found that, by substituting better grades of lubricant, in some cases power savings as high as 25 to 30 per cent could be accomplished without any change in bearings.

Up until a few years ago it was rather hard for an outsider to get some steel-mill superintendents and engineers interested in the subject of roll-neck lubricants, as they felt that no matter what lubricant was used, a large percentage would find its way to the sewer.

However, since the advent of roller bearings for this service and the possible savings in power, which are of course due to the design of the bearing, and also the fact that the roller bearing is completely enclosed and well lubricated, more interest has been taken in obtaining better lubricants for plain bearings in roll-neck service.

Also, some of the systems which have been developed for supplying grease to these roll-neck bearings have resulted in reducing power consumption greatly on the bronze or babbitt-lined bearings due to constant lubrication.

There is of course an advantage in bearings of the celoron and lignum-vitae type in that they will operate with water only, whereas brass or babbitt-lined bearings require a lubricant, which on rolling mills is usually grease. On a special test equipment which the writer has been using in making a large number of tests, it has been found that if only a very small percentage of the bearing is allowed to operate without a lubricant, power consumption increases very rapidly.

The positive system of feeding a lubricant as obtained with pressure systems will eliminate the possibility of the bearings operating partially lubricated, which condition has existed to a large extent in roll-neck service with the old method of packing the necks with a heavy grease.

Also, the writer has found in making tests that some of the heavy roll-neck greases that have been used were made very stiff and adhesive in order to stay in the openings provided and resist the action of water, and these greases would provide a heavy sticky coating, which while to some extent serving as a protection to the bearings would consume considerable power.

This is especially true if the mill was shut down and allowed to cool, as it would require considerable power to start the mill, and for the first several hours of operation the power consumption would be quite high.

Greases can be secured which will lubricate the roll necks with a much thinner film of lubricant and with a much lower power consumption.

In connection with the various cases cited, as to savings obtained by the use of bearings mentioned in the various papers, the writer would be interested in knowing if any of these comparisons are made on mills which were lubricated by pressure systems, for he is quite sure that had the mills been lubricated with a positive system, the savings would not have been as great as indicated by the comparisons given.

The writer might also mention a case where bronze bearings on a bar mill rolling tool steel had been lubricated with suet or tallow. This the mill operators felt was the best lubricant up to that time, but have changed over so that the bronze bearings have graphite inserts placed in them, and then also a graphite grease is used for lubrication, and power consumption was reduced about 25 per cent.

This is another case of power savings due to better lubrication. The graphite inserts in the bearing are somewhat of a safety feature in that if the grease did not stay up against the neck, the bearings would not be without some form of lubrication.

In fact, on the initial test brass with the graphite inserts was operated without the addition of grease, but water was of course used, and the brass ran eight weeks and then failed due to breakage, whereas brass on the opposite end of the roll, which was lubricated with suet in accordance with their previous standard practice, was worn out in three weeks' time and the neck of the roll was scored.

As mentioned in the beginning of this discussion, there are and undoubtedly will be a great many places where the bronze and babbitt-lined bearings will be used with good results, providing of course that they are lubricated, and with positive force feed much better results will be obtained than with the old method of applying the grease by packing the necks or using the paddle application.

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Production Management Applied to the Drafting Department

By WILLIAM J. KUNZ,[1] NEW YORK, N. Y.

This paper aims to bring out some of the more important factors in the management of the drafting department of one corporation. The output of this drafting department was in excess of 10,000 drawings in 1928 and 12,000 in 1929, varying from 1 to 10 sq. ft. There are three divisions, known as the boiler, stoker, and pulverized-fuel divisions. Each is divided into squads working on specific types of equipment. The management plans for a straight-line curve by moving experienced men from one division to another so that each division will carry a full allotment of work. The essence of the production is the speedy accomplishment of the work on the lowest cost basis and the unification of all interests in any specific feature of design.

IT IS NOT within the scope of this paper to give any lengthy discourse on the fundamental practices that might be followed in the production management of a drafting department, but it is the intention to bring out some of the prominent factors in the management of the drafting organization of Combustion Engineering Corporation.

This company, in the design and fabrication of its equipment, employs in its drafting department a personnel of as many as 280 men, the number employed being dependent on the amount of work on hand. The combined effort of the department resulted in a production in excess of 10,000 drawings in 1928 and 12,000 drawings in 1929, varying from 1 sq. ft. to 10 sq. ft. per drawing.

The drafting department is divided into three separate divisions—namely, boilers, stoker, and pulverized-fuel divisions. Each of these divisions is again divided into squads in accordance with the specific type of equipment for which design and drawings they are responsible. The organization chart, Fig. 1, is illustrative of the department's formation and some of its functions.

There are two classes of design established in each of these divisions—namely, the individual or specific equipment class, comprising a boiler or stoker or any other single product manufactured by this company. The second class of equipment is composed of a combination of any or all of these products as the contract requirements may demand. In the former class the design and drawing work will permit to some extent the use of so-called "standard drawings" which illustrate typical standard products. Drawings which are reproductions, the original of which was established and used on previous contracts, are also used wherever possible in the fulfilment of drawing requirements in this design class. In the latter class the possibility of using standard drawings is remote, as in almost every case the contract involves equipment of combinations and sizes that are rarely the same, with such exceptions as true duplications of previous contracts.

With the physical limitation of building enclosures and the ever-increasing demand of greater steam output per cubic foot of building space, it is obvious that design work in this latter class is of a more or less special nature, and while a definite principle of design may be followed on more than one contract, the possibility of using standard drawings is remote.

Boiler-Drafting Division

The boiler division comprises five squads and has a personnel of from 40 to 100 men, dependent on the amount of work on hand. In this number are included squad leaders, checkers, draftsmen, detailers, and bill-of-material clerks. The function of this division consists of producing designs and drawings necessary to fabricate and erect bent-tube boilers, sectional-header boilers, box-header boilers, economizers, and water walls. Also included in the responsibilities of this division is the development work necessary to keep abreast of present-day requirements and the standardization of designs and drawings in line with this development work.

Upon receipt of an order or contract for a boiler or other item of equipment in this division, the contract capacities and guarantees are checked by the contract calculating department. Concurrently the boiler engineers establish the designs preparatory to starting draftsmen on the layout or detail drafting work. The drawing work is more or less of a routine nature and occasionally a duplication. With the exception of special designs, extensive use is made of standard drawings, form drawings, and form bills of material, which have been made to suit a variety of conditions for each type of equipment.

Stoker-Drafting Division

The stoker division is similar in structure to the boiler division and has a personnel of from 30 to 50 men. Included in this number are likewise squad leaders, checkers, draftsmen, detailers, and bill-of-material clerks. The function of this division consists of the establishment of designs and the making of drawings necessary to fabricate and erect chain-grate stokers, multiple-retort stokers, and single-retort stokers. Development and standardization work likewise is an important function of this division. The responsibilities of engineering and drawing work on a contract are similar to those of the boiler division.

Pulverized-Fuel Drafting Division

The structure of this division is radically different from either the boiler or the stoker division. Its personnel varies from 50 to 150 men, according to the work on hand, and includes in its organization squad leaders, job leaders, checkers, designers, draftsmen, detailers, bill-of-material clerks, and in addition a

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

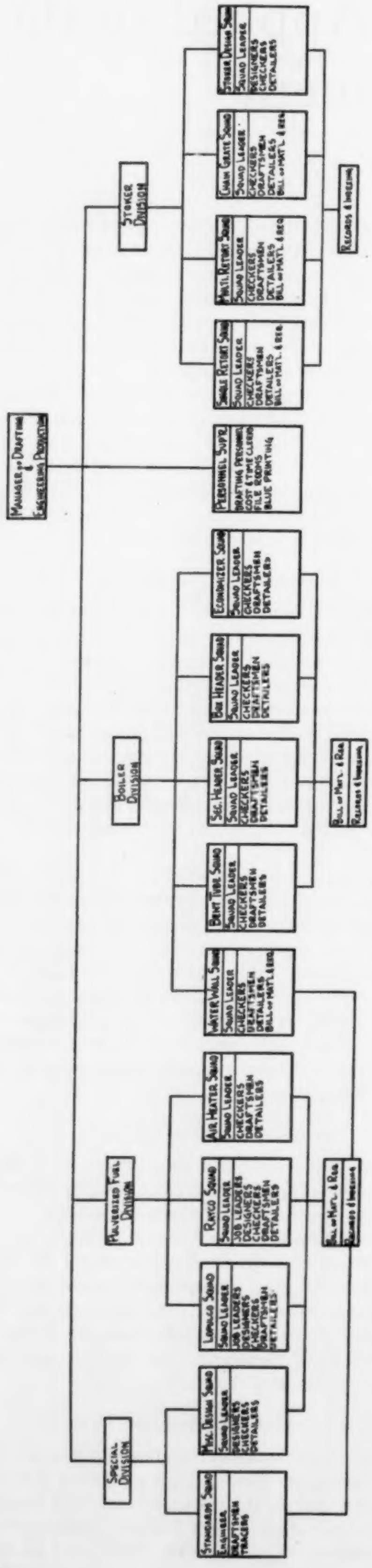

MANAGER OF DRAFTING & ENGINEERING PRODUCTION
SPECIAL DIVISION
PULVERIZED FUEL DIVISION
BOILER DIVISION
STOKER DIVISION
STANDARDS SQUAD
ENGINEER
DRAFTSMEN
TRACERS
MISC. DESIGN SQUAD
SQUAD LEADER
DESIGNERS
CHECKERS
DETAILERS
LOPULCO SQUAD
SQUAD LEADER
JOB LEADERS
DESIGNERS
CHECKERS
DRAFTSMEN
DETAILERS
RAYCO SQUAD
SQUAD LEADER
JOB LEADERS
DESIGNERS
CHECKERS
DRAFTSMEN
DETAILERS
AIR HEATER SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
WATER WALL SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
BILL OF MAT'L & REQ.
BENT TUBE SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
SEC. HEADER SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
BOX HEADER SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
ECONOMIZER SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
PERSONNEL SUP'R.
DRAFTING PERSONNEL
COST & TIME CLERKS
FILE ROOMS
BLUE PRINTING
SINGLE RETORT SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
BILL OF MAT'L. & REQ.
MULTI. RETORT SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
BILL OF MAT'L. & REQ.
CHAIN GRATE SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
BILL OF MAT'L. & REQ.
STOKER DESIGN SQUAD
SQUAD LEADER
DESIGNERS
CHECKERS
DETAILERS
BILL OF MAT'L. & REQ.
RECORDS & INDEXING
BILL OF MAT'L. & REQ.
RECORDS & INDEXING
RECORDS & INDEXING

FIG .1 ORGANIZATION CHART OF DRAFTING DEPARTMENT

clerical force for the proper compiling of departmental indexes. This group is divided into squads having the names of "Lopulco" "Rayco," and "air heater."

Unlike the boiler and stoker groups, the percentage of standard design in this group is relatively small, and the possibility of using standard or form drawings is very limited. Advantage is taken, of course, of the use of drawings established on previous contracts wherever possible, but such occasions are rare.

"Lopulco," a trade name, also designates a squad of men whose efforts are concentrated on producing designs and drawings that combine a number of products when used on indirect-fired boiler and furnace installations. "Rayco" designates a similar squad, but concentrates on direct-fired boiler and furnace installations.

The air heater squad is responsible for all layouts and detail drawing work required for the fabrication and erection of all types of air heaters and duct work.

SPECIAL DRAFTING DIVISION

Under this heading is included two squads, one of which is the machine-design squad. The efforts of this squad are concentrated on the designing of burners, feeders, blast gates, dampers, and numerous other parts that are required for a pulverized-fuel installation.

The second squad under this heading is the standards squad, which is a coordinating unit between all other squads and confines its design and drawing work to the establishment of standards and of standardization as it pertains to the products.

While a portion of the time in the Lopulco and Rayco squads is devoted to the establishment of designs of individual items of equipment such as boiler and furnace settings, pulverized-fuel transport systems, roll- and impact-mill systems, etc., by far the major portion of the work consists of making designs and drawings combining these various products.

While the boiler section is interested solely in the specific design of boiler equipment, and the stoker section is likewise interested in the specific design of the stoker equipment, it is within the scope of the Lopulco and Rayco sections to combine the boilers, stokers, pulverized-fuel equipment, furnace, steelwork, brickwork, ducts, fans, air heaters, water walls, casing, and other sundry items into one completed steam-generating installation. This requires the making of drawings which are not only suited for fabrication and erection purposes, but also are important to the customer in showing exactly of what the entire design consists, so far as the grouping of the products is concerned and their relation to each other.

Contracts for a combination of products are assigned to contract engineers who are the coordinating factors between customers and the corporation in the establishment of designs throughout the life of the job in the drafting department.

There is also assigned to this work a job leader who is responsible for all of the drawings made, and likewise is the coordinating factor between all other squads and the distributing agent for the dissemination of all information pertinent to the drafting work of the design as a whole.

As in the stoker and boiler work, the guarantees and capacities that have been established in the contract requirements are checked by a separate section of the engineering department, but it is the contract engineer under the supervision of the chief engineer who is vested with the responsibility of supervising the establishment of the designs and drawings of the job in its entirety.

The relation of the contract engineer and job leader to all squads, to the chief engineer, and to the manager of drafting on a combination contract is illustrated in Fig. 2.

It is obvious, from the nature of the work in this division, that the requirements demand skilled designers of steel, brickwork, furnaces, and a multitude of machine parts. However, this does not eliminate the fact that these men are draftsmen since their designs are worked out in drawing form at a drafting table, and as such they are included in the drafting organization.

Control of Production

In a drafting department such as this, the control of production does not involve alone the knowledge of what is being done by groups of individuals or squads, but also it is essential that the capabilities of, as well as the work on hand for each individual in such groups or in the department as a whole must be known.

In order to be fully informed as to the amount of work on hand for each man in the department, we have for ready reference a drafting department log sheet, Fig. 3, which is brought up to date as frequently as is deemed necessary. This log sheet contains the name of every draftsman, checker, job leader, and squad leader, and they are shown under the heading of their respective squads.

Periodically a survey is made of the work on hand, and against each individual is allotted an estimated time that is deemed necessary to bring to completion such drawings as are assigned to each man. This time, which is plotted in days, is then incorporated on the log sheet and forms a rather accurate estimate of the amount of work on hand for the individual or the particular group to which each individual is assigned and for the department as a whole.

The amount of work on hand for each squad can become very much unbalanced when considered in terms of man-day work for each squad, unless there are ways for correcting this condition.

The significance of this statement is brought out on the log sheet in the variation of the average day's work shown for each particular squad, and unless prompt steps are taken to overcome this condition, it will result in a part of the department being overloaded with work, and possibly necessitating overtime, while another part will be idling, with a scarcity of work.

There are two ways in which this unbalanced condition can be overcome: (1) by assigning such new work as may come in to those squads that are not fully occupied or by transferring a part of the work from one squad to another, or (2) by changing individuals from a squad that is not fully occupied to one that is overloaded. In both of these methods there are apparent handicaps, and it is only after diligent study as to the capabilities of an individual as well as to the nature of the work that either of these methods can be followed.

A condition must be taken into consideration where the designs of some product must be drawn essentially to completion within a definite squad. There is, however, a certain amount of leeway in most work of this nature which permits the interchange of individuals and squad facilities to other squads in the department than just the one more or less specialized on one specific product.

To permit transferring an individual from one squad to another, it is not sufficient that he have a good background of experience, but also that he has a fundamental understanding of his employers' products or equipment. A comprehensive knowledge of numerous products made by one company is seldom acquired except over a considerable period of time. There are, however, certain features of the equipment that are similar in nature of material or fabrication, such as structural steel, casings, brickwork, etc., and in such instances the interchanging of individuals from one squad to another can be done without reflecting increased drawing costs or decreased production.

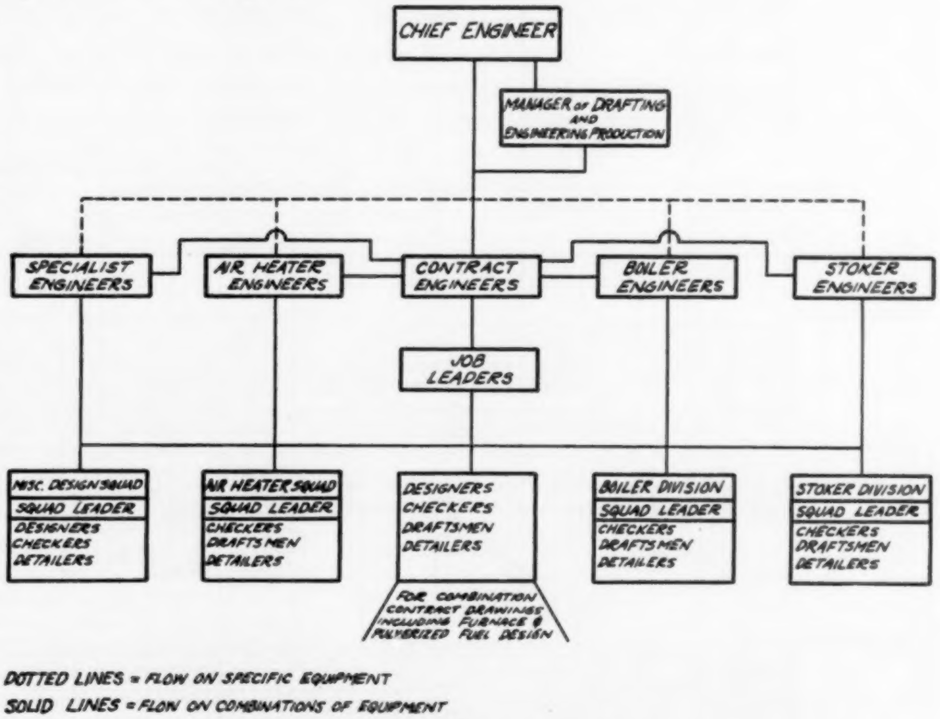

CHIEF ENGINEER
MANAGER OF DRAFTING AND ENGINEERING PRODUCTION
SPECIALIST ENGINEERS
AIR HEATER ENGINEERS
CONTRACT ENGINEERS
BOILER ENGINEERS
STOKER ENGINEERS
JOB LEADERS
MISC. DESIGN SQUAD
SQUAD LEADER
DESIGNERS
CHECKERS
DETAILERS
AIR HEATER SQUAD
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
DESIGNERS
CHECKERS
DRAFTSMEN
DETAILERS
FOR COMBINATION CONTRACT DRAWINGS INCLUDING FURNACE & PULVERIZED FUEL DESIGN
BOILER DIVISION
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
STOKER DIVISION
SQUAD LEADER
CHECKERS
DRAFTSMEN
DETAILERS
DOTTED LINES = FLOW ON SPECIFIC EQUIPMENT
SOLID LINES = FLOW ON COMBINATIONS OF EQUIPMENT

Fig. 2 Flow Chart for Combination Contracts
(Dotted lines, flow on specific equipment; solid lines, flow on combinations of equipment.)

Over a period of time we have been able to determine which of our men are most versatile and flexible. These men can be interchanged between squads as conditions demand.

There are certain squads which can be definitely termed "specialist squads," functioning in the design of equipment, the major portion of which holds no relation to any other item of equipment. In such cases, the possibility of interchanging men is a difficult one and should be avoided. The individuals of such squads have been trained along certain definite lines and have grasped the short-cut methods of their individual problems, which are duplicated, if not entirely, to a great extent, on the contracts that are received over a period of time.

The transferring of work from one squad to another is also important in the maintenance of a balanced work curve throughout the department. Those features of the equipment that permit the interchanging of men likewise permit the transferring of portions of a contract, if not the contract as a whole, from a squad that under ordinary conditions would complete all such contract drawings to another squad that is less occupied. This change is made without increased costs or delays in production.

On the aforementioned procedures depend to a great extent the straightening up of the work curve so that all squads will be carrying a full allotment of work and so that no squad which in-

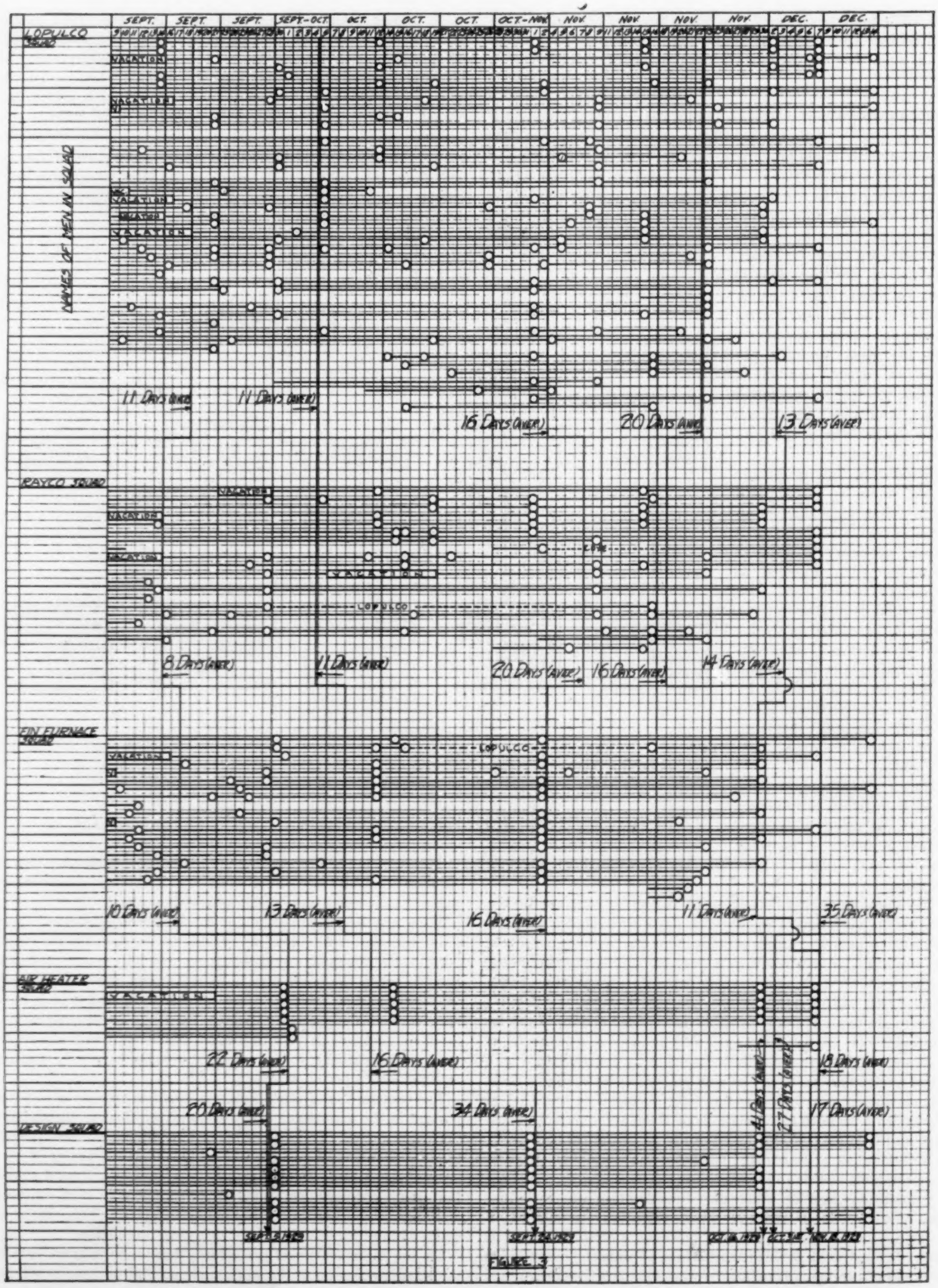


Fig. 3 Log Sheet of Drafting Department, Covering Time From September 9 to December 14, 1929

cludes the individuals of that squad will be suffering from lack of work. By following these two methods, the work curve of the department will more or less be made into a straight line, Fig. 3.

In the foregoing there has been one point definitely established—namely, that in an industry such as this, with the magnitude of the work and the diversification thereof, it is of paramount importance that the engineering talent, as well as the drafting talent, be located in one central place and be responsible to one management.

If it were only a question of designing the specific products without relation to any of the other products, it would be practical to maintain at each shop an engineering department.

But with the boiler shops located at St. Louis, Mo., and at Chattanooga, Tenn., the plate and structural shop at Phoenixville, Pa., the machine shop at Chicago, Ill., and a combined machine shop and foundry at Monongahela, Pa., it would be a difficult matter to design combination contracts in shop offices without jeopardizing drawing costs or production and would result in lack of united responsibility in establishing overall designs.

The essence of production is the speedy accomplishment of work on the lowest cost basis and the unification of all interests concerned in any specific feature of design or the design as a whole.

Control of Drawing Costs

The control of drawing costs in a drafting department of this size, with the variety of products that we are called on to design and the diversification in the combining of these products, is no small problem in the management of the department.

In order to have an accurate cost record of all individual drawings as well as the total drawings required for a job, we have in use a time record system that gives us, not alone cost data, but also an accurate story of what takes place on the job throughout its life in the drafting department.

It is required that each individual in the department make out a detailed time card, which, in addition to the date and the name of the individual, includes the contract or job number, the type of work that the individual is doing on that day (namely, checking, drafting, or tracing), the title of the drawing or a description of the work done, and the number of hours chargeable to such work. These cards are issued in two colors to distinguish regular time from overtime. They are given to the time clerk at the end of each day. From the information contained on these daily time cards, there is compiled a drafting-room cost card, as shown in Fig. 4. These cost cards are made up for a period of one month, allowing a line for each day of the month. They show the regular time, the overtime, and the total time in hours charged against the particular contract for which the card is issued.

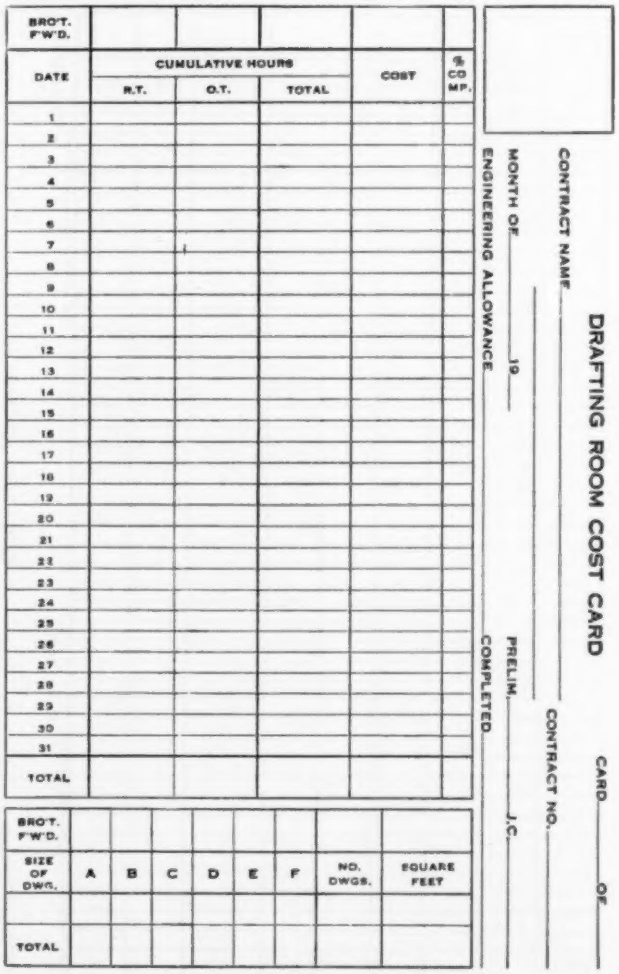
With this time and a given hourly rate, the cost of the drawing work on any job is continuously kept up to date, the cost as well as the hours being given in accumulating figures. On this card is also shown the engineering allowance that has been set up in the estimate for the making of drawings.

An estimate is made each week on each job of the percentage of drafting work completed based on the knowledge of drawing requirements for the entire job. With this information we are able to strike a balance weekly to determine whether the drawing costs are within the estimated allowance or whether this allowance is being exceeded.

On this cost card there is also kept a record of the number and sizes of drawings made within that month to enable us to arrive at the cost of these drawings per square foot at the end of the month or on completion of the job. The significance of the information contained on this card is apparent at any given time.

There is one possibility of error that enters into this index, however, which cannot be avoided as it is directly dependent on the ability of the man in charge of the work to establish the correct percentage of the amount of drawing work that has been done as compared to all of the drawings that ultimately will have to be made on the job. It may be necessary at times to reconsider the percentage established for the work completed, although with our specific products the amount of drawing work is definitely known and the percentages established will in most instances be consistent with the work done.

In order to cope with this situation, however, and also as a check on the job, a notification card is sent to all squad leaders once a week notifying them of those jobs on which the drafting costs are in excess of what they should be on the basis of the percentage established for work completed and the allowance set up in the estimate. In such instances where these cards are sent out it is necessary for the squad leader or job leader who is responsible for the work to fill out a drafting-cost report, giving the reasons why the costs are excessive.



BRO'T. F'W'D.					
DATE	CUMULATIVE HOURS			COST	% COMP.
	R.T.	O.T.	TOTAL		
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					
21					
22					
23					
24					
25					
26					
27					
28					
29					
30					
31					
TOTAL					

BRO'T. F'W'D.								
SIZE OF DWG.	A	B	C	D	E	F	NO. DWGS.	SQUARE FEET
TOTAL								

DRAFTING ROOM COST CARD

CARD______ OF______

CONTRACT NAME______ CONTRACT NO.______

MONTH OF______ 19___ PRELIM.______ J.C.______

ENGINEERING ALLOWANCE______ COMPLETED______

Fig. 4 Drafting-Room Cost Card

It has been found in the cost analyses of drafting work that there are a number of factors that contribute toward increasing costs. Some of these factors are common to all drafting departments and can be contended with readily by changing the personnel, revising systems, eliminating superfluous details, avoiding complicated designs, and so forth. One of the more prominent factors that tend to increase drawing costs on some jobs is the numerous changes to drawings that are necessary either for some one drawing or for all of the drawings required to fulfil contract obligations.

In the fabrication work of materials and equipment in general, one fact is never lost sight of, and this is that changes to the material itself or changes affecting the machine work that has been done or is to be done require an outlay of additional money by the fabricator over and above what was figured on originally for doing such work, and it is customary that the company authorizing such changes compensate the shop for the additional expenditure.

This likewise holds good in such instances where it is necessary to scrap material and to do a job over because of some change in design, and it is recognized by the industry in general that changes requiring the scrapping of materials necessitate additional compensation for the loss involved.

In the scrapping of materials or in changing the fabrication methods of such materials or equipment in general, we are dealing with something tangible, something that represents mass or weight or machine production, and something that is comprehensible in dollars and cents. In the drafting department the same feeling does not seem to exist, and at times one is led to believe that drawings have no monetary value and, therefore, can be changed at will or scrapped entirely without regard to cost.

DRAWING NO.____

DRAWING COST CARD

CONTRACT NO.____

Card___of___

Bro't F'w'd								
Date	Regular		Revision		Trac.	Check Cust. Dwg.	B/M	Sup.
	Dwg.	Ch'k.	Dwg.	Ch'k				
Total								

FIG. 5 CARD TO DETERMINE COST OF DRAWINGS

Quite often in the normal course of design and drafting work, and particularly after one or more drawings have been more or less completed, a change of idea takes place for a better design. This is perfectly justifiable, for regardless of drawing costs or the number of drawings that have to be changed, superseded, or scrapped, it is an obligation of all industries to furnish the best possible in the way of designs and equipment; and the establishment with clarity of these designs on paper, so that they can serve as the true interpretation of the designer's ideas to the purchaser, fabricator, erector, or operator, as the case may be, is of paramount importance.

The fact holds good, however, that although design changes are necessary they are reflected in increased costs of drawings, with the result that there are instances when the estimated allowance for drawing work is insufficient, resulting in some cases in not only depleting the allowance but encroaching on the profits of the completed product.

High cost of drawings is ofttimes a result of an individual not having established designs that were based on thorough investigations or considerations of the problem at hand. There are times when groups of individuals cannot agree. It is then necessary that a series of study drawings be made so that the various angles and phases of the problem in question can be considered by all individuals concerned.

There are instances where the purchaser himself may have individual opinions of a design, either by virtue of experience or operation of such equipment, and in order to have all parties interested agree, it requires extensive study drawings or drafting work in general, with numerous revisions, to reflect ultimately the ideas of every one concerned.

All of the foregoing conditions are proper and within the natural function of the engineering and drafting departments, but it is a function that accounts for much of the excessive cost in the making of drawings.

There are other factors that account for excessive drawing costs, but those mentioned, we have found, are by far the more predominant ones.

To enable us to determine the actual cost of all drawings made in the department and to maintain a definite record of these costs for future reference, we have in use a card system, Fig. 5.

A card is issued when work starts on a drawing. In addition to the drawing number and contract or job number, there is recorded on this card the dates on which time has been charged against the drawing and the distribution of such time. Under the heading of "regular" is shown such drafting or checking time in hours as is considered normal and chargeable against the drafting allowance set up in the estimate for the job. Under the heading of "revision" is shown such drafting or checking time in hours as was spent in making drawing revisions due to changes by interests outside of our organization or for reasons beyond our control. With this method we can readily determine the proper distribution of drafting charges for any one drawing or all drawings on a given job. The remainder of the headings on this card are required to complete the cost analysis of a job in its entirety.

CONCLUSION

As in other industries, the establishing of overall designs on some contracts requires the combining of our products with those of other manufacturers. In such cases the control of drafting production, as well as drawing costs, is influenced to a great extent by the cooperation received from the outside manufacturers, and in some instances by the purchasers.

On such jobs a certain amount of interchanging of designs and drawings between companies is necessary so that the arranging of all of the component products of the overall design can be accomplished without infringing on each other, either by physical interferences or capacity characteristics. In all instances the purchaser of the combined unit must be consulted in the establishing of definite locations and clearances, within building enclosures, and with respect to auxiliary power-plant equipment being furnished under separate contracts.

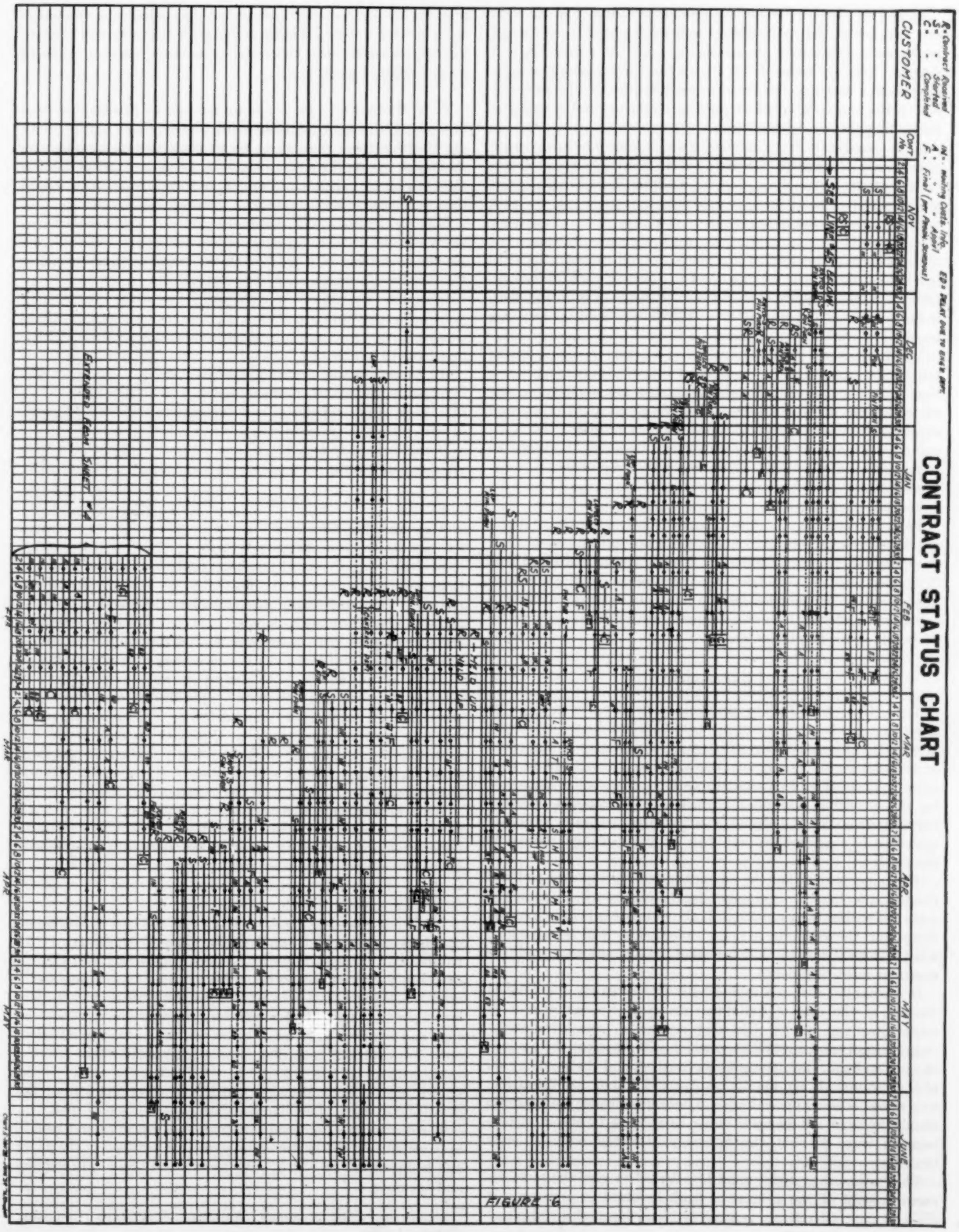


Fig. 6 Contract Status Chart

Unless all parties involved in a combined contract of this kind are working at the same speed, each coping with their design problems in the proper sequence, there will be difficulties and delays. Dilatory methods by any of the parties usually results in extensive drawing changes of a costly nature to all concerned, and in the end is reflected in delayed shipments and erection of the equipment as a whole.

As an illustration of the manner in which some jobs have passed through our department in a period of six months, we have drawn up a contract status chart, Fig. 6. On this chart, where letters are shown over solid lines, it indicates that although we were awaiting information or approvals to complete our drawing work it was possible for us to continue work on some drawings, but that delays in giving us this information would necessitate, within a known period, the holding up of all drawing work on the contract. In instances where the letters are shown over dotted lines, it indicates that drawing work was definitely held up awaiting required information or approvals.

Due to the short time that is allowed on some contracts for the fabrication and erection of equipment it is of utmost importance that design and drafting work be started immediately on the award of such contracts so as to give the shops and the erection department the benefit of as much time as possible. Such instances are indicated on the chart where the letter "S" is shown previous to the letter "R." This procedure is necessary, as sometimes two weeks or more elapse before the actual signed contract papers are received.

From this chart it is readily apparent that the control of drafting department production and costs is not entirely within our province, but to a great extent is directly dependent on others. With the exception of contracts that are true duplications, or such jobs requiring our smaller specific products, one job is rarely a criterion for another in terms of square feet of drawings, number of drawings, or drawing costs.

Discussion

T. G. Crawford.[2] The problem of drafting in a large organization has been very well presented. Flexibility is extremely essential in an organization which must of necessity be composed of different sections or groups more or less independent of each other. There should be some discussion in the outline suggested for the recording of costs of drawings. The amount of bookkeeping involved in keeping a sufficiently accurate record of drawings is considerable, and the accuracy of the records is debatable.

The paper states that the amount of work completed and still remaining to be done is closely checked each week, to see if the costs are within the estimate or if the estimate is being exceeded. Supposing the estimate is being exceeded or will be exceeded, what is done about it? Regardless of the amount estimated, the drawings must be finished.

It seems that it is advisable to eliminate entirely the detailed records of the cost of individual drawings by square foot or group drawings per job. In each section a "measuring stick" of some kind can be established which will represent the amount of work on hand, the amount being finished, and the total payroll of that group. It can readily be observed if the cost of drafting, as a whole, is going up or down. The section leader of this group should know if his men are doing their work in the most economical way, without the necessity of detail cost on each drawing.

The "measuring stick" must be selected to give the most accurate record of the volume of work involved in a group. It may be the number of drawings, but only where the amount of work on each drawing is easily comparable. It also may be drawing lists, material lists, kilowatts, horsepower, number of panels on switchboard work, etc.

F. A. Widdows.[3] As to elimination of the checker, we have been doing this, up to a certain point, for some time back. While our drafting force is small in comparison to the organizations discussed, we use the squad idea as much as is practical, making the designer the squad chief, and he in turn is responsible for the drawings, design, and accuracy.

Making the designer and even the detailer responsible, causes them to take more interest and pride in their work. We also like to keep them in contact with the building of the machine; this has been productive of very good results.

Of course, we run across some men who, while they may be good designers, have good ideas, and are fast, yet cannot be depended upon for accuracy, interferences, etc., and all their drawings have to be checked.

As to scheduling drafting, we have done nothing as yet; it probably will be rather difficult in our work, but not impossible. Our work is not repetition and the placing of standard parts; it calls, rather, for considerable inventing, new ideas, etc.

It will be hard to establish a time limit on work of this nature; yet a schedule could be made up after the design is approved.

Author's Closure

Answering Mr. Crawford's discussion, the salary expenditures for clerical work in keeping records of drawing costs, either individually or collectively for entire contracts, are not excessive in terms of percentage when compared to the drafting department payroll. (This percentage varies between 0.5 and 0.7.) The value of the information derived from these records cannot be compared to the cost of keeping them when considered in the light of accurate estimates of engineering costs for new work.

These records enable us to analyze drawing costs in terms of contract expenditures and extra expenditures necessitated by changes made and beyond our control or for work done as an extra to contract requirements. From these records we can determine what drawing work is actually costing in terms of dollars or hours per ton of structural steel, per square foot or per pound of plate work, per pound of castings, or per thousand brick.

While drawings must be completed, it has been found, with the possibility of a periodical analysis of drawing costs, that intricate designs requiring extensive detailing can at times be modified if all concerned are confronted with the cost of work as it is proceeding. Frequently excessive design costs lead to excessive fabrication costs.

While our records automatically give us the cost of individual drawings or groups of drawings on the square-foot basis, this is by no means a "measuring stick" when considering the cost of drawings for a contract in its entirety. The "measuring stick" used for the cost of drawings on combination contracts was established by analysis of numerous contracts over a period of time and is known in terms of hours per ton, per thousand brick, etc. For contracts requiring specific equipment where drawing work is more or less repetitious, the "measuring stick" is the known cost of each drawing which was determined from the detailed cost cards of similar drawings over a period of time.

The control of a balanced condition between work and men in each squad or the department as a whole is outlined in the paper and is illustrated by Fig. 3. We have found this record definite and accurate.

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[3] Assistant Chief Engineer, American Multigraph Company, Cleveland, Ohio.

MAN-54-2

Improving Drafting Management

By F. D. NEWBURY,[1] EAST PITTSBURGH, PA.

A system of improvement in drafting management extending over four years of experience is described, the use of these methods having cut drafting costs in half, while average salary rates have been increased, design quality has been improved, and drafting accuracy has been maintained. A feature of the plan is that each draftsman checks his own work, this resulting in increased responsibility and improved accuracy. Record charts showing the results attained are given.

DURING the past four years some new ideas have been applied to drafting management in several design departments of the Westinghouse company that may be of general interest. The use of these methods has cut drafting costs in half, while average salary rates have been increased, design quality has been improved, and drafting accuracy has been maintained.

These results might not be significant if they had been obtained in only one instance. The same methods have been successively applied in three different departments and have worked successfully in each.

The department in which these ideas were first developed handles the design of alternating- and direct-current machinery of medium and large size, and contains about 120 people, mainly technically trained engineers and draftsmen. This new plan of operation was started early in 1927. In the fall of 1929 the same plans of organization and operation were started in a transformer-design department, which at that time contained nearly 200 engineers and draftsmen. A few months later the plan was introduced in a third plant that builds steam turbines and a wide variety of other mechanical equipment. This department contains about 250 engineers and draftsmen. All of these design departments serve factories that in normal times have from 1500 to 2000 workmen, and so may be classed as large plants. More recently still, these ideas have been introduced in the reorganization of a much smaller design department serving a 400-man plant.

Much of the work in all of these departments is special, in the sense that it must be designed after receipt of sales contracts and to meet purchasers' specifications. Very little apparatus can be manufactured for stock in advance of sales. Much time and thought, however, have been given to the design of standard lines of parts from which a wide variety of special requirements of customers can be met with only moderate drafting and other special development expense. On account of the nature of the business a large amount of experimental engineering work is continuously under way. Approximately 30 per cent of total engineers' time and 20 to 30 per cent of draftsmen's time is devoted to this standard development effort that is not related directly to work on current customers' orders. The fact that as much as 70 to 80 per cent of drafting time is required by current orders, which must be handled as received, makes the problem of maintaining an adequate standard development drafting force a particularly difficult one and one that was not handled successfully in these departments until the present management methods were developed.

Before these new ideas were introduced, these departments were organized along conventional lines. Each department was divided into a number of separate engineering sections or groups, each of which was responsible for the initial design work connected with a specific class of apparatus. There was one common drafting section for the entire department in charge of a chief draftsman who usually was responsible for the structural or mechanical design of the product. In these several departments these single drafting sections each contained as many as 60 to 140 draftsmen.

This general type of organization results in divided design responsibilities that are not joined in a complete piece of equipment until the position of manager of the entire department is reached. In these particular departments, their size was too large to make this arrangement as effective as could be desired.

In the type of organization that has been developed, each of these departments is divided into a suitable number of design units (called divisions), each complete with electrical or mechanical engineers (or both), design draftsmen, and detail draftsmen, and in charge of one engineer-executive who directs and coördinates the work of both engineers and draftsmen. Everything possible is done to break down separation lines between engineers and draftsmen in the same division; these names carry no significance as to technical qualifications. Many of the engineers working with drafting equipment are better qualified as engineers by education and experience than many of the younger technical assistants who may be rated as engineers. It is important that all engineers and draftsmen in a division report to one executive; hence the division cannot be too large (15 to 50 people constitute good limits); and there is no place for a chief draftsman, either in the department or in the division.

A second distinctive feature of this type of organization is the grouping of draftsmen of each division into relatively small squads containing one to five men. Each squad is in charge of a squad leader, who is a qualified designer. He makes the general layouts assigned to his squad, assigns the work to his assistants, and supervises their work. The squad leader has a definite design and production responsibility for the work assigned to his squad. The squad is set up as a permanently established group, and the members, as far as possible, are located at adjacent tables. The size of the squad depends upon the character of work assigned to it. Changes in squad personnel, when necessary, usually affect the least experienced detail draftsmen; i.e., men are added to or subtracted from the bottom. Squads are specialized both as to the kind of work assigned to the squad and as to the kinds of work performed by the individual members of the squad. A broad distinction is

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

made between design work, such as covered by layout drawings, and detail drafting work, represented by the production of working drawings. It is not efficient to have designers make working drawings, nor is it wise to intrust very much designing (beyond that required by the individual working drawing) to less qualified detail draftsmen. There has been a marked improvement in quality of design since this specialization has been carried out and squad leaders of small groups have been given this definite design responsibility. Furthermore, the improved methods affecting drafting, to be described later, have been possible only because of this distinction between design drafting and detail drafting.

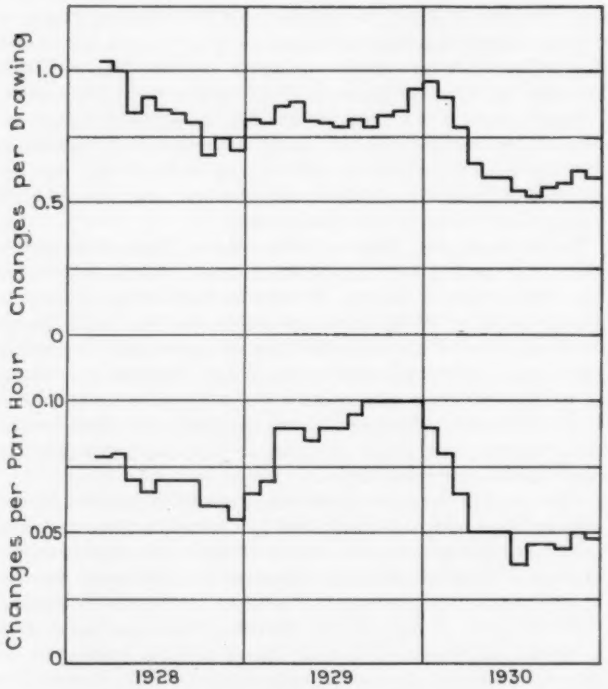


Fig. 1 Curves Represent a Three-Month Moving Average of Both Quantities

The flow of work through the department (and particularly that part directly related to customers' contracts and orders) is controlled by a production supervisor who has no responsibility for drafting management. Under the direction of the division head he assigns new work to the squad leaders, assigns completion dates for each detail drawing, and conducts all negotiations with the works production men as to production schedule dates for manufacturing information. His first duty is to set up production schedules for manufacturing information that will enable the factory to meet contract shipping dates. Bear in mind that no complete machines, and few parts, can be built in advance of customers' orders. During the past four years there have been very few instances where a schedule satisfactory to the factory could not be set up. The standard development schedule may be delayed temporarily to provide for emergencies in the customers' order schedule, and occasional overtime on a specific job may be resorted to. The rather common practice of working an entire force on a definite overtime schedule of several nights a week has not been used and is not looked on as good management.

Under this plan of organization, the head of the design division must take a considerable interest in drafting personnel and production, if the desired results are to be attained. Too frequently the supervision of the drafting part of the design organization is delegated to an executive who lacks close touch with the commercial, technical, and manufacturing aspects of the whole design problem. The division engineer has these broad contacts and is expected to handle directly all personnel and management problems connected with drafting, as well as those connected with engineers. Obviously this one-man plan is practicable only when the division is kept small in total size (15 to 50 people, as mentioned previously). The plan is feasible, even with these numbers, only because of the grouping of engineers and draftsmen in small operating units, with each unit in charge of a squad leader having definite responsibilities, and because of performance records that have been developed which give the division head, regularly, a quantitative measure of each draftsman's performance. A requirement for success of this plan is, quite obviously, the availability of good executives for division heads and, conversely, an advantage of the plan is that good material is given a chance to develop.

A large design department of the conventional type is apt to be unwieldy and slow to respond to changing conditions. When this is true, it is frequently the fault of the type of organization. Even good men can function successfully and with speed only when they are free to act in relatively small groups. If a large group must act as a unit, the inevitable inertia makes it slow to move. Speed of action, in this sense, includes mental ability to quickly change ideas and methods, as well as the more obvious physical ability to produce results.

The proper size of a design unit, for best results, cannot be defined by a single figure; this best size depends upon the quality of leadership and the character or type of work. Whether a given design unit is too large can be answered only by the results it achieves.

It is possible, in our experience, for a large design organization to combine the strength and ability inherent in the large group with the speed and mobility inherent in the small group, if the large group is divided into a sufficient number of small groups, each constituting a complete self-contained design unit.

In producing a specific design required by a customer's order, the engineer prepares a written design specification which is intended to give the draftsman-designer complete information so that he can immediately start the general layout drawing of the structure. If this design specification is not complete, much time can be lost by the draftsman while obtaining necessary information verbally, and because the engineer, under such circumstances, is apt to give ill-considered information which, later on, may be changed. In connection with lines of machines based on standard parts, the design specification covering a new combination of standard and variable parts can be made so complete and so specific that very little, if any, general layout drafting is required.

Practically all checking of drawings by separate checkers has been eliminated by training detail draftsmen to check their own work. Careful records of drawing changes are maintained to show the class of change and the responsible individual. This practice of draftsmen checking their own work may be questioned by those who believe all drawings should be checked by an independent checker. Carefully maintained records have shown no increase in errors per drawing due to this system. Records have shown a very considerable difference in accuracy among men; some men are inherently accurate and others are not. This difference in men is much greater than any difference in results due to checking methods, and indicates that good results can be attained most certainly by keeping individual accuracy records and by releasing or transferring men who are not sufficiently accurate to do satisfactory drafting work.

Experience has shown the weakness inherent in the divided responsibility for accuracy that exists when separate checkers are used. When separate checking is a part of the established routine, the detail draftsman leans more and more on the checker with an increasing deterioration in his own thoroughness and accuracy. When the proposal to eliminate separate checkers was first discussed (in July, 1927) an experiment was tried to discover the accuracy of detail draftsmen independently of the accuracy of the checker.

In a division containing 25 draftsmen, the first week's record showed five errors and omissions per drawing discovered by checkers. Within ten weeks, this figure dropped to two-tenths of a change per drawing. Out of this experience, the plan was developed whereby a detail draftsman was allowed to check his own work as soon as he had demonstrated his own accuracy by the inability of the separate checker to discover errors or omissions. It was found that good draftsmen welcomed this added responsibility and tried hard to earn it. A real check on individual accuracy was maintained by recording all errors and omissions (however minor) discovered during the factory work. This record for the electrical machinery department is shown for the past three years in Fig. 1. Fairly complete elimination of separate checking had been accomplished by the middle of 1928. It will be noted that, during the introductory period, there was an improvement in accuracy; there was considerable deterioration during 1929, caused by a 50 per cent increase in size of force, and a gratifying improvement during 1930 with a reducing size of force.

Unless the cost of separate checking is visualized by a study of the different kinds of drafting operations, its magnitude and cost are apt to be overlooked. In these three large departments the percentage of total time used in separate checking before the conventional practice of separate checking was modified and the relation between checking time and time used in making new detail drawings is as follows:

	Initial Conditions			
	Percentage of total hours			
Department	Detail drafting making new drawings, per cent	Separate checking time, per cent	Ratio of checking time to detail drafting time	Separate check, end of 1930, total hours, per cent
1	42.4	14.3	0.34	0.3
2	44.8	11.6	0.26	0.3
3	32.5	11.8	0.36	3.5

Tracing has been eliminated (except for special cases, such as Navy jobs) by making tracings from pencil drawings by the photolithoprint process, and in the case of variable parts, used only on one order, by making blueprints directly from pencil drawings made on special tracing cloth.

The work of making new detail drawings has been reduced greatly by providing so-called master tracings (only partially complete) from which lithoprints are made, and the new lithoprint "tracing" is completed by the detail draftsman. In many cases of drawings of standard parts, only dimensions need to be filled in to complete the lithoprint from the master tracing. The photolithoprint process was first used in all divisions of one department in May, 1927. In December, 1927, 188 of 424 new drawings (44 per cent) were made from existing tracings; in December, 1929, two years later, 352 of 511 new drawings (69 per cent) were made from existing tracings. This increase in the use of existing tracings for making new tracings is one of the major reasons for reduction in drafting costs.

Distribution of Drafting Time

Accurate record is kept daily of draftsmen's time by work orders, by drawing number (for certain operations), and by drafting operations. The kind of operations recorded and the distribution of total drafting time among these kinds of work are shown for a recent month as follows:

		Percentage of total hours		
No.	Operation	Dept. 1	Dept. 2	Dept. 3
1	Layout design and supervision of detail draftsmen (mainly by squad leaders)	25.8	34.2	38.8
2	Making new pencil drawings	25.7	32.7	25.1
3	Making new drawings from lithoprints	24.1	14.8	7.4
4	Changing old tracings; this may be changing old tracings for new orders, making customers' changes, or correcting mistakes	13.4	7.9	14.4
5	Checking new drawings (separate checkers only)	0.3	0.3	3.5
6	Tracing	2.2	5.5	5.0
Misc.:	Vacations and holidays	8.2	4.5	5.8

Par Time Plan

In measuring the useful output of a drafting organization, the single drawing—large or small, simple or complex—is most frequently taken as the unit of measurement. Sometimes the total square feet of drawings produced is used, but this measure also has obvious shortcomings. In the plan developed in the first design organization under discussion, the range of working drawings produced is divided into nine classes, depending upon the amount of work involved in making the working drawings, ranging in time from 2 hours to 40 hours. This time allowance, covering making and separately checking the pencil drawing, is intended to represent the time the drawing should require under ideal conditions. This means that the detail draftsman should be an experienced draftsman and a specialist on that particular kind of drawing. He should have, at his elbow, all necessary design information, and should not be compelled to repeat work because of changed information; in short, he should be able to work continuously at his drafting table. Each drawing, after it is completed, is given a "par-hour" rating by the production supervisor. This figure is analogous to the "allowed hours" for factory jobs.

In the beginning an attempt was made to give an accurate estimate of par hours to each new drawing when it was completed. It was found that a limited number of "classes" and corresponding par-hour values served the purpose with sufficient accuracy and were more satisfactory in use.

The allowed hours is called "par time" to distinguish it from the analogous standard time (or piecework rate) in factory operations. The par time is set up as an ideal time that the average draftsman is not expected to attain, at least for some time. This permits a difficult scale of par times to be adopted in introducing the plan, and a difficult mark is set up that can be used to stimulate improvement over a long period. In the corresponding factory standard-time plan, the "allowed hours" represents a practical limit that the average workman is expected to meet after a relatively short instruction period.

In the operation of the par-time plan, the actual time charged by draftsmen to the operations of making and checking new detail drawings (operations 2 and 5 or 3 and 5 in the list previously given) is recorded daily, and after the drawing is completed, the par time (i.e., drawing class) is set by the production supervisor, who is not directly responsible for drafting performance. For each completed detail drawing there is available the actual hours charged to the drawing and the par hours allowed; the ratio of actual hours to par hours for a drawing, a draftsman, or any group of draftsmen is the measure of performance—a low ratio indicating good performance.

It will be observed that the par-time plan affects, most directly, production of new working drawings. This amounts to one-third to one-half the total drafting hours (see table of distribution of drafting time). The entire drafting time is brought within the influence of the plan, however, by considering the ratio of total hours (instead of operations 2-3-5 connected with the production of new detail drawings) to par hours of new drawings produced. This ratio is the sum of the "new drawing production" ratio and an overhead or expense ratio which includes design and layout, supervision, changing old tracings, etc. (operations 1-4-6-7). This total hour or overall ratio is the summation of all performance factors. It is the ratio of total input (represented by total hours paid for) to total useful output (represented by par hours of new drawings produced). It is analogous to the total cost per standard hour in the standard cost plan as applied to factory operation.

Both ratios (new drawing ratio and total ratio) may be applied to individual draftsmen, to squads, divisions, and departments.

The results of the operation of this par-time plan in the electrical machinery department are shown in the accompanying charts, Figs. 2 and 3. Fig. 2 shows the various performance ratios that have been described. The first curve, showing the ratio of actual hours to par hours for new pencil drawings, is of most value as a measure of individual man efficiency. In the case of the second curve, for new drawings made from lithoprints, the par-time allowance is the same as for a completely new pencil drawing. This ratio, therefore, should be materially better (lower) than the ratio for new pencil drawings. If it is not, it means that poor judgment was used by the squad leaders in using existing tracings as a basis for new drawings. This second curve and the third, which is the combination of the first and second, is of value mainly as a measure of squad, division, and department efficiency. Satisfactory performance, as it is now judged, can be achieved only by producing a major part of the required new drawings from existing tracings (operation 3).

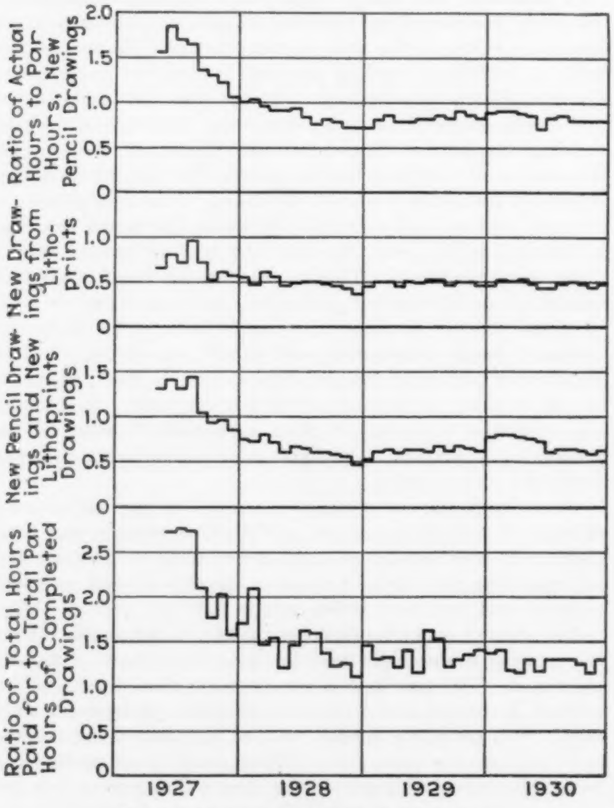


FIG. 2 VARIOUS PERFORMANCE RATIOS

The fourth curve represents the total or "overall" ratio, which is the most important. This curve is more irregular than the others because it includes absent time paid for. This is the reason for the relatively higher peaks in July and August of each year.

It is interesting to note that initial performance (in the first curve of Fig. 2) was approximately 75 per cent above par (poorer than par), but that par performance (ratio 1.00) was achieved in nine months, or by January, 1928. Since then, the ratio has been continuously under par, and in 1930 the average ratio for the year was 0.84. It is possible to "shoot better than par," for two reasons. The original par scale, which has not been changed, was too lenient and was based on separate checking. A draftsman capable of checking his own work is given credit for the saving.

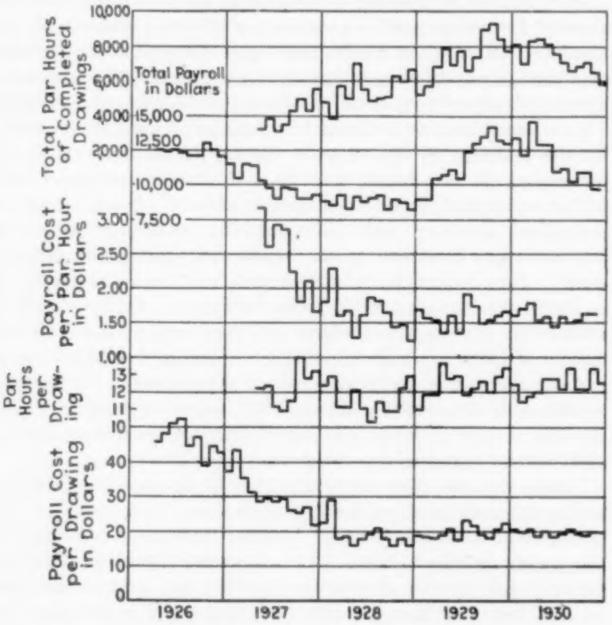


FIG. 3 RESULTS IN TERMS OF COSTS OF DRAFTING

Fig. 3 gives results in terms of costs of drafting. The total useful output is shown in the curve of total par hours of completed drawings; the total payroll cost is shown, and the other curves are derived from these.

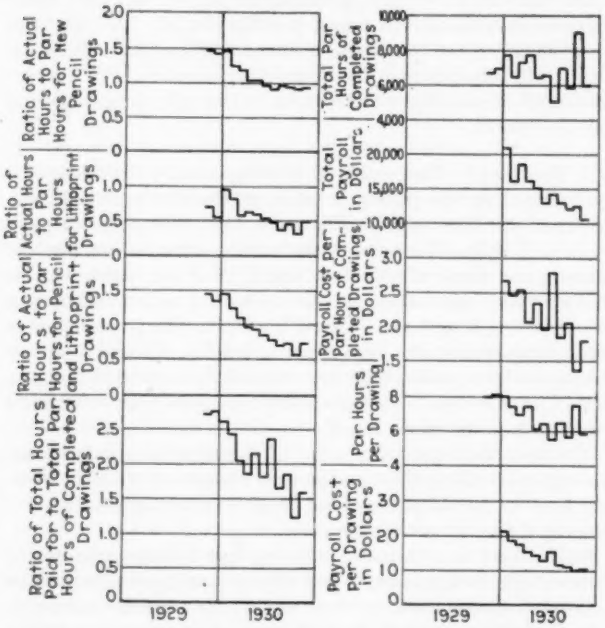
The curve of average par hours per drawing is interesting in showing how the "work content" of drawings may vary from month to month, even in the same department. When these same data are taken from different departments handling different types of apparatus, the differences are much greater, as shown by the following examples taken from divisions of three different departments (the figures are averages of several months):

	Par hours per drawing
Instrument transformers	4.0
Small distribution transformers	5.5
Large power transformers	8.1
Steam condensers	8.8
Alternating-current rotating machines	10.5
Direct-current rotating machines	15.5
Large steam turbines	17.7

The curves of Fig. 3 illustrate four distinct periods or phases occurring in successive years. During 1927 there was a slightly increasing drafting load and a substantially decreasing payroll.

During this period operating ratios were improving, and the payroll costs per par hour and per drawing were considerably reduced. This was a period of rising standards, due to improvement in individuals and to the elimination of men who were not able to meet the new standards of performance.

During 1928 there was a considerable increase in drafting output due largely to increased production of standard-part drawings as a part of a standard development program. The payroll and force did not change materially. There was continued improvement in performance, but at a decreased rate.



Figs. 4 and 5 Results From the Transformer Engineering Department

During 1929 there was a further considerable increase in drafting load (mainly caused by increased customers' orders) and a corresponding increase in payroll and personnel. The notable fact is that, in spite of this large influx of new draftsmen, the performance ratios indicated only a slight loss in efficiency. The effective assimilation of new employees is to be attributed to the small squad type of organization. Ten new men distributed among ten squads, having ten good leaders, can be assimilated without loss of efficiency where the same ten new men added at one time to a single large drafting section, and attempting to work independently, might do more harm than good.

This experience during 1929 brought out an interesting fact that has been illustrated many times in similar factory situations: new men brought into the group met the existing average pace of the group in a much shorter time than was required for the original group to achieve the faster pace.

During 1930 there was a decreasing load, a decreasing force, and an improved efficiency. This illustrates the most difficult administrative problem: to maintain individual output and efficiency with a seriously falling load.

Figs. 4 and 5 show corresponding results from the operation of the transformer engineering department. They are interesting in that they confirm the initial results of Figs. 2 and 3.

It will be noted that the performance ratios for new drawings started at 1.5 (as compared with approximately 1.75 in Fig. 2) and came down to 1.0 in nine months, which was the period in Fig. 2. The ratio of total hours paid for to par hours started at 2.7 (which was approximately the same value in Fig. 2) and came down to 1.5 in twelve months. In Fig. 2 the overall performance ratio was reduced to 1.5 in ten months.

Another indication of the greater value of "par hours" as a unit of output is afforded by the close agreement in these two departments in "cost per par hour" ($1.35 minimum value in Fig. 5 as compared with $1.25 in Fig. 3), while minimum "cost per drawing" varies from $16 in Fig. 3 to $10 in Fig. 5. The greater difference in cost per drawing is caused by the difference in average par hours per drawing.

It cannot be over-emphasized that these records do not automatically produce improvement; improvement has resulted only because the several division engineer-executives have used these records as a tool to achieve results. They have analyzed the records of each draftsman and have used them effectively in carrying home to the individual his position in his group.

Probably the most important factor in administering this plan has been the individual basis of all records. Each draftsman producing detail drawings has been advised each month of his production ratios and the errors affecting his drawings. He has been given his own standing and the maximum, minimum, and average figures of his division. A sample individual record is shown in Fig. 6. This record is one of the best, but is typical as to improvement during the first 18 months of the period this par-time plan has been in operation.

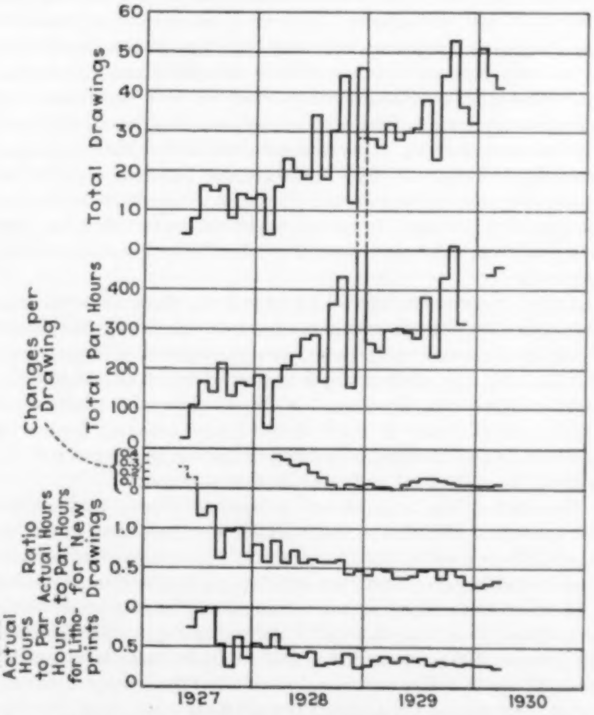


Fig. 6 Production Ratios and the Errors of an Individual Draftsman

The results have been used, even more effectively, to improve methods and working conditions. Probably a greater share of the improved performance has been possible because of improvements in the sphere of management than in those things for which the individual draftsmen could be held responsible.

Incentives

The performance ratios obtained by the par-time plan have not been used directly to determine salary payments. However, there has been an indirect relation that has been effective. Salary rates of individuals have been increased with due regard to production and error records. Individuals falling considerably below average performance have been released or transferred to other work.

It may be possible to base salary payments directly on these performance ratios, but up to the present time the dangers inherent in such an inflexible, mathematical application of the ratios have been considered too great to risk. When a plan of measurement directly affects the pay envelope, every figure used in the formula will be questioned. This par-time plan has been effective partly because it has been used with good judgment. Draftsmen have accepted occasional "bad breaks" because they have known that they would be judged only on the basis of long-term averages and with a complete understanding of circumstances and conditions. The plan has been looked upon as a valuable guide to judgment and not as an inflexible measure that could be applied with a slide-rule.

Discussion

J. P. Jordan.[2] For a great many years management of progressive companies, aided to a considerable extent by the professional industrial engineer, has done much in the betterment of manufacturing operations, but very little has been accomplished in the other departments, particularly the selling and engineering.

In the selling function only the surface has been scratched.

In the engineering division there has been even less of constructive betterment effort. One reason for this is that the engineering function, including all of its ramifications, has been regarded as measurable only in the results of the devices which have been designed and in respect to the aid which the department has been able to give in the betterment of product from a manufacturing standpoint.

But whenever any suggestions have been made as to the possible reduction of engineering expense through careful analysis of the detailed operations necessary to any engineering department, there usually has arisen a storm of protest which would indicate that an engineering department is a precocious sort of institution which must of course be more or less temperamental, topped off with a halo, and in many other ways regarded as a portion of the business to be treated with care and deep respect.

The author has brought out a common-sense, businesslike, and perfectly practical scheme which not only looks well on paper and on graphs, but to the writer's knowledge has actually worked and has produced very satisfactory and profitable results.

It strikes the writer that any comment on this paper may better be confined to an appeal to all engineering departments to take the ideas at 100 per cent value and to apply them in every-day life. The author has very clearly exhibited the results of managing an engineering department by methods other than the consideration of the awesome nature of the people that inhabit the average engineering office. He has proved distinctly that engineering departments are just as susceptible to cost and volume control as any other department. He does not in the slightest degree depreciate engineering as a profession. But he does prove conclusively that the various functions within an engineering department are subject to just as much control as the functions of any other department. In fact, if any one will analyze the operations of the average engineering department, it will almost invariably be found that a considerable portion of the cost is for work which, under proper regulation, can be done at a considerably less cost. Much of the work of any engineering department is in a way similar to the work of other departments, and there is no reason whatever to think that in such common operations there should be any excess cost.

The author's plan of measuring the output of draftsmen is a workable plan, as is not only proved by his own graphs, but as also has been proved through the observation of the writer and his associates.

Therefore, the main object of this discussion is not so much to comment on the author's plan as to urge engineers in general to take the contents of the paper as something other than theory—in fact, to take it as honest-to-goodness practical stuff which can be and should be applied to the management of any engineering department.

The times which we must go through from now on, where sharp competition demands the lowest possible cost, are just the times where engineers can contribute their share to the lowering of costs if they will seriously look their own department right in the face and apply the principles as set forth in this very interesting and valuable paper.

A. D. Hunt.[3] The system of drafting-room control, described by the author, was placed in effect at the South Philadelphia Works of the Westinghouse Electric & Manufacturing Company on Feb. 1, 1930. The type of engineering organization, also described, was made effective on March 17 of the same year, so that a revised organization in both engineering and drafting began to operate as a unit about April 1. Up to the present time a rapid improvement has been made in drafting efficiency, and on the basis of the results thus far obtained it is expected that the rate of improvement experienced over the past year will extend for many months in the future.

The new plan establishes a line of demarcation between engineering and drafting effort and permits a control of drafting work, which under the old plan was not possible on account of the overlapping of the two activities.

While there is a clean-cut dividing line between engineering and drafting work, it is important that all men in a design division be located in one room, as close contact between all engaged in a particular branch of the work tends toward cooperative effort, and permits close supervision.

As the author emphatically points out, the chief draftsman of the conventional engineering organization is missing in the new order, in which the division engineer directs the activities of both the engineers responsible for design and the draftsmen who translate the engineers' ideas into detailed drawings for manufacture. The control of both engineering and drafting by the division engineer is of prime importance, but as personal guidance throughout the group is essential, the total number under one division head should be relatively small. In such an organization the younger outstanding engineer may assume greater responsibility than in larger conventional design groups by reason of the investing of greater responsibility in a relatively small number of men.

To insure high efficiency in the new drafting system, the engineering instructions (design specifications) must be definite and sufficiently complete to permit the steady flow of drafting work. The par-time plan provides a means of measuring group or individual capacity and efficiency and is extremely valuable in scheduling drafting work. It is invaluable in relating personnel to increasing or decreasing loads.

The individual drafting records are of material assistance in determining the relative ability of draftsmen, as these records show drafting efficiency, number of errors made, and cost of such errors. Emphasis is laid on the importance of small drafting squads under capable squad leaders. It is the experience at South Philadelphia that a squad of four to six men forms the most efficient drafting-room unit.

The use of master drawings and the employment of parts of old drawings, from which lithoprints are made, are important means of reducing the time in making detailed drawings. The par-time drafting system encourages the use of these short process drawings, as their use improves the performance records o the individual and the drafting room as a whole. At South Philadelphia the lithoprint process has been used since 1927, but a determined effort during 1930 to make a greater use of this labor-saving method doubled the percentage of lithoprints in eight months, and it is expected to greatly increase this percentage in the future. In May, 1930, the drafting room produced 1041 drawings, of which 177, or 17 per cent, were lithoprints, while in January, 1931, production had fallen off to 626 drawings, but of these 215 were lithoprints, or about $34^1/_2$ per cent.

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[2] Stevenson, Jordan & Harrison, New York, N. Y. Mem. A.S.M.E.

[3] Westinghouse Elec. & Mfg. Co., South Philadelphia, Pa.

Experience with the new system of drafting management is limited to one year, and during this time drafting costs have been greatly reduced, as the author points out. However, it is believed that the future will show still greater reductions in drafting costs, as we have, in reality, only begun effectively to use the principles of management as set forth in the paper.

It has been definitely proved that this kind of drafting management may be successfully applied to the design of both electrical and mechanical machinery, and in the writer's opinion this same management may be successfully applied to many other engineering organizations.

John M. Russ.[4] The paper suggests two points, to either one of which the writer cannot entirely apply the author's fundamental premise, that of dispensing with the checker.

In the first place, it is wondered if the available supply of inherently capable men would be sufficient, if business conditions were what one would like to think of as normal and if the author's policy as outlined were generally followed.

It has been the writer's experience in the engineering offices of the machine-tool industry that to train the rank and file of the drawing-room personnel for the position of checker is an expensive process, often painful, and to pin this responsibility on the average draftsman in addition to his normal load seems potentially dangerous.

This would suggest the reiteration that the supply of individuals whose personal equation incorporates both accuracy and the acceptance of responsibility is limited—possibly too limited to permit all the engineering offices in the country to follow the author's plan.

In the second place, the writer questions the applicability of the principle of the draftsman checking his own work in either of the following fields:

1 In the plant producing a large output of interchangeable parts over a considerable period of time, the jig and fixture investment is high. It is also reasonable to assume that the same drawing may be used again and again, and the repetition of shop operations becomes enormous. It would seem here to be of the highest importance that the drawings and specifications should be as accurate when leaving the engineering office the first time as it is humanly possible to make them.

2 In the case of the consulting or contracting engineer, where various units are purchased from different plants geographically separated, it would appear to be of vast importance that the boundary or connecting dimensions of these individually fabricated units should tie in. This could be effected only by careful checking, and if the volume of business were great enough, it would take the entire time of one or more men, i.e., special checkers.

R. A. McCarty.[5] The author has pointed out that seldom during the past four years has a schedule for new drawings been unsatisfactory to the works department. He also might have said that during the five years preceding the period it was rather the exceptional case, in at least one of the four departments in question, when a drafting production schedule was wholly satisfactory to the manufacturing department.

This unsatisfactory situation arose from a lack of flexibility in the working force which made it fail to respond to the day-by-day changing demands of the load. If there existed a current load of new drawings which would keep the drafting force busy for 30 days, the "starting time" (the elapsed time before any new work could be started) was 30 days. Obviously, such a condition made it impossible to satisfactorily handle new orders for short-delivery apparatus.

With the responsibility for overall results placed in the hands of a single head, he was in a position to review the entire program each day, to readjust schedules and shift men—impossible under the old form of divided organization. This direct control obtained the flexibility necessary to satisfactory drawing dates.

On the matter of accuracy of drawings, it may be argued that Fig. 1 of the paper does not give a correct picture of the results, if a fairly large number of standard development drawings is made which may not go into immediate production. In order to check this possibility, errors discovered in the manufacturing department have been plotted against man-hours of productive labor in the factory using the rotating machinery department's drawings. This curve, covering the latter part of 1929, all of 1930, and the first two months of 1931, follows the same general trends shown in Fig. 1. It is believed that this method of comparing accuracy offers, perhaps, the most reliable basis yet developed.

Experience has shown that if a man's accuracy record is appreciably poorer than the average, the expenses of correcting his errors will later prove him unsuited for the work. Such records as these, therefore, make it possible to determine in a shorter period the accuracy qualifications of a new employee.

Author's Closure

The author appreciates highly the discussion presented by Mr. Jordan and all the more because of his wide experience in this field. After all, no plan of organization or management can be moved bodily from one plant to another and be expected to function, nor is any plan automatic in its operation. The success of a plan depends very largely on the skill with which it is adapted to local conditions and to personnel and the skill with which it is administered. The author has no thought that the details of his plan can be adopted without change, but he is confident that the principles involved and the more accurate method of measuring drafting output can be adopted in many kinds of design work with improved quality of design and with lower cost.

Mr. Russ questions the wisdom of eliminating separate checkers on two counts. He questions, first, whether the available supply of competent draftsmen is sufficient if this plan were generally adopted. This is not a matter of interest to the progressive manager interested primarily in his own operations. Certainly, no one would decide against a desirable action for the reason that, if every one took the same action, it would become impracticable. But answering this question more directly, no difficulty was found in obtaining a sufficient number of competent detail draftsmen from the existing forces in these three departments to eliminate separate checking as a routine operation. It should be remembered that, with separate checkers, there will usually be one checker for every three detail draftsmen, which at once provides a source of high-grade draftsmen to replace detail draftsmen who cannot measure up to this new opportunity and responsibility.

Mr. Russ questions, also, whether separate checking should be omitted in certain cases. This is one of those questions that should be settled by local conditions. The methods of operation advocated by the author do not prevent separate checking from being used in special cases, and a small amount of separate checking is usually being done in the design departments under his supervision. The fact that it is a special operation, and not a routine operation, makes this occasional separate checking more effective, because the detail draftsman does not know that his work will be separately checked.

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[4] Associate Professor of Engineering Drawing, Ohio State University, Columbus, Ohio.

[5] Manager, Power Engineering Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

The author has been interested in finding a confirmation of his experience in that of the author of a similar article.[6] By properly selecting detail draftsmen, who proved their ability to check their own work, the number of errors was not increased, and the cost of separately checking drawings was reduced from 35 per cent to 15 per cent of the drafting time.

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[6] "Reducing the Cost of Checking Drawings," H. Kaufman, *Machinery*, August, 1930.

MAN-54-3

Variations in Maintenance Costs and Procedure

By E. V. Stoody[1] and G. I. Ross,[2] Boston, Mass.

The paper treats in a new way the important function of maintenance in industry. It is an analysis of the ratio of the maintenance forces in 20 industrial plants. The authors give reasons for the discrepancies shown by the analysis. The paper should be of assistance to managers wishing to conduct their maintenance activities economically. The charts indicate by the wide dispersion of ratios that there is a large field for improvement in the general treatment of the maintenance problem.

E. V. Stoody

G. I. Ross

MAINTENANCE, as a function of plant operation, is often regarded by management as a necessary evil. It is too frequently considered to be non-controllable and is permitted to function independently within what are regarded as reasonable limits of expense. These limits are usually determined by guess. Maintenance departments are enlarged for emergency maintenance or for construction, and are allowed to operate with increased personnel long after the emergency work has been completed. On the other hand, due to the hazy conception of what maintenance costs should be, the maintenance department, in periods of economy, is too often depleted beyond justification and with eventual detriment to the company.

If two identical plants manufacturing the same product were placed side by side under unified executive direction, it is safe to assume that any inconsistencies in operating costs between the two plants would be quickly discovered. If the maintenance force in one plant were twice as large as that in the other, the management would soon take steps to determine the reason. The variation would warrant this attention, and each plant would serve as a yardstick to measure the other. Yet this condition exists in more than 75 per cent of the plants which we are called to study, and many of these plants enjoy the very best of managerial talent.

True, the plants studied are not identical, nor are they side by side under unified direction, but they need not be if studied carefully and the variables be given due consideration.

This relatively large difference in maintenance ratios is illustrated by the accompanying charts. The 20 plants which are used for purposes of comparison are typical of the many studied. The grand total of the plant forces represented by the chart, Fig. 1, is 28,872 persons, and the total of the maintenance forces is 1797. There may be some difference of opinion regarding one or two of the groups selected as pertaining to the maintenance function, but such difference would be small and would not alter the general relation of the forces.

The variations in maintenance forces expressed as a per cent of total employees may be explained by:

1 Different standards of plant maintenance
2 Variations in the efficiencies of maintenance forces
3 Physical differences in plants, equipment, and processes.

The chart, Fig. 1, shows the 20 plants arranged according to number of employees and indicates that, in the group selected, the size of plant alone has no bearing on the maintenance ratio.

In plants in which no maintenance employees are listed under any particular classification, the total absence of such classification can be explained by a variation in definition, the work is handled by employees classed as productive, or, due to the nature and size of the plant, the work is so minor that it is handled by employees listed under other classifications. Consequently, in the discussion of any particular classification, such plants are not considered if the classification is totally absent.

The chart, Fig. 2, indicates that the greatest variations occur in the following order:

Class	Groups	Minimum per cent, exclusive of 0 per cent	Maximum per cent
J	Janitors, sweepers, porters, etc...	0.22	4.12
E	Machinists, machine repairmen...	0.24	3.59
L	Yardmen........................	0.12	2.82
B	Belt repair, oilers, millwrights....	0.32	2.42
H	Masons, plumbers, tinsmiths.....	0.11	1.69
K	Watchmen.......................	0.16	1.59

The wide variation in the janitor-sweeper group can be easily understood when it is considered how widely the opinions of managements differ with respect to the cleanliness and tidiness of their plants and offices. Another factor which affects this classification is the type of industry. A factory with a large machining department requires the disposal of many turnings and chips which may be done by this force. The highest ratio in this group is attained by a plant of 170 persons and is due to the fact that the plant is housed in an extremely poor group of buildings and the maintenance force is poorly supervised.

This brings forth an important point, namely, that inherent property conditions may exact a heavy penalty from the company in the way of high maintenance charges. These would include not only such items as sweepers and janitors, but also building repairmen, watchmen, etc. This consideration is something

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[2] Chief Engineer, MacDonald Bros. Mr. Ross is a graduate of Sheffield Scientific School of Yale University. He has a background of research, plant engineering, production, and technical supervision in non-ferrous, hardware, and automotive industries, also consulting work in these and allied fields.

Presented at the "Three-M" (Management, Materials, Maintenance) Congress held in Cleveland, Ohio, April 13 to 17, 1931, under the auspices of the A.S.M.E. and the S.I.E.

Note: Statements and opinions advanced in papers are to be understood as individual opinions of their authors, and not those of the Society.

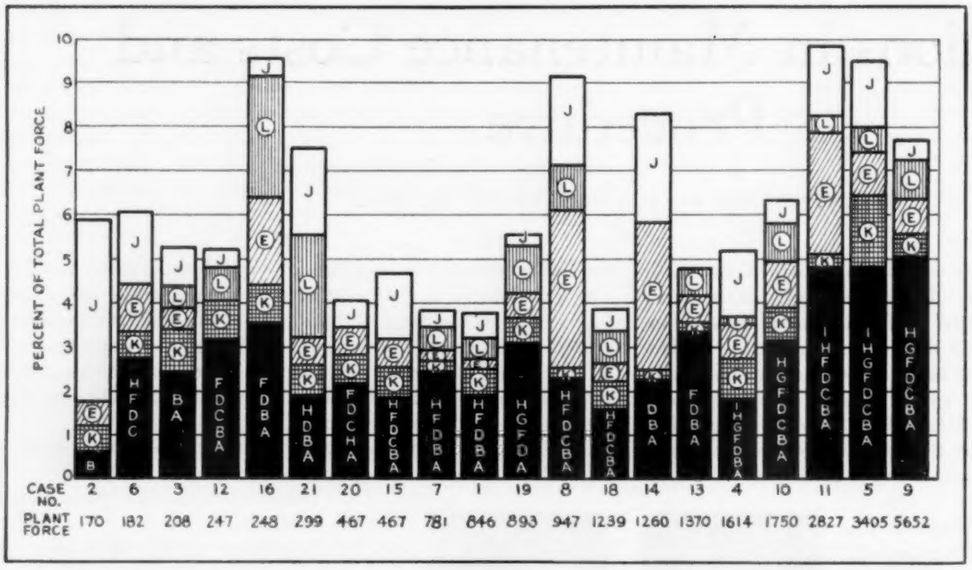

Fig. 1 Ratio of Maintenance to Plant Forces in 20 Typical Plants Arranged According to Size of Plant

(Legend: A, supervision and clerical; B, belt repair, oilers, millwrights; C, blacksmiths and welders; D, electricians; E, machinists, machine repairmen; F, carpenters, pattern makers; G, painters; H, masons, plumbers, tinsmiths; I, truck repairmen; J, janitors, porters, sweepers, etc.; K, watchmen; L, yardmen.)

which may be accepted or equated against the capital cost of a change.

In group E, machinists and machine repairmen, the variation is undoubtedly due to a large extent to the disparity between repair requirements in different industries and to the policy in some companies of letting out major machine-repair work to outside contractors.

In the case of the largest variation, case No. 8, the maintenance force was found to operate very inefficiently, and this fact, coupled with the character and age of the plant equipment, was conducive to a high force. The next largest ratio is found in case No. 14, where an excellent operating department was found to exist. The plant in question, however, is very highly mechanized, and therefore is inclined to run high on machine-repair work.

Group L, yardmen, is bound to be a large variable due to the differences in industries. Plants which have a considerable amount of loading and unloading of sheet metal often call upon the maintenance yard crew to perform this work. This is exactly the reason for the high ratios in cases No. 16 and No. 21.

The variation in group B, belt repairmen, oilers, and millwrights, is not so readily explained. Of course there is a certain amount of dispersion due to a difference in the nature or condition of the plant equipment. Other variations will creep in due to the difference in definition of the trades in the various plants. In some plants the electricians take care of the motor oiling on their routine electrical inspection, while the millwright work may be done by machinists, carpenters, etc. In this group the effectiveness of the force probably plays as large a part as any other cause.

The masons, plumbers, tinsmiths, etc., group H, seem to vary principally due to plant and equipment differences. Obviously, however, maintenance standards and the efficiency of the department play a large part. The highest ratio in the group of plants under consideration obtained this questionable honor due to all three causes, but principally to plant conditions and departmental inefficiency. The plant is a large one carried over from the war period and is encumbered with many more buildings than are required. Operating at productive capacity, the ratio would probably have been lower.

In the watchmen, group K, the same plant, case No. 5, had the highest ratio. This is again traceable to many buildings to be policed, many gates to be guarded, and street and railroad crossings to be watched.

Inefficiency in maintenance departments can be largely attributed to the following causes:

1 Poor or no supervision of operating force
2 Poor physical arrangement of quarters
3 A lack of adequate inspection routines and standards
4 The absence of a suitable order routine for flagging expenditures before they are made
5 Faulty organization, preventing an economic distribution of maintenance labor
6 Too many straw bosses.

In many cases, more than one of these causes is evident. Thus it is found that scattered maintenance shops in outlying buildings are often conducive to operating inefficiencies due to the impossibility of good supervision. Likewise, the absence of adequate inspection routines is often accompanied by frequent and unexpected machine failures. This places the maintenance function in the "corrective" class rather than the "preventive," and thereby introduces sudden demands for specialized labor. In addition, such a laxity is often responsible for interruptions in productive output.

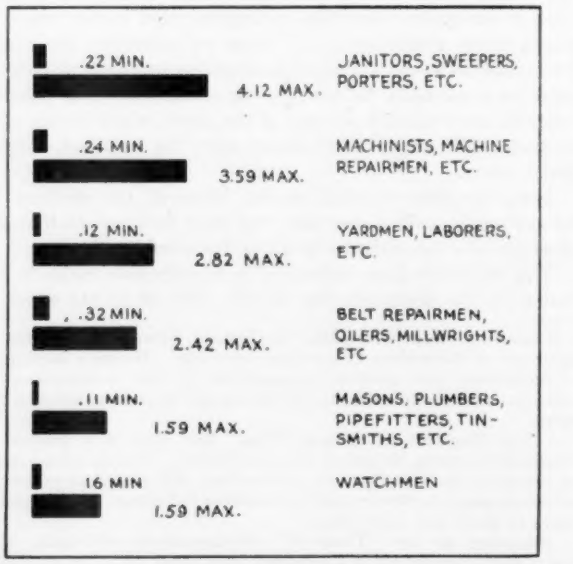

Fig. 2 Graphical Analysis of Variations in the Various Classifications

(Figures shown are percentages of the group to total plant force. Zero percentages not included.)

High maintenance costs are often charged to the inefficiency of the department when the real fault lies in the absence of suitable maintenance standards. This means that much of the work done may be unnecessary. It means that the standard of upkeep is decided by the individual worker and that the state of repair of both plant and equipment varies with the ideas of the individuals performing the repair work. In busy times, the general tendency is to slight "precautionary work" and in times of depression to overdo work of this nature.

The importance of plant maintenance cannot be emphasized too strongly, for it is the maintenance function that insures against interruption in production due to machinery breakdowns. A slight production tie-up may involve losses far greater than any maintenance cost. Furthermore, neglected equipment will very often wear out before its useful life has expired, thereby introducing hidden losses which will ultimately appear to the consternation of the management.

While maintenance must not be neglected, it also must not be overdone, for the plant which maintains within practical limits will outstrip quickly its competitor who goes in for "frills and fancies," all other things being equal. In other words, there is a point at which an economic balance exists between maintenance effort and operating costs, and this point is different for every industry. The determination of this economic balance involves the setting of maintenance standards, and these should be determined as carefully as product standards, for the maintaining thereof involves a definite charge against the operating income.

Because of the difference in standards, maintenance forces are not strictly comparable between plants, but if an unfavorable comparison is not interpreted to reflect inefficiency, it can be regarded to show an unwise difference in standards or an inherent penalty due to the condition of the plant property. The previous discussion of the chart, Fig. 1, shows how variations from the average can be explained in an analysis of the maintenance ratio.

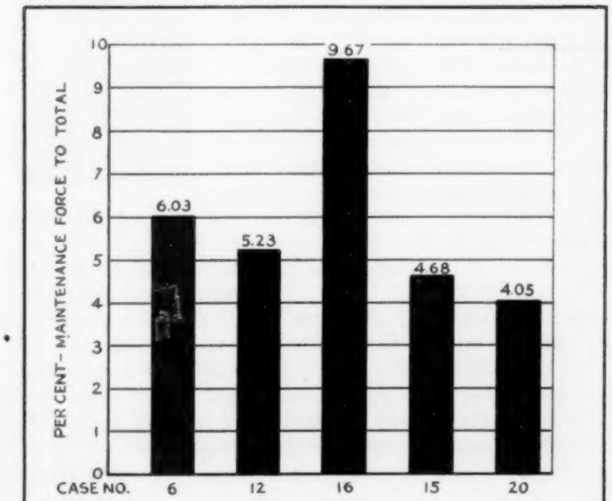

FIG. 3 RELATION OF MAINTENANCE FORCE TO TOTAL PLANT FORCE IN FIVE SIMILAR PLANTS PERFORMING STAMPING AND ASSEMBLY WORK

This analysis can be made even more searching by segregating the plants according to type of equipment and the character of the product. For example, let the same plants be examined as follows:

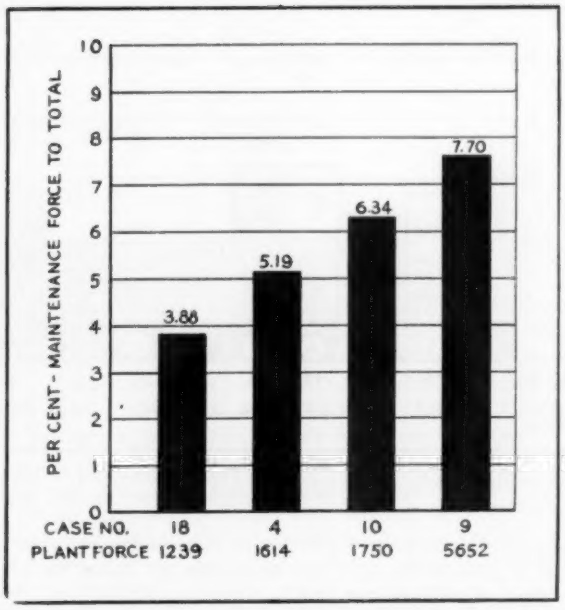

FIG. 4 RELATION OF MAINTENANCE FORCE TO TOTAL PLANT FORCE IN FOUR SIMILAR PLANTS FABRICATING AND FINISHING SMALL COPPER-ALLOY PARTS

STAMPINGS AND ASSEMBLY TO COMPLETED UNITS

Plant	Maintenance per cent to total plant	Total plant
6	6.03	182
12	5.23	247
16	9.67	248
15	4.68	467
20	4.05	467

This group is depicted by the chart in Fig. 3.

The five plants listed in the foregoing classification are considered as being comparable from the viewpoint of similarity of manufacturing processes. It is interesting to note that plants 15 and 20, with the greatest number of employees, show the lowest maintenance ratios. Plant 16 has the highest ratio, and when compared to the other plants listed in this class shows glaring inconsistencies. Reference to the chart indicates that in plant 16, classifications E, L, and B are the most out of line. A study of the plant reveals that great quantities of sheet steel are handled in the yard, which factor explains the excessive percentage in class L, yardmen. Further study fails to reveal any explanation for the excessive ratios in class E, machinists and machine repairmen, and in class B, belt repair, oilers, and millwrights. Consequently, in considering a reduction in maintenance costs, it is reasonable to look to class E and class B.

Plant 6 can also be considered to be somewhat out of line. It employs the smallest number of employees and shows the second highest maintenance ratio. In this plant, class J, class E, and class H show the greatest ratios of the total. Class J, janitors, sweepers, and porters, can be explained by the fact that the management takes extreme pride in plant appearance, and consequently the plant and grounds are maintained in excellent condition. Even so, the justification for this extra expense should be questioned. Class E and class H, although apparently high, cannot logically be considered for reduction due to the size of the plant. Although as percentages they are high, when reduced to numbers of employees, each shows less than two workmen.

Considering the points for reduction that appear to be possible in plant 16 and to a slight extent in plant 6, and considering

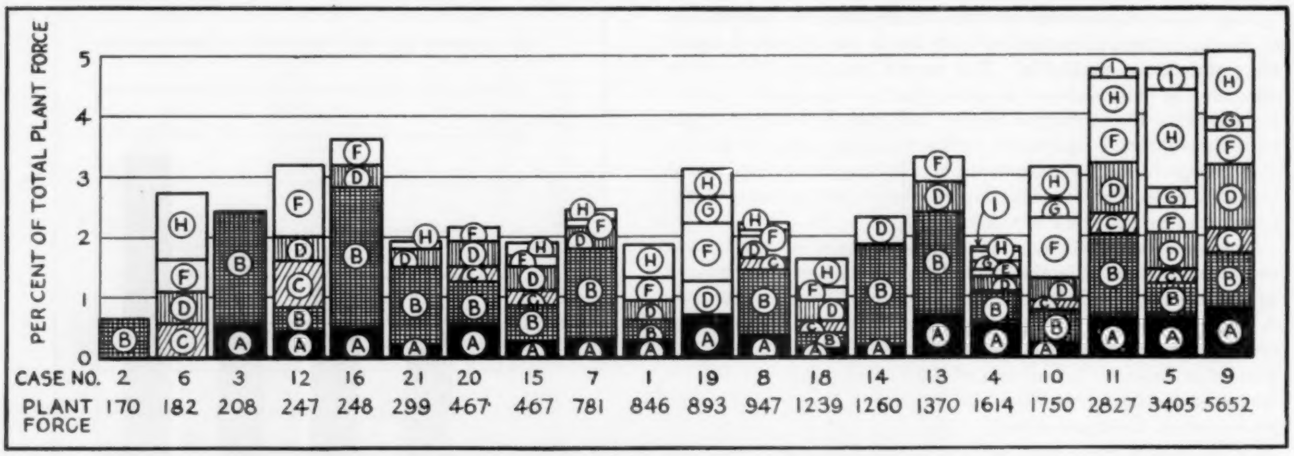

Fig. 5 Relation of Maintenance to Total Plant Forces

(Janitors, sweepers, watchmen, yardmen, etc., excluded. Legend for groups A to I, inclusive, same as in Fig. 1.)

that these reductions have been made, then the five plants in this industry classification become approximately equal from the viewpoint of maintenance ratio.

Another interesting comparison can be made of the four plants depicted on the chart, Fig. 4.

FABRICATING AND FINISHING SMALL COPPER-ALLOY PARTS

Plant	Maintenance per cent to total plant	Total plant
18	3.88	1239
4	5.19	1614
10	6.34	1750
9	7.70	5652

The four plants listed in this group are definitely comparable from the viewpoint of similarity of manufacturing processes and products, and also from the viewpoint of many old buildings housing the various manufacturing divisions. Reference to the chart fails to reveal any startling inconsistencies in any classification among the four plants listed in this grouping.

A detailed analysis of the maintenance departments in these four plants revealed that in plants 4, 10, and 9 the maintenance departments were too decentralized and split up into too many subdivisions, etc. All of this resulted in an uneconomical distribution of maintenance labor. Reductions were possible in each of these three plants.

In conclusion it can be said that maintenance costs are controllable. Standards can be determined and made to operate as effectively as production standards. Yardsticks are available for measuring the effectiveness of the plant maintenance department, and when applied intelligently, are of immeasurable benefit in controlling this function. Experience in numerous manufacturing plants throughout the country has proved these facts.

Coordination of Research and Engineering With Production and Sales

By CARL L. BAUSCH,[1] ROCHESTER, N. Y.

This paper is largely an exposition of the organization of the Bausch & Lomb Optical Company, Rochester, New York, in so far as it affects the function of research and engineering. This company went through a thorough reorganization several years ago and was fortunate in being of sufficient size to justify a distinct research and engineering division, and still small enough to make it possible to tie the functioning of these departments to that of the other departments without becoming involved in too much machinery and red tape. The company employs about 3000 people and has a research and engineering force of about 100. It manufactures a very large variety of small-quantity-production items, although there are a few items that are manufactured on what might be considered a quantity-production basis. It has, naturally, tried to minimize the amount of development work on articles that are expected to be sold in small quantities; still experience shows that quantity can have no great influence on the thoroughness with which the development work must be done, the company being often forced by circumstances to spend large amounts on work which will never result in large production.

The head of the Research and Engineering Department reports directly to the general manager and assumes full responsibility for all expenditures for development work. A yardstick has been set up by which accomplishments can be measured. This yardstick is the relationship between money spent on development work and the value of new business created, which consists of the amount of work of a regular production nature placed in the plant for manufacture, plus all special customers' orders requiring engineering-department design and follow-up. Figures are shown of this ratio for different classes of product.

Although the Research and Engineering Department assumes full responsibility for development work, sales and production are very definitely tied into the picture and must assume part of the responsibility when manufacture is finally started. Effective means are used to bring this condition about.

IT SEEMS quite necessary in presenting a paper of this kind to define the terms "Research" and "Engineering." A study of the dictionary convinces one that it is possible to make them mean very much the same thing or quite different things as one wishes. In industrial work the two functions are quite widely separated in the very large concerns and combined in the small ones. In the medium-sized concern there is a certain amount of overlapping and a flexible organization is necessary so that full advantage can be taken of the personnel available. The term "development" is probably a good one to use in describing the dual function of research and engineering in such a case.

Because of the flexibility needed in the organization great care has been taken to see that responsibility and authority are so vested as to bring about accomplishments without undue delay and friction. Organization charts, Fig. 1, help to visualize the problem involved in maintaining coordination between the activities of the sales, research and engineering, and production departments. The Scientific Bureau is the optical research and engineering department. All the optical designing is done here, regardless of the nature of the product involved or the division of the sales department it is for. Within the Bureau a division of work is made according to the type of design involved. In general these divisions are, ophthalmic lens, photographic and projection lens, instrument lens, ophthalmic instruments, and measuring instruments.

The Technical Bureau serves very much the same function as the Scientific Bureau except that it has to do with the mechanical problems in optical-instrument design. The divisions here are quite different, there being in general four separate divisions: (1) measuring instruments, (2) microscopic and ophthalmic instruments, (3) electrical and projection apparatus, and (4) industrial instruments.

The Optical-Machinery Development Department is a unit in itself having to do with the development of machines that are sold to the optical prescription trade. The Military Department is an individual one developing fire-control instruments such as range finders and submarine periscopes.

The department designated as Instrument Development is an experimental and sample department working with the Scientific Bureau and Technical Bureau in the practical development of projects that have reached the sample stage. The Spectacle Development Department is a unit by itself whose function is to develop new lines of spectacle wear. It also concerns itself with magnifying lenses and similar products.

The Drafting and Specifications Department is really what in most plants is known as the Engineering Department. It whips the creations of the other development departments into practical shape for manufacture. The Patent Bureau and the Final Inspection Department come under the Engineering Division for various obvious reasons.

Another function of the Research and Engineering Division is investigation of the possible uses to which the product might

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[1] Manager of Research and Engineering, Bausch & Lomb Optical Company. Mem. A.S.M.E. Mr. Bausch was graduated from Syracuse University in 1909 with the degree of Mechanical Engineer, and entered the employ of the Bausch & Lomb Optical Company. He has served as foreman, assistant superintendent, superintendent, and mechanical engineer, and has had his present position since 1926.

Contributed by the Management Division and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

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be put. This is a sort of general function of the entire division, and is only crystallized definitely in one department of the division. The Scientific Bureau maintains a chemical and physical laboratory where experimental work is done in an endeavor to find uses for the product.

It will be seen that the division of responsibility shown in the charts of the sales and manufacturing organizations is not the same as in the Research and Engineering Division. In the Sales Division there is an Educational and an Industrial Sales Department that may be handling the very same products, but selling them to different markets. This form of organization where the departmental classifications differ in the different divisions of the business, makes it rather difficult in some respects to bring about proper coordination of results, but the advantages of the set-up more than offset the disadvantages.

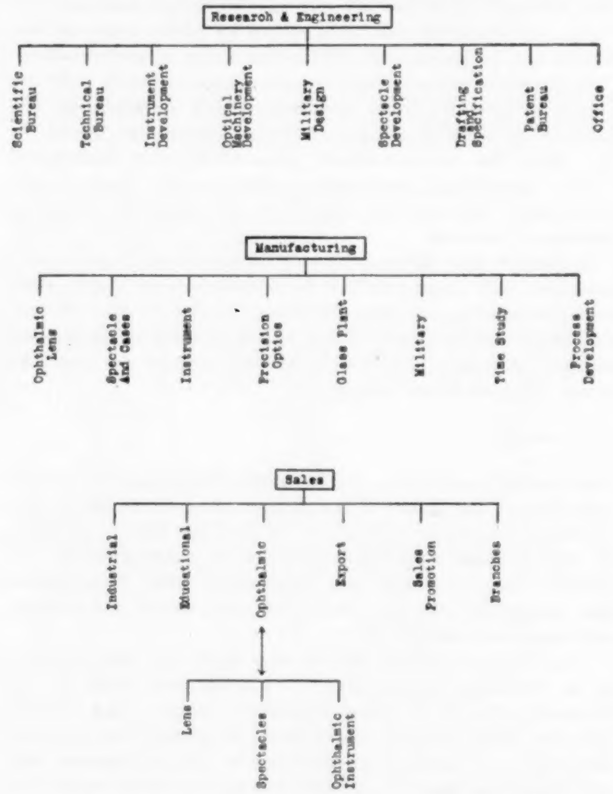
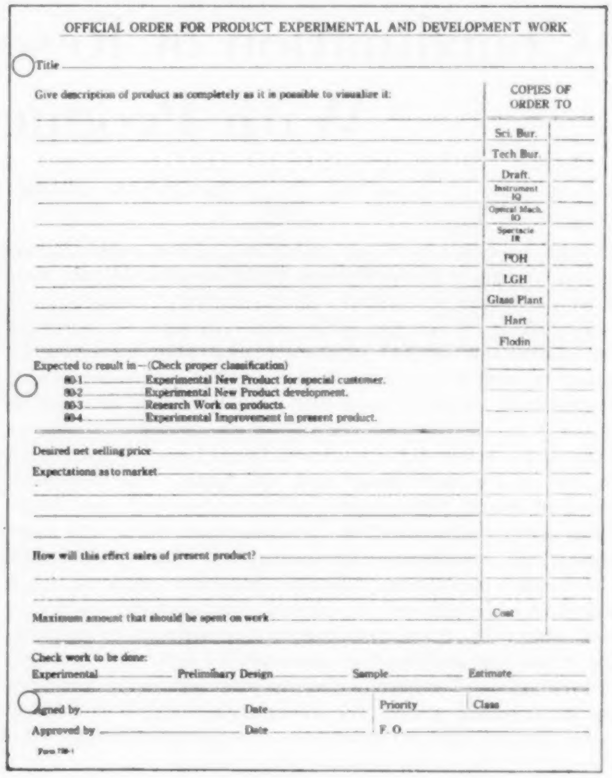

Research & Engineering
Scientific Bureau
Technical Bureau
Instrument Development
Optical Machinery Development
Military Design
Spectacle Development
Drafting and Specification
Patent Bureau
Office
Manufacturing
Ophthalmic Lens
Spectacle and Cases
Instrument
Precision Optics
Glass Plant
Military
Time Study
Process Development
Sales
Industrial
Educational
Ophthalmic
Export
Sales Promotion
Branches
Lens
Spectacles
Ophthalmic Instrument

FIG. 1 ORGANIZATION FOR RESEARCH AND ENGINEERING, MANUFACTURING, AND SALES DEPARTMENTS

In order to bring about this coordination without involving the head of the division in too much detail a department is set up directly under him and keeps in touch through records with all the work that is going on, and crystallizes its findings in reports which are very helpful in maintaining control and expediting the progress of work.

The business of Bausch & Lomb is run on a very definite budget basis and the appropriation for the program of the Research and Engineering Division is included in this budget in some detail. This appropriation includes money for development work on special products for customers, development work on new regular products, development work of a research nature, development work on the improvement of old products, service work for the manufacturing department, service work for the sales department, study of further uses for these products, literary work on catalogs, papers, sales manuals, etc.



OFFICIAL ORDER FOR PRODUCT EXPERIMENTAL AND DEVELOPMENT WORK

Title

Give description of product as completely as it is possible to visualize it:

COPIES OF ORDER TO

Sci. Bur.
Tech Bur.
Draft.
Instrument 10
Optical Mach. 10
Spectacle 18
POH
LGH
Glass Plant
Hart
Flodin

Expected to result in—(Check proper classification)
80-1 Experimental New Product for special customer.
80-2 Experimental New Product development.
80-3 Research Work on products.
80-4 Experimental Improvement in present product.

Desired net selling price

Expectations as to market

How will this effect sales of present product?

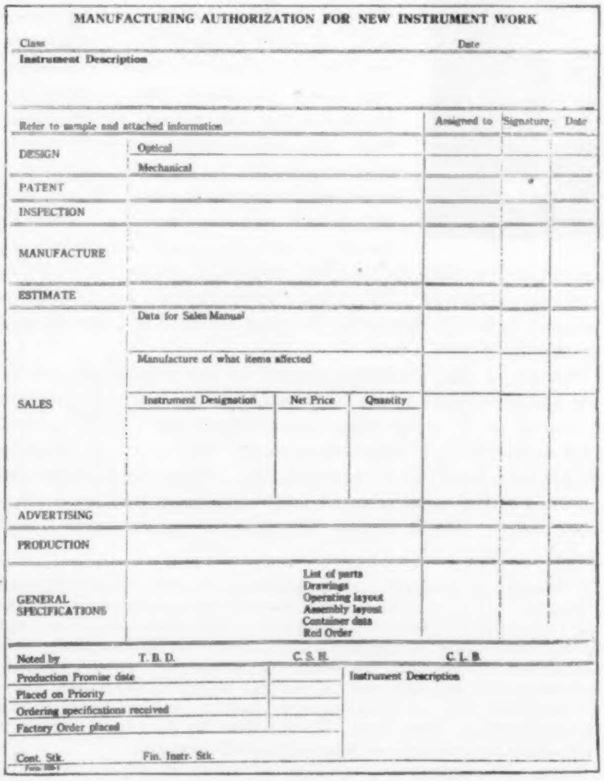
Maximum amount that should be spent on work

Cost

Check work to be done:
Experimental Preliminary Design Sample Estimate

Signed by Date Priority Class
Approved by Date F. O.

FIG. 2 EXPERIMENTAL ORDER FORM



MANUFACTURING AUTHORIZATION FOR NEW INSTRUMENT WORK

Class Date

Instrument Description

Refer to sample and attached information		Assigned to	Signature	Date
DESIGN	Optical			
	Mechanical			
PATENT				
INSPECTION				
MANUFACTURE				
ESTIMATE				
SALES	Data for Sales Manual			
	Manufacture of what items affected			
	Instrument Designation / Net Price / Quantity			
ADVERTISING				
PRODUCTION				
GENERAL SPECIFICATIONS	List of parts Drawings Operating layout Assembly layout Container data Red Order			

Noted by T. B. D. C. S. H. C. L. B.

Production Promise date
Placed on Priority
Ordering specifications received
Factory Order placed
Cont. Stk. Fin. Instr. Stk.

Instrument Description

FIG. 3 PRODUCT AUTHORIZATION FORM

To get a detail control of development work, no actual work is done without a form (see Fig. 2) being made out and an attempt made by the engineer in charge to answer the questions enumerated to the best of his ability. Looking over this form, it will be seen that there are questions which ordinarily would be put up to the Sales Department to answer, but in order to expedite action the engineer is made to feel the responsibility for getting the answers. He will naturally go to the different sales heads or others involved and get the facts that he needs to assume this responsibility. In work of a research nature, it is of course very difficult to answer the questions involved, but an attempt must be made in order to force thought on the subject.

The Engineering and Research Divisions are absolutely responsible for the money spent and for the results obtained, and it is their job to get the necessary cooperation to put the stunt over. Records are kept of expenditures against each of 20 different product classifications, and at the end of the year a comparison is made with accomplishments. For the purpose of accounting, accomplishments are assumed to consist of the dollar value of new products taken in work during the year including regular standard products and special customers' orders, minus the yearly sales value of products discontinued in the different lines during the year. This report for the year 1930 showed a ratio of value of new work created for the year against cost of development that varied from $58 to $0.50, with an average of $8.50 new products per dollar of development expenditure. Another figure that may be of interest is the ratio of expense of development to total sales for the year, which varied on the different classes from 0.2 per cent to 13 per cent, with an average of 2.8 per cent.

The Development Division does not carry the absolute responsibility beyond the creating of the product. In order to get the stamp of approval of the whole organization on new products resulting from development work a form (see Fig. 3) is used which is signed by all the departments interested before work is turned over to the factory for manufacturing. Here, under the head of design, the approval of the Scientific Bureau and Technical Bureau is secured on the optical and mechanical features, respectively. The Patent Bureau approves the project from its standpoint. The Inspection Chief indicates his approval and draws up inspection specifications for the production of the item. Under the heading of manufacture the superintendents involved sign the authorization. The Specifications Department gets from the superintendents an estimate of cost of manufacture.

The Sales Department signs up, indicating what data are required for sales manuals and catalogs, what items of manufacture will be affected, and indicates the prices to be charged based on estimates and market conditions. It also states the quantity wanted for one year's sales. The Advertising and Production Departments sign the authorization and the chief draftsman indicates what sort of specifications will be given to the factory to manufacture from. It is then noted by the managers of manufacturing, service, and engineering, and then, and not until then, is it a bona fide Bausch & Lomb product ready to be turned over to the factory.

Although up to this point the Engineering Division was entirely responsible, now the organization as a whole accepts the new product and its creation is charged up as a credit to the Engineering Division.

Discussion

Charles M. Mills.[2] The author has certainly rendered a service in the definition of "research," "engineering," and "development." He has pointed out that although in his own company the words used are, "Research and Engineering Department," the main function is that of development. From considerable corporation experience, the writer believes that the word "development" cannot be emphasized too much. Frequently the word "research" connotes a more or less isolated group of men divorced from the activity and heart of the company's business. To many line executives the word "research" is dissociated with everyday affairs.

Likewise there have been frequent clashes between groups known as "research" and "engineering." In some corporations the word "engineering" means an active group of men engaged on construction and replacement. In other companies "engineering" may include these men plus groups engaged in pure research. The word "development," therefore, appears to be a happy solution of the problem.

The author has clearly outlined his own organization. The various subdivisions of his department, Scientific Bureau, Technical Bureau, Optical Machinery, Development Department, Military Development Department, are clearly divided as to their functions. Further it is easy to see how the proper cooperation can be established between these departments and sales and manufacturing.

However, a general organization chart showing the relation between the heads of the larger departments would have been helpful. How do the heads of manufacturing sales and research and engineering tie in with the top executives of the company? Just where is limitation given to the research and engineering department to perform solely as a staff group? Is there a distinct ranking of the personnel that top the three major departments?

Joseph Geschelin.[3] Experience in the automotive field indicates that coordination is incomplete unless it includes the service field. The user has much of value to offer the research department. And the engineer must find the opportunity to contact frequently with this group.

Research is concerned not only with the development of new products, but also with improvements in the present line. Certainly no one knows better than the user just what the limitations of your product are. He can frequently suggest slight changes which may improve performance and acceptance profoundly. In a similar fashion the user may be a prolific source of ideas for new products.

Another reason for closer contact with the users and with one's own service organization is that in many cases the service man does not know as much about the product as he should. This should be a real source of concern to the engineer. In many cases observed personally, modifications are made in the field which are not reported to the factory and consequently do not appear in the products. Thus, slight inconveniences to the user are perpetuated at the expense of good will.

Accordingly for complete coordination of research and other company functions, include service as one of the most important elements.

[Discussion of this paper, jointly with that of Richard F. Wilder, by Walter E. Lee, B. Townshend, E. C. Rack, and L. A. Hawkins, will be found following the latter paper.]

Author's Closure

Mr. Charles M. Mills has suggested that a general organization chart, showing the relation between the heads of the larger departments, would have been helpful in this paper. He asks how the heads of manufacturing, sales, and research and engineering tie in with the top executives of the company. There are five divisions of the business heading up to the general manager. These are sales, research and engineering, manufacturing, service and maintenance, and comptroller. The five managers of these divisions are directors of the company and report directly to the general manager.

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[2] Assistant Director, Twentieth Century Fund, Inc., New York, N. Y.

[3] Engineering Editor, *Automotive Industries*, Philadelphia, Pa.

Mr. Joseph Geschelin calls attention to the fact that the user of products has much of value to offer the research department. This is undoubtedly true. It has been the experience of our company that contacts with the user are absolutely necessary, if one wishes to bring about ideal designs. In our organization this contact is made mostly through our salesmen, who are practically all technically trained men. These salesmen often suggest that an engineer be sent out on the job to get first-hand information, which is often done. This hooks up pretty closely with what Mr. Walter E. Lee had to say; that is, that most of the problems recommended for research come from the field through the sales department. His description of the sales department as the ears and eyes of the whole organization is very well taken. We must cater to the department that produces the revenue and makes possible all the other activities of the organization. In the last analysis ability to sell the sales department the results of research is the true measure of the efficiency of the research and engineering departments.

Mr. L. A. Hawkins takes exception to the statement that the words "research" and "engineering" are sometimes used to mean much the same thing. The author tried to make clear that this was the point of view of a medium-sized concern. It is easy to understand how Mr. Hawkins, because of his connection with a very large corporation, feels that it is a pity that these two activities cannot be kept separated. Although it is true that the search for new scientific knowledge is a different thing from the application of scientific knowledge in developing new or improved processes of product, still it is found that in a medium-sized corporation the knowledge gained by the people in research can often be applied more efficiently by them than by passing it on to a separate department. An organization such as described makes it possible to function in both ways, depending upon the circumstances surrounding the project.

Coordination of Research, Sales, and Production

By RICHARD F. WILDER,[1] CAMBRIDGE, MASS.

This paper seeks to demonstrate how the various research activities are coordinated one with another, and then as a unit with sales and production, so as to consider related factors in the instigation and carrying forward of research projects. Likewise it aims to bring out how definite mechanisms of coordination may be established whereby the greatest benefits may be derived from the research activities through the development and manufacture of products which are both commercially and technically sound.

COORDINATION of research, sales, and production—what does this expression mean to company executives throughout industry? In the first place, "coordination" is the "process whereby advance agreement is secured regarding a policy and general methods of execution from several functionaries necessarily involved in its carrying out and the further process whereby their execution of it is secured in such a manner as to harmonize the special points of view of the several functionaries to the end that there is willing, orderly, and effective carrying out of the policy."[2] The principle of coordination is found to be, "Definite mechanisms and routines must be provided to the end that all groups within an organization function harmoniously in the attainment of a common objective."[3] We may take this definition and principle of coordination in connection with research, sales, and production to mean the marshaling of all forces connected with product and process development, the consideration of all related factors, and the combination of these elements into a smoothly working mechanism with the objective of manufacturing and selling a line of products that are commercially and technically sound.

The author has attempted to determine how such coordination is effected in a number of representative companies, and from these cases to draw certain general conclusions. Some fifty organizations, over a wide geographical range and representing many different types of industries, were studied. Manufacturers catering to the style element were excepted.

The expressions "centralized" and "decentralized" research functions will be used frequently. Centralized research is defined as the separation of the research activities from the line activities, combining all such into a department reporting directly to an officer not directly connected with a line activity. Decentralized research means that the research functions are found in the respective line departments to which they are related.

Justification of Research

Present-day industry is much more sympathetic with research of a technical nature than it is with such studies relating to markets. Research of the former type is a force for improvement in both manufacturing processes and products. It appeals to the average company executive because of its tangible nature. On the other hand, despite keen competition, a fastidious public, and the addition of hundreds of new products to an already diversified list, market research has not been generally accepted as a vital function of industry. There is a definite time lag between the results obtained from market research and those obtained from technical research, inasmuch as the former are usually far removed from the original research project, the product itself being usually the result of that work. Market research may not show results until many months later, when they may possibly take the form of increased sales and profits brought about through the development of a product for a market which was known to exist before the technical development work was ever started or had progressed to any large degree.

The predominant method of interpreting market conditions, reactions, potentials, etc., is through the use of sales executives and salesmen. It would appear, generally speaking, that such a practice is basically unsound. The function of the salesman is to sell the company's products and not to spend a large amount of time in market field work. Another factor is the method by which the sales force is paid. If this is on a commission basis, these men will not spend a large amount of time on work that will cause their sales volume to decrease. Also, the inherent characteristics of a salesman will hardly fill the requirements of a successful market research man, as one of the basic characteristics of a good salesman is optimism, and his market investigations may not be based upon impartial facts.

The responsibility of deciding as to the feasibility of instigating research work usually rests with a major executive or a group of such. In order to advance sound conclusions, sound information, reflecting every angle of the problem, is necessary. We find engineering, research, patent, and manufacturing research departments each contributing their part. Yet in many cases the only consideration made of the markets will be a round-table discussion of four or five executives, the reaction of several sales managers, and the suggestions of a number of salesmen.

In many cases, particularly in those companies manufacturing industrial goods, market research is viewed as a less necessary adjunct than in those companies manufacturing consumer goods. However, in these cases there is the danger that the company

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[1] Research Assistant in Department of Business and Engineering Administration, Massachusetts Institute of Technology. The author was born in Sumter, S. C., in 1906. He attended Sumter High School, and was graduated from Furman University, Greenville, S. C., in 1928 with the B.S. degree and from Massachusetts Institute of Technology in 1931 with the S.B. degree in business and engineering administration. He was appointed to his present position in June, 1930.

[2] See discussion to "Organization and Operating Principles," American Management Association, General Management Series no. 112, p. 24.

[3] See "Notes on Measures of Management," by Prof. F. E. Raymond, Massachusetts Institute of Technology, section 5, p. 38.

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executives will overestimate the degree of concentration of their market.

Need for Coordination of Research Activities With Sales and Production

There are generally two broad mediums through which research may be conducted—either by the research departments in an organization or by the employment of outside concerns. This paper will deal with the former, concerning the policies existing within an organization, and those types of research which have a specific objective.

Consider, for example, the development of a new product or a radical change in an existing product. Generally speaking, we must determine upon:

1 The product itself—what it is to be made from, the physical and chemical characteristics necessary, cost of material, its form, design, etc.
2 The manufacturing methods and processes—whether it can be economically produced, the production facilities necessary, the new equipment needed, the cost of equipment, the cost of manufacture, etc.
3 The market—whether there is one, the quantities it will absorb, the competitors in the field, distributing channels, type of market, etc.
4 The patent situation.

Here we have four distinct problems in four separate fields to investigate, with the view of making an addition to the regular line of products or of improving the process of manufacture. These activities have to be coordinated with each other and later to be translated into routine production and selling. Each of these is analogous to four sub-assembly lines feeding a main assembly line. Obviously, a delay in any one affects the whole production schedule. Yet there are those companies that will allow one of these research activities to proceed at a wide tangent to primary objectives and accept such a condition without taking definite steps toward remedying the trouble. Is not the development of new products, improvements in existing products, and improvements in manufacturing methods and processes important enough so that all activities can be coordinated to permit speed of operation and low operating costs, and at the same time to attain technically and commercially sound results?

Some of the outstanding dangers attendant to this so-called lack of coordination of these different functions are:

1 Spending money in development work on a project that turns out to be a failure owing to improper consideration of all phases in the initial investigation
2 Development of products that are not commercially sound, because of lack of markets or high selling price due to high manufacturing costs, etc.
3 Waste of time and money on research projects that have no application to the business
4 Product design developed without consideration of the manufacturing department
5 Failure to direct the activities of the different research functions into those channels giving the greatest benefits
6 Danger of personnel difficulties arising out of apparent conflicts of authority
7 Progress delays on a project due to the lack of cooperation of departments
8 Omission of periodic follow-up of research work to determine the advisability of continuance
9 Danger of not routing research projects, in relation to types of research to be performed, so as to eliminate impractical ideas in the early stages.

In addition there is the patent situation to consider. This is usually handled by having all projects approved by the patent department before any large expenditures, either in time or money, are made.

The Elements of Coordination

Granted that there is a need for coordinating these various industrial activities, how can it be accomplished? The principle of coordination is to establish definite mechanisms and routines whereby correlation of all functionaries and their activities in complete and harmonious operation can be maintained. The essential factors would seem to be organization, personnel, and the placing of authority, each with modifications and additions as individual cases might warrant. Before attempting to solve this problem, company executives should ask the following questions:

What is the personnel situation?
What is the size of the company?
What are the types of research with which we are most concerned?
What is the nature of our product?
What is the nature of our market?
Are we in a style industry?
What is the specific object of our research activities?
What seems to be the difficulty in our present method of operation, etc.?

Inasmuch as such great variations exist between different companies, no attempt has been made to analyze and compare different policies as to the individual steps which make up the development procedure and coordinating machinery. Rather let us consider what representative companies are doing, treating each one as an individual case. From these considerations, general deductions can be made. It should be borne clearly in mind that these cases represent a pattern of the company policy from which there may be certain divergencies as individual situations demand.

The companies interviewed have been divided into four general classifications on a basis of the position of the research functions in the organization, as follows: (1) Decentralized research functions, (2) partially centralized research functions, (3) completely centralized research functions, and (4) research in the largest type of organization.

Decentralized Research Functions

These organizations range in number of employees (as of the fall of 1930) from approximately 1500 to 11,000, while the range of all interviewed companies in this classification is from approximately 750 to 45,000 (only one company above 11,000).

Attention is first called to a large company manufacturing both consumer and industrial goods, the products of which are not manufacturing equipment and not of a mechanical nature. Fig. 1 shows a partial organization chart of this general type of company. In this case the market research function is divided into two general fields (not shown on Fig. 1), each being handled by a department reporting to the vice-president in charge of sales—namely, the study of further uses for the company products and the study of these newly discovered markets in relation to potential value, buying power, etc. Such a division is necessary due to the scientific nature of the investigations carried on by the department looking for new uses.

Generally speaking, all ideas and suggestions requiring technical research are referred to the vice-president in charge of manufacture, and in all probability the advice of the sales executive is sought by him before making a decision. If the suggestion seems to present worth-while characteristics, a preliminary investigation is made with respect to product research and manufacturing research. This investigation is not expensive and may eliminate impractical suggestions at the start. If the results are satisfactory, the project is turned over to the sales executive, who instigates a market analysis. On these two investigations it is decided whether additional development work is feasible. If decided in the affirmative, the manufacturing executive instigates further development work, and the project is completed up through the determination of manufacturing processes. If possible, a small-scale test of the market is made before any new development is released for regular production.

During this development period reports covering hours of work spent, progress being made, and cost are submitted monthly to the two vice-presidents. Each executive has authority to stop work on any project on a basis of the work done in his department. After all work has been completed a recommendation of the two vice-presidents is submitted to the president.

The policy of this company presents a clear-cut example of individual management of individual functions, with all functions being coordinated through the consultation and cooperation of the two executives concerned. Under such an arrangement the relationship between the technical research function and the line manufacturing function is likely to be close, as both are under the control of the same executive. This is true for the market research and the line selling. On the other hand, it is noted that a major staff function and a major line function are controlled by a single executive, and this condition brings the personal equation to the forefront. In some cases a line executive may have all the characteristics necessary for the control of line work and not be capable of administering the associated staff activities.

A direct contrast to this method of research control is found in another company of the same general type of organization. While the products vary in nature from those in the preceding case, they are not different enough to require another type of set-up and operation. In both instances the management feels that the heads of the different research functions should report to the head of the line departments to which they are related. The control of research work is vested in a committee composed of the heads of the merchandising, market research, manufacturing methods, engineering, and chemical departments, and a special staff executive who, as chairman of the committee, reports to the president. Research projects relating to changes in existing products or the development of new products usually arise in one of the merchandising departments, and are referred to the research committee for approval after the project has been investigated superficially by a staff division of the merchandise department with respect to markets, production methods, and means. If research work is approved, the committee will determine the sequence of operations (what type of research is to be carried on first, etc.), and then assign it to one man who will follow the work through. This development work is carried on up to the point of practical usefulness, and is then turned over to the appropriate line department. Every week a report on every project being studied is submitted to the committee covering (1) the title, (2) the progress being made, and (3) the amount of time spent in the study. Once a month these reports are condensed into a formal report, and the situation is reviewed by the committee.

A great deal can be said both for and against the committee system of management. The effectiveness of a committee would depend to a large extent upon whether it is for control or advisory purposes.

In a company which has the same general type of organization as the two preceding cases, but which manufactures consumer goods to a large extent, the market research is in the sales department, and all technical research is controlled by the vice-president in charge of manufacture. The procedure for handling research projects depends upon the nature of the problem and the point of origin. For instance, some one in technical research may suggest a project to the director of the department. This man may be authorized to perform the work, provided a large expenditure of time and money is not involved. In the event of considerable cost, the vice-president in charge of manufacture must be consulted. The same situation exists in the market research department, which is ultimately controlled by the vice-president in charge of sales. In either case, if a project arising in one department concerns the other, the contacts are made through the two vice-presidents in charge.

All research work in a department is listed each month, and the lists are exchanged between the manufacturing and sales departments.

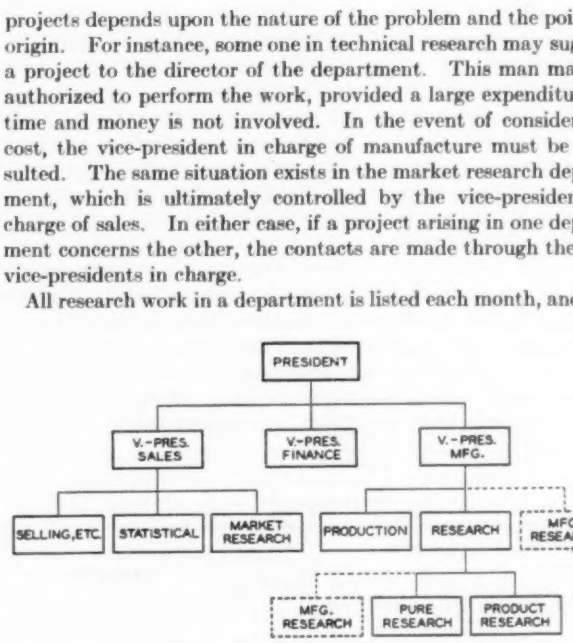

FIG. 1 PARTIAL CHART OF DECENTRALIZED TYPE OF ORGANIZATION
(Dotted lines indicate function may appear as indicated.)

Research projects for the development of a new product or a radical change in an existing product are referred to the president. If he sees merit in it, he turns it over to the vice-president in charge of sales. A sales committee of the higher executives in the sales department decide on the project. If they think it has commercial value, the market research department investigates competing products, potential markets, etc., and reports to the sales committee. There is also a committee of the administrative officers which is kept posted of all work being done (the vice-presidents in charge of sales and manufacturing serve on this committee). If the investigations up to this point have been satisfactory, the project is turned over to the vice-president in charge of manufacture for further development work. If this should uncover factors making a change desirable, the administrative committee is consulted through the manufacturing executive. After all product research is completed, the product is manufactured on a small scale and given to the sales department for a field test. If a market exists, the sales committee recommends to the administrative committee that the product be manufactured and marketed.

There is constant contact between the president and the vice-presidents in charge of manufacture and sales during the development period of any new product, a practice which carries down to the various departments, which frequently supplement one another with necessary information.

Let us now consider an organization that has no definite market research department. This company is organized along the same general lines as the three preceding cases, but the products are industrial goods of a mechanical nature.

The majority of the research projects come from the sales department through its contact with the trade. Problems relating to the products are referred to the sales manager, who, if he approves, contacts with the chief engineer. The chief engineer is in charge of research and design (except the design of tools, etc., and manufacturing research) and reports to the vice-president in charge of operations (this vice-president is also in charge of production and manufacturing research). It is the function of the sales manager to "sell" the suggestion to the chief engineer so as to have a budget set up for preliminary investigations. If the project meets with his approval, a preliminary investigation is made by the engineering and research departments, and rough sketches are submitted to the sales department, where consideration is given to the market. In some cases several salesmen go into the field to determine the potential market, competition, etc. If the results are satisfactory, the vice-president in charge of sales decides if further work is to be done. The engineering department then completes final specifications and the design of the product, after which the factory manager considers questions relating to the manufacture. A manufacturing committee, of executives of the engineering and production departments, is often used, and it is through this committee that all difficulties of design are eliminated. Sometimes a test product is made up to be sure that all production "bugs" are eliminated and also to test the market by offering this sample. This test is under the chief engineer, who has the cooperation of the sales manager.

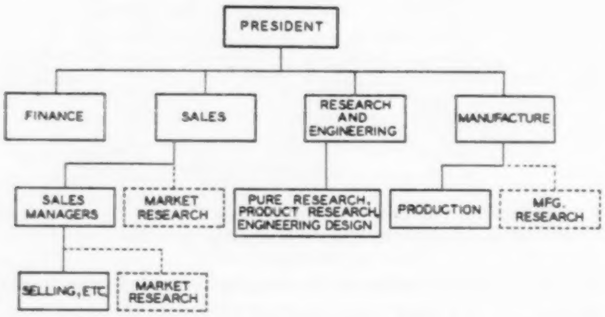

Fig. 2 Partial Chart of Partially Centralized Type Organization

(Dotted lines indicate function may appear as indicated.)

After consideration by all departments, the board of directors has final approval, in case of a new product or radical change in an existing product.

Whatever sales research is undertaken is carried on by the regular line salesmen. In cases where the product requires technically trained men to sell it, this plan has certain merits. Another point is that the relationship between the design and production departments is relatively closer than in the three preceding cases.

Partially Centralized Research Functions

The size of the companies interviewed, in terms of number of employees as of the fall of 1930, which fall into this section range from approximately 700 to 20,000.

In these companies one or more of the research departments are in a separate division apart from a line department and the remaining research activities are in some line department. In this type, the product and pure research functions are carried on by the separate division while the manufacturing and sales research are in their related line departments, although there are certain divergencies from this, as will be shown.

In this type there is greater specialization in the research and development departments, with closer direct contact with the administrators of the company. On the other hand, the relationship between the manufacturing and the product research departments is not likely to be as close from the point of control as where these two departments are responsible to the same executive.

Likewise, it would seem that the different activities should be coordinated through a committee, as the three functions of product research, manufacturing research, and market research are responsible to different executives.

An illustration of such an organization is a company manufacturing industrial goods of a mechanical and electrical nature. In this case the distinction is made between product research and engineering. See Fig. 2 for a partial organization chart of this type.

The work of engineering, research, and development is controlled by a development committee composed of the president, the vice-president in charge of sales, vice-president in charge of research and engineering, factory superintendent, research director, chief engineer, treasurer, and sometimes the three assistant sales managers in charge of the three major divisions of the sales department. In addition there is a smaller development control committee composed of the vice-president in charge of research and engineering, director of research, chief engineer, vice-president in charge of sales, and sometimes one or more other officials. It correlates the work of the research, engineering, and sales departments and saves the time of the larger committee by thrashing out points in connection with development work.

There is no market research department, as all such information is collected by the sales department. There is a feeling, though, that a market research department is advisable, and such an organization is contemplated.

All ideas and suggestions requiring research are first referred to the development control committee for a preliminary examination. If the project originates in the sales department, it is usually accompanied by a statement as to the probable market.

If the problem originates elsewhere, the sales departmen is requested (by the development control committee) to prepare such a statement. If the project seems to present merit, it is turned over to the development committee.

If desirable to go ahead with the project, the development committee orders the research or the engineering department to commence work. Periodical progress reports are submitted to the department head by the men actually doing the work, and he submits a summary to the development committee. If found satisfactory, it is sent back to the engineering department to be put into salable shape. During this period reports similar to the ones used in the development stage are submitted by the engineering department.

When all design work is completed, the development committee reviews the project again; and either sends it back for further work or turns it over to the production department, where the shop engineer (who reports to the factory manager) supervises the design of tools, production methods, etc.

The next step is to test on a small scale. The control is in the development committee or the production department, depending on the size and nature of the project. Sometimes the product of the small-scale operations is offered to the market for further test.

The salesmen often are accompanied by some one from the research or engineering departments. The sales department assigns a catalog number and submits an estimate on the amount that will be sold annually and the desirability of manufacture for stock. It is on a basis of this estimate that production schedules, etc., are set.

Another illustration of this general type is a somewhat larger company manufacturing a line of industrial products of a mechanical nature of relatively high unit values. The committee system is again used here, there being three committees acting on a major development before it is adopted. These are the executive committee, composed of the president and all the vice-presidents; the senior committee, composed of the three vice-presidents of manufacture, sales, and research and engineering; and a junior committee composed of sub-executives of the manufacturing, sales, and research and engineering departments. This company also maintains a market research department, which reports to the sales manager and through him to the vice-president in charge of sales.

The source of development and research problems has an influence on the initial investigations, but after a certain point has been reached, the procedure is the same in almost all cases.

Problems originating in the sales department are referred to the vice-president in charge of sales, who, if he approves, refers them to the market research department. Its report is submitted to the sales executive, and through him reaches the senior committee.

At this point the procedure becomes standardized for all projects. Therefore let us consider a project originating elsewhere than the sales department Such a project is referred to the vice-president in charge of manufacture, who, if he approves, turns it over to the engineering department. At this point it may be sent to the market research department. Then, if the manufacturing executive still believes the project to have merit, he refers it to the senior committee. The project may then be assigned to one of the junior committees for further study, without going to any expense. The junior committee refers the project again to the senior committee. If the senior committee thinks that further work is warranted, the executive committee can decide the extent to which funds are to be allotted to this development work.

Then the project passes to the engineering department, where the development work is commenced. If changes in the original plan become necessary, further authority must be obtained from the executive committee.

After all development work has been completed, the engineering department samples are taken out in the field. This field test is under the control of the engineering department, but the sales and manufacturing departments scrutinize the consumer reaction.

At the completion of the field test, the matter is again referred to the executive committee for final approval and the order for regular production.

Another example in this general classification is a company that manufactures industrial and consumer goods of a mechanical nature having a relatively low unit cost in comparison with the preceding case. We have here a product development department which lies between the manufacturing and sales departments, looking to the manager of manufacturing operations for control (issuance of orders, etc.), and to the vice-president in charge of sales for approval of work done and responsibility for money spent in the development department. Generally speaking, the vice-president in charge of sales has control over all development projects, but a committee composed of approximately six executives, representing every major phase of the business, acts in an advisory capacity in projects involving large capital expenditures.

Improvements in existing products or the development of new products are referred to the sales executive before any work is done. If the market is doubtful, he authorizes the market research department to make an investigation, and on this basis the product development department is authorized, through the manager of operations, to commence work. Progress reports are submitted monthly to the vice-president in charge of sales, and his approval is necessary for the continuance of work. Upon the completion of the development work the project is turned over to the vice-president in charge of manufacture, who sees that the manufacturing processes are organized in the manufacturing research department of the production division. After all production "bugs" have been eliminated, the project goes into the semi-works state, where it is further tested. This is in the production department, and the sample from these operations must be approved by the product development department. Sometimes these samples are offered to the market for a further test.

The two vice-presidents in charge of manufacture and sales are in close contact, and once a year they make out a request for a budget to cover the development work for the coming year, which is based on current work and a certain amount for additional projects that may arise.

The status of the development department is an apparent case where a man is serving two masters. It is obvious that such a policy would be possible only where personnel conditions permit, but wherever it is possible it would seem to be effective in closely relating all development work to the line activities of sales and production. An effective check is maintained, as both are represented at the outset and all along the development line through this dual contact. Then, too, any excessive enthusiasm radiating from the sales department is likely to be short-lived, since this department is responsible for the amount of money spent on development work.

This case also points out the principle that as the various research functions are drawn closer together, the importance of committees lessens.

In addition to the cases cited are those companies that are approaching centralization in a still greater degree. Some of these have all research functions, except those relating to the markets, in one department reporting to the president of the company, while others have all functions except manufacturing research so segregated. In these companies a broader initial investigation is usually made before accepting research projects. This investigation is in the hands of the originator of the idea and consists of gathering such information as expected cost of development, approximate number to be sold, expected selling price, assumed profit, the effect of this new development upon the existing line of products, etc. Such projects as these are first approved by the executive in charge of the development function, unless the individual department budgets are so constructed as to absorb this preliminary expense as part of their operating expenditure.

There are also those companies that have the market research function reporting directly to the president and the technical research under control of the production executive.

Completely Centralized Research Functions

The size of the companies interviewed (in terms of number of employees as of the fall of 1930) which fall into this section ranges from approximately 5000 to 17,000.

"Completely centralized research functions" is defined as the concentration of all research activities into one department which reports to the president or administrative committee. Under such an organization there is no connection with the line departments.

An excellent example of this type of organization and operation is a large company manufacturing industrial goods of wide distribution and a variety of uses. Fig. 3 shows a partial organization chart of this type of company. The research laboratory is interested in research of a pure or fundamental nature. The remaining divisions of the planning and development department are self-explanatory.

All ideas requiring research are turned over to the manager of the planning and development department. He first calls the proper project supervisor into consultation. It so happens that the heads of the divisions of the planning and development department are project supervisors. These two officials confer with others as they see fit and decide upon the value of the suggestion. If the suggestion appears to have merit, the project supervisor endeavors to develop information on its market possibilities and the necessary expenditure and work to produce preliminary samples for test purposes.

If the idea or product holds promise of value, the project supervisor will recommend that the manager of the planning and development department carry out the necessary development work. From this point on, the project is recognized as an organized development project.

Upon the approval of the latter, the project supervisor confers with technical and operating executives. From these conferences he can determine the scope and sequence of the investigation. He then goes to the department heads in the planning and development department and works out a plan of procedure. This plan incorporates the precise information desired and an estimate of the time necessary to obtain it. In other words, all work is preplanned within each department entering into the investigation.

When all programs have been laid out to the satisfaction of the supervisor and the department heads involved, the development work is commenced. In some cases, work is being done on the same project in two or more departments simultaneously.

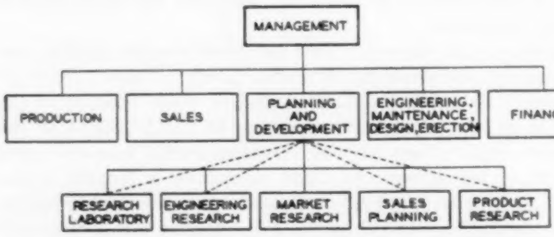

Fig. 3 Partial Chart of Complete Centralized Type of Organization

As the work progresses, constant follow-ups are made by the supervisor, who keeps the manager of the planning and development department informed as to the progress being made. Weekly reports are submitted to the supervisor on all organized development projects and monthly reports on all projects for investigation. If these reports show unlooked-for complications, the supervisor submits a report to the manager of planning and development. Likewise all planning and development department reports are routed to a group of company executives. The manager of the department also makes a brief report to the board of directors at their monthly meeting.

At the completion of the organized development project, a report is submitted incorporating the recommendations of the supervisor. The project must first be approved by the heads of the engineering research department, the research laboratory, the market research department, the project supervisor, and the manager of the planning and development department, and then later approved by an executive group composed of the vice-president in charge of sales, the vice-president in charge of manufacture, and the first vice-president.

If these approvals are forthcoming, the supervisor prepares the production plan, through the cooperation of the engineering research department and the production department. The engineering department is called upon to design and build the production equipment in accordance with the specifications of the engineering research department. The production plan must then be approved by the head of the research laboratory, the head of the engineering research department, the project supervisor, the manager of the planning and development department, the general plant superintendent, and the vice-president in charge of manufacture. Then follows the approval of the standard specifications (prepared by the research laboratory) by the chief inspector of the plant, the plant superintendent, the standards of production office, and the vice-president in charge of manufacture.

Next is the transition into production and experimental operation under the direction of the project supervisor. Sometimes a small quantity of the product is tested upon the market. The product must then be approved by the sales department, followed by the acceptance of the manufacturing unit by the general plant superintendent and the vice-president in charge of manufacture.

The preparation of the sales plan is then undertaken by the project supervisor in cooperation with the market research department, and submitted to the general sales manager (of the division of products into which the new product falls) and the vice-president in charge of sales. Approval is followed by the transition into sales operation under the supervision of the project supervisor. After this has been accomplished, the supervisor submits a final report to the manager of planning and development, and the file is closed.

In contrast to the cases cited in the two preceding sections, there are the following outstanding features. The authority of accepting or rejecting all ideas at the outset is vested in one department which initially consults with those in the organization who have information to offer. Then if the idea is accepted, it is turned over to one individual who controls all work on the project until it is absorbed in the regular schedules of the company. There is also a definite policy whereby all suggestions are investigated in a preliminary way before a definite schedule of development work is made up. Likewise there is a noticeable lack of the use of committees. Responsibility is well placed, and each person's duties are clearly defined. Attention is also directed to the presence of communication channels whereby contact can be made with anyone in the organization who might have information bearing on a given project.

In addition to this case there are other organizations operating along the same general lines, with variations as the cases may warrant. One of the companies goes so far as to include the sales development department in the research and development department. Indeed, every staff department relating in any way to the research function and the development of new products is thus incorporated.

There are also those companies similarly organized that have placed the responsibility for results on one man or a small group, whose function it is to "sell" ideas to the appropriate line organization rather than have them accepted through the approval of a group of officers of the company. In such cases the line departments concerned are consulted from time to time, but there is the possibility that friction will be developed if a mutual agreement cannot be reached.

Research in the Largest Type of Organization

Very large corporations are taken to mean those that act more or less as a holding company with a number of operating subsidiaries, or branch plants, employing upward of 50,000 men. Fig. 4 shows a partial chart of this general type of organization.

The close personal relationships that are found in the smaller companies cannot be here maintained, and consequently a greater degree of organization necessarily has to exist. Likewise it is practically impossible for any executive or group of executives to be actively in touch with all research and development work. The executive or committee in charge of the research function knows what is going on, but cannot handle the detail that is possible to the executive in the smaller company. This condition naturally leads to a greater delegation of authority.

In some cases there is a sharp distinction here between engineering, design, and research. The fact that these three should be closely related with each other, and then as a unit with sales and production is not questioned, but the problem of so doing is much more difficult. Often in these organizations the objectives of the research activities are much more varied and inclusive, and in some cases it is not necessary to investigate the markets in relation to technical research work because of the peculiar nature of the latter. The author has in mind one particular organization that carries on practically no market research in relation to the other research functions because the findings of the latter, when applied, usually change only an integral part of the product as a whole and do not warrant any detail sales consideration. Then, too, in organizations of this type the actual research work is sometimes of a more fundamental nature, which, although it might have a specific object in view, must be supplemented by engineering development work in an operating division in order to put the new development into usable and salable shape.

Attention is first called to the methods employed by a large electrical manufacturing concern, particularly in those branches with which the engineering department has the closest association—namely, the commercial and manufacturing departments and the research laboratory.

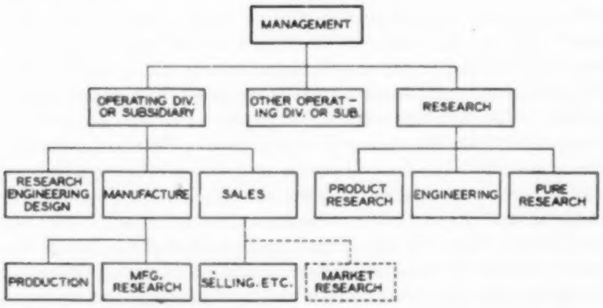

Fig. 4 Partial Chart of Largest Type of Organization
(Dotted lines indicate function may appear as indicated.)

The commercial department is headed by a vice-president and has the responsibility for the sales policy and the selling activities. The manufacturing activities are under the control of two vice-presidents. The engineering department is under the control of two vice-presidents and is responsible for all engineering work except that which affects one line of products and the activities of the research laboratory. One of these vice-presidents is in charge of the commercial engineering group and the other of the designing engineering group. The research laboratory is headed by a vice-president. This laboratory is equipped to assist the designing engineers in solving their problems, and also initiates many new developments of which advantage may be taken by the designing engineers. In addition to this central research laboratory, each branch maintains its own research laboratory, which is responsible to the vice-president in charge of the branch.

The designing engineers are responsible for the design of apparatus and the engineering supervision of its manufacture. Although in most cases they are not responsible for the methods used in manufacture or for following production, they see that the products are manufactured in accordance with the design as evolved. In some instances the manufacturing and designing responsibilities are combined in one individual who is given the title of managing engineer and who is jointly responsible through the works manager to the engineering and manufacturing vice-presidents.

The commercial engineer acts in the capacity of engineering advisor to the commercial or sales department and is engaged in applying the apparatus designed by the designing engineer to specific uses. The commercial engineer studies the requirements of the market from an engineering standpoint and determines upon the best and most economical method of meeting them. In some cases these requirements can be met by products already designed, but when this is not possible, the matter is referred to the appropriate designing engineer to make layouts and submit cost estimates covering the apparatus. There are three main commercial engineering departments, each associated with one of the main commercial departments, these divisions being made on the line of products handled. While the prime function of the commercial engineering departments is to make applications of apparatus, they often become interested in introducing improvements in the design of apparatus which they utilize.

In general, ideas relating to product development and changes in existing products are submitted to the commercial engineer in charge of the department into which this product naturally falls. This engineer investigates in a preliminary way the feasibility of the contemplated move, and if favorably impressed, refers it to one man for follow-up. Each division in the engineering department has a budget for the development work to be carried on for the coming year, and the engineer in charge is responsible for its proper disbursement. A market analysis by the appropriate division in the commercial department may be called for. After all such investigations, the project is sent to the designing department to have layouts prepared. If a project is approved by the engineering and designing departments in which it is being developed, it is sent to the works laboratory, where the manufacturing methods and processes are determined upon and a sample is made up. After leaving this department the product goes back to the designing department, where it is ordered into production in accordance with the demand.

Constant checks of progress on all current developments are made by standardizing committees. One of these committees, composed of a representative from the design, commercial engineering, and commercial departments, is appointed for each main line of products. In addition there is a general standardizing committee having jurisdiction over these individual committees.

The activities of the central research department do not form an essential part of this procedure. This department stands ready to assist any of the engineering or designing departments in the solution of their problems.

Particular attention is called to the function of the commercial engineering department and the part it plays in the relating of company products to the commercial or sales department. Here we have a large unit acting as a coordinator of the different development activities. Attention is likewise called to the close relation existing between design departments and production departments through which development activities can be closely related to line production.

In some cases the proper coordination of development activities can be brought about through a system of budgetary control, which, while giving a certain amount of freedom to the executives involved, holds them for results. This principle is illustrated by a company which is similar in general type of organization to the preceding case. This is a large corporation operating a number of subsidiaries manufacturing a wide and diversified line of products, both of an industrial and consumer nature. A central laboratory, in which all types of research are carried on, reports directly to the management. The problems are those submitted by one of the subsidiaries and those originating within the laboratory or coming from outside the corporation. Problems from the two latter sources are developed up to a certain point and then turned over to one of the subsidiaries for further development and subsequent manufacture. In considering the organization of one of the subsidiaries, there is a technical department (all research and development work except market research is carried on here) headed by a general manager, who reports to the president of the subsidiary. There is an operating division for each major line of products headed by a general manager reporting to the president. These operating divisions maintain their own sales and production departments.

In relation to research and development work, the budget system operates along the following general lines. The general manager of the technical department can spend a certain amount of money on any development work, but to get a larger appropriation after the development work has started, the executive committee of the subsidiary has to be notified. The executive committee likewise has a limit on the requests that it can approve, and all in excess of this must be approved by the executive committee of the corporation (parent company), which likewise has an upper limit, all in excess of which must be referred to the board of directors. From this it follows that, as a project grows in magnitude and importance, the scrutiny to which it is subjected is much more inclusive. In consequence, all factors having any influence on the project are properly investigated in the initial stage.

All development work is under the control of the technical director so far as dispatch, supervision, etc., are concerned, but projects have to be accepted by the general manager of the operating division concerned before adoption. This means that the technical department would have to consider the attitude of the operating division heads in order to insure the acceptance of the results. Throughout the development period these operating departments are in informal contact with the work being done by the technical department. The first formal discussion which involves them, however, takes place after all the development work has been completed and the matter is referred to a process committee. This process committee is composed of the technical superintendent from each plant (the operating divisions sometimes have several plants), the managers of the sales-service divisions, and one or more representatives from the technical department. This committee is an ideal medium whereby consideration is given to production troubles, plant difficulties as a whole, trade trends, process problems, customer reaction to the existing line of products, etc. The findings of this committee are reported in the form of recommendations to the individuals concerned, but they do not have to be adopted unless so desired by those interested. It should be added here that the sales-service departments are very active in making market tests and the like in conjunction with the development work carried on by the technical department.

Follow-up of progress on all research and development projects is accomplished through the medium of reports. Reports from the department heads in the technical department are submitted to the general manager of the department once a month (these department heads being in daily contact with the work done in their department), and the technical department reports once a month to the executive committee. All projects are considered as individual units, and the progress on each approved by the general manager of the department. Likewise all work done on any project in other departments is charged against the department that obtained the original appropriation.

A review of this case would seem to indicate that there are three factors that tend toward coordinating the various activities engaged in development work: The system of budgetary control which definitely places the responsibility for research expenditures and thereby plays its part in insuring sound conclusions based on careful consideration of all factors involved; the work of the process committee incorporating the point of view of research, sales, and production; and the organized application of executive judgment to specific projects.

Let us turn attention to the operation of a central laboratory of one of these larger corporations. This company is one in which the function of market research does not play so important a part in connection with research projects. The nature of the product is such that the projects are not concerned with the development of a new product, but with the improvement of integral parts of the existing product.

This company maintains a central research laboratory and many operating divisions. The research laboratory reports, through a technical committee, to the president of the corporation, and the operating divisions likewise report to the president. This technical committee is composed of all the chief engineers of the operating divisions, and the function of the committee is that of coordination between the operating divisions and the laboratories. There is a general research director who controls all research work done in the laboratories and coordinates all activities. The laboratories are divided into various sections in accordance with the service rendered, and these activities are directly controlled by a technical director who has charge of the technical work and a business director who has charge of the business activities. Then there is a control committee for advisory purposes, composed of the general director, the technical director, and the business director.

Problems requiring research are usually referred to the proper section head. Upon approval, the problem is sent to the technical director, the business director, and the general director. At this point the problem is thrashed out by all concerned, and the advice and opinions of a great many executives and engineers in the corporation are sought. If approved by the head of the research laboratories, the project then goes to the management of the corporation for final approval and the necessary appropriation. An appropriation is then set aside, a project number assigned, and the project sent back to the original section for the work to be done. If the project had originated further up the line or outside the laboratories (except in an operating division), the general director would turn it over to the section best equipped to handle it. Under the direct supervision of the section head, the work is carried forward until completion. In some cases it is necessary to have other sections perform a certain amount of work on a project, and under these circumstances the proper contacts are made through the technical director, and all expense is charged against the project number for which the original section is responsible. After all development work is completed, contact will be made with the operating division interested, and an attempt is made to "sell" the new development to it. The operating division does not hear of a project only upon completion and the subsequent attempt to "sell," for operating divisions are usually acquainted with the work of the laboratories through the technical committee. If the project is accepted, the research and engineering department of the operating division puts the new development into a shape whereby it can be used. Close contact is maintained between the product design department and the manufacturing department so as properly to consider the production point of view. Information concerning the markets is obtained from the sales committee (composed of the sales managers of the operating divisions and reporting to the president of the corporation) or from the sales department of the operating division, as the case may demand.

In addition to performing work on projects originating in the research laboratory, work is also done for the operating divisions on specific problems submitted by them. Under such circumstances all expense is charged against the operating division for which the work is being done.

The research projects in the central laboratory are followed up in this way. Each project is reported on periodically by the man doing the work. This report is approved by the section head and sent to the technical director, who sends it to the one whom he thinks should see it. The function of seeing that progress reports are sent to the right parties is in the hands of a separate section of the laboratories. The head of this section is notified to whom the reports are to be sent in the different operating divisions and elsewhere (here again it is seen how the various officials in the operating divisions are notified of development work in process in the laboratories). This system of reports has the necessary supporting authority to insure proper operation, in that all appropriations are made to cover a certain period (this is fixed regardless of the project), and if a project comes up for reappropriation, this request cannot pass unless the project has been reported on since the last appropriation. The responsibility for seeing that this is done is definitely fixed, because reports must clear through the technical director and all requests for reappropriation require the approval of this executive.

General Trends

While the specific object of this paper was to demonstrate how the different research activities could be coordinated with sales and production, it likewise brings out the fact that this industrial activity of coordination is in an embryonic stage. The direction of the development is clearly toward a greater cognizance of the problems of coordination, with more thoughtful attempts toward their solution. An increasing number of companies are accepting the research functions as vital to the perpetuation of their business. These new activities are usually first associated with production, and we find the technical research functions established in the manufacturing department—a decentralized form of organization.

As the research function grows in importance and the problem of coordination becomes paramount, we find the different activities being segregated into an independent department. This movement is usually initiated by removing the product research, pure research, and engineering functions into a separate department to give a partially centralized type of organization. This is later followed by the advent of the market and manufacturing research into this research and development department to give a completely centralized type of organization.

Then as the size of the individual companies increase, difficulties of coordination do likewise. This leads to a departure from the completely centralized organization and a trend toward the establishment of separate research, engineering, and pure research departments, each having a specific function to perform. Manufacturing research activities are returned to the manufacturing department, and market research to the sales department. In consequence we find a greater delegation of authority, a higher degree of organization, and an increase in importance of adequate systems of records of research work.

Conclusions

The following general conclusions may be presented:

1 In the development of a new product or a major change in an existing product, the following types of research have ordinarily to be performed: product research, manufacturing or process research, and market research. The relative emphasis to be placed on each will depend upon the peculiar needs of the business. They should be coordinated with one another and as a unit, with, sales and with production. Formal methods for such coordination are now in embryonic form in a limited number of American industries.

2 The specific product manufactured, while important, does not have as great an influence on the structure or form of coordination as is frequently attributed.

3 The technical research activities, in general, have been accepted as vital to industry and are the most widely used types. When the technical research activities are found in the manufacturing department, the relationship existing between them is usually close and the problem of coordination relatively simple. Pure research does not require coordination until the findings of the research are to be applied.

4 The use of organized market research is becoming accepted as an essential phase of present-day industry, but it frequently has not been recognized as a specific function, nor has it been as well organized or controlled as has technical research.

5 Coordinated research activities should be performed in a preplanned sequence based upon the extent of initial knowledge regarding the project and the precise relationships existing between the various necessary research activities. In many cases it has been found that all significant elements of the research problem were not given adequate weight when a project was under initial consideration.

6 The lack of proper coordination may result in:

(*a*) The spending of large sums of money in development work on a project which turns out later to be a failure owing to improper consideration of all phases in the initial investigation.

(*b*) The development of products which are not commercially sound from the point of view of either lack of markets, etc., or high selling price due to high manufacturing costs, etc.

(*c*) The waste of time and money on research projects which have no direct application to the business as a whole, either present or future.

(*d*) Product design developed without proper consideration of the manufacturing department.

(*e*) Failure to direct the activities of the different research functions into those channels from which the greatest benefits might be obtained.

(*f*) Danger of personnel difficulties arising out of apparent conflicts of authority.

(*g*) Progress delays on a given project due to the lack of cooperation on the part of the various departments concerned.

(*h*) Omission of periodic follow-up of research work to determine the advisability of continuance.

(*i*) Danger of not routing research projects, in relation to types of research to be performed, so as to eliminate impractical ideas in the early stages.

7 The factors influencing the structure or form of coordination are:

(*a*) The size of the company. (For example, small companies do not require such a high degree of organization.)

(*b*) The type of organization. (For example, highly functionalized organizations may have greater need for committees.)

(*c*) The personalities of the executives involved with research work. (For example, the extent of informal contact affects the degree of conference routine which is essential.)

(*d*) The attitude of the line executives toward research work. (For example, the importance of frequent contacts with progress of research varies inversely with cooperative attitude.)

(*e*) The types of research most used. (For example, the importance of coordination varies directly with number of research activities essential to adequate product development.)

(*f*) The specific object of the research activities demanding greatest emphasis. (For example, the relative importance of low unit cost or of high quality, or of wide sales appeal, etc., will determine the direction of coordinative emphasis.)

(*g*) The importance of research to the organization and to the perpetuation of business. (For example, synthetic products find in research perhaps their greatest insurance against competitive defeat.)

(*h*) Nature of the product. (For example, products whose form or characteristics may be radically varied by processing are particularly susceptible to benefit through coordinated research.)

(*i*) The nature of the markets. (For example, coordination with market research is particularly important when dealing with new or undeveloped markets.)

8 No best method of coordinating research, sales, and production applicable to all companies can be evolved, since the coordinating machinery should be devised to fit the organization and not vice versa. There are, however, certain general principles or rules of action which manufacturers have found helpful:

(*a*) The control of coordinated research and development requires the organization of personnel, methods, and facilities in no less degree than does production control, sales control, or financial control.

(*b*) The authority for the formal initiation of research and development work should be specifically assigned.

(*c*) Avenues of informal contact should be maintained between executives and sub-executives concerned with research, development, and allied line activities.

(*d*) Arrangements must be made whereby a single executive shall have authority to expedite any given research project to its completion through the necessary processes and departments.

(*e*) Prior to important research expenditures, a coordinated research project should have the benefit of the judgment of executives representing the major phases of the business. Such judgment may be gained through the organization of committees or conferences or through solicitation on the part of a single executive.

(*f*) After the approval of research appropriations, a coordinated research project should have, at periodical intervals, the benefit of executive judgment as to its continuing practicability and value. The activities of a single executive, committee, or conference have been found effective in making such judgment available.

(*g*) Periodical approval of coordinated research projects should be assigned to specific individuals or committees, and the "approval points" should be definitely fixed. Where different types of research work are carried on in sequence, the approval of progress of each type should be delegated to subordinates rather than the executive or committee giving the original approval. In those cases where the different types of research are carried on concurrently, the responsibility of approving progress should be vested in the executive or committee giving the original approval.

(*h*) The presence of budgetary control in the organization will be found to be of material assistance to any coordinated research project. When budgetary control is not in general use, its principles should be applied, as far as possible, in the departments affected by this coordination.

(*i*) In the larger companies the activities of an entire department may sometimes afford an effective means of coordinating the sales and product research functions. This usually takes the form of a study by this department of markets from an engineering point of view and an interpretation of these market requirements for the product design departments.

(*j*) As in production or sales control, a coordinated research project requires the establishment of a definite system of follow-up or inspection of progress.

(*k*) Where possible, the originator of any research project should be required to make an initial investigation relating to manufacturing cost, value to the company, effect on other products, expected cost of development, expected sales potential, etc. In cases where the collection of these data requires important expenditures, the project should have the approval of the executive or committee in charge of the research and development function.

(*l*) Coordinated research work is as susceptible to the benefits of planning and scheduling as is any other function of the business. The preplanning of an individual research project, however, is affected by (1) the types of research to be performed, (2) the extent of initial knowledge, and (3) the degree of centralization of the research functions.

(*m*) A fundamental in the successful conduct of coordinated research work is the definite placement upon individuals of responsibility for results.

(*n*) As a coordinative influence, as well as a research device, small-scale production and market tests are particularly valuable and should be undertaken as soon as development work has sufficiently progressed.

(*o*) Where the centralized type of research organization is in use, special care should be taken that the research activities be not too far removed from their related line activities, if the valuable constructive suggestions from line officials are to be encouraged.

(*p*) The informal placement of too much reliance in one executive as a source of either technical or commercial information of a research nature is unwise in the light of possible changes in personnel.

(*q*) The official ultimately determining the coordinative policies of the research and development function should be a man of broad business vision with balanced interest in sales and production problems.

Discussion

Charles M. Mills.[4] This paper gives a clear-cut analysis of the research problem, and points out succinctly the up-to-date progress of general research in various types of organizations. It has been exceedingly interesting to see the development in this paper between line and staff administration in various types of industries and organizations. One reads the paper feeling that there is no single or ideal set-up for the proper coordination between these three great activities of business and that each must be fitted into the particular individual staff. Behind the mere facts that are presented in the paper lie intimate histories of corporate life. Undoubtedly some of the more particular types of management are due, not to any lack of knowledge of the proper type of standard organization, but to differences in the personal equation and in the quality and type of personnel in the executive group. Thus one cannot say exactly whether completely decentralized, partially centralized, or centralized organizations are ideal.

The emphasis on the lack of market research is by no means overemphasized. It is astounding to find the lack of attention in some of the larger corporations given to the proper analysis of their markets. Too frequently, as is pointed out, the interpretation of market conditions is through sales executives and salesmen who are engaged in line administration or actual selling in the field. Such executives cannot have visualized the basic problems of markets. They are too near to see the objectives to which the efforts of the company should be directed. It is practically impossible for them to rightly interpret the facts of the situation without coloration and partiality.

Another point that the author very properly emphasizes is the lack of attention given to the patent situation. Not a few of us know of circumstances in various companies where new types of products have actually been devised and even manufactured without clearance as to patents. Line executives too often ignore the advice and counsel that may be contributed by the legal dedepartment.

It is easy to see in the author's analysis of the first large company, under "decentralized research," the distinct cross in functions between line and staff. It is exceedingly hard for the average line executive, not only to control, but to appreciate the work of staff groups under him. In the second place the planning of research is invested in a committee. The writer is inclined to think that the author has been generous in his word about committees when he says, "A great deal can be said both for and against the committee system of management. The effectiveness of a committee will depend to a large extent upon whether it is for control or advisory purposes." If one goes behind the scenes very far, he certainly will learn that unless there is an unusually high caliber group of executives who have forgotten personal jealousies, the committee system in regard to research is extremely cumbersome and irritating rather than an efficient factor in the activities of the company.

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It is hard to see from the outside how partially centralized research functions can be fully successful. The writer would be glad to hear the author enlarge upon this type of organization. Perhaps the writer is a bit prejudiced toward completely centralized research, based largely upon his experience. As the author rightly sums up, there are two great advantages—authority is invested in one department which clears with other heads in the organization who must be properly consulted, and in the second place there is a lack of committees. The paper presents a good synopsis of the growth of research in business and its coordination both with sales and production.

W. L. Waters.[5] The paper indicates that there is no standard practice in regard to the organization of research and development activities and that every concern must work out its own system. This is in accord with the general opinion on the subject.

Market and patent surveys are of course required, but they are not necessarily final, because six months after a new product has been placed on the market, some new basic development may render the product quite obsolete; or a patent, the application for which has been in the Patent Office for five years, may be issued, which covers some of the essential features of the product.

Much the most important elements in any research or development organization are the personnel at the head of it and the morale of the department. If these are of the highest grade, details of the organization, etc. become only of minor importance.

Walter E. Lee.[6] Listening to presentation of these papers, the writer is inclined to believe that what is really being considered is the coordination of *development work* to production and sales, rather than research as it is generally understood and accepted, for often there is also involved, especially in the case of the manufacturer of industrial units, the matter of engineering and product design.

The term "research" hardly embraces this necessary phase in the development of a new product or improvement of an existing product. On the other hand, "development" is a sufficiently broad term to embrace pure research, product research, manufacturing research, patent research, and market research.

The type of development organization will vary with the type of corporate organization, depending on the location of its manufacturing units, the extent of its marketing territory, and the nature of research undertaken. In one case, as in small concerns, a decentralized type of research, undertaken by each department, may be more effective than a completely centralized research department acting as a staff function and most effective in large organizations having wide-flung operations.

While research organizations may have a part of their efforts directed toward pure research regarding the physical and chemical nature or structure of raw and finished products and their characteristics relating to desirable service performance, nevertheless the majority of the problems recommended for research generally comes from the field through the sales department.

Not only does the sales department produce the revenue which makes possible all other activities of an organization, but it is also the ears and eyes of the whole organization, having a constant touch on the pulse of the consuming public and knowing the change in their tastes, requirements, and demands. Moreover, the sales department knows when it is losing ground because existing products need improvements or because it sees opportunities and markets if only products were available to fulfil them.

While a research project can arise either within the research organization itself or be suggested by manufacturing or sales, the project will generally fall into one of the following classifications:

1 Pure research into physical and chemical nature of composition of raw products or finished products
2 Development of new products for an existing need in an established market, not now suitably supplied by existing products
3 Development of new products for an existing need in a market not yet established and which need is not fulfilled by any existing product
4 Development of synthetic products having better characteristics than existing products manufactured from raw supplies
5 Improvement of existing products to improve quality or to change form or structure to lower production in cost
6 Development of experimental products, new uses for which are afterward to be found
7 Development of new products to replace competitive products now being furnished by a different type of noncompetitive industry
8 Development of standards and specification formulas
9 Change in type of product to meet new consumer demand or service needs.
10 Meeting the demands of the sales organization for a new or more suitable product to meet the field demands of the user or consumer
11 Market research into:
  a Location of possible markets for product outlets
  b Requirements of each type of user or industry in any given market
  c Trend in consumers' requirements to anticipate future demand for products to meet more severe duty or severe conditions
  d Development of new uses for existing products
  e Substitution of existing products in markets now supplied by a non-competitive industry
  f Competitive sales methods and practices and prices structure
  g Determination of volume of potential business by market and product and reasonable quota which can be expected of each sales unit.

In times such as exist during the present business depression, research projects are generally approved only when the object is likely to produce tangible results which can be commercialized so as to give the greatest sales dollar return for each research laboratory dollar spent.

Once an object has been approved, much time and expense can be saved and the pitfalls of other previous investigators avoided by a full literature survey on the subject. Likewise, the loss of many months of intensive work through patent conflict may be avoided by a parallel patent research undertaken at the same time when the literature survey is made. After these two preliminaries have been completed, the complete research program may then be drawn, approved, and executed. Cooperative programs between research laboratories of the manufacturer and the industrial user are also very advantageous.

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[6] Assistant Director, Research Laboratories, Standard Oil Development Company, Linden, N. J.

Usually the largest advertising agencies handling the accounts of large or national manufacturers perform, as a service to their clients, market forecasts as to territory season or period, product or grade, type of outlet, size of town, class of buyer, sales method and quota, and generally act as a marketing counselor.

However, this can hardly equal a full-time, 100 per cent market research within the organization itself, gathering and interpreting facts for the advice and guidance of management to predict future demand and effect of trends in market change on existing products and revenue. The writer understands that among the 200 largest organizations in this country, but seven of them have a regularly organized group to undertake and prosecute this important phase of business administration.

B. Townshend.[7] The papers by Mr. Bausch and Mr. Wilder represent careful thought on one of the fundamental problems of modern industry. Mr. Bausch's paper is a detailed description of a particular case and is valuable as a record of something which works well in a certain environment. Mr. Wilder's paper is an analysis by the author of different systems and a classification of these based on what it was possible to learn by an outside observer.

All too often systems are not sufficiently definite and classified to communicate them to another so thoroughly that he can reach accurate conclusions on their merits. In one case cited at least, with which the writer is connected, the organization of research is undergoing change in a constant effort to increase its effective power and usefulness.

In a new company starting under intelligent technical direction, the organization and control of research and other technical activities should be simple. The more general case, with which most of us are concerned, is the grafting of research in the modern sense into organizations already developed. Every manufacturing concern has done a certain amount of technical development work, usually scattered and uncontrolled, whether such work was called "research" or not. The problem is to collect such past activity, round it out and fortify it where necessary, and place it under definite management. Technically, this department must best serve the needs of the particular organization into which it is introduced, and practically it must be adjusted to peculiar conditions of habit and personalities.

Before starting any discussion of the place of research in modern industrial organizations, the meaning of the term must be defined, since it is one of the most abused words in the language. Mr. Bausch appears to have technical development (in this case of optical devices) particularly in mind, while Mr. Wilder's apparent definition appears far broader. Under "research" can be placed any or all of the following activities according to taste: Investigations in the sciences underlying the technical field of the organization, development of materials, development of processes, engineering of processes, testing of materials, control of production quality, investigation of economic conditions and markets, and general technical service to other departments and to customers. Nearly all of the foregoing activities could well be under a single direction, but it would be a "vice-president in charge of engineering and development," and "research" would be a department whose functions would be restricted to scientific investigations, and development of materials and processes, with only such testing of materials as was implied in intelligent material development. Engineering of processes, general testing of materials, control of quality, and general technical service could well be placed in an "engineering, development, and control department" and would have to work in close contact and harmony with production and sales. Economic and "marketability" research should be placed in the sales division.

The question of committee management of certain functions has been brought up by Mr. Wilder, and he cites one case with which the writer is familiar where this is supposed to be in usage. It is less true now than when the information for the paper was obtained. It is true that committees are cumbersome and inefficient. A committee generally functions as a brake, not a motor—it can stop most anything and start almost nothing. A committee is only as strong as its leader, and those committees which are efficient are really only specially appointed departments under the management of the chairman.

Obviously in the foregoing the needs of a large organization has been kept in mind. With only 200 corporations now controlling 40 per cent of the non-financial assets of this country, it is natural to restrict oneself to consideration of large units in discussing general principles for future application.

E. C. Rack.[8] Mr. Wilder has given a most interesting and comprehensive presentation descriptive of the mechanism employed by management in the coordination of the four functional divisions of industrial organization.

The organization with which the writer is connected has only in the past three years become intensively active in centralizing research as an independent function. Results of this segregation, as measured by accomplishments to date, have generally borne out those conclusions summarized by the author. It is not felt that a more detailed exposition of the operation of the research, engineering and development activities of Johns-Manville need be attempted at this time, especially in view of the fact that an outline of our methods of coordination and control, although not specifically cited by the author, are given as an example in his paper.

Convictions regarding details which must be considered by management are generally obtained as a result of experience. For this reason the writer feels that more studies of research coordination with resulting data should be reported. Contributions of this nature will undoubtedly be valuable data for study by management. It is also felt that various phases of the research function and its control as practiced, and the experience gained by industrial organizations, will be of value. As an example, with the rapid development of research new managerial methods are constantly being tried. It is desirable and necessary that management have some form of recognized judgment for approving progress. This calls for a measure with which to compare operational effectiveness, both as regards general accomplishments and those accomplishments of subdivisions under various commodity or product classification. Such an exposition is briefly given in the paper by Mr. Bausch.

The writer further feels that more descriptive outlines of the set-up and experiences of the various sectional divisions of research work would also be of much interest for round-table discussion and further study; such as for example, a detailed outline, together with an explanation of how those sections function to the best advantage which act as seekers and finders of ideas and developments which are fostered by individuals who may not be connected directly with that organization or branch of enterprise concerned. The period is past when secret processes were the order of things. When results of research are utilized and protected, data are very generally distributed for the benefit of the industry as a whole. The same policy can well be adopted by management. Results and experience of operation of research organizations when published will be received and acclaimed with as much interest as are the results of research in pure science and engineering.

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Other divisions of this Society act most efficiently as permanent clearing houses for efforts and data accumulated as a result of research work. This division can do much in the encouragement of a further discussion of the principles of management in the field of scientific research, and in this way the fundamentals of this particular branch of management will be coordinated into a specialized set of principles or laws which regulate the scientific functions of finance, sales, and manufacturing management.

L. A. Hawkins.[9] As Mr. Bausch says, the words "research" and "engineering" are sometimes used to mean much the same thing. This seems a pity, for in industrial laboratories and engineering departments there are two distinct kinds of activity which need two different words to name them. The search for new scientific knowledge is a different thing from the application of scientific knowledge in developing new or improved processes or product. The former may well be described as research; the latter, as engineering development.

A developmental project may require research before it can be successfully initiated, or in its course may reveal unknowns which need research for their explication. The research and development may be performed by the same personnel, but the two activities are different in their natures, and aptitude for one does not necessarily imply aptitude for the other. That is one of the reasons why, in a large organization, it is advantageous to have distinct engineering and research departments, in spite of the obvious advantage of closely linking activities so directly complementary in their objectives.

Now, while engineering development should be closely coordinated with production and sales, that is not wholly true of research. Production problems may call for research in their solutions, or market research may reveal the need of scientific research to determine whether a desirable development is possible or practicable, but neither production nor market studies can disprove in advance the wisdom of undertaking a given research project, since the very nature of research makes it impossible to predict its results or their applications. The only criterion is whether the project presents a promising path to new knowledge in the field of interest. Wisdom in applying that criterion is an essential qualification for success in directing research.

Mr. Wilder says, "Pure research does not require coordination until the findings of the research are to be applied." If the term "pure research" be taken to mean, as it is thought the author intended, researches other than those definitely directed toward meeting specific needs of production or sales, then his sentence is a concise equivalent of the preceding paragraph.

Ben Nash.[10] When coordinating the various factors of a product-development project, modern competitive conditions require that greater consideration be given to the appearance of the product. The appearance or visual aspect of a new product should consist of:

1 A visual exterior which adequately expresses the function, benefit, and value of the product

2 A practical visual conception so organic, simple and aesthetic as to:

a Indicate an advanced product

b Secure the greatest share of consumer preference at the time of purchase

c Continue to invite a lasting regard.

The reasons for the increasing importance of the visual aspect of a product are:

1 The necessity of meeting current trends

2 The necessity of expressing values and benefits

3 The necessity of attracting markets.

The trends indicate:

a That the pioneering functional era is now augmented by an era of good appearance.

As long as our products remained in the pioneering utilitarian era, during which each new product invention or development merely served a utilitarian need, consideration of the product's visual aspect was incidental. If a new product which functioned effectively possessed a visual crudity, this visual crudity went unnoticed because the product's ability to function was of major importance. As long as there was a ready non-competitive market for inventive talents along functional lines, visual aspects could be ignored. But now, two conditions have arisen which demand that visual aspects receive their share of attention too. There has been an increasing education in style and good taste throughout the country, and competition has learned how profitable it is to respond to the desire for products in better taste. Within our own time we have seen the automobile, the radio, and the electrical appliances change from mere assemblages of functional parts into attractive articles, which invite possession. By reducing the emphasis on their mechanisms and increasing the attention given to their complete design unity and beauty factors, the marketability of these products has been greatly enhanced. While the search for exterior design simplicity in mechanical products was going on, a new product possessing simplicity of tone design appeared upon the scene and pointed the way to new possibilities. The aeroplane, through force of circumstances, had to be simple in its design form to function successfully. As a result, we make this product without gadgets, protusions, layers of attached parts, nuts or bolts to deter its functioning or to spoil its smooth design silhouette or distinctive design character. Design simplicity was necessary in an aeroplane to enable it to fly. Design simplicity is just as essential to other products to make them sell more readily.

b That consumers are measuring values through appearances.

Present conditions are proving that consumer buying is strongly influenced by appearances. The new method of open-counter merchandising of products which possess self-selling qualities is but one evidence of this trend. The increase in the purchase of attractive novelty merchandise in this immediate era of curtailed buying of staples is another evidence of the importance of the value of product appearance.

c That products must bear a more harmonious relation to their surroundings. The consumer is interested in the harmonious ensemble idea in the home.

The educational programs carried on by magazine publishers and by the style service departments of manufacturers have caused the consumer to look for and choose products which fit into the ensemble idea. Many staple products have been given a new visual treatment or have been grouped in ensemble combination with related products in order to attract this market trend. Products which formerly sold upon their functional merits alone have now been developed to fit into the visual ensemble.

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[10] Products Development Company, New York, N. Y.

d That new living conditions, smaller living quarters, and more hand-to-mouth buying require compactness of product design to attract consumers.

More compact organization of functioning parts is showing itself in new products designed to fit more exactly into the new type living quarters. Products designed to meet these modern conditions require a new design simplicity through the force of these new circumstances.

e That new architectural treatments, new building materials, and modern color treatments require harmoniously appearing products.

Both old and new materials are being used to furnish products which fit into the home or office. Color, fittingly and adroitly applied to products, is being used to conform to consumers' desires. Consumers continue to want products to work well, but they also want them to look well and to fit appropriately into their surroundings.

f That the increasing number of purchases of mechanical appliances by housewives requires product design which reduces the mechanical and laborious-looking features.

The assemblage of mechanical parts which bespeak mechanism to the housewife is not as desirable as the necessary appointments to her daily tasks. Design unity of assembled parts, less evidence of nuts and bolts, and an aesthetic organization of materials will better secure the housewife's preference.

g That there is an increasing interest in products which will perform with the least amount of maintenance effort.

Products with less places to catch the dirt, with smoother, plainer surfaces, with enclosed parts and a minimum of protrusions furnish a visual expression of easy maintenance to the consumer. A recent consumer investigation of a mechanical product clearly indicated one outstanding consumer desire, "a product which was easy to keep clean." Another product research program indicated that one essential of the visual aspect of the product was "easy maintenance." In both of these cases these requirements were made a major specification in the product development project.

h That competitive conditions require a product to be competitively adequate in appearance as well as to be adequate functionally.

A product which functions with competitive superiority may find itself giving ground to a competitor which appears superior; for by appearing superior it furnishes a sales resistance to its more adequate competitor. What a disappointment it is to the engineer who knows the technical differences between the two products, to see the consumer influenced by appearances. There is but one certain way to meet this condition, and that is to have the visual aspect of the product wholly in keeping with its mechanical excellence. A mechanical disadvantage can often be transformed into a visual selling advantage through the adroit use of exterior design styling. The visual character of the product should seek to express or feature the attributes which particularly appeal to the consumer. As an engineer you may know that a mechanical part is the big feature, while the consumer may be influenced to buy because of a particular result. Therefore, you should feature the result and not the mechanical detail.

i That the exterior of a product must help express the unseen inherent values.

The exterior shell or dress of a product can glorify its contents so as to express its unseen worth, through design, color, a touch of embellishment, and greater appropriateness to the consumer's present-day desires. Products whose values are not visually apparent cannot express these values.

j That effective visual treatment can endow a product with exclusive features. These features can aid in securing the full benefit of sales promotion effort.

Many products which enjoy an exclusive competitive advantage accomplish this through visual characteristics. The resulting distinctiveness separates them and lifts them above comparable competition and gives the full benefit of the advertising to these features.

k That the exterior of a product can secure a more lasting good will through a continued high regard.

A product which is conceived in organic simplicity and which possesses aesthetic qualities will always be good to look upon. The more beautifully it is designed, the greater and more lasting regard it will receive. No one tires of something which is well designed. Those whose products require human attention in the fulfilment of their performance will find that a better-looking machine will perform better because it will receive greater respect and better treatment than one which is less attractive.

To attract markets:

l The visual aspect can create obsolescence. It can indicate a product advance and can reopen saturated markets.

The use of this designing policy in the automobile industry indicates the value of its application in other fields. New arrangements of functional parts or new exterior design or the use of new materials can serve to express advanced products to reattract lethargic markets.

m It can aid in attracting wider markets.

A product can find its usefulness extended into new places when made more attractive or aesthetically fitting to a wider range of surroundings. Products whose present function does not require them to be seen or to possess visual attractiveness may unfold new places for usefulness and wider markets if made visually attractive. One example of this treatment is the attractively designed furnace which has replaced the old coal furnace and transformed dirty cellars into game rooms. This opportunity is open to many other products.

A well-designed product pays from the moment it is seen by the consumer on through its life by earning and retaining a high regard.

AUTHOR'S CLOSURE

The author again wishes to emphasize a point brought out by Mr. Mills when he stated, "One reads this paper feeling that there is no single or ideal set-up for proper coordination between these three great activities of business and that each must be fitted into the particular individual staff." This is a consideration of utmost importance and should constantly be in the minds of executives undertaking the solution of this problem of coordination.

The practicability of the partially and completely centralized types of organization was questioned. Here again one is confronted with the nature and size of the enterprise and the type of personnel as the limiting factors. The outstanding advantages of the completely centralized type are (*a*) the lessening need for committees, (*b*) the definite placing of responsibility, (*c*) speeding up of the work, and (*d*) a complete segregation of line and staff work, both from the actual operation and the control points of view. An obvious disadvantage is the danger of removing these staff functions too far from their related line activities and thereby widening the breach between them. Partially centralized research organizations generally have only the technical research (except manufacturing research) in a separate department. This is usually the first step toward the completely centralized type. It is taken primarily because of the increasing importance of the research function and the broadening of its scope to include a multitude of problems. As this work grows it becomes too much for a line executive to control, as there is a direct cross between the line and staff functions. While the partially centralized type has certain of the advantages offered by the completely centralized type, it has the additional advantage of closely relating the manufacturing and market research to the production and sales departments, respectively. However, there is a danger that these two research functions will be too far removed from the remainder of the research activities.

The author heartily agrees with Mr. Mills's statement about committees and would like to add that when a committee does operate efficiently it is usually dominated by one man, in addition to having a personnel of high caliber. Unfortunately space restrictions prevented a more detailed treatment of this phase of the subject.

Mr. Waters brings out a very good point in his statement to the effect that market and patent surveys are not necessarily final. This substantiates the fact that a program of constant revaluation of research projects in the light of changing business conditions and newly discovered facts should be maintained.

Mr. Lee's criticism of the use of the term "research" can best be answered by defining the different types of research as the author used them. This will also answer one of Mr. Townshend's criticisms. Pure research is taken to mean that which has to do with scientific research of a general nature, research into fundamental scientific principles with no specific commercial objective in view—this type is sometimes known as "blue sky research." Product research has to do with the development of new products and new uses for existing products (on a basis of technical investigation), changes in existing products, and search for and development of raw materials. Very closely related to product research, and sometimes included in product research itself, are the engineering and design functions. In many cases these two functions comprise all the product research which is undertaken in a given organization. Again, product research may consist of research of a pure or fundamental nature on a specific problem, this being followed by engineering and design in the natural sequence of putting the product into salable form. Manufacturing or process research is taken to mean the development of manufacturing methods and processes, both in regard to more efficient operation and reduction of manufacturing costs, in relation to the manufacture of existing products, and to the manufacture of new products. Control research has to do with the control of the quality of raw materials, stock in process, and the finished product. This is more or less a routine inspection operation and has not been considered in this paper. Market research is concerned with finding out whether there is a market for a given product, what type of market exists and where it is, what quantities it can absorb, the number of competitors in relation to a given product, the buying powers of different markets, distribution facilities, etc.

Mr. Lee also points out that the majority of research problems come into an organization through the sales department. In the light of this it would appear very essential to properly coordinate this phase of the business with research and production, so that through these departments the problems presented could best be translated into factors of commercial and technical value.

The definition of pure research, as given, will answer the question brought out by Mr. Hawkins in his last paragraph.

Mr. Nash's discussion brings out very clearly the necessity of market consideration in product development and shows how important should be the relationship between the technical and marketing functions in American industry.

MAN-54-5

Dividend Programs Related to Depreciation

By E. G. Field,[1] Atlanta, Ga.

Figures are presented to show the working out of five selected dividend programs and to demonstrate that depreciation funds can be and often are misused as a result of the dividend program chosen. Several related subjects also are discussed, such as historical statements for interpretation of results, possibility of a current investment to current liability ratio, dividends that return parts of the original investment, and depreciation funds spent by management.

Among the factors contributing to the decline of the textile industry in New England, there has been included the lack of adequate provision for depreciation. This reason may be stated in two ways: first, because of the general failure to provide for depreciation, and second, as indicating the general lack of complete policies and programs directed to the perpetuation of the manufacturers. This same defect may be noted in the administration of Southern cotton mills and in certain other industries, wherever the value of the plant investment represents a large proportion of the assets.

It is commonly admitted that there was a lack of regular recognition of depreciation as an element of cost prior to the introduction of the Federal income tax. Some mills did not provide for any depreciation, under the assumption that the appreciation would exceed the depreciation. More often, the importance of providing for replacements had been experienced, and it had become customary to charge depreciation during good years and to omit depreciation in years of poor earnings. Thus depreciation came to be used as a means to level earnings in a more uniform relation to the dividend program. But in the long run this method did not prove satisfactory in that sufficient depreciation was not charged.

During the last 15 years there have been few failures to include depreciation as a deduction in figuring net income for Federal and state tax purposes. In fact, depreciation has become so well established that several associations have promulgated uniform rates. Depreciation is an acknowledgment of the depletion of original value, but the reserve for depreciation is intended to provide for replacement. This feature has been ignored or overlooked in the failure of management to keep available the funds so intended for the replacement of worn-out or obsolete properties. Thus the industry has continued the error of failure to provide for its future. In the omission it has encouraged abuses that have had far-reaching effects in undermining the financial position of the industry.

The depreciation reserves have taken on further significance in the late years of declining profit margins. A review of mill financial statements, issued for fiscal years ended in 1929, was printed in "Comparing Balance Sheets," in the August, 1930, number of *Cotton*, where it was stated: "Part of the difficulties within the textile industry can be traced to inadequate financing being desperately continued by loans. In turn this shortage in capitalization may have resulted from improper dividend policies, etc."

Among the significant facts about the depreciation reserve, as indicated in the tabulations in that article, were these: (1) The industry may have a depreciation reserve equivalent to one-half of the combined common stocks and surpluses, and (2) the depreciation reserve may be almost 50 per cent larger than the net current capital available for carrying on the businesses. Therefore the depreciation reserve does become of importance in the financial structure of any business requiring a large investment in plant facilities.

In considering ways for the funds represented by the depreciation reserves to be applied to other purposes, it was noticeable that the more prevalent cases included the diversion of funds in doing these things: (1) to reduce current liabilities, (2) to buy in the company's bonds, (3) to provide property additions, and (4) to pay cash dividends. In a discussion on "Depreciation Reserves," in the November, 1930, issue of *Factory and Industrial Management*, the last three diversion cases were illustrated and a method of identification was offered for detecting depreciation dividends.

As stated in that discussion: "Seldom have directors fully considered the results that may arise from following some dividend plans. This example has been cited because of the way in which the textile industry has been affected by controlling stockholders' greed for earnings, directors' lack of dividend policy, and administrative unintentional mismanagement."

The diversion of depreciation funds through the paying of cash dividends is of interest because of the common belief that any cash can be used to distribute surplus. This may arise from the usual legal requirements that dividends be paid from earnings. Nevertheless it is possible that dividends improperly paid in cash may impair the credit, and it is a warning signal to those who have loaned funds to the company. On the other hand, dividends paid from the depreciation funds and not so marked may have been in reality a return to the stockholder of a part of his original investment. Thus the amount of dividend may have been misleading, and it might have resulted in the taxpayer having paid unnecessary surtaxes. There are times when dividends should not be paid in cash, but may be paid by distributing stock.

To illustrate the effect of different dividend policies upon the depreciation funds and the company's plans for its continuation in the manufacturing business, five dividend programs will be reviewed.

In the statements to illustrate this discussion it will be noticeable that the conventional order of the items in the balance sheet has been reversed. This was done purposely to keep attention upon the net plant and the surplus.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

As a starting point, it is assumed that the balance sheet on April 30, 1930, when the plant was put in operation, disclosed the following uses and sources of funds, in dollars:

Plant at cost	1,000,000
Current assets	1,000,000
Total assets	2,000,000
Surplus	200,000
Capital stock	800,000
Bonds	500,000
Current liabilities	500,000
Total liabilities	2,000,000

In order to reduce the variations to a relatively few items so that one may more readily compare the cases to be discussed, certain features will be taken alike in all five dividend programs. These conditions include:

(1) The plant will be increased by a 60 per cent addition

(2) Assuming that the current assets will be necessary in an amount equivalent to or more than the investment in gross plant, as at the start, and also continuing the use of current liabilities at one-half of the necessary current assets, the current liabilities will be increased proportional to the change in the plant investment

(3) An arbitrary earning program proportional to the plant size will be selected

(4) Bonds will be retired on a definite schedule

(5) The reserve for depreciation will be increased by annual amounts at a fixed percentage of gross plant, except that the depreciation charges will be discontinued when a definite portion of the plant cost becomes fully depreciated.

For use in showing the points to be emphasized, each program is demonstrated. The examples continue the use of hypothetical and somewhat extreme balance sheets at the close of consecutive years.

Accompanying each set of these statements, there will be found a cumulative solution arranged to locate the changes that have taken place since the start of the program.

Program 1. In this case the dividends paid in cash have equaled the net earnings, except when funds had to be retained in anticipation of or to pay for property extensions. The position statements show, in dollars:

As on April 30	1931	1932	1933	1934	1935
Gross plant at cost	1,000,000	1,000,000	1,000,000	1,600,000	1,600,000
Depreciation reserve	200,000	400,000	600,000	900,000	1,100,000
Net plant	800,000	600,000	400,000	700,000	500,000
Current assets	1,000,000	1,000,000	1,300,000	1,600,000	1,600,000
Total assets	1,800,000	1,600,000	1,700,000	2,300,000	2,100,000
Surplus	200,000	200,000	400,000	700,000	500,000
Capital stock	800,000	800,000	800,000	800,000	800,000
Bonds	300,000	100,000	......	......	......
Current liabilities	500,000	500,000	500,000	800,000	800,000
Total liabilities	1,800,000	1,600,000	1,700,000	2,300,000	2,100,000

To observe what has taken place to the date of each balance sheet, a historical statement is now furnished:

Cumulative to April 30	1931	1932	1933	1934	1935
Additions to plant at cost	......	......	......	600,000	600,000
Necessary increase in current assets	......	......	......	600,000	600,000
Bonds retired	200,000	400,000	500,000	500,000	500,000
Cash dividends paid	300,000	600,000	700,000	880,000	1,560,000
Distributed	500,000	1,000,000	1,200,000	2,580,000	3,260,000
Increase in surplus	......	......	200,000	500,000	300,000
Dividends in cash	300,000	600,000	700,000	880,000	1,560,000
Net earnings	300,000	600,000	900,000	1,380,000	1,860,000
Necessary increase in current liabilities	......	......	......	300,000	300,000
Depreciation funds diverted	200,000	400,000	300,000	900,000	1,100,000
Provided	500,000	1,000,000	1,200,000	2,580,000	3,260,000

As a result of this dividend program, four features are noticeable: (1) The surplus was not increased as the bonds were retired; (2) the only year in which a part of the depreciation reserve was available occurred in the third year when earnings were retained in anticipation of other uses; (3) in general the depreciation policy failed to provide for any replacements, although the accounting formality had been gone through and such a charge recorded in the books of account; (4) in paying these high dividends on top of the additions to plant and working capital, the management has entirely ignored the limitations of the net earnings after depreciation.

(In working out the five programs, the depreciation reserve has been carried on the asset side of the statement and treated as a deduction from gross plant. This method has been generally adopted and is to be recommended, since it prevents the corresponding inflation of the total assets.)

Program 2. This dividend plan starts out by limiting the cash dividend to about two-thirds of the earnings so as to build up a surplus for some future use. The balance sheets are as follows:

As on April 30	1931	1932	1933	1934	1935
Gross plant	1,000,000	1,000,000	1,000,000	1,600,000	1,600,000
Depreciation reserve	200,000	400,000	600,000	900,000	1,100,000
Net plant	800,000	600,000	400,000	700,000	500,000
Current assets	1,100,000	1,200,000	1,300,000	1,600,000	1,700,000
Total assets	1,900,000	1,800,000	1,700,000	2,300,000	2,200,000
Surplus	300,000	400,000	400,000	700,000	600,000
Capital stock	800,000	800,000	800,000	800,000	800,000
Bonds	300,000	100,000	......	......	......
Current liabilities	500,000	500,000	500,000	800,000	800,000
Total liabilities	1,900,000	1,800,000	1,700,000	2,300,000	2,200,000

What has really taken place is shown by this historical statement:

Cumulative to April 30	1931	1932	1933	1934	1935
Additions to plant at cost	......	......	......	600,000	600,000
Necessary increase in current assets	......	......	......	600,000	600,000
Bonds retired	200,000	400,000	500,000	500,000	500,000
Cash dividends paid	200,000	400,000	700,000	880,000	1,460,000
Distributed	400,000	800,000	1,200,000	2,580,000	3,160,000
Increase in surplus	100,000	200,000	200,000	500,000	400,000
Dividends in cash	200,000	400,000	700,000	880,000	1,460,000
Net earnings	300,000	600,000	900,000	1,380,000	1,860,000
Necessary increase in current liabilities	......	......	......	300,000	300,000
Depreciation funds diverted	100,000	200,000	300,000	900,000	1,000,000
Provided	400,000	800,000	1,200,000	2,580,000	3,160,000

In this instance four results are outstanding: (1) The surplus was increased by a part of the amount paid out for the bonds bought in; (2) about one-half of the depreciation reserve was available until the plant was extended, at which time this reserve was fully diverted; (3) while the intention in regard to depreciation was better than in Program 1, the final result was almost as bad a failure to provide for replacements; (4) it is quite apparent that the management has tried to spend the same earnings for both expansion and dividends.

Program 3. Here a good start was made by limiting dividends in order to pay off the bonds, but no provision was made for the sudden expansion. The annual reports show:

As on April 30	1931	1932	1933	1934	1935
Gross plant	1,000,000	1,000,000	1,000,000	1,600,000	1,600,000
Depreciation reserve	200,000	400,000	600,000	900,000	1,100,000
Net plant	800,000	600,000	400,000	700,000	500,000
Current assets	1,200,000	1,400,000	1,600,000	1,900,000	2,100,000
Total assets	2,000,000	2,000,000	2,000,000	2,600,000	2,600,000
Surplus	400,000	600,000	700,000	1,000,000	1,000 000
Capital stock	800,000	800,000	800,000	800,000	800,000
Bonds	300,000	100,000	......	......	......
Current liabilities	500,000	500,000	500,000	800,000	800,000
Total liabilities	2,000,000	2,000,000	2,000,000	2,600,000	2,600,000

The facts about what has been done are given historically:

Cumulative to April 30.	1931	1932	1933	1934	1935
Additions to plant at cost	.....	.....	.....	600,000	600,000
Necessary increase in current assets	.....	.....	.....	600,000	600,000
Bonds retired	200,000	400,000	500,000	500,000	500,000
Cash dividends paid	100,000	200,000	400,000	580,000	1,060,000
Distributed	300,000	600,000	900,000	2,280,000	2,760,000
Increase in surplus	200,000	400,000	500,000	800,000	800,000
Dividends in cash	100,000	200,000	400,000	580,000	1,060,000
Net earnings	300,000	600,000	900,000	1,380,000	1,860,000
Necessary increase in current liabilities	.....	.....	.....	300,000	300,000
Depreciation funds diverted	.....	.....	.....	600,000	600,000
Provided	300,000	600,000	900,000	2,280,000	2,760,000

In consequence of this program, four points may be observed: (1) The surplus has increased in proportion to the bonds retired; (2) the depreciation reserve was kept available until the sudden expansion, for which no provision had been made, and at that time the original depreciation policy was canceled; (3) to the extent that less cash dividends have been paid out, the financial position is better than in the previous programs, although it is apparent that both the expansion and the cash dividends could not all be paid out of the same earnings; (4) the relatively constant total assets, including the jump for property additions, are indicative of the intention to fully maintain the property.

Program 4. Starting with a definite plan for expanding the business, no earnings were distributed to the stockholders until the plant additions with necessary increase in working capital had both been provided. On this program the annual positions were reported:

As on April 30	1931	1932	1933	1934	1935
Gross plant	1,000,000	1,000,000	1,000,000	1,600,000	1,600,000
Depreciation reserve	200,000	400,000	600,000	900,000	1,100,000
Net plant	800,000	600,000	400,000	700,000	500,000
Current assets	1,300,000	1,600,000	2,000,000	2,480,000	2,700,000
Total assets	2,100,000	2,200,000	2,400,000	3,180,000	3,200,000
Surplus	500,000	800,000	1,100,000	1,580,000	1,600,000
Capital stock	800,000	800,000	800,000	800,000	800,000
Bonds	300,000	100,000	......	......	......
Current liabilities	500,000	500,000	500,000	800,000	800,000
Total liabilities	2,100,000	2,200,000	2,400,000	3,180,000	3,200,000

And the history followed these steps:

Cumulative to April 30.	1931	1932	1933	1934	1935
Additions to plant at cost	.....	.....	.....	600,000	600,000
Necessary increase in current assets	.....	.....	.....	600,000	600,000
Bonds retired	200,000	400,000	500,000	500,000	500,000
Cash dividends paid	.....	.....	.....	......	460,000
Reserved for plant extension	100,000	200,000	400,000	......	......
Distributed	300,000	600,000	900,000	1,700,000	2,160,000
Increase in surplus	300,000	600,000	900,000	1,380,000	1,400,000
Dividends in cash	.....	.....	.....	......	460,000
Net earnings	300,000	600,000	900,000	1,380,000	1,860,000
Necessary increase in current liabilities	.....	.....	.....	300,000	300,000
Depreciation funds diverted	.....	.....	.....	20,000	......
Provided	300,000	600,000	900,000	1,700,000	2,160,000

As it was intended to accomplish certain purposes, it will be of interest now to observe what took place. Four items are most noticeable: (1) The surplus grew by the annual earnings, except in the last period, when a cash dividend was paid; (2) the reserves for depreciation were fully maintained for replacements, except in the fourth period, when a very small part was temporarily diverted; (3) the earnings were retained for the plant extensions without attempt also to pay them out in cash dividends; (4) it is noteworthy that the increases in total assets properly preceded or accompanied the expansion of the business.

Program 5. Finally a plan is reviewed that combines the good points of the last three arrangements. The balance sheets are presented as follows:

As on April 30	1931	1932	1933	1934	1935
Gross plant	1,000,000	1,000,000	1,000,000	1,600,000	1,600,000
Depreciation reserve	200,000	400,000	600,000	900,000	1,100,000
Net plant	800,000	600,000	400,000	700,000	500,000
Current assets	1,300,000	1,600,000	2,000,000	2,480,000	2,700,000
Total assets	2,100,000	2,200,000	2,400,000	3,180,000	3,200,000
Surplus	400,000	600,000	700,000	700,000	700,000
Capital stock	900,000	1,000,000	1,200,000	1,680,000	1,700,000
Bonds	300,000	100,000	......	......	......
Current liabilities	500,000	500,000	500,000	800,000	800,000
Total liabilities	2,100,000	2,200,000	2,400,000	3.180,000	3,200,000

The accompanying historical statement follows:

Cumulative to April 30.	1931	1932	1933	1934	1935
Additions to plant at cost	.....	.....	.....	600,000	600,000
Necessary increase in current assets	.....	.....	.....	600,000	600,000
Bonds retired	200,000	400,000	500,000	500,000	500,000
Cash dividends paid	.....	.....	.....	......	460,000
Reserved for plant extensions	100,000	200,000	400,000	......	......
Distributed	300,000	600,000	900,000	1,700,000	2,160,000
Increase in surplus	200,000	400,000	500,000	500,000	500,000
Dividends in cash	.....	.....	.....	......	460,000
Dividends in stock	100,000	200,000	400,000	880,000	900,000
Net earnings	300,000	600,000	900,000	1,380,000	1,860,000
Necessary increase in current liabilities	.....	.....	.....	300,000	300,000
Depreciation funds diverted	.....	.....	.....	20,000	......
Provided	300,000	600,000	900,000	1,700,000	2,160,000

The important considerations are four in number, thus: (1) The surplus has been increased by the amount of bonds retired; (2) the depreciation funds have been kept available except for one small borrowing; (3) the earnings retained for plant extension have been turned over to the stockholders by means of stock dividends, while further earnings can be paid in cash; (4) the increase in total assets accompanies the planned growth of the business attended by adequate financing.

Following the brief study of these programs it may be set down that a dividend policy is fundamentally wrong when it later may require the reduction or omission of future dividends in order to raise the funds necessary for replacements. Perhaps the dividends can be continued by selling stock to the unsuspecting public in order to release the replacement funds, but this will correspondingly cut down the amount of dividend to the old stockholders. Moreover, it is not convenient to sell stocks or bonds at certain times, and especially when the physical properties are well consumed and the industry is in the doldrums. No bank is going to knowingly make short-term loans for machinery replacements, and many commission sales agents are going to learn that they should not make advances for release of depreciation reserves.

Stock dividends offer a fair and equitable distribution to the stockholders in that these dividends acknowledge the extension of the facilities, permit the retention of depreciation funds, and give the stockholders an opportunity to continue their proportion of the investment or to sell their stock dividend and accept profits in lieu of the extra shares.

When a balance sheet is placed before an executive, banker, or stockholder, how is he going to sense the indications of inability to make replacements that must come eventually and even now may be responsible for the high costs that leave little or no profits? As the current assets include the cash, inventories, marketable securities, etc., that should apply in part to the depreciation reserve, the status of the working assets exclusive of the depreciation fund may be noted. At the close of the last period just reviewed, the working assets as a result of the five different policies show (losses are in italic figures):

Program	1	2	3	4 and 5
Current assets	1,600,000	1,700,000	2,100,000	2,700,000
Depreciation funds	1,100,000	1,100,000	1,100,000	1,100,000
Current investment	500,000	600,000	1,000,000	1,600,000
Current liabilities	800,000	800,000	800,000	800,000
Current working investment	*300,000*	*200,000*	200,000	800,000
Current working capital	800,000	900,000	1,300,000	1,900,000
Ratio of current assets to current liabilities	2.000	2.125	2.625	3.375
Ratio of current investment to current liabilities	0.625	0.750	1.250	2.000

From this tabulation it should be apparent that the usual current-asset–current-liability ratio requires considerable interpretation at all times. While the accountant may have stressed the importance of this ratio, the experienced banker has learned that the desirable risk is not foretold by any such computation. The current-asset–current-liability ratio is affected by the depreciation reserve, and also it should be observed for the relation of the current assets to the total assets.

In the last illustration, it is to be noted that this company started out with a current-asset–current-liability ratio of 2 to 1, and as a result of the several dividend programs, this ratio at the close of the fifth year would be 2 or better. Such a 2 to 1 ratio might indicate that the customary credit terms may be continued to the company for merchandise shipped to them in the usual way, as long as the company discounts or remits promptly.

But the banker and the investor should be more interested in the relative maintenance of the current-investment–current-liability ratio as indicative of sound financial policies supported by ability to make replacements, and especially to take advantage of improved manufacturing processes or the opportunities that may attend changes to more profitable lines or specialties.

Returning to the last illustration, it may be observed that this company started out with a current-investment–current-liability ratio of 2 to 1 and that this condition was continued at the close of the fifth year only in the cases of dividend programs 4 and 5. In the other programs large proportionate departures were shown, going as low as 5 to 8 (or 0.625) in program 1. And this was going on while the current asset ratio continued as high as 2!

Whether the annual depreciation charge is sufficient and the total depreciation reserve is proportional to the consumption of the assets plus reasonable allowance for obsolescence cannot be established directly by reference to the financial statements of any single company. But the ability to spend the replacement funds, set aside by the creation of the depreciation reserve, is indicated in the balance sheet, and it may be further determined by examination of the past history of the company. This ability, in combination with records of annual replacements made, is convincing proof that the company has officers who are looking forward into the future and planning to continue in active business.

Because the historical interpretation of financial statements can develop these significant facts, it also contributes to the study of stock values. This is not to be construed as an attempt to foretell market prices, but rather as a secondary step in judging the desirability of future ownership in a specific company.

It is a well-known fact that there is no relation between so-called book value and market value. Future dividend prospects help set the market prices. Should the future earnings be required for replacements, future dividends become improbable.

When a historical statement is available, an illuminating study of indicated value may be made. This is demonstrated by an observation of the final results from the five dividend program (losses are in italics), as follows:

Program	1	2	3	4	5
Net earnings	1,860,000	1,860,000	1,860,000	1,860,000	1,860,000
Distributed in dividends	1,560,000	1,460,000	1,060,000	460,000	1,360,000
Increase in surplus account	300,000	400,000	800,000	1,400,000	500,000
Depreciation diverted	1,100,000	1,000,000	600,000	......	......
Enhancement or *shrinkage*	*800,000*	*600,000*	200,000	1,400,000	500,000
Original surplus	200,000	200,000	200,000	200,000	200,000
Indicated value or *lack* of book surplus	*600,000*	*400,000*	400,000	1,600,000	700,000
Capital stock	800,000	800,000	800,000	800,000	1,700,000
Indicated value of net worth	200,000	400,000	1,200,000	2,400,000	2,400,000
Indicated net worth relative to original capital stock, per cent.	25	50	150	300	300
Actual book value	1,300,000	1,400,000	1,800,000	2,400,000	2,400,000

Attention is directed to the difference between the indicated net worth and the book value. This difference represents the amount of additional money that must be put back into the business in order to restore the full values. For instance, to restore the cost of the original plant facilities it will be necessary to obtain funds equivalent to the depreciation reserve diverted. If new purchasers should take over the shares they will have to expect to raise the additional funds if they intend to continue all of the business. These funds might be borrowed temporarily or by mortgage, but they will eventually have to be put in by the stockholders who may have (1) additional stock sold to them to pay for the plant addition and thus release the depreciation reserve for replacements, or (2) the future dividends must cut down until the future earnings will supply the replacements.

In the first instance, it will be apparent that the possible sale value of the stock should be below the book value by the additional amount which the stockholders must put in to restore the original cost value. The market value as usually applied to investments is dependent upon the future earning ability, since those earnings should control the dividend possibilities. It is common to see the earnings of the last five years averaged to outline the future possibilities. But do not pay any such price without further consideration! The deficit from depreciation diverted should be deducted to establish what should be paid for the stock, as was done to arrive at the indicated net worth.

In the second instance, it will be observed that the shareholders must expect the dividend rate to be cut or maybe suspended until the future earnings will repay the depreciation funds so necessary to restore the plant facilities. This point cannot be overemphasized. It explains how the big dividends of one generation may have been advance payments out of future earnings and why the second generation must go without dividends. The future earnings have been discounted to the extent that future dividends have been anticipated. Accordingly, that discounting has been accomplished by reducing the present or indicated value of the net worth. Thus, a part of the past dividends must have reduced the real investment, which is not shown by the book value.

This point is also of importance to the bondholder and banker, for it increases the risk, and in case of default it may be expected to necessitate the bondholder and banker putting in restorative cash running into large amounts that were never contemplated at the time of making the original loans. It often takes large sums of money to rehabilitate old properties that might better have been left alone.

Referring to the last tabulation, it will be clear that the first program would have given the stockholder the largest immediate return and would have left the least real or indicated value to the stock. The fifth program would have given the same cash dividend as in program 4, plus a sizable block of stock that might have been sold for quick cash return or held for the increased investment value. The first three programs give the stockholder fictitious dividends that are in reality repayments of the original investment.

As provided in the Federal Income Tax Regulations, certain companies using natural resources, such as coal mines, oil wells, timber cutting, etc., experiencing depletion, inform their stockholders concerning the portions of the dividend that were paid from earnings and by investment return. No such relief is definitely specified for the stockholders in a manufacturing company who personally may have paid a tax on cash dividends that used funds from the depreciation reserve and therefore also were a return of the investment. In Regulations 74 (1928), section 115, article 626, provision is made for "distributions from depletion or depreciation reserves." The last sentence in that article states, "No distribution, however, can be made from such a reserve until all the earnings or profits of the corporation have first been distributed." In the examples of the dividend programs 1, 2, and 3, part of the earnings was left in surplus because of the plant additions made out of earnings. But the shareholders did pay a tax on the return of that part of the investment distributed through the dividends using cash from the depreciation reserve. Whether the income tax provision is correct or is not, need not be argued here. The point to be made is that improper dividend policies may be expensive to the taxpayer who must consider surtaxes.

All investors should give more attention to guard against improper dividends that will attract unjustified competition which cannot be repelled after the financial position has become greatly weakened. The individual mills in the textile industry are responsible for the large and often unwarranted cash dividends which prompted the building of new plants that greatly extended the available capacity, and thus have brought competitive unprofitable sales.

This explanation may help account for some of the repetitive cycles of business activity, especially the sequence of good profits, expansion with cash dividends, and idle plants with lack of earnings. It certainly does explain why many stock and bond estates, formerly very attractive, have in another period deteriorated quite rapidly.

Some people may be of the opinion that too much emphasis has been given to the misuse of depreciation funds. Foremost among their arguments may be mentioned the exceptional companies that have been very profitable and apparently have never found it necessary to give much thought to the form of dividend paid. These exceptions on closer examination often may furnish conclusive evidence of the importance of the depreciation funds.

For example, an examination of the foremost automobile companies will show the frequent large expenditures of cash in providing the most economical facilities to produce new models. Here the depreciation funds are in actual use as intended, so cannot be used to distort dividend considerations.

The leaders in other lines frequently are maintaining their progressive positions because of their willingness to spend depreciation funds for more economical processing. In some instances a rule-of-thumb plan has been adopted. This may provide for an average replacement each five years equivalent to the average depreciation charge so as to keep the book value at a figure approximately one-half depreciated, but with facilities thoroughly modern. This practical solution uses the depreciation funds before that money can be misapplied. Therefore, the success of these companies has been assisted by the good management that has acted before the depreciation funds have been allowed to accumulate and become a source of trouble to unsuspecting directors.

This point brings out the value of more consideration being given to the use of depreciation funds. The banker and investor should inquire more into the expenditures from those funds. The management should be expected to average certain rates of replacements, and any slighting of this provision can be accepted as evidence of lack of sound policies. The engineer watching for signs of healthy management will insist upon more detailed reports about the depreciation reserve and especially the history of all depreciation expenditures.

The better relations between engineers and bankers makes it possible for the engineer to bring forth advance proofs that may be of great benefit. This is especially true where the engineer can show the necessity for sufficient original capital to provide all necessary funds essential to properly rehabilitate old properties into conditions of practical economic use. Worn-out properties should not be bought and returned to competition unless they are adequately refinanced to permit complete modernization within a reasonable time. Thus, many of the troubles that follow lack of sufficient own capital can be avoided.

In an unusually clear discussion of business ills, Fred W. Shibley, Vice-President, Bankers Trust Company, commented upon depreciation in his article, "Look to Your Working Capital," appearing in the May, 1931, issue of *Nation's Business*. In his presentation of the lessons taught in this last depression, he says:

> One fact has been driven into the minds of the commercial bankers who lend money and of the manufacturing corporations which look to banking credit for additional working capital. That is how seldom the item "depreciation," annually charged to expense, has been soundly treated. This item should be regarded as a true reserve set up in the form of cash or its equivalent to provide not only for depreciation, but also for obsolescence of plant and machinery. Such a reserve should be conserved as a sacred fund to be employed for no purpose except the replacement of plant and equipment, the addition of new and improved machinery, and, in an emergency, as working capital under an implied obligation that restitution shall be made of such working capital from profits in subsequent years or from the sale of capital securities.
>
> In this machine age depreciation and obsolescence cannot be ignored. Maintenance and repairs are well enough in their place, but the current pace of improvements in mechanical processes is too swift for them. A plant maintained in a high degree of efficiency may become obsolete in comparatively few years. It is necessary, therefore, that the manufacturers should reserve depreciation as a cash asset, not available for dividend requirements, a use to which it has been prostituted too frequently in recent years, particularly when payment of dividends was required to maintain the market value of corporate securities.

CONCLUSION

Dividend programs are related to depreciation. The figures that were presented to show the working out of five selected dividend programs have demonstrated that the depreciation funds can be and too often are misused as a result of the dividend program chosen. In those instances, the intention of the depreciation reserve is accordingly destroyed.

Following the demonstration of this relation between dividend and depreciation and its consequence, discussion has been given to the following related, special features:

Historical statements for interpretation of results
Dividend programs wrong if provision has not been made for replacements
When stock dividends rather than cash
Lack of significance in current-asset-current-liability ratios

Possibility of current-investment–current-liability ratio
Ability to make replacements indicates plans for continuity
Indicated net worth relative to original capital stock
Dividends that return parts of the original investment
Improper dividends promote unprofitable competition
Management should spend depreciation funds.

It is proper to suggest that more attention upon the retention of depreciation funds as intended by that reserve would help to prevent extreme business depressions such as have been experienced. Should the state laws relative to the payment of dividends be further defined to make the directors liable for dividends distributed by misuse of depreciation funds, a strong influence would be exerted in restraint of unwarranted plant expansion. It is hoped that such restraints will be developed as protection, both to the general purchaser of investment issues and to the great number of trusting workers who have so much suffering thrust upon them during the periods of long business depressions.

In the textile industry, late years have shown remarkable improvement in machinery and processing arrangements. Individual investors may advisably examine the depreciation policies and quietly withdraw from participation in those companies that are not now taking the necessary mechanical steps to lower their operating costs.

In all industries where the investment in plant values is proportionately high, it is important that the dividend program shall not be contingent upon the lack of a complete depreciation policy.

Discussion

Ernest F. DuBrul.[2] The paper gives a clear analysis of the effect of various policies in connection with the payment of dividends. Entirely too little attention generally is paid by management to the question of the handling of depreciation accounting and depreciation-reserve funds. It is thought that the author should have made it a little clearer that when he refers to a depreciation reserve he means a depreciation-reserve fund. The item called depreciation reserve is frequently used in balance sheets merely as a bookkeeping entry against which there is no actual cash or securities fund. Only by using the word fund can the meaning be made clear. The author shows how an analysis of the figures will disclose what has happened to the accounting entry.

Mr. Shibley, as quoted by the author, wants the depreciation reserve conserved as "a sacred fund to be employed for no purpose except the replacement of plant and equipment, the addition of new and improved machinery, and, in an emergency, as working capital under an implied obligation that restitution shall be made of such working capital from profits in subsequent years or from the sale of capital securities."

It is noted that Mr. Shibley himself tolerates a temporary emergency diversion of the sacred depreciation-reserve funds into working capital. To do so gives the "sacred obligation" a very weak hold on the conscience of the management. The difficulty is that so-called emergencies usually become chronic conditions. When a departure from a sound standard is made in the case of emergency, it is usually exceedingly difficult to bring the minds of directors back to the standard practice after the emergency has passed.

While the author has rendered a real service in calling attention to these conditions and showing a practical method of analysis, his paper unfortunately does not go deep enough, because his presentation is true only if one blindly accepts the accounting fallacy which assumes stability in the purchasing power of the money unit. The cold, inescapable facts are that the dollar varies in purchasing power, and therefore the actual cost of acquisition on which the author's analysis is based is of very limited use from a managerial point of view. Therefore the method of analysis needs to be amplified and clarified. In other words, the dollar measurements must be adjusted to the actual purchasing power. Unfortunately, few accountants have as yet seen the need of calibrating the dollar marks on their books to the actual current facts of purchasing power of the dollars which those dollar marks represent.

Every condition cited in the paper showing a dollar deficiency of the depreciation reserve fund is accentuated to whatever extent there has been an increase over original cost in the prices of the property which must replace property that is wearing out or becoming obsolete. Traditional accounting takes no responsibility for disclosing this cold economic fact. There was an enormous rise in prices from the low prices of 1896 to the high prices of 1920. This rise in price enormously accentuated every condition of incorrect, fallacious, and useless depreciation accounting that is set out in the paper.

Then, quite to the contrary, we have had a falling price level since 1920. This fall destroys the validity of the author's analysis in the other direction. If a plant had been built at the peak of prices of 1920, its depreciation accounting, based on the author's method, might appear very bad. Still it might have presented no financial difficulties to the company, because it may not cost as many dollars in 1931 to replace an equivalent amount of productivity which was purchased in 1920. Traditional accounting simply does not tell management these economic facts which management and owners ought to know.

A large trouble with discussions of depreciation lies in the fact that accountants are actually thinking of their book entries, although they talk as though they were thinking of the physical property. The author falls into this error of confused thought. On the fourth page of the paper he says, "To restore the cost of the original plant facilities it will be necessary," etc. Again he says, "The possible sale value of the stock should be below the book value by the additional amount which the stockholders must put in to restore the original cost value." But the inescapable managerial fact is that the thing which must be restored is the equivalent productivity *regardless* of the original cost value of the property.

The depreciation fund automatically diverted by the change in purchasing power of the dollar should be deducted to establish what should be paid for the stock if the costs of property have risen since the date of their acquisition, and on the contrary there may in fact be no real deficit from a diverted depreciation which would be shown by the author's formula, if prices of property have in the meantime fallen to such a degree as to make replacements cost less than the calculated diversion.

The writer fully agrees with the author's statement of the principle that "a dividend policy is fundamentally wrong when it later may require the reduction or omission of future dividends in order to raise the funds necessary for replacements." His own statement is perfectly true *regardless* of the original cost of acquisition of the asset. But when his accounting is based on the erroneous assumption of the stability of the purchasing power of the dollar, he vitiates the application of this perfectly sound principle which he enunciates.

Our accountants, cost consultants, industrial engineers, management consultants, financiers, bankers, and others deal with physical property which is represented by dollar symbols on books of account. We must bring more of these people to think of the preservation of the substance of the asset rather than the balancing of the accounting symbols representing that substance. All of these factors should carefully study depreciation principles such as are set out by the author, but expressed in dollar symbols calibrated to the purchasing power of the dollar.

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[2] Management Engineer, Cincinnati, Ohio. Assoc. A.S.M.E.

What should be done, of course, is the actual valuation of all balance-sheet items at current replacement values, the taking of depreciation and the setting up of depreciation-reserve funds based on current replacement value, revaluation of all balance-sheet items as often as important changes in purchasing power of the dollar justify the revaluation, and a setting up of enough adjustment accounts which will indicate the difference between the original cost and current values of not only the properties but of the depreciation sums set aside to take care of the replacement of those properties.

The usual cry is that this is entirely too complicated a procedure. But complex problems cannot be solved by simple procedures, and this problem of preserving the substance of the asset is a complex problem which kindergarten methods of accounting cannot possibly solve.

Nevertheless, it also may be pointed out that methods in actual use which attain the very objectives that the writer has set out are not excessively difficult, in spite of the cry to the contrary, so that the cry of excessive complexity is merely a confession of ignorance or worse.

Judging by the slow progress that traditional accounting practice has made in the way of giving management true pictures of the real facts of business, it will be many years before management can expect to get the true facts from the ordinary accountant. So we ought to be very grateful for any minor improvements that may come along, even though based on the inadequate facts presented by ordinary accounting.

Some improvement of that sort is possible by the use of the author's formula. Much advance would be made if these methods were carried out by accountants and managers who now violate the principles that he lays down. Half a loaf is better than no bread at all, so we should be glad to see management generally adopt the author's method of analysis. Even though it is defective, it is far ahead of anything that is usually done, and replacement programs might be improved somewhat compared to what they are.

The writer takes exception to the paragraph on the third page which says, "Stock dividends offer a fair and equitable distribution to the stockholders in that these dividends acknowledge an extension of the facilities, permit the retention of depreciation funds, and give the stockholders an opportunity to continue their proportion of the investment or to sell their stock dividend and accept profits in lieu of the extra shares."

Here again mere dollar marks on the accountant's books are taken as meaning something, whereas they have no meaning at all. The author's statement would apply only if par values on stocks meant anything, and we know that they mean nothing at all. The statement would not be applicable in the case of a company with no-par stock. Whether the company has par stock or no-par stock, the distribution of stock dividends is merely a distribution of pieces of paper evidencing some fractions of ownership. So there is no validity to the author's conclusions in connection with stock dividends on that account. It is granted that the stock dividends would balance certain columns of figures, according to the author's formula, but such balancing would not automatically do what he says. It is simply a part of the formalism of traditional accounting which confuses the issue instead of throwing any light upon it.

AUTHOR'S CLOSURE

Most of the discussion has proposed additional ideas. At the outset the author can admit the omissions cited because it was of course necessary to place a limit on the length of this paper.

Mr. H. F. Coes, in his oral discussion, emphasized the necessity that depreciation fully include both deterioration and obsolescence, and stressed the importance of depreciation (a rental charge for use of equipment) as an element of cost. In reply to his inquiry about the use of the proposed current-investment–current-liability ratio, the author will explain that this contribution has been suggested for its ready indication of past dividends and financial policies that may directly lead to an unsatisfactory financial condition.

He also helped to stress the importance of distinguishing between the depreciation reserve (an allowance for depreciation accrued) and the depreciation funds (the dollars provided to make replacements). This point was very well taken, for it is essential that these two features of depreciation be kept clearly in mind when considering the accounting for property. In the paper, the illustrations were used to keep track of the disposition of the depreciation funds that should be retained to accompany the depreciation reserve. Failure in this particular damages the financial situation and leads to economic troubles. Many of these failures have in the past been brought about by questionable dividend policies that exhausted the depreciation funds. Until management fully recognizes these consequences and acts to prevent this form of waste, the financial situation is relatively but little better than in the old days before depreciation was considered.

Mr. DuBrul has added the economic theory of fluctuating dollar values and presented it in his usual enthusiastic manner. Much could be said on both sides of this question. While we recognize this feature, it is well also to remember that it is not the only variable. In many lines of business the changes of customer desires, the competition of other products, the improvement in methods, etc. make plants partially unprofitable and have much more effect upon the plant values to going concerns than does that of change in the purchasing power of isolated dollars. Admittedly, one variable can be identified, but that does not detract greatly from the risk of doing business, which in the last analysis must include all the uncontrolled variables.

Mr. DuBrul exposes the theory he so ably presents, by announcing "the inescapable managerial fact is that the thing which must be restored is the equivalent productivity *regardless* of the original cost," etc. The periods of rising dollar values have been accompanied by remarkable improvements in the relative productivity. In many instances the rate of output from new machines has contributed to lower costs through reducing the number of the machines or the floor space required. This point was well taken by Mr. Coes, who so aptly remarked that "replacements are not *in kind*, but are by improvements." The variable dollar value cannot fairly be applied in fact finding unless the variable productivity, plant desirability, and other factors also be considered. It is not essential to include such variables in the customary financial statements, at least not before the present form of report is much better understood and used more effectively.

Finally, it is apparent that we are all striving for the reduction of economic ills, and it is generally agreed that the depreciation funds should be retained available for the replacements which must be provided in accordance with the accrued depreciation set out in the depreciation reserve. That this move is in the proper direction was stated by Mr. DuBrul: "Some improvement of that sort is possible by the use of the author's formula. Much advance would be made if these methods were carried out by accountants and managers who now violate the principles that he lays down."

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MSP-54-1

Automatic Internal Grinder

By H. L. BLOOD,[1] WORCESTER, MASS.

Internal grinding, originally a toolroom operation, has been demanded for such a large number of parts that it has become necessary to develop machines that would grind the holes at the lowest possible cost. This development has culminated in the automatic internal-grinding machine which positively determines the size of the hole and which requires no gaging on the part of the operator, who merely puts the work into the chuck and starts the cycle of operations. The improvements on the machines and their adaptation for special grinding jobs are described.

INTERNAL GRINDING was originally a toolroom operation, but the demand for large numbers of parts having accurately finished holes made it necessary to develop machines that would grind the holes at the lowest possible cost. The culmination of years of development in internal grinding is the automatic machine which positively determines the size of the hole being ground and which requires no effort on the part of the operator except to put work into the chuck and start the cycle of operations. In order to understand and appreciate the various features of the automatic internal grinder the steps by which it was developed should be reviewed.

The first type of machine used for internal grinding was a universal grinder, Fig. 1, which was designed for external and face as well as for internal grinding. The spindle and wheel for internal grinding may be seen at x.

This machine was followed by one designed especially for internal work (Fig. 2). The workhead, which rotated the work, was carried on a reciprocating table, and a high-speed grinding spindle was mounted on a cross-slide supported from the back. This was a bench machine and was adapted for grinding only small holes up to a maximum of $^1/_4$ in. diameter.

An internal grinder for holes larger than $^1/_4$ in. was brought out in 1906 (Fig. 3). Its capacity was for holes of $^1/_2$ to $8^1/_2$ in. in diameter and up to 11 in. long. When it was introduced, one of the leading machine-tool builders expressed the opinion that there would be a very small market for such a machine adapted only for internal grinding. He did not foresee the general use of hardened gears, rolls, bushing, and anti-friction bearings and the need for accurate holes in softer materials that have made internal grinding indispensable. Neither could he foresee the developments that have made the modern internal grinder an economical producer.

A basic improvement in internal grinders was made by utilizing a hydraulic cylinder to reciprocate the wheel, in place of the gears and clutches previously employed. The hydraulic system, Fig. 4, is worthy of note because of its simplicity. It consists of a gear pump, with a relief valve, and a combined throttle and reverse valve which admits oil first into one end of a cylinder and then into the other. This system was incorporated in an early machine, with the addition of a bypass valve opened by a cam bar on the table. When the operator lifts the table dog and the wheel beings to move away from the work, the bypass valve opens, allowing oil to bypass the throttle and run the table out at high speed to the rest position. After the operator unloads the chuck and puts in a new piece, he throws over the reverse valve, causing the table to run in at high speed. Just before the wheel reaches the work the bypass valve closes and the wheel reciprocates at the proper speed for roughing, as governed by the setting of the throttle valve.

When the hole is nearly to size, the operator lifts a latch, which allows the table to run out far enough to permit the diamond to drop into position for truing the wheel, and after being trued the wheel returns into the work for finish grinding. For truing the wheel, the operator reduces the speed by means of the throttle, a slow, uniform speed being necessary to allow the diamond to true the wheel properly instead of merely cutting a thread on it. Still another speed is used for finishing. From this it will be seen that the hydraulic system permits the use of the most economical speed for each part of the cycle, greatly increasing the productive possibilities of the machine. It eliminates gears and clutches, is smooth and quiet in operation, and is extremely reliable and durable, because the moving parts are few and run in oil under light loads. The hydraulic system also provides forced lubrication for the table ways and forms a foundation for the automatic features which will be described.

The same hydraulic system is also used on rotary surface grinders (Fig. 5) and in internal grinders of the planetary type, commonly known as cylinder grinders (Fig. 6). Cylinder grinders are well adapted for grinding pieces which cannot be rotated to advantage, such as large cylinders and locomotive connecting rods. The work is supported on an auxiliary table which can be adjusted horizontally and vertically to bring the work into the proper relation with the spindle, the vertical adjustment being obtained by means of a wedge member upon which the auxiliary table rests. The grinding spindle is carried in two eccentrics, one inside the other, which revolve together and carry the wheel in a circular path. Relative adjustment of the two eccentrics can be made while the machine is running, to increase the radius of operation and to enlarge the hole until the desired size is secured.

As the development of internal grinders progressed, other improvements were added, such as high-speed ball-bearing workheads and wheelheads, automatic starting of the workhead when the wheel approaches the work, automatic stopping of the workhead as the wheel runs out, and automatic starting and stopping of the flow of coolant at the proper time, but the greatest problem in internal grinding remained.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

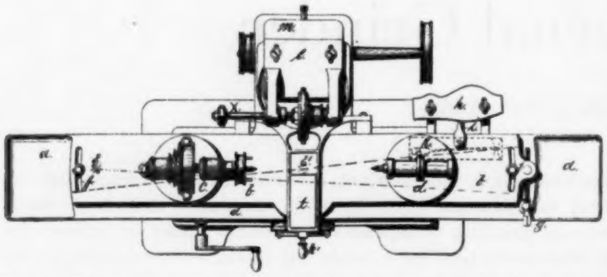
m
l
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FIG. 1. UNIVERSAL GRINDER, THE FIRST TYPE OF MACHINE USED FOR INTERNAL GRINDING

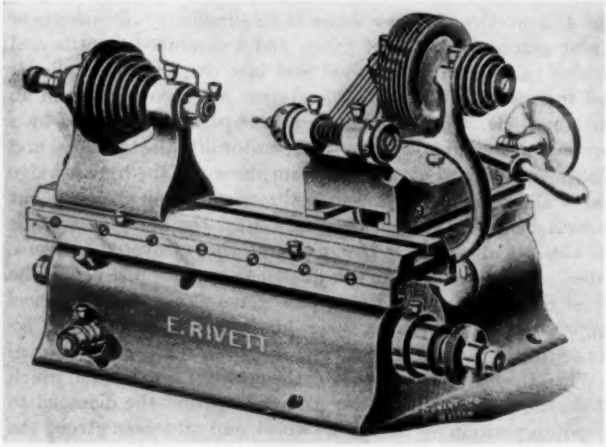

FIG. 2 RIVETT GRINDER FOR INTERNAL GRINDING
(Capacity, $^1/_{100}$ to $^1/_4$ in. diameter, $^1/_2$-in. stroke.)

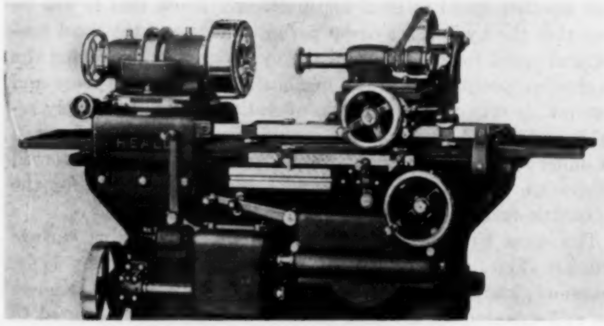

FIG. 3 INTERNAL GRINDER FOR LARGER HOLES
(Capacity, $^1/_2$ to $8^1/_2$ in. diameter, up to 11 in. long.)

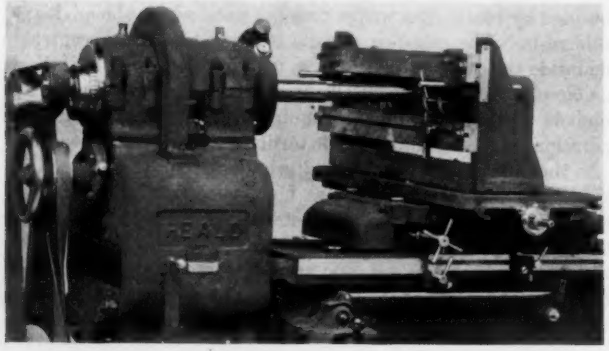

FIG. 6 INTERNAL GRINDER OF PLANETARY TYPE, OR CYLINDER GRINDER, USING HYDRAULIC SYSTEM SHOWN IN FIG. 4

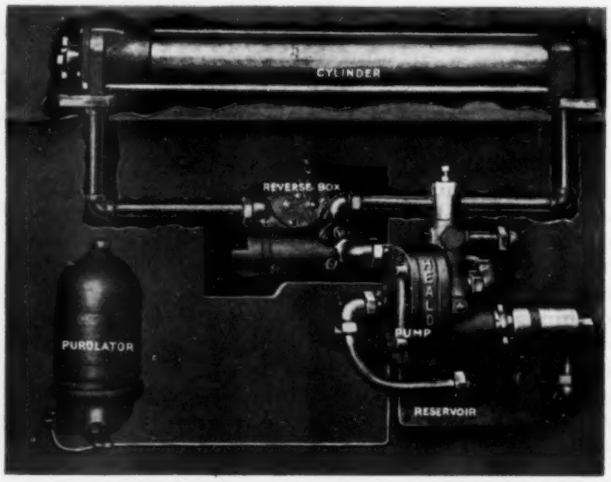
CYLINDER
REVERSE BOX
HEALD
PUROLATOR
PUMP
RESERVOIR

FIG. 4 HYDRAULIC CYLINDER TO RECIPROCATE THE WHEEL, IN PLACE OF GEARS AND CLUTCHES

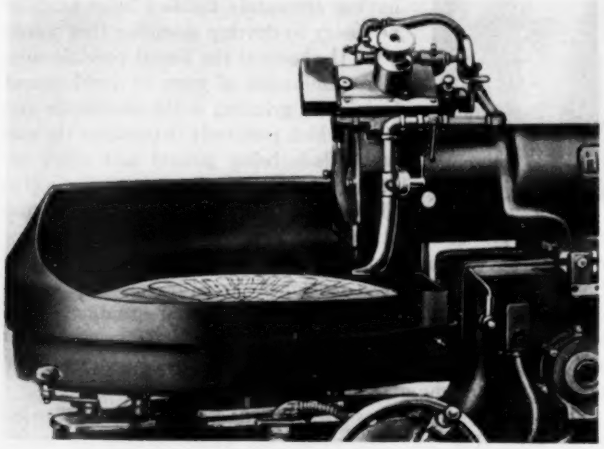

FIG. 5 ROTARY GRINDER USING HYDRAULIC SYSTEM SHOWN IN FIG. 4

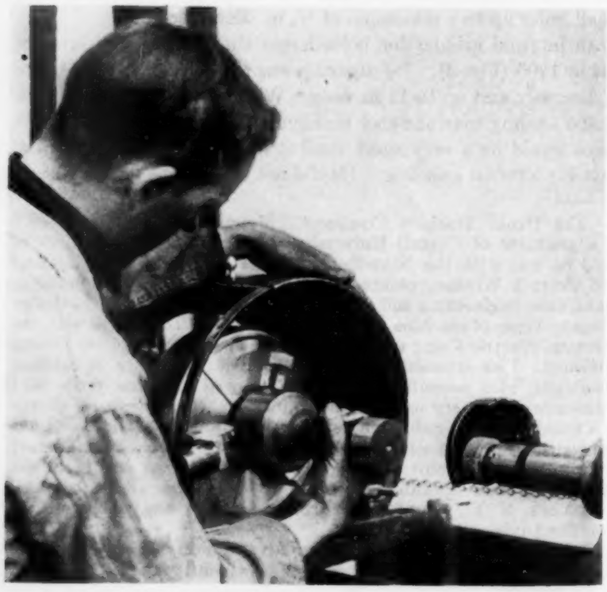

FIG. 7 INTERNAL GRINDING MACHINE WITHOUT AUTOMATIC SIZING
(With flexible idler drive and automatic wheel guard. Wheel guard gives protection to the operator when plugging the work.)

In the machines so far described it was necessary to run the wheel out of the work and try with a plug gage to see whether the hole was to size (Fig. 7). Internal grinding was a guessing contest in which the operator would guess that the hole was nearly to size, run the wheel out, try the plug, grind some more, guess that the hole was about right, run out and try again, until proper size was reached. In order to make a close guess it is necessary to estimate the effect of many variables, such as the amount of stock to be removed, which may vary widely from piece to piece, the amount of wheel wear, the condition or sharpness of the wheel, and the spring of the wheel spindle. An experienced demonstrator could make a fair showing in spite of this handicap, but training operators to obtain a reasonable production was a different matter, and at best the percentage of scrap was high, which added materially to the cost of the product. These difficulties are now becoming still more serious because the demand for closer tolerances is increasing.

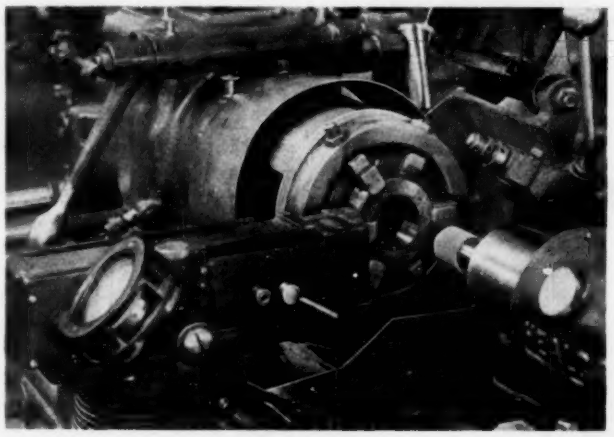

FIG. 8 DIAL INDICATOR WITH DIAMOND-POINTED FINGER BEARING AGAINST THE WORK
(This device does away with hand gaging.)

Three methods have been developed by which holes can be brought to size without the necessity for hand gaging. The first method is to arrange a dial indicator with a diamond-pointed finger bearing against the inside of the hole (Fig. 8). The indicator swings away from the work automatically as the table goes to the rest position so that the work may be unloaded and loaded quickly. By reading the indicator the operator can follow the change in size, true the wheel at the proper time, and stop the machine when finish size has been reached.

The dial indicator is not satisfactory, except for large work, because the indicator finger occupies part of the hole, making it necessary to use a comparatively small wheel and wheel spindle. The spring of a small spindle prevents the use of a fast feed and reduces the accuracy of the hole. The operator must give the indicator close attention and make a large number of motions to control the machine. He must increase the stroke of the machine for truing, reduce the table speed, pull down the diamond, reverse the table, change the table speed again for finishing, run the wheel away from the work when finish size is reached, compensate for wheel wear by turning a knob on the handwheel, and change the speed again for roughing the next piece. The motions which the operator must make not only take time, but their repetition at short intervals requires considerable physical exertion, resulting in fatigue and consequent loss in production. When the operator merely loads an automatic machine, the slow manual motions are eliminated, and it is easy for him to keep up indefinitely with the pace set by the machine. For these reasons machines of the dial-indicator type and those which require hand gaging are being superseded to a large extent by automatic machines, several types of which will be described.

Automatic sizing was first accomplished by adding two electric contacts to the dial indicator, making it an "electric indicator" (Fig. 9). The two contacts are connected to two magnets which lift the table dogs at the proper time and also open an oil valve, causing the diamond to swing down into position for truing the wheel. As the diamond swings down it moves a valve which reduces the table speed for truing, and as the diamond swings up again it moves the valve to give the desired speed for finishing. The cycle of operations is shown diagrammatically in Fig. 10. When the hole has been roughed out and approaches finish size, the first contact is made, the diamond swings down, the wheel runs out past the diamond at truing speed, and immediately re-enters the hole at finishing speed and with reduced feed. When the hole is to size, the second contact is made, and the wheel runs out to the rest position at high speed.

The electric indicator is used mainly on large work such as airplane cylinders. It is not adapted for holes having splines or keyways, or for smaller work, where it is important to keep the wheel and wheel spindle as large as possible.

Automatic interruption of the grinding cycle for truing, as just described, has proved indispensable for many jobs and economical for most jobs, because it keeps the wheel sharp, clean, and straight, making it cut rapidly and produce smooth work, practically overcoming the tendency to grind bell-mouthed holes. Automatic truing for every piece takes a very short time, and is highly desirable, even when tolerances are coarse and a smooth finish is not required, because of the increased production obtained. It might be thought that truing the wheel for every piece ground would increase the consumption of wheels. As a matter of fact, it often reduces wheel consumption because it permits the use of a comparatively hard wheel which breaks down slowly. Usually there is an increase in wheel consumption, which, however, is small compared with the advantages gained.

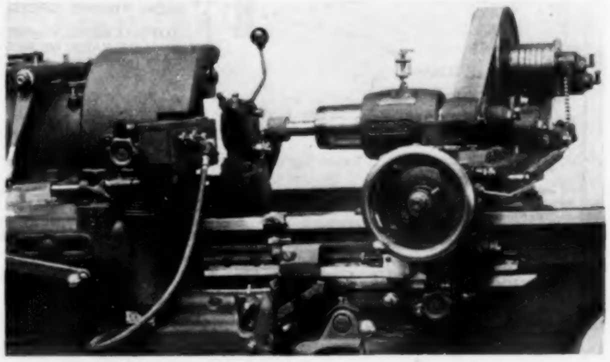

FIG. 9 AUTOMATIC SIZING, WITH TWO ELECTRIC CONTACTS ADDED TO THE DIAL INDICATOR

Automatic interruption and truing just prior to finishing the work forms the basis for another method of automatic sizing.

It has been seen that holes of a definite size cannot be produced by merely feeding the wheel a certain distance into the work, because the wheel wears down while roughing out the hole, and the amount of wheel wear depends upon several variables. However, after the work has been rough-ground to nearly the desired size, the wheel may be trued, bringing its working face into a definite location, and then fed into the work a short distance beyond this location. The desired size can be duplicated in this manner within very close limits. The wheel wear after truing is slight, because only a small amount of metal is removed, and remains practically constant from piece to piece, so that it does not interfere with duplication of size.

This method of sizing, generally known as the Size-Matic principle, is embodied in the machine shown in Fig. 11. The cycle of operations is the same as in Fig. 10, but the two contacts, instead of being in the indicator, are operated by a cam on the cross-feed handwheel. In order to have the handwheel come to the same position for each piece so that the contact will be made at the proper point, a device is incorporated which automatically advances the cross-slide at each cycle without moving the handwheel. In other words, it compensates for the wear of the wheel.

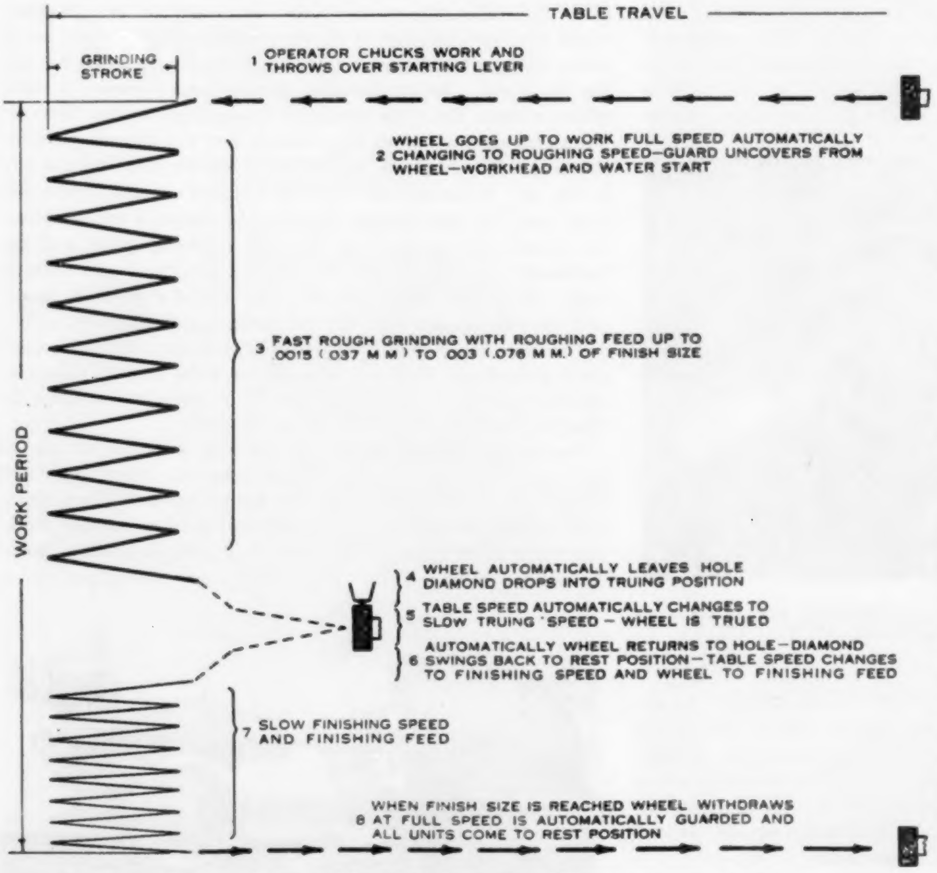
TABLE TRAVEL
GRINDING STROKE
1 OPERATOR CHUCKS WORK AND THROWS OVER STARTING LEVER
2 WHEEL GOES UP TO WORK FULL SPEED AUTOMATICALLY CHANGING TO ROUGHING SPEED—GUARD UNCOVERS FROM WHEEL—WORKHEAD AND WATER START
3 FAST ROUGH GRINDING WITH ROUGHING FEED UP TO .0015 (.037 M M) TO .003 (.076 M M.) OF FINISH SIZE
WORK PERIOD
4 WHEEL AUTOMATICALLY LEAVES HOLE DIAMOND DROPS INTO TRUING POSITION
5 TABLE SPEED AUTOMATICALLY CHANGES TO SLOW TRUING SPEED – WHEEL IS TRUED
6 AUTOMATICALLY WHEEL RETURNS TO HOLE—DIAMOND SWINGS BACK TO REST POSITION—TABLE SPEED CHANGES TO FINISHING SPEED AND WHEEL TO FINISHING FEED
7 SLOW FINISHING SPEED AND FINISHING FEED
8 WHEN FINISH SIZE IS REACHED WHEEL WITHDRAWS AT FULL SPEED IS AUTOMATICALLY GUARDED AND ALL UNITS COME TO REST POSITION

Fig. 10 Cycle of Grinding Operations With Automatic Sizing

In developing a machine to operate on the Size-Matic principle numerous refinements were necessary in the workhead, cross-slide, and diamond unit. The cross-slide must move a little as 0.0001 in. at a time and follow the motion of the feed screw accurately, and the diamond must be adjusted easily by very small amounts. The diamond is therefore mounted in two sets of preloaded ball bearings, one set being used when the diamond is swung into and out of position, while the other set is mounted eccentrically so that the diamond may be moved as little as 0.00005 in. by simply turning a graduated knob at the top of the unit, making a change of 0.0001 in. in the size of the holes being ground.

There are several reasons for the use of a swinging diamond instead of a fixed diamond. A diamond held in a fixed location necessitates a longer and more flexible wheel spindle in order to reach past the diamond, resulting in a loss in production. The swinging diamond overcomes this objection, is more accessible, and has the further advantage that the adjustments are more easily made, including the means for rotating the diamond to present a new truing surface.

Still another method of automatic sizing is to hold a plug gage against the back of the work by spring pressure, so that it will enter the hole as soon as the latter is to size and stop the machine. The essential parts may be seen in Fig. 12.

In practice it is usually desirable to use two gages *A*. These are mounted at the end of a rod *D*, which extends through the spindle of the workhead and holds the gages against the back of the work by spring pressure. The rod and the gage rotate with the work. As the wheel approaches the gages, the table pushes rod *F*, moving rod *D* and the gages out of the way of the wheel. When the work approaches finish size, the first gage, which is the smaller of the two, enters and allows adjusting screw *H* to make a contact which starts the truing cycle. The wheel then makes a few more passes at the proper speed and feed for finishing, and when the hole is to size, the second gage enters, allowing screw *G* to make a contact which causes the wheel to run out at high speed and which stops the machine. This is known as the Gage-Matic principle.

The method is not adapted for grinding blind or taper holes, but is ideal for ordinary straight open work. The work is not only produced automatically, but is inspected by the same machine and manual inspection is eliminated except for checking an occasional piece. A high rate of production can be maintained and scrap is practically eliminated regardless of variations in wheel wear, hardness of the work, amount of stock to be removed,

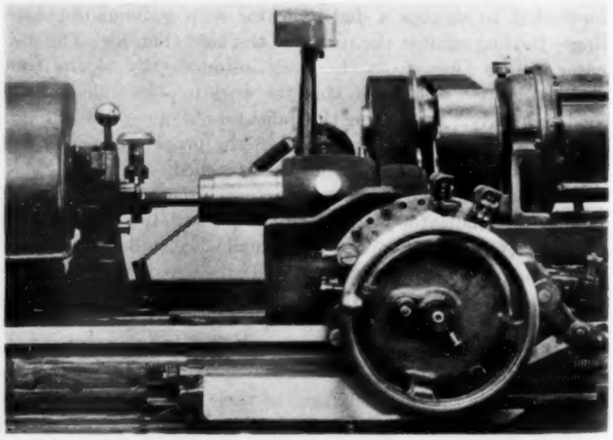

Fig. 11 Automatic Sizing, With Two Electric Contacts Operated by a Cam on the Cross-Feed Handwheel

and inattention or lack of skill on the part of the operator.

These machines, although automatic, can be used economically on small lots, as they require no more operations for setting up than plain machines. In the following comparison it is assumed that each machine is provided with a suitable chuck and wheelhead:

To set up plain internal	To set up Gage-Matic
(1) Set dogs for table stroke	(1) Set dogs for table stroke
(2) Set cross-slide feed	(2) Set cross-slide feed
(3) Grind hole to hand plug	(3) Mount and set gage
(4) Set cross-slide stop	(4) Set cross-slide cam
(5) Set diamond to true wheel	(5) Set diamond to true wheel

A number of interesting special features have been developed in connection with these machines; for instance, attachments for facing and internal grinding at the same setting, for chamfering, and for grinding pieces having two concentric holes of different diameters; a "backing-off" attachment for separating the wheel and work automatically before withdrawing the wheel, to obtain an improved finish; a short-stroke device for work requiring a short, rapid oscillation of the wheel, and "sparking out" devices which allow the wheel to make any desired number of passes after the feed has been stopped, giving extreme accuracy and especially smooth finish in connection with any of the automatic sizing methods which have been described.

Workhead bearings have now been improved to such an extent that when a "sparking out" device is used holes can be ground on a production basis with an average out-of-roundness of only 0.00005 in. (commonly known as "half a tenth"). It is now much easier to grind a hole to this degree of accuracy than to measure the error.

Recent developments in wheelheads have greatly increased the life of their ball bearings, in spite of the extremely high speeds at which they are run to secure the greatest efficiency in grinding. The usual speeds are:

Diameter of spindle, in.	Speed of wheelhead, r.p.m.
$1\,/_4$ to 1	10,300
$^3/_4$	15,000
$^9/_{16}$	19,500
$^7/_{16}$ and less	25,000

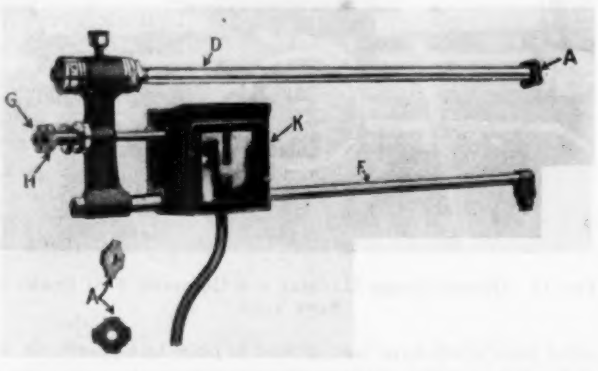

FIG. 12 AUTOMATIC SIZING, WITH PLUG GAGE OPERATED BY SPRING PRESSURE

It is not always appreciated that quick-acting means for holding the work are of more value in internal grinding than in many other operations because of the shortness of the machine cycle. A typical example illustrates this, as follows:

	Slow chuck	Quick-operating chuck
Grinding time, sec.	30	30
Chucking time	15	3
Total time (average), sec.	45	33
Production per year of 2000 hours, pieces	160,000	218,000
Increase in production, per cent	..	36
Cost of machine and chuck	\$5300.00	\$5500.00
Interest and depreciation at 16 per cent	\$ 848.00	\$ 880.00
Labor and other elements of burden at \$3000 a year	3000.00	3000.00
Total cost per year	\$3848.00	\$3880.00
Total cost per piece	0.0245	0.0177
Saving per piece		0.0068
Savings per year (218,000 pieces)		\$1482.40

Thus an investment of \$200 (the difference in cost between a slow- and a quick-acting chuck) brings a yearly return of 741 per cent. This is only an average return and is based upon conservative figures.

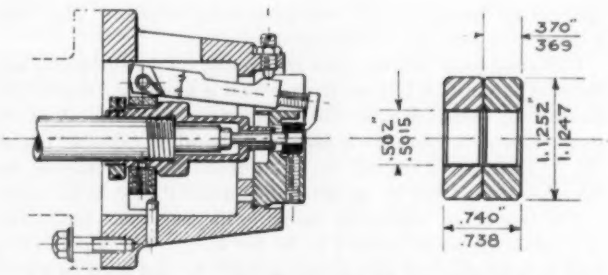

FIG. 13 CHUCK WHICH HOLDS THE WORK LATERALLY

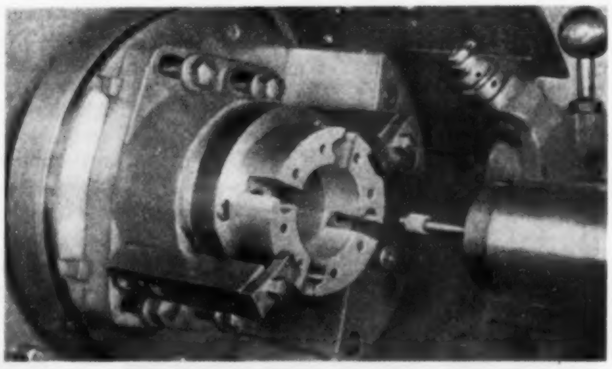

FIG. 14 GRINDING HOLES IN DIE HEAD

The following examples of typical work, chucks, and production times show what may be expected from modern internal grinders:

Parts such as valve-lifter rolls can be ground two at a time in a simple chuck which holds the work laterally (Fig. 13). Surrounding the gage is an ejector tube which pushes the finished work into a chute as the chuck opens. While the machine is grinding, the operator, having no other duties, slips two rough pieces on a hand plug, which makes it possible to load them into the chuck without loss of time. These hardened-steel rollers have 0.006 to 0.008 in. of stock to be removed, and a production of 200 an hour is maintained, although the total tolerance is 0.0005 in. on the diameter, 0.0005 in. for roundness, and 0.0001 in. for taper.

An example of a tool job where accuracy and finish are the important factors is shown in Fig. 14. The job, a die head, consists of grinding four holes which break out into slots for a part of their length. The diameters of the holes are held to a tolerance of 0.0002 in. They must be parallel to each other, and the chordal distances must have practically no error. This work is done on a plain or non-automatic machine.

Universal joint bearing pins are ground on an automatic machine at the rate of 170 per hour, which is more than double the production obtained with non-automatic equipment. A short-stroke reverse mechanism is used, and the wheel takes a "plunge cut." From 0.010 to 0.017 in. of hardened steel is removed, the tolerance being 0.001 in. on the diameter and 0.0005 in. for taper.

The use of an automatic machine increased the production of drill-chuck bodies from 6 to 60 per hour. The taper hole is held to a tolerance of plus or minus 0.001 in. and must be a good fit on a taper plug. These bodies come in several sizes, which are accommodated by simply changing adapter rings.

Cold heading dies were formerly lapped in 1 to 2 hours apiece, but are now ground in 10 to 35 min., depending upon the size. Grinding permits the use of much harder dies and gives a straighter and rounder hole with a finer finish and no bell mouth, which results in greatly increased life of the dies and improves the quality of the bolts. Chuck jaws are changed to accommodate different sizes. The holes are small in proportion to their length, and the dies are therefore ground half-way through, using as short a spindle as possible, and then turned end for end and the other half of the hole ground. The holes are held to a tolerance of 0.001 in., and the fact that it is possible to get them straight in this way shows the degree of accuracy obtained with this type of chuck.

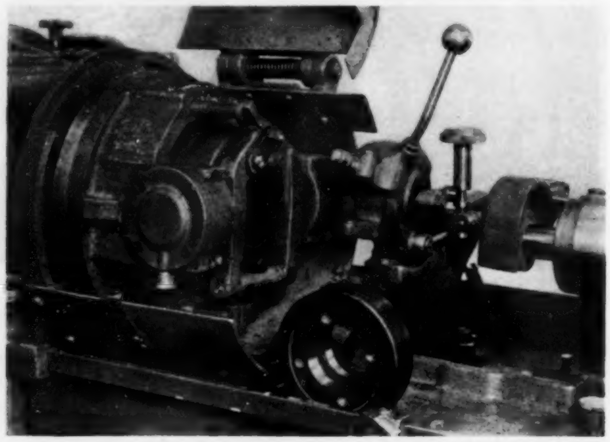

Fig. 15 Indexing Fixture for Grinding Both Ends of Steam-Shovel-Scoop Roll

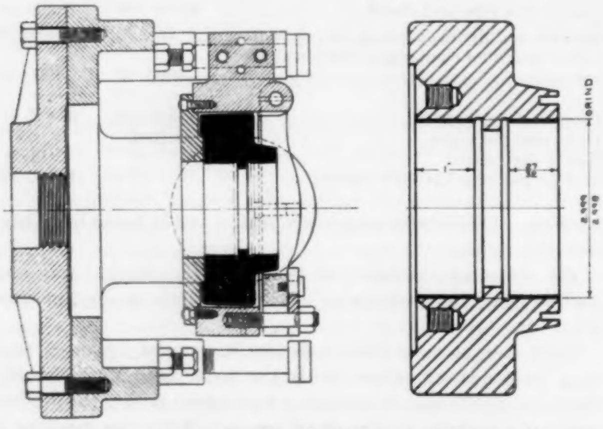

Fig. 16 Diagram of Set-Up and Work as Shown in Fig. 15

On hydraulic brake cylinders, an unusually good finish is required to prevent wear of the rubber piston. Such a finish is obtained with a production of 40 pieces per hour, although the work is long and 0.012 to 0.015 in. of stock is removed from the diameter by an unusually slim spindle, $^7/_8$ in. in diameter and 5 in. long. The tolerance for taper is 0.0005 in. The work is of cast iron and is ground dry, the dust being exhausted through the rear of the chuck. The work is loaded through the side of the chuck, conically centered and held by end pressure.

Figs. 15 and 16 show a steam-shovel-scoop roll. This job illustrates the growing use of hardened steel parts ground to close limits, even in rough machinery. The ground surfaces form seats for roller bearings and are held to a total tolerance of 0.001 in. The rolls must be ground from both sides without rechucking, to insure perfect alignment of the holes. This is accomplished with a trunnion fixture at the rate of 30 complete rolls per hour.

A large variety of sizes of shackle-bolt bushings can be handled by simply adjusting the jaws. From 0.008 to 0.012 in. on the diameter is removed from these hardened-steel parts. Holes 1 in. in diameter by $1^3/_4$ in. long are ground at an average of 165 an hour. Holes $1^1/_4$ in. in diameter by 2 in. long are finished almost as rapidly, with a tolerance of 0.001 in. on the diameter and 0.0003 in. for taper.

Spur and helical gears may be quickly and accurately chucked by means of pins held in a spring cage. The cage allows the pins to be slipped over the gear and to center themselves between the teeth. The gear, with pins and cage, is slipped between plain collet jaws which have been ground in place to a true circle, locating the gear accurately by the pitch line. In some cases a larger number of pins are used, or the pins are enlarged in the center, to minimize the effect of distortion of the teeth.

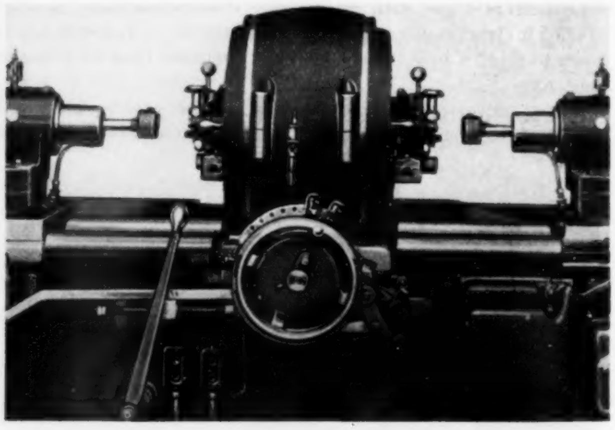

Fig. 17 Double-Ended Machine for Grinding Two Holes at Same Time

Double gears are held by means of two sets of jaws and two sets of pins. One set of pins is loosely held against the rear jaws while the other set of pins is held in a cage. The pressure on the two sets of jaws is equalized by a coil spring in the chuck. The production is 45 to 55 an hour, with a tolerance of 0.001 in.

The method of chucking cluster gears is similar to that for double gears, but a double-ended or duplex machine, Fig. 17, is used to grind the two holes at the same time and in perfect alignment. This machine operates on the Size-Matic principle and grinds gears of the size shown at the rate of 26 per hour. On some similar work the operator runs both a duplex internal and a rotary surface grinder, facing both ends of the gears while the duplex is grinding the holes. The practice of having an operator run two machines is becoming more common and is carried out quite extensively in some shops. In some cases where the operator runs only one machine he performs a secondary operation, such as burring or filing, while the machine is working.

Work of the nature of airplane cylinders, including all long tubes, is best held in a hollow workhead similar to that used on the duplex machine. Otherwise the work overhangs a long distance beyond the spindle, and the necessary rigidity cannot be obtained. The ball bearings used in these workheads are interesting because of their unusual size, the outside diameter being as large as $21^1/_2$ in. in some cases. Airplane cylinders are loaded from the rear of the workhead. Average production is four per hour, and in some cases is very much higher, although a high finish is required, the tolerances for out-of-roundness and taper are very close, and a large amount of metal is removed.

Still larger work, such as railway gears, roller-bearing races and cylinders, is handled in a heavy-duty machine which weighs 18,000 lb. and is driven by motors totaling 31 hp. These large machines are also made with hollow workheads for certain work.

Bevel gears are located from the pitch line by holding against an internal spur gear. On a bevel gear requiring a hole 0.755 in. in diameter and $^{13}/_{16}$ in. through the gear, the remarkable production of 254 pieces an hour is maintained, one piece every 14 sec.

Two automatic machines turn out the work to a tolerance of 0.001 in. as fast as an operator can load them, working between them.

The grinding of drive-shaft flanges and hubs is a good example of a chuck with adapters to accommodate a variety of work. Automatic equipment increased the production from 15 to 50 pieces per hour.

Grinding motor stators results in a uniform air gap, which is highly desirable, and makes it possible to allow comparatively wide tolerances in punching the laminations. The grinding is done dry, and the dust is exhausted through the back of the chuck. The production is 50 or 60 per hour.

For hardened-steel chain blocks, such as are used in coal-mining machinery, the work is centered by a taper plug which enters the rough bore and then recedes, leaving a uniform amount of stock to be removed from all sides of the hole. The work is reversed for grinding the second hole. Fully 50 complete blocks are ground per hour.

Universal-joint yokes are held between hollow cones having teeth which interlock with the splines. This method of holding practically eliminates the scrap which was formerly produced when the parts were held by the outside diameter, which is not always true with the splines. One operator running two machines produces 40 pieces per hour.

Cast-iron drive-shaft housings are typical of a large number of parts which are internally ground to form a good seat for ball and roller bearings. A short, rugged wheel spindle is used on this job, for grinding the hole and facing the shoulder. Facing is accomplished by allowing the table to come up to a cam stop, against which it is held by the hydraulic cylinder while the cam is turned by hand, feeding the wheel against the shoulder. One operator runs two machines, performing two operations on each, and produces 30 parts per hour. The holes are more than 3 in. in diameter by $1^1/_2$ in. deep.

The term "automatic" in connection with internal grinders has been so loosely used as to be confusing. Screw machines are commonly provided with devices for feeding work into the chuck and are then known as automatic machines. Milling machines and internal grinders, on the other hand, are not built with standard loading devices, and are called automatic when the operator has merely to load them and start the cycle of operations, the machine performing all other functions and finishing the work to the required size.

Internal grinders with hopper feeds have been developed and are in successful operation, but these are special machines.

It is obviously incorrect to call an internal grinder automatic unless it sizes the work automatically.

Performance of Cutting Fluids

Progress Report No. 3 of the Subcommittee on Cutting Fluids of the A.S.M.E. Special Research Committee on Cutting of Metals

By O. W. BOSTON,[1] ANN ARBOR, and C. J. OXFORD,[2] DETROIT, MICH.

This paper presents the results of a series of experiments to determine the influence of eleven different cutting fluids on the power required to plane and drill an annealed S.A.E. 3150 steel.

This investigation was conducted as a project of the Subcommittee on Cutting Fluids of the A.S.M.E. Special Research Committee on the Cutting of Metals, by the Department of Engineering Research of the University of Michigan, with funds provided by Engineering Foundation. The object was to secure definite performance data for certain limited conditions as a preliminary step to a comprehensive study of cutting fluids.

The force required to cut metal by planing for a single tool shape was determined for a variety of feeds and depths of cut. A formula for the planing force is developed as a function of the cutting fluid, the depth of cut, and the feed as $P = CD^{0.95}f^{0.79}$.

The torque, thrust, and power required to drill the S.A.E. 3150 steel are determined for a variety of drill diameters and feeds for each of the eleven cutting fluids. Formulas for each of these factors are developed as a function of the cutting fluid, the feed, and the drill diameter. Torque $T = Cf^{0.78}d^{1.8}$ thrust $B = Kf^{0.87}\left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$. The results show clearly the relative advantages from a power standpoint of the various oils studied, and the constants C and K provide for the influence of the cutting fluid used. All data are presented graphically.

An attempt is made to correlate various physical and chemical properties of the cutting fluids with the results of their cutting performance. It appears that cooling is the primary requisite of a cutting fluid rather than lubrication.

THIS paper is a report covering an investigation of cutting fluids. An attempt was made to determine the characteristics of each oil as determined by its influence on the cutting action of a tool. These characteristics are then correlated with the various properties of the oils.

Because of the limited funds provided for this investigation, only one material was cut. Eleven different cutting fluids, including dry cutting, were selected as representing the many types in commercial use. The cutting tests were confined to the determination of force or power values when drilling and planing.

O. W. Boston

C. J. Oxford

THE MATERIAL CUT

An S.A.E. 3150 steel was selected as the one most favorable for these tests. This high-carbon chrome-nickel steel was annealed to provide uniformity between various bars and between different locations in the same bar. About two tons of this material, 2 in. × 6 in. in section, were purchased, with 40 pieces each 14 in. long for drilling and 30 pieces each 24 in. long for planing. The material had a chemical analysis as follows:

Carbon	0.50	Phosphorus	0.022
Manganese	0.68	Chromium	0.62
Sulphur	0.023	Nickel	1.16

It was furnished in the annealed condition giving approximate physical properties as follows:

Elastic limit	66,140 lb. per sq. in.
Ultimate strength	114,200 lb. per sq. in.
Per cent elongation in 2 in.	26
Per cent reduction in area	53

The Brinell hardness was 170 on a smooth surface 1/2 in. beneath the skin.

THE CUTTING FLUIDS

The cutting fluids selected for these tests consisted of the following (they are listed with their properties in Table 1):

1 Dry cutting.

2 An aqueous solution consisting of distilled water to which was added 1 1/2 per cent borax by weight. Actual analysis of

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[1] Professor, College of Engineering, University of Michigan; Mem. A.S.M.E. Professor Boston was graduated from the University of Michigan Engineering College in 1913, received a master's degree in 1917, and the degree of mechanical engineer in 1926. After graduation he was engaged at the university as instructor in engineering mechanics and mechanical engineering for four years. In October, 1917, he was commissioned in the U. S. Navy and assigned to duty in the Bureau of Ordnance on design and manufacture of submarine mines used in the North Sea blockade. From 1919 to 1921 he was engaged in industrial engineering work for the Cleveland Tractor Co., in Cleveland, as assistant to the vice-president and works manager. In the fall of 1921, he returned to the University of Michigan, where he is now professor of shop practice and director of the department of engineering shop. He is author of many papers dealing with the subject of metal cutting and is serving as A.S.M.E. representative on the Tool Steel Committee of the American Society for Testing Materials.

[2] Chairman, Subcommittee on Cutting Fluids; Chief Engineer, National Twist Drill & Tool Co., Detroit, Mich.; Assoc-Mem. A.S.M.E. Mr. Oxford was born in Norway and came to the United States in 1909. After two years in the engineering department at the University of Michigan, he served as tool draftsman with the Packard Motor Car Co., Dodge Brothers, and Nordyke & Marmon until 1916, when he became designing engineer for the Wilt Twist Drill Company. The following year he became associated with the National Twist Drill & Tool Co.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 PROPERTIES OF CUTTING FLUIDS USED

No.	Composition	Base of mineral oil	Per cent compounding by analysis	Viscosity at deg. fahr. 100	Viscosity at deg. fahr. 130	Viscosity at deg. fahr. 210	A. P. I.[4] gravity at 60 deg. fahr.	*d* specific gravity	Temperature, deg. fahr. Pour	Temperature, deg. fahr. Flash	Temperature, deg. fahr. Fire	N. P. A. color[5]	*C* Specific heat[2] 100 deg. fahr.	*C* Specific heat[2] 210 deg. fahr.	*C* Specific heat[2] 430 deg. fahr.	*k* Thermal conductivity[3] 100 deg. fahr.	*k* Thermal conductivity[3] 210 deg. fahr.	*k* Thermal conductivity[3] 430 deg. fahr.
1	Dry cutting..........	......	....	...	...	..	..	....	......	.....	...	...	....	....	....	....	....	....
2	1½ per cent by weight of borax in water...	......	1.19 borax	...	...	..	..	....	......	.....	...	...	....	....	....	....	....	....
3	1 part soluble oil in 50 parts water.........	......	2.3 soluble oil	31	...	..	..	....	......	.....	...	...	....	....	....	....	....	....
4	1 part soluble oil in 10 parts water........	......	8.37 soluble oil	32	...	..	..	....	......	.....	...	...	....	....	....	....	....	....
5	No. 1 lard oil.........	......	12.23–13.6 F.F.A.[1] 86.4 fat	200	110	..	23.7	0.9117	45	445	...	4	0.4510	0.5149	0.6438	0.8738	0.8448	0.7852
6	Light mineral oil......	Mixed	....	111	69	40.5	23.5	0.9129	0 to −30	320–305	375	2–2¼, 2½–3	0.4500	0.5142	0.6430	0.8724	0.8440	0.7845
7	Heavy mineral oil.....	Mixed	....	209	104	45.5	23.0	0.9159	0 to −15	340–350	390	2¾–3, 3	0.4490	0.5137	0.6417	0.8695	0.8400	0.7818
8	10 per cent No. 1 lard oil in mineral oil....	Paraffin	10.12 lard oil	101	67	41	29.4	0.8794	25	355	...	3	0.4630	0.5290	0.6612	0.9052	0.8750	0.8145
9	5 per cent oleic acid in mineral oil.........	Paraffin	4.90 oleic acid	97	65	40.5	29.2	0.8805	25	295	...	3.5	0.4624	0.5284	0.6610	0.9048	0.8748	0.8138
10	Sulphurized mineral oil	Asphaltic	4.38 sulphur	105	67	40	21.6	0.9242	0 to −25	285	...	Black	0.4459	0.5096	0.6374	0.8620	0.8344	0.7752
11	Sulphurized lard oil blended with 5 parts mineral oil.........	Mixed	2.34 sulphur 4.43 F.F.A. sulphurized 13.94 fat sulphurized	102	67	41.5	25.4	0.9018	25 to 20	355	...	Black	0.4542	0.5190	0.6490	0.8837	0.8548	0.7942

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[1] F.F.A. is free fatty acids.

[2] Specific heat C computed from formula $C = (t + 670)(2.10 - d)/2030$, in which t = temperature of fluid deg. fahr., and d = specific gravity of oil at 60 deg. fahr.

[3] Thermal conductivity k computed from formula $k = (0.813/d)\,[1 - 0.0003(t - 32)]$, in which d = specific gravity of the oil at 60 deg. fahr., t = temperature of fluid in deg. fahr.; k is in units of B.t.u. per hr. per sq. ft. per in. per deg. fahr.

[4] American Petroleum Institute.

[5] National Petroleum Association.

a sample used in the drilling tests showed 1.19 per cent borax by weight.

3 A soluble oil consisting of one part oil to fifty parts distilled water. Actual analysis of a sample used in the drilling tests showed 2.3 per cent of soluble oil and a viscosity of 31 Saybolt seconds at 100 deg. fahr.

4 A soluble oil consisting of one part oil to ten parts water. Actual analysis of a sample used in the drilling tests showed 8.37 per cent of soluble oil and a viscosity of 32 Saybolt seconds at 100 deg. fahr.

5 A No. 1 lard oil commonly used for machine-shop work. This oil was pressed from inedible hog fat. An analysis of a sample from the drum of oil after being used intermittently for three weeks in the planer tests showed 12.23 per cent oleic acid and 24.3 milligrams of potassium hydroxide per gram of oil. A later analysis of a sample used for a longer period in the drilling tests showed 13.6 per cent free fatty acid and 86.4 per cent fat. The lard oil viscosity was 200 Saybolt seconds at 100 deg. fahr.

6 A light mineral oil having a final viscosity of 110 Saybolt seconds at 100 deg. fahr. This oil is a distillate from a naphthenic Texas crude oil and is refined sufficiently to render it reasonably stable. Subsequent analysis of a sample used in the drilling tests showed the oil to have some properties of both paraffin and asphaltic and so is listed as mixed in Table 1.

7 A mineral oil having a final viscosity of 210 Saybolt seconds at 100 deg. fahr. This mineral oil is the same type as No. 6.

8 A mineral-lard oil consisting of 90 per cent mineral oil and 10 per cent No. 1 lard oil, having a final viscosity of 100 Saybolt seconds at 100 deg. fahr. The mineral oil is a fully refined neutral or paraffin oil.

9 A 95 per cent mineral oil plus 5 per cent double-distilled oleic acid, having a final viscosity of 100 Saybolt seconds at 100 deg. fahr. The mineral oil is the same as that used in No. 8. Analysis of a sample used for several weeks in the drilling tests showed 4.90 per cent oleic acid.

10 A sulphurized straight mineral oil having a final viscosity of 100 Saybolt seconds at 100 deg. fahr. This was reported by the manufacturer to contain 4 per cent sulphur. Subsequent analysis of a sample used for several weeks in the drilling tests showed 4.38 per cent sulphur as given in Table 1. An asphaltic-base oil was used.

11 One part of sulphurized lard oil blended with five parts of mineral oil, having a final viscosity of 100 Saybolt seconds at 100 deg. fahr. The manufacturer reported that the oil contained 1.6 per cent sulphur. The subsequent analysis showed the sulphur content to be 2.34 per cent. This blend contains a considerably higher percentage of sulphurized lard oil than is customarily used as a cutting oil. It was felt desirable for these tests, however. Sulphurization greatly increases the viscosity of the lard oil. Rather than use a mineral oil of too low a flash to get the required final viscosity, a lower sulphur content of the lard oil was used.

Table 1 summarizes the properties of these oils. A drum of each oil was furnished by the manufacturer and reported to have the viscosities given above. Table 1 shows the viscosities as determined from later tests. They differ very little from the former. A drum of each of the cutting fluids 2, 3, and 4 were prepared, using distilled water.

Five gallons of each cutting fluid were taken from the drum for use in the drilling tests. The remainder was used in the planer tests. Every precaution was taken to avoid the mixing of one oil with another at the time of changing oils.

In Table 1 two values are given for power, flash, and color for some of the oils. The first values were given by the manufacturer when the oils were submitted, and the second values are the results of subsequent tests.

The specific heats C at temperatures of 100, 210, and 430 deg. fahr. were computed for oils 5 to 11, inclusive, using the generally accepted formula $C = (t + 670)\ (2.10 - d)/2030$, in which t is the temperature of the oil in deg. fahr. and d is its specific gravity at 60 deg. fahr. (Table 1). This formula has been proved to be reliable within the temperature range 50–430 deg. fahr., so long as the product is a liquid. The error is less than 2.5 per cent for a specific-gravity range of 0.75 to 1.0. The specific-heat values are tabulated in Table 1.

Again for the determination of thermal conductivity for the various oils 5 to 11, inclusive, the formula developed by the Bureau of Standards (Bureau of Standards Miscellaneous Publication No. 9) was used as follows: $k = (0.813/d)\ [1 - 0.0003\ (t - 32)]$, in which k is the thermal conductivity of the oil in B.t.u. per hour per square foot per inch per deg. fahr., d the specific gravity of the oil at 60 deg. fahr. (Table 1), and t the temperature of the oil in deg. fahr. The values of thermal conductivity for the various oils at 100, 210, and 430 deg. fahr. are given in Table 1.

A general discussion of the characteristics of the oils is given near the end of the paper. Temperature-viscosity curves are shown in Fig. 46. The scale used is such as to give straight lines for each of the temperature-viscosity curves. The circles indicate actual experimental determinations. Other characteristics of the cutting fluids tabulated in Table 1 are plotted in Figs. 48, 49, and 50.

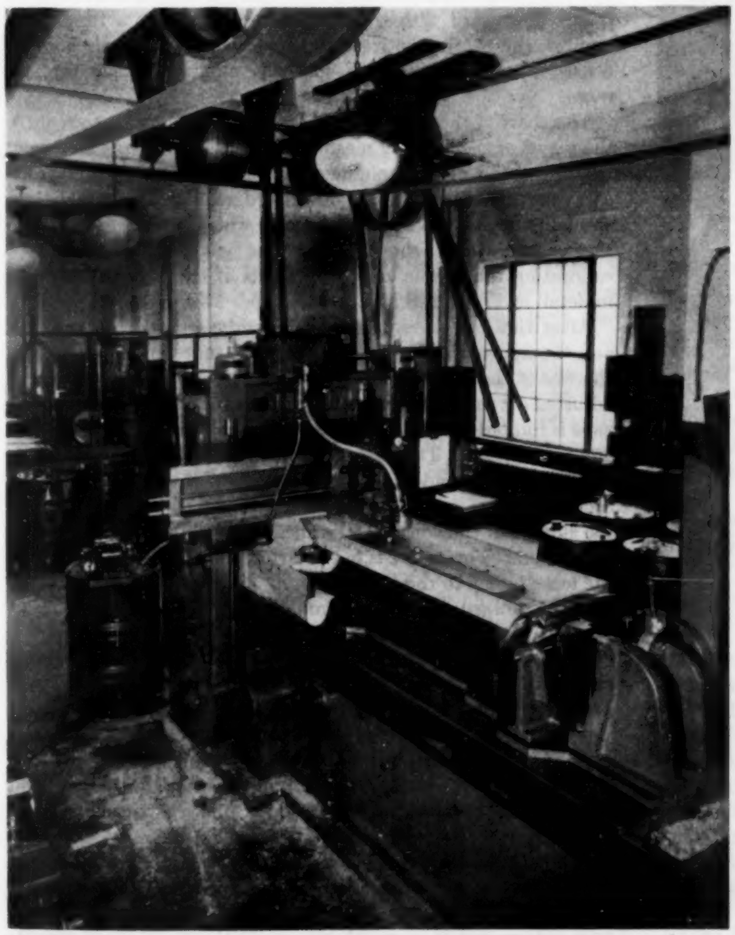


Fig. 1 Planer and Accessories Used in Tests

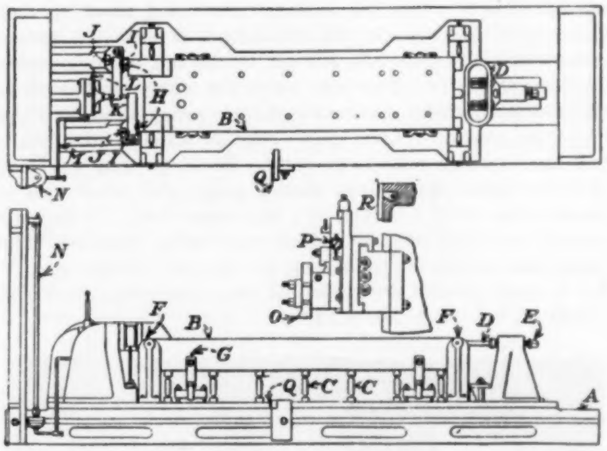


Fig. 2 Line Diagram of Planer Dynamometer

THE PLANER—ACCESSORIES AND TOOLS

Fig. 1 shows the planer and accessories used in determining the power to cut the S.A.E. 3150 steel under like cutting conditions for each of the eleven cutting fluids. On the bed of this 30-in. × 36-in. × 8-ft. planer was mounted a dynamometer, the line diagram of which is shown in Fig. 2. On the bed of the dynamometer table is mounted a large galvanized-iron pan in the bottom of which the test piece being cut is securely fastened. The various cutting fluids in drums are shown about the planer. The drum containing the coolant to be used is brought to the planer and placed as shown at the left. A portable motor-pump unit is placed on the drum so that, during the test, the oil is pumped to a reservoir shown on the top of the planer housing. This reservoir has three drain pipes, one, an overflow pipe leaving the reservoir on the rear side near the top, draining back to the drum. At all times liquid flows through this overflow pipe, insuring relatively constant head. The coolant flows from the reservoir through the flexible tube and is discharged at low velocity but in large quantities directly on the tool face. The coolant flows from the tank through a drain pipe at the left into a trough and thence to the drum. Various fine-mesh screens are placed in the returning pipes to make sure that no chips or foreign solid matter are returned to the drum. The third drain pipe from the reservoir emerges from the bottom of the reservoir and is used only for draining when the set-up is changed from one coolant to another. During this process all tanks and pipes are thoroughly cleansed with water and then wiped dry. Considering the large quantity of the cutting fluid being circulated, any smaller quantity of one oil remaining in the system, it is believed, is not detrimental to the next oil used.

Power was supplied to the planer from a 15-hp. motor driving through a reversing variable-speed transmission to the standard countershaft above the planer. The cutting speed was maintained at 20 ft. per min. in all tests, except where speeds were purposely varied for specific purposes.

Referring to Fig. 2 the dynamometer consists essentially of a cast-iron bed B, weighing about 1300 lb., mounted on seven 14-in.-long horizontal floating pins shown in the plan view supported by the brackets *F*. The bed is kept from being lifted by four knife edges shown at *G*. An initial load of 2000 lb., tending to force the bed to the left, was put on the bed by compressing the loop spring *D*. This load keeps the whole mechanism in compression, thereby eliminating chatter during a cut, and also keeps the mercury-column gage under an initial load which is indicated as a zero point on the scale. The loop spring was made of $^3/_4$-in. square spring steel having major and minor axes to center of bar of $11^1/_2$ in. and $4^3/_4$ in., respectively. It had previously been calibrated on a Universal testing machine. This spring also served for calibrating the mercury-column gage *N*. For a more detailed description of this equipment, see Trans. A.S.M.E., Vol. 48 (1926), p. 749.

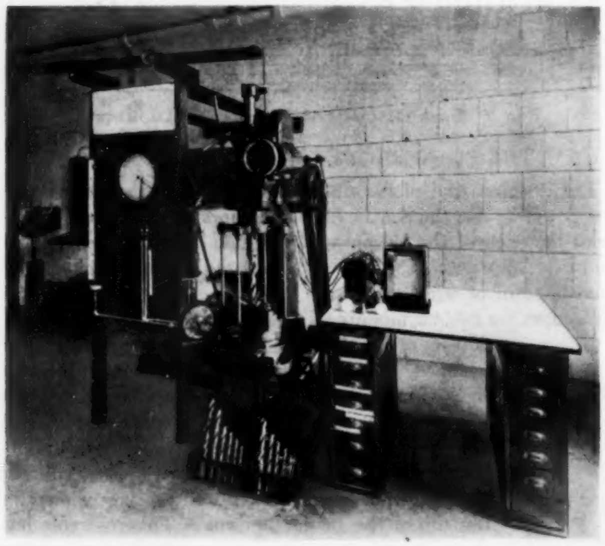


FIG. 3 DRILL PRESS AND EQUIPMENT USED IN TESTS

The Planer Cutting Tools. Tools were ground on each end of twenty $^3/_8$ in. × $^3/_4$ in. × 5 in. long high-speed-steel tool bits. The tool had a shape as follows: an 8-deg. front-rake angle; a 14-deg. side-rake angle; a 6-deg. front and side-clearance angle; a 6-deg. end-horizontal-clearance angle; and a 90-deg. plan-cutting angle with the work. The nose was ground to a $^1/_{32}$-in. radius. After being ground, all tools were honed smooth on all ground surfaces so that they would retain their geometric shape and condition throughout a series of tests. The tools were changed frequently, so that all test values are based on sharp tools. Inasmuch as only the power to cut the metal was determined as a function of the cutting fluid, no data were accumulated regarding tool-life values.

THE DRILL PRESS SET-UP AND DRILLS

The testing equipment for determining the influence of the cutting fluids in drilling is shown in Fig. 3. This equipment was devised so that torque and thrust as developed at the drill point could be determined, as well as the input power developed by the motor. The testing equipment, consisting of a drill press, a drill dynamometer with gages, and the recording wattmeter, is shown in Fig. 3. The drill press was a No. 2 manufacturing-type Colburn press, which was driven by a 5-hp., 220-volt, 3-phase, 60-cycle motor attached to the rear of the column. The motor drove the tight and loose pulley through a short belt. A gear pump was provided for raising the coolant from the tank at the side of the column to the drill. As in the case of planing, the cutting fluid was supplied to the drill in as large a quantity as possible. It was dumped at a low velocity upon the drill just above the work.

The drill dynamometer was of such a design that the torque and thrust could be measured independently. The pressure developed by the drill was exerted directly on a plate resting on a rubber diaphragm, just above the dynamometer base. Water underneath this rubber diaphragm was subjected to compression, due to the thrust of the drill, and the hydrostatic pressure was transmitted to one pen of the gage through the $^1/_8$-in. pipe. The plate resting on the rubber diaphragm which carried the thrust load was made to form the lower race of a large ball bearing consisting of sixty-four $^1/_2$-in. balls, so that the table of the dynamometer was free to rotate except for the arm extending to the left, which had a bearing against a piston resting on a second and smaller rubber diaphragm. This arm was 1 ft. long, measured from the center of the dynamometer, which latter corresponded with the center line of the drill spindle. The pressure on this small rubber diaphragm or torque diaphragm subjected the water back of the rubber to compression. This pressure was transmitted through a $^1/_8$-in. pipe, leading both to the mercury-column base and the second pen of the gage. The mercury column had a scale which had units larger (more sensitive) than those of the gage, but which did not have the range permitted by the gage. To prevent tilting of the dynamometer table on the thrust rubber diaphragm, a spider fitted around the spindle housing was attached to the table of the dynamometer by four vertical rods. To eliminate friction and to permit the spider to rotate freely with the table as the torque was applied or to be forced downward with the dynamometer table as a thrust was applied, the bearing between the spider and the spindle housing consisted of twenty-four $^1/_2$-in. balls mounted in appropriate races.

The chart of the recording gage was made to rotate, to register the travel of the drill, by means of a fine wire which passed from the upper end of the spindle over pulleys to a position vertically above the gage, thence downward, making one turn about a special pulley on the shaft of the gage which carried the chart to the counterweight shown below the gage. The method of calibrating the torque and thrust pens of the gage is described in detail in Trans. A.S.M.E., 1930, paper MSP-52-2.

After the drill press has been properly set up with the drill and the appropriate speed and feed, the material is adjusted in the vise of the dynamometer. The mercury-column gage is next adjusted for zero reading. At this point the chart of the pressure gage is rotated so as to draw the base or zero lines for the torque and thrust pens. This insures having the same torque zero reading in all tests, and the exact torque and thrust base lines for each individual test. The drill is brought to within $^3/_8$ in. of the surface of the work to be drilled, after which the feed clutch is engaged. The motor is next started and the belt shifted from the loose pulley to the tight pulley. The drill is thereby rotated and fed toward the work through the $^3/_8$-in. travel, without cutting. During this time the wattmeter is recording the power developed by the motor in driving the machine with all speed and feed gears engaged, but not cutting. This permits the wattmeter to record the tare power, and later, when the drill is cutting, to record the gross power. The tare deducted from the gross gives the net input power, which corresponds to the power output at the drill point except for the additional frictional load on the gears and bearings of the machine due to the cutting load.

Celluloid scales prepared from the calibration are used to measure the torque and thrust values from the chart. During the drilling, the actual rotating speed of the spindle is measured with a cutmeter in order that proper allowance may be made for belt slippage, reduced speed of the motor, etc.

The Drills Used

The drills used in these experiments were selected as being representative of the drills corresponding with standard specifications for taper-shank, high-speed-steel twist drills. They were selected in sizes $^1/_2$, $^3/_4$, 1, and 1 $^1/_4$ in. in diameter and correspond to the R drills described in the previously mentioned paper (MSP-52-2). The drills were ground in an SPK Blau drill grinder in such a manner that each could be ground and reground without changing the various drill-point angles. All drills had a point included angle of 121 deg., a helix angle of 30 deg., and a clearance angle near the periphery of about 5 deg. and adjacent to the chisel edge of about 13 deg. Other dimensions of the drills used are as shown in Table 2. This table also shows the feed in inches per revolution for each drill diameter in the commercial tests, as well as the actual cutting speed in r.p.m. and feet per minute.

TABLE 2 DRILLS USED IN TESTS, WITH MEASUREMENTS

Drill		Web thickness at point, in.	Helix angle, deg.	Point angle, deg.	Chisel-edge angle, deg.	Clearance at periphery, deg.	Commercial		
Diam., in.	No.						R.p.m.[1]	Ft. per min.	Feed, in. per rev.
$^1/_2$	R	0.0825	29	121	129	5–20	449	58.8	0.009
$^3/_4$	R	0.1055	31	121	134	5	300	59.0	0.012
1	R	0.147	31	121	136	5	231	60.5	0.013
1$^1/_4$	R	0.182	30	121	138	5	174	57.1	0.015
1	O.H.	0.1377	32	121		5	230	60.0	0.013
1	44	0.142	33	121		5	231	60.5	0.013

[1] This r.p.m. to give as nearly as possible 60 ft. per min.

Two additional 1-in.-diameter drills were used for special tests in which the speed, feed, and quantity of oil were varied separately. One of these 1-in.-diameter drills, indicated as O.H. in Table 2, was an oil-hole drill provided for the purpose of studying the influence of the quantity of oil supplied to the drill point as discussed below. The second 1-in.-diameter drill No. 44 was used in special tests as described below. The measurements of both drills are given in Table 2.

THE RESULTS OF THE DRILLING TESTS

Before running the commercial drilling tests to determine the torque, thrust, and power required to drill under different conditions with each of the oils, some preliminary tests were thought desirable.

Influence of Dulling During Use

The condition of the cutting edge and of the point of a drill naturally changes while cutting. In order to determine definitely just what this change was and its influence on the test results, some so-called dulling tests were run. Fig. 4 shows the results for determining the dulling effect on the torque for each of the R drills used when operated at their commercial feeds and speeds. The speed was as near as possible to 60 ft. per min. When cutting dry this speed was excessive. The chips were blued, and there was evidence of the generation of excessive heat.

Fig. 4 shows that the torque for the $^1/_2$-in.-diameter R drill operating at 0.009 in. feed and at 449 r.p.m. in the S.A.E. 3150 steel averages 14 ft-lb. for the first hole of 1$^1/_2$ in. depth; 14$^1/_2$ ft-lb. for the second 1$^1/_2$ in. of depth; and 15 ft-lb. at the end of the next 3 in. of depth, or at a total depth drilled of 6 in. Further drilling showed only a slight increase to a maximum value of 15$^1/_2$ ft-lb. for a total depth drilled of 36 in. It is seen that the maximum rate of increase of torque due to dulling is during the first 6 to 9 in. of metal drilled.

A similar dulling curve for the $^3/_4$-in.-diameter R drill operating at 0.012 in. feed and at 300 r.p.m. is shown. This shows that the torque for the newly sharpened drill is only 33 ft-lb. At the end of the fourth hole, the torque is 47 ft-lb., after which it remains practically constant. Similar curves are shown for the 1-in.-diameter drill operating at 0.013 in. feed and at 231 r.p.m., and the 1$^1/_4$-in.-diameter drill operating at 0.015 in. feed and 175 r.p.m. It is seen that the smaller diameter drills become dull more gradually than the larger diameter drills. At a total depth of 6 in. drilled, the keen cutting edge of the newly sharpened drill is apparently removed and fairly uniform conditions maintain beyond this depth. The per cent increase in torque from that of the first 1$^1/_2$-in. depth to that at the end of the 6-in. depth is approximately 8, 11, 6 $^1/_2$, and 8 for the $^1/_2$-, $^3/_4$-, 1-, and 1$^1/_4$-in.-diameter drills, respectively. Undoubtedly the smaller rate of feed accounts for the slower dulling of the smaller drills.

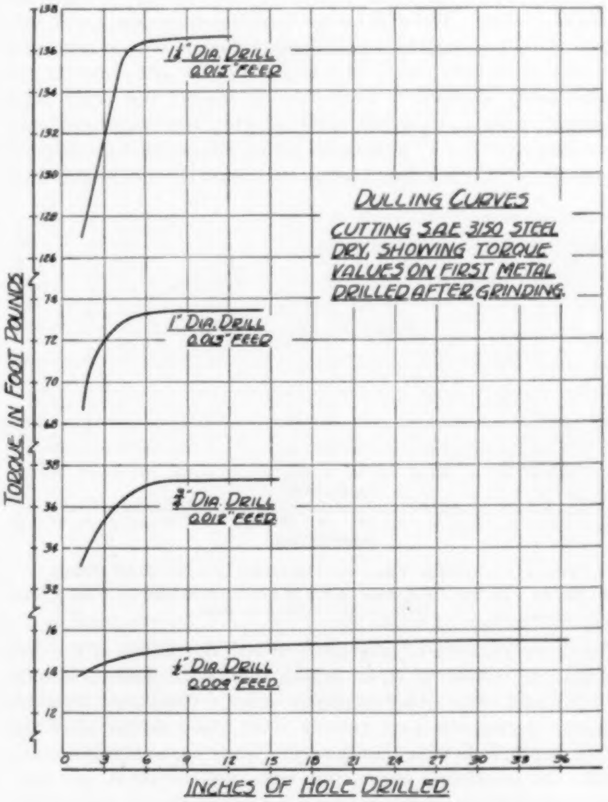


Fig. 4 Dulling Curves

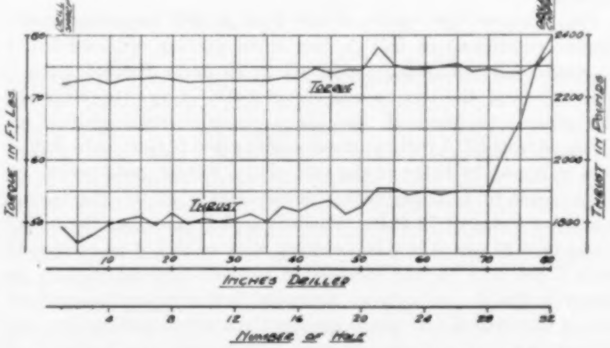


Fig. 5 Dry Dulling Tests in Drilling S.A.E. 3150 Steel
(1-in. drill, No. 44; feed, 0.013 in.; 231 r.p.m. Each hole 2$^1/_2$ in. deep.)

A similar test was run with the 1-in.-diameter drill No. 44

with a feed of 0.013 in. per revolution and a speed of 231 r.p.m. in drilling dry the annealed S.A.E. 3150 steel. Each hole was drilled to a depth of $2^1/_2$ in. The values of the drill torque and thrust are shown in Fig. 5. The value of the torque for the first hole $2^1/_2$ in. deep is 72.4 ft-lb., which agrees fairly well with the value of 72 ft-lb. for the 1-in.-diameter drill of Fig. 4 at a depth of 3 in. The drill torque increases very slowly with additional drilling, until at a total depth drilled of 55 in. it measures a value of 75 ft-lb., or a total increase of 3.3 per cent. It remains fairly constant or even falls off beyond this point until the drill chipped at a depth of 80 in., when the torque suddenly increased to 80 ft-lb. This again would indicate that the original keen edge, due to grinding, is removed during the first hole drilled.

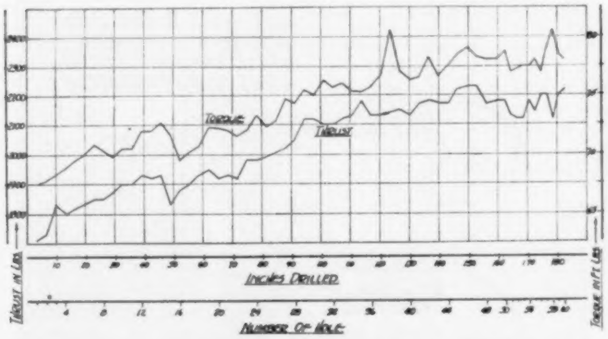


FIG. 6 DULLING TEST IN DRILLING S.A.E. 3150 STEEL
(Oil No. 4; 1-in. drill "R;" feed, 0.013 in.; 231 r.p.m. Holes 1–50, $3^1/_4$ in. deep; holes 51–60, 2 in. deep.)

The thrust values gradually increase with the dulling of the drill so that, at the end of 70 in. drilled, there is an increase of only 6.3 per cent, or a rather negligible value. The thrust increases rapidly during the next two or three holes drilled after the 70-in. depth, indicating that the drill already has suddenly become dull. As mentioned above, this high speed of 60 ft. per min. when cutting dry is excessive, and it is interesting to see that the dulling does not influence the torque and thrust to a greater extent.

Fig. 6 shows the results of the 1-in. R drill operating under similar conditions of Fig. 5, but when cutting with oil No. 4 (a soluble oil containing 1 part oil to 10 parts distilled water). The values of the torque and thrust both are erratic, although the general tendency of the torque curve shows a gradual increase of from $67^1/_2$ ft-lb. at the end of the first $3^1/_4$-in. hole drilled to a value of 79 ft-lb. at the end of the 180-in. penetration, or an increase of 14.6 per cent. After drilling 75 in., the torque reaches a value of 72 ft-lb., which is an increase over the initial value of 6.25 per cent when cutting with oil No. 4 as compared with 2 per cent for the increase in torque when cutting dry as shown in Fig. 5. It is shown, however, that when cutting with oil No. 4 the drill life is much greater than when cutting dry, although the torque increases more rapidly and to a higher final value. At the end of the 180 in. drilled in Fig. 6, the drill still was in a satisfactory condition for further drilling. The thrust curve in Fig. 6 practically parallels the torque curve, which seems to agree fairly well with the relation shown in Fig. 5.

As a result of these dulling tests, after grinding each drill, two or three holes were drilled before using the drill for test purposes, in order to remove the original keenness of the ground edge. In some subsequent tests where it was desirable to have the drill retain its condition during several tests, the cutting edge was honed. It was felt that this honing produced a condition of the cutting edge which remained more nearly constant for a considerable length of time.

TESTS TO CONFIRM TORQUE AND THRUST FORMULAS

An equation for torque as a function of variable feed and drill diameter was reported in a paper, "Power Required to Drill Cast Iron and Steel," Trans. A.S.M.E., 1930, MSP-52-2, and reported on further in a paper before the Society of Automotive Engineers at its annual meeting in Detroit on January 20 (*Journal S.A.E.*, Vol. 28, No. 3, p. 378). These equations were determined from tests on a wide variety of steels by using a soluble cutting oil one part oil to 16 parts water.

It was felt desirable to confirm these equations for torque and thrust, so some tests were run on the annealed S.A.E. 3150 steel, using a number of the cutting fluids under consideration. The results of the tests using oil No. 3 (a soluble oil consisting of one part oil to 50 parts water) are shown in Fig. 7. The variable-feed tests were run with the 1-in.-diameter R drill, whereas the variable-diameter tests were run with a constant feed of 0.009 in. per revolution. The torque values at the 0.009-in. feed for various drill diameters are shown to be practically a straight line, the slope of which is 1.8. This value represents the tangent of the angle included between the horizontal and the line. The torque values for various feeds for the 1-in.-diameter drill also indicate a rather satisfactory straight line, the slope of which is 0.78. These values then would indicate that the equation for torque is

$$T = Cf^{0.78}d^{1.8}$$

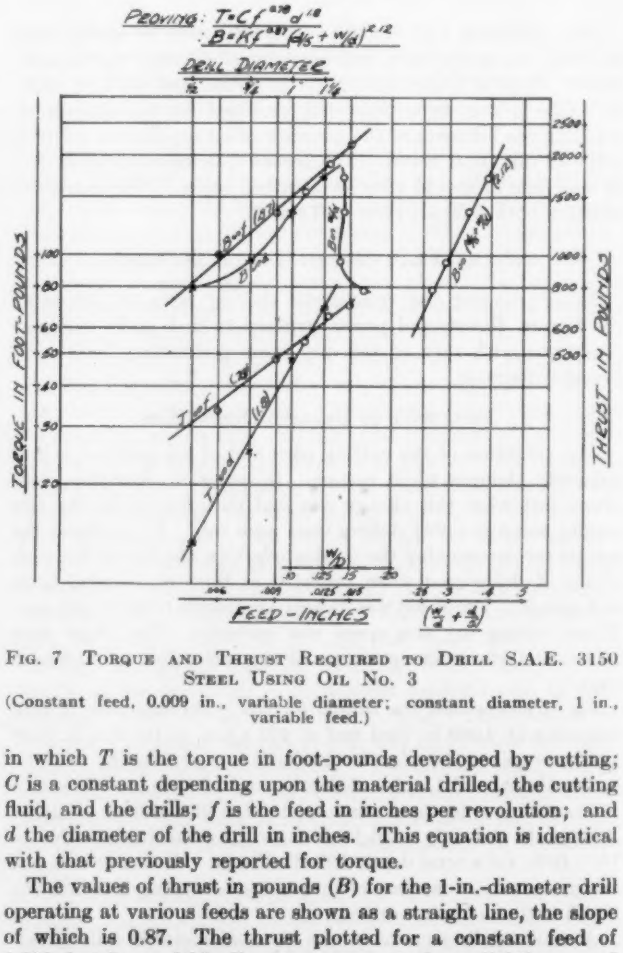


FIG. 7 TORQUE AND THRUST REQUIRED TO DRILL S.A.E. 3150 STEEL USING OIL NO. 3
(Constant feed, 0.009 in., variable diameter; constant diameter, 1 in., variable feed.)

in which T is the torque in foot-pounds developed by cutting; C is a constant depending upon the material drilled, the cutting fluid, and the drills; f is the feed in inches per revolution; and d the diameter of the drill in inches. This equation is identical with that previously reported for torque.

The values of thrust in pounds (B) for the 1-in.-diameter drill operating at various feeds are shown as a straight line, the slope of which is 0.87. The thrust plotted for a constant feed of 0.009, but for variable diameter appears to be a slightly curved line. For the drills when new, this line was more nearly straight.

After being ground several times the web thickness increased, causing the thrust to increase for each drill size, but in proportion to the increase in web thickness. Another almost vertical curve is shown representing the thrust for various values of w/d, where w is the web thickness at the point. Further to the right a straight line is obtained indicating the thrust B as a function of $[(d/5) + (w/d)]$. The slope of this line is 2.12. This indicates then that an equation for thrust is a function of the feed ($f^{0.87}$) and the value of $[(d/5) + (w/d)]^{2.12}$, resulting in the equation

$$B = K f^{0.87} \left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$$

The equation previously reported for thrust was more simple, as

$$B = K f^{0.78}\, d$$

and gives values considered fairly satisfactory for commercial use. In view of the nature of the present investigation, the first formula, which is more complicated, is being used in this paper, inasmuch as it is considered more accurate.

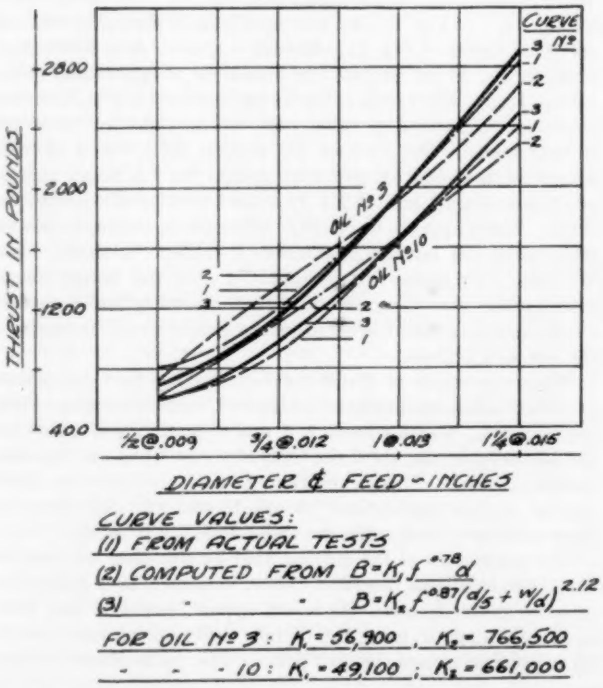


Fig. 8 Comparison of Actual and Computed Values of Thrust of Commercial Tests

A set of data similar to those for oil No. 3 shown in Fig. 7 was also obtained for oils Nos. 5 (lard oil), 6 (light mineral oil), 8 (10 per cent lard in mineral oil), and 10 (4 per cent sulphurized mineral oil). Values quite similar to those represented by the confirmed formula were obtained for oils Nos. 6, 8, and 10. Oil No. 5 (lard oil) gave an exponent of d in the torque formula of 2.3 instead of 1.8. It also gave an exponent of f of 0.92, which was the highest of all. Taking an average of the results of the tests for variable feed and variable diameter for the five oils, the following equations would result:

$$T = Cf^{0.84}d^{1.9}$$

$$B = K f^{0.84} \left(\frac{d}{5} + \frac{w}{d}\right)^{2.56}$$

It is felt that additional information of this nature, as well as confirming information, should be obtained. The limited funds available do not permit this detailed investigation, so that in the remainder of this paper the following formulas will be used:

$$T = Cf^{0.78}d^{1.8}$$

$$B = K f^{0.87} \left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$$

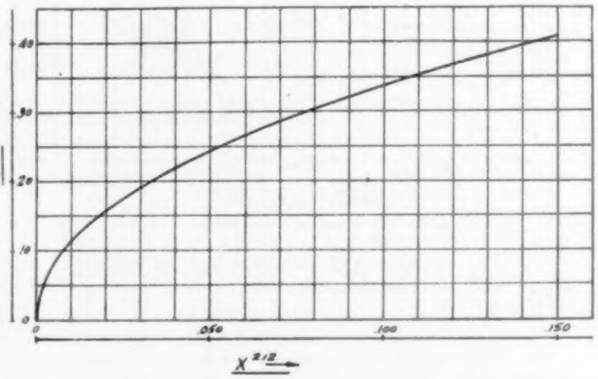


Fig. 9 Curves Showing Values of $X^{2.12}$ Which Are to Be Used in the Equation: $B = Kf^{0.87}(d/5 + w/d)^{2.12}$, Where $X = (d/5 + w/d)$

Fig. 8 has been prepared to show graphically the values of thrust in drilling the S.A.E. 3150 steel under the commercial test conditions, that is, when each drill diameter was operated at its respective commercial feed. The actual test values are plotted as curve 1 for oils Nos. 3 and 10. Curve 2 represents the values of thrust computed from the formula

$$B = K_1 f^{0.78} d,$$

while curve 3 represents the thrust values computed from the equation

$$B = K_2 f^{0.87} \left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$$

It is seen that for each oil the thrust values from the actual tests for the 1-in.-diameter drill, at 0.013 in. feed, correspond very closely with the values computed from the equations for curves 2 and 3. The same is generally true for the ½-in. drill at 0.009 in. feed. The greatest discrepancy occurs for the ¾-in. drill operating at 0.012 in. feed, for which the values of thrust computed by the equation for curve 3 agree quite satisfactorily with the actual test value, but the values computed from the equation for curve 2 are high.

In order that computations for torque and thrust can be made easily, values of the various feeds, diameters, etc. to required exponents are given in Table 5. Various values of $[(d/5) + (w/d)]^{2.12}$ may be determined from Fig. 9, in which X represents the value $[(d/5) + (w/d)]$ plotted as ordinates, and the values of $X^{2.12}$ are plotted as abscissas.

Influence of Quantity of Cutting Fluid

The commercial drilling tests as described below were all run with the coolant or cutting fluid dumped on the drill just above the work. A large volume of oil was supplied at a low velocity. In order, however, to obtain some idea as to the influence of other methods of applying the oil, a series of tests were run in which the oil was applied in various quantities and by different methods.

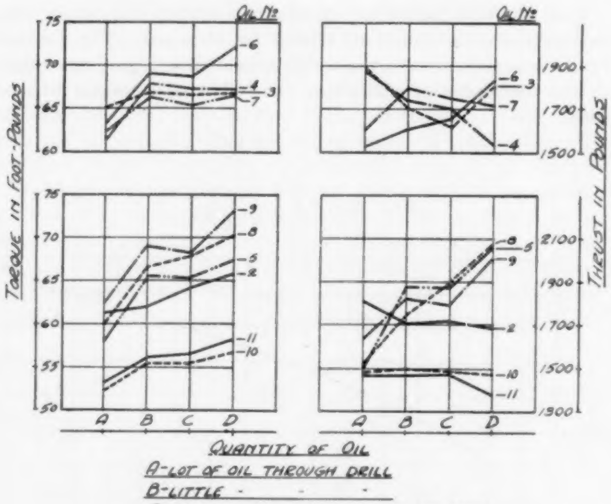


FIG. 10 INFLUENCE OF QUANTITY OF CUTTING FLUID ON TORQUE AND THRUST IN DRILLING S.A.E. 3150 STEEL
(1-in. oil-hole drill at 230 r.p.m. and 0.013 in. feed.)

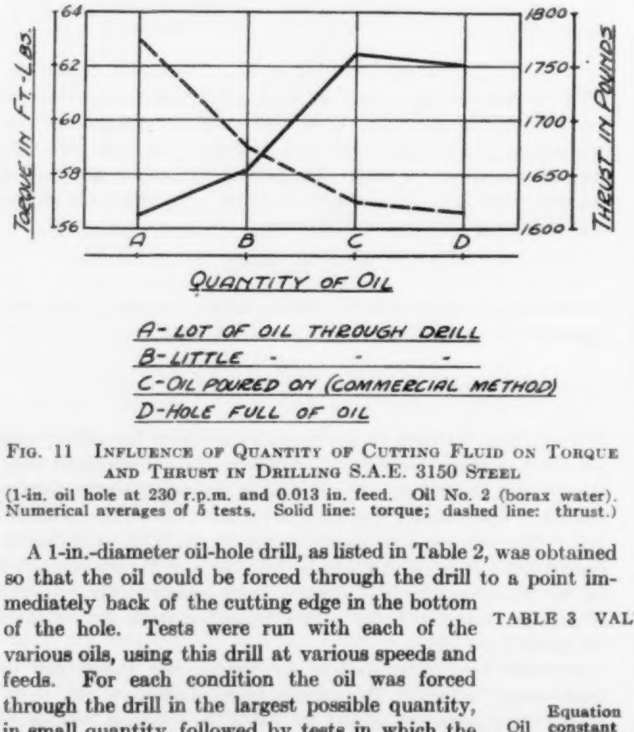


FIG. 11 INFLUENCE OF QUANTITY OF CUTTING FLUID ON TORQUE AND THRUST IN DRILLING S.A.E. 3150 STEEL
(1-in. oil hole at 230 r.p.m. and 0.013 in. feed. Oil No. 2 (borax water). Numerical averages of 5 tests. Solid line: torque; dashed line: thrust.)

A 1-in.-diameter oil-hole drill, as listed in Table 2, was obtained so that the oil could be forced through the drill to a point immediately back of the cutting edge in the bottom of the hole. Tests were run with each of the various oils, using this drill at various speeds and feeds. For each condition the oil was forced through the drill in the largest possible quantity, in small quantity, followed by tests in which the oil was dumped on the work above the drill, and finally by discontinuing the addition of oil, allowing the drill to continue cutting only with the hole full of oil. In this last condition, drilling could not be continued for long because the oil was carried out of the hole with the chips, and it steamed away rapidly because of the intense heat developed.

The average values of torque and thrust for five tests under each condition of oil application are shown plotted in Fig. 11. Cutting fluid No. 2, consisting of $1^1/_2$ per cent by weight of borax in water, was used with the 1-in.-diameter oil-hole drill operating at 230 r.p.m., and at a feed of 0.013 in. per revolution. S.A.E. 3150 steel was being cut. The results show that the lowest torque is developed when the oil is supplied in the greatest quantity at the drill point. The value of the torque for this condition is 56.5 ft-lb. The torque is increased to 58.2 ft-lb. when only a little of the cutting fluid is forced through the drill. The average torque increases to 62.5 when the oil is dumped on, and drops off slightly to 62 ft-lb. when no oil is added but when the hole is full when the cut starts. The thrust, on the other hand, starts at a maximum of 1775 lb. when the cutting fluid is applied in greatest quantity, drops to 1675 lb. when only a little is forced through the drill, and drops still further to 1625 lb. when dumped on. Only a slight further decrease is noted for the thrust when drilling with the hole full and no additional cutting fluid added.

The results of torque and thrust for a single test under each condition of cutting-fluid application, using each of the cutting fluids, are shown in Fig. 10. The curve of torque for cutting fluid No. 2 is quite similar to, but slightly higher than, the curve for torque shown in Fig. 11. That the curve for torque of Fig. 10 is higher than that of Fig. 11 is probably due to the fact that the drill was duller for the values given in Fig. 10. The thrust curve for oil, No. 2 of Fig. 10, also corresponds quite favorably with the curve for thrust of Fig. 11, although a greater drop in thrust is noted in Fig. 11 for the last two conditions of application of the cutting fluid. The values of the torque as shown in Fig. 10 show a tendency to increase in value from left to right for all of the cutting fluids. The increase for cutting fluid No. 3 (1-to-50 soluble oil) is least of all, with cutting fluid No. 7 (a heavy mineral oil) a close second, and oil No. 4 (a one-to-ten soluble oil) a close third. There appears to be little difference in torque values for these three oils for the conditions of "little," "dumped," and "in hole." It seems rather surprising that the lowest torque should occur when the greatest amount of cutting fluid is applied. Under this condition the temperature is more nearly normal and the viscosity highest.

When the values of thrust for each cutting fluid are plotted over the various conditions of application, rather confusing results are obtained. Cutting fluids 2, 3, and 4, consisting of water and the two soluble oils, show the highest thrust when an abundant amount is used. There appears to be little difference in thrust for the various applications for oils 10 and 11. All other oils show a tendency to increase the thrust from left to right.

The application of the cutting fluid by dumping was used in the commercial test described below, inasmuch as it appears to be more generally used. It would appear, however, that there is an advantage in using the oil-hole drill and forcing as much oil as possible to cool the drill point. The values shown in Figs.

TABLE 3 VALUES OF TORQUE REQUIRED TO DRILL S.A.E. 3150 STEEL, USING VARIOUS CUTTING OILS

(Arranged according to increasing value of factor.)

			Drill diameters and feeds							
			$^1/_2$ in. diam. at 0.009 in.		$^3/_4$ in. diam. at 0.012 in.		1 in. diam. at 0.013 in.		$1^1/_4$ in. diam. at 0.015 in.	
Oil No.	Equation constant C	Factor	Tests[1]	Computed[2]	Tests[1]	Computed[2]	Tests[1]	Computed[2]	Tests[1]	Computed[2]
10	1665	0.765	12.17	12.16	30.17	31.40	57.60	56.50	94.58	94.2
11	1707	0.784	12.47	12.45	31.15	33.00	62.32	60.1	101.05	99.5
2	1944	0.894	14.10	14.20	32.90	36.5	67.02	64.0	106.60	105.5
3	1958	0.899	14.32	14.30	34.43	37.4	69.10	66.3	110.05	110.0
4	1992	0.915	14.45	14.55	34.60	38.0	68.03	67.4	112.73	110.0
7	2027	0.932	15.27	14.80	35.23	38.4	68.40	68.6	114.07	113.5
5	2068	0.951	15.37	15.10	35.70	39.2	68.63	70.0	115.50	115.0
6	2089	0 960	15.50	15.25	35.83	39.5	69.47	70.4	116.57	116.4
8	2138	0.982	15.63	15.60	37.08	40.4	71.43	72.0	119.40	118.0
1	2178	1.000	15.98[3]	15.90	38.27[4]	41.35	74.47[5]	74.9	122.70[6]	122.0
9	2210	1.015	16.13	16.40	37.72	42.60	75.96	76.0	125.14	125.0

[1] Numerical average of test values.
[2] Computed from equation $T = C f^{0.78} d^{1.8}$ and corresponding to curve values in Fig. 12.
[3] Average of 37 tests.
[4] Average of 22 tests.
[5] Average of 15 tests.
[6] Average of 18 tests.
All other test values average of 6 tests.

10 and 11 are the results of only a short investigation and are not intended as final. Further work along this line, employing higher pressures, might prove advantageous.

THE COMMERCIAL DRILLING TESTS

The commercial drilling tests were carried out with each of the cutting fluids as follows:

A 1/2-in.-diameter drill rotating at 449 r.p.m. with a feed of 0.009 in.

A 3/4-in.-diameter drill rotating at 300 r.p.m. with a feed of 0.012 in.

A 1-in.-diameter drill rotating at 231 r.p.m. with a feed of 0.013 in.

A 1 1/4-in.-diameter drill rotating at 174 r.p.m. with a feed of 0.015 in.

TORQUE VALUES

After running a number of tests with all drills in each oil but in different orders, the values for torque were averaged numerically as shown in Table 3. Similarly, numerical averages of thrust are shown in Table 5. Gross power input and net power input are shown in Tables 6 and 7, respectively.

The numerical averages for torque were plotted on log-log paper as shown in Fig. 12. The ordinates equal the average torque in foot-pounds from the drilling experiments, while the abscissas equal $L = f^{0.78} d^{1.8}$ as taken from the equation developed for torque as a function of feed and drill diameter, namely, $T = Cf^{0.78} d^{1.8}$. Straight lines were obtained by connecting the points for a given cutting fluid as shown. All lines seem to be nearly parallel. That for cutting fluid No. 11 (sulphurized lard oil) has a slightly greater slope than the others, although it does not seem to diverge sufficiently from the others to warrant a separate equation.

The values of the torque required for drilling the S.A.E. 3150 steel, using the various cutting fluids as indicated by the straight-line log-log curves, Fig. 12, are tabulated for each drill diameter and feed in Table 3 in the column headed "computed." They represent the values computed from the formula $T = Cf^{0.78} d^{1.8}$, and correspond with values taken from the torque lines on the log-log scale given in Fig. 12. These values differ but slightly from the numerical averages given in the same table. This table affords a favorable opportunity to compare the numerical averages as determined from the test with the computed or smoothed-out values as read from the curves.

The highest line for torque given in Fig. 12 is for oil No. 9, a 5 per cent oleic acid and 95 per cent mineral oil, and the lowest is for oil No. 10, which is a 4 per cent sulphurized mineral oil. This indicates that oil No. 9 causes the highest torque when drilling and oil No. 10 the lowest. The other oils cause torque values in between as indicated.

Substituting a value of the torque for a given feed and diameter for cutting fluid No. 1, the constant C is determined as 2178, as shown in Table 3. Similarly, substituting values of torque for the other cutting fluids, constants for the torque equation for each oil are computed as listed in the second column of Table 3. With cutting fluid No. 1 taken as unity, factors indicating the ratio of the constant for each oil to the constant for oil No. 1 are computed and tabulated in the third column of Table 3. The oils are arranged so that the one having the lowest constant, and consequently the lowest factor, is listed at the top. The other

TABLE 4 VALUES OF $f^{0.78}$, $f^{0.87}$, AND $d^{1.8}$ FOR DRILLING TORQUE AND THRUST FORMULAS

Feed in inches	$f^{0.78}$	$f^{0.87}$	Diameter, inches	$d^{1.8}$
0.006	0.0185	0.0116	3/8	0.1710
0.007	0.0209	0.0133	1/2	0.2875
0.008	0.0231	0.0149	5/8	0.4290
0.009	0.0254	0.0167	3/4	0.5960
0.010	0.0276	0.0182	7/8	0.7860
0.011	0.0297	0.0198	1	1.0000
0.012	0.0318	0.0214	1 1/8	1.2362
0.013	0.0339	0.0229	1 1/4	1.4940
0.014	0.0358	0.0244	1 3/8	1.7740
0.015	0.0378	0.0260	1 1/2	2.0740
0.016	0.0398	0.0274	...	....
0.017	0.0417	0.0288	...	....
0.018	0.0436	0.0304	...	....
0.019	0.0454	0.0319	...	....
0.020	0.0473	0.0332	...	....
0.021	0.0491	0.0348	...	....

TABLE 5 VALUES OF THRUST REQUIRED TO DRILL S.A.E. 3150 STEEL, USING VARIOUS CUTTING OILS

(Arranged according to increasing value of factor.)

			Drill diameters and feeds							
			1/2 in. diam. at 0.009 in.		3/4 in. diam. at 0.012 in.		1 in. diam. at 0.013 in.		1 1/4 in. diam. at 0.015 in.	
Oil No.	Equation constant K	Factor	Tests[1]	Computed[2]	Tests[1]	Computed[2]	Tests[1]	Computed[2]	Tests[1]	Computed[2]
10	661000	0.7400	633	615	917	992	1641	1665	2381	2500
11	668500	0.7485	634	620	955	1002	1662	1685	2578	2520
2	750000	0.8395	725	692	1178	1125	1796	1880	2629	2840
4	760000	0.8505	716	702	1170	1140	1927	1920	2891	2890
3	766500	0.8580	817	708	1200	1150	1969	1928	2850	2900
6	784000	0.8770	764	725	1062	1175	1990	1970	3067	2950
7	793800	0.8885	765	735	1198	1190	1828	2000	3004	3010
5	849000	0.9500	799	786	1297	1272	2133	2152	3165	3210
8	880000	0.9850	807	810	1362	1320	2130	2210	3364	3320
1	894000	1.000	797	828	1374	1340	2000	2250	3427	3400
9	912000	1.021	840	842	1310	1368	2175	2285	3550	3435

[1] Numerical average of test values.

[2] Computed from equation $B = K f^{0.87} \left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$ and corresponding to curve values in Fig. 15.

TABLE 6 GROSS POWER INPUT REQUIRED TO DRILL S.A.E. 3150 STEEL, USING VARIOUS CUTTING FLUIDS

	1/2-in. drill at 0.009 in. feed			3/4-in. drill at 0.012 in. feed			1-in. drill at 0.013 in. feed			1 1/4-in. drill at 0.015 in. feed		
	Numerical average		From curve	Numerical average		From curve	Numerical average		From curve	Numerical average		From curve
Oil No.	Kw.	Hp.	Hp.	Kw.	Hp.	Hp.	Kw.	Hp.	Hp.	Kw.	Hp.	Hp.
1	1.914[1]	2.564	2.55	2.528[2]	3.391	3.39	3.432[3]	4.559	4.55	4.260[4]	5.707	5.74
2	1.783	2.387	2.39	2.350	3.147	3.13	3.133	4.191	4.18	3.840	5.147	5.15
3	1.819	2.436	2.43	2.350	3.147	3.15	3.215	4.307	4.26	3.971	5.320	5.36
4	1.858	2.488	2.48	2.403	3.221	3.25	3.270	4.380	4.37	4.090	5.485	5.48
5	1.870	2.506	2.51	2.468	3.306	3.31	3.300	4.424	4.41	4.088	5.483	5.50
6	1.850	2.479	2.48	2.427	3.253	3.25	3.265	4.375	4.37	4.265	5.715	5.48
7	1.978	2.651	2.53	2.523	3.381	3.37	3.415	4.575	4.49	4.181	5.597	5.63
8	1.910	2.559	2.56	2.548	3.414	3.42	3.420	4.582	4.59	4.418	5.915	5.91
9	2.068	2.770	2.76	2.712	3.636	3.67	3.685	4.943	4.90	4.583	6.141	6.14
10	1.500	2.091	2.01	2.061	2.762	2.76	2.783	3.730	3.64	3.413	4.567	4.57
11	1.513	2.176	2.17	2.220	2.976	2.99	2.965	3.972	3.93	3.640	4.873	4.91

[1] Average of 37 tests. [2] Average of 18 tests. [3] Average of 13 tests. [4] Average of 16 tests. All others are an average of 6 tests.

TABLE 7 NET POWER (KW. AND HP. INPUT) REQUIRED TO DRILL S.A.E. 3150 STEEL, USING VARIOUS CUTTING FLUIDS

	1/2-in. drill at 0.009 in. feed			3/4-in. drill at 0.012 in. feed			1-in. drill at 0.013 in. feed			1 1/4-in. drill at 0.015 in. feed		
	Numerical average		From curve	Numerical average		From curve	Numerical average		From curve	Numerical average		From curve
Oil No.	Kw.	Hp.	Hp.	Kw.	Hp.	Hp.	Kw.	Hp.	Hp.	Kw.	Hp.	Hp.
1	1.245[1]	1.670	1.625	2.035[2]	2.727	2.755	2.940[3]	3.943	3.947	4.020[4]	5.391	5.280
2	1.035	1.388	1.412	1.730	2.321	2.398	2.660	3.565	3.417	3.410	4.572	4.600
3	1.075	1.441	1.462	1.785	2.394	2.475	2.730	3.611	3.560	3.570	4.785	4.770
4	1.090	1.461	1.480	1.765	2.318	2.510	2.750	3.687	3.615	3.670	4.920	4.837
5	1.145	1.536	1.526	1.915	2.567	2.590	2.825	3.788	3.725	3.690	4.948	4.980
6	1.195	1.602	1.578	1.885	2.527	2.675	2.805	3.762	3.830	3.790	5.084	5.135
7	1.125	1.509	1.503	1.865	2.500	2.524	2.865	3.842	3.665	3.710	4.976	4.900
8	1.195	1.602	1.590	1.990	2.668	2.700	2.940	3.944	3.852	4.000	5.362	5.190
9	1.295	1.737	1.685	2.060	2.760	2.860	3.105	4.164	4.095	4.135	5.544	5.495
10	0.870	1.167	1.165	1.535	2.058	2.035	2.290	3.070	2.992	3.045	4.081	4.100
11	0.875	1.174	1.178	1.570	2.105	2.103	2.395	3.211	3.110	3.220	4.318	4.324

[1] Average of 35 tests. [2] Average of 18 tests. [3] Average of 13 tests. [4] Average of 16 tests. All other values are an average of 6 tests.

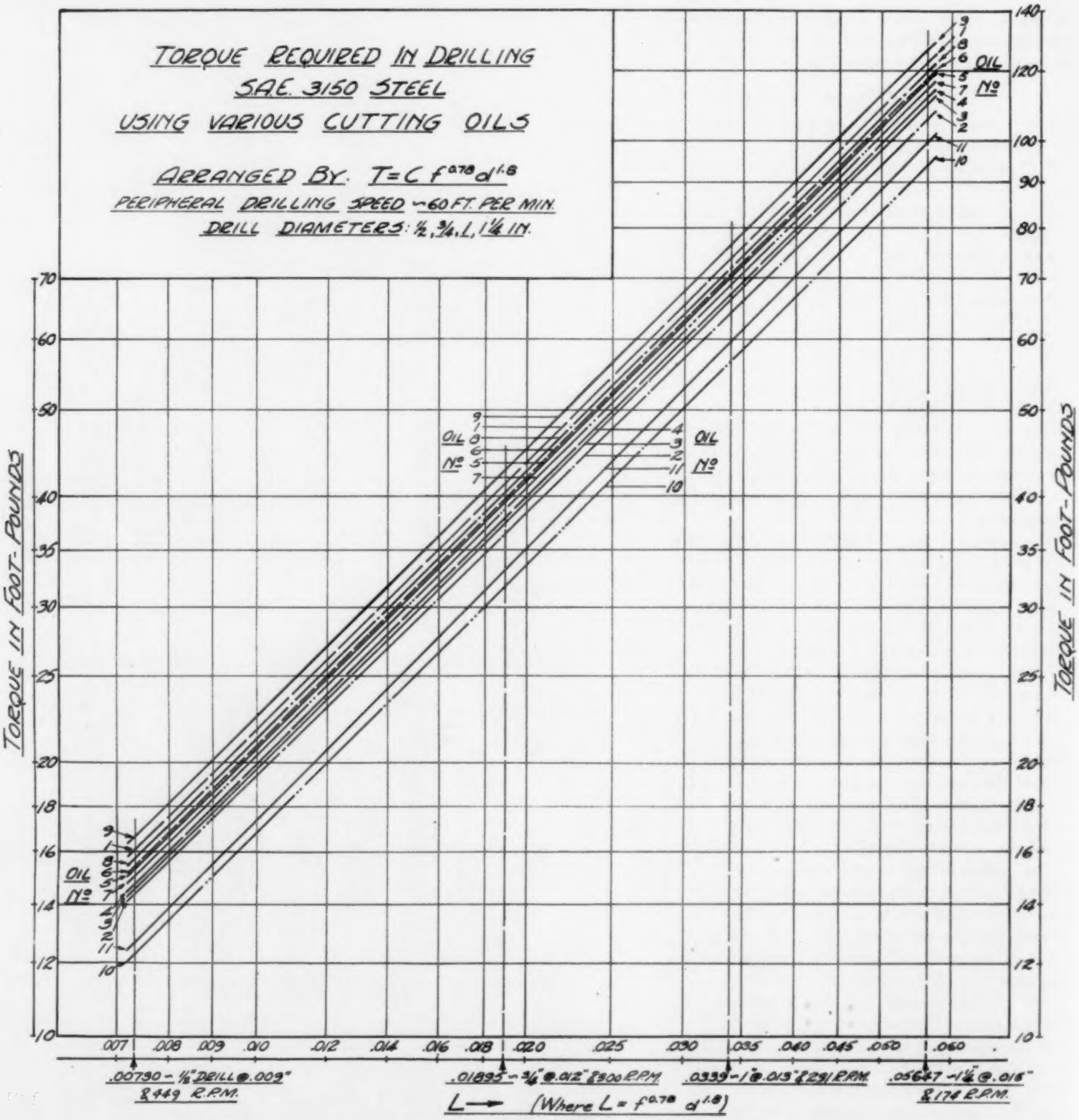


FIG. 12 TORQUE REQUIRED IN DRILLING S.A.E. 3150 STEEL, USING VARIOUS CUTTING OILS

oils are arranged in order of increasing torque constant and factor. It is seen from these factors that oil No. 9 produces a torque $1^{1}/_{2}$ per cent greater than that produced by oil No. 1, dry cutting, while oil No. 10 produces a torque of only 76.5 per cent of that for dry cutting.

The torque for any drill diameter operating at any feed using any of the oils covered by these experiments may be determined from the curves of Fig. 12 simply by computing the value of L, locating it on the abscissa axis, and projecting it vertically until the torque line for the desired oil is intersected.

Better to illustrate the relation between the torque values for the different drill diameters at their respective feeds between dry cutting and oil No. 10 (a 4 per cent sulphurized mineral oil), Fig. 13 has been prepared. Fig. 14 shows in a general way the relation between the torque required to drill the S.A.E. 3150 steel, using the various cutting oils for each cutting condition. The torque for the $1^{1}/_{4}$-in.-diameter drill operating at 0.015 in. feed is shown to be about 94 for oil No. 10, which is the lowest value. The oils are arranged in order of increasing torque values from left to right so that the torque of the $1^{1}/_{4}$-in.-diameter drill, operating at 0.015 in. feed, increases gradually to a value of 125 for oil No. 9 at the right. For the $1^{1}/_{4}$-in.-diameter drill the difference in torque between oil No. 1, dry cutting, and oil No. 10, the 4 per cent sulphurized mineral oil, is 22.8 per cent of the torque value for dry drilling. For the 1-in.-diameter drill, operating at 0.013 in. feed, this value is 24.6; for the $^{3}/_{4}$-in.-diameter drill at 0.012 in. feed, it is 24.1, and for the $^1/_2$-in.-diameter drill at 0.009 in. feed, 23.5 per cent. This shows a rather constant influence of oil No. 10, regardless of the value of the drill diameter and feed.

Knowing the torque T for any value of $L = f^{0.78}d^{1.8}$, the horsepower developed at the drill point due to the torque may be determined from the equation

$$HP_T = \frac{2\pi NT}{33{,}000} = 1904\ TN$$

in which T is the torque in foot-pounds and N is the revolutions per minute of the drill.

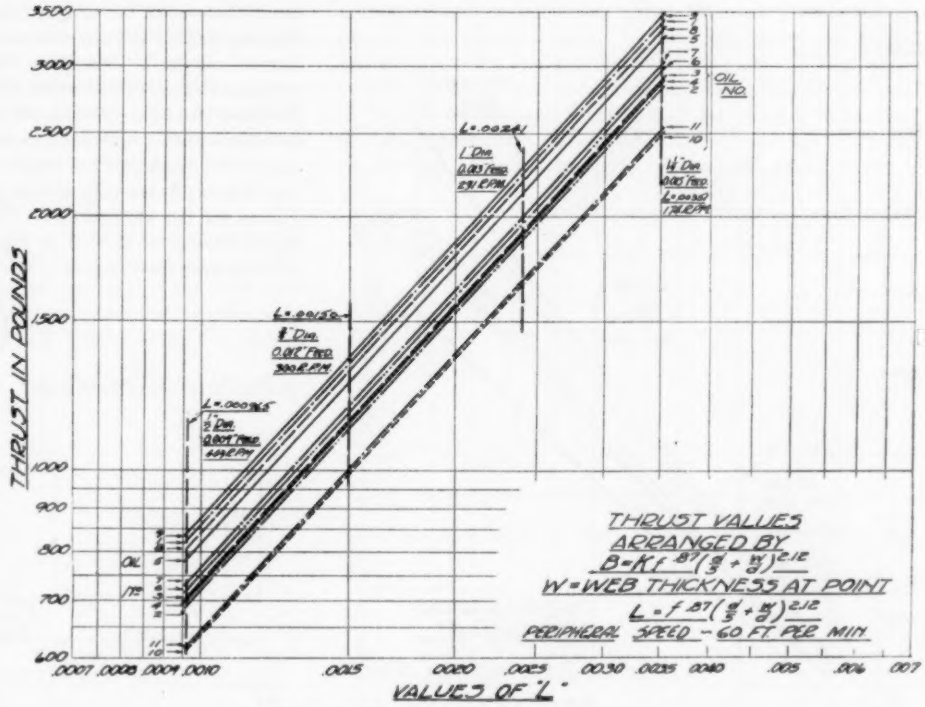


FIG. 15 THRUST FOR VARIOUS VALUES OF L

THRUST VALUES

Numerical averages of thrust required to drill the S.A.E. 3150 steel, using various cutting oils as determined from the commercial tests, are given in Table 5. As stated earlier, it was found that the best equation to represent a relation between thrust, feed, and drill diameter was such that the web thickness should be considered. The equation

$$B = K f^{0.87} \left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$$

was found to be satisfactory, in which B is the thrust in pounds, K a constant, f the feed in inches per revolution, d the drill diameter in inches, and w the web thickness in inches at the point of the drill.

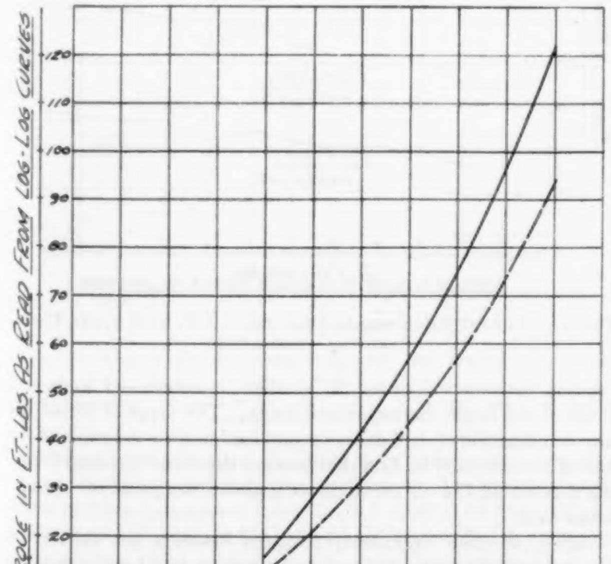


FIG. 13 COMPARISON OF TORQUE REQUIRED TO DRILL S.A.E. 3150 STEEL

(Solid line: dry; dashed line: oil No. 10.)

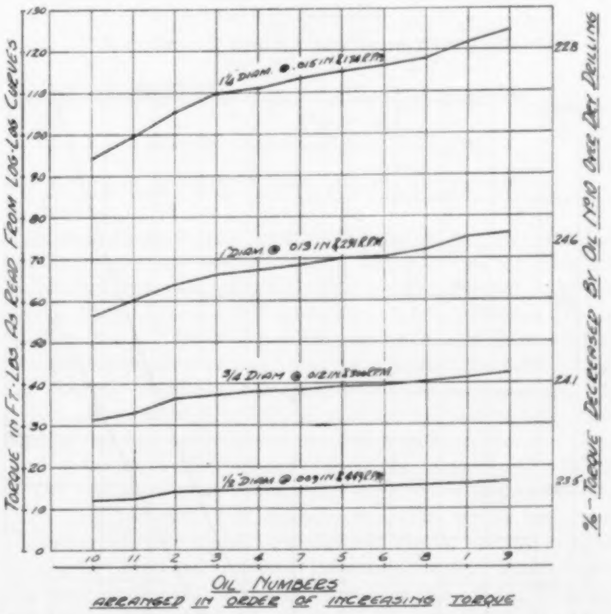


FIG 14 TORQUE REQUIRED TO DRILL S.A.E. 3150 STEEL, USING VARIOUS CUTTING OILS

The numerical averages of thrust were plotted on log-log paper as ordinates over abscissa values equal to $L = f^{0.87}\left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$ as shown in Fig. 15. The various thrust lines for the different oils are not only straight but appear to be very nearly parallel.

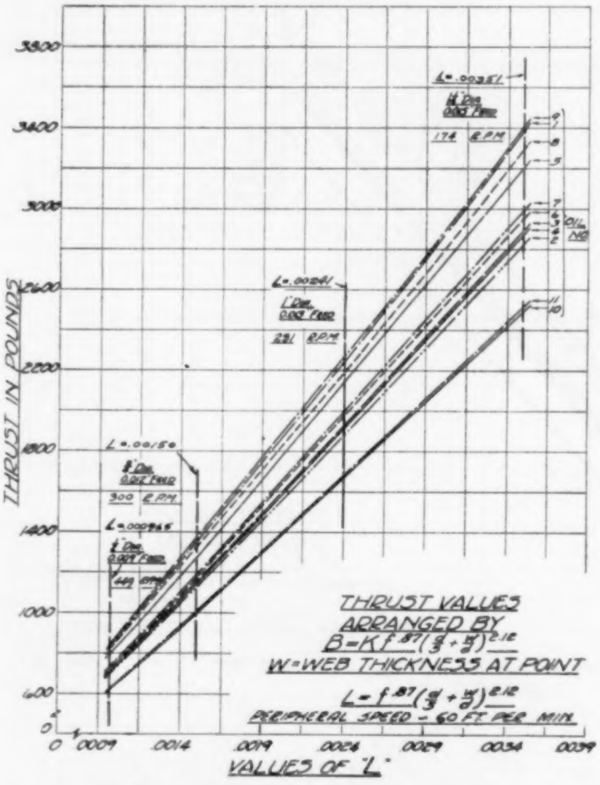


FIG. 16 THRUST FOR VARIOUS VALUES OF L

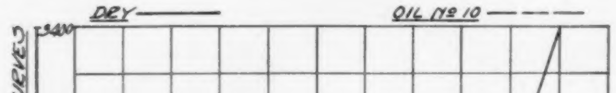


FIG. 17 COMPARISON OF THRUST REQUIRED TO DRILL S.A.E. 3150 STEEL

As in the case of the torque values, Fig. 12, the thrust values are highest for oil No. 9 and lowest for oil No. 10. It is seen that the thrust values for oils Nos. 10 and 11 are very nearly equal, whereas there is considerable difference between the torque values for these two oils. The thrust for oil 6, Fig. 15, is lower than that for oils 5 and 7, whereas the torque for oil 6 is higher than the torque for oils 5 and 7. In general, however, these thrust curves for various oils are quite similar in their arrangement to the torque curves for the various oils. The values for thrust for the commercial tests, as plotted in Fig. 15 on log-log paper, are shown plotted over the value of

$$L = f^{0.87}\left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$$

in Fig. 16 on coordinate paper. This indicates more definitely the relation between values for the smaller diameters and feeds and those of the larger diameters and feeds. The value of thrust for any combination of drill diameter and feed may be determined for any of the oils used by first determining the value of L and locating a point on the oil curve directly above the point on the abscissa axis.

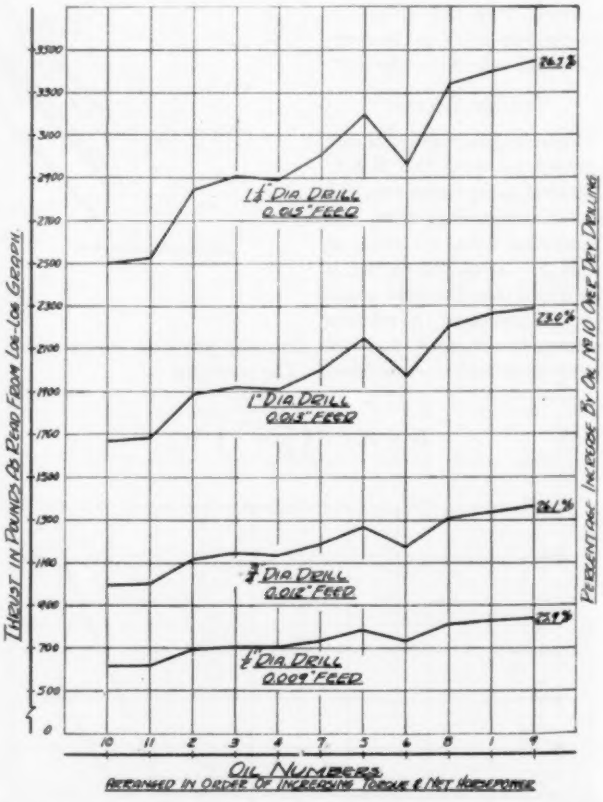


FIG. 18 THRUST REQUIRED IN DRILLING S.A.E. 3150 STEEL USING VARIOUS CUTTING OILS

Again, to show comparative values between the thrust for the different drill diameter and feed combinations for dry cutting and for oil No. 10, the values of thrust for these oils are shown plotted on coordinate paper in Fig. 17. The thrust values produced by each drill diameter at its respective feed are shown plotted over the oil numbers arranged in order of increasing torque (same as Fig. 14) in Fig. 18. It is seen that the thrust for the $1^1/_4$-in.-diameter drill, operating at 0.015 in. feed, with oil No. 10, is but 2500 lb., whereas for dry cutting it is about 3400 lb., and for oil No. 9 (5 per cent oleic acid in mineral oil) it is 3435 lb. For the 1¼-in.-diameter drill there is a saving of 26.5 per cent in thrust over dry cutting when oil No. 10 is used. For the 1-in.-diameter drill, this saving is 23 per cent; for the ¾-in.-diameter drill, 26.1 per cent; and for the ½-in.-diameter drill, 25.9 per cent. This saving appears to be practically constant in spite of the wide range of drill diameters and feeds.

Gross Power Input

The gross horsepower required to drive the drill press during the cutting operations was recorded by a wattmeter connected with the electric motor mounted on the rear of the drill-press column. The numerical averages from the various tests gave the gross kilowatts required to drill S.A.E. 3150 steel for the various cutting oils as tabulated in Table 6. Corresponding values of gross horsepower are also given in Table 6. The numerical averages of gross horsepower from Table 6 were plotted on cartesian coordinates as shown in Fig. 19, and are represented by smoothed-out curves. The values of the gross horsepower as determined from these smoothed-out curves are tabulated in Table 6.

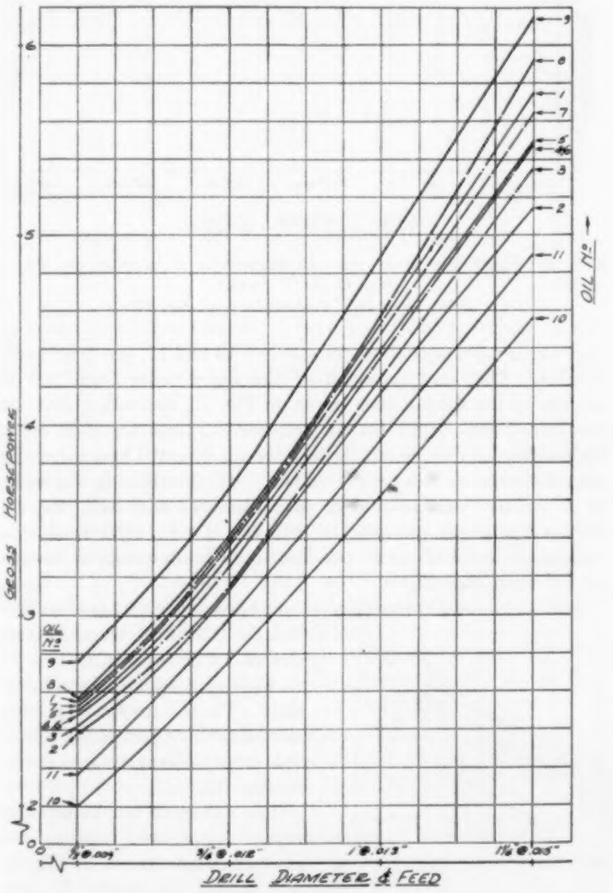


Fig. 19 Gross Horsepower Required to Drill S.A.E. 3150 Steel

(Drills operating at commercial feeds and at a cutting speed of approximately 60 ft. per min.)

These values are not shown plotted on log-log paper over the values of $L = f^{0.78} d^{1.8}$ as the torque values were, inasmuch as there is a constant in the form of tare power which would make the results misleading. The value of L would naturally hold for any drill diameter and feed, and yet the tare for one value of L would be small if a large-diameter drill were used and large if a small-diameter drill were used. The net-power values are plotted over these values of L, inasmuch as the net values are almost a direct speed function of the torque. The gross-horsepower values required to drill the S.A.E. 3150 steel as presented in Table 6 may be read directly from the various oil curves only at four points on the abscissa axis, namely, over the ½-in.-diameter drill at 0.009 in. feed, the ¾-in. drill at 0.012 in. feed, the 1-in. drill at 0.013 in. feed, and the 1¼-in. drill at 0.015 in. feed.

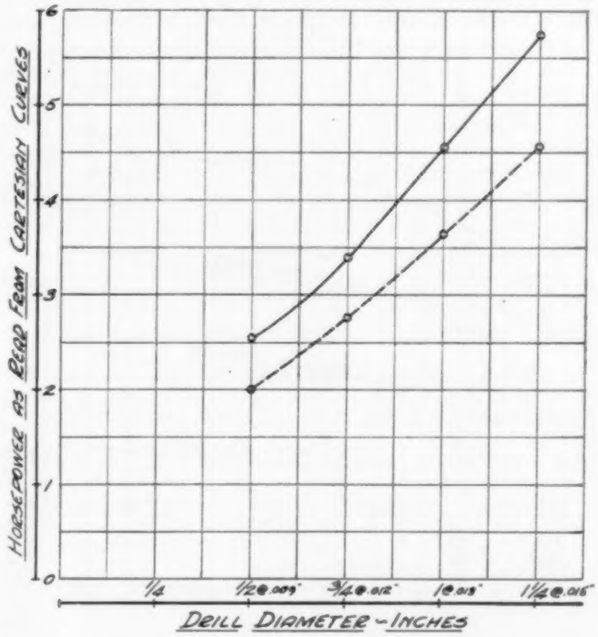


Fig. 20 Comparison of Gross Horsepower Required to Drill S.A.E. 3150 Steel

(Solid line: dry; dashed line: oil No. 10.)

A comparison of gross horsepower required to drill the S.A.E. 3150 steel dry and with oil No. 10 (the 4 per cent sulphurized mineral oil) is shown in Fig. 20. The gross horsepower developed by the ½-in.-diameter drill operating at 0.009 in. feed is 2.01 for oil No. 10 and 2.55 for dry cutting oil No. 1. This shows that dry cutting requires 21.2 per cent more power than cutting with oil No. 10. Similarly, the gross horsepower required for the 1¼-in.-diameter drill operating at 0.015 in. feed is 5.74 when cutting dry and only 4.57 when cutting with oil No. 10. This gives a saving of 20.4 per cent for oil No. 10 over dry cutting.

The values of gross horsepower required to drill the S.A.E. 3150 steel for each drill diameter operating at its respective commercial feed and speed are shown in Fig. 21 plotted over the oil numbers arranged in order of increasing torque. These curves are very similar to the torque curves, except for the constant tare power introduced for each combination of drill diameter, feed, and speed. The fact that the gross power is high for oil No. 7 and low for oils Nos. 6 and 1 may be due to an error in zero setting on the part of the wattmeter in recording the tare-power values. This irregularity is eliminated when the net-power values are determined as shown below. The percentage values along the right-hand edge of Fig. 21 are an indication for each drill size, feed, and speed of the saving in gross power over dry cutting when oil No. 10 is used.

Net Power Input

In every test the machine was started with the drill point

$^1/_4$ to $^3/_8$ in. from the surface of the metal to be drilled. This allowed the wattmeter to record the input power to drive the machine with the speed and feed gears engaged, but with no cutting. This tare power was subtracted from the gross values, resulting in the net power input. The numerical averages of the net power input are listed in Table 7 as both kilowatts and horsepower. The horsepower values were plotted on log-log paper, Fig. 22, over abscissa values equal to L, in which L equals $f^{0.78}d^{1.8}$. A series of straight lines are obtained which,

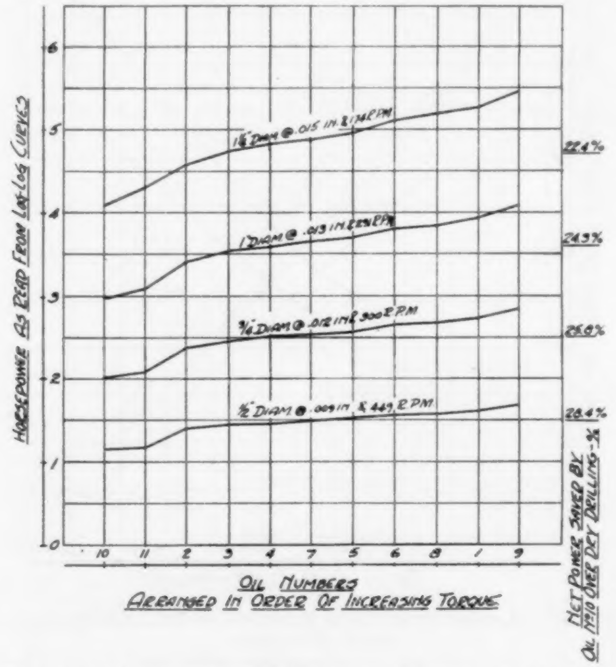


FIG. 21 GROSS HORSEPOWER REQUIRED TO DRILL S.A.E. 3150 STEEL

(Drills operating at commercial feeds and at a cutting speed of approximately 60 ft. per min.)

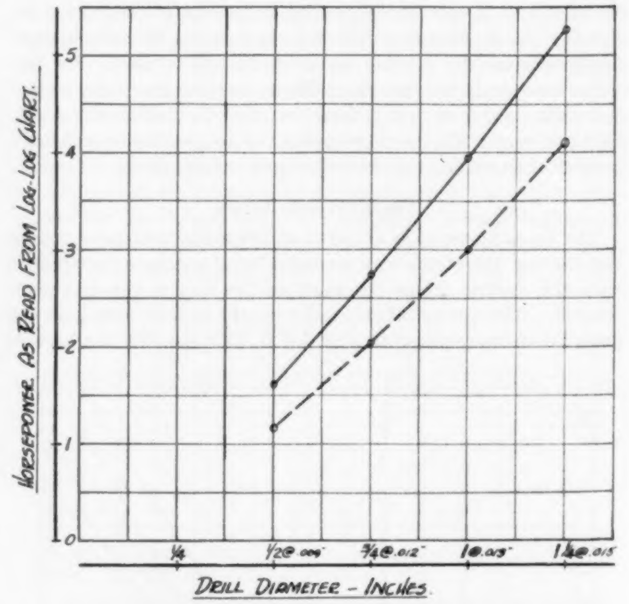


FIG. 23 COMPARISON OF NET HORSEPOWER REQUIRED TO DRILL S.A.E. 3150 STEEL

(Solid line: dry; dashed line: oil No. 10.)

with the exception of those for oils No. 10 and 11, are practically parallel. They are arranged in the same order from top to bottom as the torque lines shown in Fig. 12, indicating that the net horsepower input for all commercial tests for each oil is highest for oil No. 9 and lowest for oil No. 10, and otherwise placed similar to the torque values. By determining the value of L for any combination of drill diameter and feed, the net horsepower input required to drill the S.A.E. 3150 steel at a peripheral speed of 60 ft. per min. may be determined for any oil by using Fig. 22.

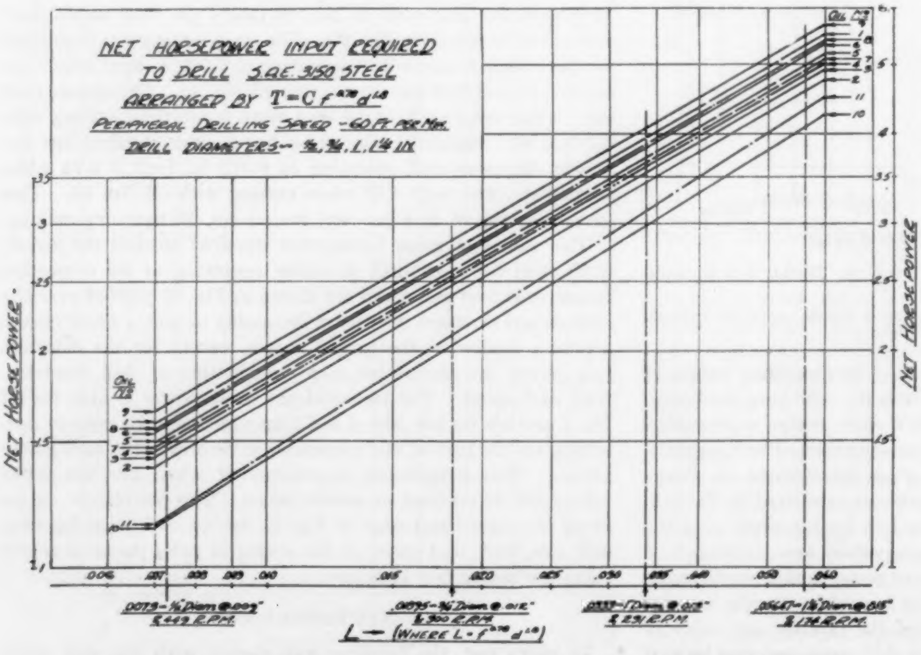


FIG. 22 NET HORSEPOWER INPUT REQUIRED TO DRILL S.A.E. 3150 STEEL

Fig. 23 shows a comparison of net horsepower required to drill S.A.E. 3150 steel when cutting dry and when using oil No. 10 (a 4 per cent sulphurized mineral oil). These curves are very similar to those comparing torque values for these two oils as shown in Fig. 13.

The values of net horsepower input as taken from the smoothed-out curves of Fig. 22 are shown in Table 7. The values of net horsepower of Table 7 are shown plotted in Fig. 24 for each drill diameter, feed, and speed combination over the various oil numbers arranged in order of increasing torque. These curves are very similar to the torque curves arranged over oil numbers in Fig. 14.

CHIPS FROM THE DRILLING EXPERIMENTS

Fig. 25 shows representative chips from each of the drill sizes operating at their respective feeds when the peripheral cutting

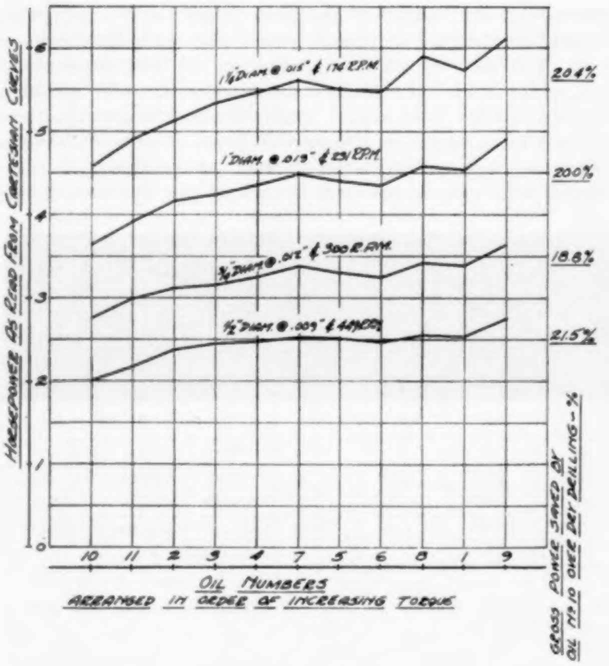


FIG. 24 NET HORSEPOWER INPUT REQUIRED TO DRILL S.A.E. 3150 STEEL

(Drills operating at commercial feeds and at a cutting speed of approximately 60 ft. per min.)

speed is approximately 60 ft. per min. The chips when cutting dry (oil No. 1) are all blued, indicating the development of excessive heat. The chips for oils Nos. 10 and 11 are browned, apparently due to the action of the sulphur in the oils rather than to high temperatures. The chips from oils Nos. 7, 8, and 9 appear to be well broken up. There were very few continuous chips. The chips from oil No. 6, which is the mineral oil having a viscosity of 110 Saybolt seconds at 100 deg. fahr., are broken up, but not so completely as are those for oil No. 7, which is a mineral oil having a viscosity of 210 Saybolt seconds at 100 deg. fahr. The chips from oil No. 6 also are colored, evidently due to high temperatures.

The chips from oil No. 2 (water containing $1^1/_2$ per cent borax) appear to be of the brightest steel-like color, giving the appearance of aluminum. This may be an indication of superior cooling qualities.

The chips from the 1-in.-diameter and $1^1/_4$-in.-diameter drills for oil No. 10 did not seem to be well broken up, but came out in large, long coils, throwing the oil about the room.

THE RESULTS OF THE PLANER TESTS

The planer cutting tests were conducted on the annealed S.A.E. 3150 steel also used in the drilling tests. The single tool shape described above was used on all cuts.

A cutting speed of 20 ft. per min. was maintained in all tests reported here. Four depths of cut were used: 0.030, $^1/_{16}$, $^1/_8$, and $^3/_{16}$ in. For each depth of cut three feeds were used, 0.0089, 0.020, and 0.036 in., respectively. In some instances feeds as light as 0.004 in. were employed, but for the shallow depths of cut the forces measured were so small that they were not considered wholly reliable.

The material being cut, which was a piece 2 in. × 6 in. in section and 24 in. long, was clamped rigidly in the bottom of the tank mounted on the dynamometer table. It was placed on its face with the length in the direction of the stroke so that cuts 24 in. in length could be started at one edge and the tool fed across the 6-in. face in taking subsequent cuts. Fig. 26 is a photograph of a short length of this top surface after a 0.030-in. depth of cut has been taken over the whole width. Starting from the left, the tool was given a feed of 0.0089 in. for about twenty cuts with cutting oil No. 1 (dry cutting). The feed was then increased to 0.020 in. for about ten cuts and then to 0.036 in. for about 6 cuts. The cutting force for each cut was measured and later averaged for each separate condition. The set-up depth of cut of 0.030 in. was maintained for the feeds of 0.0089 and 0.020 in., but was reduced 0.0005 in. for the 0.036-in. feed.

After the series of dry cuts at the 0.030-in. depth of cut, oil No. 5 (No. 1 lard oil) was used. Fig. 26 shows the actual depths for each feed for the No. 5 oil, as well as for each oil subsequently used. For each oil and depth of cut, the three feeds mentioned were employed as indicated by the resultant finish. The cuts shown in Fig. 26 are the beginning of the cuts 24 in. in length.

Fig. 27 shows a similar set of feed and actual depth readings with the resultant finishes for the same tool operating at 20 ft. per min. for the various cutting fluids when the depth of cut remained set at $^1/_{16}$ in. The bottom of the photograph represents the end

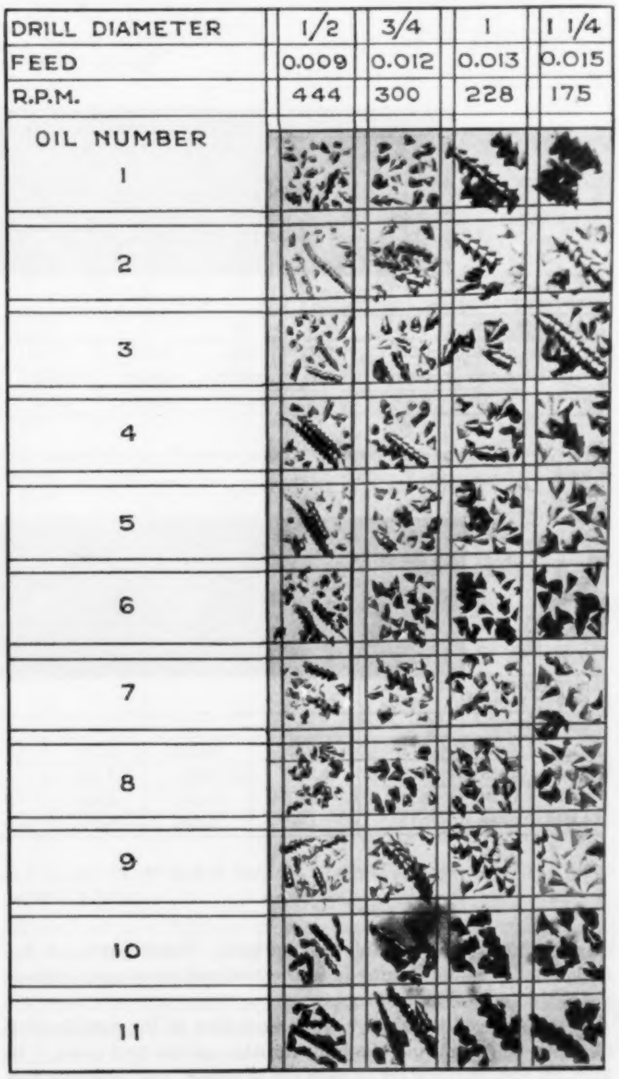
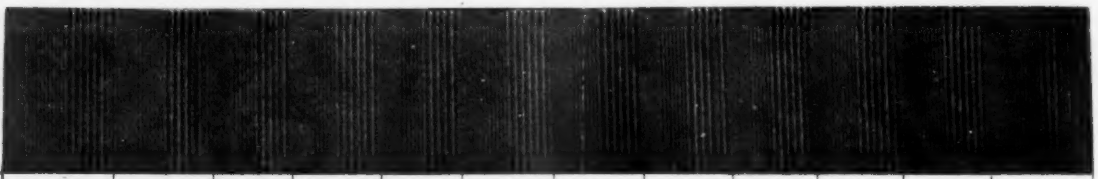


FIG. 25 DRILL CHIPS FROM VARIOUS CUTTING CONDITIONS

of the 24-in. length of cut and clearly shows how the chip was broken off at the end of the cut.

Fig. 28 shows the surface left on the bar for the beginning of each 24-in. length of cut when the set-up depth was $^1/_8$ in. The upper edge represents the beginning of each cut. This set-up depth was actually obtained by lowering the tool by means of a dial indicator. The actual depth differed from the set-up depth very little for the shallow cuts, but for the heavier cuts the difference became more noticeable and marked. This difference is due undoubtedly to the digging-in of the tool and subsequent springing of the tool holder and support.

It was noticed that every chip, regardless of the combination of depth and feed, had the built-up edge on the tool point. In Figs. 26 and 28 the start of each cut is shown, and after the first $^1/_2$ in. or so, a condition is reached which appears to be constant throughout the balance of the 24-in. length of cut. It is believed that the built-up edge is formed during the first $^1/_8$ in. of cut. This distance might be greater on the 0.030-in. depth of cut, as the finish left in some instances appears to be burnished at intervals for a somewhat longer length. In general, there appears to be very little difference in finish for cuts of the same dimensions but made with the different cutting fluids. In other words, these tests do not seem to indicate that the cutting fluid has any marked influence on the finish produced at the cutting speed of 20 ft. per min. This subject will be investigated further at a later date.



CUT NUMBER (LEFT TO RIGHT)		1	2	3	4	5	6	7	8	9	10	11	12
OIL NUMBER		1	5	8	9	7	6	10	11	4	3	2	1
ACTUAL DEPTH FOR FEEDS	0.0089	0.030	0.0305	0.0305	0.0305	0.0305	0.0305	0.030	0.030	0.030	0.030	0.0295	0.0295
	0.020	0.030	0.030	0.0305	0.0305	0.0305	0.0305	0.0295	0.0295	0.0295	0.0295	0.029	0.029
	0.036	0.0295	0.0295	0.030	0.0295	0.0300	0.0295	0.029	0.029	0.029	0.029	0.0285	0.0285

Fig. 26 Showing the Finish on the S.A.E. 3150 Steel and Giving Actual Depth of Cut for Each Feed and Oil for a Set-Up Depth of Cut of 0.030 In.

CUT NUMBER (LEFT TO RIGHT)		1	2	3	4	5	6	7	8	9	10	11	12
OIL NUMBER		1	11	10	2	3	4	7	6	9	8	5	1
FEED IN INCHES	0.0089	0.065	0.065	0.065	0.0645	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065
	0.020	0.066	0.066	0.0655	0.0655	0.0655	0.0655	0.066	0.0655	0.0655	0.0655	0.0655	0.066
	0.036	0.066	0.066	0.066	0.0655	0.066	0.066	0.066	0.066	0.066	0.066	0.066	0.066

Fig. 27 Showing the Finish on the S.A.E. 3150 Steel and Giving Actual Depth of Cut for Each Feed and Oil for a Set-Up Depth of Cut of $^1/_{16}$ In.

CUT NUMBER (LEFT TO RIGHT)		1	2	3	4	5	6	7	8	9	10	11
OIL NUMBER		1	11	10	2	4	3	7	6	9	8	5
ACTUAL DEPTHS FOR FEEDS	0.004	0.129	0.1285	0.1285	0.128	0.128	0.128	0.128	0.128	0.128	0.128	0.1275
	0.0089	0.130	0.1295	0.129	0.129	0.1295	0.129	0.129	0.1285	0.129	0.129	0.130
	0.020	0.131	0.1305	0.1305	0.130	0.130	0.130	0.1305	0.1305	0.1305	0.1305	0.1305
	0.036	0.133	0.1325	0.1315	0.1315	0.135	0.131	0.131	0.131	0.1305	0.1305	0.1305

Fig. 28 Showing the Finish on the S.A.E. 3150 Steel and Giving Actual Depth of Cut for Each Feed and Oil for a Set-Up Depth of Cut of $^1/_8$ In.

PLANING-FORCE CURVES

After running four complete sets of tests giving force values for each combination of depth of cut and feed, these values were plotted on coordinate paper as shown in Fig. 29. The force values for the actual depth were plotted and smooth lines were drawn through these points, representing force values for constant depth at the left in Fig. 29 and for constant feed at the right. The solid lines represent the force curves for different depths or feeds when cutting dry (cutting fluid No. 1), while the dashed lines represent corresponding values when cutting fluid No. 10 (4 per cent sulphurized mineral oil) was used.

Curves similar to those of Fig. 29 were drawn for the actual test data on the planer tool for each of the cutting fluids and the values thought most representative read off and plotted on log-log paper, as shown in Figs. 30 to 40, inclusive, for cutting fluids 1 to 11, inclusive. Fig. 30 represents the planer-force curves in cutting the S.A.E. 3150 steel at 20 ft. per min. when cutting dry or with cutting fluid No. 1. Two sets of curves are shown: the dashed lines are for the constant values of feed plotted over variable depths of cut, while the solid lines are for constant depths of cut for various values of feed. The straight lines on log-log paper presented in Figs. 30 to 40, inclusive, were studied for the purpose of arriving at an equation representing the force on the planer tool as a function of each cutting oil, the depth of cut, and the feed. From all of the figures nineteen lines of constant depth had a slope equal to 0.79, that is, the tangent of the angle made with the horizontal equaled 0.79. Seven lines had a slope of 0.78, while ten had a slope of 0.80. The average for the thirty-six lines is 0.792, which represents the value of y in the expression $P = CD^xf^y$. Similarly the slopes of the various constant-feed lines were averaged in order to determine the value of x in the

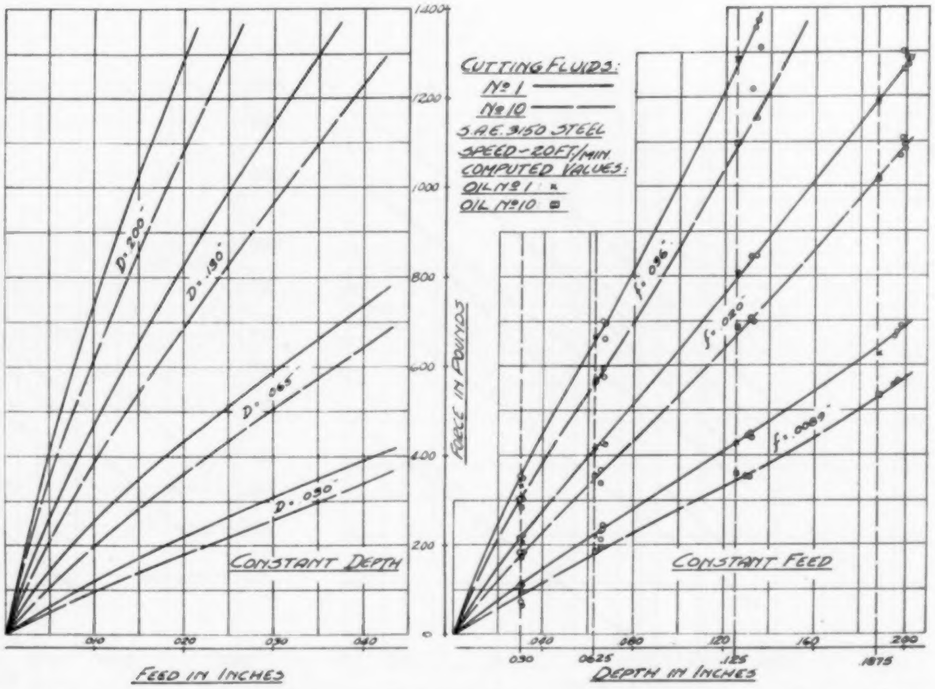


FIG. 29 PLANER-FORCE TEST DATA AND COMPUTED VALUES FOR ACTUAL DEPTHS OF CUT AND FEED

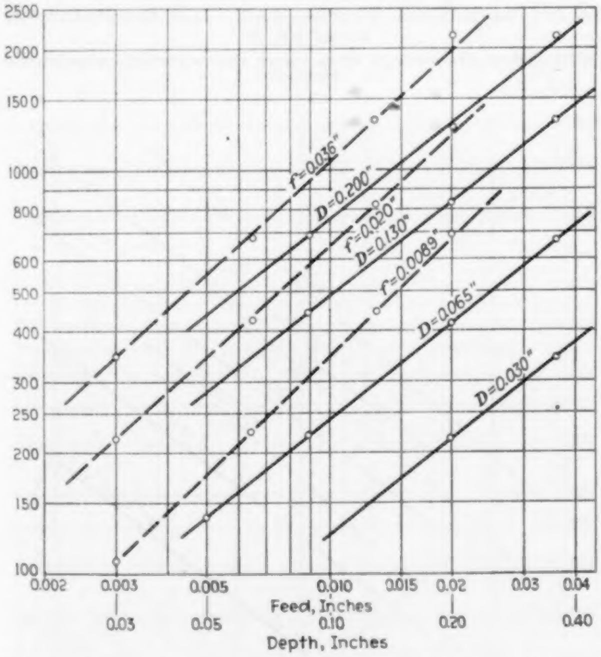


FIG. 30 PLANER-FORCE CURVES, S.A.E. 3150 STEEL—CUTTING FLUID No. 1

(Values plotted are averages from curves, Fig. 29, drawn through experimental points.)

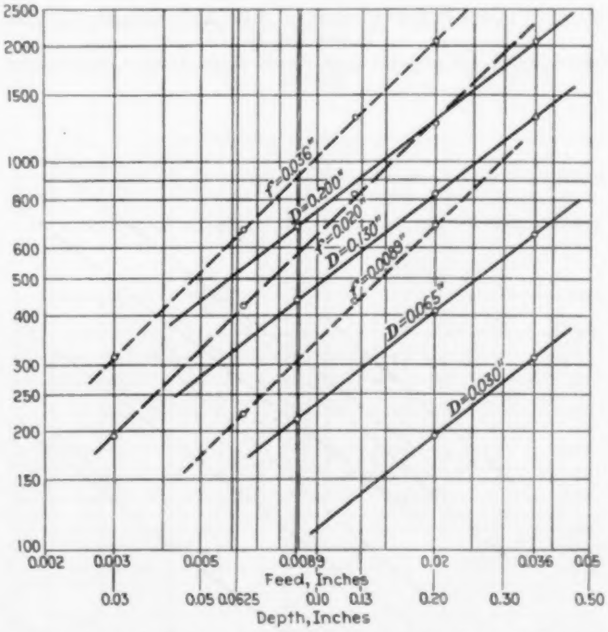


FIG. 31 PLANER-FORCE CURVES, S.A.E. 3150 STEEL—CUTTING FLUID No. 2

(Values plotted are averages from curves drawn through experimental points.)

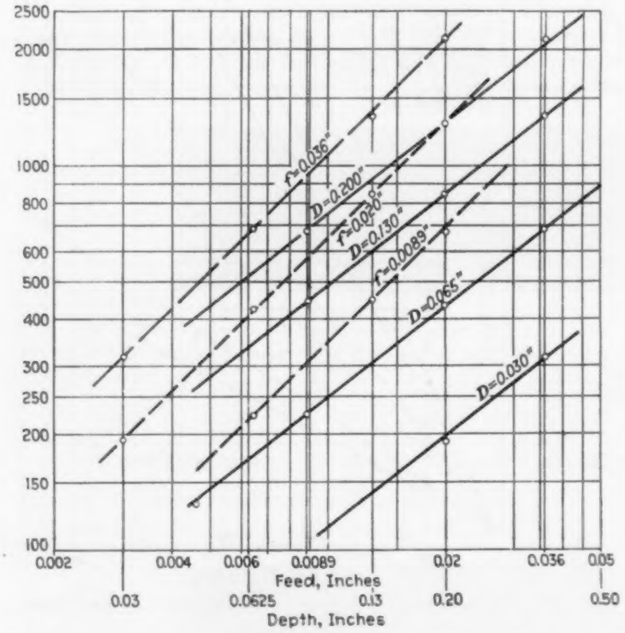


Fig. 32 Planer-Force Curves, S.A.E. 3150 Steel—Cutting Fluid No. 3

(Values plotted are averages from curves drawn through experimental points.)

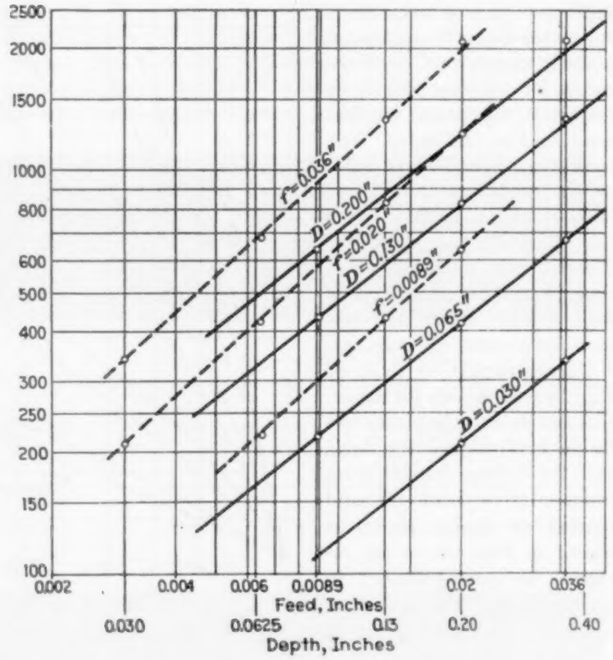


Fig. 34 Planer-Force Curves, S.A.E. 3150 Steel—Cutting Fluid No. 5

(Values plotted are averages from curves drawn through experimental points.)

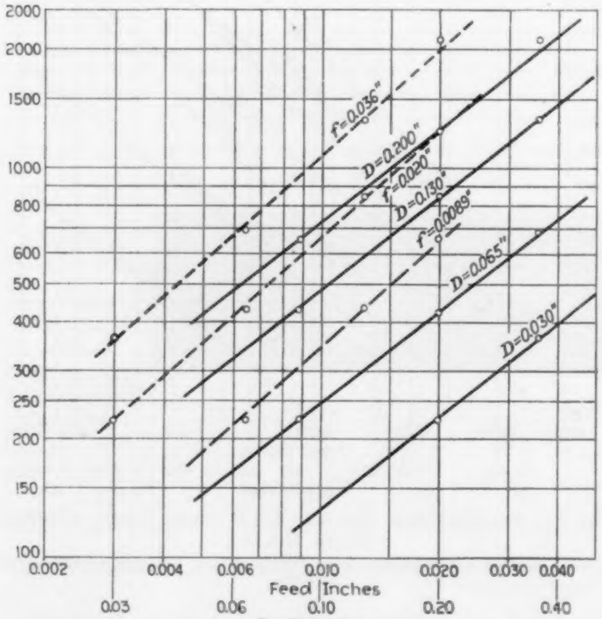


Fig. 33 Planer-Force Curves, S.A.E. 3150 Steel—Cutting Fluid No. 4

(Values plotted are averages from curves drawn through experimental points.)

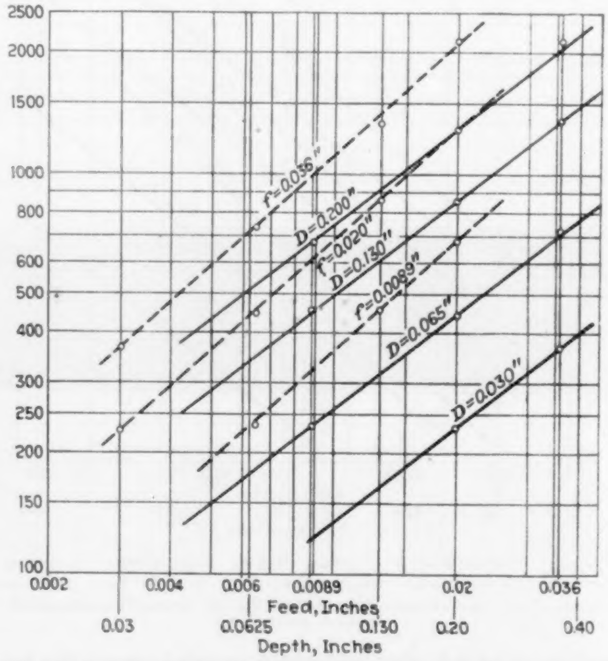


Fig. 35 Planer-Force Curves, S.A.E. 3150 Steel-Cutting Fluid No. 6

(Values plotted are averages from curves drawn through experimental points.)

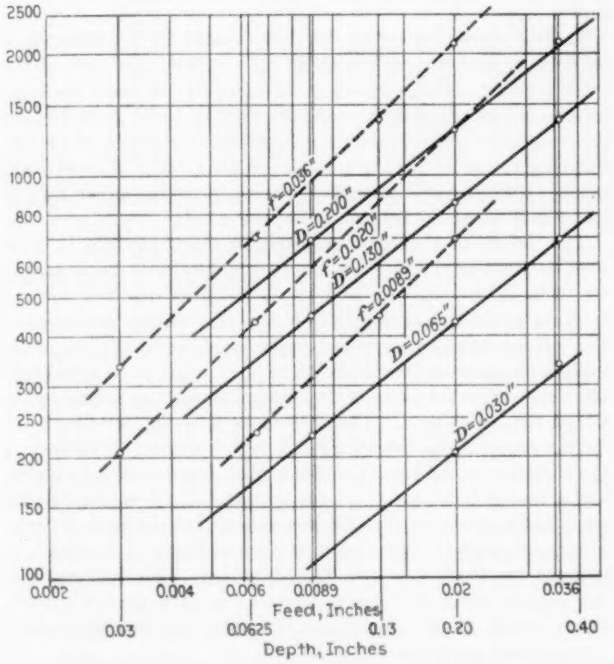


Fig. 36 Planer-Force Curves, S.A.E. 3150 Steel—Cutting Fluid No. 7

(Values plotted are averages from curves drawn through experimental points.)

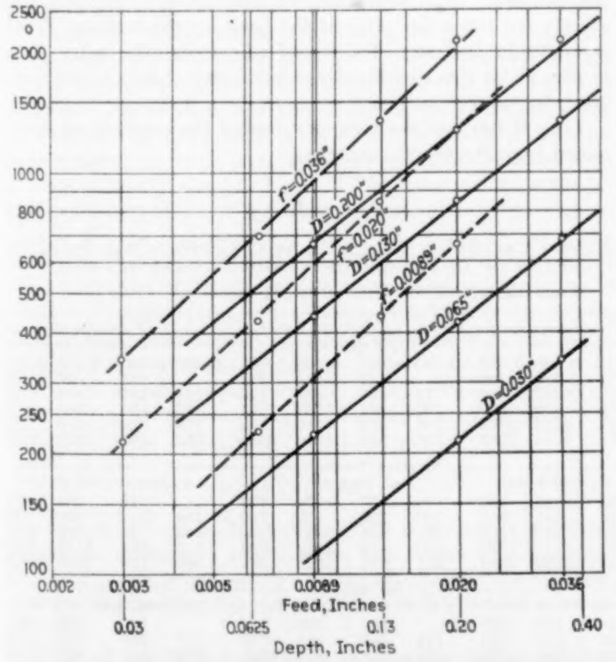


Fig. 37 Planer-Force Curves, S.A.E. 3150 Steel—Cutting Fluid No. 8

(Values plotted are averages from curves drawn through experimental points.)

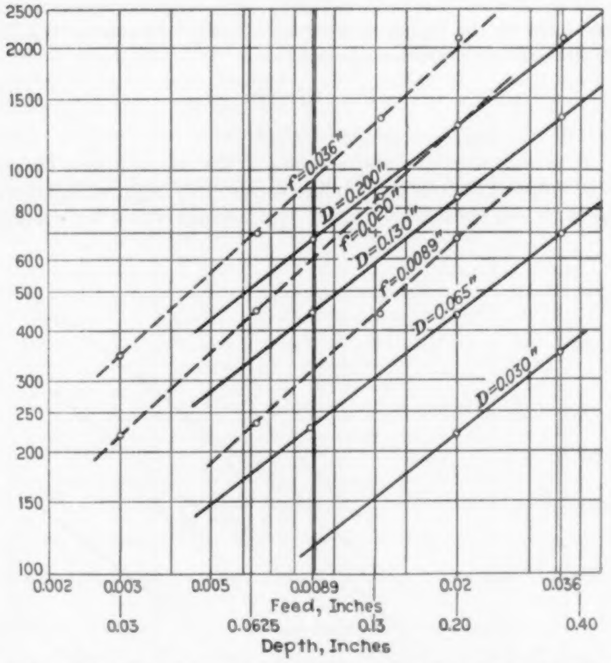


Fig. 38 Planer-Force Curves, S.A.E. 3150 Steel—Cutting Fluid No. 9

(Values plotted are averages from curves drawn through experimental points.)

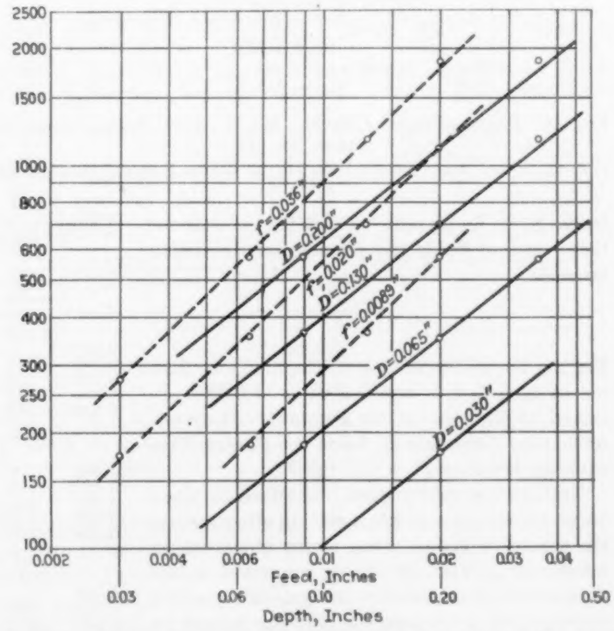


Fig. 39 Planer-Force Curves, S.A.E. 3150 Steel—Cutting Fluid No. 10

(Values plotted are averages from curves drawn through experimental points.)

foregoing equation. Average values for all of the constant-feed lines for oils 1 to 11 are, respectively, 0.926, 0.976, 0.993, 0.90, 0.92, 0.91, 0.97, 0.93, 0.92, 0.97, and 1.00. This gives a final average, including all oils, of 0.945. The equation representing the force on the planer tool for all cutting fluids on the S.A.E. 3150 steel could then be expressed as

$$P = CD^{0.95}f^{0.79}$$

Taylor's formula for steel is $P = CDF^{14/15}$

It is seen from the above slopes of the constant-feed curves that some of the oils have values consistently high, while others have values consistently low. The average value for cutting fluids 2, 3, 7, 10, and 11 is 0.98, so that for these cutting fluids the planing-force equation becomes

$$P = CD^{0.98}f^{0.79}$$

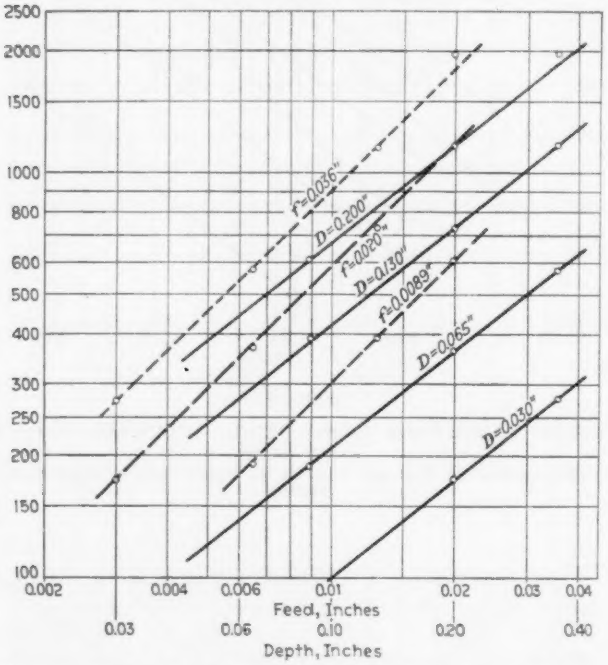


FIG. 40 PLANER-FORCE CURVES, S.A.E. 3150 STEEL—CUTTING FLUID NO. 11

(Values plotted are averages from curves drawn through experimental points.)

For the remainder of the cutting fluids consisting of 1, 4, 5, 6, 8, and 9 a value of 0.92 is obtained as the slope of the constant-feed curves, so that for these cutting fluids the planing-force equation becomes $P = CD^{0.92}f^{0.79}$.

Until further information is obtained for these various cutting fluids when cutting other metals, the general equation given above is considered satisfactory. Fig. 29 shows computed values determined from the first or general equation indicated as a cross on the solid-line curves for dry cutting, oil No. 1, and as a square on the dashed curves for which cutting fluid No. 10 was used. It is seen that these computed values as given by the general equation are very close to the curve obtained by smoothing out the experimental data. These are considered satisfactory in spite of the fact that cutting fluid No. 1 falls in the group having low exponents of depth, while cutting fluid No. 10 falls in the group of cutting fluids having the high exponents of depth.

Table 8 shows tabulated the force values on the planer tool for each combination of depth of cut and feed, for each of the cutting fluids. These values were read directly from the logarithmic curves of constant depth in Figs. 30 to 40, inclusive, for the respective cutting fluids. Numerical averages of corresponding values for oils 1 to 9, inclusive, are given at the end of the table. Differences between these averages and the values for any one of individual oils are seen to be very small.

The values of the force required to plane the S.A.E. 3150 steel as given in Table 8 are shown graphically in Figs. 41 and 42. The force values for each combination of depth of cut and feed are plotted as ordinates over the oil numbers as abscissas. The oil numbers are arranged in order of increasing torque in drilling as obtained from Figs. 12 and 14. It is to be noted that the force values for some of the oils are lower than others to the left on the same curve. The differences, however, are very small for the group of oils consisting of 2 to 9, inclusive. Considering the highest curve in Fig. 41, which is for a depth of cut of 0.200 in. and a feed of 0.036 in., the force for oils 4 and 5 is less than for oils 2 and 3, so that if the oils were arranged in the figure in order of increasing force, they would be arranged from left to right as follows: 10, 11, 5, 4, 8, 2, 6, 7, 9, 3, and 1. This indicates that dry cutting gives the largest force by a very narrow margin. Force values on the planer tool for additional combinations of cut and feed are given in Fig. 42.

By adding all the force values given for each oil in Table 8, a total sum is obtained as indicated in Table 9. Opposite this sum for each oil is listed the force on the planer tool required to remove a chip 0.130 in. in depth and 0.036 in. feed for the respective oils. It is seen that the sum for each oil is almost exactly ten times the value of the force for the 0.130-in. depth and 0.036-in. feed cut. This would indicate that the values given in Fig. 41 for this size of cut represent very closely a true force curve for the various oils.

Table 9 also lists the various values of the constant as determined from the general equation

$$P = C\,D^{0.95}\,f^{0.79}$$

TABLE 8 TABULATED VALUES OF FORCE ON PLANER TOOL FOR VARIOUS FEEDS AND DEPTHS OF CUT AND FOR VARIOUS CUTTING FLUIDS

(From log-log curves, Figs. 30 to 40, inclusive.)

	Cutting fluids											
	1 (dry)				2 (1.5% borax water)				3 (1-50 soluble oil)			
Feed	Cut depth, in.				Cut depth, in.				Cut depth, in.			
	0.030	0.065	0.130	0.200	0.030	0.065	0.130	0.200	0.030	0.065	0.130	0.200
0.005	...	138	280	440	...	136	276	435	...	144	280	430
0.010	124	240	485	760	112	238	475	745	124	247	485	740
0.020	216	409	818	1300	195	405	808	1275	197	435	845	1290
0.030	297	575	1150	1800	266	563	1140	1780	270	583	1160	1780
0.036	345	675	1340	2080	310	646	1310	2040	312	695	1330	2050
	Cutting fluids											
	4 (1-10 soluble oil)				5 (lard oil)				6 (mineral 100 at 100)			
0.005	...	143	280	420	...	137	270	410	...	148	285	430
0.010	132	246	485	720	120	238	470	710	134	256	490	740
0.020	228	425	840	1250	209	416	815	1260	230	442	850	1290
0.030	314	585	1150	1730	290	575	1140	1730	315	605	1170	1760
0.036	363	685	1330	2000	335	670	1320	1995	369	710	1360	2040
	Cutting fluids											
	7 (mineral 200 at 100)				8 (90% mineral—10% lard)				9 (95% mineral—5% oleic)			
0.005	...	140	282	430	...	142	280	435	...	147	280	420
0.010	118	242	492	750	124	243	485	735	127	252	485	730
0.020	205	423	850	1290	214	424	840	1280	220	440	845	1280
0.030	282	580	1160	1780	294	580	1150	1750	303	600	1160	1760
0.036	328	670	1340	2040	344	670	1340	2020	349	690	1350	2040
	Cutting fluids											
	10 (sulphurized min.)				11 (sulphur, animal)				Ave. (oils 1 to 9 inc.)			
0.005	...	117	230	365	...	120	241	385	...	142	280	429
0.010	103	204	400	630	99	208	418	670	123	234	484	736
0.020	178	353	700	1100	173	363	718	1150	215	424	835	1279
0.030	243	485	960	1520	240	495	1010	1600	292	584	1152	1762
0.036	280	567	1140	1750	273	575	1180	1850	339	678	1332	2035

TABLE 9 PLANER-FORCE DATA

Cutting fluid number	Planer Forces[1]	Specific Force[2]	Constant[3] C	Values for general equation[4] D	$D^{0.95}$	f	$f^{0.79}$
10	11,305	1130	108,200	0.030	0.0358	0.004	0.0128
11	11,768	1180	112,500	0.040	0.0471	0.007	0.0199
2	13,155	1310	125,300	0.050	0.0580	0.010	0.0263
3	13,397	1340	128,200	0.060	0.0690	0.012	0.0304
4	13,326	1330	127,200	0.070	0.0800	0.014	0.0343
7	13,402	1340	128,200	0.080	0.0908	0.016	0.0382
5	13,105	1320	126,300	0.090	0.1012	0.020	0.0455
6	13,624	1360	130,100	0.100	0.1121	0.025	0.0542
8	13,350	1340	128,200	0.130	0.1441	0.030	0.0663
1	13,462	1340	128,200	0.150	0.1649	0.036	0.0725
9	13,478	1350	129,100	0.180	0.1960	0.040	0.0786
Average 2 to 9, inclusive	13,367	1337	.....	0.200	0.2166	0.045	0.0864

[1] Sum of all forces given for each oil in Table 8.

[2] Force for 0.130 in. depth and 0.036 in. feed.

[3] Constant from general equation $P = CD^{0.95} f^{0.79}$

[4] D = depth of cut in inches; f = feed in inches.

in which P is the force on the planer tool in the direction of cut in pounds; C a constant depending on the material cut, the tool shape, and the cutting fluid used; D the depth of cut in inches; and f the feed per stroke in inches.

The Planer-Tool Chips

Representative chips retained from the various planing tests when cutting dry and with each of the cutting liquids are shown in Fig. 43. At the head of each vertical column is indicated the feed and set-up depth of cut in inches. The cuts taken were for four different set-up depths: 0.030, $^1/_{16}$, $^1/_8$, and $^3/_{16}$ in., respectively. For each depth, four feeds were used: namely, 0.004, 0.0089, 0.020, and 0.036 in., respectively. In some instances, chips were not obtained on the lightest feed. These tests were not run, inasmuch as it seemed that the force produced was too irregular to be considered reliable. The surfaces from which these chips were removed are shown in Figs. 26, 27, and 28.

It is obvious that the size and form of the chip is a function of the feed as well as the depth when cutting dry or with any of the coolants. When cutting dry, cutting fluid No. 1, there appears to be little evidence of heat, that is, chip coloration, except for the heaviest cut which had a depth of $^3/_{16}$ in., and a feed of 0.036 in. For all fluids and cuts there was a well-defined built-up nose on the cutting edge of the tool. For a feed of 0.004 in. and depths of $^1/_8$ and $^3/_{16}$ in., the chips appear to be in the form of small-diameter, low-pitch coils. The chips for the 0.004-in. feed and the $^1/_8$-in. depth are broken up for oil No. 3, but are continuous for the same feed and the $^3/_{16}$-in. depth of cut. The same condition applies for oils Nos. 9 and 10.

All chips from oil No. 10 are dark in color, apparently due to the action of sulphur rather than because of high temperatures. Most of these chips for oil No. 10 appear to be well broken up. This is of interest inasmuch as the force for oil No. 10 on all cuts is the lowest of similar cuts of all other fluids. The chips from oil No. 11 also were slightly brown, due evidently to the action of sulphur rather than high temperatures.

BEARING TESTS

A sample of each of the cutting fluids as used in the drilling tests was subjected to a lubrication test in order to determine the value of each of the cutting fluids as a lubricant. Dry lubrication was not included in these tests, although clear water was added.

These cutting fluids were tested on a bearing testing machine in which a journal rotates in a bearing. A new combination of journal and bearing was used for each test. The diameter of the journal was 1.000 in. The bearing was 1.007 in. in diameter and 1.000 in. long. Two sets of tests were run: one in which the journal was rotated at 25 r.p.m., and a second in which the journal was rotated at 100 r.p.m. The load in all cases was increased at a rate of 1000 lb. per sq. in. bearing pressure every 10 seconds.

Curves for each cutting fluid, which show the relation between the bearing pressure in pounds per square inch and the beam load on a lever arm 15.75 in. long, are given in Fig. 44 for 25 r.p.m. journal speed, and in Fig. 45 for 100 r.p.m. journal speed. Much greater loads are carried safely at the lower speed. The number adjacent to the curve indicates the number of the cutting fluid used. The curve ends at the point at which bearing seizure occurred. In Fig. 44, seizure occurred for cutting fluid No. 6 (light mineral oil) for a beam load of 300 lb. and a bearing pressure of 19,000 lb. per sq. in. for 25 r.p.m. of the journal. Fig. 45 shows that seizure occurred for oil No. 6 when the beam load was 130 lb. and the bearing pressure was 5000 lb. per sq. in. at 100 r.p.m. In Fig. 44 cutting fluids Nos. 5, 3, 2, 4, 11, and 10 caused the bearing to function satisfactorily up to and including the maximum load of 25,000 lb. per sq. in., at a speed of 25 r.p.m. Fig. 45 shows that for a speed of 100 r.p.m. none of the cutting fluids caused the bearing to function satisfactorily at a bearing pressure of over 14,000 lb. per sq. in. The actual bearing pressures at which seizure occurred for 25 r.p.m. and 100 r.p.m. are shown for each of the cutting fluids in Table 10.

Fig. 47 shows the journals at the completion of the tests referred to in Table 10. For the 100 r.p.m., seizure occurred in

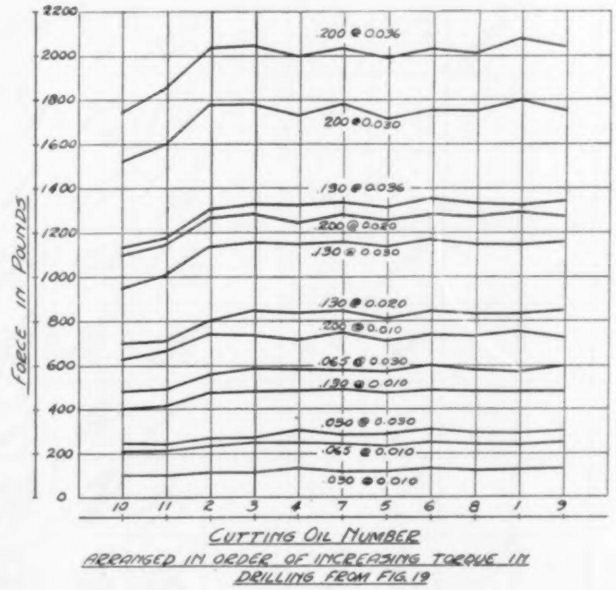


Fig. 41 Force Required to Plane S.A.E. 3150 Steel, Using Various Cutting Oils at Various Depths of Cut and Feeds

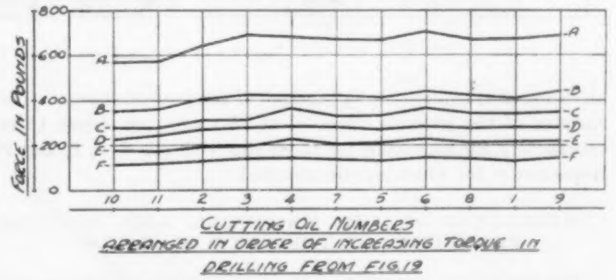


Fig. 42 Force Required to Plane S.A.E. 3150 Steel, Using Various Cutting Oils at Various Depths of Cut and Feeds (Depths (and feeds), inches: A, 0.065 (0.036); B, 0.065 (0.020); C, 0.030 (0.036); D, 0.130 (0.005); E, 0.030 (0.020); F, 0.065 (0.005).)

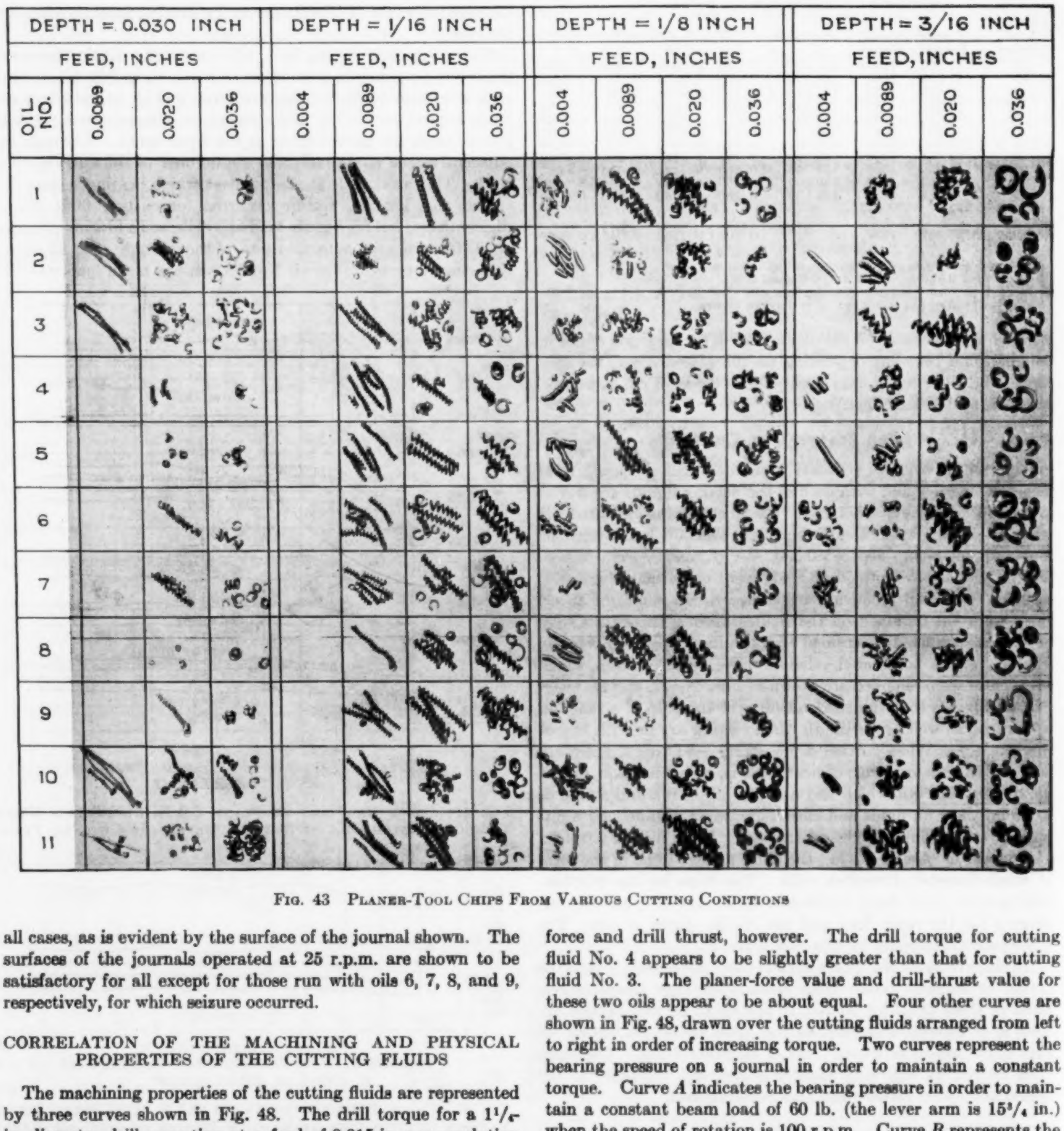


Fig. 43 Planer-Tool Chips From Various Cutting Conditions

all cases, as is evident by the surface of the journal shown. The surfaces of the journals operated at 25 r.p.m. are shown to be satisfactory for all except for those run with oils 6, 7, 8, and 9, respectively, for which seizure occurred.

CORRELATION OF THE MACHINING AND PHYSICAL PROPERTIES OF THE CUTTING FLUIDS

The machining properties of the cutting fluids are represented by three curves shown in Fig. 48. The drill torque for a $1^1/_4$-in.-diameter drill operating at a feed of 0.015 in. per revolution and a speed of 174 r.p.m. is shown plotted over the cutting fluids arranged in order from left to right of increasing torque. The thrust for the $1^1/_4$-in. drill corresponding to the torque curve is also shown plotted over each cutting fluid. The force on the planer tool in the direction of cut, when taking a cut 0.130 in. in depth by 0.036 in. feed, is shown plotted as being representative of the planer-force curves. These three machining curves by themselves show a rather close agreement.

In the drill-thrust curve, oil No. 6 shows a low value, whereas in the planer-force curve it shows an unusually high value. Also, oil No. 5, lard oil, shows a low value on the planer-force curve which seems to be out of proportion. These inconsistencies seem to occur for all other combinations of cut for the planer force and drill thrust, however. The drill torque for cutting fluid No. 4 appears to be slightly greater than that for cutting fluid No. 3. The planer-force value and drill-thrust value for these two oils appear to be about equal. Four other curves are shown in Fig. 48, drawn over the cutting fluids arranged from left to right in order of increasing torque. Two curves represent the bearing pressure on a journal in order to maintain a constant torque. Curve *A* indicates the bearing pressure in order to maintain a constant beam load of 60 lb. (the lever arm is $15^3/_4$ in.) when the speed of rotation is 100 r.p.m. Curve *B* represents the

TABLE 10 BEARING PRESSURES AT WHICH SEIZURE OCCURRED

Cutting fluid	Bearing pressure at which seizure occurs, lb. per sq. in. 25 r.p.m.	100 r.p.m.
Water	22,000	12,000
2	O.K.[1]	8,000
3	O.K.	11,000
4	O.K.	7,000
5	O.K.	13,000
6	19,000	5,000
7	23,000	7,000
8	19,000	8,000
9	17,000	6,000
10	O.K.	14,000
11	O.K.	11,000

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[1] O.K. means that seizure had not occurred at the maximum load of 25,000 lb. per sq. in.

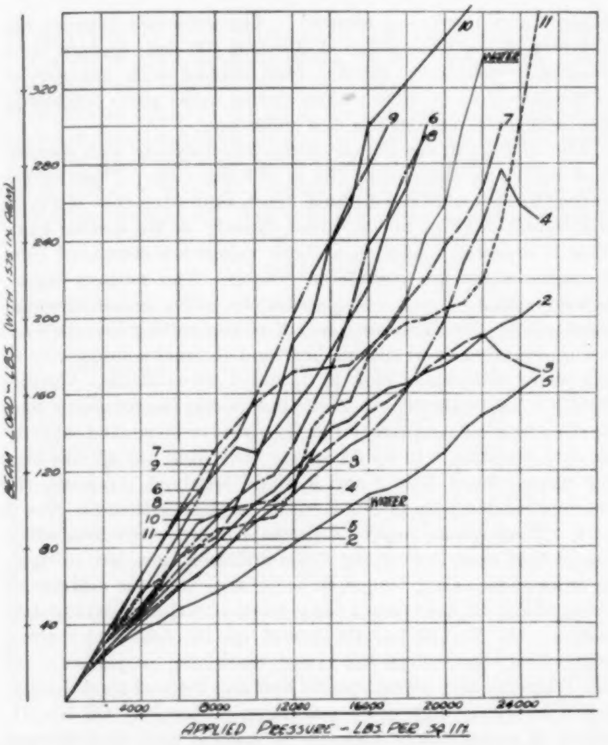

FIG. 44 BEARING-TEST CURVES FOR VARIOUS CUTTING FLUIDS (25 r.p.m. Load increased 1000 lb. each 10 sec. Journal 1 in. in diameter; bearing 1 in. long.)

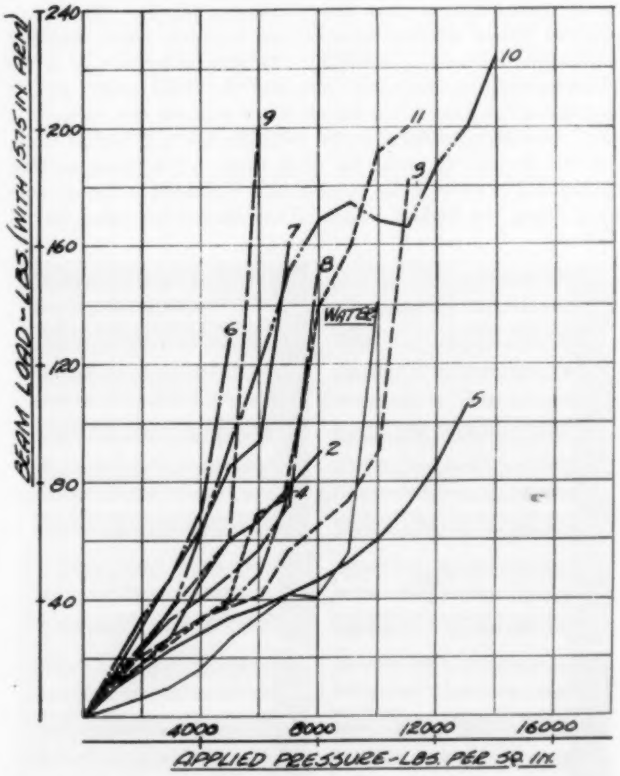

FIG. 45 BEARING TEST CURVES FOR VARIOUS CUTTING FLUIDS (100 r.p.m. Load increased 1000 lb. each 10 sec. Journal 1 in. in diameter; bearing 1 in. long.)

bearing pressure on the bearing in order to maintain a constant beam load of 160 lb. when the speed is 25 r.p.m. These two curves are very similar except for a few points. Values on curves *A* and *B* are close together for cutting fluids 10, 7, and 9. For oil 9, a lower bearing pressure is required to produce a 60-lb. beam load at 100 r.p.m. than to produce a 160-lb. beam load at 25 r.p.m.

Another curve in Fig. 48 represents the value of beam load at a journal speed of 25 r.p.m. in order to maintain a constant bearing pressure of 8000 lb. per sq. in. This curve appears to be the reciprocal of curve *B* which represents the bearing pressure for a journal speed of 25 r.p.m. in order to produce a constant beam load of 160 lb. It also appears to be more closely the reverse of curve *A*, which represents the bearing pressure at a journal speed of 100 r.p.m. for a constant beam load of 60 lb.

A curve representing the viscosity of each oil at 280 deg. fahr. is also shown in Fig. 48. This viscosity curve seems to follow the bearing-pressure

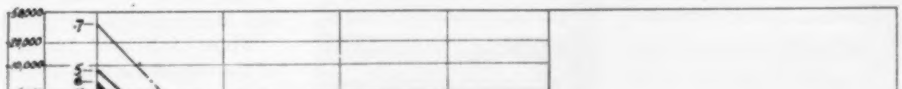

FIG. 46 TEMPERATURE-VISCOSITY RELATIONSHIP FOR CUTTING FLUIDS NOS. 5 TO 11, INCLUSIVE

curve better than it does the machining curves. There appears to be no relation between the viscosity curve and the drill-torque curve, for example. The only indication of a relation between the viscosity curve and the drill-thrust curve is that the drill-thrust is low for oil No. 6, whereas the viscosity is low. In the planer-force curve, both oils 5 and 6 appear to be out of order, as the point for oil 5 would fall between cutting fluids 2 and 3, whereas the value of oil 6 would fall at the extreme right, being the highest value. The planer-force value for oil 5 is low, while the beam load at 25 r.p.m. and a constant bearing pressure of 8000 lb. per sq. in. is low, and the viscosity and bearing pressures for constant beam load are high. The reverse is true for oil No. 6.

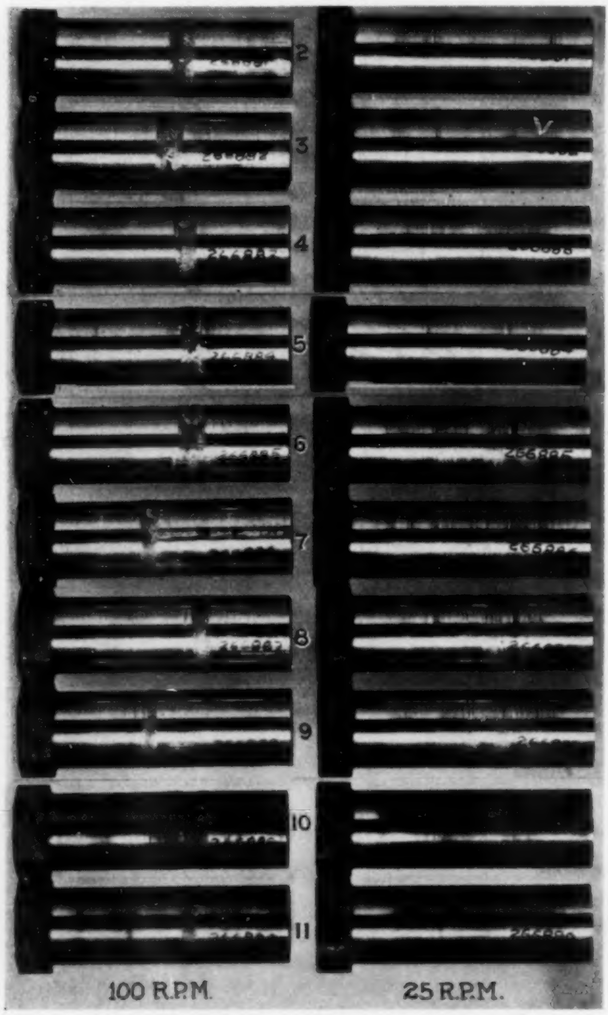

Fig. 47 Journals Used in Bearing Tests
(Figures at right refer to the cutting fluids used.)

The viscosity of the various oils used in the cutting tests at 100 deg. fahr. and 210 deg. fahr. is shown plotted in Fig. 49. The curve for viscosity at 210 deg. has no values for cutting fluids 2, 3, 4, and 1. The curve as shown has the same general shape, however, as the viscosity curve in Saybolt seconds for the oils at 280 deg. fahr., as shown in Fig. 48. The curve connecting the points of viscosity at 100 deg. fahr. is somewhat different, however. Viscosity values in Saybolt seconds for the soluble oils 3 and 4 are included in this curve, so that values for cutting fluids 2 and 1 only are omitted. The difference between the viscosity of the various oils at 210 and 100 deg. appears to be practically constant. Further tests dealing with the subject of temperatures developed when cutting under given conditions, but using these various oils, are under way.

Also in Fig. 49 are shown thermal-conductivity and specific-heat curves for the various oils at 210 deg. fahr. These curves appear to be a constant distance apart, indicating that the thermal-conductivity of an oil varies directly as its specific heat. This is expected inasmuch as both values are computed from formulas which are considered reliable. The authors regret, however, that it was not possible to make actual thermal-conductivity determinations for all of the cutting liquids used. No information as to specific heat and thermal conductivity of the water compounds Nos. 2, 3, and 4 are available. Cutting fluid No. 2 appears to have merits in reducing the force on a tool, or the power consumption, either by virtue or its cooling capacity or its lubrication. It would appear that while the specific heat for cutting fluids Nos. 2 and 3 would be about the same, the thermal conductivity of No. 2 would be greater than for Nos. 3 or 4. These specific-heat and thermal-conductivity curves which are plotted over the cutting fluids arranged from left to right, in order of increasing torque, in accordance with the drill-torque curve of Fig. 48, have only a vague relation to the machining-data curves. Oil No. 10 has the lowest specific heat and thermal conductivity as well as the lowest machining properties. Oil No. 6 has the next lowest specific heat and thermal conductivity, but is relatively high in its machining properties. Oil No. 11, which is exceptionally high in its specific heat and thermal conductivity, is exceptionally low in its machining properties. Also oils 8 and 9, which have high specific-heat and thermal-conductivity values, have the highest machining properties. Oils 7, 5, and 6, however, seem to be exceptions to the rule, inasmuch as they have low values of specific heat and thermal conductivity, but high machining properties.

Three additional curves representing lubricating properties of the various cutting oils are shown in Fig. 50. The upper curve *C* shows the bearing pressure exerted at the seizing point for the journal rotating at 100 r.p.m. This shows oils No. 10 and 5 to have approximately equal values and they are the highest of all. Cutting fluid No. 3, which is one part soluble oil in 50 parts of water, appears to be third best. The bearing pressure for cutting fluid No. 4, which is one part soluble oil to ten parts of water, is considerably lower than the value for oil No. 3. The values of the water compounds Nos. 2 and 4 appear to compare favorably with the corresponding values of the oils 7, 6, 8, and 9, which seems rather hard to explain. Small circles over cutting fluids Nos. 7, 6, 8, and 9 represent the values of bearing pressure in pounds per square inch at which seizure occurred when the journal was rotating at 25 r.p.m. Oils 7 and 6 have seizing pressures for 25 r.p.m. considerably above those for 100 r.p.m., whereas the seizing pressures for oils Nos. 8 and 9 at 25 r.p.m. are correspondingly lower.

Two beam-load curves are shown in Fig. 50 which represent for each oil the beam load at a radius of 15$^3/_4$ in. in curve *A* for a 4000-lb. bearing pressure and a journal speed of 100 r.p.m. and in curve *B* for a constant bearing pressure of 17,000 lb. for a journal speed of 25 r.p.m. The beam load representing the torque shows that the lowest torque or friction in the bearing is obtained for oil No. 5 (lard oil) in each curve. Oils Nos. 3 and 4 seem to provide the next lowest frictional loss. For the high speed of 100 r.p.m. the sulphurized lard oil No. 11 is very low, having a value equal approximately to that of cutting fluids Nos. 3 and 4. At the lower speed of 25 r.p.m. cutting fluid No. 11 has a value equal to that of No. 7, and considerably higher than the values for cutting fluids Nos. 2, 3, and 4. For the low speed of 25 r.p.m.

the beam load for oil No. 10 is slightly higher than that of oil No. 9, and yet in the machining curves oil No. 10 is shown to be superior in every case to all other oils. From curve *A*, representing beam-load values for a 4000-lb. load on the journal rotating at 100 r.p.m., the beam load for cutting fluid No. 10 is about equal to that of No. 7 and slightly less than that of No. 6. Nos. 10, 7, and 6, however, give the greatest beam loads. This does not seem to conform in any way with the results of the machining tests.

·CONCLUSIONS

Conclusions as to the results of the metal-cutting tests and the attempt to correlate the physical properties of the oils, with the results of the cutting tests, may be summarized briefly as follows:

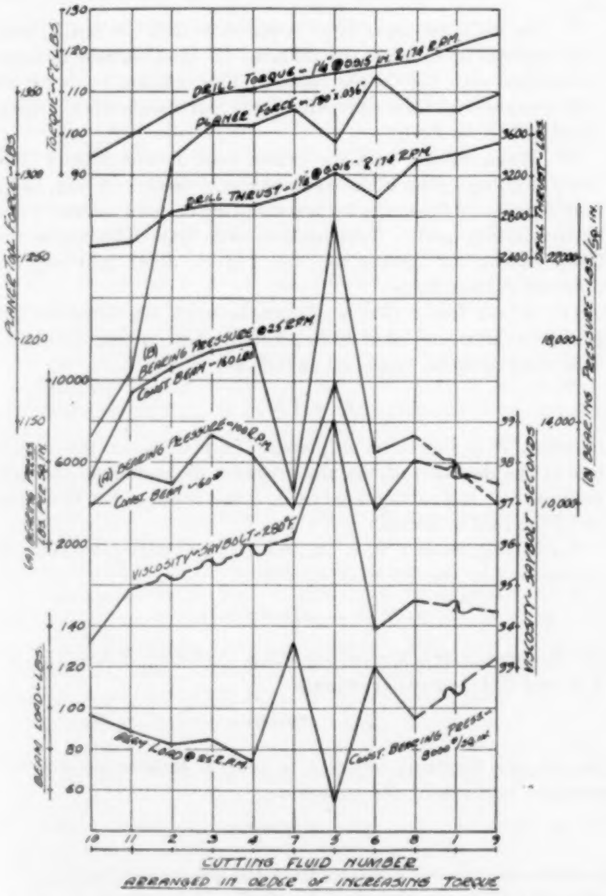

FIG. 48 MACHINING AND PHYSICAL PROPERTIES OF THE CUTTING FLUIDS

1 A newly sharpened drill produces a torque somewhat below its normal value during the first few inches of metal drilled. A normal value, or the value at which the torque remains practically constant for a considerable period of cutting, is reached sooner for a large drill diameter operating at a heavy feed than for a small drill diameter operating at a comparatively light feed. With this point in mind, newly sharpened drills were first used to drill several inches of metal before being used in tests.

2 The increase in torque and thrust appears to be less marked when drilling to almost the endurance limit of the drill if the cuts are made dry rather than with a cutting liquid. The torque and thrust are lower at the beginning of the cut when a coolant is used, but both the torque and thrust increase in value faster than when the cutting is done dry. The torque and thrust seem to be higher just before tool failure occurs, that is, after drilling many inches of metal, when a cutting fluid is used, than when cutting dry. These are conclusions which require further proof.

3 The formula representing the torque as a function of the material cut, the feed, and the drill diameter was found to be

$$T = Cf^{0.78}d^{1.8}$$

in which *T* is the torque in foot-pounds, *C* a constant depending

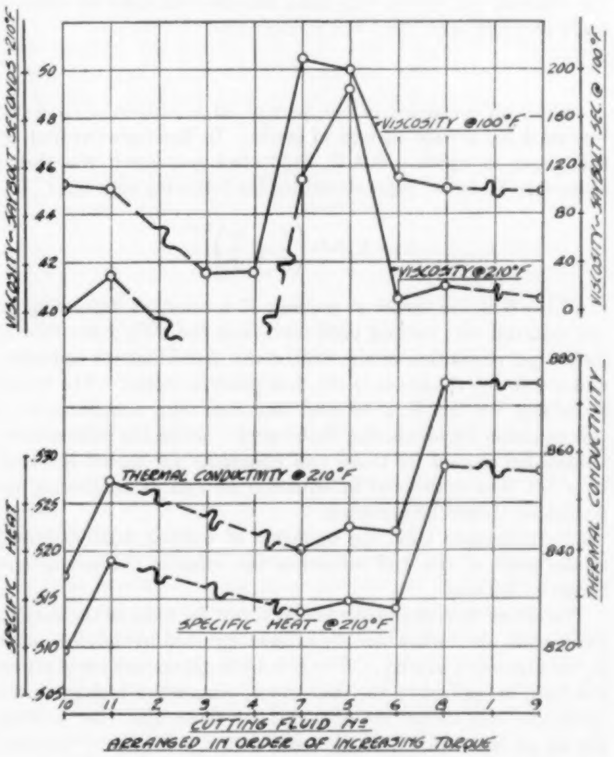

FIG. 49 PHYSICAL PROPERTIES OF THE CUTTING FLUIDS

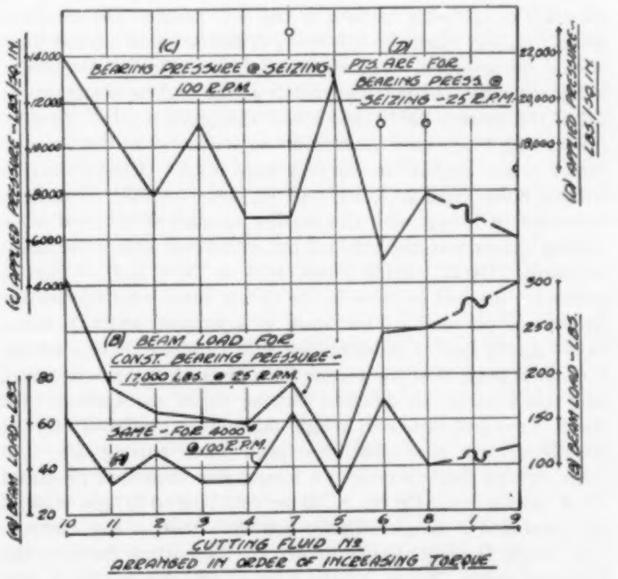

FIG. 50 PHYSICAL PROPERTIES OF THE CUTTING FLUIDS

on the material cut, the drill used, and the cutting fluid, f the feed in inches per revolution, and d the drill diameter in inches. This equation seems to hold for many types of steel and all cutting fluids used in the test.

4 In previous tests a formula for thrust as a function of the material cut, the cutting fluid, and the drill used for various feeds and drill diameters was found to be

$$B = Kf^{0.78}d$$

A soluble oil consisting of one part of oil to sixteen parts water was used for a wide variety of steels. In the tests covered by this paper in which a S.A.E. 3150 steel was used, the thrust seemed to be better represented by the following equation:

$$B = Kf^{0.87}\left(\frac{d}{5} + \frac{w}{d}\right)^{2.12}$$

in which B is the thrust in pounds, K a constant depending on the material cut, cutting fluid used, and the drill, f the feed in inches per revolution of the drill, d the drill diameter in inches, and w the web thickness at the drill point in inches. The thrust in drilling the S.A.E. 3150 steel conforms very satisfactorily to this equation for all cutting fluids used. While the difference in values determined by these two equations for thrust is small, it is felt that additional information as well as confirming information should be obtained.

5 It appears that the quantity of cutting fluid delivered at the point of the drill influences the value of the torque and thrust as follows:

The lowest torque and the lowest thrust for most of the cutting fluids were obtained when the oil was supplied to the drill point in the greatest quantity. If only a little oil, enough for continuous flow, is applied to the drill point, somewhat higher values of torque and thrust are obtained for most oils. By pouring the oil on the drill above the work at the point where the drill penetrates the work, values of torque and thrust are obtained which correspond quite favorably to the condition of a small quantity of oil being applied to the drill point. The torque is usually highest when the hole being drilled is full of oil, but when none is being added. The above would indicate that the torque is highest when the cutting fluid is at the highest temperature. All of the cutting fluids used follow this general rule. Some of the cutting fluids have practically no influence on the value of thrust when applied in varying quantities. Others, such as cutting fluids Nos. 2, 4, 10, and 11, seem actually to cause a reduction in thrust with the smaller amount of oil, and when drilling is done with the hole full, but with no oil added, the thrust is lowest. Other cutting fluids, such as Nos. 5, 6, 8, and 9, produce a marked increase in the thrust value with the smaller amount of oil used. This again is a subject which it seems would justify further investigation with a more elaborate set-up.

6 It appears that the torque for a given drill diameter, feed, and speed varies as different cutting fluids are applied. Oil No. 9 (5 per cent oleic acid in light mineral oil) produced a torque slightly greater than that developed when cutting dry. All other cutting fluids produced a torque less than that produced when cutting dry. Oil No. 8 (10 per cent lard oil in light mineral oil) produced a torque slightly less than that of dry cutting. The torque is reduced still further for the cutting fluids in the following order: No. 6 (a light mineral oil), No. 5 (No. 1 lard oil), No. 7 (a heavy mineral oil), No. 4 (a 1 to 10 soluble oil), No. 3 (a 1 to 50 soluble oil), No. 2 ($1^1/_2$ per cent borax in water), No. 11 (sulphurized-lard mineral oil), and lastly, oil No. 10 (a sulphurized mineral oil). The two sulphurized oils give the lowest values of torque under all conditions. The fact that the borax water gives the third lowest torque might indicate that the cutting fluid exerts a cooling influence rather than one of lubrication.

7 The order of the oils when arranged in order of increasing torque for a given drill diameter, feed, and speed, remains the same for all drill diameters at their respective feeds and speeds.

8 The thrust on a drill operating under given conditions of speed and feed varies in accordance with the specifications of the cutting fluid used. The thrust is reduced with the oils arranged in the same order as when the torque is reduced, except for oil No. 6 (light mineral oil), which seems to produce a thrust somewhat below the expected value for all drill sizes. The thrust for this oil is considerably less than that produced by oil No. 5, lard oil, and slightly less than that for oil No. 7, a heavy mineral oil.

9 The net horsepower input required to drill the S.A.E. 3150 steel appears to vary for the different oil specifications in exact accordance with the torque values. This relation holds for all drill diameters at their respective feeds and speeds when drilling the S.A.E. 3150 steel.

10 When cutting with the planer tool, it was noticed that every chip, regardless of the combination of depth and feed, had a built-up edge on the tool point when cutting dry and with all of the cutting liquids used. There appears to be little difference in the finish left by the cutting tool for a given depth and feed for different cutting fluids.

11 It was found that a relation between the force on the planer tool at the speed of 20 ft. per min. when cutting the S.A.E. 3150 steel could be expressed as follows:

$$P = CD^x f^y$$

in which P is the force or pressure in pounds on the planer tool in the direction of cut, C a constant depending on the tool, material cut, and cutting fluid used, D the depth of cut in inches, and f the feed in inches.

For cutting fluids 2, 3, 7, 10, and 11, this relation was closely represented by the following equation:

$$P = CD^{0.98}f^{0.79}$$

For the remainder of the cutting fluids consisting of Nos. 1, 4, 5, 6, 8, and 9 this equation becomes

$$P = CD^{0.93}f^{0.79}$$

For all oils together, however, a general satisfactory relation seems to be given by the equation

$$P = CD^{0.95}f^{0.79}$$

in which the value of C ranges from 108,200 for cutting fluid No. 10 to 129,100 for cutting fluid No. 9 (See Table 9).

12 In planing with any given cut there appears to be little difference in the planing force for the cutting fluids as a group with Nos. 10 and 11 omitted. For this same group, the drilling torque and thrust for all drill diameters at their respective feeds and speeds showed a definite influence of the cutting fluid.

13 The curve representing the values of drill torque over the cutting fluids seems to conform very closely to the curves representing planer force and drill thrust; that is, the properties of the cutting fluids seem to be equally well represented by any one of the machining-data curves.

14 The torque or beam-load curves, representing the frictional resistance of any cutting fluid used as a lubricant in a bearing, show similar characteristics for various speeds and loads. The beam-load curve at 25 r.p.m. for a constant pressure of 8000 lb. shown in Fig. 48, when plotted to the same scale as the beam-load curve for 25 r.p.m. and 17,000 lb. bearing pressure shown in Fig. 50, appears to be much more horizontal. The

beam load for the first five cutting fluids at the left consisting of Nos. 10, 11, 2, 3, and 4 present nearly a horizontal line. The values for all of these five cutting fluids are lower than all other values, except that for oil No. 5. Except for oils Nos. 10 and 11 these beam-load curves show an approximate relation to the machining-data curves.

15 The bearing-pressure curves at a given r.p.m. and constant beam load seem to show no relation to the machining-data curves.

16 The oil viscosity curves at various temperatures show similar characteristics. Lard oil and those oils containing as little as 10 per cent lard oil show a tendency to change their viscosity less for a given change of temperature than the other oils. The slopes of the viscosity-temperature lines for oils Nos. 5, 8, and 11 in Fig. 46 are less than the others.

17 The bearing pressures required to cause seizing of the bearing when plotted over the cutting-fluid numbers arranged in order of increasing torque from left to right, show a general slope downward to the right for speeds of 25 and 100 r.p.m. While the curve for bearing pressures for a journal speed of 100 r.p.m. fluctuates a great deal, it slopes down generally to the right, whereas the machining curves slope generally up to the right. This might indicate that those oils which permit seizing at a low pressure produce high power consumption in metal cutting.

18 The specific-heat and thermal-conductivity curves shown in Fig. 49 appear to be of no assistance in the solution of the metal-cutting problem. The fact that the sulphurized oil No. 10 has the lowest specific heat does not seem to agree with the fact that it produces the lowest power consumption in metal cutting, particularly in view of the fact that the borax water which presumably would have the highest specific heat also produces a very low power consumption.

ACKNOWLEDGMENTS

Grateful acknowledgment is made to Messrs. Charles E. Kraus, R. Paul Harrington, and William V. Foster, who ran the tests and prepared the data for final analysis.

Materials for the project in the form of drills, planer-tool bits, and cutting fluids were furnished gratis by National Twist Drill and Tool Company, Detroit; Columbia Tool Steel Company, Chicago; Standard Oil Company of Indiana, Chicago; Sun Oil Company, Philadelphia; Atlantic Refining Company, Philadelphia; Swift & Company, Chicago; D. A. Stuart Co., Chicago.

The authors wish also to express their appreciation to Mr. H. J. Saladin, of the Standard Oil Company of Chicago, and to Mr. W. C. Mitchell, of the General Motors Company, Detroit, for many valuable suggestions received and for furnishing experimental data dealing with physical properties of the oils.

The funds for carrying out this project were made available to The American Society of Mechanical Engineers by the Engineering Foundation. These funds then were turned over to the Department of Engineering Research of the University of Michigan, where the work was conducted as a research project under the direct supervision of the Subcommittee on Cutting Fluids of the A.S.M.E. Special Research Committee on the Cutting of Metals. The experimental work was carried on in the Department of Engineering Shop of the University of Michigan.

Discussion

F. T. HARRINGTON.[3] This paper is a valuable piece of engineering investigation, and the limited field that it covers serves to emphasize the enormity of the task which confronts the Subcommittee on Cutting Fluids of the Society's Special Research Committee on the Cutting of Metals. The paper covers very completely the effects of the commonly used cutting fluids on the power required to drill and plane this one material, i.e., annealed S.A.E. 3150 steel.

The writer will attempt to confine his discussion to a few questions relative to the paper.

(1) What explanation can be offered for the fact that both torque and thrust increase more rapidly during the first 70 in. of drilling when No. 4 cutting fluid is used than when drilling dry? This is covered by graphs in Figs. 6 and 5 and on the sixth page of the text. This is also mentioned in conclusion 2.

	Dry	With oil No. 4
Thrust of 1-in. drill at start of test, lb.	1790	1700
Thrust of same at end of 70-in. drilling, lb.	1900	1925
Torque of 1-in. drill at start of test, ft-lb.	72.5	67.5
Torque of same at end of 70-in. drilling, ft-lb.	74.5	72.0

(2) The two endurance or "breakdown" tests just cited confirm actual practice, for it is common knowledge that the life of a drill is prolonged by the presence of a cutting oil as compared with running dry, but they also show that the drill can be run at increased torque and thrust loads without failure because of some property of the cutting fluid used.

(3) The paper points out that the formulas derived for torque and thrust when using oil No. 3 by the method illustrated in Fig. 7 and explained on the sixth and seventh pages, are not theoretically correct for all of the other cutting compounds used when drilling S.A.E. 3150 steel. Are these formulas sufficiently accurate for other steels? It is stated in item (3) in the conclusion that the equation for torque seems to hold for many types of steel. Were any tests conducted using other steels to verify this conclusion? Is it believed that these formulas will hold for cast iron or nonferrous metals? Will the formulas be applicable for other cutting compounds and other ratios of coolant constituents? Based on the findings to date, do the authors expect the constants and factors for torque values as listed in Table 3 to apply when the respective coolants are used with other steels?

(4) Will the research project be extended to cover other coolants and other proportions of the coolant constituents already used, in an effort to determine what combinations approach the ideal compound for the material and machining method employed?

(5) Is the formula for torque ($T = Cf^{0.78}d^{1.8}$) applicable at peripheral drilling speeds other than 60 ft. per min.? Were any tests made to confirm this fact? This question is asked because in most instances where the formula is shown, reference is made to the cutting speed of 60 ft. per min. Is cutting speed necessarily an essential factor in the torque formula or does it only enter into power considerations?

(6) How was the slope of the expression $(d/5+W/d)$ determined in Fig. 7 where it forms a part of the line represented by the equation $B \sim (d/5 + W/d)^{2.12}$?

(7) What explanation may be given for the fact that values of gross horsepower for oil No. 8 as represented graphically in Fig. 19 and in Table 6 are greater than for compound No. 1 when computed values for both torque and thrust for No. 1 fluid exceed those for No. 8?

(8) These remarks will not include the portion of the paper devoted to the planer tests, but the writer would like to inquire as to whether the cuts showing examples of the planer work contained in Figs. 26, 27, and 28 are to exact size. With reference to these same cuts, the text on the fifteenth and sixteenth pages explains that both Figs. 26 and 28 show the start of the tool cut. What explanation may be offered for the fact that with a depth of cut of $^1/_8$ in., as used in Fig. 28, there is little indication of the tearing of the metal at the start of the cut when compared with the pronounced condition along the lower edge of Fig. 26? The text also explains that the lower edge of Fig. 17 represents clearly how the chips where broken off at the end of the cut, but the photo-

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[3] Whitman & Barnes, Inc., Detroit, Mich.

graph from all appearances is the same along both edges. What does the text have reference to in this instance? It appears that the lower edge of Fig. 26 is the finish of a cut and shows how the tool breaks out of the steel.

(9) Based on tests using the oil-hole drill, conclusion 5 states that, with certain mentioned exceptions, the application of large quantities of coolant tends to reduce the torque and thrust together with the heat produced by drilling. What explanation could be given for the tendency for borax water (No. 2 compound) to show the exact opposite results in so far as thrust to this more general axiom? This is illustrated in Fig. 11.

(10) In the lubrication tests cited, from what materials were the bearings and journals made? Would the tests have been of more value if the journals were made of hardened high-speed steel and the bearing of the annealed S.A.E. 3150 material?—if the journal had been revolved at 231 r.p.m. or 60 f.p.m. to approximate a 1-in.-diameter drill? Were the bearings completely immersed in the oil during the test?

(11) In the first part of the twenty-third page, in a comparative discussion of constant beam loads and variable bearing pressures, the statement is made, "For oil 9, a greater bearing pressure is required to produce a 60-lb. beam load at 100 r.p.m. than to produce a 160-lb. beam load at 25 r.p.m." This statement is in error, because data taken from the curves on Figs. 44, 45, and 48 show the bearing pressure to be 10,000 lb. for the 160-lb. beam load at 25 r.p.m. and 5000 lb. for the 60-lb. beam load at 100 r.p.m. This statement no doubt originated from the fact that the two constant beam-load curves cross on Fig. 48, but they are plotted to different scales.

(12) From the constant bearing pressure curves in Fig. 50 it appears that No. 10 oil is among the poorest oils in so far as lubricating qualities are concerned, and yet it withstands the greatest bearing pressure before seizure, and also is the best coolant for drilling because it gives minimum values for torque, thrust, and power. How can the high bearing pressure at seizure be associated with the poor lubricating qualities of this oil?

In conclusion 17 it is stated that the seizure curve indicates that oils which permit seizure at low pressures produce high power consumption during drilling. This deduction appears to be quite logical. Could this fact be associated with a tendency for the material being drilled to build up on the cutting edges and margins of the drills while working, which might be compared with a similar action which takes place in bearing seizure? The photograph in Fig. 47 shows this action in the case of journals used during the bearing tests. If this reasoning is correct, then the coolant, which in this project proves to be No. 10, that withstands the highest bearing load before seizure, will tend to keep the steel being drilled from building on the cutting edges and margins, which in turn would roughen these surfaces and increase torque and power during drilling.

(13) It might be suggested that an attempt be made to determine torque, thrust, and power values for drilling when the drill approaches the point of failure or "breakdown" instead of when it is newly pointed. It is believed that the project might have more practical value if these characteristics were secured when the drill approached a "dull" condition, because it is these maximum values, as acknowledged in conclusion 2, that would benefit machine-tool design. It is appreciated that this would increase the work on the project tremendously, but the writer is merely making the suggestion for what it is worth. In this same connection the determination of the cutting fluid which gives ultimate tool life would be of considerable value from a practical standpoint.

(14) The writer might also suggest that during the course of further investigations of cutting fluids less attention be directed to planing, as the majority of planing and shaping is done without a coolant. For this reason it is believed that the project would be of greater value if more emphasis were placed on the common methods of machining metals where coolants are normally used in practice. These include turning, milling, and drilling.

C. H. Casberg.[4] This is a very interesting and valuable paper and the authors should be commended for their good work.

On the eighth page, second column, middle of second paragraph in the sentence, "The values of the thrust as shown in Fig. 10 show a tendency to increase in value," it is believed that a typographical error has been made, and it should read, "The values of the torque," etc. [Corrected.]

It would have been better to have left out the discussion on what the authors call "net power input."

On the bottom of the thirteenth page and the top of the fourteenth it is explained what is meant by the "net power input."

The tare power is obtained by running the machine at the speed and feed and the reading taken of the wattmeter. If the authors assume that this is all power consumed by the machine while drilling, the assumption is not correct. In fact, as soon as the drill starts cutting at this given speed and feed the machine consumes more power because the load on the bearings and the gears increases. Losses in the motor also change with the load.

The power consumed by the machine if the machine is loaded to its capacity will be surprising because thrust of the drill will cause some of the bearings to be thrown out of alignment. If the authors wished to obtain the power at the drill point, it should have been computed from the torque and thrust. By using the values of the torque and thrust given in Tables 3 and 5 the power consumed at the drill point can be computed. The power consumed by the drill point of the $^1/_2$-in. drill is, approximately, for oil No. 10, 1.05 hp. The net horsepower for oil No. 10 in Table 7 is 1.174, or a difference in power consumption of about 0.12 hp.

The net power as shown in Table 7 for oil No. 10 for the $1^1/_4$-in. drill is 4.081, while the power consumed at the drill point is approximately 3.14, a difference of approximately 1 hp.

For oil No. 9 from Table 7 the net horsepower is 5.544 hp. and the power at the drill point is approximately 4.15 hp., a difference or about 1.4 hp., which goes to show the greater the load on the machine the more power loss in the machine due to friction of gears and bearings and distortion of the machine. The curves of net power (Fig. 22) pertain only to the machine upon which the tests were run and would not be of much value if applied to other machines. If the values of the power at the drill point were plotted on this curve, it is doubtful if the curves would be a straight line.

The foregoing is of minor importance compared with many interesting facts brought to light by this paper.

Fred H. Colvin.[5] One factor which affects the performance of cutting fluids and many other things is imagination on the part of the operator or foreman, or both. There are many instances where putting the same fluid in a different container has made it either much better or decidedly worse than it was before, in the opinion of the operator.

Authors' Closure

By O. W. Boston. The authors are indeed pleased to note the interest in this investigation as evidenced by the numerous and valuable comments. They wish to express their appreciation for those who have assisted in bringing out points which were not clear or in emphasizing various phases of the paper, or in adding the results of their own experiences.

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[5] Editor, *The American Machinist*, New York, N. Y. Mem. A.S.M.E.

The discussion by Mr. F. T. Harrington raises a number of points, which are answered corresponding to the question numbers as follows;

(1) The fact that torque and thrust values increase more rapidly when drilling with cutting fluid No. 4, 1 to 10 soluble oil, than is the case when drilling dry, cannot be explained at this time. The results as shown in Figs. 5 and 6, together with those in Fig. 4, appeared sufficient for the purpose for which they were intended; that is, to determine how long a drill may be used and still be considered sharp. The two similar dulling tests referred to, as shown graphically in Figs. 5 and 6, were run with different drills, both 1 in. in diameter, the "R" drill being used for the cutting dry test and the No. 44 drill for the tests with the No. 4 cutting fluid. The drills were nearly identical in shape, and the authors feel that the variance in results is due to the difference in cutting fluids used rather than any difference between the drills. The results indicate that while the torque and thrust values are less at the start, there is a greater rate of increase when the No. 4 cutting fluid is used than when cutting dry. The authors feel that extensive tests to determine the influence on the rate of dulling of various cutting fluids would be of considerable value applied to drilling, inasmuch as technical information regarding this subject as well as that of drill life is inadequate. The limited funds available for this project did not permit diverting too freely from the main object of the paper.

In some recent tests comparing the cutting qualities of the same 11 cutting fluids used in these tests, a hacksaw was used. It has been found that the time required to saw a given specimen increases at varying rates as the saw dulls, using the different oils. With the data so far at hand, the conclusion is reached (perhaps prematurely) that borax water (cutting fluid No. 2) has more effect than all other cutting fluids in increasing the time required to saw a given cross-sectional area. These results will be presented later.

(2) No comment necessary.

(3) The equations given for torque and thrust have been found to hold quite accurately for some 18 carbon and alloy steels. The derivation of these formulas was presented in two previous papers by the authors—"Power Required to Drill Cast Iron and Steel," Trans. A.S.M.E., MSP-52-2, January to April, 1930, and "Torque, Thrust, and Power for Drilling," *Jour. S.A.E.*, Vol. 28, No. 3, March, 1931. These formulas do not hold for cast iron or malleable iron, non-ferrous or non-metallic materials, each of which seems to have formulas characteristic of the materials. For drilling and in general for planing and milling, the effect of the cutting fluid used is only to change the constants in the equation as long as the material is constant. The exponents of feed and depth of cut do not change. When materials other than S.A.E. 3150 steel are cut, a different order of the oils is found which indicates the ease of machinability, and different constants and factors are therefore necessary. (A report of results on a wide variety of metals being drilled with each of the 11 cutting fluids is now in preparation and will be published soon.)

(4) It is intended, if funds permit, to extend in the near future the investigation to other coolants in the hope of learning more of the influences of various constituents.

(5) Torque and thrust are independent of speed, as reported on in the paper MSP-52-2. In order that the so-called commercial tests might be made on a basis comparable to commercial practice, the drills of different diameters were all rotated to give a peripheral speed as near as possible to 60 ft. per min. Other tests, however, in which the speed was varied from 74 to 500 r.p.m. for various drill diameters indicate that the torque and thrust remain constant throughout this range. When power such as net power output is considered, however, the speed must be known, inasmuch as the power varies directly as the speed. The torque times the angular velocity of the drill gives about 99 per cent of the net power output.

(6) The exponent of a variable is found by taking a tangent of the angle made by the line through the experimental points plotted on log-log paper with the horizontal. The expression $(d/5 + W/d)$ was determined by the cut and try method.

(7) Differences in gross horsepower as determined on the wattmeter usually can be traced to an error in reading the value on a comparatively small scale. Also, frequently the datum line of the wattmeter varies between one test and another. These errors usually are small. Often the zero reading on the wattmeter changes slightly because of the weight of the ink in the pen and the friction between the pen and the chart. The consistency of the grease and oil used in lubricating the motor and drill press as affected by temperature and use affect the input power value. In our experience, the wattmeter readings are not as accurate as those representing the output power at the drill point, such as the torque and thrust.

(8) The photographs represented by Figs. 26, 27, and 28, as given in the preprinted paper, were mixed and also mutilated. These have been corrected and now agree with the comments on the manuscript.

(9) The author's knowledge of this phenomenon is confined to the experimental data presented, and no additional explanation can be given at this time.

(10) The pins and bearings used in the lubrication tests were made of carburized low-carbon steel hardened and ground. Changes in composition of the steel, according to other tests in the laboratories of the General Motors Corporation, would make very little difference in the results, providing the steel was hardened to an equal degree in all cases. If the steel should be run against other materials, such as bronze or cast iron, the results would be radically different.

(11) Mr. Harrington's statement is correct, inasmuch as Fig. 44 shows that for a 160-lb. beam load at 25 r.p.m., the applied pressure is 10,000 lb., whereas for a 60-lb. beam load for oil No. 9 at 100 r.p.m., as indicated in Fig. 45, it is about 5000 lb.

(12) The fact that oil No. 10, a sulphurized mineral oil, gives the greatest pressure at seizure indicates good lubricating qualities, while the fact that it gives almost the highest beam load for a 17,000-lb. bearing load at 2500 r.p.m. indicates that this is a poor lubricant. It apparently follows that two things are involved—frictional resistance and oil film. The oil appears to be good from the fact that it does maintain an oil film at high pressure, but it is poor from a power-lubrication standpoint. This may be due to the condition of the sulphur in the sulphurized mineral oil, as the sulphurized lard-mineral oil No. 11 has somewhat the reverse influence; that is, for high bearing load at seizure, it still gives low frictional resistance.

(13) No comment.

(14) The planer was used in these tests inasmuch as it was already equipped with a dynamometer for measuring the force in the direction of cut. Further, this dynamometer measures with a considerable degree of accuracy.

Additional equipment for tests of this nature have now been installed, such as a heavy-duty engine lathe on which is mounted a Losenhausen force dynamometer which measures three components of the cutting force and records them graphically. This same lathe is equipped with a potentiometer device so that temperatures between the tool and work can be measured by the tool-work thermocouple. Simultaneously with the recording of these two values, tool life can be obtained. It is expected that the lathe will be used extensively in the future for tests of this nature, and it will replace the planer. Already considerable information on the use of cutting fluids in milling has been determined as a separate project and will be published soon.

Professor Casberg's correction in his second paragraph is correct. The second paragraph of the second column on the eighth page should read "The values of the torque" rather than "The values of the thrust."

Professor Casberg's suggestion that the discussion on "net power input" be omitted is not in agreement with the authors. This information was obtained for all of the tests, and it was felt that it was all valuable inasmuch as it represented, nearer than any other value, the net power output. This close agreement is shown throughout. It naturally is a little higher due to frictional resistance, etc. The method of obtaining the tare was completely described in the paper, Trans. A.S.M.E., MSP-52-2. It was therefore not thought desirable to repeat that in this paper, inasmuch as this paper is primarily to show by comparison the merits of various cutting fluids when cutting one steel.

Professor Casberg has mentioned the influence of losses in the motor as well as increased friction in the machine due to change in the load. These statements are correct. Actual efficiency of net output over net input power is given in the first paper for various drill sizes operating at different r.p.m. This shows the highest efficiency to be for the smallest drills operating at the highest rotating speeds. The rotating speeds were based on a peripheral speed of 60 ft. per min. This indicates that the increase in net input power is due primarily to load rather than speed.

The computed values of efficiency based on net power output as compared with the net power input for the $^1/_2$-in. drill and the $1^1/_4$-in. drill agree very favorably with the values given in Table 6 of the paper MSP-52-2.

The authors do not understand Professor Casberg's statement that the values of power at the drill point as plotted would not give straight lines. The torque values do give straight lines as indicated in Fig. 12. The net output power at the drill point is, approximately, $99^1/_2$ per cent due to the torque. Therefore, the net power output values should give approximately straight lines. It apparently is a coincidence that the net power input curves shown in Fig. 22 are straight lines, due to the fact that for all values there is a slight frictional or motor horsepower loss included.

MSP-54-3

Preloaded Ball Bearings as Applied to Machine-Tool Spindles

By H. E. BRUNNER,[1] NEW YORK, N. Y.

In attaining precision of operation of machines fitted with ball bearings, after all initial looseness had been removed and dimensional tolerances held to very close limits, it was found that another factor remained which prevented attaining the high degree of accuracy of dimension and finish required of precision machine tools. This was that elastic deformation in the ball bearing had a great effect on quality. The cure was effected by preloading, this initial loading serving to remove all looseness between the balls and the raceways, as well as the more objectionable effects of early deformation that develops in a ball bearing under load. The proper value of the preload can be determined only after full consideration of the work loads involved.

UNTIL recent years the use of anti-friction bearings in machine tools was restricted to those parts which did not directly affect the work done. On the work spindles the high degree of accuracy to which plain bearings can be built had invariably militated against the use of anti-friction bearings built to the less accurate dimensions and tolerances laid down by the different ball- and roller-bearing standardization bodies. It was not until 1920 that earnest and well-considered efforts were made to produce anti-friction bearings with finer tolerances and a closer degree of precision than are required for the general run of machine applications. The advance made in this period of approximately ten years, however, has increased the appreciation of the machine-tool manufacturers for anti-friction bearings and warrants a fuller understanding by the industry in general of the accuracy and advantages that are now obtainable from a properly designed mounting.

The anti-friction bearing, by virtue of its construction, introduces an element of error that is not present in the plain sleeve type of bearing; viz., "run-out." In an anti-friction bearing some form of raceway must be applied to the spindle, with rolling elements interposed between the inner and outer races. Though the utmost care be exercised in the manufacture of the raceway and rolling elements, some degree of "run-out" is inevitable. On the other hand, with a sleeve type of bearing, no parts need ordinarily be interposed between the spindle and the bearing surface, thus eliminating any error due to run-out.

As a starting point, therefore, in the production of anti-friction bearings that would have precision qualities comparable with the well-designed plain bearings, attention was first directed to the determination of the type of raceway and rolling elements that would lend themselves best to minimizing run-out. It was soon apparent that a ball was the logical choice, as it can be produced today with exceptionally fine equipment enabling the restriction of all dimensional errors to within 0.000025 in. and at the same time producing a smooth surface. Having settled upon the selection of the ball as the most desirable rolling element from the standpoint of dimensional accuracy, a further advantage was recognized in that it was the one practicable geometric shape that did not require some form of guiding aside from some conventional shape of raceway. Attention was next directed toward improving the tolerances on all dimensions of the ball bearing that might affect the accuracy of operation, such as bore, outside diameter, width, and parallelism of grooves with face and squareness of faces with bore, along with a better finish on the surfaces of raceways.

The improved tolerances obtained were a decided advance in the right direction, but there still remained further opportunity for improvement. Experience derived from the use of bearings showed that looseness also seriously affected the quality of finish on work produced. The next step was to remove all looseness or clearance between the balls and races so as to obtain an internal fit approximating a line-to-line contact. When this was done, a further improvement was noted, but still this tighter and very precisely made ball bearing fell short of meeting the exacting demands of present-day machine tools where comparatively heavy cuts and a high quality of finish under light cuts were required.

Necessity for Preloading

Although all initial looseness had been removed from the bearings at the time of manufacture and the dimensional tolerances held to very close limits, it was found that still another factor entered into the problem to militate against success and which was responsible to a large extent for the failure of the bearings to meet the high degree of accuracy of dimension and finish required of precision machine tools. It became evident that the elastic deformation in the ball bearing had its effect on the quality of finish and dimensional accuracy of work produced. Where internal looseness existed, chatter resulted on the finished work due to vibration in the spindle.

It will readily be appreciated that a bearing having radial or axial looseness will reflect all existing run-out error. By applying proper preloading, all early deformation or what may be termed "sponginess" is removed. This initial loading serves to correct or minimize the run-out error and increase the rigidity of the spindle because of the elastic character of the materials. Consequently the dimensional accuracy and quality of finish fully compare with results obtained from sleeve bearings.

Methods of Preloading

Preloading a bearing has therefore come to mean the applica-

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Presented before a meeting, Cincinnati, Ohio, January 20, 1931, of the Cincinnati Section of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

tion of an initial load that will remove all looseness between the balls and raceways as well as the more objectionable effects of early deformation that develops in a ball bearing under load. The proper value of the preload can of course be determined only after a full consideration of the work loads involved. A variety of methods were developed to load the bearings initially, but with varying degrees of success, due to the fact that almost all of them involved the necessity of manual adjustments that were dependent upon the skill of a mechanic or his sense of feel. To load a ball bearing excessively through the adjustment was equally as objectionable as an adjustment which failed to cause sufficient initial yield.

A few of the principal methods that have been tried and found not fully satisfactory will be touched upon.

Loading by Threaded End Covers. This was one of the earliest methods of imposing an internal load on bearings. Applying one bearing at each end of a spindle, the outer races were displaced either inwardly or outwardly by using threaded end covers. It was found that although this might prove satisfactory occasionally when the adjustment was carefully made, the method would not permit the application of an exact predetermined load, and in many cases the bearings were loaded beyond their safe capacities.

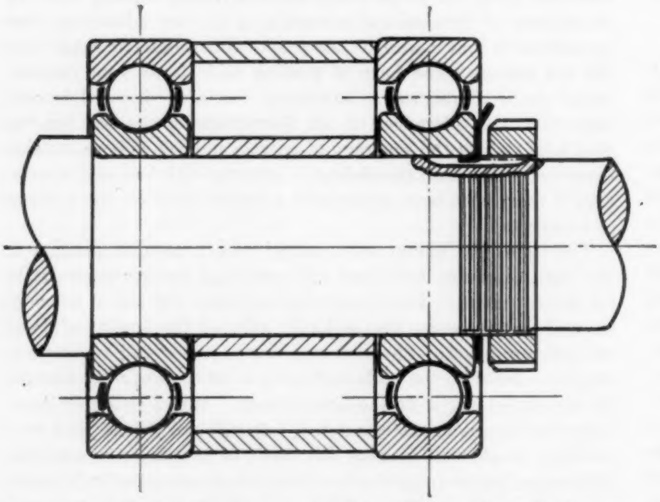

FIG. 1 TWO PRELOADED BEARINGS WITH SPACING SLEEVES PREPARATORY TO BEING LOCKED IN PLACE

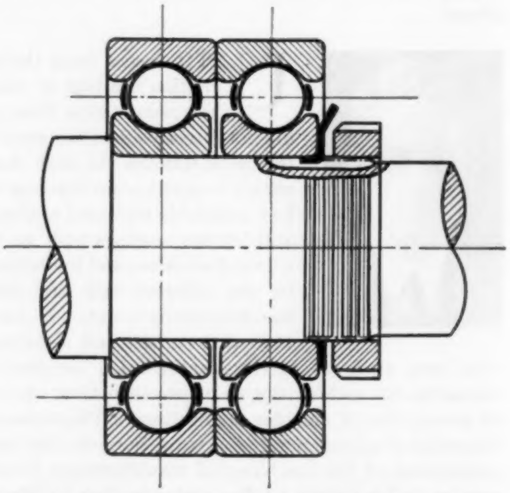

FIG. 2 TWO PRELOADED BEARINGS WITHOUT SPACING SLEEVES PREPARATORY TO BEING LOCKED IN PLACE

Loading by Springs. This method aimed to accomplish the same result by loading the bearing through the outer races. With this method a small initial load could be imposed sufficient for light duty or where the load was uniform, but it was totally inadequate on applications where the load was fluctuating owing to the varying characteristics of springs.

Loading by Oversize Balls. A small initial load can be applied by using oversize balls, but this method is greatly limited by the danger of damaging the carefully made balls and raceway surfaces during the assembly of the bearing. A further limitation of this method is the weak control the manufacturer will have in producing the required deformation through having to resort to measurements—in other words, being dependent upon manual adjustments. Hence, although occasionally successful on light-duty work, such bearings were entirely unsuited to the average run of precision applications.

Loading by Unequal Length Spacing Sleeves. This method in the development of a satisfactory preloaded ball bearing utilized two single-row ball bearings at each end of a spindle with steel sleeves of unequal length between the outer and inner races. In this way the races were displaced axially and an internal load was imposed on the bearings. The method has been found wanting for a number of reasons. It imposes on an ordinary assembly department in a plant the responsibility of machining and fitting the sleeves which is properly the work for a precision-machine department. It also necessitates relying on the judgment of the workman assembling the unit to determine the proper adjustment, resulting in a wide range of preloading with a consequent effect on the uniformity of operation. A further weakness is the fact that if replacement of bearings is required there is no positive assurance that the same degree of loading as originally built into the unit by the machine manufacturer will be duplicated, because each particular pair of bearings possesses its own individual tolerance characteristics. In any event the method requires calculations and measurements to determine the proper length of sleeves necessary to produce the desired preloading.

EXACT PREDETERMINED PRELOADING

For a ball-bearing spindle mounting to be entirely satisfactory it must not only be possible to load it with an exact predetermined load, but it should also be possible to freely interchange bearings of the same type without altering the proved requirements of the spindle. These essential requirements are met only in the type of ball bearing about to be described which is loaded at time of manufacture with a definite predetermined load. This advance in bearing design has resulted in removing the problem of adjustment from the field of uncertainty so that it is now possible to obtain the exact degree of tightness necessary for smooth and accurate spindle operation.

The principle followed in the manufacture of the preloaded ball bearing is to displace the inner race with respect to the outer race under a predetermined thrust load and to surface-grind the face of the projecting inner race until it is flush with the side of the outer race. The bearing is then reversed and the procedure repeated by applying the fixed thrust load to the inner race again, but in the opposite direction. When two such bearings are assembled on a spindle and drawn up until the inner races are in contact, the predetermined thrust under which the bearings were ground is reproduced exactly within them.

Fig. 1 shows two bearings on a spindle preparatory to being locked in place, with spacing sleeves of equal length between the outer and inner races. Fig. 2 illustrates the same two bearings mounted on a spindle with the outer races in contact preparatory to being drawn up by a locknut and lock washer. Fig. 1 is the more preferable method through the increased rigidity obtained, but where space limitations prevail Fig. 2 can be used.

Analysis of Deflection and Stresses

An analysis of the deflections and stresses in a pair of preloaded ball bearings will make clear the principles involved. If a ball bearing under thrust load acted the same as a piece of steel under compression in a testing machine, the yield under increasing loads would follow a straight line up to the elastic limit of the material—Hooke's Law (see Fig. 3). However, a ball bearing has a distinct advantage in this respect because as the load increases the supporting areas also increase from point contact at no load to comparatively large ellipses at higher loads. Hence, the deflection of a ball bearing will become less and less under equal increments of loads.

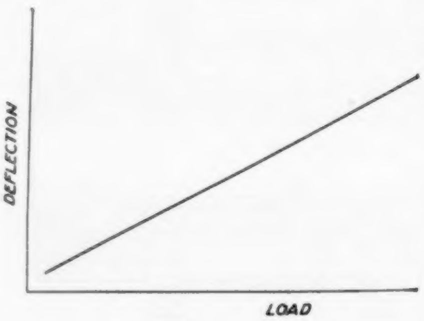

Fig. 3 Hooke's Law

Fig. 4 shows the axial deflection under thrust of a single-row preloaded ball bearing having no initial looseness. Reference to the curve will show that when carrying a thrust load P the axial deflection is D—in this case, 0.0015 in. By doubling the load ($2P$), a deflection F results, but instead of being 0.003 in., it is only 0.00225 in. An inspection of the curve will indicate clearly that the greater the initial thrust load the less the added deflection by doubling it, because the curve flattens out decidedly with increase in load.

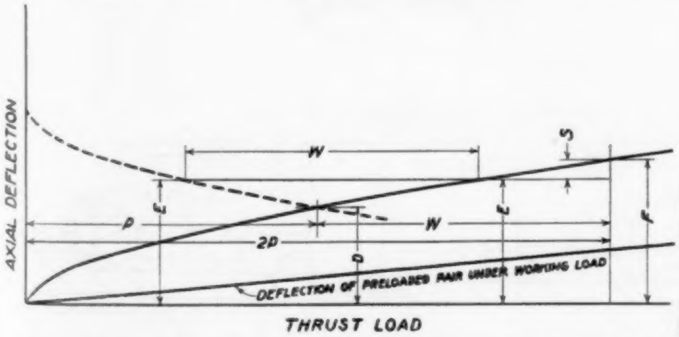

Fig. 4 Load-Deflection Curve for a Pair of Preloaded Ball Bearings

Further, if the preload is established by a second bearing opposed to the first, a still smaller increase in deflection will result from the addition of a working thrust upon the two. A study of Figs. 5 and 6 will show why this is so.

Fig. 5 shows the forces set up by tightening the locknut against two bearings designed for preloading. The preload P produces equal compression of both sets of balls and races, mutual compression between the two outer races, and tension in the shaft between the shoulder and nut.

If there is now applied a working thrust W, for sake of convenience made equal to P (see Fig. 6), the stresses in bearing A will be increased, but those in bearing B will be decreased, because the increased deflection of the balls in bearing A allows the inner races to move axially, thus relieving the pressure on bearing B. This relief will reduce the shaft tension by an amount R, and likewise decrease the preload pressure on bearing A by the same amount R. It will thus be seen that a working load does not add nearly so much to the burden borne by bearing A nor to its axial deflection as it would were A not preloaded.

The effect of the working load discussed is graphically shown in Fig. 4, where the continuous curve represents bearing A carrying the working load, while the dotted curve, exactly the same, but inverted for convenience, represents the opposed bearing B.

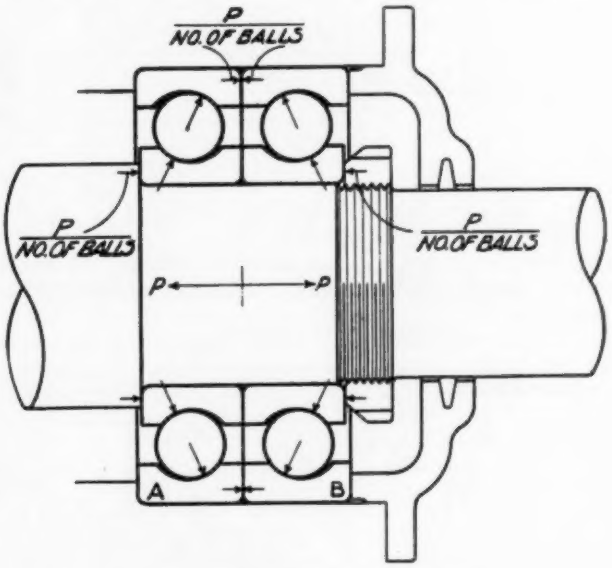

Fig. 5 Pair of Bearings Opposed to Effect Preloading

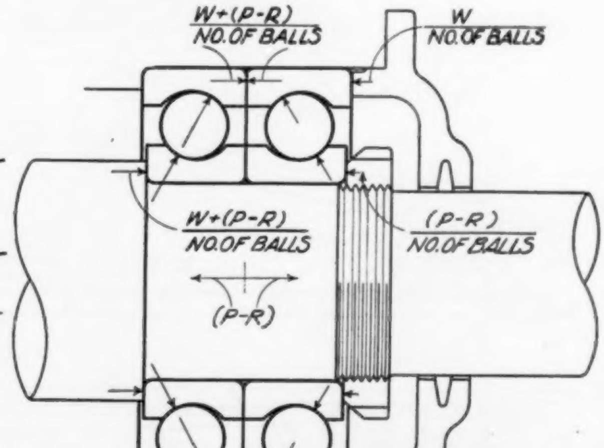

Fig. 6 Pair of Bearings Carrying Preload and Working Load

When the working load W is applied, it will take from B as much of the deflection due to the preload as it adds to A, because the "action" of B carried through the shaft by tension must equal the "reaction" of A. Hence one must determine total deflection by placing the working load W horizontally across the two curves and not by adding it to P. The resulting deflection of bearing A from its original unloaded position is shown by E, and the saving in deflection S will be evident by comparing E

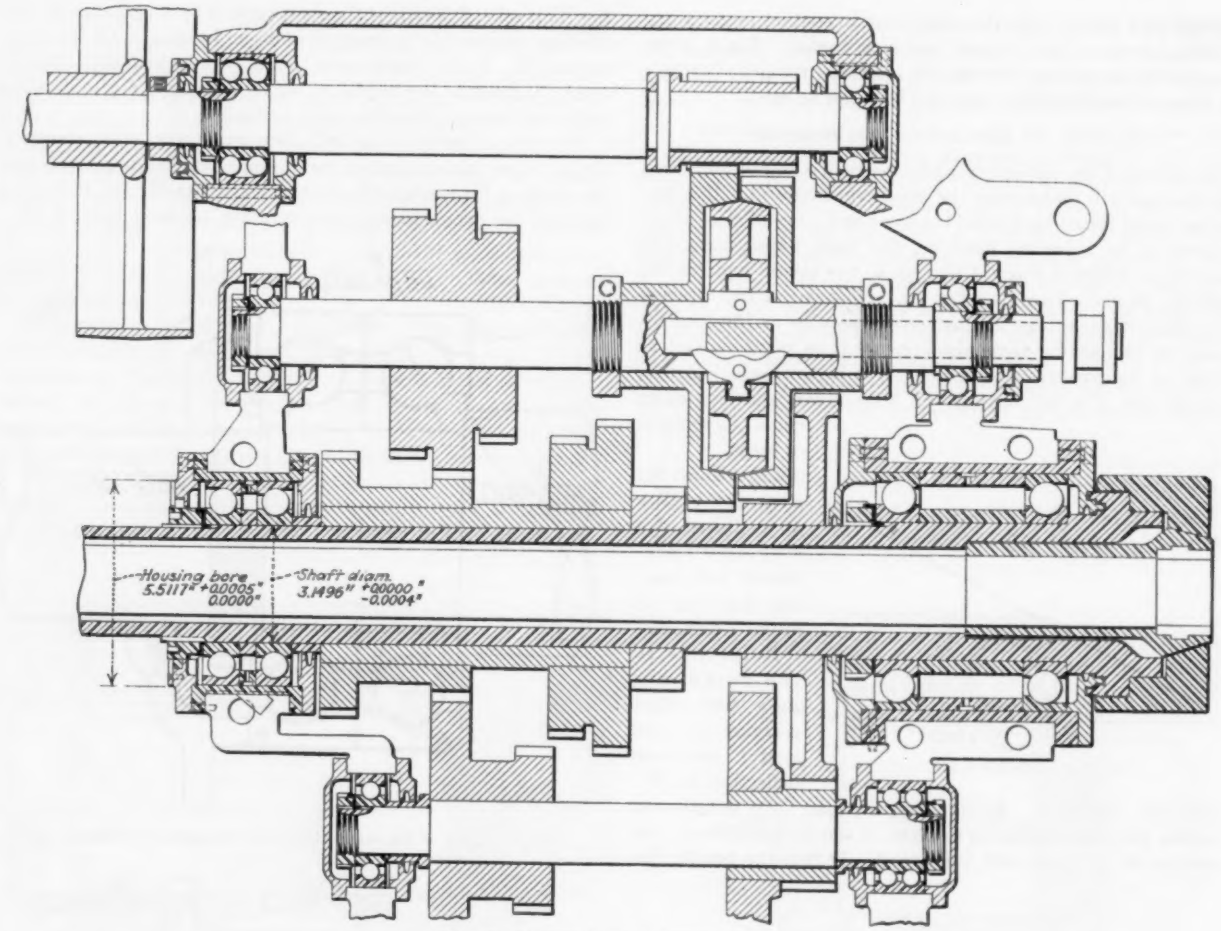

FIG. 7 HEAVY-DUTY TURRET LATHE

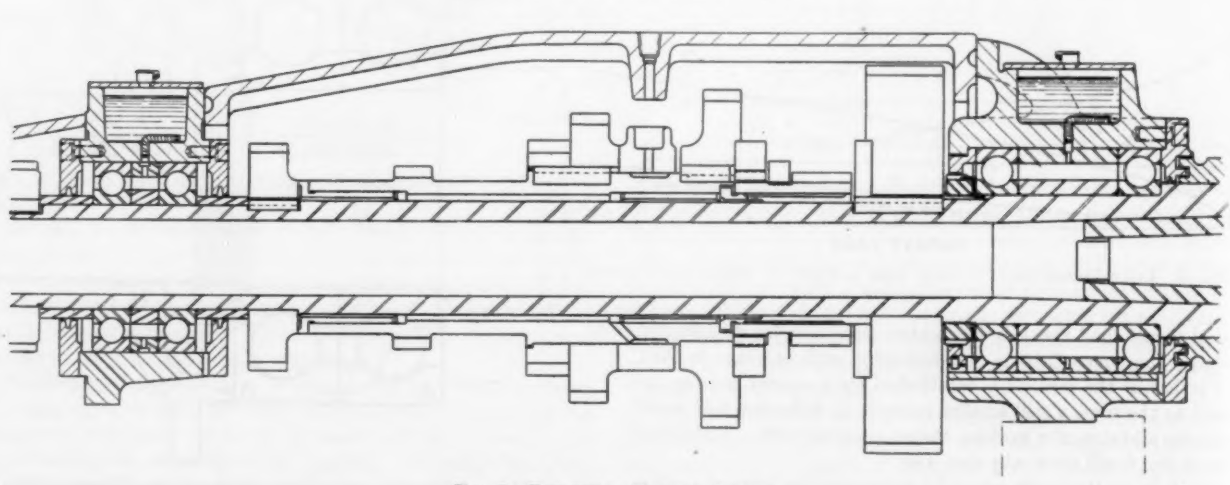

FIG. 8 PRECISION ENGINE LATHE

with *F*. Zero position of the spindle is that which it assumes when the two bearings are drawn together applying the preload. The line marked "Deflection of Preloaded Pair Under Working Load" shows the axial deflection of the spindle under various working loads.

ADVANTAGES OF PRELOADED BALL BEARINGS

Preloaded ball bearings give the following advantages:

(1) Any two such bearings can be separated with spacing sleeves of equal length, which is preferable, or if not convenient, they can be placed side by side.

(2) The necessary preloading within the safe capacity of the bearing can be definitely established for any particular application. This avoids possibility of premature failure being caused by excessive preloading through improper adjustment.

(3) The bearings are fully interchangeable as they are not

matched in pairs. If, therefore, they should be removed for any reason after mounting, it is not necessary when replacing them to have the same faces in contact or adjacent to one another. The original preload will be maintained regardless of the bearing faces in contact.

(4) The bearings do not need to be locked in the housing in order to subject them to the predetermined load. Once the locknut on the spindle is properly tightened, the predetermined load is reproduced.

(5) A spindle can be located axially on one pair of bearings and the other pair can be allowed to float in the housing without affecting in any way the initial adjustment.

Bearing Types Best Suited to Preloading

The single-row Conrad and the open-side angular-contact types of ball bearings are well suited to preloading and are the two types generally used in this manner for precision-machine spindles. The choice between them is one of design solely. For certain heavy-duty machines where thrust loads predominate and are of large magnitude, the open-side bearing has an advantage as a thrust carrier due to its greater number of balls.

Design of Application

In the preloaded ball bearing the bearing manufacturer has succeeded in developing a product that is as accurate in dimension and finish as the most modern grinding processes will produce. It is therefore essential that the machine manufacturer exercise an equivalent degree of care in the design, machining, and assembly of all other machine parts in order to obtain the full benefit of the precision that is built into the bearings. Unless this is done errors will be introduced into the finished work and for which the bearings cannot be expected to compensate.

The preloaded ball bearing is being used today with uniform

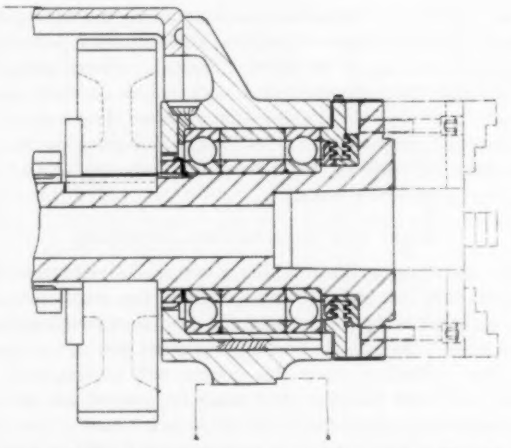

Fig. 9 Alternative Lubrication Design for Fig. 8

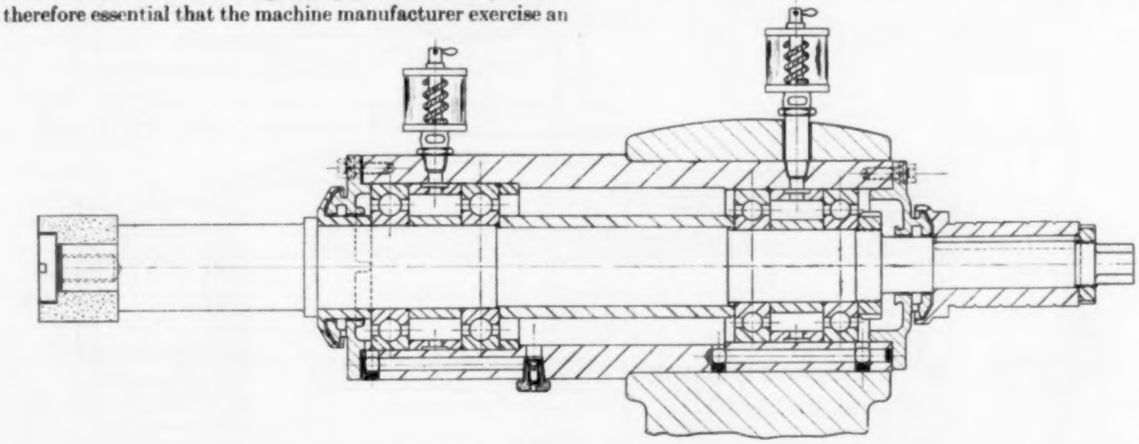

Fig. 10 High-Speed Internal Grinder

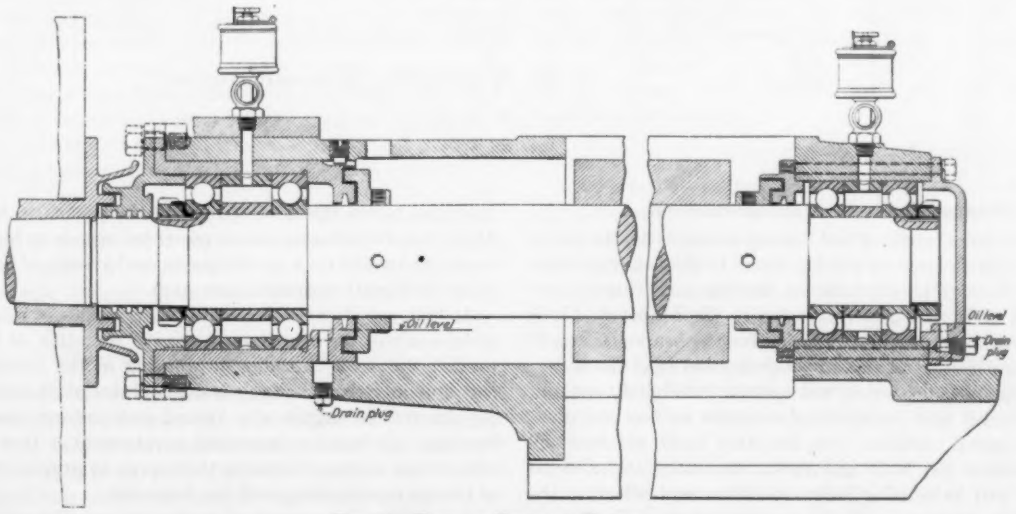

Fig. 11 Rotary Surface Grinder

success on a variety of machines such as milling machines, lathes, grinders, and other types of machine tools where a positive predetermined loading is required. Smooth, vibrationless, and accurate operation can be relied upon when properly loaded, mounted, and applied. In precision grinders they have proved their dependability on almost every type of machine from the heavy-duty roll grinder to the lighter duty high-speed units operating up to 30,000 r.p.m.

SHAFT AND HOUSING FITS IMPORTANT

Shaft and housing fits must be given special attention where preloaded ball bearings are used. These fits must not be too heavy on the spindle or otherwise the predetermined loading will be disturbed. Again, the housing fits must not be too loose in order that vibration from this source will be avoided. The bearing seat and housing bore must be ground accurately to dimensions that will result in a fit of the inner race on the spindle ranging from a line-to-line to not more than 0.0002 to 0.0003 in. press fit. The fit between the outer race and the housing should range from a line-to-line to not more than 0.0002 to 0.0003 in. loose.

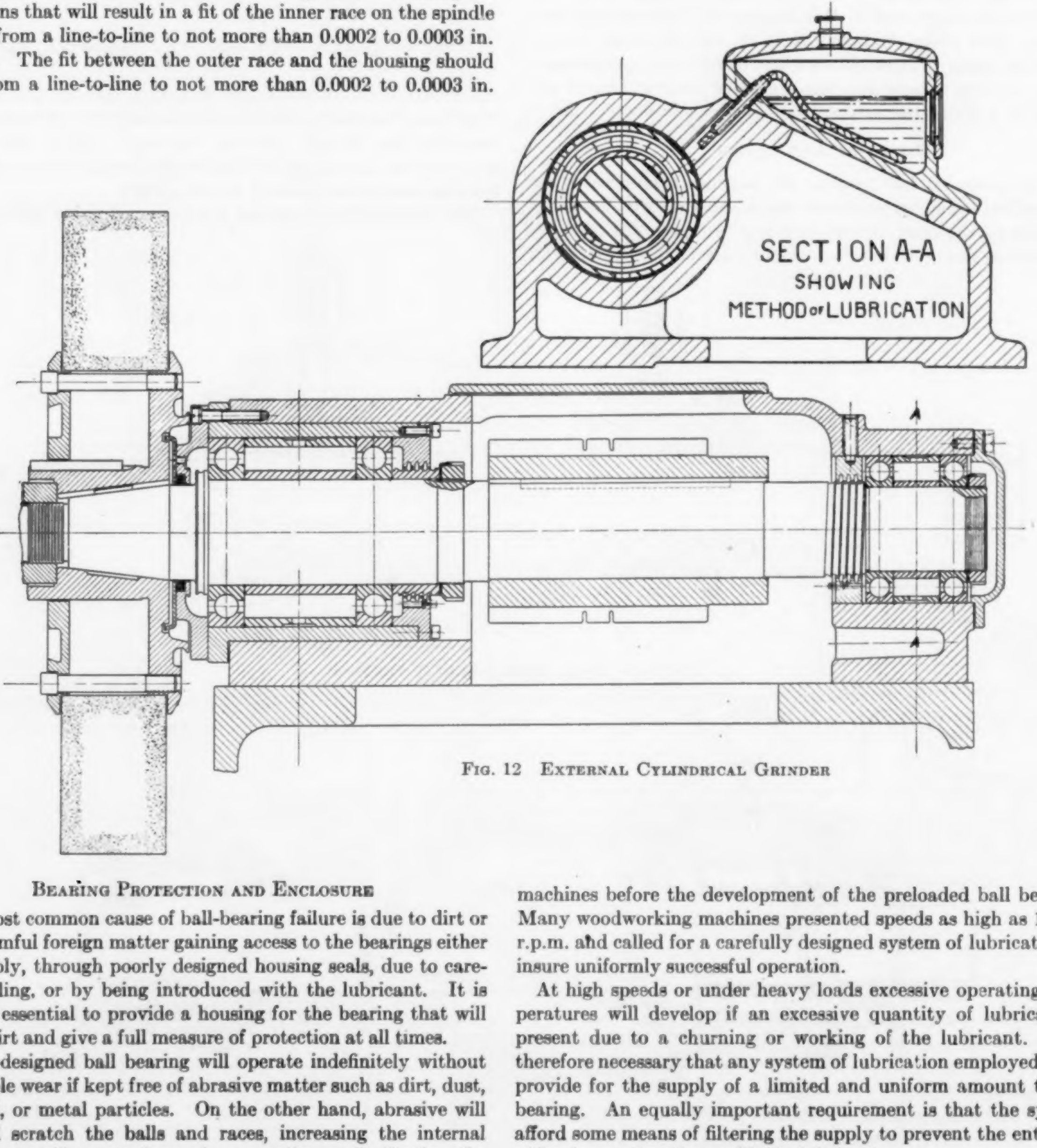

FIG. 12 EXTERNAL CYLINDRICAL GRINDER

BEARING PROTECTION AND ENCLOSURE

The most common cause of ball-bearing failure is due to dirt or other harmful foreign matter gaining access to the bearings either at assembly, through poorly designed housing seals, due to careless handling, or by being introduced with the lubricant. It is therefore essential to provide a housing for the bearing that will exclude dirt and give a full measure of protection at all times.

A well-designed ball bearing will operate indefinitely without appreciable wear if kept free of abrasive matter such as dirt, dust, core sand, or metal particles. On the other hand, abrasive will wear and scratch the balls and races, increasing the internal looseness as well as causing noisy operation and affecting the quality of work produced. Water or moisture must likewise be kept out of the bearing housing. If this is not done, the balls and races will corrode, causing unsatisfactory operation and premature failure. Bearings should be left in their original cartons until they are to be applied to the spindles. The slushing grease used by the bearing manufacturers is always thoroughly clean and has been chosen as a good protective agent. It is compounded from high-grade mineral products and will readily mix with any additional lubricant introduced afterward.

LUBRICATION

The importance of correct lubrication was recognized in the early efforts to apply anti-friction bearings to machine tools and other precision equipment. The necessity of a clean supply of lubricant was immediately observed. In fact, this requirement became evident in the use of bearings on other classes of precision machines before the development of the preloaded ball bearing. Many woodworking machines presented speeds as high as 10,000 r.p.m. and called for a carefully designed system of lubrication to insure uniformly successful operation.

At high speeds or under heavy loads excessive operating temperatures will develop if an excessive quantity of lubricant is present due to a churning or working of the lubricant. It is therefore necessary that any system of lubrication employed must provide for the supply of a limited and uniform amount to the bearing. An equally important requirement is that the system afford some means of filtering the supply to prevent the entrance of foreign matter along with the lubricant.

A considerable amount of study and experimental work was done in an effort to develop systems that would meet these essential requirements in a practical manner. We eventually succeeded in utilizing various systems of wick feed for oil lubrication in ways that have produced most gratifying results on a great many applications. By means of the wick it is possible to feed a limited quantity of lubricant into the bearing chamber and at the same time filter the supply.

There are two principal methods by which this has been accomplished. The first is to provide in the machine design for a reservoir or chamber of adequate size from which the oil will be fed by gravity into the bearing chamber. This reservoir should be so located that renewal of lubricant can be readily effected and at the same time insure that a sufficient head of oil will always be present to cause a continuous flow to the bearings. It is not entirely safe to depend upon capillary action. Where capillary action is depended upon, there is always the danger of the supply becoming choked or suspended.

In the second method wicks are also used, but gears or ring oilers are employed to lead the lubricant to a storage reservoir formed in the machine frame. A design of this type is indicated in Fig. 9.

The point at which the oil is brought into the housing is very important with either method, because where two or more bearings are to be lubricated from the one reservoir or source of supply the wick should enter midway between the bearings so as to insure an equal flow through the bearings on either side. Further, the drain should be so arranged that the oil will be carried off only after it has passed through the bearing. The various designs which follow illustrate these principles.

Where grease is used it is equally important that the system employed in applying it provide some means for supplying a proper quantity to the bearings and at the same time afford protection against dirt intrusion. This is not as easily accomplished as in the case of oil, especially from the standpoint of the quantity supplied. However, systems have been worked out that accomplish the fundamental requirements as to quantity and cleanliness, but on the majority of machine-tool applications the use of grease has been confined to those applications where the effect of deterioration of balls and grooves was not of serious concern.

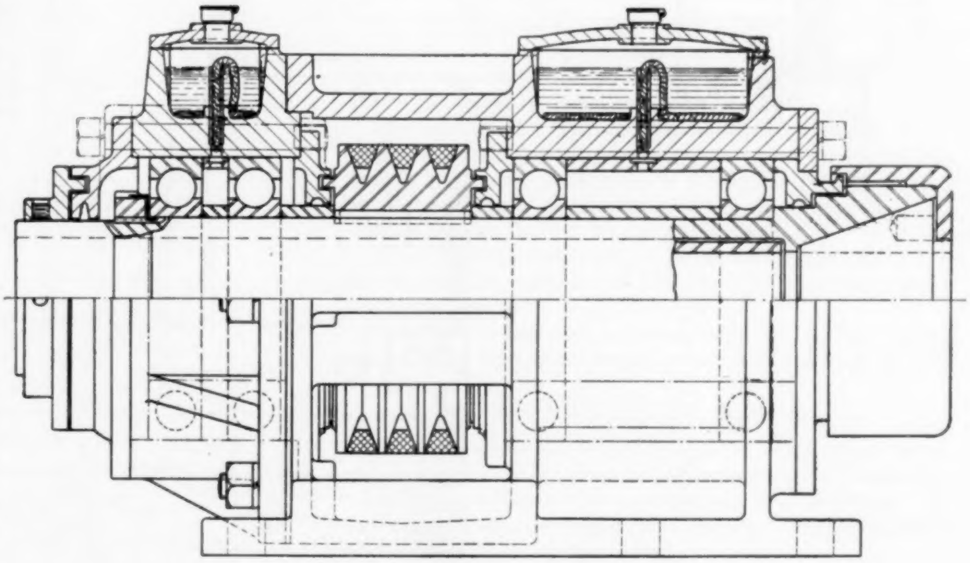

FIG. 13 DIAMOND BORING MACHINE

TYPICAL MOUNTING DESIGNS

Some typical mounting designs are presented, not only to illustrate the variety of application, but to bring out the more important design features upon which depends to a large extent the success to be obtained.

HEAVY-DUTY TURRET LATHE

Fig. 7 shows a heavy-duty, low-speed, turret lathe equipped throughout with ball bearings. In this design the frame is split so that it would be difficult to machine it with sufficient accuracy to receive the outer races of the preloaded bearings on the work spindle. This can be readily overcome, however, and the assembly facilitated by utilizing a sub-housing around each pair of bearings. As a result the work spindle, bearings, and sub-housings can be assembled on a bench and inserted as a unit in the frame, assuring a maximum of protection to the bearings. The bearings at each location are spread apart by means of equal length spacing sleeves to secure maximum rigidity.

The work spindle is located axially by holding the outer races of the two bearings at the front end between the lips of the end covers, while the pair at the rear have ample freedom to position themselves laterally in the sub-housing without affecting in any way the predetermined adjustment.

The housing seal at the front of the main spindle is of a labyrinth type to prevent the entrance of dirt or water into the bearings. At the opposite end the housing seal need not be so elaborate, and a split brass ring is used held in place by a plate screwed to the end cover.

The lubrication system is not fully indicated in Fig. 7. For the work spindle an oil trap is incorporated in the upper half of the frame, and from it the lubricant is fed to each pair of bearings through a wick which not only controls the rate of feed but acts as a filter also. The lubricant enters each bearing housing midway between the two bearings, and after passing through them is drained back to the gear casing through the plug shown. The plug is drilled and fitted with a wick to prevent dirt or foreign matter gaining access through the drain hole and working its way into the bearings.

PRECISION ENGINE LATHE

The precision engine lathe is called upon to perform heavy roughing work as well as work requiring extreme accuracy. Preloaded ball bearings are being used with very satisfactory results on this class of equipment.

Alternative mounting designs are illustrated by Figs. 8 and 9 which differ only in the system of lubrication employed. In Fig. 8 an oil reservoir is cast integral with each housing and from which the lubricant is supplied to the bearings through wicks. After passing through the bearings the oil is returned to the main reservoir. This provides a simple system for circulating and filtering the oil.

A modification of the foregoing system of lubrication that has been used on engine lathes is shown by Fig. 9. In this design an oil trap is cast integral with the gear casing at each end and entraps oil thrown by the gears. This oil is conducted through wicks to the bearings and is returned to the gear casing after passing through the bearings.

In both of the designs shown two bearings are mounted at each end of the spindle spaced apart by means of sleeves of equal length, and the spindle is located axially by holding the outer races of the bearings at the chuck end between lips on the housing end covers. The two bearings at the opposite end are free to position themselves axially in the housing without affecting the initial adjustment.

A labyrinth housing seal is incorporated at the chuck end to prevent the entrance of grit or moisture into the bearings.

High-Speed Internal Grinder

Fig. 10 illustrates a high-speed internal grinding spindle design which operates at speeds as high as 25,000 to 30,000 r.p.m. and is a splendid example of the success attained with ball bearings in high-speed precision machine tools. In designs of this nature it is particularly essential that they be thoroughly protected against the intrusion of any abrasives or other harmful substances. Also where such high speed prevails, the lubrication system must be such as to introduce clean lubricant in a limited and uniform quantity. In this design oil cups equipped with wicks are shown, which introduce a clean supply of lubricant between each pair of bearings, insuring uniform distribution. The oil is drained out at the housing through a pipe plug fitted with a filter to prevent the entrance of foreign matter through this opening.

Rotary Surface Grinder

A mounting for the work spindle of a rotary surface grinder is shown in Fig. 11. This design has two preloaded bearings at each end of the spindle mounted with spacing sleeves of equal length between the inner and outer races. In order to permit the spindle, bearings, and pulley to be assembled as a unit and inserted in place, the bearings at the grinding wheel end are mounted in a sub-housing. This permits boring the frame to a diameter slightly larger than the pulley diameter, thus allowing the pulley to pass through.

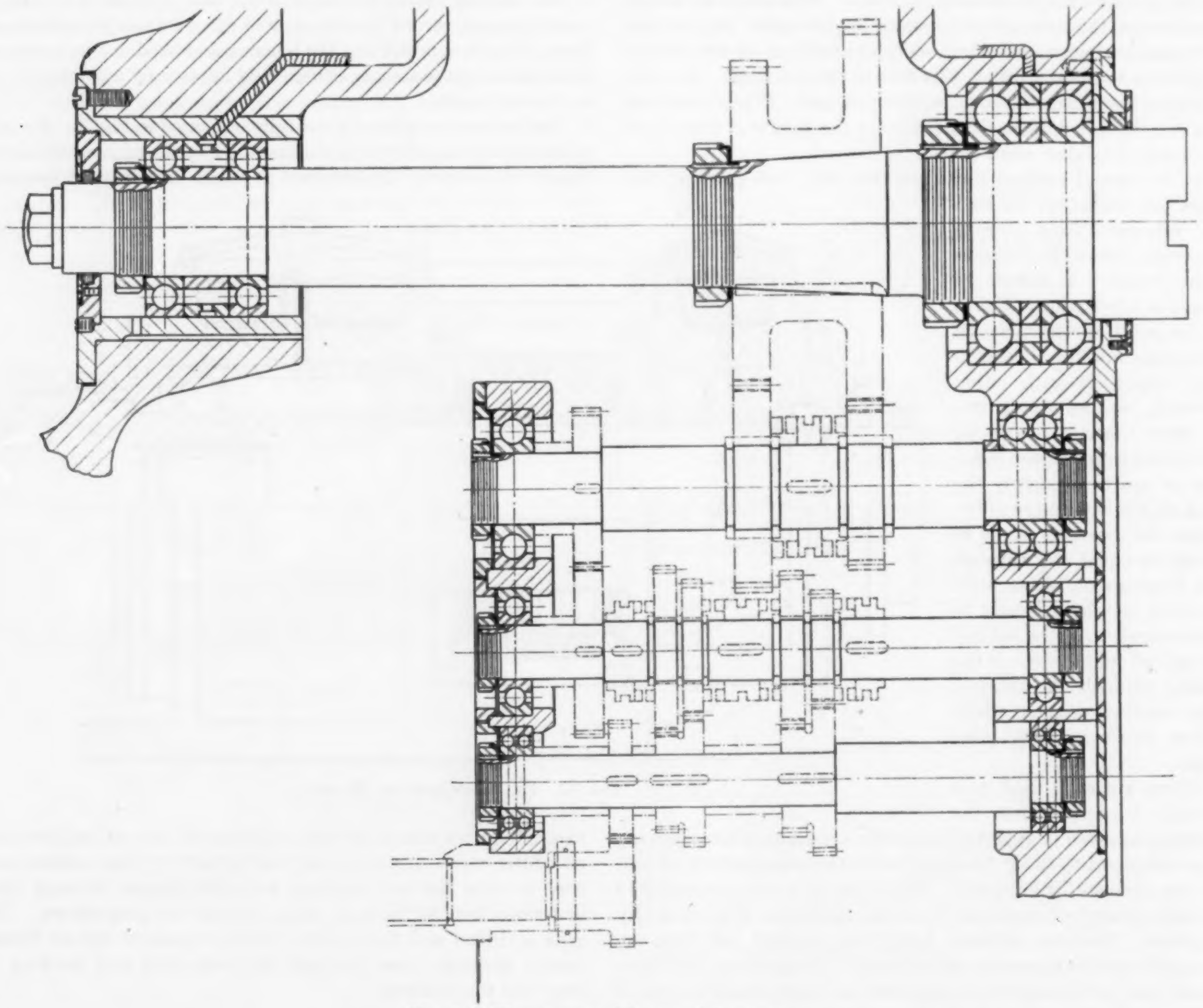

Fig. 14 Horizontal Milling Machine

The housing at the end away from the wheel is closed on one side as the spindle terminates in it. Each of the other housing enclosures consists of a labyrinth flinger and oil groove. This arrangement provides an effective seal against the intrusion of dirt or water into the bearings. As an added precaution the flinger at the enclosure next to the wheel is shaped so that any water or cooling compound that may gather on it will be drained off without getting into the clearances of the labyrinth.

Lubrication is accomplished by means of sight-feed wick oilers from which the lubricant is fed into the housings midway between each pair of bearings. The oil is allowed to accumulate in each housing, but when the level rises to the center of the lowermost balls in the bearings it is drained off through the overflow plug.

In this design the spindle is located against end movement by holding the outer races of the two bearings adjacent to the wheel, between a shoulder in the sub-housing and a lip on the outside end cover. The pair of bearings at the end away from the wheel have ample clearance on either side of the outer races so that they are free to move laterally in the housing without disturbing in any way the initial adjustment.

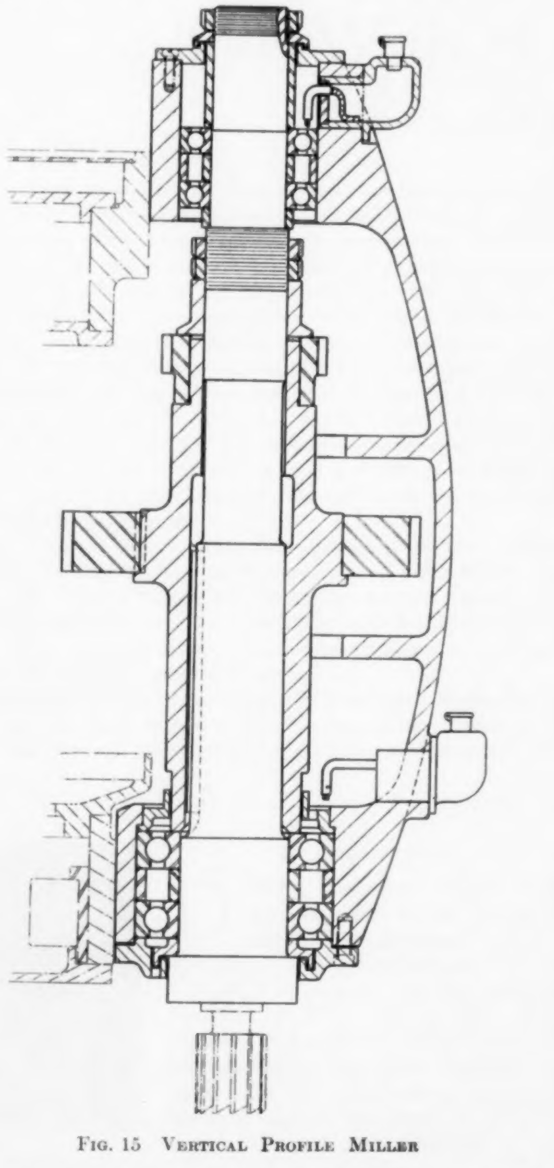

Fig. 15 Vertical Profile Miller

External Cylindrical Grinder

An external cylindrical grinding spindle is illustrated by Fig. 12. This type of machine has a wide range of application and is called upon to produce work to very close tolerances. Two preloaded ball bearings are employed at each end separated by equal length spacing sleeves to secure maximum rigidity. The spindle is located axially by holding the two bearings at the wheel end while the two bearings at the opposite end are free to float in the housing under any changes in length of spindle due to temperature variations without affecting the initial positioning of the grinding wheel.

The bearings are lubricated by means of oil that is fed to each pair of bearings from a reservoir cast integral with the frame. The rate of feed is controlled and the supply filtered by a wick. The oil enters each housing midway between the bearings and after passing through each bearing is drained out.

Particular care must be paid to the enclosures, as these machines usually operate under dirty conditions. In the design shown a labyrinth seal, together with a packing gland, is used at the wheel end.

Diamond Boring Machine

Fig. 13 is another mounting design which illustrates the wide variety of uses to which preloaded ball bearings are adaptable. In this design the chuck spindle of a diamond boring machine is shown supported on preloaded bearings.

The bearings in a machine of this kind are subjected to very severe service, not only because it operates at moderately high speeds, but in very short cycles of starting and stopping. It is also equipped with an automatic loading device that passes the product through the hollow spindle into the chuck.

A feature of this design is that the two bearings at each end are held in place by means of one lock nut only, located at the rear end of the spindle.

Suitable labyrinth seals are incorporated, as is also a wick-feed system of oil lubrication similar to others described in previous designs.

Horizontal Milling Machine

The design of bearing mounting suitable for application to the main cutter spindle of horizontal milling machines is illustrated in Fig. 14. In addition, the intermediate drive shafts are also shown supported on deep groove ball bearings, assuring maximum power being applied to the cutter.

The bearings on the work spindle are spaced apart at each end by means of sleeves of equal length to assure maximum rigidity. The pair at the tool end are held at the outer races between shoulders to locate the spindle axially, while the pair at the opposite end are free to float, obviating the necessity of extreme accuracy in the distance between spindle shoulders.

Vertical Profile Miller

Preloaded ball bearings are as readily adaptable to vertical spindles as to horizontal spindles. Fig. 15 shows the spindle of a vertical profile milling machine supported on these bearings. It will be seen that the mounting is similar to that used on horizontal spindles.

The upper seal consists of an external labyrinth flinger which prevents dirt or other foreign matter from working down the spindle and into the bearings. At the lower enclosure an external labyrinth flinger is provided to prevent leakage of lubricant.

Lubrication is accomplished by means of oil that is supplied to the bearings through wicks leading from reservoirs attached to the frame.

MSP-54-4

Influence of Chemical Composition and Heat Treatment of Steel Forgings on Machinability With Shallow Lathe Cuts

By T. G. Digges,[1] Washington, D. C.

The tests described in this report were made primarily as a study of lathe-tool performance with shallow cuts as affected by variations in chemical composition and heat treatment of the steels cut. The cutting tests were made dry with high-speed-steel tools of a selected size, form, composition, and heat treatment, with a feed of 0.0115 in. per rev. and 0.010 in. depth of cut. Comparisons were made of the Taylor speeds on the basis of equal tensile strengths when cutting 0.4 per cent carbon (S.A.E. 1040), chromium-vanadium (S.A.E. 6140), nickel-chromium (S.A.E. 3140 and 3435), chromium-molybdenum (S.A.E. 4140), and $3^1/_2$ per cent nickel (S.A.E. 2340) steel forgings heat treated to give tensile strengths between 75,000 and 220,000 lb. per sq. in.

The study also included consideration of the surface finishes of the various steel forgings as affected by the test conditions, the microstructures of the steels cut, and tool performance as affected by the additions of 3.5 to 11.7 per cent cobalt to the customary 18 per cent tungsten type of high-speed tool steel.

If machinability is measured by the cutting speed permitting the tools to last a definite time, then measurable differences were observed between the various steels cut in the lathe test with shallow cuts. The fact, however, that some given steel permits a higher cutting speed than another steel for some tensile strength which is the same for both materials does not necessarily indicate that the two steels maintain the same relationship for another tensile strength.

Of the different steels cut in the lathe tests, the plain carbon steel was the most difficult to machine other than an annealed nickel-chromium steel.

The surface finish on the plain carbon steel was also considered to be inferior to the surface finish of the alloy steels.

The results showed that the effect of changes in chemical composition of steel forgings upon their cutting speeds was dependent upon the tensile strength at which the comparisons were made. In the different steels cut with shallow cuts the most effective special alloying elements for improving machinability were the combinations of nickel and chromium or chromium and vanadium for the high tensile strengths in the neighborhood of 180,000 lb. per sq. in., while chromium and molybdenum were the most effective in the lower range of about 90,000 lb. per sq. in.

I—Introduction

During the past ten years a study has been made at the Bureau of Standards of lathe-tool performance with both roughing and shallow cuts. The lathe breakdown test for roughing cuts was used in studying the effects upon tool performance of changes in chemical composition and heat treatment of commercial and experimental high-speed steels[2,3,4] and of variations in chemical composition and heat treatment of the steels cut.[5] The lathe breakdown test has also been extended to include cutting tests with cemented tungsten-carbide tools.[6]

A method for testing lathe tools with shallow cuts was developed and used for studying the effects upon tool performance of changes in chemical composition and heat treatment of the tools. Relations between the cutting speed, feed, and depth of cut and the tool life for carbon and high-speed-steel tools were also determined.[7,8] Most of the tests with shallow cuts were made in cutting $3^1/_2$ per cent nickel-steel forgings, heat treated to have tensile strengths of about 80,000 to 100,000 lb. per sq. in.

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[2] H. J. French and Jerome Strauss, "Lathe Breakdown Tests of Some Modern High-Speed Tool Steels," Technologic Paper of the Bureau of Standards No. 228, also Trans. A.S.S.T., vol. II (1922), p. 1125.

[3] H. J. French, Jerome Strauss, and T. G. Digges, "Effect of Heat Treatment on Lathe-Tool Performance and Some Other Properties of High-Speed Steels," Trans. A.S.S.T., vol. IV (1923), p. 353.

[4] H. J. French and T. G. Digges, "Experiments With Nickel, Tantalum, Cobalt, and Molybdenum in High-Speed Steels," Trans. A.S.S.T., vol. VIII (1925), p. 681.

[5] H. J. French and T. G. Digges, "Rough Turning With Particular Reference to the Steel Cut," Trans. A.S.M.E., vol. 48 (1926), p. 533.

[6] T. G. Digges, "Cutting Test With Cemented-Tungsten Carbide Lathe Tools," Research Paper No. 206; *Bureau of Standards Journal of Research*, vol. 5 (1930), p. 365; also Trans. A.S.M.E., vol. 52 (1930), paper MSP-52-13, p. 155.

[7] H. J. French and T. G. Digges, "Effects of Antimony, Arsenic, Copper, and Tin in High-Speed Tool Steel," Trans. A.S.S.T., vol. XIII (1928), p. 919.

[8] H. J. French and T. G. Digges, "Turning With Shallow Cuts at High Speeds," Research Paper No. 120; *Bureau of Standards Journal of Research*, vol. 3, p. 829; also Trans. A.S.M.E., vol. 52 (1930), paper MSP-52-6, p. 55.

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The tests described in this report were made primarily as a study of high-speed-steel lathe-tool performance with shallow cuts as affected by variations in the chemical composition and heat treatment of the steels cut. The study also included consideration of the surface finishes of the various steel forgings as affected by the conditions of cutting and of the microstructures of the steels cut.

The lathe tests were made in cutting a plain carbon and various alloy-steel forgings heat-treated to have tensile strengths between 75,000 and 220,000 lb. per sq. in. The form, size, heat treatment, and composition of the high-speed-steel tools were not varied, except that a study was made of tool performance as affected by the addition of cobalt to the 18 per cent tungsten type of tool steel.

In most cases the limiting factor of machinability is tool failure, and machinability can best be measured in terms of the cutting speed permitting a definite tool life. That is, machinability is proportional to the cutting speed permitting a definite tool life, and it is upon this basis that the term is used in this report. From this it follows that those materials, which under otherwise fixed conditions permit the longest cuts without regrinding of the tools, are described as the most readily machinable or to have the highest degree of machinability.

With finishing cuts, close adherence to dimensions and the nature of the finish left on the work piece are probably as important factors as tool life. However, no very satisfactory method has yet been worked out for the evaluation of the appearance or type of finish produced by a cutting tool.[9] The method of test selected made possible a close adherence to the desired dimensions.

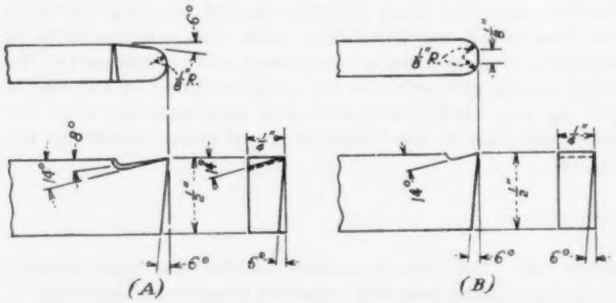

FIG. 1 DIMENSIONS AND FORMS OF THE TOOLS USED IN THE LATHE TESTS

[(*A*) Tool for roughing cuts. (*B*) Tool for shallow cuts.]

II—PREVIOUS INVESTIGATIONS

The results of previous lathe tests[8] made with high-speed-steel tools with shallow cuts showed that with a constant feed and depth of cut, there was a continuous increase in tool life as the cutting speed was decreased which could be represented approximately by the equation

$$VT^n = V_0T_0^n = c \qquad [1]$$

in which V = cutting speed, feet per minute
T = tool life, minutes
T_0 = 20 minutes
V_0 = Taylor speed; that is, the speed which would give a tool life of 20 minutes
c = a constant for the metal (with specified heat treatment) which is being cut
n = a constant experimentally determined by varying V.

The tests made at different cutting speeds with a feed of 0.0115 in. per revolution and depths of cut of 0.010 and 0.020 in. showed that within a range of tool life of from 2 to 125 min., the experimental results could be closely represented by Equation [1] with $n = 1/10$. This equation with $n = 1/10$ has therefore been used in the present report for all computations involving the relation between the cutting speed and tool life, and gives results sufficiently accurate for all practical purposes except when extrapolating for very long tool life from tests of short duration.

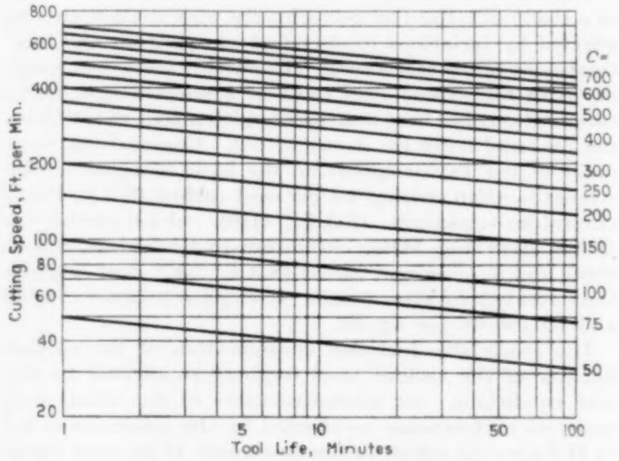

FIG. 2 CHART GIVING THE RELATION BETWEEN THE CUTTING SPEED AND THE TOOL LIFE WITH SHALLOW CUTS

(This chart applies only to "dry" turning of steels with shallow cuts with high-speed-steel tools. Lines drawn according to equation $VT^{1/10} = c$, in which V = cutting speed, T = tool life, and c = a constant.)

Equation [1] with $n = 1/10$ was established with broad-nose high-speed-steel tools, illustrated in Fig. 1(*B*), in dry cutting $3^1/_2$ per cent nickel-steel forgings having tensile strengths between 80,000 to 100,000 lb. per sq. in. This equation is represented graphically by straight lines when logarithmic coordinates are used The chart shown in Fig. 2 for finishing cuts is of a convenient form for computing the life of tools at various speeds. Numerically the constant c is the cutting speed corresponding to a tool life of one minute.

III—METHOD OF TEST

The method used for testing lathe tools with shallow cuts has been described in detail in a previous report.[8] With this method the test and indicating tools were set at equal depths in a special tool holder at the start of the test. The indicating tool began to cut when the wear on the test tool was from 0.001 to 0.002 in., and this was considered as the point of failure of the test tool. In most cases it was found that the wear of 0.001 to 0.002 in. coincided with a complete breakdown of the tool comparable to that found with heavy cuts in rough turning.

From six to ten tools were tested for each condition investigated, and only average values of tool life were used for computation and comparison purposes.

The lathe tests with shallow cuts were made with high-speed-

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[9] R. E. W. Harrison, "A Survey of Surface Quality Standards and Tolerance Based on 1929–30 Precision Grinding Practice," paper presented at the A.S.M.E. annual meeting, 1930. The finish calibrator described in this paper could possibly have been adapted to the work described in the present report and might have given results of more value than did the method used for evaluation of the surface finish. However, the experimental work of the present investigation was practically completed before the report by Harrison was published.

TABLE 1 CHEMICAL COMPOSITION AND HEAT TREATMENT OF THE TOOLS USED IN THE LATHE TESTS

Steel No.	Chemical composition, per cent: C	Mn	P	S	Si	Cr	W	V	Co	Mo	Quenched from[1] (deg. fahr.)	Tempered at[2] (deg. fahr.)
AA	0.68	0.20	0.031	0.013	0.21	3.79	18.2	0.99	..	..	2400	1100
BB	0.70	0.30	0.030	0.031	0.14	3.68	17.6	1.18	4.5	0.05	2450	1100
CC	0.69	0.17	...	...	0.34	4.23	18.3	1.53	7.8	0.96	2450	1100
Y	0.79	0.42	0.030	0.005	0.15	4.57	21.5	1.47	11.7	0.61	2450	1100

[1] All tools were first annealed by heating for 2 to 3 hours at 1600 to 1650 deg. fahr. and furnace cooled. They were preheated for 20 min. at 1600 deg. fahr. and then held 1½ min. in the high-temperature furnace at the temperatures indicated.

[2] Tools were heated in the furnace to the temperature indicated, held 30 min., and then air cooled.

TABLE 2 CHEMICAL COMPOSITION OF THE FORGINGS CUT IN THE LATHE TESTS

Forging No.	S.A.E. No.	Chemical composition,[1] per cent: C	Mn	P	S	Si	Cr	V	Ni	Mo
46	6140	0.39	0.71	0.019	0.029	0.20	0.95	0.17	..	..
47	4140	0.39	0.56	0.020	0.023	0.22	0.86	..	..	0.17
48	3435	0.39	0.59	0.017	0.019	0.26	0.76	..	2.96	..
49	3140	0.37	0.64	0.021	0.024	0.20	0.50	..	1.26	..
50	2340	0.36	0.73	0.026	0.025	0.20	..	..	3.43	..
51	1040	0.42	0.65	0.020	0.015	0.19	..	..	..	..

[1] Chemical-analysis data reported by the manufacturers.

TABLE 3 HEAT TREATMENT AND AVERAGE MECHANICAL PROPERTIES OF THE FORGINGS CUT IN THE LATHE TESTS

Forging No.[1]	Heat treatment: Quenching[3] Temp., deg. fahr.	Medium	Tempering[4] Temp., deg. fahr.	Time, hr.	Prop. limit, 1000 lb. per sq. in.	Yield point, 1000 lb. per sq. in.	Tensile strength, 1000 lb. per sq. in.	Elongation in 2 in., per cent	Reduction of area, per cent	Rockwell hardness B	Rockwell hardness C	Shore hardness No.	Brinell hardness No.
Carbon Steel (S.A.E. 1040)[2]													
51	1650	Water	500	6.5	69.0	84.7	123.6	17.8	55.5	100.5	20.6	37	241
51A	1650	Water	800	4	64.0	76.0	111.8	20.5	57.3	96.0	15.2	36	229
51B	1650	Water	1100	4	52.5	60.5	95.7	31.3	64.2	90.0	9.5	25	184
51C	1650	Water	1300	4	47.0	54.0	87.5	31.5	57.8	87.0	..	27	167
51D	1650	Water	1550	4*	47.5	48.5	78.0	34.0	53.5	81.5	..	26	149
Chromium-Vanadium Steel (S.A.E. 6140)[2]													
46	1650	Oil	500	3	98.5	114.0	145.4	15.0	55.8	106.5	30.1	50	300
46A	1650	Oil	850	4	98.0	110.0	135.5	15.0	52.3	105.0	26.4	41	290
46B	1650	Oil	1100	4	87.0	90.0	114.3	20.5	60.8	100.0	21.2	38	230
46C	1650	Oil	1350	4	55.0	61.5	96.0	30.8	60.8	92.0	10.0	31	192
46D	1650	Oil	1550	4*	42.5	49.8	90.4	29.5	53.5	89.0	..	28	168
46E	1625	Water	500	4	95.0	164.2	181.8	10.0	33.5	110.0	37.3	55	372
Chromium-Molybdenum Steel (S.A.E. 4140)[2]													
47	1625	Water	700	4	79.0	121.5	137.2	18.0	59.4	104.5	27.8	41	287
47A	1625	Water	900	4	83.0	96.2	120.7	18.5	59.5	102.0	23.9	38	255
47B	1625	Water	1100	4	70.0	76.6	108.8	26.5	63.1	95.5	15.2	31	204
47C	1625	Water	1350	4	48.0	54.3	93.7	30.0	60.7	90.0	7.9	30	179
47D	1625	Water	1550	4*	35.0	41.0	89.0	28.0	43.7	87.5	..	26	166
47E	1625	Water	500	4	127.0	165.8	210.9	6.5	31.8	112.5	42.5	59	388
47F	1625	Water	750	4	110.0	143.8	163.3	13.8	47.8	108.5	33.8	53	321
Nickel-Chromium Steel (S.A.E. 3435)[2]													
48	1540	Oil	700	4	142.0	195.0	215.5	9.8	37.0	114.5	42.7	61	429
48A	1540	Oil	900	4	112.5	130.5	149.0	19.0	52.9	107.5	32.6	50	321
48C	1540	Oil	1300	4	48.5	97.5	166.0	14.3	23.0	108.0	33.9	51	321
48D	1540	Oil	1500	4*	65.0	70.0	109.5	24.5	55.0	97.0	15.5	32	204
48F	1500	Oil	700	4	171.5	188.2	207.0	7.3	33.5	113.5	42.5	61	415
48G	1500	Oil	800	4	160.0	166.5	183.1	12.0	42.5	111.5	39.5	55	388
48H	1500	Oil	1050	4	113.5	118.7	135.0	16.7	55.9	104.5	27.4	47	269
Nickel-Chromium Steel (S.A.E. 3140)[2]													
49	1500	Water	700	4	81.0	85.3	120.6	21.7	58.7	101.5	23.2	41	257
49A	1500	Water	900	4	75.0	76.7	109.3	26.0	62.6	97.0	17.0	37	217
49B	1500	Water	1200	4	56.0	62.8	92.5	29.0	68.3	90.0	9.1	30	176
49C	1500	Water	1325	4	60.0	61.0	94.7	31.7	64.0	90.5	..	28	178
49D	1500	Water	1500	4*	55.0	56.0	89.0	31.2	55.0	87.5	..	27	170
49E	1500	Water	500	4	112.5	184.5	202.8	7.8	24.6	113.0	42.2	58	406
49F	1500	Water	750	4	96.7	122.7	140.0	14.4	50.7	104.5	28.0	45	282
Nickel Steel (S.A.E. 2340)[2]													
50	1525	Oil	500	3	71.5	125.5	150.0	13.2	45.3	106.5	30.7	42	302
50A	1525	Oil	800	4	89.5	94.0	121.5	22.0	56.4	101.5	24.7	39	255
50B	1525	Oil	1100	4	72.0	78.3	105.8	27.3	64.8	97.5	18.5	34	215
50C	1525	Oil	1325	4	49.0	66.8	116.0	22.5	42.7	98.0	18.3	34	226
50D	1525	Oil	1500	4*	61.7	63.5	101.6	24.8	44.0	93.5	12.9	28	193
50E	1500	Oil	500	4	106.0	136.1	178.8	6.5	28.8	110.0	36.5	52	363

[1] Forgings Nos. 46, 47, 48, 49, 50, and 51 were first annealed by heating at 1400 to 1420 deg. fahr. and cooled in furnace to 1100 to 1200 deg. fahr. and then cooled in air to room temperature. These forgings were approximately 8 in. in diameter and hollow bored with holes 3 in. in diameter. Forging No. 46A had the same chemical composition and heat treatment as No. 46 except for the retempering treatment, 46B has the same heat treatment as 46A except for the retempering treatment, etc. This procedure was followed in numbering and heat-treating the forgings used in the tests with shallow cuts. The forgings at the time of the second quenching treatment were smaller in diameter than at the time of first quenching treatment. This change in size accounts for the higher tensile strengths and hardness obtained in the latter case.

[2] Refer to Table 2 for chemical composition.

[3] Forgings were heated with the furnace to the hardening temperatures in 3 to 4½ hr., held at temperatures from 1½ to 3½ hr. and cooled as indicated.

[4] Forgings were cooled in air from the tempering temperatures except those marked with an asterisk (*), which were cooled slowly in the furnace.

steel tools of the broad-nose type, having dimensions and form shown in Fig. 1(*B*). A round-nose tool, Fig. 1(*A*), was used for the indicator. Chemical compositions and heat treatments of the high-speed steels used are given in Table 1.

The pieces on which the cuts were made, commonly referred to as forgings, were selected with the view of obtaining representative steels that come to the ordinary shop for machining operations. The chemical compositions of the forgings are given in Table 2. The forgings were initially about 8 in. in diameter and hollow bored with holes approximately 3 in. in diameter. Some of the forgings, after being used for a series of cutting tests, were heat-treated to higher tensile strengths and hardness values and used for further tests. The forgings having the same S.A.E. number were from the same original forging.

The heat treatments and corresponding average mechanical properties of the forgings are given in Table 3. Average mechanical properties were determined from two or more specimens cut longitudinally from the forgings so as to represent the average properties of the metal removed during cutting. Likewise the micrographs were made on sections representing the metal that had been removed during the cutting tests.

The hardness of some of the forgings after heat treatment was above the range of what is considered commercially machinable with the present high-speed-steel tools. This condition made

TABLE 4 SUMMARY OF LATHE TESTS ON DIFFERENT STEELS AT 0.0115 IN. PER REVOLUTION FEED AND 0.010 INCH DEPTH OF CUT[1]

Forging No.	Tensile strength, 1000 lb. per sq. in.	Cutting speed, ft. per min.	Number of tests made	Average tool life, min.	Taylor[2] speed, ft. per min.
		Carbon Steel (S.A.E. 1040)			
51	123.6	180	8	10.5	169
51A	111.8	200	8	7.4	181
51B	95.7	250	7	17.1	246
51C	87.5	310	8	19.9	310
51D	78.0	325	8	12.0	310
		Chromium-Vanadium Steel (S.A.E. 6140)			
46	145.4	200	8	13.4	192
46A	135.5	230	8	14.8	223
46B	114.3	280	7	14.1	270
46C	96.0	290	8	32.3	304
46D	90.4	315	8	18.5	313
46E	181.8	120	7	26.0	123
		Chromium-Molybdenum Steel (S.A.E. 4140)			
47	137.2	180	8	41.2	194
47A	120.7	230	7	24.1	234
47B	108.8	320	8	11.2	302
47C	93.7	310	8	26.0	318
47D	89.0	340	8	24.4	347
47E	210.9	60	8	19.1	60
47F	163.3	150	8	8.1	137
		Nickel-Chromium Steel (S.A.E. 3435)			
48	215.5	110	8	5.5	97
48A	149.0	200	8	2.3	161
48C	166.0	165	7	7.8	150
48D	109.5	215	8	8.7	198
48E	108.8	230	4	5.0	200
48F	207.0	110	8	10.4	103
48G	183.1	140	10	8.8	129
48H	135.0	190	8	9.0	175
		Nickel-Chromium Steel (S.A.E. 3140)			
49	120.6	200	7	20.2	200
49A	109.3	250	7	14.6	242
49B	92.5	320	10	21.9	323
49C	94.7	320	8	16.8	314
49D	89.0	330	8	19.6	329
49E	202.8	110	8	17.9	109
49F	140.0	180	8	16.4	177
		Nickel Steel (S.A.E. 2340)			
50	150.0	180	8	19.4	179
50A	121.5	230	6	16.8	226
50B	105.8	300	8	13.8	290
50C	116.0	240	6	22.7	243
50D	101.6	280	9	12.1	266
50D	101.6	300	7	4.6	259
50E	178.8	100	8	17.0	98

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[1] The lathe tests were made dry with high-speed-tool steel numbered AA. Composition and heat treatment of tools are given in Table 1. Selected size and form of tool shown in Fig. 1(*B*).

[2] Computed from the average tool life by means of Equation [1] of the text with $n = {}^{1}/_{10}$, or obtained from Fig. 2.

possible a survey of the entire machinable range from the soft or annealed steel to the upper limit of the usual machinability range.

All cutting tests were made "dry" with a constant feed of 0.0115 in. per revolution, depth of cut of 0.010 in., and variable speed depending upon the properties of the metal cut. The cutting speed was varied with the different steel forgings, but was kept constant for each series of tests on a given forging with given mechanical properties. In general the cutting speeds selected were such as to give a tool life of from 10 to 30 min. as is shown in Table 4. From the data thus obtained the cutting speed permitting a 20-min. tool life (Taylor speed) was computed from the average tool-life value by means of Equation [1] with $n = {}^{1}/_{10}$, or by extrapolating by using the chart shown in Fig. 2.

IV—Results of Cutting Tests

The machining properties of a steel forging are recognized as being affected by (1) its chemical composition, (2) heat treatment after working, and (3) quality of the metal cut. The term "quality" refers to those details of composition and constitution not defined by ordinary chemical analysis or by a statement of the heat treatment, but which nevertheless may play a part in making a metal well suited for a particular service. So-called high quality for one service may be inferior for a different service; the term is general in nature as used in this report. It simply implies recognition of the fact that there are other factors beside chemical composition and heat treatment which may contribute to the final properties of the metal.

In the following experiments most attention was given to the first variable, chemical composition. However, the data throw some light upon the second variable, heat treatment; the quality of the metal was assumed to be practically constant in so far as each type composition was concerned.

As is shown in Table 2, most of the forgings selected were alloy steels which contained approximately 0.4 per cent carbon. A plain carbon steel of similar carbon content was included so that comparisons could be made of the effects on tool performance or machinability of chromium, molybdenum, vanadium, and nickel, either alone or in combination.

The alloy steels were supplied through the courtesy of the Central Alloy Steel Corporation, now the Central Alloy Division of the Republic Steel Corporation, Massillon, Ohio. The plain-carbon-steel forging was contributed by the Illinois Steel Company, Chicago, Ill.

It is not practicable to select a single cutting speed with a fixed feed and depth of cut for tool testing on steel forgings with tensile strengths varying over the wide range of 75,000 to 220,000 lb. per sq. in. For example, if a cutting speed was selected to give a tool life of 10 to 30 min. on a forging heat-treated to give a tensile strength of 90,000 lb. per sq. in. and then an attempt was made to use the same cutting condition on a forging of similar composition but with a tensile strength of 200,000 lb. per sq. in., the result would be immediate tool failure. Similarly, if the cutting speed was selected to give the desired tool life on the forging with high strength, then the tool life on the forging of lower strength would be extremely long. As already shown, the relation between the cutting speed and tool life of high-speed-steel tools, under fixed feed and depth of cut, can be closely represented by an empirical equation, and this fact was used in selecting the speeds at which the tool tests were made.

The comparisons of the different steels cut may conveniently be made on the basis of equal tensile strengths and the Taylor speed, such a comparison being taken as a measure of machinability.

The results of the tests with shallow cuts are given in Table 4 and summarized in Fig. 3. These data show that if machinability with shallow cuts is measured by the cutting speed permitting the tool to last a definite time, then measurable and consistent differences are observed in the machinability of the carbon and various alloy steels used in the experimental work. The fact, however, that some given steel permits a higher cutting speed than another steel for a selected tensile strength which is the same for both materials, does not necessarily indicate that the two steels maintain the same relationship for another tensile strength. This is illustrated in Fig. 3 in that, for the conditions of cutting under consideration, the chromium-molybdenum steel (S.A.E. 4140) had the best machinability of the entire group of steels at a tensile strength of 90,000 lb. per sq. in. At a tensile strength of about 110,000 lb. per sq. in. the chromium-vanadium steel (S.A.E. 6140) had the best machinability of the group, and retained this superiority up to a tensile strength in the neighborhood of 175,000 to 180,000 lb. per sq. in. At this point the tensile-strength–Taylor-speed curve for the chromium-vanadium steel crosses the curves for the nickel-chromium steels (S.A.E. 3140 and 3435), and above this range the latter steels permit the higher cutting speeds.

As is also shown in Fig. 3, the slopes of the tensile-strength–Taylor-speed curves vary for the different steels. The curves for any two compositions may cross, and so reverse the order of superiority. In some cases the slopes were not very different, and there was an appreciable range in tensile strength at which, for all practical purposes, the cutting speeds were the same.

The plain carbon steel (S.A.E. 1040), within the range of tensile strengths obtained by heat treatments, was the most difficult of the group to machine other than the annealed nickel-chromium steel (S.A.E. 3435). This would indicate that the special alloying elements, nickel, chromium, molybdenum, and vanadium, either alone or in combination, when added to the plain carbon steel, improve machinability from the standpoint of the cutting speed, permitting a definite tool life with shallow cuts. However, the superiority in machinability of the alloy steels over the plain-carbon type is not to be attributed solely to any single alloying element, but rather to the combined effects of the alloying elements present in any particular steel, considering carbon also as one of the alloying additions. This is evident from the fact that the same alloying elements which in one case increased the cutting speed, under other conditions, as, for example, when present in different relative proportions and requiring different heat treatments to give comparable tensile strengths, may be without any marked effect on the cutting speed.

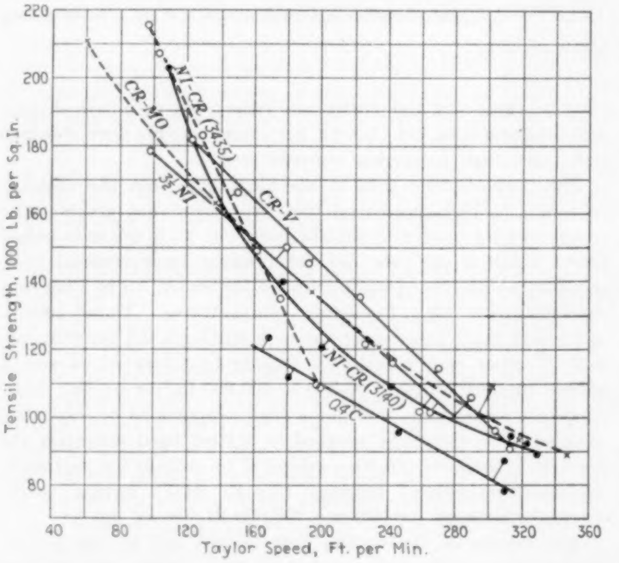

FIG. 3 RELATION OF THE TAYLOR SPEED TO THE TENSILE STRENGTH OF THE SPECIAL STEELS CUT IN THE LATHE TESTS

[For details of chemical composition and heat treatment of steel cut, see Tables 2 and 3. The composition and heat treatment of high-speed-steel tools No. AA used in the tests are given in Table 1. The lathe tests were made "dry" with the size and form of tool shown in Fig. 1(*B*), with 0.0115 in. per revolution feed and 0.010 in. depth of cut.]

This is illustrated by comparisons of the Taylor speeds of the nickel-chromium steels with that of the carbon steel at a tensile strength of 110,000 lb. per sq. in. The nickel-chromium steel containing 0.37 per cent carbon, 0.50 per cent chromium, and 1.26 per cent nickel had a Taylor speed of about 244 ft. per min., while the nickel-chromium steel containing 0.39 per cent carbon, 0.76 per cent chromium, and 2.96 per cent nickel showed the same Taylor speed as the plain 0.42 per cent carbon steel, namely, 198 ft. per min. Thus, at this particular tensile strength the addition of 0.5 per cent chromium and 1.26 per cent nickel increased the Taylor speed by 46 ft. per min., but higher proportions of nickel and chromium additions did not improve the Taylor speed over that of the plain carbon steel.

A comparison of the permissible cutting speeds or machinability of the nickel-chromium steels with those of the other alloy steels used in the experiments is of particular interest. At the higher tensile strengths superior machinability was secured with nickel-chromium alloy steels, but as already stated, certain combinations of these elements may cause a lowering of the cutting speeds of the annealed steels. As shown in Fig. 3, the slope of the tensile-strength–Taylor-speed curve for the steel containing the higher proportions of nickel and chromium was such as to permit a very wide range of tensile values without a marked change in cutting speeds. This fact may possibly be used to advantage in the industrial application of this steel.

With the exception of the nickel-chromium steels, the differences in the Taylor speeds of the alloy steels were within a range of about 20 ft. per min. when considering steels of 100,000 lb. per sq. in. tensile strength. The range increased slightly with increase in tensile strength and reached a value of about 30 ft. per min. when cutting steels of 170,000 lb. per sq. in. The Taylor speed for the 3½ per cent nickel steel was on the low side throughout the entire range. Thus, the 3½ per cent nickel steel was more difficult to machine than the chromium-vanadium steel, and was either more difficult or as difficult to machine as the chromium-molybdenum steel.

The results shown in Fig. 3 indicate that the effect of changes in chemical composition of steel forgings upon their cutting speeds was dependent upon the tensile strength at which comparisons were made. Since the highest tensile-strength values considered in these tests are only produced by quenching, with or without subsequent tempering, changes in chemical composition may act in opposite directions, depending upon whether the steel cut is close to the fully hardened (martensitic) or the annealed (pearlitic) condition.

Elements which cause a lowering in the cutting speeds of the annealed steels may improve the cutting speeds when considering higher tensile strengths. Of the special elements that improved machinability of the different steels cut with shallow cuts, the most effective were the combinations of nickel and chromium or chromium and vanadium for the higher tensile strengths in the neighborhood of 180,000 lb. per sq. in., while the combustion of chromium and molybdenum was the most effective in the lower range of about 90,000 lb. per sq. in.

The mechanical properties of the forgings, given in Table 3, and the cutting-test results, given in Table 4, permit comparisons to be made other than on the tensile-strength–Taylor-speed basis as given in the text. The relative order of machinability of the different steel forgings when comparisons were made on the Brinell-hardness-numbers–Taylor-speed basis agree very closely with the order as given by the tensile-strength–Taylor-speed basis. This agreement might be expected. The order of machinability as given by the tensile-strength–Taylor-speed was changed when comparisons were made on the basis of the Rockwell-hardness (B or C scale)–Taylor-speed, or Shore-hardness–Taylor-speed.

The heat treatments of the forgings, as given in Table 3, show that the desired range of tensile properties was obtained by annealing, or by quenching followed by tempering at different temperatures. The results also show that in some cases approximately equal tensile strengths were produced with a given forging by varying the methods of heat treatment.

The data given in Table 5 show that, in general, the cutting speeds were not appreciably affected by the method of heat treatment by which a given tensile strength was produced. The cutting speeds are slightly higher with the high tempering temperature when comparisons are made of the different methods of quenching and tempering to produce approximately equal strengths. In two cases, namely, with the plain carbon steel (S.A.E. 1040) and 3½ per cent nickel steel (S.A.E. 2340), the better machinability was produced by the heat treatment con-

TABLE 5 EFFECT OF METHOD OF HEAT TREATMENT ON THE CUTTING SPEEDS OF STEEL FORGINGS HAVING EQUAL TENSILE STRENGTHS

Test-Forging No.	Type composition, per cent	Heat treatment	Tensile strength, 1000 lb. per sq. in.	Taylor speed, ft. per min.
		COMPARISON OF DIFFERENT METHODS OF QUENCHING AND TEMPERING		
50A	0.36 Carbon, 3.4 Nickel	1525° F., oil; 800° F., air	121.5	226
50C	0.36 Carbon, 3.4 Nickel	1525° F., oil; 1325° F., air	116.0	243
49B	0.37 Carbon, 0.5 Chromium, 1.3 Nickel	1500° F., water; 1200° F., air	92.5	323
49C	0.37 Carbon, 0.5 Chromium, 1.3 Nickel	1500° F., water; 1325° F., air	94.7	314
48X	0.39 Carbon, 0.8 Chromium, 3.0 Nickel	1500° F., oil; 850° F., air	166.0[1]	145[2]
48C	0.39 Carbon, 0.8 Chromium, 3.0 Nickel	1540° F., oil; 1300° F., air	166.0	150
		COMPARISON OF QUENCHED AND TEMPERED STEELS WITH ANNEALED STEELS		
51C	0.42 Carbon	1650° F., water; 1300° F., air	87.5	310
51D	0.42 Carbon	1550° F., cooled slowly in furnace	78.0	310
46C	0.39 Carbon, 1.0 Chromium, 0.17 Vanadium	1650° F., oil; 1350° F., air	96.0	304
46D	0.39 Carbon, 1.0 Chromium, 0.17 Vanadium	1550° F., cooled slowly in furnace	90.4	313
47C	0.39 Carbon, 0.9 Chromium, 0.17 Molybdenum	1625° F., water; 1350° F., air	93.7	318
47D	0.39 Carbon, 0.9 Chromium, 0.17 Molybdenum	1550° F., cooled slowly in furnace	89.0	347
49C	0.37 Carbon, 0.5 Chromium, 1.3 Nickel	1500° F., water; 1325° F., air	94.7	314
49D	0.37 Carbon, 0.5 Chromium, 1.3 Nickel	1500° F., cooled slowly in furnace	89.0	329
50B	0.36 Carbon, 3.4 Nickel	1525° F., oil; 1100° F., air	105.8	290
50D	0.36 Carbon, 3.4 Nickel	1500° F., cooled slowly in furnace	101.6	266

The lathe tests were made dry with high-speed tool steel AA (Table 1), with a feed of 0.0115 in. per revolution and 0.010 in. depth of cut. Selected form and size of tool shown in Fig. 1(*B*).

[1] Estimated from the known relation between tempering temperature and tensile strength.

[2] Value obtained from tensile strength—Taylor speed curve of Fig. 3.

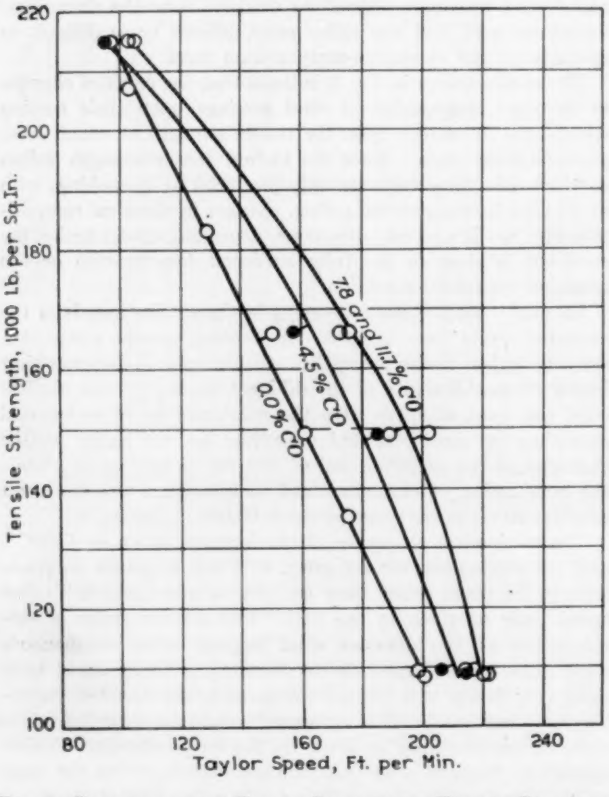

FIG. 4 RELATION OF THE TAYLOR SPEED TO THE TENSILE STRENGTH OF THE NICKEL-CHROMIUM S.A.E. 3435 STEEL CUT WITH HIGH-SPEED-STEEL TOOLS CONTAINING DIFFERENT PROPORTIONS OF COBALT

[Chemical composition and heat treatment of the high-speed-steel tools are given in Table 1. Chemical composition, heat treatment, and mechanical properties of the steel cut are given in Tables 2 and 3. The lathe tests were made "dry" with the size and form of tool shown in Fig. 1(*B*), with feed of 0.0115 in. per revolution and 0.010 in. depth of cut.]

sisting of quenching and subsequently tempering at a high temperature than with the annealing treatment used.

V—TESTS WITH COBALT HIGH-SPEED-STEEL TOOLS

High-speed-steel tools containing considerable proportions of cobalt have been reported to give excellent performance when cutting hard metals, and to have made possible the commercial machining of high-manganese steel. In previous tests made at the Bureau of Standards,[8] it was found that cobalt improved the performance of high-speed-steel lathe tools, with both shallow and roughing cuts, but that the maximum benefits were obtained only with high hardening temperatures.

The proportionate gain in machinability from the addition of cobalt to high-speed-steel tools was somewhat greater with rough turning than with shallow cuts, but an increase in cobalt above about 5 per cent did not produce improvements commensurate with those resulting from additions of 3.5 to 5 per cent together with high hardening temperatures. These conclusions were made from experiments in cutting a 0.3 per cent carbon, $3^1/_2$ per cent nickel-steel forging heat treated to give a tensile strength of about 90,000 to 100,000 lb. per sq. in. Since high-speed-steel tools containing high proportions of cobalt are claimed to be especially adapted to cutting hard materials, the tests with shallow cuts were extended to include the cutting of nickel-chromium-steel forgings (S.A.E. 3435) having tensile strengths between 100,000 and 220,000 lb. per sq. in.

The results of the cutting test carried out in the present investigation with high-speed-steel tools, with and without cobalt, are summarized in Fig. 4. The data show that lathe-tool performance with shallow cuts was improved by the additions of cobalt (together with higher hardening temperatures) to the customary 18 per cent tungsten type of high-speed steel. Maximum gain in performance was obtained when cutting the forgings with tensile strengths up to about 170,000 lb. per sq. in.; above this strength the gain in performance by increasing the cobalt additions was not so marked. Confirmation was also obtained of the previous test results, namely, that the increase in cobalt above about 5 per cent did not produce improvements of the same order as those resulting from 3.5 to 5 per cent together with high hardening temperatures.

VI—MICROSTRUCTURES AND SURFACE FINISHES

Although most attention was given to tool life with shallow cuts in studying the machinability of the different forgings cut in the lathe tests, it was also recognized that the structure and appearance of the surface finish are important factors in finishing cuts, and possibly may be considered as factors in any index to machinability.

Photographs were made of representative surface conditions while cutting with sharp tools for each series of cutting tests, and these photographs were later compared with the appearances as recorded by the observer during the machining operations. There was a fair agreement between the classification by the two methods. It was found that the photographs were a convenient but not an entirely satisfactory basis for making comparisons. All photographs were "natural size" and no attempts were made to show the surface conditions at greater magnifications.

The samples for microscopic examination were etched in 2 per cent nitric acid in alcohol and all micrographs are given at 500 magnifications. The microstructure shown in Fig. 5G corresponded to the best surface finish (shown at 5-H) of the entire series of plain carbon steels (S.A.E. 1040) used in the experiments with shallow cuts. The resulting surface was fairly smooth and polished, with some tool marks, and while not so good as some of the surfaces produced on the different alloy-steel forgings, it may be classed as satisfactory. The finishes as shown in Fig. 5-B, D, F, and J were dull, badly torn, with small bright particles embedded in the surface. None of the latter condition was considered satisfactory so far as the appearance of the finish was concerned.

An interesting feature is shown by comparing the heat treatments, microstructures, tensile properties, finishes, and machinability of forgings 51C and 51D. Forging 51C, quenched from 1650 deg. fahr. in water and subsequently tempered at 1300 deg. fahr. and cooled in air shows some spheroidization or agglomeration of the cementite (Fig. 5-G), satisfactory surface finish, 87,500 lb. per sq. in. tensile strength, and a Taylor speed of 310 ft. per min. Forging **51D**, quenched from 1650 deg. fahr. in water and subsequently annealed at 1550 deg. fahr., slowly cooled in the furnace, shows distinctly lamellar pearlite (Fig. 5-I), 81,500 lb. per sq. in. tensile strength, and a Taylor speed of 310 ft. per min. Forging 51C, therefore, showed the best machinability both from the viewpoint of the cutting speed and appearance of the surface finish, provided comparisons of cutting speeds were made on the basis of equal tensile strengths.

For the cutting conditions investigated, the chromium vanadium, chromium-molybdenum, nickel-chromium (3135), and 3$^1/_2$ per cent nickel-steel forgings were considered as producing surfaces of approximately equivalent smoothness and were slightly superior to the nickel-chromium steel (3435). However, there were no very marked differences observed in the appearance of the surfaces of the forgings of the different alloy compositions, and all may be classified as satisfactory. Each type of alloy steel showed surfaces that were considered much superior to that of the plain carbon steel.

The characteristic finish of the different alloy steels may be described as smooth, medium to highly polished, with but little tearing and showing some tool marks.

The microstructures and surface finishes of all the alloy-steel forgings with the varying heat treatments were examined, but no correlation between structure, surface finish and machinability appeared possible. Fig. 6 showing the microstructures and surface finishes of the chromium-vanadium forgings is representative of the results obtained with the alloy groups.

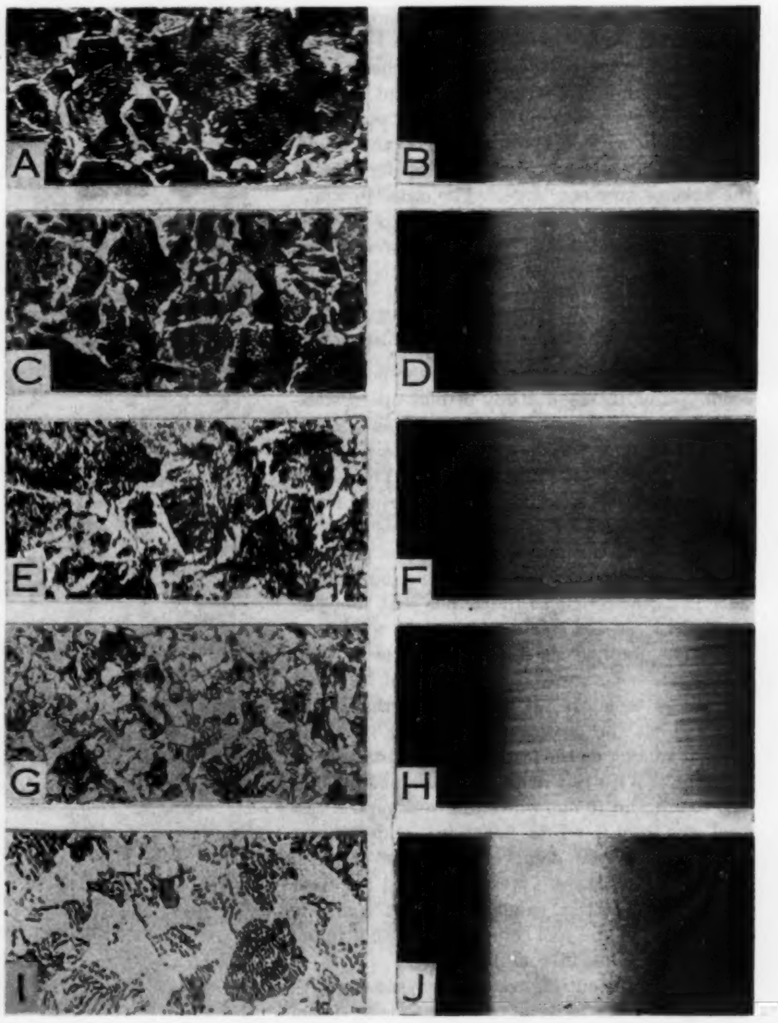

FIG. 5 MICROSTRUCTURES AND SURFACE FINISHES OF PLAIN CARBON S.A.E. 1040 STEEL CUT IN THE LATHE TESTS

[For details of chemical composition and heat treatment of the forgings see Tables 2 and 3. The lathe tests were made "dry" with high-speed-steel tools (AA) of size and form shown in Fig. 1(*B*), with 0.0115 in. per revolution feed and 0.010 in. depth of cut.]

Photomicrograph (×500) (Etched with 2 per cent nitric acid in alcohol)	Surface finish (×1)	Forging No.	Tensile strength, 1000 lb. per sq. in.	Cutting speed, ft. per min.	Taylor speed, ft. per min.
A	B	51	123.6	180	169
C	D	51A	111.8	200	181
E	F	51B	95.7	250	246
G	H	51C	87.5	310	310
I	J	51D	78.0	325	310

Vanick and Wickenden[10] have shown in some lathe tests of plain and alloy low-carbon steels of the carburizing type that for each steel and its particular heat treatment there was a critical range of volume removal rates within which a rough finish was obtained. By avoiding this critical range, smoothly finished surfaces could be obtained. It was found that cutting conditions leaving a rough surface could be changed to give a smooth finish by: (1) either lowering, or preferably increasing, the cutting speed until it is outside the critical range; (2) maintaining the speed but changing the cut or feed; (3) sharpening the cutting angle of the tool and maintaining speed and shape of chip; (4) changing the hardness of the steel being cut; usually increasing it in order that a good finish is produced at an easily obtainable speed.

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[10] J. S. Vanick and T. H. Wickenden, "Smooth Finish Machining of Low Carbon Plain and Alloy Steels," Trans. A.S.S.T., vol. 11 (1927), p. 551.

Rapatz,[11] in a study of the surface conditions of plain carbon and nickel-chromium steels of different strengths and hardness in some lathe turning tests, found that higher tensile strengths, higher speeds, and greater depths of cut favored the production of smooth surfaces, assuming the turning is performed with perfect tools. Yield point, elongation, and reduction of area were less important for obtaining smooth surfaces.

Although some of the surface finishes of the alloy-steel forgings used in the present investigation were more highly polished and came nearer to being smooth than others, the differences observed in the general characteristics of the finishes were not large. The order in which the forgings were arranged in regard to the surface appearance, under the conditions of test used, possibly might be different if the cutting conditions were changed.

VII—SUMMARY AND CONCLUSIONS

1 The tests described in this report were made primarily as a study of high-speed-steel-tool performance with shallow cuts as affected by variations in the chemical composition and heat treatment of the steels cut. The study also included consideration of the surface finish of the various steel forgings, the microstructures of the metals cut, and also of the tool performance as affected by additions of from 3.5 to 11.7 per cent cobalt to the 18 per cent tungsten type of high-speed steel.

2 The method used for testing lathe tools with shallow cuts was based upon the fact that when two tools are set at equal depths in one tool holder, the second, or indicating tool, will not cut so long as the leading, or test tool, shows no wear. With this method of test, the indicating tool began to cut when the wear on the test tool was from 0.001 to 0.002 in., and this was considered as the point of failure of the test tool. In most cases it was found that the wear of 0.001 to 0.002 in. coincided with a complete breakdown of the tool comparable to that found with heavy cuts in rough turning.

3 The lathe cutting tests were made dry with high-speed-steel tools of a selected size, form, composition, and heat treatment, with a fixed feed of 0.0115 in. per revolution, 0.010 in. depth of cut, and variable cutting speeds depending upon the properties of the material cut.

4 The metals cut included a plain carbon and various alloy-steel forgings heat treated to give tensile strengths between 75,000 and 220,000 lb. per sq. in.

5 The measure of machinability was the cutting speed permitting a definite tool life.

6 Measurable and consistent differences were observed in the machinability of the carbon and alloy steels used. The fact, however, that some given steel permits a higher cutting speed than another steel for some tensile strength which is the same for both materials, does not necessarily indicate that the two steels maintain the same relationship for another tensile strength.

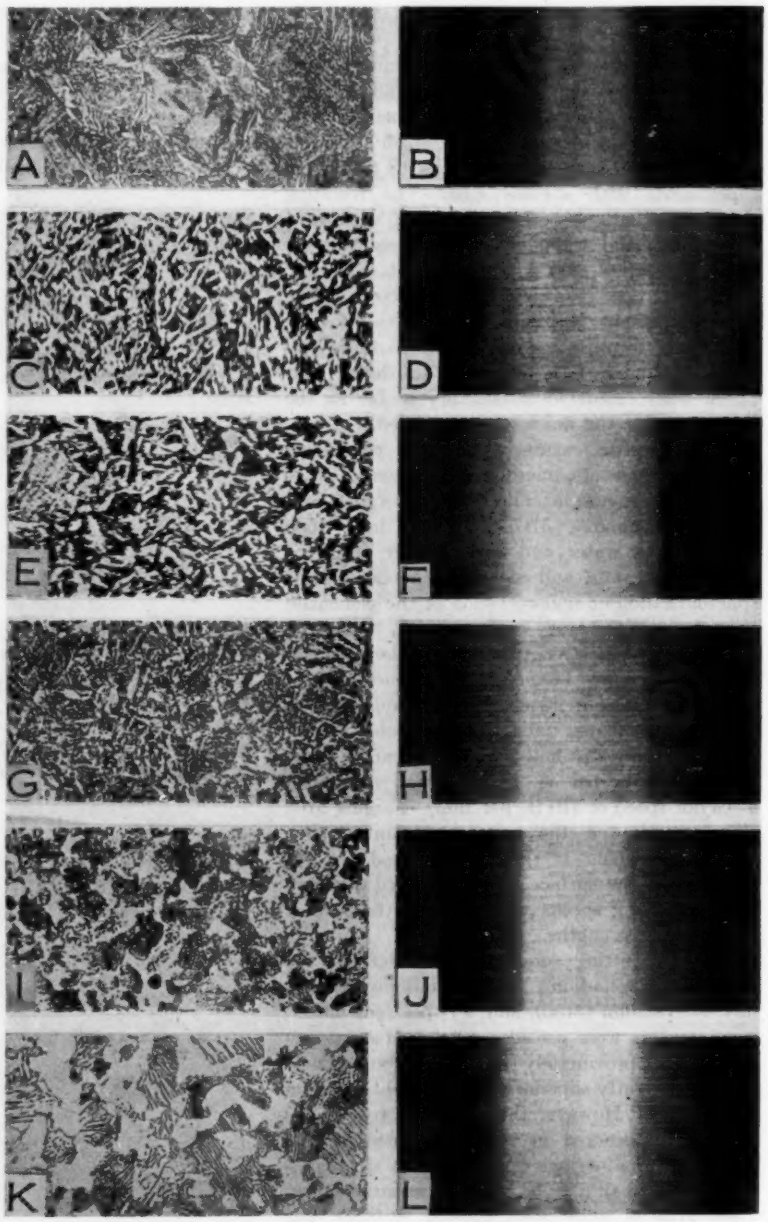

FIG. 6 MICROSTRUCTURES AND SURFACE FINISHES OF CHROMIUM-VANADIUM S.A.E. 6140 STEEL CUT IN THE LATHE TESTS

[For details of chemical composition and heat treatment of the forgings see Tables 2 and 3. The lathe tests were made "dry" with high-speed-steel tools (AA) of size and form shown in Fig. 1(*B*), with 0.0115 in. per revolution feed and 0.010 in. depth of cut.]

Photomicrograph (×500) (Etched with 2 per cent nitric acid in alcohol)	Surface finish (×1)	Forging No.	Tensile strength, 1000 lb. per sq. in.	Cutting speed, ft. per min.	Taylor speed, ft. per min.
A	B	46E	181.8	70	123
C	D	46	145.4	200	192
E	F	46A	135.5	230	223
G	H	46B	114.3	280	270
I	J	46C	96.0	290	304
	L	46D	90.4	315	313

7 The 0.4 per cent plain carbon steel, within the range of tensile strengths obtained by heat treatments, was the most difficult to machine with shallow cuts other than an annealed

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[11] F. Rapatz, "The Surface Conditions of Materials in Machining, Especially Turning," *Archiv. für Eisenhuttenwesen*, vol. 3 (1930), p. 717.

nickel-chromium steel. The surface finishes of the carbon-steel forgings were also inferior to those produced on the alloy-steel forgings. However, with a particular heat treatment of the plain carbon steel, showing a microstructure of partially spheroidized or agglomerated cementite, a medium smooth and satisfactory finish was obtained with the cutting condition used. This same heat treatment also resulted in the best machinability of the group of carbon steels when comparisons were made on the basis of the cutting speeds at equal tensile strengths.

8 The superiority in cutting speeds of the alloy steels over the plain carbon type is not to be attributed solely to any single alloying element, but rather to the combined effects of the alloying elements present in any particular steel, considering carbon also as one of the alloying additions.

9 The results of the present tests with shallow cuts compared with previously reported tests with roughing cuts[5] show that the effect of change in chemical composition of steel forgings upon their cutting speeds is dependent not only upon the tensile strength at which comparisons are made, but also upon the conditions of cutting. That is, steel forgings that show superior machinability with shallow cuts at some tensile strength do not necessarily show a similar superiority with roughing cuts.

10 Of the special elements that improved machinability of the different steels cut with shallow cuts, the most effective were the combinations of nickel and chromium or chromium and vanadium for the higher tensile strengths, in the neighborhood of 180,000 lb. per sq. in., and chromium and molybdenum in the lower range of tensile strengths, about 90,000 lb. per sq. in.

11 In general, the cutting speeds were not appreciably affected by the method of heat treatment by which a given tensile strength was produced. The cutting speeds were slightly higher with the higher tempering temperature when comparisons were made of different methods of quenching and tempering to produce approximately equal tensile strengths. In two steels, namely, with the plain carbon steel and $3^1/_2$ per cent nickel steel, the better machinability was produced by the heat treatment consisting of quenching and subsequently tempering at a high temperature than with the annealing treatments used.

12 Lathe-tool performance with shallow cuts was improved by the additions of cobalt (together with higher hardening temperatures) to the customary 18 per cent tungsten type of high-speed-tool steel. The maximum gain in performance was shown when cutting the forgings with tensile strengths up to about 170,000 lb. per sq. in., but above this strength the gain in performance was not so marked.

13 Confirmation was also obtained of previous test results, namely, that the increase in cobalt above about 5 per cent did not produce improvements in tool performance of the same order as those resulting from $3^1/_2$ to 5 per cent and higher hardening temperatures.

14 The differences observed in the surface finish of the different types of alloy-steel forgings were not large, and all were considered as being satisfactory and of about equivalent smoothness.

15 A correlation of the cutting speeds, tool life, surface finish, etc., shows that, with the test method used, the machinability of the carbon- and different alloy-steel forgings used in the experiments may be properly determined or measured by the cutting speed permitting the tool to last a definite time.

VIII—ACKNOWLEDGMENTS

The author acknowledges the assistance of E. C. Smith, chief metallurgical engineer, formerly of the Central Alloy Steel Corporation, Massillon, Ohio, for supplying the alloy-steel forgings and of S. Epstein, metallurgist, formerly of the Illinois Steel Company, So. Chicago, Illinois, for supplying the carbon-steel forging used in the cutting tests.

Grateful acknowledgment is also due to W. V. Magruder, laboratory aid, formerly of the Bureau of Standards, for his assistance in carrying out the many tests; to Louis Jordan, senior metallurgist, for his suggestions in the presentation of the test data, and to S. E. Sinclair, junior metallurgist, for the micrographs given in this report.

Discussion

Joseph Geschelin.[12] The formal methods of testing procedure, correlation of data, and other details involved need no comment. Any research job coming from the Bureau of Standards carries its own seal of approval.

What interests more than anything else is the value of the results to the production men and other investigators in the field of the cutting of metal. Much has been said in the past about machinability. In fact, at least two different machines have been introduced during the last two years for measuring machinability. But the simplest statement heard thus far is the one made by the author—namely, that the measure of machinability was the cutting speed permitting a definite tool life. This might well be linked with the quality of the surface finish, inasmuch as this is frequently a factor of prime importance.

It is very important to know that alloy steels are superior in machinability to the plain carbon type from the standpoint of cutting speed, permitting a definite tool life with shallow cuts. At higher tensile strengths, superior machinability was secured with nickel-chromium alloy steel, and as shown in Fig. 3, the steel containing the higher proportions of nickel and chromium permitted a wide range of tensile values without a marked change in cutting speed. This is, undoubtedly, valuable information to the production executive.

Another very important point is the value of combinations of chromium and vanadium, or nickel and chromium, to improve machinability for the higher tensile strengths in the neighborhood of 180,000 lb. per sq. in. Vanadium has many desirable qualities in alloying, most important of which are in the field of drop-forged steel for engine crankshafts in automotive works. It is certainly noteworthy that this element also aids machinability.

The author raised an interesting point about the possibility of using R. E. W. Harrison's method for comparing surface finishes. The criteria established by Mr. Harrison for measuring surface quality standards may have much wider application than any of us realized at the time the paper was first read.

W. S. Huson.[13] There is much in this paper that invites discussion. The data are valuable additions to those already published and bring out features in tests that may answer to some degree why in the machining of steels a certain tool will stand up longer on one grade of steel than on another generally classed in the shop as this or that alloy steel. Study of the life of a tool is desirable, say where black-bar stuff is bought on the basis of as little stock as possible to be removed to bring to size on a production basis. How much of these data can be absorbed is conjectural; each high-speed cutting tool claims superiority in some particular over the other, usually in that the cutting edge will stand up longer at higher speeds; when it does not perhaps the cutting edge is not shaped just right, and so on—shop practice often being governed by empiric finding rather than real investigation.

Maybe this thought is born of an excerpt from R. W. Kent's paper (Trans. MAN-52-2b), "Use of Economic Manufacturing

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[12] Engineering Editor, *Automotive Industries*, Philadelphia, Pa.

[13] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

Quantities—Obstacles Encountered in Introducing Mathematical Formulas for Economic Manufacturing Quantities."

While the inference of stricture can be put aside, the remark that "Since the average manufacturing executive does not appear to have sufficient mathematical mental agility to clearly, thoroughly, and readily grasp the development of formulas for mathematical determination of economic production lots or manufacturing quantities, he should prepare himself to settle down and really seriously and patiently set about understanding how the particular formula recommended for his use was derived," carried some truth with it and can be broadened into the consideration of Digges' paper.

With ten years' background of study and the ample bibliography quoted, the analyses of tool action on various grades of steel can well be put into service if time is given them, not only that dependable practice will result, but also a grasp of how the laboratory is endeavoring to help in the better, quicker, and less arduous production of finished material.

O. W. Boston.[14] The paper is a valuable contribution to the interesting subject of metal cutting. It gives the results of a great amount of painstaking experimental work, for which the Society should be grateful. The data are of value to hundreds of industries interested in finish machining.

It seems, however, that a word of caution might be offered to stress the point that the data presented apply to a particular shape of tool and to certain cutting conditions as to size. It is believed that the exponent of tool life in Equation [1] is dependent both on the shape and size of the tool and upon the depth of cut and feed per revolution. The value of this exponent has been taken as $^1/_{10}$ for the specific conditions indicated. If the tool shape were changed, it is believed that this exponent would change correspondingly. Further, if the size of tool, depth of cut, and feed were changed, it is believed that the exponent would again be changed. The author has previously shown that in rough turning, the value of this exponent is $^1/_7$ for a different tool (see author's reference 5). It seems that one important missing link in our knowledge of metal cutting is the transition from this $^1/_7$ to $^1/_{10}$. In other words, it seems that still more information of an experimental nature should be presented for different shapes and sizes of tools, each covering the whole range from heavy to finishing cuts.

It is perhaps unfortunate that the author has not attempted to correlate the results of tool life as a function of tensile strength for various steels as shown in Fig. 3 for finishing cuts with similar data obtained by taking roughing cuts, as presented in an earlier paper. The same applies to the influence of the amount of cobalt in the high-speed steel of which the tools were made.

The use of the term "machinability" has been referred to in the discussion of this paper. The writer is of the opinion that there should be a number of definitions for the word machinability, and that the word machinability has a proper place in our vocabulary. He deplores, however, the random use of the word, such as indicating that this steel has good machinability and that bad machinability, unless the meaning of machinability in the particular case is made perfectly clear.

The author has used the term machinability with reference to tool life in such a way that the material having the greatest or best machinability permits the greatest life of the standardized tool. In the latter part of the paper a study of surface finish is made in which, naturally, that structure which gives the best finish would be the most machinable in this particular sense. There are many methods of designating machinability, all of which are of peculiar interest. They may be outlined in part as follows:

(1) Tool life in turning, milling, drilling, broaching, tapping, etc.
(2) Torque in drilling
(3) Thrust in drilling
(4) Cutability as measured by drill penetration, or the penetration of a hacksaw subjected to a given feeding load, rather than the rate of feed
(5) The surface produced by a given tool
(6) Power required when cutting by various processes.

There is no known relation between these various processes; 13 per cent manganese steel cuts with great difficulty, and yet the torque in drilling with a certain size and shape of drill is no more than that produced by a number of other commonly used steels; the thrust, however, is much greater. Truly then, we are interested in this phase of machinability as it affects the drill shape and size for strength and as it affects the design of the machine tool.

A coolant on a tool may produce longer tool life and lower power in turning, drilling, etc., but when used in a hacksaw with gravity feed, it may slow up the cutting action.

These are only a few of the complexities of metal cutting, and the writer pleads for tolerance in the use of the word machinability in its various specific applications.

G. A. Bouvier.[15] The paper furnishes new proof that an understanding of the problem of machinability is extremely difficult. This is at least partly due to the fact that the definition of the term machinability is still subject to personal opinion and depends, to a large extent, upon the practical purpose of the investigation.

Since F. W. Taylor first published his book on the art of cutting metals, much work has been done in this country and abroad to throw new light on this problem. It is significant that the tests described in this paper deal with shallow cuts; i.e., cuts which Mr. Taylor did not consider as giving conclusive information on the machinability of a metal. Our manufacturing methods have, however, changed to such an extent that a knowledge of the machinability at shallow cuts is imperative. In the application of tools of the tungsten-carbide type, for instance, it has been found that increased benefit from these tools can be derived from higher speeds rather than from heavier cuts.

The trailer-tool method employed by the author for measuring tool wear seems to correspond to actual shop conditions, to the extent that a machined diameter must be held within certain limits. There is, however, doubt in my mind that this method can be consistently relied upon to give a tool life or end point comparable to that obtained with the breakdown test.

As an illustration, let us assume that a hard and a soft material are to be investigated. It seems rather certain, in this case, that the time at which the trailer tool starts to cut will be influenced not only by the wear on the cutting tool but also by the mechanical properties of the material investigated, such as ductility, drag on the tool, etc., which need not necessarily have an important bearing on the wear of the tool, but will show up on the trailer. No doubt the author has considered this factor as well as the variation due to spring and radial displacement of the tool resulting from the chip's pressure.

It would be interesting to know in this connection what reductions in the machined diameter of the various test pieces were necessary to make the trailer tool cut and also what the thickness of the chip removed by the trailer tool was in each case. The writer would expect that on some materials there would be a slight burnishing action from the trailer tool before it "bit."

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During some tests, it has apparently been found that the wear on the cutting edge of 0.001 to 0.002 in. coincided with the complete breakdown of the tool comparable to that experienced with heavy cuts in rough turning. Comparisons between the results of tests with shallow and roughing cuts seem, in this case, justified.

As far as comparisons are concerned between tests with shallow cuts where the tool life was based entirely on the time when the trailer tool started to cut, and roughing tests, where the complete breakdown of the tool was used as a basis for tool life, it may be that the inconsistency in the results is partly due to a fundamental difference in the two testing methods.

It is interesting to note that the author has also made an attempt to grade the finish and to correlate it with the material's machinability. The author has made reference to the phonograph method described by Mr. R. E. W. Harrison in one of last year's papers, which is intended as a mechanical means for grading the finish of ground surfaces. Certainly it is desirable that further research be done to find a satisfactory grading method which eliminates the subjective factor and is based entirely upon electrical or mechanical principles. The work already done by Mr. Harrison may point to a step in the right direction. Personally, the writer has not abandoned hope that the method employing reflected light will prove practicable. It seems quite reasonable to him that machinability studies involving finish should use a predetemined change in finish rather than tool wear to indicate the end point.

The term machinability as used by the author is taken to mean tool life measured either with the trailer-tool method or the breakdown test. In both cases a definite point has first to be reached before a test can be evaluated.

As noted by the author, a number of investigators have measured the cutting pressure on the tool for a definite set of cutting conditions and used the results as a measure of machinability. Others have used the increase in cutting pressure during the cutting action of the tool, since this increase bears a definite relationship to the tool life. The first method is misleading, since it does not disclose all the characteristics of a material which tend to wear out the cutting edge of a tool. The second method is basically correct, but has in its practical application in most cases suffered through the inadequacy of the measuring devices, usually consisting of hydraulic gages which, due to their sluggishness, do not register all the details of the cutting action. Suitable rigidity of the tool mounting is difficult if not impossible to obtain, and it is questionable whether any worth-while improvements can be made in devices of this type.

The Western Electric Company has overcome the greater part of the above-mentioned difficulties in a device which permits the measurement of all three components of the tool pressure. The device makes use of the piezo-electric property of quartz crystals when subject to pressure in certain directions. Electrostatic charges are created which within a wide pressure range are directly proportional to the pressures. On account of the relatively high natural frequency of the mechanical system comprising the crystals and tool mounting, the charges follow very closely all changes in the applied pressures. A record of the amplified charges in connection with calibration curves will therefore give a clear picture of the cutting action. The writer expects that it will be possible to report on the device and some of the results obtained at the next annual meeting.

The tests conducted by Mr. Digges have shown a certain consistency between tensile strength and Brinell hardness on one hand and tool life on the other hand, if one and the same material is considered. Where different materials were compared, a change in the order of machinability was found in some cases for various tensile strengths. This seems very confusing. The results of the tests, together with those obtained in previous tests by Mr. Digges and other investigators, make it seem as though we are far from a general solution of the problem of machinability and that only through a competent correlation of all the work done so far and cooperation on a national or even international scale can we hope to make satisfactory progress. There are an infinite number of variables, and the Bureau of Standards would render a splendid service by compiling a list of them and if no more than on a "guesstimate" basis indicate their relative importance.

The matter of chemical composition alone is obviously of tremendous importance, and the writer appreciates very much the author's contribution to the subject.

AUTHOR'S CLOSURE

The word "machinability" is in every-day use, and it now occupies a permanent place in our vocabulary. It is therefore unfortunate that so much confusion exists from the use of this term. As Professor Boston has pointed out, this confusion is due to the various criteria used by different investigators and machinists to evaluate machinability. Probably in the near future some one will define machinability so that its meaning will be clear to all concerned and will set aside the existing confusion. Until such a time one can only use the term for a specific application.

It is indeed gratifying to have Mr. Geschelin's and Mr. Huson's remarks on the value of the results to the production men.

Professor Boston's word of caution in regard to the application of the test data is important. The application of the results to conditions differing widely from those used in the tests are likely to be misleading. The order of superiority in machinability of the different steel forgings would probably be different if, for example, the tools had been of the new cutting material, cemented tungsten carbide. Reference has been made to the results of tests with roughing cuts, presented in an earlier report. It was felt that it would be needless repetition to include these results in the present report.

In reference to Mr. Bouvier's discussion, when the test conditions are such as to permit the tool to cut for a long period, the trailer method does not always give an end point corresponding to a complete failure of the tool with roughing cuts. In the majority of the tests here reported, the end point coincided with a complete breakdown of the tool.

It is believed that the variation in size of the forgings used in the cutting tests had no influence on the time at which the trailer tool began to cut. The design of the tool holder was such as to eliminate errors due to the mechanical properties of the materials investigated, spring of tool, etc. No attempts were made to measure the thickness of the chip removed by the trailer tool, and no burnishing action from the trailer tool was observed in the cutting tests.

The author is looking forward with considerable interest for the presentation of Mr. Bouvier's report of his experiments in measuring the forces on cutting tools with quartz crystals. Some Japanese investigators also have made use of the piezo-electric properties of quartz crystals in measuring the force on cutting tools. Some data on this subject are contained in a report by M. Okochi and M. Okoshi, Scientific Papers of the Institute of Physical and Chemical Research, Vol. 5 (No. 84), 1926–1927, p. 261.

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Small Tools and Gaging for Interchangeable Manufacture

By E. J. BRYANT,[1] GREENFIELD, MASS.

To secure interchangeable parts in continuous production, proper gaging is necessary. The author describes the specifications by which the design of the correct gage can be greatly simplified. The methods by which the manufacture of reamers, taps, threading dies, drills, etc., can be held to uniformity, within proper tolerances, are covered in the paper.

ONE of the first, and probably the most important, requirements before proceeding with the manufacture of an interchangeable product is a set of drawings of the parts which have been dimensioned to give working tolerances on all dimensions, with particular emphasis on the fact that all dimensions should have a tolerance. The less important dimensions can be satisfactorily covered by standard or printed specifications on the drawing form, which gives a tolerance for all dimensions for which the tolerance is not given between the dimension lines. Each engineering department can vary this fixed limit according to the product with which it has to deal.

These general specifications or tolerances should cover only those dimensions that designate unmachined surfaces, such as the general dimensions of castings or forgings. The machined surfaces should be dimensioned with the limits permitted for that particular dimension. The practice in the way of designating these limits should be standardized; as a matter of fact, it would be highly advantageous if the practice in this regard could be standardized throughout the world. Certainly there is no reason why all of the drawings originating from one engineering department should not be consistent in this respect, but it is very rare that they are, the reason being that different draftsmen brought up in different organizations have different methods of designating tolerances.

Thorough engineering practice in dimensioning the drawings with tolerances will bring to light many points in construction and possible interferences that might otherwise be overlooked until much time and material have been wasted by incorrect machining. This is not a theory—the case has been proved time and time again to the sorrow of many.

In bringing out a new product it is highly desirable and economical to have some parts made to the maximum metal condition and other parts made to the minimum metal condition, and to have these parts tested in actual performance before proceeding with the designing and making of tools for producing the parts. Reasonable tolerances should be allowed on all dimensions. It is well to emphasize "reasonable" rather than "maximum" or "liberal," because an excessive amount of tolerance—that is, tolerance beyond that required for ordinary careful workmanship—has been found to lead to more spoiled work through carelessness than is the case where reasonable tolerances are used. As a matter of fact, present-day production is demonstrating that so-called "close" tolerances take no more time to produce and result in less spoiled work, and in general better product, than would be the case if the maximum functional tolerance is allowed on the product.

Of course, the value of interchangeable product is not only from the economy in assembling, but in service to the consumer in being able to supply replacement parts, and this is, in most industries, an absolute necessity.

In the case of thread sizes, it is desirable for the engineering department to adopt a list of preferred sizes to be used on all product. The individual engineer in designing his product should use no other sizes except in the case of an absolute necessity, in which case the necessity for their use should be decided by the chief engineer or by a board. Economies resulting from this procedure will result in very material savings, as it materially affects inventories and availability of production tools.

Fixtures and Jigs

The next consideration after the specifications and models or sample parts is the necessary jigs and fixtures. The author will not touch on this, as his subject covers small tools and gages, and he does not interpret this to include jigs and fixtures.

The general principle in this field was well outlined by Joseph W. Roe in his contribution to the Machine Shop Practice Division at the Annual Meeting on Dec. 3, 1928.

The author only emphasizes in this connection the importance of working from one locating or registering point on the product, the necessity of clamping the work directly over a supported surface, and also that the jigs be so designed that their own clamps do not distort them. There is no metal that is being worked with in any of the industries today that is not flexible to a far greater extent than is generally realized, which results in many inaccuracies that are difficult to account for. One should provide for ample chip clearance and easy cleaning.

Gages

With the proper specifications, the designing of proper gages for the part is greatly simplified. Usually it is only when it has been decided to put limit gages on the job that serious consideration is given to the tolerances that can be allowed. A large percentage of the drawings received with instructions to supply limit gages do not have sufficient information from which to set the proper limits, so either general practice must be applied or the job be delayed until more information in regard to the functional requirements of the part are obtained. Gages answer the workman's question regarding the accuracy to which he must work.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

With the drawings properly dimensioned and gages made to these dimensions, no time is wasted by the workmen or foreman in guessing at the accuracy required for the particular dimension of the part on which he is working. The author cannot refrain from emphasizing this fact, because he knows from experience that in thousands of shops the management does not insist on the necessary time being taken to produce proper drawings and tools before proceeding with production, and they never know what their final losses are from this procedure. These losses are continuous from the time the incomplete drawing leaves the engineering department until the shop has discontinued the manufacture of the part to that drawing. The sum total of losses from this procedure runs into millions of dollars a year.

The general subject of gaging is covered by Prof. Earle Buckingham's paper contributed to the Machine Shop Practice Division at the Annual Meeting on Dec. 2, 1929.

The author would emphasize the recent trend of general practice, which is the very general use of class 2 and class 3 specifications for screw-thread work. Class 2 specifications meet the requirements of general thread assemblies, and the tolerances are as liberal as are necessary for nearly all classes of work, and with the possible exception of some of the small numbered sizes for which class 1 is more practical. Class 3, which was originally designed for the better grade of automotive work, has proved to be generally applicable to all of the better grades of screw-thread requirements.

The economical production of threaded holes to class 3 specifications has been made practical by the general use of ground thread taps, which can now be obtained in all sizes. External threads are generally produced with adjustable tools, either adjustable dies or thread-milling machines, and there is no loss of production involved in holding these parts to reasonable limits. In many instances this member can be held to class 3 specifications where it is desirable and economical to produce the hole to class 2 specifications. The combination of class 2 and class 3 assembly naturally makes a better or closer condition than the application of class 2 to both members, and it is an economical solution of many problems.

It is not economical to check too many dimensions with one gage. Gages are designed to disclose that the product is within the specifications, and except in the case of receiver gages, which check the relation of all of the important dimensions of a part, it is desirable that the gages should check the individual dimensions, as the operator has the tendency to overlook or miss some point that is outside of the gage dimension.

Automatic and semi-automatic gaging machines with electrical indicating and selecting devices are being more generally used. This type of gaging equipment requires a very large volume of production to justify the outlay involved, but where the production is continuous it results in important savings.

In closing the subject of gages, there should be emphasized the importance of a periodical inspection, not only of the gages used on the product, which of course may be subject to inspection at intervals or as they are put into use, but inspection of the master or reference-gage equipment of the factory, as a necessity for good production.

REAMERS

Reamers for interchangeable manufacture can be obtained in adjustable designs for most of the large sizes of the more expensive reamers, thus permitting the reamer to be kept well within the tolerance. Smaller sizes, of course, are used in the solid type, and must be discarded when worn below the limits of the particular job on which they are used.

The hole tolerances set up by the General Motors organization, as outlined by Mr. D. W. Ovaitt in his articles in *American Machinist* of June 5 and 12, 1930, provide a practice which can be economically used in many factories, as when the solid or non-adjustable tools of these small sizes are worn below the limits for one class, they can still be used for other classes. The establishing of the classes of limits for plain holes is now before the Sectional Committee on Allowances and Tolerances for Cylindrical Parts and Limit Gages, and undoubtedly classes of fits embodying the principles outlined by Mr. Ovaitt will be considered by this committee.

The increased efficiency of grinding permits of the more economical production of holes to closer requirements, also the use of the new steel alloys for reamer blades has increased the life of reamers, and gives economical production of holes to close limits in large quantities. Cemented tungsten carbide can economically be used for reamer blades in some cases.

It has been interesting to note the change from spiral to straight and then from straight to spiral flutes in reamer designs. The old straight fluted reamers were generally superseded by the spiral fluted to prevent chatter and give smooth holes. With the adoption of staggered spacing, the straight fluted reamer came again in vogue, as the straight fluted reamer will always produce a straighter hole than will the spiral fluted reamer; and now, probably because of better machine alignment and machining, the spiral fluted reamer is in more general use than the straight fluted reamer, although the fact still remains that the straight fluted reamer will produce a straighter hole.

TAPS

Probably one of the most troublesome problems in the past of interchangeable manufacture has been the thread parts, particularly the tapped holes. The ground threaded tap has reduced the trouble along this line to the minimum, and today ground thread taps of high-speed steel are available that are made more accurately than the old cut thread taps. They are correct for lead and form, and are within the same pitch diameter specifications as the cut thread tap. Their cost is no greater than was previously paid for the cut thread tap. These taps are generally suited to produce holes to class 2 tolerances. Then for the close specifications, such as the class 3 tolerances, the close-limit ground thread tap is available and at a very reasonable price. Both types of taps are available in various styles. The so-called hand tap of the plug type is the one most generally used in production. The various forms of the so-called "gun" tap, which curls the chip ahead of the tap, are very popular for machine tapping, particularly for through holes or where the hole can be drilled deeper than the depth of thread required. This type of tap requires considerably less power, and is used to advantage on the multiple tapping machines, automatic tappers, and when tapping with electric motors.

Ground taps can be obtained with different types of relief, and special taps with interrupted threads are a necessity for economy production of threaded holes, particularly on the tough, stringy metals, such as tubing.

The length of chamfer on a tap has an important bearing on the production, not only in the size of hole produced, but also in the life of the tap. One length of chamfer will not produce the best results in all materials under all conditions. In general, the short chamfered tap has more tendency to free itself—that is, cut oversize—and the long chamfered tap cuts closer to size. In cast iron and brass, the short chamfered tap—that is, a tap with one to two thread chamfer—is the best. On steels a longer chamfered tap is usually desirable and frequently a necessity. The alignment of the machine—that is, the alignment of the spindle with the hole—is an important feature, several production engineers going so far as to assert that the major part of the tap breakage is due to this defect. The producing of a good thread in most tough tapping jobs is usually obtained with a combination of the correct chamfer, cutting hook of the tap, and the use of the proper lubricant.

The necessity of the proper lubricant cannot be overemphasized. Where the very best possible design of tap is used, it is at times impossible to obtain anything like a decent thread or only a very limited number of good holes, until the proper lubricant is used. This means in most cases that a special lubricant adapted to the work must be found. There are some on the market at the present time that are especially adapted for the tapping of holes in tough material. Another known effect of lubricants is on the cutting size of the tap. Lubricants with a high sulphur base will cause the tap to cut considerably over in size.

The actual size that a tap will cut is controlled by a number of different conditions. This fact is not generally appreciated by the purchaser of taps, but the fact is that the size of the holes produced by a tap will vary all the way from one or two thousandths of an inch under to several thousandths over the actual size of the tap used. Of course, the undersize condition is due either to shrinkage of the metal or distortion after the tap has been removed, but as this is a condition that is frequently encountered it must be taken into consideration. Under average conditions, in steel and iron, a tap does not cut closer than 0.001 in. over its own size, and frequently it cuts from 0.002 to 0.003 in., or even more, larger than itself due to conditions under which it is used.

THREADING DIES

As previously pointed out, the threading dies used on production manufacture can nearly always be adjusted or set. This is true of the round die, spring die, "acorn" die, self-opening die, as well as thread milling. Efficient production of these tools is mainly dependent on the correct grinding of the chamfer and its proper cutting angle. The cutting angles are usually established by the maker for the class of work for which the die is recommended. This varies from radial for cast brass to a very acute cutting angle for copper, with other grades of metal ranging in between.

The proper clearance at the throat of the die is one thing that most affects the production life of the die. The chamfer of the throat should be ground with just sufficient clearance and provide the fullest amount of support to the cutting edge. If the cutting edge is not supported, it will chip when the die becomes a little dull.

DRILLS

The drill is called on to do a great deal of work in production. Little is said about it in general, but there has been a great deal of improvement made in it, and today this company is obtaining far more production from its drills than ever before. Undoubtedly most of this is due to improved steels supplied by the steel maker and to better heat treatment, but to some extent to the improvements in the design and manufacturing methods of the drills. Drills today are more accurately finished than ever before. The credit for this should go to the developers of the centerless grinding process.

INSPECTION

One of the most important phases of interchangeable manufacture is the set-up for inspection. In general the inspection should be under the management and not under the production control. The primary purpose of the inspection should be to assist the production to produce acceptable parts.

The so-called first-piece inspection method, which required the first piece produced on any given operation to be passed by the inspector, and the continued inspection or provision for the continued assurance that the work being produced by the machine is correct, rather than waiting until all of the parts have been completed, is usually the most satisfactory and economical inspection method to pursue.

Here, again, the importance of correct specifications from the engineering department results in substantial saving of time and material. Where the product is to go to a consumer who has his own inspection for material purchased, it is highly desirable that the consumer and producer get together on their inspection methods. Much time is wasted and much money spent in shipping rejected material where the chief difficulty is that the inspectors are not checking in the same manner. Producers are as anxious to make material that passes the consumer's inspection as the consumer is to have it pass his inspection, and rejections are usually the result of misunderstandings or misinterpretation of specifications. When the user finds out the particular inspection requirements and methods of the consumer, his troubles are usually eliminated.

Present-day inspection for interchangeable manufacture calls for a continuous inspection program from the time the raw material is received—and in some cases even an inspection before the material is shipped; that is, at the source—and through all the production operations. Heat-treated parts today frequently are receiving 100 per cent inspection on each operation. All of this has been found economical to insure the proper production of an interchangeable finished product.

Discussion

JOHN GAILLARD.[2] The writer would like to say a few words with regard to the question of unilateral and bilateral tolerances raised by the previous speaker. In fact, there is something in the historical development of the art of fitting which has caused an evolution from bilateral to unilateral tolerances. In the early stages of industry when handicraft was supreme, the craftsman would take care of each fit individually, removing metal until the fit was correct. He did not know about tolerances. When the rate of production increased and some degree of interchangeability between parts became necessary, an effort was made to hold individual parts to "exact" dimensions. For example, as it has just been said, "a one inch shaft was supposed to measure one inch." An "exact" dimension being something ideal, it was soon realized that it could only be aimed at and this resulted in giving a permissible plus and minus variation. Thus bilateral tolerances were established. In the course of time, however, it became evident that unilateral tolerances were preferable for a general system of tolerances as they secure the maximum degree of interchangeability in general, combined with greatest ease of visualization for the designer.

It is significant that the British Engineering Standards Association as far back as 1906 adopted a national system of fits based on bilateral tolerances, and in 1924 revised this standard to the effect that unilateral tolerances are recommended as the best practice. If you look up the British standard, you will find that bilateral tolerances are still given in addition to unilateral tolerances, as an alternative. However, it is stated that this is simply a concession to those manufacturers who had adopted, in the period between 1906 and 1924, the bilateral tolerances given in the original standard. To those who still have to adopt a system, the British standard explicitly recommends unilateral tolerances.

In addition to the British standard, eleven other national standards on fits between cylindrical parts have been established in the course of the last ten years, including the American Tentative Standard B4a-1925. All of these standards are based on unilateral tolerances.

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In the fall of 1929 somebody well acquainted with the automotive industry called my attention to the fact that some production men in that field had not felt inclined to adopt the American Tentative Standard because commercial stock reamers would not economically produce holes with unilateral plus tolerances. This is due to the fact that the present commercial stock reamers are made with a small plus tolerance on the nominal size. For example, a 1-in. reamer may be held between the limits 1.0000 and 1.0003 inch. Such a reamer wears down pretty soon to a point where it begins to produce undersize holes, and these, of course, are not acceptable, according to the American standard. Those criticizing the standard on this account, would therefore prefer bilateral tolerances which permit one to go on using the reamer for a time after it has begun to produce undersize holes. Reasoning along this line, some organizations have gone one step further, such as General Motors, by adopting hole tolerances extending *downward* from a constant reference line, instead of *upward* from the nominal size, as the American standard holes do. The General Motors system has been explained in *American Machinist*, June 5 and 12, 1930 by Mr. Ovaitt, and I discussed this system in *American Machinist* of November 6, 1930. As the chairman has mentioned, this question has been brought to the attention of the sectional committee on Allowances and Tolerances for Cylindrical Parts and Limit Gages, which no doubt will give it due consideration, and I would not have mentioned this question again, if reference had not been made to it in Mr. Bryant's paper, where it states that in the General Motors system reamers may be used subsequently for different grades of holes. In this connection, I want to call attention to the fact that such subsequent use of a reamer can also be applied to American standard holes if only the new reamer is held to a larger size. The reamer may then be used first for the hole with the largest tolerance, and as the reamer wears down, it is subsequently transferred to the production of holes with smaller tolerances.

The only thing I wanted to emphasize is that the essential requirements of a system of fits, such as general interchangeability of mating parts and clarity to the designer, are much more important than the reamer question, especially as a slight change in the present commercial reamer sizes would make it possible to combine such advantage as is claimed for non-standard systems as referred to above, with those presented by the American standard system, or in general, by unilateral plus tolerances on the holes.

W. S. Huson.[3] This paper enters quite fully into the manufacture of present-day reamers, taps and threading dies and the better results obtained by intelligent selection in these cutting units of those best accepted to the work in hand. In the past there were few small tools that could stand the abuse of those in question and while some endeavor was made to analyze the best forms little was done until the advent of quantity production to size compelled closer attention and longer life to the shapes and cutting qualities; so vital, where threading is involved, to interchangeability. The paper is a desirable addition to data already at hand and of which the author has taken note and used in its preparation.

In one part he says "the management does not insist on the necessary time being taken to produce proper drawings and tools before proceeding with production." This is being overcome in the realization that as machinery is designed, preliminary considerations must be given to the tools involved, their bearing on the final production in cost and uniformity. In other words the question of tools for interchangeable products is being taken from the tool-room and placed upon the drafting department. This is, as it should be, done in a department where the correlation of parts, often having a bearing on tooling equipment, can be studied and the best solution formed at the start. It may be said that more ingenuity is sometimes displayed in the inception and design of suitable fixtures and tools than is needed for the parts themselves. This is brought out in the instance given in Mr. McLaren's paper, "Conventional Gages and Their Application to Duplicate Manufacture" to be presented at this session, in the gage for hook and hook needle, the accuracy of which is the measure of the ability of tools to produce the work within the limits demanded.

The cost of making fixtures, etc., for production and inspection runs, often deeply, into money and there is at times hesitancy as to the advisability of incurring it; nevertheless there is an intrinsic value, which should not be overlooked, in the satisfaction that comes from the present methods of duplication, inspection and repairs servicing. This bears on the closing paragraphs of the paper where the importance of correct specifications from the engineering department is manifest in time and material savings.

The preliminary and auxiliary means are a legitimate charge in processes of production where exactitude is of prime importance.

L. D. Burlingame.[4] The author refers to the classes of screw thread work which have become standard American practice, and calls attention to the fact that under some circumstances economy of manufacture and satisfactory fits can be obtained by combining different classes of fit. He says:

"The combination of class 2 and class 3 assembly naturally makes a better or closer condition than the application of class 2 to both members, and it is an economical solution of many problems."

In his oral presentation of the paper Mr. Bryant goes further in pointing out that class 4 fit of the N.S.T.C. Report is materially closer than class 3, allowing very little for wear of gages and of tools, and states that the matter is under consideration of either liberalizing the tolerances for class 4 or of establishing an intermediate class between 3 and 4.

The question may be asked why the same plan as suggested for use between classes 2 and 3 is not applicable here also and that by making one of the mating parts to class 3 and the other to class 4, and using the class 3 tolerances for the part where difficulty of maintaining the close limits is greatest, may not give conditions practically equivalent to establishing intermediate sizes suggested and without a change from present adopted standards and without the addition of new sizes.

This suggestion would be applicable to gages as well as to the finished work, and would tend toward making the product in high-grade work more nearly interchangeable, and thus reduce not only the cost but also the requirements for selective fits.

G. A. Bouvier.[5] The writer is in general agreement with Mr. Bryant that it is good engineering practice to dimension drawings with tolerances. However, if any of the younger engineers get the idea that this is an ideal solution to the problem of attaining complete interchangeability, they are due to be bitterly disappointed. Theoretically, of course, if a part is properly dimensioned, interchangeability should result. The difficulty is that we cannot dimension everything, which has an effect on interchangeability. It is generally impracticable to specify on a drawing a tolerance for the amount of out-of-roundness and straightness of a hole, and the exact character of the surface. Holes supposed to be parallel are never exactly parallel, but I have seldom seen any tolerances for parallelism. I don't blame the draftsmen though, because it is difficult to determine the amount and direction a hole is out of parallel. Drawings on almost all the parts made in a shop depict right-angled surfaces, but we seldom see any limits for these angles.

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[3] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

[4] Legal Technician, Brown & Sharpe Mfg. Co., Providence, R. I. Mem. A.S.M.E.

[5] Western Electric Company, Hawthorne Works, Chicago, Ill.

Picture an automatic screw machine spindle and the spindle shoulder against which the thrust is taken. This shoulder must not only be square with the axis of the spindle but in addition its surface must be flat. We all know how difficult it is to obtain flatness in a small block, let alone attempt its attainment in the case of a shoulder on a shaft. When we couple this with the necessity for having the surface at right angles to the axis of the shaft, we have a problem on our hands. It is unfortunate that most draftsmen who dimension their drawings assume that the factors mentioned are perfect. However, these factors and many others naturally will have an effect on the quality of the final product. Obviously, the more complete the information given in specifications and drawings, the more intelligently will the shop be able to produce what is required, but I feel that for some time to come, we are going to have to depend on the process or method used, and upon the cleverness and skill of the craftsman, to produce a satisfactory product.

Mr. Bryant has said that present-day production has demonstrated that so-called close tolerances take no more time to produce and result in less spoiled work than would be the case if the maximum functional tolerances were allowed. I cannot agree with a literal interpretation of this statement. I am quite confident that with proper planning, more liberal tolerances would result in greater outputs. However, I believe I understand the intent of Mr. Bryant's remark to mean that the close tolerance often forces concentration on a job which otherwise might not receive proper attention. As a result better methods are used, there is more respect for the job, and carelessness and indifference are eliminated. In other words, close tolerances compel and insure good management, and in this sense most of us know of reductions in cost which have resulted from closer tolerances.

W. H. Gourlie.[6] There is no question that the points raised by Mr. Bryant and Mr. MacLaren are to the point and desirable to bear in mind on this important subject of gages. The writer would like to supplement their comments, grouping such remarks under three general headings, all of which pertain to gages: (1) Design, (2) use, (3) measurement.

It is important, even absolutely essential, that the gage be designed in the light of the requirements of the piece it is to check. The component is the important thing, and the gage must be made to best accomplish the purpose desired. At the same time there is more and more effort being made in the standardization of gage design, with the object in view of lowering the cost of gages of the simpler types where standard designs answer every need of the job. The writer refers particularly to the work of the American Gage Design Committee. Standards have been adopted by that committee, and since by the leading gage manufacturers, for thread plugs, thread rings, plain plugs, and plain rings, in the popular sizes. If the engineer will use, in so far as practical, the standard length of blanks, or standard handles, or other parts of the design, considerable saving will result in a user's gage cost. It is well worth while to bear this in mind.

There are many who use gages, and there are many who abuse them. Sometimes the distinction between use and abuse is a question of the cost of properly cleaning the piece preparatory to gaging. It costs money to clean a piece properly, and in some cases this expense is greater than the saving which would be effected by longer gage life. The writer has in mind an incident that came up just a few days ago of a thread gage used for checking tapped holes in a windshield pillar. The pillar was made from stainless iron and was highly polished after tapping, but before gaging. An appreciable amount of the polishing compound lodged in the threads, resulting in rapid gage wear, and in some cases chipping. After a thorough investigation it was found that the addition of a longitudinal chip groove, which cleaned out the bulk of the lapping compound, was the most economical solution, even though gage wear was much more rapid than would result were it possible and practical to thoroughly clean the thread before checking.

Precision gage blocks are another type of gage frequently abused because of misunderstanding of their proper use. Frequently improper cleaning of blocks before wringing results in a scratch and burr on one or two blocks. Failure to remove these will result in the scratching of additional blocks as varying combinations are made up. The writer has known of a new and complete set of blocks being scratched in just this manner, the set being less than two weeks old. When properly used, blocks will last for months, but when abused their life is short. These same principles are true in all types of gages. When used properly, gages will have a long accurate life. As abuses are permitted, life is cut down and cost of gages is increased. The user must balance cost of gages against cost of properly preparing parts for gaging. He cannot expect minimum cost with maximum accuracy unless proper precautions are taken when preparing the parts for gaging.

Methods of measurement of gages must be clarified, and set, in order to duplicate measurements to the accuracy demanded by present-day conditions by inspectors at various and sundry points throughout the country. It is comparatively easy to measure a plain cylindrical plug gage, 1 in. diameter, to 0.0005 in. A micrometer in the hands of most skilled mechanics will easily repeat to a figure well within that. When the tolerance is reduced to 0.00005 in. the story is entirely different. Temperature assumes importance of the first order. Measuring pressure is a factor, and hardness and stability of the metal in the gage must be considered. There is comparatively little dispute in such cases, however, as those manufacturers requiring such fine accuracy recognize the difficulties which must be overcome and are equipped to overcome them.

The measurement of screw threads and thread gages is an entirely different proposition. Almost every one manufacturing threaded products uses gages, and most users endeavor to measure them. Most inspectors feel that their readings are accurate within 0.0002 in., and yet the equipment used varies from a hand micrometer with wires 4 or 5 years old to the latest in measuring machines with pressure control. It is no exaggeration to say that variations in the same gage on pitch diameter are as great as 0.001 in., according to the different methods and skill of the user. The gagemaker is equipped to properly measure under suitable conditions, and the writer knows from having carried on some experiments that the leading gagemakers and the Bureau of Standards tie in very closely with measurements of pitch diameter. It is necessary, therefore, that users of gages who wish to measure to ten-thousandths on the pitch diameter of new or used gages, have a suitable measuring machine with pressure control, and wires which have been calibrated at the proper pressure, crossed over a cylinder 0.750 in. in diameter, as recommended by the Bureau of Standards.

It must be remembered, too, when discussing the measurement of pitch diameter, that that is the simplest element of the screw thread to measure. It is probably for that reason that most users check that single element, paying little attention to lead and angle. The gagemaker, on the other hand, checks all three, and maintains them within tolerances commensurate with the product tolerance.

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[6] Gage Division, Pratt & Whitney Company, Hartford, Conn.

MSP-54-5b

Conventional Gages and Their Application to Duplicate Production

By THOMAS F. MacLAREN,[1] PROVIDENCE, R. I.

The attainment of greater accuracy in present-day machines has necessitated improved gaging facilities. This article briefly treats of some important considerations in the design of gages, a description of a few special applications, and the need for standardization in gaging methods relative to machine-shop practice.

THE need for interchangeable, duplicate parts is now generally appreciated by manufacturers, and the benefits derived are shared alike by consumer and manufacturer. Parts that fit accurately in an assembly give better service and longer wear. This materially decreases assembly and repair costs.

No two things are the same, but machine parts may be duplicated to within very small tolerances in every dimension. This attainment becomes increasingly difficult with increased production rates and closer tolerances.

Duplicate production in large quantities has necessitated a revolution in methods of inspection especially with respect to inspections performed by the machine operator. Frequently the time allotted for the gaging process is limited by the schedule of machine operations, and a delay in any part of the schedule for one piece might result in considerable lost time.

The shop practice of 20 years ago in many plants was to have but one accurate measuring apparatus, and it was usually placed in a special room. Today in order to facilitate production and provide accurate work and machines, it is necessary to have accurate gages and tools in the hands of the operator. The older machines were not so accurate as those of today, and it was of little value to have gages with a degree of accuracy which could not be approached by the machines. Today, as machines are more accurate, it is necessary to produce more accurate test tools. Every forward step in accuracy by the machine manufacturer is a challenge to the gage maker.

The machine of 20 years ago which would duplicate work to within 0.001 in. has been superseded by the machine which will duplicate to within 0.0001 in. in less time. Likewise the gage department which considered 0.0001 in. to be the last word in accuracy is now measuring to a much greater degree of accuracy and with less difficulty.

In duplicate manufacturing the degree of accuracy which it is possible to attain and maintain is limited by the gages and test tools employed in the manufacturing and inspection processes. In much of the duplicate work being done at the present time, however, the accuracy demanded is not so great that the inspection need be considered as precision work. The accuracy required in the work is merely another consideration in the design of the gage.

The importance of correct design in gages and test tools cannot be overemphasized. It is of course impossible to formulate any complete list of rules which will suffice as an infallible guide to the designer. Apart from the comparatively few standard instruments for measuring, such as micrometers, vernier calipers, height gages, etc., individual gages must frequently be developed for each piece or operation. It has been said that the secret of good gaging is the faculty of seeing clearly "how big a little bit is."

For shop use it is impractical always to provide an absolute measuring device for each job, and for duplicate production plants the cost of the time required to operate the more accurate precision measuring instruments is prohibitive. It is then necessary to provide for the mechanic a dependable tool—precise, if precision is required, simple, durable; a tool with which he can carefully check his work with a minimum expense of time and effort.

The accuracy of the shop gage is dependent upon the precision desired, and it should be remembered that the most accurate gage is not necessarily the best for the job. If the gage controls the work to a tolerance less than is required by the design of the piece, with a resulting decrease in production and increase in costs, the gage is not well designed. The nature and number of pieces to be tested are factors which should determine the nature of the design with reference to the durability of the gage.

Different metals have different abrasive qualities. Aluminum is apt to wear a steel gage more rapidly than cast iron. For large productions of many thousand pieces it would be necessary to make the gage points of metal which can be heat-treated to stand the wear to which they will be subjected, while for small productions a less durable grade of metal can be used. The problem, then, is to find a steel which will have the combined features of maximum durability and lowest cost.

The gage points should be easily accessible to facilitate the gaging process and the detection of any foreign matter which might cause an error. Losses due to an interference at the gage points may amount to hundreds of dollars in a few minutes. Trick designs should be avoided. Generally speaking, the gage which is simple to operate is easier to manufacture. Too often the gage designer forgets that while it is a simple job to place a straight line on paper, it may require a difficult machine operation to reproduce it on the gage. A straight line demands accuracy, where a liberal limit might be permissible.

The gage maker's tolerance should be placed on the "wear side." Every gage is in error to some extent. Where the gage maker is given a tolerance, commercial gages should be so designed that as much metal as possible may be left on the gage points. If reliable data regarding the wear of a gage under certain conditions can be obtained, it is permissible to use smaller tolerances in its manufacture to allow for this wear, but such a

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

procedure should be authorized only when the dimension limits are relatively large and the proposed use of the gage is limited.

Fig. 1 shows a sewing-machine hook and hook needle, with a gage which is now being used to test the composite piece. This gage is one of over 30 gages of similar design which are used in the machining process of over 60 operations required by this part. Several other methods of testing have been resorted to in an attempt to reduce the manufacturing time, but in each case the results were no better than those obtained with the present gage. A projector type necessitated too many holding devices, and a gage with fixed gage points gave no indication of the magnitude of the error. It is true that for many jobs the plunger or finger type gage is obsolete, yet until another proves to be superior in every instance it cannot be discarded. In the department where this gage is used there are hundreds of similar gages which give better service than any other types which have yet been tried. The dimensions for the piece are distributed in many planes, and practically the entire contour of the hook end is held to close limits. Each of the plungers is brought up against its mating surface in turn, and the final one *A* has a standard plug which must be in alignment with the hole in the hook needle. In gages of this type the error is indicated by the relation of the outer ends of the plungers to the outside surface of the gage, and if a limit of 0.0005 or more is permitted, the outer end of the plunger is made with a high and a low side. If the piece is correct, the high side will extend above the gage surface and the low side will be below the surface. This can be detected by passing the finger over the surface.

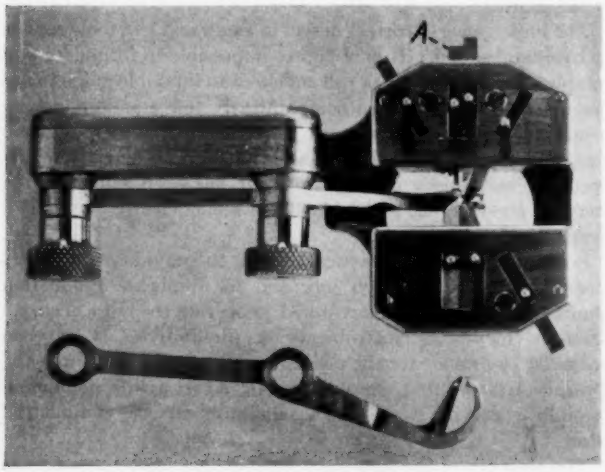


Fig. 1 Finger or Plunger-Type Gage With Seven Gage Points Used on Hook and Hook Needle

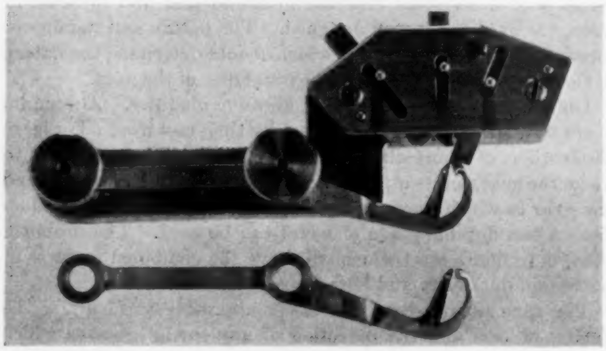


Fig. 2 Finger Gage for Use on Sewing-Machine Hook and Hook Needle

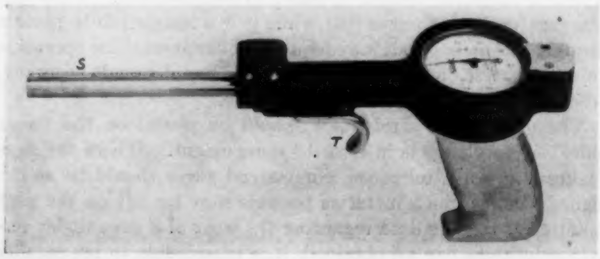


Fig. 3 Comparator Gage for Internal Measurements

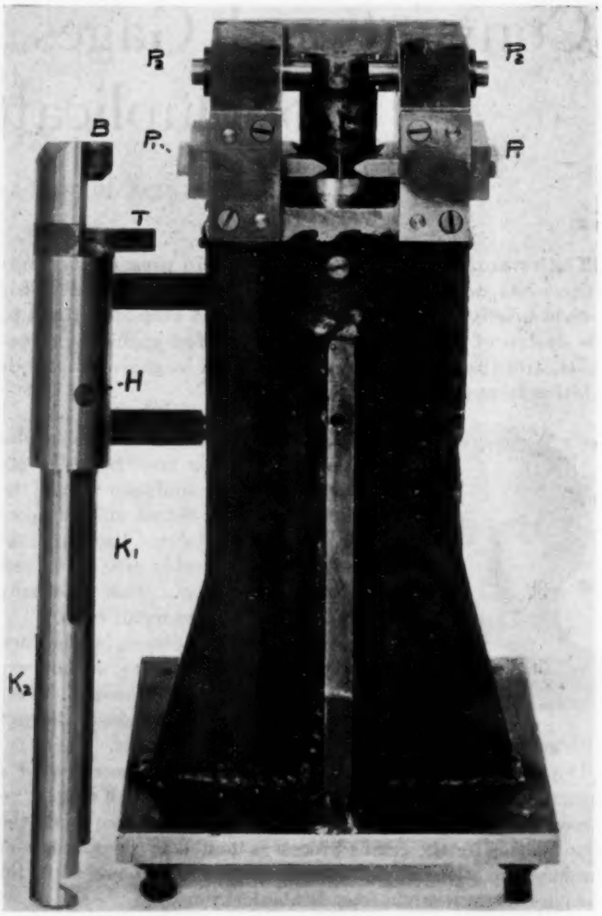


Fig. 4 Control Shaft and Rear View of Gage

Fig. 2 shows another gage which is used on this same piece. With many gages, where the gage surface is kept smooth and well polished, limits of 0.0005 are being held on jobs similar to this.

Selective assembly methods have opened a field for two types of gages which resemble the regular plug and snap gages in principle, but which have adjustable gage points. Fig. 3 shows a gage used for gaging the diameter of a hole or similar dimensions. The spreaders *S* in the shaft are controlled by the trigger *T*, and when the gage shaft is inserted in the piece to be gaged the trigger is released. The spreaders expand, and this motion actuates the dial indicator which indicates the deviation of the piece from a master.

A gaging device similar in principle to the one just described is used for external work to good advantage in many jobs. It

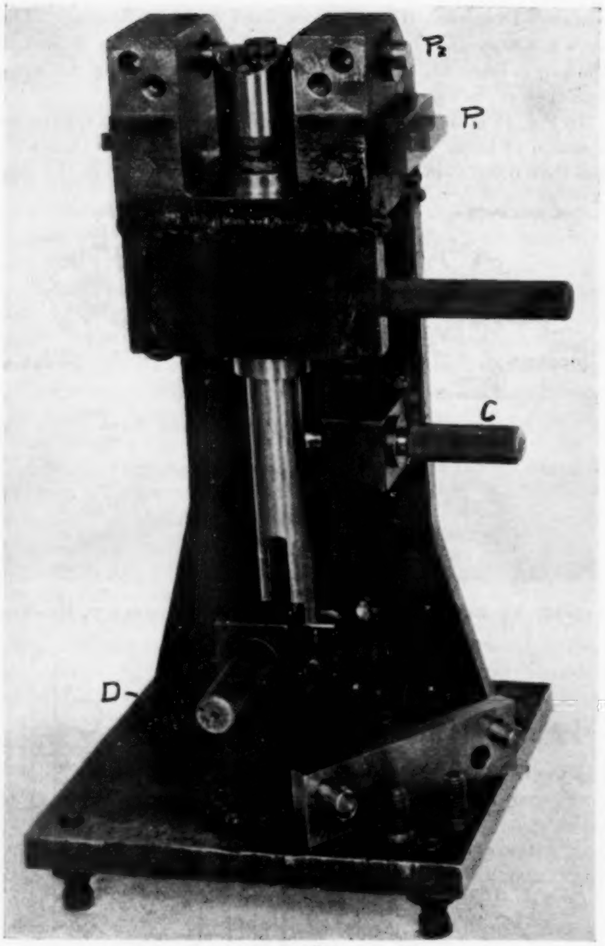


Fig. 5 Front View of Control-Shaft Gage, Showing Locating Pin and Keyway Plungers

consists of one fixed jaw and one floating jaw, the motion of which actuates a dial indicator. When the trigger is released, the jaws close on the piece to be gaged, and by comparing the reading with that of a master, any error is easily noted.

For each of these gages provision is made for their use over a wide range of sizes.

Arc welding is now being extensively used in the manufacture of gages, with a resultant saving of time in many instances. The gage used for testing the control shaft (Figs. 4 and 5) was constructed from boiler plate. As can be seen from the outline of the piece (Fig. 6), it is a type of gage which is adaptable to a large percentage of machine-shop work. The shape of the arrow- or carrot-shaped tooth *T* and its relation to the reamed hole *H*, the two keyways K_1 and K_2 and the knob *B* are most important. Fig. 4 shows the two sets of plungers. The lower set P_1 mates with the contour of the "carrot" *T*, and any deviation over 0.0001 in. can be seen with the naked eye. The size is checked by the high and low sides on the plungers. The knob form does not have to be as accurate as the "carrot," and is checked for size by high and low sides of the two upper plungers P_2.

The two keyways are checked by the keyway plungers *C* and *D*, which gage for size as well as location with reference to the "carrot." The hole is not necessary for the operation of the assembled machine, but it is drilled for use as a locating point in the jigs, fixtures, and gages used in the manufacturing and inspection operations.

In many operations used in duplicate production it is desirable to check work to less than 0.0005 without removing it from the machine. Several devices have been manufactured to do this with varying degrees of success. One micrometer on the market has incorporated in the frame an indicator which has a constant-pressure spring actuated by a floating anvil. By adding or subtracting the reading of the indicator dial from the reading on the thimble it is possible to gage to 0.0001 in.

Another micrometer designed to read to 0.0001 in. is shown in Fig. 7. To obtain the size, it is necessary only to add the reading of the small auxiliary thimble to the reading of the main barrel. The small thimble turns independently of the main thimble and does not affect the spindle movement. With this micrometer and similar measuring devices where the graduations are widely spaced, it is possible to estimate to parts of a ten-thousandth, a valuable feature in many instances where an accuracy greater than that afforded by many standard measuring tools is necessary.

In many machine operations in which there is a relative change in the location of the cutting member and its work, it is desirable to have a gage which will make it possible to note this change.

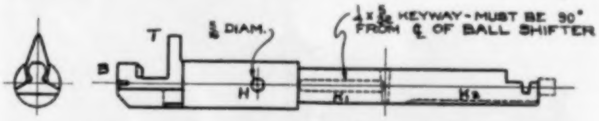


Fig. 6 Outline Sketch of Control Shaft

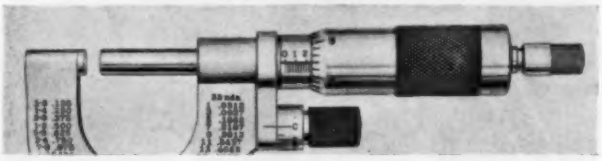
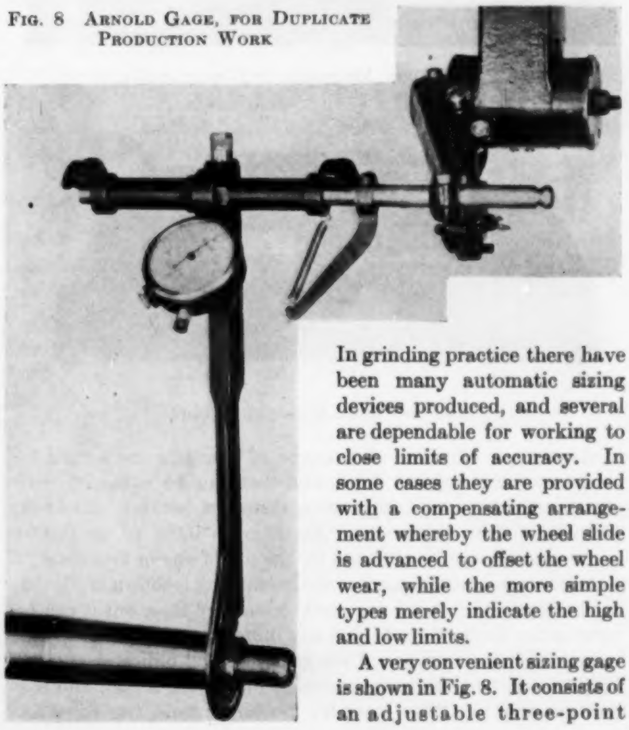


Fig. 7 Micrometer With Auxiliary Thimble for Reading Directly to 0.0001 In.

Fig. 8 Arnold Gage, for Duplicate Production Work



In grinding practice there have been many automatic sizing devices produced, and several are dependable for working to close limits of accuracy. In some cases they are provided with a compensating arrangement whereby the wheel slide is advanced to offset the wheel wear, while the more simple types merely indicate the high and low limits.

A very convenient sizing gage is shown in Fig. 8. It consists of an adjustable three-point spreader, with two of the points fixed and the third floating against a plunger, the motion of which is recorded by a dial indicator. It does not read to size directly and is in reality a comparator rather than a measuring tool. One piece of work is ground correctly, and the gage is then set to the high limit with the dial at zero. Fig. 9 shows a similar gage mounted on a grinding machine. When the grinding operation is completed, the gage is conveniently swung clear to permit the operator to change the work unhindered. Diamond or Carboloy points on the spreader of these gages eliminate the danger of changing the setting by wear or of injuring the finish, and they can be equipped with attachments for measuring tapers, distances between shoulders, and many other dimensions. Readings to 0.001 in. or 0.0001 in., as desired, can be obtained by the use of one or two dials.

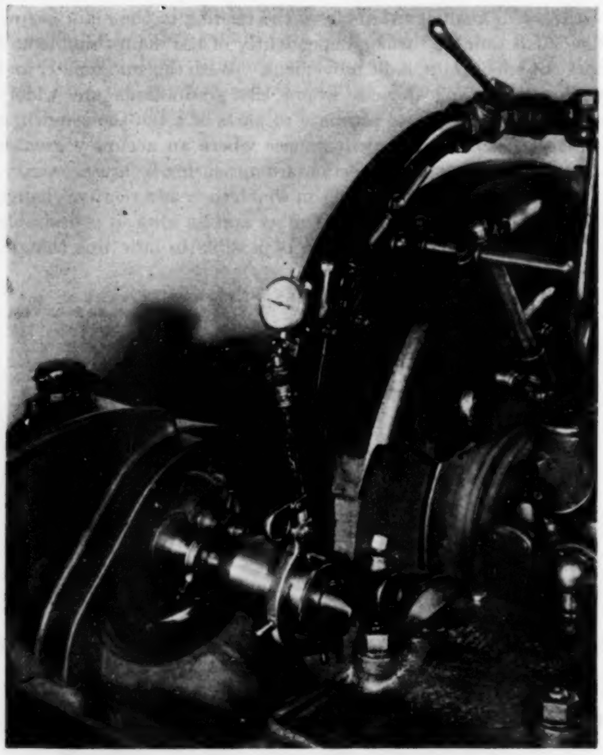


FIG. 9 PRATT GAGE, MOUNTED ON GRINDING MACHINE

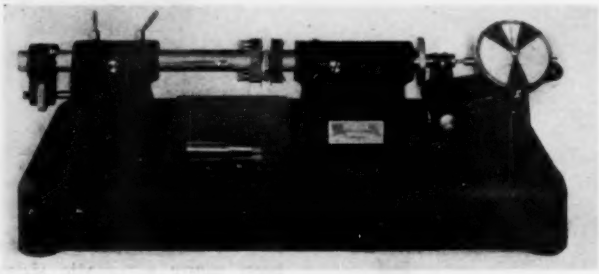


FIG. 10 WIRE-MEASURING GAGE

A convenient bench gage for wire measuring is shown in Fig. 10. In the illustration it is being used to measure taps, but it can be used on bar stock, threads, and any dimensional work which will come within the capacity of the gage. A dial indicates any deviation, and two blinds are provided for setting a high and low limit, as shown. When used for measuring taps, two wires are suspended between the jaws, one fixed and one floating. The jaws are separated by a lever located under the dial indicator, the tap is inserted, and the jaws are released, forcing the wires into their correct position.

In Fig. 11 is shown a gage which was manufactured for the inspection of hubs. So accurately was this gage made that with less than 0.00025-in. tolerance in the keys or keyways in the plug or ring, it is possible to insert the plug in any one of the 16 possible ways. This gage illustrates an item in the cost of gage manufacture which must be considered by the designer, since in order to check a gage of this type it is usually necessary to make a mating gage which may require more time to build than the original.

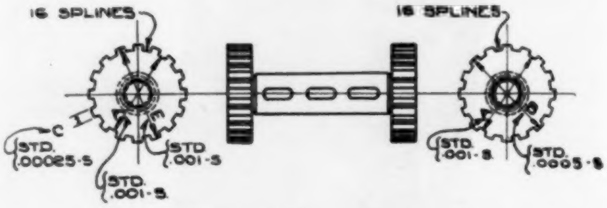


FIG. 11 GAGE FOR TESTING SPLINED HUBS

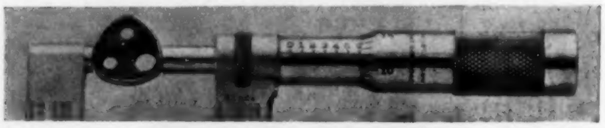


FIG. 12 ECCENTRIC WHICH GIVES CONSTANT MICROMETER READING

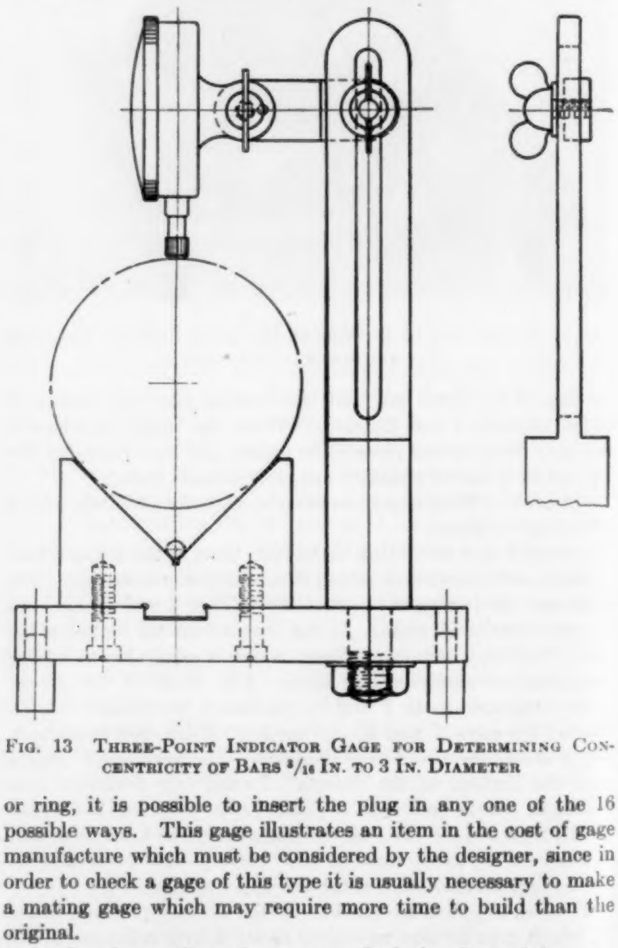


FIG. 13 THREE-POINT INDICATOR GAGE FOR DETERMINING CONCENTRICITY OF BARS $^{3}/_{16}$ IN. TO 3 IN. DIAMETER

A misunderstanding of proper methods for determining concentricity has resulted in many inferior machine tools. In order that machine operators may not be misled, it should be understood that a micrometer, vernier, or any similar two-point measuring instrument is not adaptable for measuring or establishing the concentricity of a bar. If the bar is round, the two-point measuring tool will not indicate otherwise. But it is possible to secure duplicate readings at intervals of small increments over the entire periphery of the bar with the micrometer spindle and anvil surfaces tangent to the bar, and yet be able to tell with the naked eye that the piece is not concentric. An exaggerated illustration of this is shown on Fig. 12. This piece will give duplicate readings on the micrometer as long as the gage surfaces are tangent to the piece at the gage points.

In order to determine the concentricity of a piece, the ideal measurement would be from the outside to the plane center of gravity. If the work has accurate centers, this measurement can be made by rotating the piece on centers using a dial indicator. In centerless grinding work, however, it is not convenient to use this method, and some different but equally satisfactory method must be used.

Fig. 13 shows a simple design which will overcome this difficulty. The same principle can be used in a variety of ways with a varying degree of precision. By revolving the piece in the V-shaped base, any eccentricity will be noted on the dial indicator. Micrometers have also been manufactured with a V-shaped anvil for the purpose of measuring cylindrical stock.

It is impossible to list the many gages on the market. Comparator blocks have displaced many devices, and frequently they are less expensive and have a wider range of application than the gages displaced. Fluid gages, optometers, many types of comparators, and other new gaging devices are being produced to meet the demand of modern production methods.

The need for standardization in gaging is becoming more and more essential in many branches of shop work. Perhaps in no phase of the industry is there a greater natural opportunity for variation in practice of design, manufacture, and inspection than in gearing, and along this line Mr. F. W. England, chairman of the inspection committee of the American Gear Manufacturers Association, has edited a most valuable paper, in which he recommends in exact detail a code of inspection for the member companies of the association. With his discussion he has included descriptive drawings and photographs of many testing devices which are giving good service in the inspection of splines, tooth parts, and all dimensions which are common to the manufacturers of gears and their customers. Similar codes of gaging practice might well be adopted by other engineering organizations.

For practically every job there is more than one gage, but there is one which is superior. The limitations and adaptability of the gage, coupled with the design of the piece, the dimensions to be checked, and the accuracy required, will determine which is the gage for the job.

Discussion

[Note: Discussion of this paper jointly with that of E. J. Bryant, by W. H. Gourlie, will be found following the latter paper.]

W. S. Huson.[2] Accuracy in gages, leading to accuracy in sizing and inspection, is well brought out in this paper, for inspection has often been handicapped because of lack of testing tools properly designed and made and held in exact consonance with the gages and tolerance allowances used in production. There is more than just a modicum of truth in the author's remark, "Every forward step in accuracy by the machine manufacturer is a challenge to the gage maker," for while skill can be shown in the making of intricate gages, the use for convenient testing is sometimes lost sight of, and perhaps because of cost, limitation to the opportunity for a gage to test to the fullest extent is overlooked. There is a practical side to testing as well as to other tools, in that the cost to make them fully efficient is incurred but once, whereas if they fall short of all that could be done, there is continuing expense in the need for additional gages or the like as well as more time required. The realization that work can be produced to close limits cheaper than otherwise has not penetrated fully into manufacture, but as papers like this one are put forward the reluctance to grasp the value of sizing to fixed standards is breaking down in sensing that it has become the modern way of doing and proving work economically.

The illustrations show examples of gages that bear out the facility with which gaging can be done. The increasing use of arc welding for making jigs, fixtures, and testing apparatus has been a material help in enabling them to be made more efficient, because castings cannot always be as fully made, as there may be molding limitations. The welded tool may not look as pretty perhaps as a casting, but if it can be more efficient that is the main point. Figs. 4 and 5 show this and are worth studying by any who have work where a keyway is the locating unit.

This paper ties in well with the companion paper by Bryant; they are somewhat parallel in import and should be read into each other. The importance of what is set forth is emphasized by the paper by C. R. Reiman (Trans. PI-52-10) setting forth the sizes and tolerances for metal fits and graphically showing by charts the practical use in every-day production of shafting made and held to very close limits.

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[2] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

MSP-54-6

Improved Device for Recording Instantaneous Tool Pressures in Machinability Studies

By O. F. GECHTER[1] and H. R. LAIRD,[2] CHICAGO, ILL.

O. F. Gechter

The device described permits the measurement of the three tool-pressure components during the machining of short test pieces. The advantages of the device as compared to similar devices used in the past are outlined. It is shown that the use of the piezo-electric pressure-measuring method makes possible a rigid tool mounting and the recording of relatively sudden pressure changes. The mechanical and electrical limitations are discussed, and improvements are suggested. A discussion of some of the test results illustrates the value of the device for machinability studies.

H. R. Laird

THE device described in this paper is the fundamental part of equipment being employed in one plant of a large industrial concern[3] to determine the relative machinability of die-cast aluminum alloys. The results of the tests, together with those of an investigation of rates of corrosion, adaptability to casting, and other physical and chemical properties, will be used to select the alloy most suitable for the manufacture of the case shown in Fig. 1. The tests are being made on parts cast in blank form in order to obtain results which will, as far as possible, reflect the properties of the alloys as encountered under actual manufacturing conditions.

The complexity of the problem of machinability is well illustrated by the fact that, in spite of the large amount of research already devoted to the problem, no method has been developed which makes it possible to draw conclusions regarding all the phases involved. It is true that attempts in this direction have been made. E. G. Herbert,[4] for instance, measured the hardness in various sections of the chip, the tool, and the material cut, and as a result has expressed the opinion that the only true measure of machinability is the hardness of the chip. F. Schwerd[5] took high-speed cinematographic records of the chip formation during the cutting action of the tool and expects that a procedure along this line will finally solve the problem. Both methods require further research to be of practical value.

The mechanical grading of the finish of machined surfaces, one phase of the problem of machinability, has in recent years been brought closer to solution by a number of methods. Promising examples, in the opinion of the authors, are a phonographic finish-measuring instrument described by R. E. W. Harrison[6] and a photoelectric device developed by F. Firestone,[7] and used in one case for the detection of objectionable flaws in wristpins.

The major part of research in connection with the problem of machinability has been based on tool life and tool wear. This is logical, because these two quantities play an important part in the economic manufacture of machined products. The method most widely used is probably the standard tool-life test originated by F. W. Taylor.[8] This type of test gives results that are especially applicable to roughing cuts. The trailer-tool method used by T. G. Digges and H. J. French[9] compares the machinability on the basis of actual tool wear rather than ultimate tool life and corresponds therefore to modern manufacturing requirements with their close dimensional limits.

It may be interesting at this time to mention the results of tests conducted by A. Wallichs and H. Opitz[10] on automatic screw ma-

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[2] Engineer, Western Electric Company, Hawthorne Works. Mr. Laird attended the University of Wisconsin, from which he received his B.A. degree in 1922 and his Ph.D. degree in physics in 1928. From 1926 to 1929 he was engaged in process control work in connection with the manufacture of hot cathode X-ray tubes with the General Electric X-Ray Corporation at Chicago. In the fall of 1929 he was employed as an engineer by the Western Electric Company, where he has since been doing investigation work.

[3] Western Electric Company, Chicago, Ill.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] Edward G. Herbert, "The Problem of Machinability Measurement," *Metallurgia*, September, 1930.

[5] Friedrich Schwerd, "New Investigations of the Theory of Cutting and the Problem of Machinability," *Stahl und Eisen*, April 16, 1931.

[6] R. E. W. Harrison, "A Survey of Surface Quality Standards and Tolerance Costs Based on 1929–1930 Precision Grinding Practice." Paper presented at the A.S.M.E. Annual Meeting, December, 1930.

[7] Floyd Firestone, machine for detecting flaws, U. S. patent 1,640,567.

[8] F. W. Taylor, "On the Art of Cutting Metals," A.S.M.E. Annual Meeting, December, 1906.

[9] H. J. French and T. G. Digges, "Turning With Shallow Cuts at High Speed," A.S.M.E. Annual Meeting, December, 1929.

[10] A. Wallichs and H. Opitz, "Machinability Tests on Screw Stock," *Archiv. fuer das Eisenhuettenwesen*, November, 1930.

chines. These tests have apparently proved that interrupted cuts, such as performed on automatic screw machines, and continuous cuts will result in the same tool life as long as the tests are made under otherwise identical conditions. This conclusion was verified by roughing tests made both on a planer and a lathe.

In view of the relative importance of removing metal with the least amount of energy, it is only natural that the problem of chip pressure received early attention. Some investigators have even gone so far as to use this pressure as a measure of machinability. It has been repeatedly pointed out that this procedure is misleading because it does not take into account various factors, such as abrasive action of the material, etc., which result in a shorter tool life, but which may not affect the tool pressure in a way conducive to the measurement of machinability.

The true value of tool-pressure measurements has been demonstrated by tests conducted by Schlesinger[11] and others. It was found that especially the radial and feeding components (P_1 and P_2, respectively, in Fig. 4) increase as the tool wears, and become abnormally high at the point of complete tool failure. Obviously, the conclusion could be drawn that tool-pressure measurements represent a means for determining wear and life of a tool. Similar results were obtained in more recent tests, during which the tool temperature was measured by using the contact point of tool and material as the hot junction of a thermocouple.

In deciding upon a suitable method for measuring the machinability of the die-cast aluminum alloys, it was considered advisable to deal separately with the problem of grading the surface finish. Main consideration was therefore given to the following points:

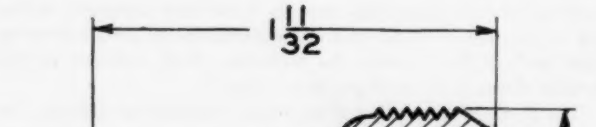

Fig. 1 Die-Cast Aluminum-Alloy Case on Which Machinability Tests Were Made

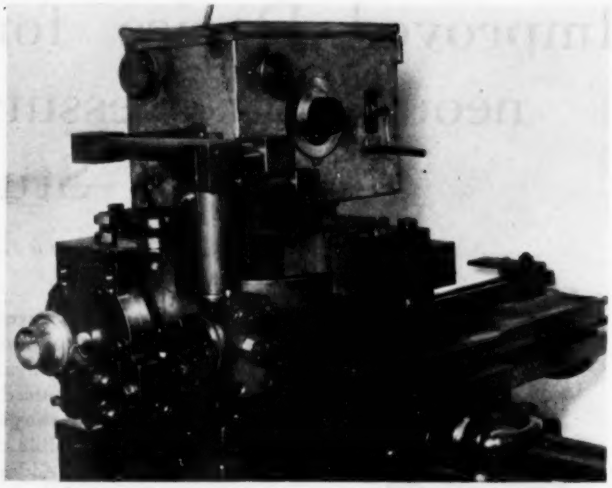

Fig. 2 Tool-Shank Mounting, With Pressure Units and Shielding Box for Special Vacuum Tube

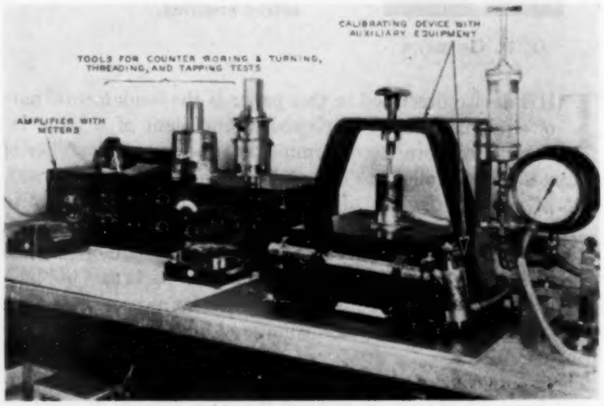
CALIBRATING DEVICE WITH AUXILIARY EQUIPMENT
TOOLS FOR COUNTER BORING & TURNING, THREADING, AND TAPPING TESTS
AMPLIFIER WITH METERS

Fig. 3 Amplifier and Calibrating Device for Tool-Pressure Measuring Device

It was desired, first of all, to obtain as much detailed information as possible regarding all those properties of the alloys which affect tool wear and tool life. It was felt, for instance, that comparative data on the hardness and uniformity of structure of the individual parts and of parts of the same and of different alloys would be of great value. These properties cannot be detected with ordinary tool-life tests, which measure only an accumulative effect. Their knowledge is helpful, however, as an indication of one of the causes of tool wear and of casting qualities.

The form, the number, and the material of the test pieces also were taken into consideration. The form excluded the possibility of using the trailer-tool method of Digges, since the total length of cut during the turning tests amounts to only $^7/_{16}$ in. The number and the material of the test pieces made it highly desirable to select a method that would require the least number of parts for one test in order to be able to vary the cutting conditions as much as possible.

A device of the type to be described, which permits the measurement of the three components of the tool pressure, was finally considered as best meeting the requirements. It was expected that tool-pressure measurements would considerably shorten the duration of the tests by permitting a comparison of the machinability of the alloys on the basis of the increase in tool pressure or the trend in this increase during the machining of a fixed number of parts rather than on the basis of ultimate tool life. It was further assumed that any changes in the uniformity of the parts and the alloys could be detected by the necessary resulting changes in the tool pressure. An important factor was also the desire to investigate the relationship between tool pressure, cutting angles, tool life, coolant, and other variables. We have in mind in this connection especially the investigation of the problem as to whether, for a definite set of test conditions, the tool with the least cutting resistance will show also the longest life.

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[11] G. Schlesinger, "Reports of the Research Laboratory for Machine Tools at the Technical University of Berlin," nos. 1, 7, 8.

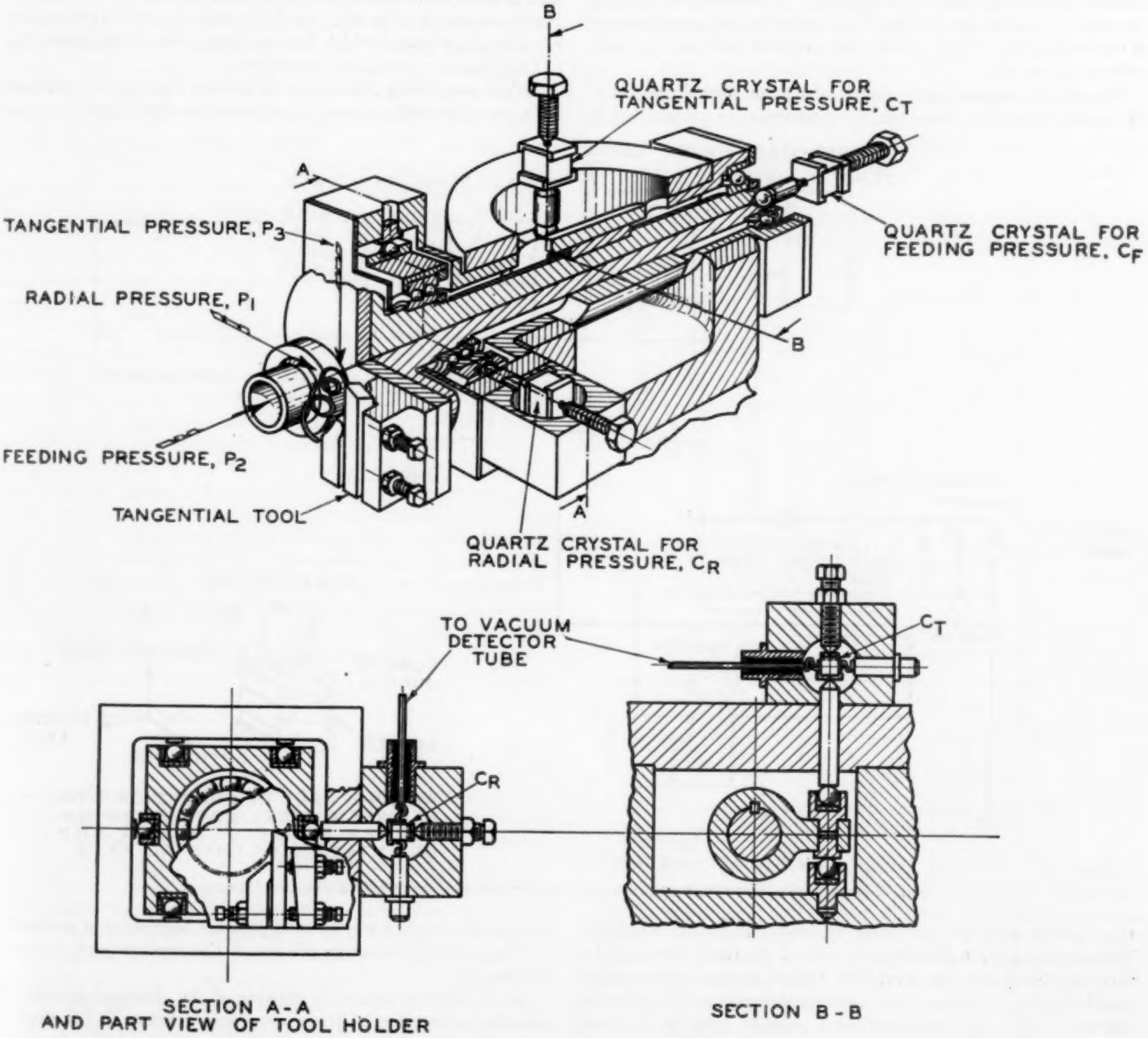
B
QUARTZ CRYSTAL FOR TANGENTIAL PRESSURE, C_T
A
TANGENTIAL PRESSURE, P_3
QUARTZ CRYSTAL FOR FEEDING PRESSURE, C_F
RADIAL PRESSURE, P_1
B
FEEDING PRESSURE, P_2
TANGENTIAL TOOL
A
QUARTZ CRYSTAL FOR RADIAL PRESSURE, C_R
TO VACUUM DETECTOR TUBE
C_T
C_R
SECTION A-A AND PART VIEW OF TOOL HOLDER
SECTION B-B

FIG. 4 TOOL-SHANK MOUNTING OF CUTTING-PRESSURE MEASURING DEVICE

It was realized that the device, in order to give satisfactory results, would have to avoid the disadvantages common to practically all similar devices used in the past. An investigation was therefore conducted with the purpose of finding or developing a method of measuring pressure that would permit a rigid tool mounting and the recording of relatively sudden pressure changes. From the description of a simple test set-up[12] used for recording the tangential component of the cutting pressure on a lathe tool, it appeared that the piezo-electric method would best meet the demands. This method was therefore adopted. It is based on the property of quartz crystals to produce electrostatic surface charges when subjected to pressure in certain directions. The charges produced are proportional to the applied pressures and can therefore be used for pressure measurements. These charges appear or disappear practically instantaneously as the pressure is applied or removed.

DESCRIPTION OF THE DEVICE

The mechanical part of the device consists of a tool shank with holder plate mounted in ball bearings and between balls in such a way that the three components of the tool pressure are transmitted to three separate quartz crystals. The electrostatic charges produced on a given crystal by the corresponding pressure component are collected on electrodes of copper foil and allowed to charge a condenser. The voltage produced across the condenser by these charges is impressed on the grid of a special vacuum tube having a very high input impedance. The output of this tube is amplified sufficiently to be read on a rugged milliammeter or recorded by an oscillograph. The meter readings are calibrated directly in terms of applied pressure by the use of a hydraulic device and a dummy tool.

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[12] J. Kluge and H. E. Linckh, "Piezo-Electric Measurement of Compression and Acceleration Forces," *Z.D.V.I.*, Sept. 14, 1929.

Fig. 2 shows the tool-shank assembly with pressure units and the metal box containing the condenser and special vacuum tube mounted on the turret of a No. 6 Brown & Sharpe hand-screw machine. This machine was selected for the cutting tests on account of its availability and rigidity. It was modified to give a maximum cutting speed of 800 ft. per min. for the large diameter of the test pieces. Fig. 3 shows the amplifier and the hydraulic calibrating device.

Fig. 4 is a diagrammatic sketch of the tool-shank assembly which illustrates how the pressure components are transmitted to the crystals with the minimum amount of frictional resistance. The mounting block for the outer race of the front bearing is located vertically between steel balls housed in cups with approximately $^1/_{64}$ in. clearance. The location sideways is fixed between one steel ball which rests against a pressure disk in the front housing and another ball which transmits the radial pressure to the quartz crystal C_R. The bearing housing is clamped to the machine turret.

The rear end of the shank is mounted in a single-row open-type ball bearing. This type of bearing was selected because it permits slight angular and longitudinal movements of the inner race. The outer race of this bearing is clamped in a housing mounted on the machine turret. The inner race is clamped to the shank. The shank is located longitudinally between the shoulder of the outer race and a pressure ball which transmits the feeding pressure to the quartz crystal C_F.

The magnitude of the tangential pressure component is determined by measuring the torque produced on the tool shank. A torque arm is mounted on the shank for this purpose which transmits the pressure to the quartz crystal C_T. The radial location of the torque arm and therefore also the shank is fixed between one ball, bearing against a pressure disk in the machine turret, and another ball which transmits the resulting torsional force to the quartz crystal C_T.

All quartz crystals are mounted between hardened and ground holder plates in pressure units of identical construction. The design of these units can be seen in detail from Fig. 4. The adjusting screws provided in each one of the units are used to preload the crystals to an extent which insures elimination of all static play in the pressure transmission members.

Before assembling the device all housing cups for the pressure balls were filled with grease. Great care was then taken to mount the shank in such a way that all pressure balls were in proximity to their correct location in relation to the pressure pins, to avoid binding.

Fig. 5 shows a schematic drawing of the electrical circuit and also the orientation of the crystal axes relative to the applied pressure. The pressure is applied along the neutral axis of the crystal, and the electrostatic charges are collected from the surfaces perpendicular to the electric axis. The precautions necessary when dealing with other electrostatic measurements must be taken care of here. Hence, all the connections between the quartz crystals and the grid of the pressure tube, as well as this tube itself, are electrostatically shielded. Furthermore, all of the insulation in this part of the circuit must be of the best, red sealing wax and hard-rubber blocks having been found suitable. The pressure-detecting tube itself must have a very high input impedance. The tube used here has an input impedance of the order of 10^{16} ohms. Insufficient insulation in this part of the circuit will cause objectionable drifts in the pressure readings.

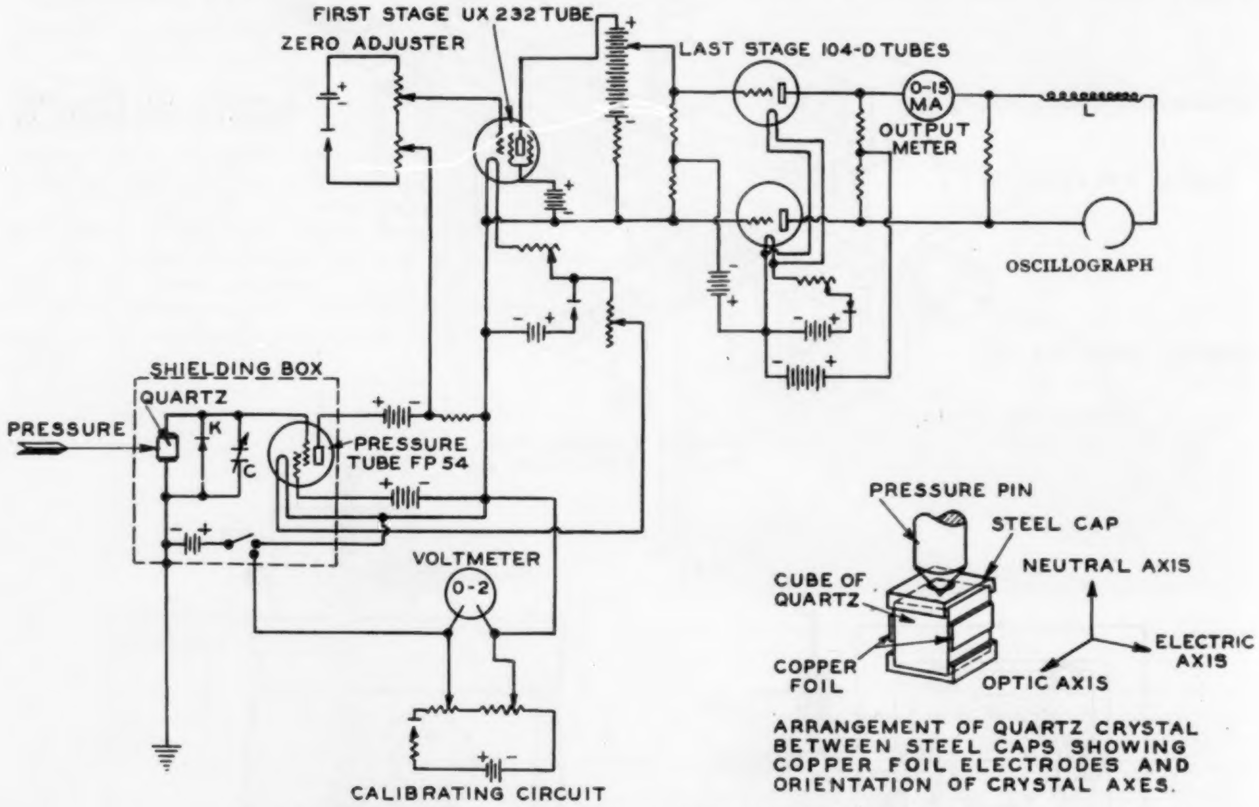

Fig. 5 Schematic Wiring Diagram for Piezo-Electric Pressure-Recording Device

The values of the drifts here were negligibly small. The effect of the actual numerical values of the circuit and tube leakage resistances on these drifts will be discussed farther on.

In Fig. 5 all parts of the circuit which must be shielded are shown as included within the shielding box. This is true of all except the crystals and leads from the crystals, which are all mounted in separate shields. A switch (not shown) is also mounted in the shielding box, which allows each crystal to be connected singly to the condenser and pressure tube. A grounding key is provided which places the grid of the pressure tube at the definite bias of —4.5 volts before starting a series of measurements. This key must of course be open when pressures are being measured.

The variable condenser *C* allows the sensitivity of the output of the amplifier in relation to applied pressure to be varied as desired. This is evident from the following considerations. With a given amount of charge, corresponding to a given applied pressure, the voltage produced across the condenser varies inversely with the condenser capacity. Since, as will be shown farther on, the output of the amplifier is proportional to this condenser voltage, it follows that the output for a given applied pressure will also vary inversely with the condenser capacity.

In the device as described the change in the grid bias of the pressure tube, or ΔEg, for maximum output is approximately 0.8 volt. The total charge produced on the side of a cube of quartz of any size is 1.04×10^{-11} coulombs per lb. of total force. This was calculated from the relation $Q = d_{11}W$, where Q is the total charge in electrostatic units and W the total force in dynes, using the value 69×10^{-9} for the piezo-electric coefficient d_{11}. (See Sosman,[13] pages 559 and 560.) The available charge, due to the electrodes not covering the whole of the side of the cube, is 0.65×10^{-11} coulombs per lb. Hence, from the relation that voltage equals the ratio of charge to capacity, the pressure values required to give 0.8 volt for different capacities may be calculated. These pressure values will then give the maximum output for the corresponding capacities. Such pressure values, for the maximum and minimum capacities obtainable with the condenser used here, are as follows:

Pressure value, lb.	*C* (micro-micro farads)
130.0	1060
9.9	80

The amplifier for the output of the pressure tube is two-stage and resistance coupled. The first stage utilizes the screen-grid UX-232 vacuum tube, and the last stage two 104D vacuum tubes connected in push-pull. A zero adjuster, working on the grid bias of the first stage, is provided to retain the zero of the output milliammeter against slow changes due to battery-voltage variations. An adjustable voltage source and voltmeter inserted in the grid circuit of the pressure tube make it possible to calibrate the output in terms of the voltage supplied by the crystal. A switch is provided to cut out this calibrating circuit when measuring pressures. This provision was made because the long leads from the shielding box to the meter, even though encased in a grounded metal sheath, picked up electrical disturbances caused by the starting and stopping of electric motors. These disturbances, while producing no appreciable effect while the grounding key was closed, caused large permanent changes in output when the key was open.

By the use of the push-pull circuit in the last stage and suitable choice of the operating ranges of the grid voltages of the pressure tube and the first stage of the amplifier, the output has been made linear with the grid voltage of the pressure tube from 1.0 to 9.0 milliamperes, these outputs corresponding to grid voltages of —4.5 and —3.7 volts. A typical calibration curve is shown in Fig. 6, where "output" *MA* or ΔMA is plotted against positive voltage increments applied to the grid of the pressure tube. By "output" *MA* is meant the reading in excess of 1.0 *MA*, since 1.0 *MA* is taken as the zero point. During use the sensitivity of the amplifier decreases slowly, about $^1/_4$ per cent per hour, and this may be instantly checked when desired by applying a single known voltage increment to the grid of the pressure tube. This decrease is mainly due to changes in B-battery voltage. The sensitivity recovers on standing for a few hours.

STATIC CALIBRATION OF DEVICE

In order to be able to convert the output of the amplifier into actual pressure in pounds the calibration device shown in Fig. 3 has been designed and found very practical. It permits a complete calibration of all three pressure components in approximately 30 minutes.

The device consists of a hydraulic pressure generating unit, a portable hydraulic unit connected to the former by flexible metal hose, and auxiliary equipment for applying the calibration pressures to a dummy tool. Light mineral oil is used as pressure medium. The hydraulic cylinder of the generating unit is mounted on the platform of a 200-lb. scale. A pressure screw used to apply pressure to the piston of the cylinder is mounted in a bracket above the scale. In calibrating, the desired pressure is set on the beam of the scale in the customary manner. The approximate adjustment of the pressure is then made by means of the grease gun and the manometer. The accurate adjustment is made with the pressure screw. The portable unit, consisting mainly of cylinder and piston, is applied to the dummy tool in the directions of the three components of the cutting pressure by means of suitable back stops.

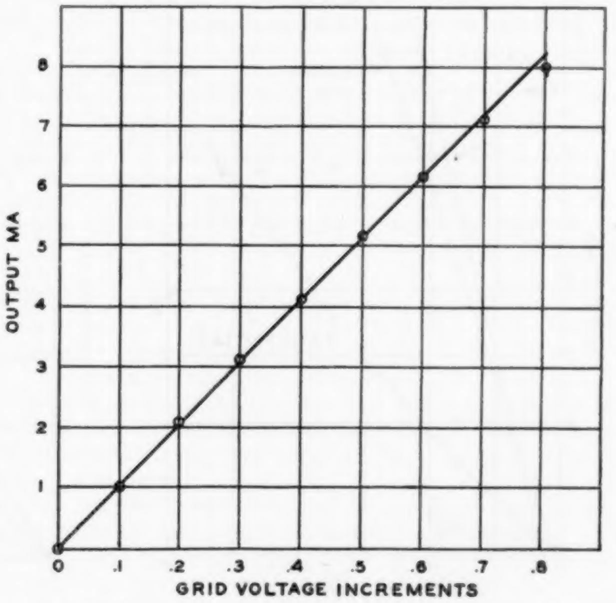

FIG. 6 VARIATION OF OUTPUT WITH GRID VOLTAGE OF PRESSURE TUBE

The accuracy of the calibrating device has been found to be within a fraction of 1 per cent at high pressures, while at low pressures it is less.

This calibration does not take into account the simultaneous action of all three components of the cutting pressure on the cutting tool, since the frictional resistances produced during the cutting action on the ball bearings and balls by the two pressure components not being calibrated, cannot be duplicated. To do this, three complete calibration units would be required. The error in the calibration is so small, however, due to the small coefficient of rolling friction, that in this case it can be neglected for all practical purposes.

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[13] Sosman, "The Properties of Silica," Am. Chem. Monograph Series, no. 37.

In Fig. 7 are shown calibration curves of the variation of "output" milliamperes with tool pressure for each of the three components. Two sets of curves are shown—one for the condition of maximum sensitivity which is obtained with the minimum condenser capacity and the other for the condition of minimum sensitivity which is obtained with the maximum condenser capacity.

On close analysis it was found that these calibration curves consist of two portions—a portion in the lower pressure range which will be referred to as *A* and one in the higher pressure range which will be referred to as *B*. The portion *A* corresponds to the range in which, as the tool pressure is being increased, the tool shank moves gradually away from the back stops, and thus the preloading pressure is continuously being decreased. The actual increments in crystal pressure therefore correspond to the difference between the increments of tool pressure and the decrements of preloading pressure. For this reason the slopes of the *A* portions depend upon the magnitudes of the preloading pressures. Furthermore, there may be a slight curvature to the *A* portion, since the back stops are balls, and hence their elastic deflections are not strictly linear with pressure.

The *B* portions extend over the higher pressure range from the tool pressure values at which the preloading pressures are just removed. Here the increments in crystal pressure correspond to the increments in tool pressure. This portion of the curves, therefore, is always straight, and the slopes over this portion are independent of the magnitudes of the preloading pressures. The slopes would be the same for all three components if it were not for the fact that the ratio of tool pressure to crystal pressure is different for each component.

In adjusting the preloading pressures it is desirable to keep them as low as possible and still remove all static play. Such procedure will result in the slopes of the *A* portions of the curves being only slightly less than the slopes of the *B* portions. This will prevent the noticeable bend upward or increase in slope which occurs at the point where the preloading pressure is just removed in case this pressure is excessive. In Fig. 7 this point occurs at about 20 lb. pressure in the case of the radial component, where a slight bend upward can be seen. In the case of the feeding component the bend cannot be distinguished, while in the case of the tangential component only the *A* portion of the curve is shown, the bend occurring at a pressure of more than 200 lb.

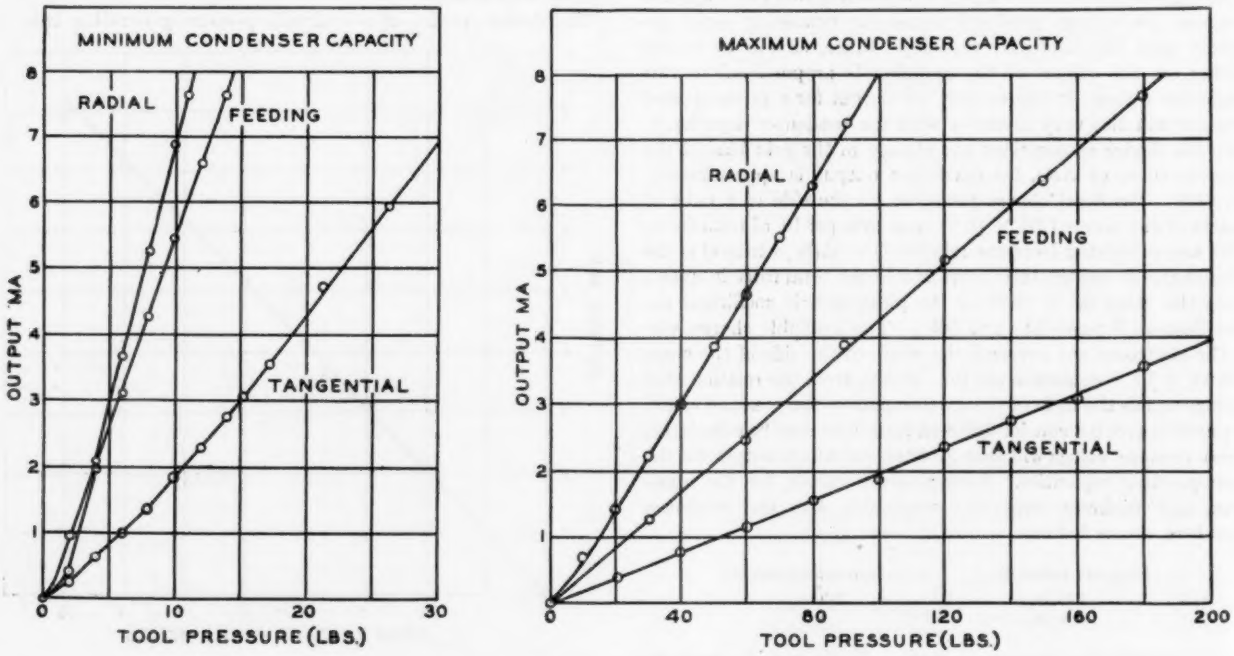


FIG. 7 VARIATION OF OUTPUT WITH TOOL PRESSURE

DYNAMIC BEHAVIOR OF THE DEVICE

In general, in order to record the fluctuations of any quantity without distortion, the recording instrument must have a natural frequency several times as great as that of any pulses or vibration trains occurring. If this condition is satisfied, the damping of the recording instrument is not important. The damping is of importance, on the other hand, when the natural frequency of the recording instrument is as low or lower than that of some of the pulses encountered. In this case an insufficiently damped instrument will overshoot and exaggerate the magnitudes of the fluctuations, while a critically damped instrument will, if anything, underestimate them.

In order to get an idea of what frequency of fluctuations of cutting pressure this device is capable of recording, it is necessary to know the natural frequencies of vibration and also the damping of the mechanical system comprising tool-holder plate, shank, balls, and quartz crystals. From computations on the device it was found that the elastic vibrational frequencies corresponding to deformation of the tool shank, torque arm, etc. due to forces applied in the directions of the three components of cutting pressure were much higher than the frequencies of vibration of the shank assembly considered as a rigid body, with the balls which transmit the pressure to the quartz crystals acting as restoring springs. Hence the natural frequencies corresponding to the application of forces in the directions of each of the components had to be computed on the latter basis.

These frequencies were computed in this manner, from the moments of inertia and the masses and the forces corresponding to unit displacements in these three directions. These forces were determined both by measurement and by computations on the compressibility of steel balls, both of which lead to the same values. The natural frequencies were also determined experimentally by taking oscillograms of the vibrations of the tool shank produced by suddenly released forces applied in the directions of each of the three components.

It was found that the vibrations of the tool shank were damped exponentially corresponding to viscous damping, and the values of k corresponding to the tangent exponential curve e^{-kt}, were determined from the oscillograms.

These experimental values of frequency n and damping k are as follows:

Pressure component	n (cycles per sec.)	k
Tangential..................	190	44.0
Radial......................	145	149.0
Feeding.....................	160	91.0

These experimental values of n agree with those computed, as described, within 5 to 10 per cent. With these values of the damping factor k, 5 to 15 vibrations of the tool holder occur on sudden release before they are damped down to a negligible value, so it is seen that the damping is far from critical.

If the device were used to record steady harmonic vibrations of a particular frequency, even with this insufficient damping, the response would not be exaggerated badly up to frequencies of 75 to 100 cycles per sec., although above these frequencies resonance would cause considerable exaggeration. However, the fluctuations in cutting pressure occur as an entirely irregular series of impulses. Hence we are interested in the behavior of the device under transient rather than steady state conditions.

The transient behavior is best illustrated by considering the response of the device to a force which rises from zero to maximum value in a given interval T, after which the force remains constant. If the interval T is several times greater than the natural period, the device will respond correctly and reproduce the actual pressure-time curve. If, on the other hand, the interval T is less than the natural period, the device will overshoot and execute its own damped vibration about the constant value.

Thus it is seen that when subjected to an irregular series of sufficiently rapid impulses such as the cutting pressure, the device will respond with a series of its own damped vibrations interrupted at irregular intervals. Actually the response is further complicated by impulses transmitted to the tool-shank assembly by the vibration of the lathe itself. It is obvious that oscillograms taken of the cutting pressure under these conditions will considerably exaggerate the magnitude of the pressure fluctuations.

The most satisfactory solution of this problem would be the critical damping of the tool shank for all three of its motions. This would cut out the overshooting and the bringing in of the natural vibrations of the tool-shank assembly.

However, the critical damping of such a heavy vibrating member appeared to be a problem of considerable magnitude. As a compromise, it was decided to retard the action of the oscillograph so that the time taken for it to reach full deflection was of the same order as that required for the oscillations of the tool holder to damp out. It was felt that this would in the main prevent it from following the natural vibrations of the tool holder, and hence smooth out these vibrations from the cutting-pressure oscillograms.

This result was accomplished by the use of a choke in the shunt circuit of the oscillograph which, by its inductive action, retards the rate at which the current rises to full value. Oscillograms were then taken, both of the vibrations produced by the sudden release of the tool holder and of cutting pressures. In all cases the natural vibrations of the tool holder were practically smoothed out, and other fluctuations were then apparent.

TEST PROCEDURE

The turning tests are being conducted with high-speed steel tools of the type indicated in Fig. 4. These tangentially cutting tools can be ground accurately and cheaply and therefore have an advantage as compared to radially cutting tools usually employed during machinability studies.

It is intended to investigate the suitability of the alloys for threading with chaser tools.

An objection may be raised against the use of tangentially cutting tools on account of their cutting action, which is somewhat different from that of radial tools. It should be kept in mind, however, that tangential tools are quite frequent in screw-machine work and are successfully applied in a number of commercial threading tools. One expects of course a fundamental difference in the life of both types of tools. This difference will be determined for similar test conditions by a series of tests with radial tools. It has no bearing, however, on the ultimate purpose of the tests—namely, the determination of the comparative machinability of the alloys.

The procedure during the turning tests is as follows. The parts are chucked on the small diameter and trued to a fixed dimension on the large diameter by means of a side tool. During the test cuts readings with a milliammeter are then taken alternately of the electrical output produced by each pressure component. The curves plotted from these readings represent a true picture of the changes in the pressure components due to tool wear and are used for the determination of the comparative machinability of the alloys. Indicative values of the cutting pressure in pounds per square inch of chip section are recorded on charts by means of the calibration curves.

Oscillograms are taken at various intervals to obtain a comprehensive picture of the cutting action. They are also used to determine the relationship between milliammeter readings (which are of course average output values) and the instantaneous output values of the records.

The machined diameters are frequently checked. Chip samples are also taken at fixed intervals for later reference.

Mechanical calibration tests are made about twice every week. Electrical calibration tests are made more frequently.

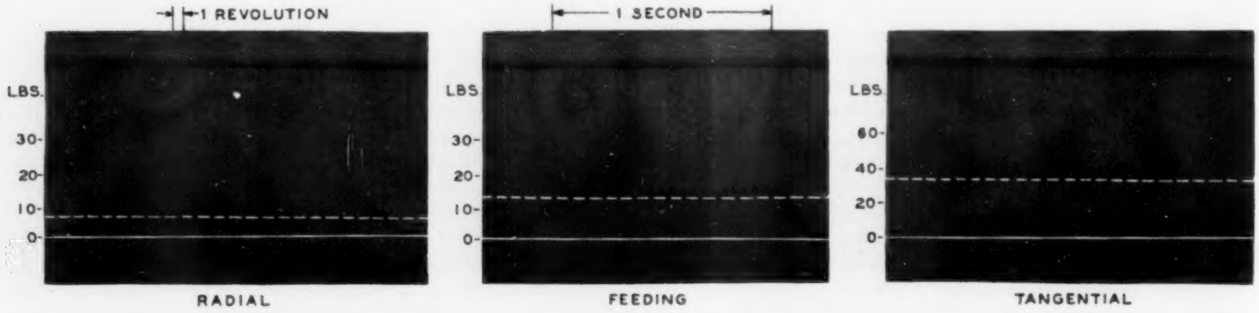
Visual records of the wear of the tools after each test are obtained by means of a Hartness screw-thread comparator. These records are used for a comparison with the results obtained with the cutting-pressure measuring device.

RESULTS OF TESTS

The authors do not, as yet, consider the tests sufficiently complete to give detailed information on the behavior of the individual alloys under various test conditions. They present therefore only the following general results, which were obtained during turning tests made without a coolant:

1 Oscillograms of the three pressure components gave a good indication of the uniformity of the material. It was found that the large-diameter section of the test pieces was invariably softer in the middle than at the ends. This condition showed up best in the records of the radial pressure and seems to indicate that this component best reveals any non-uniformity of the material. (See Fig. 8.)

2 A comparison of pressure curves of the three tool-pressure components verified the findings of previous investigators already referred to—that the tool wear affects the radial and feeding pressure much more than the tangential pressure. This is illustrated by the curves shown in Fig. 9 of the tool pressures during the dry machining of 250 parts of four alloys.



UPPER TRACE - SPINDLE REVOLUTIONS.
LOWER TRACE - TOOL PRESSURE MILLIAMETER READINGS SHOWN BY DOTTED LINES.
SPINDLE SPEED - 1250 R.P.M. OTHER CUTTING CONDITIONS SAME AS IN FIG. 9 ALLOY NO. 6

FIG. 8 OSCILLOGRAMS OF THE THREE COMPONENTS OF TOOL PRESSURE

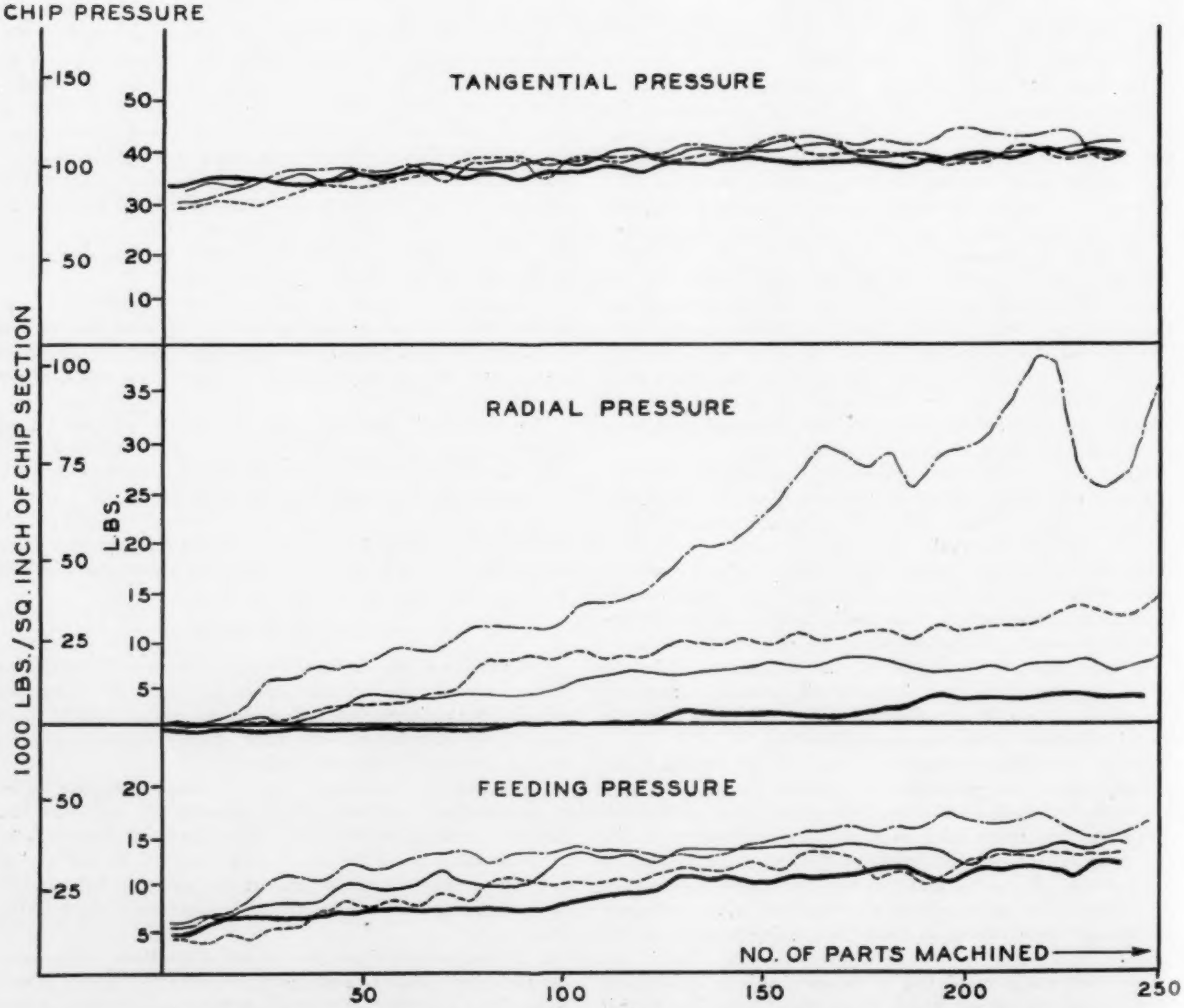


FIG. 9 VARIATION OF THE THREE COMPONENTS OF TOOL PRESSURE WITH NUMBER OF PARTS TURNED

(Heavy line: No. 8 alloy, 95.5 Al, 2.25 Ni, 0.75 Si, 1.5 Cu. Broken line: No. 5 alloy, 87.0 Al, 13.0 Si. Light line: No. 6 alloy, 95.0 Al, 3.0 Si, 2.0 Cu. Dot-and-dash line: No. 11 alloy, 90.0 Al, 8.0 Si, 2.0 Cu. Cutting conditions: Tool used, tangential 50-deg. cutting angle, material high-speed steel. Cutting speed, 765 ft. per min. Feed, 0.012 in. per revolution. Depth of cut, 0.031 in. No coolant used.)

3 The pressure curves of the radial and feeding components permitted a fair comparison of the machinability of the alloys for dry cutting conditions. (See Fig. 9.) For some of the alloys results were already obtained after machining 250 parts. For other alloys with apparently similar machining qualities the tests had to be extended.

4 A comparison of corresponding values of the tangential pressure during the machining of 250 parts each of two alloys, one with high and the other with low silicon content, under various test conditions, demonstrated that this pressure, usually referred to in a general way as cutting pressure, is in itself no conclusive measure of machinability. It was found that at the beginning of the tests the high silicon alloy required less pressure than the low silicon alloy, whereas at the end of the tests this condition was just reversed. It is obvious that wrong conclusions would have been reached if the machinability of these alloys had been based on the starting values of the tangential pressure and not, as had been done here, on the increase in the radial and feeding components of the tool pressure.

5 The starting values of the tangential cutting pressure ranged from approximately 75,000 lb. to 105,000 lb. per sq. in. of chip section for cutting speeds of 500 to 750 ft. per min., 0.012 in. feed per revolution, 0.031 in. depth of cut, and cutting angles of from 35 to 50 deg. The corresponding radial pressures amounted in most cases to only a few thousand pounds per square inch and were sometimes even negative. The corresponding feeding pressures ranged from approximately $^1/_4$ to $^1/_8$ of the magnitude of the tangential pressures. The end values depended upon the wear of the tool.

WORKING LIMITS OF THE METHOD AND OF THE DEVICE ITSELF

Applications of the piezo-electric property of quartz have been made by many in the field of the measurement of rapidly changing pressures. Work has been done along this line by Karcher[14] in measuring peak pressures, and more recently by Webster[15] and also by Kluge and Linckh[12] in measuring instantaneous pressure values. The work of Webster was on the high pressure produced by the firing of a gun, while that of Kluge and Linckh was on lower pressures, such as tool pressures and the vibration of a stretched wire.

The device that has been just described was designed along similar electrical lines to that of Kluge and Linckh, but has been improved from the standpoint of electrical leakage and also in that the output is linear with crystal voltage. Mechanically it represents the application of this method to the special problem of measuring the three components of cutting pressure during the machining of the short test pieces available. By providing suitable tool holders, it can be used for tests on practically any type of tool, such as turning tools, drills, etc.

The following is a short discussion of the controlling factors that enter in to determine the range of usefulness of this method as it might be applied to the measurement of other types of instantaneous pressures, and also of the device as described.

(1) *Lower Pressure Limit.* Even with more amplification than used here, there is an electrical limit to the lowest pressure that can be satisfactorily measured. This is because even with the best obtainable insulation in the circuit comprising the crystal, condenser, and grid of the pressure tube there will be a small leakage current which will produce a drift in the reading of the meter indicating the pressure. This drift may be divided into two parts:

(*a*) The drift due to the grid current of the pressure tube.

(*b*) The drift due to leakage over the quartz crystal and over the insulation of the condenser, grounding key, etc.

The first of these drifts has the same value whether the reading is at zero or full scale and is in the direction of increasing pressures. In the device as described this drift amounts to 0.03 lb. per min., corresponding to a grid current of 3×10^{-15} amp.

The second drift is back toward the zero reading with constant applied pressure. This drift varies with the magnitude of the pressure reading and amounts to a certain percentage of the reading. This percentage is proportional to the sensitivity of the device as determined by the condenser value. The greatest percentage observed was 0.5 per cent per minute, this being with the highest sensitivity—namely, full scale for 9.9 lb. crystal pressure. This value of drift corresponds to a leakage resistance over the insulating surfaces of the order of 10^{14} ohms.

For the work done here both of these drifts were sufficiently small. They amount to about $^1/_{20}$ of the drift present in the apparatus of Kluge and Linckh. It is probable that the tube drift could be cut down to 0.01 lb. per min. by special drying of the glass surface, since the manufacturer states a grid current corresponding to a drift of this magnitude. Special precautions might produce an improvement in the other insulating surfaces also. As it was, it was found necessary to put a drier consisting of calcium chloride in the compartments containing the quartz crystals. This was done because the shellac used to cement the foil to the crystals was quite hygroscopic, and caused a large leakage current without the drier present.

From the mechanical standpoint, the main point in determining the lowest pressure that can be satisfactorily measured is the friction on the members transmitting the forces to the quartz crystals. With the device as described, with the greatest sensitivity, a full-scale reading was obtained with the following values of the three components:

Tangential......	34.0 lb.
Radial..........	11.5 lb.
Feeding.........	14.5 lb.

There were no appreciable errors traceable to friction. As mentioned previously, due to different leverages and preloading pressures, these are quite different from the actual pressures on the quartz crystals.

(2) *Higher Pressure Limit.* There should be no upper limit electrically to the pressure that can be measured by this method. The shunting condenser can be made large enough to keep the working voltage of the correct magnitude. The pressure-charge relationship has been found linear up to 12,500 lb. on a crystal of the size used here (computed from Karcher's[14] data). It may be well to mention here that the piezo-electric constant is practically independent of temperature in the range from 0 deg. cent. to 100 deg. cent. (See Sosman,[13] pages 565 and 566.)

With the device as described, the full-scale reading with the maximum condenser value was for 130 lb. crystal pressure, the values for the three components being somewhat different due to leverages and preloadings. By substituting a larger condenser the range could be easily extended.

From the mechanical viewpoint, considering the quartz itself, the crushing strength of a $^1/_2$-in. cube such as is used here amounts to 75,000 lb., as calculated from data given on page 481 of Sosman.[13] The linear compression of such a cube amounts to 15 $\times 10^{-6}$ inches for 100 lb. or about twice that for a steel cube. This was calculated from Bridgman's value of the linear compressibility given on page 465 of Sosman.[13] The upper limit of the method as far as the working stresses are concerned would then depend on these figures as well as on the stresses produced in the steel balls and the other parts of the tool-shank mounting.

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[14] J. C. Karcher, "A Piezo-Electric Method for the Instantaneous Measurement of High Pressures," Scientific Paper of Bureau of Standards (1922), 18, pp. 257–264 (No. 445).

[15] R. A. Webster, "A Piezo-Electric Gage and Amplifier," Franklin Inst. Jour., vol. 211, no. 5, May, 1931, pp. 607–616.

The upper pressure limit will also depend on the maximum displacements at the cutting point of the tool which can be tolerated as far as the limits of accuracy of the particular turning operation are concerned. For the device as described, the approximate displacement for an applied tool pressure of 100 lb. in the three different directions are:

Tangential....	0.0010 in.
Radial........	0.0018 in.
Feeding.......	0.0005 in.

These deflections were found to be largely due to the compression of the balls used to transmit the pressure to the quartz crystals and were considered not excessive for the work in hand. The use of rollers instead of balls is now being considered for transmitting the pressures to the crystals. This construction will require a much greater accuracy of the working parts than the present design to make it successful. However, as computations show, it will reduce the deflections of the tool point considerably.

(3) *Rapidity of Response.* In applying any method for measuring instantaneous pressure values occurring in any mechanical system it is essential that the building in of the recording device does not materially change the mass or the rigidity of the system, or the vibration characteristics of the system will be changed. Due to the high modulus of elasticity of quartz, this method is especially suitable for application to mechanical systems of high rigidity.

In applying any method to the problem of measuring instantaneous tool pressures, it is necessary for distortionless recording, as mentioned previously, that the system comprising tool, tool holder, and measuring device have a natural frequency several times that of any pressure pulses that will be encountered. To satisfy this condition, the masses must be kept to a minimum and the rigidity to a maximum.

The desire to be able to measure all three force components necessitated the design of a tool holder and mounting of rather larger mass and lower rigidity than otherwise would have been the case.

The device as described would satisfy these requirements if no pressure pulses corresponding to a frequency of more than 50 cycles per sec. were encountered. As the maximum frequencies corresponding to chip formation under the conditions of the tests appear to be over 5000 cycles per sec. (by chip size and surface speed), even the added rigidity of rollers, which would raise the natural frequencies of the device considerably, probably would not be sufficient. Hence, to prevent undue exaggeration of the response to pressure fluctuations, damping of some kind is necessary, and that which is in effect produced electrically, as described, appears at present the most practicable. Under these conditions the records obtained are reliable in every respect except for fineness of detail in regard to rapid fluctuations.

ACKNOWLEDGMENTS

The authors wish to acknowledge their indebtedness to Mr. G. A. Bouvier, under whose direction and supervision this work was done. They further wish to acknowledge the suggestions of Mr. R. J. Beavers during the earlier work and the assistance of Mr. F. J. Bigler in the carrying out of the tests.

Discussion

R. Poliakoff.[16] The authors rightly mention the importance of the problem under their investigation as based on tool life and tool wear. The methods used, however, by various experimenters are limited practically to roughing cuts, whereas finishing cuts still wait to be given at least part of the attention given to roughing cuts.

The writer would therefore refer to a method of studying the wear and life of finishing cuts used by him as far back as 1913, a description of which is given in the "Report on Past Experimental Work on Cutting Tools" of the Tool Steel Research Committee of the Manchester Association of Engineers (England), published in 1916. The method was favorably commented upon, also, in the "Proceedings of the Institution of Mechanical Engineers," London, 1913, p. 1150.

The principle employed was to work with the tool until it reached a predetermined bluntness. The latter was observed as the tests progressed by means of an indicator fixed in the rest immediately below the tool. (See Fig. 10.) The indicator was set to zero at the commencement of the cut, and it pressed against the bar so that as the tool became blunt and lost its cut, the radius of the bar increased by a like amount, and this was shown by the indicator. The indicator was graduated in 1/2000ths of an inch and the degree of tool bluntness decided upon for the particular test was 3/2000ths of an inch, i.e., about 25 per cent of the depth of the cut. The method, when proper instruments are used, affords a very accurate way of studying wear of finishing tools.

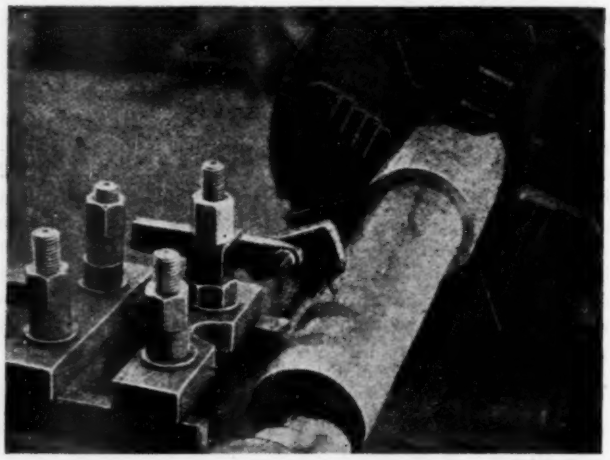

Fig. 10 Indicator in Position for Measuring Wear While Cutting

The writer cannot agree with the authors' statement that the problem of chip pressure received early attention in view of the relative importance of removing metal with the least amount of energy. It is important to study chip pressures, as they affect many elements both in the tool and the machine, but even with great chip pressures the amount of energy necessary to remove the metal can be very small and of minor importance, both in itself and as money expense. This, of course, is particularly true in the cases of problems which the authors had under investigation, where the chips removed were not heavy.

O. W. Boston.[17] The device described for recording instantaneous tool pressures is obviously the result of painstaking efforts of a high scientific order. The application of the quartz crystal seems superior to the hydraulic type of dynamometers because of the rigidity introduced in the measuring mechanism. The use of a liquid to transmit pressure of such high frequency as suggested would undoubtedly introduce additional difficult calibrations due to the tendency on the part of the liquids and containers to absorb and delay the transmission of the high-frequency forces.

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The use of this device to measure pressures in order to anticipate the degree of tool wear and tool life for the different materials machined is unique. It is not new in principle, but does indicate great possibilities. A complete relation between the cutting force and tool wear must first be determined carefully in order that tool life may be based on tool wear. As the tool wears, the force may actually become less and remain less until the point of tool failure, due to the cupping action of a chip on high-speed-steel tools, as when cutting ductile steel. On the other hand, the tool wear may be by abrasion on the flank below the cutting edge, as when cutting cast iron with high-speed-steel tools or when cutting steel with carbon-steel tools. The various die-casting alloys studied in the paper might react differently in this respect on a given tool.

The writer is not in a position to comment intelligently on the influence of the dampening effect of the heavy vibrating member which carries the part being machined. It appears that this is a problem, as the authors have stated, of considerable magnitude and one requiring careful calibrations. The authors make the statement that the fluctuations in cutting pressure occur as an entirely irregular series of impulses. The writer has found[18] that most metals when removed by the cutting edge of a tool are formed into small laminations which group themselves into elements of unequal size. On the other hand, on some metals, such as free-cutting screw-stock brass and free-cutting steel, the sections formed are quite equal. When viewing the formation of the sections through a microscope as the chip is being formed at very slow speeds, it was found that these sections occurred regularly at, say, every 0.015 in. of tool travel for certain cutting conditions. Presumably, the die-casting alloys being studied fall in the group which give chip sections of unequal size, producing the irregular series of impulses. It might be of help to study these impulses by trying to machine a free-cutting brass or free-cutting steel, such as those which have been found by the writer to give regular impulses.

The writer is interested in the statement made in paragraph 1, under the heading of "Results of Tests," that "It was found that the large-diameter section of the test pieces was invariably softer in the middle than at the ends." He would like to ask, Does this mean softer as measured by a hardness-testing machine or does it cut easier, that is, with lighter pressure?

In paragraph 2 under the same heading the statement is made that tool wear affects the radial and feeding pressure much more than the tangential pressure, as illustrated by the curves in Fig. 9 of the tool pressures during the dry machining of various parts. Inasmuch as these die castings will probably eventually be machined by the assistance of a coolant, it seems only reasonable that data should be obtained confirming this statement when cutting with a cutting fluid. The writer has found that the wear of a tool, particularly drills, is influenced by the nature of the cutting fluid.

It is of unusual interest that the radial and feeding pressures indicate more pronouncedly the increase in cutting force because of tool dulling than does the tangential pressure. Presumably, one conclusion is that the tool is failing by abrasion of the flank reducing the clearance angle below the cutting edge to zero, rather than by the cupping method. The writer has experienced this same condition when cutting cast iron and semi-steel with high-speed-steel tools.

Fig. 9 would indicate that the tool pressure when cutting a No. 8 alloy (the heavy line) is negative for the first 80 pieces produced. If this interpretation is correct, might this be due to the fact that the back-rake angle of the tool for this material is such as to cause the tool to dig in and take deeper cuts?

It is hoped that the authors will find it convenient to present the results of machinability. Certainly that part of the report dealing with instantaneous pressure is interesting and may throw considerable light on compositions with irregular impulses and tend to minimize or completely eliminate chatter.

The writer is looking forward anxiously to further progress dealing with this interesting study.

M. Stone.[19] Although the ultimate aim of the work undertaken by the authors is not the development of a dynamic-pressure-measuring instrument, this occupies the major part of the paper, and is of general interest as well.

In the field of instruments for measuring very small vibratory motions and forces, the writer has had much to do with the application and improvement of dynamic extensometers of the variable-coupled transformer (magnetic) and carbon-pile types. These instruments have very desirable accuracy, down to 0.00001 in., and have provided means for investigating a number of dynamic phenomena that otherwise could have been investigated only inappropriately—such as measurement of instantaneous and vibratory torque in Diesel drives, short-circuit forces and motions of turbo-generators, variable stresses in oil-pumping mechanisms, acceleration of railway-truck parts, etc. One reservation always has been present, however, and that concerns the inertia of the instrument to exceedingly rapid motions.

In this respect, the piezoelectric pressure-measuring device provides something that takes us far above the range of natural frequencies that other types of devices have. The fact, too, that the front of this device is merely a flat, parallel-faced crystal allows it to be used in places where other types of instruments would be decidedly disadvantageous, due to their size. In making up their instrument, however, the authors were not able to take advantage of the first property apparently because of the relative flexibility of the mounting carrying the crystals. Concerning the second, however, the reason for their compact instrument is made possible only through the small size of the crystals.

The crystal measuring device that the authors have followed in their design is that of Kluge and Linckh at the Technische Reichsanstalt in Berlin, and rightfully so. Although they were not the originators of the idea of using piezoelectric effects for mechanical measurement, these two men have perfected the application of this device to several types of measuring instruments. The writer recently had the opportunity of discussing with them several possibilities of its further use. At the beginning it was thought that the care necessary in measuring the exceedingly small electrostatic charges would hinder its being successful, but this has been to a large extent overcome by careful shielding and the use of the tube voltmeter.

Due to its high natural period, this type of instrument can do two things probably more successfully than other instruments—(1) record impactive forces, of explosion rapidity, and (2) record even the highest harmonic existent in normal elastic structures, which escape the other types of instruments.[20]

The authors have furthered the application of this type of measuring device in an instrument designed for a type of work that has been little investigated in this country. This discusser agrees with the authors that it is only through the type of investigation that they are planning that any real knowledge on the relative machining characteristics of various materials and tools can be based.

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[18] "What Happens When Metal Is Cut," Trans. A.S.M.E., 1930, paper MSP-52-11.

[19] Engineer in Charge of Development, Power Engineering Department, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[20] See "Piezoelectrische Messung mechanischen Grössen," by Kluge and Linckh, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, 1931, p. 95, where high harmonics are shown in the measurement of the starting torque of a direct-current motor.

R. E. W. Harrison.[21] This contribution to knowledge regarding the stresses involved in the cutting of metal is valuable and of special interest at the present time. The availability of the apparatus described by the authors makes it possible for the designer of machine-tool and small-tool equipment to more thoroughly appreciate the stresses involved, and therefore to adequately take care of them in the designs which he produces.

The paper brings out a number of points of exceptional personal interest, inasmuch as the writer spent some years at the Manchester College of Technology, where the tests were made in England, which determined the relative merits of high-speed and carbon-steel tools.

It would seem, from the graphical records on the eighth page, that the piezoelectric pressure-measuring method is more complete, inasmuch as there does not appear to be the slight lag and rounding of the curves at the points of high and low pressures which characterize the graphs produced by hydraulic-pressure-recording devices.

The authors are to be congratulated upon the careful analytical thought which has been put into the design of their tool carrier as well as the complete calibration of the apparatus before the tests were run.

As a record of an extremely interesting investigation into methods of recording tangential, radial, and feeding pressures, it is doubtful if anything of a more reliable nature has been made available. However, on the general question of machinability—and it is understood that this was the major motive which actuated the experiments—the writer would suggest the desirability of further investigation into the processes through which the raw material is passed prior to the machining operations. The reason for this recommendation is that during the past year various phases of the problem of machinability have been presented, and the results obtained from a series of very careful tests indicate that in almost every case where difficulty is experienced in machining a particular object, the most complete and permanent solution of the problem can be reached only by going back to the chemical or metallurgical analysis of the material being worked on, and then following through by a most accurate control over subsequent heat treatments such as normalizing, annealing, etc.

In one case in particular which the writer has in mind, it has been proved that to secure reliable machinability it is necessary to revise all previous ideas regarding permissible variations in chemical analysis and permissible working range in pouring, annealing and normalizing, etc. The results of the tests show that there is every indication that the older and more generally accepted methods of testing for machinability with a file, with the drill point, with the scleroscope, and with the Brinell machine do not produce the full story. The detection of inclusions, hard crystalline formations, and laminations cannot be accomplished reliably by any of the older methods, and it has been shown that these defects can be most adequately brought to light by cross-sectioning and a deep and rapid acid etching. In the particular case in question the results obtained included:

(*a*) Increase in cutting speed of from 20 to 40 per cent
(*b*) Increase in tool life of at least 50 per cent
(*c*) Almost entire elimination of rejections due to unmachinability
(*d*) Overall increase in manufacturing efficiency with practically no increase in the cost of raw material.

This latter statement serves to illustrate the fact that it is many times possible to do the job rightly at no increase in cost over and above the cost of the wrong or what might be termed the wide-range method. Whether figures such as those quoted could be made applicable to the particular piece of work shown in Fig. 1, the writer does not know. However, it is hoped that they will serve to draw attention to the importance of the control of specifications affecting the material before it is presented to the machine.

It is to be hoped that the authors will continue their experiments, as it would seem that their apparatus is particularly well suited for the recording of the effects of laminations in forged-steel products, and further experimentation would undoubtedly bring to light a definite relationship between forging and annealing temperatures and tool life.

E. O. Waters.[22] Fig. 9 of the paper is an interesting commentary on previous measurements of tool pressures, in showing that the radial and feeding pressures, rather than the tangential pressure, increase during the use of a tool. Such being the case, it appears that, contrary to early surmises, there is no correlation between the energy of cutting and the life of a tool, at least within the limits of duration in Fig. 9. In other words, as long as the removal of metal is the only criterion, and surface finish is disregarded, it would have been possible for the machine to keep on running, producing pieces of the specified diameter, well beyond the "250 parts" which apparently terminated the tests. If roughing cuts only are being taken, what difference does it make if the radial and feeding pressures rise to extreme values, as long as the tool and the machine are strong enough to stand up against these pressures?

On the other hand, if a high quality of surface finish is desired, it may well be that a sharp rise in the radial-pressure curve can be associated with a dulling of the tool sufficient to make it unsuitable for finish cuts. Such a correlation, however, would have to be arbitrarily established before the pressure measurements would be of any value.

The foregoing is not offered as a criticism of the authors' work, but of tool-pressure measurements in general. The method of measurement described in this paper is noteworthy for its sensitivity and for the practical absence of motion in the parts being weighed. It should be studied attentively by all engineers who may have occasion to make pressure measurements of a similar order of refinement in other fields of research.

Authors' Closure

The indicator method referred to by Mr. Poliakoff for the direct measurement of tool wear during finishing cuts is comparable to the trailer-tool method used by Digges and French for the same purpose. The authors are sure that both of these methods can be used to advantage and will furnish good results in all those cases where the test pieces are available in comparatively long bars of fairly uniform structure. As mentioned in the paper, however, it was for their investigations considered important to obtain not only data on the machinability of the alloys as expressed in tool wear or tool life, but also as much information as possible regarding such causes contributing to tool wear as variation in the hardness of the parts, presence of blowholes, and other physical properties. This reasoning led to the adoption of the method described.

The authors admit that for most modern machine tools the problem of removing metal with the least amount of energy is relatively unimportant from the standpoint of expense for the power required. They can well imagine, however, that investigators, during the early stages of machine-tool design, were much concerned with this problem also from the point of view of keeping the stresses and the wear developed in the relatively coarse machine elements down to a minimum. From the broader aspect of tool-pressure measurements in general, the authors are certain that even today a more accurate knowledge of chip pressures for the various materials and machining operations is desirable.

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The authors concur with the opinion advanced by Professor Boston that the relationship between tool wear and the change in the cutting forces must be established before tool-pressure measurements are used for machinability studies. This was realized, and visual observations of the condition of the cutting edge of the tools were therefore made with a Hartness screw-thread comparator at the beginning of the tests, to check this relationship. The results obtained were considered satisfactory.

The irregular fluctuations in the magnitude of the cutting forces referred to in the paper should be regarded in the light of the natural frequencies of vibration of the device. These frequencies, while being relatively high, were yet not high enough to permit the device to follow the very rapid pressure changes resulting from the separation of the individual chip laminations which, according to calculations, occurred in some instances at the rate of 5000 per sec. The oscillograms of the cutting pressures, therefore, do not reveal all the fine details of the cutting action, but are a record rather of such irregular occurrences as blowholes and hard and soft spots. It can easily be seen that, to bring out the details of the cutting action, the cutting speeds used would have had to be decreased to values far below any practical shop speed.

The statement made in the paper regarding the variation in the hardness of the middle and end portions of the large-diameter section of the test pieces referred to the hardness as it expressed itself in the magnitude of the cutting pressure. While it would probably have been more correct, as suggested, to speak of the ease of cutting rather than of hardness, it was felt that the use of the latter term was justified in view of the greater chilling effect of the die on the end portions as compared with that on the middle portion.

Concerning the use of a cutting fluid, it was the intention of the authors at the beginning of the investigation to conduct tests under both dry and wet cutting conditions. However, a number of tests, during which a coolant was used, indicated that the respective rates of tool wear would be so slow as to make it advisable to restrict the investigation to dry-cutting conditions.

The authors share the opinion advanced regarding the interpretation of the negative radial pressures at the beginning of one test with alloy No. 8. The radial component of the chip pressure against the back of the tool in the direction toward the axis of the work must, in this case, have been greater than the radial pressure produced at the contact point of tool and work against the front face of the tool.

The authors appreciate the comments made by Mr. Stone regarding the electrical aspects of the device. They regard, with high esteem, the work done by Kluge and Linkh in Germany in connection with a number of applications of the piezoelectric pressure-measuring method, and are satisfied if their own work tends to further the adaptation of modern electrical developments to purely mechanical research such as machinability studies.

The point brought out by Mr. Harrison regarding the importance of chemical and metallurgical analyses of the materials to be tested in connection with machinability studies is well taken. While the authors expected valuable information in this respect from a separate study conducted on the physical, chemical, and metallurgical properties of the alloys referred to, they were of the opinion that the testing method for the investigation of the machining qualities should, if possible, furnish data also on some of those metallurgical properties of the alloys, which, it was expected, would affect the machinability. This latter consideration was therefore one of the factors which led to the adoption of the method described.

Regarding Professor Waters' comment, the rates of increase in the radial and feeding components of the chip pressures were taken as an indication of tool wear on the basis of the results of tests by previous investigators and of preliminary tests during which the effect of tool wear on the three pressure components was investigated. From a general point of view the authors see no reason why the method described cannot be applied equally well to roughing and finishing cuts. They desire to call attention, however, to the fact that they regard the problem with which they were confronted as a special one, in view of the shape and character of the parts, and ask that the method adopted be considered partly in this light.

MSP-54-7

Belt Drives With Cast-Iron Pulleys and Paper Pulleys

By C. A. NORMAN[1] AND G. N. MOFFAT,[2] COLUMBUS, OHIO

C. A. NORMAN

This paper gives data on transmission efficiency of various types of belts on cast-iron and paper pulleys and goes thoroughly into the transmissive power of oak-tanned and rubber belts on such pulleys with varying contact angle, a subject of importance in view of the prevalence of short-center drives with large transmission ratio. It is found that commonly used formulas for the effect of contact angle on transmissive power are not dependable for all combinations of belts and pulleys.

G. N. MOFFAT

PREVIOUS tests at the Ohio State University have covered a great variety of belt materials and have given the comparative degree of slackness at which belts of these materials could be run.[3] Data were also given with regard to stretch. These tests were principally on cast-iron pulleys and sheet-steel pulleys. Belts were also run at very high speed on pulleys with wood rims, and conclusions were presented regarding the effect of centrifugal force on transmissive power. Data on transmissive power from tests on a static slip test machine were given for comparison with the data from running tests, and the relative applicability and value of static and running tests were discussed.

No data were given, however, on transmission efficiency, i.e., the ratio of output at the driven end to input at the driving end; and effective pulls in excess of 50 lb. per inch of width were dealt with only for pulleys of a diameter of 6 in. Many inquiries having been received regarding transmission efficiency, it was concluded that it might still be well to publish some data of this kind. Also it appeared that the tests on pulleys with diameters of only 6 in. gave an advantage to very flexible belts. It appeared desirable to investigate high effective pulls with larger pulleys, on which even somewhat stiffer belts might develop sufficiently intimate contact. Data are furnished herewith for effective pulls as high as 100 lb. per inch of width for both oak-tanned and rubber belts. While on 6-in. pulleys in the earlier tests only a certain mineral-tanned belt was found capable of carrying effective pulls of this magnitude, the tests here presented show that on 18-in. pulleys both oak-tanned and rubber belts are likewise able to carry such loads.

In the earlier tests, for rim speeds of more than 8000 ft. per min., wood-rimmed pulleys were used, but no thorough investigation of the transmissive power of other than cast-iron or sheet-steel pulleys was undertaken. Impregnated paper pulleys have, however, long been known as an extremely effective means of transmitting power, and they have been gaining increasing acceptance in many fields. It appeared of value to investigate their properties rather fully.

The investigation was carried out not only at 180-deg. contact angle, but at a variety of contact angles. This appeared of particular importance in view of the prevalence of drives with short center distances and large transmission ratios, i.e., small contact angle on one pulley unless an idler is used. It is well known that V-belts operate well under such conditions. It was of interest also to find out what flat belts might do.

Well-known formulas purport to make possible an allowance for variations in contact angle. These formulas are, however, based on the assumption of dry, solid friction between belt and pulley. Actually, with fillers, impregnations, and belt dressings in general use, the friction may well be of a semi-viscous character. Tests, for instance, by Friedrich[4] in Germany and the generally accepted influence of slip on friction render this highly plausible. Under such conditions the formulas in general use are not dependable.

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[2] Assistant Professor of Mechanical Engineering, Ohio State University. Assoc-Mem. A.S.M.E. After graduation from the Mechanics Arts High School, St. Paul, Minn., Mr. Moffat attended the University of Minnesota, receiving his B.Sc. degree in 1918 and his M.E. degree in 1919. In 1917 he entered the U. S. Army Ordnance and served as Engineer of Tests, testing shells and gun forgings. Mr. Moffat was with the General Motors Corporation, and then went to the Harrison Radiator Corporation as research engineer, working on problems of the cooling of automobile engines. In January, 1923, he went to the Mechanical Engineering Department of the Ohio State University, where he has charge of materials of engineering and also assists in the instruction of machine design.

[3] Published as Bulletin 41, "Transmissive Power and Stretch of Belting," by the Engineering Experiment Station, Ohio State University, 1928.

Contributed by the Machine Shop Practice Division and presented at the meeting, Boston, Mass., Sept. 22, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, during the National Metal Congress.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] See *Zeit. Ver. deut. Ing.*, July 3, 1915, p. 537; also Norman, "Principles of Machine Design," p. 397.

APPARATUS USED

Tests were performed on the 40-hp. belt dynamometer set in the laboratory of the Mechanical Engineering Department of the Ohio State University. The set is shown in Fig. 1. The letter *A* denotes a motor, cradled so as to oscillate around its axis and mounted in a frame which rolls on ball bearings on a track *B*. This motor carries the driving pulley *C*. The torque is weighed on a platform scale *D*. The total pull of the belt, the sum of the tight-side and slack-side tensions, is carried from the frame of the motor through a wire rope and a bell crank to a platform scale *E*. The driven pulley *F* is mounted on an electric dynamometer *G*. The dynamometer torque is weighed on a scale arm *H*. The speed is measured by speed counters connected by means of flexible shafts *I* and *J* to the driving and the driven shafts. On the flexible shafts are two clutch couplings operated by electromagnets in series so that the speed counters can be coupled in and out at the driving and the driven end at the same instant. In consequence, speed differences and creep or slip may be determined very accurately.

For tests on the influence of contact angle a frame *K* and an adjustable idler *L* were added to this equipment as shown in Fig. 2. By moving the idler along the frame the contact angle could be varied from 90 deg. on the driving end and about 150 deg. on the driven end to 150 deg. on the driving end and 90 deg. on the driven end. The idler was carried on ball bearings and consumed a practically negligible amount of power.

The transmissive power of the belt is expressed as effective pull and as tension ratio. The effective pull is the difference between the tight and the slack tension, $T_1 - T_2$, where T_1 is the tight tension and T_2 the slack tension. Multiplied with the belt speed and divided by 33,000 it gives the horsepower transmitted. The same effective pull, however, may be transmitted with either a tight belt or slack belt. Evidently a belt capable of running quite slack has superior transmissive power. For any given effective pull, the tight tension T_1 is comparatively small and the belt is not very highly stressed. This should be conducive to long life and but little stretch. Moreover, the bending pull on the shaft is low and the load and friction in the bearings are low. The belt gives the most favorable and economical conditions of power transmission all around.

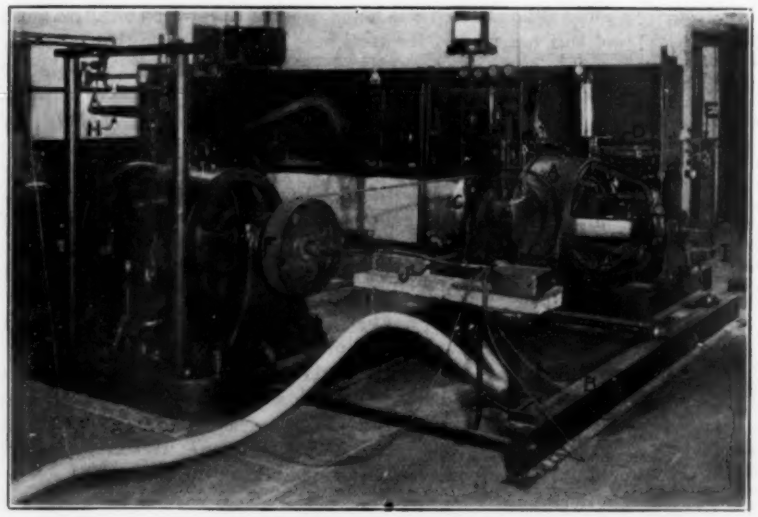

FIG. 1 40-HP. DYNAMOMETER SET

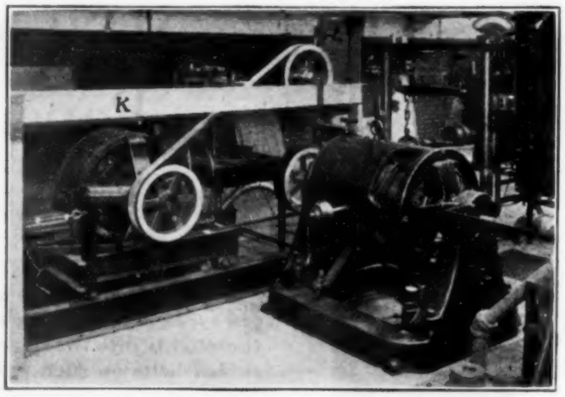

FIG. 2 DYNAMOMETER SET WITH FRAME FOR VARYING CONTACT ANGLE

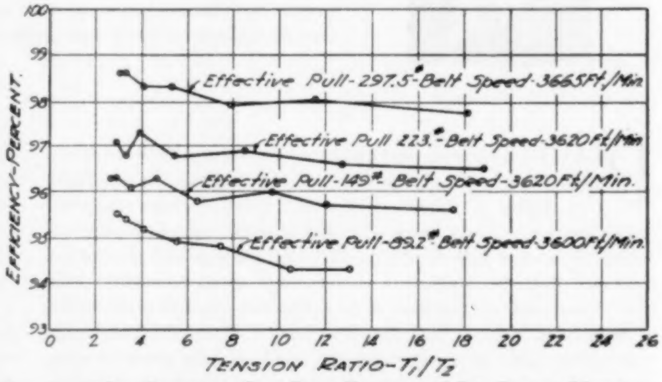

FIG. 3 18-IN. CROWNED CAST-IRON PULLEYS; 3-IN. SINGLE MINERAL-TANNED BELT; 180-DEG. CONTACT ANGLE

The best way to indicate condition of slackness seems to be by the ratio of the tight tension to the slack tension, i.e., by T_1/T_2. If T_1 is high compared with T_2, i.e., if the ratio T_1/T_2 is high, then the belt runs very slack, and if the slip is not great, it is an economical and efficient means for transmitting power. On the other hand, if T_1/T_2 is low, the belt is tight, and while it still may show a high transmission efficiency, i.e., a high ratio of output to input, yet it runs with comparatively high bearing friction, high shaft pull; it may stretch considerably and require frequent take-ups, and the life will not be as long as if it could be run slacker. For practical purposes a tension ratio of 2 may be considered indicative of a very tight belt, while a tension ratio of 10 means a quite slack belt.

To present transmissive power, tension ratio has been plotted against slip, i.e., the curves show to what degree of slackness the belt can be reduced before serious slip commences. In some cases serious slip commences suddenly; the curves break off sharply. In others the slip increases gradually. Before the belt is in danger of losing hold entirely, the slip becomes greater than can be permitted in a drive that is operating rationally.

It should be stated that the method of measuring speed has an influence on the extent to which slip readings can be carried. If speed measurements were instantaneous, one might get readings even if the belt was slipping very seriously. Revolutions were, however, counted over two minutes of time, and it was necessary to

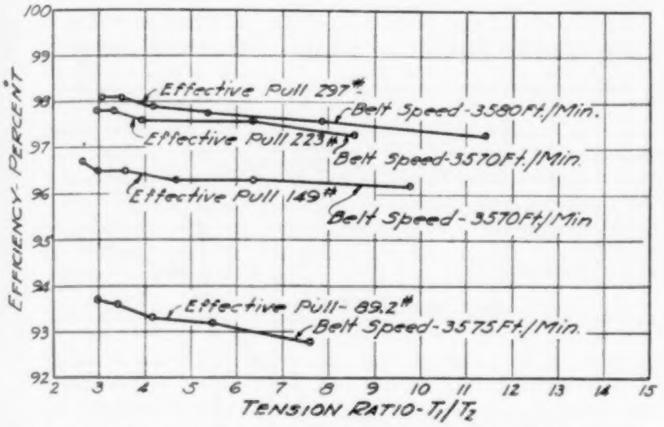

FIG. 4 18-IN. CROWNED CAST-IRON PULLEYS; 3-IN. SINGLE OAK-TANNED BELT; 180-DEG. CONTACT ANGLE

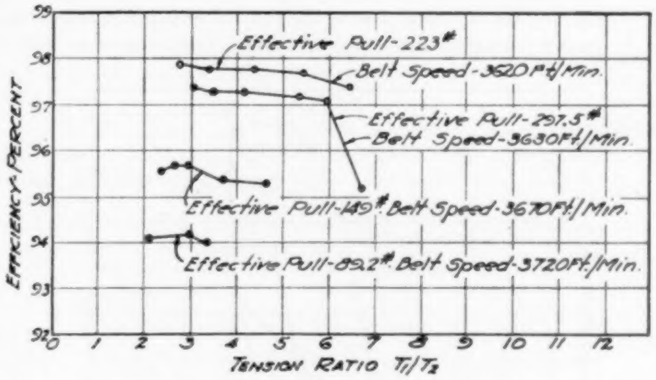

FIG. 7 18-IN. CROWNED CAST-IRON PULLEYS; 3-IN. FOUR-PLY RUBBER BELT; 180-DEG. CONTACT ANGLE

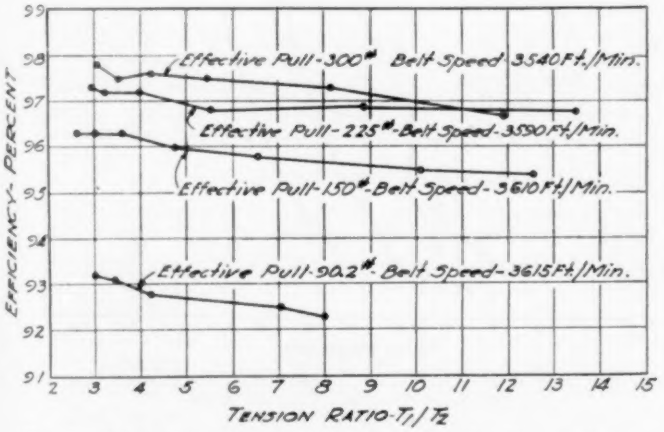

FIG. 5 18-IN. CROWNED PAPER PULLEYS *A*; 3-IN. SINGLE OAK-TANNED BELT; 180-DEG. CONTACT ANGLE

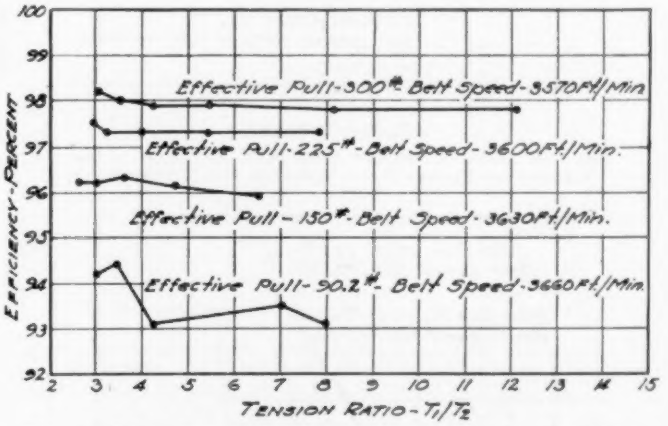

FIG. 8 18-IN. CROWNED PAPER PULLEYS *A*; 3-IN. FOUR-PLY RUBBER BELT; 180-DEG. CONTACT ANGLE

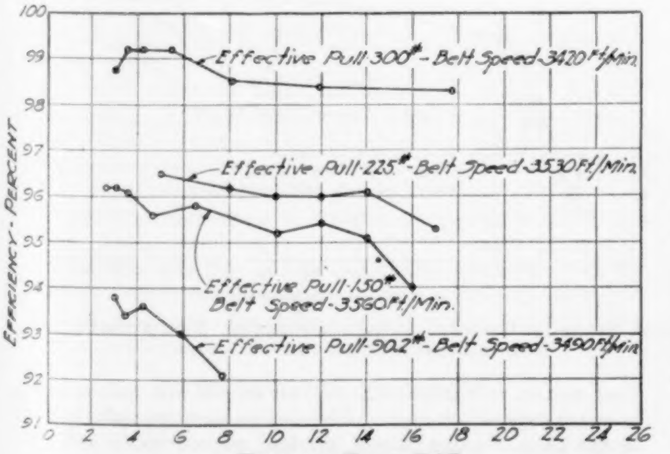

FIG. 6 18-IN. CROWNED PAPER PULLEYS *B*; 3-IN. SINGLE OAK-TANNED BELT; 180-DEG. CONTACT ANGLE

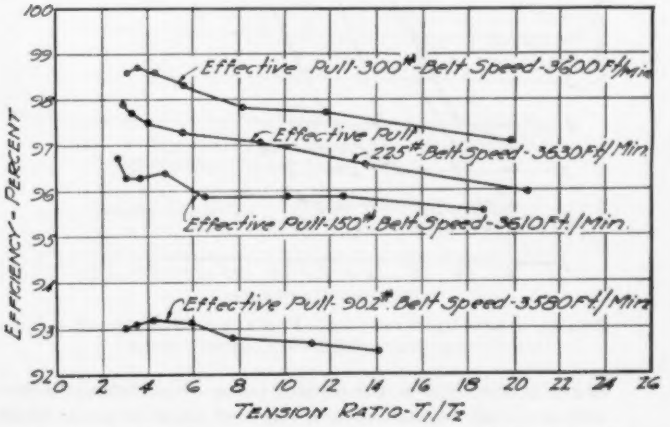

FIG. 9 18-IN. CROWNED PAPER PULLEYS *B*; 3-IN. FOUR-PLY RUBBER BELT; 180-DEG. CONTACT ANGLE maintain steady running conditions for this period. This meant that slip could not be carried to a point where the drive was not perfectly positive. An attempt was always made to get the last point on the curves as close to the point of serious slip as possible. In this connection it is well to remember that belts often behave rather erratically near the slipping point. Sometimes it is possible gradually to work up to a degree of slackness under which the belt could not possibly be started up under load, and one which could not possibly be reset after the belt has once begun to slip seriously. The final breakdown point of the drive is therefore one that cannot be stated with absolute exactness.

The ultimate points given on the curves are the last at which the observers could still get dependable torque and speed readings over two minutes of time.

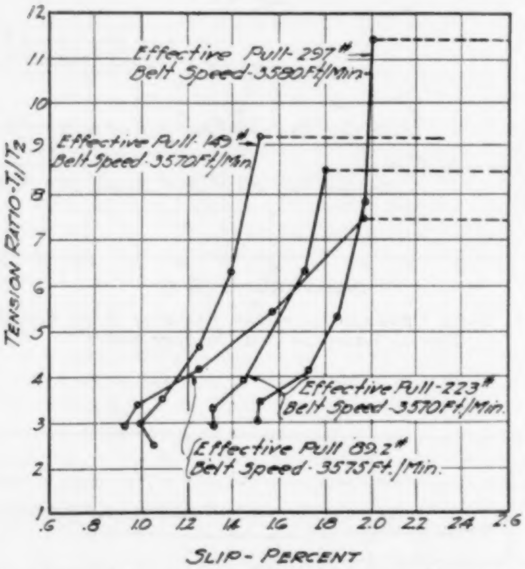

Fig. 10 18-In. Crowned Cast-Iron Pulleys; 3-In. Single Oak-Tanned Belt; 180-Deg. Contact Angle

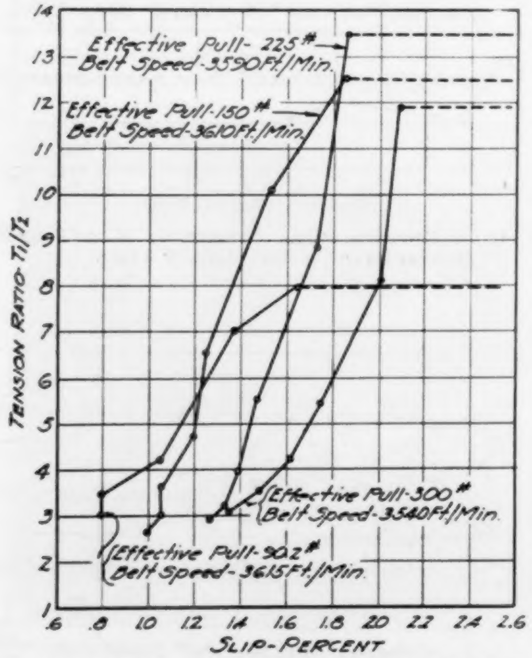

Fig. 11 18-In. Crowned Paper Pulleys *A*; 3-In. Single Oak-Tanned Belt; 180-Deg. Contact Angle

Efficiency Data

The efficiency data are presented in Figs. 3 to 9, inclusive. They pertain to a mineral-tanned belt on cast-iron pulleys and to an oak-tanned belt and a rubber belt on cast-iron pulleys and on two kinds of paper pulleys, marked, respectively, *A* and *B*.[5] The leather belts were somewhat-used single belts, and the rubber belt a four-ply belt of a recent type brought out by a well-known rubber concern. All belts were 3 in. wide. All pulleys had a di-

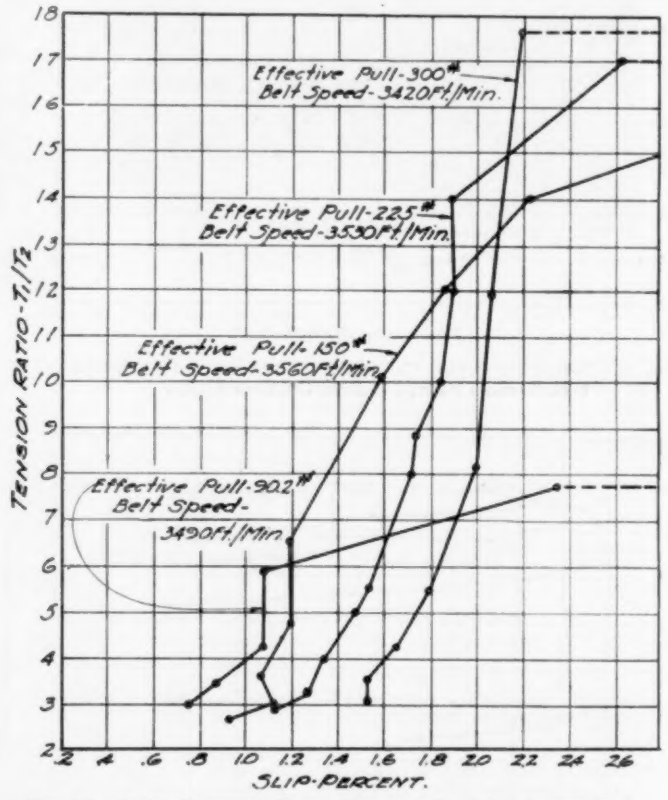

Fig. 12 18-In. Crowned Paper Pulleys *B*; 3-In. Single Oak-Tanned Belt; 180-Deg. Contact Angle

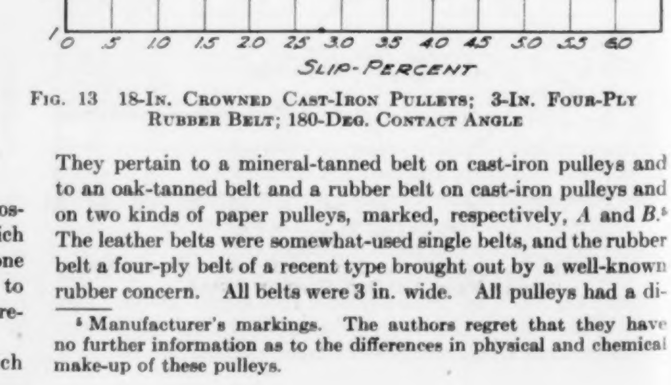

Fig. 13 18-In. Crowned Cast-Iron Pulleys; 3-In. Four-Ply Rubber Belt; 180-Deg. Contact Angle ameter of 18 in. and were crowned. The slack strand was run on top. The belt speed was approximately 3600 ft. per min. in all cases. It is given on the curves. So is likewise the effective pull. It will be seen that four different effective pulls were held. They are approximately 90, 150, 225, and 300 lb. total, or 30, 50, 75, and 100 lb. per inch of width. A normal effective pull for these belts would be about 50 lb. per inch of width. It may then be said that the loads were carried to 100 per cent overload.

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[5] Manufacturer's markings. The authors regret that they have no further information as to the differences in physical and chemical make-up of these pulleys.

No attempt was made to smooth out the curves. The actual points obtained in any one test are simply joined together.

It will be seen that the efficiency values obtained vary but little from belt to belt. At a normal load of 50 lb. per inch of width the efficiency is about 95 or 96 per cent for all belts. As the load increases, the efficiency goes up, and reaches values around 98 per cent for 100 per cent overload. As the load drops, the efficiency drops. This is due to the fact that the power consumed in flexing the belt over the pulleys and the windage loss on belts and pulleys remain the same at all loads; consequently, these losses constitute a larger percentage of a small power input than they do of a large one. Naturally a flexible belt should show a somewhat higher efficiency than a stiff one, and this should show up particularly at light load. But to judge by the results here shown, the difference is not great on 18-in. pulleys. All told, there is little justification to discriminate between belts and between pulleys on account of differences in mechanical efficiency. All belts when rationally run and loaded are an efficient means for transmitting power. If any lesson is to be learned, it is that one should, as far as possible, avoid idling belts.

TRANSMISSIVE POWER

Data for transmissive power at 180-deg. contact angle are given in Figs. 10 to 15, inclusive. The data refer to the pulleys mentioned in the preceding section. Only oak-tanned and rubber belts are included, however.[6] On pulleys of 18-in. diameter both oak-tanned and rubber belting are shown to be capable of transmitting an effective pull of 100 lb. per inch of width, and this condition holds true on cast-iron pulleys as well as on paper pulleys. However, the degree of slackness, i.e., the tension ratios to which the belt can be carried, is much higher on paper pulleys, particularly of type *B*, than on cast-iron pulleys.

The possibility of running rubber belts at tension ratios as high as 14 to 15 should be of considerable convenience and importance. The necessity of running rubber belts tighter than leather belts on cast-iron pulleys has been held as a great drawback with rubber belting. The drawback may not be quite as serious, at least with sufficiently large contact angles, as has sometimes been stated. Nevertheless, pulleys giving a decided improvement in the tractive power of rubber belts should be welcomed.

A peculiarity shown by the tests is that with the lowest effective pull, 30 lb. per inch of width, the belts start slipping at much lower tension ratios than with higher effective pulls and do not attain nearly as high maximum ratios. The reason is to be seen in the stiffness of the belt. A low tension ratio with a very low effective pull means that there is not enough total tension to pull the belt snugly onto the pulley, the slack-side tension being very low and particularly lacking in this respect.

INFLUENCE OF CONTACT ANGLE

The results of the tests with varying contact angle are plotted in Figs. 16 to 27, inclusive. As above only oak-tanned leather belts and friction-surfaced rubber belts were used in these tests, and only two effective pulls, 50 lb. and 75 lb. per inch of width, were held. Even so, with four idler positions for each belt-pulley combination and each effective pull, the data gathered are somewhat voluminous.

The commonly used formula for connecting tension ratio with angle of contact and coefficient of friction is

$$T_1/T_2 = e^{fa}$$

where e is the basis of the natural logarithms, f the coefficient of friction, and a the contact angle.

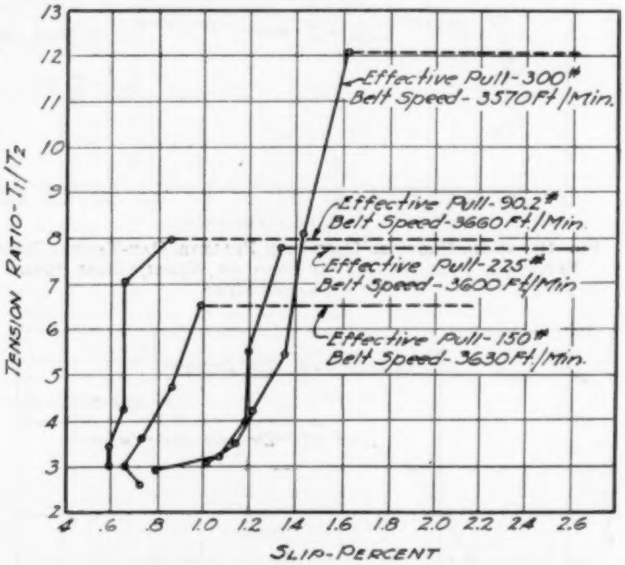

FIG. 14 18-IN. CROWNED PAPER PULLEYS *A*; 3-IN. FOUR-PLY RUBBER BELT; 180-DEG. CONTACT ANGLE

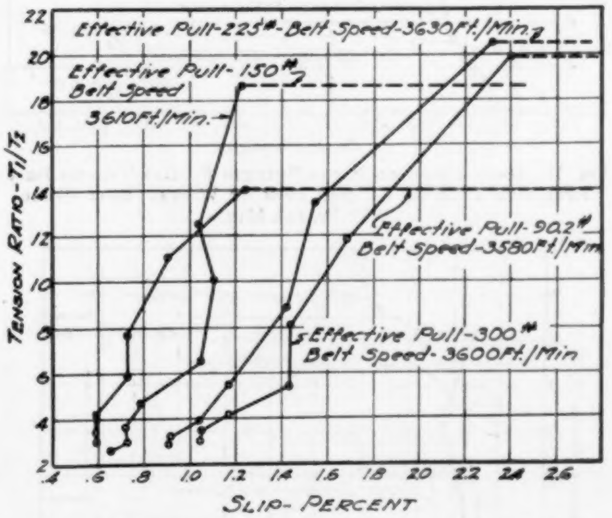

FIG. 15 18-IN. CROWNED PAPER PULLEYS *B*; 3-IN. FOUR-PLY RUBBER BELT; 180-DEG. CONTACT ANGLE

By means of this formula the coefficient of friction was figured for the maximum tension ratio obtained with 180-deg. contact angle. With the coefficient of friction so obtained, the tension ratios corresponding to angles of 140, 120, and 90 deg. were computed, and these theoretical values were plotted into the diagrams in Figs. 16 to 27 so that they can be directly compared with the experimental values obtained.

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[6] Transmissive power of high-grade mineral-tanned belting was covered in Bulletin 41 of the Engineering Experiment Station of the Ohio State University previously referred to. It was there shown to be capable of transmitting an effective pull of 100 lb. per inch of width even on pulleys of 6 in. diameter.

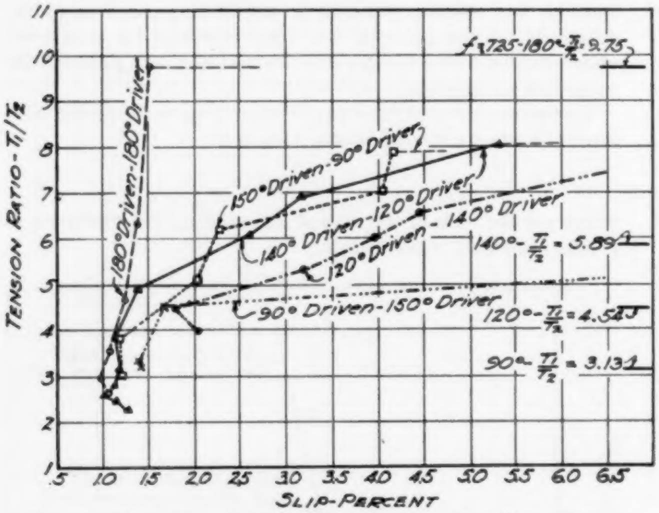

FIG. 16 18-IN. CROWNED CAST-IRON PULLEYS, OAK-TANNED BELT; EFFECTIVE PULL 50 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

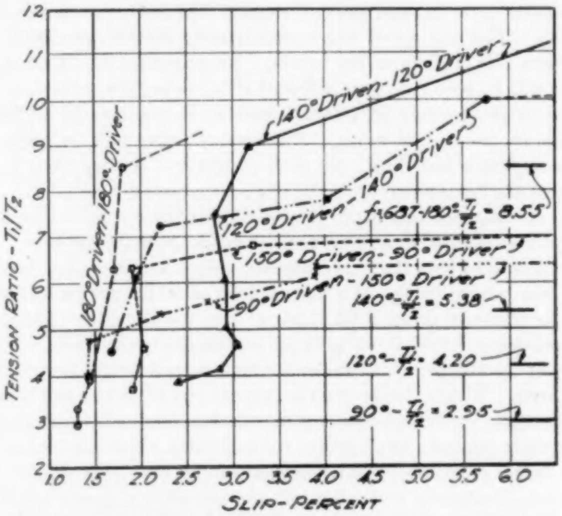

FIG. 17 18-IN. CROWNED CAST-IRON PULLEYS; OAK-TANNED BELT; EFFECTIVE PULL 75 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

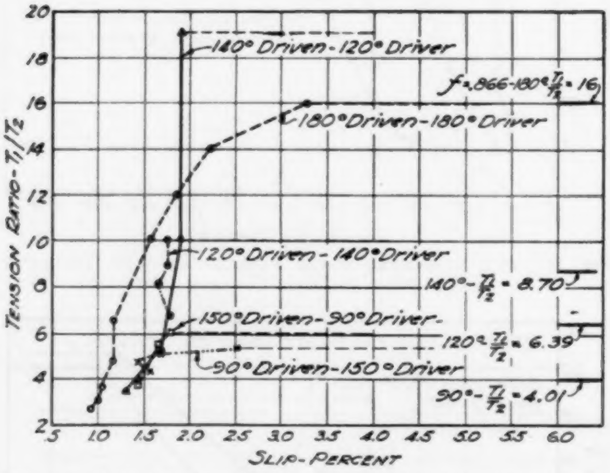

FIG. 18 18-IN. CROWNED PAPER PULLEYS *B*; OAK-TANNED BELT; EFFECTIVE PULL 50 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

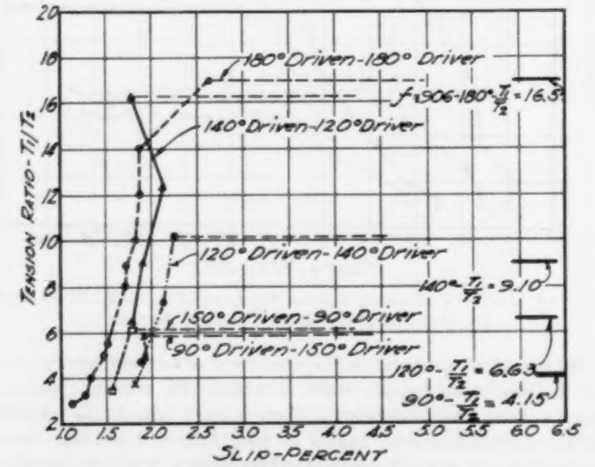

FIG. 19 18-IN. CROWNED PAPER PULLEYS *B*; OAK-TANNED BELT; EFFECTIVE PULL 75 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

Turning first to the classical combination—oak-tanned belt on cast-iron pulley—the results are given in Figs. 16 and 17. The fact is brought out very clearly that with reduced contact angle the transmissive power is as a rule improved by a certain amount of slip. The improvement is, however, quite irregular, and to be on the safe side, one should hardly assume a maximum permissible tension ratio of over 4 with an effective pull of 50 lb. per inch of width, and over 5 with 75 lb. The lower value with 50 lb. effective pull is due to the stiffness of the belt. As already explained, a certain total pull is needed to secure a sufficiently close contact between belt and pulley, and this necessary pull is a heavier charge against the lower effective pull than it is against the higher.

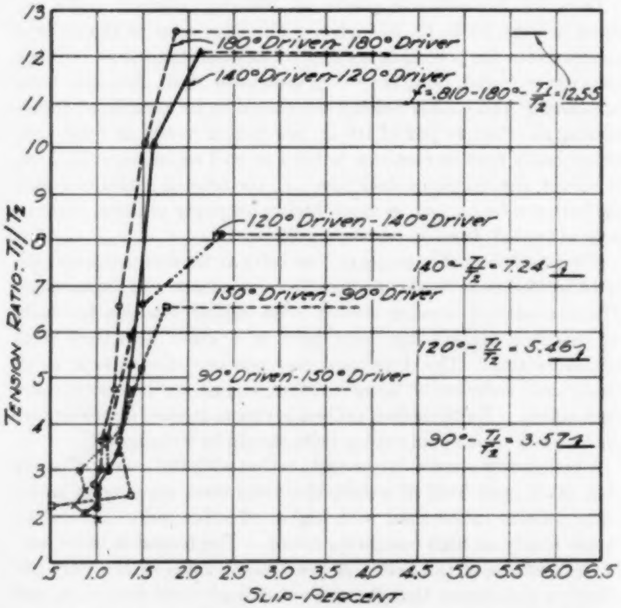

FIG. 20 18-IN. CROWNED PAPER PULLEYS *A*; OAK-TANNED BELT; EFFECTIVE PULL 50 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

The tension ratios obtained may be said to exceed those derived from the theoretical formula. However, the irregularity of the results is such that it would hardly be advisable to try to draw a curve to take the place of the theoretical relation. This is as might be expected for a belt the transmissive power of which depends in an intricate fashion on both solid and viscous friction. Particularly the latter would no doubt be influenced in a very complicated manner by temperature, humidity, slip, age, and stiffness of belt, etc. One would almost be tempted to suggest that for oak-tanned belts on cast-iron pulleys no attempt to correct for contact angle should be made at all, but that the minimum tension ratios found with 90-deg. contact angle should be made the basis for all belt calculations.

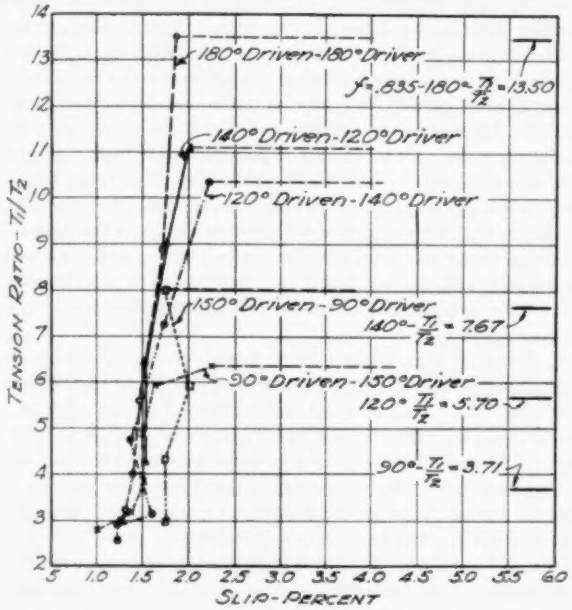

FIG. 21 18-IN. CROWNED PAPER PULLEYS *A*; OAK-TANNED BELT; EFFECTIVE PULL 75 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

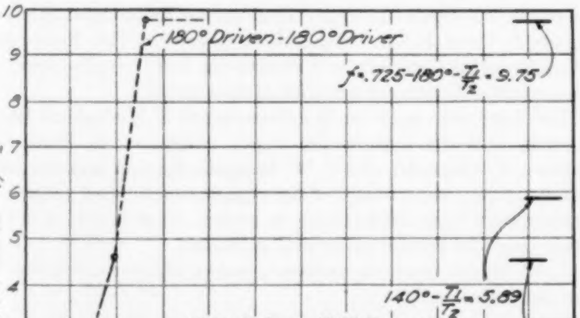

FIG. 22 18-IN. CROWNED CAST-IRON PULLEYS; RUBBER BELT; EFFECTIVE PULL 50 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

Substituting paper pulleys of type *B* for the cast-iron pulleys, a very different picture of the transmissive power of oak-tanned belting is presented. The data are given in Figs. 18 and 19. The friction coefficients with 180-deg. contact angle are higher, and there is a much more decided dropping off of transmissive power with reduced contact angle, the minimum values, however,

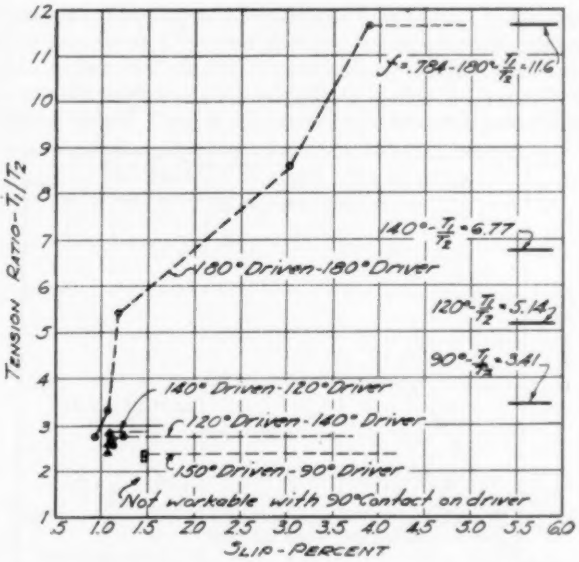

FIG. 23 18-IN. CROWNED CAST-IRON PULLEYS; RUBBER BELT; EFFECTIVE PULL 75 LB. PER INCH OF WIDTH, BELT SPEED 3800 FT. PER MIN.

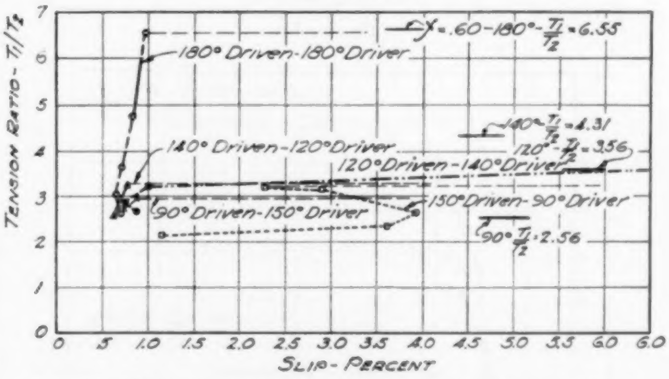

FIG. 24 18-IN. CROWNED PAPER PULLEYS *A*; RUBBER BELT; EFFECTIVE PULL 50 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

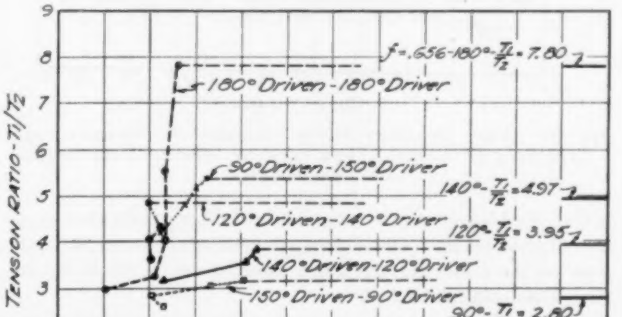

FIG. 25 18-IN. CROWNED PAPER PULLEYS *A*; RUBBER BELT; EFFECTIVE PULL 75 LB. PER INCH OF WIDTH; BELT SPEED 3650 FT. PER MIN.

remaining at least as good as with cast-iron pulleys and the intermediate values remaining decidedly better. A peculiarity is the abnormally high values obtained for both at 50 lb. and at 75 lb. effective pull per inch of width for a contact angle of 140 deg. on the driven pulley and 120 deg. on the driver. The probability is that the highest points were obtained after a somewhat excessive slip had set in. This slip may have developed a slight "tackiness" of the pulley surface, which asserted itself after the equilibrium was again established.

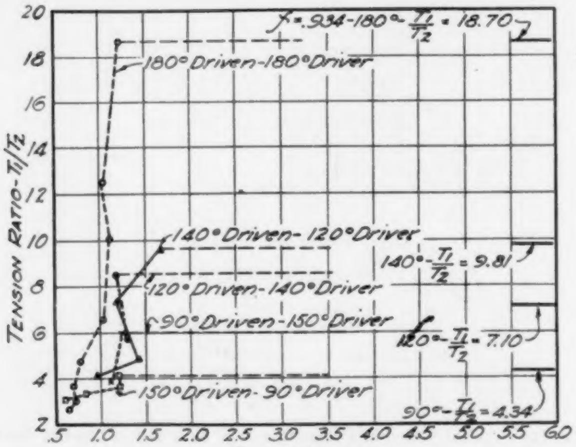

FIG. 26 18-IN. CROWNED PAPER PULLEYS *B*; RUBBER BELT; EFFECTIVE PULL 50 LB. PER INCH OF WIDTH; BELT SPEED 3770 FT. PER MIN.

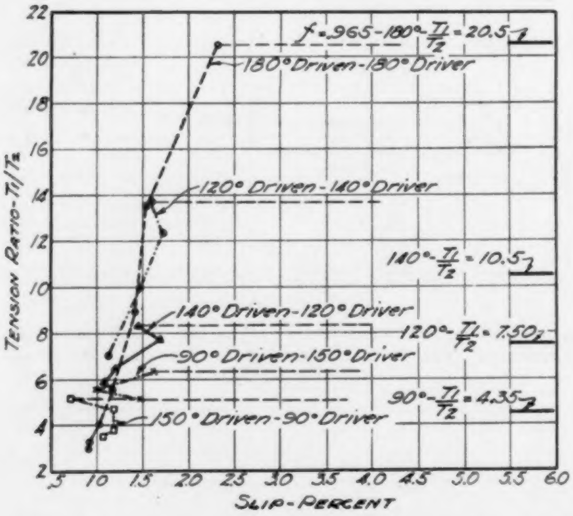

FIG. 27 18-IN. CROWNED PAPER PULLEYS *B*; RUBBER BELT; EFFECTIVE PULL 75 LB. PER INCH OF WIDTH; BELT SPEED 3900 FT. PER MIN.

A more regular behavior is displayed by paper pulleys of type *A* as shown in Figs. 20 and 21. The maximum values are not quite so high as with the type *B*, but the minimum values are as good. The actually obtained tension ratios with reduced contact angle are always higher than the theoretical ones, and the full transmissive power is developed with much less slip than with cast-iron pulleys. From the nature of the curves one would be tempted to say that neither tackiness of the pulley surface nor oiliness of the belt surface has anything to do with the favorable results obtained. In consequence, the conditions observed should be of an unvarying and dependable character.

For rubber belts on cast-iron pulleys the situation is as depicted in Figs. 22 and 23. With large contact angles, quite high transmissive powers may at times be obtained, but with reduced contact angles the transmissive power drops very seriously, and it is not much better at 120-deg. contact angle than it is at 90-deg. It should be remarked that the maximum tension ratios plotted in this graph are exceptional. They were obtained for several effective pulls in the series of tests with 180-deg. contact angle, which forms the basis of this paper. Such values were, however, not obtained in the tests published in 1928, nor were they obtained in check tests carried out late in 1930 and here published as Fig. 13. Maximum tension ratios of 4 to 5 are more likely than maximum ratios of 9 to 11. It is possible that these high ratios were brought out by stickiness caused by beginning slip. If lower ratios are used for maximum values, then the relative reduction in transmissive power with reduced angle is not so severe. However, the actual tension ratios at small angles will be no better than before, and the conclusion remains that with small angles of contact the combination of rubber belt and cast-iron pulley is not the best possible.

As shown in Figs. 24 and 25 an improvement is obtained by using paper pulleys of type *A*, particularly at fairly high effective pull (Fig. 25). However, as indicated in Figs. 26 and 27, to obtain really excellent ratios, paper pulleys of type *B* should be used. The reduction in tension ratio with reduced contact angle follows here rather closely the one derived from the formula, but even the lowest ratios are a considerable improvement over those obtained with rubber belts on cast-iron pulleys.

It would seem then that by a judicious use of paper pulleys the transmissive power both of oak-tanned and of rubber belts may be considerably improved.

ACKNOWLEDGMENTS

The investigations here presented were carried out as a project of the Ohio State University Engineering Experiment Station, of which Dean E. A. Hitchcock is director. The Rockwood Manufacturing Company, of Indianapolis, Ind., supplied some of the pulleys and defrayed a portion of the expense.

The tests were made in the Department of Mechanical Engineering, and the authors are under obligation to Professors William T. Magruder and F. W. Marquis, the past and the present chairman, respectively, of the department. They gratefully acknowledge their indebtedness to several other members of the department for helpful advice and assistance.

S. M. Marco, graduate assistant, was an efficient aid in testing and computation.

Editorial assistance has been furnished by Dr. K. Y. Tang, Department of Electrical Engineering, F. W. Marquis, and J. M. Weed, assistant director of the station.

Discussion

J. C. LINCOLN.[7] As to power transmission by belts, the speed multiplied by the difference in tension between the tight and slack sides equals the power transmitted. Consequently, the higher the belt speed the greater the power transmitted with a good allowable tension. The maximum tension which can be held with long life can be determined with reasonable accuracy, and consequently the only method of increasing the power transmitted is to increase the speed. The effect of centrifugal forces will be to decrease the pressure between the pulley and the belt, especially on a small pulley; with a given belt speed the centrifugal force decreases as the diameter of the pulley increases. Thus, at 5000 ft. per min. belt speed the centrifugal force on the

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[7] The Lincoln Electric Company, Cleveland, Ohio.

6-in. pulley is twice as great as it is on a 12-in. pulley; consequently to transmit the maximum amount of power, the pulley sizes should be as large as possible.

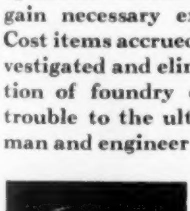
AUTHORS' CLOSURE

Mr. Lincoln is correct in stating that the centrifugal force reduces the pressure between belt and pulley. This reduction is, however, not dependent on pulley diameter, but only on belt speed. The mass of the belt on the pulley increases in direct proportion to the pulley radius, and the centrifugal force increases in direct proportion to the mass. On the other hand, the centrifugal force with unchanged mass decreases with increased radius. The two effects neutralize each other, and the formula for the centrifugal force on a belt is simply wV^2/g, where w is the weight of the belt per unit length, V the belt speed, and g the acceleration due to gravity.

Correlation of Casting Design and Foundry Practice

By ALEX TAUB,[1] DETROIT, MICH.

Correlation of design and foundry is a necessary part of any sound engineering program. However, it is quite often neglected, adversely affecting the ratio of the result per dollar. In considering the foundry the fact that individual foundry practices vary must be borne in mind. Castings are virtually assembled in the liquid state within the mold, and this principle must be borne in mind, but not overdone. Open-mindedness on the part of the designer is essential. Foundrymen must be urged to express new foundry ideas, since in new designs provision might be made to permit the use of new foundry development. Machine-made green-sand coring is a new practice being successfully used. The true bulk of a design is in the flask, not in the individual casting. Wall thickness is a general target on weight reduction. This is bad practice. Weight must be eliminated by design. The designer can help in obtaining clean castings by providing ample space for cores. Driers are a necessary evil, and the design should eliminate them wherever possible. Elimination of fins still requires development on the part of the foundry. A large proportion of foundry and engine-operation difficulties can be traced to improper determination of engine length quite often by a non-technical executive. Impatience on the part of engineers to see their designs in operation results in insufficient time for the foundry to gain necessary experience with experimental castings. Cost items accrued during productive periods should be investigated and eliminated from any new design. Elimination of foundry difficulties also means elimination of trouble to the ultimate owner. Therefore the foundryman and engineer are equally interested in the final result.



ENGINEERING and economics are quite often considered separate sciences. However, although economics is broad, standing firmly on its own feet, engineering cannot be so separated. Engineering in terms of pure research may sometimes be separated from economics, but as applied to a product engineering is merely the mechanicalizing of economics. Before a product is ready for manufacture it must include many economic considerations, the most outstanding yet most often neglected being that branch represented by the foundry. Designers have been notorious for their neglect of foundry consideration, and it is with the hope of improving these conditions that the following generalizations are presented.

The fundamental that must be borne in mind in correlating design and foundry is the universal law of "result per dollar." To meet the competitive conditions prevailing today, product change must be considered from the angle of progress. Is the change that is being contemplated an improved result? Will this result cost more than the improvement can be worth? Is it possible to obtain the desired result for the prevailing cost, or by simplification can the desired result be obtained with a lower cost?

The following considerations are in the spirit of "result per dollar." On new products or new designs consideration must be given, not only to known foundry practice, but to specific foundry practice available to the particular designer. Foundry practices vary. Some large institutions will readily consider the green-sanding of every possible part of a job, particularly if they are short of core-oven space. Other foundries that have an abundance of core-oven space will not give much consideration to green-sand coring, since they are able to produce their castings at satisfactory prices with the regular baked-sand process.

The designer must bear in mind that a casting, particularly such castings as automobile cylinder blocks, are virtually being assembled in the liquid state within the mold. The principle of liquid assembly must ever be borne in mind in the shaping of the design to insure that unnecessary projections, uneven sections, and uneconomic bulk shall be avoided.

It would seem a simple proposition, with the principle of liquid assembly in mind, to arrive at the maximum in economics if one arranges to assemble in the mold the maximum amount of material, including all brackets possible that are now being bolted on. The fallacy of this argument is obvious when one considers that quite often the inclusion of a bracket in a cylinder-block casting may easily raise the price per pound of the entire block. Increasing the price per pound of a block weighing 200 lb. becomes serious.

The price per pound of a casting is the true measure of the extent of designer-foundry correlation. The difference in cost between iron at the cupola and the cost per pound of the casting on the foundry floor is the final arbiter as to whether a job has been well done.

The designer does not always comprehend the foundry as an institution housing and using the sciences. It seems to be difficult for some engineers to get away from the idea that a foundry is a place where commercial mud-pie making is going on.

One does not expect a designer to be a foundry expert, since the foundry art is a lifetime's study. However, he is expected to know approximately how his casting should be made in order that his design may not create unnecessary cost-producing hazards. At least his design should incorporate the proper draft considerations and the fewest possible external cores.

The external cores are the first sign of design weakness. They must be tolerated where necessary, but forever remain a red-flag point to be eliminated at the first opportunity. Elimination or reduction of internal cores should be given consideration, but only with the cooperation of the particular foundry available to the designer, since, as stated earlier, an abundance of core-oven space may make the elimination of certain cores uneconomic.

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[1] Research Engineer, Chevrolet Motor Company. Mr. Taub has been associated with the automotive industry since 1906. He was educated in England. He has been with General Motors Corporation since 1917. Mr. Taub has presented several papers before the Society of Automotive Engineers.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Cooperation of Departments

Open-mindedness on the part of the designer is a most important requisite. The foundryman must not only be heard and heeded, but quite often must be tempted to discuss his ideals. Perhaps a small modification might permit the development of new foundry practice. Such a practice may be awaiting just such an opportunity. The author knows of no better example of this than the recent practice of the Chevrolet foundry at Saginaw.

In Fig. 1 *a* indicates a casting used as a clutch housing. Such parts have been used in the industry for many years. At Chevrolet this design of clutch housing was preceded by a design as shown in the construction pictured in *b* Fig. 1. The casting portion of the *b* construction is obviously a simple open housing easily completely molded with green sand without the aid of any dry-sand cores. In fact, the construction is so simple that in this particular case two of these pieces were cast in one mold.

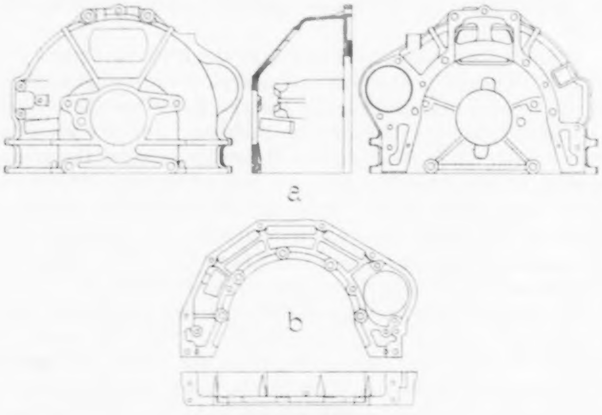

Fig. 1 Clutch Housing
(*a*, new design, requiring a housing; *b*, former design, cast two in a mold.)

When the construction was changed, it made necessary the use of a housing as at *a*. The question of cost immediately entered, since this housing was not suited to the casting of two pieces to the mold, and neither was it suited to complete green-sanding. Precedent dictated that the inside should be dry-sand cored. However, the Saginaw foundry thought differently, and advised the engineering department that if certain principles were adhered to they could completely mold this casting in green sand. Therefore the exact design as indicated was evolved—not all at once, but gradually, small changes being made continually as problem after problem confronted the foundry.

The inner mold of this piece was made of a special sand under fairly high compression, and is fitted into place within the flask without being removed from the molding machine on which it is made. The flask containing the impression of the inner back flange is then put in place, and the mold is ready for pouring. Note the direction of the various contours, the sharp corners representing the face of the mold. All of this detail was necessary to make this molding process a success. The success attained by this method has encouraged this organization to attempt its application to other and more complicated castings.

Due to the fact that the green-sand core under these circumstances is machine-located, the castings obtained by this process have been accurate and highly satisfactory. The price per pound of this casting undoubtedly was lowered considerably, due to the successful development of this method.

It must be remembered that it required the fullest cooperation and confidence of the engineering department in the foundry to complete this process. Due to the many difficulties encountered by the foundry, the project would have been put on the shelf had the engineering department objected at all to the many minor changes requested by the foundry. In fact, this is one instance where the foundry came very close to losing patience with itself before any signs of impatience had appeared in the engineering department.

In spite of the foregoing, it is well to bear in mind the over-ingenious foundryman, who will undertake to make anything. This type of individual is just as troublesome as the individual who will not cooperate. He permits his enthusiasms to carry him into the realms of impracticability, resulting in hazards for himself and unsatisfactory final cost to his customers. Designers do not realize that actual disaster may follow ill-considered designs of castings.

Recently a cylinder block that necessitated a complete dry-sand coring of the inside and outside was designed and put into production. So heinous an offense against progress could not have been perpetrated had the foundry been given any consideration or the slightest voice in the matter of design.

True Meaning of Bulk

Bulk to the designer may mean only the total size and weight of a given casting. The true bulk is the overall size of the flask necessary to make that casting. If the general dimensions of the casting are such as to require a considerable amount of waste sand, the molding equipment must be larger and therefore slower than necessary.

An example of this is shown in Fig. 2, which illustrates two types of exhaust manifold. The upper left figure and the upper right figure are, respectively, top and end views of the same manifold in the mold, and the lower left and lower right views are a redesign manifold, permitting a reduced volume of sand over that shown in the upper views. One requires a considerable amount of unused green sand in a large flask difficult to handle, therefore requiring heavier and slower equipment. The other requires considerably less sand; likewise is easier handled, and consequently lighter molding machines can be used.

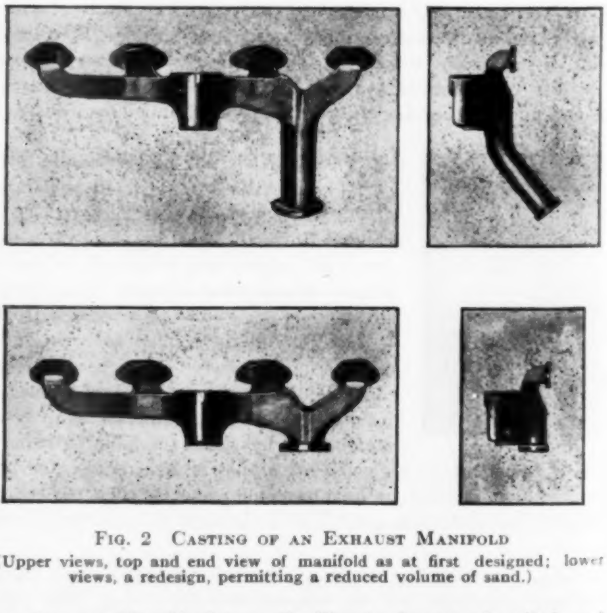

Fig. 2 Casting of an Exhaust Manifold
(Upper views, top and end view of manifold as at first designed; lower views, a redesign, permitting a reduced volume of sand.)

The principle of flask bulk should be borne in mind with every type of casting. Sometimes a projecting boss or point may require 2 in. or so additional flask, placing molding of the casting just outside the capabilities of a lighter and faster machine, thus increasing the cost burden of the final product.

IMPORTANCE OF WALL THICKNESS

Wall thickness is the haven for the designer who wishes to reduce weight; yet actually this is the one place where weight cannot be reduced unless the walls prior to change were thicker than good practice would dictate. Light walls breed every known type of foundry loss, and during the process of manufacturing quite often give trouble in handling. Further than this, light walls are the source of resonance under operation.

It has been said many times that "the more cast iron, the less brains," but it is obviously better to have 110 lb. of cast iron at 4 cents a pound than to have 100 lb. of cast iron at 5 cents a pound. The 10 lb. eliminated might be something for the engineer to be proud of, but it is nothing that the engineer can offer his customers with any degree of assurance that the customer will obtain more result for the money he is expending.

If weight is to be eliminated, it must be done by shortening walls and eliminating parts; in other words, by designing less weight into the product, using care that structure has not been impaired and that the machine shop has been interfered with in no way.

There is no casting in use today in any industry wherein walls have a more important function than in the cylinder and crankcase unit casting as used in the automotive industry. In this instance the walls are not used to cover space, but have the most important structural functions, and these walls must be thick enough to answer the purpose. They must be thick enough to be easily cast and thick enough to stand for rough handling of cleaning and fabricating.

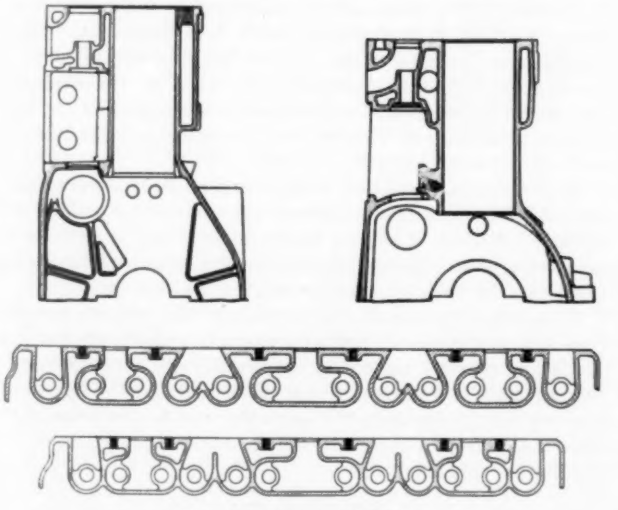

FIG. 3 CYLINDER BLOCKS
(Transverse sections at top—at left, design with little cooperation from the foundry; at right, design with fullest cooperation. Longitudinal sections at bottom—upper, good design; lower, poor design.)

CLEAN CASTINGS

The designer seldom feels that it is within his scope to aid with the matter of producing clean castings. He feels that this is the business of the foundry, and if the castings are not clean the foundry is at fault. There is no doubt that good foundry practice goes far in insuring clean castings; nevertheless, this ideal could be more easily attained if the designer would cooperate with the foundry in this matter. The foundry gives considerable thought in the method of molding to enhance this point.

Dirt has a tendency to float upward with the iron; therefore the most important part of the cylinder block is usually at the bottom of the mold when the casting is being poured. The dirt rises to an unimportant surface which is later machined, and the difficulty from dirt is more or less mitigated.

However, if core walls are very thin and small projections of sand are permitted to exist, there is always the possibility that such spots will easily wash away and the sand find a resting place in a critical location, causing the costliest kind of scrap. A cylinder block, for instance, might be almost completely machined before it is found that one of the bore walls is loaded with sand.

In considering sand cores and green-sand impressions, one should visualize a torrent of boiling iron rushing through this mold, which torrent has weight and therefore pressure, sufficient to break off corners and projections. One should consider where cores are thin and that there is molten iron on both sides of this thin piece of sand, and therefore there is a possibility of burning through. This may come in a spot not easily detected until the casting has been manufactured and delivered into the hands of a customer, who will be constantly annoyed by the recurrence of, perhaps, heating troubles that will baffle even a willing service station.

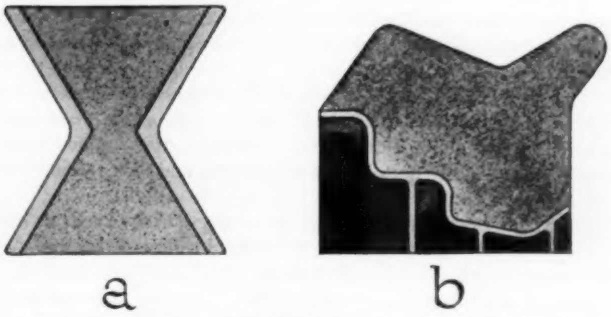

FIG. 4 UNSUPPORTED DESIGN THAT CANNOT BE MOVED
(*a*, hour-glass shape, with narrow neck that cannot support the upper body of sand; *b*, design with no supporting surface.)

Fig. 3 shows various critical sections of cylinder blocks. Good and poor designs are indicated in the longitudinal sections. Generosity at this location means the difference between good and poor operating engines and trouble or no trouble in the foundry. The transverse sections through the blocks indicate one design made with very little collaboration on the part of the foundry and the other design made with the fullest collaboration.

The cylinder block as indicated with the generous water space is still being used and is being produced in large quantities, whereas the other block has gone the way of many faulty designs and is no longer being produced.

Small water-jacket core holes may not appear small on the design, but when the designer considers the fact that supporting wires may be at the center of these small cores, then he will realize that his space is indeed small. The fundamental to bear in mind is that the size of the core must always be such as to insure the designer that space will always exist.

DRIERS FOR THE SAND

In the core making itself, the designer must be made to realize that a dry-sand core, though in itself capable of sustaining its own weight without bending, is baked into this condition, and before baking it is a loose, soft, oily pile of sand. Core-sand mixtures are such as to permit almost unbelievable overhanging of the sand. Nevertheless, there are limits to which wet core sand can be piled up and carried from bench to oven.

An obviously impossible design to produce and carry around unsupported would be an hour glass, such as shown in Fig. 4 at *a*. Evidently the narrow center neck cannot support the upper body of sand. Further, it is clearly impossible to carry a pile of wet sand that has been squeezed into the shape as shown in Fig. 4 at *b*. This shape allows no supporting surface to permit carrying from bench to oven.

Automotive foundries are asked to make cores of this type every day, and do so. However, it requires additional equipment in the form of driers, which are usually castings with the necessary impressions that are left in the wet sand until it is baked and hard enough to permit withdrawal. Driers are a necessary evil and can never be considered otherwise, and wherever possible the efforts of the designer and the foundryman should be directed toward their elimination.

Mobility for change has become an industrial necessity. All large organizations have learned that it is necessary that they be able to follow the trend by changes as quickly and as readily as possible.

With a casting which is being produced in large quantities, necessitating a considerable number of driers, engineering changes become tremendously expensive, and even minor changes become important.

Recently a minor change was made in the water jacket of a cylinder head. The changes in the pattern equipment were simple enough and not expensive. However, this simple change scrapped the driers used in this equipment, and this necessitated an expense of $25,000.

In the automotive industry the water-jacket cores in the cylinder block represent the most fruitful source of trouble and expense. Here one may find driers in abundance, small core sections, and every known evil of the foundry. This condition applies more particularly to the L-head type of cylinder block.

The overhead-valve water-jacket core is simple, and, as is the case with the present Chevrolet six-cylinder engine, with the proper design-foundry cooperation, can be made in one piece, thus positively eliminating fins from within the block casting.

Fins Are Troublesome

One of the most troublesome by-products of the foundry is the fin, particularly when these fins locate themselves in critical water passages, obstructing circulation. There is very little that a designer can do to obviate this nuisance, with the exception of providing ample space so that fins may not bridge across.

For the elimination of this evil the designer must rely mostly upon the skill and knowledge of his foundry associates. Fins usually occur where two dry-sand cores are pasted together, the iron penetrating the split, burning away the supposedly adequate seal. This is one point in foundry development that has been slow, and a point that is worthy of more serious consideration. Improvement in sealing of split cores would undoubtedly prolong engine-valve life.

Of course, the root of this evil is inadequate core or water space brought about mainly because engine length is quite often determined by front-office executives who know little of engine fundamentals, yet do not hesitate to state how long a given engine must be. The engineer endeavors to fit his engine length to the space allotted. Wheelbase having been placed upon a sacred pedestal and body space being a sales argument, there appears to be nothing to do but to crowd the engine.

Obviously, the only possible space within the engine itself that can be absorbed is core space, particularly since on the drawing board this core space is just space. The engineer quite often overlooks the fact that this space must first be sand and later water.

It has been said by an eminent radiator engineer that "One inch of wheelbase would cure most of our engine troubles, including the troubles of manufacturing and the foundry," and this is undoubtedly true. Wheelbase during the last three years, and even to the present, is being increased 3 to 4 inches at a time, for appearance or for competitive reasons. Yet there are few executives who would not condemn the engineer who had the temerity to request an inch more of wheelbase to permit the building of a better engine through the provision of an improved core space. It is certain that when executives realize that the fundamentals of engine success are irrevocably tied up with adequate core space, new progress will be made with foundry problems.

Overcoming Engineering Impatience

To obtain the fullest benefits of design-foundry cooperation, it is first necessary to overcome engineering impatience. Too often an engineer who has given every possible consideration to his foundry problems in his design considers his job done. He immediately hounds the foundry for sample castings overnight. These castings are quickly machined and converted into an experimental unit for development work.

This is a great mistake. First, the foundryman has not been given sufficient time to review the feasibility of the suggestions that they may wish to have incorporated in the casting. Further, they have had no opportunity to check even near-production methods for obtaining the casting. If the casting incorporates practice that is new to the particular foundry, they have not gained any experience that may serve at a later date when they are called on to determine how the product can be profitably produced.

It is important that the foundry be given opportunity to make a sufficient number of castings. This permits them to determine its peculiarities, so that castings submitted to the experimental department at least represent their best judgment. These castings should not be machined, but should be cut up, and the sections very carefully analyzed. In this way the engineers can determine whether wall thicknesses are as desired in their various locations and whether the core space and the internal contours are what they really expected.

Quite often such analysis brings out the fact that cores have shifted. This may be due to methods adopted for making the castings. Perhaps it may be due to faulty design of patterns or core boxes, or an important foundry fundamental may have been overlooked by both the engineer and the foundryman.

A careful going-over of such castings by the engineer and the foundryman together will bring the engineer nearer to the foundry ideals and create a respect in the foundryman's mind for engineering ideals. When a casting is cut up and found to be perfect in every way, a duplicate of this casting can be machined, with some assurance that the results obtained therefrom will represent the future product.

Cost Barnacles

In spite of the greatest care on the part of both engineer and foundryman, cost barnacles are bound to be found clinging to the ship of progress of castings. These cost barnacles are created by the things that have been overlooked and by changes made during the productive period. Unfortunately, a new crop of cost barnacles will find their way into the new product during its productive period. For that reason alone, the foundry is entitled to be rid of the old difficulties.

A most healthy practice for any engineering organization is to go through the foundry practice of the past product, isolating the various annoyances met with during the year and carefully eliminating them from the new product.

True progress can be measured only by tangible results that can be handed to the customer in the form of freedom from difficulty, increased life, quietness, or smoothness. We are too quick to congratulate ourselves for making a casting shorter or "getting by" with smaller core space, due to some clever trick on the part of the foundry. However, one cannot sell these things to a customer. It means nothing to a man who drives an automobile that his cylinder block is one-half inch shorter than it might be. It means even less if the core spaces are smaller than necessary. It is interesting to note, in the final analysis, that elimination of foundry difficulties means greater satisfaction to the ultimate owner.

The foundryman and the engineer are equally interested in the final results of their joint product. Alone their scope is limited; united their progress will be continuous.

Discussion

F. A. Melmoth.[2] While referring specifically to cast iron, the author's comments could with equal accuracy be applied to the steel branch of the founding industry—perhaps even with greater value, as higher temperatures and greater shrinkages being common to steel castings, the call for designer consideration and cooperation is even more imperative.

The author's statement, "Designers have been notorious for their neglect of foundry consideration," is unfortunately true, and in the opinion of the writer is due almost entirely to the fact that few designers have been given the opportunity to familiarize themselves with the fundamentals of foundry practice and do not possess a working knowledge of the solidification problems associated with the metal in which they are designing. In other words, the designer is likely to be merely one of form and shape, intent upon the evolution of a part to perform some mechanical requirement or to transmit so much power effectively, regardless of the fact that his conclusions are theoretical only until his design is available in a solid form in the metal required. The least result of such an attitude may be increased cost of production, owing to difficulty in the foundry. The most dangerous possible result is that the part, produced by an over-optimistic foundryman, may not be able to exhibit, even approximately, the theoretical strength values on which it was designed, owing to localized weaknesses arising out of unconsidered solidification factors due to design.

A sentence that is striking and apt is, "The designer does not always comprehend the foundry as an institution housing and using the sciences." Foundries as an industrial body are not good advertisers. They do not with sufficient insistence bring before the engineering industry as a whole the fact that the industry of metal casting is making a determined effort to utilize modern scientific knowledge applicable to its own product. Research to produce better steel castings and better malleable and gray-iron castings, not to speak of the multitude of non-ferrous products, is steadily going on within the industry. It is realized that to produce a representative casting of any design we must understand the constitution and behavior of the metal going to make it. Foundries are no longer dirty ramshackle places, content to be looked upon as more or less necessary evils, or the Cinderellas of the metal trades. They are literally becoming homes of practical applied science. But unless that applied science of the metals involved is also permitted to exert its influence on design, much of its effect is lost, and progress must be retarded.

The following quotation from the paper is also important, "Wall thickness is the haven for the designer who wishes to reduce weight." The development of alloy steels in the steel foundry is materially affected by this mental attitude. We know that at very small increased cost many steel castings can be made with greatly improved physical properties, by means of alloy additions allied to adequate heat treatment. Such steels are rarely more fluid than plain carbon steels, very often distinctly the opposite, at a given temperature. They are not less affected by liquid and solid shrinkage—again rather otherwise. Indiscriminate reduction of wall thickness, therefore, increases the hazard of production out of all reason, causes an increased likelihood of defects, and entirely prohibits the casting from manifesting that increased service value indicated by its higher cost per pound.

When castings are designed as castings, by men whose training involves an intimate familiarity with all that fundamentally goes to make a casting, they will quickly occupy their real place in the engineering industry, being produced more cheaply and without that degree of hazard which is the bane of the casting industry.

Finally, engineers might ponder the fact that were it not for design in the shape of the ingot, it would be impossible to produce a satisfactory steel forging. How much more so, then, does design affect the foundry, where an individual casting may and does contain a conglomerate of varying sections and contours, outside the control of the foundry, but defying every natural law of metal solidification if complete soundness is expected?

E. R. Glenn.[3] The author has covered the subject admirably for a plant having its own foundry. However, there are many plants requiring castings which are purchased from a jobbing factory. Our plant uses many carloads per year of light gray and malleable iron castings purchased from outside foundries.

We are also confronted with the problem of designing parts for the most efficient handling in the foundry. In many cases the engineering department can proceed on the basis of past performance where new designs are practically a duplication of other designs now in use. In other cases we submit either the drawing or an experimental pattern to the foundry for criticism. We find the foundries eager to assist us along this line.

One problem that we have periodically is in changing foundries and finding that our equipment does not meet with the approval of the new foundry, which may request changes in methods of gating, coring, etc. The fact that different foundries have different thoughts and requirements is one we cannot alter and must consider when we make a change.

Many of our castings are not machined and must fit to each other or to steel parts. We therefore must consider an allowance for rapping, swelling, core shrinkage, etc., requiring on the part of the engineering department a knowledge of foundry practice, inasmuch as it concerns what must be allowed in such cases.

In our plant the pattern shop is under the supervision of the engineering department. Close cooperation is possible in the making of the pattern as the engineering department and the foundry want it. Where such a connection between the engineering department and pattern shop is possible, it will be found to be most profitable.

At every opportunity the writer visits the foundries and spends some time with the foundry officials talking over foundry problems and checking on the latest developments in foundry practice. This not only keeps the department up to date on foundry practice, but makes for ideal cooperation of engineering department and foundry.

Mr. Schwartz.[4] The writer wishes to expand one of the ideas which the author presents in what appears to be in incomplete form. This is that "the price per pound of casting is the true measure of the extent of designer-foundry correlation."

It is the writer's conception that the object of correlation of design and production is not to produce a given article at the lowest pound price, but rather to produce an object of certain required properties for the least overall cost in the form in which it finds its final application. Thus the best cooperation in the manufacture of a cylinder block will have been secured when the cheapest cylinder block in its machined state and of acceptable quality is available for delivery.

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[2] Detroit Steel Casting Company, Detroit, Mich.

[3] In Charge of Engineering, S. L. Allen & Co. Inc., Philadelphia, Pa. Assoc-Mem. A.S.M.E.

[4] American Foundrymen's Association, Chicago, Ill.

In the design of any cast part there are in general two subjects to be considered, both equally important—the physical characteristics of the metal out of which the object is to be made and the amount and disposition of this metal in the object. It is quite generally true that, in changing from one cast metal to another or from one grade of a given cast metal to another, foundry conditions change sufficiently so that changes of design may be imperative.

The material is usually selected from the viewpoint of certain known physical properties. Frequently the suitability of a material for a given purpose involves the selection of several physical properties which may be incompatible; for example, high strength and high ductility or shock resistance in the same steel castings may be attainable only by unusual alloys or heat treatments. In the case of malleable iron high machinability is desired in order to minimize fabrication costs, and yet certain minima of strength or ductility may be requisite to make the castings adequate for their intended use. Many metallurgists hold that machinability and strength are contradictory and that in such cases a suitable compromise must be made. Instances exist where for foundry reasons a sufficient decrease in cross-section to take advantage of an increased strength for a changed material is impracticable, and it may therefore happen that the substitution of a stronger material is of no merit since it involves no corresponding decrease in weight.

Again there are instances in which the substitution of a more expensive light metal for a denser cheap metal is practicable under circumstances where the accompanying reduction in strength is of no consequence or, as may perhaps be less generally true, the light metal will produce an equally strong object of a lessened weight sufficient to make up the difference in cost.

Apparently accurate data have been shown the writer by others tending to prove that of two grades of the same material the one of the stronger ultimate strength may not produce the stronger casting; this under conditions where the use of the stronger material involves increased foundry faults.

In conclusion the writer would urge that consultation with the foundry should always be had before the design has approached completion. It quite frequently happens that after the designer has determined upon details in themselves unimportant, it proves impossible to change these details on account of tool conditions or on account of assembly with other parts. If the foundry engineer be given an opportunity to advise regarding the design of a casting while these unimportant details are not yet fixed by such considerations as these, he will be in a much better condition to decrease the ultimate cost of the part in question.

Max Sklovsky.[5] Our experience has indicated that it is difficult to obtain constructive criticisms from operating foremen on matters of design prior to the completion of the design. Usually it has been found that the foremen hesitate to offer opinions, as they are uncertain as to whether they fully understand the designs, even if they are certain of the difficulties that may be experienced in production of the specifically designed patterns. Also there has been observed an unwillingness to take the responsibility for criticisms and an unwillingness to offer suggestions for corrections that would imply a final responsibility for manufacturing methods. Consulting on the part of designers on this basis has not therefore been totally satisfactory.

Another plan which was adopted a few years ago has proved highly satisfactory—namely, the plan of assigning to the designing department a man from the production department to act as consultant to the designers. Such a man, having in mind the available methods and knowing the specific difficulties, constantly reviews designs and advises of difficulties that may be expected in production in carrying out specific designs. The individual so assigned is responsible directly to the production manager and is kept on the payroll of the production department. This plan has worked out with eminent success and has removed considerable friction and the bulk of the difficulties that had been encountered.

It has been found that when such an individual is part of the designing department and on the payroll of that department, he is likely to be shifted to work, when emergency arises, other than that of consultant as to manufacturing methods. With such an individual acting solely as a representative of the production department, there is no object to utilize him to serve otherwise in the designing department and remove him from the chief function for which he has been appointed.

J. H. Lansing.[6] The author has made a valuable contribution to the correlating of casting design and foundry practice. His illustrations are of decided interest and well develop the points made.

Of comparable interest to engineers in the automotive and many other industries is the correlation of casting design and foundry practice in the case of malleable-iron castings. The writer will venture to outline some considerations involved in malleable-iron-casting design.

The writer does not wish to stress the problems of any individual foundry, so will not therefore consider this subject from the standpoint of foundry equipment, available core capacity, etc., but rather from the standpoint of producing the most satisfactory castings in the most economical manner, assuming, as the great majority of designers must, that the customary foundry equipment and facilities will be available in the foundry which is to produce their malleable-iron castings.

He wishes also to caution against unqualified application of the statement that "the price per pound of a casting is the true measure of the extent of designer-foundry correlation." Although in many cases this is true, there are also many instances where, by designer-foundry correlation, the weight of a casting may be so decreased that a higher price per pound is involved, at the same time decreasing the cost per casting or part and increasing its adaptability for the required purpose.

To follow the application of some of the recommendations which will be outlined, one must appreciate some few of the characteristics of the subject material. Malleable iron, in the preliminary white-iron form, may be cast in intricate shapes and within such close limits as to reduce machining to a minimum. It is nevertheless subject to contraction strains, which are harmful only when sufficiently severe to cause the casting to crack in the hard state, and therefore to be scrapped in the foundry. The more severe strains may be eliminated by proper design, and the less severe ones, as stated, are of no importance, due to the fact that, in the normal process of manufacture, the castings are heated to a high temperature, held there for a sufficient period of time, and slowly cooled, entirely eliminating all casting strains and producing a refined grain structure. The resultant malleable-iron castings are composed entirely of ferrite containing nodules of temper carbon, and therefore machine with the greatest ease of any ferrous product. Good-quality malleable castings have an average elongation of approximately 20 per cent, yield point of 36,000 lb., and utimate strength of 54,000 lb. per sq. in.

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[5] Chief Engineer, Deere & Co., Moline, Ill. Mem. A.S.M.E.

[6] Works Manager, Danville Malleable Iron Division, Allith-Prouty Company, Danville, Ill.

In connection with some work for the American Foundrymen's Association and the Malleable Iron Research Institute, the writer has been working on the revision of "Recommendations to Designers of Malleable Iron Castings." Members of the former organization originated the rough primary draft of these recommendations, some having already appeared in the symposium presented at the A.F.A. and A.S.T.M. joint meeting in June, and several members of the Malleable Iron Research Institute have collaborated in the revision and clarification. Although their present form may not necessarily be considered final, they may well be outlined at this time to indicate factors worthy of consideration in the designing of malleable castings. The present draft is as follows:

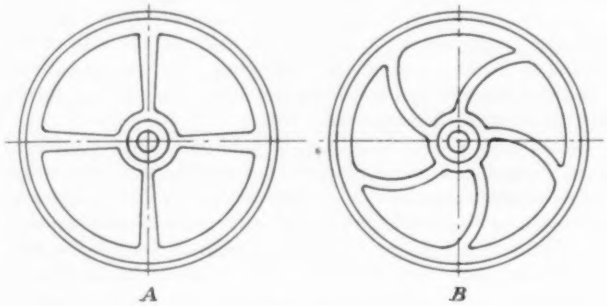

FIG. 5

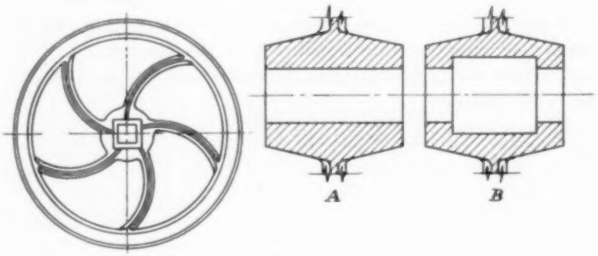

FIG. 6

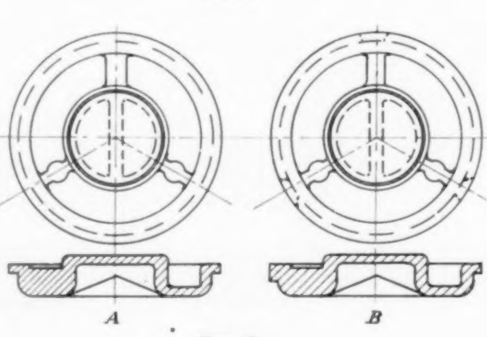

FIG. 7

1 To avoid unnecessary expense and delay, submit tentative designs to your malleable foundry for suggestions as to possible changes which, while not affecting utility, will aid in producing better and less expensive casting.

2 Thickness of casting should be as near uniform as possible, avoiding abrupt changes in section. Where it is necessary to vary metal from light to heavy sections, a gradual taper of the joint makes for greater economy in production and for greater strength. If this is impracticable, ribs of sufficient thickness to facilitate feeding may be placed on the light sections so as to connect with and feed into the heavy section or boss.

3 Where heavy sections such as bosses or lugs are located on the interior, it is desirable to have heavy sections of metal leading to feeding source.

4 The location and size of ribs should be such that they can be fed, or, if connecting light and heavy sections, they should be of such thickness that they can be used to feed metal through into the heavy parts.

5 Provision should be made of suitable place for attaching gates and feeders to heavy sections.

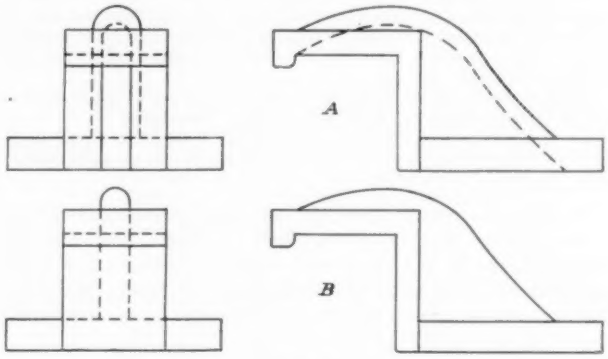

FIG. 8

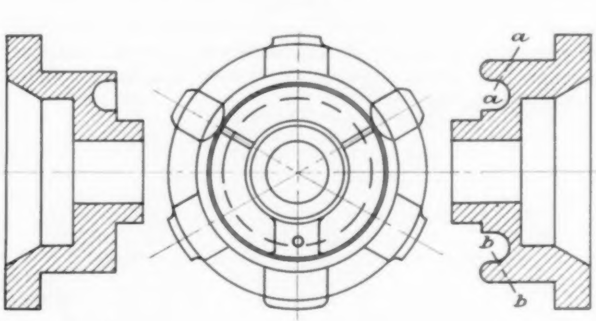

FIG. 9

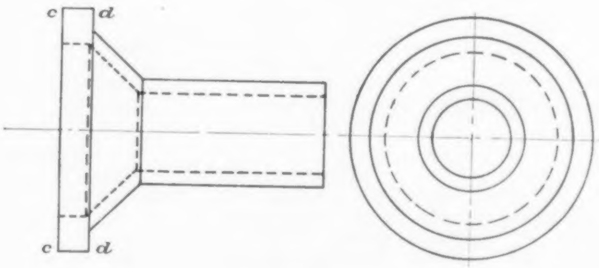

FIG. 10

6 Use as large fillets as possible and avoid the use of reentrant angles. The reasons for rounded corners and large fillets are (1) easier molding, (2) less scrap from shrinkage and cracks, and (3) less danger of cutting away of sand by flowing iron.

7 Where possible, avoid use of dry-sand cores, thus eliminating core-making expense and producing a cleaner casting.

8 Use ram-up cores in preference to set cores where practicable.

9 Parting lines should be as near even as possible.

10 Designs should provide ample draft for easy molding.

11 Try to avoid all deep pockets and small recesses that would complicate drawing the pattern or lifting the cope.

12 Sufficient finish allowance should be given when castings are to be machined, and the amount of such finish should be indicated. For small parts $^{1}/_{16}$-in. finish is satisfactory, with larger parts requiring a proportionately greater allowance. In determining the necessary amount of finish, factors to be considered are (a) size

of casting, (*b*) whether a heavy-sectioned compact casting or a large light-sectioned casting, the latter having of course greater tendency to warp, (*c*) whether part will be made in sufficiently large production quantities to justify the making of straightening dies, (*d*) the fact that many finishing operations on malleable castings can be performed in a coining press which practically eliminates finish allowance in the case of these particular operations.

13 Locating points should be held to one side of the parting to eliminate trouble in machining.

14 Dimensions which have no finish allowance and are to be held to very close limits should be considered as the proper place from which to start development of tooling fixtures.

15 When castings are to be machined, the blueprints sent to the patternmaker should always show machine-shop locating points. If these are not shown, the patternmaker will either not consider these points or may assume them to be other than those which will be used in machining the part.

16 For economical production, dimensions of castings should not be held to closer tolerance limits than is necessary.

17 If there is an opportunity to choose between a single large light-weight intricate casting and an assembly of two or more smaller ones, the latter is preferable.

To illustrate some of these points, several instances are given:

Item 1. A 20-in.-diameter, 70-lb. casting. (See Fig. 5.) This is a redesign of a shop truck wheel, showing in *B* a spoke arrangement eliminating the probability of cracks and high foundry loss inherent in the diametrically opposed straight spokes shown in *A*.

Item 2. A 15-in.-diameter, 11-lb. casting. (See Fig. 6.) Design *B* affords greater uniformity of section than design *A*, particularly when its section is considered in relation to the lighter section of spokes of the hand-brake wheel. This permits more uniform cooling of metal, thereby reducing possibility of shrinkage cracks.

Item 5. An 8-in.-diameter, 4-lb. casting. (See Fig. 7.) Part *B* illustrates the addition of a small amount of stock at points where heavy bosses join the thin flange, thus making it possible to attach a sufficiently heavy gate or feeder to produce a sound casting. This part is a spare-wheel carrier plate.

Item 7. A 5-lb. casting. (See Fig. 8.) Part *B* illustrates the elimination of the unnecessary core shown in the hatch-batten cleat *A*, also simplifying the molding.

Item 11. A 4-in.-diameter, $3^{3}/_{4}$-lb. casting. (See Fig. 9.) By removing the metal projecting into the cope above *a–a* and *b–b*, the lifting of the cope is greatly simplified or a ram-up core is eliminated. This is a differential-case casting.

Item 13. A $4^{1}/_{2}$-in.-diameter, 3-lb. casting. (See Fig. 10.) Locating points should if possible be held on one side of the parting to eliminate trouble in machining. In the case of this transmission-drive pinion-bearing retainer, the parting should be at *c–c* rather than at *d–d*.

In outlining factors that may be advantageously considered when designing parts to be cast in malleable iron, the writer wishes to discourage no one in the specification of this material by reason of any feeling that the part under consideration is one that will not adapt itself to a design in accordance with recommendations which have been listed. Malleable-iron foundrymen are as resourceful as any and will produce excellent castings for parts where it may be impossible to follow much of the recommended practice in design. The purpose has therefore been to cover certain factors which, when possible for the designer to consider them, will aid in the production of the most satisfactory castings from the standpoint of quality, service, and economy.

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Yokohama.......Library of Yokohama

Mexico
Mexico City......Asociacion de Ingenieros y Arquitectos de Mexico
Library of the Escuela de Ingenieros Mecanicos y Electricistas

Norway
Oslo.............Universitetsbibliothek den Polytekniska Forening

Poland
Warsaw..........Bibljoteka Publicazna

Porto Rico
Mayaguez........University of Porto Rico

Portugal
Lisbon...........Institute Superior Technico

Roumania
Bucharest........Scoala Polytechnica din Bucharest

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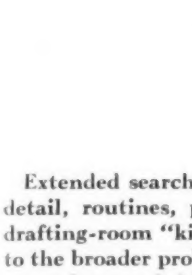
Turkey
Constantinople...Robert College

MSP-54-9

Machine-Design Management

By J. L. ALDEN,[1] CHICAGO, ILL.

Extended search reveals many papers on drawing-room detail, routines, printed forms used in drafting rooms, drafting-room "kinks," etc., but almost nothing relating to the broader problems of management which the design supervisor and his superiors must face. The present paper is a discussion of design-management practices, and is an attempt to record such of these practices as concern the Hawthorne Machine-Design Division of the Western Electric Company, Inc. The author's views are influenced largely by the past and present problems confronting him. Many of these, doubtless, are peculiar to the Western Electric Company.



THE TERM "management" implies control of expenditures, personnel, and technical practices, and, as used in this paper, it must be interpreted in the light of the ultimate purpose of machine-design organizations: namely, to design the best possible machine for the least possible design cost. The "best possible machine" is one in which that economic balance is reached between first cost, operating cost, maintenance cost, etc., which results in the lowest overall cost of the manufactured product consistent with the required quality level of that product. Proper management is the brand of control which achieves these things by orderly constructive thinking and planning.

How can such a brand of management be displayed in connection with machine-design work? To answer this question, management requirements of a design organization will be discussed.

There is no more important duty of management than the selection of capable supervisory personnel. Any machine-design management system, no matter how well chosen, will not be successful unless the men administering it have the proper managerial and technical qualifications. While the higher ranking executives lay down the broad policies of control, the execution of these policies is in the hands of the immediate supervisors, who, if poorly selected, can nullify the best efforts of their superiors. A large part of the successful administration of a sizable machine-design organization depends upon the character of the engineering executive in charge. In the Western Electric Company, this man is ranked as a development engineer. The payroll under his control is as large as the total payroll of many substantial businesses. The annual construction cost of machinery built, in normal times, from his designs is several millions of dollars. Therefore it is obvious that the first requirement for such a division chief is that he be an orderly thinker with sound business sense. Engineering ability and experience are necessary, but it is absolutely essential that he have the ability satisfactorily to take care of the business phases of his job. The division chief who is too close to detail, who is more interested in the size of the shaft than in the relation between payroll and output, lacks the necessary management viewpoint. Since this man must execute the management's policies it is essential that he think in terms of management rather than in terms of machinery and design. With the proper viewpoint at the head of the design organization, changes in policies and methods or even drastic reorganizations are effected with the minimum of trouble. The general supervisor of a large group must necessarily guide his organization by the use of reports and statistical data. It is not possible for him to be in close touch with detail or personally to judge the work of individual designers. He must make decisions based on the reports of others and must evaluate these reports in the light of his knowledge of the individuals reporting. He must, therefore, be continually checking, cross-checking, questioning, and employing that sense of skepticism which is necessary for all supervisors remote from the details of their jobs.

The first-line supervisors, department chiefs, and squad leaders must be skilled designers and engineers, and must be well equipped to lead, direct, train, and discipline their men. The source of supply from which design-department heads are drawn is ordinarily the design organization itself. However, the author believes that, before promotion in the design division, a candidate should be transferred to other development work for a period of training. It is well to have, as a design supervisor, a man who at some time or other has been a customer of the design division and is thus able to look at his organization with less prejudice and self-satisfaction, and is less likely to overlook and excuse faults in his group.

Work should flow through the design division with the minimum of delay or expense caused by lost motion, "red tape," divided authority, personnel peculiarities, etc. This means that the groups handling the various phases of the work must be so set up and supervised that there will be little or no overlapping of function, complete responsibility of each group for its work, and harmonious relations insisted upon and maintained throughout the organization.

Supervisors should not be forced to neglect important phases of their work on account of having to handle too many people. Responsibility for each phase of the work must be clearly centered in some individual. Office routines should be as few and as simple as possible without sacrifice of financial control, work scheduling, and necessary supervisory checks. It must not be forgotten that the primary purpose of a machine-design division is to design machinery and not to support an elaborate and cumbersome clerical and accounting system. The control routine put into effect should assist, at least, in evaluating the kind of a job the individual designers, draftsmen, engineers, and clerical people are doing, as well as develop the overall efficiency of the organization.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The foregoing statements are axiomatic and apply to all organizations, but the specific methods adopted must, of course, be selected to suit the nature of the work. For instance, some design organizations may find it desirable to maintain a separate group to handle all detailing, while others may keep the designer on the job until it has been completely detailed. As a matter of fact, both methods are used at Hawthorne. The system should never become so firmly established that flexibility is lost or that changes are difficult to effect.

The control of the technical phases of machine designing must insure that new designs are based upon correct engineering fundamentals, and that the machinery built therefrom will be reasonable in first cost and economical to maintain. Fads and prejudices must be avoided as far as possible, yet the designers' initiative and ingenuity must not be stifled. Advantage should be taken of contemporaneous design developments in the same or allied fields, and when a satisfactory mechanism or unit can be purchased at a reasonable price it should be bought rather than designed. A comprehensive system of design and drawing standards and suggested practices should be compiled and maintained, so that similar jobs will be done at different times in the same way. The accumulated experience of the office will thus be readily available to younger designers, and the man in the shop will always see like information presented upon different drawings in the same way.

The financial control of any machine-design organization should be both comprehensive and definite. A machine-design group may be managed in such a way that good equipment, economical to build and operate, is produced with a reasonable design cost; or it may be managed in such a way that not only are the design costs high, but, what is far more serious, the costs of equipment built in accordance with the designs furnished by such organizations are higher than they should be, and costly and embarrassing delays in equipment deliveries are sometimes traceable to work done in the drawing rooms.

It is essential, therefore, that some method be provided whereby the operating efficiency of the entire organization can be gaged. If there is not too much variation in the types of equipment designed, the work may be divided into relatively small output units such as the drawing sheet or the detail, and a careful record maintained of the average cost per unit over an appreciable period of time. This provides an efficiency measuring stick, which, while not entirely adequate, is perfectly definite, is independent of human judgment, and is not easily tampered with. It probably is more satisfactory, for the set of conditions outlined, than systems depending upon an arbitrary evaluation, by some individual or group, of the work done.

In addition to this, it is essential that the cost of each individual design job be accurately known. These costs and supporting data should be so maintained that they are readily available to facilitate the preparation of reasonably accurate design-cost estimates. Such cost estimates, supplemented by the necessity for securing proper approvals before money is expended for machine-design work, provide partial insurance against the designing and building of "sour jobs;" in other words, equipment which has no sound economic foundation is sometimes complicated and expensive, and should never be built. Detailed information should also be available at all times in connection with the costs of routine and supervision, the costs of those phases of engineering development work which are particularly related to design; and finally, the cost of drawing changes and changes to manufactured equipment caused by drawing inaccuracies should be definitely known.

The different factors mentioned can be integrated to provide a composite picture of the financial side of the organization which, embodied in suitable reports, will convey to the management a definite idea concerning the kind of job the design organization is doing.

The proper solution of the many personnel problems arising in connection with the management of a design organization constitutes no small part of the duties of the design supervisor. Adequate personnel records should at all times be maintained for the use of the supervisors, and so that the management, when endeavoring to fill a specific position requiring certain qualifications, may quickly find qualified prospects from the information available in the personnel file. Before hiring new people the design supervisor should carefully consider the amount of design work on hand and the amount which will become available within a reasonable period. This information should influence the policy followed when new people are hired, as it will help to avoid the expensive practice of hiring new people, breaking them into their new duties, and then laying them off in a few months because of lack of work. Very complete and carefully compiled work-load charts, work schedules, and other data should be maintained at all times so that approaching periods of reduced design activity may be detected and suitable adjustments made. Such records are also essential to enable the force to be increased at the beginning of rush periods before the design organization becomes so loaded with work that extensive overtime programs, etc., are necessary.

It is, perhaps, easier for a design force to become stagnant and ingrown than for any other group of technical men of equal ability. Broadening contacts are often absent, habits of thought become fixed, and an attitude of self-sufficiency is developed which is a bar to the progress not only of the organization but of the individual as well. It is a responsibility of management to avoid the growth of such a situation. Men for all technical jobs must be carefully selected and must be given opportunity for training. Training, both technical and non-technical, is doubly necessary for designers and draftsmen. Their progress must be watched, and selected individuals must be given opportunities to develop and to display such ability as they may have along other than design lines. Latent ability of this nature is difficult to recognize in a drafting organization unless the management makes a deliberate effort to seek it out. In the main, the work is not spectacular and the individual has not the chance to display his talents to supervisors with desirable vacancies to fill in other divisions. It is because so many employers of draftsmen have failed to recognize their responsibility to the men on the board that drafting has acquired the name of being a "blind alley" job. Because drafting has this reputation, the best of the young engineering graduates of the type the drafting room needs most are reluctant to enter this field. The effect of this reluctance is twofold. First, it deprives the design organization of valuable men, and, second, the men who avoid this field miss a type of training which they cannot afford to be without.

In order to attract the highest type of personnel the job must be recognized as being entitled to the same respect as other engineering work commands. It must offer definite opportunity for advancement. Alert and energetic designers who give promise of supervisory ability should be transferred to other divisions for upgrading and further opportunity. As soon as it becomes recognized that good men are adequately rewarded it is no longer difficult to get young engineers into the drafting room. After these men are recruited it is necessary that a definite effort be made to train them. The number of good machine designers now working on the board is wofully small. Many can design mechanisms which will function properly, and many are clever at designing intricate mechanical movements. However, the designer who can, with only nominal supervision, lay out a machine which is good enough for the job, but not too good, which can be easily built and easily maintained, which is safe and easy to operate and control, is a rare individual indeed, and most of the men who can do these things successfully are soon advanced to supervisory positions. Therefore, there is the necessity of constantly developing new men, and this must be recognized as being one of the most important jobs of a design supervisor. It is not felt that a formal or organized course of instruction is either necessary or desirable. It is believed that the best way to develop engineers is by personal attention, advice, criticism, and correction. The division chief must pay personal attention to this work because it will be found that many department chiefs are reluctant to spend the necessary time on instruction of their men. It is necessary that all department chiefs recognize that the building up of a group of energetic, self-reliant, capable designers relieves the supervisor of much of the tedious and disagreeable parts of his work.

It is the function of the management to determine how designers and draftsmen shall be given the required training in shop practices. It is not thought that a formal course is particularly effective. It will be found that plenty of shop contact will be reflected in a higher grade of design, particularly if that shop contact is acquired in correcting shop difficulties encountered during the construction of a machine. Whenever possible, the designer should be required to contact not only with the shop foreman, but with the machinist at the bench on all machining or erecting troubles. The draftsman should be encouraged to become personally acquainted with shop mechanics. Supervisors should be enjoined to make a definite effort to use shop difficulties as examples in the training of their people.

Management Methods Used by a Large Company

As the "proof of the pudding is in the eating," the operation of any set of theories or suggested practices can best be illustrated by actual example. A brief description will therefore be given of the machine-design organization located at the Hawthorne plant of the Western Electric Company, together with an outline of the control policies, systems, and routines used in directing and guiding its activities. It should be clearly understood that no attempt is being made to present this organization as an example of a perfectly controlled and perfectly functioning unit. There are admittedly faulty features in the management and operation of all industrial organizations, and the Hawthorne Machine-Design Division is no exception. However, the controlling and guiding policies used represent an attempt to secure efficiency in a design organization by the application of orderly thinking and sound business principles supplemented by common sense, and engineers and supervisors are requested to read this description with that thought in mind.

General Organization

The Engineer of Manufacture organization, of which the Hawthorne Machine-Design Division is a part, is responsible for all engineering activities conducted by the manufacturing department of the Western Electric Company. One of the most important of these activities, the development of equipment, processes, etc., to improve the quality or reduce the manufacturing cost of company products, is handled by the development branches of the Engineer of Manufacture. The skeleton chart, Fig. 1, shows the relation between these organizations and the Machine-Design Division. In order to avoid needless repetition, only part of the organizations have been shown on the chart.

Economic studies and development work done in connection with specific projects involving mechanical equipment are handled by specialized groups of development engineers, as shown on the chart. When one of these engineering groups has investigated a specific project sufficiently to decide that machine equipment of special design should be provided, a brief description is prepared outlining the machine requirements, operating features, etc. It is sometimes necessary to have preliminary designs worked up by the Machine-Design Division before a specification covering the final design can be written. The specification and a request for the necessary machine drawings are then forwarded to the Machine-Design Division. If doubt exists concerning the feasibility of certain machine constructions or operations, a rude model to demonstrate their performance is made and experimentally operated.

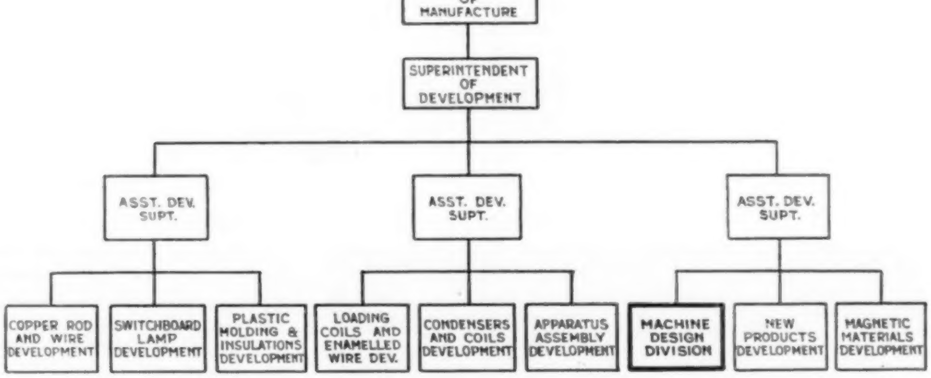

ENGINEER OF MANUFACTURE
ASST. ENG. OF MANUFACTURE
SUPERINTENDENT OF DEVELOPMENT
ASST. DEV. SUPT.
ASST. DEV. SUPT.
ASST. DEV. SUPT.
COPPER ROD AND WIRE DEVELOPMENT
SWITCHBOARD LAMP DEVELOPMENT
PLASTIC MOLDING & INSULATIONS DEVELOPMENT
LOADING COILS AND ENAMELLED WIRE DEV.
CONDENSERS AND COILS DEVELOPMENT
APPARATUS ASSEMBLY DEVELOPMENT
MACHINE DESIGN DIVISION
NEW PRODUCTS DEVELOPMENT
MAGNETIC MATERIALS DEVELOPMENT

Fig. 1 Chart Showing Relation Between Engineer of Manufacture and Machine-Design Division

The special equipment designed for Western Electric manufacturing operations is of a diversified character, ranging from small and intricate mechanisms such as automatic assembling machinery for small telephone apparatus piece parts, to heavy machinery such as hydraulic extrusion presses, telephone-cable stranding machines, copper-wire drawing machines, etc. Western Electric machinery is designed to operate under conditions ranging from ordinary to most adverse with the minimum of attention and maintenance. The great diversity of manufacturing operations performed, and the large number of kinds of products made in rather limited quantities, means that in relatively few cases does the number of machines of a given type exceed four to six. Since none of the machinery designed is for sale, and since most of it is built in the shops of the Western Electric Company, it will be recognized immediately that some of the problems with which the Hawthorne design organization is confronted are not parallel to those of other companies. It should be borne in mind, of course, that standard purchased equipment is utilized wherever possible.

The Hawthorne machine-design organization also handles considerable work related to machine designing, such as the preparation of specifications from which special machinery is constructed by outside firms, special engineering studies and laboratory work for other company organizations, the handling of all safety activities in connection with machine equipment coming under the jurisdiction of the Hawthorne Engineer of Manufacture, etc. This work, while interesting, is beyond the scope of the present paper, and will not be dealt with here.

Divisional Organization

Fig. 2 illustrates, in chart form, the set-up of the Hawthorne Machine-Design Division. (See also Fig. 3.) The division is composed, essentially, of four separate machine-design departments, a machine-design engineering and checking department, a safety investigation and coordination department, and a clerical and tracing department. Each one of the machine-design departments contains an average of 20 machine designers and draftsmen. The work in each machine-design department is in charge of a department head assisted by one or more squad leaders who are in immediate charge of the different design projects. To a certain extent, each design department specializes in certain classes of design work. This has proved to be a profitable practice, as design work in connection with essentially similar mechanisms is handled by people who are proficient in that particular kind of work. The designers, including the squad leader, generally stay with each design project until it is completed, doing their share of the detail work, as well as minor layouts, etc.

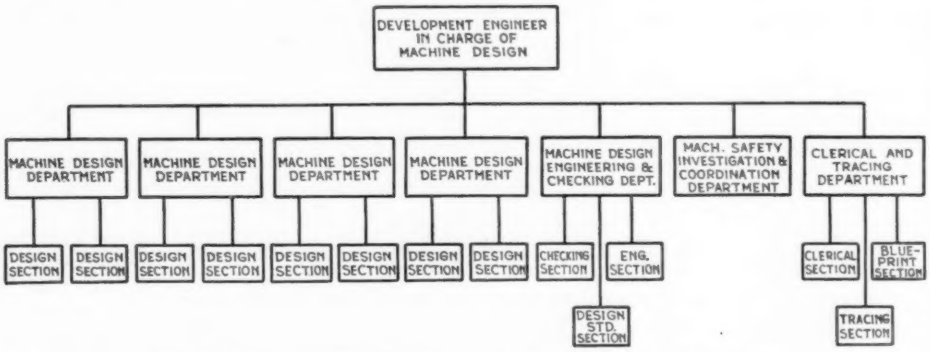

DEVELOPMENT ENGINEER IN CHARGE OF MACHINE DESIGN
MACHINE DESIGN DEPARTMENT
MACHINE DESIGN DEPARTMENT
MACHINE DESIGN DEPARTMENT
MACHINE DESIGN DEPARTMENT
MACHINE DESIGN ENGINEERING & CHECKING DEPT.
MACH. SAFETY INVESTIGATION & COORDINATION DEPARTMENT
CLERICAL AND TRACING DEPARTMENT
DESIGN SECTION
DESIGN SECTION
DESIGN SECTION
DESIGN SECTION
DESIGN SECTION
DESIGN SECTION
DESIGN SECTION
DESIGN SECTION
CHECKING SECTION
ENG. SECTION
DESIGN STD. SECTION
CLERICAL SECTION
BLUE-PRINT SECTION
TRACING SECTION

Fig. 2 Organization of Hawthorne Machine-Design Division

Fig. 3 Draftsmen at Work in the Machine-Design Division

Design and Clerical Routine

The design and clerical routine to be described is graphically shown in Fig. 4. When the Machine-Design Division receives a machinery specification and request for drawings from one of the development organizations, the development engineer in charge of machine design assigns the work to the design department best able to handle it. The design-department head analyzes the specification, and with the squad leader who is expected to carry on the design work, discusses the matter with the development engineer in charge of the project from whom he receives an economic picture of the job. An estimate of the drawing cost is next prepared by the design supervisor and the squad leader, and forwarded to the development engineer on the design estimate and authorization form shown in Fig. 5. The development engineer then obtains the required approvals for design expenditure and returns the approved form to the Machine-Design Division, so that design work may be started.

Upon receipt of the approved authorization an order number is assigned to the job, a record taken, and a folder is prepared by the clerical organization. This folder consists of a copy of the design authorization, a copy of the specification, any other correspondence pertaining to the job, and a time sheet used for cost-recording purposes, and upon which the job or order number is shown. A daily log, in which are recorded items of importance decided upon in discussions between the development engineer and members of the Machine-Design Division, is also included. The job folder is forwarded through the department head to the squad leader in immediate charge of the project as his authorization to start the design. As soon as the work has progressed sufficiently to permit a fairly accurate estimate of the time required to complete the drawings, the information is furnished to the clerical department, which submits the completion date to all interested organizations, and also uses the information for the compilation of work-load charts, etc., to be described later. As design work proceeds, consulting-engineering service is furnished by the engineering and checking department when required, as later described in connection with the technical control of design work. Close contact is maintained at all times with the development engineer responsible for the project in order to insure that floor-space requirements, operating conditions and requirements, etc., have been adequately taken care of.

Drawings of special machines to be built in appreciable numbers, or unusually intricate designs from which only one or two machines will be built, are sent to the checking section for examination. Drawings of the simpler machines are given a rather rough check by the designers, especially if the work is urgent. Checking routines will be outlined in a subsequent paragraph under "Technical Control of Design Work." Upon completion of the drawings, the development engineer in charge of the project, and the development engineer in charge of machine design approve all assembly drawings. The delivery form shown in Fig. 6 is prepared, on which is listed the job number, drawing number, sheet number, and issue number of all drawings that are to be sent to the Machine Ordering Department for manufacturing purposes. This form is approved by the design-department head and is forwarded with the drawings and job folder, to the blueprint-section order clerk.

Upon receipt of tracings and requests for blueprints, the order clerk originates a "record card" of organizations which are to be kept up to date on subsequent issues of blueprints. Next a requisition for blueprints is originated and sent to the blueprinting department with the tracings. Upon receipt of prints from the blueprinting department, the blueprint clerk checks the

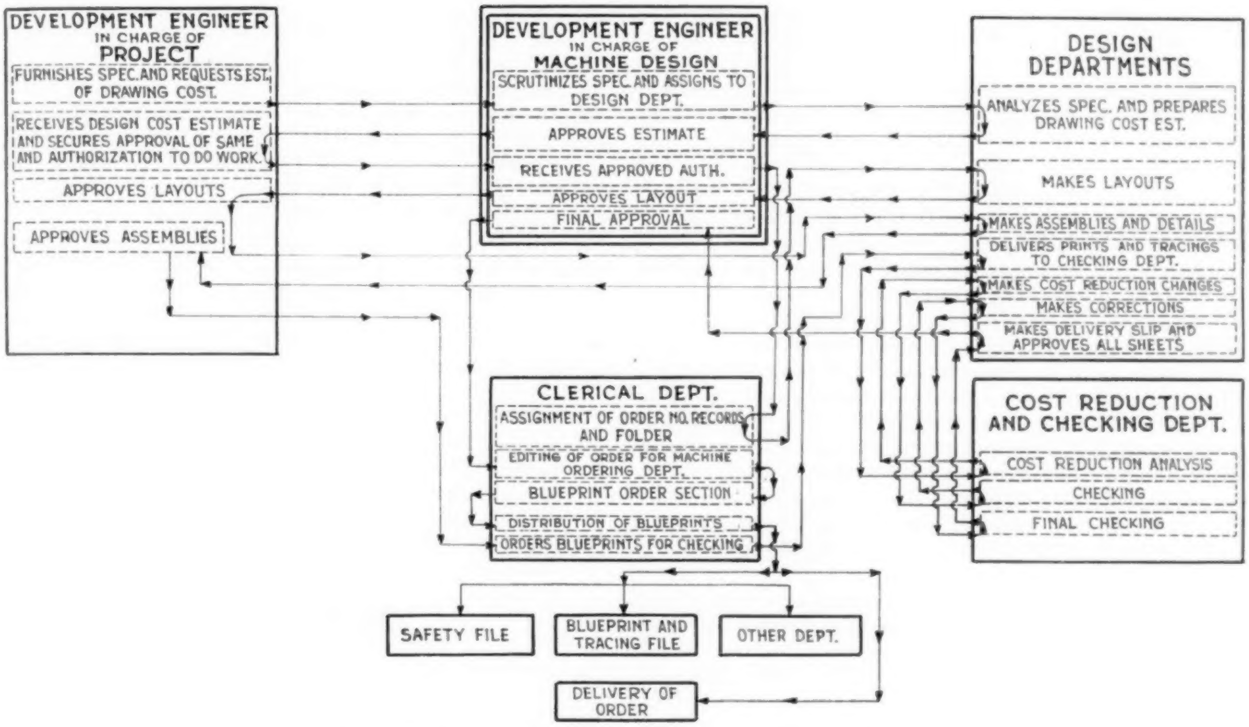
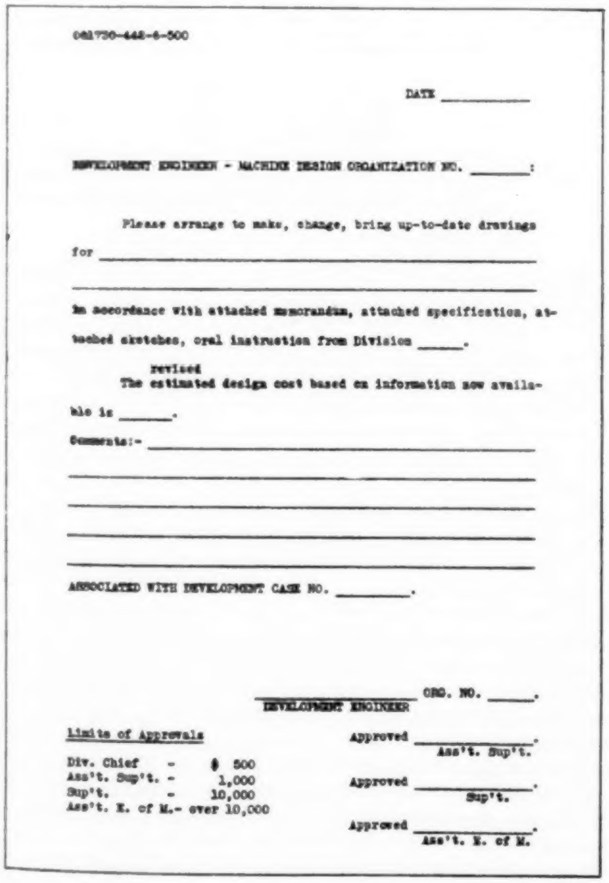

DEVELOPMENT ENGINEER IN CHARGE OF PROJECT
FURNISHES SPEC. AND REQUESTS EST. OF DRAWING COST.
RECEIVES DESIGN COST ESTIMATE AND SECURES APPROVAL OF SAME AND AUTHORIZATION TO DO WORK.
APPROVES LAYOUTS
APPROVES ASSEMBLIES
DEVELOPMENT ENGINEER IN CHARGE OF MACHINE DESIGN
SCRUTINIZES SPEC. AND ASSIGNS TO DESIGN DEPT.
APPROVES ESTIMATE
RECEIVES APPROVED AUTH.
APPROVES LAYOUT
FINAL APPROVAL
DESIGN DEPARTMENTS
ANALYZES SPEC. AND PREPARES DRAWING COST EST.
MAKES LAYOUTS
MAKES ASSEMBLIES AND DETAILS
DELIVERS PRINTS AND TRACINGS TO CHECKING DEPT.
MAKES COST REDUCTION CHANGES
MAKES CORRECTIONS
MAKES DELIVERY SLIP AND APPROVES ALL SHEETS
CLERICAL DEPT.
ASSIGNMENT OF ORDER NO. RECORDS AND FOLDER
EDITING OF ORDER FOR MACHINE ORDERING DEPT.
BLUEPRINT ORDER SECTION
DISTRIBUTION OF BLUEPRINTS
ORDERS BLUEPRINTS FOR CHECKING
COST REDUCTION AND CHECKING DEPT.
COST REDUCTION ANALYSIS
CHECKING
FINAL CHECKING
SAFETY FILE
BLUEPRINT AND TRACING FILE
OTHER DEPT.
DELIVERY OF ORDER

FIG. 4 DESIGN AND CLERICAL ROUTINE SHOWN GRAPHICALLY



061730-442-6-500

DATE ________

DEVELOPMENT ENGINEER - MACHINE DESIGN ORGANIZATION NO. ______:

Please arrange to make, change, bring up-to-date drawings for ____________________

In accordance with attached memorandum, attached specification, attached sketches, oral instruction from Division _____.

revised
The estimated design cost based on information now available is ______.

Comments:- ____________________

ASSOCIATED WITH DEVELOPMENT CASE NO. ________.

________ ORG. NO. _____.
DEVELOPMENT ENGINEER

Limits of Approvals

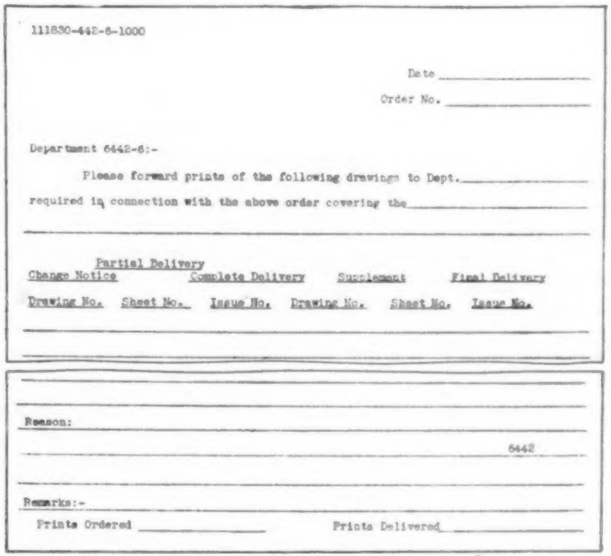
Div. Chief - $ 500
Ass't. Sup't. - 1,000
Sup't. - 10,000
Ass't. E. of M.- over 10,000

Approved ________. Ass't. Sup't.

Approved ________. Sup't.

Approved ________. Ass't. E. of M.

FIG. 5 DESIGN ESTIMATE AND AUTHORIZATION FORM



111830-442-6-1000

Date ________

Order No. ________

Department 6442-6:-

Please forward prints of the following drawings to Dept. ________ required in connection with the above order covering the ________

Partial Delivery
Change Notice Complete Delivery Supplement Final Delivery

Drawing No. Sheet No. Issue No. Drawing No. Sheet No. Issue No.

Reason:

6442

Remarks:-

Prints Ordered ________ Prints Delivered ________

FIG. 6 PRINT AND DRAWING DELIVERY FORM

prints to make sure that the correct quantity has been made. The prints and the original request as a delivery sheet with the drawing, sheet, and issue numbers listed thereon, are delivered to the proper organization by messenger. A duplicate copy of the delivery sheet is then filed as a record of delivery of prints.

TECHNICAL CONTROL OF DESIGN WORK

The technical phases of the machine designing done at Hawthorne are controlled by the design executive and supervisory force, assisted by the machine-design engineering and checking department. The engineering department is composed of a checking section, and engineering section containing mechanical engineers, electrical engineers, a section chief, and a standards engineer with the rank of section chief, reporting to the department head.

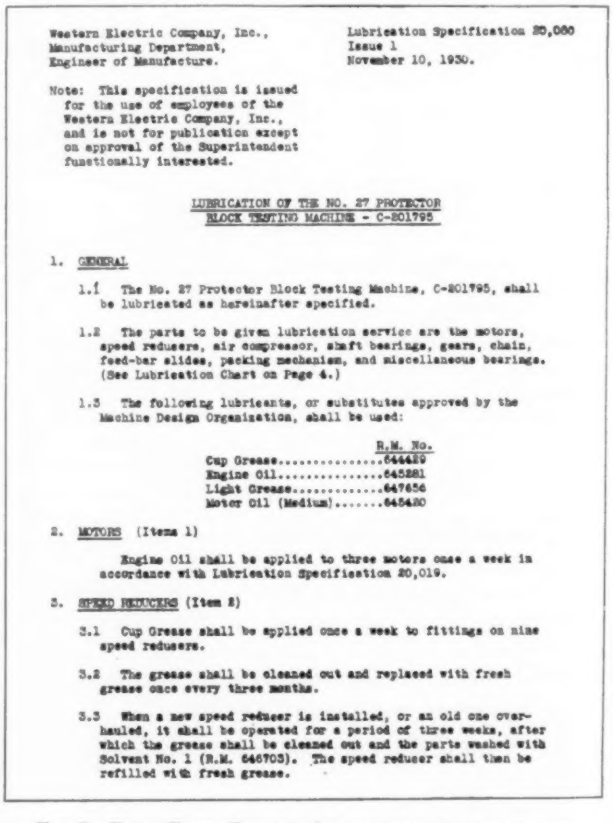
As checking routine is substantially similar in all design offices, Western Electric checking methods will not be dealt with to any extent. However, particular care is taken to insure that standard practices are carried out with respect to materials, the use of commercial units, electrical features, etc. Where economically justified, it is by all means desirable to have drawings examined by the checking organization, as the experience gained by the individual checker in handling a large volume of work from a considerable number of designers tends to develop in him a varied fund of information, orderly methods of thought and action, and balanced judgment to a greater extent than is possible on design work. Also, the advantages of an independent check in eliminating both dimensional inaccuracies and errors of technique and judgment are well known. Full cooperation is given the design departments so that the experience mentioned will be readily available and frequently applied through consultation while machine designs are still in the layout stage. By these means the checking section contributes quite largely to the quality of the work turned out by the division and to economies in the construction of special machinery.

It is obvious that careful work by the detailer will keep down to a minimum the number of errors on detail drawings, and consequently the work the checker must do and the number of mistakes allowed to slip through. The checker, when examining detail drawings, records the number of errors he finds. Based upon this information, a simple bar chart having a bar for each month showing the average "percentage of accuracy" for that month is maintained for each man in the division who turns out detail drawings. Planographed copies of the entire chart series with numbers substituted for the names of individuals are distributed to the designers and draftsmen periodically, and those who have an undue number of mistakes per sheet are interviewed personally. By this means a check is kept upon the operating efficiency of those members of the organization who turn out detail drawings, and more accurate work is done. This practice has resulted in a substantial reduction of both drawing and machine-building costs.

In an effort to reduce, by proper design, the cost of equipment built from Hawthorne designs, all drawings turned out by the design departments are carefully and critically examined by the head of the checking section in an endeavor to eliminate complicated or expensive constructions from the different designs. This inspection does not constitute either a functional or dimensional check, but is handled solely from a cost-reduction standpoint. The results of this work have been gratifying, as periodic investigations of the work done indicate that cheaper constructions are quite frequently recommended and accepted by the design supervisor.

Engineering analysis of new and untried machine designs is one of the most important functions of the engineering department, as such analyses furnish assurance to the management that the new machinery turned out is in substantial conformity to correct engineering principles, and that no element or mechanism has been used which can be determined to be defective by an analysis based on conditions known at the time the machine is designed. These analyses are sometimes quite extensive and embrace an independent check of the designers' estimate of the performance of electrical mechanisms, as well as the forces caused by acceleration and retardation, the loads upon cam rollers, Geneva mechanisms, gear and bearing reactions, etc. They also include kinematic studies of linkage systems, etc., to insure that the combination of elements shown upon the drawing will produce the required motions, timing, etc. Information is supplied to the design organizations concerning material properties, safe stresses, methods of processing and fabricating, sources of supply, commercial stocks available, correct specifications, etc.—in short, anything pertaining to materials of machine construction that will help to get the best possible material for a given purpose and predict its action in service. Considerable time is saved which would otherwise be spent by each designer in trying to secure this information from various sources, not to mention the lack of uniformity that would result from the latter practice.

The Engineering Section furnishes consulting service on lubricants and lubrication problems to the several machine-design organizations, analyzes machine designs from a lubrication standpoint while still on the drawing board, and writes lubrication specifications for all special machinery and some purchased machinery. These lubrication specifications are used by the operating and maintenance organizations, and are an important item in the keeping of machinery maintenance costs at the lowest possible figure. A page of a typical lubrication specification is shown in Fig. 7.



Western Electric Company, Inc.,
Manufacturing Department,
Engineer of Manufacture.

Lubrication Specification 20,000
Issue 1
November 10, 1930.

Note: This specification is issued
for the use of employees of the
Western Electric Company, Inc.,
and is not for publication except
on approval of the Superintendent
functionally interested.

LUBRICATION OF THE NO. 27 PROTECTOR
BLOCK TESTING MACHINE - C-201795

1. GENERAL

1.1 The No. 27 Protector Block Testing Machine, C-201795, shall be lubricated as hereinafter specified.

1.2 The parts to be given lubrication service are the motors, speed reducers, air compressor, shaft bearings, gears, chain, feed-bar slides, packing mechanism, and miscellaneous bearings. (See Lubrication Chart on Page 4.)

1.3 The following lubricants, or substitutes approved by the Machine Design Organization, shall be used:

	R.M. No.
Cup Grease	644429
Engine Oil	645281
Light Grease	647656
Motor Oil (Medium)	645420

2. MOTORS (Items 1)

Engine Oil shall be applied to three motors once a week in accordance with Lubrication Specification 20,019.

3. SPEED REDUCERS (Item 2)

3.1 Cup Grease shall be applied once a week to fittings on nine speed reducers.

3.2 The grease shall be cleaned out and replaced with fresh grease once every three months.

3.3 When a new speed reducer is installed, or an old one overhauled, it shall be operated for a period of three weeks, after which the grease shall be cleaned out and the parts washed with Solvent No. 1 (R.M. 646703). The speed reducer shall then be refilled with fresh grease.

FIG. 7 PAGE FROM TYPICAL LUBRICATION SPECIFICATION

The standards section takes care of work in connection with design and drafting standards and allied matters. The section chief functions as chairman of a standards committee which meets about once a month and discusses all suggestions received by as well as originated in the division, relative to changes and additions to drawing standards. Where changes seem desirable new or revised standard sheets are prepared under the supervision of the standards engineer, and submitted to the department heads and development engineer in charge of machine design for further action.

Any specialized line of manufacture requires certain classes of equipment which, through the process of development over a period of time, become characteristic of that line of manufacture and are particularly suited to the class of work they perform. Sketches and short descriptions of such characteristic Western Electric mechanisms are shown in the standards book. Their purpose is to acquaint machine-design employees with the work that has been done, and to furnish a reference to satisfactory mechanisms which may be either adopted in principle or used outright in new designs. By this means, at least a part of the accumulated experience of the organizations is made available to its newer people.

The standards book contains the usual information about drafting conventions, standard mechanisms, machine elements to be incorporated wherever possible, dimensional limit systems, etc.; it also incorporates information concerning welding practices, cost comparisons of various alternative machine constructions, electrical data, and other things useful to the designer.

The standards section handles all outside contacts related to commercial machine elements, materials, etc., maintains up-to-date catalog files, and handles all telephone calls requesting engineering information from outside parties. The information secured as a result of these interviews and telephone conversations is so filed that it is available for future reference, thereby saving much of the machine designers' time when information is needed concerning commercial units, etc.

The choice of the proper machine elements, materials, and mechanisms to incorporate in Western Electric special machine designs is based not only upon experience, but also upon research, which quite often involves fairly extensive laboratory tests which are conducted by members of the engineering section at the request and with the collaboration of the design supervisors. This is true not only of special machine elements and mechanisms such as distributors, transfer devices, cam-driven mechanisms, etc., but is also carried out with respect to commercial machine elements such as clutches, couplings, lubrication devices, speed reducers, etc. As a result of these tests, new machine elements are developed and information is obtained by means of which savings are effected, as well as a reduction of the shop troubles encountered when proving in new machines.

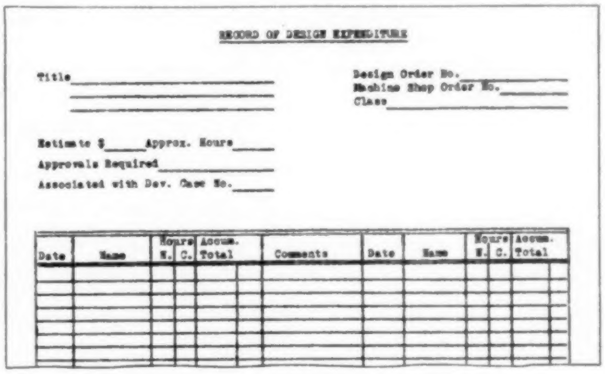
FINANCIAL AND CLERICAL CONTROL

Some of the more important general-accounting and budgetary practices related to the work of the Hawthorne machine-design division are outlined herewith.

Operating expenses of the design division are broken down into three classes; namely, productive expense, non-productive expense, and other direct expense. The machine-design division is classed as a productive-expense organization, and all of the designers, draftsmen, checkers, and tracers in the division are classed as productive-expense employees. Supervisory, engineering, and clerical people are classed as non-productive expense employees. Other direct expense is that expense incurred by the division for supplies, traveling, conducting experimental tests, work in the laboratory, etc.

A budget is provided for each of these three classes of expense, and preparations are made as early as September to determine the budgetary requirements for the following year. Estimates for proposed expenditures are based upon: A survey of the probable amount of design work which the development engineers will require during the following year; a survey of the cost of machines which it is planned to build during the same period, and a careful consideration of past performances. This information enables a reasonably accurate estimate to be made of the average number of designers and draftsmen that will be required by the machine-design division during the coming year. The size of the non-productive force is governed mainly by the size of the productive organization, and by the anticipated work load, etc. On the same basis of calculation, the budget required for other direct expense is determined.

The non-productive drafting expense and other direct expense is recovered through loadings applied against productive drafting labor. The designers, draftsmen, checkers, and tracers, as productive—expense employees, charge their time to definite job or order numbers. Included in the charges against the jobs are the salaries of the productive-expense employees, plus a loading rate, or overhead, which consists of the non-productive salary expense and the other direct expense, as well as a portion of the works service charges.



RECORD OF DESIGN EXPENDITURE

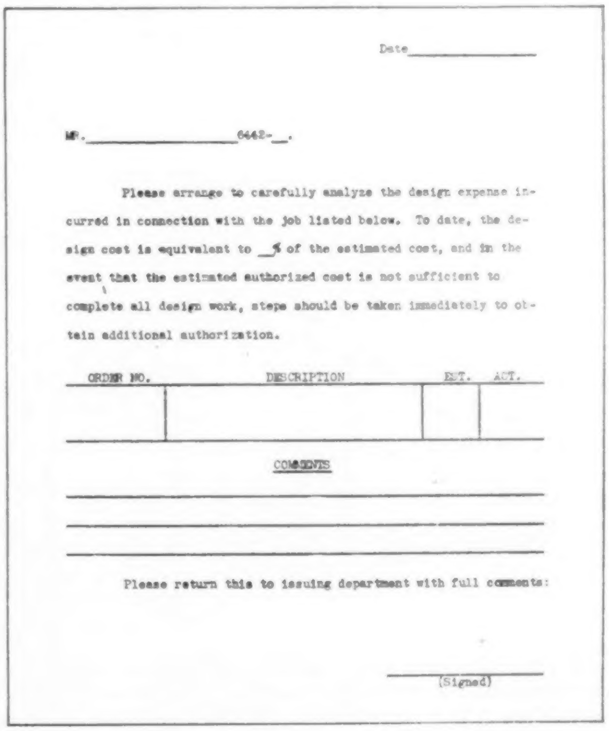
Title______________________

Design Order No.__________
Machine Shop Order No.________
Class______________

Estimate $_____ Approx. Hours_____
Approvals Required______________
Associated with Dev. Case No._____

Date	Name	Hours		Accum. Total		Comments	Date	Name	Hours		Accum. Total	
		W.	C.						W.	C.		

FIG. 8 SHEET FOR RECORDING DESIGN EXPENDITURES



Date______________

MR.________________6442-__.

Please arrange to carefully analyze the design expense incurred in connection with the job listed below. To date, the design cost is equivalent to __% of the estimated cost, and in the event that the estimated authorized cost is not sufficient to complete all design work, steps should be taken immediately to obtain additional authorization.

ORDER NO.	DESCRIPTION	EST.	ACT.

COMMENTS

Please return this to issuing department with full comments:

____
(Signed)

FIG. 9 NOTIFICATION FORM FOR CHECKING PROGRESS OF WORK

Financial and clerical control activities within the machine-design division, together with their related accounting practices, are handled mainly by the clerical and tracing department. A reasonably accurate idea of the operating efficiency of the organization is secured by detailed and periodic checks of the cost per unit of divisional output. The drawing sheet has been chosen as the design-output unit, and unit costs are kept comparable from year to year by periodic checks of the number of details shown per sheet. The cost of all design layout work, checking, etc. is included in these unit costs. This method of measuring efficiency works out rather well for an organization like the Hawthorne group where the engineering problems encountered over a period of time are quite similar. It has the added advantage of requiring the minimum of clerical and supervisory effort.

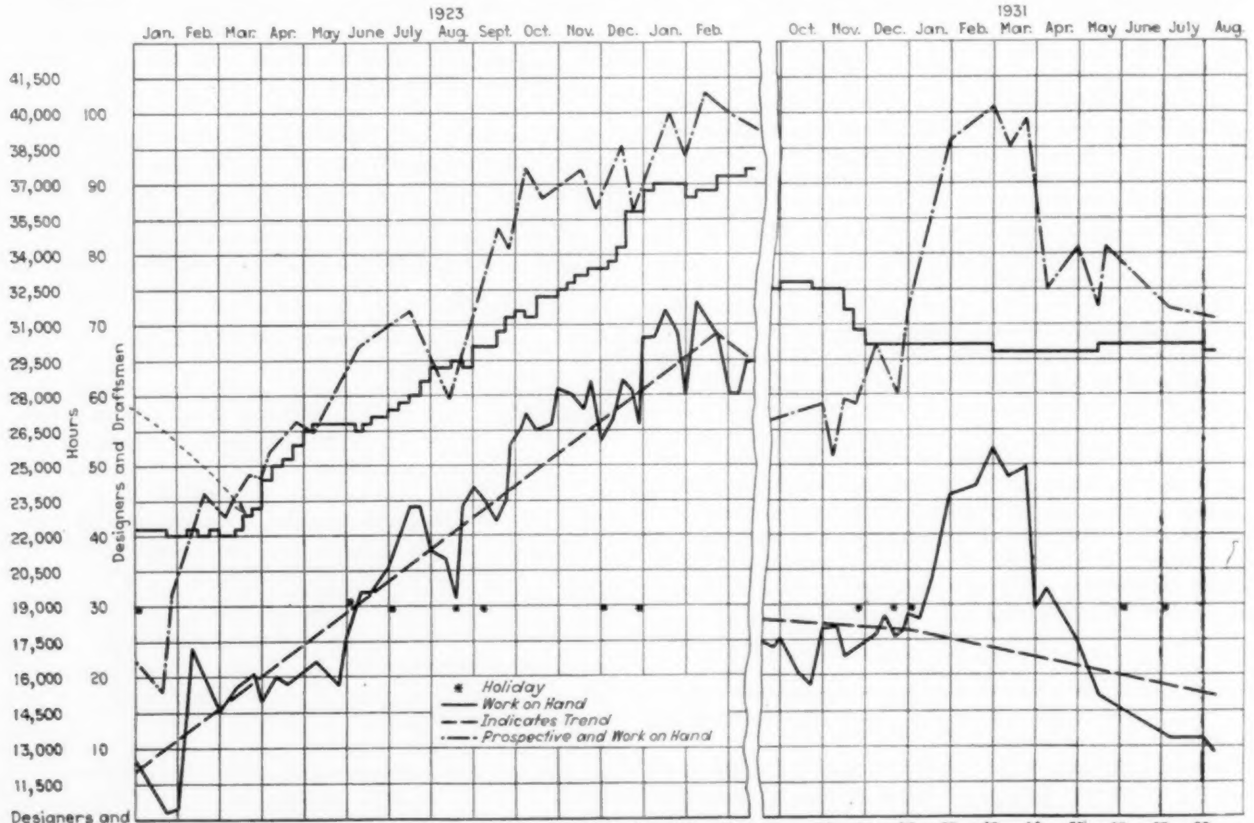
Cost records of each individual design job are also maintained for cost-control purposes, so that information may be given to interested organizations to aid in the preparation of new cost estimates, and to furnish material from which monthly and special cost reports are compiled. All time spent by the designer or draftsman is posted to the cost sheet, Fig. 8, which is issued for each job. The time clerk carefully watches the charges accumulating against each design project, and when such charges reach 80 per cent of the estimated drawing cost, the notification from Fig. 9 is sent to the design-department supervisor, who immediately checks the progress of the work, and determines whether or not an increase in the estimate will be necessary. If more money is needed to complete the design, additional approvals covering the increase are immediately obtained on the form shown in Fig. 5, which is reissued to cover the increase.

At the completion of each new-design job a detailed report of the charges incurred is submitted to the design supervisor and the development engineer in charge of machine design for their information and possible analysis. Monthly reports showing the cost status of each design project are submitted to the management. Special reports, principally in chart form, showing design expense broken down into proper classifications, non-productive payroll, unit costs, budgets, expenditures, etc., are frequently prepared by the clerical organization for the information of the supervisors and management, especially at the beginning of fiscal periods. Experience has shown that these reports are a great aid in properly controlling design activities.

Personnel Control

Not every person has the qualifications necessary to design machinery successfully. In addition to engineering training, the machine designer should possess orderly ways of thought, and have a large fund of the common sense necessary successfully to determine that middle course between conflicting factors which constitutes such a large proportion of machine-design work. When hiring new people a prospective employee is shown existing equipment similar to that he will be expected to design; he is interviewed by several people and every effort is made to ascertain his inclinations, capabilities, and probable future value to the company. The performance of each employee is under the constant observation of his supervisors, and periodic checks are conducted by the Engineer of Manufacture personnel organization.

In an earlier paragraph it was stated that in order to encourage young men to enter design work it is necessary to offer opportunities for advancement. The organization of the Engineer of Manufacture force at Hawthorne is so constituted that there are



Designers and Draftsmen	40	41	44	53	56	57	62	64	71	74	78	86	90	91	75	67	67	67	67	66	66	67	67	66
Tracers	8	8	8	8	8	8	8	10	9	11	13	15	20	21	7	6	6	6	6	6	66	6	6	6
Adm. and Clerical	21	21	21	20	20	21	20	22	21	23	23	23	27	28	27	26	26	26	26	25	25	25	25	25
Total	69	70	73	81	84	86	90	96	101	108	114	124	137	140	109	99	99	99	99	97	97	98	98	97

NOTE: The Amount of Work on Hand for 1931 is not Comparable with 1923 Due to Transfer of Design Work to Other Plants Placed in Operation since 1923

Fig. 10 Long-Time-Trend Chart

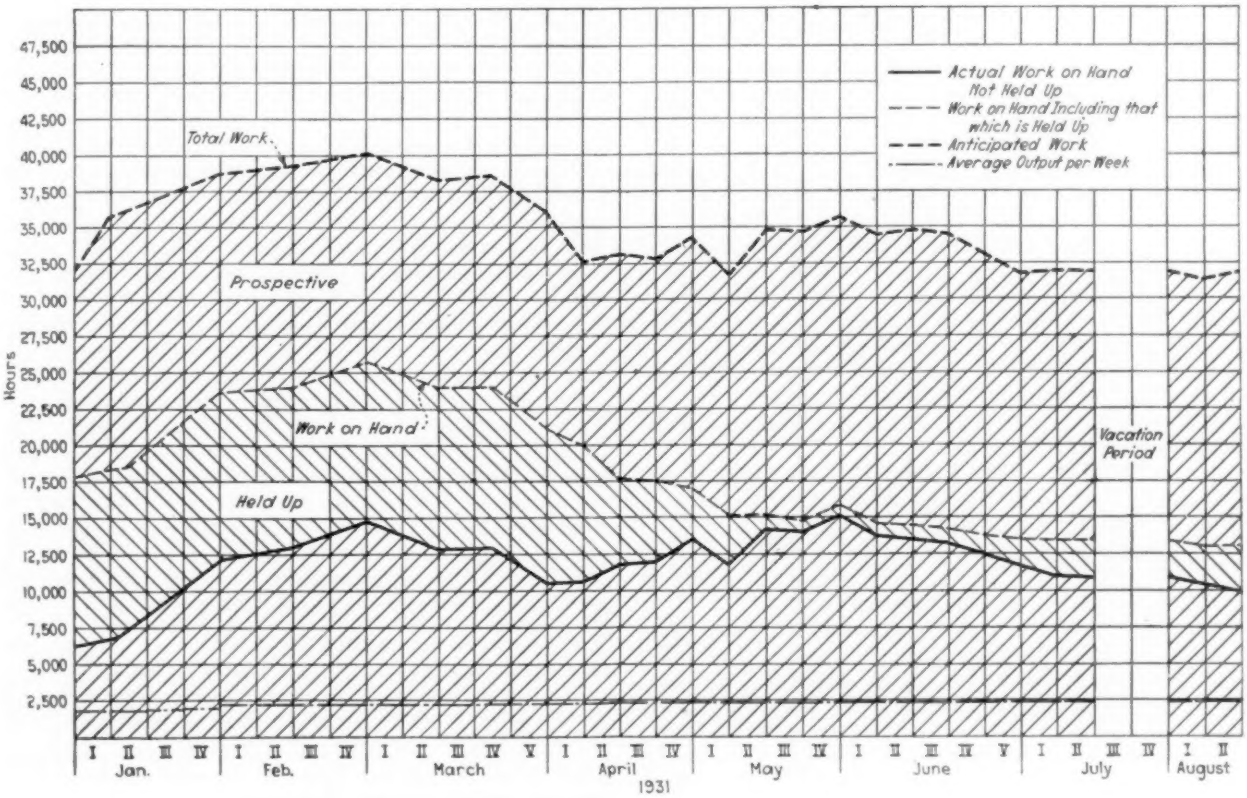


FIG. 11 CHART SHOWING BREAKDOWN OF WORK IN THE MACHINE-DESIGN DIVISION

many places where former designers and draftsmen can be absorbed. Therefore, a practice has been established in the Hawthorne machine-design division of taking in superior types of young men, training them in machine-design practices, and, after three or four years of productive work, transferring them for further training and upgrading to other development divisions. The advantages are twofold. In the first place, the machine-design division is able to attract a higher type of man, and, in the second place, the development divisions gain men by transfer who have had good design training and experience. An incidental benefit to the design division is that when former machine designers feed work into the division, it is generally found to be better conceived and more fully completed than when such work originates with men with non-design training.

Among the important duties of any executive or supervisory staff is that of regulating the size of the organization with respect to its work load. Information upon which control of the size of the Hawthorne machine-design personnel is based is gathered from at least four sources, namely; a long-time-trend chart, Fig. 10; a breakdown chart, Fig. 11, giving more detailed information concerning the work load at given times; a graphic schedule, Fig. 12, showing the work ahead of each designer and draftsman in the division, arranged by departments; and by consultation with the individual development engineers who have equipment designed at Hawthorne.

The long-time-trend chart, Fig. 10, which was started several years ago, has:

A curve showing, in man-hours, the work on hand for which actual orders have been received in the machine-design division.

A curve showing, in man-hours, the sum of work on hand and prospective design work which is quite certain to be released in the near future.

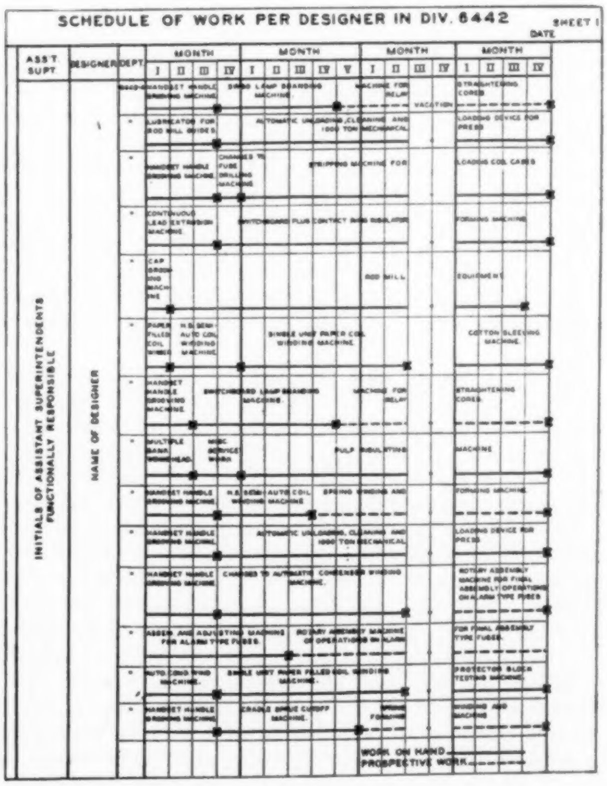

SCHEDULE OF WORK PER DESIGNER IN DIV. 6442
SHEET 1
DATE
ASS'T SUPT
DESIGNER
DEPT
MONTH
MONTH
MONTH
MONTH
I II III IV
I II III IV V
I II III IV
I II III IV
INITIALS OF ASSISTANT SUPERINTENDENTS FUNCTIONALLY RESPONSIBLE
NAME OF DESIGNER
VACATION
WORK ON HAND
PROSPECTIVE WORK

FIG. 12 SCHEDULE SHEET OF WORK TO BE DONE

A curve showing the number of designers and draftsmen in the division.

A line showing the general trend of the work-on-hand curve.

Tabulated figures showing the number of designers and draftsmen, tracers, and administrative and clerical people on the roll at the end of each month.

This chart is brought up to date each month by the machine-design clerical department from divisional personnel records, and an inspection of the others, memoranda, etc., received in the division. It not only presents a most interesting visual record of machine-design activity over a period of years, but is also a valuable aid in determining the general policy to be followed with respect to the size of the force to be maintained in the machine-design division. It should be noted that the curves showing work on hand and the number of designers and draftsmen are substantially parallel throughout their length, indicating that, in general, the working force is adjusted to follow machine-design business conditions quite closely.

The breakdown chart, Fig. 11, presents curves showing in man-hour units: (1) The total amount of prospective design work, (2) work on hand, (3) that portion of the work on hand which has been held up for one reason or another by different development engineers, and (4) a curve showing the average output per week of the productive machine-design force.

This chart is made to a fairly large scale and shows curve fluctuations in much more detail than the long-time-trend chart. It is brought up to date by the machine-design clerical department every third day during the periods of reduced design activity when there may be a possibility of a shortage of work. During busy periods such frequent revisions are, of course, not necessary, and the chart is brought up to date each week.

The graphic schedule of work per designer, Fig. 12, is arranged by departments, and shows the number of weeks' work ahead of each productive person in the division, together with a brief description of the equipment to be designed and whether it is work on hand or prospective work. This chart is reissued each month with minor pencil revisions at such shorter periods as appear advisable. It is frequently used by the design supervisory force at conferences, and is always referred to when adjusting the personnel or the work load between the machine-design departments, so that each department will have the proper amount of work ahead. This distribution of design work and men between departments minimizes the possibility of design jobs, being delayed because of an overload in one department, when another department might be able to handle the job much sooner. When it becomes necessary to schedule rush design work ahead of work already in the division, this chart aids in selecting the job to be deferred.

While this chart is used principally as an aid to the design supervisor in scheduling his work, it is also useful in connection with the other charts in determining the size of the design personnel, as it presents a detailed picture of the different types of work in the division at a given time and also provides information concerning drawing-checking activities that the other charts do not cover.

When deciding upon the proper design personnel to be maintained, the long-time-trend chart is critically examined in an endeavor to visualize the design activity outlook for the next few months. Supporting facts concerning the probable amount of future machine-design activity are gathered by means of consultations with the different development engineers, and by considering budgetary matters, etc. The breakdown chart and graphic schedule are then used to determine the machine-design personnel required efficiently to take care of what may be termed the seasonal fluctuations of machine-design work. The number of man-hours of work ahead shown on the breakdown chart, excluding that portion of the work on hand which has been held up by different development engineers, are carefully plotted and an arbitrary low deadline set. This deadline is not a fixed quantity, but is set differently each time, depending upon those consultations, analyses, etc. mentioned before. Should the available number of man-hours of work fall below this deadline, arrangements are immediately made to adjust the productive working force.

During rush periods an examination of the different charts sometimes shows the work curves steadily rising; in other words, the inflow of work to the machine-design organization is greater than its output. When this condition reaches a point where design work is retarded and reasonably prompt drawing deliveries cannot be made, the usual design-activity forecast is made. If this forecast indicates that the inflow of work to the design organization is likely to be maintained or increased during the not-far-distant future, action is started toward increasing the personnel.

The particular methods exemplified in the control of the Hawthorne design organization probably cannot be used in their entirety by organizations widely different in size, type of work handled, etc. though they are believed to be fundamentally sound and adaptable, with suitable modifications, to either large or small design groups. It is possible, too, that they are already in use, at least partially, by other design offices. However, in view of the dearth of design-management literature, it is hoped that this paper will promote a beneficial exchange of ideas.

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Discussion

E. G. Scott.[2] There are not many companies so fortunately situated with regard to their machine design division as the Western Electric. Most companies that maintain an engineering office for design work pertaining only to their own product do not have a big enough organization to subdivide the functions so clearly drawn up in the paper. Usually the head of the department has to supervise, as well as furnish, detail working information for work of a wide and varying character, and this applies to many large industries that are non-technical in their nature.

The experience of the writer has been largely with industries whose basic business has not been engineering. He has experienced most of the things mentioned in this paper and believes that the author has outlined a comprehensive and workable scheme for the management of a large design office. It shows an arrangement toward which one might strive if the organization was large enough to justify it.

However, even in much smaller organizations they have to go through the same motions. One individual may perform many of the separate functions. While it may not be necessary to make a written record of everything, someone has to make at least a mental note of them all.

The writer has tried many schemes to keep track of jobs during the process of development. We have now worked out one quite similar to that of the author. We have a job folder in which are filed the original or a copy of correspondence bearing on a particular job, engineers' calculations, pencil sketches, and a blueprint of all drawings through the process of development. It is helpful when working up new jobs to have a complete record of what was done in past years on similar jobs.

The system of indexing, in use for a number of years, is simple and infinitely expansible. In handling the new construction work and maintenance work for four organizations, it is desirable to keep these separate, and yet the information collected must be easily locatable for any one or all of the four. The writer has organized his index as follows:

The key numbers 1000, 1100, 1200, and 1300 designate the four organizations. Each is divided into departments, such as the administrative, productive, and non-productive departments, and they in turn are subdivided so that every major operation lends itself to classification. The key number has a prefix of the last two numerals in the year. The key number appears on each document, whether it is filed with related papers or not.

In using the system, the key number representing each of the four units ends with two numbers for the department. The productive departments are symbolized by numbers from 00 to 34, the non-productive by 35 to 67, and the administrative by 68 to 93. To further coordinate the papers relating to a particular job, a specification might have the letter number S-31-1027-1 or a bill of material would receive BM-31-1017-1, etc.

The author relates how the Western Electric has solved problems similar to those that have been bothering the writer for many years. One especially troublesome one is the promotion of draftsmen. In industries where engineering is quite remote from the main business, promotion to other departments is not as simple as with an industry such as the Western Electric, but even in these something can be done for the advancement of the draftsmen by transfers to other departments.

The keeping of cost records of engineering reports is quite important. It is easy to ask for reports on certain projects without realizing what such reports cost. When the report is completed, the one asking for it should be advised of the cost. If the department heads have this information, they will use more care in making requests of the engineering department. Of course, it is understood that the engineering department should give an estimate of the cost of the work before starting, but untrained people do not always believe these estimates. The engineering department has the dual responsibility of promoting progress and at the same time of not unduly running up the engineering expense.

F. O. Hoagland.[3] It is difficult to keep a force as large as the one described occupied efficiently. It is interesting to follow the author's description of the duties of the division chief, in continually checking and cross-examining, seeing that standard machinery is procured in the open market when available, that records are kept of past experiences, that designs are standardized, and that costs are kept within reason.

He probably finds that the checking section is the "neck of the bottle" and that in order to rush work through it is often necessary to short-cut it. The writer has found that it works out to advantage to have the group that prepares the designs and layouts and details also to check each other's work, and to have the detail drawings go through the standards engineers in order to check standard parts, means for lubrication, tolerances and allowances for various classes of fits, with the chief designer or a responsible engineer checking for general and functional designs.

The author's budgeting and averaging of the costs of designs per drawing turned out reminds of the mail-order house practice of weighing the first-class mail each morning and thereby determining the dollars represented.

He makes a strong point of the training of men. How about apprentices? Should not the advantage of actual shop training be emphasized? Nor has he overlooked the importance of giving the men opportunities for advancement.

The author's charts show in a comprehensive way the general organization, the distribution of duties and responsibilities, the work on hand, and the men available. It is hoped that he is successful in keeping these charts up to date, as too frequently it is found that this matter is apt to lag, the usefulness of the charts being thereby considerably reduced.

The writer regrets to note that even in the author's line the amount of work and the men employed fluctuate considerably, and if charts could in some way be laid out to eliminate this factor, it certainly would be very desirable.

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R. E. W. Harrison.[4] Having regard to the large part played by economics in the determination of managerial policy, it is surprising how little literature there is available on this subject for the guidance of those confronted with similar problems. Most of the executives handling assignments of this nature have developed a technique which is almost entirely the result of their own progressive experience, but to the best of the writer's knowledge, this paper represents the first effort of its kind to place on record the policies and practices of one of the larger engineering corporations.

While the author would probably be the last person to suggest that any scheme of control could be taken and applied in blanket form to another business, there are without doubt many valuable ideas in the paper which can be readily adapted to almost any existing engineering department, even though only a relatively small number of men be employed.

As a result of many years' contact with work of this nature, the writer can confirm that the budget and appropriation method of handling engineering design assignments is the only businesslike way of insuring control over costs, and while experience proves that it is almost impossible to estimate with accuracy the cost of solving a new engineering problem, particularly from the design standpoint, it has been shown that the wise course to follow is to make the appropriations in steps and to bring the whole matter up for review when the financial limit of each step has been reached.

In these days of rapidly changing economic situations, it has not been unknown for a design to become economically unattractive during the period of its development. Hence, step-by-step appropriation brings the management and the designer into closer contact and insures that degree of control which will obviate the spending of money on an unprofitable project.

Relatively few concerns keep accurate cost records of their designs, and it is the writer's belief that this is due, more than anything else, to lack of appreciation of the value of accurate cost records as a basis for estimates of the expense involved in future work.

Too much stress cannot be laid upon the importance of the overall-cost-reducing virtues of a well-organized and up-to-the-minute engineering standards department. This department, when properly run, not only automatically reduces the cost of manufacture, but also tones up the quality of all designs produced by the engineering department as a whole. The writer's experience also indicates that it is just as important to carefully plan the work schedule in the machine-design department as in the works-production department. The management, in consultation with the sales and engineering department executives, should call for a date of completion, as only in this way can the engineer in charge so dispose his forces that they most accurately interpret the business policy of the company.

It would seem that there is too little appreciation of the fact that the drafting room provides a good training ground for potential executives and that a period of not less than two years in this department should be regarded as an essential part of the training of every man destined for a major executive position. The assignment during this period should not be that of a looker-on or even a supervisor, but as a man who is expected to produce and who will be expected also to put up an outstanding performance if he is to be regarded as potential executive material.

It is a regrettable fact that many outstanding designers make poor executives. The trouble would appear to be largely psychological and in a measure can be traced to the type of training which a man must receive to become a first-class designer. As in most things, a compromise is the most effective expedient, and it is usually up to the executive head of the department to supply the economic balance and background and to keep sufficiently closely in touch with the progress of the work to insure that after laying down the principles on which the design is to be made, he also can exercise control over those features which involve manufacturing costs, potential life, and overall efficiency of performance.

Still another unfortunate aspect to this matter is the fact that the generally better remuneration earned by men in executive positions induces some good designers to make every effort to abandon this end of the work in the belief that they can do better for themselves in an executive position. The remedy is obviously a better appreciation by the management of the fact that good design is the starting point of all successful engineering enterprises.

With regard to training the draftsmen in workshop practice, experience shows that no man can become a really efficient designer unless conversant with resources at the disposal of the manufacturing end of the business. It sometimes happens that the most brilliant designs are wrecked in the economic sea because they cannot be economically translated into simple manufacturing procedure.

In this connection much benefit can be accrued from providing facilities for the designers and draftsmen to contact not only with their own manufacturing divisions, but with the user as well. Nothing has quite so completely constructive an effect on a designer as to have to service the machine which is his own creation or to talk over its merits and demerits with a busy production superintendent who has had to live with it for a year or more. While it sometimes happens that the solutions for the defects put forward by the operating end of the business are not all in accordance with the accepted principles of applied mechanics and good design generally, these complaints and suggestions serve a very useful purpose in keeping before the designer's mind a vivid picture of the difficulties which he can create by inattention to the service and operation angles of the design.

In so far as actual drawing-office control of the work schedule is concerned, the writer has found that a very complete picture can be obtained by keeping a master record of each job in a loose-leaf folder. This record is in the form of a chart, at the head of which is the designation of the work, order numbers, dates, promises and estimated time, etc. The chart is ruled vertically into days and weeks, and by utilizing the services of a clerk to make a daily record, indicating the time put in on the job by the designer, detailer, and checker, a very effective running balance can be maintained. Such a picture enables the supervisor to give a reasonably accurate estimate of the time required to deliver the drawings and an absolutely reliable statement of the time already charged to this particular piece of work. Also, it has the merit of being cheap to maintain, and furthermore gives that detailed control which is so necessary to insure the 100 per cent interpretation of the management's requirements.

F. D. Newbury.[5] The writer wishes to discuss some aspects of the design problem that are related to the form of organization of the design department. While discussing organization, there is no wish to overemphasize its importance. It is not as important, for example, as quality of personnel nor as important as skill in administration. The success of any organization is the net result of all three of the factors—form of organization, quality of personnel, and skill in administration; but good organization permits the other two factors to function most effectively.

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[4] Engineering Sales Director, Cincinnati Milling Machine and Cincinnati Grinders, Inc., Cincinnati, Ohio. Mem. A.S.M.E.

[5] General Manager, Machinery Engineering, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa. Mem. A.S.M.E.

The design work connected with the design of any machine or factory equipment may be divided into several parts as follows:

(1) The functional design of the apparatus, the determination of the correct proportions and arrangement of elements to accomplish the desired results—as, for example, the design of the "steam path" of a turbine to perform satisfactorily as a heat engine or the design of the magnetic and electrical circuits of a motor or generator.

(2) The mechanical design of the structure of the machine or apparatus, consisting usually of the combinations of well-known mechanical elements. In the case of the steam turbine, this part of the design problem is the problem of designing a turbine that will function acceptably as a mechanical machine.

(3) The translation of the design into working drawings suitable for use in the factory.

These strictly design operations are, in the most highly technical industries, backed up by a research organization, investigating principles of operation, design methods, and materials.

In connection with all of these research and design activities, each merges more or less into the next, and the best form of organization and administration is the one that provides for the closest cooperation between these various groups interested in the design of the same product.

It is generally agreed that the functional design work requires specialized engineers—electrical engineers of various kinds, steam turbine engineers, pump engineers, etc.

It does not follow, however, that all of these engineers are necessarily of high quality and long experience. This functional design operation includes original work of the highest type and greatest difficulty, but it also includes, in a large company, a great deal of strictly routine "slide-rule" work that requires no more original thinking or ability than does detail drafting.

The mechanical design of the structure requires mechanical engineers, who may be engineers highly trained in mechanics and mathematics and capable of analyzing stress and vibration conditions encountered in high-speed machinery; or this phase of design may require men skilled in kinematics and in the design of machine elements. Usually, the general layout and structural design is drawn up on a paper as a part of this portion of the design operation.

In this second division of the work, also, we are concerned with men of a wide range of talent and education; some must be highly trained in theory, others must be ingenious in devising new mechanical arrangements and others may be merely skilled draftsmen.

In the third or detail drafting division of design work there is the relatively simple job of making suitable drawings of detail parts in accordance with the general layout drawing. This work requires well-trained men who are naturally accurate.

The difficulties and differences of opinion and practice in engineering design departments are encountered largely in the second division of design work concerned with the mechanical structural design. In this division "desk design" and "drafting-table design" overlap considerably, and it is difficult to effectively conduct this part of the design job if the form of organization draws a sharp line between engineers who work at desks and designers who work on the board.

There is a natural step-by-step progression in the design organization from detail draftsman to layout or design draftsman, to mechanical engineer or specialized design engineer, and if the form of organization recognizes and provides for all of these positions in one design group, there will be little difficulty in giving each draftsman and engineer the work he is best able to do, and advancing men as their abilities increase. There need be no blind alleys. On the other hand, if there is a sharp division, arising from the form of organization, between designers who work on the drafting board and engineers who work at desks, there will be difficulty in keeping high-grade engineers and designers, who should do their designing on the drafting board, in the drafting organization.

In the form of organization in which the functional design and the structural or machine-design problems are handled by different major groups, as in the organization described by the author, and also in organizations in which the "functional" type of designing is largely absent, the design department or division becomes entirely a "drafting department," and designer and draftsman become synonomous titles. In design organizations, in which the functional type of designing is combined with the others, more closely related to drafting, so that all grades of specialized engineers, designers, and detail draftsmen are associated together under a single engineer-executive, the problems of administration are greatly reduced. This statement is based on considerable practical experience with both types of organization.

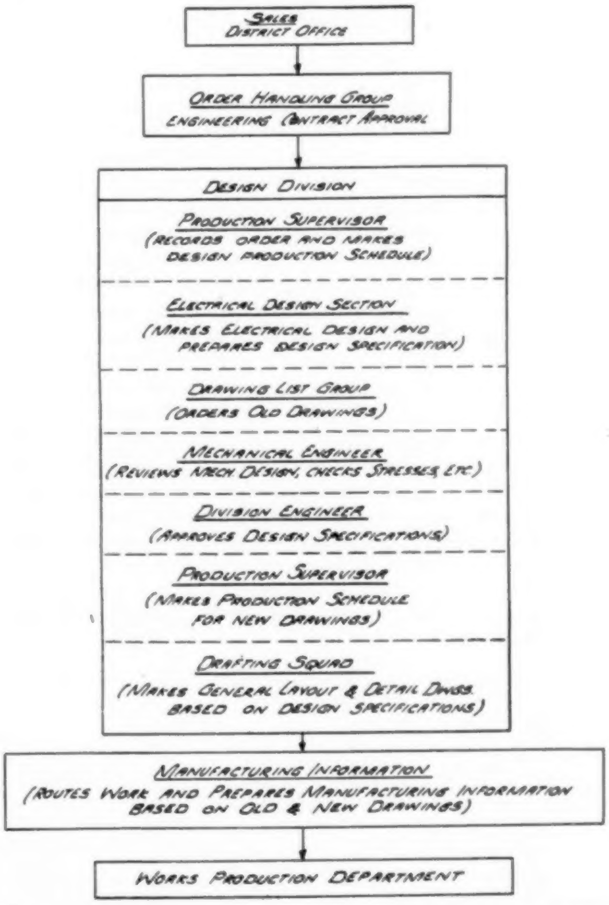
SALES
DISTRICT OFFICE
ORDER HANDLING GROUP
ENGINEERING CONTRACT APPROVAL
DESIGN DIVISION
PRODUCTION SUPERVISOR
(RECORDS ORDER AND MAKES DESIGN PRODUCTION SCHEDULE)
ELECTRICAL DESIGN SECTION
(MAKES ELECTRICAL DESIGN AND PREPARES DESIGN SPECIFICATION)
DRAWING LIST GROUP
(ORDERS OLD DRAWINGS)
MECHANICAL ENGINEER
(REVIEWS MECH. DESIGN, CHECKS STRESSES, ETC.)
DIVISION ENGINEER
(APPROVES DESIGN SPECIFICATIONS)
PRODUCTION SUPERVISOR
(MAKES PRODUCTION SCHEDULE FOR NEW DRAWINGS)
DRAFTING SQUAD
(MAKES GENERAL LAYOUT & DETAIL DWGS. BASED ON DESIGN SPECIFICATIONS)
MANUFACTURING INFORMATION
(ROUTES WORK AND PREPARES MANUFACTURING INFORMATION BASED ON OLD & NEW DRAWINGS)
WORKS PRODUCTION DEPARTMENT

Fig. 13 Order Route Chart of Westinghouse Company Power Engineering Department

To illustrate this difference in type of design organization, the writer wishes to compare the organization described in this paper with that of the Power Engineering Department of the Westinghouse Company. This department is responsible for the electrical and mechanical design and drafting connected with heavy rotating electrical machinery. It is divided into a number of self-contained design divisions, each completely responsible for the design and drafting of a definite class of machinery and small enough to be administered by one engineer-executive.[6] This comparison can be most readily made by comparing the author's graphical chart (Fig. 4) with the accompanying chart (Fig. 13) showing the travel of an order, for the production of a new machine, through the design department. The essential point is the division of responsibility, in the author's organization, among three major groups (development department, machine-design department, and machine-design engineering and checking department), and the concentration of complete design responsibility (in the power engineering department) in one relatively small design division.

Author's Closure

Mr. Hoagland's questions regarding training of men strike at a recognized weakness of design personnel. The Western Electric Company maintains an apprentice course for toolmakers, development machinists, machine-building machinists, tool designers, and machine designers. The machine-design apprentices receive the same fundamental shop training as do the shop apprentices, and it has been our experience that, in general, these young men make very satisfactory designers. It is difficult, of course, in a large organization to provide the close contact between shop and drafting room that is possible in a small company. Nevertheless, we make a definite effort to keep our men in as close touch with the shop as possible. This is accomplished by requiring the designer to correct shop difficulties which arise during the construction of the machines and to contact with the foremen and machinists on the job. It cannot be said, however, that we have completely solved the problem of educating designers in shop practice.

A greater amount of system and routine is necessary in a large drafting room than in a small one. Nevertheless, it is essential that the system be as flexible as possible. Therefore, we make every effort not to take our departmental routines too seriously, and we depart from them just as often as our judgment dictates that such departure is advisable. For example, certain jobs are carefully checked by the regular checkers. On other jobs, the designer checks the work of his detailers. On still others, the designer details his own work, which is sometimes checked and sometimes not. Moreover, at times of reduced load and activity we completely discard some portions of our system with the expectation of resuming these portions whenever the need of the work requires. We feel very strongly that a large design group may easily become overorganized, and therefore take more than ordinary precautions to avoid substituting system and routine for responsibility of personal supervision.

It is because of the foregoing facts that the author believes that the differences between the organization described in the paper and that illustrated by Mr. Newbury are not as great as appears on the surface. In the first place, the Hawthorne Machine Design Division designs nothing but machinery used within the Western Electric Company for the manufacture of telephone apparatus. It designs no machinery for sale. Moreover, the various development groups shown in the organization chart of Fig. 1 are closely associated with the design division. Because of this close association and because the machinery designed is built and installed in the Western Electric Company's plants, the machine designers are in close touch with operating requirements and the practical phases of their jobs. It is certainly a fact that the Hawthorne machine designers are designers rather than draftsmen and that they have full opportunity to exercise all of the ingenuity they possess. From Mr. Newbury's comments, it appears probable that the author's chart, Fig. 1, gives a false picture of functional rigidity which does not actually exist. To be effective, of course, any design group must adapt itself to the conditions surrounding its job. The Hawthorne design division functions smoothly and fits in well with the general scheme of organization of the Western Electric Company, although it is recognized that to meet a different set of conditions, a radical change in type of organization might be necessary.

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[6] This organization is described in "Improving Drafting Management," by F. D. Newbury, presented before A.S.M.E., Cleveland, Ohio, April 13 to 17, 1931.

MSP-54-10

Oxygen Machine Cutting in Great Britain

By C. G. BAINBRIDGE,[1] LONDON, ENGLAND

Cutting of metal by the oxygen flame has been practiced commercially for more than 25 years by means of hand-guided blowpipes. The hand cutter is largely used for scrap cutting and by concerns whose work involves the use or manipulation of steel plate, forgings, or sections. The work produced by the hand cutter is rough and inaccurate. The paper describes the use of the machine designed to hold the cutter rigidly and that propels the cutter by a simple feed motion at a uniform speed. Great progress has been made in the past 10 years in the development of these machines, and they now are considered as reliable, quick, and accurate shop tools. The paper describes the various machines and their uses, and gives instances of their application to difficult work, with the resulting reduction in costs.

MOST of us remember the laboratory experiment in which a piece of iron or steel wire or strip is heated in a bunsen flame and then passed through a stream of oxygen; the metal is rapidly oxidized and disappears in a shower of sparks. What has happened is that a chemical action has taken place between the iron or steel and the oxygen, forming a magnetic oxide of iron (Fe_3O_4), the sparks being not pieces of molten metal, but particles of red-hot oxide. The bunsen burner merely supplies the necessary heat to bring the iron or steel to the ignition temperature. The process is therefore one of oxidation of the iron or steel.

It is a comparatively simple matter to calculate that 1.17 cu ft of oxygen is required for the complete combustion of 1 cu in. of iron under ideal conditions, assuming the metal to be 100 per cent pure iron and the gas 100 per cent pure oxygen.

In practice, steel is generally the material to be cut, and the oxygen is generally not more than 99.5 per cent pure. Moreover, complete combustion does not always take place, as some of the particles of the iron in the path of the cut are loosened and blown out by the force of the oxygen stream; in other words, we do not always get pure magnetic iron oxide, but probably something between Fe_2O_2 and Fe_3O_4.

In the laboratory experiment the bunsen burner heated the metal before it reached the oxygen jet. In commercial practice this heating flame is furnished by the same blowpipe that carries the oxygen jet. The heating flame consists of a mixture of oxygen and fuel gas, such as acetylene, coal gas, or hydrogen. Some blowpipes are fitted with two oxygen connections—one for the heating flame and one for the cutting jet. Other types have only one oxygen connection in addition to the fuel gas, the oxygen being diverted inside the blowpipe in order to supply both heating flame and cutting jet.

The heating flame of course takes no active part in the actual cutting operation, the heat provided by it being relatively small compared with the heat evolved by the combustion of the metal. It is necessary, however, to maintain the heating flame while cutting in order to compensate for heat losses by radiation and conduction.

In practice, the heating jet or jets must surround the cutting jet. The first blowpipes or cutters had the jets arranged as at *A*, in Fig. 1. In this arrangement, as the cutting jet follows the heating jet, only straight-line cuts could be made satisfactorily. For shape cutting, the cutting jet has to make abrupt changes of direction; therefore, present-day cutter nozzles are usually arranged as at *B* or *C*, where it will be seen that no matter in what direction the cutting jet is moved it is traveling over material that already has been heated by the heating jet.

Oxygen cutting has been applied commercially for more than 25 years by means of hand-guided blowpipes. The hand cutter is largely used for scrap cutting and by concerns whose work involves the use or manipulation of steel plate, forgings, or sections; free-hand cutting is very crude, and when the cut is to be used for production jobs, it is usual to improvise suitable guides, trammels, or templets, with which to steady the cutter.

There can be no doubt that the hand cutter saves a tremendous amount of time and considerably simplifies the roughing out of large or intricate shapes that would be impossible to shear and for the production of which the only alternative would be drilling.

The work produced by the hand cutter is, however, at its best very rough and inaccurate. It is difficult to obtain uniformity over a number of cuts, and a considerable amount of machining is necessary in order to obtain a clean cut.

It is obviously a very short step from a guided hand cutter to a simple machine that will hold the cutter rigidly and that will by means of a simple feed motion propel the cutter at a uniform speed. The first machines introduced were, in fact, merely devices for holding a hand cutter. Some users, seeing the good results that would be obtained if the cutter were mechanically driven and guided, converted old lathes and planing machines into oxygen cutting machines.

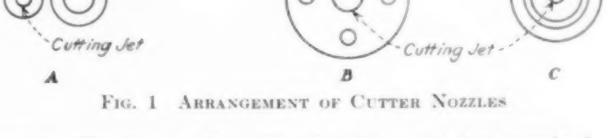

FIG. 1 ARRANGEMENT OF CUTTER NOZZLES

The past 10 years or so has witnessed great progress in the development of these machines, and now there are machines on the market for a very wide range of applications of oxygen cutting. These machines may be definitely considered as reliable, quick, and accurate shop tools. Indeed, the large machines especially designed for profile cutting are machine tools in every sense of the term.

Owing to the regular feed and the steady travel of the cutter, the cut surface is so smooth and accurate that often no further machining is necessary. If, however, the use to which the article is to be put necessitates a machined surface, the amount of metal

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[1] British Oxygen Company, Ltd. Assoc-Mem. Instn. of Mech. Engrs.

Contributed by the Machine Shop Practice Division and presented at the Semi-Annual Meeting, Bigwin Inn, Lake of Bays, Ontario, Canada, June 27 to July 1, 1932, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

left on for machining can be reduced to a minimum; $^1/_{32}$ to $^1/_{16}$ in. is ample for jobs in good-quality material up to about 4 in. thick, and $^1/_8$ in. on material up to about 8 in. thick. On material thicker than this it is advisable to leave a little more metal for machining to size, as there is often the risk of slight drag, causing the bottom of the cut to be out of register with the top; also, it is more difficult to control expansion and contraction. Again, as articles of this thickness probably will be cut from forgings, it is as well to allow for slight defects in the forging.

Reference to the illustrations will indicate some present-day commercial applications of the process.

One of the most frequently applied uses of oxygen cutting is for gapping crankshafts and for cutting out complete crank webs. (See Fig. 2.) The crankshaft shown here is for an air compressor; the thickness of the cut part is about 7 in., and the time for cutting out the shape shown is about 4 min, only $^1/_8$ in. over the diameter of the crank journal being left for machining off. The smoothness of the cut can be judged from the middle throw which has been cut.

Separate crank webs are usually cut from slab forgings, if fairly thick, or from forged steel bar when practicable. Fig. 3 shows a machine shaping a crank web from a forged slab about 10 in. thick. Fig. 4 shows a machine cutting crank webs from a forged bar about 5 in. thick.

Firms that produce large forgings and that have installed an oxygen cutting machine now find it convenient in many cases to eliminate expensive intricate forging, which cannot be conveniently or accurately done under the hammer, leaving the intricate shaping to be done by the oxygen cutting machines.

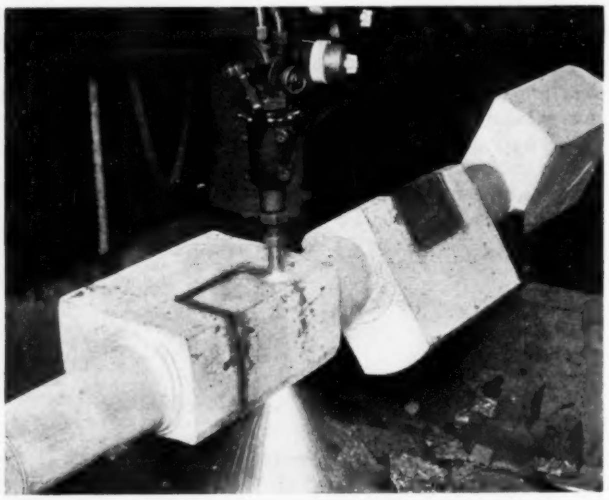

FIG. 2 OXYGEN CUTTING OF CRANKSHAFT WEBS

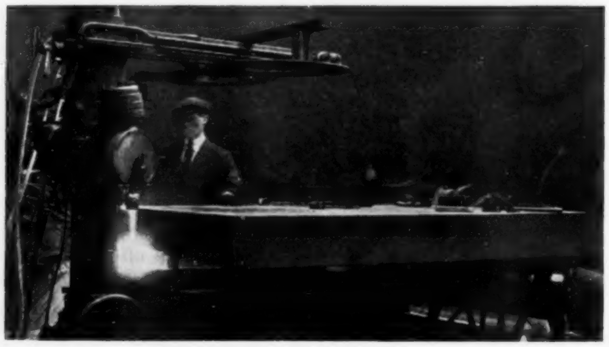

FIG. 3 MACHINE CUTTING CRANKSHAFT WEBS FROM FORGED SLAB

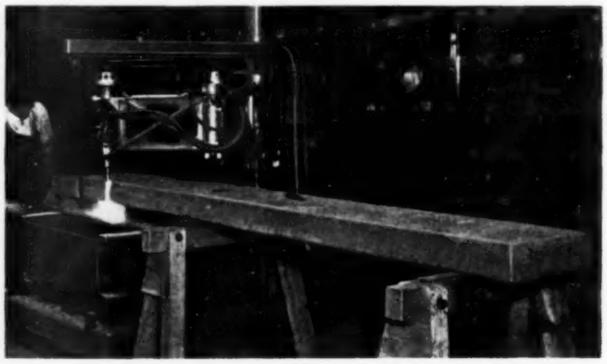

FIG. 4 MACHINE CUTTING CRANKSHAFT WEBS FROM FORGED BAR

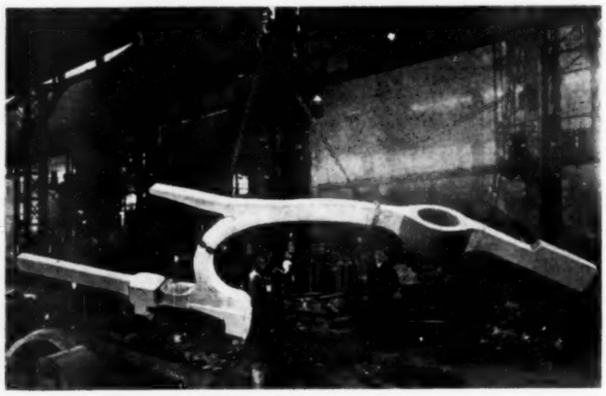

FIG. 5 INTRICATE SHAPING DONE BY OXYGEN CUTTING MACHINE

A remarkable instance of this class of work is shown in Fig. 5. The forging shown is a complete stern frame for a ship. The steel varies in thickness from 12 in. to 14 in., and the total length of cutting on the job was about 13 ft. The oxygen cutting machine used on this job is seen in the background.

EFFECT OF OXYGEN CUTTING ON MATERIAL

The usual question asked by those unacquainted with the process is, "Does oxygen cutting affect the steel?" The answer is that, generally speaking, it does not. That is to say, oxygen cutting has no detrimental effect on the material provided it is a steel not adversely affected by the application of heat. This means that all carbon steels containing carbon up to 0.3 per cent can be quite easily cut and the surface of the cut will be readily machinable after cutting.

The general effect of oxygen cutting is to change normal ferrite and pearlite condition to sorbitic pearlite with ferrite envelopes at the cut edge. This results in a slight hardening at the cut surface, the total increase in Brinell hardness on mild steel being 30 to 40 points. There is also an increase of carbon near the surface of the cut, but this carbon change usually penetrates only for little more than half the total depth of hardening effect.

This is a structure and effect similar to that produced by water quenching and tempering steel of the same composition. On ordinary mild steels of 20 to 30 tons tensile, 0.18 to 0.25 carbon, the change of structure does not extend inward from the cut surface for more than $^1/_8$ in., and generally only for $^3/_{32}$ in. This applies to material up to 3 or 4 in. thick, but the depth of effect varies with the material's thickness and may extend nearly $^1/_4$ in. on material 6 in. thick. The depth given, of $^1/_8$ and $^1/_4$ in., re-

resents the point where the steel becomes normal in structure and hardness; i.e., the hardness diminishes from a maximum at the cut surface to this point.

This change of structure is largely due to the chilling effect of the comparatively cold mass of metal at the back of the cut surface. Cuts made on metal that has been preheated show only a superficial film of toughened metal.

The depth of effect will be increased if, because of vibration of the machine or dirty or damaged jets, the cut surface is rough.

If the article is normalized after cutting, all traces of cutting effect will be removed, but unless the use of the article makes machining unnecessary, there is no reason why jobs cut from steel containing up to 0.3 per cent carbon should not be put into service exactly as produced.

Above the 0.3 per cent carbon the increase in carbon and surface hardening may possibly cause a little inconvenience from the machining point of view; therefore, it is recommended that carbon steels of more than 0.3 per cent carbon content be preheated to about 500 C (visible red in daylight) before cutting. Alternatively, normalizing may be carried out after cutting with thicker material containing 0.4 per cent carbon. There may possibly be some chances of the oxygen-cut surfaces developing small cracks, etc., so that material of this composition always should be preheated before cutting in order to eliminate the possibility.

This cracking does not necessarily occur, but would largely depend on the shape and dimensions of the article being cut. The cracking is probably brought about in the following manner: The heating of the steel causes the cut face to expand, and this expansion is resisted by the colder metal at the back of the face. The metal at the cut face tends to be compressed by this resistance, and during cooling the compressed metal endeavors to resume its original length, but cannot without stressing the metal beyond the yield point.

The stress may be so severe as to crack material, such as high-carbon steel, that has not sufficient ductility to accommodate itself to any concentration of stress. The preheating precautions are not required for carbon steels of carbon content below 0.4 per cent or for parts that are not subjected to severe stresses in service.

Alloy steels, such as nickel, nickel-chrome, nickel-chrome-molybdenum, and high-carbon steels, should be preheated before oxygen cutting to avoid hardness of the cut faces and internal stresses that may lead to cracking.

It is important to mention that great advantages can be obtained by employing oxygen for cutting manganese steel. As is well known, manganese steel cannot be machined by the ordinary mechanical methods available to the engineer. Oxygen cutting has no such detrimental effect on steel, and perfectly smooth accurate cuts are obtainable, with no hardening of the cut surface.

Fourteen tests carried out to ascertain the effect of oxygen cutting on the tensile strength of mild steel gave the following average results:

	Oxygen cutting	Machine-tool shaping
Yield, tons per sq in.	17.50	16.50
Tensile strength, tons per sq in.	25.95	24.40
Elongation, per cent in 8 in.	21.95	23.2
Reduction of area, per cent.	59.0	58.8

It will be seen that, with oxygen cutting, ductility is not affected to any great extent, while the yield point and tensile strength are slightly increased. This is probably due to the slight toughening effect of the cutting on the surface of the metal.

In order to further investigate the effect of oxygen cutting on the ductility of the material, some bend tests were made in the following manner: A piece of mild steel 2 in. thick was cut into a strip about $^{5}/_{8}$ in. wide with oxygen cuts on both sides of the strip; the strip was then bent around a pin $^{3}/_{4}$ in. in diameter so that the cut surfaces were in tension and compression, respectively. No signs of cracking occurred until the piece had been bent through almost 180 deg.

It should not be overlooked that oxygen cutting depends on the accurate direction of a jet of gas through the depth of the metal, and that characteristically this jet will tend to take the path of least resistance. Cavities and porous spots, laminations, or slag inclusions will therefore almost certainly cause defects in the cutting due to diversion of the oxygen stream. Only good quality steel should be used, and forgings should be free from any of these defects.

Equipment for Oxygen Machine Cutting

There are several different types of oxygen shape-cutting machines on the market, but they may be divided into two classes—those that employ a templet representing the shape to be cut and those that cut the required shape direct from a drawing; some machines also embody a combination of the two systems.

The machines that cut direct from a drawing are undoubtedly very convenient and useful where only a few shapes have to be cut; for instance for repair work, where it may be necessary to cut a replacement shape, only a sketch of the original article being available. In this case, unless a high degree of accuracy is required, it would hardly be necessary to make a templet. (See Fig. 6.)

Machines employing templets are generally most suitable for quantity production when uniformity and accuracy are desired.

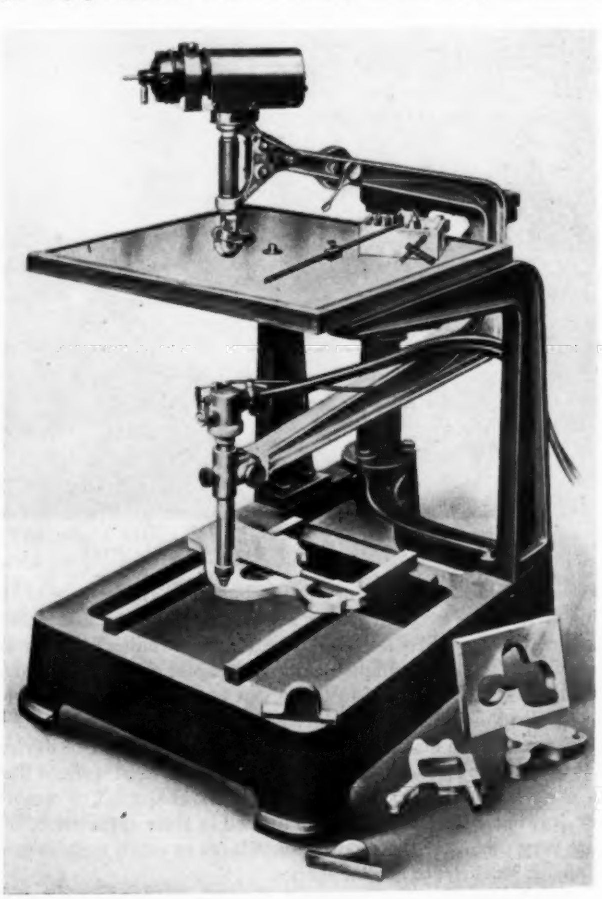

Fig. 6 Small Model of Direct Profiling Machine

Machines of this type are usually automatic, the templet-tracing device being arranged to follow the templet shape either magnetically or mechanically by gripping rollers. (See Fig. 7.) It should not be assumed that an automatic oxygen shape-cutting machine working from a templet is suitable only for quantity production; the process can be used to considerable advantage for the production of small quantities of irregular shapes. It is often asserted that the cost of making the templet is out of proportion to the job when only small quantities are required. This is probably true if the article required is only in thin plate, but as the thickness of plate increases, so the cost of making a templet decreases when compared with the cost of producing the required article by other methods. Obviously on material 1 in. thick and upward the saving by oxygen cutting is considerable, even including the time taken to make the templet.

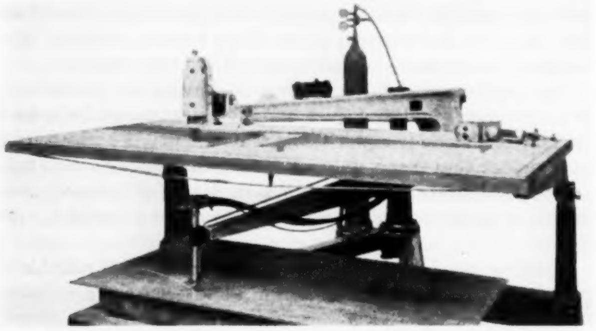

FIG. 7 DIRECT PROFILING MACHINE, SHOWING AUTOMATIC HEAD

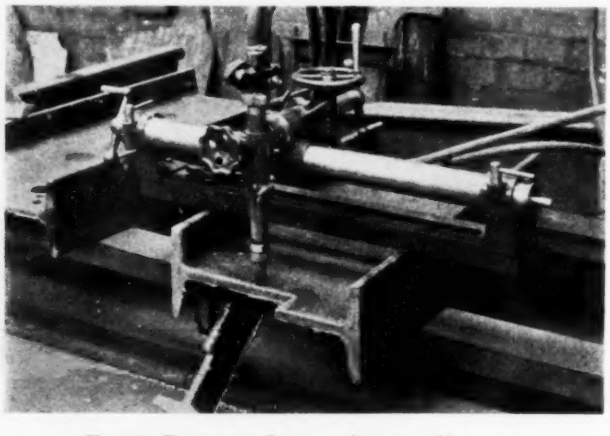

FIG. 8 PORTABLE OXYGEN CUTTING MACHINE

The templet has the advantage that it can be stored and used at any subsequent time for further production. In the case of the metal templets necessary with a very popular type of machine in Great Britain there is no deterioration due to climatic or storing conditions, wear is negligible, and the templet can be adjusted so that a very fine degree of accuracy can be obtained. Such templets also take up very little space.

Some of these machines employ mechanical guides and devices by means of which shapes that are a combination of straight lines and circles can be cut automatically without the aid of templet or drawing. Such machines are limited in their application, but are very useful for the production of large or small quantities of flanges, brackets, or similar shapes.

Apart from the vast potential applications of the automatic shape-cutting machine, there are many less obvious applications of the process which have become established in a large number of industries.

For instance, oxygen cutting in connection with constructional steel work is developing rapidly. Apart from the problem of shaping plates, gussets, brackets, cleats, reams, column bottoms, and other items, the process can be used for shaping steel joists and other sections used in this class of work.

There are several machines suitable for this purpose, some of them portable (see Fig. 8), enabling cutting to be done on site or in the stock-yard, while others are shop tools equivalent to the hacksaw, which can perform a variety of operations, such as parting off, beveling, notching, etc., after the joist has been brought into the shop. Such machines also are suitable for cutting up bar and strip plate stock. Further than this, holes now can be cut by means of a special oxygen cutting machine.

Of special interest to railway companies is a machine that has recently been introduced for parting off rails. Such a machine is extremely portable and weighs very little more than a hand cutter, but of course the cut has all the finish of a machine cut. As previously mentioned, oxygen cutting will deal with manganese steel just as easily as ordinary mild or carbon steel, and this is a decided advantage when cutting rails. Whereas it is almost impossible to drill or saw manganese rails on the track, an oxygen hole-cutting or parting-off machine will do the work easily and quickly.

There are also many machines on the market for straight and circle cutting, the variation being chiefly in the capacity of the machine. Such machines are extremely useful to shipbuilders, tank and boiler makers, etc., especially where cuts have to be made in plates that would be too big to go on the usual automatic machines. Such machines will generally cut bevel edges, so that if the article is being cut for welding it can be passed direct to the welders from the cutting machine.

Oxygen cutting machines are not subject to the strains and excessive wear of the mechanical tool, and should therefore have a fairly long life if properly attended to. Compared with the extraordinary saving that often can be made by employing this process in place of other methods, the cost of the equipment is very low. According to size an oxygen shape-cutting machine will cost between £75 and £1500 ($275 and $5500),[2] the cheaper machines being suitable for the small workshop, while the higher priced ones are machine tools fitted with every possible refinement and capable of producing a complete locomotive sideframe at one setting.

Portable appliances, such as those described for constructional and railway work, do not cost more than £30 or £40 ($110 or $147), while appliances for hole cutting and parting off rails are at the bottom of the price scale at £12 to £15 ($44 to $55).

It is usual in the case of oxygen cutting machines which are installed in the workshop as permanent production units, to convey the oxygen at suitable pressure by pipe lines and cylinders located outside the workshop in a convenient position for the replenishment of supplies. Other gases such as acetylene or coal gas also can be piped to the machine from storage, so that there need be none of the inconvenience or risk of having cylinders or acetylene generators located in the machine shop.

SPEED AND QUALITY OF CUT

As mentioned before, the chief features that make oxygen machine cutting so totally different from hand cutting are the rigidly held cutter and the uniform rate of travel, resulting in a remarkably smooth and accurate cut, which is, moreover, produced at a much faster speed than hand cutting.

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[2] In converting the pound sterling into the American dollar, the exchange rate of £1 = $3.67, prevailing on May 12, 1932, has been taken. The shilling is taken as equivalent to $18^1/_2$ cents and the penny as $1^1/_2$ cents.

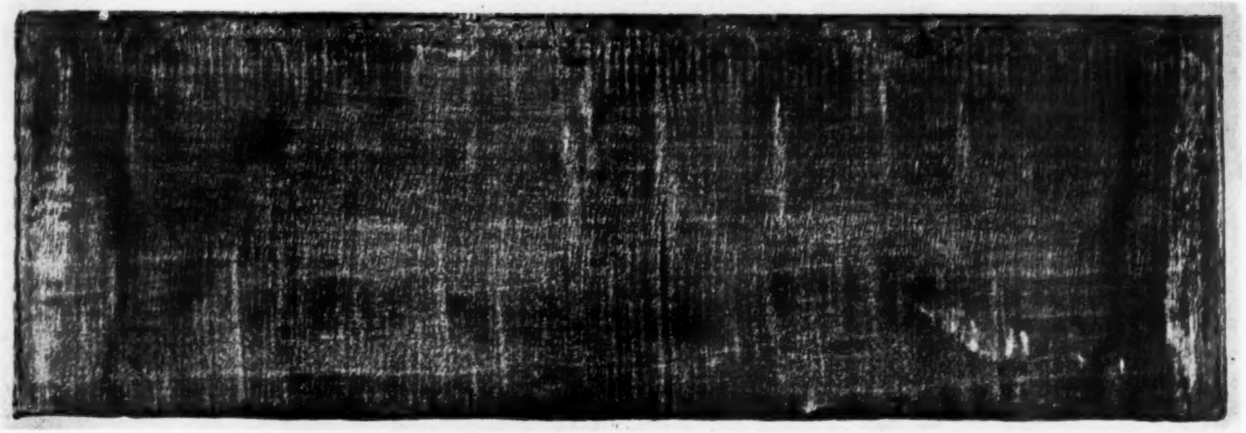

FIG. 9 FACE OF CUT MADE AT NORMAL SPEED

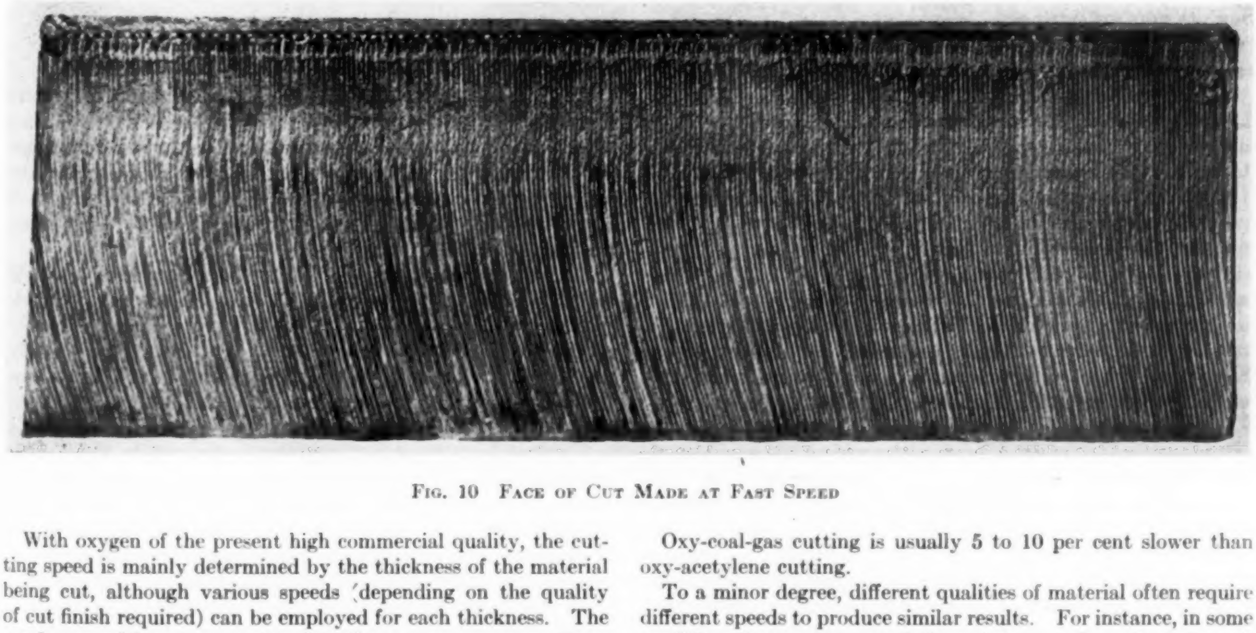

FIG. 10 FACE OF CUT MADE AT FAST SPEED

With oxygen of the present high commercial quality, the cutting speed is mainly determined by the thickness of the material being cut, although various speeds (depending on the quality of cut finish required) can be employed for each thickness. The cutting speed is not proportional to the oxygen pressure; 10 or 15 per cent increase in oxygen pressure may allow the speed to be increased 40 or 50 per cent on thin material, but on thick material it may be necessary to increase the pressure 30 per cent to obtain a 20 per cent increase in speed.

If the article is to be machined after cutting, a roughing cut can be made at a fairly fast speed, but if the article is to be used as produced, a good smooth finish is generally desired, in which case it is advisable to use a slower speed. Straight and large circular cuts always can be made at a faster speed than profile cuts, as shape cutting to a templet often involves abrupt changes in direction and a fast speed will cause the cut to drag, so that the bottom of the cut may not register quite accurately with the top. Fig. 9 shows the face of a cut made at normal speed and Fig. 10 at fast speed. The following table shows typical oxy-acetylene cutting speeds:

	Thickness of steel, in.																
	$^1/_2$	1	$1^1/_2$	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Normal speed, ft per hour.......	60	46	40	35	26	23	22	21	21	20	20	19	18	16	15	14	14
Fast speed, ft per hour.......	90	65	55	50	40	35	30	25	—	—	—	—	—	—	—	—	—

Oxy-coal-gas cutting is usually 5 to 10 per cent slower than oxy-acetylene cutting.

To a minor degree, different qualities of material often require different speeds to produce similar results. For instance, in some qualities of manganese steel the cutting speed can be increased to almost double the speed required for cutting mild steel in order to produce results similar to those usually obtained on mild steel of the same thickness.

The speed attainable in oxygen machine cutting facilitates increased production in comparison with other shaping methods, such as sawing, slotting, or shaping. This applies especially to large shapes which are often drilled out of plates, afterward being smoothed up to the required size by slotting or milling.

To give an example, concerning locomotive sideframes, one firm using an oxygen cutting machine is now able to produce a complete locomotive sideframe in 6 hours, compared with 30 hours by the previously used mechanical shaping methods. This example is for ordinary frames $1^1/_4$ in. thick; when bar frames about 4 in. thick are being dealt with, the saving is increased proportionally.

FUEL GAS

An oxygen cutting machine requires to be supplied with two gases, oxygen and a fuel gas. The function of the fuel gas is to provide the necessary heat value in the flame which is used for

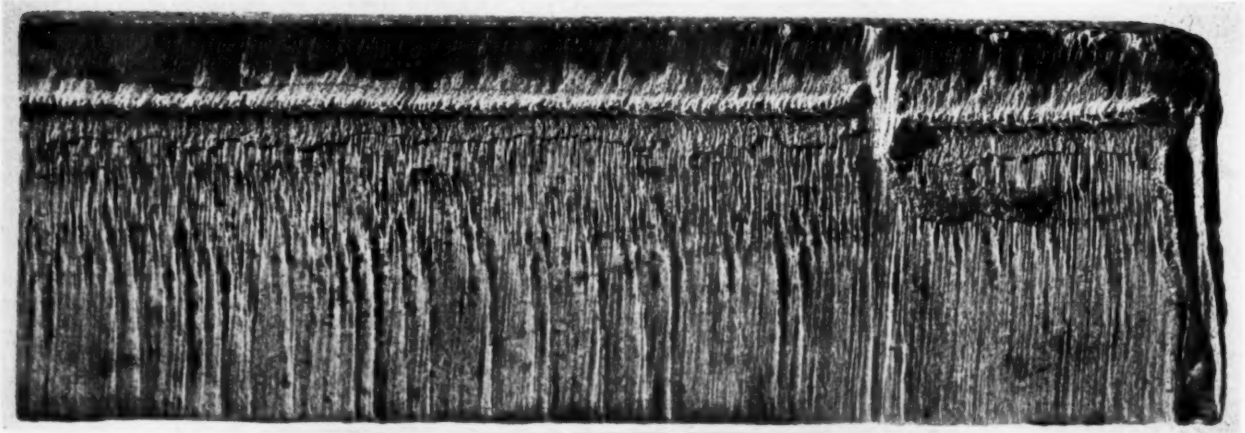

Fig. 11 Bad Cutting as Result of Cutter Nozzles or Jets in Poor Condition

preheating prior to commencing the cut and for maintaining the requisite heat for cutting.

Three fuel gases are, as a rule, commercially available—namely, acetylene, coal gas, and hydrogen; and the question not unnaturally arises, which is the best to use? No hard-and-fast ruling can be made on this point, as the choice of a suitable fuel gas will probably be decided by the conditions under which the machine is to be used.

From the point of view of convenience, coal gas is often preferred, as not only is coal gas usually available in an engineering shop, but it provides sufficient heat for the process consistent with economy and a fair cutting speed. The oxy-coal-gas flame is not easily mishandled, and is therefore specially suitable for use by comparatively inexperienced operators. It is especially suitable for use when cutting thin material, say between $^1/_8$ and 3 in., and its limit of usefulness is generally considered to be reached on material 8 in. thick.

A popular alternative to coal gas is acetylene; this has the advantage that it enables a faster cutting speed to be employed, while the oxygen consumption is generally somewhat less. The flame may be a little more difficult for an inexperienced operator to adjust, and there is a risk in such cases of burning the metal, especially on thin material.

Acetylene is available in two forms: that produced by a generator on the spot and that compressed in cylinders. Generated acetylene is cheaper, but a disadvantage is that a generator is not very suitable for intermittent operation, and this not infrequently leads to a considerable waste of gas. The acetylene generator cannot be housed inside a factory building, and an outside shed or other covering is usually necessary. Water supplies and drainage and means for disposal of carbide sludge also must be available. Compressed acetylene has the advantage that there is no mess and there is no waste when not in use. If sufficient supplies are kept in hand, it is always ready when required, and so is suitable for intermittent operation.

Acetylene is suitable for cutting all thicknesses up to 18 in., but if not skilfully employed, is likely to produce too much heat for very thin plate.

Hydrogen is an excellent fuel gas for cutting, is similar to coal gas in its application, and has the advantage that it is equally suitable for cutting thin or thick sections. Its employment usually is prohibited by the fact that it is only available compressed in cylinders, and its freightage makes it expensive unless it is employed near a compressing station. It is not so easily obtainable as coal gas or acetylene.

Tests have proved that, provided the flame is properly regulated, no difference in depth or intensity of effect on the cut metal is noticeable whatever fuel gas is employed.

Cost of Cutting

The power required to drive a mechanically operated machine is usually supplied by a motor of $^1/_8$ to $^1/_4$ hp, so that the running costs are extremely small, the maintenance being correspondingly low. Therefore, the main factors entering into the cost of cutting are the oxygen, fuel gas, and operator's time. Of these the gases form the largest item; the operator's time usually costs much less than the gases.

Assuming oxygen to be available at the cutting machine at the rate of 25 shillings ($4.58) per 1000 cu ft and the proportionate quantity of fuel gas at about one-quarter of this amount, and the operator's time to be 1 shilling 6 pence (27 cents) per hour, the average cost of cutting per inch thickness per foot run of material varies from $1^1/_2$ pence ($2^1/_4$ cents) on thin material (say 1 in. thick) to $3^1/_2$ pence ($5^1/_4$) cents on material about 14 in. thick.

The first foot of cutting is generally assumed to cost a little more than this, as the time and the gas involved in preheating the metal immediately prior to cutting have to be considered. As the thickness of the material increases, this preheating cost becomes a considerable item when compared with the cost of cutting per foot run. Whereas the cost of preheating on material 1 in. thick could almost be neglected, the preheating cost on material about 14 in. thick may amount to as much as 6 pence (9 cents).

The foregoing figures are of course averages taken over a number of cuts under varying conditions; and variations in the cost of gases, operator's time, etc., will affect them to a slight extent. These figures of course refer only to running costs; idle time, depreciation of machine, setting up costs, etc., must be added to obtain a real cost figure.

It is not usually necessary to clamp down material that is being cut with oxygen; the material has only to be laid on the cutting table supports. Therefore the time taken to shift the material between cuts is reduced to a minimum.

Rigid clamping of work to be cut might result in buckling and distortion due to expansion of the metal while cutting, but there is no reason why this expansion should not be controlled by means of suitable stops.

Apart from the low cost of the actual cutting operation, considerable economy in material can be effected. Many jobs are so shaped that by careful planning one can be cut within the other; in fact, it often is possible to make arrangements so that part of the cut on one shape suffices for part of the next shape to be cut.

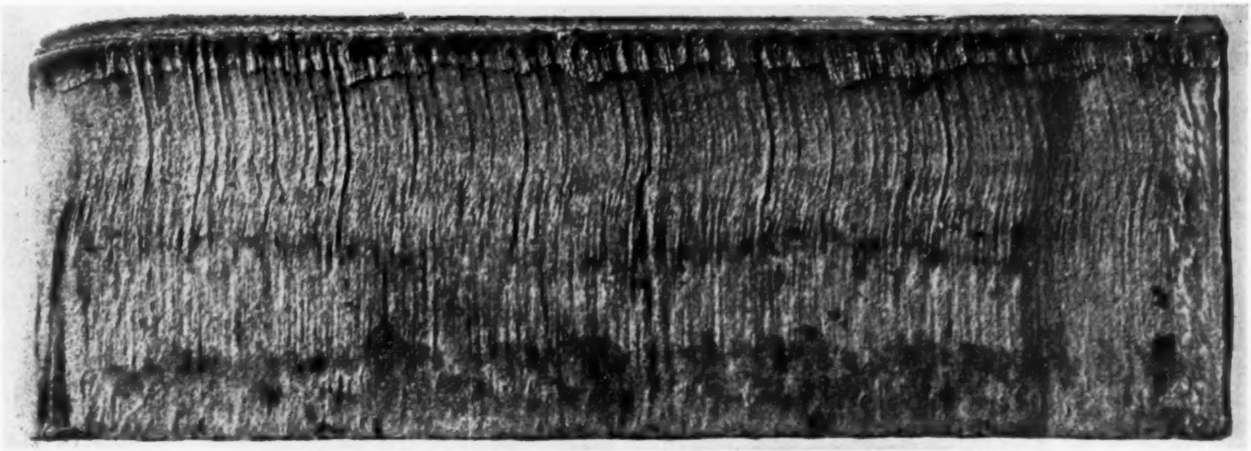

Fig. 12 Bad Cutting as Result of Cutter Nozzles or Jets in Poor Condition

Apart from this method of working one job in with another, it will be found that the scrap left over from one job can be utilized for the production of smaller shapes. For instance, the pieces cut from the crankshaft shown in Fig. 2 are found very useful for toolroom blocks. Of course, the only material that really is scrap and unrecoverable is that discharged from the small kerf left by the cutting jet, and the width of this varies from about $^1/_{10}$ in. in thin material, say $^1/_2$ in. thick, to about $^1/_4$ in. on 8-in. and 10-in. material.

Practical Considerations

In the same way that a mechanical tool will not give good service unless it is maintained in good condition, with sharp tools and operated intelligently, so it is necessary to exercise the same care with an oxygen cutting machine. The best oxygen cutting machine in existence will not produce good smooth cuts unless the cutter nozzles or jets are kept in good condition, free from burrs or particles of dirt or oxide, which will tend to partially obstruct or divert the gas stream. Some idea of the bad results caused by dirty jets will be gained from Figs. 11 and 12.

There has been a tendency, largely due to earlier experience gained with the hand cutter used as a scrapping tool, to regard the oxygen cutting machine as nothing more than a roughing process, with consequent carelessness in fixing the machine suitably. Fortunately, the true value of the process as an accurate and reliable production method is now realized, but even now it is necessary to point out that machines should not be placed in positions where they are subject to floor tremors or vibration caused by the operation of nearby hammers, punches, presses, or guillotines (shears). The vibration will be evident in the cut, and the operator will find it impossible to produce the smooth clean cuts that are characteristic of good machine cutting.

The manufacturers of oxygen cutting machines have now done a considerable amount of experimental and research work in connection with the correct oxygen pressures and cutting speeds suitable for various thicknesses of material and charts, and these and other details necessary for the operation of the machine are supplied by the makers. Provided the operator reads his instruction book and carries out the instructions, there is no reason why he should not be able to produce good cuts right from the start. The only item that is left to the operator's judgment is the adjustment of the cutter to produce the correct heating flame, but here again charts usually are provided, and a little practice soon enables the operator to judge the correct flame for any particular class of work.

Cutting must start from an edge, and when cutting a quantity of shapes from a piece of steel plate, this is quite easy. The first cut being started from the edge of the plate, subsequent cuts are started from the hole left by the previous cut. If it is necessary to cut a hole within a shape or a piece of plate, such as the hole in a flange or spring buckle or a manhole in a piece of boiler plate, it is necessary to drill the hole for starting. A hole $^3/_8$ in. or $^1/_2$ in. in diameter is usual, but the larger the hole the easier it is to start; $^3/_8$ in. is suitable for material 1 in. thick, but anything from $^1/_2$ in. to $^3/_4$ in. would be desirable on material over 6 in. thick.

Blowing through or piercing the plate by means of the cutter itself is sometimes done, but this is recommended only up to 1 in. thick. Greater thicknesses than this can be pierced with the hand cutter, but it is not recommended that this be done with the machine, owing to the risk of burning the nozzles through particles of molten metal or slag being blown upward; moreover, some of the slag is likely to adhere to the nozzle, thus diverting the oxygen stream and causing bad cuts. Starting on round surfaces, such as the edges of forged slabs or round bar, sometimes causes difficulty. To minimize this difficulty a piece of iron welding wire should be applied to the point where the cut is to be started. If this is done, with the heating flame alight, the wire will melt and a blob of metal will adhere to the material, quickly forming an incandescent spot, which will start burning readily when the cutting oxygen is turned on.

Industrial Applications

The remarkable result obtainable with the oxygen cutting process has led to its adoption as a recognized production stage in most of the industries employing steel plate or forgings in the course of manufacture of their products.

The ability of the cutting machine to produce intricate shapes from steel plate or slabs of almost any thickness has induced many manufacturers to dispense with forgings, with a corresponding decrease in overhead charges, machining, and waste-material costs. Moreover, steel plate or slab forgings are useful stock, and factories not possessing their own forge are able to produce special jobs at short notice with the oxygen cutting machine.

As an instance of this class of work, a firm employing an oxygen cutting machine now produces a brake arm in 10 min that previously took a smith and striker $2^1/_2$ hours at 2 shillings 6 pence (46 cents) per hour to forge, and after that the forging had to be machined. The oxygen-cut article is not machined, and the scrap metal can be used, whereas previously the articles were cut off from the solid bar and bored out, leaving no usable metal.

In another instance concerning the cutting of two mast rings out of material over 9 in. thick, the rings were $4^{1}/_{2}$ ft over the lugs, and it is believed that the wage cost in forging would have been about £36 ($132), whereas the oxygen cutting cost was about 70 shillings ($12.83).

Apart from the possibility for some classes of work of substituting forgings by oxygen cutting, parts which are better forged can often be produced more economically by leaving intricate shaping to the cutting machine. The ship stern frame shown in Fig. 5 is an instance of this class of work.

Some firms are cutting blanks to profile prior to shape forging, notably for locomotive components.

One of the greatest fields of development for the oxygen cutting process, in conjunction with oxyacetylene or electric welding, is the fabrication of parts to substitute castings; a cutting machine is indispensable to firms undertaking this class of work.

Fabricated parts compared with castings have many advantages; for instance, no blowholes or sponginess of metal are encountered. More reliable finished parts are obtained, pattern-shop charges are eliminated, and the necessity of carrying heavy stocks is obviated, on account of the fact that any part can be manufactured at short notice. The designer is not restricted by limitations of casting, steel fabrication affording him considerable latitude and scope. Changes in design, alterations, and additions can be made at any time; generally speaking, fabricated parts are lighter than corresponding castings; the reduction in weight lessens transport and freightage costs, while the danger of breakages occurring in transit abroad is considerably reduced.

A printing-machine manufacturer who is now fabricating his machines entirely with steel claims that owing to the additional strength and rigidity of the machine it would now stand up to a much higher speed than hitherto obtainable.

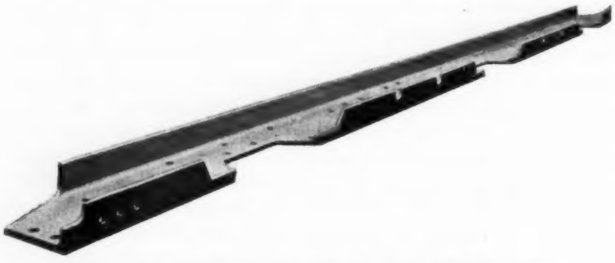

Fig. 13 Side Frame Cut on Universal Machine

Electrical-equipment manufacturers have adopted fabrication to a very large extent. Some of the parts that they now are fabricating are a-c machine sideplates, stator and alternator frames, wrapper plates, rings for armature end shields, rotor end plates, foundation plates, switch covers, switch cubicles, and bearing housings.

A particularly noteworthy job recently executed is the cover for a large oil-immersed circuit-breaker switch, measuring over 5 ft in diameter, weighing about 1 ton, and tested to a pressure of 500 lb per sq in.

Apart from the savings resulting from the substitution of castings and forgings by oxygen cutting, there are many instances where oxygen machine cutting replaces operations previously carried out by drilling, sawing, and shaping, with considerable reduction in production time and cost. The instance of locomotive frames given previously is a case in point. The ability to cut plain or intricate profiles with equal facility enables many jobs to be done that otherwise could not be undertaken. Another instance can be given of a firm cutting slots in large-diameter steel pipes, with a tube thickness of $1^{1}/_{4}$ in. The cost by oxygen cutting works out at $2^{1}/_{4}$ pence ($3^{1}/_{3}$ cents) per slot, whereas the cost previously was 2 shillings 3 pence (41 cents).

The railway companies and railway-equipment manufacturers oxygen-cut large quantities of spring buckles. These are generally cut from bar, and what was originally a fairly expensive forging, which had to be machined, is now produced by the oxygen cutting machine and taken direct to the smith's shop to have the spring leaves fitted. This is some indication of the consistent accuracy that is obtainable by oxygen cutting.

Many machines are in use for cutting for die work, forgers and stampers finding it a great advantage to be able to rough out the centers of their stripping dies by this method.

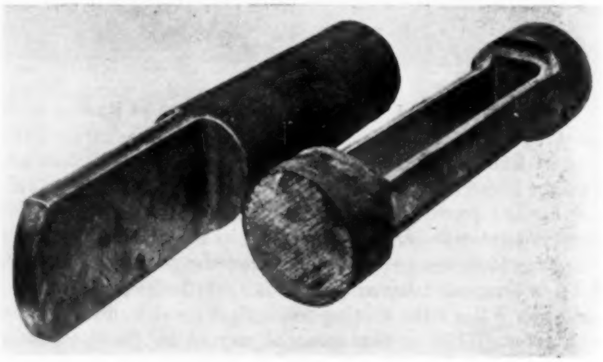

Fig. 14 Round Bar as Shaped for Turnbuckles, Yoke Ends, Eyes, Forks, Etc.

The use of oxygen machine cutting is not limited to cutting from flat steel plate or forgings; it is quite common practice to employ the process for shaping brackets, links, and similar parts on stock angle, tee, and channel sections. (See Fig. 13.) Round bar also can be employed with advantage for many jobs, such as lifting eyes, turnbuckles, connectors, etc.; a square end can be cut on a round shaft much quicker than it would be forged. (See Fig. 14.)

In many engine shops it is common practice to employ the process for cutting out the ends or jaws of connecting rods after they have been forged. On this class of work there may be considerable variation of thickness throughout the cut.

Ship repairers owning oxygen cutting machines have used them very effectively on urgent ship repairs. During the winter gales a cargo boat put into a South Coast yard with damaged steering gear. To have obtained from the Midlands the large forgings that were required would have meant several days' delay, but the oxygen cutting machine was brought into service and the parts were produced in about two days.

Another job recently recorded was the cutting out of certain sections of a three-throw crankshaft; the sections had faulty shaft extremities. The crankshaft weighed 8 tons and was in three sections, the shaft and pin diameter was 14 in., and the web thickness was 9 in. Three cuts through the 14-in.-diameter shaft were made at various places, each cut taking 7 min, whereas $2^{1}/_{2}$ hours would be required for sawing.

To remove the pieces of shaft remaining in the web, cores were cut out, leaving shells approximately $^{3}/_{8}$ in. thick, which could easily be withdrawn, leaving the web undamaged.

MSP-54-11

Researches on Steels and Forgings for Greater Density, Machinability, and Durability

By FRED W. CEDERLEAF[1] AND W. E. SANDERS,[2] MUNCIE, IND.

F. W. CEDERLEAF

In this paper the authors give details of a research carried out in an endeavor to find means of improving the machinability of gear blanks upset from bar steel, as well as the durability of the finished gears after heat treatment. The investigation shows that a steel of coarse grain upsets into a denser product than one of finer grain, and that this denser material is more readily machined, and with much less resultant tool wear. Specifications are given for the steel finally adopted and also details of its heat treatment, as well as figures from dynamometer tests which indicate a longer life for the wearing surfaces of the teeth on the gears made therefrom, and a marked decrease in pitting and granulation.

W. E. SANDERS

THE purpose of this paper is to deal with the improved machinability and durability obtained by the greater working or compressing of steel in the process of forging; and also to study the various steel characteristics affecting the development of greater density in forgings. The result of this working or compressing is referred to as density, but in the sense of greater compression rather than that of specific gravity.

The steels used in these experiments, other than those specifically mentioned, were S.A.E. 2320, 4615, 5140, 1315, 3140, 3240, 2515, and 2520.

I—THE THEORY OF POROSITY IN DEEP-ETCHED FORGINGS

Nearly every one having to deal with steels and forgings has at some time or other cut pieces in two, polished the faces and etched them, and after etching has found a number of tiny cavities or voids occurring at various places, sometimes more prominent at the center or core and sometimes near the surface. The location, nature, and extent of these voids or cavities, as brought out by the etch test, determine the degree of density of any forging. The authors' theory is, substantially, that in forging it is possible to produce internal slipping or tearing of the fibers in the steel.[3,4,5,6,7] This slipping, or tearing, is indicated by the voids or cavities brought out by acid etching. Some previous papers on the subject attribute the general porosity in etched forging sections entirely to the quality of the steel, this quality being determined by the amount and nature of the non-metallic substances present.[8] With this the authors cannot quite agree, although they recognize that sulphur and other impurities existing in the steel to too great an extent will make the grain appear coarse or open. In addition to this, however, they are endeavoring to bring out the fact that the largest percentage of openness or coarseness is due to torn fibers; also, that there are some steels of the same analysis, containing approximately the same amount of non-metallic impurities, which after proper forging do not show this porosity.

Fig. 1 represents an etched forging section in which it is believed that non-metallic substances are entirely responsible for the appearance. On the other hand, Fig. 2 is a typical example in which it is thought that forging has played an important part in developing the open etched condition. Fig. 3 exhibits the same characteristics observed in Fig. 2 at ten times higher magnification.

Having observed a large number of etched forgings in which the porosity or density was almost negligible in some and quite pronounced in others, the authors were inclined to associate these conditions with the variables in machinability in the machine shop.

About this time, in machining a number of 3.50 per cent

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Taro Ueda. "The Effect of Cold-Working on the Density and Electric Resistance of Metals." Science Reports of Tohoku University (Japan), vol. 19, 1930, pp. 473–498.

[4] Zay Jeffries. "The Plasticity of Metals." MECHANICAL ENGINEERING, vol. 53, April, 1931, pp. 262–266.

[5] Earle E. Schumacher. "The Nature of Metals in Relation to Their Properties." *Scientific Monthly*, January, 1932, pp. 22–30.

[6] Herbert F. Moore and Tibor Ver. "A Study of Slip Lines, Strain Lines, and Cracks in Metals Under Repeated Stress." University of Illinois Bulletin, no. 208, June 3, 1930.

[7] A. T. Adams. "Wire Drawing and the Cold Working of Steel."

[8] E. R. Frost. Article in National Forging Machine Talks, No. 72. National Machinery Co., Tiffin, Ohio.

nickel (S.A.E. 2315) gears, it was noticed that some of them, after being drilled and broached, showed a very smooth finish in the hole, broaching with considerably more ease than others that had been forged and annealed at the same time. Some of the gears tore badly in the drilling. The broach, being hydraulically actuated, pulled very unevenly, showing that greater energy was required to force it through, and the holes were torn and very rough, as shown in Fig. 4.

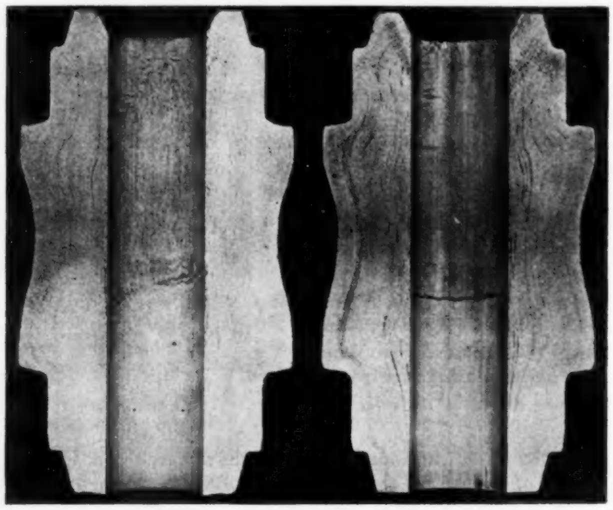

Fig. 1 Forging Showing Open, Non-Metallic Inclusions

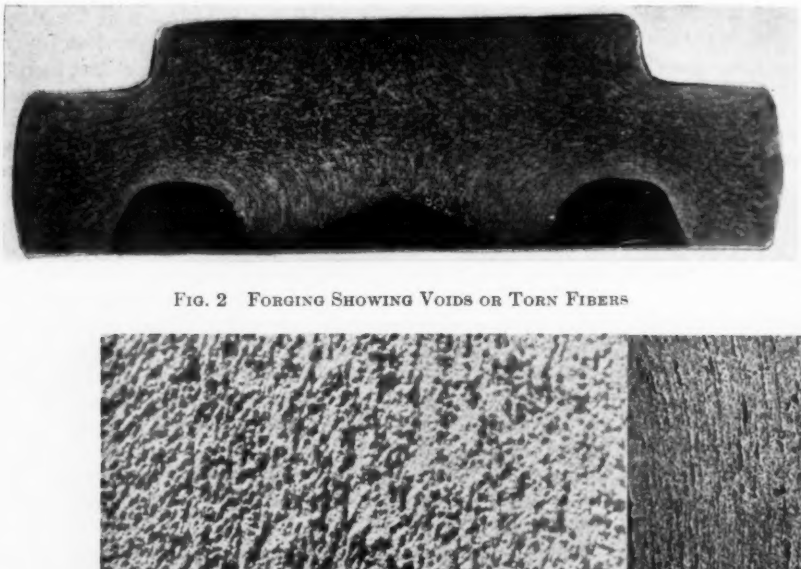

Fig. 2 Forging Showing Voids or Torn Fibers

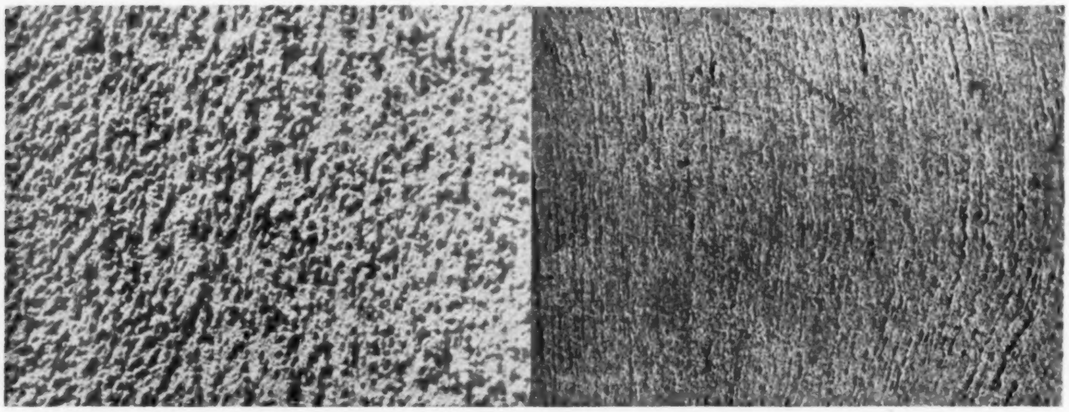

Fig. 3 Improved Structure by Greater Compression

Characteristics Observed in Fig. 2 at 10×

Improved Structure by Greater Compression

After studying these forgings, it was found that some of them had round corners while others had square ones. This indicated a difference in the filling of the die at the time the pieces had been forged. It was then found that by machining a round-cornered, or unfilled, piece a torn condition in the hole was produced, but when machining a square-cornered, or filled, piece a very smooth surface was obtained. The same procedure was repeated several times with both filled and unfilled forgings, and the same results were obtained. This suggested a study of forge-shop conditions.

II—Forging Practice and Its Relation to Density

Investigating conditions in the forge shop, it was found that a higher and more uniform temperature, namely, 2150 F, must be maintained, and that this temperature must not vary more than 100 F; the dies must be properly designed so that in no operation would a flash be created which would minimize the compressing of the metal by the dies in the subsequent operations; and the flash must be longitudinal with the heading tool, or there must be a flash pocket to prevent the metal from flowing between the dies so that the compression would not be on the flash but on the forging.

After correcting the dies and forging temperatures, a marked improvement was found in machinability. The next thing was to determine whether, by filling the dies and controlling the flash, maximum machinability had been attained, and whether anything further could be accomplished by compression of the metal. It was also desired to learn the actual effect of the forging temperature on machinability. As far as could be determined, the only effect accomplished by proper heating for forging was that of permitting satisfactory flow of the metal to fill the die.

To determine further whether still greater compression would improve machinability, pieces were upset, reheated, and struck several blows by a board hammer. These pieces were then used for machinability tests. Like pieces were also upset, reheated, and struck several blows by the hammer; the flashes were then ground off and the pieces reheated and again put under the hammer, and the same operation repeated the third time. In each of these tests it was found that the repeated hammering or compressing of the metal improved its machinability. The pieces which had been repeatedly heated and hammered showed upon etching that the original cavities had not been entirely closed.

However, there was an evidence of adhesion at the cavity surfaces, but not of cohesion. This confirms Emil Gathmann's statement[9] to the effect that rolling of an unsound ingot can never result in more than intermittent adhesion of the metal at the piped or blowhole portion, and cohesion, not adhesion, is necessary for strength and reliability in any portion. It has been shown that it is only possible to effect reasonable adhesion of the original cavities produced in the first forging by further hammering and compression. The next thought was to study the characteristics of the steel prior to forging that might produce more ductile fibers, thus promoting flow of the metal without tearing.

This suggested experiments having as their object a coarsening of the structure of the forging bars prior to heating for forging. It was decided to accomplish this by annealing at high temperatures. A bar of 0.50 carbon, 1.00 chromium steel was selected which had a medium-fine structure in the rolled condition. Forgings were made from one end of this bar, after which the remainder was cut into lengths suitable for making the same kind of forgings. These pieces were then annealed by furnace cooling after soaking for one hour at 1650 F. Half of these forging lengths were then upset into forgings like those which had been produced from the original bar. The remaining pieces were then reannealed by furnace cooling from 1750 F after soaking one hour, and were then upset into the same kind of forgings. Microscopic specimens, prepared from the bars in each group before forging, showed the structure to be coarsened considerably after each anneal.

The three groups of the forgings were then cut and etched. Typical examples of each group are shown in Fig. 5, *A* representing the structure of the forging made from the untreated bar; *B*, after the first anneal; and *C*, after the double anneal. It will be observed from this figure that there is a progressive decrease in the amount of torn fibers from *A* through to *C*, and a like increase in the continuity of the flow lines. The increased density of forgings *B* and *C* made it necessary to etch them 10 minutes longer to bring out the flow lines. This experiment is significant in that it suggested that a coarse structure has more ductile fibers, leading to further research on steels which are inherently coarse-grained.

III—Relation of McQuaid-Ehn Grain Size in Steel to Density in Forgings

It is desirable at this point to explain the meaning of inherent grain size, or, as it is commonly spoken of, the McQuaid-Ehn grain size. This is the grain size which is developed in steel when it is subjected to carburizing at 1700 F for a minimum of eight hours and allowed to cool slowly. This procedure brings out a carbide network in the case outlining the grain boundaries, which, when examined under the microscope at 100 magnification, shows the number of grains per unit area. This number, or the size of the grains at the specified magnification, is compared with certain standard charts prepared for the purpose of rating the grain size of any particular lot of steel. An example of one of these charts is shown in Fig. 6. For instance, if a sample is carburized as stated above and if upon examination it is found to look very much like photomicrograph No. 3 in Fig. 6, this steel would then be given a McQuaid-Ehn grain-size rating of No. 3.

In view of the previous statements in regard to the higher

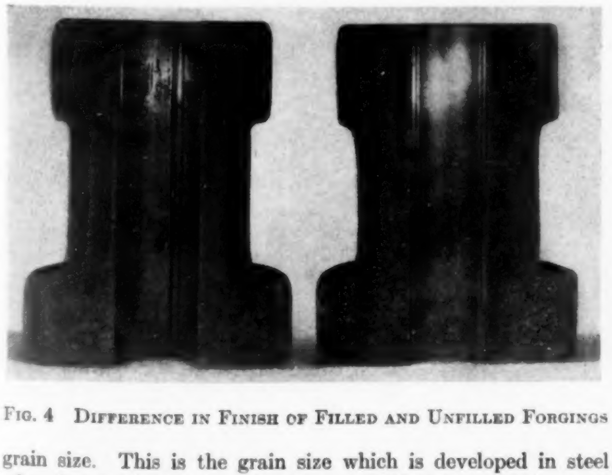

FIG. 4 DIFFERENCE IN FINISH OF FILLED AND UNFILLED FORGINGS

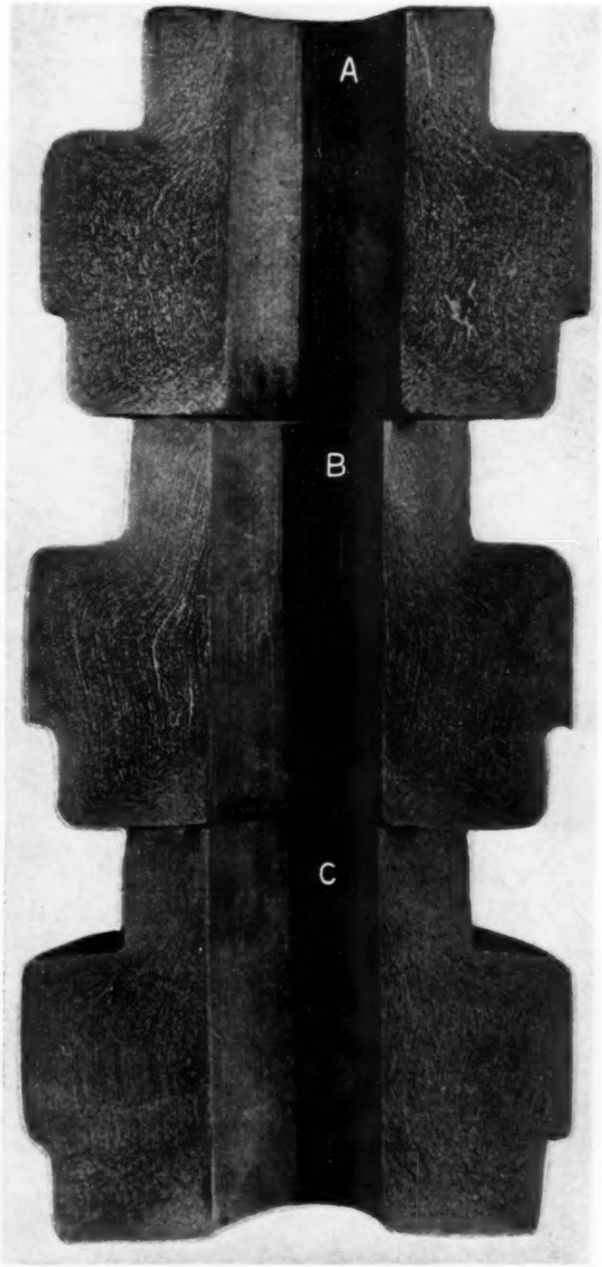

FIG. 5 STRUCTURE OBTAINED BY ANNEALING BAR BEFORE FORGING
(*A*—As forged from bar. *B*—As forged after first anneal. *C*—As forged after double anneal.)

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[9] Emil Gathmann. "Ingot Contour and Its Relation to Sound Steel."

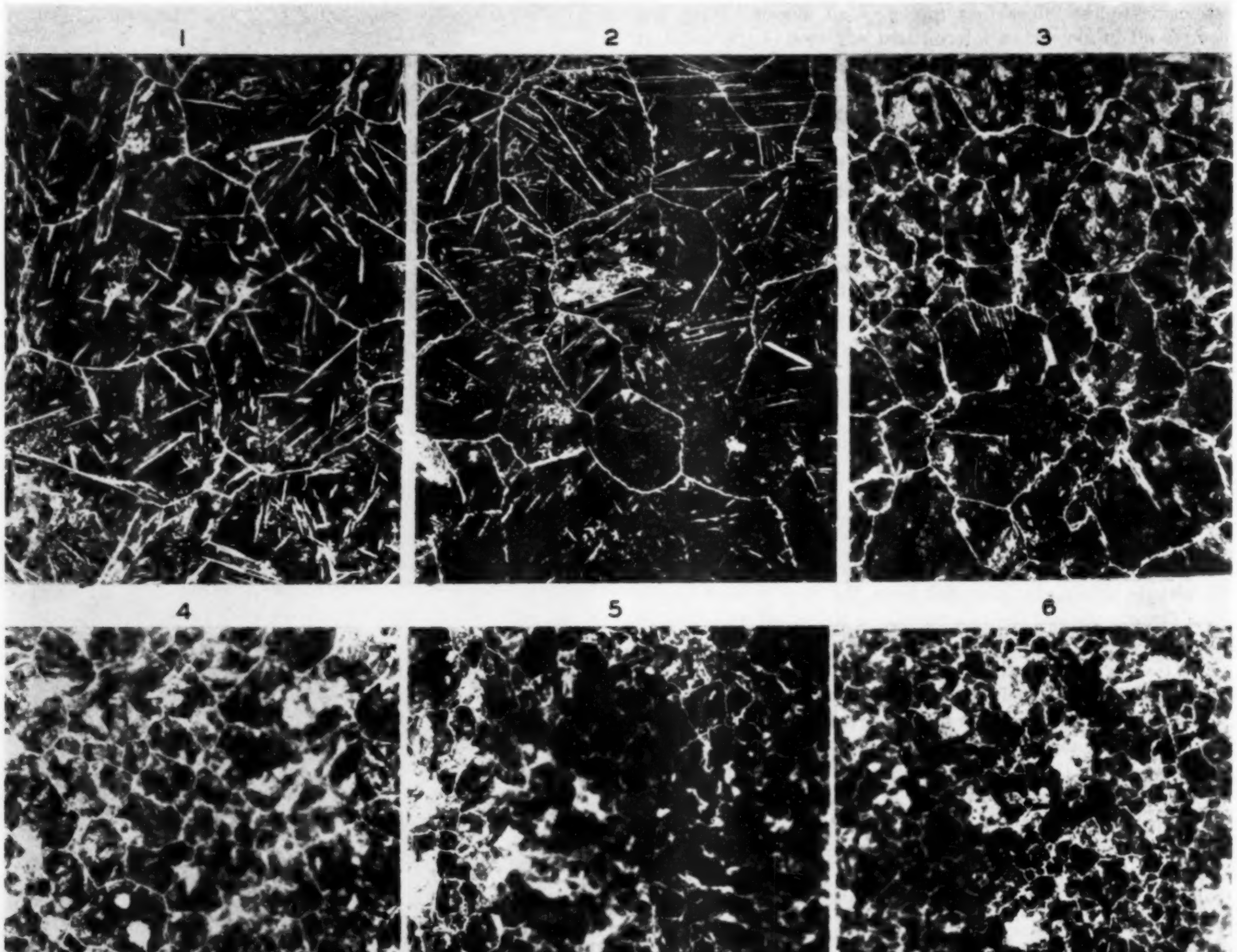

Fig. 6 The McQuaid-Ehn Grain-Size Chart (×100)

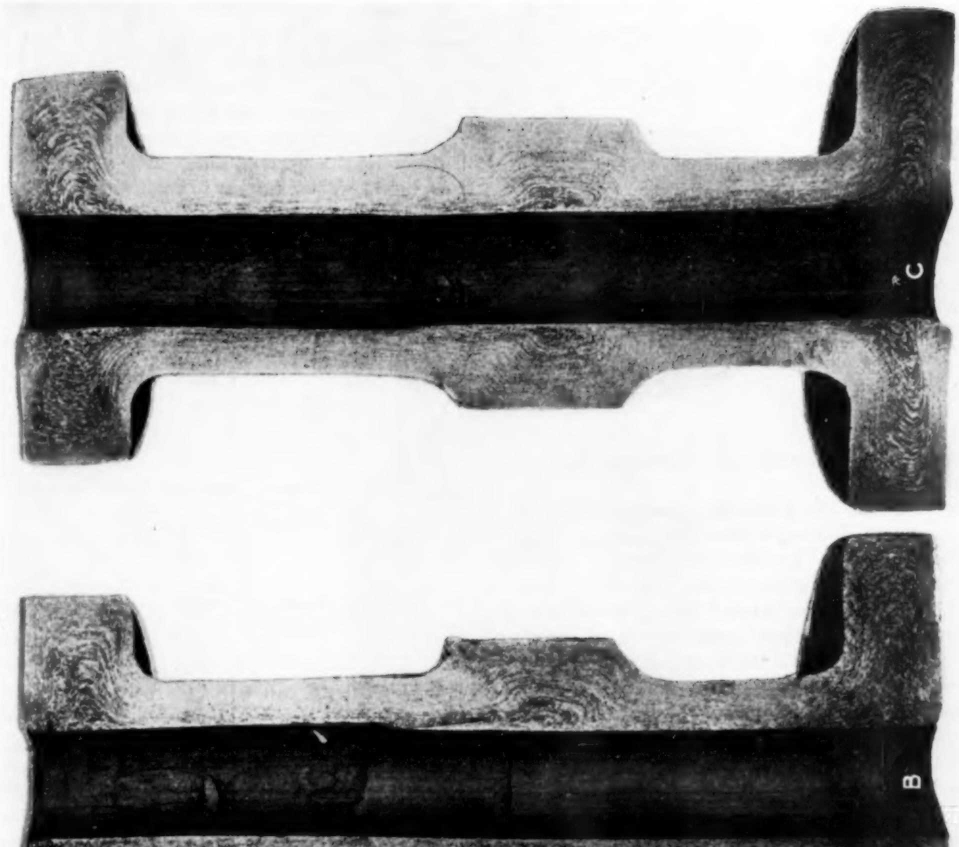

FIG. 7 EFFECT OF GRAIN SIZE IN FORGINGS

ductility of the coarser structures as produced by high annealing, it was thought advisable to investigate a variety of steels having different McQuaid-Ehn grain sizes. This was done on the assumption that the coarser-grained steels would exhibit forging properties different from those of finer-grained steels. Arrangements were made at this time with the steel companies to furnish a few bars having McQuaid-Ehn grain-size ratings from 4 to 9. These tests were made to determine the effect, if any, of the grain size of steel on upsetting. The bars were heated to 2100 F and different-sized upsets were made on the end of the bar in ratios of $1^1/_2$ to 1, in $^1/_2$-in. steps, up to 7 to 1. It was noted in these tests that the coarse-grained steels were more ductile and did not tear in upsetting, while the finer-grained steels tore and became more porous. At this time two forgings were received from different gear makers. These were cut in two and etched. One of them showed a very dense structure, while the other had porous sections with torn fibers. The one having a dense structure was rated a coarse grain, and the other a fine grain. Grain-size tests were made on different heats of steel in stock, and it was found that these varied from 4 to 8. The bars of coarse-grained steel were upset and again checked, all showing dense structures. This research proved that the fibers were more ductile in the coarse-grained steel than in the fine-grained, and that these more ductile fibers afford greater density in the forging.

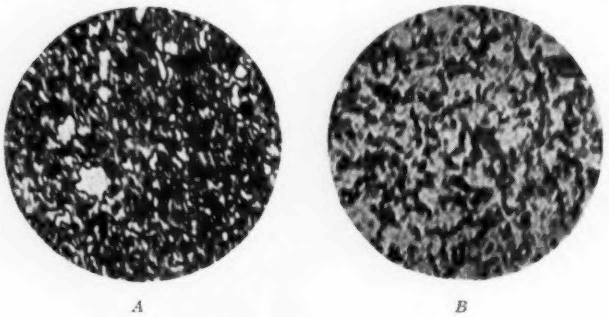

Fig. 8 Photomicrographs of Filled and Unfilled Forgings

A good illustration of the difference in upsetting fine- and coarse-grained steel can be seen in Fig. 7. Here bars of three different grain sizes, namely, 4, 6, and 7, were upset into cluster-gear forgings. *A*, representing the No. 7 grain size, shows tearing in the piercing of the hole as well as tearing of the fibers in the upsets; *B*, representing the No. 6 grain size, denotes less tearing in the hole, with less tearing of the fibers and slightly greater density; while *C*, representing the No. 4 grain size, shows a clean, well-pierced hole with very little, if any, tearing of fibers and much greater density.

It will be noted from these photographs that the ductility of the fibers of the coarse-grained steel not only makes it possible to obtain greater density of the metal in upsetting, but also permits piercing of smaller holes of greater depths or lengths.

A larger quantity of steel of the coarse-grained variety, namely, 4 to 6, was then obtained, from which forgings for test gears were made. For test this steel was cast into 23-in. ingots. It was noticed that the forgings were materially improved, but there was still some evidence of torn fibers. It was then that the authors began to consider the ingot size from a different angle, recognizing the fact that a smaller-sized ingot, with fewer fibers over a given area to be controlled or made ductile, might lead to further improvement. To substantiate this hypothesis, arrangements were made to have a test run on steel of the coarse-grained variety, namely, 4 to 6, cast in different-sized ingots. The ingot molds were all of the Gathmann type: big-end up, fluted molds, with hot tops.

One heat of steel of the 4-to-6-grain size was cast into 12-, 17-, 19-, 20-, and 21-in. ingots. Another heat was made and cast in 23-, 26-, and 28-in. ingots. Bars were rolled from each of these ingots and tests made much the same as described heretofore on the upsets used for showing the effect of grain size, with the exception that greater steps were taken between upsets. The best results were obtained with a No. 4 grain size and a 19-in. ingot. Then, further to confirm this test, a large number of gear sets were forged from steel having a No. 4 grain size which had been cast in 19-in. ingots, the object being to use some of these forgings for etch testing and others later for annealing, machining, hardening, and dynamometer tests for durability.

Being fully aware of the more or less prevalent idea that fine-grained steels have practically the same values as coarse-grained steels for the yield point, tensile strength, and elongation, but that fine-grained steels are tougher, have higher impact values, less distortion in heat treatment, and wider hardening ranges, the authors naturally proceeded with this research on

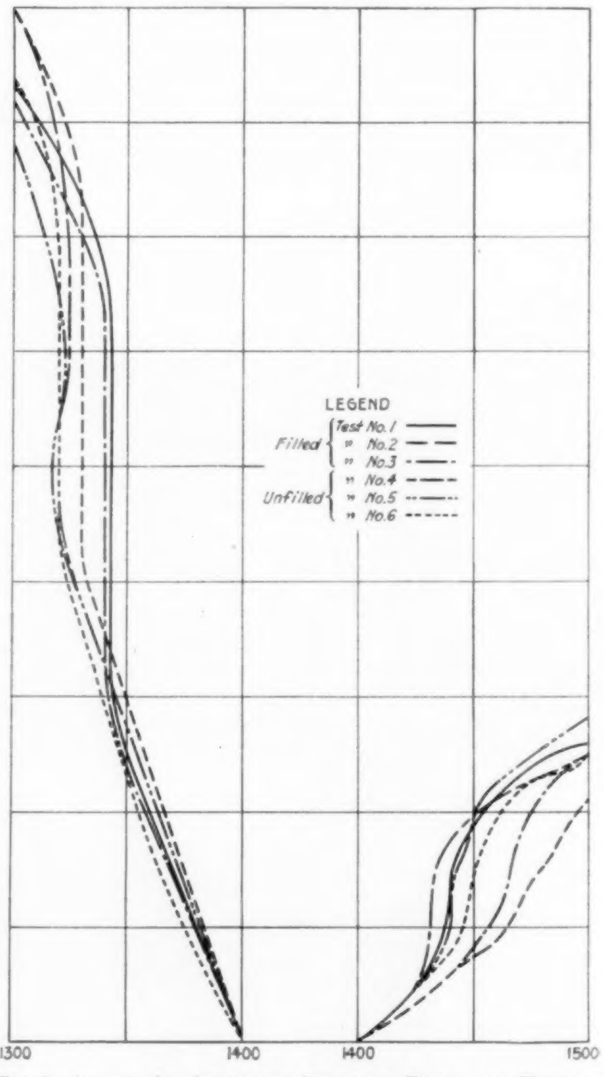

Fig. 9 Ac_3 and Ar_3 Annealing Curves of Filled and Unfilled Forgings

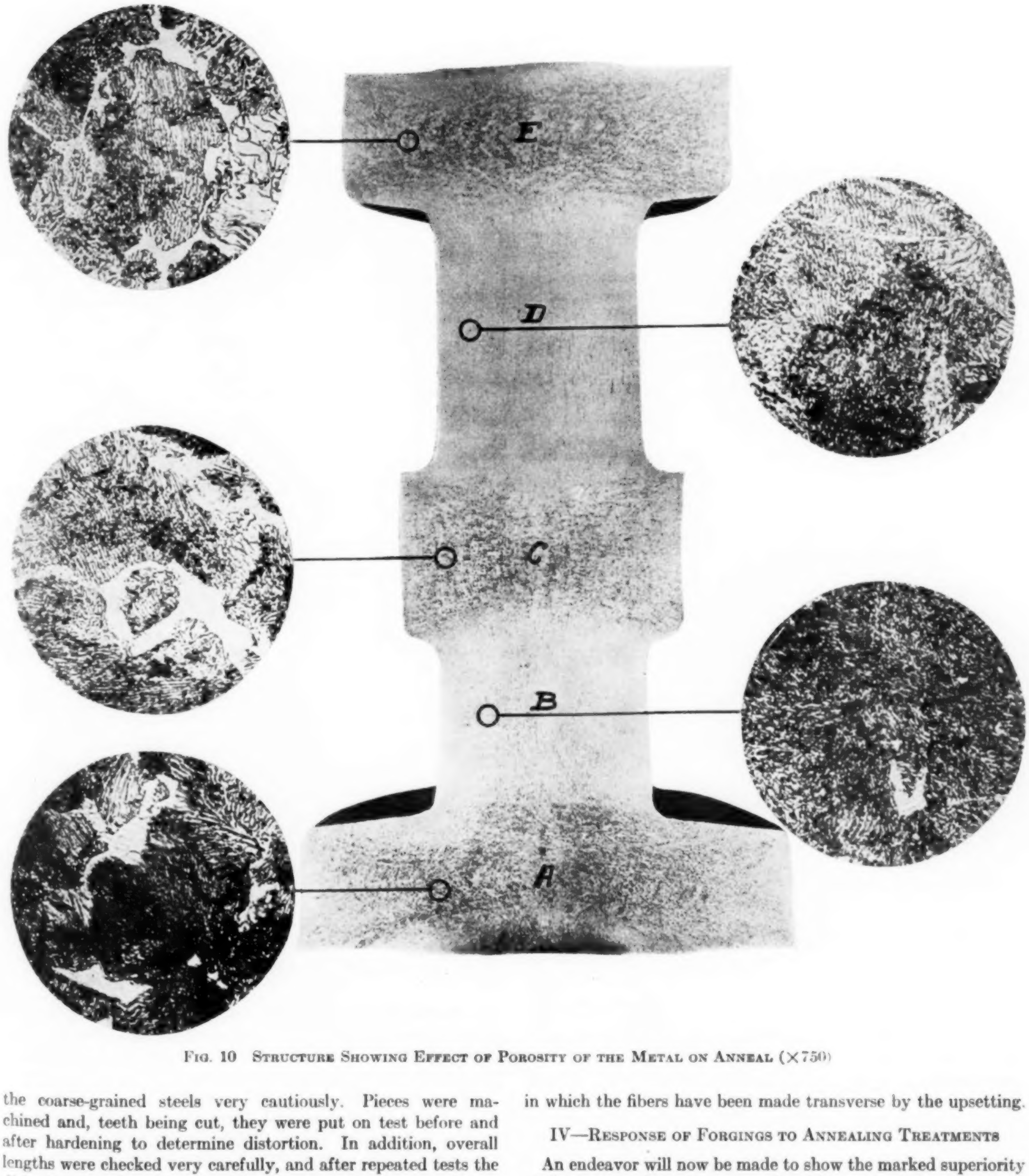

FIG. 10 STRUCTURE SHOWING EFFECT OF POROSITY OF THE METAL ON ANNEAL (×750)

the coarse-grained steels very cautiously. Pieces were machined and, teeth being cut, they were put on test before and after hardening to determine distortion. In addition, overall lengths were checked very carefully, and after repeated tests the distortion was found to be of little consequence because it was uniform and could be allowed for in the cutting. While the same thing is true of the fine-grained steel, the authors have found greater uniformity in the distortion of the coarse-grained metal. It was observed when the metal was properly compressed the impacts were no lower on the coarse-grained than on the fine-grained steels. In one particular test, in which gears made from steel of three different grain sizes were heated and quenched on the same fixture, the impacts were found to be the same. This of course refers to actual tooth impacts of the gear in which the fibers have been made transverse by the upsetting.

IV—RESPONSE OF FORGINGS TO ANNEALING TREATMENTS

An endeavor will now be made to show the marked superiority of the improved forgings over the old type in their response to standardized annealing treatments.

Referring to Fig. 4, one of each of these gears of the filled and unfilled type was cut up for microscopic analysis. The photomicrographs, Fig. 8, show the difference in the microstructure of the two forgings after the same annealing treatment, *A* representing the unfilled forging and *B* the filled forging. Observing this difference, the authors become interested in determining the actual effect of the annealing treatment upon the spongy material. Tests were made on filled and unfilled gear forgings in a

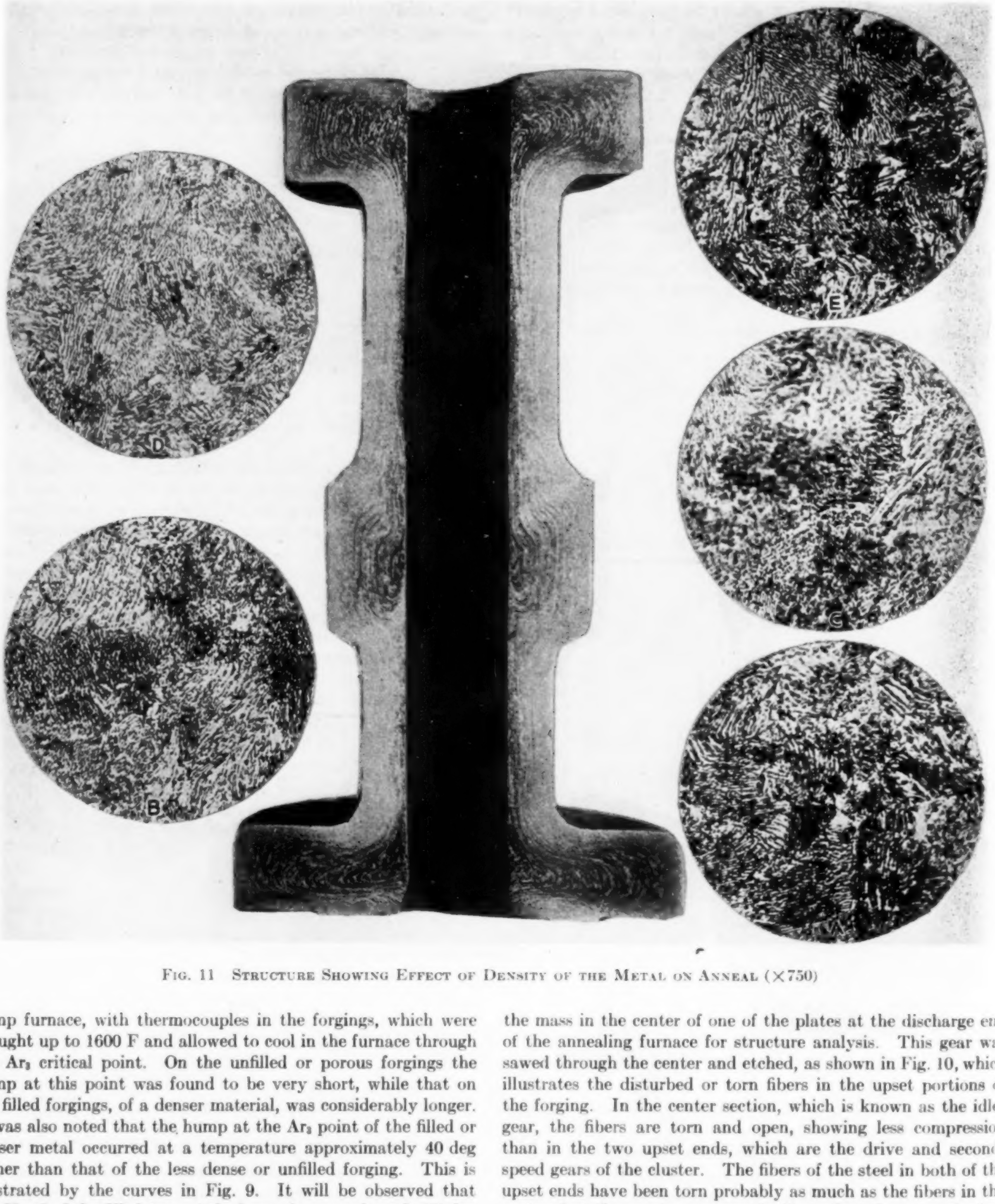

FIG. 11 STRUCTURE SHOWING EFFECT OF DENSITY OF THE METAL ON ANNEAL (×750)

hump furnace, with thermocouples in the forgings, which were brought up to 1600 F and allowed to cool in the furnace through the Ar_3 critical point. On the unfilled or porous forgings the hump at this point was found to be very short, while that on the filled forgings, of a denser material, was considerably longer. It was also noted that the hump at the Ar_3 point of the filled or denser metal occurred at a temperature approximately 40 deg higher than that of the less dense or unfilled forging. This is illustrated by the curves in Fig. 9. It will be observed that line No. 2 of the filled type at the hump is approximately 20 deg below lines Nos. 1 and 3, but the duration through the recalescence is as long as in lines Nos. 1 and 3. The explanation for this is that while the forging had the appearance of being filled and well compressed, its density was not as great as that of Nos. 1 and 3.

In additional tests that were made to determine, if possible, the reason why the material was unresponsive to the annealing treatment, a cluster-gear forging was taken from the center of the mass in the center of one of the plates at the discharge end of the annealing furnace for structure analysis. This gear was sawed through the center and etched, as shown in Fig. 10, which illustrates the disturbed or torn fibers in the upset portions of the forging. In the center section, which is known as the idler gear, the fibers are torn and open, showing less compression than in the two upset ends, which are the drive and second-speed gears of the cluster. The fibers of the steel in both of the upset ends have been torn probably as much as the fibers in the center, but the greater compression has effected some adhesion.

Photomicrographs were then taken of the various sections of this cluster gear, as shown in Fig. 10, *A*, *C*, and *E* designating the different upsets, and *B* and *D* the undisturbed portions of the bar. These photomicrographs show clearly the non-uniform response of the less dense or porous sections to the annealing treatment. *A*, *C*, and *E* show a considerable precipitation of free ferrite and some large islands of massive ferrite—a very poor structure for machinability, while *B* and *D* show a fairly uniform structure of lamellar pearlite with very small islands of free ferrite—a more suitable structure for machinability. These structures seem to prove definitely that the sections of the cluster which were disturbed in forging do not readily respond to the anneal, while those which were undisturbed exhibit a homogeneous lamellar pearlite, due, in all probability, to the denser metal prevalent at these locations.

For comparative purposes a similar analysis was then made of one of the improved cluster-gear forgings made from steel having a No. 4 grain-size rating. Fig. 11 shows structural microphotographs taken in positions comparable to those in Fig. 10. These structures are all quite alike, indicating a very uniform response of the entire forging to the annealing treatment. This uniformity is no doubt the result of the denser structure made possible by the present upsetting methods applied to the coarse-grained steel.

V—Improved Machinability Resulting From Denser Forgings

In this research the authors think it has been proved beyond a doubt that these cavities or voids which affect machinability are due to a spongy surface or metal at the cutting edge of the tool; the tool has double duty to perform; first, to close the cavities with adjacent metal so that it has proper backing, and then to cut—see Fig. 12.

After correcting the forging practice, an improvement in machinability of 50 to 400 per cent was found, depending on the operation. Speeds and feeds had been increased 11 to 60 per cent, and tool life increased in like proportion. As an example, figures on the cutting of gear teeth on gear-hobbing machines may be cited showing the number of pieces per shift of hob, the hob having four shifts per grind. The following table compares machining results today with those obtained a year ago on the old-type gears. At that time the steel was purchased without grain-size specification, and no precautions were taken in forging to develop increased density.

	—Pieces per shift of hob—	
Part name	December, 1931	A year ago
M. D. gear	35	27
Idler (15-tooth)	120	65
Idler (19-tooth)	64	58
Second-speed	35	24
Sleeve	14	8 to 10
Low slider	70	60
Cluster (17-tooth)	35	20
Cluster (15-tooth)	90	52
Cluster (20-tooth)	30	20

Note the variation in figures. With the exception of the 15- and 19-tooth gears, all are finished in one cut with 0.060 in. feed per revolution of blank.

In addition to the improved machinability shown, it might be added that the overall tool cost of the plant per job has been reduced from 84 cents to 44 cents within the last year.

To a gear manufacturer there are other things of major importance besides machinability, that is, changes in the fire. If these changes can be controlled and made uniform, the change in cutting can be allowed for and becomes of little consequence. To show how uniform these changes are after settling on steel of a definite grain size and making forgings of improved density, Mr. Stanard, gear supervisor of the company with which the authors are associated, will be quoted:

> Since we have been getting forgings with greater density we have maintained an accuracy of spacing on hobbing machines within 0.0002 in., contour within ±0.00025 in., and when they come out of fire, we can expect to get about 0.0004 to 0.0005 in. change in contour and no apparent change in spacing. All checks were made with teeth marked and starting from the same position and carrying through in the same direction. Gears going to the fire with a concentricity of less than 0.001 in., come back with one of about 0.0015 in.
>
> Those who cut gears perhaps know that a great deal of the deformation of tooth contour is caused by cold working of the metal when the hob loses its keen, sharp edge. In going through the fire the strain is relieved and the portion cold-worked will rise above the remainder of the profile. As the portion around the point of the hob tooth removes the most stock, it is important that the metal being cut shall be dense and not cause undue wear at this point, as this portion forms the flanks of the teeth, their most sensitive area. The flanks are difficult to correct by burnishing or lapping, and are the contact points of the top edge of the mating tooth when two teeth are in contact in each gear. If we can prevent the rapid wear of the hob tooth around the top, the gear will go to be hardened without cold working, and the distortion due to hardening will be uniform.
>
> With the control now maintained on the steel and forgings, it is a simple matter to correct the hob by grinding more or less rake in the cutting face, thus compensating for the regular fire distortion. To say the least, it is very gratifying to a gear man to have his product come back in the shape it went away. This year's production has shown without any chance of dispute that the contours, spacings, and concentricity maintained before the fire, changed as expected. This has proved of infinite value to us.

VI—Improved Durability Resulting From Denser Forgings

Under this heading, reference will be made to some dynamometer tests which were run to determine the difference in durability of gears made from old- and new-type forgings. Enough forgings were made for two complete sets each of gears of the unfilled and the filled type. These were machined, hardened, and assembled for dynamometer tests. The four assembled transmissions were then mounted on a fan dynamometer and 40-hp loads were applied with the transmissions in second gear. They were operated for periods of 15 min each. In the intervals between they were allowed to cool for approximately 15 min while the wearing surfaces of the gears were being examined. The transmissions were torn down periodically and the shafts and bushings examined for wear.

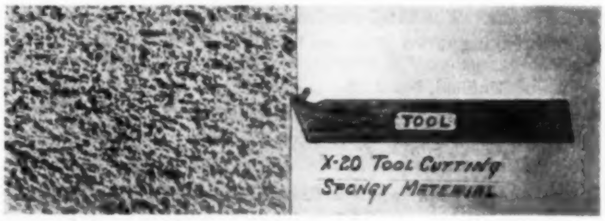

Fig. 12 Illustrating Theory of Effect of Porosity on the Cutting Tool

Accurate records were kept of the operating temperatures and air temperatures, as well as of the time required to show granulations, pitting, and, finally, breakdown. Examination of these gears, after they finally broke down, disclosed that the pits on the filled-forging gears were considerably smaller than those on the unfilled ones.

Due to the fact that these tests required a considerable length of time, it was impossible to maintain exactly the same atmospheric temperature throughout the test. Therefore records were kept to permit checking the average temperature rise in the 15-min intervals during which the transmissions were operated. The average temperature rise for each of the four transmissions was as follows:

Unfilled, No. 1	146 F
Unfilled, No. 2	159 F
Filled, No. 1	116 F
Filled, No. 2	126 F

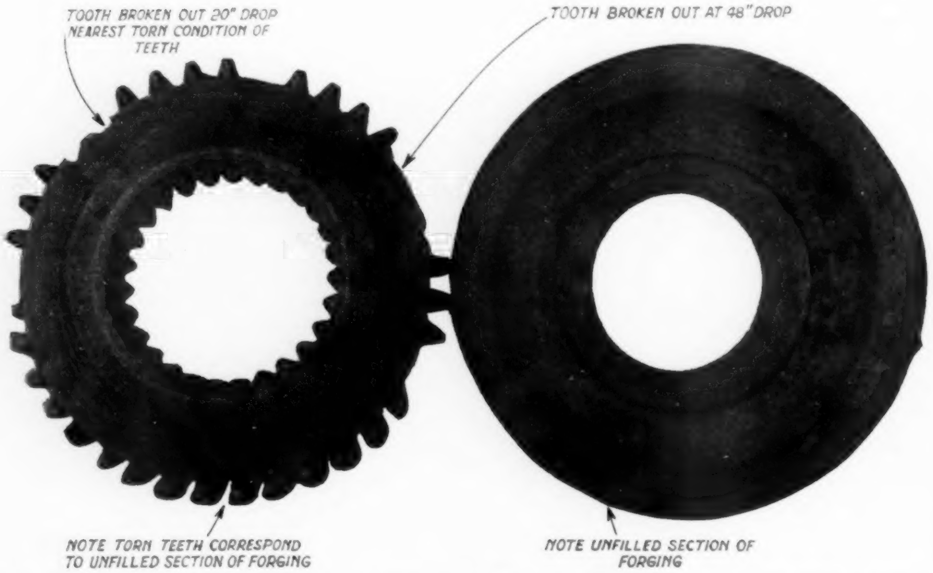
TOOTH BROKEN OUT 20" DROP NEAREST TORN CONDITION OF TEETH
TOOTH BROKEN OUT AT 48" DROP
NOTE TORN TEETH CORRESPOND TO UNFILLED SECTION OF FORGING
NOTE UNFILLED SECTION OF FORGING

Fig. 13 Sliding Gear Broken in Dynamometer Tests and Forging Illustrating Reasons for Failure

It is apparent that the transmissions made from filled forgings operated at a lower temperature difference, and this means that their efficiencies were higher because of smaller frictional losses.

All four transmissions were operated as outlined above until a breakdown prevented further testing. It happened in each case that one or more teeth broke out of the second-speed countershaft gear. The total number of hours each transmission ran before breaking down is shown below:

Unfilled, No. 1	80 hr
Unfilled, No. 2	70 hr
Filled, No. 1	126 hr
Filled, No. 2	128 hr

These hours are equivalent to like numbers in thousands of miles of service. It may be safely concluded from the foregoing tests that the filled-forging gears excelled the unfilled-forging ones on the basis of hours operated before granulations and pitting took place, cooler operation, high efficiency, and longer life.

At this time another discovery was made on the dynamometer which showed that the durability of the metal was affected by its porosity. Among the dynamometer tests that are constantly being made is one which the authors designate as the slip-out test. This test is conducted by having the dynamometer mounted at an angle and duplicating the effect of the transmission in a car being propelled up or down hill. The test has as its object the detection of errors or defects in the transmission which might cause a slip-out in first or second speed on a grade. On one of these tests the horsepower was built up to approximately 300, at which point the transmission broke down. Upon opening the latter it was found, as shown in Fig. 13, that the teeth on one side of the sliding gear had been torn out, while those on the opposite side showed no pitting or wear. After disassembling, it was also noted that the broached internal teeth opposite the missing peripheral teeth had been slightly torn in machining, while those on the opposite side had machined without tearing, and with a good finish. This gear was then submitted to an impact test, and it was found that the tooth nearest the torn section broke out with a 20-in. drop of the hammer, while the tooth on the opposite side broke out with a 48-in. drop.

In operating this test the gear is clamped in a fixture on the base of the testing machine, and an anvil is placed on a tooth of the gear; a 10-lb hammer is then raised with a rope until, coming in contact with a dog set on a vertical rail, it is tripped and allowed to drop upon the anvil. The dog is originally set to trip at 1 in., which height is increased an inch at a time until the gear tooth is broken out.

Investigation of the gear blanks disclosed that the forgings showed less compression or density on one side than on the opposite side. This was caused by the gathering of the metal forming the cone in the first operation being eccentric to the bar, and, in forging, one side failed to fill the die because of this unequal distribution of metal. This is shown in Fig. 13.

VII—Conclusions

Regardless of analysis, the same beneficial results were obtained with all types of steel used.

It has been shown that it is possible to produce denser forgings, in which slipping and tearing of the fibers are eliminated by employing a steel of a specified grain size, accurate forging temperatures, and specially designed dies for controlling the flash to insure greater compression of the forging.

Employing this procedure the authors have been able to produce forgings with a more uniform anneal, thereby effecting improved machinability, i.e., greater feeds and speeds, and longer tool life, resulting in lower tool costs. In addition, these denser forgings have proved highly responsive to the hardening treatment, resulting in uniform distortion, and dynamometer tests run for durability have shown that the life of the gears has been increased more than 50 per cent, with marked decrease in pitting and granulations, the maintenance of smoother surfaces, and quieter operation.

Discussion

S. D. Williams.[10] Much of the authors' work can be explained if we will remember that all steels have what is known as "fiber" to a greater or less degree. The fibrous structure in forged or rolled steels is a deformed structure of the dendrites in which the segregation of impurities is present. Phosphorus is an impurity that is one of the chief causes of the fibrous structure. The desirability of fibrous steel is one of the bitterly disputed subjects. Severely fibrous steel is almost always associated with a steel that shows fine grain in the McQuaid-Ehn test, but the latter does not necessarily imply a fibrous steel.

A steel made fine grained according to the McQuaid-Ehn test, assuming the same ingot size, is usually more fibrous, or rather the fibers are finer and shorter, than a steel that is made inherently coarse grained according to the McQuaid-Ehn test. The fine-grained steel, when given the same heat treatment as the coarse-grained steel, will show approximately the same values for tensile

[10] Timken Steel and Tube Company, Canton, Ohio.

strength and elastic limit, but decidedly superior ductility values in the static test piece taken longitudinally to the direction of rolling. The dynamic test piece machined longitudinally to the direction of rolling and broken across the fiber or grain will show decidedly superior values under impact for the steel made inherently fine grained. When, however, a static test is made transversely to the direction of rolling, the tensile strength and elastic limit are again approximately the same, but the ductility values as shown by the elongation and reduction of area are now decidedly greater in the case of the steel made inherently coarse grained. The transverse impact piece machined transversely and broken parallel to the direction of rolling will show much better resistance to shock in the case of the coarse-grained steel.

In the authors' forging operation on upset gears, the steel is subjected to severe transverse stresses, and there is less tearing on large-grain steel than on fine-grain steel, simply because there are fewer planes of weakness and less chance for a parting of the metal. It is certainly a very interesting fact that this work seems to be the first that has tied up actual density of forgings with the fibrous condition in steel which has been recognized for some time to contain planes of weakness where a slippage or parting of the metal is possible in the upsetting operation.

The authors have proved that a dense forging lends itself more readily to the normalizing cycle, resulting in a more suitable structure for machining than a less dense forging. This, in the writer's opinion, is due to the fact that a dense forging is more fiber-free or has less non-metallic film material between the fibers than a less dense forging, and consequently there is less of an insulating effect to the transfer of heat. In other words, the transformations that take place in the normalizing operation with an evolution of heat occur more uniformly because of the relative absence of insulating non-metallic films between the fibers.

The work of Charpy and others has shown that excessive reduction from ingot to bar lowers the transverse test values of steel, especially the dynamic test values. One of the reasons that a crucible-melted steel performs better than a similar grade of steel cast in a larger ingot is due to the fact that with a smaller ingot a more uniform and quicker freezing occurs with smaller dendritic growth and less non-uniform distribution of carbon, etc. The smaller the ingot, the fewer and also shorter will be the number of fibers in a bar rolled from that ingot. Also, the larger the ingot, the more is the likelihood of the non-metallic particles, carbides, etc. being thrown into the center before freezing occurs. The authors have proved that there is a more dense condition on the surface of hot-rolled steel bars than in the center. The non-metallic material thrown into the center of the slowly cooling large steel ingot causes a pronounced fibrous condition, resulting in planes of weakness where tearing occurs in the upsetting operation. It seems, therefore, that for quality steel our trend should be to work to the smallest commercially practical steel-ingot size in order to get the quickest possible freezing of the ingot. The quicker the freezing that occurs in the ingot, the more uniform will be the fiber condition from skin to center of the bar and the more uniform will be the density of the bar, as the authors understand it, from outside to center of the bar.

Incidentally, it might be mentioned that the direction of the fibers has an intimate relation to the lines of quenching cracks in steel.

Recent researches have shown that the fibrous structure decreases gradually in distinctness as the heating temperature rises above 1100 C, but complete disappearance seems to be almost impossible even though the heating be conducted as high as 1300 C (2372 F). It might be advisable for the authors to conduct further tests on steels that have previously been heated for some time at these high temperatures and see if the diffusion that takes place has some bearing on their results.

Some years ago the English metallurgist, Brearley, made the following statement:

"The relation between the figure obtained on the longitudinal impact piece and the figure obtained in the transverse impact piece is, roughly, a measure of the cleanness of the steel; the transverse figure is never as high as the longitudinal figure, and sometimes it is ten times less. If the steel is for high-class purposes, such as ball races, where the transverse strength of the material is very important, the transverse impact figures should not be less than one-half of the longitudinal."

It would be interesting and probably instructive if the authors in their future work would get some data on the ratio between transverse impact and longitudinal impact and the relation of this ratio to their findings regarding density, machinability, and durability.

R. E. W. Harrison.[11] The authors have accomplished an extremely useful basic manufacturing research, and it is remarkable that this vitally important subject has not hitherto been throughly probed. The conclusions are as basically important in their influence on manufacturing procedure as the introduction of the new carbide cutting materials. The remarkable results obtained from what is nothing more or less than the application of painstaking care in preliminary operations and the admixture of common sense to drop-forging practice indicate the need for the trained analytical machine-shop type of mind in all the operations preceding the actual machining of the forging.

It is an unfortunate fact that many of the manufacturers preparing the raw materials do not appreciate the difficulties that they can create in the machine shop as well as in the performance of the finished product by hit-and-miss methods of chemical analysis, temperature control, die practice, pickling, and in fact all the operations that are necessary before the forging is delivered to the first machine tool.

The most remarkable result obtained from the researches of the authors, apart from the increased efficiency of machine-shop operations, lies in the increased strength of the gear teeth with the possible influence of this factor on the design of transmissions. In these times, when more and more mechanisms are being introduced into automobile transmissions, the question of compactness, which is directly related to strength of materials, is paramount.

The photomicrographs remind the writer of some interesting models made to indicate fiber structure, in which an ingot or slug of laminated colored wax was used and then sectioned. The availability of this method of obtaining a quick picture of fiber placement should not be lost sight of by those responsible for the design of forged structures.

There is an interesting thought in connection with this matter regarding the applications of carbide material-cutting tools to the superior forgings produced by the authors' method, and the writer would like to suggest to the makers of these tools that some research along these lines would probably well repay them, especially having regard to the relatively poor showing hitherto made by all carbide tools on steels as against cast irons.

The pressure of economics compels all process engineers to be constantly on the lookout for practices that reduce the number of operations necessary to fabricate a given piece, and the writer wonders if the authors could state whether their more accurate dies, and forging practice in general, would permit of an increased number of precision-grinding operations directly from the forging without the necessity for the intermediate turning operation.

It is understood that in certain European factories which manufacture anti-friction bearings the most popular sizes are ground direct from the forging, this practice having been made economically possible by the provision of really accurate forgings. It is also understood that the practice is not altogether unknown in one or two of the larger manufacturers of this commodity in the United States.

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[11] Chief Engineer, Cincinnati Grinders, Inc., Cincinnati, Ohio. Mem. A.S.M.E. Secretary, Machine Shop Practice Division, A.S.M.E.

The writer would also like to inquire if the authors have investigated the merits of the rolling die, and whether they believe this would enable them to obtain the same density of internal structure; also if this type of die could be utilized effectively for the rolling of gear teeth to such proportions that the gashing operation could be eliminated and the gear-cutting machinery employed for finish cutting only. It would seem that this thought might have some merit, inasmuch as it would reduce the tendency to cold working, which the authors have observed when hobs are used that need sharpening.

The writer was privileged to see the results of some of this research work prior to the publication of the paper, and he can testify, both from the point of view of the machine-tool manufacturer and the user of the product, that the results of this improvement in manufacturing technique will be far-reaching indeed. This paper is but another evidence of the value of the well-hammered anvil, a phenomenon with which most engineers have been conversant, but which we all apparently have been slow to apply.

It is suggested that some special means be taken to circulate this paper to all concerns making forgings, either for their own product or for the trade.

Authors' Closure

Mr. Williams may recall the results that the authors obtained by annealing the bar before upsetting, as by annealing before upsetting greater density was obtained. The authors are now working on tests that have been run at high temperatures for diffusion before upsetting.

The authors are unwilling to say much of these tests, but will say that in the 18 tests that have been run very gratifying results have been recorded, and it is the authors' opinion that the work from these results will terminate in still greater accomplishment.

Mr. Harrison may be interested to know that from this work the authors have been able to reduce the weight of the transmission by 12 lb, with greater strength and longer life by dynamometer tests. With this method of forging and material there is less need of the application of carbide material-cutting tools.

All gear-blank forgings are forged with the holes in them. They are taken from the forge shop annealed, broaching the hole as forged without core drilling, and then machined from hole, with some surfaces ground to size without turning.

The authors have investigated the merits of the rolling dies for rolling the teeth in a ring gear. In this case the fibers were torn in the center of the teeth, but the forged ring before rolling showed considerable porosity. It may be accomplished, but it is believed that continued rolling on the surface of the teeth to size would cause porosity in the center of the tooth, as you are pulling metal out of the teeth and using it to compress the metal at the root of the teeth or in the body of the gear.

MSP-54-12

Life of Tools Machining Free-Cutting Brass

By L. D. Spence[1] and J. A. Hall,[2] Providence, R. I.

L. D. Spence

J. A. Hall

The authors present a considerable amount of data on the life of tools under shop conditions machining free-cutting brass in automatic screw machines. The analysis of this data shows that larger feeds and higher cutting speeds than those used in current practice can be combined with a reasonable tool life. The type of relation existing between cutting speed, feed, and life of tool found by Frederick W. Taylor and others for rough-turning of steel does not fit these tests of forming and drilling free-cutting brass. Within the limits of these tests an increase of the cutting speed is accompanied by greater economy. When machining free-cutting brass with ground circular-form tools, the life of the cutting tool between grinds appears to be nearly independent of the feed and of the cutting speed, and to depend primarily on the linear distance the work moves in contact with the tool. This indicates that the wear is primarily a lapping action, which is influenced more by the amount of movement of the edge of the tool against the work than by the pressure between them. Within the limits of the tests the maximum economy in machining free-cutting brass is secured by using the largest feed that the work and tools will stand and by running the stock at the highest spindle speed available.

This paper gives a summary of data collected on the life of tools during the past five or six years in the screw-machine laboratory of the Brown & Sharpe Mfg. Co. The figures are a by-product of the main work of the laboratory, in which the study of automatic screw machines under extreme operating conditions and the experimental investigation of new developments are made. In carrying out these investigations, each machine was equipped with a counter that recorded the number of pieces completed. Whenever adjustments or replacements were necessary, the information concerning what was done was entered against the counter reading in a log book. Thus a definite basis for the comparison of different designs was secured. In connection with this study of machines it was easy to obtain accurate data on the life of tools between grindings by requiring an entry in the log book, together with the counter reading, each time that a tool was ground.

The tools were ground whenever it was necessary in getting the specified finish in maintaining manufacturing limits and so forth. Undoubtedly much longer time between grindings could have been secured if every effort had been exerted toward securing the greatest possible tool life. Regular shop operating conditions were maintained in the laboratory; so these results can in no way be considered stunt cuts. The results showed considerable variation in the successive records of life of tools between grindings, as might be expected where finish plays a part. However, no figures were included in this study unless there were a sufficient number of grindings to give a reasonable average. While it may be objected that the method does not give scientifically exact results, it should be noted that the machines were operated under shop conditions, and that the results should therefore be close to what may be expected in manufacturing plants.

In all cases the material cut was free-cutting brass rod. Circular-form tools made of high-speed or of carbon tool steels were used in the forming and cutting-off operations. In all cases the cutting lubricant was a commercial mineral-lard oil.

Form Cutting

The data on the life of a considerable number of circular-form tools are given in Table 1. In all cases the forms of these tools had been ground after the hardening operation. The maximum cutting speed is the surface velocity of the work at the outside where the tool starts cutting, while the same figure for the smallest diameter to which the tool cuts gives the minimum. In cases where the feed per revolution was changed during the cut, the weighted average is given. The feeds vary through a rather wide range, as their choice depends on the width of the cut and on the diameter of the work. A heavy, wide cut may cause excessive deflections of small-diameter stock. If an excess of time is available for forming due to the requirements of the turret operations, a feed smaller than the maximum may be used.

The last three columns of Table 1 give calculated data for the average number of pieces per grind. From the work of others in this field, it was at first felt that the most significant figure would be the cutting life of the tool. This was calculated by multiplying the length of time during which the tool was cutting on each piece by the average number of pieces per grind, and is given in the third from the last column. Another figure, which roughly corresponds to the amount of work done, was the total cutting depth, which is entered in the next column. This is the product of the depth of cut per piece times the average number of pieces.

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[1] In charge of tool designing, Brown & Sharpe Mfg. Co. Mem. A.S.M.E. Following a public-school education, Mr. Spence became a machinist's apprentice at the Brown & Sharpe Mfg. Co., completing the course in 1902. Since that time he has been in charge of various departments at that plant, including both engineering and manufacturing work. For the past 12 years he has been in charge of screw-machine tool design and screw-machine sales.

[2] Professor of Mechanical Engineering, Brown University. Mem. A.S.M.E. James A. Hall was graduated from Brown University with the degree of A.B. in 1908, and received the degree of Sc.B. in M.E. from the same institution. He remained there as a laboratory instructor for four years, and then entered the Engineering Department of the Link-Belt Company in Philadelphia. In 1915 he returned to Brown University as an assistant professor, teaching machine design and industrial management, and is now Professor of Mechanical Engineering. During the past ten years he has held a position with Brown & Sharpe as part-time consulting engineer.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The third column gives the linear distance that the surface of the work moved in contact with the tool while being cut. This figure is found by multiplying the average of the maximum and minimum cutting speeds by the cutting time. Where hexagonal or square stock was being formed, the maximum cutting speed and depth of cut were based on the radial distance to the corners. No attempt was made to correct for the reduced distance on the flat sides, as it was felt that the shock of hitting the corners more than balanced the saving in wear while passing the spaces where no metal was removed.

Referring to Table 1, test No. 1 was a wide cut on a piece of small diameter, and this accounts for the very fine feed. A fine finish was required, which probably made it necessary to grind the form tool more frequently than on average work. Nos. 2, 3, and 4 were all on the same piece. The cut was fairly wide, and the maximum tolerance on the diameter over part of the piece was but 0.001 in. A very fine finish had to be maintained as the piece was to be plated. In test No. 2 a high-carbon-steel tool was used, and this was replaced by high-speed steel in No. 3. A ground circular-form tool that had been chromium plated was used in test No. 4.

The limits were not particularly close on Nos. 5 and 6, although a fair finish was required. On job No. 7 the finish was not so important, as the formed surface had a thread cut on it by a die. Nos. 6 and 7 were made from hexagonal bars. Nos. 8 and 9 are the same piece, the first test being run with a carbon-steel tool and the second with high-speed. The stock was square, and a very good finish had to be maintained on the turned surface. On No. 10 the cut was rather narrow, and a liberal tolerance was permitted on the diameter. No. 11 was similar to No. 10, except that less attention was paid to the finish, because the formed surface was later knurled.

Nos. 12 and 13 are alike in that the form tool also acted as a cutting-off tool. The back of the first piece and the front of the next were formed and the cutting off was completed all at the same time. Possibly these two should be considered as cutting-off rather than as forming tools.

In trying to find some correlation between the results, the first thought is that the life of the tool in minutes should have some relation to the cutting speed. Frederick W. Taylor, in his "Art of Cutting Metals," found for rough-turning that the cutting speed and the life of the tool for a given feed varied by the equation $V = K/\sqrt[8]{T}$, where V is the cutting speed, T the time, and K a constant.

Some of the German authorities have urged that the 3d root rather than the 8th should be used. Prof. W. A. Knight, in an article entitled "Cutting Speeds and Feeds," published in *American Machinist* in 1916, applied the results of the researches of Taylor and others to screw-machine work and suggested the use of the 10th root of the time in the foregoing equation. For any desired life of tool, Professor Knight gave, as an approximation based on the work of Taylor and others, the following equation for the relation between feed, depth of cut, and cutting speed: $V = K/\sqrt[3]{(DF^2)}$, where V is the cutting speed, D the depth of cut, F the feed, and K a constant.

A cursory examination shows that the data given in Table 1 cannot be correlated by these equations. For instance, the feeds in tests Nos. 3 and 5 are practically the same, while the cutting speed is twice as great in the latter. The life of the tool was reduced only about a third by the increase in speed, while it should have dropped to less than 15 per cent by any of the suggested relations. Furthermore, nearly doubling the feed between tests Nos. 5 and 7 with practically the same cutting speed resulted in an increased rather than a decreased life of tool. Similar comparison of the other tests shows that those results do not fit into the equations given.

It should be remembered that the equations referred to were all developed from tests of lathe tools rough-turning steel. In the present studies a relatively new material, free-cutting brass rod, was used. Furthermore, the tool movement is radial, a reasonable surface finish must be secured, and manufacturing limits have to be maintained. It is therefore doubtful if any equations for rough-turning with lathe tools can be adapted to this type of form-cutting.

In a further attempt to secure some correlation between the data, the last columns, entitled "Total cutting depth" and "Linear distance of cutting," were calculated. The first of these gives some idea of the total amount of work done between grinds by the tool, while the second gives the linear distance through which the tool cut. If lapping action of the work on the point of the tool is a major factor, the last figure may be of importance, as it really gives the linear distance of rubbing of the point of the tool against the surface of the work.

An examination of the figures for "Linear distance of cutting" raises the question if this lapping action is not the primary consideration in form-cutting of this type of brass. The figures for all of the tests are fairly close together, while they vary widely for cutting time and total cutting depth. For instance, in tests Nos. 1 and 3, where finish was highly important, the values for "Linear distance of cutting" were 31,800 and 32,500, respectively. Where the finish was somewhat less important, the amounts go up, but fall rather close together, while the highest figure of all, 77,100, is for the job where the finish was of the least importance.

In the range of these tests, the average cutting speeds were changed by a more than 5-to-1 ratio from the largest to the smallest value, while the feeds per revolution varied through a 15-to-1 range. Yet the "Linear distance of cutting" for high-speed steel tools came within a $2^1/_2$-to-1 range, and this variation is largely explained by the difference in finish required.

It should be remembered that all of these tools had ground forms. It is quite possible that the same relation would not be found with tools which were finish-formed prior to the hardening operation and polished afterward. The latter type of tool is likely to have a soft film at the surface, and hence is not as resistant to the lapping action.

While the data are not sufficiently extensive to come to any final conclusion, it appears that within the range of these tests variations in cutting speed and in feed per revolution have a negligible effect on the life of the tool between grinds as compared with the "Linear distance of cutting." In other words, when using a ground form tool on free-cutting brass rod, the linear distance through which the edge of the tool contacts with the rod while cutting, appears to be the primary factor in the life of such a tool between grindings.

If the foregoing conclusion is accepted, the maximum production rate and the greatest output between tool grinds will be secured by conforming to the two following rules: First, the maximum possible feed should be used. This is limited by the power of the drive and the ability of the stock to keep from deflecting unduly under the pressure of a partly worn tool. Second, the cutting speed should be as high as possible. There is undoubtedly a limit beyond which increase in speed results in a loss rather than a gain, but it had not been reached in these tests at 600 fpm.

Another conclusion has to do with the relative merits of carbon- and high-speed-steel tools. At the time of the first tests there was still a strong feeling among users of screw machines that carbon steel was preferable for form tools when a fine finish was required on the work. Probably this was due to the fact that high-speed steel tended to deform more in hardening at the high temperature required and the practice of grinding form tools after hardening had not been generally adopted. These tests show definitely that longer life can be expected from high-speed steel even in this field. Chromium plating was tried on one tool without causing appreciable improvement. In this case the tool was plated but once, so that when freshly ground the top did not present a plated surface. Better results might have been secured if the whole tool could have been replated after each grinding, but this would be out of the question from a production standpoint where a single form tool was available.

TABLE 1 FORM CUTTING

No.	Spindle rpm	Cutting speed, fpm Max	Cutting speed, fpm Min	Feed, in. per rev	Average pieces per grind	No. of grinds	Tool material	Data per grind: Cutting time, min	Data per grind: Total cutting depth, in.	Data per grind: Linear distance of cutting, ft
1	5000	314	197	0.00022	4,220	15	h.s.s.	124.5	164.5	31,800
2	2700	287	182	0.0012	3,442	12	c.s.	78.7	255.0	18,500
3	2700	287	182	0.0012	6,000	131	h.s.s.	137.0	440.0	32,150
4	2700	287	182	0.0012	6,170	38	h.s.s.[a]	141.0	456.0	33,100
5	2400	576	481	0.00125	3,715	15	h.s.s.	92.9	278.0	49,000
6	2400	454	294	0.0013	4,047	13	h.s.s.	165.0	514.0	61,700
7	2400	590	384	0.0023	4,760	23	h.s.s.	135.0	749.0	65,700
8	5000	462	269	0.0008	4,150	8	c.s.	74.8	299.0	27,300
9	5000	462	269	0.0008	9,680	34	h.s.s.	174.0	699.0	63,500
10	3600	570	421	0.0038	20,340	141	h.s.s.	119.0	1625.0	59,000
11	3600	602	438	0.0022	13,700	35	h.s.s.	148.5	1178.0	77,100
12	5000	164	0	0.0023	69,916	24	c.s.	380.0	4370.0	31,900
13	5000	174	0	0.0018	88,652	48	h.s.s.	662.0	5900.0	57,600

[a] Chromium plated.

TABLE 2 CUTTING OFF

No.	Spindle rpm	Cutting speed, fpm Max	Cutting speed, fpm Min	Feed, in. per rev	Average pieces per grind	No. of grinds	Tool material	Data per grind: Cutting time, min	Data per grind: Total cutting depth, in.	Data per grind: Linear distance of cutting, ft
1	2400	392	298	0.0025	37,000	4	h.s.s.	450.0	2700	155,000
2	2400	450	292	0.003	13,610	4	h.s.s.	231.0	1660	87,000
3	2400	595	255	0.0033	5,474	19	h.s.s.	189.0	1470	80,500
4	3600	382	338	0.0045	52,800	55	h.s.s.	93.0	1510	33,900
5	3600	382	338	0.0045	30,000	5	c.s.	51.8	858	19,300
6	2700	231	122	0.0038	26,200	18	c.s.	199.0	2040	35,200
7	3600	438	338	0.0026	66,500	7	c.s.	384.0	3590	149,000
8	2400	392	154	0.0023	11,133	8	c.s.	383.0	2120	109,000
9	2400	392	208	0.0036	13,300	5	c.s.	225.0	1970	67,500
10	2400	392	208	0.0036	14,470	3	h.s.s.	243.0	2140	73,000

CUTTING OFF

The action of a cutting-off tool is similar to that of a form tool except that it does not have to leave a smooth surface on the face. On the other hand, there must always be an acute corner at the point where it finished the back of the piece cut off. This may increase the tendency to wear. Furthermore, it is essential that this corner does not throw up a burr if it is cutting to the edge of a drilled hole. Such a burr may be the factor which determines when the tool must be reground.

The record of a number of cutting-off tools is shown in Table 2. The meaning of the column headings is the same as in the form-tool records. Possibly tests Nos. 12 and 13 of Table 1 should be included with the cutting-off group.

In general these tests show higher values than for forming in the "Linear distance of cutting" column. This is as would be expected, since slight wear near the point of the cutting-off tool is not as serious as on the face of a form tool. One result which is out of line is test No. 4, but here there may have been a greater tendency to throw up a burr at the hole due to the high feed.

Taking the group as a whole, the advantages of high-speed steel over carbon steel are not as obvious as in the form-tool series. However, two cases are given where both types of material are used in cutting the same piece, Nos. 4 and 5 and Nos. 9 and 10. In the former there is a decided advantage in the use of high-speed steel, while it is relatively small in the latter.

While the conclusions concerning the life of these tools are not as definite as with the forming group, the data do give a considerable number of cases where high cutting speeds and large feeds have been combined with long tool life.

DRILLING

Table 3 gives the data on a number of cases of drilling. Cutting speed refers to the surface velocity of the outside of the drill. The feed is given in both inches per minute and inches per revolution. The last two columns record the "Cutting time," which is given by the time spent in drilling one piece multiplied by the number of pieces per grind, and the "Total depth," which is the product of the distance drilled in each piece and the number of pieces.

In the tests reported, the feeds per revolution varied from 0.0078 in. to 0.030 in., while the feeds per minute went from a minimum of 36.7 in. to a maximum of 81 in. All of the drills were of high-speed steel. Most of them were standard twist drills designed for brass, except No. 1, which was a combination drill and counterbore, and No. 8, which was a special steep-spiral drill. There was quite an amount of breakage on test No. 4, so that the figures for 4a are inserted, which give the results of six successive grindings on one drill.

One point of interest in these tests is that large feeds can be combined with high cutting speeds and fairly long tool life can still be attained. Another conclusion is purely negative, as the tests show that high speeds are not accompanied by greatly increased wear.

TABLE 3 DRILLING

No.	Drill rpm	Diam, in.	Cutting speed, ft per min	Feed, in. per rev	Feed, in. per min	Average pieces per grind	Data per grind: No. of grinds	Data per grind: Cutting time, min	Data per grind: Total depth, in.
1	2400	$^{9}/_{16}$	350	0.027	64.8	5,750	21	71.8	4,660
2	3600	0.358	338	0.0173	62.3	22,800	21	179.0	11,200
3	2400	0.358	226	0.018	43.2	30,000	38	270.0	14,700
4	2700	0.173	122	0.030	81.0	11,260	70	92.2	8,700
4a	2700	0.173	122	0.030	81.0	13,000	6	106.5	10,100
5	3600	0.358	338	0.020	72.0	30,400	87	210.0	15,150
6	2400	0.406	255	0.022	52.8	7,200	15	77.6	4,100
7	5000	0.181	237	0.0078	39.0	72,500	5	242.0	9,420
8	2400	0.246	156	0.0153	36.7	13,290	6	305.0	11,200

A number of writers have felt that in general the relation between cutting speed and tool life in drilling was practically the same as for rough turning. In test No. 1 the cutting speed is nearly three times that of test No. 4 and the feeds per revolution are nearly alike, yet the cutting time is reduced by less than 25 per cent, while according to the theories advanced it should have dropped over 90 per cent. This and other comparisons indicate that laws for the rough turning of steel do not apply.

While it seems impossible to deduce any law of drill life from the data of these tests, it does seem apparent that when using free-cutting brass, feeds much in excess of average practice can be used, and that high cutting speeds do not result in a rapid increase of wear.

CONCLUSIONS

This paper has given a considerable amount of data on the life of tools used in machining free-cutting brass in automatic screw machines. The analysis of this data shows that larger feeds and higher cutting speeds than those used in average current practice can be combined with a reasonable tool life.

It is found that the type of relation between cutting speed and life of tool determined by Frederick W. Taylor and others for the rough turning of steel does not apply to the forming and drilling of free-cutting brass, but that within the limits of these tests an increase in the cutting speed is accompanied by greater economy.

When machine free-cutting brass with ground circular-form tools, the life of the tool was determined primarily by the "Linear distance of cutting" and appears to be practically independent of the feed and of the cutting speed. The wear of the form tool seems to be due mainly to the lapping action of the work on the tool. This lapping action appears to be influenced much more by the amount of movement of the edge of the tool against the work than by the pressure between them.

The maximum economy in machining free-cutting brass will be secured by using the greatest feed that the work and tools will stand and by running the stock at the highest spindle speed available.

OGP-54-1

Report on Oil-Engine Power Cost for 1930[1]

THIS report presents information covering 94 oil-engine generating plants, containing 283 engines, totaling 161,583 rated brake horsepower. The previous report (1929) covered data on 36 plants, containing 107 engines, totaling 68,775 rated brake horsepower.

The questionnaires for 1930 were sent to over 1000 plants. Owing to the fact that many additions to the mailing list were made late in the investigation, some plants did not receive the questionnaires in time to have the data in their replies included in the main report. Data pertaining to these plants are given in Supplementary Tables VII, VIII, IX, and X.

Engines listed in the report are full-Diesel, vertical type, direct-connected to generators, unless otherwise noted.

Of the 81 plants in the main report, all but one are located in the United States. The one exception is in Mexico. Of the 13 plants listed in the supplementary tables, one is in Mexico, the remainder being in the United States.

Plant Numbers. The system used in former reports of designating plants by numbers has been retained. Numbers identifying plants previously reported correspond to the same plants in this report.

Period Covered. Most of the data submitted refer to the calendar year 1930. Some plants, however, have reported for periods of three and four years, and a few have presented figures for periods of less than one year.

Output Represented. The total *net* output for the 94 plants listed in this report amounts to 309,369,930 kw-hr.

Bases for Costs and Performances. Unit costs referred to in this report were calculated on the basis of net kilowatt-hours. The net kilowatt-hour output is found by subtracting the power used for plant auxiliaries and station lights from the total gross output of the plant.

Figures given for power output per gallon of fuel oil and of lubricating oil were calculated on the basis of the gross outputs of the individual units and plants.

Load and capacity factors were calculated on the basis of the gross outputs.

Formulas defining running engine capacity factor, running plant capacity factor, annual plant load factor, and plant service factor are as follows:

$$\text{Running engine capacity factor, per cent} = \frac{\text{Engine output in gross kw-hr.} \times 100}{\text{Kw. rating} \times \text{number of hours operated}}$$

$$\text{Running plant capacity factor, per cent} = \frac{\text{Plant output in gross kw-hr.} \times 100}{\text{Total rated kw-hr. of individual units}}$$

$$\text{Annual plant load factor per cent (see below)} = \frac{\text{Plant output in gross kw-hr.} \times 100}{\text{Peak load in kw.} \times \text{number of hours in period}}$$

$$\text{Plant service factor per cent} = \frac{\text{Total rated kw-hr. of individual units} \times 100}{\text{Total installed kw.} \times \text{number of hours in period (see below)}}$$

The expression "rated kw-hr." refers to the kilowatt rating of an engine-generator set multiplied by the number of hours operated. For example, if a unit having a rating of 200 kw. was operated 4000 hours, the rated kw-hr. equals 800,000, no matter what the actual output may have been. Thus the denominator of the expression for "Running plant capacity factory" and likewise the numerator for the "Plant service factor" are arrived a by totaling the rated kilowatt-hours of all plant units.

In the strict sense of its definition, the annual plant load factor cannot be correctly applied to data covering any period other than one year. However, the committee extended the application of this to plants operated 8760 hours plus or minus 2 per cent, using the actual number of hours in the denominator in each case.

The formula for "Plant service factor" requires further explanation for special cases. The expression is an index of the actual number of hours of operation as compared to the total number of hours installed for operation. Therefore, when some units have been installed for longer periods than have others in the same plant, account must be taken of the fact in the calculation. For example, assume that a plant is reported for a twelvemonth period, during which time one unit rated at 200 kw., installed before the start of the period, was operated 5000 hours. Six months after the start of the period, a unit rated at 300 kw. was installed, and subsequently operated 2500 hours. The plant service factor in per cent is therefore:

$$\frac{200 \times 5000 + 300 \times 2500}{200 \times 8760 + 300 \times 4380} \times 100$$

Since the 300-kw. unit was not installed during the entire 8760 hours, but only for 4380 hours, this adjustment must be made. In comparing this year's plant service factors with those contained in the previous reports, it should be borne in mind that these adjustments were not always made in the past. The comparative table for the two years' operation (Table II) included in this report contains only those figures that are in accord with the preceding definitions.

Fuel- and Lubricating-Oil Data. This year the committee summarizes the tabulated figures on fuel- and lubricating-oil economy in graphical form in Figs. 1 and 2. In these charts the gross kilowatt-hours per gallon, using plant averages only, are plotted against running plant capacity factors. The type of the plant was judged to be that of the engines generating 95 per cent of the gross output. The following types of plants are illustrated:

Diesel four-stroke cycle, air injection
Diesel four-stroke cycle, mechanical injection
Diesel two-stroke cycle, air injection
Diesel two-stroke cycle mechanical injection, separate scavenging
Diesel two-stroke cycle mechanical injection, crankcase scavenging
Mixed-type full Diesel
Mixed Diesel and semi-Diesel.

A median line was drawn on each chart, the number of points above the line being equal to the number below. High and low boundary lines were drawn to include all but 10 per cent of the full-Diesel plants on the high and low sides, respectively.

Unless it is otherwise noted in the tables, plants received fuel oil in tank cars.

The fuel- and lubricating-oil costs presented this year include costs for the handling of oil from the cars to the tanks. It was the intention of the committee to have these handling charges included in the 1929 report also. However, some operators may not have included these expenses; this year they were all specifically asked to do so.

This year the committee extended its questions on lubricating oil by asking that the number of gallons of new oil used be segregated as follows:

(*a*) Lubricating oil used to replace shrinkage
(*b*) Lubricating oil used to replace oil discarded.

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[1] Submitted by the Sub-Committee on Oil Engine Power Cost; Oil and Gas Power Division, A.S.M.E., Franz Eder, Chairman, Engineer, Robert W. Hunt Company, New York, N. Y.

Presented at the Fourth National Oil and Gas Power Meeting, Madison, Wis., June 23 to 26, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

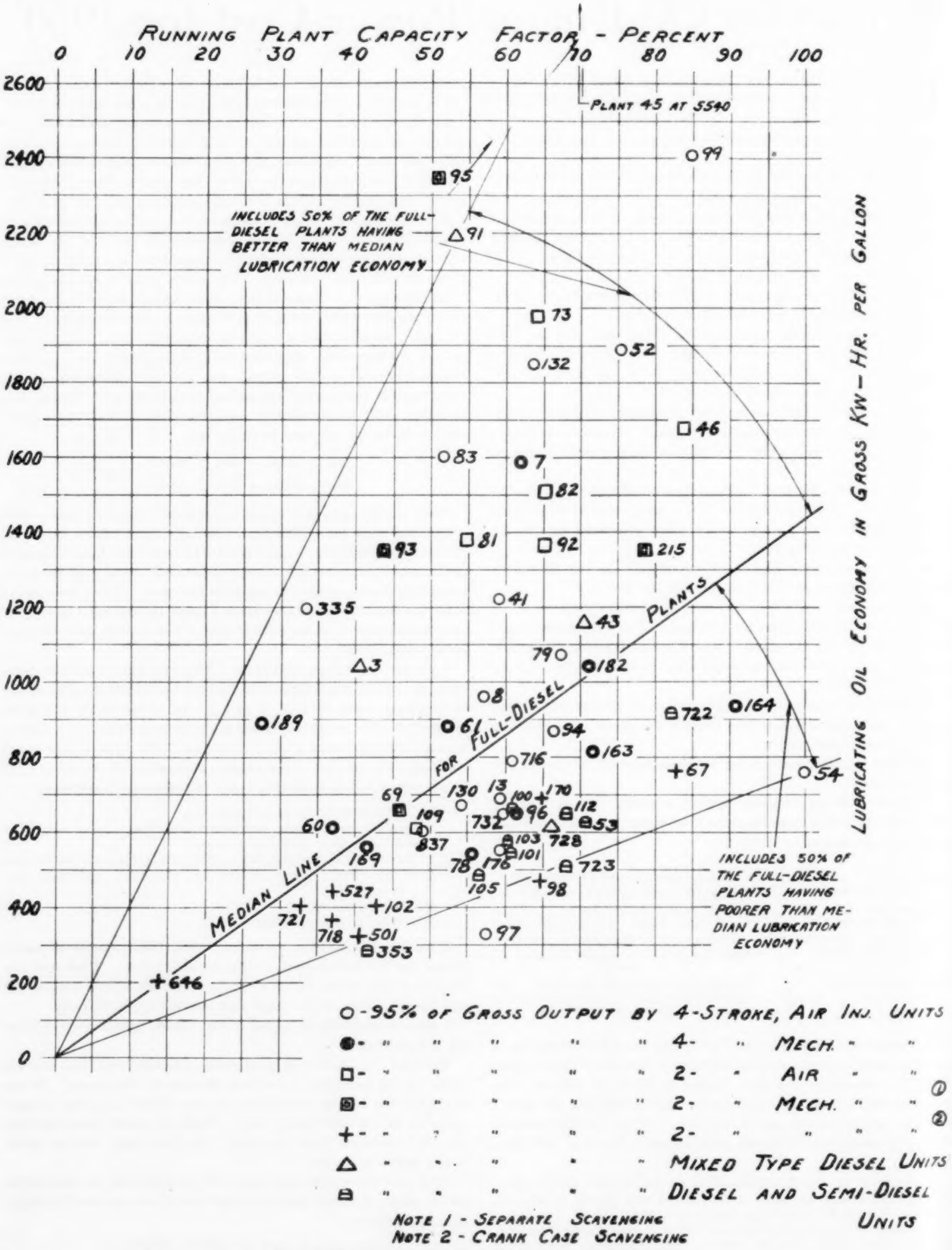

FIG. 1 LUBRICATING-OIL ECONOMY OF OIL-ENGINE PLANTS

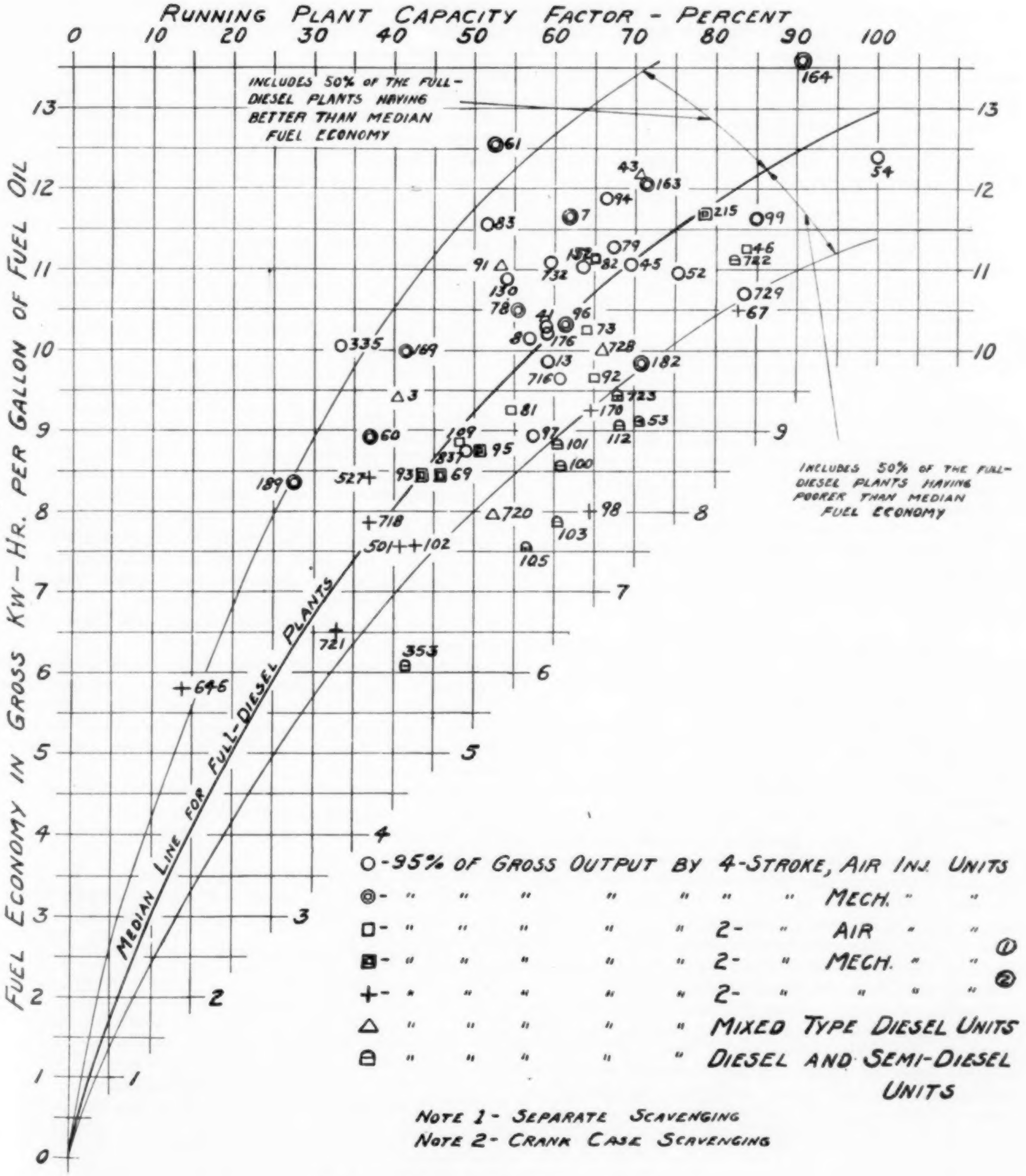

Fig. 2 Fuel-Oil Economy of Oil-Engine Plants

As applied to crankcase oil, this segregation could be easily accomplished; but interpretation was more difficult in the cases of cylinder and compressor oil. Therefore the committee in setting up the tables changed the classification to the following:

(*a*) Gallons of new lubricating oil used
(*b*) Gallons of unfit lubricating oil discarded.

Cooling Water. The various water-cooling systems are indicated by the following symbols:

System A, raw water going to waste after one pass
System B, raw water recirculated after passing over cooling tower or spray pond
System C, soft water continuously recirculated, cooled by raw water going to waste after one pass through heat exchanger
System D, soft water continuously recirculated, cooled by raw water also recirculated after cooling by cooling tower or spray pond
System E, any of the foregoing systems with engine circulating water treated (added as a suffix).

Investment Costs. Last year the committee presented a short table of fixed charges. This year it was considered more informative to present investment costs. Table VI sets forth the actual expenditures for land, buildings, and equipment in whole or part by plants, specifying, where possible, in what years the expenditures were made. It is hoped that in next year's report more complete data can be presented on these items.

The committee invites discussion on the following items which it suggests for the 1931 questionnaire:

Enforced Shutdowns. The committee proposes to ask in future for:

(*a*) Number and total duration of enforced shutdowns due to engine failures
(*b*) Number and total duration of enforced shutdowns for all failures other than engine failures.

Peak Loads. The duration of peak loads has heretofore not been specified. Some operators report instantaneous peaks, others 15-minute peaks, a few hourly peaks, and one operator reported 24-hour peaks. The committee proposes to define peak load as the highest average load sustained for 15 minutes.

Liner Wear. No specification has been previously given for the measurement of liner wear. The committee proposes to stipulate in the future that this measurement be the average of the wear in inches in all the cylinders of the engine, the wear in each cylinder to be found by averaging two measurements, one taken parallel to the crankshaft, the other at 90° to the crankshaft, both taken at the point of greatest wear, which in most cases will be at the top of the piston-ring travel.

Supplies and Repairs. There still exists some confusion as to what is included in the terms "supplies," "repairs," and "miscellaneous." Rather than set up another standard, the committee suggests definitions presented in the "Uniform Classification of Accounts for Electrical Utilities," published by the National Association of Railway and Utilities Commissioners.

Under "Account A707.22" "Production Supplies" are defined as items "used in the power generating plant which are consumed in the operating process, the replacement of which does not constitute a repair or renewal." The principal items are listed as "waste, packing, wipers, gage glasses, gaskets, bolts, screws, nails, dynamo and motor brushes, cans for containing rags and waste, transformer oil and hand oil cans." A definition is given under "Account A707.23" which the committee suggests for "miscellaneous." The items listed are "lighting, heating, and cleaning systems, fire-protection systems, janitor's supplies, ice water, meals and carfares, stationery, telephone and toilet service, care of streets, yards, and sidings."

Investment Cost. There seems to be some uncertainty as to what items belong properly under "land," "buildings," and "equipment." The committee suggests the following divisions:

Land—real estate, clearing, grading, draining, sodding, roads, walks, fences, legal fees.

Buildings—drainage or unwatering, excavation and backfill, piling, concrete work (except equipment foundations), structural steel, walls, floors, roof, doors, windows, skylights, millwork, painting and finishing, heating and ventilating provisions, plumbing and toilet facilities, lighting and building wiring, fire protection, offices, shops, warehouses, and other outbuildings.

Equipment—crane, crane runway, cooling system including pond or tower, oil storage and handling facilities, engines, generators, auxiliaries, and their foundations, lubricating, oil handling, and purifying equipment, air ducts and filters, exhaust mufflers, station piping, switchboards, control and protective equipment, but not including outside electrical structures.

Type of Load. The committee suggests the following definitions:

Complete Power—The plant is run regularly alone when needed, without assistance from any base- or peak-load service.

Base Load—The plant is run at substantially full load whenever its capacity can be used, usually supplemented by a peak-load service. When full or nearly full capacity cannot be used, the plant is shut down.

Peak Load—The plant is run only when the load exceeds the capacity of the regular source of power.

Standby—The plant is run only when the regular source of power is interrupted.

Seasonal—Any of the foregoing types of loads may apply to a "seasonal" plant; i.e., one which is operated normally for annually recurring periods, between which it is shut down.

SUB-COMMITTEE ON OIL ENGINE POWER COST

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TABLE I - INFORMATION ON PRODUCTION COST (Page 1)

Plant Number	Character of Plant (See Notes)	Type of Load (See Notes)	Number of Engines	Total Installed B.H.P.	Total Installed K.W.	Total Plant Hours Operated in Reported Period	Number of Months in Reported Period	Total Gross Output-K.W. Hrs.	Total Net Output-K.W. Hrs.	Per Cent of Gross K.W. Hrs. for Plant Purposes	Annual Plant Load Factor (See Text)	Running Plant Capacity Factor (See Text)	Plant Service Factor (See Text)	Average Cost of Fuel Oil-Cents Per Gallon	Average Cost of Lubricating Oil- Cents Per Gallon	Costs Per Net K.W. Hr. - Mills								Plant Number
																Fuel Cost	Lubricating Oil Cost	Attendance Cost, Including Superintendence	Cost of Supplies and Miscellaneous, Including Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Combined Cost of All Repairs, Supplies & Misc.	Total Production Cost	
43	U	C	8	11,540	8,100	8,760	12	28,875,816	26,221,176	9.2	46.1	70.4	57.8	4.37	56.2	3.95	0.53	0.94	0.21	0.67	0.05	0.93	6.35	43
46	I	B	5	9,250	6,200	8,760	12	28,708,340	27,444,480	4.4	60.7	83.6	67.2	3.62	37.3	3.35	0.23	0.61	0.18	0.96	0.25	1.38	5.58	46
82	M-W	C	6	5,660	3,964	8,760	12	8,107,305	7,958,851	1.8	31.3	65.0	35.9	5.71	76.5	5.22	0.52	3.73	1.38	1.91	1.08	4.37	13.86	82
91	U	C	5	5,250	3,675	8,760	12	8,696,300	8,167,025	6.1	53.7	53.1	52.4	4.09	58.9	3.95	0.29	1.06	0.16	0.33	0.04	0.53	5.83	91
73	M	C	4	5,000	3,400	8,760	12	8,473,300	8,016,000	5.4	37.9	64.0	44.4	1.30	58.8	1.34	0.31	2.62	0.01	0.10	0.01	0.12	4.39	73
92	U	B	3	4,900	3,400	8,760	12	10,454,100	10,027,739	4.1	56.8	65.0	61.1	2.09	43.0	2.25	0.33	0.92	0.26	0.26	0.06	0.58	4.08	92
52	M	C	4	4,580	3,110	8,760	12	7,391,500	7,257,703	1.8	33.5	75.2	45.4	5.70	71.1	5.30	0.38	2.83	0.74	0.56	0.48	1.78	10.29	52
45	M	C	5	4,250	2,900	8,760	12	7,492,325	6,908,500	7.8	34.9	69.5	42.4	5.29	48.0	5.18	0.09	1.77	0.04	1.01	0.10	1.15	8.20	45
54	I	B	7	4,200	2,800	8,760	12	24,221,000	22,910,000	5.4	94.0	99.9	98.9	2.53	39.7	2.16	0.55	1.18	1.16	0.81		1.97	5.86	54
130	M	C	4	3,600	2,524	8,760	12	4,760,800	4,484,700	5.8	41.8	54.1	50.7	4.72	56.7	4.59	0.89	1.36	0.77	0.82	0.17	1.76	8.60	130
164	I	B	4	3,360	2,200	8,760	12	15,884,500	14,709,450	7.4	81.4	90.7	72.7	3.66	27.2	2.91	0.31	1.38	0.16	1.12	0.19	1.47	6.07	164
81	W-M	C	3	3,250	2,260	---	12	5,986,230	5,643,399	5.7	43.5	54.6	55.3	2.79	32.2	3.20	0.25	1.94	0.29	0.37		0.66	6.05	81
3	M	S	4	3,125	2,105	8,760	12	2,433,000	2,357,153	3.1	27.8	40.4	32.7	5.00	46.6	5.49	0.46	4.71	0.23	1.84	0.03	2.10	12.76	3
720	U	-	5	3,080	2,040	8,760	12	3,344,700	3,226,050	3.6	34.7	52.5	35.7	3.99	--	5.21	0.94	1.37	0.37	0.26	0.35	0.98	8.50	720
7	M	C	5	2,760	1,795	8,760	12	3,526,330	3,211,821	8.9	38.0	61.8	53.1	4.79	51.1	4.52	0.35	3.17	1.19	1.57	0.31	3.07	11.11	7
732	U	-	5	2,500	1,720	8,760	12	5,494,770	4,848,480	11.8	37.4	59.7	61.1	3.65	52.0	3.73	0.91	1.61	--	--	--	2.08	8.33	732
60	I	C	3	2,400	1,620	8,760	12	1,770,900	1,658,900[1]	6.3	24.1	37.0	33.7	4.62	54.1	5.54	0.94	4.81	--	--	--	3.24	14.53	60
41	M-W	C	3	2,370	1,800	8,760	12	4,118,290	4,068,290	1.2	39.2	59.0	44.2	5.19	58.6	5.09	0.48	4.08	0.37	0.63	0.15	1.15	10.80	41
79	M-W	C	5	2,170	1,470	8,760	12	3,359,600	3,184,025	5.2	42.6	67.3	38.8	3.33	49.3	3.12	0.46	4.13[3]	0.08	0.49	0.31	0.88	8.59[3]	79
5	U	S	3	2,160	1,440	---	12	1,184,450	1,013,269	14.6	8.1	--	--	4.04	53.9	4.86	1.25	5.56	1.14	7.03		8.17	19.84	5
80	M	C	4	2,150	1,410	8,760	12	2,403,000	2,280,120	5.1	41.9	--	--	3.49	45.0	3.54	0.55	4.09	0.74	1.18	0	1.92	10.10	80
6	U	C	4	2,100	1,424	---	12	1,614,100	1,576,234	2.4	18.8	--	--	4.51	49.0	3.93	0.57	2.07	0.40	1.63		2.03	8.60	6
728	U	-	3	2,040	1,385	8,760	12	3,393,340	3,104,140	8.5	41.6	66.0	42.4	5.90	40.8	6.44	0.73	2.62	0.84	1.09	0.10	2.03	11.82	728
716	U	C	4	1,965	1,440	8,672	12	3,645,000	3,601,253	1.2	41.2	60.8	47.5	3.60	51.1	3.78	0.65	2.15	0.25	3.13	0.32	3.70	10.28	716
93	U	C	3	1,960	1,336	8,616	12-	2,928,200	2,659,910	9.2	30.9	43.4	58.6	4.09	47.4	5.35	0.39	2.65	0.16	0.51	0.02	0.69	9.08	93
215	U	S	2	1,960	1,352	2,445	10+	1,395,600	1,372,941[1]	1.6	--	78.5	17.5	5.40	77.0	4.68	0.58	1.12	0.33	0.01	0.10	0.44	6.82	215
729	U	-	5	1,925	1,380	---	12	2,257,260	2,133,920	5.5	19.8	83.5	22.7	4.71	--	4.66	0.40	3.23	0.55	0.21	1.11	1.87	10.16	729
837	U	C	5	1,875	1,310	8,760	12	2,181,045	2,016,465	7.6	33.2	49.0	38.8	3.07	37.7	3.80	0.68	7.39	0.66	3.32	1.12	5.10	16.97	837
109	M	S	2	1,800	1,200	2,295	13	893,690	885,399	0.9	--	48.1	16.3	5.53	75.7	6.33	1.26	3.78	0.88	0.60	0.11	1.59	12.96	109
176	I	-	3	1,800	1,230	35,040	48	18,773,800	18,612,300	0.9	--	59.2	85.8	7.70	75.7	7.58	1.38	2.81	0.79	0.57		1.36	13.13	176
722	U	S	4	1,705	1,167	---	12	942,550	796,450	15.5	9.3	82.1	11.2	5.61	54.0	5.97	0.69	7.99	1.49	1.19	0.90	3.58	18.23	722
18	U	C	3	1,650	1,120	---	12	2,502,301	2,406,690	3.8	27.2	--	--	4.49	53.5	4.51	0.61	2.08	0.21	4.59		4.80	12.00	18
8	M-W	C	3	1,560	1,075	8,760	12	2,099,100	2,049,000	2.4	34.2	57.0	39.1	3.37	54.9	3.40	0.58	4.68	0.70	0.27	0	0.97	9.63	8
132	M	C	3	1,500	1,030	8,760	12	1,710,500	1,446,160[1]	15.4	33.9	63.5	29.9	4.64	50.0	4.98	0.32	4.51	0.34	0	0	0.34	10.15	132
154	U	C	3	1,400	950	---	12	1,617,985	1,571,594	2.9	26.8	--	--	4.48	50.2	3.79	0.32	3.95	0.35	0.38		0.71	8.77	154
163	I	C	3	1,400	1,050	8,760	12	5,384,200	5,125,835	4.8	60.4	71.4	82.1	2.44	16.6	2.13	0.21	1.12	0.29	3.15		3.44	6.90	163
723	U	S	3	1,320	880	---	12	1,381,230	1,270,520	8.0	19.7	68.0	26.3	5.62	54.0	6.46	1.15	4.01	0.39	2.37	0.97	3.73	15.35	723
725	U	-	4	1,310	876	6,480	12	---	1,976,000	-	--	--	--	4.40	61.1	4.39	0.84	2.58	0.38	0.46	0.20	1.04	8.85	725
13	M-W	C	3	1,225	800	8,760	12	1,291,100	1,131,661	12.4	32.7	59.2	31.2	5.69	39.7	6.60	0.65	4.88	0.54	0.39	0.08	1.01	13.14	13
17	U	C	2	1,200	820	---	12	2,217,200	2,148,910	3.1	27.5	--	--	4.45	54.8	4.23	0.50	2.06	0.36	1.60		1.96	8.75	17
94	U	B	2	1,200	820	9,000	12+	3,485,210	3,315,550	4.9	39.9	66.4	71.1	3.12	49.2	2.76	0.59	1.69	0.30	0.51	0.05	0.86	5.90	94
616	M	-	3	1,150	785	8,760	12	1,694,180	1,553,500	8.3	37.2	--	--	5.10	70.4	4.83	0.85	4.81	--	--	--	0.39	10.88	616
95	U	B	2	1,120	750	6,520	12	1,512,700	1,408,900	6.9	23.3	50.7	45.4	3.12	45.3	3.83	0.21	3.54	0.38	0.68	0.02	1.08	8.66	95
96	U	C	3	1,100	736	8,520	12-	1,241,800	1,215,181	2.2	24.3	61.5	32.2	3.56	41.8	3.54	0.66	4.26	0.43	0.78	0.21	1.42	9.88	96
182	M-W	C	3	1,090	775	4,344	6	1,231,900	1,072,530	13.0	--	71.0	51.1	6.00	54.0	6.99	0.59	1.34	0.18	0.33	0	0.51	9.43	182

TABLE I - INFORMATION ON PRODUCTION COST (Page 2)

Plant Number	Character of Plant (See Notes)	Type of Load (See Notes)	Number of Engines	Total Installed B.H.P.	Total Installed K.W.	Total Plant Hours Operated in Reported Period	Number of Months in Reported Period	Total Gross Output-K.W. Hrs.	Total Net Output-K.W. Hrs.	Per Cent of Gross K.W. Hrs. for Plant Purposes	Annual Plant Load Factor (See Text)	Running Plant Capacity Factor (See Text)	Plant Service Factor (See Text)	Average Cost of Fuel Oil-Cents Per Gallon	Average Cost of Lubricating Oil-Cents Per Gallon	Costs Per Net K.W. Hr. - Mills								
																Fuel Cost	Lubricating Oil Cost	Attendance Cost, Including Superintendance	Cost of Supplies and Miscellaneous, Including Water	Cost of Engine Repairs	Cost of All Other Plant Repairs	Combined Cost of All Repairs, Supplies & Misc.	Total Production Cost	Plant Number
721	U	-	3	1,080	720	8,760	12	809,700	749,476	7.4	23.1	32.9	39.0	6.52	60.0	11.48	1.60	4.61	1.05	-4.16	1.69	6.90	24.59	721
78	I	C	2	1,060	800	---	36	4,954,516	4,867,305	1.8	--	55.5	88.7	5.08	43.7	4.92	0.82	2.79	0.08	0.61	0.13	0.82	9.35	78
97	U	B	3	1,020	693	7,552	12	1,876,040	1,816,136	3.2	38.9	57.4	53.9	3.36	50.9	3.87	1.61	3.98,	0.19	0.26	0.08	0.53	9.99	97
83	M-W	C	2	1,000	686	8,760	12	1,647,900	1,512,520	8.3	33.6	51.6	53.1	6.75	68.2	6.37	0.46	7.34^4	0.54	1.00	0	1.54	15.71^4	83
69	M-W	C	2	960	640	8,760	12	1,268,358	1,123,340	11.4	41.4	45.9	49.3	3.50	56.0	4.69	0.96	5.45	0.71	0.09	0	0.80	11.90	69
53	U	B	3	900	806	8,709	12	2,369,280	2,300,000^1	3.0	50.4	70.6	63.7	3.26	44.4	3.68	0.73	2.12	0.43	1.76	0.05	2.24	8.77	53
98	U	B	3	900	600	8,712	12-	1,910,800	1,774,521	7.1	39.9	64.5	56.7	4.73	39.1	6.37	0.90	3.28	0.60	1.00	0.31	1.91	12.46	98
74	M-W	C	3	885	600	8,760	12	---	1,010,212	--	--	--	--	4.53	64.0	5.32	0.80	3.98	0.29	0.94		1.23	11.33	74
189	M	C	3	850	595	8,760	12	850,976	800,000	6.0	32.4	27.4	59.5	4.10	77.8	5.22	0.93	8.63	1.03	1.50		2.53	17.31	189
153	U	S	1	840	560	---	12	97,970	49,340	49.7	2.2	--	--	4.08	49.9	7.70	1.01	44.30	11.92	12.51		24.43	77.44	153
100	U	B	3	740	496	8,784	12+	1,539,450	1,459,540	5.2	35.1	60.9	58.1	3.50	42.6	4.32	0.68	3.70	0.47	0.82	0.06	1.35	10.05	100
112	M-W	C	3	740	491	8,760	12	1,235,500	1,165,300$_1$	5.7	29.7	68.1	42.2	4.43	51.0	5.19	0.83	5.46	0.19	0.54	0.12	0.85	12.33	112
67	P	C	2	720	500	5,438	12	1,099,500	1,098,900^1	0.1	58.4	82.9	30.3	4.97	54.1	4.73	0.71	1.70	0.23	1.03	0.02	1.28	8.42	67
170	I	C	2	720	480	2,980	12	889,910	860,110$_1$	3.3	19.5	64.7	32.7	6.51	60.0	7.28	0.89	2.50	0.12	1.30	0	1.42	12.09	170
527	M-W	C	2	720	480	7,344	10	733,300	673,300^1	8.2	--	37.0	56.2	3.87	65.0	5.02	1.59	3.97	0.25	0.04	0.04	0.33	10.91	527
718	U	C	2	720	480	7,344	10	670,490	604,312-	9.9	--	36.9	51.5	4.27	44.0	6.05	1.33	6.50	0.83	1.70	0.58	3.11	16.99	718
19	U	C	3	700	460	---	12	764,380	710,280$_2$	7.1	34.9	--	--	4.13	57.7	5.72	1.26	4.96	1.07	2.98		4.05	15.99	19
61	I	C	2	600	440	8,760	12	1,603,900	1,539,900^2	4.0	--	52.3	79.6	6.39	49.9	5.31	0.59	5.53	0.33	0.61	0.06	1.00	12.43	61
106	U	-	1	600	410	---	12	1,498,970	1,435,434	4.2	35.6	--	--	4.49	50.3	4.18	0.27	3.09	0.65	1.87		2.52	10.06	106
103	U	C	3	590	420	8,760	12	1,013,570	995,145	1.9	38.0	60.2	61.2	4.20	52.5	5.43	0.93	4.60	0.49	1.73	0.43	2.65	13.61	103
102	U	-	3	580	380	8,760	12	660,060	591,620	10.4	27.4	43.7	55.1	3.25	49.0	4.78	1.36	8.32	0.34	0.23	0.35	0.97	15.43	102
101	U	C	3	540	380	8,760	12	817,825	768,521	6.0	49.1	60.5	40.6	3.79	46.0	4.56	0.90	5.62	0.35	0.74	0.03	1.12	12.20	101
501	M	C	3	480	315	8,760	12	421,022	403,022	4.3	32.5	40.6	37.5	5.50	55.3	7.58	1.80	10.58	0.40	0.14	0.31	0.85	20.81	501
335	M-W	C	2	430	295	8,760	12	489,420	432,940	11.5	38.5	33.2	57.0	4.85	59.9	5.45	0.57	11.01	1.25	0.19	0.07	1.51	18.54	335
99	U	S	1	400	275	1,607	11+	375,100	365,005	2.7	--	84.9	19.4	3.18	55.3	2.81	0.24	3.12	0.63	0.61	0.02	1.26	7.43	99
247	M-W	C	3	390	275	8,760	12	429,890	428,290^1	0.4	28.9	--	--	6.00	55.0	8.12	1.10	10.79	0.11	0	0	0.11	20.12	247
353	M	C	3	390	250	8,371	12	282,575	268,077	5.1	25.6	41.6	31.0	5.50	44.0	9.52	1.74	19.93	0.35	0.61	0	0.96	32.15	353
105	U	C	3	370	232	9,120	12+	425,800	372,330	12.6	33.4	56.5	35.7	3.14	46.3	4.77	1.09	11.00	0.97	0.44	0.17	1.58	18.44	105
152	U	S	1	360	240	---	12	28,500	25,948	8.9	21.7	--	--	5.35	41.8	6.55	0.69	81.00	12.28	10.00		22.28	110.52	152
857	U	C	2	330	197	---	12	349,340	307,163	12.1	44.3	--	--	3.39	40.8	5.91	2.06	11.50	1.75	1.22	0.84	3.81	23.28	857
27	U	C	2	315	198	---	12	210,630	190,375	9.7	30.1	--	--	4.93	50.0	9.00	2.42	12.68	0.77	5.91		6.68	30.78	27
515	M	C	3	305	200	8,760	12	---	193,000	--	--	--	--	5.63	54.0	11.65	3.36	20.73	0.78	0.52	0.23	1.53	37.27	515
169	I	C	1	240	160	2,378	12	157,780	151,850	3.8	11.3	41.5	27.1	5.98	51.0	6.23	0.94	2.56	0.55	0.51	0	1.06	10.79	169
265	M	C	2	180	110	8,760	12	---	184,000	--	--	--	--	9.14	59.9	13.48	1.76	13.58	1.10	0.71	0	1.81	30.63	265
858	U	C	2	180	127½	---	12	133,000	123,538	7.1	23.0	--	--	3.41	38.4	6.87	3.26	31.90	4.02	4.66	2.08	10.76	52.79	858
646	M	P	1	120	72	1,016	12	16,690	15,893	6.6	3.8	13.7	7.0	7.00	60.6	12.65	3.15	147.25	11.26	0	0	11.26	174.31	646

NOTES: Character of Plant
U - Private Power Company
P - Private Pumping Company
M - Municipal Power Plant
W - Municipal Pumping Plant
I - Industrial Power Plant

Type of Load
C - Complete Power
B - Base Load
S - Standby
P - Peak Load

Numbered Notes
1 - Estimated by Operator.
2 - Estimated by the Committee.
3 - Includes Water Department Labor also.
4 - Includes Steam Plant Labor also.

TABLE II - COMPARATIVE COSTS - 1929 REPORT AND 1930 REPORT (Corrected)

Plant Number	Total Installed B.H.P.		Total Net Output-K.W. Hrs.		Annual Plant Load Factor		Running Plant Capacity Factor		Plant Service Factor		Average Cost of Fuel Oil-Cents Per Gallon		Costs Per Net K.W. Hr. - Mills																Plant Number
													Fuel Cost		Lubricating Oil Cost		Attendance & Superintendance Cost		Cost of Supplies & Msc. Including Water		Cost of Engine Repairs		Cost of All Other Plant Repairs		Total of All Supply, Repair & Msc. Costs		Total Production Cost		
	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	1929	1930	
43	11,540	11,540	22,591,027	26,221,176		46.1	5.99	70.4		57.8	4.25	4.37	3.82	3.95	0.69	0.53	0.60	0.94	-	0.21	-	0.67	-	0.05	1.04	0.93	6.14	6.35	43
46	6,900	9,250	24,909,500	27,444,480	69.4	60.7	83.3	83.6	76.3	67.2	4.92	3.62	4.48	3.35	0.19	0.23	0.83	0.61	0.32	0.19	1.33	0.95	0.22	0.25	1.87	1.39	7.38	5.58	46
91	3,450	5,250	7,585,725	8,167,025	46.6	53.7	52.0	53.1	72.8	52.4	3.90	4.09	3.72	3.95	0.48	0.29	1.18	1.06	0.16	0.16	0.22	0.33	0.07	0.04	0.45	0.53	5.82	5.83	91
92	2,500	4,900	6,662,389	10,027,739	44.1	56.8	71.6	65.0	61.6	61.1	2.15	2.09	2.20	2.25	0.55	0.33	1.24	0.92	0.17	0.26	1.85	0.26	0.12	0.06	2.14	0.58	6.13	4.08	92
52	3,330	4,580	5,722,032	7,257,703	31.1	33.5	65.3	75.2	45.2	45.4	5.58	5.70	5.38	5.30	0.55	0.38	3.83	2.83	0.88	0.74	0.74	0.56	0.40	0.48	2.02	1.78	11.77	10.29	52
45	4,250	4,250	7,442,350	6,908,500	36.8	34.9	72.5	69.5	43.6	42.4	5.23	5.29	5.04	5.18	0.09	0.09	1.65	1.77	0.06	0.04	-	1.01	-	0.10	0.92	1.15	7.70	8.20	45
2	3,450	3,450	16,479,500	16,077,095	83.2	78.8	94.4	94.3	84.5	77.1	2.71	2.18	2.62	1.98	0.33	0.45	1.40	1.14	0.13	0.09	-	1.56	-	0.09	0.48	1.74	4.83	5.31	2
3	3,125	3,125	2,341,730	2,357,153	32.1	27.8	42.2	40.4	38.2	32.7	4.53	5.00	5.95	5.49	0.54	0.46	5.62	4.71	0.47	0.23	1.20	1.84	0.15	0.03	1.82	2.10	13.93	12.76	3
7	1,830	2,760	3,231,938	3,211,821	36.0	38.0	64.8	61.8	53.2	53.1	5.77	4.79	5.31	4.52	0.36	0.35	3.06	3.17	1.34	1.19	-	1.57	-	0.31	3.98	3.07	12.70	11.11	7
41	2,370	2,370	3,798,810	4,068,290	39.6	39.2	-	59.0	-	44.2	5.51	5.19	5.43	5.09	0.48	0.48	4.11	4.08	0.24	0.37	0.22	0.63	0.07	0.15	0.53	1.15	10.54	10.80	41
5	2,160	2,160	319,120	1,013,269	3.7	8.1	-	-	-	-	4.15	4.04	5.45	4.86	1.77	1.25	11.61	5.56	3.56	1.14	-	-	-	-	17.99	8.17	36.81	19.84	5
6	2,100	2,100	1,318,898	1,576,234	10.0	18.8	-	-	-	-	4.70	4.51	4.17	3.93	0.72	0.57	2.67	2.07	0.47	0.40	-	-	-	-	1.61	2.03	9.16	8.60	6
93	1,960	1,960	2,838,950	2,659,910	27.4	30.9	44.8	43.4	59.4	58.6	4.16	4.09	5.25	5.35	0.52	0.39	2.50	2.65	0.15	0.16	0.38	0.51	0.09	0.02	0.62	0.69	8.88	9.08	93
18	1,650	1,650	2,351,123	2,406,690	26.4	27.2	-	-	-	-	4.71	4.49	4.52	4.51	0.41	0.61	2.58	2.08	0.38	0.21	-	-	-	-	2.30	4.80	9.81	12.00	18
10	1,500	—	No Reply To Inquiries This Year																										
11	1,350	—	No Reply To Inquiries This Year																										
13	1,225	1,225	1,169,971	1,131,661	33.8	32.7	55.9	5.92	34.1	31.2	6.00	5.69	7.44	6.60	0.57	0.65	4.74	4.88	1.00	0.54	0.89	0.39	0.77	0.08	2.66	1.01	15.41	13.14	13
17	1,200	1,200	1,611,876	2,148,910	20.7	27.5	-	-	-	-	4.52	4.45	4.69	4.23	0.53	0.50	2.64	2.06	0.67	0.36	-	-	-	-	2.11	1.96	9.96	8.75	17
94	1,200	1,200	2,982,340	3,315,550	39.6	39.9	67.0	66.4	64.9	71.1	3.17	3.12	2.83	2.76	0.55	0.59	1.93	1.69	0.36	0.30	0.25	0.51	0.19	0.05	0.80	0.86	6.11	5.90	94
68	1,160	—	No Reply To Inquiries This Year																										
95	1,120	1,120	1,675,810	1,408,900	28.0	23.3	48.0	50.7	57.8	45.4	3.23	3.12	3.76	3.83	0.32	0.21	3.60	3.54	0.32	0.38	0.36	0.68	0.15	0.02	0.83	1.08	8.51	8.66	95
96	1,100	1,100	1,133,027	1,215,181	17.0	24.3	49.1	61.5	37.7	32.2	3.61	3.56	4.30	3.54	0.99	0.66	3.87	4.26	0.54	0.43	4.39	0.78	0.19	0.21	5.12	1.42	14.26	9.88	96
97	1,020	1,020	2,027,967	1,816,136	39.9	38.9	51.1	57.4	67.7	53.9	3.18	3.36	3.48	3.87	1.51	1.61	3.01	3.98	0.30	0.19	1.56	0.26	0.10	0.08	1.96	0.53	9.96	9.99	97
98	1,000	900	1,677,817	1,774,521		39.9	63.2	64.5		56.7	5.57	4.73	7.31	6.37	1.06	0.90	3.06	3.28	0.45	0.60	1.68	1.00	0.21	0.31	2.34	1.91	13.75	12.46	98
100	740	740	1,247,150	1,459,540	49.7	35.1	59.9	60.9	50.2	58.1	3.38	3.50	4.14	4.32	0.73	0.68	4.69	3.70	0.41	0.47	0.37	0.82	0.20	0.06	0.98	1.35	10.53	10.05	100
67	720	720	1,163,600	1,098,900	64.0	58.4	79.1	82.9	33.7	30.3	5.94	4.97	6.03	4.73	0.55	0.71	1.60	1.70	0.21	0.23	0.04	1.03	0	0.02	0.25	1.28	8.43	8.42	67
19	700	700	603,260	710,280	29.8	34.9	-	-	-	-	4.50	4.13	6.32	5.72	1.40	1.26	5.40	4.96	1.07	1.07	-	-	-	-	2.24	4.05	15.36	15.99	19
106	600	600	750,419	1,435,434		35.6	-	-	-	-	4.66	4.49	4.75	4.18	0.66	0.27	2.66	3.09	0.61	0.65	-	-	-	-	2.62	2.52	10.69	10.06	106
103	390	590	772,985	995,145	40.8	38.0	53.3	60.2	67.8	61.2	4.63	4.20	6.04	5.43	0.99	0.93	5.82	4.60	0.56	0.49	0.85	1.73	0.28	0.43	1.69	2.65	14.54	13.61	103
102	460	580	577,920	591,620	24.7	27.4	39.8	42.7	58.1	55.1	3.41	3.25	5.22	4.78	1.61	1.36	8.51	8.32	0.57	0.34	0.56	0.28	0.30	0.35	1.43	0.97	16.77	15.43	102
101	540	540	666,871	768,521	36.5	49.1	53.7	60.5	40.3	40.6	3.56	3.79	4.85	4.56	1.43	0.90	6.39	5.62	0.83	0.35	2.67	0.74	0.57	0.03	4.07	1.12	16.74	12.20	101
104	415	—	Not Operated In 1930; Load Served By High Tension System																										
99	600	400	1,358,689	365,006	31.7	-	72.9	84.9	49.9	19.4	3.09	3.18	2.68	2.61	0.41	0.24	3.04	3.12	0.42	0.63	0.60	0.61	0.21	0.02	1.23	1.26	7.36	7.43	99
24	400	—	No Time To Work Up Information To Fill Out Questionnaire																										
105	370	370	272,138	372,330		33.4	51.7	56.5		35.7	3.20	3.14	5.97	4.77	3.89	1.09	14.60	11.00	1.76	0.97	6.59	0.44	0.90	0.17	9.25	1.58	33.51	18.44	105
26	330	—	Not Operated In 1930; Using Purchased Power																										

TABLE III INFORMATION COVERING LUBRICATING OIL (Page 1)

Plant Number	Engine Designation	Engine Cycle	Injection System - See Notes	Scavenging System - See Notes	Trunk Piston or Cross Head	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Rated Horsepower Hours in Reported Period	Gallons of New Lubricating Oil Used	Gallons of Unfit Lubricating Oil Discarded	Rated H.P. Hrs. per Gallon of New Lubricating Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Gross Output - K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Lubricating Oil Treatment - (See Notes)	Average Cost of Lubricating Oil - Cents per Gallon
43	a	4	A	-	TP	500	3	22	29-1/2	150	300	1915	637-1/2	318,750	438	0	728	75.0		143,606	328	BC& CC& C& S	
	b	4	A	-	TP	500	3	22	29-1/2	150	300	1916	433-1/2	216,750	364	0	596	73.8		95,848	263		
	c	4	A	-	TP	520	4	19	24-1/2	200	350	1919	848	440,960	359	0	1,228	87.2		258,660	720		
	d	4	A	-	TP	520	4	19	24-1/2	200	350	1920	880-1/2	457,860	335	0	1,365	74.6		229,870	686		
	e	4	A	-	CH	1,000	6	20-1/2	36	150	700	1922	1,033	1,033,000	719	0	1,437	63.2		456,610	635		
	f	4	A	-	CH	1,000	6	20-1/2	36	150	700	1925	876	876,000	705	0	1,242	63.0		386,020	547		
	g	2	A	S	CH	3,750	6	30	42	124	2,600	1928	6,982-1/2	26,184,375	10,126	0	2,586	68.7		12,480,300	1,232		
	h	2	A	S	CH	3,750	6	30	42	124	2,800	1929	7,363	27,611,250	11,820	0	2,335	71.9		14,824,900	1,254		
	Plant					11,540					8,100			57,138,945	24,866	0	2,300		70.4	28,875,816	1,160		56.2
46	a	2	A	S	CH	1,250	5	20-3/4	26	180	850[2]	1922	3,772	4,715,000	—	-	—	78.5		2,517,890	—	BC& S& F	
	b	2	A	S	CH	1,250	5	20-3/4	26	180	850[2]	1922	4,600	5,750,000	—	-	—	79.2		3,094,050	—		
	c	2	A	S	CH	2,250[1]	4	28	44	124[1]	1,500[2]	1922	6,162	13,864,500	—	-	—	81.0		7,494,500	—		
	d	2	A	S	CH	2,250[1]	4	28	44	124[1]	1,500[2]	1922	6,507	14,640,750	—	-	—	84.9		8,293,500	—		
	e	2	A	S	CH	2,250	4	28	44	124	1,500[2]	1930[3]	5,461	12,287,250	—	-	—	89.2		7,306,400	—		
	Plant					9,250					6,200			51,257,500	17,078	-	3,000		83.6	28,706,340	1,680		37.3
82	a	4	A	-	TP	360	4	16-1/4	21	225	250	1921	844	303,840	—	-	—	61.3		129,355	—	BC	
	b	2	A	S	CH	750	4	17	27	180	517	1923	840	630,000	—	-	—	57.3		246,725	—	"	
	c	2	A	S	CH	750	4	17	27	180	517	1924	601	450,750	—	-	—	58.5		181,925	—	"	
	d	2	A	S	CH	1,150	6	17	27	180	800	1927	2,785	3,202,750	—	-	—	64.7		1,442,700	—	"	
	e	2	A	S	CH	1,150	6	17	27	180	800	1927	2,779	3,195,850	—	-	—	65.0		1,446,100	—	"	
	f	2	A	S	CH	1,500	8	17	27	180	1,080	1929	6,536	9,804,000	—	-	—	66.0		4,656,500	—	"	
	Plant					5,660					3,964			17,587,190	5,356	-	3,280		65.0	8,107,305	1,512		76.5
91	a	4	M	-	TP	1,125	6	22	30	180	800	1927	7,756	8,725,500	—	-	—	50.4		3,125,850	—	CC&S	
	b	4	M	-	TP	1,125	6	22	30	180	800	1928	7,683	8,643,375	—	-	—	51.7		3,178,575	—	"	
	c	4	M	-	TP	600	6	17	24	225	400	1927	3,905	2,343,000	—	-	—	63.0		983,675	—	"	
	d	4	M	-	TP	300	6	17	24	225	400	1927	3,327	1,996,200	—	-	—	61.0		811,900	—	"	
	e	4	A	-	TP	1,600	6	24	32	225	1,275	1930[4]	889	1,600,200	—	-	—	52.6		596,300	—	"	
	Plant					5,250					3,675			23,308,275	3,963	-	5,880		53.1	8,696,300	2,193		58.9
73	a	2	A	S	CH	1,250	5	20-3/4	26	180	850	1923	5,332	6,665,000	1,193	0	5,586	63.1		2,857,800	2,396	BC&F	
	b	2	A	S	CH	1,250	5	20-3/4	26	180	850	1924	6,155	7,693,750	1,325	0	5,800	64.3		3,364,400	2,530	"	
	c	2	A	S	CH	1,250	5	20-3/4	26	180	850	1928	2,606	3,257,500	1,135	0	2,865	65.2		1,443,600	1,271	"	
	d	2	A	S	CH	1,250	3	20-3/4	26	180	850	1928	1,473	1,841,250	620	0	2,970	64.5		807,500	1,302	"	
	Plant					5,000					3,400			19,457,500	4,273	0	4,555		64.0	8,473,300	1,982		58.8
92	a	2	A	S	CH	1,250	5	20-3/4	26	180	900	1928	4,444	5,555,000	—	-	—	67.6		2,704,400	—	BC&S	
	b	2	A	S	CH	1,250	5	20-3/4	26	180	900	1928	6,203	7,753,750	—	-	—	64.2		3,581,200	—	"	
	c	2	A	S	CH	2,400	8	21	24	225	1,600	1930[5]	4,062	9,748,800	—	-	—	64.1		4,168,500	—	"	
	Plant					4,900					3,400			23,057,550	7,644	-	3,015		65.0	10,454,100	1,367		43.0
52	a	4	A	-	CH	830	4	23	32	164	560	1928	1,250	1,037,500	258	0	4,020	73.4		513,600	1,986	BC&S	
	b	4	A	-	CH	1,250	6	23	32	164	850	1928	4,875	6,093,750	1,718	0	3,545	75.0		3,109,700	1,806	"	
	c	4	A	-	CH	1,250	6	23	32	164	850	1928	4,791	5,988,750	1,584	0	3,780	75.2		3,061,500	1,932	"	
	d	4	A	-	CH	1,250	6	23	32	164	850	1930[5]	1,080	1,350,000	353	0	3,820	77.0		706,700	2,000	"	
	Plant					3,580					3,110			14,470,000	3,913	0	3,695		75.2	7,391,500	1,887		71.1
45	a	4	A	-	TP	600	6	16-1/2	24	200	400	1923	1,047	628,200	196	0	3,206	53.6		224,550	1,145	— BC&S &C &F	
	b	4	A	-	TP	600	6	16-1/2	24	200	400	1923	1,028	616,800	223	0	2,765	55.0		226,300	1,015		
	c	4	A	-	CH	750	4	23	32	150	500	1923	2,725	2,043,750	162	0	12,620	63.0		857,725	5,290		
	d	4	A	-	CH	1,150	6	23	32	150	800	1926	5,610	6,451,500	429	0	15,040	72.3		3,245,250	7,565		
	e	4	A	-	CH	1,150	6	23	32	150	800	1926	5,128	5,897,200	342	0	17,250	71.6		2,938,500	8,590		
	Plant					4,250					2,900			15,637,450	1,352	0	11,560		69.5	7,492,325	5,540		48.0
54	a	4	A	-	TP	600	6	17	25	200	400[6]	1925 to 1926	—	—	—	-	—	-		—	—	CC&S	
	b	4	A	-	TP	600	6	17	25	200	400[6]		—	—	—	-	—	-		—	—	"	
	c	4	A	-	TP	600	6	17	25	200	400[6]		—	—	—	-	—	-		—	—	"	
	d	4	A	-	TP	600	6	17	25	200	400[6]		—	—	—	-	—	-		—	—	"	
	e	4	A	-	TP	600	6	17	25	200	400[6]		—	—	—	-	—	-		—	—	"	
	f	4	A	-	TP	600	6	17	25	200	400[6]		—	—	—	-	—	-		—	—	"	
	g	4	A	-	TP	600	6	17	25	200	400[6]		—	—	—	-	—	-		—	—	"	
	Plant					4,200					2,800		60,613	36,367,800	31,777	-	1,145		99.9	24,221,000	762		39.7
130	a	4	A	-	TP	600	6	16-1/4	23	225	412	1924	4,075	2,445,000	—	-	—	68.8		1,154,400	—	CC&BC	
	b	4	A	-	TP	600	6	17	24	200	412	1926	3,780	2,268,000	—	-	—	65.5		1,021,200	—	"	
	c	4	A	-	TP	1,200	6	20	24	300	850	1929	4,996	5,995,200	—	-	—	39.6		1,683,800	—	"	
	d	4	A	-	TP	1,200	6	20	24	300	850	1930[7]	1,563	1,875,600	—	-	—	67.8		901,400	—	"	
	Plant					3,600					2,524			12,583,800	7,059	1,832	1,762		54.1	4,760,800	674		56.7

TABLE III INFORMATION COVERING LUBRICATING OIL (Page 2)

Plant Number	Engine Designation	Engine Cycle	Injection System - See Notes	Scavenging System - See Notes	Trunk Piston or Cross Head	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Rated Horsepower Hours in Reported Period	Gallons of New Lubricating Oil Used	Gallons of Unfit Lubricating Oil Discarded	Rated H.P. Hrs. per Gallon of New Lubricating Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Gross Output - K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Lubricating Oil Treatment - (See Notes)	Average Cost of Lubricating Oil - Cents per Gallon
164	a	4	M	-	TP	840	6	19-1/2	24	214	550	1926	7,899	6,635,180	---	-	---	89.2		3,875,300	---	CC&S &F	
	b	4	M	-	TP	840	6	19-1/2	24	214	550	1926	8,304	6,975,360	---	-	---	87.5		3,998,000	---		
	c	4	M	-	TP	840	6	19-1/2	24	214	550	1926	7,575	6,363,000	---	-	---	92.6		3,858,000	---		
	d	4	M	-	TP	840	6	19-1/2	24	214	550	1926	8,059	6,769,560	---	-	---	93.7		4,153,200	---		
	Plant					3,360					2,200			26,743,080	16,932	1,200	1,579		90.7	15,884,500	938		27.2
81	a	2	A	S	CH	750	3	20-3/4	26	180	500	1921	3,518	2,638,500	---	-	---	56.5		993,330	---	CC&S &F	
	b	2	A	S	CH	1,250	5	20-3/4	26	180	880	1924	4,718	5,897,600	---	-	---	54.8		2,275,000	---		
	c	2	A	S	CH	1,250	5	20-3/4	26	180	880	1928	5,733	7,166,250	---	-	---	53.9		2,717,900	---		
	Plant					3,250					2,260			15,702,250	4,322	-	3,634		54.6	5,986,230	1,384		32.2
3	a	4	M	-	TP	400	6	14-1/2	18	277	275	1923	700	280,000	169	-	1,657	90.4		174,100	1,030	BC&S &C&F	
	b	4	A	-	TP	800	8	17	24	200	540	1924	2,868	2,294,400	695	-	3,300	36.7		568,000	818		
	c	4	A	-	TP	800	8	17	24	200	540	1926	1,274	1,019,200	307	-	3,320	47.2		325,000	1,057		
	d	4	M	-	TP	1,125	6	22	30	180	750	1927	4,791	5,389,875	1,165	-	4,625	38.0		1,365,900	1,172		
	Plant					3,125					2,105			8,983,475	2,336	-	3,842		40.4	2,433,000	1,041		46.6
720	a	4	M	-	TP	380[8]	2	22-1/2	36	150	240	1922	1,643	624,340	---	-	---	56.9		224,300	---	CC	
	b	4	M	-	TP	300	3	17-3/4	22	225	200	1926	4,250	1,275,000	---	-	---	52.2		443,400	---	"	
	c	2	M	S	TP	840	6	16	20	257	560	1926	5,070	4,258,800	---	-	---	52.2		1,481,800	---	"	
	d	2	M	S	TP	720	6	16	20	257	480	1927	4,480	3,225,600	---	-	---	52.2		1,122,000	---	"	
	e	2	M	S	TP	840	6	16	20	257	560	1928	250	210,000	---	-	---	52.3		73,200	---	"	
	Plant					3,080					2,040			9,593,740	---	-	---		52.5	3,344,700	---		-
7	a	4	M	-	TP	360	6	11-1/2	16	277	225	1925	4,617	1,734,120	---	0	---	64.7		701,380	---	CC	
	b	4	M	-	TP	360	6	11-1/2	16	277	225	1925	4,625	1,665,000	---	0	---	57.6		599,150	---	"	
	c	4	M	-	TP	360	6	11-1/2	16	277	225	1925	4,894-1/2	1,762,020	---	0	---	62.0		683,300	---	"	
	d	4	M	-	TP	750	6	16	22	240	500	1927	4,656-1/2	3,492,375	---	0	---	62.2		1,448,600	---	"	
	e	4	M	-	TP	930	6	18	22	257	620	1930[9]	237-1/2	220,875	---	0	---	63.8		93,900	---	"	
	Plant					2,760					1,795			8,874,390	2,218	0	4,000		61.8	3,526,330	1,589		51.1
732	a	4	A	-	TP	240	4	14	17-1/2	257	160	1916	1,613	387,120	---	-	---	-		---	---	BC	
	b	4	A	-	TP	240	4	14	17-1/2	257	160	1916	2,738	657,120	---	-	---	-		---	---	BC	
	c	4	A	-	TP	520	4	19	24-1/2	200	400	1921	6,920	3,598,400	---	-	---	-		---	---	BC	
	d	4	A	-	CH	750	4	23	32	150	500	1923	5,789	4,341,750	---	-	---	-		---	---	CC	
	e	4	A	-	CH	750	4	23	32	150	500	1925	5,687	4,265,250	---	-	---	-		---	---	CC	
	Plant					2,500					1,720			13,249,640	8,462	-	1,565		59.7	5,494,770	649		52.0
60	a	4	M	-	TP	800	8	19-1/2	22	200	540	1924	2,090	1,672,000	---	-	---	35.3		398,700	---	CC	
	b	4	M	-	TP	800	8	19-1/2	22	200	540	1924	4,362	3,489,600	---	-	---	36.5		859,700	---	"	
	c	4	M	-	TP	800	8	19-1/2	22	200	540	1924	2,409	1,927,200	---	-	---	39.4		512,500	---	"	
	Plant					2,400					1,620			7,088,800	2,897	-	2,447		37.0	1,770,900	611		54.1
41	a	4	A	-	TP	520	3	22	29-1/2	164	400	1922	3,766	1,958,320	---	-	---	54.2		816,990	---	CC&S &F	
	b	4	A	-	TP	700	4	22	29-1/2	164	500	1923	3,188	2,231,600	---	-	---	56.4		900,000	---		
	c	4	A	-	TP	1,150	6	23	29-1/2	164	900	1928	4,302	4,947,300	---	-	---	62.0		2,401,300	---		
	Plant					2,370					1,800			9,137,220	3,369	100	2,710		59.0	4,118,290	1,222		58.6
73	a	4	A	-	TP	225	3	16	24	164	150	1911	926	208,350	202	-	1,031	57.6		80,000	396	S	
	b	4	A	-	TP	225	3	16	24	164	150	1912	269	60,525	41	-	1,476	58.2		23,500	573	"	
	c	4	A	-	TP	600	6	16-1/4	23	225	410	1929	5,186	3,111,600	1,061	-	2,934	66.9		1,422,100	1,340	"	
	d	4	A	-	TP	520	4	19	24-1/2	200	350	1923	2,557	1,329,640	677	-	1,963	71.4		639,000	944	"	
	e	4	A	-	TP	600	6	16-1/4	23	225	410	1925	4,368	2,620,800	1,140	-	2,297	66.7		1,195,000	1,048	"	
	Plant					2,170					1,470			7,330,915	3,121	-	2,347		67.3	3,359,600	1,076		49.3
5	a	2	M	S	TP	720	6	16	20	257	480	--	---	---	---	-	---	-		---	---	-	
	b	2	M	S	TP	720	6	16	20	257	480	--	---	---	---	-	---	-		---	---	-	
	c	2	M	S	TP	720	6	16	20	257	480	--	---	---	---	-	---	-		---	---	-	
	Plant					2,160					1,440			---	2,348	-	---		-	1,184,450	504		53.9
80	a	4	A	-	TP	365	4	16-1/4	23	225	250	1921	---	---	---	-	---	-		---	---	None	
	b	4	A	-	TP	520	4	19	24-1/2	200	350	1921	---	---	---	-	---	-		---	---	"	
	c	4	A	-	TP	600	6	16-1/4	23	225	400	1929	---	---	---	-	---	-		---	---	"	
	d	4	M	-	TP	665	4	21	30	180	410	1926	---	---	---	-	---	-		---	---	"	
	Plant					2,150					1,410			---	2,804	-	---		-	2,403,000	856		45.0
6	a	2	M	CC	TP	300[10]	6	14	17	257	200	1926	---	---	---	-	---	-		---	---	-	
	b	4	A	-	TP	600	6	17	25	200	400	1926	---	---	---	-	---	-		---	---	-	
	c	4	A	-	TP	600	6	17	25	200	412	1927	---	---	---	-	---	-		---	---	-	
	d	4	A	-	TP	600	6	17	25	200	412	1928	---	---	---	-	---	-		---	---	-	
	Plant					2,100					1,424			---	1,825	-	---		-	1,614,100	884		49.0

TABLE III INFORMATION COVERING LUBRICATING OIL (Page 3)

Plant Number	Engine Designation	Engine Cycle	Injection System - See Notes	Scavenging System - See Notes	Trunk Piston or Cross Head	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Rated Horsepower Hours in Reported Period	Gallons of New Lubricating Oil Used	Gallons of Unfit Lubricating Oil Discarded	Rated H.P. Hrs. per Gallon of New Lubricating Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Gross Output - K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Lubricating Oil Treatment - (See Notes)	Average Cost of Lubricating Oil - Cents per Gallon
728	a	4	A	-	TP	600	6	16-1/4	23	225	410	1926	4,000	2,400,000	---	-	---	59.0		967,700	---	BC	
	b	4	A	-	TP	600	6	16-1/4	23	225	410	1926	4,000	2,400,000	---	-	---	61.9		1,014,100	---	"	
	c	2	M	S	TP	840	6	16	20	257	565	1929	3,300	2,772,000	---	-	---	75.7		1,411,540	---	"	
	Plant					2,040					1,385			7,572,000	5,550	-	1,363		66.0	3,393,340	611		40.6
716	a	4	A	-	TP	600	6	16-1/4	23	225	410	1927	4,321	2,592,600	---	-	---	62.3		1,104,000	---	BC& S& F	
	b	4	A	-	TP	365	4	16-1/4	21	225	250	1922	3,521	1,285,165	---	-	---	68.6		604,000	---		
	c	4	A	-	TP	500	4	18-7/8	28-3/8	164	380	1918	6,219	3,109,500	---	-	---	56.6		1,337,000	---		
	d	4	A	-	TP	500	4	18-7/8	28-3/8	164	400	1925	2,434	1,217,000	---	-	---	61.6		600,000	---		
	Plant					1,965					1,440			8,204,265	4,600	-	1,783		60.8	3,645,000	793		51.1
93	a	2	M	S	TP	700	5	16	20	257	480	1928	5,532	3,872,400	---	-	---	40.7		1,080,200	---	CC	
	b	2	M	S	TP	700	5	16	20	257	480	1928	4,930	3,451,000	---	-	---	42.2		998,000	---	"	
	c	2	M	S	TP	560	4	16	20	257	376	1928	4,586	2,567,600	---	-	---	49.3		850,000	---	"	
	Plant					1,960					1,336			9,891,000	2,167	-	4,560		43.4	2,928,200	1,351		47.4
215	a	2	M	S	TP	980	7	16	20	257	676	1930[4]	1,361	1,333,780	---	-	---	77.1		709,100	---	BC	
	b	2	M	S	TP	980	7	16	20	257	676	1930[4]	1,270	1,244,600	---	-	---	80.0		686,500	---	"	
	Plant					1,960					1,352			2,578,380	1,030	-	2,500		78.5	1,395,600	1,353		77.0
729	a	4	A	-	TP	225	3	16	24	164	160	1912	74-3/4	16,819	---	-	---	-		---	---	BC& S& F	
	b	4	A	-	TP	225	3	16	24	164	160	1914	53-1/2	12,038	---	-	---	-		---	---		
	c	4	A	-	TP	225	3	16	24	164	160	1916	85-3/4	19,294	---	-	---	-		---	---		
	d	4	A	-	TP	500	4	18-7/8	28-3/8	164	380	1920	1,897-3/4	948,875	---	-	---	-		---	---		
	e	4	A	-	CH	750	4	23	32	150	500	1922	3,898-1/2	2,923,875	---	-	---	-		---	---		
	Plant					1,925					1,360			3,920,901	---	-	---		83.5	2,257,260	---		-
837	a	4	A	-	CH	750	4	23	32	150	500	1922	8,079	6,069,250	2,452	-	2,470	49.3		1,991,720	812	S&F	
	b	4	A	-	TP	450	6	16	24	164	330	1918	548	246,600	1,163	-	495	50.6		91,560	163	"	
	c	4	A	-	TP	225	3	16	24	164	160	1904	733	164,925		-		40.3		47,210		"	
	d	4	A	-	TP	225	3	16	24	164	160	1904	521	117,225		-		45.1		37,585		"	
	e	4	A	-	TP	225	3	16	24	164	160	1906	208	46,800		-		39.0		12,970		"	
	Plant					1,875					1,310			6,634,800	3,615	-	1,835		49.0	2,181,045	603		37.7
109	a	2	A	S	CH	675	4	16-3/4	22	240	450	1927	1,973	1,331,775	---	-	---	44.3		393,350	---	F	
	b	2	A	S	TP	1,125	8	14-3/4	21	257	750	1929	1,295	1,456,875	---	-	---	51.5		500,340	---	"	
	Plant					1,800					1,200			2,788,650	1,470	-	1,895		48.1	893,690	608		75.7
176	a	4	A	-	TP	600	6	17	25	200	410	1926	30,755	18,453,000	---	-	---	61.6		7,765,300	---	BC&S	
	b	4	A	-	TP	600	6	17	25	200	410	1926	32,174	19,304,400	---	-	---	55.1		7,287,600	---	"	
	c	4	A	-	TP	600	6	17	25	200	410	1928[7]	14,370	8,622,000	---	-	---	63.5		3,740,900	---	"	
	Plant					1,800					1,230			46,379,400	34,001	-	1,363		59.2	18,773,800	552		75.7[11]
722	a	2	M	CC	TP	200[10]	4	14	17	257	136	1923	352	70,400	---	-	---	66.8		31,950	---	BC	
	b	2	M	CC	TP	300[10]	6	14	17	257	200	1923	354	106,200	---	-	---	81.6		57,730	---	"	
	c	2	M	S	TP	840	6	16	20	257	565	1929	1,460	1,226,400	---	-	---	86.1		710,000	---	"	
	d	4	A	-	TP	365	4	16-1/4	21	225	266	1916	766	279,590	---	-	---	70.1		142,870	---	"	
	Plant					1,705					1,167			1,682,590	1,024	-	1,642		82.1	942,550	920		54.0
18	a	2	M	CC	CH	450	5	15-1/4	16	275	300	1922	---	---	---	-	---	-		---	---	-	
	b	4	A	-	TP	600	6	17	25	200	410	1926	---	---	---	-	---	-		---	---	-	
	c	4	A	-	TP	600	6	17	25	200	410	1928	---	---	---	-	---	-		---	---	-	
	Plant					1,650					1,120			---	2,754	-	---		-	2,502,301	908		53.5
8	a	4	A	-	TP	400	4	16-1/2	24	200	275	1921	5,524	2,209,600	---	0	---	45.1		684,970	---	F&CC	
	b	4	A	-	TP	560	3	22-1/2	22-1/2	225	400	1922	1,126	630,560	---	0	---	64.1		288,840	---	CC	
	c	4	A	-	TP	600	6	17	25	200	400	1926	4,280	2,568,000	---	0	---	65.7		1,125,290	---	CC	
	Plant					1,560					1,075			5,408,160	2,182	0	2,480		57.0	2,099,100	962		54.9
132	a	4	A	-	TP	375	3	17-1/2	24-1/2	225	255	1929	5,492	2,059,500	---	0	---	58.3		816,800	---	BC&S	
	b	4	A	-	TP	500	4	17-1/2	24-1/2	225	345	1929	2,860	1,430,000	---	0	---	70.3		693,600	---	"	
	c	4	A	-	TP	625	5	17-1/2	24-1/2	225	430	1929	714	446,250	---	0	---	65.2		200,100	---	"	
	Plant					1,500					1,030			3,935,750	924	0	4,260		63.5	1,710,500	1,852		50.0
154	a	4	A	-	TP	600	6	17	24	200	410	1927	---	---	---	-	---	-		---	---	-	
	b	4	A	-	TP	400	4	17	24	200	270	1926	---	---	---	-	---	-		---	---	-	
	c	4	A	-	TP	400	4	17	24	200	270	1925	---	---	---	-	---	-		---	---	-	
	Plant					1,400					950			---	994	-	---		-	1,617,985	1,628		50.2

TABLE III INFORMATION COVERING LUBRICATING OIL (Page 4)

Plant Number	Engine Designation	Engine Cycle	Injection System - See Notes	Scavenging System - See Notes	Trunk Piston or Cross Head	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Rated Horsepower Hours in Reported Period	Gallons of New Lubricating Oil Used	Gallons of Unfit Lubricating Oil Discarded	Rated H.P. Hrs. per Gallon of New Lubricating Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Gross Output - K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Lubricating Oil Treatment - (See Notes)	Average Cost of Lubricating Oil - Cents per Gallon
163	a	4	M	-	TP	400	4	17-3/4	22	225	275	1924	5,180	2,072,000	1,089	219	1,902	66.6		948,200	870	BC	
	b	4	M	-	TP	400	4	17-3/4	22	225	275	1927	8,458	3,383,200	2,573	100	1,314	79.5		1,849,600	719	"	
	c	4	M	-	TP	600	6	17-3/4	22	225	500	1929	7,587	4,552,200	2,951	388	1,543	68.2		2,586,400	876	"	
	Plant					1,400					1,060			10,007,400	6,613	707	1,512		71.4	5,384,200	814		16.6
723	a	2	M	CC	TP	300[10]	6	14	17	257	200	1925	1,712	513,600	---	---	---	73.4		251,370	---	BC	
	b	2	M	CC	TP	300[10]	6	14	17	257	200	1925	998	299,400	---	---	---	71.9		143,620	---	"	
	c	2	M	S	TP	720	6	16	20	257	480	1927	3,101	2,232,720	---	---	---	66.2		986,240	---	"	
	Plant					1,320					880			3,045,720	2,706	---	1,126		68.0	1,381,230	510		54.0
725	a	4	A	-	TP	600	6	17	25	200	400	1926	6,480	3,888,000	---	---	---	-		---	---	CC&S	
	b	4	A	-	TP	400	4	17	25	200	270	1925	5,760	2,304,000	---	---	---	-		---	---	"	
	c	2	M	CC	TP	200[10]	4	14	17	257	136	1924	0	0	0	0	---	-		---	---		
	d	2	M	CC	TP	110[10]	2	14	17-1/2	257	70	1923	0	0	0	0	---	-		---	---		
	Plant					1,310					876			6,192,000	2,729	---	2,268		-	---	---		61.1
13	a	4	A	-	TP	275	4	13-1/2	19-1/2	257	175	1923	6,751	1,856,525	1,060	---	1,765	64.9		767,500	731	CC& S&F	
	b	4	A	-	TP	200	3	13-1/2	19-1/2	257	125	1923	2,578	515,600	411	---	1,255	66.0		213,100	519		
	c	4	A	-	TP	750	4	23	29-1/2	164	500	1925	1,356	1,017,000	407	---	2,500	45.8		310,500	762		
	Plant					1,225					800			3,389,125	1,868	---	1,815		59.2	1,291,100	691		39.7
17	a	4	A	-	TP	600	6	17	25	200	410	1926	---	---	---	---	---	-		---	---	-	
	b	4	A	-	TP	600	6	17	25	200	410	1926	---	---	---	---	---	-		---	---	-	
	Plant					1,200					820			---	1,947	---	---		-	2,217,200	1,139		54.8
94	a	4	A	-	TP	600	6	17	24	200	410	1927	6,498	3,898,800	---	---	---	65.7		1,751,890	---	CC&S	
	b	4	A	-	TP	600	6	17	24	200	410	1927	6,293	3,775,800	---	---	---	67.1		1,733,320	---	"	
	Plant					1,200					820			7,674,600	3,998	---	1,918		66.4	3,485,210	872		49.2
616	a	2	M	CC	TP	100[10]	2	14	17	257	60	1920	---	---	---	0	---	-		12,000	---	CC	
	b	4	A	-	TP	300	3	16-1/2	24	200	200	1923	---	---	---	0	---	-		775,870	---	"	
	c	4	A	-	CH	750	4	23	32	150	525	1925	---	---	---	0	---	-		906,310	---	"	
	Plant					1,150					785			---	1,867	0	---		-	1,694,180	907		70.4
95	a	2	M	S	TP	560	4	16	20	257	375	1929	4,016	2,248,960	331	---	6,800	50.0		752,500	2,275	CC	
	b	2	M	S	TP	560	4	16	20	257	375	1929	3,936	2,204,160	312	---	7,070	51.5		760,200	2,435	"	
	Plant					1,120					750			4,453,120	643	---	6,930		50.7	1,512,700	2,350		45.3
96	a	4	M	-	TP	600	6	17	24	225	400	1925	2,400	1,440,000	---	---	---	48.9		469,000	---	S	
	b	4	M	-	TP	300	3	17	24	225	200	1923	4,717	1,415,100	---	---	---	76.2		719,100	---	"	
	c	2	M	CC	TP	200[10]	4	14	17	257	136	1921	844	168,800	---	---	---	46.8		53,700	---	"	
	Plant					1,100					736			3,023,900	1,906	---	1,585		61.5	1,241,800	652		41.8
182	a	4	M	-	TP	270	4	14-1/2	18	300	175	1930[12]	2,704	730,080	523	0	1,396	73.6		348,100	665	BC&F	
	b	4	M	-	TP	410	6	14-1/2	18	300	300	1930[12]	2,511	1,029,510	388	0	2,650	69.8		525,900	1,355	"	
	c	4	M	-	TP	410	6	14-1/2	18	300	300	1930[12]	1,679	688,390	268	0	2,570	71.0		357,900	1,335	"	
	Plant					1,090					775			2,447,980	1,179	0	2,065		71.0	1,231,900	1,045		54.0
721	a	2	M	CC	TP	360	6	14	17	257	240	1925	3,405	1,225,800	---	---	---	33.3		272,400	---	CC&F	
	b	2	M	CC	TP	360	6	14	17	257	240	1925	4,540	1,634,400	---	---	---	33.0		360,400	---	"	
	c	2	M	CC	TP	360	6	14	17	257	240	1926	2,300	828,000	---	---	---	32.0		176,900	---	"	
	Plant					1,080					720			3,688,200	2,000	---	1,844		32.9	809,700	405		60.0
78	a	4	M	-	TP	400	4	17	24	225	300	1928[13]	24,136	9,654,400	8,952	750	1,078	57.4		4,151,314	464	CC	
	b	4	M	-	TP	660	6	17	24	240	500	1930[7]	3,386	2,234,760	202	0	11,063	47.5		803,202	3,976	"	
	Plant					1,060					800			11,889,160	9,154	750	1,298[14]		55.5	4,954,516	541		43.7
97	a	4	A	-	TP	520	4	19	24	200	333	1915	4,854	2,524,080	---	---	---	-		---	---	S	
	b	4	A	-	TP	250	4	13-3/4	17-1/2	257	150	1914	4,752	1,188,000	---	---	---	-		---	---	"	
	c	4	A	-	TP	250	4	13-3/4	17-1/2	257	210	1914	4,478	1,119,500	---	---	---	-		---	---	"	
	Plant					1,020					693			4,831,580	5,744	---	841		57.4	1,876,040	327		50.9
83	a	4	A	-	TP	600	6	16-1/4	23	225	416	1929	4,571	2,742,600	627	460	4,442	55.3		1,051,300	1,703	BC	
	b	4	A	-	TP	400	4	16-1/4	23	225	270	1927	4,780	1,912,000	411	300	4,650	46.2		596,600	1,452	"	
	Plant					1,000					686			4,654,600	1,038	760	4,525		51.6	1,647,900	1,602		68.2
69	a	2	M	S	TP	480	4	16	20	257	320	1927	4,125	1,980,000	---	---	---	-		---	---	BC	
	b	2	M	S	TP	480	4	16	20	257	320	1927	4,515	2,167,200	---	---	---	-		---	---	"	
	Plant					960					640			4,147,200	1,919	700	2,160		45.9	1,268,358	661		56.0

TABLE III INFORMATION COVERING LUBRICATING OIL (Page 5)

Plant Number	Engine Designation	Engine Cycle	Injection System - See Notes	Scavenging System - See Notes	Trunk Piston or Cross Head	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Rated Horsepower Hours in Reported Period	Gallons of New Lubricating Oil Used	Gallons of Unfit Lubricating Oil Discarded	Rated H.P. Hrs. per Gallon of New Lubricating Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Gross Output - K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Lubricating Oil Treatment - (See Notes)	Average Cost of Lubricating Oil - Cents per Gallon
53	a	2	M	CC	TP	200^{10}	4	14	17	257	136	1923	2,794	558,800	633	75	882	72.1		274,110	433	S	
	b	2	M	CC	TP	300^{10}	6	14	17	257	200	1925	4,386	1,315,800	1,219	131	1,175	71.0		624,980	558	"	
	c	4	A	-	TP	400	4	16-1/4	23	225	270	1925	7,758	3,102,400	1,905	60	1,630	70.2		1,470,190	772	"	
	Plant					900					606			4,977,000	3,757	266	1,325		70.6	2,369,280	631		44.4
98	a	2	M	CC	TP	360	6	14	17	257	240	1926	5,469	1,968,840	—	—	—	64.8		850,900	—	BC	
	b	2	M	CC	TP	360	6	14	17	257	240	1929	5,083	1,829,880	—	—	—	65.3		796,400	—	"	
	c	2	M	CC	TP	180	3	14	17	257	120	1926	3,557	640,260	—	—	—	61.8		263,500	—	"	
	Plant					900					600			4,438,980	4,064	—	1,092		64.5	1,910,800	470		39.1
74	a	2	M	CC	CH	225	3	15-1/4	16	277	150	1924	588	132,300	97	—	1,364	-		—	—	C&S	
	b	4	M	-	TP	260	4	14-1/2	18	277	175	1928	5,656	1,470,560	803	—	1,831	-		—	—	"	
	c	4	M	-	TP	400	6	14-1/2	18	277	275	1928	2,680	1,072,000	366	—	2,929	-		—	—	"	
	Plant					885					600			2,674,860	1,266	—	2,113		-	—	—		64.0
189	a	4	M	-	TP	550	5	17	24	240	375	1929	8,000	4,400,000	—	0	—	-		—	—	S&C	
	b	4	M	-	TP	200^{6}	2	17	24	200	145	1922	700	140,000	—	0	—	-		—	—	"	
	c	4	M	-	TP	100^{6}	1	17	24	200	75	1915	60	6,000	—	0	—	-		—	—	"	
	Plant					850					595			4,546,000	955	0	4,760		27.4	850,976	891		77.6
153	a	2	M	S	TP	840	6	16	20	257	560	1929	—	—	100	—	—	-		97,970	980	-	49.9
100	a	2	M	CC	TP	300^{10}	6	14	17	257	200	1923	4,408	1,322,400	—	—	—	59.2		521,620	—	BC	
	b	2	M	CC	TP	240	4	14	17	257	160	1924	5,602	1,344,480	—	—	—	57.6		516,200	—	"	
	c	2	M	CC	TP	200^{10}	4	14	17	257	136	1924	5,507	1,101,400	—	—	—	67.0		501,630	—	"	
	Plant					740					496			3,768,280	2,362	—	1,600		60.9	1,539,450	652		42.6
112	a	2	M	CC	TP	200^{10}	4	14	17	257	136	1925	4,719	943,800	594	—	1,588	64.8		415,900	700	BC& S&F	
	b	2	M	CC	TP	240	4	14	17	257	155	1927	4,292	1,030,080	733	—	1,406	64.0		425,500	580		
	c	2	M	CC	TP	300^{10}	6	14	17	257	200	1925	2,545-1/2	763,650	572	—	1,333	77.4		394,100	689		
	Plant					740					491			2,737,530	1,899	—	1,440		68.1	1,235,500	651		51.0
67	a	2	M	CC	TP	360	6	14	17	257	250	1925	2,590	932,400	625	—	1,491	83.2		539,000	863	F	
	b	2	M	CC	TP	360	6	14	17	257	250	1925	2,720	979,200	819	—	1,195	82.5		560,500	684	"	
	Plant					720					500			1,911,600	1,444	—	1,323		82.9	1,099,500	761		54.1
170	a	2	M	CC	TP	360	6	14	17	257	240	1929	2,946	1,061,280	650	10	1,630	-		—	—	C&F	
	b	2	M	CC	TP	360	6	14	17	257	240	1929	2,778	1,000,080	633	10	1,580	-		—	—	"	
	Plant					720					480			2,061,360	1,283	20	1,605		64.7	889,910	694		60.0
527	a	2	M	CC	TP	360	6	14	17	257	240	1930^{4}	3,672	1,321,920	—	—	—	36.7		323,400	—	F	
	b	2	M	CC	TP	360	6	14	17	257	240	1930^{4}	4,590	1,652,400	—	—	—	37.2		409,900	—	"	
	Plant					720					480			2,974,320	1,652	250	1,800		37.0	733,300	444		65.0
718	a	2	M	CC	TP	360	6	14	17	257	240	—	3,926	1,414,080	829	139	1,705	37.0		348,280	420	CC&S &F	
	b	2	M	CC	TP	360	6	14	17	257	240	--	3,641	1,310,760	997	129	1,314	36.9		322,210	323		
	Plant					720					480			2,724,840	1,826	268	1,492		36.9	670,490	367		44.0
19	a	2	M	CC	TP	100^{10}	2	14	17	257	60	—	—	—	—	—	—	-		—	—	-	
	b	2	M	CC	CH	240	4	12-1/2	13-1/4	325	160	1927	—	—	—	—	—	-		—	—	-	
	c	2	M	CC	CH	360	4	15-1/4	16	275	240	1927	—	—	—	—	—	-		—	—	-	
	Plant					700					460			—	1,553	—	—		-	764,380	492		57.7
61	a	4	M	-	TP	300	3	17-3/4	22	225	220	1928	8,169	2,450,700	—	—	—	54.8		984,100	—	CC&F	
	b	4	M	-	TP	300	3	17-3/4	22	225	220	1928	5,779	1,733,700	—	—	—	48.8		619,800	—	"	
	Plant					600					440			4,184,400	1,818	50	2,300		52.3	1,603,900	882		49.9
106	a	4	A	-	TP	600	6	17	25	200	410	1929	—	—	761	—	—	-	-	1,498,970	1,970	-	50.3
103	a	2	M	CC	TP	240	4	14	17	257	160	1927	6,151	1,476,240	—	—	—	63.1		620,860	—	BC&S	
	b	2	M	CC	TP	150^{10}	3	14	17	257	100	1923	5,398	809,700	—	—	—	56.3		303,610	—	"	
	c	2	M	CC	TP	200^{10}	4	14	17	257	160	1920^{15}	1,006	201,000	—	—	—	55.4		89,100	—	"	
	Plant					590					420			2,486,940	1,768	—	1,406		60.2	1,013,570	573		52.5
102	a	2	M	CC	TP	240	4	14	17	257	160	1928	4,729	1,134,960	—	—	—	43.7		330,250	—	BC&S	
	b	2	M	CC	TP	240	4	14	17	257	160	1928	4,818	1,156,320	—	—	—	41.0		315,650	—	"	
	c	2	M	CC	TP	100^{10}	2	14	17	257	60	1922^{15}	304	30,400	—	—	—	77.6		14,160	—	"	
	Plant					580					380			2,321,680	1,638	—	1,417		42.7	660,060	403		49.0

TABLE III INFORMATION COVERING LUBRICATING OIL (Page 6)

Plant Number	Engine Designation	Engine Cycle	Injection System - See Notes	Scavenging System - See Notes	Trunk Piston or Cross Head	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Rated Horsepower Hours in Reported Period	Gallons of New Lubricating Oil Used	Gallons of Unfit Lubricating Oil Discarded	Rated H.P. Hrs. per Gallon of New Lubricating Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Gross Output - K.W. Hrs.	Gross K.W. Hrs. per Gallon of New Lubricating Oil	Lubricating Oil Treatment - (See Notes)	Average Cost of Lubricating Oil - Cents per Gallon
101	a	2	M	CC	TP	240	4	14	17	257	160	1926	5,367	1,288,080	---	---	---	64.4		553,315	---	BC&S	
	b	2	M	CC	TP	150^{10}	3	14	17	257	120	1919	2,342	351,300	---	---	---	46.3		130,190	---	"	
	c	2	M	CC	TP	150^{10}	3	14	17	257	100	1923	2,105	315,750	---	---	---	63.8		134,320	---	"	
	Plant					540					380			1,955,130	1,495	---	1,308		60.5	817,825	547		46.0
501	a	2	M	CC	TP	120	2	14	17	257	80	1925	4,480	537,600	---	---	---	-		---	---	CC& S&F	
	b	2	M	CC	TP	120	2	14	17	257	80	1925	4,481	537,720	---	---	---	-		---	---		
	c	2	M	CC	TP	240	4	14	17	257	155	1929	2,057	493,680	---	---	---	-		---	---		
	Plant					480					315			1,569,000	1,311	---	1,197		40.6	421,022	321		55.3
335	a	4	A	-	TP	250	4	13-3/4	17-1/2	257	175	1924	7,660	1,915,000	334	---	5,730	32.4		433,870	1,298	S	
	b	4	A	-	TP	180	4	11-3/4	15	300	120	1919	1,100	198,000	75	---	2,640	42.1		56,550	741	"	
	Plant					430					295			2,113,000	409	---	5,160		33.2	489,420	1,197		59.9
99	a	4	A	-	TP	400	4	17	24	200	275	1927	1,607	642,800	155	---	4,150	64.9	64.9	375,100	2,410	BC&S	55.3
247	a	2	M	CC	TP	240	4	14	17	257	160	1928	---	---	---	---	---	-		---	---	C&F	
	b	2	M	CC	TP	100^{10}	2	14	17	257	75	1920	---	---	---	---	---	-		---	---	"	
	c	2	M	CC	TP	50^{10}	1	14	17	257	40	1915	---	---	---	---	---	-		---	---	"	
	Plant					390					275			---	860	---	---		-	429,890	500		55.0
353	a	2	M	CC	TP	50^{10}	1	14	17	257	30	1916	2,470	123,500	---	0	---	68.9		51,050	---	S&F	
	b	2	M	CC	TP	100^{10}	2	14	17	257	80	1920	3,715	371,500	---	0	---	46.7		104,100	---	"	
	c	2	M	CC	TP	240	4	14	17	257	160	1927	2,386	572,640	---	0	---	33.4		127,425	---	"	
	Plant					390					250			1,067,640	1,015	0	1,052		41.6	282,575	278		44.0
106	a	2	M	CC	TP	150^{10}	3	14	17	257	100	1922	4,325	648,750	---	---	---	62.4		270,180	---	BC	
	b	2	M	CC	TP	120	2	14	17	257	72	1926	3,889	466,680	---	---	---	47.8		133,880	---	"	
	c	2	M	CC	TP	100^{10}	2	14	17	257	60	1923	701	70,100	---	---	---	51.7		21,740	---	"	
	Plant					370					232			1,185,530	879	---	1,348		56.5	425,800	484		46.3
152	a	2	M	CC	TP	360	6	14	17	257	240	1925	---	---	43	---	---	-	-	28,500	663	-	41.8
857	a	2	M	CC	TP	180	3	14	17	257	125	1929	---	---	---	---	---	-		185,240	---	S&F	
	b	2	M	CC	TP	150^{10}	3	14	17	257	72	1927	---	---	---	---	---	-		184,100	---	"	
	Plant					330					197			---	1,550^{17}	---	---		-	349,340	225		40.8
27	a	2	M	CC	TP	75^{10}	2	12	15	300	48	1926	---	---	---	---	---	-		---	---	-	
	b	2	M	CC	CH	240	4	12-1/2	13-1/4	325	150	1926	---	---	---	---	---	-		---	---	-	
	Plant					315					198			---	924	---	---		-	210,630	228		50.0
515	a	2	M	CC	TP	180	3	14	17	257	120	1928	---	---	---	---	---	-		---	---	F	
	b	2	M	CC	TP	75^{10}	2	12	15	300	48	1915	---	---	---	---	---	-		---	---	"	
	c	2	M	CC	TP	50^{10}	1	14	17	257	32	1915	---	---	---	---	---	-		---	---	"	
	Plant					305					200			---	1,200	---	---		-	---	---		54.0
169	a	4	M	-	TP	240	6	10-3/4	13-3/4	327	160	1928	2,377-1/2	570,600	281	26	2,030	41.5	41.5	157,780	561	CC	51.0
266	a	2	M	CC	TP	120	2	14	17	257	72	1927	2,920	35,040	---	---	---	-		---	---	F	
	b	2	M	CC	TP	60	1	14	17	257	38	1928	5,840	350,400	---	---	---	-		---	---	"	
	Plant					180					110			385,440	540	170	714		-	---	---		59.9
858	a	4	M	-	TP	60^{8}	1	14-1/2	16	325	37½	1929	---	---	---	---	---	-		98,800	---	S&F	
	b	2	M	CC	TP	120	2	14	17	257	90	1929	---	---	---	---	---	-		34,200	---	"	
	Plant					180					127½			---	1,050^{17}	---	---		-	133,000	127		38.4
646	a	2	M	CC	TP	120	2	14	17	257	72	1927	1,016	121,920	82½	15	1,478	13.7	13.7	16,690	202	BC&F	60.6

NOTES:

INJECTION SYSTEM

A - Air
M - Mechanical

SCAVENGING SYSTEM

S - Separate (i.e., Pump or Blower)
CC - Crankcase (for 2-Stroke Cycle Only)

LUBRICATING OIL TREATMENT

CC - Continuous Centrifuging.
BC - "BATCH" Centrifuging
C - Chemically
F - Filtering
S - Settling

NUMBERED NOTES

1. Speeded up and Connected to New Generator since last Report
2. Name Plates are 700 and 1400 K.W.
3. April 4. February 5. October
6. Rated 450 KVA. 7. September
8. Horizontal Engine. 9. December
10. Semi-Diesel Engine
11. Cost includes Import Duty
12. August
13. March - the start of the Period.
14. No. 1 Engine has Plain Rings; No. 2 has Special Rings
15. Installed in Plt. #103 in August, 1930
16. Installed in Plt. #102 in October, 1930
17. Estimated by Operator

TABLE IV - INFORMATION COVERING FUEL OIL (Page 1)

Plant Number	Engine Designation	Engine Cycle	Injection System-See Notes	Scavenging System - See Notes	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Gross Output - K.W. Hrs.	Fuel Oil Used - Gallons	Gross K.W. Hrs. per Gallon of Fuel Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Nature of Fuel Oil Used - See Notes	Average Cost of Fuel Oil - Cents per Gallon	Is Fuel Centrifuged?	Plant Altitude - Feet Above Sea Level	Purpose for Which Jacket Water Heat is Utilized - See Notes	Purpose for Which Exhaust Heat is Utilized-See Notes
43	a	4	A	-	500	3	22	29-1/2	150	300	1915	637-1/2	143,608	14,765	9.73	75.0		25.9° API		No			
	b	4	A	-	500	3	22	29-1/2	150	300	1916	433-1/2	95,848	9,792	9.79	73.8		19,135 BTU/lb.		No			
	c	4	A	-	520	4	19	24-1/2	200	350	1919	848	258,660	29,653	8.72	87.2		12° Say.F @ 77° F		No			
	d	4	A	-	520	4	19	24-1/2	200	350	1920	880-1/2	229,870	26,970	8.52	74.6		0.7% S		No			
	e	4	A	-	1,000	6	20-1/2	36	150	700	1922	1,033	456,610	48,134	9.49	63.2		1.0% BS&W		No			
	f	4	A	-	1,000	6	20-1/2	36	150	700	1925	876	386,020	40,403	9.55	63.0		0.3% Asph. by Holde Method		No			
	g	2	A	S	3,750	6	30	42	124	2,600	1928	6,982-1/2	12,480,300	1,097,042	11.38	68.7		19+° API		No			
	h	2	A	S	3,750	6	30	42	124	2,800	1929	7,363	14,824,900	1,105,160	13.41[1]	71.9		18,910 BTU/lb. 61.3° Say.F @ 77° F		No			
	Plant				11,540					8,100			28,875,816	2,371,939	12.17[1]		70.4	13.6% Asph. Res. - 1/2 hr. @ 400° 0.023% Ash 0.03% BS&W	4.37		2,500	N	N
46	a	2	A	S	1,250	5	20-3/4	26	180	850[3]	1922	3,772	2,517,890	247,301	10.18	78.5		21° API		No			
	b	2	A	S	1,250	5	20-3/4	26	180	850[3]	1922	4,600	3,094,050	293,697	10.54	79.2		18,954 BTU/lb.		No			
	c	2	A	S	2,250[2]	4	28	44	124[2]	1,500[3]	1922	6,182	7,494,500	682,923	10.97	81.0		20° Say.F @ 122° F		No			
	d	2	A	S	2,250[2]	4	28	44	124[2]	1,500[3]	1922	6,507	8,293,500	761,158	10.90	84.9		1.7% S		No			
	e	2	A	S	2,250	4	28	44	124	1,500[3]	1930[4]	5,461	7,308,400	561,178	13.03	89.2		0.2% BS&W		No			
	Plant				9,250					6,200			28,708,340	2,546,257	11.27		83.6		3.62		5,000	F	N
82	a	4	A	-	360	4	16-1/4	21	225	250	1921	844	129,355	13,249	9.76	61.3		28-32° API		No			
	b	2	A	S	750	4	17	27	180	517	1923	840	248,725	27,218	9.14	57.3		U.S. Navy Standard		No			
	c	2	A	S	750	4	17	27	180	517	1924	601	181,925	19,694	9.24	58.5		Diesel Fuel		No			
	d	2	A	S	1,150	6	17	27	180	800	1927	2,785	1,442,700	130,124	10.98	64.7				No			
	e	2	A	S	1,150	6	17	27	180	800	1927	2,779	1,446,100	133,949	10.79	65.0				No			
	f	2	A	S	1,500	8	17	27	180	1,080	1929	6,536	4,658,500	403,790	11.54	66.0				No			
	Plant				5,660					3,964			8,107,305	728,024	11.13		65.0		5.71		23	N	N
91	a	4	M	-	1,125	6	22	30	180	800	1927	7,756	3,125,850	---	--	50.4		25° API		No			
	b	4	M	-	1,125	6	22	30	180	800	1928	7,683	3,178,575	---	--	51.7				No			
	c	4	M	-	600	6	17	24	225	400	1927	3,905	983,675	---	--	63.0				No			
	d	4	M	-	600	6	17	24	225	400	1927	3,327	811,900	---	--	61.0				No			
	e	4	A	-	1,800	6	24	32	225	1,275	1930[5]	889	596,300	---	--	52.6				No			
	Plant				5,250					3,675			8,696,300	787,921	11.03		53.1		4.09		6,000	N	F
73	a	2	A	S	1,250	5	20-3/4	26	180	850	1923	5,332	2,857,800	271,884	10.51	63.1		11-14° API		Yes			
	b	2	A	S	1,250	5	20-3/4	26	180	850	1924	6,155	3,364,400	329,847	10.19	64.3		18,150 BTU/lb.		Yes			
	c	2	A	S	1,250	5	20-3/4	26	180	850	1928	2,606	1,443,600	143,416	10.07	65.2		20° Say.F @ 122° F		Yes			
	d	2	A	S	1,250	5	20-3/4	26	180	850	1928	1,473	807,500	80,635	10.01	64.5		0.4% S 1.0% BS&W		Yes			
	Plant				5,000					3,400			8,473,300	825,782	10.26		64.0	0° F Pour	1.30		500	F	N
93	a	2	A	S	1,250	5	20-3/4	26	180	900	1928	4,444	2,704,400	---	--	67.6		20° API 19,050 BTU/lb. 78° Say.U @ 100° F		No			
	b	2	A	S	1,250	5	20-3/4	26	180	900	1928	6,203	3,581,200	---	--	64.2		1.49% S - 0.11% Ash		No			
	c	2	A	S	2,400	8	21	24	225	1,600	1930[4]	4,062	4,168,500	---	--	64.1		15% Asp. Res.		No			
	Plant				4,900					3,400			10,454,100	1,082,092	9.66		65.0		2.09		2,560	F	N
52	a	4	A	-	830	4	23	32	164	560	1928	1,250	513,600	47,712	10.75	73.4		0.9045 Spec. Grav. @ 60° F 19,300 BTU/lb.		No			
	b	4	A	-	1,250	6	23	32	164	850	1928	4,875	3,109,700	287,454	10.81	75.0		82° Say.U @ 100° F		No			
	c	4	A	-	1,250	6	23	32	164	850	1928	4,791	3,061,500	274,781	11.13	75.2		0.42% S Trace Ash		No			
	d	4	A	-	1,250	6	23	32	164	850	1930[6]	1,080	706,700	64,019	11.03	77.0		0.08% BS&W		No			
	Plant				4,580					3,110			7,391,500	673,966	10.97		75.2		5.70		30	N	B
45	a	4	A	-	600	6	16-1/2	24	200	400	1923	1,047	224,550	22,412	10.02	53.6		35° API 19,000 BTU/lb.		No			
	b	4	A	-	600	6	16-1/2	24	200	400	1923	1,028	226,300	23,655	9.57	55.0		43° Say.U @ 79° F		No			
	c	4	A	-	750	4	23	32	150	500	1923	2,725	857,725	83,083	10.32	63.0		0.35% S		No			
	d	4	A	-	1,150	6	23	32	150	800	1926	5,610	3,245,250	286,615	11.32	72.3		0.06% CC		No			
	e	4	A	-	1,150	6	23	32	150	800	1926	5,128	2,938,500	261,002	11.25	71.6		Trace BS&W		No			
	Plant				4,250					2,900			7,492,325	676,767	11.07		69.5	1/2 Year by Tank Wagon	5.29		50	N	L
54	a	4	A	-	600	6	17	25	200	400[7]		---	---	---	--	-		27.1° API		Yes			
	b	4	A	-	600	6	17	25	200	400[7]		---	---	---	--	-		280° Say.U @ 60° F		Yes			
	c	4	A	-	600	6	17	25	200	400[7]	1925	---	---	---	--	-		0.7% S		Yes			
	d	4	A	-	600	6	17	25	200	400[7]	to	---	---	---	--	-		By Pipe Line from Refinery		Yes			
	e	4	A	-	600	6	17	25	200	400[7]	1928	---	---	---	--	-				Yes			
	f	4	A	-	600	6	17	25	200	400[7]		---	---	---	--	-				Yes			
	g	4	A	-	600	6	17	25	200	400[7]		---	---	---	--	-				Yes			
	Plant				4,200					2,800		60,613	24,221,000	1,957,776	12.39		99.9		2.53		500	N	N
130	a	4	A	-	600	6	16-1/4	23	225	412	1924	4,075	1,154,400	199,760	10.88	68.8		16-30° API		No			
	b	4	A	-	600	6	17	24	200	412	1926	3,780	1,021,200			65.5		18,700 BTU/lb.		No			
	c	4	A	-	1,200	6	20	24	300	850	1929	4,998	1,683,800	154,450	10.90	39.6				No			
	d	4	A	-	1,200	6	20	24	300	850	1930[8]	1,563	901,400	82,550	10.91	67.8				No			
	Plant				3,600					2,524			4,760,800	436,760	10.89		54.1		4.72		750	N	N
164	a	4	M	-	840	6	19-1/2	24	214	550	1926	7,899	3,875,300	---	--	89.2		21° API 18,620 BTU/lb.		No			
	b	4	M	-	840	6	19-1/2	24	214	550	1926	8,304	3,998,000	---	--	87.5		57° Say.U @ 100° F		No			
	c	4	M	-	840	6	19-1/2	24	214	550	1926	7,575	3,858,000	---	--	92.6		1.3% S		No			
	d	4	M	-	840	6	19-1/2	24	214	550	1926	8,069	4,153,200	---	--	93.7		0.1% BS&W		No			
	Plant				3,360					2,200			15,884,500	1,167,150	13.61		90.7	By Pipe Line from Refinery	3.66		30	N	N
81	a	2	A	S	750	3	20-3/4	26	180	500	1921	3,518	993,330	---	--	56.5		.9374 Spec. Grav. @ 60° F		No			
	b	2	A	S	1,250	5	20-3/4	26	180	880	1924	4,718	2,275,000	---	--	54.8		19,650 BTU/lb. 1.26% S		No			
	c	2	A	S	1,250	5	20-3/4	26	180	880	1928	5,733	2,717,900	---	--	53.9		8.48% CC 0.19% Ash 0.02% BS&W		No			
	Plant				3,250					2,260			5,986,230	646,979	9.25		54.6		2.79		900	F-B	B

TABLE IV - INFORMATION COVERING FUEL OIL (Page 2)

Plant Number	Engine Designation	Engine Cycle	Injection System-See Notes	Scavenging System - See Notes	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Gross Output - K.W. Hrs.	Fuel Oil Used - Gallons	Gross K.W. Hrs. per Gallon of Fuel Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Nature of Fuel Oil Used - See Notes	Average Cost of Fuel Oil - Cents per Gallon	Is Fuel Centrifuged?	Plant Altitude - Feet Above Sea Level	Purpose for Which Jacket Water Heat is Utilized - See Notes	Purpose for Which Exhaust Heat is Utilized-See Notes
3	a	4	M	-	400	6	14-1/2	18	277	275	1923	700	174,100	17,000	10.24	90.4		22.8° API 19,320 BTU/lb.		No			
	b	4	A	-	800	8	17	24	200	540	1924	2,868	568,000	61,500	9.23	36.7		28" Eng. @ 212° F		No			
	c	4	A	-	800	8	17	24	200	540	1926	1,274	325,000	34,500	9.42	47.2		0.39% S;0.18% CC		No			
	d	4	M	-	1,125	6	22	30	180	750	1927	4,791	1,365,900	145,927	9.36	38.0		0.03% BS&W		No			
	Plant				3,125					2,105			2,433,000	258,927	9.40		40.4		5.00		20	N	N
720	a	4	M	-	380[9]	2	22-1/2	36	150	240	1922	1,643	224,300	---	--	56.9		28-32° API		No			
	b	4	M	-	300	3	17-3/4	22	225	200	1926	4,250	443,400	---	--	52.2		19,000 BTU/lb.		No			
	c	2	M	S	840	6	16	20	257	560	1928	5,070	1,481,800	---	--	52.2		1.5% S		No			
	d	2	M	S	720	6	16	20	257	480	1927	4,480	1,122,000	---	--	52.2				No			
	e	2	M	S	840	6	16	20	257	560	1928	250	73,200	---	--	52.3				No			
	Plant				3,080					2,040			3,344,700	421,000	7.94		52.5		3.99		4,685	N	N
7	a	4	M	-	360	6	11-1/2	16	277	225	1925	4,817	701,380	---	--	64.7		36-40° API 19,500 BTU/lb.		Yes			
	b	4	M	-	360	6	11-1/2	16	277	225	1925	4,625	599,150	---	--	57.6		45" Say.U @ 100° F		Yes			
	c	4	M	-	360	6	11-1/2	16	277	225	1925	4,894-1/2	683,300	---	--	62.0		0.23% S		Yes			
	d	4	M	-	750	6	16	22	240[11]	500	1927	4,656-1/2	1,448,600	---	--	62.2		0.016% CC		Yes			
	e	4	M	-	930	6	18	22	257	620	1930[10]	237-1/2	93,900	---	--	63.8		0.012% Ash 1.3% BS&W		Yes			
	Plant				2,780					1,795			3,526,330	302,870	11.64		61.8		4.79		1,230	N	N
732	a	4	A	-	240	4	14	17-1/2	257	160	1916	1,613	---	---	--	-		22-28° API		No			
	b	4	A	-	240	4	14	17-1/2	257	160	1916	2,738	---	---	--	-		0.8% S		No			
	c	4	A	-	520	4	19	24-1/2	200	400	1921	6,920	---	---	--	-				No			
	d	4	A	-	750	4	23	32	150	500	1923	5,789	---	---	--	-				No			
	e	4	A	-	750	4	23	32	150	500	1925	5,687	---	---	--	-				No			
	Plant				2,500					1,720			5,494,770	496,202	11.09		59.7		3.66		760	N	L
60	a	4	M	-	800	8	19-1/2	22	200	540	1924	2,090	398,700	---	--	35.3		28° API		No			
	b	4	M	-	800	8	19-1/2	22	200	540	1924	4,362	859,700	---	--	38.5				No			
	c	4	M	-	800	8	19-1/2	22	200	540	1924	2,409	512,500	---	--	39.4				No			
	Plant				2,400					1,620			1,770,900	198,865	8.91		37.0		4.62		4,000	N	N
41	a	4	A	-	520	3	22	29-1/2	164	400	1922	3,788	816,990	88,190	9.26	54.2		34° API 18,900 BTU/lb.		No			
	b	4	A	-	700	4	22	29-1/2	164	500	1923	3,188	900,000	91,030	9.89	58.4		0.58% S		No			
	c	4	A	-	1,150	6	23	29-1/2	164	900	1928	4,302	2,401,300	220,660	10.88	62.0		Trace BS&W		No			
	Plant				2,370					1,800			4,118,290	399,880	10.30		59.0		6.19		20	N	B
79	a	4	A	-	225	3	16	24	164	150	1911	928	80,000	9,492	8.43	57.6		23° API 18,500 BTU/lb.		No			
	b	4	A	-	225	3	16	24	164	150	1912	269	23,500	2,538	9.26	58.2				No			
	c	4	A	-	600	6	16-1/4	23	225	410	1929	5,166	1,422,100	120,325	11.82	66.9				No			
	d	4	A	-	520	4	19	24-1/2	200	350	1923	2,557	639,000	61,324	10.41	71.4				No			
	e	4	A	-	600	6	16-1/4	23	225	410	1925	4,368	1,195,000	103,789	11.52	66.7				No			
	Plant				2,170					1,470			3,359,600	297,468	11.28		67.3		3.33		370	N	N
5	a	2	M	S	720	6	16	20	257	480	--	---	---	---	--	-				-			
	b	2	M	S	720	6	16	20	257	480	--	---	---	---	--	-				-			
	c	2	M	S	720	6	16	20	257	480	--	---	---	---	--	-				-			
	Plant				2,160					1,440			1,184,450	121,798	9.72		-		4.04		-	-	-
80	a	4	A	-	385	4	16-1/4	23	225	250	1921	---	---	---	--	-		28° API 19,300 BTU/lb.		No			
	b	4	A	-	520	4	19	24-1/2	200	350	1921	---	---	---	--	-		0.6% S		No			
	c	4	A	-	600	6	16-1/4	23	225	400	1929	---	---	---	--	-				No			
	d	4	M	-	665	4	21	30	180	410	1928	---	---	---	--	-				No			
	Plant				2,150					1,410			2,403,000	231,636	10.36		-		3.49		1,134	N	B
6	a	2	M	CC	300[12]	6	14	17	257	200	1926	---	---	---	--	-				-			
	b	4	A	-	600	6	17	25	200	400	1926	---	---	---	--	-				-			
	c	4	A	-	600	6	17	25	200	412	1927	---	---	---	--	-				-			
	d	4	A	-	600	6	17	25	200	412	1928	---	---	---	--	-				-			
	Plant				2,100					1,424			1,614,100	137,210	11.76		-		4.51		-	-	-
728	a	4	A	-	600	6	16-1/4	23	225	410	1928	4,000	967,700	---	--	59.0		25° API 18,780 BTU/lb.		No			
	b	4	A	-	600	6	16-1/4	23	225	410	1928	4,000	1,014,100	---	--	61.9		0.4% S;0.175% CC		No			
	c	2	M	S	840	6	16	20	257	565	1929	3,300	1,411,540	---	--	75.7		0.07% Ash Trace - BS&W		No			
	Plant				2,040					1,385			3,393,340	339,260	10.00		66.0		5.90		8	N	N
716	a	4	A	-	600	6	16-1/4	23	225	410	1927	4,321	1,104,000	---	--	62.3		34°-36° API		No			
	b	4	A	-	365	4	16-1/4	21	225	250	1922	3,521	604,000	---	--	68.6		19,500 BTU/lb.		No			
	c	4	A	-	500	4	18-7/8	28-3/8	164	380	1918	6,219	1,337,000	---	--	56.6				No			
	d	4	A	-	500	4	18-7/8	28-3/8	164	400	1925	2,434	600,000	---	--	61.6				No			
	Plant				1,965					1,440			3,645,000	378,640	9.63		60.8		3.60		4,000	N	N
93	a	2	M	S	700	5	16	20	257	480	1928	5,532	1,080,200	---	--	40.7		28°-30° API		No			
	b	2	M	S	700	5	16	20	257	460	1928	4,930	998,000	---	--	42.2				No			
	c	2	M	S	560	4	16	20	257	376	1928	4,586	850,000	---	--	49.3				No			
	Plant				1,960					1,336			2,928,200	347,778	8.42		43.4		4.09		15	N	N
215	a	2	M	S	980	7	16	20	257	676	1930[8]	1,361	709,100	---	--	77.1		33° API 19,490 BTU/lb.		No			
	b	2	M	S	980	7	16	20	257	676	1930[8]	1,270	686,500	---	--	80.0		0.22% S;0.3% Ash		No			
	Plant				1,960					1,352			1,395,600	119,236	11.69		78.5		5.40		2,150	N	N

TABLE IV - INFORMATION COVERING FUEL OIL (Page 3)

Plant Number	Engine Designation	Engine Cycle	Injection System—See Notes	Scavenging System - See Notes	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Gross Output - K.W. Hrs.	Fuel Oil Used - Gallons	Gross K.W. Hrs. per Gallon of Fuel Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Nature of Fuel Oil Used - See Notes	Average Cost of Fuel Oil - Cents per Gallon	Is Fuel Centrifuged?	Plant Altitude - Feet Above Sea Level	Purpose for Which Jacket Water Heat is Utilized - See Notes	Purpose for Which Exhaust Heat is Utilized—See Notes
729	a	4	A	-	225	3	16	24	164	160	1912	74-3/4	---	---	--	-		27° API		No			
	b	4	A	-	225	3	16	24	164	160	1914	53-1/2	---	---	--	-		19,030 BTU/lb.		No			
	c	4	A	-	225	3	16	24	164	160	1918	85-3/4	---	---	--	-				No			
	d	4	A	-	500	4	18-7/8	28-3/8	164	380	1920	1,897-3/4	---	---	--	-				No			
	e	4	A	-	750	4	23	32	150	500	1922	3,898-1/2	---	---	--	-				No			
	Plant				1,925					1,360			2,257,280	210,893	10.70		83.5		4.71		1,097	N	N
837	a	4	A	-	750	4	23	32	150	500	1922	8,079	1,991,720	222,312	8.96	49.3		By Tank Wagon		No			
	b	4	A	-	450	6	16	24	164	330	1918	548	91,560	27,307	8.93	50.6				No			
	c	4	A	-	225	3	16	24	164	160	1904	733	47,210			40.3				No			
	d	4	A	-	225	3	16	24	164	160	1904	521	37,585			45.1				No			
	e	4	A	-	225	3	16	24	164	160	1908	208	12,970			39.0				No			
	Plant				1,875					1,310			2,181,045	249,619	8.74		49.0		3.07		5	N	F
109	a	2	A	S	675	4	16-3/4	22	240	450	1927	1,973	393,350	---	--	44.3		24° API 18,500 BTU/lb.		No			
	b	2	A	S	1,125	8	14-3/4	21	257	750	1929	1,295	500,340	---	--	51.5				No			
	Plant				1,800					1,200			893,890	101,202	8.83		48.1		8.53		4,500	N	N
176	a	4	A	-	600	6	17	25	200	410	1928	30,755	7,765,300	---	--	61.6		24° API (15,000 gals. of		No			
	b	4	A	-	600	6	17	25	200	410	1928	32,174	7,267,600	---	--	56.1		14° API)		No			
	c	4	A	-	600	6	17	25	200	410	1928[8]	14,370	3,740,900	---	--	63.5				No			
	Plant				1,800					1,230			18,773,800	1,832,210	10.25		59.2		7.70[13]		6,500	N	N
722	a	2	M	CC	200[12]	4	14	17	257	136	1923	352	31,950	4,215	7.58	66.8		32°-36° API 19,700		No			
	b	2	M	CC	300[12]	6	14	17	257	200	1923	354	57,730	6,234	9.26	81.6		BTU/lb.		No			
	c	2	M	S	840	6	16	20	257	565	1929	1,460	710,000	61,390	11.56	86.1				No			
	d	4	A	-	365	4	16-1/4	21	225	266	1916	766	142,870	12,880	11.09	70.1				No			
	Plant				1,705					1,167			942,550	84,719	11.12		82.1		5.61		1,286	N	N
18	a	2	M	CC	450	5	15-1/4	16	275	300	1922	---	---	---	--	-				-			
	b	4	A	-	600	6	17	25	200	410	1928	---	---	---	--	-				-			
	c	4	A	-	600	6	17	25	200	410	1928	---	---	---	--	-				-			
	Plant				1,650					1,120			2,502,301	241,517	10.36		-		4.49		-	-	-
8	a	4	A	-	400	4	16-1/2	24	200	275	1921	5,524	684,970	---	--	45.1		18°-32° API		No			
	b	4	A	-	560	3	22-1/2	22-1/2	225	400	1922	1,126	288,840	---	--	84.1		(Mostly 24°-26°)		No			
	c	4	A	-	600	6	17	25	200	400	1928	4,280	1,125,290	---	--	85.7				No			
	Plant				1,560					1,075			2,099,100	206,579	10.15		57.0		3.37		1,160	N	B
132	a	4	A	-	375	3	17-1/2	24-1/2	225	255	1929	5,492	816,800	---	--	58.3		34° API;0.5% S 19,000 BTU/lb.		No			
	b	4	A	-	500	4	17-1/2	24-1/2	225	345	1929	2,860	693,600	---	--	70.3		Trace - CC		No			
	c	4	A	-	625	5	17-1/2	24-1/2	225	430	1929	714	200,100	---	--	65.7		35" Say.U @ 100° F Less than 1.0% BS&W		No			
	Plant				1,500					1,030			1,710,500	155,197	11.02		63.5		4.64		1,100	N	B
154	a	4	A	-	600	6	17	24	200	410	1927	---	---	---	--	-				-			
	b	4	A	-	400	4	17	24	200	270	1928	---	---	---	--	-				-			
	c	4	A	-	400	4	17	24	200	270	1925	---	---	---	--	-				-			
	Plant				1,400					950			1,617,965	133,015	12.16		-		4.48				
163	a	4	M	-	400	4	17-3/4	22	225	275	1924	5,180	948,200	79,484	11.92	66.6		24° API 19,064 BTU/lb. 203" Say.U @ 100° F		No			
	b	4	M	-	400	4	17-3/4	22	225	275	1927	8,458	1,849,600	154,780	11.94	79.5		0.52% S;0.026% Ash		No			
	c	4	M	-	600	6	17-3/4	22	225	500	1929	7,587	2,586,400	212,936	12.14	68.2		5.95% CC;0.45% BS&W Via Pipe Line from		No			
	Plant				1,400					1,050			5,384,200	447,200	12.04		71.4	Refinery	2.44		1,020	N	N
723	a	2	M	CC	300[12]	6	14	17	257	200	1925	1,712	251,370	27,400	9.17	73.4		32°-36° API		No			
	b	2	M	CC	300[12]	6	14	17	257	200	1925	998	143,620	15,415	9.32	71.9		19,700 BTU/lb.		No			
	c	2	M	S	720	6	16	20	257	480	1927	3,101	986,240	103,200	9.56	66.2				No			
	Plant				1,320					880			1,381,230	146,015	9.46		68.0		5.62		1,206	N	N
725	a	4	A	-	600	6	17	25	200	400	1928	6,480	---	---	--	-				No			
	b	4	A	-	400	4	17	25	200	270	1925	5,760	---	---	--	-				No			
	c	2	M	CC	200[12]	4	14	17	257	136	1924	0	---	---	--	-				No			
	d	2	M	CC	110[12]	2	14	17-1/2	257	70	1923	0	---	---	--	-				No			
	Plant				1,310					876			---	197,292	--		-		4.40			N	N
13	a	4	A	-	275	4	13-1/2	19-1/2	257	175	1923	6,751	767,500	74,637	10.29	64.9		36° API; 0.45% S 19,500 BTU/lb.		No			
	b	4	A	-	200	3	13-1/2	19-1/2	257	125	1923	2,578	213,100	20,995	10.15	66.0		0.04% CC		No			
	c	4	A	-	750	4	23	29-1/2	164	500	1925	1,356	310,500	35,618	8.72	45.8		Trace Ash & BS&W		No			
	Plant				1,225					800			1,291,100	131,250	9.84		59.2		5.69		79	N	N
17	a	4	A	-	600	6	17	25	200	410	1928	---	---	---	--	-				-			
	b	4	A	-	600	6	17	25	200	410	1928	---	---	---	--	-				-			
	Plant				1,200					820			2,217,200	204,329	10.85		-		4.46				
94	a	4	A	-	600	6	17	24	200	410	1927	6,498	1,751,890	---	--	65.7		24°-36° API		No			
	b	4	A	-	600	6	17	24	200	410	1927	6,293	1,733,320	---	--	67.1				No			
	Plant				1,200					820			3,485,210	293,156	11.88		66.4		3.12		1,000	N	N
616	a	2	M	CC	100[12]	2	14	17	257	60	1920	---	12,000	1,200	10.00	-		22° API		No			
	b	4	A	-	300	3	16-1/2	24	200	200	1923	---	775,870	66,851	11.60	-		less than 0.5% S		No			
	c	4	A	-	750	4	23	32	150	525	1925	---	908,310	79,039	11.47	-				No			
	Plant				1,150					785			1,694,180	147,090	11.52		-		5.10		20	N	N

TABLE IV - INFORMATION COVERING FUEL OIL (Page 4)

Plant Number	Engine Designation	Engine Cycle	Injection System-See Notes	Scavenging System - See Notes	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Gross Output - K.W. Hrs.	Fuel Oil Used - Gallons	Gross K.W. Hrs. per Gallon of Fuel Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Nature of Fuel Oil Used - See Notes	Average Cost of Fuel Oil - Cents per Gallon	Is Fuel Centrifuged?	Plant Altitude - Feet Above Sea Level	Purpose for Which Jacket Water Heat is Utilized - See Notes	Purpose for Which Exhaust Heat is Utilized-See Notes
95	a	2	M	S	560	4	16	20	257	375	1929	4,016	752,500	---	--	50.0		24°-26° API		No			
	b	2	M	S	560	4	16	20	257	375	1929	3,936	760,200	---	--	51.5				No			
	Plant				1,120					750			1,512,700	172,801	8.75		50.7		3.12		746	N	N
96	a	4	M	-	600	6	17	24	225	400	1925	2,400	469,000	---	--	48.9		25° API		No			
	b	4	M	-	300	3	17	24	225	200	1923	4,717	719,100	---	--	76.2				No			
	c	2	M	CC	200[12]	4	14	17	257	136	1921	844	53,700	---	--	46.8				No			
	Plant				1,100					736			1,241,800	120,449	10.31		61.5		3.56		605	N	N
182	a	4	M	-	270	4	14-1/2	18	300	175	1930[14]	2,704	348,100	35,477	9.81	73.6		28.8° API;0.18% S 19,510 BTU/lb. 33" Say.U @ 77° F 0.15% BS&W		No			
	b	4	M	-	410	6	14-1/2	18	300	300	1930[14]	2,511	525,900	53,378	9.85	69.8				No			
	c	4	M	-	410	6	14-1/2	18	300	300	1930[14]	1,679	357,900	36,210	9.89	71.0				No			
	Plant				1,090					775			1,231,900	125,065	9.85		71.0		6.00		1,800	N	N
721	a	2	M	CC	360	6	14	17	257	240	1925	3,405	272,400	---	--	33.3		28.3° API;19,000 BTU/lb. 1.5% S		No			
	b	2	M	CC	360	6	14	17	257	240	1925	4,540	360,400	---	--	33.0				No			
	c	2	M	CC	360	6	14	17	257	240	1926	2,300	176,900	---	--	32.0				No			
	Plant				1,080					720			809,700	132,000	6.52		32.9		6.52		2,500	N	N
78	a	4	M	-	400	4	17	24	225	300	1928[13]	24,136	4,151,314	388,704	10.68	57.4		20°-35° API		No			
	b	4	M	-	660	6	17	24	240	500	1930[6]	3,386	803,202	83,102	9.66	47.5				No			
	Plant				1,060					800			4,954,516	471,806	10.50		55.5		5.08		173	N	N
97	a	4	A	-	520	4	19	24	200	333	1915	4,854	---	---	--	-		24°-26° API		No			
	b	4	A	-	250	4	13-3/4	17-1/2	257	150	1914	4,752	---	---	--	-				No			
	c	4	A	-	250	4	13-3/4	17-1/2	257	210	1914	4,478	---	---	--	-				No			
	Plant				1,020					693			1,876,040	209,765	8.94		57.4		3.38		322	N	N
83	a	4	A	-	600	6	16-1/4	23	225	416	1929	4,571	1,051,300	83,970	12.51	55.3		50" Say.U @ 100° F;0.5% S Trace BS&W Trace Ash		No			
	b	4	A	-	400	4	16-1/4	23	225	270	1927	4,780	596,600	58,750	10.15	46.2				No			
	Plant				1,000					686			1,647,900	142,720	11.54		51.6		6.75		12	N	BF
69	a	2	M	S	480	4	16	20	257	320	1927	4,125	---	---	--	-				No			
	b	2	M	S	480	4	16	20	257	320	1927	4,515	---	---	--	-				No			
	Plant				960					640			1,268,358	150,580	8.42		45.9		3.50		1,092	N	F
53	a	2	M	CC	200[12]	4	14	17	257	136	1923	2,794	274,110	---	--	72.1		23.5° API 18,900 BTU/lb. 50" Say.U @ 100° F		No			
	b	2	M	CC	300[12]	6	14	17	257	200	1925	4,386	624,980	---	--	71.0				No			
	c	4	A	-	400	4	16-1/4	23	225	270	1925	7,756	1,470,190	---	--	70.2				No			
	Plant				900					606			2,369,280	259,906	9.12		70.6		3.26		1,650	N	N
98	a	2	M	CC	360	6	14	17	257	240	1926	5,469	850,900	---	--	64.8		36°-38° API		No			
	b	2	M	CC	360	6	14	17	257	240	1929	5,083	796,400	---	--	65.3				No			
	c	2	M	CC	180	3	14	17	257	120	1926	3,557	263,500	---	--	61.8				No			
	Plant				900					600			1,910,800	238,875	8.00		64.5		4.73		2,951	F	N
74	a	2	M	CC	225	3	15-1/4	16	277	150	1924	586	---	---	--	-		32°-36° API 19,600 BTU/lb. 30" Say.U @ 100° F 0.75% S Trace BS&W		No			
	b	4	M	-	260	4	14-1/2	18	277	175	1928	5,656	---	---	--	-				No			
	c	4	M	-	400	6	14-1/2	18	277	275	1928	2,680	---	---	--	-				No			
	Plant				885					600			---	116,631	--		-		4.53		475	N	N
189	a	4	M	-	550	5	17	24	240	375	1929	8,000	---	---	--	-		28°-30° API		No			
	b	4	M	-	200[9]	2	17	24	200	145	1922	700	---	---	--	-				No			
	c	4	M	-	100[9]	1	17	24	200	75	1915	60	---	---	--	-				No			
	Plant				850					595			850,976	101,869	8.35		27.4		4.10		1,800	N	N
153	a	2	M	S	840	6	16	20	257	560	1929	---	97,970	9,310	10.52	-	-		4.06	-	-	-	-
100	a	2	M	CC	300[12]	6	14	17	257	200	1923	4,408	521,620	---	--	59.2		30°-32° API		No			
	b	2	M	CC	240	4	14	17	257	160	1924	5,602	516,200	---	--	57.6				No			
	c	2	M	CC	200[12]	4	14	17	257	136	1924	5,507	501,630	---	--	67.0				No			
	Plant				740					496			1,539,450	180,315	8.54		60.9		3.50		1,286	N	N
112	a	2	M	CC	200[12]	4	14	17	257	136	1925	4,719	415,900	48,102	8.65	64.8		33° API		No			
	b	2	M	CC	240	4	14	17	257	155	1927	4,292	425,500	45,819	9.29	64.0				No			
	c	2	M	CC	300[12]	6	14	17	257	200	1925	2,545-1/2	394,100	42,533	9.27	77.4				No			
	Plant				740					491			1,235,500	136,451	9.06		68.1		4.43		845	N	N
67	a	2	M	CC	360	6	14	17	257	250	1925	2,590	539,000	49,519	10.88	83.2		27½° API;18,675 BTU/lb. 88" Say.U @ 100° F 0.75% S;0.5% CC		No			
	b	2	M	CC	360	6	14	17	257	250	1925	2,720	560,500	55,163	10.15	82.5				No			
	Plant				720					500			1,099,500	104,682	10.50		82.9		4.97		860	F	N
170	a	2	M	CC	360	6	14	17	257	240	1929	2,946	---	---	--	-		34° API;0.4% S 143,000 BTU/Gal. 42"-46" Say.U @ 100° F Trace BS&W By Tank Truck		No			
	b	2	M	CC	360	6	14	17	257	240	1929	2,778	---	---	--	-				No			
	Plant				720					480			869,910	96,190	9.25		64.7		6.51		50	N	N
527	a	2	M	CC	360	6	14	17	257	240	1930[5]	3,672	323,400	---	--	36.7		28°-32° API		No			
	b	2	M	CC	360	6	14	17	257	240	1930[6]	4,590	409,900	---	--	37.2				No			
	Plant				720					480			733,300	87,240	8.41		37.0		3.87		750	N	B
718	a	2	M	CC	360	6	14	17	257	240	--	3,926	348,280	---	--	37.0		32°-36° API;19,530 BTU/lb. 36" Say.U @ 100° F 0.7% S;0.01% CC Trace Ash Trace BS&W		No			
	b	2	M	CC	360	6	14	17	257	240	--	3,641	322,210	---	--	36.9				No			
	Plant				720					480			670,490	85,470	7.84		36.9		4.27		1,200	N	N

TABLE IV - INFORMATION COVERING FUEL OIL (Page 5)

Plant Number	Engine Designation	Engine Cycle	Injection System–See Notes	Scavenging System - See Notes	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Gross Output - K.W. Hrs.	Fuel Oil Used - Gallons	Gross K.W. Hrs. per Gallon of Fuel Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Nature of Fuel Oil Used - See Notes	Average Cost of Fuel Oil - Cents per Gallon	Is Fuel Centrifuged?	Plant Altitude - Feet Above Sea Level	Purpose for Which Jacket Water Heat is Utilized - See Notes	Purpose for Which Exhaust Heat is Utilized-See Notes
19	a	2	M	CC	100^{12}	2	14	17	257	60	--	---	---	---	--	-				-			
	b	2	M	CC	240	4	12-1/2	13-1/4	325	160	1927	---	---	---	--	-				-			
	c	2	M	CC	360	4	15-1/4	16	275	240	1927	---	---	---	--	-				-			
	Plant				700					460			---	784,380	7.78		-		4.13		-	-	-
61	a	4	M	-	300	3	17-3/4	22	225	220	1928	8,169	984,100	---	--	54.8		29.5° API;0.25% S 19,450 BTU/lb. 60° Say.U @ 100° 0.2% CC;0.1% BS&W		No			
	b	4	M	-	300	3	17-3/4	22	225	220	1928	5,779	619,800	---	--	48.8				No			
	Plant				600					440			1,603,900	127,754	12.55		52.3		6.39		651	N	N
108	a	4	A	-	600	6	17	25	200	410	1929	---	1,498,970	133,470	11.23	-	-		4.49	-	-	-	-
103	a	2	M	CC	240	4	14	17	257	160	1927	6,151	620,860	---	--	63.1		24°-26° API		No			
	b	2	M	CC	150^{12}	3	14	17	257	100	1923	5,398	303,610	---	--	56.3				No			
	c	2	M	CC	200^{12}	4	14	17	257	160	1920^{18}	1,006	89,100	---	--	55.4				No			
	Plant				590					420			1,013,570	128,440	7.89		60.2		4.20		3,100	N	N
102	a	2	M	CC	240	4	14	17	257	160	1928	4,729	330,250	---	--	43.7		24°-26° API		No			
	b	2	M	CC	240	4	14	17	257	160	1928	4,818	315,650	---	--	41.0				No			
	c	2	M	CC	100^{12}	2	14	17	257	60	1922^{17}	304	14,160	---	--	77.6				No			
	Plant				580					380			660,060	87,105	7.58		42.7		3.25		568	N	N
101	a	2	M	CC	240	4	14	17	257	160	1928	5,367	553,315	---	--	64.4		30°-32° API		No			
	b	2	M	CC	150^{12}	3	14	17	257	120	1919	2,342	130,190	---	--	46.3				No			
	c	2	M	CC	150^{12}	3	14	17	257	100	1923	2,106	134,320	---	--	63.8				No			
	Plant				540					380			817,825	92,611	8.83		60.5		3.79		992	N	N
501	a	2	M	CC	120	2	14	17	257	80	1925	4,480	---	18,040	--	-		37°-40° API		No			
	b	2	M	CC	120	2	14	17	257	80	1925	4,481	---	19,880	--	-				No			
	c	2	M	CC	240	4	14	17	257	155	1929	2,057	---	17,690	--	-				No			
	Plant				480					315			421,022	55,610	7.57		40.6		5.50		1,108	N	N
335	a	4	A	-	250	4	13-3/4	17-1/2	257	175	1924	7,660	433,870	43,124	10.05	32.4		32°-36° API 18,500		No			
	b	4	A	-	180	4	11-3/4	15	300	120	1919	1,100	55,550	5,533	10.00	42.1		BTU/lb. 0.5% S		No			
	Plant				430					295			489,420	48,657	10.04		33.2		4.85		867	N	N
99	a	4	A	-	400	4	17	24	200	275	1927	1,607	375,100	32,229	11.63	84.9	84.9	25° API	3.18	No	700	N	N
247	a	2	M	CC	240	4	14	17	257	160	1928	---	---	---	--	-				No			
	b	2	M	CC	100^{12}	2	14	17	257	75	1920	---	---	---	--	-				No			
	c	2	M	CC	50^{12}	1	14	17	257	40	1915	---	---	---	--	-				No			
	Plant				390					275			429,890	58,000	7.41		-		6.00		5,000	N	N
353	a	2	M	CC	50^{12}	1	14	17	257	30	1916	2,470	51,050	---	--	68.9		37° API		No			
	b	2	M	CC	100^{12}	2	14	17	257	60	1920	3,715	104,100	---	--	46.7				No			
	c	2	M	CC	240	4	14	17	257	160	1927	2,386	127,425	---	--	33.4				No			
	Plant				390					250			282,575	46,380	6.09		41.6		5.50		1,100	N	N
106	a	2	M	CC	150^{12}	3	14	17	257	100	1922	4,325	270,180	---	--	62.4		32°-36° API		No			
	b	2	M	CC	120	2	14	17	257	72	1926	3,889	133,880	---	--	47.8				No			
	c	2	M	CC	100^{12}	2	14	17	257	60	1923	701	21,740	---	--	51.7				No			
	Plant				370					232			425,800	56,421	7.55		56.5		3.14		1,580	N	N
152	a	2	M	CC	360	6	14	17	257	240	1925	---	28,500	3,172	8.99	-	-		5.35	-	-	-	-
857	a	2	M	CC	180	3	14	17	257	125	1929	---	165,240	---	--	-				No			
	b	2	M	CC	150^{12}	3	14	17	257	72	1927	---	184,100	---	--	-				No			
	Plant				330					197			349,340	53,506	6.53		-		3.39		4,200	F	N
27	a	2	M	CC	75^{12}	2	12	15	300	48	1926	---	---	---	--	-				-			
	b	2	M	CC	240	4	12-1/2	13-1/4	325	150	1926	---	---	---	--	-				-			
	Plant				315					198			210,630	34,795	6.05		-		4.93		-	-	-
515	a	2	M	CC	180	3	14	17	257	120	1928	---	---	---	--	-		38°-40° API		No			
	b	2	M	CC	75^{12}	2	12	15	300	48	1915	---	---	---	--	-				No			
	c	2	M	CC	50^{12}	1	14	17	257	32	1915	---	---	---	--	-				No			
	Plant				305					200			---	40,000	--		-		5.63		485	N	N
169	a	4	M	-	240	6	10-3/4	13-3/4	327	160	1928	2,377-1/2	157,780	15,819	9.98	41.5	41.5	34° API;0.315% S 19,670 BTU/lb. 0.1% BS&W less than 40° Say.U @ 100° F	5.98	No	20	N	N
285	a	2	M	CC	120	2	14	17	257	72	1927	2,920	---	---	--	-		36°-40° API		No			
	b	2	M	CC	60	1	14	17	257	38	1928	5,840	---	---	--	-				No			
	Plant				180					110			---	27,160	--		-		9.14		970	N	N

TABLE IV - INFORMATION COVERING FUEL OIL (Page 6)

Plant Number	Engine Designation	Engine Cycle	Injection System-See Notes	Scavenging System - See Notes	Rated B.H.P.	Number of Cylinders	Cylinder Bore - Inches	Stroke - Inches	Rated R.P.M.	Generator Rating - K.W.	Year Engine Started to Work	Engine Hours Operated in Reported Period	Gross Output - K.W. Hrs.	Fuel Oil Used - Gallons	Gross K.W. Hrs. per Gallon of Fuel Oil	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Nature of Fuel Oil Used - See Notes	Average Cost of Fuel Oil - Cents per Gallon	Is Fuel Centrifuged?	Plant Altitude - Feet Above Sea Level	Purpose for Which Jacket Water Heat is Utilized - See Notes	Purpose for Which Exhaust Heat is Utilized-See Notes
858	a	4	M	-	60[9]	1	14-1/2	18	325	37-1/2	1929	---	98,800	---	--	-				No			
	b	2	M	CC	120	2	14	17	257	90	1929	---	34,200	---	--	-				No			
	Plant				180					127-1/2			133,000	24,864	5.35		-		3.41		4,800	F	N
646	a	2	M	CC	120	2	14	17	257	72	1927	1,016	16,690	2,877	5.80	13.7	13.7	32°-34° API	7.00	Yes	1,000	N	N

NOTES:

INJECTION SYSTEM
A - Air
M - Mechanical

SCAVENGING SYSTEM
S - Separate (i.e., Pump or Blower)
CC - Crankcase (for 2-Stroke Cycle Only)

HEAT UTILIZATION
N - Not Utilized
B - To Heat Building
F - To Heat Fuel
L - To Heat Lubricating Oil
BF - To Heat Boiler Feed Water

NATURE OF FUEL OIL
API - American Petroleum Institute
BTU/lb. - British Thermal Units per Pound
"Say.U - Seconds Saybolt Universal
"Say.F - Seconds Saybolt Furol
°Eng. - Degrees Engler
S Sulphur
CC - Carbon by Conradson Method
BS&W - Bottom Sediment and Water
Spec. Grav. - Specific Gravity
Asph. Res. - Asphaltic Residue

NUMBERED NOTES
1. No deduction made for motor-driven Scavenging blower.
2. Engine speed increased and new generators installed since 1929 report.
3. Name plate ratings are 700 and 1400 K.W.
4. April 5. February 6. October
7. Nominal Rating 450 K.V.A.
8. September
9. Horizontal Engine
10. December 11. Reported 257 R.P.M. last year in error.
12. Semi-Diesel Engine
13. Cost includes import duty.
14. August 15. March - Start of 3 year period.
16. Installed in Plt. #103 in August, 1930.
17. Installed in Plt. #102 in October, 1930.

TABLE V - INFORMATION COVERING MAINTENANCE AND OPERATING LABOR (Page 1)

Plant Number	Engine Designation	Engine Cycle	Injection System—See Notes	Scavenging System - See Notes	Number of Cylinders	Rated B.H.P.	Generator Rating - K.W.	Year Engine Started to Work	Peak Load During Reported Period - K.W.	B.Mep. at Rated B.H.P. - Lbs. per Sq. In.	B.Mep. at Peak Load (90% Generator Efficiency)	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Piston Cooling—See Notes	Are Air Filters Used?	Average Liner Wear per 1000 Hrs. Operation - Ins.	Operating Period Over Which Liner Wear Was Measured - Hrs.	Type Cooling System (See Text)	Cost of Regular Engine Upkeep - Total Dollars - Material - Extra Labor	Cost of Repairs for Engine Accidents - Total Dollars-Material - Extra Labor	Total Engine Repairs in Dollars per Rated B.H.P. per Year	Number of Enforced Shutdowns	Total Duration of Enforced Shutdowns - Hours	Number of Shifts per Day	Number of Hours per Shift	Number of Attendants per Shift
43	a	4	A	-	3	500	300	1915	310	78.4	72.4	75.0		W	No	--	---		308-308	0	0.62	-	--			
	b	4	A	-	3	500	300	1916	300	78.4	70.1	73.8		W	No	--	---			0		-	--			
	c	4	A	-	4	520	350	1919	340	74.0	72.1	87.2		W	No	--	---		309-737	0	1.01	-	--			
	d	4	A	-	4	520	350	1920	310	74.0	65.8	74.6		W	No	--	---			0		-	--			
	e	4	A	-	6	1,000	700	1922	650	73.9	71.7	63.2		W	No	--	---		444-1,360	0	0.90	-	--			
	f	4	A	-	6	1,000	700	1925	710	73.9	78.4	63.0		W	No	--	---			0		-	--			
	g	2	A	S	6	3,750	2,600	1928	2,600	67.2	69.4	68.7		W	Yes	.0096	6,982-1/2		6,849-7,180	0	1.87	-	--			
	h	2	A	S	6	3,750	2,800	1929	2,800	67.2	74.8	71.9		W	Yes	.0100	7,363			0		-	--	2	8	2
	Plant					11,540	8,100		7,150				70.4					B-E						1	8	3
46	a	2	A	S	5	1,250	850[1]	1922	---	62.5	-	78.5		W	No	--	---		-25,983-		2.81	-	--			
	b	2	A	S	5	1,250	850[1]	1922	---	62.5	-	79.2		W	No	--	---					-	--			
	c	2	A	S	4	2,250	1,500[1]	1922	---	66.2	-	81.0		W	No	--	---					-	--			
	d	2	A	S	4	2,250	1,500[1]	1922	---	66.2	-	84.9		W	No	--	---					-	--			
	e	2	A	S	4	2,250	1,500[1]	1930[2]	---	66.2	-	89.2		W	No	--	---					-	--	1	8	7
	Plant					9,250	6,200		5,400				83.6					B-E						2	8	2
82	a	4	A	-	4	360	250	1921	---	72.7	-	61.3		A	No	.0039	844		6,872-8,350	0	2.74	0	--			
	b	2	A	S	4	750	517	1923	---	67.3	-	57.3		W	Yes	.0039	840			0		0	--			
	c	2	A	S	4	750	517	1924	---	67.3	-	58.5		W	Yes	.0040	601			0		0	--			
	d	2	A	S	6	1,150	800	1927	---	68.8	-	64.7		W	Yes	.0040	2,785			0		0	--			
	e	2	A	S	6	1,150	800	1927	---	68.8	-	65.0		W	Yes	.0040	2,779			0		0	--			
	f	2	A	S	8	1,500	1,080	1929	---	67.3	-	66.0		W	Yes	.0040	6,536			0		0	--	2	8	2
	Plant					5,660	3,964		2,960				65.0					D-E						1	8	4
91	a	4	M	-	6	1,125	800	1927	550[3]	72.3	52.6	50.4		A	Yes	--	---		2,721-0	0	0.52	-	--			
	b	4	M	-	6	1,125	800	1928	600[3]	72.3	57.4	51.7		A	Yes	--	---			0		-	--			
	c	4	M	-	6	600	400	1927	350[3]	64.6	56.2	63.0		A	Yes	--	---			0		-	--			
	d	4	M	-	6	600	400	1927	325[3]	64.6	52.2	61.0		A	Yes	--	---			0		-	--			
	e	4	A	-	6	1,800	1,275	1930	1,000[3]	72.9	60.4	52.6		O	Yes	--	---			0		-	--			
	Plant					5,250	3,675		1,850[3]				53.1					D						3	8	1
73	a	2	A	S	5	1,250	850	1923	---	62.5	-	63.1		W	Yes	--	---		-825-	0	0.17	0	--			
	b	2	A	S	5	1,250	850	1924	---	62.5	-	64.3		W	Yes	--	---			0		0	--			
	c	2	A	S	5	1,250	850	1928	---	62.5	-	65.2		W	Yes	--	---			0		0	--			
	d	2	A	S	5	1,250	850	1928	---	62.5	-	64.5		W	Yes	--	---			0		0	--	2	8	2
	Plant					5,000	3,400		2,550				64.0					C-E						1	8	5
92	a	2	A	S	5	1,250	900	1928	800[3]	62.5	59.6	67.6		W	Yes	--	---		1,787-838	0	0.54	-	--			
	b	2	A	S	5	1,250	900	1928	800[3]	62.5	59.6	64.2		W	Yes	--	---			0		-	--			
	c	2	A	S	8	2,400	1,600	1930[2]	1,600[3]	63.5	63.1	84.1		O	Yes	--	---			0		-	--	1	8	2
	Plant					4,900	3,400		2,100[3]				65.0					D						2	8	1
52	a	4	A	-	4	830	560	1928	605	75.3	81.8	73.4		W	No	--	---		494-3,594[5]	0	0.68	0	--			
	b	4	A	-	6	1,250	850	1928	910	75.7	82.0	75.0		W	No	.0017	10,676			0		0	--			
	c	4	A	-	6	1,250	850	1928	915	75.7	82.5	75.2		W	No	--	---			0		0	--			
	d	4	A	-	6	1,250	850	1930[4]	905	75.7	81.6	77.0		W	No	--	---			0		2	2	1	8	3
	Plant					4,580	3,110		2,520				75.2					A						2	8	2
45	a	4	A	-	6	600	400	1923	400	77.2	76.6	53.6		A	Yes	.0010	1,047		638-4,906[7]	1,429-31[6]	1.65	0	--			
	b	4	A	-	6	600	400	1923	400	77.2	76.6	55.0		A	Yes	.0010	1,028					0	--			
	c	4	A	-	4	750	500	1923	550	74.5	81.4	63.0		W	Yes	.0004	2,725					0	--			
	d	4	A	-	6	1,150	800	1926	850	76.1	83.8	72.3		W	Yes	.0004	5,610					0	--	2	8	2
	e	4	A	-	6	1,150	800	1926	850	76.1	83.8	71.6		W	Yes	.0004	5,128					0	--	1	8	4[7]
	Plant					4,250	2,900		2,450				69.5					B						1	8	1[7]
54	a	4	A	-	6	600	400[8]	1925 to 1926	---	69.8	-	-		A	No	--	---					-	--			
	b	4	A	-	6	600	400[8]		---	69.8	-	-		A	No	--	---					-	--			
	c	4	A	-	6	600	400[8]		---	69.8	-	-		A	No	--	---					-	--			
	d	4	A	-	6	600	400[8]		---	69.8	-	-		A	No	--	---					-	--			
	e	4	A	-	6	600	400[8]		---	69.8	-	-		A	No	--	---					-	--			
	f	4	A	-	6	600	400[8]		---	69.8	-	-		A	No	--	---					-	--			
	g	4	A	-	6	600	400[8]		---	69.8	-	-		A	No	--	---					-	--			
	Plant					4,200	2,800		2,940				99.9					B-E						3	8	3
130	a	4	A	-		600	412	1924	---	73.8	-	68.8		A	No	--	---		-3,687-		1.02	-	--			
	b	4	A	-		600	412	1926	---	72.7	-	65.5		A	No	--	---					-	--			
	c	4	A	-		1,200	850	1929	---	70.1	-	39.6		O	Yes	--	---					-	--			
	d	4	A	-		1,200	850	1930[9]	---	70.1	-	67.8		O	Yes	--	---					-	--			
	Plant					3,600	2,524		1,300				54.1					A						3	8	1
164	a	4	M	-	6	840	550	1926	538[10]	72.4	69.2	89.2		O	No	--	---		-16,480-		4.91	-	--			
	b	4	M	-	6	840	550	1926	525[10]	72.4	67.5	87.5		O	No	--	---					-	--			
	c	4	M	-	6	840	550	1926	592[10]	72.4	76.1	92.6		O	No	--	---					-	--			
	d	4	M	-	6	840	550	1926	612[10]	72.4	78.7	93.7		O	No	--	---					-	--			
	Plant					3,360	2,200		2,230[10]				90.7					A						3	8	4
81	a	2	A	S	3	750	500	1921	---	62.5	-	56.5		W	Yes	--	---					-	--			
	b	2	A	S	5	1,250	880	1924	---	62.5	-	54.8		W	Yes	--	---					-	--			
	c	2	A	S	5	1,250	880	1928	---	62.5	-	53.9		W	Yes	--	---					-	--			
	Plant					3,250	2,260		1,570				54.6					D-E						3	8	2

TABLE V - INFORMATION COVERING MAINTENANCE AND OPERATING LABOR (Page 2)

Plant Number	Engine Designation	Engine Cycle	Injection System—See Notes	Scavenging System - See Notes	Number of Cylinders	Rated B.H.P.	Generator Rating - K.W.	Year Engine Started to Work	Peak Load During Reported Period - K.W.	B.Mep. at Rated B.H.P. - Lbs. per Sq. In.	B.Mep. at Peak Load (90% Generator Efficiency)	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Piston Cooling—See Notes	Are Air Filters Used?	Average Liner Wear per 1000 Hrs. Operation - Ins.	Operating Period Over Which Liner Wear Was Measured - Hrs.	Type Cooling System (See Text)	Cost of Regular Engine Upkeep - Total Dollars - Material - Extra Labor	Cost of Repairs for Engine Accidents - Total Dollars-Material - Extra Labor	Total Engine Repairs in Dollars per Rated B.H.P. per Year	Number of Enforced Shutdowns	Total Duration of Enforced Shutdowns - Hours	Number of Shifts per Day	Number of Hours per Shift	Number of Attendants per Shift
3	a	4	M	-	6	400	275	1923	270	64.1	64.3	90.4		A	Yes	.0029	700		75-0	0	0.18	1	1/15			
	b	4	A	-	8	800	540	1924	550	72.7	74.3	36.7		A	No	.0028	2,668		361-0	3,300-0^{11}	4.60	0	—			
	c	4	A	-	8	800	540	1926	540	72.7	73.0	47.2		A	No	.0039	1,274		210-0	0	0.26	0	—			
	d	4	M	-	6	1,125	750	1927	755	72.3	72.3	38.0		A	Yes	.0010	4,791		370-0	0	0.33	0	—	1	8	3
	Plant					3,125	2,106		1,000				40.4					D						2	8	2
720	a	4	M	-	2	380^{12}	240	1922	225	70.1	61.8	56.9		A	No	—	—		-626-		0.27	-	—			
	b	4	M	-	3	300	200	1926	200	64.7	64.2	52.2		A	No	—	—					-	—			
	c	2	M	S	6	840	560	1928	475	53.6	45.2	52.2		O	Yes	—	—					-	—			
	d	2	M	S	6	720	480	1927	425	46.0	40.5	52.2		O	Yes	—	—					-	—			
	e	2	M	S	6	840	560	1928	475	53.6	45.2	52.3		O	Yes	—	—					-	—			
	Plant					3,080	2,040		1,100				52.5					B-E						3	8	2
7	a	4	M	-	6	360	225	1925	—	103.2	-	64.7		A	Yes	—	—		3,762-1,276	0	1.83	0	—			
	b	4	M	-	6	360	225	1925	—	103.2	-	57.6		A	Yes	—	—			0		0	—			
	c	4	M	-	6	360	225	1925	—	103.2	-	62.0		A	Yes	—	—			0		0	—			
	d	4	M	-	6	750	500	1927	—	93.2	-	62.2		A	Yes	—	—			0		0	—			
	e	4	M	-	6	930	620	1930^{13}	—	85.2	-	63.8		A	Yes	—	—			0		0	--			
	Plant					2,760	1,795		1,060				61.8					D^{14}						3	8	2
732	a	4	A	-	4	240	160	1916	—	68.5	-	-		A	Yes	—	—					0	—			
	b	4	A	-	4	240	160	1916	—	68.5	-	-		A	Yes	—	—					0	—			
	c	4	A	-	4	520	400	1921	—	74.0	-	-		W	Yes	—	—					0	—			
	d	4	A	-	4	750	500	1923	—	74.5	-	-		W	Yes	—	—					0	—			
	e	4	A	-	4	750	500	1925	—	74.5	-	-		W	Yes	—	—					0	—			
	Plant					2,500	1,720		1,680				59.7					B-E						3	10^{15}	2
60	a	4	M	-	8	800	540	1924	—	60.3	-	35.3		A	No	—	—					1	1			
	b	4	M	-	8	800	540	1924	—	60.3	-	36.5		A	No	—	—									
	c	4	M	-	8	800	540	1924	—	60.3	-	39.4		A	No	—	—							1	8	3
	Plant					2,400	1,620		840				37.0					D						2	8	1
41	a	4	A	-	3	520	400	1922	400	74.6	65.3	54.2		W	No	—	—		-2,567-		1.08	-	—			
	b	4	A	-	4	700	500	1923	500	75.3	60.2	56.4		W	No	—	—					-	—			
	c	4	A	-	6	1,150	900	1928	900	75.5	68.0	62.0		W	No	—	—					-	—	1	8	3
	Plant					2,370	1,800		1,200				59.0					A						2	8	2
79	a	4	A	-	3	225	150	1911	110	75.0	54.6	57.6		A	No	—	—		1,570-0	0	0.72	0	—			
	b	4	A	-	3	225	150	1912	110	75.0	54.6	58.2		A	No	—	—			0		0	—			
	c	4	A	-	6	600	410	1929	450	73.8	62.5	66.9		A	No	—	—			0		0	—			
	d	4	A	-	4	520	350	1923	350	74.0	74.2	71.4		W	No	—	—			0		0	—			
	e	4	A	-	6	600	410	1925	425	73.8	77.9	66.7		A	No	.0023	4,368			0		1	1/6	1	8	5^{16}
	Plant					2,170	1,470		900				67.3					B						2	8	2^{16}
5	a	2	M	S	6	720	480	—	—	46.0	-	-		O	-	—	—					-	—			
	b	2	M	S	6	720	480	—	—	46.0	-	-		O	-	—	—					-	—			
	c	2	M	S	6	720	480	—	—	46.0	-	-		O	-	—	—					-	—			
	Plant					2,160	1,440	—	1,675				-					B-E						-	-	-
80	a	4	A	-	4	365	250	1921	—	67.3	-	-		A	No	—	—		-2,684-	0	1.25	0	—			
	b	4	A	-	4	520	350	1921	—	74.0	-	-		W	No	—	—			0		0	—			
	c	4	A	-	6	600	400	1929	—	73.8	-	-		A	No	—	—			0		0	—			
	d	4	M	-	4	665	410	1926	—	70.4	-	-		A	Yes	—	—			0		0	—	1	8	2
	Plant					2,150	1,410		655				-					B-E						2	8	1
6	a	2	M	CC	6	300^{21}	200	1926	—	29.4	-	-		A	-	—	—					-	—			
	b	4	A	-	6	600	400	1926	—	69.8	-	-		A	-	—	—					-	—			
	c	4	A	-	6	600	412	1927	—	69.8	-	-		A	-	—	—					-	—			
	d	4	A	-	6	600	412	1928	—	69.8	-	-		A	-	—	—					-	—			
	Plant					2,100	1,424		960				-					-						-	-	-
728	a	4	A	-	6	600	410	1926	420	73.8	77.0	59.0		A	No	—	—		500-372	2,000-500	1.65	-	—			
	b	4	A	-	6	600	410	1926	420	73.8	77.0	61.9		A	No	—	—					-	—			
	c	2	M	S	6	840	565	1929	550	53.6	62.3	75.7		O	Yes	—	—					-	—			
	Plant					2,040	1,385		930				66.0					D						3	8	2
716	a	4	A	-	6	600	410	1927	—	73.8	-	62.3		A	No	—	—		-11,281-		5.53	34	840^{17}			
	b	4	A	-	4	365	250	1922	—	73.8	-	68.6		A	No	—	—					16	72^{17}			
	c	4	A	-	4	500	380	1918	—	76.0	-	56.6		A	No	—	—					65	75^{17}			
	d	4	A	-	4	500	400	1925	—	76.0	-	61.6		A	No	—	—					0	—			
	Plant					2,040	1,440		1,010				60.8					B						3	8	2
93	a	2	M	S	5	700	480	1928	400	53.6	45.6	40.7		O	Yes	—	—		1,344-0	0	0.69	-	—			
	b	2	M	S	5	700	480	1928	500	53.6	57.0	42.2		O	Yes	—	—			0		-	—			
	c	2	M	S	4	560	376	1928	400	53.6	57.0	49.3		O	Yes	—	—			0		-	—			
	Plant					1,960	1,336		1,100				43.4					D						3	8	1
215	a	2	M	S	7	980	676	1930^{18}	700	53.6	57.0	77.1		O	Yes	—	—		12-0		—	1	36			
	b	2	M	S	7	980	676	1930^{18}	700	53.6	57.0	80.0		O	Yes	—	—					0	—			
	Plant					1,960	1,352		1,400				78.5					A						3	8	1^{19}

TABLE V - INFORMATION COVERING MAINTENANCE AND OPERATING LABOR (Page 3)

Plant Number	Engine Designation	Engine Cycle	Injection System-See Notes	Scavenging System - See Notes	Number of Cylinders	Rated B.H.P.	Generator Rating - K.W.	Year Engine Started to Work	Peak Load During Reported Period - K.W.	B.Mep. at Rated B.H.P. - Lbs. per Sq. In.	B.Mep. at Peak Load (90% Generator Efficiency)	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Piston Cooling-See Notes	Are Air Filters Used?	Average Liner Wear per 1000 Hrs. Operation - Ins.	Operating Period Over Which Liner Wear Was Measured - Hrs.	Type Cooling System (See Text)	Cost of Regular Engine Upkeep - Total Dollars - Material - Extra Labor	Cost of Repairs for Engine Accidents - Total Dollars - Material - Extra Labor	Total Engine Repairs in Dollars per Rated B.H.P. per Year	Number of Enforced Shutdowns	Total Duration of Enforced Shutdowns - Hours	Number of Shifts per Day	Number of Hours per Shift	Number of Attendants per Shift
729	a	4	A	-	3	225	160	1912	160	75.0	79.4	-		A	No	--	---		446-0	0	0.23	-	--			
	b	4	A	-	3	225	160	1914	160	75.0	79.4	-		A	No	--	---			0		-	--			
	c	4	A	-	3	225	160	1916	160	75.0	79.4	-		A	No	--	---			0		-	--			
	d	4	A	-	4	500	380	1920	380	76.0	86.0	-		A	No	--	---			0		-	--			
	e	4	A	-	4	750	500	1922	500	74.5	74.0	-		W	No	--	---			0		-	--	2	8	1
	Plant					1,925	1,360		1,300				83.5					B-E						1	8	2
837	a	4	A	-	4	750	500	1922	---	74.5	-	49.3		W	No	--	---		3,414-3,284[8]	0	3.57	-	--			
	b	4	A	-	6	450	330	1918	---	75.0	-	50.6		A	No	--	---			0		-	--			
	c	4	A	-	3	225	160	1904	---	75.0	-	40.3		A	No	--	---			0		-	--			
	d	4	A	-	3	225	160	1904	---	75.0	-	45.1		A	No	--	---			0		-	--			
	e	4	A	-	3	225	160	1906	---	75.0	-	39.0		A	No	--	---			0		-	--			
	Plant					1,875	1,310		750				49.0					A						3	8	2
109	a	2	A	S	4	675	450	1927	---	57.4	-	44.3		O	Yes	--	---		532-0		0.30	2	25			
	b	2	A	S	6	1,125	750	1929	---	60.4	-	51.5		O	Yes	--	---					2	40			
	Plant					1,800	1,200		1,070				48.1					A						2	8	1
176	a	4	A	-	6	600	410	1926	---	69.8	-	61.6		A	No	.0016	29,755					-	--			
	b	4	A	-	6	600	410	1926	---	69.8	-	55.1		A	No	.0016	31,174					-	--			
	c	4	A	-	6	600	410	1928[9]	---	69.8	-	63.5		A	Yes	.0028	13,370					-	--			
	Plant					1,800	1,230		1,020				59.2					B-E[20]						3	8	2
722	a	2	M	CC	4	200[21]	136	1923	140	29.4	30.7	66.8		A	No	--	---		-946-	0	0.55	0	--			
	b	2	M	CC	6	300[21]	200	1923	200	29.4	29.2	81.6		A	No	--	---			0		0	--			
	c	2	M	S	6	840	565	1929	560	53.6	53.2	86.1		O	Yes	--	---			0		0	--			
	d	4	A	-	4	365	266	1916	260	73.8	78.3	70.1		A	No	--	---			0		0	--	2	8	1
	Plant					1,705	1,167		1,160				82.1					C						1	8	3
16	a	2	M	CC	5	450	300	1922	---	44.3	-	-		A	-	--	---					-	--			
	b	4	A	-	6	600	410	1926	---	69.8	-	-		A	-	--	---					-	--			
	c	4	A	-	6	600	410	1928	---	69.8	-	-		A	-	--	---					-	--			
	Plant					1,650	1,120		1,050				-					B-E						-	-	-
8	a	4	A	-	4	400	275	1921	250	77.2	71.9	45.1		A	No	--	---		58-0	0	0.15	0	--			
	b	4	A	-	3	560	400	1922	300	73.4	58.6	64.1		O	No	--	---		0	0	0	0	--			
	c	4	A	-	6	600	400	1926	450	69.8	78.0	65.7		A	Yes	--	---		0	491[22]-0	0.91	1	2			
	Plant					1,560	1,075		700				57.0					B-E						3	8	2
132	a	4	A	-	3	375	255	1929	250	74.7	74.2	58.3		A	Yes	--	---		0	0	0	0	--			
	b	4	A	-	4	500	345	1929	350	74.7	77.9	70.3		A	Yes	--	---		0	0	0	0	--			
	c	4	A	-	5	625	430	1929	420	74.7	74.7	65.2		A	Yes	--	---		0	0	0	0	--	1	11	2
	Plant					1,500	1,030		575				63.5					D-E						1	13	1
154	a	4	A	-	6	600	410	1927	---	72.7	-	-		A	-	--	---					-	--			
	b	4	A	-	4	400	270	1926	---	72.7	-	-		A	-	--	---					-	--			
	c	4	A	-	4	400	270	1925	---	72.7	-	-		A	-	--	---					-	--			
	Plant					1,400	960		690				-					-						-	-	-
163	a	4	M	-	4	400	275	1924	270	64.7	65.0	66.8		A	No	--	---					-	--			
	b	4	M	-	4	400	275	1927	288	64.7	69.4	79.5		A	No	--	---					-	--			
	c	4	M	-	6	600	500	1929	460	64.7	73.8	68.2		A	No	--	---					-	--			
	Plant					1,400	1,050		1,018				71.4					C						3	8	1
723	a	2	M	CC	6	300[21]	200	1925	190	29.4	27.8	73.4		A	No	.0012	1,712		-3005-	0	2.28	0	--			
	b	2	M	CC	6	300[21]	200	1925	190	29.4	27.8	71.9		A	No	--	---			0		0	--			
	c	2	M	S	6	720	480	1927	480	46.0	45.7	66.2		O	Yes	--	---			0		0	--	2	8	1
	Plant					1,320	880		800				66.0					C						1	8	3
725	a	4	A	-	6	600	400	1926	---	69.8	-	-		A	No	--	---		808-0	100-0	0.09	-	--			
	b	4	A	-	4	400	270	1925	---	69.8	-	-		A	No	--	---					-	--			
	c	2	M	CC	4	200[21]	136	1924	---	29.4	-	-		A	No	--	---					-	--			
	d	2	M	CC	2	110[21]	70	1923	---	31.5	-	-		A	No	--	---					-	--	2	10	1
	Plant					1,310	876		---				-					D						1	4	1
13	a	4	A	-	4	275	175	1923	185	75.8	75.8	64.9		A	No	.0006	6,751		248-0	0	0.90	1	1/3			
	b	4	A	-	3	200	125	1923	145	73.5	79.3	66.0		A	No	.0010	2,578		125-0	0	0.63	1	1/3			
	c	4	A	-	4	750	500	1925	450	73.8	65.9	45.8		W	No	.0022	1,356		70-0	0	0.09	0	--			
	Plant					1,225	800		450				59.2					A						3	8	1
17	a	4	A	-	6	600	410	1926	---	69.8	-	-		A	-	--	---					-	--			
	b	4	A	-	6	600	410	1926	---	69.8	-	-		A	-	--	---					-	--			
	Plant					1,200	820		920				-					B-E						-	-	-
94	a	4	A	-	6	600	410	1927	480[3]	72.7	86.7	65.7		A	Yes	--	---		1,034-600	0	1.40	-	--			
	b	4	A	-	6	600	410	1927	490[3]	72.7	88.5	67.1		A	Yes	--	---			0		-	--			
	Plant					1,200	820		970[3]				66.4					D						3	10[18]	1
616	a	2	M	CC	2	100[21]	60	1920	50	29.4	21.9	-		A	No	--	---					-	--			
	b	4	A	-	3	300	200	1923	200	77.2	76.7	-		A	No	--	---					-	--			
	c	4	A	-	4	750	525	1925	520	74.5	76.9	-		W	No	--	---					-	--	1	8	2
	Plant					1,150	785		520				-					B						2	8	1

TABLE V - INFORMATION COVERING MAINTENANCE AND OPERATING LABOR (Page 4)

Plant Number	Engine Designation	Engine Cycle	Injection System—See Notes	Scavenging System - See Notes	Number of Cylinders	Rated B.H.P.	Generator Rating - K.W.	Year Engine Started to Work	Peak Load During Reported Period - K.W.	B.Mep. at Rated B.H.P. - Lbs. per Sq. In.	B.Mep. at Peak Load (90% Generator Efficiency)	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Piston Cooling—See Notes	Are Air Filters Used?	Average Liner Wear per 1000 Hrs. Operation - Ins.	Operating Period Over Which Liner Wear Was Measured - Hrs.	Type Cooling System (See Text)	Cost of Regular Engine Upkeep - Total Dollars - Material - Extra Labor	Cost of Repairs for Engine Accidents - Total Dollars - Material - Extra Labor	Total Engine Repairs in Dollars per Rated B.H.P. per Year	Number of Enforced Shutdowns	Total Duration of Enforced Shutdowns - Hours	Number of Shifts per Day	Number of Hours per Shift	Number of Attendants per Shift
95	a	2	M	S	4	560	375	1929	360[3]	53.6	51.3	50.0		O	Yes	--	---		955-0	0	0.85	-	--			
	b	2	M	S	4	560	375	1929	400[3]	53.6	57.0	51.5		O	Yes	--	---			0		-	--			
	Plant					1,120	750		740[3]				50.7					B						3	10[15]	1
96	a	4	M	-	6	600	400	1925	440[3]	64.6	70.4	48.9		A	No	--	---		945-0	0	0.86	-	--			
	b	4	M	-	3	300	200	1923	220[3]	64.6	70.4	76.2		A	No	--	---			0		-	--			
	c	2	M	CC	4	200[21]	136	1921	100[3]	29.4	25.2	46.8		A	No	--	---			0		-	--			
	Plant					1,100	736		600[3]				61.5					B[23]						3	10[15]	1
182	a	4	M	-	4	270	175	1930[24]	206	60.0	68.9	73.6		A	No	--	---		349[25] -0	0	0.32	0	--			
	b	4	M	-	6	410	300	1930[24]	320	60.7	70.6	69.8		A	No	--	---			0		0	--			
	c	4	M	-	6	410	300	1930[24]	320	60.7	70.6	71.0		A	No	--	---			0		0	--			
	Plant					1,090	775		560				71.0					C						3	9[15]	1[26]
721	a	2	M	CC	6	360	240	1925	200	35.3	29.2	33.3		A	Yes	--	---		-3,122-		2.89	0	--			
	b	2	M	CC	6	360	240	1925	200	35.3	29.2	33.0		A	Yes	--	---					1	1464			
	c	2	M	CC	6	360	240	1926	200	35.3	29.2	32.0		A	Yes	--	---					0	--			
	Plant					1,080	720		400				32.9					B-E						3	8	1
78	a	4	M	-	4	400	300	1928[27]	300	64.6	72.3	57.4		A	Yes	.0029	24,136		1,888-784		2.23	5	40			
	b	4	M	-	6	660	500	1930[9]	470	66.7	70.6	47.5		A	Yes	--	---		165-113		0.21	0	--	2	12	1
	Plant					1,060	800		720				65.5					A						3	8	1
97	a	4	A	-	4	520	333	1915	340[28]	75.6	73.6	-		W	No	--	---		465-0	0	0.46	-	--			
	b	4	A	-	4	250	150	1914	225[28]	74.1	99.4	-		A	No	--	---			0		-	--			
	c	4	A	-	4	250	210	1914	225[28]	74.1	99.4	-		A	No	--	---			0		-	--			
	Plant					1,020	693		550[28]				57.4					B						3	10[15]	1
63	a	4	A	-	6	600	416	1929	400	73.8	73.3	55.3		A	Yes	.0007	4,571		600-915	0	1.52	0	--			
	b	4	A	-	4	400	270	1927	300	73.8	82.4	46.2		A	Yes	.0013	4,780			0		0	--			
	Plant					1,000	686		560				51.6					B						3	8	2[19]
69	a	2	M	S	4	460	320	1927	300	46.0	42.8	-		O	No	--	---		-106-		0.11	1	1/12			
	b	2	M	S	4	480	320	1927	350	46.0	49.9	-		O	No	--	---					0	--			
	Plant					960	640		350				45.9					B-E						3	8	1
53	a	2	M	CC	4	200[21]	136	1923	140	29.4	30.6	72.1		A	No	.0020	2,064		-4,051-		4.51	-	--			
	b	2	M	CC	6	300[21]	200	1925	190	29.4	27.7	71.0		A	No	.0015	3,383					-	--			
	c	4	A	-	4	400	270	1925	280	73.8	76.8	70.2		A	No	.0013	9,067					-	--	1	12	2
	Plant					900	606		540				70.6					B-E						1	12	1
98	a	2	M	CC	6	360	240	1926	225[3]	35.3	32.8	64.8		A	Yes	--	---		1,788-0	0	1.97	-	--			
	b	2	M	CC	6	360	240	1929	225[3]	35.3	32.8	65.3		A	Yes	--	---			0		-	--			
	c	2	M	CC	3	180	120	1926	100[3]	35.3	29.2	61.8		A	Yes	--	---			0		-	--			
	Plant					900	600		550[3]				64.5					D						3	10[15]	1
74	a	2	M	CC	3	225	150	1924	150	36.7	36.4	-		O	No	--	---					0	--			
	b	4	M	-	4	260	175	1928	200	62.5	71.6	-		A	Yes	--	---					0	--			
	c	4	M	-	6	400	275	1926	300	64.1	71.6	-		A	Yes	--	---					0	--			
	Plant					885	600		375				-					D						3	8	1
189	a	4	M	-	5	550	375	1929	300	66.7	54.1	-		A	Yes	--	---					0	--			
	b	4	M	-	2	200[12]	145	1922	125	72.6	67.6	-		A	No	--	---					0	--			
	c	4	M	-	1	100[12]	75	1915	60	72.6	64.9	-		A	No	--	---					0	--			
	Plant					850	595		300				27.4					D-E						3	8	1
153	a	2	M	S	6	840	560	1929	500	53.6	47.5	-	-	O	-	--	---	-				-	--	-	-	-
100	a	2	M	CC	6	300[21]	200	1923	200	29.4	29.2	59.2		A	Yes	--	---		1,088-100	0	1.61	-	--			
	b	2	M	CC	4	240	160	1924	160	35.3	34.9	57.6		A	Yes	--	---			0		-	--			
	c	2	M	CC	4	200[21]	136	1924	140	29.4	30.6	67.0		A	Yes	--	---			0		-	--			
	Plant					740	496		500				60.9					B						3	10[15]	1
112	a	2	M	CC	4	200[21]	136	1925	130	29.4	28.5	64.8		A	No	--	---		-634-		0.86	-	--			
	b	2	M	CC	4	240	155	1927	155	35.3	34.0	64.0		A	No	--	---					-	--			
	c	2	M	CC	6	300[21]	200	1925	190	29.4	27.7	77.4		A	No	--	---					-	--	2	10	1
	Plant					740	491		475				68.1					B						1	4	1
67	a	2	M	CC	6	360	250	1925	215	35.3	31.4	83.2		A	No	.0018	15,600		-1,135[30] -	0	1.58	0	--			
	b	2	M	CC	6	360	250	1925	215	35.3	31.4	82.5		A	No	.0017	16,500			0		1	24			
	Plant					720	500		215				82.9					A						3	8	1[19]
170	a	2	M	CC	6	360	240	1929	260	35.3	38.0	-		A	No	--	---		171-946[31]	0	1.55	1	11			
	b	2	M	CC	6	360	240	1929	260	35.3	38.0	-		A	No	--	---			0		3	4			
	Plant					720	480		520				64.7					B						1	10	2[19]
527	a	2	M	CC	6	360	240	1930[18]	220	35.3	32.1	36.7		A	Yes	--	---		10-20	0	0.04	1	1/6			
	b	2	M	CC	6	360	240	1930[18]	220	35.3	32.1	37.2		A	Yes	--	---			0		0	--			
	Plant					720	480		320				37.0					D						3	8	1
718	a	2	M	CC	6	360	240	--	210	35.3	30.7	37.0		A	No	--	---		-1,025-		1.42	16	60			
	b	2	M	CC	6	360	240	--	200	35.3	29.2	36.9		A	No	--	---					7	2/3			
	Plant					720	480		315				36.9					B						2	12	1

TABLE V - INFORMATION COVERING MAINTENANCE AND OPERATING LABOR (Page 5)

Plant Number	Engine Designation	Engine Cycle	Injection System—See Notes	Scavenging System - See Notes	Number of Cylinders	Rated B.H.P.	Generator Rating - K.W.	Year Engine Started to Work	Peak Load During Reported Period - K.W.	B.Mep. at Rated B.H.P. - Lbs. per Sq. In.	B.Mep. at Peak Load (90% Generator Efficiency)	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Piston Cooling—See Notes	Are Air Filters Used?	Average Liner Wear per 1000 Hrs. Operation - Ins.	Operating Period Over Which Liner Wear Was Measured - Hrs.	Type Cooling System (See Text)	Cost of Regular Engine Upkeep - Total Dollars - Material - Extra Labor	Cost of Repairs for Engine Accidents - Total Dollars - Material - Extra Labor	Total Engine Repairs in Dollars Per Rated B.H.P. per Year	Number of Enforced Shutdowns	Total Duration of Enforced Shutdowns - Hours	Number of Shifts per Day	Number of Hours per Shift	Number of Attendants per Shift
19	a	2	M	CC	2	100[21]	60	—	—	29.4	-	-		A	-	—	—					-	—			
	b	2	M	CC	4	240	160	1927	—	45.0	-	-		A	-	—	—					-	—			
	c	2	M	CC	4	360	240	1927	—	44.3	-	-		-	-	—	—					-	—			
	Plant					700	460		250				-					B-E						-	-	-
61	a	4	M	-	3	300	220	1928	—	64.7	-	54.8		A	No	—	—		9-198	5-3	0.72	2	4			
	b	4	M	-	3	300	220	1928	—	64.7	-	48.8		A	No	—	—		140-255	300-27	2.41	8	154			
	Plant					600	440		—				52.3					B						3	60[32]	1
106	a	4	A	-	6	600	410	1929	480	69.8	83.0	-	-	A	-	—	—	-				-	—	-	-	-
103	a	2	M	CC	4	240	160[3]	1927	130	35.3	28.5	63.1		A	No	—	—		1,721-0	0	2.92	-	—			
	b	2	M	CC	3	150[21]	100[3]	1923	100	29.4	29.2	56.3		A	No	—	—			0		-	—			
	c	2	M	CC	4	200[21]	160[3]	1920[33]	130	29.4	28.5	55.4		A	No	—	—			0		-	—			
	Plant					590	420[3]		305				60.2					D						3	10[15]	1
102	a	2	M	CC	4	240	160[3]	1928	125	35.3	27.4	43.7		A	Yes	—	—		166-0	0	0.29	-	—			
	b	2	M	CC	4	240	160[3]	1928	125	35.3	27.4	41.0		A	Yes	—	—			0		-	—			
	c	2	M	CC	2	100[21]	60[3]	1922[34]	50	29.4	21.9	77.6		A	Yes	—	—			0		-	—			
	Plant					580	380[3]		275				43.7					D						3	10[15]	1
101	a	2	M	CC	4	240	160	1926	160[3]	35.3	35.1	64.4		A	Yes	—	—		571-0	0	1.06	-	—			
	b	2	M	CC	3	150[21]	120	1919	80[3]	29.4	23.4	46.3		A	No	—	—			0		-	—			
	c	2	M	CC	3	150[21]	100	1923	85[3]	29.4	24.9	63.8		A	No	—	—			0		-	—			
	Plant					540	380		190[3]				60.5					B						3	10[15]	1
501	a	2	M	CC	2	120	80	1925	—	35.3	-	-		A	Yes	—	—		58-0	0	0.12	1	1/6			
	b	2	M	CC	2	120	80	1925	—	35.3	-	-		A	Yes	—	—			0		0	—	1	10	1
	c	2	M	CC	4	240	155	1929	148	35.3	32.4	-		A	Yes	—	—			0		0	—	1	9	1
	Plant					480	31b		148				40.6					B-E						1	5	2
335	a	4	A	-	4	250	175	1924	145	74.1	64.1	32.4		A	No	—	—		84-0	0	0.20	0	—			
	b	4	A	-	4	180	120	1919	115	73.0	69.5	42.1		A	No	—	—			0		0	—	2	10	1
	Plant					430	295		145				33.2					A						1	4	1
99	a	4	A	-	4	400	275	1927	340	72.7	92.0	84.9	84.9	A	Yes	—	—	B	224-0	0	0.56	-	—	1	-	1
247	a	2	M	CC	4	240	160	1928	—	35.3	-	-		A	No	—	—		0	0	0	0	—			
	b	2	M	CC	2	100[21]	75	1920	—	29.4	-	-		A	No	—	—		0	0	0	0	—			
	c	2	M	CC	1	50[21]	40	1915	—	29.4	-	-		A	No	—	—		0	0	0	0	—			
	Plant					390	275		170				-					A						3	8	1
353	a	2	M	CC	1	50[21]	30	1916	—	29.4	-	68.9		A	No	—	—		0	0	0	-	—			
	b	2	M	CC	2	100[21]	60	1920	—	29.4	-	46.7		A	No	—	—		36-0	0	0.36	-	—	1	5	1
	c	2	M	CC	4	240	160	1927	—	35.3	-	33.4		A	No	—	—		128-0	0	0.53	-	—	1	9	1
	Plant					390	250		126				41.6					B						1	10	1
105	a	2	M	CC	3	150[21]	100	1922	100[3]	29.4	29.2	62.4		A	Yes	—	—		162-0	0	0.44	-	—			
	b	2	M	CC	2	120	72	1926	50[3]	35.3	21.9	47.8		A	Yes	—	—			0		-	—			
	c	2	M	CC	2	100[21]	60	1923	50[3]	29.4	21.9	51.7		A	Yes	—	—			0		-	—			
	Plant					370	232		140[3]				56.5					D						3	10[15]	1
152	a	2	M	CC	6	360	240	1925	150	35.3	21.9	-	-	A	-	—	—	-				-	—	-	-	-
657	a	2	M	CC	3	180	125	1929	—	35.3	-	-		A	No	—	—		-375-	0	1.14	-	—			
	b	2	M	CC	3	150[21]	72	1927	—	29.4	-	-		A	No	—	—			0		-	—			
	Plant					330	197		90				-					B						3	8	1
27	a	2	M	CC	2	75[21]	48	1926	—	29.1	-	-		A	-	—	—					-	—			
	b	2	M	CC	4	240	150	1926	—	45.0	-	-		A	-	—	—					-	—			
	Plant					315	198		80				-					—						-	-	-
515	a	2	M	CC	3	180	120	1928	90	35.3	26.3	-		A	No	—	—		100-0	0	0.33	2	1/3			
	b	2	M	CC	2	75[21]	48	1915	40	29.1	23.1	-		A	No	—	—			0						
	c	2	M	CC	1	50[21]	32	1915	26	29.4	22.8	-		A	No	—	—			0						
	Plant					305	200		90				-					B-E						3	8	1
169	a	4	M	-	6	240	160	1928	160	77.6	77.0	41.5	41.5	A	Yes	.0022	4,271	B	23-55	0	0.32	1	3	1	8-1/2	1[19]
265	a	2	M	CC	2	120	72	1927	80	35.3	35.0	-		A	No	—	—		100-30	0	0.72	2	1/4			
	b	2	M	CC	1	60	38	1928	35	35.3	30.6	-		A	No	—	—			0		0	—	1	12	1
	Plant					180	110		80				-					B-E						1	12	2

TABLE V - INFORMATION COVERING MAINTENANCE AND OPERATING LABOR (Page 6)

Plant Number	Engine Designation	Engine Cycle	Injection System-See Notes	Scavenging System - See Notes	Number of Cylinders	Rated B.H.P.	Generator Rating - K.W.	Year Engine Started to Work	Peak Load During Reported Period - K.W.	B.Mep. at Rated B.H.P. - Lbs. per Sq. In.	B.Mep. at Peak Load (90% Generator Efficiency)	Running Engine Capacity Factor (See Text)	Running Plant Capacity Factor (See Text)	Piston Cooling-See Notes	Are Air Filters Used?	Average Liner Wear per 1000 Hrs. Operation - Ins.	Operating Period Over Which Liner Wear Was Measured - Hrs.	Type Cooling System (See Text)	Cost of Regular Engine Upkeep - Total Dollars - Material - Extra Labor	Cost of Repairs for Engine Accidents - Total Dollars - Material - Extra Labor	Total Engine Repairs in Dollars per Rated B.H.P. per Year	Number of Enforced Shutdowns	Total Duration of Enforced Shutdowns - Hours	Number of Shifts per Day	Number of Hours per Shift	Number of Attendants per Shift
858	a	4	M	-	1	60[12]	37-1/2	1929	---	52.8	-	-		A	No	--	---		-575-	0	3.19	-	--			
	b	2	M	CC	2	120	90	1929	---	35.3	-	-		A	No	--	---			0		-	--			
	Plant					180	127-1/2		66				-					B						3	8	1
646	a	2	M	CC	2	120	72	1927	50	35.3	21.9	13.7	13.7	A	No	--	---	B-E	0	0	0	4	7-3/4	2	12	1

NOTES:

INJECTION SYSTEM
A - Air
M - Mechanical

SCAVENGING SYSTEM
S - Separate (i.e., Pump or Blower)
CC - Crankcase (for 2-Stroke Cycle Only)

PISTON COOLING
A - Air. W - Water
O - Oil

NUMBERED NOTES

1. Name Plates are 700 and 1400 K.W.
2. April 3. 60 Minute Peak
4. October 5. Total Labor; Operating Crew does not Maintain 6. Expense Replacing Cylinder Heads Cracked in 1928 Through Sabotage
7. Part Time on Operation; Part Maintenance 8. Rated 450 K.V.A.
9. September 10. 24 Hour Average
11. Repaired Crankshaft Broken in 1928
12. Horizontal Engine 13. December
14. Soft Water Makeup 15. Overlapping Shifts 16. Includes Water Dept. Labor 17. Further Information Requested but not Received
18. February 19. Part Time 20. Treated Last Two Years 21. Semi-Diesel Engine 22. Operator Failed to Start Oil Pump 23. Changed in July from "A" 24. August
25. Repair Parts into Stock 26. Half time Against Water Dept.
27. March, the start of the Two Year Period 28. Instantaneous 29. Same Labor Also Operates Steam Units
30. Five Year Overhaul
31. Total Labor; All Repairs in Overtime 32. Hours per Week 33. Installed in Plt. #103 in August, 1930
34. Installed in Plt. #102 in October, 1930

TABLE VI - INFORMATION ON INVESTED COSTS (Page 1)

Plant Number	Number of Engines	Total Installed K.W.	Year in which Investment was Made	Cost of Land		Cost of Buildings		Cost of Equipment		Engineering Cost		Total Cost of Plant		Plant Number
				Cost in Dollars	Cost in Dollars Per K.W.	Cost in Dollars	Cost in Dollars Per K.W.	Cost in Dollars	Cost in Dollars Per K.W.	Cost in Dollars	Cost in Dollars Per K.W.	Cost in Dollars	Cost in Dollars Per K.W.	
82	6	3,964	Total	15,210	4	---		---		---		941,172	237	82
73	4	3,400	1923					95,000						
			1924					91,000						
			1927					159,000						
			Total					345,000	101					73
52	4	3,110	1915	10,584[1]		---		---						
			1928	---		166,167		334,487		Included in Equipment Costs				
			1929	---		1,899		15,265						
			1930	---		3,032		110,431						
			Total	10,584	3	171,098	55	460,183	148			641,865	206	52
45	5	2,900	1922	25,000		46,396		209,704						
			1926	---		---		248,523						
			Total	25,000	9	46,396	16	458,227	158	---		529,623	183	45
130	4	2,524	1923	500[1]		8,486[1]		---						
			1924	10		8,672		94,948						
			1925	1,993		3,657		---						
			1926	284		3,407		62,336						
			1927	---		138		1,532						
			1928	---		11		1,383						
			1929	---		2,576		63,674						
			1930	---		104		56,525						
			Total	2,787	1	27,051	11	240,398	95	---		270,236	107	130
3	4	2,105	1923					3,200						
			1924					68,000						
			1926					61,000						
			1927					71,000						
			Total					203,200	97					3
7	5	1,795	Total	3,000	2	10,434	6	155,665	87	(Msc.) 3,925	2	173,024	96	7
41	3	1,800	1912	5,785		10,309		54,357						
			1922	---		20,000		---						
			1923	---		---		60,000						
			1928	---		20,000		100,000						
			Total	5,785	3	50,309	28	214,357	119	3,023	2	273,474	152	41
215	2	1,352	1930	1,250	1	10,800	8	126,910	94	---		138,960	103	215
109	2	1,200	1927	257		11,318		48,926						
			1929	---		6,387		70,840						
			Total	257	-	17,705	15	119,766	100	---		137,728	115	109
176	3	1,230	1926	174		17,029		116,535						
			1928	---		---		58,267						
			Total	174	-	17,029	14	174,802	142	---		192,005	156	176
132	3	1,030	1929			39,262	38	102,046	99	21,500	21			132
13	3	800	1900	2,000		---		---						
			1923	---		6,000		56,500						
			1925	---		8,000		65,000						
			Total	2,000	3	14,000	18	121,500	152	---		137,500	172	13

TABLE VI - INFORMATION ON INVESTED COSTS (Page 2)

Plant Number	Number of Engines	Total Installed K.W.	Year in which Investment was Made	Cost of Land		Cost of Buildings		Cost of Equipment		Engineering Cost		Total Cost of Plant		Plant Number
				Cost in Dollars	Cost in Dollars Per K.W.	Cost in Dollars	Cost in Dollars Per K.W.	Cost in Dollars	Cost in Dollars Per K.W.	Cost in Dollars	Cost in Dollars Per K.W.	Cost in Dollars	Cost in Dollars Per K.W.	
616	3	785	Total	5,000	6	56,000	71	128,900	164	Incl. in Equip.Cost		189,900	242	616
182	3	775	1920	600		---		---						
			1930	---		13,000		117,480						
			Total	600	1	13,000	17	117,480	152	---		131,080	169	182
1061	1	680	1930	1,046	2	20,308	30	85,202	125	---		106,556	157	1061
69	2	640	1927	100	-	12,000	19	109,000	170	---		121,100	189	69
74	3	600	-	1,000		---		---						
			1924	---		5,000		34,000						
			1928	---		11,000		60,000						
			1929	1,430		---		---						
			Total	2,430	4	16,000	27	94,000	157	---		112,430	187	74
1056	1	565	1929	1,623	3	32,597	58	112,556	199	---		146,776	260	1056
67	2	500	1887	100		---		---						
			1925	---		9,000		49,815						
			Total	100	-	9,000	18	49,815	100	---		58,915	118	67
527	2	480	1929	3,500	7	5,920	12	63,000	131	---		72,420	151	527
501	3	315	1924	1,000		---		---						
			1925	---		6,000		24,000						
			1929	---		2,000		24,500						
			Total	1,000	3	8,000	25	48,500	154	2,000	6	59,500	189	501
335	2	295	-	Gift		5,750		---						
			1915			1,200		---						
			1919			---		28,735						
			1924			---		31,525						
			Total	0	0	6,950	24	60,260	204	3,061	10	70,271	238	335
247	3	275	Total	10	-	7,000	25	36,000	131	500	2	43,510	158	247
353	3	250	1916			1,500[2] [1]		5,000						
			1920			---		8,000						
			1927			7,000[2]		32,000						
			Total			8,500[2]	34	45,000	180	1,600	6	55,100	220	353
265	2	110	1927	500	5	3,416	31	27,000	245	338	3	31,254	284	265

NOTES:

1 - Appraisal Value as of Year Indicated

2 - Includes Land Cost

SUPPLEMENTARY TABLE VII INFORMATION ON PRODUCTION COST

																COST PER NET K.W.HR. — MILLS								
PLANT NUMBER	CHARACTER OF PLANT (SEE NOTES)	TYPE OF LOAD (SEE NOTES)	NUMBER OF ENGINES	TOTAL INSTALLED B.H.P.	TOTAL INSTALLED K.W.	TOTAL PLANT HOURS OPERATED IN REPORTED PERIOD	NUMBER OF MONTHS IN REPORTED PERIOD	TOTAL GROSS OUTPUT K.W.HRS.	TOTAL NET OUTPUT K.W. HRS.	PERCENT OF GROSS K.W.HRS FOR PLANT PURPOSES	ANNUAL PLANT LOAD FACTOR (SEE TEXT)	RUNNING PLANT CAPACITY FACTOR (SEE TEXT)	PLANT SERVICE FACTOR (SEE TEXT)	AVERAGE COST OF FUEL OIL-CENTS PER GALLON	AVERAGE COST OF LUBRICATING OIL-CENTS PER GALLON	FUEL COST	LUBRICATING OIL COST	ATTENDANCE COST, INCLUDING SUPERINTENDANCE	COST OF SUPPLIES AND MISCELLANEOUS INCLUDING WATER	COST OF ENGINE REPAIRS	COST OF ALL OTHER PLANT REPAIRS	COMBINED COST OF ALL REPAIRS, SUPPLIES & MISC.	TOTAL PRODUCTION COST	PLANT NUMBER
2	M	B	3	3450	2400	8705	12	16,213,990	16,077,095	8.4	78.8	94.3	77.1	2.18	35.0	1.98	0.45	1.14	0.09	1.56	0.09	1.74	5.31	2
865	U	B	4	1450	----	8621	12	3,700,550	3,544,230	4.2	49.1	----	----	1.94	51.4	1.93	0.53	2.04	1.00	2.55		3.55	8.05	865
70	M-W	---	3	1420	950	8760	12	----	1,980,400	----	----	----	----	3.68	42.0	3.81	0.44	3.84	0.13	0.12	0.05	0.30	8.39	70
12	P	---	3	1350	1125	8760	12	2,753,000	2,449,083	11.0	----	55.8	50.1	2.67	22.7	3.92	0.22	9.70		3.56		3.56	17.40	12
863	U	---	2	1100	700	8781	12^+	2,247,900	2,136,540	5.0	41.0	69.4	52.7	3.55	77.2	3.29	0.91	1.76	0.46	2.32	0.06	2.84	8.80	863
801	U	B	1	1000	735	8760	12	4,557,200	4,110,052	9.81	74.3	74.6	94.9	3.82	66.6	4.19	0.43	1.25	0.79	0.89	0.17	1.85	7.72	801
856	U	---	2	1000	700	----	11	----	2,669,022	----	----	----	----	----	----	4.54	1.58	5.43	1.01	0.61	0.32	1.94	13.49	856
1061	U	B	1	980	680	4452	6^+	1,447,563	1,405,473	2.9	----	96.7	49.4	4.32	56.4	3.22	0.56	0.68	0.10	0.43		0.55	5.01	1061
1056	U	B	1	840	565	8760	12	2,299,407	2,122,057	7.7	46.5	92.5	50.2	4.10	56.5	4.13	0.56	1.61	0.15	0.22		0.37	6.67	1056
731	U	S	1	825	600	1641	12	484,278	454,641	6.2	9.2	49.2	18.7	5.75	46.6	5.96	1.84	9.43	0.37	1.56	0	1.93	19.16	731
862	U	B	4	588	401	8760	12	664,510	627,820	5.5	25.3	47.7	39.7	4.43	----	4.62	0.83	8.84	0.42	1.63	1.88	3.93	18.22	862
246	M	B	3	540	----	8760	12	377,870	364,500[1]	3.5	28.8	----	----	6.47	60.0	11.42	2.69	9.80	.69	0.14	0	0.83	24.74	246
733	U	S	2	175	120	210	12	15,600	14,800	5.1	1.6	65.0	1.6	4.82	----	5.54	2.91	8.92	2.91	0	0	2.91	20.28	733

1 = Estimated by Operator

CHARACTER OF PLANT

U = Private Power Plant
P = Private Pumping Plant
M = Municipal Power Plant
W = Municipal Water Works

TYPE OF LOAD

B = Base Load
P = Peak Load
S = Standby

SUPPLEMENTARY TABLE VIII INFORMATION COVERING LUBRICATING OIL

PLANT NUMBER	ENGINE DESIGNATION	ENGINE CYCLE	INJECTION SYSTEM - SEE NOTES	SCAVENGING SYSTEM - SEE NOTES	TRUNK PISTON OR CROSS HEAD	RATED B.H.P.	NUMBER OF CYLINDERS	CYLINDER BORE - INCHES	STROKE - INCHES	RATED R.P.M.	GENERATOR RATING - K.W.	YEAR ENGINE STARTED TO WORK	ENGINE HOURS OPERATED IN REPORTED PERIOD	RATED HORSEPOWER HOURS IN REPORTED PERIOD	GALLONS OF NEW LUBRICATING OIL USED	GALLONS OF UNFIT LUBRICATING OIL DISCARDED	RATED H.P. HOURS PER GALLON OF NEW LUBRICATING OIL	RUNNING ENGINE CAPACITY FACTOR (SEE TEXT)	RUNNING PLANT CAPACITY FACTOR (SEE TEXT)	GROSS OUTPUT - K.W. HRS.	GROSS K.W. HRS. PER GALLON OF NEW LUBRICATING OIL	LUBRICATING OIL TREATMENT - SEE NOTES	AVERAGE COST OF LUBRICATING OIL - CENTS PER GALLON
2	a	4	A	-	TP	1,150	6	23	29½	167	800	1925	7,131	8,200,680	-	-	-	94.4		5,383,200	-	CC	
	b	4	A	-	TP	1,150	6	23	29½	167	800	1925	7,151	8,223,680	-	-	-	94.3		5,394,450	-	"	
	c	4	A	-	TP	1,150	6	23	29½	167	800	1925	7,207	8,288,050	-	-	-	94.3		5,436,540	-	"	
	Plant					3,450								24,712,350	20,694	-	1,195		94.5	16,213,990	783		35.0
866	a	4	A	-	TP	225	3	16	24	164	-	-	-	-	-	-	-	-		47,630	-	S	
	b	4	A	-	TP	225	3	16	24	164	-	-	-	-	-	-	-	-		51,910	-	"	
	c	4	A	-	TP	500	4	18-7/8	28-3/8	164	-	-	-	-	-	-	-	-		2,044,510	-	"	
	d	4	A	-	TP	500	4	18-7/8	28-3/8	164	-	-	-	-	-	-	-	-		1,556,200	-	"	
	Plant					1,450					-				3,624	-	-		-	3,700,550	1,021		51.4
70	a	2	M	S	TP	560	4	16	20	257	375	1928	3,840	2,150,400	-	0	-	-		-	-	BC	
	b	2	M	S	TP	560	4	16	20	257	375	1928	2,190	2,226,400	-	0	-	-		-	-	&	
	c	2	M	CC	TP	300¹	6	14	17	257	200	1920	3,476	1,042,800	-	0	-	-		-	-	S	
	Plant					1,420					950			5,419,600	2,053	0	2,640		-	-	-	"	42.0
12	a	2	M	CC	CH	450	6	15½	16	275	375	1926	4,725	2,126,250	733	-	2,900	51.5		913,400	1,246	-	
	b	2	M	CC	CH	450	6	15½	16	275	375	1926	4,013	1,805,850	865	-	2,087	66.8		1,006,000	1,162	-	
	c	2	M	CC	CH	450	6	15½	16	275	375	1926	4,420	1,989,000	742	-	2,682	50.3		833,600	1,123	-	
	Plant					1,350					1,125			5,921,100	2,340	-	2,530		55.8	2,753,000	1,176	-	22.7
863	a	4	A	-	TP	350	3	17	24	200	200	1925	3,842	1,344,700	-	-	-	64.0		492,100	-	CC	
	b	4	A	-	TP	750	6	17	24	225	500	1927	4,939	3,704,250	-	-	-	71.1		1,755,800	-	"	
	Plant					1,100					700			5,048,950	2,522	-	2,002		69.4	2,247,900	891		77.2
801	a	2	A	S	CH	1,000	4	20½	24	180	735	1927	8,309	8,309,000	2,680	-	3,100	74.6	74.6	4,557,200	1,700	BC	66.6
856	a	4	A	-	TP	500	5	17	25	225	350	1927	-	-	-	-	-	-		-	-	S	
	b	4	A	-	TP	500	5	17	25	225	350	1927	-	-	-	-	-	-		-	-	"	
	Plant					1,000					700			-	-	-	-		-	-	-		-
1061	a	2	M	S	TP	980	7	16	20	257	680	1930²	2,200	2,156,000	1,403	-	1,536	96.8	96.8	1,447,563	1,031	BC & S	56.4
1056	a	2	M	S	TP	840	6	16	20	257	565	1929	4,400	3,696,000	2,103	-	1,757	92.5	92.5	2,299,407	1,093	BC & S	56.5
731	a	4	A	-	TP	825	4	23	27	164	600	1926	1,641	1,353,825	1,796	530	754	49.2	49.2	484,278	270	F	46.6
862	a	2	M	CC	TP	75	2	12	15	300	48	-	68		-	-	-	-		-	-	F	
	b	2	M	CC	TP	75	2	12	15	300	48	-	198		-	-	-	-		-	-	"	
	c	2	M	CC	TP	180	3	14	17	257	125	-	3,992		-	-	-	-		-	-	"	
	d	4	A	-	TP	258	3	16½	24½	180	180	-	4,850		-	-	-	-		-	-	"	
	Plant					588					401				-		-		47.7	694,510	-		-
246	a	2	M	CC	TP	100¹	2	14	17	257	-	1918	-	-	-	-	-	-		-	-	S & C	
	b	2	M	CC	TP	200¹	4	14	17	257	-	1922	-	-	-	-	-	-		-	-	& F	
	c	2	M	CC	TP	240	4	14	17	257	-	1929	-	-	-	-	-	-		-	-	"	
	Plant					540					-			-	1,631	-	-		-	377,870	232		60.0
733	a	2	M	CC	CH	100	2	12½	13½	327	60	1922	210	21,000	-	-	-	-			-	S & F	
	b	2	M	CC	TP	75¹	2	12	15	300	60	1914	190	14,250	-	-	-	-			-	"	
	Plant										120			35,250	-	-	-		65.0	15,600	-		-

1 = Semi-Diesel
2 = Installed June 27

Injection System

A = Air
M = Mechanical

Lubricating Oil Treatment

CC = Continuous Centrifuging
BC = Batch Centrifuging
S = Settling
C = Chemical
F = Filtration

Scavenging System

S = Separate Scavenging
CC = Crank Case Scavenging

SUPPLEMENTARY TABLE IX INFORMATION COVERING FUEL OIL –

PLANT NUMBER	ENGINE DESIGNATION	ENGINE CYCLE	INJECTION SYSTEM-SEE NOTES	SCAVENGING SYSTEM-SEE NOTES	RATED B.H.P.	NUMBER OF CYLINDERS	CYLINDER BORE -INCHES	STROKE -INCHES	RATED R.P.M.	GENERATOR RATING-K.W.	YEAR ENGINE STARTED TO WORK	ENGINE HOURS OPERATED IN REPORTED PERIOD	GROSS OUTPUT -K.W.HRS.	FUEL OIL USED -GALS.	GROSS K.W.HRS. PER GALLON OF FUEL OIL	RUNNING ENGINE CAPACITY FACTOR-(SEE TEXT)	RUNNING PLANT CAPACITY FACTOR-(SEE TEXT)	NATURE OF FUEL OIL USED - SEE NOTES	AVERAGE COST OF FUEL OIL -CENTS PER GAL.	IS FUEL CENTRIFUGED?	PLANT ALTITUDE -FEET ABOVE SEA LEVEL	PURPOSE FOR WHICH JACKET WATER HEAT IS UTILIZED (SEE NOTES)	PURPOSE FOR WHICH EXHAUST HEAT IS UTILIZED (SEE NOTES)
2	a	4	A	—	1,150	6	23	29½	167	800	1925	7,131	5,383,000	485,762	11.10	94.4		27° API-0.004% Ash		No			
	b	4	A	—	1,150	6	23	29½	167	800	1925	7,151	5,394,450	486,795	11.09	94.3		19000 Btu/lb.		No			
	c	4	A	—	1,150	6	23	29½	167	800	1925	7,207	5,436,540	490,693	11.09	94.3		40° Say. Univ @ 100°F		No			
	Plant				3,450					2400			16,213,990	1,463,250	11.09		94.3	0.5%S: 1.5% C.C.	2.18		735	N	L
866	a	4	A	—	225	3	16	24	164	—	—	—	—	—	—	—		14°-20° API					
	b	4	A	—	225	3	16	24	164	—	—	—	—	—	—	—		18,800 Btu/lb.					
	c	4	A	—	500	4	18-7/8	28-3/8	164	—	—	—	—	—	—	—		1.8% S					
	d	4	A	—	500	4	18-7/8	28-3/8	164	—	—	—	—	—	—	—							
	Plant				1,450					—			—	3,624	—		—		1.94	No	2,400	N	L
70	a	2	M	S	560	4	16	20	257	375	1928	3,840	—	99,840	—	—		—					
	b	2	M	S	560	4	16	20	257	375	1928	2,190	—	56,940	—	—		—					
	c	2	M	CC	300	6	14	17	257	200	1920	3,476	—	48,665	—	—		—	3.68	No	1,200	N	N
	Plant				1,420					950			—	205,445	—		—						
12	a	2	M	CC	450	6	15½	16	275	375	1926	4,725	913,400	—	—	51.5		—		—			
	b	2	M	CC	450	6	15½	16	275	375	1926	4,013	1,006,000	—	—	66.8		—		—			
	c	2	M	CC	450	6	15½	16	275	375	1926	4,420	833,600	—	—	50.3		—		—			
	Plant				1,350					1,125			2,753,000	359,234	7.66		55.8		2.67		20	N	N
863	a	4	A	—	350	3	17	24	200	200	1925	3,842	492,100	46,211	10.21			24°-26°API 19000 Btu/lb.					
	b	4	A	—	750	6	17	24	225	500	1927	4,939	1,755,800	149,924	11.71			0.75% S: 1.0% C.C.					
	Plant				1,100					700			2,247,900	196,135	11.38			1.0%Ash:0.5% BS&W	3.55	No	932	F	B
801	a	2	A	S	1,000	4	20¾	24	180	735	1927	8,309	4,557,200	451,000	9.90	74.6	74.6	1% S:22°-26° API	3.82	Yes	591	N	N
																		15,000 Btu/lb.					
856	a	4	A	—	500	5	17	25	225	350	1927	—	—	—	—	—							
	b	4	A	—	500	5	17	25	225	350	1927	—	—	—	—	—		—					
	Plant				1,000					700			—	—	—	—	—		—	No	35	N	N
1061	a	2	M	S	980	7	16	20	257	680	1930^{2}	2,200	1,447,563	104,706	13.82	96.8	96.8	32°API;19,500 Btu/	4.32	No	1,200	F	N
																		lb;1.25% S;0.02 CC					
1056	a	2	M	S	840	6	16	20	257	565	1929	4,400	2,299,407	213,470	10.77	92.5	92.5	32°API;19,500 Btu/	4.10	No	1,800	N	N
																		lb;1.25% S;0.02 CC					
731	a	4	A	—	825	4				600	1926	1,641	484,278	47,213	10.25	49.2	49.2	30°-32°API;18,300	5.75	No	865	N	N
																		Btu/lb;0.7% Ash;Less than 0.5% S					
862	a	2	M	CC	75	2	12	15	300	48	—	68	—	—		—		—					
	b	2	M	CC	75	2	12	15	300	48	—	198	—	—		—							
	c	2	M	CC	180	3	14	17	257	125	—	3,992	—	—		—							
	d	4	A	—	258	3	16½	24½	180	180	—	4,850	—	—		—							
	Plant				588					401			664,510	65,496	10.14		47.7		4.43	—	221	N	N
246	a	2	M	CC	100^{1}	2	14	17	257	—	1918	—	—	—		—		36°-40° API					
	b	2	M	CC	200^{1}	4	14	17	257	—	1922	—	—	—		—							
	c	2	M	CC	240	4	14	17	257	—	1929	—	—	—		—							
	Plant				540					—		—	377,870	64,337	5.87		—		6.47	No	5,000	B	N
733^{3}	a	2	M	CC	100	2	12¼	13¼	327	60	1922	210	—			—		35°API					
	b	2	M	CC	75^{1}	2	12	15	300	60	1914	190	—			—		19,275 Btu/lb					
	Plant				175					120			15,600	1,700^{2}	9.18		65.0	0.5% S	4.82	No	780	N	N

FUEL OIL

1 = Semi-Diesel
2 = Installed June 27
3 = Estimated

WASTE HEAT UTILIZATION

B = Heating Boiler Water
F = Heating Fuel Oil for Treatment
L = Heating Lubricating Oil for Treatment

NATURE OF FUEL OIL

BTU = British Thermal Unit
S.Univ.= Saybolt Universal
S = Sulphur
CC= Conradson Carbon
F = Fahrenheit

INJECTION SYSTEM

A = Air
M = Mechanical

SCAVENGING SYSTEM

S = Separate
CC = Crank Case

SUPPLEMENTARY TABLE X INFORMATION COVERING MAINTENANCE & OPERATING LABOR

PLANT NUMBER	ENGINE DESIGNATION	ENGINE CYCLE	INJECTION SYSTEM - SEE NOTES	SCAVENGING SYSTEM - SEE NOTES	NUMBER OF CYLINDERS	RATED B.H.P.	GENERATOR RATING - K.W.	YEAR ENGINE STARTED TO WORK	PEAK LOAD DURING REPORTED PERIOD - K.W.	B.MEP. AT RATED B.HP. - LBS. PER SQ. IN.	B.MEP. AT PEAK LOAD (90% GENERATOR EFFICIENCY)	RUNNING ENGINE CAPACITY FACTOR (SEE TEXT)	RUNNING PLANT CAPACITY FACTOR (SEE TEXT)	PISTON COOLING - SEE NOTES	ARE AIR FILTERS USED?	AVERAGE LINER WEAR PER 1000 HRS. OPERATION - INS.	OPERATING PERIOD OVER WHICH LINER WEAR WAS MEASURED - HRS.	TYPE COOLING SYSTEM (SEE TEXT)	COST OF REGULAR ENGINE UPKEEP TOTAL DOLLARS - MATERIAL - EXTRA LABOR	COST OF REPAIRS FOR ENGINE ACCIDENTS TOTAL DOLLARS - MATERIAL - EXTRA LABOR	TOTAL ENGINE REPAIRS IN DOLLARS PER RATED B.H.P. PER YEAR	NUMBER OF ENFORCED SHUTDOWNS	TOTAL DURATION OF ENFORCED SHUTDOWNS - HRS.	NUMBER OF SHIFTS PER DAY	NUMBER OF HOURS PER SHIFT	NUMBER OF ATTENDANTS PER SHIFT
2	A	4	A	-	6	1150	800	1925	-	74.5	-	94.4		W	No	.0042	7131		15,679	9,343	7.25	34	256			
	B	4	A	-	6	1150	800	1925	-	74.5	-	94.3		W	No	.0039	7151					27	210			
	C	4	A	-	6	1150	800	1925	-	74.5	-	94.3		W	No	.0032	7207					27	220	1	8	2
	Plant					3450	2400		2350				94.3					A						3	8	2
865	A	4	A	-	3	225	-	-	-	75.1	-	-	-	A	No	-	-					-	-			
	B	4	A	-	3	225	-	-	-	75.1	-	-	-	A	No	-	-					-	-			
	C	4	A	-	4	500	-	-	-	76.0	-	-	-	A	No	-	-					-	-	-	-	-
	D	4	A	-	4	500	-	-	-	76.0	-	-	-	A	No	-	-									
	Plant					1500			860									D-E								
70	A	2	M	S	4	560	375	1928	400	53.6	57.0	-		O	Yes	-	-		- 247 -		0.17	0	-			
	B	2	M	S	4	560	375	1928	380	53.6	54.2	-		O	Yes	-	-					0	-			
	C	2	M	S	6	300	200	1920	210	29.4	30.6	-		A	No	-	-					0	-			
	Plant					1420	950		480				-					A						3	8	1
12	A	2	M	CC	6	450	375	1926	-	36.9	-	51.5		O	Yes	-	-					-	-			
	B	2	M	CC	6	450	375	1926	-	36.9	-	66.8		O	Yes	-	-					-	-			
	C	2	M	CC	6	450	375	1926	-	36.9	-	50.3		O	Yes	-	-					-	-			
	Plant					1350	1125		-				55.8					C						3	8	2
863	A	4	A	-	3	350	200	1925	175	84.7	63.0	64.0		A	No	.0002	21,131		4522 -	0	4.51	-	14			
	B	4	A	-	6	750	500	1927	450	80.6	72.0	71.1		A	No	.0015	67,286		440	0		-	6			
	Plant					1100	700		625				69.4					D-E						3	8	1
801	A	2	A	S	4	1000	735	1927	700	67.7	70.7	74.6	74.6	W	Yes	-	-	A	3646 -	978	4.62	0	-			
856	A	4	A	-	5	500	350	1927	-	62.0	-	-		A	No	-	-		1319 -	0	1.62	0	-			
	B	4	A	-	5	500	350	1927	-	62.0	-	-		A	No	-	-		- 297	0		0	-			
	Plant					1000	700		800				-					D						3	8	1
1061	A	2	M	S	7	980	680	1930^2	680	53.6	55.4	96.8	96.8	O	No	-	-	D				-	50	2	10	1½
1056	A	2	M	S	6	840	565	1929	565	53.6	53.8	92.5	92.5	O	No	.015	6,233	D				-	100	2	10	1½
731	A	4	A	-	4	825	600	1926	600	-	-	49.2	49.2	-	Yes	-	-	-	709-0	0	0.86	-	-	3	9	1
862	A	2	M	CC	2	75	48	-	-	29.1	-	-		A	No	-	-		- 1025 -		1.74	-	-			
	B	2	M	CC	2	75	48	-	-	29.1	-	-		A	No	-	-					-	-			
	C	2	M	CC	3	180	125	-	-	35.3	-	-		A	No	-	-					-	-			
	D	4	A	CC	3	258	180	-	-	72.2	-	-		A	No	-	-					-	-			
	Plant					588	401		300				47.7					B						3	8	1
246	A	2	M	CC	2	100^1	-	1918	50	29.4	21.9	-		A	No	-	-		50 - 0	0	0.09	0	-			
	B	2	M	CC	4	200^1	-	1922	100	29.4	21.9	-		A	No	-	-			0		0	-			
	C	2	M	CC	4	240	-	1929	130	35.3	28.5	-		A	No	-	-			0		0	-	2	10	1
	Plant					540			150				-					B						1	4	1
733	A	2	M	CC	2	100	60	1922	60	44.7	40.0	-		A	No	-	-		0	0	0	0	0			
	B	2	M	CC	2	75^1	60	1914	50	29.1	28.9	-		A	No	-	-		0	0	0	0	0			
	Plant					175	120		110				65.0					B						-	-	-

1 = Semi-Diesel
2 = Installed June 27

INJECTION SYSTEM

A = Air
M = Mechanical

SCAVENGING SYSTEM

S = Separate
CC = Crank Case

OGP-54-2

Gear Drives for Motorships

As Affecting the Development of an Efficient American Merchant Marine

By J. BARRAJA-FRAUENFELDER,[1] PHILADELPHIA, PA.

This paper has been written with the object of clarifying a much-misunderstood situation. While the conclusions arrived at represent the author's own deductions from current accomplishments, they are based on known facts and data, and other authorities are quoted extensively in order to present unbiased views from engineers who have no direct interest in the production of any particular type of engine and to avoid the impression of a one-man idea. Where opportunity has permitted it, reliable data have been quoted; where not so permissible, data have been omitted; but the author will gladly corroborate any statement or quotation which is thought by others to be doubtful or not sufficiently substantiated. It is suggested that the bibliography appended to the paper be carefully consulted in order that the scope of this effort may be fully digested. In substance, the paper should be considered a compendium of practically all that has been accomplished to date in this line which is, in the author's opinion, worthy of note.

THE arrival in New York of the Hamburg-American Line's new motorships *St. Louis* and *Milwaukee* opened a new era in motorship drives. The author refers to geared Diesel engines as exemplified by the power plants of those most up-to-date ships. These plants are composed of two-cycle, double-acting Diesel engines geared to the propellers at an approximate ratio of 2 to 1. Two engines per shaft are installed, or a total of four engines, the ships being twin-screw vessels. As it is believed that two-cycle, double-acting engines are the logical type to be used in connection with gear drives for large ships, and as these ships represent the latest development following a long line of successful predecessors equipped with gear drives, the author will refrain at this point from commenting on them and go back to the early days of gear drives, from whence he will trace their progress down to the present.

Historical

The earliest known commercial applications of gear drives to Diesel-propelled ships date back to 1909. Several paddle-wheel towboats were fitted with gear transmissions for service on the River Volga in Russia. The gears served to reduce the speed of the engines, of about 250 r.p.m., to that of the paddle wheels, which were to turn at 40 to 50 r.p.m. Seiliger (31),[2] p. 141, Fig. 82, shows such an arrangement, but with Krupp six-cylinder, four-cycle engines of 120 b.hp. turning at 400 r.p.m.

Table 1 gives a list of geared ships known to exist and in successful operation at the present time. This list is not complete, as very small vessels are not included because they are of no interest to shipowners engaged in the operation of large ships, and the picture the author intends to unfold to view involves only such ships. General characteristics of the ships listed are given, including those of the machinery.

As may be noted, the earliest applications, especially in Germany, involved four-cycle, single-acting engines of the high-speed submarine type. This was simply the result of an economic condition. As Dr. Dreves states (8):

> The desire for maximum efficiency in the upbuilding and operation of the new merchant marine, dictated by the economic conditions existing in Germany right after the war, was responsible for the adoption of indirect drive with oil engines for the propulsion of ships, which already had proved successful with turbines.

Also, after the war a number of German Diesel-engine manufacturers found themselves with submarine engines on their hands, either completely or partially finished or with sufficient stocks of parts, which together with existing patterns and jigs enabled them to complete those unfinished and to produce new ones of the same type at reasonable prices. So that such engines as escaped destruction at the dictum of the allied disarmament commission were thus devoted to peaceful purposes. Again quoting Dr. Dreves:

> As far as the necessity of driving the oil engine at a higher rotative speed was concerned, there could have been no fundamental objection against their use in merchant ships since their reliability has been proved, if not in continuous service, at any rate under the most difficult conditions of submarine service.

Still they possessed rather extreme characteristics for commercial use, and to suit the requirements of the merchant marine for relatively higher reliability and longer life, which is more important, both engine speed and mean effective pressure were suitably reduced. In addition, certain parts which did not require extreme lightness were made more rugged or heavier. The type nevertheless was preserved and was undoubtedly the most logical one to be so utilized at that time.

These earlier applications so showed their superiority over direct drives that at least one builder adopted them as standard, redesigning them for out-and-out commercial applications; hence the vessels which followed show this new trend, as will be seen later.

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[1] J. Barraja-Frauenfelder & Co., consulting engineers in oil engines. Consulting Diesel engineer of the Sun Shipbuilding & Dry Dock Company. Mr. Barraja-Frauenfelder was born in Italy and was graduated from Naples Nautical College and Milan Polytechnic. He came to this country in 1901, and from 1905 to 1911, while with the Holland Torpedo Boat Company, he devoted considerable time to the study of Diesel engines. From 1911 to 1913, he was assistant to the chief engineer of the Fore River Shipbuilding Corporation, where he conducted investigations on Diesel engines and made a European investigation trip. During 1913 and 1914 he was chief engineer of the Spezia, Italy, works of the Fiat-San Giorgio Company, in charge of design and construction of Diesel engines. He returned to this country, and from 1914 to 1923 was chief engineer of the Lake Torpedo Boat Company. He has taken an active part in the development of high-speed submarine engines of both two- and four-cycle types.

Presented at the National Oil and Gas Power Meeting, held at the Pennsylvania State College, State College, Pa., June 12 to 14, 1930.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to Bibliography at end of paper.

From four-cycle, single-acting, we note a step forward to include four-cycle, double-acting, then two-cycle, double-acting, which seems at the present moment the ultimate type of engines to be used in gear drives.

Brief comments covering some of the most notable installations listed are in order.

The *Libby Maine*, built in 1918, represents one of the earliest attempts at gear drives in the United States. She was one of two wooden sister ships. Her mate, the *James Timpson*, was lost at sea in a storm in 1924 after covering 190,000 miles. The *Libby Maine*, as far as is known, is still operating successfully; see (13). The only difference in the *James Timpson* was in the main engines, they being Winton engines as against Dow-Willans engines in the former. Both ships had Falk gears.

The next notable application was that of the German ships *Havelland* and *Munsterland*, built in 1921 for the Hamburg-American Lines. In this connection it is interesting to note that while America pioneered in this type of drive, the Germans were the ones to realize its possibilities to the fullest extent. These ships were, to the author's knowledge, the first large motorships put in actual service at the time; the *Havelland* made her maiden voyage at the end of 1921.

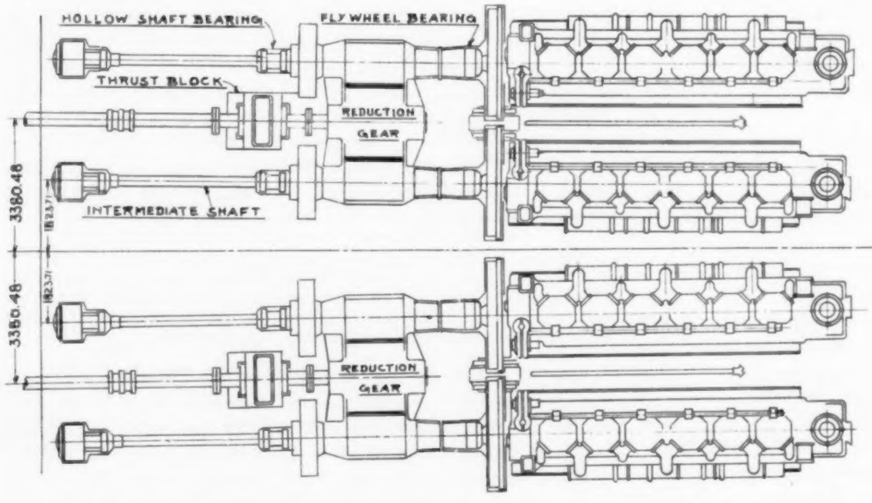

Fig. 1 Machinery of the Motor Passenger Liner "Monte Sarmiento"

Two ten-cylinder, four-cycle, single-acting engines of the M.A.N. submarine type were installed in these boats; the engines had a cylinder bore of 530 mm. (20$^{7}/_{8}$ in.) and a stroke of 530 mm. also, and were rated at 1700 b.hp. each, at 230 r.p.m. (original submarine rating 3000 s.hp. at 390 r.p.m.). Each engine was coupled to a propeller shaft by direct gear drive at a ratio of 1:2.7 (85 r.p.m. of propeller). The connections between the crankshaft and the intermediary shaft, as well as between the latter and the pinion, consisted of rigid flange couplings. The installation has become rather famous on account of a special shaft extension of great elasticity between the engines and the pinions. There has been a widespread belief that this long and flexible shaft was chosen with the idea of keeping the irregular torque of the engine away from the teeth of the gear, but this is not the case. From the beginning, the designers (Blohm & Voss) were absolutely convinced that the impulses resulting from the piston forces would have no detrimental effect on the gears. The reason for adopting the long intermediary shaft was to shift the critical range of the 5th order of the main vibrations as low as possible.

The machinery and, particularly, the gears have proved very successful. The Hamburg-American Line had ordered, at the same time, two other motorships, the *Rheinland* and the *Ermland*, of the same size and description, but equipped with slow-speed, direct-coupled, four-cycle, six-cylinder Diesel engines of the crosshead type, running at 95 r.p.m. It is the best proof of the reliability of the geared installations that the Hamburg-American Line, when further ships of the same size and for the same purpose had to be ordered, decided on duplicates of the *Havelland*. These two newer boats were the *Vogtland* and *Friesland*. The installation in the latter boat varies, however, from the original *Havelland* arrangement, inasmuch as both engines in the *Friesland* are driving one propeller shaft over one common large gearwheel. The long and flexible intermediary shafts have been eliminated in this ship, with the result that the critical range of the 5th-order two-noded vibrations is located in the neighborhood of 65 r.p.m. of the propeller, which is less convenient than in the other installations. Regarding the reasons for this alteration and regarding the couplings used in the M.S. *Friesland*, see the description of the *Monte Sarmiento* given below.

M.S. *Hans Lonehardt* (ex-*Vulcan*). This was an experimental ship built by the Vulcan Werke in order to demonstrate the Vulcan-Foettinger hydraulic reduction gear; likewise the Bremer-Vulcan Works built a tugboat for the same purpose. These ships have been successful as demonstrators, except that there does not seem to be sufficient justification for this expensive type of reversing gear, as later developments will show.

M.S. *Monte Sarmiento* and *Monte Olivia* (see 7). During the year 1923, the Hamburg Süd-amerikanische Dampfschifffahrts Gesellschaft placed an order for two motorships of nearly 14,000 British register tons with the firm of Blohm & Voss, Hamburg. The desired speed of these ships required a total of 5400 b.hp.; Blohm & Voss suggested twin-screw ships with four single-acting, four-cycle Diesel engines of six cylinders each and with 600 mm. bore and 700 mm. stroke, running at 215 r.p.m.; each propeller shaft was driven by two of these engines over one common large gearwheel, the propeller shafts running at 77 r.p.m. During the course of construction it was agreed to raise the speed of the ships to 14.25 knots, which required a total of 7000 b.hp. per engine; this increased output was obtained by installing turbo blowers for supercharging.

The first engines (for the M.S. *Monte Sarmiento*) were built as trunk-piston engines, but owing to many difficulties with the pistons, it was decided to use crossheads for the four engines of the *Monte Olivia*, cylinder bore, stroke, and r.p.m. remaining the same. At first, long and flexible intermediary shafts between the engines and the pinions, similar to the installation in the *Havelland*, were provided. The first trials showed that, at certain speeds, rather heavy oscillations took place between the two engines coupled to one propeller shaft, so that for the long intermediary shafts there had to be substituted very short shaft connections. The natural frequency of oscillations became thereby much higher, which was admittedly less convenient, but the gears worked just as satisfactorily as before. As a matter of fact, with all the gear-driven installations mentioned here, there has never been the least complaint about the gears; the wear and tear has proved entirely negligible. Fig. 1 shows the original layout of the engines, and Fig. 2 the gear set with quillshafts.

With the twin-engine arrangement, the necessity arises to provide couplings between the crankshaft and pinion that can be quickly and easily disconnected. The engineers of Blohm & Voss designed a very simple flange coupling in which a small number of very strong pins were rigidly fastened in one half of the coupling and fitted to a running fit into corresponding holes in the other half, some clearance existing, of course, in the axial direction between the two flanges. After reliable lubrication of the bearing surfaces of the pin was provided, this coupling proved quite satisfactory. It has been stated by the operators that it takes only from 8 to 10 minutes to disconnect one engine. During the trial runs and during the first voyages of the *Monte Sarmiento*, several piston seizures occurred which brought the engines to a sudden stop. Contrary to the prophecies of the advocates of the hydraulic gear, the toothed gears stood these trying tests exceptionally well; no damage whatever of the teeth could be detected. The gears are of the herringbone type, with 45-deg. angle; the diameters are 818 mm. (32 in.), with 124 teeth for the pinions, and 2295 mm. (90 in.), with 348 teeth for the large wheels; circular pitch, about 0.816 in.

M.S. *Heidelberg*. In contrast with the foregoing motorships, the *Heidelberg* is equipped with Vulcan hydraulic couplings between the engines and pinions. It must be understood that the gear here mentioned, as made by Vulcan, is not entirely hydraulic. The transmission ratio is obtained by ordinary toothed gears of the herringbone type, but the connection with the engine is made through a Foettinger-type hydraulic coupling which is an outgrowth of the old Foettinger transformer of turbine fame. When reversibility is desired, another hydraulic coupling designed to run in the opposite direction is installed at the opposite end of the pinion.

Reversing is accomplished by pumping the fluid out of the ahead coupling and into the astern coupling. This process naturally required time; hence the reversing type of hydraulic coupling is not favored.

Two single-acting, four-cycle, eight-cylinder engines of the crosshead type are installed in this ship, each driving an individual propeller shaft; the engines have cylinders of 640 mm. ($25^1/_4$ in.) bore by 700 mm. ($27^9/_{16}$ in.) stroke; they are rated at 2050 b.hp. each at 215 r.p.m. With an eight-cylinder, single-acting, four-cycle engine, the most dangerous vibrations are those of the 4th order. When a hydraulic coupling is installed, the entire shafting is divided, as far as torsional vibrations are concerned, into two independent systems; one consisting of the engine with flywheel and the primary parts of the coupling, and the other of the secondary parts of the coupling, the reduction gear composed of pinions and large wheel, the line shafting, and the propeller. The use of the hydraulic coupling, in the case of the *Heidelberg*, resulted in the oscillations of the main critical range of the 4th order being shifted above the maximum running speed, which is certainly of great advantage.

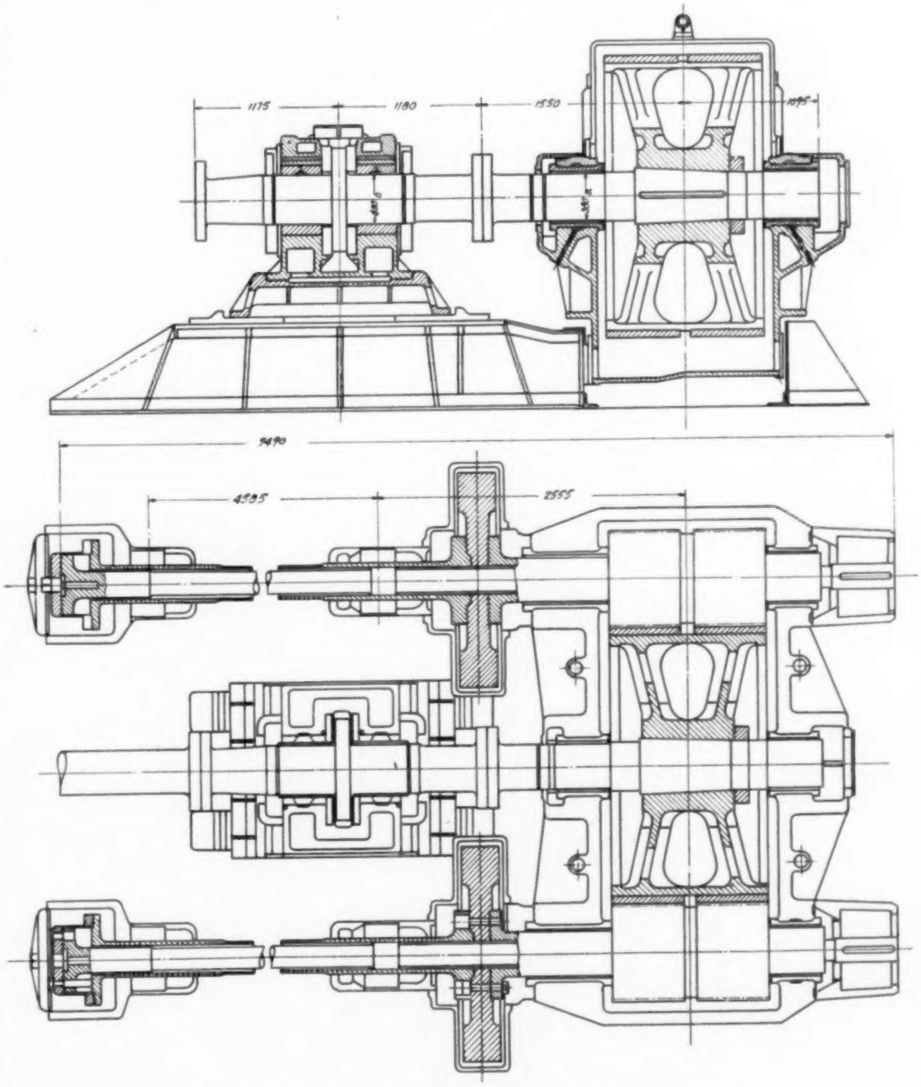

Fig. 2 Gear Set of Motorship "Monte Sarmiento"

During the first voyages, several difficulties were encountered with the hydraulic gear, mostly due to the inability to fully eject the air carried in by the lubricating oil used for filling the couplings, and further, due to defects in the auxiliary pumps belonging to the hydraulic gear. After various alterations these difficulties were overcome to the satisfaction of the owners. Two similar plants have been installed in the M.S. *Ruhr* and *Rhein*. One of the author's correspondents, who is acquainted

TABLE 1 LIST OF MOTOR PASSENGER AND CARGO SHIPS EQUIPPED WITH GEAR DRIVES

Date	Name of ship	Owners	Tonnage Gross	Tonnage Dead weight	Make and type of engines[a]	Number of engine	Cylinders per engine	Bore and stroke	R.p.m. Engine	R.p.m. Propeller	B.hp., each engine	S.hp., total	Type of gear	Speed knots
1918	*Libby Maine*	Libby, McNeil & Libby	1811	2250	Dow-Willans 4C-SA-TP	2	6	12" x 18"	250	100	320	640	Plain, Nuttall couplings	..
1918	*City of Tachoon*	Piaggio & Co.	2250	...	Winton 4C-SA-TP	2	6	13" x 18"	275	...	...	800	Unknown	..
1921	*Havelland*......	Hamburg-American Line	6308	10235	Blohm & Voss-M.A.N. 4C-SA-TP	2	10	20 7/8" x 20 7/8"	230	85	1700	3300	Plain, with quillshafts, twin screw	12 1/2
1921	*Munsterland*....	Hamburg-American Line	6308	10235	Blohm & Voss-M.A.N. 4C-SA-TP	2	10	20 7/8" x 20 7/8"	230	85	1700	3300	Plain, with quillshafts, twin screw	12 1/2
1924	*Hans Lonehardt* (ex-*Vulcan*)..	Lonehardt & Blumberg	1314	2000	Vulcan-M.A.N. 4C-SA-TP	2	6	13 3/4" x 13 3/4"	300	85	325	620	Vulcan hydraulic gear	..
1924	Tugboat.......	Bremer-Vulcan	...	...	Bremer-Vulcan 4C-SA-TP	2	4	....	300	100	260	490	Reversible-single screw	..
1925	*Friesland*......	Hamburg-American Line	6252	10000	Blohm & Voss-M.A.N. 4C-SA-TP	2	10	20 7/8" x 20 7/8"	230	85	1700	3300	Plain, with rigid-connection single screw	..
1925	*Vogtland*.......	Hamburg-American Line	6252	10000	Blohm & Voss-M.A.N. 4C-SA-TP	2	10	20 7/8" x 20 7/8"	230	85	1700	3300	Plain, with quillshafts, twin screw	..
1925	*Heidelberg* (ex-*Duisburg*)....	Hamburg-American Line	6530	9400	Vulcan-M.A.N. 4C-SA-CH	2	8	25 3/16" x 27 9/16"	215	77	2050	4100	Vulcan hydraulic gear, reversible twin screw	13
1925	*Monte Sarmiento*....	Hamburg-South American Line	13750	8600	Blohm & Voss-M.A.N. 4C-SA-TP	4	6	23 1/2" x 27 1/2"	215	77	1600	6400	Plain, with rigid-connection, twin screw	14 1/2
1925	*Monte Olivia*...	Hamburg-South American Line	13750	8600	Blohm & Voss-M.A.N. 4C-SA-CH	4	6	23 1/2" x 27 1/2"	215	77	1600	6400		14 1/2
1925	*Rendsburg*......	German Australian	8000	10000	Vulcan-M.A.N. 4C-SA-CH	2	8	25 3/16" x 27 1/2"	215	77	2000	4000	Vulcan hydraulic gear, reversible twin screw	..
1925	*Altenfels*.......	Hansa S.S. Co.	...	11300	Vulcan-M.A.N. 4C-SA-CH	2	8	25 3/16" x 27 1/2"	210	78	1900	3800		
1925	*Cornell* (tug)...	Cornell S.S. Co.	79	...	Nelseco 4C-SA-TP	1	6	13" x 18"	230	102	..	360	Plain, Falk	..
1925	*Peres Puchol*...	Cia. Trasmediterranea	1270	...	M.A.N. 4C-SA-TP	2	10	20 7/8" x 20 7/8"	200	85	..	2800	Unknown, twin screw	..
1926	*Rhein*.........	Hugo Stinnes	6013	9500	Vulcan-M.A.N. 4C-SA-CH	2	8	22 1/16" x 24 7/16"	210	77	1900	3800	Vulcan hydraulic gear, reversible twin screw	14 1/2
1926	*Ruhr*..........	Hugo Stinnes	6013	9500	Vulcan-M.A.N. 4C-SA-CH	2	8	22 1/16" x 24 7/16"	210	77	1900	3800		14 1/2
1926	*Wulsty Castle*..	William Beardmore	...	6300	Beardmore 4C-DA-CH	2	3	20" x 20 3/4"	240	90	850	1600	Vulcan hydraulic gear, non-reversible single screw	..
1926	Suction dredge.	Dutch Indies	...	...	M.A.N.	2	..	....	190	...	300	510	Hydraulic gear between engines and pump, with flexible couplings	..
1926	*Barbara*.......	German Admiralty	1585	2830	Weser-M.A.N. 4C-SA-TP	2	6	14 3/16" x 21"	300	80	530	1010	Vulcan hydraulic gear, reversible single screw	..
1927	*Herman Falk*...	Oil Transport Company	...	10200	Falk-Banner 4C-SA-TP	2	8	18" x 22 1/2"	275	90	1425	2600	Plain, with Bibby coupling	14 1/4
1928	*Monte Cervantes*	Hamburg-South American Line	14000	8000	Blohm & Voss-M.A.N. 4C-SA-CH	4	6	24 7/8" x 27 1/2"	210	75	1500	6000	Plain, with rigid-connection twin screw	14 1/4
1928	*Mary Pinchot*..	Gifford Pinchot	246	...	Winton 4C-SA-CH	1	8	8" x 10"	700	...	....	300	Plain, Falk	
1929	*St. Louis*......	Hamburg-American Line	17000	10100	Vulcan-M.A.N. 2C-DA-CH	4	6	19 1/8" x 26"	225	110	3100	12000	Vulcan hydraulic gear, non-reversible twin screw	16
1929	*Milwaukee*.....	Hamburg-American Line	17000	10100	Blohm & Voss-M.A.N. 2C-DA-CH	4	6	19 5/8" x 25 1/2"	220	110	3000	12000	Plain, with rigid connection twin screw	16
1929	*Ersats Preussen*	German Navy	10000[b]	...	M.A.N. 2C-DA-CH	..	..	....	...	...	....	50000	Vulcan hydraulic gear, non-reversible twin screw	26
1929	*Kota Agoeng*...	Rotterdam Lloyd	...	9500	Fijenoord-M.A.N. 2C-FA-CH	2	5	21" x 27 5/8"	215	...	2750	5400		..
1930	*Monte* class....	Hamburg-South American Line	14000	8000	Blohm & Voss-M.A.N. 4C-SA-CH	4	6	24 3/4" x 27 1/2"	210	75	1800	7000		14 1/2
1930	*Monte* class....	Hamburg-South American Line	14000	8000	Blohm & Voss-M.A.N. 4C-SA-CH	4	6	24 3/4" x 27 1/2"	210	75	1800	7000		14 1/2

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[a] Abbreviations: 4C, four-cycle; 2C, two-cycle; SA, single-acting; DA, double-acting; TP, trunk piston; CH, crosshead.
[b] Displacement net without fuel.

with the installation in one of these ships, expresses his opinion as follows:

> The plant works quite satisfactorily, including the hydraulic reversing part, but the space occupied, the weight, and cost of the couplings and the reversing gear, including all the auxiliaries (pump, tanks, piping), appear to be much out of proportion to the advantage obtained over the plain gear drive, whereby it can be considered today a proved fact that the simple gear drive is absolutely reliable if everything is correctly designed.

M.S. *St. Louis* and *Milwaukee.* These vessels have already made several voyages to and from New York. With the exception of the engines, their installations are similar to that of the M.S. *Monte Sarmiento.* The engines, four in number, are of the double-acting, two-cycle type with six cylinders each, and so rated that each ship will obtain 12,000 s.hp. normal rating. The difference between the two plants is shown in Table 1, to the effect that the *St. Louis*, built by the Bremer-Vulcan, has Vulcan non-reversible hydraulic couplings, while the *Milwaukee*, built by Blohm & Voss, has pin couplings. In the former the engine rating is slightly higher to make up for the losses of the hydraulic couplings. Both groups of engines have been built by the respective shipbuilders under license from M.A.N. A layout of the *St. Louis* engine spaces is shown in Fig. 3. The *Milwaukee* is slightly different in minor details.

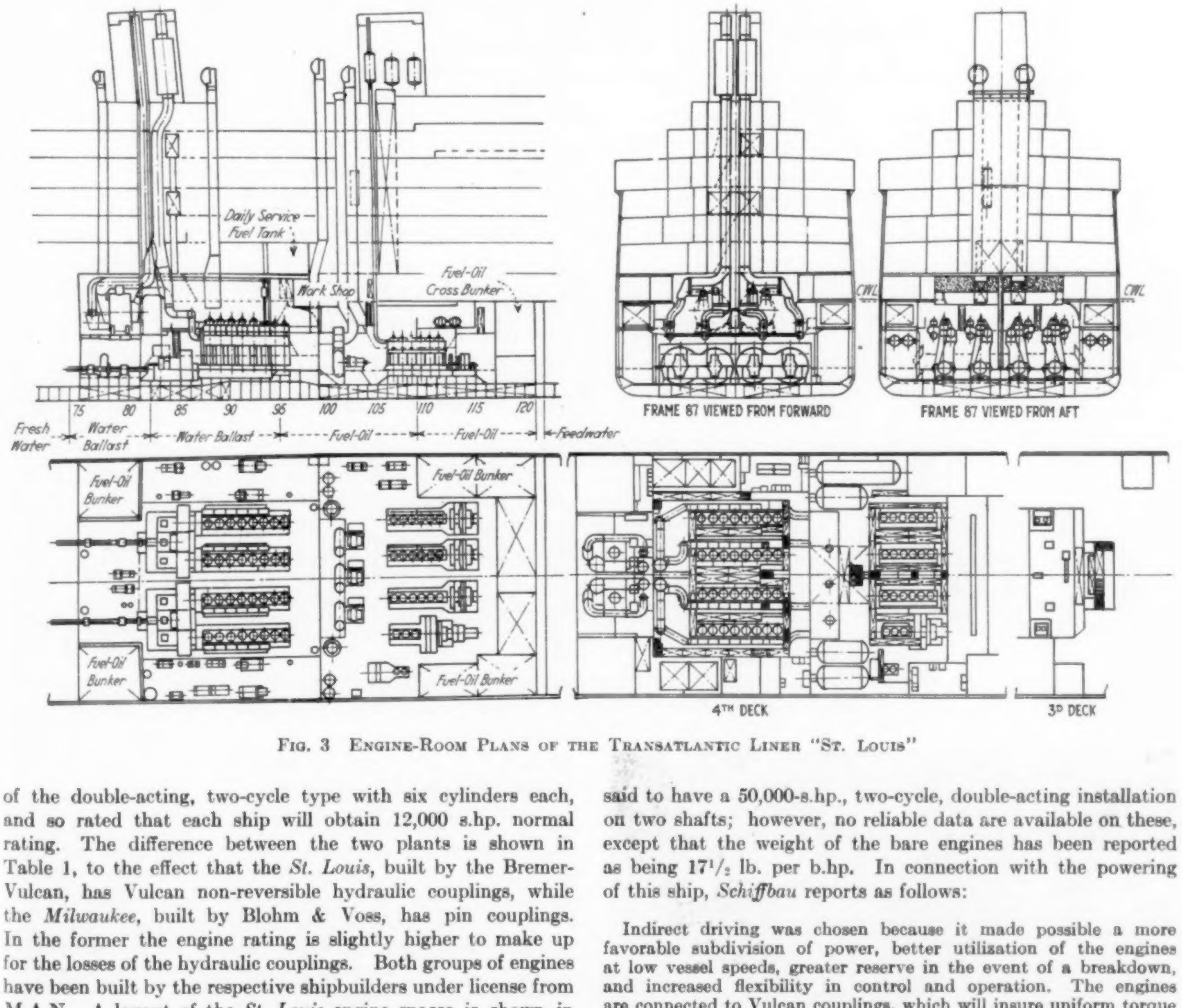


Fig. 3 Engine-Room Plans of the Transatlantic Liner "St. Louis"

Antedating the M.S. *St. Louis* and *Milwaukee* with double-acting engines is the M.S. *Wulsty Castle*, owned by the Beardmore Company and engined by the Vulcan Werke, Hamburg, with four-cycle double-acting engines of the Tosi type, and Vulcan hydraulic couplings. Reports are to the effect that this ship is functioning satisfactorily.

Another notable installation is that of the famous rotor ship *Barbara*, the power plant of which consists of four-cycle, single-acting engines coupled with hydraulic reversible gear, shown in Fig. 4.

Next in interest is the American ship *Herman Falk*, formerly *Tuxpanoil*, a very good description of which appeared in the October, 1927, issue of *Motorship* (14). Fig. 5 shows the forward end of the engines and gear. This ship has so far operated in excess of 100,000 miles, and the installation has fully justified its existence.

In addition to the merchant ships listed in the table, it is known that several naval ships have been and are being equipped with gear drives between high-speed Diesel engines and propeller shafts, among which may be mentioned the much-talked-of all-Diesel German "treaty" ship *Ersatz Preussen*, said to have a 50,000-s.hp., two-cycle, double-acting installation on two shafts; however, no reliable data are available on these, except that the weight of the bare engines has been reported as being $17^{1}/_{2}$ lb. per b.hp. In connection with the powering of this ship, *Schiffbau* reports as follows:

> Indirect driving was chosen because it made possible a more favorable subdivision of power, better utilization of the engines at low vessel speeds, greater reserve in the event of a breakdown, and increased flexibility in control and operation. The engines are connected to Vulcan couplings, which will insure uniform torque on the reduction gears and propeller shafts, besides adding the enormous advantage of enabling any one engine to be disconnected instantly without stopping the propeller shaft. The total weight of the propelling machinery, everything included, still remains less than that of a modern turbine plant of equal rating, leaving the elimination of the boilers and the saving of fuel as a clear gain to the motor vessel.

It is evident from Table 1 and the foregoing description that much history has already been accumulated on this interesting type of drive. There remains to analyze the reasons why, in

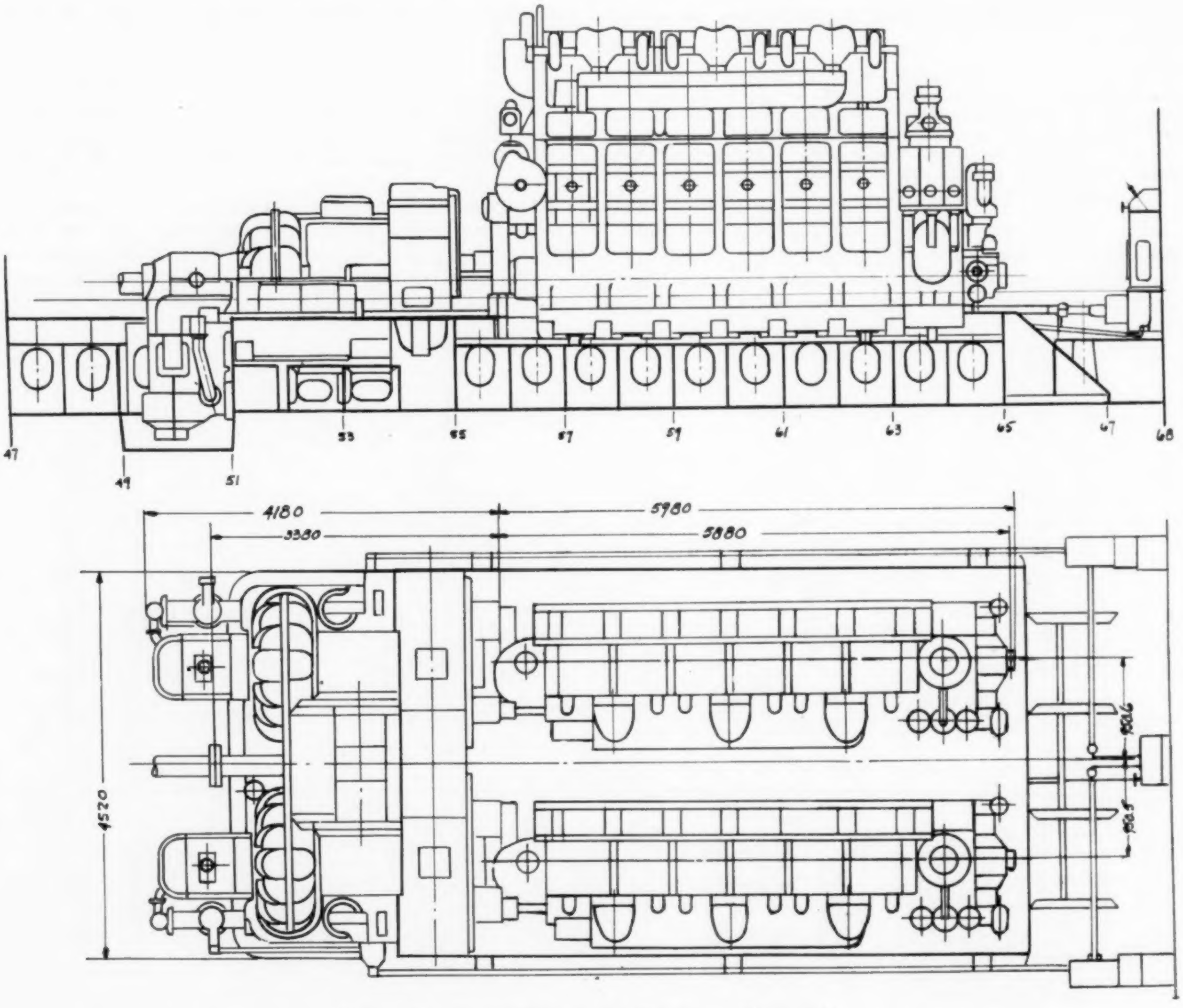


FIG. 4 PROPELLING UNIT OF THE ROTORSHIP "BARBARA"

(Two Weser-M.A.N. four-cycle, single-acting engines, trunk-piston type; bore, 14$^3/_{16}$ in.; stroke, 21 in.; r.p.m., 300; propeller r.p.m., 80; total s.hp., 1010.)

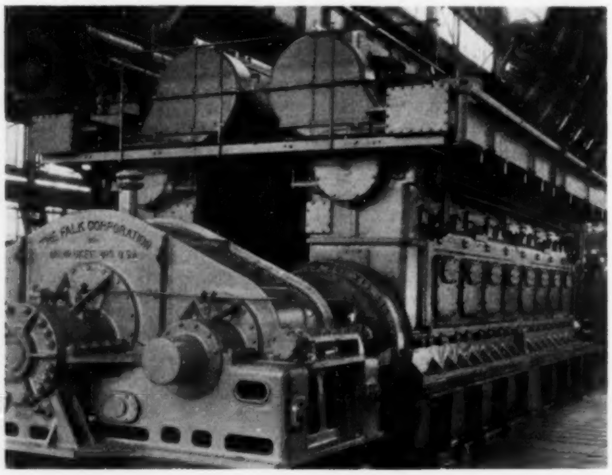


FIG. 5 FORWARD END OF ENGINES AND GEAR SET OF MOTORSHIP "HERMAN FALK"

the face of its obvious advantages, more rapid progress toward its application had not been made.

GEARS

The first and most innate prejudice is against the gears. There is a widespread misconception that the so-called pounding due to the piston impulses in a Diesel engine is detrimental to the life of the gears. It would be so if pounding existed, but since such a condition is greatly exaggerated in the minds of the critics, there does not seem to exist any real foundation for apprehension in this respect.

To begin with, gear drives of ratios approximately like those required in Diesel ship drives have been used in land industries for years, and under conditions much more severe than those encountered on board ship. To illustrate a common case, take the numerous rolling-mill installations which have been running for years without the slightest upkeep expense, as far as the gears are concerned. The owners of one such installation of about 15,000 i.hp. have stated:

> So far as we can learn, the only repairs to the bearings during the life of this engine were the replacement of some of the cushion

> springs and cross-tierods. No change was made over in the crankshaft, and the bearings ran for sixteen years without being changed. When this engine was dismantled, due to the motorizing of the mill, the gears were still in good condition. The reversing of the engine varies with the length of the bar in the rolls, but ran as high as two reversals per minute with short bars.

Any one can imagine the tremendous shocks that the gears must have withstood every time an impact was received from the ingot entering the rolls. No such condition could ever materialize in a ship unless the propeller struck solid rock.

One could keep on with this subject indefinitely, but any one with a fair knowledge of the modus operandi of various major industries can easily check up the different types of gears in use and acquire valuable knowledge.

The technique of gear manufacture has advanced considerably in the last twenty years, and especially in the last ten. The resonance troubles, due to faulty machining, experienced in the early turbine gears, are no longer present today, principally because of better workmanship. If such difficulties have been mastered with gear ratios around 1:20, then the problem becomes infinitely easier with gear ratios of 1:2 up to 1:4, such as are used in geared Diesel drives, because the stresses set up by such resonances are in the approximate order of $^1/_{100}$ of those set up in a 1:20 ratio gear set, as will be later explained.

It has been noted that many writers at home and abroad who have discussed the subject of indirect drive for motorships have considered the gear drive as unsuitable because the torque of an oil engine varies considerably and rather suddenly and the gear transmission could hardly be expected to be reliable under such conditions. The fallacy of such an assumption has been proved by many of the above-described successful installations, in some of which the torque characteristics were far from being favorable.

It is entirely up to the designer to so fix and select engine characteristics as to obtain the best possible operating conditions, the most important of which are positive torque at all times and proper lubrication. The former will be elaborated on hereafter; the latter can be easily met by very little attention to the lubricating equipment.

Comparing again turbine and Diesel gears, Mr. Banner, of the Falk Corporation, has said:

> The former operate at very high pitch-line velocities, and the number of tooth contacts per unit time is so high that shrill sounds can be generated which may become quite disturbing if resonance effect can obtain in or about the gear case. Vibrations of high, and therefore disagreeable, frequencies are also sometimes observed due to the distortion of turbine rotors.
>
> In contradistinction to those well-known and much-dreaded conditions, the oil-engine gears are operating at such low peripheral speeds and such small numbers of tooth contacts that they do not emit any noise. Balancing of pinions and gears is simple and relatively unimportant due to the low speeds. The pinions are extremely massive and do not distort.
>
> Thus inherently oil-engine units are noiseless and vibration-free, and experience with them proves this deduction to be an established fact. Such units can therefore be used with utmost advantage on passenger vessels. Since the oil engines can be made in the majority of cases vibration-free, within the running range, by design, propelling engines of this type are the given power for passenger vessels.

This is not mentioning the economic side, of which we will speak later.

The best proof of the reliability of gear-driven motorships is the fact that the Hamburg-American Line, which is in possession of most of the geared ships in operation and which therefore ought to know more about geared Diesel installations than any other concern, decided on this form of propulsion for the new transatlantic liners *St. Louis* and *Milwaukee.*

Magg (33) states:

> Direct drive means engines of extremely low speed compared to stationary engines of equal output, where, for instance, an engine of 2000 b.hp. runs at a speed of 167 r.p.m. and a 3000-b.hp. engine at 150 r.p.m. corresponding to generators of 36 and 40 poles.

There is an inherent disadvantage in direct drive where, due to propeller requirements, the maximum capability of a Diesel engine cannot be utilized; hence, *as in steam turbines, a gear transmission can even up things for the Diesel.*

Furthermore, where two engines are coupled to one propeller shaft, twin-screw safety is obtained with single-screw structural simplicity. Since modern, correctly cut gears show an efficiency of 98 per cent at least, the losses in this respect are practically negligible, while the additional expense of the gear transmission is more than offset by the reduction in engine weight and cost.

The question of vibrations must be well studied, of course, and here is where a little common-sense will steer the designer away from difficulties.

Professor Lewis (10) states:

> Just as in the steam turbine, the first condition of success may be said to lie in having good gears, accurately cut, and of sufficient size to carry the load.
>
> The second condition of the operation of a geared Diesel drive may be stated as being "the condition of positive torque." It is essential that the torque at the gear faces shall not pass through zero, otherwise the teeth will separate and pound.

The sometimes frequent and annoying troubles experienced with certain geared turbine installations, especially during and shortly after the war, have been traced back to inaccuracies in the gear-cutting process. For instance, a minute difference in pitch or in the shape of the individual teeth due either to small cyclical variations in the driving mechanism of the cutting machine or some similar disturbing factor, while of such a minute character that the resulting inaccuracies in the finished gears could hardly or not at all be found by any direct mechanical method of checking, nevertheless produced destructive effects under running conditions. It was found that force reactions of a rhythmic or pulsating nature could be produced by these small variations in the gears. If these periodically varying forces synchronized with some order of a natural frequency of the mass elastic system, which for a given installation is a fixed characteristic now easily determined in the designing stage of any machine aggregate, forces of a value many times the original ones resulted by additions of each individual vibration and caused the destruction of the gears.

While these troubles have now been eliminated in the geared turbine installations by perfection in the cutting process, wherein extremely low cutting speed and utmost rigidity are the main requirements, it will be evident from the following considerations that the geared Diesel engine is affected to a far lesser degree by possible defects in the manufacturing process. In fact, the geared Diesel drive may be considered as immune against such troubles as were experienced in the geared steam turbine.

This will be readily understood when it is considered that the forces in question will be proportional to the squares of their respective frequencies. The higher the frequency or the number of vibration cycles per unit time, the higher their accelerations and consequently their force.

A Diesel pinion running at $^1/_{10}$ the speed of a turbine-driven pinion will have acceleration forces of only $^1/_{100}$ of the latter. "Consequently," Dr. Dreves states in view of the above facts "*these exciting forces are of no practical importance in the oil-engine drive.*"

Careful and expert attention must, however, be given to the periodic torque variation inherent in all reciprocating engines.

As previously mentioned, the influence of these harmonic exciting forces emanating from the Diesel engine itself may now be evaluated for any type of installation before construction. The study of the dynamics of a mass elastic system with respect to torsional vibrations has reached a state where no excuse can be found for serious trouble of this nature, whether it may be actual breakage in the shafting system or excessive vibrations accompanied by destructive deformations in the shafting at any vital point, such as in the gear connection.

The possibility of perfect control of these conditions in the initial stages of design contrasts sharply with the uncertainty of the possible disturbing forces in the turbine gears. The latter forces being results of chance influence, preclude analysis and treatment before installation.

Finally, with regard to treatment of the two forms of gear vibrations, one characteristic of the turbine installation is more difficult, owing to the fact that the exciting forces originate in the gears themselves, whereas the characteristic pulsations in the Diesel drive have their source outside the gears, making possible various effective and easily provided means of diverting the impulses of an exciting nature from reaching the gears.

Concerning the torque variations without reference to the special state of synchronization at certain speeds, their influence on the gears is easily disposed of by fitting a flywheel having inertia according to the characteristics of the type of engine under consideration, and which will eliminate variations at the point of the gears to any desired degree.

From what has already been said it is evident that the design of a geared Diesel drive must be laid down with the exact knowledge of its characteristics with respect to critical speeds.

Practical Considerations

To quote Dr. Dreves:

> The first problem of the designer is therefore not to permit such synchronism or equality between the frequency of the exciting forces and the natural period, or if that is impossible, to place the critical speed outside of the operating range of engine speeds. It becomes necessary, therefore, to analyze the system with a given set of dimensions for cylinder bore, crankshaft, and lineshaft diameters, as well as given masses of flywheel and gear drive, for torsional vibrations, which necessitates knowledge of modes of vibration and form of excitation.
>
> It seems necessary at this time to refer to the originally projected elastic couplings of the engines of the *Monte Sarmiento* to their respective drive pinions by means of a quillshaft.

The following is by F. M. Lewis (10), p. 135, sec. 65:

> It will be found extremely difficult in many cases, while using connecting shafting of ordinary dimensions and keeping the criticals above the operating range, to satisfy the condition of positive torque. This difficulty has led to the use of the so-called "quillshaft drive." The device has been previously used for geared turbines. The pinion shaft is bored hollow and extends aft alongside the lineshaft. The reduced diameter quillshaft is fastened to the engine aft of the flywheel, passes through the hollow pinion shaft, and is connected to it by an arrangement permitting slight longitudinal motion at their after extremities. It is generally necessary to fix an extra flywheel to the pinion shaft. The effect of the arrangement is that the critical speeds are reduced far below the operating range. The engine and flywheel can respond to the torque variations by variations in the flywheel speed. The twist is taken up in the quillshaft, and the gears can run at a nearly constant speed and torque. This arrangement has met with complete success when applied to a single-engine driving gear. Examples of such a drive are the motorships *Havelland* and *Munsterland*. A quillshaft drive involving the use of step-up gears to a centrifugal pump has been recently constructed and is now in operation on the dredge *Norfolk* of the Atlantic, Gulf, and Pacific Company.
>
> The purpose of the quillshaft can be accomplished in some cases by the use of a torsionally elastic coupling. A coupling could be used in all cases were it practical to build one with sufficient torsional elasticity. As it so happens, more elasticity can be introduced by means of the quillshaft than in any other way.

In the case of *Monte Sarmiento*, however, Dr. Dreves goes on to state:

> It is characteristic of the reliability of the gear drive that during various extraordinary experiments, undertaken to broaden our knowledge of the phenomena involved and under very unfavorable circumstances, the gear teeth did not suffer the slightest damage.
>
> Fears have been also expressed for the gear drives, especially when two engines are coupled to one gear drive, in case one of the pistons seizes. Even this has been "practically tried out." Certain circumstances contributed to a seizure of one of the pistons of the *Monte Sarmiento* on its first trip. The gear drive, however, did not suffer the least. In this case the possibility of quickly disconnecting one of the engines proved to be a decided advantage.
>
> Disconnection of individual cylinders has also been repeatedly tried without the slightest effect on the gear drive.

As an example of what to expect in a well-designed drive, we may mention again the M.S. *St. Louis*, in which criticals are located as reported—one at around 45 r.p.m. on the propeller shaft and another at 170 r.p.m. on the crankshaft. The latter is exclusively confined to the primary system, comprising only the crankshaft and its masses, together with the primary parts of the hydraulic coupling. This critical has consequently no influence or effect on the performance of the gears.

It may be of interest to note that this particular critical is of the 9th order, one-noded variety, contradicting the implications of several writers on the subject of torsional vibrations that only even-numbered criticals of any noticeable magnitude are present in an installation of this type.

The critical at 45 r.p.m. in the propeller shaft receives its excitation from the four-bladed propeller. The one-noded natural frequency of this shaft system lies around 188 per min.; consequently the critical is a major one of the 4th order.

Arrangements having a more direct method of coupling engines with propeller shaft would of course have an entirely different characteristic. No such separation into two independent systems would exist. The one-noded and probably also the two-noded critical would be pushed as far down the running range as possible, while trying to keep all criticals of higher frequency above the normal speed range.

So, then, *the success of any geared Diesel drive, assuming correctly executed gears, depends chiefly upon the proper evaluation of the influence of the periodic variations in the torque from the various cylinders of the engine with regard to synchronization with one or the other of the natural frequencies of the elastic system as given by the entire engine aggregate, or in other words, with the so-called critical speeds.*

A sine qua non for a successful gear drive is the demand that at no time must the variation in torque at the point of the shafting where the gears are located reach a value of more than double the mean torque of the engine and thus at intervals produce a negative torque. This would cause periodic separations of the faces of the gear teeth, causing noisy operation and destructive pounding, with consequent rapid deterioration.

There are many and various means, as we have seen, available to conform to the foregoing conditions and insure effectivity within the operating range. It is required, however, that an exact analysis of the installation, with respect to critical speeds, be made beforehand, and the distribution of masses and shaft lengths made according to the requirements of the condition as stated. Such an analysis can be made with close approximation or even with exactness in the light of present knowledge, and a satisfactory solution may be found for almost any type of Diesel engine. In this connection, it may be noted, without going into details, that certain types of engines are inherently more advantageous in this respect than others (see Fig. 6). Thus the two-cycle, double-acting engine is most eminently suited, on account of its smooth torque characteristics, necessitating small flywheel masses, to insure uniform torque, and also because of its limited number of criticals of possible disturbing influence—the latter a valuable characteristic of this type of

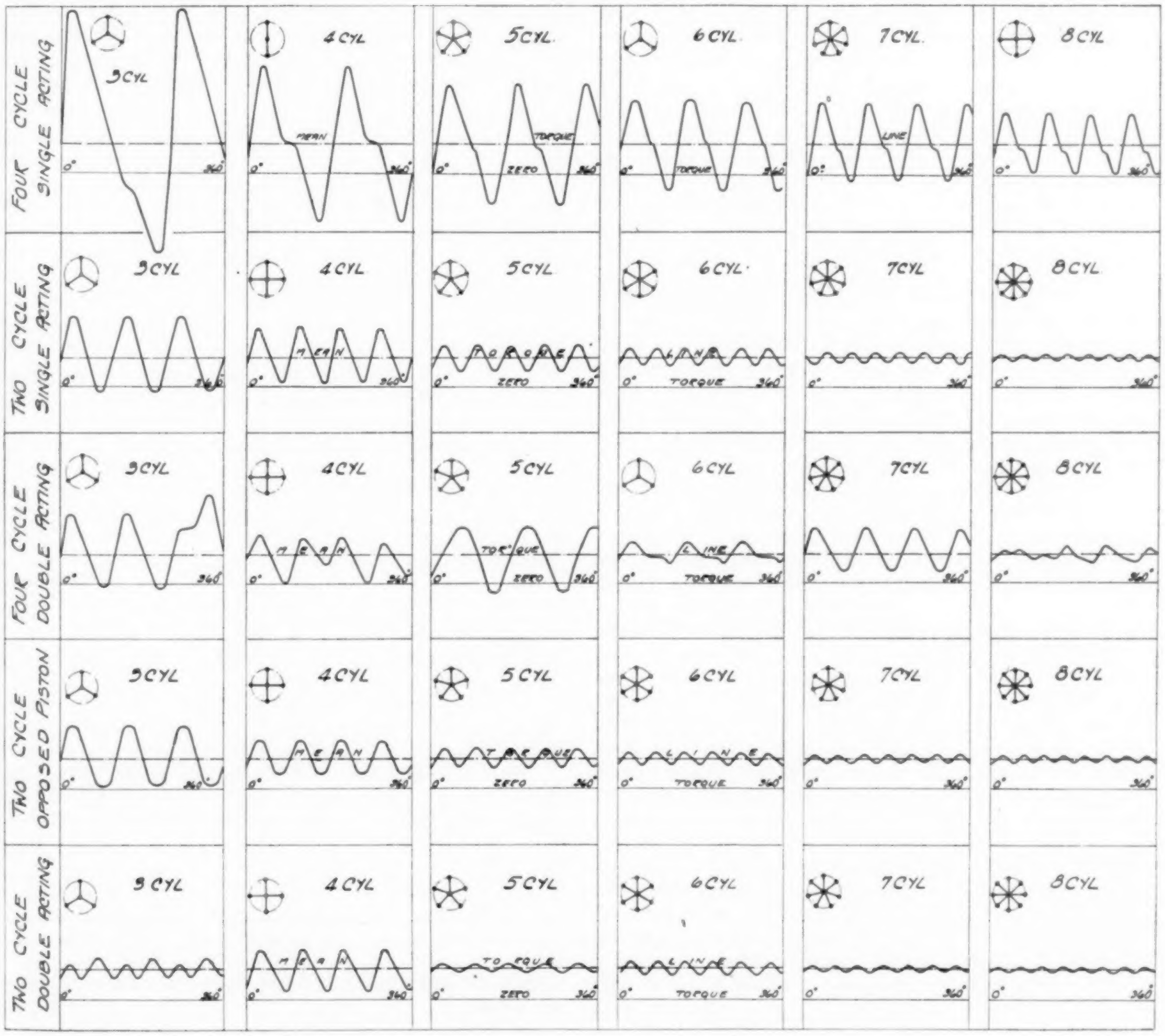


FIG. 6 TORQUE-VARIATION DIAGRAMS FOR DIESEL ENGINES OVER ONE REVOLUTION
(All engines are 100 i.hp., 100 r.p.m., and 100 lb. per sq. in. m.i.p.)

engine. Assuming approximate equal loads in top and bottom cylinders, the two-cycle, double-acting engine may have a compensating effect on many minor criticals as compared to the large number of possible disturbing points of resonance in the single-acting, four-cycle engines.

By correctly dimensioned shafts and properly arranged masses, many of the criticals of a minor order may be eliminated or reduced in such a way as to insure positive torque at the point of the gears under all circumstances. This leaves only a few more severe criticals to be dealt with. These can and usually must be put outside the operating range by a suitable arrangement of various variables.

Hydraulic transformers will eliminate the transmission of large torque variations to the gears under all circumstances, but do not act as effective damping devices in every case; at least they cannot dampen the vibrations within the engine shaft. They are, however, per se, no guarantee of successful operation, many statements in the technical literature to the contrary notwithstanding, as each case must be analyzed on its merits.

In substance, there are four ways of transmitting the power of the engines to the gears:

1 By direct rigid connection
2 By semi-rigid connection, as represented by quill-shafts and elastic couplings
3 By hydraulic or mechanical coupling
4 By damping couplings.

All of the foregoing can be supplemented by clutches or clutching features enabling the engine to be disconnected at will.

The first and second methods need no explanation, as they already have been amply discussed.

The third method includes such types of couplings as the Vulcan hydraulic, reversible and non-reversible. This coupling has the additional virtue of being capable of quick disengagement by draining the oil contained in the coupling case so that an engine can be quickly disconnected without stopping the system and can be reconnected by pumping the oil back again. This type of transmission, as we have learned, has the advantage of entirely separating the mass elastic system of the engine from that of the lineshafting, but it does not act as a vibration damper for the engine if run into the critical range except to a very small degree and under very favorable circumstances.

The fourth method includes that class of couplings which act as vibration dampers for the engine, and which also transmit power and allow disconnection when wanted. This class of coupling is necessary when it is impossible to work all major criticals out of the system. It is a comparatively new development and promises to be the most effective remedy for vibrations.

ECONOMICS

The economics of the oil-engine gear drive is not parallel to that of the turbine gear drive. In the latter high turbine speed is necessary to obtain the desired thermal efficiency, which was unobtainable when using direct connection of turbine and propeller. A compromise had to be struck before the advent of the geared turbine whereby the turbine was driven at the lowest possible speed consistent with fair commercial economy, while the resulting propeller speed was as high as compatible with the latter's efficiency. Both units were at a disadvantage. The geared turbine corrected this.

As a consequence this resulted in a smaller space being occupied by the turbine, which was somewhat offset by the space occupied by the gear, a lower weight, and consequently a lower first cost.

With the oil engine there is no necessity for increase in speed to obtain thermal efficiency. Only the questions of space, weight, and cost are involved, with certain other minor desirable features, and for these, only speeds about double those now used in direct drives are necessary. The first cost of Diesel engines is still too high for successful competition with steam installations in spite of the thermal superiority of the former. Costs must come down, and gearing offers the most desirable avenue for reduction, as not only does the higher speed bring about a greater output per unit weight, but the consequent reduction of cylinder sizes and their greater number facilitate standardization to a higher degree than with direct drive, production processes can be more readily applied, spare parts are reduced in number and bulk, and repairs are facilitated by the smaller parts which are much more easily handled.

Frahm (16) makes the comparison of Diesel drives of 8000 s.hp. at 110 r.p.m. shown in Table 2.

TABLE 2

	Cylinder			M.e.p., lb. per sq. in.	Piston speed, ft. per min.	Stroke volume, cu. ft.	Weight, lb.	Weight, lb. per s.hp.
	No.	Bore, in.	Stroke, in.					
D.A., four-cycle	8	33	59	74	1080	235	1,650,000	207
D.A., two-cycle	8	$28^3/_8$	$43^3/_8$	68	790	127	1,430,000	179
2 D.A., two-cycle geared	2x6	$22^1/_2$	$29^1/_2$	68	835	81	1,180,000	$147^1/_2$

NOTE: Two-cycle includes turbo blowers and generating sets for same. Indirect drive includes gear transmission.

The basis of comparison is a common propeller speed of 110 r.p.m. receiving the power of 8000 hp. for a single-screw ship, covering approximately the data given for the motorship *Asturias* of the Royal Mail.

Also in (4), comparative figures, referred to a single-screw ship of 2000 b.hp. and m.i.p. of 92 lb. per sq. in. for all engines, are given as follows:

(*a*) Single-acting, four-cycle, direct-drive, eight-cylinder engine, 28 in. × 95 r.p.m. (crosshead). Weight, 965,000 lb. (482 lb. per b.hp.) and cost, 418,000 r.m. ($52.25 per b.hp.).

(*b*) Single-acting, four-cycle, geared-drive, two six-cylinder engines, $21^5/_8 \times 23^5/_8$ in. × 235 r.p.m. per 70 r.p.m. (trunk piston). Weight, 480,000 lb. (240 lb. per b.hp.) and cost, 291,000 r.m. ($36.40 per b.hp.).

While these costs are not indicative of the cost in the United States, they are relative and represent a saving of 50 per cent in weight and 30 per cent in cost for the gear drive, which cannot easily be disregarded, and while they refer to single-acting, four-cycle engines, they may form a criterion of comparative values between two-cycle, double-acting engines, direct-connected and geared.

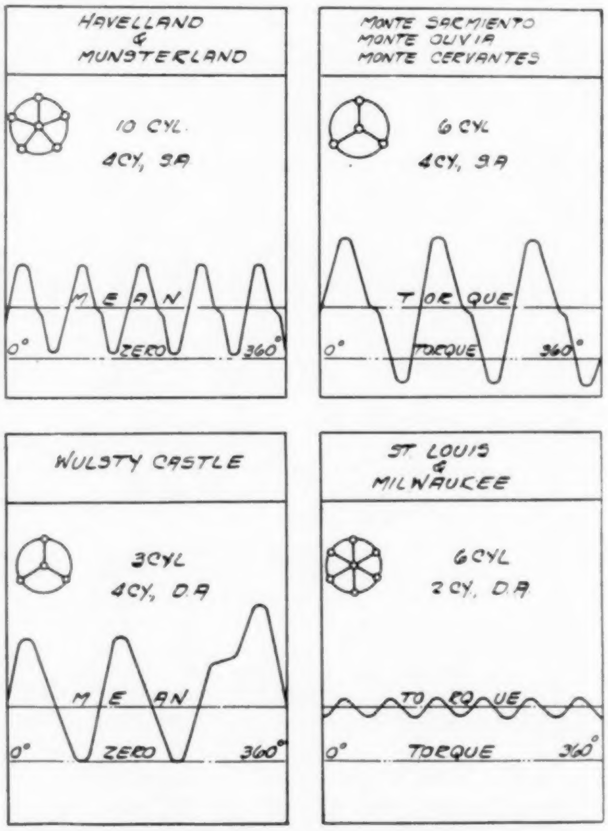


FIG. 7 COMPARATIVE TORQUE VARIATIONS OVER ONE REVOLUTION
(Based on 100 i.hp., 100 lb. per sq. in. m.i.p., 100 r.p.m. for all engines.)

CHOICE OF ENGINE

We have already seen that the most important requisite of a successful gear drive is uniformity of torque and positive character at all times. Fig. 6 shows that multi-cylinder engines, preferably of six cylinders or more, will give the desired uniformity, but above all the two-cycle, double-acting engine is the type which gives the greatest amount of uniformity with the minimum number of cylinders, and which therefore gives the least amount of fluctuation between maximum and mean torque, which in turn is the greatest safeguard against the ordinary or accidental occurrence of negative torque.

In order to bring this more in evidence, Fig. 7 has been prepared showing the torque conditions of some of the typical Diesel-geared ships in service. The *Havelland* and *Munsterland* seem almost to fulfil the condition of positive torque; the *Monte* class of ships does not. They show that theoretically there exists a small amount of negative torque; this was probably corrected by proper flywheel masses, otherwise the ships could not have performed satisfactorily. The *Wulsty Castle* shows that in spite of double action her engines are just about inside the danger line, while the *St. Louis* and *Milwaukee* show their superior characteristics by the minimum amount of torque fluctuation inherent in this class of engines.

Thus we can say that the two-cycle, double-acting engine is the logical choice for adaptation to geared drives for large ships.

But there is another quality which favors this type of engine above all others.

We must also consider the fact that the two-cycle, double-acting engine represents the logical evolution of the ideal prime mover on the basis of various practical reasons. As it is known, the simplest and cheapest type of Diesel engine is the one where a trunk piston also serves as a crosshead and depends merely upon air circulation for cooling. In modern four-cycle practice this method is used in cylinder sizes up to 16 in. and even larger, where speed and load conditions are favorable; with two-cycle engines, only the smallest cylinder sizes permit such an arrangement. In order to prevent seizure and cracking of pistons, and to always insure efficient lubrication in continuous operation under heavy loads (such as in marine practice), larger pistons require artificial cooling by oil or water circulation. The conveying system for this cooling liquid (usually telescopic pipes), while in itself comparatively simple, introduces mechanical complications on account of the ever-present danger of excessive misalignment due to the inevitable wear and deformations of trunk pistons.

On account of this, the crosshead arrangement, which virtually eliminates these complications, is more and more coming into favor in those sizes that require artificial piston cooling; this is particularly true in two-cycle practice where engines of 15 in. cylinder bore and even smaller are now more often built in the crosshead form. Experience has shown that the higher all-around reliability of such engines more than offsets the higher first cost.

With the crosshead type of single-acting engines, a form of construction is obtained which in weight and space requirements will not be appreciably exceeded by a double-acting engine of the same bore and stroke. Identically the same crosshead and guides will of course serve either type; and since it is the upper cylinder that exerts the maximum load upon the structure, the entire engine framing can also be the same. The upper cylinder end, cylinder head, valves, and valve gear will also be the same for either type, their operating conditions being in nowise affected by whatever use is made of the lower cylinder end. Likewise, in two-cycle engines the same exhaust and scavenging ports, and the control means for the latter, can be made to serve the upper as well as the lower cylinder end, so that in this respect also there need be no difference between either type.

The only outstanding feature of distinction in the double-acting engine is the lower cylinder end, with the necessary valves, the water-cooled cylinder head, and the stuffing box for the piston rod. This adds approximately 15 per cent to the height of the double-acting engine, with a corresponding increase in cost of from 10 to 12 per cent. Against this, however, we have a power output from 80 to 90 per cent greater than that of the single-acting engine, so that, reduced to a horsepower basis, both the weight and the cost of the double-acting type are at least 40 per cent lower than those of the former.

Analyzing the matter from the basis of a given capacity, it follows that, for the same number of cylinders and piston speed, the area of the double-acting cylinder reduces to about 0.55 that of the single-acting cylinder (inverse ratio of double-acting to single-acting cylinder capacity of fixed bore and stroke). Increasing this figure by 10 to 15 per cent to make allowance for the addition of the lower cylinder head, etc., gives a fair approximation of the relative weight and cost of a double-acting engine as compared with a single-acting one of equal capacity.

A smaller cylinder or a piston of shorter stroke can reciprocate a greater number of times than one of longer stroke with the same piston speed; hence the number of revolutions of the former will be greater for the same power. But the two-cycle engine has already a smaller cylinder than the four-cycle engine for the same output, so that the two-cycle, double-acting engine has the smallest cylinder of any type of engine for equal outputs. The most efficient propeller speed can now be combined with the most convenient engine speed. Conservative piston speeds compatible with sound marine practice can be adopted, thus obtaining a safe combination of engine and gear which will not be open to criticism on account of extreme conditions.

In actual practice the cylinder bore of the double-acting engine would not be arrived at by the foregoing method of reasoning. Other considerations affecting the bore-stroke ratio would enter into the problem, but the underlying idea, as given, remains; so that on the strength of careful analysis it can be stated that *the cost of a double-acting engine will always be lower in units of such capacity which in the single-acting form require to be built in the crosshead arrangement.*

Since we are only concerned here with large installations, we need not elaborate on the lower limit of size for a double-acting engine, but for the sake of defining the field, it might be stated that if a lower combustion efficiency for the lower cylinder is accepted by using one fuel valve, a cylinder as small as 12 in. bore can be used. Better combustion will be obtained with two fuel valves, and for such an arrangement more than 12 in. bore is necessary if air injection is used. From this it might be inferred that the benefits to be derived from the application of geared drives are only confined to such power units as may be obtained from the use of double-acting cylinders of 12 in. bore or more giving approximately 250 i.hp. per cylinder. Such is not the case, as where smaller powers are needed and the benefits of double action cannot be secured, the lower cost of manufacture obtainable with such engines must be secured in other ways; and this brings us back to the foregoing paragraph in which it is stated that the simplest and cheapest type of Diesel engine is the one where a trunk piston also serves as a crosshead, and depends merely upon air circulation for cooling.

In addition to the cheapness of trunk-piston construction, additional reduction in weight and cost can be obtained by the use of the highest possible rotative speed, and this can be obtained to a higher degree with four-cycle engines; hence, below the powers served by two-cycle double-acting engines, the four-cycle, single-acting, high-speed type of engine is the logical choice, except that there is an overlapping zone between 1000 and 2000 b.hp. in which trunk-piston engines with cooled pistons will very conveniently fit. The choice between the two lies primarily in space considerations.

This does not preclude the use of other types of engines so long as torque conditions are satisfied, as there may occur cases where special manufacturing advantages enjoyed by some builders may make the use of their particular type more desirable. Among these may be mentioned the two-cycle, opposed-piston type of engine, the torque characteristics of which are nearly equal to those of the double-acting two-cycle type, and which lend themselves admirably to the scheme, assuming that weight and cost conditions are met.

As to the upper limit, considering what follows, there does not seem to be any immediate need for double-acting cylinders suitable for gear drives above 24 in., or possibly a little larger. Since cylinders suitable for direct drive have been built up to 33 in. bore, we need not worry about this phase at the present moment.

TABLE 3 PROPOSED TYPICAL PLANTS. WITH GEARED TWO-CYCLE, DOUBLE-ACTING ENGINES, ATTACHED COMPRESSORS, AND SEPARATE SCAVENGE BLOWERS; GEAR EFFICIENCY, 98 PER CENT

	Item	Single-screw		Single-screw		Twin-screw		Twin-screw	
1	S.hp., total	5000		10,000		25,000		50,000	
2	S.hp., per shaft or gear	5000		10,000		12,500		25,000	
3	Number of engines per gear	2		2		2		4	
4	Total number of engines	2		2		4		8	
5	Number of cylinders per engine	6		6		7		7	
6	Total number of cylinders	12		12		28		56	
8	Bore and stroke, in.	18 x 26		24 x 32		24 x 33		24 x 33	
9	R.p.m. of engines	200		180		175		175	
10	Piston speed, ft. per min.	867		960		963		963	
11	M.i.p., lb. per sq. in.	81		85		86		86	
12	I.hp. per engine	3075		6,145		7,690		7,690	
13	Mechanical efficiency, per cent	83		83		83		83	
14	B.hp., per engine	2550		5,100		6,380		6,380	
		Weight, lb.	Cost, dollars	Weight, lb.	Cost, dollars	Weight, lb.	Cost, dollars	Weight, lb.	Cost, dollars
	Total weight and cost of engines	648,000	254,625	1,128,000	428,640	2,632,000	973,840	5,264,000	1,947,680
	Per b.hp.	127.0	39.92	110.6	42.02	103.0	38.15	103.0	38.15
	Cost per pound	.....	0.39	......	0.38	......	0.37	......	0.37
	Reduction gear without thrust	95,000	24,000	120,000	30,000	300,000	75,000	540,000	120,000
	Thrust bearings	11,300	4,110	20,000	8,000	22,000	8,500	40,000	15,000
	Blowers	26,000	17,000	49,000	33,500	126,000	72,000	252,000	144,000
	Generating sets for blowers	57,000	28,000	108,500	53,500	240,000	129,000	480,000	258,000
	Total weight and cost of plant	837,300	327,732	1,425,500	553,640	3,420,000	1,258,340	6,576,000	2,484,680
	Per s.hp.	167.4	65.55	142.5	55.36	136.8	50.35	131.4	49.70
	Airless injection	150.9	58.07	128.1	49.90	123.4	45.39	118.0	44.74
	S.hp.	5,350	.....	11,300	.....	28,250	......	56,500	......

AIR VERSUS AIRLESS INJECTION

The question of air versus airless injection is still the subject of much discussion or controversy in spite of the fact that the latter has made very rapid strides during the past few years and especially during the last two. Airless-injection engines have been growing in size very rapidly, so much so that one prominent builder, the M.A.N. company, is now ready to fulfil contracts for such engines for any size previously built in the air-injection type.

The alleged lesser perfection in atomization attributed to the airless-injection method and the limitation in fuel grades which can be utilized have had to be overlooked in favor of the greater simplicity, lesser weight, and cost of the engines built of this type. It must be admitted, however, that recent results of large airless-injection engines have shown fuel-consumption figures fully equal and in some cases superior to those of similar types of air-injection engines, which indicate efficient atomization and combustion, provided the proper grade of fuel is used.

Not wishing to enter the controversy on this subject, but fully realizing that the demands of the ship owners for simplicity and low cost will eventually force the air-injection engine to retirement, the author has computed the hypothetical plants which follow for air-injection engines, but showing the improvement to be expected from airless injection in output, reduction of weight, and cost wherever convenient.

As a general rule, by omitting the air compressor, a gain of about 7 per cent in effective output is obtained, with a reduction in weight of about the same amount. This results in a reduction of weight and cost per brake horsepower of about 13 per cent. Such a gain is certainly worth while even if the fuel consumption does not improve.

CHARACTERISTICS OF ENGINES

In order to arrive at satisfactory basic characteristics preliminary to a computation of weights and costs which will not be open to criticism, let us assume that the mean indicated pressure of a two-cycle, double-acting engine should be maintained at not more than 85 lb. per sq. in., at least until wider experience with this class of engines warrants going higher.

This coincides with present practice by the foremost builder of this class of engines at home and abroad. In the matter of piston speed, since this is a question involving only rigidity of structure and proper lubrication, the author will limit it to the highest so far obtained in high-grade single-acting engines, i.e., not over 1000 ft. per min.; say, 950 ft. per min. as a limit.

A check-up of the latest accomplishments in the matter of weight reduction of two-cycle, double-acting engines discloses the fact that a suitable structure capable of withstanding ordinary and overload stresses of this class of engine can now be built within 165 lb. per b.hp. for conservatively rated direct-drive engines of the air-injection type down to 15 lb. per b.hp. for highly rated naval types of the airless-injection type, with the value of 100 to 125 lb. per b.hp. as a good average for commercial types of engines suitable for gear drives.

The engines of the motorship *St. Louis* have been reported (20) as weighing 110 lb. per b.hp. without the blowers and 115 lb. per b.hp. with the blowers. The gears are reported as weighing 40.5 lb. per s.hp., so that the total weight of the propelling units would be 155.5 lb. per s.hp. If a plain reduction gear were used, fully 25 lb. per s.hp. could be deducted, resulting in a weight of 130.5 lb. per s.hp. and 123 lb. per s.hp. if the compressor is omitted.

The M.A.N. company offers engines suitable for direct drive, of 2000 b.hp. at 125 r.p.m., weighing 113 lb. per b.hp. (19).

On the foregoing basis, careful analyses have been made of designs which can be carried out in commercial practice, embodying all the requirements of structural strength and reliability. The designs analyzed have been correlated to existing designs as carried out by the leading builders and have been found to be in keeping with their accepted limits of strength and reliability.

The weights have been calculated from completed shop working drawings. A greater degree of refinement in execution has been allowed than in single-acting engines, avoiding, however, the use of special materials.

Costs have been calculated on the basis of these weights, allowing necessary machining wastage, but with no allowance for recovery of scrap. The man-pound-hour rate of labor has been fixed from an average drawn from a large number of factors as obtained over a period of 20 years and from a large number of commercial shops. Overhead has been fixed at 100 per cent over labor costs. Profit and contingencies have been figured in the usual way to meet present financial conditions and competition, with the understanding that the costs of drawings and patterns should be absorbed by at least ten engines.

On this basis, costs and other characteristics of propelling units of 5000-b.hp. and 10,000-b.hp. single-screw and 25,000-b.hp. and 50,000-b.hp. twin-screw have been worked out by way of examples. See Figs. 8–11 and Table 3.

It will be noted from Table 3 that the cost per pound is 20 to 25 per cent higher than that of high-grade single-acting plants, which should be sufficient guarantee of its reasonableness.

This has been checked with the costs of many existing plants of equal characteristics as well as with all bona-fide commercial quotations which have come to the author's attention, including those submitted to the Shipping Board, and except in cases where bids showed unmistakable evidence of having been fixed to the utmost that the traffic would bear, has been found to be substantially in accord with present-day practice, allowing of course for the infancy of the art.

TABLE 4

	Single-screw	Single-screw	Twin-screw	Twin-screw
S.hp.	5,000	10,000	25,000	50,000
Total weight, prime mover only, tons	374	636	1,530	2,937
Lb. per s.hp.	167.4	142.5	136.8	131.4
Airless injection	150.9	128.1	123.4	118.0
Total cost, dollars	327,732	553,640	1,258,340	2,484,680
Dollars per s.hp.	65.55	55.36	50.35	49.70
Airless injection	58.07	49.90	45.39	44.74
S.hp.	5350	11,300	28,250	56,500

In Table 3 the weights and costs of reduction gears, blower sets, and generating sets are those obtained in bona-fide quotations.

Let us now see how these data work out when concrete cases are analyzed. To begin with, Table 3 has been condensed, as shown in Table 4, to contain the same items in the same form as Table 5 for direct comparison.

Table 5 deals with two typical steam plants as described by J. Johnson in (28). The Diesel plants directly comparable to the steam plants have been evolved from Table 4 by using the 10,000-s.hp. plant factors for the 18,000-s.hp. plant and the 25,000-s.hp. plant factors for the 30,000-s.hp. plant, which is obviously a handicap against the Diesel.

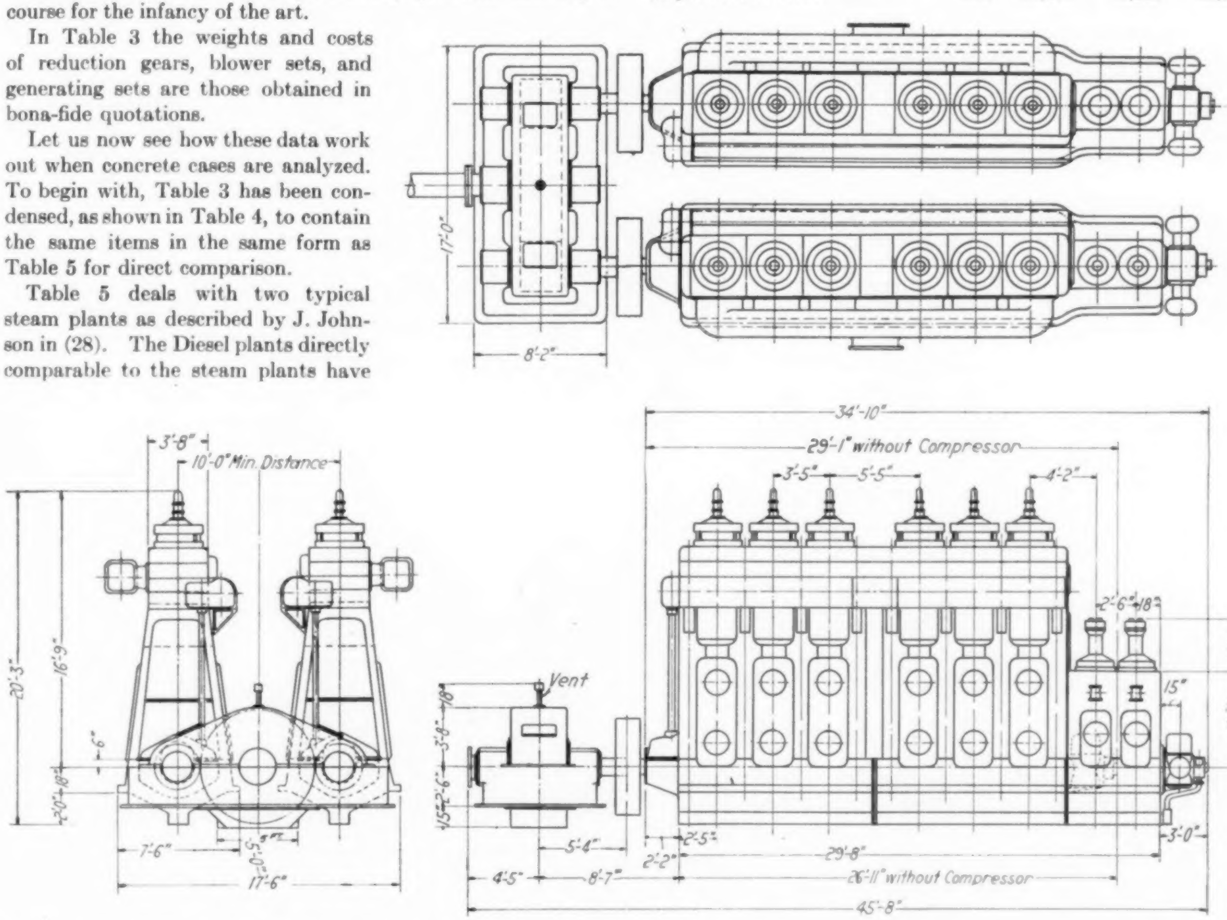


Fig. 8 Propelling Unit for Two-Cycle, Double-Acting Oil Engines With Reduction Gear, 5000 S.hp., for Single-Screw Installation

Item	Weight, lb.	Cost, dollars
Two 2550-b.hp., 6-cylinder engines, 18 × 26 in., 200 r.p.m., with compressor only	648,000	254,625
One reduction gear with thrust bearing, ratio 2 to 1	106,300	28,110
Two blowers 11,000 cu. ft. each, with dual motors	26,000	17,000
Three 110-kw. generating sets	57,000	28,000
Total power plant, prime movers only	837,300	327,735
Per s.hp.	167.4	65.55

While Mr. Johnson does not positively state the make-up of the weights quoted, it is to be inferred that they include turbines, boilers, and direct auxiliaries.

Again in Table 6, 10,000-s.hp. plants of single- and double-geared turbine sand a geared Diesel are compared, using the data for steam plants as given by *Marine Age* (28). Greater improvement could be shown with airless-injection engines.

The differential shown over the steam plants available for the balance of the machinery weights, while not definitely indicating an advantage in favor of the Diesel plant, certainly does prove that the argument is debatable and that an intelligent choice of auxiliaries may definitely swing the balance to the Diesel's favor.

In connection with the comparison between vessels propelled by geared turbine machinery and those with geared Diesel engines, it is interesting to note what Dr. Frahm (4) has to say

TABLE 5

	18,000 S.hp.		30,000 S.hp.	
	Geared turbine	Geared Diesel	Geared turbine	Geared Diesel
Total weight, prime mover only, tons	2275	1145	3410	1836
Airless injection	....	1065	....	1708
Lb. per s.hp.	283.0	142.5	254.5	136.8
Total cost, dollars	1,244,160	997,000	1,895,400	1,510,000
Airless injection	....	928,000	....	1,405,000
Dollars per s.hp.	69.12	55.36	63.18	50.35
Saving in favor of geared Diesel	....	1130 tons $247,160	....	1574 tons $385,400
Airless injection	....	1210 tons $316,160 s.hp. 19,450	....	1702 tons $490,400 s.hp. 32,400

in regard to certain analyses made by him on bona-fide projects:

> For the purpose of comparing steam and motor propulsion for large liners, two vessels are taken, namely, the Hamburg-Amerika Line's *Hamburg*, of the *Albert Ballin* class, and the Hamburg-South Amerika liner *Cap Polonio*. Dealing first with the *Hamburg*, this is a vessel of 21,000 gross tons, equipped with turbine machinery of 13,000 s.hp., giving a speed of $16^1/_4$ knots.

Fig. 12 shows the steam machinery, Fig. 13 the same vessel with double-acting direct-drive two-stroke machinery, and Fig. 14 with double-acting two-stroke engines and gearing.

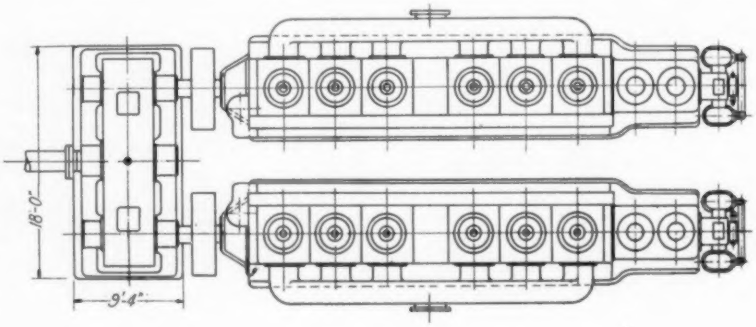
The respective weights for the different classes of machinery are indicated in Table 7.

Continuing, Dr. Frahm says:

> So far as fuel consumption is concerned, that of the steam plant is 0.385 kg. of oil per b.hp-hr. for the main turbines and 0.185 kg. per b.hp-hr. for the Diesel-driven dynamos. For the motorship the overall consumption is 0.185 kg. per b.hp-hr. The daily fuel consumption of the steamer is estimated at 122.5 tons and that of the motorship as 70 tons, it being understood that some of the heat in the exhaust gases from the propelling engines is utilized for raising steam for various purposes on the ship.
>
> The second comparison is between steam and motorship of the *Cap Polonio* class, equipped with machinery of 24,000 s.hp. [Fig. 15 shows the turbine drive as fitted to this type of vessel with two groups of four water-tube boilers, four Diesel generating sets, and geared turbine drive. The weight comparison works out as shown in Table 8, taking double-acting two-stroke machinery, both direct and with gearing.]

TABLE 6

	Duchess type single-gear turbine	*Dixie* type double-gear turbine	Single-gear 2C-DA Diesel
S.hp.	10,000	10,000	10,000
Total of prime movers only, tons	....	....	747
Total of all machinery, tons	1,181	1,076	.....
Lb. per s.hp	264.6	241.0	167.4
Total cost, dollars	1,066,000	980,000	655,465
Dollars per s.hp	106.60	98.00	65.5
Available for balance of machinery weights			329–434 tons $32.45–41.05 per s.hp.



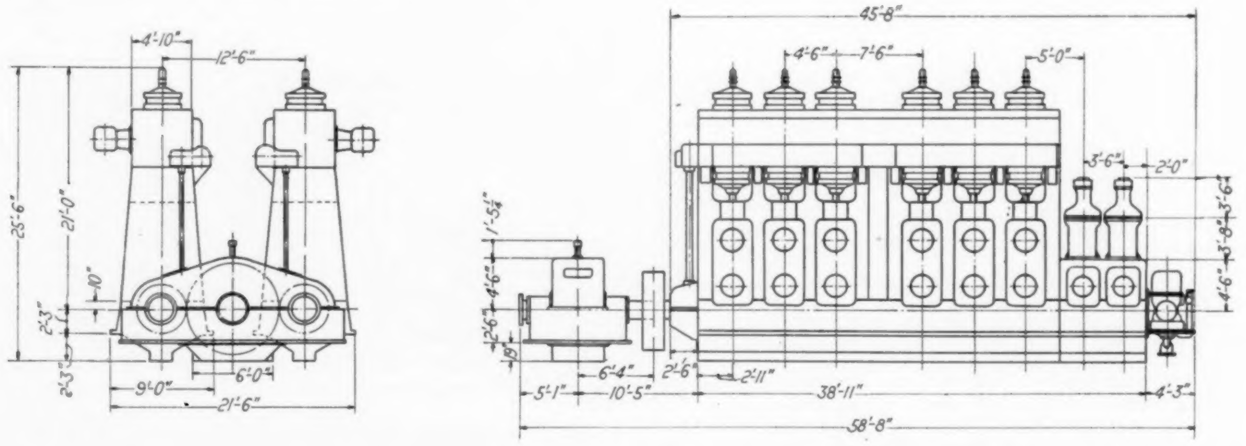


Fig. 9 Propelling Unit for Two-Cycle, Double-Acting Oil Engines With Reduction Gear, 10,000 S.hp., for Single-Screw Installation

Item	Weight, lb.	Cost, dollars
Two 5100-b.hp., 6-cylinder engines, 24 × 32 in., 180 r.p.m., with compressor only	1,128,000	428,640
One reduction gear with thrust bearing, ratio 1.8 to 1	140,000	38,000
Two blowers 21,000 cu. ft. each, with dual motors	49,000	33,500
Three 210-kw. generating sets	108,500	53,500
Total power plant, prime movers only	1,425,500	553,640
Per s.hp.	142.5	55.36

> [The general arrangements for motorships of this class with geared and direct drive are seen in Figs. 16 and 17, respectively.] The motor drive shows itself to be distinctly more favorable, as indeed is the case in regard to machinery weight. In particular, three funnels are required on the *Polonio* vessels as steamers, whereas two would be sufficient with motor drive. The respective fuel consumptions of the vessels as steamers and motorships are estimated at 211 tons and 121 tons daily. This is based on a guaranteed fuel consumption of 0.78 lb. of oil per s.hp-hr.
>
> With both these vessels, *taking into account interest on capital, repairs, insurance, depreciation, and running costs*, it is found that the motorship would show to advantage, although it was ultimately decided to order steamers, because the machinery would be similar to that in vessels already in the owner's service, and therefore familiar to the personnel; also on account of the quiet running of previous turbine plants, the wish to learn of lengthy experience with oil engines of large power before adopting them; further, because of the possibility of reverting to oil firing and the shorter time of delivery of the steamers.

TABLE 7 MACHINERY WEIGHTS FOR TOTAL OUTPUT OF 13,000 S.HP.

	Turbine drive		Oil engines	
	Cylindrical boilers, tons	Water-tube boilers, tons	D-A, T-S direct, tons	D-A, T-S geared, tons
Total machinery weight, with electrical auxiliaries	2730	2290	3000	2760
Half bunker weights for the trip New York-Hamburg-New York	1300	1300	750	750
Reserve bunker	240	240	140	140
Average voyage weight for machinery and fuel	4270	3830	3890	3650

TABLE 8 WEIGHT OF 24,000-S.HP. MACHINERY

	Turbine drive		Oil engines	
	Cylindrial boilers, tons	Water-tube boilers, tons	D-A,TS direct, tons	D-A, TS geared, tons
Total machinery weight with electrical auxiliaries..	4200	3530	4560	4200
Half bunker weight for the round trip Lisbon-Buenos Aires-Lisbon............	2580	2580	1450	1450
Reserve bunkers...........	420	420	240	240
Average voyage weight of machinery and fuel......	7200	6530	6250	5890

From an independent check of the foregoing weights it results that the engines in the geared plants have been figured at about 150 lb. per b.hp. At the time that the Frahm analysis was made, the latest accomplishments in light double-acting engines had not been either obtained or made public.

The answer to the objections raised at that time as standing against the adoption of motorships is furnished by the motorships *St. Louis* and *Milwaukee* built for the Hamburg-American Line since then.

Let us now take some ship with which we are acquainted and see what would happen if geared, two-cycle, double-acting Diesel plants were substituted for the existing ones.

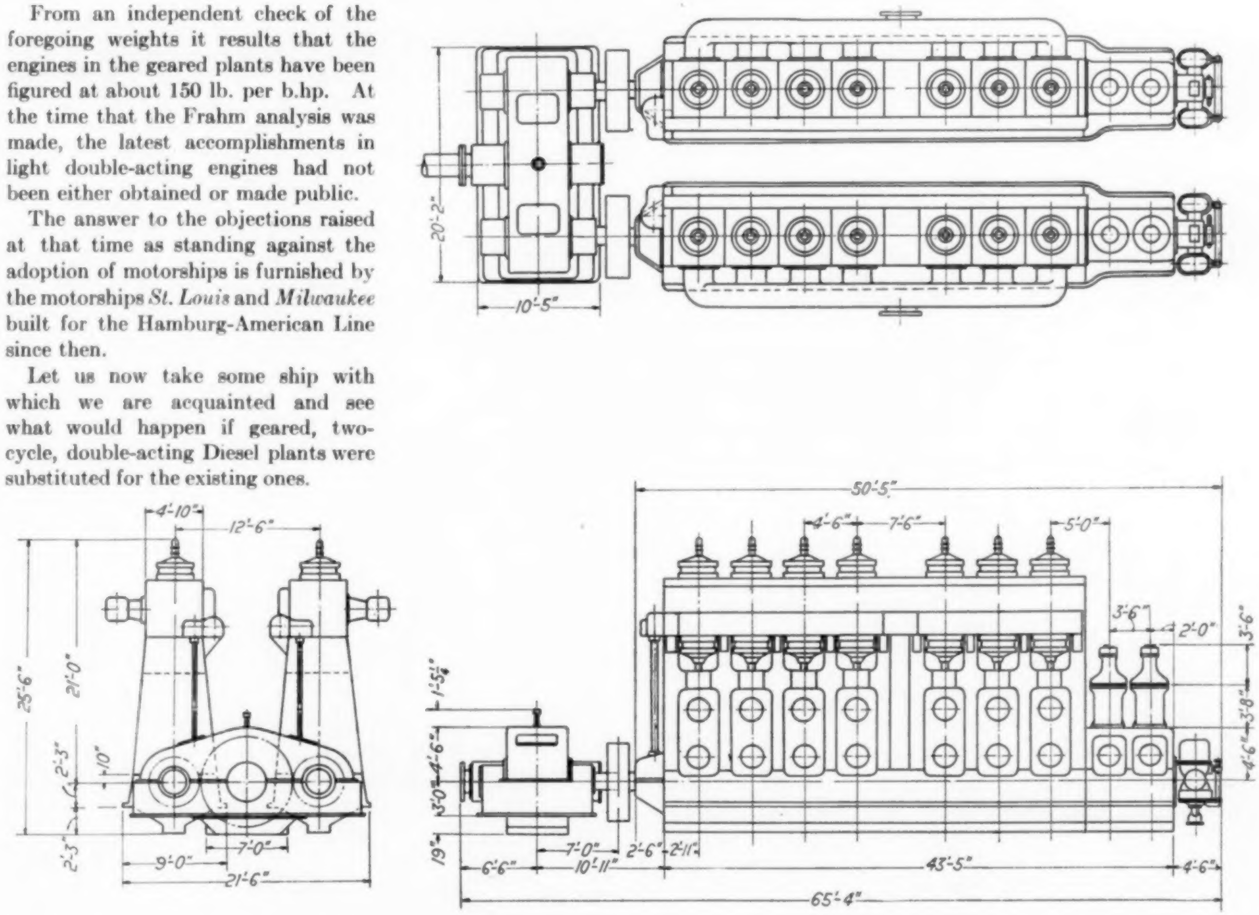


FIG. 10 PROPELLING UNIT FOR TWO-CYCLE, DOUBLE-ACTING OIL ENGINES WITH REDUCTION GEAR, 25,000 S.HP., FOR TWIN-SCREW INSTALLATION

Item	Weight, lb.	Cost, dollars
Four 6380-b.hp., 7-cylinder engines, 21 × 33 in., 175 r.p.m., with compressor only..................	2,632,000	973,840
Two reduction gears with thrust bearing, ratio 1.75 to 1..................................	322,000	83,500
Three blowers 60,000 cu. ft. each (two service, one standby).....................................	126,000	72,000
Three 500-kw. generating sets....................	240,000	129,000
Total power plant, prime movers only...........	3,420,000	1,258,340
Per s.hp.......................................	136.8	50.35

One of the much-argued-about vessel classes is the *Triumph* class, now equipped with Diesel-electric drive (see 35). Fig. 18 shows a layout of this vessel as it is at present and with a proposed geared plant of 5000 b.hp. The difference in space, weight, and cost is so striking that no amount of comment can bring it out any more clearly.

Another much-discussed conversion was the proposed one for the S.S. *Mt. Vernon* and *Monticello*. Fig. 19 shows the layout of these ships with geared Diesels, using the typical plant illustrated in Fig. 10 and adding two cylinders per engine to obtain the 28,500 s.hp. desired. Without arguing the matter of cost, it is seen that the weight is considerably less than that of an equivalent steam plant, and at least half of the length of the present engine and boiler-room spaces is saved. Greater savings will be obtained with airless-injection engines.

One must not lose sight of the fact, in comparing these hypothetical geared plants with their equivalent steam competitors, that in order to do so fairly, equivalent conservative characteristics must be used for the steam plants; in other words, as against 85 lb. per sq. in. m.i.p., which is at least 10 per cent under what could be done in cases not too extreme, an equivalent conservative pressure and superheat for steam must be used; and as against 850 to 950 ft. per min. piston speed, equivalent conservative turbine speed must be used. It would also be in the interest of fairness to arrive at a common understanding as to whether the high-pressure, high-superheat steam plants now contemplated or very recently put in operation are to be considered as conservative plants or not.

Turning to the *Mauretania*, the plan of Fig. 20 shows a triple-screw layout composed of three units, or a total of 75,000 s.hp., as illustrated in Fig. 14. Note the reduction in machinery-space length from 488 ft. 4 in., and that this is all accomplished with engines of only 24 in. bore and 33 in. stroke. It must be noted also that this plant represents a potentiality of at least 107,000 s.hp., because with very little increase in length the engines can be made with ten cylinders instead of seven, thus increasing the shaft horsepower in proportion. The possibilities of such an arrangement for a high-speed large liner are obvious.

For instance, if four shafts are used and engines of ten cylinders each, the potential shaft horsepower of such an arrangement would be 142,500, and this power could be accommodated in a ship of the *Bremen* size, with the added advantage that the spaces for public rooms and cabins, especially above the machinery space and in place of the boiler casings, could be rearranged and increased, thus enabling it to carry more passengers, and therefore bring in more revenue. In addition, the fuel to be carried, assuming a five-day load, would be decreased from about 4200 tons to 2400 tons one way. This is not an indifferent reduction; and if airless-injection engines are used, 153,500 s.hp. could be obtained in the same space.

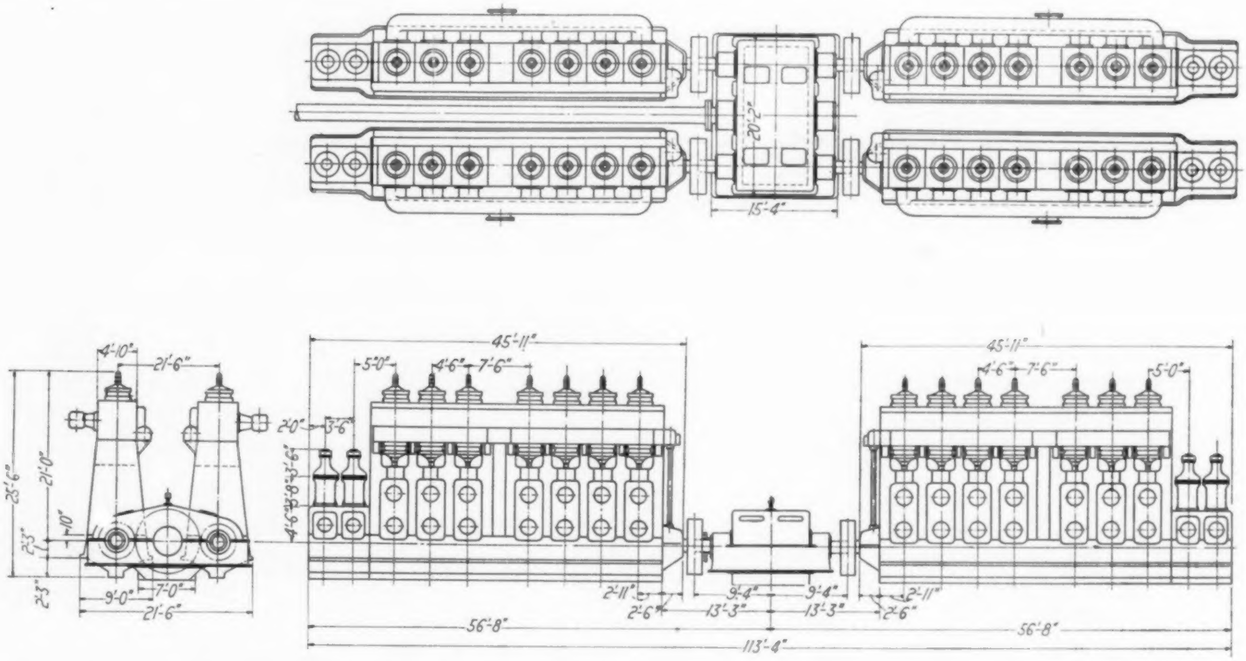


Fig. 11 Propelling Unit for Two-Cycle, Double-Acting Oil Engines With Reduction Gear, 50,000 S.hp., for Twin-Screw Installation

Item	Weight, lb.	Cost, dollars
Eight 6380-b.hp., 7-cylinder engines, 24 × 33 in., 175 r.p.m., with compressor only	5,264,000	1,947,680
Two reduction gears with thrust bearing, ratio 1.75 to 1	580,000	135,000
Six blowers 60,000 cu. ft. each (four service, two standby)	252,000	144,000
Six 500-kw. generating sets	480,000	258,000
Total power plant, prime movers only	6,576,000	2,484,680
Per s.hp.	131.4	49.70

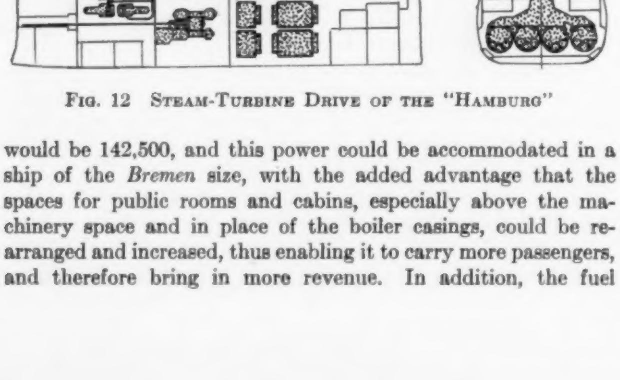


Fig. 12 Steam-Turbine Drive of the "Hamburg"

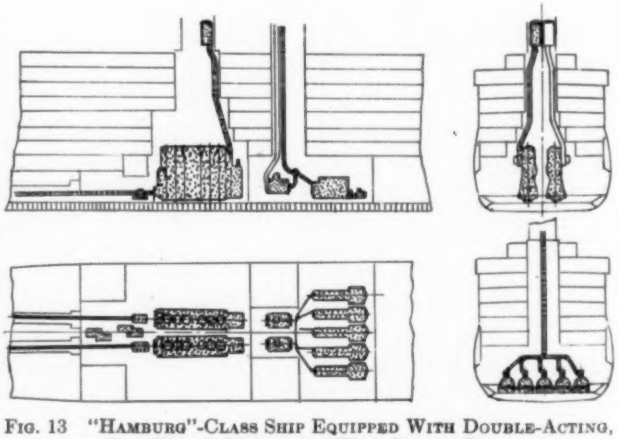


Fig. 13 "Hamburg"-Class Ship Equipped With Double-Acting, Two-Stroke Machinery With Direct Drive

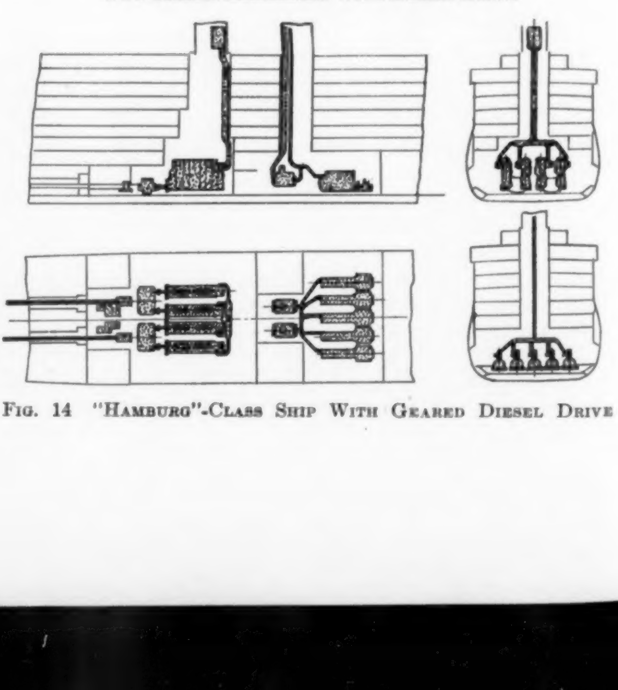


Fig. 14 "Hamburg"-Class Ship With Geared Diesel Drive

It must be borne in mind that if a ship is designed outright for Diesel propulsion, especially for geared Diesels, the aforementioned savings of space and weight can be made use of to provide a smaller ship to carry the same number of passengers at the same speed; hence the capital investment for the smaller ship and for the lesser power required will be considerably reduced (Hillhouse).

Another factor worthy of note is the element of safety of such a plant.

With four engines per shaft, coupled with readily disconnectable hydraulic clutches, the breaking down of one engine would mean only a reduction of 25 per cent of the power of one shaft, about $8^{1}/_{2}$ per cent of the total power for a three-shaft arrangement, and $6^{1}/_{4}$ per cent for a four-shaft arrangement, which would reduce the speed of the vessel by only a fraction of a knot. As a corollary, this ability to disconnect engines at will could be made use of to provide reserve power, thus obtaining an appreciable advantage over the method of obtaining reserve power by means of extra boiler capacity.

The argument is often put forward that in installations such as have been described, a large number of engines is necessary, which from the point of view of many is undesirable. Attention is called to the fact that the *Bremen* is equipped with three units per gear, which in substance are equivalent to three engines, without considering the reverse turbines. This is a total of twelve engines for the ship. Similar arrangements of three Diesels per shaft could be worked out, but they would not be convenient or desirable; hence arrangements with either two or four engines per shaft or gear must be adopted, so that in a four-shaft layout there would be sixteen engines as against the *Bremen* twelve, which is not so objectionable. In fact, the advantage is on the Diesel side, because while the turbine gear has three turbine units of different size, the Diesel unit is composed of four engines, each of the same design and size, with a number of cylinders all of the same size, which renders the spare-part and the overhauling-labor problems much simpler, and in addition full power astern is obtained.

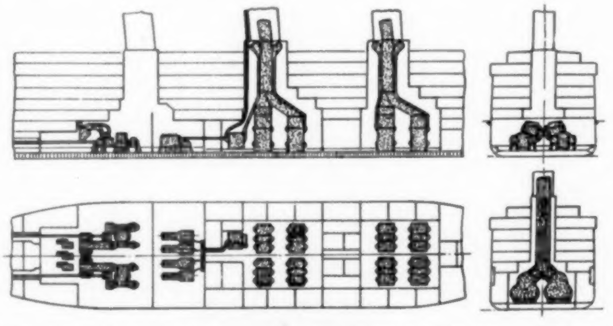


FIG. 15 "CAP POLONIO" FITTED WITH 24,000-S.HP. GEARED-TURBINE MACHINERY

Table 9 summarizes all these examples. The cost of the *St. Louis* engines has been computed by making use of the same factors used in Table 2 and the same cost per pound.

POSSIBILITIES OF THE FUTURE

Reverting to the *St. Louis* and her sister ship and to some of the accomplishments of the German builders, let us look at another situation which now presents itself.

A 15,000-b.hp., two-cycle, double-acting engine of M.A.N. design and of nine cylinders was built for the Hamburg Electric Company. This engine has been performing satisfactorily for over three years and weighs 125 lb. per b.hp., bare engine. The cylinders have a bore of 33.8 in. and a stroke of 59.1 in., and at 94 r.p.m. the piston speed is accordingly 925 ft. per min. The m.i.p. is 79 lb. per sq. in. This engine would represent a conservative direct-acting marine engine, but it would not serve our purpose except to show what has been done in the way of large power in one unit.

Recently two 11,700-b.hp. engines of a modified and lighter type have been delivered to the Berlin Electric Company and have now been in operation for some time. They represent the latest design evolved by the M.A.N. and are similar to those delivered on the second order by their American licensees to the Shipping Board, except that the latter are air-injection while the former are airless-injection engines. The cylinders are of $23^{5}/_{8}$ in. bore and $35^{7}/_{16}$ in. stroke, and at 215 r.p.m. the piston speed is 1270 ft. per min. The m.i.p. is 81 lb. per sq. in. This rating is too high for marine purposes, but if rated down to 950 ft. piston speed, but with 85 lb. per sq. in. m.i.p., they would still show a b.hp. of 8550, and since their weight is 55 lb. per b.hp. at full rating, the marine rating would bring this to about 75.4 lb. per b.hp., which is considerably less than anything the author has suggested, and is an accomplished fact. Adding to this a gear at 10.8 lb. per b.hp., the total unit weight of engines and gear would be 86.2 lb. per b.hp.

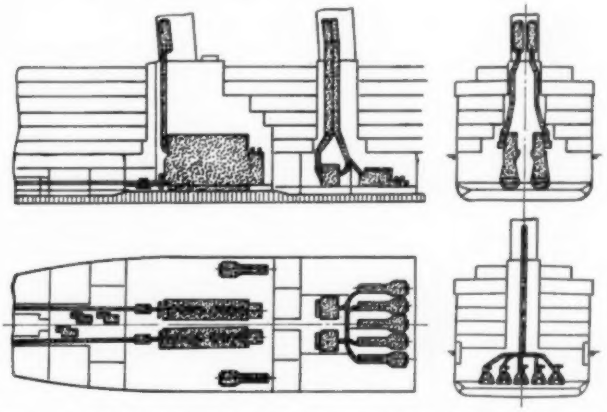


FIG. 16 "CAP POLONIO"-CLASS VESSEL WITH GEARED DIESEL DRIVE

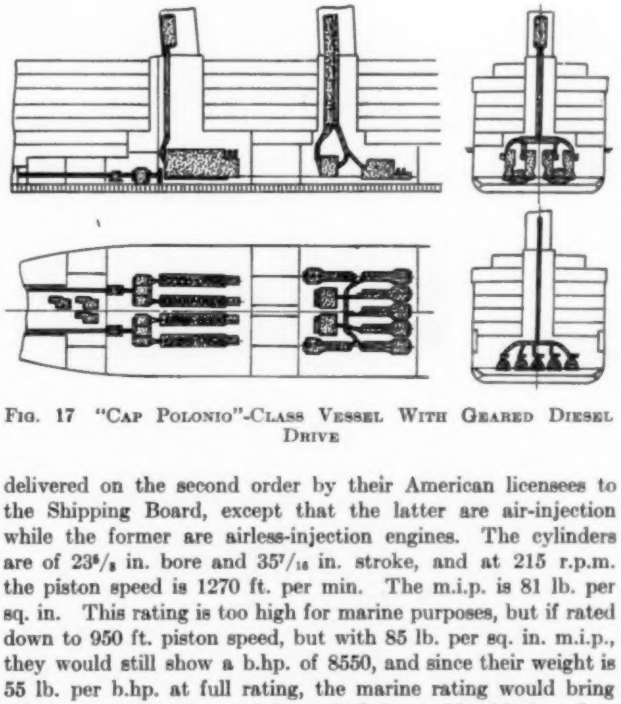


FIG. 17 "CAP POLONIO"-CLASS VESSEL WITH GEARED DIESEL DRIVE

The M.A.N. company has produced engines for naval use weighing $17^{1}/_{2}$ lb. per b.hp., which if rerated for commercial purposes would show not more than double this unit weight, but still with special materials used. If common materials are sub-

TABLE 9 TYPICAL INSTALLATIONS

	Triumph class			*St. Louis*	*Mt. Vernon*	*Mauretania*
	Diesel-electric 4 C-S.A.	Geared Diesel 2 C-D.A.	Geared Diesel 2 C-D.A.[a]	Geared Diesel 2 C-D.A.	Geared Diesel 2 C-D.A.	Geared Diesel 2 C-D.A.
S.hp.	4000	5000	6800	12,000	28,500	75,000
Total weight of prime movers only, tons	554	374	483	833	1,685	4,395
Lb. per s.hp.	310.0	167.4	159.0	155.5	132.4	131.4
Total cost of prime movers, dollars	458,000	327,730	447,500	630,000	1,433,655	3,726,165
Dollars per s.hp.	114.00	65.55	65.85	52.50	50.30	49.70

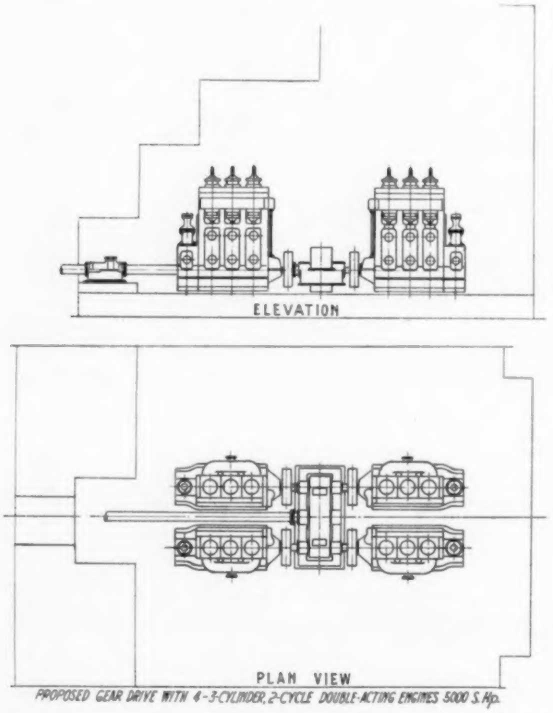
[a] This plant, described in (35), will go in same space.

stituted, the weight would then approach the aforementioned 55 lb. per b.hp., but at marine rating.

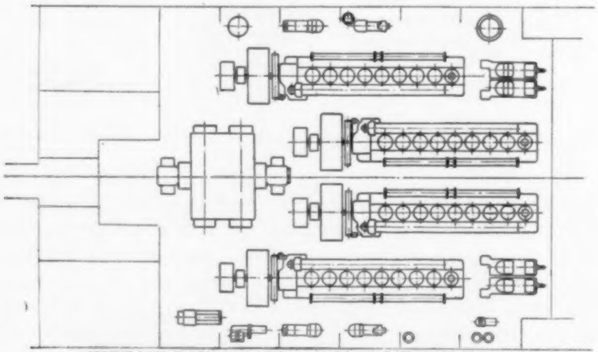
It is obvious from this that not only are the plants that have been used as examples very conservative, but that in the light of what already has been accomplished we may face in the near future more and bigger liners of the *St. Louis* type.

The aforementioned 11,700-b.hp. engines rerated to 8550 b.hp. could well be employed in a modern liner which could carry the same number of passengers at the same or better speed on a smaller displacement than the *Bremen*, assuming that the correct proportions for speed can be obtained. This is easily accomplished in view of the lesser fuel requirements and the lesser space required for machinery, which means a smaller ship, less capital investment, less cost of operation, and more profits.

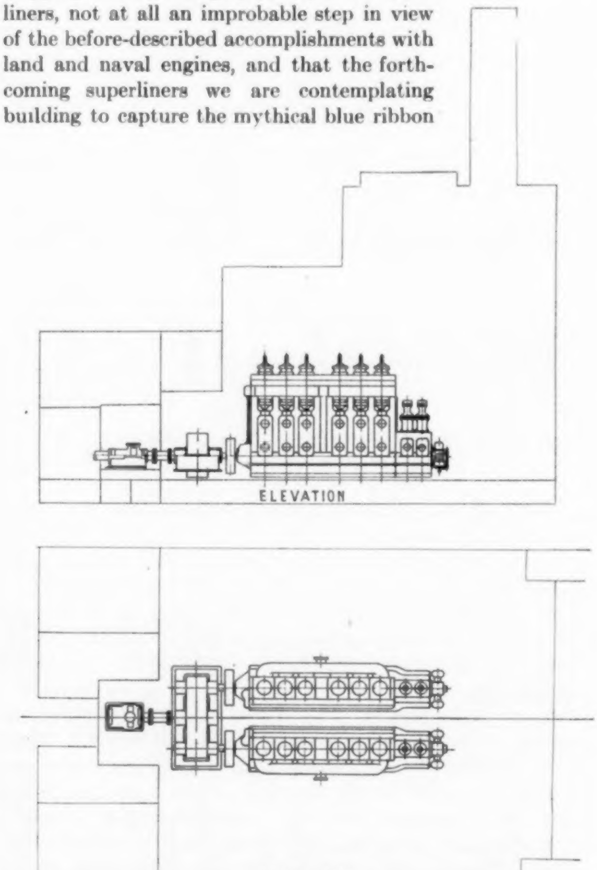
It is not to be assumed that the Germans are asleep, in view of the rapid comeback they have so far made on the sea, and the *St. Louis* should be considered only as a forerunner and as a stepping stone to larger, faster, and better liners, not at all an improbable step in view of the before-described accomplishments with land and naval engines, and that the forthcoming superliners we are contemplating building to capture the mythical blue ribbon



PROPOSED GEAR DRIVE WITH 4-3-CYLINDER, 2-CYCLE DOUBLE-ACTING ENGINES 5000 S.Hp.



PRESENT DIESEL-ELECTRIC LAYOUT WITH 4-4CYCLE SINGLE-ACTING ENGINES 4000 S.Hp.



PROPOSED GEAR DRIVE WITH 2-6 CYLINDER, 2 CYCLE DOUBLE-ACTING ENGINES 5000 S.Hp.

FIG. 18 COMPARISON OF DIESEL-ELECTRIC AND GEAR DRIVES OF MOTORSHIPS "COURAGEOUS," "DEFIANCE," AND "TRIUMPH"

Diesel-Electric

Item	Weight,[a] lb.	Cost, dollars
Four main engines, 8 cylinders, 20 × 24 in., 250 r.p.m., 1200 b.hp. each	758,000	288,000
Four generators, 800 kw. each	168,000	170,000
One propelling motor, 4000 s.hp.	300,000	
One controlling system	16,000	
Total power plant, prime movers only	1,242,000	458,000
Per s.hp.	310.0	114.00

Gear Drive

Item	Weight, lb.	Cost, dollars
Two 2550-b.hp., 6-cylinder engines, 18 × 26 in., 200 r.p.m., with compressor only	648,000	254,625
One reduction gear with thrust bearing, ratio 2 to 1	106,300	28,110
Two blowers 11,000 cu. ft. each, with dual motors	26,000	17,000
Three 110-kw. generating sets	57,000	28,000
Total power plant, prime movers only	837,300	327,735
Per s.hp.	167.4	65.55

[a] With four 3-cylinder, double-acting engines, the weight and cost increase about 5 per cent.

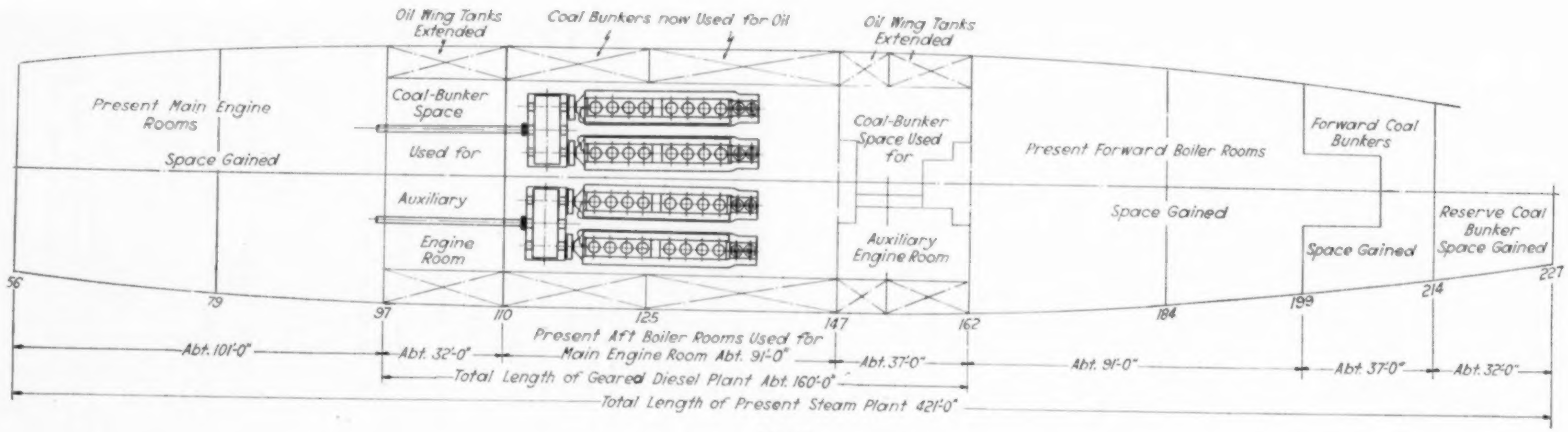

Oil Wing Tanks Extended
Coal Bunkers now Used for Oil
Oil Wing Tanks Extended
Present Main Engine Rooms
Space Gained
Coal-Bunker Space
Used for
Auxiliary
Engine Room
Coal-Bunker Space Used for
Auxiliary Engine Room
Present Forward Boiler Rooms
Space Gained
Forward Coal Bunkers
Space Gained
Reserve Coal Bunker Space Gained
56
79
97
110
125
147
162
184
199
214
227
Present Aft Boiler Rooms Used for Main Engine Room Abt. 91'-0"
Abt. 101'-0"
Abt. 32'-0"
Abt. 37'-0"
Abt. 91'-0"
Abt. 37'-0"
Abt. 32'-0"
Total Length of Geared Diesel Plant Abt. 160'-0"
Total Length of Present Steam Plant 421'-0"

Fig. 19

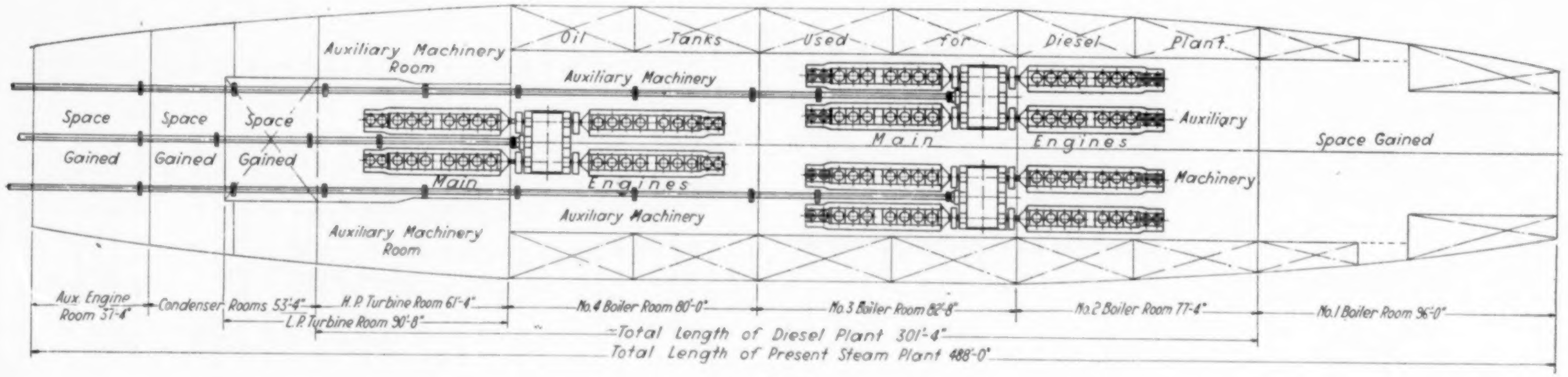

Oil Tanks Used for Diesel Plant
Auxiliary Machinery Room
Auxiliary Machinery
Space Gained
Space Gained
Space Gained
Main Engines
Auxiliary Machinery
Main Engines
Auxiliary Machinery
Space Gained
Auxiliary Machinery Room
Aux. Engine Room 31'-4"
Condenser Rooms 53'-4"
H.P. Turbine Room 61'-4"
L.P. Turbine Room 90'-8"
No. 4 Boiler Room 80'-0"
No. 3 Boiler Room 82'-8"
No. 2 Boiler Room 77'-4"
No. 1 Boiler Room 96'-0"
Total Length of Diesel Plant 301'-4"
Total Length of Present Steam Plant 488'-0"

Fig. 20

Fig. 19 Proposed Two-Cycle Double-Acting Geared Oil-Engine Power Plant of Steamships "Monticello" and "Mount Vernon," 28,500 S.hp.

Item	Weight, lb.	Cost, dollars
Four 7290-b.hp., 8-cylinder engines, 24 × 33 in., 175 r.p.m., with compressor	3,003,480	1,112,454
Two reduction gears with thrust bearing, ratio 1.75 to 1	367,500	97,700
Three blowers, 68,000 cu. ft, each (two service, one standby)	142,000	81,500
Three 550-kw. generating sets	265,000	142,000
Total power plant, prime movers only	3,777,980	1,433,655
Per s.hp.	132.4	50.30

Fig. 20 Proposed Two-Cycle, Double-Acting Geared Oil-Engine Power Plant of Steamship "Mauretania," With 75,000 S.hp.

Item	Weight, lb.	Cost, dollars
Three groups, each consisting of four 6380-b.hp., 7-cylinder engines, with compressors only	7,885,680	2,920,664
Three reduction gears, with thrust bearings, ratio 1.75 to 1	870,000	202,500
Nine blowers, 60,000 cu. ft. each (six service, three standby)	378,000	216,000
Six 750-kw. generating sets	720,000	387,000
Total power plant, prime movers only	9,853,680	3,726,164
Per s.hp	131.4	49.70

of the Atlantic will most likely have to contend, three or four years hence, when they will be in service, with German and possibly with British, French, and Italian superior motor liners equipped with geared Diesels, which in all probability will outpoint them in operating profit if not in speed. And again, quoting *Marine Engineering and Shipping Age* (36):

> An examination of the present situation reveals the fact that in Europe, at any rate, there are far more motor passenger liners being built than steam-driven vessels. Apart from the *Bremen*, which is now completed and the *Europa*, the two 46,000-ton North German Lloyd ships, and the 40,000 tonner which is being constructed on the Clyde for the Canadian Pacific Ocean services, *all of the big passenger liners now on the stocks in British and Continental yards are to be propelled by internal-combustion machinery.*
>
> This general acceptance of the Diesel motor for passenger liners is not widely known, and most engineers and shipowners, without realizing the facts, are under the impression that the steam plant is being used quite as much in new European passenger tonnage as are oil engines. *Actually, there is four times as much motor tonnage under construction as steam tonnage, although this refers only to passenger ships.*

In the matter of reliability, the following comments in *British Motorship* are of interest:

> Recently before the Brennkrafttechnischen Gesellschaft, in Germany, a lecture was delivered on "High-Pressure Steam and Diesel Engines," in which the author was unwise enough to make the statement that "the greater reliability of steam plant in ships is of the highest importance." The author, who is a member of a turbine-building firm, quoted Sir John Biles's paper on the subject to support his case, and also spoke favorably of pulverized-coal firing in a general way.
>
> Mr. Goos, who is engineering director of the Hamburg-Amerika Line, made some interesting comments on the subject. "In general," he said, "the very numerous Diesel plants on ships are extraordinarily reliable. We have ships which have been in service many years making voyages to the Far East regularly. They come back from a five months' voyage, and in three days are ready to start again on their outward trip.... I must say we have had, with turbine ships, many unfortunate experiences.... For Diesel machinery of normal power, 3000 to 6000 b.hp., it can certainly be said that the degree of reliability is remarkable and that the regularity of operation is highly satisfactory. Up to the present time no definite statement can be made regarding the reliability of high-pressure steam machinery. We do not know exactly what type of boiler should be used."
>
> Mr. Goos also referred to Sir John Biles's paper, remarking that these lectures, "not only England, but also elsewhere, have failed to find universal approval; on the contrary, they have been very severely criticized."
>
> The author had mentioned the price of 300 marks (about $75) as the cost of Diesel generators per kilowatt, including the electrical portion, building foundations, etc. Dr. Riehm, of the M.A.N., in the discussion stated that this price is too high, as his firm was quoting 200 marks to a maximum of 250 marks ($55 to $62.50) per kilowatt, including the auxiliaries, the cost of erection, foundation, the electrical portions, and the building.

More recently, Dr. Bauer in his paper on "The Four-Day Liner," read before the November, 1929, meeting of the Schiffbautechnische Gesellschaft in Berlin (37), brings out the possibilities of motor liners of the *Bremen* size by giving an example of a 120,000-s.hp. plant with geared drive, and states that for any project involving oil engines the indirect drive is to be preferred, with gears and hydraulic coupling inserted between high-speed Diesel engines and the propeller shaft. In the discussion which followed the reading of the paper, Navy Technician Laudahn also stated:

> When considering oil engines, only high-speed ones with transmission will be considered. These can very well take up the competition with turbines.... One may say that an oil-engine plant of 130,000 s.hp. on board a liner is possible.... The first liner with oil engines is no longer an utopia.

It is to be noted that Dr. Bauer is an old advocate of the indirect drive (see 9). It is presumed, therefore, that the main reason against the adoption of such a drive for the *Bremen* was the state of the art, at the time the ship was projected, in regard to Diesel engines for such a large installation, involving considerable risk in its conception.

Since then, and since the *Bremen* was designed, the *Ersatz Preussen*, the new German treaty battleship, has been evolved, the power plant of which consists of 50,000 s.hp. on two shafts, with four engines of about 7000 hp. on each shaft. Since it is possible, without much complication, to install four engines per gear and four shafts in a ship of the *Bremen* size, the *Ersatz* power plant represents a potential 112,000-s.hp. installation.

Of course, such a plant is a naval plant, and hence highly rated, but if discounted for commercial use it could very well and with safety be considered a 68,000-s.hp. plant, with the same engines rated down to more conservative speeds, pressures, and unit weight. But such more conservative rating would still class the engines, in economy of weight and possibly cost, below anything previously analyzed in this report.

We in the United States, by reason of the higher labor costs, should further economize in the cost of Diesel installations by resorting to gear drives more than do our European competitors. The author's own feeling is that a start should be made toward some such motor liner in our own merchant marine, by first establishing the probable power plant, preferably quadruple-screw with four engines per shaft, and then constructing and equipping a twin-screw intermediate liner with identically the same engines, but only two per shaft, and if such a plant would produce 30,000 s.hp., it would represent a potential 120,000-s.hp. installation.

This is desirable in view of the fact that, at this writing, the new British intermediate motor liner *Britannic* has made its appearance, thus bringing us to a realization of the kind of competition we must meet on the Atlantic in the near future. This ship cannot be beaten in capital cost and operating expenses if built in this country, unless a geared drive is adopted to even up things from the capital-cost end.

By following this course, we could be enabled to acquire some valuable operating experience before embarking in the construction of a larger and more powerful liner.

The author presents this compendium as a probable picture of the future development of Diesel-propelled ships, and would suggest as a well-meant prophecy that he who fails to consider the geared Diesel for any large vessel, whether cargo or passenger carrier, is seemingly making a mistake.

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		Société des Ingénieurs Civils de France
Germany		
	Berlin	Bücherei des Vereines deutscher Ingenieure
		Bibliothek der Technischen Hochschule
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	Bangalore	Mysore Engineers Association
	Calcutta	Bengal Engineering College
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	Mexico City	Asociacion de Ingenieros y Arquitectos de Mexico
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Turkey		
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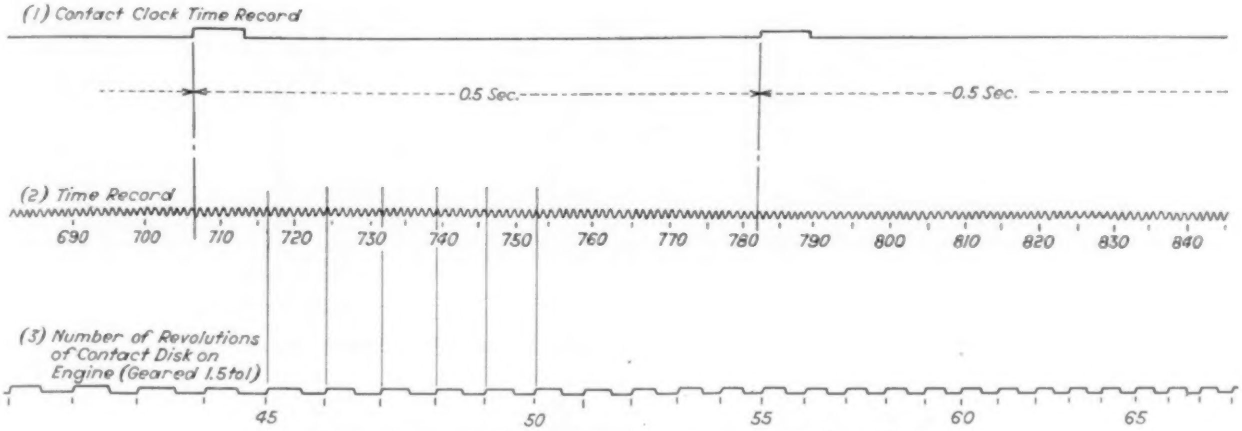

FIG. 1 PORTION OF RECORD STRIP OF ENGINE ACCELERATION TESTS

Comparative Performance of Diesel and Gasoline Engines in Motor Trucks

BY PROF. P. LANGER,[1] AACHEN, GERMANY

The results are given of tests of the comparative performance of Diesel and gasoline engines in motor trucks. The question is raised whether the carburetor is any longer justified, in view of the present high development of the fuel pump. It seems improbable that carburetor engines will ever do any better in running at low speed under load than they are now doing. The possibilities of development of the fuel pump are great. Among these is the application of the Diesel principle to light-oil engines, rendering possible a standard motor-truck engine perfect both from the thermodynamic and the automotive points of view.

THERE are in operation in the world today some 36 million automotive vehicles. These vehicles are driven by internal-combustion engines having a combined output of about 1200 million horsepower—seven times the total power developed by locomotives and nine times that of all the electric plants in existence. Indeed, a picture of the modern world would be incomplete without the automotive vehicle and its internal-combustion engine. It is a peculiar fact that the internal-combustion engine is by no means in principle the ideal prime mover for a vehicle. While it is unsurpassed for high speed and light weight, it has several serious inherent drawbacks. Above all, it is not able to start on its own power, but must be cranked before it will run, and it was only by employing an electric starter in combination with a storage battery that this disadvantage was finally overcome and the internal-combustion motor pushed to final victory over the steam engine. The modern 9 to 12 liter (550 to 730 cu. in.) capacity vehicle motor of about 100 hp. can hardly be cranked by hand at all. A further difficulty in the internal-combustion engine for vehicle propulsion is that, even at operating temperature, it is only able to pull its full or at least its approximate load upon attaining a certain minimum speed at which the fuel begins to be properly mixed and fed to the separate cylinders. The weakness of internal-combustion engines, in a word, is in operation at low speeds. The characteristic curves of both Diesel and gasoline engines show their advantages at high speeds, but these are usually limited to a minimum of about 500 r.p.m., below which the highly important range of low speeds is unprovided for.

In the investigation reported on in the present paper, particular attention was paid to the performance of engines in the lowest speed ranges.

THE TESTS

The investigations embraced Diesel automotive engines working on heavy oil with fuel pumps upon the fuel combustion principles at present most widely employed in vehicle propulsion (designated in the charts attached to this report by DSZ) for direct injection, DVK for the precombustion-chamber method, DSP for the Acro auxiliary air-chamber method, and gasoline carburetor engines (designated by VB), all of the engines considered being mounted in motor trucks. The tests were all carried out by the author at the factories, except those on engines DSZ 7c and VB 11, which were examined at the Berliner Verkehrsgesellschaft (Berlin Autobus Company) in their vehicles. All of the tests were undertaken with the engines at operating temperature.

ENGINE-ACCELERATION TESTS

With the vehicle at rest and the clutch disengaged, the engines were made to idle at the lowest speed, this being determined in the Diesel engines by the low-speed governor, and in the gasoline engines by the no-load nozzle and the butterfly-valve stop. By depressing the fuel pedal, the engine was accelerated and

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[1] Director of Engine Laboratory, Aachen Institute of Technology. Dr.-Ing. W. Thomé and Dr.-Ing. Marquard collaborated with Professor Langer in the preparation of the paper.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

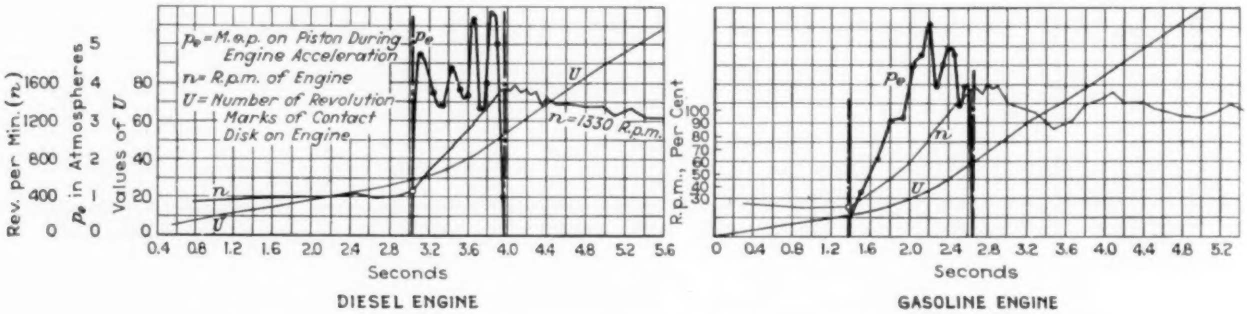


Fig. 2 Mean Effective Pressure and R.p.m. During Acceleration of Engine (Disengaged)

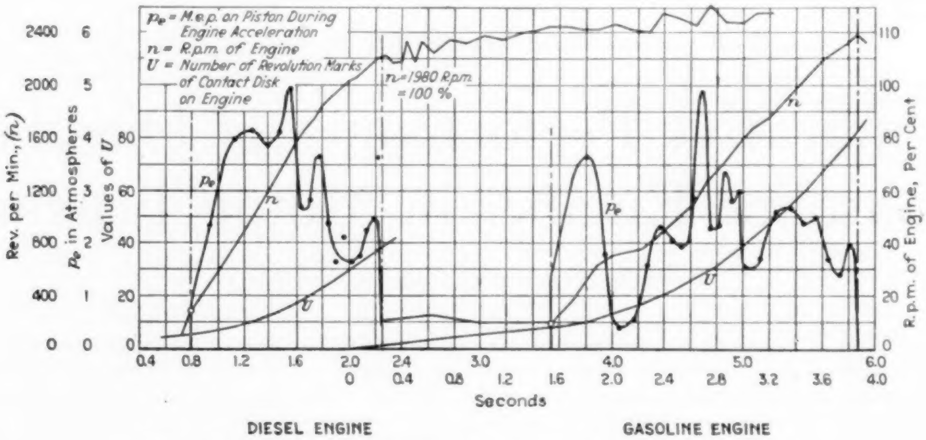


Fig. 3 Mean Effective Pressure and R.p.m. During Acceleration

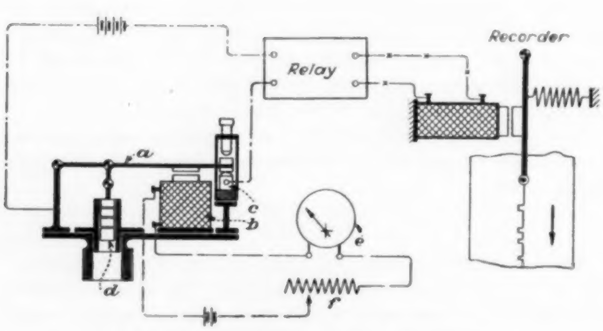


Fig. 4 Pressure-Measuring Instrument

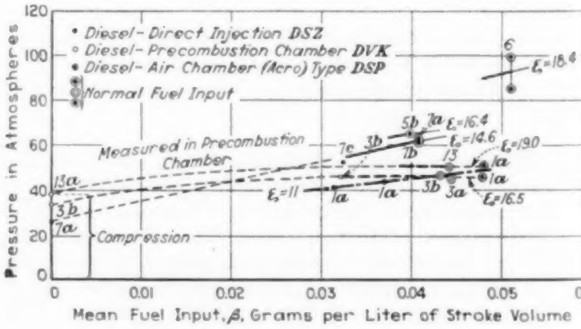


Fig. 6 Combustion Pressures

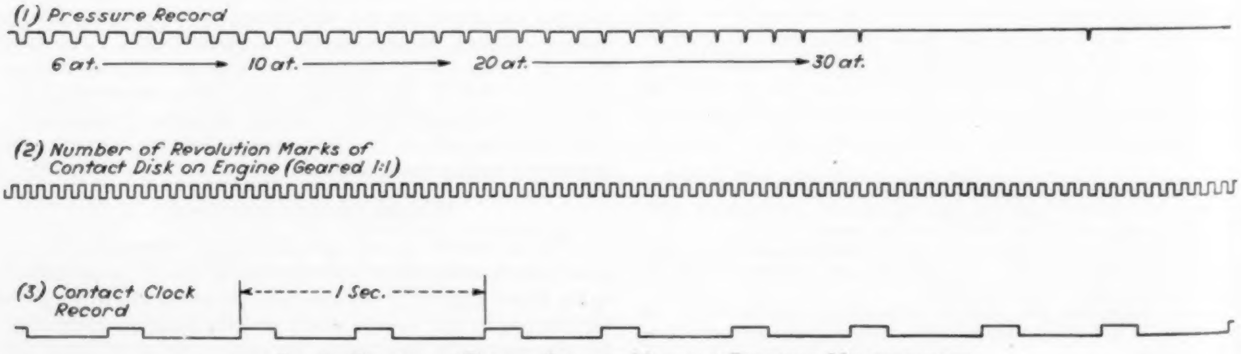


Fig. 5 Portion of Record Strip of Maximum Pressure Measurements

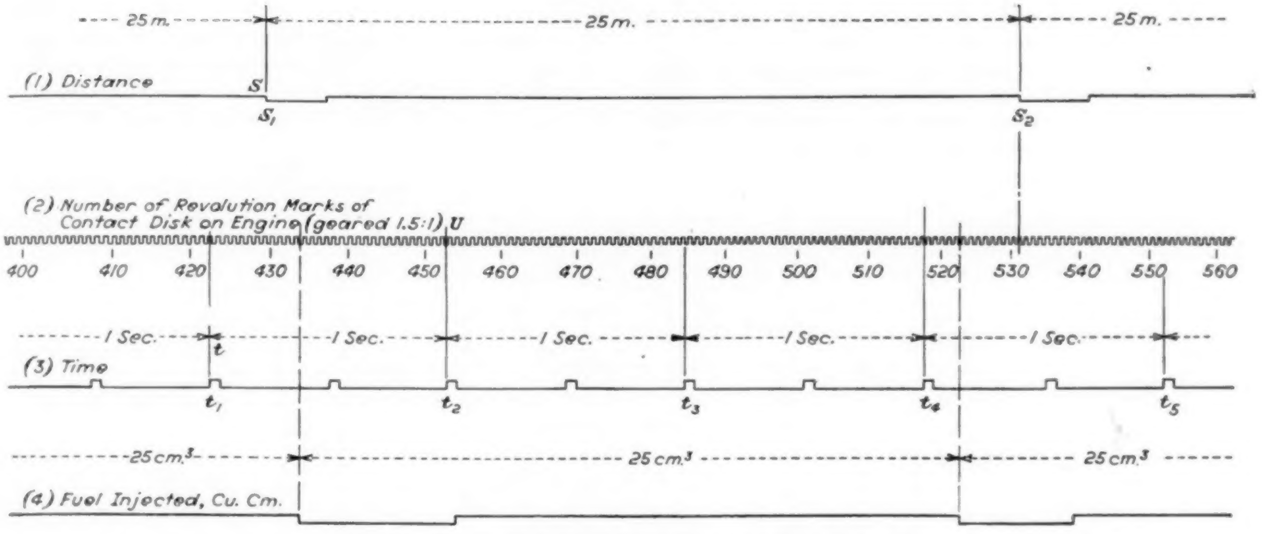


Fig. 7 Portion of Record Strip of Acceleration Tests

regulation of the fuel admission left to the speed-limiting governor.

The records were made by a magnetic recorder, shown in Fig. 1, and having three tracing points. Of the graphs registered by these points, No. 1 shows the records of a half-second contact clock; No. 2 the oscillations produced by a constant-frequency electromagnetic buzzer; and No. 3 the record of revolutions of an engine countershaft provided with rotating contacts.

The time log of the revolutions of the engine, the r.p.m., and the mean effective pressures calculated from the acceleration on the basis of the known moments of inertia of the rotating masses of the engine are recorded in Figs. 2 and 3, both for a heavy-oil Diesel engine and a gasoline carburetor-type engine of the same stroke and the same nominal maximum speed. In Fig. 2 the slow rise of the p_e curve of the gasoline engine as compared to the Diesel engine is worthy of note. Fig. 3 shows the typical carburetor holes, by which the starting time is considerably increased. In both engines the mean effective pressures were considerably lower than corresponding testing-stand values.

Maximum Pressures During Combustion

In order to obtain an idea of the maximum pressures prevailing during combustion, measurements on the engine were carried out with the truck traveling, and during the accelerating period with full fuel input by means of a maximum-pressure meter developed for the purpose and schematically illustrated in Fig. 4. Lever *a* is drawn by electromagnet *b* into contact with *c*. As soon as the pressure on piston *d* balances the attraction of electromagnet *b*, the contact pressure at *c* is relieved, and the indicating circuit is interrupted at this point. By means of a highly sensitive relay, the interruption is translated into the motion of a magnetic tracer, which thus records the interruption in graphic form. The attractive force of magnet *b*, which is located in a special circuit, is regulatable by rheostat *f*, and the pressures corresponding to the indications of ammeter *e* are denoted by marks.

A sample record is shown in Fig. 5. Of the graphs in this record, No. 1 shows the movement of the magnetic tracer caused by the circuit interruptions at *c*, No. 2 the record of the revolutions of an engine countershaft with rotating contacts, and No. 3 the registrations of a half-second contact clock.

The force of electromagnet *b* was increased until only occasional movements were made by tracer No. 1. The ammeter reading in the example under discussion gave a pressure of 30 atmospheres, which is attained and exceeded only at a few ignitions. By increasing the feed, the duration of the combustion pressures may also be exactly determined, although this question must be reserved for later investigation.

Fig. 6 is a record of maximum combustion and compression pressures measured at various full fuel inputs on Diesel engines in road operation, and in part also on the testing stand. With gasoline engines having a compression ratio of about 5.5 to 1, the pressures observed during acceleration, or in other words with wide-open throttle, were between 33 and 37 atmos. In one of the tests, made with an insufficiently knock-proof fuel, the pressure in the engine jumped to 43 atmos. with light knocking.

Road Acceleration Tests

Of the essential criteria of effective road performance, the following properties of engines were ascertained by road tests on suitable stretches:

1 The lowest running speed in high gear
2 The acceleration from the moment of giving fuel to the attainment of 90 per cent of the maximum running speed
3 Fuel consumption, as expressed by fuel input to the cylinders, during acceleration.

By the testing and measuring procedure adopted, each vehicle was started by shifting through the various speeds, and then accelerated from the lowest running speed in high gear by pushing down the fuel pedal to the limit. The recording instrument employed was the quadruple magnetic tracer. Of the four graphs shown in Fig. 7, No. 1 shows the road distance marks as registered by a hand-operated button, No. 2 the revolutions of an engine countershaft with rotating contacts, No. 3 the registrations of a half-second contact clock, and No. 4 the fuel marks as registered by a hand-operated button at the moment at which one of the consecutively arranged fuel vessels of a capacity of 15 cu. cm. (0.92 cu. in.), for heavy oil, and 25 cu. cm. (1.63 cu. in.), for gasoline, ran dry.

Comparison of the road marks with the revolution contact marks showed a clutch and wheel slippage, which was allowed for in determining the space-time curve, the velocity-time curve, and the time record of acceleration. As a rule each test was made twice, once going and once coming, in order to obtain a mean of the two runs and thereby to eliminate as much as possible the influences of grades and wind resistance.

The magnitude of the acceleration measured does not afford a direct comparison of the performance of the engine, unless piston displacement, vehicle masses, gear reductions, and wheel diameters are alike for the different engines and vehicles concerned. In addition, clutch and wheel slippage must also be considered, since these losses must be overcome by the engine with the same relative exertion as for acceleration. The mean effective pressure (p_b in kg. per sq. cm.) causing acceleration on the level road is obtained from the equation:

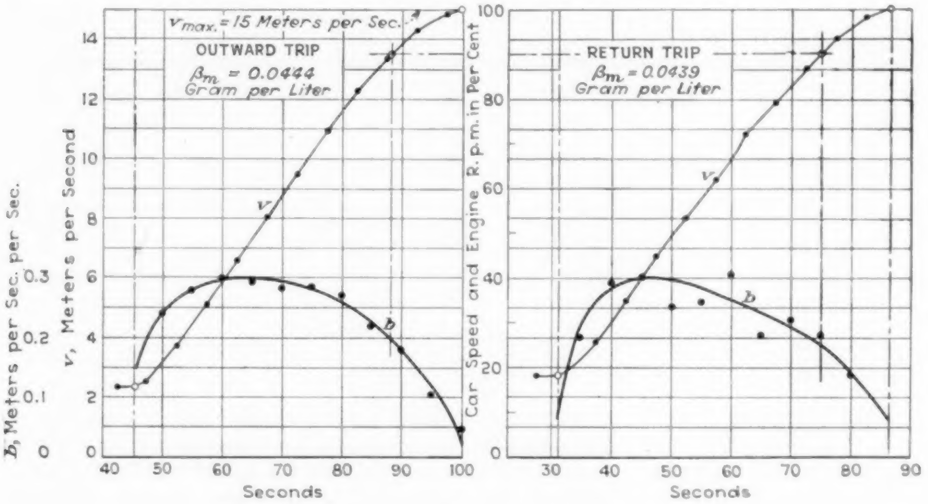


FIG. 8 CAR SPEED AND ACCELERATION IN HIGH GEAR

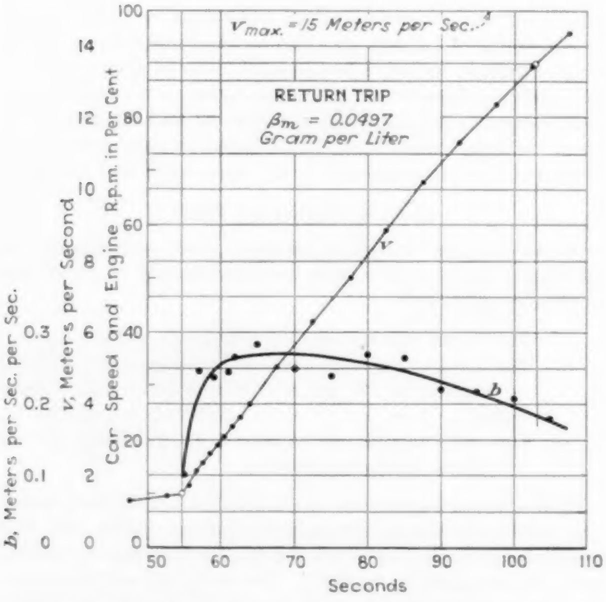


FIG. 9 CAR SPEED AND ACCELERATION IN HIGH GEAR

> Excess work of engine per revolution = work required for accelerating vehicle + wheel and clutch slippage work per revolution of engine

or:

$$10 \times p_b \times V_H \times i = m \times b \left(\frac{2R\sigma\pi}{\rho} + \frac{2R(1 - \sigma)\pi}{\rho} \right). [1]$$

where

V_H = displacement of engine in liters (1 liter = 61.02 cu. in.)

i = number of working strokes per cylinder and revolution (i = $^1/_2$ for four-cycle engines; i = 1 for two-cycle engines)

$10 \times p_b \times V_H \times i$ in kg-m. = excess work of engine per revolution

$\frac{2R\sigma\pi}{\rho}$ = distance traveled by vehicle during one revolution of engine

where

R = rolling radius in meters

ρ = driveshaft reduction figure

σ = wheel-clutch slippage

$2R\ (1 - \sigma)\pi$ = loss in travel due to wheel and clutch slippage

m = dynamic mass of vehicle —$G/g \times e$

where

g = acceleration due to gravity in m. per sec.^{2}

G = total weight in kg. of vehicle

e = number denoting rotating masses of engine and vehicle requiring acceleration in traveling

b = acceleration of vehicle in meters per sec.^{2}

From Equation [1] is found:

$$\frac{p_b}{b} = e\frac{G}{g} \times \frac{R\pi}{V_H \times \rho \times i \times 5}$$

If the value $\frac{e \times G \times R}{V_H \times \rho \times i}$ is designated by K = dynamic vehicle-engine constant, then:

$$p_b = 0.064 \times b \times K$$

The addition of the mean effective pressure p_r used for overcoming the gear friction, and rolling and air resistance on up and down grades, to p_b gives the mean effective pressure p_e of the engine during the period of acceleration:

$$p_e = p_b + p_r$$

p_r was determined by coasting tests on the various test stretches with the engine cut off, and therefore with a reduction allowed for in factor e corresponding to the elimination of the rotating engine masses. Since, in coasting, only small forces are transmitted through gears and wheels, and p_r is therefore bound to be somewhat smaller than when the full engine power is transmitted, p_e will be somewhat too small, although the error may be neglected, especially in a comparison of engines with each other.

From the equation:

> Work of engine per liter of piston displacement = fuel quantity per liter of displacement × calorific value × total efficiency

$$p_e \times 10 = \beta \times H \times \eta \times \frac{1}{A}$$

where

$p_e \times 10$ = liter work in kg-m.
$1/A = 427$ = equivalent of 1 calorie
β = fuel input in grams per liter
η (total) = total efficiency = $632/B \times H$
p_e = mean effective piston pressure in kg. per sq. cm.
H = calorific value of fuel in gram-calories
B = specific fuel consumption in grams per hp-hr.
632 = equivalent of 1 hp-hr.

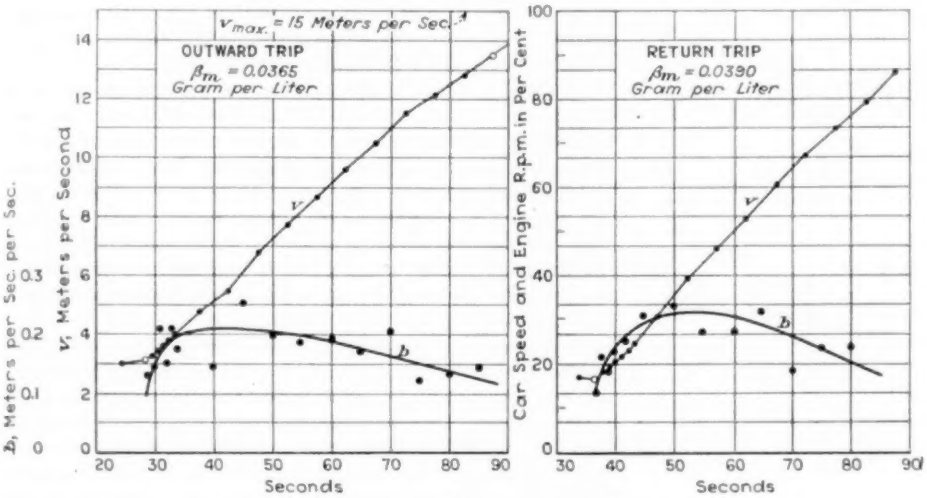


FIG. 10 CAR SPEED AND ACCELERATION IN HIGH GEAR

there follows:

$$\beta = \frac{P_e \times B}{27{,}000} \qquad \text{or} \qquad B = \frac{P_e}{\beta} \times 27{,}000$$

By this equation, the specific fuel consumption was calculated from the known mean effective pressure during acceleration and the fuel input as ascertained by measurement.

As examples Figs. 8 to 10 show the speed and the acceleration time logs of a Diesel-engine-driven truck at various fuel inputs and Fig. 11 of a gasoline-carburetor-engine-driven truck. The mean effective pressures resulting with the help of the dynamic vehicle-engine constant K are illustrated in Figs. 12 and 13. Fig. 14 presents the results of all tests performed in their relation to the average values of the mean effective pressures and the mean-specific-fuel-consumption figures as a function of the fuel input, the values of the mean effective pressures at specific-fuel-consumption figures at normal fuel input being emphasized for greater clearness by surrounding circles.

TABLE 1 COMPARISON OF DIESEL-ENGINE AND GASOLINE-ENGINE CAPACITY

Diesel Engines (fuel pumps)

Engine No.	n_{max} %	n_1 %	n_2 %	n_3 %	p_{em}	p_{emax}	B_m	β_m
DVK 3b	100	30	44	48	5.17	5.32	0.0432	226
DVK 13	100	15	19	22	5.22	5.50	0.0443	229
DSZ 6	100	21	22	29	6.10	6.60	0.0510	225
DSZ 7a	100	26	34	34	5.00	5.15	0.0410	221
DSZ 7b	100	18.5	28	28	4.99	5.40	0.0410	222
DSP 1	100	17.5	25	28	5.18	5.55	0.0442	230
Mean values		21	29	31	5.27	5.60	0.0440	225

$p_{em} = 0.94\ p_{emax}$

Gasoline Engines (carburetors)

Engine No.	n_{max} %	n_1 %	n_2 %	n_3 %	p_{em}	p_{emax}	B_m	β_m
VB 2	100	20	32	35	5.25	6.00	0.0577	297
VB 12	100	19	22	28	6.46	7.40	0.0650	272
VB 9	100	43	45	47	5.69	6.10	0.0595	282
VB 8	100	37	41	45	5.69	6.00	0.0570	271
VB 11b	100	21	27	27	5.00	5.90	0.0585	315
VB 11a	100	21	30	32	5.24	5.85	0.0585	302
Mean values		27	33	36	5.55	6.20	0.0894	289

$p_{em} = 0.89\ p_{emax}$

In order to obtain an idea of the average capacity of Diesel and gasoline engines in vehicle operation, the most important test results of the different engines are shown and the average values for Diesel and gasoline engines are given in Table 1.

As important from the point of view of vehicle operation, the following values were used in the comparison:

n_1 = lowest r.p.m. from which the car was accelerated in high gear in per cent of the maximum r.p.m. n_{max}
n_2 = lowest r.p.m. at which p_e = 5 kg. per sq. cm. was attained, in per cent of the maximum r.p.m. n_{max}
n_3 = lowest r.p.m. at which p_{em} was attained, in percentage of the maximum r.p.m. n_{max}
p_{em} = mean time value of mean effective pressures between n_1 and 0.9_{max} in kg. per sq. cm.
p_{emax} = maximum value of mean effective pressure between n_1 and 0.9 n_{max} in kg. per sq. cm.
β_m = mean value of fuel input in grams per liter for working-stroke volume during acceleration
B_m = mean value of specific fuel consumption in grams per hp-hr.

Fig. 15 presents the average comparative mean effective pressure as a function of the r.p.m. of Diesel and gasoline engines

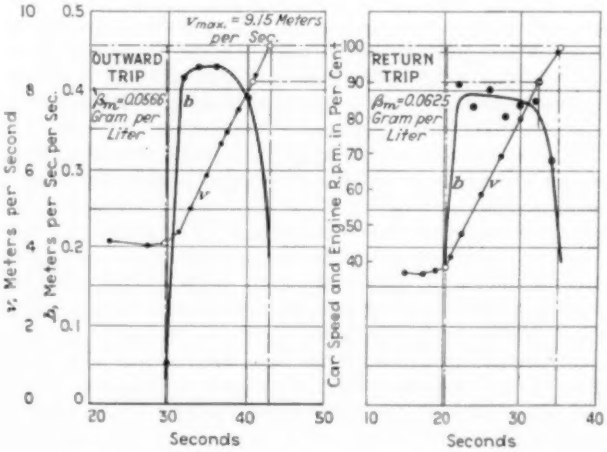


FIG. 11 CAR SPEED AND ACCELERATION IN HIGH GEAR

examined, the mean value of the maximum speed being 1475 r.p.m.

CONCLUSIONS

1 First of all it must be stated that, almost throughout, the maximum mean effective pressures occurring in the engines during acceleration of the vehicle were lower for a given fuel input than the values given in current literature and based on the results of testing-stand runs. This is true of Diesel engines, and possibly even to a greater extent of gasoline engines. The cause of this phenomenon cannot be fully explained by the fact

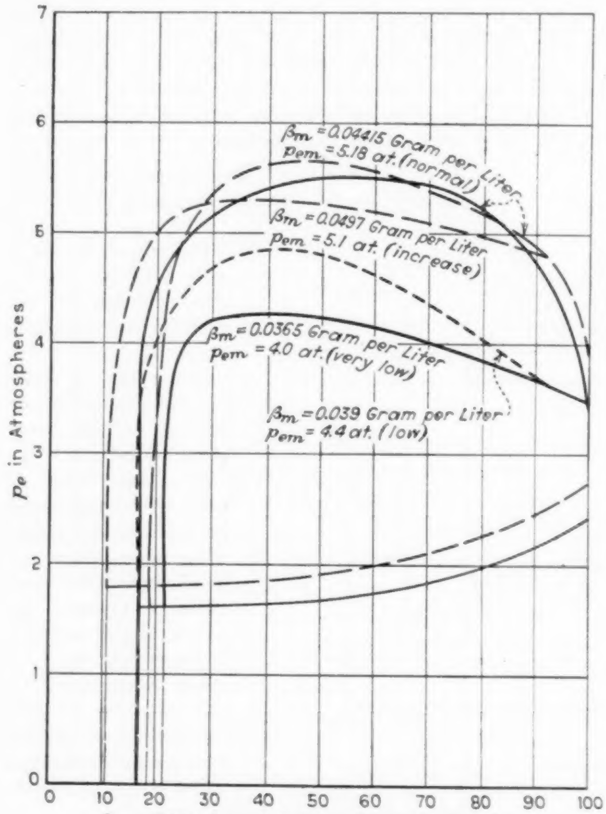


FIG. 12 MEAN EFFECTIVE PRESSURES DURING ACCELERATION

that, due to non-allowance for the losses previously referred to, the working piston pressures resulting from the present measurements on road performance are slightly lower. It would appear that engines on testing stands show a higher output at any given sustained speed than they do during operation in a vehicle, where they are subjected to rapid speed variations.

Test-stand results are of no value in judging the performance of engines in vehicles for which the fuel input is specially adjusted for each r.p.m. so as to cause the engine to be brought to the maximum output with invisible exhaust. For practical vehicle operation this would presuppose the presence of a device which would automatically adjust the ideal smokeless-fuel input for each engine speed. To the knowledge of the author such a device, which adapts the fuel input exactly to the momentary engine speed, has not up to the present been designed and introduced. It is therefore necessary to judge engine performance on the basis of the maximum fuel input giving invisible exhaust gases at the least favorable r.p.m.

2 *The maximum values of the mean effective pressures* ($p_{e\max}$) during acceleration of the vehicle are 10 per cent lower for Diesel engines than for gasoline engines, whereas *the mean values* (p_{em}) differ by only 5 per cent.

3 *The mean effective pressures at full engine speed,* therefore the excess outputs at full vehicle velocity, are the same for Diesel and gasoline engines of equal piston displacement, equal r.p.m., and equal running resistances of the vehicle.

4 Although *fuel input* is reduced by 27 per cent in Diesel over gasoline engines, the mean output of Diesel engines during the acceleration period is but 5 per cent less at full fuel input due to the thermodynamic superiority of the Diesel process. The ratio of the specific-fuel-consumption figures of Diesel and gasoline engines is as 1 to 1.27. These specific fuel consumptions which were measured during the acceleration period admit of no direct comparison of fuel consumption per 100 km. The showing is more favorable for Diesel engines, since at partial load the specific fuel consumption increases more rapidly in gasoline than in Diesel engines.

5 The strikingly quiet and slow-running characteristic of gasoline engines idling with the vehicle at rest are not in evidence with Diesel engines. On the other hand, Diesel engines attain their *stable starting torque at 21 per cent* of their maximum speed, whereas 27 per cent is required for gasoline engines, the average maximum speeds of the Diesel and gasoline engines compared being about equal—namely, about 1475 r.p.m.

6 The degree of "roughness" in the running of engines is independent of the *maximum combustion pressures.* At any rate, direct-injection engines, in spite of their higher maximum combustion pressures, do not show a greater degree of roughness than precombustion-chamber or auxiliary air-chamber engines. The cause of the rougher running of Diesel over gasoline engines may be due to the knocking properties of the gas oil, this knocking being noticeable in all known Diesel engines, though not being sufficiently marked to be heard over the vehicle noises during traveling.

In any attempt to judge the harmfulness of pressures, it is necessary to consider how and at what phase of the working process they originate. High pressures resulting from rapid though steady combustion at dead center cannot be rated as injurious. They do not cause any hard driving impacts, are controllable, and constitute an indispensable condition for minimum fuel consumption, low exhaust temperatures at full fuel input, and small cooling-water losses. These thermodynamically correct

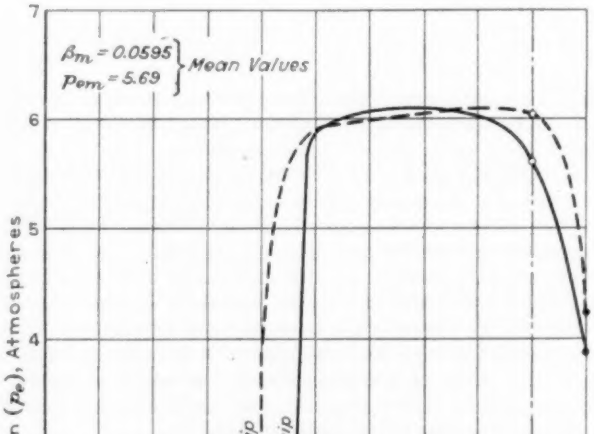


FIG. 13 MEAN EFFECTIVE PRESSURES DURING ACCELERATION

high combustion pressures might be designated as "cold high pressures," as distinguished from "hot high pressures" such as are caused by ignition through incandescence, premature ignition, or by detonation after dead center, and which are accompanied by a reduction of engine output at a given fuel input. The equivalent of the output reduction in this case becomes noticeable in an increase of the exhaust-gas or cooling-water temperature in a manner detrimental to the engine.

7 In Diesel engines the nearer approach of the average to the maximum values of the mean effective pressures, and above all the advantage, from the point of view of practical vehicle operation, of rapid attainment of the starting torque at low speeds, is not a question of the thermodynamic superiority of the Diesel principle, but of the fuel pump, which provides a sufficiently accurate fuel input to the various cylinders even at the lowest speeds.

In view of these facts, the question should be considered whether, in view of the present highly developed fuel pump, the carburetor is in any degree any longer justified. It is true that the carburetor has undoubtedly the great advantages of cheapness and practical freedom from wear. When it is considered, however, that in order to improve the running qualities of gasoline engines, especially in the low-speed range, it was found necessary generally to increase the piston displacement and simultaneously to throttle the suction, then it would seem that considerations of first cost must give way to the more important question of what is better from the point of view of vehicle operation. It seems improbable that carburetor engines will ever do any better in running at low speed under load than they are now doing. The possibilities of development opened up in this direction by the fuel pump, however, are by no means exhausted. Among these is the application of the Diesel principle to light-oil engines, which would render it possible to create a standard motor-truck engine perfect both from the thermodynamic and the automotive points of view.

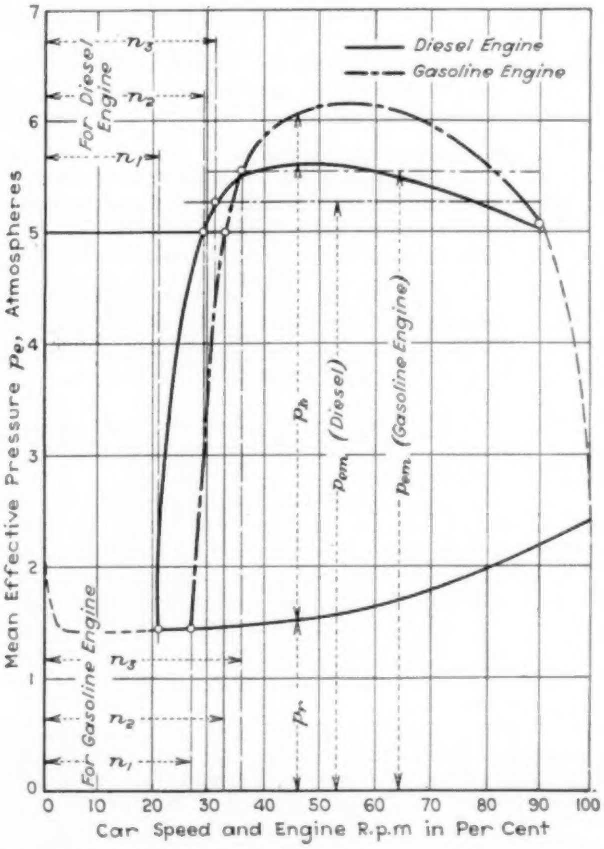


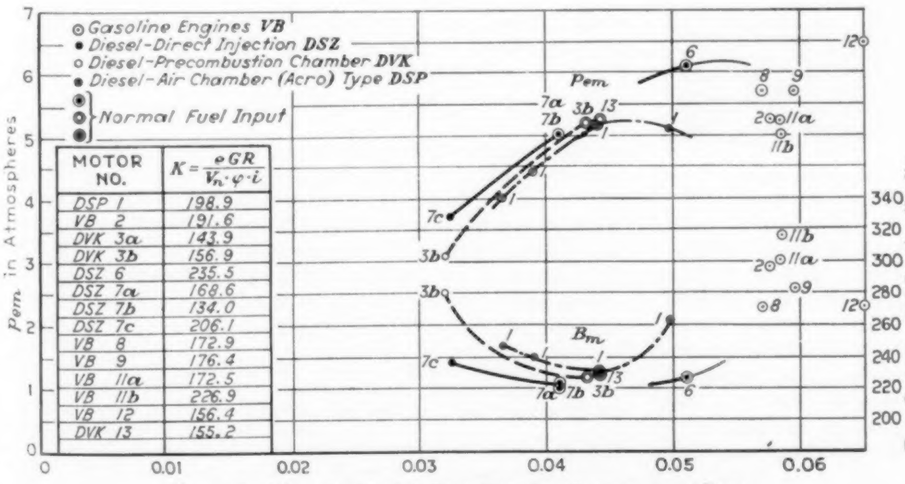
FIG. 15 AVERAGE VALUES OF MEAN EFFECTIVE PRESSURES DURING ACCELERATION

Discussion

A. A. LYMAN.[2] It is interesting to note that although Professor Langer has confined his experiments to accelerating periods, the results again demonstrate the superior fuel economy of Diesel engines and check fairly closely with general operating results with which the writer is familiar. These results cover the operation in actual service of several Diesel buses, and all show an increase in fuel mileage of 30 to 50 per cent over gasoline vehicles. This decrease in specific fuel consumption, together with the advantage of lower unit fuel costs, results in fuel savings of tremendous importance to fleet operators.

We thoroughly agree with Professor Langer's conclusions regarding the relative costs of carburetors and fuel pumps; that consideration of first cost must give way to the more important question of what is better from the point of view of vehicle operation. This difference in first cost is insignificant compared with the saving in truck and bus fuel costs resulting from the use of Diesel engines.

Our first Diesel truck engine was rather rough, but we have recently operated a Diesel bus engine that is reasonably smooth throughout the entire operating range, including idling. The slight roughness is unnoticed by the passengers and apparently does no harm. This roughness is greatly influenced by the characteristics of the fuel, and has been reduced to a negligible



MOTOR NO.	$K = \frac{\rho G R}{V_n \cdot \varphi \cdot i}$
DSP 1	198.9
VB 2	191.6
DVK 3a	143.9
DVK 3b	156.9
DSZ 6	235.5
DSZ 7a	168.6
DSZ 7b	134.0
DSZ 7c	206.1
VB 8	172.9
VB 9	176.4
VB 11a	172.5
VB 11b	226.9
VB 12	156.4
DVK 13	155.2

FIG. 14 MEAN VALUES (p_{em}) OF MEAN EFFECTIVE PRESSURES DURING ACCELERATION AND OF MEAN FUEL CONSUMPTION (B_m)

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amount by changes in the fuel that have not, however, increased its cost.

In comparing Diesel and gasoline engines, Professor Langer does not mention some of the practical advantages of Diesel automotive engines, such as:

- (*a*) Elimination of the fire hazard in vehicles as well as in the storage, handling, and transportation of the fuel
- (*b*) Appreciable reduction in road failures due to the elimination of the entire ignition system
- (*c*) Practical elimination of carbon monoxide.

These advantages, with the lower fuel costs, make the Diesel engine particularly attractive to truck and bus operators, and it is hoped that the development of light, high-speed, high-power automotive Diesel engines will be vigorously carried on.

HARTE COOKE.[3] There are a number of interesting things which this paper brings out and a number of things which would suggest further investigation.

In the first place, it should be remarked that while the comparative characteristics of the Diesel engine and the gasoline engine during the period of acceleration is an interesting phase of the use of the Diesel engine for this purpose, it does not give the full measure of the desirability of the Diesel engine for this service, for in general the operating cost of the two forms of motive power and the relative first cost of the truck so equipped would be the principal commercial factors as to which form would be the more desirable.

In connection with the method of obtaining the b.m.e.p. of the two types of engines during acceleration, it would seem that there must be some losses such as distortion of the rubber tires and the extra friction losses under the maximum loads imposed on the part doing the acceleration, which would account for the m.e.p. in actual service not coming up to what it would be on the test stand.

The difference in weight is always brought up between the Diesel engine and the gasoline engine, due to the higher pressure involved with the Diesel engine.

With modern designs the crankshaft in most cases has to be proportioned to keep the speeds at which torsional resonance occurs at a desired location, and the weight of the frame is governed usually by the ability of the foundry to cast the iron sections required, so that the lighter weights of the gasoline engines are undoubtedly due in most cases to the fact that they use an underslung crankshaft with oil pan instead of base and use a cylinder construction which permits very close center-to-center distance.

In fact, the records in the paper show maximum combustion pressures in the gasoline engines of from 550 to 630 lb., which are comparable with the pressures obtained in the Diesel engine.

Where the maximum possible acceleration is a very desirable feature, such as in local bus service, the characteristics of the different engines shown in Table 1 should be very carefully analyzed to see what the characteristics are that give some of these engines better acceleration than others.

In any case, the table shows that, as a whole, the Diesel-engine vehicles can operate in a stable manner at slower speeds than the gasoline-engine vehicles, which is a very desirable characteristic in traffic.

In regard to the roughness of the Diesel engine, it would seem that any undesirable roughness would be gradually overcome if the combustion characteristics of the engines and the fuels which they use are better understood, as it would seem to the writer that arrangements which will reduce the time lag of the ignition in the Diesel engine would make for smoother operation.

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OGP-54-4

Fuel-Spray Formation

By DANA W. LEE,[1] LANGLEY FIELD, VA.

Experiments are described in which the diameters of the fuel drops in sprays from different types of nozzles were measured and the effects of fuel-injection pressure, nozzle dimensions, and the density of the chamber air on the atomization of the sprays were determined. A simple method of determining the size and form of the high-velocity parts of sprays is also described. High-speed spark photographs of fuel sprays are shown that were made with lens systems having magnifying powers of 0.5, 2.5, and 10. These photographs reveal the process by which the fuel is transformed from the solid jet to the atomized spray, and also show the effect of several factors on spray dispersion.

ONE of the most difficult problems encountered in the development of high-speed compression-ignition engines has been the proper atomization and distribution of the fuel in the combustion chamber during the extremely short time available. Rapid combustion of the fuel does not take place as soon as it enters the combustion chamber, but a certain time, known as the ignition lag, elapses during which the temperature of the fuel is raised to its auto-ignition point by the absorption of heat from the compressed air. The duration of this ignition lag and the time required for complete combustion will decrease with a decrease in the size of the fuel drops, because of the increase in the ratio of the surface area to the volume of the drops.

However, the problems of atomization and distribution are closely connected, changes in one frequently causing changes in the other. This fact should be kept in mind during an investigation of either of these characteristics, and if possible the two should be studied simultaneously.

In this paper there are presented some of the results of the experiments conducted during the past two years in the laboratory of the National Advisory Committee for Aeronautics on the atomization and distribution of the fuel in fuel sprays. More complete accounts of these experiments may be found in the first three publications listed at the end of this paper.

Spray-Atomization Experiments

The sizes of the drops in fuel sprays were determined by catching them on smoked-glass plates, and then measuring the impressions which they made in the lampblack surface. The apparatus consisted principally of a common-rail fuel-injection system so arranged that it produced only a single injection at a time, an automatic fuel-injection valve, an airtight cylindrical chamber 18 in. long and 6 in. in diameter, smoked-glass plates, and a microscope with camera attachment. The chamber was mounted horizontally, and smoked plates 5 by 18 in. were placed on a shelf below the center line of the chamber. The injection valve was inserted through one end of the chamber, so that the spray axis was parallel to and $2^{5}/_{16}$ in. above the smoked plate. The chamber was connected to a compressed-air reservoir, so that the density of the air into which the fuel was injected could be raised to values corresponding to those in the combustion chambers of engines at the time of fuel injection. The fuel used for these atomization experiments, and for all other experiments described in this paper, was a high-grade Diesel fuel having a specific gravity of 0.86, a viscosity of 38.5 Saybolt Universal sec, and a surface tension of 0.00016 lb per in. at atmospheric pressure and at a temperature of 73 F. The injection period and the effective injection pressure were determined for each experiment from photographically recorded lift-time curves of the valve stems, using the method described by Rothrock. (See reference 4.) After each injection, ample time was allowed for the smallest measurable drops to settle to the lampblack, after which the smoked plate was removed from the chamber and placed under the microscope. By using a strong side illumination, the lampblack was made to appear as a light-gray surface, while the holes made by the fuel drops were left black. (See Fig. 1.) It was found that a magnifying power of 50 easily showed the smallest impressions that the lampblack was capable of recording, its granular structure being of such a magnitude that no impressions less than about 0.00025 in. in diameter were discernible. After a careful examination of each smoked plate, photomicrographs were taken of a series of representative areas. From these photomicrographs the impressions were measured and classified into groups, the first group containing all discernible impressions less than 0.00075 in. in diameter, the next group all those between 0.00075 and 0.00125 in., etc. These groups will be referred to hereafter by their group mean diameters—0.0005 in., 0.0010 in., etc.

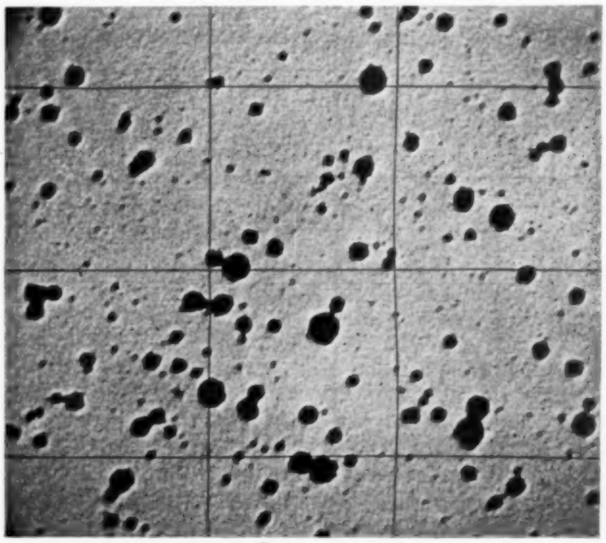

Fig. 1 Photomicrograph of Lampblack Surface Showing Impressions Made by Fuel Drops × 50

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[1] Junior Mechanical Engineer, National Advisory Committee for Aeronautics. Mr. Lee was graduated from Cornell University in 1929 with degree of Mechanical Engineer, and has been with his present connection since that time.

Presented at the Fifth National Meeting of the Oil and Gas Power Division of The American Society of Mechanical Engineers, State College, Pa., June 8 to 11, 1932.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

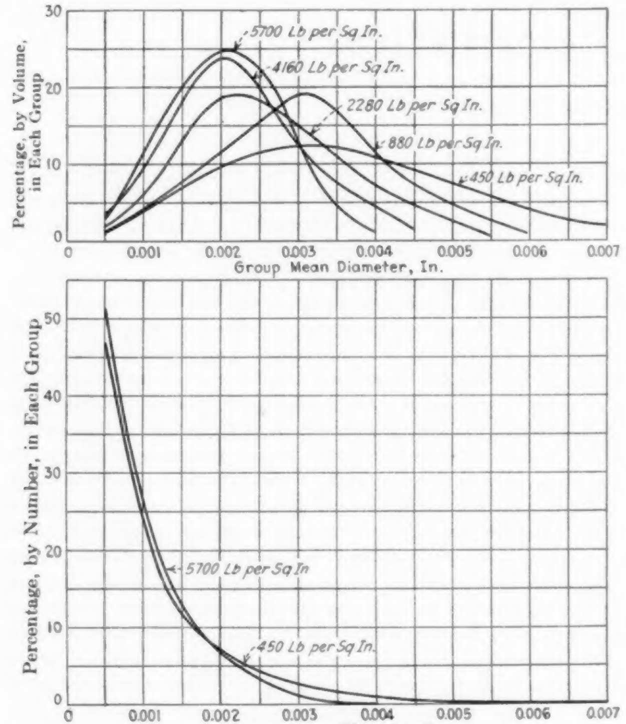

FIG. 2 EFFECT OF INJECTION PRESSURE ON SPRAY ATOMIZATION

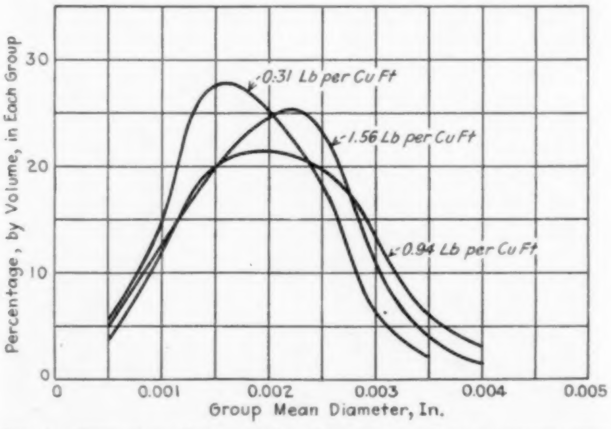

FIG. 3 EFFECT OF CHAMBER-AIR DENSITY ON SPRAY ATOMIZATION

The assumption that the impressions in the lampblack had the same diameter as the fuel drops that made them was checked by computing the weight of the fuel discharged (1) from the counted number of drop impressions and their diameters and (2) from the nozzle diameter, its coefficient of discharge, the effective injection pressure, and the injection period. The results of these computations are given in columns 10 and 11 of Table 1, which also gives the important test conditions and some of the test results. When it is considered that only about 0.1 per cent of the drop impressions was measured, the variation of about 50 per cent in the two computed weights seems surprisingly small.

The total number of drops in each spray was computed by multiplying the number of drops counted by the ratio of the area of the smoked plate to the area included in the photomicrographs. In column 12 of the table are given the arithmetical means of the drop diameters, and in column 13 are given the diameters obtained by taking the arithmetical mean of the drop volumes.

Curves plotted from the results of these atomization experiments are given in Figs. 2 to 5, and are of the type generally known as frequency curves. Group mean diameters were plotted against the percentage of the total number or total volume of the drops contained in that group. This method gave a series of independent points, rather than points on a curve, but faired curves were drawn through the points to make comparisons easier. Specific values can be obtained only at the group mean diameters. For example, at the group mean diameter 0.001 in. one may read: So many per cent of the total number (or volume) of the drops larger than 0.00025 in. in diameter were found to be between 0.00075 and 0.00125 in. in diameter.

The frequency curves of Fig. 2 show the effect of injection pressure on the atomization of fuel sprays. In the upper part of the figure the curves are based on the total volume of the fuel charge, and in the lower part they are based on the total number of drops in the spray. In the latter case the curves fall very close to each other, so that only the limiting cases were drawn, and in the following figures only the curves based on the volume of the fuel charge are shown.

Fig. 3 shows the results of the experiments made with different chamber-air densities, and Fig. 4 shows the effect of changing the nozzle diameter. In Fig. 5 the frequency curve for a centrifugal-type spray is compared with those for two sprays from plain nozzles. (Experiments Nos. 15, 3, and 13 of Table 1.)

Experiments made with a spray formed by two impinging fuel jets showed that its atomization was poorer than that of a plain spray under similar conditions, and a spray formed by a single fuel jet impinging on a metal lip was found to have the same fineness of atomization as a similarly produced plain spray. A series of tests was made using different nozzle length-diameter ratios, and the results showed no measurable differences in the atomizations.

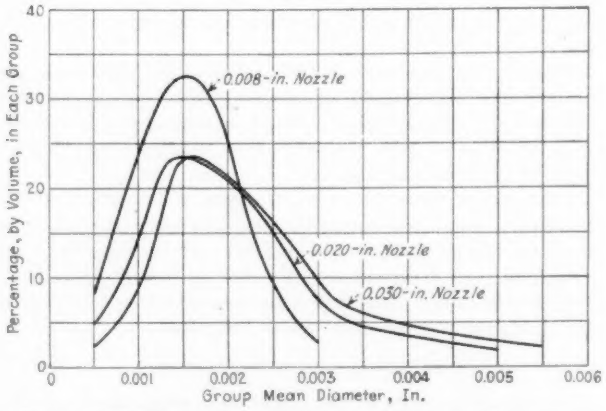

FIG. 4 EFFECT OF NOZZLE DIAMETER ON SPRAY ATOMIZATION

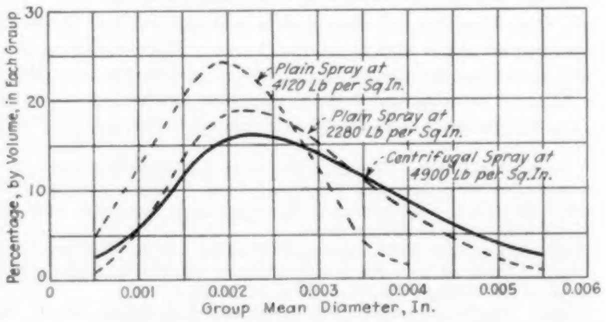

FIG. 5 ATOMIZATION CURVES FOR A CENTRIFUGAL-TYPE SPRAY AND FOR SPRAYS FROM A PLAIN NOZZLE

TABLE 1 TEST CONDITIONS AND COMPUTED RESULTS OF THE ATOMIZATION EXPERIMENTS

1 Experiment No.	2 Variable	3 Nozzle diameter, in.	4 Nozzle length-diameter ratio	5 Chamber-air density, lb per cu ft	6 Injection period, sec	7 Effective injection pressure, lb per sq in.	8 Number of drops measured N (corrected for overlapping)	9 Total number of drops in spray	10 Total weight of drops caught on lampblack, lb	11 Total weight of fuel discharged, lb	12 Arithmetic mean drop diameter $\frac{(\Sigma nd)}{N}$, in.	13 Volumetric mean drop diameter $\sqrt[3]{\frac{\Sigma(nd)^3}{N}}$, in.
1	Injection pressure	0.020	6	0.94	0.0041	450	3,240	9,580,000	0.000211	0.000125	0.00112	0.00177
2	Injection pressure	0.020	6	0.94	0.0049	880	6,225	18,400,000	0.000331	0.000210	0.00112	0.00170
3	Injection pressure	0.020	6	0.94	0.0046	2280	4,108	8,090,000	0.000441	0.000303	0.00113	0.00158
4	Injection pressure	0.020	6	0.94	0.0051	4160	16,572	16,568,000	0.000631	0.000473	0.00091	0.00128
5	Injection pressure	0.020	6	0.94	0.0046	5700	8,547	16,800,000	0.000574	0.000491	0.00095	0.00127
6	Chamber-air density	0.020	2	0.31	0.0040	4140	2,970	15,300,000	0.000337	0.000367	0.00082	0.00111
7	Chamber-air density	0.020	2	0.94	0.0040	4140	2,316	11,900,000	0.000355	0.000362	0.00088	0.00123
8	Chamber-air density	0.020	2	1.56	0.0040	4140	2,388	12,320,000	0.000311	0.000356	0.00083	0.00116
9	Nozzle diameter	0.008	6	0.94	0.0015	3920	1,049	4,060,000	0.000065	0.000021	0.00077	0.00100
10	Nozzle diameter	0.020	6	0.94	0.0015	3920	5,005	3,590,000	0.000084	0.000132	0.00094	0.00115
11	Nozzle diameter	0.030	6	0.94	0.0015	3900	4,145	4,520,000	0.000178	0.000296	0.00095	0.00134
12	Nozzle length-diameter ratio	0.020	0.5	0.94	0.0040	4140	2,185	11,280,000	0.000320	0.000362	0.00089	0.00121
13	Nozzle length-diameter ratio	0.020	6	0.94	0.0040	4120	2,560	13,200,000	0.000351	0.000361	0.00085	0.00118
14	Lip-nozzle spray	0.018	4	0.94	0.0015	3600	3,077	3,770,000	0.000078	0.000102	0.00079	0.00109
15	Centrifugal spray	0.020	6	0.94	0.0045	4900	4,231	8,330,000	0.000391	0.000175	0.00094	0.00143

When lip-nozzle, impinging-jet, centrifugal, and other types of high-dispersion sprays are injected into the atmosphere, they always appear to be more finely atomized than corresponding sprays from plain nozzles. However, these atomization experiments have shown that this is not the case and that spray atomization cannot be judged by visual observation.

Figs. 6 and 7 compare some of the results of the present experiments with those obtained by other investigators. The various experiments on the effect of injection pressure agree in general, although the range of drop diameters reported varies considerably. Sass found that an increase in the air density resulted in a finer atomization, whereas the N.A.C.A. experiments indicated that this factor had no effect. Wöltjen gives no data for the effect of air density, but states that his experiments showed that it had no effect on the atomization.

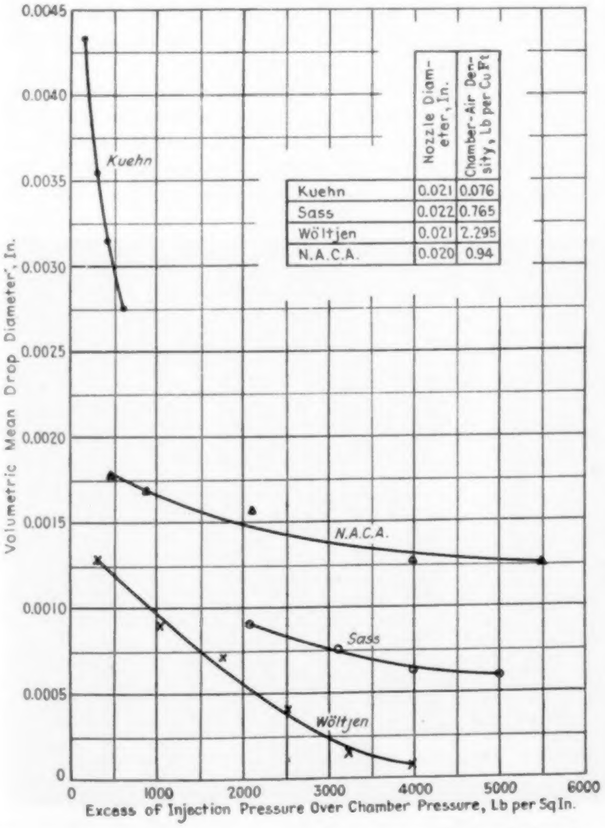

FIG. 6 COMPARISON OF RESULTS OF VARIOUS INVESTIGATORS ON EFFECT OF INJECTION PRESSURE ON VOLUMETRIC MEAN DROP DIAMETER

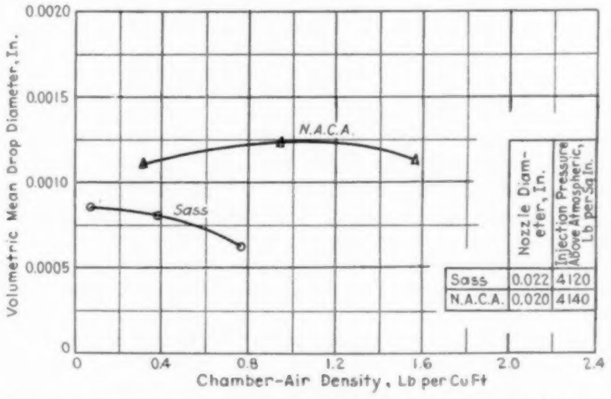

FIG. 7 COMPARISON OF RESULTS OF SASS AND THE N.A.C.A. ON EFFECT OF CHAMBER-AIR DENSITY ON VOLUMETRIC MEAN DROP DIAMETER

The results of the N.A.C.A. atomization experiments may be briefly summarized as follows:

1 Each spray is composed of several million fuel drops, whose diameters range from less than 0.00025 in. to 0.0050 in., and sometimes more. By far the greatest number of drops have diameters of 0.0010 in. or less, but those between 0.0015 in. and 0.0025 in. usually contain more than half the weight of the fuel charge.

2 Within the range of nozzle sizes and operating conditions commonly used, the variation in the mean drop diameter is small.

3 If the fuel-injection pressure is increased or the size of the discharge orifice decreased, there will be a decrease in the relative number of large drops, resulting in a more uniform atomization and a smaller mean drop size.

4 The density of the air into which the fuel is injected and the nozzle length-diameter ratio have little effect on the final sizes attained by the drops.

5 Impinging of a fuel jet against a metal surface close to the orifice results in no measurable change in the atomization.

6 Centrifugal-type sprays are less finely atomized than sprays from plain cylindrical nozzles produced at equal injection pressures.

EXPERIMENTS WITH PLASTICINE TARGETS

Single sprays from automatic valves were injected against

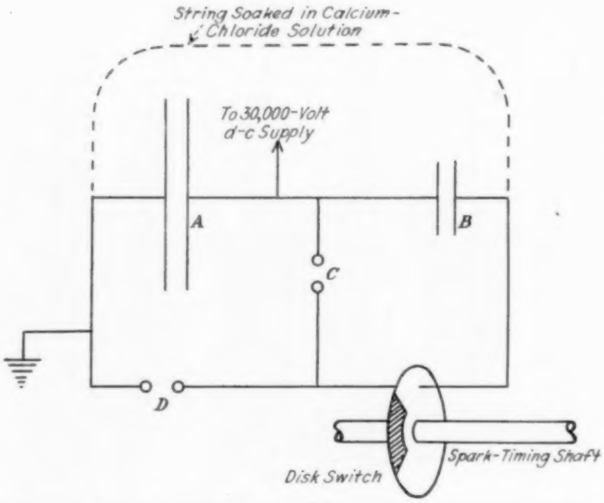
String Soaked in Calcium-Chloride Solution
To 30,000-Volt d-c Supply
A
B
C
D
Disk Switch
Spark-Timing Shaft

FIG. 8 CIRCUIT USED TO PRODUCE ILLUMINATING SPARKS OF SHORT DURATION
(*A*, *B*, condensers; *C*, *D*, spark gaps.)

pieces of Plasticine, a proprietary substitute for modeling clay, mounted in the chamber that was used for the atomization experiments. Table 2 lists the diameters of the impressions made in the Plasticine targets when they were mounted at different distances from the nozzle. At a distance of 3.5 in. the impressions were very faint, and at 4 in. no impression could be seen. Two sets of targets were used, one set thick enough to completely stop the spray and the other consisting of thin slices backed by wire screening. Table 2 also lists the corresponding outside diameters of a spray produced under the same conditions, as measured from a spray photograph. The impressions in the thin targets indicate the diameter of the high-velocity core of fuel in the center of the spray, the rest of the spray being composed of fuel particles having a much lower velocity. The deepest impression made in the thick targets was 1 in., when the distance from the nozzle to the target was 0.25 in. The bottom of each impression was roughly conical in form. The deeper ones showed evidence of having been enlarged, as if by a tiny explosion, the diameters of the holes below the surface of the Plasticine being greater than at the surface. These experiments confirm the opinion that fuel sprays from cylindrical nozzles create strong blasts of air at the center of the spray, which enable some of the atomized particles to penetrate farther from the nozzle than they could in still air.

Plasticine targets were also used to determine what parts of other types of sprays had the greatest penetrating power. A centrifugal spray made a circular impression, in the center of which was a small hole similar to those made by plain nozzles, and a slit nozzle made an impression that looked like a pair of parentheses. The use of plastic targets in this manner should be useful in the development of new types of nozzles and injection systems, as it furnishes a cheap and ready means of measuring the relative penetrating power of different sprays or parts of the same spray.

TABLE 2 RESULTS OF PLASTICINE TARGET EXPERIMENTS

Distance from nozzle to target, in.	Diameter of impression at surface of target, in.		Outside diameter of spray from photograph, in.
	Thin target	Thick target	
0.25	0.04	0.08	0.25
1.0	0.08	0.20	0.50
2.0	0.14	0.25	0.75
2.5	0.18	0.20	0.94
3.0	0.16	0.12	1.10

NOTE: 0.020-in. plain nozzle; 4000-lb-per-sq-in. injection pressure; 1.1-lb-per-cu-ft air density; 0.005-sec injection period.

SPARK PHOTOGRAPHY

The study of the process of fuel-spray formation is being continued at the present time by means of spark photography, using the spark-producing circuit shown in Fig. 8. Closing the disk switch allows the small condenser *B* to discharge across the auxiliary spark gap *C*. The air in gap *C* being ionized, the main condenser *A* then discharges across both gap *C* and the illuminating gap *D*. The duration of the discharge across *D* has been estimated to be $^1/_{10,000,000}$ sec, which is from ten to a hundred times faster than the discharges that have been used with the original N.A.C.A. spark-photography equipment. An adjustable coupling connects the spark-timing shaft to the camshaft controlling the fuel injection, so that any stage of the injection may be photographed.

No attempt is made in the present paper to cover all of the work being done along these lines, but a few of the photographs are shown, accompanied by a brief explanation.

The photographs of Fig. 9 show the effect of the density of the chamber air on the formation of fuel sprays from plain cylindrical nozzles. They were made at one-half natural size and the illumination was from the side. When different nozzles were used to produce sprays in the evacuated chamber, it was found that the

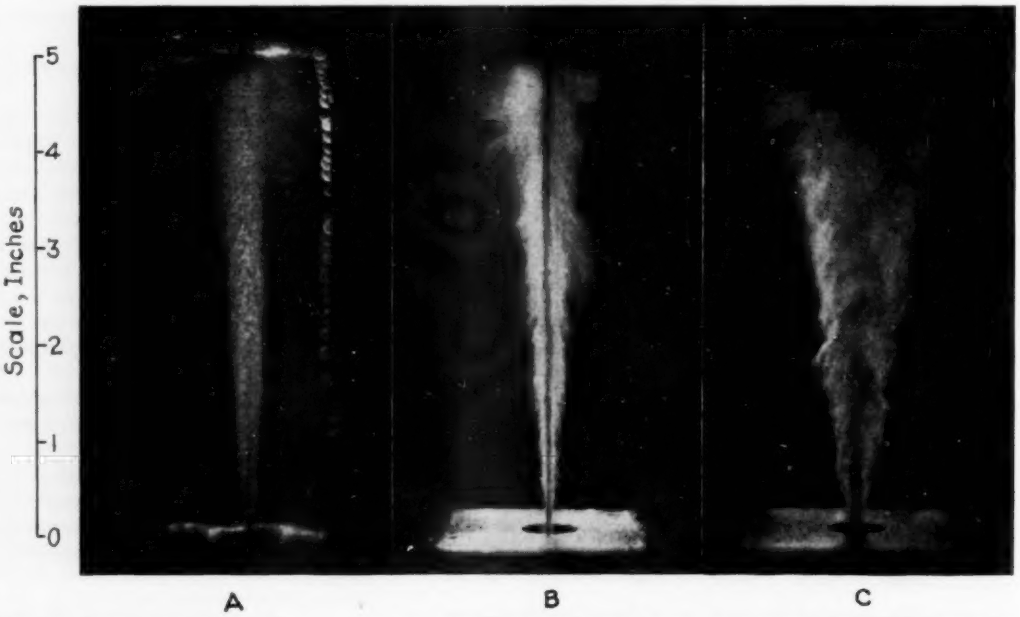

FIG. 9 HIGH-SPEED SPARK PHOTOGRAPHS OF FUEL SPRAYS INJECTED INTO AIR HAVING DIFFERENT DENSITIES
(Injection pressure, 4000 lb per sq in. Chamber-air density: *A*, 0.0005 lb per cu ft; *B*, 0.076 lb per cu ft; *C*, 1.1 lb per cu ft.)

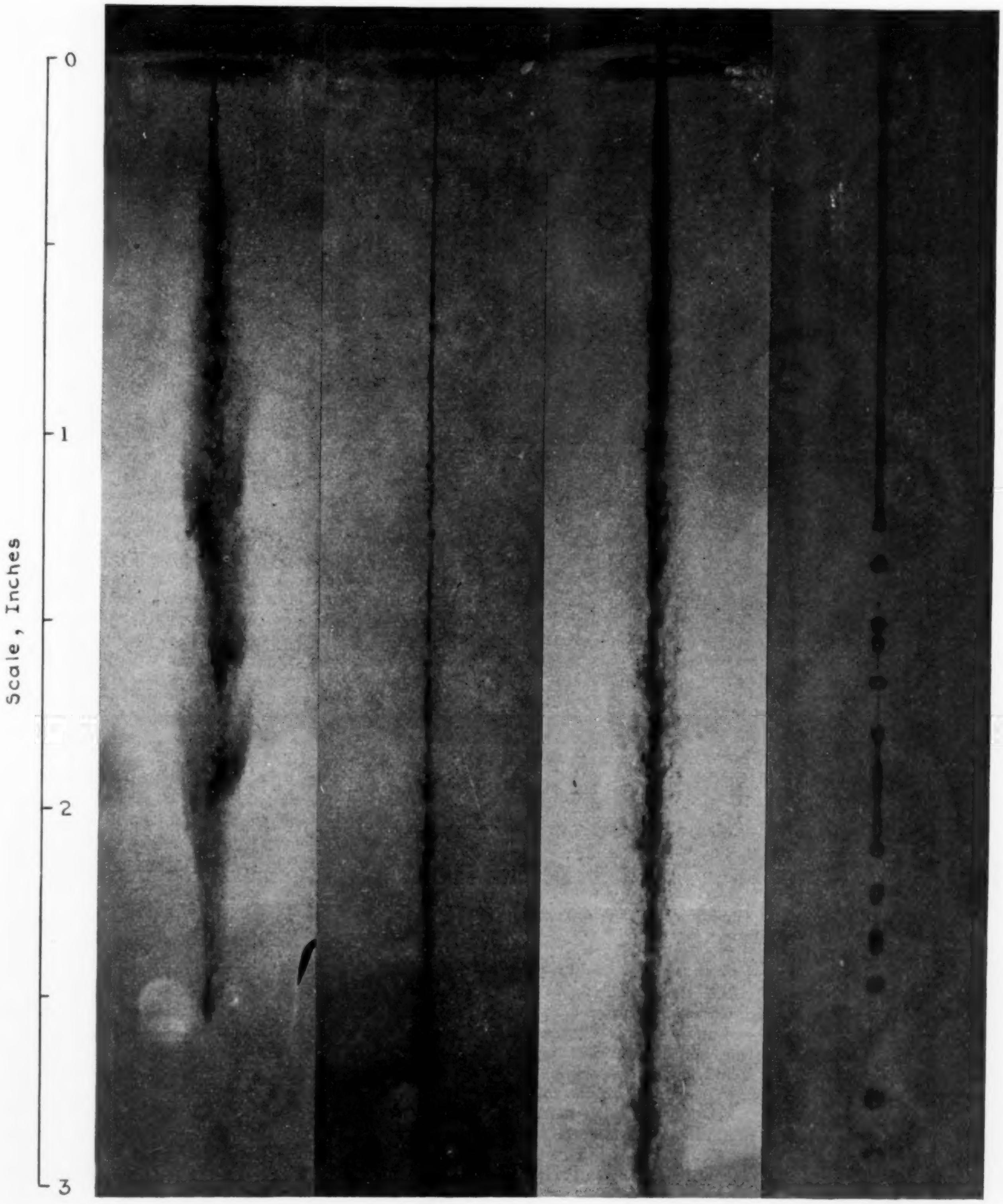

Fig. 10 Silhouette Photographs of Fuel Sprays Injected Into the Atmosphere

(Nozzle diameter: *A*, *B*, 0.008 in.; *C*, *D*, 0.020 in. Injection pressure: *A*, *B*, 1500 lb per sq in.; *C*, 1000 lb per sq in.; *D*, 10 lb per sq in.)

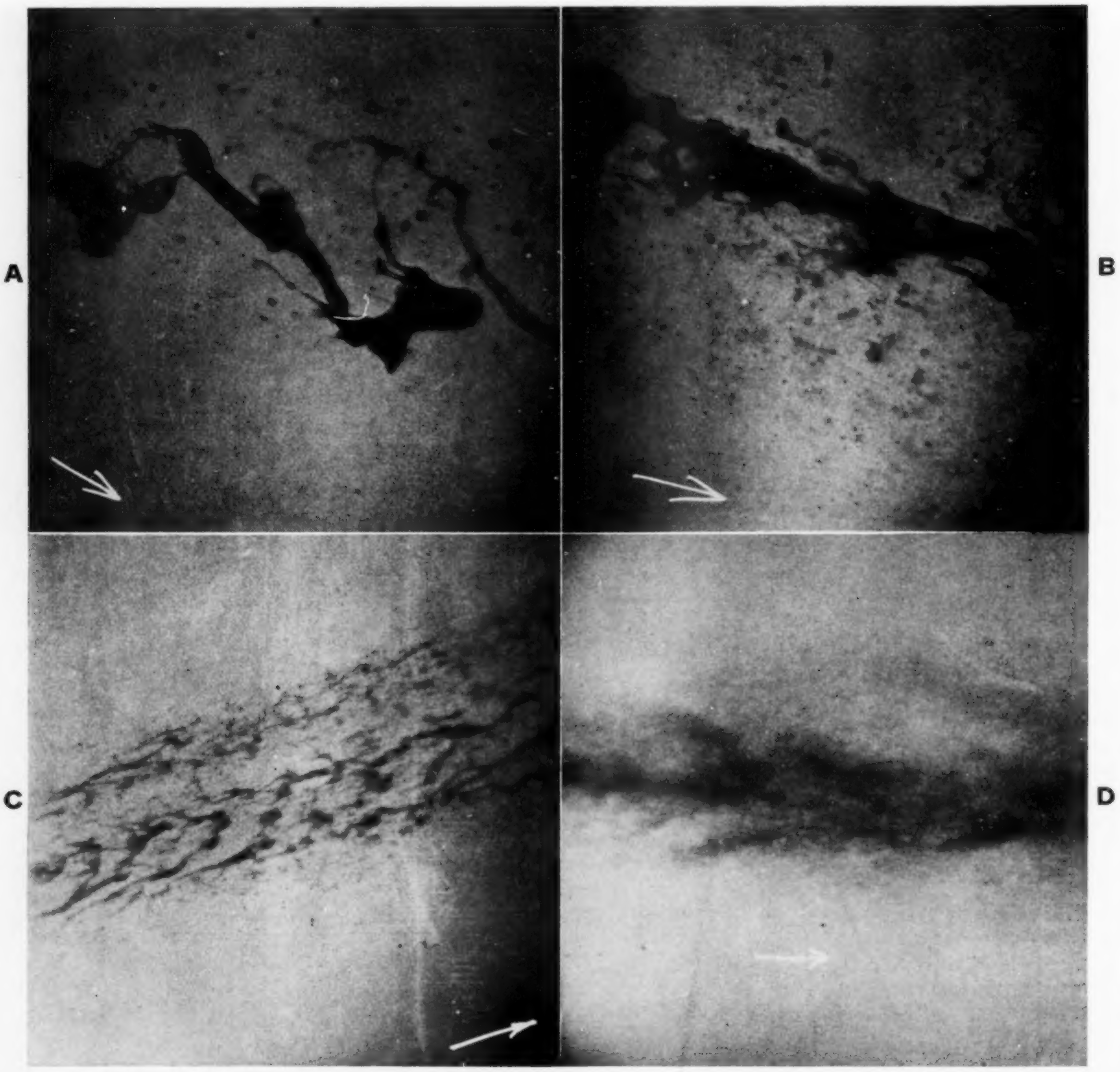

Fig. 11 Photomicrographs of Fuel Sprays Showing the Atomizing Process ×10

(*A:* Nozzle diameter, 0.020 in. Distance from nozzle, $4^3/_4$ in. Air density, 0.076 lb per cu ft. Injection pressure, 140 lb per sq in. *B:* Nozzle diameter, 0.020 in. Distance from nozzle, $4^3/_4$ in. Air density, 0.076 lb per cu ft. Injection pressure, 625 lb per sq in. *C:* Nozzle diameter, 0.008 in. Distance from nozzle, $^7/_8$ in. Air density, 0.076 lb per cu ft. Injection pressure, 1000 lb per sq in. *D:* Nozzle diameter, 0.008 in. Distance from nozzle, $1^1/_4$ in. Air density, 1.01 lb per cu ft. Injection pressure, 1000 lb per sq in.)

spray dispersion varied greatly. In some cases, such as that shown in Fig. 9A, the sprays were nearly as well dispersed as those injected into the atmosphere, but in other cases the dispersion was very slight, even at high injection pressures. The differences are thought to be due to irregularities in the nozzles. In one case a nozzle which gave a well-dispersed spray was found to be very slightly rusted. After it was polished, the spray dispersion was much less.

Photographs like those in Fig. 9 give the general outline of the spray and much other useful information, but they do not show the structure of the spray core nor teach much about the atomization process. When higher magnification powers were tried, it was found necessary to photograph the sprays in silhouette. Such photographs reveal the structure of the core, but do not show the fine cloud of spray that surrounds it. This fact should be remembered when examining the photomicrographs of the fuel sprays.

In Fig. 10 is shown a series of spray photographs made at $2^1/_2$ times natural size. The chamber air was at atmospheric conditions in each case. Fig. 10A shows the early part of a spray from an 0.008-in. cylindrical nozzle, and Fig. 10B shows a later stage of another spray made under the same conditions. Notice that the first part of the fuel discharged was more widely dispersed than that which followed. It seems likely that the more widely dispersed fuel was that which had remained in the space between the stem seat and the orifice since the previous injection. This difference in dispersion was even more pronounced in the evacuated chamber, but in air at densities greater than atmospheric it was hardly noticeable. Fig. 10C shows a spray from a 0.020-in. nozzle, the photograph being taken during the latter part of the injection period. Fig. 10D shows a stream of fuel flowing from a 0.020-in. nozzle at a very low velocity. This photograph is included to illustrate how a column of fuel is broken into different-sized drops by its surface tension.

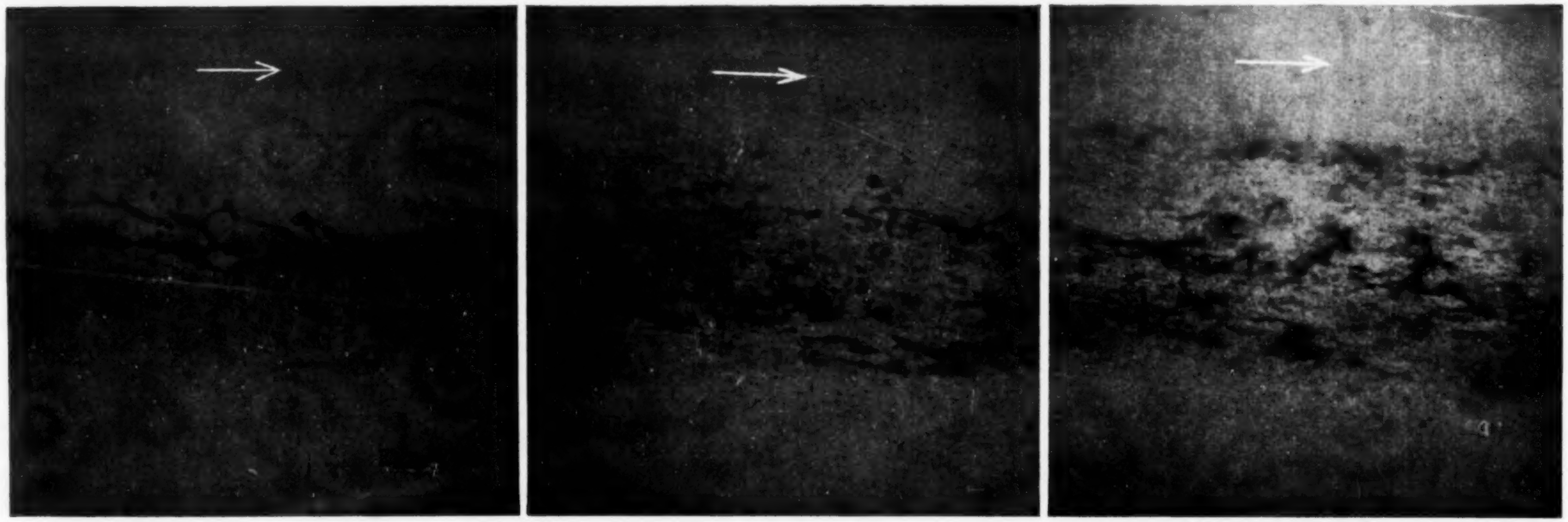

Fig. 12 Photomicrographs Showing Effect of Injection Pressure on Spray Characteristics ×10
(Nozzle diameter, 0.008 in. Distance from nozzle, $1^7/_8$ in. Air density, 0.076 lb per cu ft. Injection pressure: *A*, 160 lb per sq in.; *B*, 500 lb per sq in.; *C*, 1650 lb per sq in.)

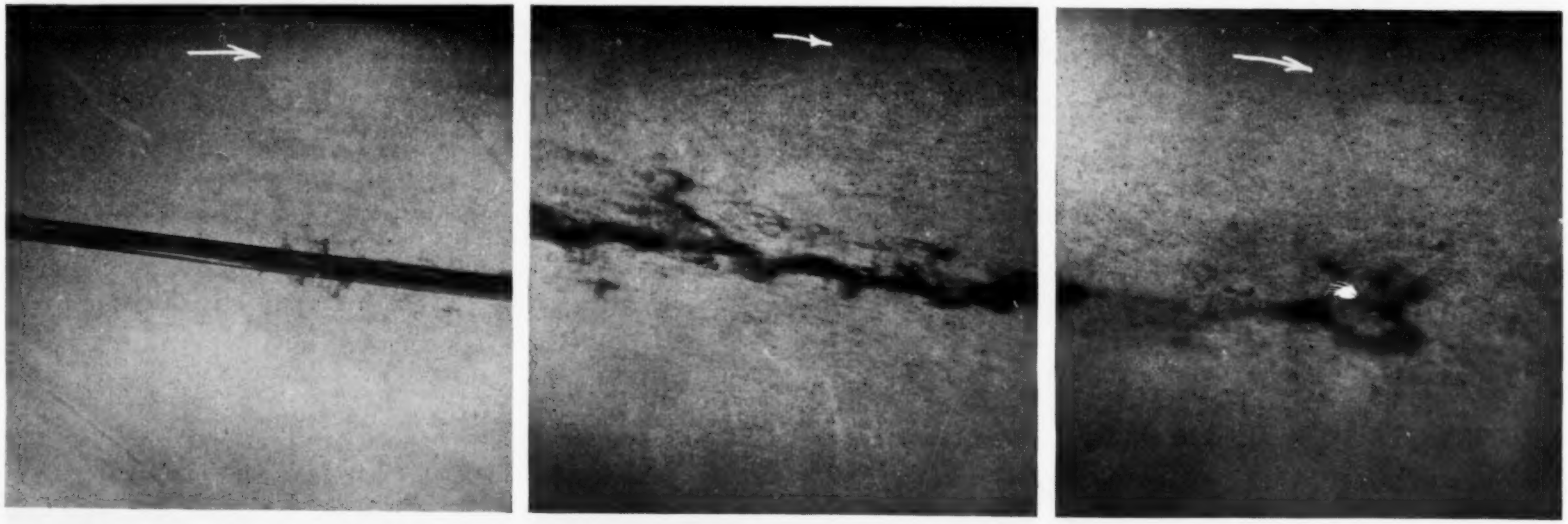

Fig. 13 Photomicrographs of Similar Sprays Made at Different Distances From the Nozzle ×10
(Nozzle diameter, 0.014 in. Injection pressure, 1000 lb per sq in. Air at atmospheric conditions. Distance from nozzle: *A*, $^1/_2$ in.; *B*, $2^1/_2$ in.; *C*, $5^1/_2$ in.)

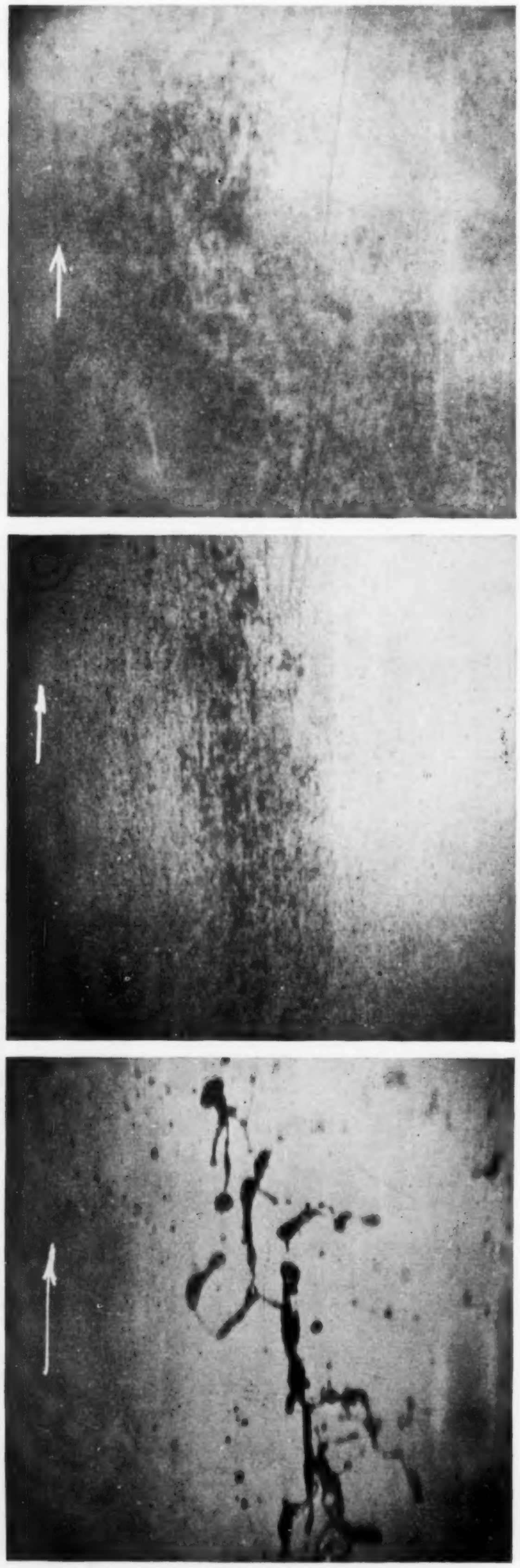

FIG. 14 PHOTOMICROGRAPHS SHOWING EFFECT OF AIR DENSITY ON FUEL SPRAYS ×10

(Nozzle diameter, 0.008 in. Injection pressure, 1000 lb per sq in. Distance from nozzle, 3 in. Air density: *A*, 0.0003 lb per cu ft; *B*, 0.076 lb per cu ft; *C*, 1.01 lb per cu ft.)

Photomicrographs of fuel sprays made at a magnifying power of 10 are shown in Figs. 11 to 14. The direction of the spray is indicated in each case by an arrow. Many of the photomicrographs are strikingly similar to the photographs made by Scheubel of fuel being atomized in a model carburetor, and indicate that the atomization processes in carburetors and in Diesel sprays are very similar, if not identical. This similarity was foreseen by R. A. Castleman, Jr., who has advanced an explanation of air-stream atomization based on the friction between the fuel and air. He says: "A portion of the large mass is caught up (say, at a point where its surface is ruffled) by the air stream and, being anchored at the other end, is drawn out into a fine ligament. This ligament is quickly cut off by the rapid growth of a dent in its surface, and the detached mass, being quite small, is swiftly drawn up into a spherical drop." All of the stages described by Castlemen may be found in the photomicrographs of Fig. 11. The ligaments are largest, and their breakdown slowest, at low fuel-jet velocities, large nozzle diameters, and low air densities.

The photomicrographs of Fig. 12 were made with different injection pressures to show the effect of fuel velocity on the disruption of the jet. In Fig. 13 the photomicrophographs show how a jet appeared at different distances from the nozzle. These two figures indicate that spray dispersion becomes greater and the size of the fuel particles becomes smaller as the injection pressure is increased or as the distance from the nozzle is increased.

In Fig. 14 a series of photomicrographs is shown for which the only variable is the density of the chamber air. They indicate that, at a given distance from the nozzle, the atomization and dispersion improve with an increase in the air density.

The atomization tests reported earlier in this paper showed that a change in the air density had no decided effect on the atomization, whereas the experiments of Sass showed that an increase in the air density resulted in a finer atomization. The photomicrographs of Figs. 13 and 14 offer a possible explanation of these different results. They indicate that the atomizing process begins as soon as the fuel leaves the nozzle and continues until the friction between the fuel and the air can no longer produce ligaments. Although at equal distances from the nozzle the atomization seems to vary as the density of the air, the total penetration of the sprays will also be dependent on the air density, so that the final atomization may be the same for all densities. Sass caught the fuel particles after they had traveled only about 8 in., and it may be that the atomizing process had not been completed at that distance, especially at the lower air densities. In the N.A.C.A. experiments, it is known that the fuel particles had lost all of their velocity relative to the air before they were caught on the lampblack.

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Discussion

R. A. Castleman, Jr.[2] It has been noted by several investigators that at a small fraction of atmospheric density a jet appears to the unaided eye about the same as at atmospheric; and this has been taken to mean that the atomization is about the same in the two cases. Fig. 14 of the paper seems to advance definite evidence of the incorrectness of this deduction; for this shows that the "atomization" at about 1/250 of atmospheric is noticeably coarser than at atmospheric.

Fig. 14 seems also to throw further light on the mechanism of atomization, and to favor the ligament theory. In the first picture, the influence of air friction being almost entirely removed, the only forces left to act seem to be those due to turbulence in the jet or the tension of its bounding surface, and the result is a rather coarse disintegration. The large drops that result can be further subdivided to form the spray only when the air friction is given a chance to cause ligaments to be formed, by raising the air density to an appreciable value, as in the next picture.

The explanation that the author offers, in the last paragraph of his paper, of the discrepancy between his results and those of Sass seems quite plausible, but it should be pointed out that it is not the only possible one. Indeed, another seems equally probable (perhaps the true explanation is a combination of this and the effect pointed out by the author). Fig. 14 indicates that increase of air density from a small fraction of an atmosphere to atmospheric (pictures *A* and *B*) causes a much greater decrease in drop size per atmosphere than further increase from 1 to 13 atmospheres (pictures *C* and *D*). Both Lee and Sass made their measurements at super-atmospheric pressure, Sass ranging from 1 to 10 atm. and Lee from about 4 to 20 atm. It seems quite possible that, methods of measurement and of presenting results not being entirely comparable in the two cases, the effect might show up better in Sass' work, which seems more direct.

An analogous result was found in the case of the air-stream atomization of alcohol by F. N. Scheubel (reference 10, pictures I_6, I_5, and I_4), who found that doubling the air velocity from a comparatively low value produced a noticeable decrease in drop size, but that again doubling it resulted in little further change.

Professor Schweitzer, before the meeting, pointed out that, in a case where there is streamline flow in the orifice and therefore a parabolic velocity distribution with zero flow at the wall, it would be some distance beyond the orifice before the relative velocity at the edge of the jet would build up to a value high enough for atomization; and therefore that in such cases internal nozzle design might affect atomization. If, however, there is turbulent flow in the orifice, the mean velocity persists very close to the wall, and although the thin wall layer of zero velocity would in this case also be left behind in the orifice, the relative velocity at the edge of the jet would soon approach that given by Bernoulli's law and the coefficient of velocity. This is probably true, and pure streamline flow in the orifice should be avoided; which does not seem to be at all difficult with plain round-hole orifices of the sizes usually employed, at injection pressures customary in practice.

That such exceptional conditions can be imagined seems no argument against the general validity of the author's results. Rather this circumstance furnishes an illustration of a fact to which the writer has called attention in an N.A.C.A. report, entitled "The Mechanism of Atomization Accompanying Solid Injection," now in press. While the mechanism of air-stream atomization and of solid injection appear identical physically, in the case of solid injection it seems to be more difficult so to arrange matters that the surface of the liquid is continually exposed to an air stream of relative velocity high enough for atomization; and that therefore, under apparently similar circumstances, air-stream atomization may be expected to produce a more uniformly fine spray. By this it is not intended to imply any general preference for air-stream atomization. Circumstances (penetration, thermodynamic, and structural considerations, etc.) must determine the preferential form in each case. The two processes form useful complements.

To Professor DeJuhasz belongs the credit for first pointing out that air-stream atomization and solid injection do not necessarily yield identical results in all cases.

It may be well to briefly summarize the status of solid injection:

In 1910 James McKechnie, seeking a useful substitute for the bulky and heavy air compressor necessary in Diesel's old form of high-pressure air injection, found that a liquid could be atomized by injecting it under high pressure into comparatively still air. This invention, however, was largely undeveloped until comparatively recently, when it was found to be quite useful in the development of the high-speed compression-ignition engine.

Early in 1930 the writer suggested in *The Physical Review*, on the basis of the very brief life periods of ligaments fine enough to break up into the drops observed in atomized sprays, that the mechanism of both air-stream atomization and solid injection could be explained by the formation and collapse of fine ligaments at the liquid's surface; and in an article in *The Journal of Research* of the Bureau of Standards about a year later, he amplified this theory and pointed out a probable means of ligament formation, due to the relative motion of gas and liquid. Dr. Scheubel, of the Aachen Technische Hochschule, published (the author's reference 10) some spark photographs that he had obtained in a model of a carburetor throat, which verified the ligament theory for air-stream atomization. While he offered no explanation, his pictures fitted the ligament theory so well that he allowed the writer to reproduce some of them in his second paper (reference 11).

Dr. Sass (reference 6) reported results of drop-size measurements which could be interpreted in a manner to throw light on the mechanism of solid injection, but he also gave no interpretation. Early in 1931 Dr. Haenlein, of the Dresden Technische Hochschule, published (reference 12—English translation in N.A.C.A. Tech. Mem. No. 659) work on the influence of air friction on the disintegration of liquid jets, which he indicated would lead to atomization at high relative air speed; and soon after this Professor DeJuhasz advanced experimental evidence which he claimed to disprove the predominating influence of surface forces, and Professor Schweitzer has shown how certain conditions of flow in the orifice may delay the action of surface forces. Mr. Heldt favors the influence of turbulence in the nozzle. The writer has discussed the works of Sass, Haenlein, DeJuhasz, and the N.A.C.A., and the turbulence and ligament theories, in the N.A.C.A. report mentioned; but the author presents more direct evidence that the small drops are formed only by the formation and collapse of fine ligaments.

He is to be commended on having done a very important and difficult piece of experimental work so well.

R. A. Castleman, Jr.[3] The first picture in Fig. 11 seems to advance definite evidence against two theories: (1) Haenlein, of the Dresden Technische Hochschule, observed that a liquid jet became wavy under the influence of relative air motion, and since this waviness increased with the relative air motion, he indicated that the result of sufficiently high relative motion would be the complete disintegration of the jet into fine particles, or its "atomization." However, the author also finds waviness, but that the small drops are detached from the large drops only by means of the formation and collapse of ligaments. (2) It was pointed out by A. L. Bird, of Cambridge University, that the flow in a small orifice is intermediate between the streamline and the turbulent condition. This has been taken by some to mean that the liquid suddenly becomes turbulent when it issues from the orifice, and therefore "blows up." The author shows that this is not the case; for this would occur immediately beyond the orifice, while the author finds atomization as much as 4 in. from the orifice.

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[2] East Falls Church, Va.

[3] Bureau of Standards, Washington, D. C.

Edward Adams Richardson.[4] Colonel Allen remarked upon the necessity for considering the various factors that control combustion, a point emphasized in the writer's paper, "Heating Fuels for Injection Engines." In this connection it might be well to suggest how such an analysis applies in the particular case of the Oberhaensli engine described by Neumann in an earlier paper.

Turbulence has two purposes in an engine; the securing of better atomization or the securing of better mixing. K. O. Keller, of the Doxford Engine Company, presented a paper, "Combustion and Its Difficulties in Marine Oil Engines," before the North-East Coast Institution of Engineers and Shipbuilders at Newcastle-on-Tyne, December 7, 1928. In this paper it was shown that one complete passage of the air through the oil jets was sufficient for mixing purposes and more was undesirable. Dr. R. A. Castleman, Jr. (Res. Paper No. 281, Bur. Stand. Jour. Research, March, 1931) and F. N. Scheubel (referred to by Castleman) have shown that about the maximum practicable degree of atomization is obtained at relative velocities of fuel and air of 350 ft per sec. It is important to separate these two functions in studying combustion.

In the Oberhaensli engine, we find air velocities, according to the air-flow diagrams, of nearly 400 ft per sec, more than sufficient for a high degree of atomization. It is interesting to note that the fuel is fed at low pressure, so advantage is taken of the turbulence to reduce fuel pressures. As regards mixing, we find that the air may be expected to pass at least 10 times through the fuel jet during combustion. This is much more than is necessary.

Mr. Kuttner has seen fit to emphasize the advantages of turbulence in the Oberhaensli engine. In order to avoid possible misunderstanding, it seems well to carry the analysis further.

It must be remembered, as has been shown in the writer's paper, that excessive turbulence, from the standpoint of mixing, is undesirable. Hence, 10 times as much turbulence as is necessary cannot be regarded as a particular recommendation for an engine.

Mr. Kuttner has stated that since there is ample turbulence at high speed, the turbulence (which he assumes to have a constant angular rate irrespective of the speed of the engine) should be sufficient at any lower speed. Now it is closely true that the degree of turbulence is proportional to the engine speed. If this were not so, we should have much more trouble than we do securing combustion in any sort of high-speed engine. Hence the excessive turbulence must continue to all lower practicable speeds, remaining roughly 10 times too great.

It must be remembered that heat losses by convection vary roughly as the 0.8th power of the linear velocity. Hence, with turbulence 10 times too great for mixing, we tend to have a heat loss nearly 7 times that necessary. It would be better, we might say, to increase the fuel pressure for atomization purposes and reduce the turbulence very materially to that just sufficient for mixing purposes.

Of course, in this particular engine, the heat losses have been minimized by a particular form of construction in which the combustion chamber walls are in poor thermal contact with the rest of the engine; hence the temperature drop is lower than normal, thereby offsetting the losses due to increased turbulence. It is most important in studying this engine, or any other for that matter, to consider the question of turbulence versus heat loss. Excessive turbulence, as in this case, has forced the adoption of a detached or divided heat-storing combustion chamber. We must not attach too much importance to surface-volume ratio, the effect of which is relatively moderate, when we are taking special pains to increase fluid velocities over those surfaces. For small engines, it is not turbulence but rather the reduction of turbulence that should receive attention. Higher fuel pressures to insure proper atomization, and the limitation of turbulence to that just sufficient to pass the air once through the jets, would appear to be a proper approach to the problem. The so-called advantages of subdivided combustion chambers would be much less apparent, provided both engines were designed for the same degree, and a very moderate one, of turbulence. Surface-volume ratio, though of some importance, would then make a less imposing case. Even in this engine it is very difficult to maintain suitable ignition conditions for any appreciable time under light load, judging by the curves for surface temperature of the combustion chamber. Of course the surface-volume ratio has been accentuated by the form of construction.

Following Colonel Allen's suggestion to consider the various factors involved in combustion, we find that there is much that we can learn concerning the mode of action of a given engine and the significance of the various features of design. What is more to the point, the step-by-step method of analyzing the factors is a relatively simple procedure. If we were to take slightly more trouble in the analysis and estimate more quantitatively the various quantities involved, we should have a much clearer picture. The writer has emphasized in his paper the necessity of sticking close to basic principles in our studies.

H. K. Cummings.[5] The experimental work at Langley Field on the physical structure of oil sprays is an important contribution to our knowledge on this subject, and the author's technical report on the atomization and distribution of fuel sprays (reference 1) will be awaited with much interest.

A translation of Haenlein's dissertation (reference 12), issued by the N.A.C.A. as Technical Memorandum No. 659, includes shadow pictures of the disintegration of a gas-oil jet which resemble the spray photographs shown in Fig. 10. As the author points out, the microphotographs in Figs. 11 to 14 support the ligament-formation theory advanced by Castleman (reference 11) as explaining both air-stream atomization and the atomization accompanying solid injection. A paper by Dr. Castleman on "The Mechanism of Atomization Accompanying Solid Injection" is to be issued by the N.A.C.A. as Technical Report No. 440.

P. H. Schweitzer.[6] The writer is interested in the paper because we have been doing work along similar lines attacking the same problems by different methods. The author's results largely check with ours.

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[4] Bethlehem, Pa. Jun. A.S.M.E.

[5] Chief, Automotive Power Plants Section, Bureau of Standards, Department of Commerce, Washington, D. C.

[6] Associate Professor, Pennsylvania State College, State College, Pa.

Regarding atomization, the lampblack receiver no doubt gives comparative information on the effect of various factors. In our atomization experiments, Lieutenant Thew, U. S. Navy, used a liquid receiving medium (Queol), and he found, as did the author, that the air pressure and the orifice diameter above 0.015 in. have little effect on atomization, which depends mostly on the oil pressure and viscosity. But as to the absolute magnitudes we are not convinced that the lampblack method does give correct droplet sizes, not because the diameters would suffer appreciable distortion, but because the small drops cannot be identified. The smallest drops that the author counted were taken as having 0.0005 in. diameter. We have used 200 magnification on the liquid samples, and our results were between Sass' and Woeltjen's. We found that in every case over 50 per cent of the droplets were smaller than 5 microns (0.0002 in. in diameter) and usually more than 30 per cent smaller than 2.5 micron (0.0001 in. in diameter).

It is feared that with the lampblack method these small globules are ignored. The error incurred certainly would not show up in the weight balance, as can be seen from the following example: In Experiment No. 5, 16,800,000 globules were determined as having a mean size of 0.00127 in. in diameter. If an equal number of globules (16,800,000) were present having a diameter of 3.75 micron, the difference in total weight would have been only about one-tenth of 1 per cent, which of course cannot be detected. But regarding the weight checking, one would like to know why the weight of the deposited oil drops was calculated, which involves some uncertainty, instead of determining the deposits on the lampblack by weighing.

The use of Plasticine for determining spray penetration is ingenious and commendable. Manufacturers should make use of this practical method for comparing sprays as to penetration. By the electrical balance method the writer has found that the jet velocity increases considerably from zero to about 0.4 in. distance from the nozzle and then it begins to drop. It is wondered if the Plasticine experiments gave any similar results.

The microphotographs of the oil sprays are extremely interesting. Fig. 14 shows, what we also found, that atomization and dispersion do take place even in vacuum; therefore the air friction cannot be the sole cause of disintegration. Our explanation of jet disintegration is described in Bulletin No. 40 of the Engineering Experiment Station, which has just been published.

Author's Closure

It is interesting to notice that two of the discussors refer to the photomicrograph of a spray in a vacuum, shown in Fig. 14. Dr. Castleman believes that it supports his ligament theory, and Professor Schweitzer cites it to show that air friction is not the sole cause of jet disintegration. The many additional photomicrographs of sprays in a vacuum and in air that have been taken since this paper was presented show that both of these views are correct. In a vacuum, the disintegration of the jet and the dispersion of the fuel particles depends on the Reynolds number for the fuel flow and on the nozzle characteristics. If the flow is turbulent, the jet spreads out into thin sheets, which later split into parts and the parts are drawn up into large drops by the force of surface tension. If the flow is non-turbulent, the jet remains as a smooth, round column for at least 3 in., even though the injection velocity be as high as 550 fps. In no case were the drop sizes small enough in vacuum to warrant consideration of fuel turbulence as a primary factor in the atomization process. When the density of the air is atmospheric or greater, the effect of turbulence is dwarfed in comparison with that of air friction. The jet is torn by the air into smaller and smaller irregular particles, and the final atomization is accomplished by ligament formation and collapse. The results of the experiments made since the presentation of this paper will be included in an N.A.C.A. technical report now being prepared.

The discussion by Mr. Richardson does not seem to be very closely connected with fuel-spray formation. The author believes that Mr. Kuttner is in a better position than he to defend the advantages of air turbulence in the Oberhaensli engine.

As Professor Schweitzer points out, the lampblack receiver method ignores all drops less than about 0.00025 in. in diameter. On the other hand, when a fuel spray is caught by a liquid receiver while it is still traveling at high velocity, as was done by Lieutenant Thew, there is probably some further atomizing action in the receiver, the extent of which may be much greater than would have taken place in air. The decision to use the lampblack was made only after a consideration of both methods, with some preliminary experimentation with each.

To check the discharge weight by weighing the plates would have been very difficult. The weight of the fuel was a very small part of the total weight, and large corrections for fuel evaporation and for absorption of moisture from the air would have been necessary. The accuracy of the calculated fuel weights was checked by catching 10 similar injections in a cup and weighing. The two values usually differed by less than 10 per cent.

No Plasticine records were made at close intervals near the nozzle, so that the changes in spray velocity found by Professor Schweitzer could not have been observed.

OGP-54-5

A Proposed Speed Characteristic for Oil Engines

By V. L. Maleev,[1] Stillwater, Okla.

The author proposes a method of readily classifying oil engines as low, medium, high, or super-high speed, by means of a "speed characteristic" equal to the square of the revolutions per second multiplied by twice the stroke in feet. To show that it provides a consistent and practical means of classifying engines, he applies it to the data of some 84 modern engines covering a wide range of sizes, speeds, and powers.

WITH the advent of high-speed oil engines it seems timely to ask where to draw the dividing line between low and high speeds. In searching for a proper yardstick, the best name for which seems to be "speed characteristic," let us briefly enumerate the respects in which an increase in rotative speed influences the behavior and hence the design and construction of an oil engine.

The principal results of an increase in rotative speed are: (1) an increased number of reversals of the reciprocating parts; (2) shortened periods for the different cycle events, such as intake of the air charge, exhaust, and particularly the injection of fuel oil; (3) an increase in the acceleration imparted to reciprocating parts; (4) the reaction of these increased inertia forces upon the rotating parts; (5) increased noisiness as a result of increased inertia forces and velocities; (6) increased danger from torsional vibrations and critical speeds; (7) increased peripheral speeds (bearings, gears, chain drives); (8) increased linear speeds of reciprocating wearing surfaces; (9) increased piston displacement per time unit, hence increased gas velocities through the intake- and exhaust-valve openings.

There are two characteristics by which engines are commonly classified in respect to speed: the rotative speed, expressed in revolutions per minute, and the so-called piston speed, in feet per minute. However, it is easy to show that neither of these characteristics can be used satisfactorily for that purpose.

Undoubtedly, the rotative speed is a very important factor influencing the majority of the results enumerated; however, it has little connection with results (8) and (9). Without going into any computations, it is easy to see that an engine of 10 hp. per cylinder of $4^{1}/_{2}$ in. bore and 6 in. stroke and running at 1000 r.p.m. presents a less difficult problem and is less of a high-speed engine than an 80-hp. engine of $8^{1}/_{4}$ in. bore and 12 in. stroke, running at the same speed of 1000 r.p.m., and that an engine builder would encounter insurmountable difficulties, at least at present, if he attempted to build an engine of 650 hp. per cylinder of 18 in. bore and 24 in. stroke, running also 1000 r.p.m. These examples show that the size of an engine must be taken in consideration when speaking of its speed.

Let us see now whether the average piston speed, usually designated by c, may not be better suited for use as a speed characteristic. Its expression

$$c = \frac{ns}{6} \qquad [1]$$

where n is the rotative speed in r.p.m. and s the length of the stroke in inches, shows that piston speed is directly proportional to s, which in turn, other things being equal, is proportional to the size or power of an engine.

Again let us compare three engines, this time with the same piston speed (e.g., 1000 ft. per min.): the aforementioned engine, $4^{1}/_{2}$ in. × 6 in. × 1000 r.p.m., a 220-hp.-per-cylinder engine, 17 in. × 23 in. × 260 r.p.m., and a 450-hp.-per-cylinder engine, 32 in. × 60 in. × 100 r.p.m. It is evident that these three engines cannot be put in the same class in respect to speed. They behave differently in regard to results (1)–(7) previously enumerated. In spite of equal piston speeds these engines belong to three different classes: the 10-hp.-per-cylinder at 1000 r.p.m. is a high-speed, the 220-hp.-per-cylinder at 260 r.p.m. a medium-speed, and the 450-hp. at 100 r.p.m. is decidedly a low-speed engine.

Thus it is evident that the piston speed c is not suitable as a speed characteristic, although it contains the two necessary basic features—the primary speed factor n and the influence of engine size s. The trouble is that multiplying n by s, formula [1], practically obliterates the influence of n. A simple means of restoring the proper influence of n and retaining the influence of s is to take the speed characteristic in the form of a product of c and n. Large numbers with many digits not being suitable as characteristics, it is better to use revolutions per second instead of revolutions per minute. Designating this speed characteristic by c_s and using for c its expression [1], we obtain

$$c_s = \frac{n^2 s}{21{,}600} \qquad [2]$$

The factor $n/60$ of [2] being revolutions per second and $2s/12$ being feet, the expression for c_s becomes feet per second per second, which is that of acceleration and very appropriate for a

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[1] Research Professor of Mechanical Engineering, Oklahoma A. & M. College. Formerly Chief Engineer, Western-Enterprise Engine Co., Los Angeles, Calif. Mem. A.S.M.E. Dr. Maleev was born in Russia in 1879. After graduating from the Imperial Technical College in Moscow in 1902 with the degree of M.E., he was instructor in mechanical engineering at the St. Petersburg Polytechnic Institute from 1903 to 1906. He was associate professor and professor of mechanical engineering at the Technological Institute of Tomsk, 1906–1917. In 1912 he was awarded the degree of Doctor of Applied Mechanics by the Polytechnic Institute of Kiev. From 1917–1919 he was President of the Polytechnic Institute of Omsk.

After coming to this country in 1920 Dr. Maleev became connected with the Western Machinery Co. of Los Angeles, Calif., first in the capacity of designing engineer, then as experimental and consulting engineer, and finally as chief engineer. Since September, 1930, he has been research professor of mechanical engineering at the Oklahoma Agricultural and Mechanical College, Stillwater, Okla. He is the author of over fifty manuals, books, papers on thermodynamics, theory, design, and testing of internal-combustion engines and steam turbines, and also on technical education.

Presented at the Kansas City Meeting, Kansas City, Mo., Sept. 7 to 9, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 2 DATA ON RECENT HIGH-SPEED OIL ENGINES

Make of engine	Cycle	Number of cylinders	B.hp.		Bore, d, inches	Stroke, s, inches	Rotative speed, n, r.p.m.	Piston speed, c, ft. per min.	Speed characteristic, c_s, ft. per sec.[2]	Abbreviated classification
			Total	Per cyl.						
A.E.C.—Assoc. Equipment Co.[1]	4	6	100	17	4.33	5.59	2500	2329	1617	S
Atlas-Imperial	4	6	120	20	7	$8\frac{1}{2}$	650	921	166	M
Attendu	2	2	85	43	$5\frac{1}{2}$	$6\frac{1}{2}$	1620	1755	790	S
Beardmore	4	8	650	81	$8\frac{1}{4}$	12	1000	2000	556	H
Beardmore[1]	4	12v	1330	111	12	12	800	1600	356	H
Benz	4	6	75	13	3.94	5.91	1600	1575	701	S
Benz-Mercedes[1]	4	6	70	12	4.13	6.49	1300	1407	508	H
Benz (MWM)[1]	4	6	300	50	8.07	10.63	1000	1770	492	H
Bessemer	4	6	200	33	$7\frac{1}{2}$	$10\frac{1}{2}$	670	1172	218	M
Brotherhood-Ricardo	4	6	300	50	$7\frac{1}{2}$	12	1000	2000	556	H
Buda-MAN	4	4	92	23	6	8	1000	1333	370	H
Buda-MAN	4	6	180	30	$6\frac{1}{2}$	$8\frac{3}{4}$	1000	1458	405	H
Colo-Diesel	4	6	60	10	5	7	750	875	182	M
Cummins	4	6	60	10	$4\frac{1}{2}$	6	1000	1000	278	H
Cummins	4	6	170	28	$6\frac{3}{4}$	9	800	1200	267	H
Cummins	4	6	500	83	10	12	800	1600	356	H
Deutz	4	6	85	14	4.53	6.69	1250	1394	484	H
Dorner	4	4	30	8	3.74	6.30	1000	1048	291	H
Fairbanks-Morse	2	4	60	15	6	$6\frac{1}{2}$	800	867	193	M
Fulton[1]	4	6	250	42	$8\frac{1}{2}$	11	750	1375	286	H
Hill[1]	4	4	40	10	$3\frac{1}{2}$	6	2000	2000	1110	S
Hill	4	6	75	13	5	7	1000	1167	324	H
Hill	4	6	120	20	6	10	800	1333	296	H
Ingersoll-Rand	4	6	300	50	10	12	550	1100	168	M
Ingersoll-Rand	4	6	800	133	$14\frac{3}{4}$	16	500	1333	185	M
Junkers	2	2	65	33	3.15	7.9 + 7.9	1000	1316	366	H
Junkers[1]	2	6	600	100	4.72	8.27 + 8.27	1500	2160	900	S
Koerting	4	6	90	15	5.1	7.09	1200	1420	473	H
Linke-Hoffman	4	4	50	13	4.53	6.49	1200	1298	433	H
Linke-Hoffman[1]	4	6	150	25	5.70	8.27	1200	1652	551	H
MAN-Augsburg[1]	4	6	68	11	4.53	7.09	1000	1182	328	H
McIntosh-Seymour[1]	4	6	350	58	$9\frac{1}{2}$	$10\frac{1}{2}$	800	1400	311	H
Morris-De La Vergne[1]	4	6	300	50	9	11	750	1375	286	H
Packard-Airplane[1]	4	9	225	25	$4\frac{13}{16}$	6	1900	1900	1002	S
Peugeot	2	2	53	27	4.72	5.91	1450	1427	575	H
Speedway	4	6	300	50	$8\frac{1}{2}$	11	700	1284	250	H
Standard Motor Const.	4	6	100	17	6	8	750	1000	208	M
Treiber[1]	4	4	65	16	5	7	1200	1400	467	H
Treiber[1]	4	6	225	38	$7\frac{1}{2}$	$9\frac{1}{2}$	900	1425	356	H
Treiber	4	12v	750	63	$9\frac{1}{2}$	12	700	1400	272	H
Treiber	4	12v	3000	250	16	16	700	1867	363	H
Westinghouse	4	6	300	50	$8\frac{1}{4}$	12	800	1600	356	H
Westinghouse	4	12v	1340	112	12	12	800	1600	356	H
Winton	4	6	150	25	7	$8\frac{1}{2}$	750	1062	221	M
Winton	4	6	250	42	8	10	750	1250	261	H
Winton	4	8	425	53	$8\frac{1}{2}$	12	700	1400	272	H

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[1] Additions made by author.

TABLE 3 DATA ON MEDIUM- AND LOW-SPEED OIL ENGINES

Make of engine	Cycle	Number of cylinders	B.hp.		Bore, d, inches	Stroke, s, inches	Rotative speed, n, r.p.m.	Piston speed, c, ft. per min.	Speed characteristic, c_s, ft. per sec.[3]	Abbreviated classification
			Total	Per cyl.						
Atlas-Imperial	4	6	200	33	9	$10\frac{1}{2}$	500	875	122	M
Atlas-Imperial	4	6	480	80	$14\frac{1}{2}$	18	277	831	64	L
Bessemer	4	6	300	50	9	12	600	1200	200	M
Bessemer	4	6	1125	187	18	22	300	1100	92	M
Cummins	4	6	110	18	$5\frac{1}{2}$	$7\frac{1}{2}$	800	1000	222	M
Deutz	2	3	40	13	5.91	8.67	600	867	145	M
Deutz	2	6	200	33	8.27	12.60	500	1050	146	M
Deutz	2	6	550	92	13.00	15.75	450	1181	148	M
McIntosh-Seymour	4	12v	800	67	14	18	325	975	88	M
McIntosh-Seymour	4	6	2700	450	32	60	95	950	25	L
Morris-De La Vergne	4	6	450	75	15	18	300	900	75	M
Morris-De La Vergne	4	6	660	110	17	24	240	960	64	L
Morris-De La Vergne	4	6	1520	253	22	30	200	1000	56	L
Nordberg	2	6	1320	220	17	23	257	986	70	L
Nordberg	2	6	1800	300	21	24	225	900	56	L
Nordberg	2	6	1500	250	$20\frac{3}{4}$	26	180	780	39	L
Nordberg	2	6	4000	667	29	44	125	917	32	L
Standard Motor Const.	4	6	180	30	$8\frac{1}{2}$	12	450	900	112	M
Standard Motor Const.	4	6	280	47	10	14	400	933	104	M
Standard Motor Const.	4	6	400	67	12	17	375	1063	111	M
Superior	4	6	105	17	$6\frac{1}{2}$	8	700	933	181	M
Superior	4	6	135	23	7	9	700	1050	204	M
Superior	4	6	210	35	$8\frac{1}{2}$	11	600	1100	183	M
Washington Iron Works	4	6	132	22	$8\frac{1}{2}$	10	450	750	94	M
Washington Iron Works	4	6	200	33	10	$12\frac{1}{2}$	350	729	71	L
Washington Iron Works	4	6	325	54	$12\frac{1}{2}$	16	280	747	58	L
Washington Iron Works	4	6	425	71	$14\frac{1}{2}$	18	250	750	52	L
Washington Iron Works	4	6	695	116	18	24	200	800	44	L
Western-Enterprise	4	4	70	17	$6\frac{1}{2}$	$10\frac{1}{2}$	550	963	147	M
Western-Enterprise	4	4	110	27	9	$12\frac{1}{2}$	330	687	63	L
Western-Enterprise	4	6	300	50	12	15	300	750	63	L
Western-Enterprise	4	6	375	62	13	18	260	780	56	L
Western-Enterprise	4	6	500	83	15	20	240	800	53	L
Winton	4	6	175	29	$8\frac{1}{2}$	11	450	825	103	M
Winton	4	6	375	62	11	15	450	1125	141	M
Winton	4	6	520	87	14	16	375	1000	104	M
Winton	4	6	430	72	14	18	275	825	63	L
Winton	4	6	500	83	$14\frac{1}{2}$	21	250	875	61	L
Winton	4	6	800	133	$16\frac{1}{2}$	26	250	1083	75	M

speed characteristic. Another argument in favor of [2] as a speed characteristic is the fact that the factor limiting the speed of a reciprocating engine is the so-called inertia loads which vary directly as the square of rotative speed and as the first power of engine stroke, or exactly according to formula [2].

For the sake of convenience in practical use, [2] can also be presented in the form

$$c_s = \frac{nc}{3600} \qquad [3]$$

which saves one slide-rule operation if *c* is already known.

It may be also useful to consider why the other dimension determining the size of an engine, namely, the cylinder bore *d*, has no direct relation to the speed characteristic. Although with an increase of *d* the weights of the reciprocating parts and the inertia forces increase, and in the same proportion, the dimensions of the parts subjected to these forces also are increased in a correct engine design, so that the stresses and unit pressures remain the same and are independent of *d*.

After having established tentatively a method of computing a speed characteristic, the next step is to determine the limits between which an engine should be classified as a low- or high-speed one, and to prove from a sufficient number of examples that the speed characteristic and scale of limits selected are correct and sound from all viewpoints.

The standard practice in classification where a certain feature or characteristic changes gradually from a low to a high value is to insert a medium group between the two main groups and additional groups with a prefix "extra" or "super" for values outside the main groups. A classification of engines selected with these points in view and the corresponding speed characteristics are given in Table 1. The speed-characteristic limits were determined by computing the characteristics c_s from expression [2] for a large number of actual oil engines and taking in consideration their behavior in regard to the nine effects of speed increase outlined at the beginning of the paper.

Obviously the dividing lines between the speed-characteristic limits for the different groups are more or less arbitrary. On the other hand, there must exist a certain regularity between these figures. The best results were obtained with an increase of the characteristic limits in a geometrical progression with the ratio 3, which, with a slight rounding off of the figures, gave the values in the third column of Table 1.

TABLE 1 SPEED CLASSIFICATION OF ENGINES

Classification of engines	Abbreviated designation	Speed characteristic limits (c_s), ft. per sec. per sec.	Limit figures in existing engines	
			n, r.p.m.	*c*, ft. per min.
Low speed	L	25– 74	95– 350	687–1000
Medium speed	M	75– 229	300– 800	750–1333
High speed	H	230– 699	600–1450	1000–1867
Super high speed	S	700–2100	1500–2500	1755–2329

The proposed method of speed characteristics, expressions [2] and [3], retains its value even if the system of classification and engine designations presented in Table 1 is changed either in respect to limits, to number of subdivisions, or to both.

For the sake of comparison, the limits of *n* and *c* of the various oil engines listed in Tables 2 and 3 and classified according to the author's proposal are given in the last two columns of Table 1. These figures follow the trend of the speed characteristics given there very well, but with considerable overlapping—which was the reason for the present investigation.

To check the method of classification described in respect to its usefulness, data for a number of existing oil engines were compiled in Tables 2 and 3. Table 2 comprises data on a number of oil engines of the latest design, and commonly regarded as high-speed engines. In its compilation a table of high-speed engines, published by E. C. Magdeburger,[2] was used as a basis, with a few changes and additions, the latter being suitably indicated; and out of several engines with the same bore, stroke, and r.p.m., only one was taken in each case. Two new columns were added: the speed characteristic c_s, and classification as to speed according to Table 1. Similar data for engines usually considered of low or medium speed are presented in Table 3. In order to obtain a short, clear table only a limited number of mechanical-injection vertical engines were considered. In several instances the choice was decided by the availability of the necessary data. The chief object was to select representative engines differing in speeds, cycles, and sizes, avoiding duplication of engines of the same bore, stroke, and r.p.m. Table 3 in connection with Table 2 makes it possible to check the consistency and soundness of the proposed speed characteristic c_s over the whole range of oil engines at present constructed.

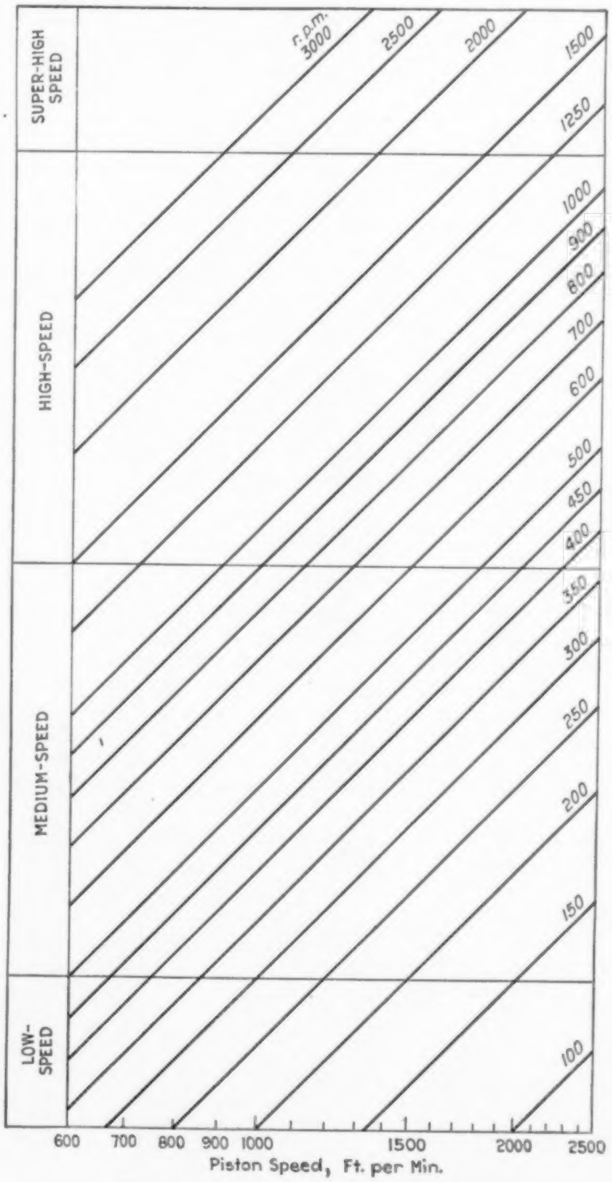

FIG. 1 SPEED CLASSIFICATION OF ENGINES

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[2] *S.A.E. Journal*, February, 1930, p. 204.

From Table 2 the following conclusions can be drawn: That not all engines commonly considered as high-speed can be classified as such, and therefore it is desirable to have a positive means of classifying them; that the introduction of a "super-high" classification makes it possible to pick out at a glance engines showing greater departure from a conventional design and that therefore this designation is desirable.

Table 3 shows that engines of certain makes have very close speed characteristics in their respective groups; for instance, the three Deutz engines, the three Standard Motor Construction engines, and the four low-speed Western-Enterprise engines. Engines of other builders show less regular speed characteristics. The speed characteristics generally fall off considerably with increase of engine size, showing clearly an excessive lowering of the rotative speed with increase of stroke and output per cylinder. This shows how an engine designer can benefit from the use of a list of speed characteristics of oil engines actually built and satisfactorily operating, as given in Tables 2 and 3.

The author believes that as a result of this discussion the following general conclusions can be drawn:

1 The use of the proposed speed characteristic c_s gives a consistent and correct means of judging oil engines in respect to speed.

2 The system of classification as presented in Table 1 is sound, and without irregularities.

3 The system of classification proposed is practical and general; it presents a means of showing in a concise way what has been done by other engine builders, and can be used as a guide by engine designers, regardless of whether the engine to be designed is a high-, medium-, or low-speed one.

Discussion

E. C. **Magdeburger.**[3] The observations of the author on the desirability of establishing a yardstick for classifying oil engines are thoroughly concurred in, and the deductions leading to his "speed characteristic" appear logical enough to make it a satisfactory means for differentiation of type of engines. There is only one suggestion that is believed to be pertinent. It would make the practical application of the proposed classification quick and easy and may thereby contribute to the adoption of the idea itself. The various division lines, as the author himself suggests, are perfectly arbitrary, but there appears to be no good reason why the width of ranges of the different classes should be so arbitrarily chosen as Table 1 would indicate.

It is therefore suggested to use for speed classification a unit, proportional to that proposed by the author, which has the advantage that it will require no slide rule or any other constants to memorize and can be obtained by a bit of mental arithmetic in all but the few cases close to the border line. The proposed "speed characteristic" is to be

$$\frac{\text{R.p.m.} \times \text{piston speed}}{100{,}000}$$

For the division lines of the various classes, which can be easily remembered, it is proposed to establish the powers of number 3, so that for low-speed engines the speed characteristic is below 3, for medium-speed engines it is to be between 3 and $3^2 = 9$, for high-speed engines it is to be between $3^2 = 9$ and $3^3 = 27$, and engines having a speed characteristic above $3^3 = 27$ are to be classed super-high-speed.

An easily made diagram which further simplifies the classifying process, once the speed in r.p.m. and piston speed in feet per minute are known, is given in Fig. 1. If the vertical line of the piston speed is followed to the inclined r.p.m. line, their intersection will give the proper classification of the engine.

It may be of interest to add that the first submarine Diesel engine of our Navy had cylinders of $12^3/_4$-in. bore and $13^1/_2$-in. stroke and ran at 400 r.p.m. (piston speed, 900 ft. per min.). It belonged, therefore, to the medium-speed class (400 × 900/100,000 = 3.6), and, in fact, all of our submarine engines are in the same class.[4] It is only with the new "experimental" Diesel engine, having cylinders of 11.024-in. bore and 14.961-in. stroke and a maximum speed of 700 r.p.m. (piston speed, 1745 ft. per min.) that the Navy ventured into the high-speed class (700 × 1745/100,000 = 12.2).

Author's Closure

The author agrees with the practical usefulness of Mr. Magdeburger's suggestion of changing the divisor in expression [3], and expresses the hope that this change, in connection with other advantages, will help in the adoption of the speed characteristic c_s by designers of oil engines in their work.

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[3] Aide on Diesel Engines, Bureau of Engineering, Navy Department, Washington, D. C. Mem. A.S.M.E.

[4] See "Diesel Engines in Submarines," by E. C. Magdeburger, *J. Am. Soc. Nav. Engrs.*, August, 1925, p. 572.

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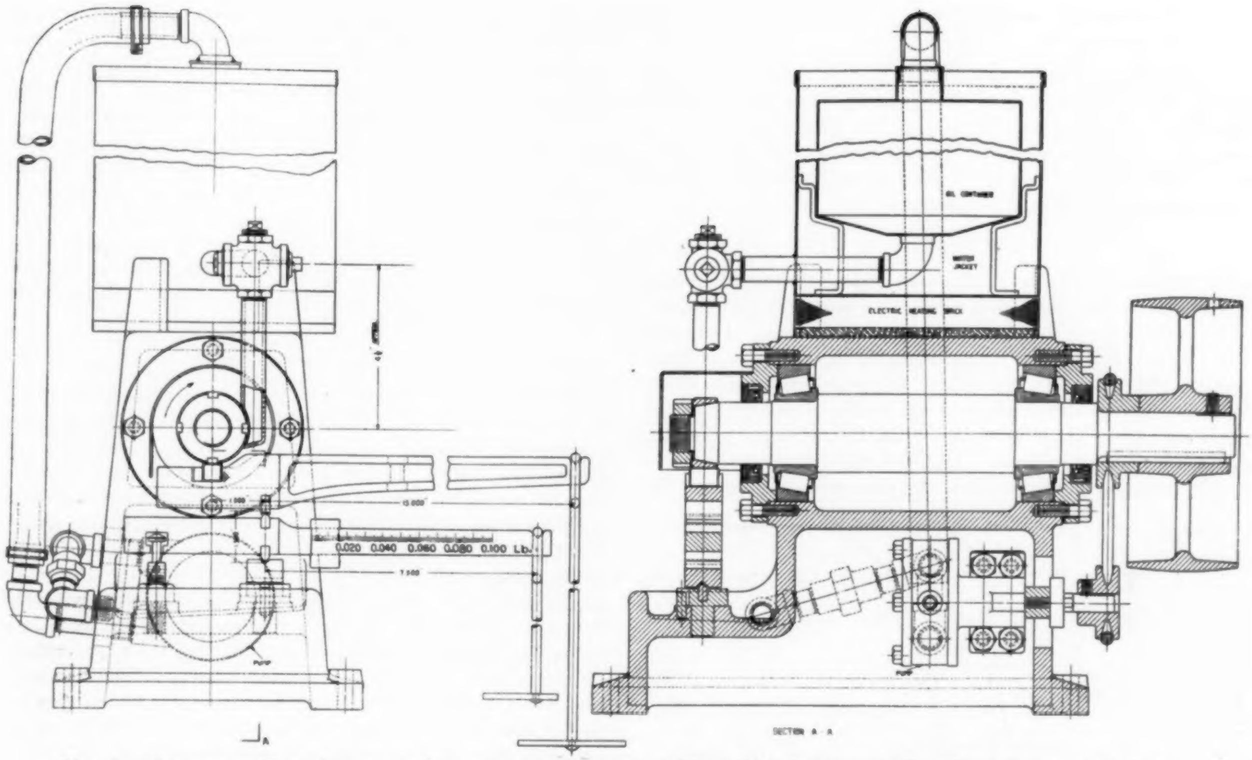

FIG. 1 MACHINE TO TEST ABILITY OF LUBRICANTS TO WITHSTAND HIGH UNIT PRESSURES WITH HIGH RUBBING SPEEDS

Lubricant Testing

By E. G. BODEN[1] AND O. L. MAAG,[2] CANTON, OHIO

A simple machine is described which was developed for the purpose of testing the power of lubricants to withstand high unit pressures with high rubbing speeds. The results of these tests are shown in photos of test blocks indicating the effect of pressure and speed when using oils of varying viscosities and varying composition.

THE complex nature of lubricants used in engineering, even in their simplest form, gives physical results which will vary, depending on their composition. A great amount of information is available on the effect of lubricants used for plain journal bearings, but this paper will be limited to the lubricants used for gearing and in conjunction with anti-friction bearings.

The ever-increasing demand for higher speeds in all forms of engineering has resulted in a limit being placed on speeds due to the lack of efficient lubrication. Considerable advancement has been made within recent years in the metallurgy of steel, and today we can, with fairly definite knowledge of the properties of metal, develop machinery which can be depended upon to produce certain given results. This development has reached its present stage by repeated physical testing and by designing the special testing means to obtain desired characteristics of the material.

GENERAL TESTS

Until quite recent years the testing of lubricants was confined to the general physical constants determined by A.S.T.M. and S.A.E. standard methods. The objects were to determine:

1 The viscosity of the lubricant, which is usually taken by means of the Saybolt viscosimeter
2 The cold test, which is to find the point at which a lubricant becomes non-fluid
3 The flash test
4 The fire test
5 The gravity test
6 Determination for ash
7 Determination for carbon residue

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[1] Experimental Engineer, Timken Roller Bearing Company. Mr. Boden's technical training was obtained at the Manchester College of Technology and in mechanical engineering and apprentice work at Crossley Motor Company, Manchester, England. Later he was design engineer for Sizaire Berwick Automotive Engineers in London, and next with the Chandler Motor Car Company in Cleveland as quality and research engineer.

[2] Lubricating Engineer, Timken Roller Bearing Company. Mr. Maag's technical training was obtained at the University of Kansas in chemical engineering and the teaching of chemistry. In the commercial field his experience has been as a chemical engineer with the Kansas City Industrial Laboratory, the U. S. Bureau of Standards, Washington, and the Galena Signal Oil Company, Franklin, Pa.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

8 A channeling test
9 A corrosion test.

With these tests, only the effects of the viscosity of the lubricant in regard to the load-carrying capacity could be used as an indication of the relative merits of different lubricants for withstanding varying degrees of unit pressure. Within certain temperature ranges the effect of viscosity will be to carry higher unit loading with increase in viscosity.

GEAR LUBRICANTS

Lubricants suitable for gearing will depend on the unit loads, gear material, and tooth design reflecting on the slippage speed, which appears to be one of the main causes of failure. Gears are generally lubricated from a fluid-oil bath, the quantity of lubricant passing the point of mesh of the gear being quite small, depending on the adhesiveness of the lubricant and the effect of centrifugal force tending to throw off the lubricant from the power-transmitting surfaces. The lack of lubricant, which has a double function in maintaining the separation of the metallic surfaces and the conduction of heat generated by the inevitable friction between parts highly loaded or subject to high rubbing speed, gives rise to scoring or a welding action due to the breakdown of the separating film.

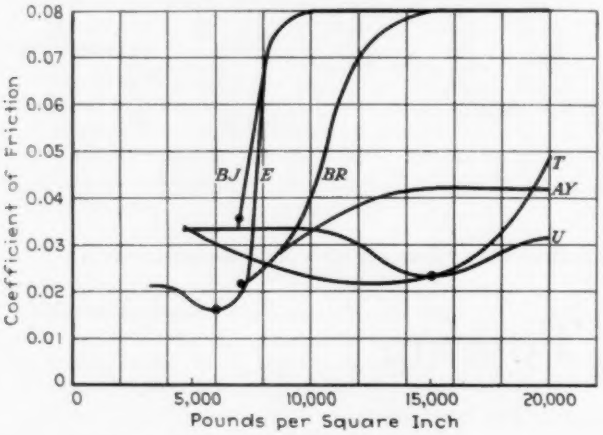

FIG. 2 RESULTS OF TESTS OF SIX OILS

(*E*, cylinder oil; *AY*, cylinder oil with 5% lime soap added; *BJ*, cylinder oil with 2% litharge; *BR*, cylinder oil with 5% aluminum stearate; *U*, straight mineral oil of 800 viscosity at 200°F.; *T*, cylinder oil with 2% free sulphur. Dot indicates maximum load before scoring starts. Rubbing speed, 320 f.p.m.)

SPECIAL TESTS

Several devices for determining the film strength of lubricants have been developed, among which may be cited the Amsler, Hershey, Mougey, McKee, Kingsbury, Herschel, and Dickinson. Most of these machines were designed for testing lubricants on plain bearings. Exception to this would be the Amsler machine, which is made in the form of rotating disks having their edges sliding against each other, and the McKee machine, utilizing balls in grooved shafts.

Owing to the lack of public information on the ability of lubricants to withstand high unit pressures with high rubbing speeds, our company developed a simple testing machine for this purpose.

DESCRIPTION OF SCUFFING MACHINE

Fig. 1 shows the cross-section and end view of this apparatus. A mandrel is mounted on two Timken bearings held in a cast-iron base and having on one end of the mandrel a tapered end to receive a standard Timken bearing cup, which is used as one of the test members. A nut having a left-hand thread securely locks this tapered bore cup to the mandrel, with no possible tendency for it to rotate. Directly beneath the revolving cup a test block $^{1}/_{2}$ in. square by $^{3}/_{4}$ in. long is mounted in a lever, shown in the end view of the apparatus. This lever is supported on a knife edge which is carried on the top of a second lever, which also is carried on a universal knife edge on the base of the machine. This arrangement assures that the test block will at all times be parallel to the revolving cup, so that the unit loading will be maintained constant over the length of the test piece. The top lever is loaded by means of weights on the end, which produces a pressure on the revolving member ten times the amount of load added to the lever.

TEST PROCEDURE

The lubricant to be tested is placed in the 1-gal. tank mounted on the top of the base casting, and brought to the required temperature by means of the electric heating pad directly beneath the oil tank. The oil is flowed by gravity from the tank over the test piece, the flow being controlled by a valve, shown in the illustration. The arbor is then driven at any desired speed, depending on the conditions to be met, either by means of a belt or by being coupled directly to an electric motor. It can be readily seen that the revolving test piece in contact with the stationary square test block will produce a frictional force in a horizontal direction, tending to move the top lever horizontally toward the left. This tends to rotate the bottom lever in an anticlockwise direction, and is resisted by means of a conical stop pin on the base of the machine. The bottom lever, known as the frictional lever, is counterbalanced with its scale pan, so that to balance the friction created by the load, it is necessary only to add weight to the pan so that when the lever just leaves the stop pin the frictional force has been counterbalanced. It is then possible to determine the coefficient of friction. The oil is returned to the tank by a pump inside the base of the machine and driven from the mandrel by a small round belt, as shown in the diagram. The machine is therefore a self-contained assembly. The test pieces can be any desired material, but for checking ordinary gears such as are found in automotive rear axles, the test pieces are made from carburized steel, case-hardened and ground, the standard cups having a Rockwell hardness of 60 on the "C" scale.

The machine, as run for checking lubricants suitable for gears, uses a square test block which gives a very high initial loading. The test is comparative as the initial areas are the same. The abrasives in the lubricant, however, will change the ultimate area of contact during test depending on the severity of the abrasives. It is more convenient and cheaper to prepare specimens having flat faces than to prepare specimens having two curved surfaces in contact. If it is desired to maintain constant unit pressure, then test pieces can be made having curved surfaces to correspond with the round test block to be used as the stationary test member.

TEST RESULTS

The results outlined in the following tests were obtained when running the machine at a rubbing speed of 320 ft. per min., using specimens of carburized steel hardened to 60 Rockwell on the "C" scale.

Curve *E* in Fig. 2 is a friction curve for a straight mineral oil of 150 seconds viscosity at 210 deg. fahr., which shows a rapid increase in friction at a load of 6000 lb. per sq. in. This is the point at which scuffing starts and is the maximum load this oil will carry at this particular speed.

Curve *U*, for a straight mineral oil of 800 seconds viscosity at 210 deg. fahr., also shows a minimum friction and then a rise to

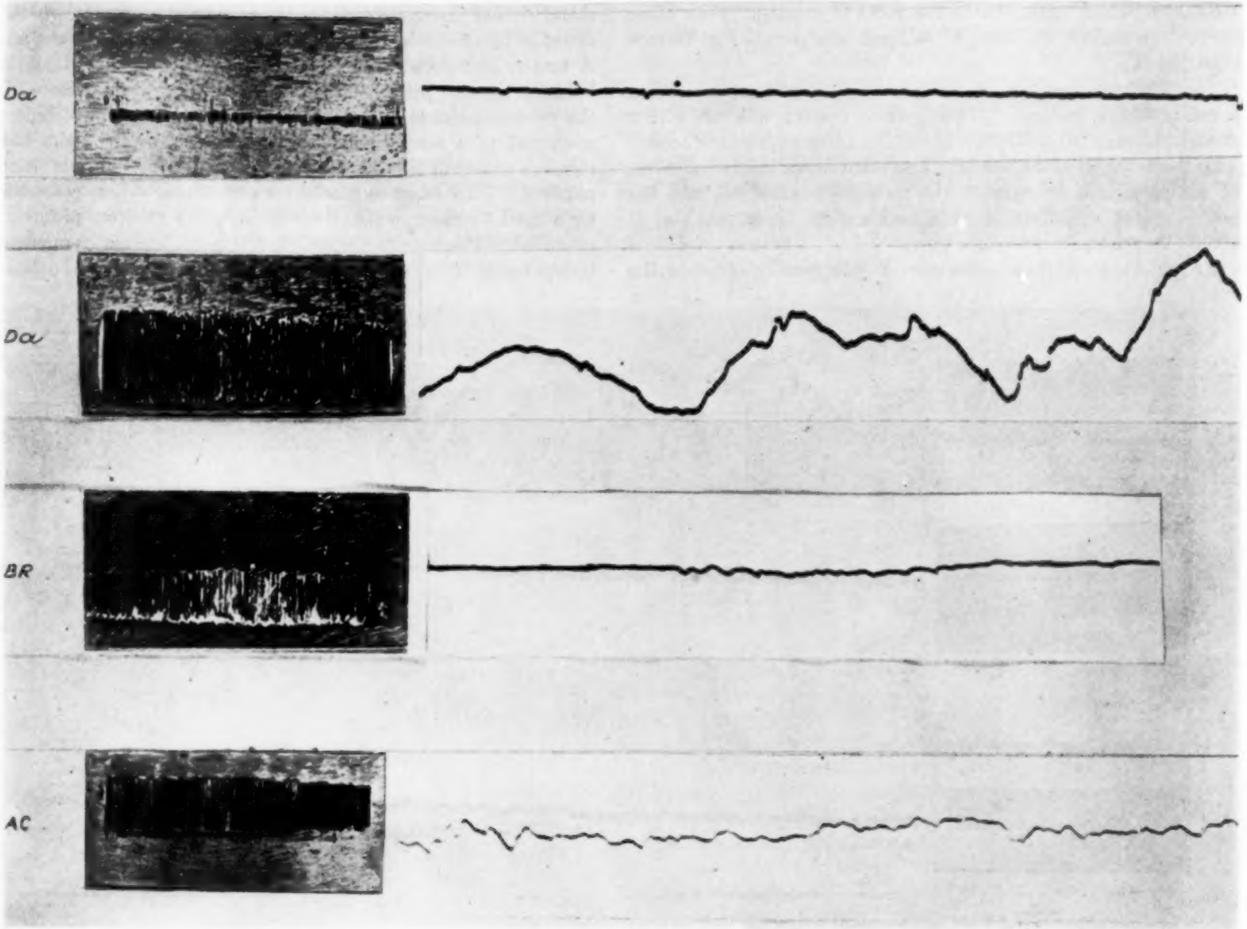

Fig. 3 Four Typical Test Blocks

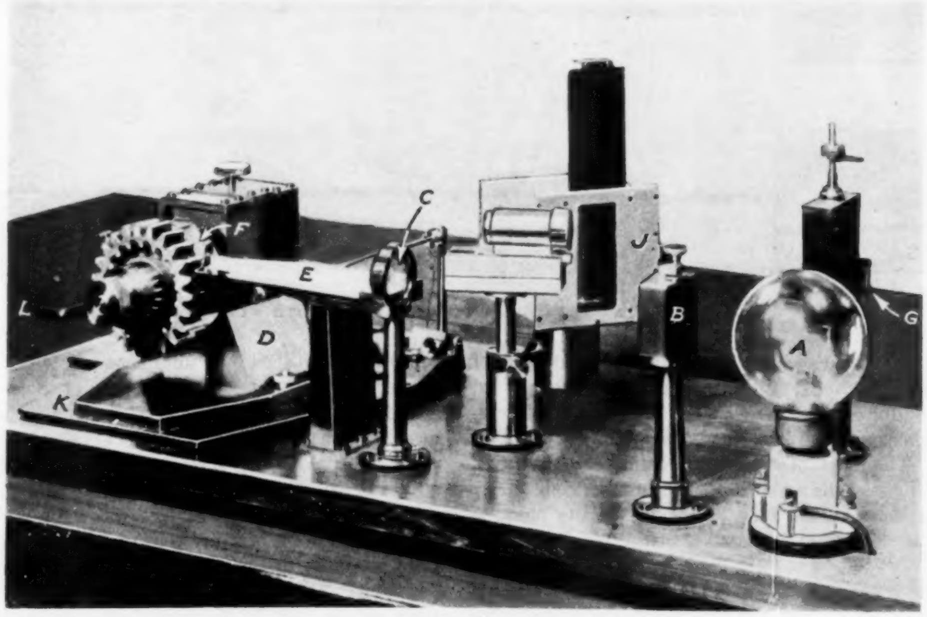

Fig. 4 Roughness-Recording Machine

15,000 lb., which again checks the point of scoring. Four other curves are shown for straight mineral oils containing various ingredients.

Fig. 3 shows four typical test blocks after test, at a rubbing speed of 320 ft. per min. The block at the top was run with a straight mineral oil at 5000 lb. per sq. in. pressure, and the second with the same oil at 10,000 lb. The third block shows the effect of adding aluminum soap to the straight mineral oil, and the bottom block the effect of adding soda soap, these two also at 10,000 lb. per sq. in. pressure.

As can be seen, these additions do not greatly improve the carrying capacity of the lubricant. The curves to the right of the block show the roughness of the face as determined by an apparatus designed for our company by the University of Michigan which can check the surface roughness of either plain or curved surfaces. The magnification is approximately 500 times, so that $^1/_4$ in. in height of the actual size of the curve produced by the machine represents 0.0005 in. The width of the block is $^1/_2$ in., so it will be seen that the scuffing of the block is rather severe.

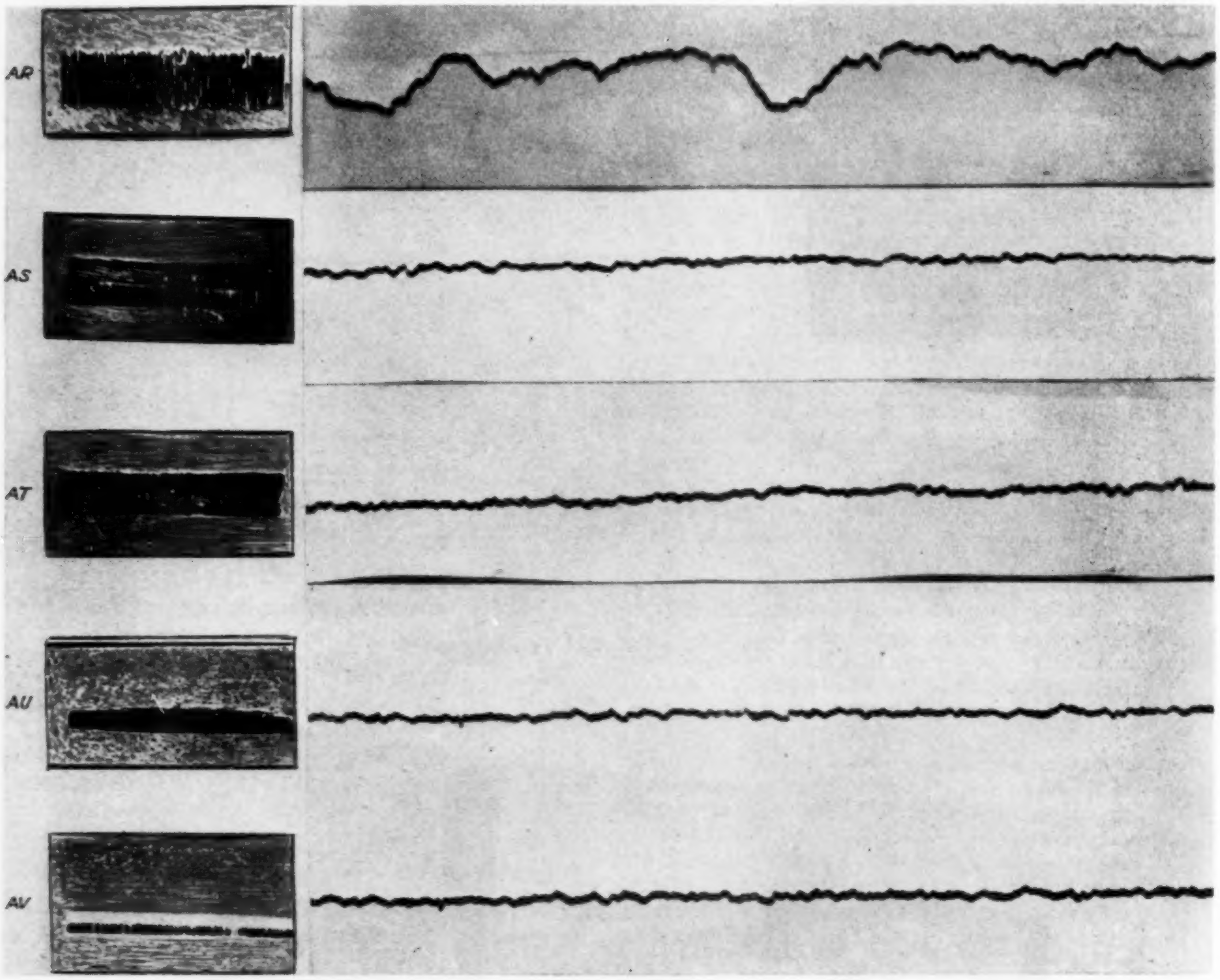

Fig. 5 Results of Tests of Straight Mineral Oil, With Various Additions

Operation of the roughness-recording machine, which is shown in Fig. 4, is as follows:

Light from a source *A* is projected through a small hole in the stand *B*, and through a lens *C* against a small mirror *D*, which is fastened to a razor blade attached to a lever *E* hinged to a stand. A master surface *F* supports this lever, so that any deviation between the test pieces and the master piece will be reflected by the movement of the mirror *D*. This reflected beam of light is projected to a second mirror *G*, which reflects the light back through a second lens *H*, not shown, on to a piece of sensitized paper *J*. This paper is placed on a revolving drum, connected by a cord to the movable base *K* on which the test pieces are mounted. The base moves only in a straight line and is operated by the handle *L* at the rear of the fixture. To test plane surfaces the arbor is replaced by a suitable stand, which also must carry a master test piece.

Fig. 5 is a photograph of test blocks after a run with experimental Pennsylvania oil, which is extremely low in natural sulphur. Oil "AR" was a straight mineral oil of 150 seconds viscosity at 210 deg. fahr. At a pressure of 10,000 lb. per sq. in., it scored very badly. Oil "AS" was the same oil as "AR," but with the addition of 2 per cent of free sulphur. Fine scratches were produced in this test. The load was carried to 20,000 lb. per sq. in. Oil "AT" had 2 per cent of lead oleate added to the same oil as "AR." No scratches were evident with this oil. The load was carried to 20,000 lb. per sq. in. Oil "AU" had

10 per cent of lead oleate added to "AR," and the load was again carried to 20,000 lb. per sq. in. Again no scratches were observed. Oil "AV" was made by adding together oils "AS" and "AT," and it therefore contained 1 per cent of lead oleate and 1 per cent of sulphur. Fine scratches were noticed, which shows that the sulphur was the cause of the scratches.

Curves of the coefficient of friction for these same oils, Fig. 6, show that the friction increases with the increase of lead oleate.

Temperatures reached by these oils in the tank are shown in Fig. 7. The sulphur oil shows the lowest temperature, as might be expected from the well-known cooling action when adding sulphur to hot journals.

Highest temperatures were reached with the high-lead-oleate oil. The friction curves in Fig. 6 followed the temperature curves in Fig. 7 in the same order.

Many oils compounded in various ways have been tested, but the most promising as regards load-carrying ability are sulphur oils, chlorinated oils, and lead-soap-base oils. Test results obtained with lead-soap oils are shown in Fig. 8. The rubbing speed was 320 ft. per min. Oil "AX" is a well-known brand of lead-soap oil containing sulphur. Oil "AJ" is a similar lead-soap oil. Oil "BF" is another containing approximately 7 per cent of lead-soap and no sulphur. All of these were tested up to 20,000 lb. per sq. in.

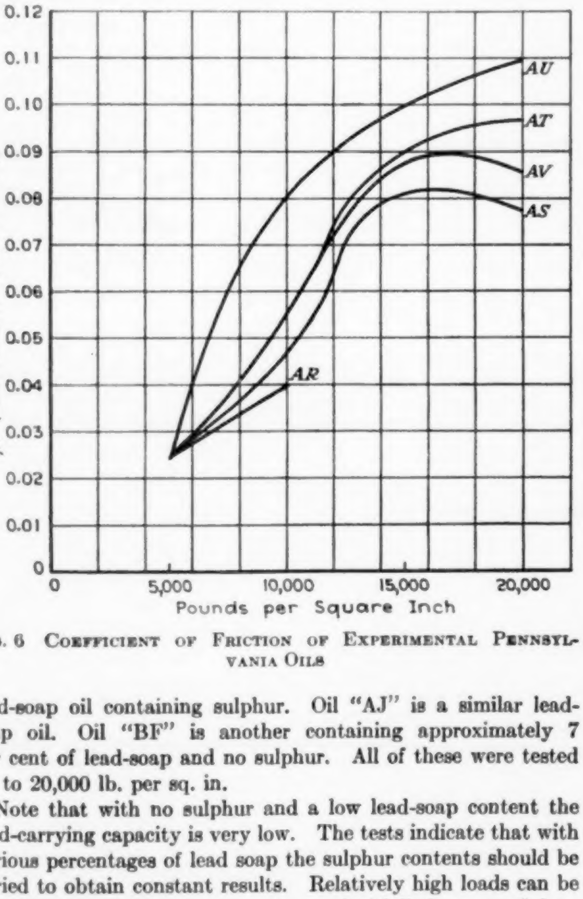

FIG. 6 COEFFICIENT OF FRICTION OF EXPERIMENTAL PENNSYLVANIA OILS

Note that with no sulphur and a low lead-soap content the load-carrying capacity is very low. The tests indicate that with various percentages of lead soap the sulphur contents should be varied to obtain constant results. Relatively high loads can be carried with a high lead-soap content and with little or no sulphur, but to hold high percentages of lead-soap in permanent suspension is very difficult, if not impossible.

Test results obtained with sulphur oils are shown in Fig. 9. Oil "BT" had 20 per cent of added sulphur. Oil "BU" had 10 per cent of added sulphur. In this case the sulphur supposedly was added on to the hydrogen atom. Oil "BV" contained 10 per cent combined sulphur, the sulphur atom replacing the hydrogen and therefore being a chemical combination. All of these were tested up to 20,000 lb. per sq. in.

The results of this test are interesting, showing the abrasive qualities as well as the carrying capacity of an oil high in free sulphur ("BT"). This is shown by the extreme width of contact surface that has been very badly lapped. Most chemically combined forms of sulphur score badly at 20,000 lb. per sq. in. and would carry loads of only 10,000 to 15,000 lb. Sulphur oils that are high in free sulphur separate badly in test.

Of the three oils represented in Fig. 9, oil "BT" separated badly in test and oil "BU" separated slightly after standing approximately one month. No separation occurred in oil "BV," which was chemically combined.

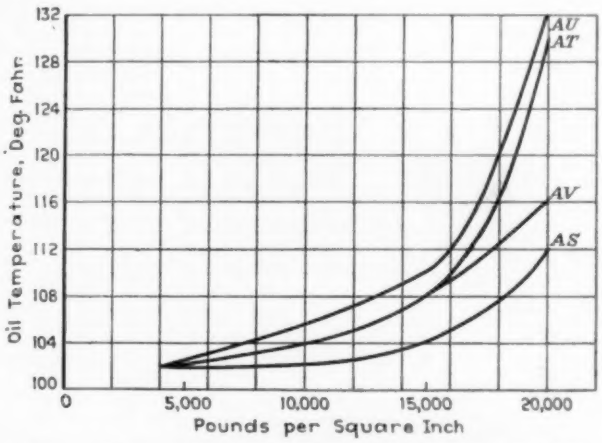

FIG. 7 TEMPERATURE OF EXPERIMENTAL PENNSYLVANIA OILS

The effect of castor oil under a load of 10,000 lb. per sq. in. is shown in Fig. 10, oil "O." It was surprising to find that this oil did not stand any more load than a straight mineral oil of approximately 150 viscosity at 210 deg. fahr. The advantage of this oil is a low coefficient of friction and low cold-test value. The result obtained with an oil having an addition of 10 per cent of a cutting oil to a straight mineral oil of 150 viscosity is also shown, oil "CL." This cutting oil is a sulphur chloride of chemically combined form. The results obtained were very surprising, as the sulphur contents of the mixture did not exceed 0.5 per cent.

The effect of rubbing speed in relation to pressure per square inch before scuffing occurs has been determined for three different oils, as shown in Fig. 11. The oils tested were: (1) Lead-soap oil "CX" (curve *A*); (2) straight mineral oil of 150 viscosity at 210 deg. fahr. (curve *B*); (3) oil *B* with 3 per cent of flowers of sulphur (curve *C*).

The curves are extremely interesting as showing the importance of rubbing speed with relation to pressure. This brings up the question of whether a standard means of testing the load-carrying capacity of an oil is desirable. It is our opinion that this is very essential in analyzing an oil for its load-carrying capacity and is perhaps more important than the usual physical test made on lubricants in the past.

Fig. 12 shows a test conducted on a bronze block of 80 per cent copper, 10 per cent tin, and 10 per cent lead, using a steam-refined straight mineral oil of 150 viscosity at 210 deg. fahr. at a rubbing speed of 320 ft. per min. It is to be noted that the point of minimum friction falls at the same place as it did when steel was used—namely, approximately 6000 lb. per sq. in. In the case of the bronze, however, instead of a scoring taking place, rapid wear begins to set in beyond this pressure, and as seen at 20,000 lb. per sq. in., the surface is relatively smooth, but the specimen has been badly worn away.

An attachment to this machine can be installed to enable non-

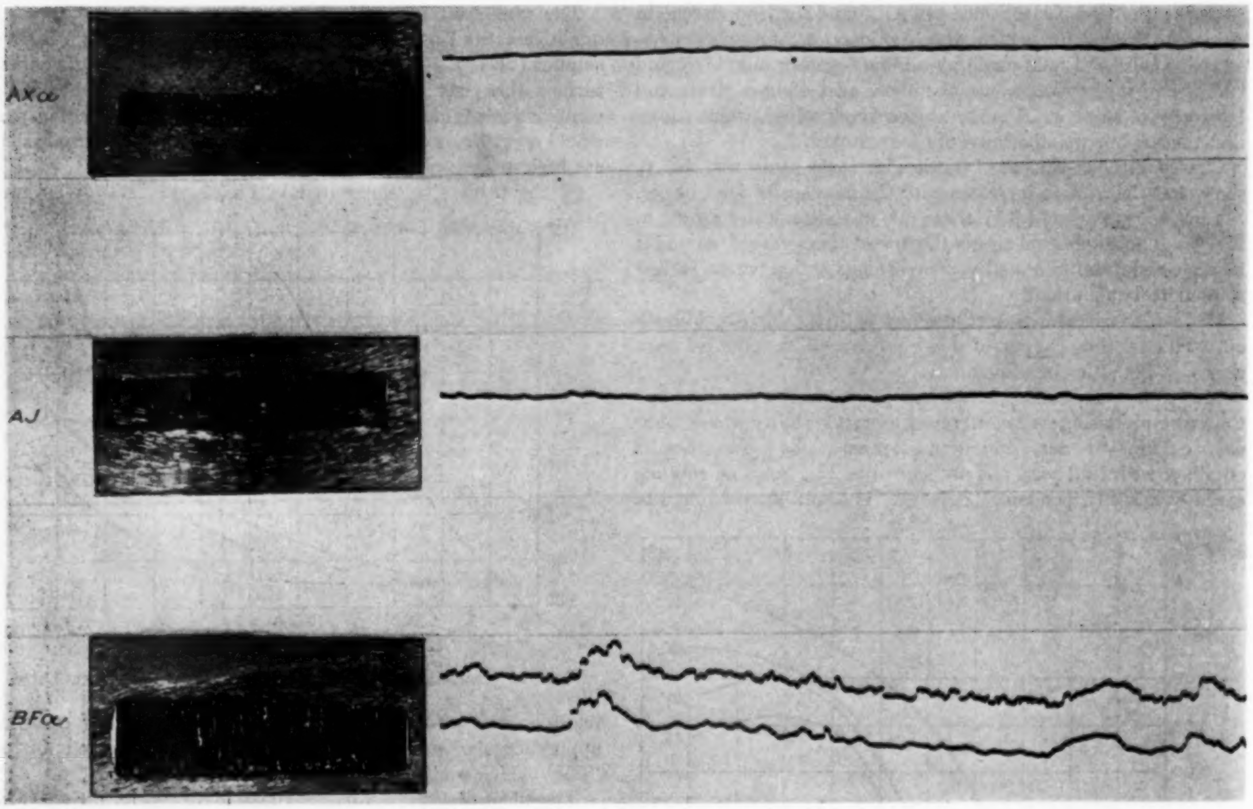

Fig. 8 Test Results Obtained With Lead-Soap Oils

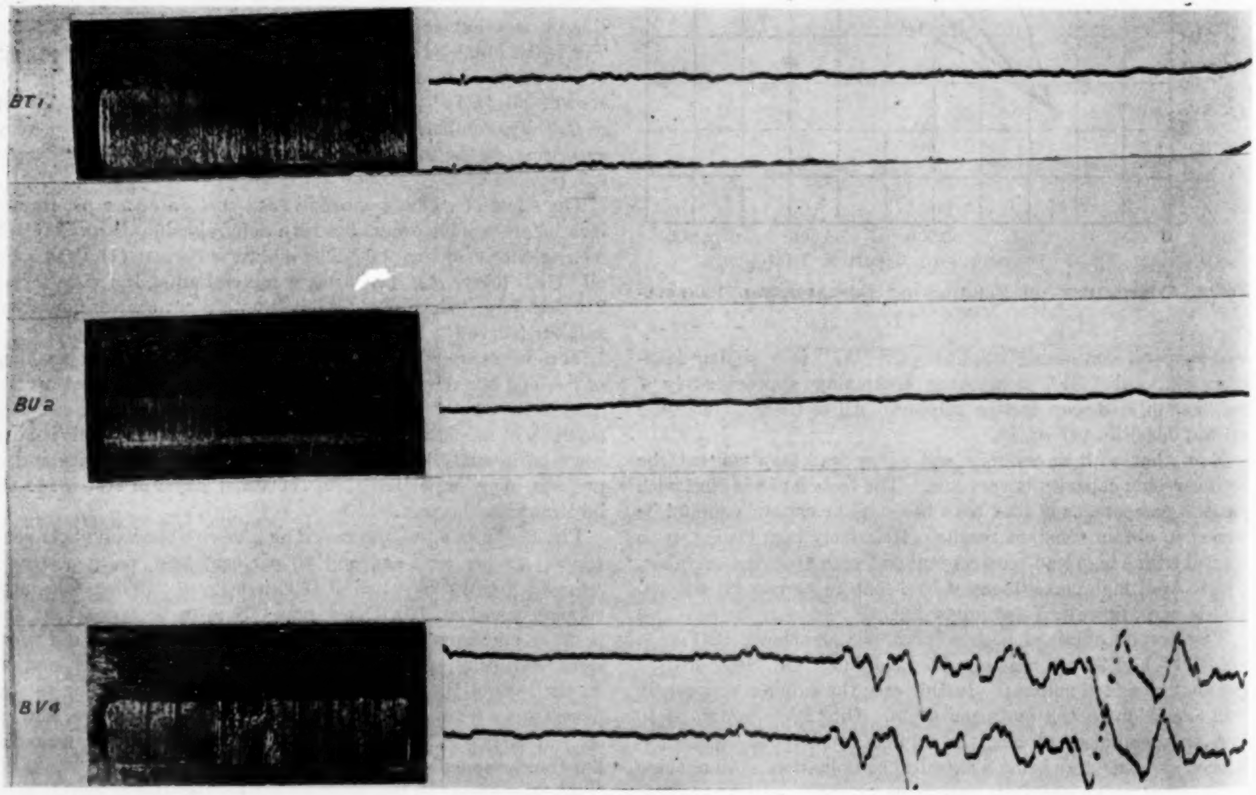

Fig. 9 Test Results Obtained With Sulphurized Mineral Oils

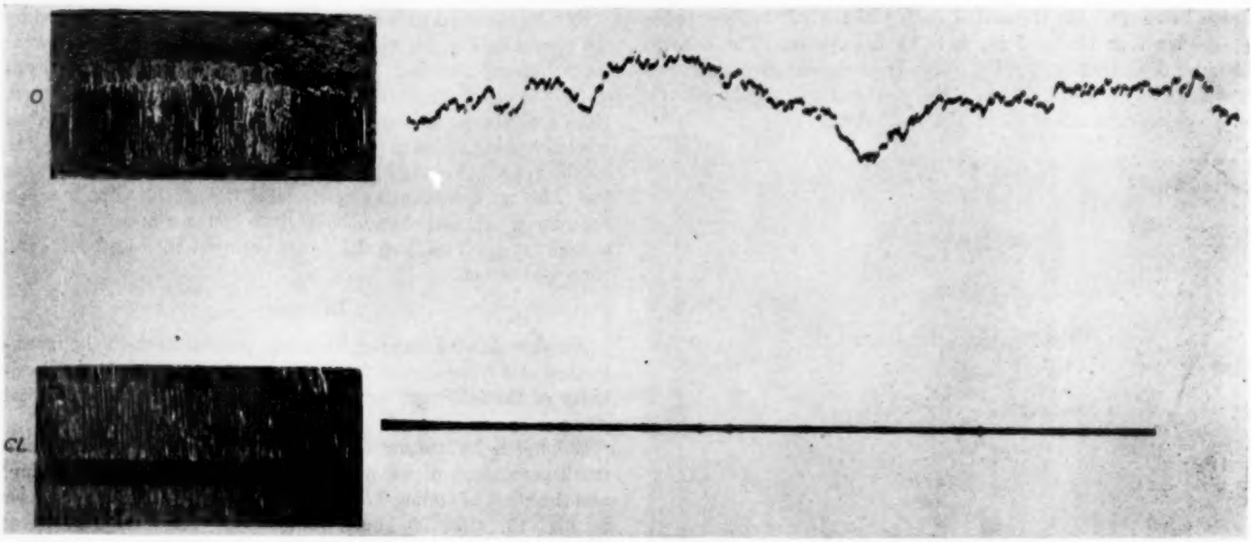

Fig. 10 Effects of Castor Oil and of a Straight Mineral Oil With Cutting Oil Added

fluid oils or greases to be tested for load-carrying capacity.

It is interesting to note the difference between greases on the adhesiveness of the grease to the test pieces. This shows up in the increased carrying capacity and appears a very important item in the selection of a grease for heavy loads.

Greases that readily channel and will not be drawn into the load zone will not carry very high pressure and will score very readily.

It should be noted that the temperatures of operation within certain limits do not give a true indication of the lubricating value of the grease, which, after all, is important only in keeping down the local temperature of the parts subject to loading.

A desirable feature of any lubricant is its heat-dissipating quality, and for this reason lubricants of high viscosity are not suitable for high-speed lubrication.

Abrasive Properties

Coupled with the efforts to obtain lubricants which will stand high unit pressures is the problem of abrasiveness of the compounded lubricants.

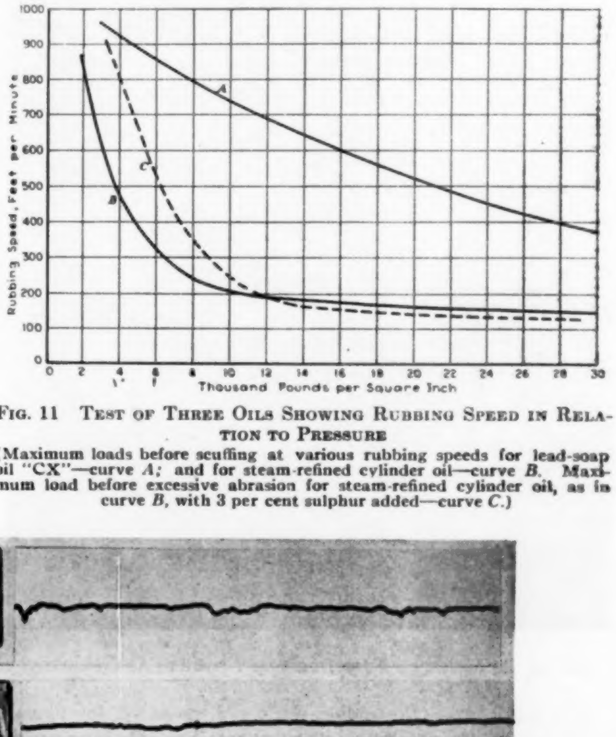

Fig. 11 Test of Three Oils Showing Rubbing Speed in Relation to Pressure

(Maximum loads before scuffing at various rubbing speeds for lead-soap oil "CX"—curve *A;* and for steam-refined cylinder oil—curve *B.* Maximum load before excessive abrasion for steam-refined cylinder oil, as in curve *B,* with 3 per cent sulphur added—curve *C.*)

It might be stated that in these investigations of lubricants there has not been found a lubricant which would permit higher unit pressures to be carried which did not at the same time have an increased abrasive action on the load-transmitting members.

To test a lubricant for its abrasiveness, a machine was built to make comparative tests, and is illustrated in Fig. 13. It embodies four pinions, each mounted on two bearings and driven by a single ring gear. This machine is mounted in a vertical position and driven by a vertical electric motor through a belt drive.

The pinions are run at 2000 r.p.m., and the lubricant is supplied to the gear box and circulated through pipes to the four

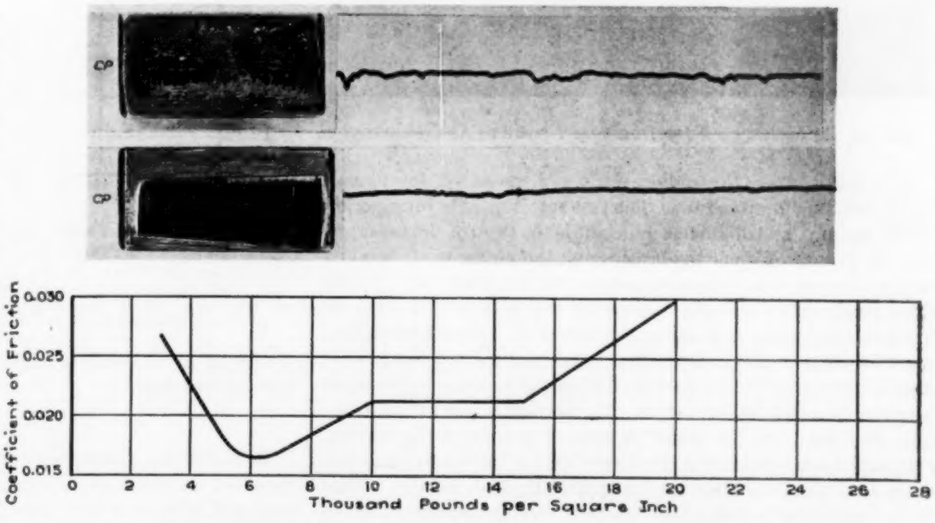

Fig. 12 Test on Phosphor-Bronze Block, Using Steam-Refined Straight Mineral Oil

pinion housings. On the end of each pinion shaft is mounted a fan having four blades 5 in. wide by 6 in. long. The outside diameter of the fan is $16^1/_2$ in. This arrangement gives a load of approximately 8 hp., and is also the means of cooling the oil to a constant temperature of about 150 deg. fahr.

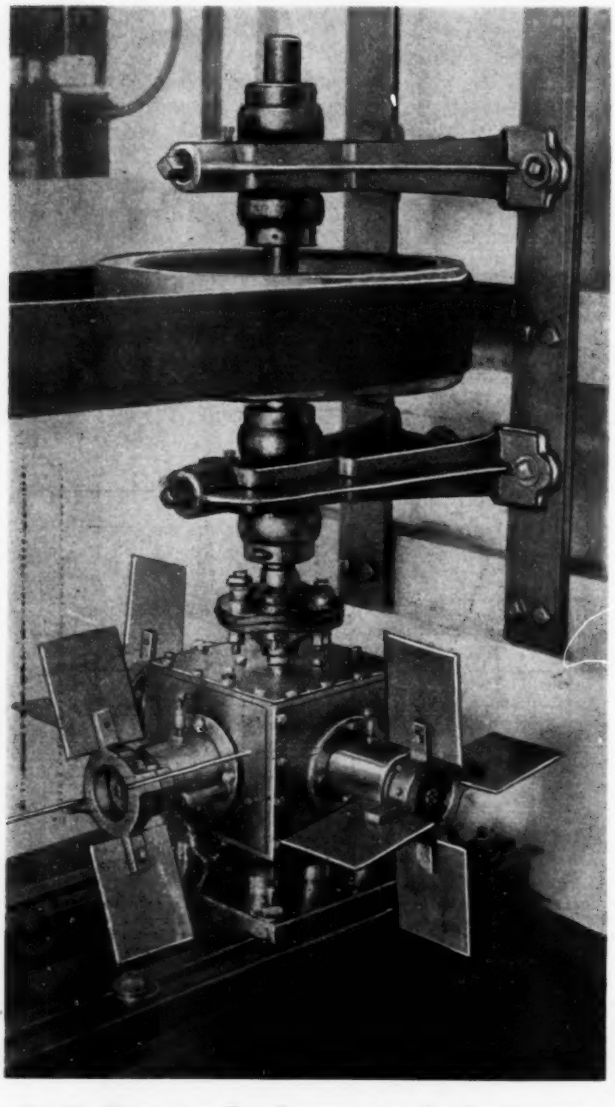

FIG. 13 MACHINE TO TEST LUBRICANT FOR ITS ABRASIVENESS

The bearings on the pinion shaft are measured for overall height before and after test. The test run is usually for a period of 200 hours. The difference in height is the drop of the bearings, and the average drop of the bearings on the pinion shaft is taken as the measure of the abrasive property of the lubricant. Under these conditions a straight mineral oil will give a total wear on Timken bearings of approximately 0.0015 in. on the two pinion-shaft bearings in 200 hours. It is important when testing bearings for wear that the load be applied on the bearings in the same way that it is applied in service. In the case of the pinion bearings the load from the pinion is applied outside of the bearing axis, and when considering the thrust load in particular produces more wear than when the load is applied through its axis. This is the reason for building this machine, and the results obtained have justified its development.

Results obtained with some of the oils tested are represented by the curves in Fig. 14, which show comparative amounts of wear for 200 hours' duration. These reveal that straight mineral oils of 150 viscosity, which was the lowest viscosity of the oils tested, gives a minimum amount of wear. Abrasion is increased when ingredients are added to the oil, the amount of abrasion depending on the type and quantity of the substances present. Note that the chemically combined sulphur oil of 3 per cent sulphur was a Smackover oil, and showed very little increase in abrasion over straight mineral oil, and that wear occurred with lead-soap and copper-oleate oils.

ETCHING

Another disadvantage which might present itself in the manufacture of a compounded lubricant for high pressure is the possibility of the lubricant causing etching of highly polished metal parts.

Sulphur in its free state, particularly when in the presence of a small percentage of water, is very active in producing etching, and the effect of adding 0.5 per cent of water to these oils is shown in Fig. 15, after 50 hours' immersion. The oils used were: (1) straight mineral oil of 150 viscosity at 210 deg. fahr.; (2) Smackover oil of similar viscosity, which is a natural sulphur oil of 3 per cent sulphur; (3) straight mineral oil (oil 1, with the addition of 2 per cent of free sulphur).

When sulphur is in its free state, considerable discoloration is produced, which is obviously undesirable in anti-friction bearings. The effect of discoloration on natural sulphur oil is much less marked, and perhaps no trouble would be experienced with this oil.

It is important, therefore, that lubricants containing sulphur should not have this element in a free or active state, which would be detrimental to highly polished surfaces such as anti-friction bearings.

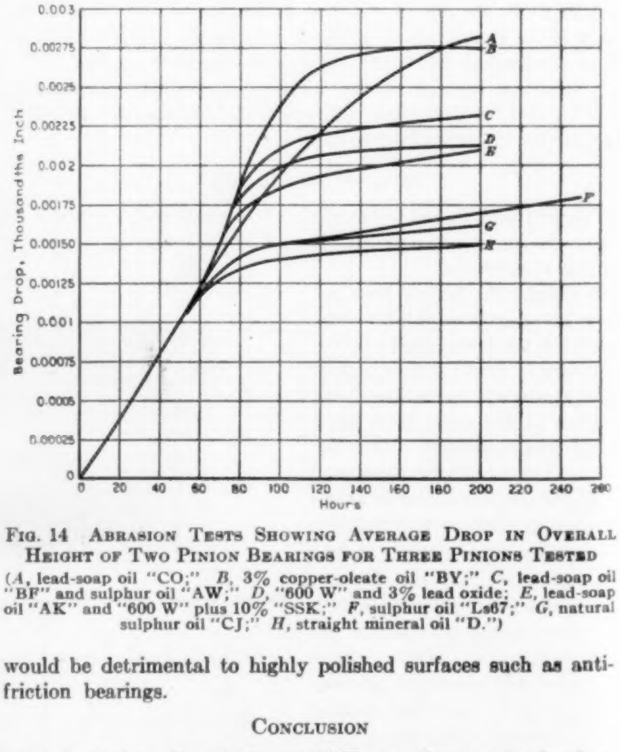

FIG. 14 ABRASION TESTS SHOWING AVERAGE DROP IN OVERALL HEIGHT OF TWO PINION BEARINGS FOR THREE PINIONS TESTED
(*A*, lead-soap oil "CO;" *B*, 3% copper-oleate oil "BY;" *C*, lead-soap oil "BF" and sulphur oil "AW;" *D*, "600 W" and 3% lead oxide; *E*, lead-soap oil "AK" and "600 W" plus 10% "SSK;" *F*, sulphur oil "Ls67;" *G*, natural sulphur oil "CJ;" *H*, straight mineral oil "D.")

CONCLUSION

In conclusion, the authors would like to advocate the development of standard apparatus for determining the carrying capacity of lubricants to be used for gearing in general and particularly for high unit loading.

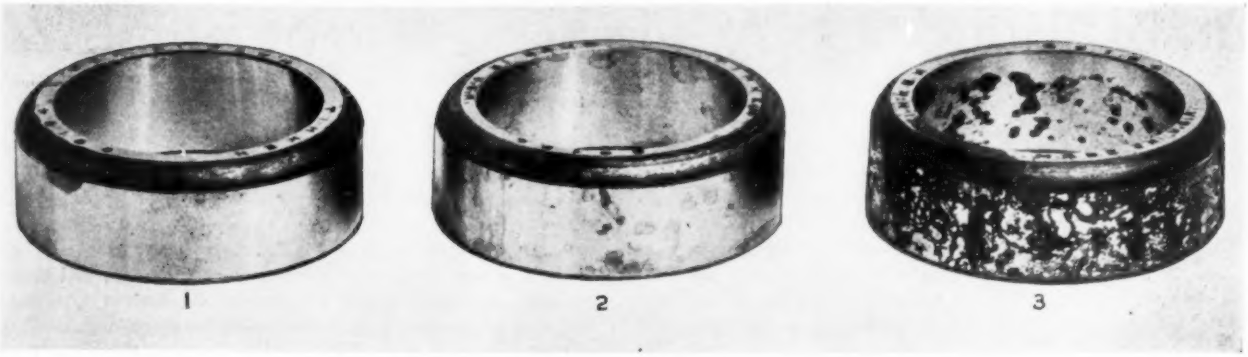

Fig. 15 Etching Produced by Sulphur, With Small Quantity of Water Added

The effect of rubbing speed and surface temperature should be taken into consideration, and also the quantity of oil flowing over the loaded members during the test.

From the results obtained on the abrasion machine the use of an abrasive filler should not be used where these lubricants must lubricate anti-friction bearings, and the etching effect of the lubricant must be held to a minimum.

A lubricant made for this type of service should have at least the following characteristics:

(a) Freedom from separation when standing and in service
(b) Minimum of abrasive action
(c) Ability to carry certain specific loads at definite rubbing speeds
(d) Freedom from channeling at low temperatures
(e) Be reasonably cheap and conveniently obtainable
(f) Shall not produce etching or discoloration of bearings and the like.

Considerable research work has been done within the last two years, and all the important oil companies are now busy developing lubricants which will be suitable for the higher speeds and heavier loads which are becoming more and more in evidence day by day.

Welding Progress at the A. O. Smith Plant

By T. McLean Jasper,[1] Milwaukee, Wis.

This paper covers the history of welding of pressure vessels in one plant covering a period of one-third of a century. The oil industry is credited with having provided the opportunity for the rapid development of welding, not being limited by code restrictions as to the size and shape of desired pressure equipment. Testing equipment is described, and tests on steel for pressure vessels are detailed.

The history of welding by the A. O. Smith Corporation dates back to the year 1897, or about one-third of a century. The first product to be welded consisted of bicycle hangers and clips. This welding was performed with acetylene gas obtained from France. In 1899 the first electric-arc welding was performed, which soon replaced the acetylene torch for this type of work. Electric butt or resistance welding was first used in 1903 for solid spindles. Then, in 1911, welding by oxygen and hydrogen gas came in use for automobile parts. Rear-axle housings first had minor parts welded with the oxyacetylene gas, and in 1913 reinforcing rings were spot-welded into these housings.

The electric arc replaced oxyacetylene for quantity production in 1917, and from that time on, this new method of welding has held the premier position in the company's progress in welding.

Since this period over 1,000,000 tons of steel have been put into electric-arc-welded products, of which the chief details are given in Table 1.

TABLE 1 WELDED PRODUCTS OF A. O. SMITH CORPORATION SINCE 1918

Type of product	Quantity of units	Total weight in tons	Years shipped
Aerial drop bombs	287,895	4,318	1918–1919
Artillery poles	52,857	3,964	1918–1919
Automobile rear-axle housings	2,797,152	48,950	1918–date
Casing couplings	1,218,626	21,326	1923–date
Large pressure vessels	1,675	52,165	1925–date
Oil and gas line pipe, miles	7,838	1,054,120	1927–date
Total		1,184,843	

The first and second items of the list in Table 1 were war materials, necessitating a rapid expansion in production. The service conditions demanded a weld as strong as the plates that were joined, without an increase in thickness of the welded section. It was at this time that the foundation was laid for the type of welding and the principles of quality which made possible the present broad application of the welding art.

From the time that welding was first employed a third of a century ago up to 1917, welds were applied only to secondary members, in which a possible failure meant little to safety of life and property. However, with the application of welding to axle housings, a failure might mean serious injury or loss of life.

The principles of quality welding established at this time demanded a weld of a material similar to the plate in strength and energy-absorbing qualities. This meant high ductility and impact values in the deposited weld. It also meant that the strength and energy-absorbing qualities of the metal adjacent to the weld be not impaired by the act of welding. These requirements necessitated great improvements in the quality of the plate steel and in what at that time were considered very stringent materials specifications for its production.

From the beginning, welding had to face opposition, chiefly from zealous code committees and from interests equipped for the older forms of fabrication, and obviously also from users that had suffered unpleasant experiences with welded materials of inferior quality. This attitude is now rapidly changing, thanks to the production of welds of high quality and to the tireless efforts of the pioneers in welding. Thirteen years of diligent application and experimental work has shown results in the company's welded-steel products which in many instances embrace the most severe types of service to which steel can be put. Users of equipment know the advantages of welding and are learning to differentiate between good and bad work.

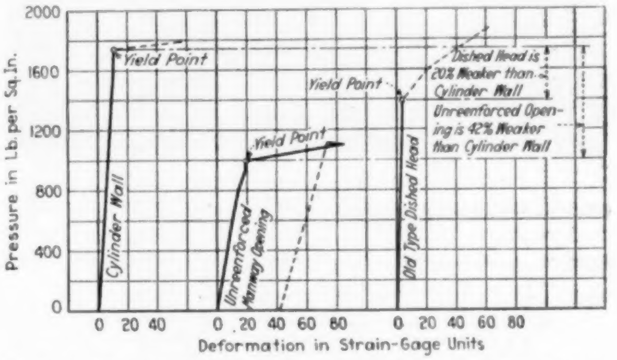

Fig. 1 Lack of Uniformity in Strength of Vessel With Dished Heads and Unreenforced Openings

(These curves were plotted from test results obtained simultaneously during destruction test on a vessel 6 ft. in diameter, 2 in. thick, and about 40 ft. long.)

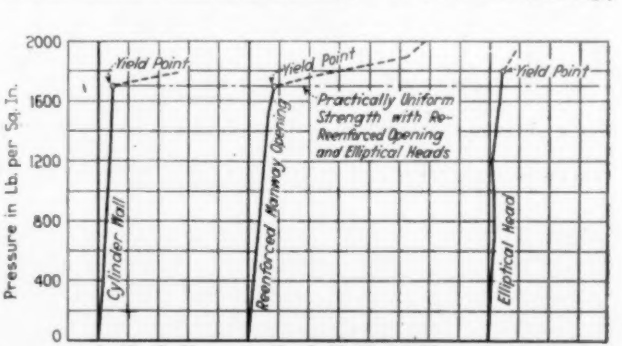

Fig. 2 Uniform Strength in Vessel With Elliptical Heads and Reenforced Openings

(These curves were obtained from tests to destruction on full-size production vessels 40 ft. long, 6 ft. in diameter, with 2-in. wall.)

It certainly was fortunate for the future of welding that, during this development, the oil industry was guided by progressive, wide-awake men. This allowed the development of welding to proceed in such a manner that it won out in a most difficult type of equipment for a hazardous service.

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[1] Director of Research, A. O. Smith Corporation. Mem. A.S.M.E.

Presented at the National Petroleum Refining Meeting, Elizabeth, N. J., May 5 and 6, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Looking back over these years of the company's welding development it is noteworthy how many full-sized vessels were tested to destruction as part of the program to perfect both design and process. It is curious that, until then, so very few tests on full-sized equipment were available on which sound engineering judgment could be based. Long-time experience was the measure of advancement practiced, and naturally very timid progress was made. This attitude produced a condition in which industries were being retarded because of their unwillingness to spend a relatively small amount of money to carry on logical testing.

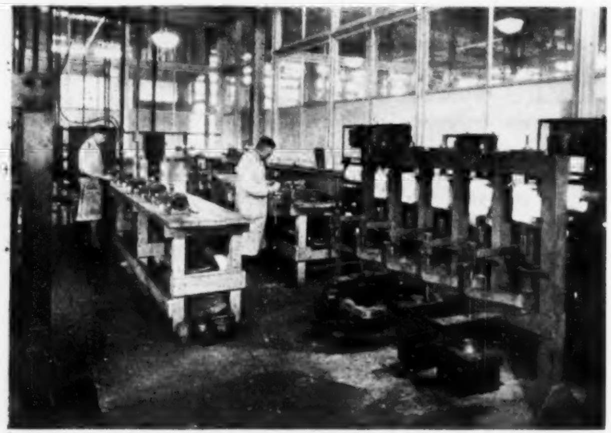

FIG. 3 A. O. SMITH RESEARCH LABORATORY

(Showing fatigue-testing machines to the left and elevated-temperature-testing machines to the right.)

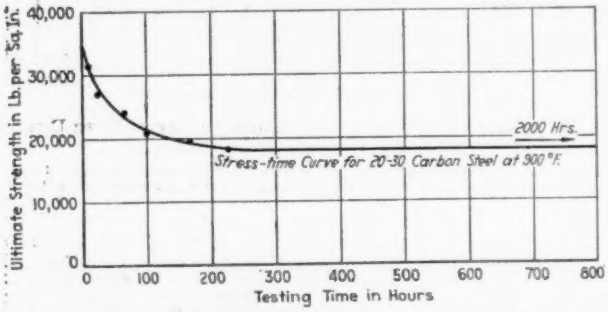

FIG. 4 METHOD IN ARRIVING AT LONG-TIME ULTIMATE STRENGTH OF STEELS

(The two last specimens were run for 2000 hours without failure at a stress of 17,000 and 18,000 lb. per sq. in. respectively.)

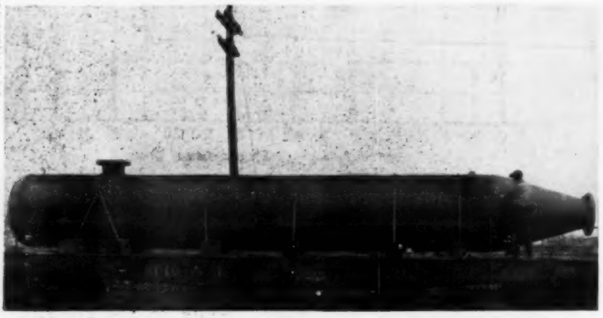

FIG. 5 ONE OF THE FIRST WELDED OIL-CRACKING STILLS

(This was 41 ft. long, 5 ft. in diameter, and $2^1/_2$ in. thick, and was shipped July 25, 1925.)

Testing is an important adjunct to proper engineering application in that it points out weaknesses in methods of construction and design, which can be taken care of only when known and actually understood. Through this means the designers of the future will and can pay more attention to safety than has been done in the past. Safety, of course, is a term which is both relative and arbitrary. Testing is the most important means for clearing up this matter.

The elements which enter into the question of safety in a vessel are as follows:

1 Design
2 Materials
3 Fabrication
4 Testing

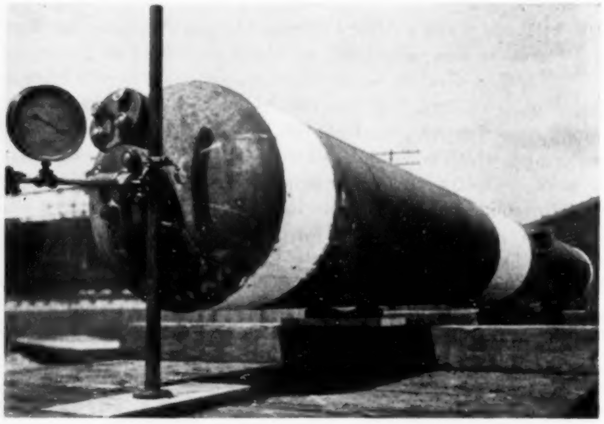

FIG. 6 WELDED REACTION CHAMBER OF CHROME-VANADIUM STEEL

(Operating pressure is 5000 lb. per sq. in. Vessel is shown under hydrostatic test pressure of 10,000 lb. per sq. in.)

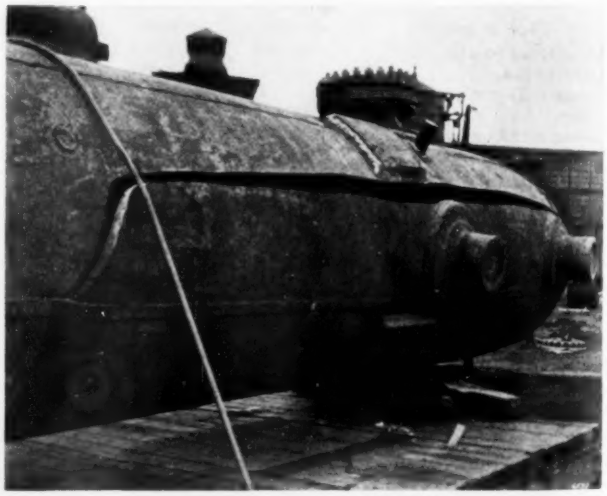

FIG. 7 RESULT OF DESTRUCTION TEST ON WELDED VESSEL AFTER $3^1/_2$ YEARS' SERVICE

(Full strength of the steel was developed, and the fracture occurred through the plate proper, away from the welds.)

Pressure-vessel design is a matter over which there has been almost as much controversy as over the acceptance of welding. This company early practiced methods of design which were based on the outcome of destruction tests on large vessels and numerous researches on the materials used for their construction. It is gratifying that today these principles of design are quite generally recognized and accepted.

Figs. 1 and 2 show how a design can be checked by means of testing. However, the design of a vessel cannot be tested in miniature. The thin walls of such a miniature test vessel will invariably give results which cannot be directly interpreted for a thick-wall vessel. Of course, it is costly to work with full-sized vessels, but nevertheless it is the least expensive method of safe development.

After all, knowing what a vessel will carry in service is much more important to the user than what the calculations of a theorist show it should be able to carry.

On the matter of materials it is interesting to observe how welding has shown up defects in steel not previously known or understood. This has resulted in improved quality of present-day steel used in welded products and in longer life of vessels made from such steel because of its greater resistance to corrosion.

Figs. 3 and 4 show testing equipment and typical long-time test results for a 0.20 to 0.30 carbon steel when used at 900 deg. fahr. A clear understanding of the strength of steel under long-time tests at elevated temperatures is essential, particularly to the refiner, and this, in conjunction with the quality of steel, is all that need be said about materials at this time.

Correct fabrication methods for large-size, heavy-wall vessels is much more important than for thin-wall vessels, and the proof of successful operation is needed to establish adequate precautions in fabrication. Fig. 5 shows such a large vessel, which has been in successful service for nearly six years.

Today it is possible to successfully weld vessels from almost any kind of appropriate steel. A considerable number of stainless steels and of alloy steels of other compositions find application in welded vessels. Even vessels made of common carbon steels lined with stainless steels were introduced by this company. Fig. 6 shows an example.

While testing cannot be carried to destruction with each vessel, there is no reason why every vessel cannot be tested to a relatively high value of stress in conjunction with lime wash, which will show up possible weak points. That this is not detrimental to large vessels is well proved by subsequent tests made on some vessels that this company built nearly six years ago for the oil industry. After three and one-half years of successful operation, these vessels were withdrawn from service and tested to destruction. They developed the full strength of the steel from which they were made, fracturing through the plate proper, as shown in Fig. 7.

From the foregoing it is evident that the oil industry is to be credited with having provided the opportunity for the rapid development of welding and in turn has greatly profited thereby. This industry was not code-restricted, and being free to buy and use equipment best suited for the requirements, naturally made use of the fabrication process which did not limit the desired pressure equipment as to size and shape.

Discussion

Walter Samans.[2] This paper rightly calls attention to the commendable leadership which the A. O. Smith Corporation has exerted in the development of rational designs for pressure vessels of welded construction.

The author's statement that the user is more interested in what a vessel will carry than in what the calculations of a theorist show it is good for, is not believed to be a reflection on the proper use of theory in design, but rather on the limited spread of theory that has been corroborated by practical tests.

In this connection, his last paragraph, pointing out that the oil industry, as regards pressure vessels, was not code-restricted and was able to profit thereby in taking advantage of modern progress in fabrication, is of particular interest. The A.S.M.E. Unfired Pressure Vessel Code has since recognized fusion-welded vessels by a new section, which, while for the present applied to service temperatures not exceeding 300 deg. fahr., will eventually, with the expected cooperation of the oil industry, have no limiting service temperatures, as far as materials permit.

There still remains much to be done on full-sized tests to determine the practical limitations of such theoretical knowledge as can be mathematically developed when taking into account the variables in the metallurgy of steel and its alloys and the tolerances that must be permissible in fabrication.

On the whole, the paper presents the high points showing need for uniform strength in all parts of the pressure vessel, to obtain maximum service life and safety.

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[2] Chief Engineer, Atlantic Refining Company, Philadelphia, Pa. Mem. A.S.M.E.

PME-54-3

New Development of Chromium-Tungsten Steel for Oil-Refining Service

By V. T. MALCOLM,[1] INDIAN ORCHARD, MASS.

Apparatus used in oil-refining service has been designed mostly from practical experience instead of experimental research, and the experience gained from operation over long periods gives notable results, among them being that certain alloys are resistant to corrosive conditions, and that if these alloys are used the so-called limiting creep stresses can be more than doubled over carbon steel. Other factors besides the strength of metals and corrosion resistance that must be considered are thermal expansion, thermal conductivity, resistance to abrasion, scaling, and oxidation. The search for an alloy steel that would resist the severe service in oil refining resulted in the development of chromium-tungsten steel. A series of tests are described in the development of properties required of a steel for high-temperature construction and service.

DURING recent years engineering interest in the design of equipment for operation at greater pressures and higher temperatures has steadily been increasing. This is true of structures of the most varied sorts, but actually has been most marked in the field of oil refining. The oil-refining industry, not perhaps as much hampered by tradition and precedent, and under greater pressure and increased efficiency, has not hesitated to construct and operate equipment, including valves and fittings, at temperatures as high as 1100 deg. fahr., as these progressive engineers and constructors have not feared to go to this temperature after experience and careful practical experiment.

However, the conservative attitude of engineers has been reasonable in view of the limited amount of information available upon the behavior of metals and alloys at high temperatures, and particularly on account of the manner in which it has been presented and regarded.

It is felt, however, that the rational interpretation of what is now known of the behavior of metals under stress and temperature justifies a less conservative attitude.

There is very little doubt that guessing and unsupported assumptions have liberally been indulged in, but it is certain that no solution of the problem is likely to command acceptance unless backed by specific established facts.

That the study of corrosion resistance and of the properties of metals at elevated temperature is receiving much attention is shown by the numerous articles on these subjects in the technical press, and from these articles valuable data have been accumulated, but it is certain that the entire investigation will be made far more simple when agreement is reached by various investigators.

Much of the apparatus now in operation in refinery service was designed from practical experience rather than from experimental research; and the experience gained from operation of certain equipment over periods of long duration gives results that are well worth considering, among which is the fact that certain alloys are resistant to peculiar corrosive conditions of refinery service and that, if these alloys are used, the so-called limiting creep stresses can be more than doubled over carbon steel when equipment is being designed for safe operation.

However, there are other factors besides the strength of metals and the corrosion resistance at elevated temperatures that must be taken into consideration, these factors being thermal expansion, thermal conductivity, resistance to abrasion, scaling, and oxidation.

The oil refinery, with its processes involving complex reactions mostly of a highly corrosive nature, presents special problems in the supply of a suitable alloy for tanks, evaporators, tubes, valves, fittings, etc., to restrain these corrosive activities from being exerted on the plant itself, either by direct contact or through other vapors.

Until recently the materials used in refinery service were cast iron, cast steel, and some limited use of bronze and brass. However, with the use of higher pressures and temperatures, many of the metals that formerly served the purpose are now no longer used except for unimportant service, and the refineries are now found to be in a state of transition toward the use of special metals and alloys, as had been experienced in other industries some years ago.

Special metals and alloys cannot fail to be of importance in affecting the destinies of refinery engineering progress; and developments in this direction should be of great interest, especially if noteworthy advances are to be made; for it has not been a difficult matter to increase the pressure of operation, because increased pressure can be taken care of by simply increasing the thickness of the metal used, but higher temperatures and greater corrosion resistance will have to be arrived at much more slowly and with greater caution.

What the refinery engineer needs is a comparatively cheap metal that will have high physical properties not impaired by high-temperature service. He needs a metal that is resistant to corrosion, a metal that will hold its surface under heavy sliding pressures, and one that will not suffer from abrasion due to the great velocity of the pressure and gases used; and in order that a metal or alloy may have this resistance to corrosion and oxidation, it must be stable at the temperatures employed and be capable of resisting for the economic life of the piece of equipment of which it forms a part the stresses to which it will be subjected.

All should like to see this ideal alloy, possessing, as it would

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[1] Metallurgical Engineer, Chapman Valve Mfg. Co. Mr. Malcolm has been in his present position for the last ten years. Previously he was associated with the Bartlett Hayward Company of Baltimore as engineer of tests. He is a member of the Joint Research Committee on the Effect of Temperature on Properties of Metals, the Heat Treatment Committee of the Iron and Steel Division of the A.I.M.E., the Foundry Committee of the A.I.M.E., the Committee on Steel Castings for High Temperature Service of the A.S.T.M., and the Committee on Pipe Flanges and Fittings of the A.S.A.

Presented at the National Petroleum Refining Meeting, Elizabeth, N. J., May 5 and 6, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society

have to, innumerable contradictory qualities, but it is not wise to sit down and wait for it. It is much more to the point to make the best of such metals as are now at hand and to believe until definite proof to the contrary, that progress is still possible with the use of existing alloy steels.

In these circumstances, the knowledge of the wonderful fertility of the field of alloys has stimulated the search for new alloys. Because of the developments of refinery processes now being pursued so vigorously and successfully in the direction of reactions carried out under high temperature and pressure, superior-strength alloys become still more desirable—in fact, a necessity.

To those engaged in the study of metallurgy, who are aware of the many directions in which alloying has been found to transform the physical properties of metals, the conferment of chemical stability by such means is entirely possible.

There was a time when manufacturers, as a general rule, did not care to bother with special steels or alloys, and particularly is this applicable to plants making specialized products, for the production of these in the regular conduct of business presented peculiar problems which could not be solved without much experimental work.

Because of demands made by the modern refinery for alloys to take the place of ordinary carbon steels (which have been used to a great extent in the past because of their relative cheapness and ease of fabrication) and on account of their recognized limitations, it has become necessary for the manufacturer of specialized products to develop particular alloys that are resistant to corrosion, abrasion, and oxidation and that at the same time maintain their strength at elevated temperature.

It is almost impossible to exaggerate the importance of the place that alloy steels are taking in the industrial world. The day of special alloy steels has certainly arrived, and the engineer who would keep in line with the march of progress should give more than momentary consideration to the spectacular advance of certain of these alloy steels.

Alloy Steel for Refinery Service

Years of laborious and costly scientific and practical research were required in the search for an alloy steel that would withstand the severe service encountered in oil refineries, and this work resulted in the development of chromium-tungsten steel. Its discovery and application have been made possible only through the coordinate help of the metallurgist and refinery engineer.

Metallurgists, as a general rule, have accepted the belief that metal consists of two states, a crystalline and an amorphous state. The crystalline phase or state is "thermally stable but mechanically unstable," and an amorphous phase is "thermally unstable but mechanically stable." This implies that the two phases are not fixed, but may vary according to given conditions.

According to White and Clark,[2] if it is desired to increase the strength of a metal consisting of two phases of unequal strength the following ways are open:

(1) The addition or formation of some substance which by its presence increases the strength of the weaker phase.

(2) The addition or formation of some substance which tends to remove the weaker of the two phases.

And from this it is first necessary to determine which of the phases is the weaker and then to take steps to strengthen or remove this weaker phase.

It is a well-known fact that at ordinary temperature the fracture of steel proceeds through the grains and not around them, this being caused by the greater strength of the amorphous boundaries. As the temperature is increased, however, the amorphous phase weakens at a more rapid rate than the crystal proper and a temperature is reached where it proceeds around the grains and not through them. This temperature, which Jeffries[3] calls the equicohesive temperature, corresponds closely to the recrystallization temperature. Therefore it is important to know over what temperature range and for what duration stability is desired before the effect of alloying elements can be properly determined.

White and Clark[2] advance the following hypothesis: If stability is desired below the equicohesive temperature, they find that—

(1) The addition or formation of any element, compound, or constituent which does not enter into solid solution with the matrix, but which by the very nature of its presence interferes with crystal slippage, will tend to increase the stability of the materials of which it forms a part.

(2) The addition or formation of any element, compound, or constituent which strengthens the grain boundaries by interfering with plastic flow will increase the stability of the material of which it forms a part.

Considering these foregoing principles and the effect of certain alloys upon the structure of metals to which they are added, it should be possible to predict and explain the influence of various alloy additions upon the stability of the resulting alloy.

Many alloying elements influence stability throughout the temperature range of the steels to which they are added, but the effect may vary depending upon the temperature under consideration.

The carbide formers, such as chromium and tungsten, increase the stability of metals below the equicohesive temperature because of carbide particles interfering with crystalline slippage, and they increase stability above the equicohesive range due to carbide formers at the grain boundaries interfering with plastic flow.

It is now generally recognized that, when a metal is alloyed with one or more elements, the properties of the metal are changed to a marked degree. Therefore, by suitably alloying a metal, for instance with chromium and tungsten, the properties of the resulting alloy possess profound qualities. One of the most important objects in the manufacture of chromium-tungsten steel is that the alloying elements enable one to decrease the injurious or undesirable properties and to intensify others that are preferred in the final product.

The effect of carbon as a constituent of steel is well understood, and the presence of chromium and tungsten intensifies the effect of carbon in all the best qualities that it is capable of producing alone. These increases take form not only in the physical results such as tensile strength, elastic limit, dynamic strength, resistance to wear and fatigue, increased hardening power, lack of brittleness, etc., but also in the purity, solidness, and homogeneity of the metal, because the alloying elements induce a freedom from occluded gases.

A quality often described as "tough hardness" is especially desired and is one of the outstanding features of chromium-tungsten steel. This so-called quality is in all probability due to the somewhat finer and denser structure of the microscopic constituents and their uniform distribution. This fine dense structure so paramount in quality steels is obtained only through the actual influence of the combination of alloying elements in producing in the steel a condition of purity and freedom from gases, and subsequently forms a metal which, at the time structural changes occur, is in the best possible condition for these changes to take place.

This dense structure is one of the major reasons for high physical properties and greatly increased resistance to fatigue, shock, erosion, and corrosion.

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[2] "Stability of Metals at Elevated Temperature," University of Michigan Engineering Research Bulletin no. 11, Nov., 1928, pp. 92–94.

[3] "Effect of Temperature, Deformation, and Grain Size on the Mechanical Properties of Metals," Trans. A.I.M.E., vol. 60 (1919), pp. 474–577.

The addition of chromium and tungsten to improve the physical properties and corrosion resistance is particularly valuable because they possess, when added to steel, all the advantages of ordinary steels as regards casting and forging. This type of steel can be cast, forged, rolled, and stamped; it machines well, and the chemical and physical properties are readily under control and enable the manufacturer to consistently comply within reasonable limits of the specifications.

As the production of clean, sound steel is the first requisite, the second of great importance is the thorough and proper heat treatment, so that the alloy steel may possess the best properties of which it is capable.

ALLOYING ELEMENTS

The use of chromium as an alloying element to retard corrosion was first mentioned about 100 years ago when Faraday, in England, in 1820, made experiments with alloys to retard oxidation and when Berthier, in France, in 1822, reported that when a small amount of chromium was added to iron he found an alloy not readily attacked by acids.

Chromium has been called the "master metal" even in these days of spectacular advances in metallurgy, because it is almost indispensable and through its characteristic properties it probably affects a more diversified range of industrial activities than any other metal in common use except iron.

Chromium, either alone or in combination with iron, forms a very stable carbide, diffusing itself throughout the metal and inducing a fine grain and correspondingly great toughness because of its reluctance to accumulate into sizable masses.

Furthermore, the chromium remaining after all the carbon is satisfied enters in solution in the iron and confers considerable resistance against corrosion to the base metal. The addition of chromium does not increase the brittleness of steel, as would be the case were additional hardness obtained by the use of a higher carbon content. Chromium steels harden much deeper than the same steels without chromium, and as chromium refines the grain of the steel and is partially dissolved in the iron, both characteristics add to the toughness and hardness of steel—a truly remarkable combination.

Chromium produces certain characteristic effects on the properties of steel, as follows:

(1) It lowers the carbon content of the eutectoid ratio.

(2) It raises the temperature of transformation Ac_1–Ar_1.

(3) It forms with carbon a series of carbides which go into solution slowly. High temperature is necessary as the speed of diffusion of carbon is diminished.

Chromium dissolves in iron in all percentages, forming solid solutions with the ferrite. In the presence of carbon, however, it forms a double carbide with the iron. These carbides that are deposited within the crystals will interfere with the process of crystalline slippage, and for that reason materially increase the stability of the alloy at all temperatures below the equicohesive temperature.

The carbides which are precipitated at the grain boundaries will interfere with plastic flow in the material, and so increase the stability of the steel above the equicohesive temperature. However, the relation to the ductility above the equicohesive temperature must constantly be borne in mind.

It is generally agreed by metallurgists that the effect of chromium in steel is progressive and that there are no areas of composition the boundaries of which indicate abrupt changes in physical properties owing to transition of structurally free solid solutions to free carbides of different composition.

With chromium between 4 and 6 per cent there is no porosity in the form of pinholes when used in high-temperature service.

Likewise, compositions which produce the free carbides show greater strength in the alloys in which they are found than compositions which throw the carbides into solid solution as evidenced by the superior results obtained by the use of tungsten over and above nickel.

Tungsten, therefore, alloyed with chromium in steel, produces the best results as to stability either above or below the equicohesive temperature of any of the series of alloy steels tested.

Tungsten combines with carbon rather than with iron and exerts a very strong effect. It is essentially a carbide former.

Tungsten lowers the eutectoid percentage of carbon; but from the work accomplished, indications are that tungsten may not necessarily raise the Ac point to a great extent, but decidedly increases the rate of sluggishness at which this change takes place.

This same sluggishness is also encountered in the cooling cycle when coming out of solution, and a much greater hardness is developed.

The presence of tungsten increases the strength even in the cast or rolled condition, but like most alloy steels, the tungsten alloyed steels require heat-treatment to develop their best properties.

Tungsten has that ability to inhibit grain growth and gives a very high elastic ratio, and for the same strength has a much higher ductility than other types of alloy steels.

Perhaps the most important property of tungsten alloyed steels is their resistance to tempering, for in the softening after normalizing treatment it requires a very much higher drawing temperature and a longer time, which of course means that the steel may be used more safely at higher operating temperatures, and also due to the high draw, internal stresses are released to a much greater extent than with other types of alloy steels.

Inasmuch as tungsten alloyed steels do not require drastic quenching and have a slower rate for deep hardening the internal stresses may be kept very low.

Tungsten alloyed steels, particularly with the addition of chromium, show marked resistance to both the single blow and repeated impact, have excellent mechanical properties, and can be machined as readily as other types of alloy steels having much lower hardness.

Tungsten alloyed steels, with chromium, are characterized by the exceptional fineness of the grain, and this fine grain is undoubtedly a reason for high elastic ratio. Temperatures of treatment that would ruin most ordinary alloy steels can safely be used in the chromium-tungsten steels.

Tungsten is a distinctly modern metal, and the industrial applications are necessarily somewhat limited, but there is little doubt that if tungsten ores were more abundant and the metal more easily manipulated, it would displace other alloying elements for many purposes. Tungsten itself is harder than quartz, it never rusts, and it is insoluble in acids. Its expansion by heat is one-third that of iron, and it has a melting point of 6600 deg., which is greater than the heat of the electric arc. It is almost three times as heavy as iron, and its strength is greater than the strongest steel. It has been worked into wire 0.0002 in. in diameter—almost too thin to be seen, yet stronger than copper wire ten times its size.

Tungsten has that ability of sustaining high temperature without a breakdown, and any one familiar with shop practice knows that a tungsten alloyed tool is at its best when working at red heat.

CHEMICAL PROPERTIES

The chemical properties of chromium-tungsten steel meet the following specifications:

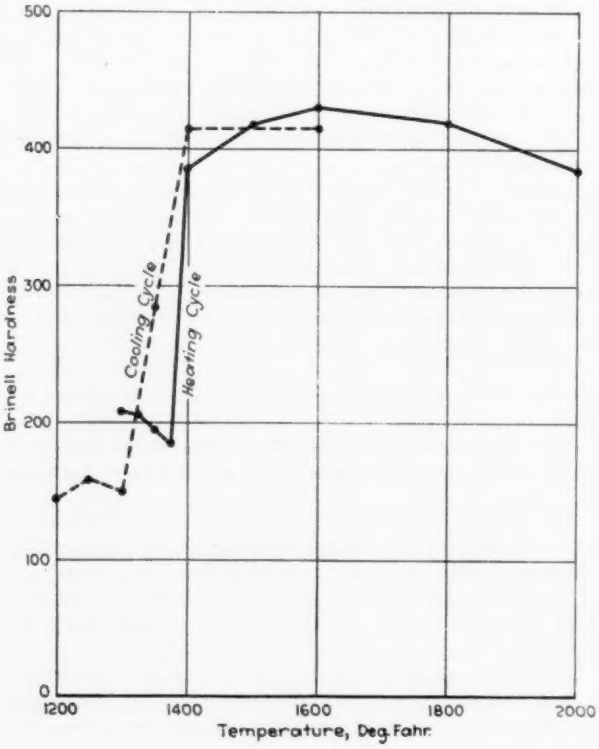

FIG. 1 HEATING AND COOLING CYCLES

Carbon	0.15–0.25
Manganese	0.55 max.
Silicon	0 50 max.
Chromium	4.50–6.50
Tungsten	0.75–1.00
Sulphur	0.05 max.
Phosphorus	0.05 max.

The chemical analysis of steels on which the following tests were made is as follows:

Rolled		Cast	
Carbon	0.153	Carbon	0.25
Manganes	0.43	Manganese	0.53
Silicon	0.12	Silicon	0.42
Chromium	5.89	Chromium	5.87
Tungsten	0.69	Tungsten	0.80
Sulphur	0.015	Sulphur	0.017
Phosphorus	0.019	Phosphorus	0.021

Test Results, Thermal Analysis, Rolled Steel. Disks were cut from the "as rolled" bars and held at the following temperatures for $^1/_2$ hour before quenching in water:

Temperature, deg. fahr.	Brinell hardness
1300	207
1325	202
1350	196
1375	187
1400	387
1500	418
1600	430
1800	418
2000	387

From this cycle the Ac transformation point appears to be between 1375 and 1400 deg., while the maximum hardness is obtained at 1600 deg., with 430 Brinell.

Quenching from higher temperature appears to retain the austenite.

COOLING CYCLE

Temperature, deg. fahr.	Brinell hardness
1600	418
1400	418
1350	286
1300	149
1250	156
1200	149

This cycle indicates the sluggishness of this type of steel in coming out of solution. A decided drop in hardness is noticed at 1350 deg. fahr., and a second is obtained at 1300 deg. fahr. where the maximum softness is developed.

From this it may be noted that in the heating cycle, employing a temperature range of 1300 to 2000 deg. fahr., there is a decidedly increased rate of sluggishness at which transformation takes place. This sluggishness is also encountered in the cooling cycle when coming out of solution, and this phase is common to tungsten alloyed steels. Maximum hardness is obtained at 1600 deg. fahr. From the hardness values as shown, it appears that austenite is retained at 1800 deg. fahr. and above. Heating and cooling cycles are shown in Fig. 1.

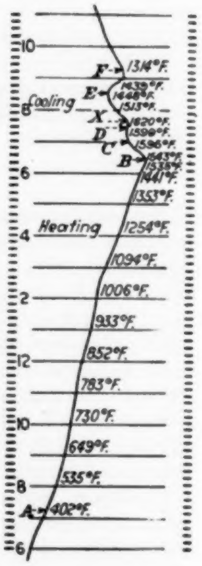

FIG. 2 TIME-DILATION CURVE

DILATOMETRIC ANALYSIS

Cast chromium-tungsten steel was studied to determine the critical points.

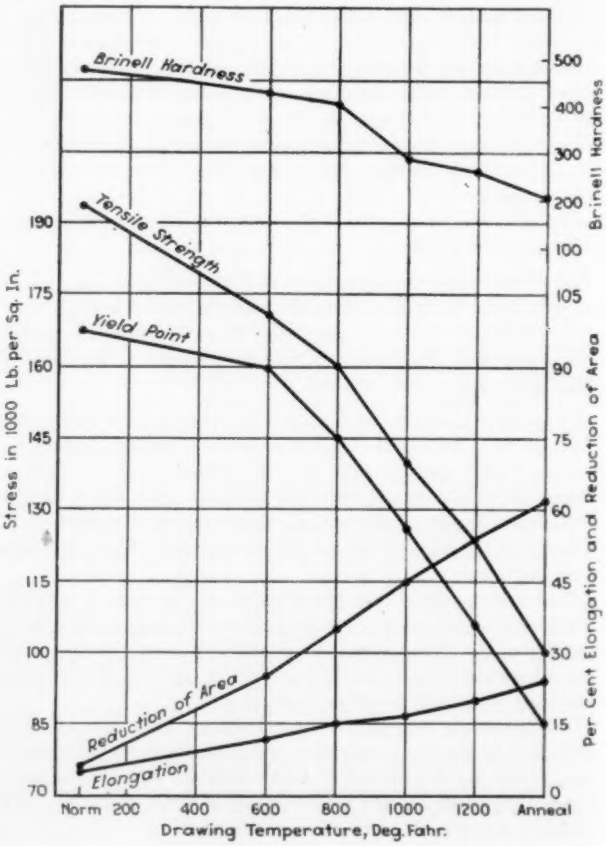

FIG 3 (RIGHT) EXPERIMENTAL HEAT-TREATMENT RESULTS

Fig. 2 shows a typical time-dilation curve as determined on chromium tungsten with the Rockwell dilatometer.

From curve *A* to *B* there is normal expansion of alpha iron. At *B* expansion stops and contraction begins, denoting the beginning of the Ac transformation.

From *B* to *C* the loss in volume due to the alpha-gamma transformation continues, as shown by the contraction, while from *C* to *D* nearly constant volume obtains, until solution of carbides is complete.

This "hold time" or period of nearly constant volume is quite characteristic of this type of steel. At *D* there is the beginning of reexpansion, denoting the completion of the Ac transformation. From *D* to *X*, at which point the furnace current was shut off, expansion of austenite takes place. Cooling at the normal furnace rate immediately starts, and there is a period of austenite contraction from *X* to *E*, where expansion marks the beginning of the Ar transformation. The increase in volume accompanying the gamma to alpha transformation continues from *E* to *F*.

Experimental Heat Treatment

Cast chromium-tungsten test bars were normalized at 1750 deg. fahr. and drawn at temperatures noted, after which three bars were tested in tension from each treatment with the following average results:

Fig. 3 shows test results, and Fig. 4 shows photomicrographs obtained from experimental bars.

Heat Treatment. The fact that steel is improved by heat treatment has been known for many centuries, but it has been only in fairly recent times that heat treatment of steel has been studied and to a great extent perfected.

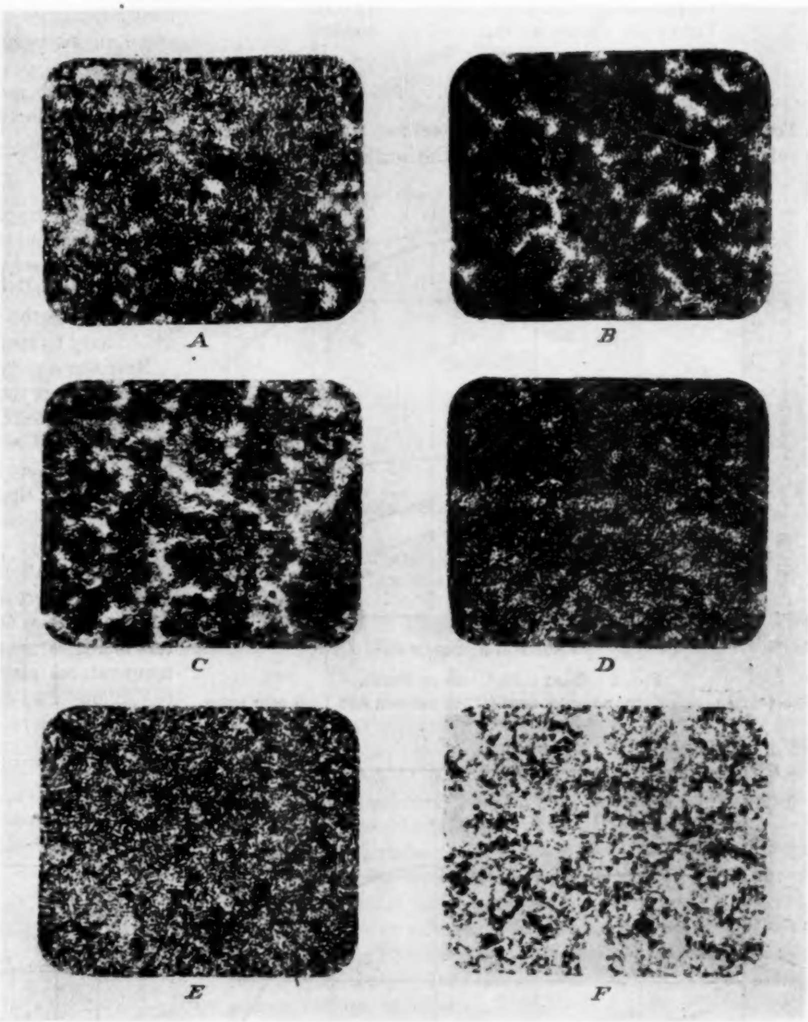

Fig. 4 Photomicrographs, Etched Nital × 100

(*A*, as normalized; *B*, drawn at 600°F.; *C*, drawn at 800°F.; *D*, drawn at 1000°F.; *E*, drawn at 1200°F.; *F*, annealed at 1600 to 1650°F.)

	Normalized	Drawn, 600 deg.	Drawn, 800 deg.	Drawn, 1000 deg.	Drawn, 1200 deg.	Annealed, 1600 to 1650 deg.
Tensile strength (psi)	193,860	171,000	160,570	140,000	124,000	100,000
Yield point (psi)	167,270	159,930	145,000	126,000	105,900	85,000
Elongation in 2 in., per cent	5	12	15	17	20	24
Reduction of area, per cent	6.7	25	35	45	54	62
Brinell hardness	490	425	401	285	260	210

Heat treatment, of course, will not give to steel the properties which it does not contain, but it does bring out and make available the properties that are inherent to the steel.

Without going into detail, the purpose of heat treatment may be summed up as follows:

(1) To increase softness and ductility.

(2) To secure a desirable combination of strength and elastic limit with fair ductility and resistance to shock.

(3) To eliminate any existing coarseness of grain and to impart to the steel a fine dense structure insuring toughness.

(4) To relieve any internal stress that may have been previously set up.

The first point to be considered is the temperature to heat steel past the critical transformation so that any existing coarse structure is broken down.

The second point is the time element or length of time that the steel should be kept at required temperature. Complete saturation of the steel is one of the most essential requisites to good heat treatment, and prolonged soaking is preferable to incomplete saturation.

The third point is the cooling, which should be as rapid as possible in order to retain the fine grain structure, after which it is necessary to draw, to such a temperature as to obtain the physical properties required.

From previous test results it was determined that a normalizing treatment at 1750 deg. fahr., followed by drawing at 1200 to 1250 deg. fahr., obtained the best physical properties and structure in order for the steel to retain its stability at temperatures of service operation.

The steel is held at the maximum temperature, so that the rate of heat penetration is $^1/_2$ in. per hour, after which it is withdrawn and cooled in still air.

After the steel has reached a black heat, it is reheated or drawn at 1200 to 1250 deg. fahr., and it is held at this temperature so that the rate of heat penetration is $^1/_2$ in. per hour, and then allowed to cool with the furnace. By this treatment, the following physical properties are obtained:

Tensile strength, lb. per sq. in...	125,000
Yield point, lb. per sq. in.......	100,000
Elongation in 2 in., per cent.....	18
Reduction of area, per cent.....	45
Brinell hardness................	240 to 260

Test Results. Just as ordinary cast steel met the requirements of refineries several years ago, so did the ordinary tensile test suffice for acceptance of steel. With the advent of high-temperature metals, however, a closer study had to be made of such properties of metals as resistance to shock, endurance limit, resistance to scaling and oxidation, resistance to corrosion, thermal expansion, short-time tests at elevated temperature, and finally the creep values when tested over long-time periods, because engineers are today demanding material that will "stay put."

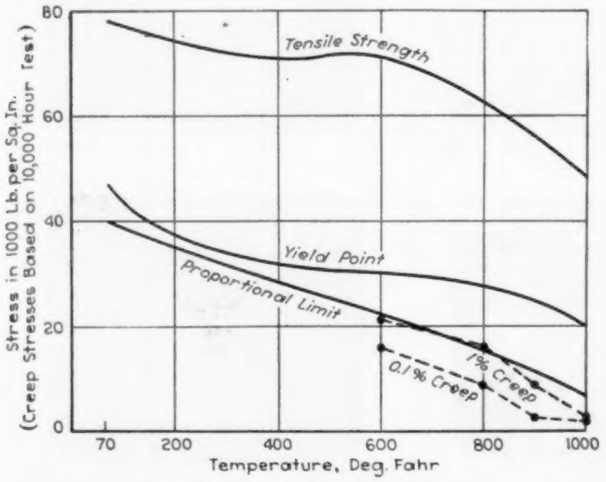

FIG. 5 CAST 0.35 CARBON STEEL

(Short-time tensile tests and creep tests for 0.1 per cent and 1 per cent creep in 10,000 hours.)

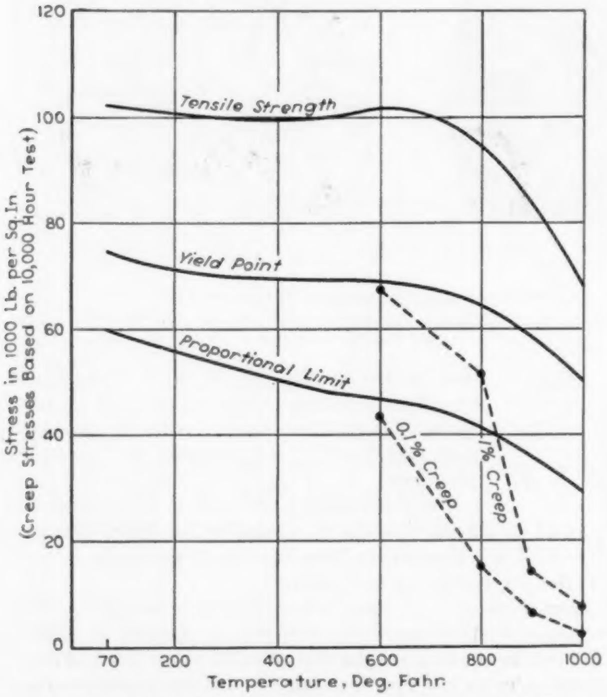

FIG. 6 CAST CHROME-NICKEL STEEL (0.35 CARBON, 2.00 NICKEL, 0.80 CHROMIUM)

(Short-time tensile tests and creep tests for 0.1 per cent and 1 per cent creep in 10,000 hours.)

The methods used for making these tests have been described in a number of previous papers[4] by the author and will not be repeated in detail.

HIGH-TEMPERATURE TESTING

What are the properties required of a steel for high-temperature construction and service? It is at once obvious that the primary characteristic acquired is the ability to carry loads continuously without appreciable deformation or at least harmful permanent deformation—the same characteristic as required in steel for use at ordinary temperature.

Now the ability of a steel to withstand stress, be it tensile, compression, or shear, at ordinary temperatures and continuously, is fairly indicated by the conventional short-time test. The standard steels will not usually fail under continued application of loads and stresses at ordinary temperatures which do not exceed one-quarter of their strength or one-half the elastic limit. Thus engineers have been able to utilize test data of this sort with adequate safety.

But the relation between the results of the conventional tests of steels of short duration and the capacity of the same steels to withstand stress changes markedly as the temperature of use and test increases; so much so that results of short-duration tests at temperatures above 700 deg. fahr. are inadequate criteria of continuous load-carrying capacity at those temperatures. At ordinary temperature steel flows or yields quickly if at all under

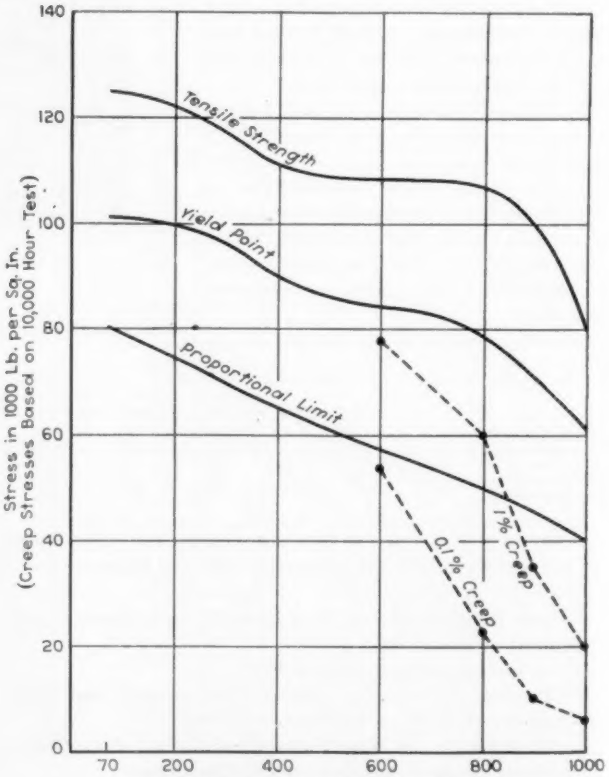

FIG. 7 CAST CHROMIUM-TUNGSTEN STEEL

(Short-time tensile tests and creep tests for 0.1 per cent and 1 per cent creep in 10,000 hours.)

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[4] "Investigation of Bolt Steels," by V. T. Malcolm and J. W. Juppenlatz, Trans. A.S.S.T., vol. 11, pp. 177–216. "Metallurgical Developments in the Valve and Fitting Industry," by V. T. Malcolm, *Mechanical Engineering*, Dec., 1925, pp. 1141–1143.

applied load—so quickly that its yielding may be easily observde and measured in the usual tensile test. But at higher temperatures steel may flow continuously under load, but so slowly that days, months, or even years are required to detect and measure it. At loads considerably below those at which in short-duration tests at high temperatures it will yield perceptibly, steel will yield slowly but continuously, and finally fail by rupture.

It is this fact, experienced from time to time, in attempts to push operating temperature limits higher, which undoubtedly has induced conservatism among engineers with respect to design of high-temperature equipment. The factor of ignorance was too great 15 years ago.

It is, however, in the author's opinion, no longer too great; in fact, our exact knowledge of the behavior of steels under continued applied load at high temperatures is in some respects better than that concerning steels at ordinary temperatures. Actual tests of flow or creep have been made on our steels, and it is confidently felt that it is known with a reasonable degree of accuracy the temperature and stress limits of their satisfactory and defined performance.

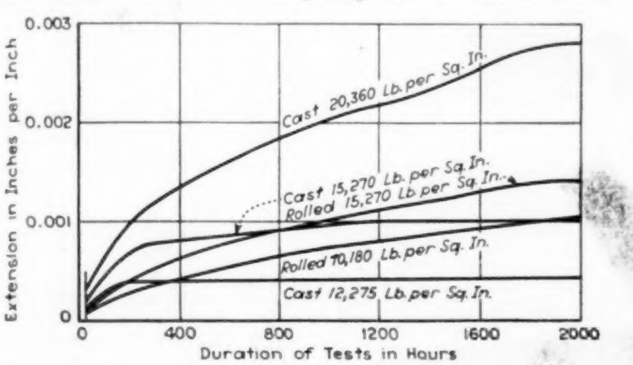

Fig. 8 Creep or Flow Tests on Chromium-Tungsten Steel at 1000 Deg. Fahr. With Various Loadings

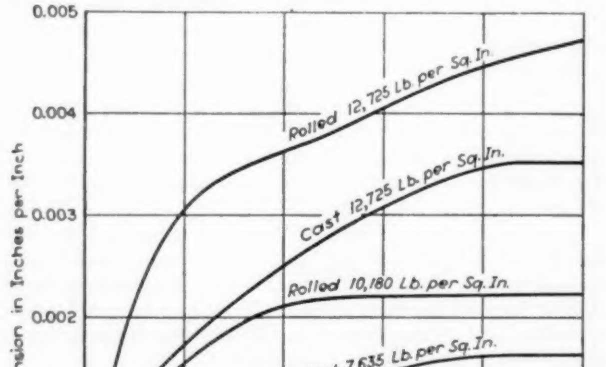

Fig. 9 Creep or Flow Tests on Chromium-Tungsten Steel at 1100 Deg. Fahr. With Various Loadings

Extensive investigation into the creep or flow of metal has demonstrated that for each composition of steel there is a range of stresses upward of zero, the continuous application of which does not produce rupture at least within reasonable periods of time, and for each of which there is, speaking approximately, a limit of deformation which also is not exceeded within reasonable periods of time.

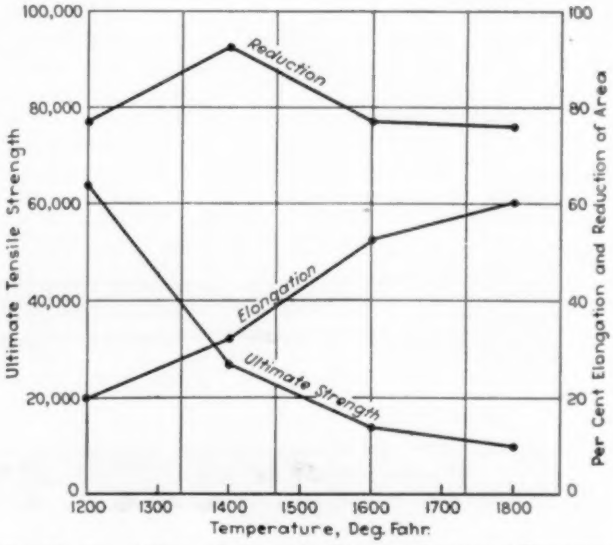

Fig. 10 Short-Time Tests on Rolled Chromium-Tungsten Steel at Temperatures From 1200 to 1800 Deg. Fahr.

One may call these ranges of stress creep-limit stresses; that is, he stresses at which certain limiting values of creep are not exceeded (within reasonable periods of time). Thus a 1-deg. creep-limit stress at 1000 deg. fahr. is that stress which may be safely applied, probably indefinitely, without rupture and without more than 1 per cent deformation.

Creep-limit stresses as determined by tests of the sort described may serve quite satisfactorily as safe design limits for high-temperature valve and fitting construction, and in fact are the only determinable stress limits which may so serve, and this fact has been amply confirmed by the experience of the author and his company in the practical design, construction, and operation of valves, fittings, and bolts. It is obvious that different types of construction will allow of different limiting values of deformation, for which different creep limit stresses would be required as safe design limits.

Figs. 5, 6, and 7 give short-time and limiting creep stresses of several steels at various temperatures with 1 per cent and 0.1 per cent deformation based on actual 10,000-hour tests and not calculated from short-duration tests.

Figs. 8 and 9 show results of creep tests carried out on chromium tungsten steel at 1000 and 1100 deg. fahr. with different loads.

Fig. 10 shows results of short-time tests on rolled steel from 1200 to 1800 deg. fahr.

Other Tests

Under this heading is included impact, both single blow and repeated bending, endurance, torsion, bend, scaling, thermal expansion, and resistance to corrosion.

Impact Tests. This method of testing steels is considered to be one of the most important of the mechanical tests, as it affords a reliable indication of the resistance to deformation (of hardness and tensile strength) and also when cohesion is small in relation to internal frictional resistance to distortion of the cohesive strength. In our work an Amsler Charpy type machine was used with capacity of 125 ft-lb.

With heat-treated chromium-tungsten steel, the rolled sustained 110 ft-lb. average and the cast 18 ft-lb. average. The cast heat-treated material was aged at 950 deg. fahr. for 20 hours and then tested, by impact, to 19 ft-lb. When aged at 850 deg. fahr. for 20 hours and tested, by impact, it sustained 19.6 ft-lb.

Repeated Bending Impact Tests. In this test on heat-treated chromium-tungsten steel an Eden Foster machine was used to reproduce stress conditions which are met with where the effect of alternating blows is produced on the material at positions where there is a reduction of the cross-section.

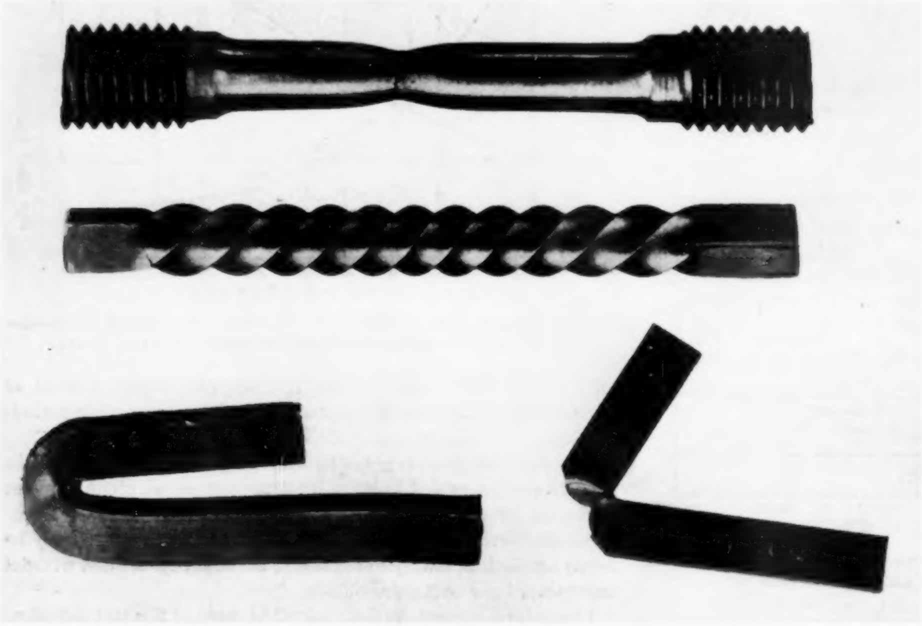

Fig. 11 Chromium-Tungsten Steel Bars After Various Tests
(Upper, tensile test, cast metal; middle, torsion test, cast metal; lower left, bend test, cast metal; lower right, impact test, rolled metal.)

The test bar is fixed in the machine where it receives the blow on a reduced section at the rate of 90 blows per minute with a hammer whose weight is 5 lb. and which falls from a height of 4 in. Between each blow the bar is turned through 180 deg., and test is continued until failure occurs. The rolled sustained 1400 blows and the cast 850 blows. The cast heat-treated material was aged at 950 deg. fahr. for 20 hours and tested to 870 blows. When aged at 850 deg. fahr for 20 hours and tested, it sustained 860 blows.

Endurance Testing. The ratio of the bend endurance limit to the tensile strength of a metal may be called the endurance ratio or, more specifically, the "bend tension static endurance ratio."

Endurance or fatigue machines are of two forms, the beam type or cantilever type. In our work we used the Farmer beam type, as modified by Prof. H. F. Moore.

Endurance or fatigue tests are made on these machines by taking a number of test pieces, beginning with a stress about one-half the tensile strength of the metal, and testing to destruction. The stress is somewhat reduced in successive tests until a load to withstand 20×10^6 is obtained without fracture. This test is very desirable for determining the endurance properties of the steel used for bolts and spindles for high-pressure and temperature service.

With heat-treated chromium-tungsten steel, tested to an endurance of 20×10^6 cycles, the rolled showed 46 per cent of tensile strength and the cast 43 per cent of tensile strength.

Torsion Tests. The ultimate strength in torsion of the heat-treated cast chromium-tungsten steel was found to be 92,000 lb. per sq. in. and the angle of twist 1050 deg.

Bend Tests. In both the heat-treated cast and rolled chromium-tungsten steel it was found that test bars would bend through an angle of 180 deg. Fig. 11 shows chromium-tungsten steel bars after various tests.

Thermal Expansion. Thermal expansion is one of the most important points that has a direct bearing on design of equipment for safe operation at high temperature, and it can reasonably be assumed that many failures taking place at high temperature are caused by the use of a combination of metals entirely unsuited for the work or from lack of knowledge on the part of the designer of certain stresses, not readily calculated, due to the non-uniformity of temperature distribution.

A test of the mean coefficient of expansion of heat-treated chromium-tungsten steel was as follows:

	Rolled, per °F.	Cast, per °F.
70°F.	....	....
300°F.	0.00000623	0.00000648
550°F.	0.00000673	0.00000692
800°F.	0.00000695	0.00000710
1000°F.	0.00000703	0.00000715

Wear and Abrasion. Hardness appreciably improves service performance, but hardness itself is no certain criterion of erosion resistance, as this depends to a great extent upon the structural composition. Hardened and drawn steels are superior for severe service conditions on account of their toughness, and this toughness is certainly a factor that tends to resist erosion and wear.

Resistance to erosion and wear does not, in general, increase directly with strength and hardness. Both hardness and toughness are important factors in determining the wearing qualities of a steel, because hardness increases resistance to penetration, scratching, and displacement of metal, while toughness makes it difficult to tear off small particles of metal.

Therefore, chromium-tungsten steel which has a considerably higher hardness and toughness for the same treatment than other alloy steels is better suited to resist erosion and wear.

It should be remembered at this time that any hardness or toughness produced in any steel by quenching and tempering will be lost in service if the tempering operation necessary to produce the required hardness and toughness has been carried out at a temperature below the operating temperature.

Chemical Stability. Excellent physical properties at elevated temperatures, however, are not the only criteria to be used in selecting metal for this service, as scaling and oxidation are decided factors. The real difficulty in connection with high-temperature work is the combining of necessary physical properties with some particular chemical stability so as to have correct design for the complete unit.

Problems of corrosion at high temperature, while not so formidable, are far more difficult to solve because of greater chemical affinity at elevated temperature.

Corrosion Testing. The ultimate aim of all corrosion tests should be to derive a comprehensive series of tests to which standard specimens of materials may be subjected and by which the rate of corrosion of these metals may be predicted for certain service conditions.

One of the most important though somewhat neglected studies of the corrosion problem is the action of various gases at high temperature.

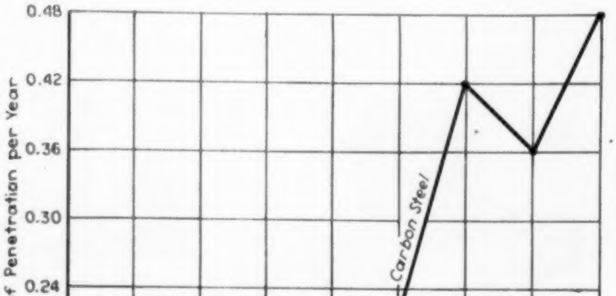

FIG. 12 TESTS ON SEVERAL STEELS IN REFINERY SERVICE AT 850 TO 930 DEG. FAHR.

(Tests 1 to 5, regular crude; 6 to 8, west Texas crude. Each test was of 10 days' duration.)

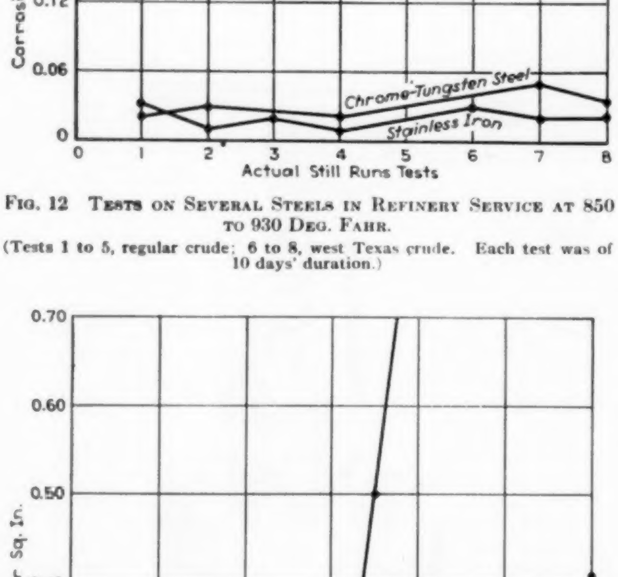

FIG. 13 SCALING TESTS ON SEVERAL STEELS

(Twenty cycles of test; each test of 12 hours' duration.)

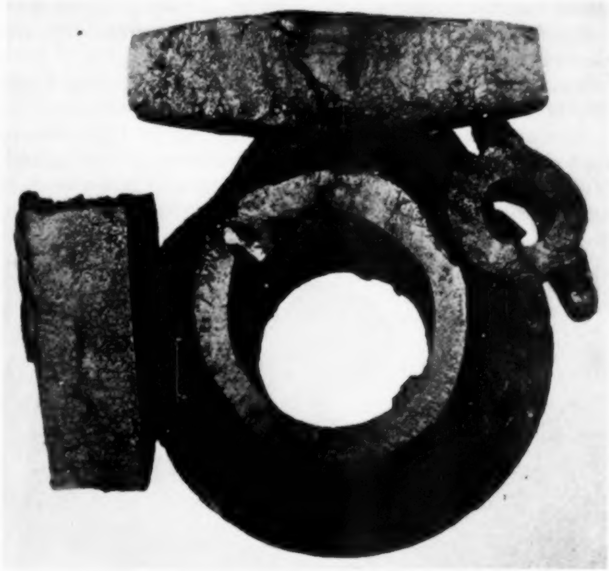

FIG. 14 BROKEN 1350-LB. STEEL VALVE BODY

(In the "as-cast" condition. Note soundness of metal.)

Therefore, in the study of corrosion there are two conditions to consider; namely, "erosion," which is the disintegration or destructive action by physical causes, and "corrosion," which is the disintegration or destructive action by chemical causes.

Little is really known about the mechanism and true nature of corrosion, and so much depends upon the kind of reaction produced that it is hard to make a prediction, based on laboratory tests, as to what will happen after a piece of equipment has been in service a month.

In our work, efforts have been made to overcome this difficulty in determining true corrosion by using apparatus designed as nearly as possible to duplicate service conditions, and by noting the film) pitting, etc., and reducing the work to inches of penetration per year or month, so that a relative figure may be obtained for different classes of metals. Only service tests, however, tell the true story. Fig. 12 shows tests of several steels in actual refinery service. Fig. 13 shows results of scaling tests on several steels.

UNUSUAL TESTS

A number of unusual tests were made to determine the fitness and solidness of chromium-tungsten-steel valves, and a résumé of certain of these tests is given.

Destruction Tests. A number of valves were broken in the "as cast" condition to determine the soundness and homogeneity of steel. Examination revealed that the castings were solid throughout, showing no porosity, cracking, or interior shrinkage due to poor molding or steel-making practice.

Fig. 14 shows a broken 1350-lb. valve body, as cast.

Valve bodies were heat-treated by normalizing at 1750 deg. and drawing at 1250 deg., and then tested in order to determine their resistance to shock and the ductility of the steel. The procedure in this test was as follows:

Valve bodies were placed on a steel bed, and a 3000-lb. ball was allowed to drop through 40 ft., hitting the casting directly in the center, each blow having an energy of 120,000 ft-lb. As many as six blows, making direct hits on the casting, were required to break it. This test shows the remarkable resistance to shock and the splendid toughness of the chromium-tungsten steel.

Part of a flange was taken from a casting and placed under a power hammer having a striking energy of 1200 ft-lb. per blow, and 200 blows were required before failure occurred. The section was reduced 33 per cent before fracture.

Fig. 15 shows a chromium-tungsten fitting after impact test. Fig. 16 shows a chromium-tungsten valve after impact test.

In connection with testing of steel castings for high-pressure, high-temperature service, the author believes that test bars are indicative only of the quality of the metal and heat treatment and do not give an idea as to the strength or solidness of the casting itself. Neither does he believe in taking test bars cut from castings, for the reason that they do not show the true conditions.

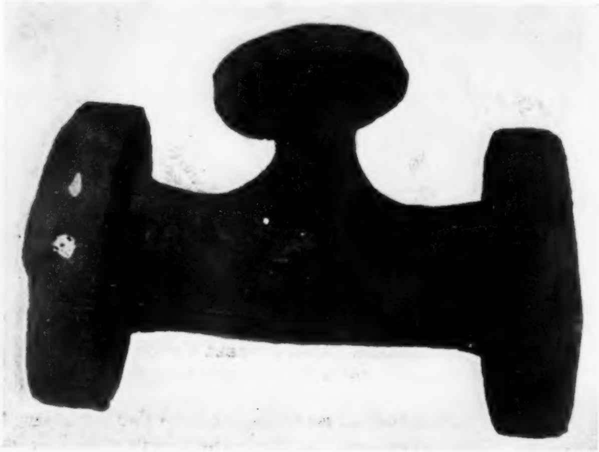

Fig. 15 Heat-Treated Chrome-Tungsten-Steel Fitting Unbroken After Test

(This was after having been struck six times by a 3000-lb. ball dropping 40 ft.)

However, with molding practice standardized in such a manner that "shrink heads" and gates are the same for each group of castings poured, it may be better practice to test a certain casting or number of castings from each group made from the same pattern.

By this method, strength, ductility, and toughness may be readily noted, as well as any other defects such as shrink, blowholes, piping, cracks, etc.

This method of test is in use by the author's company and has proved of great value.

From the results of our tests, it will be noted that the usual factors of safety as employed are sufficient, and should stresses be reached which would produce creep, there is only a remote probability of rupture until at least 20 to 25 per cent of the total creep or distortion has taken place.

This means, with the use of chromium-tungsten steel, that should factors of safety for some reason or other be neglected and a valve be subjected continuously to high temperature and great pressure, so as to build up circumferential creep of 0.03 in. per year, it would simply result in slight expansion of the valve and would do no harm as regards safe operation of the plant.

Fig. 17 shows a 1350-lb. oil-refinery valve for 1200 deg. of temperature.

Cost

That chromium-tungsten steel, on account of the costly alloys used, is necessarily more expensive than ordinary steels, is met by the fact that the length of service life has been increased and, most important, great savings are effected by much less frequent renewals, and while there is no doubt it will require time to convince some refinery engineers of the advantages gained, its use must eventually be found to be highly economical.

Conclusions

In summing up the various factors that enter the design of apparatus for high-temperature service, the most outstanding is the fact that the relation of stress to strain can no longer be represented by Hooke's Law.

In the study of steel specifications much is found on heat treatment, tests, and chemical analyses, but nothing is stated about manufacturing methods, except to specify the steel-making

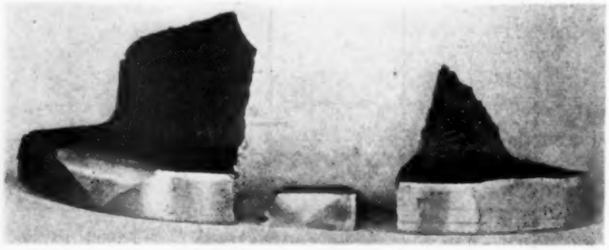

Fig. 16 Result of Power Hammer Striking Broken Flange of Chrome-Tungsten-Steel Valve

(Small piece in center was struck 200 blows with a 600-lb. hammer dropping 2 ft., before fracture occurred. Compare reduction with other flanges.)

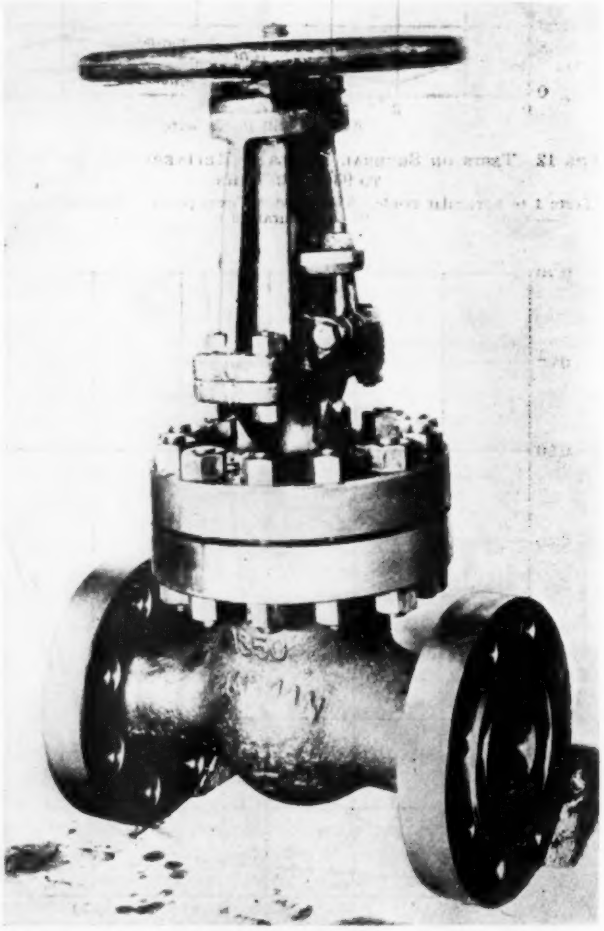

Fig. 17 A 1350-Lb. Refinery Valve for 1200 Deg. Fahr. Operation

process and that the steel must be free from injurious defects. From this one may infer that, as long as the steel meets the clauses specified, the quality of the material and method of manufacture are not questioned, and any failure in service must be from some other source than material. This inference is incorrect, for while certain steels meet specifications, manufacturing methods, if not correct, may cause a lot of trouble to the ultimate consumer.

With such conditions in mind, it is of the utmost importance to place rigid inspection and safeguards other than specifications on steel being made for high-temperature service, to insure it being correct for the purpose intended.

As previously stated, our company recognized that something better than ordinary steel was required in the oil-refining industry owing to the highly corrosive and high-temperature conditions, and it is our firm belief that chromium-tungsten steel will materially increase the life of valves and of fittings in the oil refinery.

While in other practical applications chromium-tungsten steel may be regarded as past the experimental stage, there is a strong reason to believe that with increased production and various advances now being made in manufacture, the market for this steel will become much broader.

PME-54-4

Evolution of Pressure Vessels to Meet Present-Day Refining Pressures and Temperatures

By H. LeROY WHITNEY,[1] NEW YORK, N. Y.

In the growth of the oil-refining process to meet the demand for fuels for internal-combustion engines, the pressure vessel has played a foremost part. The paper covers the progress of riveting and welding in perfecting a tight and dependable container. Failures attributed to defective plate and to defective welding led to greater experience and operating knowledge. Recognizing the future tendency to still higher temperatures and pressures, research was devoted to developing a new method of electric welding and the rolling of smooth seamless cylinders of large diameter with which to fabricate vessels of any commercial requirement.

TWO factors are responsible for the strides made in refining processes: the advent of the motor car and competition within the refining industry itself. Two decades ago the refining process consisted primarily of topping straight-run gasoline and kerosene in shell stills. Such vessels as were required were of light wall, and riveted construction was adequate. Pressures were low, temperatures were negligible, and high rate of production was not a factor. The motor car changed all of this. As internal-combustion motors became more perfect, fuel requirements not only grew tremendously, but became more exacting. The chemical engineer of the refining industry was called in to knock off a few hydrogen atoms here, add a few carbon atoms there, and generally change nature's idea of things.

To accomplish this interchange of structural atoms, both heat and pressure were necessary and the refining plant began to lose its general appearance of a disjointed steam plant and assume the appearance of the modern refinery as known today.

In the pressure-vessel industry, during the birth of "cracking," riveting was the generally accepted method of joining steel plates. Oils, and especially hot oils and vapors, have a peculiar method of rapidly and with general abandon finding their way through a riveted joint, and it required only a brief investigation to convince the operating man that something must be done.

Almost paralleling the change in refining methods was the development of forge-and-hammer lap welding. This process was not new in America 30 years ago, but as practiced was crude, uncertain, and not always satisfactory. M. W. Kellogg believed that the development of a welding process that resulted in a product bottle-tight and entirely dependable would be an essential contribution to the refining industry.

Both oxyacetylene and electric-arc welding as practiced at that time were fully investigated, considerable experimenting was done, and several vessels were built, but neither process gave such results as was believed necessary. Attention was then turned to the forge-welding process, and development was conducted until the product was dependable and satisfactory.

There were certain limitations to this process evident from the start: (1) The quality of the product depended on the human element, (2) entirely uniform steel plate was not available, (3) plate in excess of 2 in. could not be economically welded, and (4) only low-carbon plate, and consequently low-tensile-strength plate, could be welded.

For general purposes, however, the process was eminently successful, as is evidenced by the many hundreds of vessels that were produced, some of which have seen many years of service and are still in operation.

As is the history of any epoch-making development, in the development of the refining of petroleum there were pressure-vessel failures. Some could be attributed to defective plate and some to defective welds, but the greater proportion was due to lack of operating knowledge and experience. Little was known of the difference between sweet and sour crudes, and when the latter were used, vessels were operated in many instances long after they had corroded away, above the liquor line, to a thickness where failure was inevitable. The vessels which are particularly referred to were partially full of oil under pressure and direct-fired. Every operating man was endeavoring to increase his output of gasoline, and as a consequence many vessels were kept on stream until the layer of deposited coke was so thick that it acted as an insulator, causing overheating of the wall of the vessel which often caused failure. Improper design also contributed to failures.

The lessons learned from these failures resulted in a great advancement of the art of welding and in the design of the vessels. The results of the knowledge obtained were as follows:

1 More stringent material specifications.

2 Increase of the knuckle radii of dished heads until the ellipsoidal-shaped head was produced.

3 Proper reinforcement of openings.

4 The necessity for stress-relieving of the completed vessel.

5 Marked improvement in pressure-still operations.

6 Direct firing of pressure stills was an unnecessary hazard.

The art of refining was also moving forward, increasing the temperatures and the pressures, which, combined with large-diameter vessels, exceeded the welding capacity of the forge-welding process. Seamless forged vessels were being used with entire satisfaction when properly constructed, but their use was limited both for economic and physical reasons. Heavy riveted vessels also were being used, in some cases with calked seams and in many cases with seal-welded seams and rivets. Neither type of riveted vessel filled the requirements of the service.

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[1] Chief Engineer, Piping and Pressure Vessel Division, M. W. Kellogg Company, Inc. Mem. A.S.M.E. Mr. Whitney was born in New York City in 1884. He was educated at St. Paul's School, Concord, N. H., and at Sheffield Scientific School, Yale University, graduating in 1905. After a brief experience in mining, he joined the Kellogg organization in August, 1906, and has been with it ever since, except for service with the First New York Field Artillery on the Mexican border in the summer of 1916 and during the World War from June, 1917, until May, 1919; served with 302nd Field Artillery of the A.E.F. He is a director of the Kellogg company. Mr. Whitney is a member of the Boiler Code Committee of the A.S.M.E.

Presented at the National Petroleum Refining Meeting, Elizabeth, N. J., May 5 and 6, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Recognizing the future tendency of the industry toward still higher temperatures and pressures, two research programs were authorized involving the development of a new method of electric welding and the rolling of smooth seamless cylinders of large diameter with which to fabricate vessels of any commercial requirement. The reason for the development of seamless cylinders rolled from pierced ingots was that this seemed the best method by which clean, tough steel of uniform structure and analysis could be manufactured. The benefit by the elimination of the longitudinal joint is obvious.

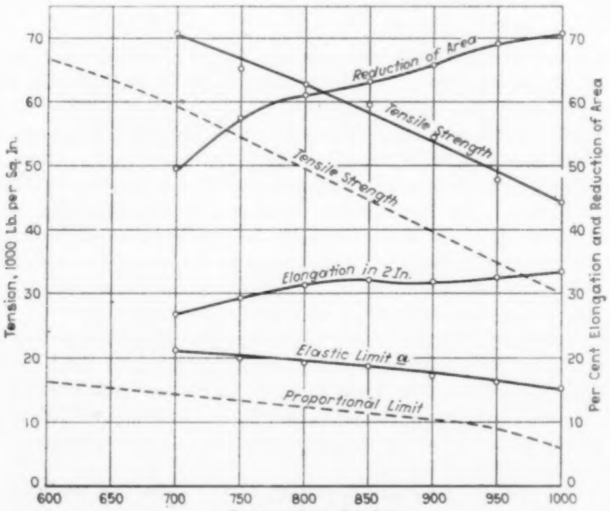

FIG. 1 SHORT-TIME TENSION TESTS AT HIGH TEMPERATURES

[Solid line, Kellogg seamless steel, 0.18 per cent carbon: broken line, 0.24 per cent carbon boiler plate, as reported by H. J. French in Bureau of Standards Technical Paper No. 296. Each plotted point represents the average of four test bars (standard 0.505-in. bars) taken from a seamless cylinder 78 in. inside diameter by 1$^3/_4$ in. thick. a (On Elastic Limit line) Comparable to the proportional limit as determined by H. J. French. Elastic limit determined with extensometer dial; one division equals 0.0001 in.]

Coincident with the use of higher operating pressures and temperatures has come the study of the properties of steels of various analyses under these conditions. A great amount of commendable work has been done, both on short-time high-temperature tests and long-time tests, to determine the proportional limit and creep values. There is a wide divergence of opinion on the subject of creep, but the important thing is to analyze the data available and to apply it correctly, taking into consideration the service for which the steel is destined. A very small amount of creep would destroy the efficiency of a turbine, whereas in a pressure vessel a considerable amount of creep is not harmful; and, further, the desired life of any particular pressure vessel must also be taken into consideration, not forgetting that where excess thickness is provided to take care of corrosion, the approach of the creep point is materially postponed.

There are two kinds of steel that ordinarily are used in the manufacture of plate, seamless cylinders, seamless forgings, and tubing—namely, rimmed and killed carbon steels. In both short-time and long-time high-temperature tests made on these kinds of steel, the manganese content has been given very little consideration, for the reason that most commercial steels are fairly low in this element. It is quite definite, however, that at high temperatures between 800 and 1000 deg. an increase in the manganese content from nominal manganese of 0.30–0.50 to 0.60–0.80 is much more beneficial than an increase in the carbon content from 0.20 to 0.30. Most authorities will concur with this statement.

Another point on which most authorities seem to agree is that the elastic limit or proportional limit of steel at high temperatures, as determined by short-time high-temperature tests, is a very fair indication of what creep value may be expected if long-time tests were run.

Fig. 1 is a chart of short-time high-tension tests at high temperatures. Kellogg seamless-cylinder material, shown by the solid lines, is compared with 0.24 per cent carbon boiler plate, shown by broken lines, as reported by H. J. French in Bureau of Standards Technical Paper No. 296.

Fig. 2 shows a transverse fracture of a rimmed steel ingot where the line of non-oxidized blowholes is very clear. Attention is called to the uniform distance of these blowholes from the ingot wall, and also to the depth, which indicates the very best rimming practice.

In addition to these non-oxidized blowholes, which theoretically are welded up when the ingot hits the blooming mill, segregation takes place in the steel while it is cooling, and not only is the dirt driven to the center and top of the ingot, but the carbon and other impurities also follow the same path to a considerable extent. The larger the ingot the more evident is this segregation, and chemical and physical analyses of any quantity of plates will show higher carbon content and higher tensile strength in the test bar taken from the end of the plate that represents the top of the cropped ingot than in the test bar taken from the end of the plate that represents the bottom of the ingot. In fact, A.S.T.M. Specification for Firebox-Quality Plate permits a 15,000-lb. variation in strength in the two test specimens from the same plate, and no limit as to the variation in chemical analysis is specified.

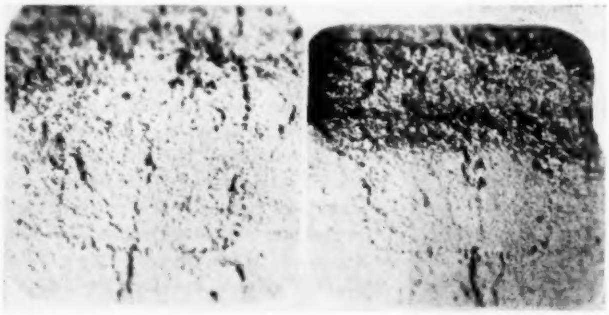

FIG. 2 RIMMED STEEL INGOT SECTION, SHOWING NON-OXIDIZED BLOWHOLES

FIG. 3 PIERCED INGOT BEFORE MACHINING

Unquestionably a certain amount of segregation goes on even in an ingot of killed steel while it is cooling, but as proof of the fact that this segregation is less than in rimmed steel, chemical analyses were made from samples taken from both ends and from the outer and inner walls of a great number of cylinders. These chemical analyses were so uniform that it is a fair statement to make that seamless cylinders rolled from pierced killed steel ingots are of uniform chemical analysis throughout. Table 1 shows the results of these analyses and gives the maximum, minimum, and average variation in carbon, manganese, etc.

TABLE 1 RESULTS OF ANALYSES

Element	Maximum	Minimum	Average
C	0.04	0.01	0.02
Mn	0.03	0.00	0.02
P	0.012	0.003	0.006
S	0.013	0.003	0.007

The cleanness and a certain amount of the uniformity of the steel is unquestionably due to the method of piercing, wherein the major portion of the center of the ingot, after it has been cropped, is punched out in the form of a slug.

Fig. 3 shows a pierced ingot at the plant of the Midvale Steel Company before being machined. The uniform wall thickness is to be noted. The reason for machining the outside of pierced ingots is to remove all possible laminations, slips, or surface cracks that always are present in the surface of rough forgings.

Fig. 4 shows a finished rolled cylinder 9 ft. inside diameter by 2 in. thick. Attention is called to the smoothness and true cylindrical shape. The only machining required on the finished cylinders is squaring up of the ends or the machining of welding grooves preparatory to welding.

The maxium dimensions of these seamless rolled cylinders are length 10 ft. and inside diameter 12 ft. The maximum weight of billet which can be handled at the present time is 25 tons. A 25-ton billet will give a cylinder 10 ft. long, approximately 3 in. thick at the maximum inside diameter of 12 ft., or 6 in. thick with an inside diameter of 6 ft. Shorter billets with a greater wall thickness can of course be handled, which would result in shorter finished cylinders of greater wall thickness. The minimum diameter to which cylinders can be rolled is approximately 36 in., but the rolling of small-diameter cylinders is not economical unless the walls are reasonably heavy.

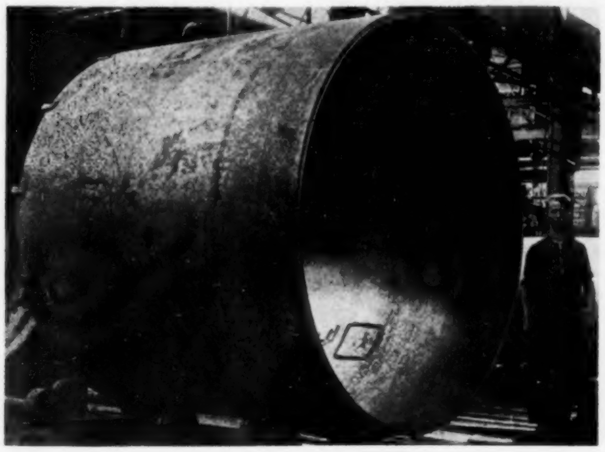

FIG. 4 ROLLED SEAMLESS STEEL CYLINDER, 9 FT. INSIDE DIAMETER BY 2 IN. THICK

Kellogg cylinders are rolled from hot-topped ingots of killed acid open-hearth steel of firebox quality, in two grades: grade 1 with a carbon content not exceeding 0.30 and grade 2 with a carbon content not exceeding 0.50. The minimum physicals guaranteed for these two classes are exactly the same as the minimum physical requirements for grades 1 and 2 seamless steel drum forgings, Specification S-4 of the A.S.M.E. Boiler Construction Code. The minimum tensile strength and elongation in 2 in. of the two grades are: grade 1, 60,000 lb. and 26 per cent; grade 2, 75,000 lb. and 24 per cent.

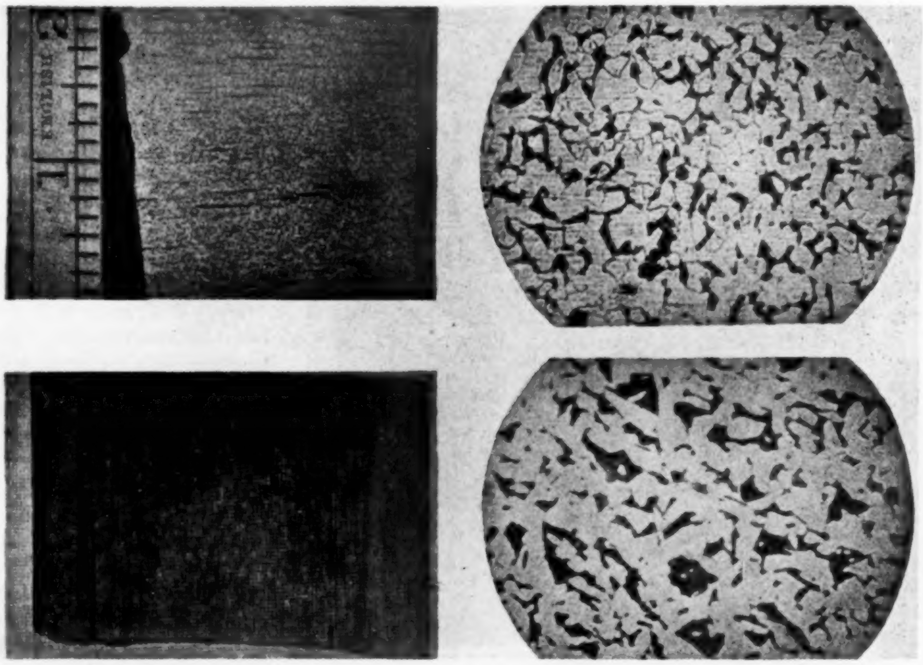

FIG. 5 MACROGRAPH OF SEAMLESS CYLINDER MATERIAL (ABOVE) AND OF A SPECIAL WELDING QUALITY PLATE (BELOW)

(The seamless cylinder material was etched 40 min. and the plate 20 min. in hot 1-to-1 hydrochloric acid. The respective micrographs at the right of each specimen show the grain structure.)

To illustrate the uniform physical characteristics, test specimens were taken from one lot consisting of 10 finished cylinders 9 ft. in diameter by 2 in. thick, having an average carbon content of 0.17 per cent. The maximum and minimum tensile strength, yield point, elongation, and reduction of area are given in Table 2.

TABLE 2 PHYSICAL CHARACTERISTICS OF TEST SPECIMENS

Tensile strength, lb.	Yield point, lb.	Elongation in 2 in., per cent	Reduction of area, per cent
69,750 (maximum)	48,350	33	61.3
65,250 (minimum)	40,450	29	48.4

The excellent physical qualities of this steel of comparatively low carbon content are due, first, to the steel from which the cylinders are made; second, to the working of the steel longitudinally in the piercing operation; third and most important, to the rolling of the cylinders which works the steel in the direction opposite to piercing and lays the fibers circumferentially and parallel to the line of greatest stress in a cylinder subjected to internal pressure.

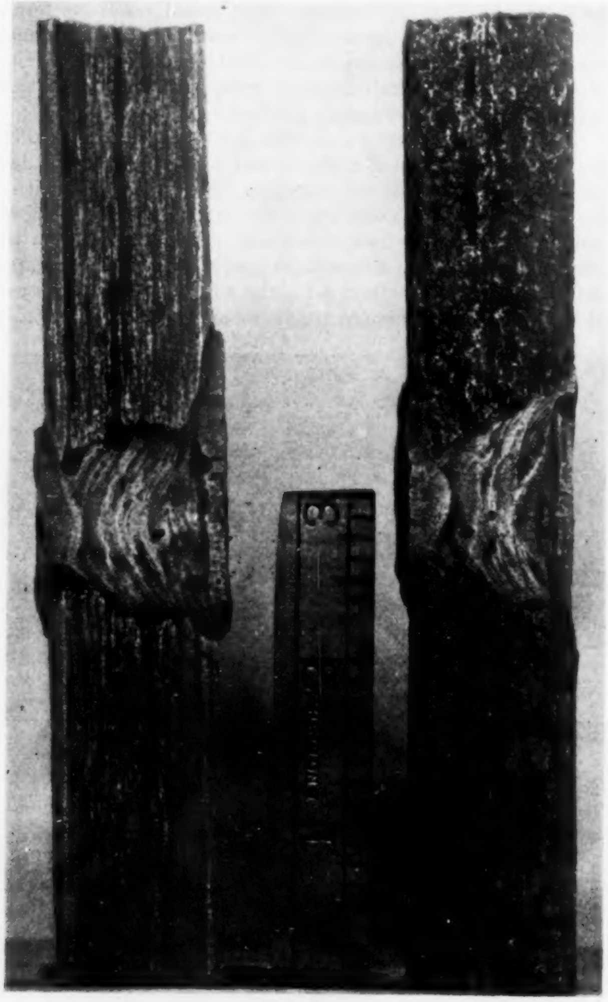

Fig. 6 Special Welding Quality Plate (Left) and Rolled Seamless Cylinder Material (Right)

(These 6-hour corrosion tests show how much more rapidly in relation to the respective welds the plate corroded than the seamless cylinder material.)

Fig. 5 shows macrographs of deeply etched sections—one (at the top) cut from welding quality seamless rolled cylinder stock 2 in. thick and one (at the bottom) a special discard welding quality plate $2^3/_8$ in. thick. The etching time on the plate sample was only 20 min., and reveals the porosity in the two surfaces of the plate and marked segregation lines between the two surfaces and the core of the plate, also the openness of the core in comparison with the two surfaces. It required 40 minutes of etching to bring out any lines whatever in the seamless cylinder materials, and the uniformly clean and dense structure throughout may be noted. Fig. 5 also shows micrographs of polished samples of the same seamless cylinder and plate material from which the macrographs (opposite to each) were taken. Both samples are of approximately the same analysis. The micrograph of the seamless material shows much better distribution of both pearlite and ferrite structure than in the plate sample.

In order to determine the relative resistance to corrosion of seamless rolled cylinder stock and special discard welding quality plate, samples of both materials welded by the Kellogg process were made, and bars were machined from these samples and immersed in a hot 1-to-1 solution of hydrochloric acid for six hours.

Fig. 7 Reaction Chamber, 60 In. Inside Diameter, $4^5/_8$ In. Thick, and 40 Ft. Long

Fig. 6 depicts the usual result from such corrosion tests. The rolled surfaces of both samples were very much more resistant to corrosion than the machined surfaces. The rolled surfaces of the plate material, however, corroded much more rapidly than the deposited metal, whereas the rolled surfaces of the seamless cylinder corroded at about the same speed as the deposited metal.

In the plate sample, segregation lines (or whatever one wishes to call these lines) are most evident, and wherever one of these lines joins the deposited metal, rapid pitting takes place. In the seamless sample corrosion is uniform over the entire machined surface, and there is no more pitting at the junction line than at any other point in this sample. The small pits in the deposited metal of both samples are but natural, as in hot etching there are always certain parts of a section that are more rapidly affected than others. In all steel practice it is known that the markings on a cross-section of an ingot will be carried down as the ingot is reduced, even if rolled to fine wire.

Fig. 7 shows a reaction chamber for use in the petroleum industry. This vessel is 60 in. inside diameter, $4^5/_8$ in. wall thickness, and 40 ft. long, made from seamless rolled-steel cylinders with ellipsoidal-shaped spun heads, joined with Kellogg circumferential seams. The weight of this vessel was approximately 150,000 lb.

PME-54-5

Suggested Specifications for Testing Tubular Heat-Transfer Equipment for Oil Refineries

By WALTER SAMANS,[1] PHILADELPHIA, PA.

This is a draft of specifications for testing tubular heat-transfer equipment after it has been installed in the refinery, this draft to serve as the basis for a more complete and general specification to apply to any equipment of this character which may be used by the refiner in oil processing. Points are enumerated that must be considered by the designer, and some mechanical details are listed which the purchaser's inspection must assure as being right, to avoid difficulties due to leakage of equipment, cleaning, and replacements in service.

THE purpose of this paper is to present a draft of specifications for testing tubular-heat transfer equipment after it has been installed in the refinery, which draft may serve as the basis for a more complete and general specification to apply to any equipment of this character which may be used by the refiner in oil processing. This draft is presented herewith as Appendix 1. It is not the purpose to make these specifications, as written, a finished product, but with the hope that a committee of the A.S.M.E. will be appointed, on which manufacturers of such equipment would be represented, and that such a committee would review the situation on this subject, so that by the next Annual Meeting of the Society something might be presented for adoption which would serve as a guide for users as well as manufacturers.

The related matter pertaining to shop inspection and actual operation, with data on field cleaning methods of such apparatus which is given herein, is contributed only as our share of experience, with the hope that it will draw out in the formal committee's deliberations such additional data from other refiners and manufacturers as will help to make the whole understandable. No one refiner can claim to have sufficient experience or be acquainted with a sufficiently varied practice in this regard to set himself up as an infallible guide. What follows, therefore, should not be taken as a complete exposition of the subject, no more than that the suggested draft of specifications is complete, and it is hoped that any discussion will be submitted to the committee aforementioned, and that the manufacturers will indicate their cooperation.

Approximately four years ago, when these specifications were first used in connection with the test of a particular group of apparatus, very few manufacturers had any means for duty-testing such units in their shops; and in fact, considering the varied nature of charging stocks, products handled, and the expensive apparatus needed for obtaining the various products in their unrefined state, together with all the attendant operating conditions, it is doubted whether even now any manufacturer can test large units in their entirety in a conclusive manner. The importance to the purchaser of having equipment meet specified duty, and considering that the manufacturer should not be bound by field conditions which cannot be expressed with certainty, particularly as to the corrosive nature of the products handled and the fouling of heat-transfer surfaces encountered, must be recognized by both parties. The few curves illustrated will indicate what the manufacturer of this equipment must face in order to guarantee satisfaction on the part of the refiner; and it would be unfair to the manufacturer to let him assume only heat-transfer values that have been obtained by tests on equipment operated with selected stocks or products, in having to fulfil a guarantee drawn up by the purchaser, without having that purchaser make all allowances for severe conditions that may be entirely unforeseen by the manufacturers' engineers unless they are expressed as completely as possible, with a frank statement on various troubles experienced in other similar apparatus previously used. Unless the refiner when purchasing such equipment is absolutely frank in giving these points proper consideration, he will not only do the manufacturer an injustice, but he will himself be disappointed in not being able to obtain the duty from the equipment when actually installed and in service, involving endless delays, trials to improve unsatisfactory apparatus, and further delays and expense incidental to obtaining additional equipment and its proper arrangement with apparatus already purchased, and which is affected by more than just the consideration of required additional heat-transfer surface as calculated.

In the following paragraphs are briefly enumerated the points the designer must consider and some mechanical details which the purchaser's inspection must assure as being right, to avoid difficulties due to leakage of equipment, cleaning, and replacements in service. Attention must be given to proper piping design in connection with these units, and it would not be out of the way for the purchaser to submit for the manufacturer's approval the piping hook-up selected, to assure the manufacturer that no improper strains will be brought upon his equipment which might contribute to leaky joints and even breaks.

In the design, the relative arrangement, mechanical construction, and fabrication methods, as well as the selection of materials, there are some markedly different problems than those encountered in the design of similar equipment in steam practice. These briefly may be summarized as follows:

1 The higher temperatures encountered.

2 Generally higher pressures.

3 The fouling of the surfaces in contact with oil in a crude state or in unrefined condition.

4 Corrosiveness of the oil and vapors.

5 Tendency on the water side to accelerate corrosion and

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

deposit of salts from cooling water on account of higher temperatures than encountered on steam condensers.

The fundamental factors which determine the design of a steam condenser must be largely discarded when these conditions have to be met.

Corrosion determines selection of materials, and this is made more complex by reactions obtained with neutralizers that may be used in oil processing. For instance, the use of soda ash, lime, or caustic, which must be used in excess to obtain the proper neutralizing effect on the sulphur compounds at the higher temperatures encountered with oil, may result in the need for steel tubes rather than non-ferrous tubes such as Admiralty. For high sulphur crudes, however, the effect of sulphur in light distillates makes Admiralty metal preferable. It is found that Muntz metal is seldom satisfactory under these conditions. However, while Admiralty metal is also subject to corrosion by alkalis, such metal in tubes and tube sheets in low-temperature service does not involve sufficiently rapid corrosion to prohibit them altogether, but the use of heavier gage tubes is advisable to give them longer life.

With steel tubes in the same low-temperature service, with the use of ammonia as a neutralizer, ammonium chloride will be deposited on the tubes, but at the same time the sulphur compounds rapidly attack steel tubes, forming a shell-like coating of iron sulphide which greatly interferes with heat transfer and which is more difficult to clean off, except by laborious hand methods.

Where cooling water is brackish, which cannot be avoided in certain localities, the tube sheets should be of the same material as the tubes, or recirculated cooling water must be provided, with cooling towers to cool it.

In general, the subject treated by this paper concerns mainly low-pressure apparatus used in connection with pipe stills, and the suggested specifications given in Appendix 1 should be taken as applying to such equipment, although they can readily be modified to suit condensers and similar units in connection with pressure equipment—that is, cracking still condensers, coolers, etc.

The fouling of tube surfaces, which on low-temperature units is more noticeable on steel tubes than on Admiralty tubes and to that extent interferes more with heat transfer, makes it unnecessary and undesirable to plan on the full benefits in heat transfer obtainable on clean surfaces with higher velocities of vapors or liquids. This condition, usually obtained by close spacing of tubes, coupled with careful baffling, increasing the effectiveness of the surfaces wiped except in isolated cases, encounters in oil-handling equipment surfaces that are so dirty that there really is a soaking heat condition, rather than a wiping action of the fluids passing over the surfaces. This is to some extent also true in water-cooled units, on the inside of tubes where dirty river water is encountered, and no economical method can be devised for overcoming that objection.

One can readily see the advantage of a wider spacing of tubes to enable surfaces to be cleaned more readily and to avoid absolute bridging of the spacing between the tubes by the salts and solids.

In the curves of heat transfer illustrated, the units consisted of 1-in.-diameter tubes in rectangular arrangement with $^3/_{16}$-in. clearance between tubes, and it was found that by omitting some of the rows of tubes, greater over-all heat transfer might be obtained by reason of having cleaner surfaces. The present standard for a number of oil refiners is to have this space between individual tubes $^3/_8$ in., but not less than $^1/_4$ in. between rows.

In connection with shop inspection of tubular equipment, which is the first phase of this subject, it is desirable to definitely limit the inspector's duties to following out construction details in accordance with plans, and to limit the type of repairs that can be made with his approval, so that if any special cases come up in which more extended alterations, repairs, or deviations from the plans appear desirable to the manufacturer's shop superintendent, either for facility in fabrication, future repair, or replacement in the field, or to avoid delay in completion, the inspector is protected from the too urgent argument by the manufacturer's shop superintendent, by having to refer such special problems to his home office. However, this is not an indictment of the manufacturer's methods, as generally they have been found whole-heartedly in favor of the purchaser's requirements.

The requirements for shop inspection by the purchaser's inspector are briefly as follows:

(1) *Prints.* Inspector to be provided with prints of apparatus sufficiently in detail to check sizes, material, nozzle tolerances, etc.

(2) *Hydrostatic Tests.* Inspector to witness these tests in manufacturer's shops. The pressure tests inside of tubes and around tubes shall be made separately and for a duration of three hours, during which time there are to be no leaks whatsoever. This length of time was confirmed in actual experience with cast shells that were thus found to have porous spots, and leaks at bolt holes and gaskets. The test around tubes is to be accomplished with ring-type closure piece between floating-head tube sheet and shell flange. This procedure indicates just which tube leaks from the outside and only leaky tubes need be rerolled. Test pressures to be applied are to be $1^1/_2$ to 2 times the designed working pressure. (Note that for naphtha units the design may be for 100 lb., although operated at lower pressures.)

(3) *Tube Bundle Removal.* All tube bundles are to be removed in the presence of the inspector, to determine definitely that correct clearance is available for readily withdrawing tube bundles from shells in a manner practicable by field methods. Where tube bundles are reversible, they must be removed and turned through 180 deg. and inserted and removed again, and there must be no indication of binding whatsoever. If slight grinding of baffles is necessary to produce freedom of withdrawal, they must again be inserted and withdrawn to the satisfaction of the inspector. Care must be taken that clearance between baffles and shells does not exceed the tolerance allowed.

(4) *Casting Inspection.* Imperfections in castings, due to core shifts, sand holes, etc., are cause for casting rejection, especially where minimum thickness specified is not met. In such cases, each instance must be referred to the purchaser's engineers. No welding of flaws will be permitted unless approved by the inspector. All porous metal and impurities must be cut out. Forged-steel pipe plugs are permitted, if the outside of the thread diameter required does not exceed three times the thickness of the metal at point of insertion.

(5) *Tube Sheet Warping.* All contact surfaces on tube sheets are to be refaced true and square if they become buckled or warped during the tube rolling operation.

(6) *Nozzle Locations.* Where nozzles and end flanges are welded to steel shells, the flanges on these nozzles should be faced and drilled true and square after the nozzle is securely welded to shell. This avoids any distortion to flanges, and hence provides for piping connections without the use of beveled fillers.

Nozzle locations are to be checked after assembly and must conform to plan within a tolerance of plus or minus $^1/_{16}$ in. for each 6 in. diameter of flange. The flanged faces must also be held within the plane specified on print with a tolerance of plus or minus $^1/_{32}$ in. per 6 in. diameter of flange. This is usually accomplished by setting up the units in a special frame.

Assuming that the foregoing methods have been followed and that the equipment has been installed in the plant, one must now be sure that it will be tested under properly controlled conditions which are well understood by both parties. It would be unfair to the purchaser to make a test for duty immediately after the equipment is installed, provided the understanding at the time of purchase contemplated a proper fouling factor. On the contrary, the purchaser should not expect, if the normal run on the equipment extends for a long period, as measured in weeks, that at the time of normal shutdown for overhauling, cleaning, and adjustment the heat-transfer equipment should have its full designed capacity. In other words, there must be some intermediate time, let it be assumed halfway through the normal period of running, at which the equipment will meet the designed duty, and that before this time arrives the equipment will have an excess duty, and that during the latter half of the run the equipment will have less than this duty. It would, further, be unfair to the manufacturer to consider that at the end of the normal run the equipment should have a definite percentage of capacity, for in some types of apparatus and with certain products that may be handled through such equipment, the variation in degree of fouling is not by any means a straight line, but may accumulate very rapidly within short periods, as will be described farther on. In the same way, both parties must expect a rather rapid drop in the heat-transfer rate in the first part of the normal running period after the equipment has been cleaned by the usual methods, and that after this, almost up to the time the equipment is shut down, the decrease in the transfer rate will be gradual but steady, with very few exceptions.

It should be agreed to by both parties, therefore, that the equipment shall have gone through at least one normal run, followed by a normal cleaning witnessed by both parties, before a formal acceptance test is made, and in this respect any specifications must be sufficiently flexible to permit this point to be properly covered for each purpose where a guaranteed duty is to be met on performance test.

With respect to corrosiveness of the materials that pass through the equipment and their effects on different materials, both parties should agree to what may happen on the basis of their joint past experience, and either approval must be given by both parties or a plain statement must be made at the time the equipment is purchased on the material selected. The author has in mind here only its effect on heat transfer and not on wear and tear or excessive corrosion due to the methods of running or the nature of the stocks run. No manufacturer should be asked to guarantee the life of equipment against corrosion, and it is not believed that this is now the generally accepted practice, but on the contrary, that it is specifically guarded against in the manufacturers' contract.

The methods of cleaning that may be followed are only outlined in the draft herewith, and it may be quite possible that certain refiners have other methods which are feasible to maintain the efficiency of heat transfer without interrupting the operation. For instance, in connection with the particular equipment for which a former draft of such specifications was made, it was found that by blowing the water side of gasoline condensers and coolers, where dirty river water was passed through the tubes, with compressed air every eight hours, a marked improvement in the reduction of temperature on the outlet of the naphtha stream was noted. Similarly, water streams injected on the vapor side help to wash off the accumulated ammonium salts where ammonia was used as a neutralizer, but care must be taken, on account of such additional water and colder reflux, that there is obtained proper separation at the point of drawing reflux and that the heat balance of the tower is not upset.

There is no doubt that much interference with heat-transfer rates is occasioned on water-cooled units by reason of muddy river water, and the refiner at the time of selecting and purchasing equipment and agreeing upon the basis for design should not forget that when using such muddy water he may be paying a premium, not only on extra surface and equipment, but also extra operating cost. An attempt should be made by him to evaluate this figure against the construction of settling basins or recirculation systems of cooling water involving cooling towers. A manufacturer of heat-transfer equipment will generally stand ready to offer all possible advice on this subject, as in many cases additional equipment for assuring a cleaner water supply that he may furnish may no more than offset the savings in first cost on heat-transfer equipment. Any interruption of water supply due to clogging lines and water-cooled sections of heat-transfer equipment will result in unbalanced operating conditions and clogging of heavy oil coolers due to decreased velocity, and on the extremely heavy oil, commonly termed high viscosity oil, the entire shell around the tubes may be filled with a jelly which cannot be removed except by taking out the tube bundle and cleaning it, which entails loss of time to the refiner.

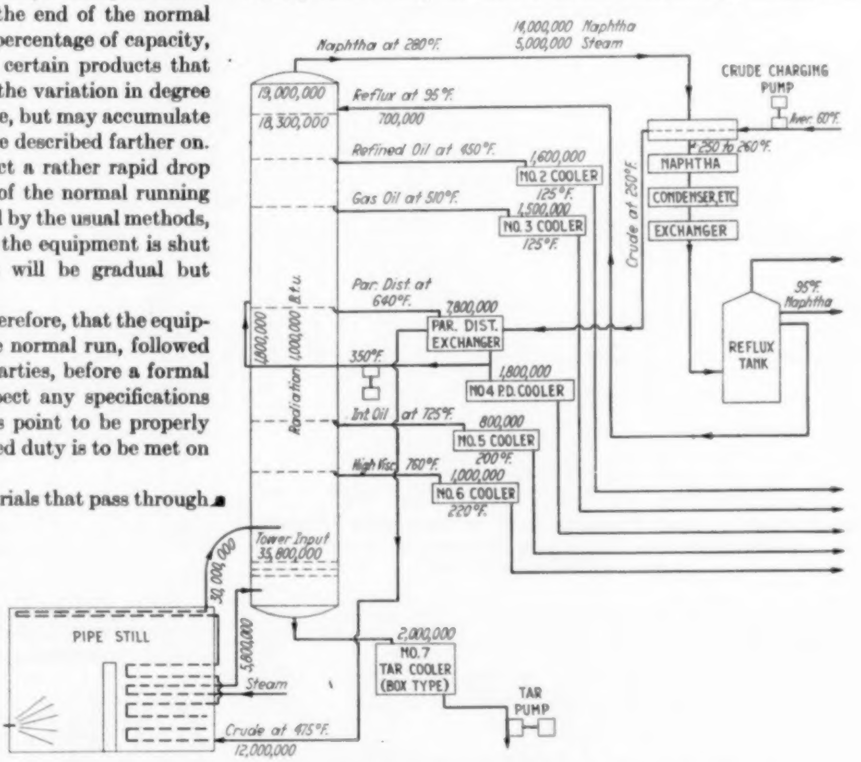


Fig. 1 Average Heat Balance of 3400-Bbl. Pipe Still, Nominal Capacity
(Heat values in B.t.u. per hour.)

It will be noted that these suggested specifications do not contain any reference to special cleaning compounds, although these may have been tried and found successful, as it is thought that the refiner will himself modify any specifications drawn up, based on his own experience.

Besides the method of cleaning tubes described in these specifications, at times the tubes have become plugged so solidly that some form of drill bit was needed to clean them out. Spare tube bundles are advisable for every group of similar units, so that the cleaning can be done at leisure, and clean bundles promptly installed to minimize shutdown time to conform to other work being done around the still battery of minor character. A special hydraulic pump may be maintained for blowing out tubes, using corks, but of course the limit of pressure possible for any diameter and gage of tube should not be exceeded. For cleaning and mechanical stability alone, the tendency has been toward larger tubes, and naturally of heavier gage to stand cleaning operations and to obtain longer life where corrosion is not entirely eliminated.

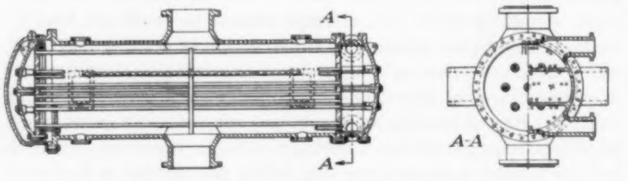


Fig. 2 Typical Vapor Condenser

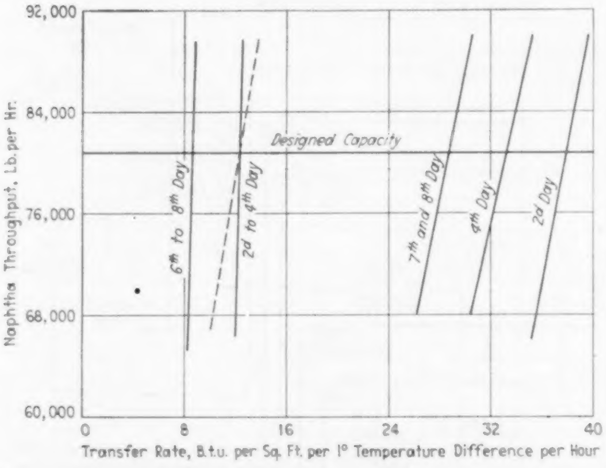


Fig. 3 Typical Heat-Transfer Rate of Exchanger (Naphtha to Crude) and of Condenser (Naphtha to Water)
(Calculated from the water side.)

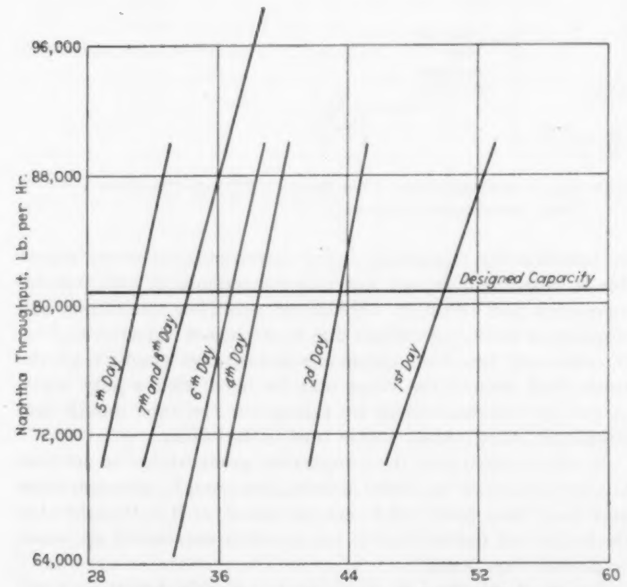


Fig. 4 Typical Heat-Transfer Rate of Condenser (Naphtha to Water)
(Calculated from oil side.)

Fig. 1 is the diagram of a crude fractionating still, indicating where the various side streams are taken off and the approximate temperatures and heat-transfer duty, on the basis of 3400 bbl. (50 gal. each) per 24-hour charge.

Fig. 2 illustrates a typical vapor condenser which is similar to the design for an exchanger, as in the latter case the crude charge is used as the cooling medium, and on account of higher pressures the floating heads and channel box may be of cast steel, whereas cast iron could be satisfactorily used for the water coolers.

Fig. 3 indicates the heat-transfer rate encountered on an installation similar to the typical shell design shown, in which steel tubes were used, and indicates the falling off in heat-transfer rate for readings taken every few days. The lines were drawn intermediate between a number of points obtained from a considerable number of observations, and in case of the condenser show a slope approximately proportional to the square root of the velocity of the vapors, the water rate being held constant. On the exchangers, however, the lines are steeper, the dotted line marked "2d to 4th day" being proportional to the square root of the vapor velocity. The solid line indicates the actual change in heat transfer with a difference in rate of vapor flow, the crude charging rate being approximately constant. The steep inclination of these curves is no doubt due to the heavy scale formation on the tubes, which consisted of iron sulphide and ammonium salts, indicating that the wiping action of the vapors did not have a material effect on what was practically a soaking heat effective through the scale on the tubes.

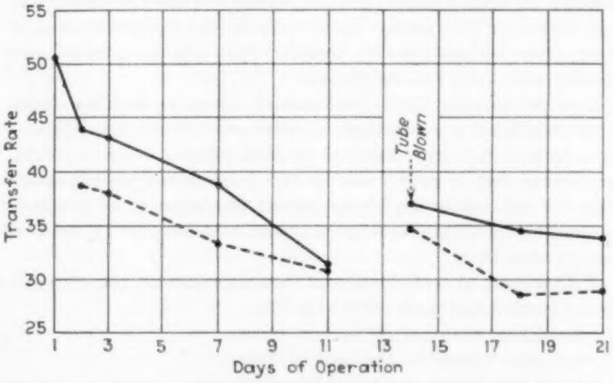


Fig. 5 Recovery of Heat-Transfer Rate After Blowing Out Inside of Tubes (Water Side) With Air and Water Mixture
(Solid line, oil side; broken line, water side.)

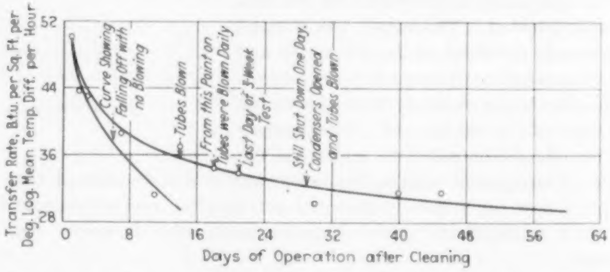


Fig. 6 Typical Heat-Transfer Rate of Naphtha Condenser (Water in Tubes)

Fig. 4 shows similar readings on the section of similar shells in which water is the condensing and cooling medium. A sudden drop on the 5th day and recovery on the 6th day is not clear, but may have been due to accumulation of mud on the water side which was automatically flushed out, when for some reason the pressure on water flow was temporarily increased.

Fig. 5, indicating the transfer rate, shows the falling off on the oil and the water sides, the difference depending upon the heat curves adopted for this test and plotted from actual points obtained by calculation. To some extent it shows the effect of blowing out the tubes by means of compressed air just before the readings taken on the 14th day, and temporary recovery, which is all that can be expected when the water is muddy.

Fig. 6 indicates similar points from another test in which a smooth curve is drawn, averaging the location of the points obtained by calculation. The actual location of the curve would vary slightly, depending upon the opinion of the one plotting it, but it shows a rapid falling off of heat transfer when the water side is not blown out and a more gradual falling off when tubes are blown. The right-hand half of this curve might be almost a horizontal line under the most favorable interpretation of the points in question.

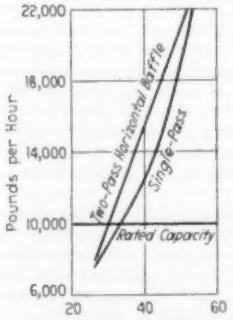
This curve is an illustration of where the manufacturer might go wrong by assuming a heat-transfer rate between 40 and 44, whereas the purchaser would expect to have the full duty when the heat-transfer rate was in the neighborhood of 32 to 34.

Fig. 7 shows the change in heat-transfer rate at various rates of flow of refined oil. In this particular instance the flow was in a zigzag direction across the tubes on the two-pass flow, and an average of an approximate ring and disk flow on the single-pass flow, and as this was a fairly clean oil, it appears that there was more short-circuiting on single-pass than on double-pass, or rather that there were more dead corners.

Fig. 8 shows a similar line for a high-viscosity oil cooler, indicating a very flat slope, and it is believed that a curve more accurately drawn would not be a straight line, but would be steeper at the upper capacity and flatter at lower rates. That is, for this particular kind of oil, the lower rate would result in overcooling of this oil and the formation of a jellylike substance blocking the spaces between the tubes.

Figs. 9 and 10 show, respectively, the paraffin distillate to crude exchanger and the paraffin distillate cooler where water was used as the cooling medium.

The points calculated from the readings are shown for these charts, and the lines are only approximate slopes, and in fact, on Fig. 10 either of two lines may be assumed as the proper one. There, again, the steeper line for the cooler indicates solidification, or at least high viscosities at lower rates of flow.



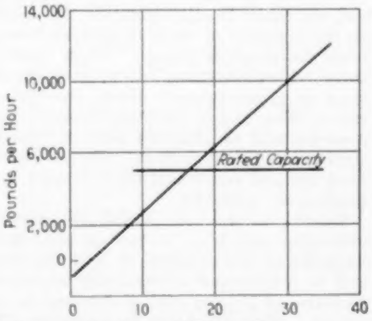


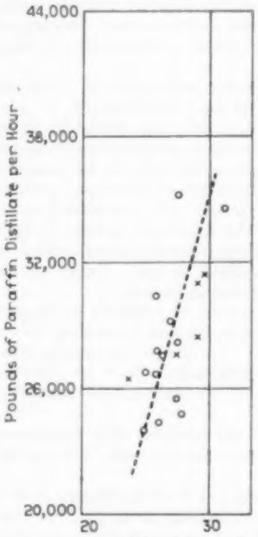
Fig. 7 Refined Oil Cooler

(Typical slope line. More points for plotting might indicate curves at upper and lower ranges.)

Fig. 8 High-Viscosity Oil Cooler

It is hoped that the various items of information given and the rather unscientific way in which the matter is presented will be considered merely as something in the way of a contribution toward what should be considered as real research work. In other words, the data that can be given by one company, based largely on tests made on one fairly large installation and being made for the purpose of obtaining approximate data for checking guaranteed duties, can only be incomplete for real research study.

As stated at the beginning, it is an effort to make a start on some form of uniform specification for performance tests to which both manufacturers and purchasers can agree, and with the hope that work can be carried to conclusion by others, cooperating toward what may be considered as a highly desirable end.



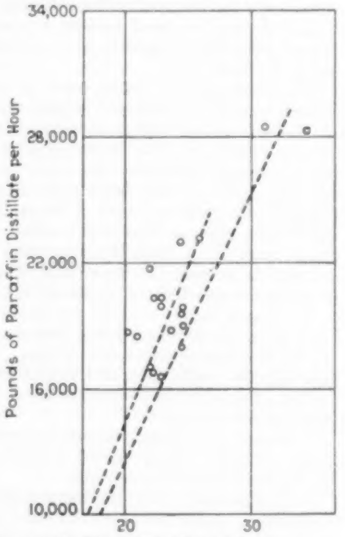


Fig. 9 Paraffin Distillate to Crude Exchanger

(Average flow, 80,000 lb. of crude per hour. Rated, 35,000 lb. of paraffin distillate per hour.)

Fig. 10 Paraffin Distillate Cooler

(Cooling into 180 gal. per min.; 17,000 lb. of paraffin distillate per hour.)

Appendix 1

PROPOSED SPECIFICATIONS FOR PERFORMANCE TEST ON PIPE STILLS, FOR EXCHANGERS, CONDENSERS, AND COOLERS, WHEN MADE IN PURCHASER'S REFINERY

(Based on original specification drawn up by E. H. Ten Broeck, The Atlantic Refining Company.)

I Cleaning

Operations, in sequence, are as follows:

(*a*) The outside of the tubes to be flushed with water injected under pressure available into the openings in the top of each unit, while the tube bundles are still in position.

(*b*) After withdrawing the tube bundles, the outside of the tubes to be scraped, if required, with thin steel strips, as thoroughly as their arrangement will allow.

(*c*) The outside of the tubes shall then be washed off with water, using preferably a fire hose and plant fire pressure.

(*d*) The inside of each tube to be brushed with wire brushes, and at the same time water shall be injected at the end of the tube opposite that which the brush enters, in order to keep the bristles effective—that is, clear from fouling.[2]

(*e*) The inside of each tube shall then be flushed with water, and air injected with the mixing nozzle, using fire pressure on the water and plant compressed air.

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[2] For the removal of slime and mud, it has not been general experience that either a straight air blowing or a straight water washing will be effective. Good results have been secured in the removal of such deposits by introducing air and water together in the same nozzle under a pressure of 100 lb. per sq. in. Very effective washing is secured in this manner. Also, effective cleaning has been secured by the use of manufactured plugs, which may be secured for standard sizes of tubes. These plugs fit inside of the tube with a fairly close clearance and would be blown through either by water or air pressure. Use of such plugs would be much more effective than the use of ordinary corks. It is expected that such of these cleaning means shall be adopted as actual inspection of the deposit on tubes indicates may be necessary. As to the cleaning of the tubes on the outside, it is expected that such foreign deposits as can be removed shall be removed and that tube bundles will be removed so that the entire length of the tubes may be observed.

The work shall be under the direction of the manufacturer's representative.

II Instruments

(*A*) *Temperature Measurement.* To be taken with glass thermometers having graduations directly on the glass, at at least 2-deg. intervals and checked against a standard thermometer (Bureau of Standards) at two points and showing errors not exceeding 1 deg., plus or minus.

Thermometers to be installed in ordinary pipe wells of 1/4-in. or 3/8-in. pipe with lower end welded shut in the usual manner.

Wells to extend 3 in. into 4-in. pipe, 4 in. into 6-in. and 8-in. pipe, and at least 6 in. into larger pipes or nozzles on shells of units under test, all measured from the outer surface. All wells to be inserted so that oil or mercury can be held in the bottom to give good contact between the bulb and wall of the well. If any well is installed at an angle in a pipe, so that it is parallel to the direction of flow, it shall be at least 1/2 in. from the inner wall of the pipe and must extend to at least 2 in. past the inner side of the elbow or tee. Temperature measurements shall be made at the following points:

On vapor side: (*a*) In vapor inlet nozzle of naphtha exchanger unit or in vapor pipe leading thereto at a point not more than 24 in. above the face of the nozzle flange. (*b*) In one of the nozzles connecting the exchanger and naphtha condenser units. (*c*) In the outlet pipe from the lowest condenser unit, the well to be installed in the horizontal pipe.[3]

Temperature measurements between the sections of a condenser unit, if it is sectional, may be made by the manufacturer, if he desires and provides the thermowells.

On liquid side: (*a*) In water inlet line to lowest condenser at any convenient point between the unit and the supply main. (*b*) In water outlet from upper condenser unit at any convenient point not more than 4 ft. from the outlet flange on the condenser. (*c*) In cold-oil inlet line to exchanger unit at any convenient point between the charging pump and the unit. (*d*) In oil outlet line from exchanger unit at any convenient point not more than 4 ft. from the outlet flange on the unit.

(*B*) *Pressure Measurements.* On vapor side, to be taken with manometers using Hg or CCl_4 for the indicating medium. If CCl_4 is used, its gravity shall be determined by comparison with water in a U-tube and the result accepted by the representative of the manufacturer.

On liquid side, the water pressures and oil pressures shall be measured by means of industrial pressure gages of approved make, having a range equal to 1 1/2 to 2 items the maximum pressure to be encountered.

Pressure gages to be calibrated either directly on a deadweight calibrating machine or by comparison with a standard gage that has been so calibrated. Gages must not vary more than 1 per cent, plus or minus, throughout the expected working range.

Pressure measurements to be made at approximately the same points as the temperature measurements, and in addition, the differential pressures across the naphtha exchanger unit and the condenser unit to be measured on the vapor side.

(*C*) *Quantity Measurements.*

On vapor side: (*a*) Naphtha Forward Flow and Reflux. To be measured by a standard thin-plate, sharp-edge orifice and manometer installed in the run-down line between the outlet of the condenser and the water separating tank at a point below the gas vent connection, making necessary correction for water (steam) measured, as described below.

(*b*) Steam. The steam to be measured by a standard sharp-edge orifice placed in the supply line to the steam superheater, if one is used. If a calibrated round-edge orifice is used in connection with steam-flow control, the pressure against this orifice may be taken as a correct indication of the quantity of steam passing, in accordance with the customary formula (Grahsof's) for such round-edge orifices. The pressure gage used is to be an ordinary pressure gage (of approved make) as regularly supplied on the still. Before starting the test, the orifices are to be removed and examined to determine whether their condition and dimensions are satisfactory and new ones supplied if necessary. The pressure gage to be calibrated or checked against a standard instrument and must be correct to plus or minus 1 per cent. The quantity of steam to be maintained at substantially the specified quantity ( ) lb. per hr. (Blank to be filled in with proper figure for each case.) A variation in this rate of flow of plus or minus 5 per cent will be permitted.

(*c*) Gas. The gas to be measured by a thin-plate sharp-edge orifice and manometer (or draft gage), and the orifice plate will be removed, inspected, and renewed if necessary, before the start of the test. While it is desirable that the gas from the still under test be segregated, this is not always possible where the still is in a battery, and in some cases the quantity of gas from the still under test should be taken in proportion of the still throughput to the entire throughput of the battery, as determined by the quantity measurement on the charging oil.

On liquid side: (*a*) Crude Oil. The quantity of crude oil passing through the unit may be determined by the number of pump strokes, where reciprocating charging pumps are installed. The pump for the still on test to be calibrated by gaging the amount pumped out of one of the regular charging tanks in a measured period of time. The amount removed from the tank must be such as to lower the level in the tank by at least 2 ft. Three calibrating runs to be made on different days and the average taken to use in the calculations of the test results. These pump calibrations need not be simultaneous with test runs on the naphtha exchanger and condenser units, but are to be made at the convenience of the still operators and must be completed within 30 days after the tests on the manufacturer's units. These calibrating runs to be witnessed and the calibration accepted by the representative of the manufacturer. If a centrifugal charging pump is used, the quantity of oil charged to still on test should be measured by the usual orifice plate.

(*b*) Water. If necessary, the water used may be heated to the specified entrance temperature by recycling a portion thereof, maintaining it as near to this specified temperature as practicable, and it may be measured by a V-notch weir in a suitable measuring tank. It will usually be more convenient to measure the water quantity by means of orifice plates placed in the measuring section.

III Procedure

The test shall be made on the crude specified in the order, if possible, or another crude accepted by both parties may be substituted during such portions of the test as may suit the convenience of the refinery superintendent. After the units are cleaned, as described above, they are to be reassembled as promptly as possible and the stills started and operated in the normal way for a period of 14 days[4] on the acceptable crude. A "set" of test readings to be taken each day for a period of 7 days. (It may be agreed that several days will intervene between "sets" of readings.)

Each "set" of test readings shall consist of three "series" of runs, one "series" of runs to be at a rate above that specified and one below, and the other at approximately the specified rate (rate to be based on the quantity of crude oil passing through the unit, as determined from the speed of pump). Each "series" of runs shall consist of three "determinations," each determination requiring four observations at approximately 5-min. intervals, of each temperature and rate of flow, and at least one observation of each pressure, and one measurement of charging rate by counting the number of pump strokes in a 15-min. period. The above readings are not to be started until uniform conditions are indicated; that is, the system must be balanced in operation.

During all the test period, the vapor temperature entering the exchanger unit is to be held as near that specified as possible. A variation of plus or minus 20 deg. will be permitted. The water rate will be maintained substantially constant at the specified rate. A variation of 2 1/2 per cent in charging rate, plus or minus, will be permitted. The actual rate as measured will be used in calculations.

IV Calculations

The method of calculation shall be as follows:

For each "determination" the average of the readings of each instrument will be taken and the transfer rate calculated as follows:

Naphtha to Crude Exchanger Unit. The heat absorbed by the crude oil will be determined by use of the .... curve of heat content of oil and vapor. Curve prepared by ....[5] It will be assumed that no vaporization takes place in the crude while passing through this unit.

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[3] It was noted in tests for which the original of these proposed specifications was formulated that the temperature in the vertical outlet pipe from the lowest condenser unit varied by 20 to 30 deg. from the temperature several feet farther on, in a horizontal run of this pipe where the pipe was full of liquid. This difference may be accounted for by the liquid falling from the tubes on to the upper thermometer being cooler than the mixture of vapor and liquid at the second location.

[4] The provision that the stills be operated for 14 days after a normal cleaning preceding the formal duty test is based on a normal run lasting during the 6 or 7 weeks following the normal cleaning time. It is important, on the basis of falling off of efficiency due to fouling and roughening of tube surface, that the manufacturer consider a proper fouling factor to take care of these features, as that is the actual working condition. The time of starting test should, however, in a general specification be left open—that is, with blank spaces to be filled in—depending upon the type of apparatus purchased and the service for which it is intended.

The logarithmic mean temperature difference between the two mediums will be calculated on the assumption that true counterflow exists.[5]

The transfer rate will be calculated by dividing the quantity of heat per hour, determined as above, by the log mean temperature difference multiplied by the effective surface of the exchanger unit.

For purposes of comparison, the transfer rate required to meet the specified conditions will be calculated from these conditions in the same manner. In this calculation, the temperature drop of the vapors through the exchanger unit will be assumed to be the same as that actually found as the average for the particular "set" of test readings under consideration.

Naphtha Condenser. The heat absorbed by the water will be determined by multiplying the quantity in pounds per hour by the temperature difference between the inlet and outlet.

The logarithmic mean temperature difference will be calculated on the assumption of true counterflow as for the exchanger unit.[5]

The transfer rate will be calculated by dividing the heat quantity, as determined above, by the logarithmic mean temperature difference multiplied by the effective surface of the condenser unit.

For purposes of comparison, the transfer rate required to meet the specified conditions will be calculated from these conditions in the same manner. In this calculation the temperature of the vapor entering the condenser unit will be assumed to be the same as the vapor outlet temperature from the exchanger unit as determined above.

V INTERPRETATION OF RESULTS

After making the calculations as described above, the transfer rates for each "determination" shall be plotted against the naphtha throughput in pounds per hour and a curve drawn in the usual manner through these points.

The point where this curve crosses the line representing the specified naphtha rate shall be taken as the transfer rate of the unit for that day. If any of these points are obviously not in agreement with the others found in that "set" of readings, these "off" points need not be considered in drawing the curve for that "set." They should be shown on the sheet, however.

If the points for any "series" are so widely distributed that there is doubt as to the position of the curve, it shall be determined by taking the average naphtha rate for the three readings of the series and the average of the three transfer rates as calculated. If the one point is obviously in error, it may be neglected and the other two used as above. If two points are obviously in error, the entire "series" may be neglected and the position of the curve determined by the remaining two "series" of the "set." If two "series" of a "set" are obviously in error, the entire "set" shall be omitted. This omission shall not be considered to affect in any way the validity of the test, provided that not more than two such omissions shall be made.

The average transfer rates for all the days of the test shall be the figures used in determining the capacity rating of the units.

If this figure is below the average of the specified transfer rates, determined as above for each set of readings, the shortage of surface shall be considered to be the difference between that now installed and that which would be required to meet the specifications calculated from the actual average transfer rates as determined from these tests by the method outlined above. This shortage shall be separately determined for each unit, that is, exchanger and condenser.

VI SIDE-STREAM COOLERS AND EXCHANGERS[6]

For the cooler to be tested, the tube bundles are to be withdrawn and cleaned in a satisfactory manner,[7] similar to that described above for the naphtha units, and inspected thereafter by the representatives of both parties, before replacing in shell.

The size of weighing tank should be as large as practical, but preferably not less than will accommodate a 10-min. flow period. This period should be taken at a time when indications of uniform conditions are evidenced.

The quantity of cooling water can be determined by connecting the outlet pipe through a hose to a section of pipe containing a standard thin-plate sharp-edge orifice, and the flow regulated at inlet valve to within plus or minus 5 per cent of the specified quantity. Where recycling of cooling water is impracticable to maintain the specified water-inlet temperature, both parties may agree to a modification in the flow rate of cooling water to compensate for any water temperature that may be available at the time of the test.

Inlet and outlet temperatures of oil to be taken by approved indicating pyrometer and/or glass thermometers, and by the latter on the water side. All such temperature-indicating devices to be located as nearby as possible to the unit being tested. Average indications taken during one "set" of test readings to be used in the calculations.

Although cooler shells are customarily not insulated, no account shall be taken of heat radiation from shell, where oil is outside of tubes.

VII GENERAL AGREEMENT

Before starting preparations for this test, the manufacturer and the purchaser must agree through their accredited representatives to the conditions as outlined above, after making such modifications as may be mutually agreed upon, and no deviations will be permitted without further written agreement between the companies.

Before starting the test, all equipment must be inspected by the accredited representative of the manufacturer and approved by him in writing. This inspection and approval must include the following items:

(1) Cleanliness of equipment.

(2) Thermometers: Type; calibration; location; wells and their installation.

(3) Pressure gages: Type; calibration; location and installation.

(4) Manometers: Type; location and installation including connections. (Note: For small rates of cooled oil a disk meter may be agreed upon.)

(5) Orifice plates: Condition; dimensions; location; position of taps; connections to manometers or pressure gages. (Note: Orifice plates to be in accordance with A.S.M.E. accepted practice.)

(6) Water-measuring equipment: Size and arrangement of tank; condition and dimensions of weir; type and position of hook gage. (Note: Pitot tube readings may be agreed upon by both parties as an alternative.)

(7) Any other instruments, equipment, or arrangements not mentioned above.

If any instruments or equipment are not approved by the representative of the manufacturer, the manufacturer should promptly furnish such substitutes as he considers suitable, subject of course to purchaser's approval.

If, through accident or oversight, any item is omitted from his inspection and approval, such omission will not vitiate any of the data or results of the test. Equipment broken or failing during the tests will be replaced with similar equipment calibrated or checked in the same manner as the original items, and approved by the manufacturer's representative.

The data sheets for each set of readings shall be initialed by the accredited representative of the manufacturer, immediately after the conclusion of the test work on that set. No erasures should be made on these data sheets at any time. If corrections or changes to the readings are necessary, the original figures shall be crossed out and initialed by the representatives of both companies, with a notation explaining the change or correction.

General note: The proposed specifications in Appendix 1 are submitted not so much for discussion at this particular meeting, but with the idea in mind that a committee might be formed in the Petroleum Division of the A.S.M.E. which could use this data as a basis for a more complete and general specification which would be acceptable to both users and manufacturers, so as to have the tests of this character on a uniform basis throughout.

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[5] The paragraph covering this should be written to permit of the latest method of calculation for each particular type of equipment and service as this work is developed by various research projects. While these original specifications, made for a particular test some time ago, assumed that no vaporization takes place in the crude oil passing through the exchanger unit, is is believed that data are now available which make it possible to more closely estimate this vaporization. Likewise, it is not always best to consider that true counterflow exists, but an agreement should be reached between the parties to the test at the time the units are purchased as to what assumptions shall be made on this factor.

[6] It is desirable that the piping arrangement at the units to be tested provide a spare run-down line from which the oil issuing from any cooler or exchanger can be drawn to a weighing tank, for determining the rate of flow.

[7] It has been found advisable in some cases to place the tube bundles in a soaking tank for a period of 10 to 20 hours, in which the contained water is heated by the continuous admission of steam, and to which certain cleaning solutions may be added, depending upon the experience of each refiner. This clause for cleaning may therefore be modified or blank lines provided for insertion of this additional requirement.

Discussion

F. C. Fyke.[8] These comments briefly outline other practices and experiences in regard to a slightly different phase of the testing of heat exchangers. These deal chiefly with the inspection of exchanger units at the manufacturers' plants, preparatory to their acceptance by the purchaser.

When the type of exchanger to be used and its design have been determined, the engineer's next function is the inspection of the material entering into the assembly and the completed exchanger prior to its acceptance and shipment. Ordinarily the manufacturer of heat exchangers must purchase his raw materials from outside sources. By raw materials is meant the steel plate for shells or completed shells, rivets, bolts, tubes, and castings. It is our practice to inspect all so-called raw materials at the various plants of their manufacture so as to be familiar with the processes employed and to see that the specifications are complied with, thereby saving time in the ultimate assembly through the non-receipt of defective or improper materials. We make one exception to this general rule in the case of castings, which are only inspected at the plant where the machine work is executed. We find that little can be learned by inspecting any reasonably good casting before machining. A thorough examination is made after machining, and whenever possible before assembly the castings are pressure-tested; in the case of steel, to four times the working pressure, and of cast iron, to twice the working pressure.

Any defects found either before or after testing are explored by chipping or burning out with a torch or carbon arc. An acid etch is used to determine whether the defect has been removed. If the number, extent, and location of the defects are not serious enough to cause rejection, permission is then given by the inspector to weld up the cavity. No castings with previously welded defects are acceptable, as our inspectors must in all cases examine the extent of the defect and be able to certify to its complete removal. After welding, castings are reannealed, wholly wherever possible; otherwise, local annealing is resorted to.

Where the size of the casting is within reason and the cost will permit, another testing method is sometimes used. This is a test to destruction by means of a drop ball. This may appear to be an antiquated method of examining castings, in the face of such modern methods as the X-ray, but it offers a sound method of locating certain types of defects, inherent to the particular pattern or foundry practice, such as shrinkage cavities, sponginess, and gas pockets, which may not be revealed by the X-ray. Obviously, the drop ball locates weak spots with a certainty that could not be approached by the slower and more laborious method of sawing up a casting.

When the size of the order warrants, a full-time inspector is assigned to the fabricator's plant to follow the fabrication of the exchangers; otherwise a thorough final inspection is made when the first unit is ready for acceptance, at which time any other units in the course of assembly are examined. Some of the chief points which are checked during the final inspection are easy removal and adequate clearance of the tube bundle, proper rolling of tubes for the full depth of the tube sheets, and evidence of overrolling or cracking of the tube ends. Minimum clearance between tubes and baffle plates should be demanded, not only to limit short-circuiting, but to prevent wear on the tubes, which may be caused by vibration where too large a clearance is allowed. A hydrostatic pressure test is applied, preferably to the tube bundle and shell unassembled. If, however, the unit is assembled for this test, the tube bundle is removed on completion of the test, and both the tube bundle and shell are carefully examined, particular attention being given to joints that may be calk-welded, as the welding is sometimes cracked in the pressure test. The pressure is usually maintained for one hour, during which time the shell is hammer-tested, and all rivets are tapped and all parts are examined for leakage.

In testing heat exchangers, two common sources of error have been noted which lead to the determination of fictitious heat-transfer coefficients. These are leakage around the baffles between tube passes and the use of the logarithmic mean temperature difference applied to the terminal temperatures.

An approximate check is available to determine the existence of leakage between the tube passes by comparing the actual pressure drop with the loss as calculated by the Fanning equation. If the actual pressure drop is materially less than the calculated amount, it is safe to assume that an appreciable amount of by-passing is occurring.

The logarithmic mean temperature difference is often used, regardless of whether the flow is strictly counter-current or not. The greater the variance from true counter-current flow, the greater the possible error that may be introduced in determining the true value of the coefficient. However, the log-mean temperature may be used in testing, and if the exchange design does not give it, then the manufacturer is penalized in that his product will only pass test if the transfer rate is sufficiently high to overcome the disadvantage imposed by the lack of counter-current flow.

Tests on various exchangers have shown that the artificial turbulence produced in the shell side of liquid-to-liquid exchangers by baffle plates materially increases the film coefficient as long as the flow is in the viscous range, but this effect disappears as the critical point is approached. Within the range of turbulent flow the only value of baffles is to increase the length of the path of flow, with the consequent decrease in the shell diameter and increases in the mean velocity. Any great amount of leakage through the shell-side baffle plates will of course tend to neutralize the above advantages; hence the advisability of careful inspection of these plates for excess clearance.

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[8] Standard Oil Development Company, Linden, N. J.

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Auburn	Engineering Library, Alabama Polytechnic Institute
Birmingham	Public Library

Arkansas

Fayetteville	Engineering Library, Arkansas University

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Berkeley	Library, University of California
Long Beach	Public Library
Los Angeles	Public Library
	University of Southern California
Oakland	Oakland City Library
	Teachers' Professional Library
Pasadena	Library, California Institute of Technology
Santa Clara	Library, University of Santa Clara
San Diego	Public Library
San Francisco	Public Library (Civic Center)
	Engineers Club of San Francisco
	Mechanics Institute
Stanford University	Library, Stanford University

Colorado

Boulder	Library, University of Colorado
Denver	Public Library
Fort Collins	Colorado State Agricultural College

Connecticut

Bridgeport	Public Library
Hartford	Public Library
New Haven	Public Library
	Library, Yale University
Waterbury	Silas Bronson Library

Delaware

Newark	University of Delaware
Wilmington	Wilmington Free Institute

District of Columbia

Washington	Scientific Library, United States Patent Office
	Library of Congress
	Bureau of Standards Library
	George Washington University
	Catholic University

Florida

Gainesville	University of Florida
Jacksonville	Free Public Library
Miami	Public Library
Tampa	Public Library

Georgia

Atlanta	Carnegie Public Library
	Georgia School of Technology
Savannah	Public Library

Idaho

Moscow	University of Idaho

Illinois

Chicago	John Crerar Library
	Western Society of Engineers
	Library, Armour Institute of Technology
	Public Library of Chicago
Moline	Public Library
Urbana	University of Illinois

Indiana

Evansville	Public Library
Fort Wayne	Public Library
Indianapolis	Public Library
	Indiana State Library
Notre Dame	Library, University of Notre Dame
Terre Haute	Rose Polytechnic Institute
West Lafayette	Library, Purdue University

Iowa

Ames	Iowa State College
Des Moines	Public Library
Iowa City	State University of Iowa

Kansas

Kansas City	Public Library, Huron Park
Lawrence	Library, University of Kansas
Manhattan	Kansas State Agricultural College
Wichita	Wichita City Library

Kentucky

Lexington	University of Kentucky
Louisville	Speed Scientific School
	University of Louisville

Louisiana

Baton Rouge	Louisiana State University
New Orleans	Howard Memorial Library
	Louisiana Engineering Society
	Public Library

Maine

Orono	University of Maine

Maryland

Annapolis	United States Naval Academy
Baltimore	Johns Hopkins University
	Engineers Club of Baltimore
	Public Library

Massachusetts

Boston	Northeastern University
	Boston Public Library
Cambridge	Harvard University (Engineering Library)
	Massachusetts Institute of Technology
Fall River	Public Library
Lowell	Lowell Textile Institute
Lynn	Free Public Library
New Bedford	Free Public Library
Springfield	Springfield City Library
Tufts College	Tufts College
Worcester	Worcester Polytechnic Institute
	Free Public Library

Michigan

Ann Arbor	University of Michigan
Detroit	Public Library
	Cass Technical High School
	Highland Park Public Library
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East Lansing	Michigan State College
Flint	Public Library
Grand Rapids	Public Library
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Jackson	Public Library

Minnesota

Duluth	Public Library
Minneapolis	University of Minnesota
	Minneapolis Public Library (Engineering and Circulating Libraries)
St. Paul	James Jerome Hill Reference Library

Mississippi

Columbus	Mississippi Agri. & Mech. College

Missouri

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Jamaica, L. I.	Queens Borough Public Library
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Rochester	Rochester Engineering Society
Schenectady	Union College
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Utica	Public Library
Yonkers	Public Library

North Carolina

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Raleigh	North Carolina State College

North Dakota

Fargo	North Dakota State College
Grand Forks	University of North Dakota

Ohio

Ada	Ohio Northern University
Akron	Public Library University of Akron
Canton	Public Library
Cincinnati	University of Cincinnati Public Library Engineers Club of Cincinnati
Cleveland	Public Library Case School of Applied Science Cleveland Engineering Society
Columbus	State of Ohio Library Public Library Ohio State University
Dayton	Engineers Club of Dayton
Toledo	Public Library University of Toledo
Youngstown	Public Library

Oklahoma

Norman	Oklahoma University
Oklahoma City	Public Library
Stillwater	Oklahoma Agricultural and Mechanical College
Tulsa	Public Library

Oregon

Corvallis	Oregon State Agricultural College
Portland	Portland Library Association

Pennsylvania

Allentown	Free Library
Bethlehem	Lehigh University
Easton	Public Library Lafayette College
Erie	Public Library
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Philadelphia	Engineers Club Drexel Institute University of Pennsylvania Franklin Institute
Pittsburgh	University of Pittsburgh Engineers' Society of Western Pennsylvania Carnegie Institute of Technology Carnegie Library (Schenley Park) Carnegie Free Library of Allegheny
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Scranton	Public Library
State College	Pennsylvania State College
Swarthmore	Swarthmore College
Villanova	Villanova College
Wilkes-Barre	Public Library

Rhode Island

Kingston	Rhode Island State College
Providence	Brown University Providence Engineering Society Public Library

South Carolina

Clemson College	Library, Clemson College

Tennessee

Kingsport	Public Library
Knoxville	University of Tennessee
Memphis	Goodwin Institute
Nashville	Vanderbilt University

Texas

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College Station	Agricultural & Mechanical College of Texas
Dallas	Public Library
El Paso	Public Library
Fort Worth	Carnegie Public Library
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PME-54-6

Effect of Vacuum on Lubricating-Oil Distillation

By R. S. DANFORTH,[1] SAN FRANCISCO, CALIF

Vacuum distillation of lubricating oils possesses many advantages, and the paper brings out some of its basic features. The benefits of the vacuum process are mainly in the reduction of cracking, in the improved separation of fractions, and the reduction of heat input and condensing requirements. The author mentions a possible fractionator design and suggests a condenser design.

FOR some years it has been known that the vacuum distillation of lubricating oils possesses many advantages, and the purpose of this paper is to bring out some of its basic features. The benefits of the vacuum process are mainly in the reduction of cracking, in the improved separation of fractions, and the reduction of heat input and condensing requirements.

In lubricating-oil distillation, cracked products are usually less desirable. The extent of cracking depends mainly on the temperature and the time of exposure, the heavier molecules cracking at a lower temperature. C. R. Burch reported[2] experiments on Pennsylvania and other distillates indicating a cracking activity starting at temperatures varying between 584 and 644 F.

In the distillation of a lubricating oil in a shell still, on account of long exposure gas evolution usually becomes quite objectionable around 640 F, while in the modern pipe still, the limit of operation from this point of view is above 700 F, dependent on the type of oil.

EFFECT OF TEMPERATURE UPON COMPOSITION OF VAPOR

An important feature in distillation is the effect of a low boiling temperature in greatly decreasing the percentage of heavy fractions in the vapor. By Raoult's law, the ratio by weight of the lighter vapor fraction to the heavier equals the ratio of their weights in the liquid multiplied by the ratio of their vapor pressures. With a lower temperature the vapor pressure ratio is greatly increased, so that the vapor has much less of the heavier fractions. In the following example, taking merely two fractions for simplicity, the percentage of heavier fraction in the vapor is reduced from 29.2 per cent when distilling at 800 F under 974 mm absolute pressure to 5.1 per cent when distilling at 550 F and 32 mm pressure, or to 1.9 per cent at 450 F and 5 mm pressure.

As this increased separation is repeated on each plate of a fractionating column, the amount of reflux in fractionation, the heat required to reevaporate the reflux, and the heat input to the condenser are all reduced with lower temperature operation. The time of exposure and extent of cracking are decreased with the lessened amount of reflux. In distilling, the heat required is reduced by the smaller amount of heavier fractions evaporated.

Fig. 1 shows the variation of vapor pressure with temperature for normal hydrocarbons of the paraffin series. Mixtures of oils of equivalent boiling points follow practically the same curves.

For example, suppose we are distilling a lubricating fraction of 296 molecular weight from a heavier fraction of 492 molecular weight, the vapor in equilibrium with a residuum containing 10 per cent by weight of the lighter fraction. At a distillation temperature of 800 F the vapor pressure of the lighter fraction is 5000 mm, the heavier fraction 230 mm, and their ratio 21.7. The ratio of the weight of the lighter fraction in the vapor to the heavier fraction equals $1/9 \times 21.7 = 2.42$. This is equivalent to 70.8 per cent lighter fraction and 29.2 per cent heavier fraction in the vapor. Distilling at 450 F, the heavier fraction in the vapor will be reduced to only 1.9 per cent instead of 29.2 per cent.

By Dalton's law, the total still pressure equals the sum of the partial pressures of the fractions. The partial pressure of a fraction is equal to its vapor pressure multiplied by its mol fraction in the liquid. (The number of mols equals the weight in pounds divided by the molecular weight.) The mol fractions in the foregoing liquid would be 0.16 and 0.84, respectively. The still pressure would be 974 mm at 800 F and 5 mm at 450 F.

The effect of reducing the boiling temperature is shown in Table 1.

TABLE 1 VAPOR COMPOSITION, RESIDUUM RETAINING 10% BY WEIGHT OF LIGHTER FRACTION

(Two fractions having molecular weights of 296 and 492)

Temp., F	Vapor pressure ratio	Ratio of weights in vapor	Heavier fraction in vapor by weight, %	Still pressure, mm mercury
800........	21.7	2.42	29.2	974
700........	43.2	4.80	17.2	281
600........	97.8	10.9	8.4	76
550........	167.0	18.5	5.1	32
500........	268.0	29.8	3.3	13
450........	470.0	52.3	1.9	5

At 450 F the ratio of the lighter fraction in the vapor to the heavier is over 20 times the ratio at 800 F.

As a further example of the benefit of low temperature upon relative volatilities, assume that it is desired to remove a light fraction of 128 molecular weight from a lubricating-oil fraction of 394 molecular weight. Assuming 0.1 per cent by weight of the light fraction retained in the residuum, the results are as follows:

Temp., F	Still pressure, mm	Heavier fraction in vapor by weight, %
750....................	808	93.5
700....................	424	90.4
600....................	115	77.0
500....................	30	48.8
400....................	9	17.4

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[2] Proc. Royal Soc., vol. A123, 1929.

Presented at a meeting of the San Francisco Section of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, San Francisco, Calif., June 26, 1931.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

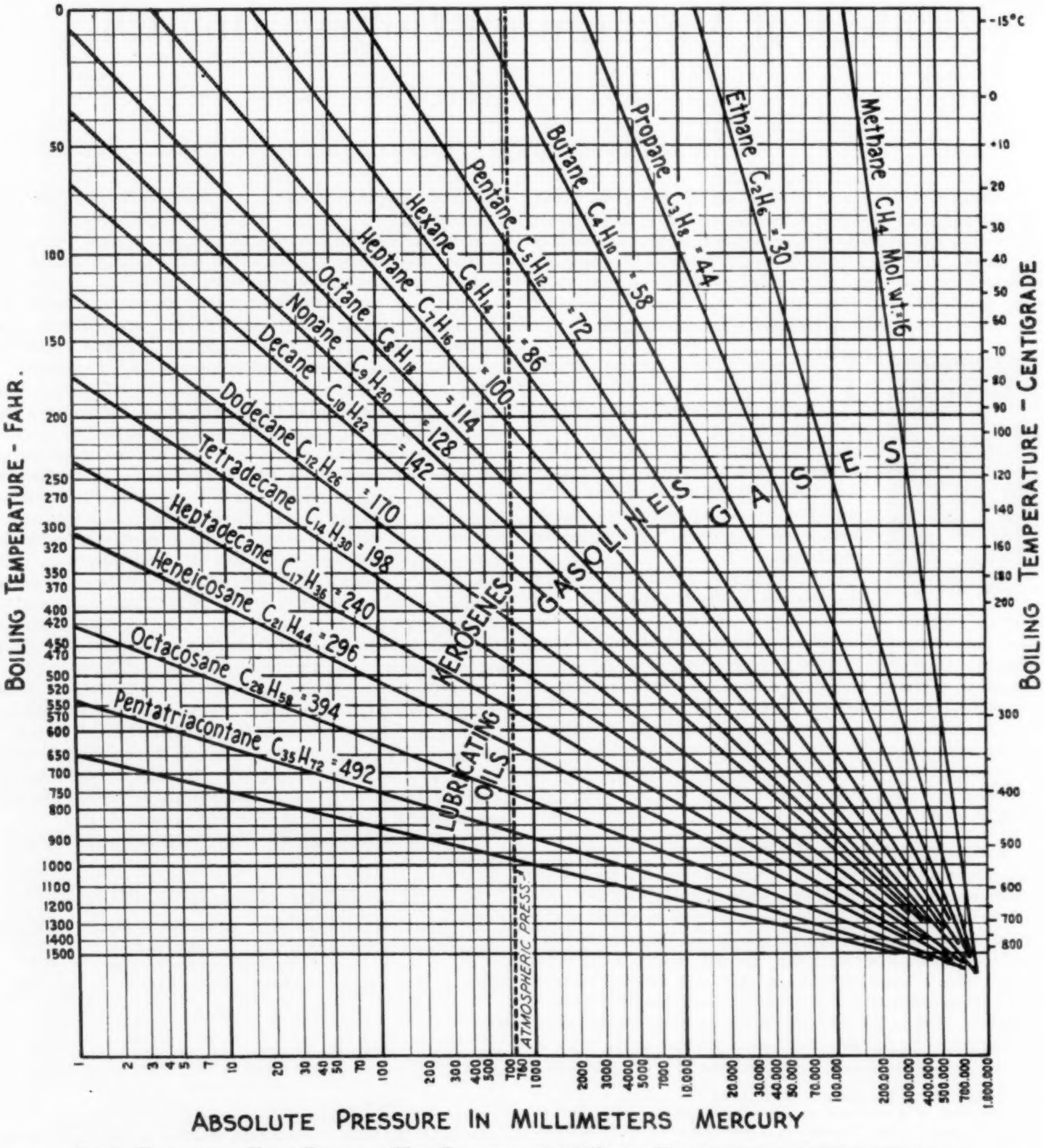

Fig. 1 Variation of Vapor Pressures With Temperature for Normal Hydrocarbons of the Paraffin Series
(Method of plotting according to E. R. Cox, *Industrial and Engineering Chemistry*, 1923, p. 592. Boiling points below 200 molecular weight from treatise on "Distillation," by C. V. Rechenberg. Above 200 molecular weight from Day, "Petroleum Industry," Vol. 1, p. 470.)

With high-vacuum distillation, sufficient separation might be secured in some cases to eliminate the use of fractionators entirely.

Distillation Temperature

The distillation temperature should therefore be as low as possible in order to improve the separation of fractions, reduce the quantity of gases formed by cracking, secure the highest quality product, and operate with the lowest heat input.

The distillation temperature can be lowered by reducing the pressure by operating under a vacuum or by indirectly reducing the partial pressure of the oil vapors by the presence of steam.

Vacuum and Steam Distillation

In dry vacuum distillation, the boiling point is lowered solely by the reduction of pressure. If the liquid is under a hydrostatic head, the low pressure will not be effective, so the main essential is to properly expose the liquid.

In steam distillation, the liquid is exposed at the surface, and

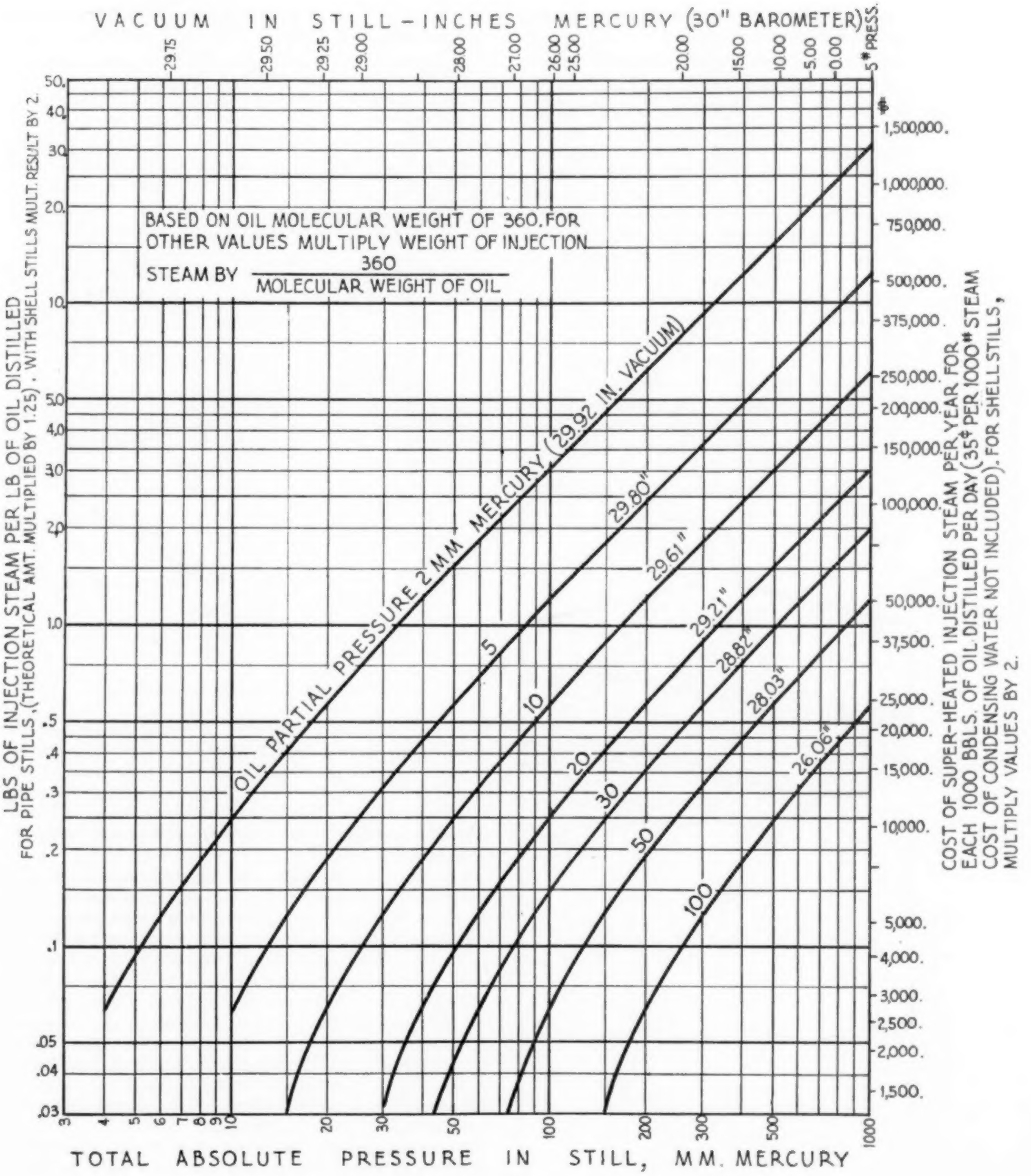

FIG. 2 INJECTION STEAM REQUIRED FOR STEAM DISTILLATION

in the steam bubbles, to a mixture of oil vapor and steam. The total pressure is divided between the partial pressures of the oil and the steam. The boiling point corresponds to the partial pressure of the oil, and is considerably lower than the boiling point corresponding to the total pressure.

AMOUNT OF STEAM REQUIRED

Assuming that the steam becomes saturated with oil vapors, the injection or distillation steam required per pound of oil distilled to secure a given boiling point is given by the following formula:

$$a = \frac{18\,(P - p)}{Mp}$$

where a = pounds of injection steam per pound of oil distilled over

P = total absolute pressure in the still

p = partial pressure of the oil corresponding to the desired boiling point (in same units as P)

M = average molecular weight of the vaporized oil.

With steam distillation in shell stills, owing to the high velocity, large volume, and slow diffusion, it is difficult to secure a uniform saturation of the steam with oil vapors, and between two and three times as much steam as theoretically necessary is often used. With pipe stills, where the steam is brought into more intimate contact, the theoretical amount is used and sometimes up to one and one-quarter times the theoretical amount. Fig. 2 shows the injection steam required for steam distillation in accordance with the foregoing, and also the yearly cost per 1000 bbl distilled over per day. The steam should be superheated above the distillation temperature, otherwise it will absorb heat from the oil being distilled. The cost of condensing water is not included.

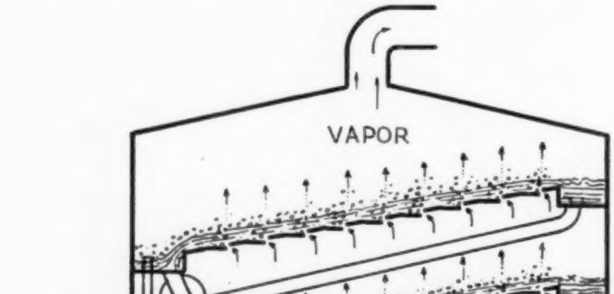

Fig. 3 Possible Fractionator Design

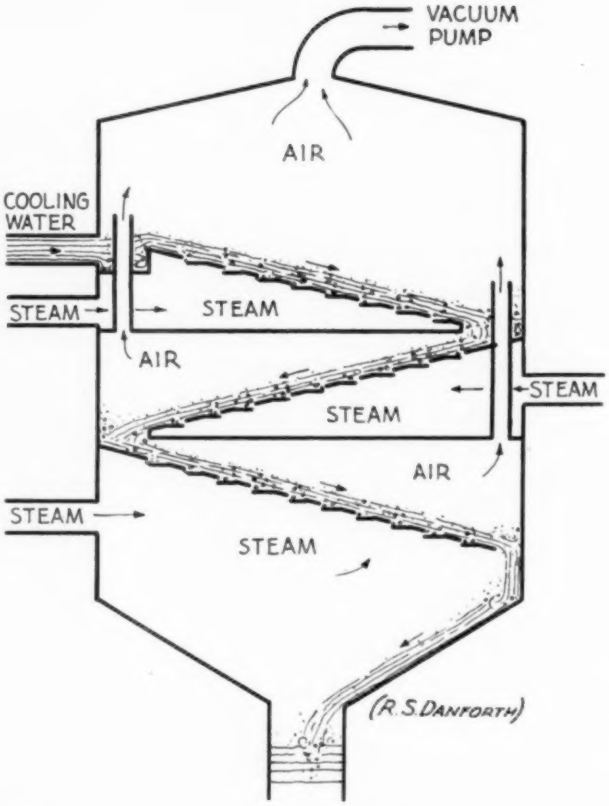

Fig. 4 Suggested Condenser Design

Considering simple distillation only, without reference to fractionating towers, in one dry vacuum plant the air leakage and non-condensable gases were less than 22 cu ft of free gas per minute for 3000 bbl per day oil distilled. To handle this volume of gas at 5 mm absolute pressure, it would require, with one type of vacuum pump, approximately 62 operating horsepower and, at $1\frac{1}{2}$ cents per kw-hr, about \$6075 per year. To secure the same boiling point by the use of live process steam in pipe stills at 50 mm pressure (28 in. vacuum), it would require a process-steam supply costing some \$60,000 or \$70,000 per year, with steam at 35 cents per 1000 lb, without considering the cost of cooling water required for its condensation or the operation of the vacuum apparatus. If low-cost steam is available, such as the exhaust from steam pumps, this cost will be greatly reduced or possibly nearly eliminated.

If no process steam is used, a high dry vacuum can be efficiently maintained, as only the non-condensable gases and air leakage have to be handled. If any process steam is used, the obtainable vacuum is limited by the cooling-water temperature in the steam condenser and is usually around 28 in. With cooling water around 60 F, $28\frac{1}{2}$ in. vacuum can be secured. With refrigerated cooling water, an absolute pressure of 20 mm could be secured.

A higher distillation vacuum can be secured with process steam by using a booster ejector[3] between the oil condenser and the steam condenser, but this booster has to handle the process steam as vapor under high vacuum besides the gases and air leakage. The steam consumption is therefore considerable, and cooling water is required for its condensation.

Where steam cost is important, it is therefore desirable to eliminate all process steam, and this could easily be done if fractionating towers were not usually required for clean separation of cuts.

Fractionating towers are usually used in commercial practice, and owing to the pressure drop through the bubble plates, it is impossible to secure the full advantages of dry high-vacuum distillation. Process steam is therefore customarily used, carrying as high a vacuum as is permitted by the condenser cooling-water temperature. Various types of fractionating towers, designed for a low vapor-pressure drop, have been used, but for various reasons they have not been extensively adopted.

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[3] Kramer patent No. 1,619,348.

Injection steam in side-stream stripping sections was formerly supposed to have a mysterious action in removing the light cracked products. The effect of steam vapor at any point is to lower the partial pressure and boiling point of the oil, but it apparently does not affect otherwise the composition of the vapor. When the boiling temperature is lowered, the ratio of the vapor pressures is increased, increasing the percentage of light fractions in the vapor.

If process steam was not needed in the fractionating tower, the lowering of the boiling point required to vaporize in the strippers could be secured by the use of steam and a booster ejector to a separate condenser or by carrying a higher vacuum on the strippers. The volume of fixed gases to be handled would be small.

A Possible Fractionator Design

As an additional possibility as yet untested, the author mentions a fractionator in which the plates are inclined and fitted with vapor slots, as shown in Fig. 3. The liquid on each plate is circulated by pumps, so that it flows over the slots between guiding partitions. The hydrostatic head slightly assists the flow of vapor instead of opposing it, and the pressure losses caused by passing through the usual bubble caps should be greatly reduced.

The tendency to bypass should be reduced as each bubble of vapor is introduced into a fresh surface of the liquid. The liquid is thoroughly agitated, keeping its concentration uniform. The difficulty found in maintaining horizontal the plates of the usual bubble towers would not be present. Entrainment should be materially reduced, owing to the vapor leaving the liquid while traveling down the plates, and therefore having a considerable horizontal velocity. A serious objection is the installation of a pump for each plate operating under vacuum at a high temperature and the consequent mechanical difficulties.

Suggested Condenser Design

The foregoing principle might also be applied in the design of barometric condensers, securing a quick intermixture of cooling water and vapor, eliminating bypassing, and reducing the pressure drop through the apparatus. Fig. 4 indicates one arrangement, circulating pumps for each plate not being required.

PME-54-7

Analysis of Stresses in Sucker-Rod Joints

By EMORY KEMLER,[1] PITTSBURGH, PA.

A recent analysis of oil-field pumping production showed that 25 per cent of the lost production was caused by sucker-rod troubles. Much research has been attempted to determine the loads carried by the rods, but heretofore no attention has been given to the stress condition existing in the sucker-rod joint. These joint failures amount to 72 per cent of sucker-rod failures. As the sucker-rod joint is the largest individual trouble maker in pumping operations, the purpose of the paper is to determine the magnitude and distribution of load in the joint and to discuss methods of studying the problem.

IN a recent analysis of oil-field pumping troubles it was found that 25 per cent of the lost production was caused by sucker-rod troubles.[2] This fact has resulted in many investigations being undertaken to determine the loads carried by the rods. Much research has been done along this line, but no one seems to have given attention to the stress condition existing in the sucker-rod joint. These joint failures, if one includes failures at the fillet connecting the rod and pin or box, amount to 72 per cent of the sucker-rod failures. The sucker-rod joint is, then, the largest individual trouble maker encountered in pumping operations. It is the purpose of this paper to determine the magnitude and distribution of load in the joint and to discuss methods of studying this important problem.

Since the only variations in sucker rods made by the different manufacturers are in the method of manufacture and in the manner by which the pin and box are connected to the rod, the standard A.P.I. joint is taken as the basis for comparisons. As this is the most-used size, it has been taken as the standard in all calculations.

DISCUSSION OF PROBLEM

Types of Joint Failures

An analysis of field data on operating wells has shown that the sucker-rod failures are distributed according to the following percentages:

Unscrewed joints	10.8
Pin breaks	25.3
Box breaks	12.2
Body breaks	28.3
Heat-treatment failures	23.4

The unscrewed-joint failures need no explanation. They result from the action of the vibration of the rod in causing the joint to loosen and then gradually to become unscrewed. This type of failure can be solved either by increasing the make-up torque, which results in a higher initial load applied to the joint, or by using some patented thread form or locking device to prevent the joint from loosening.

The pin breaks take place at the root of the last vanishing thread. A typical example of a pin failure is shown in Fig. 1. The failure is due to fatigue, and it starts in the form of a crack at the root of the thread, which spreads until it results in complete fracture. This fatigue failure is the result of the high stress concentration at the root of the last vanishing thread. As will be shown later, this thread transmits a large part of the load, and the stress concentration is therefore high.

The size of the sucker-rod joint is limited by the size of tubing used, and any increase in the size of the joint is therefore impossible. The obvious remedy, then, for reducing pin breaks is to reduce the stress concentration. One possibility is to change the design of the pin threads in such a way as to reduce the stress concentration at their roots. Another possibility would be to modify the box in such a way as to improve the load distribution along the pin. Also, a reduction of the load carried by the joint could be accomplished by using no initial tightening, and this would result in a considerable strengthening of the pin. Such a method would require some special locking device to prevent the joint from coming unscrewed.

A typical box failure is shown in Fig. 2. The box failures usually take place after the box has become worn due to rubbing against the tubing. The reduction of area due to wear results in increased unit stress, which, with a high stress-concentration factor, results in failure. These box breaks, since they are caused by excessive wear, can be reduced only by some type of protector or guide. Some changes in proportions might result in a small decrease in failures.

The body breaks take place in the uniform section of the rod, as shown in Fig. 3. They are usually caused by fatigue and are accelerated by the presence of corrosion pits. The high stress concentration at the bottom of these pits results in a fatigue crack starting below the ordinary fatigue load. The body breaks are beyond control so far as design affecting them is concerned. These failures can be reduced only by decreasing the load or increasing the corrosion endurance resistance of the steel used in fabrication or by a protective coating.

The heat-treatment breaks are so designated because they were first supposed to be a result of the method of manufacture. The pins and boxes are formed by upsetting the ends of the rods.[3] After being upset, the ends are normalized in order to remove all upsetting strains and to cause a complete recrystallization of the metal into a fine-grained condition. If the ends are not properly normalized, the material will be weakened and there will be present some initial strains due to upsetting. There is also some stress concentration present at the fillets, because the rigidity of the joint causes any bending to be thrown into the fillet connecting the rod and joint and because of the change of

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[2] Blaine B. Wescott, "Manufacture of Sucker Rods," *Petroleum Engineer*, April, 1931.

Contributed by the Petroleum Division and presented at the Annual Meeting, New York, N. Y., November 30 to December 4, 1931, of The American Society of Mechanical Engineers. (Part of the material for this paper is from a thesis presented in partial requirements for the degree of Master of Science at the University of Pittsburgh.)

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

section. A typical example of this type of failure is shown in Fig. 4.

The problem of reducing heat-treatment breaks is one of metallurgy as well as mechanics. The methods of manufacture and of heat treating are properly metallurgical problems. The part belonging to the field of mechanics is the determination of the stress distribution and concentration in the fillet connecting the box or pin and rod.

A very complete study of sucker-rod failures has been made recently by Dr. Blaine B. Wescott and Mr. C. G. zurHorst.[3]

Variation in Actual Joints

In a recent investigation it was found that not one of a series of stock-joint studies conformed to the A.P.I. specifications.[4] It was shown, however, that with proper care and the use of better tools the manufacturers could meet these specifications without having undue hardships imposed on them. Since the variations in joints have a decided bearing on the value of any theoretical calculations, they will be discussed here.

In the aforementioned investigation it was shown that a lead error of 0.001 in. per length of thread consumes 0.0017 in. in the pitch-diameter tolerance or that a total error of 0.0017 in. on a sucker-rod box or pin consumes the entire pitch-diameter tolerance. This is equivalent to 0.00028 in. per thread, since there are six full threads on the standard A.P.I. pin. This is on the basis of all the tolerance on the pitch diameter being taken up by the error in pitch. Since a large part of the tolerance on the pitch diameter must be allowed for variation in machining, only a part can be taken up by error in pitch. It could safely be said that not more than one-third of the maximum possible error in pitch could be allowed. This would amount to 0.00009 in. per thread. Table 1 gives averages of measured pitches in actual sucker-rod pins and boxes.

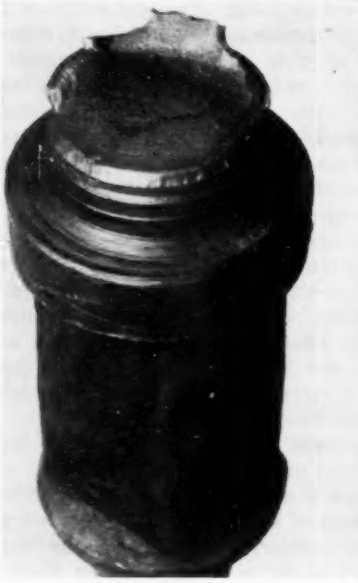

Fig. 1 Typical Pin Break

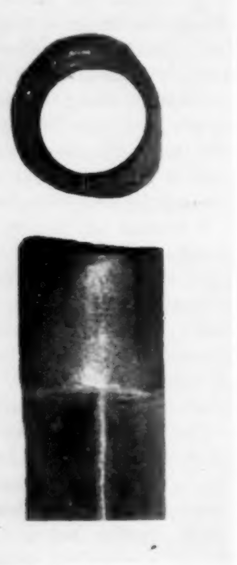

Fig. 2 Typical Box Failure

TABLE 1 AVERAGE LEADS IN $^3/_4$-IN. SUCKER-ROD BOXES AND PINS

Manufacturer	Lead per thread of pin	Error	Lead per thread of box	Error
A	0.10037	0.00037	0.10028	0.00028
B	0.09968	0.00032	0.10042	0.00042
C	0.10101	0.00101	0.10025	0.00025
D	0.10030	0.00030	0.09987	0.00013
E	0.10037	0.0037	0.09999	0.00001
F	0.10142	0.00142	0.10017	0.00017
G	0.10004	0.00004	....	....

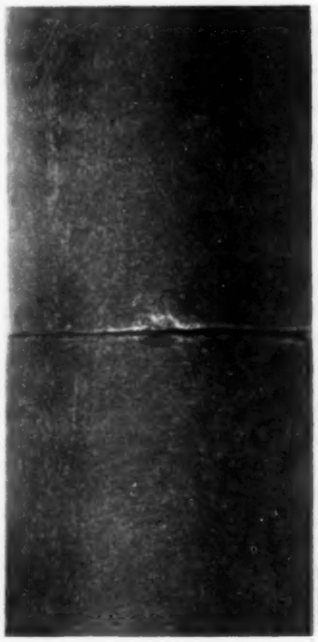

Fig. 3 Typical Body Failure

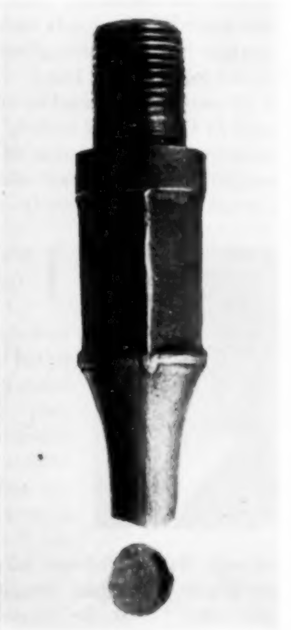

Fig. 4 Heat-Treatment Failure

From Table 1 it can be seen that the error in lead in all except a few cases is enough to consume all the tolerance in pitch diameter. The investigation carried out on these joints showed that not one of the pins or boxes approached the A.P.I. specifications. These results, therefore, cannot be used in comparing the variations in threads with the actual deflections to be obtained when the threads are loaded. Since it has been demonstrated that it is possible to make pins and boxes that will meet A.P.I. specifications without imposing undue hardships on the manufacturers, the comparison will be made on the basis of an allowable variation in pitch of 0.00009 in. per thread.

If the load were concentrated at the end of the thread, the deflection would be 0.0001 in. per 1000 lb load. It will be shown later that the first thread may carry as much as 40 per cent of the load. If the load were 20,000 lb, which would be an average value, as shown by the tensile tests on joints, the deflection of the thread would be 0.0008 in. The deflection of the first few threads is then many times greater than the maximum error in machining. Since the threads toward the end of the pin carry only a small amount of the load, the deflections caused by the load may be smaller than the errors in machining. We would therefore expect to find that the actual load distribution would vary but little from the theoretical where the maximum load occurs, but may vary considerably toward the end of the pin where there is little load transferred.

REVIEW OF WORK ON STRESS CONCENTRATION IN FILLETS, GROOVES, AND THREADS

Fillets

The stress concentration in fillets subjected to direct tension, compression, or bending has been investigated by Timoshenko and Dietz[5] for relatively small fillets. The results of this in-

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[3] *Petroleum Engineer*, November, 1931.

[4] C. N. Bowers and Blaine B. Wescott, "A.P.I. Sucker-Rod Specifications," *Petroleum Engineer*, May, 1931.

[5] Timoshenko and Dietz, "Stress Concentration Produced by Holes and Fillets," Trans. A.S.M.E., vol. 47, 1925, p. 199.

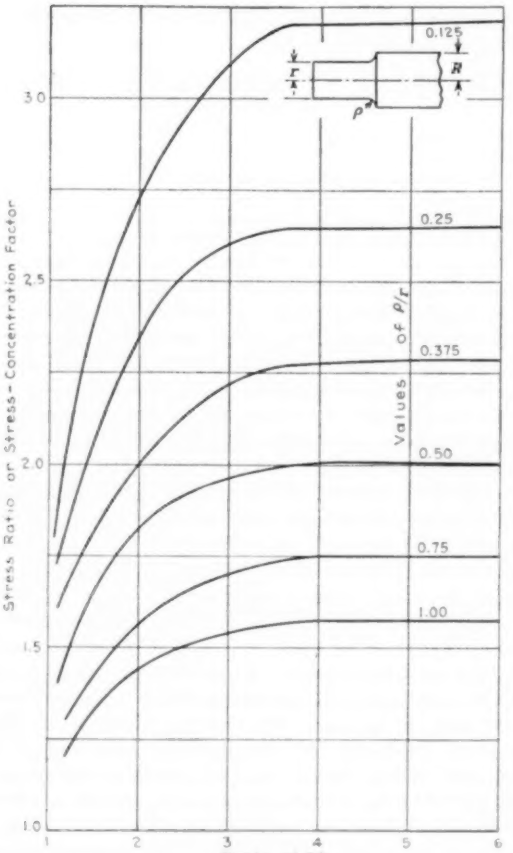

FIG. 5 STRESS CONCENTRATION IN FILLETS

vestigation are shown in Fig. 5. They show that the stress concentration may be very high in some cases. The stress concentration in cases where the radius is larger has been investigated by Moris.[6] The results of his test are given in Table 2. The proportions of the specimens are shown in Fig. 6. The catenaries were made with $BC/AB = 2$ and 4.

TABLE 2 MAXIMUM TENSION WHEN THE AVERAGE LOAD IN THE MINIMUM SECTION IS 1000 LB

Fillet radius	—Maximum tension for various values of W— 1.0	0.8	0.65	0.575	0.525
$^1/_2$ in.	1350	1210	1085	1028	1000
1 in.	1120	1120	1085	1015	1000
$1^1/_2$ in.	1060	1000	1000	1000	1000
2 in.	1000	1000	1000	1000	1000
2/1 catenary	1350	1350	1350	1350	1350
4/1 catenary	1060	1060	1060	1060	1060

There has been a great deal of work done on the stress concentration in holes and cracks.[7] This work has shown that the stress concentration in cracks or grooves may be very high. The amount depends, to a very great extent, on the shape of the crack. We would therefore expect that, if the sucker rods were corroded or pitted, the stress concentration in such pits would be high. If these corrosion pits happen to be at the fillet connecting the pin or box to the rod, the concentration would be much more severe than if they were in the body of the rod. Furthermore, the concentration due to a change of section occurs at a point only a little removed from the body of the rod, so the area of the section is very nearly the same as the body of the rod. We would therefore expect a larger number of the breaks to occur at this section. This is actually the case in sucker rods.

An increase of the fillet radius should result in a decrease in the failures at this connection between the pin or box and the body of the rod. For instance, the large diameter of the box at the connection is $1^1/_2$ in. and the diameter of the rod $^3/_4$ in., making $(R/r) = 2$. If the radius of the fillet is $^3/_8$ in., $(\rho/r) = 1$. From Fig. 5, it is seen that the ratio of the maximum stress to the actual stress is 1.45, or the stress at the section is 45 per cent greater than the average value or is 45 per cent greater than the stress in the rods. If a 1-in. radius had been used, the maximum stress would be 1.12 times the average stress, and if a 2-in. radius is used, there is no appreciable increase of stress caused by concentration. (See Table 2.)

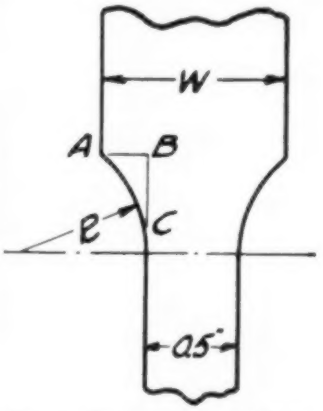

FIG. 6 PROPORTIONS OF PHOTO-ELASTIC SPECIMENS

The stress concentration in the case of torsion has been investigated by Jacobson.[8] The results of his investigation are shown in Fig. 7. This question of stress concentration in torsion caused by a change of section will be of use when the torsional stress caused by make-up is considered.

GROOVES AND THREADS

Moore[9] has investigated

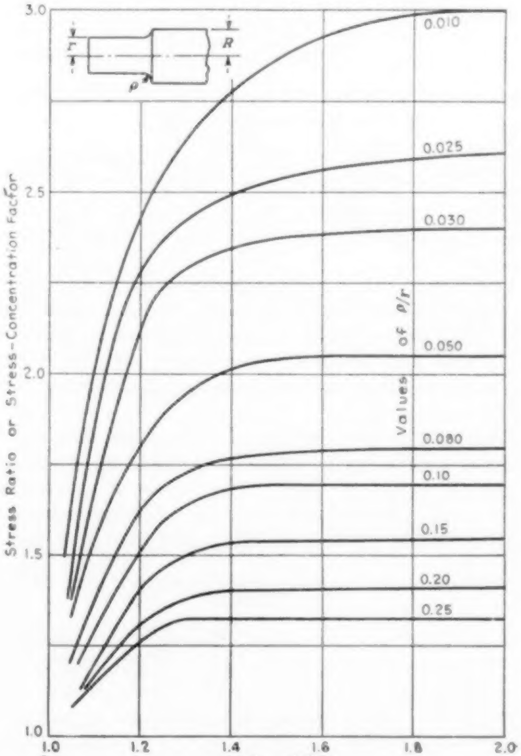

FIG. 7 TORSIONAL-STRESS CONCENTRATION

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[6] H. B. Moris, "Photoelastic Investigation of Tensile-Test Specimens," Journal of Optical Society of America, vol. 15, 1927, pp. 207–237.

[7] E. G. Coker, "Photoelasticity for Engineers," *General Electric Review*, vol. 23, no. 12, pp. 966–973.

[8] L. S. Jacobson, "Torsional Stress Concentrations, Etc.," Trans. A.S.M.E., vol. 47, 1925, p. 619.

[9] R. R. Moore, "Effect of Grooves on Fatigue of Metals," A.S.T.M., vol. 26, part 2, 1926.

the relative effect of a radial groove and a continuous thread. His results on the endurance tests are shown in Fig. 8. The results of tests are given in Table 3.

The endurance limit of the grooved specimen shows a very large decrease in strength, while the threaded specimen of the same groove size shows a much higher endurance limit. The increased strength of the continuous threaded section under flexural stress might be due to the strengthening effect of the helical thread and partly for the reason that the minimum section is greater than the root diameter of the thread because of the helix.

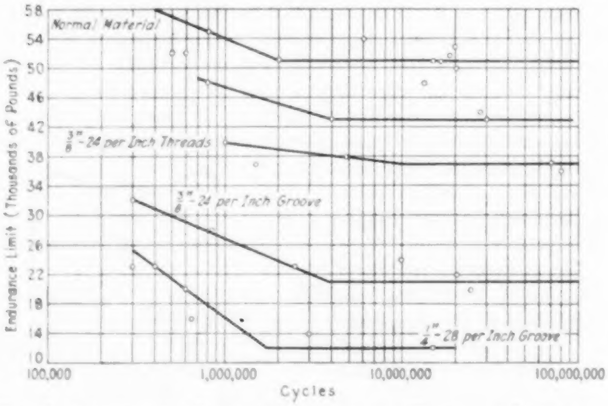

FIG. 8 ENDURANCE TESTS ON GROOVED AND THREADED SPECIMENS

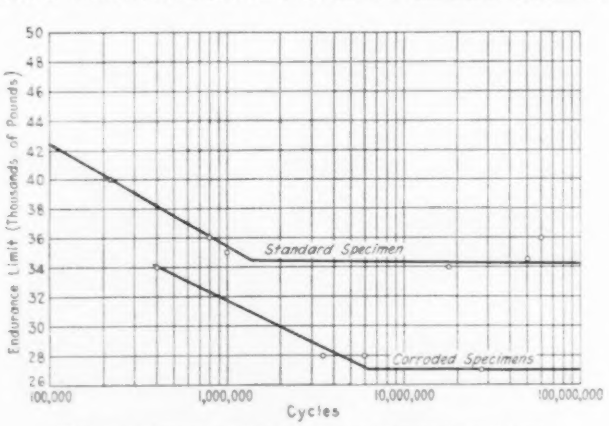

FIG. 9 EFFECT OF CORROSION ON ENDURANCE LIMIT

Mr. Moore also gives results of a specimen that had been subjected to a salt spray for 40 days. The average pitting was of the order of 0.0015 in. deep. The results of the endurance tests are given in Table 4 and Fig. 9, and show that such a small amount of pitting reduced the endurance limit 21.7 per cent. While his experiments were not extensive, they show that pitting seriously weakens the member.

TABLE 3 SUMMARY OF TESTS ON THREADED AND GROOVED SPECIMENS

	Plain	1/4 in., 28 threads per inch		3/8 in., 24 threads per inch	
		Notch	Thread	Notch	Thread
Root diameter, in.......	.....	0.203	0.203	0.321	0.321
Outside diameter, in.......	.....	0.250	0.250	0.375	0.375
Proportional limit, lb per sq in.....................	58,850	84,850	61,760	83,450	74,200
Tensile strength, lb per sq in.....................	140,600	166,000	140,600	165,000	156,300
Elongation, per cent.......	4.0	0	2.5	0	3.25
Reduction of area, per cent..................	14.9	.....	.....	.....	.....
Endurance limit, lb per sq in.....................	51,000	12,000	43,000	21,000	37,000
Ratio of tensile strength notch to tensile strength.	.....	1.181	1.000	1.172	1.112
Ratio of endurance limit notch to endurance limit.	.....	0.235	0.844	0.412	0.726

TABLE 4 RESULTS OF TESTS SHOWING EFFECT OF CORROSION

Tensile strength, lb per sq in..........................	76,000
Endurance limit, lb per sq in..........................	34,500
Ratio of endurance limit to tensile strength..............	0.454
Tensile strength after corrosion, lb per sq in..............	75,800
Endurance limit after corrosion, lb per sq in..............	27,000
Reduction in endurance limit due to corrosion, per cent....	21.7

Fig. 10 shows the results of Peterson[10] on stress-concentration factors due to U.S.S. threads by means of the plaster-of-paris specimens. These results show that the average stress concentration is nearly constant for all sizes of U.S.S. threads.

The results obtained by these investigators cannot be applied directly to our type of problem because they do not involve loading of the thread itself. When the joint is loaded, the threads transfer the load from the pin to the box, and they necessarily carry quite a load in bending, which greatly increases the stress at the root of the thread.

METHODS OF INVESTIGATION

USE OF THE EXTENSOMETER

The extensometer has been used to find the actual load that exists in a sucker-rod joint. When a string of sucker rods is placed in a well, each joint is tightened by a wrench, which may be from 18 to 60 in. long. With such wrenches, a considerable torque can be applied, and consequently quite a high initial tension will be put in the joint. In order to determine the amount of this load, it is necessary to measure the strain set up in the box, and from this calculate the value of the load. The small space available on the box for attaching an extensometer to determine this initial tension makes it necessary to use special attachments. On account of the small size of a joint, it is impossible to apply this method to finding the stress distribution in the threaded portion of the joint.

A Huggenberger tensometer with a magnification of 1010 to 1 was used in some tests made to determine the initial load in the joint caused by tightening. This instrument has a standard

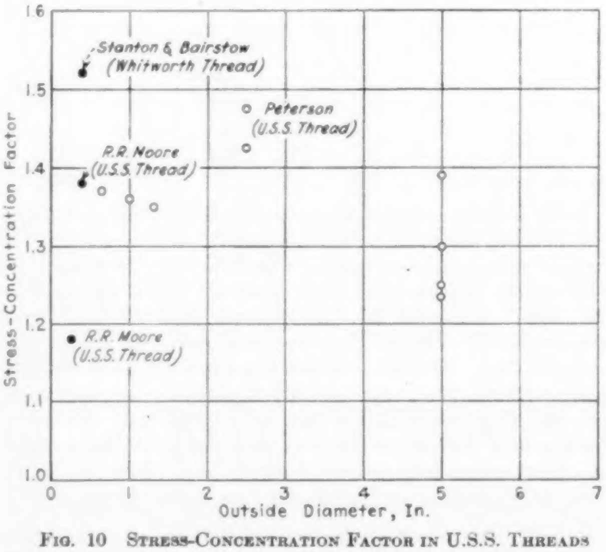

FIG. 10 STRESS-CONCENTRATION FACTOR IN U.S.S. THREADS

gage length of 1 in. In the tests on joints it was necessary to have a gage length of less than $^1/_2$ in., and so a special attachment was designed for this instrument, so that any gage length between $^3/_8$ in. and 1 in. could be obtained. The instrument with the special attachment is shown in Fig. 11 on a box that has been cut away to show the relative position of the extensometer and the threads.

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[10] R. E. Peterson, "An Investigation of Stress Concentration by Means of Plaster-of-Paris Specimens," *Mechanical Engineer*, vol. 48, no. 12, December, 1926, pp. 1449–1452.

BRITTLE-MATERIAL METHOD

The brittle-material method is very simple in principle. Assume two specimens *a* and *b*, as shown in Fig. 12, to have the same net cross-sectional area. These two specimens are then tested to failure, and the breaking load of *a* is P_1 and of *b* is P_2. The ultimate strength of the material would be $S_a = (P_1/A)$, where A is the net area of the specimens. If there were no stress concentration, P_2 would equal P_1. The stress-concentration factor for a member having variation of cross-section is the ratio of the actual stress in the member with the irregularity to the stress as computed from a straight member. If we let k be the stress-concentration factor, then $k = (P_2/P_1)$.

It has been found upon comparing the results of photoelastic, plaster-of-paris specimen tests (brittle-material method), and actual fatigue tests on specimens with fillets that in all cases the photoelastic tests gave much higher stress concentration than the actual fatigue results and that the brittle-material method was much nearer the actual results, except in the case of chrome-nickel steels, which seem to be sensitive to stress concentration. A summary of this comparison was made by Peterson[10] and is shown in Fig. 13.

For a brittle material that has a very nearly straight-line stress-strain diagram, the same relation will hold between load and maximum stress as in a member of elastic material, provided the stress does not exceed the proportional limit of the material. In a brittle material, the stress concentrations, due to imperfections in the material, are greater than in ductile materials and play a large part in determining the strength of the specimen. It is therefore necessary to run duplicate tests and to use a statistical average.

This brittle-material method is based on the maximum-stress theory. This theory says that failure will occur when, and only when, one of the principal stresses reaches the value corresponding to the limiting stress in tension or compression, as the case may be. A discussion of this theory can be found in most advanced textbooks on strength of materials or applied elasticity. The theory has been shown to be true for such a brittle material as cast iron. In such a material the fracture always occurs at right angles to the axis of stress.

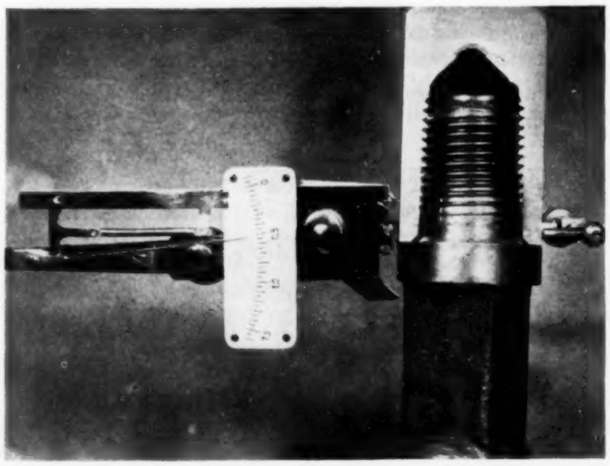

FIG. 11 HUGGENBERGER TENSOMETER, WITH SPECIAL ATTACHMENT

Because of the importance of stress concentration in connection with fatigue strength of a ductile material, such a method is very valuable. It has the further advantage of being a method that can be applied to three-dimensional stress problems.

PHOTOELASTIC METHOD

This method of stress analysis is limited to cases where the deformation is parallel to a plane. These are called two-dimensional stress problems. There are many three-dimensional problems that, because of radial symmetry, can be reduced to a two-dimensional problem for the study of certain features. An attempt is being made to use this method in the study of the sucker-rod problem. The problem of machining the sample is a very difficult one, and as soon as the machining methods are developed the analysis can be made.

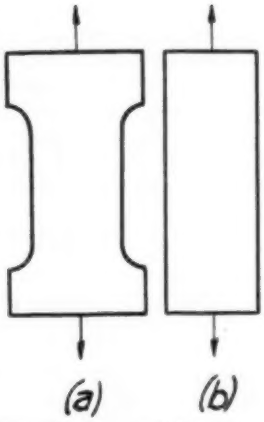

FIG. 12 BRITTLE-MATERIAL METHOD

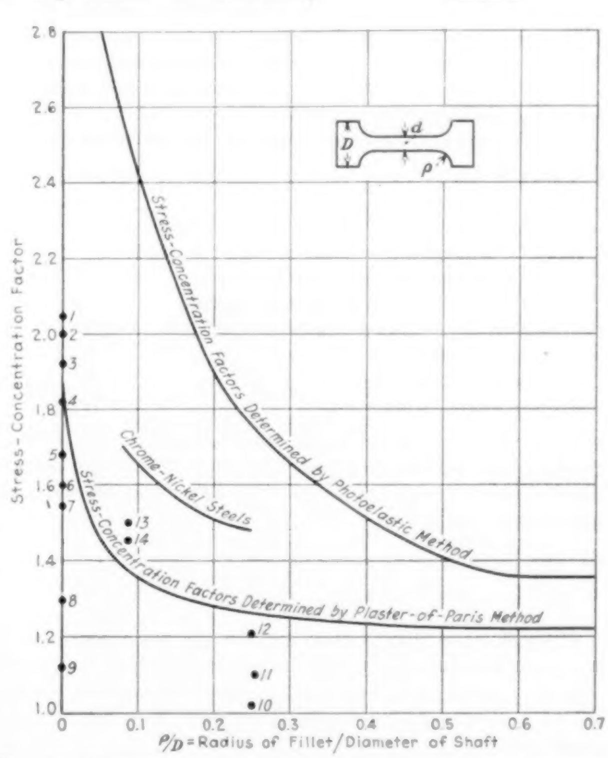

FIG. 13 COMPARISON OF RESULTS OF PHOTOELASTIC AND BRITTLE-MATERIAL METHODS

(Legend: 1, Moore and Kommers, heat-treated steels; 2, Stanton and Bairstow, hard steels; 3, Moore and Kommers, ductile steels; 4, Stanton and Bairstow, mild steels; 5, Eden, Ross, and Cunningham; 6, Wohler, axle steel; 7, Stanton and Bairstow, mild steel; 8 and 9, Wohler, wrought iron; 10, Moore and Jasper, 0.37 C steel; 11, Timoshenko and Lessells, 0.33 C steel; 12, Moore and Jasper, average; 13 and 14, Timoshenko and Lessells, carbon steel.)

THEORETICAL

This method permits a determination of the relation between the initial, applied, and total load in a joint and of the stress distribution in a joint. The method cannot be expected to give results that will represent every joint, because of the variation in the manufacture of joints. It will give results that should correspond to an average joint. Since this is all that we can expect to study, it seems that this should be of interest in a study of the general problem. In making such an analysis it is necessary to assume, as usually is the case in a solution of such problems, that the load is uniformly distributed across a section and that the material is not stressed beyond the proportionate limit.

LOADS CARRIED BY JOINT

TENSILE

In the following analysis it will be assumed that the loads will be applied by non-flexible members. This assumption causes little error in cases where the bolt or pin and compression members are relatively long, as is generally the case.

In a sucker-rod joint this is not the case. The part of the joint that takes the initial compression is short, and the elasticity of the threads will result in the actual loads as carried by the joint being less than the theoretical amount for a rigid case. It will be of interest to go through the theoretical case to determine the maximum load the joint may have to take and to determine the form of the equations.

In the derivation of these equations the following notation will be used:

A_p = area of pin
A_b = area of box
P_o = initial load in pin and box caused by make-up
F = applied load or load in rods
P = load in box when F has been applied
Q = load in pin after F is applied = $F + P$
Δ_p = unit stretch in pin
Δ_b = unit stretch in box
E = modulus of elasticity

When torque is applied to the joint in making it up, the box is compressed and the pin is put in tension. This initial tightening of the joint then causes the initial load P_o. The unit deformations caused by the load P_o are:

$$\Delta_p = \frac{P_o}{A_pE} \quad \text{and} \quad \Delta_b = \frac{P_o}{A_bE}$$

When the load F is applied, some of the compression in the box is relieved and the load in the box is reduced to some lower value P. If the compressive load in the box is relieved, the box will increase in unit length a small amount, which can be represented by $d\,\Delta_b$,

$$d\,\Delta_b = \frac{P_o}{A_bE} - \frac{P}{A_bE}$$

If the box increases in length, the pin also must increase in length. The load in the pin will be $(P + F)$, and if the change in unit length of the pin be represented by $d\,\Delta_p$,

$$d\,\Delta_p = \frac{P+F}{A_pE} - \frac{P_o}{A_pE}$$

Since these two deflections must be the same unless the joint actually opens, one can write that

$$d\,\Delta_p = d\,\Delta_b$$

or

$$\frac{F+P}{A_pE} - \frac{P_o}{A_pE} = \frac{P_o}{A_bE} - \frac{P}{A_bE}$$

$$P\left(\frac{1}{A_p} + \frac{1}{A_b}\right) = P_o\left(\frac{1}{A_b} + \frac{1}{A_p}\right) - \frac{F}{A_p}$$

$$P = P_o - \frac{F}{A_p\left(\frac{1}{A_p} + \frac{1}{A_b}\right)} = P_o - \frac{A_bF}{(A_b + A_b)}$$

$$P + F = P_o + F - \frac{FA_b}{A_b + A_p} = P_o + F\left(1 - \frac{A_b}{A_b + A_p}\right)$$

$$Q = F + P = P_o + F\left(1 - \frac{A_b}{A_p + A_b}\right)$$

For a $^3/_4$-in. standard A.P.I. joint the root area of the box and the area of the pin in the region considered are equal, so $Q = P_o + (F/2)$. The load in the joint at any time before the joint opens is then theoretically equal to the initial load plus one-half of the applied load. When the joint opens, $Q = F$, so the load to open the joint is given by $F = P_o + (F/2)$ or $(F/2) = P_o$ and $F = 2P_o$, or the load to open a joint is then twice the initial load.

RELATION BETWEEN TORQUE AND INITIAL LOAD

When a joint is made up by applying a torque to the wrench square, an initial tension is put in the pin and a compression in the box. They result in the load P_o being applied to the shoulder and the same load on the threads, as is shown in Fig. 14.

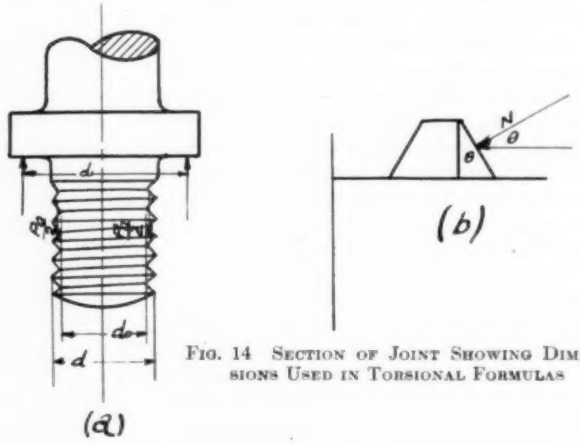

FIG. 14 SECTION OF JOINT SHOWING DIMENSIONS USED IN TORSIONAL FORMULAS

Let T = torque applied at wrench square
μ = coefficient of friction (assume the coefficient to be the same at the shoulder and thread).

The normal pressure N on the thread can be found from the relation

$$N = \frac{P_o}{\cos\theta}$$

For θ = 30 deg

$$N = 1.15\,P_o$$

$$T = \mu \times 1.15\,P_o \times \frac{d_o}{2} - \mu \times P_o \times \frac{d_1}{2}$$

$$= \mu P_o\left(\frac{1.15\,d_o}{2} - \frac{d_1}{2}\right)$$

For a standard $^3/_4$-in. rod d_o = 1 in.
d_1 = 1.28 in.

Then $T = 1.215\mu P_o$

$$P_o = \frac{T}{1.215\mu}$$

For a 3000 in-lb initial torque it was found that the average initial load was 19,500 lb. This would mean that the value of μ was 0.125 for the joints tested.

Magnitudes of Torsional Stress

When the joint is made up by applying a torque, part of the torque is taken up by friction at the shoulder and the rest by friction in the threads. The frictional torque of the threads must be transmitted by the pin, and this will then set up a torsional stress in the pin.

Let T_T = torque in the threads

Then $T_T = \mu \times 1.15\, P_o \times \frac{d_o}{2}$

If S_s = shearing stress due to torque in threads

$$S_s = \frac{T_T \times 16}{\pi d^3}$$

This is on the basis of no stress concentration. Because of the very sharp radius at the root of the threads, this stress concentration will be high. Fig. 7 shows the value of the stress concentration for different proportions. The upper curve has been extrapolated from other curves and is probably conservative.

In order to get an idea of the magnitude of this torsional stress, we can take the approximate dimensions of a threaded joint and find

$$(\rho/r) = 0.0105 \text{ approx.}$$
$$(R/r) = 1.13.$$

From Fig. 6, it can be seen that the stress-concentration factor is approximately 2.2.

As an example, take $T = 4500$ in-lb and $\mu = 0.15$.

$$P_o = \frac{4500}{1.215 \times 0.15} = 24{,}600 \text{ lb}$$

$$T_T = 0.15 \times 1.15 \times 24{,}600 \times {}^1/_2 = 2120 \text{ in-lb}$$

$$S_s = 12{,}800 \text{ lb per sq in.}$$

If the stress concentration factor is 2.2, the actual torsional stress at the root of the threads will be 28,200 lb per sq in.

The torsional stress in the joint would be of the order of magnitude of that calculated when the joint is first made up. It is probable that when some of the initial load in the box is relieved as the load is applied and the joint is subjected to vibration, a large part of this stress is relieved. The amount would depend upon the initial torque, initial tension, and the magnitude of the applied load.

Experimental

Experimental work is now in progress at the Gulf Research Laboratory on a study of the loads carried by joints under different make-up conditions. It has been found that the load carried by a joint can be represented by

$$Q = P_o + {}^1/_4 F$$

That is, the load in the joint is the initial load plus ¹/₄ of the load in the rods. The theoretical formula, on the basis of neglecting the flexibility of the threads, gave $Q = P_o + {}^1/_2 F$. This shows that the flexibility has quite an effect on such a short joint.

Some preliminary tests showed the initial load in a joint under ordinary make-up conditions to be in excess of 20,000 lb. This means that on wells 4000 ft or under and operating at ordinary speeds the initial load would be greater than the rod load and the load carried by the pin may be as much as 50 per cent greater than the load in the rods. It can be seen then that any type of joint that would eliminate the initial tightening would result in a big reduction in load carried by the joint.

LOAD DISTRIBUTION IN A THREADED JOINT

A Joint to Give Uniform Load Distribution

This example shows one method by which it is possible to get theoretical load distribution in an ordinary joint by using a special box.

Let a = effective length of nut
E = modulus of elasticity of material
P = load carried by joint
A_b = area of box
A_p = area of pin
L_b = load in box
L_p = load in pin
d_b = diameter of box
d_p = diameter of pin.

By definition of the problem the load in the pin at any section x is to be $L_p = P - \frac{Px}{a}$. The load in the box will then be $L_b = \frac{Px}{a}$. The unit elongation of the pin will be $\Delta_p = \frac{P}{EA_p}\frac{x}{(1 - a)}$ and the unit elongation of the box will be $\Delta_b = \frac{Px}{aA_bE}$.

Since they are the same, we can write:

$$\frac{P}{EA_p}\left(1 - \frac{x}{a}\right) = \frac{Px}{aA_pE} \ldots\ldots\ldots\ldots [1]$$

$$A_b = A_p \frac{x}{(a - x)} \ldots\ldots\ldots\ldots [2]$$

or

$$\frac{\pi(d_b^2 - d_p^2)}{4} = \frac{\pi d_p^2}{4}\frac{x}{(a - x)} \ldots\ldots\ldots [3]$$

$$d_b^2 = d_p^2 \frac{a}{(a - x)}$$

$$d_b = d_p \sqrt{\frac{a}{a - x}} \ldots\ldots\ldots\ldots [4]$$

The approximate shape of the box is shown in Fig. 15. A slight modification is necessary in this type of nut. When x approaches a in value, d_b approaches an infinite value, and when x approaches zero, $d_b \longrightarrow d_p$. In practice the box must have a small area at $x = 0$ and a finite area at $x = 1$.

In this arrangement no account need be taken of the elasticity of the threads, because the box deforms in such a manner as to cause each thread to take the same load, and since each thread will then have the same deflection, there will be no change in the distribution.

Theoretical Distribution in a Standard A.P.I. Joint

The theoretical distribution of the load in a standard A.P.I. pin will be given by the expression:

$$P = P_o \left(\frac{e^{Bx} - e^{B(2l - x)}}{l - e^{2Bl}}\right)\text{(footnote 11)}$$

where l is the working length of the thread, x the distance along the pin, and P_o the initial load. B is a constant involving the elasticity of the thread and is given by

$$B^2 = \frac{1}{\alpha E}\left(\frac{1}{A_b} + \frac{1}{A_p}\right)$$

α is the spring constant of the thread and is the displacement in inches between two originally coinciding sections of the pin and box caused by a unit force transmitted per unit length of the pin.

The foregoing equation is derived on the following basis:

If P is the load in the pin at any section x, the load in the box will be $P_o - P$. Let q be the load transmitted from the pin to the box per unit length along the pin, then $\frac{dP}{dx} = -q$.

If $d\Delta_p$ is the elongation of the pin in a length dx, then $d\Delta_p = \frac{Pdx}{A_pE}$ and $d\Delta_b = \frac{(P_o - P)dx}{A_bE}$.

If $d\Delta$ is the difference in elongation of the pin and box,

$$d\Delta = d\Delta_p - d\Delta_b = \frac{Pdx}{A_pE} - \frac{(P_o - P)dx}{A_bE} \ldots\ldots [5]$$

This difference must be taken up in the threads.

Then

$$\Delta = -\alpha q \ldots\ldots [6]$$

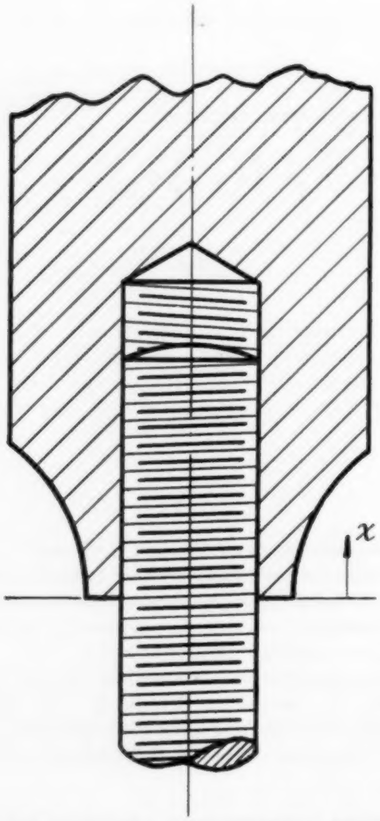

Fig. 15 Joint to Give Uniform-Load Distribution

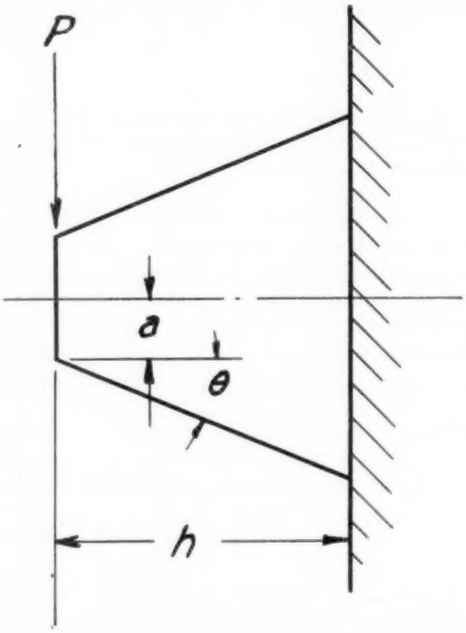

Fig. 16 Section Through Standard A.P.I. Thread

If this equation is differentiated with respect to x, we have

$$\frac{d\Delta}{dx} = -\alpha\frac{dq}{dx} = \alpha\frac{d^2P}{dx^2} \ldots\ldots [7]$$

since $q = -\frac{dP}{dx}$

From Equation [5]

$$\frac{d\Delta}{dx} = \frac{P}{A_pE} - \frac{(P_o - P)}{A_bE} \ldots\ldots [8]$$

Equating [7] and [8], we have

$$\frac{d^2P}{dx^2} = \frac{P}{\alpha E}\left(\frac{1}{A_p} + \frac{1}{A_b}\right) - \frac{P_o}{A_bE}$$

This equation when solved will give the load distribution quoted.

Deflection in Screw Threads

The section through a thread might be represented as in Fig. 16. A thread is then considered as a short cantilever beam. In such a short thread the deflection due to shear becomes of the same order of magnitude as that caused by bending, so it must be considered.

Let P = load per unit length along thread
h = height of thread
θ = $^1/_2$ angle of thread
a = $^1/_2$ width of thread at top
A = area of unit length of thread at any section x.

$$A = 2(a + x \tan\theta) \ldots\ldots [9]$$

$$S_s = \frac{P}{A} = \frac{P}{2(a + x\tan\theta)} \ldots\ldots [10]$$

If y_s represents the deflection caused by shear, we can write:

$$dy_s = \frac{S_s}{E_s}dx \ldots\ldots [11]$$

$$= \frac{P}{2E_s}\cdot\frac{dx}{(a + x\tan\theta)}$$

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[11] J. P. Den Hartog, "The Mechanics of Plate Rotors for Turbo-Generators," Trans. A.S.M.E., vol. 51, 1929, paper APM-51-1.

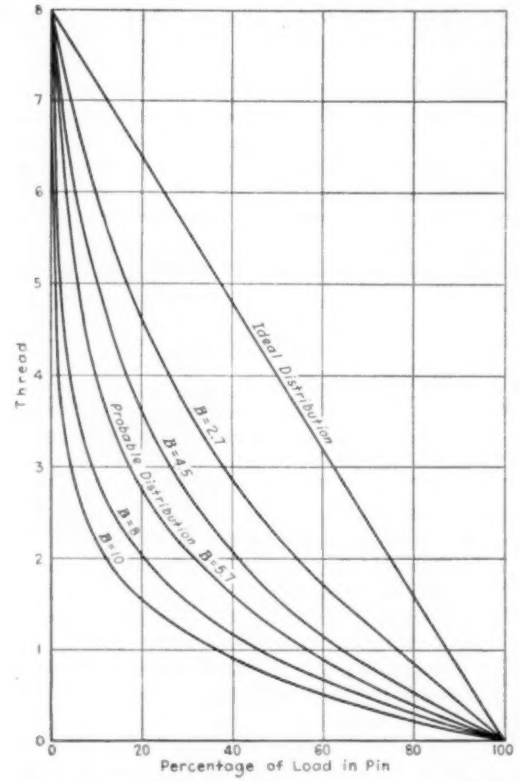

FIG. 17 LOAD DISTRIBUTION IN PIN AT ANY THREAD

$$y_s = \int \frac{P}{2E_s} \frac{dx}{(a + x \tan\theta)} = \frac{P}{2E_s \tan\theta} \log(a + x\tan\theta) + C. \quad [12]$$

For $x = h, y_s = 0$.

$$C = -\frac{P}{2E_s \tan\theta} \log_e (a + h \tan\theta) \ldots\ldots\ [13]$$

$$y_s = \frac{P}{2E_s \tan\theta} \log_e \frac{(a + x\tan\theta)}{(a + h\tan\theta)} \ldots\ldots\ldots\ [14]$$

If the deflection caused by bending is desired at several points, it would be best to determine the equation of the elastic curve from the relation

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

where

$$I = \frac{1}{12} \cdot l \cdot (a + x \tan)^3 \times 8 = \frac{2}{3} (a + x\tan\theta)^3$$

so that

$$\frac{d^2y}{dx^2} = \frac{-px}{E \cdot {}^2/_3(a + x\tan\theta)^3} \ldots\ldots\ldots\ldots [15]$$

Since the maximum deflection at the end is all that is desired, it is simpler to use the strain-energy method, because there are no constants to be evaluated in accordance with boundary conditions.

If U = energy in system due to bending

$$U = \int_0^h \frac{M^2 dx}{2EI} = \int_0^h \frac{(Px)^2 dx}{2E \cdot {}^2/_3(a + x\tan\theta)^3} \ldots [16]$$

$$= \int_0^h \frac{P^2x^2dx}{{}^4/_3E\,(a + x\tan\theta)^3}$$

If Δ_p represents the deflection due to bending at the point where P is applied, and in the direction of P, we have

$$\Delta_p = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \int_0^h \frac{P^2x^2dx}{{}^4/_3E(a + x\tan\theta)^3}$$

$$= {}^3/_2 \frac{P}{E} \int_0^h \frac{x^2dx}{(a + x\tan\theta)^3}$$

$$= {}^3/_2 \frac{P}{E \tan^3\theta} \left[\frac{a}{(a + h\tan\theta)^2} + \frac{1}{2} + \frac{2a}{(a + h\tan\theta)} - 2 + \log \frac{(a + h\tan\theta)}{a} \right]$$

$$= \frac{3P}{2\tan^3\theta} \left[\frac{a}{(a + h\tan\theta)} \left(2 - \frac{a}{2(a + h\tan\theta)} \right) - \frac{3}{2} + \log_e \left(\frac{a + h\tan\theta}{a} \right) \right]$$

If $\Delta = y_s + \Delta_p$ = total deflection due to P, then

$$\Delta = \frac{P}{2E_s} \log \left(\frac{a + h\tan\theta}{a} \right) + \frac{3P}{2\tan^3\theta} \left[\frac{a}{(a + h\tan\theta)} \left(2 - \frac{a}{2(a + h\tan\theta)} \right) - \frac{3}{2} + \log \frac{(a + h\tan\theta)}{a} \right]$$

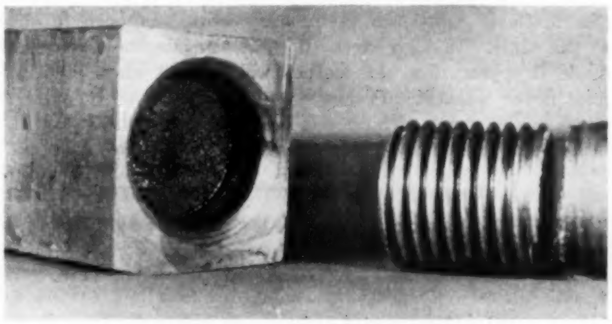

FIG. 18 FAILURE OF CAST-IRON JOINT

For a standard $^3/_4$-in. joint

$$a = 0.0057$$
$$E = 30 \times 10^6$$
$$E_s = 12 \times 10^6$$
$$\theta = 30 \text{ deg}$$
$$\tan\theta = 0.577$$
$$a + h\tan\theta = 0.0403$$

$$\Delta = 1.4 \times 10^{-7}P + 1.91 \times 10^{-7}P = 3.31 \times 10^{-7}P$$

The deflection caused by a load of 1000 lb is $\Delta_{1000} = 3.31 \times 10^{-7} \times 1000 = 3.31 \times 10^{-4} = 0.000331$ in.

The foregoing deflection is per inch of length. If we take the deflection for a complete thread, using the pitch diameter as the length,

$$\Delta_{1000} = \frac{0.000331}{\pi \times 1} = 0.0001055 \text{ in.}$$

DISCUSSION OF THE DISTRIBUTION EQUATION

Values for B have been calculated for various types of loading of the thread. For the load at the end of the thread $B = 2.70$, at $^1/_4$ the distance from the root $B = 4.5$, and at the center $B = 5.7$. In all these cases it is to be remembered that the deflection is the sum of the deflection of the pin thread and the deflection of the box thread. If, for instance, the box-thread deflection was not taken into account, the value of B for the load at the center would be $B = 8$. The most probable value for B would be for the case when the load comes at the middle of the thread. Fig. 17 shows the curves for the various values of B. These curves have been drawn to show the percentage of load in the pin at any point. It is seen that the increasing of the flexibility results in a very much better load distribution.

One other factor that affects the load distribution is the strength of the thread. If a load larger than the elastic limit of the material is to be applied to a thread, the thread will yield before this load has been reached and more load will be carried by succeeding threads. When ductile materials are used, this factor tends to cause a better distribution of load. In cases where the joint is subjected to fatigue loads, the joint is very weak if loads high enough to cause yielding are present, because fatigue failure will be rapid.

Distribution as Determined by Brittle-Material Method

The result of a test on a joint made of cast iron is shown in Fig. 18. It is seen that the joints in each case failed without any yielding and in the same identical place that fatigue breaks start. Only three tests were made, and all the joints broke in the same place. The difficulty with the use of this method was in obtaining good samples of cast iron and in getting a tensile-test specimen for comparison in which the stress-concentration factor would be known. Further work is being planned using this method. Only the preliminary work has been finished at this time. In view of the few experiments that have been made, it appears that this method will show at which point the joint is the weakest. It does not, however, show the state of stress in the region of failure as does the photoelastic test, and it may not give results as to the actual strength of the section.

Methods of Getting Better Distribution

Some of the methods that have been proposed previously in addition to the modified box to give uniform distribution are to make the threads with a variable pitch, to make the box with a parabolic surface on the inside,[11] and to increase the fillet at the root of the threads and thereby decrease the stress concentration.

The modified box has the disadvantage that it cannot be used where initial tightening must be used. The variable-pitch thread and box with a parabolic surface on the inside require such accurate machining that they are not practical. The increasing of the fillet at the root of the threads means shorter threads and consequently much higher load concentration at the first threads, as is shown in Fig. 16, because it will increase the value of B.

CONCLUSIONS

The results of this analysis show:

1 Any improvement in the endurance strength of the present joint can probably be brought about only by bettering the load distribution or by using a material with a much higher endurance limit.

2 A modification of the box by having a varying area would give a more uniform distribution of load along the pin.

3 An increase in the flexibility of the threads would give much better load distribution.

4 An increase in the radius at the root of the thread would greatly reduce the stress concentration at that point.

5 The initial load put in a joint in present practice results in the joint being under a much higher stress than if only the applied load was present.

6 The torsional stress set up in a joint by the initial tightening might be of importance in considering the strength of the joint.

7 The fillet connecting the rod and box or pin should be made with a radius of at least 2 in.

Acknowledgments

The author is indebted to Dr. Blaine B. Wescott, of the Gulf Research Laboratory, at whose suggestion this work was started, and to Prof. John A. Dent, of the University of Pittsburgh, for their help and suggestions in making this investigation.

PME-54-8

Properties of Hydrocarbon Mixtures at High Pressures

By W. K. LEWIS[1] and C. D. LUKE,[2] CAMBRIDGE, MASS.

W. K. Lewis

The paper provides a comparatively simple method for the design of apparatus handling complex hydrocarbon mixtures at extraordinary pressures. The computation methods outlined enable one to calculate the density of any vapor, mixed or otherwise, at any temperature and pressure; the heat effect of any heating or cooling operation, whether conducted at constant pressure or changing pressure; the equilibrium between vapor and liquid at any temperature and pressure; the equilibria and changes in concentration throughout the tower in any operation of rectification or absorption, at whatever pressures they may be conducted; and the isothermal reversible work of compression of vapor mixtures, whatever the pressure range. These computation methods enable the refining engineer to design high-pressure equipment and operations with a high degree of precision.

C. D. Luke

A—PRESSURE-VOLUME-TEMPERATURE RELATION OF HYDROCARBONS

This laboratory[3] recently published an analysis of the data available on the behavior of the vapors of pure hydrocarbons at pressures up to and somewhat above the critical pressures.[4] This study indicated that one could treat all hydrocarbons containing more than 3 carbon atoms per molecule as conforming to the graphical equivalent of a single reduced equation of state. In other words, when the pressure on any given hydrocarbon is expressed as the reduced pressure, i.e., the absolute pressure on the system divided by the critical pressure of the material $P_R = P/P_c$, the temperature on the material as the reduced temperature $T_R = T/T_c$, and the volume as the actual volume occupied by the substance under these conditions of temperature and pressure divided by the volume of an equal amount of the substance in the critical state $V_R = V/V_c$, V_R is a unique function of P_r and T_r, the same for all higher hydrocarbons. It is perhaps more convenient to express the same result by saying that the behavior of any such hydrocarbon can be represented by the equation $PV = \mu RT$, where μ can be looked upon as a correction factor to the gas-law equation $PV = RT$, μ being a unique function of P_R and T_R. These relationships are shown in Figs. 1, 2, and 3, which may be considered as a graphical equation of state. In all figures μ is plotted against P_R for various values of T_R. The first plot, Fig. 1, is restricted to critical temperatures and below, while the second and third plots, Figs. 2 and 3, are for critical temperatures and above. The second goes up to a value of $P_R = 6$, while the third is continued to $P_R = 12$.[5] By the use of these plots one can calculate at any pressure and temperature, within the range represented, the vapor density of any pure hydrocarbon for which the critical conditions are known. For straight-chain paraffins, the data indicate that at the critical point the value of the ratio MP_c/T_c is equal to 5.0, where the pressure is given in atmospheres absolute, the temperature in degrees Centigrade absolute, and M represents the molecular weight of the hydrocarbon in question. For aromatic compounds without side chains, the corresponding value is apparently about 6.9. For saturated-ring compounds without side chains, a value of 6.0 should be employed. It is probably allowable to use for ordinary petroleum hydrocarbons a figure of from 5.5 to 6.0. Where the

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[3] Research Laboratory of Applied Chemistry.

[4] Cope, Lewis, and Weber, *Ind. Eng. Chem.*, 23, 887 (1931).

Contributed by the Sub-committee on Petroleum Refining of the Petroleum Division at a meeting, New York, N. Y., April 8, 1932, of the Metropolitan Section, The American Society of Mechanical Engineers.

[During the summers of 1930 and 1931 there were conducted at the Massachusetts Institute of Technology two conferences of five weeks each on the engineering phases of oil refining. The conferences were attended only by practicing engineers from the staffs of 24 companies in the industry, representing organizations not only in the United States, but in Canada, the Dutch West Indies, and the Argentine. In these conferences one of the major discussion groups dealt with those operations involving interaction of gases, vapors, and liquids, including evaporation, condensation, rectification, absorption, and the like. In the 1931 conference this particular discussion group was under the leadership of Dr. W. K. Lewis, Professor of Chemical Engineering at the Massachusetts Institute of Technology, formerly head of the Department of Chemical Engineering, and Mr. C. D. Luke, Research Assistant and Graduate Student in that institution. This paper is in no small degree an outgrowth of the conferences and work of that summer school.]

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] The article referred to[4] did not extend these curves to high values of P_R. The plots here given are constructed at higher values from Amagat's data on ethylene,[6] but experimental confirmation of the points is available from data on methane,[7,8] normal pentane,[9] isopentane,[10] and benzene.[11] Inasmuch as none of the data for the higher hydrocarbons covers the desired ranges of temperature and pressure, the extensive ethylene data were used in the construction of Figs. 1, 2, 3, and 4, which satisfactorily represent all available data on the *P-V-T* relations of hydrocarbons with the possible exception of methane.

[6] Intl. Crit. Tables, vol. III, p. 14.

[7] Keyes and Burks, *Jl. Am. Chem. Soc.*, 49, 1403 (1927).

[8] Kvalnes and Gaddy, *Jl. Am. Chem. Soc.*, 53, 394 (1931).

[9] Rose-Innes and Young, *Phil. Mag.*, 47, 353 (1899).

[10] Young, Proc. Phys. Soc. (London), 13, 602 (1894–1895).

[11] Luke, S. M. Thesis, M.I.T. Library, 1931.

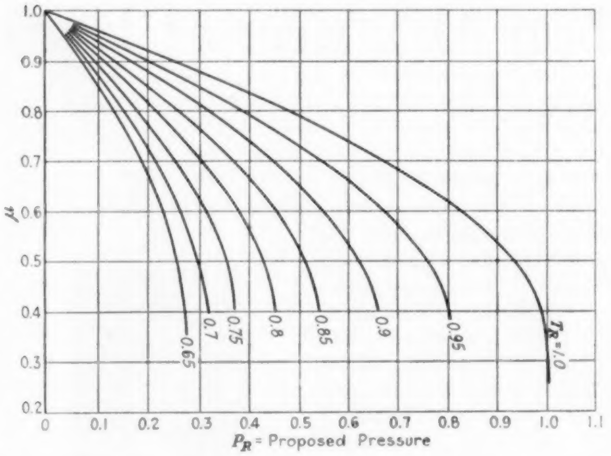

FIG. 1 RELATIONS OF HYDROCARBONS BELOW THE CRITICAL TEMPERATURE

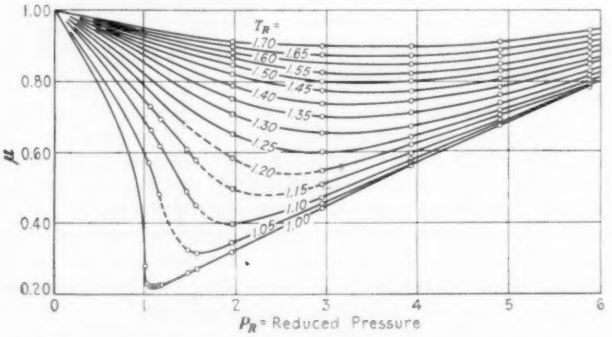

FIG. 2 *P-V-T* RELATIONS FOR HYDROCARBONS ABOVE THE CRITICAL TEMPERATURE (LOW-PRESSURE RANGE)

stock is of the type ordinarily referred to as highly paraffinic, one would use the lower value. For other stocks containing lower ratios of hydrogen to carbon, the higher figure is more probable. Granting that the value of this ratio to be employed for a given hydrocarbon can be estimated, if one knows a single point on its vapor-pressure curve, such as its boiling point, one can extrapolate its vapor-pressure curve by any of the well-accepted methods up to the point at which MP/T becomes equal to the value of this ratio. The corresponding pressure and temperature are the critical values for the compound in question. This places one in a position to estimate the vapor density of any hydrocarbon for which one knows the molecular weight together with a single point on its vapor-pressure curve.

To show the technique of employing these curves, assume that one wishes to know the density of benzene vapor at a temperature of 763 F and a pressure of 2335 lb per sq in. abs. The critical temperature of benzene is 551 F, and its critical pressure is 704.5 lb per sq in. abs. Therefore, the reduced temperature at the condition in question is $T_R = \dfrac{763 + 460}{551 + 460} = 1.209$, and the reduced pressure $P_R = 2335/704.5 = 3.315$. From Fig. 2 one reads the corresponding value of $\mu = 0.575$. If one wishes to express the result in lb per cu ft, one will use pressure expressed as lb per sq ft, volume as cu ft, temperatures as degrees Rankine (degrees Fahrenheit absolute), and quantities in pound-mols. In these units $R = 1543$. Since, however, one is interested in the volume of 1 lb, one is dealing with $1/M$ pound-mol; i.e., the volume in cu ft per lb is $\mu RT/MP = 0.04138$. In other words, the density of benzene vapor under these conditions is $1/0.04138 = 24.17$ lb per cu ft. One of the authors has measured the density of this material under these conditions and found an experimental value of 24.90 lb per cu ft, with which the calculated values agree within 3 per cent. The density calculated from the perfect gas law, $PV = RT/M$, is found to be 13.89 lb per cu ft, which is 44.25 per cent in error, compared with the observed value of 24.90. In the article referred to[4] the *P-V-T* relationship just discussed was employed to calculate the enthalpy of a hydrocarbon vapor. However, the method there developed breaks down at high pressure. It has been found that at high pressures and temperatures above the critical, the data of Figs. 2 and 3, replotted as isometrics, i.e., P_R plotted against T_R at constant values of V_R, fall on substantially straight lines. These lines are shown in Fig. 4. This plot may be used for the determination of volume relationships at high pressures and is sometimes more convenient than the μ curves of Figs. 1, 2, and 3. However, the main reason for the construction of Fig. 4 is to make possible the development of a method for

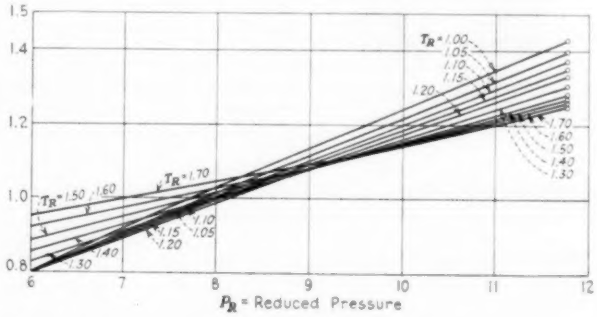

FIG. 3 *P-V-T* RELATIONS FOR HYDROCARBONS (HIGH-PRESSURE RANGE)

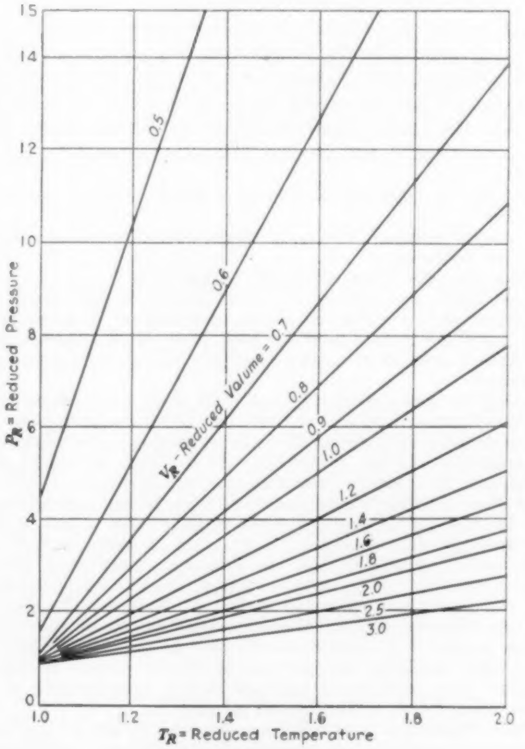

FIG. 4 REDUCED ISOMETRICS

calculating the enthalpy of hydrocarbon vapors in these high-pressure ranges. This is discussed below.

B—THERMAL PROPERTIES OF HYDROCARBON VAPORS

So long as any substance or mixture of substances exists in the form of a vapor at a pressure low enough so that it follows the gas laws, its internal energy and enthalpy are both independent of its pressure and volume and dependent upon the temperature only. This rule is the one usually used in the computation of the enthalpy of hydrocarbons, both pure and mixed, in the problems of refinery design. However, when the pressure is sufficiently high to produce deviations from the gas laws, the internal energy and total heat vary with the pressure at constant temperature. Since the coefficient μ is the measure of the deviation from the gas laws, it is clear that in any range in which μ begins to deviate appreciably from unity, it becomes necessary to apply corrections for the influence of pressure on the thermal properties. The article referred to[4] gave the following expressions for the pressure corrections of the internal energy and enthalpy:

$$E_1 - E_0 = -(S-1)FRT(1-\mu) \qquad [1]$$

$$H_1 - H_0 = -(S-1)FRT(1-\mu) - RT(1-\mu) \qquad [1a]$$

where E_0 and H_0 are the internal energy and enthalpy, respectively, at a pressure sufficiently low so that the vapor does not deviate appreciably from the gas law $PV = RT$, and E_1 and H_1 are the corresponding values at the high pressure and at the same temperature. In the foregoing expressions, S can be evaluated from Table 1. The quantity F is a complex function of the reduced temperature T_R and of a criterion $z = \dfrac{RT}{P_cV} = \dfrac{P_R}{\mu}$, and may be evaluated from Fig. 5. It should be noted that for values of z less than 2.5, F is unity. μ may be obtained from Figs. 1 or 2. T is the absolute temperature in degrees Kelvin or Rankine. R is equal to 1.985, both in calories per degree Centigrade and in Btu per degree Fahrenheit. Consistent units must be employed throughout the equation.

TABLE 1

T_R	S	T_R	S
1.50	4.36	1.05	3.82
1.45	4.36	1.00	3.71
1.40	4.36	0.95	3.66
1.35	4.36	0.90	3.50
1.30	4.36	0.85	3.16
1.25	4.36	0.80	2.74
1.20	4.36	0.75	2.74
1.15	4.34	0.70	2.74
1.10	4.07	0.65	2.74

However, Equations [1] and [1a] break down at values of P_R in the range to the right of the dotted line of Fig. 2. On the other hand, the *P-V-T* relations in this high range of P_R are expressed by the straight isometrics of Fig. 4. At these high values of P_R, the pressure corrections for the internal energy and enthalpy are given by the expression:[12]

$$E_2 - E' = -3.23T_c\left(\frac{1}{V_{R_2}} - \frac{1}{V_R'}\right) \qquad [2]$$

$$H_2 - H' = -3.23T_c\left(\frac{1}{V_{R_2}} - \frac{1}{V_R'}\right) - RT(\mu' - \mu_2) \qquad [2a]$$

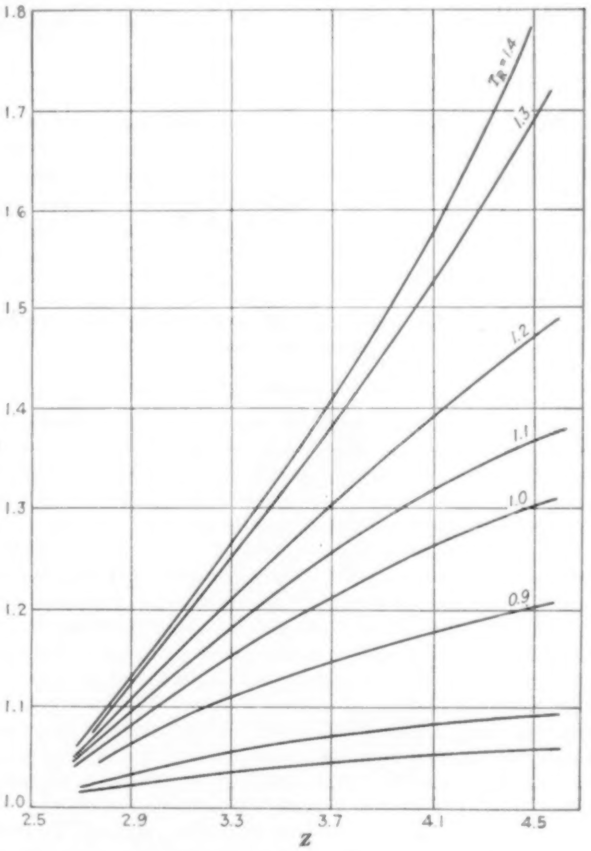

FIG. 5 PLOT OF F AS A FUNCTION OF Z AND T_R

In Equations [2] and [2a] E', H', V_R', and μ' are the internal energy, enthalpy, reduced volume, and gas-law deviation factor, respectively, at the value P_R given by the dotted line of Fig. 2, and E_2, H_2, V_{R_2}, and μ_2 are the corresponding values at the higher pressure.

Unfortunately, the isometrics of Fig. 4 do not remain straight to low values of P_R, so that Equation [2] breaks down at the low pressures, whereas exactly the reverse is true of Equation [1]. The problem of calculation can be solved by using each expression in its proper range.

Thus, assume that one wishes to obtain the enthalpy above an arbitrary reference point of benzene vapor at a temperature of 457 C and a pressure of 200 atm abs. The enthalpy of the benzene vapor at 457 C and 1 atm may be readily calculated by any of the accepted methods, most of which depend on low-temperature heat of vaporization data and low-pressure specific heat data for the vapor. From this point on, the equations developed may be used to compute the enthalpy at the high pressure.

The final conditions of the vapor correspond to $T_R = 730/561.6 = 1.30$ and $P_R = 200/47.9 = 4.18$. Inspection of Fig. 2 shows that one must use Equation [1a] up to a value of $P_R = 1.42$, and Equation [2a] from $P_R = 1.42$ to $P_R = 4.18$. Hence, by combining the two equations,

$$H_2 - H_0 = -3.23T_c\left(\frac{1}{V_{R_2}} - \frac{1}{V_R'}\right) - (S'-1)F'RT(1-\mu') - RT(1-\mu_2)$$

When $P_R' = 1.42$ and $T_R = 1.30$, $\mu' = 0.781$, $F' = 1.00$, and

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[12] Phillips, H. B., *Jour. Math. and Phys.*, Mass. Inst. of Tech. 1, 42 (1922).

$S' = 4.36$; $V_R' = \dfrac{3.78\ \mu' T_R}{P_R'} = 2.70$. When $P_{R2} = 4.18$ and $T_R = 1.30$, $\mu_2 = 0.690$ and $V_{R2} = \dfrac{3.78\ \mu_2 T_R}{P_{R2}} = 0.811$.

Substituting these values in the foregoing expression, $H_2 - H_0 = -3074$ cal per gram-mol $= -1.8(3074) = -5540$ Btu per lb-mol. Hence, if one were to calculate the enthalpy of benzene vapor at this temperature and pressure by assuming the gas laws, i.e., by assuming that pressure has no effect on the total heat at constant temperature, the result would be too high by 5540 Btu per lb-mol.

C—PROPERTIES OF VAPOR MIXTURES AT HIGH PRESSURES

1 Volume Relations of Mixtures

The properties of gas and vapor mixtures at low pressures are usually given in terms of Dalton's law, which states that each component in such a mixture exerts a partial pressure equal to the pressure that it would exert were it present alone in the same concentration, the total pressure being equal to the sum of the partial pressures of all the components. At high pressure this rule breaks down completely, and the concept of partial pressures loses its physical significance. However, an alternative way of expressing the behavior of gas and vapor mixtures at low pressures is to say that if such substances be mixed at constant pressure, the volumes are additive, and in the absence of chemical action, the heat effect of mixing is zero. The data indicate that this rule is substantially valid for the mixing of gases and vapors up to appreciable pressures. From the thermal point of view it is equivalent to saying that on mixing components at constant pressure the internal energies and enthalpies are additive. Since the curves and equations outlined in parts A and B put one in a position to calculate volumes and enthalpies of pure components, the rules just given make it possible to calculate the corresponding quantities for mixtures, however complex, when the compositions are known.

Assume that one has a mixture containing 56.8 per cent by weight of benzene and 43.2 per cent of toluene. These figures correspond to 60.8 mol per cent of the former and 39.2 mol per cent of the latter. This mixture is at a pressure of 25.4 atm and a temperature of 260 C.

The critical temperature of benzene is 288.5 C and of toluene 320.6 C; the critical pressures are 47.9 atm and 41.6 atm. The reduced temperature of the benzene is therefore $T_R = 533/561.6 = 0.949$, and of the toluene $T_R = 533/593.7 = 0.899$. The corresponding reduced pressures are $P_R = 25.4/47.9 = 0.531$ and $25.4/41.6 = 0.610$, respectively. At these values of P_R and T_R for benzene, one finds from Fig. 1 that $\mu = 0.708$ and for toluene, at $T_R = 0.899$ and $P_R = 0.610$, $\mu = 0.515$. Consider as a basis 100 grams of the mixture. This contains 56.8 grams of benzene, which is $56.8/78 = 0.728$ mol of benzene. It also contains 43.2 grams of toluene or $43.2/92 = 0.470$ mol. From the corrected gas-law equation $PV = \mu nRT$, under the pressure in question of 25.4 atm, the volume of the benzene in the mixture, in the form of pure benzene vapor, would be $V = \mu nRT/P$. Using the gas constant in cubic centimeter-atmospheres per degree Centigrade, $R = 82.07$, $V = (0.708)(0.728)(82.07)(533)/25.4 = 888$ cc. Similarly, the volume of the toluene in the mixture, measured at the same pressure and temperature, but in the pure state, is $V = 416$ cc. According to the foregoing rule, the volume of the mixture is the sum of these two individual volumes, i.e., 1304 cc. Since this is the volume of 100 grams of the mixture, the density is $100/1304 = 0.0766$ gram per cc. This is numerically equal to the specific gravity of the mixture. Since the density of water in English units is 62.3 lb per cu ft, the density of this mixture in English units is $0.0766(62.3) = 4.77$ lb per cu ft. This density might have been calculated directly by considering, not 100 grams of the original mixture, but 100 lb of that mixture, expressing the pressure in lb per sq ft, the temperature in degrees Rankine, and using the corresponding gas constant, $R = 1543$. The density of this particular mixture under these conditions has recently been measured in this laboratory and found experimentally to be 0.0785 gram per cc, corresponding to a volume of 1064 cc per mol. The difference between the experimentally determined and the calculated density is seen to be 2.42 per cent. However, had one calculated the density from the gas laws, one would have found a value of 0.0485 gram per cc, only 61.8 per cent of the actual value.

Had one used Dalton's law, one would have assumed the partial pressure of the benzene in the mixture to be its mol-fraction times the total pressure, i.e., $0.608\ (25.4) = 15.43$ atm. Similarly, the partial pressure of toluene would have been 9.97 atm. The value of μ for benzene at this partial pressure would be 0.835 and for toluene 0.856. The volume of 1 mol of the mixture calculated from the equation $v = \mu n\ RT/P$, using this new value of μ for benzene, is found to be 1437 cc. Similarly calculated for toluene, the figure is 1472 cc. It is seen that neither of the computed values checks the experimentally determined value, 1064 cc per mol.

2 Thermal Relations of Vapor Mixtures

As previously outlined, when the components of a vapor mixture are mixed at constant temperature and pressure, the heat of mixing is essentially zero and the internal energies are additive, i.e., the internal energy of the resultant mixture, at the same temperature and pressure, is the sum of the internal energies of all the components. Likewise, the enthalpies are additive.

Suppose one wishes to expand the benzene-toluene mixture of the preceding problem isothermally from the pressure there given down to a pressure sufficiently low so that the vapor obeys the gas laws. No work, mechanical or otherwise, will be done during the expansion. How much heat must be added during expansion per mol of the mixture?

Inasmuch as the operation is isothermal, only Equations [1a] and [2a] need be considered. From the preceding calculations, for benzene, $T_R = 0.949$, $P_R = 0.531$, and $\mu = 0.708$; for toluene, $T_R = 0.899$, $P_R = 0.610$, and $\mu = 0.515$. Obviously, the range here considered is covered entirely by Equation [1a]:

$$\Delta H = -(S-1)FRT(1-\mu) - NRT(1-\mu) \ldots. [1a]$$

For benzene, S at $T_R = 0.949$ is equal to 3.66, and for toluene at $T_R = 0.899$, $S = 3.50$. For both benzene and toluene, $F = 1.00$ so that Equation [1a] reduces to

$$\Delta H = -SNRT(1-\mu) \ldots\ldots\ldots\ldots. [1b]$$

Substituting the foregoing values in Equation [1b], $\Delta H = 1130$ cal per gram-mol for benzene and 1795 for toluene. Hence for 1 mol of mixture, equivalent to 0.608 mol of benzene and 0.392 mol of toluene, $\Delta H = 0.608\ (1130) + 0.392(1795) = 1390$ cal per gram-mol. In engineering units, the result is $1.8(1390) = 2502$ Btu per lb-mol.

3 Vapor-Liquid Equilibria

It is well recognized that at low pressures Raoult's law affords an excellent method of estimating the volatility of the constituents in a liquid mixture of hydrocarbons. This law demands that the partial pressure p of a given constituent shall equal its mol fraction in the liquid times the vapor pressure of the constituent in question in the pure state at the same temperature, i.e., $p = Px$. It is usually combined with Dalton's law in the form $p = \pi y = Px$. Since Dalton's law breaks down at high pressures, it is to be expected that Raoult's law will likewise become invalid. The physical chemist has proposed modifying these solution laws at high pressures by substituting for the pressure a corrected pressure, called the fugacity.[13] The fugacity f of any pure gas or vapor at any pressure whatsoever is defined by the differential equation $(\partial \ln f/\partial p)_T = V/RT$. Granting knowledge of the P-V-T relationship, this integral can be determined graphically. This has been done from Figs. 1, 2, and 3, and the results are plotted in Figs. 6 and 7. Choosing as ordinates the dimensionless ratio f/p makes it possible to represent on these two diagrams the relationships for all hydrocarbons which follow Figs. 1, 2, and 3.

NOMENCLATURE

P = vapor pressure of a pure component
π = total pressure on the system
f_P = fugacity of a pure liquid component under its own vapor pressure at the temperature of the system
f_π = fugacity of a pure vapor component at the temperature and total pressure of the system
x = mol fraction of a component in a liquid
y = mol fraction of a component in a vapor.

The fugacity of any component in a liquid must of necessity be equal to the fugacity of the same component in the vapor in equilibrium with that liquid. Hence, one can calculate the fugacity of a pure liquid f_P by calculating the fugacity of its saturated vapor. Furthermore, the fugacity f of any component in a vapor mixture may be estimated by multiplying the mol fraction of that component in the mixture y times the fugacity of the pure component at the temperature and total pressure of the mixture, i.e., $f = f_\pi y$. Raoult's law now becomes,

$$f = f_\pi y = f_P x \qquad [3]$$

instead of the more familiar form,

$$p = y\pi = xP \qquad [4]$$

It will be noted that the only change is the substitution of the fugacity for each pressure term in Raoult's law.

That which makes this equation of vital interest to the refinery engineer is the fact that it enables one to estimate the equilibrium between vapor and liquid phases at high pressure. Furthermore, these equilibria may in special cases differ greatly from those calculated by Raoult's law, although in other cases the distinction between the results obtained from the two expressions is entirely negligible.

Suppose one has a straw oil which is contacted sufficiently intimately with a gas containing ethane at 32.2 C, so that the oil comes into equilibrium with it. This particular temperature is the critical temperature of ethane. Assume three cases: In the first, the gas is pure ethane at a pressure of 1 atm abs. In the second, the gas contains 2 mol per cent of ethane and 98 per cent of hydrogen, but the total pressure on the mixture is 50 atm. In the third case, the mol per cent of ethane is 0.5 and of hydrogen 99.5, but the total pressure on the mixture is 200 atm. What will be the mol fraction of ethane in the oil in equilibrium with it in the three cases?

Using the ordinary nomenclature, inspection of the data shows that in each of the three cases the partial pressure of the ethane, i.e., the mol fraction of the ethane in the vapor times the total pressure, is 1 atm. Consequently, Raoult's law in its ordinary form would require that the solubility of the ethane in the oil be the same in all three cases. Since the critical pressure of ethane is 48.8 atm, Raoult's law indicates that the solubility of the ethane in the oil, $x = \pi y/P = (1/48.8) = 0.0205$ in each of the three cases.

Using the corrected form of Raoult's law in which the pressures are replaced by the corresponding fugacities, one obtains an entirely different result. The fugacity of pure liquid ethane f_P at this temperature is calculated as follows: It is at its critical point, and consequently from Fig. 6 one sees that $f_P/P = 0.645$. Since its saturation pressure, i.e., the critical pressure, is 48.8 atm, its fugacity in the pure state is $f_P = 0.645\ (48.8) = 31.5$ atm. The fugacity f_π must be calculated at the total pressure on the system. In the first case, this is 1 atm. Hence, $\pi_R = 1/48.8 = 0.0205$. The corresponding value of f_π/π at $T_R = 1.00$ is 0.995, or $f_\pi = 0.995$ atm. Hence, $x = f_\pi y/f_P = [(1)\ (0.995)/31.5] = 0.0316$. It will be noticed that this mol fraction, i.e., the solubility of pure ethane gas at this temperature and 1 atm pressure, is approximately 50 per cent greater than that calculated by Raoult's law.

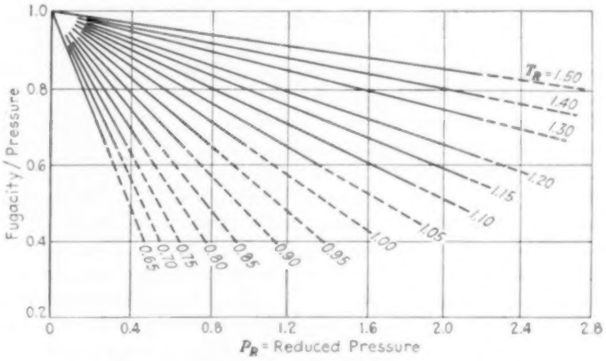

FIG. 6 FUGACITY PLOT (LOW-PRESSURE RANGE)

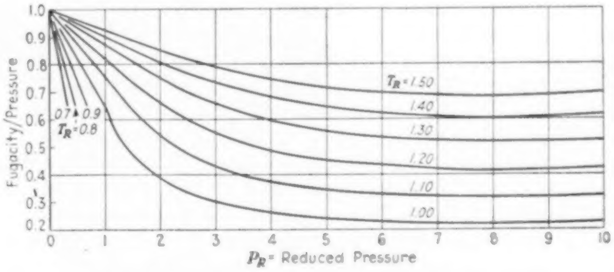

FIG. 7 FUGACITY PLOT (HIGH-PRESSURE RANGE)

Turning to the second case, the value of f_P remains unchanged, but f_π must be calculated at a total pressure of 50 atm, where $\pi_R = 50/48.8 = 1.025$. The corresponding value of f_π/π is 0.635, whence $f_\pi = 31.7$. Substituting this in the equation for x gives $x = 0.0201$. In this case the solubility is practically the same as that calculated by Raoult's law. It would have been absolutely identical had the chosen total pressure equaled the critical pressure. At the third condition mentioned, at $\pi = 200$ atm, $\pi_R = 200/48.8 = 4.10$ and, from Fig. 7, $f_\pi/\pi = 0.257$, or $f_\pi = 51.4$ atm. Hence, $x = 0.00815$. In other words, at this high pressure the solubility of ethane is roughly 40 per cent of that calculated by Raoult's law.

This illustration emphasizes the point that pressure makes a great difference in gas solubility, even at what one would normally consider identical partial pressures. Furthermore, it is well to note that at low pressures the solubility is abnormally high and at high pressures it is abnormally low. This is in a sense unfortunate, because the engineer resorts to high pressure when he wants to force a gas into a liquid and turns to low pressure when he wants to strip it out. In both cases, the direction of the deviations from the gas laws is such as to tend to defeat his purpose. Clearly, however, the magnitude of the variations is such that they can by no manner of means be ignored in engineering calculations in the refining industry.

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[13] Lewis and Randall, "Thermodynamics and the Free Energy of Chemical Substances," McGraw-Hill Book Co., New York, 1923, pp. 190–201, 221–227, 232–235.

Inspection of Figs. 6 and 7 will make it clear that corrections to Raoult's law are unnecessary so long as the total pressure on the gas π and the vapor pressures of the pure components at the temperature in question P are small, because under these conditions f/P is always nearly unity. These are the conditions under which Raoult's law has been tested most carefully in the past. Unfortunately, the amount of data available for testing deviations from it under high values of π and P are meager, but those available fully confirm the general dependability of the computation method here proposed.

Thus, suppose one wishes to calculate the vapor pressure of the liquid and the composition of the vapor in equilibrium with a liquid containing 53.0 mol per cent benzene and 47.0 per cent toluene at a temperature of 290 C. At this temperature, the vapor pressures of benzene and toluene are 49.2 and 28.4 atm, respectively. Inasmuch as 290 C is above the critical for benzene, its vapor pressure must be determined by extrapolating the vapor pressure curve to 290 C. The foregoing data correspond to $T_R = 1.001$ and $P_R = 1.028$ for benzene and for toluene, $T_R = 0.950$ and $P_R = 0.682$, whence the corresponding fugacities are $f_P = 31.5$ and 20.0 for benzene and toluene, respectively. In order to calculate the total pressure π, one must first assume a value of π, calculate f_π for each component, and then calculate the value of y for each constituent from the equation $y = f_P x / f_\pi$. The correct value of π is that which makes Σy, i.e., the summation of the calculated mol fractions in the vapor, equal unity. Following this procedure, one obtains $\pi = 39.4$ atm and mol fractions of benzene and toluene in the vapor equal to 0.596 and 0.404, respectively. The corresponding values observed in this laboratory were $\pi = 37.8$, $y = 0.608$ for benzene, and $y = 0.392$ for toluene. Thus, the calculated π and y for benzene differ from the observed by 4 per cent and 2 per cent, respectively. On the other hand, Raoult's law calculations give $\pi = 39.45$ atm, and $y = 0.661$ and 0.339 for benzene and toluene, respectively. These calculated values of π and y for benzene differ by 4 per cent and 9 per cent, respectively, from the observed values.

TABLE 2

Method	π	Deviation, per cent	y, benzene	Deviation, per cent
Fugacity..........	39.4	4	0.596	2
Raoult's law.......	39.45	4	0.661	9
Observed..........	37.8	..	0.608	..

One will see from Table 2 that the pressure calculated either by Raoult's law or by the fugacity method checks the observed pressure very satisfactorily. However, the vapor composition calculated by the fugacity method checks the observed value much more closely than that calculated by Raoult's law. The data available indicate that this is generally true, i.e., that Raoult's law is passable for calculating total pressures at moderate pressures, but unsatisfactory for calculating vapor composition. Furthermore, the error involved in using Raoult's law increases as one approaches the critical point and becomes excessive when the mol fraction of the component in question in the vapor phase is small.

Most unfortunately there are almost no data available giving experimental determinations of the fugacity of vapors at temperatures below the critical but pressures above it. Furthermore, no satisfactory method of calculation of the values to be expected in this range is known. This is the more serious because this is a range in which the modern petroleum engineer is becoming more and more vitally interested. This laboratory is making experimental determinations in this field and hopes to be able to publish data of value in the near future. However, it is the conviction of the authors that even now one can estimate the behavior of hydrocarbon mixtures in this region by extrapolating both the μ curves and f/P curves to high values of P_R, drawing them parallel to the curve corresponding to $T_R = 1$ and spacing them approximately proportionately between that curve and the horizontal axis. Certainly, this method of estimation will be far more dependable than any of the guesses hitherto employed.

In the design of equipment for both rectification and absorption, the engineer uses over and over again the slope of the equilibrium line y/x. Hitherto, he has employed P/π for this slope. Clearly, in view of the preceding discussion, the correct slope to employ is not P/π, but f_P/f_π. Furthermore, the engineer uses this slope at a given temperature and total pressure. Fortunately, under these same conditions the correct value of the slope f_P/f_π is a constant quantity. The method of calculating it has already been illustrated. In order to have this equilibrium relationship correct in all computations of design, at whatever pressure the operation may be carried out, the engineer has only to substitute this new ratio f_P/f_π, for the ratio P/π, with the use of which he is already familiar.

It is well to call attention to the fact that the fugacity offers an extremely direct method of calculating the net isothermal reversible work of either pure compounds or of mixtures, however complex. Indeed, the fugacity may be defined by the equation,

$$W = nRT \ln f_2/f_1 \quad \text{.................} [5]$$

where W is the net isothermal reversible work of compressing n mols of a gas from the condition in which the fugacity is f_1 to that in which the fugacity is f_2. Furthermore, since the volumes of vapors are additive at constant temperature and pressure, it follows that the foregoing equation can be used for a mixture by applying the equation in succession to each component, the total work of compression of the mixture being the sum of the compression work thus computed for each component.

Thus, it is desired to calculate the net isothermal reversible work of compressing 1 mol of a vapor mixture containing 40 mol per cent ethane and 60 per cent propane from 10 atm to 400 atm at a constant temperature of 120 C. The value of $T_R = 393/305.3 = 1.288$ and $393/370.5 = 1.060$ for ethane and propane, respectively. At $P_1 = 10$ atm, P_{R1} for ethane equals $10/48.8 = 0.205$, and for propane $P_{R1} = 10/44.5 = 0.225$. Hence $f_1 = 9.72$ and 9.36 for ethane and propane, respectively. At $P_2 = 400$ atm, the corresponding values of f_2 are 202.0 and 112.8, respectively. Equation [5] becomes, for this case,

$$W = n_{\text{ethane}} RT \ln \left(\frac{f_2}{f_1}\right)_{\text{ethane}} + n_{\text{propane}} RT \ln \left(\frac{f_2}{f_1}\right)_{\text{propane}}$$

For 100 mols of the mixture, one obtains $W = 2108$ cal per gram-mol, or expressed in engineering units, $W = 1.8(2108) = 3794$ Btu per lb-mol.

4 MECHANISM OF SOLUTION AT TEMPERATURE ABOVE THE CRITICAL POINT

A problem frequently met by the engineer is the necessity for estimating the solubility of gases in liquids at temperatures above the critical points of the gases. It is clear that Raoult's law in its ordinary form breaks down under these conditions. Designers have not infrequently extrapolated the vapor pressure curve, graphically or otherwise, and used the extrapolated values of P in Raoult's law for the estimation of solubilities. It is clear that any such procedure is grossly inaccurate because of the wide differences between fugacity and pressure in these ranges. However, it is interesting to consider the extrapolation of the fugacity curve of the saturation curve beyond the critical temperature.

The fugacity form of Raoult's law, as given, implicitly assumes additivity of volumes and absence of heat effects on mixing liquids. This granted, it is easy to show that, at temperatures sufficiently low that the vapor pressures of a given pure component are low enough so that deviations of the saturated vapors from the gas laws are negligible, the heat of solution of a vapor in a non-volatile solvent is numerically equal to the heat of vaporization of the pure liquid being dissolved. This is true whatever the pressure on the vapor, because the enthalpy of the vapor depends on only the temperature and is independent of the pressure. However, the moment the temperature of the vapor gets sufficiently high so that the saturated vapor deviates from the gas laws, this will no longer be true. Thus, the heat of vaporization of ethane at 32.2 C, its critical temperature is zero, but the heat of solution of ethane in a non-volatile oil at atmospheric pressure and 32.2 C is a very considerable quantity. This heat of solution is the difference in enthalpy between that of the saturated vapor and the low pressure vapor at the same temperature, as given by Equation [1a]. If ethane be compressed at its critical temperature, beyond its critical pressure, its enthalpy becomes less than the enthalpy of ethane at the critical condition. Clearly, therefore, if this super-compressed ethane be dissolved in some liquid at this temperature, the heat of solution will be negative, i.e., heat will be absorbed rather than evolved. Now suppose one considers the behavior of ethane at a temperature above the critical. If the ethane gas is at a low pressure, it becomes highly concentrated on solution, and there is a corresponding decrease in enthalpy. If, however, the original ethane at this high temperature is under an excessively high pressure, it will actually expand on solution in a liquid, and its enthalpy will therefore increase, i.e., there would be heat absorption on solution. There must be an intermediate point at which the heat effect of dissolving compressed ethane in a liquid solvent is zero. While the temperature is above the critical for the ethane, and consequently ethane cannot exist as a liquid in the ordinary sense of that term, it can be conceived as a liquid in the sense that it is in a condition such that it will mix with other liquids without heat effect or volume changes. In this sense, ethane at this pressure may be said to be saturated and the corresponding pressure may be looked upon as an extrapolated point on the vapor pressure curve of the liquid. This concept gives a physical meaning to such an extrapolation.

The small amount of data available indicate that, while this sort of interpretation and extrapolation gives results that are good approximations to the experimental facts, considerable deviations exist. It is probably true that there is not in fact a condition above the critical temperature of a given hydrocarbon at which it will mix with other liquid hydrocarbons entirely without heat effect or volume change. None the less, this method of estimation is suggested as the best at present available for purposes of engineering design.

While all the illustrations given deal with mixtures of a limited number of components, it will be clear that the same methods can be employed in the case of any mixture, however complex, for which one has the true boiling point curve, by resorting to graphical integration. The computation methods here outlined therefore put one in a position to calculate (*a*) the density of any vapor, mixed or otherwise, at any temperature and pressure, (*b*) the heat effect of any heating or cooling operation, whether conducted at constant pressure or changing pressure, (*c*) the equilibrium between vapor and liquid at any temperature and pressure, (*d*) the equilibria and changes in concentration throughout the tower in any operation of rectification or absorption, at whatever pressures they may be conducted, and (*e*) the isothermal reversible work of compression of vapor mixtures, whatever the pressure range. These computation methods enable the refining engineer to design high-pressure equipment and operations with a degree of precision hitherto unattainable.

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Design Considerations of a Natural-Gas Transmission System

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Since design is one of the chief factors in the economic success of a transmission system, the engineering principles and the requisite empirical data are discussed herein as employed in the design of a pipe-line system now completed. The maximum day demand in successive years of development is analyzed by component parts, particular emphasis in the domestic element of the load being given to previous usage of manufactured gas in terms of B.t.u. per average meter for range and water-heating purposes and to the probable space-heating element based upon the degree-day deficiency method. A brief discussion is included of basic relationships of the factors in the pipe-line flow formula. Consideration is given to relative physical advantages to be derived from application of an understanding of compressor requirements at varying ratios of pipe-line pressure expansion. The unusual features inherent in the looping of lines are shown graphically. Reasons for using a 24-hour instead of a 1-hour period as the time interval of maximum load in design are explained. A diagrammatic presentation of a part of an actual system is made, and discussion thereof is included, to illustrate the manner in which the several engineering principles apply.

OF ALL the gas utilized in this country (1)[2] in 1929, approximately 82 per cent was natural gas; in the same year over 4.4 times as much potential energy (2) was contained in natural gas utilized as was generated in all of the country's hydroelectric central stations; during 1930 of the money spent in the 16 Southern states for all construction purposes nearly 22 per cent was for the building of pipe lines (3) for the conveyance of petroleum products. These statements are indicative of the important influence that the natural-gas industry exerts on modern life. Furthermore, new systems and vast extensions of existing properties to new centers of consumption that are now in process of construction will make, by comparison, an even more pronounced impression on the economic structure. It may well be said, then, that the problems and economics of the natural-gas business are of national importance because of either the present or future effect upon users of fuel for domestic, commercial, and industrial purposes. Since transportation is one of the chief factors of the cost of gas service in which design of transmission pipe lines is so important an element of that cost, it is hoped that engineers representing either present or potential users of gas, as well as manufacturers of fuel-utilization apparatus, would be interested in a discussion of design considerations of a natural-gas transmission system.

Scope of Discussion

The scope of this paper includes a discussion of the estimation of domestic, commercial, and industrial peak day demands and load factors, an explanation of the most commonly used gas-flow formula, together with certain of its inherent relationships, and also a diagrammatic presentation of the design results of an actual pipe-line arrangement. Several valuable papers have been published (4) embodying theoretical generalizations of some phases of these problems, and it is hoped that by treating upon the specific case of a recently completed system, as illustrated herein, a clearer conception of actual conditions to be confronted may be gained. In order to design a gas-transportation system properly, a rather startling mass of information is required concerning existing plants distributing manufactured gas, the economic feasibility of supplying towns heretofore without local delivery facilities, the variable volumes of natural gas for industrial users that can be procured at different ranges of prices, and also an intimate knowledge of the character of the country as affecting pipe-line-laying costs. No equation including all factors has yet been nor probably ever will be formulated to determine the selection of a system having the maximum ratio of net return to capital investment, in consequence of which the "cut and try" method is employed. It is necessary, therefore, to assume as a basis in each trial set-up for weighing the design and relative economic advantage of each type of system that a given set of rate schedules be effective. For obvious reasons it is appropriate to deal in this paper, treating upon a specific system, only with matters of annual consumptions and peak loads probable under existing rates and with certain aspects of some general economic advantages to be secured by properly employing the several fundamental principles involved in design work.

Elements Composing Peak Demand

Analyzing the component parts of the potential gas load conditioning the line capacity requirements of a system, a general classification points to four major divisions: (1) domestic range and water heating, (2) domestic house heating, (3) commercial usage, and (4) industrial consumption. Inasmuch as the normal, fully developed natural-gas system experiences its largest demand during the household space-heating season, the greatest day of which occurs between November 15 and

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[2] Superior numbers in parentheses relate to references at the end of the paper.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 ANALYSIS OF AVERAGE DOMESTIC SALES AND LOAD CONDITIONS OF ACCESSIBLE GAS DISTRIBUTION SYSTEMS EXISTING IN 1929 IN CENTRAL ALABAMA AND WESTERN GEORGIA

	1929[a]	1931	1932	1933	1934	1935	1940
1 Population	618,901	635,000	644,000	655,000	667,000	680,000	715,000
2 Average number of meters	59,780	70,600	73,900	77,100	80,580	84,040	89,380
3 Ratio of population to active meters	10.36	8.99	8.72	8.49	8.29	8.09	8.00
4 Space-heating connections, per cent of total meters	0.14	10.0	25.0	45.0	70.0	80.0	85.0
5 Unaccounted-for, per cent of annual sales	8.62	12.0	10.0	9.0	8.0	7.5	7.0
6 Annual sales per meter, 1000 B.t.u.:							
(*a*) Cooking and water heating	21,340	23,500	25,000	25,800	26,400	27,000	28,000
(*b*) Space heating	[b]	4,416	11,040	19,872	30,912	35,328	37,536
(*c*) Unaccounted-for	1,840	3,510	3,600	4,110	4,590	4,670	4,580
(*d*) Total	23,180	31,426	39,640	49,782	61,902	66,998	70,116
7 Maximum day demand, cu. ft. of 1000 B.t.u. gas per meter:							
(*a*) Cooking, water heating, and unaccounted-for	76.2	82	87	90	92	94	95
(*b*) Space heating	[b]	68	169	308	473	540	574
(*c*) Total	76.2	150	256	398	565	634	669
8 Average daily sales of 1000 B.t.u. gas per meter	63.5	86.1	108.6	136.4	169.6	183.6	191.9
9 Ratio of loads, average daily to maximum day	1.2	1.74	2.36	2.92	3.33	3.45	3.49

[a] Last calendar year with manufactured gas.

[b] Negligible.

March 15, the total peak day for any given city is ascertained by superimposing the largest week-day industrial demand upon the domestic maximum 24-hour period. Each of the four elements of the total anticipated maximum days to be provided for in successive years of development is considered below during the four-month periods in which the maximum 24-hour heating demands occur.

RANGE AND WATER-HEATING FACTOR

Approach to the domestic range and water-heating factor in the cities with existing distribution plants is best made through available statistics in terms of B.t.u. of manufactured gas previously delivered. As shown in Table 1, records establish that in 1929 the average domestic customer in central Alabama and western Georgia consumed 21,340,000 B.t.u., in consequence of which 1,840,000 B.t.u. was lost in effecting delivery. Reports indicate that about 49 per cent of the active domestic customers had water-heating installations. The ratio of population to active meters was 10.36 to 1, which, even considering that 34.1 per cent was negro population, is far from either the normal or the obtainable meter saturation. These sales were made at an average cost to the consumer of $2.68 per 1,000,000 B.t.u. Due to the fact that, under the rate schedules applicable to each customer using natural gas, the first 2500 cu. ft. per month is sold at an average of about $1.65 per 1,000,000 B.t.u., it is estimated that ten years after natural gas has replaced manufactured gas the average annual consumption for domestic ranges and water heating will increase to 28,000 cu. ft. of 1000-B.t.u. gas per meter. As nearly as can be determined the ratio of average daily send-out to the maximum day send-out for the manufactured-gas plants was 1.2, which load factor has been used in subsequent years for the element of range and water heating in estimating natural-gas peak-day demands. It has been assumed that in ten years a ratio of population to active meters of 8 to 1 will be attained. This ratio is based upon existing ratios in other cities where natural gas is used, with due consideration having been given to the ratio of black to white population and to the relative scale of living in the respective communities.

DOMESTIC SPACE-HEATING FACTOR

Study of the domestic house-heating load is unaided by statistical data of past experience because of the inconsequential number of customers using manufactured gas for such purposes in this territory. The most authentic available data on this phase of the problem involving the experiences in the degree-day deficiency method as applied to Dallas, Texas,(5) furnish particularly valuable information. The annual deficiency is defined as the summation of the daily differences between 65 deg. fahr. and the mean temperature when that daily mean is less than 65 deg. It has been found by a six-year investigation in Dallas, whose average temperature deficiency is 2341, that 16,040 B.t.u. per year per degree-day deficiency per average meter, with a plus or minus error in any one year of 2 per cent, is consumed for domestic space heating. The Dallas results, arrived at in a district with a population of 315,000 (15.1 per cent negroes) having a total of 65,650 live meters, also show that for an extremely cold series of days the consumption per degree deficiency per meter ranges from 12,000 to 14,000 B.t.u. per day.(6) Humidity, velocity of winds, and the percentage of sunshine have an influence upon consumption, the extent of which has yet not been ascertained, although the effect due to these causes is noticeable to only a small extent. No weight has been given these latter factors in this paper.

In Table 1 the average annual domestic consumption per meter handling space-heating gas is based upon a consumption of 16 cu. ft. of 1000 B.t.u. gas per degree-day for a total average annual deficiency of 2760. The total average deficiency is based upon records of the U. S. Weather Bureau in each of the several cities now having natural gas weighted with respect to the number of meters in each. In the territory being considered it is interesting to observe the wide fluctuations of average annual deficiencies, Rome, Ga., having 3067 and Montgomery, Ala., 1886. The average annual consumption for each domestic consumer using space-heating natural gas is taken in Table 1 as 44,160 cu. ft., whereas the peak day has been assumed at 12.5 cu. ft. times the difference between 65 deg. fahr. and the coldest day's mean temperature of 11 deg. fahr., or 675 cu. ft. The percentage of domestic consumers using gas for space heating has been assumed rather arbitrarily at 10 per cent in the first full year of operation after an eight months' development program following introduction of natural gas to the old manufactured-gas systems. In the tenth year reasonable expectancy permits the assumption that 85 per cent of all live meters will be passing domestic-space-heating gas. The speed(7) with which this portion of the load is added to a pipe-line system is dependent on a number of factors, the principal of which are (1) the relative cost of other fuels, (2) the economic condition of the people, (3) the effective aggressiveness of local distribution managers in the securing of new business and the servicing of installations, and (4) the equipment-merchandizing policies adopted.

Along the route of every large natural-gas system are numerous relatively small towns unfavored with gas whose size is insufficient to support a manufactured-gas system. The sales per consumer obtainable will parallel those of the cities discussed, with a tendency to be somewhat larger initially, but smaller in later years of development, due in general to less wealth per capita. Generally speaking, financial success of small-town distribution systems is possible only when industrial consumers may be secured or when towns are so located as to permit group operation.

COMMERCIAL AND INDUSTRIAL FACTORS

An accurate forecast of possible commercial and industrial consumptions demands a detailed survey of each individual plant. The highly competitive nature of this element of the load requires adequate fuel cost analysis and, from the point of view of peak conditions, precise data on the largest probable fuel demand in week days during the domestic heating season. The difference between what a concern could afford to pay for gas on a "break even" basis with present fuel used and what a plant's average earned gas rate would be under existing price schedules is the measure of the plant management's readiness to purchase natural gas. The development of the sales of a newly constructed natural-gas system is attended usually by a marked degree of reduction in other fuel prices akin in many instances to an actual price-cutting warfare. In the determination of a "break even" gas rate for a company, costs should be surveyed in the light of the production expenses and the enhanced sales value of the output, if any, whether the commodity be pounds of steam, tons of steel, barrels of lime or cement, or the renting of hotel rooms. It is the gas industry's general experience that, in the majority of cases, industrial plants will purchase gas only upon a strictly tangible savings or direct-gain basis. In the case of boiler plants, savings normally allowed by the managers include relative fuel-handling costs, percentage of weight lost in transit of coal from point of billings, moisture in fuel as received, relative annual overall firing efficiencies of respective fuels on an "as fired" basis, differences in equipment and furnace maintenance, and interest on fuel inventories. Unless a plant's earned gas rate is appreciably under the "break even" rate based upon competitive fuel prices actually to be confronted, considerable time will be required to secure the bulk of the potential load. Generally speaking, natural gas is more quickly sold where full analysis of costs with other fuels is known. Analysis of the potential securable industrial load on the system under consideration indicates that during the domestic heating season the load factor of industrial business will be between 70 and 80 per cent, or the industrial-peak day load to be superimposed on the domestic-peak day load will be 1.41 to 1.25 times the average industrial daily load. The commercial sales, which include apartment houses, office buildings, bakeries, and other similar loads, have approximately the same load factor as the total domestic sales.

DUMP-GAS SALES

Examination of the ratios of the maximum daily demand to the average daily load of the varieties of gas sales to be made in the fifth year of development conditioning the pipe-line capacity requirements permits the following comparison: (1) The domestic and commercial is 3.45 to 1.0; (2) the industrial is about 1.3 to 1.0. The fact that the pipe-line flow capacity necessary to deliver a given annual volume of domestic or commercial gas is about 2.65 times larger than that required to deliver an equal volume of industrial gas is the principal cause of the greater expense per m.c.f. to a pipe line in transporting domestic gas. In addition to the foregoing classes of sales, that of so-called "dump" gas constitutes an important factor in the economic aspects of a transmission system. A pipe line must have sufficient flow capacity to meet the maximum coincident demand of all customers entitled, by virtue of prices paid, to continuity of service. In order to augment earnings, recourse is taken to effect the sales of gas at low prices to large industrial plants under the condition that service may be suspended temporarily on short notice in order to meet firm gas demands. To permit such conditions of dump service, industrial plants must maintain secondary fuel supply and equipment. Provided a pipe line has an installed capacity to meet firm peak demands, the only significant costs of selling off-peak gas is the cost of gas at the source, added compressor fuel and engine operation, maintenance and repair expenses, and also incidental metering and servicing charges, inasmuch as overhead, interest on investment, and pipe-line maintenance expense are sustained irrespective of dump-gas sales. Load of this character is desirable particularly from March 15th to November 15th, to raise the annual ratio of maximum load to average daily sales.

In designing a transmission system wholesaling gas at border meter stations for retail distribution inside of cities, the estimation of peak demands at any given point is the element in pipeline design most susceptible to error. Knowledge of the factors determining present and future conditions and judgment and experience on the part of the engineer are of the utmost importance in dealing with this phase of the gas design problem.

GENERAL FLOW FORMULAS

Of the several flexible and workable gas pipe-line flow formulas accepted and employed in practice, that set out below (8) has been demonstrated (9) to be the most accurate:

$$Q = \frac{K\sqrt{P_1^2 - P_2^2}}{\sqrt{L}} \qquad [1]$$

or in a more convenient form for design purposes:

$$\left(\frac{Q}{K}\right)^2 L = P_1^2 - P_2^2 \qquad [2]$$

in which

Q = flow of free gas, cu. ft. per 24 hours
L = length of pipe line, miles
P_1 = initial or starting pressure, lb. per sq. in., abs.
P_2 = terminal pressure, lb. per sq. in., abs.

$$K = \frac{433.5 \times T_o d^{8/3}}{P_o\sqrt{T\,G}} \qquad [3]$$

in which if the following assumptions, based upon probable operative conditions in times of expected peak demands, are made

T = 520 = temperature of flowing gas, deg. fahr., abs.
P_o = 14.65 = pressure base of gas measurement, lb. per sq. in., abs.
T_o = 520 = temperature base of gas measurement, deg. fahr., abs.
G = 0.6 = specific gravity of gas relative to air.

where d is the internal diameter of pipe in inches.

Then from [3]

$$K = 871.12\, d^{8/3} \qquad [4]$$

Fig. 1 indicates graphically (10) the extent to which variations from the foregoing assumed constants in the value of K will modify formulas [1] and [2]; e.g., an increase in the average flowing gas temperature of 40 deg. fahr. to 560 deg. fahr. will decrease the pipe-line flow capacity to 96.4 per cent of the previous capacity.(11) At the beginning of design work on a system, it is advisable to make a tabulation of K values for standard pipe sizes of each commercial thickness, having determined first the probable values of other functions of K, as in formula [3], under conditions to be met on the occasions of peak load.

Examination of the foregoing formulas bearing upon flow capacity reveals that

(a) Q varies directly as K; see [1]
(b) Q varies inversely as the square root of L, or, conversely, that L varies as the inverse of $\left(\frac{Q}{K}\right)^2$; see [1]
(c) Q varies directly as the square root of the difference of squares of the absolute end pressures; see [1]

- (*d*) The drop in pressure in any given length L is a function of the difference in the squares of absolute end pressures; see [2]
- (*e*) K varies as the 8/3 power of d; see [3] or [4]

LARGER FLOW OBTAINABLE THROUGH HIGHER PRESSURES

The condition of pressure drop emphasized in *d* is attributable to the compressibility of gas, a circumstance unlike that in hydraulics, in which Q varies directly as the square root of the difference in hydraulic pressures instead of as the square root of the difference in the squares of the absolute gas pressures. Upon this condition also hinges the explanation of why, at progressively higher pressures, greater volumes of gas can be transported even though the ratio of pressure expansion in a line remains constant. Transmittable quantities increase faster proportionately than actuating pressures (see *a* and *b*). The plotting of pressure conditions against distances from the starting point will illustrate graphically the more rapid falling off of pressure at successive points as the terminus of the line is approached.

QUANTITY VARIES DIRECTLY AS K

Reference to formulas [4] and [1] shows that Q, instead of varying directly as d, varies as the 8/3 power of d. In other words, the capacity of a 16 × $^1/_4$-in. line is 87.3 per cent greater than a 12$^3/_4$ × $^1/_4$-in. line. The relationship of Q as a direct function of K offers a ready means of selecting the requisite size of multiple river-crossing lines required to equal or to exceed by any given percentage the capacity of a given single main line.

To illustrate, suppose it is desired to select five multiple lines to have as nearly as possible 140 per cent of the flow capacity of a 20 × $^3/_8$-in. main line whose K = 2,318,210. Then K_{20} ÷ 5 = 463,642, which is approximately equal to K for a 12$^3/_4$-in. pipe similar to that frequently used for river-crossing work.

PIPE-WALL THICKNESS

Comparing the value of K for 20 × $^1/_4$ in. with that for 20 × $^3/_8$ in. indicates that the thinner wall pipe has 3.66 per cent greater carrying capacity. The larger internal diameter pipe also has a 2.57 per cent greater storage capacity. Carried down into terms of net revenues, the larger flow capacity of the thinner wall pipe has an appreciably beneficial effect. These are points frequently overlooked by many exponents of higher tensile-strength, thinner-wall pipe.

It is pointed out in *c* that higher average pressures result in increased flow capacity, but such procedure necessitates, of course, the use of thicker wall pipe to withstand added internal stresses. The wall thickness necessary to carry given maximum pipe-line pressures is to be calculated by Barlow's equation as follows:

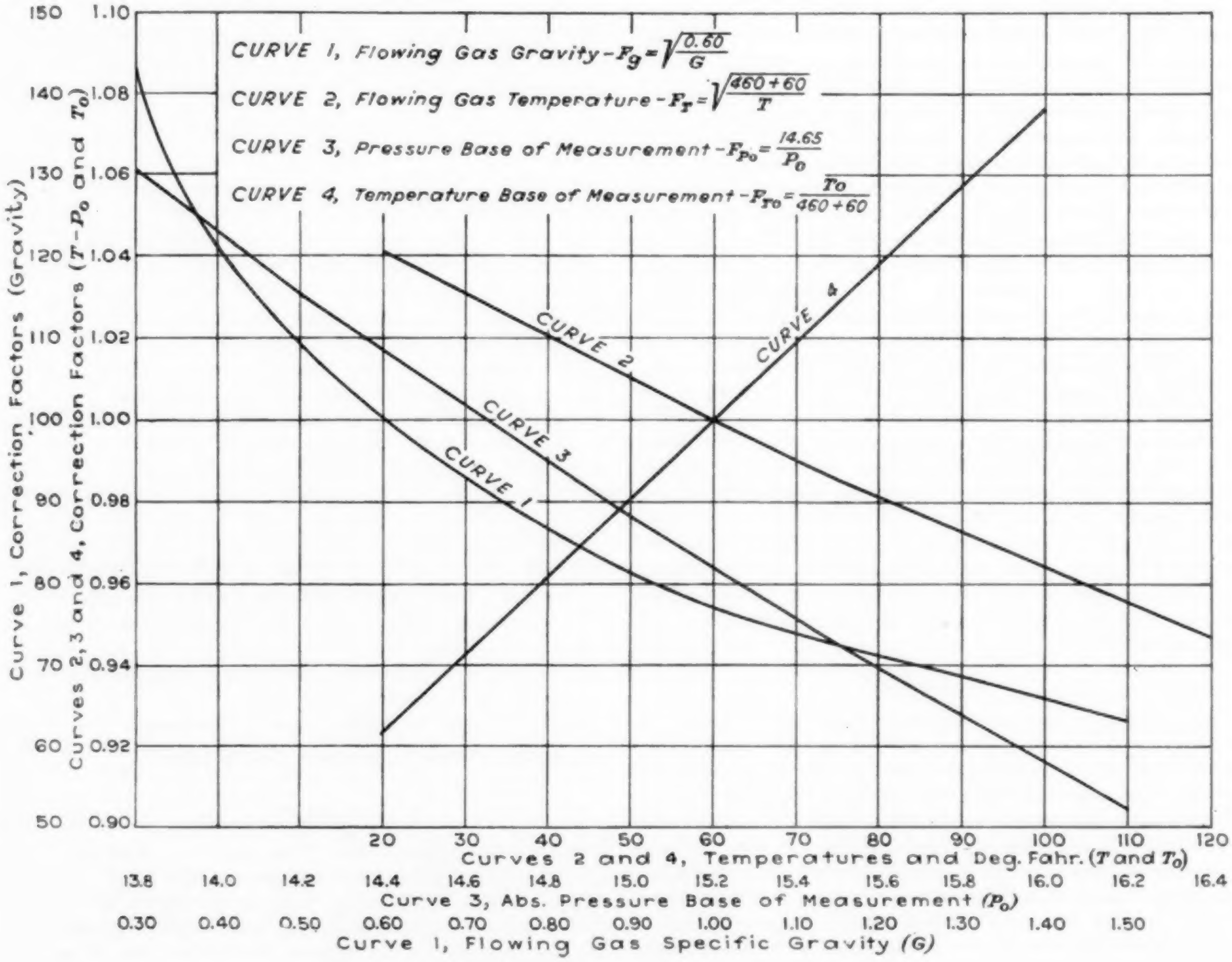

FIG. 1 GAS PIPE LINE CORRECTION FACTORS FOR FLOW CAPACITIES

$$T = \frac{DP}{2P + \dfrac{2UE}{F}} \quad \text{[5]}$$

in which

T = thickness on pipe wall, in.

D = internal pipe diameter, in.

P = maximum gas pressure, lb. per sq. in., gage

U = ultimate minimum tensile strength of pipe steel under advisement, lb. per sq. in.

E = minimum percentage of longitudinal weld and/or method of joining-pipe-lengths efficiency relative to pipe, provided the percentage < 100

F = allowable factor of safety.

The trend in the natural-gas industry is toward thinner walls, with due consideration being given to certain minimum practical limits bases on fragility and bending qualities of the pipe. Some engineers permit an allowable factor of safety in formula [5] of as low as 3. Change from one wall thickness to another due to variations in tensile strength causes a large alteration to required pipe tonnage. In the steel industry, premiums per ton are demanded for high tensile strength, thereby partially offsetting tonnage reductions secured by the use of thin wall pipe.

DOUBLING LINE

Due to the conditions mentioned in *b*, the paralleling of a gas line does not increase the flow-carrying capacity in direct proportion to the percentage of line so looped. Fig. 2, curve 1, is a plotting of the formula for looping with pipe of the same size

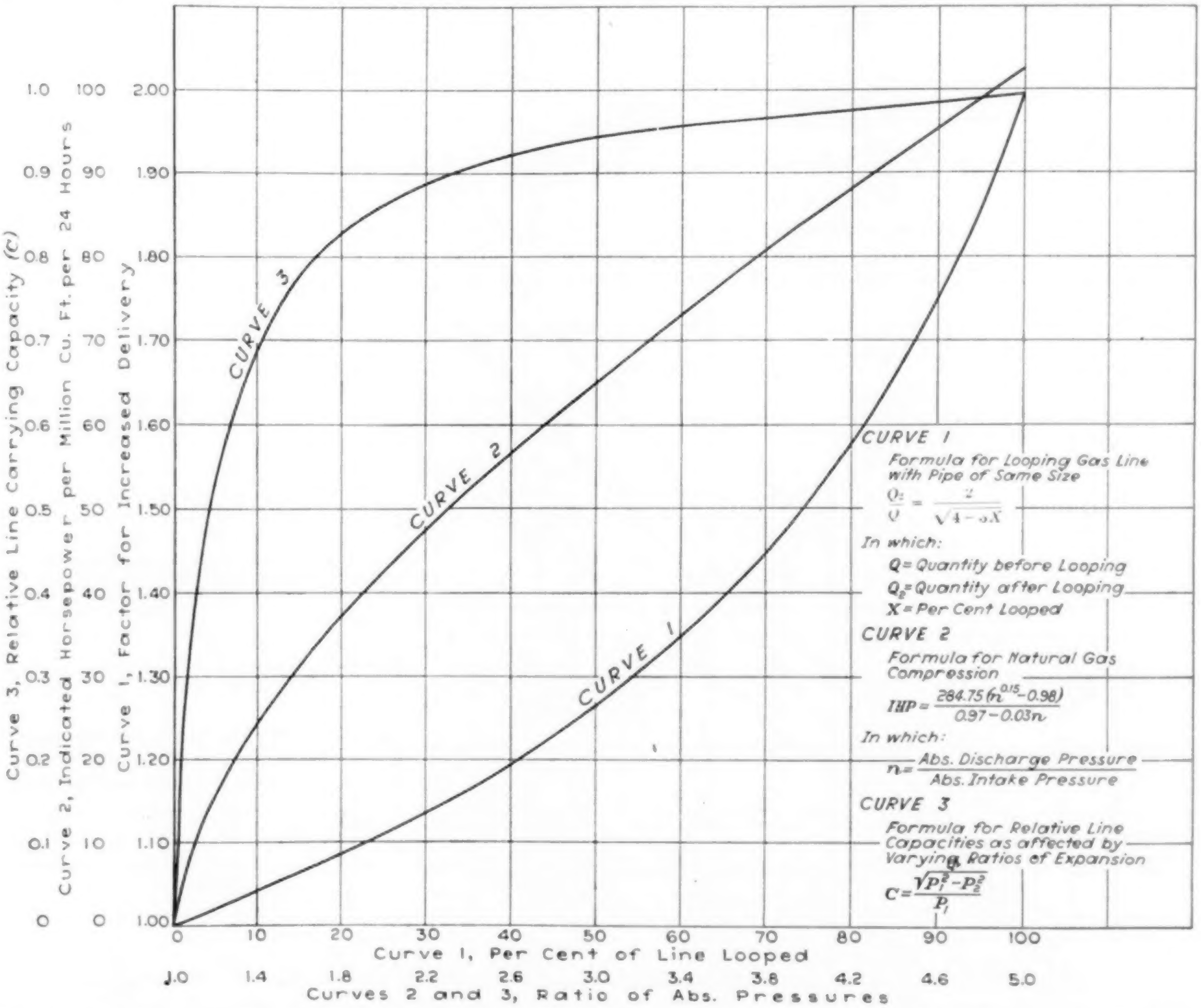

FIG. 2 POWER REQUIREMENTS AND RELATIVE FLOW CAPACITIES AT VARIABLE PRESSURE RATIOS, ALSO FLOW CAPACITIES AT VARIABLE PERCENTAGE OF LOOPED LINE

$$\frac{Q_2}{Q} = \frac{2}{\sqrt{4-3X}} \quad \text{[6]}$$

in which

Q = quantity of gas before looping

Q_2 = quantity of gas after looping

X = per cent of line looped

Curve 1 reveals that by looping 50 per cent of a line the flow capacity of the entire section is increased only 26.5 per cent. Examination of formula [6] and also of curve 1 indicates that after two-thirds of the line is doubled a further increase in the percentage looped is accompanied by a greater proportionate increase in the quantity factor until a self-evident result is apparent in that a doubled line will transmit twice the quantity of a single line. The tangential point of greatest slope in curve 1, since Q varies as $\frac{1}{\sqrt{L}}$ is at the point of the square root of 3, or at 1.732.

Whether the line is looped at the discharge of one compressor station or at the intake of the next successive station does not affect the flow-carrying capacity provided a constant volume of gas is transported over the entire line section between such compressors.

Relative Pressure Expansion and Compression Ratios

In the progress of gas through a pipe, pressure is lost in accordance with formula [2], thereby necessitating for long-distance lines the operation of stations at various points to compress to higher pressures for further transportation. Pressure loss in a gas pipe line is analogous in many respects to energy dissipation in power transmission. Curve $3^{(12)}$ in Fig. 2 illustrates the relation of the ratio of gas expansion, in terms of the ratio of initial to terminal pressure, to the relative maximum flow capacity of a pipe line. Curve 2 in Fig. 2 illustrates the relation of horsepower required to compress 1,000,000 cu. ft. of free gas per day to the ratio of intake to discharge pressures, commonly called the compression ratio. Comparison of curves 2 and 3 explains a vital element in pipe-line design. At a pressure ratio of 2.5, the horsepower required for compression is 54 and the percentage of line carrying capacity relative to the maximum 91.5 per cent; whereas at a 4 pressure ratio the values are 84.5 hp. and 97 per cent, respectively, or an increase of 69.4 per cent in compressor horsepower required to gain only 5.5 per cent in relative line flow capacity. In other words, to pass a slightly larger Q between two fixed points—see (c)—or to slightly increase the distance L between two stations—see (b)—the sacrifice to additional power requirements is disproportionately large. The compressor ratio of 2.38 has been found by experiment to be the most economical on fuel consumption. The normal

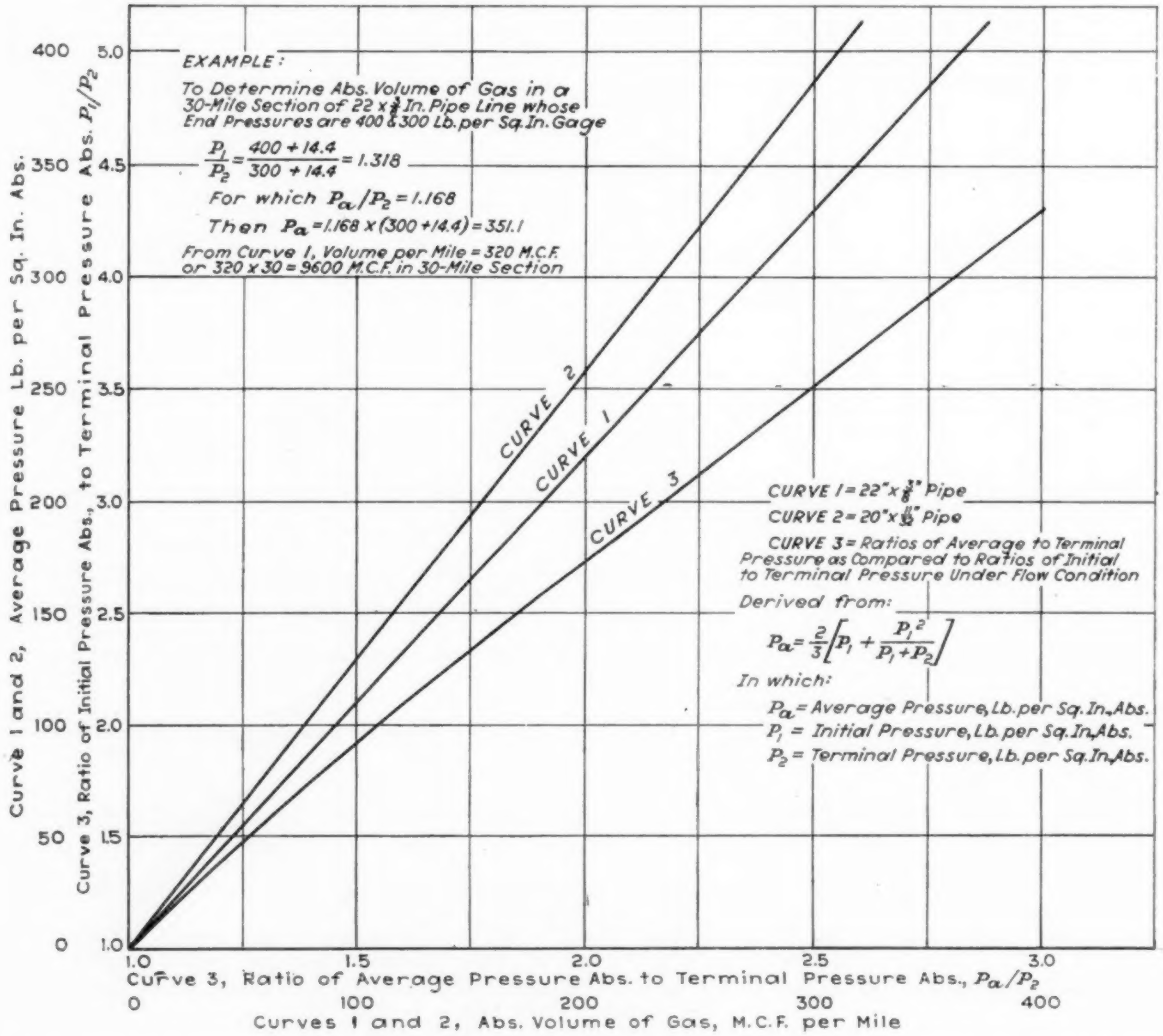

Fig. 3 Absolute Volume of Gas per Mile of Main at Variable Average Pressures, Also Average Pressure of Variable Ratios of Expansion

practice in the industry is to use natural gas out of the pipe line itself as fuel for the engines driving the compressors. Approximately 1.25 per cent of the gas pumped in any one station is used as compressor fuel for a compression ratio of about 2.5, thereby causing a cumulatively greater fuel-gas withdrawal, either as the number of stations is increased or as the distance from field to delivery point is greater.

Measure for Design Is Peak 24-Hour Period

The analysis of maximum loads in the foregoing discussion of long transmission lines has been confined to treatment on a daily basis, whereas the hourly peak is the basis used ordinarily in distribution-system design. Experience in comparable climates has indicated that at a border station of a city with practically complete domestic heating saturation the ratio of maximum hour to average hour on the coldest days is about 1.6 to 1.0 for the domestic and commercial elements of the total load. The ratio of the minimum hour to the average hour on the coldest day will range between 0.25 and 0.45 to 1.0 for the domestic and commercial business. No generalization of this nature can be made for the industrial sales, but the range is usually between 1.3 and 1.8 to 1.0. Due to the fact that inside of cities the small amount of free gas per cubic foot of pipe at the low pressures carried restricts the storage volume, distribution mains are designed for hourly gas quantities. The storage capacities per mile of normal 22-in. and 20-in. pipe at varying pressures is shown in curves 1 and 2 in Fig. 3, indicating, among other things, that the absolute volume of 22-in. pipe at 414.4 lb. per sq. in. abs. is 320,000 cu. ft. By means of "packing" the main lines to an average pressure between compressor stations greater than that required under peak 24-hr. uniform-flow conditions, sufficient excess-gas storage can be accumulated to provide for the hourly peaks. Through local weather agencies advance notice of extremely low temperature can be secured thus affording an opportunity to "pack" the lines for peak deliveries. Curve 3, Fig. 3, shows graphically the fundamental equation(13) for determining the average pressure between two points in a pipe line under uniform flow condition:

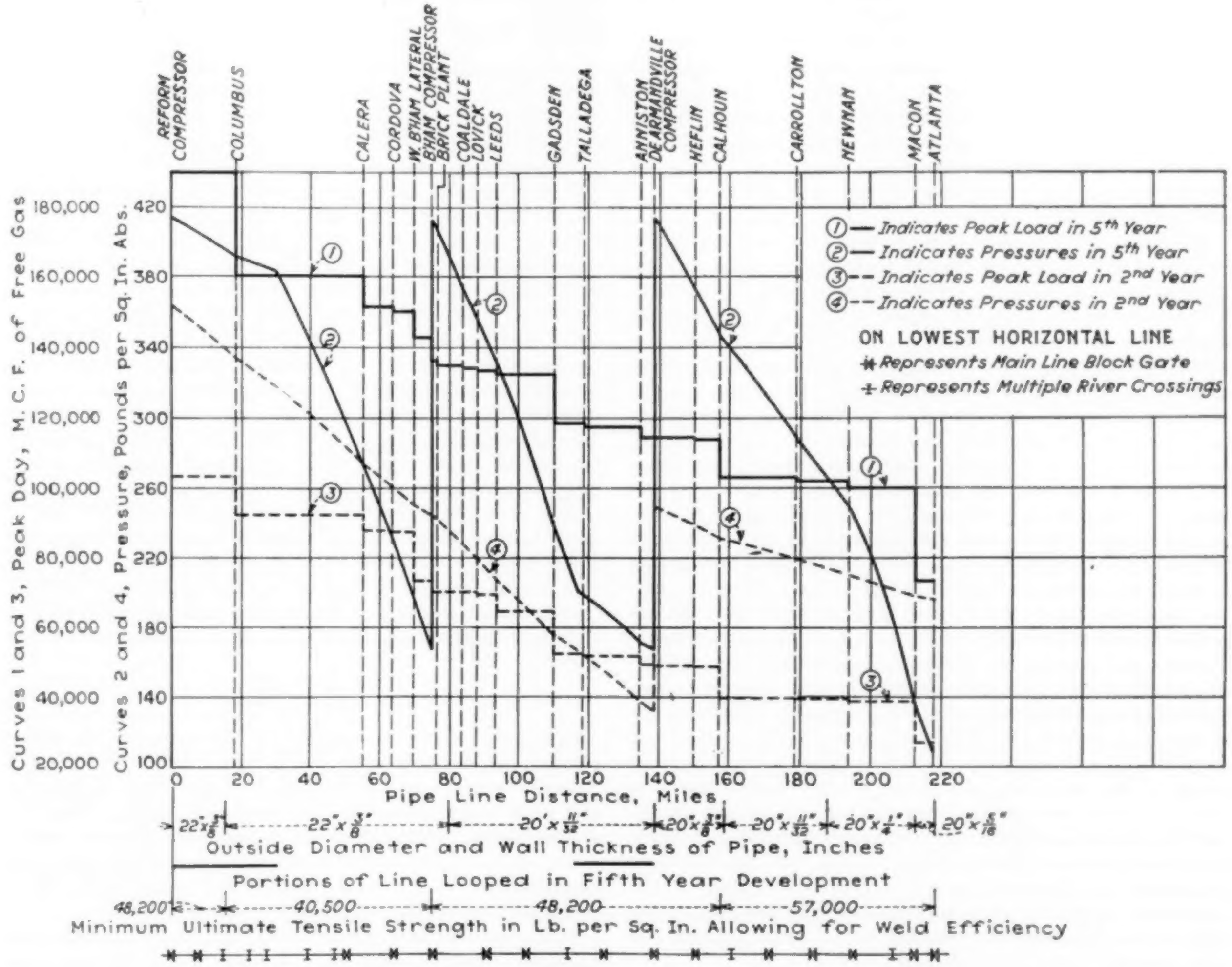

Fig. 4 Diagrammatic Plan of Pipe Sizes and Thicknesses, Pressure, and Peak-Load Conditions of a Recently Completed Section of a Southern System

$$P_a = \frac{2}{3}\left[P_1 + \frac{P_1^2}{P_1 + P_2}\right] \qquad [7]$$

in which

P_a = average pressure, lb. per sq. in., abs.
P_1 = initial pressure, lb. per sq. in., abs.
P_2 = terminal pressure, lb. per sq. in., abs.

It was stated in effect in (*d*) that the average and mean pressures differed; e.g., from curve 3 if 400 lb. per sq. in., abs., is the initial pressure and 150 lb. per sq. in., abs., the terminal pressure, then $P_a/P_2 = 1.86$, or $P_a = 1.86 \times 150 = 279$ lb. per sq. in., abs., whereas the mean pressure is 275 lb. per sq. in., abs. In view of the factors as stated, long-distance pipe lines are designed with peak demands considered in 24-hour periods.

Application to an Actual System

The basic data and formulas discussed were used in the design of a natural-gas transmission system recently completed in the South, a plan of the eastern half of which is presented in Fig. 4. This diagram depicts the anticipated progressive peak loads and pressure conditions in the second and fifth years of development plotted against pipe-line mileage. The latter is subdivided to show block valves at both main-line gates and river-crossing headers, pipe diameters and thicknesses actually installed, as well as the portions looped and the pipes' effective tensile strength. On the side opposite to the mileage are designated the successive points at which tap lines or branch systems take off from the main line.

Heretofore most published papers have been based upon the theoretical condition that all gas entering the system, other than compressor fuel, has been delivered at the pipe-line terminus. The transportation system studied here is the largest single system involving numerous branch lines ever built in a continuous construction program and which was conceived in practically all of its details before the first pipe was laid, thereby necessitating careful preliminary designing. Any variation in main-line pressures caused by the developments in the peak demands affects the required sizes of the various branch lines carrying gas from the main line to actual delivery points. As the maximum demands increase from year to year, additional compressor stations at predetermined sites along the main line are required to insure proper gas deliveries. Due to these fluctuations in pressure at any given point at different stages of development, the tap lines must be so designed as to accommodate the anticipated demand in any year on such taps with the pressure available in the main line. In some instances it is economically advisable to locate compressor stations in later years on branch systems to reduce the initial pipe-line investment. Undoubtedly on a system of such a complicated nature the sum of the peak days in any year occurring at each delivery point will be in excess of the peak day experienced at the gas-field receiving station. It has been assumed that each tap line, branch system, and the last section of main line requires pipe capacity sufficiently large to carry the calculated maximum load, but that the peaks experienced on the main line proper, due to the theory of the probability of the coincident occurrence of such maximums, will decrease progressively westward from 3 to 10 per cent less than the total sum of the calculated peaks.

Observations on Results

Among the most important features of the part of the completed system shown in Fig. 4 is that the peak in the second year at the eastern end of the pipe line is about 14 per cent of that at the starting point in the fifth year. Under maximum flow conditions the section of line shown in Fig. 4 contains approximately 66,000 m.c.f. of free gas. The ultimate minimum terminal pressure is 100 lb. per sq. in. gage, whereas the maximum compressor discharge is 400 lb., working on a 2.5 compression ratio. Although the compression ratio selected for peak day is slightly above the point of minimum fuel consumption, the theoretical horsepower requirements are reduced thereby and the stations allowed to run at a lesser ratio on off-peak days, which more nearly approaches the average operating condition. It is interesting to note that in the fifth year it is essential, in order to carry the peak load, to loop a part of the pipe line in two of the three sections between the compressors shown. With loads coming off the line progressively in a manner similar to that shown, the investment due to looping can be reduced generally by laying pipe on the discharge instead of the intake side of compressor stations, thereby accommodating the larger volume of gas. In the second section of the line shown, however, because of the extremely rugged country, it is less expensive to achieve results in the fifth year by laying, instead of 18.98 miles on the discharge side of the upstream compressor, 21.93 miles on intake side of the downstream station. The necessity for considering branch-line sizes in terms of the most advantageous relation of load to main-line pressures is illustrated in Fig. 4 at several taps, as, for example, at the 55-, 70-, 93-, and 157-mile points. In the second year of development it is necessary at the 212-mile point to have a pressure of 188 lb. per sq. in. in order to properly supply the Macon branch system.

This condition may be accomplished by several means: (1) delivering gas at the main-line terminus at the minimum of 100 lb. by operating the main-line station next upstream at a 1.133 compression ratio, thereby requiring a separate station to be installed on the branch line to operate at a 1.677 compression ratio or (2) delivering gas, as shown, at the terminus at 186 lb. by operating the main-line station next upstream at a 1.89 compression ratio. Distinct savings are to be effected by the latter method, for instead of installing two stations to operate on highly inefficient rates of fuel consumption and with unusually large diameters of compressor cylinders, one station may be installed to operate with intake and discharge pressures more nearly those to be required with its ultimate compression ratio. The discussion of (*b*) and the general flow formula [2] is illustrated forcibly by Fig. 4, in which it is established that in order to meet increased gas demands from 103,000 m.c.f. in the second year to 189,800 m.c.f. in the fifth year, or an 84 per cent increase in capacity requirements, it is necessary to increase the theoretical compressor-station power installations(14) from 1336 horsepower to 13,526 horsepower, or an increase of 1012 per cent.

Conclusion

In order to select the system installation susceptible to the greatest percentage of return on investment it is necessary to design a separate pipe-line layout for each series of volumes and peak loads procurable on various rate scales. From the system design, estimation of the approximate ultimate investment can be made with due allowance for depreciation and capitalization of operating expenses and taxes, from which investment the ratio of return may be determined.

In the operation of a pipe line the application of the engineering principles discussed assists in economically meeting peak-load contingencies by "packing" at opportune times and also in the manipulation of dump-gas transportation through expert dispatching to secure the sale of a maximum volume. Not only is an accurate survey of potential load essential in preliminary design plans, but continual study of anticipated demands during the years of development is necessary to reduce expenditures to the minimum in the providing of adequate compressors and line capacity to supply peak requirements.

References

(1) Based on U. S. Bureau of Mines estimated figures.

(2) Figures for hydroelectric generation from N.E.L.A. annual. Average thermal value of natural gas is assumed at 1000 B.t.u. per cu. ft.

(3) *Manufacturer's Record*, issue of Jan. 15, 1931.

(4) (*a*) "Economics of High-Pressure Transportation for

Natural-Gas Pipe Lines," by P. MacDonald Biddison, presented before Joint Meeting of Mid-Continent Section, A.S.M.E., and Southwest Division of Natural Gas Department of A.G.A., Jan. 15, 1929, in Shreveport, La.

(b) "Design of High-Pressure Gas-Pipe Lines," by R. E. Davis and L. F. Terry, Trans. Petroleum Development and Technology, 1928–29.

(c) "Long-Distance Gas Transmission," by E. G. Hill and G. I. Rhodes, published in *Natural Gas*, issue of October, 1930.

(d) "Factors Effecting a Minimum Cost in Gas-Pipe Lines," by W. R. Kepler, published in *Western Gas*, issue of November, 1930, with continuation.

(e) "High-Pressure Natural-Gas Transmission," by R. S. Lord, published in *Oil and Gas Journal*, issue of Oct. 2, 1930.

(5) "Application of Degree-Day Deficiency Computing Domestic and Heating Loads," by R. M. Redding, published in *Natural Gas*, issue of May, 1930, also, "Keeping Dallas Warm With Gas," same author, presented before Southwestern Public Service Association, May, 1925.

(6) It is interesting to note that the total average annual consumption in Dallas for range, water heating, and space heating per domestic meter of 79,960,000 B.t.u. per year was sold at an average rate of about $0.807 per 1,000,000 B.t.u. In this Dallas rate schedule the first 2500 cu. ft. per month cost the consumer at the rate of $0.80 per m.c.f. Due to a large part of Dallas' water supply coming from wells of unusually high temperature being used for domestic hot water, the gas requirements for this purpose are uncommonly low. The ratio of population to active meters is 4.81 to 1.

(7) "Substitution of Natural Gas for Manufactured Gas in Public Utility Operations," by George Wehrle, 1930 A.G.A. convention.

(8) "Measurement of Natural Gas," by T. R. Weymouth, Trans. A.S.M.E. (1912). This formula was used originally for quantities on an hourly basis for straight pipe and was enunciated in slightly different arrangement than that used herein. In actual practice the efficiency of the Weymouth formula has been found to diminish in varying degrees due to departure from the assumed constant coefficient of friction of $f = \frac{0.008}{\sqrt[3]{d}}$, to flattened or buckled pipe bends, to use of gates smaller than pipe size, and to the accumulations of liquids in sags. In lines where there is an appreciable difference in elevations between the source and delivery points, care should be taken in handling the factor P_o, Equation [3].

(9) "Determining the Most Satisfactory Formula for Measuring the Flow of Natural Gas Through Pipe Lines," by E. O. Hickstein, published in *American Gas Journal*, Feb. 2, 1918, also, "Flow of Natural Gas Through High-Pressure Transmission Lines," by T. W. Johnson and W. B. Berwald, published by Department of Commerce, Bureau of Mines, serial 2942, June, 1929.

(10) "A Handbook of the Petroleum Industry," vol. 1, by David T. Day, Ph.D., published by John Wiley & Sons, Inc., 1922, p. 441.

(11) This explains one of the reasons for using coolers on gas where heat content has been increased by a compressor.

(12) Used by special permission of P. MacDonald Biddison.

(13) "Natural Gas Handbook," by John C. Diehl, published by Metric Metal Works, 1927, p. 350.

(14) See Fig. 2, Curve 2.

Discussion

P. McDonald Biddison.[3] As is usually the case in actual practice, the requirement to keep the initial cost within such limits that the project can at an early date make a satisfactory return has in this case forced a design which at the larger quantities to be transported in later years is far less economical than some other design might be. It may be demonstrated that the steel required in a transmission line increases with operating pressure and that the economical pressure is the lowest pressure that will utilize the full strength of the minimum wall thickness of pipe. A suggested equation for minimum wall thickness in inches is $T = 0.00904D + 0.087$, where D is the outside diameter in inches. It cannot be claimed that this is generally accepted in the natural-gas industry as there is a wide diversity of opinion on the question of minimum wall thickness. If this equation is accepted, however, then for pipe materials suitable for 16,000 and 13,000 lb. wall stress there would be the following equations for pressure on walls of minimum thickness: For 16,000 lb. wall stress, $P = 289.2 + (2794.6/D)$. For 13,000 lb. wall stress, $P = 235.0 + (2270.6/D)$.

From the foregoing may be derived the following wall thicknesses and corresponding operating pressures:

Outside diameter, in.	Wall thickness, in.	Gage pressures	
		13,000-lb. stress	16,000-lb. stress
20	0.269	349	429
22	0.287	338	416
24	0.305	330	406
26	0.323	322	397

In any given line and at any given operating pressure the minimum horsepower requirements for compression are obtained at a compression ratio of about 1.35, corresponding to about 20 i.hp. per million cubic feet per station. With such a pressure ratio and with pressures and wall thicknesses as outlined, a capacity of 150,000 m.c.f. per day could be obtained with 13,000 lb. wall stress from a 24-in. line with compressor stations spaced 36 miles apart, or from a 26-in. line with compressor stations spaced 53 miles apart, each station requiring but 3000 operated i.hp. With 16,000 lb. wall stress, the same conditions could have been met with a 22-in. line with compressor stations spaced 35 miles apart, a 24-in. line with compressor stations spaced 54 miles apart, or a 26-in. line with compressor stations spaced 79 miles apart. Some of these alternates, especially in view of the probable development beyond a capacity of 150,000 m.c.f. per day, would doubtless have produced ultimately a more economical installation.

J. M. Naiman.[4] By far the most important "design consideration of a natural-gas transmission system" is "market," the analysis of which requires an intimate mixture of (1) careful geographical fuel survey, (2) comparative combustion engineering, (3) intelligent rate making, and (4) sales engineering.

Next in importance to the question of the "market" comes that of the "load factor." As is well known by now, long-distance natural-gas pipe lines would be economically impossible without industrial load for flattening the peak effects of domestic load. But not enough stress is being laid on the fact that industrial load has its own peaks, the ratio of such peak load to the average industrial load being in some cases a very serious handicap to project feasibility.

It is only because the problem of getting the most out of a new pipe line does not become acute until after the maximum compressor station capacity is built and utilized that the importance of storage for taking on additional load and improving the load factor is not as well appreciated as it deserves to be. Dump load does not take the place of storage capacity, but rather supplements it, by taking care of seasonal load variations. Pressure-tank storage can only be most economically used to care for hourly and weekday to week-end load variations. Subterranian storage in depleted gas fields, if close enough to market and if proved to be tight, offers the most economical means for offsetting seasonal load variations, more so than even a dump-load rate system.

A. B. Dunning III.[5] Especially interesting is the degree-day deficiency method of estimating domestic space heating load. The emphasis placed on the economics of design is commendable and of ever-increasing importance.

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[4] Consulting Research Engineer, Republic Flow Meters Company, Chicago, Ill. Assoc-Mem. A.S.M.E.

[5] Tennessee Coal, Iron & Railroad Company, Ensley, Ala.

An analysis of the gas-flow formulas used by the natural-gas industries might be of some value, as the formula set forth (Weymouth's) is of a type that has certain definite limitations in its application.

Weymouth's formula is based on the assumption that the coefficient of friction f in Unwin's formula varies only with the diameter. It is actually much more complex, being a function of the velocity or quantity of flow, the density and viscosity of the gas, and the surface condition or roughness of the pipe as well as a function of the diameter.

Quoting from Reports of Investigations, Department of Commerce, Bureau of Mines Serial No. 2942, June, 1929, "Flow of Natural Gas Through High-Pressure Transmission Lines:"

"For Weymouth's pipe-line flow formula, $Q = C\sqrt{[(P_1^2 - P_2^2)d^{5^1/_3}/GTL]}$. Comparisons of the percentage differences between the measured volumes and the volumes calculated from Weymouth's formula for different rates of delivery indicate that there is a variation in the proportional relationship between Q and $\sqrt{(P_1^2 - P_2^2)}$."

The writer has been making studies of gas-flow theory and practice for several years, and although not yet ready to make public the full results of these studies, sufficient work has been done to establish the following formula for Unwin's coefficient of friction as being dependable for air and ordinary commercial gases flowing through reasonably clean and smooth cast-iron or wrought-steel pipe:

$$f = \frac{0.00746\, d^{0.08}\, T^{0.14}}{Q^{0.14}\, G^{0.105}}$$

The entire formula follows:

$$Q = \frac{1075\, d^{2.645}}{G^{0.461}\, T^{0.613}} \left[\frac{P_1^2 - P_2^2}{L}\right]^{0.538}$$

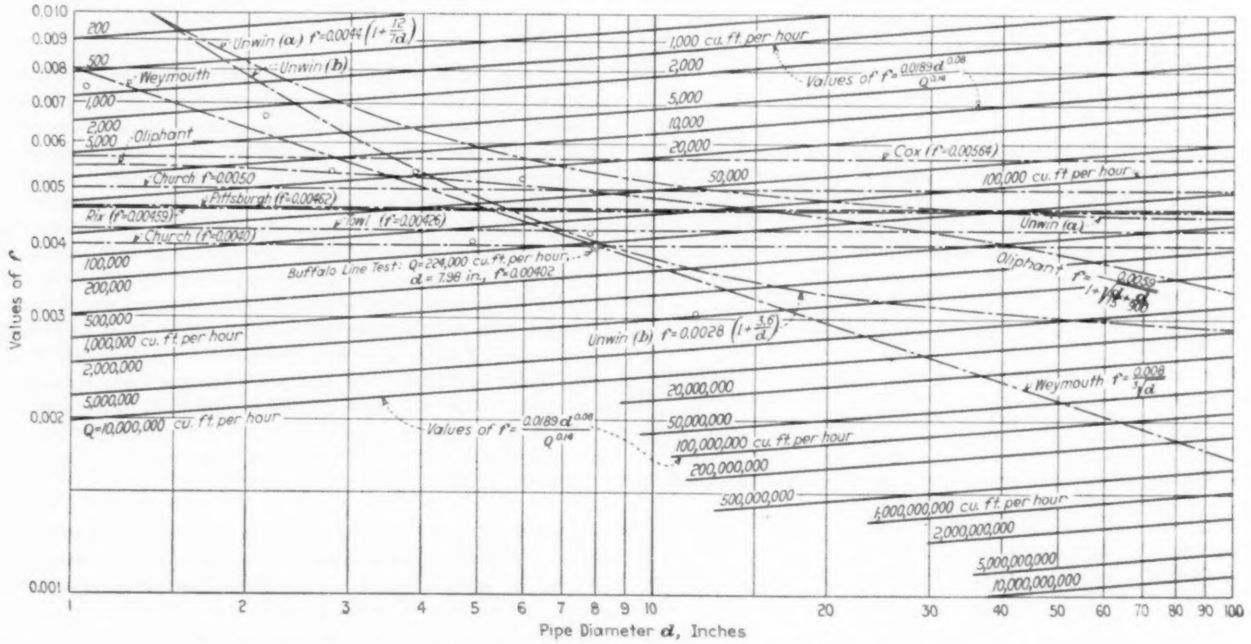

FIG. 5 VALUES OF f COMPARED WITH BEST-KNOWN FORMULAS FOR HIGH-PRESSURE GAS FLOW

[Showing values of coefficient of friction $f = \frac{0.00746\ d^{0.08}\ T^{0.14}}{Q^{0.14}\ G^{0.105}}$. $f = \frac{0.0189\ d^{0.08}}{Q^{0.14}}$ when $G = 0.60$ and $T = 519.6$ (60 deg. fahr.) compared with values of f derived from eight best-known formulas for high-pressure transmission of gas. Note: Substituting for f in $Q = \frac{\sqrt{3245\ (p_1^2 - p_2^2)d^5}}{fGTL}$ gives the complete formula $Q = \frac{1075\ d^{2.645}}{G^{0.461}\ T^{0.613}}\left[\frac{p_1^2 - p_2^2}{L}\right]^{0.539}$. Notation: Q = flow, cu. ft. per hour at 60 deg. fahr. and 30 in. Hg; d = diameter of pipe, inches; L = length of pipe, miles; p_1 = initial pressure, lb. per sq. in. abs.; p_2 = final pressure, lb. per sq. in. abs.; G = specific gravity of gas-air = 1; T = temperature of gas, fahrenheit absolute; o = tests upon which Weymouth's formula was based.]

The symbols are the same as in the paper except Q, which is expressed in cubic feet per hour at 60 deg. fahr. and 30-in. mercury pressure.

This formula is based on a carefully laid theoretical structure coordinated with the results of several hundred experiments involving pipe sizes up to and including 36 in.

Fig. 5 shows values of f as determined by the foregoing formula compared with the best-known formulas for high-pressure gas flow:

In order to show the formula graphically, temperature T is assumed at 519.6 (60 deg. fahr.) and specific gravity G at 0.60, which are average figures in natural-gas transmission practice. This leaves:

$$f = \frac{0.0189\, d^{0.08}}{Q^{0.14}}$$

The experiments on which Weymouth's formula is based are also spotted in on the chart and the value of f for the Buffalo line test is shown. It will be noted that this test gives almost the identical value as Weymouth's formula, also Church's formula with $f = 0.004$; but this is the only size pipe for which these two formulas (Church's and Weymouth's) agree.

The chart is largely self-explanatory and shows clearly why Weymouth's formula is the favorite in the gas fields where maximum flow is the rule. Weymouth's is the only one of the commonly used formulas which gives approximately correct results at large flows through the larger pipe sizes, while other formulas (Oliphant's or Unwin's for instance) are more accurate at lower velocities.

A study of Weymouth's formula indicates that reasonably accurate results may be expected when applied to gases of near 0.60 specific gravity flowing through pipes from 4 to 24 in. in diameter and with actual mean velocities from 15 to 30 ft. per sec. and temperatures near 60 deg. fahr. Any considerable departure from these limits will bring uncertain results. For example, an actual test on a 24-in. gas main carrying 420,000 cu. ft. per hour of coke-oven gas, specific gravity 0.38, and velocity about 40 ft. per sec., gave for the coefficient of friction $f = 0.00424$, whereas Weymouth's formula gives $f = 0.00278$. This difference is largely due to the low specific gravity and viscosity of the coke-oven gas. The chart indicates for any given size of pipe the flow for which Weymouth's formula is correct.

Considering everything, Weymouth's formula has served its purpose remarkably well in view of the fact that it was based on but nine rather uncertain experiments with compressed air flowing through pipes varying in size from 1 to 12 in.

Author's Closure

The tendency in the natural-gas industry has been toward the use of steel pipe walls unnecessarily thick to withstand peak operating pressures. The majority of these overly conservative selections have been attributable to uncertainty concerning the efficiency of welding processes and to engineers' natural inclination toward large factors of safety. Although the degree of effectiveness of commercial protective coatings is a much-debated subject, the use of excessive weight of pipe as a rust-resistant agent, except under extremely corrosive conditions, is uneconomical. The formula introduced by Mr. Biddison has an excellent application in large-diameter transmission lines, but for all sizes smaller than $10^3/_4$ in. o.d. would lead to the selection of wall structures too thin to withstand normal construction practices. Distribution-system engineers are the most persistent in adhering to erroneous policies of specifying steel pipe of superfluous weight.

Subsequent to the date of the discussion, Mr. Biddison presented an excellent paper to the American Gas Association entitled "Compression and Transmission of Natural Gas," embodying a mathematical demonstration that about 1.35 is the economical compression ratio. It is probable that the necessity for a standby unit in each compressor station would tend toward increasing this ultimate design ratio of a compression to about 1.7.

Too much emphasis cannot be placed on what Mr. Naiman has said with reference to "market" analysis and "rate making." Conclusions drawn from any source other than thoroughly accurate studies of the fuel economics of each plant are necessarily misleading. In situations where town distribution systems operate as entities not affiliated financially with the pipe-line system, the desirability of soliciting the load of an industrial prospect must be weighed in the light of the investment required by the distribution concern to serve the individual customer. Not the least important factor in "market" analysis is (inasmuch as a considerable period of time elapses between the preparation of initial load data and completion of large pipe-line systems) the forecast of the general economic trend of the entire area to be supplied. Due to the recent expansion of the natural-gas business, many pipe-line systems are faced with strangulation in developmental stages by adverse influences of general decline of prosperity in the service area.

The existence of "Reynolds' criterion" as a factor conditioning the coefficient of friction for the flow of fluids in pipes has been long recognized. Mr. Dunning's chart has emphasized graphically the important bearing the criterion has on this coefficient, but it should be remembered that this factor modifies Q inversely as the square root. Principal errors (in addition to those mentioned in the footnote and above) in the Weymouth formula as applied to natural-gas transmission occur when any of the following conditions obtain: (1) Welding "icicles," (2) abnormal internal pipe roughness, (3) unusual specific gravities, and (4) radical departures from ordinary gas velocities. In normal cases these refinements to the empirical coefficient of friction used in Weymouth's formula are largely academic in interest to the natural-gas industry, but the development of a technically accurate formula having mathematical facility is one of the steps of progress yet to be made.

PME-54-10

Modern Oil-Lubrication Methods as Applied to Rolling-Mill Equipment

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Jos. A. Merrill

The paper traces the improvement made in lubricant-application methods for rolling-mill machinery from earlier days to the present. The increase in size, speed, and capacity of rolling-mill machinery necessitated the use of heavier oils and of better methods of application. The modern heavy-oil circulating system is described, and common practice in the selection of oils for various types of equipment is indicated. Modern oil conditioning and application give maximum safety and protection against involuntary shutdowns and cut costs of lubrication as much as 80 per cent.

Edwin M. May

NO INDUSTRY has had greater problems of lubrication than the steel industry with its rolling mills. Massiveness of machinery; all machines in exposed positions subjected to dirt, to metal scale, and to abnormal heat and cold; enormous consumption of power; large acreage of plants; necessity of continuous operation; necessity of automatic machinery for handling hot and cold metal—all are most important factors of the problem. With this heavy equipment working under the strains of constantly increasing production, the fact has been many times evident that lubrication consists of three elements: the machine, the lubricant, and the method of application. When either the machine or the lubricant has reached its limit of endurance, further advance is possible only through the development of the systems of oil application.

Early Rolling-Mill Lubrication

The evolution of the rolling mill from the period of animal and water power to that of steam power brought along the ancient methods of hand-applied lubrication with the animal and vegetable fats and greases. Only 40 years ago the cast pinions were lubricated by hand, at intermittent intervals, with stiff tar products. Pinion and mill bearings were made with recesses or pockets in which tallow or grease could be packed. Roll necks were packed with sides of pork, with tallow, or with hard lime greases. During the period when the mineral oils were being introduced, many mill bearings were packed with wool waste saturated with the cheapest black oil. Later, due to the availability of the better mineral oils, ring-oiling bearings were introduced for the reduction gears and other large shafts. Pinion and other large bearings were lubricated by large oil cups, and cast housings permitted dip or bath lubrication by the use of the semi-plastic lubricants for both reduction gears and pinion gears. From the standpoint of lubrication this era was one of transition from the use of the animal and vegetable oils and fats and the heavy tars and greases to mineral oils; from the mill standpoint it encompasses a change from the crude, inefficient, short-lived machinery to modern highly developed, efficient machinery.

Every improvement in machine design that embodied a new method of oil application allowed of the use of a better lubricant, with greater production results for the unit. Tremendous strides have been made in this direction; otherwise production never could have grown from 5,236,000 tons of finished material in 1887 to 41,070,000 tons in 1929. Those familiar with the present trend can plainly see that the economic production of steel depends not so much upon the volume of equipment employed as it does upon the design and arrangement that will make each unit produce the greatest tonnage. Larger ingots are now going through the blooming mills with greater speed than ever before. Continuous mills are being rapidly installed wherever production can be speeded up. Electric power is rapidly replacing steam. Every such advance in the mechanics of the situation requires an advance to be made in lubrication, either through an improvement in the oils themselves or in the methods of their application.

Early Rolling-Mill Oiling Systems

The first rolling-mill oil-filtering and oil-circulating systems using light oil served only the ring-oiled bearings of the reduction gears, the bevel-gear-drive bearings, pedestal bearings, and in some cases the pinion bearings. Pinion and reduction gears were lubricated by dipping them in a bath of heavy gear lubricant.

The major objection to this practice of using light oil for the bearings and heavy-gear lubricant for the gear baths is the tendency of the light oil to work into the gear lubricant and of the

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[2] District Manager, Lubrication and Filtration Division, S. F. Bowser & Co., Inc. Assoc-Mem. A.S.M.E. Mr. May was graduated from Purdue University in 1903 with degree of Mechanical Engineer; was engineer, Traylor Engineering Company, 1905–1908; engineer, Richardson-Phenix Company, 1908–1913; manager, New York office, Richardson-Phenix Company, 1913–1921, and when this company was consolidated with S. F. Bowser & Co., he remained in same capacity. Mr. May has written on lubricating systems for various trade papers and has specialized in the study of lubrication.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Fig. 1 Gravity-Type System

(The dirty oil enters the cleaning system at 21, and after being freed from water and foreign substances, returns to the machinery through 29.)

Fig. 2 Pressure-Type System

(The size of the unit is varied to meet the conditions in the plant.)

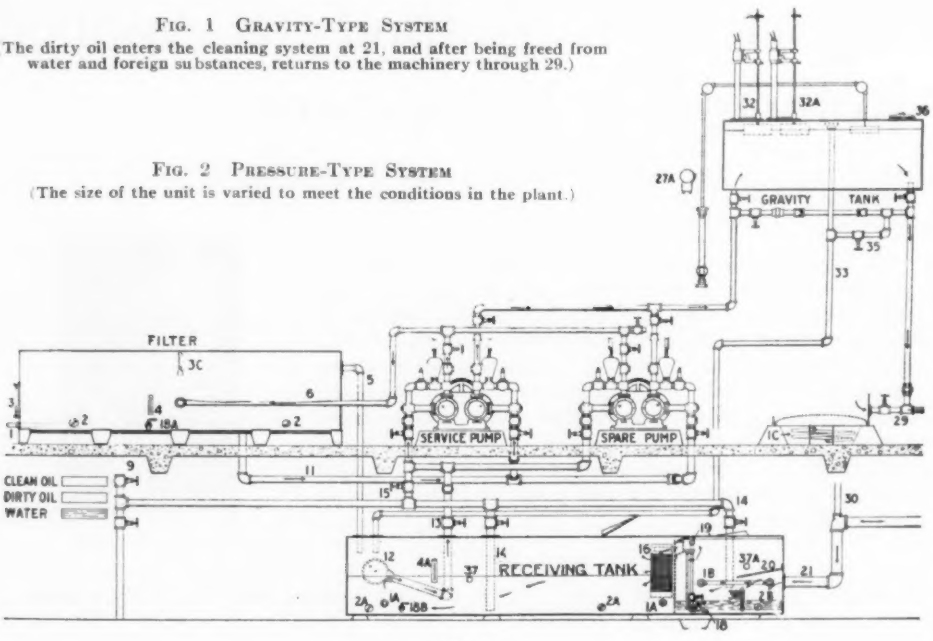

Fig. 1

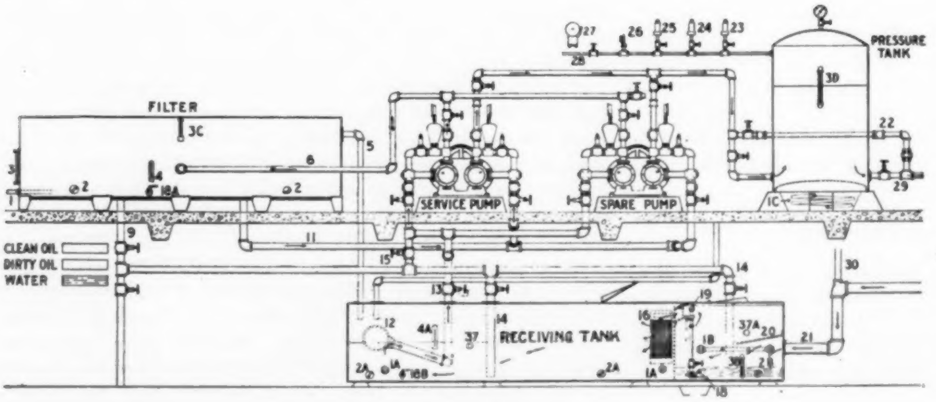

Fig. 2

gear lubricant to work into the oil, with the result that the mixture becomes too heavy for use in a ring-oiled bearing and too thin for the bath-lubricated gears. To offset this, elaborate sealing arrangements using oil slingers and grease wipers have been designed, but generally speaking, a mixture of the light and heavy lubricants is likely to occur sooner or later. In the pinions especially, where overall length must be kept down to a minimum, the extra length required for the sealing arrangements is obviously undesirable.

This practice of dipping gears in a bath of very heavy lubricant on large mills results in two other difficulties, the importance of which varies with the conditions in each mill. These are excessive power consumption, especially during the warming-up period after a shutdown in winter, and the tendency to excessive heating due to fluid friction when dipping gears in heavy lubricants. If the gear lubricant selected is too heavy or if the teeth dip too far in the bath, this heating may become so excessive as to reduce greatly the viscosity of the gear lubricant under continuous operating conditions.

Lubrication of Gears and Bearings With a Single Comparatively Light Oil

As the modern continuous mill developed, great improvement was made in the design, material, and methods of machining gear teeth, thereby providing uniform contact between accurately cut high-carbon-steel surfaces.

Hoping to provide adequate lubrication for both gears and bearings with a single grade of oil, a number of oil-application systems were installed in which pinion gears and reduction gears were sprayed with oil of 500 to 600 sec. Saybolt viscosity at 100 deg. fahr., the same oil being used for the bearings. With the introduction of the fully housed spiral-bevel-drive gears from main shafts to pinion stands, these gears were also spray-lubricated with oil of about 500 sec., the first example being the 14-in. Morgan mill for the Inland Steel Company at Indiana Harbor. As an additional safety factor, gears and pinions were permitted to dip in a bath of light oil on these early systems.

Where the gear teeth were properly cut, of suitable hardness, and run at fairly high speed, these early lubrication systems were amazingly successful, showing little wear on the gear teeth and providing ideal lubrication for the bearings. They demonstrated the advantage of a single light lubricant and a single-oil circulating and oil-filtering system for important parts of the mill.

These early circulating systems for rolling-mill gears and bearings were similar to the gravity-type filtering systems used on steam engines and on other big machines, except that they were much larger.

Increased Mill Size and Higher Tooth Pressures Demand Heavier Oils

With the rapid and tremendous increase in size, speed, and capacity of continuous rolling mills, it soon became apparent that oil of 500 to 600 sec. Saybolt viscosity at 100 deg. fahr. was not heavy enough to cushion some gear teeth properly and not nearly heavy enough for blooming-mill or universal-mill high-tooth-pressure gear lubrication. As a consequence the large oil companies developed heavier and heavier oils, with excellent cold test and good water-separating ability, to meet this increased service demand. At the present time almost any viscosity of oil desired may be obtained, and these will be suitable for nearly all normal tooth pressures. Using these heavier oils in the modern completely automatic circulating and filtering system, the largest blooming-mill, universal-mill, and continuous-mill pinions are being most successfully lubricated with the same oil for both bearings and gears.

MODERN HEAVY OIL-CONDITIONING AND APPLICATION SYSTEM

Briefly, a modern central oil-application system for any type of rolling mill consists of the same essential apparatus, varying in size and detail design with the mill served, and particularly with the viscosity of the oil selected. A typical automatic circulating system is shown in Figs. 1 and 2. A large receiving tank placed in the basement or in a pit receives all used-oil drainings from the gear cases and bearings. This tank has three distinct purposes: to remove and eject water from the oil in the system, to maintain a temperature of the oil of approximately 100 deg. fahr. minimum in cold weather, and to settle out as far as possible all heavy foreign matter from the oil. In addition, this tank must have sufficient capacity to receive the drainage of all oil piping and gear cases when the mill is shut down.

This large receiving tank is equipped with ingenious baffles, steam coils with automatic temperature control, and floating suction pipes, and is sometimes very useful when used for heating the entire batch of oil to remove traces of temporary emulsion that may occur through excessive water leaks at some point on the mill.

Service and spare pumps automatically controlled by pressure switches draw oil from the floating suction pipes in the receiving tank and deliver it to an overhead tank, as shown in Fig. 1, or with the full forced-feed systems, as shown in Fig. 2, to a cylindrical air-pressure tank of considerable size located in the oil cellar or on the mill floor above it, automatically maintaining a working pressure of about 35 lb. in this tank. The upper third of the tank capacity is occupied by air under 35 lb. pressure, and to this air space are connected pressure switches for starting and stopping the service and spare pumps, a third pressure switch for an automatic alarm, a pressure gage, and a safety valve.

In a full continuous-feed system all oil is forced through pressure filters, which remove any foreign matter that might pass the receiving tank. However, very large systems generally operate on the partial-filtration principle in which about 25 per cent of the oil actually circulated is continuously pumped through pressure filters by means of separate filter pumps. It has been found that this effectively keeps the entire batch of oil in perfect condition.

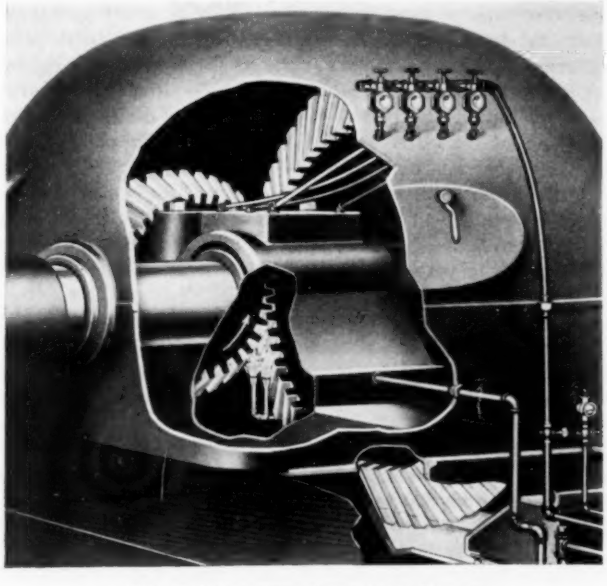

FIG. 3 FLATTENED-STEEL-PIPE SPRAYS

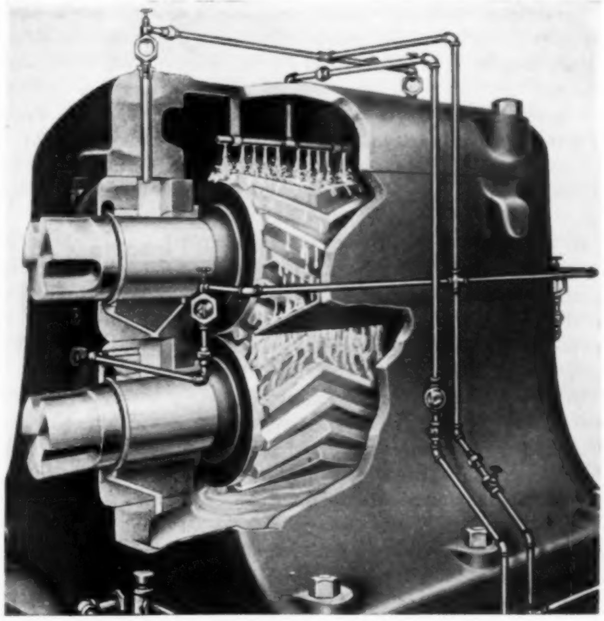

FIG. 4 PERFORATED-PIPE LUBRICATOR FOR BEVEL GEARS, PINIONS, AND BEARINGS

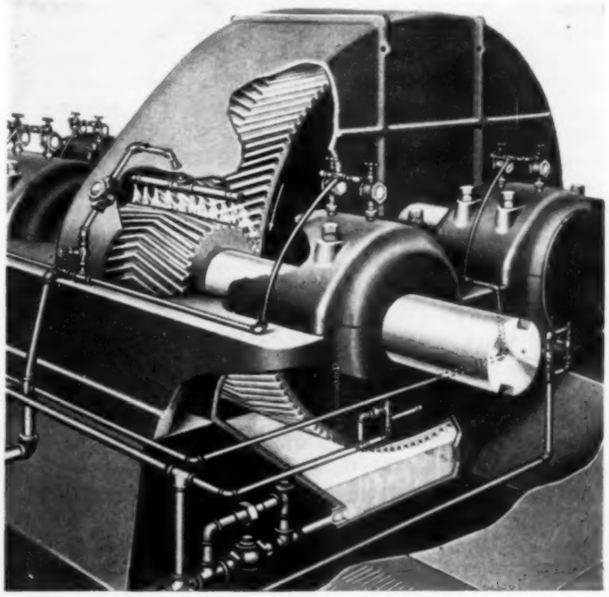

FIG. 5 FLOOD AND BATH LUBRICATION APPLIED TO LARGE HIGH-SPEED GEARS

In a partial-filtration system, therefore, the main service pump delivers oil directly through the pressure tank to the mill bearings. In either type the supply to the bearings is controlled by large needle-valve sight feeds, which are vented. The supply to the gear sprays is controlled by globe valves, using sight flow indicators to indicate whether oil is flowing to the sprays.

The continuous application of an adequate volume of cool, clean oil to the gear surfaces over their entire length and at the point of mesh is very desirable and efficient in the lubrication of gears with even heavy oil. This is accomplished by one of these methods:

1 Flattened-steel-pipe sprays (see Fig. 3) which throw a sheet of oil into the gears at the point of mesh, now generally used for all reduction-gear drives, for bevel-gear drives, and for many pinions.

2 A perforated pipe of all-welded construction (see Figs. 4 and 5) is introduced directly above the point of mesh for some down-running single-reduction gears and above the top center line of the upper pinions in pinions housings, where pinions are reversing.

In the latter case, oil from the perforated pipe is thrown off against the side of the pinion housing and led back as near to the point of mesh as possible by means of sloping baffle plates bolted to the inside of the housing. The advantage of this design on reversing pinions is that only one set of sprays is necessary, regardless of the direction the gears are traveling.

Once started, the heavy-oil-application system is entirely automatic. A pressure switch controls the service pump, automatically maintaining the desired pressure in the pressure tank; a second pressure switch cuts in on a slight reduction of air pressure in the pressure tank and starts the spare pump; and a third pressure switch is provided to operate on a further reduction in pressure to sound a horn, light a red lamp, or ring a gong to warn the operator that both service and spare pumps have stopped.

Should both pumps fail, there still remains in the air-pressure tank a large volume of oil under air pressure, sufficient to supply the gears and bearings during the interval between the final alarm and the shutting down of the mill. So far as is known, it has never been necessary to shut down a mill due to failure of both pumps.

With heavy-oil-application systems it is of course necessary to protect the drain lines, and in fact much of the supply lines, from extremely low temperatures. Where these lines are exposed, a small low-pressure steam line can be run parallel to them, all three lines being wrapped with insulating material, usually wool felt wrapped with roofing paper.

Classification of Mills for Lubricants

For purposes of selecting lubricating oil for the modern application systems, rolling mills might be classified as follows:

1 Blooming, heavy skelp, heavy structural, plate, rail, slabbing, universal, wide strip (over 30 in.).

2 Bar, billet, heavy merchant, light skelp, medium strip, medium structural, tin-plate mills.

3 Cluster, light merchant, light strip, rod, wire, both ferrous and non-ferrous.

Without going into the subject of oil analysis, it may be useful to indicate in a general way the common practice in the selection of oil on the basis of viscosity alone for the following various types of equipment:

Blooming, Heavy-Skelp, Heavy-Structural, Plate, Rail, Slabbing, Universal, Wide-Strip Mills. Gears and bearings of these heavy mills are being successfully lubricated with oil averaging 1700 to 2600 sec. Saybolt viscosity at 100 deg. fahr., either with or without dipping of the lower gear in a bath and with two sets of sprays or two baffle plates on reversing mills for each point of mesh. This oil should be a high-grade straight mineral oil with a suitable pour test and excellent demulsibility, the viscosity selected depending a good deal on the actual tooth pressures anticipated. The oil should be free from animal or vegetable fats to avoid emulsion trouble in case of water leaks.

On very large heavy-duty continuous mills where the roughing-mill pinion speeds are extremely low and the pinion-tooth pressures very high, as on certain exceedingly large recent mills, it is desirable from every standpoint to provide a separate application system using a very heavy oil, say, 2600 sec. Saybolt viscosity at 100 deg. fahr., for the pinion gears and their bearings, and a lighter oil, say, 1500 sec. Saybolt viscosity at 100 deg. fahr., for the reducing gears and their bearings. While this arrangement of two systems on a single mill costs a little more to install, it is frequently well worth while and avoids excessive tank and pump sizes. An example of this is the 60-in. continuous strip mill of the Wheeling Steel Company at Steubenville, Ohio.

On very large continuous mills tooth pressures may be so high on the roughing-end pinions as to make lubrication with mineral oil undesirable. In such cases the bearings and gears of the roughing-end pinions can be supplied with a special leaded lubricant from a separate oiling system operating at normal temperature. These parts may be also lubricated with a very heavy gear compound, using steam coils in the gear cases and automatic temperature controls. This will allow the heavy gear compound to be pumped from the bottom of the gear case into each point of tooth mesh, filtering it by means of a pressure filter in batches at intervals and at high temperature to reduce the viscosity for ease of filtering.

Generally speaking, if the gear teeth are made of a high-carbon steel, accurately machined and aligned, heavy oil lubrication of the teeth is entirely practical, provided the unit loadings are within the load-carrying capacities of the materials used. But with low-carbon-steel gears within excess of the load-carrying capacities of the materials used, it is probable that wear will occur with either heavy oil or heavy gear lubricants due to overloading. A very heavy soap-thickened oil using lead oleate or sulphur compound is suggested for such cases.

Bar, Billet, Heavy-Merchant, Light-Skelp, Medium-Strip, Medium-Structural, Tin-Plate Mills. Bearings and gears on these modern continuous mills are successfully lubricated with the continuous application system and with a straight mineral oil, viscosity ranging from 800 to 1700 sec. Saybolt viscosity at 100 deg. fahr., depending on the pinion-tooth pressures and provided the mill is of average size and speed.

Cluster, Light-Merchant, Light-Strip, Rod, and Wire Mills, Ferrous and Non-Ferrous. High-speed, light-duty rolling mills with comparatively low tooth pressures and high-carbon-steel gears properly cut are still being very successfully lubricated by central oil-circulating systems using oil of 500 to 800 sec. Saybolt viscosity at 100 deg. fahr.

Vertical Edgers, Isolated Drives, and Auxiliary Equipment, Such as Reel Winders, Scale-Breaker Drives, Etc. Vertical edgers are quite likely to have water leaks and are generally set so low that drainage from the base is difficult and the gear-tooth pressures involved are usually low. For all of these reasons a lighter oil, of 800 to 1200 sec. Saybolt viscosity, easily circulated and with excellent water-separating ability, may be very desirable, and in some recent mills the vertical edgers have been placed on a separate oil-circulating system.

If the vertical edgers are placed on a separate oiling system, there is no objection to including other small units on the same system, and on a recent very large installation at South Chicago this was done with excellent results. Sometimes an isolated gear case is best handled by a very small individual full-continuous heavy-oil application system to avoid excessive length in the drain and supply lines and to avoid lowering the basement-floor level where necessary for draining such isolated cases by gravity. On the general run of mills, however, all units should be and in most cases are successfully served by a single large central system.

Cold-Rolling Mills. In general, cold-rolling mills are being lubricated with individual or central application systems, using oils ranging from 800 to 2500 sec. Saybolt viscosity at 100 deg. fahr. The selection of oil depends very largely on the tooth pressures and on the kind of steel used in the gears.

Cold-rolling mills frequently present a combination of relatively low pressures and fairly high speed on the reduction gears, which makes this part suitable for light oil lubrication with very high tooth pressure on the pinion gears, which run at very low speed and require the heavier lubricants. In designing systems for cold-rolling mills great care should be taken to check the tooth pressures accurately, providing oil heavy enough for the pinion gears, even if it is necessary to supply these gears with a separate lubrication system. Many large cold-rolling mills are now being lubricated with a single oil in the system supplying all gears and bearings. This oil averages around 1500 sec. Saybolt viscosity at 100 deg. fahr., or lighter where possible.

Rolling-Mill Tables, Hot Saws, and Cold Saws. The gear drives used on saws and mill tables run with relatively low tooth pressures and operation is intermittent, so the oil never overheats through fluid friction. The lubricant is also exposed to the lowest temperatures in the mill. For these reasons gears and bearings of these units are satisfactorily lubricated by means of small application systems, circulating oil of about 500 sec. Saybolt viscosity at 100 deg. fahr., using bath or spray lubrication for the gears and sight-feed lubrication for the bearings. Flexible tubing is employed to take the oil to and from the sliding parts of the big saws, and the oiling system is located in a small pit adjacent to the equipment. Most table rolls are now designed for single connection to the circulating oiling system for each section, oil distribution to the gears and bearings within the table being accomplished by rather elaborate small pressure piping inside the table or by dipping the gears in a bath and providing wipers to scrape the oil off the gears and feed it to the bearings. In the latter type, oil may be continuously changed, heated or cooled, and filtered while keeping the bath at a constant level. Oil for either of these types of equipment must have a suitable cold test and should be a straight mineral oil with excellent water-separating ability.

OTHER ROLLING-MILL OIL-APPLICATION SYSTEMS

Roller-Bearing Lubrication. On modern four-high mills the large roller bearings have been successfully lubricated by means of circulating-oil systems which automatically provide a large volume of clean, cool oil averaging about 500 sec. Saybolt viscosity at 100 deg. fahr. to each bearing.

Four-high mills equipped with roller bearings are also being lubricated with light grease applied with any of a number of ingenious hand-operated or automatic grease systems.

With the development of very large and high-speed roller bearings for mill drives, it is evident that a most important function of the oil is to continuously flush the bearings, cleaning out any abrasive dirt or material, as well as to carry off the heat developed in the bearings; and for this purpose a special type of system is already in use, feeding oil to the bearings under pressure and drawing it out of the bearings under pump suction, the oil being continuously settled and filtered and afterward cooled with a surface cooler during each cycle.

Large Motor-Generators and Motor-Bearing Lubrication. On modern large flywheel motor-generator sets without water cooling in the bearings, there is a tendency toward high operating temperature in the inboard bearings, due partly to the heavy load, but particularly to induced heat along the shaft from the generator. Lubrication systems designed to provide and apply cooled oil to these particular bearings are usually extended to include the outboard bearings of the flywheel motor-generator sets and the ring-oiled bearings of the other large motors driving the mill. Ordinary motor bearing oil is usually recommended, having a viscosity of 300 sec. or less at 100 deg. fahr., and an overflow fitting is provided to maintain the oil level in each bearing.

This oil is delivered through sight feeds from a central pressure system under a pressure of about 35 lb., and instead of a large receiving tank, a special unit is provided to accomplish full, continuous filtration, cooling, and clean-oil storage in a single filter-tank shell. The first compartment of this tank contains the filter units which discharge filtered oil to a cooling compartment, where the oil passes over an ingenious system of baffled coils, finally collecting in a large clean-oil compartment, from which it is pumped through a pressure tank, with automatic pump control, to the bearings.

Besides furnishing cooled oil in large volume to the bearings that tend to operate at high temperature, this system has the advantage of providing continuous make-up and change of oil for all of the principal motor-generator and motor bearings. A considerable saving in lubricant and labor is therefore provided.

Lubrication of Prime Movers in Steel Mills. Steam turbines are frequently equipped with a continuous bypass batch-oil-filtering system in which a large percentage of the turbine oil is continuously circulated through a filter to remove water and insoluble sludge. At regular intervals the entire batch is drawn off into a large two-compartment tank and allowed to settle for about 14 days to remove any trace of moisture and sludge that would be in solution at operating temperature.

Large gas engines driving generators and blowers in steel mills are equipped with individual oil-circulating systems of large size, which automatically separate and eject water and filter the oil continuously.

CONCLUSION

In reviewing the development of rolling-mill lubrication during the last 15 years, one cannot escape the conclusion that the continuous oil-application systems that supply the oils for the lubrication of both gears and bearings offer very important economic advantages that can well be considered by all the mechanically equipped industries. In general the use of these modern systems has the following results:

(*a*) Makes lubrication of all large units an orderly automatic engineering function, thereby completely eliminating all dangers due to the careless failure to apply lubricant or to the misapplication of lubricant; eliminates also the dangers of dirt and water contamination of the oil and to damages due to these various causes.

(*b*) Eliminates the dangers resulting from the use of heavy gear lubricant which has been thinned by the light oil from the bearings; likewise prevents undue heating of the bearings, with consequent loss of power through the heavy gear lubricants working into the bearings.

(*c*) Lowers gear-case temperatures by reducing the friction, thus lowering the power required for the operation. It is well known that gear friction is almost directly proportional to the viscosity of the lubricant, temperature rise above the room temperature being a true indication of friction loss. Circulating systems will supply oil to the parts to be lubricated at any desired uniform temperature, thereby maintaining the proper oil viscosity for the job.

(*d*) Eliminates dirt and gritty material from the bearings, and through continuous lubrication with clean, cool oil the gears and pinions and all bearings have a longer life.

(*e*) Eliminates the cause of costly shutdowns due to machine and part failures through improper lubrication.

(*f*) The starting load, particularly in cold weather, is materially reduced.

(*g*) Lubrication economy. The important gears and bearings in many large mills are being adequately lubricated with a total consumption of approximately one barrel of oil per month, indicating cost savings of as much as 80 per cent against all former methods.

Generally speaking, the high-carbon heat-treated gears that have been properly machined and aligned show little or no wear over long periods when force-feed-lubricated with a clean and cool oil of suitable character. Modern heavy mineral oils, when delivered in volume and at the proper temperature at the point of tooth mesh, have remarkable good cushioning and lubricating properties for gears.

Such oil-conditioning and oil-application systems also provide maximum safety and protection against involuntary shutdowns, and it is for all of these reasons that the modern oil-application systems are in service on the majority of modern ferrous and non-ferrous mills, thus pointing out a very obvious line of lubrication development work for other branches of industry where lowering the costs of operation is desirable.

Discussion

William F. Parish.[3] There is a growing conviction that the continuous application of the proper lubricant through the modern oil-circulating system will reduce maintenance charges and lower the power required to drive production machinery. The saving in oil is slight compared to the value of the better and more continuous lubrication. The principles involved are simple. Keeping a continuous flow of clean, cool lubricant, under pressure, to all of the bearings of a machine is a rational thing to do. Up to the acceptance of these systems it was the custom to apply oil at irregular intervals to all hand-oiled bearings. Most of the oil was wasted. Dirt and contamination was never considered. No thought was given to the main function of the oil in its power of carrying the heat away from the bearing. In bearings that are not heated externally, the heat developed in the bearing is due, first, to fluid friction of the oil itself and then to the striking of the parts of the opposing bearing surfaces as the film becomes too thin for the load conditions. The heat thus generated is in the film of oil, which is immediately affected and before the heat can dissipate to the metal and through it to the air. With the ring-oiled bearing there is a continuous flow of oil to the film, but the backflow of heated oil is to a small reserve in a confined sump. This, however, allows the heat to be dissipated over a wide area. These bearings in special cases can be much improved by increasing the oil supply, having it in contact with greater surface of bearing housing so that there will be a greater path for conductivity of the heat of friction, by turbulence of the oil over a greater metal surface, or by putting a water coil in the sump.

The bearings in industrial machines that are the most difficult to lubricate are those where heating is applied to the shaft externally. This must be done in nearly all industries, either in the first process of handling heated materials, as in steel or iron rolling, or in the finishing process, as with paper and textiles. Hot rolls and their heated necks have always required the heaviest lubricants, with their heavy friction drags, their features of separation, and general danger due to thickening and drying. Shafting between a bearing and a fan that must work in a furnace flue is an extreme example of such a case. Such a shaft is always hard to lubricate. From a strictly engineering standpoint, without considering what can be done with lubrication, the first thought is to put in a water-cooled bearing. The next is to put in a hollow shaft and secure an induced draft of cool air with a blower. Both of these ideas come from a conviction that the excessive heat from the outside is the cause of the lubrication difficulties, which is right. The cure is applied after the damage has been done. The trouble is always caused by the heat effect on the working oil film, making it thinner, causing it to be reduced to a point where the high parts of the opposing bearing surfaces strike through. This sets up greater local heats, which can accumulate until there is film and bearing rupture.

As the flow of heat is always along the shaft, through the oil film to the outer bearing surfaces and then out through the housing tc the air, the hollow shaft will not entirely cut this flow from the film of oil. It will reduce it by effecting some radiation to the air from the inner surface of the shaft. There will always be heat flowing along the shaft, through the oil to the outer bearing. In the case of the water-cooled bearing, the heat flows through the oil film unrestricted, but at some increased speed due to its greater radiation through the circulating water, plus the bearing surfaces of the housing. When heat from the outside is allowed to be added to the heat of friction of the oil itself, the film of a very thick oil, which is always used on such a bearing, is not greater than the film of a very light machine oil that is kept properly cooled.

As all troubles with such heated bearings are caused by the film becoming too thin through the addition of external heat to the natural internal heat of oil friction, the obvious manner of correcting the condition is to treat the oil itself. All that is necessary is to apply a continuous stream of cool oil to the bearing surface and arrange that the oil supply be properly cooled by having a large amount of oil in the supply tanks or by putting a heat exchanger directly in the supply feed to the bearing. Then the oil should be as light as possible, so that it will work with reduced internal friction and will quickly absorb and carry the heat flowing along the shaft, this heat to be then easily liberated through the tanks or exchangers to the air.

Small circulating systems can be made locally with what is available around the plant—two tanks made out of oil drums, a small pump, a cooler (which can be a coil of pipe in one of the tanks), and the necessary piping. Hose can be used for a temporary job. The oil from the bearing flows to the under tank, which is the sump. The pump drives it to the upper tank, and from there it flows through the cooler, if an extra one is used, directly to the bearing.

In all new equipment, where it is known that extreme temperatures of operation will be encountered, it is just as well to include a circulating system properly engineered for the installation, even if there is only one bearing involved. This will pay for itself many times over and will allow the job to be put in operation with the full assurance that even if the heat encountered should be more than anticipated the flexibility of the system will fully compensate for it and there will be no danger to the operation.

In bearings where high heats are to be encountered, it is not wise practice to use grease. A bearing packed with grease retains its heat through the grease, which can only dissipate this heat through the metal of the housing to the outside air. This heat will so thin down the greases that some kinds will separate into their oils and thickeners, which may include large amounts of non-lubricating limes. Such thickeners will close the oilways or other clearances, will act as mild abrasives or polishes, and while smoothing the bearing surfaces will change the measurements gradually until they will have to be compensated for. When the grease film is heated, it becomes thin, the same as an oil film. The high spots of the bearing surfaces then strike and set up high local heats that still further reduce the film thickness. The surrounding grease that is packed in the housing of course rushes in and replaces the ruptured film, and this is the only security offered by the practice. As the grease thins through heat or separation of the solids, it leaks out of the bearing. As the amount decreases, the heat of the operation is absorbed by a smaller amount and conveyed through less housing surface, so that the changes which take place can be expected to be greater the longer the bearing is allowed to run on the single filling of grease.

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[3] Consulting Lubrication Engineer, New York, N. Y. Mem. A.S.M.E.

Consider the fundamental functioning of a circulating system attached to a bearing or a machine of many bearings. The oil is fed under pressure. The outlet of the feed pipe at the bearing is larger than the outlet of the clearances of the bearing. The clearances therefore govern the flow of oil through the bearing.

There is always a great supply back of the bearing of cool, clean oil waiting to flow through. Under normal conditions, those that exist most of the time, there is a steady flow of oil at the bearing, and its temperature is sustained in its established equilibrium. Then the temperature of the film, and of the surrounding metal, and of the oil in and out of the bearing is some degrees above the temperature of the surrounding air, with which it rises and falls.

Then comes a period of undue stress or of some interference with the film, and as a consequence the high points of the opposing bearing surfaces strike. Momentarily a very high degree of heat is possible through the pressure and rupture of the metal, as small parts of the oil will decompose, leaving a tar or carbon residue accompanied with the liberation of some gas. In the still at the refinery it is necessary to have temperatures in excess of 600 deg. fahr. at atmospheric pressure to liberate gas and form the tar and carbon residues. Few realize the existence of these high local temperatures that are the combined result of abrasion and pressure in the presence of the reduced oil film. These temperatures are not capable of measurement except by their residual effect. They are at best but momentary, but the heat is quickly absorbed by the surrounding film, raising its temperature and reducing its viscosity and film thickness. This, unfortunately, with all heating bearings lubricated by hand or drop oilers or that are packed with wool or grease, is at a time when the striking metal should be separated by a cooler and a thicker oil film instead of just the opposite. As there is no way of carrying the extra heat away or of increasing the thickness of the film, abrasion, once started, generally proceeds with these bearings until the film and then the metal ruptures.

In the case of a bearing attached to a circulating system, there is a much more favorable set of conditions existing. Should stress damage the film and allow the high points to strike and set up local heating, the oil becomes thin and flows faster in consequence. This brings in a greater amount of cool oil from the outside supply to add to the heated film at a time when it is needed. The increased outward flow of the heated oil, due to its being thinner, carries heat away just that much faster. The heated oil film, when there is a circulating system involved, has its own automatic and thermostatically controlled feed through the actual condition of the oil film itself. When this film suffers through overheating, it automatically cools down through its increased flow. When it is too cold, the oil flow is reduced, and internal friction sets up enough heat to bring the bearing back into equilibrium where the generation of internal heat exactly balances its absorption and radiation through the oil flow and bearing metals. This is one of the hidden reasons why there is so little trouble with machinery that is lubricated by the circulating system. From examination of the oil in such systems it is apparent that local heating has occurred at times. This has not been noticed by the attendants or indicated by the instruments. The automatic thermostatic control inherent in the system corrected the matter, although the oil showed every indication of having suffered severe punishment.

In the marine service, with the hand-oiled bearings and parts of the reciprocating steam engines, all bearings were fitted so that water could be turned on them to keep them cool in case of trouble. When such engines were modernized with a forced-feed system, condensed-water leakages from the steam glands and salt water from other leakages would get into the circulating systems and cause some trouble through forming emulsions that would cause stoppages of flow. There seldom were corrosion troubles on these older engines. This was undoubtedly due to the fact that the metal surfaces were so completely saturated with oil that it acted as a protection.

On many new ships fitted with circulating and forced-feed systems for the steam engines, turbines, or Diesel engines much damage was caused to the shafts and to all bright parts of the engines by a series of events that were the result of simple uncleanliness and of a lack of appreciation of the fact that the best protection against rust and corrosion is a good and penetrating film of oil.

In putting in a circulating system for a large machine of whatever type, the piping is threaded by the use of lard or similar cutting oil. The joints are red-leaded, and this carries linseed or some other vegetable oil. There also would be white lead from the shafts or cutting oils from the machined parts, with all the incidental metal chips, dirt, and foreign matter that can get into tanks and pipes during fabrication, shipment, and erection. Then upon heating the engines with steam and putting them slowly into service, leakages of water would always occur. The oil in the circulating system would churn up the combination of animal and vegetable matter, with other kinds of dirt acting as binders and possibly catalyzers, with the result that these first emulsions were very bad. They would settle on all bright parts of the shafts and machines, and the water oxygen contents, with the free fatty acids and possibly salts and other acids, would cause etching and pitting of the metal and general corrosion of all bright parts. This always happened during the first few days after starting the machinery. Examination of the bearings usually disclosed an unhappy state of affairs. In many cases several days of hard work would be necessary to stone the bearing surfaces back into shape. In other cases the original pitting and etching of the shafts, causing a change in bearing condition, undoubtedly explains some of the later trouble encountered with the bearings.

In several very large marine power plants, as well as in some of the earlier land turbine and steam plants, it was found that it was possible to prevent corrosion and serious emulsions simply by properly cleaning the entire circulating system before the power unit was started or the steam turned on. The method was to fill the system with a light spindle oil and allow it to be circulated freely by the pumps. This was done for at least a day. Then all of the oil was removed and put into the fuel. This practice had two objects: to thoroughly clean the system of all of the accumulated foreign material and to saturate all bright parts reached by the oil. On duplicate installations, where this practice would be carried out on one and not on the other, any cost involved by the cleaning and saturation process was repaid many times due to the entire lack of corrosion of parts and emulsification of oil. Where this cleaning was not done, shafts had to be taken out or stoned, lines had to be taken down and steamed out, and tanks had to be cleaned, and there was enough loss of circulating oil in the process of cleaning up afterward and in the emulsions that would not settle to more than pay for a preliminary cleaning of the system. In the study of these cases it was found that a circulating system attached to any machine, the bright parts of which have been corroded, subsequently gives continuous trouble until all of the iron oxide has been removed. Iron oxide acts as a sort of binder in the emulsion of water and oil.

In any new machine all bearing surfaces and other bright parts should be thoroughly impregnated with mineral oil before the machine is started. Any circulating or other lubricating system attached to the machine or bearing should certainly be thoroughly cleaned. The best manner of cleaning is to operate with a light mineral oil for at least 24 hours.

When erecting engineers are impressed with the necessity before starting up the machinery of having everything made as clean as hospital operating rooms, then corrosion will be almost a thing of the past so far as it affects bearings and the bright parts of all machinery reached by oil from a circulating oiling system.

A. G. **Witting.**[4] Fully realizing the importance of efficient lubrication, but on the other hand also the first cost of installation and maintenance and operating costs afterward, the writer would like to ask the question where the limit would be set for increased efficiency to offset the costs of a circulating system.

At one of our merchant mills there is a set of reduction gears running in a bath of oil. The speed at the pitch line is 1864 ft. per min. The gear teeth show no sighs of wear. The following is a comparison of the oil when new and after $2^{1}/_{2}$ years of operation:

	New oil	Used oil
Date of sample	Sept., 1927	April, 1931
Water	None	None
Sediment, per cent	None	0.08
Viscosity at 210 deg	128	134
Neutralization number	0.03	0.053
Flash, deg	470	480
Emulsion	Complete separation	

Leo Kresser.[5] For the lubrication of roll-neck bearings a simple and reliable apparatus has been built by this company. A number of small but powerful pumps are combined in one machine, with a common drive and a common container, which press the lubricant into the bearing with a pressure that will secure positive delivery. A pressure up to 3000 lb. per sq. in. can be overcome by these pumps, which is far more than the lubrication of the roll necks requires.

The amount of lubricant delivered to each feed point can be adjusted to the exact needs of the bearing. The apparatus is easy to install, requires only a fraction of a horsepower to drive and when installed and adjusted needs no further attention.

F. P. **Dahlstrom.**[6] Oil-circulating systems have been applied very extensively to mills of our manufacture for the past ten years. At first some of the main drives were lubricated with a gear-shield type of lubricant, but during the last five years practically all installations have been designed so that all gears and bearings in the mill proper are lubricated with oil from circulating systems such as those mentioned in the paper. Such systems have been operating with entire satisfaction.

Because of the comparative transparency of the oil used, it was early noticed that certain gears in these installations were pitting. In many cases this was incipient and soon stopped, and in certain other cases where pitting appeared abnormal there was the expected reaction that the use of oil as a lubricant for heavy loaded gears was the cause of such pitting. Research has shown, however, that this condition is the result of exceeding the compressive fatigue limit of the material from which the gear is made. Below such limits gears will operate indefinitely without wear, and above this limit pitting (which is oftentimes wrongly called wear) is certain to occur. The dislodged particles in being scraped across the surface of the teeth often score the teeth and thus conceal the real cause.

It is obvious that the surface stresses between the cylindrical tooth surfaces in contact are very high and may be from about 100,000 lb. per sq. in. in some of the better grades of steels used in rolling-mill work up to as high as 250,000 lb. per sq. in. where small case-hardened automobile gears are used. During the past year wear tests on different gear materials have been conducted for us with the Lewis gear-testing machine at the Massachusetts Institute of Technology under the direction of Professor Buckingham. Severe loads have been applied to test gears made of different materials. These loads cause equal reacting loads on hardened and ground master gears in the Lewis machine. These master gears have been used for several thousand tests in the course of such work during the past six years and have broken down many surfaces of various materials. During all these tests the gears have been intermittently lubricated with oil. It is interesting to note that the surface of these teeth, even under a microscope having 25 magnifications, shows only a few minor scratches.

Investigation shows that slow-speed mill pinions of heavy-duty mills are stressed beyond the compressive fatigue limit. Correction of such difficulties is largely an economic one. It is probably cheaper to discard the pinions after many years of use rather than to design disproportionately large pinions and housings which would require valuable space. The writer has examined the pinions in a sheet-bar mill which ran through the World War and which were lubricated by dipping in a light gear shield. After five years of wear the tooth thicknesses of these pinions were about from one-half to two-thirds of their original value. He also has observed the tooth thickness of mill pinions on a similar mill which also ran through the World War, but which were lubricated by dipping in a light oil, approximately 400 sec. at 100 deg. The tooth thickness of these pinions was practically the same as those in the mill first cited.

During the last few years certain special oils have been developed for highly stressed gears. The introduction of the hypoid gear with its more pronounced sliding tooth action was one of the first factors in creating a need for such lubricants. Subsequently, special testing machines have been exhibited, and the resulting data show that ordinary oils break down at about 8000 lb. per sq. in., while the recently developed lubricants apparently show a very much higher film strength.

Knowing that gears made from ordinary steels can easily withstand surface stresses of nearly 100,000 lb. per sq. in. without lubrication failure, it is our belief that high local temperatures in several of these testing machines actually break down the film and not pressure alone. A lubricating film is a very strong and tenacious thing provided we do not burn it and thus destroy it. There is no such tendency in modern, correctly designed gears.

Sulphur and other such substances that now are introduced into special lubricants tend to prevent welding, which again seems to prove that localized heat and not pressure in itself must be considered. Until these temperatures are known, existing pressure data in connection with the determination of oil film strength are not absolute, but only comparative.

It would be interesting to see the results of such pressure data recompiled with respect to the flash and fire points of the oils instead of their viscosities.

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[6] Morgan Construction Company, Worcester, Mass.

RP-54-1a

Once-Through Series Boiler for 1500 to 5000 Lb. Pressure

By H. J. Kerr,[1] New York, N. Y.

The paper covers the development of series-type boilers from the early stages which resulted in the Calumet steaming economizer type now in extensive use to the once-through series boiler. It describes the operation of the latter units from 1500 to 5000 lb. per sq. in., and test data are given. The limiting factors of this type of unit, such as equalization of flow, control with variable capacity, and operation, are discussed. Heat-transfer data on high-pressure superheated steam are presented.

An increase in steam pressure has always been accepted as a means of increasing the efficiency of power generation. The disagreement between those favoring a lower pressure of, say, 300 or 400 lb. per sq. in. and those favoring a higher pressure has been due largely to the commercial question of the final total cost. The curve in Fig. 1, which shows the cycle efficiency for various pressures, may be open to some question as to the exact value, but it is felt that the general form of this curve, which shows decreasing rates of the efficiency gain with an increase in pressure, will be accepted as correct.

Obviously if higher-pressure plants are built of apparatus designed for low pressure, with the various parts increased in thickness to withstand the higher pressure, the cost of such plants will increase materially with pressure increase. As the increase in efficiency to be obtained with increased pressure is a decreasing quantity, the highest pressure which will prove to be commercial will depend largely on the ability to design and build apparatus for the production and use of high-pressure steam at a minimum increase in cost.

Recognizing these principles, the Babcock & Wilcox Company has been engaged on the problem of developing types of boilers to meet these requirements so as to permit of the highest possible commercial pressure.

One of the first developments was the series or two-stage boiler (Fig. 2), as described by Dr. D. S. Jacobus in his paper, "Present Practices in Steam Generation in the United States," presented at the First World Power Conference in London in July, 1924.

The first commercial installation of the type of unit shown in Fig. 2 was at the Calumet Station of the Commonwealth Edison Company, the unit being put into service in November, 1926. The success of this type of unit is perhaps best demonstrated by the number of high-pressure installations now in service, such as the last units at Edgar Station in Boston, Deepwater, Holland, State Line, and other stations. Fig. 3 shows the side elevation of the new units now under construction for the State Line Station extension for 1400 lb. pressure. This illustration also shows the latest slag-tap furnace and dust screen beneath the boiler.

The next development was the drumless series boiler (Fig. 4), described by Dr. Jacobus in his paper, "Developments in High-Pressure Boilers," presented before the Engineers' Society of Western Pennsylvania in October, 1927. In this boiler all boiler drums were eliminated, forced circulation was used, and various percentages of excess water were passed to a separator. Water was drained continuously from the separator, and the rate at which the boiler was fed was adjusted so as to hold the height of the water in the gage glass within a given range. While no commercial installations of this type of boiler have been made, the test unit was operated over long periods of time at 600 to 650 lb. pressure and was of sufficient size to determine various unknown quantities so that a commercial unit of this type can be built where conditions would warrant.

It will be noted from the preceding statements that investigations were started along a certain path in boiler design leading to a boiler which would consist of a set of tubes only, into which water would be pumped and from which steam at the desired pressure and temperature would emerge.

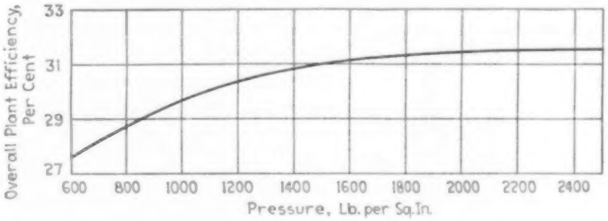

Fig. 1 Overall Efficiency of Steam Cycle for Various Pressures

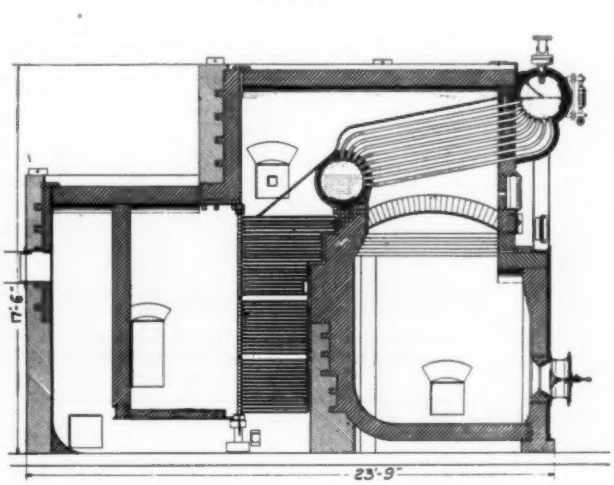

Fig. 2 One of the Series Boilers Tested

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[1] Chief Engineer, Service Department, Babcock & Wilcox Company. Mem. A.S.M.E. The author was graduated from Webb Institute in 1907. He was engineer officer, U. S. Coast Guard, in 1909. Mr. Kerr is in charge of research development and service of the Babcock & Wilcox Company, having been connected with that department since 1914.

Contributed by the Special Research Committee on Critical Pressure Steam Boilers and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

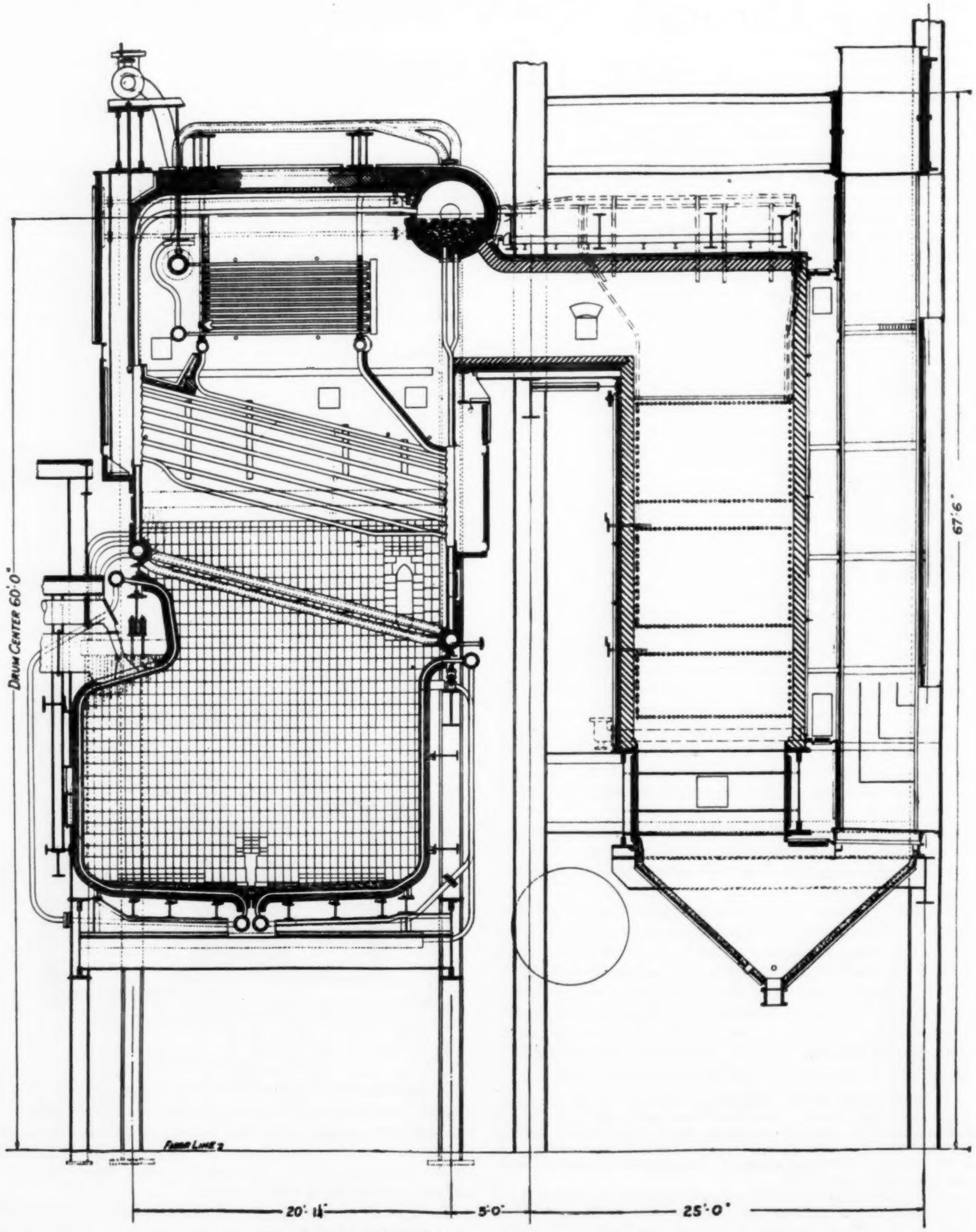
DRUM CENTER 60'0"
67'6"
FLOOR LINE 2
20' 1½"
5'0"
25'0"

FIG. 3 NEW UNIT FOR STATE LINE STATION

The purpose of this paper is to present certain facts dealing with the design and the operation of a boiler with straight-through flow without drums. While four of these straight-through units have been built for investigation, a description of the last one will serve the present purpose.

Fig. 5 shows this unit as erected and tested at Bayonne Works. It has been in successful operation at steam pressures from 1500 to 5000 lb. per sq. in. at a capacity which is limited by that of the pump. This capacity is about 5000 lb. per hour, with a final steam temperature of the order of 800 deg. fahr. The unit consists of seven circuits made of 1-in. tubes, $^1/_2$-in. inside diameter, the particulars of which are as follows:

Economizer surface (convection bank), sq. ft.	189
Exposed length of economizer tube per circuit, ft.	103
Furnace surface, semicircumference of tubes, sq. ft.	98
Furnace surface, projected area of tubes, sq. ft.	62
Exposed length of tube per circuit in furnace, ft.	107
Furnace volume, cu. ft.	87
Diameter of furnace at bottom, ft.	4
Height of furnace inside, ft.	$7^2/_3$

The flow diagram for the unit is shown in Fig. 6. The water enters the economizer section at the top, passing down through seven circuits (each circuit consisting of four double coils). At the end of the economizer section the circuits join and pass to the bottom of the furnace-wall tubes, where the flow is again subdivided into seven circuits, forming the furnace-wall tubes, which become the steaming section and superheater. There are ten coils per circuit in the furnace. Steam leaves the top of the furnace at the desired superheat temperature.

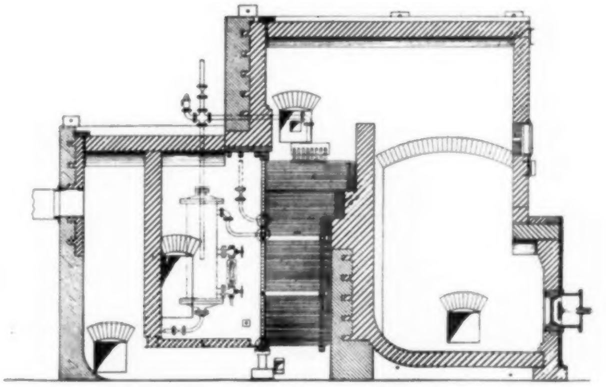

Fig. 4 Drumless Series Boiler Fired With Oil

Table 1 shows typical test data for various pressures at approximately 5000 lb. of steam per hour, 800 deg. fahr. outlet steam temperature, and pressures from 1500 to 5000 lb. per sq. in.

With reference to Table 1, the following points are of interest:

1 All temperatures (except furnace temperatures) were taken with iron-constantan thermocouples. Tube temperatures were taken with the couples peened to the tube in the usual manner. For taking steam temperatures the arrangement was virtually a thermocouple well of small diameter made integral with the tube, with the two wires welded to the bottom of the well. It seems conservative to state that steam temperatures taken in this manner are accurate to within 5 deg. fahr.; all checks that have been made indicate that the error is less than 5 deg. fahr.

2 Of interest in this table is the difference between the average steam temperature and the maximum steam temperature from any one circuit, these differences being 45, 35, 21, and 12 deg. at 1550, 2500, 3200, and 5000 lb. per sq. in., respectively. Undoubtedly this could be further improved where desirable.

This boiler has been in successful operation for a considerable period of time under the conditions noted in Table 1 and with variations in capacity of the order of 30 per cent.

Some of the problems involved in the design of this type of boiler are as follows:

Distribution of Flow

Such a unit consists of a number of tubes connecting the common feed supply with a common steam discharge, and it is desired to have equal flow in all circuits. At first thought this would seem to be quite simple with a reasonable pressure differ-

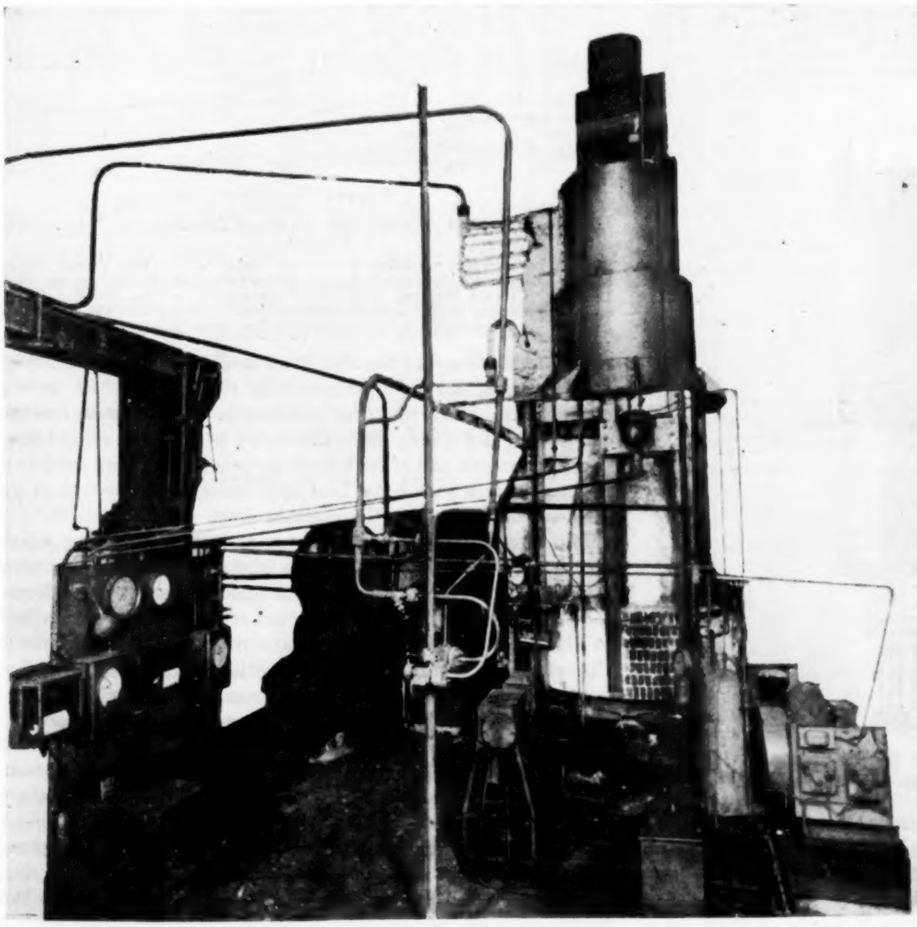

Fig. 5 Once-Through Series Boiler Tested at Bayonne Works

TABLE 1 TYPICAL TEST DATA FOR VARIOUS PRESSURES

Steam pressure, lb. per sq. in.	1,550	2,500	3,200	5,000
Steam, lb. per hr.	4,600	4,450	4,600	4,500
Mass flow, lb. of steam per hr. per sq. ft. cross-section area of tubes	480,000	465,000	480,000	470,000
Furnace temperature, deg. fahr.	2,600	2,600	2,600	2,600
B.t.u. liberated per hr. per cu. ft. furnace volume	100,000	93,000	93,000	83,000
Heat transfer to furnace tubes, B.t.u. per hour per sq. ft. projected tube area:				
(*a*) Steaming section	65,000	70,000	62,500	59,000
(*b*) Average for superheater section	30,000	36,000	40,000	31,000
Corresponding temperature difference between fire-side tube wall and steam:				
(*a*) Steaming section	56	73	68	65
(*b*) Average for superheater section	55	60	67	50
(*c*) Temperature difference for 40,000 B.t.u. per hr. per sq. ft. (basis of item *b*)	73.5	66.7	67.0	64.5
Feedwater temperature, deg. fahr.	184	175	177	178
Pressure drop through unit, lb. per sq. in.	220	165	130	80
Temperature of water in each of the seven circuits at the outlet of the convection section, deg. fahr.:				
1	598	570	550	523
2	605	670	613	560
3	607	569	545	521
4	620	605	577	553
5	621	620	591	558
6	619	670	640	611
7	620	634	596	573
Combined outlet	620	618	587	557
Temperature of steam in each of the seven circuits at the superheater outlet, deg. fahr.:				
1	826	829	830	808
2	856	856	842	815
3	860	857	842	814
4	838	849	826	808
5	824	829	814	807
6	802	793	806	797
7	790	788	803	800
Combined outlet	815	822	821	803
Difference between maximum and combined outlet	45	35	21	12

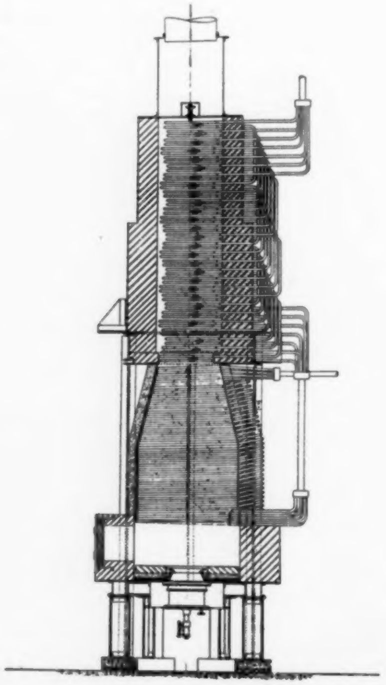

FIG. 6 ONCE-THROUGH SERIES BOILER
(Seven-circuit tube arrangement and flow diagram.)

ential, and as a matter of fact it might be expected that, if the circuits were exactly the same, such a condition would exist irrespective of the pressure differential. This, however, is not the case, as an unstable condition of flow may take place even with a uniform application of heat. To state it briefly, if there are a number of identical circuits between a common water supply and a common saturated- or superheated-steam outlet, with all of the circuits equally heated, there may be an unstable condition which will result in an unequal flow in the different circuits. Again, the amount of this inequality in the flow will vary and can reach such proportions that the temperature of the outlet of a circuit may increase to a point where the tube will be burned.

Fig. 7 shows the effect at 2500 lb. pressure of various feedwater inlet temperatures, together with various outlet-heat contents per pound. Taking the case of 200-deg. feedwater inlet, the curve on the left shows that up to an outlet-heat content of 800 B.t.u. per lb., the temporary slowing down of flow of one circuit or a number of circuits will tend to cause a decrease in pressure drop across those circuits, which would of course result in increased flow, tending to bring the circuits back to a stable condition. As the average heat content of the emerging fluid is increased—that is, as more heat is added per pound, the total remaining constant—this equalizing effect becomes less and less, until finally at an average heat-content condition of 1000 B.t.u. per lb. in the emerging fluid, it is evident from this curve that there will be no tendency toward an equilibrium condition; but on the contrary, a definite tendency for the circuits to become further unbalanced. On increasing the inlet temperature, the form of the curves changes, until finally when the feedwater-inlet temperature reaches 400 to 500 deg. fahr., the entire curve, as shown in dotted line, becomes smooth and of such form as to show that any tendency to unequal flow with equal heat application will be corrected by the tendency to change the pressure drop.

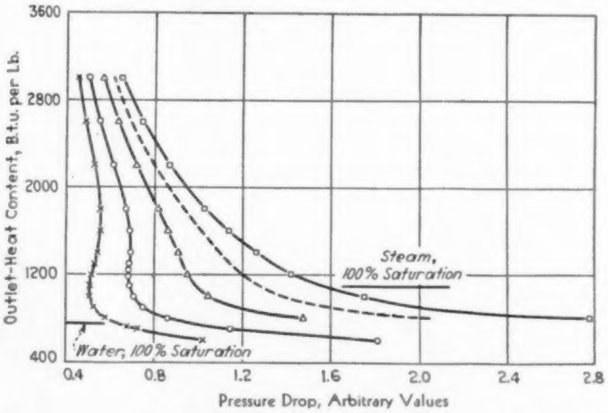

FIG. 7 PRESSURE DROPS FOR VARIOUS VALUES OF B.T.U. PER POUND

(Added in a circuit receiving a constant total B.t.u. per pound input evenly distributed along the length. Pressure is 2500 lb. per sq. in. abs. Legend: Crosses, 200 deg. fahr. inlet temperature; circles, 300 deg. fahr.; triangles, 400 deg. fahr.; squares, 500 deg. fahr.; dotted line, 460 deg. fahr. minimum inlet temperature for smooth curve.)

Fig. 8 is a similar set of curves at 3200 lb. pressure, again showing by the dotted line that an inlet temperature of between 400 and 500 deg. fahr. gives a definite smooth curve of proper form.

Fig. 9 shows the effect on a test unit of the conditions described in Figs. 7 and 8. In this particular unit four circuits were used. Circuits *A* and *B* show an outlet condition at saturation temperature, undoubtedly with some percentage of water in the steam. Circuits *C* and *D* show a superheat temperature starting around 820 deg. fahr., increasing with time to temperatures above 960 deg. fahr., with all conditions on the unit maintained as near constant as possible. In this test the feedwater-inlet temperature was 200 deg. fahr.

Fig. 10 shows the minimum inlet temperature for stable operation from 1500 to 3200 lb. This curve assumes interconnecting circuits from the inlet to the outlet of equal dimensions, the heat application being equal throughout the length of the circuits and equal for all circuits.

Under certain conditions it may not be desirable to increase the temperature of the entering material to such a value as to produce natural stability of the circuits. In such cases stability can be produced by interposing sufficient resistance at the entrance to the circuits to bring about this condition.

The foregoing deals with the natural instability of circuits caused by the change in volume of fluid which takes place as heat is applied. Another factor, however, is involved in this general problem—namely, the unequal heating of circuits. This can be eliminated in certain cases by so designing the unit that unequal heating will not take place. Data from Table 1 give a fair measure of the results that can be obtained with small oil-fired units. In many cases, however, particularly with larger units, this will not be possible. Where this unequal heating is sufficient to produce too great a difference in the heat content of the material leaving the circuits with equal flow, unequal resistance will be required so as to produce greater flow in those circuits receiving the greater amount of heat, so as to give approximately an equal heat content in the material leaving all circuits. Obviously, as the size of unit increases, the problem becomes somewhat more difficult. The problem also becomes further complicated in the case of pulverized coal, involving the question of uneven slag accumulation on the furnace walls.

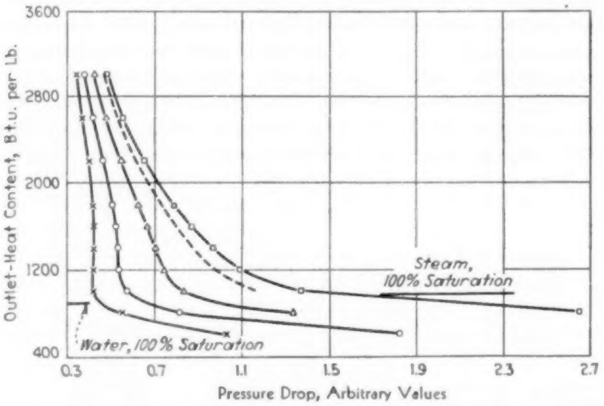

FIG. 8 PRESSURE DROPS FOR VARIOUS VALUES OF B.T.U. PER POUND

(Added in a circuit receiving a constant total B.t.u. per pound input evenly distributed along the length. Pressure is 3200 lb. per sq. in. abs. Legend: Crosses, 200 deg. fahr. inlet temperature; circles, 300 deg. fahr.; triangles, 400 deg. fahr.; squares, 500 deg. fahr.; dotted line, 473 deg. fahr. minimum inlet temperature for smooth curve.)

CONTROL OF THE UNIT

This type of unit obviously has little if any water- or heat-storage capacity. It is therefore necessary with variable loads to have a quick-acting automatic control which will maintain the supply of feedwater, fuel, and air necessary for the capacity demands. Such a control has been developed on these test units by the Bailey Meter Company.

The control is arranged as follows: The motion of the throttle valve which regulates the steam flow gives the primary impulse to change the oil, air, and feedwater supply. The steam pressure at the outlet readjusts the oil and air flow to maintain the steam pressure. The temperature at a point intermediate between the point where the steam becomes dry and the superheater outlet readjusts the water flow to maintain a constant outlet-steam temperature. The control on the experimental unit operated through Selsyn motors which actuated the hydraulic mechanisms.

With this control no difficulty was experienced in operating with 30-per cent variation in load applied in one to two seconds by opening and closing the throttle valve. The outlet-steam temperature was consistently maintained within 10 deg. with a pressure variation of 50 to 100 lb. at 2500 lb. per sq. in. pressure. It should be understood that in an actual installation variations in the steam flow would actuate a mechanism that would take the place of the throttle valve used in these experiments for giving the primary impulse for regulating the supply of the fuel, air, and feedwater. Further improvements in this control have already

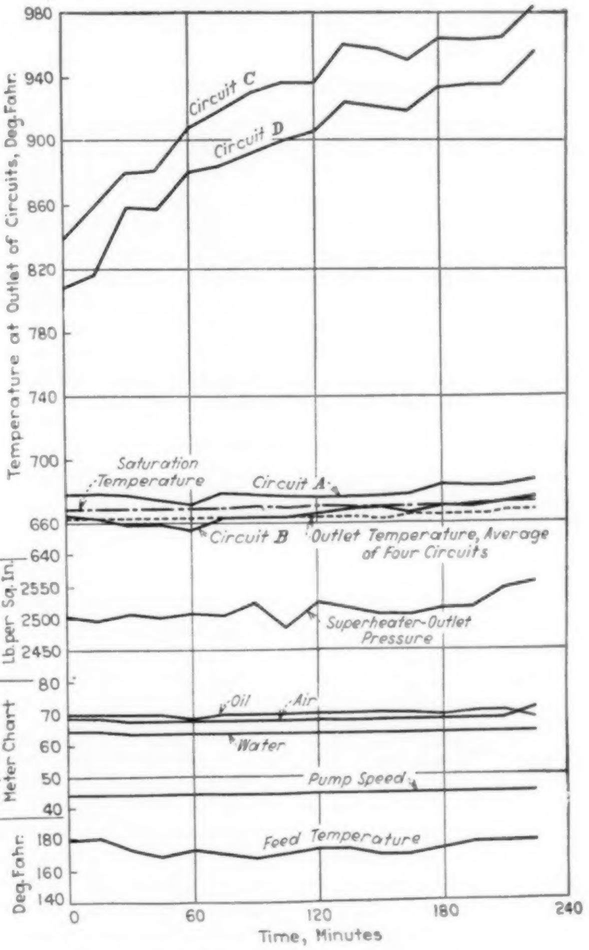

FIG. 9 EFFECT OF INSTABILITY ON THE EXPERIMENTAL BOILER

(Four-circuit layout series boiler. Circuit *A*, top coil; circuit *B*, bottom coil.)

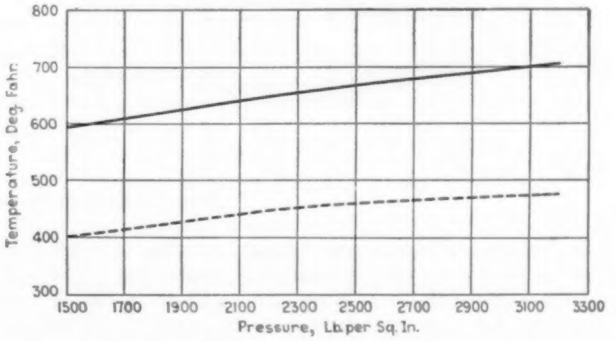

FIG. 10 MINIMUM INLET TEMPERATURE FOR STABLE OPERATION FROM 1500 TO 3200 LB.

(Dotted line, minimum inlet temperature for stable operation; solid line, saturation temperature corresponding to pressure.)

been accomplished, so that this problem has been definitely solved.

Treatment of the Feedwater

The feedwater used in these tests was deaerated condensate maintained slightly alkaline with phosphate and caustic soda. A careful inspection showed that the internal surfaces of the tubes were unaffected by corrosion or scale formation. The amount of hydrogen in the steam leaving the unit was measured at different pressures up to 3270 lb. and was found to be in line with that normally existing in low-pressure units, and of such a low quantity as to be negligible. The quantity of phosphate or caustic required to maintain the desired pH value of 10.5 did not form any scale which remained in the tubes, but was apparently blown through to the condenser and thus recirculated through the system. Thus only very small additions were required.

Conditions for Starting Up

It is obvious that a unit of the type described cannot be started up without circulating water through the tubes. At first thought this might appear to be a disadvantage, but it has a compensation that in some cases will outweigh the complication. This compensation is the ability to bring such a unit up to 3500 lb. pressure and 800 deg. temperature at the outlet in 10 minutes. When it is considered that modern high-pressure practice requires a time of from two to three hours for starting up, the gain in this respect is a valuable one.

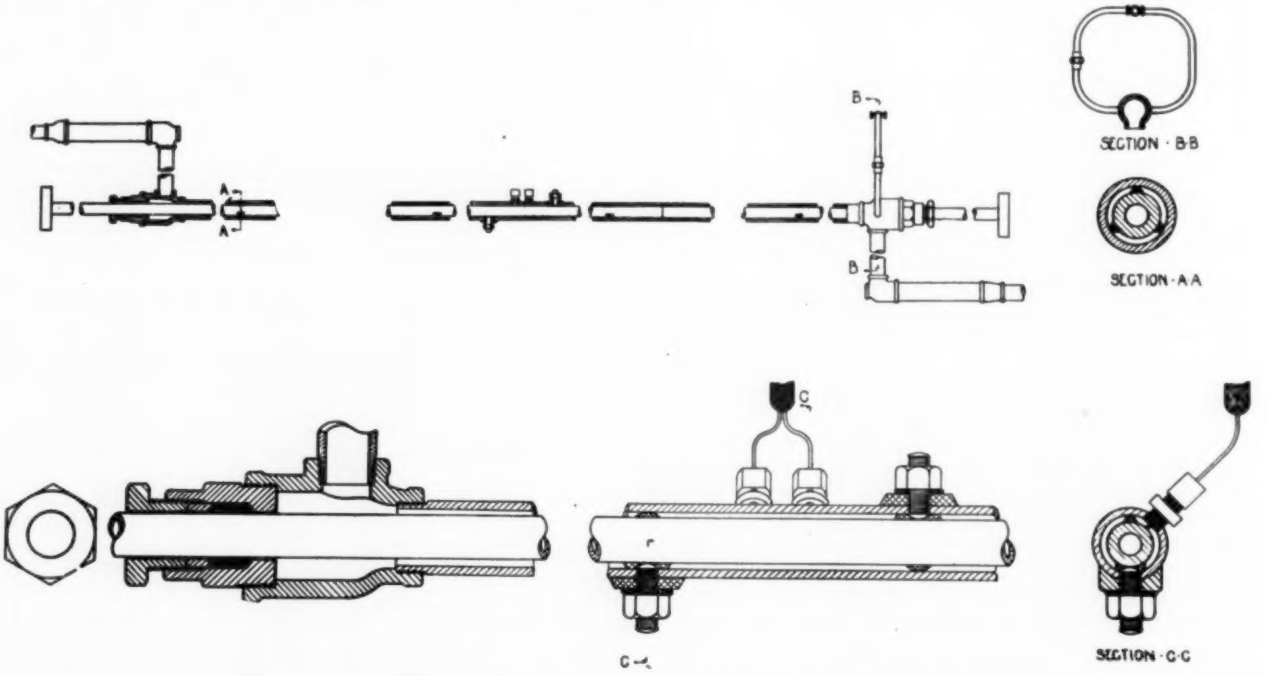

Fig. 11 Experimental Heat-Transfer Apparatus

Triplex-Plunger Feed Pump Used

Throughout this work a triplex-plunger pump with a direct-current motor drive has been used, and no difficulty has been experienced with it. The feed-pump drive must, however, be arranged for small variations in capacity. This has been accomplished in the test apparatus by using a 120-point multiple rheostat. In larger units this will represent some complication, but not a serious one.

Arrangement and Construction

The general arrangement of heating surface is that most suitable for small-sized units and would undoubtedly be entirely different for units of larger size.

The only construction detail which might be considered as worthy of mention is the elimination of all flanges, the unit being completely welded from end to end, using the welds not only for tightness but for the necessary strength. In this work resistance, arc, thermit, and acetylene welding have been used with complete success.

Superheated Steam at High Pressure

It may be of interest to add that a heat-transfer apparatus was operated in conjunction with this boiler in order to obtain heat-transfer data at the higher pressures. The apparatus, which is shown in Fig. 11, consisted of an inner tube of 1-in. external diameter and $^1/_2$-in. internal diameter, through which the high-pressure steam was passed. Surrounding this tube was an outer jacket consisting of $1^1/_4$-in. extra-heavy seamless pipe. Low-pressure superheated steam at approximately 200 lb. per sq. in. pressure was passed through the annular space between the jacket and the inner tube and absorbed the heat given up by the high-pressure steam.

The flow of both high- and low-pressure steam was measured, as well as the temperature differences and the pressure drop through both circuits of the apparatus.

In these tests the high-pressure and low-pressure steam was condensed and weighed, the heat quantity given up or absorbed being thus checked. The transfer rate of the high-pressure steam was then determined from the known data on the transfer rate of low-pressure steam.

Results with superheated steam up to 3300 lb. per sq. in. pressure and 800 deg. fahr. conform to the following formula:

$$h = 0.028446 \frac{W}{A} (T)^{-.05}\sqrt{(fC_p)}$$

h = heat-transfer rate through one fluid film in B.t.u. per hour per sq. ft. of film surface per deg. fahr. of temperature difference across the film

f_a = friction factor from curve corresponding to values of

$$\frac{Dus}{Z} = \frac{D\frac{W}{A}}{224{,}280Z}$$

D = diameter of tube in inches

$\frac{W}{A}$ = mass flow of fluid in lb. per hour per sq. ft. of cross-sectional area

C_p = means specific heat of fluid film at constant pressure

T = mean temperature of fluid film in deg. fahr. abs.

Z = mean viscosity of steam film relative to water at 68 deg. fahr.

It is not the purpose of the author to propose the immediate adoption of 2500 lb. pressure for power generation. The facts appear to be, however, that 2500 lb. steam pressure makes possible higher efficiency. The amount of this increased efficiency which will be available will depend upon the cost of obtaining it, and the type of unit described may make it possible under certain conditions to proceed further in this direction than with standard constructions. One can hardly operate the type of unit here described without developing the definite conviction that 3000 lb. steam pressure with a unit of this type represents little complication as compared with the ordinary 600-lb. to 1500-lb. units so common at the present time. Again, the information developed gives data that can be used for the proper design of units of this type and that prior to the making of these tests were not available.

Discussion

I. E. Moultrop.[2] The paper describes some interesting developments which should have a far-reaching effect in power-plant practice. There can be no question about the increased efficiency and the desirability of going to much higher steam pressures than have been considered standard practice. The increased efficiency obtained by higher pressures has been recognized for some time, but the considerably greater cost of the higher-pressure boiler has doubtless deterred many engineers from adopting it.

The once-through series boiler certainly promises to bring about a great reduction in boiler costs. The writer believes the cost of the drum for a 1400-lb.-pressure boiler is about one-half the total cost of the boiler, and if this is eliminated, it should make a considerable reduction in the cost of the unit. The drumless series boiler should be a very rugged piece of apparatus so far as the boiler itself is concerned. However, the elimination of the drum places a considerably greater responsibility on the boiler-feed pump, and a momentary failure of the pump might result in serious damage to the boiler. It is recalled that one of the principal arguments in the past against the Loeffler and the Benson boilers was the responsibility that these types of boilers placed on the pump. The boiler-feed pump is a fairly reliable piece of apparatus, but it may have a momentary interruption once in a while, and it only takes one of these to burn a drumless boiler. To make the drumless boiler commercial, it will be necessary to find means of absolutely insuring continuity of the feed.

The quality of water supplied to the drumless boiler is another important item. While all power boilers should receive proper conditioned water, this has not been general practice in the past. Using higher pressures makes the proper conditioning of the water imperative or serious trouble may ensue. This is a subject about which far too little is generally known.

This experimental work which the Babcock & Wilcox Company is doing is a very valuable contribution to the art, and in connection with the experiments being made by the Detroit Edison Company on higher temperatures should put in the hands of the power-plant engineers apparatus much more efficient than is available today.

[Further discussion of this paper, jointly with that of A. A. Potter, H. L. Solberg, and G. A. Hawkins, by A. G. Christie, N. Artsay, R. M. Gates and W. H. Armacost, Karl A. Mayr, George A. Orrok, Arthur J. Herschmann, and James M. Taggart, will be found following the latter paper.]

Author's Closure

I would like to thank those who have so kindly contributed to the value of the paper by their discussion.

It seems particularly appropriate that the discussion should be led by Messrs. I. E. Moultrop and George A. Orrok; Mr. Moultrop because he is the man that led the way in America to the use of 1400-lb. steam pressure in power plants; and Mr. Orrok because of his well-known interest in this subject over a number of years.

Mr. Moultrop does not question the gain to be obtained by using high steam pressures. Again his experience in high-pressure practice permits him to place his finger on two very important factors in connection with the unit described—namely; the need for reliability in the feed pump and the necessity for good feedwater conditions.

Mr. Orrok refers to Perkins, to whom we owe a very considerable debt for the things that he accomplished under adverse conditions. In many cases the inability to fully appreciate the advantages of a new development, or trouble that was not fully understood, has been a stumbling block in preventing progress.

An example of a development which failed because of lack of knowledge, and which later has been successful, was the steel economizer. More than 30 years ago quite a number of steel economizers were installed in plants throughout the country, all of which failed due to corrosion. Notwithstanding this experience, there has been witnessed the return to steel economizers with complete acceptance and success.

Mr. Orrok asks what operating difficulties have been experienced with steaming economizers. In the author's experience, he has never seen a new development approaching in extent that of the steaming economizer which has been so free of operating difficulties. Speaking from very close contact with this whole development, he does not know of any operating difficulties that have developed from their use, and this fact is evidence of the value of complete preliminary testing of new ideas before applying them to commercial jobs.

The operation of units of the type shown by Fig. 4 represents no difficulties, but the importance of using a reliable pump becomes greater, as does also the control of feedwater, fuel, and air.

It is true, as Mr. Orrok states, that unequal circulation in water walls, superheaters, etc. has been in some cases straightened out by the application of poorly understood principles, but this is not so in a series boiler, which requires full understanding of the factors involved, and it was necessary to determine these in order to obtain satisfactory results.

Prof. A. G. Christie refers to the question of the purity of the feedwater, which is obviously an important factor with the type of unit described. Throughout the tests and operation of the units, condensate has been used with small additions of sodium salts to maintain the desired alkalinity. Experience has shown that the salts passed through the unit.

The operation of mercury boilers has proved that it is

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[6] "Principles of Chemical Engineering," by Walker, Lewis, and McAdams, 2d ed., p. 87.

[2] Chief Engineer, Construction Branch, Edison Electric Illuminating Company of Boston, Boston, Mass. Mem. A.S.M.E.

practical to build a complete unit in such a way that none of the fluid will leak out and other materials will not enter. Units of this type have operated continuously over a period of nine months, and on being shut down show practically the same vacuum as when started and without the loss or gain of any of the mercury.

A steam unit can duplicate these conditions provided the gas formed is taken care of and the solubility of iron in pure water without oxygen is not excessive. The latter condition can be assisted by certain salts which may give little trouble in passing through the turbine.

No difficulty was experienced with the seven-flow boiler which was designed on the basis of our investigations. Figs. 7, 8, and 10 were obtained from theoretical calculations based on experimental data, and the curves are extended beyond permissible values as a matter of interest. Fig. 9 represents actual test data that were used in the theoretical calculations.

The data given on Table 1 on the seven-flow boiler, as well as the data given by Dean Potter and his colleagues on the Purdue unit, furnish a demonstration that, having determined the factors involved, series boilers can be built to give stable conditions.

Dr. Karl A. Mayr questions the values of the efficiency curve given in Fig. 1. In presenting this the author stated that the actual values might be open to question. The values given represent figures obtained from various sources and probably represent as nearly correct values as can be presented. It is not possible to give definite figures that will be ironclad, since the results are much affected by variable conditions not common to all plants.

While it is true that the equilibrium of flow through circuits remains much more constant when the water in the circuit is not permitted to change its state, the units described have shown that it is not necessary for the boiler to be operated at the critical pressure in order to obtain satisfactory equilibrium conditions. Where a pressure of 2500 lb. is desired at the turbine, it is a distinct handicap to be forced to increase the work of the feed pump by maintaining a pressure above the critical. In the case of the unit described in the paper, the maximum pressure drop from the feed pump to the steam available for the turbine is of the order of 200 lb. at 1500 lb. pressure and considerably less at 2500 lb. pressure. Obviously, such a unit will give higher net efficiencies than if it were necessary to operate the pump at a pressure of approximately 4000 lb., as in the case with a critical pressure boiler. Again, while it appears quite simple to maintain critical pressure in the water tubes and thus have no evaporating zone, the expansion of this steam through the superheater beyond the control valve, while applying heat in an amount to prevent formation of water and maintain flow through the circuits, is perhaps not so simple.

Mr. Arthur J. Herschmann, speaking from the experience with Loeffler boilers in Europe, questions the value of exceeding 1900 lb. pressure. Comparing this with Dr. Mayr's discussion, it would appear that European engineers have not as yet reached an agreement on a desirable upper limit for pressure. Perhaps this is due to different values for the efficiency curve as given in Fig. 1. The particular type of boiler involved may also affect their conclusions on the optimum pressure. It would appear as a handicap to have a feed pump operating at a pressure of approximately 4000 lb. for a pressure of 1900 lb. at the turbine, which would be the condition for the critical pressure boiler. Again, the stresses in tubes of ordinary steel containing steam at 2500 lb. pressure and 932 deg. fahr. temperature are not to be overlooked.

Mr. James M. Taggart points out the value of mingling the water and steam at different sections. A careful study of Figs. 7 and 8 will show this effect. The difficulty of applying this method to large units will be obvious.

Mr. N. Artsay questions the stability of flow and the meaning of Figs. 7 and 8, both of which points have been answered in the previous discussion.

Messrs. R. M. Gates and W. H. Armacost misinterpret the meaning of Fig. 9. This figure is simply a demonstration of the facts as brought out by the theoretically determined curves of Figs. 7 and 8 to prove the instability of circuits. Having determined the factors involved, a series boiler can be designed to eliminate this condition as shown by Table 1. The same statement is true of forced-circulation oil stills and without reducing the number of circuits.

Plain carbon steel has been used in all tubes of the series boilers which we have developed, and no growth or difficulty has been experienced with it. Consideration of the metal temperatures given in Table 1 will show that the unit has been properly designed to prevent such difficulties.

To sum up, the factors involved in the design and operation of series boilers have been investigated, and it has been shown that such units can be built for satisfactory operation. The value of such type units for high pressure has been pointed out. It remains to be seen whether the next development will be with the straight-through series boiler or along the lines of an intermediate stage with a greater amount of steaming than is common with present series-flow steaming economizers.

Characteristics of a High-Pressure Series Steam Generator

By A. A. POTTER,[1] H. L. SOLBERG,[2] and G. A. HAWKINS,[3] LAFAYETTE, IND.

The steam generator consists of two parallel circuits of seamless-steel tubing supplied with feedwater at one end, and delivering steam from the other end at any desired pressure up to 3500 lb. per sq. in. and at a maximum temperature of 830 deg. fahr. The changes which take place in the circuit are measured by means of thermocouples and manifolded pressure connections. The unit is controlled automatically by a system which uses the Thyratron vacuum-tube circuit. A set of charts is presented showing the operation of the automatic control on a swinging load. The tests reported in the paper indicate the effect of (a) varying the steaming capacity at constant pressure and temperature, (b) changing the pressure at constant capacity and temperature, and (c) increasing the steam temperature at constant capacity and two different pressures. The temperature and the pressure changes through the water and steam circuit are shown graphically. Data are also presented on the heat-transfer rates through the furnace-wall tubes. The results up to date indicate that this steam generator will operate successfully at any pressure between 1500 and 3500 lb. per sq. in. and at a maximum temperature which is limited by the materials used. Automatic control held the steam pressure and temperature nearly constant throughout the range of operation.

A. A. Potter

H. L. Solberg

G. A. Hawkins

The steam generator, shown in Fig. 1, consists of a cylindrical water-cooled furnace placed immediately beneath a counterflow economizer or convection section. This steam generator is a further development of the Calumet-type unit in that the steaming economizer, or convection element, is continued through the evaporating and superheating sections. There is no steam drum, and the water that enters the unit passes through a continuous coil and leaves it as superheated steam. It can be operated at any pressure between 1000 and 3500 lb. per sq. in. at the outlet, although in the lower pressure ranges the friction loss is excessive by reason of the small tubes used. No throttle valves or other regulating devices are installed between the boiler-feed pump and the steam-generator outlet. By varying the ratio of fuel to water at any load, the final quality of the steam may be changed from saturated to superheated steam at a maximum temperature of about 830 deg. fahr., this being limited by the tube material. The unit is constructed of 1 in. O.D. by $^1/_2$ in. I.D. seamless-steel tubing (A.S.M.E. 1927 Code boiler steel). All joints are gas-welded, using a butt type of weld.

The feedwater is delivered to the top of the convection section, where it is divided into two parallel circuits each 521 ft. in length, and flows downward countercurrent to the gas flow. Fig. 2 shows this section before it was bricked in. The long sets of coils at the bottom are provided to support the section. The vertical pipes and horizontal welded bars shown in Fig. 2 were removed after the brickwork was completed.

The water which leaves the two parallel circuits at the bottom of the convection section is carried to the bottom of the furnace coils by a single downcomer or equalizer which is made of double-extra-heavy $1^1/_2$-in. seamless-steel tubing. The lower furnace-coil section consists of two parallel circuits wound in the form of a cylindrical helix and discharging at the upper end into a common tube or equalizer. The middle and upper furnace-wall sections consist of two parallel circuits wound in the form of a conical helix, with another equalizer connection between the sections.

Fig. 3 shows the entire unit during erection. Fig. 4 illustrates the relative positions of the heating and non-heating surfaces, as well as the location of the 42 thermocouples and 7 pressure connections which have been used in studying pressure and temperature distribution. The heating surface of the steam generator is distributed as follows:

Area of convection section, sq. ft.	259
Surface area of furnace-wall section, sq. ft.	101
Projected area of furnace-wall section, sq. ft.	64.3
Volume of furnace, cu. ft.	84.4
Distance from furnace floor to convection surface, in.	110

The steam generator is fired by a single mechanical oil burner placed underneath the cylindrical furnace, the gases flowing straight upward in a single pass to the stack.

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[1] Dean of Engineering and Director of Engineering Experiment Station, Purdue University, Mem. A.S.M.E. Dr. Potter is a graduate of the Massachusetts Institute of Technology; has had 28 years' experience in teaching and practice; was connected with the General Electric Company during the earlier development of the steam turbine; was for 15 years Professor and Dean of Engineering at the Kansas State College, and since 1920 has been in his present chair. He is author of books, articles, and papers on thermodynamics and heat engineering.

[2] Associate Professor of Mechanical Engineering, Purdue University. Assoc-Mem. A.S.M.E. Mr. Solberg holds the B.S. degree from South Dakota State College and the B.S. and M.S. degrees from Purdue University. He has been for the last 10 years on the teaching staff of Purdue University. In his present position he is responsible for the instruction in power-plant engineering and thermodynamics. He is author of numerous articles on power engineering in the technical press.

[3] Assistant in Applied Mechanics, Purdue University. Jun. A.S.M.E. Mr. Hawkins received part of his college training at the Colorado School of Mines and the Bachelor's degree in Mechanical Engineering at Purdue University. In his present connection with Purdue University he is pursuing post-graduate work leading to an advanced degree.

Contributed by the Special Research Committee on Critical Pressure Steam Boilers and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

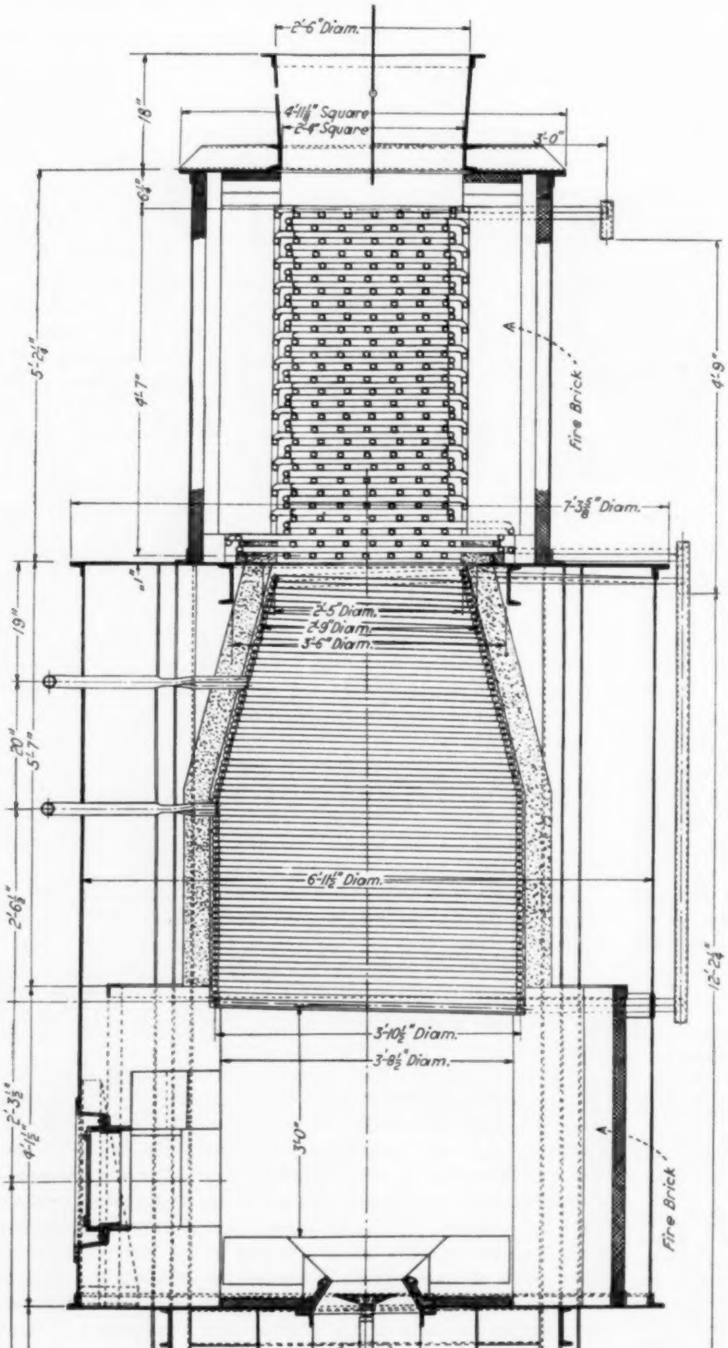


Fig. 1 Cylindrical Water-Cooled Furnace Beneath a Counterflow Economizer or Convection Section

Fig. 5 is a diagrammatic layout of the installation. Feedwater is supplied by a triplex pump driven by a direct-current variable-speed motor. The feedwater from the pump discharge is heated in a special heat exchanger by means of the steam leaving the generator. From the heat exchanger the live steam enters a condenser, which is of the steel-tube floating-head type, designed for a pressure of 100 lb. The condensate is weighed in closed steel tanks mounted on platform scales. These tanks are connected to the piping system by flexible hose. The weight of the condensate is checked by a Bailey flowmeter.

Live steam from the laboratory header is passed through an auxiliary condenser for use as make-up water. Upon the recommendation of the Babcock & Wilcox Company, trisodium phosphate is used to condition the boiler water. Tests are made on alternate days for total alkalinity, sodium phosphate, and the pH value of the water. Dissolved oxygen is also determined frequently. The condition of the boiler water is held within the following limits:

Maximum total alkalinity (expressed as sodium carbonate), p.p.m.	75
Maximum sodium phosphate (expressed as disodium phosphate), p.p.m.	40
pH value	10.0–10.5
Dissolved oxygen (per liter), cc	0.0–0.1
Maximum total solids, p.p.m.	1500

The fuel is stored in an underground tank of 9000 gal. capacity and is pumped to the oil-weighing tanks. The fuel-weighing system consists of two tanks, which are provided with flexible discharge connections and are mounted on platform scales. A constant-speed motor-driven screw-type oil pump discharges the fuel through a duplex strainer, heater, and throttle valve to the burner. Constant oil pressure is maintained ahead of the throttle valve by a Fisher bypass governor.

A motor-driven forced-draft fan is used to supply the air for combustion. The regular laboratory stack is used, being supplemented by a steam jet.

The steam and oil throttle valves are furnished by the Babcock & Wilcox Company; they are both designed to open with 180 deg. of valve-stem rotation and to give a straight-line flow characteristic. The oil throttle valve is shown in Fig. 6. The valve seat has a radial rectangular slot which is covered by the cam-shaped valve disk. The same type of valve and valve seat is also used in the steam throttle valve, but the connections are flanged and there are no screwed joints subjected to pressure.

The ring type of lap joint is used on the high-pressure valves and has proved very satisfactory. A set of 2-in. companion flanges of this type at the boiler outlet was pulled up while warm by one man without sledging, using a 24-in. socket wrench, and has shown no sign of leakage during six weeks of service, starting from cold every morning and with frequent shutdowns. No insulation has been applied to the joint so far, but it is expected to insulate the flanges and to watch their behavior under the changed conditions.

Several 6000-lb. standard screwed forged-steel tees are subjected to the full steam pressure and temperature. The $^1/_4$-in., $^3/_4$-in., and 2-in. screwed

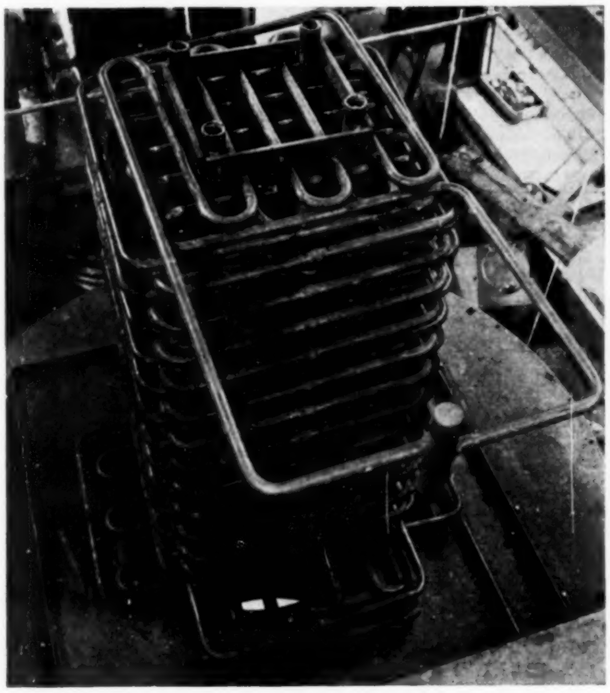


FIG. 2 CONVECTION OR ECONOMIZER SECTION

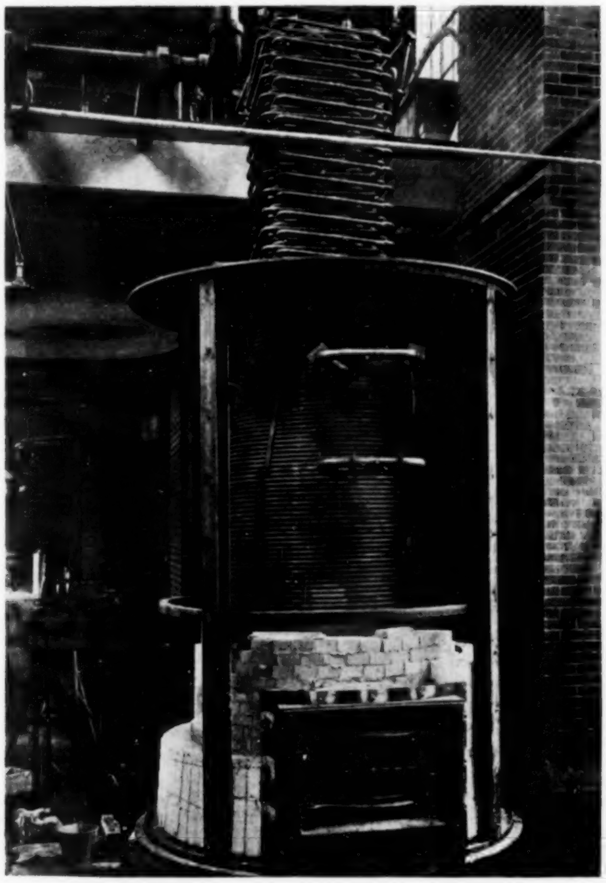


FIG. 3 GENERAL VIEW OF UNIT DURING ERECTION

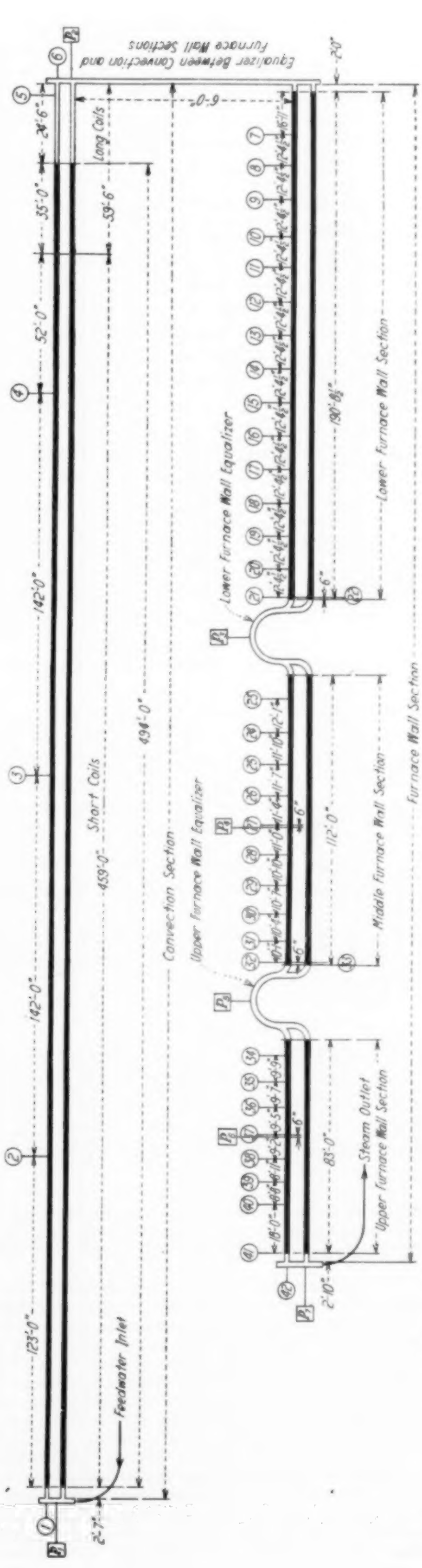

Equalizer Between Convection and Furnace Wall Sections
Long Coils
Short Coils
Convection Section
Feedwater Inlet
Lower Furnace Wall Equalizer
Upper Furnace Wall Equalizer
Lower Furnace Wall Section
Middle Furnace Wall Section
Furnace Wall Section
Steam Outlet
Upper Furnace Wall Section
123'-0"
142'-0"
142'-0"
52'-0"
35'-0"
24'-6"
459'-0"
434'-0"
53'-6"
6'-0"
2'-0"
2'-7"
130'-8½"
112'-0"
83'-0"
18'-0"
2'-10"
6"

FIG. 4 RELATIVE POSITIONS OF HEATING AND NON-HEATING SURFACES; LOCATION OF THERMOCOUPLES AND PRESSURE CONNECTIONS
(Wide solid lines, heating surface; double lines, non-heating surface; figures in circles, thermocouples; letters and figures in squares, pressure-gage connections.)

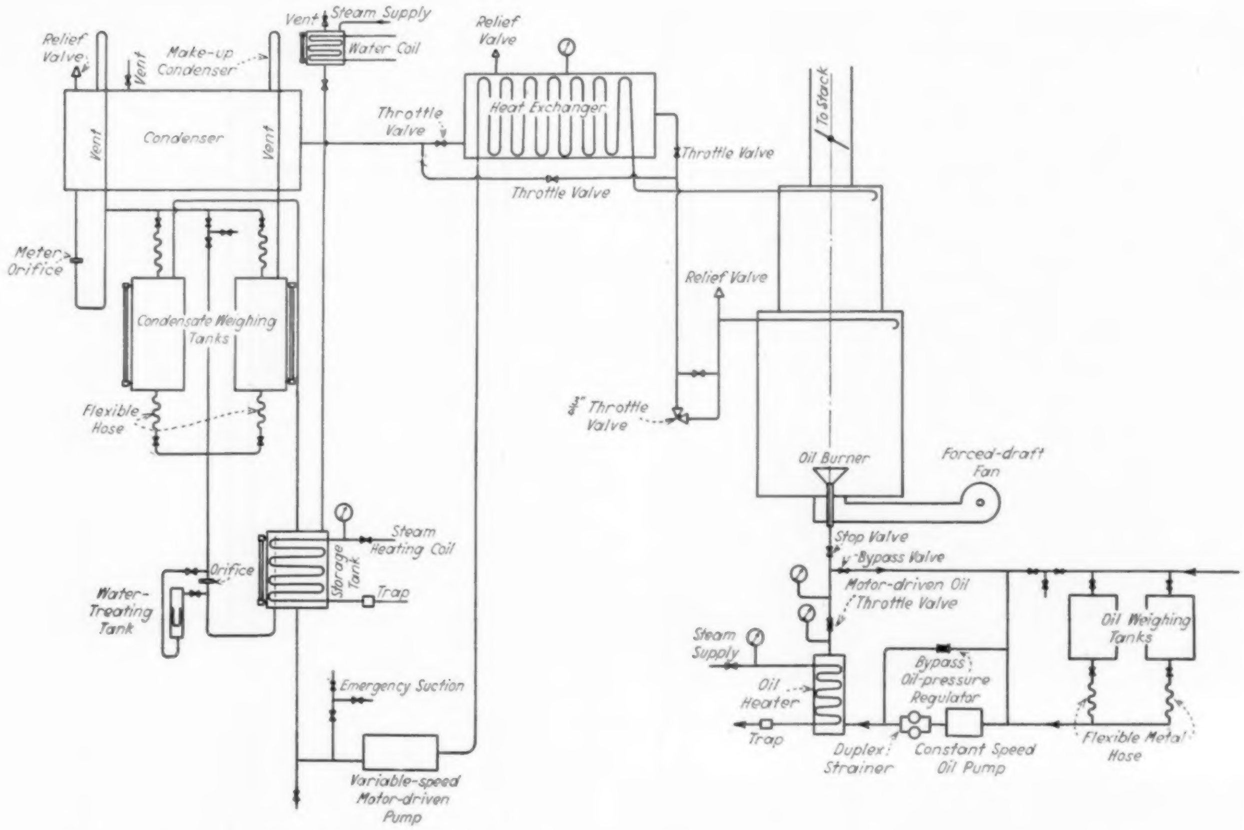

Relief Valve
Vent
Make-up Condenser
Vent
Steam Supply
Water Coil
Relief Valve
Condenser
Vent
Vent
Throttle Valve
Heat Exchanger
Throttle Valve
Throttle Valve
To Stack
Meter Orifice
Condensate Weighing Tanks
Relief Valve
Flexible Hose
3/4" Throttle Valve
Oil Burner
Forced-draft Fan
Steam Heating Coil
Stop Valve
Bypass Valve
Water-Treating Tank
Orifice
Storage Tank
Trap
Motor-driven Oil Throttle Valve
Oil Weighing Tanks
Steam Supply
Bypass Oil-pressure Regulator
Emergency Suction
Oil Heater
Trap
Duplex Strainer
Constant Speed Oil Pump
Flexible Metal Hose
Variable-speed Motor-driven Pump

Fig. 5 Flow Diagram for Series Steam Generator and Auxiliaries

connections of this type were made up with No. 3 Smooth-On iron cement without back-welding and have shown no signs of leakage at any joint.

Double-extra-heavy class B seamless-steel tubing is used for the feedwater piping, while a chromium-molybdenum-steel alloy tube is employed for the steam piping.

Automatic Control

The steam generator is fitted with an automatic control system developed by the Bailey Meter Company, which regulates the fuel, feedwater, and air supply. The problems involved in the control of this type of steam generator are indicated by P. S. Dickey, research engineer of the Bailey Meter Company, in the following statement:

> Two features of the series steam generator are important, at least in so far as automatic control is concerned, and while they are each a result of the elimination of the drum, they should be considered separately:
>
> First, with the conventional type of steam generator, in the drum and in the natural circulation system, there is an appreciable storage of water at a saturation temperature corresponding to the pressure. The water thus stored is capable of releasing or condensing steam upon a change in pressure, and this release or absorption of steam tends to counteract or rather to retard pressure variations. While in the conventional type of steam generator this tendency is reduced as the pressure is increased, the series type, with only a very small amount of water in the steaming zone, permits almost no accumulator action to take place, and it is therefore essential that practically instantaneous changes in water and heat input be made upon load changes if the pressure is to be kept within reasonable limits. The available energy storage within this type of steam generator approaches that of the flywheel effect of the turbo-generator, and the control system must accordingly resemble the turbine governor in so far as speed of operation is concerned. This requires not only

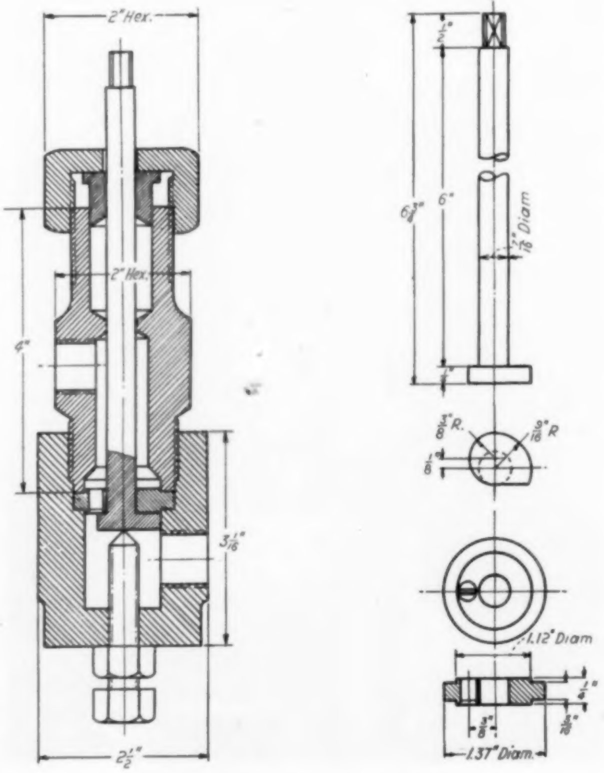

2" Hex.
2" Hex.
4"
3 1/16"
2 1/2"
6"
1.12" Diam
1.37" Diam.

Fig. 6 Oil Throttle Valve

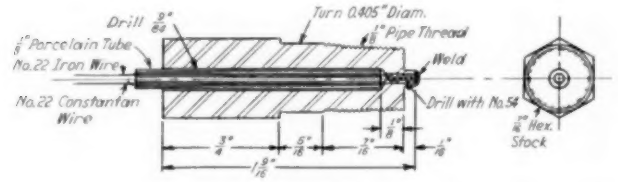
a rapidly responding control system, but a quick response from the combustion and feedwater supply systems as well. It almost demands automatic control if the boiler is to carry a fluctuating load.

The second important feature of this steam generator which differentiates it from the standard boiler is that there is no fixed steam separating surface similar to the water line of the conventional boiler, and the amount of heating surface used for superheating may be widely variable. It is this feature of the series steam generator which makes it possible to maintain any predetermined outlet steam temperature with different steam pressures and output ratings without altering the initial arrangement of the heating surface.

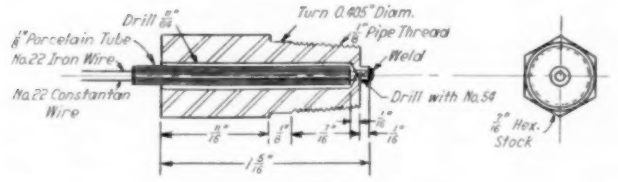
The foregoing features necessitate a control system which is suitable not only to proportion rapidly the fuel, air, and water supply with load variations, but also to maintain at all times a definite relation between the fuel and water input which will give the desired final steam temperature. Since variations in pressure affect the specific heat of the superheated steam and variations in steam temperature affect the specific volume of the steam, the control must be sensitive to small changes in order to prevent variations in pressure from affecting the temperature and vice versa.

The necessary sensitivity for pressure control is obtained by an automatic indicator of the dead-weight loaded-piston type. The dead-weight piston is rotated by a motor and has a range of 400 lb. pressure with a given loading on the piston. A scale beam and weights make it possible to load the piston for any desired pressure up to 4000 lb. Pressure variations are transmitted from the dead-weight piston to the control system by means of Selsyn motors.

The pyod type of thermocouple (Fig. 7) has proved satisfactory for rapid and fairly accurate measurement of steam temperatures. A standard Leeds & Northrup temperature recorder is used to transmit the temperature changes to the control system through Selsyn motors.



ORIGINAL DESIGN



FINAL DESIGN

Fig. 7 Pyod Type of Thermocouple

A Selsyn motor mounted on the throttle valve causes the controls to function in case of load changes.

The fuel-oil throttle valve and the boiler-feed-pump motor rheostat are driven through reducing gears by small reversing direct-current motors. Each of these motors receives its power supply through two Thyratron vacuum tubes which control the direction of rotation and will stop the motor instantly. Variations in pressure, temperature, or load will change the position of contactors in the vacuum-tube circuits and thus control the movement of the motors regulating the fuel and water supply.

A standard Bailey flowmeter is used to control the proportion of oil and air. The displacement of the oil-flow and air-flow pens on this meter shifts the position of the contactors in the circuits of the Thyratron vacuum tubes which supply power to a reversing direct-current motor geared to the fan damper.

The use of the Thyratron vacuum-tube circuit makes it possible to eliminate overtravel and to build a control system which must develop only sufficient power to open and close light electrical contactors. The system therefore responds very quickly to small changes in operating conditions.

Push-button operation is provided for manual control of the supply of oil, water, and air. In all tests reported in this paper, automatic control was used on the oil and air. Full automatic control was used on all tests numbered above ten.

Measurement of Pressures and Temperatures

The triplex boiler-feed pump used in this installation produces pressure pulsations which are quite severe and which continue well into the steaming zone. The accurate measurement of high pressures under such conditions requires a sensitive gage which will hold its calibration. A standard Bailey power-pressure-measuring device has been modified for this purpose by equipping it with a scale beam and weights to give it the necessary range of capacity. It is a motor-driven dead-weight piston-loaded gage similar to the one used on the automatic control system already described. It has a 600-lb. range and can be loaded to bring this range to any place between 1500 and 4100 lb. per sq. in. It drives a 17-in. pressure indicator through Selsyn motors. The indicator dial is graduated in 20-lb. increments with about 1 in. between graduations. This gage is very sensitive to pressure changes and has proved to be quite satisfactory. The seven pressure taps (Fig. 4) are connected to a common manifold. The Bailey dead-weight gage and a Crosby Bourdon-tube gage are connected in parallel to this manifold, while a second Crosby gage serves the boiler outlet. Pressures throughout the unit are thus measured on the same gages.

The measurement of water and steam temperatures in a small thick-walled tube subjected to high pressures requires a device which is small, mechanically rugged, and not subject to errors caused by heat conduction through the tube. The pyod type of thermocouple has been used for this purpose. Fig. 7 illustrates the design of the first thermocouples that were installed as well as the design that is now being used. When checked against the saturation temperature, as determined by pressure measurements, it was found that the original thermocouples read high by an error which increased with the rating and which amounted to as much as 6 or 7 deg. In the final design of thermocouples more metal was removed from the body of the pyod in order to improve the cooling at the hot junction. These thermocouples show little or no tendency to read high.

One of the original thermocouples was checked in the university physics laboratory against a platinum, platinum-rhodium thermocouple and was found to agree very closely with the standard Leeds & Northrup calibration curve for iron-constantan. The other thermocouples were checked against the calibrated couple in an electric furnace and indicated no appreciable error. At present, when new couples are required, they are installed without calibration and are checked by operating the unit on saturated steam at moderate pressure, comparing the observed temperatures with the saturation temperature as determined from the pressures.

All thermocouple leads are carried down to a jackboard where a Leeds & Northrup potentiometer is used to determine the temperatures throughout the system. The cold junctions on the back of the jackboard are shielded against radiation from the boiler and piping.

In the first tests which were conducted on the steam generator, the flue-gas temperatures were measured by a calibrated iron-constantan thermocouple installed between the convection surface and the damper. Since the uptake connection is made of steel plate, this thermocouple is entirely surrounded by relatively cold black-metal surfaces which are not far from the thermocouple in any direction. In order to determine the effects of

TABLE 1 DATA AND RESULTS

	Test Series											
	Series I									Series		
Item	2	3	4	6	8	9	10	27	29	11	12	13
1 Date of test (1931)	7–23	7–23	7–23	7–24	7–27	7–27	7–27	8–12	8–13	8–8	8–8	8–8
2 Time	10:10– 11:40	1:00– 2:30	3:05– 4:35	1:10– 2:40	8:36– 10:06	10:27– 11:57	2:08– 3:38	4:10– 5:40	11:12– 12:42	8:17– 9:47	10:54– 12:24	1:12– 2:42
Gas Analysis:												
3 CO_2, per cent	12.7	12.0	12.3	11.6	12.0	11.2	10.5	10.0	10.9	11.6	12.6	10.4
4 O_2, per cent	4.8	5.8	5.6	6.0	6.0	7.4	8.0	7.3	7.4	6.8	3.3	8.0
5 CO, per cent	0.1	0.1	0.04	0.05	0.1	0.0	0.0	0.04	0.02	0.0	0.3	0.02
6 N_2, per cent	82.4	82.1	82.2	82.3	81.9	81.4	81.5	82.7	81.7	81.6	84.0	81.6
7 Dry gas per pound of fuel, lb	17.5	18.3	18.1	19.0	18.4	19.9	21.3	22.05	20.3	19.2	17.3	21.3
8 Moisture from combustion of hydrogen per lb. fuel, lb	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97
9 Moisture in air per lb. of fuel, lb	0.29	0.34	0.35	0.32	0.33	0.38	0.43	0.28	0.32	0.45	0.42	0.52
10 Total products of combustion per lb. of fuel, lb	18.76	19.63	19.4	20.3	19.7	21.2	22.6	23.3	21.6	19.6	18.7	22.7
11 Air supplied per lb. of fuel, lb	17.33	18.2	17.86	18.95	18.18	19.52	20.95	22.1	20.1	18.92	17.48	21.06
12 Per cent of excess air	23.6	29.8	27.4	35.1	29.6	39.2	49.4	57.6	43.0	34.9	23.9	50.1
Pressures and Drafts:												
13 Barometer, in. hg	29.29	29.29	29.29	29.51	29.58	29.55	29.5	29.62	29.72	29.49	29.47	29.44
14 Feedwater pressure, lb. per sq. in. abs. P_1	2764	2790	2844	3367	3572	3478	3099	2884	3023	2877	3329	3679
15 Pressure at end of convection section, lb. per sq. in. abs. P_2	2714	2724	2765	3176	3331	3255	2969	2800	2910	2774	3224	3597
16 Pressure at lower equalizer, lb. per sq. in. abs. P_3	2667	2673	2695	3011	3099	3039	2844	2734	2823	2696	3158	3546
17 Pressure at upper equalizer, lb. per sq. in. abs. P_4	2608	2605	2609	2798	2828	2801	2703	2620	2701	....	3072	3475
18 Pressure at superheater outlet, lb. per sq. in. abs. P_7	2554	2560	2536	2606	2574	2567	2580	2541	2588	2512	3016	3419
19 Oil pressure at burner, lb. per sq. in. gage	155	182	229	114.9	153.6	130.7	211.9	129	217.6	166	158	130
20 Wind-box pressure, in. of water	+0.17	+0.21	+0.29	+1.1	+1.88	+1.84	+0.84	+0.29	+0.62	+0.52	+0.50	+0.37
21 Draft in furnace, in. of water	−0.07	−0.01	−0.02	−0.11	−0.01	−0.02	−0.04	−0.09	−0.06	−0.07	−0.07	−0.09
22 Draft at boiler outlet, in. of water	−0.08	−0.10	−0.12	−0.42	−0.41	−0.38	−0.23	−0.22	−0.23	−0.22	−0.22	−0.21
Temperatures:												
23 Dry-bulb temp. of air for combustion, ° F.	93.5	97	94	91.5	94.3	96.2	98.6	89.7	89	94	97.6	100.9
24 Wet-bulb temp. of air for combustion, ° F.	...	...	81	76.8	80	81.1	83.4	78.7	76.2	85.8	86.1	87.3
25 Relative humidity, per cent	50	50	57	53	53	53	52	61	55	70	63	60
26 Temp. of oil, ° F.	161	165	165	157	153	157	163	152	155	159	163	164
27 Temp. of gases leaving boiler, ° F.	801	854	894	1156	1205	1126	935	831	888	823	804	819
28 Furnace pyrometer, ° F.	...	...	...	...	2650	2724	2603	2545	2614	2525	2530	2525
29 Temp. of feedwater, ° F.	245	245	247	249	248	248	246	242	245	241	240	238
30 Temp. of water at end of convection section, ° F.	586	600	600	660	692	694	677	657	660	666	652	641
31 Temp. of steam at superheater outlet, ° F.	751	751	747	741	741	744	737	752	751	741	748	745
Hourly Quantities:												
32 Duration of test, min	90	90	90	90	90	90	90	90	90	90	90	90
33 Fuel per hour, lb	158.7	171.2	191.3	328	387	358	250	189	233	217.8	209.5	189.5
34 Fuel per cu. ft. of furnace volume per hour, lb	1.8	2.0	2.3	3.9	4.7	4.2	2.7	2.1	2.8	2.6	2.4	2.2
35 Water evaporated per hour, lb	2003	2150	2358	3778	4280	4059	3022	2317	2783	2600	2676	2605
Unit Quantities:												
36 Enthalpy of feedwater, B.t.u. per lb	219	219	221	226	224	224	222	216	220	215	215	214
37 Enthalpy of water leaving convection section, B.t.u. per lb	593	614	614	708	776	788	746	707	711	726	692	671
38 Enthalpy of steam leaving superheater, B.t.u. per lb	1247	1246	1244	1229	1232	1238	1225	1250	1244	1239	1195	1131
39 Heat absorbed in convection section, B.t.u. per lb	374	395	393	482	552	564	524	491	491	511	477	457
40 Heat absorbed in furnace wall section, B.t.u. per lb	654	632	630	521	456	450	479	543	533	513	503	460
41 Total heat absorbed, B.t.u. per lb	1028	1027	1023	1003	1008	1014	1003	1034	1028	1024	980	917
Rate of Heat Absorption:												
42 Rate of heat absorption in convection section, kb. per hr	748	850	927	1820	2360	2290	1585	1140	1365	1330	1275	1190
43 Rate of heat absorption in furnace wall section, kb. per hr	1310	1360	1485	1970	1950	1830	1450	1260	1485	1335	1345	1195
44 Rate of heat absorption in steam generating unit, kb. per hr	2058	2210	2412	3790	4310	4120	3035	2400	2850	2665	2620	2385
45 Heat absorbed in furnace wall, per cent of total	63.7	61.5	61.5	52.0	45.3	44.4	47.8	52.4	52.2	50.2	51.3	50.2
Evaporation:												
46 Heat absorbed per lb. of fuel, kb	12.9	12.9	12.6	11.5	11.1	11.5	12.1	12.7	12.2	12.2	12.5	12.6
47 Rate of heat absorption per sq. ft. of convection surface, kb. per hr	2.88	3.28	3.57	7.03	9.10	8.82	6.11	4.40	5.27	5.13	4.92	4.58
48 Rate of heat absorption per sq. ft. of furnace-wall surface, kb. per hr	13.0	13.5	14.7	19.5	19.3	18.1	14.4	12.5	14.7	13.2	13.3	11.8
49 Efficiency of steam-generating unit, per cent	69.3	69	67.5	61.7	59.5	61.5	64.9	68.0	65.5	65.4	66.8	67.3
Heat Balance:												
50 Heating value of fuel, B.t.u.	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650
per cent	100	100	100	100	100	100	100	100	100	100	100	100
51 Heat absorbed by unit, B.t.u.	12,910	12,850	12,600	11,500	11,100	11,450	12,090	12,700	12,200	12,200	12,460	12,550
per cent	69.3	69.0	67.5	61.7	59.5	61.5	64.9	68.0	65.5	65.4	66.8	67.3
52 Heat loss due to water from combustion of H_2, B.t.u.	1250	1274	1291	1415	1441	1402	1311	1302	1300	1265	1252	1260
per cent	6.7	6.8	6.9	7.6	7.7	7.5	7.0	7.0	7.0	6.8	6.8	6.8
53 Heat loss due to moisture in air, B.t.u.	94.4	118	128	154	168	178	164	95.5	117	143	136	172
per cent	0.5	0.6	0.7	0.8	0.9	1.0	0.9	0.5	0.6	0.8	0.7	0.9
54 Heat loss due to dry chimney gas, B.t.u.	2970	3330	3470	4850	4930	4910	4260	3920	3890	3365	2930	3670
per cent	15.9	17.9	18.6	26.0	26.4	26.3	22.8	21.0	20.9	18.0	15.7	19.7
55 Heat loss due to CO, B.t.u.	49.5	74	26	39	74	00.0	00.0	36	17	0.0	189	17
per cent	0.3	0.4	0.1	0.2	0.4	0.0	0.0	0.2	0.09	0.0	1.0	0.10
56 Radiation unaccounted for, B.t.u.	1376	1004	1136	692	946	710	825	597	1127	1677	1683	981
per cent	7.3	5.3	6.2	3.7	5.1	3.7	4.5	3.3	6.0	9.0	9.0	5.3

OF EFFICIENCY TESTS

Number																
II					Series III							Series IV				
14	15	16	17	19	20	21	22	23	24	25	26	32	33	34	35	36
8-8	8-8	8-10	8-10	8-11	8-11	8-11	8-11	8-11	8-12	8-12	8-12	8-14	8-14	8-14	8-14	8-14
3:00-	4:52-	9:40-	12:55-	8:15-	10:56-	1:10-	3:00-	4:50-	8:45-	10:32-	12:27-	9:05-	10:53-	12:50-	2:39-	4:20-
4:30	6:22	11:10	2:25	9:45	12:26	2:40	4:30	6:20	10:15	12:02	1:57	10:35	12:23	2:20	4:09	5:50
10.5	10.8	12.2	11.1	11.1	11.2	10.9	11.0	10.6	11.1	11.2	10.8	10.1	10.2	10.2	10.1	10.3
8.1	8.0	5.8	...	6.8	7.0	7.1	6.5	6.4	7.0	4.8	6.0	7.1	8.1	7.7	7.0	7.5
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.02	0.04	0.0	0.0	0.0	0.0	0.0	0.02	0.0	0.0
81.4	81.2	82.0	...	82.1	81.8	82.0	82.5	83.0	81.9	84	83.2	82.8	81.7	82.1	82.9	82.2
21.2	20.6	18.2	...	20.0	19.9	20.4	20.2	20.9	20.0	19.8	20.4	21.9	21.7	21.7	21.9	21.5
0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97
0.54	0.51	0.33	...	0.37	0.35	0.38	0.39	0.36	0.30	0.34	0.36	0.38	0.38	0.41	0.42	0.39
22.7	22.1	19.5	...	21.3	22.2	21.8	21.6	22.2	21.3	21.1	21.8	23.2	23.1	23.1	23.3	22.9
20.84	20.2	18.12	...	19.82	19.65	20.25	20.80	21.0	19.80	20.20	20.60	22.50	22.40	22.50	22.50	22.80
48.6	44.0	29.2	...	41.3	40.1	44.4	48.3	49.7	41.2	44.0	46.8	60.4	59.7	60.4	60.4	62.6
29.41	29.41	29.43	29.41	29.53	29.53	29.51	29.52	29.54	29.65	29.67	29.64	29.83	29.83	29.79	29.77	29.77
3782	3573	2214	2543	3470	2878	2878	2902	2943	3000	3019	3042	3563	3603	3604	3571	3547
3693	3486	2124	2456	3404	2751	2763	2792	2833	2900	2929	2937	3481	3518	3504	3462	3451
3639	3433	1928	2328	3340	2676	2673	2706	2749	2801	2830	2833	3399	3441	3422	3391	3390
3578	3354	1737	2205	3279	2586	2598	2609	2647	2679	2682	2680	3323	3354	3335	3317	3313
3527	3300	1537	2084	3233	2507	2517	2534	2547	2539	2552	2564	3253	3281	3256	3245	3254
125	126	161	141	137	163	152	153	180	223	231	225	143	145	149	129	117
+0.37	+0.39	+0.67	+0.48	+0.35	+0.45	+0.42	+0.47	+0.56	+0.70	+0.70	+0.68	+0.43	+0.46	+0.48	+0.39	+0.30
−0.09	−0.09	−0.50	−0.09	−0.09	−0.08	−0.09	−0.08	−0.06	−0.06	−0.05	−0.06	−0.09	−0.09	−0.09	−0.09	−0.10
−0.20	−0.20	−0.24	−0.24	−0.22	−0.23	−0.23	−0.22	−0.23	−0.25	−0.24	−0.24	−0.22	−0.22	−0.22	−0.21	−0.21
102.2	100	89.3	89.7	82.6	84.8	85.7	89.4	88.6	86.8	89.0	90.3	86.3	88.6	90.2	90.5	89.1
88.2	88	78.3	78.5	76.1	76.6	77.8	79.1	77.6	75.8	77.6	78.6	76.5	77.8	78.7	79.2	77.5
59	62	62	61	80	70	71	64	60	57	59	58	64	62	60	62	60
162	157	109	117	147	161	155	154	159	159	160	161	157	159	161	161	162
823	830	890	878	764	848	837	851	865	890	888	877	797	796	788	751	741
2528	2528	2525	2505	2525	2548	2523	2515	2660	2595	2621	2622	2520	2497	2489	2498	2480
239	239	248	241	240	240	241	244	246	243	246	246	242	244	244	243	243
634	648	646	654	635	642	639	655	667	677	682	680	676	682	668	653	643
744	748	756	731	742	712	700	739	763	784	819	807	788	796	768	745	736
90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
192	194	235	229	207	216	210	218	224	248	244	241	209	206	209	193	188
2.2	2.3	2.8	2.8	2.6	2.6	2.5	2.6	2.6	3.0	2.9	2.9	2.5	2.4	2.5	2.3	2.3
2685	2580	2635	2675	2578	2656	2692	2610	2647	2691	2658	2671	2462	2486	2638	2590	2573
215	215	221	215	215	215	215	218	220	217	221	221	219	220	220	219	219
659	683	693	705	662	680	675	704	726	745	760	754	734	746	719	692	675
1110	1156	1328	1266	1152	1197	1175	1233	1261	1286	1323	1307	1233	1245	1201	1157	1135
444	468	472	490	447	465	460	486	506	528	539	533	515	526	499	473	456
451	473	635	561	490	517	500	529	535	541	563	553	499	499	482	465	460
895	941	1107	1051	937	982	960	1015	1041	1069	1102	1086	1014	1025	981	938	916
1190	1210	1240	1310	1150	1235	1240	1270	1340	1420	1430	1425	1265	1310	1315	1220	1175
1210	1220	1675	1500	1265	1370	1345	1380	1415	1455	1495	1475	1230	1240	1270	1210	1185
2400	2430	2415	2810	2415	2605	2585	2650	2755	2875	2920	2900	2495	2550	2585	2430	2360
50.3	50.2	57.3	53.3	52.3	52.5	52.0	52.0	51.3	50.6	51.2	50.8	49.3	48.7	49.1	49.8	50.2
12.5	12.5	12.4	12.3	11.6	12.0	12.3	12.1	12.3	11.6	11.9	12.0	11.9	12.4	12.3	12.6	12.5
4.58	4.67	4.78	5.06	4.43	4.77	4.78	4.90	5.17	5.47	5.52	5.50	4.88	5.06	5.07	4.71	4.53
12.0	12.0	16.6	14.9	12.5	13.6	13.3	13.7	14.0	14.4	14.8	14.6	12.2	12.3	12.6	12.0	11.7
66.9	66.8	66.2	65.6	62.4	64.5	65.7	64.9	65.8	62.0	64.1	64.5	63.8	66.5	66.0	67.3	67.0
18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650	18,650
100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
12,460	12,450	12,350	...	11,600	12,020	12,260	12,100	12,260	11,550	11,940	12,000	11,870	12,400	12,300	12,550	12,480
66.9	66.8	66.2	...	62.4	64.5	65.7	64.9	65.8	62.0	64.1	64.5	63.8	66.5	66.0	67.3	67.0
1262	1270	1341	...	1251	1275	1280	1282	1284	1297	1293	1289	1255	1252	1250	1233	1227
6.8	6.8	7.2	...	6.7	6.8	6.9	6.9	6.9	7.0	6.9	6.9	6.7	6.7	6.7	6.6	6.6
178	168	108	...	116	123	130	138	129	112	126	130	124	126	133	128	118
1.0	0.9	0.6	...	0.6	0.7	0.7	0.7	0.7	0.6	0.7	0.7	0.7	0.7	0.7	0.7	0.6
3660	3610	3500	...	3265	3800	3690	3690	3890	3855	3790	3860	3740	3690	3600	3480	3370
19.6	19.4	18.8	...	17.5	20.4	19.8	19.8	20.8	20.6	20.3	20.7	20.1	19.8	19.3	18.7	18.1
00.0	00.0	00.0	...	00.0	00.0	00.0	16.0	34.0	00.0	00.0	00.0	00.0	00.0	22.0	00.0	00.0
00.0	0.0	0.0	...	0.0	0.0	0.0	0.09	0.2	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
1090	1152	1351	...	2418	1432	1290	1424	1054	1836	1501	1371	1661	1182	1367	1259	1455
5.8	6.1	7.3	...	12.8	7.7	7.0	7.6	5.6	9.8	7.9	7.2	8.7	6.3	7.2	6.7	7.7

radiation to the cold surfaces, a suction gas thermometer was constructed by placing a thermocouple in a piece of $^3/_8$-in. pipe with the hot junction of the couple drawn back into the pipe a short distance. A steam ejector was used to draw the hot gases through the pipe. The suction gas thermocouple was used in most of the subsequent tests, although in some cases both were used. The difference in the reading of the two thermocouples is indicated in Fig. 8. When the hot junction of the suction thermocouple was pushed some distance beyond the end of the pipe and set close to the junction of the bare thermocouple, the two were found to check. It should be understood that in testing commercial boilers it is not customary to have the flue-gas thermocouple so closely and completely surrounded by cold surfaces, and consequently such discrepancies in flue-gas temperatures as are shown in Fig. 8 are seldom encountered. The flue-gas temperatures reported in tests 2 to 10 inclusive (Table 1) were obtained by correcting the bare thermocouple reading in accordance with Fig. 8. In some cases this involved extrapolation of the curve which the authors realize to be only approximate, but it is believed that these results are more nearly correct than the temperatures as determined by the bare thermocouple.

OPERATING PROCEDURE

In bringing the steam generator up to operating conditions from cold, the feed pump is started and its speed is adjusted according to the flowmeter in order to develop the capacity desired for the test. With the throttle valve wide open, the fire is lighted and the oil flow is regulated so as to raise the pressure to 1500 lb. and give saturated steam in about 20 min. The throttle valve is then closed sufficiently to bring the pressure to within about 400 lb. of the desired operating pressure, after which the oil flow is gradually increased in order to produce superheated steam. As the steam becomes superheated, the pressure increases also. An experienced operator can so manipulate the controls as to bring the unit up to operating pressure and temperature by the adjustment of the oil feed once the throttle valve is set. When the desired pressure and temperature are reached, the automatic control is cut in and the unit will take care of itself. Normally, it takes about 40 minutes to bring the cold boiler up to operating conditions and put it on automatic control, although it can be done by one man in 20 minutes.

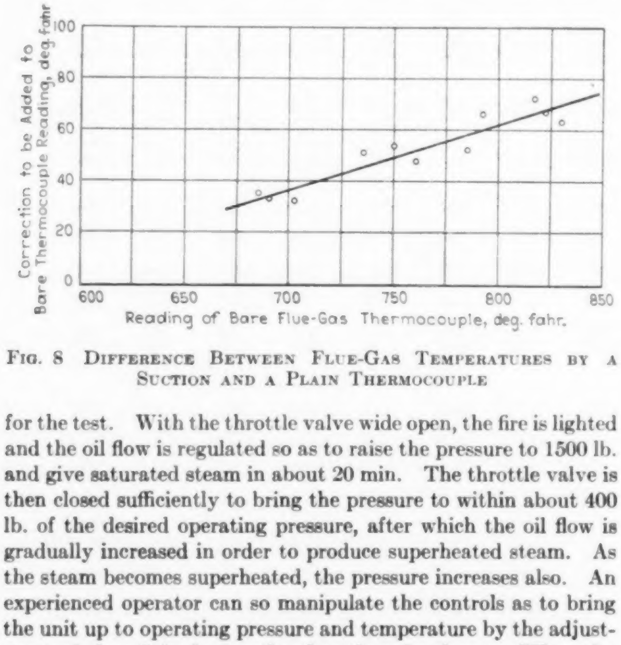


FIG. 8 DIFFERENCE BETWEEN FLUE-GAS TEMPERATURES BY A SUCTION AND A PLAIN THERMOCOUPLE

In shutting down the unit, the oil valve is closed, the dampers are shut, and the boiler-feed pump is kept in operation. Within five minutes the steam gage will register the friction loss through the throttle valve and piping due to the flow of water. The pump is kept in operation for about $1^1/_2$ hours or until the brick furnace is black and the final water temperature is under 250 deg. fahr.

Insufficient draft has proved to be the factor which limits the rating obtainable from the unit. This matter is now being taken care of. The desired flexibility of operation has not been secured as yet because of the limited speed range obtainable from the boiler-feed-pump motor. This is a 230-volt d.-c. motor which is operated with either 115 volts or 230 volts on the field circuit. The maximum speed obtainable with the lower voltage is less than the minimum obtainable with the higher voltage, and the motor becomes unstable in operation when the field voltage is reduced too much. On account of automatic control, it is desirable that the motor speed shall be stable at any given rheostat position and should not be materially affected by changes in pump discharge pressure. A new field winding is being installed on the motor to overcome this difficulty and to give a 3-to-1 speed range. The maximum range of capacity obtainable with a given oil-burner tip is about 2 to 1.

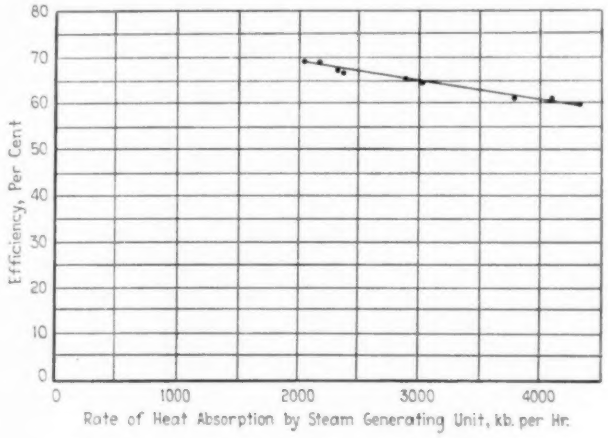


FIG. 9 STEAM-GENERATOR EFFICIENCY, SERIES I

OUTLINE OF INVESTIGATIONS

All tests that are reported in this paper have been conducted with a reasonably constant feedwater temperature and with sufficient excess air to maintain a clean stack. These tests are grouped as follows:

Series I—2550 lb. per sq. in. discharge pressure; 750 deg. fahr. final steam temperature; variable capacity of 2000 to 4280 lb. of steam per hour.

Series II—750 deg. fahr. final steam temperature; 2600 lb. of steam per hour; variable pressure of 1500 to 3520 lb. per sq. in.

Series III—2500 lb. per sq. in. discharge pressure; 2600 lb. of steam per hour; 700 to 820 deg. fahr. final steam temperature.

Series IV—3250 lb. per sq. in. discharge pressure; 2600 lb. of steam per hour; 730 to 800 deg. fahr. final steam temperature.

The steam generator was fired up each morning about two hours before it was planned to begin the testing. Test conditions were established within one hour after starting from cold, and these conditions were maintained constant for at least an hour before the first test readings were taken. Each test was continued for a period of $1^1/_2$ hours, with about $^1/_2$ hour allowed between tests for changing conditions. Since the unit is oil fired and has very little energy storage in the water and steam circuits as compared to the conventional types of boilers, it is believed that the time allowed for tests and for changes in conditions is ample.

Condensate and oil were weighed in tanks on calibrated platform scales. Pressure and temperature readings were recorded at 10-min. intervals, thus giving ten readings to be averaged for

TABLE 2 TEMPERATURES AND ABSOLUTE PRESSURES IN THE STEAM-GENERATOR CIRCUIT

(NOTE: See Fig. 4 for location of thermocouples and pressure-gage connections)

Pressure-gage connection or thermocouple	Test Number																												
	Series I									Series II								Series III							Series IV				
	2	3	4	6	8	9	10	27	29	11	12	13	14	15	16	17	19	20	21	22	23	24	25	26	32	33	34	35	36
P_1.....	2764	2790	2844	3367	3572	3478	3099	2885	3023	2877	3329	3679	3782	3579	2214	2543	3471	2879	2879	2903	2944	3001	3019	3043	3563	3602	3603	3571	3547
P_2.....	2714	2724	2765	3176	3331	3255	2969	2801	2910	2774	3224	3597	3693	3486	2124	2456	3405	2752	2764	2793	2834	2901	2929	2938	3481	3518	3504	3462	3451
P_3.....	2667	2673	2695	3011	3099	3039	2844	2735	2823	2696	3158	3546	3639	3433	1928	2328	3341	2677	2684	2707	2750	2802	2830	2838	3399	3441	3422	3391	3390
P_4.....	..	..	..	..	..	..	..	2682	2760	2643	3119	3511	3608	3392	1832	2271	3329	2629	2643	2664	2694	2734	2753	2763	3362	3399	3380	3354	3355
P_5.....	2608	2605	2609	2798	2828	2801	2703	2621	2701	..	3072	3475	3578	3354	1737	2205	3280	2587	2599	2610	2648	2660	2682	2681	3323	3353	3335	3317	3313
P_6.....	..	..	..	..	..	..	..	2585	2639	2547	3040	3451	3559	3338	1643	2140	3268	2547	2560	2570	2600	2607	2622	2622	3292	3322	3310	3291	3297
P_7.....	2554	2560	2536	2606	2574	2567	2580	2541	2588	2509	3016	3419	3527	3300	1537	2084	3234	2508	2518	2532	2548	2540	2552	2565	3252	3281	3256	3245	3254
t_1......	245	245	247	250	247	248	246	242	245	241	240	238	239	239	248	241	240	240	241	244	246	243	246	246	242	244	244	243	244
t_2......	290	294	296	303	313	314	310	299	304	303	298	292	291	295	309	301	295	298	296	301	306	306	309	309	306	309	305	301	299
t_3......	360	368	368	398	423	427	415	392	400	405	394	383	378	386	408	397	383	388	386	395	402	408	412	411	411	414	403	396	388
t_4......	467	478	476	534	574	581	558	526	534	546	531	613	505	518	545	533	510	517	512	528	540	552	557	554	555	559	545	530	518
t_5......	577	592	592	653	686	689	671	652	655	662	652	635	628	642	648	649	629	636	634	651	661	673	678	676	676	678	664	649	637
t_6......	585	600	600	660	692	693	676	656	660	666	652	641	634	647	646	654	635	642	639	655	667	677	682	680	676	681	668	653	643
t_7......	607	621	622	676	699	702	687	676	676	..	665	654	649	662	646	661	644	652	648	666	678	686	692	692	691	697	685	672	661
t_8......	636	646	645	689	705	704	695	685	687	679	684	673	668	679	646	662	669	671	669	682	687	691	692	693	704	708	700	688	678
t_9......	657	656	658	695	706	705	695	686	692	683	694	690	686	692	645	662	688	682	682	684	687	691	692	693	712	716	710	701	693
t_{10}.....	678	680	682	702	707	705	694	685	691	683	700	696	693	698	643	661	695	682	682	684	686	689	691	691	714	716	712	704	698
t_{11}.....	680	680	682	702	707	705	694	686	692	682	704	705	704	706	642	662	706	682	682	684	686	690	691	691	720	721	718	715	711
t_{12}.....	680	680	682	701	706	705	693	685	692	682	704	707	707	708	642	661	708	680	681	684	685	689	690	690	721	721	719	717	714
t_{13}.....	680	680	682	701	706	704	693	680	691	683	704	..	714	709	641	660	..	681	..	684	686	688	689	691	722	722	720	718	716
t_{14}.....	680	680	681	701	706	704	692	684	689	682	704	712	715	711	640	660	710	681	681	683	685	688	689	689	722	722	720	717	716
t_{15}.....	680	680	681	701	706	703	691	684	689	681	703	714	717	711	640	660	711	680	681	682	685	688	689	690	722	722	720	717	717
t_{16}.....	679	680	681	701	705	703	691	682	690	680	703	715	719	712	637	659	711	680	680	682	684	687	690	690	723	726	723	719	719
t_{17}.....	679	679	681	700	705	702	690	682	690	679	703	715	719	712	637	658	712	680	679	682	682	687	694	693	724	728	724	720	720
t_{18}.....	678	679	681	700	704	702	689	683	688	679	702	715	722	713	636	657	713	680	679	682	684	692	701	698	724	..	724	720	719
t_{19}.....	678	679	681	699	703	701	689	682	689	679	702	717	723	714	633	657	713	680	679	681	685	695	..	710	726	730	725	720	719
t_{20}.....	678	679	681	698	703	701	688	687	690	679	702	719	724	715	632	656	714	680	679	681	688	701	713	712	..	734	726	720	720
t_{21}.....	678	678	680	698	702	700	688	690	694	679	704	719	724	716	631	657	714	680	679	681	691	705	719	717	731	738	729	721	720
t_{22}.....	678	678	680	697	702	699	688	685	689	679	702	719	724	715	631	657	714	680	679	681	685	690	700	697	728	729	726	720	719
t_{23}.....	678	678	680	697	702	699	688	687	691	679	702	720	724	716	635	657	714	680	679	681	690	..	712	708	..	736	727	720	719
t_{24}.....	678	678	680	697	701	700	692	687	692	679	702	720	724	716	638	656	715	679	678	682	..	700	714	712	734	738	727	719	..
t_{25}.....	681	681	680	701	705	702	694	693	696	..	706	721	725	718	649	657	716	678	678	685	697	709	724	720	737	742	730	721	719
t_{26}.....	686	688	685	705	710	705	697	697	700	688	708	724	729	722	659	661	719	678	678	..	702	714	733	727	742	747	..	724	722
t_{27}.....	690	693	689	707	712	708	699	702	703	690	712	726	730	723	667	665	721	679	678	693	707	718	739	733	745	750	735	724	721
t_{28}.....	694	697	692	709	712	710	700	706	706	694	715	726	731	724	674	669	723	677	678	695	708	724	746	740	748	754	738	725	722
t_{29}.....	700	704	697	712	715	713	703	712	712	700	720	729	732	727	686	677	..	683	677	701	716	731	755	749	753	761	741	728	723
t_{30}.....	705	710	702	716	718	718	706	715	716	704	723	730	734	729	693	683	726	685	680	706	723	738	762	755	756	765	743	730	724
t_{31}.....	710	716	706	719	721	721	710	719	721	708	730	..	..	..	701	687	727	688	681	709	729	743	770	762	762	768	748	733	725
t_{32}.....	715	721	711	723	724	724	714	727	725	713	729	734	736	735	709	694	730	691	684	713	734	749	776	767	765	772	752	734	726
t_{33}.....	710	717	711	717	720	723	712	722	721	710	726	732	736	733	702	693	727	..	686	711	727	746	774	765	762	769	749	734	726
t_{34}.....	715	723	714	720	721	723	714	725	725	717	732	735	737	736	710	697	729	695	688	716	735	752	781	771	768	775	753	735	726
t_{35}.....	721	727	719	723	724	727	716	727	728	720	734	737	738	737	717	702	732	697	688	719	738	756	785	775	769	775	753	735	726
t_{36}.....	725	733	724	725	725	728	717	731	732	721	735	738	739	738	724	705	733	698	..	721	742	761	790	782	772	779	755	736	727
t_{37}.....	728	737	727	727	726	730	721	736	737	725	738	739	739	739	730	710	734	701	693	725	748	766	797	787	776	782	757	737	727
t_{38}.....	734	742	733	731	730	733	725	738	739	727	739	740	740	741	737	715	735	702	694	728	751	770	802	791	778	785	758	..	728
t_{39}.....	741	748	739	734	735	737	729	743	744	733	743	742	742	744	745	722	739	707	697	733	756	776	810	798	782	789	763	741	730
t_{40}.....	745	754	742	738	739	741	732	746	746	736	744	743	742	745	749	725	739	709	697	734	757	779	814	800	782	790	763	742	731
t_{41}.....	749	756	746	739	740	741	735	751	751	741	748	746	743	748	756	731	742	711	700	739	765	786	820	808	787	796	767	744	732
t_{42}.....	750	757	747	741	737	739	736	751	751	741	748	745	744	748	756	731	742	712	700	739	763	784	819	807	787	796	768	745	732

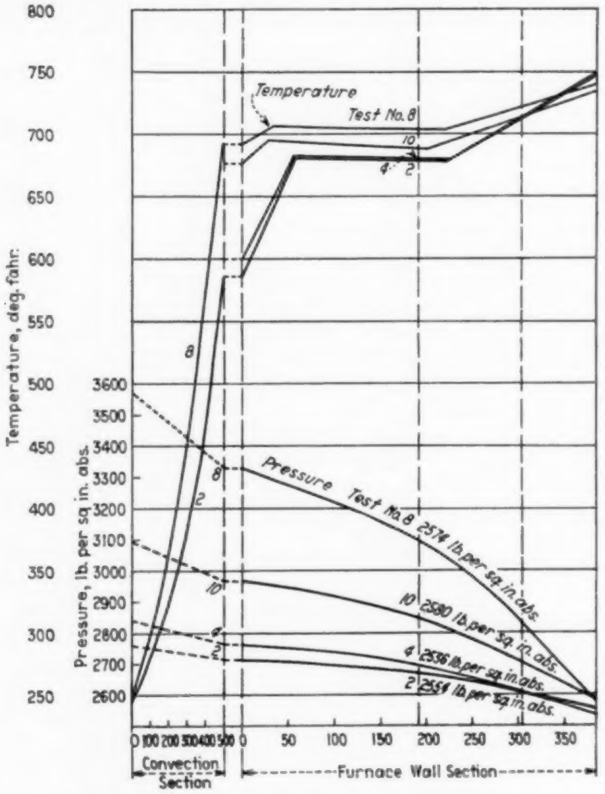


FIG. 10 PRESSURE-TEMPERATURE RELATIONSHIPS, SERIES I (Tests 2, 4, 8, and 10.)

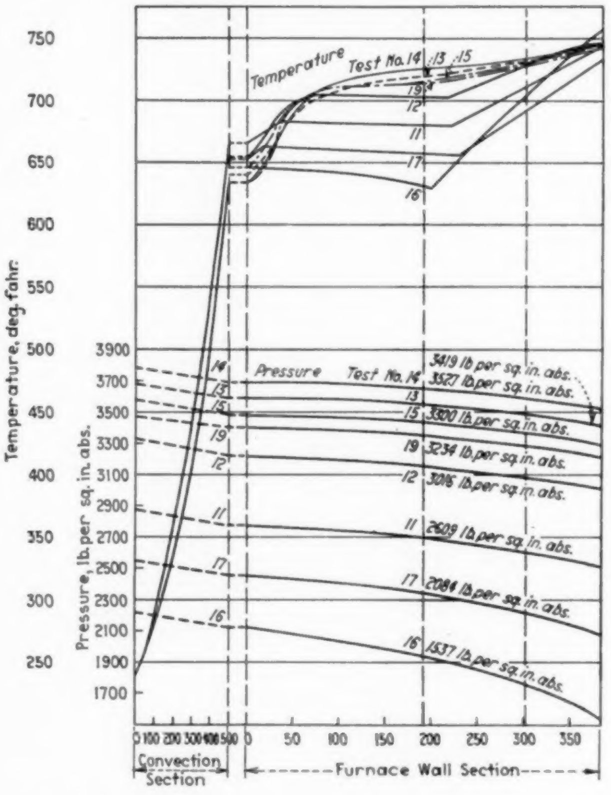


FIG. 11 PRESSURE-TEMPERATURE RELATIONSHIPS, SERIES II (Tests 11-17 and 19.)

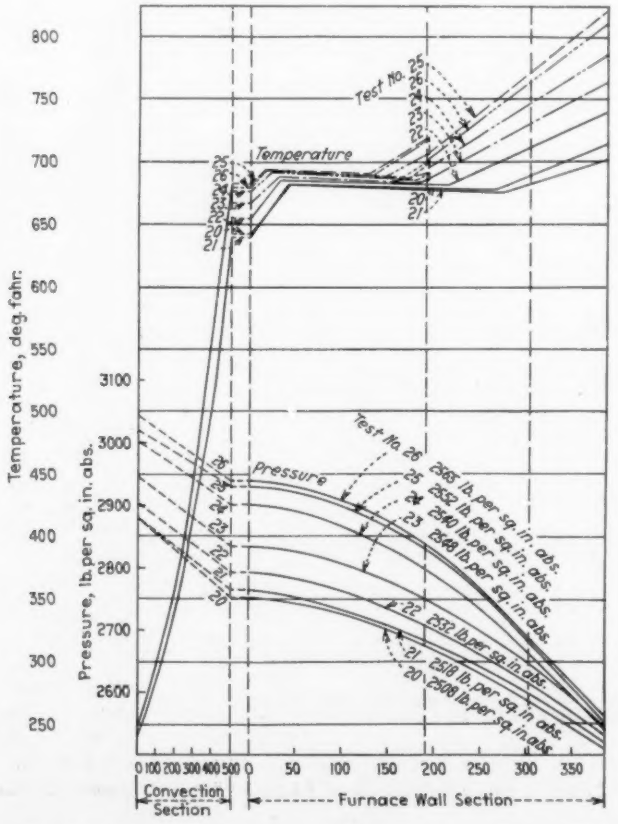


FIG. 12 PRESSURE-TEMPERATURE RELATIONSHIPS, SERIES III (Tests 20-26.)

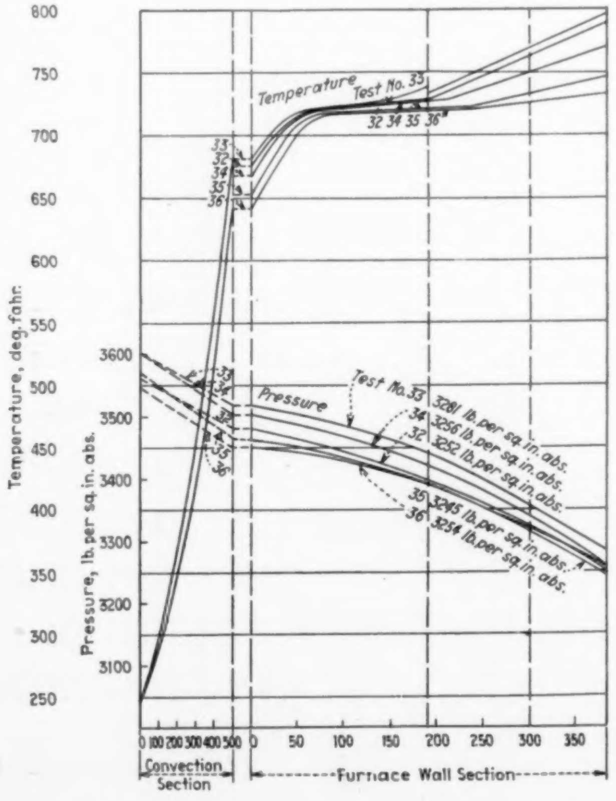


FIG. 13 PRESSURE-TEMPERATURE RELATIONSHIPS, SERIES IV (Tests 32-36.)

each test. The data, together with the calculated results of these tests, are recorded in Tables 1 and 2.

A single shipment of oil was used in all tests reported in this paper. The analysis of the oil was as follows:

Carbon, per cent	88.83
Hydrogen, per cent	10.75
Nitrogen, per cent	0.14
Sulphur, per cent	0.57
Gravity (A.P.I.)	15.9
Viscosity at 100 deg. fahr. (Saybolt Universal), sec	172
Pour point, deg. fahr	Below 0
Flash point (Cleveland open cup), deg. fahr	260
Fire point (Cleveland open cup), deg. fahr	320
Heating value, B.t.u. per lb	18,650

Discussion of Results

The efficiency of the steam generator, as reported in Table 1 and Fig. 9, is between 60 and 70 per cent within the load range at which it was operated. High efficiencies cannot be expected from a unit of this size having a short single-pass convection section. In a commercial installation this type of steam generator would undoubtedly be equipped with air heaters.

The pressures and temperatures in the circuit of the steam generator, while operating at several different conditions, have been plotted in Figs. 10 to 13. The abscissa represents the length of the circuit in feet in each of the various sections of the steam generator. It should be noted that the furnace-wall section has been plotted on a scale which is five times that of the convection section and that a break exists on the diagram between the two sections. The pressure curves in the convection section have been shown as dotted lines, since only the initial and final pressures have been measured. The shape of this curve can be determined by connecting the pressure gage into the thermocouple taps. The pressure-temperature relationships which exist in the various parts of the steam generator under several different conditions of operation indicate the following:

The temperature curves for the variable-load tests (series I) show the position of the evaporating zone as a straight line in which the temperature decreases slightly due to the effect of friction. The furnace and heating surfaces are so proportioned that the length of tubing utilized to superheat the steam is practically constant within the range of capacities at which the unit was operated. As the output is increased, a larger percentage of the total heat absorption takes place in the convection section, with the result that at the higher capacities the water enters the furnace section at a temperature not far from the saturation temperature. The increased temperature in the evaporating zone at higher ratings is due to the additional pressure drop in the circuit.

The effects of operating at a variety of pressures (series II) and the consequent pressure-temperature relationships in the circuit are illustrated in Fig. 11. The difference between maximum and minimum flow in pounds per hour during this series of tests is about 4 per cent, but the difference in heat absorption is considerably greater. The pressure drop through the system varied from about 700 lb. at 1500 lb. final steam pressure to 260 lb. at 3500 lb. final steam pressure. While the steam generator operates as satisfactorily and is as easily controlled automatically at the lower pressures as at the high pressures, it would be desirable for operation at the lower pressures to divide the surface into a greater number of parallel circuits of shorter length or to use tubes of larger diameter. Four tests were conducted with final steam pressures below 3200 lb. and the same number at final pressures above 3230 lb. At the four lower pressures a distinct evaporating zone is noticeable where the temperature curve is a straight line which drops off slightly due to a decrease in the saturation temperature caused by friction in the tubing. Above 3230 lb. final pressure the temperature curves show a gradual and continuous increase in temperature, indicating the apparent absence of an evaporating zone. Additional thermocouples will have to be placed in the first 50 ft. of furnace-wall tubing in order to locate the temperature curve more definitely in this section. However, the tests up to date seem to indicate that the critical pressure is between 3200 and 3300 lb. per sq. in. The pressure range between 3000 and 3500 lb. per sq. in. will be investigated in the near future much more extensively than has been done.

In Fig. 12 is illustrated the effect of varying the final steam temperature at 2500 lb. pressure with a fairly constant evaporation in pounds per hour (series III). Increasing the superheat means burning more fuel. The friction loss through the furnace-wall section is increased considerably, as are also the pressure and temperature in the evaporating section and the temperature of the water leaving the convection surface. Any degree of superheat is obtainable from zero to that which is limited by the materials of the tubes, and the unit operates equally well under the conditions reported. The principal difference is an increase in the amount of furnace-wall tubing used for superheating the steam as the outlet temperature is changed. Some unbalance apparently exists in the two parallel circuits of the lower furnace-wall section. It is not sufficient to be serious, and from the experience gained in variable-load operation it has been found that it becomes less at the higher ratings where the furnace temperatures are the highest and when the heat-transfer rate through the tubes is the greatest. All tests were run on automatic control, and it is noticed that an increase in steam temperature tends to raise the final pressure slightly. The pressure control could have been adjusted between tests to avoid this tendency, but it was not considered of sufficient importance.

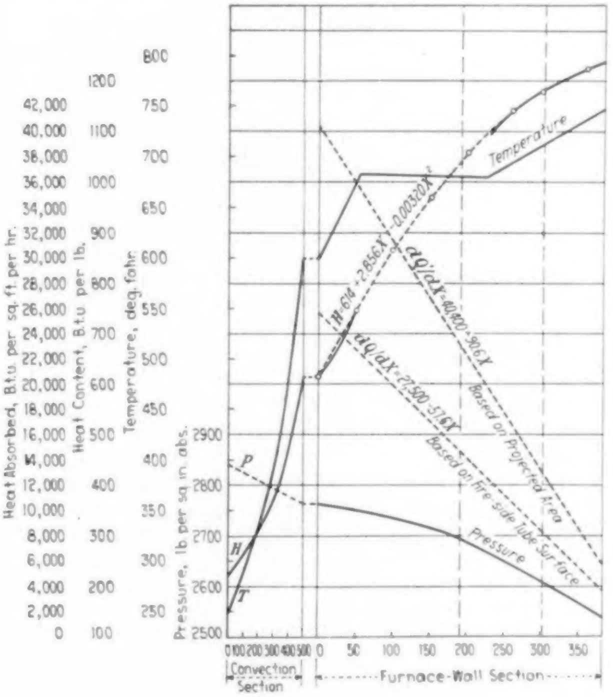


Fig. 14 Heat-Transfer Curves
(Test 4.)

The pressure and temperature changes when operating at about 3250 lb. per sq. in. final pressure, fairly constant flow, and final temperatures between 730 and 800 deg. fahr. (series IV) are illustrated in Fig. 13. The friction loss through the unit is of course increased at higher steam temperatures, due to the greater specific

TABLE 3 HEAT-TRANSFER RATES IN FURNACE-WALL TUBES

(Results Are Given in B.t.u. per Square Foot per Hour, Expressed as a Function of the Length of Furnace-Wall Circuit X in Feet)

Test	No.	Final steam pressure, lb.	Final steam temperature, deg. fahr.	Evaporation, lb. per hour	Enthalpy per pound	Heat-transfer rate based on external surface	Heat-transfer rate based on projected area
Series I	2	2554	751	2003	$H = 593 + 3.074X - 0.00364X^2$	$dQ/dX = 23520 - 55.7X$	$dQ'/dX = 37000 - 87.5X$
	4	2556	747	2358	$H = 614 + 2.856X - 0.00320X^2$	$dQ/dX = 25700 - 57.6X$	$dQ'/dX = 40400 - 90.5X$
	10	2580	737	3022	$H = 746 + 2.058X - 0.00214X^2$	$dQ/dX = 23750 - 49.4X$	$dQ'/dX = 37300 - 77.5X$
	8	2574	741	4280	$H = 776 + 1.656X - 0.00123X^2$	$dQ/dX = 27050 - 40.2X$	$dQ'/dX = 42500 - 63.2X$
Series II	16	1537	756	2635	$H = 693 + 2.945X - 0.00337X^2$	$dQ/dX = 29600 - 67.7X$	$dQ'/dX = 46500 - 106.4X$
	17	2084	731	2675	$H = 705 + 2.338X - 0.00228X^2$	$dQ/dX = 23850 - 46.5X$	$dQ'/dX = 37450 - 73.0X$
	11	2512	741	2600	$H = 726 + 2.197X - 0.00223X^2$	$dQ/dX = 21800 - 44.3X$	$dQ'/dX = 34250 - 69.6X$
	12	3016	748	2676	$H = 692 + 2.236X - 0.00244X^2$	$dQ/dX = 22800 - 49.8X$	$dQ'/dX = 35800 - 78.2X$
	19	3233	742	2578	$H = 662 + 2.147X - 0.00226X^2$	$dQ/dX = 21150 - 44.5X$	$dQ'/dX = 33200 - 70.0X$
	15	3300	748	2580	$H = 683 + 1.872X - 0.00173X^2$	$dQ/dX = 18450 - 34.1X$	$dQ'/dX = 29000 - 53.6X$
	13	3419	745	2605	$H = 671 + 1.761X - 0.00149X^2$	$dQ/dX = 17510 - 29.6X$	$dQ'/dX = 27550 - 46.3X$
	14	3527	744	2685	$H = 659 + 1.792X - 0.00163X^2$	$dQ/dX = 18390 - 33.4X$	$dQ'/dX = 28900 - 52.5X$
Series III	21	2517	700	2692	$H = 675 + 2.175X - 0.00234X^2$	$dQ/dX = 22350 - 48.1X$	$dQ'/dX = 35100 - 75.6X$
	22	2534	739	2610	$H = 704 + 2.386X - 0.00263X^2$	$dQ/dX = 23800 - 52.5X$	$dQ'/dX = 37400 - 82.5X$
	24	2539	784	2691	$H = 745 + 2.433X - 0.00269X^2$	$dQ/dX = 25000 - 55.3X$	$dQ'/dX = 39300 - 87.0X$
	25	2552	819	2658	$H = 760 + 2.484X - 0.00267X^2$	$dQ/dX = 25200 - 54.1X$	$dQ'/dX = 39600 - 85.0X$
Series IV	36	3254	736	2573	$H = 675 + 2.081X - 0.00239X^2$	$dQ/dX = 20450 - 47.0X$	$dQ'/dX = 32150 - 73.9X$
	34	3256	768	2638	$H = 719 + 2.159X - 0.00235X^2$	$dQ/dX = 21700 - 47.3X$	$dQ'/dX = 34100 - 74.4X$
	32	3253	788	2462	$H = 734 + 2.253X - 0.00250X^2$	$dQ/dX = 21200 - 47.0X$	$dQ'/dX = 33300 - 73.8X$
	33	3281	796	2486	$H = 746 + 2.246X - 0.00250X^2$	$dQ/dX = 21350 - 47.5X$	$dQ'/dX = 33500 - 74.6X$

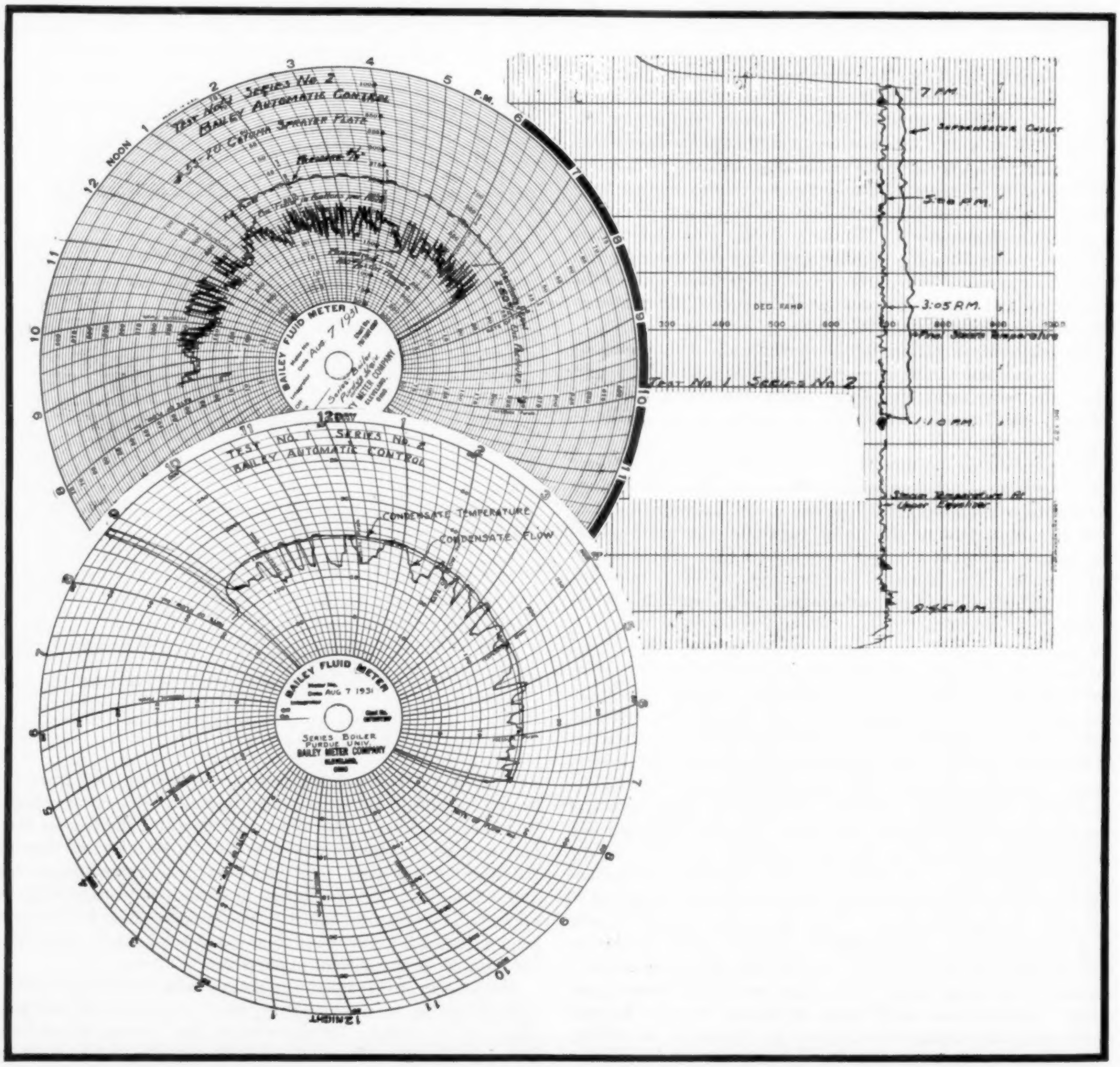


FIG. 15 PERFORMANCE OF BAILEY AUTOMATIC CONTROL WITH A SWINGING LOAD

volume of the steam. Some unbalance exists in the lower furnace-wall section at the highest final steam temperature. In the furnace-wall section there is a rapid rise in temperature, followed by a section in which the temperatures increase quite slowly and then a more rapid rise to the final temperature. If the steam tables are correct at these pressures, an isothermal process, which takes place with decreasing pressures, involves the absorption of considerable energy by the steam. Consequently, although the temperature is seen to increase throughout the length of the furnace-wall section, the absence of such a temperature rise might not indicate the presence of an evaporating zone.

Studies on heat transfer in the furnace-wall section have been handled as follows:

In Fig. 14, the curves of pressure and temperature relations for test 4 have been plotted against tube length. The enthalpy, or heat content, per pound of the liquid and of superheated steam has been calculated from these data, using Keenan's Steam Tables and the data on Compressed Liquid Water as published by Professor Keenan in *Mechanical Engineering* for February, 1931. The solid line labeled H was thus obtained. Two points were selected in the superheat region and one in the liquid region of the furnace-wall section, from which a quadratic equation of enthalpy was obtained, expressed as a function of the length of the circuit X. The first derivative of this equation, when multiplied by the steam flow in pounds per hour and a constant (3.819), gives, as a function of the tube length, the rate of heat transfer in B.t.u. per sq. ft. of (gas side) furnace-wall surface per hour. This equation has been called dQ/dX. The heat-transfer rate based on the projected area of the furnace-wall surface (dQ'/dX) is obtained by multiplying the equation by $\pi/2$. Both heat-transfer curves are plotted in Fig. 14. Heat-transfer equations were obtained in a similar manner for most of the tests reported in this paper, and are given in Table 3.

With 240 deg. fahr. feedwater, the water temperature at the entrance to the furnace-wall section is so high that the curve of enthalpy of the liquid is quite short. For this reason only one point was selected in the liquid region when determining the enthalpy equation as a function of the tube length. Additional data with lower feedwater temperatures may make it possible to develop a cubic equation instead of the present quadratic equation and with possibly more accurate results.

The information on heat-transfer rates secured in the tests reported in this paper does not include all of the variables which enter into this problem. Additional tests will be undertaken in the near future to cover this phase of the investigation.

The experience with the Bailey automatic control indicates that the system functions satisfactorily at any final steam pressure between 1500 and 3500 lb. per sq. in.; also at any final steam temperature ranging from a few degrees of superheat to 830 deg. fahr., which is considered the maximum temperature at which it is desirable to operate the unit.

In Fig. 15 is exhibited a set of meter records obtained with automatic control on a swinging load over a period of six hours. These charts show that it is practical to operate with instantaneous load changes of approximately 20 per cent of the maximum possible rating without abnormal variation in pressure and with small variations in final steam temperature.

It has not been possible to determine the maximum instantaneous load changes which can be handled by the control system because of limitations in the speed variation obtainable from the boiler-feed-pump motor, but this is now being corrected.

This investigation was made possible through the generous cooperation of the Babcock & Wilcox Company, the Bailey Meter Company, the Crane Company, the Commonwealth Edison Company, the Crosby Steam Gage and Valve Company, and the Fisher Regulator Company.

Discussion

D. Robert Yarnall.[4] The high-pressure boiler described in Dean Potter's paper is of great interest to steam engineers everywhere, not only because of its high pressure and temperature range, but also because of its unique design, which enables it to be operated for experimental purposes at a wide range of pressures. In this same connection perhaps it will be of interest to engineers to know that another high-pressure boiler has just been installed and placed in service in our new steam laboratory in Chestnut Hill, near Philadelphia, which has still other characteristics. This boiler has been designed for the purpose of developing apparatus for the power plant and for industrial plants. Its characteristics must be such as to provide a controllable water level at the highest pressures and temperatures likely to be met by such apparatus in the large steam-power stations now built and likely to be built in the next ten years.

The boiler for this research laboratory, built for us by the Superheater Company, is of an unusual design. To the best of our knowledge it is the only boiler built to date for 2500 lb. working steam pressure, depending only on thermal circulation for the protection of evaporating surface.

This boiler consists of a mud drum, evaporating surface, and a steam drum. The evaporating surface is in the form of an ascending multi-loop construction, the terminals of which are connected to the mud drum and steam drum. The evaporating surface is in the form of a multi-loop construction so as to eliminate any strain where the evaporating surface is connected to the mud drum and the steam drum, due to contraction and expansion. The downcomers from the steam drum to the mud drum for feeding this evaporating surface are located out of the gas pass, but inside of the setting, so as to insure against any steam generation at this point, thus eliminating any tendency for the changing of the direction of flow in the evaporating surface.

In the generation of steam at 2500 lb. pressure the steam bubbles are considerably smaller due to this pressure than at the lower pressures; therefore considerable care must be exercised in the design of a boiler of this high pressure in order that the metal temperature be kept within a safe operating range when taking into consideration the metal used. This protection of the metal is obtained by fabricating the surface, as mentioned, in ascending multi-loop construction. The water is compelled to circulate continuously in the same direction, and by doing this those parts of the heating system exposed to the greatest heat always have water present, and not until the gases have cooled down do they come in contact with the tube containing the mixture of steam and water. Thus a positive circulation is established.

The evaporating surface is joined to the mud drum by means of a rolled joint with handhole openings opposite, whereas the surface is joined to the steam drum by means of a special clamp construction, making a metal-to-metal joint and also eliminating the necessity of handhole openings for making this joint. This type of joint was used on the steam drum to cut down the number of joints necessary to avoid the difficulties which may develop due to the changes in temperature which may be expected on the steam drum, as it has been found by practice that joints under these conditions should be kept down to the minimum, especially so when working under such high pressures.

The design of the high-pressure metal parts in this boiler was so arranged as to make all joints on the outside of the gas pass accessible for inspection, as well as for maintenance, and at the same time making the complete boiler as simple as possible in construction.

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[4] Vice-President, Yarnall-Waring Co., Philadelphia, Pa. Mem. A.S.M.E.

The steam drum, having outside length of 5 ft. 10 in., outside diameter 18 in., inside diameter 12 in., or 3 in. wall thickness, is made from a solid forging such as is used on most high-pressure boilers. The mud drum is also bored from a solid forging.

The safety valve, made for 2500 lb. relieving pressure, is of the conventional type used on present high-pressure boilers, but in this case for a series of tests at lower pressures, extra springs were provided for relieving pressures of 750, 1500, and 2000 lb.

The boiler is provided with a series of different types of forged-steel water columns and high-pressure flat and round glass water gages. The blowoff valves installed for test purposes are of the forged-steel seatless type, designed for these high pressures.

This boiler has been designed with 130 sq. ft. of heating surface for maximum working steam pressure of 2500 lb.; a furnace volume of 82 cu. ft. has been provided. The boiler is fired by city gas fuel, the rate of combustion being automatically controlled by the steam pressure for the particular test being made at the time, the steam pressure itself being controlled for each test by an adjustable regulator.

In view of the small bore of the tubes which form the heating surface of the boiler, elimination of scale is very important in the operation of this boiler. For this purpose an evaporator has been provided, the condensate from which is stored in a vertical tank, from which the triplex motor-operated feed pump provides the necessary boiler feed.

Later on we shall be glad to present the results of the year's research resulting from the operation of this high-pressure steam testing laboratory. We will be glad to have others who may desire to conduct tests on high-pressure steam make use of this steam laboratory for their own tests, provided of course such tests do not conflict with our own researches.

Discussion following was given jointly with the paper by H. J. Kerr)

A. G. Christie.[5] Mr. Kerr's boiler has seven parallel circuits for the water and steam between feed inlet to the boiler proper and the outlet manifold. He had considerable trouble with instability of flow under certain conditions with these multiple circuits. The Purdue boiler, on the other hand, had only two parallel circuits, and these were provided with equalizer connections at intervals. Profs. Potter and Solberg do not mention any instability difficulties with any feed temperature. Evidently if multiple circuits are desired—and these will be necessary if large outputs are to be obtained—numerous equalizer connections should be provided to remove the danger of instability and to keep temperatures within moderate limits in all circuits.

Mr. Kerr might give a more complete discussion of the derivation and use of the data presented in Figs. 7 and 8. Heat contents of 3000 B.t.u. per lb. of steam, as here indicated, would require temperatures of 4000 deg. fahr. or thereabouts. The analysis of and deductions from these curves are not clearly stated in the paper.

It is claimed for this type of boiler that no scale is formed in the tubes. If absolutely pure water were used at all times, this boiler would serve admirably. But condenser leakage, and the desirability of maintaining proper sulphate-alkalinity ratios in the feed, will lead to the introduction of some solids into the boiler. If these pass on with the steam, there will be deposits of such materials in turbines at those stages near the dew point. Such deposits close the blade passages and lower the capacity of the turbine. It is a difficult matter to remove such deposits, especially if silica is present. This trouble may be greater with the through type of boiler than with present designs.

Boiler-feed pumps of the plunger type have proved most economical for pressures over 3000 lb. With such pumps the power at light loads becomes of such magnitude as to consume a large portion of the gain from going to such higher pressures. The question of auxiliary power will be an important consideration with high pressures.

This boiler has demonstrated the workability and reliability of high-pressure equipment and has opened the way for commercial developments. With materials under development for high temperatures as well, it seems reasonable to expect plants to be built for, say, 2500 lb. pressure and 1000 deg. fahr. steam temperature, requiring only one stage of steam reheating.

N. Artsay.[6] The two papers supplement one another, and it is best to consider them together. Enough has been said on the general aspects of the problem of coil boilers, and it is desired to comment on some important details.

The instability of flow in parallel circuits has been mentioned in both papers. The word instability implies that in some of the parallel circuits the resistance in presence of evaporation tends to decrease in spite of an increase in the rate of flow. The writer has made a theoretical investigation of stability of flow in such conditions and has found that the real instability is utterly impossible unless water passes through some metastable phase. What actually happens is that the changes in resistance at changing rates of flow are substantially reduced by the opposite changes in specific volume of the mixture, and so a considerable variation in flow is necessary to balance a slight change in hydraulic resistance, hence large steam temperature differences at the outlets from uniformly heated parallel circuits. In a given circuit the resistance of water is proportional to the square of mass flow, and in presence of evaporation the resistance increases also in proportion to the increasing specific volume, the latter in turn varying in opposite direction as the mass flow. Thus in evaporating coils the resistance will vary as a power of mass flow lesser than two but always positive. Therefore the true instability is impossible, but quite a range of conditions exists for the *uneven distribution* of flow among the parallel circuits.

Diagrams for 2500 and 3200 lb. per sq. in. pressures (Figs. 7 and 8), given in Mr. Kerr's paper to show the inherent instability of flow in some conditions, are very difficult to understand if the coordinates and the caption are correct. The relation between the curves is not clear, because it is not stated what factor has been kept constant in drawing the curves. Enthalpies of 3000 B.t.u. per lb. shown on the diagrams mean steam temperatures of the order of 5000 deg. fahr., and the pressure drops are so low that one is tempted to refer them to a small stretch of coil.

The work done at Purdue University, presented by Messrs. Potter, Solberg, and Hawkins, is excellent material for studies of coil-boiler possibilities. It seems that the character of the work undertaken by the authors invites suggestions on lines of further experimentation. So as to avoid misunderstandings in this serious feature of coil boilers, the diagram should be thoroughly explained.

The traced position of the point at which all water is converted to steam is rather important, as bearing not only on the correctness of thermocouple readings in their actual setting, but also as a possible means to detect the presence of metastable states of water. Therefore it would be of interest to weld two nipples before and after the average position of said point for connecting Professor Donovan's electrical calorimeter recently developed and checking the calorimetric determinations of steam quality at the pressure against temperature readings. This ingenious calorimeter could be of great value also in other ranges of evaporation. Great variations of heat content at slight temperature changes are indicated around the critical point, and the calorimeter would serve to check them.

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[6] Principal Assistant Engineer of Geo. A. Orrok, Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

Another line of experimentation is suggested. At some point, let us say about 80 per cent vaporization, the boiler should be tapped and a small fraction of the mixture of steam and water led into a steel test tube heated strongly by a gas or oil burner. Concentrated solutions of various salts could be injected into the test tube and the deposition of scale in various conditions studied. The effect of velocity, pressure temperature, and concentration on scale formation could be studied at a low expense and in the range of conditions so far almost unexplored. Operation of Benson boilers seems to indicate that troubles due to scale deposits are reduced to a great extent if the vapor phase is eliminated. On the other hand, operation of high-fluid-velocity coil boilers also indicates the reduction of scale deposition. Thus a very interesting field may be opened up by such an investigation.

R. M. Gates[7] and W. H. Armacost.[8] The experiences outlined in the two papers should be of considerable value to those interested in this field of development. Analyzing the data reported in these papers, there are several points which do not seem clear.

Referring to Fig. 9 in Mr. Kerr's paper, it appears that this type of boiler unit is very sensitive, due to the wide variation of steam conditions from the several different circuits. It would be interesting to know whether this was due to unit design features, furnace conditions, or the type of feed pump used. It has been demonstrated in the operation of cracking units, as well as oil-heater units, which had several parallel flows, that wide variations in temperature resulted. This was of a serious consequence, due to the coking of oil in some of the circuits from overheating. Therefore, in some instances, separate pumps were used for individual circuits to enable uniform outlet conditions to be maintained. However, due to the improvements in design of heat exchangers, it was possible to reduce the number of streams and revert to the use of a single pump, at the same time obtaining sufficiently uniform temperatures at the outlets of the circuits.

There is another factor to which all power engineers are giving a great amount of consideration, this being the effect of the tremendous work which the metal must do and the temperatures which it must stand in high-pressure, high-temperature power-plant apparatus. To make this problem still more difficult there are the different sources of heat to deal with—of radiation, convection, or a combination of the two.

The preliminary results which have been obtained and are given in the report on the boiler at Purdue lead to much speculation regarding certain characteristics of water and steam at the place of transformation from one to the other when located in the radiant-heat section of the boiler.

Referring to the paper by Messrs. Potter, Solberg, and Hawkins, upon the high-pressure series boiler installed at Purdue University, it will be noted that Fig. 4 shows the relative positions of the sections of convection and radiant-heat absorbing surfaces, as well as location of thermocouples and pressure connections. Also, test No. 4 (quantity readings), tabulated on the sixth page, together with the thermocouple readings on the ninth page, shows that a uniform temperature in the radiant section is obtained from thermocouple position 10 to thermocouple position 25, which apparently denotes the transformation sections where liquid is being changed into vapor. This section contains approximately 45.5 sq. ft. of furnace exposed surface. Assuming that the temperatures in this section correctly denote those of water and saturated steam at the corresponding pressures and that latent heat only is added at this point, the heat-transfer rate is approximately 15,300 B.t.u. per sq. ft. of furnace exposed surface.

Referring to test No. 8, it will be noted that practically the same square feet of surface is in the transformation section, the temperature varying from 707 to 701 deg., whereas the absorption per square foot of furnace exposed surface would be 7140 B.t.u. if based upon the steam tables. It appears this absorption should be considerably higher than in test No. 4, as approximately twice as much fuel is fired, also the gas temperature leaving the boiler is 50 per cent higher.

Therefore in checking through the recorded thermocouple readings in Table 2, it appears that these readings in the transformation section are not giving the true temperatures, since practically a uniform temperature is recorded over a considerable length of tubing. In analyzing this further, it appears that this might be due to the fact that the furnace structure has a circular cross-section, and with the water flowing at the rate of 7 to 15 ft. per sec. through the tubes, the water holds to the opposite wall of the tubes from the furnace, due to the centrifugal force set up, whereas the steam being liberated in the form of small bubbles would probably have a tendency to slide along the inner surface of the tubing next to the furnace, thus causing the steam and water to travel along a greater length of tube before a thorough mixture results and complete transformation has taken place. Due to this reasoning the furnace side of the tubing in the transformation section would be overheated and the growth in the tube diameter would be greater in this section over a period of time than in any other part of the equipment. It would be interesting to know whether the authors of these papers have experienced any unusual growth or failure of the tubing in this particular section of the equipment.

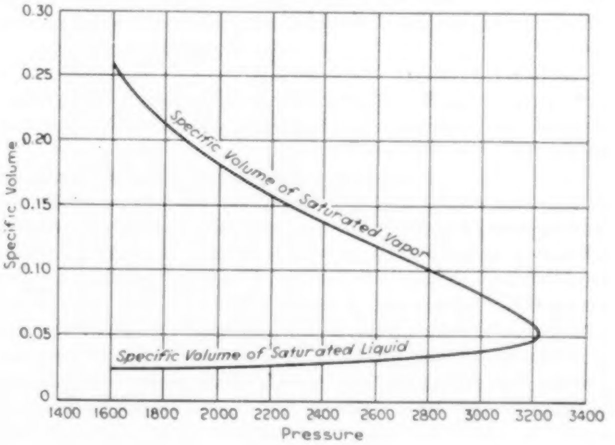

Fig. 16

Referring to Fig. 13, which shows that the temperature curves as plotted are practically a horizontal curve from 75 to 175 ft. on the furnace-wall section, it appears that this might be due to the foregoing reasoning, as the thermocouple readings are probably taken on the outside wall, where the water in the transformation section is always in contact with these thermocouples due to the centrifugal force resulting from the velocity of water. It would be interesting to know how all of these thermocouples were located in the tubing relative to the furnace.

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[7] Vice-President, The Superheater Company, New York, N. Y. Mem. A.S.M.E.

[8] Engineer, Industrial Department, The Superheater Company, New York, N. Y. Assoc-Mem. A.S.M.E.

To substantiate this reasoning, it is only necessary to draw the curves showing the specific volume of saturated vapor and the specific volume of saturated liquid, using pressure and cubic feet per pound as abscissas and ordinates, respectively. By doing this the relatively small change in volume is readily visualized when saturated liquid changes to saturated vapor at these pressures. (See Fig. 16.)

Mr. Yarnall, in his discussion, has told that the particularly novel feature of his testing plant is the gravity circulation, or the fact that circulation was natural and not forced or positive in the boiler.

There are other features of design which are unique, but it must be borne in mind that the installation at the Yarnall-Waring plant was made for a specific condition, while the equipment for the Purdue plant was designed for capacity as well as for temperatures and pressures. It will be interesting to hear Mr. Yarnall's further remarks regarding the operation, not only of the boiler equipment, but also of the apparatus which will be tested.

It is encouraging to note the interest that manufacturers are taking in testing laboratories of their own. It is further interesting to note that there are facilities open at universities as well as at certain corporations where other manufacturers can have instruments and apparatus tested under conditions approaching those which will be attained in the field of operation.

The writers feel, however, that laboratory results will not give the entire answer and that we must still utilize the operating plants to give final results, but the laboratory work should eliminate much of the expense and time required to do all such work in the field.

The tests as reported are comprehensive and the equipment installed lends itself to very interesting tests as to absorption at high pressures by radiation as well as by convection. The authors are to be complimented on the great amount of test work accomplished and recorded in such a short interval of time.

Karl A. Mayr.[9] The systematic work done with the so-called once-through series boiler will be greatly appreciated by every one interested in steam generation.

Fig. 1 of Mr. Kerr's paper, showing over-all plant efficiencies at various steam pressures, is misleading. The efficiencies shown on this paper are obviously theoretical efficiencies based on the amount of heat available at straight adiabatic expansion down to a certain percentage of moisture, if steam of various pressures but of equal temperature is used.

When employing higher steam pressures, higher steam temperatures can be used effectively. If one would use, for instance, a steam temperature of 1000 deg. fahr. in connection with a steam pressure of only 300 or 400 lb. per sq. in., superheated steam would be blown into the condenser. A part of the valuable superheat could not be converted into power by any means and would be carried away by the condenser cooling water. If the temperature of 1000 deg. fahr. is used in connection with 1000 lb. per sq. in. steam pressure, a highly efficient cycle is obtained.

With higher steam pressures, more turbine bleeding and feedwater heating stages can be employed. Valuable heat can be transmitted to the boiler feedwater instead of carrying it to waste in the condenser cooling water.

The higher the steam pressure the more often or more extensively the steam can be resuperheated between prime-mover stages.

A paper by the writer was published in the April 1, 1930, issue of *Power*. The influence of steam pressure, steam temperature, the number of bleeder stages and the number and extent of reheater stages on the over-all plant efficiency was investigated in this paper. A turbine was taken as prime mover with very conservative efficiency figures for the individual stages. The influence of various efficiencies of the high-pressure turbine stages was also considered and of steam-heated reheaters versus reheaters in the gas path.

The investigation revealed that a plant with a steam flow of 100,000 lb. per hour at 2500 lb. per sq. in. pressure and 840 deg. fahr. temperature at the turbine admission valve, having a reheater in the gas path delivering steam at 240 lb. and 750 deg. fahr. to the lower pressure stage of the prime mover and having four feedwater heating stages, has an over-all efficiency of almost 34 per cent. This figure is based on an average stage efficiency of the high-pressure turbine stages between 570 and 2500 lb. per sq. in. pressure of only 70 per cent. Pressures in the neighborhood of 2500 lb. steam pressure and at least 800 deg. fahr. temperature, if used in connection with one reheater, give a decided advantage over cycles with lower initial pressures, also if such cycles use much higher initial temperatures. The use of pressures above 2500 lb. shows further improvements only if much higher initial temperatures are used or in the case of non-condensing plants.

Figs. 7 and 8 of Mr. Kerr's paper show that the disturbances caused by the evaporation process in tubes with a high rate of heat transmission disappear if the evaporating process is carried out over a larger part of the heat-absorbing tubes and when the flow resistance or the velocity is increased. They also disappear when the conversion of water to steam is carried out at the critical point. At this point no boiling takes place. If steam is generated at the critical point, it is 100 per cent dry. This is not assured if steam is generated in tubular once-through boilers at lower pressures and without special arrangements to separate the steam from the water.

These facts were realized by the Siemens-Schuckertwerke A.-G. in Germany in 1923, when they started to develop the forced-circulation boiler on a large scale. No unbalanced conditions need to be in the circuits of a boiler under forced circulation. The flow through the individual sections can be arranged in a way that most favorable heat absorption, velocity, and pressure-drop conditions are obtained. Conditions in every part of the generator are absolutely controlled. This is much in contradistinction to the thermo-syphon circulation boilers, where much is left to nature, and the distribution and velocity, particularly in the water walls of the combustion chambers, are almost the opposite of what they should be to assure equal and efficient heat-absorption work of all tubes and safe and controlled temperatures of the individual tubes.

The heat-transfer figures revealed in the paper by Messrs. Potter, Solberg, and Hawkins in combination with the respective gas and water or steam temperature give evidence that the heat transmission per square foot of heating surface and degree temperature difference to tubes containing water or a liquid is much higher than the heat transmission to tubes containing steam or water in a boiling state. The heat transfer from tube wall to the medium inside the tube depends roughly on the 1.2 power of the velocity of the medium. In a forced-circulation boiler, water and steam velocity can be considerably increased over that in natural-circulation boilers. If such a boiler is designed and operated so that the medium to be vaporized is as long as possible in a liquid state and under high velocity, the tubes are cooled very effectively. The maximum proportion of tubes containing water is in a boiler which generates steam at the critical point. Such a generator contains only water-heating and steam-superheating elements.

The design of the first forced-circulation critical-pressure steam generator of a capacity of 10,000 lb. of steam per hour was started in 1923. This boiler was in operation early in 1924.

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[9] Chief Engineer, Siemens-Schuckertwerke A.-G., New York, N. Y. Assoc-Mem. A.S.M.E.

A pulverized-coal-fired boiler for 60,000 lb. of steam per hour was built in 1926. Another for twice this capacity followed two years later. A boiler which can be fired with gas, pulverized coal, or oil and of a capacity of up to 300,000 lb. of steam per hour has been in operation more than one year in Belgium. Rotary feed pumps are used in combination with the larger boilers.

An automatic control was developed in connection with these boilers which maintains the steam temperature within 3 per cent and the steam pressure within 2 per cent at load variations from 60 to 100 per cent. This temperature at the outlet of the generator is maintained high enough to secure sufficient superheat in the steam, so that no matter to what low pressure the steam might be reduced in the following superheating sections, there will be always 100 per cent dry steam in the superheater. A temperature of 760 deg. fahr. would be sufficient for this purpose according to the Mollier diagram. The boilers, however, are operated so that the steam temperature at this point is 800 to 850 deg. fahr.

In the ordinary boiler, the fuel supply, etc., must be adjusted to maintain a certain pressure in the boiler drum, and since the fireman can serve only one master, the steam temperature at the superheater outlet naturally cannot be steady at variable-load conditions. In boilers of the described type, fuel, air, and water supply are all adjusted for one purpose only—to obtain the desired steam pressure and temperature in the steam main at any output condition. A steady steam temperature, however, is most important in a high-pressure and high-temperature steam plant. Leaky joints and excessive stresses in the piping material occur only if the temperature changes.

The writer believes that the forced-circulation boilers will soon replace the boiler with natural circulation, at least where pressures above 500 lb. per sq. in. are used. The time is near when no one will leave nature such a free hand as she now has in the water walls and convection sections of a natural-circulation boiler, certainly not in equipments involving investments of hundreds of thousands of dollars.

Geo. A. Orrok.[10] It is very gratifying to see the presentation of the extremely interesting experimental results obtained in testing a series or coil boiler. This work has been delayed a full hundred years. When Jacob Perkins designed and built his high-pressure series boiler in 1828, only the lack of suitable instruments of measurement prevented him from proving to the world the great step he had made in the steam-generation problem. His engineering genius saw far ahead of his time. The work of Herreshoff in 1877, afterward in 1879, and Serpollet in 1880 showed that such designs were successful and could give good efficiency and real service. Much later, in 1897, DeLaval startled the forgetful engineering world by his high-pressure (2800 lb. per sq. in.) coil boiler and turbine at the Stockholm Exposition. A score or more of forgotten inventors have designed and built such devices, often very ingenious. The Siemens-Schuckert Company in Europe had had considerable success with the Benson boiler, while recently Doble in America has attained practical results in the automotive field.

In Mr. Kerr's paper the evolution of the steaming economizer into a coil boiler is well shown, and it would be very interesting to know the comparative operating difficulties for types shown by Fig. 2 and Fig. 4.

The next step was to build a plain coil boiler, and the engineering staff of the company should be congratulated on this enterprise. It seems that at last belated steps are being taken to investigate the problem of evaporation in a tube, the theoretical knowledge of which has lagged many years behind the study of other lines of development.

A year ago, in Chicago, the writer presented a paper entitled "The Boiler Room of the Future," in which he ventured the idea that the boilers of the future would be of the drumless coil type, cellular in their construction, absorbing the greater part of the heat as radiant heat, and fired with gas. Therefore, he doubly appreciates the present papers.

Let us hope that the able staff of Purdue University which has just given this exhaustive account of their first series of experiments on this boiler will carry the work much farther and investigate the processes of scale formation, corrosion, stability of tube materials, and effect of variable loads, so that the construction of the ultimate type of a high-pressure coil boiler will be soon a practical possibility.

The paper by Messrs. Potter, Solberg, and Hawkins contains a wealth of material for anybody desiring to study coil boilers. Sufficient observations of pressure drop in various sections of the boiler are given, to be a subject of a separate paper. So little is known about resistance of steam-water mixtures in tubes that the working out of this material is highly desirable.

The writer is not so sure about the seriousness of the question of instability in parallel circuits. He had the privilege of supervising several tests of the Benson boiler at the Siemens-Shuckert plant in Gartenfeld, Berlin, and the distribution of water among the twelve parallel circuits in that boiler was fairly good and never gave any trouble. We have a similar problem in low-pressure-drop superheaters, in reheaters, and in water walls, but these things have a way of straightening themselves out with the help of certain poorly understood principles which might be termed secondary effects.

There are of course many problems which must yet be solved before we have a really commercial coil boiler. The tube size should be large enough to allow turbining. The boiler must be adaptable for short banking periods and be able to stand rough handling with various fuels and various operating conditions. Automatic controls are necessary, as Mr. Kerr has pointed out. The operating man must forget the old shibboleths of "releasing surface," "water level," "steam storage space," and "water content," and go back to the principle laid down by Perkins of feeding the boiler when and as steam is required. The writer pointed out in an earlier paper that in high-pressure boilers the water content of the drum would not exceed 10 per cent of the total water contents. The coil boiler has cut out even this small proportion. It is believed that it will be found that high pressures, high temperatures, and large outputs can be secured with boilers of this type with the maximum of safety and at reasonable expense.

Arthur J. Herschmann.[11] Much in these two papers has been covered by able discussion. The over-all efficiency curve, Fig. 1 in Mr. Kerr's paper, shows, as can be verified by the Mollier chart, that there is little advantage in raising the steam pressure over 1900 lb. Operating engineers will be quick to appreciate the fact that while there is little to be gained in economy beyond 1900-lb. pressure there may be additional worry and anxiety. Mr. Kerr also states in regard to the once-through type of boiler that "whatever goes in must go out" and this must be especially well remembered where the tubes are so small in diameter and the pressure drop so great. The Loeffler boiler meets these conditions exceedingly well and gives besides, with great constancy, the highest superheat which tubes of reasonable cost permit. Loeffler boiler tubes are of carbon-steel with about 0.60 molybdenum. Mr. Moultrop, in his discussion of Mr. Kerr's paper, reminded us that highest superheat should receive as much attention as a reasonably high operating pressure. Loeffler boilers have now operated for some time without extraordinary difficulty being experienced, in units of 160,000 lb. of steam per hour, and we now build two boilers each for 330,000 lb. per hour. They always operate at 1900-lb. steam pressure with 932 deg. fahr. total temperature. Serious high-temperature trouble in Loeffler superheaters has never been experienced, and that without the use of expensive alloy steel, because we have always very ample cooling of these tubes by forced circulation. Even if the load should drop entirely, the temperature would rise but very little, because the steam which is circulated has little relation to the steam of the line. Only the pressure would rise for a short period.

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[10] Orrok, Myers & Shoudy, Consulting Engineers, New York, N. Y. Mem. A.S.M.E.

[11] United States Agent, Vitkovice Steel Works, New York, N. Y. Mem. A.S.M.E.

James M. Taggart.[12] These two papers suggest the desirability of a simple, expressive, and uniform name for this class of steam generators. In addition to the titles given by the authors, boilers of the type described have been called flash, non-water line, and drumless boilers. One simple and expressive name appears to be "uniflow." This term, it is true, has been used to a limited extent and somewhat incorrectly for a form of fire-tube boiler with a special grouping of tubes. Accordingly where the designation must be specific the name could be extended to "uniflow water-tube boiler."

Small uniflow boilers similar to those described in the papers have been roughly tested and used with a fair degree of success for automobiles and some special installations. The pressures used have been from 300 to 500 lb. The principal objectives have been small size and freedom from danger of explosion. Larger sizes tried out have been either complete commercial failures or limited to special service. The reasons for failure have been the attempts to use them with contaminated feedwaters, faulty combustion, and flow controls, and in most cases with no provision for inspection, cleaning, or replacement of tubes.

The boilers described have avoided the use of contaminated feedwater and have proved the feasibility of combustion control. They do not, however, promise any improvement in respect to cleaning or easy replacement of tubes. It is true that if absolutely suitable feedwater is always used there would be no necessity of cleaning nor any damage. However, experience and the nature of the service prove that at times it will be necessary to operate on raw or contaminated water, even with the risk of burnt tubes. Where cleaning is possible, such operation over limited periods is feasible. With short tube lengths, easily removed, burnt spots would constitute only a minor maintenance charge.

Pressure drops through uniflow boilers require careful consideration. In the boilers described, as well as in most of the types of uniflow boilers formerly tried out, a constant area of flow is used. Apparently a better design relative to heat absorption and pressure drop over a wide load range would provide an increasing flow area over the steaming section.

The differential flowage encountered by Mr. Kerr in his tests is common to the type of equipment described. Where the length of parallel circuits is broken up by mixing headers, as would be required in an arrangement suitable for cleaning and easy replacement and also for providing an increasing flow area, the differential flow would cease to be accumulative. With proper design it could then be made a negligible factor.

The tests recorded are very interesting and provide new data relative to high-pressure steam generation. It is satisfactory to see that they check fairly well with the new steam tables. The results present some discrepancies, as is natural in a test of this character. The furnace temperature is shown as approximately constant for widely varying fire conditions. This would seem to be in error. Some heat-flow variances otherwise unaccountable may be due to this feature. In general, heat flow to water walls is above that shown by empirical and semi-theoretical calculations generally used for boiler furnaces. This probably indicates the importance of taking into careful consideration the shape and size of the furnace and the nature and location of the flame.

It is noteworthy that the heat flow in the steaming section was so much higher than for the superheating section. Making due allowance for the temperature difference, it is apparent that even with high densities dry steam has a much reduced heat-absorbing capacity.

The efficiency curve for varying steam pressures submitted by Mr. Kerr, like all similar general curves, is more deceptive than informative. Thus any increase in power-generating efficiency resulting from an increase in steam pressure depends quite largely on the increased range of regenerative effect resulting. Reheating and machine efficiency must also be considered.

After all, the real value of a commercial uniflow boiler consists in its safety from explosion, small size, and its low cost compared to any standard form of high-pressure boiler. When developed, it will open up a new field for high steam pressure in motive power and for certain industries where standard high-steam-pressure boilers are not now economical. It is probable that for central stations some drum effect will always be advisable.

Authors' Closure

The authors are particularly interested in the description by Mr. Yarnall of the 2500-lb.-pressure drum-type boiler which has been installed in the laboratory of the Yarnall-Waring Company by the Superheater Company of America. In the design of this boiler careful consideration has been given to the problem of natural circulation, which becomes serious at the higher pressures because of the increased density of the steam. It is hoped that Mr. Yarnall will investigate the circulation in this boiler under a number of pressures and steaming capacities.

Professor Christie states that it is reasonable to expect plants to be built in the future for 2500 lb. pressure and 1000 deg. fahr. steam temperature with one stage of steam reheating. The authors are in agreement with this prediction, and this fact has in part influenced their decision to carry on most of their testing at 2500 lb. pressure.

Mr. Artsay has apparently misunderstood the authors when he stated that the word "instability" had been used in their paper. No evidence of instability has been encountered during the operation of the Purdue University boiler. Some inequality of flow, or unbalance, exists in the two parallel circuits of the lower furnace-wall section due to unequal resistances in the circuits, but the outlet temperatures are stable at all times.

Messrs. Mayr and Taggart call attention to the greater heat-transfer rate in the furnace-wall tubes containing water than in the superheater tubes and attribute this to the better film coefficient of steel to water. In the opinion of the authors, the controlling factor of the heat-transfer rate is the arrangement of the surface with respect to the flame. The tubes in the lower furnace-wall section are in a better position to absorb the maximum amount of radiant energy.

Mr. Taggart states that "the furnace temperature is shown as approximately constant for widely varying firing conditions." He is probably referring to item 28 in Table 1, which the authors purposely called "furnace pyrometer reading," since they doubt the existence of a "furnace temperature" in a cavity in which combustion and heat transfer are proceeding simultaneously.

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[12] Consulting Engineer, New York, N. Y. Assoc-Mem. A.S.M.E.

The pyrometer readings were obtained by sighting an optical pyrometer, through the inspection hole in the furnace door, on the flame at a point about 16 in. above the burner. Since the paper was prepared, experience on tests with variable excess air has indicated that this point is too near the burner, and as it is not convenient to change the opening in the furnace wall, these readings have been discontinued.

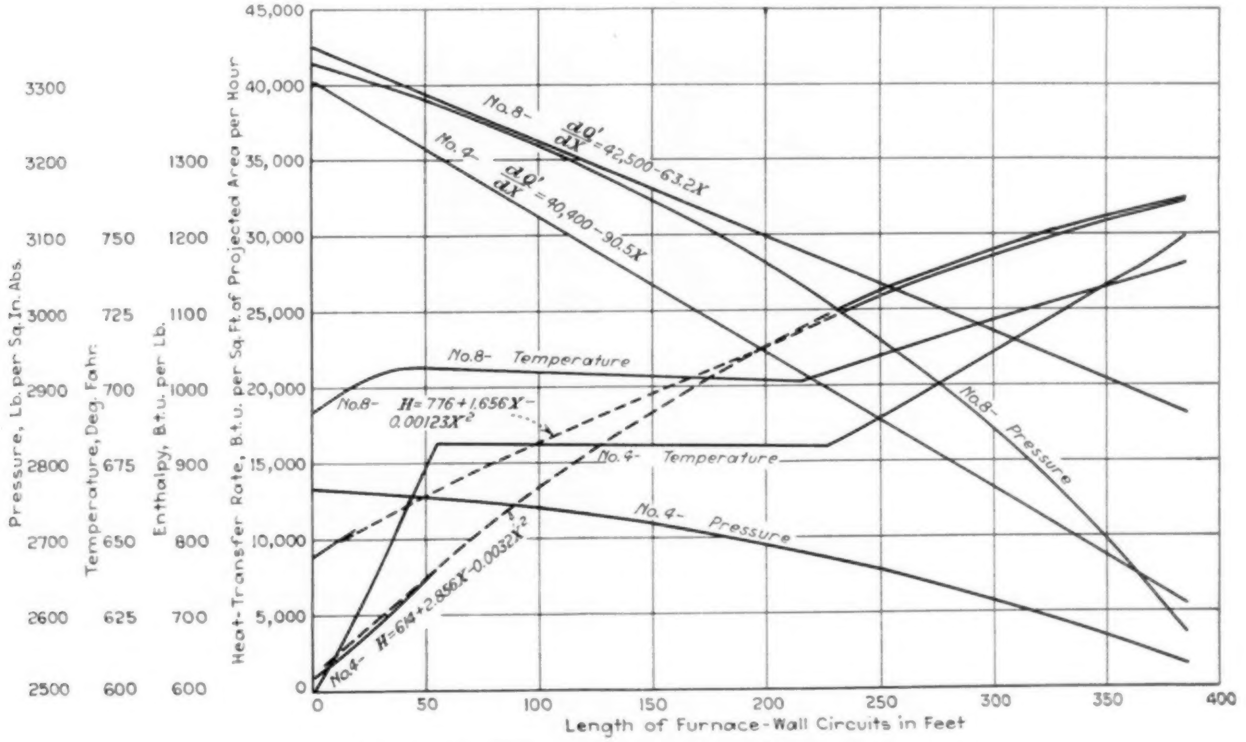

Fig. 17 Heat-Transfer Rates for Tests Nos. 4 and 8

Messrs. Gates and Armacost question the accuracy of thermocouple readings and state that, on this account, the heat-transfer rate in the evaporating zone in test No. 8 is only half of that in test No. 4, although the unit is operating at about double the rating. The authors have plotted the temperatures and pressures for these tests in Fig. 17 with the length of furnace-wall circuits in feet as the abscissa. The heat-transfer rates have been determined as outlined in the paper and plotted on the same abscissa. Apparently, Messrs. Gates and Armacost have made an error in their calculations. This is probably due to the fact that they had neglected to consider the effect of the pressures and of large pressure drops on the thermodynamic properties of compressed water and steam. In Test No. 8 the pressures pass through the critical point, and the specific heat of water is consequently very high. Furthermore, a plot of Keenan's data on compressed liquid water at temperatures between 680 deg. fahr. and the critical temperature will show not only big changes in enthalpy with either temperature or pressure variations, but an insufficient number of points to locate the curves accurately in this region. Consequently, large errors can be introduced by basing conclusions on one or two thermocouple readings in the region of the critical pressure.

If there is a marked tendency for centrifugal force to separate the water and steam and to prolong the apparent evaporating section, as suggested by Messrs. Gates and Armacost, such a condition should be noticed by a sudden rise in steam temperature immediately beyond the point where evaporation is complete. The method of calculating the heat-transfer rates should eliminate errors due to this factor, and should show at the same time the existence of such a condition. This possibility could not affect the curves of Fig. 13, as suggested by Messrs. Gates and Armacost, since the pressures are above the critical pressure at the points in question, and there is no evaporating zone. In this region the absorption of a large amount of energy can occur at nearly constant temperatures with decreasing pressures; this accounts for the shape of the curves in Fig. 13.

Messrs. Gates and Armacost raise the question of overheating of the furnace side of the tubes due to the possible separation of water and steam under the influence of centrifugal force. The furnace-wall tubes are clean internally and show no external evidence of overheating at any point. Thermocouples have been peened to the fire side of the tubes at a number of points and indicate a fire-side metal-surface temperature of from 40 to 80 deg. fahr. above the fluid in the tube.

In addition to the foregoing, the following may clear up questions which have been asked outside of the meeting:

Condition of Brickwork. An examination of Table 1 indicates that item 56 (radiation unaccounted for) is greatest during the first tests made in any one day. This is due to too little time having been allowed for the brickwork to be heated up to the proper temperature. While these tests are not intended primarily to check the efficiency of the unit, care is now being taken to reduce such errors to a minimum.

Air Supply. Table 3 shows considerable variation in the furnace heat-transfer rates. To check the effect of excess air supply upon heat-transfer rates in the furnace wall, a number of tests have been carried on to study the effect of varying the air supply. Briefly, the results indicate that variation in the maximum heat-transfer rates may be accounted for by changes in the air supplied. Complete details concerning these tests will be presented in the near future.

RP-54-2

Economic Life of Equipment

Derivation of Formulas for the Selection and Replacement of Manufacturing Equipment Which Depend Upon the Theory of Economic Utilization of Capital Resources

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H. O. Vorlander

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The correct solution of the problems of how to select and when to replace equipment is vital to industry. In fact, it revolves about the larger problem of how to invest economically and profitably the capital resources of any enterprise. Many rule-of-thumb as well as a number of somewhat more exact methods are now being employed as a basis for determining equipment policies; however, no one of them is suitably adapted to the present needs of industrial executives, process engineers, plant managers, and sales executives. Accordingly the A.S.M.E. sponsored a research project to investigate thoroughly this subject and formulate the principles upon which any equipment policy should be based, and out of which practical technique can be evolved for the ordinary purposes of industry. As a result, this paper, which constitutes the formal report on the project, presents criteria for the economic or useful life of equipment, for the determination of depreciation policies, for the ultimate ability of equipment to produce earnings or profits, for protection from the hazard of impending obsolescence, and for the guidance of equipment manufacturers in the evolution of new designs and in anticipating the needs of industry. All of these criteria depend upon the relative economic superiority of some proposed type of equipment over some equivalent existing type or one chosen as a standard of comparison, and the evaluation of each can be achieved through formulas which are equally applicable to machine tools, jigs and fixtures, labor-saving or material-handling devices, process and power equipment, and even buildings or structures. Moreover, in the course of the investigations, many other important facts were discovered which relate to financial policies, accounting practice, the origin and proper methods of treating overhead, plant layout, maintenance, productivity and idle time, salvage and resale policies, machine design, and, in general, the earning power of the enterprise; all of which are discussed in detail.

SINCE the advent of the present era of widespread mechanization of industry, industrial executives have been continually confronted in the formulation of equipment policies with the questions: What type of equipment is it best to install to perform a given task or service? When should existing equipment be replaced? How can the capital resources of the enterprise be most profitably employed so that all productive fixed assets in the business can not only pay for themselves in the shortest possible time but also earn an adequate return on the investment—how to protect the enterprise from the hazards of obsolescence? In order that these executives may be provided with facts and a broader understanding of the issues at stake, this Society has instigated and sponsored an extended research project with the objective of investigating and revealing all aspects of the problem as well as formulating eventually practical techniques of approach.[3]

In its essence the whole problem resolves itself into the fact that those in industry who are entrusted with the expenditure of the funds of an enterprise must guard themselves against

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Now that the first portion of this research project has been completed and the resulting criteria, formulas, and techniques, in their fundamental form, have been presented to the membership of the Society, it is essential that the cooperation of a large number of corporations and their executives be obtained in order to carry out the second portion of the project relating to the evolution of individual formulas and techniques applicable to specific types of equipment such as all kinds of machine tools, jigs, and fixtures, material-handling and labor-saving devices, as well as power and process equipment and the like, which will be appropriately adapted to the current accounting procedure employed in any case. It is hoped that the value of this paper will be appreciated throughout industry to the extent that many executives will be willing to signify their desire to aid in the completion of this project by communicating directly with the secretary of the Main Research Committee of the A.S.M.E.

undue extravagance, as the greatest utility and satisfaction from their use can only be attained when such resources are adequately conserved and employed with economy. Accordingly if this is to be accomplished some fundamentally sound basis must be evolved which will permit the original selection of the most appropriate type of equipment for the performance of a given kind of service, and later its replacement by some more highly developed counterpart which can perform the same or even a more difficult service with sufficient additional economy of operation or maintenance to justify the cost of removing the existing equipment and the investment of the same or larger amounts of capital in that of the latest design. Obviously the foundation on which such a method of solution rests is the ability to obtain an economic balance between the costs of securing and carrying the proposed investment of capital and the anticipated economies of operation. It so happens that the actual amount of the capital investment is really of less consequence in the formulation of a policy for the installation or replacement of equipment, as the justifiable investment is controlled entirely by the economies which can be effected by their use. Therefore, since the fulfilment of all obligations of a financial or operating nature must be achieved within the serviceable life of the equipment, this measure of its ultimate economic utility will necessarily be a function of *time*.

Utility of Formulas in Evaluating Equipment Policies

Naturally, since the formulation of any equipment policy involves definite values of capital, costs, and quantities of production as well as time, the fundamental relations of these items in their various combinations can best be expressed in formulas, the resulting solution of which will yield the desired criteria for the guidance of industrial executives, or else they can be translated into a variety of practical techniques for use by plant managers, process engineers, or sales executives. If in this manner a means can be evolved for evaluating and rating the relative economic advantages of one type of equipment over another in the performance of a specific task or service, industrial executives will then be able to determine their equipment policies on a rational basis in terms of an economic utility which comprehends automatically all obligations arising from any given situation. Thus, they will be able to foretell when a piece of equipment has lost a sufficient amount of its original utility to make it worth while either to replan the methods of production or replace the machine with one of a radically new and more efficient type, to select the equipment with the highest economic utility, and to translate the sales arguments of the equipment manufacturer into terms of utility for the specific task involved. On the other hand, the equipment manufacturer can establish ratings of performance for each item or design in his catalog which will show clearly the relative economic advantage to be realized by the adoption of any one. Moreover the manufacturer can so schedule improvements in the design of equipment that he can stimulate the maximum uniform sales volume without discouraging his prospective customer by too frequent design change and an unreasonable sales policy of progressive obsolescence. In addition to the advantage gained by the process engineers through the aid such methods give them in their work of cost reduction, the facts thus derived will also permit the more rational distribution of earnings to the owners, workers, and consumers as well as to the enterprise itself in the form of a higher return on investment, more generous wage incentive systems, lower prices, or a greater margin for expansion or improvement, as each case may justly deserve.

Criteria for Determining an Equipment Policy

As a result of the present investigation it has been found that there are certain definite periods of time in the life of any equipment, when measured from the date of its original installation, whose completion marks the instant that certain primary obligations have been fulfilled. When such corresponding criteria for alternative types of equipment are compared with each other they provide a measure of the relative state of economic utility that the most efficient type has achieved over the other.

1 *The True Accounting Life* (H_t). The prime requisite in all equipment problems is the time within which a given type of equipment can pay for itself. This criterion can be evaluated by equating the summation of all costs, expenses, and obligations for the proposed equipment, whether of positive or negative characteristics, to a similar summation for some standard basis of comparison when making an initial installation, or to a similar summation for the existing equipment when there seems to be an advantageous opportunity to replace it. In making such a comparison it is assumed that the above criterion is satisfied when there are no excess savings or when all savings over and above the costs incurred by other obligations are devoted to the depreciation of the initial or gross investment in the proposed equipment. This condition has been commonly termed the "break-even point." Since in this case there can be no further savings achieved, the time that it will take to accumulate in a reserve fund the original capital outlay or investment in the proposed equipment from yearly payments or allocation of assigned savings in lieu of depreciation will be the minimum time for the equipment to pay for itself, and as such it has been termed the "true accounting life." Accordingly, this criterion should be employed as the basis for all rational depreciation policies.

2 *The True Economic Life* (H_r). In the ordinary course of business it can never be assumed that any type of equipment has achieved a full state of economic utility when it has been able through its operation over a period of years merely to pay for itself, because there is the implied obligation upon all productive capital assets that they must in some manner pay back to the investor of that capital, whomsoever he may be, an adequate return as a reward for assuming the risks of business through his investment. Thus, in only satisfying the first criterion this second consideration will have been omitted. Accordingly the ultimate criterion for any equipment policy demands that not only shall the original investment be returned to its owner together with the normal cost for its use, but also an adequate return must be earned as well. The amount of this return will vary in all cases, and depends upon the profitableness of the industry and the ability of the management. The minimum return for any single unit of equipment will be determined by the average earning power of every dollar invested in productive assets. Accordingly the *minimum utility life* (H_u) of equipment will be that period of time which is required for any single equipment to fulfil all direct and implied obligations as outlined above.

However, this situation does not account for that which ultimately involves the relative economic utility of alternative types of equipment. In this case it is of primary importance to ascertain whether the proposed type of equipment can achieve an earning power equal to that of the present or existing type within a reasonable length of time. Of course if the relative economic superiority of the proposed type is so low that it cannot become at least as profitable to operate as the existing type before impending obsolescence occurs, there is little relative advantage in actually installing new equipment of that type. Furthermore there may be reason to ascertain whether the proposed type can enhance the earning power already realized from the present type, when the performance of both is projected into the future, so that a desired increase in the return can be obtained after a certain number of years of operation from the present have elapsed, or so that these savings can be transferred from the owners of the business to its customers in the form of lowered prices. All of this can be determined through an inspection of the relative trends of the curves for the *functional rate of return* (r_H) for both types of equipment over the desired number of years. This functional rate of return is a yearly rate factor dependent upon the total net savings available, when computed for any definite number of years and averaged for the elapsed time, after all fixed and variable obligations have been met. Now the ultimate time criterion for the relative economic utility of alternative types of equipment for the same task is measured by the period of time (H_r) required for the functional rate of return for one equipment to equal that for the other. Hence this time when these conditions are satisfied is termed the *true economic life*.

3 *The Critical or Minimum Life* (H_s). In contrast with the utility life and comparable with the true accounting life, there is a period of time when only a single unit of equipment is considered, at the end of which the accumulated margin between all obligations and the net revenue from the sale of the products produced or processed by the equipment is just equal to the original outlay of capital invested in the equipment with no consideration for the return expected on the investment. In other words, the equipment unaided by any outside considerations can just pay for itself in this minimum amount of time. Hence, in an emergency the equipment can be replaced at this moment without leaving any fixed obligations to be borne by the superseding equipment, although all of its obligations to earn any return whatsoever on its investment must be carried over into the future. This instant of time has been designated as the *critical or minimum accounting life*.

4 *The Serviceable Life*. In opposition to the various time criteria just discussed, which depend upon the performance of the equipment, there is the *serviceable life*, which is governed by the occurrence of obsolescence. Obsolescence is in most cases an uncertain factor, but it may be largely influenced by a more rational control over design change, or by the development of better production methods and processes, except where the whim of the customer influences the utility of the product or service and in turn influences the utility of the equipment employed in its production or performance. When an arbitrary accounting life is set there is always the risk that impending obsolescence will occur before that life has expired. The case is the same with the true life, but if the true life is shorter than the arbitrary one, which it often is, there is no need of running an undue risk of impairing capital resources irreparably through the occurrence of obsolescence before the equipment has paid for itself on the arbitrary basis. The risk of impending obsolescence can be measured by the ability to forecast future conditions, or in this case the probable rapidity and intensity of design change, and its relation to the true life. If the true accounting life or even the true economic life exceeds the apparent serviceable life, then the proposed machine is not sufficiently highly developed to justify its installation; that is, the increment in the transfer of skill to the machine is not great enough to warrant replacing the present equipment with that which is under consideration on the basis of anticipated economy of operation. If the true life approaches the supposed time for the occurrence of obsolescence or the serviceable life, but does not surpass it, then the interval of time between these two points when related to the true accounting life is the measure of the risk. The severest test is that between the critical life and the useful life when compared with the anticipated serviceable life.

Thus, if equipment be classified according to design cycles and compared upon the basis of some standard task, the dividing line between partially obsolete and totally obsolete equipment can be determined by a comparison of the true accounting or minimum utility life for the most modern design with some basic criterion for the time in which a machine can pay for itself, probably some fraction of a year. In this way a measure of the relative utilities of the two designs can be obtained which depends upon the savings that accrue through the marked superiority of the most modern one. For the minimum condition just mentioned, the superiority of the latest design will be sufficiently great to warrant the complete obsolescence of the earlier design, with which it is being compared, and of course all previous models as well.

5 *The Physical Life* (H_p). Furthermore there is the *physical life* of the equipment, which depends upon the wear and tear from actual operation and deterioration of its substance. This life is the measure of the depletion of the physical characteristics and attributes of the equipment. Without maintenance the physical life may be short, so that maintenance in a sense is insurance against a too early depletion of the actual utility of service.

Criteria for Depreciation Policies

At this point it is advisable to digress for a moment to discuss depreciation policies, because the bases for such policies underlie also the whole theory of the economic utilization of capital resources and its measure, the economic life of equipment. Current accounting practice differs radically from the methods proposed in these pages for depreciation because normally an arbitrary rate of depreciation is set and then there is no actual accumulation of a reserve. In the determination of the *true accounting life* a fixed rate of depreciation cannot be employed, because then one automatically sets up an arbitrary time in which the equipment is supposed to pay for itself whether the conditions will permit it or not, and then there is no relation expressed between the savings, which can be legitimately diverted to depreciation, and actual depreciation. In such cases this estimated time is called the *arbitrary accounting life* (H_a) of the equipment. Furthermore current accounting practice rarely makes any provision for maintaining the original investment intact. Ordinarily, at any time in the normal life of the equipment its actual capital value is represented by the book value as it stands upon the records of the company, that is, the initial investment less depreciation allowance. But such a value at no time can represent the initial investment since depreciation takes no tangible form; for the most part it is only a writing down of asset values, with a corresponding adjustment on the balance sheet from earnings. One can never say that the savings which theoretically go to depreciation are available when the time comes for replacement as so much cash or its equivalent to be used in the purchase of new equipment. To be correct and maintain the original investment intact, depreciation payments should be made by setting aside periodically cash or securities in a reserve for replacement, so that for any item of equipment at any time the sum of those funds derived from depreciation plus the book value of the equipment will always equal the initial investment, with due credit for interest on the funds thus reserved. In passing, it should be stated that there is no definite obligation to set aside more funds for depreciation in lieu of increased future capital expenditures at the time of eventual replacement, because one need not provide for expansion or improvement in this manner if the return on the investment will be an ample inducement in securing additional capital when the appropriate time comes. This factor of a reserve for replacement demands immediate consideration by all industrial executives.

Other Criteria in the Problem of Selection and Replacement of Equipment

In addition to the foregoing criteria there are several others which are of importance in dealing with any problem of selection and replacement of equipment. The first is the determination of the *maximum justifiable investment* in any type of equipment which is suited to a specific task. This will depend upon the economies of operation which it is desired to obtain, assuming certain standard conditions. The second is the determination of the *minimum resale value* which it will be permissible to anticipate, and whether or not the equipment is so productive of economies that it will be permissible to scrap the equipment at the end of its useful life instead of having to depend somewhat on second-hand values to obtain within a reasonable time the full return of the initial capital investment. The third is the *critical total production* which will have to be effected before there will be sufficient total revenue out of which adequate funds for depreciation can be obtained. If prices fluctuate or the volume of business declines it is quite probable that, unless some unforeseen circumstance arises, the equipment will not be able to pay for itself in the appointed economic life. Furthermore it is advisable to learn the *minimum average yearly output* or capacity under which the equipment can operate and meet all obligations, or, in other words, *the maximum allowance for idle time.*

The Basis for an Equipment Policy

In the subsequent analysis from which these criteria have been derived, certain conclusions stand out as guiding principles to be followed in formulating an equipment policy and in evaluating the criteria which govern a particular situation. They are:

a The capital resources of the enterprise must at all times be conserved, and the initial investment must at no time be impaired, for any cause, even impending obsolescence.

b All equipment must be maintained in the most efficient state, and at least ready to operate.

c An adequate return must be earned on the investment over and above the cost of capital, not only as a reward for risk, but also to provide some opportunity for growth either through earnings put back into the business or through the ability to attract new sources of capital whenever such funds are required.

d Adequate provision must also be made for estimating and accounting for certain contingencies arising from the normal risks of doing business or the too sudden occurrence of obsolescence.

e In order to account properly for the nature, occurrence, and behavior of any and all direct or implied obligations, all comparisons and computations must be based upon the total of the entire group over some definite or indefinite period of time. The comparison of annual costs has failed to reveal all the vital complexities of the problem.

f Finally, since provision has been made in the original cost estimates for products to be produced or processed by the equipment, or established by the performance of equipment now in operation for the eventual fulfilment within a definite time of all obligations belonging to a specific type, all additional obligations must be paid for out of savings which can be achieved within the limits prescribed by these provisions and still within a reasonable time, as it is impossible to suppose that a reduction in the normal rate of return on the investment or an increase in selling prices will be permissible to offset any such increase in the total obligations incurred by a change in equipment.

Fundamental Considerations for the Evaluation of Equipment Policies

For practical purposes of comparison, out of all the formulas and methods proposed and employed heretofore, only two groups have been found worthy of attention. The first, which are recommended for use in determining policies for the selection and replacement of equipment, depend upon the total accumulated costs, expenses, and obligations for the entire anticipated life of both types under consideration. The second, which are applicable to the determination of the effect of an equipment policy upon the final cost of the product, depend upon the translation of all yearly costs to unit costs at the normal capacity of the equipment or some adjusted capacity equivalent to the current sales demand. The remainder of this paper will accordingly be devoted to the evolution of appropriate formulas and the development of practical techniques for daily use in industry. However, in order to avoid certain erroneous conclusions, the following injunctions must be observed.

a One must assume that all types of equipment are identical as far as the general problem is concerned, and that in making any application of the technique all possible combinations of factors may be encountered.

b To arrive at the correct solution of the problem, all comparisons employed must be fundamentally based upon equal amounts of service rendered by both types of equipment, with an additional provision for modifying this assumption only when the circumstances involve a change in the amount of service as well. If the comparison is made on the basis of unequal amounts of service, a false advantage is gained for one or the other piece of equipment which may or may not at some future time be realized.

c Similarly all obligations which are dependent upon or vary with time must be reduced to a common or equivalent basis. This will be necessary when there is some future obligation which will occur in its entirety at some one specific time in advance and for which a provision for its eventual payment must be made each year prior to the time when the full obligation becomes due, in order that the eventual burden which it will place upon the economy of operation may be recognized and accounted for among those obligations which are a constant burden upon the equipment at the present time.

d All comparisons of the cost characteristics of alternate types of equipment cannot be made to reveal the correct economic advantage of that type which is superior unless the resulting relationships are so formulated as to expose the differential between each individual cost characteristic for both types. Only in this way can the significant factors be exposed and those for which there is no relative advantage eliminated.

e It must be recognized that the introduction of a new item of equipment will require a readjustment of the burden rates commonly employed in the distribution of overhead, and that a new method of allocating the burden of these items must be devised so as not to be unfair to either piece of equipment involved in the exchange. Even though the total overhead may not be changed, especially if those items which relate to the equipment are treated separately, the burden rates will have to be corrected because the use of the old ones in connection with the evaluation of the obligations of the new machine may not under the changed conditions account for the entire burden of overhead which must be distributed in full.

f Depreciation must be based not on the gross initial investment but only on the out-of-pocket investment; that is, the difference between the gross investment and the net salvage value which it is possible to anticipate at the time when the economic life of the equipment comes to an end, with due allowance for removal costs and similar changes, including any unamortized investment left over from previous types of equipment because of their replacement prior to the expiration of their true accounting life.

g Finally, simplification of the resulting formulas and techniques must and can be made only when all elements of the problem are thoroughly revealed and understood. Equipment will vary in accordance with the service for which it is designed, and in no case will all of the elements of the problem apply to the same degree. Hence eventually separate formulas must be evolved for machine tools, jigs, and fixtures, and labor-saving devices or material-handling equipment.

Application of the Theory of Economic Utilization of Capital Resources

With these considerations in mind let us turn for a moment and see what this whole problem of selection and replacement of equipment comprehends if a rational and workable solution is to be achieved. The problem itself is not a difficult one; in essence, the sole objective is to justify the expenditure of capital resources for some form of facility or equipment on the basis of the anticipated economic utility or superiority of the proposed type over the existing type or that which may have been adopted as a standard of comparison. Primarily it is a financial problem dealing with the investment of funds, the cost of capital, depreciation, the usual financial charges pertaining to taxes, insurance, and the like, income from reserve funds, return on the investment, and savings from operations or services performed. Thus the principles which have just been set forth are in reality applicable to problems of much larger scope than those found in industry alone, and hence when taken as a whole they constitute the *theory of economic utilization of capital resources* which normally appear in the form of facilities employed to promote our daily business activities. The economic superiority of one facility over another, whatever its nature may be, can then be demonstrated by a comparison between the magnitude of the financial obligations incurred by the installation and operation of any two facilities designed or appropriate for a specific purpose, and the relative ability of either one to pay for itself in the shortest possible time out of operating economies, with due regard for the quality of output or the service performed. Thus in industry the problem may become more than a financial one, because if the ultimate economic superiority of one form of a facility depends upon its operating costs, it must take into consideration as well all factors such as materials, labor, set-up or preparation costs, tools, maintenance, repairs, rent, power, indirect labor, and all overhead items, including supervision and service charges, which naturally pertain to industrial equipment. In fact, the problem revolves about the characteristic differences between alternate types of equipment for each item of financial and operating costs, especially those which vary with the quantity or magnitude of the service and the passage of time. Accordingly, if an adequate solution is to be attained it is essential that the elements of the problem be investigated, the nature of each contributing cost factor be ascertained, and its behavior and influence revealed.

Elements of Investment

Since interest is centered at this moment entirely upon the selection and replacement of manufacturing equipment, the investigation so far has been confined to a study of the industrial aspects of the problem, though it will require only slight adjustment in the conclusions to apply them to any facility, whatever its useful purpose may be. Before entering upon a consideration of the relationships which underlie and control any sound equipment policy, it is essential that a thorough understanding be obtained as to the precise interpretation of each factor relating to the problem. First of all are the investment factors. Normally, the original investment is regarded as the capital expended for the equipment either through purchase or through development and construction. Nevertheless in the final analysis this does not represent the entire investment as there must be added thereto transportation and installation costs as well as any investment of a similar nature for auxiliary equipment and accessories, and any expense that may be incurred through testing and perfecting the performance of the equipment when first placed in service. Of all these various forms of capital outlay some portions are characteristically an asset which will have to be depreciated over the life of the equipment, while others are in the nature of transitory expense which has no real, lasting value and which must be charged to the cost of operation and paid for out of the potential savings during the first year the equipment is in use before depreciation and realization of operating savings can be provided for. Furthermore, everything considered, it is not the actual investment for the proposed equipment which must be justified, but rather the obligation to maintain intact the total outlay of capital incurred either by the installation of the new equipment or by the present or previous equipment, in which case the obligation must be carried over and imposed upon the new equipment as an additional burden. In other words, it is the differential or out-of-pocket investment, the algebraic sign of which determines the nature of the obligation, because only that portion of the total obligation upon the new equipment needs to be depreciated which cannot be offset by the resale or scrap value for both the present and proposed equipment and any income on the replacement reserve.

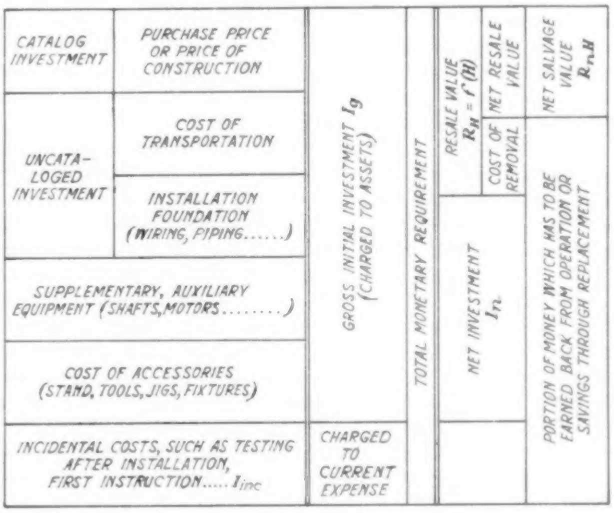
CATALOG INVESTMENT
PURCHASE PRICE OR PRICE OF CONSTRUCTION
UNCATA-LOGED INVESTMENT
COST OF TRANSPORTATION
INSTALLATION FOUNDATION (WIRING, PIPING......)
SUPPLEMENTARY, AUXILIARY EQUIPMENT (SHAFTS, MOTORS........)
COST OF ACCESSORIES (STAND, TOOLS, JIGS, FIXTURES)
INCIDENTAL COSTS, SUCH AS TESTING AFTER INSTALLATION, FIRST INSTRUCTION..... I_{inc}
GROSS INITIAL INVESTMENT I_g (CHARGED TO ASSETS)
CHARGED TO CURRENT EXPENSE
TOTAL MONETARY REQUIREMENT
RESALE VALUE $R_H = f'(H)$
NET RESALE VALUE
COST OF REMOVAL
NET INVESTMENT I_n
NET SALVAGE VALUE R_{nH}
PORTION OF MONEY WHICH HAS TO BE EARNED BACK FROM OPERATION OR SAVINGS THROUGH REPLACEMENT

Fig. 1 Elements of Prime Investment

The *prime investment* in a situation of replacement involves all cost from the time of the first consideration to the time of placing the equipment into useful operation. J. Chas. Fish in his "Engineering Economics"[4] defines prime investment as the sum of the costs of investigation, promotion (financing, organizing), and construction or purchase and installation.

The investment obligations due to the installation of any piece of equipment are defined by the *gross investment* (I_g) of capital, the sources and ultimate disposition of which can be demonstrated by the accompanying chart (Fig. 1).

Each situation rquires a different set-up for the elements of prime investment. Other items must be considered, as, for example, cost of removal, cost or expenses for temporary equipment during the installation of main equipment, the purchase of patents and rights if in the nature of assets, the interest cost on capital tied up during the time of investigation, promotion, or purchase (installation) of equipment, loss of net income due to interruptions during period of installation, cost of accidents to persons employed in the installation, cost of accidents to equipment during time of installation, and similar contingencies.

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[4] **McGraw-Hill Book Co., 1915, p. 34.**

That portion of the total outlay of capital, herein designated as *incidental costs* (I_{inc}), which is expended in the testing of equipment and the training of the operator is not ordinarily considered a part of the gross investment because it is an intangible non-recurring item and should be written off as an element of current expense. All cost incidental to the installation of new equipment is usually charged to current expense or depreciated during the first year. Whatever the accounting method is in regard to such expenses, they should not be omitted in the equation for total accumulated cost (C_H) during the whole life. If such costs are debited to current or operating expense, they immediately lose their connection with the particular equipment under consideration; if they are charged off as depreciation during a shorter predetermined but arbitrary time, the result in the total-cost formula is an addition to the gross initial capital value. It is then obvious that the new equipment has to carry its own incidental expenses that must be earned back in some indeterminate time.

The main point is to know on just what portion of the prime investment interest and depreciation are to be paid, and upon which taxes are to be based. The second factor to determine is the sum of all capital costs incidental to the acquisition of the equipment. Owing to the prescribed basis for the computation of taxes, the ever-existing obligation to pay a cost for the use of capital resources and insure the equipment from all common hazards, the major purpose of depreciation in maintaining the original capital outlay intact and possibly the contractual obligation to pay royalties on patents which are specified on some arbitrary basis, there may be distinct differences in the items of investment employed in computing these various obligations. The cost of capital must be computed on that portion of the total outlay which is eventually to be capitalized as an asset and which normally is the gross investment I_g defined above. If, on the other hand, the incidental costs I_{inc} are not to be written off in the first year but depreciated along with the gross investment I_g, then the cost of capital must be computed upon the basis of $I_g + I_{inc}$. Since there are legally prescribed methods of computing taxes, the basis for yearly tax payments will be different from the gross investment or even the book value; however, by means of appropriate coefficients the actual basis for tax computations can be related to the gross investment, hence one must use $I_t = a \cdot I_g$. The basis for insurance will depend either upon the gross investment or the book value at the time of renewal of the policy, but it is conceivable that the actual premiums paid will not conform exactly to the payment of cost of capital. Here again an adjustment by means of coefficients will be required if the gross investment is to be employed as the actual basis for such computations. Since the funds set aside annually for a replacement reserve are only of such an amount as at the expiration of the economic life will equal the original outlay less funds received from resale (S_r) or salvage (S_v) plus any obligation (K) carried over from previous equipment, depreciation must be based on the summation of these investment factors which for any type of equipment will be expressed by $I_g + I_{inc} - S_v$ if the incidental costs are not written off in the first year. Finally, the payment of royalties may be based upon some arbitrary figure depending upon the particular advantage obtained from the use of patents or designs belonging to others; however, the basis of such payments can again be probably related to the gross investment by means of a suitable coefficient.

OTHER VALUES RELATING TO THE INVESTMENT

During the useful life of the equipment the value of the original investment is gradually altered through a natural process of exhaustion of its useful qualities until a time arrives when it is sold or discarded as junk. The various terms applied to designate the actual value at any one of these stages may be interpreted as follows:

a Book Value (I_b) is the arbitrary value assigned to any piece of equipment at any moment during its useful life, and may or may not correspond with the value derived from the gross investment less accumulated depreciation.

b "Wearing" Value[5] is an arbitrary term employed to designate the value remaining in any piece of equipment, as evaluated by appraisal of remaining utility through the difference between the net resale value and the scrap value at any time.

c Salvage Value (S_v) is that value which can be obtained in some manner for the equipment at the time of its removal or displacement. When the equipment is not sold the salvage value is computed upon the net factory *utility value*, or that value which, by appraisal, the equipment will retain and carry over to the new use to which it will be put in some other location in the plant.

d Net Resale Value (S_r) is that value which at any time can be realized by the sale of equipment in the second-hand market, and is greater than the scrap value of the equipment but approaches that value as its ultimate limit. Due allowance must be made, however, for the costs or expenses of removal, etc. Actually its real value at any instant of time is a function of the elapsed time from date of purchase.

e Scrap Value (S_c) is the minimum realizable value of the equipment expressed in terms of the "junk" value of the materials employed in its construction at the current market quotations for scrap-metal prices, after there ceases to be any utility value in the equipment as such for any purpose whatsoever.

f Unamortized Value (K) is that portion of the prime initial investment which remains at any time when a piece of equipment is replaced, as the difference between depreciated gross investment or true book value and the net proceeds received from salvage or transfer of the equipment to some other division or department of the enterprise with due allowance for costs of removal or transfer, and which could only be paid back were the equipment retained in operation throughout its full accounting life.

The unamortized value can be expressed mathematically as follows:

Let C_R = cost of removal or transfer of present equipment
I_b = book value of present equipment
S_v = salvage value of present equipment.

Then

$$\begin{aligned} K &= I_b - (S_v - C_R) \\ &= I_b + C_R - S_v \end{aligned}$$

If the sale of the present discarded equipment does not cover the book value I_b, the loss K is usually dealt with in three ways:

a It can be written off to profit and loss, either at once or in the course of two or more years. It can also be charged to special replacement reserves, if such are set aside from surplus.

b It can be included in overhead and charged to cost that way.

c It may be added to the cost of the new equipment and treated as an addition to the investment, eventually being charged to cost through depreciation.

The first two methods are commonly practiced in accounting for losses from replacing industrial equipment, while the third method is followed in the case of real estate and the buildings erected upon it. But whatever method is followed, one should not forget that charging the unamortized value to profit and loss or to overhead takes the burden away from the particular equipment that causes the loss, thus loading the whole business or all the facilities with the responsibility of carrying the loss. A replacement formula is independent of the accounting practice when it has to scrutinize the economic factors of the equipment charge. The relation of such a loss to the new equipment must be upheld by charging the full unamortized value of K to the new equipment. Then this loss is added to the capital value and depreciated along with it.

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[5] J. C. L. Fish, "Engineering Economics," p. 47.

It is well to note at this point that the salvage value when the equipment is sold will be equal to the gross resale value, or when junked it will equal the scrap value.

CAPITAL VALUES EMPLOYED IN REPLACEMENT PROBLEMS

Owing to the fact that the formulation of a policy for the selection and replacement of equipment depends upon the differential relationship between the total obligations for both items of equipment over their entire lives, respectively, one will be required to draw eventual conclusions from certain capital values which express their relative characteristics, as follows: Whenever such differences are to be employed in the subsequent analysis, as may occur between some item relating to the present or existing equipment (indicated by subscript 2) and the same item for the new or proposed equipment (indicated by subscript 1), the symbol Δ will be prefixed to the symbol for the item itself to signify that a comparison of this nature is intended.

a Additional Capital is the increment in capital outlay between the gross investment for each type of equipment, and can be expressed as

$$I_{g1} - I_{g2} = -\Delta I_g$$

This value is of considerable importance because provision must be made to earn the normal expected return on this increment as well as on the original investment before the remaining savings from operations can be even applied to amortize the investment; otherwise the earning power of the corporation will be impaired.

b Differential Prime Investment is the measure of the relative basic investment obligations entering into any equipment problem, and can be expressed by

$$(I_{g1} + I_{\text{inc1}}) - (I_{g2} + I_{\text{inc2}}) = -\Delta(I_g + I_{\text{inc}})$$

c Out-of-Pocket Investment is that portion of the combined investment obligations for both types of equipment under consideration which remains after balancing the individual obligations for each, and can be represented by the relation

$$(I_{g1} + I_{\text{inc1}} - S_{v2}) = (I_{g1} + I_{\text{inc1}} - I_b + K)$$

d Invested Capital That Must Be Earned Back is that portion of the combined investment obligations which can only be replaced through depreciation in order to maintain the total outlay of capital unimpaired, and can be represented by

$$(I_{g1} + I_{\text{inc1}} - S_{v2}) - S_{v1} = \text{capital} - f(H)$$

This equation is set up upon the assumption that the out-of-pocket value on the day of replacement is to be reduced by the salvage value at any time. The knowledge of such a function of salvage value is therefore of great importance for the computation of the true accounting life during which the suggested equipment will pay for itself out of savings effected from replacement. Heretofore no attempt has been made to introduce this function into the problem. The present simplified solution deals with the scrap value (S_c) at the end of the useful life rather than with the functional salvage value,

$$(I_{\text{inc1}} + I_{g1} - S_{v2}) - S_{v1}$$

Thus the amount of capital to be earned back is in excess of the "wearing value," or the difference between salvage and scrap value. If the present equipment has reached its inherent scrap value $S_{c2} = S_{v2}$, then $S_{c2} = I_b - K$, and consequently the *refunding investment* becomes

$$(I_{\text{inc1}} + I_{g1} - S_{c1}) - (I_b - K)$$

or

$$(I_{\text{inc1}} + I_{g1} - S_{c1}) - \left[I_{g2}\left(1 - \frac{H_2}{H_{e2}}\right) + \frac{H_2}{H_{e2}} S_{c2} \right] + K$$

when time differences are accounted for and where

H = any period of time in the life of the equipment and,
H_e = an estimated life for the equipment.

ORIGIN AND NATURE OF COST FACTORS

Each equipment represents a unit of investment with all the obligations, financial, physical, and managerial, necessary to carry the equipment. Since the prime object is its useful operation, either for productive or other purposes, all items of cost as well as charges relevant to the invested capital must be taken into account. These obligations naturally divide themselves into three distinct classes.

The first, straight financial charges (F_I)—mainly interest taxes, insurance, and depreciation—are those connected with the maintenance of capital. They arise from the obligations incurred by the employment of capital in the form of facilities having physical shape or form, which are derived in turn from the necessity to maintain intact the original outlay of capital and the fact that all capital, no matter how it is employed, carries with it certain costs, not only that due lenders of capital but also those which are assessed upon it in the form of taxes, insurance, royalties, and the like. It is well to note that in the fulfilment of these obligations certain minor sources of income may develop which, along with the major items of expense, will produce positive and negative obligations.

The second class consists of operating expenses (F_o) which are understood to include only such items as maintain the equipment in an operating condition, as, for example, direct labor, power, direct lubrication, preparation, and set-up costs. Inasmuch as any equipment can be operated at the above-named costs without fulfilling its function, it is not necessarily productive and this involves in addition not only the cost of material, but also that for the lubrication of the tool.

The third class of expenses are usually referred to as overhead (F_B), the origin of which is not always clearly related to any particular equipment.

The combined annual financial, operative, and general costs are considered to be a characteristic for any equipment. They may be called *producing equipment cost* if connected with production, or in general, *operating equipment cost* (C_y).

DEPRECIATION

First in importance in the class pertaining to financial charges (F_i) is the yearly depreciation (D), as the whole problem of replacement revolves about the ability of the equipment to pay for itself in a reasonable period of time. Normally an arbitrarily specified amount is charged off from the original investment or the book value each year to account for a natural and gradual loss in utility. This amount represents in the eyes of executives that portion of the investment which the equipment is expected to earn back during any year. In any isolated case which does not involve replacement or a comparison between two similar types of equipment, the investment to be depreciated will of necessity be that portion of the original investment which cannot be regained through resale or which cannot be absorbed in the first year as a charge to operations. When a comparison is to be made, the differential investment value must be employed as previously defined. The magnitude (D_a) of the yearly depreciation is usually determined by dividing the out-of-pocket investment which must be written off with due consideration for salvage values by a predetermined number of years (H_a) in which it is expected that the equipment may be maintained in profitable operation. Hence in general

$$\frac{(I_g + I_{\text{inc}} - S_v)}{H_a} = D_a$$

$$D_a = I_d \cdot d_a$$

$$I_d = (I_g + I_{\text{inc}} - S_v)$$

and

$$d_a = \frac{1}{H} \text{ when } H = H_a$$

Now in the case where some new equipment supersedes some existing equipment, the depreciation of which has not already been fulfilled, it is insufficient to base the investment to be depreciated solely upon the out-of-pocket investment because the new equipment must in some way bear the burden of the undepreciated portion (K_1) of the existing equipment or any unamortized investment (K_2) which it is obliged to carry itself from its own time of installation on still earlier types of equipment. Accordingly the actual depreciation charge (D) for both types of equipment as used in eventual computations will be

$$D_1 = I_{d1} \cdot d_1 = \frac{I_{g1} + I_{\text{inc1}} - S_{v1} + K}{H_a} \quad \text{if } d_1 = \frac{1}{H_{a1}}$$

and

$$D_2 = I_{d2} \cdot d_2 = \frac{I_b - S_v + K_2}{H_{a2}} \quad \text{if } d_2 = \frac{1}{H_{a2}}$$

where $K = K_1 + K_2$. It is well to note that, in most of the equations which follow, K_2 will be disregarded and K will be assumed to be K_1.

This arbitrarily determined yearly depreciation on the basis of H_a has, however, no relation to the ability of the equipment to earn back the investment in this number of years, and gives no guarantee that the original investment will always remain unimpaired. Furthermore, unless funds are definitely set aside periodically equal to the amount of yearly depreciation, whatever it may be, there will be no money available to accomplish the replacement of the equipment when the appropriate time arrives. Hence a reserve fund for replacement is of the utmost importance if the equipment of a plant is to be maintained in a high state of efficiency. Otherwise how can the earnings of any equipment be accumulated so that in time its original investment will be actually returned to the owner and equivalent modern equipment be substituted for the present one when its utility has ceased to exist? The fact is that the ability of any equipment to pay for itself is dependent solely upon the savings which it can effect through economies in operation, except in such cases where it has been possible to reduce the aggregate of all yearly financial charges at the time of installation through careful purchasing and finance. Hence, as will be emphasized later, the time which it will thus take to earn back the out-of-pocket investment with due regard for all direct and contingent obligations will be the measure of the ability of the equipment to pay for itself, and as such it becomes the only rational basis for determining the annual depreciation rate (d). This time will hereafter be designated as the true accounting life (H_t), the evaluation of which is one of our major objectives.

INTEREST

The second important element among the financial charges is the cost of capital. It is universally accepted that the employment of capital in any manner implies a charge for the use of that capital at an annual rate of interest (i) as determined by current money-market operations. Thus, when capital is expended for equipment of any sort there is immediately created an obligation for that equipment to bear the cost of the capital invested in it, regardless of the source of the capital. The investment I_i upon which the cost of capital is to be computed will in general be that amount (I_g) of the prime investment in any equipment which has been capitalized as an asset and as such is carried on the books of the company. In the case of new equipment which supersedes some existing equipment, the asset value of which (K) has not been fully depreciated, the new type must also assume the burden of paying the cost of this capital until it has been fully amortized. Accordingly

$$I_{i1} = I_{g1} + K$$

where $K = K_1$ and K_2 is assumed equal to zero. Similarly if the present equipment is still carrying the burden of some equipment which it displaced, provision already exists for the fulfilment of this obligation, so that in special cases

$$I_{i2} = I_b + K_2$$

Ordinarily K_2 may be neglected in this case as well. Hence the interest charge, which in this case is computed on the principal alone and not compounded, may be expressed as

$$i \cdot I_{i1} = i_p(I_{g1} + K)$$

or

$$i \cdot I_{i2} = i_p(I_b + K_2)$$

where $i = i_p$ in both cases.

It so happens, however, that interest is not only payable but also receivable, because through the operations of an adequate depreciation policy a reserve fund is accumulated, the interest charge on which should be credited to the equipment for the time that the money is idle awaiting eventual reinvestment in some superseding equipment. Thus, in addition to the cost of capital, income from the reserve fund applicable to any single equipment must be accounted for in determining its accounting or economic life. Accordingly, if the estimated life of the equipment be H_e for depreciation purposes, the interest charges payable for any period H are

$$\sum_0^H (iI)_p = Hi_pI$$

and the interest charges receivable for the period H are[6]

$$\sum_0^H (iI)_r = H_e \frac{(H_e - 1)i_rD}{2} + (H - H_e)i_rI$$

$$= H_ii_rI - {}^1/_2\, i_rI - {}^1/_2\, H_ei_rI$$

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[6] The derivation of these expressions depends on a time series which is developed in full in the original report on this research project on file at the Massachusetts Institute of Technology.

where $D = I_d/H_e$. The net difference or total effective interest charge for the period of H years then becomes

$$(iI)_e = \sum_{0}^{H} (iI)_p - \sum_{0}^{H} (iI)_r$$

$$= \frac{iI_i}{2}(H_e + 1)$$

assuming that the rates of interest payable and receivable are identical. Now if the time H_e within which the equipment is expected to pay for itself is made the variable as in the case of the determination of the true accounting life, $H_e = H$ and

$$(iI)_e = \frac{iI_i}{2}(H + 1)$$

The average net interest charge for any year within the period may now be computed for use in determining the yearly financial charges; whereupon

$$(iI)_{\text{avg}} = \frac{(iI)_e}{H} = \frac{i \cdot I_i}{2} + \frac{i \cdot I_i}{2H_e}$$

Since the second term in the equation decreases progressively as the numerical value of H increases it can be disregarded for practical purposes, so that the approximation

$$(iI)_{\text{avg}} = \frac{i \cdot I_i}{2}$$

will be acceptable for the moment.

Unfortunately no such simple expression can be employed in the determination of the net average yearly investment charge, because the capital values employed in computing interest payable and receivable are not identical and because in any comparison between two types of equipment the elements of investment differ, depending upon the extent to which depreciation of the present equipment has been achieved. Furthermore there are two means of computing the interest charges for the present equipment, one based on the original investment and the other on the book value with due regard for previously accumulated reserve fund. An analysis[7] of this situation shows that there is no difference whether interest is computed on a decreasing book value or as a constant on the original capital investment corrected by a credit from current depreciation provisions or with an average interest percentage rate, provided the interest and depreciation charges are accounted for in the proper time sequence. As a result of all derivations for interest the following two equations for the new and present equipment will be used in the later discussion:

$$H\,(iI)_{c1\text{avg}} = \frac{i(I_{g1} + K)}{2}(H + 1) + \frac{iS_{c1}}{2}(H - 1)$$

$$= \frac{i}{2}H\,(I_{g1} + S_{c1} + K) + \frac{i}{2}(I_{g1} - S_{c1} + K)$$

and

$$H\,(iI)_{c2\text{avg}} = \frac{i}{2}(H + 1)(I_b + S_{c2})$$

Total Financial Charges

Naturally in computing the total financial charges (F_1), items such as taxes and insurance must be included. However, all such items are independent of time in the determination of their annual amount since both of the above are derived from the application of a fixed rate upon a certain portion of the prime investment which is set arbitrarily by tax laws or insurance policies. Furthermore any rights relevant to the ownership of the equipment may justifiably be included in this category as well. They include royalties on patents, franchises, licenses, etc. The character of these charges has duly to be considered as either an additional cost to the original capital (prime investment) or as a yearly charge without specific time limitation, or as a charge proportional to the output of the equipment. Their treatment in the problem depends upon the characteristic of each item as established by prior agreement and the method of prorating each, either to the unit of production, on the basis of prime investment, or as an indeterminate charge over the life of the equipment. Accordingly this total accumulated value for any period of H years can be simply indicated by

$$F'' = H(tI_t + i_sI_s + \ldots.\ \text{etc.})$$

all of which can be accurately determined for any year from factory records and can be considered as a distinct group of *invariable charges.* Hence for any single equipment the *total financial charges* can be expressed as

$$HF_1 = HD + H(iI)_{c\text{avg}} + F''$$

Direct Labor, Materials, and Supplies

Turning now to the second class of obligations, relating to operating expenses (F_o), the cost of direct labor and materials will first be considered. In general these two factors need but slight treatment as their relation to the problem is obvious and their nature should be quite well understood.[8] Both factors when expressed as a yearly cost vary directly with the quantity produced (XN) and their total value for a period of H years is likewise a direct multiple.

Briefly, *material expense* is based on the required number of pieces at full capacity (N) passing through the equipment, and may be expressed as ($M + A$) in dollars per year where M represents the annual monetary equivalent of material actually needed for the finished product while A is the allowance for spoilage and waste material which normally accumulates during any year. If there is a change in the nature of the service performed, due to the introduction of a higher quality of material, the advantage thus gained will be evidenced by a reduction in waste or number of pieces failing to pass inspection, by the salvage costs as represented by the term A, or by the operating savings due in general to better methods of manufacture made possible in this manner. Otherwise the cost of materials will cancel out when making a comparison between two types of equipment.

Direct Labor Cost (L) equals the annual amount of dollars spent for direct labor on a given task at full capacity (N) during the year. If there is a gain due to the new equipment from a reduction of accidents or a lessening of workers' fatigue, it will show up eventually in the final operating savings as the general efficiency of human effort will be greater. *Direct supplies* (M_s) behave in a manner similar to either of the above items, and the same conclusions apply since only those items which are consumed in the course of production such as tool points, waste, cutting oils, lubricants, and other similar materials are to be considered a part of it. Its value for purposes of computation will be the summation of the normal yearly amount of each contributory item.

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[7] Again the derivation of these expressions depends on a certain time series which is developed in full in the original report on this research project.

[8] See "Quantity and Economy in Manufacture," by F. E. Raymond, McGraw-Hill Book Co., 1931, for a more extended treatment of the composition of these two elements.

PREPARATION COSTS

The items of this group are distinct from direct production costs because they are independent of the quantity produced on any order and have a constant value for any lot being processed by the equipment. These costs include all charges relating to machine change over, tool preparation, order writing, production control, control laboratory, and technical engineering work pertaining only to the order or any other overhead items having similar characteristics. In the long run it cannot be said that the preparation costs (P_p) per order are entirely independent of quantity as their accumulation over a period of years depends upon the relation of the economic lot size to the average annual consumption (S_y) or demand for the product or products processed by the equipment. To account for these costs properly special formulas must be employed which have been specifically derived for the purpose.[9] Now if P' be allowed to represent the yearly accumulation of the preparation charges where P_p equals the constant cost per lot produced, differing for each product and each type of equipment, and f' and f'' are certain unit cost factors, it is possible to use the expression

$$P' = \sqrt{\frac{XNP_p}{f'}} \quad \text{for a large dollar value of inventories and storage costs}$$

or

$$P' = \sqrt{\frac{P_p}{f''}} \quad \text{for small inventories and a high work in process value}$$

where X = productivity factor, and
N = normal number of pieces produced per year at capacity, both of which will be explained more fully later on.

POWER COST

Even though the cost of power is usually treated as an item of overhead, it is in reality much more of a direct charge against production when an analysis of the cost structure is centered upon the equipment, and therefore in this case it must be considered on the latter basis as distinct in itself. Unlike the foregoing cost elements which are directly related to the quantity produced, the annual power charge (E) has two definite characteristics: one being the actual cost relating to the power consumed by the equipment, which varies directly with production; and the other the fixed capital charges relating to the power plant, transmission lines, supplementary control equipment, as well as line losses, all of which are of a general nature and do not vary with production. The first may be easily computed on the basis of the power required to perform a specific task and reduced to a unit charge as a function of time of operation or of quantity produced. The difficulty comes with the invariable charges as these are constant for the plant, department, or burden center, and have no relation to the specific equipment in any group. Thus when old equipment of one type is replaced by a more modern type there is naturally a change in the proportionate allocation to each unit of equipment in the new arrangement. If no change in the power plant or its equipment belonging to the factory is required because of new productive equipment, the influence of these invariable charges may be neglected, provided that their allocation through machine-burden rates is not unfair to all equipment individually in the group affected. On the other hand, if a change in the power equipment is necessitated by replacement or installation of new equipment in the factory, the latter must carry the burden of the increased invariable charges or else this increase must be prorated to all equipment thus affected by a change in burden rates. It is recommended that the annual invariable charges (V_i) be allocated on a basis of the rated horsepower of each unit of production equipment. Hence, if E equals the rated horsepower demand for the factory, the unit charge per rated horsepower will be

$$v = \frac{V_i}{E}$$

and then the annual demand charge for a piece of equipment having a horsepower rating of e_{eq} will become $W_i = v_i e_{eq}$ in dollars per year. The variable power charges (V_v) can then be allocated on the basis of hourly or yearly production if

v_v = power consumed per piece or
v_v' = power consumed per hour
NX = pieces produced per year
t_o = the time to process one piece in hours, and
XT_N = actual working hours per year
$= NXt_o$

Hence, the general power-consumption charge for any equipment will be

$$\left.\begin{aligned} W_c &= v_v XN \\ &= v_v' XT_N \end{aligned}\right\} \text{in dollars per year.}$$

UPKEEP EXPENSE

In ordinary accounting practice, upkeep expense is treated entirely as an overhead item; however, since the magnitude of this form of expense is completely controlled by the nature of the equipment, it must be given separate consideration. Upkeep is composed of three elements which incur expenses over a period of years in quite different ways. The first is that arising from the daily attention given to the machine by its operator or millwright in the form of lubrication, cleaning, adjustments, and the like, the annual cost of which is directly related to production. This element is naturally a function of quantity [$f(X)$]. The second is maintenance, which must be carried on regardless of the use of the machine in order to insure its being in a ready-to-run condition. Its cost over a period of years is a direct function of time [$f(H)$]. The third is repairs, which are necessary to replace worn out parts and to keep the equipment up as nearly as possible to its original state of efficiency, and consequently its cost may increase year by year as more and more parts have to be replaced. The cost of this element throughout the life of the equipment is undoubtedly a complex function of time [$f(H^\mu)$].

Accordingly, the yearly cost of maintenance becomes

$$M_y = m_1 + m_2 f(X) + m_3 f(H)$$

where m is any constant.

Then the total cumulative maintenance cost for H years is

$$\sum_0^H (M_y) = M_H$$

Now if the yearly increment of any element in maintenance cost for which m is the coefficient is a linear function ($M_y = mH$), then

$$\sum_0^H (M_y) = M_H = \frac{m}{2} H(H + 1)$$

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[9] See "Quantity and Economy in Manufacture," by F. E. Raymond, p. 172 for generalized formulas, and p. 37 for their various derivatives for special cases, from which an evaluation of the factors f' and f'' can be obtained.

$$= \frac{m}{2} H^2 + \frac{m}{2} H$$

In this case the total accumulated charge is represented by the sum of all the rectangles for the reason that each one equals the yearly increment. Evidently (see Fig. 2) the area under the curve is smaller by $Hm/2$, bringing up the total to

$$M_H = \frac{m}{2} H^2 + \frac{m}{2} H$$

However, if the yearly cost of maintenance becomes a more complex function $[f(H)]$,

$$M_y = m \cdot f(H)$$

or, for example,

$$M_y = m_1 + m_2 X + m_3 H^{\mu}$$

where

$$\sum_{0}^{H} (M_y) = M_H$$

Here again the summation of M_y is not only the area under the curve but an additional summation of a series of triangles standing beyond the curve, so

$$M_H = H(m_1 + m_2 X) + m_3 \frac{H^{\mu} + 1}{\mu + 1} + \frac{m_3}{2} - H^{\mu}$$

In combination with a functional depreciation charge and its accumulation at any time H in accordance with the resale value of the equipment, it is well to note that such equations might lead to the determination of a minimum point of yearly cost at a future time. This investigation, however, was not intended to pursue all the characteristics of the individual equipment, but only those for alternative equipment with the view of comparing them.

SPACE CHARGES

Similarly this group of charges normally belongs in overhead, and is composed of rent items such as property taxes, insurance, interest, depreciation, upkeep, repairs on buildings and fixtures, and service items such as heat, light, ventilation, and the like. However, since it is common practice to allocate such items through machine-burden rates where the proportion of the whole borne by each unit of equipment is prorated on the basis of the floor area occupied by it, a change in equipment where greater or less floor space is utilized will seriously alter the distribution of this part of the overhead. Furthermore, if no cognizance of the importance of this item is taken, no opportunity will be provided for accounting for any savings which may arise through a reduction in the area devoted to production. This is important because in many instances processes may be consolidated by new equipment and other work may then be performed in space thus made available, with a marked saving in overhead and the possible avoidance of having to add to the factory buildings in the near future. Even though no immediate saving can be made in rental and space charges, unless the property is not owned outright and the contemplated changes are extensive, it seems illogical to penalize a product by continuing, as a part of its ultimate cost, charges which do not properly belong to it. With the new equipment, if less space is occupied, the burden of the remaining unoccupied space left over from the previous equipment should become a general charge against the management until such a time arrives that it can be properly utilized, so that the product affected by the change can derive its full share of the benefit. When greater space is required than formerly, there is a deduction from the general burden of unoccupied space borne by the particular department or burden center in which the equipment is located, and then a greater burden of space charges must be transmitted through the equipment to the product. Now if:

A_t = total space or area of a productive department in square feet
A_{eq} = total space occupied by equipment including space for its operator, work bench, raw and finished material, etc.
A_w = all space required for aisles and other general departmental purposes
A_u = all unoccupied space, and
b_s = summation of all space charges per year.

$$A_t = A_{eq} + A_w + A_u$$

and the unit space-charge factor becomes

$$s_p = \frac{b_s}{A_t} \text{ (dollars per year per square foot)}$$

Then if a_t = area occupied by the unit a_q plus that assigned to it, $a_w = a_q + a_w$, both in square feet, where $a_w = A_w a_q / A_{eq}$, the annual burden carried by any unit of equipment will be

$$B_s = s_p a_t + s_p A_u \frac{a_t}{A_{eq} + A_w}$$

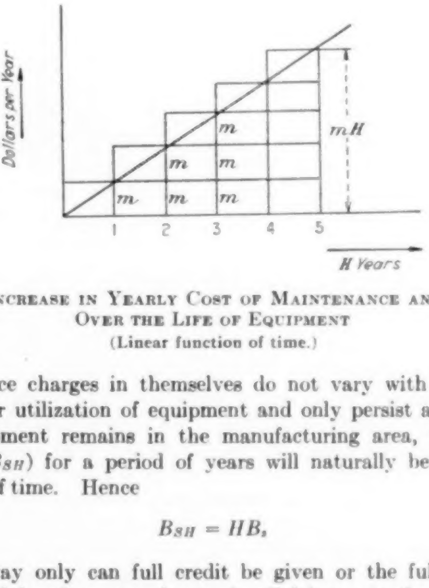

FIG. 2. INCREASE IN YEARLY COST OF MAINTENANCE AND REPAIR OVER THE LIFE OF EQUIPMENT
(Linear function of time.)

Since space charges in themselves do not vary with the production or utilization of equipment and only persist as long as the equipment remains in the manufacturing area, the total charge (B_{SH}) for a period of years will naturally be a direct function of time. Hence

$$B_{SH} = HB_s$$

In this way only can full credit be given or the full burden distributed for the space charges in all fairness to the product, and then it is up to the management to see that the second term in the expression for B_s is made to approach a value of zero by profitably utilizing the unoccupied space. If this is not done the burden of the space charges is increased upon all equipment within the department or production center.

OVERHEAD EXPENSES

The third and last class of obligations contain all expenses and charges which are normally considered a part of overhead (F_B), exclusive in this case of such items as have been previously given separate treatment because of the fact that their magnitude is largely dependent in some manner upon the specific type of equipment. In general those items which will now be considered under this classification will be those that are derived from some service supplementary or complementary to actual production, and whose origin is in no way connected with the direct operation of the equipment or the function it performs. Normally such items are budgeted for the year and allocated to departments, production centers, productive units, or the product itself, and distributed through burden rates computed either on man-hours or machine-hours according to their association either with the product or the equipment and their individual origin or behavior. Those which vary with production will be combined into a man-hour burden rate and those which continue irrespective of production will be combined into a machine-hour rate in the sense that they accumulate as long as the plant is in a ready-to-run condition. Thus an accounting can be made of the cost of idleness. The all-important point in the computation of the natural obligations which are incurred in the course of manufacture or performance of a service is that every element of expense must be accounted for and related eventually to the product in some manner characteristic of its particular behavior at the point in the production sequence nearest to that where the expense originated. Normally such refinement in accounting practice is unessential; however, in the determination of an equipment policy where one picks out and studies a particular operation and the facilities employed to perform the task, all of the obligations must be recognized and properly related if the economic advantage of one type of equipment over some other is to be ascertained.

The great weakness of burden charges made in the formulations by several authors and in the formula of The American Society of Mechanical Engineers is that they all contemplate charging the burden at a percentage rate on some direct cost related to the amount of work done. As a matter of fact, various kinds of equipment in operation call for widely varying investment charges, operating expenses, and all the remaining items making up burden as well. This fact is entirely disregarded if a uniform burden rate per hour, or a percentage of the labor cost, is added to the strictly financial and prime operating expense. The essential object of the new equipment being to reduce either labor cost or production time, the old methods of applying overhead are apt to charge a smaller amount of burden to the superior equipment, although this equipment might in reality be the source of higher general expenses. Furthermore there is the danger of duplicating costs which have already been accounted for separately. This use of such an ideal cost-accounting scheme is possible, as will be shown later through the introduction of "Differential Costs." Thus automatically from all productive overheads, general burden, or even administrative burden, only those are selected which in any way originate or disappear by virtue of the adoption of a new type of equipment.

The allocation of all items of overhead expense must be given very careful consideration in this particular problem as there is a serious element of danger connected with the use of ordinary burden rates, because, if they be employed without regard to the origin or nature of the specific expense items relating to the new equipment on an equal basis with the old, the changes in relative performance between the two types may make it impossible to adequately distribute the total burden of overhead which should rightly apply in each case. Some overhead items apply directly to the product, while others apply only to the machine. Furthermore some items accumulate on the basis of space or operating time, while others accumulate on the basis of the time the machine is actually installed, whether in use or not, the horse power rating, the number of machines under one operator, the lot size, the quantity produced, or the capacity of the machine in pieces per hour. Moreover, overhead may accrue to the machine or the product not only from the use of the equipment but also from the allocation of general or departmental expenses which apply to all machines in the department. Charges of this nature are derived from general administration expense, cost of control, time keeping and records, supervision, service, supplies consumed in the course of departmental operations, transportation, indirect labor, and helpers. These items may each have specific characteristics of its own as described above, or if not they can be treated as a lump-sum item of general overhead. At all events, in figuring the obligations of this nature which must be borne by a new piece of equipment, the treatment of overhead must be so worked out that the summation of all obligations prorated to each equipment item in a department after any proposed change will equal the summation of all obligations before the change or the total obligations for the department in either case. For instance, a change in machine capacity may require the readjustment of existing burden rates for all items accumulating on time and independent of the quantity produced, even if there is no change in the total departmental overhead burden due to the change in equipment. In the computation of burden rates for use in problems relating to equipment policies it is recommended that the total annual expense for any overhead item be divided into its variable and invariable items and reduced to two hourly rates as follows:

(*a*)

$$\frac{\text{Total fixed charges}}{\text{Year}} \times \frac{1}{\text{Total rated capacity}} = \frac{\text{Unit demand rate}}{\text{Year}}$$

$$\frac{\text{Unit demand charge}}{\text{Year}} \times \frac{\text{Year}}{\text{Standard Working Hours}} = \frac{\text{Unit demand rate}}{\text{Hour}}$$

(*b*)

$$\frac{\text{Total Variable charges}}{\text{Year}} \times \frac{\text{Year}}{\text{Total production}} = \text{Unit consumption rate}$$

(*c*)

$$\frac{\text{Unit consumption rate}}{\text{Hours}} + \frac{\text{Unit demand rate}}{\text{Hours}} = \text{Total hourly service rate}$$

(*d*)

$$\text{Man-hour rate} + \text{machine-hour rate} = \text{Total burden rate}$$

or

$$b_{mn} + b_{egn} = b_r$$

Accordingly, if T_x equals the actual operating man-hours per year and T_N the normal available working hours per year, the yearly variable burden related to any piece of equipment will be

$$O_x = b_m T_x = b_m X T_n$$

and the yearly invariable burden born by the equipment will be

$$O_f = b_{eg} T_N$$

Elements of the Annual Equipment Cost

The foregoing analysis of all cost elements which are directly or indirectly incurred by the introduction of equipment of any sort to facilitate production or the performance of some service, is of utmost importance if correct conclusions as to the true economic utility of any single piece of equipment are to be determined. Only in rare cases will each item discussed above be required in its entirety, and in the solution of any problem only those elements or that portion of their mathematical expression need be used which the circumstances demand. In the further development of this subject little attention need be paid to these individual elements as long as their specific characteristics are recognized, so that when application of any resulting formulas is desired, simple substitutions of values in the correct locations can be made. In the case of operating expense (F_o), it will be seen that certain yearly costs vary directly with production $f(X)$, and all such items will be collected into a group to be known as *Quantitative Costs in Direct Proportion* (XC_q), where X stands as the factor of productivity. If N equals the standard number of pieces to be produced in a year or the capacity of the equipment and N_z the actual production in a year to meet the normal sales demand (S_{yz}),

$$X = \frac{N_z}{N}$$

Hence

$$XC_q = X(M + A + L + M_s + W_c + O_z + m_2 + \ldots\text{etc.})$$

A similar group which varies in a more complex manner with production will be termed *Quantitative Costs in Functional Proportion* [$f(X)C_p$]. Typical of these is the preparation cost P', so that

$$f(X)C_p = \sqrt{\frac{XNP_p}{f'}} + \ldots\ldots\ldots\ldots \text{ etc.}$$

A third group under this classification are those which can be designated as *Time-Related Costs in Functional Proportion* [$f(H)C_t$], which for the most part are characterized by repairs under upkeep costs (M_y) when they increase from year to year. Hence

$$f(H)C_t = \frac{m_3}{H}\,\frac{H^\mu + 1}{\mu + 1} + \frac{m_3}{2}\,H^{\mu - 1}$$

The other portions of upkeep such as regular attendance (m_2x) and regular maintenance (m_1) naturally will fall under other groups. If repairs be uniform over the life of the equipment, the average yearly cost can be included directly with the maintenance expense.

A fourth grouping may also be established that will comprise all annual charges or expenses which do not vary from year to year, and this will be composed of certain cost elements contained in each of the three classes of obligations. This group will be known as *Invariable Costs* (C_f), and its value may be computed from the expression

$$C_f = F_I + O_s + W_i + m_1 + B_s + \text{etc.}$$

Now the total yearly operating or *Producing-Equipment Cost* (C_y) can be expressed by

$$C_y = C_f + XC_q + f(X)C_p + f(H)C_t$$

and evaluated when the correct expressions and their equivalent numerical values are introduced, as this equation contains every element of the direct obligations which are incurred through the use of equipment. If annual depreciation and any other similar obligations be deducted from the total yearly financial charges (F_I), they can be combined into a fifth group, the elements of which will have the characteristic of being *constants throughout the entire life of the equipment.*

PREVIOUS TYPES OF SOLUTION

Since an expression can be obtained through which the entire obligations may be appropriately accounted for with regard to any specific type of equipment, it now becomes possible to compare various types of equipment which are adapted to the performance of certain tasks and determine which is the most suitable in terms of its economic utility. Most often in the mind of the industrial executive the criterion for judging the utility of any type of equipment is expressed by the ability of the equipment to perform a certain task as to quality at least as well as the present equipment or one that is taken as a standard of comparison, to effect certain economies in operation, and to pay for itself in the shortest possible time. The question now arises as to the best manner in which to determine that this criterion can be satisfied by one type of equipment more satisfactorily than by any other. Numerous attempts have been made to derive a formula for this purpose, but none of them have proved entirely suitable to the problem, and in no case was any one universal in application. To summarize the existing formulas, they can be classified according to methods of solution as follows:

a Based on yearly cost of service[10]
b Based on capitalized cost of service[11]
c Based on unit cost of service[12]
d Based on capitalized unit cost of service[13]
e Miscellaneous.[14]

The fact is that all of these formulas fail in some respect to meet the criteria for an appropriate solution of the problem as set forth at the beginning of this paper, although each formula arrives at an approximate value for the *true accounting life.*[15]

THEORY OF DIFFERENTIAL COSTS

Accordingly a new basis of comparing the economic utility of alternative types of equipment was evolved upon the theory that only through the relation of the total obligations incurred by any two types of equipment over an indefinite number of years such that at some time the obligations of both would become equal could the economic superiority of one type be established. Hence the elapsed time or period (determined by the solution of this relation) required for one type to pay back to its owner all obligations incurred by him in its original purchase and expected of it while in operation, becomes one measure of the economic utility of that equipment for the specified task, and as such it has been termed the *True Accounting Life* (H_t).

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[10] A.S.M.E. Formula for Labor-Saving Devices, by J. A. Shepard and G. E. Hagemann, *Mechanical Engineering*, vol. 47, no. 5, May, 1925, p. 403.

Formula for Tools, Jigs, and Fixtures, by J. W. Roe, Trans. A.S.M.E., MSP-51-11, p. 79.

Equipment Formula, by C. H. Porter, *Accounting Review*, 1928, p. 369.

[11] Equipment Formula, J. J. Berliner, *Factory*, April, 1928, p. 785.

[12] Equipment Formula for Sales Purposes, by E. P. Blanchard, sales literature of the Bullard Co., of Bridgeport, Conn.

[13] Formulas for Cost of Service, by J. C. Fish, Sec. 110 and Table VII, in "Engineering Economics," McGraw-Hill Book Co., 1915, pp. 112 and 186.

[14] Warner & Swasey Co., *American Machinist*, vol. 70, no. 20, p. 764.

SKF Industries, *American Machinist*, vol. 62, no. 19, p. 719.

H. Weiss Mfg. Co., *Manufacturing Industries*, vol. 15, p. 27.

R. M. Barnes, *Factory and Industrial Management*, June, 1928, p. 1188.

B. A. Franklin, *Industry*, vol. XXV, no. 10, May, 1930, p. 1.

P. P. Buckey, *Mining and Metallurgy*, vol. 11, p. 99, Feb., 1930.

[15] For the analysis of previous types of solution see "A Study of Formulas for the Selection and Replacement of Manufacturing Equipment," by H. O. Vorlander, Graduate Thesis 1930, Course XV, Department of Business and Engineering Administration, Massachusetts Institute of Technology, or the official report on this research project.

In such a comparison it is assumed that a new type of equipment is to be compared with some type now in use or with some standard type chosen as representative of a desired standard of performance. Now to express mathematically, so as to obtain the desired solution, and measure this relationship the concept of *differential costs* was introduced. By this is meant the difference between the total obligations (C_{H2}) of the present or existing equipment over the period of time ($H = H_t$) and the total obligations (C_{H1}) for the new or proposed equipment over the same period of time. Thus if the symbol ΔC_H be employed to indicate the differential cost it may be evaluated by the expression

$$\Delta C_H = C_{H2} - C_{H1}$$

As a result of this relationship differential costs will appear among all the elemental cost items, so that hereafter a standard notation will be employed where, for instance, in the case of material costs

$$\Delta(M + A) = (M_2 - M_1) + (A_2 - A_1)$$

or

$$-\Delta(M + A) = (M_1 - M_2) + (A_1 - A_2)$$

or

$$\Delta M - (-\Delta A) = (M_2 - M_1) - (A_1 - A_2)$$

as might be the case if $A_1 > A_2$.

The Break-Even Point and the Reward for Risk

Thus to satisfy the foregoing criteria for the true accounting life as the measure of the economic utility of some proposed type of equipment, it becomes obvious that the total obligations incurred by both types over the period H_t must be equal and that $\Delta C_H = 0$. This condition will be designated as the *break-even point.* Naturally if either type of equipment be continued in operation beyond the time H_t where this situation occurs, that type having the lower operating and overhead costs will have a distinct economic advantage over the other type with higher costs. This is, of course, due to the fact that the savings in such costs are sufficient to pay back to the investor or owner of the equipment all charges incurred through the adoption and installation of the higher priced equipment in the time H_t, including the original outlay of capital. Therefore it would seem that if all financial operating and overhead costs are contained in the value of the yearly producing-equipment costs for the existing type (C_{y2}) and for the proposed type (C_{y1}), the break-even point, and hence the true accounting life, could be computed from the fact that

$$\Delta C_y = C_{y2} - C_{y1} = 0$$

However, there is one more obligation which, though of an implied nature, is very real, and that is the necessity for maintaining unimpaired the earning power of the capital facilities employed. This is due to the fact that all capital invested in manufacturing or productive operations must return to its owner not only the cost for its use but also a reward for the risks involved in its investment. This reward for risk when it has accumulated eventually appears as that portion of the dividends paid over and above that equal to the cost of capital at the current market rate. Hence if the investment in equipment for the performance of some task or service is increased so as to reduce the operating and overhead costs through better production methods, an equal return or reward must be earned upon the added capital invested in the new equipment. Accordingly, if the current rate of return on all capital invested in the productive assets of the enterprise be r_a, the extra reward for the risk of investing this additional capital will become

$$H\,\Delta C_r = r_a(I_{g2} - I_{g1})H$$

and then the total direct and implied obligations for both types of equipment will be

$$C_{H1} = HC_{y1} + r_a I_{g1} H$$

and

$$C_{H2} = HC_{y2} + r_a I_{g2} H$$

The true break-even point (see Fig. 3) can now be expressed as

$$\Delta C_H = H(\Delta C_y + \Delta C_r) = H\,[(C_{y2} - C_{y1}) + (C_{r2} - C_{r1})] = 0$$

Evaluation of the True Accounting Life

Since the time in which some proposed type of equipment can pay for the increased financial obligations out of operating economies is the means of justifying in the eyes of individual executives the adoption of this new type over some existing type, the true accounting life (H_t) will be the time in which full depreciation of the original capitalized investment (I_g) plus any write-off of incidental installation charges (I_{inc}) less any legitimately anticipated salvage value (S_v) can be achieved. Hence the time H_t will be the logical basis for establishing a sound depreciation policy and the annual rate of depreciation, such that $d = 1/H_t$. In later portions of this study this value will be recommended as that for the depreciation rate in place of an arbitrary accounting rate d_a which has no relation to the existing obligations. Accordingly, with these facts in mind, the time H_t can be determined from the following equation, if the two financial items for interest and depreciation be written out in full and the appropriate expression for any case be introduced for the remaining items. If

$$C_{H1} = H\left[\frac{(I_{g1} + I_{\text{inc1}} - S_{e1} + K)}{H} + \frac{i}{2}(I_{g1} + S_{e1} + K) + \frac{i}{2H}(I_{g1} - S_{e1} + K) + C_{y1}'' + C_{r1}\right]$$

where $C_{y1}'' = C'_{f1} + X_1 C_{q1} + f(X_1)C_{p1} + f(H)\,C_{t1}$ and

$$C'_{f1} = C_{f1} - D_1 - (iI)_{\text{cavg1}}$$

and if

$$C_{H2} = H\left[\frac{I_b - S_{e2}}{H} + \frac{i}{2H}(H + 1)(I_b + S_{e2}) + C_{y2}'' + C_{r2}\right]$$

where $C_{y2}'' = C'_{f2} + X_2 C_{q2} + f(X_2)C_{p2} + f(H)C_{t2}$ and

$$C'_{f2} = C_{f2} - D_2 - (iI)_{\text{cavg2}}$$

and since for the break-even point $C_{H1} = C_{H2}$,

$$H = H_t = \frac{I_{g1} + I_{\text{inc}} - I_b + K + (S_{e2} - S_{e1}) + \frac{i}{2}[(I_{g1} - I_b + K) - (S_{e2} - S_{e1})]}{(C''_{y2} - C''_{y1}) + (C_{r2} - C_{r1}) - \frac{i}{2}[(I_{g1} - I_b + K) - (S_{e2} - S_{e1})]}$$

In order to avoid compound functions of time $f(H)$, which would greatly complicate the use of this equation in ordinary practice, the salvage value S_v for either type of equipment has been taken as the ultimate scrap value S_e instead of $S_v = f(H)$ as is actually the case when the situation $H = H_t$ depends upon the break-even point. Similarly, the group of cost items which, if they existed, would appear in the item of time-related costs in functional proportion $[C_t f(H)]$, have been disregarded, assuming that in ordinary cases these items would be directly proportional to time and would be included under the group of invariable yearly charges C_f. Of course, if a true mathematical picture is to be obtained of the actual situation as governed by known events where, for instance, the annual sales curve could be projected accurately into the future $[S_{yH} = f(H)]$, it would be necessary not only to insert these functional values for S_y and C_t but also to relate all quantitative costs $[XC_{qH} + f(X)C_{pH}]$ for the entire period (H) to time through the expression

$$[XC_{qH} + f(X)C_{pH}] = HXC_q + f(HX)C_p$$

where

$$HX = \frac{\sum_0^H N_s}{N} = \frac{\int_0^H S_y f(H)dH}{N}$$

and the total sales for the period H is $S_{yH} = S_y f(H)$. However, if these relations be introduced into the equation $C_{H1} = C_{H2}$ and solved for H, the resulting value for H_t will only have a theoretically useful value. Fig. 3 will assist the reader in visualizing the relationship of all obligations or cost elements and the reason that there exists a point H_t where these obligations for alternative types of equipment become equal and are satisfied.

BASIS FOR DETERMINING THE TRUE ECONOMIC LIFE

In specifying the criteria for the true accounting life within which any proposed equipment can return its original cost and pay for all obligations out of operating economies, no consideration was given to the fact that normally the installation of new equipment implies that these operating economies should be sufficiently adequate to permit cost reduction, the realization of a larger earning power, and eventually price reductions to customers. This situation changes quite radically the aspect of the problem. Now the criterion becomes the time H_r, the true economic life within which all obligations can be met, including depreciation plus an accumulation of savings equivalent over a period of years H to the total cost reduction or extra profits desired on all items produced or services performed by the equipment in the same period. This entails a complete analysis of the entire price-cost structure of each of these products or services, and the introduction of the sales price or its equivalent. The total annual revenue from the sale of any one of these may be expressed by

$$pS_y = \Sigma R_{eq} + R_w + C_b + \Sigma C_y$$

where p = unit net average sales price
S_y = average annual consumption or sales demand
ΣR_{eq} = total annual return on the investment in all equipment employed in a specific process
R_w = total annual return on the investment in inventories and work in process for a specific product
C_B = total annual portion of the cost of doing business allotted to a specific product (general financial, sales, and administrative expense), and
ΣC_y = total annual cost of operation for all operations and equipment employed in that process. (Previously described and defined.)

As developed in another study,[16] the total revenue from any product or service can be broken down into its elements and an equivalent piece price (p') established for the product at any point in the process sequence, so that

$$p'S_y = R_{eq} + C_y$$

When multiple products are handled by a given unit of equipment a summation in this equation of identical elements for each product can be made in order to obtain the total equivalent yearly revenue. This whole development is made possible by the fact that the total profit from annual sales of a product can be expressed as so many dollars of return on each dollar of capital invested productively in either equipment of all kinds, work in process, or inventories. Accordingly if $(\Sigma I_{eq} + I_w)$ dollars are invested in the production of an article or the performance of a service, there will be a uniform rate of return r_a on that capital so that if

$$\Sigma R_{eq} + R_w = pS_y - C_B - \Sigma C_y$$

$$r_a = \frac{\Sigma R_{eq} + R_w}{\Sigma I_{eq} + I_w} = \frac{pS_y - C_B - \Sigma C_y}{\Sigma I_{eq} + I_w}$$

or for any equipment in the process sequence,

$$R = R_{eq} = r_a I_g = p'S_y - C_y$$

THE EARNING POWER THROUGHOUT LIFE OF ANY EQUIPMENT

Since it has been established that all equations for determining the economic utility of equipment must be based on relations involving all obligations, now of any sort, upon their accumulated totals over an indefinite period of H years, it can be stated that

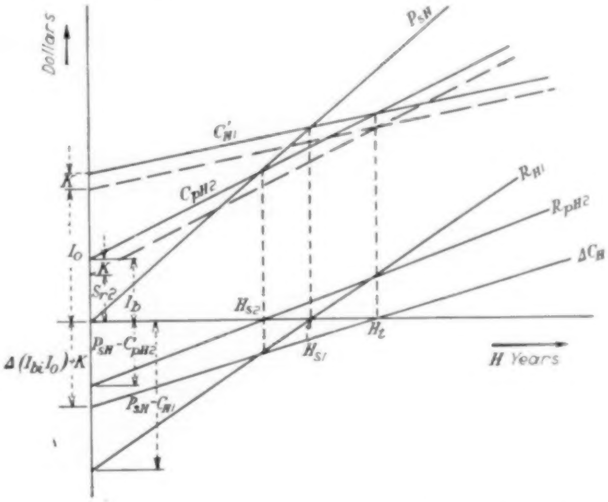

FIG. 3 RELATION OF ALL OBLIGATIONS AT THE BREAK-EVEN POINT FOR DETERMINING THE TRUE ACCOUNTING LIFE (H_t)

$$P_{SH} = Hp'S_y = [r_a I_g + D_a + C_y']H$$

when for convenience $D_a = I_d/H_a$ and $C'_y = (iI)_c + C''_y$, where C''_y has been already defined, have been inserted. The value to be used for H_a in computations may be either an arbitrarily established accounting time for depreciation to become completed or the mathematically correct time H_t just previously derived. Hence if provision for the financial obligations of depreciation and reward for risks of doing business is to be made within the period H,

$$HR + D_a = P_{SH} - C'_y H$$

or in any average year

$$R + D_a = p'S_y - C'_y$$

Now if the portion R_H of the total revenue for the period H above all accumulated current yearly charges or obligations contained in HC'_y normally available for depreciation and a return on the investment is

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[16] See "Quantity and Economy in Manufacture," by F. E. Raymond, McGraw-Hill Book Co., 1931, chapters X, XI, and XII.

$$R_H = H(R + D_a) - (I_o + K)$$

because if $R_H = P_{SH} - C_H$ by definition, $R_H = (HR + HD_a + HC'_y) - (I_o + K + HC'_y)$ where $I_d = I_g + I_{\text{inc}} - S_v + K$ in general, and $I_o = (I_g + I_{\text{inc}} - S_v)$ in the original expression for C_H, then for the proposed or new equipment,

$$R_{H1} = H(R_1 + D_{a1}) - (I_{o1} + K_1)$$
$$= H(p'_1S_{y1} - C_y'_1) - (I_{o1} + K_1)$$

and for the existing or present equipment

$$R_{H2} = H(R_2 + D_{a2}) - (I_{o2} + K_2)$$
$$= H(p'_2S_{y2} - C'_{y2}) - (I_{o2} + K_2)$$

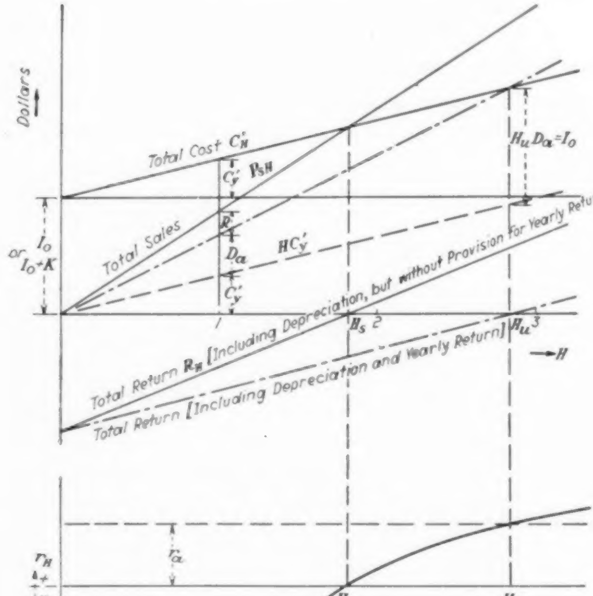

FIG. 4 RELATIONSHIP OF TOTAL SALES REVENUE AND ALL DIRECT AND IMPLIED OBLIGATIONS FOR THE DETERMINATION OF THE FUNCTIONAL RATE OF RETURN (r_H), THE CRITICAL LIFE (H_s), AND THE UTILITY LIFE (H_u) FOR ANY SINGLE UNIT OF EQUIPMENT

However, since this last expression R_{H2} refers to the time the existing equipment was installed and inasmuch as a definite time H_r has elapsed since then, it is advisable also to express R_{H2} at the present time at which the replacement policy is being determined. Accordingly if the investment in the existing machine now has only a depreciable value of $I_b - S_{c2}$ remaining due to an accumulation of $d_{a2}(I_{g2} + I_{\text{inc}2} - S_{r2} + K_2)H_r$ dollars in a reserve fund over this elapsed time,

$$R_{H2} = H(R_2 + D_{a2}) - (I_b - S_{c2})$$

TIME CRITERIA OF EARNING POWER

Now with these facts in hand it will be possible to penetrate into the situation still farther, as it is well to know what takes place with regard to the economic status of any equipment whether it has been in use a short or relatively long time. In the course of business a situation might arise where it was essential that the equipment policy be revised, and it would be necessary to determine the immediate economic status of some particular single unit actually in use. In other words, has it been possible in the elapsed time for the equipment to have earned enough money to pay for itself, assuming that all savings over cost are devoted to this purpose? From the equation for R_{H1} it is evident under these conditions that if the elapsed time $H = H_r = H_s$, the critical life for the equipment will be expressed by

$$H_s = \frac{I_o + K}{R + D_a} = \frac{I_g + I_{\text{inc}} - S_r + K}{R + D_a}$$

In this case original cost besides all obligations will be paid, except that no return on the investment can be earned.

On the other hand, if the elapsed time $H = H_r = H_u$, where

$$H_u = \frac{I_o + K}{D_a} = \frac{I_g + I_{\text{inc}} - S_r + K}{D_a}$$

the normally anticipated return on the investment at a rate r_a can be earned besides the payment of all financial and operating expenses. This time H_u, the *utility life*, will coincide with the time H_t, the true accounting life, only in the special case where $d_a = d = 1/H_t$. Fig. 4 will aid in visualizing these conditions. In all such determinations it must be assumed that the actual value of the term $H(R + D_a) > (I_o + K)$ and that $P_{sy} > C_y'$ if any valid interpretation of the solution of any problem is to be obtained.

THE FUNCTIONAL RATE OF RETURN

Now if, in the latter case especially the total accumulated return on the investment be analyzed year by year from the time of the original installation of the equipment forward, it will be found that the rate r_H, obtained by dividing the annual average R_H/H of the total profits applicable to a return on the investment up to a given date by the capitalized gross investment I_g on which the return is to be earned, will vary (see Fig. 4) from minus infinity ($-\infty$) at the start to zero at the time $H_r = H_s$, and will reach a value of $r_H = r_a$ at the time $H_r = H_u$. Below the point H_s the value of r_H is negative, because some of the revenue normally applicable to a return on the investment is really devoted to depreciation. Above this point the value of r_H is positive. Now this *functional rate of return* is of much significance in the whole problem of the selection and replacement of equipment, as it is the measure of the potential earning power of any type of equipment performing any task or service at any instant of time throughout its actual serviceable life. If, for instance, the actual functional-rate curve for a given equipment is known, and it is advisable to ascertain whether a certain earning power (r_{des}) can be achieved after a given period of operation, one can trace along the curve until the ordinate representing the rate desired (r_{des}) is reached, and then read off from the corresponding abscissa the time required to attain it. If the time thus found is equal to or less than that in mind, the situation is satisfactory. Furthermore, if it is important that a reduction in price be brought about, a similar procedure may be employed to ascertain whether the functional rate of return will permit the revision without lowering the value of $r_{H'}$ below that of r_{des}, or whether it will so alter the position of the curve, its general shape remaining the same, that the rate r_{des} or at least the rate r_a can be still attained in a given time without the danger of equipment obsolescence interfering with the new price policy. Accordingly, if such information can be obtained from the functional rate of return its value at any time H can be determined for

$$r_H = \frac{H(R + D_a) - (I_o + K)}{HI_g}$$

$$= \frac{R + D_a}{I_g} + \frac{(I_o + K)}{HI_g}$$

$$= \frac{p'S_y - C'_y}{I_g} - \frac{I_o + K}{HI_g}$$

As a corollary to this situation the time H_{des} at which any desired total accumulation of the return on investment as expressed by the functional rate r_{des} can be attained may be determined from

$$H_{des} = \frac{I_o + K}{(p'S_y - C'_y) - r_{des}I_g}$$

$$= \frac{I_o + K}{R + D_a - r_{des}I_g}$$

Furthermore, in order to connect this portion of the subject with that devoted to the determination of the true accounting life, it can be shown[17] that $\Delta R_H = -\Delta C_y$.

Relative Earning Power of Equipment

Now when it comes to the comparison of alternative types of equipment especially adapted to the performance of a certain task to determine which has the greatest economic utility, the relation of the functional rate of return for each type under the prescribed conditions has its most valuable application. Through a comparison of the two functional rate curves (see Fig. 5) it will be possible to determine at exactly what time (H_r) the overall performance of one type in terms of economic criteria will become identical with that of the other and beyond which the first equipment will excel, not only in having met its greater obligations in a shorter time and at a lower production cost, but also with a greater cumulative return even on a larger investment, or the same cumulative return at least from a lower price level. In making such a comparison the concept of relating the total accumulated obligations respectively incurred by the two types of equipment over their entire serviceable life is most essential in order that accurate conclusions may be arrived at. Otherwise unfulfilled obligations of one type of equipment cannot be properly transferred to the type which supersedes it and appropriately accounted for. To obtain an adequate picture of the circumstances which surround the problem, Fig. 5 should be referred to.

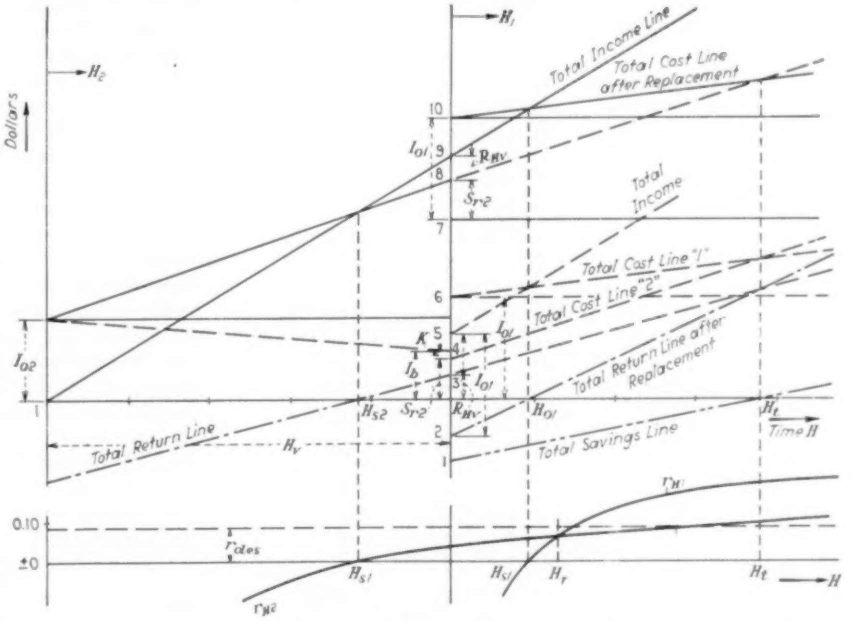

FIG. 5 RELATIONSHIP OF TOTAL SALES REVENUE AND ALL DIRECT AND IMPLIED OBLIGATIONS FROM THE TIME OF INSTALLATION OF THE ORIGINAL EQUIPMENT FOR THE DETERMINATION OF THE TRUE ECONOMIC LIFE (H_r) WHEN ALTERNATIVE TYPES ARE INVOLVED

TRANSFERENCE OF OBLIGATIONS FROM THE PAST, PRESENT, OR FUTURE

In approaching the problem care should be exercised to differentiate as well as to properly translate the various obligations of either equipment from the time at which they occur, either past, present, or future, to the time at which the comparison is being made. In so doing it must first be recognized that over the period H_v during which the existing equipment has been in service, the excess of revenue over all obligations applicable as a total return on the investment has accumulated to R_{Hv}, whereupon the functional rate of return after H_v years of operation is

$$r_{Hv} = \frac{R_{Hv}}{H_v I_{g2}}$$

In the second place, when the existing equipment is replaced the salvage value (S_v), whether in the form of net resale value S_r or scrap value (S_e), must be added to the net income over the time H_v, which will automatically enhance the net remainder between revenue and total obligations so that

$$r'_{Hv} = \frac{R_{Hv} + S_r^2}{H_v I_g^2}$$

In the third place, if a deficiency exists in the attainment of a desired average rate (r_{ydes}) of return on the investment in the old equipment over the period of H_v years, or in the fulfilment of depreciation, it must be transferred to the new equipment just as any credits due to an excess in return can be used to help justify the new type. Accordingly, if L be used to designate the total deficiency necessary to increase $r_a I_{g2}$ to $r_{ydes} I_{g2}$, the new functional rate will become[18]

$$r''_{Hv} = \frac{R_{Hv} + S_{r2} + L}{H_v I_{g2}}$$

where

$$R_{Hv} = H_v(R_2 + D_{a2}) - (I_{o2} + K_2)$$

and

$$L = I_{inc2} + K_2 + K_1 - S_{r2} + C_R + (r_{ydes} - r_a)I_{g2}H_v$$

TIME CRITERIA FOR THE NEW EQUIPMENT REFERRED TO ORIGINAL DATE

Now turning to the new or proposed equipment the economic utility of which is to be justified, these new considerations pertaining to the transfer of unfulfilled obligations will necessitate a revision of the time criteria H_s and H_u already evaluated under the simplest conditions. Out of these, three alternative criteria appear, each dependent upon the nature of the obligations transferred. First, if an excess R_{Hv} of revenue over all obligations has accumulated over H_v years from the old or present equipment, a deduction can be made in the total obligations of the new equipment throughout its life. Second, if only a certain salvage value is received upon the disposal of the old equipment, a smaller sum will have to be credited instead. Third, if a deficiency has occurred, then these unfulfilled obligations of the old equipment will appear as a debit against the anticipated performance of the new equipment. Hence it will be necessary to rewrite the expressions for H_{s1} and H_{u1} as follows, where, for the critical life,

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[17] For definite proof of this statement see official report on this research project (Part II, p. 59), on file at the Massachusetts Institute of Technology.

[18] For derivation of the appropriate expression, as given below, see official report on this research project part II, pages 65–66.

$$H_{s1} = \frac{I_{o1} - S_{v2} - R_{Hv}}{R_1 + D_{a1}}$$

$$H'_{s1} = \frac{I_{o1} - S_{v1}}{R_1 + D_{a1}}$$

$$H''_{s1} = \frac{I_{o1} - S_{v1} + L}{R_1 + D_{a1}}$$

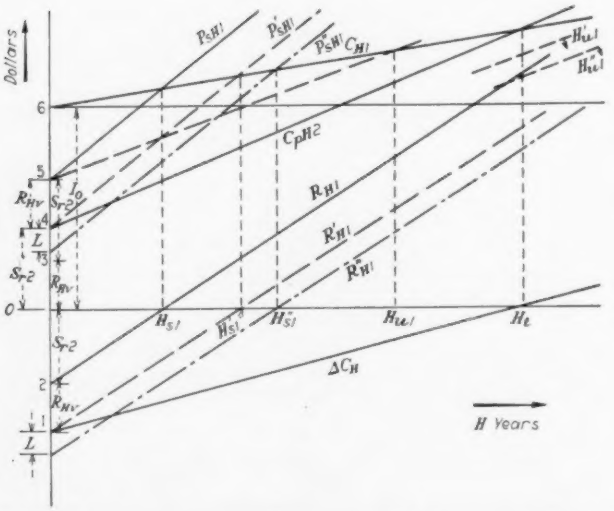

FIG. 6 INFLUENCE OF AN EXCESS OR DEFICIENCY IN TOTAL REVENUE OVER THE LIFE OF THE EXISTING EQUIPMENT UPON THE TOTAL RETURN ON INVESTMENT (R_{H2}) AND THE OBLIGATIONS (L) TO BE TRANSFERRED TO SOME PROPOSED TYPE OF EQUIPMENT

Similarly for the utility life,

$$H_{u1} = \frac{I_{o1} - S_{v2} - R_{Hv}}{D_{a1}}$$

$$H'_{u1} = \frac{I_{o1} - S_{v2}}{D_{a1}}$$

and

$$H''_{u1} = \frac{I_{o1} - S_{v2} + L}{D_{a1}}$$

It will be noticed in Fig. 6 that the effect of imposing these credits and debits on the new equipment is either to raise or depress the curve R_H representing the actual accumulation of excess revenue over all obligations available for a return on the investment. Thus when the functional rate r_{H1} for the new equipment is to be determined for any time H, it represents the composite situation for the entire serviceable life $H_v + H$ of both equipments combined.

EVALUATION OF THE TRUE ECONOMIC LIFE

At the beginning of this section on the relative earning power of alternative types of equipment it was stated that the particular type having the greatest overall economic utility would be that one which could achieve in the time $H = H_r$ an earning power, as measured by the functional rate of return r_{Hr1} at that moment, which would equal the functional rate r_{Hr2} that would have been attained by the original or existing equipment if continued in operation up to the same moment, $H_v + H_r$ years from its installation, with due consideration of the logical transfer of all unfulfilled obligations at the time H_v of actual replacement. Therefore it is important for all interested in replacement problems to know the exact time H_r when this situation occurs as it is the ultimate criterion, even superseding that of the true accounting life H_t. Because of this fact, the elapsed time H_r is designated as the *true economic life.* Accordingly, the basis for evaluating H_r is derived from the fact that, as seen in Fig. 5, the above criterion is satisfied only when the functional-rate curves for both types of equipment intersect. Nevertheless, before an equation can be established for the true economic life, a correction will have to be made in the general expression for R_{H1} to account for the transfer of obligations from the old to the new equipment. Hence if $+L$ equals all debit items (see L above) and $-L$ all credit items (see R_{Hv} above) exclusive of the salvage value S_{v2}, and the total accumulated revenue from the new equipment be corrected for these adjustments such that

$$P''_{SH1} = H(R_1 + D_{a1} + C'_{y1}) + (S_{v2} \pm L)$$

and

$$R''_{H1} = P''_{SH1} - C_{y1}$$

or

$$= H(R_1 + D_{a1}) - (I_{o1} - S_{v2} + L)$$

then

$$r''_{H1} = \frac{R_1 + D_{a1}}{I_{g1}} - \frac{I_{o1} - S_{v2} \pm L}{HI_{g1}}$$

or, disregarding L,

$$r'_{H1} = \frac{R_1 + D_{a1}}{I_{g1}} - \frac{I_{o1} - S_{v2}}{HI_{g1}}$$

Now if

$$r_{H2} = \frac{R_2 + D_{a2}}{I_{g2}} - \frac{I_{o2} + K_2}{(H_v + H)I_{g2}}$$

and $r_{Hr} = r_{H2} = r_{H1}$, for $H = H_r$,

$$H_r = \frac{I_{g1}(I_{o2} + K) - I_{g1}(I_{o1} - S_{v2}) - H_v I_{g2} I_{g1}}{2I_{g1}I_{g2}\Delta\alpha}$$

$$\pm \sqrt{\left[\frac{H_v I_{g1} I_{g2} - I_{g1}(I_{o2} + K_2) + I_{g2}(I_{o1} - S_{v2})}{2I_{g1} I_{g2}\,\Delta\alpha}\right]^2 - \frac{(I_{o1} - S_{v2})H_v}{I_{g1}\Delta\alpha}}$$

where

$$\Delta\alpha = \frac{R_2 + D_{a2}}{I_{g2}} - \frac{R_1 + D_{a1}}{I_{g1}}$$

$$= \Delta r_a + \Delta d$$

$$d = \frac{1}{H}$$

$H = H_t$ when depreciation is based on the true accounting life

H = H_a when depreciation is based on an arbitrary accounting life

r_a = rate of return on the gross capitalized investment in any single equipment as determined by the profit from the sale of products processed over it.

INFLUENCE OF OBSOLESCENCE UPON THE USEFUL LIFE OF EQUIPMENT

Since definite stages of economic utility for any type of equipment can be established in accordance with certain criteria, it now becomes possible to show what effect the occurrence of obsolescence within one of these stages will have on the utility with regard to the obligations and anticipated earning power which have yet to be fulfilled. The fact is that the hazard of impending obsolescence, whether it arises from design change or economic conditions, introduces an element of risk that obsolescence may occur at some time when all obligations incurred by the installation of new equipment cannot be completely fulfilled. If it occurs before the accounting life H_t has expired, the original capital outlay will be impaired, causing a loss in investment to the owners of the enterprise. If it occurs before the equipment has been able to earn an adequate return on the investment and thereby increase the earning power through realization of the actual economies of operation, the anticipated gain through a reduction in costs cannot be achieved, and on the basis of this obligation as well the equipment has not been able to fulfil its purpose. Furthermore, if depreciation is based upon an arbitrary accounting life H_a, the occurrence of obsolescence will again make it impossible for the machine to pay for itself in the appointed time. Accordingly, the intensity of the risk of impending obsolescence can be measured by the ratio of the time H_s, the critical life, and H_u, the utility life, at which the investment is earned back through depreciation and yields a satisfactory return, to any assumed time of obsolescence H_{ob}, at which all reasonable estimates of the ability to forecast show that obsolescence will in all probability occur.

$$\text{then } \rho_a = \frac{H_u - H_s}{H_u\left(1 - \dfrac{H_s}{H_{ob}}\right)}$$

The basis for constructing such a ratio measure becomes evident when we consider that each equipment passes through these various life-time stages, and that the chance of loss from obsolescence is diminished for two reasons: one being the capability of the equipment to attain H_s within a minimum of time, and the other its capability under the circumstances of so improving the functional rate of return between H_s and H_u that the time H_u comes as close as possible to H_s. Whenever time of impending obsolescence may occur, the ratios H_s/H_{ob} and H_s/H_u must become a minimum in order to avert the risk of obsolescence. If, as an example, $H_s = 1$, and if obsolescence is not likely to occur before ten years, the risk factor depends upon the time H_u. The factor ρ_a for H_s/H_{ob} = const., then increases when H_u comes nearer to H_{ob}, and decreases when H_u approaches H_s. The limiting values are (see Fig. 7)

$$\begin{array}{ll} H_u = H_s & \rho_a = 0 \\ H_u = H_{ob} & \rho_a = \dfrac{H_u - H_s}{H_{ob} - H_s} = 1 \\ H_s = H_{ob} & \\ & \rho_a = \infty \end{array}$$

As a result it becomes evident that any equipment, the design of which is susceptible to rapid changes or improvements, should have as short a critical life H_s as possible. Furthermore, since any equipment is also obligated to return over a period of years an appropriate profit on the investment, it is essential that the utility life H_u, at which point the return is measured by the normal anticipated rate of return r_a, should approach the time H_s as closely as possible. In fact, the new equipment should be able to attain a designated earning power at a time as much as possible in advance of the time when the existing equipment would achieve the same earning power. Thus the ratio ρ_{ob} provides a measure of the ability of any type of equipment to meet all of its direct and implied obligations before obsolescence occurs. It should be noted that this ratio is in no way a measure of design obsolescence itself, as such a factor[19] depends upon the degree to which additional entities of designers' and workers' skill have been transferred and embodied into the mechanism of the equipment.

SECONDARY CRITERIA FOR THE SELECTION AND REPLACEMENT OF EQUIPMENT

Now that definite time criteria have been established for the guidance of industrial executives in formulating an equipment policy when the specifications for two alternative types of equipment are known as well as for the tasks or services which the equipment is to perform, there are a number of other criteria, in some instances of equal importance, which can be derived from the major ones just developed.

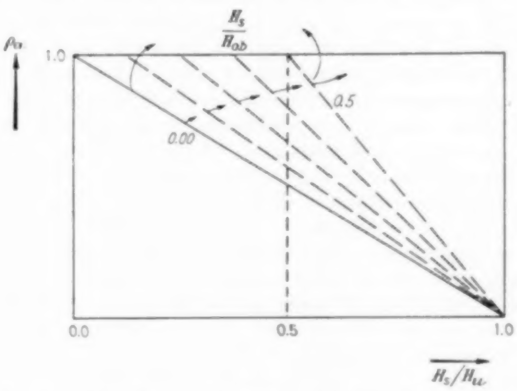

FIG. 7 CHART FOR COMPUTING THE RISK FACTOR (ρ_a) OF IMPENDING OBSOLESCENCE

1 *The Critical Sales Volume for the Economic Life.* Since the volume of business may fluctuate during the expected life of the equipment and if it should, as a result, not amount to the total volume which was anticipated as being necessary to provide the gross revenue from which adequate funds for meeting the obligations incurred by the adoption of the new equipment can be obtained, it is essential that some means of detecting any changes in the original economic situation shall be devised. Accordingly, if the total output required to meet all obligations be computed on the basis of the normal yearly capacity N_s, a figure can be obtained which, when compared with cumulative sales from year to year, will indicate whether the conditions underlying the true accounting life will be met. Hence the total economic or critical quantity to be produced during H_t years at a cost of C_{Ht} will be $S_{yHt} = N_s H_t$.

2 *Maximum Justifiable Investment.* In making the selection of equipment it is often important to know the maximum outlay of capital which the anticipated savings from operations or other sources will justify, because if there be any excess savings in favor of one particular type of equipment, a better type even

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[19] The development of such a factor is still in a preliminary stage. See Graduate Theses (1931) of C. A. Bartells and T. R. Wigglesworth, Course XV, Department of Business and Engineering Administration, Massachusetts Institute of Technology.

than that might still be justifiable provided the opportunity for its adoption could have been foreseen in some manner. Now, if after computing the value of H_t for what appears to be the most suitable equipment available not exceeding certain limits of capital outlay, it may be found that the time H_t could within reason be extended to an estimated life H_e without incurring any danger from obsolescence and thereby permit the purchase of a more expensive type which otherwise could not have been justified. Accordingly, since the excess savings ΔC_{He}, which could be obtained from the type of equipment first selected if continued in service up to H_e after all ordinary obligations have been satisfied, will be dependent upon the relation of H_e to H_t, and since any obligations incurred by an additional investment will have to be paid for out of these savings, the maximum justifiable investment is

$$(I_{g1} + I_{\text{incl}} + K - I_b)_{\max} = \frac{H_e}{H_t}(I_{g1} + I_{\text{incl}} + K - I_b)$$

3 *Minimum Permissible Resale Value.* On the other hand, if circumstances similar to those in the previous case exist, except that it is definitely decided to purchase a certain type of equipment for an investment of $I_{g1} + I_{\text{incl}}$ at some time, and if it be advisable to postpone the purchase for the moment provided that the same price and installation costs will hold in the future, the postponement will be justifiable for such time as the second-hand resale value of the equipment to be replaced remains above a certain fixed minimum. Of course the determination of this exact time is in all probability indeterminate unless there is a rather well-recognized resale curve for the specific type of equipment. Nevertheless, if a limiting minimum resale value can be determined, it can be used through a comparison with periodic second-hand price quotations to ascertain when the replacement should be made so that the resale value actually received will not be less than this predetermined minimum. Accordingly, on the assumption that $K = I_b - S_{r2}$ and that the excess savings ΔC_{He} can be earned in the estimated time H_e, the minimum resale value for the equipment to be discarded can be determined from the expression

$$(S_{r2})_{\min} = (I_1 + I_{\text{incl}})\left(\frac{H_e}{H_t} - 1\right) - \frac{H_e}{H_t}(I_b - K)$$

Furthermore, it might be mentioned that another equation could be developed, though possibly more complicated, to express the situation where part of the savings ΔC_{He} are employed to increase the justifiable investment and the other part to reduce the minimum permissible resale value at the same time.

4 *Minimum Productivity.* It often happens in establishing a policy of equipment replacement that a new machine can be obtained and operated at a considerable economic advantage, and yet there is the danger that over protracted periods it cannot be operated at capacity N or even at a normal annual rate of productivity $N_s = S_y$. However, if the equipment can pay for itself in a comparatively short time H_t, and if there is reason to believe that it will not become obsolete before some estimated time H_e, it may be advisable to find out what minimum rate of productivity X_m will be permissible in order to properly compensate for the cost of idle time so that the equipment can still pay for itself in a reasonable period. From an analysis of the total obligations C_{yH} for any period H, it becomes evident that the portion of these obligations which continue regardless of production, namely, those elements which drop out while $X = 0$, will constitute the cost of idle time (C_{LH}). Then the yearly cost when $X = X_m$ will be

$$C_{Ly} = \left(\frac{I_d}{H} + C_f'\right)(1 - X_m)$$

From this situation it is possible to develop an expression for the minimum productivity permissible for the proposed equipment on the basis of a comparison between its performance (X_m) under these conditions and that (X_s) for the existing equipment under standard conditions, whereupon

$$X_m = \left(\frac{C_p}{C_q + \beta}\right)^2 + (X_s C_q + \sqrt{X_s} C_p)$$

$$\pm \frac{C_p}{(C_q + \beta)}\sqrt{\left[\frac{C_p}{4(C_q + \beta)}\right]^2 + (X_s C_q + \sqrt{X_s} C_p)}$$

where

$$\beta = \frac{I_d}{\dfrac{H_t}{2} \pm \sqrt{\dfrac{H_t^2}{4} - \dfrac{I_d}{C_f'(C'_{ye} - C'_{yn})}}}$$

and the yearly obligations less depreciation under standard conditions are

C'_{ye} for the existing equipment

and

C'_{yn} for the proposed equipment

if H_t = true accounting life for the proposed equipment at normal productivity X_n.

Furthermore the time H_{tXm} which it will take to complete full depreciation at minimum productivity can be computed from the equation for H_t when X_m is substituted for X.

UNIT-COST COMPARISONS

In many instances the ultimate economic utility may have a greater significance to industrial executives when the circumstances which surround any equipment problem have been translated into terms of unit costs rather than retained in their original form as derived directly from a consideration of the relative overall economic advantages of alternative types of equipment over a period of H years. Accordingly, to permit this transformation the total annual operating equipment cost C_y must be reduced to the unit equipment cost c_x, where

$$c_x = \frac{C_y}{N_x} = \frac{u_f}{X} + u_q + u_n f'(X) + f'(H)\frac{u_t}{X}$$

For simplicity in the following analysis it will be permissible to neglect the term $u_n f'(X)$ as its value when reduced to a unit basis ordinarily becomes relatively insignificant, if N_x is reasonably large, and to assume that the cost elements in the term $f'(H)u_t/X$ are constant for any year and therefore can be combined with those belonging to the term u_f/X. As a result the following deductions can be made.

1 *Savings From Changes in Unit Cost Based on Equal Amounts of Service* (see Fig. 8).

Case A. Neither equipment is fully employed:

$$\Delta C_y = N_{xa}(c_{x2} - c_{x1}) = N_{xa}\Delta c_x$$

Case B. Present equipment is fully employed:

$$\Delta C_y = N_2(u_{\min 2} - c_{x1})$$

Case C. Suggested equipment is fully employed while present equipment is overloaded:

$$\Delta C_y = N_1(c_{x2} - u_{\min 1})$$

Case D. Capacity of suggested equipment can be fully utilized, while the required amount of output necessitates additions to the present equipment.

$$\Delta C_y = N_1(c_{x2a} - u_{\min 1})$$

No attempt has as yet been made to study the cost curves of additional equipment, and there remains as well the question regarding the influence of utilizing the equipment in excess of normal capacity. The unit cost curves in Fig. 8 are plotted on the assumption that the equipment remains idle for the remainder of the allotted time. Only when the equipment is adapted to other tasks can the savings per unit of service be computed on a minimum unit cost basis which neglects the unit cost of idleness.

2 *The Number of Pieces to Maintain Same Unit Cost* that must be obtained per year from the proposed equipment can be computed as follows (see Fig. 9):

$$N_{x2}c_{x2} = N_{x1}c_{x1}$$

$$N_{x1} = N_{x2}\frac{c_{x2}}{c_{x1}} \text{ and}$$

$$X_1 = \frac{N_{x2}c_{x2}}{N_1 \ c_{x1}}$$

For any required number of pieces N_{x2} there is a corresponding number of pieces (and an equivalent time) for the suggested equipment to operate at the same unit cost.

TABLE 1 TABLE FOR THE COLLECTION OF DATA—YEARLY COST C'_y

(All in dollars per year)

Cost items	Machine tools		Jigs and fixtures		Handling equipment	
	Present	New	Present	New	Present	New
F^s						
Taxes	x	x	x	x	x	x
Insurance	x	x	x	x	x	x
Rights, licenses	x	x	x	x	x	x
$(iI)_c$						
Interest	x	x	x	x	x	x
F_o						
Maintenance during idleness	x	x	x	x	x	x
XC_x						
Direct material (or semi-finished product)	x	x	..	..	..	..
Direct labor	x	x	x	x	x	x
Preparation	x	x	x	x	..	..
Equipment change-over cost	..	..	..	..	..	..
Tool cost (unless separately)	..	..	..	..	..	..
Production control unless in supervision	..	..	..	..	..	..
Indirect material, equipment supplies	x	x	x	x	x	x
Indirect labor	x	x	x	x	x	x
Inspection	x	x	x	x	..	..
Consumption charges for						
Power	x	x	..	..	x	x
Transportation	x	x	..	..	x	x
Light, heat, ventilation, unless in space charge	x	x	..	..	x	x
Jigs and fixtures cost, unless taken as addition to capital	x	x	..	..	..	..
Tool cost	x	x	x	x	..	..
Any other	..	..	..	..	..	..
F_B						
Charges direct to the order—departmental supplies, unassigned space, etc	x	x	..	..	..	..
General departmental charges	x	x	..	..	..	..
Service charges and service demand charges for						
Organization	x	x	..	..	..	..
Supervision	x	x	..	..	..	..
Power	x	x	..	..	x	x
Transportation and storage	x	x	..	..	..	..
Space (assigned space charge)	x	x	..	..	..	..

3 *Capacity of New Equipment Required to Obtain Equal Unit Cost at the Break-Even Point:*

$$N_b = \frac{u_{f1}N_1 - u_{f2}N_2}{\Delta u_q}$$

also

$$X_{b1} = \frac{N_b}{N_1}$$

This number of pieces is the critical or minimum number in order to obtain an economic advantage from the suggested equipment.

4 *Minimum Time* on proposed equipment to turn out the minimum number of pieces:

$$T_{\min} = \frac{N_b T_n}{N_1} = X_{\min} T_n \text{ (hours per year)}$$

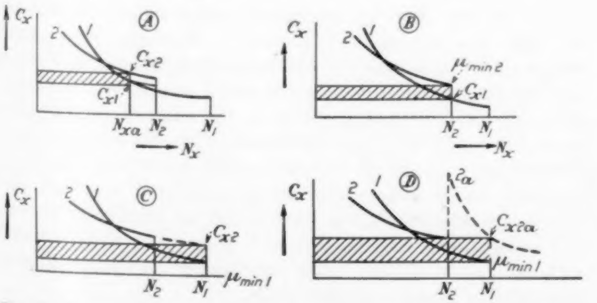

FIG. 8 SAVINGS FROM CHANGES IN UNIT COST DUE TO THE REPLACEMENT OF EQUIPMENT BASED ON EQUAL AMOUNTS OF SERVICE

where T_n equals the normal working hours per year at capacity.

5 *Yearly Cost of Idle Equipment.* If equipment is operated only a portion of the available time at a normal rate of operation but with resulting idleness during the rest of the year,

$$C_{is} = u_f(N_{x1} - N)$$

where N_x is the reduced output and N the normal amount thereof. This item, however, cannot be an absolute figure unless the unit cost of invariable charges (u_f) comprehends all original expenses.

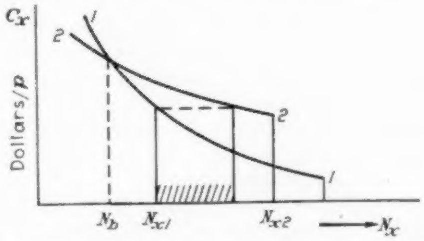

FIG. 9 RELATIONSHIP OF UNIT COSTS FOR THE DETERMINATION OF THE PRODUCTIVITY OF THE PROPOSED EQUIPMENT SUCH THAT UNIT COSTS ARE UNALTERED

6 *Technical Sales Arguments.* The information derived from the foregoing analysis and comparisons of relative operating proficiency for alternative types of equipment will be of value to the manufacturers of equipment because the technical advantages of each type can be presented in terms of the unit cost of the product which are familiar to the prospective purchaser as a user of equipment. Thus, a technique can be developed for use in the sale of any particular type from these relationships, by means of which a rapid comparison can be obtained between existing and proposed types, on a standard basis of comparison, without involving any of the intricacies of the users' accounting system or discrepancies which might exist between two dissimilar methods.

APPLICATION OF FORMULAS TO VARIOUS TYPES OF EQUIPMENT PROBLEMS

Much emphasis has been laid throughout the foregoing discussion upon the necessity of knowing what factors required by a general solution of the problem are essential in the determination of a suitable equipment policy for some specific type. Naturally when any of the formulas evolved in this study are to be applied to machine tools or process equipment, jigs and fixtures, and labor-saving devices or material-handling equipment, specific attention must be paid to the origin and behavior of the cost elements and financial obligations incurred in each case, as no two situations will require identically the same elements. For this reason much space has been devoted to a consideration of the nature of all obligations and costs so that their relation to time and quantity could be thoroughly understood. Furthermore, it is essential that the distribution of overhead be carefully analyzed so as to separate that portion which may be affected by changes in equipment from that which is invariable with such changes. It was pointed out that there is considerable danger in the treatment of these fixed overhead items if their basis of distribution be altered by a change, as in no case is it permissible to show a reduction in the total burden placed on any equipment when, for instance, the new equipment can achieve a reduction in operating time but when the total annual burden of this sort for the specific equipment remains unaltered by the change. The point is that the difference between a reduction in such burden per unit of product and the invariable total annual burden placed upon the equipment must be recognized. Accordingly, as a result of a preliminary survey of a number of cooperating industrial enterprises, it has been shown that the elements which are indicated by a cross (x) in Table 1, must be considered when formulating an equipment policy for any of the general types specified at the top of the columns.

Discussion

Harold V. Coes.[20] It would be very illuminating if the authors were to secure the necessary basic information from some well-established company or companies, and apply the data to their formulas by comparing what actually was done by a given company in its replacement of equipment over a period of years, as compared with what might have been done under a more scientific basis as developed by the authors, and then compare what might have been done with the business cycles that prevailed at that time.

In other words, the writer is anxious to see whether it is possible to so utilize this scientific selection of equipment as to be able to place orders for equipment in off years or on the low side of the business cycle. He is thinking of long-range planning and the application of the process of scientific selection of equipment to long-range planning and the adaptation of these plans to business cycles. We have just opened the door to this type of thinking and planning in the papers that have been presented.

W. H. Rastall.[21] The writer would like to see the committee, in connection with its further work, go into some of the collateral phases of the problems it has been attacking.

In various parts of the report the authors use the expression "depreciation policy." It is suggested that they change this and attack the problem from the point of view of a broad "equipment policy."

American industry appears to be very much in need of an adequate equipment policy which, obviously, should cover depreciation, as has so long been discussed. It also should cover obsolescence, which is a somewhat newer phase of the old problem.

As yet the committee in its reports does not appear to have taken up the matter of calculating these factors on the basis of "replacement value," a point which was emphasized by Mr. Du Brul, and which the writer endeavored to second, because from our privileged position here we can see quite clearly that, in certain foreign countries particularly, this matter seems to be overlooked, with disastrous consequences. For example, quite recently Britain has abandoned gold and taken up sterling, yet British manufacturers still seem to be quoting the same old sterling prices, which is another way of saying that overhead costs have not been increased. Unconsciously they are giving their products away at less than cost. When a similar situation developed in Germany some years ago and the value of the mark collapsed, manufacturers there made the same mistake, and prominent Germans will today confess that they are not at all proud of the volume of export business Germany secured during those months. American manufacturers drifted into the same error during the period 1914 to 1920, when the purchasing value of the dollar was practically cut in two, and any manufacturer figuring his overhead costs simply in dollars unconsciously fritters away the assets of his enterprise.

It is true that certain able business men contend that we should not calculate these items on the basis of replacement values, for they contend that by using the present value of a 20-in. by 10-ft. lathe where the actual machine they might be employing was purchased, say in 1915, we are really using an entirely different thing as a basis for our calculations because of the progress in machine-tool design during the interval; that is to say, we are setting up a standard that is definitely higher than the one from which we started. Undoubtedly there is a certain validity in this argument, and the extreme case could probably be found in the experience of the telephone company, which finds it necessary to attract millions or hundred of millions of new capital annually in order to keep equipment down to date and to render the best possible service.

However, the typical small enterprise finds itself in a very different position from the telephone company, and if it fails to cover in its costs and accounting methods allowances that will enable it to maintain its factory equipment in an up-to-date condition, the enterprise will probably drift from a position of leadership into one of secondary standing. For this reason it is the writer's personal impression that an adequate equipment policy not only should require that depreciation and obsolescence be calculated on replacement values, but also should allow for the progress of the art in order that a position of leadership may be maintained.

An adequate study of equipment policy should also have to do with the second-hand problem. Obviously management suffers a great temptation in this connection and wishes to secure the highest possible price for discarded equipment. However, it is quite easy by such methods to encourage the development of an undesirable form of competition which serves to demoralize the industry. A recent experience that came to my attention covers a transaction where an inexperienced workman purchased second-hand knitting machinery at practically no cost, and consequently felt he was in a position to make sweaters without any overhead costs whatsoever—obviously a most demoralizing influence in that trade.

As is well known, the British textile industry is today suffering seriously from the same thing because of the export of second-hand textile machinery from Lancashire to those who compete in the markets of India and China. An adequate equipment policy would also seem to need to include allowances for the influence of the business cycle. Experience shows that the equipment industries suffer more from these fluctuations than do industries in general. The chart (see Fig. 10) illustrates this point. When industrial production as a whole rose, say, 20 per cent, in 1919 and 1920, machine-tool orders rose 200 per cent, and a somewhat similar condition existed in 1928 and 1929. Conversely, following collapse, there is a paralysis of buying, and in 1921 and 1931 it simply was not possible to sell machine tools. The conditions in 1919 and 1929 can only be described as equipment anarchy, a buying orgy, and conditions in 1921 and 1931 as a paralysis of buying, representing the absence of any adequate policy. At any rate, there was no continuity of policy, but merely a drift into a situation, but which resulted in the creation of a condition that greatly aggravated the distress resulting from unemployment. Certainly it would be difficult to justify the purchase of factory equipment under conditions existing as they did in 1919 and 1929, and if we are to claim a continuity of policy, it would be difficult to justify the paralysis of 1921 and 1931.

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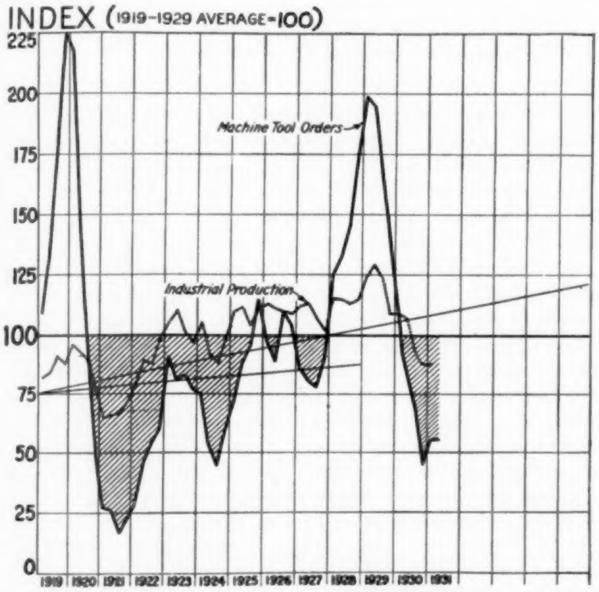


Fig. 10 Post-War Fluctuations of Machine Tools Compared With General Industry

All of the foregoing is based on the assumption that the committee will find it possible to continue its work which is indicated in the reports submitted at New York, and also on the assumption that they are in a position to broaden their discussion and include these aspects of an equipment policy, in addition to the previous discussion, which seems to have been confined to the factors related to depreciation and obsolescence in their simpler forms.

Authors' Closure

In the light of the foregoing discussions, the authors wish to emphasize that, in undertaking a research project which deals with a subject of so large a scope, they have recognized the magnitude of their problem and that they are not merely confining it to a consideration of depreciation or any other single factor, but are in fact taking up in sequence each phase of the problem in order to develop fundamentally sound principles upon which adequate "equipment policies" for industry can be formulated. There has been much confusion of thought over depreciation and amortization of the investment in plant and equipment which must be explained and eliminated before the problem of formulating true equipment policies can be reliably approached. Furthermore, the authors thoroughly appreciate that obsolescence is a major factor in all equipment policies, and it was their purpose in the presentation of this subject to show that whatever the accounting or economic life of some equipment may be according to the conditions under which the equipment is used, no proposed equipment could be justified if obsolescence should occur in the extreme case before the true accounting life had expired or in the normal case before the true economic life had expired. The fact is that the serviceable life based on the occurrence of obsolescence and the useful life based upon the economic utility of any type of equipment are inseparable and mutually dependent.

Similarly, the authors anticipate to extend their present work to include a study of resale values and second-hand markets, replacement values, the influence of steadily increasing cost of repairs with the passage of time; in fact, every risk of doing business which may arise from the use of equipment by industry. On the basis of such facts gained from an intimate knowledge of the equipment problems, not only of the user of equipment, but also of the manufacturer of and dealer in equipment, it is hoped that it will be possible to reveal the correct principles and methods of determining an equipment policy for industry. To accomplish this result, the authors wish to reiterate their desire that industry and particularly all members of this Society will cooperate with them in carrying forward this research project to a successful conclusion by presenting to them every equipment problem with which each executive or industry is faced.

RP-54-3

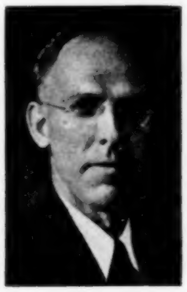
A Study of Primary Metering Elements in 3-In. Pipe

By S. R. BEITLER,[1] PAUL BUCHER,[2] and T. C. BARNES, JR.,[3] COLUMBUS, OHIO

This paper presents part of the results of a study of the performance of a complete series of primary metering devices in a 3-in. pipe line. The devices tested were as follows: Three types of orifices (concentric, eccentric, and segmental) with diameter ratios varying from 20 to 90 per cent; flow nozzles with diameter ratios varying from 30 to 88 per cent; also two venturi tubes with three throat sections each, with diameter ratios varying from 33 to 50 per cent. Discharge coefficients are given for these elements, using water as the metered fluid, with a wide range of differential heads. Coefficients are given also on some of the elements where superheated steam at various temperatures was the metered fluid. The effect of sharpness of orifice edge, of roughness of approach pipe, and of surface finish of flow nozzle on water coefficients is shown. The effect of temperature upon the diameter of the primary device and upon the coefficient of discharge for steam is also given.

THE work covered by this paper represents the continuation of the studies on the flow of fluids past metering elements in pipes, the results of which were presented in a paper by the authors at the 1929 Annual Meeting and published later in the Transactions of the Hydraulic Division.

S. R. Beitler



Paul Bucher

T. C. Barnes, Jr.

This first paper covered the work done on metering elements in a 6-in. pipe. Since the relatively small size of the condensing equipment made it impossible to secure high velocities in the pipes, it was decided to carry on the investigation in a 3-in. pipe line. With this line it was possible to get flows far beyond the values used at present in commercial practice and to cover more completely the range in use at present.

This work has been carried out as a project of the Engineering Experiment Station of the Ohio State University in cooperation with the Bailey Meter Company and has been financed jointly by the two organizations. The actual taking of the data on 3-in. pipe was started late in the summer of 1929 and continued until about the first of April, 1931. During the early part of the work R. L. Galley, Junior Engineer of the Bailey Meter Company, was in charge of the taking of data, while after December, 1930, Mr. Barnes took over this work and carried it to completion. All metering elements used, excepting the venturi tubes, were furnished by the Bailey Meter Company. The venturi tubes were furnished by the Builders Iron Foundry.

The authors wish to express their appreciation for the services of Mr. Galley, the help and advice of R. E. Sprenkle and J. C. Albright of the Bailey Meter Company, and of Profs. F. W. Marquis and Horace Judd of the Department of Mechanical Engineering of the Ohio State University, also to Ed. S. Smith, Jr., of the Builders Iron Foundry, for his cooperation, and to Dean E. A. Hitchcock and J. M. Weed of the Engineering Experiment Station of the university, and R. S. Coffin of the Bailey Meter Company, whose cooperation made it possible to finance this work.

Scope of the Work

When the work was first planned, it was decided to attempt to make it cover as far as possible all the usual metering elements and conditions that might be obtained in practice. This was carried out as far as possible. Data were taken with orifices, flow nozzles, and venturi tubes, using steam and water, and within the limits of the apparatus everything was varied which might influence the flow and which might vary in practice.

The metering elements tested were as follows:

(1) Three sets of concentric orifices with the ratio of orifice diameter to pipe diameter ranging from 20 to 80 per cent.

(2) One set of eccentric orifices with diameter ratios varying from 70 to 88 per cent.

(3) One set of segmental orifices with equivalent diameter ratios varying from 70 to 90 per cent.

(4) One set of steel flow nozzles, cadmium plated, with diameter ratios varying from 65 to 88 per cent.

(5) One set of bronze flow nozzles with diameter ratios of from 30 to 73 per cent.

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[1] Assistant Professor in Mechanical Engineering, Ohio State University. Jun. A.S.M.E. Mr. Beitler received the degree of Bachelor of Mechanical Engineering at Ohio State in 1920. In 1920–1921 he served as apprentice engineer with the Whiting Corporation of Harvey, Ill. From 1921 he has been at Ohio State University as Assistant and Instructor in Mechanical Engineering. From February, 1927, to date he has been Consulting Research Engineer for the Bailey Meter Company, Cleveland, Ohio.

[2] Associate Professor of Steam Engineering, Ohio State University. Assoc-Mem. A.S.M.E. Mr. Bucher received the B.M.E. degree from Ohio State in 1918. He spent three months as assistant superintendent of power for the Kimberley-Clarke Paper Company, Neenah, Wis. From 1918 he has served as Instructor and Associate Professor at Ohio State. He has also, during vacation periods, served as test engineer for the Appalachian Power Company at Glen Lyn, Va., and from February, 1927, to date has served as Consulting Research Engineer for the Bailey Meter Company, Cleveland, Ohio.

[3] With S. R. Beitler, Ohio State University. Jun. A.S.M.E. Mr. Barnes received the M.E. degree in 1929 from Rensselaer Polytechnic Institute. He has been employed since 1929 as Junior Research Engineer by the Bailey Meter Company, Cleveland, Ohio, in experimental work on metering elements and in general engineering work on fluid meters.

Contributed by the A.S.M.E. Special Research Committee on Fluid Meters and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

(6) One set of steel flow nozzles unplated with diameter ratios varying from 35 to 88 per cent.

(7) Two venturi tubes, with three throats each, with diameter ratios varying from 35 to 50 per cent.

(8) In the foregoing were several sets of three or four "similar" orifices of the same diameter ratio.

All were tested at various differential heads, and the coefficient of discharge was determined, using water as the calibrating fluid. Tests were made with differential heads as low as 0.05 ft. of water and as high as 49 ft. of water.

They were also tested using steam as the calibrating fluid. They were tested at two upstream pressures, 85 lb. per sq. in. absolute and at 20 lb. per sq. in. absolute. Each element was tested at various pressure drops at each pressure condition.

Certain elements were tested also at varying steam temperatures.

Tests were made also to determine the effect of sharpness of the edge of an orifice and of the roughness of the upstream pipe and of the effect of roughness on the inner surface of a nozzle.

All of these results have not yet been completely analyzed, and so in this paper it will be possible to present and discuss only a few of the results. These will be the values of the coefficients for water, the effects of orifice edge, pipe roughness, nozzle roughness, and the effect of temperature on coefficient and element size.

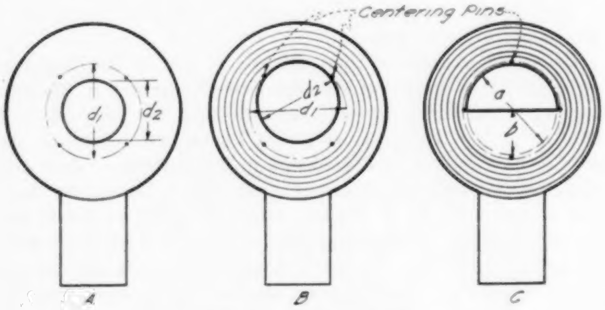
Centering Pins
d_1
d_2
d_2
d_1
a
b
A
B
C

FIG. 1 ORIFICE PLATES
(*A*, concentric orifice; *B*, eccentric orifice; *C*, segmental orifice.)

APPARATUS USED IN THE TESTS

Fig. 1 shows the types of orifices used in this work. In this figure *A* is a concentric orifice, *B* an eccentric orifice, and *C* a segmental orifice. These orifice plates were, with only one exception, made of Monel metal plate. The eccentric and seg-

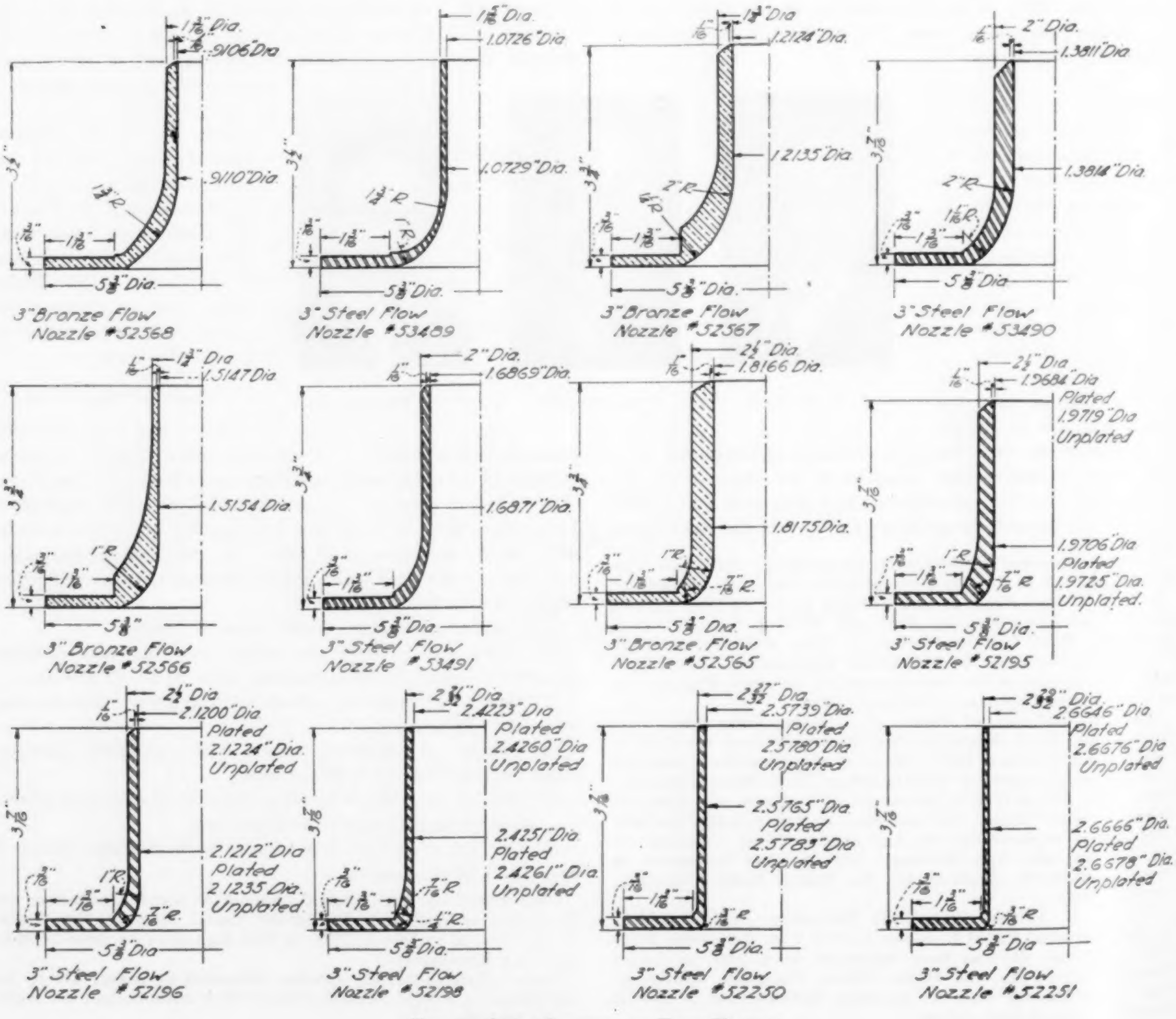
1 3/16" Dia.
.9106" Dia
.9110" Dia.
1 3/4" R
3 1/4"
3/16"
1 3/16"
5 3/8" Dia.
3" Bronze Flow Nozzle #52568
1 5/16" Dia.
1.0726" Dia
1.0729" Dia.
1 3/4" R
1" R
3 1/4"
5 3/8" Dia.
3" Steel Flow Nozzle #53489
1 3/4" Dia
1.2124" Dia.
1.2135" Dia.
2" R
1 1/16" R
3 1/4"
5 3/8" Dia.
3" Bronze Flow Nozzle #52567
2" Dia.
1.3811" Dia
1.3814" Dia.
2" R
1 1/16" R
3 7/16"
5 3/8" Dia
3" Steel Flow Nozzle #53490
1 3/4" Dia
1.5147 Dia
1.5154 Dia.
1" R
3 1/4"
5 3/8"
3" Bronze Flow Nozzle #52566
2" Dia.
1.6869" Dia.
1.6871" Dia.
3 1/4"
5 3/8" Dia.
3" Steel Flow Nozzle #53491
2 1/2" Dia.
1.8166 Dia.
1.8175 Dia.
1" R
7/16" R
3 1/4"
5 3/8" Dia.
3" Bronze Flow Nozzle #52565
2 1/2" Dia
1.9684" Dia Plated
1.9719" Dia Unplated
1.9706" Dia Plated
1.9725" Dia. Unplated.
1" R
7/16" R
3 7/16"
5 3/8" Dia.
3" Steel Flow Nozzle #52195
2 1/2" Dia
2.1200" Dia Plated
2.1224" Dia. Unplated
2.1212" Dia Plated
2.1235 Dia. Unplated.
1" R
7/16" R
3 7/16"
5 3/8" Dia
3" Steel Flow Nozzle #52196
2 21/32" Dia
2.4223" Dia Plated
2.4260" Dia Unplated
2.4251" Dia Plated
2.4261" Dia. Unplated
7/16" R
1/4" R
3 7/16"
5 3/8" Dia.
3" Steel Flow Nozzle #52198
2 27/32" Dia
2.5739" Dia. Plated
2.5780" Dia Unplated
2.5765" Dia Plated
2.5783" Dia Unplated
3/16" R
3 7/16"
5 3/8" Dia.
3" Steel Flow Nozzle #52250
2 29/32" Dia
2.6646" Dia Plated
2.6676" Dia Unplated
2.6666" Dia. Plated
2.6678" Dia. Unplated
3/16" R
3 7/16"
5 3/8" Dia
3" Steel Flow Nozzle #52251

FIG. 2 CROSS-SECTIONS OF FLOW NOZZLES

mental orifices were made of plate 0.038 in. thick. This plate was corrugated with five concentric corrugations around the outside of the plate. The purpose of these corrugations was to enable the plate to be used without, or rather in place of, a gasket between two flanges. In all there were seven segmental and six eccentric orifices.

There were three series of concentric orifices used. The first series was made up of nine orifices of two types of construction. The first four orifices of diameter ratios 50 per cent and below (diameter of 50 per cent = 1.515 in.) were of $^1/_{16}$-in. plate with the edge of the orifice on the downstream sides beveled so that the thickness was $^1/_{32}$ in. at the edge. The other five orifices of 50 per cent diameter ratios and above were made of the 0.038-in. plate corrugated, as were the eccentric and segmental plates.

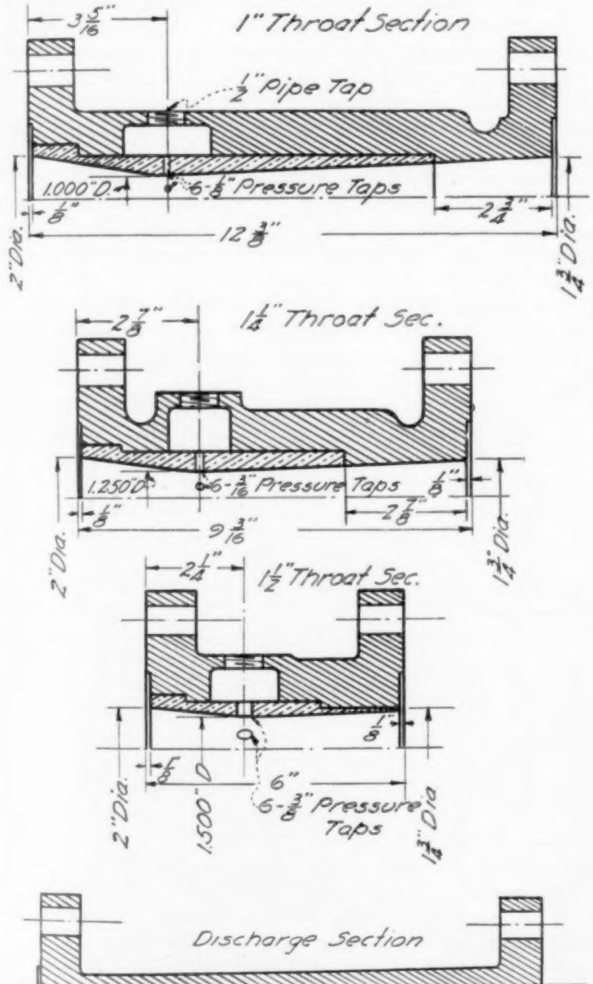
3 5/16"
1" Throat Section
1/2" Pipe Tap
1.000" D.
6-1/8" Pressure Taps
1/8"
2 3/4"
12 3/8"
2" Dia.
1 3/4" Dia.
2 7/8"
1 1/4" Throat Sec.
1.250" D.
6-3/16" Pressure Taps
1/8"
2 7/8"
9 3/16"
2" Dia.
1 3/4" Dia.
2 1/4"
1 1/2" Throat Sec.
1/8"
6"
6-3/8" Pressure Taps
2" Dia.
1.500" D
1 3/4" Dia
Discharge Section
1/8"
14"
1 3/4" Dia
3" Dia.
Inlet Section
1/2" Pipe Tap
3 7/16"
6-1/4" Pressure Taps
1/8"
8 5/8"
3" Dia
2" Dia.

FIG. 3 CROSS-SECTIONS OF VENTURI-TUBE APPROACH AND THROAT

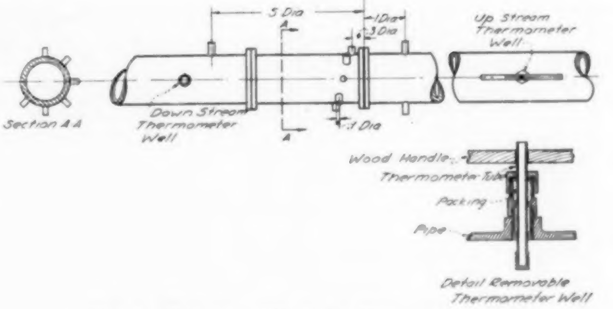
5 Dia
1 Dia
3 Dia
A
Up Stream Thermometer Well
Section A A
Down Stream Thermometer Well
3 Dia
A
Wood Handle
Thermometer Tube
Packing
Pipe
Detail Removable Thermometer Well

FIG. 4 DETAIL OF PIPE AT ORIFICE FLANGE

The second series was quite similar to the first except that all plates were of $^1/_{16}$-in. metal with the orifice edges beveled to $^1/_{32}$ in. There were six orifices in this series. All of the orifices in these two series were made by fastening the plate on a face plate and boring the orifice hole with an ordinary lathe tool and then polishing with some very fine emery cloth.

The third series was made of $^1/_{16}$-in. plate, as was the second, with the edges beveled, but the holes instead of being just bored, were bored and then honed to give a smooth finish and a smooth, sharp edge.

One concentric orifice plate was made of sheet brass, $^1/_{16}$ in. thick and beveled as were the others. This orifice was bored, but not honed.

All orifices were centered in the pipe by means of four pins riveted into the plate. A stamped "ear" showing the orifice number and other data was fastened to each orifice plate on the edge of the plate which was to be placed toward the pressure taps.[4]

Fig. 2 shows longitudinal sections of the various nozzles tested. These nozzles were made of steel, fabricated by flanging and welding, or were bronze castings. They were carefully bored to size and were finished very smooth. When first tested, the first five nozzles with diameter ratios above 65 per cent were cadmium plated. This plating was later removed by heating, and all nozzles were used with a smooth surface. All nozzles were carefully measured and inspected in order to have the throat section as nearly as possible a true cylinder. No nozzles were used which were out of round more than 0.0032 in. nor which had any divergence in the throat section from entry to exit.

Fig. 3 shows the general shape of the venturi tubes tested. These are approximate only, as no attempt was made to determine accurately the curvature at the throat and entry of the tube. There were two similar sets of tubes. They were cast-iron tubes with bronze inserts, as shown in the figure.

The orifices and nozzles were mounted between the flanges of two special sections of pipe which are shown in detail in Fig. 4. The upstream section consisted of a piece of seamless-steel tube $3^1/_2$ in. outside diameter and 3 ft. 6 in. long. Standard flanges were screwed on this tube, pressure connections and thermometer wells were welded in place, and then the tube was bored to an inside diameter of 3.027 in., which was the diameter of the pipe preceding the special sections. The tube was then cadmium plated to resist corrosion. It was thought this would keep the upstream pipe section the same at all times and thus eliminate

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[4] NOTE: There were 13 tables accompanying the paper, which may be consulted by any one so desiring who may wish to obtain data without working directly from the curves.

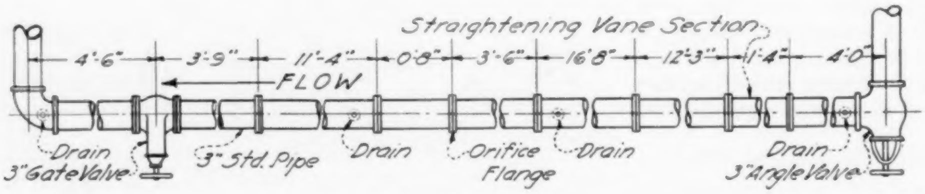

Fig. 5 Layout of Test Line

rusting or corrosion which might affect the orifice flow. However, the high-temperature steam used melted the cadmium, and for the last set of tests the pipe was not protected. This loss of plating, however, made no appreciable change in the diameter of the pipe.

The pressure taps used are shown in detail in Fig. 9. In this pipe two were used, both at a distance of one pipe diameter upstream from the orifice face—one for a determination of the differential head and one for the static pressure in the line. The thermometer well used in determining steam temperature was placed in the pipe at five pipe diameters upstream. This well could be withdrawn from the pipe, leaving no disturbance to flow when taking readings of differential head. The well used was the standard A.S.M.E. finned-wall type. It was in most cases filled with mercury to assure rapid heat transfer to the thermometer bulb. For some of the tests, however, cylinder oil was used as a fluid while for the highest temperatures solder was used. The thermometer used in determining the temperature of the water flowing through the line was located in a well five pipe diameters downstream.

The downstream section of pipe directly below the orifice flange was made of a piece of 3-in. extra-heavy pipe 9 in. long. This also had welded pressure connections and screwed flanges and was bored to 3.027 in. inside and cadmium plated to match the upstream section. The pressure taps were placed at each 0.1 pipe diameter from 0.3 to 0.8 diameter downstream from the orifice face. These connections were placed around the pipe at angles of 45 deg. to each other.

The flanges of the two special sections of pipe were then drilled with eight holes, the two sections lined up on a mandrel, the various pressure taps lined up along the pipe, and the flanges doweled. With this arrangement it was possible always to have the two pieces of pipe in line and also to have any downstream pressure taps lined up with the upstream taps. When the venturi tubes were being tested, the entry section was bolted to the flange of the upstream section of pipe.

The location and general arrangement of these sections is shown in Fig. 5. This arrangement gave approximately 32 ft. 5 in., or 130 diameters of straight pipe upstream from the orifice. In the 16 in. preceding this straight section there were straightening vanes consisting of pieces of $^3/_8$-in. pipe 14-in. long welded into the 16-in. section of 3-in. pipe.

A pressure tap and thermometer well were located at a point five pipe diameters below the orifice in the discharge pipe. These were for the purpose of measuring the temperature drop and restoration of pressure. The thermometer well was placed about one diameter below the pressure connection.

When water was being used as the calibrating fluid, the upstream flange in the section of pipe containing the straightening vanes was bolted directly to a 3-in. elbow, and this was connected through a valve to the laboratory service line. Water for calibration purposes was supplied by means of a 6-in. centrifugal pump having a capacity of about 1500 g.p.m. at 100 lb. per sq. in. discharge pressure. This pump was driven by a steam turbine, the speed of which was controlled by means of a fairly sensitive governor which gave a relatively constant pressure for any capacity, so that for any test run the flow was fairly constant during the whole run. The quantity of water flowing was controlled by the discharge valve of the pump.

When steam was used, the inlet flange of the first section of pipe was connected to a 3-in. angle valve through a 4-ft. length of pipe, giving the flow a chance to straighten out somewhat before striking the straightening vanes. This valve was connected to the laboratory superheated-steam header. Steam was supplied to this header at a pressure of 140 to 150 lb. per sq. in. gage and with quality varying from about 98 per cent dry to about 100 deg. of superheat. The steam was piped through a gas-fired superheater and then into the header. There was one pressure-reducing valve between the boilers and this line. This arrangement made it possible to secure a fairly constant pressure and temperature at the metering element for any one run, and to vary either the pressure or temperature within the limits of the apparatus.

For steam, the entire pipe line with the exception of the flanges and the section downstream from the measuring section was lagged with 85 per cent magnesia pipe covering of standard thickness.

The quantity of water flowing was determined either by weighing or by volumetric measurement. For weighing, the water was piped through a deflector pipe into either of two weighing tanks on platform scales. These tanks were of about 1200 lb. capacity and were fitted with quick-opening valves for dumping. This method was used for measuring quantities up to about 0.2 cu. ft. per sec.

For quantities above 0.2 cu. ft. per sec., the volumetric method of measurement was used. For measurement, one of the cisterns or bays of the laboratory was used. These bays are concrete tanks set below the floor of the laboratory. They are connected to $^3/_4$-in. gage glasses in a central measuring pit in the center of the laboratory by 2-in. pipes, so that the level of the water in the bay can be read on a scale placed behind the gage glass. Thermometers were placed in these gage glasses at about 4-ft. intervals in order to correct for difference in density between the water in the bay and that in the glass. The volume present in the bay for any gage reading was determined by calibration, and was read from a calibration chart.

When the measuring bays were used, the discharge from the test line was piped to a point directly over the wall between two of the bays and a suitable deflector pipe arranged to enable the water to be discharged into either bay. One bay could then be used as a spill and supply bay and the other as a measuring bay.

The pressure in the pipe was in all cases regulated by means of a 3-in. valve placed near the discharge end of the test line.

When steam was being used as the calibrating fluid, the quantity was determined by condensing the exhaust and weighing the condensate. To do this the exhaust line was piped to a surface condenser. This pipe was valved to control the pressure in the test line. The condenser used was an Admiralty type of surface condenser with a cooling surface of approximately 465 sq. ft. It was fitted with a wet vacuum pump and a circulating pump of the reciprocating type driven by a central steam cylinder. The cooling water was obtained from an outdoor reservoir and was returned to the reservoir for cooling. The water from the condensate pump was discharged into an open overhead tank for the elimination of air. This tank was fitted with a gage glass, to show the water level in the tank

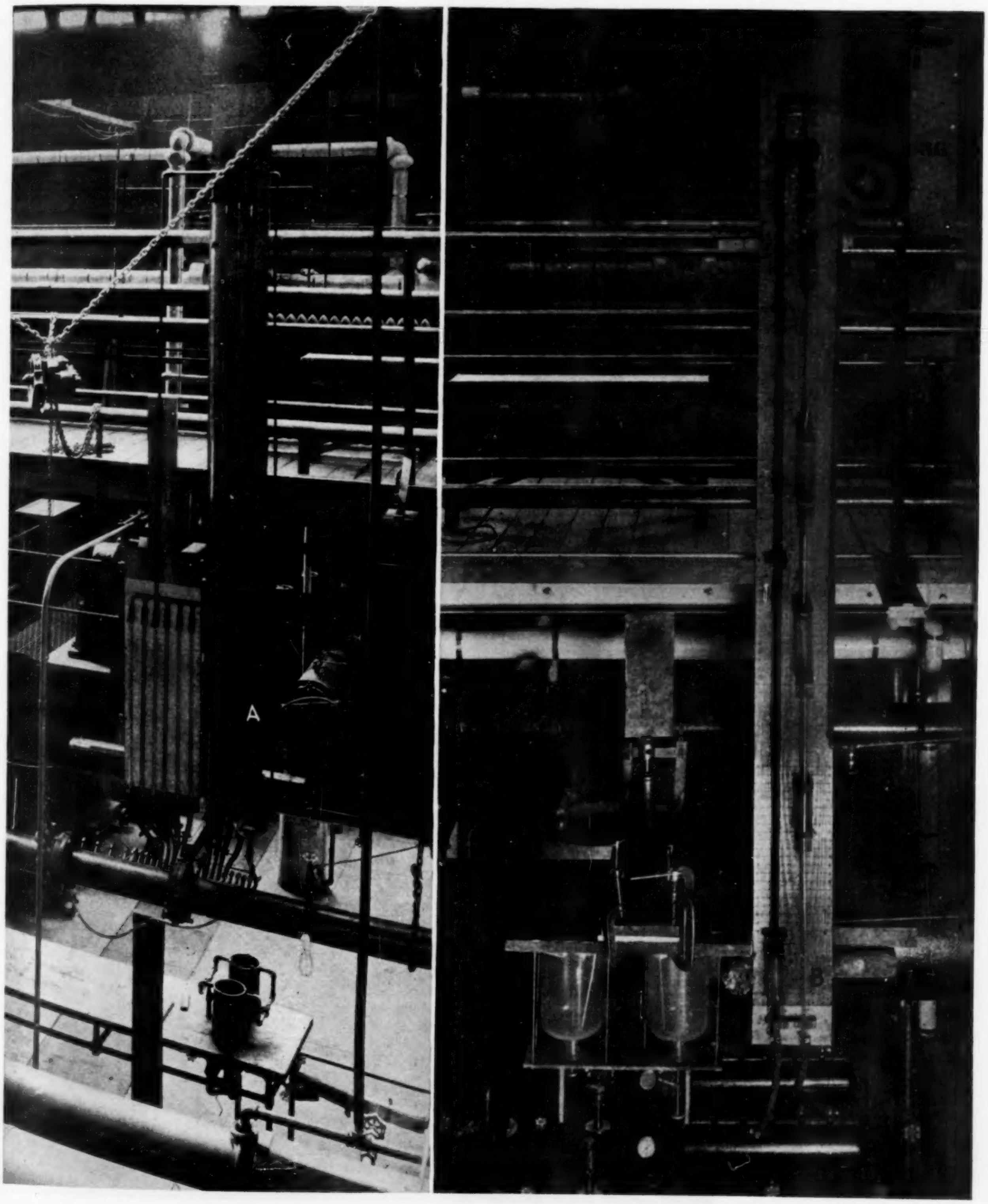

Fig. 6 Manometers for Water

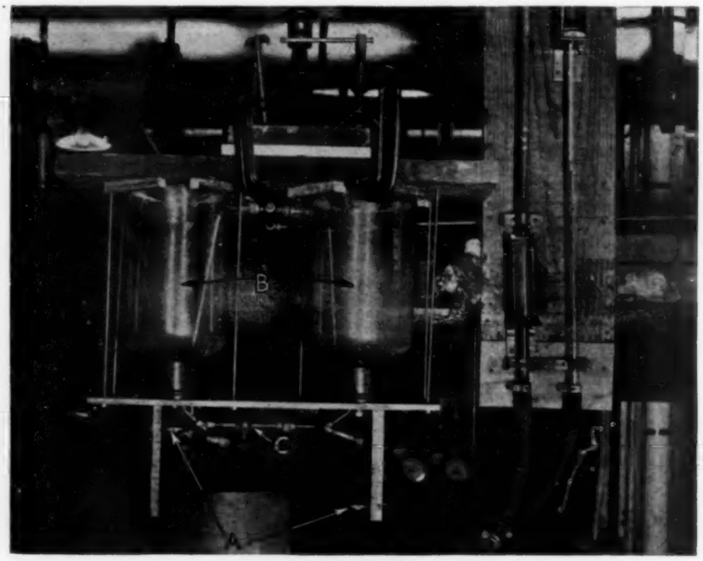

FIG. 7 DIFFERENTIAL HOOK GAGE

and the condensate was carried away through a pipe tapped into the side about 1 ft. from the bottom of the tank. The condensate then flowed by gravity through a suitable deflector pipe to one of two tanks of about 1200 lb. capacity set on calibrated platform scales.

For the water tests, two sets of manometers were used for measuring the differential pressure across the metering element. These manometers are shown in Figs. 6 and 7. The one first used was similar to the one shown at *A* in Fig. 6. This was a multiple inverted water manometer. It was made of one 10 ft. length of $^5/_8$-in. o.d. glass tubing connected to the upstream pressure tap and six $2^1/_2$-ft. lengths connected to the downstream pressure taps. The tops of the short tubes were manifolded and connected to the top of the long tube with $^3/_8$-in. pipe. A $^1/_4$-in. air line was piped into this line at the top of the long tube for supplying air under sufficient pressure to balance the water pressure in the manometer. The valves in this line were located at a convenient height and were arranged so that the air pressure could be varied as desired, and thus the level of the water in the two sets of tubes could be set at any convenient position for observation. This manometer was used for measuring pressure differences of from 1 to 10 ft. of water.

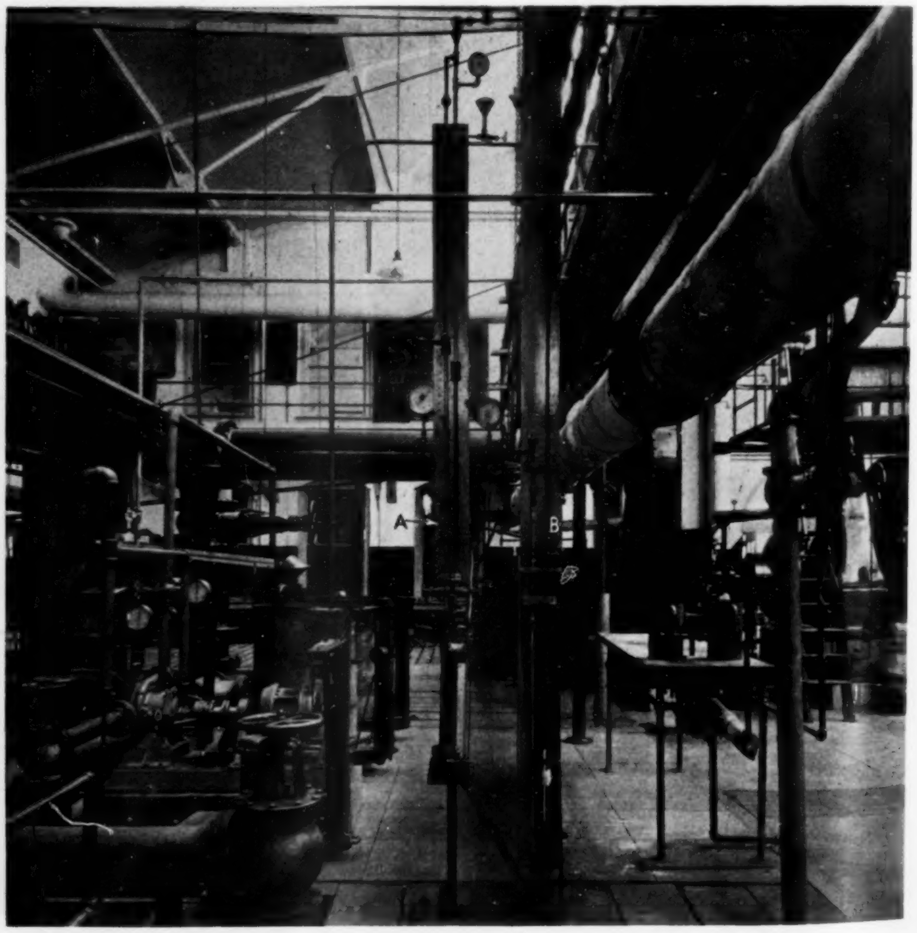

FIG. 8 MANOMETERS USED FOR DIFFERENTIAL AND STATIC PRESSURES FOR STEAM

For the later tests the manometer shown at *B* in Fig. 6 was substituted for the one at *A*. The difference in the two was the addition of the 2-in.-diameter glass tubes as shown at the zero level on the downstream side and at the 2-ft., 4-ft., 6-ft., and 9-ft. levels on the upstream tubes. The purpose of these volumes was to absorb the small pulsations in manometer readings and so make throttling unnecessary.

The scales for both of these manometers were laid off directly on the board behind the tubes, and the boards were carefully shellacked to protect them from moisture. They were graduated in feet and hundredths, and the zeros were carefully leveled. Thermometers were placed in the tubes of both manometers to determine the density of the water in the tubes. These were arranged at heights of 2, 4, 6, and 9 ft. in the upstream tube and at zero in the downstream tubes.

For measuring differential pressures of greater than 10 ft. of water an ordinary 50-in. mercury manometer was substituted for the water manometer and piped up with the necessary vents.

For differential pressures of 1 ft. or less the double differential hook gage shown in Fig. 7 was used. This was made of two glass aspirator bottles about 10 in. in diameter and 15 in. high. They were inverted and connected together at the top, while the bottoms were connected to the pressure taps. A special hook gage was mounted at the bottom of each bottle with micrometer scale *A* reading directly in 0.001 in. The hook *B* consisted of a straight piece of rod sharpened on the end. By using a light behind the bottle it was possible to read this as accurately as the scale could be read. A valved bypass *C* was provided between the two pressure connections for checking the zero reading of the two gages, and an air connection was made to the top connecting pipe to make it possible to balance the pipe-line static pressure.

A mercury manometer was also used to measure the difference in pressure between one pipe diameter upstream and five diameters downstream to determine the pressure loss across the metering elements.

Fig. 8 shows some manometers similar to the ones used for measuring differential head and static pressure when steam was being used.

The manometer shown at *A* was the one used for measuring differential heads. It was a 10-ft. inverted water manometer. It was made from gage glass and piping as shown in the figure. The top of the manometer was connected to a high-pressure air line so that it was possible to adjust the pressure in the manometer to approximately the same pressure as that within the pipe before the manometer valves were opened. A gage was used to indicate the pressure at the top of the manometer. The manometer scale was laid off on the oak supporting board, and the board was shellacked to prevent any change in the scale length due to atmospheric conditions. The bottom of the manometer was connected to the pressure taps in the pipe with $^1/_2$-in. copper tubing and radiators, which will be described later. The face of the manometer was covered with non-shatterable glass during the tests, so that the observers would be protected in case of tube failure.

Fig. 9 shows the arrangement of the bull's eye and radiator reservoir as connected to the pressure tap. The manometer legs were connected to the radiators by $^1/_2$-in. copper tubing and compression fittings.

The radiators were standard high-pressure radiator condensers used by the Bailey Meter Company. They are built of welded steel and are arranged as shown in the figure. The reservoir serves to hold a sufficient quantity of water to fill the tubes during any slight change of head, while the fins on the steel tube serve to condense enough steam to keep the reservoir filled with water. A vent is provided in the top of each reservoir to permit the venting of air. They were operated manually while warming up the apparatus. A special offset gate valve was placed at the end of the reservoir and was connected to the pressure tap on the pipe, through a special fitting. This fitting was made with two flat glass bull's eyes in the sides so that the level of water in the pressure tap could be observed. Care was taken always to have the reservoirs level with each other and also horizontal.

The static-pressure tap was taken off from the opposite side of the pipe, as shown in Fig. 4. This pressure tap was connected through a radiator reservoir and offset gate valve to two of three manometers. One of these, a 15-in. high-pressure mercury manometer, was connected also to a point five diameters downstream from the orifice flange and was used to measure the head loss through the metering element. One of the other manometers was used to measure the static pressure in the pipe. If this pressure was less than 20 lb. per sq. in., a standard 50-in. mercury manometer was used. Most of the runs were made at higher pressures, and for these the special manometer shown at *B*, Fig. 8, was used. This was a two-leg mercury manometer with two operating ranges, one from 6 ft. to 11.4 ft. of mercury, corresponding to pressures of 37 lb. per sq. in. to 68 lb. per sq. in., and the other from about 14.5 ft. to 19.9 ft. of mercury, corresponding to pressures of 86 lb. per sq. in. to 117 lb. per sq. in. It could be used for either range by changing the amount of mercury used. This manometer was made of three sections of $^1/_4$-in. glass tubing connected by $^1/_8$-in. pipe and fittings. A mercury reservoir made of $^1/_2$-in. pipe was mounted on the high-pressure side to prevent mercury from running into the pipe, and a trap was mounted at the top of the atmospheric leg to prevent any high pressure blowing out the mercury. The piping and tube packing glands were mounted on 1-in. by 6-in. boards fastened to an angle-iron frame, and the manometer scale was laid out directly on this board. The boards were well coated with shellac.

Since the low side of this manometer was higher than the center of the pipe line, it was necessary to pipe up to it from the radiator reservoir and to provide a vent at the highest point, which was at the top of the short leg of the manometer.

A pressure gage was placed on the pipe about ten pipe diameters above the pressure taps and was used for determining the upstream pressure when water was flowing and for adjusting only for steam flow.

METHOD OF TAKING DATA

The method of taking the data was quite similar in all cases. For the volumetric tests with water, it was as follows: With the orifices or nozzle properly centered and bolted between flanges, the pumps were started and flow was established at a rate to give the desired pressure drop across the metering element. This water was passed through the deflector pipe to the suction bay until a steady flow had been established and all of the air eliminated from the apparatus. The level of water in the measuring bay was then read. After all of the conditions were satisfactory, at a definite time the deflector pipe was snapped to a position discharging into the measuring bay. Readings of heads and temperatures were taken at the desired intervals throughout the run. The heads were read by two observers reading simultaneously. The test run was ended by deflecting the stream of water again into the suction bay at the time decided upon for the end of the run. After a sufficient length of time had elapsed for surging to cease in the measuring bay, a final reading of elevation of water in the bay was taken. All readings of the bay elevation were checked by two observers. The length of the test was controlled by several factors. First, in order to get accuracy, the minimum quantity of water in the bay was limited to a depth of 4 ft. If the level of water in the bay increased more than 4 ft. in a half hour, the run was limited to 30 min. in duration. Otherwise the test ended when the measuring bay was completely filled. Observations were taken so that the minimum number of head readings was 20, while the maximum time between readings was 2 min. Temperature readings and static-pressure readings in the test line were ordinarily taken at the beginning and ending of each run.

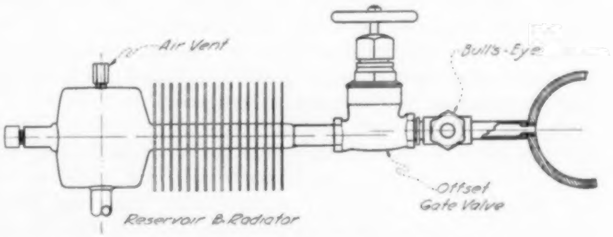

FIG. 9 DETAIL OF PRESSURE TAP AND CONNECTIONS

For the weighed-water test, the procedure in testing was similar to the displacement tests except that the water was run into one of the weighing tanks. At the beginning of a run the discharge was shifted into the other tank, and the two tanks were alternately filled, weighed, and emptied in the usual manner throughout the duration of the test. A calibrated stop watch was used to check the length of these runs, as some of them were of relatively short duration.

When steam was used, the condenser and superheater were started, and steam was turned into the line usually three or four hours before the beginning of a test. When the line was thoroughly warmed, the flow was adjusted to give approximately the desired pressure drop across the metering element, and the pressure in the pipe was adjusted to the approximate static pressure desired, and the apparatus was allowed to run at this rate for sufficient time to allow all temperatures to come to a point where they would remain practically constant. While this was going on the discharge from the condensate pump was allowed to flow into the waste-water trench; and while the line was warming, care was taken to eliminate all air and steam pockets from the manometer connections, and the water in the manometer tubes was allowed to cool approximately to room temperature. When the temperatures were constant, the test was started by throwing the deflector pipe to discharge the condensate into one of the weighing tanks, and the time of start was noted on a reliable chronometer.

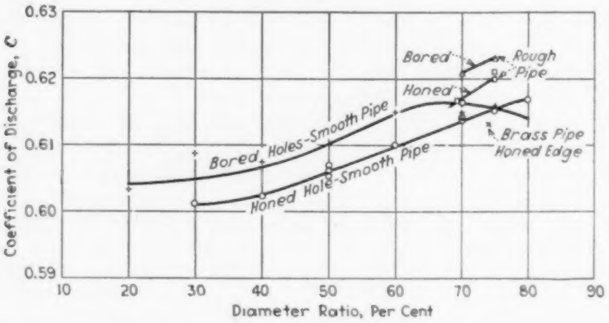

FIG. 10 VARIATION OF AVERAGE COEFFICIENTS WITH DIAMETER RATIOS FOR CONCENTRIC ORIFICES

Readings of the differential and static heads, temperatures, and pressures were taken at the desired intervals throughout the run. The differential and static-head readings were made by two observers reading simultaneously. The test was ended at a certain time by shifting the condensate discharge into the waste trench. The relative elevation of the water in the bulls' eyes at the pressure connections was recorded for each run. During the runs the cans were alternately filled, weighed, and emptied as with the weighed-water runs. Each run was made of sufficient length to get at least two tanks of water, the minimum amount in a tank being about 225 lb. The maximum length of run was 90 min., and the minimum was 30 min. Readings were taken of all pressures and temperatures at such intervals that the minimum number of readings was 20, and the maximum time between readings was 2 min.

CALIBRATION OF INSTRUMENTS

All instruments used were calibrated carefully. The platform scales were calibrated before and after using, over the entire range, by means of standard weights. These weights were then checked by the Ohio State Sealer of Weights and Measures, and found to be accurate within $^3/_4$ oz. in 100 lb., and the errors were found to be practically compensating.

The scales could be read to about $^1/_4$ lb., and were found to be so accurate that no correction was necessary. Care was taken at all times to keep the knife edges clean and to keep dirt and water from all working parts of the scales.

The measuring bay was calibrated by filling it with water. The water was weighed as it was put in, the height of the water in the bay gage glass noted, and the temperature of the water taken. The weight of water was found by using tanks on calibrated platform scales. A reading of the height of water in the bay was taken at increments of approximately 2000 lb. of water. A curve was then plotted showing the amount of water present in the bay for any gage height.

All thermometers were calibrated, the low-reading ones against a standard thermometer, and the high-reading ones by being placed in a well similar to the one in which they were used, and checked against temperatures corresponding to observed saturated-steam pressures.

The Bourdon gages used were checked against a dead-weight gage tester.

All manometer scales were checked periodically against a steel tape or a steel scale, while the zeros of the inverted water manometer were checked with a level at the same time. All manometers were checked for perpendicularity at frequent intervals. The mercury manometer was calibrated by comparison with a water column. The stop watches used were checked at intervals against a reliable chronometer, which in turn was checked against Western Union time.

The orifice and nozzle diameters were measured carefully with inside micrometers. The diameters were measured at four places, and the average considered as the average diameter of the opening. The nozzle diameter was measured at the outlet and also near the entry to the cylindrical portion. The diameter of the pipe was measured at the upstream pressure tap, and at a point about one-half pipe diameter downstream.

The condenser and measuring bays were checked for leakage at regular intervals.

DISCUSSION OF RESULTS

Water Coefficients. The results for runs using water as the calibrating fluid are shown graphically in Figs. 10 to 21, inclusive.

An attempt was made to secure at all times as great accuracy as possible in obtaining the data. All metering elements were installed with great care and inspected at regular intervals. When any change was made from one head or quantity measuring device to another, overlapping runs were made with both devices to insure that no errors due to equipment were affecting the results.

The usual practice was to make four runs on each primary element at differential pressures of approximately 2, 4, 6, and 9 ft. of water. The values of the coefficients for these runs were averaged, and this average was used for comparison with the coefficients for other orifices.

Fig. 10 shows these average coefficients plotted against orifice diameter for the concentric orifices, while Fig. 12 shows the same thing for all orifices, and on this curve are plotted the curves for concentric and eccentric orifices in 6-in. pipe as published in the 1929 Transactions of the A.S.M.E. Hydraulic Division.

The orifices calibrated being of two different finishes, gave two different curves, as shown in Fig. 10. The original orifices were made by boring the orifice holes with an ordinary lathe tool and then polishing the edges of the hole with fine emery paper. This did not give a uniform edge to the various orifices, leaving the upstream edge slightly rounded. This rounding was invisible to the naked eye, but could be detected under the microscope. This amount of rounding varied with different orifices, being the greatest on orifice No. 25,176, 30 per cent diameter ratio, and being very much smaller on the 75 per cent to 80 per cent diameter ratios.

In an attempt to secure more uniform results another set of orifices was made with the holes finished by honing instead of using emery cloth. This method of finishing gave sharp, smooth edges with no apparent rounding even under the microscope. When calibrated, these orifices gave the ratio-coefficient curve marked "honed hole." This curve is about $^2/_3$ of 1 per cent lower than that for the rounded-edge orifices up to about 65 per cent diameter ratio, and then the two curves cross at about 75 per cent diameter ratio, and the "honed hole" coefficient is about $^1/_2$ of 1 per cent higher at 80 per cent diameter ratio. The lower parts of these curves are about parallel to each other and to the curve for 6-in. concentric orifices, while the curve for the "bored holes" parallels the 6-in. curve throughout. The curve for the bored orifices is slightly above the 6-in. curve, while that for the "honed holes" is below it except beyond the 70 per cent diameter ratio.

The curve for the "bored holes" has about the relative shape and location with respect to the 6 in. as would be expected from the previously published work of Spitzglass.[5]

The lower portion of the curve for the "honed holes" is also as would be expected, as the sharper edge should give greater contraction and so a smaller coefficient. However, the upper portion of this curve is not at all as expected. No reason for this divergence from previous results was found, and apparently more data must be secured before any reasons can be found for this divergence. In an attempt to discover if rounding was the only thing affecting this, some of the honed orifices were chucked in a lathe and the edges rounded by holding fine emery cloth in the orifice while it was revolving. One 80 per cent and one 50 per cent orifice treated in this way were then calibrated, and in both cases the coefficient raised about $^1/_2$ of 1 per cent. The dulling caused by this treatment could not be detected by the naked eye.

During the latter part of the calibration the effect of scale in the upstream pipe was also investigated. The earlier tests were made with this pipe cadmium plated, and there was no rust or scale formed, but while running the steam tests the cadmium plate melted. While making some check runs later it was found that the coefficient for some of the orifices was from $^1/_2$ to 1 per cent higher than on previous calibrations. Cleaning the upstream tube to as nearly as possible the original finish, however, restored the coefficient to the original value. The two short curves marked "rough pipe" show the values of the average coefficients for the 70 per cent and 75 per cent ratio orifices with rough pipe.

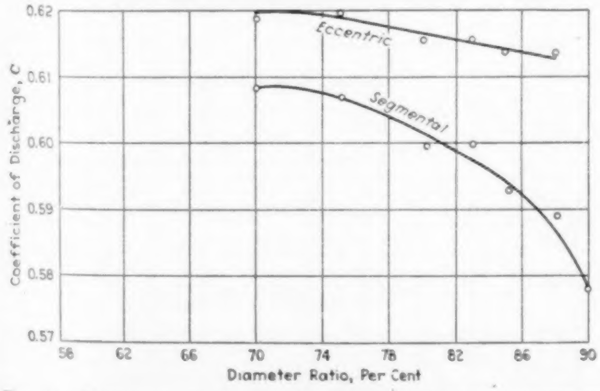

FIG. 11 VARIATION OF AVERAGE COEFFICIENTS WITH DIAMETER RATIOS FOR DIFFERENT ORIFICES

Fig. 11 shows the variation of average coefficient of discharge with orifice ratio for eccentric and segmental orifices. These are ordinary bored orifices and are subject to the variation in rounding of hole which accounts for the relative irregularity of the points on these curves. The curve for the eccentric orifices is higher and the values of R do not drop as rapidly with increase in diameter ratio as do those for the segmental orifices. This is probably due to the fact that the holes in the eccentric orifices are round and therefore similar, while those in the segmental orifices are not similar.

When plotted in Fig. 12, together with the curves for the concentric orifices and the curves for the 6-in. pipe, two things were apparent concerning the curve for the eccentric orifices. First, the curve for eccentric orifices seems to be about parallel and 0.2 per cent below that for the concentric orifices of the same type of hole finish, and, second, that there is no apparent similarity between the curves for the 6-in. eccentric orifices and those of the 3-in. The curve for the 6-in. eccentric orifices is above the curve for the 6-in. concentric orifices at all times, and instead of being parallel, the curvature is in exactly the opposite direction. Of these two curves the 3-in. appears to be much more logical. This would tend to throw some doubt on the accuracy of the determination of the eccentric orifice coefficients for the 6-in. pipe. The only apparent difference in the taking of data for the two different sizes of pipe was that, for the 6-in. pipe, the arrangement of the piping was such that the upstream and downstream pressure taps were not exactly in line along the pipe and this would probably affect the coefficient for an eccentric orifice.

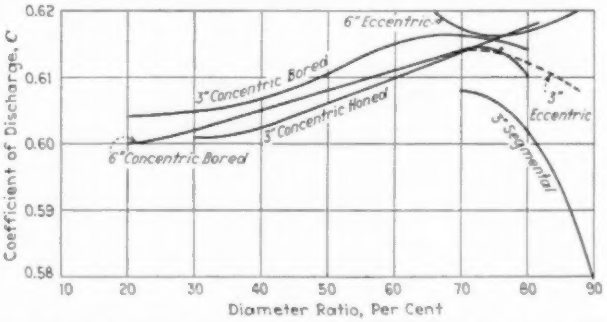

FIG. 12 VARIATION OF AVERAGE COEFFICIENT WITH DIAMETER RATIOS FOR ALL ORIFICES

Since no segmental orifices were calibrated in the 6-in. pipe, there is nothing with which to compare the curve for the 3-in. pipe; however, the values seem to have a logical relation to those for concentric and eccentric orifices.

Fig. 13 shows the values of the coefficient plotted against differential head for the various concentric orifices. A study of this figure shows that, for the 24 concentric orifices tested, the lower ratios showed a practically flat curve above a differential head of 1 ft., but below this head the coefficient of discharge rises very rapidly. As the diameter ratio of the orifice increases, this tendency of the coefficient to increase seems to start at a higher head; however, this is just a tendency, as not all do this—notably the coefficients for the 80 per cent honed orifice are almost the same down to 0.5-ft. differential head.

The 70 per cent orifice was tested up to 50 ft. of water differential head, and the results are plotted in Fig. 14. This curve shows practically no variation in the coefficient beyond the range shown for the rest of the orifices.

Figs. 15 and 16 show the variation of coefficient with head for the eccentric and segmental orifices. These curves are very similar to those for concentric orifices in that they are flat except for a sharp rise in coefficient for heads of less than 1 ft.

Fig. 17 shows the variation of the vena contracta and pressure

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[5] Trans. A.S.M.E., vol. 45, 1922.

tap downstream from the orifice with orifice ratio. These values were determined previously to these tests, using data on other sizes of pipe, since the arrangement of downstream pressure connections made it impossible to get simultaneous readings of pressure drop at more than three points downstream for the concentric orifices or more than one point for the others. This curve shows the advantage of the eccentric and segmental orifices in throwing the vena contracta and therefore the pressure tap farther downstream.

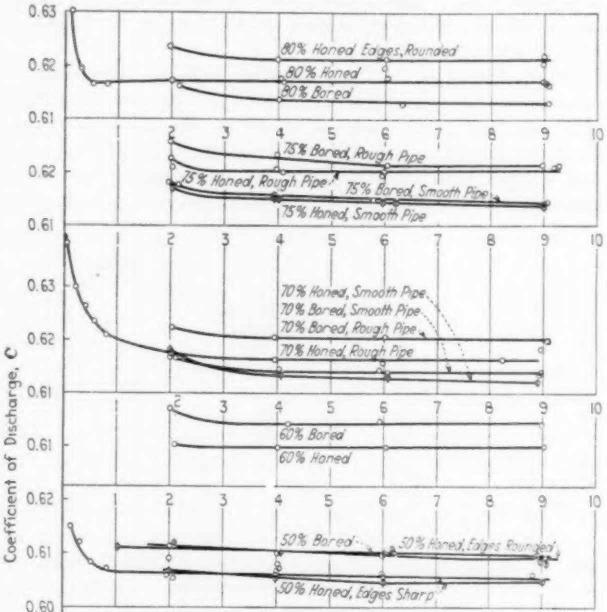

FIG. 13 VARIATION OF COEFFICIENT OF DISCHARGE WITH DIFFERENTIAL HEAD FOR CONCENTRIC ORIFICES IN 3-IN. PIPE

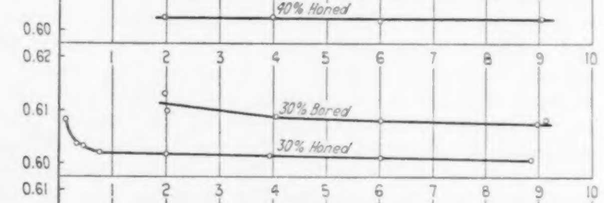

FIG. 14 VARIATION OF COEFFICIENT OF DISCHARGE WITH DIFFERENTIAL HEAD FOR CONCENTRIC ORIFICES

Fig. 18 shows the variation of average coefficient with ratio for the flow nozzles and venturi tubes. There are two curves for the flow nozzles—one for the cadmium-plated nozzles and one for the unplated nozzles. The difference in the two types of nozzles was only in the finish, as the same nozzles were used, first plated, and then with the plating removed. The plating gave the nozzle a somewhat rougher surface than the plain steel finish. These rough nozzles had a higher coefficient of discharge than did the smooth ones.

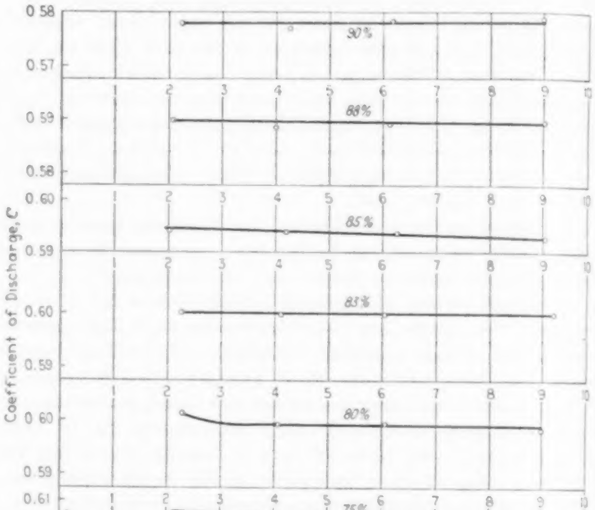

FIG. 15 VARIATION OF COEFFICIENT OF DISCHARGE WITH DIFFERENTIAL HEAD FOR SEGMENTAL ORIFICES

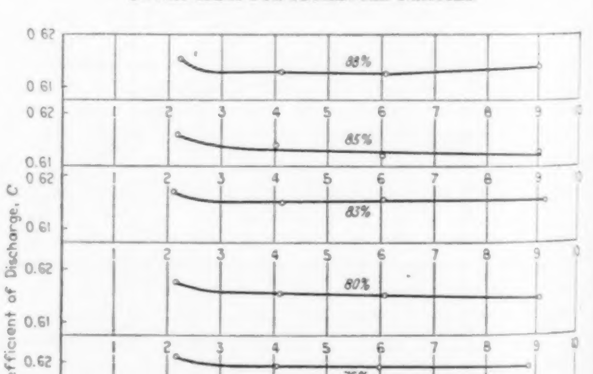

FIG. 16 VARIATION OF COEFFICIENT OF DISCHARGE WITH DIFFERENTIAL HEAD FOR ECCENTRIC ORIFICES

The points on the flow-nozzle curves do not line up as well as do those for the sharp-edged orifices. In fact, the variation from the line drawn is in the neighborhood of $^1/_2$ of 1 per cent plus or minus for the nozzles, while the maximum variation for the orifices was $^1/_2$ of 1 per cent for one point, and no other points were more than $^1/_4$ of 1 per cent off. This wide variation is probably due to differences in the approach curvature of the various nozzles. An examination of Fig. 2 shows that no two of these nozzles had exactly the same relative approach contour in respect to their throat diameter, and apparently that this contour has some effect on the magnitude of the coefficient.

For this reason the values of the coefficients given in this curve cannot be compared with the values published in the A.S.M.E. Fluid Meters Committee Report, part No. 1, as the contours of the nozzles whose coefficients are plotted there are not given.

One thing which is very apparent is the very rapid dropping of the coefficient with ratio for ratios greater than 75 per cent.

The curve for the venturi tubes is comparatively short, as only three tubes with different ratios were tested (33 to 50 per cent ratio). For this range the curve is apparently a straight line sloping at a somewhat greater angle than the curve for flow nozzles and slightly above it. Of the six throats tested only one showed values of coefficient off this line more than 0.05 per cent, and this one was so far off (0.65 per cent) that it appears to be due to some error of installation rather than any inherent difference in coefficient.

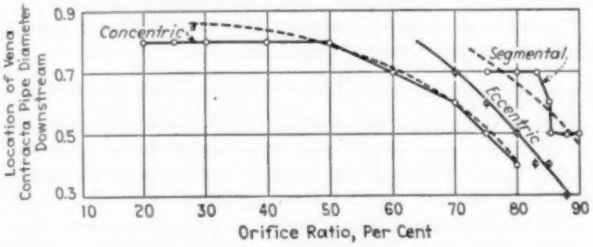

FIG. 17 LOCATION OF VENA CONTRACTA WITH REFERENCE TO PIPE DIAMETER AND ORIFICE RATIO

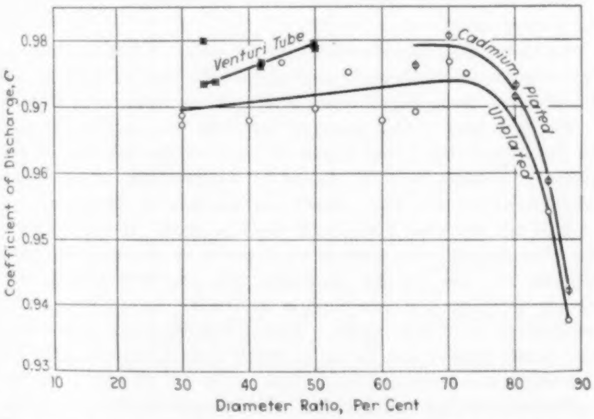

FIG. 18 VARIATION OF AVERAGE COEFFICIENTS WITH DIAMETER RATIOS FOR FLOW NOZZLES AND VENTURI TUBE

The curves, Fig. 19, giving the variation of coefficient with head for the flow nozzles, show an entirely different slope than do the similar curves for the orifices. In general they are not as flat as the orifice curves, but show a decreasing coefficient for decreasing head over the whole range of heads used, with the rate of change in coefficient increasing very rapidly for low differential heads.

Certain curves on Fig. 19 call for further explanation. The two curves for the 30 per cent nozzle show the effect of roughness

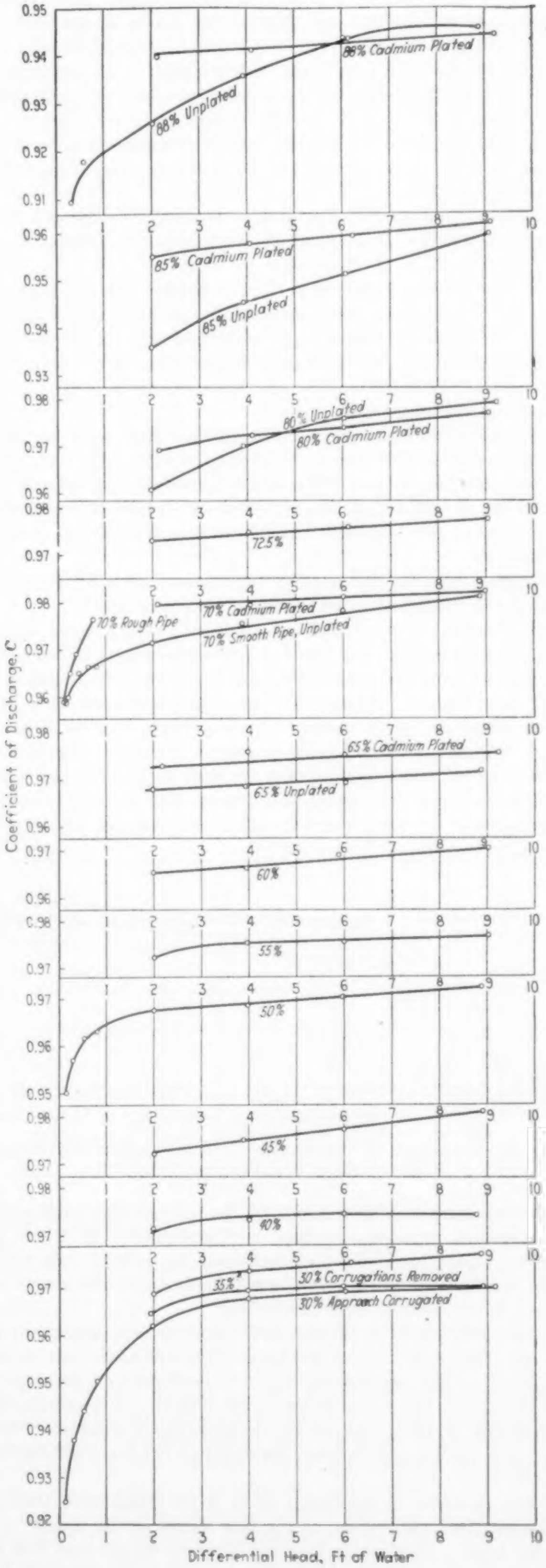

FIG. 19 (RIGHT-HAND COLUMN) VARIATION OF COEFFICIENT OF DISCHARGE WITH DIFFERENTIAL HEAD FOR FLOW NOZZLES

on the nozzle flange. On this particular nozzle part of the flange was exposed to pipe flow. The points shown on the short section of the dotted curve at low heads for the 70 per cent nozzles are the only tests made with a nozzle in the rough pipe and tend to show that roughness also increases the coefficient of a nozzle. However, these four points are not conclusive.

In general, the curves for the unplated nozzles seem to have a greater slope than do those for the plated nozzles, though this is not always true.

Pressure taps for the nozzles were taken at a point 0.8 of a pipe diameter downstream, which is about 1 in. upstream from the discharge end of the nozzle.

For the venturi tubes the coefficient head curves are given in Fig. 20. These show the same general curvature and shape as those for the flow nozzles except for the fact that the 50 per cent ratio curve is flat for differential heads above 4 ft. of water. The rest of the coefficient curves, however, are very similar to nozzles of the same ratio.

Fig. 20 shows the variation of coefficient with head for high differential heads for a 50 per cent ratio venturi tube and for a 70 per cent ratio nozzle. The nozzle curve changes slope until about 35 ft. differential head is reached, while as mentioned before the 50 per cent ratio venturi-tube curve is flat above 4 ft. differential.

Pressure taps for the venturi tubes were taken by standard piezometer rings and connections, as shown in Fig. 3.

The results presented make it possible to compare the action of three of the most used types of primary elements in the same set up and using the same calibrating apparatus. It is apparent that any type of element has certain advantages over the other elements. Any number of sharp-edged orifices if carefully constructed can be made to reproduce results consistently within a very small fraction of a per cent if the upstream pipe conditions are at all similar and proper care is taken in their installation. Nozzles, however, apparently are not as easy to reproduce, and it is the opinion of the authors that until some standard of approach contour and smoothness of nozzle is decided upon, any values of coefficient for nozzles will apply only to a nozzle which is an exact duplicate of the nozzle on which the coefficients were determined.

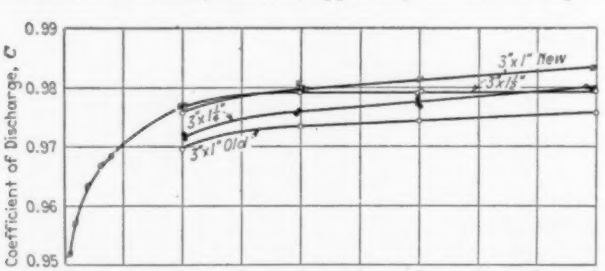

FIG. 20 VARIATION OF COEFFICIENT OF DISCHARGE WITH DIFFERENTIAL HEAD FOR VENTURI TUBES

The variation of coefficient with head is very important in all metering work, as for mechanical reasons most meters are designed on the assumption that the coefficient of discharge is constant over the whole range of the meter. This assumption apparently is erroneous, as all of the primary elements tested showed a variation with head, especially at the lower differential heads.

This variation of coefficient is of more importance than is apparent from the curves, as the flow varies as the square root of the head. This means that the head for 25 per cent flow in a meter using 9-ft. maximum head would be about 0.56 ft., and if an orifice were used with a coefficient which was correct at from 50 per cent to full flow (2.25 ft. to 9 ft. differential head), the meter would read low for 25 per cent flow, and the lower the flow the greater the error. Due to this variation at low heads the meter selected for any particular job should be, for greatest accuracy, designed to operate in the ranges of higher differential head.

For commercial metering the flat head-coefficient curve for the orifice, as compared with the sloping curve for the other two elements, should give the orifice meter a greater accuracy over a wider range of flow.

Both nozzles and orifices were apparently very sensitive to upstream pipe roughness, and great care must be taken in their installation if great precision is to be obtained when used as metering devices. By great precision is meant a degree of accuracy of the order of less than plus or minus 1 per cent.

The range of venturi tubes tested was so small that no general conclusions can be drawn about them.

EFFECT OF TEMPERATURE UPON ELEMENTS AND COEFFICIENTS

As a part of this work all of the primary elements calibrated on water were also tested, using superheated steam as the calibrating fluid. While at the present time all of the results of these runs are not ready for presentation, there were a few interesting facts discovered which it is possible to describe.

These steam tests included many tests to determine experimentally the values for the correction for compressibility, and also a great deal more work on the effect of orifice edge and nozzle roughness on coefficient. This material will be published at a later date.

Another phase investigated was the effect of temperature on the primary element and upon the coefficient of discharge of the element using superheated steam as the calibrating fluid.

The first part of this phase of the work was an investigation to determine the actual change in size of the opening in the primary element with a change in temperature, and to find whether or not the heat caused any warping of the element.

A 70 per cent steel flow nozzle was first used. It was clamped between flanges with gaskets as it would be in an actual pipe installation, and an electric heater was provided to heat the nozzle directly and the flanges indirectly by radiation and conduction from the nozzle. Thermocouples were mounted in the nozzle throat and flange in small drilled holes in order to determine the temperature of these parts. It was felt that this approximated the arrangement of the nozzle in the pipe when heated by steam flowing through the pipe and that it would show if the change in actual diameter would check the calculated value and whether or not the nozzle would warp when heated.

The nozzle was heated to about 1000 deg. fahr. and then allowed to cool slowly, and the diameter of the throat and entry was measured at regular temperature intervals. These measurements were taken by means of a telescoping gage and outside micrometers.

There was no apparent change in the relative sizes of throat or entry section of the nozzle and evidently little or no warping of the nozzle due to heat or stresses from differences in expansion between nozzles and retaining flanges. The increase in diameter of the hole was, as closely as could be measured, the same as the calculated values for this increase based on the coefficient of expansion of soft (0.12 per cent carbon) steel. Fig. 22 shows the results of these measurements. The solid line shows the theoretical diameter of this nozzle figured from the coefficient of expansion of steel of 0.12 per cent carbon, while the points shown are the actual measurements. The test points are all within 0.0008 in. of the theoretical and fall on both sides of the line, showing that the increase in diameter of the nozzle hole for any temperature can be calculated.

The same method was used in checking the expansion of a Monel metal orifice of 70 per cent ratio, except that the orifice was heated by gas rather than by electricity. It was thought that possibly the differences in coefficient of expansion between Monel and cast iron would cause the orifice plate to buckle, and so it was also checked for flatness.

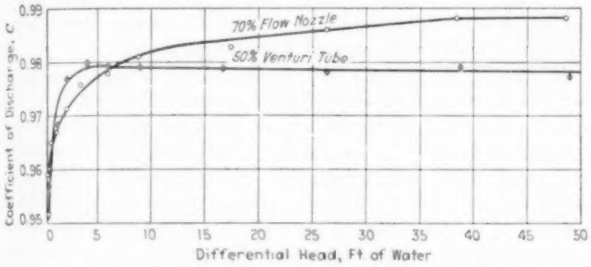

Fig. 21 Variation of Coefficient of Discharge With Differential Head for Flow Nozzle and Venturi Tube

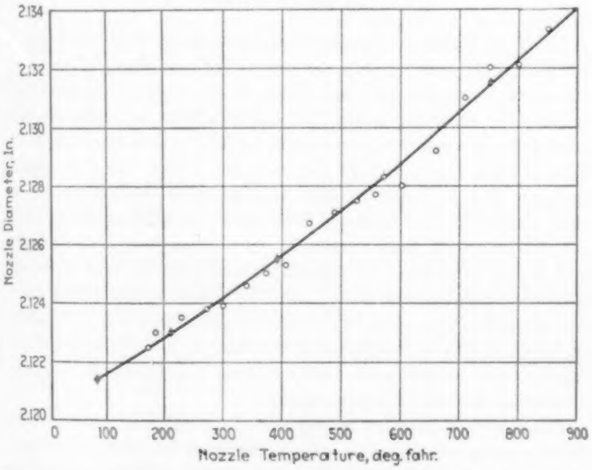

Fig. 22 Variation of Diameter of Throat With Temperature for Steel Flow Nozzle

(Circles, test points; crossed circles, calculated points.)

It was found that the orifice remained flat and that, as in the case of the nozzle, the increase in the diameter was very close to the calculated value using the theoretical coefficient of expansion. Fig. 23 shows the experimental points plotted together with the line showing the calculated diameter. The maximum variation of any point from this line was about 0.0011 in., and the test points fall on both sides of the theoretical line.

An attempt was made to do the same thing to a venturi tube, but it was found impossible with the equipment available.

After the effect of temperature in the element itself was determined, the next step was to determine the effect upon the coefficient of changing the temperature of the steam flowing through the element. To determine this effect the 70 per cent orifice was first put in place in the pipe and a series of calibration runs were made, varying the temperature and keeping all other variables as nearly constant as possible. These runs were made at approximately 5 lb. per sq. in. gage pressure and at 9 ft. of water differential head.

When the runs on the orifice were completed, the same thing was done for the 70 per cent flow nozzle and the 50 per cent venturi tube.

The results of the tests on the orifices are shown graphically in Fig. 24. The coefficients of discharge were calculated in two ways—first, based on the original diameter of the orifice and, second, based on the calculated diameter of the orifice at the temperature of the steam in the pipe. Both of these assumptions are probably erroneous. However, the second should be very nearly correct, as the temperature at the orifice should be within a few degrees of the temperature five diameters upstream.

The values for the coefficient based on the diameter of the orifice at room temperature averaged 0.5698 and varied from this average about 0.6 per cent minus and about 0.3 per cent plus. The points seem to be independent of temperature; that is, the curve of coefficient versus steam temperature is a flat, straight line. The test points do not fall exactly on the line, and there is as much variation as noted above, due to the fact that it was impossible to get exactly the same absolute pressure or the same differential head, for any two runs. Since the coefficient used here is not corrected for compressibility, any change in the compressibility factor which varies with a variation of ratio of differential head to absolute pressure would change the coefficient. However, there were enough runs made to average out this variation, and the average values should be very nearly correct for the average head.

When the values of the coefficient based on the diameter of the orifice at steam temperature were used, the curve was found to be not a flat straight line, but to have a very definite slope, with the coefficient decreasing with an increase in temperature. The amount of this variation was about 0.7 per cent for a change of temperature from 250 deg. to 650 deg. fahr. This variation of coefficient is rather difficult to account for, as there is nothing varying except temperature, volume, and viscosity.

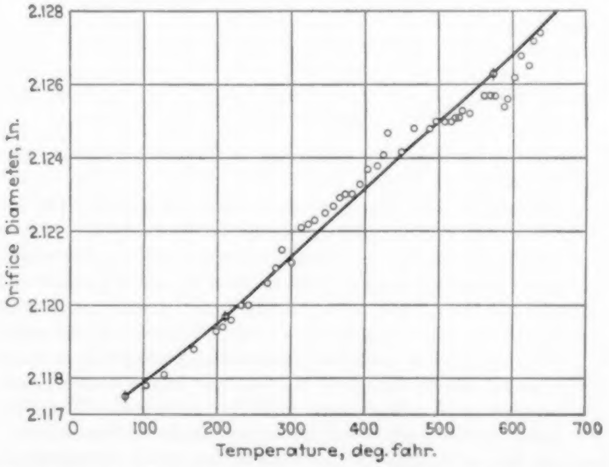

Fig. 23 Variation of Diameter of Hole With Temperature of Monel Metal Orifice Plates Between Cast-Iron Flanges

(Circles, test points; crossed circles, calculated points.)

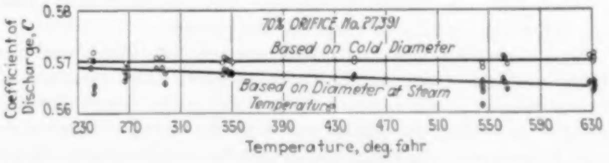

Fig. 24 Variation of Coefficient With Temperature for a 70 Per Cent Concentric Orifice

To see if these other variables could be affecting the coefficient in any way, the Reynolds number was calculated for a few of these runs and the coefficient was plotted against the Reynolds number, as shown in Fig. 25. This curve shows the coefficient increasing for an increase of the Reynolds number, but no longer as a straight-line relation, the coefficient dropping more rapidly for the lower Reynolds numbers which correspond to the higher temperatures. Since the previously published work of Ed. S. Smith, Jr. (Trans. A.S.M.E., 1930, paper HYD-52-7*b*) shows a practically flat curve for this range of Reynolds numbers, this does not seem to help any.

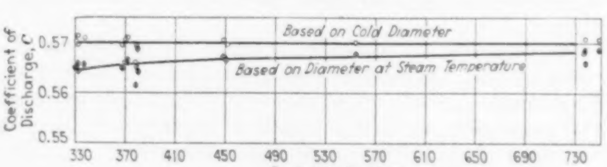

FIG. 25 VARIATION OF COEFFICIENTS OF DISCHARGE WITH REYNOLDS NUMBER FOR 70 PER CENT CONCENTRIC ORIFICE

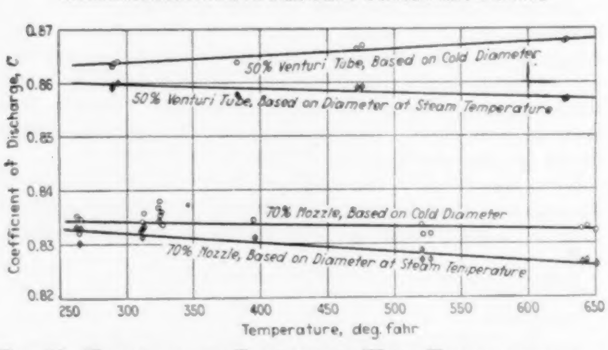

FIG. 26 VARIATION OF COEFFICIENT WITH TEMPERATURE FOR VENTURI TUBE AND FLOW NOZZLE

In connection with this analysis the authors feel that the use of the Reynolds number as a parameter for the study of metering elements is impractical, especially in the case of a square-edged orifice for metering gases. This is because, to determine the values of the Reynolds number, the diameter, pressure, and temperature at the measuring section must be known. If the point of highest velocity is used, the diameter and temperature must be estimated, and if based on the velocity at the orifice plane, the pressure and temperature must be estimated. The fact that these factors must be estimated means that all items entering into the calculation of this number are based on estimates, which makes the value of the number itself very doubtful.

The 70 per cent ratio flow nozzles and the 50 per cent venturi tube were tested also at varying temperatures. The results of these tests are plotted in Fig. 26. The results in both cases are somewhat similar to those for the orifice.

The curves for the flow nozzle are not flat, but sloped for both methods of calculating the coefficient. The variation of the coefficient based on the hot diameter of the nozzle shows a greater slope than the similar curve for the orifice. This variation is, however, only about 0.8 per cent for a similar temperature difference (250 deg. to 650 deg. fahr.).

The curves for the venturi tubes were different and more nearly according to theory. The values of the coefficient based on the cold diameter increase with temperature, while those based on the hot diameter decrease, but at a much slower rate than those for the orifices and nozzles. The variation of the "hot" coefficient for the tube is, for the range from 250 deg. to 650 deg. fahr., less than 0.5 per cent.

It is also possible that the determination of the "hot" coefficient for the venturi is not as accurate as the others, since no actual measurements of the increase of size of the venturi with temperature were made. Then, too, the fact that the venturi tested was of a different ratio than the orifice and nozzle makes direct comparison impossible.

This variation of coefficient with temperature is of importance in the design of meters for steam, as it might mean a difference of almost 1 per cent in the coefficient of discharge of the primary element if it is designed for one operating temperature and used at some other temperature. However, it appears from the data presented that, except for the venturi tubes, the effect of the expansion of the orifice can be neglected in design and the resulting coefficient will not vary with temperature.

In closing, the authors wish to state their disappointment in not being able to present at this time more of the results of the steam calibration runs. However, the mass of data collected is so great that there has not been sufficient time to study it and get it ready for presentation. It is hoped that the additional material will be ready for presentation soon and that the bulletin of the Ohio State University Experiment Station covering the complete project to date will be ready for publication shortly.[6]

Discussion

M. P. O'Brien.[7] The authors' statement that "the use of the Reynolds number as a parameter for the study of metering elements is impractical, especially in the case of square-edged orifices for metering gases," is disconcerting to engineers who have come to regard it as the most important contribution in the field of fluid mechanics in the last fifty years. If it is applied properly and is found to lead to erroneous conclusions in one case, it will make necessary a revision of our ideas on the dynamical similarity of fluid systems. In aeronautics, heat transfer, and many branches of hydraulics, especially in pipe flow, the Reynolds number has been found by many investigators to be the simplest method of coordinating data, and it is difficult to understand why it should fail in the case of flow through a sharp-edged orifice, which is so closely related in principle to the fields in which it has been found to apply.

The Reynolds number is only one of a number of dimensionless criteria upon which dynamical similarity of fluid flow depends, of which the most evident is the scale ratio of linear dimensions. This geometrical similarity of the boundaries of the fluid system extends even to the surface roughness. Other criteria are necessary to include the effects of the expansion of gases and the transfer of heat in the case of liquids or gases, and as far as the theory is concerned, all these criteria must have identical values in each of two fluid systems if dynamical similarity is to exist. Whether some of these criteria are irrelevant can only be determined by experiment, and the present paper does not appear to present sufficient evidence to warrant the conclusion that they are inapplicable. As to whether the Reynolds number is impractical or not, it is necessary that we consider the particular problem involved. If the orifices calibrated are to be used only for steam and water, then there is probably no real advantage in using the Reynolds number, but if they are to be used for metering other liquids and gases, there appears to be no method of applying these data other than by the use of the Reynolds number and a criterion representing the expansion effects. Considering the number of different fluids which are being metered, it seems desirable that all experimental data be represented in a manner sufficiently general that it can be used for computing the flow of any fluid, provided its physical properties can be specified.

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[6] For a complete discussion of the method of calculating the results, see the appendix of the paper, "The Flow of Fluids Through Orifices in Six-Inch Pipes," Trans. A.S.M.E., 1930, paper HYD-52-7a.

[7] Associate Professor of Mechanical Engineering, University of California, Berkeley, Calif. Assoc-Mem. A.S.M.E.

In applying the Reynolds-number method, it is necessary that the viscosity and density be known, and these quantities, particularly the viscosity, seem to be somewhat in doubt in the case of superheated steam. Errors in these quantities might shift the experimental points in Fig. 25 so as to give a straight line, but this straight-line relation is not necessary from any theoretical considerations. The theory of dynamical similarity states that, if the only important variables are the velocity, viscosity, and density, the coefficient should be identical for geometrically similar orifices at the same Reynolds number. The description of the orifice used states that the plates were corrugated, and if these corrugations were not geometrically similar in the case of the 3-in. and 6-in. pipes, there is no reason to expect identical coefficients. For steam, the Reynolds number is not sufficient as a criterion of similarity, and can be applied only at equal values of the dimensionless group representing the effect of the expansion. The objection that the quantities involved in the Reynolds number are difficult to estimate seems to be without foundation, since they can all be measured for similar conditions at an equal number of diameters upstream from the orifices.

To the writer, the question of the validity of the Reynolds number appears far more important than the coefficient of discharge of steam and water, and it is hoped that this question will be thoroughly discussed in the final report on this extensive and valuable series of experiments.

Edgar Buckingham.[8] The authors' statements, near the end of the paper, regarding the Reynolds number, suggest the following remarks:

The Reynolds number may be, and is, defined in various ways according to circumstances, convenience, and personal preference; but unless something is said to the contrary, the Reynolds number for any section A_1 of a stream of fluid along a round pipe of diameter D is always understood to be the value of the expression R_D, defined by the equation

$$R_D = \frac{DS_1\rho_1}{\mu_1} \qquad [1]$$

where ρ_1 and μ_1 are the density and viscosity of the fluid, and S_1 is the mean linear speed parallel to the axis of the pipe, all referred to the section A_1, and all expressed in terms of normal units, such as "British Absolute" or C.G.S.

The speed S_1 is a fictitious one which cannot be measured directly but has to be computed from the measured rate of volume or mass flow. It is, in fact, defined by the equation

$$M = \frac{\pi}{4} D^2\rho_1 S_1 \qquad [2]$$

in which M is the mass flow, and Equation [1] may be put into the entirely equivalent form

$$R_D = \frac{4M}{\pi D\mu_1} \qquad [3]$$

If an orifice of diameter d is installed in the pipe, it is natural (though not necessary) to define the Reynolds number for the orifice by the analogous equation

$$R_d = \frac{4M}{\pi d\mu_2} \qquad [4]$$

in which μ_2 is the viscosity in the plane of the orifice; and since M is a constant, for steady flow, we have

$$R_d = R_D \frac{D\mu_1}{d\mu_2} \qquad [5]$$

It must, of course, be assumed that the viscosity-temperature curve of the fluid is known, for otherwise it is useless to discuss Reynolds numbers. If it is known, there is no difficulty in computing R_D from the observed mass flow and initial temperature, and the only doubt about the value of R_d concerns the change from μ_1 to μ_2 caused by the change in temperature.

In all ordinary cases the viscosity may be regarded as dependent only on the temperature. If the fluid is a liquid, there is no appreciable fall of temperature and Equation [5] reduces to

$$R_d = R_D \frac{D}{d} \qquad [6]$$

With a gas, the temperature falls, but the effect on R_d and on μ is usually insignificant. For example: with air at an initial temperature of 70 deg. fahr., and with a 20 per cent differential—a value seldom exceeded in metering practice—the fall of temperature all the way to the vena contracta is 32.7 deg., at most. Any dissipation by turbulence and skin friction raises the lower temperature, and the fall to the plane of the orifice is less than to the vena contracta, so that 32.7 deg. is an outside limit.

The temperature coefficient of the viscosity of air is about 0.0015, so that cooling the air 33 deg. lowers its viscosity in the ratio $(1 - 0.0015 \times 33) = 0.95$; and if we ignore the change and compute R_d from [6] instead of [5], the error can not be as much as 5 per cent. Except under extreme conditions, seldom encountered in practice, an error of this magnitude is absolutely insignificant, because the variation of the discharge coefficient caused by a 5 per cent change of viscosity is within the errors of measurement.

It is, however, perfectly legitimate to take [6] as an *exact* definition, for the following reasons:

With a given apparatus, a given gas, and given initial pressure and temperature, the whole state of affairs near the orifice is fixed by the rate of flow; and for any given value of M, the distribution of velocity, static pressure, and temperature, and therefore of density and viscosity, is fixed. Consequently, the value of R_D fixes the value of R_d for any other section of the stream, even though we may be quite unable to compute that value.

Hence it is entirely permissible to study our results by plotting them against R_D, or against R_d defined by [6]—which turns out in practice to give simpler results. We are under no obligation whatever of plotting discharge coefficients against values of the Reynolds number "at the measuring section."

Whether there is anything to be gained by going to the trouble of computing the Reynolds number is another question. Witte's experiments have shown that when R_d is greater than about 150,000, the effects of viscous forces on the motion through the orifice are within the errors of measuring the coefficient, of about $\pm$ 0.5 per cent. In the experiments to which the authors' Fig. 25 refers, the lowest value of R_d was considerably higher than this. There was therefore, in this case, no new information to be expected from plotting results in terms of the Reynolds number; and the fact that the lower curve in Fig. 25 is not a horizontal straight line is not due to viscosity but to some other cause, either physical or algebraic.

J. O. Feldmark.[9] A thorough analysis of the various curves shown in the paper brings out the following facts:

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[8] Bureau of Standards, Washington, D. C.

[9] Flow Meter Engineer, The Brown Instrument Co., Philadelphia, Pa. Mem. A.S.M.E.

Fig. 10 shows conclusively that a sharp-edged honed orifice has a coefficient of discharge which is about two-thirds of 1 per cent lower than that for a round-edged or bored orifice. This difference can readily be attributed to the effect of the sharpness of the orifice edge. However, the writer did not find any justification for the crossing of the two curves at a diameter ratio of about 0.75.

If the writer were to draw any conclusions with regard to this discrepancy, he would say that the reason for this behavior is probably due to the unreliability of results when the orifice reaches 75 per cent of the pipe diameter in small pipe lines. In this instance a 75 per cent ratio is equal to a 2.270-in. orifice opening, as against a 3.027-in. internal pipe diameter. The orifice rim inside the pipe is therefore only $^3/_8$ in., which no doubt upsets the true differential readings.

Fig. 11 would have been of much greater value had it contained also the average concentric-orifice values for the same ratio of diameters. A comparison would then be possible between the three types of orifices (concentric, eccentric, and segmental).

Furthermore, the results of the present tests should have been plotted against the curves used up to the present time, either as published in part 1 of the A.S.M.E. Fluid Meter Report, or compared with tests made by other experimenters. In this manner a direct comparison would have been possible between the various results, bringing out clearly where the difference is, if any.

In Fig. 12 there is an attempt to compare the present tests, but it is handicapped by the fact that the three types of orifices are not extended to the same maximum- and minimum-ratio limits. Further, the comparison should have been made between the curves corresponding to the same pipe size. The introduction of a curve for 6-in. pipe, while indicating the general direction and shape, does not, however, show how much difference there is between the present 3-in. coefficient and those used previously for a 3-in. pipe.

Fig. 13, as plotted, requires a tedious selection of values in order to make a comparison between the coefficients corresponding to the various orifice ratios. It seems to the writer that by making a step further, all the honed-orifice ratios should have been plotted separately on one curve, combining all the tested ratios. The same arrangement could be made with all bored orifices. The change of the coefficient with the size of the orifice ratio would then stand out boldly for each group. The same comments apply to Figs. 15 and 16.

Fig. 14, dealing with a 50-ft. differential head, proves only that for non-compressible fluids, such as used in this test, the coefficient does not change with an increase in the differential. This, however, does not hold true for steam and gases. In this particular case, with a pressure of 85 lb. per sq. in., the compressibility effect would probably be quite an item when measuring the flow of steam. It would be worth while to repeat the same test when presenting the promised results of the steam tests.

Referring to Fig. 17, the broken line is not explained and the conclusion drawn therefrom applies only to the same limits of orifice ratios.

As to Fig. 18, the writer agrees with the authors that tests made with flow nozzles depend on the contour of the curvature. Therefore there can actually be no comparison between any two nozzles unless special pains are taken to duplicate them in every respect.

The authors state that seamless steel tubing was used on the upstream side, and extra heavy pipes on the downstream side, both bored to an exact internal diameter of 3.027 in. In going over the paper, the writer was unable to find where this arrangement was changed to the standard commercial size found in a practical application. It would be interesting to know how much difference a rough commercial pipe would make in the results as compared to those employed in this ideal laboratory installation.

Another important fact brought out was the change in the orifice coefficient with a variation in the flow temperature. The writer hopes that this will be further investigated and reported on, as promised by the authors, when the results of their steam tests are published. He fully agrees with the authors as to the importance of selecting larger differential heads for more accurate metering results, as this brings the coefficients within the range of the straight line over a wider range of flow.

Raymond P. Genereaux.[10] The authors state on the 14th page that "the use of the Reynolds number as a parameter for the study of metering elements is impractical" due to the difficulty of measuring accurately the velocity and density at the measuring section. This implies measuring separately velocity s, in feet per second, and density γ, in pounds per cubic foot, for use in Reynolds number $Ds\gamma/\mu$. The product, $s\gamma$, equals mass velocity, which is independent of pressure and temperature since its units are pounds per second per square foot. Mass velocity is obtained directly from the weight rate of flow (weight of the condensate in the case of steam) divided by the area in square feet of cross-section at any point in the equipment where the Reynolds number is to be used as the parameter. Thus, if the Reynolds number had been written as DG/μ, where D is the diameter in feet at the chosen point, G is the mass velocity in pounds per second per square foot, and μ is the viscosity in pounds per foot per second, it might have been more readily recognized as a practical tool in studying metering elements.

George L. Tuve.[11] One is much impressed with the care used by the authors in controlling all those conditions which are known to have an effect on the coefficient of discharge for primary metering elements. It is rather disappointing to note how great an effect was produced by the variation in certain conditions which have usually been considered unimportant. For instance, the flow-nozzle curves of Fig. 19 show that by such a slight change in surface conditions as that produced by cadmium plating, the coefficient may be affected to the extent of 1 to 2 per cent if the meter has a high diameter ratio and is operating at low rates of flow. A somewhat smaller difference was produced by plating on a venturi tube. Similarly, by different methods of finishing the face of a concentric orifice plate, the results of which cannot be seen except under a microscope, a change of almost 1 per cent in the coefficient of discharge is produced (see Fig. 12). In both of the above cases the general shape of the curve of coefficient against diameter ratio was materially altered.

These variations of coefficient with surface finish bring up a major question with regard to the correlation of earlier data. Presumably the primary elements used in the determination of the orifice-coefficient curves of Fig. 14 in Part 1 of the Fluid Meters Report were all finished in a similar manner, but if important differences cannot be detected with the naked eye, perhaps such differences actually existed. The same applies to the derived curves of venturi-meter coefficients of Figs. 6 and 7 in the Fluid Meters Report. This leads to the question: Are we certain that the variation of discharge coefficient with pipe diameter is correctly shown in Figs. 14, 7, and 6 in Fluid Meters, Part 1?

The effect of pipe diameter is of course taken into account when coefficients are plotted against the Reynolds number, as in the original determination of Figs. 6 and 7 in the Fluid Meters Report. The authors of the present paper reject the use of the Reynolds number for correlating data on compressible fluids, but it surely has a great advantage when dealing with liquids. Even when so applied, however, it cannot be used to correlate the data on different orifices with an accuracy much greater than 1 per cent unless the orifices in question were finished in identical manner.

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[10] Chemical Engineer, E. I. du Pont de Nemours & Co., Inc., Experimental Station, Wilmington, Del. Jun. A.S.M.E.

[11] Associate Professor of Mechanical Engineering, Case School of Applied Science, Cleveland, Ohio. Assoc-Mem. A.S.M.E.

At Case School of Applied Science some experimental work is being done on the variation of orifice coefficients with the Reynolds number, and correspondence is invited from those who may be working along the same lines. In the light of the data given by the authors it would appear that all those experimenting with primary elements had best get together on methods for finishing and for specifying the finish of orifices, nozzles, and venturis if their data are to be expected to be comparable within a limit of less than 1 to 2 per cent.

H. S. Bean.[12] One very striking feature of the authors' work is the apparent pains taken in making their various measurements, particularly in the pressures and pressure differences.

A careful comparison of the numerical results shown for concentric orifices in Figs. 10 and 12 with the values from other sources has not been made, partly because it is difficult to make a satisfactory comparison with the small-scale plots. Also the exact pressure-tap locations used with these tests are not given. However, from Fig. 12, these later 3-in.-line tests appear to be in fair agreement with the earlier 6-in.-line tests. It may be mentioned here that the results of the 6-in.-line tests, as given in Tables 1 and 3 of an earlier paper by Messrs. Beitler and Bucher,[13] have been found to agree very well with the results of numerous other tests when the effects of pressure-tap location have been taken into account. It is to be regretted that the 14 tables mentioned in footnote 4 were not printed with the rest of the paper.

Figs. 13 and 14 show very clearly the effect of the orifice-edge condition upon the value of the discharge coefficients. The magnitude of this edge effect of course varies with the edge condition. In direction, and possibly in magnitude, the results shown agree with those from other tests.

In regard to the nozzle coefficients shown in Fig. 18, there is no doubt but that scattering of the test results for different nozzles is largely due to the contour of the entrance section.[14] The higher coefficient of the "rough" cadmium-plated nozzle is contrary to what would normally be expected. Possibly, as one of the authors suggested, the coefficient of friction of the cadmium surface is enough lower than that for the steel to more than offset the apparently rougher surface.

In regard to the effect of thermal expansion of the orifice plate upon the coefficient, it is interesting to note that according to Fig. 23, the orifice area at 650 deg. fahr. is 0.7 per cent greater than at 250 deg. fahr. According to Fig. 24, the coefficient of the plate based on the diameter at a temperature of 650 deg. is about 0.7 per cent lower than when based on the diameter at 250 deg. Also, for the nozzle, the area increase by Fig. 22 is about 0.6 per cent over the same temperature range, and the coefficient decrease is about 0.6 per cent.

Of course the full explanation of the change in the coefficient with changes in the orifice diameter due to the fluid temperature is by no means as simple as the above. In order to make a full explanation it would be necessary to also take account of such changes as may occur in the mass rate of flow, the different pressure, the density of the fluid, and possibly its viscosity, simultaneously with the change of fluid temperature. This would present a rather complicated problem.

The writer will await with interest the publication of the mass of data to which the authors refer in their closing paragraph.

R. E. Sprenkle.[15] Fig. 10 of the paper shows clearly the effect of orifice-edge sharpness on the coefficient. These data are borne out by additional tests, referred to on the 9th page, which were obtained by rounding the edges of some of the honed orifices previously used in determining the data shown in Fig. 10. The peculiar thing, however, is the relatively small effect that this edge rounding, or lack of sharpness, has on accuracy. Tests made by the Bailey Meter Company Laboratory on orifices having a very marked burr on the inlet side—that is, a burr at least $^1/_{64}$ in. in height—are in very close agreement with the deviation shown by Fig. 10 between sharp and bored orifices, that is, about 0.6 per cent higher value. While this shows the necessity for accuracy in orifice manufacture, nevertheless it indicates that even though the orifice edge should become slightly dull in service, the meter accuracy would not be appreciably affected.

The very "flat" coefficient curves with differential-head variation for honed or sharp-edged orifices shown in Fig. 14 indicate that for the majority of cases a very simple meter mechanism can be used without employing cams or other devices for correcting the variable coefficient to a "flat" or uniform basis. In this respect the orifice presents a distinct advantage over other types of differential pressure-producing devices. It will be noted from Figs. 19 and 20 that the variation of coefficient with head for flow nozzles is about the same as that for venturi tubes.

Referring to the authors' comment on the use of the Reynolds number as a parameter, the writer would agree that little or no good has resulted from such application. In fact, the writer and his associates have tried putting all the data obtained from these tests on a Reynolds-number basis, and have proved its futility. Reynolds' numbers showed such a wide variation in coefficient between different pipe sizes as to render them useless. Possibly this variation was due to the change in absolute roughness from one size of pipe to another, or to a lack of complete similarity. Whatever the reason, the method failed when applied to practical metering.

Two innovations or decided improvements are noted in conducting these tests, namely the use of hook-gage-type manometers for measuring small differentials accurately, and of bull's-eyes for visualizing the exact height of the condensate columns above the manometer. These are refinements not usually associated with tests of this kind, but the results obtained more than justify their use. The writer would earnestly suggest that other experimenters in the field of fluid-flow measurement take equal pains with small details such as these, as only by minute attention to the little things can really worth-while results be obtained.

Authors' Closure

The statement that "the use of the Reynolds number as a parameter is impractical," which has been questioned by Professor O'Brien and Mr. Genereaux, was made only after a large amount of time had been spent attempting to use the Reynolds number in analyzing the results of our work, not only that published, but that which is as yet unpublished. The authors feel that the conclusions stated by Dr. Buckingham in the final paragraph of his discussion cover very satisfactorily the results of our attempt to use the Reynolds number in analyzing these results. It might be stated here that the calculation of the values of the Reynolds number becomes very cumbersome when any number of computations are involved. As in all of the tests reported upon, the values of the Reynolds number are relatively high, this would tend to show that some other factor than viscosity was effecting the results.

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[12] Physicist, U. S. Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

[13] "The Flow of Fluids Through Orifices in 6-In. Pipes," Beitler and Bucher, Trans. A.S.M.E. (1930), paper HYD-52-7a.

[14] See Fig. 12, Bureau of Standards Research Paper 49.

[15] Mechanical Engineer, Bailey Meter Company, Cleveland, Ohio. Assoc-Mem. A.S.M.E.

Professor O'Brien's question about the corrugations in orifice plates can be answered by saying that the corrugations were at all times between the flanges and not on the parts of the orifice which came into contact with the flowing fluid.

Replying to Mr. Feldmark's question about the pipe, the bored tubing was used in all of this work. The inner surface of this tubing was very uniform, but had toolmarks which would cause it to be somewhat comparable to commercial pipe. This pipe was finished in this manner in order to give as nearly ideal conditions as possible. This surface, however, would become rusty and scaly much more quickly than would rolled pipe.

The authors wish to acknowledge with thanks the clarifying discussions of Dr. Buckingham, H. S. Bean, Professor Tuve, and Mr. R. E. Sprenkle.

RP-54-4

O. W. Boston

The Elements of Milling

By O. W. Boston[1] and Charles E. Kraus,[2] Ann Arbor, Mich.

C. E. Kraus

This paper presents the results of laboratory milling tests on a variety of materials, using a pendulum-actuated single-tooth milling dynamometer. Tools of various widths having front-rake angles of 15 deg., but no side-rake or helix angles, were used. Tests were run both dry and with a number of cutting fluids. Except where deliberately allowed to dull, the cutters were considered sharp at all times.

The cutting was done with the cutter rotating in a plane perpendicular to the surface of the work, and in all cases data for two fundamental methods of cutting are shown: first, milling up in the regular way with the work being fed into the cutter, and second, milling down with the work fed with the cutter. In nearly all cases a difference in energy is found between these two methods of cutting.

Data giving the energy in foot-pounds necessary to remove a single chip are shown for thirteen widely different materials with constant feed and various values of depth, and then with constant depth and various values of feed. Several different cutting fluids were used. These data are plotted and energy equations are derived for each material, of the form $E = Cwf^x d^y$, where E is the energy in foot-pounds per chip, C a constant for the specific condition, w the width of the cutter in inches, f the feed in inches, and d the depth of cut in inches. Values of the constants are given. Corresponding formulas and values for the horsepower per cubic inch per minute are also given. No two materials give the same values of the exponents for the feed and depth variables.

Data are also shown for milling with the cutter in the sharp, dull, and very dull conditions, to determine the effect of the sharpness of the tool on the milling-energy formulas. It is found that dulling does change the formula, but has greater effect on feed than on depth of cut.

Tests were run on four materials using eleven different cutting fluids to determine the effect of cutting fluids on milling energy. It is concluded from the nature of the tests that lubrication rather than cooling has greater effect. The data show a different order for the fluids for each material, but in nearly all cases a power saving is made by their use.

Milling-energy data are also presented for a large variety of materials when the cut was held constant at 0.125 in. depth, 0.010 in. feed, and the cutter was 0.250 in. wide and had a 15-deg. front-rake angle, in order to compare these materials from a milling-energy standpoint. It is concluded that there is no way, other than actual test, to determine the milling properties of a material.

A discussion of previous milling papers has been added, and an analysis of the form of a milling chip is presented. It is concluded from this analysis that, theoretically, the coarse-tooth cutter is better than the fine-tooth cutter, when milling up, keeping the linear feed per tooth constant. When milling down, however, the fine-tooth cutter removes a chip of greater average thickness than the coarse-tooth cutter, and consequently is considered to be more efficient from a power standpoint when diameter and feed per tooth are constant.

The fine-tooth cutter cutting down produces the greatest average thickness of chip of all combinations, and is thought to be the most efficient way of milling metal when power is the chief consideration. It is shown that the average thickness of chip when both feed and depth of cut are varied is not a satisfactory basis for an energy equation.

THIS paper presents the results of a series of tests in milling a variety of ferrous and non-ferrous metals. The energy required to remove a chip of a given metal when milling "up," when the work is fed into the cutter, and that when milling "down," when the work is fed with the cutter, were determined and compared. Cutting was done wet and dry. A number of different types of cutting fluids were used, and these are listed in Table 2. Data showing the influence of variable feed with constant depth and variable depth for constant feed are shown. Formulas representing the energy required to mill a given chip as a function of the feed and depth are developed for a number of materials listed in Table 1, when cutting both wet and dry and when cutting up and cutting down. Tests were run with the cutting edge of the tool in various conditions of sharpness, such

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[1] Professor, College of Engineering, University of Michigan. Mem. A.S.M.E. Professor Boston was graduated from the University of Michigan Engineering College in 1913, received a master's degree in 1917, and the degree of mechanical engineer in 1926. After graduation he was engaged at the university as instructor in engineering mechanics and mechanical engineering for four years. In October, 1917, he was commissioned in the U. S. Navy and assigned to duty in the Bureau of Ordnance on design and manufacture of submarine mines used in the North Sea blockade. From 1919 to 1921 he was engaged in industrial engineering work for the Cleveland Tractor Co., in Cleveland, as assistant to the vice-president and works manager. In the fall of 1921, he returned to the University of Michigan, where he is now professor of shop practice and director of the department of engineering shop. He is author of many papers dealing with the subject of metal cutting and is serving as A.S.M.E. representative on the Tool Steel Committee of the American Society for Testing Materials.

[2] After graduation from the high school and junior college in Grand Rapids, Mich., Mr. Kraus entered the University of Michigan in October, 1929, and is now a senior in the College of Engineering, specializing in mechanical engineering, and a member of its A.S.M.E. Student Branch. He holds an appointment as student assistant to Professor Boston in the Department of Engineering Shop.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

as very sharp, dull, and very dull, to find the influence of this variable on the energy required to remove a given chip.

Experimental data are given for several metals, showing the influence of eleven different cutting fluids of various types on the energy required to remove a chip for each method of milling, that is, milling up or down. These materials are listed in Table 1.

TABLE 1 MATERIALS USED IN THE MILLING TESTS

Bar No.	Material	Chemical analysis	Heat treatment	Hardness Brinell	Hardness Rockwell "B"	Hardness Scleroscope
A-1	S.A.E. 1020 steel	Commercial	None	137	84	29
	Cold-rolled steel 1" × 2 1/2"	Commercial low-carbon	None	...	..	..
KF	S.A.E. 3150 steel	0.50 C, 0.022 P, 0.68 Mn, 0.62 Cr, 0.023 S, 1.16 Ni	Annealed	170	..	..
A-13	S.A.E. 6140 steel	Commercial	Anneal 1500, reanneal 1400	163	85.5	28
414	Free-cutting screw-stock steel 1 1/16 round	0.10 C, 0.098 P, 0.66 Mn, 0.195 S	Cold drawn	207	93	29
	Annealed high-speed steel		Annealed	...	..	..
CF	Cast iron 2 × 4 in. (soft)	1% Ni		...	..	..
Cu	Pure copper	100% Cu	Annealed	40.4	28	6
BF	Leaded screw-stock brass	62% Cu, 34% Zn, 3% Pb	Cold drawn	...	..	..
187-A	Leaded screw-stock brass	62% Cu, 35% Zn, 3% Pb	Annealed	49.9	63	12.6
50-A	Unleaded brass	0.25% Pb, 65.5% Cu, 34.15% Zn	Annealed	50.9	61	12.7

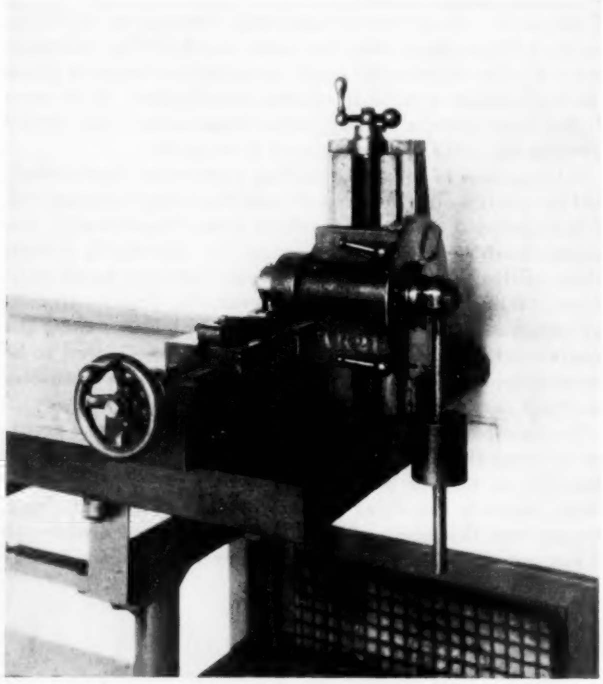

FIG. 1 THE PENDULUM-TYPE MILLING DYNAMOMETER

Finally, comparative data are given showing the average energy required to remove a given chip by the milling-up and milling-down processes for a large variety of metals when cutting dry. Additional information showing the influence of a mineral lard oil on these data is also presented. These materials are listed in Tables 22, 23, and 24.

THE MILLING CUTTERS AND TESTING EQUIPMENT

The machine used was a milling dynamometer of the pendulum type, constructed by Carl Oxford,[3] and was first described in the paper, "The Art of Milling."[4] It is illustrated in Fig. 1. It was calibrated to read directly on the graduated dial the energy in foot-pounds required to remove a given chip.

This milling machine is constructed similarly to the knee type. A single-tooth cutter is held in one end of the arbor overhanging the table. It is driven by a heavy pendulum attached to the other end of the arbor. If the pendulum be allowed to fall freely from the top vertical position, it will swing again almost to the same height, that is, nearly complete one revolution. A freely rotating pointer is carried forward with the pendulum as it rotates and indicates on a steel disk the highest position to which the pendulum swings. This position is taken as the zero reading to allow for friction in the bearings.

If the material is bolted to the table and the cutter adjusted for depth of cut and feed, a subsequent fall of the pendulum will cause a chip to be removed, and the pendulum will not swing so high after taking the chip as when taking no chip. This difference in height is a result of the energy in foot-pounds given up in forming the chip. It is the same principle as that used in impact testing machines. The face of the steel disk is graduated at its periphery, so that the reading of energy absorbed by taking the chip of any given material and size is read directly.

The single-tooth milling cutters were adjusted so as to have a radius of 1.75 in. For most tests these cutters had a rake angle or hook of 15 deg., and end- and side-clearance angles of 4 deg. After adjusting the material to be cut on the milling-machine table, the cutter was fed vertically downward from the surface to the required depth of cut as successive cuts were taken. The work was then fed horizontally the required distance for each chip. All data given in this paper were obtained by taking cuts in this manner, so that a groove was formed in the work.

THE MATERIALS TESTED

The materials tested have been reported in previous papers. They are summarized, however, in Table 1. Chemical analyses, heat treatments, and Brinell, Rockwell, and scleroscope hardness numbers are given.

THE CUTTING FLUIDS USED

The cutting fluids used in these tests are listed with their properties in Table 2. They were previously reported on in a paper entitled "Performance of Cutting Fluids."[5] They were selected to represent a wide variety of types commonly used.

The influence of the cutting fluids in tests of this nature is due probably more to lubrication than to cooling. The cutting is intermittent; that is, considerable time elapses between successive cuts, so that there is no perceptible rise in temperature of the tool, work, or cutting fluid as the tests are continued, because of the heat generated between individual cuts. The time in removing a chip is so small that there can be little heat transfer during the cutting action.

THE RESULTS OF TESTS

The tests run for this paper covered a variety of sub-classifications and are, therefore, treated separately below.

THE EFFECT OF CUTTER WIDTH

The cutter resembled an ordinary cut-off tool of the end-cutting type, being square on the end. Tests were run with cutters varying in width from 0.22 in. to 0.71 in. in an effort to determine the influence of width on the energy required to

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[3] Chief Engineer, National Twist Drill and Tool Company, Detroit, Mich. Assoc-Mem. A.S.M.E.

[4] Trans. A.S.M.E., vol. 43 (1921), p. 549.

[5] Trans. A.S.M.E., vol. 53 (1931).

TABLE 2 PROPERTIES OF CUTTING FLUIDS USED

[From "Performance of Cutting Fluids," by O. W. Boston and C. J. Oxford, Trans. A.S.M.E., vol. 53 (1931)]

No.	Composition	Base of mineral oil	Per cent compounding by analysis	Viscosity at deg. fahr. 100	Viscosity at deg. fahr. 130	Viscosity at deg. fahr. 210	A.P.I.[4] gravity at 60 deg. fahr.	d specific gravity	Temperature, deg. fahr. Pour	Temperature, deg. fahr. Flash	Temperature, deg. fahr. Fire	N. P. A. color[5]	C Specific heat[2] 100 deg. fahr.	C Specific heat[2] 210 deg. fahr.	C Specific heat[2] 430 deg. fahr.	k Thermal conductivity[3] 100 deg. fahr.	k Thermal conductivity[3] 210 deg. fahr.	k Thermal conductivity[3] 430 deg. fahr.
1	Dry cutting	......	....	...	...	..	..	....	....	.....	...	...	....	....	....	....	....	....
2	$1\frac{1}{2}$ per cent by weight of borax in water	......	1.19 borax	...	...	..	..	....	....	.....	...	...	....	....	....	....	....	....
3	1 part soluble oil in 50 parts water	......	2.3 soluble oil	31	...	..	..	....	....	.....	...	...	....	....	....	....	....	....
4	1 part soluble oil in 10 parts water	......	8.37 soluble oil	32	...	..	..	....	....	.....	...	...	....	....	....	....	....	....
5	No. 1 lard oil	......	12.23–13.6 F.F.A.[1] 86.4 fat	200	110	..	23.7	0.9117	45	445	...	4	0.4510	0.5149	0.6438	0.8738	0.8448	0.7852
6	Light mineral oil	Mixed	....	111	69	40.5	23.5	0.9129	0 to −30	320–305	375	$2-2\frac{1}{4}$, $2\frac{1}{2}-3$	0.4500	0.5142	0.6430	0.8724	0.8440	0.7845
7	Heavy mineral oil	Mixed	....	209	104	45.5	23.0	0.9159	0 to −15	340–350	390	$2\frac{3}{4}-3$, 3	0.4490	0.5137	0.6417	0.8695	0.8400	0.7818
8	10 per cent No. 1 lard oil in mineral oil	Paraffin	10.12 lard oil	101	67	41	29.4	0.8794	25	355	...	3	0.4630	0.5290	0.6612	0.9052	0.8750	0.8145
9	5 per cent oleic acid in mineral oil	Paraffin	4.90 oleic acid	97	65	40.5	29.2	0.8805	25	295	...	3.5	0.4624	0.5284	0.6610	0.9048	0.8748	0.8138
10	Sulphurized mineral oil	Asphaltic	4.38 sulphur	105	67	40	21.6	0.9242	0 to −25	285	...	Black	0.4459	0.5096	0.6374	0.8620	0.8344	0.7752
11	Sulphurized lard oil blended with 5 parts mineral oil	Mixed	2.34 sulphur 4.43 F.F.A. sulphurized 13.94 fat sulphurized	102	67	41.5	25.4	0.9018	25 to 20	355	...	Black	0.4542	0.5190	0.6490	0.8837	0.8548	0.7942

[1] F.F.A. is free fatty acids.

[2] Specific heat C computed from formula $C = (t + 670)(2.10 - d)/2030$, in which t = temperature of fluid deg. fahr., and d = specific gravity of oil at 60 deg. fahr.

[3] Thermal conductivity k computed from formula $k = (0.813/d)\ [1 - 0.0003(t - 32)]$, in which d = specific gravity of the oil at 60 deg. fahr., t = temperature of fluid in deg. fahr.; k is in units of B.t.u. per hr. per sq. ft. per in. per deg. fahr.

[4] American Petroleum Institute.

[5] National Petroleum Association.

mill metal when other factors were constant. Due to the difficulty of maintaining the same degree of sharpness and, with the wider tools, the necessity of correcting resulting data because of deflections permitting actual change in depth of cut, it was practically impossible to get favorable comparative data. It can be said, however, that when cutting in a groove, the energy required varies not as the first power of the width but as a slightly lower power. It is felt that, if the cutting were done from the top of the land, the width of the land determining the width of chip, the energy would vary more directly with the width of the cutter. The energy required to cut along the side or edge of the chip is probably not a function of the width.

In an earlier paper,[6] it was found that, when cutting on a land with a planer tool, the force expressed in pounds per square inch of chip removed was constant for a given depth of cut and variable widths, but dropped off slightly when the width of cut became less than from 0.100 to 0.200 in.

In this paper the assumption is made that the energy varies directly as the width. For the purpose of comparison, data from cutters varying in width from the 0.231 to 0.249 in. have been corrected to conform with a cutter of 0.250 in. in width. The error thus introduced is believed to be negligible.

VARIABLE-FEED-AND-DEPTH DATA

S.A.E. 1020 Steel. A number of variable-feed-and-depth tests were run on bar A-1, S.A.E. 1020 steel, using different cutting fluids, to determine the milling-energy formula per chip for this metal, and to find the effect of cutting fluids on this formula. The data are given in Tables 3 and 4, and are shown plotted in Figs. 2 and 3.

It is found that the slope of the lines drawn on log-log paper, showing the relation between energy as a function of feed for constant depth and as energy for variable depth and constant feed, is the same for all of the wide variety of cutting fluids used as for dry cutting. These lines for different cutting fluids are merely displaced vertically. This has been found to be true for the cutting fluids on a number of metals which have been cut both wet and dry.

A discussion of the effect of cutting fluids is presented later.

In Table 3 are given the results of various tests for a number of cutting fluids when cutting the S.A.E. 1020 steel with the 0.250-in.-wide cutter having 15 deg. front rake. The upper part of the table shows the feed constant at 0.010 in. and the depth varied from 0.010 to 0.150 in. in seven steps. These data are shown plotted in Fig. 3. The milling-energy values in foot-pounds per chip are plotted as ordinates, with the depth in inches as abscissas. Parallel straight lines are obtained with these data plotted on log-log paper. The solid lines represent the values for milling up, while the broken lines represent the milling energy when cutting down. One exception to this is the milling-down curve of cutting fluid No. 6 (light mineral oil), which is a broken line superimposed on the milling-up or solid line of cutting fluid No. 8 (10 per cent lard in mineral oil). In all cases the broken line for a given oil is below the solid line for that oil, indicating that for this material the energy consumed in milling down is less than when milling up. In milling up, as expressed by the solid lines, No. 10 (sulphurized mineral oil) gives the lowest values, with No. 5 (lard oil) next, then No. 8 (10 per cent lard in mineral oil), while No. 4 (a 10-to-1 soluble oil) and No. 6 (a light mineral oil) are identical at the top. The energy for dry cutting is shown still farther up. Similarly, in considering, the energy values in milling down, No. 5 cutting fluid (lard oil) is the lowest, followed in order by Nos. 10 (sul-

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[6] "The Elements of Metal Cutting," Trans. A.S.M.E., vol. 48 (1926), p. 749.

TABLE 3 ENERGY REQUIRED IN MILLING S.A.E. 1020 STEEL, USING VARIOUS CUTTING FLUIDS

(Cutter, 0.250 in. wide, with 15-deg. front-rake angle. Energy values given in foot-pounds required per chip for milling up and down. Values are shown plotted in Figs. 2 and 3)

Feed in inches	Depth in inches	Oil No. 1 (Dry)		Oil No. 4		Oil No. 5		Oil No. 6		Oil No. 8		Oil No. 10	
		Up	Down	Up	Down	Up	Down	Up	Down	Up	Down	Up	Down
0.010	0.010	2.00	1.67	1.57	1.34	1.43	1.17	1.60	1.51	1.52	1.38	1.40	1.25
0.010	0.025	4.25	3.40	3.13	2.76	2.97	2.36	3.10	3.05	3.12	2.90	2.72	2.57
0.010	0.050	6.99	6.00	5.40	4.58	5.25	4.19	5.45	5.36	5.32	4.80	4.85	4.13
0.010	0.075	9.80	7.98	7.51	6.50	7.05	5.66	7.48	7.41	7.26	6.70	7.20	5.94
0.010	0.100	12.20	10.50	9.14	8.21	8.57	7.13	9.28	9.10	9.21	8.25	8.19	7.21
0.010	0.125	14.40	11.95	11.05	9.90	10.50	8.70	11.14	10.97	10.85	10.10	9.63	8.98
0.010	0.150	16.30	14.00	...	...	12.00	9.96	...	...	12.65	11.79	11.20	10.00
0.002	0.125	4.71	4.41	4.10	3.71	3.90	3.26	...	...	...	...	3.61	3.31
0.005	0.125	8.23	7.32	7.34	6.39	6.90	5.85	...	...	...	...	6.50	5.90
0.010	0.125	13.10	11.15	11.05	9.90	10.85	9.16	...	...	...	...	9.63	8.98
0.015	0.125	16.60	15.00	14.10	13.63	13.69	11.68	...	...	...	...	12.85	11.95
0.020	0.125	20.00	17.80	18.40	17.02	16.87	14.35	...	...	...	...	15.80	14.75

TABLE 4 ENERGY REQUIRED IN MILLING S.A.E. 1020 STEEL, CUTTING DRY (NO. 1 OIL)

(Cutter 0.347 in. wide, 15-deg. front-rake-angle. Energy values given in foot-pounds required per chip, milling up and down)

Feed in inches	Depth in inches	Milling	
		Up	Down
0.010	0.010	2.60	1.97
0.010	0.025	4.94	4.20
0.010	0.050	9.12	6.60
0.010	0.100	14.95	12.20
0.002	0.100	5.50	4.71
0.005	0.100	10.10	8.43
0.010	0.100	14.95	12.20
0.015	0.100	20.23	16.85

Cutting with a 10-to-1 soluble oil (No. 4), with a 0.250-in. tool, 0.050 in. depth

Feed in inches	Up	Down
0.005	3.53	2.98
0.010	5.52	4.75
0.015	7.27	6.38
0.020	8.70	7.59

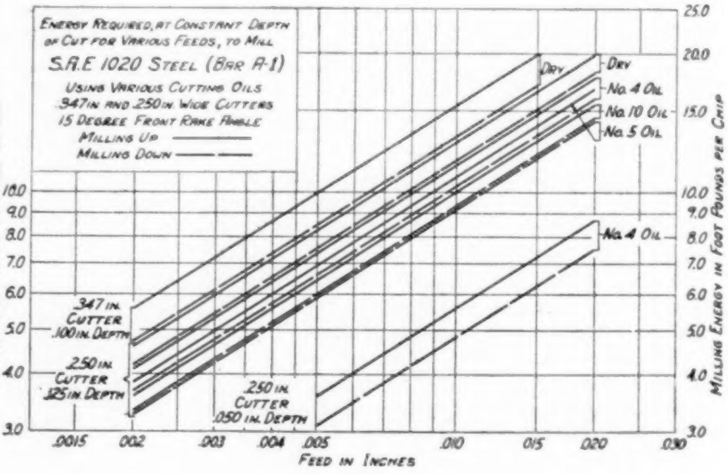


FIG. 2

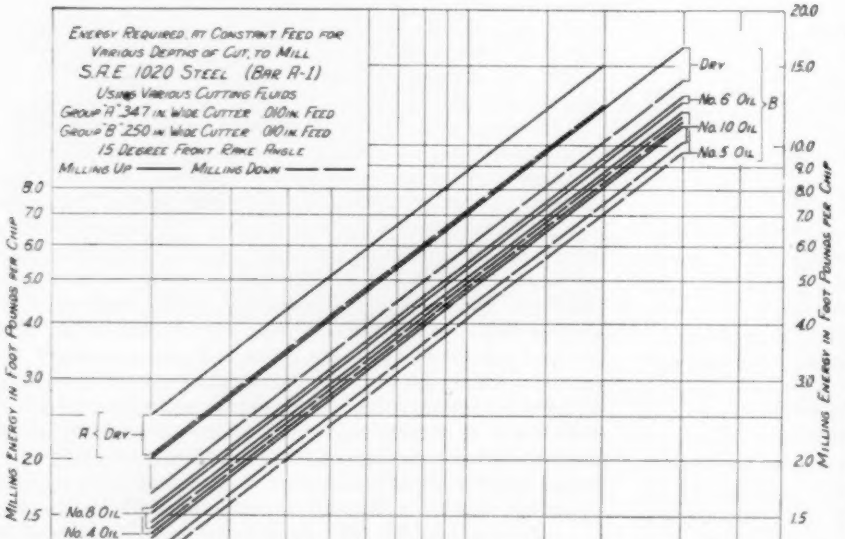


FIG. 3

phurized mineral oil), 4 (a 10-to-1 soluble oil), 8 (10 per cent lard in mineral oil), 6 (a light mineral oil), and dry.

It is interesting to note that these lines as plotted on log-log paper are parallel, so that an equation, representing the energy required to remove a chip when expressed as a function of depth of cut and feed, can be modified by changing the constant to hold for all oils.

At the bottom of Table 2 is shown another series of tests on S.A.E. 1020 steel using the 0.250-in.-wide cutter in which the depth of cut is constant at 0.125 in., while the feed is changed from 0.002 in. to 0.020 in. in five steps. These energy values are shown plotted in Fig. 2 for the 0.250-in. cutter at 0.125 in. depth. These data for cutting fluids Nos. 5, 10, 4, and dry cutting, when plotted on log-log paper, again give parallel straight lines. The broken lines represent the down-cutting energy values, while the solid lines represent the up-cutting values. Of the up-cutting lines, No. 10 is lowest, followed by Nos. 5, 4, and dry cutting, in the order of increasing value. Similarly, for down cutting No. 5 gives the lowest energy value, followed in order by Nos. 10, 4, and dry cutting. In all cases the down-cutting values are less than the up-cutting ones.

In Table 4 are listed the results of another series of milling-cutter tests on the S.A.E. 1020 steel. The upper part of the table shows tests in which the depth was varied from 0.010 in. to 0.100 in. for a constant feed of 0.010 in. These energy values are for a tool 0.347 in. wide but having the same 15-deg. front-rake angle. The cutting was done dry. These constant-feed data are shown plotted in Fig. 3 (indicated at *A*), for dry cutting.

Again, two straight but parallel lines are obtained, the broken line, representing down-cutting values, being far below the solid line which represents the up-cutting values. It is seen, however, that these lines for the 0.347-in.-wide tool are parallel to but higher than the corresponding dry-cutting lines of the 0.250-in.-wide tool, showing that the width does not influence the exponents of the variable. Table 4 in the center shows another set of milling-energy values when cutting dry with the 0.347-in. wide cutter when the depth is constant at 0.100 in. and the feed is increased from 0.002 to 0.015 in. in four steps. These data are plotted in Fig. 2 at the top, and again lines parallel for the up- and down-cutting values are obtained which also are parallel to the up- and down-cutting values of the 0.250-in.-wide cutter, showing that the exponent of feed does not vary with the width of cut.

At the bottom of Table 4 is shown still another set of data in which the 0.250-in.-wide tool was used with cutting fluid No. 4 (a 10 per cent soluble oil), with the depth of cut reduced to 0.050 in. These values are shown plotted in the lower portion of Fig. 2. Even with this shallow depth, lines parallel to the other milling-energy curves are obtained, with the milling-up-energy curves well above the milling-down-energy curves, showing that a variation in depth does not influence the exponents of the variable.

The formula for energy in foot-pounds required to remove a chip of S.A.E. 1020 steel, bar A-1, may be expressed as

$$E = Cwf^{0.64}\,d^{0.78} \qquad \ldots\ldots [1]$$

in which C is a constant for the 0.250-in.-wide tool having a 15-deg. front-rake angle, and the particular material, w the width of cutter in inches, f the feed in inches per chip, and d the depth of cut in inches. The values of C are given for the several conditions in Table 5, and summarized with other values in Table 18.

TABLE 5 VALUE OF C IN MILLING-ENERGY FORMULA ($E = Cwf^{0.64}\,d^{0.78}$), FOR S.A.E. 1020 STEEL (A-1)

(Using a 15-deg. front-rake-angle tool with no side rake)

Cutting fluid		Milling		Ratio of $\frac{\text{Down}}{\text{Up}}$
Type	No.	Up	Down	
Dry	1	5520	4650	0.843
10-to-1 emulsion	4	4320	3790	0.878
Lard	5	3980	3170	0.796
Light mineral	6	4320	4150	0.960
10 per cent lard	8	4150	3900	0.940
Sulphurized mineral oil	10	3700	3400	0.919

Also, the equation giving the horsepower per cubic inch of metal cut per minute may be expressed as

$$\text{Hp. per cu. in. per min.} = \frac{E}{33{,}000 \times \text{vol. of chip}} = \frac{E}{33{,}000\,wfd}$$

or, substituting for E from Equation [1],

$$\text{Hp. per cu. in. per min.} = \frac{Cwf^{0.64}\,d^{0.78}}{33{,}000\,wfd} = \frac{C}{33{,}000\,f^{0.36}\,d^{0.22}} \quad \ldots [2]$$

Cold-Rolled Low-Carbon Steel. A series of variable-feed-and-depth tests were run on a cold-rolled low-carbon steel. The data are given in Table 6. Lard oil (No. 5) was used, and milling cuts both up and down for each condition were taken. The data are tabulated under depths of cut of 0.025, 0.050, and 0.075 in. for a feed per chip of 0.005 and 0.010 in., respectively. In Fig. 4, curves A, B, and C show the data plotted over variable feeds for constant depths of 0.025, 0.050, and 0.075 in., respectively. These data are obtained with the cutter in a sharp condition. It is noted that there is a slight angular divergence between the solid and broken lines, that is, the cutting-up and down curves, for each depth. This appears to be an influence of degree of tool sharpness, as is discussed below. It is not of particular importance as long as the cutting tool is sharp, but when it is dull, the angular divergence becomes greater. (See Fig. 15.) For each depth the milling-down curve values are seen to be less than those for milling up by practically a constant percentage.

TABLE 6 ENERGY REQUIRED IN MILLING COLD-ROLLED LOW-CARBON STEEL

[Cutter, 0.243 in. wide; using lard oil (No. 5), milling up and down. Values plotted in Fig. 4]

	0.025 Depth		0.050 Depth		0.075 Depth	
	0.005 feed	0.010 feed	0.005 feed	0.010 feed	0.005 feed	0.010 feed
Up	1.42	2.32	2.53	4.25	3.60	6.04
Down	1.25	1.95	2.24	3.58	3.17	5.05

Lines D and E in Fig. 4 show the milling-energy values plotted for constant feeds of 0.005 and 0.010 in., respectively. For each depth, cutting-up and cutting-down lines are shown. The cutting-up and cutting-down lines are parallel in themselves, and the energy lines for different feeds also are parallel in themselves. The slope of the constant-feed lines, however, is somewhat greater than that of the constant-depth lines. Using these curves as a basis, in which lard oil was used as a lubricant, a milling-energy formula is obtained as follows:

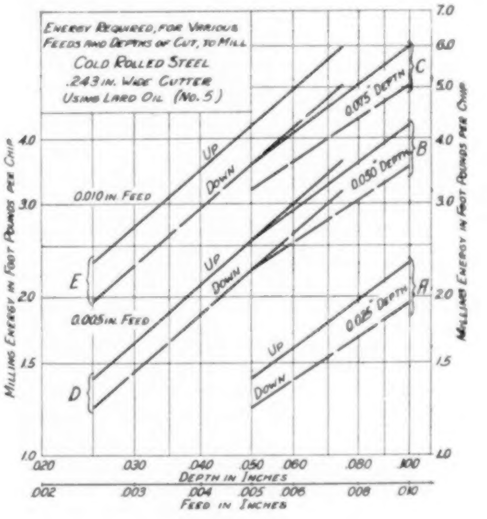


FIG. 4

$$E = Cwf^{0.73}\,d^{0.87} \qquad \ldots\ldots [3]$$

For this condition the value of C when milling up, using lard oil, is found to be 6800. In milling down, the energy equation becomes:

$$E = Cwf^{0.67}\,d^{0.87} \qquad \ldots\ldots [4]$$

For this condition the value of C, using lard oil as a lubricant, becomes 4260. These equations and constants are summarized in Table 18.

S.A.E. 3150 Steel. Milling-energy values for a constant feed of 0.010 in. for various depths of cut ranging from 0.010 to 0.125 in. are shown in the upper part of Table 7. These values are for dry cutting and are given for both up and down cutting. They are shown plotted in Fig. 5 as curves indicated at A. The down-milling-energy values are consistently lower than the corresponding values for milling up. The down and up lines, however, are parallel. The slope of these variable depth curves is unusual, being 1.01, which signifies that deep cuts are removed less efficiently from a power or energy standpoint.

for this material. The energy values for a constant depth of 0.125 in. and variable feeds for both dry cutting (No. 1) and when cutting with a sulphurized mineral oil (No. 10) are shown for both up and down cutting in the lower part of Table 7. These values are shown plotted in Fig. 5 at *B*. Four parallel lines are obtained, with the down-cutting-energy values lower than the up-cutting values for both cutting fluids Nos. 1 and 10. From these curves, a formula representing milling energy is obtained as follows:

$$E = Cwf^{0.7}\, d^{1.00} \qquad [5]$$

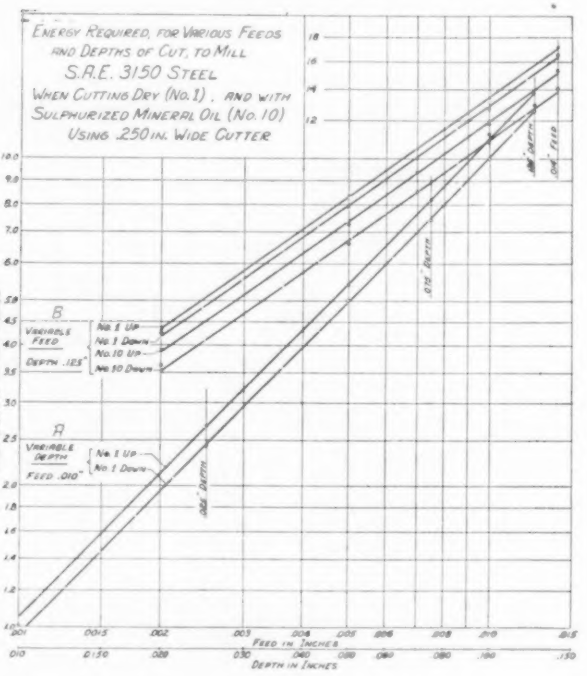


FIG. 5

which holds for both up and down cutting. The value of *C* for dry cutting (No. 1) is 11,000 for milling up and 10,600 for milling down. For the sulphurized mineral oil (No. 10) it is 9780 for milling up and 8980 for milling down. These formulas and constants are summarized in Table 18.

S.A.E. 6140 Steel. Additional milling-energy data for various depths of cut and feeds are shown in Table 8. For a constant feed of 0.010 in., the depth was varied from 0.010 to 0.150 in. when milling both up and down with a 10 per cent lard, 90 per cent mineral oil (No. 8). These values are shown plotted at *A* in Fig. 6. The broken line, representing the milling-energy values on the down stroke, is seen to be parallel to, but above, the corresponding energy values for milling up. This is the first instance in the data presented that the milling-down values have appeared to be above the milling-up values.

The lower part of Table 8 gives the results of energy values for a constant depth of 0.125 in., but for various values of feed when cutting dry and with oil No. 8. These values are shown plotted at *B* in Fig. 6. The up-and-down energy values for each of the two oils gave four parallel lines when plotted over variable feed. Again, in each instance the milling-down lines are above the corresponding milling-up lines, although the percentage difference is not large. The formula representing milling energy as a function of cutter width, feed, and depth of cut as determined from the curves in Fig. 6 may be written as

$$E = Cwf^{0.72}\, d^{0.90} \qquad [6]$$

TABLE 7 ENERGY REQUIRED IN MILLING S.A.E. 3150 STEEL

(Cutter, 0.250 in. wide; cutting dry and with sulphurized mineral oil. Values plotted in Fig. 5)

Feed		Depth in inches					
0.010		0.010	0.025	0.050	0.075	0.100	0.125
Oil No. 1....	Up	1.11	2.69	5.40	8.16	11.25	12.98
	Down	1.01	2.43	5.02	7.48	10.05	12.65

Depth		Feed in inches				
0.125		0.002	0.005	0.007	0.010	0.014
Oil No. 1.........	Up	4.31	8.37	10.54	13.40	17.20
	Down	4.23	7.91	10.07	12.70	16.56
Oil No. 10........	Up	3.90	7.31	9.39	11.89	15.40
	Down	3.62	6.61	8.52	10.88	14.15

TABLE 8 ENERGY REQUIRED IN MILLING S.A.E. 6140 STEEL

[Cutter, 0.250-in. wide; cutting with a 10-per cent lard–90-per cent mineral oil (No. 8), milling up and down. Values plotted in Fig. 6]

Feed in inches	Depth in inches	Oil No. 1 (Dry) Up	Oil No. 1 (Dry) Down	Oil No. 8 Up	Oil No. 8 Down
0.010	0.010	...	...	1.45	1.55
0.010	0.025	...	...	3.40	3.57
0.010	0.050	...	...	6.45	6.75
0.010	0.075	...	...	9.60	9.70
0.010	0.100	...	...	12.35	12.60
0.010	0.125	...	...	14.80	15.30
0.010	0.150	...	...	17.15	17.39
0.001	0.125	2.90	3.00	2.65	2.71
0.003	0.125	6.45	6.69	5.85	6.37
0.005	0.125	9.65	9.55	8.77	9.10
0.010	0.125	14.91	15.85	14.51	15.22

in which the value of *C* for dry cutting (No. 1) is 10,630 for milling up and 11,220 for milling down. For oil No. 8 it is 9950 for milling up and 10,520 for milling down. These values are summarized in Table 18.

Bessemer Screw Stock. Data showing the milling energy in foot-pounds to mill a free-cutting cold-drawn steel, bar 414, are shown listed in Table 9 for a constant feed of 0.010 in. and depths of cut from 0.010 in. to 0.100 in. in four steps. Values are given for milling both up and down. Also energy values are given for a constant depth of cut of 0.100 in. and for three additional feeds of 0.005, 0.015, and 0.020 in., respectively.

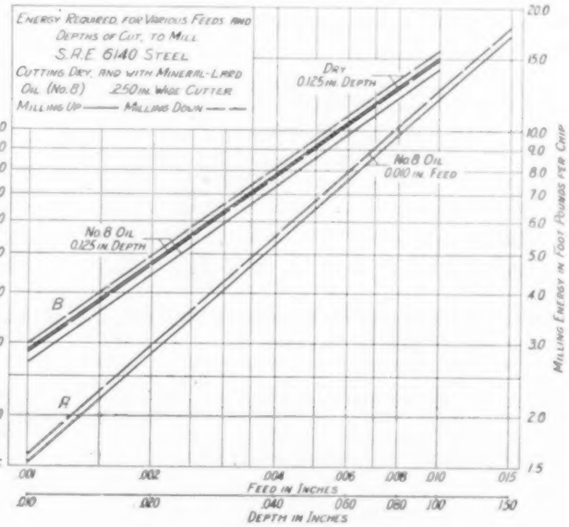


FIG. 6

These values are shown graphically in Fig. 7. The cutter width for these tests was 0.250 in., and a light mineral oil (No. 6) was used. It is seen that for the cutter width of 0.010 in. the milling-down curve is lower but parallel to the milling-up curve. Also the milling-energy curve for various feeds with a constant depth of cut of 0.100 in. shows the milling-down line lower but parallel to the milling-up line. The equation representing the energy in foot-pounds required to remove one chip of this free-cutting screw stock is found to be:

$$E = Cwf^{0.77}\,d^{0.86} \qquad [7]$$

in which C for oil No. 6 when cutting up is 8180 and when cutting down is 6720. This equation is given with others in Table 18.

High-Speed Steel. Table 10 gives data for various combinations of feed and depth when cutting both dry (No. 1) and with mineral lard oil (No. 8) when milling up and down. The values are plotted for constant depth and constant feed, respectively, on log-log paper in Fig. 8. For this combination the cutting-down-energy values are greater than those for cutting up for both constant feed and depth. Also there appears to be little difference in energy values per chip whether the annealed high-speed steel is cut dry or with a mineral lard oil (No. 8).

The equation representing the milling energy as a function of the cutter width, feed, and depth was found to be

$$E = Cwf^{0.73}\,d^{0.84} \qquad [8]$$

in which the values of C for milling up with both dry cutting (No. 1) and mineral lard oil (No. 8) were found to be practically the same at 9700. The value when milling down was found to be 10,400 for both oils Nos. 1 and 8, as given in Table 18.

Cast Iron. The milling-energy data were obtained from a set of cast-iron test bars 2 in. × 4 in. × 14 in. These bars contained 1 per cent of nickel. Readings were taken when milling dry for cuts both up and down. The milling-energy data for various cuts are given in Table 11 and plotted on log-log paper in Fig. 9.

Again it is seen that the milling-energy values when cutting down for both the constant feed of 0.010 in. and constant depth of 0.150 in. are greater than when cutting up, as was found in the case of the annealed high-speed steel. The milling-energy formula was found to be:

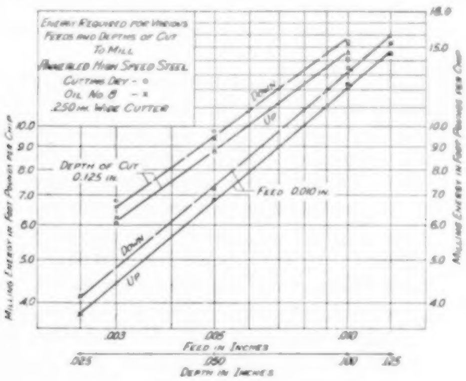


Fig. 8

$$E = Cwf^{0.41}\,d^{0.56} \qquad [9]$$

in which values of the constant C when cutting dry were found to be 595 when cutting up and 643 when cutting down. The exponents of the feed and depth in this equation are the lowest for all the metals tested. This material caused the tool to dull quickly so that considerable precaution was needed in keeping the tool cutting edge in the sharp condition.

Pure Copper. Further tests to determine the influence of variable feed and depth when milling were run on pure copper. The results of these tests are shown in Table 12, and the data are plotted in Fig. 10. Results of two series of tests are shown, the first having a constant feed of 0.010 in. with variable depths from 0.010 in. to 0.150 in. in seven steps. For these tests oil No. 8, consisting of 10 per cent lard oil in 90 per cent light mineral oil, was used. The second series of experiments were varied for values of feed between 0.001 and 0.010 in. per chip for a constant depth of cut of 0.125 in. when cutting dry (cutting fluid No. 1), and when cutting with a 10 per cent lard oil in 90 per cent light mineral oil (No. 8). For each cutting condition, cuts were made both on the up stroke and down stroke.

The results presented in Table 12 for the constant feed of 0.010 in. for variable depth are shown plotted as the two lowest lines,

TABLE 9 ENERGY REQUIRED IN MILLING FREE-CUTTING SCREW-STOCK STEEL NO. 414

[Cutter, 0.250-in. wide; cutting with light mineral oil (No. 6), milling up and down]

Feed in inches	Depth in inches	Milling Up	Milling Down
0.010	0.010	1.12	0.88
0.010	0.025	2.42	2.06
0.010	0.050	4.52	3.64
0.010	0.100	8.11	6.67
0.005	0.100	4.75	4.05
0.015	0.100	11.18	9.44
0.020	0.100	13.89	12.05

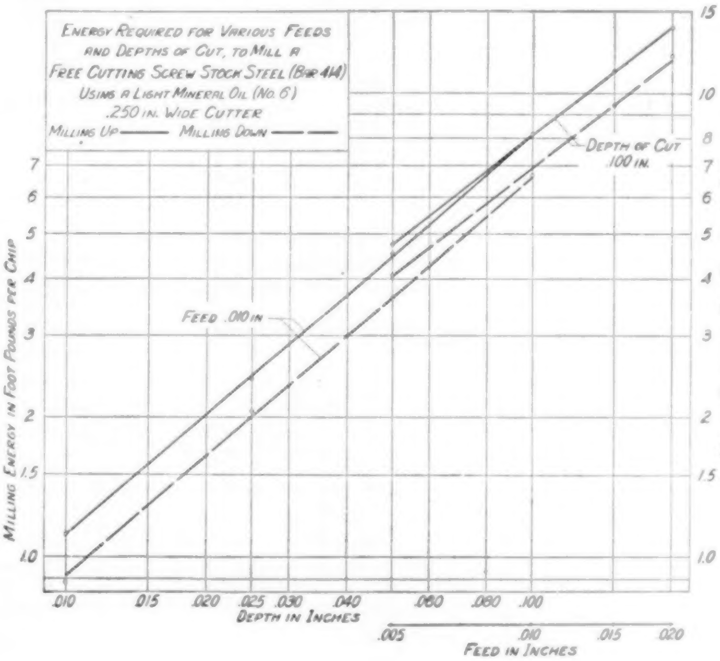


Fig. 7

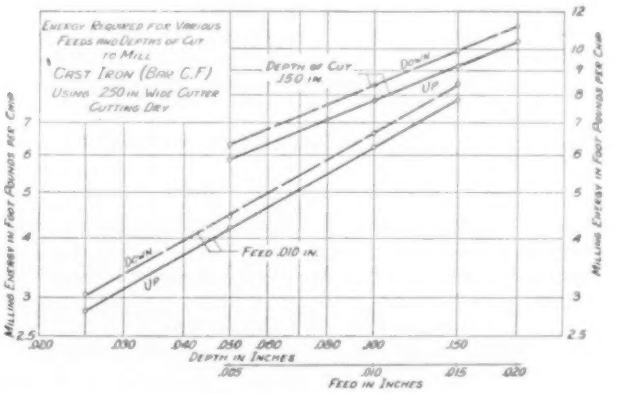


Fig. 9

TABLE 10 ENERGY REQUIRED IN MILLING ANNEALED HIGH-SPEED STEEL

[Cutter, 0.250-in. wide; cutting dry (No. 1) and with mineral lard oil (No. 8); milling up and down]

Feed in inches	Depth in inches	Oil No. 1 (Dry) Up	Oil No. 1 (Dry) Down	Oil No. 8 Up	Oil No. 8 Down
0.010	0.025	...	...	3.78	4.12
0.010	0.050	...	...	6.81	7.2
0.010	0.100	...	...	12.40	13.48
0.003	0.125	6.20	6.80	6.1	6.6
0.005	0.125	8.73	9.73	8.83	9.37
0.010	0.125	14.08	15.45	14.55	15.25

TABLE 11 ENERGY REQUIRED IN MILLING CAST IRON (CF)

[Cutter, 0.250 in. wide; cutting dry (No. 1); milling up and down]

Feed in inches	Depth in inches	Milling Up	Milling Down
0.010	0.025	2.82	3.04
0.010	0.050	4.20	4.43
0.010	0.100	6.20	6.62
0.010	0.150	7.80	8.40
0.005	0.150	5.85	6.30
0.015	0.150	9.20	9.90
0.020	0.150	10.40	11.20

indicated at *A*, in Fig. 10. The experimental points indicate the two lowest straight lines in the figure, the upper or solid one of which represents the energy required to remove the chip on the up stroke, while the broken line represents the energy required to remove the chip on the down stroke. In both instances oil No. 8 was used. It is seen that the experimental points conform quite closely to the indicated lines. The two lines are parallel, indicating that the coefficient of the variable depth in the energy or power equation remains constant.

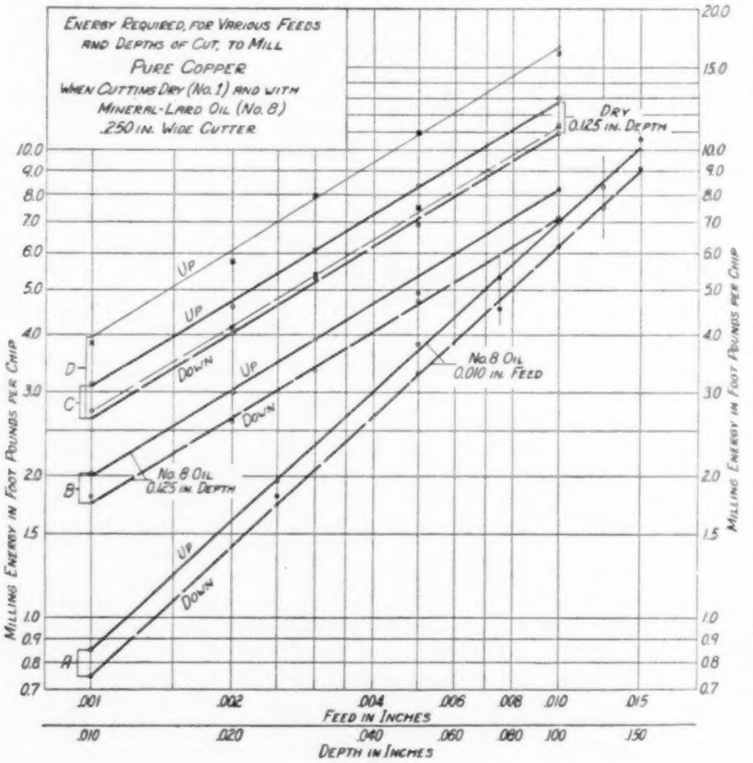


FIG. 10

TABLE 12 ENERGY REQUIRED IN MILLING PURE COPPER

(Cutter, 0.250 in. wide; cutting dry and with No. 8 oil; milling up and down. Values are shown plotted in Fig. 10)

Feed in inches	Depth in inches	Oil No. 1 (Dry) Up	Oil No. 1 (Dry) Down	Oil No. 8 Up	Oil No. 8 Down
0.010	0.010	...	...	0.85	0.75
0.010	0.025	...	...	1.95	1.81
0.010	0.050	...	...	3.83	3.29
0.010	0.075	...	...	5.30	4.53
0.010	0.100	...	...	7.00	6.10
0.010	0.125	...	...	8.30	7.55
0.010	0.150	...	...	10.15	9.10
0.001	0.125	3.13	2.75	2.02	1.80
0.002	0.125	4.60	4.02	3.00	2.61
0.003	0.125	6.10	5.30	3.91	3.35
0.005	0.125	8.35	6.85	4.93	4.69
0.010	0.125	12.90	10.75	8.10	7.12

TABLE 13 EFFECT OF SLIGHT AMOUNT OF OIL ON CUTTING ENERGY WHEN MILLING PURE COPPER

(Cutter, 0.250 in. wide; feed, 0.010 in.; depth, 0.125 in.)

	Energy in foot-pounds per chip: Dry, Clean [a]	Dry, Not clean [b]	Oil No. 8
Up	16.75	14.50	7.47
Down	11.15	10.89	6.91

[a] Tool and work cleaned with alcohol. Adhering chips removed with a clean screwdriver.

[b] Same as [a], except chips removed with fingers. Oil film on tool was not visible.

The data of Table 12 for the variable feed values are shown plotted in Fig. 10 for both dry cutting and when cutting with oil No. 8, as indicated by *B* and *C*, respectively. The depth of cut in each case was constant at 0.125 in. Uniform performance was obtained when cutting with oil No. 8 with both the up and down cuts. Each experimental point represents the average of ten successive cuts. The curves at *C*, representing the energy when cutting up and down when no lubricant was used (cutting fluid No. 1), are the results of three different tests; that is, each point represents the average energy value of 30 individual cuts. There was a noticeable lack of uniformity when cutting pure copper dry until it was noticed that the operator was removing with his finger the chips which, after being severed from the work, remained attached to the cutting edge. This led to some additional experiments, the results of which are given in Table 13. The same cutter was used, but it had been reground, and the tests were run at a later date. Cuts were successively taken as follows:

1 With the tool and work thoroughly cleaned with alcohol, for which condition, chips adhering to the cutter were pushed off with a clean screw driver.

2 The tool and work were thoroughly cleaned with alcohol, but the chips adhering to the tool were pushed off with the finger, for which condition no film of oil on the tool was detectable.

3 Tests in which oil No. 8 was employed.

Table 13 shows that on the up stroke, the energy per chip is 16.75 ft-lb. in the thoroughly clean condition, 14.5 ft-lb. when the chip was removed by the finger, and 7.47 ft-lb. when cutting oil No. 8 was used. Similarly, when cutting down, the milling-energy values were 11.15 for the clean condition, 10.89 for the condition when the chips were removed with the finger, and 6.91 when cutting fluid No. 8 was used. The differences for dry cutting between conditions *a* and *b* on the upward stroke are large. In this condition the chips were rigidly attached to the cutting edge, so that it was an effort to pry them off with the finger. There was, therefore, a noticeable contact between the finger and the cutting edge which undoubtedly left a certain moisture film. On the down stroke, however, the differences in energy values under the conditions *a* and *b* as shown in Table 13 appear to be small. In this type of cut, frequently the chips were removed with ease, and in many instances did not adhere at all after the cut.

It appears that the readings for condition *b*, in which the chips adhering to the cutting edge were removed by the finger

TABLE 14 ENERGY REQUIRED IN MILLING UNLEADED BRASS 50-A

[Cutter, 0.347-in. wide; cutting with lard oil (No. 5); milling up and down]

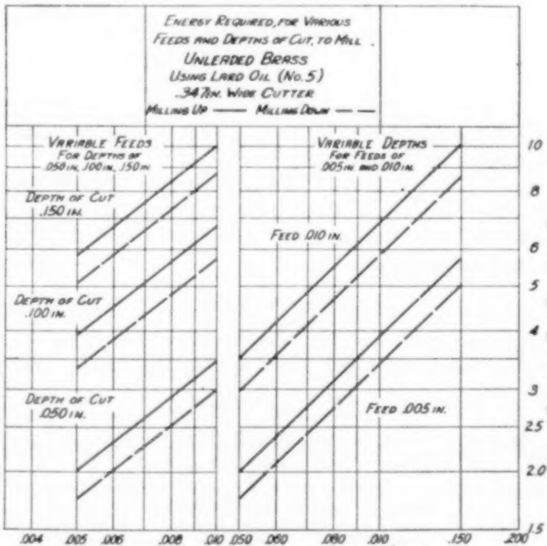
	—0.050 depth—		—0.100 depth—		—0.150 depth—	
	0.005 feed	0.010 feed	0.005 feed	0.010 feed	0.005 feed	0.010 feed
Up	2.01	3.55	3.89	6.77	5.72	10.07
Down	1.77	2.99	3.37	5.69	5.09	8.71

TABLE 15 ENERGY REQUIRED IN MILLING LEADED-BRASS SCREW STOCK (187-A)

[Cutter, 0.243 in. wide; cutting with lard oil (No. 5); milling up and down]

	—0.050 depth—		—0.075 depth—		—0.125 depth—	
	0.010 feed	0.020 feed	0.010 feed	0.020 feed	0.010 feed	0.020 feed
Up	1.39	2.50	1.97	3.53	3.05	5.53
Down	1.35	2.37	1.99	3.49	3.05	5.55

on both the up and down stroke, would conform more nearly with the values indicated by the two curves C in Fig. 10. Inasmuch as the test data shown in Table 13 were obtained at a considerable time interval after the tests shown by the curves B and C in Fig. 10, the cutting tool had been reground and was unquestionably not in the same ideal condition. It seems certain, however, that if the chips had not been removed by the finger, the cutting-up curve at C for dry cutting, as shown in Fig. 10, actually would be moved upward vertically by about 30 per cent. Such a set of values in which the work and tool were thoroughly cleaned and the cutter not touched by the hand is indicated by the light solid and broken lines D, which were obtained as a final check on this controversial question. It appears that the milling-down curve at C would be affected but little by the method of removing the chip from the cutting edge.



F.G. 11

From Tables 12 and 13 energy values on the up- and down-cutting strokes for oil No. 8 are available when the cut was 0.125 in. in depth and with a feed of 0.010 in. For the up stroke from the variable-feed tests, this value is 8.3 ft-lb., as compared with 8.1 from the variable-feed tests, and with 7.47 ft-lb. as given in Table 13. Naturally these values should all be the same, if they were taken with the cutter in the same condition. This was not the case, inasmuch as the constant-feed and constant-depth data of Table 12 were each obtained in tests run at different times, and the check tests of Table 13 were run at still a later date. The discrepancies in values are due, it is believed, to the different conditions of the cutter rather than to lack of uniformity of tests or to errors.

Curves A, Fig. 10, were obtained with a cutter in a slightly duller condition than in the case of curves B and C. From curves A and B it appears that with the former, which were obtained with the cutter in a slightly duller condition, the difference varies between energy values for a given cutting condition on the up and down stroke. All this has been investigated further and is reported on later. No other material tested shows such sensitiveness to cut, quantity of oil, or mixture as this pure copper.

Two things may be noted from Fig. 10: the inclination of the constant-depth line is decidedly less than that of the constant-feed lines, and a considerable saving in energy is found for milling down when the cutter is in a reasonably sharp condition.

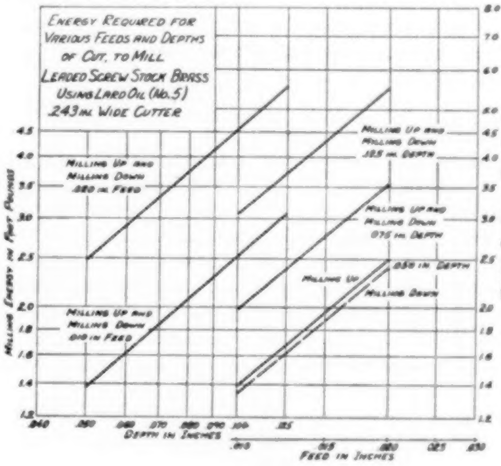


FIG. 12

The milling-energy formula for pure copper is found to be:

$$E = Cwf^{0.62} d^{0.92} \ldots\ldots\ldots\ldots [10]$$

in which C is 5980 for oil No. 1 (dry cutting) cutting up and 5090 for cutting down. For oil No. 8 (10 per cent lard oil in a light mineral oil) this constant is 3860 for cutting up and 3375 for cutting down. Also, the horsepower per cubic inch of metal removed per minute may be expressed as

$$\text{Hp. per cu. in. per min.} = \frac{C}{33{,}000\, f^{0.38}\, d^{0.08}} \ldots\ldots [11]$$

Unleaded Brass. The milling-energy tests when milling up and down for a combination of depths of cut and feeds were extended to an unleaded brass 50-A with a 0.347-in. cutter when using lard oil (No. 5). These data are given in Table 14 and are shown plotted in Fig. 11. Feeds of 0.005 and 0.010 in. were used for three depths, 0.050, 0.100, and 0.150 in., respectively.

The constant-feed variable-depth energy lines plotted in Fig. 11 show the milling-down-energy values to be below the corresponding values when milling up. For the constant-depth curves the milling energy when cutting down is also below the energy value when milling up.

The equation representing the energy as a function of cutter width, feed, and depth of cut when cutting with lard oil (No. 5) was found to be

$$E = Cwf^{0.77} d^{0.96} \ldots\ldots\ldots\ldots [12]$$

for which the value of the constant when cutting up is 6040 and when cutting down is 5170. See Table 18.

Annealed Leaded-Brass Screw Stock. An annealed leaded-brass screw stock, 187-A, was next selected as the material of interest in this study. A cutter 0.243 in. wide was used when cutting with lard oil (No. 5). Energy values when cutting up and down for feeds of 0.010 and 0.020 in., for depths of cut of 0.050, 0.075, and 0.125 in., respectively, are listed in Table 15 and shown graphically in Fig. 12.

Table 15 shows that the energy values when milling up and down are practically identical for any given cutting conditions. For a depth of cut of 0.050 in. and a feed of 0.020 in., the energy when cutting up is 2.50 ft-lb. as compared with 2.37 when cutting down. This is the greatest divergence in values noted and, in view of the consistent results otherwise obtained, is negligible.

The curves in Fig. 12 show the energy values for milling up and down and indicate the equality of the two values except for the milling-down line for the 0.050-in. depth of cut, which is shown slightly below that for milling up. The milling-energy formula for this annealed leaded brass, bar 187-A, when cutting with lard oil (No. 5) is

$$E = Cwf^{0.84}\, d^{0.87} \qquad [13]$$

When cutting with lard oil (No. 5) the constant C was found to be 3700 when cutting up or down. When cutting with a light mineral oil (No. 6) the values of C were found to be 3730 cutting up and 3700 cutting down. See Table 18.

Cold-Drawn Leaded-Brass Screw Stock. Using the 0.250-in.-wide cutter having a 15-deg. front-rake angle, energy values were determined when milling up and down with lard oil (No. 5) and mineral oil (No. 6), a cold-drawn leaded-brass screw stock, BF. The energy values for the various cuts are given in Table 16, and are shown plotted on log-log paper in Fig. 13.

The milling-down-energy values are consistently below those for milling up, and there appears to be little difference in energy values as a result of using different oils. The milling-energy lines resulting from these data give the formula

$$E = Cwf^{0.76}\, d^{0.96} \qquad [14]$$

for which the value of C, when milling up, is found to be 3830 for both lard oil (No. 5) and light mineral oil (No. 6), and 3600 for both oils when milling down. See Table 18.

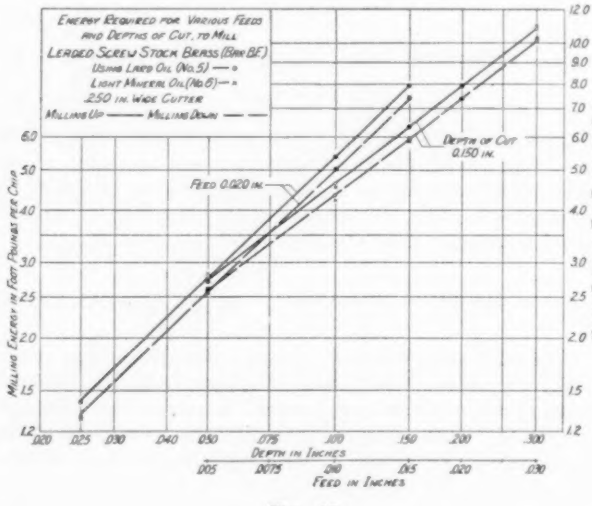


FIG. 13

Bakelite and Wood-Fiber Rod. It was thought desirable to extend these milling tests to some non-metallic materials, and for this purpose a bar of bakelite $^3/_4$ in. in diameter and another of wood fiber 1 in. in diameter were selected. The energy values are listed in Table 17 and are shown plotted on log-log paper in Fig. 14.

As shown in Fig. 14 bakelite machines with more difficulty from the energy standpoint than wood fiber. The distinctive characteristic of these lines seems to be that for the first time in the investigation, with a sharp cutter, the energy lines, when cutting up and down for a given material, are not parallel. For variable depth the energy lines for bakelite are nearly parallel but tend to converge at the lower left. The milling-down-energy values in all cases are above those when milling up. It appears, however, that, if the depth of cut were small enough or the feed were reduced sufficiently, the milling-down-energy value would fall below that for milling up.

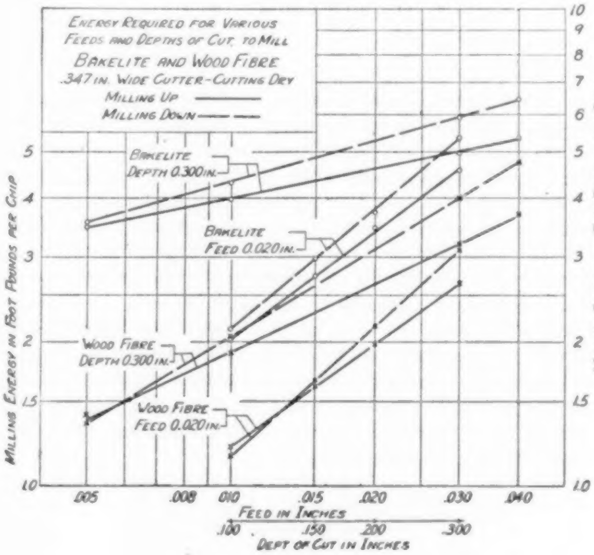


FIG. 14

For bakelite it seemed desirable to give two milling-energy equations, one representing the energy required to mill on the up stroke, and the other to mill on the down stroke. When milling up, the equation is

$$E = Cwf^{0.21}\, d^{0.75} \qquad [15]$$

When cutting dry (No. 1) the value of C is found to be 74. When milling bakelite on the down stroke, the energy equation is

$$E = Cwf^{0.29}\, d^{0.83} \qquad [16]$$

TABLE 16 ENERGY REQUIRED IN MILLING LEADED-BRASS SCREW STOCK (BF)

[Cutter, 0.250 in. wide; cutting with lard oil (No. 5) and mineral oil (No. 6); milling up and down]

Feed in inches	Depth in inches	Milling with lard oil (No. 5)		Milling with mineral oil (No. 6)	
		Up	Down	Up	Down
0.020	0.025	1.42	1.30	1.42	1.29
0.020	0.050	2.77	2.56	2.82	2.60
0.020	0.100	5.39	5.03	5.37	5.00
0.005	0.150	2.75	2.60	2.75	2.54
0.010	0.150	4.64	4.34	4.52	4.22
0.015	0.150	6.36	5.96	6.33	5.85
0.020	0.150	7.93	7.46	7.92	7.39
0.030	0.150	10.78	10.56	11.00	10.20

TABLE 17 ENERGY REQUIRED IN MILLING BAKELITE AND WOOD FIBER

(Cutter, 0.347 in. wide; cutting dry; milling up and down)

Feed in inches	Depth in inches	Bakelite		Wood fiber	
		Up	Down	Up	Down
0.020	0.100	2.02	2.12	1.20	1.15
0.020	0.150	2.74	2.97	1.61	1.66
0.020	0.200	3.40	3.74	1.97	2.16
0.020	0.300	4.59	5.35	2.63	3.10
0.005	0.300	3.49	3.56	1.41	1.35
0.010	0.300	3.97	4.30	1.88	2.04
0.030	0.300	4.98	5.90	3.19	4.00
0.040	0.300	5.35	6.42	3.70	4.75

When cutting bakelite dry the value of C was found to be 130.

For wood fiber, the energy formula when cutting up was found to be

$$E = Cwf^{0.49}\, d^{0.72} \qquad [17]$$

When cutting dry, the value of C was found to be 121. When cutting the wood fiber on the down stroke, the formula was

$$E = Cwf^{0.61}\, d^{0.91} \qquad [18]$$

When cutting dry, the constant C was found to be 281. These formulas are summarized in Table 18.

Summary. The various milling-energy formulas when milling up and down with a variety of cutting fluids on thirteen different materials are summarized in Table 18. The milling energy is given in foot-pounds per chip and is given as a function of a constant C which depends upon the material being cut; the condition of cutting, whether up or down; the shape, sharpness, and size of the cutter; the feed; and the depth of cut. The general formula is

$$E = Cwf^{x}\, d^{y} \qquad [19]$$

in which E is the energy in foot-pounds per chip, C the constant for the cutter, material, and cutting fluid; w the width in inches of the cutter having a 15-deg. front-rake angle and 0-deg. side-rake angle; f the feed in inches per chip; and d the depth of cut in inches.

TABLE 18 MILLING-ENERGY AND HORSEPOWER FORMULAS AND VALUES OF THE CONSTANT C FOR VARIOUS MATERIALS AND FOR MILLING UP AND DOWN WITH VARIOUS CUTTING FLUIDS

(Formula: $E = Cwf^{x}d^{y}$, in which, C = constant for cutter, material, and cutting fluid; E = energy in foot-pounds per chip; w = width of cutter in inches having 15-deg. rake; f = feed in inches per chip; and d = depth of cut in inches)

Material cut	Formulas: Energy, ft-lb. per chip	Formulas: Hp. per cu. in. per min.	Hp./cu. in./min.[a] Up	Hp./cu. in./min.[a] Down	Oil No.	Values of C: Up	Values of C: Down
S.A.E. 1020 steel, A-1	$E = Cwf^{0.64}\, d^{0.78}$ both up and down	$= \frac{C}{33{,}000\, f^{0.36}\, d^{0.22}}$	1.388	1.169	1	5520	4650
			1.084	0.952	4	4320	3790
			1.000	0.796	5	3980	3170
			1.084	1.043	6	4320	4150
			1.043	0.980	8	4150	3900
			0.930	0.854	10	3700	3400
Cold-rolled low-carbon steel	Up: $E = Cwf^{0.73}\, d^{0.87}$	$= \frac{C}{33{,}000\, f^{0.27}\, d^{0.13}}$ (Up)	0.936	....	5	6800	..
	Down: $E = Cwf^{0.67}\, d^{0.87}$	$= \frac{C}{33{,}000\, f^{0.33}\, d^{0.13}}$ (Down)	....	0.775	5	..	4260
S.A.E. 3150 steel (KF)	$E = Cwf^{0.70}\, d^{1.00}$	$= \frac{C}{33{,}000\, f^{0.30}}$	1.327	1.278	1	11000	10600
			1.180	1.082	10	9780	8980
S.A.E. 6140 steel (A-13)	$E = Cwf^{0.72}\, d^{0.90}$	$= \frac{C}{33{,}000\, f^{0.28}\, d^{0.10}}$	1.44	1.52	1	10630	11220
			1.348	1.427	8	9950	10520
Free-cutting screw-stock steel (No. 414)	$E = Cwf^{0.77}\, d^{0.86}$	$= \frac{C}{33{,}000\, f^{0.23}\, d^{0.14}}$	0.954	0.784	6	8180	6720
High-speed steel	$E = Cwf^{0.73}\, d^{0.84}$	$= \frac{C}{33{,}000\, f^{0.27}\, d^{0.16}}$	1.437	1.525	1	9700	10400
			1.437	1.525	8	9700	10400
Cast iron (CF)	$E = Cwf^{0.41}\, d^{0.56}$	$= \frac{C}{33{,}000\, f^{0.59}\, d^{0.44}}$	0.685	0.740	1	595	643
Leaded screw-stock brass (BF)	$E = Cwf^{0.76}\, d^{0.96}$	$= \frac{C}{33{,}000\, f^{0.24}\, d^{0.04}}$	0.381	0.358	5	3830	3600
			0.381	0.358	6	3830	3600
Annealed leaded screw-stock brass (187-A)	$E = Cwf^{0.84}\, d^{0.87}$	$= \frac{C}{33{,}000\, f^{0.16}\, d^{0.13}}$	0.307	0.307	5	3700	3700
			0.3095	0.307	6	3730	3700
Annealed unleaded brass (50-A)	$E = Cwf^{0.77}\, d^{0.96}$	$= \frac{C}{33{,}000\, f^{0.23}\, d^{0.04}}$	0.573	0.491	5	6040	5170
Pure copper	$E = Cwf^{0.62}\, d^{0.92}$	$= \frac{C}{33{,}000\, f^{0.38}\, d^{0.08}}$	1.233	1.049	1	5980	5090
			0.795	0.695	8	3860	3375
Bakelite	Up: $E = Cwf^{0.21}\, d^{0.75}$	$= \frac{C}{33{,}000\, f^{0.79}\, d^{0.25}}$ (Up)	0.1432	....	1	74	..
	Down: $E = Cwf^{0.29}\, d^{0.83}$	$= \frac{C}{33{,}000\, f^{0.71}\, d^{0.17}}$ (Down)	....	0.1475	1	..	130
Wood fiber	Up: $E = Cwf^{0.49}\, d^{0.72}$	$= \frac{C}{33{,}000\, f^{0.51}\, d^{0.28}}$ (Up)	0.0685	....	1	121	..
	Down: $E = Cwf^{0.61}\, d^{0.91}$	$= \frac{C}{33{,}000\, f^{0.39}\, d^{0.09}}$ (Down)	....	0.0618	1	..	281

[a] Feed in inches, 0.010; depth of cut in inches, 0.125.

Where only one formula is given, the presumption is that it represents the energy for both cutting up and down with sufficient accuracy. The constant C in the formula, however, may differ for cutting up or down, but the energy lines, when plotted on log-log paper for a constant depth or constant feed, are parallel. It is seen that the exponent of the depth d ranges in volume from a minimum of 0.56 for cast iron containing 1 per cent nickel to a maximum of 1.00 for the S.A.E. 3150 steel annealed. This indicates that for cast iron there would be a great saving in power when heavy depths of cut are taken, but no saving when cutting steel.

The exponents of the feed f ranges from 0.21 for bakelite milling up to 0.84 for annealed leaded screw-stock brass. For cast iron it is 0.41, whereas the average for the metals appears to be approximately 0.7.

Inasmuch as the average exponent of the depth appears to be much higher (approximately 0.85) than that of the feed, which averages about 0.7, it is obvious that greater economy of power is obtained when taking heavy feeds than when taking heavy cuts. The formulas indicate that the depth should be as great as possible, but it is of more importance that the feed per tooth be as great as possible.

The energy formulas for each material cut have been reduced to power formulas giving the horsepower required to cut one cubic inch per minute. These formulas are given as functions of the constant C, the feed f, and the depth of cut d. The values of the constant C are the same as those for the energy formulas and are listed at the right in Table 18 for various cutting fluids and for milling up and down.

Specific values of the horsepower per cubic inch per minute when cutting up and down with one or more cutting fluids are given for each of the materials cut. These horsepower values are for the specific cutting conditions when the cutter is $^1/_4$ in. wide, which value does not appear in the horsepower formula. The cutter was sharp and had a front-rake angle of 15 deg. and no side-rake angle. The feed, however, is 0.010 in. per chip, and the depth of cut 0.125 in. It is seen that the horsepower per cubic inch per minute when cutting the S.A.E. 1020 steel by milling up is 1.388, but when milling down is only 1.169. For this metal more power is required to mill up the usual way for all cutting fluids than when milling down. It seems desirable, therefore, from the viewpoint of power, to mill S.A.E. 1020 on the downstroke, that is, with the feed. For the S.A.E. 1020 steel, the same formulas with different constants hold for milling both up and down.

When milling the cold-rolled low-carbon steel, it was found that the energy formula for milling up is somewhat different from that when milling down. The exponent of the feed is 0.73 when milling up and only 0.67 when milling down. The horsepower per cubic inch per minute when milling up with lard oil is 0.936, and when milling down is only 0.775. This shows that the saving when milling down for this material increases with increased feed. The horsepower value when cutting the S.A.E. 6140 steel dry on the up stroke is 1.44, but when cutting down is 1.52, showing that more power is required for this material when cutting down than when cutting up. The same relation holds true when cutting this steel when 10 per cent lard oil in mineral oil (No. 8) is used. More power is required on the down stroke than on the up stroke for both dry cutting and cutting fluid No. 8 when annealed high-speed steel is cut. It is therefore seen that some materials cut easier on the down stroke, while others cut easier on the up stroke.

The Effect of Dulling

Tool-life tests are not practical using the dynamometer employed in these tests, so that further investigations of the nature studied in this paper, carried out by commercial cutters on standard milling machines both in the laboratory and by actual practice, are indicated.

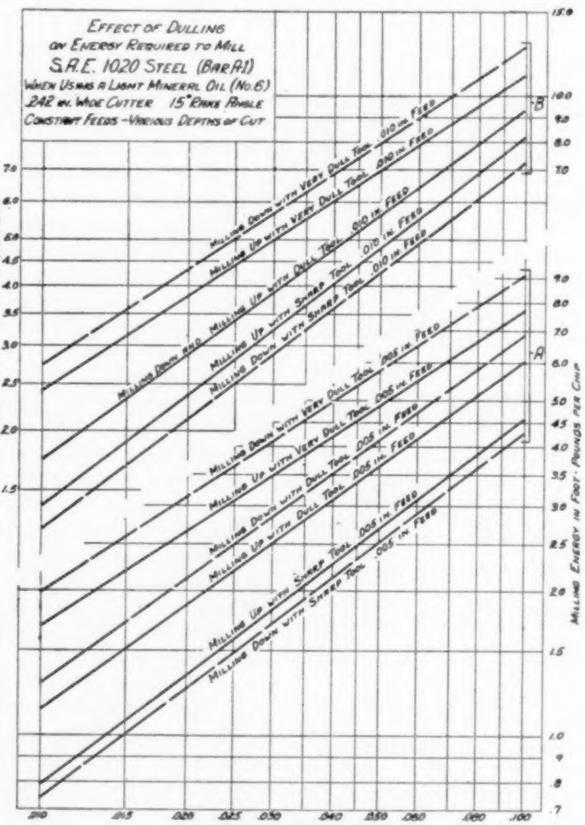


Fig. 15

It was felt desirable, however, to determine the effect of certain degrees of dullness of the cutter on the energy required in milling by the up and down processes. For this purpose a series of tests were undertaken in which the degree of dullness was made the variable.

Table 19 gives the results of a series of tests in which the milling cutter first was given a constant feed of 0.005 in. per cut and a depth of cut ranging from 0.010 to 0.100 in. in four steps. In the first test the cutter was ground and then honed so as to be as sharp as possible. For each of the depths indicated, the energy required to remove a chip is seen to be greater when cutting up than when cutting down. In the second and third series of tests, *B* and *C*, the cutter was dull and very dull, respectively.

By using a micrometer and magnifying glass, it is estimated that, in the very dull condition, the cutting edge was honed so as to form a segment of a cylinder from 0.006 to 0.007 in. in diameter. For the intermediate or dull condition the diameter is estimated to be approximately one-half that for very dull. For the dull and very dull conditions the energy values are greater when cutting down than when cutting up, which is the reverse of that indicated for the tool in the sharp condition. These values are shown plotted in Fig. 15, giving a series of lines indicated at *A*. Again, the solid line in each instance indicates milling up, while the broken line indicates milling down.

The lowest two lines of group *A* are for the tool in the sharp condition when the feed was 0.005 in. and the depth varied from 0.010 to 0.100 in. These values plotted on log-log paper give straight parallel lines, with the milling-up line above the milling-down one. The values for milling down are about 6.8 per cent below those for milling up.

The intermediate set of lines of group *A* show the milling-energy curves for the tool in a dull or intermediate condition with the feed still at 0.005 in. per chip. This shows the milling-up line to be below the milling-down one. For the dull condition the milling-down-energy values are 11.1 per cent above the milling-up-energy values.

The upper two lines of group *A* represent the milling-energy values for the 0.005-in. feed when the tool is in a very dull condition. Here again it is to be noted that the milling-down-energy values are above the milling-up values by 18.2 per cent.

The effect of dulling is very apparent from this determination. As the cutter becomes dull, the energy required to remove a chip increases. The lines become more nearly horizontal as the dullness increases, which indicates that dullness has less influence on heavier feeds and depths of cut. It is also apparent that the energy required to remove a chip increases more rapidly with dullness when cutting down than when cutting up, so that, while less energy is required in milling down with a sharp tool, much more is required as the tool becomes dull.

The variable-depth tests were repeated with a feed per cut of 0.010 in. These values for each condition of sharpness are given in the lower portion of Table 19. They are shown plotted in the upper part of Fig. 14, resulting in the group of straight lines indicated at *B*. Again the energy values for the tool in the sharp condition, when taking a feed of 0.010 in. per chip, show that the milling-up values are higher than the milling-down ones. With the tool in the dull condition at this feed, the milling-up and milling-down values become equal. The energy values for the tool in the very dull condition are reversed in that the milling-down-energy values are well above the milling-up-energy values. Again it is seen that the slope of the energy lines for the tools in the very dull condition are less than those for the tool in the sharp condition, indicating again that for heavy depths of cut the dullness has less effect.

When the feed is 0.005 in. the total increase in milling energy per chip when milling up with a very dull cutter over the milling up with a sharp cutter, is 114 per cent for a depth of cut of 0.010 in. For a depth of cut of 0.100 in. the milling-up energy for the very dull cutter is 69 per cent greater than the milling-down energy for the sharp cutter. Also, for the 0.005-in. feed, the milling-down-energy value with the very dull tool is 168 per cent greater than when milling down with a sharp tool for a depth of cut of 0.010 in. Also, for a depth of cut of 0.100 in., the milling-down-energy value for the very dull tool is 112 per cent higher than the corresponding energy value when milling down with a sharp tool.

When the feed is 0.010 in., the increase in milling-down-energy values of the very dull tool over the sharp tool is 122 per cent, as compared with a corresponding increase of 73 per cent for the milling-up values at a depth of 0.010 in. Similarly, the increase of the milling-down-energy values for the very dull tool over the milling-down values with the sharp tool for the 0.010 in. feed and 0.100 in. depth is 68 per cent, as compared with 34 per cent for the corresponding milling-up-energy values.

From Fig. 15 it is obvious that increased dulling has less influence on both heavier feeds and depths of cut.

The values of the energy required to remove a chip by milling for various degrees of dullness of the cutter, as given in Table 19, are shown in Fig. 16 plotted over variable feed for each condition of tool sharpness and for each of the four depths of cut. In the lower left-hand corner of Fig. 16 are shown the milling-energy values when cutting up and down for a depth of cut of 0.010 in. when a light mineral oil (No. 6) is used. For milling up with the sharp tool, the energy value for the depth of cut of 0.010 in. and a feed of 0.005 in. is 0.79 ft-lb., as indicated at the left. The energy value when milling up with a feed of 0.010 in. and 0.010 in. depth of cut is 1.39 ft-lb. The corresponding values for down milling are 0.74 and 1.24 ft-lb., respectively, for the 0.005-in. and 0.010-in. feed. Above these lines are shown other lines for depths of cut of 0.025, 0.050, and 0.100 in., respectively. In the center of Fig. 16 are shown the various energy values when milling up and down for the dull tool for each of the four different depths of cut. At the right are shown increasing energy values for the very dull tool for each of the four depths of cut. For the sharp tool the milling-up-energy values, as indicated by the solid lines, are above the milling-down-energy values, as indicated by the broken lines, for each of the four depths of cut.

For the cutting edge in the intermediate or dull condition, the milling-down-energy values are higher than the milling-up values for all depths of cut until the feed of 0.010 in. is reached, when they coincide. When the cutting edge of the tool is in a very dull condition, the milling-down-energy values are shown above the milling-up-energy values for all depths of cut and for all values of feed between 0.005 and 0.010 in. It is interesting to note that the lines representing energy when milling up for each degree of tool sharpness are parallel for the different depths of cut. The same statement holds true for the milling-down-energy lines, although there appears to be a great variation in the slope of the milling-down-energy lines as the sharpness of the tool is changed from sharp to dull. There is a little additional change in slope of the milling-down-energy lines as the tool sharpness is changed from dull to very dull.

The effect on the depth exponent in the milling-energy formula as a function of depth and feed is not nearly so pronounced as on the feed exponent. This seems due to the fact that the thickness of the chip is more a function of the feed than of the depth and the thicker the chip the less effect dullness might be expected to have. The effect of dulling on milling energy is quite different from that in drilling or planing where continuous chips are formed. This is probably because the duration of cut in milling is too short for the built-up edge, if it is formed, to exert an appreciable influence on the formation of the chip.

Effect of Dulling on S.A.E. 3150 Steel. As the effect of dulling is more apparent on feed than on depth, an additional test to study dulling was made on S.A.E. 3150 steel in an annealed condition, bar KF, in which a sulphurized mineral lard oil (No. 11) was used and in which the feed only was varied. The depth of cut was maintained at 0.075 in., while feeds of 0.005 and 0.010 in. were used.

Energy values were obtained when cutting both up and down with the cutting tool in the sharp, dull, and very dull conditions. These values are shown in Table 20. The conditions of dullness as shown in Fig. 17 were approximately the same as referred to in Figs. 15 and 16.

TABLE 19 EFFECT OF DULLING ON ENERGY IN MILLING S.A.E. 1020 STEEL (BAR A-1)

[Cutter, 0.242 in. wide; cutting with light mineral oil (No. 6); milling up and down with various feeds and depths of cut. Values plotted in Figs. 14 and 15]

Feed in inches	Depth in inches	A (sharp)		B (dull)		C (very dull)	
		Up	Down	Up	Down	Up	Down
0.005	0.010	0.79	0.74	1.14	1.30	1.69	1.98
0.005	0.025	1.56	1.47	2.01	2.42	3.08	3.67
0.005	0.050	2.71	2.55	3.78	4.30	4.88	5.83
0.005	0.100	4.59	4.30	6.04	6.75	7.69	9.09
0.010	0.010	1.39	1.24	1.79	1.72	2.41	2.75
0.010	0.025	2.89	2.46	3.24	3.38	4.31	4.95
0.010	0.050	4.56	4.22	5.58	5.56	7.06	8.12
0.010	0.100	8.2	7.30	9.50	9.65	11.00	12.6

In Fig. 17 the energy-value lines for different degrees of sharpness, plotted against the feed in inches for milling down, are above the milling-up-energy lines for each degree of dullness. The same general relations between energy and dullness are to be noted.

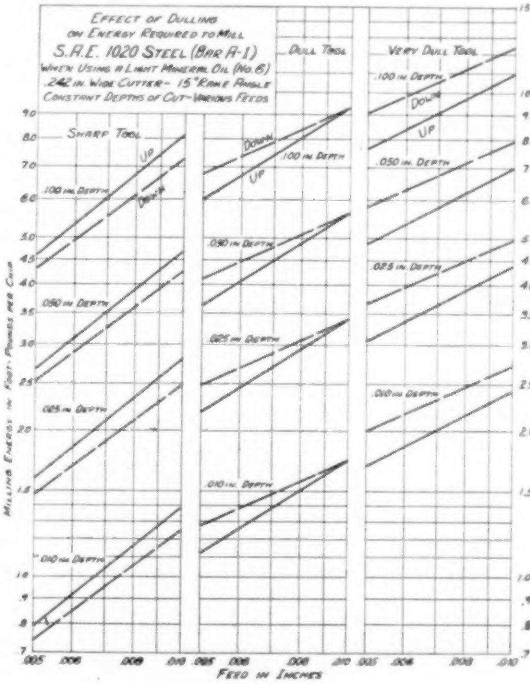


FIG. 16

EFFECT OF CUTTING FLUIDS ON MILLING ENERGY

Data for use in determining the effect of various cutting fluids on the two methods of milling, that is, milling up and milling down, were obtained when cutting four metals: S.A.E. 1020 steel (bar A-1), cold-rolled low-carbon steel, free-cutting screw-stock steel, and a leaded brass, and are given in Table 21. They are shown plotted in Figs. 18, 19, 20, and 21, respectively.

Effect in Milling S.A.E. 1020 Steel. In Table 21 the energy values are given for both the up and down cuts when the 0.347-in.-wide cutter is used, operating at 0.100-in. depth and 0.010-in. feed, in the numerical order of the oils. These energy values are shown plotted over the oil numbers in Fig. 18 for the S.A.E. 1020 steel, arranged in the order of increasing energy required in milling up. From this it is seen that the lowest value in milling up is obtained with the sulphurized mineral oil (No. 10), with lard oil (No. 5), sulphurized mineral lard oil (No. 11), mineral lard oil (No. 8), and oleic acid mineral oil (No. 9) following

TABLE 20 EFFECT OF DULLING IN ENERGY IN MILLING S.A.E. 3150 STEEL

[Cutter, 0.250 in. wide; depth, 0.075 in.; using a sulphurized mineral-lard oil (No. 11) and milling up and down. Values plotted in Fig. 16]

	A (sharp)		B (dull)		C (very dull)	
Feed	Up	Down	Up	Down	Up	Down
0.005	5.83	6.28	7.95	8.40	9.88	10.52
0.010	8.55	8.88	10.80	10.85	13.10	13.00

closely in the order named. Cutting fluids 1-to-50 soluble oil (No. 3), heavy mineral oil (No. 7), 1-to-10 soluble oil (No. 4), and light mineral oil (No. 6) follow with almost equal energy values. Borax water (No. 2) shows a material increase in energy required, but all of the oils and water compounds are distinctly below the energy values for dry cutting (cutting fluid No. 1) for milling up.

The energy values when milling down show little conformity with those for milling up. Lard oil (No. 5) shows the lowest values of energy for milling down, with the sulphurized mineral oil a close second. It is interesting to note that the water compounds, consisting of borax water (No. 2), 1-to-50 soluble oil (No. 3), and 1-to-10 soluble oil (No. 4) give the next lowest values in the order named. Again, when cutting down, the highest energy values are obtained for dry cutting (cutting fluid No. 1).

The lower part of Fig. 18 shows plotted over the cutting-fluid numbers, arranged in the order of increasing value of the ratio, the ratio of energy values when cutting down to those when cutting up. This ratio is smallest for the borax water

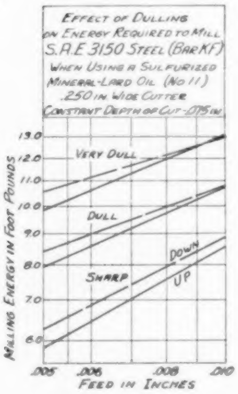


FIG. 17

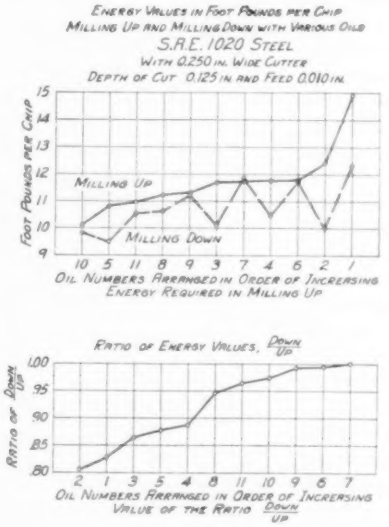


FIG. 18

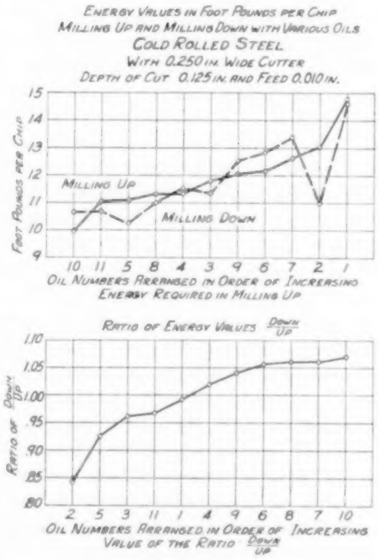


FIG. 19

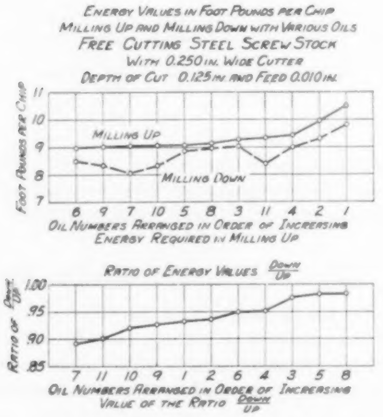


FIG. 20

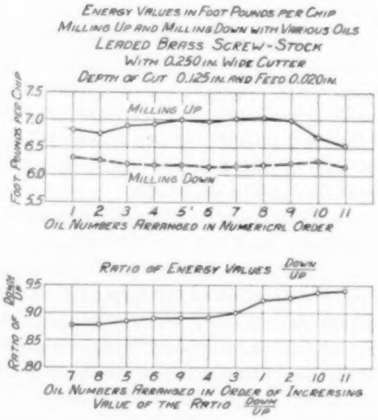


FIG. 21

TABLE 21 ENERGY VALUES IN FOOT-POUNDS PER CHIP IN MILLING UP AND DOWN SEVERAL METALS WITH VARIOUS OILS

(Average of thirty readings; values plotted in Fig. 17)

Metal cut	Cutter width, in.	Depth, in.	Feed, in.		Cutting-fluid numbers										
					1	2	3	4	5	6	7	8	9	10	11
S.A.E. 1020 steel (A-1)	0.347	0.100	0.010	Up	14.89	12.44	11.72	11.78	10.80	11.80	11.77	11.23	11.30	10.10	10.93
				Down	12.31	10.02	10.13	10.46	9.50	11.71	11.87	10.63	11.21	9.83	10.54
Cold-rolled low-carbon steel	0.250	0.125	0.010	Up	14.77	13.05	11.81	11.32	11.07	12.15	12.63	11.32	12.05	9.95	11.05
				Down	14.65	11.00	11.35	11.55	10.25	12.85	13.40	12.00	12.55	10.65	10.70
Free-cutting screw-stock steel (414)	0.250	0.125	0.010	Up	10.50	9.95	9.25	9.45	9.05	8.95	9.03	9.15	9.00	9.04	9.33
				Down	9.80	9.33	9.03	8.99	8.85	8.50	8.05	8.95	8.33	8.31	8.40
Leaded screw-stock brass (BF)	0.250	0.125	0.020	Up	6.83	6.75	6.89	6.93	7.00	6.93	7.02	7.05	6.99	6.69	6.55
				Down	6.31	6.27	6.20	6.18	6.18	6.15	6.16	6.18	6.22	6.27	6.17

(cutting fluid No. 2), at a value of approximately 0.8. Dry cutting (No. 1) gives the next lowest value, 0.825, whereas the mineral oils, 9, 6, and 7 show the highest values of this ratio of nearly unity.

Effect in Milling Cold-Rolled Low-Carbon Steel. The energy values in foot-pounds per chip, when milling up and down the cold-rolled low-carbon steel with each of the eleven cutting fluids, are shown in Table 21. In Fig. 19 the energy values for the cold-rolled low-carbon steel are shown plotted over the oil numbers, arranged in the order of increasing energy required when milling up.

There are comparatively slight differences between the milling-up and milling-down-energy values for the various oils, except perhaps those for borax water (No. 2). The trend for milling up and down appears to be consistent. Again, the lowest energy values are obtained for both milling up and down for the sulphurized oil (No. 10), the sulphurized mineral lard oil (No. 11), and the lard oil (No. 5). Also, the highest values for both milling up and down are obtained when cutting dry (No. 1).

In the lower part of Fig. 19 is shown plotted over the oil numbers, arranged in the order of increasing value of the ratio, the ratio of the milling-up-energy value to the milling-down value.

In this instance the lowest ratio, approximately 0.84, is obtained for borax water (No. 2), while the highest ratios are obtained for cutting fluids 6, 8, 7, and 10, each equal approximately to 1.06.

A careful study of Figs. 18 and 19 shows that the results in milling the S.A.E. 1020 steel and the cold-rolled low-carbon steel of approximately the same analysis, correspond, in a general way, for the various cutting fluids. Arranged in the order of milling energy up, cutting fluids 10, 5, 11, and 8 give the four lowest energy values for each steel. The two mineral oils No. 6 and 7 are in about the same relative position for each steel, though reversed in order. In both cases, cutting fluids No. 2 (borax water) and 1 (dry) are at the right hand of the curve, showing that for them the greatest amount of energy is required. Dry cutting (No. 1) requires considerably more energy in milling up than the other cutting fluids. The greatest difference in the two materials is perhaps manifest in the value of the ratio of energy values of down to up. Those for cold-rolled low-carbon steel are consistently higher by about 6 per cent than those for the S.A.E. 1020 steel. Here again the order of the cutting fluids, when arranged from left to right in the order of increasing value of the ratio, as shown in Figs. 18 and 19, is about the same.

For both steels borax water (No. 2) is the lowest, and the emulsions (Nos. 3 and 4) and lard oil (No. 5) are in the same relative position. Oils 6, 7, and 10 are, on the other hand, high for both steels.

Effect in Milling Free-Cutting Steel. The third material on which these various cutting-fluid milling tests were run was a free-cutting screw-stock steel (No. 414) of the bessemer type. The milling-energy values for both cutting up and cutting down for each of the cutting fluids are listed in Table 21. The values are shown plotted in Fig. 20.

It is obvious that the energy required to remove this material is, for both milling up and down, considerably less for all of the cutting fluids than when milling the S.A.E. 1020 or cold-rolled low-carbon steels. The material machines in such a way that differences between the energy values, resulting from the use of various cutting fluids, is small, and it seems worthy of note that the sulphurized oils, which usually give low energy values, do not seem to be outstanding in this respect when cutting this free-cutting steel. In fact, the sulphurized mineral lard oil gives an energy value, when milling up, higher than all of the other oils, and even higher than soluble oil (No. 3). Oils 7, 10, and 11, however, show up to advantage on the down milling. As in the case of the two low-carbon steels, cutting fluids Nos. 2 and 1, borax water and dry cutting, respectively, give the highest energy values for the free-cutting steel when milling up. The borax water (No. 2) gives the second highest energy values when milling the free-cutting screw-stock steel, whereas they give relatively small values for down milling on the two low-carbon steels.

The value of the ratio of the energy required to mill down to that required to mill up is shown at the bottom of Fig. 20. These values cover a rather narrow range as compared with the corresponding values for the low-carbon steels, the lowest being 0.89 for heavy mineral oil (No. 7), and the highest value being 0.98 for the mineral lard oil (No. 8).

Effect in Milling Leaded Brass. The energy values per chip when milling a leaded screw-stock brass (BF) with a 0.250-in.-wide cutter having a 15-deg. front-rake angle when taking a depth of cut of 0.125 in. and a feed of 0.020 in. per chip for various cutting fluids, are shown at the bottom of Table 21. Chips were removed by milling both up and down. The values are shown plotted in Fig. 21.

While the feed has been increased over that used on the steels, the energy values are seen to be relatively low. Also, the differences in energy values for various cutting fluids are small. The energy values when milling up and down, as shown in Fig. 21, are plotted over the cutting fluid numbers, arranged in numerical order, due to the small differences in values for the different fluids. The milling-down curve is practically a straight horizontal line.

The highest energy value developed when milling down, however, is when cutting dry (No. 1), and the second highest when cutting with borax water (No. 2) and sulphurized mineral oil (No. 10). The lowest values are obtained with the straight mineral oils (Nos. 6 and 7) and the sulphurized mineral lard oil (No. 11). When milling up, however, the lowest values are obtained with the sulphurized oils (Nos. 10 and 11), with borax water (No. 2) a distant third. Little difference appears between the energy values for the other oils.

In the lower part of Fig. 21 are shown plotted the ratios of the energy values when cutting down to those when cutting up. The total variation is small, being 0.877 for the heavy mineral oil (No. 7) and 0.94 for the sulphurized mineral lard oil (No. 11).

Finish and Tool Life

While it was not the authors' intention to present data on the subject of finish and its relation to tool life in this paper, a number of observations were made which consistently point to certain conclusions. Perhaps the finish and tool life obtained on a comparatively frail dynamometer, such as that used in the tests made, may not check with similar findings on a regular commercial machine and operations.

It was noticed that the tendency to chatter when milling up is pronounced, especially as the tool becomes dull. In only a few instances were chatter marks visible on the finished surface when cutting was done on the down stroke. Milling up invariably gives a shinier finish, due probably to a burnishing action at the very start of the cut. Milling down seems to give a surface which is flatter and freer from ridges resulting from the feed, and the resulting cut surfaces often had a dull, torn appearance. Ridges were invariably found when milling up with heavy feeds.

Dulling seemed to have little effect on the condition of the surface when milling down, and it may follow that if the cutter life is based on finish, its life will be greater when milling down than when milling up.

TABLE 22 MATERIALS USED FOR COMPARING MILLING ENERGY WHEN MILLING UP AND DOWN[1]

(Showing chemical analysis and hardness numbers)

Bar No.	Material	Size	Chemical analyses	Hardness Brinell	Hardness Rockwell B	Hardness Scleroscope
	Cold-rolled	1 × 2½ in.	....	...	...	..
BB	Cast bearing bronze	2 in. diam.	80 Cu, 10 Pb, 10 Sn	67	62	19
187-A	Annealed leaded screw-stock brass	1½ in. hex.	62 Cu, 35 Zn, 3 Pb	49.9	63	12.6
MB	Cast manganese bronze	2 in. diam.	50–60 Cu, 0.75 Sn, 30–45 Zn, 0.4 Al, 0.05 Mn	130	95	25
A1-31	Cast No. 31 alloy	2 in. diam.	3 Cu, 15 Zn, 82 Al	73	88	22
Cu	Pure annealed copper	1½ in. hex.	100 Cu	40.4	28	6
M	Monel, rolled	2 in. diam.	28–29 Cu, 2.5 Al, 67–68 Ni, 1 Mn	130	106	29
DM	Dow metal, cast	1½ in. diam.	6 Al, 0.25 Mn, 93.75 Mg	48	54	20
11-1	Cast iron	1 in. sq.	Comb. C 0.53, 0.52 Mn, 2.26 Si, 0.056 S, 0.32 P	207	97.5	39
23-B	Malleable iron	1¼ × 3 in.	....	...	...	..
H.S.	High-speed steel, annealed	¾ × ¾ in.	....	...	...	..
30	S.A.E. 2320 steel	5 × 6 in.	0.20 C, 0.65 Mn, 0.18 Si, 0.018 S, 0.014 P, 3.37 Ni	162	55	24
31	S.A.E. 3120 steel	5 × 6 in.	0.17 C, 0.60 Mn, 0.31 Si, 0.019 S, 0.014 P, 1.28 Ni, 0.67 Cr	155	48	24
32	Carbon tool steel	5 × 6 in.	1.03 C, 0.28 Mn, 0.018 S, 0.017 P, 0.18 Si	155	52	25
33	Die steel "X"	5 × 6 in.	0.64 C, 0.64 Mn, 0.132 Si, 0.016 S, 0.017 P, 1.07 Cr	190	94	30
T-1	Tool steel	5 × 6 in.	0.93 C, 0.33 Mn, 0.187 Si, 0.018 S, 0.010 P	188	87.6	24.2
T-2	Tool steel	5 × 6 in.		189	87.1	26.0
A-1	Axle steel	1 × 1½ in.	0.49 C, 0.55 Mn, 0.030 S, 0.01 P	145	74.3	21.2
S-1	Screw-stock steel	1 × 1½ in.	0.3 C, 0.7 Mn, 0.275 Si, 0.063 S, 0.083 P	146	77.9	20.2

[1] Materials used in "Methods of Tests for Determining the Machinability of Metals in General, With Results," by O. W. Boston, Trans. A.S.S.T., vol. 16 (1929), no. 6.

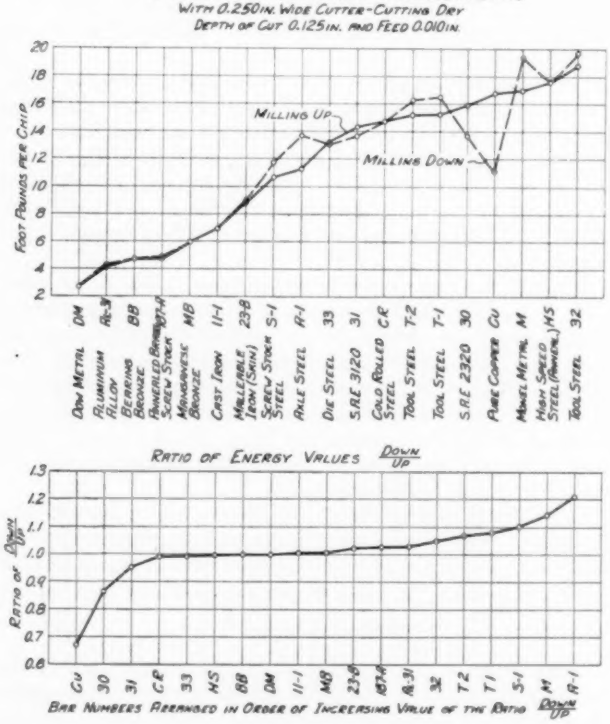


FIG. 22

COMPARING A VARIETY OF MATERIALS BY MILLING ENERGY

For the purpose of obtaining additional information on the influence of milling up or down on the energy values in foot-pounds required to remove a given chip, comparative tests have been made on a wide variety of ferrous and non-ferrous materials. All tests were run dry (No. 1) with a cutter 0.250 in. wide when taking a depth of cut of 0.125 in. and a feed of 0.010 in.

THE MATERIALS TESTED

The materials used in this study are listed in Tables 22, 23, and 24. Those in Table 22 included a variety of steels formerly reported on in a study of machinability.[7] The size of the bar, its chemical analysis, and the Brinell, Rockwell, and scleroscope hardness values are given.

TABLE 23 MATERIALS USED IN COMPARING MILLING ENERGY WHEN MILLING DRY, UP AND DOWN[1]

(Showing chemical analyses and hardness numbers)

Bar No.	S.A.E. specifications	Chemical analysis	Heat treatment, deg. fahr.	Hardness Brinell Dia.	Hardness Brinell No.	Hardness Rockwell B	Hardness Scleroscope No.
1	1020	Commercial	None	5.1	137	84	29
2	1045	Commercial	Normalize 1600 Anneal 1475	4.75	159	82	28
3	2320	Commercial	Anneal 1600	4.7	163	85.5	25
4	3120	Commercial	Anneal 1600	5.0	143	75	24
5	3135	Commercial	Normalize 1575 Anneal 1500	4.35	192	88.5	29
6	6150	C 0.50 Mn 0.20 Cr 0.80 V 0.20	Normalize 1650 Anneal 1550	4.4	187	86.5	29
	3250	C 0.50 Cr 1.00 Ni 1.75	Normalize 1650 Anneal 1475	4.2	207	92.5	30
8	3140	C 0.50 Cr 0.60 Ni 1.25	Normalize 1650 Anneal 1500	4.3	196	92	30
9	2330	Commercial	Anneal 1500 Reanneal 1400	4.25	202	93.3	32
10	2340	Commercial	Anneal 1500 Reanneal 1400	4.2	207	93.5	32
11	6120	Commercial	Anneal 1500 Reanneal 1400	5.1	137	76.5	25
12	6130	Commercial	Anneal 1500 Reanneal 1400	4.8	156	81.5	28
13	6140	Commercial	Anneal 1500 Reanneal 1400	4.7	163	85.5	28
14	9250	Commercial	Anneal 1500 Reanneal 1400	4.25	202	93	32
15	Cr Mo	C 0.33 Si 0.20 Mn 0.50 P 0.011 S 0.010 Cr 0.66 Mo 0.18	Anneal 1500 Reanneal 1400	4.8	156	77.5	27
16	Cr Mo	C 0.28 Si 0.23 Mn 0.52 P 0.028 S 0.017 Cr 0.65 Mo 0.19	Anneal 1500 Reanneal 1400	4.8	156	79	27

[1] Materials used in "Power Required to Drill Cast Iron and Steel," Trans. A.S.M.E., vol. 52 (1930), MSP-52-2. Paper shows photomicrographs.

Table 23 lists those steels reported on in a study of drilling.[8] These bars were mostly forgings 2 in. × 4 in. in section, and were all in an annealed condition.

A number of cold-drawn steels are shown with their chemical analyses and hardness numbers in Table 24. These have previously been reported on in a study of machining properties.[9]

Photomicrographs (of all of these materials) are given in the respective references.

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[7] "Methods of Tests for Determining the Machinability of Metals in General, With Results," by O. W. Boston, Trans. A.S.S.T., 1929, vol. 16 (1929), no. 6.

[8] "Power Required to Drill Cast Iron and Steel," by O. W. Boston and C. J. Oxford, Trans. A.S.M.E., vol. 52 (1930), MSP-52-2.

[9] "Machining Properties of Some Cold-Drawn Steels," by O. W. Boston, Trans. A.S.M.E., vol. 53 (1931), MSP-53-6.

TABLE 24 MATERIALS USED IN COMPARING MILLING ENERGY WHEN MILLING DRY, UP AND DOWN[1]

(Showing chemical analyses and hardness numbers)

Bar No.	Material	Chemical analysis, per cent: Carbon	Manganese	Phosphorus	Sulphur[a]	Chrome or nickel	Vanadium	Si	Mo	Hardness number: Brinell	Rockwell B	Scleroscope
412	Stainless chromium iron annealed	0.06	0.31	0.012	0.022	13.35[d]	..	0.18	..	...	...	..
413	Free-machining rustless steel	0.09	0.30	0.016	0.46	15.15[d]	..	0.14	0.25	...	...	..
414	$1\frac{1}{16}$-in. round super-free cutting screw-stock steel	0.10	0.66	0.098	0.195	....	..	..	..	207	93	29
415	1-in. round free-cutting screw-stock steel (imported)	0.05	0.48	0.066	0.192	....	..	..	..	180	93	32
416	$1\frac{1}{4}$-in. square S.A.E. 1112 cold-drawn, $\frac{1}{16}$-in. draft	0.09–0.13	0.70–0.90	0.08–0.11	0.085–0.12	....	..	..	..	196	94.5	30
417	$\frac{3}{4}$-in. square S.A.E. 1120 cold-drawn, $\frac{1}{16}$-in. draft	0.18–0.22	0.60–0.80	0.050	0.075–0.12	....	..	..	..	156	90	26
418	$1\frac{1}{2}$-in. square S.A.E. 1120 cold-drawn, $\frac{1}{16}$-in. draft	0.18–0.22	0.60–0.80	0.050	0.075–0.12	....	..	..	..	163	85	28
419	$\frac{3}{4}$-in. square S.A.E. 1112 cold-drawn, $\frac{1}{16}$-in. draft	0.09–0.13	0.70–0.90	0.08–0.11	0.085–0.12	....	..	..	..	207	98	34
420	$1\frac{1}{8}$ round manganese screw-stock cold-drawn, $\frac{1}{16}$-in. draft (1)	0.10–0.20	1.00–1.30	0.05	0.09–0.13	....	..	..	..	170	92	29
421	1-in. round S.A.E. 1315 cold-drawn, $\frac{1}{16}$-in. draft (2)	0.10–0.20	1.25–1.65	0.05	0.10	....	..	..	..	179	92	28
422	$1\frac{3}{16}$-in. round S.A.E. 6140 cold-drawn, $\frac{1}{16}$-in. draft	0.35–0.45[b]	0.50–0.80	0.04	0.04	0.80–1.0[d]	0.15	..	..	228	99.5	35
423	$1\frac{1}{8}$ in. round S.A.E. 2320 cold-drawn, $\frac{1}{16}$-in. draft	0.15–0.25	0.50–0.80	0.04	0.045	3.25–3.75[e]	..	..	..	205	96	33
424	$1\frac{1}{4}$-in. round manganese-carbon alloy cold-drawn, $\frac{1}{16}$-in. draft	0.30–0.40[c]	1.60–1.90	0.04	0.045	....	..	..	..	255	103	42
425	1-in. round special manganese-carbon alloy cold-drawn, $\frac{1}{16}$-in. draft (4)	0.25–0.40	1.25–1.65	0.05	0.10	....	..	..	..	199	96.5	33
426	$1\frac{1}{2}$-in. round S.A.E. 1320 cold-drawn, $\frac{1}{16}$-in. draft (3)	0.15–0.25	1.25–1.65	0.05	0.10	....	..	..	..	168	92	31

[1] Materials used in "Machining Properties of Some Cold-Drawn Steels," Trans. A.S.M.E., vol. 53 (1931), MSP-53-6. Paper shows photomicrographs.

[a] The sulphur on all the high-sulphur steels may run above the limits indicated.
[b] Actual carbon content, 0.336 per cent.
[c] Actual carbon content, 0.365 per cent.
[d] Chrome.
[e] Nickel.

Results of Tests

The various values of energy in foot-pounds required to remove a specific chip are shown plotted over the different materials in Fig. 22. The materials are arranged from left to right in the order of increasing energy values for the condition of milling up. The energy values for milling down are superimposed and are shown to follow the milling-up line quite favorably. A number of interesting facts are evident. For the non-ferrous metals—cast iron, malleable cast iron, as well as the cold-rolled low-carbon steel, the die steel, and annealed high-speed steel—there appears to be very little difference between the milling-energy values when milling up and down. The value of the ratio of the energy required when milling down to that for milling up is practically unity.

Copper and monel metal are exceptions and show opposite characteristics. Of all the metals tested, copper has the lowest value of the ratio of energy down to up, this being only 0.668. Monel metal requires more energy when cutting down than when cutting up and has a ratio of energy values equal to 1.14. The three tool steels as well as the axle steel and screw-stock steel (S-1) also show greater energy values for milling down than for milling up, all giving values ranging from 1.05 to 1.21.

It is interesting also to note that the S.A.E. 2320 and 3120 steels, the only alloy steels except the die steel, show the values of the ratio considerably below unity, which indicates that in machining these metals it is desirable to mill down rather than up from a power standpoint.

The energy values when milling the standard chip 0.250 in. wide and 0.125 in. deep at a 0.010-in. feed with a 15-deg. front-rake cutter, for the group of carbon- and alloy-steel forgings are shown plotted in Fig. 23. The materials are arranged as abscissas from left to right in the order of increasing energy when milling up. This curve indicates the possibility of a relation to the energy values of two factors for each series of metals, namely, the carbon content and the temperature of annealing.

In the S.A.E. 61 series, for example, the energy value for S.A.E. 6120 is greater than that for S.A.E. 6130, which in turn is greater than that for 6140, which also in turn is greater than that for 6150. This indicates that the energy values increase as the percentage of carbon decreases and, with the exception of S.A.E. 6150, the value of the ratio of milling energy down to up increases as the percentage of carbon increases. The S.A.E. 6150 has a low ratio value of only 0.93. The S.A.E. 6150 steel, as shown in Table 23, was normalized at 1650 deg. fahr. and annealed at 1550 deg. fahr., giving a Brinell number of 187, whereas S.A.E. 6120, 6130, and 6140 all were annealed at 1500 deg. fahr. and reannealed at 1400 deg., giving Brinell numbers of 137, 156, and 163, respectively. This shows the Brinell number to be in direct proportion to the carbon content. This might indicate that the higher annealing temperature causes the material to machine easier. The arrangement for the S.A.E. 61 series confirms the arrangement previously shown in a paper on drilling[10] in which the S.A.E. 6150 has the lowest torque, followed in order by S.A.E. 6140, 6130, and 6120.

The same general tendency is shown for the S.A.E. 23 series. S.A.E. 2320 which was annealed at 1600 deg. fahr. shows lower milling-energy values than S.A.E. 2330 and 2340, which were annealed at 1500 deg. and reannealed at 1400 deg. This same relation was found also in the drilling experiments. The ratio value of energy required to mill down as compared with that for milling up is below unity for the S.A.E. 2320, equal to unity for S.A.E. 2340, and slightly above unity for S.A.E. 2330.

It will be noted that for the majority of these steels, less energy is required for milling down than for milling up.

As a conclusion, similar milling data for a variety of cold-drawn steels are shown plotted in Fig. 24. Again the bars are arranged as abscissa from left to right in the order of increasing milling energy when milling up. In the majority of cases, a saving is found in milling down and, in general, of about the same proportion.

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[10] Trans. A.S.M.E., vol. 52 (1930), MSP-52-2, p. 19.

Bars 420, a manganese screw-stock steel, and 422, an S.A.E. 6140 steel, seem to be exceptions in that they are the only ones to show a material increase in energy in milling down over milling up. A curve showing the value of the ratio of milling down to milling up is also shown in Fig. 24, plotted over the materials arranged from left to right in order of increasing value of the ratio. The lowest ratio value is 0.885 for bar 413, a free-machining rustless steel. The ratio is fairly uniform at 0.95 for the majority of the metals, and only the last four to the right show ratio values above unity.

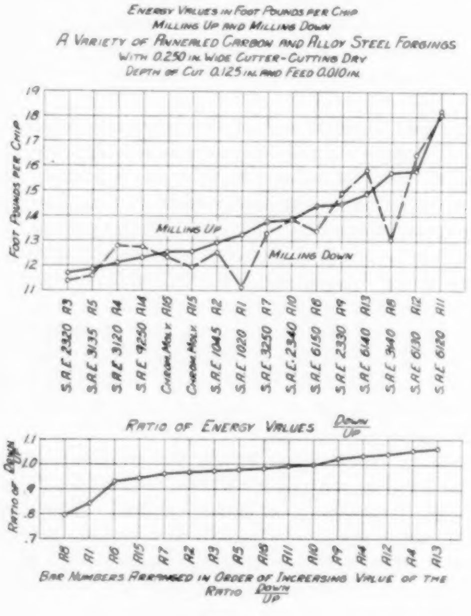

Fig. 23

Supplementary Milling-Energy Data Comparing Metals Cut With a Cutting Fluid

In order that some comparison of milling-energy data when using a lubricant with those when cutting dry might be made, a variety of metals were remachined, using a 10 per cent lard oil in 90 per cent mineral oil (No. 8) instead of dry cutting (No. 1). These milling-energy values in foot-pounds per chip when milling up and down are shown in Table 28. In making these check runs, the tool was supposedly in a sharp condition, although it was not freshly ground, so that the energy values under the same cutting conditions did not vary appreciably. It is to be noted, however, that with this mineral-lard oil, the energy values when milling up and down on S.A.E. 6150 steel (A-6) are both higher than when milling dry, as shown in Table 26. Also, the milling-up-energy value for the S.A.E. 6140 steel (A-13), in Table 28, when using cutting fluid No. 8 is slightly higher than the corresponding value when milling dry, as shown in Table 26.

TABLE 25 ENERGY REQUIRED TO MILL A VARIETY OF FERROUS AND NON-FERROUS METALS

[Cutter, 0.250 in. wide; 0.125 in. depth; 0.010 in. feed; ft-lb. per chip when cutting dry (No. 1); cutting up and down]

Bar No.	Material	Milling Up	Milling Down
	Cold-rolled steel	14.77	14.65
187-A	Annealed brass screw stock	4.72	4.85
BB	Bearing bronze	4.71	4.71
MB	Manganese bronze	5.95	5.98
Al-31	Aluminum alloy	4.16	4.29
Cu	Pure copper	16.72	11.16
M	Monel	16.97	19.40
DM	Dow metal	2.65	2.65
11-1	Cast iron	6.88	6.91
23-B	Malleable iron (skin)	8.77	8.91
H.S.	Clarite high-speed steel, annealed	17.55	17.50
30	S.A.E. 2320	15.86	13.70
31	S.A.E. 3120	14.30	13.65
32	Tool steel, 1.03 per cent carbon	18.75	19.70
33	Die steel	13.20	13.09
T-1	Tool steel	15.25	16.45
T-2	Tool steel	15.20	16.28
A-1	Axle steel	11.29	13.65
S-1	Screw-stock steel	10.65	11.75

In nearly all cases where the milling-down-energy values were less for dry cutting than those for milling up, they are less when cutting fluid No. 8 was used. The annealed high-speed steel and monel metal, however, are glaring exceptions. For the high-speed steel when cutting dry, the energy value was 17.5 when cutting up and down. When cutting fluid No. 8 was used, the energy value when milling up is only 14.55, and when milling down is 15.25, a considerable saving because of the lubricant.

For monel metal, using the mineral-lard oil, 16.11 ft-lb. are required when milling up and 12.50 when milling down as compared with 16.97 when milling up dry and 19.40 when milling down dry, again showing a great saving, and particularly a greater saving when cutting down. A great saving is shown in the energy value required to mill copper when using the mineral-lard oil over that when cutting dry. In general, considerable and consistent savings in energy are made when milling metals both up and down when a cutting fluid is used.

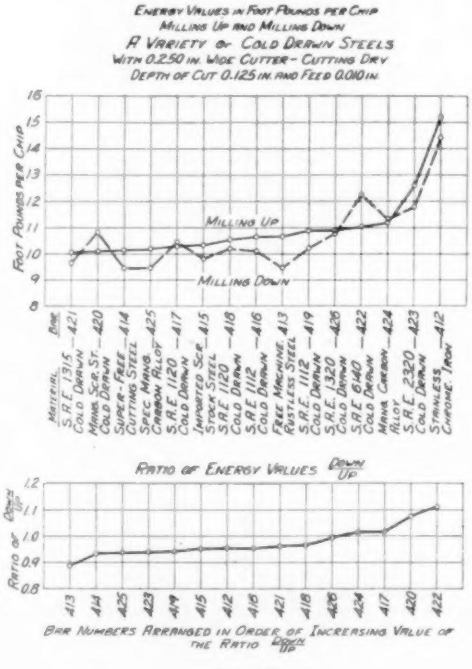

Fig. 24

Discussssion of Milling Data of This and Other Papers

Considering the importance of milling operation in modern production work, it seems strange that more has not been done to put milling on a scientific bases. Early cutters had a large number of teeth, each ground radially, that is, they had no front rake or side rake. Such cutters were used extensively until the last few years.

In 1911, when a paper on "Milling Cutters and Their Effi-

TABLE 26 ENERGY REQUIRED TO MILL A VARIETY OF ANNEALED CARBON AND ALLOY-STEEL FORGINGS

[Cutter, 0.250 in. wide; 0.125 in. depth; 0.010 in. feed; ft-lb. per chip when cutting dry (No. 1); cutting up and down]

Bar No.	Material	Milling Up	Milling Down
A1	S.A.E. 1020	13.24	11.15
A2	S.A.E. 1045	12.95	12.56
A3	S.A.E. 2320	11.70	11.38
A4	S.A.E. 3120	12.11	12.80
A5	S.A.E. 3135	11.85	11.60
A6	S.A.E. 6150	14.41	13.42
A7	S.A.E. 3250	13.80	13.30
A8	S.A.E. 3140	15.75	12.55
A9	S.A.E. 2330	14.50	14.90
A10	S.A.E. 2340	13.89	13.89
A11	S.A.E. 6120	18.20	18.10
A12	S.A.E. 6130	15.80	16.45
A13	S.A.E. 6140	14.90	15.85
A14	S.A.E. 9250	12.31	12.75
A15	Chrome-molybdenum	12.57	11.95
A16	Chrome-molybdenum	12.53	12.35

TABLE 27 ENERGY REQUIRED TO MILL A VARIETY OF SCREW-STOCK STEELS

[Cutter 0.250 in. wide; 0.125 in. depth; 0.010 in. feed; ft-lb. per chip when cutting dry (No. 1); cutting up and down]

Bar No.	Material	Milling Up	Milling Down
412	Stainless chromium iron, annealed	15.20	14.45
413	Free-machining rustless steel	10.69	9.46
414	$1^{1}/_{16}$-in. round super-free cutting	10.15	9.47
415	1-in. round imported	10.35	9.82
416	$1^{1}/_{4}$-in. sq. S.A.E. 1112 cold-drawn	10.65	10.13
417	$^{3}/_{4}$-in. sq. S.A.E. 1120 cold-drawn	10.30	10.45
418	$1^{1}/_{2}$-in. sq. S.A.E. 1120 cold-drawn	10.59	10.20
419	$^{3}/_{4}$-in. sq. S.A.E. 1112 cold-drawn	10.90	10.25
420	$1^{1}/_{8}$-in. round mang. screw stock cold-drawn	10.10	10.85
421	1-in. round S.A.E. 1315 cold-drawn	10.05	9.65
422	$1^{3}/_{16}$-in. round S.A.E. 6140 cold-drawn	11.05	12.25
423	$1^{1}/_{8}$-in. round S.A.E. 2320 cold-drawn	12.60	11.80
424	$1^{1}/_{4}$-in. round mang. carbon alloy	11.20	11.35
425	1-in. round special mang. carbon alloy	10.20	9.55
426	$1^{1}/_{2}$-in. round S.A.E. 1320 cold-drawn	10.90	10.80

TABLE 28 ENERGY REQUIRED TO MILL A VARIETY OF FERROUS AND NON-FERROUS METALS

[Cutter, 0.250 in. wide; 0.125 in. depth; 0.010-in. feed; ft-lb. per chip when cutting with mineral lard oil (No. 8); cutting up and down]

Bar No.	Material	Milling Up	Milling Down
BB	Bearing bronze	3.70	3.61
Al-31	Aluminum alloy	3.48	3.80
Cu	Pure copper	7.74	7.11
M	Monel	15.11	12.50
11-1	Cast iron	5.53	5.75
	High-speed steel (annealed)	14.55	15.25
T-1	Tool steel	14.07	16.35
	Cold-rolled steel	13.32	12.00
414	$1^{1}/_{16}$-in. round super-free-cutting screw-stock steel	9.05	8.35
422	$1^{3}/_{16}$-in. round S.A.E. 6140, cold-drawn	10.95	11.40
A-1	S.A.E. 1020	9.83	9.25
A-6	S.A.E. 6150	15.27	14.40
A-13	S.A.E. 6140	15.10	15.20

ciency"[11] by A. L. DeLeeuw was published, some coarse-tooth cutters with moderate rake angles and helix angles made their appearance. From that time until a paper by Prof. John Airey and Carl J. Oxford, entitled "The Art of Milling,"[12] was presented in 1921, little improvement seems to have been made in industrial practice. Many valuable data were given in this latter paper as to the effect of feed and depth of cut as well as the influence of rake and helix angles on the power required to mill metals. Cutter design was considered, and formulas were suggested for determining the number of teeth in a cutter. The paper aroused interest in the subject, and, in the next few years, several additional research projects were carried out.

Milling data, showing the energy in foot-pounds per chip as a function of the feed in inches, have been taken from Tables 1, 2, and 3 of Airey and Oxford's paper and plotted on log-log paper in Fig. 25. This has been done to compare these data with the results obtained by the authors. Curves are drawn for three materials: cast iron, machine steel, and tool steel. For each material, four tools were used, having zero, 10, 20, and 30 deg. front rake, respectively. In a number of instances it seems desirable that the experimental data should have been checked. For instance, in cutting machine steel with a feed of 0.0025 in., the 20-deg. side cutter gave energy values below the 30-deg. cutter. For all other feeds, the energy value for the 20-deg. cutter is higher. This would therefore indicate that the value for the 20-deg.-rake tool is low.

It seems obvious, however, that the influence of the rake angle is more to raise or lower the energy lines than it is to change their direction. Also, the exponent of the feed in the energy equation for cast iron is shown by the slope of the energy-feed lines to be very near to the value for machine steel. The authors did not find this to be the case. It may be that there is a difference between the cast irons used in the two sets of tests. In the case of machine steel, the exponent of the feed indicated in Fig. 25 is about 0.65, which compares very favorably with 0.64 for the S.A.E. 1020 steel shown in the authors' Table 18.

In general, it appears that the energy-feed lines for the three materials shown in Fig. 25 are parallel.

In 1923 a paper by Fred A. Parsons, entitled "Power Consumed in Milling,"[13] presented perhaps what are the most complete data up to the present time on the power required to mill various metals. Formulas on the basis of average thickness of chip were given for soft steel, several cast irons, yellow brass castings, and commercial aluminum castings. The general formula for milling was developed as follows:

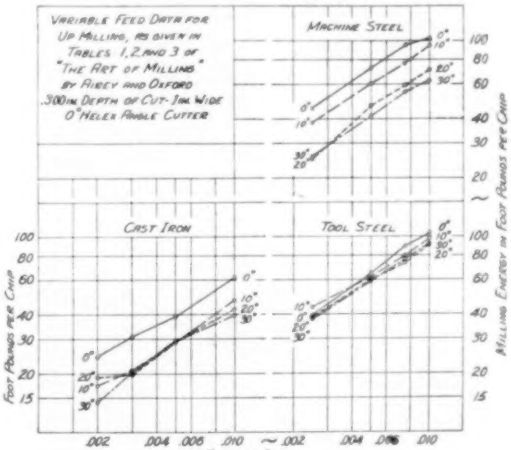

FIG. 25

$$\text{Cubic inches per horsepower per minute} = C\sqrt[x]{\text{A.T.C.}}$$

An attempt has been made to check Mr. Parsons' results, using the authors' data as given in Table 18. Mr. Parsons' data are based on the A.T.C. (average thickness of chip), which is computed from the feed f, for the depth of cut d, and the diameter of the cutter D, all expressed in inches, from the following equation:

$$\text{A.T.C.} = f\sqrt{\frac{d}{D}\left(1-\frac{d}{D}\right)}$$

Values of average thickness of chip were calculated for feeds of 0.003, 0.006, 0.009, and 0.012 in. for each of three depths of 0.010, 0.100, and 1.000 in., respectively. The diameter of the cutter used by the authors is 3.5 in. These values of average thickness of chip are shown in the right-hand column of Table 29, for the various combinations of feed and depth. The cubic inches of metal removed per horsepower-minute were then calculated for these feeds and depths from the horsepower formulas for a number of metals similar to (but probably not exactly like) those tested by Mr. Parsons. These values are shown in Table 29.

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[11] Trans. A.S.M.E., vol. 33 (1911), pp. 245–277.

[12] Ibid., vol. 43 (1921), pp. 549–587.

[13] Trans. A.S.M.E., vol. 45 (1923), pp. 193–227.

TABLE 29 CALCULATED VALUES OF AVERAGE THICKNESS OF CHIP (A.T.C.) AND CUBIC INCHES OF METAL REMOVED PER HORSEPOWER PER MINUTE FOR VARIOUS MATERIALS, FEEDS, AND DEPTHS OF CUT

[When milling up only with cutting fluid No. 1 (dry) and lard oil (No. 5). Diameter of cutter, 3.5 in.]

Feed in inches	Depth in inches	Cold-rolled, fluid No. 5	Bar A-1, S.A.E. 1020 Fluid No. 1	Bar A-1, S.A.E. 1020 Fluid No. 5	BF screw-stock brass, fluid No. 5	CF cast iron, fluid No. 1	Average thickness of chip, in.
0.003	0.010	0.565	0.266	0.369	1.77	0.252	0.00016
0.006	0.010	0.67	0.332	0.461	2.1	0.358	0.00032
0.009	0.010	0.747	0.395	0.548	2.32	0.454	0.00048
0.012	0.010	0.808	0.432	0.600	2.48	0.538	0.00064
0.003	0.100	0.762	0.445	0.617	1.94	0.696	0.0005
0.006	0.100	0.904	0.556	0.772	2.3	0.980	0.001
0.009	0.100	1.007	0.661	0.918	2.54	1.25	0.0015
0.012	0.100	1.087	0.723	1.004	2.72	1.49	0.002
0.003	1.00	1.03	0.740	1.027	2.14	1.91	0.00136
0.006	1.00	1.22	0.924	1.28	2.54	2.72	0.00271
0.009	1.00	1.36	1.10	1.53	2.81	3.45	0.00407
0.012	1.00	1.47	1.20	1.67	3.00	4.08	0.00542

These data are shown plotted graphically on log-log paper over average thickness of chip in Fig. 26. A number of facts are immediately apparent. The cubic-inches-per-horsepower-per-minute values of constant depth of cut but with variable feed appear, in all cases, to fall very nearly on a straight line, one line for each depth. Only in the case of cold-rolled steel is it possible to approximate all computed values with one straight line. When cutting free-cutting brass with lard oil (No. 5), for instance, Fig. 26 shows three distinct lines, one for each constant depth of cut indicated immediately below the line, for its range of feeds. These three straight lines appear to be almost parallel.

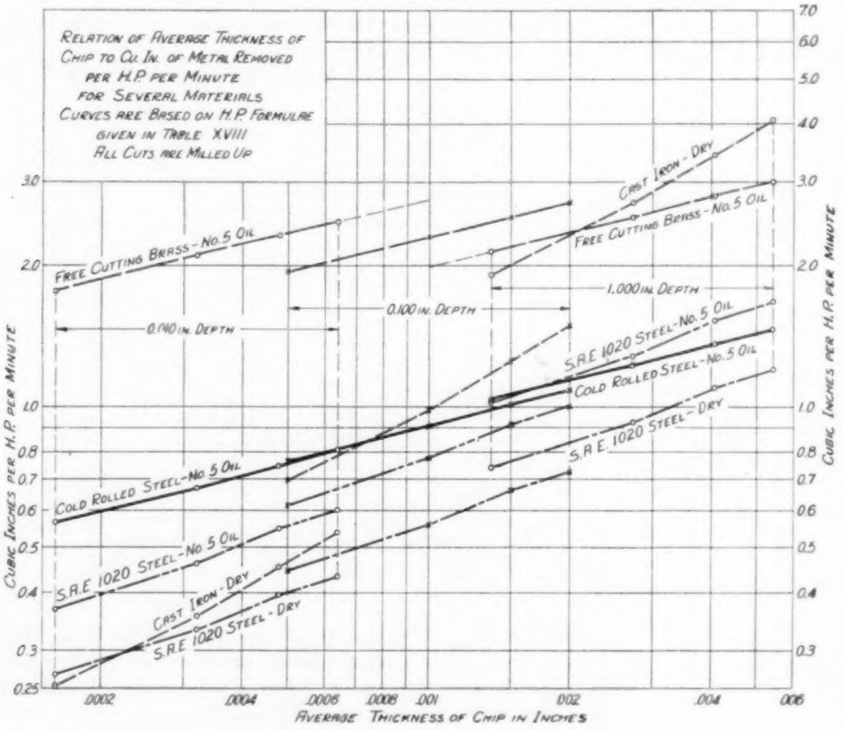

FIG. 26

Considering these three lines, it is seen that, when the depth of cut is 0.010 in., approximately $2^1/_2$ cu. in. of free-cutting brass are removed per horsepower-minute when the average thickness of chip is 0.00064 in. When the depth of cut is 0.100 in., $2^1/_2$ cu. in. are removed per horsepower-minute when the average thickness of chip is about 0.00136 in. When the depth of cut is 1.000 in., the same $2^1/_2$ cu. in. are removed per horsepower-minute when the average thickness of chip is about 0.0027 in. Therefore, it is seen that by varying the depth the same value of cubic inches per horsepower per minute is obtained for three distinctly different average thicknesses of chip.

Similarly, if the three straight lines shown for free-cutting brass when cutting with lard oil (No. 5) were extended to intersect the vertical line over the average thickness of chip of 0.001 in., the cubic inches removed per horsepower-minute are about 2.75 when the depth is 0.010 in., about 2.3 when the depth is 0.100 in., and about 2.0 when the depth of cut is 1.000 in.

The corresponding relations of the cubic-inch-per-horsepower-minute values for the three depths of cut when cutting cast iron diverge much more than those just illustrated for brass. Cold-rolled low-carbon steel, however, when cut with lard oil (No. 5) gives three lines which are almost identical. This one material, then, in a general way, confirms Mr. Parsons' results, although the others, even the S.A.E. 1020 steel, show a material divergency.

The reason for this divergence is apparent from a logical consideration of the variables involved in the formula. From

$$\text{A.T.C.} = f\sqrt{\frac{d}{D}\left(1 - \frac{d}{D}\right)}$$

it is seen that for relatively small values of d/D, the A.T.C. varies as $f\sqrt[2]{d/D}$, or as $fd^{0.5}$ since D is constant. Also, from the authors' formulas, the cubic inches per horsepower per minute equal a constant times $f^a d^b$, or (cu. in. per hp-min.)$^{1/a}$ varies as $fd^{b/a}$.

Therefore, in order for an equation of the formula

$$\text{Cubic inches per horsepower per minute} = C\sqrt{\text{A.T.C.}}$$

to hold for any given metal, it is necessary that the ratio of b/a equal 0.5. Values of b/a are seen from Table 18 to vary from zero for S.A.E. 3150 (KF) to 0.746 for cast iron (bar CF), so it is obviously inaccurate to use the A.T.C. as the variable in a general milling formula for cuts of this type. This is unfortunate, because the average thickness of chip seems to be a logical basis for determining cutting qualities.

A GRAPHICAL ANALYSIS OF CHIP SHAPE

Referring again to Airey and Oxford's paper on "The Art of Milling,"[14] one of the conclusions reached is that, given the same feed per tooth, two cutters with different numbers of teeth will have the same efficiency. The horsepowers per cubic inch removed per minute will be identical, although the cutter with the larger number of teeth will remove more metal in a given period of time because of the greater accumulated feed per revolution of the cutter per minute.

General practice at that time favored the coarse-tooth cutter. This led Prof. J. H. Hall and B. P. Graves to undertake a series of investigations at the plant of the Brown & Sharpe Manufacturing Company.[15] The conclusions which they reached were that coarse-tooth cutters were quite decidedly superior to fine-tooth cutters even when the feed per tooth was held constant.

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[14] Trans. A.S.M.E., vol. 43 (1921), pp. 549–587.

[15] Ibid., vol. 45 (1923), pp. 165–191.

In the milling machine used by the authors in accumulating the data presented in this paper, the work was held stationary while a single chip was removed. In actual practice, as a cutter rotates uniformly, the work is fed uniformly into the cutter so that, in some respects, the data presented by the authors do not conform with practical milling. An analysis was therefore made as to the amount of the error introduced when the work is held stationary as compared with results when the work is fed into the cutter. This analysis becomes more important when the cutting up or cutting down is considered. A graphical analysis of the shape of the undistorted chip removed by a single tooth of each of two cutters having 4 and 20 teeth, respectively, is shown both for up cutting at A and C and for down cutting at B and D in Fig. 27. These illustrations represent the conditions which would be maintained if the cutter had an outside diameter of 3.5 in. and in every case a feed of 0.250 in. per tooth. This excessive feed is used for purpose of graphic illustration.

At A the 4-tooth cutter is taking a chip in the conventional manner, that is, milling up against the feed. At B the same cutter is shown when milling down or cutting with the feed. Also, at C is shown the 20-tooth cutter milling up, while at D it is shown cutting down. In all cases the length of the chip parallel to the finished surface of the work is indicated by L_A, L_B, etc. For purpose of comparison, this horizontal projection of the true length of the chip will suffice.

Since metal is removed more efficiently for the greater thicknesses of chip, it is apparent from Fig. 27 that the thickest and shortest chip occurs at position D where the 20-tooth cutter is milling down. It is also apparent, when milling down with a 4-tooth cutter, that a thicker chip is obtained than when milling up with a 4-tooth cutter. Increasing the number of teeth in the cutter when milling down increases the chip thickness and so logically increases the efficiency of the cutter.

It is also seen that when milling up, the chip is thinnest and longest when the number of teeth is greatest. Summarizing, it is seen that, comparing cutters A and C, that is, the coarse-tooth and the fine-tooth, both when milling up in the conventional manner, that the thicker chip is obtained with equal feeds per tooth for the coarse-tooth cutter, resulting in greater efficiency for the coarse-tooth cutter. This seems to confirm the conclusions set forth by Graves and Hall, inasmuch as all of their conclusions were based on the conditions for milling up. It appears, then, that A is more efficient than C, that B is more efficient than A, and that D is more efficient than B. In other words, the fine-tooth cutter cutting up is the worst possible condition, and the fine-tooth cutter cutting down is the best, keeping the feed per tooth and depth of cut constant.

In many instances the data presented by the authors show that down milling is more efficient than up milling from an energy or power point of view. In such instances these differences would be increased in favor of down milling by virtue of the explanation just given when applied to practical milling in which the work moves while being cut.

Conclusions

In arriving at conclusions covering the experimental data presented in this paper, it is to be understood that the cutters were supposed to have been in an uniformly sharp condition at all times. A small amount of dulling has such a decided effect on the energy values that some inaccuracy, for this reason, is inevitable. The data have been accumulated over a period of time and have not always been taken methodically. Data determined in one series of tests may show slight differences when compared with corresponding data determined for other purposes. For this reason, check runs on many tests have been made and, where close agreement was not found, three and sometimes four individual checks were made. Standard practice was to make each experimental point the average of at least ten successive cuts. It is felt that the data presented are accurate to a very close degree and that the following conclusions are justified:

1 When cutting a groove it is found that energy values do not increase directly with cutter width for narrow widths, but at a slightly lower rate. Data are not available to determine exactly what this rate is, but the assumption is made, which is not far from correct, that the energy values do vary directly as the width of the cutter.

2 In all cases the energy values expressed in foot-pounds per chip are found to vary with the feed at rates, varying with different materials, from $f^{0.21}$ for bakelite to $f^{0.84}$ for annealed free-cutting brass.

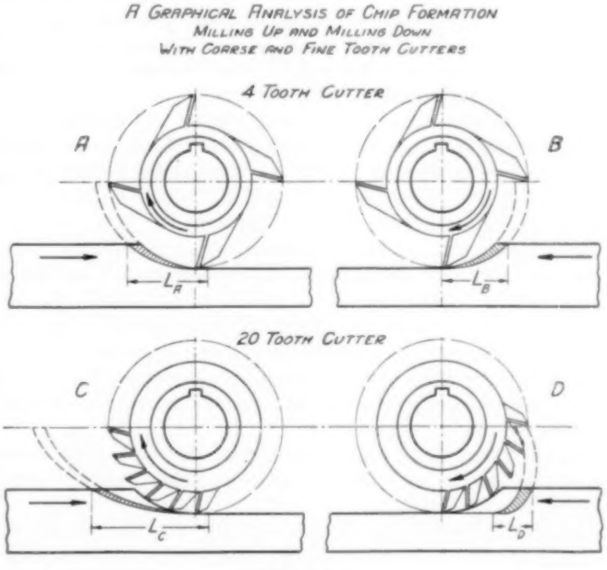
A Graphical Analysis of Chip Formation
Milling Up and Milling Down
With Coarse and Fine Tooth Cutters
4 Tooth Cutter
A
B
L_A
L_B
20 Tooth Cutter
C
D
L_C
L_D

Fig. 27

The exponents of feed for a given material are not always the same when milling up as when milling down. They appear to be close enough, however, for practical purposes.

Varying the cutting fluids, other conditions remaining the same, does not change the exponents of the feed for any of the materials tested. It simply moves the straight line on log-log paper up or down.

3 Energy values in foot-pounds per chip vary with the depth of cut for the different materials tested from $d^{0.56}$ for cast iron to $d^{1.00}$ for S.A.E. 3150 steel annealed.

The exponent is the same when milling up or down and regardless of the cutting fluid used when cutting all metals, but is not strictly so when cutting bakelite and wood fiber.

4 No two materials have been found which give the same milling-energy formula, and the authors know of no way of predetermining either the constants or the exponents of the feed and depth of cut except by making actual milling tests under given conditions for the particular metal.

5 Contrary to many other types of metal cutting, the energy value in foot-pounds per chip increases rapidly and quite uniformly as the milling cutter becomes dull.

6 The effect of cutting fluids is variable, as they react differently for various materials. Borax water, which tends to produce a low torque in drilling or low power in turning, gives consistently high energy values in milling.

7 The finish of the work is, in general, better when milling down than when milling up. The surface is flatter when milling down but less shiny, appearing to be torn rather than burnished.

8 Very little tendency to chatter was noticed when milling down, but in many cases chatter was objectionable when milling up. The data on the subject are too limited in the work reported for final conclusions to be drawn.

9 When milling a wide variety of materials, using the same width of cutter, depth of cut, and feed per chip, values of the ratio of energy when cutting down to that when cutting up were found to vary from 0.668 for copper to 1.21 for axle steel. For the majority of metals, some saving was found for milling down over milling up. On many materials, however, the reverse is true.

10 The value of the ratio of milling down to milling up for the S.A.E. 61 series and the S.A.E. 23 series steels was found to increase for increase in carbon content and Brinell hardness, and decrease for an increase in normalizing and annealing temperatures. The actual energy values in foot-pounds per chip are found to decrease with increasing carbon content and increased normalizing and annealing temperatures.

11 With the exception of pure copper and monel metal, small differences were found between the milling-energy values when milling down and up for the non-ferrous metals tested.

12 For most of the free-cutting high-sulphur steels, a rather uniform saving was found when milling down.

13 Formulas have been presented representing the energy required to remove a single chip under standardized conditions when milling up and down for a number of ferrous, non-ferrous, and non-metallic materials. These are summarized in Table 18. Corresponding formulas, representing horsepower per cubic inch per minute for these standardized conditions of milling up and down, also are given with actual horsepower-per-cubic-inch-per-minute values for each material.

14 In general, considerable and consistent savings in energy are made when milling metals both up and down with a mineral-lard oil as a lubricant.

15 It is believed that the average thickness of chip is not a sufficiently definite basis on which to base energy or power values in milling, due to the fact that under similar cutting conditions, such as feed per tooth, the thickness of chip will vary as the number of teeth in the cutter varies for the same diameter of cutter when milling up or down. Also, the average thickness of chip varies under otherwise equal cutting conditions for the same number of teeth in the cutter when cutting up or down. As Dr. Salomon has shown, the average thickness of chip under otherwise equal cutting conditions is greatest for cutters of small diameter and least for cutters of greatest diameter.

16 From a graphical analysis of chip shape, the following is found: (*a*) In practice, down milling gives thicker chips and should give greater efficiency than milling up, all other conditions being the same.

(*b*) As the number of teeth in the cutter increase with the same cutter diameter, the thickness of chip for milling up decreases, indicating that the cutter with few teeth would be more efficient when the feed per tooth is constant.

(*c*) As the number of teeth increase, the thickness of chip for down milling increases, indicating that the cutter with few teeth would be less efficient.

17 As a final conclusion, the authors feel that the results of this paper should be substantiated, in regard to finish, power, and tool life, by records secured from various jobs in industrial plants. In these tests, records should be kept for similar cutters or the same cutter when milling up and milling down, and for cutters of the same diameter having different numbers of teeth when milling up and milling down. Additional information from industrial practice also should be obtained, using cutters with varying helix, rake, and clearance angles. These tests should cover a variety of materials, cutting fluids, feeds, depths of cut, and types of cut.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to Carl J. Oxford, chief engineer of the National Twist Drill and Tool Company, for the use of the milling dynamometer. Mr. Theodore Thoren, a student in the Department of Mechanical Engineering, has been of real assistance in preparing the drawings and in working up the data. The work covered in this paper was done in the laboratories of the Department of Engineering Shop at the University of Michigan, Ann Arbor, Mich.

Discussion

O. W. Winter.[16] There is probably no field of engineering wherein more has been accomplished in the way of results and with less known from a scientific viewpoint than the field of machine-shop practice. In this particular case, it is believed, there is less known about milling than any other operation, with the possible exception of grinding. There have been exhaustive tests made on turning, drilling, and planing. This in a way is to be expected.

There are more variables that enter into a milling operation than in any of the aforementioned operations. This circumstance makes any milling research work complicated and utterly prevents any general assumptions being made. This necessitates a comparatively large number of tests in order to tie in all variable factors, and hence the concurrent expense and time required. Consequently, past investigations on this subject, while commendable and contributing, were not thorough enough.

The authors have made a worth-while advance. It is apparent they intend to get at the elements of milling. They will agree, however, that there is still much work to be done.

For some time the writer has been connected with a project of "milling-cutter efficiency." There have been considerable time, energy, and money spent on this work. This research uses standard-design milling cutters of various types on several common materials, noting the milling action, power consumed, etc. In the first year's work over 20,000 tests were run on cast iron and nickel-chromium steels, covering as many variable factors as possible. The following year about 10,000 tests were run on aluminum alloys. The results are not yet ready for publication.

In the meantime, it should be both interesting and enlightening to compare the results obtained by the authors and the writer. This cooperative comparison, coupled with further investigations, should do much to bring the subject of milling out of the darkness of guesswork and into the light of exact science.

James A. Hall.[17] There have been presented many interesting and valuable data on the energy required for milling a large variety of materials under a number of different conditions. All of these tests have been made with a single-tooth cutter which moves with a variable speed as the energy of the falling weight of a dynamometer is dissipated by the resistance of the cut, while in practice the milling cutter moves at a constant speed and several teeth may be cutting at one time. The work also moves with relation to the center of the cutter in practice, while there was no relative movement of these parts while these test cuts were in progress. There is no apparent reason why these variations from practical milling should make an appreciable difference in the relative values of the results. With reference to their absolute accuracy, it is noted that the power required in practice is influenced by the machine as well as by the cutter, but the data of these tests should not vary widely from results secured in the shop.

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[16] Detroit, Mich. Jun. A.S.M.E.

[17] Professor of Mechanical Engineering, Brown University, Providence, R. I. Mem. A.S.M.E.

The equations for horsepower required on the eleventh page are of particular interest. One of the conclusions to be drawn from them may be illustrated by a milling machine which is working to the limit of its power capacity, using a 24-tooth cutter with a feed of, say, 0.006 in. per tooth. If this cutter is replaced by one with eight teeth, leaving everything else (width, depth of cut, feed per revolution, speed, etc.) the same, the feed per tooth will be increased to 0.018 in. If the material being cut is S.A.E. 1020 steel, the equations of the paper show that the power required for the same machine output as before will have dropped over 32 per cent. The machine will then carry with ease a cut that formerly may have been carried only with difficulty.

The authors refer to tests made some years ago by Benjamin P. Graves and the writer in which the advantages of coarse-tooth cutters were demonstrated. These tests were made with standard cutters on a standard milling machine, and great care was taken to have accurate values of the power delivered to the cutter. Savings comparable to that given in the foregoing illustration were found when commercial fine-tooth cutters were replaced by others with fewer teeth. On the other hand, when a series of feeds per tooth were secured by changing the speed of coarse-tooth cutters, a saving in power due to the larger chips was apparent, but the amount was considerably less than the value given by the equations of this paper.

If cast iron were substituted for steel in the foregoing illustration, the energy equation would give the horsepower needed for the coarse-tooth cutter as approximately half that required for the fine-tooth cutter, the work output being the same for both. If bakelite were the material being cut, the saving in power due to the use of the coarse-tooth cutter in the illustration would be over 57 per cent. These are greater savings than would normally be expected.

The authors present an interesting discussion of the effect of cutting with and against the work on the relative merits of fine- and coarse-tooth cutters. Due to the differences in the relative motion of the cutting edge and the work when the comparison is made on a chip-per-tooth basis, the chip will have a greater average thickness cutting down with the fine-tooth cutter, while the reverse will be true when cutting up. As the power required should vary roughly in proportion to the average chip thickness, the authors come to the conclusion that coarse-tooth cutters have greater advantages when cutting up and fine-tooth when cutting down. They apply their reasoning to the tests made by Mr. Graves and the writer to explain why the cutters with fewer teeth gave better results even compared on a chip-per-tooth basis. This may well be the answer to this question, as the differences in those tests were comparatively slight when cutters of sufficient coarseness were being used.

The advantages of fine-tooth cutters when hook or climb milling, as noted in the preceding paragraph, are realized only if the feed per tooth is the same as with the coarse cutters. To get the same feed per tooth with finer-tooth cutters, the feed in inches per minute must be increased, resulting in greater pressures between the cutter and work and more power input into the machine, or the surface speed of the cutter must be reduced, which also results in greater pressure between the work and cutter, although the power requirement remains the same. For instance, if the power capacity of the machine is reached with a coarse-tooth cutter running at a surface speed of 100 ft. per min. and taking the maximum permissible feed per tooth, a fine-tooth cutter with twice the number of teeth would have to run at 50 ft. per min. This would result in doubling the pressure between work and cutter.

If the proper number of teeth in the cutter is to be chosen for any job, the first and most important question is to determine the largest possible chip per tooth consistent with the structural strength of the cutter and the strength of the work. The number of teeth would then depend on the power capacity of the machine and on the strength and rigidity of the work, fixture, arbor, etc. Higher speeds and fewer teeth in the cutter will result in lower pressures on these parts. The finish required or danger of chatter or shock may be modifying conditions. The choice of the proper number of teeth depends on the proper evaluation of these factors. Under any condition the problem cannot be properly settled without taking the machine into account.

This paper emphasizes the great importance of using the maximum feed per tooth. The experience at the Brown & Sharpe plant indicates this may be fully as important from the standpoint of the life of the cutter between grinds. It would be interesting to have a discussion at some future time of the factors limiting chip size as a further step to putting cutter design on a scientific basis.

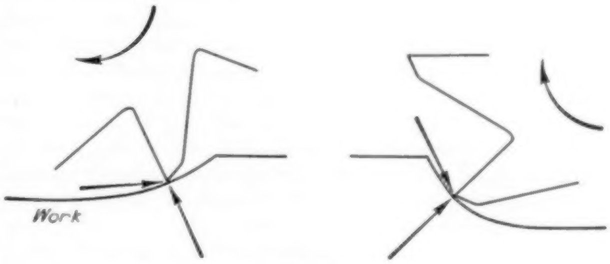

Fig. 28

In the tests described in this paper all of the cuts were made with a blade the outer edge of which moved in an arc of 1$^3/_4$-in. radius corresponding to a 3$^1/_2$-in. cutter. If a 2-in. cutter had been used, the length of the arc moved through by the cutter surface would have been less and the average thickness of the chip would have been greater for any given feed and depth of cut. Conversely, the opposite would have occurred if a 5-in. cutter had been used.

The writer assumes that the authors intended the formulas given to apply to a 3$^1/_2$-in. cutter only, and wonders if they made any investigation to show how the constants and coefficients were modified by varying the cutter diameter. In this connection it is unfortunate, from the shop standpoint, that they could not find the average chip thickness a suitable standard of comparison, as this would have combined all feeds, depths of cut, and cutter diameters on a common basis.

The maximum depth of cut used on any of the metals tested was 0.150 in. The writer wonders if the energy requirements would still follow a straight line on logarithmic paper if very deep cuts, $^3/_4$ in. or more, had been tried, and would like to inquire if any experiments were made along this line.

The discussion of the differences in power requirements cutting up and cutting down on the work is of particular interest. Which method to use on any particular job is also a problem that cannot be finally settled without taking into account the effect on the machine and fixture of the forces acting between the cutting and the work. These may be separated into tangential and radial components, as shown in the diagrams (Fig. 28). In average milling the radial force will exceed 50 per cent of the tangential force. If the vertical components of these forces are added, it will be noted that the pressure tending to push the cutter upward away from the work may be many times greater when cutting down than when cutting up. This pressure will result in greater deflections when cutting down, and increases the difficulty of milling flat surfaces on work of varying width. There is also more trouble of running the cutter back over the work. To take care of this condition, heavier arbors and more rigid set-ups will be required with this new type of milling.

In the usual type of milling, the forces always tend to push the work along the table and away from the cutter. In milling down, the horizontal components of the tangential and normal pressures may tend to pull the work along under the cutter. If this happens, it may result momentarily in excessive chips per tooth and in extreme pressures between work and cutter, resulting possibly in breakage of cutters or fixtures. The cutter must therefore be very well supported, have a foolproof fastening to the table, and the table must have enough drag on its ways or such a rigid mounting that there is no tendency for it to jump ahead. These represent some typical problems that must be met if the practice of milling down on the work is to be adopted.

A. N. GODDARD.[18] Any one carefully reading the paper will have some appreciation of the great amount of time and effort it represents on the part of the authors, even though it presents thought on but a very small unit of a very large subject, and only an elementary study of that unit, namely, "cutting up" versus "cutting down," and the energy required by each method in the process of removing metal by milling. Data have been gathered and tabulated, curves plotted, and results noted for quite a wide range of metals. This at least is a worthy attack on the problem, and goes farther than many previous papers have in a study of this general problem.

At the outset it is noted that these tests were run with tools in the condition of "sharp," "dull," and "very dull," and that energy measurements were attempted under these three conditions. These terms mean nothing in a scientific description, and one is quite "at sea" until he gets well beyond many tables of data and curves and finds in the middle of the paper an attempt at a description of the two latter conditions, but no definition of a *sharp* tool. It is a simple matter to determine whether a cutting edge has a 0.001-in. radius or one of 0.006 in. or 0.010 in., and this should be predetermined in such a test. In actual practice the edge of a metal-cutting tool, such especially as a milling cutter, when dull is not in the condition of an edge that has been stoned to a good radius, but many times when seeming to be that way microscopic inspection indicates that the process of dulling was first the crumbling away of small particles of the keen edge, and then the jagged points wore down by abrasion.

Again, in the Conclusions, it is stated that "a small amount of dulling has such a decided effect on the energy values that some inaccuracy for this reason is inevitable." Most of these data represent work done with comparatively small tools averaging from $^1/_4$ to $^3/_8$ in. in width, and with these almost infantile tools the energy values were so small, it seems, that slight inaccuracies would possibly produce large errors in results. It is hard to harmonize the thoughts in this paper as to the condition of the cutting edge with the theory expounded by the author in a previous paper in which it was shown that the cutting edge of a tool did not actually do the cutting, but rather built up at once a cold-weld wedge of the metal being cut on the nose of the cutting tool, this wedge being pried off the metal from then on.

Many tables and plotted curves throughout the paper speak of "constant" feed. The writer cannot appreciate how this is possible in any hand-fed mechanism and would question the use of this term and the data based on it. Doubt as to this term is further increased by the statement at the top of the 21st page that the data presented were gathered by holding the work stationary while a single chip was being removed. It is granted that "this is contrary to actual practice," so an analysis was made to determine the amount of error this introduced. This would seem to be quibbling with theory in the absence of practice and makes one reluctant to accept results.

All the experiments upon which the milling data are supposed to be based were made with a single-point tool, cutting one chip at a time through a limited arc of swing, with no feed of the work while the tool was cutting and with variable power applied to the tool. From these experiments and by graphical analysis the paper proceeds to *deduce* what would *probably* happen with a *real* milling cutter in a *real* milling machine. One deduction is that "increasing the number of teeth in a cutter when "cutting down" increases the chip thickness and so logically increases the efficiency of the cutter."

By how much can one increase the number of teeth? Where does this deduction lead us to? Would a rotary file be more efficient "cutting down" than "cutting up?" Chip clogging is a prominent source of cutter inefficiency and that is caused by fine-tooth cutters. The writer is only trying here to show what abstract theoretical reasoning may lead to if one goes too far; and why the need of such theorizing when practical means are at hand from which to obtain practical data?

Referring back to the equipment used in gathering these data, "the pendulum-type milling dynamometer" as described is not a mechanical power-operated machine. It has no constant source of energy actuating the cutting tool; the work is not mechanically fed under the cutting tool during the actual cutting. In fact, the machine is described briefly as having the same principle as an impact testing machine. In this machine the tool is not rotating at a constant speed as it approaches the cut nor as it passes through the cut. Being actuated by a partially impeded falling weight, it approaches the cut at a constantly accelerated speed; it strikes the cut with an impact, and its progress through the cut is not constant, but rather a constantly decelerated speed due to the dying energy of the falling body whose fall it serves to brake. It is being supplied with a dying-out energy due not only to its braking effect on the weight, but further to the fact that the weight constantly being diverted from its free-falling path, varies the energy supply from maximum to zero.

The effects so produced are more in accord with a cold chisel receiving the stored energy of a hammer blow. It seems, however, that this method is very far from correct in endeavoring to arrive at correct data on the practice of milling as performed on a machine supplying constant power at constant speed with constant table feed, all these constants being variable but controllable at the will of the operator and using a multiple-point rotary cutting tool.

The conclusions of the paper are quite detailed at the end, but are very well and more briefly summarized in the abstract given at the opening. The writer will therefore consider them briefly as there stated:

(*a*) "In nearly all cases a difference in energy is found between these two methods of cutting—cutting down and cutting up."

(*b*) "No two materials give the same values of the exponents for the feed and depth variables."

(*c*) "It is concluded from the nature of the tests that lubrication rather than cooling has greater effect."

(*d*) "In nearly all cases a power saving is made by their use."

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[18] President, Goddard & Goddard Company, Inc., Detroit, Mich. Mem. A.S.M.E.

(*e*) "It is concluded that there is no way, other than actual test, to determine the milling properties of a material."

(*f*) "It is concluded from this analysis that, theoretically, the coarse-tooth cutter is better than the fine-tooth cutter, when milling up, keeping the linear feed per tooth constant. When milling down, however, the fine-tooth cutter removes a chip of greater average thickness than the coarse-tooth cutter, and consequently is considered to be more efficient from a power standpoint when diameter and feed per tooth are constant.

(*g*) "The fine-tooth cutter cutting down is thought to be the most efficient way of milling metal when power is the chief consideration."

The first five of these conclusions are borne out by practice; in fact, are common knowledge in the industry, and would be generally true in any machine using rotating tools.

The conclusion (*f*) is not borne out in practice.

The conclusion (*g*) is doubtful as to relative pitch of cutter and inconsequential as to power efficiency.

Under the caption "Discussion of Milling Data of This and Other Papers," it is stated that "from the time of Mr. De Leeuw's paper on 'Milling Cutters and Their Efficiency,' which was presented in 1911, until a paper on 'The Art of Milling' by Prof. John Airey and Carl J. Oxford in 1921, little improvement seems to have been made in industrial practice." This refers to improvement in design of rake angles, tooth form, tooth spacing, and results obtained. A correction of this statement is suggested, as the writer personally made a very intensive study of milling cutters and made many improvements in their design which very materially increased the efficiency of milling practice. This work was begun in 1918, and by 1921 was widely known by advertising, by catalogs, and by application in many shops throughout the country. Our whole output of cutters was along these improved lines before any research was undertaken by Professor Airey and Mr. Oxford. Papers were not presented to the A.S.M.E. on the subject as the writer was too busy, but the work was done.

In conclusion, the writer is led to ask from the practical viewpoint, What does it all mean—what real knowledge as to the art or practice of milling is here contributed? Here is a very long and serious paper on "milling down" versus "milling up" with the relative power requirements by the two methods as the chief consideration. "Power" never is the chief consideration in milling, nor for that matter in the very large majority of metal-machining operations; it is quite the minor factor.

Keen advertising of machine tools today is certainly not far off the main point in directing prospects to the superiority of one's product. Note then the current ads of two of the largest manufacturers of milling machines. The slogan of one is "If it can be milled, mill it faster." The symbol of the other is a milling cutter with wings denoting speed of operation and the saving of time.

The time of doing the work; character of finish; accuracy of dimension; freedom from distortion when work is removed from machine; down time of machine for redressing tools and changing set-ups due to dulling—these are the important considerations in the manufacturing plant that is going to survive today. Then why not study such problems both scientifically and practically? The authors just glimpse this thought in the final (17th) conclusion, but why not concentrate study on some of these vital problems in our university laboratories that are constantly asking for funds for research from manufacturers? The authors feel that "little has been done to put milling on a scientific basis." While there is much that is not yet known, there is a very great amount of explicit and factual information regarding milling. There are established practices in various industries in the field of milling today that are astounding. To cite but a simple example, milling and sawing brass and tough tin bronze is done at speeds and feeds that would leave woodworking machinery far in the rear, and not with tungsten-carbide tools but with straight high-speed steel.

Milling-machine manufacturers and milling-cutter manufacturers have a fund of information that would be surprising reading if compiled in a single book, but by the time the book was compiled it would be ancient history, such is progress today.

It would be a most worthy effort for a professor of engineering to attempt to collect and classify, from the many sources available, such data. It would be a real contribution to the metal-working industries. At least it would be a very informative basis of departure for further research. Can we not thus get a closer contact between university research and industrial need?

This discussion may seem like a harsh and unkind criticism, but nothing is further from the spirit of it. If it be criticism, it is hoped it may be constructive.

Fred A. Parsons.[19] The comparative up-cutting and down-cutting tests, as well as the tests with different cutting fluids, are particularly timely. The apparatus used, however, appears to be open to some criticism in that it does not duplicate the operation of commercial milling machines. Particularly it appears that in the authors' machine the speed of the cutting tool was not constant during chip removal; also that the work piece was not advanced during chip removal; also that the cuts were all grooving cuts. It may be that these departures from usual practice without proper allowance therefor account at least in part for the failure of the authors' tests to check more nearly with the conclusions and formulas given in the writer's paper of 1923, with which a comparison is drawn by the authors. The nature of a grooving cut is such that the writer's 1923 formulas for average thickness of chip would not apply as the authors have applied them. The true average chip thickness for grooving cuts must take some account of the sides of the cut as well as the bottom width, similar to the writer's 1923 formulas for form cutters. When so corrected, the authors' Fig. 26 will correspond much more closely to the writer's 1923 formulas. These were in any event only advanced as approximations making it possible to put the power computations on a usable slide rule. Several well-recognized factors were therefore not taken into consideration. It appears probable that in computing the true average chip thickness for milling, the number of teeth in the cutter should be taken into account as well as the up or down nature of the cut, and perhaps other factors as well; but it is thought not proved that the true average chip thickness is not the most usable basic measure of power required for metal cutting. It may be added that in milling practice the conditions when cutting up where considerable of the power used is transmitted through the relatively inefficient feed train may cause for this reason alone the decidedly different total machine power from that when cutting down.

Hans Ernst.[20] Any investigation which leads to a better understanding of the numerous factors involved in the complex process of milling, should be highly commended. In the present instance, the authors are to be congratulated on the contribution which they have made by the investigation of some of the more obscure factors, such as direction of cut with respect to feed and the action of cutting fluids.

The pendulum type of dynamometer used by the authors undoubtedly provides a simple method of determining (with considerable accuracy) the energy dissipated in the removal of a given chip under the conditions imposed, and eliminates many of the difficulties and uncertainties presented by other methods; nevertheless, it also introduces limitations in the application of the data obtained.

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[19] Engineer in Charge of Patents, Kearney & Trecker Corporation, West Allis, Wis. Mem. A.S.M.E.

[20] Research Engineer, The Cincinnati Milling Machine Company, Oakley, Cincinnati, Ohio. Mem. A.S.M.E.

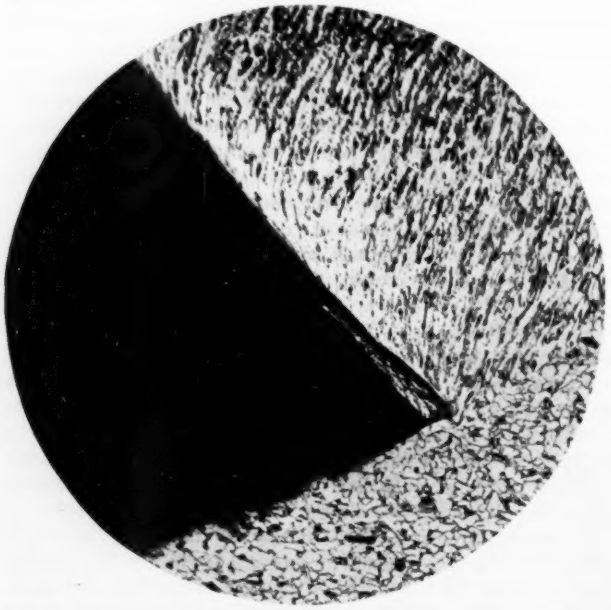

FIG. 29 PHOTOMICROGRAPH OF CHIP PARTIALLY REMOVED FROM WORK (MILLING UP) × 75

(Cutter stopped suddenly near end of cut. Material, S.A.E. 1112 steel; depth of cut, $^1/_4$ in.; feed per tooth, 0.020 in. Note built-up edge preceding nose of cutting tool.)

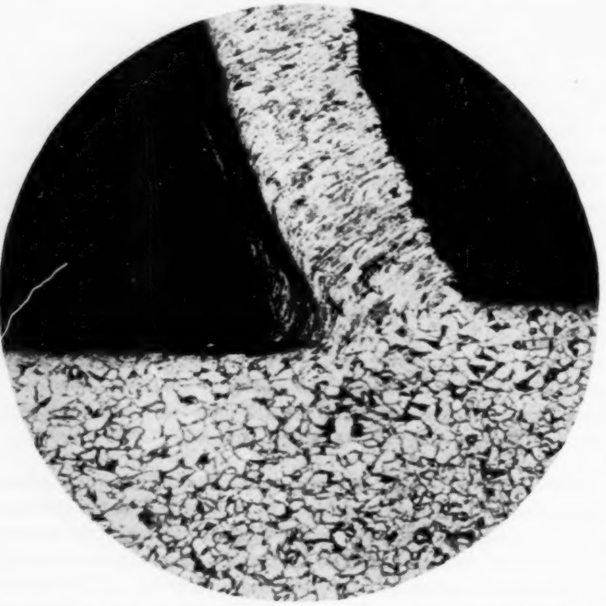

FIG. 30 SAME AS FIG. 29, BUT CUTTING DOWN × 100

(Note presence of built-up edge.)

With this type of dynamometer no information can be obtained in regard to the relative direction and magnitude of the several components of the cutting force; consequently, the data secured do not materially help in the design of a milling-machine structure, except in so far as they may indicate the maximum power which must be transmitted to the cutter for any given set of conditions. For the same reason they provide no guide to a rational explanation or analysis of the many unaccountable relationships which have been observed by the authors and presented in this paper.

For instance, in Figs. 3 and 18, which illustrate the results of tests of S.A.E. 1020 steel, there is shown a relatively small difference between the energy required for milling up and milling down when using cutting fluids Nos. 6 and 8, but a very large difference when using No. 5 fluid. When milling the free-cutting screw-stock steel, however, the difference in energy up and down is about equal for both No. 5 and No. 8, but considerably greater for No. 6. Similarly, the sequence of the fluid numbers, when arranged in order of the ratio Down/Up is totally different for each of the materials tested. This includes even the condition of cutting dry.

An interesting and useful addition to this investigation, which may shed some light on these perplexing issues, would be the micrographic study of the chips resulting from each series of tests. A careful comparison of chip structure will usually reveal much of importance, particularly in regard to the formation and effect of the "built-up edge" and the extent to which chatter will affect the amount of energy dissipated in chip removal.

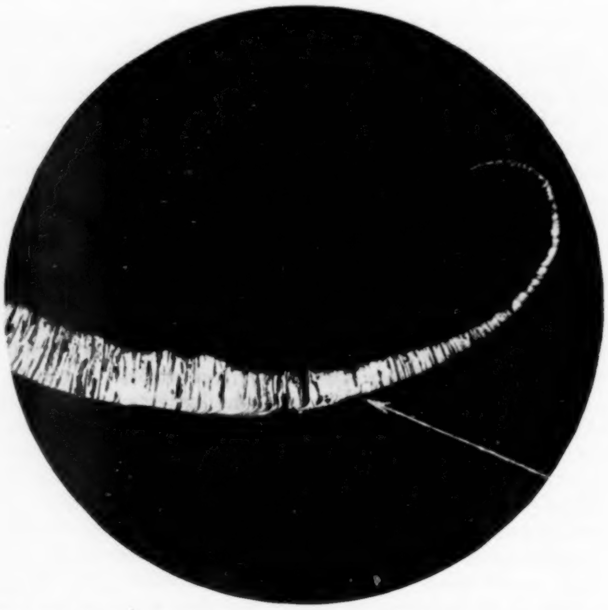

FIG. 31 "THIN END" PORTION OF CHIP REMOVED BY CUTTING UP (BEGINNING OF CHIP) × 30

(Material, S.A.E. 1020 steel; depth of cut, $^{13}/_{32}$ in.; feed per tooth, 0.013 in. *A*, built-up edge begins to form here.)

On the 13th page the authors state that "the duration of the cut in milling is too short for the built-up edge, if it is formed, to exert an appreciable influence on the formation of the chip." This is entirely contrary to our own experience. Under almost all normal milling conditions the built-up edge begins to form shortly after the beginning of each chip, and thereafter profoundly affects the formation of the remainder of the chip and the relationship and magnitude of the cutting-force components.

The existence of the "built-up edge" during both upward and downward milling is clearly shown by the accompanying photomicrographs Figs. 29 to 32.

Fig. 29 shows a chip partially removed from the work by milling up, while Fig. 30 shows the same condition by milling down; in each case the cutter was stopped suddenly while in the act of removing the chip. In each picture the small wedge-shaped portion, or built-up edge, is seen preceding the edge of the cutting tool and protecting the extreme edge from abrasion by the flowing chip. This effect continues until the chip is finally severed from the work, with the built-up edge firmly attached at the end.

Figs. 31 and 32 show the "thin end" portions of chips removed by milling up and milling down, under otherwise identical conditions. In each case the depth of cut was $^{13}/_{32}$ in., the feed per tooth 0.013 in., and the material S.A.E. 1020. The difference in chip shape and structure is striking. The downwardly milled chip is flatter and of much more uniform thickness toward the leaving end than the upwardly milled chip. In upward milling, the initial thickness of the chip is zero, and its rate of increase of thickness relatively slow. Hence, at the beginning of this chip (as shown in Fig. 33) there is no built-up edge, and the extreme edge of the cutting tooth is in contact with the work. The resistance to chip flow against the smooth face of the tooth is therefore small, and the chip is cleanly removed from the work and freely curled. As the chip thickness increases, the built-up edge is developed, the resistance to flow increases, and the chip section becomes thicker and flatter. Thus the chip from upward milling assumes the characteristic wedge shape, closely curled at its thin end, as shown in Figs. 31 and 33.

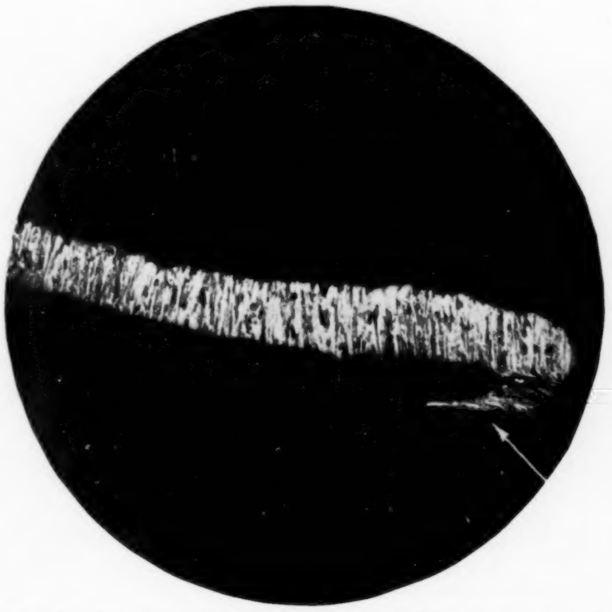

Fig. 32 "Thin End" Portion of Chip Removed by Cutting Down (End of Chip) × 30

(*B*, built-up edge attached to chip. Material, depth of cut, and feed per tooth same as in Fig. 31.)

In downward milling the thickness of the chip is a maximum almost at its beginning; consequently, the built-up edge is formed much more rapidly, and when once formed, persists to the extreme end. In Fig. 32 the remnant of built-up edge is shown attached to the leaving end of the chip. Thus the extreme edge of the cutting tooth is protected during a considerable portion of its engagement with the work.

Many questions are suggested by a study of this paper. On S.A.E. 1020 steel the authors found that the energy required for milling down is less than that for milling up when the cutter is sharp, but that the situation is reversed when the cutter becomes dull; the down energy being then greater than the up. What, therefore, is the effect of dulling upon the ratio of "down" to "up" energy on materials such as S.A.E. 6140, or high-speed steel, where the down energy was found to be greater than the up, even with a sharp cutter? It would also be interesting to know the effect of the various cutting fluids on materials such as these. It is unfortunate that not one of these materials was included in the tests reported in Table 21.

The most definite result of this investigation appears to be the establishment of formulas for the energy required per chip.

On the third page the authors state that, for any particular material, the energy curves for constant depth with respect to feed and for constant feed with respect to depth (for all the cutting fluids tested) could be represented by parallel straight lines when drawn on log-log paper. This is equivalent to stating that the exponents a and b are constant for any one material. This is substantiated by a large number of such charts on a variety of materials; consequently, we must assume that under the particular conditions of each test, such a relationship did hold true. Nevertheless there are some inconsistencies for which it is difficult to account.

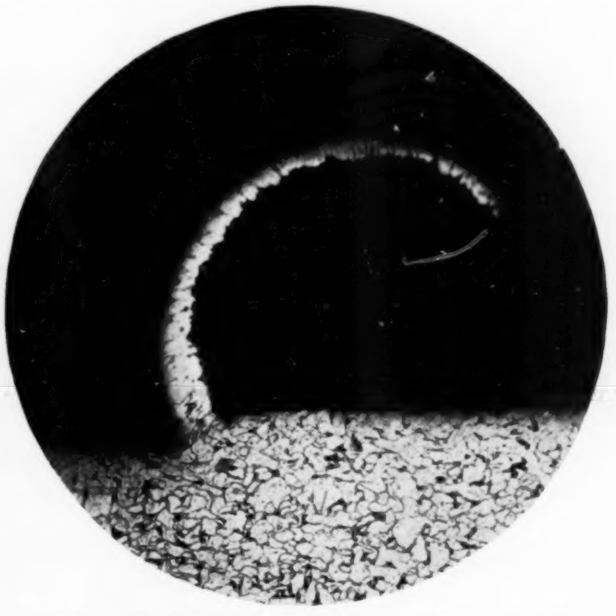

Fig. 33 Chip Partially Removed From Work × 100

(Beginning of chip, cutting up. Note absence of built-up edge.)

On the basis of the data shown in Fig. 5, which represents the results of tests on S.A.E. 3150, the exponent of f is found to have the value 0.70 for both the cutting fluids tested, and for both up and down cutting. Furthermore, the energy required for up cutting, with both the fluids, is appreciably higher than for down cutting. In Fig. 17, on the other hand, where the effect of cutter dulling is shown for the same material, the value of the exponent of f is approximately 0.56 when cutting up and approximately 0.50 when cutting down. In this series of tests, even when the cutter is sharp, the energy for cutting down is shown to be somewhat higher than for cutting up. This discrepancy between the two series of tests can scarcely be explained on the basis of the difference in cutting fluid, as all the other tests have shown the exponent of f to be independent of this factor.

The difference between the energy required for cutting up and cutting down, under the different conditions, as well as the effect of dulling (as shown in Figs. 15 and 16) may be explained

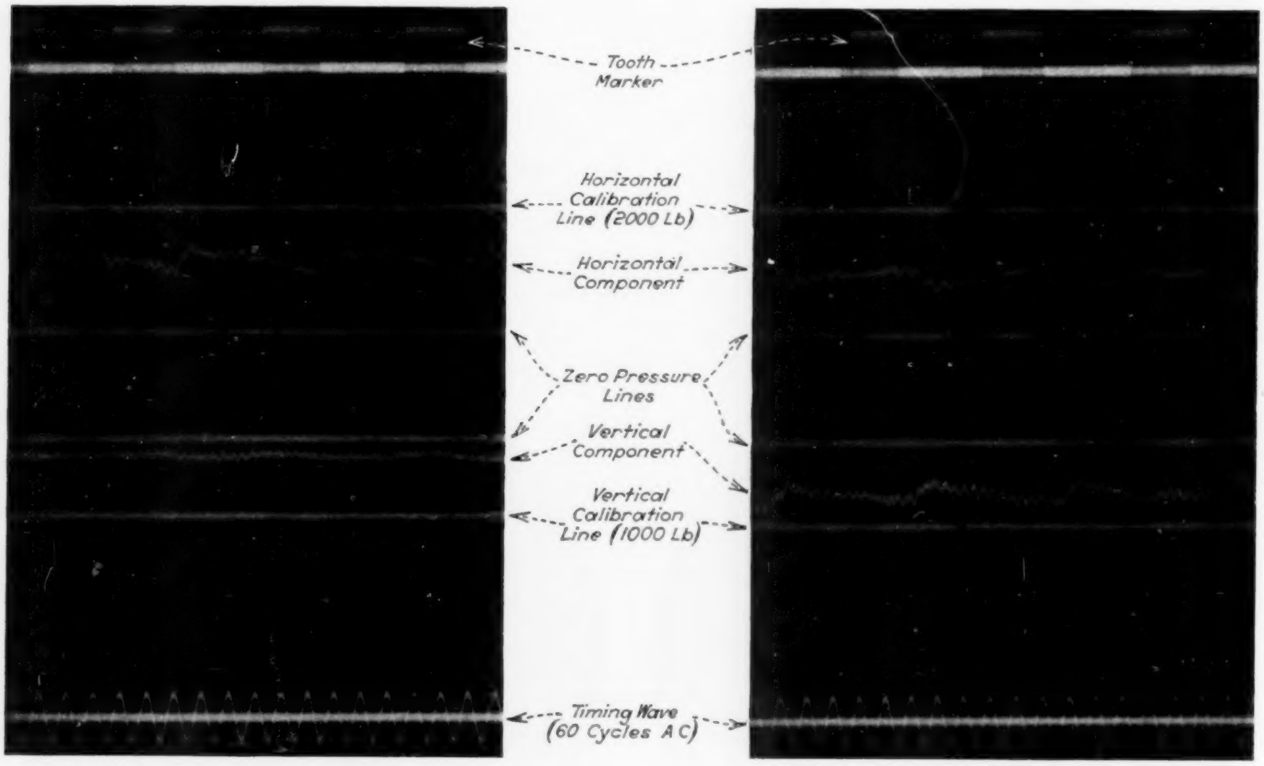

Fig. 34 Cutting Up

Fig. 35 Cutting Down

FIGS. 34 AND 35 OSCILLOGRAMS OF HORIZONTAL AND VERTICAL CUTTING-FORCE COMPONENTS
(Material, cast iron; depth of cut, 0.125 in.; feed per tooth, 0.012 in.)

on the basis of the radical difference between the relative magnitude of the cutting-force components for the two types of cut.

The accompanying oscillograms (Figs. 34 and 35) show the instantaneous values of the horizontal and vertical components in a particular case of milling cast iron with a feed per tooth of 0.012 in. and a depth of 0.125 in. Fig. 34 shows the condition of cutting up, and Fig. 35 that of cutting down. These records were made with an oscillograph operating in conjunction with a special dynamometer, designed to record extremely rapid force fluctuations in both a horizontal and a vertical plane. Calibration, tooth marker, and timing lines are carried on each chart.

A comparison of these records will show a marked difference in the character of the force-component curves for the two types of cut.

By integrating these curves, and subsequently recombining them to obtain the resultant tangential force on the cutter, the energy per chip may readily be calculated. Further work with this instrument will enable a definite comparison to be made between energy values under normal milling conditions and those obtained by the authors with the pendulum dynamometer.

G. A. Bouvier.[21] The writer cannot agree with some of the previous discussers that the authors' data are of little practical use because the method of test is so different from conventional milling practice, but feels, rather, that they are to be commended for recognizing that the only way to obtain an understanding of the fundamentals of milling is to break the general problem up into its elements in such a way that detailed studies of the subject can be made. It should make little difference if, in the analyses of the elements, simple devices are used for test purposes, because carefully gathered data can be analyzed with due consideration for the variations from practice. Furthermore, under these conditions far more accurate conclusions can be drawn than would be possible if highly complicated steps involving many variables were attempted. A thorough understanding of any engineering problem can be obtained only by careful isolation one by one of the many factors in the problem. It is surprising that so much really fundamental information has been obtained by the authors with a comparatively simple testing device. In further and more extensive work, they can be assured at least that they will be following the proper research road. The complexity of our work is too often increased by our insistence on making the problem complicated, with the false idea that we must do so in order to simulate ordinary shop practice.

Roger D. Prosser.[22] One of the conclusions reached in this interesting paper is that milling down is more efficient than milling up, as far as the power requirements are concerned. The tests were run with steel cutters, but if we consider the application of cemented-carbide cutters, which is becoming more and more important every day, it will probably be necessary to modify this conclusion, on account of the fact that the slight increase in power consumption will doubtless become a very minor factor in comparison with the greatly increased cutting speeds and length of tool life obtainable with the cemented-carbide-tipped cutters.

With reference to Fig. 27, showing the chip formation in milling up and milling down, it will be seen that, when milling down, the tooth must start with quite a heavy cut, which tapers off to nothing at the bottom. When milling up, the tooth starts with a very light cut, which increases up to the top. Furthermore, the average chip thickness when milling up is smaller than when milling down, taking the same feed and depth of cut. Thus milling up would appear to be much better for cemented-carbide cutters than milling down, as the tooth starts to cut with less shock, and the pressure on the cutting edge is less throughout the cutting. This is important, due to the very hard and comparatively brittle nature of the cemented carbide as compared with high-speed steel.

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[21] Development Engineer, Western Electric Company, Inc., Hawthorne Station, Chicago, Ill.

[22] Thomas Prosser & Son, New York, N. Y. Assoc. A.S.M.E.

It would be interesting to see further tests conducted, with the idea of extending the scope of this paper to include the cemented-carbide cutting materials.

Edward G. Herbert.[23] More than 20 years ago the writer's attention was drawn to the fact that file teeth were always cut with a negative rake, generally between 0 and —15 deg. and the teeth of milling cutters with zero rake. The writer's knowledge of the action of cutting tools, or what he then believed to be knowledge and now knows to have been ignorance, prompted him to suggest to the makers of files and milling cutters that these tools would cut much better if the teeth had a positive rake. The file makers and cutter makers acknowledged that the tools might be more efficient if made so, but pointed out that the tools did in fact cut in a highly efficient manner as made, and they gave clearly to understand that they did not propose to abandon a tooth shape which ought not to cut well but did, in favor of a shape which ought to cut better but might not.

After nearly a quarter of a century the writer finds that file teeth still have negative rake, that power hacksaws have zero rake, that many, perhaps most, milling cutters have zero rake, and that all of them cut metal in a highly efficient manner.

The explanation of this anomaly, the efficient cutting action of apparently inefficient tooth forms, is now known. It is illustrated by the two micrographs of undetached chips made by a hacksaw cutting through a steel bar $1^1/_2$ in. in diameter. (Figs. 36 and 37.) They clearly show that the cutting is not done by the tooth of inefficient form, but by the very efficient "built-up edge." In some cases it has been found that the built-up edge was specially acute and well formed and the cutting was correspondingly efficient. In other cases the edge was absent or ill formed and cutting was inefficient.

The chief feeling after reading the paper is one of wonder, first, as to what was happening in front of the tools, and second, why the authors did not use the simple and obvious method of sectioning undetached chips to find out what was happening.

The chips shown in Figs. 36 and 37 were formed by a tooth which had traversed less than 1 in. It is known that fully developed built-up edges may be formed and discarded as many as 50 times per second. And the built-up edge is not merely an incident of cutting; it is the cutting agent. Surely a study of the elements of milling should include an investigation of chip formation under the conditions adopted.

A great part of this investigation was directed to comparing the efficiency of "cutting up" and "cutting down" on various metals and with various lubricants, efficiency being measured by power absorbed.

This method of measuring efficiency calls for comment, because power consumption is by no means the first concern of the practical engineer, and because it is not at all certain that the power consumed in the Oxford dynamometer would correspond at all closely with the power consumed in a milling machine.

In practical milling it is usual to provide ample power for the work in hand, and, what is more important, ample rigidity in tool and work. The chief concern is the rapid removal of metal with adequate accuracy of finish; the second concern is the saving from damage of expensive cutters; and the third, if it is a concern at all, economy of power.

In milling up, a milling machine is required to drive the cutter through the work and also to drive the work against the cutter. In milling down, the work is not driven against the cutter. The action of the feed mechanism is rather to regulate the rate at which the work is dragged back by the cutter. The feed mechanism is practically running light. In the Oxford dynamometer this distinction is absent, and the power consumption takes account of only one of the two factors. It seems probable that in actual milling the saving in power by cutting down would be in all cases greater than these experiments indicate. There might be a net saving on all materials and under all cutting conditions. Whether this saving of power would justify the general adoption of a practice which is not free from difficulties and dangers is open to question.

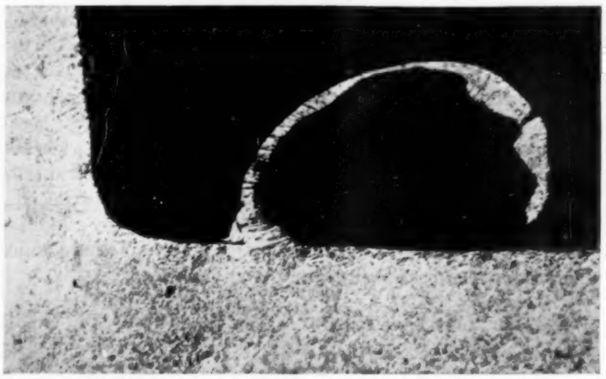

Fig. 36 Hacksaw Chip in Steel × 20

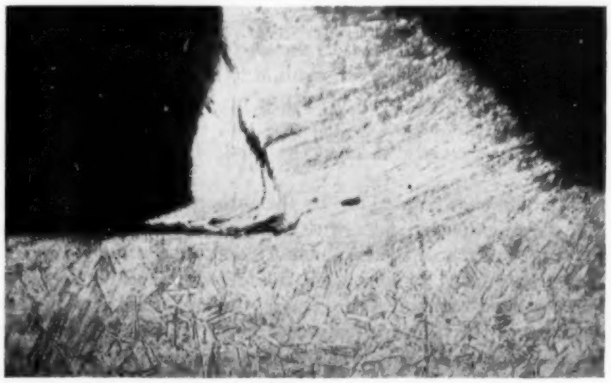

Fig. 37 Hacksaw Chip in Steel × 100

The method of cutting adopted by the authors was virtually that of hitting the specimen with an axe and taking off a wedge-shaped chip. The blow was heavy enough to remove the chip, but a slowing down of the axe was essential to the method of measurement. The action somewhat resembled that of a milling machine with such a slack belt or such a deficiency of power that it would slow up every time a tooth engaged with the work. Is it certain that data obtained from a limping milling machine could be directly applied to normal milling with ample power and uniform speed of rotation?

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[23] Director, Edward G. Herbert, Ltd., Atlas Works, Levenshulme, Manchester, England. Mem. A.S.M.E.

This is by no means intended as destructive criticism. In investigating a complex operation like milling, a simplification is necessary; otherwise the tests would become so unwieldy and the results so involved that any useful deductions would be rendered difficult or impossible. One such simplification, the use of a single-tooth instead of a multiple-tooth cutter, seems entirely justified. The other simplification, whereby the energy absorbed in a single pass of a pendulum-driven tool is taken as a measure of the efficiency of milling in a power-driven machine with power feed, may or may not be justified. It would be easy to trace a logical connection, thus: low absorption of energy; slight deformation of the cut material; low evolution of heat; the possibility of running the cutter at a high speed without damage by overheating; hence high production at low cost.

The validity of the connection thus suggested could be tested by a study of chip formation, which might be expected to show why, on a given metal, cutting from the thin end of the wedge takes more or takes less power than cutting from the thick end.

Its validity could be established by running some check tests with a normal cutter and machine, to find whether the condition indicated by the experiments as efficient did in fact permit a greater output than those indicated as inefficient; whether, in short, the results of the experiments could be translated into dollars and cents. If contact could thus be established, if only at a few points, between the artificial laboratory conditions and the conditions of the workshop, the authors could indeed claim to have laid the industry under an obligation.

Geh. Regierungsrat Prof. A. Wallichs[24] **and Dr.-Ing. H. Schallbroch.**[25] In order to make it possible to evaluate the magnitudes presented in this report dealing with the chip removal for various materials in comparison with earlier or later experiments of the same kind, it would be necessary to have a drawing showing the exact shape of the cutter. It is particularly essential to have the profile of the cutter, showing whether a side curvature is present as well as the dimensions of the arbor. If there were to be given, in addition to a drawing of the cutter, also a drawing of the place of cutting on the piece being worked (in two projections, all in perspective), this would enable the reader to see clearly at once the kind of chip removal being employed.

The difference pointed out by the authors between the method of cutting employed here with the work piece stationary and moved forward only by the amount f, and the continuous moving of the piece being worked, while the tooth turns as on the actual milling machine, is much greater than the authors believe. Because of this the application of down milling, recommended partly because of the lower work in foot-pounds per single chip, is not justified. As a matter of fact this forward motion ("down" in the direction of the rotation of the miller) is in practice not common, because of the unequal work and other objectionable features due to undesirable backlash of the work against the tooth.

A further difference may be established as due to the fact that the cutter tooth on an ordinary miller cuts the work at a constant velocity, while the single tooth of the device employed here lags in the matter of velocity while going through the work. As admitted by the author of the original paper, there is a further objection to this method from the point of view of practical conditions in milling, and that is the limited efficiency of the work coolant (namely, lubrication, but no cooling); a further objection lies to the evaluation of tests made with cutting teeth which, before running the test, were deliberately and uniformly dulled at the edge. The usual dulling of the cutting edges on actual millers unquestionably causes inequalities of operation which are much more difficult to observe. The application of the conclusions of the authors to actual milling practice will encounter certain difficulties due to the fact that the influence of cutting speed, so important in all processes of chipping, has been left out of consideration. It was recognized by F. W. Taylor as the most important variable in all processes making chips, and investigations made within the last decade have brought out particularly clearly how strongly the speed of cutting affects the life of the tool as well as the characteristics of the surface on the work.

It has been shown on radically different processes producing chips (turning, boring, milling) that the force of cutting proper (not the energy consumption of the machine, which naturally increases with the velocity of cutting) does not increase or decrease in any regular manner with the speed of cutting, but remains practically the same with all velocities of cutting in the same process of producing chips. Because of this it is difficult to draw, from the energy consumed in cutting, any reliable conclusion as to the life of the tool.

If we consider the chip cutting of the work only from the point of view of consumption of energy, then the data obtained with the simple experimental set-up used by the authors are in good accord with the data obtained in shop tests on milling published by Fr. Beckh.[26] The observations of Fr. Beckh to the effect that the energy consumption of the millers tested by him with eight to sixteen teeth is proportional to the width of cutting W and the depth of cut d, and increases in a somewhat lower ratio than the increase of feed f, would seem to be supported by the results of the authors with the single-tooth equipment.

At the beginning of each chapter of their work, the authors of the paper are very careful to point out the limitation of their method and caution the reader against too general applications of their data to actual milling processes. This makes their investigation very valuable, and one may congratulate the authors on having carefully done this work and clearly arranged the results.

Dr.-Ing. Georg Schlesinger.[27] Referring to the writer's own experimental work at the Experimental Laboratory for Machine Tools at Charlottenburg and on the basis of these tests, together with the data obtained by other German research workers, the writer has endeavored to combine the theoretical and experimental information hitherto available in reference to the process of milling in such a manner as to obtain general, very comprehensive, and yet simple equations having the advantage (as compared with other equations previously proposed) that they present truly geometrical relations unassailable from the mathematical point of view and yet in good accord with data obtained experimentally.

In so far as they cover the calculation of the work consumption of a single tooth these formulas read as follows. The work of the tooth is:

$$A_Z = K_M \times a \times b \times s_Z \quad [20]$$

The following notation is here used:

A_Z = work of a single tooth in kg-mm.
K_M = specific cutting force in kg. per sq. mm.
a = depth of cut in mm.
b = width of cut in mm.
s_Z = feed per tooth in mm.

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[24] Aachen, Germany. Mem. A.S.M.E.

[25] Aachen, Germany.

[26] Fr. Beckh, "Die Metallbearbeitung mittels Walzenfräsern," *Maschinenbau*, 5 (1926), pp. 497 to 504 and 557 to 561.

[27] Professor of Mechanical Engineering, Technische Hochschule, Berlin, Germany. Mem. A.S.M.E.

The specific cutting force is

$$K_M = f(h_M) \quad \ldots\ldots\ldots\ldots\ldots\ldots [21]$$

in which

$$h_M = s_Z \times \sqrt{(a/d)} \quad \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

is the thickness of the chip midway in the cut and d is the diameter of the milling cutter in millimeters.

Whether the shape of this functional graph plotted on log log paper is a straight line or a curve has not yet been determined, but the specific cutting force K_M may be expressed in the form of an experimental curve, as has been done by the discusser himself in *Werkstattstechnik*, 1931, p. 413. (See Fig. 38.)

If in this figure the slightly curved line plotted on log log paper for the type of steel St. 50.11 (notation according to the German Standards DIN 1611) is approximately replaced by a straight line which is, however, valid only for the range from $h_M = 20\mu$ (microns) to 100μ, Equation [21] becomes

$$K_M = k \times h_M{}^Z = 150\ h_M{}^{-0.28} \ldots [21a]$$

Here $k = 150$ has been obtained by extrapolation of the experimental line in Fig. 38 assumed to be straight and extended up to the point 1 mm. $= 1000\mu$; $Z = 0.28$ is obtained as tangent α = the inclination of the experimental curve (straight line). It would have been more precise and less subject to objection if one could read the values off the experimental curve itself, i.e., in its curved shape.

By thickness midway in the cut h_M is meant the thickness of the chip at that point determined for the average force and hence for the work of removing the chip by a single tooth as has been done by Boston and Kraus.

From these three general equations the Boston and Kraus equation can be obtained by a simple mathematical operation. It is $E = C \times w \times f^x \times d^y$. Here E is the work of a single tooth in foot-pounds; C, the constant for the milling tooth (0.250 in. wide with a 15-deg. cutting angle) and given material of the work; w, width of cut in inches; f, path of a single tooth in the direction of feed in inches; d, depth of cut in inches.

If the discusser's notation and the metric system are used and the values changed to the American system of units only at the end, the following may be derived from the three fundamental equations as stated above:

$$h_M = s_Z \times \sqrt{(a/d)} = s_Z \times a^{0.5} \times d^{-0.5} \ldots\ldots [25]$$

From [22], and from curve in Fig. 38 for the type of steel referred to above ("St. 50.11") can be obtained approximately:

$$K_M = 150 \times h_M{}^{-0.28} \ldots\ldots \text{compare } [21a]$$

Then, by combining the two, there is obtained

$$K_M = 150(s_Z \times a^{0.5} \times d^{-0.5})^{-0.28} = 150 \times s_Z{}^{-0.28} \times a^{-0.14} \times d^{0.14} \ldots\ldots\ldots [24]$$

But it has been stated above:

$$A_Z = K_M \times a \times b \times s_Z \ldots\ldots \text{compare } [20]$$

Hence we obtain by substituting the values of [24] in Equation [20]:

$$A_Z = (150 \times s_Z{}^{-0.28} \times a^{-0.14} \times d^{0.14}) \times a \times b \times s_Z$$
$$= 150 \times d^{0.14} \times b \times s_Z{}^{0.72} \times a^{0.86} \ldots\ldots\ldots\ldots\ldots [25]$$

Equation [25] corresponds exactly to the Boston and Kraus equation for cold-rolled low-carbon steel, as is evident by comparing the two equations:

$$\text{Schlesinger:}\quad A_Z = (150 \times d^{0.14}) \times b \times s_Z{}^{0.72} \times a^{0.86}$$
$$\text{Boston:}\quad E = 6800 \times w \times f^{0.73} \times d^{0.87}$$

(6800 and the exponents have been taken from Table 18 of the Boston and Kraus paper.)

The following correspondence in the two formulas has been found:

Schlesinger	Boston and Kraus	
A_Z in kg-mm.	E in foot-pounds	= work of a single tooth
b in mm.	w in inches	= width of cut
s_Z in mm.	f in inches	= feed per tooth
a in mm.	d in inches	= depth of cut

From the above comparison the following may be derived:

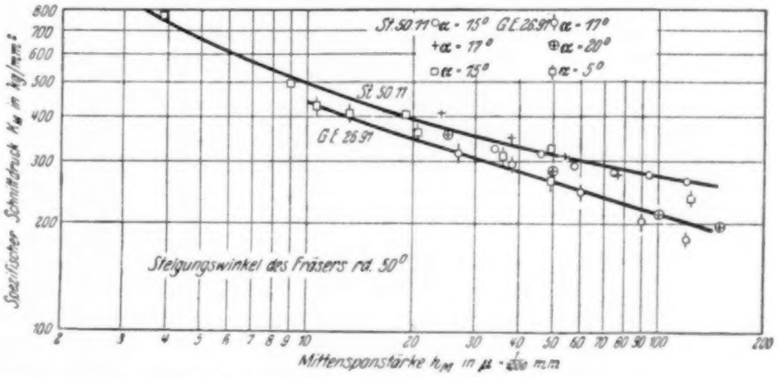

FIG. 38 RESULTS OF MEASUREMENTS OF THE SPECIFIC CUTTING PRESSURE IN MILLING STEEL AND CAST IRON (FROM THE ADB INVESTIGATIONS)

(*Specifischer Schnittdrock K_M in kg/mm.²* = Specific cutting pressure K_M in kg. per sq. mm. *Steigungswinkel des Fräsers rd.* 50° = Helix angle of milling cutter about 50 deg. *Mittenspanstärke h_M in $\mu \cdot {}^1/_{1000}$ mm.* = Chip thickness h_M midway in cut in thousandths of a millimeter.)

The exponents in the two equations set side by side are practically the same; namely, for tooth feed 0.72 and 0.73 and for depth of cut 0.86 and 0.87. But from the generalized and therefore more comprehensive equations of the discusser one may assume that there must exist a certain relation between these two exponents, which in his case may be written as follows:

$$\left.\begin{aligned} 1 - 0.72 &= 0.28 \\ 1 - 0.86 &= 0.14 \end{aligned}\right\} 2 \times 0.14 = 0.28$$

These relations as shown by the more comprehensive equations must, however, necessarily apply for all materials. The differences between the exponents and unity must be in the ratio of 1 to 2. Hence the experiments carried out by Boston and Kraus must be capable of being expressed by a single characteristic curve of the type $K_M = f(h_M)$. The specific cutting pressure is functionally related to the thickness of chip midway in the cut.

This experimental curve may, with precision sufficient for practical purposes, be expressed by a straight line on a double logarithmic paper, and a simple algebraic formula may be then derived for this line in the usual manner. Under no conditions, however, can it be said that the curves given by Boston and Kraus represent two experimental curves independent of each other. $A_Z = f(s_Z)$, energy consumption of a tooth as depending on the tooth feed, and $A_Z = f(a)$, energy consumption of a tooth as depending on depth of cut.

In many cases, but not always, the necessary connection between the two exponents in the Boston and Kraus equations is present with a certain amount of precision. Apart from the slight variations which might have been due to experimental error, but which in view of the very great care exercised by Boston and Kraus may amount to comparatively little, we may be dealing here perhaps with the fundamental weakness of the experimental apparatus used. The results obtained with the pendulum dynamometer do not in every instance coincide with the amounts of energy expended in actual milling, as will be shown more completely later in this discussion. In the first place we may ask in how far the constants in the two juxtaposed equations coincide with each other. There must be an agreement between $150 \times d^{0.14}$ in the metric system and 6800 in the American system of units.

It will be noted from the more comprehensive equations of discusser that the constant comprises the diameter of the milling cutter and is affected by an exponent which is dependent on the experimental curve, and in the present case is about the seventh root of the milling-cutter diameter. The influence of the diameter, however, is slight, as when the diameter is doubled the constant increases by only about 10 per cent. The more comprehensive formula, however, brings out this influence.

Boston and Kraus were working with an installation in which the milling-cutter diameter corresponded to $d = 3.5$ in. $= 89$ mm. From this the constant in the discusser's modified equation would be:

$$150 \times 89^{0.14} = 150 \times 1.87 = 280$$

In recomputing this constant in terms of the American system of units, b, sz, and a must be converted from millimeters into inches by using the factor:

$$25.4^{1+0.72+0.86} = 25.4^{2.58} = 4220$$

and furthermore kg-mm. must be change to ft-lb. by dividing by 138. This gives

$$280 \times \frac{4220}{138} = 8550$$

which compares with 6800 in the American experiments.

The constants of the German experiments are therefore about 25 per cent higher. Apart from the difference in the materials of the work which were used in the experiments, the discusser is inclined to explain this in part by the fact that, contrary to the American experimental apparatus, in German tests on milling, cutters with inclined teeth were used, which is in accordance with present machine-shop practice. And according to his experience, this type of cutter possesses a higher specific cutting force, which means that it must be granted constants of higher values. This proof of the two sets of experimental values shows a notable agreement between them. It might be therefore recommended to use in evaluating the experimental results the more general and simpler equations, as this would increase the practical value of the material obtained in the experiments. It may be added that in the Charlottenburg tests actual milling cutters were used; that is, a practice more approaching the actual operations.

As regards the employment of a pendulum dynamometer, it may be pointed out that it creates conditions in the removal of the chip which are different from those occurring in actual milling. The difference pointed out in the paper—namely, that in one instance the work is stationary during the process of cutting, while in the other is fed forward—is, in the discusser's opinion, not of predominating practical importance.

He considers as more important the fact that where the pendulum dynamometer is used the speed of cutting does not remain constant during the process of cutting (passage of tooth through the work). The pendulum in full swing has at the two peak positions of the weight a velocity of zero, while at the lowest position of the weight it attains the highest speed of cutting. Moreover, the removal of the chip additionally retards the pendulum and thereby causes a variation on the speed of cutting; moreover, this variation is different in milling *in* the direction of feed from what it is *against* the direction of feed.

There are materials which exhibit only a slight dependence of the specific cutting force on the velocity of cutting which applies to very many of the ferrous metals. With these metals there is the best chance of obtaining an agreement between the values measured with the pendulum dynamometer and those obtained in actual milling, and this agreement is shown on an example referred to. In other cases, the application of the pendulum dynamometer for the determination of milling coefficients seems to be objectionable. It does not appear to be a mere accident that the correspondence between the exponents in the expression for feed and depth of cut, as indicated in Table 18 of the paper, is found to exist in the majority of ferrous metals, but not in the case of other materials, such as brass. In the latter, however, the discusser has been able to demonstrate a strong influence of the speed of cutting, particularly when the angle of cutting was unfavorable, and he did it with the measuring apparatus set directly on the machine tool and leaving the state of operation of the machine unaffected as far as was possible. It may be worth noticing in this connection that according to Fig. 22 of the Boston and Kraus paper the greatest inequality in energy consumption in milling in both directions is found to be present in the case of metals in which the influence of the speed of cutting has been demonstrated, especially with an unfavorably selected angle, and the angle of 15 deg. mostly used by Boston and Kraus in the cases of light metals unfortunately belongs to this category.

A further objection to the use of the pendulum dynamometer is seen in the circumstance that it is impossible to determine by calibration the additional friction produced in the bearings during cutting. It becomes necessary to exercise the greatest care in order to keep this friction as low as possible. Because of this, the discusser considers it preferable to keep the path of the pendulum weight, as is done in the pendulum hammer in an impact testing machine, in the same plane with the path of the cutting tooth. Apart, however, from these considerations against the application of the values measured by means of the pendulum dynamometer to actual milling, without having previously established by comparative tests directly on a working milling machine the permissible limits of such an application with irreproachable precision, the pendulum dynamometer can solve only a small part of the problems arising in the milling process. For a complete solution of the milling process, it is not sufficient to know the peripheral force on the milling tool, the average value of which may be determined by calculation from the measurement of energy consumption of a milling cutting, providing this value is applicable to actual working conditions. In addition to this, it is necessary to know the three components of the cutting force perpendicular to each other, as it is only then that the magnitude and direction of the cutting force can be determined.

With the measuring apparatus designed by the discusser (*American Machinist*, 1930, p. 578, Fig. 7), these three components can be read off directly or plotted on an operating milling machine. It is only when one knows all of these that one can determine how to execute a milling operation in the best way on a given milling machine, being able to form a correct judgment not only of the energy required for driving the spindle, but also—and this is perhaps of greater importance—of the strength of the machine frame and of its deformation, which is highly important as regards accuracy of the work machined. In this connection the discusser speaks briefly on the Boston and Kraus tests with dull cutters which once more confirm in a beautiful manner that in the case of milling the same relations hold good as in other forms of cutting. When the cutting edges are dull, the force required becomes greater, and this increase of force (and hence of the energy consumption of a milling tooth in milling) is shown according to the discusser's tests in a much higher degree by the radial component of the cutting force than by the peripheral, which latter component only is measured by Boston and Kraus.

Referring to the Berlin experiments on turning, the discusser hopes that it will be possible to establish a criterion for the dulling of a milling cutter by measuring the *radial* component of the cutting force. He is working in this direction in his Berlin laboratory.

Authors' Closure

The authors are grateful indeed to those who have discussed this paper with such thoroughness. It is gratifying to see that real interest has been aroused on the several phases of milling discussed. Before answering the various questions raised, it appears proper to outline the conditions under which this work was undertaken.

The authors started this investigation on milling with a single objective—to learn something definite about milling. This paper is the first of four contemplated. The second paper, now nearing completion, deals with additional elements, such as the influence of land or groove cutting, variable rake, variable side rake or helix, etc., using the same single-tooth pendulum tester. Work for the third paper is well under way. A No. 3-B plain milling machine with a 10-hp. direct-drive motor is equipped with a wattmeter for measuring the gross and net input power, and a dynamometer that registers graphically the two vertical and the one horizontal components of the cutting force is mounted on the table. This dynamometer will permit the determination of the magnitude and direction of the resultant cutting force. The wattmeter will be used only as a check. Several sets of cutters of different types, each type with a single variable, such as number of teeth, rake angle, helix angle, etc., have been supplied by cutter manufacturers. Eleven cutting fluids, representing typical types in use, have been supplied also. Eight metals, representing different types, already are available. It is hoped that results may be obtained with this equipment that will confirm those resulting from the study of elements.

The fourth paper, as planned, is to present milling data from actual manufacturing conditions that will confirm or throw additional light on the several problems investigated. Judging from preliminary attempts, it will be most difficult to obtain reliable information for the fourth paper.

Milling equipment used in manufacture represents millions of dollars, yet almost every one will admit that milling is an art based on cut-and-try methods. The authors are not connected with the milling industry, and, without remuneration, have sought to learn something definite, something that could be duplicated, and something that could be used for predeterminations in practice. This paper sets forth conclusions based on experimental data. Personal opinions have been suppressed.

The pendulum-type tester used was considered adequate for the preliminary study of "The Elements of Milling." It has been used for several years in the study of machinability and has been found to be a simple and quick machine to operate and sufficiently accurate for the purpose. Milling energy per chip for various metals has been found to compare quite favorably with data obtained from the same metals by other standardized methods of tests.

Most milling data are based on wattmeter readings for gross power input. Sometimes the machines have been calibrated to compare the prony brake power on the spindle with the gross input. In most of such tests, the feeding power of the table is neglected. The authors have found that the efficiency based on brake horsepower at the spindle divided by the gross power input determined by a wattmeter is as low as 25 per cent on two available machines of different makes. The small difference in power, due to the variable being studied, may be less than normal variations in the wattmeter readings. Under the most favorable conditions with such a set-up, satisfactory results cannot be expected. This is one of the reasons why the pendulum-type dynamometer involving a single-tooth cutter is believed to be more satisfactory in this elementary work.

The type of machine used in the tests has been commented on by several. This type of machine is particularly valuable, inasmuch as both the head, after feeding the cutter to depth, and table, after advancing the work each time equal to the tooth feed, are locked in position. This feature is favorable particularly for the down-milling experiments in which rigidity, omission of back lash, etc. are required.

Several discussers have raised the question that the pendulum-type dynamometer used does not permit the work to move with or against the cutter while the chip is being removed, as is done in a regular milling machine. It is true that the work was stationary when the chip was removed. Fig. 27 was prepared to show that the projected length of the chip is changed when a feeding motion of the work is introduced. In cutting up, as shown at *A* and *C* in Fig. 27, this feeding motion of the work would cause the values of L_A and L_C to be increased slightly over those for no feeding motion. When the depth of cut is $^1/_8$ in. and the feed per tooth 0.010 in., the value of L_C shown at *C* in Fig. 27 is approximately 0.7 in. for a 3.5-in.-diameter cutter with no feeding motion of the work. With the 20-tooth cutter, as shown, if the work moved uniformly to provide the feed per tooth, the value of L_C would be increased by approximately 0.0124 in., or 1.7 per cent, while the cutter tooth sweeps through an angle, from the bottom to the top of the cut, of approximately 22 deg. and 20 min., which is 6.2 per cent of a complete revolution. This small increase in the value of L_C would reduce the average thickness of chip but slightly and thereby increase the energy consumption but slightly. Similarly, in down cutting, the values of L_B and L_D, as shown at *B* and *D*, respectively, in Fig. 27, would be reduced slightly from those for no feeding motion, consequently thickening the chip. Increasing the length of the chip, reducing its average thickness when cutting up, and reducing the length of the chip, increasing the average thickness when cutting down, as would be the case if the pendulum machine were replaced by a milling machine, would tend to increase slightly the difference between the energy values when cutting up and when cutting down. It was felt desirable, however, in these preliminary experiments to keep the work stationary so as to study better the influence of up cutting versus down cutting. It is hoped that this statement can be confirmed by the more practical tests in the work now being done for the third paper.

The fact that constant speed is not obtained during the cut by the single tooth of the pendulum-type dynamometer has been referred to by several. It is thought there is a probability that a variable speed which differs from commercial practice would affect the value of the energy readings. The authors appreciate this fact. It has been proved previously by them that drilling torque and thrust do not vary over a wide range of speed variation. (Trans. A.S.M.E., Vol. 52, 1930, paper, MSP-52-2.) Gottwein has shown that, in turning, there is practically no change in cutting force over a wide range of speed. The authors have confirmed this using a Losenhausen dynamometer. These results would seem to be confirmed in Figs. 4 and 11, in which cuts both up and down have been taken for the same range of variable feeds but for three different depths of cut. Without a doubt there is a greater slowing up of the cutter for the greater depths of cut than for the shallow depths of cut. The slopes of all six lines in each case, as well as those where the depth has been varied for feed values of 0.005 and 0.010 in., respectively, are the same, indicating that the exponents of the formula are not changed by the variable speed.

An analysis of the speeds of the single-tooth cutter show the following:

Velocity at the start of cut $= 70.2/h\sqrt{(2h)}$

Velocity at the finish of cut $= 70.2/h\sqrt{(2h - d - x)}$

in which h is the distance from the center of the shaft to the center of gravity of the pendulum, d is the vertical distance through which the center of gravity of the pendulum moves while engaged in a cut, and x is the vertical distance of the center of gravity below the original top dead-center or vertical starting position of the pendulum after making a swing, when taking a chip.

When 1 ft-lb. of energy is given up by the pendulum, $d = x$ in the formula for velocity at the finish of the cut. It is found that the velocity at the start is 98.5 ft. per min., and the velocity at the finish is 95 ft. per min. for up milling. For down milling, with the same assumption as above, it is found that the velocity both at the start and finish of the cut is equal to 96.7 ft. per min.

When the energy given up by the pendulum is equal to 10 ft-lb., the value of x in the equations is 0.728 ft. For this condition the velocity at the start is found to be 98.5 ft. per min., and the velocity at the finish is found to be 76.8 ft. per min. for up milling. For down milling under the same conditions, the velocity of the cutter at the start is found to be 96.7 ft. per min., and the velocity at the finish is found to be 79 ft. per min. The 10 ft-lb. of energy given up by the pendulum would correspond to a depth of cut of 0.150 in. and a feed of 0.030 in. when cutting the free-cutting brass, or a depth of 0.150 in. and a feed of 0.020 in. when cutting cast iron, or approximately 0.100 in. depth of cut and 0.015 in. feed when cutting free-cutting steel. All of these cuts represent the heavier ones used in the experiments with the 0.250 in. wide cutter, so that the speed variations would represent nearly the maximum. It is believed that these small speed changes do not appreciably affect the results obtained.

Professor Hall raises several questions not covered in the foregoing discussion. It is believed that the large savings in power when cutting various materials by using heavy feeds rather than fine feeds may be due to the fact that the savings indicated are based on the net energy values at the tool point. Presumably those used by Professor Hall in his comparison are based on net gross power in savings, which would not be so high in per cent. Professor Hall is correct in assuming that the formulas presented in the paper are for a cutter $3^1/_2$ in. in diameter. No other diameter was experimented with. Dr. Schlesinger states that "The influence of the diameter, however, is slight, as when the diameter is doubled, the constant increases but only about 10 per cent." Inasmuch as the tests used in the paper had a maximum depth of 0.150 in., the authors are not in a position to predict what might happen at greater depths. Experiments now in progress will involve this point, however.

Mr. Goddard has raised the question as to the condition of a sharp cutter. All tests, except those run under the heading of sharp, dull, and very dull cutters, were run with cutters supposedly sharp. They were ground on a tool-room surface grinder, so that, from all appearance, keen cutting edges were obtained. The conditions of the dull and very dull cutters definitely are outlined under that heading. Mr. Goddard's discussion of the built-up nose formed on the face of the cutting tool is very ably discussed by Mr. Ernst. The views expressed by Mr. Ernst are in complete accordance with those of the authors. By constant feed was meant that, for a series of tests, the work was advanced the same amount for each chip removed. During such a series of tests, the depth, the width of the tool, the cutting fluid, etc. might have been varied.

The discussion by Mr. Ernst is decidedly enlightening. The authors did not intend to convey the idea that they believed the built-up edge was not formed in milling up, but rather that it was not formed from the beginning of the cut as when cutting down. Their observations in removing chips by up and down cutting agree with those expressed and illustrated by Mr. Ernst. The authors have not as yet made photomicrographs of chips, but have had that in mind.

The remarks of Mr. Herbert add materially to the value of the paper. His opinions agree substantially with those of Mr. Ernst and the authors as far as the formation of the built-up edge is concerned.

It is believed that the questions raised by Professor Wallichs and Dr. Shallbroch have been answered in part in the general discussion at the beginning of the closure. Apparently, milling down is not being used as generally in Europe as in the United States. It is interesting to note that the results of Fr. Beckh, in using regular milling cutters having eight to sixteen teeth, agree substantially with those of the authors determined with the single tooth equipment. It also is encouraging to note, from Dr. Schlesinger's discussion, the complete agreement between the results of tests with regular milling cutters on standard milling machines and those obtained by the authors with their single-tooth cutter. This agreement tends to confirm the authors' statements regarding the negligible effect of the variable speed and lack of movement of the work during the cut, which have been criticized.

RP-54-5

Foaming and Priming of Boiler Water

By C. W. FOULK,[1] COLUMBUS, OHIO

A rapidly increasing interest in the subject of the foaming and priming of boiler water is shown by the fact that as many papers have appeared during the last three or four years as in the preceding 20 or 30 years. New information about the behavior of suspended solids and of dissolved salts has been brought out, and a helpful theory of foam formation has also been advanced. It would be inferred from recent work that too broad generalizations in regard to foaming and priming of boiler water should not be made, because in some cases the design of the boiler profoundly affects the behavior of the water.

A PAPER presented by the author under the title, "Present Knowledge of Foaming and Priming of Boiler Water With Suggestions for Research," constituted the 1926 report of Subcommittee No. 3 of the Boiler Feedwater Studies Committee (1),[2] and was published early in 1927 in the Journal of the American Water Works Association (2). The 1927 annual report of Subcommittee No. 3 was a review of two papers on foaming and priming (3). These two reports brought the subject fairly well to date and therefore the papers reviewed here will begin where the 1927 report ended.

A Physico-Chemical Theory of Foams

The major cause of wet steam is often more physical or mechanical than chemical, but even in such cases the physico-chemical properties of the water play an important role, and it is worth noting that in all cases that have come to the author's attention the most important chemical property of the water is the degree to which it is capable of forming liquid films. This formation of films is sometimes an obscure, but nevertheless a frequent determining factor in wet steam. Film formation, particularly a mass of spherical films, or a foam, is therefore one of the most important characteristics of a water, and has been so recognized from the first. Obviously then, the study of the causes of wet steam without a physico-chemical theory of foams, or film formation, was like a game of blind man's buff because there was nothing to serve as guide. The earlier general papers on foams confused the formation of films with the question of their stability after formation, and these two points were so intermingled that it was impossible to disentangle them. The author felt this so keenly in his own researches that efforts were directed toward finding a general theory, and in the 1927 report of Subcommittee No. 3 (3), such a theory was offered. It was subsequently published in a more elaborated form (4), from which the following brief statement of the theory is quoted:

> "Liquid films are always formed by the approach to each other, usually with an extension of area, of two already formed liquid surfaces. In the case of solutions the mechanical force acting to bring the surfaces together meets with resistance when the layer of liquid between the surfaces becomes very thin, with the result that a film is produced. In the case of pure liquids no such resistance is encountered and consequently the two surfaces merge and vanish. The reason for the counter force in the case of solutions is the difference in concentration between the surface layer of the liquid and the mass of the liquid. Solute either concentrates in the surface or recedes from the surface. In either case this movement of matter takes place against diffusion and it will, therefore, require work to restore the equality of concentration; in other words, thin surface layers will resist a force to mix them. This theory harmonizes the puzzling facts that both positively and negatively adsorbed dissolved matter causes foam in liquids and explains why pure liquids do not foam." The statement is also made that, "film formation and film stability are separate questions which have often been confused in the literature."

This first paper contained little or no experimental evidence, a lack now remedied by the quite recent publication (4*a*) of a paper in *Industrial and Engineering Chemistry*. In this second paper data and graphs are offered in support of the theory.

The mechanical part of the theory, namely, that liquid surfaces on approaching each other cease their approach when very close together if the liquid is a solution, is proved by a simple experiment. Two bubbles are caused to issue from orifices placed near each other in the liquid the film-forming properties of which are being examined. If the liquid is a solution the bubbles flatten against each other, that is, they cease their approach when very close because a liquid film is formed between them. If, however, the two bubbles are in a pure liquid, like distilled water, or in a very dilute solution, they do not cease approaching, because no film is formed between them and the two bubbles merge together into a larger one as soon as they touch each other. By counting a large number of such contacts of two bubbles in the liquid and noting the number of times the bubbles do not coalesce on touching, that is, the number of times a film is formed between them, a quantitative measure of the film-forming tendency of the liquid is obtained. It will be observed that in this experiment the tendency to film formation is the dominant property measured. The stability of the films plays a minor role.

The stability of a film, however, plays the chief role—the role of the villain—in a boiler-water drama, and therefore, there should be some way of measuring it. This seems in a fair way of being accomplished in a simple manner with an instrument described in this experimental paper. It was designed in the author's laboratory by Hansley (5) who called it a dynamic foam meter because the measurements are made while the foam is being produced.

A Dynamic Foam Meter

The apparatus in question is nothing more than a glass tube closed at one end with a porous septum, and so mounted that a current of air can be blown through the septum into the liquid

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[2] Numbers in parentheses apply to literature references at the end of the article.

Contributed by the Joint Research Committee on Boiler Feedwater Studies and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

under examination in the tube. If a pure liquid or a very dilute solution is in the tube the bubbles of air issuing from the pores of the septum merge at once into larger bubbles and these larger bubbles on touching each other coalesce again to form still larger ones. The column of liquid in the tube is thus full of large bubbles and there is little or no foam on the surface. If now some soluble matter is added, sodium sulphate for example, in gradually increasing concentration, it will be seen that the bubbles rising through the liquid gradually become smaller, that is, the coalescence of small bubbles as they emerge from the pores of the septum, gradually becomes less because of the resistance to mixing offered by the layers of solution constituting the interior surfaces of the bubbles, until at a certain concentration the coalescence ceases and the solution takes on a "white" appearance due to the mass of tiny air bubbles in it. In other words, the bubbles retain the size they have on emerging from the pores of the septum. In addition to this phenomenon a layer of foam forms on the surface of the liquid. The thickness of this foam layer is determined by the steady state between the rate of formation of the bubbles and the stability of the bubble films. The height of the foam layer is not the measure of the stability of the foam, but it is proportional to it, and the instrument therefore serves to compare the foam-forming tendency, or the foaminess of different liquids.

It is proposed, in the author's laboratory, to make a thorough study of this instrument, particularly with the view of standardizing it so that different investigators can obtain comparable results.

The direct measurement of bubble stability, which, of course, must be stated in units of time, has recently been made in a very simple manner by Talmud and Suchowolskaja (6). They determined the number of seconds during which individual bubbles lasted on the surface of the solutions tested. The results in so far as it was possible to compare them agreed very well with those obtained with the foam meter.

A Theory of Antifoams

Boiler-water chemists will also be interested in one of the corollaries from the theory since it offers a general theory for guiding an investigator in search of antifoams. Since the theory of film formation states that films or foams are formed only in those cases in which the dissolved matter either concentrates in the surface (positive adsorption) or recedes from the surface (negative adsorption), the question naturally arises, what happens if two substances are dissolved in the same liquid, one exhibiting positive adsorption and the other negative adsorption? A corollary from the theory answers this question by stating that in such cases there will be certain mixtures that do not foam because positive and negative adsorption cancel each other, if the amounts of the two substances present are in the right proportion.

Various non-foaming mixtures of foam-forming substances were found and are recorded in the paper. Perhaps the most interesting is the pair sodium sulphate and soap. A solution of sodium sulphate in the dynamic foam meter described foams vigorously because the salt is negatively adsorbed. The foaming is instantly stopped by the addition of a minute amount of soap, which is strongly positively adsorbed.

This experiment shows the danger of hasty generalization in the study of foaming. As a rule the addition of soap to a water solution makes it foam, assuming of course that there is no chemical action on the soap, but it is seen, there are exceptions.

It is also worth recording at this point that four or five years ago the author was appealed to by the chemists engaged in concentrating some of the heavy brines that were being worked up in the west for potassium salts. These brines produced a great deal of foam which, it was accidentally discovered, could be dispelled by the addition of a little soap. The reviewer was asked to explain this strange phenomenon, and at that time was unable to do so. Now the situation is clear on the basis of the corollary referred to above, and it is interesting to observe that here was an actual instance of the commercial use of soap as an antifoam.

Particle Size of Suspended Solids

German engineers and chemists have lately turned their attention to foaming and priming, and recently Niehaus (7) has published a general paper entitled "Physico-Chemical Considerations on the Problem of Boiler-Water Foaming." This interesting article aims to set forth the existing general facts and theories, but it contains no references to American investigations. Niehaus lays special emphasis on particle size as a factor in the stabilization of foam by suspended solids and states that there is a certain size which has the maximum stabilizing power. Anything above or below this has less effect. He gives this optimum size as that lying on the border of colloidal and gross dispersion. The effect of various soluble substances in the boiler water is ascribed by Niehaus to their influence on the particle size of the solids. The specific effects in this direction of such substances as caustic soda, sodium carbonate, phosphates, magnesium sulphate, and sodium chloride are discussed. One of the most interesting items in the summary at the end of the paper is the statement that soluble phosphates should be kept below 15 to 20 mg. per l. This is a very low concentration and the reviewer wonders whether American experience is in agreement with Niehaus.

Projection of Droplets by Bursting Bubbles

In 1927 (2) the author pointed out that apparently no one had yet studied the well-known fact that a bubble bursting on the surface of a liquid frequently throws a droplet of the liquid upward, sometimes to the height of 4 or 5 in. This phenomenon, which might well have an effect on the amount of water in steam, is a part of the mechanism of effervescence and has been studied under that title by Craven and Stuhlman (8). The author unfortunately has seen only an abstract of their paper. The upward projection of liquid droplets is mentioned in this abstract, but is not explained, nor is there any detail on the effect of such changing conditions as occur in a boiler water. The author has also studied this phenomenon and is able to say that the droplet thrown upward is not part of the ruptured bubble film as is commonly supposed, but has its origin in the center of the depression in the liquid surface on which the bubble exists just before bursting. When the bubble film bursts the pressure which causes this depression is relieved and the liquid rises. In doing so the center of the depressed area rises faster than the rest with the result that a drop necks off and is thrown up. The phenomenon can be produced by directing a current of air downward against a liquid surface and then suddenly interrupting the air. The depressed area flies up and the necking off of the projected droplet can be observed. The author has not seen Craven and Stuhlman's complete paper and therefore cannot say whether they have explained the projection of droplets the same way or not. This question of projected droplets is worthy of study from the standpoint of boiler-water foaming and priming, and is one of the topics to be investigated at some future time in the author's laboratory.

This concludes the more theoretical information that has come out during the last few years. Whatever the verdict of the future may be, the author is finding this theoretical matter a serviceable guide in his present work. For example, the theory of film formation furnishes an explanation for the strange results obtained in the investigation next to be described.

An Unusual Cause of Water in Steam

Through the courtesy of the Paige-Jones Company of Hammond, Indiana, the author has at his disposal a somewhat elaborate experimental boiler consisting of a horizontal drum 4 in. in diameter and 5 ft. 6 in. long, into the ends of which is connected a water tube. Both of these connections are close to the lower side of the drum. A year ago the Engineering Experiment Station of The Ohio State University furnished funds for hiring an experienced chemist to help in the work on the problem of foaming and priming. It was first proposed to repeat the experiments of Joseph and Hancock (9) and later of Hancock (10), who it will be recalled, operated an experimental steel boiler working at pressures up to 150 lb. and found that the solid matter which they used had no effect on the amount of water thrown into the steam. The outcome of these experiments is so contrary to general opinion that it seemed desirable to repeat them with a different type of boiler. The boiler which Joseph and Hancock used, by the way, was a simple drum lying on its side and heated along the bottom with a gas flame.

In order to furnish an appropriate background for the proposed experiments with suspended solids the water-tube boiler was first operated a number of times with distilled water and then with solutions of various salts at different concentrations, but with no solid matter present. The results of these preliminary experiments were more astonishing by far than the findings of Joseph and Hancock, because it was found that the throwing of water into the steam decreased regularly as the concentration of salt in the water increased. This was such an amazing situation that its study was pursued instead of investigating the effect of solid matter as first proposed.

In brief a large number of experiments were performed with solutions of sodium chloride, sodium sulphate, sodium carbonate, and sodium hydroxide. The amount of water thrown into the steam was measured by the chemical method, allowance being made in all cases for the progressive concentration in the boiler. The experiments showed, first that the level of water in the drum had perhaps the greatest effect, and in the sense that the lower this was the less water, other things being equal, appeared in the steam. Secondly, other things being equal, there was considerable difference among the salts. Sodium sulphate and sodium carbonate and hydroxide gave less water in the steam than the chloride. This, it will be observed, is also contrary to popular belief and other experimental evidence. Scores of experiments were made so that there is no doubt about the observations. The author is compelled to say that at the time he had no idea as to why such peculiar results were obtained. It was necessary, however, to find an explanation and accordingly a pyrex glass boiler was constructed of precisely the same design and almost the same size as the steel one. This glass boiler was operated a number of times at pressures of 12 or 13 lb., but since it had been shown in the work with the steel boiler that the same effects were produced at atmospheric pressure as at higher pressure the glass boiler was operated at atmospheric pressure. It was at once seen why the peculiar phenomenon described above took place. Waves were propogated along the length of the drum and the bulk of the water appearing in the steam was due to the crests of these waves hitting the steam outlet which was on the top of the drum about two-thirds of the length of the drum from the end at which the steam entered from the water tube. These waves were highest in the case of distilled water and dilute salt solutions. As the concentration of the salts increased, the waves flattened out and not so many crests hit the steam outlet. This then was a simple physical explanation of the anomalous results. There remained, however, the problem of why the dissolved salt had this peculiar effect on the height of the waves. Observation of the glass boiler in operation seemed to show that the steam bubbles in the case of the distilled water and dilute solutions were very much larger than in the cases of the more concentrated salt solutions. The words "seemed to show" are used advisedly because the rate at which the steam bubbles moved through the tube was so high that visual observation was not conclusive, particularly in the case of the more concentrated salt solutions.

The theory of film formation outlined calls for small steam bubbles in salt solution because the bubbles do not merge together to form larger ones as would be the case with distilled water. It was evident that if such a difference in bubble size could be proved the higher waves in the distilled water are then simply explained by the greater surges or pulsations set up by the larger bubbles issuing from the water tube. Instantaneous photographs of the tube would, of course, answer the question and consequently such were taken. Exposures of about 0.002 sec. were necessary to catch the individual bubbles. The outcome was conclusive. The steam bubbles were very much smaller in the salt solutions and consequently they set up smaller waves along the length of the drum when they emerged from the water tube.

The author wishes to point out that in this obscure case of foaming and priming, the theory of film formation suggested the explanation which was then proved to be correct by the instantaneous photographs.

It may possibly be said that the state of affairs observed was so unusual that it could never occur in a commercial installation. This, however, is probably not true because two practical engineers, who heard the paper presented at the Indianapolis meeting of the American Chemical Society (1930), told the author of experiences of a similar character, namely, of boilers that tolerated an enormous concentration of dissolved matter in the water.

The author also wishes to make another observation. Suppose that this series of experiments had been made by one with no particular background of knowledge and experience in the peculiar behavior of boiler water. Such a hypothetical investigator would probably have generalized on the basis of the data obtained with the steel boiler, and would have reported that the generally held opinion that foaming and priming is promoted by increased concentration of salts in the water is all wrong, because he had experimented and found the contrary to be true. It merely shows once more the enormous complexity of the problem. Indeed, it is doubtful if any complete generalization may be made. What actually happens is perhaps in all cases a certain combination of the physico-chemical properties of the water modified by the design of the boiler and the manner in which the boiler is operated.

These results will soon be offered for publication together with diagrams of the apparatus employed and reproductions of the photographs.

Effect of Suspended Solids

The effect of suspended solids in the boiler has received new attention since the publications of Joseph and Hancock (9 and 10). The author has also made a beginning on this problem. It will be recalled that his first paper in 1924 (11) recorded many experiments on the boiling of salt solutions, with and without solid matter, in glass vessels in the laboratory. Six widely different kinds of solids were used and since they all stabilized the foams of the boiling salt solutions it was natural to infer that all kinds of solids would have the same effect. The author now knows that such a generalization is too broad, and in a paper presented at the Indianapolis Meeting (1930) of the American Chemical Society new data and points of view were advanced. The essential points of this investigation are as follows: Boiling experiments were carried out in glass vessels, and also determinations were made of the surface viscosity and surface tension of cold solutions on the surface of which finely divided solid matter was floated. These latter experiments with surface viscosity and surface tension were made largely to satisfy scientific curiosity, because it seemed desirable to leave no stone unturned in the pursuit of the problem. All that need be said here about the outcome is that the results were in harmony with existing theories. One simple experiment, however, is worth mentioning. By the use of a special technic, the surface of water or of salt solutions could be kept covered by a layer of floating solid particles, and it was found that under these conditions bubbles a centimeter or more in diameter could be formed on the surface and lifted out on paper. The paper would, of course, absorb the excess of liquid and if the bubbles were then protected under a bell jar until all the liquid had evaporated, the solid particles were left behind like a vaulted ceiling of fairy workmanship. This bit of laboratory by-play showed that under favorable conditions the stabilization of a bubble film by solid matter is 100 per cent.

Two apparently new pieces of information were developed by the boiling experiments. First, it was clearly shown that different kinds of solid matter exhibit greatly different effects in stabilizing the foams of boiling salt solutions. Finely ground flint, for example, had absolutely no stabilizing effect, that is, the specimen of flint used did not. It is a reasonable hypothesis to assume that stabilization by solid matter is proportional to the tendency of the solids to float on the surface, and that the flotability is in turn inversely proportional to the tendency of the solid to be wet by the solution. Ground flint for example is instantly wet by the solutions used and instantly sinks. It had no effect on the foaming.

Experimental data from the art of ore flotation suggested the probability that absorbed oil in boiler scale is the cause of its high stabilizing effect. Experiments were, therefore, made by extracting with ether and benzene the oil from a specimen of boiler scale which in its original form greatly increased the foaming of a boiling salt solution. The extracted scale was found to have lost nearly all of its foam-stabilizing properties. Other experiments were made by strongly igniting an oil-bearing scale in order to destroy the oil and then after shaking it with water to extract the soluble mineral matter produced by the ignition, it was found that the residue had very little effect in stabilizing foams.

Experiments were also made with specimens of dolomite, limestone, and boiler scale, all of which were very efficient in increasing foaming, in order to see what effect prolonged boiling at atmospheric pressure would have. The following interesting results were obtained. The dolomite lost its foam-stabilizing properties on about 3 hr. boiling, the limestone after about 10 hr. boiling, and boiler scale after about 11 hr. After these boiling solutions had ceased to foam the boiling was stopped and the liquid filtered. In all cases fresh solid matter added to the salt solution which had been used in the experiment produced the usual amount of foam thus showing that the part of the foaminess which was due to the solution had not been affected. On the other hand, when the filtered off solid matter, which had been washed several times, was added to a fresh salt solution no foaming was produced, thus showing conclusively that the solid matter had lost its foam-stabilizing properties.

The author has the following suggestions to make about this experiment. If a given type of solid matter loses its foam-stabilizing power in 10 hr. boiling at atmospheric pressure, it will probably lose it much faster in the same solution boiling at the temperature of a high-pressure boiler. At any rate it is a good working hypothesis to assume that in some cases at least the suspended solids in a boiler water may at first have some stabilizing power, but will lose this property after some time in the boiling water. The rates at which different kinds of solid matter lose their foam-stabilizing powers will in all likelihood differ greatly. Let it be supposed then that in a given boiler a given type of solid matter is losing its foam-stabilizing property about as fast as it is being formed in the water. In such a case the solid will have little or no effect on the foaming. Another boiler using the same water may be operated at a lower pressure and consequently at a lower temperature with the result that the solid matter precipitated out of the water will not lose its stabilizing effects so fast and therefore will promote foaming. These are merely suggestions or working hypotheses that may in some cases lead to the explanation of an obscure case of wet steam.

Still another hypothetical situation might be suggested. Two boilers of identical design and using the same water are found to behave very differently. One foams over badly and must be blown down frequently while the other one tolerates a high concentration of both solids and dissolved matter. It might happen that one boiler is operating a reciprocating engine with a line for returning condensed steam. The scale or sludge in this boiler will then contain adsorbed oil making it more flotable, and giving it a high foam-stabilizing power, thus producing heavy foaming and priming. The other boiler of this hypothetical pair has no return line to carry in oil and consequently the water in it will not foam up as in the other case.

The above are purely hypothetical cases. The author cites them as part of an argument in favor of experimentation in glass vessels in the laboratory. Scores of such experiments can be made in a few days and the results stimulate the scientific imagination of the investigator engaged on a commercial problem, so that he will think of things that otherwise might not occur to him.

Effect of Solid Matter at Higher Pressures

The author's experiments referred to were all made at atmospheric pressure. There is also a growing list of papers on the effect of solids in experimental and commercial boilers operating at 150 lb. pressure or higher. Hancock (10) repeated some of the work of Joseph and Hancock and found again that finely divided solid matter had no effect on the moisture in the steam. Hancock generalizes as follows from these experiments: "It has been confirmed that at normal pressures (160 lb.) suspended solids have no influence on priming." This of course means only that no effect was obtained with the particular solids used and under the particular conditions employed. It is useless, however, to speculate about what might be the cause. The thing to do is to experiment under different conditions and in all cases seek to harmonize the experimental results with existing theory.

Other references on the effect of suspended solids in promoting foaming are as follows:

Chr. Eberle (12) states that in his experiments solid matter had a very marked effect in increasing foaming. His work was done in an experimental boiler operated at pressures as high as 150 lb.

The Water Service Committee of the American Railway Engineers Association, in its report for 1929 (13), gives data on the behavior of certain commercial boilers which show that solids greatly increased the water in the steam.

The preponderance of evidence is on the side of the opinion that suspended solids in the boiler water increases the moisture in the steam. In view, however, of the work of Joseph and Hancock and some of the work of the author, it is likely that there are exceptions. The author is willing, however, to go on record as saying that finely divided solids are still on trial for causing thousands of cases of wet steam, even if it can be proved in a few instances that the accused, though present, was not guilty.

Effect of Alkalinity

In 1927-1928 Mumford (14) made an extensive series of experiments at the Kip's Bay Station of the New York Steam Corporation and came to the conclusion that the hydroxyl-ion concentration played the chief role in the production of wet steam. He found a value for this concentration above which the production of dry steam was exceedingly difficult. For the boilers in question Mumford places this critical value at 220 to 250 p.p.m. The effect of this critical value of hyroxyl "was the same in boilers just filled with raw water as it was in boilers which had been on the line for weeks so that the effect of hydroxyl must have been independent of the concentration of suspended matter which must have varied under the above conditions." Finally it is pointed out that the particular critical hydroxyl-ion concentration in a given case is a function of the boiler design. The author naturally observed the following passage: "It may be said at this point, to forestall possible criticism, that several hundred concentrations of the raw water showed no organic matter when made acid and subjected to ether extraction, so that saponification is not the obvious reason for the action of hydroxyl in this case." Mr. Mumford adds that the findings make "a good question to lay at the door of the laboratory specialists in these subjects." The author takes this to heart, but is compelled to say that he has not yet had time to look into the parentage of this foundling on his door-step. That too will make a topic for future study if some other laboratory worker does not first adopt the child.

Mumford's article is outstanding in that it contains a wealth of engineering detail, which makes it especially interesting to chemists.

Effect of Organic Matter

No one seems to question the opinion that organic matter is often a cause of wet steam and recently there have been some investigations bearing directly on this point. Hancock (10) reports experiments in which he removed the organic matter from a surface supply (River Nile) and then restored the other constituents. The original water foamed badly, but after removal of the organic matter the foaming ceased. In connection with these studies Hancock makes the statement, "It is important that the organic matter be in solution." Many chemists believe that organic matter in colloidal suspension also causes foam.

Behrman (15) reports the case of a water that foamed badly on account of organic matter. This was proved by removing the organic material and then restoring the original alkalinity of the water. The foaminess was enormously decreased.

Smoothness of Boiling

Hall (16) in a paper entitled "Present Tendency of Boiler-Water Conditioning," states that calcium has a marked influence on smoothness of boiling, but gives no data. The following quotation indicates his point of view: "The fact must be recognized that a fully conditioned water does not boil as smoothly as one not conditioned; absence of calcium in the former is the cause." In other words, calcium promotes smoothness of boiling. The author has no other information on this point and can offer no theoretical explanation.

Some attention should be given to this problem of "smoothness of boiling." At present there are no available data; indeed there is not even a definition, so that few know just what the expression means. To the author the term means literally boiling smoothly, that is, uniformly and without bumping. Smoothness of boiling may and sometimes does accompany foaming. There has been some dabbling with the problem in the author's laboratory, but nothing definite is ready for publication. It might be added as a final note that solid matter frequently promotes smooth boiling.

Boiler Water Circulation

The author professes no knowledge of boiler-water circulation, that is, the rate and direction of the flow of steam with respect to the water. One of the experiences cited "An Unusual Cause of Water in Steam" suggests, however, that such circulation may have a pronounced influence on the production of wet steam Eberle (12) gives some interesting data obtained with one of his experimental boilers operated at atmospheric pressure, which show that the water in the steam increased with the depth of water in the drum through which the steam bubbles generated in the water tube were compelled to pass.

Conclusion

The author is impressed with the rapidly increasing interest in the subject of wet steam. During the last four or five years there have been as many experimental investigations bearing on the phenomenon as were made in the preceding 15 or 20 years. These investigations have increased the list of known specific conditions that affect the degree of moisture in steam and will consequently help in the solution of such problems in commercial installations. During the last few years there has also been advanced a theory of liquid film formation that is proving a helpful guide in researches on foaming. Those then who are interested in the foaming and priming of boiler water, whether for economic or scientific reasons, may feel that the curve of progress toward a better understanding of the subject is rising rapidly.

The author is more and more impressed with the complexity of the subject. The structural details of the boiler, the distance of the water from the steam outlet, and the physico-chemical properties of the water appear to be the controlling conditions. These factors do not, however, function independently, but rather in varying relations to each other. Indeed, these relations are such that in the experimental boiler used in the investigation entitled "An Unusual Cause of Water in Steam" it was shown that distilled water gave by far the wettest steam. Ordinarily the opposite would be true.

It is a pleasure to announce that the author's studies have been aided by a grant from the Boiler Feedwater Studies Committee. It is proposed to experiment with the effect of suspended solids of various kinds in experimental boilers of different design, operated at pressures of 150 lb.

Finally, the author asks the indulgence of his readers if any papers have been omitted. Necessity compelled the preparation of this review in a hurry, and hurry brings certain faults in its train. One paper by Müller (17) in 1927 was omitted because time did not permit access to it. It is entitled "The Present Status of the Question of Foaming and Priming of Steam Boilers."

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Discussion

J. A. Holmes.[3] The theories and ideas advanced by Professor Foulk, in this and in previous meetings, have been the basis for many of our company's research problems on carryover and foaming.

In order to verify and elaborate on some of these suggestions, we recently constructed a small boiler with a capacity of 1 gal. of water and that could be operated at pressures up to 1000 lb., with sight glasses so that boiling characteristics could be observed. The rate of evaporation could be carried as high as 500 per cent.

More than 90 runs have been made in this boiler, using almost as many different types of treatment. Much information has been gained therefrom, but this discussion will simply point out a few of the results that are applicable to the points mentioned in the paper.

The physico-chemical theory of foam advanced by Professor Foulk has been adopted as a fact (since it has not been disproved), and research work has proceeded along this line. Since there are considerable data on the effect of pure chemical substances on the formation of foam, this work has been passed up, and only natural waters have been experimented with.

The dynamic foam meter has been used, and it is surprising how closely the measurement of the degree of foaming in this tube corresponds with plant experience. We have not only been able to measure the degree of foam, but in some cases have been able to measure the "explosiveness" or violence of bubble breaking.

Very little work has been done on anti-foam, except on the effect of suspended solids on carryover. This will be mentioned later.

Professor Foulk mentions the projection of droplets by bursting bubbles which would cause an increase in moisture in the steam. We have noticed one phenomenon allied to this. When operating our experimental boiler, as the pressure was increased to 450 lb. we could not see through the water, but could see through the steam by placing a light back of the sight glasses. Above 450 lb. there was a gradual clearing up of the water and a darkening of the steam, so that at 600 lb. pressure or above we could see through the water easily but could not see through the steam, which had a very black appearance. Until reading the paper we were at a loss to account for this change. However, we did notice that at the lower pressures there were a large number of steam bubbles in the water and that at 250 lb. pressure a foam was formed averaging $1^1/_2$ in. thick. As the pressure was increased, the number of steam bubbles passing through the water decreased, until above 600 lb. pressure there were practically none, and the water seemingly was boiling only at the surface, the thickness of the foam decreasing to less than $^1/_2$ in. The disappearance of the bubbles would allow us to look through the water and accounts for its being clear. Possibly the reduction of the thickness of the film of foam on the water, thereby eliminating a cushioning effect, and the boiling at the surface projected the small droplets mentioned in the paper and formed a sort of mist in the steam area, thereby causing it to appear dark. Condensed steam during this operation showed no carryover by chemical tests. Conductivity tests of the condensed steam were not made at that time.

The condition of varying bubble size described in Professor Foulk's experimental glass boiler has also been observed in ours and also in the dynamic foam meter. Also we have noted that different types of suspended solids can cause the same phenomena. This point should, we believe, be investigated further. Not only does the breaking of bubbles play an important part in the prevention of foam, but also possibly the formation of the bubbles may have some effect, although we are inclined to believe that there is less variation of bubble size at pressures encountered in boiler-plant operation.

Most of our work has pertained to the effect of suspended solids on carryover. Our experimental boiler was operated with various types of water and treatment, at a definite rating of 300 per cent and at 250 lb. pressure. Operation would continue until chemical tests of the condensed steam showed definite evidence of carryover due to foam. Most of the work was done on Chicago tap water, which varies but little in composition. In order to determine the effect of suspended solids on carryover, a series of runs were made using various chemicals commonly used in water treatment. The same amount of alkalinity was carried in the feedwater in each treatment, so that the alkalinity, chlorides, and sulphates of the boiler water would increase at the same ratio with each other and with the total dissolved solids. The major changes would be in the sludge—whether it was predominantly carbonate, phosphate, or aluminate. These runs showed that straight soda ash as internal treatment was the worst offender. Sodium bicarbonate was little better. As a comparison the concentration of dissolved solids at carryover with soda ash was considered as 100. Treatment with sodium bicarbonate showed carryover at 110, sodium diphosphate (no soluble excess) at 140, a combination of sodium aluminate and sodium phosphate 160, a combination of sodium bicarbonate and aluminate 195, a 15 per cent solution of sodium aluminate plus 20 per cent of caustic soda 205, and straight sodium aluminate with no excess caustic 275.

If the runs on the Chicago water, approximately 25, are plotted in the order of their increasing concentration, the percentage of alumina in the sludge, which varies from 0.5 per cent to 10 per cent, will rise in proportion to the dissolved-solids curve. The percentage of alumina in the sludge is a measure of floc formation which tends to absorb the more finely divided precipitates of carbonates and phosphates. These series of runs would tend to prove that the nature of the suspended solids had considerable to do with the point at which carryover resulted. A further experiment was made by using a water containing 3.5 grains of hardness and treating with disodium phosphate alone in sufficient quantities to carry an excess. Carryover resulted at 150 grains per gallon soluble solids. In a second run the water was treated with disodium phosphate and sodium aluminate, carryover resulting at 245 grains per gallon. A third treatment, a combination of sodium diphosphate, sodium aluminate, and magnesium sulphate, gave carryover at 355 grains per gallon or 225 per cent of the straight disodium phosphate treatment. Alkalinities, etc. were kept practically the same. Treatment with phosphates alone showed no coagulation, and the water was very difficult to filter clear. The treatment with phosphates and aluminate showed very little coagulation, but the water could be filtered fairly easily, whereas the treatment with phosphates, aluminate, and magnesium sulphate showed visible good floc formation and was very easily filtered. Here again only the nature of the suspended solids were changed, but a radical increase in the concentration that could be carried was noted.

Another experiment was made to verify the effect of aging of the sludge by continued boiling. This was done by operating the boiler to carryover and blowing down a sample for analysis. Operation was continued, keeping just below a concentration to give noticeable carryover in the condensed steam. In some cases by this aging process or nursing of the boiler we could increase the concentration at which carryover would result as much as 75 per cent. Professor Foulk mentioned the effect of continued boiling at atmospheric pressure, and our experiments would tend to indicate that such might be the case under pressure. However, in actual plant practice there must be a continuous supply of feedwater added to the boiler, which of course will be creating new suspended solids which have not been aged.

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[3] National Aluminate Corporation, Chicago, Ill.

One experiment on the effect of alkalinity on carryover was made. Lake Michigan water was softened with lime and soda ash to 2.0 grains hardness with practically no excess alkalinity. Carryover resulted at 200 grains per gallon. Another run was made by softening the water to 0.5 grains per gallon, and a high excess of alkalinity was carried. Carryover then resulted at 275 grains, or an increase of 35 per cent. The alkalinity in the latter case or with the higher concentration was 390 per cent of the former. Therefore higher concentrations were carried with the high alkalinity and low hardness than with the high hardness and low alkalinity in the treated water. This would tend to indicate that the suspended solids were the prime cause for carryover and that alkalinity in this case effected foaming only as it contributed to the total amount of dissolved solids present.

It is also noted that in the runs on Chicago water the alkalinity increased in direct proportion to the dissolved solids, so that the higher the concentration at carryover the higher the alkalinity. If alkalinity was the prime cause of foaming, then the runs showing the highest concentration should have the lowest alkalinity, but the reverse was the case.

In conclusion, research work in regard to foaming in boilers should, as Professor Foulk states, be done with a clear understanding of boiler operation and conditions. Wet steam is not always caused by the formation of a definite foam, and consequently any work done along this line should definitely state and produce proper data to show conclusively whether foam is formed and what is its degree of stability, and all the conditions and analyses noted should accompany the experiments. Also experiments should be made in vessels so equipped that boiling, etc. can be observed and so that the tests will duplicate to some degree actual plant practice.

A. R. Mumford.[4] In offering discussion of the paper, the writer feels that it is better to submit additional data on the ways the quantity of moisture in steam has been reduced at the plants of the New York Steam Corporation than to offer imaginative descriptions of what might be taking place in commercial boilers to compare with what has been seen in the laboratory.

As operators of boilers we may set down three factors which we have found will influence the quantity of moisture in the steam:

1 The level of water indicated by the gage glass has a definite, although not a fixed, relation to the water level in the steaming drums, and all know that if the indication of the gage glass is too high we will have wet steam.

Boiler No. 5 at Kips Bay Station was thoroughly tested to determine the relation of water level to steam quality. The results of the tests are shown in Fig. 1, on which the temperatures indicated by each of four calorimeters are plotted against the average level indicated by the four gage glasses. One calorimeter was installed in a vertical run on each of the four steam outlets from the dry drun, and each of the two 60-in.-diameter steaming drums had a gage glass at each drum head. An examination of Fig. 1 and of Table 1 shows that as the average water level was decreased the temperatures indicated by the calorimeters rose until we were able to say that an indicated average water level of 2 in. would correpond to the calorimeter temperature which ought to result from dry steam. The operation of a boiler with only 2 in. of water showing in the glasses would require so much man power that it was impractical, and the water columns and gage glasses were lowered by providing lower connections. The check tests made after the columns were lowered are plotted as large open circles on the curves of Fig. 1. The ratings of the boiler varied about 15 per cent between some of the tests, the results of which are plotted in Fig. 1, but the data indicate that this variation did not influence the results.

TABLE 1 VARIATION OF CALORIMETER TEMPERATURES WITH AVERAGE WATER LEVELS

Test	Calorimeter temperature deg. fahr. East	E. center	W. center	West	Water level, in.	Conc. Cl, p.p.m.	Conc. OH, p.p.m.	Conc. sludge, p.p.m.	Rating 1000 lb. per hr.
1	280	238	306	212	10.5	145	80	300	721
2	292	266	310	238	9.2	125	90	1000	632
3	295	264	311	227	9.0	130	105	1200	769
4	304	282	311	253	8.5	135	135	1400	745
5	305	281	311	251	7.0	150	120	1750	770
6	311	290	312	271	6.5	165	175	1650	760
7	303	283	312	253	5.5	160	160	1400	751
8	304	283	311	251	7.0	155	130	1600	740
9	305	293	311	265	7.5	180	145	1550	756
10	311	301	311	276	5.0	225	125	1900	741
11	306	297	316	284	3.5	135	70	1030	799
12	311	303	316	294	2.5	175	130		842
13	313	305	314	300	1.8	185	150	2130	860
14	317	306	314	300	1.3	230	180	3150	879
15		313	314	313	7.65	91	165	655	496
16		313	315	312	7.45	93	165	760	725
17		313	314	314	8.25	77	146	560	637

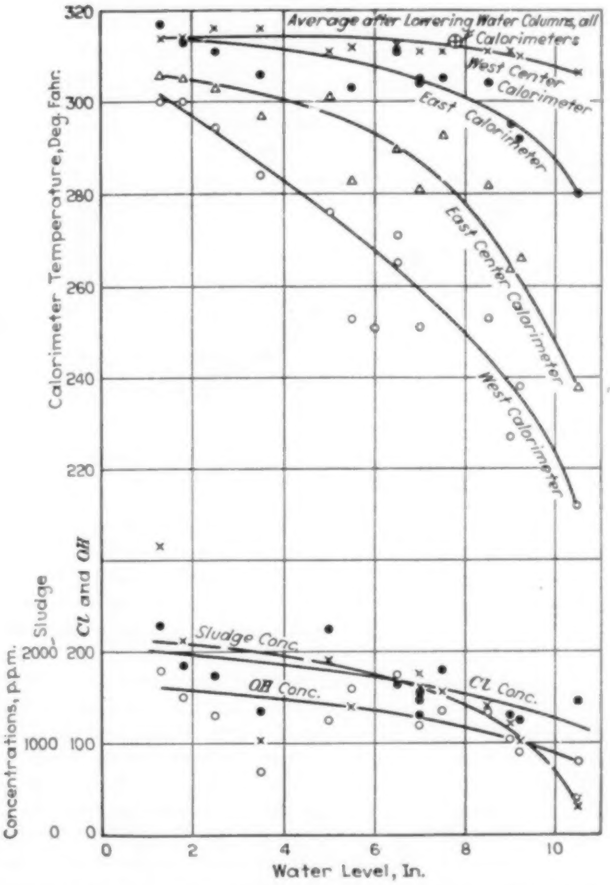

Fig. 1 Variation of Calorimeter Temperatures With Average Water Level

The new location of the water column brought forward an interesting problem in densities. An average level of 2 in. in the gage glasses at the old location would provide a safety factor in that the level in the drum would be sufficiently high to submerge all tubes if the fluid inside the drum were dense water. However, we all know that because the steam is formed beneath the water level the density of the fluid in the drum is less than that of dense water, and therefore the level in the drum will always be higher than that in the glass in a steaming boiler. In lowering the water columns it is necessary to know the average density in the drum in order to compute what level in the glass at the new

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TABLE 2 VARIATION OF CALORIMETER TEMPERATURES WITH WATER LEVEL, WITH AND WITHOUT PERFORATED BAFFLE

Test	Water level, in.	Cal. temp., °F.	Conc. OH p.p.m.	Conc. Cl, p.p.m.	Conc. sludge, p.p.m.	Conc. PO$_4$, p.p.m.	Rating, 1000 lb. per hr.	
4	6.3	294	8.1	20.2	1060	Tr.	101.0	Baffle, low conc.
5	4.7	298	17.5	24.0	715	Tr.	104.5	
6	3.0	300	29.4	26.9	?	10	102.7	
7	5.9	275	124.0	89.9	1777	118	106.3	Baffle, high conc.
8	4.5	298	135.0	95.6	1801	118	105.9	
9	2.85	300	122.0	87.6	3020	100	107.4	
10	6.4	214	149.6	64.1	2285	150	103.8	No baffle
11	4.6	290	166.0	68.3	?	137	101.3	
12	3.0	296	50.7	30.7	753	17	100.2	
13	5.75	212	119.0	76.0	—	80	56.5	No baffle, check test
14	4.4	293	119.0	73.4	—	90	101.0	
15	2.9	302	39.0	38.6	—	55	101.0	

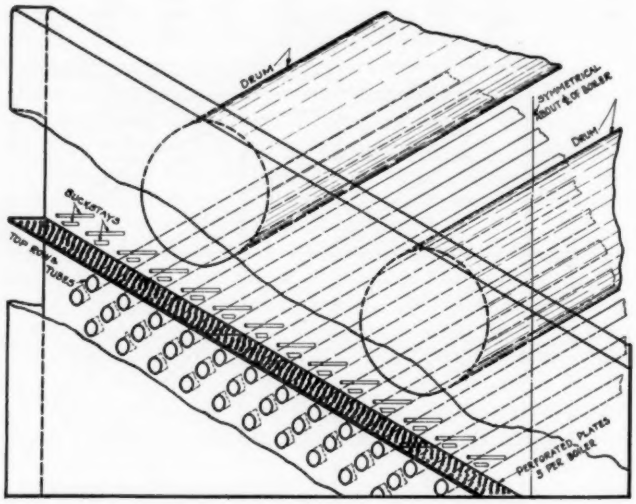

FIG. 2 INSTALLATION OF PERFORATED BAFFLE
(Water leg and horizontal water-tube boiler.)

location will correspond with a given level in the glass at the old location. No information was readily available on this point, so attempts were made to determine the density by studying the rate of concentrating the Cl ion. These tests indicated densities of from 10 to 50 per cent that of dense water and were suspended when the results began to cause loss of sleep.

At another station we found that the upper tubes in a horizontal water-tube boiler were sagging during a period of perhaps five years, probably due to lack of water. After comparing the circulation in this boiler with others of similar type on which the circulation had been studied, it was decided that the mixture of water and steam rising through the front water leg ought to be restricted so that some of the water would be deflected through the upper tubes to the rear water leg, while the steam with some water could pass on upward to the drums. For this purpose a perforated plate was installed in the front water leg at a point above that at which the upper tubes entered the tube sheet.

The presence of water in the upper tubes was determined qualitatively by inserting a sampling tube into several of the upper boiler tubes through a specially made cap. The character of the discharge from a throttling valve on the outer end of the sampling tube indicated qualitatively the amount of water in the tube.

The perforated baffle had little or no effect upon the amount of water in the upper tubes, but it did make it possible to operate the boiler at a high enough water level to increase materially the density of the fluid in the upper tubes and yet have no more moisture in the steam than at the low level without the perforated baffle. The data summarizing this investigation are given in Table 2, and a sketch of the installation is shown in Fig. 2.

In the foregoing the mechanical action of a perforated plate in the circulation was to reduce the effect of high water level on moisture in the steam so that full protection could be afforded to tubes which otherwise might have sagged over long periods due to the cumulative effect of occasional repeated overheating.

2 The second factor is the rate at which steam is generated in the boiler. Without any provision for the elimination of moisture in the steam, installed in the circulation or the steam path, we find that as the rating is increased a point is reached where moisture appears in the steam, and on further rating increase the amount of moisture increases as an approximate linear function of the rating. This condition is indicated by the curve in Fig. 3. This condition may be corrected by lowering the water level progressively with increase in rating, by installing moisture removing equipment, or by the installation of a mechanical means for rupturing the steam bubble films so that the bubbles may coalesce into larger ones from which the water may be more readily separated by gravity. We have found that the perforated baffle is incomparably more preferable to other means as reported herewith and as reported in the writer's paper quoted by Professor Foulk.

TABLE 3 CONCENTRATION AND TREATMENT FIGURES, NOVEMBER, 1931

1	Chemical added:	
	(*a*) To feedwater...........	NaOH, 11 lb. per million
	(*b*) To boiler water.........	Na_2HPO_4 and NaH_2PO_4 (10–1) To give pH value of 7.5 in 1% solution, 34 lb. per million) Na_2SO_4, 14.5 lb. per million
2	Boiler-water concentrations:	
	(*a*) (OH)	78 p.p.m.
	(*b*) (PO$_4$)	124 p.p.m.
	(*c*) (CO$_3$)	101 p.p.m.
	(*d*) (SO$_4$)	457 p.p.m.
3	Per cent blowdown:	
	(*a*) Continuous,	3.5%
	(*b*) Total,	5.7%

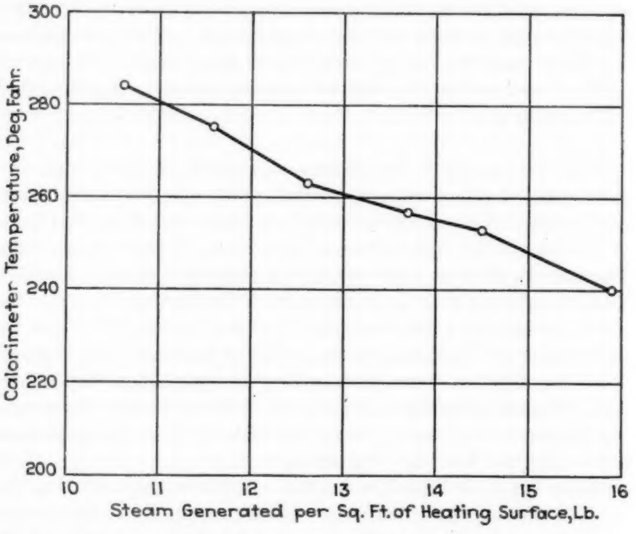

FIG. 3 CALORIMETER TEMPERATURE VARIATION WITH RATING
(No particular provision for moisture removal.)

The action of a perforated baffle is probably the mechanical rupturing of the bubble films, but this point can best be explained by Professor Foulk.

3 Within the limits of concentrations normally existing in our boilers we have not found any factor other than hydroxyl ion which specifically affects the quality of the steam. We know that there is a value of the concentration of hydroxyl ion, for those of our boilers which we have studied, above which the producton of dry steam is practically impossible, and we believe that a critical value of this concentration, which varies with the design, exists for all boilers. We understand that other investi-

gators have confirmed the existence of a critical value of the hydroxyl-ion concentration for their own boilers. Our efforts over the past three years have been directed to the adjustment and control of our feedwater treatment so that the normal building up of the hydroxyl-ion concentration would be offset by the neutralizing action of selected phosphate salts. We have not yet applied the maximum available neutralizing power, but all steps taken to date have been successful.

We find considerable variations in the sludge content of our boiler waters, and yet we find no noticeable variation in steam quality. Sludge or suspended matter may influence the permanence of steam bubble films, but in our boilers, if the sludge concentration is a factor, it is masked by other factors of more power.

We would like to call Professor Foulk's attention to the sampling error which influences reports from the power plants and which can be eliminated in the laboratory. In determining concentrations in boiler waters we select a sampling point which we have found by test to be fairly representative of average conditions, but we have not yet been able to find a sampling point which represents the average of several sampling points. The obvious conclusion is that uniform distribution of dissolved or suspended matter does not exist in our large boilers, and the corollary to that conclusion is that reported values of occasional samples cannot be depended upon for important conclusions.

In order that Professor Foulk may have figures representing what may be called normal concentrations, in Table 3 are given the average of daily tests of boiler waters at the Kip's Bay Station.

Summarizing the foregoing, we have indicated the relation of visible water level to quality of steam in one of our boilers. Whether the turbulence caused by the escape of steam from beneath the water level in the steam space when that space is restricted by too high a water level is the governing factor of this moisture relation or not is hidden from us by two inches of steel.

We have indicated that, as the rate of steam generation increases, the quality of steam decreases, unless a mechanical means for counteraction is provided. We believe that the perforated baffle is effective for this purpose, and we presume that its effectiveness is based on the rupture of bubble films.

We have not found as yet any other factor than hydroxyl-ion concentration which, under our conditions, will critically influence moisture, and we respectfully ask if Professor Foulk has ever met a foaming condition in an acid water.

Author's Closure

It is needless to say that the author is encouraged by the way that men in the commercial field are using his modest contributions to this subject of foaming and priming of boiler water. There are points in the discussions of both Mr. Holmes and Mr. Mumford which have suggested new laboratory experiments, and the author can assure these gentlemen that he probably got more out of their papers than they got out of his.

Mr. Mumford asks the specific question if the author has "ever met a foaming condition in an acid water." It can be said in answer, Not in a steam boiler, but then the author has had no experience with commercial boilers. In laboratory experiments in glass vessels acid solutions foam as readily as alkaline ones.

RP-54-6

Flow of Steam Through Square-Edged Orifices in a 4-In. Line

By W. W. FRYMOYER[1] and A. HERSCHEL,[2] FOXBORO, MASS.

W. W. Frymoyer

The flow of steam through square-edged orifices has little in the way of available data. Tests were therefore run to secure the necessary data on steam-flow measurement. The objects were to determine coefficients for steam flow through square-edged orifices under conditions favorable to securing basic coefficients; to determine effects of various types of disturbances, both upstream and downstream, on basic coefficients; to determine effects of straightening vanes with 0.5 pipe diameter from the upstream edge of the orifice plate on coefficients. The general statement of the results may be made that, for the disturbances covered in the report, and with the orifices installed with vanes $^1/_2$D from the upstream edge of the orifice plate, the actual coefficients will be near to 1 per cent of the 0.99 standard coefficient, for d/D ratios not over 0.75.

A. Herschel

THERE is little available in the way of accurate data on the flow of steam through square-edged orifices under the conditions met with in steam-flow measurements. Tests, therefore, were run to secure the necessary data for steam-flow measurement. Prof. J. J. Eames, in charge of the Steam and Hydraulic Laboratories of the Massachusetts Institute of Technology, made it possible for these tests to be run. The objects of the tests were as follows:

1. To determine coefficients for steam flow through square-edged orifices under conditions favorable to securing basic coefficients
2. To determine effects of various types of disturbances, both upstream and downstream, on basic coefficients
3. To determine effects of straightening vanes with 0.5 pipe diameter from the upstream edge of the orifice plate on coefficients.

It was soon found that tests near disturbances without straightening vanes produced serious and unpredictable results, so that these tests were confined almost entirely to conditions where straightening vanes were used. Furthermore, it is frequently necessary to install orifices where a large distance between the orifice and the upstream straightening vane cannot be maintained. All tests where disturbances were encountered were run with 0.5 pipe diameter between the vane and the upstream edge of orifice.

All work was done on flanges of the type shown in Fig. 1, as other comparative data applied to this type of flange.

It was originally planned to run tests covering the entire series of d/D ratios from 0.25 to 0.84. However, tests on the orifices under 1.6 in. in diameter (a d/D ratio of 0.420) gave wet steam on many of the flanges, so that they were omitted. On the 1.6 in. diameter, only high velocities were run to assure dry steam. On the larger orifice diameters only the lower velocities were run because of the limitation of the condenser capacities.

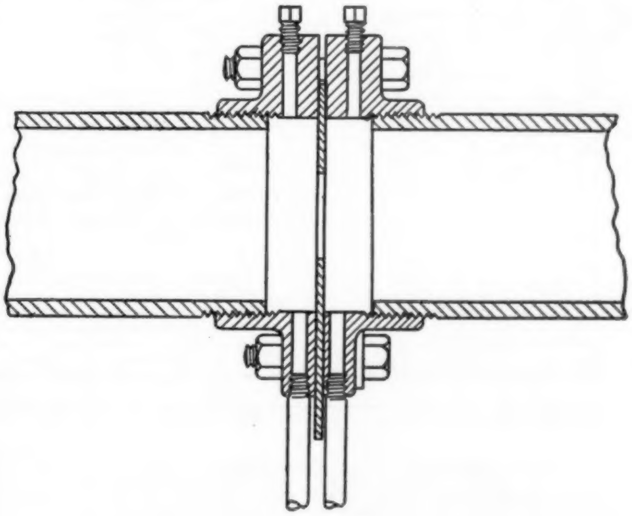

Fig. 1

In planning the program of tests it was decided to endeavor to maintain constant $(P_1 - P_2)P_2$ ratios for all pressures varying from 30 to 150 lb. per sq. in. At the lowest pressure only the largest $(P_1 - P_2)/P_2$ ratio (approximately 0.030) was run; at approximately double this pressure, $(P_1 - P_2)/P_2$ ratios of approximately 0.015 and 0.030 were run; while at four times the

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[1] Director of Research for The Foxboro Company. He attended the Carnegie Institute of Technology, and in 1921 received the degree of B.S. from the Massachusetts Institute of Technology. He joined the Bureau of Standards as an assistant engineer in 1921. He has been with The Foxboro Company since 1926.

[2] Research Department, The Foxboro Company. He received the degree of B.S. in M.E. from New York University in 1925 and the degree of M.E. in 1927. He worked for D. Saunders' Sons Inc., Yonkers, and in the Turbine Research Department of the General Electric Company, Lynn, Mass., previous to joining The Foxboro Company.

Contributed by the A.S.M.E. Special Research Committee on Fluid Meters and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

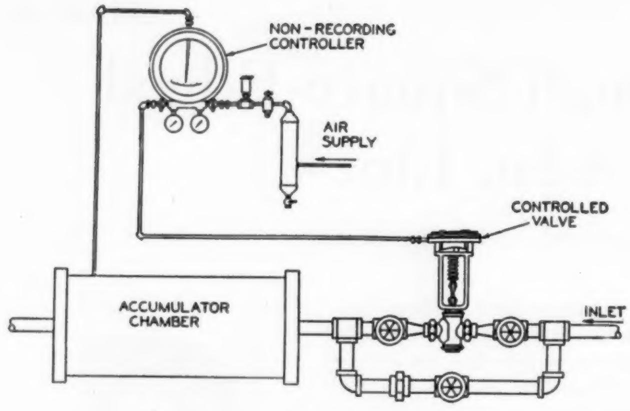
NON-RECORDING CONTROLLER
AIR SUPPLY
CONTROLLED VALVE
ACCUMULATOR CHAMBER
INLET

FIG. 2

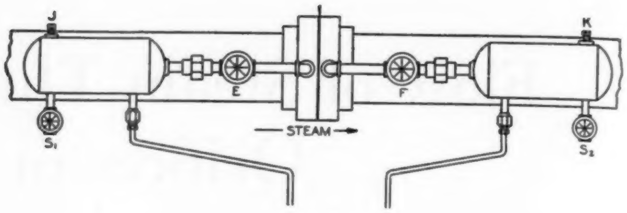
J
K
E
F
S_1
S_2
STEAM

FIG. 3

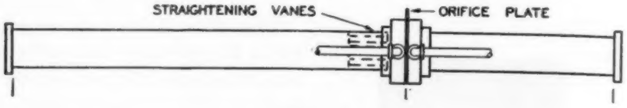
STRAIGHTENING VANES
ORIFICE PLATE
I
I'
I

FIG. 4

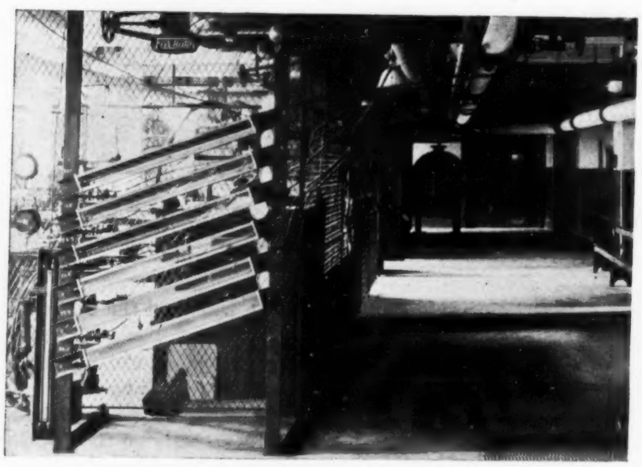

FIG. 7

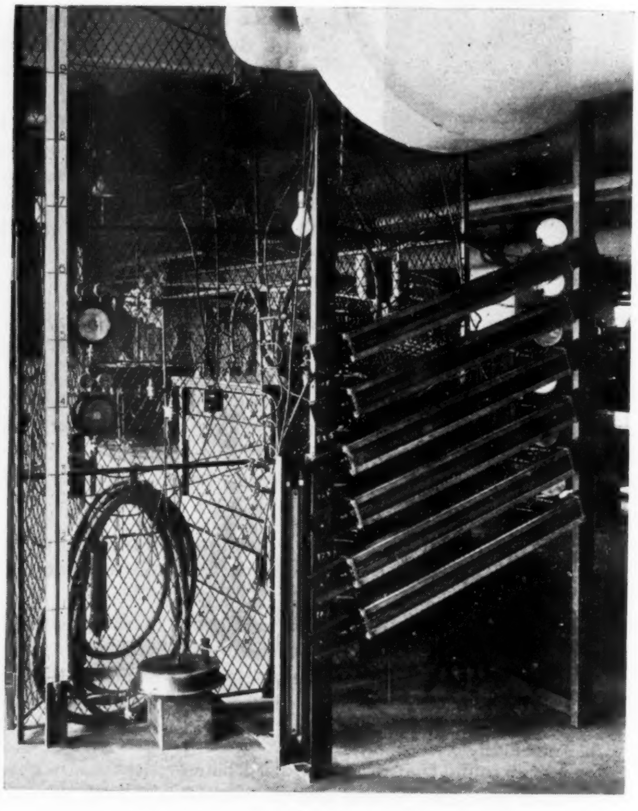

FIG. 5

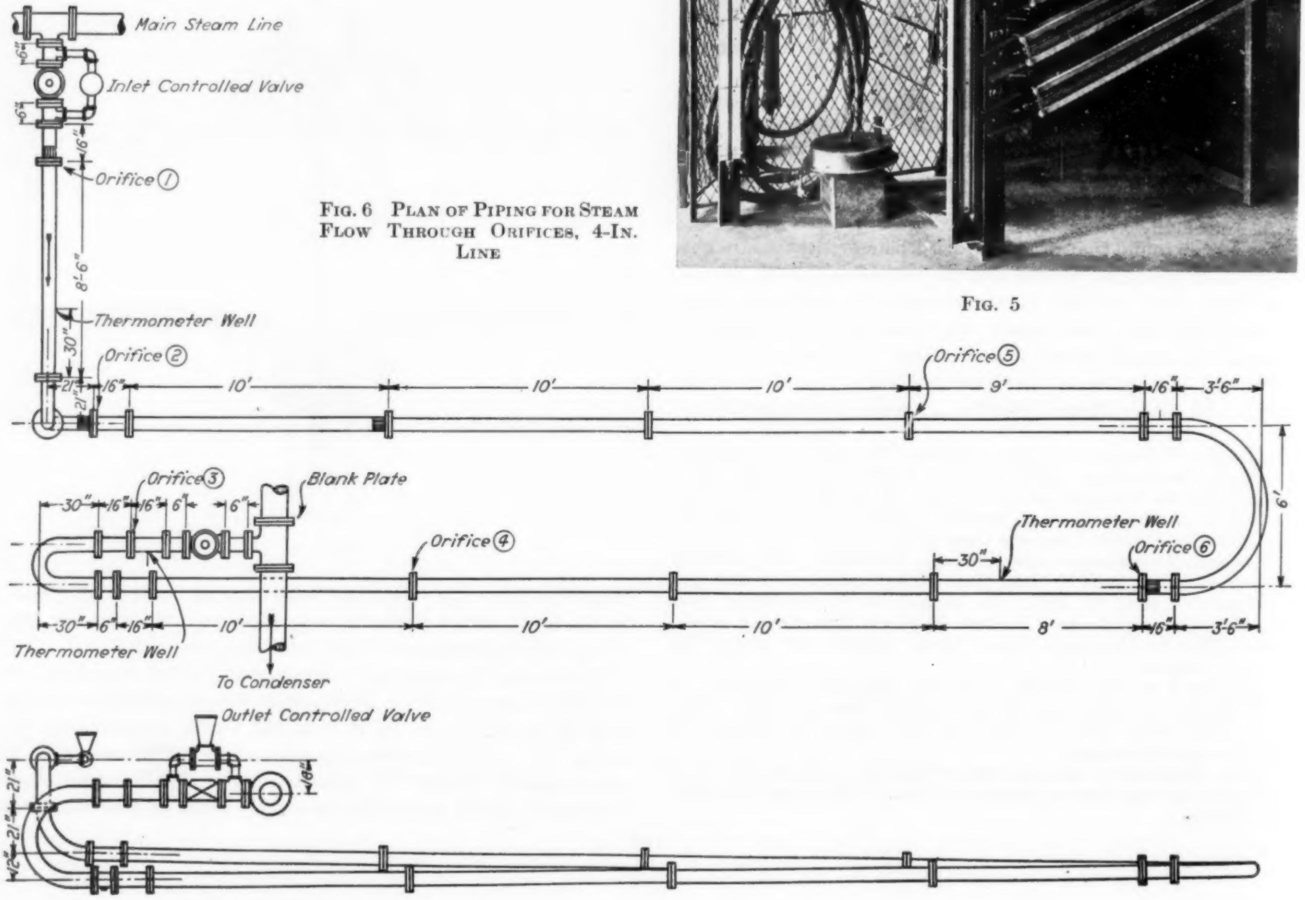
Main Steam Line
6"
Inlet Controlled Valve
6"
16"
Orifice ①
8'-6"
Thermometer Well
30"
Orifice ②
21"
16"
10'
10'
10'
Orifice ⑤
9'
16"
3'-6"
6'
Orifice ③
Blank Plate
30"
16"
16"
6"
6"
Orifice ④
Thermometer Well
30"
Orifice ⑥
30"
6"
16"
10'
10'
10'
8'
16"
3'-6"
Thermometer Well
To Condenser
Outlet Controlled Valve
21"
21"
12"
18"

FIG. 6 PLAN OF PIPING FOR STEAM FLOW THROUGH ORIFICES, 4-IN. LINE

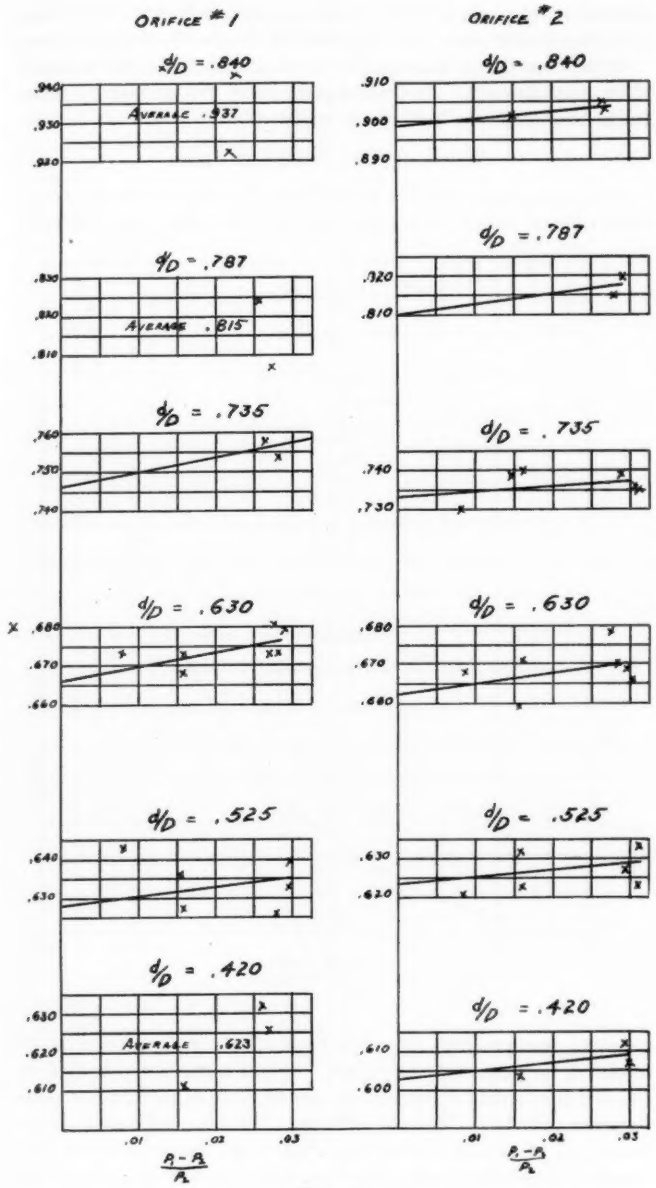

FIG. 8

lowest pressure, $(P_1 - P_2)/P_2$ ratios of 0.008, 0.015, and 0.030 were run.

EQUIPMENT USED IN THE TESTS

The 6-in. steam main, furnishing steam to the steam laboratory at 165 lb. and 100 deg. superheat, was available as a source of steam. One Alberger condenser of surface type with 1200 sq. ft. of condensing surface, with a Wheeler aftercooler, was available for condensing steam. Both condensers were carefully checked for tightness. All water trapped in the drip was collected in a long, slender pipe equipped with a gage glass and calibrated to give the measurement of water in pounds. The condensate was pumped to weighing tanks by a Worthington duplex pump.

The condensate was discharged into weighing tanks on Howe scales. The scales were checked for accuracy at the beginning of the test, once in middle of the test, and after completion of the tests. The discharge pipe had facilities for throwing the discharge from one weighing tank to another by simultaneous operating of two valves.

Control of flow was secured by pressure control at the entrance to the test piping and by a duplicate pressure control at the exit. The control was adjusted so that no surging occurred at any time. (See Fig. 2.)

Straightening vanes were made of $^3/_4$-in. tubing 4 in. long, welded into a tight unit. They were forced into the line, and a bead was turned up in front by means of a cape chisel to prevent movement during tests. They were inspected at regular intervals to determine whether any changes in position or appearance had occurred. (See Fig. 4.) All piping except orifice flanges were covered with 2 in. of 85 per cent magnesia. Condensers on meter lines were of the usual commercial standard fitted with the usual air vents, but with special gage glasses to show the water level. (See Fig. 3.)

Lines to the meters were a combination of steel and copper tubing. Both high- and low-pressure sides of each differential were tied adjacent to each other to insure temperatures as similar as possible. Differential gages were of inclined mercury type, calibrated while under actual conditions of liquid head by comparison with a water column. (See Fig. 5.)

Pressure gages were Foxboro standard steel-tube test gages, carefully calibrated at frequent intervals on a dead-weight tester. The thermometer wells were standard Foxboro thermometer wells, using a carefully checked mercury thermometer for measurement. The conditions under which the various orifices were tested are shown on the layout of the test line in Figs. 6 and 7.

Orifice No. 1 was preceded by a 16-in. straight pipe, a drip tee

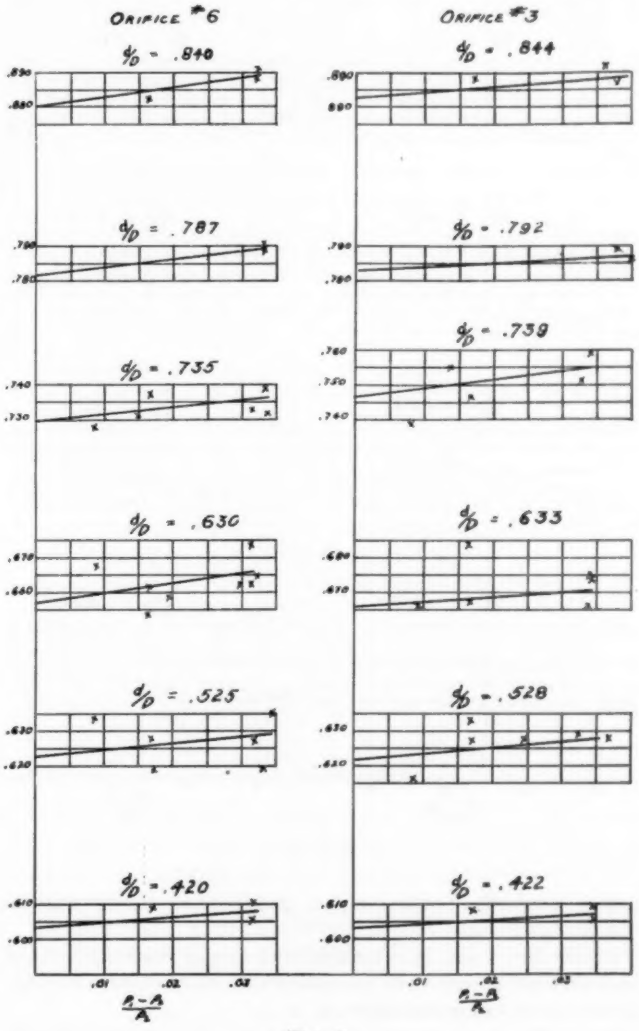

FIG. 9

into which the controlled valve of the pressure control discharged, and a gate valve open until the $1^1/_4$-in. valve in the bypass controlled the pressure. A straightening vane was installed 1.9 in. from the upstream face of the orifice plate. (See Fig. 6.)

Orifice No. 2 was preceded by a 16-in. straight pipe, a double-plane bend 21 in. on each leg, and another straight run of pipe. A straightening vane was also installed 1.9 in. from the upstream face of the orifice plate.

Orifice No. 3 was preceded by a 16-in. straight pipe and a single-plane return bend of about 25 in. radius. Following the orifice was a further 16-in. length, a drip tee going to the downstream controlled valve, and a gate valve. There was no straightening vane.

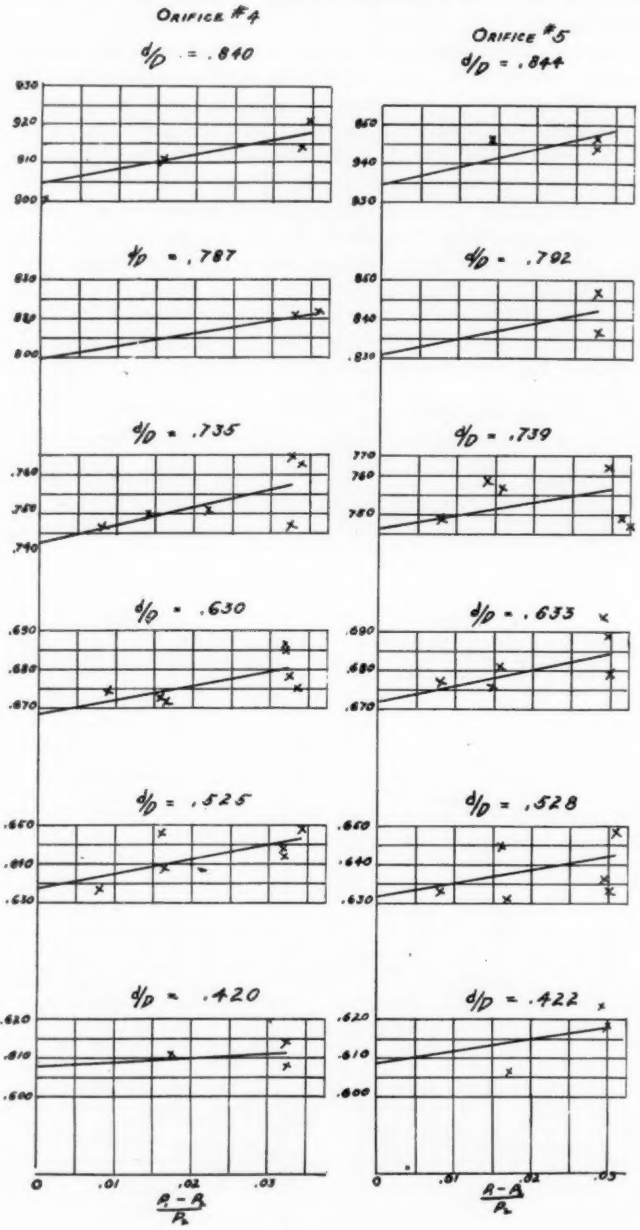

FIG. 10

Orifice No. 4 was in a straight run of pipe with $22^1/_2$ ft. of unobstructed straight run preceding and 11 ft. 6 in. following. There was no straightening vane.

Orifice No. 5 also was in a straight run of pipe with $22^1/_2$ ft. of unobstructed straight run preceding and 10 ft. 6 in. following. A straightening vane was installed 20 ft. ahead of this orifice.

Orifice No. 6 was preceded by a bend with 3 ft. radius and a 16-in. straight run. A straightening vane was installed 1.9 in. above the upstream face of the orifice plate.

METHOD OF RUNNING THE TESTS

In running these tests the gate valves at both ends of the piping were opened until something slightly less than the required quantity was passing. Further regulation was then secured by means of the pressure controllers. Steam was allowed to flow in the test line until for 30 min. no appreciable change in the temperature occurred. Then a minimum of 30 min. run was made on each type of set-up. At 10-min. intervals the condensate was checked to determine whether there had been any more than 0.5 per cent change in the weight flowing during any 10-min. interval. Where discrepancies larger than this occurred, further runs were made.

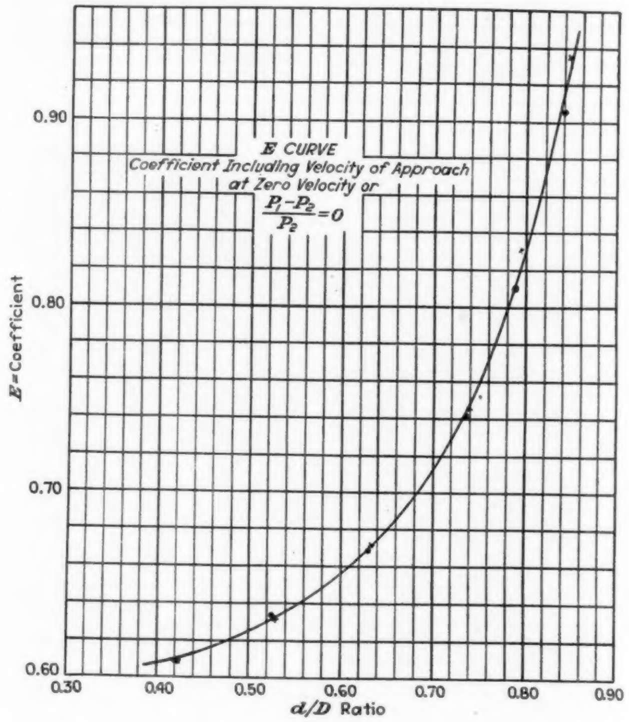

FIG. 11

The differential gages and the pressure gages were read at 2-min. intervals by two observers. The convenience in the arrangement of the gages and the arrangement of the scales to eliminate parallax enabled accurate readings to be made quickly. Owing to the use of pressure controls, the readings remained practically constant throughout any one run. Before each run, pains were taken to eliminate all air from connecting piping. A third observer made the readings of the thermometers while a fourth observer took care of the condensate tanks. The quantities were timed by the individual in charge who gave the signals for all observers.

MEASUREMENT OF ORIFICE PLATES

All orifice plates were checked by three observers with inside micrometers. The temperatures at which these observations were made were noted. Inasmuch as the greatest discrepancy between the averages of the three individual readings did not exceed 0.0003 in., and since the average of all readings in every plate was within 0.0001 in. of the nominal orifice sizes, comparisons between the various coefficients were greatly facilitated. The pipe used in the tests was carefully selected for uniformity of inside diameters and concentricity. The average dimensions of each of the pipes preceding and following the individual orifices were used in calculating the d/D ratios.

Discussion of Results

Preliminary tests showed that the thermometer wells placed five diameters downstream from the orifice plates had little effect on the coefficients. However, these tests gave the temperature gradient throughout the pipe line which was subsequently used in determining the temperature for the individual orifices at five diameters downstream.

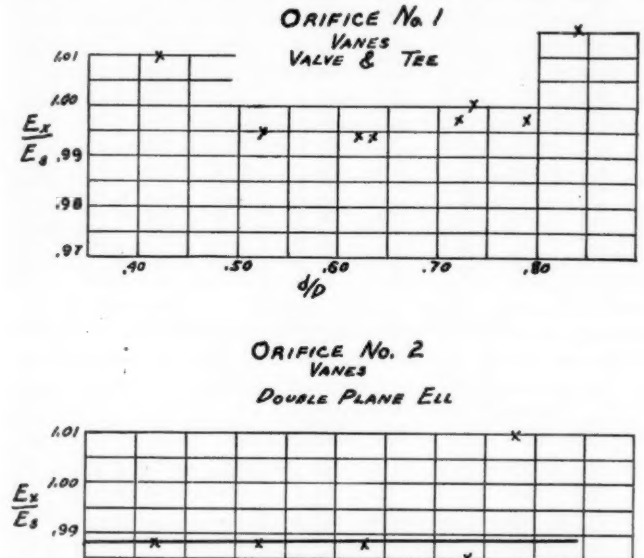

Fig. 12

Figs. 8, 9, and 10 show graphs of coefficients against $(P_1 - P_2)/P_2$ ratio. The lines are based on the average trend for the entire series of tests. Fig. 11 shows the E value based on plotting the coefficients of orifices 4 and 5 for zero velocity taken from Fig. 10. No effort was made to transfer these values into a graph of the coefficient of discharge, as the E graph gives information that the average user of orifices can conveniently use.

Fig. 12 shows the ratios of the coefficients of the various orifices under various types of disturbances as compared with the coefficient for the standard orifices 4 and 5. No attempt was made to plot a curve for this ratio for orifice 1, as two points at either end of the d/D ratio may have been produced by a large difference in the conditions existing when the two runs were made. For instance, during some tests the gate valves at either end of the piping were wide open; for others they were completely closed, with the entire flow being handled by the controlled valve in the bypasses. This analysis applies equally well to orifice No. 3. Furthermore, orifice No. 3 was installed without straightening vanes.

However, the general statement may be made that for the disturbances covered in this report, and with the orifices installed with vanes $^1/_2D$ from the upstream edge of the orifice plate, the actual coefficients will be within ± 1 per cent of the 0.99 standard coefficient, for d/D ratios not over 0.75.

Appendix

CALCULATIONS

The formula used in calculation of the E value was as follows:

$$E = \frac{W}{0.0997\, d^2 \sqrt{(h/v)}}$$

where

E = coefficient of discharge, including velocity of approach
W = weight of steam flowing in pounds per second
d = the diameter of the orifice in inches
v = the specific volume of the steam in cubic feet per pound
h = the differential in inches of water
E_s = the coefficient for standard conditions
E_x = the coefficient for any non-standard condition.

Inasmuch as the water in the legs of the differential gages was kept at nearly the same temperature, no corrections were made for differences in densities due to temperature. In the calculation of the coefficient, allowance was made for the effect of temperature on the dimension of the various orifice plates. No allowance was made for the 0.101-in. drain hole in the orifice plates. The density of steam was determined by using the values for v given in Keenan's Steam Tables.

It is desired to express appreciation for the interested cooperation, the many suggestions, and helpful criticisms of Prof. J. J. Eames and L. K. Spink, manager of Flow Meter Division, Foxboro Company, and for the assistance of H. D. Hibbard and A. B. Brand, students at Massachusetts Institute of Technology, and C. B. Ives, Research Department, Foxboro Company.

Discussion

George D. Conlee.[3] This paper is very interesting and the data obtained will prove of great value. The authors are to be congratulated on being able to condense so much information in a small space.

There is really very little known about the effect of various pipe fittings in front of an orifice or other differential medium. Various opinions are held by different engineers, but there is so

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[3] Chief Engineer, Republic Flow Meters Company, Chicago, Ill. Mem. A.S.M.E.

TABLE 1 FOR ORIFICE LOCATION NO. 1

(See Fig. 6 for installation)

Run No.	Orifice size, in.	Steam pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Diff., inches water	Weight steam, lb. per sec.	*d/D*	*E*	*C*
211	2.000	151.2	439	128.1	1.4870	0.525	0.626	0.602
221	2.000	141.6	417	32.0	0.7785	0.525	0.643	0.618
231	2.000	146.0	434	63.5	1.0930	0.525	0.636	0.611
241	2.000	75.9	409	63.3	0.7876	0.525	0.639	0.614
251	2.000	73.0	371	32.6	0.5605	0.525	0.627	0.603
261	2.000	40.2	352	32.5	0.4204	0.525	0.633	0.608
271	2.400	38.4	340	30.9	0.6242	0.630	0.679	0.623
291	2.400	79.3	405	60.3	1.1966	0.630	0.673	0.617
301	2.400	74.4	388	32.0	0.8499	0.630	0.668	0.613
311	2.400	153.5	431	121.1	2.3596	0.630	0.673	0.617
321	2.400	147.1	425	63.7	1.6800	0.630	0.673	0.617
331	2.400	153.3	426	117.5	2.3637	0.630	0.681	0.625
341	2.400	142.4	423	32.1	1.2141	0.630	0.674	0.618
351	2.800	39.0	391	31.2	0.9255	0.735	0.754	0.634
361	2.800	78.2	409	57.7	1.7861	0.735	0.758	0.637
371	2.800	75.0	401	29.1	1.2410	0.735	0.754	0.634
381	2.800	154.8	451	117.6	3.5432	0.735	0.760	0.639
391	2.800	148.7	451	62.4	2.5141	0.735	0.756	0.636
401	2.800	144.5	440	31.8	1.7533	0.735	0.745	0.627
411	3.200	42.8	351	26.4	1.4609	0.840	0.923	0.654
421	3.200	42.0	395	26.8	1.4491	0.840	0.943	0.668
431	3.200	76.4	418	28.1	1.9904	0.840	0.945	0.670
511	3.000	40.5	360	29.0	1.1632	0.787	0.824	0.646
521	3.000	79.5	390	60.1	2.2806	0.787	0.806	0.632
701	1.600	149.0	401	103.8	0.9283	0.420	0.632	0.623
711	1.600	149.0	390	110.8	0.9452	0.420	0.626	0.617
721	1.600	143.8	392	61.4	0.6837	0.420	0.611	0.602

TABLE 2 FOR ORIFICE LOCATION NO. 2

(See Fig. 6 for installation)

Run No.	Orifice size, in.	Steam pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Diff., inches water	Weight steam, lb. per sec.	*d/D*	*E*	*C*
212	2.000	148.0	435	120.0	1.4870	0.525	0.627	0.603
222	2.000	140.8	415	34.3	0.7784	0.525	0.621	0.597
232	2.000	144.8	430	64.4	1.0930	0.525	0.632	0.607
242	2.000	74.3	405	64.8	0.7876	0.525	0.633	0.608
252	2.000	72.2	365	33.1	0.5605	0.525	0.623	0.599
262	2.000	39.9	343	33.3	0.4204	0.525	0.623	0.599
272	2.400	38.4	332	31.7	0.6242	0.630	0.666	0.611
292	2.400	77.2	400	61.8	1.1966	0.630	0.670	0.614
302	2.400	74.2	383	32.6	0.8499	0.630	0.659	0.604
312	2.400	150.0	428	124.7	2.3596	0.630	0.669	0.613
322	2.400	145.4	423	65.0	1.6800	0.630	0.671	0.615
332	2.400	149.9	424	121.5	2.3637	0.630	0.679	0.623
342	2.400	141.9	420	32.8	1.2141	0.630	0.668	0.613
352	2.800	38.5	386	32.7	0.9255	0.735	0.735	0.618
362	2.800	77.2	407	61.5	1.7861	0.735	0.739	0.621
372	2.800	74.5	399	30.5	2.5141	0.735	0.739	0.621
382	2.800	151.5	450	127.9	3.5432	0.735	0.736	0.619
392	2.800	146.8	449	65.5	1.2410	0.735	0.741	0.623
402	2.800	143.8	438	33.2	1.7533	0.735	0.730	0.614
412	3.200	40.3	348	29.1	1.4609	0.840	0.905	0.641
422	3.200	40.7	391	30.0	1.4491	0.840	0.903	0.640
432	3.200	75.4	416	31.2	1.9904	0.840	0.902	0.639
512	3.000	39.4	356	30.5	1.1632	0.787	0.820	0.643
522	3.000	77.5	389	60.3	2.2806	0.787	0.815	0.639
702	1.600	145.0	396	118.0	0.9283	0.420	0.607	0.599
712	1.600	145.0	384	118.3	0.9452	0.420	0.612	0.603
722	1.600	141.5	387	64.0	0.6837	0.420	0.604	0.596

TABLE 3 FOR ORIFICE LOCATION NO. 3

(See Fig. 6 for installation)

Run No.	Orifice size, in.	Steam pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Diff., inches water	Weight steam, lb. per sec.	*d/D*	*E*	*C*
213	2.000	134.6	416	128.9	1.4870	0.528	0.628	0.603
223	2.000	137.6	385	34.2	0.7784	0.528	0.616	0.591
233	2.000	137.6	407	66.0	1.0930	0.528	0.633	0.608
243	2.000	66.6	378	70.2	0.7876	0.528	0.628	0.603
253	2.000	68.1	333	33.2	0.5605	0.528	0.627	0.602
263	2.000	35.3	302	35.9	0.4204	0.528	0.629	0.604
273	2.400	34.9	290	33.0	0.6242	0.633	0.666	0.609
293	2.400	69.9	376	67.2	1.1966	0.633	0.665	0.608
303	2.400	69.9	360	32.9	0.8499	0.633	0.667	0.610
313	2.400	137.0	412	132.2	2.3596	0.633	0.673	0.616
323	2.400	140.2	420	64.3	1.6800	0.633	0.684	0.626
333	2.400	129.5	406	130.8	2.3637	0.633	0.675	0.618
343	2.400	138.4	398	35.1	1.2141	0.633	0.666	0.609
353	2.800	33.9		Defective gage				
363	2.800	69.4	390	64.8	1.7861	0.739	0.751	0.629
373	2.800	70.4	380	30.1	2.5141	0.739	0.755	0.633
383	2.800	137.0	436	131.9	3.5432	0.739	0.759	0.636
393	2.800	139.8	436	66.9	1.2410	0.739	0.747	0.626
403	2.800	140.1	424	32.7	1.7533	0.739	0.739	0.619
413	3.200	34.2	330	34.9	1.4609	0.844	0.887	0.623
423	3.200	34.8	370	35.2	1.4491	0.844	0.892	0.626
433	3.200	69.7	400	34.2	1.9904	0.844	0.888	0.625
513	3.000	34.4	334	35.9	1.1632	0.792	0.786	0.612
523	3.000	68.5	374	71.7	2.2806	0.792	0.789	0.615
703	1.600	131.1	366	126.2	0.9283	0.422	0.605	0.595
713	1.600	131.0	352	126.4	0.9452	0.422	0.609	0.599
723	1.600	134.0	351	63.1	0.6837	0.422	0.608	0.598

TABLE 4 FOR ORIFICE LOCATION NO. 4

(See Fig. 6 for installation)

Run No.	Orifice size, in.	Steam pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Diff., inches water	Weight steam, lb. per sec.	*d/D*	*E*	*C*
214	2.000	138.2	423	120.1	1.4870	0.525	0.644	0.619
224	2.000	138.2	393	32.6	0.7784	0.525	0.634	0.609
234	2.000	139.6	414	62.8	1.0930	0.525	0.648	0.623
244	2.000	68.2	385	65.7	0.7876	0.525	0.649	0.624
254	2.000	69.2	341	31.9	0.5605	0.525	0.639	0.614
264	2.000	36.6	313	32.9	0.4204	0.525	0.642	0.617
274	2.400	35.1	305	31.3	0.6242	0.630	0.686	0.629
294	2.400	71.3	382	64.1	1.1966	0.630	0.678	0.622
304	2.400	70.8	369	32.3	0.8499	0.630	0.673	0.617
314	2.400	140.0	416	129.3	2.3596	0.630	0.675	0.619
324	2.400	140.2	410	65.8	1.6800	0.630	0.672	0.616
334	2.400	140.4	410	125.2	2.3637	0.630	0.685	0.628
344	2.400	139.4	404	33.9	1.2141	0.630	0.675	0.619
354	2.800	35.2	364	32.7	0.9255	0.735	0.763	0.642
364	2.800	70.7	394	64.7	1.7861	0.735	0.747	0.628
374	2.800	71.0	385	30.1	2.5141	0.735	0.750	0.631
384	2.800	140.3	440	126.9	3.5432	0.735	0.765	0.643
394	2.800	141.5	440	65.7	1.2410	0.735	0.751	0.632
404	2.800	140.7	428	32.0	1.7533	0.735	0.747	0.628
414	3.200	35.3	335	32.1	1.4609	0.840	0.914	0.648
424	3.200	35.5	373	32.4	1.4491	0.840	0.921	0.653
434	3.200	71.0	403	32.0	1.9904	0.840	0.911	0.646
514	3.000	35.6	339	32.4	1.1632	0.787	0.822	0.644
524	3.000	64.5	377	64.5	2.2806	0.787	0.821	0.644
704	1.600	134.7	374	122.8	0.9283	0.420	0.608	0.599
714	1.600	134.5	362	122.8	0.9452	0.420	0.614	0.605
724	1.600	135.8	364	64.6	0.6837	0.420	0.611	0.602

TABLE 5 FOR ORIFICE LOCATION NO. 5

(See Fig. 6 for installation)

Run No.	Orifice size, in.	Steam pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Diff., inches water	Weight steam, lb. per sec.	*d/D*	*E*	*C*
215	2.000	144.9	431	117.7	1.4870	0.528	0.636	0.611
225	2.000	140.1	407	32.8	0.7784	0.528	0.633	0.608
235	2.000	143.4	425	62.7	1.0930	0.528	0.645	0.619
245	2.000	72.1	399	63.2	0.7876	0.528	0.649	0.623
255	2.000	71.2	359	32.5	0.5605	0.528	0.631	0.606
265	2.000	38.7	333	32.8	0.4204	0.528	0.633	0.608
275	2.400	37.1	324	29.9	0.6242	0.633	0.694	0.635
295	2.400	76.3	395	61.0	1.1966	0.633	0.680	0.622
305	2.400	72.9	378	31.4	0.8499	0.633	0.676	0.619
315	2.400	147.3	425	123.6	2.3596	0.633	0.679	0.621
325	2.400	144.2	419	62.9	1.6800	0.633	0.681	0.623
335	2.400	146.2	420	120.6	2.3637	0.633	0.689	0.630
345	2.400	141.3	414	34.0	1.2141	0.633	0.677	0.619
355	2.800	37.3	379	32.5	0.9255	0.739	0.749	0.628
365	2.800	70.5	394	64.7	1.7861	0.739	0.747	0.626
375	2.800	73.2	394	29.2	2.5141	0.739	0.759	0.636
385	2.800	147.8	447	122.1	3.5432	0.739	0.762	0.639
395	2.800	145.3	446	63.6	1.2410	0.739	0.757	0.634
405	2.800	141.7	435	31.9	1.7533	0.739	0.749	0.628
415	3.200	38.5	344	27.8	1.4609	0.844	0.946	0.664
425	3.200	38.6	385	28.8	1.4491	0.844	0.944	0.662
435	3.200	72.5	411	29.2	1.9904	0.844	0.948	0.665
515	3.000	38.2	349	28.8	1.1632	0.792	0.847	0.660
525	3.000	75.1	385	58.9	2.2806	0.792	0.836	0.651
705	1.600	141.5	387	113.0	0.9283	0.422	0.624	0.614
715	1.600	141.4	372	116.8	0.9452	0.422	0.618	0.608
725	1.600	139.2	377	65.5	0.6837	0.422	0.606	0.596

TABLE 6 FOR ORIFICE LOCATION NO. 6

(See Fig. 6 for installation)

Run No.	Orifice size, in.	Steam pressure, lb. per sq. in. abs.	Temp., deg. fahr.	Diff., inches water	Weight steam, lb. per sec.	*d/D*	*E*	*C*
216	2.000	142.2	426	126.1	1.4870	0.525	0.627	0.603
226	2.000	139.5	400	32.6	0.7784	0.525	0.634	0.609
236	2.000	141.5	418	66.2	1.6930	0.525	0.628	0.604
246	2.000	70.5	393	68.2	0.7876	0.525	0.635	0.610
256	2.000	70.8	351	34.0	0.5605	0.525	0.619	0.595
266	2.000	38.1	324	34.6	0.4204	0.525	0.619	0.595
276	2.400	36.4	316	33.2	0.6242	0.630	0.662	0.607
296	2.400	73.9	389	65.3	1.1966	0.630	0.662	0.607
306	2.400	72.4	374	33.8	0.8499	0.630	0.654	0.600
316	2.400	143.9	421	130.6	2.3596	0.630	0.665	0.610
326	2.400	142.4	417	67.2	1.6800	0.630	0.662	0.607
336	2.400	143.4	414	167.9	2.3637	0.630	0.674	0.618
346	2.400	140.0	410	35.1	1.2141	0.630	0.668	0.613
356	2.800	36.1	372	34.8	0.9255	0.735	0.732	0.615
366	2.800	73.2	400	64.9	1.7861	0.735	0.733	0.616
376	2.800	73.0	390	31.4	2.5141	0.735	0.731	0.615
386	2.800	143.6	444	133.3	3.5432	0.735	0.739	0.621
396	2.800	143.3	444	67.7	1.2410	0.735	0.737	0.620
406	2.800	141.8	432	33.6	1.7533	0.735	0.728	0.612
416	3.200	36.8	340	32.8	1.4609	0.840	0.888	0.629
426	3.200	37.2	380	33.3	1.4491	0.840	0.891	0.631
436	3.200	72.5	408	33.6	1.9904	0.840	0.883	0.626
516	3.000	36.9	345	34.2	1.1632	0.787	0.789	0.619
526	3.000	72.6	381	67.8	2.2806	0.787	0.791	0.620
706	1.600	138.5	382	122.2	0.9283	0.420	0.605	0.597
716	1.600	138.1	367	122.1	0.9452	0.420	0.610	0.601
726	1.600	138.0	372	65.1	0.6837	0.420	0.608	0.599

much divergence in opinion that they cannot all be right. In practice, it is customary to obtain all of the straight pipe possible in front of the primary element and hope for the best.

The data on the effect of bends are not conclusive, even in this paper. It is doubtful whether definite rules can be established that can be used to determine the correction to be applied under various disturbed conditions.

The data on straightening vanes are of much real value because of the indications that straightening vanes, immediately in front of a primary element, will affect the coefficient only slightly.

It has been generally believed that straightening vanes should be at least six diameters in front of an orifice, if accurate results were to be obtained. This opinion has made it necessary to place the straightening vanes in a flange some distance ahead of the one used for the measuring orifice and has often made it difficult to install vanes. The data obtained during these tests indicate that a straightening vane may be placed within one diameter of the measuring orifice without serious effect on the coefficient. This simplifies the problem greatly and will probably result in the use of many more straightening vanes in the future.

J. O. Feldmark.[4] The claim is made by the author that there is little of accurate data available on the flow of steam through square-edged orifices under the conditions met with in steam-flow measurement. From this statement the conclusion may be drawn that the object of their test was to secure the necessary reliable data which can be applied to the general measurement of steam flow under conditions actually met with in any industrial application.

However, such is not the case, as proved by the special type of flanges used for these tests. What was probably meant by the experimenters was to test only one particular application of steam-flow measurement. This becomes evident from the fact that no mention is made or consideration given to the "vena contracta" and "restoration taps," nor was any other form of flanges tested, such as commonly used in steam-flow measurement.

But such a conclusion can be reached only by men familiar with the various forms of flow metering. Hence, it is believed that, in order to avoid any possible misunderstanding, the paper ought to be corrected in such a way as to bring out clearly to the average reader that the results of these tests apply exclusively to the unique application shown in Fig. 1 and are not meant to cover the multitude of conditions met with in the wide field of steam measurements.

With this correction in mind, the present tests become valuable as a general contribution to that particular phase of metering and may serve as a base for comparison between coefficients used previously under identical conditions and those obtained by the present tests. However, for some reason nothing of this comparison is shown in this paper, so that it is impossible to determine whether the results differ from figures previously used under the same arrangement.

When one analyzes next the results of the other two intended objects, with regard to the effects of disturbances and straightening vanes on the basic coefficients, he finds the condition in such a shape that it is hard to reach the same conclusion that the experimenters did.

In the first place, there is very little given in the paper, either in the form of tables or curves, to show what happens when an orifice plate is installed near disturbances. Furthermore, no mention is made nor does the test diagram show of any disturbances on the downstream side, as intended according to the outline of object 2, while those on the upstream side are done away with quickly at the very beginning with a statement reading as follows: "It was soon found that tests near disturbances without straightening vanes produced serious and unpredictable results, so that these tests were confined almost entirely to conditions where straightening vanes were used."

Such a statement, important as it may be, is hardly what the metering industry would expect from experimenters. If serious and unpredictable results were obtained, why not show them, in order to demonstrate the disadvantage of such orifice locations or the beneficial effect of straightening vanes? Otherwise, an entirely different picture may be drawn, based on a comparison of the coefficients *E* gathered from the printed tables.

However, in order to facilitate any comparison between the results of the various orifice locations with and without straightening vanes, there is listed in Table 7 a condensation of the average coefficients *E* for each ratio, as given in Tables 1 to 6 in this paper. These are arranged in Table 7 in numerical order of orifice ratios.

TABLE 7 DISCHARGE COEFFICIENT *E* FOR ORIFICE LOCATION

Orifice ratio d/D	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
0.420	0.623	0.607	0.607	0.611	0.616	0.607
0.525	0.634	0.626	0.627	0.643	0.638	0.627
0.630	0.674	0.669	0.669	0.678	0.682	0.664
0.735	0.754	0.737	0.750	0.754	0.754	0.733
0.878	0.815	0.818	0.788	0.821	0.842	0.790
0.840	0.937	0.903	0.889	0.921	0.946	0.887

Comparing, for instance, the results of orifices No. 3 and No. 6 to No. 4 orifice, located in a straight run of pipe, $22^1/_2$ ft. upstream and $11^1/_2$ ft. downstream, and taken as a standard, one finds that orifice No. 3, although located after a sharper bend than No. 6 and without a straightening vane, comes closer to the results of orifice No. 4 than orifice No. 6, located with a vane after a bend of a greater radius.

If one is to draw any conclusions from this comparison, he may be inclined to consider the presence of the straightening vane as detrimental or at least unnecessary. This certainly was not the object in the experimenters' minds.

In the same manner, one could attribute the differences between No. 4 and No. 5 orifice coefficients to the restriction created in the pipe area by the presence of a straightening vane. A rough calculation shows that close to 30 ${}^3/_4$-in. tubes are required to make up the vane. The walls of those tubes occupy an area of approximately 1 sq. in., which introduces an error of 0.7 per cent in the pipe area. Slight as this may be, the effect is nevertheless there and may explain why the coefficients are different in the two cases, although both are located in ideal straight lengths of pipe.

However, notwithstanding the few shortcomings mentioned, the authors deserve the thanks of the metering industry for their valuable contribution to the field of steam-flow measurement.

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RP-54-7

Optimum Conditions in Journal Bearings

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This paper presents in practical, serviceable form the most important results obtainable from the hydrodynamic theory of the oil film. The general theoretical treatment is supplemented by numerical evaluations whereby the conditions for the maximum load and for the minimum friction are determined for the ideal bearing; and these results are reduced to apply to practical cases by means of approximate methods. Tables and graphs are given, by means of which the loads, the losses by friction, and other characteristics of specified bearing forms may be determined, and special consideration is given to the optimum conditions for more or less arbitrary bearing proportions.

I—INTRODUCTORY

NO ABSOLUTE criterion of bearing safety has thus far been derived from practice, experiment, or theory. It is well known that bearings under laboratory conditions may be operated with speeds and loads far exceeding those applicable in practice; the latter remain, and probably must always remain, matters of judgment and experience. Nevertheless the mathematical theory offers valuable aid for attaining satisfactory results in practice.

Under running conditions, the minimum thickness of the oil film is the obvious feature determining safety. It does in some way fix the degree of safety, but it offers no possibility of establishing a definite "factor of safety," since danger of failure occurs only when contact of the metals occurs; and even then failure may be obviated by plasticity and good heat conductivity in the bearing metal. The most that can be said on this point is that the greater the minimum film thickness, the greater the safety. The film thickness is difficult to measure, even under laboratory conditions; but it is hoped that the present discussion may be of assistance in determining suitable practical values for this important quantity.

With a complete film and clean oil, there is no wear under running conditions; but when starting and stopping under load, there is inevitably more or less wear, depending upon the load, the area, and the properties of the oil, as well as upon surface conditions and upon the rates of acceleration. In many cases, bearings must serve as brakes for the ultimate stopping of rotating masses of great inertia. In general, reduction of wear is obtainable by increasing the viscosity or the area, or both; but either of these steps involves increased power loss when running. Here, again, there is no absolute criterion; the permissible loading must be based upon experience and judgment. It may, however, be said that the small wear occurring under the moderate journal loads of current practice does not in general cause increased friction or reduced safety under running conditions; it may have the contrary effects, as shown by the known beneficial effects of "running in" newly fitted bearings.

The present discussion is based primarily upon the minimum film thickness as a critical quantity, by whatever means its value appropriate to a given set of conditions may be determined; whether directly by physical conditions at the point of nearest approach of the metals when at speed, or indirectly by the limitations of loading imposed by starting and stopping conditions. No attempt is made to establish a law for the values of permissible film thicknesses. The general method of procedure in the research is as follows:

(*a*) Formal integration of the differential equations for pressure, load, and friction, in films between both plane surfaces and cylindrical surfaces, for the hypothetical condition of "infinite width" (or "no side leakage") and assuming uniform viscosity

(*b*) Determination of film forms giving respectively maximum load capacity and minimum coefficient of friction, for a given minimum film thickness, by means of trial numerical substitutions in the integral equations of (*a*)

(*c*) Approximate determination of the factors necessary for modifying the results of (*a*) and (*b*) to apply to bearings of finite widths; that is, covering the effects of "side leakage" and of varying viscosity

(*d*) Preparation of tables and graphs for convenient practical use, with special reference to the predetermination of optimum conditions, rather than the analysis of any arbitrarily chosen conditions.

The procedure thus outlined has been applied to three types of conditions of fit and of loading, as follows:

Class A Journal Bearings: Those in which the direction of the load and the clearance, that is, the difference of the radii of the journal and the bearing, are not arbitrarily conditioned, but are determined completely by the requirement of a maximum load or of a minimum coefficient of friction, for a given minimum thickness of the oil film, and for one direction only of rotation

Class B Journal Bearings: Those in which the clearance is assumed as zero, that is, journal and bearing of exactly the same radius, and the center of pressure is determined by the requirement of maximum load or of minimum coefficient of friction, for one direction of rotation and for a given minimum thickness of the oil film. This class also includes plane surfaces, considered as journal bearings of infinite radius and zero bearing angle, yet of finite arc length; but these are most readily treated as in Section II

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[1] President, Kingsbury Machine Works, Inc. Mem. A.S.M.E. Mr. Kingsbury studied mechanical engineering at the Ohio State University and Cornell University, receiving the degree of M.E. from the latter institution in 1889. He was instructor in mechanical engineering and physics at New Hampshire College during the following year, after which he became mechanical engineer for the H. B. Camp Co., Cuyahoga Falls, Ohio. In 1891 he returned to New Hampshire College as professor of mechanical engineering, remaining until 1899, when he was appointed professor of applied mechanics at Worcester Polytechnic Institute. In 1903 he joined the Westinghouse Elec. & Mfg. Co. as mechanical engineer, remaining there until 1910. Since that year he has practiced as consulting engineer and headed his present organization. He has been a frequent contributor to the transactions of engineering societies on bearings, lubrication, and other mechanical subjects.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Class C Bearings: Those in which the loading is symmetrical, that is, the center of pressure is at the center of the bearing arc, and the clearance is determined by the requirement of a maximum load or of a minimum coefficient of friction, for a given minimum thickness of the oil film; rotation to be in either direction.

The foregoing procedure appears to be the best method available at present. Exact calculation for practical bearing conditions in general has not been accomplished; certain solutions for plane bearing surfaces have been made by Michell (1),[2] by Martin (2), and by Duffing (3); and for fitted journal bearings by Boswall (4); but no mathematical results for the important case of journal bearings with running clearance have been obtained.

The latter case, along with others, is fairly well covered by the approximate experimental method of Kingsbury (5). In view of the great difficulty of mathematical solutions, the only hope of reducing theory to practice at present lies in approximate solutions. Those offered in the present discussion appear to be as nearly exact as are justifiable at present, in view of the uncertainties as to the actual film thicknesses and temperatures in practice, dependent as they are upon the circumstances of workmanship, of wear, and of distortion of the metals by heat and by pressure.

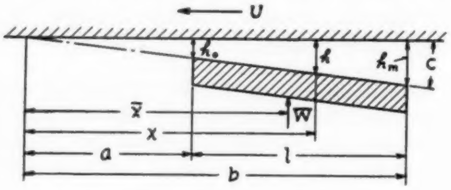

FIG 1

Mathematical analyses for the cases of the "infinitely wide" bearings are in general only qualitatively applicable in practice, although some of the results, such as the line of action of the load, the required volume of oil supply, and the nominal adiabatic heating of the oil, are directly usable for good practical forms of bearings without serious error. The main importance of such analyses is that they determine the maximum capacities theoretically attainable, and furnish the most convenient starting points and bases of reference for solutions for bearings of finite widths, whether these solutions be analytical or experimental.

Throughout this discussion the term "length" (l) is used for the dimension of the bearing surface in the direction of the motion, and the term "width" (w) at right angles to the motion; all bearing surfaces being assumed rectangular, either actually or as developed, with edges respectively parallel to the motion and at right angles thereto. The quantities calculated refer only to the active portion of the whole surfaces involved, taking no account of actions in enlarged clearance spaces and oil-supply channels; and all conditions are assumed as steady. Unless otherwise specifically stated, the film thickness is assumed to vary only in the direction of the motion, not in the transverse direction. The friction, being small compared with the pressure, is neglected as affecting the amount and direction of the load.

The oil supply is assumed sufficient to maintain the film, and any small pressure of the supply is neglected as affecting the film pressures. The viscosity of the oil in the film is taken as uniform at an average value, unless otherwise noted.

II—Theory of Plane Bearing Surfaces of Infinite Width

This case was first treated[3] by Reynolds (6). A more extended treatment is given by Boswall (4), including cases of varying viscosity. Although it does not exactly represent any case in practice, its mathematical simplicity and ease of calculation render it useful in furnishing qualitative results, applicable in kind to practical cases.

Fig. 1 shows the geometrical notation.

U is the velocity of the moving surface
c is the small angle of inclination of the planes
μ is the coefficient of viscosity, assumed constant
p is the pressure in the film at any value of x, above atmospheric pressure
h is the film thickness at any value of x
W is the resultant pressure, or load, per unit of width, and is taken normal to the moving surface
f is the shear stress in the oil at the moving surface
F is the integrated shear stress, opposed to U, per unit of width, and
λ = F/W is the coefficient of friction.

The differential equation of Reynolds (6), neglecting the transverse flow, is

$$\frac{d}{dx}\left(h^3 \frac{dp}{dx}\right) = -6\mu U \frac{dh}{dx}$$

or, since $h = cx$,

$$\frac{d}{dx}\left(x^3 \frac{dp}{dx}\right) = -\frac{6\mu U}{c^2} = -C \ldots\ldots\ldots\ldots [1]$$

By integration

$$x^3 \frac{dp}{dx} = -Cx + K_1 \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

Again integrating,

$$p = Cx^{-1} - \frac{K_1}{2} x^{-2} + K_2 \ldots\ldots\ldots\ldots\ldots [3]$$

For determining the integration constants K_1 and K_2 we have the conditions $p = 0$ when $x = a$ and when $x = b$, whence, by substituting these values in [3],

$$K_1 = 2C \frac{ab}{(a+b)} \qquad K_2 = -C\left(\frac{1}{a+b}\right)$$

From [2],

$$\frac{dp}{dx} = \frac{C}{(a+b)}\left[\frac{2ab - x(a+b)}{x^3}\right] \ldots\ldots\ldots\ldots [4]$$

Inspection of [4] shows that $dp/dx = 0$ and that the pressure is therefore a maximum where $x = x_1$ and $h = h_1 = cx_1$, and

$$x_1 = \frac{2ab}{a+b} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

Differentiating [4] and writing $d^2p/dx^2 = 0$ shows that dp/dx has a minimum value, and therefore the pressure curve has a point of inflexion, at a point where $x = \dfrac{3ab}{a+b} = \dfrac{3}{2} x_1$; and therefore, from [5], this can occur only if $b > 2a$ as shown in Section V. If h_0 and l be considered constant, the value of W [8] is a maximum when $b = 2.189\,a$, and the value of λ [13]

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[2] Numbers in parentheses refer to items in bibliography at end of paper.

[3] In Reynolds' original paper, this short section, covering less than two pages, contains ten misprints of subscripts and exponents, in addition to a real error in neglecting the part of the friction attributable to pressure. All these defects are reproduced in his "Collected Papers."

is a minimum when $b = 2.533\,a$; thus for either of these conditions the pressure curve has a point of inflexion near the leading edge, but this point would not be apparent in a graph except by large-scale plotting of exactly calculated pressures.

Substituting for K_1 and K_2 in [3],

$$p = \frac{C(b-x)(x-a)}{(a+b)x^2} \qquad [6]$$

The maximum pressure, from [5] and [6], is

$$p_1 = \frac{C(b-a)^2}{4ab(a+b)} = \frac{Cl^2}{4ab(a+b)} \qquad [7]$$

The load per unit of width is

$$W = \int_a^b p\,dx = C\left[\log_e \frac{b}{a} + 2\,\frac{(a-b)}{(a+b)}\right] \qquad [8]$$

and the line of action of W is at

$$\bar{x} = \frac{\int_a^b px\,dx}{W} = \frac{\dfrac{b-a}{2} - \dfrac{ab}{(a+b)}\log_e \dfrac{b}{a}}{\log_e \dfrac{b}{a} - 2\,\dfrac{(b-a)}{a+b}} \qquad [9]$$

The shearing stress in the oil at the moving surface is

$$f = \frac{\mu U}{h} + \frac{h}{2}\frac{dp}{dx} \qquad [10]$$

which reduces to

$$f = \frac{\mu U}{c}\left(\frac{3x_1}{x^2} - \frac{4}{x}\right) \qquad [11]$$

The total shearing resistance per unit width is

$$-F = \int_a^b f\,dx = \frac{\mu U}{c}\left[4\log_e \frac{b}{a} - 6\,\frac{(b-a)}{(a+b)}\right] \qquad [12]$$

The negative sign with F corresponds with the fact that F is opposed to U.

The coefficient of friction is

$$\lambda = \frac{F}{W} = \frac{c}{6}\left[\frac{4(a+b)\log_e \dfrac{b}{a} - 6(b-a)}{(a+b)\log_e \dfrac{b}{a} - 2(b-a)}\right] \qquad [13]$$

III—THEORY OF JOURNAL BEARINGS OF INFINITE WIDTH, WITH RUNNING CLEARANCE

Fig. 2 shows the geometrical notation. The axes of the cylindrical surfaces are assumed to be parallel, and at a distance $c\eta$. η is the difference of the radii (the radial clearance) and c is the eccentricity factor. With the exception of the angles α and β, all angles are measured from the line of centers, positive in the direction of rotation.

U is the linear velocity of the journal surface
h_m is the greatest thickness of the film
h_0 is the least thickness.

Neither h_m nor h_0 falls necessarily in all cases at the extremity of the bearing arc.

h_1 is the film thickness at the point where the pressure is a maximum
μ is the coefficient of viscosity of the lubricant, taken as constant
p_0 is the unit pressure referred to the projected area of the journal, and is equal to $W/2a$.

Reynolds' differential equations

$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right) = -6\mu U\frac{dh}{dx} \qquad [14]$$

and

$$\frac{dp}{dx} = 6\mu U\left(\frac{h-h_1}{h^3}\right) \qquad [15]$$

apply to this case, by writing $a\theta$ for x, $\eta(1 + c\cos\theta)$ for h, and $\eta(1 + c\cos\theta_1)$ for h_1. For convenience the negative sign in [14] is disregarded. [15] thus becomes

$$\frac{dp}{d\theta} = \frac{6\mu Uac\,(\cos\theta - \cos\theta_1)}{\eta^2(1 + c\cos\theta)^3} \qquad [16]$$

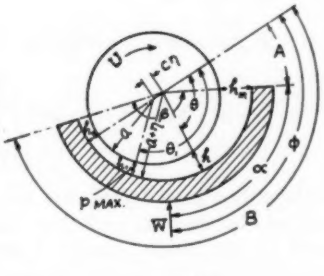

FIG 2

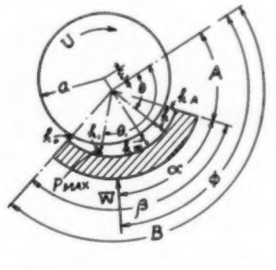

FIG. 3

The integral of [16] may have various forms; the form here employed is that of Harrison (7):

$$\begin{aligned} p = C + \frac{6\mu Ua}{(1-c^2)^2\eta^2}\Bigg[&\frac{c\sin\theta}{(1+c\cos\theta)^2}\left\{-(1-c^2)(1+c\cos\theta) + \frac{1}{2}(1+c\cos\theta_1)(1-c^2+3[1+c\cos\theta])\right\} \\ &+ (1-c^2)^{-1/2}\left\{2(1-c^2) - (1+c\cos\theta_1)(2+c^2)\right\}\tan^{-1}\left\{\sqrt{\frac{1-c}{1+c}}\tan\frac{\theta}{2}\right\}\Bigg] \qquad [17] \end{aligned}$$

C is here a constant of integration.

For further procedure, the following abbreviations for quantities in [17] are convenient:

$$\left.\begin{aligned} S &= (1-c^2) \\ M &= (1+c\cos\theta_1) \\ T &= (1+c\cos\theta) \qquad T_A = (1+c\cos A),\ \text{etc.} \\ K &= \frac{6\mu Ua}{(1-c^2)^2\eta^2} \\ D &= (1-c^2)^{-1/2}\tan^{-1}\left(\sqrt{\frac{1-c}{1+c}}\tan\frac{\theta}{2}\right) \\ D_A &= \text{the value of } D \text{ when } \theta = A,\ \text{etc.} \\ E &= \frac{c\sin\theta}{T^2}; \qquad E_A = \frac{c\sin A}{T_A{}^2},\ \text{etc.} \end{aligned}\right\} \qquad [18]$$

By the substitution of these expressions, [17] becomes

$$p = C + K\left[E\left\{-ST + \frac{M}{2}(S+3T)\right\} + D\left\{2S + M(S-3)\right\}\right] \qquad [19]$$

For finding C and θ_1, thus far unknown, we have the conditions $p = 0$ when $\theta = A$ and when $\theta = B$. The substitution of these conditions in [19] leads to

$$M = \frac{S[2(D_B - D_A) + E_A T_A - E_B T_B]}{\frac{E_A}{2}(S + 3T_A) - \frac{E_B}{2}(S + 3T_B) + (S - 3)(D_A - D_B)} \quad \ldots\ldots\ldots [20]$$

$$C = -K\left[M\left\{\frac{E_A}{2}(S + 3T_A) + D_A(S - 3)\right\} + S(2D_A - E_A T_A)\right] \ldots\ldots\ldots [21]$$

and from [18],

$$\cos\theta_1 = (M - 1)/c \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

The definite integral for the pressure at any value of θ thus becomes, from [17], [20], and [21],

$$p = K\left[M\left\{-\frac{E_A}{2}(S + 3T_A) - D_A(S - 3)\right\} + S(E_A T_A - 2D_A) + \frac{EMS}{2} + ET\left(\frac{3M}{2} - S\right) + D\left\{S(M + 2) - 3M\right\}\right] \ldots\ldots\ldots [23]$$

In this equation M, S, and the subscripted factors are constant for a given film of assumed dimensions, and thus the constant terms may be replaced by the single symbol L for convenience in numerical substitutions and for further integrations: thus

$$p = K\left[L + \frac{EMS}{2} + ET\left(\frac{3M}{2} - S\right) + D\left\{S(M + 2) - 3M\right\}\right] \ldots\ldots\ldots [24]$$

The quantity L is in fact the value of $-C/K$, C being the constant of integration in [19], [21].

For finding W, the load per unit of width, and the angle $\phi = A + \alpha$, Fig. 2, the procedure is that of resolving the pressures p into components respectively normal to the line of centers and parallel thereto, and integrating for the respective components of W. Briefly indicated,

$$\left.\begin{aligned} W_V &= a\int_{\theta = A}^{\theta = B} p\sin\theta\, d\theta \qquad W_H = a\int_{\theta = A}^{\theta = B} p\cos\theta\, d\theta \\ W &= \sqrt{W_V^2 + W_H^2} \qquad \tan\phi = \frac{W_V}{W_H} \end{aligned}\right\} [25]$$

Omitting here the somewhat extensive details of the integrations indicated in [25] and writing only the final results,

$$W_V = Ka\left[{}_{\theta = A}^{\theta = B} - L\cos\theta + \frac{MS}{2c}\left\{\frac{c\sin\theta}{1 + c\cos\theta} - \theta + \frac{2}{\sqrt{S}}\tan^{-1}\left(\sqrt{\frac{1 - c}{1 + c}}\tan\frac{\theta}{2}\right)\right\} + \left(\frac{3M}{2} - S\right)\left\{-\sin\theta + \frac{\theta}{c} - \frac{2\sqrt{S}}{c}\tan^{-1}\left(\sqrt{\frac{1 - c}{1 + c}}\tan\frac{\theta}{2}\right)\right\} + \left\{\sqrt{S}(M + 2) - \frac{3M}{\sqrt{S}}\right\}\left\{\left[-\tan^{-1}\left(\sqrt{\frac{1 - c}{1 + c}}\tan\frac{\theta}{2}\right)\right]\left(\frac{1 + c\cos\theta}{c}\right) + \frac{\theta\sqrt{S}}{2c}\right\}\right] \ldots\ldots\ldots [26]$$

$$W_H = Ka\left[{}_{\theta = A}^{\theta = B} L\sin\theta - \frac{MS}{2c}\left\{\log_e(1 + c\cos\theta) + \frac{1}{1 + c\cos\theta}\right\} + \left(\frac{3M - 2S}{2c}\right)\left\{\log_e(1 + c\cos\theta) - (1 + c\cos\theta)\right\} + \left\{\sqrt{S}(M + 2) - \frac{3M}{\sqrt{S}}\right\}\left\{\tan^{-1}\left(\sqrt{\frac{1 - c}{1 + c}}\tan\frac{\theta}{2}\right)\sin\theta + \frac{\sqrt{S}}{2c}\log_e(1 + c\cos\theta)\right\}\right] \ldots\ldots\ldots [27]$$

The algebraic value of W is not required for numerical computations, since the numerical value of W is most readily derived by [25], W_V and W_H being evaluated separately in order to find ϕ.

To find the frictional resistance per unit width at the surface of the journal, we proceed from Reynolds' equation for the shear stress in the fluid at that surface

$$f = \frac{\mu U}{h} + \frac{h}{2}\frac{dp}{dx}$$

Substituting $a\theta$ for x, and $\eta(1 + c\cos\theta)$ for h, this reduces to

$$f = \frac{\mu U}{\eta}\left[\frac{1}{1 + c\cos\theta} + 3c\frac{(\cos\theta - \cos\theta_1)}{(1 + c\cos\theta)^2}\right] \ldots\ldots [28]$$

The total friction per unit width at the surface of the journal is

$$F = a\int_{\theta = A}^{\theta = B} f d\theta$$

which upon integration becomes

$$F = \frac{\mu Ua}{\eta(1 - c^2)}\left[{}_A^B \frac{3Mc\sin\theta}{1 + c\cos\theta} + 2\left\{\frac{4(1 - c^2) - 3M}{\sqrt{1 - c^2}}\right\}\tan^{-1}\left(\sqrt{\frac{1 - c}{1 + c}}\tan\frac{\theta}{2}\right)\right] \ldots\ldots\ldots [29]$$

The coefficient of friction is

$$\lambda = F/W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [30]$$

The complete algebraic expression for λ would be complicated and would not facilitate numerical computation; F and W being computed separately, λ readily follows from their numerical values. In the numerical evaluation of the preceding equations, the quantities that determine the form of the film are assumed as known, and the following procedure avoids ambiguities and multiple values. (*a*) The radical factors are considered as positive throughout. (*b*) The angle θ is limited physically to a complete circle, that is, its whole range of values is limited to 2π, whatever its initial and final values, whether positive or negative. The whole range of $\theta/2$ is thus limited to π, and thus as θ varies throughout its range, $\tan\theta/2$ passes through all its possible values. Writing $y = \tan^{-1}\left(\sqrt{\frac{1 - c}{1 + c}}\tan\frac{\theta}{2}\right)$, the value of y is taken as that nearest in value to $\theta/2$; it equals $\theta/2$ when $\theta = 0$, π, or 2π. The film form being taken as known, there can be but one physically permissible solution for any of the equations. The solutions reached by the above considerations are checked by those obtained by the graphical methods of Howarth (13), based directly on the differential equations.

IV—Theory of Fitted (No-Clearance) Journal Bearings, Infinite Width

The geometrical notation for this case is indicated in Fig. 3. The differential Equation [15]

$$\frac{dp}{dx} = 6\mu U\left(\frac{h - h_1}{h^3}\right)$$

applies to this case also, by substituting $a d\theta$ for dx, and $\epsilon \sin\theta$ for h;

$$\frac{dp}{d\theta} = \frac{6\mu U a}{\epsilon^2}\left(\frac{\sin\theta - \sin\theta_1}{\sin^3\theta}\right) \ldots\ldots\ldots\ldots [31]$$

As in the preceding cases, h_1 and θ_1 here refer to the point where $dp/d\theta = 0$ and the pressure is a maximum.

Integration of [31] leads to

$$p = C + \frac{3\mu U a}{\epsilon^2}\left\{\sin\theta_1\left(\frac{\cot\theta}{\sin\theta} - \log_e \tan\frac{\theta}{2}\right) - 2\cot\theta\right\} \ldots [32]$$

To find the integration constant C and the unknown θ_1, we have the conditions $p = 0$ when $\theta = A$ and when $\theta = B$, leading to

$$\sin\theta_1 = \frac{2(\cot A - \cot B)}{\dfrac{\cot A}{\sin A} - \dfrac{\cot B}{\sin B} + \log_e \dfrac{\tan B/2}{\tan A/2}} \ldots\ldots [33]$$

and

$$C = -\frac{3\mu U a}{\epsilon^2}\left\{\sin\theta_1\left(\frac{\cot A}{\sin A} - \log_e \tan\frac{A}{2}\right) - 2\cot A\right\} \ldots [34]$$

For brevity we may write

$$\left.\begin{aligned} \sin\theta_1 &= M \\ \frac{3\mu U a}{\epsilon^2} &= K_1 \\ -\frac{C}{K_1} &= L \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [35]$$

These substituted in [32] give the definite integral for the pressure at any value of θ

$$p = K_1\left\{L + M\left(\frac{\cot\theta}{\sin\theta} - \log_e \tan\frac{\theta}{2}\right) - 2\cot\theta\right\} \ldots [36]$$

The load W per unit of width, and the direction angle ϕ of its resultant, are found from the integrals of the components;

$$\left.\begin{aligned} W_V &= \int_{\theta = A}^{\theta = B} p \sin\theta \, a\, d\theta \qquad W_H = \int_{\theta = A}^{\theta = B} p\cos\theta\, a\, d\theta \\ W &= \sqrt{W_V^2 + W_H^2} \qquad \tan\phi = \frac{W_V}{W_H} \end{aligned}\right\} [37]$$

Omitting the details of the integrations,

$$W_V = K_1 a\left\{L(\cos A - \cos B) + M\left(\cos B \log_e \tan\frac{B}{2} - \cos A \log_e \tan\frac{A}{2}\right) - 2(\sin B - \sin A)\right\} \ldots [38]$$

$$W_H = K_1 a\left\{L(\sin B - \sin A) + M\left(\cot A - \cot B + \sin A \log_e \tan\frac{A}{2} - \sin B \log_e \tan\frac{B}{2}\right) + 2\left(\cos A - \cos B + \log_e \tan\frac{A}{2} - \log_e \tan\frac{B}{2}\right)\right\} \ldots [39]$$

For the shear stress at the surface of the journal, the equation

$$f = \frac{\mu U}{h} + \frac{h}{2}\frac{dp}{dx}$$

becomes, by the substitution of $a\theta$ for x, and $\epsilon \sin\theta$ for h,

$$f = \frac{\mu U}{\epsilon\sin\theta} + \frac{\epsilon\sin\theta}{a} K_1 \frac{\sin\theta - \sin\theta_1}{\sin^3\theta}$$

$$= \frac{\mu U}{\epsilon\sin\theta}\left(4 - \frac{3\sin\theta_1}{\sin\theta}\right) \ldots\ldots\ldots\ldots\ldots\ldots [40]$$

The total friction per unit of width, at the journal surface, is

$$F = \int_{\theta = A}^{\theta = B} f a\, d\theta = \frac{\mu U a}{\epsilon}\int_{\theta = A}^{\theta = B}\left(\frac{4}{\sin\theta} - \frac{3\sin\theta_1}{\sin^2\theta}\right) d\theta \ldots [41]$$

and by integration

$$F = \frac{\mu U a}{\epsilon}\left[4\left(\log_e \tan\frac{B}{2} - \log_e \tan\frac{A}{2}\right) + 3\sin\theta_1(\cot B - \cot A)\right] \ldots\ldots [42]$$

The coefficient of journal friction then is

$$\lambda = \frac{F}{W} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [43]$$

V—Maximum Load Capacity and Minimum Coefficient of Friction for a Given Minimum Film Thickness

The natures of these relationships are typified in Fig. 4, which is calculated for the hypothetical case treated in Section II, for planes of infinite width and with constant viscosity. Assuming the length l as constant, h_0 as fixed, and h_m increasing from an initial value = h_0, the load per unit of width W reaches a maximum when h_m = 2.189 h_0; the friction per unit width, F, decreases continuously; and the coefficient of friction reaches a minimum when h_m = 2.533 h_0. Reynolds (6) showed the existence of such a maximum for W in his treatment of plane surfaces; and Boswall (4) has further shown the minima for λ with constant viscosity and with several assumed laws of variation of viscosity.

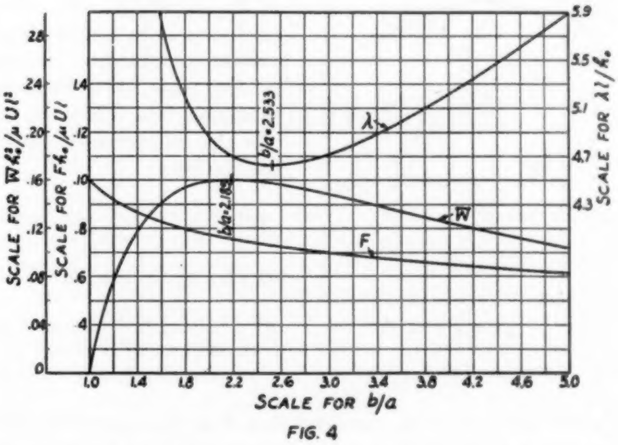

FIG. 4

It has been found that closely similar relationships exist in journal bearings, but the corresponding numerical calculations present far greater difficulty, as may be inferred from the complexity of the formulas relating to journal bearings. However, the calculations have been made for a number of cases, and the results are shown in the accompanying tables and graphs. The graphs relate only to the conditions of minimum coefficients of friction; these conditions differ but slightly from those for maximum loads.

The conditions for the minimum coefficient of friction, with a fixed minimum film thickness, are quite distinct from the analogous conditions based on a fixed running clearance in the hypothetical 360-deg. bearing, as treated by Sommerfeld (8) and by Harrison (7). A still different minimum is that obtained on a test of a partial bearing by increasing load, or by reducing speed, or by reducing the oil supply; a minimum found in this way depends largely on the accuracy of form and of finish of the bearing surfaces, and occurs when the film thickness is reduced to nearly zero at some point or points, and further reduction of speed or increase of load causes contact of the metals.

For plane surfaces, the maximum-load conditions and the minimum-friction conditions are readily obtained by numerical substitutions in Equations [8] and [13], varying the ratio b/a.

For clearance journal bearings, the general procedure for finding the conditions for the maximum load capacity is as follows: Referring to Fig. 2 as typical, a bearing arc β is chosen; then with a fixed value of $c = c_1$, Equations [26] and [27] are evaluated for various values of A and the dependent values of $B = A + \beta$, the factors μ, U, a, and η being constant. Three or more such calculations yield a curve 1, Fig. 5, which is found to be nearly a parabola with its axis vertical. To find the maximum W_1, a parabola is calculated, passing through the three points nearest the apparent maximum, and the exact vertex of this parabola is then calculated as giving the values of W_1 and A_1 corresponding to the particular value of $c = c_1$. The same procedure with other values of c yields values of W_2, A_2, W_3, A_3, etc. The next step is to reduce these several intermediate values to the basis of a common minimum film thickness, h_0. The general expression for W has the form

$$W = \frac{\mu U a^2}{\eta^2(1 - c^2)^2} f(c, A, B)$$

which may be written

$$W = \frac{\mu U a^2}{\eta^2(1 - c)^2(1 + c)^2} f(c, A, B) \quad \ldots. [44]$$

in which $\eta(1 - c) = h_0$, provided that B is not appreciably less than 180 deg., which is found to be generally true for optimum film conditions in bearings with running clearance. Since h_0 is to be the same for all values of c, it follows that η must vary inversely as $(1 - c)$. Thus for reducing the values of W_1, W_2, etc. to the same h_0 as that of W_3, the products $W_1 \left(\frac{1 - c_1}{1 - c_3}\right)^2$, $W_2 \left(\frac{1 - c_2}{1 - c_3}\right)^2$, etc. furnish ordinates for the curve 4; this curve, treated by the parabolic method, gives W_4, the maximum load capacity for the given bearing arc β and a given h_0; also the value of A_4 corresponding thereto. The axis of the parabola 4 intersects the curve $W_1W_2W_3$ at a point corresponding to c_4, the required eccentricity ratio; the value of c_4 is obtained by interpolation. The values of $W_4 \frac{h_0^2}{\mu U a^2}$, A_4, and c_4, together with other non-dimensional quantities consistent therewith, derived from the formulas in sections III and IV, are shown in plates I, IV, and VII.

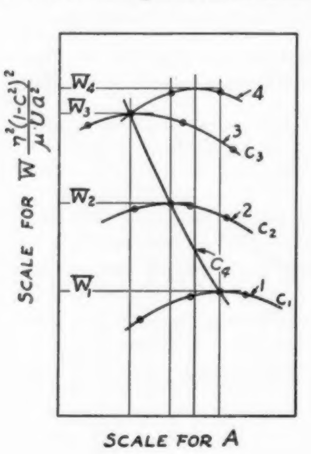

FIG. 5

For the condition of a minimum coefficient of friction, the procedure is analogous to that just described. The results of the calculations are shown in plates II, III, V, VI, VIII, and IX. These conditions differ but little from those for maximum load.

A comparison of the infinitely wide bearings of three chosen cases, for various angles subtended by the bearing arc, is shown in Fig. 6. It is seen that class A is superior to classes B and C for the same N and h_0, except that it is practically the same as class B up to about 85 deg. Under these optimum conditions for infinitely wide bearings, plots of the pressure curves show that with atmospheric pressure at the ends of the bearing arc, no subatmospheric pressures occur in any case, even when there are divergent films of large arc length. But experimental integrations (5), as well as numerous direct tests of bearings, show that when there is side leakage, as in all practical cases, the divergent films, if of sufficient length, cause negative pressures. Examples of this are shown in Figs. 7, 8, and 9, in which the curves show the average pressures for the respective widths, derived by the electrical method (5); w being the axial width, and l the arc length $a\beta$.

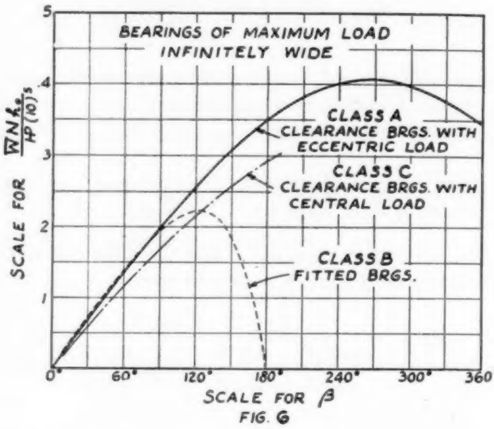

FIG. 6

These average pressures include the effects of the theoretical negative pressures, which may be considerable in amount; yet in practice these pressures are uncertain, and may not go below absolute zero, owing to the lack of tensile strength of the lubricant. In the present discussion, however, the theoretical load factors, Fig. 10 and plate X, are derived as if the theoretical negative pressures actually existed; hence some small uncertainties occur with large arcs and small widths.

In all that precedes Figs. 7, 8, and 9, the formulas and the calculated constants are based upon ideal conditions, including constant viscosity and the absence of side leakage. Under practical conditions, the pressures are in all cases materially less than those that have been derived; and all other quantities are changed, although to a less extent than the pressures, in general. Up to this time no exact general method of mathematical treatment of practical journal-bearing films has been developed; and in view of the complexity of the subject there is no prospect of such a method. It is evident that the derivation of constants and of simple rules for practical use can be accomplished only through approximate methods, with due regard to the relative importance of the factors involved, and necessarily with disregard of factors that may be considered as of secondary influence. The remainder of this paper is devoted to the derivation of approximation methods, with the minimum film thickness as the primary critical factor; with the allowable pressure per unit of projected area of the journal as a secondary critical factor; and with special reference to the conditions for minimum power loss by friction.

VI—Effects of Side Leakage on Pressure

Fig. 10 shows the results of calculations that have been made to date, reduced to a common basis, namely, the ratio L_1 of the mean load per unit width to the load that would be carried by the same film if there were no side leakage, with varying ratios of length to width; all with the assumption of uniform viscosity.

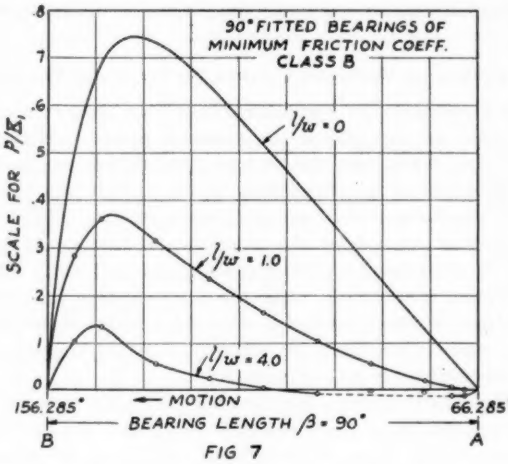

FIG 7

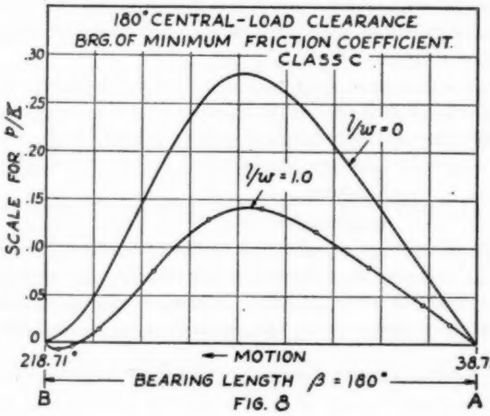

FIG. 8

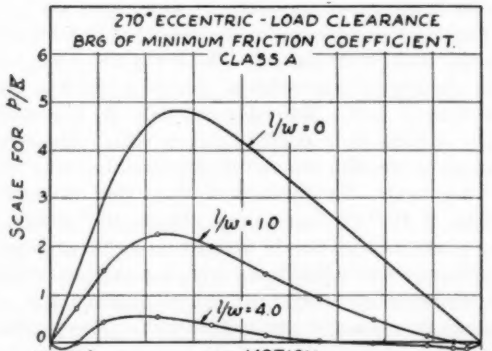

FIG. 9

The film forms for all the cases shown are either those giving the minimum coefficients of friction or not far therefrom, under the assumption of infinite width. The plot shows an average curve; in what follows, this curve, shown for greater convenience in Plate X, is used for approximate correction for the values of W in bearing films under nearly optimum conditions for load or for friction. An application of this kind has been made by Karelitz (9) on the basis of Michell's (1) and of Martin's (2) calculations for plane surfaces.

VII—Effect of Side Leakage on Friction

In the general differential equation for the shear stress at the moving surface

$$f = \frac{\mu U}{h} + \frac{h}{2}\frac{dp}{dx}$$

the first term on the right corresponds to a state of uniform shear across h, without reference to the flow corresponding to the pressure gradient, and is independent of the width of the bearing. The second term, being dependent on the pressure gradient, thus depends on the effect of side leakage upon the pressure and therefore upon the width of the bearing.

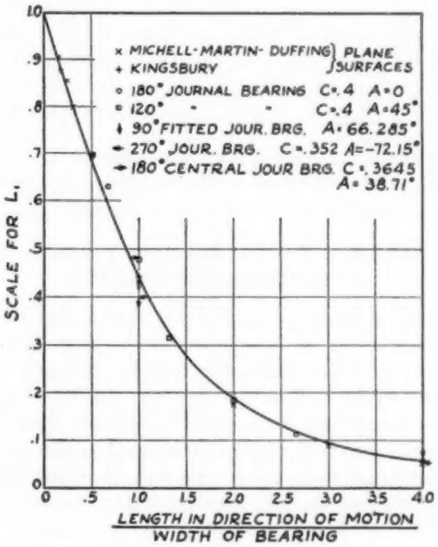

FIG. 10

The integrals for the separate terms on the right have been calculated for the film forms for minimum coefficient of friction, for the ideal bearings of infinite width. There is no great variation in the proportions of the total friction per unit of width due to shear and to pressure, respectively, for the several bearing arcs of infinite width.

Examination of cases of finite widths that have been integrated experimentally in detail (5) (180-deg. and 120-deg. bearings of finite widths) shows that the part of the friction due to pressure per unit of width is closely proportional to the pressure total per unit of width, whatever the width may be. Thus taking an average value for the friction due to simple shear as 84 per cent of the friction in the infinitely wide bearings, and the ratios L_1 in plate X as applying to the part of the friction due to pressure, plate X shows also the factors F_1 for reducing the friction of the infinitely wide bearing to apply to cases of finite length/-width ratios. For finite widths, the correction factor L_1 is hereafter used with the W of the "infinitely wide" theory, and the factor F_1 with the F of the theory.

The ratios L_1 and F_1 are further combined in plate X, giving a factor L_1/F_1, which is convenient for use in correcting for the coefficient of friction and quantities dependent thereon. Thus in what follows, plate X is used throughout as giving the approximate correction factors for W, F, and λ, for all bearings of Classes A, B, and C.

In the dynamical tables and curves, plates I to IX, inclusive, the numerical coefficients are for the infinitely wide condition. Thus for correction for bearings of finite width and given l/w ratio, these coefficients are to be multiplied by the proper factor from plate X, as below:

Tabular Dynamical Quantity	Multiplying Factor for Coefficient
W	L_1
F	F_1
$\lambda\left(\text{or } \frac{F}{W}\right)$	$\frac{F_1}{L_1}$
HP	F_1
$\frac{HP}{W}$	$\frac{F_1}{L_1}$
$\frac{W}{HP}$	$\frac{L_1}{F_1}$
$\lambda \Big/ \sqrt{\frac{\mu N}{p_0}}$	$\frac{F_1}{\sqrt{L_1}}$

With these operations, the quantities resulting from the tabular formulas all relate to actual loads, frictions, etc., including the "pressure per unit of projected area of journal," p_0.

VIII—Effect of Side Leakage and Viscosity Change on the Center of Pressure

Various calculations (1, 4, 5) have shown that one effect of increasing the ratio of length to width, thus increasing the relative amount of side leakage, is to shift the center of pressure toward the trailing end of the bearing surface. On the other hand, both theory and experiment (10) show that there is an increase in temperature of the oil in passing through the film, with a consequent reduction of viscosity; and one effect of this viscosity change is to shift the center of pressure toward the leading end. An exact account of these opposing influences would be difficult, if not impossible, and in any event would introduce complications. Experiments with pivoted-block thrust bearings show that the friction is not greatly affected by shifting the center of pressure by moderate amounts; they even operate well if the center of pressure coincides with the center of figure, which is theoretically impossible on the assumptions that the viscosity is constant and that the surfaces are plane.

For the foregoing reasons it is assumed that the optimum positions of the center of pressure, calculated as for infinite width and for constant viscosity, may be taken as fair averages for the ordinary practical conditions of finite width and varying viscosity.

IX—Effect of Side Leakage and Viscosity Change on the Optimum Ratios of h_m/h_0

The following results of calculations for the hypothetical rectangular plane surfaces may serve to show the relatively small effects:

$\frac{\text{Length}}{\text{Width}}$	Viscosity change	h_m/h_0 for $W_{\text{max.}}$	h_m/h_0 for $\lambda_{\text{min.}}$
0	0	2.189	2.533
0	2:1	2.22	...
1	2:1	2.325	...
2	2:1	2.335	...

In the first two of these cases, the results were obtained through exact integrations; in the third and fourth the integrations were effected by the electrical method (5).

These figures indicate that both the side leakage and the viscosity change require that for a maximum W the ratio h_m/h_0 should be somewhat greater than that calculated for infinite width and constant viscosity; and the same appears to be true as regards the minimum λ. But for ordinary bearing proportions, the required increase in h_m/h_0 is not great; and taking the view that similar conditions exist in journal bearings, it is assumed that the calculated values of h_m/h_0 for minimum coefficients of friction are actually very close to optimum conditions.

X—Effect of Viscosity Change on Load and Friction

It has been shown by Boswall (4) that for a moderate change in viscosity, for example, if the viscosity at the outlet end is one-half that at the inlet, the load and the friction may be fairly well approximated by calculating as for a constant viscosity taken as the mean of the inlet and outlet values. This conclusion has been verified by the experimental method of Kingsbury (5) in the case of a plane surface of square form. Thus in what follows, a numerical value for the coefficient of viscosity implies that an average value is assumed. Exact uniformity in practical cases is inconceivable, although it might be approached in water-cooled bearings, for example.

For the reasons given above (sections VI-IX), it is assumed that the geometrical quantities given in plates I–IX, defining the forms of the films for maximum loads and for minimum coefficients of friction, for the infinitely wide bearings, may also be used without great error for bearings of ordinary finite widths. These quantities are non-dimensional, and may be used with any system of units.

The dynamical groups in plates I to IX, inclusive, are also non-dimensional, but the units employed in deriving the tabulated constants are the English units—pound, inch, second—with N = revolutions per minute and HP = frictional horsepower. The ratio $\frac{\text{load}}{\text{horsepower}}$ is the same for the whole bearing as for unit of width. Application of the dynamical constants in plates I–IX requires that the geometric relations for eccentricity, clearance, and center of pressure be complied with, since these conditions form the basis for the dynamical tables and graphs.

XI—Experimental Check on the Preceding Theory

An approximate check of the theory, as regards the 360-deg. bearing, is furnished by the author's tests (11) of an air-lubricated journal 6 in. in diameter and 6.25 in. wide, load, 50.5 lb. In this case, the negative pressures in the upper 180 deg. are roughly similar to the positive pressures in the lower 180 deg.; and for this reason the l/w ratio is taken as that of a 180-deg. bearing, that is $3\pi/6.25 = 1.51$. Referring to plate X, this ratio determines $F_1 = 0.88$, $L_1 = 0.273$, $L_1/F_1 = 0.31$. The temperature of the air in the film was not determined, but was approximately 20 deg. cent.; the viscosity of air at this temperature is taken as 2.62×10^{-9} (in-lb-sec.), or 1.81×10^{-4} poises. The differences of the measured radii were by no means uniform; the average difference was 0.00079 in., with a maximum of 0.00163 in. and a minimum of 0.00062 in.; for the middle 5 in. of the width, the mean clearance was 0.000694 in. The journal was "out of round" by 0.00025 in., and the bearing by about 0.00012 in. By assuming the effective clearance as $\eta = 0.000705$ in., the experimental load and friction conform exactly to the theoretical values at 247 r.p.m.; but the experimental eccentricity ϵ at this speed was 0.0003 in. ($c = 0.426$), while the theory requires $\epsilon = 0.000258$ ($c = 0.366$). This discrepancy is 6 per cent of the nominal clearance, or 0.000042 in. Thus in the above respects the experiment agrees reasonably with the theory; but owing to the elasticity of air, the pressure distribution and the several critical angles differ considerably from those indicated by theory. These latter points have been discussed by Harrison (7).

The experiment illustrates the difficulties of determining effective film thicknesses in experimental bearings; and where heat and pressure affect the surface forms, the film dimensions are even less closely determinable than in the experiment described, in which these effects were negligible. With non-uniform clearance, both η and h_0 of the formulas must be regarded as nominal or effective values only, for purposes of calculation.

XII—Optimum Bearing Proportions

Plates XI, XII, and XIII are plots showing the values of the non-dimensional group WNh_0/HP as affected by side leakage, with the film forms for the minimum coefficient of friction, for bearings of the three principal classes.

The derivation of the curves is as follows: The curves marked $w = \infty$ are plotted directly from the constants for WNh_0/HP for the several values of β, plates III, VI, IX. The curves marked with finite values of w/a are derived from the curves $w/a = \infty$ by multiplying their ordinates by the proper values of L_1/F_1, plate X, for the various ratios of $\beta a/w$, that is, l/w. In each case these curves for finite values of w/a are tangent to that for $w = \infty$, at the origin. For a given w/a, the group WNh_0/HP has a maximum value at a certain value of β, which is therefore considered as an optimum value of β for the given w/a; and the curve H connecting the several maximum points, comprises all such optimum values of β for values w/a. An additional curve J is drawn, connecting the optimum values of β from the curve H with the corresponding values of w/a, with a linear scale for the latter at the right. If fixed values for N and h_0 be assumed, plates XI, XII, and XIII furnish direct measures of the relative merits of different proportions for bearings. While W and HP by themselves relate to unit width of the bearing, their ratio is the same as for the whole load and the whole power loss. To attain the desirable end of making this ratio high, with given values of N, h_0 must be made small, while fulfilling the geometrical conditions, as well as providing the required viscosity. The number and variety of the circumstances that affect the permissible values of h_0 render it impracticable to derive a general rule for it from theory alone.

If, however, h_0 be supposed to vary with N, β, w, or a, or all of these, according to some more or less arbitrary law, corresponding curves may be drawn, which will indicate optimum forms differing from those shown. But since h_0 ultimately remains to be fixed by experience and judgment, as well as mechanical skill, the simplest method at present is to use plates XI, XII, and XIII with chosen values of N and h_0.

XIII—Examples

Example 1. Required, a bearing of Class A, of minimum friction, to support a load of 25,000 lb. at 1500 r.p.m., with $h_0 = 0.003$ in., and with 200 lb. per sq. in. on the projected area of the journal. This requires a projected area of 125 sq. in.

Referring to plate XI, assume for this example that $w/a = 3$. The projected area being $2a^2(w/a) = 125$, it follows that $a = \sqrt{(125/6)} = 4.564$ in. This radius may or may not be sufficient for strength in bending or in torsion, or both; but it will be assumed that it is sufficient. The least power loss is incurred when $\beta = 153.6$ deg. (2.68 rad.), for which the factor (L. H. Scale) is 16,900; hence the ratio of load to power loss being the same for the whole width as for unit width,

$$\text{Power loss} = \frac{25{,}000 \times 1500 \times 0.003}{16{,}900} = 6.66 \text{ hp.}$$

As previously noted, this does not include losses in the clearance spaces outside of the arc β.

The clearance η (plate II, curve 7) for $\beta = 153.6$ deg. is $\eta = 1.72\, h_0 = 0.00516$ in.

The bore of the bearing is therefore larger than the 9.128 in. diameter of the journal by $2 \times 0.00516 = 0.01032$ in., or 0.00113 per inch of diameter.

The arc length of the bearing surface is

$$l = 2.68 \times 4.564 = 12.23 \text{ in.}$$

The width is

$$w = 3.00 \times 4.564 = 13.69 \text{ in.}$$

The ratio l/w is therefore 0.893. By reference to plate X, this determines the several correction factors,

$$L_1 = 0.487 \qquad F_1 = 0.916 \qquad L_1/F_1 = 0.532$$

The required mean viscosity in the oil film may now be determined from plate III, curve 10. For the infinitely wide bearing and $\beta = 153.6$ deg., curve 10 gives the factor 0.117. The actual mean load per unit of width is

$$\frac{25{,}000}{13.69} = 1826 \text{ lb.} = L_1W$$

hence

$$W = \frac{1826}{0.487} = 3749 \text{ lb.}$$

and therefore (10)

$$\mu = \frac{3749 \times 0.003^2}{0.117 \times 1500 \times (4.564)^3} = 2.022 \times 10^{-6} \text{ (in-lb-sec.)}$$

With the conversion factor 69,000, this viscosity is 0.1395 poise, which is that of a light machinery oil at about 130 to 140 deg. fahr. (12). The coefficient of friction (plate III, 12) is

$$\lambda = \frac{F_1}{L_1} \times 2.00 \times \frac{h_0}{a} = \frac{2.00 \times 0.003}{0.532 \times 4.564} = 0.00247$$

Inspection of plate XI indicates that by increasing the ratio w/a, other things being equal, the relative power loss will be reduced; this is shown more specifically in the following example. The extent to which w/a may be increased must be limited eventually by considerations of space, cost, and the strength or elasticity of the journal.

In the present example, the angle β might be increased, securing thereby a somewhat greater bearing area; but this would involve a relatively greater friction loss, as is obvious on inspection of plate XI.

Example 2. In Example 1, suppose that w/a is taken as 4 instead of 3, the other data remaining unchanged. We then have, referring to the same plates,

$$a = \sqrt{\frac{125}{8}} = 3.953 \text{ in.}$$

$\beta = 180.8$ deg. $= 3.155$ radians, for the least power loss.

$$\text{Power loss} = \frac{25{,}000 \times 1500 \times 0.003}{20{,}700} = 5.435 \text{ hp.}$$

Clearance $\eta = 1.622\, h_0 = 0.004866$ in.
$= 0.00123$ in. per inch of diameter.
Length $l = 3.155 \times 3.953 = 12.47$ in.
Width $w = 4 \times 3.953 = 15.81$ in.
Ratio $l/w = 0.789$

This determines

$$L_1 = 0.533 \qquad F_1 = 0.924 \qquad L_1/F_1 = 0.577$$

$$\frac{25{,}000}{15.8} = 1581 \text{ lb.} = L_1 W \qquad \therefore\ W = 2967 \text{ lb.}$$

$$\mu = \frac{2967 \times 0.003^2}{0.1517 \times 1500 \times (3.953)^3} = 1.90 \times 10^{-6} \text{ (in-lb-sec.)}$$
$$= 0.131 \text{ poise}$$

The coefficient of friction is:

$$\lambda = \frac{F_1}{L_1} \times 1.762 \times \frac{h_0}{a} = \frac{1.762 \times 0.003}{0.577 \times 3.953} = 0.002318$$

The procedure for examples under classes B and C is similar, having reference to the appropriate plates corresponding to those referred to in Example 1.

Conclusion

In the preparation of the material for this paper, the large amount of necessary detail work has been carried out by Mr. S. J. Needs.[4] The analytical work was for the most part performed independently by Mr. Needs and by the author, and identical results were obtained. Mr. Needs alone carried through the numerical evaluations leading to the tabulated results; the experimental integrations by the electrical method (5); and the preparation of the tables and graphs.

The numerical evaluations for the infinitely wide bearings were in all cases carried to at least six figures; the tabulated results are given to the nearest unit in the fourth place, which is believed to be amply accurate for any present or future purposes. For bearings of finite width, all results are based on the approximations indicated in plate X. Owing to the lack of complete and precise test data, the degrees of approximation cannot be estimated closely, but the present indications are that the method does not involve errors of serious practical importance.

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(Discussion follows on p. 146)

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[4] Research Engineer, Kingsbury Machine Works, Inc., Philadelphia, Pa. Jun. A.S.M.E.

CLEARANCE BEARINGS OF INFINITE WIDTH, ECCENTRIC LOAD AND UNIFORM VISCOSITY.

CONDITIONS FOR MAXIMUM LOAD ($\overline{W}$) PER UNIT WIDTH FOR A GIVEN MINIMUM FILM THICKNESS (h_o)

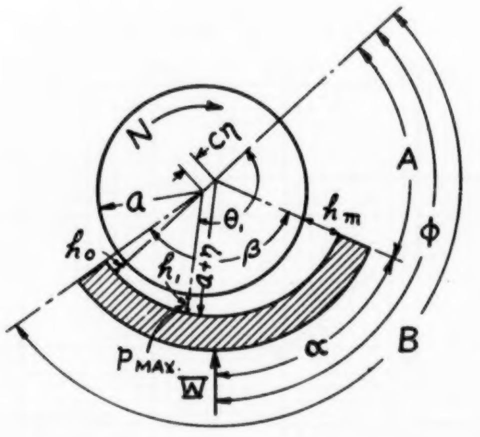


TABLE I - GEOMETRICAL

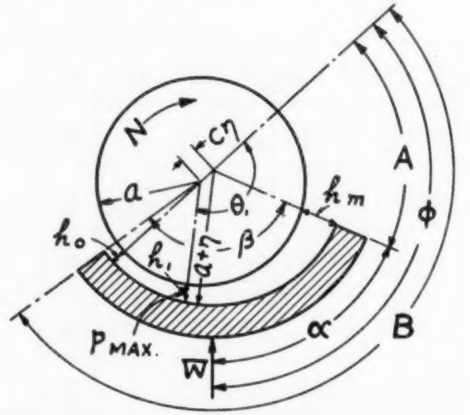
	β	60°	120°	180°	270°	360°
1	A	122.3°	66.33°	12.17°	-62.48°	-118.6°
2	α	32.94°	66.49°	101.3°	156.6°	208.6°
3	α/β	.5490	.5541	.5627	.5799	.5796
4	C	.7481	.4821	.3655	.3310	.3379
5	h_m/h_o	2.382	2.305	2.139	1.990	2.021
6	h_1/h_o	1.200	1.209	1.226	1.257	1.266
7	η/h_o	3.970	1.931	1.576	1.495	1.510
8	θ_1	158.8°	140.8°	127.4°	118.8°	118.6°
9	ϕ	155.3°	132.8	113.5°	94.09°	90.00°

TABLE II - DYNAMICAL

	β	60°	120°	180°	270°	360°
10	$\overline{W}\frac{h_o^2}{\mu N a^3}$	.0201	.0759	.1543	.2635	.2939
11	$F\frac{h_o}{\mu N a^2}$	.0936	.1863	.2768	.4080	.5379
12	$\lambda\frac{a}{h_o}$	4.660	2.453	1.794	1.549	1.830
13	$\lambda\frac{\ell}{h_o}$	4.880	5.138	5.637	7.297	11.50
14	$HP\frac{h_o(10)^6}{\mu N^2 a^3}$	1.485	2.956	4.392	6.473	8.534
15	$HP\frac{(10)^5}{\overline{W}Nh_o}$	7.394	3.893	2.847	2.457	2.904
16	$\frac{\overline{W}Nh_o}{HP(10)^5}$	.1352	.2569	.3513	.4070	.3444
17	$\lambda/\sqrt{\frac{\mu N}{P_o}}$	.4671	.4780	.4983	.5620	.7016

PLATE I CLASS A

CLEARANCE BEARINGS OF INFINITE WIDTH, ECCENTRIC LOAD AND UNIFORM VISCOSITY.



CONDITIONS FOR MINIMUM COEFFICIENT OF FRICTION FOR A GIVEN MINIMUM FILM THICKNESS(h_o)

TABLE III - GEOMETRICAL

	β	60°	120°	180°	270°	360°
1	A	117.9°	58.04°	1.119°	-72.15°	-121.0°
2	α	33.89°	68.54°	104.7°	161.4°	211.0°
3	α/β	.5649	.5711	.5815	.5977	.5860
4	C	.7496	.4911	.3844	.3520	.3660
5	h_m/h_o	2.586	2.476	2.249	2.086	2.155
6	h_1/h_o	1.248	1.263	1.283	1.292	1.280
7	η/h_o	3.986	1.965	1.624	1.543	1.577
8	θ_1	156.4°	136.7°	123.1°	117.5°	121.0°
9	ϕ	151.8°	126.6°	105.8°	89.24°	90.0°

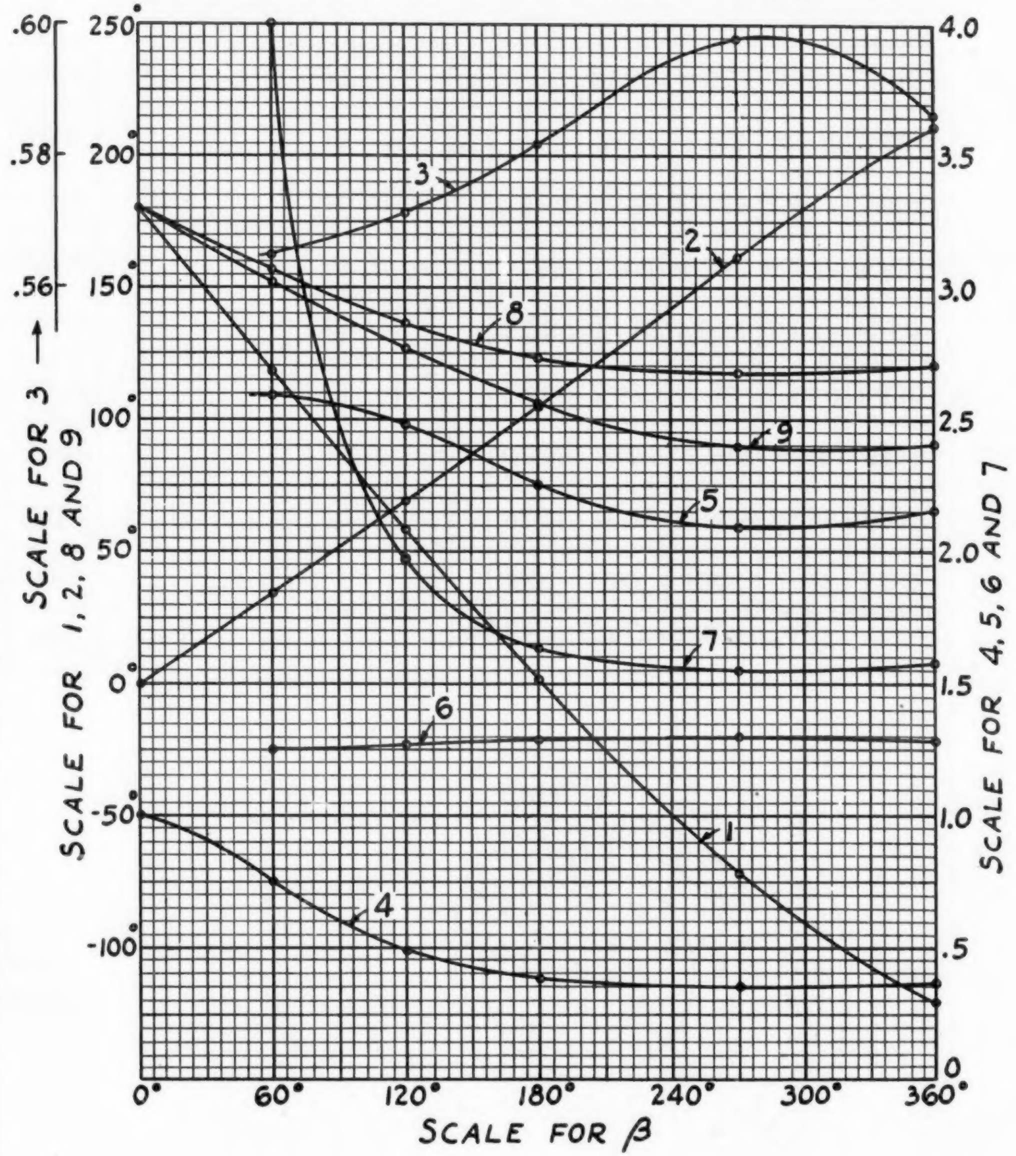
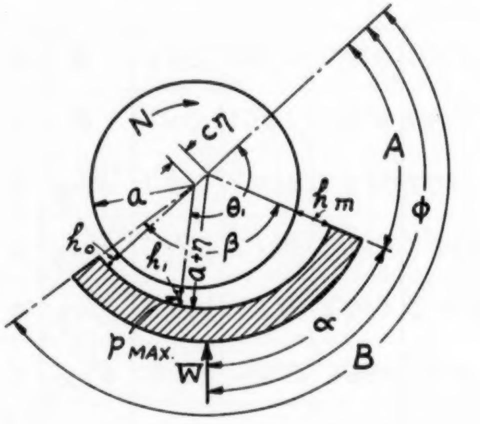


PLATE II CLASS A

CLEARANCE BEARINGS OF INFINITE WIDTH, ECCENTRIC LOAD AND UNIFORM VISCOSITY.



CONDITIONS FOR MINIMUM
COEFFICIENT OF FRICTION FOR A
GIVEN MINIMUM FILM THICKNESS(h_o)

TABLE IV-DYNAMICAL

	β	60°	120°	180°	270°	360°
10	$\overline{W}\frac{h_o^2}{\mu N a^3}$	.0197	.0742	.1507	.2602	.2925
11	$F\frac{h_o}{\mu N a^2}$	.0902	.1790	.2658	.3985	.5327
12	$\lambda\frac{a}{h_o}$	4.584	2.413	1.763	1.531	1.821
13	$\lambda\frac{\ell}{h_o}$	4.801	5.054	5.540	7.217	11.44
14	$HP\frac{h_o(10)^6}{\mu N^2 a^3}$	1.432	2.840	4.217	6.323	8.452
15	$HP\frac{(10)^5}{\overline{W}Nh_o}$	7.274	3.829	2.798	2.430	2.890
16	$\frac{\overline{W}Nh_o}{HP(10)^5}$	.1375	.2612	.3574	.4115	.3460
17	$\lambda/\sqrt{\frac{\mu N}{p_o}}$	.4548	.4647	.4841	.5524	.6965

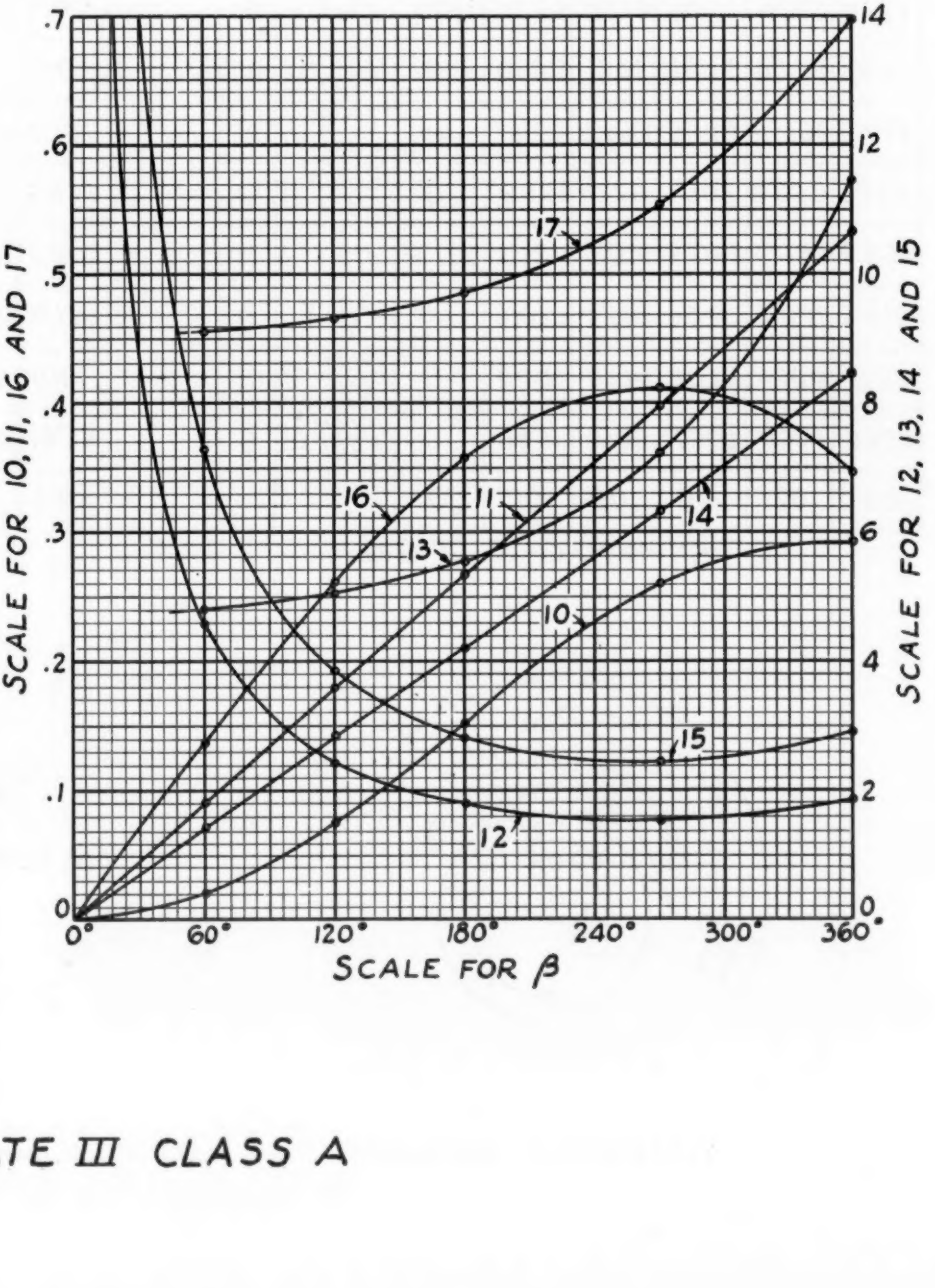


PLATE III CLASS A

FITTED BEARINGS OF INFINITE WIDTH AND UNIFORM VISCOSITY.

CONDITIONS FOR MAXIMUM LOAD($\overline{W}$)
PER UNIT WIDTH FOR A GIVEN
MINIMUM FILM THICKNESS (h_o)

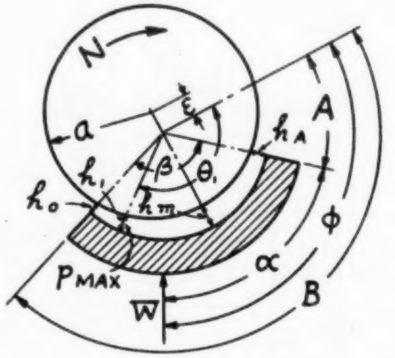


TABLE IX-GEOMETRICAL

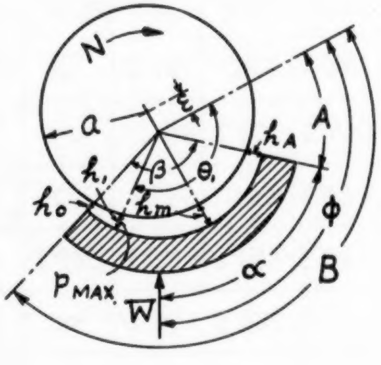
	β	0°	30°	60°	90°	120°	150°
1	A	180°	129.1°	92.21°	64.30°	40.67°	19.39°
2	α	0°	17.47°	35.79°	55.72°	77.76°	101.1°
3	α/β	.5774	.5822	.5964	.6191	.6480	.6739
4	h_A/h_o	2.189	2.176	2.143	2.078	1.969	1.804
5	h_m/h_o	2.189	2.176	2.143	2.306	3.021	5.433
6	h_1/h_o	1.373	1.398	1.480	1.624	1.843	2.131
7	θ_1	180°	150.1°	136.4°	135.2°	142.4°	156.9°
8	ϕ	180°	146.6°	128.0°	120.0°	118.4°	120.5°

TABLE X-DYNAMICAL

	β	0°	30°	60°	90°	120°	150°
9	$\overline{W}\frac{10h_o^2}{\mu N a^3}$	0	.0451	.1685	.3309	.4534	.3924
10	$F\frac{h_o}{\mu N a^2}$	0	.0407	.0774	.1073	.1284	.1339
11	$\lambda\frac{a}{h_o}$	∞	9.027	4.597	3.242	2.831	3.413
12	$\lambda\frac{\ell}{h_o}$	4.706	4.727	4.814	5.093	5.930	8.935
13	$HP\frac{h_o(10)^6}{\mu N^2 a^3}$	0	.6455	1.229	1.702	2.037	2.125
14	$HP\frac{(10)^5}{\overline{W}Nh_o}$	∞	14.32	7.293	5.144	4.492	5.415
15	$\frac{\overline{W}Nh_o}{HP(10)^5}$	0	.0698	.1371	.1944	.2226	.1847
16	$\lambda/\sqrt{\frac{\mu N}{P_o}}$	.4311	.4285	.4219	.4171	.4263	.4781

PLATE IV CLASS B

FITTED BEARINGS OF INFINITE WIDTH AND UNIFORM VISCOSITY



CONDITIONS FOR MINIMUM COEFFICIENT OF FRICTION FOR A GIVEN MINIMUM FILM THICKNESS (h_o)

TABLE XI - GEOMETRICAL

	β	0°	30°	60°	90°	120°	150°
1	A	180°	133.0°	95.59°	66.29°	41.73°	20.08°
2	α	0°	17.84°	36.26°	55.95°	77.45°	99.41°
3	α/β	.5914	.5946	.6043	.6217	.6455	.6627
4	h_A/h_o	2.533	2.501	2.408	2.276	2.122	1.993
5	h_m/h_o	2.533	2.501	2.408	2.486	3.189	5.804
6	h_1/h_o	1.434	1.457	1.528	1.662	1.873	2.165
7	θ_1	180°	154.8°	140.8°	138.1°	144.0°	158.1°
8	ϕ	180°	150.8°	131.9°	122.2°	119.2°	119.5°

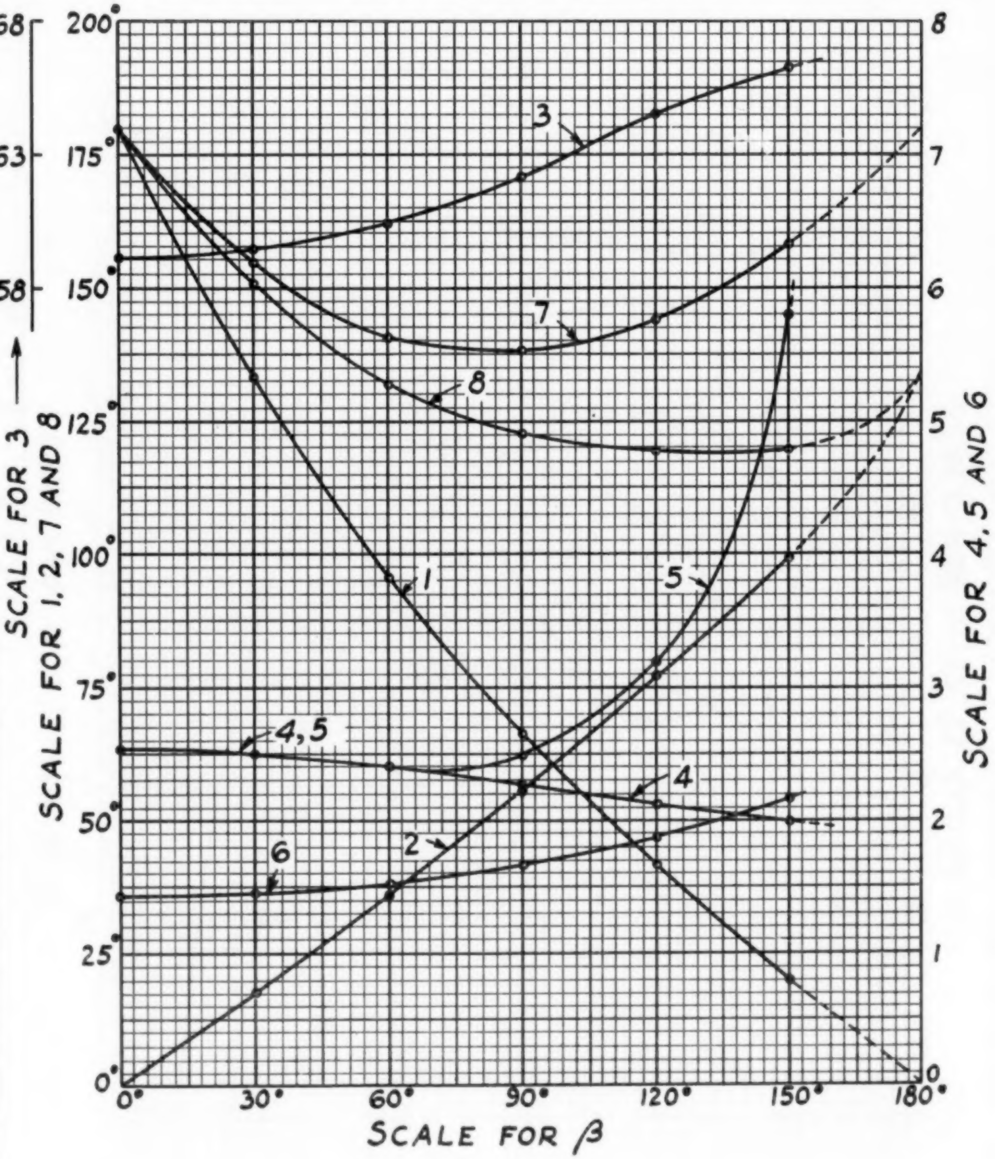
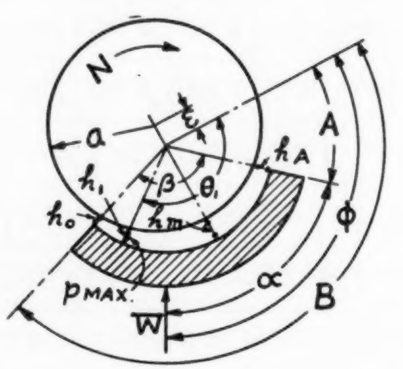


PLATE V CLASS B

FITTED BEARINGS OF INFINITE WIDTH AND UNIFORM VISCOSITY



CONDITIONS FOR MINIMUM COEFFICIENT OF FRICTION FOR A GIVEN MINIMUM FILM THICKNESS(h_o)

TABLE XII-DYNAMICAL

	β	0°	30°	60°	90°	120°	150°
9	$\overline{W}\frac{10h_o^2}{\mu N a^3}$	0	.0443	.1663	.3281	.4504	.3876
10	$F\frac{h_o}{\mu N a^2}$	0	.0393	.0755	.1054	.1267	.1307
11	$\lambda\frac{a}{h_o}$	∞	8.877	4.538	3.214	2.813	3.372
12	$\lambda\frac{l}{h_o}$	4.622	4.648	4.752	5.049	5.892	8.829
13	$HP\frac{h_o(10)^6}{\mu N^2 a^3}$	0	.6238	1.197	1.673	2.010	2.074
14	$HP\frac{(10)^5}{\overline{W}Nh_o}$	∞	14.08	7.201	5.100	4.463	5.351
15	$\frac{\overline{W}Nh_o}{HP(10)^5}$	0	.0710	.1389	.1961	.2240	.1869
16	$\lambda/\sqrt{\frac{\mu N}{p_o}}$	.4194	.4177	.4138	.4117	.4221	.4695

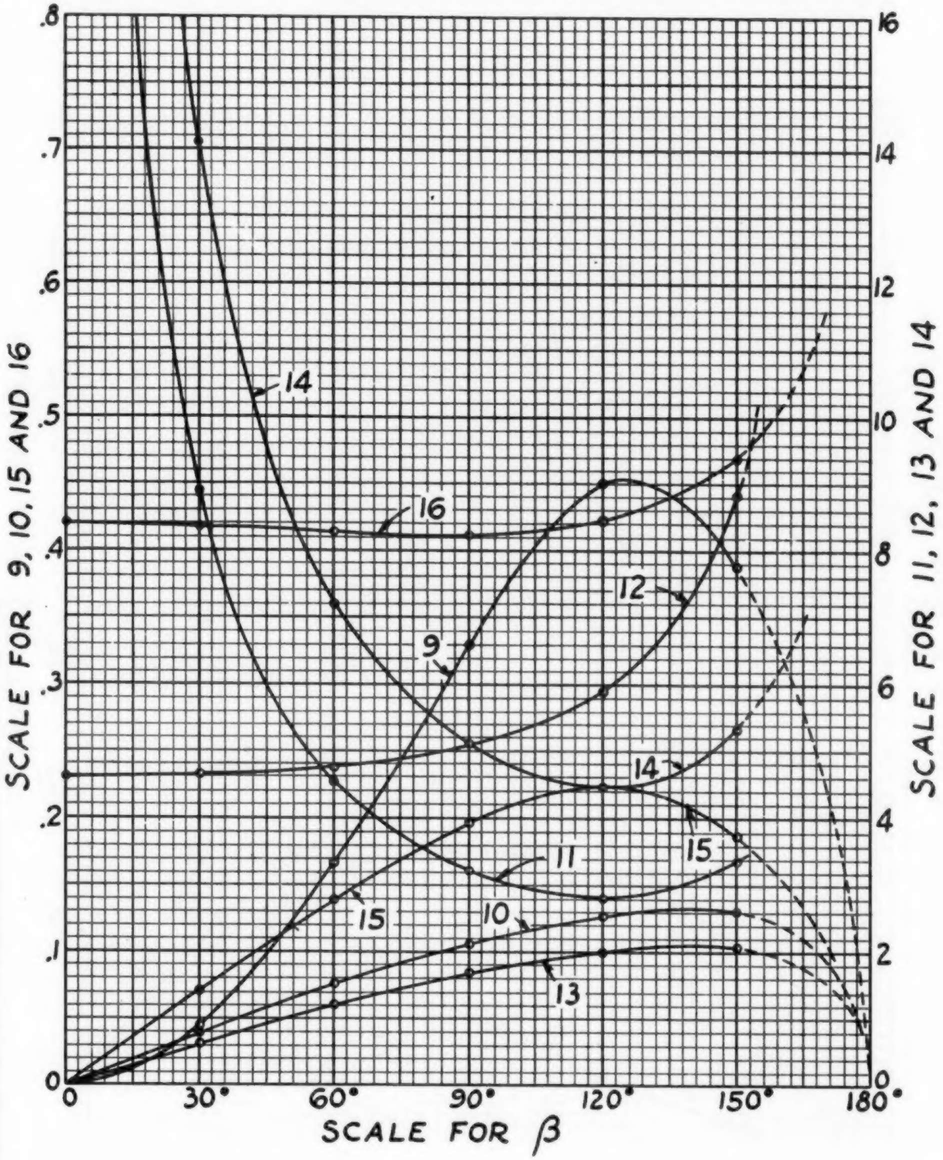


PLATE VI CLASS B

CLEARANCE BEARINGS OF INFINITE WIDTH, CENTRAL LOAD AND UNIFORM VISCOSITY.

CONDITIONS FOR MAXIMUM LOAD ($\overline{W}$) PER UNIT WIDTH FOR A GIVEN MINIMUM FILM THICKNESS (h_o)

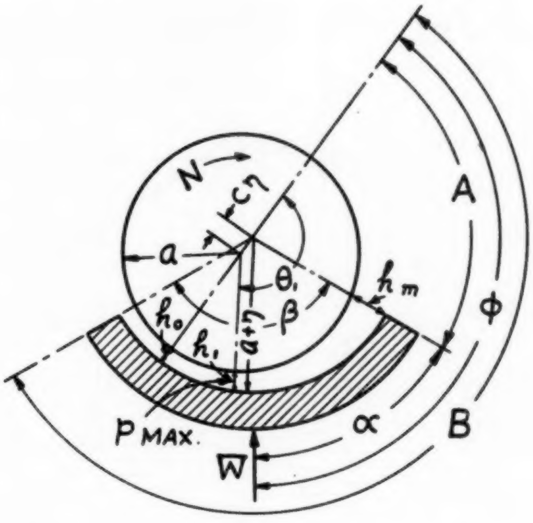


TABLE V - GEOMETRICAL

	β	60°	120°	180°
1	A	131.2°	83.66°	37.83°
2	α	30°	60°	90°
3	α/β	.50	.50	.50
4	C	.7466	.4688	.3437
5	h_m/h_o	2.004	1.980	1.937
6	h_1/h_o	1.137	1.144	1.158
7	η/h_o	3.946	1.883	1.524
8	θ_1	162.5°	146.8°	134.3°
9	ϕ	161.2°	143.7°	127.8°

TABLE VI - DYNAMICAL

	β	60°	120°	180°
10	$\overline{W}\frac{h_o^2}{\mu N a^3}$	.0180	.0673	.1352
11	$F\frac{h_o}{\mu N a^2}$	.0982	.1954	.2902
12	$\lambda\frac{a}{h_o}$	5.464	2.906	2.147
13	$\lambda\frac{\ell}{h_o}$	5.722	6.087	6.744
14	$HP\frac{h_o(10)^6}{\mu N^2 a^3}$	1.559	3.101	4.605
15	$HP\frac{(10)^5}{\overline{W}Nh_o}$	8.670	4.611	3.406
16	$\frac{\overline{W}Nh_o}{HP(10)^5}$	.1153	.2169	.2936
17	$\lambda/\sqrt{\frac{\mu N}{p_o}}$	.5181	.5329	.5581

PLATE VII CLASS C

CLEARANCE BEARINGS OF INFINITE WIDTH, CENTRAL LOAD AND UNIFORM VISCOSITY.

CONDITIONS FOR MINIMUM COEFFICIENT OF FRICTION FOR A GIVEN MINIMUM FILM THICKNESS (h_0)

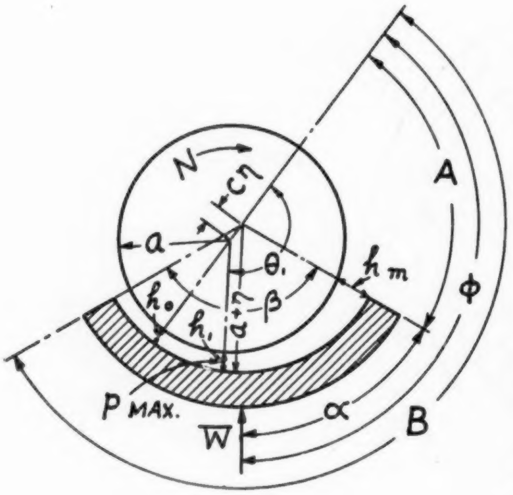


TABLE VII – GEOMETRICAL

	β	60°	120°	180°
1	A	131.6°	84.39°	38.71°
2	α	30°	60°	90°
3	α/β	.50	.50	.50
4	c	.7615	.4904	.3645
5	h_m/h_0	2.072	2.056	2.021
6	h_1/h_0	1.140	1.148	1.163
7	η/h_0	4.193	1.962	1.574
8	θ_1	163.0°	147.8°	135.7°
9	ϕ	161.6°	144.4°	128.7°

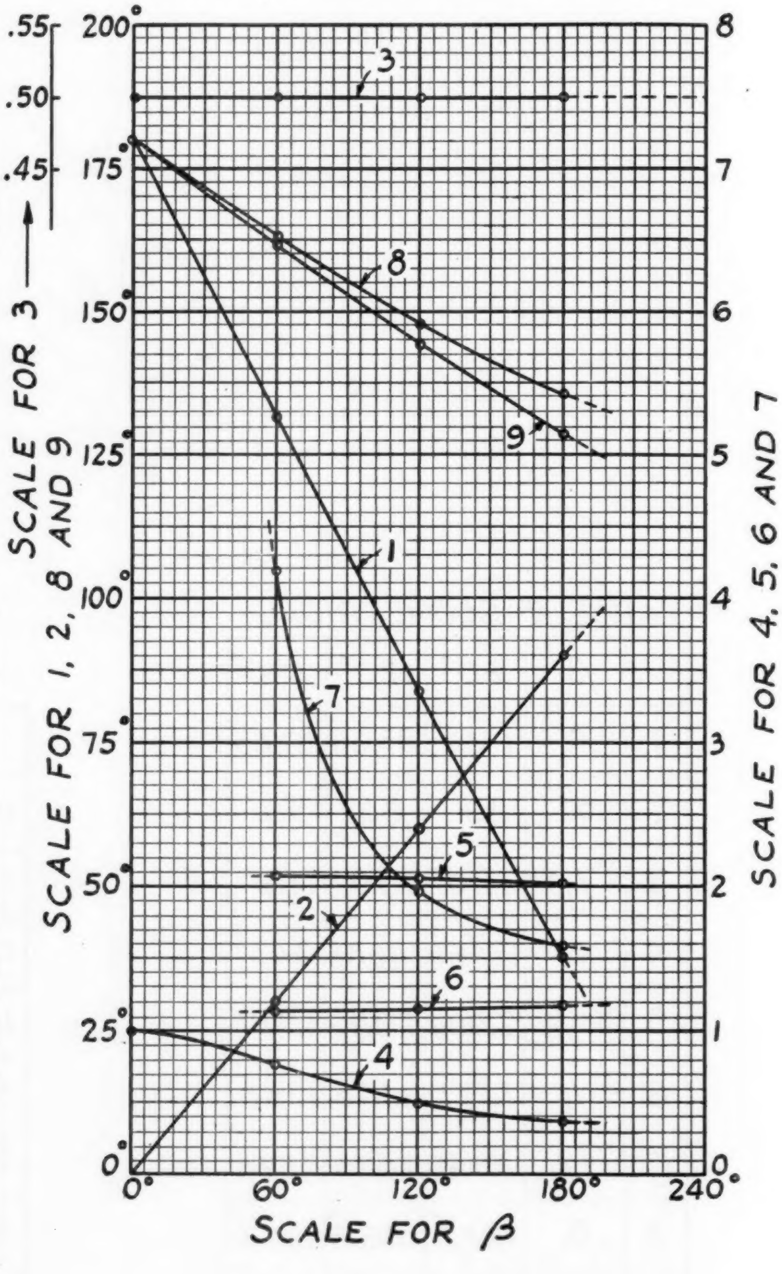
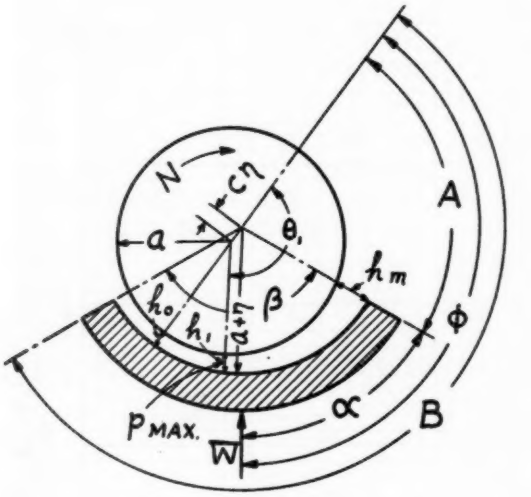


PLATE VIII CLASS C

CLEARANCE BEARINGS OF INFINITE WIDTH, CENTRAL LOAD AND UNIFORM VISCOSITY.

CONDITIONS FOR MINIMUM COEFFICIENT OF FRICTION FOR A GIVEN MINIMUM FILM THICKNESS (h_o)



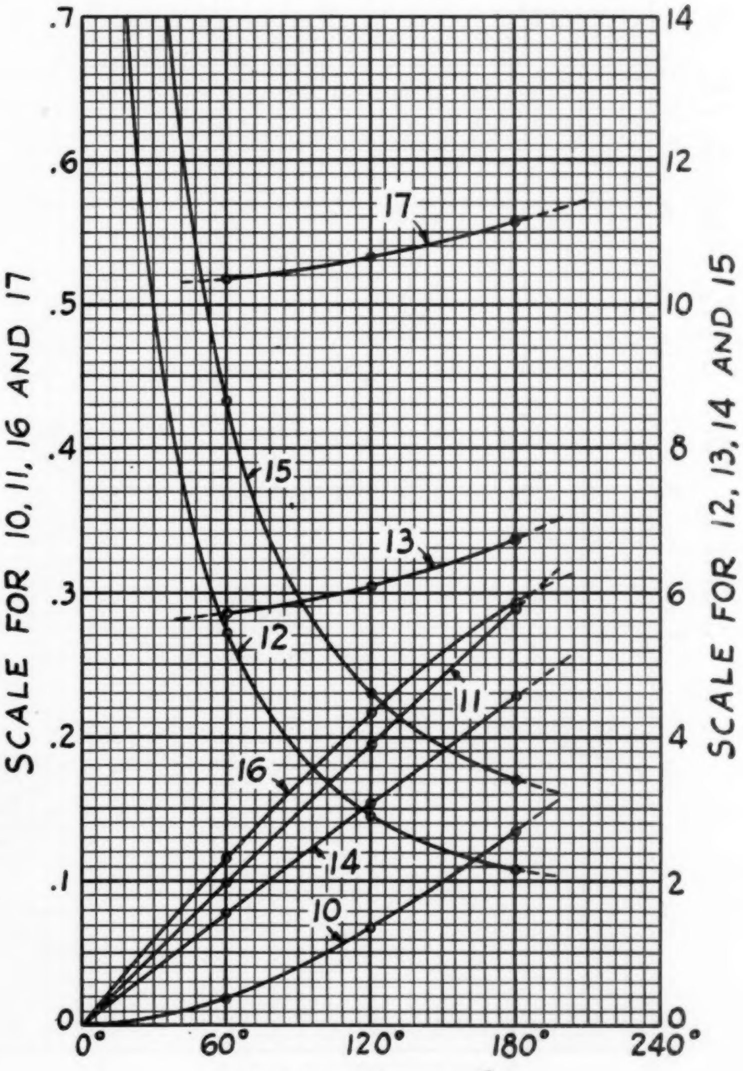


TABLE VIII—DYNAMICAL

	β	60°	120°	180°
10	$\overline{W}\frac{h_o^2}{\mu N a^3}$	.0180	.0671	.1349
11	$F\frac{h_o}{\mu N a^2}$	.0980	.1948	.2890
12	$\lambda\frac{a}{h_o}$	5.458	2.902	2.142
13	$\lambda\frac{l}{h_o}$	5.716	6.078	6.731
14	$HP\frac{h_o(10)^6}{\mu N^2 a^3}$	1.555	3.091	4.586
15	$HP\frac{(10)^5}{\overline{W}N h_o}$	8.660	4.604	3.399
16	$\frac{\overline{W}N h_o}{HP(10)^5}$	.1155	.2172	.2942
17	$\lambda/\sqrt{\frac{\mu N}{p_o}}$	.5171	.5317	.5564

PLATE IX CLASS C

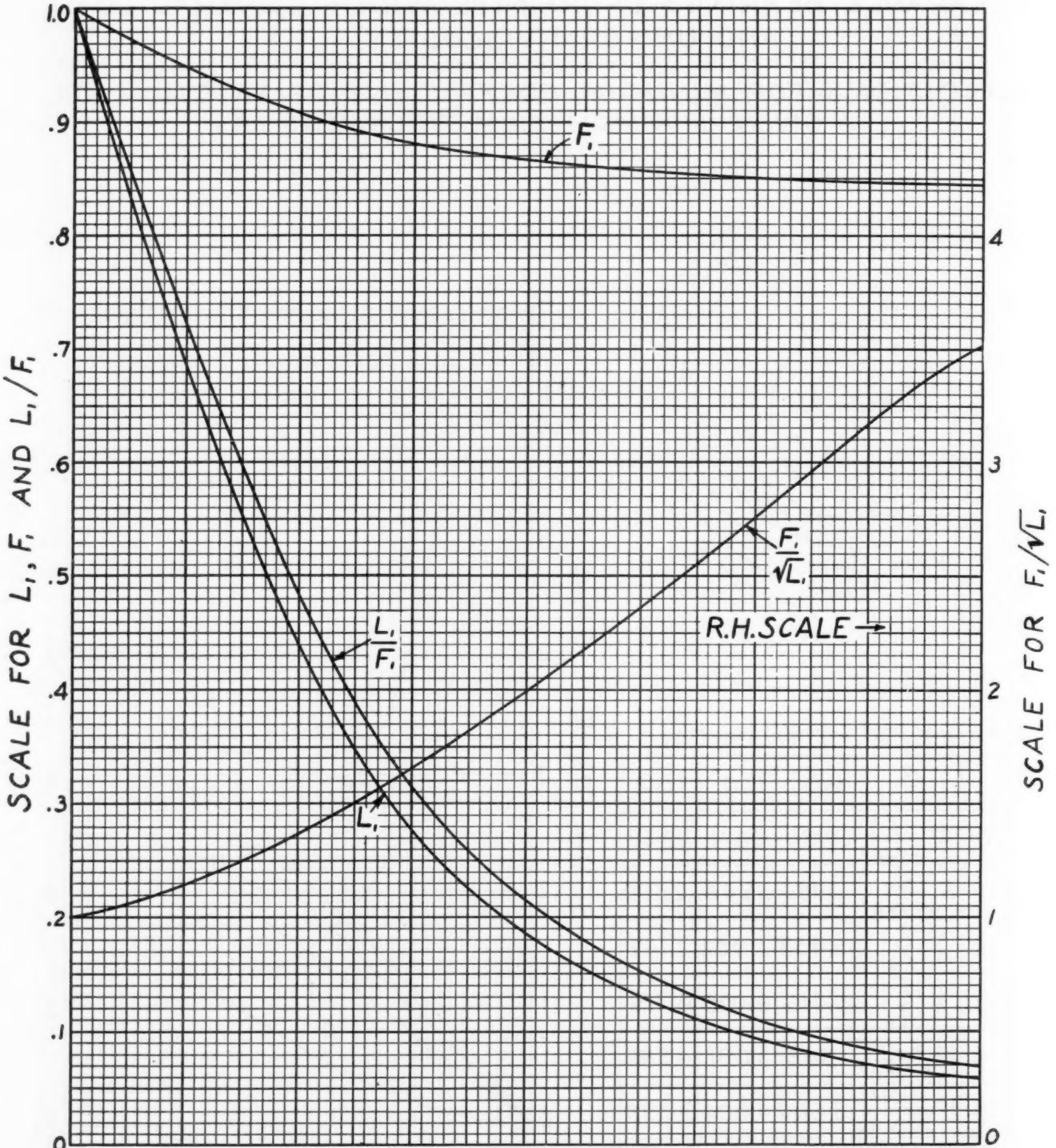

1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
SCALE FOR L_1, F_1 AND L_1/F_1
F_1
$\frac{L_1}{F_1}$
L_1
$\frac{F_1}{\sqrt{L_1}}$
R.H. SCALE →
4
3
2
1
0
SCALE FOR $F_1/\sqrt{L_1}$
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
LENGTH IN DIRECTION OF MOTION / WIDTH OF BEARING

PLATE X

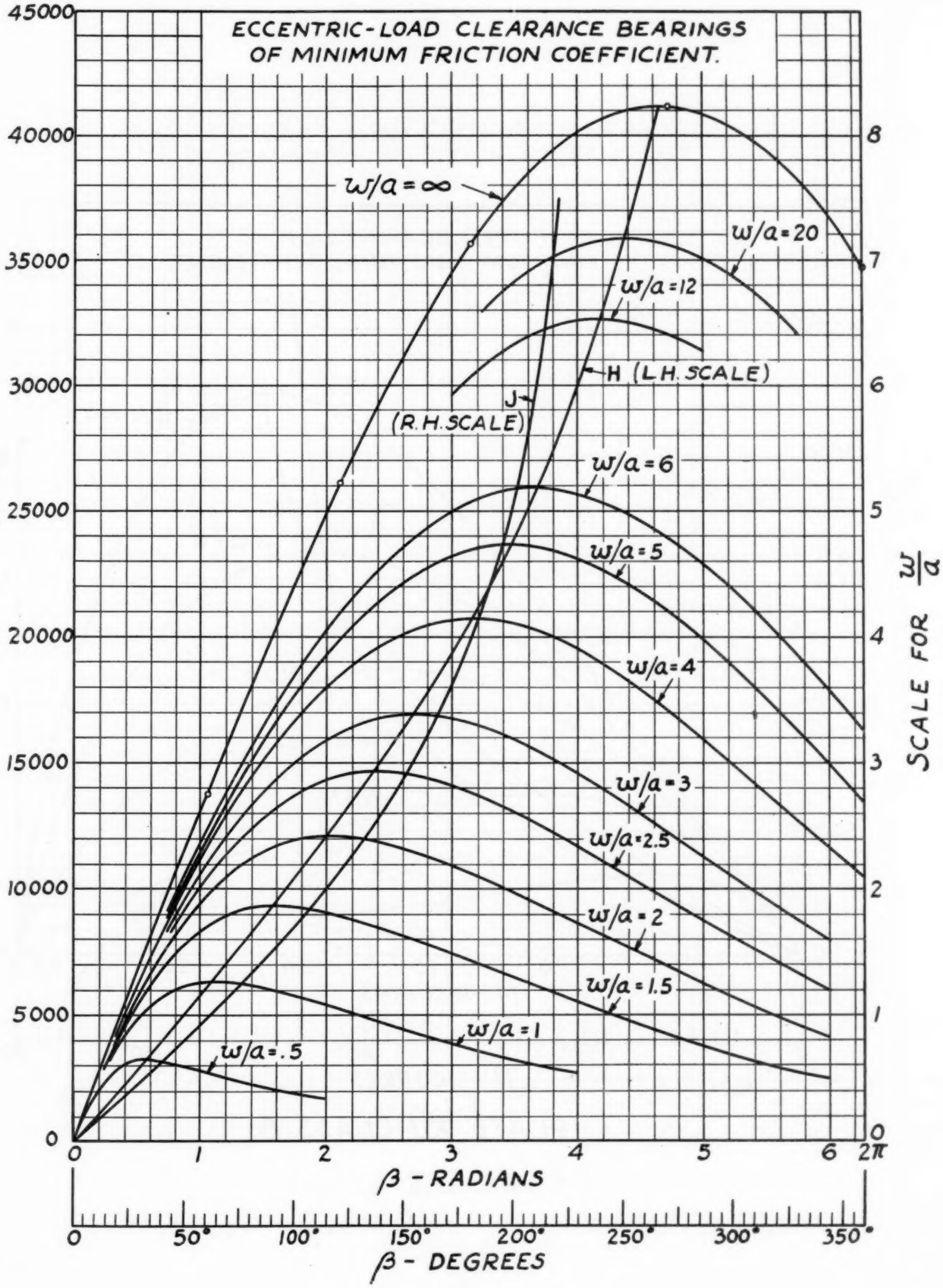


PLATE XI CLASS A

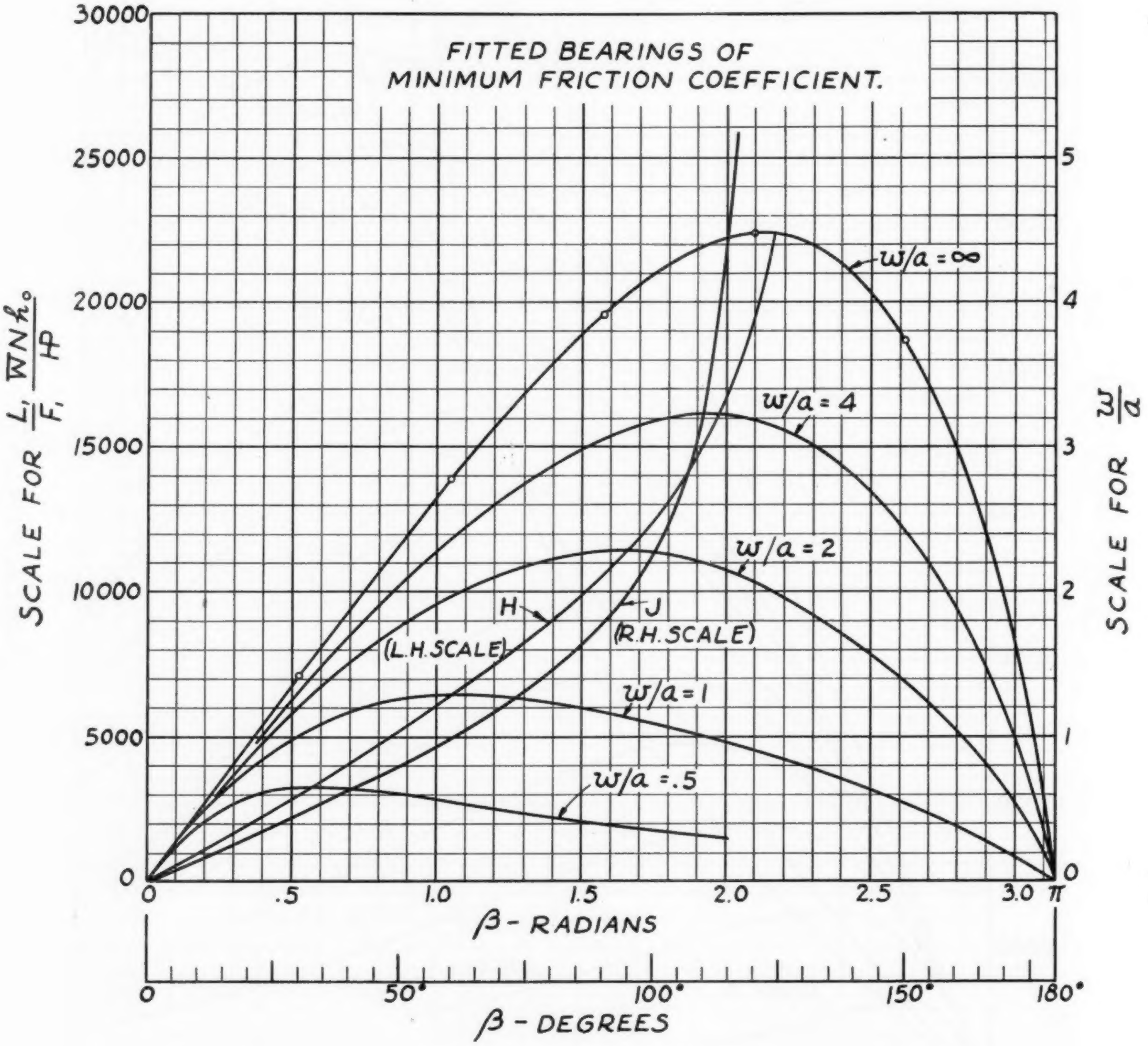


PLATE XII CLASS B

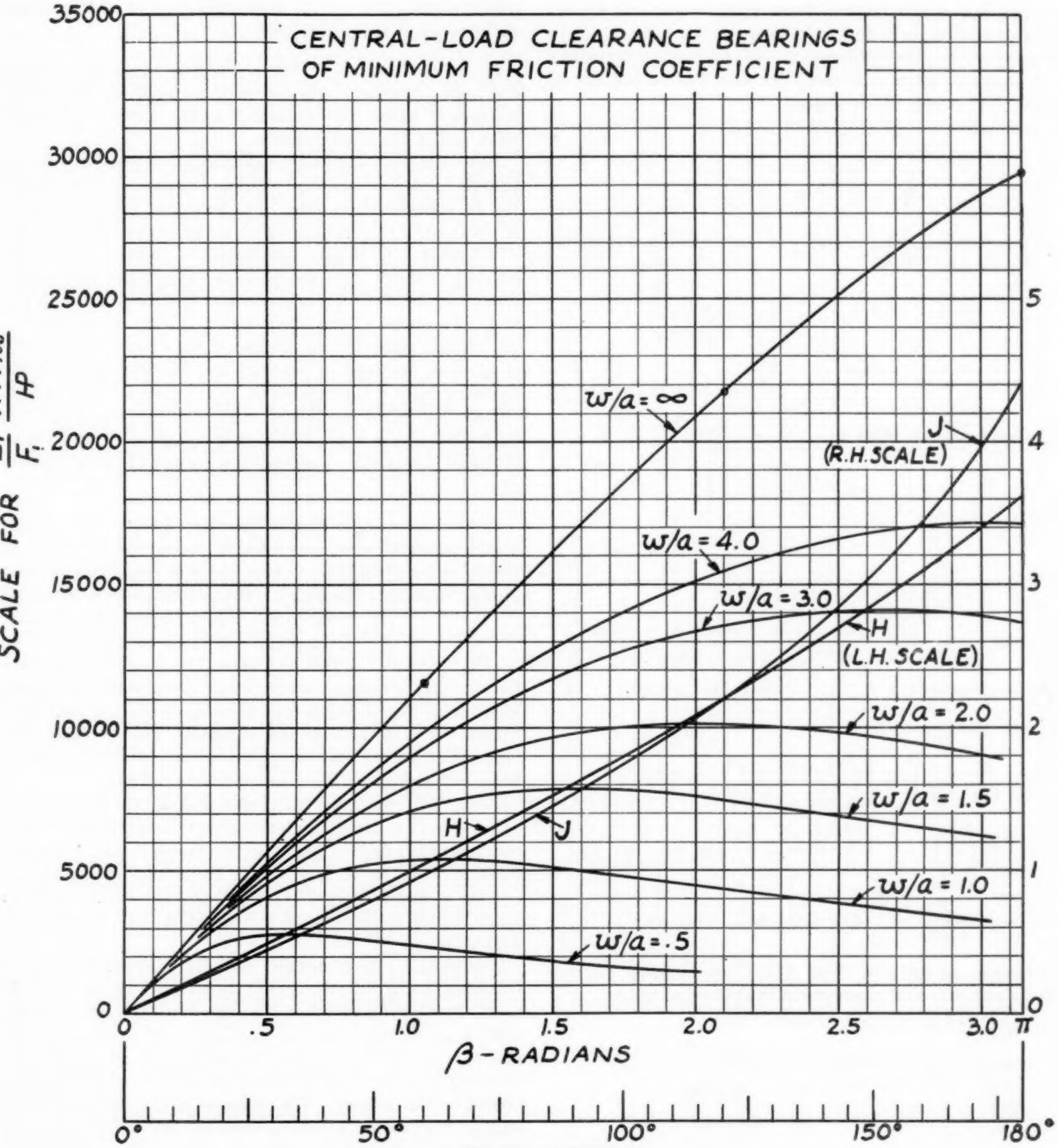


PLATE XIII CLASS C

Discussion

Georg Duffing.[5] The author's work will be a valuable help to the engineer in designing power plants. These well-arranged tables give indeed a clear view of the whole matter.

Although the results presented are widely supported by experimental research and the experience of the author, many engineers may nevertheless desire to perform the mathematical work themselves, which can easily be done by making use of Equations [17] et seq. of the paper.

The calculation will be simplified by making the substitution[6]

$$\sqrt{\frac{1-c}{1+c}}\tan\frac{\theta}{2} = \tan\frac{\eta}{2}$$

where η is a new variable (not the η of [16], which is equivalent to the writer's Δ). Then

$$\tan\theta = \sqrt{1-c^2}\,\frac{\sin\eta}{\cos\eta - c};\quad \cos\theta = \frac{\cos\eta - c}{1 - c\cos\eta};$$

$$\sin\theta = \sqrt{1-c^2}\,\frac{\sin\eta}{1-c\cos\eta};\qquad 1 + c\cos\eta = \frac{1-c^2}{1-c\cos\eta}$$

and [17] will be transformed into

$$(1-c^2)^{3/2}\frac{p}{6\mu U}\frac{\Delta^2}{a} = \left[1 - \lambda\frac{2+c^2}{1-c^2}\right]\eta - c\left[1 - \frac{4\lambda}{1-c^2}\right]\sin\eta - \frac{\lambda}{2}\frac{c^2}{1-c^2}\sin 2\eta + K$$

where

$$\lambda = (1-c^2)\,\frac{\eta_1 - \eta_2 - c(\sin\eta_1 - \sin\eta_2)}{(2+c^2)(\eta_1-\eta_2) - 4c(\sin\eta_1 - \sin\eta_2) + \frac{1}{2}c^2(\sin 2\eta_1 - \sin 2\eta_2)} = \frac{1}{2}\frac{h_1}{\Delta}$$

η_1 = value for $\theta = B$; η_2 = value for $\theta = A$.

It follows, then, that

$$(1-c^2)^{3/2}\frac{W_v}{6\mu U}\left(\frac{\Delta}{a}\right)^2 = \left[\left(\frac{1-c^2}{c} - \frac{2\lambda}{c}\right)\eta + 2\lambda\sin\eta\right]_{\eta=\eta_2}^{\eta=\eta_1}$$

$$\frac{(1-c^2)^2}{c}\frac{W_H}{6\mu U}\left(\frac{\Delta}{a}\right)^2 = \left[-\left(\frac{1-c^2}{c} - \frac{2\lambda}{c}\right)\cos\eta - \frac{\lambda}{2}\cos 2\eta\right]_{\eta_2}^{\eta_1}$$

$$(1-c^2)^{3/2}\frac{F\Delta}{\mu U a} = \left[(4 - 4c^2 - 6\lambda)\eta + 6\lambda c\sin\eta\right]_{\eta_2}^{\eta_1}$$

For example, referring to Plate VII, Class C (page 17), for $\beta = 180°$ and $c = 0.3437$, there result the following values:

Given, $\theta_2 = 37°\,50'$, $\theta_1 = 217°\,50'$

then $\eta_2 = 26°\,56'$, $\eta_1 = 232°\,14'$, $\lambda = 0.3800$

$K = -0.1330$

$h_0 = 0.6563\,\Delta$

Further,

$$\frac{W_v}{6\mu U}\left(\frac{\Delta}{a}\right)^2 = 0.3936;\quad \frac{W_H}{6\mu U}\left(\frac{\Delta}{a}\right)^2 = 0.3009$$

$$\frac{W}{6\mu U}\left(\frac{\Delta}{a}\right)^2 = 0.4954;\quad -\tan\phi = 1.3080;\quad \phi = 127°\,24'$$

$$\frac{F}{\mu U}\frac{\Delta}{a} = 4.3636;\quad \frac{Wh_0^2}{\mu N a^3} = 0.1340;\quad \frac{Fh_0}{\mu N a^2} = 0.2998$$

in accordance with the paper.

The tables given in the paper save a lot of labor, not only in common use, but if for any reason the most favorable conditions of the paper cannot be realized.

H. A. S. Howarth.[7] For about four years Mr. Kingsbury has been intensively engaged in investigations, the important features of which were presented to the Society in December, 1931, and in the present session.

Now that these analyses are available, it is necessary that practical problem after practical problem be solved and that test after test be interpreted, in order to demonstrate the practical utility of the families of curves to which we have now been introduced.

We must now acquire new habits of thought about oil-film bearings. Many of us are perhaps still playing with such children's toys as Morin's coefficients and the thumb rule of $PV = C$. If so, we ought to put them away. They have been passé for several years among engineers who have closely followed progress in the field of bearings and lubrications. We should therefore make a supreme effort to get abreast of the times and become "optimum conditions" minded. This is not said lightly. Up-to-dateness is a habit that may be acquired by hard work, provided one is still young-minded enough to accept something new.

G. B. Karelitz.[8] During the past several years there has been exerted a continuous effort to bring the hydrodynamical theory of lubrication to a state where it could be conveniently applied to design of bearings. The present paper is an important step in the logical development of the oil-film study. It really consists of two parts. The first gives the analysis and methods for choosing a bearing which, under given working conditions, will provide the maximum load-carrying capacity or the minimum friction for a desired oil-film thickness, this analysis treating bearings of infinite width. Means of taking into account finite width of bearings are discussed in the second part.

From daily contact with practical designers it seems to the writer that they are still quite reluctant to consider actually built bearings in the light of the hydrodynamical theory. Apparently there is doubt as to how closely the actual performance of a bearing is described by this theory, and the various methods of experimental verification of this question may be discussed to advantage.

The most common method used by experimenters has been that of exploring the pressures in the oil film over the bearing surface. It can be readily seen that this method cannot be reliable. Indeed, no matter what the distribution of pressures in the oil film of a partial bearing may be under normal conditions, the resultant of all the pressure forces will always be equal to the load carried by the bearing. The pressures at the inlet and the outlet edges of the bearing, as well as those at the bearing ends, must be equal to the atmospheric pressure. Under these circumstances, the maximum pressure will always be of the order from two and one-half to three and one-half times the average pressure over the projected area of the bearing. The shape of the pressure curve can only be compared with the theoretical curve, and any deviations from the latter can be very well charged to inaccuracy of observations.

The location of the journal center in the bearing clearance has been observed by a number of experimenters. Generally speaking, the location of this center, otherwise the minimum thickness of the oil film, checks quite well with the calculated values when the end leakage is taken into consideration. This circumstance is rather gratifying since it justifies the application of the hydrodynamical theory to an analysis of safety of bearing designed. It emphasizes once more the importance of the present paper.

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[5] Berlin, Germany.

[6] Duffing, *Zeit. f. angew. Math. u. Mech.*, no. 4, 1924, p. 314; Auerbach-Hort, "Handbuch der phyiskalischen u. technischen Mechanik," Bd. V, p. 846.

[7] Vice-President and General Manager, Kingsbury Machine Works, Inc., Philadelphia, Pa. Mem. A.S.M.E.

[8] Associate Professor, Columbia University, New York, N. Y. Mem. A.S.M.E.

Experiments were completed recently at the laboratory of the Westinghouse Company by the writer and Mr. S. E. Amos, during which the end leakage was measured in a 6-in. × 10-in. partial bearing under different conditions of load, speed, and inlet oil temperature. The end leakage observed was of the same order as that which could be approximately estimated on basis of the hydrodynamical theory. The results have been submitted in the form of a preliminary report to the Lubrication Committee. It appears that, on the basis of all the experimental information available at present, there can be no doubt that the hydrodynamical theory represents the phenomena in the oil film correctly.

In connection with correction factors for bearings of finite length, Fig. 10 of the paper is very interesting. It will be noted that the values of the ratio L_1 giving the effect of finite length lie very close to an average curve, whether a plane bearing or a journal bearing be considered. This justifies the application of conclusions arrived at, when analyzing one kind of bearing, such as a plane bearing, to any other kind, such as partial journal bearings, and vice versa. This is particularly important with the present trend toward shorter journal bearings.

In conclusion, the writer wishes to call attention to the tremendous amount of work, analytical, numerical, and experimental, which is involved in the preparation of the paper and the tables given in it. Part of the work was made possible only by the ingenious device for experimental integration of the hydrodynamic equation of an oil film invented by Mr. Kingsbury and reported before the Society last year. This device, consisting of an electric analogy, will be used, no doubt, extensively in laboratories concerned with the mechanism of fluid lubrication. It must be remembered that there are numerous questions to be solved yet, such as the oil-film performance under a rotating load, the behavior of oil films in bearings under shock loads, oil flow in bearing clearances, etc.

M. Stone.[9] In this paper the author has provided the engineer interested in designing bearings beforehand rather than explaining phenomena in an existing bearing, with procedures for creating optimum conditions that he has hitherto lacked. The important influence of side leakage has been quantitatively introduced. Throughout the paper attention has been called to the importance of knowing what the minimum film thickness is, and that this be the prime consideration in bearing design. Some three years ago, the writer had occasion to study the dynamic stability conditions in a bearing oil film, with the purpose of determining the actual motion of the shaft in the bearing during rotation and at different speeds.

The measurements made were extremely accurate—down to 10^{-5} in.—specially developed coupled transformers being used, with the shaft acting as the movable armature. The results—the actual motion of the shaft, plotted inside the clearance circle—are shown in Fig. 11. The tests were made in a 10,000-hp. generator running at 900 r.p.m. and having a 9-in. diameter 90-deg. partial bearing, with a projected-area pressure of 102 lb. per sq. in. From these plots, by graphing the equilibrium position of the shaft center, we can obtain two important verifications of the hydrodynamical theory: (1) minimum oil-film thickness vs. ZN/P, and (2) locus of shaft center as a function of shaft speed.

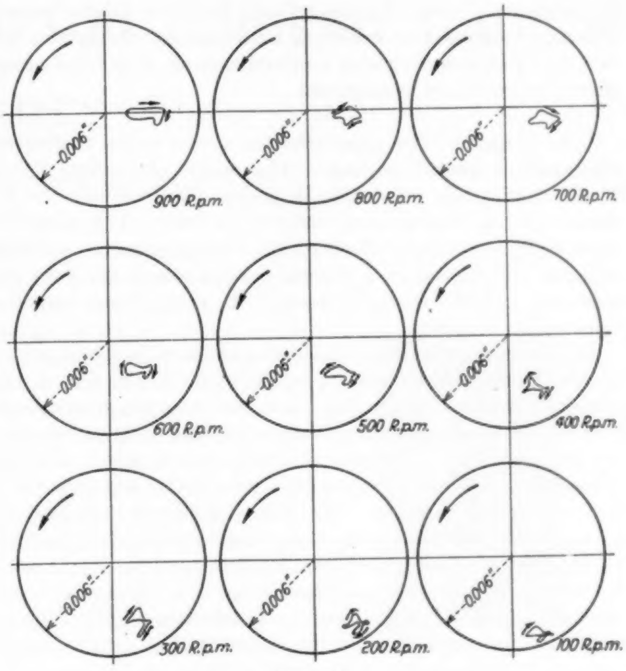

Fig. 11

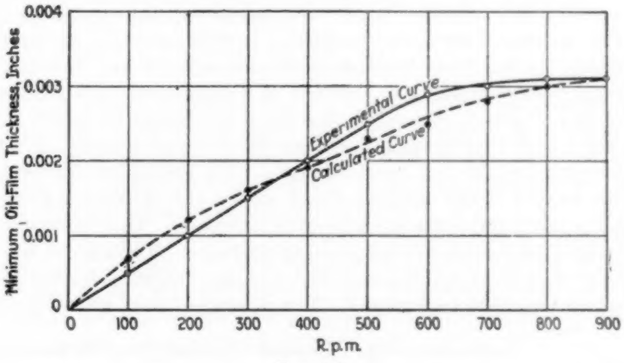

Fig. 12

The first curve is shown in Fig. 12, where it is compared with the theoretical one calculated by G. B. Karelitz in his paper "Charts for Studying the Oil Film in Bearings" [Trans. A.S.M.E., vol. 47 (1925), p. 1101], slightly modified to include side-leakage effects. This indicates that the major design premise in bearings—the minimum oil-film thickness—has every assurance of experimental justification for all types of practical bearings.

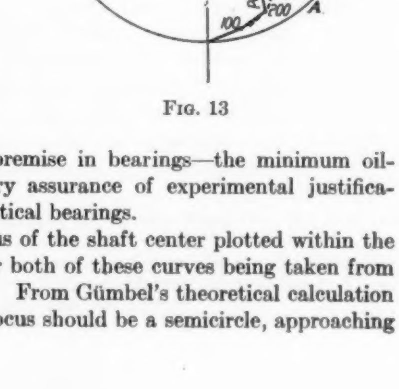

Fig. 13

Fig. 13 shows the locus of the shaft center plotted within the clearance circle, data for both of these curves being taken from results shown in Fig. 11. From Gümbel's theoretical calculation for total bearings, this locus should be a semicircle, approaching the geometrical center for theoretically infinite rotational speed. The departure from the semicircle is accounted for mainly by the fact that we are dealing with a partial bearing, with side-leakage effects that may not be neglected.

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I. A. TERRY.[10] The paper presents a very useful theoretical treatment of journal bearings. The results of Sections VI to X are particularly interesting in giving quantitatively the influence of side leakage and variable viscosity. The summary given in Plate 10 of the effects of side leakage upon the carrying capacity and friction of a journal bearing makes the data immediately available for use in design if the value of minimum film thickness h_0 is known.

Three classes of bearings have been considered in the paper, all of which have been based upon an initial knowledge of the minimum thickness of oil film. A fourth class which represents a large portion of the bearings used on rotating electrical machinery has conditions as follows: A definite load is to be carried by a bearing at a certain speed with a minimum journal diameter and a given oil viscosity. The journal diameter may be fixed by torque requirements or mechanical limitations. It is required to find the bearing proportions for minimum loss or, stated in different terms, minimum temperature rise. This set of conditions has led in recent years to a reduction of the ratio of length in an axial direction to journal radius (w/a), with a somewhat corresponding increase in unit pressures because of the fact that the coefficient of friction λ decreases faster than the unit pressure p increases. It has also led to a study to determine the optimum clearances for such a journal bearing. One procedure for this study has been to use the curves of Mr. Howarth's paper[11] corresponding to the type of bearing used, that is, complete or partial, then, with the given conditions of speed, viscosity of oil, and assumed pressure, the loss in a bearing for different clearances has been obtained. The results are similar to the λ-curve of the author's Fig. 4, except the ordinate is loss in kilowatts and the abscissa is bearing clearance. This shows some latitude in the selection of running clearance. Tests which have been made on large journals with various clearances have given rather unsatisfactory results. In a number of cases where curves showed that an increased clearance would give a lower loss, the bearings were bored out and the operating temperature was actually decreased. In other cases where the curves showed that a reduced loss should be obtained on increased clearances, enlarging them made practically no difference. Evidently other factors than those considered by Mr. Howarth enter the problem, and perhaps the effects of side leakage is one of the more important theoretical ones. Probably the greatest difficulty in the whole study is the fact that small differences in journal and bearing surfaces seem to make rather radical differences in bearing operation. It would be interesting to know if the author considers that the curves given by Mr. Howarth could be modified by means of Plate 10 to form a basis for a more complete theoretical study of this problem.

AUTHOR'S CLOSURE

The simplification offered by Professor Duffing effects an appreciable saving in the labor of calculation. The small discrepancies between his numerical results and those shown in the paper appear to be due to less exact calculation from his formulas.

Professor Duffing has also pointed out that in Equation [27] of the paper, the logarithmic terms cancel out, and therefore may be omitted, with a helpful saving in numerical work. This work is at best a serious burden, as suggested by Professor Karelitz; and any and all means for reducing it will aid progress in the study of the subject.

Referring to Mr. Terry's question regarding the graphical solutions of Mr. Howarth [13]: The graphical work has been checked at numerous points by the analytical forms of the present paper, and no appreciable discrepancies have been found. It may therefore be said that the graphical results are trustworthy, and may be used instead of the analytical solutions for the "infinitely wide" bearings. As to their application in connection with plate X of the present paper, this figure is derived only for the conditions of eccentricity etc. that are assumed to be the "optimum" ones. For other conditions of eccentricity etc. the effects of side leakage indicated in plate X, especially as regards the load factor L_1, could not be expected to apply closely. In fact, recent researches by Mr. Needs, using the electrical method, indicate that the L_1 curve itself should be regarded as a first approximation, not as a finality; but its use gives load values that are on the safe side. The F_1 curve does not appear subject to as great corrections.

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[10] A. C. Engineering Dept., General Electrical Co., Schenectady, N. Y.

[11] "Graphical Analysis of Journal-Bearing Lubrication," Trans. A.S.M.E., vols. 45–47 (1923–1925).

RP-54-8

Pressure Distribution in Oil Films of Journal Bearings

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S. A. McKee

T. R. McKee

One fact which is becoming widely recognized by users of journal bearings is that under proper conditions of operation the load applied to a journal bearing is supported by the hydrostatic pressures built up in a film of lubricant wedged between and completely separating the journal and bearing. This paper describes an investigation at the Bureau of Standards the purpose of which was to determine the distribution of pressure in the oil film of a full journal bearing for various conditions of operation and of bearing construction. A specially designed apparatus provided a means for measuring the film pressures at practically all points on the surface of a bearing when operating under different conditions of load, speed, and viscosity of the lubricant. The results of nine tests made under conditions involving changes in clearance, load, speed, and viscosity are presented graphically in the form of pressure-distribution curves. In conclusion the characteristics of the film-pressure distribution are summarized, and further steps toward a better understanding of the pressure distribution are outlined.

ONE method of procedure in the design of a journal bearing suitable for a given machine and operation is based upon the hydrodynamic analysis of the load-carrying oil film. This analysis originated in the work of Tower, Reynolds, and Sommerfeld, and was made more complete by Kingsbury, Harrison, Howarth,[3] and others.

Another method, originated by Hersey,[4] is based upon certain general relations derived by dimensional analysis, and may be used for the planning of experiments and for the interpretation of the data obtained.

While in principle the former method requires no special experiments, the mathematical treatments are in general approximations involving simplifying assumptions which may not be applicable to an actual bearing. Both methods are therefore dependent to a considerable degree upon direct experiments. With the former method it is necessary to check the correctness of the assumptions and in the latter to obtain suitable coefficients for the dimensionless relations developed.

The problem may be attacked experimentally by observing the effects of changes in the construction of the bearing and in the conditions of operation upon (1) the friction developed in the bearing, (2) the minimum thickness of the oil film, and (3) the hydrostatic pressures developed in the film.

Each of these factors is a variable dependent upon construction and operation, and it is probable that a thorough knowledge of all three will be necessary to eliminate guesswork in bearing design. The first, the friction developed, provides a convenient means for studying the economic question of power losses in bearings, the heat developed, and with proper correlation of data may be used for the determination of the carrying capacity of bearings. It is of special use also in the study of bearing performance when conditions of operation are such that a complete film of lubricant is not maintained between the rubbing surfaces. The second, the minimum thickness of the oil film, is of special significance in the determination of the carrying capacity or factor of safety. The third, the hydrostatic pressure distribution in the film, possibly offers the most direct means for comparison with analytical studies based upon the principles of hydrodynamics. Notable progress in the study of the problem from this standpoint has been made in a recent development by Kingsbury[5] in which the number of cases where solutions to the analytical treatment are possible has been greatly increased by an ingenious experimental method of making electrical measurements upon enlarged models of the film.

The bearing herein described provides a means for the study of

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[3] Howarth, H. A. S., "A Graphical Analysis of Journal Bearing Lubrication," Trans. A.S.M.E., vol. 45 (1923), pp. 421–448; Part II, vol. 46 (1924), pp. 809–832; Part III, vol. 47 (1925), pp. 109–114; reprinted by The American Society of Mechanical Engineers, 29 West 39th Street, New York, N. Y.

[4] Hersey, M. D., "On the Laws of Lubrication of Journal Bearings," Trans. A.S.M.E., vol. 37, 1915, pp. 167–202.

Contributed by the A.S.M.E. Special Research Committee on Lubrication and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers. Published by permission of the Director of the Bureau of Standards, U. S. Department of Commerce.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] Kingsbury, Albert, "On Problems in the Theory of Fluid-Film Lubrication, With an Experimental Method of Solution," Trans. A.S.M.E., 1931, paper APM-53-5, pp. 59–75.

this third factor—namely, the hydrostatic pressures developed in the oil film under various conditions of actual operation.

THE APPARATUS AND ITS OPERATION

The apparatus as originally designed was built in 1926. At that time, however, it was capable of measuring the film pressures only at the longitudinal center of the bearing. Changes in design were made in 1930 to permit determinations of the film pressure at practically any point on the whole bearing surface.

A photograph of the complete apparatus set up for use is shown in Fig. 1, and a drawing showing the arrangement of the principal working parts in Fig. 2.

The hardened and ground steel test journal (*A*, Fig. 2) is provided with a Morse taper for attaching to a vertical spindle mounted on the frame of a portable drilling apparatus and driven through belt and pulleys by an electric motor. This special drive is not essential, since an ordinary power drill press may be used if convenient. The actual working surface of the journal is 1 in. long and is located at the extreme lower end.

The bearing consists of two parts which can be rotated relative to each other around the axis of the bore. One part includes the bearing proper *B*, the oil bath *C*, and the thermometer well *D*. The bearing proper is a bronze bushing 2 in. long with a $^7/_8$-in. bore provided with the pressure tap *E* (a small hole 0.025 in. in diameter) at its longitudinal center and radial to its axis. The other part includes the yoke *F* and the adjustable nut *G* for connecting a horizontally mounted spring balance (see Fig. 1) used for applying the load. The yoke is held in place longitudinally by the collars *H* and *J*. The pressure tap is connected by copper tubing, manifold, and needle valve to a small combined vacuum and pressure gage (—15 to 100 lb. per sq. in. gage) mounted on the thermometer well. The weight of the bearing is supported by a horizontal table through the ball *K* set in the socket at the base of the oil bath. Counterweights are provided on the oil bath and yoke so that the gravitational forces are symmetrical about the ball. By this means unknown gravitational moments are eliminated and the sole force acting on the bearing surfaces is that caused by the pull of the spring balance.

By means of this apparatus the film pressures over practically the entire working surface may be observed with one pressure tap and gage. The angular location of the tap relative to the line of application of the load may be changed by releasing the wing bolt *L* and revolving the bearing within the yoke, the thermometer well acting as a convenient handle. Marks on the upper collar at 6-deg. intervals from the pressure tap in conjunction with a mark on the yoke at the center of load provide an indication of the location of the tap. The location of the tap relative to the ends of the working surface of the bearing may be changed as desired by moving the journal up or down in the bearing. This is accomplished by the "feed" on the portable drilling apparatus, a locking device with screw adjustment being attached for maintaining the journal in a given position. A mark on the upper part of the journal together with a special scale for fitting to the top of the oil bath provide a means for determining the longitudinal position of the tap. The nut *G* and the adjustable table (see Fig. 1) for holding the spring balance provide a means for centering the load for any position of the journal. A mark on the nut and a scale on the yoke are used for indicating the position of the load. These adjustments may be used also for making tests with the load off center.

The level of the lubricant in the bath is kept above the top of the working surfaces. The hollow arms from the bath to the thermometer well allow for circulation of the lubricant and prevent any pressure being built up in the lower portion of the bath other than that caused by the slight head of oil. By the use of the vertical journal and bath lubrication, oil is at all times automatically admitted to the proper point on the periphery of the bearing

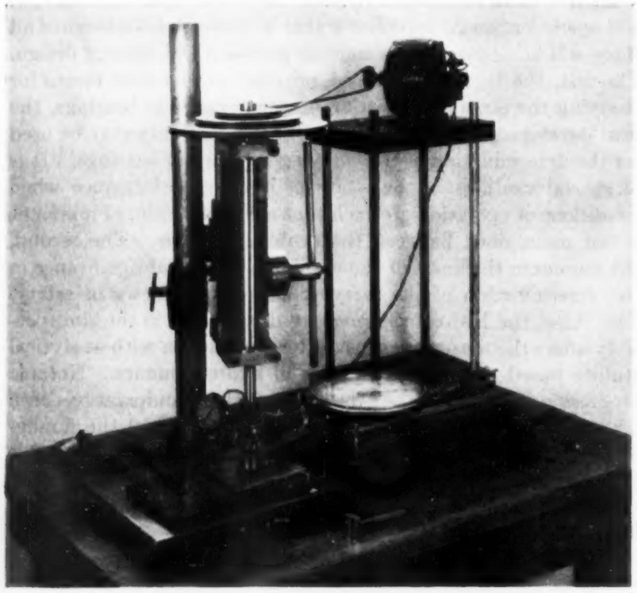


FIG. 1 JOURNAL-BEARING FILM-PRESSURE APPARATUS, GENERAL ARRANGEMENT

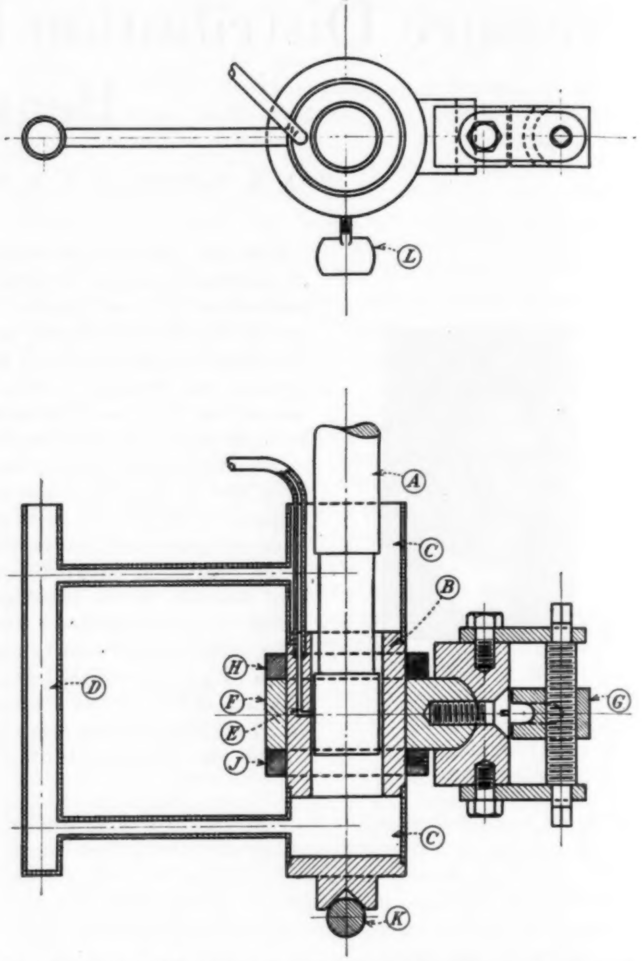


FIG. 2 JOURNAL-BEARING FILM-PRESSURE APPARATUS, ARRANGEMENT OF PRINCIPAL WORKING PARTS

The time required for the gage to indicate the pressure in the film at the bearing tap is kept small by making the pressure connections as short as convenient and by evacuating the air from gage and connections and filling with lubricant.

Trial runs indicated that accurate alignment of the load and the journal was necessary for obtaining a symmetrical longitudinal pressure distribution. It was found more convenient to secure the proper alignment by observing the pressure at certain points rather than by ordinary mechanical methods. A suitable screw adjustment for tilting the journal in relation to the line of the load was attached to the portable drilling frame. (If a regular drill press had been used, an adjustable table would probably have been more convenient.)

Since it is necessary to release the load and stop the journal in order to change and observe the longitudinal position of the pressure tap, it was found more convenient to make all the observations around the periphery in a given plane perpendicular to the journal before changing the longitudinal position of the tap. In each of the tests conducted with a given load, speed, and lubricant, observations were made in 11 different planes perpendicular to the length of the bearing surface.

Test Conditions

Of the tests made, each gave an indication of the distribution of pressure in the oil film for a particular condition of operation or construction. The factors changed were clearance, viscosity, speed, and load. Changes in the clearance were made by using two journals, one having a clearance (difference of diameters) of approximately 0.005 in. and the other of 0.002 in. Viscosity changes were made by the use of two lubricants, one having a viscosity of 125 sec. Saybolt Universal at 100 deg. fahr. and the other of 73 sec. Saybolt Universal at 100 deg. fahr. Large changes in speed were made by using pulleys of different sizes, and speed regulation was provided by a rheostat in the motor circuit. The load was limited to a certain extent by the range of the pressure gage used on the apparatus, and with the exception of one test a load of 25 lb. was applied. No attempt was made to control the temperature of operation, which was measured by a thermometer inserted in the well. However, the apparatus was allowed to come to steady conditions at a given load and speed before pressure readings were started.

The conditions of operation and construction in the respective tests are given in Table 1.

TABLE 1 TEST CONDITIONS

(Diameter of bearing, 0.8753 in.; length of bearing, 1 in.)

Test No.	Diameter journal, in. (*D*)	Diametral clearance, in. (*C*)	Load, lb.	Speed, r.p.m. (*N*)	Viscosity lubricant, centipoises (*Z*)	ZN/P	Application of load
1.......	0.8703	0.005	25	565	42.9	848	Center
2.......	0.8703	0.005	25	592	39.4	817	Center
3.......	0.8703	0.005	25	244	36.5	310	Center
4.......	0.8703	0.005	20	244	34.4	368	Center
5.......	0.8733	0.002	25	561	42.3	832	Center
6.......	0.8733	0.002	25	227	42.5	338	Center
7.......	0.8733	0.002	25	531	18.4	342	Center
8.......	0.8733	0.002	25	218	18.6	142	Center
9.......	0.8733	0.002	25	553	20.0	389	0.1 in. off center

Symbols and abbreviations used for expressing the various factors pertaining to the bearing and its performance are as follows:

D = journal diameter, inches
C = diametral clearance (difference between bearing diameter and journal diameter), inches
L = bearing length, inches
P = bearing pressure (total load/DL), pounds per square inch
Z = absolute viscosity lubricant, centipoises
N = journal speed, revolutions per minute
p = hydrostatic pressure in film, pounds per square inch
θ = angle to point where p is measured as shown in Fig. 3. θ is measured in the direction of rotation of the journal and expressed in degrees.

Graphical Presentation of Results

The results obtained in test No. 1 are shown graphically in Fig. 4.

The circumferential pressure distribution is given in *A*. These curves are obtained by plotting the observed pressures against the circumferential location of the tap as measured by the angle θ. Each curve is for a different position of the tap along the journal axis as indicated in terms of decimal parts of *L*, the bearing length. The curve labeled 0.5*L*, for example, indicates the pressure distribution around the circumference of the bearing on a plane perpendicular to the axis of the journal and at the longitudinal center of the bearing surface.

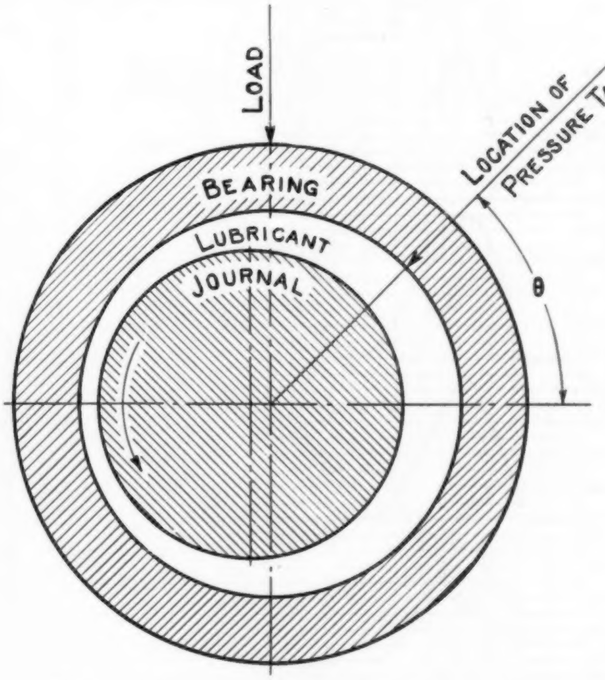


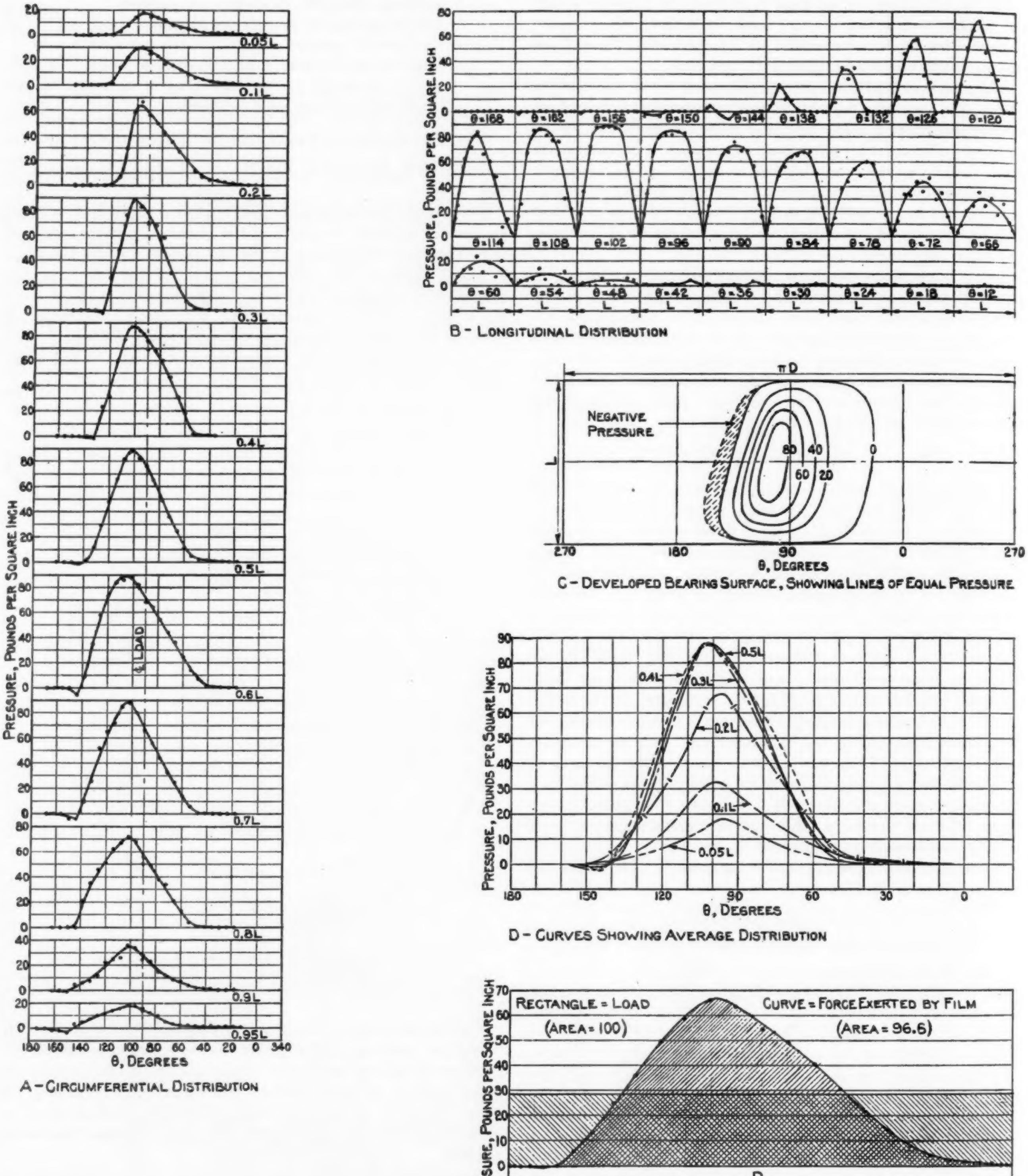
Fig. 3 Diagrammatic Representation of Bearing

(Angle θ, measured in the direction of the rotation of the journal, is used to indicate any particular circumferential position of the pressure tap.)

Curves showing the longitudinal distribution are given in *B*. These are made by plotting the observed pressures against the position of the tap relative to the length of the bearing for the given angle θ as noted below each curve. For example, the curve labeled $\theta = 90$ shows the pressure distribution along an element of the bearing passing through the point where the load is applied.

An isobaric plot of the film is shown in *C*. In this figure the bearing surface is developed into a plane with a length equal to the circumference (πD) and width equal to *L*. Upon this, isobars (loci of points of equal pressure) are plotted for gage pressures of 0 (atmospheric), 20, 40, 60, 80, and 100 lb. per sq. in. The shaded portion indicates roughly the region where the film pressures are negative (below atmospheric). Except for this region of negative pressure, the entire area outside the zero isobar is at atmospheric pressure.

Curves showing the average circumferential distribution are


A – Circumferential Distribution

B – Longitudinal Distribution

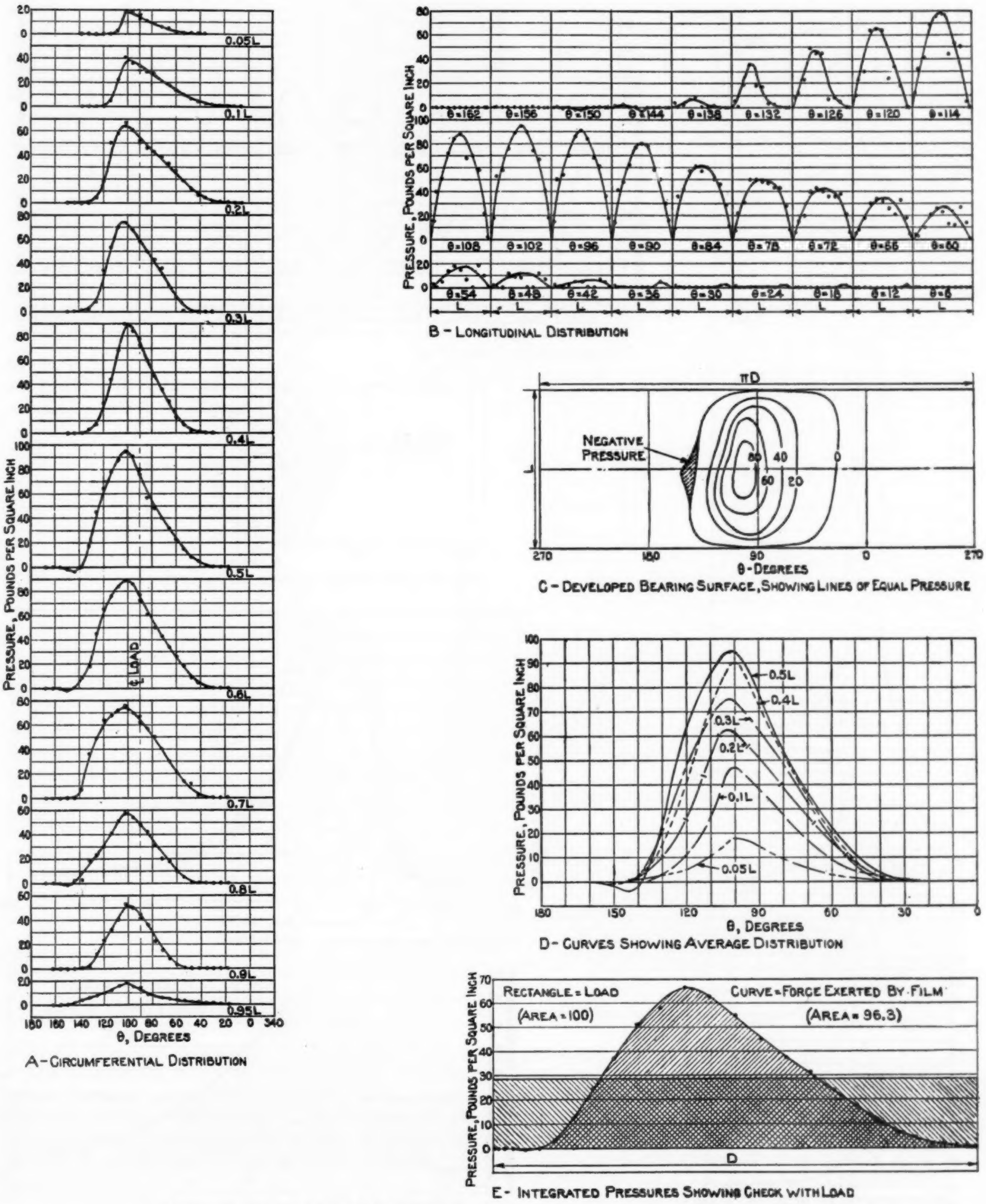
C – Developed Bearing Surface, Showing Lines of Equal Pressure

D – Curves Showing Average Distribution

E – Integrated Pressures Showing Check with Load

Fig. 4 Curves Showing Distribution of Pressure in Oil Film of Journal Bearing

(Diameter of journal, 0.8703 in.; length of bearing, 1 in.; clearance, 0.005 in.; load, 25 lb.; speed, 565 r.p.m.; viscosity of oil, 42.9 centipoises; $ZN/P = 848$.)



A – CIRCUMFERENTIAL DISTRIBUTION

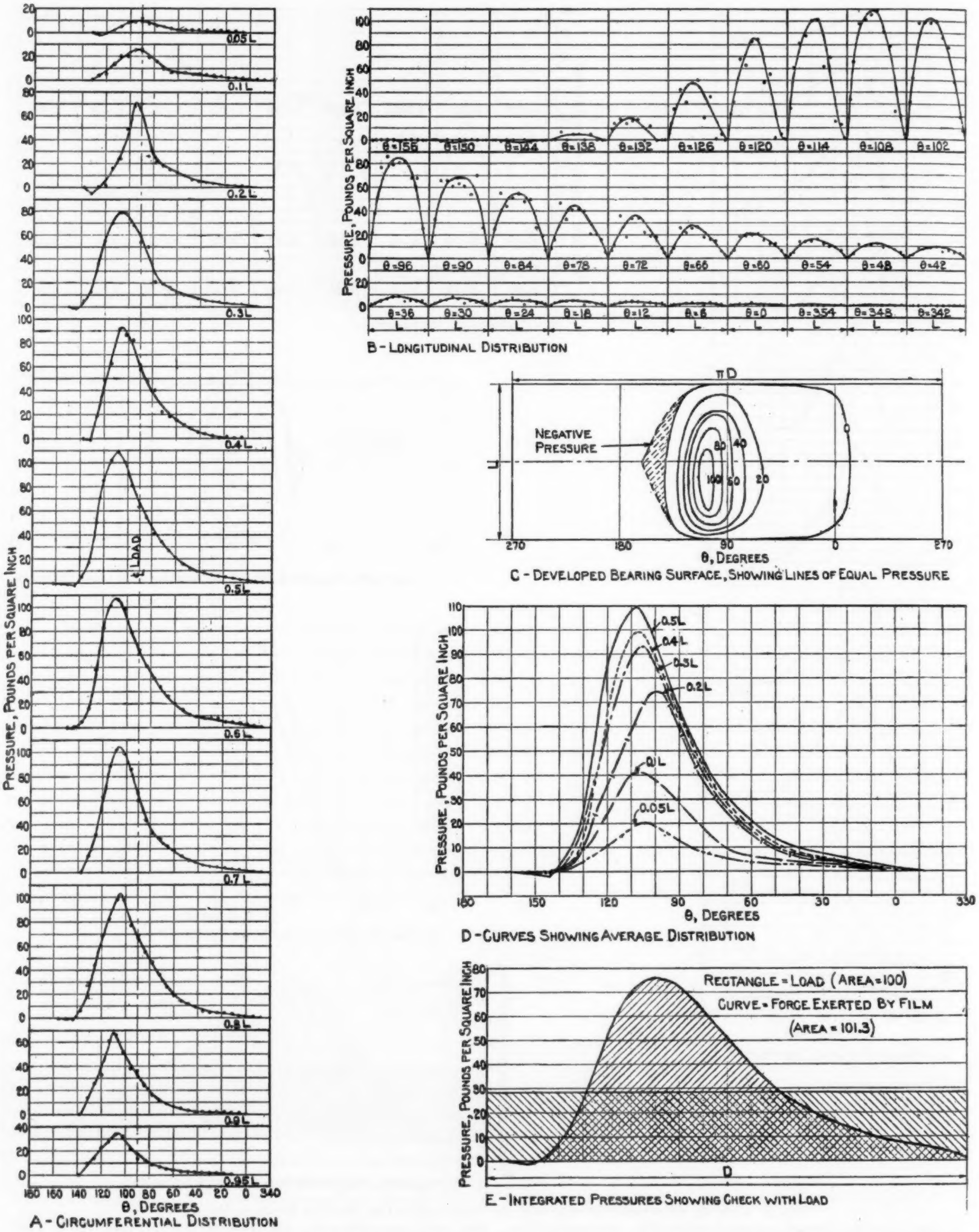
B – LONGITUDINAL DISTRIBUTION

C – DEVELOPED BEARING SURFACE, SHOWING LINES OF EQUAL PRESSURE

D – CURVES SHOWING AVERAGE DISTRIBUTION

E – INTEGRATED PRESSURES SHOWING CHECK WITH LOAD

FIG. 5 CURVES SHOWING DISTRIBUTION OF PRESSURE IN OIL FILM OF JOURNAL BEARING
(Diameter of journal, 0.8703 in.; length of bearing, 1 in.; clearance, 0.005 in.; load, 25 lb.; speed, 592 r.p.m.; viscosity of oil, 39.4 centipoises; $ZN/P = 817$.)



A - Circumferential Distribution

B - Longitudinal Distribution

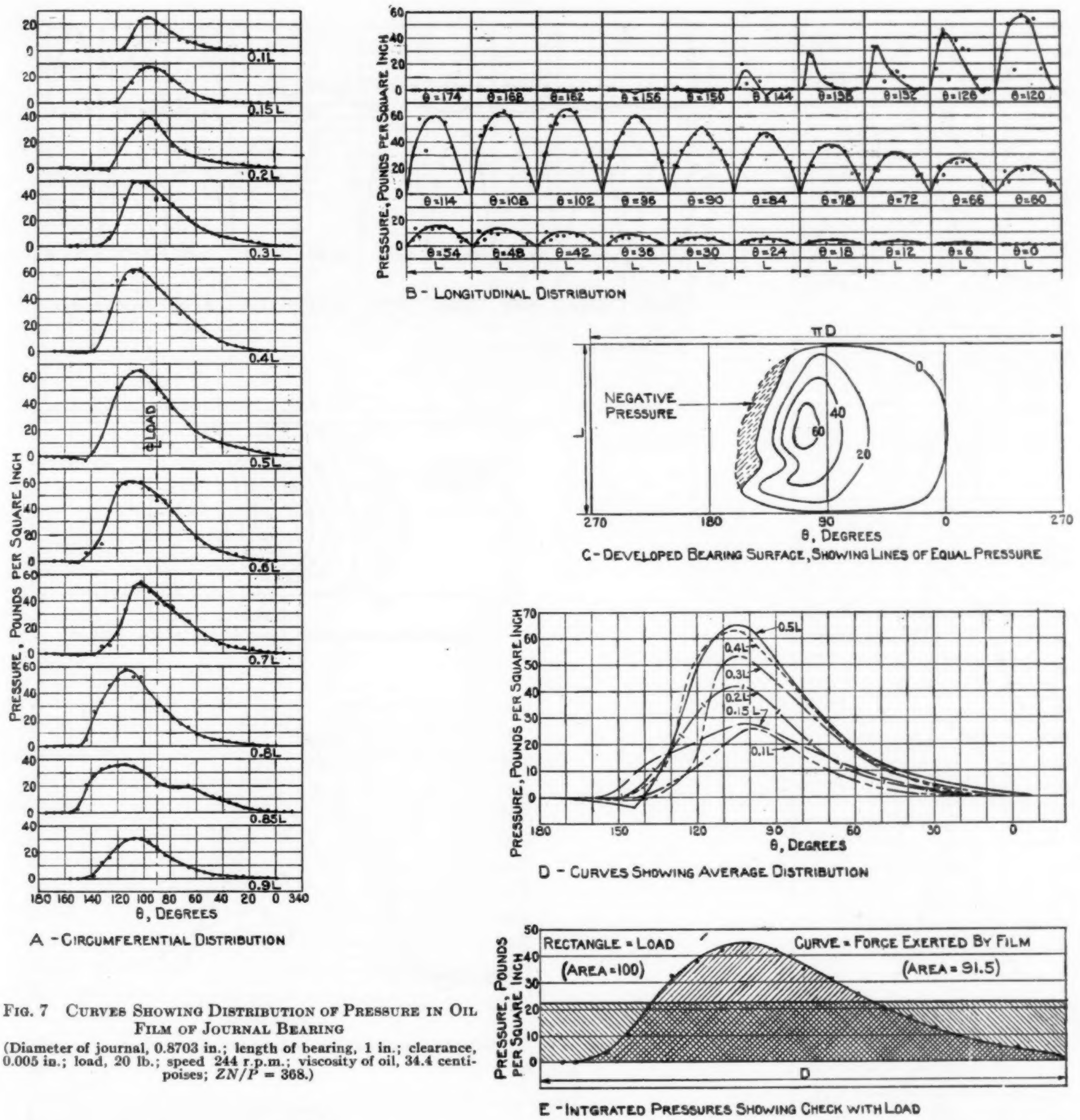
C - Developed Bearing Surface, Showing Lines of Equal Pressure

D - Curves Showing Average Distribution

E - Integrated Pressures Showing Check with Load

Fig. 6 Curves Showing Distribution of Pressure in Oil Film of Journal Bearing

(Diameter of journal, 0.8703 in.; length of bearing, 1 in.; clearance, 0.005 in.; load, 25 lb.; speed, 244 r.p.m.; viscosity of oil, 36.5 centipoises; $ZN/P = 310$.)



A - Circumferential Distribution

B - Longitudinal Distribution

C - Developed Bearing Surface, Showing Lines of Equal Pressure

D - Curves Showing Average Distribution

E - Intgrated Pressures Showing Check with Load

Fig. 7 Curves Showing Distribution of Pressure in Oil Film of Journal Bearing

(Diameter of journal, 0.8703 in.; length of bearing, 1 in.; clearance, 0.005 in.; load, 20 lb.; speed 244 r.p.m.; viscosity of oil, 34.4 centipoises; $ZN/P = 368$.)

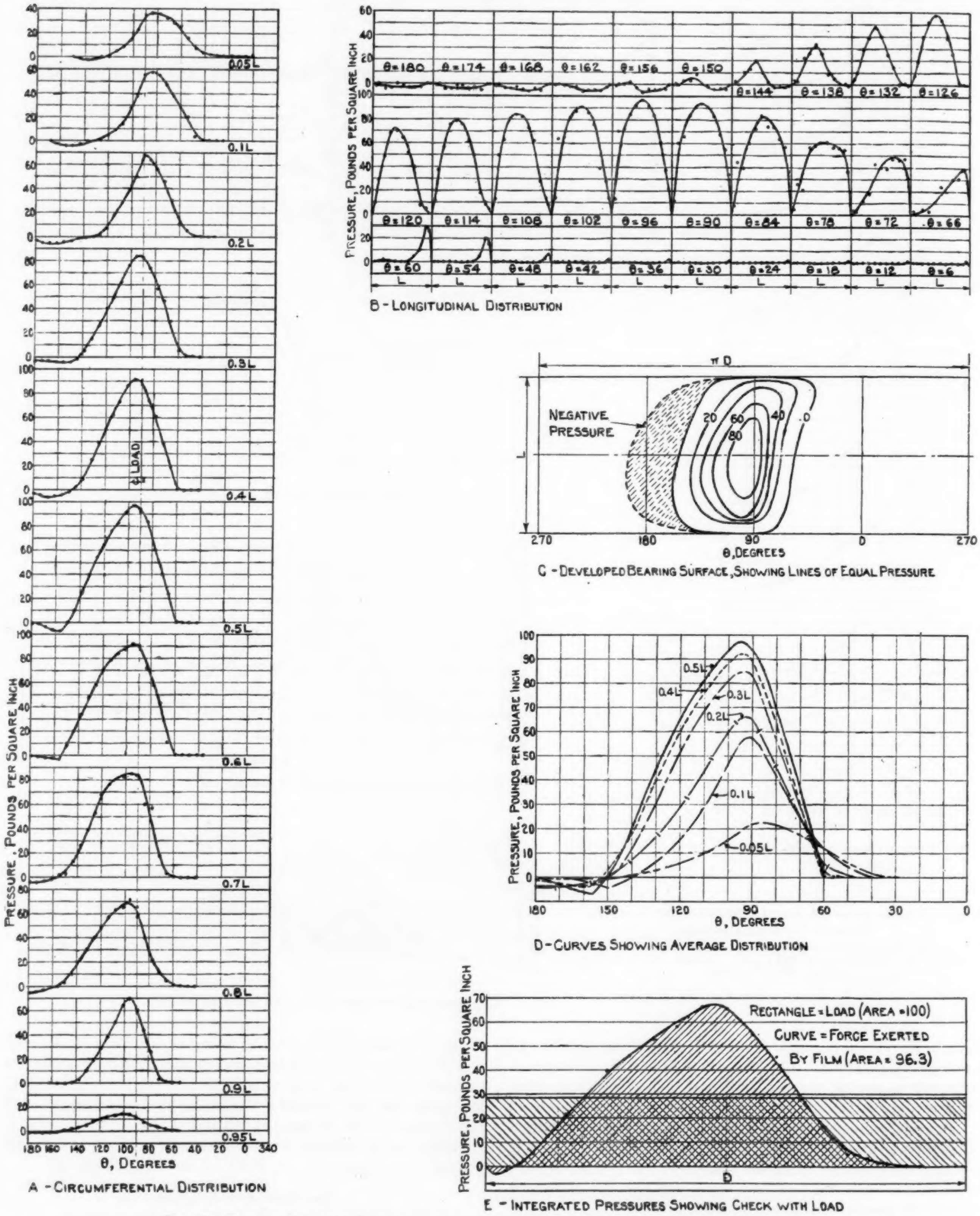
shown in *D*. These curves are similar to those shown in *A* except that those labeled 0.05*L*, 0.1*L*, 0.2*L*, 0.3*L*, and 0.4*L* in *D* are the arithmetic means of curves 0.05*L* and 0.95*L*, 0.1*L* and 0.9*L*, 0.2*L* and 0.8*L*, 0.3*L* and 0.7*L*, and 0.4*L* and 0.6*L*, respectively, in *A*. In other words, the curves in *D* represent the circumferential distribution at the respective planes along the length of the bearing if the pressures along any element at two points the same distance from each end had been equal and also the arithmetic mean of the pressures observed at these points.

The curve in *E* is obtained by plotting the mean effective pressures for the various values of θ against the diameter of the journal. The mean effective pressure for any value of θ was obtained by integrating with a planimeter the curves in *B*. The area inclosed by this curve is therefore a measure of the resultant of all the pressures acting parallel to and against the load imposed on the bearing. The area indicated by the shaded rectangle is a measure of this load; thus a comparison of the areas inclosed by the curve and the rectangle, respectively, gives an indication of the accuracy of the pressure observations.

Similar curves showing the results of tests 2, 3, 4, 5, 6, 7, 8, and 9 are given in Figs. 5, 6, 7, 8, 9, 10, 11, and 12, respectively.

Discussion of Results

Comparison With Hydrodynamical Equations. From the mathematical analyses by Sommerfeld or by Harrison it can be shown that in a complete journal bearing free from end leakage and operating under steady conditions such that a complete film of the lubricant separates the journal from the bearing, the



A - Circumferential Distribution

B - Longitudinal Distribution

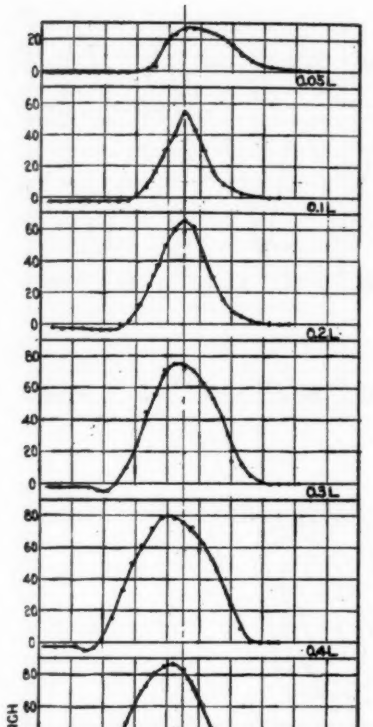
C - Developed Bearing Surface, Showing Lines of Equal Pressure

D - Curves Showing Average Distribution

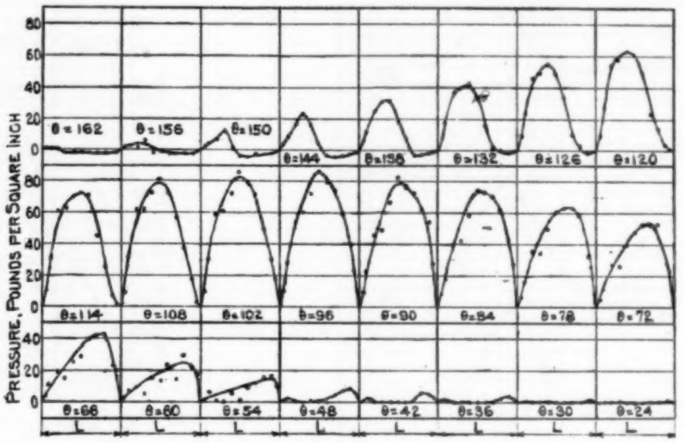
E - Integrated Pressures Showing Check with Load

Fig. 8 Curves Showing Distribution of Pressure in Oil Film of Journal Bearing

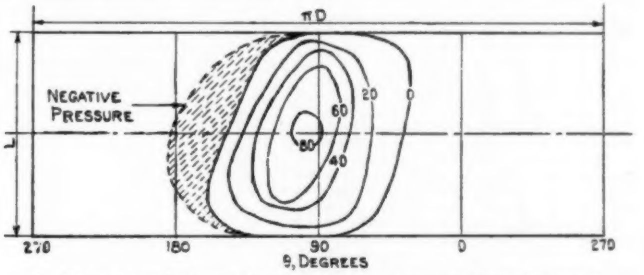
(Diameter of journal, 0.8733 in.; length of bearing, 1 in.; clearance, 0.002 in.; load, 25 lb.; speed 561 r.p.m.; viscosity of oil, 42.3 centipoises; *ZN/P* = 832.)



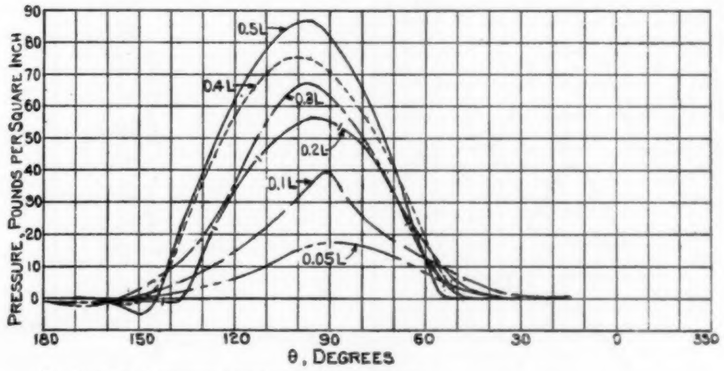
A - CIRCUMFERENTIAL DISTRIBUTION



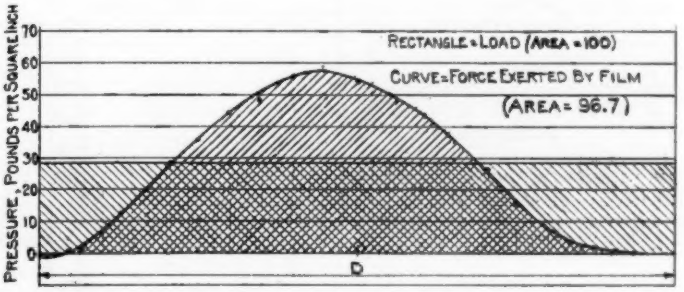
B - LONGITUDINAL DISTRIBUTION



C - DEVELOPED BEARING SURFACE, SHOWING LINES OF EQUAL PRESSURE



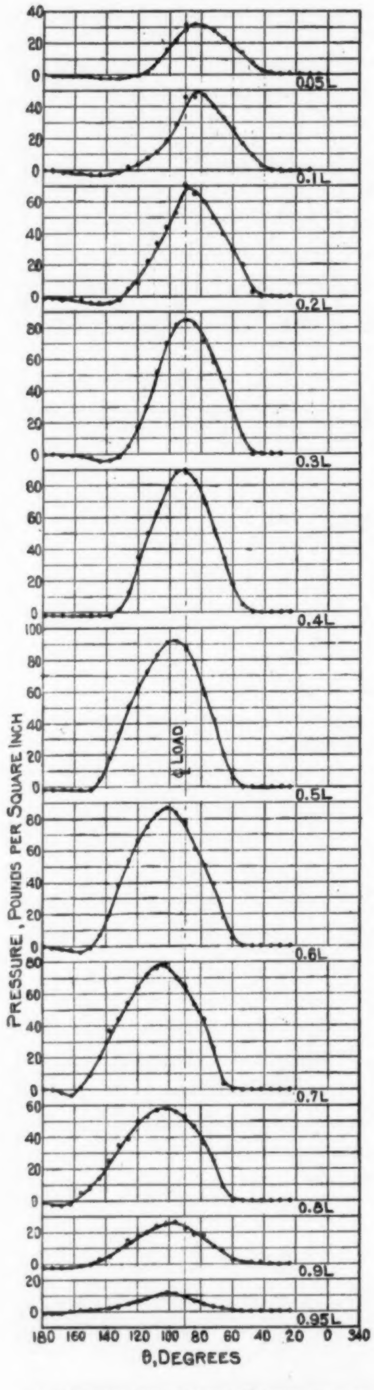
D - CURVES SHOWING AVERAGE DISTRIBUTION



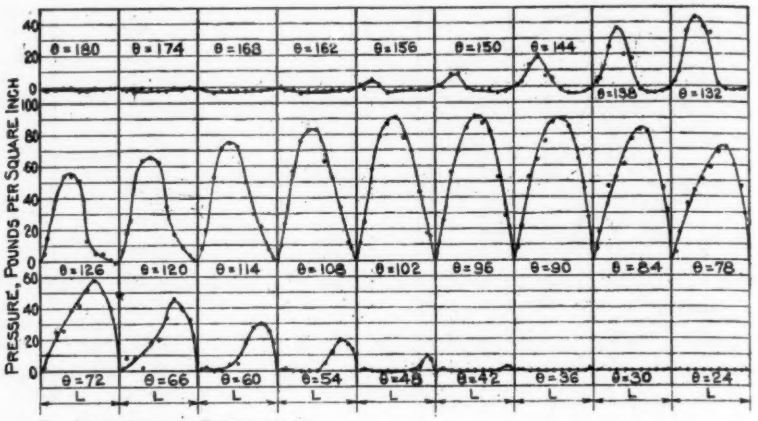
E - INTEGRATED PRESSURES SHOWING CHECK WITH LOAD

FIG. 9 CURVES SHOWING DISTRIBUTION OF PRESSURE IN OIL FILM OF JOURNAL BEARING

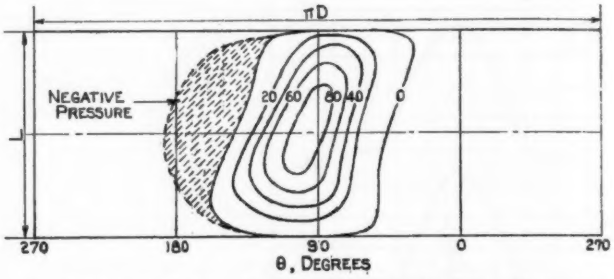
(Diameter of journal, 0.8733 in.; length of bearing, 1 in.; clearance, 0.002 in.; load, 25 lb.; speed, 227 r.p.m.; viscosity of oil, 42.5 centipoises; $ZN/P = 338$.)



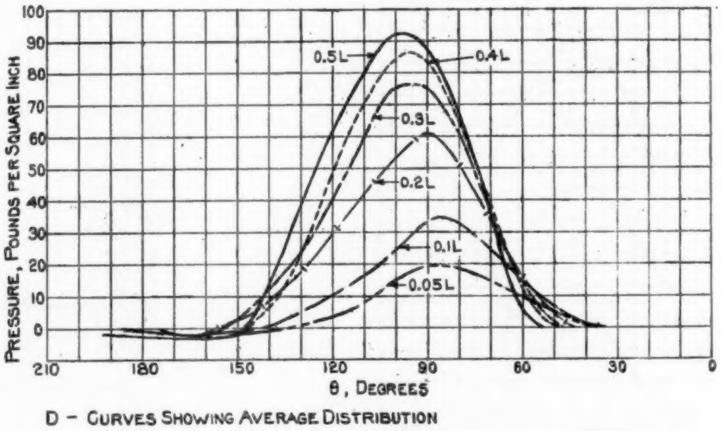
A - Circumferential Distribution



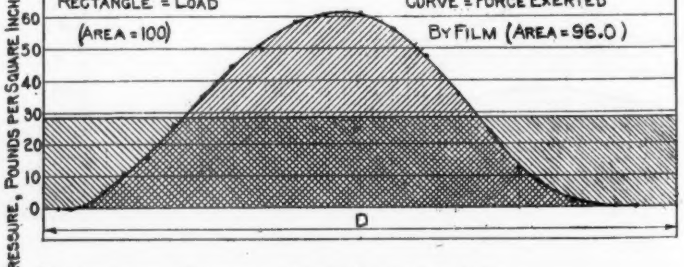
B - Longitudinal Distribution



C - Developed Bearing Surface, Showing Lines of Equal Pressure



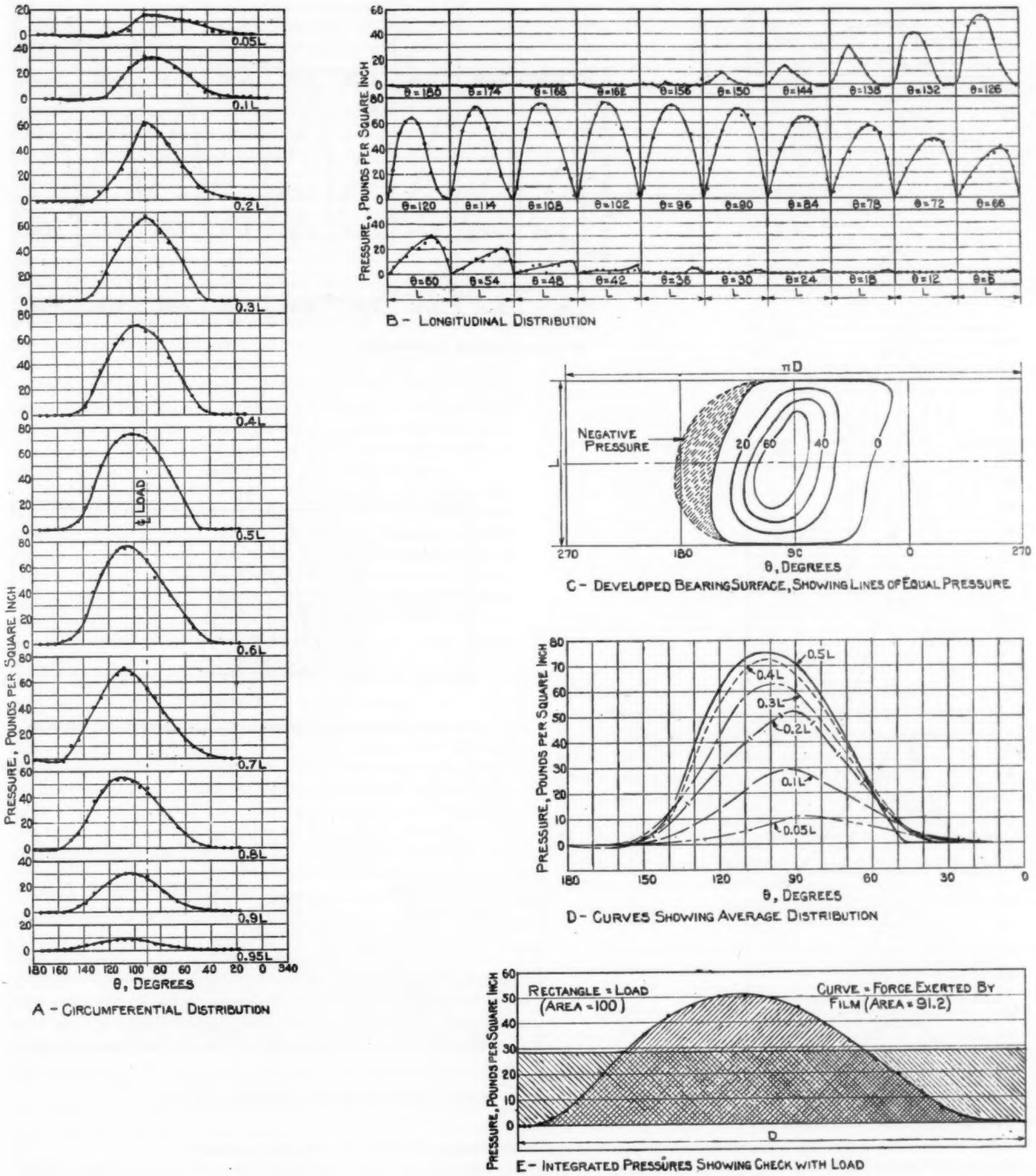
D - Curves Showing Average Distribution



E - Integrated Pressures Showing Check with Load

Fig. 10 Curves Showing Distribution of Pressure in Oil Film of Journal Bearing

(Diameter of journal, 0.8733 in.; length of bearing, 1 in.; clearance, 0.002 in.; load, 25 lb.; speed 531 r.p.m.; viscosity of oil, 18.4 centipoises; $ZN/P = 342$.)



A – Circumferential Distribution

B – Longitudinal Distribution

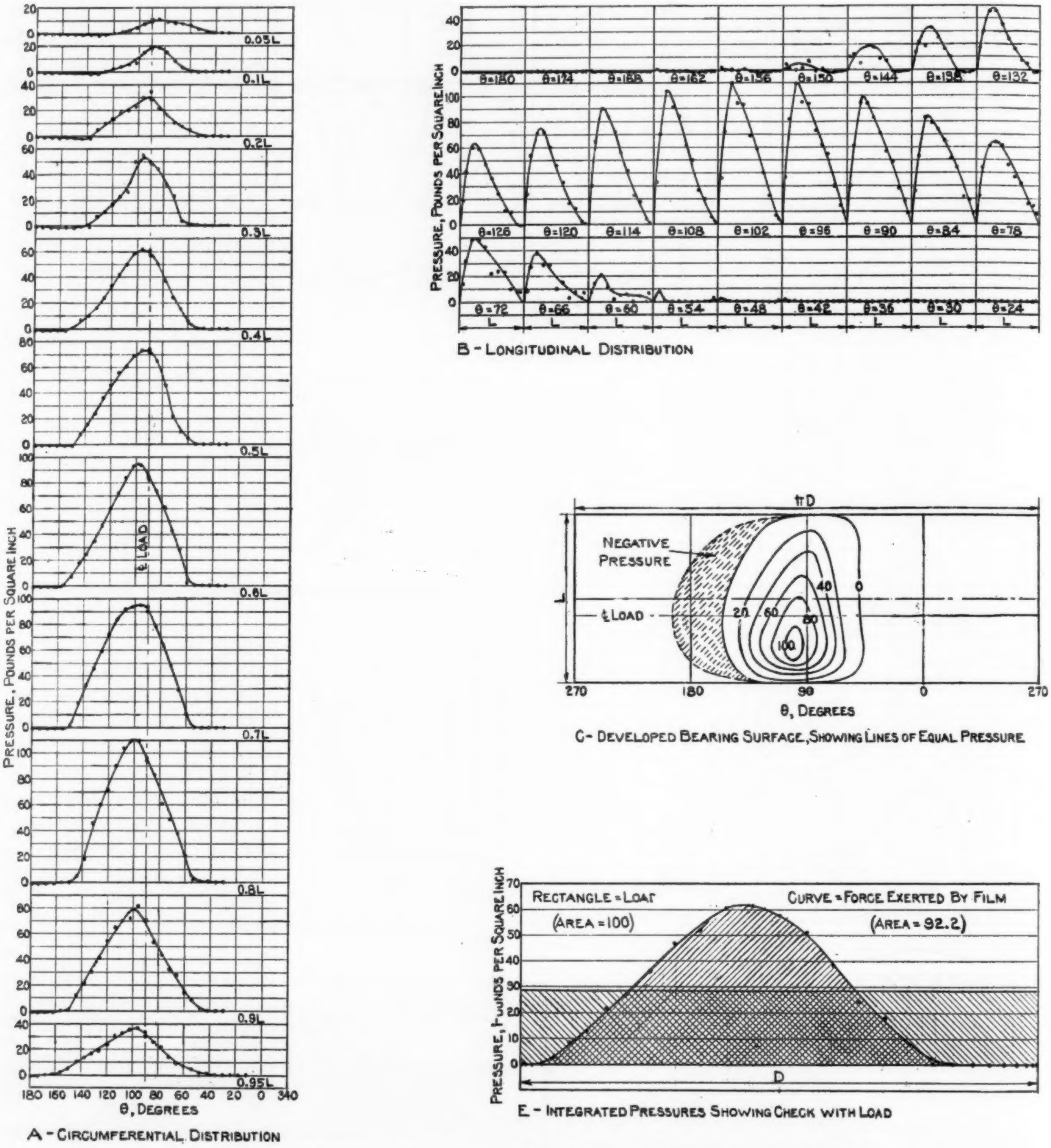
C – Developed Bearing Surface, Showing Lines of Equal Pressure

D – Curves Showing Average Distribution

E – Integrated Pressures Showing Check with Load

FIG. 11 CURVES SHOWING DISTRIBUTION OF PRESSURE IN OIL FILM OF JOURNAL BEARING

(Diameter of journal, 0.8733 in.; length of bearing, 1 in.; clearance, 0.002 in.; load, 25 lb.; speed, 218 r.p.m.; viscosity of oil, 18.6 centipoises; $ZN/P = 142$.)



A - CIRCUMFERENTIAL DISTRIBUTION

B - LONGITUDINAL DISTRIBUTION

C - DEVELOPED BEARING SURFACE, SHOWING LINES OF EQUAL PRESSURE

E - INTEGRATED PRESSURES SHOWING CHECK WITH LOAD

FIG. 12 CURVES SHOWING DISTRIBUTION OF PRESSURE IN OIL FILM OF JOURNAL BEARING

(Diameter of journal, 0.8733 in.; length of bearing, 1 in.; clearance, 0.002 in.; load, 25 lb. (0.1 in. off center); speed, 553 r.p.m.; viscosity of oil, 20.0 centipoises; $ZN/P = 389$.)

journal will take up some such position as is shown by the exaggerated cross-section in Fig. 3. In a given bearing of this type, the coefficient of friction, the eccentricity, and when the load is constant the pressure distribution all are dependent upon the operating variable ZN/P. Since there is no end leakage, the pressure will be constant along any element of the bearing; thus a single curve will represent the circumferential pressure distribution for a given bearing operating under a given load and at a given value of ZN/P.

A comparison of the pressure distribution for such a bearing and the pressures observed in one of these tests is shown in Fig. 13. In this figure curve A shows the circumferential distribution at the longitudinal center of the bearing obtained in test No. 6. Curve B represents the distribution as calculated from Harrison's equations for a bearing with no end leakage operating under the same conditions of load, speed, viscosity, and clearance and with the assumption that the lubricant is fed at atmospheric pressure to the point of minimum pressure on the bearing periphery.

From this figure it is apparent that the actual bearing has the pressure characteristics of a partial rather than a complete bearing. This fact together with the drop in pressure toward each end is the obvious explanation for the maximum pressure in curve A being so much greater than that of curve B.

Effect of Changes in Clearance. From Table 1, it is seen that the load, speed, and viscosity of the lubricant in test 3 are practically the same as in test 6, but that the clearance was much greater. Thus a comparison of the curves in Figs. 6*D* and 9*D* should give some indication of the effect of a change in clearance. These curves show that the pressure distribution in the bearing with the large clearance differs from that with the small clearance in the following characteristics: (1) The region of varying pressures extends over a greater arc, (2) the maximum pressure is greater and at a larger value of θ, (3) the rise in pressure as θ increases is more gradual and the drop more abrupt.

It should be noted that with the method used for supplying the lubricant, it is necessary for the oil to be drawn from the ends toward the center of the bearing. Thus a decrease in the width of the orifice through which the oil must flow might be expected to have a noticeable effect upon the amount of oil available to the bearing surfaces and may possibly be the reason why the area of varying pressure with the small clearance is less than that with the large. In some of the tests, however, an oil of lower viscosity was used. One reason for the use of this oil was the expectation that the lighter oil would flow more readily toward the center of the bearing and possibly give a comparative indication of the magnitude of this throttling effect. In this respect it is of interest to note that the pressure curves obtained with both oils with the same journal extended over approximately the same portion of the bearing surface.

Effect of the Value of ZN/P. Tests 6 and 7 were made with the same clearance and load and at practically the same value of ZN/P. In test 6 this value of ZN/P was obtained by a relatively low speed and high viscosity, while in test 7 a relatively high speed and low viscosity were used. A comparison of the curves shown in Figs. 9 and 10 for the respective tests indicates that the pressure distributions are practically alike. This is of interest in that it is what would be expected from dimensional analysis or the hydrodynamic equations.

Longitudinal Distribution. From the general trend of the longitudinal distribution curves it would appear that if perfect journal and bearing surfaces and perfect alignment had been attained the longitudinal pressure distribution at any value of θ in a region of positive pressure would have approximated a parabola symmetrical about the center of the bearing when centrally loaded. This of course assumes that the irregularities and scattering of observations as shown by the curves are caused by irregularities of the journal and bearing surfaces and by accidental misalignment in addition to experimental error.

Eccentric Loading. In tests 7 and 9 the conditions of operation were approximately the same, except that in test 9 the load was shifted to a point 0.1 in. from the longitudinal center of the bearing. A comparison of the pressure distribution curves for the respective tests shown in Figs. 10 and 12 indicates that the effect of eccentric loading is to increase the maximum pressure in the oil film and to move the point of maximum pressure away from the longitudinal center of the bearing in the same direction as the shift in the point of application of the load.

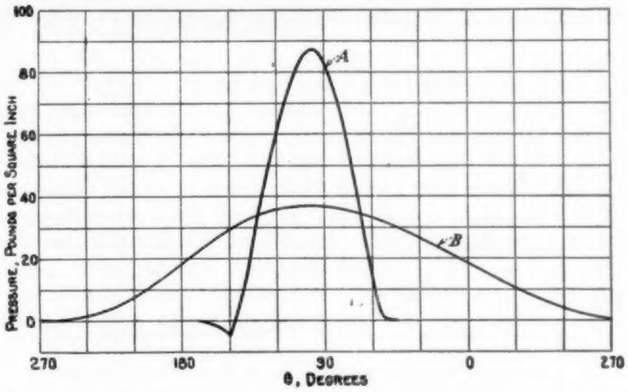


FIG. 13 COMPARISON OF EXPERIMENTAL AND CALCULATED DISTRIBUTION

(Curve A, circumferential distribution at center of bearing, test 6; curve B, circumferential distribution computed from hydrodynamical equations for same bearing under same operating conditions, but with no end leakage.)

SUMMARY

The characteristics of the film-pressure distribution indicated by this investigation may be summarized as follows:

1 The load applied to the bearing was in most cases wholly supported by the hydrostatic pressures in a film of the lubricant between the surfaces of journal and bearing.

2 In all the tests the pressure distribution was typical of a partial rather than a complete bearing.

3 The maximum pressure observed in the bearing for given operating conditions was usually considerably greater than the maximum pressure computed from hydrodynamic equations for a bearing with no end leakage operating under equivalent conditions.

4 The maximum pressures observed for the various conditions of operation were from two and one-half to four times the load per square inch of projected area.

5 The curve showing the longitudinal distribution along any element of the bearing in the region of positive pressure approximated a parabola in shape when the bearing was centrally loaded.

CONCLUSION

The results obtained in these tests of course apply only to the particular conditions present, and it would not be justifiable to predict that they are a reliable indication of the pressure distribution characteristics of bearings where the factors of construction or operation are markedly different.

Further steps toward a better understanding of the pressures developed in the oil films of bearings in actual service suggested by these tests are to determine the pressure distribution under various conditions favorable for a study of the following factors: the absolute size of the bearing, the bearing length, journal deflection, distortion, temperature gradients in the oil film, method of applying the lubricant, and partial or complete rupture of the oil film.

Discussion

D. P. Barnard.[6] From this work it seems evident that pressure-distribution studies represent the most effective tools for investigating bearing performance. Of particular interest to the writer are the virtually identical pressure distributions obtained at different speeds and oil viscosities, but at the same values of ZN/P. It would appear that as this work progresses it should lead to the deathblow for the time-honored pV-product or "rubbing factor," particularly for bearings operating at the higher rubbing speeds. Figs. 8 and 11 are of particular interest in that the longitudinal pressure gradients observed tend to corroborate the influence of ZN/P values on oil-pumping efficiency as observed by the writer several years ago.[7] Fig. 14 is a reproduction of the Fig. 2 presented at that time and shows an observed reduction in pumping efficiency from 0.3 to 0.2, which is about as might be expected by comparing the pressure contour diagrams in Figs. 8 and 11. The similarity of the pressure distributions observed in Figs. 9 and 10 also tends to verify this effect.

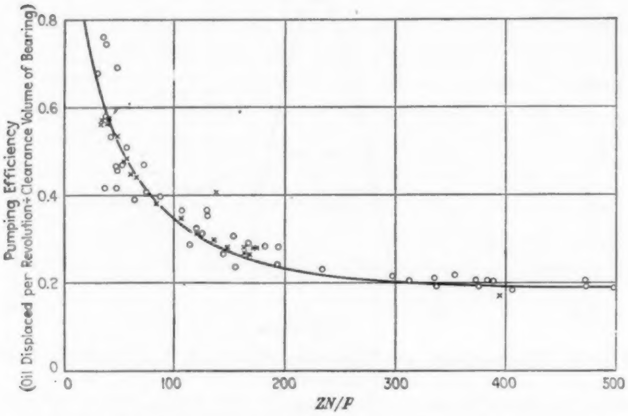


Fig. 14 Oil-Flow Characteristics of Complete Journal Bearing

(Oil-feed pressure, 10 lb. per sq. in.; diameter, 1 in.; length, 2 in.; ○ clearance of 0.0006 in.; ×, clearance of 0.0011 in.)

F. E. Cardullo.[8] The paper admirably serves to point out the great importance of end leakage in the hydrodynamic theory of journal bearings. The method used by the authors to obtain the pressure of the oil films at different points on the bearing surface is ingenious, but it necessarily produces hydrodynamic conditions different from those encountered in an ordinary bearing, and these conditions are unfavorable to lubrication.

In all practical bearings the oil is plentifully supplied by a longitudinal groove at a point where the pressure is such that the oil can readily enter the bearing and be swept into the region of maximum pressure by the revolving journal. In the case that the authors describe, such a groove is necessarily absent, and the oil has to find its way into the bearing through the clearance space. The conditions of pressure distribution which the authors have found therefore differ somewhat from the conditions of pressure distribution which would be found if an oil-distributing groove were provided. The large clearances used in the authors' tests are made necessary by the absence of such a groove.

It would be very interesting if the authors would add test results of the effect of length of bearing on the distribution of the pressure. All tests apparently were made with a fixed ratio of length to diameter.

In view of the forms shown by the circumferential distribution of pressure, it would be well for the authors to continue their experiments using a fairly wide distribution groove approximately opposite to the point of minimum of clearances, and to have available, not one, but a number of holes in the box, through which the oil pressure could be measured. The pressure could be read simultaneously or tiny valves be provided in the body of the box which would serve to connect any one of the several openings with the pressure gage. It would be a matter of great interest to compare the authors' results with the results obtained when an oil-distribution groove is used.

M. D. Hersey.[9] The A.S.M.E. Special Research Committee on Lubrication asked to have a limited number of experiments made with an oil-hole bearing also, reproducing conditions in the preceding investigation (effect of clearance and length). This was done, and showed agreement with dimensional analysis, also fair agreement with the submerged bearing. Can the authors supply particulars?

Dr. G. F. Rouse was asked by the committee to compute the hydrodynamic theory of the McKee test, neglecting circumferential flow instead of neglecting longitudinal flow, and found closer agreement than is shown by Fig. 13.

The writer believes the McKee data furnish the best expression available of full bearing characteristics, and therefore he questions the statement that tests agree better with partial bearing.

Dimensional analysis predicts the same relative pressure distribution for larger diameter bearings if strictly geometrically similar and running at the same value of ZN/P.

H. A. S. Howarth.[10] It seems that the authors "builded better than they knew" when they devised their testing machine. Their apparatus is beautifully simple and effective, and work upon it should be continued. A larger journal would have some obvious advantages, and higher unit loads should be tried. No amount of argument over the advantages of ZN/P will convince the layman so well as tests on a full-size model.

The authors are very modest in suggesting a limited practical value of such tests as they have conducted. They need not regret their inability to measure film thicknesses and friction. After all is said, are not the pressure distribution and loads safely carried the most important data of all for checking the hydrodynamical theory?

The exploration of pressure distribution by means of one hole has its advantages and its disadvantages. First, the resulting curves of recorded pressures will appear consistent, whether right or wrong, because the perfection or imperfection of the bearing surface about the hole will be present for every observation. Second, the bearing surface opposed to the journal is not the same for all axial positions of the one observation hole. A great degree of accuracy is necessary, therefore, in the bearing surface and also the journal.

The symmetry of the curves of "longitudinal distribution" of pressure indicates a fair degree of accuracy in the second respect. The θ-values below the curves of "circumferential distribution" of pressure, however, seem to indicate some defect in the bearing surface near the observation hole, which means that the first requirement is not fulfilled.

An examination of the curves of pressure distribution around the journal should enable one to place the position of the resultant with fair accuracy by inspection. The directions of such resultants appear to be about 10 deg. farther around the journal than the authors place their applied-load lines.

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[6] Research Laboratory, Standard Oil Company (Indiana), Whiting, Ind.

[7] "Oil Flow in Complete Journal Bearings," by D. P. Barnard, *S.A.E. Journal*, vol. 17, no. 2, August, 1925.

[8] Chief Engineer, G. A. Gray Company, Cincinnati, Ohio. Mem. A.S.M.E.

[9] Research and Development Department, Vacuum Oil Company, Inc., Paulsboro, N. J. Mem. A.S.M.E.

[10] General Manager, Kingsbury Machine Works, Inc., Philadelphia, Pa. Mem. A.S.M.E.

One cannot explain this by the turning of the bearing shell to balance the friction, because such turning would serve only to increase the apparent discrepancy. The only explanation that occurs to the writer is the presence of a bulge ahead of the observation hole, tending to reduce the gage pressure below the average pressures that actually existed along that element of the cylindrical surface.

Further evidence of the possibility of the presence of the defect mentioned is the moderate length of the arc between the zero and 20-lb. isobars in Figs. 4-*C* to 7-*C*, and the considerably reduced arc that appears in Figs. 8-*C* to 11-*C* for the case of smaller clearance. One would expect the smaller clearance to increase the range of the positive pressures. Also, if there were a bulge producing negative effect just ahead of the pressure-measuring hole, its influence would be more pronounced with the smaller clearance.

Now with regard to Fig. 13, the authors have in it taken their shot at the hydrodynamical theory, as it relates to a full bearing. Why bring that up? Unless an experimenter is able to produce a film with measurable pressures throughout the whole circumference of the journal, he should let the poor old full bearing alone. He should compare his test results with theoretical films that are really comparable with them. There are now plenty of data on theoretical oil films of partial bearings, especially if the influence of side leakage be neglected.

The practical value of tests of this kind is considerable, when we are sure our measured pressures give a fair picture of those in the film. The authors have shown, for example, the parabolic transverse distribution of pressures. One can therefore say that if side leakage did not take place, the test load could have been carried by pressures two-thirds as great as those measured. A reference curve could therefore be drawn with ordinates two-thirds as high as those found in the mid-plane of the bearing.

To this reference curve we could fit a theoretical pressure curve of a suitable partial bearing, produced by similar speed and viscosity. The eccentricity which this theoretical curve represented would be another reference point. We would know that the actual eccentricity was greater than this reference value, because if it were not greater, the test pressures could not have been produced.

One could go a step further in analysis of the test results, and assume, what is probably close to the fact, that the shape of the actual test film would be close to the optimum investigated by Mr. Kingsbury. One could therefore apply his charts to the tests and determine what was the probable film shape and minimum thickness. By doing this one would discover how closely the most fully developed theory will fit in with actual test conditions.

It is for these reasons that, as said in the beginning, the authors "have builded better than they knew." They have shown an easy way to obtain test data that can be used to determine the degree of reliability of the hydrodynamic theory as extended and interpreted by Mr. Kingsbury and other investigators.

Adolph A. Moline.[11] The authors have presented a very interesting method for determining hydrostatic pressures in oil films. It gives a very close approximation to the conditions in an actual bearing.

Comparing curves C, Figs. 4, 6, and 12, the lines indicating the higher pressures in C, Fig. 4, are fairly symmetrical about the center line of the load application, but are shifted a considerable distance away from the center line of load application in C, Figs. 6 and 12. Possibly this is due to a slight deflection in the shaft. Will the authors please comment on this point?

The writer has observed peak pressures of from two to three times the load per square inch of projected, which agrees with the results found by the authors.

On large machines, in order to reduce the starting torque, oil under high pressure is fed under the journal through a small opening in the bearing shell. If a gage is placed in this oil-feed line, the pressure indicated, after the starting period, is that near the peak load. Any lowering or fluctuating of this pressure would indicate trouble in the bearing. Some operating companies use this idea and claim it is superior to that of the more commonly used thermostat or thermometer.

S. J. Needs.[12] The idea of shifting the journal to make an entire exploration with one pressure hole is quite ingenious, but it has the undesirable feature that each different circumferential pressure reading is made at a different position of the journal in the bearing, hence on a different bearing. The effect of this on the pressure curves is probably slight at the loads and speeds under which the experiments were conducted, particularly if the bearing was round and of uniform diameter.

The general slope of the isobaric plots would seem to indicate that the axes of the bearing and journal were not parallel in the running position.

Apparently, convection is relied upon for oil circulation. The position of the thermometer would hardly afford an accurate indication of the bearing temperature. However, if there is no change during a run, knowledge of the temperature is not necessary except for comparison with other runs and theory.

In Fig. 13 the authors compare the circumferential pressure distribution at the longitudinal center of the bearing, as found in a particular test, with the calculated theoretical distribution curve. At first glance the theory appears inadequate. Curve *A*, however, represents the maximum pressure values found, while *B* represents the mean pressures, constant over the bearing width at a given value of θ. It would seem more reasonable to the writer to compare the mean of the test results with the theoretical mean, bearing in mind that the theory neglects side leakage. This the writer has done in Fig. 15. The curve *B* is taken directly from Fig. 13, while the curve *A′* represents the mean of the longitudinal distribution curves given in Fig. 9. The mean values were found by planimeter.

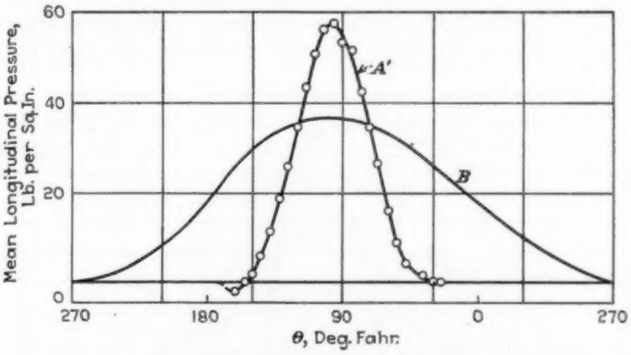

Fig. 15 Comparison of Calculated and Experimental Distribution

Neglecting side leakage, curves *A′* and *B* are now directly comparable, and their lack of similarity opens a most interesting field of discussion. The classical hydrodynamic theory appears inadequate, yet Professor Kingsbury's experiments on an air-lubricated bearing have established the theory on a rather firm foundation, at least for an incompressible lubricant. All the available test results on complete journal bearings, using an incompressible lubricant, indicate that the oil film has not the strength to resist the tension set up by diverging surfaces on the "off" side of the bearing. Hence, rupture and zero pressures. The greater the clearance and load, the shorter the useful or load-carrying arc of bearing. Possibly with reduced clearances and loads, the authors will find an increased angular length of bearing having positive pressures, the oil being introduced at the theoretical point of minimum pressure. This conceivably could be extended to a check of the hydrodynamical theory.

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[11] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[12] Research Engineer, Kingsbury Machine Works, Inc., Philadelphia, Pa. Jun. A.S.M.E.

It is interesting to further examine the curves A' and B. The area under each represents their respective load-carrying capacity. A' from the test bearing, subject to side leakage, has an area of 2.70 sq. in. to a certain scale. B was computed from an equation uncorrected for side leakage. The area of B is 5.48 sq. in. to the same scale as A'. From A' the effective bearing arc is about 128 deg. This gives a length-to-width ratio of 0.977. The side-leakage factor for partial cylindrical bearings of this length-to-width ratio has been found to be 0.494. Correcting B by this factor gives $5.48 \times 0.494 = 2.71$ sq. in., about the same capacity as the test bearing.

It is the writer's belief that the classical theory is an unfailing guide when its requirements of film continuity are met.

A. E. Flowers.[13] It would appear from the omission of any mention of oil grooves in either the bearing or journal that no oil grooves were used. The authors might clarify this point and also answer the question as to whether they observed any experimental evidences of instability of the pressure readings.

It would also be of interest to know whether any of their test conditions approached the critical conditions for film breakdown and whether any checks were made of surface smoothness or efforts made to get the bearing well run-in before taking observations.

R. Plank.[14] The extensive research revealed in the paper means a significant progress in the knowledge of the phenomenon of lubrication, for there was until now but little known about the end leakage and the real distribution of the pressure in bearings. At the same time this research gives a new confirmation of the hydrodynamical theory, particularly if its results are supplemented by the observations of E. Schneider, who in my institute compared some years ago the coefficients of friction of real bearings with the results of the theory. Both papers are in accordance also in confirming better the theoretical equations of Gümbel than those of Sommerfeld, which disagree by different conditions of integration. According to the last research there are only very small negative pressures in the bearings, while the equations of Sommerfeld led to greater negative pressures. Indeed, the theoretical curve B in Fig. 13 runs only in the sphere of positive pressures, but it deviates more from the distribution of pressure experimentally ascertained than the values calculated according to Gümbel. The writer has prepared another Fig. 13 (see Fig. 16) which contains besides the former curves A and B a new one C following the equations of Gümbel.

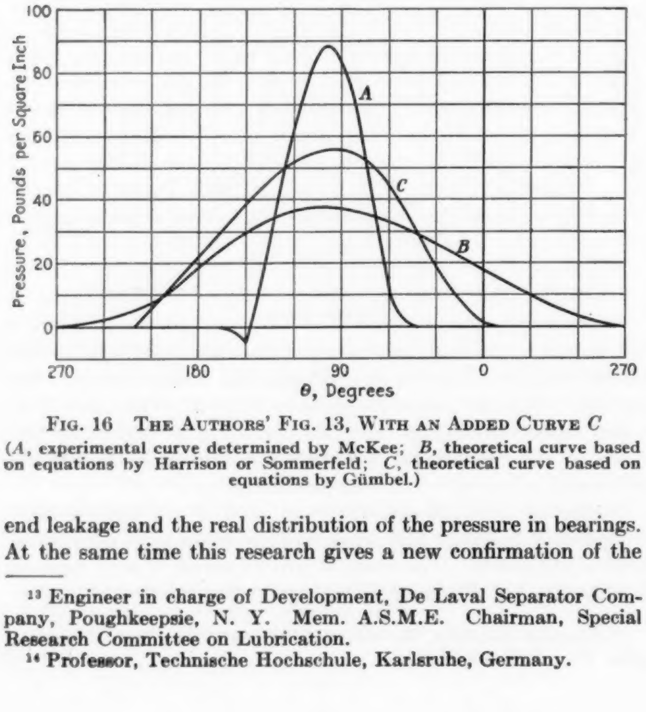

Fig. 16 The Authors' Fig. 13, With an Added Curve C

(A, experimental curve determined by McKee; B, theoretical curve based on equations by Harrison or Sommerfeld; C, theoretical curve based on equations by Gümbel.)

Henri Weiss.[15] The writer would like to ask the authors the following questions: (1) They have shown the existence of a region of negative pressure in the neighborhood of $\theta = 150$ deg. Is this in accord with the fact previously deduced from the study of the parabola of speed of liquid filaments—namely, that at the exit of the wedge of oil certain filaments of oil attain a speed greater than the peripheral velocity of the journal? This fact has been emphasized particularly in a paper by Brillié. (2) The study of the authors shows that the complete journal bearing acts as regards the distribution of hydrostatic pressures as foreseen by theory for incomplete bearings Does not this explain a result found by Mr. S. A. McKee in a previous paper in a study of frictional torque? With small clearance the frictional torque is greater than indicated by hydrodynamic theory; the more so, the less the clearance. Would not the fraction of the

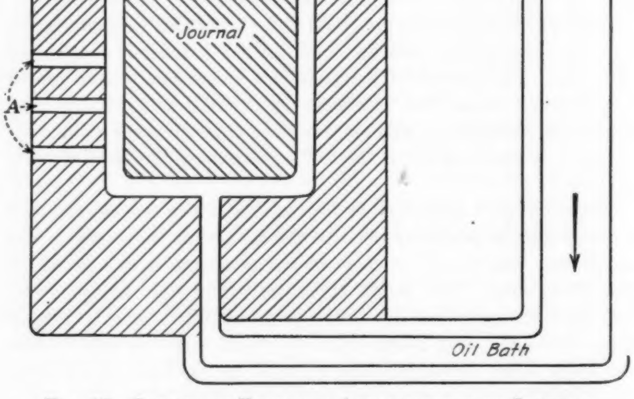

Fig. 17 Sketch of Proposed Arrangement of Journal

(A, holes for pressure gage; only one open during a measurement.)

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[13] Engineer in charge of Development, De Laval Separator Company, Poughkeepsie, N. Y. Mem. A.S.M.E. Chairman, Special Research Committee on Lubrication.

[14] Professor, Technische Hochschule, Karlsruhe, Germany.

[15] Director, L'École Nationale Supérieure du Petrole, University of Strassbourg, France.

torque indicated by the theory be produced by the oil between $\theta = 0$ deg. and $\theta = 180$ deg.? Would not the excess of torque be produced by the effect of the oil situated between $\theta = 180$ deg. and $\theta = 360$ deg., other things being equal, the influence being the greater the narrower the clearance. In other words, could not this phenomenon be thus explained, without including any effect of oiliness? (3) Given that the main fact put in evidence by the authors is that the distribution of pressure occurs as theory predicts for incomplete bearings, and given that the authors attribute this in all probability to the loss of oil at the sides. It would be of the greatest interest to confirm this explanation by a test. Would it not be possible for the authors to have constructed an arrangement where the loss of oil would be considerably reduced at one side without changing anything else in the apparatus? Perhaps that would permit seeing whether near this side the pressure was distributed in a less localized manner around the periphery of the journal, as Fig. 13 of the paper would indicate. A sketch of the proposed arrangement is given in Fig. 17.

A. Sommerfeld.[16] It is very interesting to see from Fig. 13 how markedly the actual pressure distribution deviates from the theoretical. Besides, the theoretical pressure distribution in my old work (Fig. 15) is not symmetrical about $\theta = 90$ deg., except at extremely high velocities, but is somewhat displaced toward larger angles; i.e., just as in Fig. 13 of the paper. It is especially noteworthy that the region of negative pressure, which can in fact also be indicated by the theory, can be found experimentally. It would perhaps be advantageous, even if somewhat difficult, to extend the theory to the case of a journal of finite length, in order thus to obtain the end effect also theoretically.

May the writer point out that his papers on friction of lubricants, together with papers of N. Petrow, O. Reynols, and A. G. M. Michell, are reprinted in Ostwald's "Klassiker" No. 218, Leipzig, Academy Press, 1927, price about 3 marks.

Authors' Closure

With reference to the discussion regarding the type of apparatus used, it should be noted that the design of a practical apparatus for measuring film pressures in a journal bearing must include some compromise between maintaining conditions as in actual service and obtaining sufficient data for a complete picture of the pressure distribution.

It was realized that with the apparatus used in these experiments the significance of the data was dependent to a considerable extent upon whether or not the journal and bearing were true cylinders, as was mentioned by Mr. Howarth. It was realized also that the lines of flow in the oil film were changed to some extent by the method of applying the lubricant. Its chief advantage, however, is that a sufficient number of observations could be taken to obtain a more complete picture of the pressure distribution than could be advantageously obtained with the two types of apparatus that have been commonly used in this kind of work—namely, the fixed bearing with numerous pressure taps and the swiveled bearing with several taps along the length.

Some indication that the method of applying the lubricant from the ends instead of at the center of the bearing might not materially affect the pressure distribution was obtained in the experiments mentioned by Mr. Hersey. In these tests pressure observations were taken at one point directly opposite the load ($\theta = 90, 0.5\, L$) at various loads and speeds on an oil-hole bearing of different diameter but of approximately the same L/D ratio. The results indicated that the ratio of film pressure at this point to the pressure on the projected area was of the same order of magnitude in both bearings.

It was not the intention of the authors to attempt either to credit or discredit any one of the various mathematical interpretations of the hydrodynamic theory. The chief reason for including curve B in Fig. 13 was to emphasize the fact that the observed film pressures (other than atmospheric) extended over only a portion of the bearing periphery, although from the standpoint of construction the bearing extended the full 360 deg. and had no oil grooves or relieved spaces.

With reference to curves C in Figs. 4, 6, and 12, it is believed that the differences in symmetry in the pressure lines in Figs. 4 and 6, noted by Mr. Moline, are due chiefly to differences in alignment. It was stated in the paper that the alignment between load and journal was obtained by observing the pressures at various points; thus it was chiefly a matter of "cut and try." It is quite probable that better alignment was obtained in test No. 1 (Fig. 4) than in test No. 3 (Fig. 6). With reference to Fig. 12, this test was run with the load off-center, and some of the lack of symmetry is possibly due to the fact that there was more surface available for carrying the load on one side of the center line of load than on the other.

Referring to Dr. Flowers' questions, there were no oil grooves in either the bearing or journal and the bearing was not run in before the tests. Also no evidences of instability of the pressure readings were observed. No attempts were made to check the surface smoothness. In one test, however, it was found that one part of the journal had become scratched to an extent hardly visible to the eye. This abrasion was discovered by noting the presence of the sharp "hooks" in the pressure lines shown in Fig. 7, curve C. These "hooks" were at the same position as the scratch relative to the length of the journal and did not occur in the tests that were made after the journal had been polished to remove the abrasion. With reference to the question of film breakdown, in test No. 4 (Fig. 7) there was evidence of abrasion, in test No. 8 (Fig. 11) the value of ZN/P was lower than in the other tests, and in test No. 9 (Fig. 12) a low-voltage lamp placed in series with journal and bearing gave an indication of film breakdown at some settings of the pressure tap. It is of interest to note that in curves E of Figs. 7, 11, and 12 the ratios of the area under the curve to the area of the rectangle are appreciably lower than in the other figures.

A possible explanation for the presence of negative pressures other than that given by Mr. Weiss is that the negative pressures were developed beyond the point of closest approach where there was a diverging rather than a converging wedge of lubricant. With reference to the question by Mr. Weiss regarding the frictional torque, the results of previous experiments[17] by the authors seem to indicate that end leakage or drop in pressure in the film toward the ends of the bearing is one possible explanation as to why the experimental friction curves should be higher than those indicated by the theoretical equations of Sommerfeld, since the observed increase in coefficient of friction is greater for the short than for the long bearings.

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[16] Professor, Institut fur Theoretische Physik, University of Munich, Germany.

[17] See Figs. 3 to 7 in paper on "Friction of Journal Bearings as Influenced by Clearance and Length," by S. A. McKee and T. R. McKee, *Trans. A.S.M.E.*, paper APM-51-15, pp. 161–171; *Mech. Eng.*, vol. 51, p. 593.

RP-54-9

Fluxing of Ashes and Slags as Related to the Slagging Type Furnace[1]

Progress Report of the A.S.M.E. Special Research Committee on Removal of Ash as Molten Slag From Powdered-Coal Furnaces[2]

By P. NICHOLLS[3] and W. T. REID,[4] PITTSBURGH, PA.

This paper is presented as a report for the Special Research Committee on the Removal of Ash as Molten Slag From Powdered-Coal Furnaces. The primary object of the investigation was to obtain data on the fluxing of coal-ash slags to enable users of slagging-type boilers to increase the fluidity of the deposited ash should it not be possible to tap at available furnace temperatures. Studies were made using a platinum-wound furnace, supplemented by a gas furnace with a 32-in. by 28-in. hearth. A standard of fluidity desirable for easy tapping was established, and the results were expressed as the temperature necessary to produce this fluidity, designated the flow temperature. Values for flow temperature in terms of the chemical composition of the ash, or ash plus flux, were determined for a range of ashes found in American coals, and for additions of the following fluxes: iron ore, limestone, dolomite, fluorspar, saltcake, steel slags, and some less important minerals. It is deduced that limestone or dolomite would be the most economical material to use for slagging-type boilers as now built, and from a diagram the quantity of flux can be computed. The investigation was carried beyond present requirements for slagging-type boilers and to flow temperatures below 2400 deg. fahr. A complete diagram is given for coal ash-lime-iron showing the flow temperature plotted against composition.

Introduction

The work here reported may be considered as the Second Progress Report presented for the A.S.M.E. Special Research Committee on Removal of Ash as Molten Slag From Powdered-Coal Furnaces. The first report[5] covered a limited investigation at the Toronto Station of the Ohio Light and Power Company; the main object of that investigation was to study the slagging factor when burning coals, the ashes of which had higher fusion temperatures than that of the strip-mine coal used in the regular operation. In these tests it was not found possible to tap the furnace when using a semi-bituminous coal having an ash-softening temperature of 2700 deg.,[6] and a bituminous coal with an ash-softening temperature of 2470 deg. was on the border line.

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At the time when these tests were made there was only one other station, the Huntley Station, Buffalo, N. Y., in which slagging boilers were in operation, although a number were being installed. The boilers at Toronto in which these tests were made had been converted from the dry-bottom type, and the installation was more or less experimental. While it was recognized that the furnace temperatures could be increased and the location of the tap hole and the method of tapping improved, yet it was felt desirable that there should be the added safeguard of the ability to add some material which would act as a flux and increase the fluidity of the bed so that the slag could be tapped at available furnace temperatures, or which could be used in emergencies. As a consequence, at the request of the committee, the Bureau of Mines undertook to make the study of fluxing the second phase of the investigations of this method of utilizing coal; from the bureau's standpoint this was a logical continuation of other studies of coal ash.

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[5] Sherman, Nicholls, and Taylor, "Study of Some Factors in Removal of Ash as Molten Slag From Powdered-Coal Furnaces," Trans. A.S.M.E., 1929, paper FSP-51-51, p. 399.

[6] Fahrenheit scale used throughout this report.

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OUTLINE OF PROBLEM

The original conception of what amount of work would be required and how far it should be carried was quite indefinite. The plan agreed to with the committee was to have three stages, as follows:

(1) *Small-scale laboratory tests*, by which the fluxing effects of admixtures were to be determined under the ideal conditions of intimate mixture and uniform temperature.

(2) *Large-scale laboratory tests* in a gas-fired furnace having a slag hearth $2^1/_2$ ft. by 3 ft. to check the conclusions arrived at from the small-scale tests and to study the mass action.

(3) *Plant tests*, to confirm the laboratory tests and to develop methods for adding the flux.

Whether they should be completed in the order given or should be overlapped was left to development, but it was desirable that some rough ideas of the possibilities of fluxing should be obtained as early as possible. Fortunately, the need for the use of fluxes has not arisen, and although two stations have asked for advice, yet so far as is known all stations have been able to raise the slag to the flow temperature, and lack of fluidity in the slag has not been a major difficulty. As a consequence, the expense involved in conducting full-scale tests at the stations has not been necessary.

The question arose early in the investigation, as to what limitations should be put on the order of fluxing required. Based on the experience at the Toronto Station, it could be said that if the fluid temperature of the deposited ash as determined by the standard cone test was 2550 or lower, then it could be tapped from a furnace the design of which permitted a temperature of the bed not less than that obtained at Toronto. This limited the immediate study to slags more refractory than the 2550 deg., and the fluxing to that necessary to reduce refractory ashes to fluidity at 2550 deg. It was evident, however, that a knowledge of the fluxing of coal ash was desirable for contingencies other than that of a general fluxing of the whole of the slag bed. The following possible applications were suggested:

(*a*) That a flux would be useful, and perhaps sometimes necessary, to make more fluid the portion around the tap hole.

(*b*) That a flux might be used occasionally over the whole bed to make it possible to drain the bed more thoroughly before the boiler was shut down for repairs; the use of a flux would permit this to be done without the risk of heating the permanent structure of the bed abnormally.

(*c*) That the slagging-bottom principle might be applied to smaller furnaces and boilers in which the furnace temperature was materially lower than is customary in larger furnaces.

(*d*) That a knowledge of the possibilities of fluxing coal ash was of importance in fields other than that of boiler furnaces.

In addition to the foregoing, it was early seen that the insight these studies gave on the properties of coal ash and their slags, and the reactions of other materials with them, would be very useful and would supply data on a subject on which knowledge was very meager. A direct application is the problem of absorbing the fly ash in the bed, and an indirect one the reaction between slags and refractories.

The investigation has therefore been broader than what were considered the immediate necessities, and studies have been made of slags having relatively low fluid temperature, the fluxing studies often being continued to fluidity at 2400 deg. and lower.

When this investigation was started there was very little information available, and this only of a general type. The most pertinent were the deductions which could be made from the fusibility of coal ash as determined by the cone method of test, a large number of values for which have been determined by the Bureau of Mines and others. These data have been, however, of little use unless accompanied by the chemical analysis of the ash, and relatively few analyses have been made. In addition, so far, no close relationship has been established between the chemical analysis and the fusibility; also, the only fluxing constituents in coal ash which vary over more than a few per cent are the lime and iron oxide.

Although many workers have studied slags associated with other industries, such as those of pottery, glass, iron, and other metals, yet each field is peculiar to itself, and data obtained for one are of little help to the others. The relationships between the properties of all types of slags are quite complex, and no general laws have been established. Also, determinations at high temperatures are time-consuming and expensive in apparatus, and if work is done on a truly fundamental basis the time may run into years. This is true where the slags studied have only three constituents, and would be more so for coal ashes which contain a larger number of constituents, independent of others which may be added as fluxes.

It has therefore been the endeavor to limit the work to the order of accuracy that would accompany its application and, as far as possible, to avoid investigations on the reactions or underlying causes. This was considered sufficient for the purpose since in no application of fluxes to slagging-bottom furnaces could the exact analysis of the deposited ash be known, and the composition and the temperature will vary over the area of the bed.

This does not mean, however, that a more thorough study of slags and their properties is not desirable and would not be useful to the operator, and the temptation has continually been present to extend the investigation into side lines. The following section briefly discusses some of the general properties of slags and may be helpful in the better understanding of the problem.

PROPERTIES OF SLAGS

All slags contain silica (SiO_2) or alumina (Al_2O_3), or both, together with other materials, depending on the industry the slags belong to; the distinction between a glass and a slag is merely one of name. A slag resulting from the fusing of a coal ash will show by chemical analysis iron, either as ferric oxide (Fe_2O_3), if the slag has been subjected to an oxidizing atmosphere, or as ferrous oxide, if reducing, and usually both; also metallic iron may be present if the atmosphere be highly reducing; lime (CaO), magnesia (MgO), and the alkalis (Na_2O and K_2O); small quantities of sulphur and phosphorus or their oxides; and also, perhaps, small quantities of oxides of other metals may be present, of which titanium oxide (TiO_2) is the more common.

A slag is not a mixture of these oxides, but some portions of them are combined, and form simple or complex minerals; some such compounds in coal-ash slags may be mullite ($3Al_2O_3.2SiO_2$) fayalite ($2FeO.SiO_2$), anorthite ($CaO.Al_2O_3.2SiO_2$), and many others. The determination of the minerals in a slag is directly accomplished by microscopic examination of the cold slag, which will show their crystals embedded in the matrix—that is, glass. The forms in which the oxides exist in the glass cannot be determined directly, but sometimes may be deduced; in general, the glass can be thought of as a solution of materials in each other, the materials either being in compound or simple form.

When a slag is heated, the crystals and glass do not necessarily melt at the same temperature, and consequently its softening (that is, its fluidity) may not change uniformly with increase in temperature. Also, new combinations may be formed and existing ones broken into simpler forms, and heat may be required or evolved for each such change. Little is known on this subject, and it becomes very complex as the number of oxides increase beyond two. It is of interest here only as it underlies the action of melting.

Slags in general soften and increase their fluidity gradually, and the relation of their increase in fluidity to increase of temperature differs materially. A small change in composition may cause a large change of the temperature to produce the earlier softening, but the effect of small change is less as the whole composition of the slag is more complex. Such small changes have less effect at stages of higher fluidity, such as that at which the slag will run freely.

Fig. 1 represents diagrammatically some forms of melting which different slags may have. The vertical scale represents fluidity; the axis 0 represents a solid material; the line marked Flow is the fluidity which was standardized for these tests as necessary to permit of easy tapping of the slag; the line Penetration is the fluidity necessary to permit a rod to be poked into a slag without too much effort; the line Oil is the fluidity of Mobiloil B at 68 deg., which is a rather heavy summer oil.

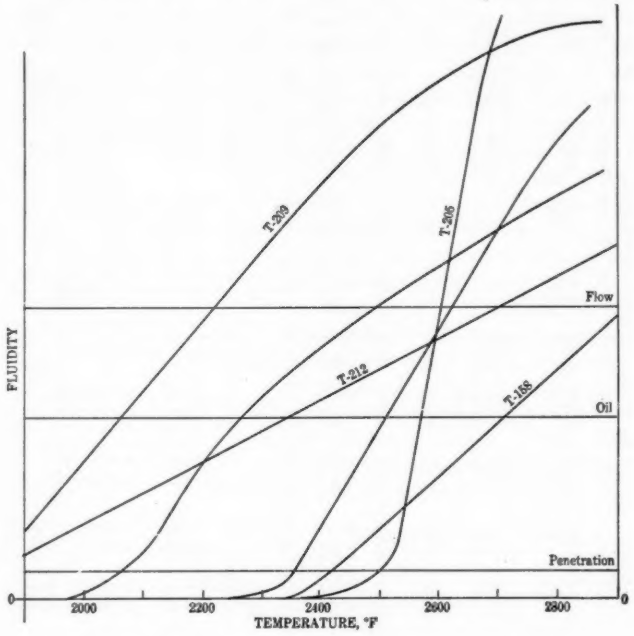

FIG. 1 DIAGRAM SHOWING HOW SLAGS MAY MELT

The melting curves are based on the values for Penetration and Flow obtained for some of the slags used in this investigation; the shapes of the curves are not based on definite data. They show how some slags may soften gradually and others rapidly; thus, T-212 starts to melt at 1520 deg., but does not flow freely till it reaches 2700; slag T-205 flows at 2590 deg., but the temperature only has to fall to 2505 for the slag to become stiff enough to make penetration difficult. These characteristics materially affect operation and tapping.

METHOD USED IN THE SMALL-FURNACE INVESTIGATIONS

The first problem in the small-furnace work was to devise a method for determining the temperature at which a slag, or slag with flux, would be fluid enough to flow when tapped; it therefore required the fixing of a standard fluidity (or viscosity) and a method for its measurement. Several methods previously used were considered, particularly the apparatus developed by Feild[7] for determining the viscosity in absolute units, but it was believed that a simpler method involving less expense would suffice. Fulton,[8] working on copper slags, states that he determined the flow point by stirring with a heavy platinum rod, but makes no statement on the order of accuracy he obtained. Such a method was tried, using a sirup for the liquid, and it was shown that two observers could closely duplicate their own and the other's results. This method was therefore adopted, because of its simplicity and promise of relatively rapid progress.

Fig. 2 shows the platinum-wound electric furnace as now constructed. It has no particularly novel features. The currents through the inner and outer windings are under full control, and the temperature at the outer winding can be measured and thus avoid burning out of the nichrome wire. Originally there were nichrome windings on the two flat ends, but these gave trouble and were abandoned.

The platinum crucible is shown in place. It is $^{3}/_{4}$ in. in diameter and $1^{1}/_{2}$ in. high; it has two sets of three buttons welded on, which center the crucible in the tube, the nominal diameter of which is 1 in. There are two crucibles to a furnace, so that one can be cleaned while the other is in use. A Tork clock has been used to switch the current on at some set time so as to heat up the furnace before the investigator arrived. A temperature regulator has also been used in some of the tests, but normally work can be advanced quicker by hand adjustment of the outside resistances.

The measurement of the temperature of the slag is an important factor. The straightforward method would be to insert a thermocouple into the slag. There are, however, a number of objections to this as a regular practice, as the temperature must be followed continuously; it would mean that the couple would be in all the time and would be in the way. The temperature was measured as shown in Fig. 2; a platinum, platinum-rhodium couple threaded through a $1^{1}/_{4}$-mm. inside and 2-mm. outside porcelain tube lay next to the inner wall, with the junction at the average height of the slag in the crucible. It follows that the temperature of the slag in the crucible would lag behind and be lower than that registered by the couple. The relationship between the two

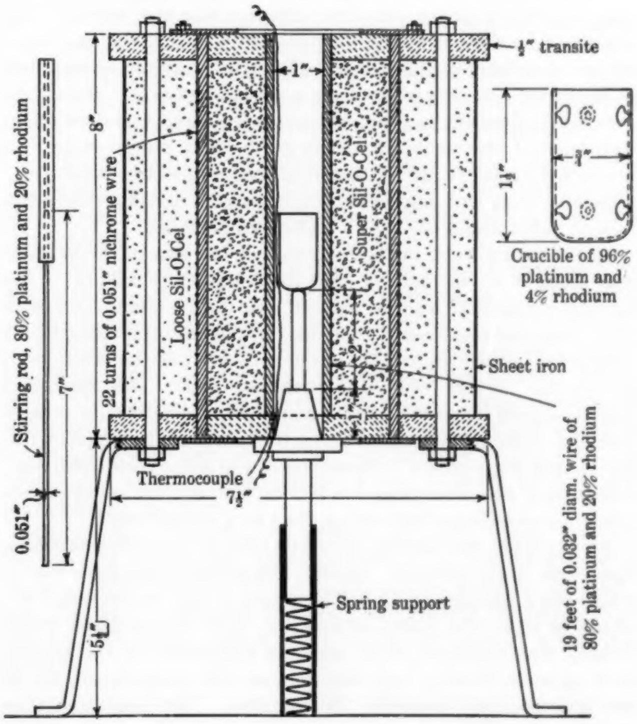
½" transite
1"
8"
Stirring rod, 80% platinum and 20% rhodium
22 turns of 0.051" nichrome wire
Loose Sil-O-Cel
Super Sil-O-Cel
2
1
7"
1½"
¾"
Crucible of 96% platinum and 4% rhodium
Sheet iron
19 feet of 0.032" diam. wire of 80% platinum and 20% rhodium
Thermocouple
7½"
0.051"
Spring support
5¼"

FIG. 2 PLATINUM-RESISTANCE FURNACE

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[7] A. L. Feild, "A Method for Measuring the Viscosity of Blast-Furnace Slag at High Temperature," U. S. Bureau of Mines Tech. Paper 157, 1916.

[8] C. H. Fulton, "The Constitution and Melting Points of a Series of Copper Slags," Trans. A.I.M.E., vol. 44, p. 764, 1912.

temperatures was determined at intervals, so that the recorded value could be corrected to that of the slag.

Test Procedure With Small Furnace

It was first necessary to establish a standard of fluidity, which should be such that slag would run freely when tapping. The standard was based on a sample of a slag collected during the tests at the Toronto Station which gave satisfactory flow and the temperature of which was known at the time of tapping. Some of this slag was melted in the crucible, brought to the afore-stated observed temperature, and its "feel" when stirred studied. From this a standard feel was learned. The feel at which the fluidity reached the value determining the temperature at which the slag would flow was judged by the ease of stirring, the rapidity with which the slag flowed in behind a cold stirring rod drawn through the melt, and by the quantity of slag adhering to the platinum rod when withdrawn from the melt.

This in effect established a standard of fluidity and viscosity, but the method of test does not itself give numerical values. The corresponding viscosity was determined by an indirect method, and found to be 5 poises.[9] The determination was made by using castor oil, of which the viscosity-temperature relation is known. The 5 poises is the viscosity of castor oil at 83 deg.

In subsequent studies of slags the operation consisted of determining the temperatures at which each slag had the standard fluidity. This temperature is designated as the flow temperature.

In addition to the flow temperature for each slag, the temperature at which it showed the first softening was also noted. This point can be defined as the temperature at which the slag has a degree of softening which would allow the platinum stirring rod to penetrate the slag when using a pressure of 6 oz. This temperature is termed the penetration temperature; this determination has not the same exactitude as the flow temperature. In many melts a penetration point would occur at a certain temperature, and then the slag would harden again, and another penetration point would occur at a higher temperature.

In the initial work the temperature of the slag in the crucible was brought to as high as 2800 deg., but early failure of the furnace required this to be reduced and the maximum temperature to be used was fixed at 2630 deg., which limitation lengthened the period between failures to as much as five months' continuous use of the furnace when the wire was new. This limitation seemed logical because at that time it was thought that reasonable operation would require that a furnace could be tapped without having the average temperature of the slag exceed 2630 deg., although it was known that a higher temperature could be obtained by carrying a high rating for a long enough time.

All the slags and fluxing materials used in the electric-furnace tests were finely ground. Briefly, the general procedure was to weigh out a sample, usually of 10 grams, of ash or slag, add two-thirds of it to the platinum crucible, heat to the point where fluidity was obtained, cool, add the remainder of the charge, heat again to fluidity, and maintain at that temperature till all gas was evolved—generally 20 minutes. The melt was then cooled until solid. In reheating for determinations of points, a high rate was used until the approach of the penetration temperature was indicated by a stickiness of the slag surface, after which the heating rate was decreased to about 25 deg. per min. Further heating was continued at the same rate until approaching the flow temperature, when the rate was decreased to about 10 deg. per min. The melt was then allowed to cool, a similar heating

TABLE 1 LIST AND PROPERTIES OF SLAGS AND ASHES USED IN THE INVESTIGATION

1 Laboratory No.	T-66	T-102	T-136	T-157	T-158	T-185	T-190	T-195	T-197	T-198	T-199	T-205	T-209	T-211	T-212	T-227	T-228	T-242	T-246	F	G	K	N	P	S
2 Received (mo., year)	11–28	11–28	2–29	5–29	6–29	6–29	6–29	11–29	6–29	7–29	7–29	10–30	10–30	12–30	12–30	3–31	4–31	5–31	6–31	....	....	....	....	....	...
3 Quantity, lb.	2	2	2	2	1000	1000	600	600	600	600	500	1000	10	150	150	10	1000	180	50	....	....	....	....	....	...
Ash fusions, deg. fahr.:																									
4 Initial	2130	2000	2200	1950	2715	2240	2055	2270	2110	2560	2760	2260	....	1925	1520	1920	....	....	2210	2020	1960	2090	2110	2100	1930
5 Softening	2360	2080	2350	2020	2835	2400	2225	2470	2210	2660	2870	2320	2050	2130	1860	2010	....	....	2320	2190	2070	2210	2280	2270	1990
6 Fluid	(2630)	2430	2440	2320	+2940	2640	2680	2730	2470	2820	2940	2590	2260	2700	2320– —2490	2070	....	....	2680	2390	2290	2330	2460	2430	2170
7 Penetration temperature, deg. fahr.	2425	2465	2355	2450	2425	2430	2430	....	2335	....	....	2505	1890	2215	1885	2290	2115	2355	2345	2365	2305	2445	2385	2295	2145
8 Flow temperature, deg. fahr.	2590	2590	2615	2580	+2800	+2630	+2630	+2630	2520	+2630	+2630	2590	2230	+2630	+2590	2545	+2590	+2590	+2630	2565	2450	2595	+2590	+2590	2285
Chem. analysis:																									
9 Loss on ignition	8.0[a]	1.0	—0.3[b]	—0.9	1.1	1.0	0.7	1.6	1.5	1.1	2.6	0.8	1.6	0.1	—0.1[b]	....	....	....	....	....	....	....	....	....	...
10 SiO_2	48.2	46.9	51.5	40.8	57.5	50.6	57.7	67.7	47.6	53.9	58.1	38.1	39.2	54.2	61.2	38.6	54.1	52.8	52.8	46.3	44.3	47.9	56.4	54.4	45.7
11 Al_2O_3	24.2	21.9	28.2	22.0	30.9	29.5	17.4	18.5	24.0	30.6	28.9	21.3	17.5	22.8	13.4	15.4	15.4	27.1	28.7	24.5	17.1	23.5	23.0	24.2	14.5
12 Ratio SiO_2 to Al_2O_3	1.99	2.14	1.86	1.85	1.86	1.72	3.32	3.66	1.98	1.76	2.01	1.79	2.24	2.38	4.57	2.51	3.51	1.95	1.84	1.89	2.59	2.06	2.44	2.25	3.14
13 Total Fe as Fe_2O_3	13.5	27.7	13.1	36.4	5.4	13.5	18.1	7.3	17.1	9.2	5.9	42.0	5.6	8.8	8.5	37.5	10.0	12.9	8.5	19.9	24.9	22.2	12.9	11.9	16.5
14 Fe_2O_3	....	....	....	....	....	....	....	....	....	....	....	9.04	....	....	....	11.7	....	....	0.8	....	....	....	....	....	...
15 FeO	....	....	....	....	....	....	....	....	....	....	....	22.63	....	....	....	19.3	....	....	6.9	....	....	....	....	....	...
16 Fe	....	....	....	....	....	....	....	....	....	....	....	5.46	....	....	....	3.1	....	....	....	....	....	....	....	....	...
17 CaO	4.4	1.5	4.7	2.3	0.9	1.8	1.7	1.3	6.5	1.7	0.9	1.4	18.3	1.3	1.6	7.6	2.5	3.1	5.4	3.6	10.1	2.4	2.1	3.8	17.5
18 MgO	0.4	0.4	0.9	0.9	0.6	1.1	1.1	0.9	0.9	1.2	1.0	0.1	6.6	0.7	0.6	0.8	2.5	2.0	1.8	0.8	0.4	0.6	1.3	1.0	0.7
19 Alkalis, by difference	1.2	0.5	....	....	3.3	....	2.6	....	....	2.2	2.5	....	....	....	....	3.5	7.3	1.8	3.3	....	....	....	....	....	....
20 Na_2O	....	....	....	0.2	....	1.2	....	0.8	0.3	....	....	0.3	4.4	8.6	11.8	....	....	....	....	0.4	0.6	0.4	0.5	0.6	0.9
21 K_2O	....	....	....	0.2	....	0.9	....	0.7	0.4	....	....	0.6	3.9	0.8	1.0	....	....	....	....	2.1	2.0	1.6	2.5	1.9	2.0
22 SO_3	....	....	....	0.2	0.3	0.2	0.6	....	....	....	....	0.1	0.3	0.1	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.2	0.2
23 Cr_2O_3	....	....	....	....	....	....	....	....	....	....	....	....	....	1.1	0.2	....	7.5	....	....	....	....	....	....	....	...
24 V_2O_5	....	....	....	....	....	....	....	....	....	....	....	....	....	0.3	0.2	....	Nil	....	....	....	....	....	....	....	...
25 TiO_2	....	....	....	....	....	....	....	....	....	....	....	0.8	....	....	....	....	....	....	....	1.0	0.9	1.0	1.0	1.0	0.9
26 P_2O_5	....	....	....	....	....	....	....	....	....	....	....	0.12	3.0	....	....	....	....	....	....	0.17	0.13	0.22	0.05	0.27	0.24

[a] Mainly carbon. [b] Gain.

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[9] A poise is that viscosity which a liquid possesses when a force of 1 dyne is necessary to cause two parallel liquid surfaces of unit area and unit distance apart to slide past one another with unit velocity. ("Outline of Theoretical Chemistry," Getman, p. 91, 4th edition.)

TABLE 2 ORIGIN OF SLAGS AND ASHES IN TABLE 1

Laboratory No.	T-66	T-102	T-136	T-157	T-158	T-185	T-190	T-195	T-197	T-198	T-199
From station	Toronto	Toronto	Toronto	Huntley	Columbia	Trenton Channel	14th Street	Montaup	Lakeside	River Rouge	Fordson
Location	Toronto, Ohio	Toronto, Ohio	Toronto, Ohio	Buffalo, N. Y.	Cincinnati, Ohio	Detroit, Mich.	New York, N. Y.	Somerset, Mass.	Milwaukee, Wis.	Detroit, Mich.	Stone, Ky.
Type of furnace	Wet	Wet	Wet	Wet	Dry	Dry	Dry	Dry	Dry	Dry	Dry
Kind of fuel	Pittsburgh No. 8	Stripmine, Ohio	Pittsburgh No. 8	Pittsburgh seam	Coal	Coal	Semi-bituminous	Sewall seam and Roderfield seam	Coal	Coal	Bituminous Kentucky

Laboratory No.	T-205	T-209	T-211	T-212	T-228	T-227	T-242	T 246	P	F-K	G-N-S
From station	Deepwater	Incinerator	Louisiana Steam Products, Inc.	Louisiana Steam Products, Inc.	Louisiana Steam Products, Inc.	State Line	Hell Gate	Glen Lyn	Mine	Mine	Mine
Location	Deepwater, N. J.	Buffalo, N. Y.	Baton Rouge, La.	Baton Rouge, La.	Baton Rouge, La.	Chicago, Ill.	New York	Glen Lyn, Va.	....	....	....
Type of furnace	Wet	Wet (?)	Wet	Wet	Wet	Wet	Wet	Wet	....	....	....
Kind of fuel	Miller "B" seam	Refuse	Oil waste and coke	Oil waste and coke	Oil waste and coke	Kincaid, Ill.	Semi-bituminous	Pocahontas and Clinch Valley	Bituminous Pennsylvania	Bituminous Ohio	Bituminous Illinois

cycle made, and the average of the values for three such cycles taken as the penetration and flow temperatures for that slag.

When fluxing, each addition of flux was added to the cold solid-slag surface, the melt brought not more than 30 deg. higher than the flow temperature of the previous run, and the points determined in triplicate. The first flux addition, when the flow temperature of the slag had been determined in a previous run, was made by intimately mixing the finely ground slag or ash with the flux on glazed paper before transferring to the crucible. Obviously, this was not possible when the slag had once been melted down in the crucible.

Cleaning was accomplished by mechanically removing as much slag as possible while fluid; soaking in hydrofluoric acid for 36 hours at room temperature generally removed the remainder. Fusion with potassium pyrosulphate was resorted to in extremely stubborn cases.

Slags and Ashes Used

There was the option of making synthetic mixtures of slags or of using slags or ashes from boiler plants. The latter method was adopted, as it would give more direct data and also because of the size of the samples that would be required. Table 1 lists the slags and ashes which have been used; Table 2 is supplementary and gives their origin. They were selected to cover a range of fusibility from low to high. Since few slagging-bottom boilers were in operation at that time, those samples collected at the start of the investigation were from dry-bottom furnaces; it is not known how far the selective action in the combustion chamber may affect the composition of the ash deposited, but the ash from a dry-bottom furnace served equally well.

Table 2 shows that all the ashes and slags were from coals, except T-209, which was from a refuse incinerator, and T-211, T-212, and T-228, from oil sludges and coke. The samples designated by letters were obtained by ashing coals selected from a series previously used in a clinkering investigation.[10]

An examination of the chemical analyses illustrates the difficulties involved in classifying by composition; however, for the coal ashes, the three principal factors are the silica-alumina ratio, total iron as Fe_2O_3, and the CaO.

The silica-alumina ratio of the coal ashes listed in Table 1 ranges from 1.72 to 3.66, with the greater number having a ratio from 1.8 to 2; this is representative of ash in coals from the Eastern and Central states; no coals from the Pacific Coast states are included, but from the few analyses of their ashes which are available, it would appear that the silica-alumina ratio is much lower and sometimes as low as 0.7. The conclusions reached in this report must not be applied to such ashes.

Materials Considered for Fluxes

In considering possible fluxes the main consideration was the price. The use of a flux would be an additional cost, not only in its purchase and handling, but also in most plants in the increased weight of slag to be tapped and disposed of. The desirable characteristics can be listed as:

(1) Low price
(2) Well distributed to avoid cost of a long haul
(3) Effectiveness to be greater as the cost is higher
(4) Should not volatilize at furnace temperatures
(5) Should not be a fine powder so that the loss to the stack would be high
(6) Should not damage refractories
(7) Should store well, and preferably in the open
(8) Should be delivered preferably in the size to be used, and thus avoid crushing
(9) It might be an additional economic advantage if the flux were normally a waste material, but it should be obtainable without investment for its reclamation.

All known sources were consulted for information on materials that might be considered. The following are those which were given consideration, and are discussed without reference to their effectiveness:

Iron Oxide (Fe_2O_3). The forms in which this is obtainable are mill scale and iron ore. Mill scale, however, is not a regular article of commerce, and the steel mills reuse it; it would only be available in limited amounts and under special conditions, and would be more suited for experimental purposes than for regular use. What is known as the red ore would be more suitable as a flux because it is more friable than the black ore and would be ready for use as delivered. The Superior ore is about 74 per cent Fe_2O_3, with the remainder SiO_2 and Al_2O_3 in varying proportions and moisture.

It is quoted at $4.50 a ton at the lower lake ports; the cost per ton of Fe_2O_3 would therefore be $6.10, or, say, $7 delivered. Since a certain portion of the Fe_2O_3 will be required to satisfy the silica and alumina in the ore, the available effectiveness as Fe_2O_3 is still further reduced. A typical analysis of a Superior ore is:

Fe_2O_3	SiO_2	Al_2O_3	CaO+MgO	MnO	Moisture
74	10	2	1	0.6	11

An average cost per ton of available Fe_2O_3 can be taken as $7.20.

A slag formed in connection with the Aston process of manufacturing wrought iron has properties which should make it a quick acting flux when iron oxide is required. It contains about 85 per cent iron oxide, mainly FeO, and 10 per cent silica, so that the slag has about 40 per cent fayalite (Fe_2SiO_4). It melts at about 2150 deg. and is very fluid. It was not tested and it is likely that its cost would prohibit its use except for special purposes.

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[10] Fieldner, Selvig, and Nicholls, "The Clinkering of Coal Ash as Related to Laboratory Fusibility Determinations," Trans. A.S.M.E., 1928, paper FSP-50-6, p. 53.

Limestone ($CaCO_3$). Only the CaO would enter into the slag, and the CO_2 would be driven off; the calcining absorbs heat; the heat used per pound of limestone would be 771 B.t.u. in addition to the sensible heat. Limestone is a material readily purchasable in all sizes and is good for handling and storage. The $^3/_4$-in. or $^1/_2$-in. screenings should be suitable; prices are variable, but in carload lots can be taken as 75 cents per ton at the quarry or as $1.50 delivered. The CaO forms about 55 per cent of the limestone, so that the cost per ton of CaO would be about $2.75.

Dolomite. Dolomite ranges from $CaCO_3.MgCO_3$ to deposits which contain only a small percentage of the magnesium carbonate. It is in the same class as limestone, and the relative availability of the two materials depends on the location; MgO forms 47.8 per cent of $MgCO_3$. The cost of dolomite at the quarry can also be taken as 75 cents, but the screenings tend to be somewhat cheaper. Assuming $1.50 delivered, the cost per ton of equivalent CaO.MgO would be about $2.95.

Lime. As calcined stone, whether as CaO or CaO.MgO, lime need not be considered, since it is not suitable for handling for use as a flux. There are, however, a number of industries which have lime waste as a hydrate or carbonate. Such industries are soda works, magnesia manufacturers, the sugar industry, and others. Some plants might be so situated that it would be economical to use lime in these forms.

Magnesite ($MgCO_3$). Magnesite is mined mainly in the West, and its cost is high compared with limestone and dolomite.

Fluorspar (CaF_2). Fluorspar is used as a flux in the steel industries. It is mined in Illinois and Kentucky and is quoted regularly in the trade journals at this time at from $17 to $18 per ton at the mine. It is also mined in Nevada and New Mexico, where it is quoted at $12 to $13 because of the location relative to demand. The greater part is shipped in gravel size; the ore averages about 95 per cent CaF_2; another grade with 85 per cent CaF_2 is quoted at $13. It melts at 2510 and has a flow temperature of 2600 deg.; its density when solid is 3.1, which is higher than that of coal-ash slags, and therefore it tends to sink in molten slag. It decrepitates when thrown in the furnace, and there is a tendency for the fluorine to combine with the silica to form SiF_4, which is a gas; these two properties introduce the possibility that the use of fluorspar as a flux may cause some slagging of the refractories of the walls.

Silica (SiO_2). If silica was found to be effective, it would not be necessary to use the pure grade employed in the glass industry, but rather sands high in silica. The composition of sands is very variable in different localities. It is desirable that the sand should be low in alumina, but the presence of iron would be an advantage. The percentage of silica in some molding sands is as high as 97; in sea sands 90 and in river sands 83 are average values. The cost of sand would depend on whether a suitable one were produced in the district. Quotations on building sands vary by location, but $1.50 per ton might be expected.

Low-Grade Clays. It was thought that low-grade clays high in iron were good possibilities, but no records were found except for clays too high in silica and alumina. There may be deposits better suited, but since they are of no value for other purposes they have not been investigated or mined. Two well-known clays used for glazing are the Albany and Michigan slips; the latter contains 6.5 per cent Fe_2O_3, 8.5 CaO, and 5.3 alkalies. These clays are, however, too expensive for consideration, and no clays were used in the tests.

Saltcake (Na_2SO_4). Saltcake is used in the soda and glass industries. It is obtained as a by-product of the manufacture of hydrochloric acid by the action of sulphuric acid on common salt. It does not volatilize, but melts at 1600 deg., is quite fluid, and being less dense than molten coal-ash slag, will float on the top of the bed. Its cost has been taken at $17 per ton delivered. The Na_2O is the fluxing constituent of saltcake and can be taken as 46 per cent of the weight of the saltcake.

Common Salt (NaCl). Salt was considered, but rejected as a possibility because of the ease with which it volatilizes, which would mean loss of flux and, in addition, possibility of added trouble in erosion. There is, however, some doubt whether these deductions are justified, and the salt might be added in such size that it would not volatilize as it does in small granules. The price of salt at the mines is given as $3 to $4 a ton, and in containers at New York as $12.

Borax ($Na_2B_4O_7.10H_2O$). Borax was considered because it is commonly used as a flux in laboratory work. Its cost is too high for use in service.

Steel Slags. The slags from the open-hearth process are high in metallic oxides and low in alumina. One, termed "tapping slag," is stated to be a waste product, and so its fluxing effect was studied. Representative analyses are given later, and it will be noted that it introduces two oxides not included in the foregoing fluxes—namely, those of manganese and phosphorus. No price can be assigned to this waste material, but to the cost of reclamation would be added that of crushing.

Coal-Ash Slags. The obvious solution of trouble caused by an insufficiently fluid slag from a given coal might seem to be to mix with that coal one having a low-fusion ash, or, as was done in the Toronto tests, to use the latter coal at intervals. The committee considered that this solution might be objectionable from an operating standpoint. The same end would be obtained by using as a flux the slag from a furnace burning the low-fusion ash coal, but again it is not likely that the two coals would be used in the same plant or that another plant using such a coal would be located within economic shipping distance.

TESTS MADE IN SMALL ELECTRIC FURNACE

The greater part of the work has consisted of determination of flow temperatures in the small electric furnace. A number of months were required at the start on more or less guess trials to obtain some preliminary knowledge of the fluxing effect of economically possible material on typical ashes. As data were accumulated it became possible to systematize the order of the tests to some extent, but it was not possible to completely cover the area of the field—that is, to try each flux with a complete range of possible ash compositions. As the work progressed it also became necessary to make side investigations to check previous results or to try to obtain explanations for disagreements, or for departures from the expected. The results obtained from the various tests will be dealt with independent of the order in which the tests were made.

RELATION BETWEEN FLOW TEMPERATURE AND ASH FUSIBILITIES

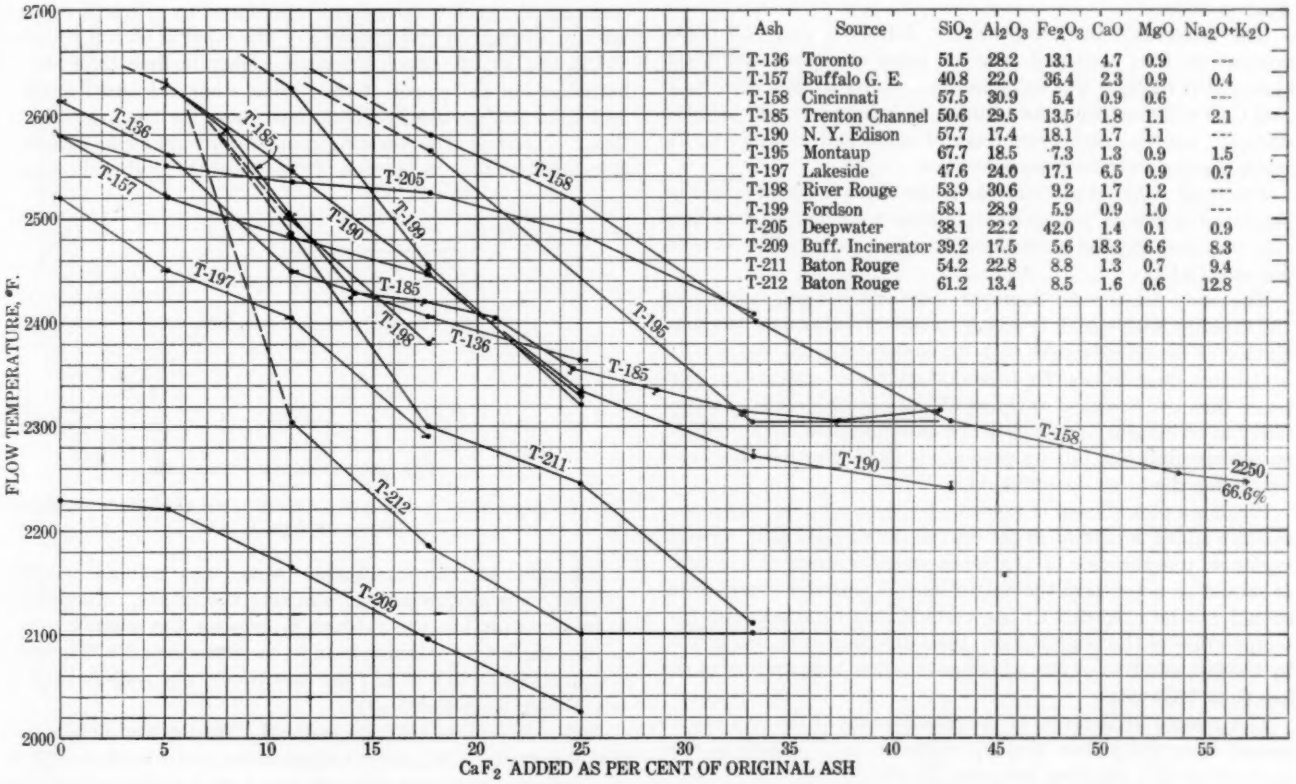
It would be convenient if one could predict the flow temperature—that is, the temperature the slag must attain for free flow—from the standard ash-fusion temperatures. Analyses of the values in Table 1 for items 4, 5, and 6 for the ash fusions and 7 and 8 for the measures used in this investigation do not show good relationships. That these relations cannot be predicted without a full knowledge of the properties of the slag can be understood from the discussion on Fig. 1. Although the fluid temperature of the cone test is nearer to the condition of the slag represented by the flow temperature, yet, because of the technique of the test method, neither it nor the initial temperature is as reliable a measurement as is the softening temperature, which is used in the comparisons shown in Table 3.

Table 3 includes all the ashes of Table 1 in which both values for the softening and flow temperatures are given; a number of the flow-temperature values of Table 1 are marked +2630, because to prolong the life of the furnace the temperature was

TABLE 3 COMPARISON OF FLOW AND SOFTENING TEMPERATURE

Ash	Flow temperature	Softening temperature	Difference	Fe_2O_3, per cent	Computed flow temperature
T-136	2615	2350	265	13	2505
T-66	2590	2360	230	14	2630
S	2285	1990	295	16	2290
T-197	2520	2210	310	17	2525
F	2565	2190	375	20	2550
K	2595	2210	385	22	2590
G	2450	2070	380	25	2405
T-102	2590	2080	510	28	2560
T-157	2580	2020	560	36	2620
T-205	2590	2320	270	42	..

not run higher. Table 3 shows that the flow temperature averages from 280 to 580 deg. greater than the softening temperature, which means that at the softening temperature the slag is far from the free-flowing state. Feild[11] experimented on a limited number of coal ashes and found that the viscosity represented by the softening temperature was about 20 poises, but he recognized that the state of the slag might not be the same in the two methods of test.



Ash	Source	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	Na_2O+K_2O
T-136	Toronto	51.5	28.2	13.1	4.7	0.9	---
T-157	Buffalo G. E.	40.8	22.0	36.4	2.3	0.9	0.4
T-158	Cincinnati	57.5	30.9	5.4	0.9	0.6	---
T-185	Trenton Channel	50.6	29.5	13.5	1.8	1.1	2.1
T-190	N. Y. Edison	57.7	17.4	18.1	1.7	1.1	---
T-195	Montaup	67.7	18.5	7.3	1.3	0.9	1.5
T-197	Lakeside	47.6	24.0	17.1	6.5	0.9	0.7
T-198	River Rouge	53.9	30.6	9.2	1.7	1.2	---
T-199	Fordson	58.1	28.9	5.9	0.9	1.0	---
T-205	Deepwater	38.1	22.2	42.0	1.4	0.1	0.9
T-209	Buff. Incinerator	39.2	17.5	5.6	18.3	6.6	8.3
T-211	Baton Rouge	54.2	22.8	8.8	1.3	0.7	9.4
T-212	Baton Rouge	61.2	13.4	8.5	1.6	0.6	12.8

FIG. 3 EFFECT OF ADDITIONS OF FLUORSPAR ON FLOW TEMPERATURE OF VARIOUS COAL ASHES

The fifth column of Table 3 gives the iron in the ashes as per cent Fe_2O_3, and the table lists the ashes in order of increasing Fe_2O_3, which shows that the difference increases with increasing Fe_2O_3. A plot showed that the flow temperature is fairly closely given by the empirical equation: flow temperature = softening temperature + 60 + 15 (per cent Fe_2O_3).

The last column of the table gives the computed values, which are close to the experimental values considering the variations in the composition of the ashes. Ash G departs most, but it contains 17.5 per cent CaO. Ash T-205 (Deepwater) is entirely out of line, but it will be shown later that this ash is peculiar in all respects.

Tests of more ashes would be required to confirm the truth of this empirical formula, particularly for ashes having low iron; it should be noted that the softening temperature of the deposited ash must be used, and not of the ash of the coal burned, because that ash will not be of the same composition as the ash which is deposited on the slag bed.

Results Under Headings of Fluxes

The less important fluxes will be covered first. The difficulty of presenting data which cover as many variables as are included in the composition of the ashes and the fluxes will be recognized, especially as the material used as a flux may also be in the original ash, and as combinations of fluxes were also tried.

There are also two optional ways in which the proportions of the constituents of the ash and the flux may be expressed:

(*a*) To call the original ash 100 and to express the flux as "per cent of original ash."

(*b*) To express in any mix the ash plus flux as 100, so that each addition of flux is evaluated as "flux, per cent of total mix."

Method (*a*) is best suited to service application, but method (*b*) is the better to show relationships, especially when the material used as the flux is already in the ash.

There is a further complication when the flux contains material which is lost, such as $CaCO_3$, which loses its CO_2. Formulas for converting from one system to another will be appended to the report.

Borax. The tests with borax were limited to its use with ash T-212 (Louisiana). The flux was added as $Na_2B_4O_7.10H_2O$, but was calculated as the dry sodium biborate, $Na_2B_4O_7$. On this basis, two additions of flux were made, each being of 5$^1/_2$ per cent of the ash weight. The first addition did not produce fluidity at the limiting temperature of 2630; the second gave a slag fluid at 2425; but as doubt existed as to the absorption of the borax, the melt was maintained at 2590 for two hours. At that time a considerable quantity of borax had volatilized from the melt and condensed on the cooler parts of the furnace, badly slagging the porcelain protecting sheath around the thermocouple. A redetermination of the flow temperature showed it to have increased to 2545. This volatility of the flux showed that it would be unsuited for use in a furnace, and it was not studied further.

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[11] A. L. Feild, "A Method for Measuring the Viscosity of Blast-Furnace Slag at High Temperature," U. S. Bureau of Mines Tech. Paper 157, 1916.

Feldspar. The feldspar used analyzed:

SiO_2	Al_2O_3	Na_2O	K_2O	CaO	MgO
67.4	20.0	10.03	1.04	0.5	0.20

Feldspar was tried early in the investigation with ash T-158 (Columbia), but with additions of feldspar up to 11 per cent the flow temperature was still higher than 2630. Later it was tried with T-197 (Lakeside), which had a flow temperature of 2520 deg.; however, each addition of the feldspar increased the flow temperature until, with 17 per cent added, the slag was no longer fluid at 2630.

The ashes used had, respectively, 5.4, 17.1, and 11.9 Fe_2O_3 content, so that a normal range of ashes was tried and it was shown that feldspar was not effective. Later studies with Na_2O and CaO will show that the addition of limestone would help the feldspar, but the high percentage of silica and alumina in the feldspar removes it from consideration.

Fluorspar. Although fluorspar is the mineral which would be employed in service, yet in the small-scale tests c.p. calcium fluoride was used because commercial fluorspar contains but 85 to 95 per cent CaF_2.

The early trials with fluorspar were encouraging because of the rapidity with which it acted, its effectiveness, and the continuity of the effectiveness with increasing amounts of flux.

Calcium fluoride has the disadvantage for investigational purposes that the chemical analyses of slags containing it are very likely to be in error; not only is the determination of the CaF_2 in such slags difficult, but its presence is likely to cause incorrect values for the other constituents.

Fig. 3 is a plot of most of the tests using calcium fluoride alone; the flux added is plotted as per cent of the original ash; for convenience, the analyses of the ashes are also given on the plot. It should be noted that the dotted lines in this and other figures should not be extended to the vertical axis to obtain the flow temperature of the original ash; their direction as shown is fixed by the last addition of flux which gave a flow temperature of not less than 2630 deg.

The general irregularity of the effectiveness of the flux can be judged from the jumble that the curves make. Attempts were made to obtain relations between the curves and the chemical composition of the ashes, but if there are direct relationships they are too complicated to express the flow temperature as a function of the composition.

The effectiveness of a flux in a plot such as Fig. 3 is shown by the slope of its curve, and is greater as the slope is greater. Considering the upper portions of the curves T-195, T-199, T-211, and T-212, all have steeper slopes, and all also have a higher silica-alumina ratio. T-158 and T-199 have the same iron and lime, but the latter has the higher ratio and the steeper slope. Little is known about the form the calcium fluoride takes in a slag, but it would seem that higher silica increases the fluidity for moderate additions of CaF_2, though with larger additions the curves show that each pair meets.

If one considers only the coal ashes and omits T-211, T-212, and the incinerator ash T-209, and also again excludes the Deepwater ash T-205, the best approximate rule that can be given from the tests—which are few compared with the number of variables—is:

The lowering of flow temperature by the addition of fluorspar up to 20 per cent is 12 deg. for each 1 per cent of fluorspar added for ashes having up to 2 silica-alumina ratio and 16 deg. if the ratio is 3.

The foregoing statement is based on using a commercial fluorspar containing 80 per cent calcium fluoride. Such a rule is very rough and presupposes a knowledge of the flow temperature of the ash. If the composition of the ash is known, one should be able to make a fair estimate from the data given in Fig. 3.

In all tests calcium fluoride was found to be quick acting, and its effectiveness tends to remain more constant with increase in the amount added than occurs with other fluxes. The additions for ash T-158 were continued to determine the lower limit of temperature, which was about 2200 deg. Calcium fluoride in excess of that required to give the standard fluidity gave a very fluid slag, indicating that this flux is useful for flushing the hearth.

Some tests also were made in which a mixture of fluorspar and limestone was used. These tests did not show any marked increase of effectiveness because of the combined flux. That the lowering of the flow temperature produced by either is not materially affected by the presence of the other is shown by ashes T-136 and T-197. Both these are higher in lime than the remainder, but the slopes of the curves when calcium fluoride is added do not materially differ from those for the ashes low in lime. Therefore, the lowering produced by fluorspar and limestone can be taken as the sum of the lowerings produced by each.

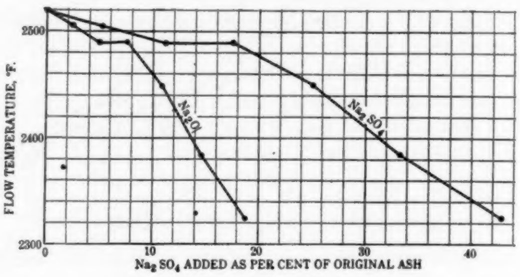

FIG. 4 EFFECT OF ADDITIONS OF Na_2SO_4 ON THE FLOW TEMPERATURE OF ASH T-197

Saltcake. In the small-scale tests, c.p. sodium sulphate (Na_2SO_4) was used in place of commercial sulphate. Commercial saltcake as used in the Leblanc process for the manufacture of sodium carbonate contains from 94 to 98 per cent Na_2SO_4, which is equivalent to 40.9 to 42.7 per cent Na_2O; c.p. sodium sulphate contains 43.6 per cent Na_2O.

Although sodium sulphate melts at 1700 deg. and does not decompose by itself at furnace temperatures, yet in contact with the slag definite reactions take place, the most notable one being the formation of sodium silicates by combination with silica. The speed with which this reaction takes place is dependent largely on temperature; below 2230 deg. no reaction occurs, and below 2460 deg. reaction is very slow. The rate of reaction between sodium sulphite and silica is greater at low temperatures than with sodium sulphate, so that in a reducing atmosphere or in contact with carbon higher reaction velocities will probably occur. This fact is made use of in the manufacture of glass, where charcoal or a high-grade coal is added to accelerate the saltcake-silica reaction.

From a knowledge of the system $CaO\text{-}SiO_2$, minimum viscosity

TABLE 4 SILICA-SALTCAKE

—Percentage composition—				
SiO_2	Na_2O	CaO	Molecular ratio	Flow temperature, deg.
74.6	25.4	..	$Na_2O.3SiO_2$	2490
72.6	27.4	..	$Na_2O.2.75SiO_2$	2430
70.6	29.4	..	$Na_2O.2.5SiO_2$	2380
66.0	34.0	..	$Na_2O.2SiO_2$	2360
57.2	29.5	13.3	$Na_2O.0.5CaO.2SiO_2$	2355
50.5	26.0	23.5	$Na_2O.CaO.2SiO_2$	2280
60.4	20.8	18.8	$Na_2O.CaO.3SiO_2$	2250

above the temperature for complete solution has been shown to exist at the compound $CaSiO_3$, and if similar conditions exist in the system Na_2O-SiO_2, minimum viscosity should occur at the compound sodium-disilicate ($Na_2Si_2O_5$).

Melts to indicate the reliability of this argument were made up; the flow temperatures were determined in the usual manner. Table 4 indicates the results.

Steadily decreasing flow temperatures are observed with each step toward the expected ideal composition. In a slag such combination may be difficult to obtain, but no doubt does occur when lime and alkalis are in the proper proportions to combine with silica not otherwise held in combination with other slag constituents. Attempts made to procure reaction between saltcake and limestone were unsuccessful.

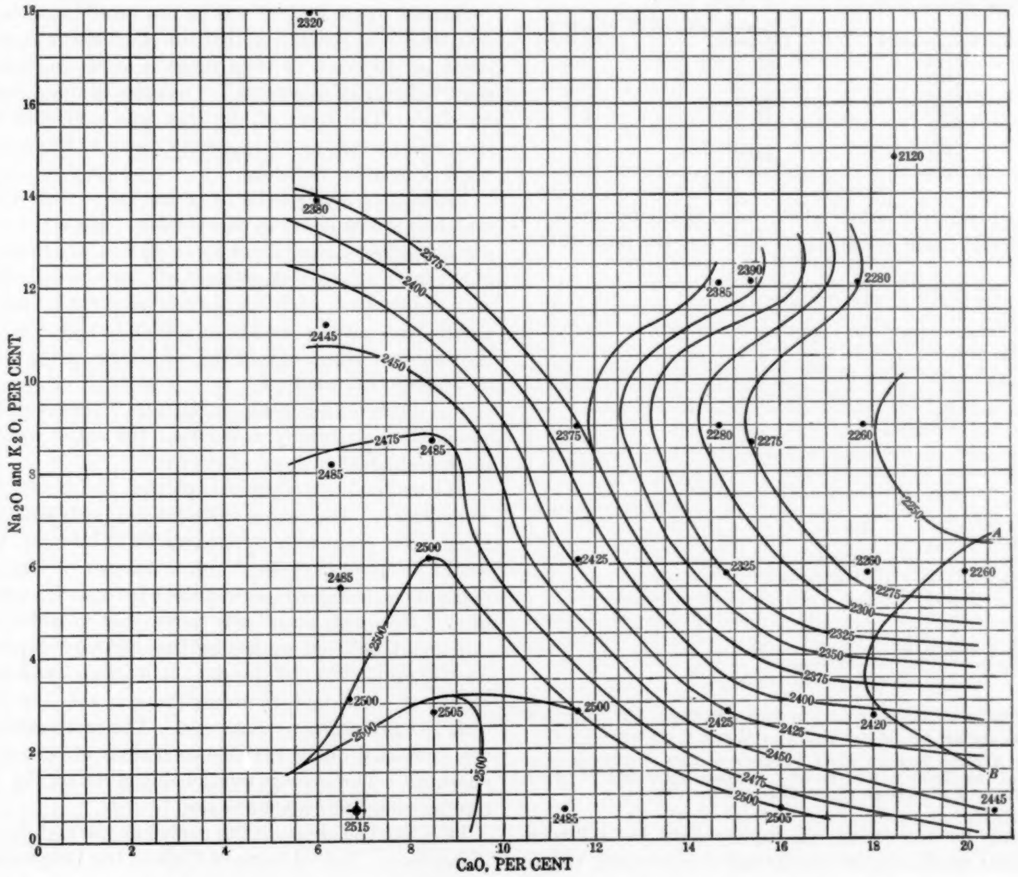

FIG. 5 PLOT SHOWING EFFECT OF CHANGE IN CaO AND Na_2O CONTENT FOR ASH T-197
(Having a constant ratio of SiO to Al_2O_3 to Fe_2O_3 of 2.78 to 1.40 to 1.00. SiO_2 + Al_2O_3 + Fe_2O_3 + CaO + Na_2O + K_2O = 100 per cent. MgO limited to less than 0.8 per cent; K_2O less than 0.4 per cent.)

Only a few tests were made using sodium sulphate alone. Fig. 4 shows the fluxing with ash T-197, in which the Fe_2O_3 was 17.1 per cent and the CaO 6.5 per cent; the flux added is expressed as per cent of original ash. The one curve shows the weight of sulphate added, and the other the effective part as Na_2O which combines with the slag.

The flux was effective from the start and maintained its rate for large additions. As a rough average, 1 per cent of Na_2SO_4 lowered the flow temperature 3.8 deg., or 1 per cent of Na_2O, 8.7 deg., which is not high compared with the cost of saltcake.

Since the effectiveness of sodium oxide appeared to be materially dependent on the lime present, a survey was made of the field (ash + Na_2O + CaO), the lime present varying from 6.4 to 20 per cent and the Na_2O up to 13 per cent. Ash T-197 was used. It contained initial 17.1 per cent Fe_2O_3 and 6.5 per cent CaO. The fluxes were used as Na_2SO_4 and $CaCO_3$.

Fig. 5 shows a plot of the results; the axes represent the percentages of Na_2O and CaO in the mix and therefore the percentages of the other constituents in the original ash, $SiO_2 + Al_2O_3 + MgO = 100 - (Na_2O + CaO + K_2O)$. The curves represent the mixtures which had the flow temperatures indicated by their markings. The test points are shown, and these were used in drawing the probable curves.

The method of plotting generalizes the data more than does plotting each set of tests in curves as in Fig. 3, and it will probably be better understood by most than would a trilinear plot. It should be clearly recognized that the curves in Fig. 5 show the percentages of the two fluxes which must be in the mix to produce a required flow temperature and not the percentages that must be added. Formulas for conversion are given in the appendix.

Such a set of curves show complex relationships which cannot be expressed in words. The position of the orginal ash is shown by a cross; assuming one wanted to flux to a flow temperature of 2450 deg., starting from the cross one has the choice of a number of relative proportions of the two fluxes which could be used and yet obtain the required flow temperature. In making a choice one desires that which would give the minimum cost for the total flux added.

An exact but simple method for determining the economic quantities is given as an appendix. Using that method, curve *AB* was found which represents the economic compositions;

the intersection of curve *AB* with each temperature line gives the economic composition for that flow temperature. The equivalent cost of CaO was taken as $2.75 per ton and the Na_2SO_4 as $17.

Because of the high cost of saltcake relative to limestone the economic curve *AB* comes at the high-lime end. As indicated in the appendix, a full consideration of economics would involve a consideration of the curves at the left of the diagram for flow temperature of 2500. With fluxing curves of the complex form of Fig. 5, a full knowledge of the whole area might result in material savings in cost of flux.

From the formulas 1 and 2 the weights of Na_2SO_4 and $CaCO_3$ were computed, and Table 5 shows the pounds of these fluxes required per 100 lb. of ash T-197 for a series of flow temperatures.

TABLE 5 WEIGHTS OF LIMESTONE AND SALTCAKE REQUIRED PER 100 LB. OF ASH T-197

	Flow temperature	Pounds per 100 lb. of ash Limestone	Pounds per 100 lb. of ash Saltcake
1	2425	29.3	3.2
2	2400	25.9	6.0
3	2375	25.2	7.6
4	2350	25.7	8.9
5	2325	26.2	10.3
6	2300	27.5	11.7
7	2275	28.4	13.1
8	2250	33.7	17.2

Fig. 5 is good only for ashes which have the same ratio of silica-alumina-ferric oxide as has ash T-197, but the lime in the original ash may be any value. Complete data for saltcake-limestone as fluxes would require similar sets of curves with ranges of ratios of silica-alumina-ferric oxide sufficient to cover all the possible compositions of coal-ash slags; this would require an amount of work which was not considered necessary at this time. That done was restricted to a determination of the effect of variations in the ferric oxide. To keep the determinations in the neighborhood of the economic mixture of the fluxes, a ratio of 1 Na_2O to 2.8 CaO (that is, 1 of saltcake to 2.17 limestone) was adopted for the flux and the ferric oxide varied. The results are shown in Fig. 6.

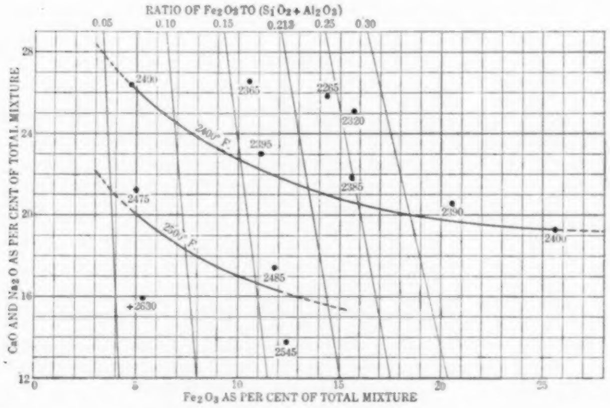

FIG. 6 PLOT SHOWING EFFECT OF CHANGE IN Fe_2O_3 CONTENT ON FLOW TEMPERATURE OF SLAGS FLUXED WITH CaO AND Na_2O

(Based on $SiO_2 + Al_2O_3 + Fe_2O_3 + CaO + Na_2O + K_2O = 100$ per cent. Average ratio CaO to $Na_2O + K_2O$ is 2.8. Average ratio SiO_2 to Al_2O_3 is 1.85. K_2O limited to less than 0.4 per cent.)

There are not enough values to plot many flow-temperature curves, but those for 2500 and 2400 show the trend of the effect of the quantity of iron in the mix. The sloping lines show the paths of composition followed by ashes having the Fe_2O_3 to (SiO_2 + Al_2O_3) ratios indicated for each line.

Fig. 6 gives a rough guide for estimating the percentages of Na_2O + CaO having a ratio by weight of 1 to 2.8 that would be required as dependent on the ferric oxide content of the original ash. Some computation would be necessary to obtain actual weights for a given ash, but tables could be constructed.

Iron Oxide and Limestone. Because the iron oxides and lime occur in all coal ashes, it is simpler to treat them together. Since the addition of iron oxides and lime did not add a mineral not already present in coal ashes, this phase of the investigation, in addition to being a study of fluxing, was also in great part a study of coal ashes themselves. The problem was, however, none the less complex because—

(1) The silica and alumina vary in quantity and in ratio

(2) The effect of the three possible forms of iron, Fe_2O_3, FeO, and Fe, had to be considered

(3) There are the minor constituents of magnesia, the alkalis, possibly titanium oxide and phosphorus, and always some sulphur.

Some of these factors will be discussed later, but as a fluxing problem it was considered that the effect of the minor constituents could be neglected as their range is small and somewhat of the same order in all coal ashes. The silica-alumina ratio was always watched. The form of the iron was neglected to a large extent, since additions were always made as Fe_2O_3 and the atmosphere was never reducing.

No tests on the special or incinerator ashes, T-209, 211, 212, and 228, are included in this section. Since the coal ashes gave a good range of equivalent Fe_2O_3 (5.4 to 42) to start from, most of the tests consisted in adding CaO, for which $CaCO_3$ was always used. Details of each set of tests need not be given, and Fig. 7 summarizes all the results in a plot having CaO and Fe_2O_3 as coordinates; this plot is of the same form as Fig. 5, the principles of which were discussed.

The limitations accompanying Fig. 7 are given in its title. It should be particularly noted that the ashes were selected with a silica-alumina ratio of 1.7 to 2.0.

The test points having been plotted, the curves to best fit them were drawn. The curves as drawn are fairly smooth and regular. If each test point were to be given its full weight, then the curves would be more irregular; also a number of the points conflict. Some of the possible causes of these irregularities will be discussed later. However, every new factor that is added introduces another variable that multiplies the difficulty of presentation, and for the application of the data it is of no avail to evaluate the effects of small factors unless there is also the ability to state exactly the conditions of service. The composition of slags from the same coal under variable conditions of burning will never be constant. The average relationship given by Fig. 7 should therefore be sufficient for general use.

Deciding the shape of the curves at the high-iron end involved difficulties. The addition of CaO to the Deepwater ash with its 42 per cent equivalent Fe_2O_3 should have given good points, but if all these had been used there would be big jogs in the curves. The points used were obtained by increasing the iron in T-102, which, however, had a high silica-alumina ratio, and 25 deg. was added to these indicated values when drawing the curves.

An inspection of the plot shows that the fluxing effect of the Fe_2O_3 decreases as the amount present increases, and that after 20 per cent it is negligible. This action is important, since it means that slags with high iron can absorb silica and alumina without reducing their fluidity.

To a less extent, the effectiveness of increase of CaO decreases with increase of Fe_2O_3. There is a material difference, however, in the fluxing requirements of a slag having high iron and of one having high lime. With high iron, the flow curves are far apart, so that large percentages of lime would be required. With high lime, they tend to gather together, and a small addition of iron will produce a large change.

It also will be noted that in the absence of lime the flow temperature cannot be reduced below about 2600 deg., however much the iron is increased.

No data were obtained for curves having a flow temperature higher than 2625 deg. One probably could plot two more without much error, but further work is required, as this area of the plot increases in importance with increase in furnace temperatures, and the curves should be carried to 2800 deg. at least.

To use the plot one must know the analysis of the original ash, reduce it to the form specified in the title, and locate the corresponding point on the diagram. An inspection will show whether an addition of lime or iron or a combination of the two will be more effective for that point.

In fixing the economics, the cost of the CaO, as previously developed, may be taken as $2.75 per ton, and of Fe_2O_3 as $7.25, or 2.6 times the cost of the CaO. To obtain the exact economic composition one would use the method of the appendix on the chart, but a good guess can be made by locating the point for the composition of the original ash and then judging by eye the point on the new flow-temperature curve which would require 2.6 times more CaO than Fe_2O_3.

The economic curves were determined, but were not added to the figure because of its crowded state. At a future time, when further data have been collected and the test points can be omitted, a more workable chart will be prepared.

For convenience, and as an indication of how such a chart should be used, the scales at the top and right-hand side have been added. These lines show the composition resulting from changes in one flux only; for instance, the line joining the 20 on the lower and the 20 on the upper scale gives at its intersection with the CaO horizontal lines the percentage of Fe_2O_3 in the mix corresponding to the CaO for all mixtures in which the ratio of Fe_2O_3 to $SiO_2 + Al_2O_3$ is 20 to 80—that is, when lime only is added to an ash the analysis of which falls on this line.

Unless Fe_2O_3 in some cheaper form than iron ore were available it would be desirable to use limestone alone, rather than introduce the complication of having the two fluxes to handle. Lime, however, becomes less effective as a flux as the iron in the slag is low, and the weight required may become large.

The question as to what would happen if the CaO was increased to large amounts in excess of what had been used arose when considering how it might affect the absorption of the flux at the surface of a slag bed when the flux was placed on the surface and the slag might become supersaturated with CaO.

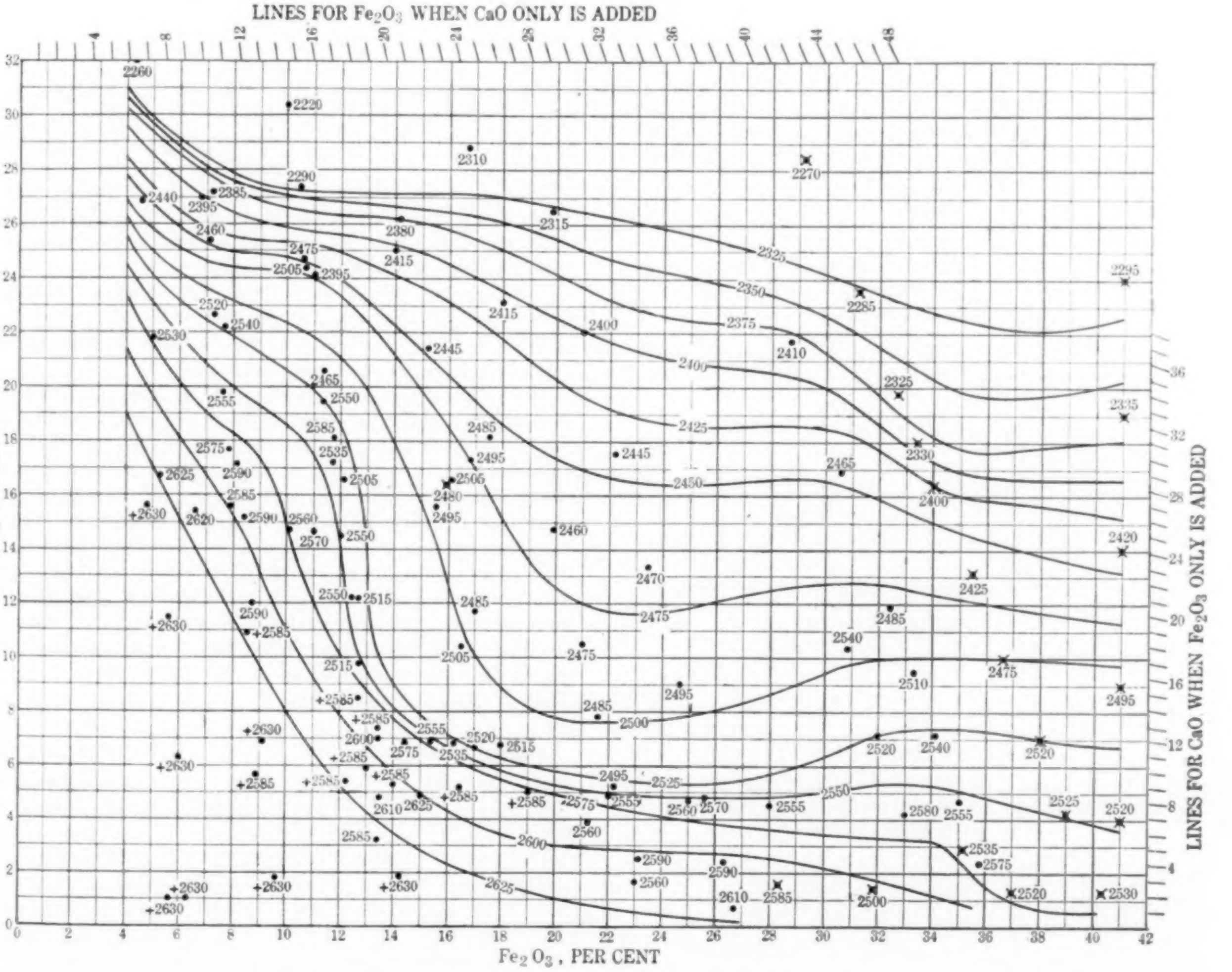

FIG. 7 PLOT SHOWING FLOW TEMPERATURES IN OXIDIZING ATMOSPHERES OF COAL ASHES HAVING A SILICA-TO-ALUMINA RATIO OF 1.7 TO 2.0, WITH VARYING CAO AND FE_2O_3 CONTENT

(CaO and Fe_2O_3 calculated on the basis of $SiO_2 + Al_2O_3 + Fe_2O_3 + CaO = 100$ per cent. Original ashes limited to a maximum of 2.0 per cent MgO. Points X have a silica-to-alumina ratio of 2.15.)

Fig. 8 shows the results from adding $CaCO_3$ to ash T-228 (Louisiana); this ash was chosen because at that time it was being used in the large gas furnace. Note that the percentage of flux added is shown as $CaCO_3$; the fluxing effect was uniform until 40 per cent $CaCO_3$ (22 per cent CaO) had been added; the flow temperature then remained at about 2190 deg. until 90 per cent $CaCO_3$ (50.4 CaO) was reached, after which the flow temperature increased at about the same rate as it decreased for the earlier additions.

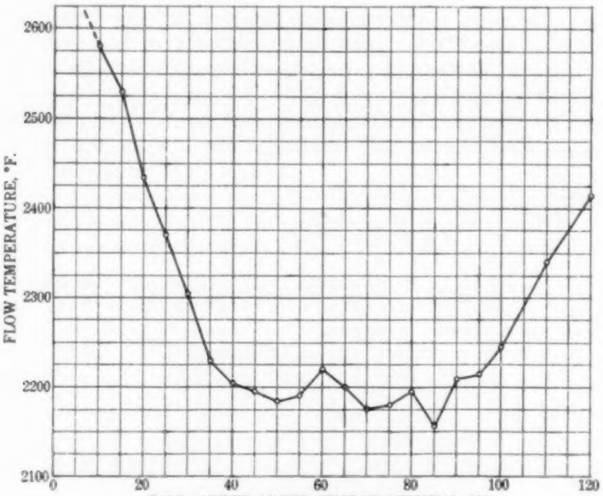

FIG. 8 PLOT SHOWING CHANGE IN FLOW TEMPERATURE OF SLAG T-228 WHEN FLUXED WITH CALCIUM CARBONATE ($CaCO_3$)

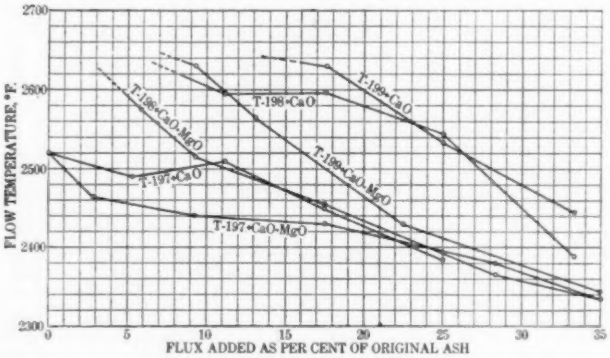

FIG. 9 PLOT SHOWING RELATIVE EFFECTIVENESS OF CaO AND CALCINED DOLOMITE (CaO.MgO) IN REDUCING THE FLOW TEMPERATURE OF THREE COAL ASHES

After about an 80 per cent addition of $CaCO_3$ the mixture began to have a crystalline feel. This test does not give a definite answer, because each addition was stirred in and not absorbed naturally, but it showed the limit of the fluxing effect of CaO.

Dolomite. Less work was done with dolomite than with limestone. In many districts dolomite and limestone are equally available and cost about the same, and it is therefore important to know the relative effectiveness of the two. There are several references in the literature in which opinions are expressed based on general observation. Bowles[12] discusses it in connection with metallurgical slags and points out the conflict of opinions.

Feild[13] made a series of tests on steel slags in which he changed the ratio of MgO to CaO within the limits of the MgO varying from 1.5 to 7.5 per cent of the total MgO + CaO and found that 5 per cent gave the greatest fluidity. The range he covered, however, is small compared with that of high-grade dolomite in which the percentage of MgO is 41.5 of the MgO + CaO.

Fig. 9 shows the effectiveness of dolomite and limestone on three ashes. The iron, lime, and silica-alumina ratios in the three ashes were:

Ash	Fe_2O_3	CaO	SiO_2/Al_2O_3
T-199	5.9	0.9	2.01
T-198	9.2	1.7	1.76
T-197	17.1	6.5	1.98

The figure shows that for ash T-199 the CaO.MgO was much more effective than the CaO, and for a flow temperature of 2600 deg. the weight of limestone required would be 1.7 times that for dolomite.

With ash T-198, dolomite had a still greater relative average effectiveness, and yet more for some portions of the curves for ash T-197. All three pairs of curves tend to converge, however, for large additions of flux, and even to cross.

The foregoing were all coal ashes. Fig. 16 shows a comparison of the CaO.MgO and CaO curves for T-212, an ash from oil sludges. The superiority of the dolomite is material, but not as great as for the other three ashes.

The evidence from these tests would appear to indicate conclusively that dolomite is decidedly superior to limestone, but it is not considered to be sufficient to generalize for ashes of all compositions; in particular, it has not been proved that it is true for the more refractory ashes when it is desired to lower their flow temperature a small amount, because the field of flow temperatures above 2630 deg. has not been studied.

An attempt was made recently to determine what percentage of MgO the dolomite should have to make it most effective. In these tests ash T-185, having 13.5 per cent Fe_2O_3, was used. Synthetic dolomites were made from $CaCO_3$ and basic magnesium carbonate. The compositions of the fluxes used, based on MgO + CaO = 100, were:

Number	1	2	3	4	5	6
MgO, per cent	0	7.2	14.6	22.2	32.0	41.9

Fig. 10 shows the results; they appear erratic and do not indicate as great a superiority for high MgO as is shown by Fig. 9. It is proposed to make further tests, and the results will

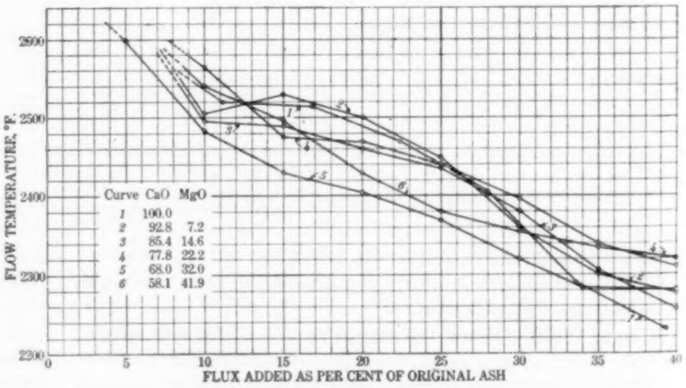

FIG. 10 PLOT SHOWING EFFECT OF SYNTHETIC DOLOMITES ON FLOW TEMPERATURE OF ASH (T-185)

(Flux additions made on calcined basis.)

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[12] Bowles, Oliver, "Metallurgical Limestones," U. S. Bureau of Mines Bull. 299, 1929.

[13] Feild, A. L., and Royster, P. H., "Slag Viscosities for Blast-Furnace Work," U. S. Bureau of Mines Tech. Paper 187, 1918, p. 22.

not be discussed further; however, in general, the curves show superiority for the high-magnesia fluxes.

Silica. The use of silica as a flux would be advantageous since it is obtainable generally and is of low cost. A comparison of ashes having high and low silica content and a given flux concentration usually showed a greater fluidity for the higher silica ashes. An example is ash S, having a silica-alumina ratio of 3.15 and a flow temperature of 2285 deg., whereas Fig. 7 indicates that with a ratio of 1.85 the flow temperature for the same lime and iron is 2450 deg. The difference of 165 deg. is evidently caused by the greater silica content of ash S.

The fluxing of ash P to the same composition as ash S by addition of silica, as well as iron and lime, lowered the flow temperature to 2355 deg. Similarly, fluxing to the same composition as S gave with ash T-185 a flow temperature of 2390 deg., with T-199 2380 deg., and with T-205 slag, where alumina was one of the added materials, 2370 deg. The average flow temperature for these four tests is 2370 deg., which is 80 deg. lower than indicated for a silica-alumina ratio of 1.85, but 85 deg. higher than the value for S. The reason that the flow temperature of S was not obtained may possibly be the inability of all the added silica to combine with the slag. In spite of prolonged thermal treatment, in which the slag, after addition of silica, was held at 2630 deg. for 2 hours, very small grains of silica were sometimes disitnguishable upon the slag surface.

The results of adding silica to ash T-205 are shown in Fig. 13. Here the addition of silica was effective up to a certain point, and further addition increased the flow temperature. The minimum flow temperature coincided with about a 3 to 3.2 silica-alumina ratio.

Other examples of lower flow temperatures occurring with higher sillica are pointed out in the section on steel slags.

From the limited data available it can be stated that the fluxing effect of silica with the coal ashes tested is limited, but it may be useful in slightly lowering the flow temperature of slags having a low silica-alumina ratio. It has been proved that this is so for slags having a high iron content, but not for those with low iron, and from the known relationship it is somewhat questionable whether it would help low-iron slags. Work should be done to settle this, but it would require the use of higher temperatures than those employed in this investigation.

Steel Slags. Table 6 gives the analyses of the steel slags tried. Slags A and B are waste products; it is understood that the flushing slag D, because of its high manganese content, is reused; slag C was made up to determine the effect of omitting the manganese and phosphorus. It will be noted that they all have a low alumina content and that all the other constituents except manganese have been mentioned as possible fluxes.

TABLE 6 ANALYSES OF STEEL SLAGS

Slag	A	B	C	D
Type of slag	Tapping	Tapping	Synthetic tapping	Flushing
SiO_2	14.88	17.68	18	29.40
Al_2O_3	4.08	...	..	2.68
Fe_2O_3	5.34	5.52	28	1.01
FeO	17.28	17.0	..	12.61
MnO	10.40	5.07	..	26.74
CaO	35.45	38.60	40	17.19
MgO	12.95	11.02	14	9.32
P_2O_5	1.78	1.87	..	0.76

Fig. 11 shows the results plotted as per cent of original ash. Note that to avoid confusion the temperature scale is twice that of all the other curves, and consequently, for comparison, the slopes are twice what they should be. The table on the figure arranges the ashes in the order of the increasing action of the fluxes on them; that is, the first one is the highest curve.

The deductions from a study of the curves are:

1 Considering curves 1 and 3, fluxes A and B are about the same; ash T-195 has more iron and lime than T-158. One would therefore expect that the result of fluxing the two ashes would be about the same, but curve 3 has a much greater slope than 1. The explanation must be in the higher silica in ash T-195 so that, when the additional silica which is in the flux is added to it, the silica is in excess and the fluxing components are rendered less effective.

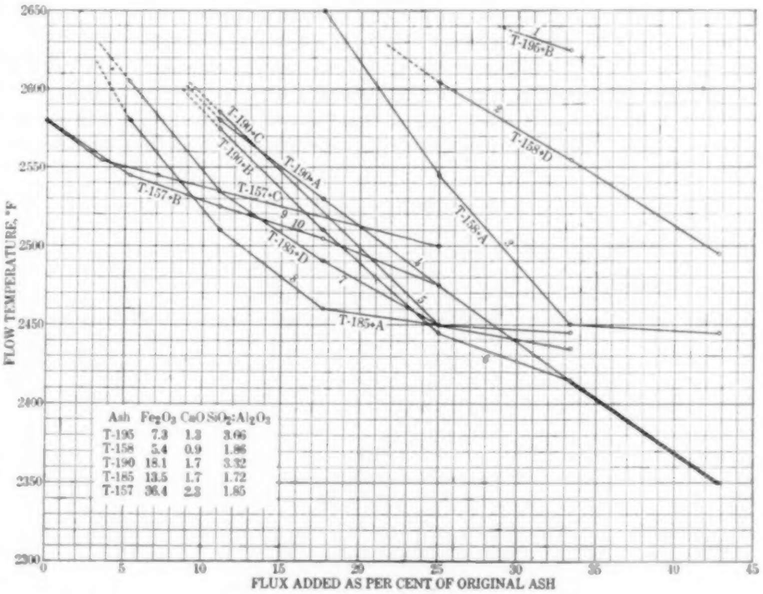

Fig. 11 Plot Showing Effect of Various Steel Slags on Flow Temperatures of Five Coal-Ash Slags

2 The same deduction can be made from curves 4 and 8. Both use flux A, and the iron in T-185 of curve 8 is lower than that in T-190 of curve 4. In spite of that fact, T-185 fluxes better than T-190 because of its lower silica.

3 Curves 2 and 3 are both with ash T-158; curve 2 uses flux D, and curve 3 flux A. Therefore flux A was more effective than D. Comparing fluxes A and D one sees that this effect may be partly due to the higher silica in D, but it is probably that the greater manganese oxide in D is not as effective as the greater iron oxide and lime in A.

4 Curve 10 is with ash T-157, which has a low flow temperature; the effectiveness of flux B as expressed by the slope of 10 was not as great as flux B used with ash T-190 in curve 6. This is explained by Fig. 7, which shows that the lime in the flux will be less effective with the high-iron ash T-157 than with ash T-190 having only 18.1 per cent Fe_2O_3.

5 Curves 5 and 6, and also 9 and 10, compare the effectiveness of fluxes C and B, the first curve in each pair being for flux C. In both pairs flux B was somewhat more effective than C. The idea in these tests was to determine whether the MnO had much influence; the curves show that the flux with the MnO was somewhat more effective, but the method of comparison does not justify assured conclusions. A further study was made on MnO by adding it in 1 per cent quantities to the slag as it existed at the last point of curve 5. The resulting

curve for flow temperature undulated somewhat, but up to 8 per cent MnO the average slope of the line was the same as the last section of curve 5, showing that the MnO under the test conditions had the same effectiveness as slag C and that small amounts did not introduce marked improvement.

These tests show that the tapping slags are effective fluxes, and the data given are sufficient to indicate the effect of the composition of the ash to be fluxed on their usefulness.

Forms of Iron

All the tests in the electric furnace were made in an atmosphere of air. Under reducing conditions platinum ware is quickly attacked and has a short life. The slags and ashes used in the investigation were in various stages of oxidation when received, and performing tests under oxidizing conditions was expected to remove one source of error by maintaining all tests under uniform conditions. Fieldner and others[14] have shown that cone-softening temperatures are largely influenced by the surrounding atmosphere, being higher for both highly reducing and highly oxidizing conditions than when the two effects are balanced. The effect on flow-temperature determinations as made in this investigation could not accurately be foretold, and therefore the following studies were made.

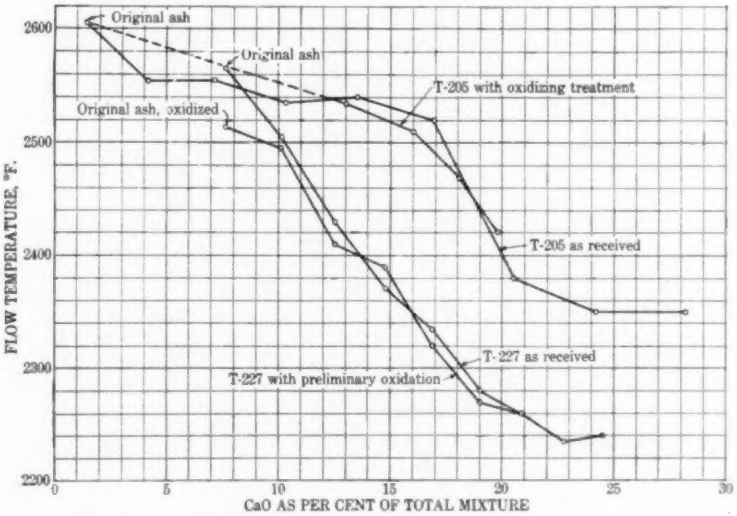

Fig. 12 Plot Showing Effect of Increase in CaO Content and Change in Forms of Iron for Two Similar Coal-Ash Slags

(Oxidation at 2650 deg. fahr.)

Analyses for forms of iron on T-205 and T-227 slags showed the reduction to be about the same. Oxidation by heating each slag at 2590 deg., with oxygen being blown over the molten slag surface, reduced the flow temperature of T-227 slag by 50 deg. in 13 hours and of T-205 by 15 deg. in 11 hours, therefore indicating the better fluidity with oxidized slag. That such results may be a function of internal rearrangement of the slag constituents is possible, but is hardly thought to be the explanation. No petrographic studies of the slags have been made, since it was felt doubtful that useful information on fluxing could be obtained without a long investigation.

Some of the ashes, of which T-227 was one, showed a greater drop in the flow temperature for the first addition of $CaCO_3$ than for the second and third additions, and it was thought that this might be due to oxidation taking place at a higher rate than when no flux was added. That $CaCO_3$ hastens oxidation is shown by tests on two ashes. Using ash T-205, one sample was maintained at 2590 deg. with oxygen blown over the surface; a second sample was mixed with $CaCO_3$ and heated for two hours without oxygen; and the third was mixed with limestone and heated for 12 hours with oxygen. Ash T-227 was given one test when mixed with $CaCO_3$ and heated for 13 hours with oxygen. The forms of the iron at the end of these tests are shown in Table 7.

TABLE 7 OXIDATION OF SLAGS

	FeO	Fe_2O_3	Fe
Ash T-205:			
1 Original ash	22.6	9.0	5.5
2 Oxidized 11 hours at 2590 deg.	11.1	28.5	Trace
3 Added CaO to 21.0 per cent, 1½ hr. at 2590 deg., without oxygen	1.6	30.7	Trace
4 Added CaO to 20.7 per cent, oxidized 12 hours at 2590 deg.	2.9	29.5	Trace
Ash T-227:			
5 Original ash	19.3	11.7	3.1
6 Added CaO to 21.8 per cent, oxidized 13 hours at 2590 deg.	2.1	28.6	Trace

Item 1 shows that, even with pure oxygen over the surface, the oxidation of the FeO is very slow and that there was 11 per cent at the end of 11 hours, whereas with $CaCO_3$ there was only 1.6 per cent left at the end of 1½ hours. The other items confirm this, although they were made for another purpose.

It can therefore be taken that, when determining the flow point of a slag having the iron in a reduced form, little oxidation occurred during the time required to make the determination.

Fig. 12 probably shows this more clearly. Two tests were made with slag T-227. The flow-temperature determination of the slag as received made in the standard method is shown on the curve to be 2565 deg.; the first addition of $CaCO_3$ reduced it to 2505, and further addition to values as shown by the curve. In a second test with the same slag, it was given a preliminary oxidation before adding lime, which reduced the flow temperature to 2515. The first addition of $CaCO_3$ was then made and resulted in a flow temperature of 2495; further additions were made as in the previous test. The figure shows that, except for the effect of the first addition, the two curves are identical within the order of difference to be expected in such tests. The tests with T-205, as shown by its curves, indicate the same result.

The conclusions from these tests are:

(1) That a slag having iron in a reduced form has a higher flow temperature than one in which the iron is largely oxidized.

(2) That the first addition of $CaCO_3$ to a slag having the iron in a reduced form causes an oxidation of the iron which results in a greater drop in flow temperature than occurs when the iron is initially in an oxidized state.

(3) That after the first addition of $CaCO_3$ the fluxing is the same independent of the form of the iron in the original ash.

Temperature-Time Tests

The description of the technique employed on the electric-furnace tests pointed out that after putting an addition of flux into the crucible the mixture was brought up to a temperature not more than 30 deg. higher than the flow temperature found for the previous addition of flux, and then a determination was made of the new flow temperature. This was done to save time and to make the mixture fluid enough that the flux could be thoroughly mixed with the slag; if this were not done there would be no assurance that the flux was distributed through all the slag in the crucible. Following this procedure, a slag which originally had a flow temperature of 2700 deg. might, by successive additions of the flux, finally have a flow temperature of 2300.

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[14] Fieldner, A. C., et al., "The Fusibility of Coal Ash and the Determination of the Softening Temperature," U. S. Bureau of Mines Bull. 129, 1918.

In service and in the large gas-furnace tests the imposed conditions would be quite different; then one might desire to flux the same ash when its temperature could not exceed 2300 deg. Would this be possible, would the amount of flux required be the same as that found necessary in the small-scale tests, and how long would it take?

One of the objectives of the large-furnace tests was to obtain answers to these questions. As far as they involve action of large masses they can be answered only by large-scale tests, but since such tests are time-consuming and expensive, some studies were made in the electric furnace. It could at least be proved whether the before-mentioned refractory ash and slag, mixed in the proportion to give a flow temperature of 2300 deg., would melt and attain the flow fluidity if kept at not above 2300 deg.; also the time required to attain fluidity would give some indication as to what the relative times would be in the large-scale tests.

To obtain this information, a series of tests were made in the electric furnace, and these tests are briefly summarized. Quantities of flux previously determined from a run in the normal manner were intimately mixed with ash and brought slowly to the temperature where the melt should be fluid; care was taken to avoid a higher temperature. Constant watching and occasional thorough stirring indicated when fluidity was finally attained. In almost every run, "gassing" of the charge seemed to increase viscosity, but on the final evolution of gas the charge became quiet and fluidity was easily judged.

Although 21 tests in all were made, a brief summary should suffice to enable conclusions to be drawn. With ash T-197 an addition of limestone, previously known to have been fluid at 2450 when overheated, became fluid at 2455 in 15 minutes when that temperature was not exceeded. Similarly, enough lime to produce an expected flow temperature of 2245 became fluid at 2245 in 2$^1/_2$ hours; the time was required mainly to work out bubbles of gas which were continuously forming, indicating a slower reaction.

Using T-198, which by itself is not fluid at 2630 deg., sufficient lime was added to produce fluidity at 2535; it became fluid at a maximum temperature of 2545 in 20 minutes. With this same refractory ash and solid flux, fluidity was obtained in 35 minutes at 2390 with a mixture previously known to be fluid at 2390 deg.

Similar results were obtained with T-185, Detroit Edison ash; for a mixture of lime and ash previously found to be fluid at 2215 when the ash had been heated in contact with lime to 2630, standard fluidity was obtained in 65 minutes at 2230 when that was the maximum temperature to which the new mixture was brought.

These tests show the possibility of fluxing reactions at temperatures where both constituents are solid. For instance, with T-185, the penetration point does not occur till 2430 while the initial deformation by the cone test occurs at 2240. In general, it should not be necessary, therefore, to raise the temperature of the ash-flux mixture to a point more than a few degrees higher than the finally attained flow temperature unless unusual speed of fluxing is desired.

Although these tests show that fluxing can be accomplished at very low temperatures, yet to date the minimum limit of temperature has not been determined. Further information could be obtained by a series of flux additions and temperatures coinciding with the curve of Fig. 8.

PENETRATION TEMPERATURE

Little analysis has been made of the penetration temperatures. Their values convey more meaning if expressed as the difference between the flow temperatures and the penetration temperatures, in that these values give some indication of the change in fluidity with temperature. As the difference is large, tapping will be easier as the slag will not stiffen as quickly; small values will indicate tendencies to clog the spout.

Some of the differences for lime and also fluorspar were plotted, and they confirm the effectiveness of the fluorspar in giving a fluid slag which is easy to tap. The comparisons were taken from the tests of ash T-185. With lime the interval between the two temperatures was approximately the same for percentages of lime added up to 40 per cent, the value being about 125 deg. With fluorspar there was a difference of from 350 to 400 deg., showing that the fluorspar gives a long range of fluidity. When a flux consisting of lime and 20 per cent fluorspar was used, the difference was about 180 deg.

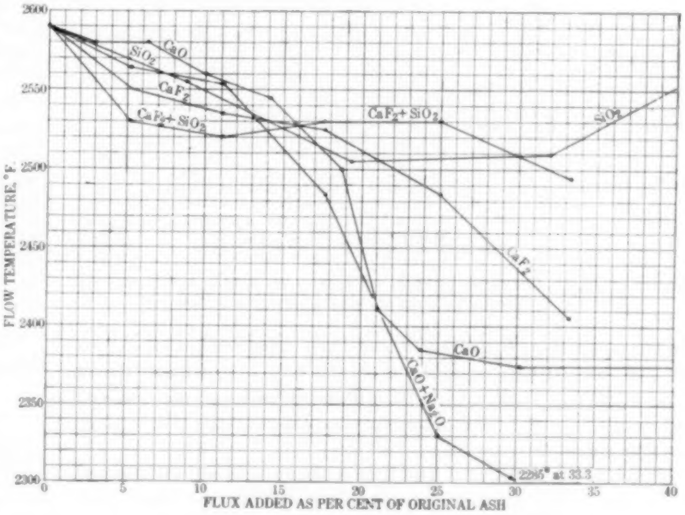

FIG. 13 PLOT SHOWING EFFECT OF VARIOUS FLUXES ON DEEPWATER SLAG (T-205)

(For CaF_2 + SiO_2, weight ratio of fluxes is 0.87 CaF_2 to 1.00 SiO_2. For CaO + Na_2O, weight ratio is 3.4 CaO to 1.0 Na_2O.)

A peculiarity has been apparent when fluxing with lime—namely, that with some ashes the first or second addition of flux produces a relatively low penetration temperature; the meaning of this has not been determined.

DEEPWATER SLAG, T-205

The samples of slag received from the Deepwater Station, T-205, are of special interest because of their high iron content and their highly reduced state; also they are interesting because the deposited slag contains 42 per cent iron as Fe_2O_3, whereas the percentage in the ash of the coal is only 26 per cent.

The ash had a very short fluid range, and the difference between the flow temperatures and the penetration temperatures was only 85 deg. The sudden increase in fluidity was very noticeable.

In the fluxing tests the slag had the peculiarity that in the earlier stages it was resistant to the action of the fluxes; therefore the chemical analysis was carried further than is usually necessary, and the titanium dioxide and phosphorus were determined; these, shown in Table 1, were low, and chemically the analysis showed nothing specially noteworthy other than the low Fe_2O_3 and high FeO and metallic iron, which is discussed under the section on forms of iron.

Because T-205 was at the high-iron end of the ashes, it was tried with a number of fluxes to obtain points for Fig. 7. Fig. 13 shows a number of the tests made; note that this plot also has twice the temperature scale; the flux is given as per cent of original ash.

The CaO curve shows that the action of the flux was not great for the early additions, but after 6 per cent the effectiveness increased uniformly up to 21 per cent. Even the action of fluorspar as shown by the CaF_2 curve was sluggish up to 10 per cent, as also was the action of lime and saltcake, the CaO + Na_2O curve, but both became more active as the percentage of added flux increased. The action of silica in the SiO_2 curve is interesting because, of all the fluxes tried, its effectiveness was constant up to a certain point and then it reversed itself, and a 10 per cent further addition took the flow temperature above the 2630 deg. limit. The silica-alumina ratio of the original ash was 1.79, and at the minimum flow temperature of the curve it was about 3.

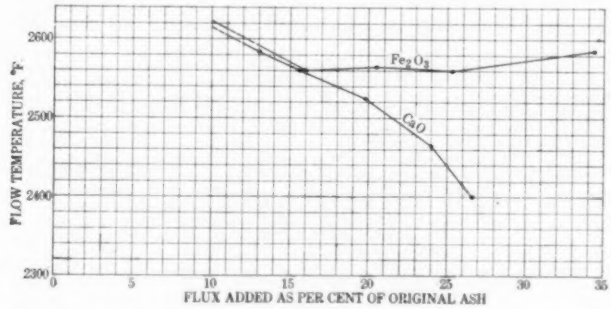

FIG. 14 PLOT SHOWING EFFECT OF CaO AND Fe_2O_3 ADDITIONS ON THE FLOW TEMPERATURE OF GLEN LYN SLAG (T-246)

The CaF_2 + SiO_2 curve is of interest because it was an attempt to utilize a low viscosity ratio of lime and silica. Herty and others[15] have shown that these oxides in the proportions to give calcium meta-silicate, $CaSiO_3$, have a low viscosity with rapid increase in fluidity with increase in temperature. In the tests made the ratio of CaF_2 and SiO_2 used for the flux was that to give SiF_4 and $CaSiO_3$, the former being a gas. The first 5 per cent addition was more effective than any of the other fluxes, but after that, for some unexplained reason, there was no action for the following 20 per cent addition.

It was thought that the peculiarities of the Deepwater slag might be due to the mineralogical forms taken by the constituents because of the high iron content, and that if the percentage of iron present was reduced by adding silica and alumina, then it would act as other slags with a similar iron percentage. To test this, silica and alumina in the same 1.79 ratio of the original slag were added; curves for these tests will not be given. After reducing the Fe_2O_3 in the mix to 20 per cent, lime was added; the resulting flow temperatures were higher than they should be from Fig. 7. Explanations have not been found, and it is probable that there are other factors affecting the high-iron end that have not been recognized.

GLEN LYN SLAG, T-246

Two small samples of slag were received from the Glen Lyn Station with the report that they were on the border line of tapping. They are of interest because they are the most refractory slags received from a slagging-bottom furnace. The second sample was received some months after the first, but the analyses of the two do not differ much. The analyses on the basis of the major constituents are as given in the following table, but because of the extra high MgO in these ashes, the lime has been increased 1 per cent:

	First	Second
SiO_2	54.5	55.3
Al_2O_3	30.8	30.2
Fe_2O_3	9.7	8.9
CaO	5.0	5.6
	100.0	100.0

Using the foregoing analyses it will be found that the points lie outside the 2625-deg. curve of Fig. 7; the flow-temperature determination on the first ash confirmed this. A flow temperature of about 2700 deg. might be expected.

The first ash was tested with lime and ferric oxide. Fig. 14 shows the results, which check well with those computed from Fig. 7. From what is known of the success in tapping this slag, if a flux were used it would be necessary only to lower the flow temperature a little to make the tapping somewhat easier; because of the limitation of test temperatures, Fig. 14 therefore does not give the exact values, but probably 7 per cent of either flux would be required to reduce the flow temperature to 2650 deg. The curves of Fig. 14 visualize what can be derived from Fig. 7; they show that for slags of this type, iron oxide and lime are equally good down to a flow temperature of 2580 deg., but after that the iron is not as effective.

INCINERATOR SLAG, T-209

This slag was from the combustion chamber of an incinerator, the bottom of which gradually filled up with a glassy slag; the operators desired to know whether anything could be added to keep it in a loose form. The opportunity was taken to study its reactions with various fluxes, and it might be that the slag could be run out instead of shutting down to remove it. Fig. 15 shows the tests made. The slag itself had a flow temperature of 2230. Dolomite lowered the temperature 40 deg., but all the other fluxes raised it, except fluorspar, which was quite effective. The fluorspar was not used beyond 25 per cent.

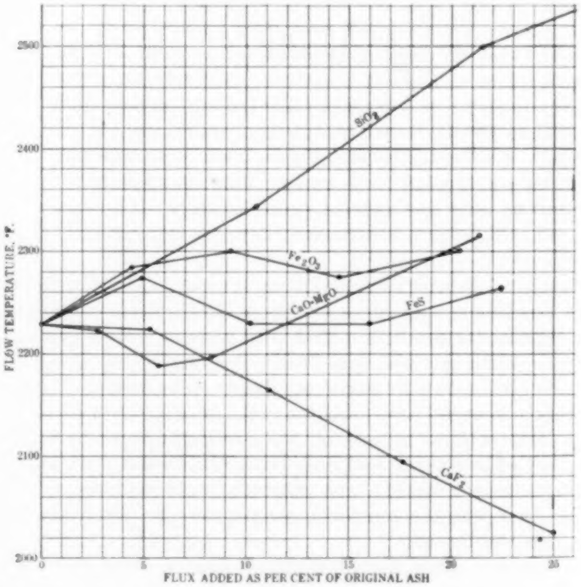

FIG. 15 PLOT SHOWING EFFECT OF VARIOUS FLUXES ON FLOW TEMPERATURE OF INCINERATOR SLAG (T-209)

but 2025 degrees was reached; this is the lowest flow temperature attained by any mix during this investigation.

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[15] Herty C. H., Jr., et al., "Temperature-Viscosity Relations of the Lime-Silica System," Coop. Bull. 47, U. S. Bureau of Mines and Carnegie Institute of Technology, 1931, p. 15.

Louisiana Slag, T-211, T-212, and T-228

The analyses of the three slags are given in Table 1. The ash deposited in the furnace differs materially from those from coals, and because of changes in the fuels used, the deposited slag will vary from time to time.[16] The samples received were of the same class; they differed materially as lumps, but when averaged for each lot were more nearly alike. The widest differences were evidently dependent on the quantity of extraneous matter mixed with the coke when burned.

Because of the high silica and alkalis, these slags tend to have properties like glass and a long melting range. Thus T-212 started to soften at 1520 degrees, but its flow temperature was above 2630 and probably about 2700.

Tests were made on all these slags. A comparison of fluorspar on two of them is shown in Fig. 3. Fig. 16 shows the relative action of several fluxes on T-212. Assuming that it was necessary to reduce the flow temperature to 2600 deg. for tapping, then the equivalent of about 6 per cent CaO or of calcined dolomite would be required—that is, 10 lb. of limestone or 11 lb. of dolomite per 100 lb. of slag.

These slags flux down to low flow temperatures with constant effectiveness for all fluxes. For lower values dolomite is more effective than limestone, as was also found for the coal ashes.

Tests in Large Gas Furnace

Fig. 17 shows the large gas-fired furnace which was built for this investigation. The desirability of constructing a furnace in which coal could be burned and the ash from it deposited in the furnace was considered, but the additional complications and expense offset the possible advantages. The combustion chamber aimed to draw the gases over the surface of the bed and to give uniform distribution. The four stacks have damper bricks to permit of individual control, and in addition the draft on the common flue to which the stacks are connected was controllable. The upper sheet-iron dampers gave further control and admitted air which mixed with the gases so that the sheet-iron flues could be kept cooler.

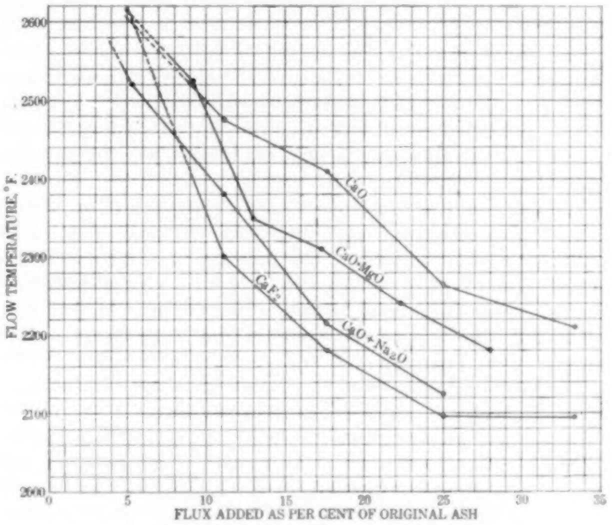

Fig. 16 Plot Showing Effect of Various Fluxes on Flow Temperature of Louisiana Slag (T-212)

(For added CaO and Na_2O, weight ratio is 3.4 CaO to 1.0 Na_2O.)

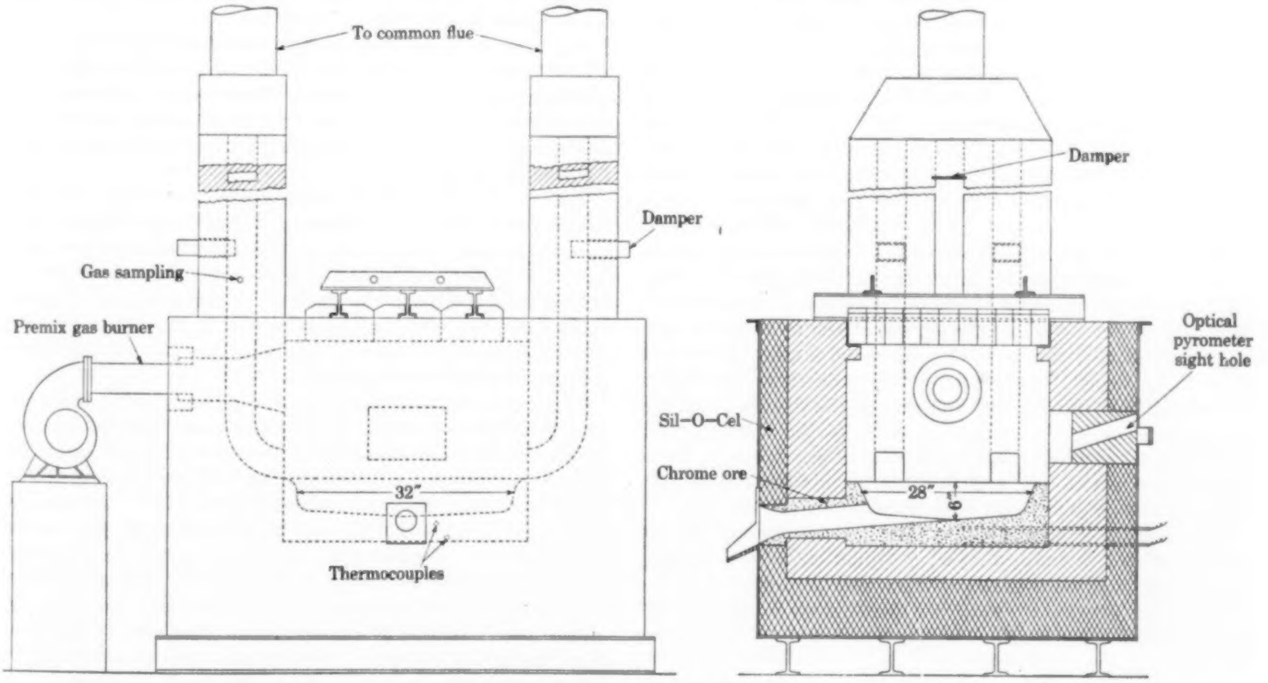

Fig. 17 Large Gas Furnace

The best material to use for the hearth caused some worry. It was important that it should not be absorbed by the slag, and also that the slag could be run out completely and the hearth used for the next test without renewal; also, it should not be too thick. It was decided to try chrome ore first, although at that time this material had not been used for boiler furnaces, as has been done since. It was also proposed to try carborundum, but this has not been used to date. Analyses have shown that the amount of the ore absorbed into the body of the slag is small for any one test, and the most damage has occurred in trying to remove all the slag at the end of a test when excess fluorspar is used.

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[16] Klotz, H. S., "Use of Oil-Refinery Waste Fuels in Modern Steam Boilers," A.S.M.E., Fuels Division, Chicago Meeting, Feb., 1931.

The temperature of the surface of the slag was observed through a sight hole in the door. The optical pyrometer was carefully calibrated, and observers were trained by checking each other. A further check was made by sighting on a rare-metal couple inserted through the opposite wall, the junction being encased in an alundum ball.

The gas flow was controlled through a meter and an orifice; a continuous measure of the composition of the stack gas was given by a CO_2 indicator. The pressure in the furnace was measured and kept to as near zero as possible without too much leakage at the top of the furnace. There was no difficulty in controlling the burning to maintain the temperature of the surface of the bed within 20 deg. of the desired value.

The two couples buried in the hearth permitted of a fair estimate of the temperature gradient in the slag.

It was not proposed that the ease of tapping this furnace should be used as a measure of the fluxing. The small quantity of slag to be removed, the relatively thick wall, and the necessity of keeping the hole small were not favorable to ease of tapping, but yet the arrangement acted very satisfactorily. The slag was caught on sheet-iron trays.

The general procedure was as follows: The furnace was heated for some hours at a temperature higher than that to be used in the test. The ash or slag was broken to pass a $^1/_2$-in. mesh; to save time the required weight of ash was added in two or three instalments, with the objective of fusing it into one mass. When all the ash was more or less fused, the temperature of the surface was reduced to the desired value. The addition of flux and the subsequent procedure depended on the objectives of the test.

The fluidity of the slag at any time was an important measurement. Several methods which would give numerical, though probably arbitrary values, were considered, but one which simulated the "feel test" of the small electric furnace was adopted. A tool was made consisting of a piece of $^1/_2$-in. pipe 8 ft. long, with a $^1/_2$-in. by 1-in. ell at one end. A piece of 1-in. carbon rod 9 in. long was screwed into the ell, and it was therefore at right angles to the pipe; the end of the carbon rod was kept sharpened to a 45-deg. angle. The carbon rod was used in a manner similar to the platinum stirring rod, and though the fluidity could not be measured with the precision possible in the small furnace, yet the method proved satisfactory; care was taken to avoid stirring the molten slag and thus not to mix it mechanically. Welding carbon was found very satisfactory for the rod.

The samples of slag collected from the bed during the test were restricted to the top layer; it would have been useful if they could have been collected from various depths, but this would have disturbed the slag, and there always would be doubt as to where it came from. The slag which clung to the carbon rod was usually sufficient.

Readings were made every 30 minutes, but the temperature of the surface of the bed was observed as much oftener as was necessary to maintain constancy.

Ten tests have been made to date. More were desirable, but as a choice between that and other work, they were put off to the future. The first two tests were made early in the investigation to obtain an idea of what possibilities there were in the method; at that time there were few data from the small furnace, and it was recognized that the large one could not be used effectively without knowing more about the relations derived from the small tests; consequently, no further tests were made till that work was well advanced. The feeling rod was not developed until after the second test, and the thermocouples were not put in until after the sixth test.

There is one factor which is common to all tests that should be borne in mind—namely, the temperature gradient through the fuel bed and its effect upon the actions which occur. In the electric furnace all the slag in the small crucible was at approximately the same temperature, but a large gradient was possible through the bed of the gas furnace, and it was not known what allowance should be made when deciding on the quantity of flux to be added.

Test No. 1. Ash T-185 was used; it was aimed to keep the surface at 2600 deg. To 100 lb. of ash 18.5 lb. of fluorspar was added. Trouble occurred because the molten slag ate through the taphole. Tapping was tried after $2^3/_4$ hours, but the bottom of the bed was not fluxed, and that tapped was so fluid that it worked behind the sheet-iron lining and into the insulation. Then 10 lb. more of fluorspar was used to clean the bed.

This test illustrated the rapidity of the fluxing effect of fluorspar and its high fluidity when in excess.

Test No. 2. Test No. 2 was made on the most refractory ash, T-158, having 5.4 per cent Fe_2O_3 and 0.9 CaO. The ash and the flux were mixed before putting in the furnace; 100 lb. of ash and 25 lb. of fluorspar—which was equivalent to 21 lb. of CaF_2—were used. The mixture was added in three lots, and the surface temperature was kept at 2600 deg. It was successfully tapped $9^1/_2$ hours after the first additions, and 134 lb. was removed; the extra 9 lb. tapped was probably some of the slag left from the previous test in addition to some chrome ore from the hearth.

Fig. 3 shows that 21 per cent of added flux gives ash T-158 a flow temperature of 2550 deg. Since neither the temperature gradient in the bed nor the fluidity was measured, exact comparisons cannot be made, but the successful tapping showed that the prediction from Fig. 3 was on the safe side for the successful tapping of this ash.

Test No. 3. The slag from the Louisiana Station was used. Ashes T-211 and T-212 represented slag from different locations in the bed; as there was not enough of either for the large-scale tests, they were mixed, and an average curve had to be assumed for predicting the quantity of flux required from the small-scale data. The object of this test was to study the action of fluxing this ash with limestone when the surface temperature was kept at 2550 deg.

The hearth thermocouples were not in place, and the temperature gradient was not known. Later work showed that the average temperature assumed for the bed was too low and therefore the predicted flux too small, but the test was informative.

Ash and limestone in the amounts of 250 lb. and 78 lb., respectively, were used; this was equivalent to 31 per cent of added $CaCO_3$. The limestone was added to the top of the slag in one charge; this made a too thick insulating layer and lowered the temperature of the slag, so a portion was removed and added later. It took $6^3/_4$ hours for the limestone to disappear from the surface of the bed; 9 hours more was allowed to elapse before tapping, and as the slag was not sufficiently fluid, the temperature was increased to 2575, when it was tapped with some difficulty.

A number of the samples collected were analyzed, and the results were very puzzling. Details need not be given, but they acted as guides for future work. The chief difficulty was to explain what had become of the lime, as a sample taken at the surface of the bed contained only 12 per cent of the lime added, and one taken during tapping only 21 per cent. The explanation of this was not discovered until later.

Two analyses were made to determine if much of the hearth was absorbed by the slag. They gave Cr_2O_3 as 0.5 and 0.3 per cent, showing that it was negligible for this ash and flux.

Test No. 4. Ash T-205, Deepwater, was used with limestone, with the aim of fluxing with a surface temperature of 2500 deg. The slag used was 250 lb.; a total of 105 lb. of limestone, or 23 per cent CaO, was added in stages; the first charge was too heavy and materially lengthened the test; from Fig. 13 there should be sufficient flux for 2390 deg. Heating for 20 hours failed to produce a fluidity low enough for tapping; fluorspar was therefore added, and the slag was tapped three hours later.

The slow absorption of the lime was very noticeable, and the progress of the softening through the bed was very slow; after $22^{1}/_{2}$ hours from the first addition of flux the bed was solid 1 in. below the surface. The short interval between the penetration temperature and the flow temperature has been referred to and would be unfavorable for rapid fluxing. Only three of the samples collected were analyzed; two were taken from very fluid pools in the bed and showed an excess of CaO. The third was an attempt to obtain a sample from near the bottom of the bed; it also contained excess lime, but the method of collecting was questionable.

Test No. 5. The object of this test was to determine whether fluorspar would flux ash T-205 more readily than did limestone in test No. 4; the surface temperature was the same, 2500 deg.; the spar was 83 lb., equivalent to 26 per cent CaF_2, which Fig. 13 shows should give a flow temperature of 2455 deg.; the flux was placed on the surface. The melting progressed regularly, but not abnormally fast; after 16 hours the bed was fluid for a depth of $3^{1}/_{2}$ in. and was tapped at 20 hours, although it was not considered to have the standard fluidity.

This test showed a more rapid action for fluorspar than for limestone, but not as great as was expected from the small-scale tests. Again, from later knowledge, it was known that the flux added was not enough to take care of the temperature gradient in the bed. As mentioned before, chemical analyses with fluorspar present are difficult to make and are likely not to be dependable; the three samples analyzed showed a very uneven distribution of the flux.

Test No. 6. This test was a duplication of test No. 4, except that the ash and flux were mixed before being put into the furnace; the object of this was to simulate the action in a slagging-bottom furnace in which it is assumed that the flux is added continuously and is thus thoroughly mixed with the ash as it is deposited. There were 200 lb. of ash T-205 and 85 lb. of limestone used, and the surface temperature was maintained at 2580 deg. The mixture was put in the furnace in three lots, and it took a total of $9^{1}/_{2}$ hours to obtain fluidity of the whole bed.

The mixing of the slag and flux thus gave better action, though some of the improvement would be due to the higher temperature.

Three samples were analyzed. The estimated CaO in the mix was 23.7; a sample from the liquid at the surface at $4^{1}/_{2}$ hours gave 25 per cent CaO; one immediately before tapping, 27 per cent; a composite sample of all the slag tapped gave 23.1 per cent. The limestone was therefore accounted for and was slightly higher at the surface.

Test No. 7. It was necessary to replace the chrome-ore bed after test No. 6, and the thermocouples were added when doing this. This test used 200 lb. of ash T-228 (Louisiana) and 68 lb. of limestone, with a surface temperature of 2400 deg. The object was to attempt to flux at a low temperature and to try to account for the CaO missing in test No. 3. Fig. 8 shows the fluxing effect of $CaCO_3$ on T-228. The test ran 60 hours from the first addition of flux, which was added in three lots spread over the test; the surface temperature was also carried at several values. These changes were not premeditated, but resulted from the knowledge of the temperature gradient in the bed.

Finally the slag was successfully tapped with a surface temperature at 2400 deg., at which time the mean temperature of the slag bed was estimated as 2260 deg. The 68 lb. of limestone added to the 200 lb. of ash, and allowing for 1 per cent impurities in the limestone, gives 33.7 per cent $CaCO_3$ added; from Fig. 8 the predicted flow temperature would be 2250 deg., which is as close a check as could be expected.

Five of the samples taken during the test were analyzed. Those taken from the top of the bed showed very low CaO content; this will be discussed later. An average sample of the whole of the tapped slag gave the equivalent of 33.9 per cent added of $CaCO_3$, which closely checks the 33.7 actually added.

This test showed that the temperature gradient through the bed was greater than had been allowed for in previous tests, and if the correct amount of flux were added, the laboratory results were confirmed.

Test No. 8. Test No. 8 was run immediately after test No. 7 without allowing the furnace to cool and was a test of ash T-198 with limestone and a surface temperature of 2580 deg. The ash is refractory, having 9.8 per cent Fe_2O_3 and 1.7 CaO, and the bed was therefore quite solid when the flux was added. To study what would happen, only enough limestone was added at first to produce fluidity at 2580 deg. The reasons for such a study are discussed later. The absorption of the flux was slow; after 17 hours the top was lumpy, and there were only a few pools about $^{1}/_{2}$ in. deep. More limestone was added, but after 30 hours the surface was lumpy, although the underneath slag was plastic. To test whether an intimate fresh mixture would become fusible under these conditions, 40 lb., in the same proportion of flux to ash, was added to the surface; it fused relatively quickly. The test was run for a total of 50 hours; during the last 15 there was little change, the top being thickly fluid with floating lumps in it and the stiffness increasing with depth. As it was necessary to close the test, fluorspar was added, and it was tapped after $2^{3}/_{4}$ hours.

Fluidity of the whole bed was not expected in this test because the lime content of the mix was only 22.3 per cent, with a resulting Fe_2O_3 of 7.8. Locating this point on Fig. 7 shows that the corresponding flow temperature is 2520, whereas the mean temperature of the bed was about 2380 deg.

Five of the samples collected were analyzed. Some floating lumps which had looked like unabsorbed lime gave only 5.36 CaO, showing that they were mainly ash.

Test No. 9. The purpose of test No. 9 was to determine if an ash having a flow temperature above 2700 deg. could be fluxed in a furnace which could not raise the temperature of the surface of the bed higher than 2500 deg., with the additional limitation that the flux was to be added to the surface of the bed.

For the ash, 100 lb. of T-198, the same as for test No. 8, was used. The total dolomite added was 59 lb., or 30 per cent CaO.MgO; Fig. 9 shows that the resulting flow temperature should be 2360 deg. The final mean temperature of the bed was 2340.

The absorption of the dolomite was slow, but seemed more rapid than the action in the previous test with the limestone, in which the surface temperature was higher but the percentage flux too low. Lumps floating on the surface dissolved very slowly; because these were not all dissolved after 17 hours, and because they would have clogged the taphole, fluorspar was added. Even then solution of the lumps was slow, and it required five hours before it was safe to tap.

The test showed that a refractory ash could be fluxed according to the predictions from the laboratory tests at a mean temperature of 2340 deg., which would be below its penetration temperature.

The test required 59 per cent of dolomite, and the increase in weight of the slag was 31 per cent. Also it showed that the flux could be added to the surface, but better action would result if the flux was added in proportion to the ash deposited.

Some lumps taken from the surface gave by analysis the same CaO.MgO content as the fluid material at the surface, in contradiction to the lumps taken in test No. 8.

Test No. 10. Test No. 10 was to obtain additional data on mass action when using fluorspar, and in particular to determine how the spar tended to distribute through the bed. There was not enough of ash T-198 for this test, so others were mixed to give one of about the same iron and lime content. [Note: Three additional tests are given in the Authors' Closure.—Ed.]

For 125 lb. of ash, 36.5 lb. of fluorspar, or 25 per cent of CaF_2, was used, and the surface temperature was kept at 2500 deg.; this should have been sufficient to flux at 2320. The results did not come up to expectations, either in rapidity of the action or the fluidity. After 33 hours there still were lumps and the slag at the bottom was stringy, so the temperature was raised to ensure ease of tapping.

The cause of the lack of fluidity was an underestimate of the flux required, because the small-scale test with this flux and ash had not been carried low enough and also because the CaF_2 in the fluorspar was taken as greater than it was, and allowance was not made for the silica and alumina which formed the rest of the spar.

Discussion of Large Gas-Furnace Tests

As far as they have been carried, the tests in the large furnace show that the data obtained in the electric furnace could be used to predict the quantity of flux that would be required to produce fluidity, although the work has not been carried far enough to give predictions on the time which would be required. No large-scale tests were made using saltcake and some of the other possible fluxes. It has been thought possible that the use of a small percentage of saltcake with the limestone might accelerate the action, and it was the intention to make some trials.

It will be noted from the reports of the individual large-scale tests that it was acknowledged that full information was not obtained in all tests because of incorrect quantities of flux or other causes. However, each test increased the knowledge of the investigators on what happened and how slags melted. It may be that similar studies have been made, but no such account has been found, and the authors were unable to find answers to a number of questions which suggested themselves, but the answers to which have an important bearing on the investigation. Some of these were as follows:

Assume that a fluxing material is spread on the surface of a bed of slag which is sufficiently solid that the flux will not sink into it, and assume that the surface of the bed is kept at a fixed temperature by combustion above it, and that consequently there is a temperature gradient through the bed, then:

(1) As reactions occur between the slag and the flux, what order of saturation of the flux will occur in the top layers of the slag; will it tend to become supersaturated?

(2) At what rate will the flux progress through the bed? What will be its final distribution?

(3) Will the upper and hotter layers tend to retain a higher percentage of flux and thus deprive the lower layers of the opportunity to become more fluid, so that the fluidity will decrease with distance below the surface?

(4) Will a flux which is molten at the temperature of the surface have an advantage because of its molten condition?

It is possible, of course, to suggest certain chemical, thermodynamical, and hydrodynamical actions or laws that will fix or control the sequence of events that will occur, but because the actions will of necessity be very complex, it would seem that answers can be obtained only by experimentation.

There was curiosity on these questions because it was feared that, unless the bed became fluid enough for convection to occur in the bed, there would be a tendency for the flux to segregate in the upper part of the bed; also the answers to them have an important bearing on operation.

Distribution of Flux in the Bed

Under test No. 3 it was stated that the samples taken from the surface showed a deficiency in CaO and that in later tests this feature was carefully watched and a number of samples were taken and analyzed.

All the data collected will not be given in this report, but the evidence shows that although limestone or dolomite is placed on the top of a nearly solid bed of slag, yet ultimately the concentration at the surface is much below the average, and because a composite sample shows a quantity of flux equal to that known to be in the bed, therefore the concentration of flux at the bottom must be above the average.

The same was found with dolomite and with fluorspar. Against this evidence, in test No. 6, where the limestone and slag were mixed before placing in the furnace, the concentration at the top was about normal.

The deficiencies in flux at the top were quite large, 50 per cent in tests Nos. 3 and 7, and 40 per cent in test No. 8, all when using limestone.

To check this, a special test was made in a small gas furnace, but under similar conditions of heating to the large furnace. The average computed CaO was 17 per cent; the CaO at the surface, middle, and bottom was 4.6, 13.4, and 22.4, respectively.

No assured explanation can be given. The natural one is that the porous calcined stone absorbs heavy slag, perhaps by preference taking the iron, and then gradually eats its way to the bottom of the bed faster than it is dissolved into the slag; but one would not have predicted such a course of events and movement toward the lower temperature.

Whatever the cause, as far as this may occur it is all to the good from the standpoint of fluxing. Because the temperature decreases downward, more flux will be required to produce the same fluidity as that at the upper portion; consequently, the fluidity through the bed will be more uniform and a greater depth can be run off at one tapping than would be possible if the flux were uniformly distributed through the bed.

Results as Applied to Slagging-Bottom Furnace

As stated in the introduction, no advice has been received since the investigation started that any station burning coal in a slagging-bottom furnace has found it impossible to remove the slag by tapping. It is not known how many stations are on the border line, or whether many have trouble in starting the flow—which trouble could be lessened by a partial fluxing.

Also, it has been pointed out that when planning the investigation, for lack of more definite information, it was assumed that the slag should be such that it would flow with the necessary freedom when at a temperature of 2550 deg., this value being the temperature of the bed at the middle of the thickness which it was desired to run off in one operation; consequently, to prolong the life of the test furnace the maximum temperature used was fixed at 2630 deg., and when testing a refractory slag, the smallest order of fluxing measured was the quantity of flux to lower the flow temperature to 2630.

It has turned out that 2630 is low, and that in many installations the slag is tapped at a higher temperature; if in such plants a certain coal gave a slag with a flow temperature of 2800 which could not be tapped, then it would be necessary only to add sufficient flux to reduce the flow temperature to 2700, for example. An assured value for the quantity of flux required for the foregoing illustration is not given by the work done to date, but a knowledge of the quantity to flux the slag to 2625 is probably sufficient; the minimum quantity to overcome difficulties with average operation would under any circumstances be fixed by experience.

If a flux were used as a regular procedure, then in the type and size of boilers installed to date the economics would be the controlling factor. On this basis, the investigation has shown that, for slagging-bottom furnaces, limestone, dolomite, and iron ore, or their equivalent in waste products, are the only fluxes that need be considered.

In the choice between limestone and dolomite, the investigation does not give assured data for flow temperatures above 2600, but it is probable that their effectiveness is approximately the same down to 2600 deg. and that they could be used interchangeably; therefore, only limestone and iron will be discussed.

All predictions for quantity of flux necessarily assume that the chemical analysis of the slag is known. Knowing the analysis and using Fig. 7, the flow temperature of the slag as deposited can be found; having this point, its position relative to the line representing the flow temperature it is desired to reach can be studied, and a point on it selected, the composition of which will probably entail the lowest cost of flux. The costs used in this respect are \$2.75 for CaO as limestone and \$7.25 for Fe$_2$O$_3$ as iron ore; thus for the same cost one can use 2.6 times as much CaO as Fe$_2$O$_3$. Knowing the desired new composition of the slag, the weights of limestone and iron ore required per 100 lb. of slag can be computed from formulas [1] and [2] of the appendix.

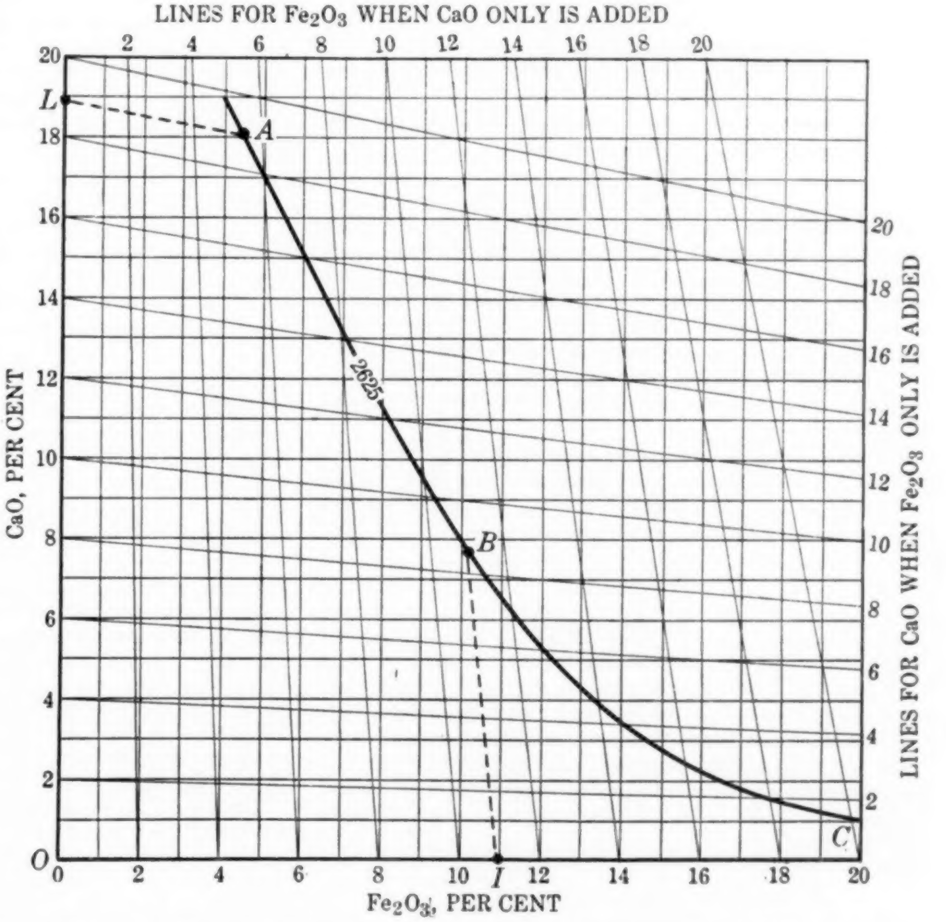

FIG. 18 PLOT SHOWING THE DERIVATION OF THE ECONOMIC QUANTITIES OF CaO AND Fe$_2$O$_3$
(For ashes having flow temperatures greater than 2625, when fluxed to 2625.)

The economic curves were not plotted on Fig. 7 because of the confusion they would cause. If the conditions are limited to fluxing slags having a flow temperature greater than 2625 deg., and make the problem that of desiring to reduce it to 2625, the economics can be simplified.

Fig. 18 shows a portion of Fig. 7, and only the 2625 flow-temperature curve is plotted; the functions of the sloping lines were explained in connection with Fig. 7. Using the foregoing costs, it happens that the economic tangent derived by the method in the appendix coincides with the straight portion of the 2625 curve. The following facts result:

(1) If the point corresponding to the composition of the deposited ash falls within the area *OLABIO*, then any point on *AB* can be taken for the composition of the fluxed slag, and all points on this line will give the same and also the lowest cost for the total flux.

(2) If the point falls in the area *BIC*, then the lowest cost of flux will result from using lime alone and no iron.

(3) The continuation of the 2625 line *BA* upward has not been determined, but if we assume that no deposited ash will contain less than 5 per cent Fe$_2$O$_3$, then the general statement can be made that, if a deposited ash has a flow temperature above 2625, there is no cheaper way of fluxing it than by using limestone.

(4) Although limestone will give a low cost in flux, yet the total weight added to the bed will usually be greater than if the same fluxing were done by also using iron.

Limestone can be used also for occasional fluxing, such as around the taphole. The absorption of limestone is slow, and quicker action results from using fluorspar. Information on this feature is not complete, and further tests will be made.

GENERAL SUMMARY

The relations shown by Fig. 7 for iron and lime are the most important outcome of this investigation. It gives the relations for only one state of fluidity, but a rough idea of the relations for the more plastic condition represented by the penetration temperature will be obtained when their values are analyzed.

The relations for the other fluxes were not worked out as completely, but the data given are probably sufficient for general use.

The investigation as here presented is not considered complete, and a limited extension of it is planned, including that of obtaining flow-temperature curves for Fig. 7 for higher temperatures.

It is expected that additional data on fluxing that can be added to those here given will be obtained in the next phase of this general investigation related to the use of fuels in slagging-bottom furnaces. It has been agreed with the committee to study the selective action in the combustion chamber of such furnaces as fixing the quality and quantity of the ash deposited. This will necessitate the testing of a large number of ashes and slags collected from the slagging furnaces.

The success obtained in this next phase will be dependent largely on the help that is received from operating stations, and the demands that it will be necessary to make on them will be far greater than for the fluxing phase.

ACKNOWLEDGMENTS

This investigation has been carried out under the authorization of O. P. Hood, chief engineer, mechanical division. R. A. Sherman, fuel engineer, and E. Taylor, assistant fuel engineer, conducted some of the earlier work; M. E. Bessemer, junior fuel engineer, has worked on the investigation for the greater part of its life. The many chemical analyses required were made in the chemical laboratory under the direction of W. A. Selvig. Helpful suggestions were received from C. H. Herty, Jr., physical chemist.

Acknowledgment is made of the support given to the investigation by the members of the committee and its chairman, P. B. Rice; also to the engineers of a number of stations who have supplied information and samples.

Appendix

PERCENTAGES

WORK such as that reported requires frequent changes from one basis of percentages to another. Usually these are straightforward, but the following two formulas are helpful for a system in which there are two fluxes and both vary.

Call one flux X and the other Y and assume that they are used in plots such as Figs. 5 and 7. Assume also that the slag to which the fluxes are to be added contains originally some of both fluxes.

Let

A_x = per cent of flux X in the original ash
A_y = per cent of flux Y in the original ash
x = per cent of flux X in the slag after it is fluxed
y = per cent of flux Y in the slag after it is fluxed
W_x = weight of flux X added per 100 lb. of original ash
W_y = weight of flux Y added per 100 lb. of original ash

(Note: Both W_x and W_y are the flux as it appears in the slag—that is, CaO and not limestone.)

Then

$$W_x = \frac{(100 - A_y)x - A_x\,(100 - y)}{100 - (x + y)} \qquad [1]$$

$$W_y = \frac{(100 - A_x)y - A_y\,(100 - x)}{100 - (x + y)} \qquad [2]$$

ECONOMIC AMOUNT OF FLUX

A study was made to derive the simplest method of determining the economic amounts of flux. The derivation of the formula and method need not be given, but it results in the following: Use the foregoing symbols, and, in addition, let

C_x = cost of X per unit weight
C_y = cost of Y per unit weight

Then for a minimum cost

$$\frac{dy}{dx} = -\frac{100\,\dfrac{C_x}{C_x - C_y} - y}{100\,\dfrac{C_y}{C_x - C_y} + x} \qquad [3]$$

If the costs of X and Y are the same,

$$dy/dx = -1, \text{ or } 135 \text{ deg.} \qquad [4]$$

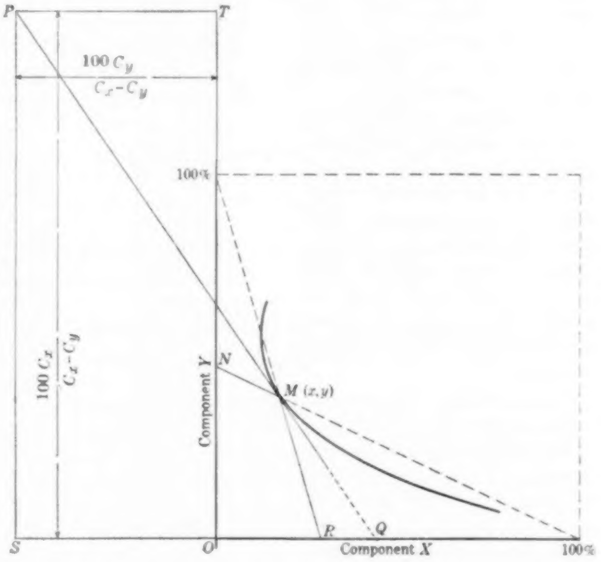

FIG. 19 DIAGRAM SHOWING PRINCIPLE OF DETERMINATION OF ECONOMIC MIXTURE

Since the curves are irregular, Equation [3] cannot be solved, but there is a simple way of doing it graphically for assembly of curves like Figs. 5 and 7. Fig. 19 shows this for the condition that cost of X, C_x, is greater than C_y.

The point P is found as shown, and the tangent from it to any one of the curves gives the point M. The quantity of fluxes at M (x, y) are those which will make the total cost of flux less than that of any other point on the curve.

The meaning of the point M is that if the point for the composition of the original ash lies in the area $ONMR$, then the correct quantity of fluxes must be added to bring the composition to that of M; formulas [1] and [2] would be used.

If the point for the composition of the original does not lie within the area $ONMR$, the point M cannot be reached by adding fluxes; if it lies above the line NM, then the lowest cost will result if only flux X is added; if the point lies to the right of MR, then only flux Y would be added.

If the flow-temperature curve is not a simple one, solutions can still be obtained, but the discussion would be rather long.

Discussion

ARTHUR J. HERSCHMANN.[17] It would be interesting to know how the fluxing material would be prepared for, and how introduced into the furnace chamber. Would it be conditioned as to size and temperature?

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[17] Agent for Vitkovice Steel Works, New York, N. Y. Mem. A.S.M.E.

Blast-furnace slag was used many years ago for cement manufacture in Bohemia. The writer does not know whether slag from pulverized coal would be less suited for that use nor is he sufficiently posted on cement manufacturing to know the reasons governing such possibilities at the present time.

W. W. Boyd.[18] Chrome ore (chromite $FeCr_2O_4$) is a mixture of oxides of iron and chromium; it is the most inert of all refractory materials, having no tendency to unite with anything.

The use of chrome ore as a hearth lining is open to the objection that a certain amount of iron is absorbed from the charge by the lining, and transfer may occur between the charge and lining according to the equilibrium characteristics of the ash under test. Since all of the ash analyses show iron present, the lining eventually will contain iron, which might give erroneous results.

To clean the hearth, it may be useful to use a jet of compressed air played upon the hearth by an iron pipe in the hands of the operator. With the furnace up to temperature and the slag out, the jet can be used to "wash" out the remainder. On this small gas furnace, oxygen might be useful.

The suggestion to use carborundum (silicon carbide) as a lining brings up a characteristic which is troublesome—the slags freeze at high temperatures; likewise, it acts opposite to the bath—if the bath is acid, the carborundum acts basically.

G. K. Saurwein.[19] These experiments have been carried out to find answers to the problems and serious difficulties that are encountered in the operation of the slag-tap type of furnace. As the granular form of ash is much simpler to handle, the question arises as to whether it is worth while to continue the use of the wet-bottom furnace. Of course, if an outlet for the refuse from either type of furnace could be found, the question would easily settle itself. When powdered coal first began to be extensively used, say 8 or 10 years ago, operators were hoping that satisfactory outlets for ash would be developed, with perhaps some profit, but at any rate that the refuse could be disposed of without imposing too much of a burden on the operators or causing a nuisance to the public. The ash-disposal question will become more acute as time goes on. Are we coming any nearer to the solution of that problem?

This question of what to do with the ash interests greatly all powdered-coal users and is without doubt of sufficient importance to be assigned for study to a sub-committee of the Research Committee.

Authors' Closure

Replying to Mr. Herschmann, in the original plans made by the committee it was proposed to carry out an additional phase of the investigation, which would have been conducted at a station on a full-size operating boiler, but because no station to date has found it necessary to use fluxes extensively, this additional work has not been done. At that time several methods were suggested for introducing the flux into the furnace. It was agreed that it would probably not be advisable to add the flux to the coal before pulverizing because, apart from the hardness of most possible fluxes, the flux should not be introduced as a fine powder, but the pieces should be at least large enough to ensure that they would not be carried out of the furnace with the gases. Two methods looked feasible; one, to pulverize to a large-mesh size and add to the pulverized coal on its way to the burner, but this would probably be inadvisable because of the extra wear on pipes and burners; the other, to crush to a mesh of $^1/_2$ in. or less, and to distribute the flux over the bed at intervals through one or more nozzles by means of an air blast. For occasional use good distribution could be obtained by injecting through sight holes situated well above the bed.

The study of possible uses for or means of disposing of the slag as tapped has not been a part of this investigation, but it is understood that several companies and individuals are giving this subject careful consideration.

Replying to Mr. Boyd, it was necessary to replace the chrome-ore lining after the test no. 6 because the relatively thin sides were eroded. Although the slags which had been used contained high iron, particularly the Deepwater, yet the penetration into the material of the lining was not deep. However in tests nos. 11 and 12, in which iron ore was used as a flux, the analyses of the resulting slag showed a deficiency in iron, and absorption by the bed may account for this. The composition of the lining will be determined when it is renewed.

Considerable additional work has been done since this paper was written. This has included three further tests on the large gas furnace to study the action of iron ore and of salt cake. Brief reports on these tests have been added. They confirmed that the data from the small-scale tests given in the paper can be safely applied in predicting the action of all the fluxes used.

Additional Tests on Large Gas Furnace

The following three tests were made after the paper was written. They are required to complete the data given in the paper.

Test No. 11. The purpose of this test was to study the action when using iron ore alone with a slag low in CaO, and to follow this by adding limestone; it was in effect a study of changes over the plot of Fig. 7. A charge of 125 lb. of ash T-185 was used. On the basis of Fig. 7 it had a content of 1.9 per cent of CaO and 14.1 of Fe_2O_3 and a flow temperature of +2630; the point corresponding to it is shown on the figure. The iron ore used had a Fe_2O_3 content of 78 per cent and the limestone 51 per cent of CaO. A reducing atmosphere was maintained throughout the test.

The ash was heated as in previous tests with a surface temperature of 2680, but though fused it was solid. The first addition of ore was 17.2 lb., with the surface temperature maintained at 2660. Action was very slow, and after 13 hours the bed was pasty in spots, but mainly covered with layers of unmelted ore. A second addition of 22.5 lb. of iron ore was equivalent to an average of 31.5 per cent Fe_2O_3 in the charge. At the end of 25 hours more there was a fluid layer of 1 in. with a hard surface below; the mean temperature of the bed at that time was 2340. Samples taken at this time gave:

	FeO	Fe_2O_3	as Fe_2O_3
Fluid surface	12.8	15.5	29.7
Floating lumps	6.2	40.6	47.5

The conclusion is that the reduction of the iron ore to FeO was slow, as was its absorption into the bed, but yet that the majority of the ore was below the fluid layer. The iron distribution through the fluid layer was not determined, nor is it known how far the added iron had penetrated into the more solid part of the bed. This slow action would be expected because Fig. 7 shows that the fluidity is not increased materially by the addition of iron.

In a boiler furnace some unconsumed carbon will be deposited on the bed and might help the reduction of the Fe_2O_3 to FeO. To simulate this action a mixture of silica sand, iron ore, and anthracite fines was added to one-half the bed; it contained 8.6 lb. of Fe_2O_3 and 0.9 lb. of SiO_2. To the other half of the bed was added 10 lb. of limestone.

On the iron ore-silica half there was vigorous bubbling, and at the end of 1 hour the surface was again fluid for 1 in., but with the same abrupt change from liquid to solid. On the limestone half, the surface was also fluid at the end of the hour, but the transition from liquid to solid was not as abrupt. At $7^1/_2$ hours the iron ore-silica side was fluid to $1^1/_2$ in. depth, with abrupt transition; the limestone side was fluid for $2^1/_2$ in., with gradual transition.

To equalize the lime distribution, 10 lb. of limestone was added to the iron ore-silica side, and 22 lb. distributed evenly over the whole surface. Vigorous boiling occurred with rapid increase in fluidity. At the end of 4 hours the whole bed showed an approximately uniform fluidity through its depth, but this fluidity was below the standard because not enough lime had been added for the mean temperature of 2460 deg. To rectify this, 15 lb. of limestone was added, mainly to the center and iron-silica side. The slag was successfully tapped 3 hours later.

An analysis of a composite sample of the tapped slag gave 17.9 per cent CaO, 14.7 FeO, and 9.5 Fe_2O_3, or an equivalent of 25.8 per cent Fe_2O_3. Fig. 7 gives a flow temperature of 2435 deg. for this composition, against an estimated mean temperature for the bed of 2460; this agreement shows that, for the range of temperatures covered by this test at least, the values given by Fig. 7 can be depended on for the quantity of flux required.

This text confirmed that iron ore alone is not a suitable flux for ashes low in lime; that the rate of absorption of the iron ore is slow; that with a high iron content the rate of action of CaO is rapid.

Test No. 12. Test no. 12 was a repetition of test no. 11 except that the iron ore and limestone were premixed instead of all the ore being added first and the limestone later. The ash and procedure used were the same as in test no. 11. The first addition of flux was 22 lb. of iron ore and $35^1/_2$ lb. of limestone. The surface temperature was kept at 2660.

At the end of 1 hour a thin layer of slag covered most of the surface, and at the end of 3 hours the entire bed was covered with $^3/_4$ in. of fluid slag, but the bed beneath was quite solid. At the end of 6 hours from the application of the flux the entire bed was penetrable and of approximately the same fluidity; the layer of very fluid slag on the surface had disappeared.

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[18] Hamilton, Ohio. Assoc-Mem. A.S.M.E.

[19] Engineer, Lehman Hall, Harvard University, Cambridge, Mass. Mem. A.S.M.E.

A second addition of flux of 15 lb. of iron ore and 25 lb. of limestone was added, the total flux being that estimated to give fluidity at 2430 deg. One hour later there was again a layer of fluid slag over part of the surface, but the greater part was lumpy. From this time, the bed gradually became more fluid and was tapped 13 hours from the second addition of flux.

This test showed that when the iron ore and limestone are applied at the same time, the fluxing action is relatively rapid.

The analysis of a composite sample of the slag tapped gave 17.1 per cent CaO, 9.6 Fe_2O_3, and 9.2 FeO. The Fe_2O_3 equivalent was thus 19.8 against 24 per cent estimated from the materials used. This discrepancy may be partly due to the material added not having the estimated iron content, but the weight of the slag tapped in both this and in test no. 11 was below the weights estimated from the materials put in the furnace; it is probable that some slag, high in iron, was absorbed by the lining of the bed.

The evidence collected from tests nos. 11 and 12 on the distribution of the iron oxides in the bed is conflicting. Samples taken by the feeling rod before the final fluidity was attained indicated that the iron oxide content was greater in the lower layers. On the other hand, samples taken from the top layer of the bed previous to tapping gave the following:

	Fe_2O_3	FeO	as Fe_2O_3	CaO
Test 11, surface-*R*	17.5	21.4	41.3	3.1
Test 11, surface-*L*	8.4	24.0	35.1	10.5
Test 11, composite	9.5	16.3	25.8	17.9
Test 12, surface	25.4	10.4	37.0	13.9
Test 12, composite	9.6	9.2	19.8	17.1

The surface sample *R* in test no. 11 was from the side to which the iron ore-silica mixture had been added, and *L* from the other side. The reason for the high iron content in the surface samples has not been determined. These and all other samples of these two tests confirmed the tendency of the lime content to increase with depth. The lime in the composite samples agrees with the estimate of that added, 17.2 per cent.

Test No. 13. The object of this test was to determine whether an addition of a small proportion of salt cake to limestone would help the fluxing. The ash used was 125 lb. of T-185, the same as for tests nos. 11 and 12. The surface temperature was kept at 2660 deg., and the mean temperature of the bed was about 2460 deg.

The first flux addition was 27 lb. limestone and $5^1/_2$ lb. salt cake. After $2^1/_4$ hours the bed was soft nearly to the bottom and of nearly uniform fluidity, although the surface was uneven.

To watch the relative action, the second addition consisted of 17 lb. of limestone to the left half and 17 lb. of limestone and 5.1 lb. of salt cake to the right. After 1 hour the left half showed less absorption than the right. After $4^1/_2$ hours from the second addition the left half showed more lumpy material at the bottom than did the right and was not quite as fluid. After $6^1/_2$ hours the whole bed was of the same fluidity and free from lumps, and it was tapped.

Because salt cake is a relatively thin fluid at 2625 deg., it was not possible to ensure that the distribution aimed at was maintained. However, the test showed that the salt cake increased the rate of fluxing, but not enough to warrant its use because of its high cost.

The analyses of samples from this test are not too reliable, because 11, 12, and 13 were run consecutively and the tap hole had become plugged so that the bed could not be thoroughly drained; also the distribution in the bed might not have been uniform. The analyses were as follows:

	CaO	$Na_2O + K_2O$
Sample from top before tapping	19.6	4.2
Composite	18.9	4.2

This indicates uniform distribution of the CaO.

The conclusion from the three tests nos. 11, 12, and 13 with ash T-185 is that limestone by itself would not only be more economical, but the addition of iron ore would increase the weight of slag to be tapped and would lengthen rather than shorten the time required for fluxing. The use of salt cake, although not economical, increased the rate of action and decreased the time required to produce the required fluidity.

RP-54-10

The SO_4–CO_3 Ratio for the Prevention of Sulphate Boiler Scale

By EVERETT P. PARTRIDGE,[1] NEW BRUNSWICK, N. J., W. C. SCHROEDER,[2] ANN ARBOR, MICH., AND R. C. ADAMS, JR.,[3] ANNAPOLIS, MD.

The investigation described in this paper is a logical development of R. E. Hall's well-known work. The authors felt that the true value of the SO_4–CO_3 ratio must be appreciably greater than the value calculated by Hall, and, as a result, a program of research at the University of Michigan was sponsored by The Detroit Edison Company to determine more accurately, if possible, the solubility relations of boiler waters. The work done on this project is described by the authors.

E. P. PARTRIDGE

W. C. SCHROEDER

R. C. ADAMS, JR.

WHEN R. E. HALL applied physical chemistry to the control of boiler-water conditions he rendered an inestimable service to boiler operators. His work described in his well-known bulletin[4] and in numerous publications has stimulated many other investigations. The one described in the present paper is a logical development from Hall's pioneering efforts. While the authors believe that the present paper modifies the numerical value of Hall's SO_4–CO_3 ratio to an important degree, they feel that their investigations in no way detract from the brilliance of Hall's earlier efforts.

It seemed probable to the authors that the true value of the SO_4–CO_3 ratio must be appreciably greater than the value calculated by Hall. As a result a program of research at the University of Michigan was sponsored by the Detroit Edison Company in order to determine more accurately, if possible, the solubility relations in boiler waters. The research on this project was carried on continuously from February, 1929, to July, 1931.

METHODS OF DERIVING THE SO_4–CO_3 RATIO

Only a brief comparison of Hall's derivation of his SO_4–CO_3 ratio and of the present authors' method of evaluating this ratio will be given in this paper, since the subject has been discussed in a more complete form in a paper presented before the American Chemical Society which is to be published soon. The purpose of the present paper is to place the results before boiler operators for discussion.

Hall's Derivation. Hall evaluated the SO_4–CO_3 ratio for the simultaneous existence of solid anhydrite ($CaSO_4$) and solid calcite ($CaCO_3$) in equilibrium with a boiler water at a given temperature by using the concept of the solubility product. His method of derivation is open to three criticisms. The first of these is that a boiler steaming at a high rating may not actually approach an equilibrium condition, but instead the boiler water may be more or less supersaturated with respect to either anhydrite or calcite or both substances. The second criticism is that Hall made a long extrapolation of Kendall's uncertain data for the solubility of calcite in the range up to 100 deg. cent. in order to obtain the solubility product for calcite at boiler temperatures. The third point is that Hall's use of the ionization data of Noyes involves considerable uncertainty, since it leads to rather large variations in the value of the solubility product, which theoretically should be a constant. Hall himself noted these variations,[5] but since the fluctuations for both anhydrite and calcite were in the same direction, he assumed that they would largely cancel out in the ratio.

A New Method of Evaluating Hall's Ratio. Most boiler waters fall in the range of dilute solutions as usually defined by the physical chemist. It seemed, therefore, that it might be profitable to apply some of the newer developments in the field of dilute solutions to the study of boiler-water equilibria. Some years ago Lewis and Randall[6] had discussed their development of the activity concept with relation to dilute solutions, and Debye and Hückel subsequently supplied a highly mathematical theoretical substantiation of the empirical conclusions reached by Lewis and Randall. It is interesting to note that the Debye-Hückel theory has recently been applied to the field of boiler-water chemistry by McKinney.[7]

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[1] Supervising Engineer, Nonmetallic Minerals Experiment Station, U. S. Bureau of Mines, Rutgers University. Mr. Partridge received the degree of B.S. in Chemical Engineering from Syracuse University in 1925, and was a graduate student and Detroit Edison Fellow in Chemical Engineering at the University of Michigan for three years, receiving the degrees of M.S.E. in 1926 and Ph.D. in 1928. He acted as Associate Editor of *Industrial and Engineering Chemistry* from 1928 to 1931 and also acted as research engineer in the Department of Engineering Research of the University of Michigan during the same period. He assumed his present position in 1931.

[2] Fellow in Boiler Feedwater Studies, Department of Chemical Engineering, University of Michigan, and Detroit Edison Fellow in the study of boiler-scale formation. He received the degrees of B.S. in Chemical Engineering and M.S. in Engineering from the University of Michigan.

[3] Chemical Engineer engaged in boiler-water studies, U. S. Naval Experiment Station. Mr. Adams studied at the Colorado Agricultural College and the University of Michigan, receiving the degree of A.B. from the latter in 1927. He was Detroit Edison Fellow in Chemical Engineering, 1929–1930, and subsequently supervisor at the experimental plant of the Du Pont Ammonia Corporation.

[4] R. E. Hall, et al., Carnegie Institute of Technology, Mining and Metallurgical Investigations, Bulletin 24, 239 pp., 1927.

Contributed by the Joint Research Committee on Boiler Feedwater Studies and presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] R. E. Hall, *Ind. and Eng. Chem.*, vol. 17, pp. 283–290, 1925.

[6] Lewis and Randall, "Thermodynamics and the Free Energy of Chemical Substances," McGraw-Hill Book Co., New York, 1923, pp. 326–385.

[7] D. S. McKinney, *Ind. and Eng. Chem.*, Anal. Ed., vol. 3, 192–197, 1931.

The present authors started from Lewis and Randall's definition of the "ionic strength" as a measure of the total concentration of a dilute solution and of the "mean molality" as a measure of the solubility of a saturating substance in such a solution. The ionic strength, represented by the symbol μ, is defined by the equation, $\mu = \frac{['] + 4[''] + 9['''] + \ldots}{2}$ where the brackets represent the respective sums of the concentrations of all the uni-valent, bi-valent, and tri-valent ions present in the solution under consideration. Concentrations are customarily expressed in mols per 1000 grams of solution and are stoichiometric values involving no theoretical considerations of ionization. The mean molality for a substance such as $CaSO_4$ or $CaCO_3$ is the square root of the product $[Ca][SO_4]$ or $[Ca][CO_3]$, stoichiometric values again being used. In considering the equilibrium between $CaSO_4$ and $CaCO_3$ it is possible to save one step in the calculation of data by using, instead of the mean molality, merely the products $[Ca][SO_4]$ and $[Ca][CO_3]$.

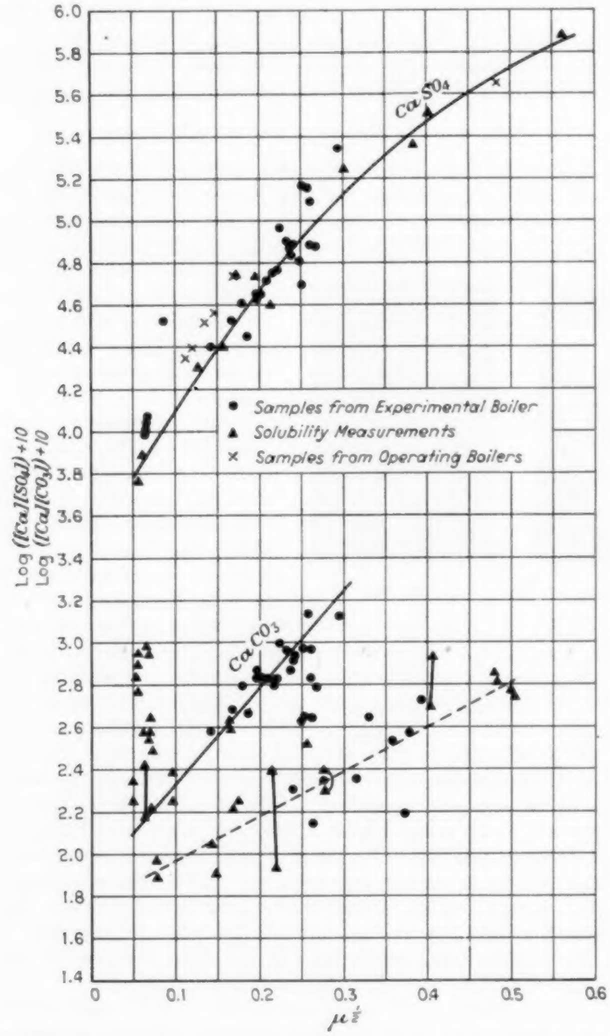

FIG. 1 CURVES REPRESENTING SOLUBILITIES OF $CaSO_4$ AND $CaCO_3$ AT 185 DEG. CENT. (150 LB. PER SQ. IN.) AS A FUNCTION OF IONIC STRENGTH OF SOLUTION

Concentrations expressed in mols per 1000 grams H_2O were used in calculating values of log ($[Ca][SO_4]$) and log ($[Ca][CO_3]$).]

For all dilute solutions saturated with respect to anhydrite the values of some function of the product $[Ca][SO_4]$ plotted against the values of some function of the ionic strength should define a curve or a narrow band. In other words, the solubility of anhydrite as expressed in terms of $[Ca][SO_4]$ should be approximately constant for all solutions of the same ionic strength regardless of the specific composition of these solutions. For actual use it is convenient to plot log ($[Ca][SO_4]$) and log ($[Ca][CO_3]$) against $\mu^{1/2}$. If a sufficient number of complete analyses were available for solutions saturated at a given temperature with either anhydrite or calcite or both solid phases, it would be possible by plotting the results in the manner described to define a range within which points for anhydrite fall and another range in which the points for calcite fall. If these ranges were narrow, they might be represented by single curves. The equilibrium value of the SO_4-CO_3 ratio at any given ionic strength might then be determined from the difference in ordinate of these curves, since

$$\frac{[Ca][SO_4]}{[Ca][CO_3]} = \frac{[SO_4]}{[CO_3]}$$

for a solution saturated with respect to both anhydrite and calcite. Converting this equation to logarithmic form yields,

$$\log\Big([Ca][SO_4]\Big) - \log\Big([Ca][CO_3]\Big) = \log\frac{[SO_4]}{[CO_3]}$$

The difference in ordinate of the curves is then the logarithm of the desired equilibrium SO_4-CO_3 ratio.

THE EVALUATION OF THE SO_4-CO_3 RATIO

The method outlined was used in determining the SO_4-CO_3 ratio at 185 deg. cent. corresponding to a boiler pressure of 150 lb. per sq. in. gage. The resulting graphs of data are shown in Fig. 1. This includes solubility values for calcium sulphate in pure water and in solutions of Na_2SO_4 and NaCl,[8,9] boiler-water analyses from actual boilers[9] which were known to be depositing either anhydrite or calcite or both substances, and a large amount of data from the experimental boiler operated in the laboratory by the authors. The individual data are presented and discussed in another paper[10] and only the general significance of the diagram will be treated here.

Discussion of Data. In spite of wide variations in the composition of the various solutions the points for anhydrite define a curve fairly well. The points for calcite, however, spatter considerably. The authors see three possible explanations for this result. In the first place, two crystal forms of calcium carbonate have been reported in boiler scale. These are aragonite and calcite, the latter being the more stable form. Presumably these two forms have differing solubilities. While aragonite is probably the less stable form under boiler-water conditions, it might persist for considerable periods of time. In addition, it is possible that fluctuations in boiler-water composition might cause reversal from calcite to aragonite, although this is improbable. The second explanation, which seems plausible to the authors, is that the various solutions even at a temperature of 185 deg. cent. may remain supersaturated with respect to calcium carbonate for long periods of time, and that the degree of super-saturation may vary considerably. The third explanation would be inaccuracy in the sampling methods and analytical procedure used in con-

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[8] R. E. Hall, et al., Carnegie Institute of Technology, Mining and Metallurgical Investigations, Bulletin 24, pp. 59–67.

[9] E. P. Partridge, "Formation and Properties of Boiler Scale," University of Michigan Engineering Research Bulletin 15, 1930, pp. 42–43, 159–160.

[10] E. P. Partridge, W. C. Schroeder, and R. C. Adams, Jr., paper on "Direct Determination of the Sulphate-Carbonate Ratio for the Prevention of Sulphate Boiler Scale," submitted to *Industrial and Engineering Chemistry.*

nection with the experimental boiler. Sampling was carried out with an internal filter and a cooling coil to reduce the filtered solution to room temperature before it was released from boiler pressure. All samples were guarded with the utmost care from contamination by the atmosphere. The authors believe that the values for Ca, obtained by the method involving precipitation as oxalate and titration with $KMnO_4$, and the values for CO_3, obtained by an evolution method rather than the customary titration methods, are accurate to within 1 p.p.m. Due to the small concentrations involved, the percentage error in these determinations is in many cases appreciable. The maximum analytical error in locating a point for $CaCO_3$ on the logarithmic ordinate of Fig. 1 should, however, not be more than ±0.15, while the actual vertical spread reaches a value of 1.

While the possibility of two crystal forms and uncertainty in analysis, coupled with the probability of supersaturation, render it impossible to draw a single curve representing the solubility of calcite in complex solutions, it is possible to draw a straight line through the upper range of the values. Since this line represents the maximum concentrations with respect to $CaCO_3$ obtained in the investigation, the use of values on this line in the calculation of the SO_4–CO_3 ratio will yield a value of the latter which is safe, although it may not be the precise equilibrium value which might be obtained by extremely refined methods.

The broken line drawn through the lower range of points for $CaCO_3$ may possibly define approximately the solubility of the less soluble of the crystal forms of $CaCO_3$, but the authors do not wish to stress this suggestion. The solubility curves of both aragonite and calcite may, however, be reasonably assumed to lie within the region bounded by the solid and the broken line.

At the lowest values of the ionic strength there are a number of high points for $CaCO_3$ which represent solubility measurements in the closed experimental boiler used as an autoclave at 150 lb. pressure. This was initially charged with solid $CaCO_3$ and distilled water. During the course of the experiment the value of log ([Ca][CO_3]) dropped irregularly to a value on the solid line and then increased again. On addition of Na_2CO_3 with corresponding increase in the ionic strength of the solution, much lower values were obtained along the broken line. A second solubility experiment yielded the sets of paired values shown which indicate less initial supersaturation than was evident in the first experiment.

A New Value for the SO_4–CO_3 Ratio

At 150 lb. per sq. in. gage pressure the value of the SO_4–CO_3 ratio calculated by Hall is 11. Much higher values are obtained from Fig. 1 by using the curve for $CaSO_4$ and the solid line through the upper range of points for $CaCO_3$. These values are indicated in Table 1 for various values of $\mu^{1/2}$. Due to the arbitrary use of a straight line through the $CaCO_3$ data, the calculated values of the ratio vary with the ionic strength. While the exact value of the ratio is in doubt, the authors feel quite safe in stating that it is at least 100, and may be appreciably greater.

The conclusion that the SO_4–CO_3 ratio has a value greater than 100 is substantiated by the results from controlled scale-formation tests made in the experimental boiler. The significant data from three runs are shown in Table 2. During runs *D* and *E*, which lasted 12 and 13 days, respectively, the solution fed to the boiler contained approximately 55 p.p.m. $CaSO_4$ and 55 p.p.m. Na_2CO_3, or slightly more soda ash than was theoretically equivalent to the concentration of Ca. After this water entered the boiler, $CaCO_3$ was precipitated as sludge, but the reduction in Ca concentration thus effected was not sufficient to prevent the formation during both runs of very thin layers of anhydrite scale on the hot tube, across which heat was being transferred at the rate of 39,000 B.t.u. per sq. ft. per hr. No attempt was made during these runs to control the SO_4–CO_3 ratio, which varied during run *D* from slightly above 100 up to 162 and then down to slightly less than 100, and during run *E* from slightly above 100 to 265 and back down to 179. Since at some time during each of these runs the boiler water must have been depositing not only $CaCO_3$ sludge, but also $CaSO_4$ scale, the "equilibrium" SO_4–CO_3 ratio must lie somewhere in the range of values calculated from the boiler-water analyses made during the runs.

TABLE 1 VALUES OF SO_4–CO_3 RATIO FOR PREVENTION OF SULPHATE SCALE AT 150 LB. PRESSURE AS CALCULATED FROM FIG. 1

$\mu^{1/2}$	10 + log([Ca][SO_4])	10 + log([Ca][CO_3])	Δ log	SO_4–CO_3 ratio mols	SO_4–CO_3 ratio p.p.m.
0.10	4.10	2.34	1.76	58	93
0.15	4.39	2.56	1.83	68	109
0.20	4.67	2.79	1.88	76	122
0.25	4.91	3.02	1.89	78	125
0.30	5.13	3.25	1.88	76	122

TABLE 2 DATA FROM SCALE-FORMATION RUNS IN EXPERIMENTAL BOILER AT 150 LB. PRESSURE

Run	Time, hr.	SO_4, p.p.m.	CO_3, p.p.m.	SO_4–CO_3, ratio	Composition and thickness of scale produced
D	32	537	5.1	105	Ca 27.9
	78.5	734	6.1	120	CO_3 3.3
	102.5	865	8.4	103	SO_4 66.8
	175	1580	12.0	132	R_2O_3 2.7
	198	1625	11.3	144	Thickness 0.003 in.
	223	1550	9.6	162	
	244.5	1485	10.8	138	
	271.5	1105	10.6	104	
	289	1070	10.2	105	
	289	1050	11.2	94	
E	100	1018	9.9	103	Ca 26.1
	124.5	1537	10.0	154	CO_3 2.9
	144.5	1875	11.2	167	SO_4 66.0
	198	1933	9.1	212	R_2O_3 8.7
	220	2517	9.5	265	Thickness 0.006 in.
	246	1810	8.2	220	
	268	2060	10.5	196	
	312	1810	10.1	179	
	312	1968	11.0	179	
F	49	1400	10.2	137	Ca 26.4
	73	1330	9.1	146	CO_3 40.8
	93	1250	10.2	122	SO_4 3.8
	144	2360	15.5	152	R_2O_3 20.0
	168	2950	29.4	100	Thickness 0.002 in.
	192	3400	36.6	93	
	213	3720	37.4	100	
	240	3780	43.3	88	
	258	4150	36.6	113	
	313	2650	27.6	96	
	334	1430	25.8	56	
	358	1950	19.9	98	
	384	2025	26.0	78	

During run *F* an attempt was made to hold the SO_4–CO_3 ratio in the boiler at a value of 100 by varying the amount of soda ash added to the feedwater, which was a solution of calcium sulphate containing 48 p.p.m. $CaSO_4$. In spite of the fact that the ratio value was rather high during the first six days, the hot tube at the end of the 16-day run showed practically no calcium sulphate on its surface, the very thin deposit being almost entirely $CaCO_3$ and iron rust.

The fact that $CaCO_3$ was the solid deposited in run *F*, while the thin scales of runs *D* and *E* were almost entirely $CaSO_4$, seems definite proof that the safe value of the SO_4–CO_3 ratio is above 100. The authors feel that the next step should be experimental operation of a large boiler under close control to test the practical effects of maintaining high ratio values. Any work of this nature will be wasted effort, however, unless the SO_4 and CO_3 concentrations are determined more accurately than has been the custom in boiler-water control. The authors have elsewhere[11] described the development of an evolution method for total carbon dioxide with which very accurate values for CO_3 may be obtained, in contrast to the high results indicated by the phenolphthalein-methyl orange or modified Winkler titrations. They have also noted the fact that turbidimetric determination of SO_4 may give high values for SO_4 in a boiler water even though the instrument indicates correctly with sulphate solutions containing no other substances.

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[11] W. C. Schroeder, E. P. Partridge, and L. F. Collins, "Determination of Carbonate and Hydroxide in Boiler Waters," submitted to *Industrial and Engineering Chemistry*.

Significance of the New Ratio Value

The great objection to the use of soda ash as a chemical for the internal conditioning of boilers has been the high alkalinity produced in the boiler water by the hydrolysis of carbonate. The difficulty encountered in maintaining the SO_4–CO_3 ratio set by Hall for the prevention of sulphate scale without overstepping the Na_2SO_4–alkalinity ratio recommended for the inhibition of caustic embrittlement has led in many cases to the substitution of various phosphate salts in place of soda ash as a conditioning chemical. It now seems probable, however, that the CO_3 concentration required by Hall's original calculation of the SO_4–CO_3 ratio was considerably larger than necessary, and there is a chance that soda ash may still hold an important place in boiler-water treatment.

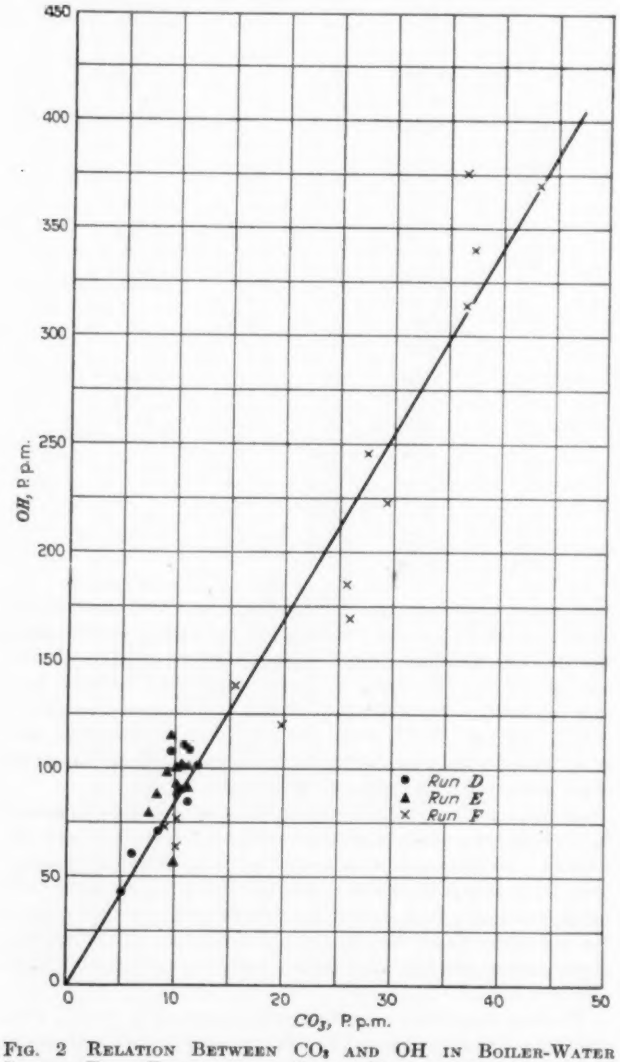

Fig. 2 Relation Between CO_3 and OH in Boiler-Water Samples From Experimental Boiler During Scale-Formation Tests

The soda ash added as conditioning chemical to a feedwater may be thought of as divided into two portions. The first portion is that amount which is chemically equivalent to the Ca in the feedwater and which will be removed from solution in the boiler by precipitation as solid $CaCO_3$. The second portion is the excess of Na_2CO_3 which is necessary to maintain a given CO_3 concentration and hence a given SO_4–CO_3 ratio in the boiler water. It is this Na_2CO_3 which is largely responsible for the development of alkalinity, since the precipitated $CaCO_3$ is quite effectively removed from the scene of action. Obviously, if it is possible to decrease the amount of excess Na_2CO_3 used in a boiler, the alkalinity produced by hydrolysis will be correspondingly decreased, other factors remaining the same. Since the rate of hydrolysis of dissolved carbonate is rather rapid, it is much better practice to add Na_2CO_3 continuously at as even a rate as possible than it is to make large additions intermittently. In the latter case the concentration of CO_3 is temporarily raised to a high value, but drops rapidly to a very low value, resulting in an increase in OH concentration with but brief protection as far as the SO_4–CO_3 ratio is concerned. On the other hand, when the feed of Na_2CO_3 is accurately and continuously proportioned to the flow of feedwater to the boiler, the maximum benefit from the standpoint of scale prevention may be obtained with the minimum increase in OH concentration.

The results obtained in the experimental boiler were quite encouraging. If the OH concentration in the boiler water is plotted against the CO_3 concentration, for all of the boiler-water samples taken during runs *D*, *E*, and *F*, Fig. 2 is obtained. A straight line represents the data within its limits of accuracy. From this line $OH/CO_3 = 8.5$. That is, with the experimental boiler operating continuously at 150 lb. gage pressure and with an intermittent blowdown due to sampling of less than 0.5 per cent of the feed, the boiler water contained on an average less than 9 p.p.m. OH for each p.p.m. CO_3. If a boiler were operated under good control with an average ratio $SO_4/CO_3 = 100$ to prevent sulphate scale formation, and if, by careful proportioning of the soda ash used for conditioning, the ratio $OH/CO_3 = 8.5$ obtained in the experimental work were realized, then the embrittlement ratios[12] would have the values $Na_2SO_4/NaOH = 7.4$ and $Na_2CO_3/NaOH = 0.087$. The Na_2SO_4–$NaOH$ ratio is far above the value of 2.5, which is believed to be the upper limit of the embrittlement range.

Looking at it from the other view-point, an SO_4–CO_3 ratio of 100 would allow the development of 24 p.p.m. OH in the boiler water for each p.p.m. CO_3 before the edge of the embrittlement range would be reached.

While these calculations are based upon small-scale work in an experimental boiler, and thus are subject to the criticism that plant operation never does succeed in duplicating laboratory results, the authors feel that there is no fundamental reason why any boiler operated at up to 150 lb. pressure under competent chemical control cannot be held in the region of boiler-water composition limited on the one hand by the maximum SO_4–CO_3 ratio allowable for scale prevention and on the other by the minimum Na_2SO_4–$NaOH$ ratio which is believed to inhibit embrittlement. To the authors it even seems possible that boilers might be operated at higher pressures with soda ash as a conditioning chemical if any considerations made this desirable. In this connection, the original data upon which the curves of Larson[13] are based would be extremely valuable, since from them it should be possible to calculate rates of hydrolysis.

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[12] F. G. Straub, Univ. Ill. Engr. Expt. Sta., Bull. 216, 1930, p. 77, Fig. 41.

[13] Larson, R. F., preprint "Decomposition of Sodium Carbonate in Steam Boilers," Hartford, Conn., Meeting, The American Society of Mechanical Engineers, June, 1931.

Discussion

Frederick G. Straub.[14] It is interesting to note that the authors of this paper recognize the errors involved when the solubility product was used to calculate this ratio. However, the writer wonders if, in using the newer method of evaluating the SO_4-CO_3 ratio which involves the concept of ionic strength, mean molalities, and other assumptions, we might not be led astray again.

Since we are working at temperatures where we have very meager information as to the real solid phase which exists in equilibrium with the liquid phase, it appears as though the less calculating we do and the more actual determinations we run, the better off we will be. Due to the fact that the authors have not furnished the data from which were calculated the datum points in Fig. 1, it is not known whether other salts were present when these solubility data were collected. This is very important, since if these tests were made in the presence of sodium hydroxide, it would be found that the effect of the hydroxide on the solubility of the calcium would not be compensated for by the increase in ionic strength. This in turn would reflect in the ratios calculated in Table 1. The results obtained in Table 2 should give a fairly correct idea as to the ratios necessary, since they involve actual tests and the assumptions made are reduced in number. However, it is believed that the presence of sulphate scale in tests *D* and *E* and its absence in *F* do not prove that a ratio below 100 will prevent sulphate scale. Unfortunately in run *F* the CO_3 was much higher than in the previous tests, and it is believed that the sulphate scale was prevented by the increase in CO_3. In other words, if the CO_3 is kept above 20 p.p.m. and NaOH is present, sulphate scale would be prevented irrespective of the sulphate content.

The results of research work which we have been conducting along this line, but mainly at higher pressures (that is, up to 3000 lb. steam pressure), indicate that there is no SO_4-CO_3 ratio. The amount of CO_3 necessary is independent of the sulphate present. Thus in these tests, with about 25 p.p.m. of CO_3 present, sulphate scale would have prevented scale even if the sulphate had been twice as high as it was.

The data plotted in Fig. 2 are interesting, since they show a definite relationship between the CO_3 and the OH in the operating boiler. This is similar to the results experienced in our tests on the decomposition of sodium carbonate in steam boilers. The author's results, which indicate that boilers can operate with soda ash, should certainly be good news to many boiler operators.

Authors' Closure

The authors heartily concur with Mr. Straub on some points and just as heartily disagree with him on others. For example, let us always have more actual experimental data that are reliable; these data will, however, mean very little unless they can be satisfactorily correlated. Due to the complex composition of boiler waters, such correlation of results has previously been extremely difficult. In the paper a method was presented which allows interpretation of results from any boiler water, no matter how complex, provided its composition is completely determined and the solid material deposited from it is accurately identified.

The method presented by the authors is just as applicable to a boiler water high in hydroxide as to one high in sulphate. Mr. Straub's conclusion to the contrary is evidently based upon his study of the solubility of $CaSO_4$ in NaOH solutions.[15] As the present authors have noted in their discussion of his work, the apparently anomalous decrease in Ca concentration which he found with increase in OH was probably due to the formation of $Ca(OH)_2$ in the solid phase and Na_2SO_4 in solution when the initial concentration of NaOH at 360 deg. fahr. was greater than about 2 millimols per 100 grams H_2O. The authors venture the opinion that if Mr. Straub will calculate values for log ($[Ca][SO_4]$) and $\mu^{1/2}$ from the actual analyses of his final solutions, which should have been in equilibrium with $CaSO_4$, instead of from his initial compositions, his points for 360 deg. fahr. will fall reasonably near the upper curve in Fig. 1 of the paper.

The data presented in Fig. 2, which indicated hydroxyl concentrations during runs D and E, and in the upper curve of Fig. 1, which shows the relation between the ionic strength and the stoichiometric solubility product of anhydrite during these same runs, confirm the statement that hydroxide has no appreciable specific effect upon solubility equilibria.

What increasing hydroxyl concentration actually does is to hasten the approach to equilibrium by tending to prevent supersaturation with respect to calcium carbonate. Most of the lower points shown for $CaCO_3$ in Fig. 1 represent samples containing appreciable concentrations of OH, while most of the higher points represent samples low in OH. As noted in the paper, a more complete discussion of the data is to appear in *Industrial and Engineering Chemistry*.

The authors would like to call attention to the fact that the comparison of solubility data by the use of the ionic strength and the stoichiometric solubility product is empirical rather than theoretical. In fact, the concept of these particular functions and their use has been based entirely on experimental results. Plotting of solubility relations through the use of these functions does not demand that the results follow any hard and fast mathematical formula, but simply gives a means of comparing the solubility of a salt in one solution with the solubility of the same salt in any other solution. To paraphrase an old saying, the proof of the plotting is in the reading. The fact that data for many individual substances in any number of different complex solutions in the dilute solution range have been correlated for a number of years by a method similar to that used by the authors is good evidence for the value of this method. Furthermore, deviations of any considerable magnitude serve as warnings to the investigator to check his analytical methods, to investigate his control of experimental conditions, or to anticipate supersaturation or undersaturation, as the case may be.

Possible confusion of ideas may be prevented if it is emphasized at this point that a knowledge only of the Ca concentration of a boiler water does not allow one to predict whether anhydrite or calcite will be deposited as solid by evaporation. The Ca concentration merely tells how much of one or the other substance may be thrown out of solution by concentrating the boiler water a certain amount. In this way, the Ca concentration represents a solubility value, but in order to predict which substance will be deposited, the relation between the SO_4 and CO_3 concentrations, that is, the SO_4-CO_3 ratio, must be known.

When Mr. Straub says that "if the CO_3 is kept above 20 p.p.m. and NaOH is present, sulphate scale would be prevented irrespective of the sulphate content," he is making a dangerously broad claim, which he qualifies somewhat in his next paragraph. Actually it is quite possible that a concentration of 20 p.p.m. CO_3 would prevent sulphate scale in the presence of much more than 2000 p.p.m. SO_4, but there is some SO_4 concentration at which sulphate scale would form. The value of 100 for the SO_4-CO_3 ratio was given by the authors merely as a safe lower limit. Some of the data indicate ratio values greatly in excess of this.

Most of the lack of agreement between Mr. Straub and the authors is a matter of point of view and definitions. Mr. Straub says, in effect, that if the SO_4-CO_3 ratio has a very large value it does not exist. The authors prefer to admit on one hand the fundamental hypothesis of chemical equilibrium and on the other the proverbial difficulty of attaining this equilibrium, while attempting to measure dynamically rather than statically a limiting value for the SO_4–CO_3 ratio which will be amply safe as a criterion for boiler control.

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[14] Special Research Assistant Professor, Department of Chemical Engineering, University of Illinois, Urbana, Ill.

[15] F. J. Straub, "Solubility of Calcium Salts in Boiler Water," Fig. 10, paper presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of the American Society of Mechanical Engineers.

RP-54-11

Design and Investigation of a Spring in Which All Coils Nest Simultaneously

By J. B. Reynolds[1] and O. B. Schier,[2] Bethlehem, Pa.

J. B. Reynolds

The purpose of this study was to design a spring of constant slope, the lower coils of which would not nest until loads larger than those causing corresponding coils to nest in constant pitch springs were placed on the spring. The results obtained when the spring of constant slope was compared with one of constant pitch made it seem desirable to try to design a spring in which all coils would nest simultaneously. Work with this spring and with those previously investigated leads to the conviction that end effects considerably influence the amount of deflection. This spring, having a complete dead coil at each end, must be somewhat affected by this influence, though no experiments have been made to determine how much. The result of this influence would be that the experimental deflections would be greater than the theoretical, which agrees with the results found.

O. B. Schier

Introduction

In a study[3] of the design and characteristics of conical springs, experiments showed that when a conical spring of constant pitch, supported on a plane, deflects under load, the lower coils are the first to nest. The added deflection in the remaining coils under additional load results in corresponding additional stresses that are not fully shared by the lower coils. The purpose of that study was to design a spring of constant slope, the lower coils of which would not nest until loads larger than those causing corresponding coils to nest in constant pitch springs were placed on the spring.

Interesting results were obtained when the spring of constant slope was compared with one of constant pitch; hence, it seemed desirable to try to design a spring in which all coils would nest simultaneously. In a spring with this characteristic, it appeared evident that there would be a greater uniformity in the division of work throughout the coils and an increase of stress in the lower coils of the spring. Also the design formula for such a spring would be easy to apply in practical design, since it would readily predict the load-deflection characteristic and corrections for change of base radius, due to nesting of coils as in other types of conical springs, would be unnecessary.

Description of Spring and Spring Materials

The spring as designed consists of four active coils with one complete dead coil at each end (see Fig. 1). The bottom diameter is $6^1/_2$ in., while the top diameter is $3^1/_2$ in. These diameters were chosen to allow a clearance of $^1/_8$ in. between coils when the spring is fully compressed. The height is 10 in. All dimensions are those of the neutral axis.

The spring was designed with what may appear an exaggerated size so that large deflections might be obtained for comparatively small loads, thus permitting a more accurate determination of deflections in the spring.

The material used is a chrome-vanadium stock of the following analysis:

C	Mn	P	S	Si	Cr	Va
0.45	0.71	0.020	0.029	0.20	1.01	0.17

This stock consisted of eight $^1/_4$-in. round rods each 14 ft long. This length was sufficient to make one spring and allow for two test specimens. These specimens were used to determine the yield point and ultimate stresses and the moduli of elasticity and of shear for the steel.

Mathematical Design

The mathematical design of a spring of the kind described is based upon two requirements. The first is that the projection of the neutral axis of the spring upon a plane perpendicular to its geometric axis shall be the spiral of Archimedes, the equation of which is of the form $r = r_o + k\theta$. The second is that the angle formed by the tangent to the neutral axis and the base plane shall vary in such a way as to have all the coils nest simultaneously.

In the derivation of the formula for this conical spring the following notation is used:

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[1] Supervisory Member in Charge of Research at Lehigh University and Professor of Mathematics and Theoretical Mechanics, Lehigh University. Joseph B. Reynolds was born on May 17, 1881, near New Castle, Pa. He attended country school and New Castle High School; attended Lehigh University on competitive scholarship, 1903–1907; took B.A. degree, 1907; was instructor in mathematics and astronomy, 1907–1913; took M.A. degree at Lehigh, 1910; assistant professor of mathematics and astronomy, 1913–1922; granted degree Ph.D. at Moravian College, 1919; associate professor, 1922–1927; guest professor, Princeton University, 1926–1927; professor of mathematics and theoretical mechanics at Lehigh University, 1927. Mr. Reynolds is the author of many articles and textbooks.

[2] Research Assistant, Lehigh University. Oscar B. Schier was born in Baltimore, Md., on July 18, 1907. He attended the public grade schools for eight years, and then entered the Baltimore Polytechnic Institute; upon graduation in 1926, was admitted to Lehigh University with advanced standing; was graduated in 1929 with M.E. degree, and was appointed the James Ward Packard Fellow.

[3] "The Design and Investigation of Conical Springs With Coils of a Constant Slope," by J. B. Reynolds and O. B. Schier, presented at the same meeting as this paper.

Presented at the Open Meeting of the A.S.M.E. Special Research Committee on Mechanical Springs, held in connection with the Annual Meeting, New York, N. Y., November 30 to December 4, 1931, of The American Society of Mechanical Engineers.

r_o = base radius of active portion of spring
r_n = top radius of active portion of spring
n = number of active coils in spring
θ = vectorial angle of base spiral for point on axis of spring
$k = (r_n - r_o)/\theta_n$
θ_n = vectorial angle corresponding to r_n
P = applied load
A = area of cross-section of the spring material
E = Young's modulus of elasticity of the spring material
G = shearing modulus of the spring material
L = total length of active spring material
D = axial deflection
p = variable slope angle
z = height of any point (r, θ, z) above the base of the neutral axis of the spring in the unloaded state

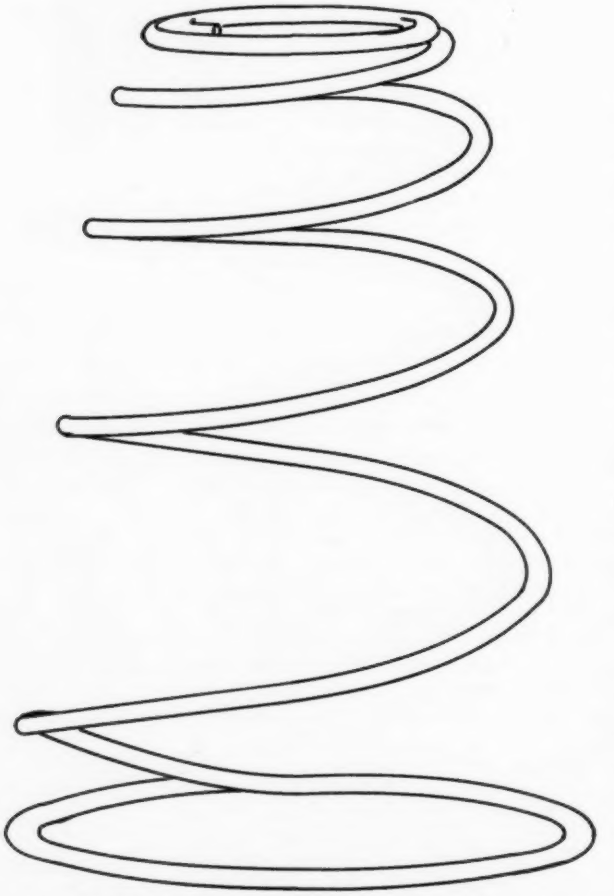

FIG. 1 SPRING WITH FOUR ACTIVE COILS AND ONE DEAD COIL AT EACH END

z_n = total height of the neutral axis above the base under no load
f = reciprocal of the load causing complete compression
I_v = moment of inertia of A with respect to the tangent to the neutral axis
I_u = moment of inertia of A with respect to a line in the cross-section and also in a vertical plane through the center of the cross-section
I_w = moment of inertia of A with respect to a horizontal line in the cross-section through its center.

The mathematical analysis (see Appendix) leads to the following formulas:

$$f = 42\,(I_v/A^4G)(r_n^4 - r_o^4)/4kz_n \qquad [1]$$

$$z = (r^4 - r_o^4)z_n/(r_n^4 - r_o^4) \qquad [2]$$

$$D = fPz \qquad [3]$$

DESIGN OF MANDREL

In the design of the mandrel upon which the springs were wound, use was made of Equation [2]. This equation shows that the neutral axis of the spring lies upon a frustum of a fourth degree surface of revolution. Therefore, the mandrel required a curved profile instead of a straight one such as is ordinarily used for winding springs of a constant pitch. Previous experience with the construction of springs with coils of constant slope emphasized the value of using a full semi-circular groove around the mandrel in which to place the spring steel. To facilitate the removal of the completed spring from this groove a collapsible mandrel was designed.[4]

METHOD OF WINDING SPRINGS

To wind the steel into springs it was thought best to have it heated to a plastic state. This method was followed in order to prevent the spring from changing shape due to initial stresses as it would if the steel were wound cold. Provision had to be made for heating the 14-ft lengths of stock to a uniform heat. Previous experience in heating with an oxyacetylene torch had proved unsatisfactory. It was therefore decided to use a current-resistance method for obtaining this uniform heat.

To wind a spring, a length of stock is placed in a tight-fitting brass terminal at the small end of the mandrel. From the mandrel the stock passes approximately horizontally through a tension tool with adjustable rolls which gives the necessary tension for winding. A rope attached to a counterweight over a pulley helps to support the heated rod. This serves to hold it nearly straight and so aids in feeding the material through the tension tool. The second electric terminal is a brass clamp fastened near the end of the rod.

With a current of 300 amp and a flow of 33 v, the required winding temperature of 1400 F was reached after heating for 1 min. This current was obtained by stepping a current of 15 amp at 220 v up to 660 v, and then stepping this voltage down to 33 v, giving 300 amp of current. An auto-transformer was used in the line to maintain a constant current as the resistance of the steel varied with the temperature. The temperature was measured with an optical pyrometer. With the temperature held constant at 1400 F, the steel was wound in the groove on the mandrel. The remaining steel was then cut off, the mandrel collapsed, and the spring was removed from the mandrel. The remaining stock was cut into standard lengths for tension and torsion specimens and marked for identification.

HEAT TREATMENT ARBOR

For the heat treatment of the springs it was desired to use the current-resistance method of heating, since no furnace of suitable size was available for this purpose. It was essential that all coils be supported at the quarter points to prevent deflection under their own weight at the high temperatures necessary for normalizing and quenching. Since it also was essential that all the current flow through the steel in the spring, some form of material other than metal had to be used to form the fins of the arbor. A commercial form of sheet asbestos was used. It withstands high temperatures and is a very poor conductor of electricity.

The arbor consists of four fins, the profiles of which are cut to correspond to the profiles at the quarter points on the mandrel.

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[4] For detailed description see "The Design and Investigation of a Spring in Which All Coils Nest Simultaneously," a thesis by O. B. Schier, Lehigh University, 1931.

These fins are bolted together between 1-in. angle irons, which give strength to the arbor. It was necessary to clamp the dead coil at the top and bottom of the spring to prevent deformation by expansion when the spring was heated.

HEAT TREATMENT

From experience in heat treating springs of chrome-vanadium steel, it was known that normalizing and quenching temperatures of 1600 and 1550 F, respectively, give good results. However, because of the smaller cross-section of the material used in these springs, the tempering temperature was reduced to 750 F to try to improve the elastic properties of the spring.

The spring, on the arbor, was heated to 1600 F by the current-resistance method and then allowed to cool in air. It was then heated to 1550 F and quenched in an oil bath. In each case the temperature was measured with an optical pyrometer. Had there been a suitable way of accurately measuring the tempering temperature, the current-resistance method would have been used for the final treatment. As the temperature of 750 F gave no visible color, the optical pyrometer could not be used. Therefore, a salt bath consisting of sodium nitrate and potassium nitrate fused in equal parts was heated to the temperature of 750 F. This temperature was measured by using a thermoelectric couple immersed in the bath and connected to a millivoltmeter with a direct-reading temperature scale. The spring was placed in this bath and held for 30 min.

The finished spring when placed under a load of 90 lb for 24 hr acquired a permanent set of 0.05 in. in a total deflection of 10 in.

TESTING REQUIREMENTS

In testing the spring it was desirable that the following requirements be met: (1) that there be an axial application of load, (2) that the top of the spring be permitted to rotate or not as desired, and (3) that the spring be prevented from tipping. By the design and construction of special attachments it was possible to use an Olsen testing machine for the tests. The tests were as follows: Loading and unloading of the spring in 10-lb increments of load with—

1 the top of the spring unconstrained from rotation
2 the top of the spring constrained from rotation
3 the top of the spring unconstrained under repeated loading.

Sufficient runs were made in each test to determine whether the spring characteristics were constant under repeated testing. In every series of tests and for each increment of load up to the maximum load of 90 lb, the following data were recorded: (1) Axial deflection of spring, (2) angular deflection, and (3) net height of each coil; that is, the distance from the bottom of the spring to the top of each coil.

The axial deflections were measured with an inside micrometer from a fixed reference plane to the top of the spring. The angular deflections were obtained by the use of the attachment[4] designed for the Olsen testing machine. The measurements of the net height of each coil were made with a surface gage. A radial line was scribed across the top of each coil so that the measurement would be made to the same point for each reading. These lines were placed 360 deg apart and directly above the heel of the spring.

A Ewing tensometer was used to measure the elongation of the specimens in tension in determining the modulus of elasticity of the steel. The specimens were not necked because of their small diameter of $^1/_4$ in. This prevented the determination of the yield point of the steel, since failure occurred outside of the gage length before the yield point was reached. An average value of 28,000,000 lb per sq in. was obtained for the modulus of elasticity. This value agrees with that obtained in previous tests on chrome-vanadium steel subjected to a similar heat treatment.

The modulus of shear could not be determined for the $^1/_4$-in. steel because of difficulty encountered in the use of the torsion machine. However, as the modulus of elasticity agrees with the value obtained in a previous study, a value for the modulus of shear of 11,300,000 lb per sq in., as taken in the previous study, was used in computing deflections.

RESULTS OF TESTS

Fig. 2 is a graphical representation of the coil actions of the spring and clearly indicates the uniformity of action for the various end constraints, since the curves cannot be separated. From the figure it is evident that the requirement that all coils nest simultaneously has been nearly realized and that a load of about 90 lb will completely compress the spring. It is also evident that the different coils of the spring have the same characteristic, since the deflection curves are of the same general type. It will be noted that the load-deflection curves are approximately straight lines up to one-half the total load. At half the total load each curve begins to take a curvature which increases as the load increases. This indicates that for ranges of deflection equal to or less than half of the total, the deflection is directly proportional to the load. However, for larger deflections this condition has not been realized.

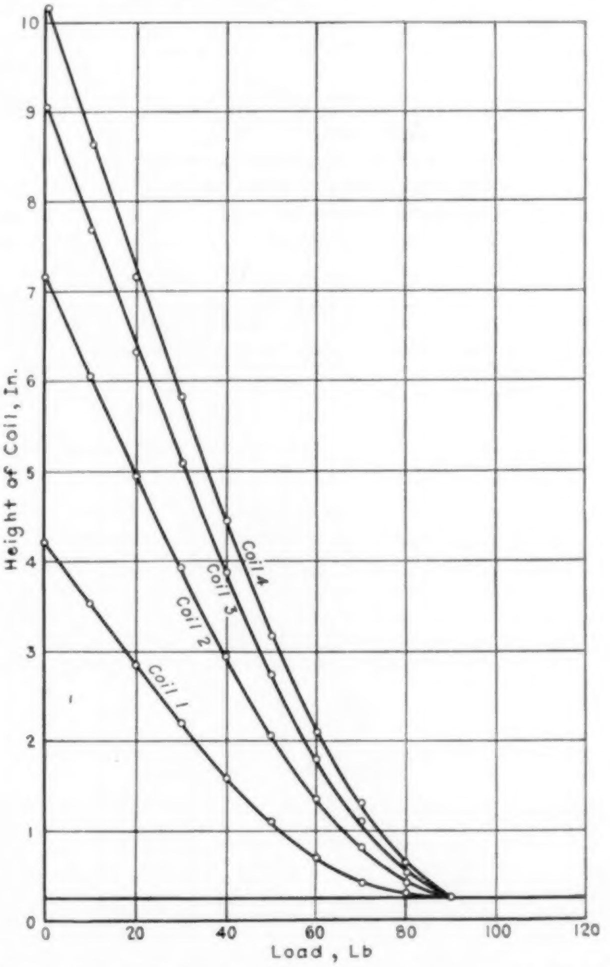

FIG. 2 CURVES REPRESENTING ACTION OF COILS IN A CONICAL SPRING IN WHICH ALL COILS NEST SIMULTANEOUSLY

In computing the theoretical values of deflection for comparison with the experimental values, the following equation, as developed by J. B. Reynolds, was used:

$$D = fPz$$

where D = deflection of any point on the spring initially at a height z above the base
f = the reciprocal of the load causing complete compression
P = load.

The value of f is obtained from Equation [1], which is as follows:

$$f = (42\, I_v/A^4G)(r_n^4 - r_o^4)/\, 4\, kz_n$$

where

I_v = moment of inertia of A with respect to the tangent to the neutral axis
A = area of cross-section of spring material
G = shearing modulus of the material
r_n = top radius of the spring
r_o = base radius of the spring
k = $(r_n - r_o)/\theta_n$
z_n = total height of the spring.

Fig. 3 compares deflections for the full height of spring as determined experimentally with those computed by the foregoing formula. The variation is considerable, and possible explanations of this will be presented later in the report.

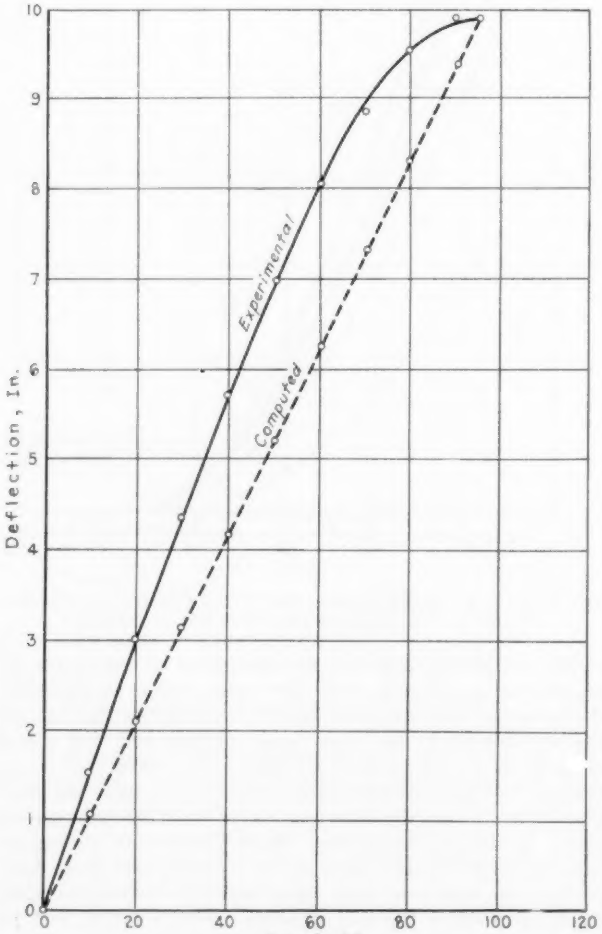

FIG. 3 COMPARISON OF LOAD-DEFLECTION CURVES FOR COMPUTED AND EXPERIMENTAL VALUES

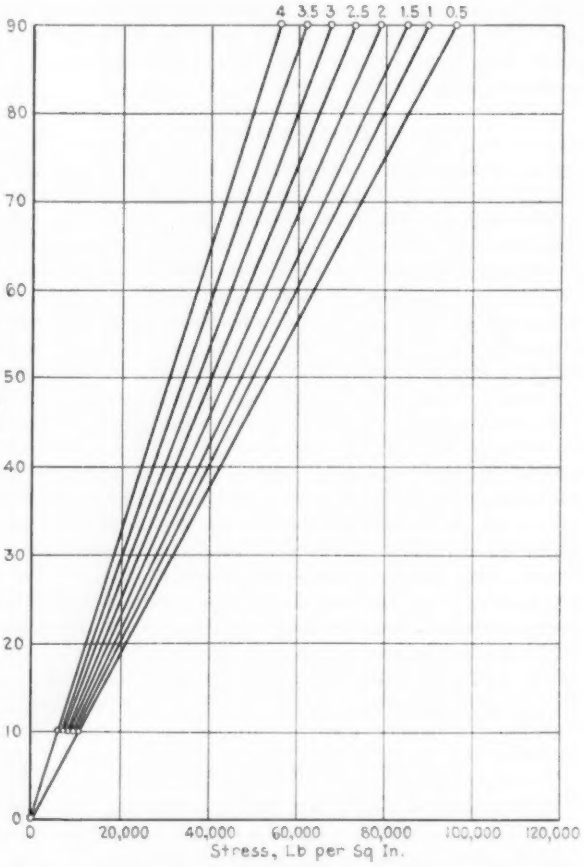

FIG. 4 LOAD-STRESS CURVES FOR EACH HALF COIL IN THE CONICAL SPRING
(Figures indicate numbers of coils.)

In the theoretical determination of stresses, use was made of the following formula given by A. M. Wahl:[5]

$$S = \frac{16\ Pr_1}{\pi d^3}\ k$$

where

S = maximum shear stress, lb per sq in.
P = axial load on spring
r_1 = mean base radius of each half coil
d = diameter of stock
k = stress multiplication factor
$= \dfrac{4c - 1}{4c - 4} + \dfrac{0.615}{c}$
c = $2r_1/d$

In calculating the stresses the load was considered axial, and the change in the base radius, when the first half of the first coil nest was not considered.

Fig. 4 represents the distribution of stresses on the inside of the coils for each half turn. These are theoretically the maximum stresses in the section at these points. The figure shows that the stresses increase directly with the loads and by equal intervals for successive half coils; that is, the stresses in this spring are proportional to the radii of the coils. The greatest stress is therefore found at the heel of the bottom coil and amounts to 101,000 lb per sq in., which is within the range of allowable shearing stress.

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[5] "Stresses in Heavy Closely Coiled Helical Springs," by A. M. Wahl, Trans. A.S.M.E., vol. 51, 1929, paper APM-51-17.

Conclusions

Work with this type of spring and those previously investigated leads to the conviction that end effects considerably influence the amount of deflection. This spring, having a complete dead coil at each end, must be somewhat affected by this influence, though no experiments have been made to determine how much. The result of this influence would be that the experimental deflections would be greater than the theoretical, which agrees with the results found. It also is to be noted that the slope of the theoretical load-deflection line varies inversely with the modulus of shear. Thus a reduction in the modulus of shear would rotate the straight line in Fig. 3 counterclockwise about the origin and make it more nearly coincide with the experimental curve. As pointed out in a previous report, the modulus of shear is somewhat uncertain.

It seems advisable to make experimental studies of various methods of constraining the ends of the spring to determine, if possible, the best practical method. Different methods of end constraint result in different eccentricities of loading. Hence, a study of end constraints will involve a study of eccentricity of loading. Maximum stresses in the spring are affected by the eccentricity of loading.

Manufacturers are still skeptical of results obtained by Wahl in connection with maximum stresses in helical springs. It therefore seems advisable to make experimental stress determinations on conical springs to bring further evidence to bear on this matter.

In developing the mathematics on which the design is based, it was necessary for simplification to neglect many terms which, though small, would have some effect. No effort has been made to determine the possible amount of this effect. If accuracy of investigation justifies it, further refinements in mathematics can be made.

Appendix

The purpose of this discussion is to derive formulas for the design of a conical spring in which all coils nest simultaneously. The results will be obtained in terms of constants for the material of which the spring is made, the load applied, and the dimensions of the spring.

In the derivation of the formulas for this spring the following notation is used:

r_o = base radius of active portion of spring
r_n = top radius of active portion of spring
n = number of active coils in spring
θ = vectorial angle of base spiral for point on axis of spring
$k = (r_n - r_o)/\theta_n$
θ_n = vectorial angle corresponding to r_n
P = applied load
A = area of cross-section of the spring material
E = Young's modulus of elasticity of the spring material
G = shearing modulus of the spring material
L = total length of active spring material
D = axial deflection
p = variable slope angle
z = height of any point (r, θ, z) above the base of the neutral axis of the spring in the unloaded state
z_n = total height of the neutral axis above the base under no load
f = reciprocal of the load causing complete compression
I_r = moment of inertia of A with respect to the tangent to the neutral axis
I_u = moment of inertia of A with respect to a line in the cross-section and also in a vertical plane through the center of the cross-section
I_w = moment of inertia of A with respect to a horizontal line in the cross-section through its center.

The requirements are, first, that the projection of the neutral axis of the spring upon a plane perpendicular to its geometric axis shall be the spiral of Archimedes, the equation of which is of the form $r = r_o + k\theta$, and second, that the angle formed by the tangent to the neutral axis and the base plane shall vary in such a way as to have all the coils nest simultaneously.

From the relations developed in the Appendix of a former report,[3] a spring of constant slope, is found the general formula for conical springs:

$$\left.\begin{aligned} dD = rd\theta_r = [42\, I_r P \cos p/A^4G + P \sin p \tan p/EI_u] \\ r^2(r^2+k^2)^{\frac{1}{2}}d\theta + [Pk^2 \sec p/EI_w - 42\, I_r Pk^2 \cos p/A^4G - \\ Pk^2 \sin p \tan p/EI_u](r^2+k^2)^{\frac{1}{2}}d\theta. \end{aligned}\right\} .[1]$$

Letting $a = 42\, I_r/A^4G$ (for a circular section $a = 1/I_rG$), $b = 1/EI_u$, $c = 1/EI_w$, Equation [1] may be written:

$$\left.\begin{aligned} dD = P(a \cos p + b \sin p \tan p)r^2(r^2+k^2)^{\frac{1}{2}}d\theta \\ + Pk^2(c \sec p - a \cos p - b \sin p \tan p)\,(r^2+k^2)^{\frac{1}{2}}d\theta \end{aligned}\right\} \ldots [2]$$

The variable angle p may be eliminated from [2] by using the differential right triangle of height dz, base $(r^2 + k^2)^{\frac{1}{2}}d\theta$ and hypotenuse dL, giving

$$\left.\begin{aligned} dD/d\theta = P[a(r^2+k^2)^{\frac{1}{2}}d\theta/dL + (bdz/(r^2+k^2)^{\frac{1}{2}}d\theta)dz/dL] \\ r^2(r^2+k^2)^{\frac{1}{2}} + Pk^2[cdL/(r^2+k^2)^{\frac{1}{2}}d\theta - a(r^2+k^2)^{\frac{1}{2}}\,d\theta/dL \\ - (bdz/(r^2+k^2)^{\frac{1}{2}}d\theta)dz/dL](r^2+k^2)^{\frac{1}{2}} \\ = Pr^2[a(r^2+k^2)d\theta/dL + (bdz/d\theta)dz/dL] + Pk^2[cdL/d\theta \\ - a(r^2+k^2)d\theta/dL - (bdz/d\theta)(dz/dL)] \end{aligned}\right\} .[3]$$

It is required that the spring shall compress so that every point on its neutral axis at an original height z above the base of the neutral axis shall be at a compressed height Cz, where C is a constant having different values for different amounts of compression; that is, varying directly with the load P, but not for different points on the neutral axis of the spring. If D_z is the deflection of a point at a height z, then

$$Cz + D_z = z \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

or

$$D = D_z = (1 - C)z = fPz$$

in which f is the reciprocal of the load which will completely compress the spring. From Equation [4]

$$dD/dz = fP \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

Now, $dD/dz = (d\theta/dz)\, dD/d\theta = (dD/kd\theta)\, dr/dz$, so that by Equations [3] and [5]

$$\left.\begin{aligned} fP = (dr/kdz)\, P[a(r^2+k^2)\, d\theta/dL + (bdz/d\theta)\, dz/dL]r^2 \\ + Pk^2[cdL/d\theta - a(r^2+k^2)d\theta/dL - (bdz/d\theta)dz/dL] \end{aligned}\right\} \ldots[6]$$

and putting for $d\theta$, dr/k,

$$\left.\begin{aligned} f = (dr/kdz)[a(r^2+k^2)dr/kdL + (bkdz/dr)\, dz/dL]r^2 \\ + k^2[ckdL/dr - a(r^2+k^2)\, dr/kdL - (bkdz/dr)\, dz/dL] \end{aligned}\right\} \ldots[7]$$

Since

$$dL = [r^2 + k^2 + (dz/d\theta)^2]^{\frac{1}{2}}d\theta = [r^2 + k^2 + k^2(dz/dr)^2]dr/k$$

or

$$dL/dr = [r^2 + k^2 + k^2(dz/dr)^2]^{\frac{1}{2}}/k$$

we get:

$$\left.\begin{aligned} f[r^2 + k^2 + k^2(dz/dr)^2]^{\frac{1}{2}}/k = (dr/kdz)a(r^2+k^2)r^2/k \\ + bk(dz/dr)^2r^2 + k^2[c(r^2 + k^2 + k^2(dz/dr)^2)/k - a \\ (r^2+k^2)/k - bk(dz/dr)^2] \end{aligned}\right\} \ldots\ldots[8]$$

or

$$\left.\begin{array}{l} kf[r^2 + k^2 + k^2(dz/dr)^2]^{\frac{1}{2}}dz/dr = (r^2 + k^2)[ar^2 + \\ \quad (c - a)k^2] + k^2[br^2 + (c - b)k^2](dz/dr)^2 \end{array}\right\} \ldots\ldots [9]$$

When powers of k above the third are dropped, there remains:

$$kf[r^2 + k^2 + k^2(dz/dr)^2]^{\frac{1}{2}}(dz/dr) = ar^4 + ck^2r^2 + bk^2r^2(dz/dr)^2 \ldots\ldots\ldots [10]$$

Squaring both sides of Equation [10] gives:

$$\left.\begin{array}{l} k^2f^2[r^2 + k^2 + k^2(dz/dr)^2](dz/dr)^2 \ (ar^4 + ck^2r^2)^2 \\ \quad + 2bk^2r^2(ar^4 + ck^2r^2)(dz/dr)^2 + b^2k^4r^4(dz/dr)^4 \end{array}\right\} \ldots\ldots\ldots [11]$$

Again dropping k^4 and higher powers, we have:

$$k^2f^2r^2(dz/dr)^2 = a^2r^8 + 2ack^2r^6 + 2abk^2r^6(dz/dr)^2 \ldots\ldots [12]$$

or

$$k^2(f^2 - 2abr^4)(dz/dr)^2 = r^4(a^2r^2 + 2ack^2)$$

whence

$$k(f^2 - 2abr^4)^{\frac{1}{2}}dz/dr = r^2(ar + ck^2/r - c^2k^4/2ar^3) \ldots\ldots [13]$$

and, therefore,

$$\left.\begin{array}{l} kdz = ar^3dr/(f^2 - 2abr^4)^{\frac{1}{2}} + ck^2r\,dr/(f^2 - 2abr^4)^{\frac{1}{2}} \\ \qquad - c^2k^4dr/2ar(f^2 - 2abr^4)^{\frac{1}{2}} \end{array}\right\} \ldots\ldots [14]$$

Neglecting the term in k^4 and integrating [14], we find:

$$\begin{array}{l} kz = (^1/_4b)[(f^2 - 2ab)^{\frac{1}{2}}r_o^2 - (f^2 - 2ab)^{\frac{1}{2}}r^2] + [ck^2/2 \\ \quad (2ab)^{\frac{1}{2}}]\{\sin^{-1}[(2ab)^{\frac{1}{2}}r^2/f] - \sin^{-1}[(2ab)^{\frac{1}{2}}r_o^2/f]\} \ldots\ldots\ldots [15] \end{array}$$

Now the factor ab/f is small, so we expand in powers of this fraction, and dropping powers above the first, get:

$$\left.\begin{array}{l} kz = (^1/_4b)[f - abr_o^4/f - f + abr^4/f\} + [ck^2/2(2ab)^{\frac{1}{2}}] \\ \qquad [(2ab)^{\frac{1}{2}}r^2/f - (2ab)^{\frac{1}{2}}r_o^2/f\} \end{array}\right\} \ldots\ldots [16]$$

or

$$kz = a(r^4 - r_o^4)/4f + ck^2(r^2 - r_o^2)/2f \ldots\ldots\ldots [17]$$

The second term on the right is very small, so for an equation of the approximate surface upon which the neutral axis lies, the equation becomes:

$$r^4 = r_o^4 + 4kfz/a \ldots\ldots\ldots\ldots\ldots\ldots [18]$$

or, in rectangular coordinates, for the surface of revolution:

$$(x^2 + y^2)^2 = r_o^4 + 4kfz/a \ldots\ldots\ldots\ldots\ldots [19]$$

To determine f, Equation [18] gives:

$$f = a(r^4 - r_o^4)/4kz \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [20]$$

so that if r_n is the top radius and z_n the total height,

$$f = a(r_n^4 - r_o^4)/4kz_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [21]$$

This gives:

$$z = (r^4 - r_o^4)z_n/(r_n^4 - r_o^4) \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

or

$$r^4 = r_o^4 - (r_n^4 - r_o^4)z/z_n \ldots\ldots\ldots\ldots\ldots\ldots [23]$$

RP-54-12

Design and Investigation of Conical Springs With Coils of a Constant Slope

By J. B. Reynolds[1] and O. B. Schier,[2] Bethlehem, Pa.

J. B. Reynolds

The development of design and the investigation of the characteristics of conical springs with coils of constant slope rather than constant pitch are presented, and also the paper compares the actual performance of these springs with those of constant pitch as well as with the theoretical performance. The conclusions drawn from the investigation are: It is possible with existing formulas to predict the behavior of conical springs. For chrome-vanadium steel of the analysis given, the best heat treatment is to normalize at 1600 F, cool in air, quench at 1550 F in oil, and temper at 900 F in air. Conical springs of constant slope behave more like helical springs than do conical springs of constant pitch as regards load-deflection characteristics.

O. B. Schier

Introduction

In recent years there has been a growing demand for more information regarding all types of mechanical springs. Considerable study has been given to helical springs and much experimental work has been done. Improved formulas for their design have been developed, though these formulas have not been generally accepted by the industries. Recently, the helical spring has been replacing the conical spring in many of its practical applications. This is true even in cases where the form of the conical spring is decidedly advantageous. One reason for this radical change is to be found in the many instances of failure in conical springs. It is thought that these failures are caused by improper design, resulting in high stresses in some coils of the spring. These high stresses may be materially increased by improper heat treatment, repeated loading, and a synchronism of the period of vibration of the spring with that of the motion of the machine. Limited space requirements, likewise, have an important bearing upon the design of springs and hence in this problem of high stresses.

The conical springs most frequently used in the past have been characterized by coils of constant pitch. It is the object of this paper to present the development of design and the investigation of the characteristics of conical springs with coils of constant slope rather than constant pitch; also to compare the actual performance of these springs with those of constant pitch as well as with the theoretical performance.

Description of Springs and Spring Material

The general plan of the investigation was to prepare two series of springs that differ only in respect to pitch and slope of coils. Each type of spring was made in two cross-sections, a rectangular and a circular. All springs, regardless of type or cross-section, are of four active coils and one complete dead coil at each end. The height, which is the same for all springs, is 10 in., and the top diameter, likewise the same for all springs, is $3^{1}/_{2}$ in. The bottom diameter of the springs of rectangular stock is $6^{1}/_{2}$ in., but for the springs of circular stock this diameter was increased to $9^{1}/_{2}$ in. to allow for the same clearance between coils when nested as that between the coils of the lighter rectangular stock. All dimensions of spring size are those of the neutral axis.

The diameters and height of the springs, considering the number of free coils and the dimensions of the material used, indicate an exaggerated size. The springs were designed with these dimensions so that large deflections might be obtained for comparatively small loads, thus permitting a more accurate determination of deflections in the spring coils.

The material used is a chrome-vanadium steel rolled from two heats, the analyses of which are as follows:

	C	Mn	P	S	Si	N	Cr	Va
Rect.	0.46	0.78	0.027	0.027	0.24	0.00	1.04	0.16
Round	0.47	0.70	0.015	0.030	0.20	0.05	1.03	0.16

The dimensions of the rectangular stock are $^{9}/_{16}$ in. by 0.235 in., and the diameter of the circular stock is $^{1}/_{2}$ in. The stock consisted of six bars of each cross-section, each 14 ft long. This length was sufficient to make one spring and allow for two test specimens. These specimens were used to determine the yield point and ultimate stresses and the moduli of elasticity and of shear for the steel.

Mathematical Design

The mathematical analysis for the spring of constant slope, which is the subject of this investigation, is based upon two requirements. The first is that the projection of the neutral axis of

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[1] Supervisory Member in Charge of Research at Lehigh University and Professor of Mathematics and Theoretical Mechanics, Lehigh University. Joseph B. Reynolds was born on May 17, 1881, near New Castle, Pa. He attended country school and New Castle High School; attended Lehigh University on competitive scholarship, 1903–1907; took B.A. degree, 1907; was instructor in mathematics and astronomy, 1907–1913; took M.A. degree at Lehigh, 1910; assistant professor of mathematics and astronomy, 1913–1922; granted degree Ph.D. at Moravian College, 1919; associate professor, 1922–1927; guest professor, Princeton University, 1926–1927; professor of mathematics and theoretical mechanics at Lehigh University, 1927. Mr. Reynolds is the author of many articles and textbooks.

[2] Research Assistant, Lehigh University. Oscar B. Schier was born in Baltimore, Md., on July 18, 1907. He attended the public grade schools for eight years, and then entered the Baltimore Polytechnic Institute; upon graduation in 1926, was admitted to Lehigh University with advanced standing; was graduated in 1929 with M.E. degree, and was appointed the James Ward Packard Fellow.

Presented at the Open Meeting of the A.S.M.E. Special Research Committee on Mechanical Springs, held in connection with the Annual Meeting, New York, N. Y., November 30 to December 4, 1931, of The American Society of Mechanical Engineers.

the spring upon a plane perpendicular to its geometric axis shall be the spiral of Archimedes, represented by the equation $r = r_o + k\theta$. The second is that a tangent to the neutral axis at every point shall make the same angle a with this plane.

In the derivation of the formulas for this spring the following notation is used:

r_o = base radius of active portion of spring
r_n = top radius of active portion of spring
n = number of active coils in spring
θ = vectorial angle of base spiral for point on axis of spring
k = $(r_n - r_o)/\theta_n$
θ_n = vectorial angle corresponding to r_n
P = applied load
A = area of cross-section of the spring material
E = Young's modulus of elasticity of the steel
G = shearing modulus of the steel
L = total length of active spring material
D = axial deflection
a = constant-slope angle
z = height of any point (r, θ, z) of the neutral axis of the spring above the base plane
z_n = total height of the neutral axis above the base plane under no load
θ_z = angular deflection about the geometric axis
I_v = moment of inertia of A with respect to the tangent to the neutral axis
I_u = moment of inertia of A with respect to a line in the cross-section and also in a vertical plane through the center of the cross-section
I_w = moment of inertia of A with respect to a horizontal line in the cross-section through its center
S_1 = an integral
$= {}^1/_4[\theta_n(r_n + r_o)(r_n^2 + r_o^2) + k^2\theta_n(r_n + r_o) - (k^3/2)\log_e(r_n/r_o)]$
S_2 = an integral
$= {}^1/_2(r_n + r_o)\theta + (k/2) \log_e (r_n/r_o)$
Q = $42\, PI_v(S_1 - k^2S_2)/A^4G$
B = $P(S_1 - k^2S_2)/EI_u$
C = Pk^2/EI_w

The mathematical analysis (see Appendix) leads to the following formulas:

$$D = z_n + (Q + C - z_n)/[1 - (Q + C - z_n)(2B + C - Q)/2L^2] \ldots\ldots [1]$$

$$Q_z = P\theta_n \sin a[1/EI_u - 42\, I_v/A^4G]\, (r_n^2 + r_or_n + r_o^2)/3 \ldots [2]$$

$$z = (r_n^2 - r_o^2) \tan \theta/2k \ldots\ldots\ldots\ldots\ldots [3]$$

DESIGN OF MANDREL

In the design of the mandrels upon which the springs of constant slope were wound, use was made of Equation [3]. This equation shows that the neutral axis of the spring lies upon a frustum of a paraboloid of revolution. Therefore the mandrel required a curved profile instead of a straight one, such as was used for winding the springs of constant pitch.

Four mandrels,[3] machined from low-carbon steel, were required for winding the four types of springs to be used in the tests. Accuracy in the construction of the mandrels was considered important, and for this reason the precaution was taken to have all dimensions accurate to one-hundredth of an inch.

METHOD OF WINDING

To wind a spring the stock was fed through a tension tool, mounted on the lathe carriage, and then gripped in a slot at the small diameter of the mandrel. As the stock was heated with an oxyacetylene torch, the mandrel was slowly turned in the lathe, while the carriage and tension tool were moved by hand at a speed just sufficient to feed the stock against the shoulder cut in the surface of the mandrel. When the spring was cool, the remaining stock was cut off and the spring was removed from the mandrel. The remaining stock was then cut into standard tension and torsion specimen lengths, necked, and marked for identification.

HEAT TREATMENT

Although information was available for the heat treatment of chrome-vanadium steel, it was decided to try to determine the treatment best adaptable to conical springs. Four small test pieces were cut from the rectangular stock for the determination of the critical temperature and the Rockwell C hardness. Three of the specimens were normalized at 1600 F and then quenched in oil at a temperature of 1550 F. These temperatures were high enough above the determined critical temperature of 1405 F to assure the relieving of all stresses created during the winding of the spring. The specimens were then tempered at 850, 950, and 1050 F, respectively, for the determination of the Rockwell C hardness. The results were:

Specimen No.	Tempering temperature, F	Rockwell C hardness
1	850	45
2	950	43
3	1050	41–42

A comparison of these results with the standard[4] established by the American Society for Steel Treating revealed that tempering temperatures of 950 F and 1050 F were too high to give sufficient hardness to this steel. However, as a precaution two springs of rectangular stock and uniform pitch were treated at the temperatures of 950 F and 1050 F, respectively. The spring tempered at 1050 F was too soft to give a true permanent set. To avoid extreme end effects at the top of the spring, the spring tempered at 950 F was loaded to only 450 lb for 24 hr and was found to have acquired a permanent set of 1.62 in. A third spring of the same type and material was then tempered at 900 F and placed under a load of 450 lb for 24 hr. The set was found to be 1.23 in. All sets are for a total deflection of approximately 9 in. under the maximum load. Though this set was still large, the tempering temperature was not reduced further pending the testing of a spring of rectangular stock and constant slope given the same treatment. This procedure was followed because this type of spring has a greater pitch in the first coil, which led to the belief that less set would occur. It was found that this type of spring acquired only 0.55 in. permanent set under the same loading conditions.

Though no identical comparison of characteristics could be made between the springs of rectangular stock and those of circular stock because of the increase in the base diameter of the latter springs, it was desirable to have these springs receive the same heat treatment to permit comparison of yield-point stresses for the two stocks. Therefore two springs of circular stock, one of constant pitch and one of constant slope, were normalized at 1600 F, quenched in oil at 1550 F, and tempered at 900 F. When loaded to 900 lb for 24 hr, the spring of constant pitch acquired a permanent set of 0.10 in., while the spring of constant slope acquired a set of 0.29 in. under an equal load during the same time. Here, again, the total deflection at maximum load was approximately 9 in.

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[3] For a detailed description of construction, see Progress Report No. 8 of the A.S.M.E. Special Research Committee on Mechanical Springs, "The Design of Conical Springs With Coils of a Uniform Slope," by J. B. Reynolds and O. B. Schier, presented at Annual Meeting, New York, N. Y., December 1 to 5, 1930, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

[4] A.S.S.T. Handbook, pp. 149–152.

These results were judged to be satisfactory, so that the treatment given to all springs and test specimens was as follows:

Treatment	Temperature, F	Cooling medium
Normalize	1600	Air
Quench	1550	Oil
Temper	900	Air

It was essential that there should not be any change in the shape of the spring when it reached a plastic state during the heat treatment. Therefore, an arbor was designed to hold every coil of the spring in place. The arbor was constructed of steel plates welded together to form four fins, the profiles of which corresponded to the inside dimensions of the spring. Each coil was supported at four points at the lower edge, while the top and bottom dead coils were held by shoulders on the fins. Three springs of each type were treated individually on one arbor. The tempering was done in an electric furnace, providing positive control of the tempering temperature, so that each spring received the same final treatment.

TESTING REQUIREMENTS

In testing the springs it was desirable that the following requirements be met: (1) that there be an axial application of load, (2) that the top of the spring be permitted to rotate or not as desired, and (3) that the spring be prevented from tipping. By the design and construction of special attachments it was possible to use Riehle and Olsen testing machines for the tests. These tests were as follows: Loading and unloading of the spring in 50-lb increments of load with—

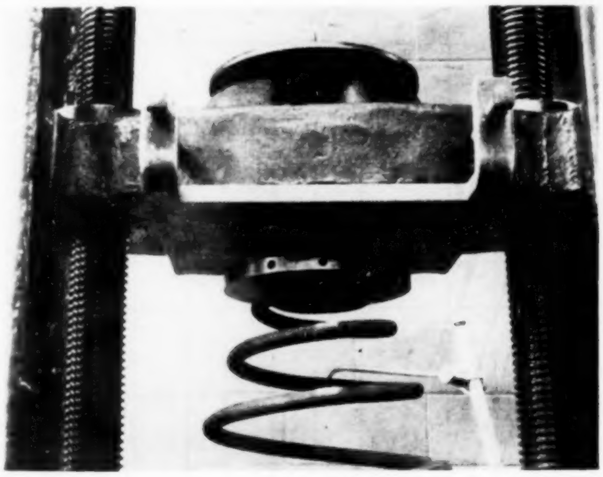

FIG. 1 THRUST BEARING WITH TORQUE PINS IN PLACE

(1) the top of the spring unconstrained from rotation
(2) the top of the spring constrained from rotation
(3) the top of the spring unconstrained under repeated loading ten times as rapidly as possible.

Sufficient runs were made in each test to determine whether the spring characteristics were constant under repeated testing. In every series of tests and for each increment of load up to the maximum of 500 lb for the springs of rectangular stock and 900 lb for the springs of circular stock, the following data were recorded: (1) axial deflection of spring, (2) angular deflection, and (3) net height of each coil; that is, the distance from the bottom of the spring to the top of each coil.

The axial deflections were measured with an inside micrometer from a fixed reference plane to the top of the spring, as shown in Fig. 2. The angular deflections were obtained by using an attachment designed to fit both the Olsen and Riehle testing machines, used in testing the springs of rectangular stock and circular stock, respectively. This attachment, shown in Fig. 1, consists of an S-K-F thrust bearing, one race of which is fastened to a plate which clamps on the movable platen of the machine and the other race of which is fastened to a circular plate which centers the top of the spring under the bearing. Through this

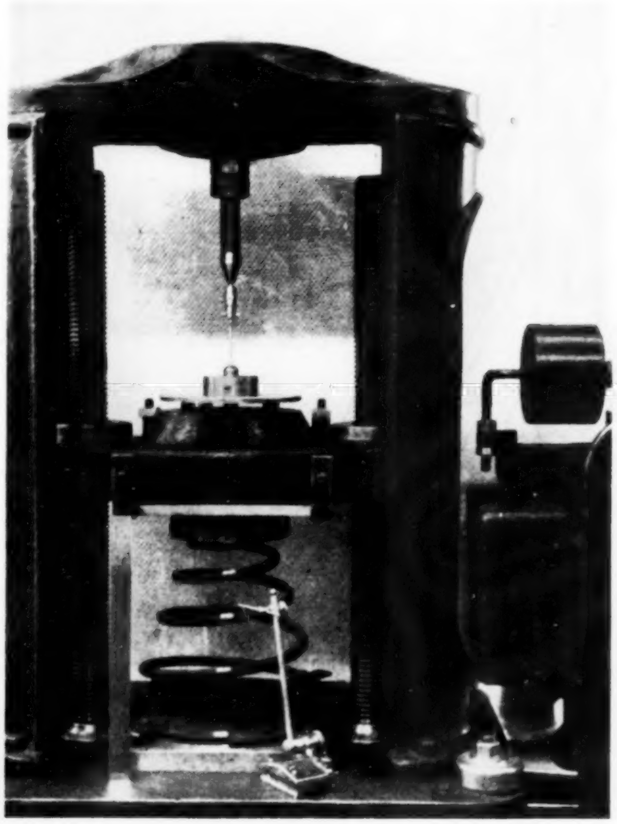

FIG. 2 SPRING UNDER INITIAL LOAD

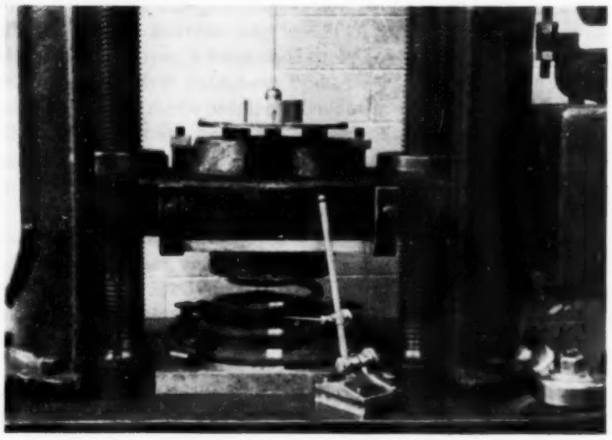

FIG. 3 SPRING PARTLY COMPRESSED, SHOWING NESTING OF COILS

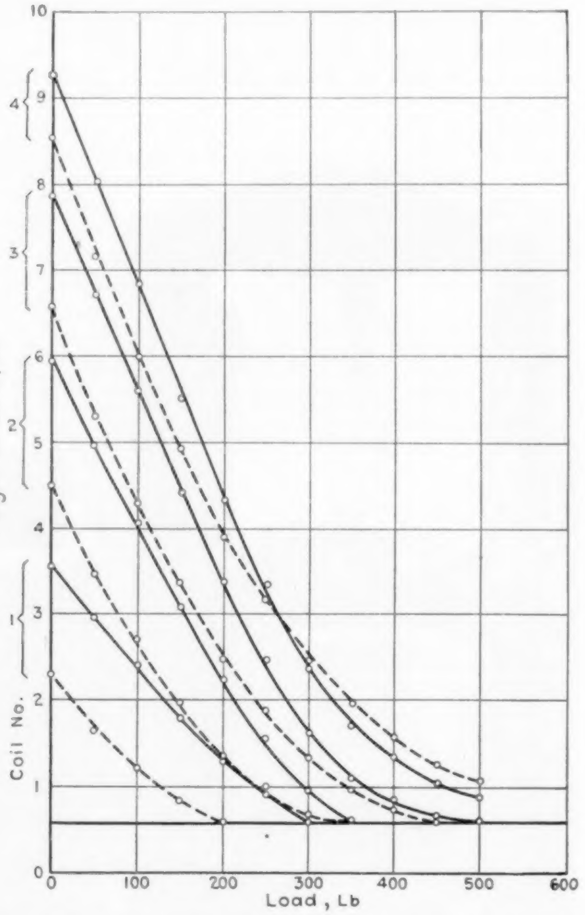

FIG. 4 COMPARISON OF COIL ACTIONS IN TWO CONICAL SPRINGS OF RECTANGULAR STOCK, WITH TOP FREE TO ROTATE

(Solid line, coils of constant slope; broken line, coils of constant pitch.)

plate are two holes in which torque pins can be inserted and screwed into the platen. These pins prevent turning of the bearing. To this circular plate is screwed a post which projects through the bearing, platen, and graduated disk on top of the platen. To the post is attached a pointer which moves over the disk, graduated in degrees, as the spring rotates under the application of load. Little change results in the tare of the machine, since the torque pins are the only change made in the apparatus when changing from constrained to unconstrained rotation.

The measurements of the net height of each coil was made with a surface gage as shown in Fig. 3. With the gage sitting on the machine table, the pointer was adjusted until it touched the top of the coil. The gage was then swung around, and the distance of the pointer above the table was measured with a steel scale. A radial line was scribed across the top of each coil so that the measurement would be made to the same point for each reading. These lines were placed 360 deg apart and directly above the heel of the spring.

A Ewing tensometer was used to measure the elongation of the tension specimens for determining the modulus of elasticity of the steel. The specimens of the rectangular stock were necked to an area $^3/_8$ in. by 0.235 in., while those of the circular stock were necked to a $^3/_8$ in. diameter. The results obtained from the tests on the rectangular specimens tempered at 1050, 950, and 900 F show the importance of proper heat treatment. The results are as follows:

Tempering temperature, F	Yield-point stress, lb per sq in.	Ultimate stress, lb per sq in.
1050	159,000	164,800
950	162,000	165,000
900	189,000	198,000

From the specimen with a temper of 900 F the modulus of elasticity was found to be 28,000,000 lb per sq in. The circular stock, which was of a slightly different analysis, had a yield point stress of 199,000 lb per sq in. and an ultimate strength of 206,000 lb per sq in. for a temper temperature of 900 F. This steel also had a modulus of elasticity of 28,000,000 lb per sq in.

Torsion tests were made on specimens from the circular stock to determine the modulus of shear of the chrome-vanadium steel. An average of several tests gave a modulus of shear of 11,300,000 lb per sq in. The shear modulus was not determined for the rectangular stock, because the small section of the necked specimen would not permit an accurate determination. The value was assumed to be the same as for the circular stock, but later evidence shows that this assumption was probably in error.

RESULTS OF TESTS

Fig. 4 gives a comparison between the coil actions of a spring of rectangular stock with coils of constant slope and one with coils of constant pitch while the top of the springs are free to rotate. It will be noticed that the coils of constant slope more

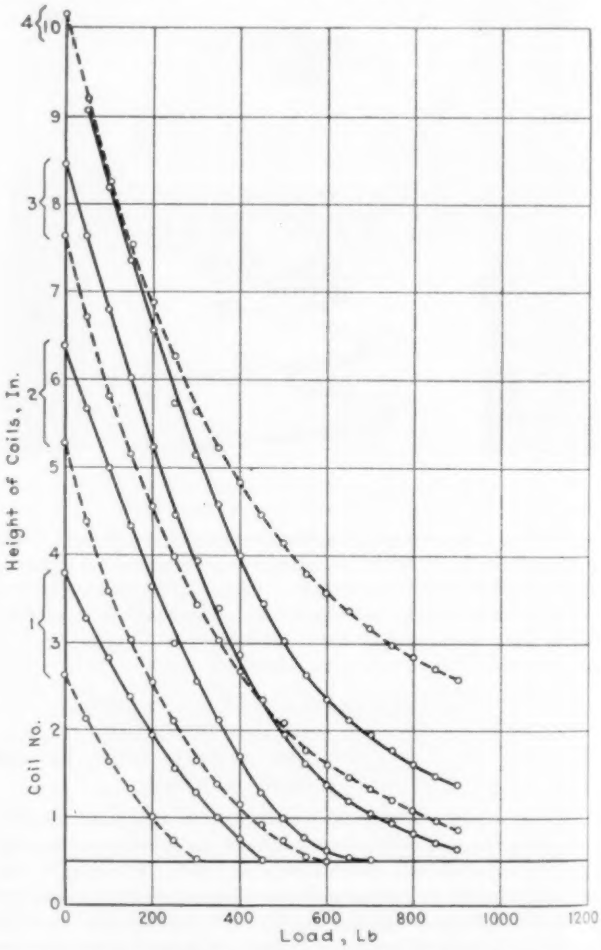

FIG. 5 COMPARISON OF COIL ACTIONS IN TWO CONICAL SPRINGS OF CIRCULAR STOCK, WITH TOP FREE TO ROTATE

(Solid line, coils of constant slope; broken line, coils of constant pitch.)

nearly approach the straight-line characteristic of the helical spring than do the coils of constant pitch. This is due principally to the delayed nesting of the lower coil, which does not occur until a load is applied that is greater than that making the lower coil of the constant-pitch spring nest. This prolongs the free action of the upper coils. The permanent set in the spring of constant slope is less than that in the spring of constant pitch, there being a difference of $^3/_4$ in. in their final heights.

There was made a similar comparison of the same springs, with the exception that the top of the spring was constrained from rotation. The actual difference between the two types of end conditions proved so small for these springs that there seems to be little advantage in the use of unconstrained rotation.

Fig. 5 is a comparison between the coil actions of a spring of circular stock with coils of constant slope and one with coils of constant pitch. As in the springs of rectangular stock the coils of constant slope approach the straight-line characteristic of the helical spring. It is thought that the sharp angle between the dead coil and the lower active coil has considerable effect on the action of the spring coils. This effect, here called end effect, is far more pronounced in the spring of circular stock because of the larger base diameter and correspondingly larger pitch. The departure of the curves from a straight line up to a load of 450 lb, where the first coil nests, is thought to be largely caused by this end effect.

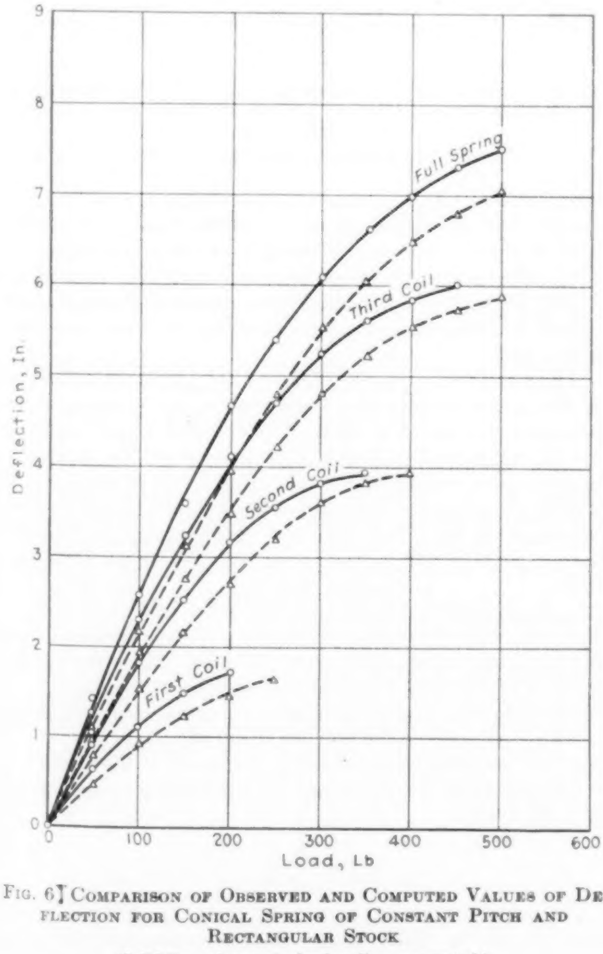

FIG. 6 COMPARISON OF OBSERVED AND COMPUTED VALUES OF DEFLECTION FOR CONICAL SPRING OF CONSTANT PITCH AND RECTANGULAR STOCK

(Solid line, observed; broken line, computed.)

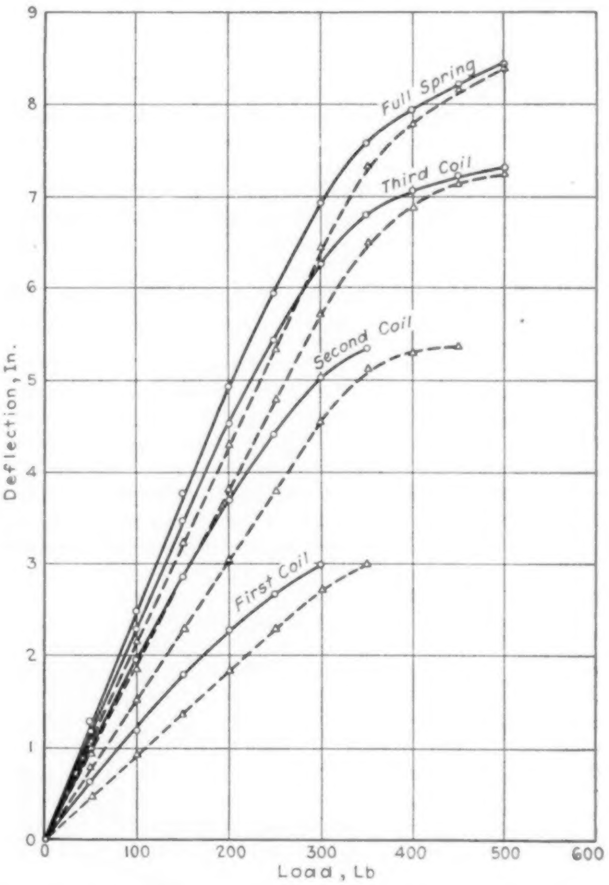

FIG. 7 COMPARISON OF OBSERVED AND COMPUTED VALUES OF DEFLECTION FOR CONICAL SPRING OF CONSTANT SLOPE AND RECTANGULAR STOCK

(Solid line, observed; broken line, computed.)

In computing deflection for the spring of rectangular stock and constant pitch, the formula for rectangular stock as adapted to conical springs by J. K. Wood[5] has been used, with the results shown in Fig. 6. This formula is as follows:

$$D = K_c K_f \frac{19.6\, P r_1{}^3 n}{G t^3 (b - 0.56t)}$$

where

D = unit deflection of each half coil, in.
P = load, lb
r_1 = base radius of each half coil
r_2 = top radius of each half coil
n = number of coils = $^1/_2$
G = modulus of shear, lb per sq in.
t = width of stock, in.
b = depth of stock, in.
K_c = $(1 + e)(1 + e^2)/4$
K_f = $1 + 2(1 + e + e^2)/3c(1 + e)(1 + e^2) - 1/c^2(1 + e^2)$
e = r_2/r_1
c = $2r_1/t$.

A similar formula is used in computing the curves for circular stock in Fig. 8. This formula is as follows:

$$D = K_c K_f \frac{64\, P r_1{}^3 n}{G d^4}$$

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[5] "Conical Spring Design; Some New Factors and Formulas," by J. K. Wood, 1929, *Am. Mach.*, vol. 71, pp. 620–624.

where the values are the same as above and

d = diameter of circular stock, in.
c = $2r_1/d$.

The unit deflection for each half coil was obtained by calculating a constant for each half coil and multiplying this by the increments of load. The constants vary with the base radius and top radius of each half coil. Thus by varying the values of c and e and in turn K_c and K_f by using a varying base radius, the total deflections of each coil were similar to the experimental deflections.

To compute the deflections for the springs of constant slope the formula developed by J. B. Reynolds was used. It is as follows:

$$D = z_n + (Q - z_n)/[1 - (Q - z_n)(2B - Q)/2L^2]$$

where

D = total deflection of the spring, in.
z_n = total height of spring after permanent set, in.
Q = $4\pi^2 P I_v (S_1 - k^2 S_2)/A^4 G$, where $I_v = \pi d^4/32$ for the circular stock and $I_v = bt(t^2 + b^2)/12$ for the rectangular stock
B = $P(S_1 - k^2 S_2)/EI_u$
L = $(\sec a)[(r_o + r_n)\theta_n + k \log_e (r_n/r_o)]$.

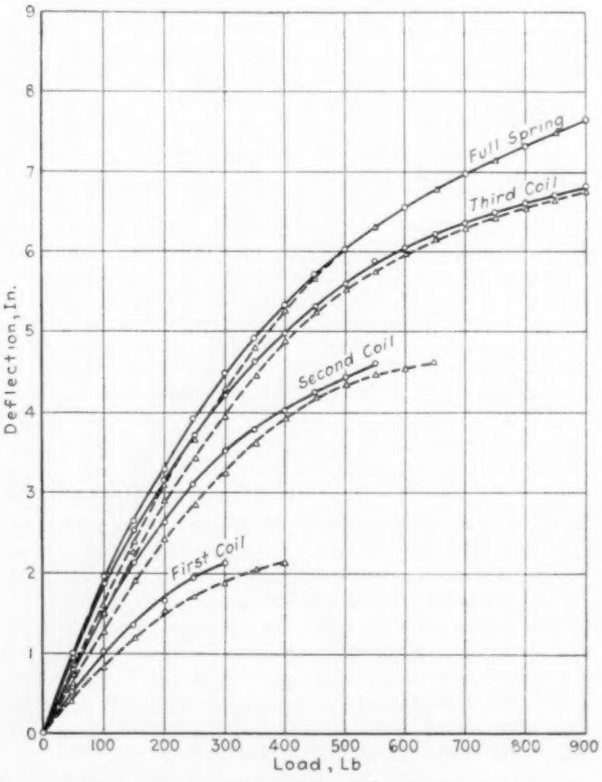

Fig. 8 Comparison of Observed and Computed Values of Deflection for Conical Spring of Constant Pitch and Circular Stock
(Solid line, observed; broken line, computed.)

Figs. 6 and 7 give the comparison between observed and computed deflections for two conical springs of rectangular stock. Fig. 6 compares the deflections in a spring of constant pitch, while Fig. 7 compares the deflections in a spring of constant slope. The lack of agreement in Figs. 6 and 7 is probably partly due to the value of the shear modulus used in the formula. As previously mentioned, this value was not determined for the rectangular stock, and so the value of 11,300,000 lb per sq in. is likely to be in error. This error is cumulative, since the deflections as calculated are unit half-coil deflections which are then added to give the total. A decrease of the modulus of shear would increase the deflection, and it is believed that the value, as chosen, may be high.

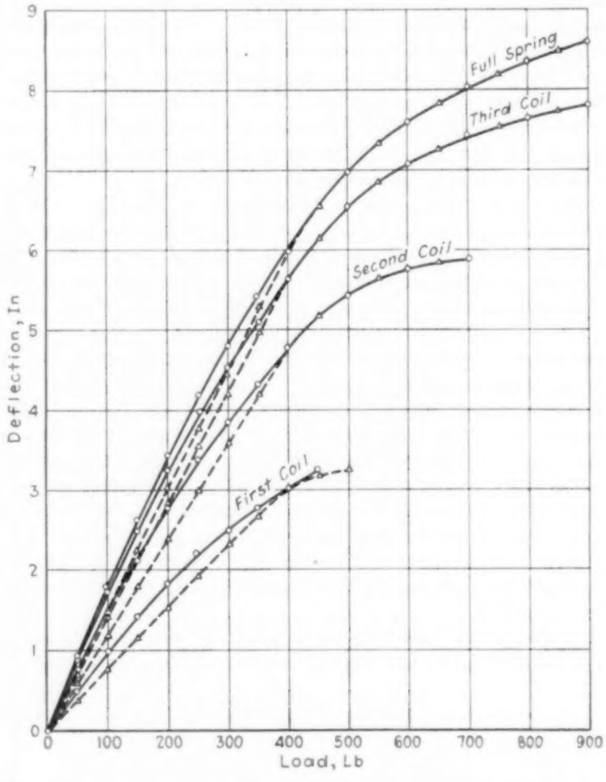

Fig. 9 Comparison of Observed and Computed Values of Deflection for Conical Spring of Constant Slope and Circular Stock
(Solid line, observed; broken line, computed.)

Figs. 8 and 9 compare the observed and computed deflections for two conical springs of circular stock. Fig. 8 compares the deflections in a spring of constant pitch, while Fig. 9 compares the deflections in a spring of constant slope. The agreement detween these deflections is good. The variation that appears between the curves up to the load of 400 lb is probably due almost entirely to the end effect at the base of the spring; for after the first coil nests at 450 lb, the curves coincide for the remaining loads.

In the springs of both rectangular stock and circular stock those with coils of constant slope give a better agreement than do the springs of constant pitch. Whether this is due to the accuracy of the deflection formula developed in the paper or to the differences of end effect cannot be stated until further research is conducted on the problem of end effects.

In the theoretical determination of stresses in the springs of rectangular stock, use was made of the following formula given by A. M. Wahl:[6]

$$S = Pr_1 \frac{(3b + 1.8t)k}{b^2 t^2}$$

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[6] "Further Research on Helical Springs of Round and Square Wire," by A. M. Wahl, Trans. A.S.M.E., 1930, vol. 52, paper APM-52-18.

where

S = maximum shear stress, lb per sq in.

P = axial load on spring

r_1 = mean base radius of each half coil

b = depth of rectangular section

t = width of rectangular section

k = stress multiplication factor

$= \dfrac{4c-1}{4c-4} + \dfrac{0.615}{c}$

c = $2r/t$

This formula was used for computing stresses in springs of both constant pitch and constant slope. The factor k is the multiplication factor for stresses on the inside of the coil where the maximum stresses occur. Fig. 10 represents the computed stresses in each half coil in springs of rectangular stock. The dotted lines show the increase in stress in the coils of constant slope over the stress in the coils of constant pitch. From Fig. 10 it will be seen that the maximum theoretical stress occurs in the middle section of the fourth coil in both types of springs. It follows that failure is most likely to occur here. This has been found to be the case in numerous examples of failure in springs in operation.

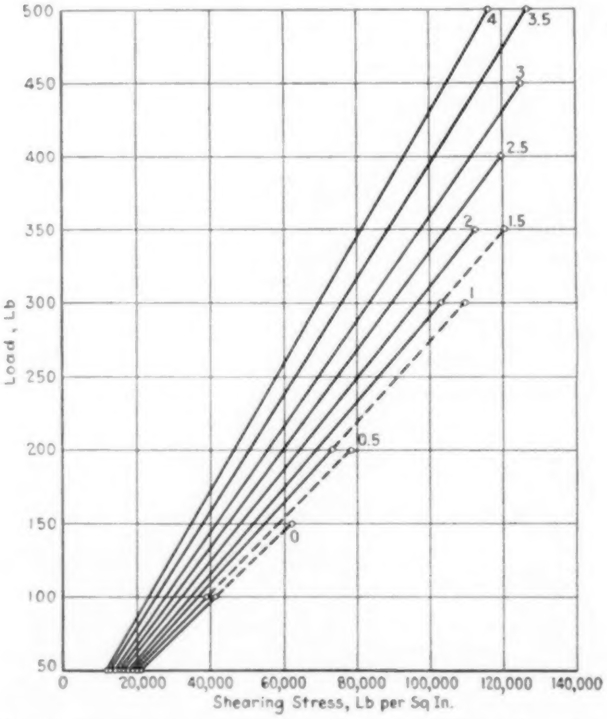

Fig. 10 Comparison of Load-Stress Curves for Each Half Coil in Two Conical Springs of Rectangular Stock

(Solid line, coils of constant pitch; broken line, coils of constant slope. Figures indicate number of coil.)

The stresses in springs of circular stock have been computed from the following formula given by A. M. Wahl:[7]

$$S = \frac{16\,Pr_1}{\pi d^3}\,k$$

where

S = maximum shear stress, lb per sq in.

P = axial load on spring

r_1 = mean base radius of each half coil

d = diameter of stock

k = stress multiplication factor

$= \dfrac{4c-1}{4c-4} + \dfrac{0.615}{c}$

c = $2r/d$

Fig. 11 represents the computed stresses at each half coil in springs of circular stock. The maximum theoretical stress occurs in the middle section of the third coil in both types of springs. However, the stress in the full third coil is almost exactly equal to that in the middle section, and failure may occur at either section.

In connection with the determination of stresses it should be pointed out that there is yet much to be done. The work was begun with a definite plan in mind for the development of a spring which would acquire higher stresses in the lower coils than have been previously obtained in springs with coils of constant pitch. That is, an endeavor was made to obtain a more uniform division of stresses throughout the spring coils. The study of stresses, however, was not carried to the extent first planned because of the unmeasured variation from the testing requirement of an axial application of load. It was thought that one full dead coil at both the top and bottom of the spring would approximately meet this requirement. However, the results of this investigation seem to imply considerable end effects and corresponding eccentricity of loading.

The load as actually applied to the spring acts as a combination of a force and a couple. The effective force on the top coil is applied somewhere between the axis of the spring and the point where the active coil joins the dead coil. This point shifts as the load is increased, since the bearing point or heel at the bottom of the spring moves as the coils nest. That is, as the coils nest the base radius decreases and the heel of the spring

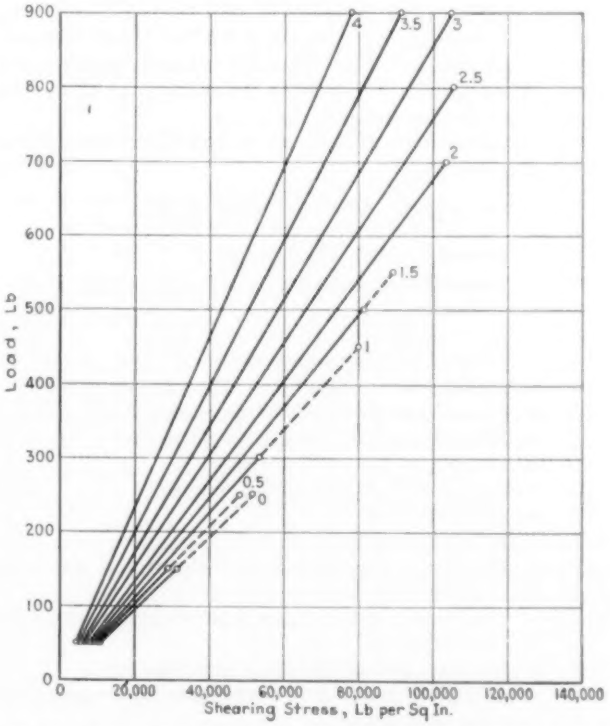

Fig. 11 Comparison of Load-Stress Curves for Each Half Coil in Two Conical Springs of Circular Stock

(Solid line, coils of constant pitch; broken line, coils of constant slope. Figures indicate number of coil.)

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[7] "Stresses in Heavy Closely Coiled Helical Springs," by A. M. Wahl, Trans. A.S.M.E., 1929, vol. 51, paper APM-51-17.

rotates about the axis of the spring. Thus for a light load, with all active coils free, the heel at the top of the spring is in an axial plane over the heel at the base so that both forces lie a maximum distance to one side of the spring axis. This condition gives maximum stresses in the coils at the half points directly opposite from the heel. With a load on the spring sufficient to move the heel around to the half point in any coil, the forces have now been shifted to the opposite side of the spring axis. However, the distance of the forces from the spring axis is not as great as before because of the reduced base radius; thus the increase in stresses to be added to those existing is less than for the above condition.

It seems, then, that the maximum stresses can occur only at the half points in the coils directly opposite the heel of the spring under full load.

CONCLUSIONS

The following conclusions can be drawn from the results of the investigation:

1 It is possible with existing formulas to predict with considerable accuracy the behavior of conical springs.

2 For chrome-vanadium steel of the analysis given in this paper the best of several heat treatments tried was: Normalize at 1600 F; cool in air; quench at 1550 F in oil; temper at 900 F in air.

3 Conical springs of constant slope behave more like helical springs than do conical springs of constant pitch as regards load-deflection characteristics.

4 The coils of conical springs of constant slope seat less quickly under load than do those of constant pitch, but develop higher theoretical stresses.

5 There is need of research work on end effects in conical springs.

Appendix

The purpose of this discussion is to derive formulas for the design of a conical spiral spring and for the angular deflection under a known load. The results will be obtained in terms of constants for the material of which the spring is made, the load applied, and the dimensions of the spring.

In the derivation of the formulas for this spring the following notation is used:

r_o = base radius of active portion of spring
r_n = top radius of active portion of spring
n = number of active coils in spring
θ = vectorial angle of base spiral for point on axis of spring
k = $(r_n - r_o)/\theta_n$
θ_n = vectorial angle corresponding to r_n
P = applied load
A = area of cross-section of the spring material
E = Young's modulus of elasticity of the steel
G = shearing modulus of the steel
L = total length of active spring material
D = axial deflection
a = constant slope angle
z = height of any point (r, θ, z) of the neutral axis of the spring above the base plane
z_n = total height of the neutral axis above the base plane under no load
θ_z = angular deflection about the geometric axis
I_v = moment of inertia of A with respect to the tangent to the neutral axis
I_u = moment of inertia of A with respect to a line in the cross-section and also in a vertical plane through the center of the cross-section
I_w = moment of inertia of A with respect to a horizontal line in the cross-section through its center.

It is assumed that the projection of the neutral axis of the spiral spring upon the plane of its base is a plane spiral whose equation is $r = r_o + k\theta$ and that the tangent to the neutral axis at all points makes a constant angle a with the plane of the base. The angle a will differ for different loads on the spring, but will be assumed constant for a given load. The initial radius of the plane spiral is r_o. It is the value of r for $\theta = 0$. If r_n is the final radius of the plane spiral and θ_n equals 2π times the number of coils n of the spring which act freely, then $r_n = r_o + k\theta_n$, from which k may be obtained. By that part of the spring which acts freely is meant the portion which does not come in contact with the base or with the mechanism by which the pressure is applied.

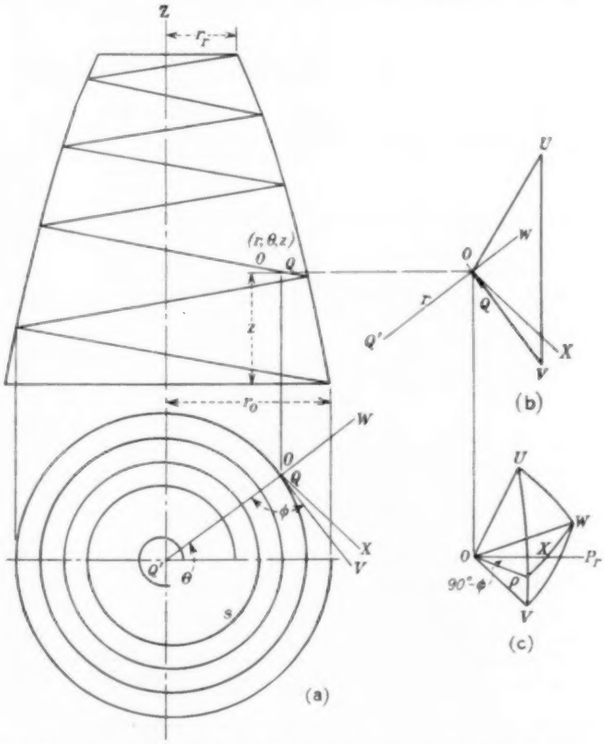

FIG. 12 NEUTRAL AXIS OF THE SPRING IN HORIZONTAL AND VERTICAL PLANES

Let ϕ be the angle between the radius vector r and the tangent to the plane curve $r = r_o + k\theta$, then $\tan\phi = r/(dr/d\theta) = r/k = \sin\phi/\cos\phi$. Since ϕ is nearly 90 deg, $\sin\phi$ is nearly unity, so we may write $\cos\phi = k/r$ and $\sin^2\phi = 1 - k^2/r^2$. If the length of the plane curve is called s and that of the neutral axis L, then $ds = dL\cos a$, for L makes a constant angle a with the plane of s. From this $dL = [r^2 + (dr/d\theta)^2]^{\frac{1}{2}}d\theta \sec a = (\sec a/k)(k^2 + r^2)^{\frac{1}{2}}dr$. Since k is small, dL is expanded in powers of k, giving $dL = (\sec a/k)[r + k^2/2r - k^4/8r^3 + \ldots]dr$. Integration between the limits r_o and r_n leads to (1) $L = (1/2)(\sec a)[(r_o + r_n)\theta_n + k\log_e(r_n/r_o) - k^4\theta_n(r_o + r_n)/8r_n^2r_o^2]$, a formula from which to get L that will be needed later.

Fig. 12(*a*) shows projections of the neutral axis of the spring in horizontal and vertical planes. Consider a small portion QO of the plane curve s. $Q'OV$ is the angle ϕ. OX is drawn perpendicular to $Q'O$ and OW perpendicular to the tangent OV.

Fig. 12(*b*) shows a small portion QO of the neutral axis of the spring as seen obliquely. *OV* is tangent to the neutral axis. *OU* is perpendicular to *OV* in the vertical plane through *OV*, and *OW* is perpendicular to both *OV* and *OU*. *OX* is perpendicular to *OQ'* in the horizontal plane *XOQ'*, as in Fig. 12(*a*).

Fig. 12(*c*) is a schematic diagram showing right spherical triangles, on a sphere with *O* as center, used to assist in getting angular relations for resolution of the vector *Pr* along *OX* into components along *OV*, *OU*, and *OW*. The vector *Pr* arises from the consideration that for the point *O* the pressure *P* may be thought of as a vertical force applied at *Q'* producing a torque *Pr* about *OX* which may be represented by a vector in the direction of *OX*. The assumption that the resultant of the pressure applied to the spring to compress it is equivalent to a force *P* along the axis of symmetry of the spring is, in effect, assuming that the pressure is symmetrically distributed about this axis where it is applied.

A length dL of the spring may be thought of as a beam subject to moments which tend to bend it an angular amount $d\theta_w$ about the axis *OW* and an angular amount $d\theta_u$ about the axis *OU* and to twist it through an angle $d\theta_v$ about *OV*, its neutral axis. The resultant of all these angular displacements must be an angular displacement $d\theta_x$ about *OX*. The amount that *O* will move parallel to the axis of symmetry of the spring due to this angular displacement is then $rd\theta_x$. This is dD. The integral of dD throughout the length of the spring gives *D*, the total deflection or compression of the spring.

Let the torque *Pr* be resolved into components T_w, T_u, T_v about *OW*, *OU*, *OV*, respectively; then from Fig. 12(*c*):

$$T_w = Pr \cos XOW = Pr \cos \phi,\ T_u = Pr \cos XOU = Pr \sin a \sin \phi$$

$$T_v = Pr \cos XOV = Pr \cos a \sin \phi.$$

Also,

$$d\theta_x = d\theta_v \cos XOV + d\theta_u \cos XOU + d\theta_w \cos XOW.$$

By the principles of mechanics of materials,

$$d\theta_w = T_w dL/EI_w,\ d\theta_u = T_u dL/EI_u,\ d\theta_v = 42\, I_v T_v dL/A^4G$$

in which *E* is Young's modulus for the material of which the spring is made and *G* is its modulus of shear. I_w is the moment of inertia of a cross-section of the spring material with respect to the axis *OW* and I_u the corresponding moment of inertia with respect to the axis *OU*. I_v is the moment of inertia of a cross-section about *OV* and *A*, its area.

From the relations so far given:

$$\left.\begin{aligned} dD = rd\theta_x = {} & [42\, I_vP \cos a/A^4G + P \sin a \tan a/EI_u] \\ & r^2(r^2 + k^2)^{\frac{1}{2}}d\theta + [Pk^2 \sec a/EI_w - 42\, I_uPk^2 \cos a/A^4G \\ & - Pk^2 \sin a \tan a/EI_u](r^2 + k^2)^{\frac{1}{2}}d\theta \end{aligned}\right\} \ldots\ [1]$$

Replacement of $d\theta$ by dr/k and integration between the limits r_o and r_n yields:

$$\left.\begin{aligned} D = {} & P(42\, I_v \cos a/A^4G + \sin a \tan a/EI_u)S_1 + Pk^2 \\ & (\sec a/EI_w - 42\, I_v \cos a/A^4G - \sin a \tan a/EI_u)S_2 \end{aligned}\right\} \ldots\ [2]$$

in which S_1 and S_2 are integrals defined as follows:

$$S_1 = 1/k \int_{r_o}^{r_n} r^3(1 + k^2/r^2)^{\frac{1}{2}}dr$$

$$= 1/k \int_{r_o}^{r_n} (r^3 + k^2r/2 - k^4/8r + \ldots)dr$$

$$= (r^4_n - r^4_o)/4k + k(r^2_n + r^2_o)/4 - (k^3/2) \log_e (r_n/r_o) \ldots$$

$$= (1/4)[\theta_n(r_n + r_o)(r_n^2 + r_o^2) + k^2\theta_n(r_n + r_o) - (k^3/2) \log_e (r_n/r_o)]$$

and

$$S_2 = 1/k \int_{r_o}^{r_n} r(1 + k^2/r^2)^{\frac{1}{2}}dr = 1/k \int_{r_o}^{r_n} (r + k^2/2r \ldots)dr$$

$$= 1/k[(r_n^2 - r_o^2)/2 + (k^2/2) \log_e(r_n/r_o) \ldots]$$

$$= 1/2(r_n + r_o)\theta_n + (k/2) \log_e(r_n/r_o)$$

When regrouped, the result for *D* may be written:

$$D = Q \cos a + B \sin a \tan a + C \sec a \ldots\ [3]$$

in which

$$Q = 42\, PI_v(S_1 - k^2S_2)/A^4G,\ B = P(S_1 - k^2S_2)/EI_u$$

$$C = Pk^2/EI_w$$

Now the angle *a* depends upon the compression *D* in such a way that if z_n is the free height of the spring, $\sin a = (z_n - D)L$, and hence to second powers of $(z_n - D)L$, which is always a small quantity,

$$\sin a \tan a = (z_n - D)^2/L^2,\ \sec a = 1 + (z_n - D)^2/2L^2$$

$$\cos a = 1 - (z_n - D)^2/2L^2$$

Hence the deflection may be written:

$$D = Q + C + (2B + C - Q)\,(z_n - D)^2/2L^2$$

and for a close approximation:

$$D = z_n + (Q + C - z_n)/[1 - (Q + C - z_n)\,(2B + C - Q)/2L^2] \ldots\ [4]$$

If the term in $(z_n - D)^2/2L^2$ be neglected, then $D = Q + C$, and since *Q* and *C* vary directly with *P*, to this degree of accuracy, *D* varies directly with *P*. Experiments show that this is nearly true for considerable ranges of *P*.

Instead of using the approximate result from [4], one may use [3], solving it by trial. First let $a = 0$, giving $D = Q + C$; with this value of *D* determine *a* by $\sin a = (z_n - D)/L$; with this value of *a* determine a new value D_1 of *D*. With D_1 get a new value a_1 of *a*, and with this another value D_2 of *D*, etc. The values *D*, D_1, D_2, etc., will rapidly converge toward the exact value of *D*.

The angular deflection θ_z of the spring about the vertical is given by

$$\begin{aligned} d\theta_z & = d\theta_u \cos a - d\theta_v \sin a \\ & = T_u \cos a dL/EI_u - 42T_vI_v \sin a dL/A^4G \\ & = (1/EI_u - 42\, I_v/A^4G)Pr \sin a \cos a \sin \phi dL \end{aligned}$$

Upon expanding and integrating, it is found that

$$\theta_z = P \sin a[1/EI_u - 42\, I_v/A^4G]\theta_n(r_n^2 + r_nr_o + r_o^2)/3 \ldots\ [5]$$

in which powers of *k* above the third have been dropped.

The height *z* of any point (r, θ, z) on the neutral axis above the base, since $d_z = dL \sin a$, is $z = L \sin a$, or by [1]

$$z = \tan a[r_n^2 - r_o^2 + k^2 \log (r_n/r_o) - k^4(r_n^2 - r_o^2)/8r_n^2r_o^2]/2k \ldots\ [6]$$

This is the equation of the surface upon which the neutral axis of the spring lies. If k^3 and higher powers are dropped, the equation reduces to that of a paraboloid of revolution with respect to the axis of the spring. Thus

$$z = (r_n^2 - r_o^2) \tan a/2k \ldots\ [7]$$

or in rectangular coordinates

$$z = (x^2 + y^2 - r_o^2) \tan a/2k$$

RP-54-13

Determination of Carbonate, Hydroxide, and Phosphate in Boiler Waters

Progress Report of Subcommittee No. 8 of the Joint Research Committee on Boiler-Feedwater Studies[1]

By W. C. SCHROEDER,[2] ANN ARBOR, MICH., and C. H. FELLOWS,[3] DETROIT, MICH.

Work of Subcommittee No. 8 of the Committee on Boiler-Feedwater Studies this year has involved critical examination of the methods generally used for examining boiler water, with a view of establishing methods for plant control and referee work that later might be recommended for adoption as standard in the industry. There has been much dissatisfaction with several methods for determining certain constituents in boiler water. This is because the methods used were developed for examining other types of water and consequently are not always applicable for accurate testing of water in steam boilers.

Those methods which, in the opinion of the subcommittee, required immediate consideration because they had to do with the determination of constituents important in control work in most plants, and because no generally accepted scheme of procedure had been found to yield satisfactory results, were: (1) Methods for the determination of alkalinity, especially that due to carbonate salts; (2) those for dissolved oxygen and carbon dioxide; and (3) those for the determination of phosphates and silicates, inasmuch as these salts influence alkalinity and, in some cases, interfere with its determinations.

It was felt that the most logical way to clear up the questions arising in connection with these methods was to put the matter before a competent technician who would devote his entire time to the problem. The subcommittee was authorized to establish a fellowship in some university to carry out the work, and one was established at the University of Michigan. The first problem taken up was methods for determining alkalinities and some of those ions influencing alkalinity and its determination. The report tells of the progress made in carrying out this work.

AT THE MEETING of the Committee on Boiler-Feedwater Studies held in Pittsburgh, Pa., in May, 1931, the proposed work of the reorganized Subcommittee No. 8 was discussed. The work involved critical examinations of the methods generally used for examining boiler water, with the view of establishing methods for plant control and referee work that later might be recommended for adoption as standard in the industry.

W. C. Schroeder

C. H. Fellows

It was brought out in general discussion that a great deal of dissatisfaction with several methods for determining certain constituents in boiler water existed. This was due principally to the fact that the methods had been developed for use in the examination of other types of water and consequently were not always applicable for accurate testing of the water in steam boilers.

Those methods which, in the opinion of the subcommittee, required immediate consideration because, first they had to do with the determination of constituents important in control work in most plants, and second, because no generally accepted scheme of procedure had been found to yield satisfactory results, were (*A*) methods for the determination of alkalinity, especially that due to carbonate salts; (*B*) those for dissolved oxygen[4] and carbon dioxide; and (*C*) those for the determination of phosphates and silicates inasmuch as these salts influenced alkalinity and, in some cases, interfered with its determinations.

It was felt that the most logical way to clear up the questions arising in connection with these methods was to put the matter before a competent technician who would devote his entire time to the problem. The subcommittee was authorized to establish a fellowship in some university to carry out the work. As a result, a fellowship was established at the University of Michigan, and the first problem to be taken up was that involving the methods for the determination of alkalinities and some of those ions influencing alkalinity and its determination. This paper sets forth, as briefly as is consistent, the progress made to date in carrying out this work.

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[1] Sponsored by the American Boiler Manufacturers Association, American Railway Engineering Association, American Water Works Association, National Electric Light Association, American Society for Testing Materials, and The American Society of Mechanical Engineers.

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Presented at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

[4] Although this subcommittee published an acceptable method for dissolved oxygen several years ago, that method appears not to have yielded entirely satisfactory results in all cases, and it was felt further study would be justified.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

It happened that, previous to the establishment of this fellowship, a very considerable amount of work dealing with the determination of carbonates had already been carried out at the University of Michigan. Some of the results of this work have been submitted to *Industrial and Engineering Chemistry*, and the first part of the present report has been condensed from that paper.

These earlier studies were carried out in connection with the analysis of solutions taken from an experimental boiler. As the results of this work offered at least a tentative method for the accurate determination of carbonate it was decided to give a greater amount of attention to the determination of hydroxide, and, if possible, some consideration to the determination of phosphate.

The results of this earlier work and those of the present investigation may be summarized briefly to show that: First, all of the methods generally in use for the determination of carbonate are open to large errors at low concentrations of carbonate and high concentrations of hydroxide. It is at such concentrations that most boilers operate in which the carbonate determination is important. A method which is very reliable and which is fairly rapid has been worked out for the determination of carbonate. Second, the determination of the hydroxide concentration by either the A.P.H.A. or the Winkler method is not greatly in error. Methods have been worked out for increasing the accuracy of this determination. Third, a method for the determination of phosphate has been investigated which seems to be extremely accurate as well as very rapid. The determination is not interfered with by any of the ions ordinarily found in samples of boiler water.

The determination of carbonate was studied in both laboratory and plant. A discussion of the findings of each of these investigations will be taken up and this will be followed by a discussion of the determination of hydroxide and of the determination of phosphate.

DETERMINATION OF CARBONATE

The literature on the analysis for carbonate makes it very apparent that investigators other than those interested in boiler-water analysis have found difficulties in determining small amounts of carbonate. Endless details, modifications, and rediscoveries of various methods have been given. In general the methods of analysis may be divided into two classes, (*a*) those involving the titration of the original sample, and (*b*) those involving an initial evolution of carbon dioxide.

The first class in general depends on either the color change of indicators to determine the end point of the titration or changes in electrical potential. The A.P.H.A. and the Winkler method, which are generally used for the determination of hydroxide and carbonate in boiler waters, belong in this class. Titration of the original sample with indicators is open to very many errors as the work of many investigators has shown. It would seem that some improvement might be made by an electrometric method for the determination of the end points. Such end-point determination has not been very generally tried in plant practice due to the complexity of the equipment involved. It has been used by several investigators with good success in the laboratory.

Methods of analysis involving the initial evolution of the carbon dioxide seem to be free of many of the very serious errors involved in the titration of the original sample. This is especially true if there is any danger of the original sample being contaminated by unknown impurities such as may exist in a boiler water. A variety of methods has been used for measuring the total carbon dioxide evolved from a sample. These various methods utilize measurement of gas pressure or volume, the freezing-out of solid carbon dioxide, and absorption in various solid or liquid media. Many of these methods have been developed in fields rather distant from boiler-water chemistry. Their possible application to the latter depends upon how simply and how accurately they may be used for the determination of the small amounts of carbon dioxide present in boiler waters.

The present work has concerned itself to some extent with an investigation of the accuracy of the A.P.H.A. or the phenolphthalein-methyl orange method for the determination of carbonate, and the Winkler method. An attempt has been made to develop equipment which will make it possible to use the evolution method for the determination of carbonate without inconvenience in plant practice. The results of this work will be discussed under the headings, "Laboratory Investigation and Results," and "The Improved Evolution Apparatus."

Laboratory Investigation and Results

Two pieces of apparatus were designed for the determination of carbonate in boiler waters. The first of these was used in the laboratory in connection with work on an experimental boiler. The second apparatus, which was both simpler and more accurate, was subsequently developed for the plant studies described in the second part of this report. The fundamental idea was the same for both pieces of apparatus. The total carbon dioxide was liberated by addition of acid and heating, and this carbon dioxide was carried by an air stream into an excess of standard barium hydroxide, precipitating barium carbonate. The residual barium hydroxide was then titrated with standard acid. This method is not at all new and its original application may date back to Dalton. Its successful application to the determination of carbonate in solution depends upon the following factors:

1 Complete evolution of all carbon dioxide in the sample.

2 Complete absorption of the liberated carbon dioxide in barium hydroxide.

3 Complete precipitation of barium carbonate equivalent to the amount of carbonate liberated and absorbed.

4 Titration (of residual barium hydroxide) to definite reproducible end points.

5 Freedom from contamination by carbon dioxide of the atmosphere.

In general, the second condition is the one least likely to be satisfied by ordinary apparatus. Since it is necessary to sweep the carbon dioxide out of the original sample by means of an air stream initially free of carbon dioxide, a considerable volume of gas must be passed through whatever absorbing unit is used. As a bubble of gas containing carbon dioxide on passing through barium hydroxide immediately forms a film of barium carbonate at the gas-liquid interface, it is highly improbable that complete absorption can be attained except by prolonged contact or intimate mixing.

The first absorption apparatus consisted essentially of a train for freeing the air of carbon dioxide, a flask into which the sample was introduced and in which a regular phenolphthalein-methyl orange titration or a Winkler titration could be carried out, and an absorption system which would sweep all of the carbon dioxide from the air passed through it. In the actual determination of the total carbon dioxide of the sample by evolution, the carbon-dioxide free air was passed through the heated acidified sample, sweeping out all of the carbonate of the sample in the form of carbon dioxide and carrying it into the barium-hydroxide absorption system, where it was precipitated as barium carbonate. After all of the carbon dioxide had been liberated and absorbed the excess barium hydroxide was titrated with standard acid.

In this system complete removal of the carbon dioxide from the acidified and heated sample could be insured by passing the air stream through it for a sufficiently long time. A period of 15 min. was found to be ample for solutions containing up to 90 p.p.m. of CO_3. Practically complete absorption was insured by passing the gas stream first through a flask containing some of the barium hydroxide and then through a long absorption tower containing the rest of the hydroxide which was filled with glass beads. The completeness of absorption was further checked by passing the gas through a sight bulb before it left the system. In case any carbon dioxide had passed through the absorption system this bulb became cloudy.

Complete precipitation was insured by using standard barium hydroxide which was initially saturated with barium carbonate. By carrying out the titration of the excess barium hydroxide with uniform illumination, which was secured by placing the flasks in which the titration was carried out in an illuminated box, closely reproducible results were obtained. Blank tests indicated that all of the carbon dioxide was scrubbed from the entering air by the train for removing the carbon dioxide, and further blank tests showed that hydrochloric acid could be used for acidifying the sample without danger of carrying over acid in the gas stream to the barium hydroxide.

It will be remembered that means were provided on this apparatus for carrying out a regular phenolphthalein-methyl orange titration or a Winkler titration. Since the evolution method only determines the carbonate it was necessary to do this in order to secure a value for the hydroxide.

The accuracy of the apparatus was tested and the method was compared with both the phenolphthalein-methyl orange and the Winkler titrations, using samples very carefully prepared to contain known concentrations of carbonate and hydroxide. Every precaution was taken to insure an accurate knowledge of the concentrations of these samples.

The values obtained with a series of hydroxide-carbonate samples titrated by the A.P.H.A. method or the method using phenolphthalein and methyl orange are shown in Table 1.

TABLE 1 TESTS OF THE TITRATION WITH PHENOLPHTHALEIN AND METHYL ORANGE ON SOLUTIONS OF KNOWN COMPOSITION

Reference number	OH—p.p.m. Known	OH—p.p.m. Found	CO_3—p.p.m. Known	CO_3—p.p.m. Found
1	55	47	6.0	24.7
2	55	45	6.0	29.7
3	81	77	6.0	9.1
4	81	66	26.1	31.3
5	81	78	46.1	47.9
6	81	77	86.2	89.0
7	191	188	26.1	31.6
8	191	187	46.1	53.9
9	191	189	46.1	53.9
10	285	277	6.0	16.5
11*	285	277	6.0	19.0
12*	285	276	26.1	34.9
13*	285	277	46.1	57.0
14	285	284	86.2	85.5
15*	407	411	6.0	0.0
16	407	403	6.0	11.1
17*	407	403	6.0	6.4
18*	407	398	26.1	34.8
19*	407	400	46.1	47.6
20*	407	395	86.2	101.2

* $CaSO_4$ added in solution.

The titrations were carried out under conditions superior to those usually surrounding control titrations of boiler waters. Loss of carbon dioxide from the sample during titration was obviated by introducing the acid near the bottom of the sample. Since the titration was carried out in a closed system all error due to atmospheric carbon dioxide was prevented. The values calculated for CO_3 by the customary method of interpretation (1)[5] are, however, consistently high and rather irregular, particularly for the lower concentrations. As a result of this arbitrary method of calculation, high values for CO_3 lead to slightly low values for OH.

The titrations shown in the table were made on distilled-water solutions containing no impurities. In the analysis of boiler waters it is doubtful if the relative accuracy shown here could be expected, since salts of slightly dissociated acids other than carbonic acid may be present, and the solutions may be in general more or less colored.

TABLE 2 TESTS ON BARIUM-CHLORIDE TITRATION ON SOLUTIONS OF KNOWN COMPOSITION

Reference number	Procedure in titration	OH—p.p.m. Known	OH—p.p.m. Found	CO_3—p.p.m. Known	CO_3—p.p.m. Found
1	(1)	81	86	26.1	0.0
2	(1)	81	90	46.1	3.3
3	(1)	81	101	86.2	5.0
4	(1)	191	194	26.1	9.5
5	(2)	191	188	26.1	31.6
6*	(2)	191	188	26.1	34.8
7	(3)	191	190	26.1	26.9
8	(4)	191	186	26.1	41.2
9*	(4)	191	182	26.1	53.9
10*	(3)	191	186	26.1	41.2
11	(2)	191	189	46.1	49.1
12	(5)	191	189	46.1	49.1
13	(6)	191	192	46.1	39.6
14	(1)	191	193	46.1	36.4
15	(1)	191	196	46.1	23.8
16*	(2)	191	187	46.1	55.5
17*	(3)	191	186	46.1	61.7
18*	(4)	191	183	46.1	71.4
19	(3)	191	190	46.1	46.0
20	(1)	285	290	26.1	0.0
21	(1)	285	292	46.1	6.3
22	(1)	285	308	86.2	0.0

(1) 10 cc. of 0.2N $BaCl_2$ added, sample titrated immediately.
(2) 10 cc. of 0.2N $BaCl_2$ added, sample boiled, titrated hot.
(3) 10 cc. of 2N $BaCl_2$ added, sample titrated immediately.
(4) 10 cc. of 2N $BaCl_2$ added, sample boiled, titrated hot.
(5) 10 cc. of 0.2N $BaCl_2$ added, sample boiled, titrated cold.
(6) 10 cc. of 0.2N $BaCl_2$ added, sample let stand one hour, then titrated.

* Solution of calcium sulphate used to make sulphate concentration of sample about 700 p.p.m. SO_4.

The results of titrations by the barium-chloride or Winkler method on a series of samples of known concentration are shown in Table 2. These samples were treated in a variety of ways as indicated by the number in the column headed "Procedure in titration." The details of each treatment and the reasons for the results noted need not be considered here, but it should be noted that in all cases where calcium sulphate was added to the solution the results are high.

In general the barium-chloride titration will yield the best results when a considerable excess of the chloride is used. Letting the sample stand or heating it before titration does not increase the accuracy. When a considerable amount of sulphate is present in the solution, the barium-chloride titration gives high values for carbonate, and consequently, slightly low values for hydroxide.

The results of the direct-absorption method for the determination of carbonate are shown in Table 3. These results were se-

TABLE 3 TESTS OF DIRECT ABSORPTION METHOD ON SOLUTIONS OF KNOWN COMPOSITION

Reference number	CO_3—p.p.m. Known	CO_3—p.p.m. Found
1	6.0	5.7
2	6.0	5.7
3	6.0	4.5
4	6.0	6.4
5	6.0	7.1
6	6.0	4.4
7	6.0	9.6
8	6.0	5.8
9*	6.0	6.5
10	6.0	6.8
11*	26.1	26.7
12	26.1	25.2
13*	26.1	25.6
14	26.1	27.7
15*	26.1	25.6
16	46.1	43.8
17	46.1	47.5
18*	46.1	51.0
19*	46.1	43.8
20*	46.1	43.9
21	86.2	80.0
22	86.2	80.0
23	86.2	93.6
24	86.2	86.3
25*	86.2	83.5

* $CaSO_4$ added to the sample.

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[5] Numbers in parentheses are for references at the end of this article.

cured on the first absorption apparatus which has just been described. They are very close throughout the range of concentrations investigated, the method is accurate at low-carbonate concentrations, and the presence of calcium sulphate introduces no error. It is to be expected that the presence of most impurities would have little effect when it is remembered that the actual titration for the carbonate is conducted in a distilled-water solution containing only barium hydroxide and suspended barium carbonate resulting from the absorption of carbon dioxide evolved from the original sample.

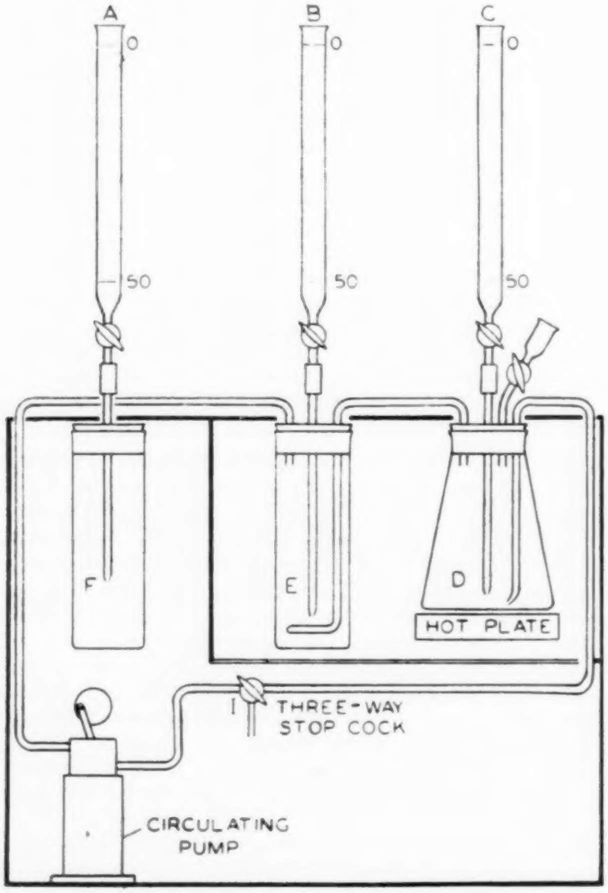

FIG. 1 EVOLUTION CARBONATE DETERMINATION APPARATUS

In contrast to the uncertain results obtained with the customary titration methods, the absorption method for total carbon dioxide shows consistent results, particularly in the case of the lower concentrations of carbonate, and it may be expected to serve as satisfactorily for actual boiler waters as for pure solutions. The only source of possible misinterpretation of the results would be the presence in the boiler water of sulphides, sulphites, or some organic salts which might yield a volatile acid in the direct absorption determination.

As carried out in the apparatus described the chief disadvantage of the absorption method was the complicated equipment and the time required for analysis, which was approximately one hour for each sample.

IMPROVED EVOLUTION APPARATUS

The disadvantages of the absorption method just described arose from the complicated apparatus used. An effort at improvement or simplification resulted in the development of an improved apparatus which requires a relatively short time for an analysis of total carbon dioxide, and which may be operated satisfactorily by a person not familiar with the chemistry of the process or particularly skilled in the manipulation of such apparatus.

The essential difference between the original and the simplified absorption apparatus is the manner in which the carbon dioxide is absorbed. In the simplified apparatus the sample flask and the absorption flask are a part of a closed system in which the air containing carbon dioxide is circulated continuously by a small pump until absorption is complete, thus eliminating the need for a scrubbing tower on the absorption flask. Absorption is hastened by the use of a device to produce many small bubbles in the absorption flask, although this is not essential to the satisfactory operation of the apparatus.

The apparatus consists of a box equipped with daylight bulbs and a ground-glass panel for uniform illumination. Inside of the box are fastened three stoppers connected to the buretted and air-circulating systems as shown in Fig. 1. The evolution flask *D* containing the sample is attached to the right-hand stopper and is supported in place by a small electric hot plate which is swung up beneath it. The absorption tube or flask *F* is filled with 0.03 N barium hydroxide from the burette on the left-hand stopper and is then attached as at *E* to the middle stopper for the absorption and subsequent titration.

The air outlet from the absorption tube *E* is connected directly to the suction side of a small air pump. The delivery side of the pump is connected back to the sample flask *D* as shown. The air line from the pump to this flask is provided with a three-way stopcock with one connection open to the atmosphere to make it possible to vent air from the system before and after an absorption in order to relieve pressure differences. The bubbling device shown inside the absorption tube is a perforated platinum disk sealed in glass.[6] The purpose of this disk is to break up the gas stream into fine bubbles so that the carbon dioxide may be more readily absorbed by the barium hydroxide. A small air condenser, not shown in Fig. 1, is placed between *D* and *E* to prevent condensate from *D* from collecting back of the disk on the bubbling device and stopping the air flow. The circulating pump, which may be driven by any convenient means, consists of two sliding sleeves sealed with mercury to prevent the possibility of any air escaping.

The supply bottles for acid and barium hydroxide are mounted on a shelf formed by the projecting base of the box, and are connected to the burettes by glass tubes. Rubber bulbs are also connected to the supply bottles to allow rapid filling of the burettes by air pressure. Excess acid above the zero mark of the burette is automatically siphoned back into the supply bottle on release of pressure. The outfit is entirely self-contained, and may be moved readily.

The operation of this apparatus is extremely simple. The measured sample is placed in flask *D*, and this flask is held in place by the heater as shown in Fig. 1. The absorption tube is filled with 0.03 N barium hydroxide and put in position at *E*. The sample in *D* is then titrated to give a value for the hydroxide. Excess hydrochloric acid equivalent to 2 cc. of normal acid is then added to the sample and it is heated just to the boiling point. The carbon dioxide evolved from the sample is swept by the circulating air into the barium hydroxide, where it is precipitated as barium carbonate. During a 10-min. absorption period the gas is passed through the hydroxide about 15 times, insuring practically complete absorption. The excess hydroxide in *E* is then titrated, the phenolphthalein used as indicator being contained in the 0.03 N acid added. From the acid equivalent of the hydroxide originally introduced into tube *E* and the amount of acid used for the final titration, the amount of carbon dioxide evolved from the sample is calculated. In alkaline solutions where the pH value is in the range above 11, the total carbon dioxide as determined by the absorption method may be assumed to be equivalent to the CO_3 concentration of the solution. For less alkaline solutions, the concentrations of CO_3 and HCO_3 may be calculated conveniently from the total carbon dioxide and pH value by the method outlined by McKinney (6).

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[6] Supplied by the Fisher Scientific Company.

Eight samples of known concentration prepared from pure standard sodium-carbonate solutions were analyzed in the manner described. The results as shown in Table 4 were very consistent and considerably more accurate than were the results obtained on the more complicated apparatus first described.

TABLE 4 RESULTS OF ANALYSIS OF SOLUTIONS OF KNOWN CARBONATE CONCENTRATION

CO_3—p.p.m.	
Known	Found
8.8	8.6
	8.8
22.0	21.6
	22.0
44.0	42.0
	42.5
87.0	83.0
	82.0

TABLE 5 RESULTS OF ANALYSIS ON SOLUTIONS CONTAINING KNOWN CONCENTRATIONS OF SO_4, PO_4, OH, AND CO_3

	Known concentrations in p.p.m.				Conc. found
Sample number	SO_4	PO_4	OH	CO_3	p.p.m. CO_3
4	2000	20	69	9.7	10.5
5	2000	20	69	9.7	9.7
6	2000	20	69	24	24
7	2000	40	69	24	23
8	2000	20	69	92	90
9	2000	40	69	92	88
10	2000	20	172	10.5	10
11	2000	20	172	10.5	10
12	2000	20	172	47	47
13	2000	40	172	47	50
14	2000	20	172	93	96
15	2000	80	172	93	92
16	2000	20	344	11.4	11
17	4000	20	344	11.4	11
18	2000	20	344	25	25
19	4000	80	344	26	25
20	2000	20	344	94	94
21	4000	20	344	94	88
22*	2000	40	172	11	10.5
23*	2000	20	172	24	26

* $CaSO_4$ added to sample to give 298 p.p.m. $CaSO_4$.

A series of determinations was also run on solutions containing sodium sulphate, sodium phosphate, and sodium hydroxide as well as the carbonate. The results for these analyses are shown in Table 5. The accuracy of the method, as is seen, is extremely good.

All of the solutions used in these analyses were made up with double-distilled water which had been boiled for 20 min. and cooled in a flask protected from carbon dioxide by ascarite. The bottles in which stock solutions were stored were also protected by ascarite, and the portions taken out to make up the samples for analysis were pipetted from the bottle through a stopcock, which was immediately closed when the pipette was full. All solutions other than the carbonate solution were made up to be carbonate free and blanks were run to make sure that they were free of carbonate. The sodium hydroxide was made from metallic sodium by dissolving it in water under a layer of ether as described in Clark's "Determination of Hydrogen Ions" (3). The sodium carbonate was made by heating bicarbonate in a jacketed crucible at 270 to 300 deg. cent. for $^1/_2$ hr.

The apparatus was standardized to make allowance for the carbon dioxide in the air in the system at the start of each analysis and tests were run to determine if there were any danger of boiling hydrochloric acid from the sample flask over into the barium hydroxide. With hydrochloric-acid concentrations up to twice that used in the regular analysis no evidence of acid being boiled over was noticed.

The results obtained in the direct absorption determination are probably more accurate than those given by any standard titration method, especially when applied to impure waters. Although approximately 20 min. of elapsed time is required for one complete determination, in many cases it would seem better for control purposes to have a single accurate analysis rather than any number which cannot be relied upon.

It should be remembered that the absorption apparatus determines only the total carbon dioxide present in a sample of water, regardless of the form in which it may exist in solution. Where precision is necessary in experimental work, the activities or concentrations of HCO_3 and CO_3 (in the original sample) may be calculated from the dissociation constants of carbonic acid. For practical purposes the total carbon-dioxide content of the alkaline water from a boiler may be assumed to have been present in that water usually as carbonate, although in the case of natural waters in contact with the air such an assumption would lead to highly inaccurate conclusions.

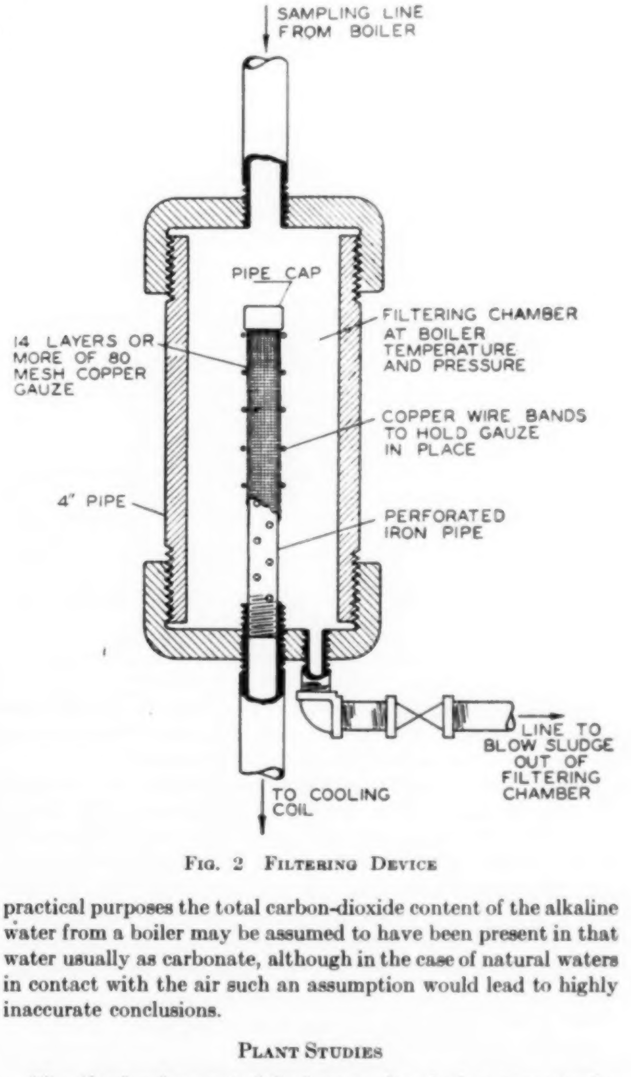
SAMPLING LINE FROM BOILER
PIPE CAP
FILTERING CHAMBER AT BOILER TEMPERATURE AND PRESSURE
14 LAYERS OR MORE OF 80 MESH COPPER GAUZE
COPPER WIRE BANDS TO HOLD GAUZE IN PLACE
4" PIPE
PERFORATED IRON PIPE
LINE TO BLOW SLUDGE OUT OF FILTERING CHAMBER
TO COOLING COIL

FIG. 2 FILTERING DEVICE

PLANT STUDIES

After the development of the improved evolution apparatus for the determination of carbonate, permission was obtained to carry out a series of tests on one of the 125-lb. pressure W-type Stirling boilers in the Congress Street heating plant of The Detroit Edison Company. This boiler was conditioned with sodium carbonate for the prevention of scale. The make-up was about 70 per cent, and Detroit city water was used as the feedwater. As a result of the rapid hydrolysis of carbonate in the boiler water, it contained a low concentration of CO_3 and a relatively high concentration of OH.

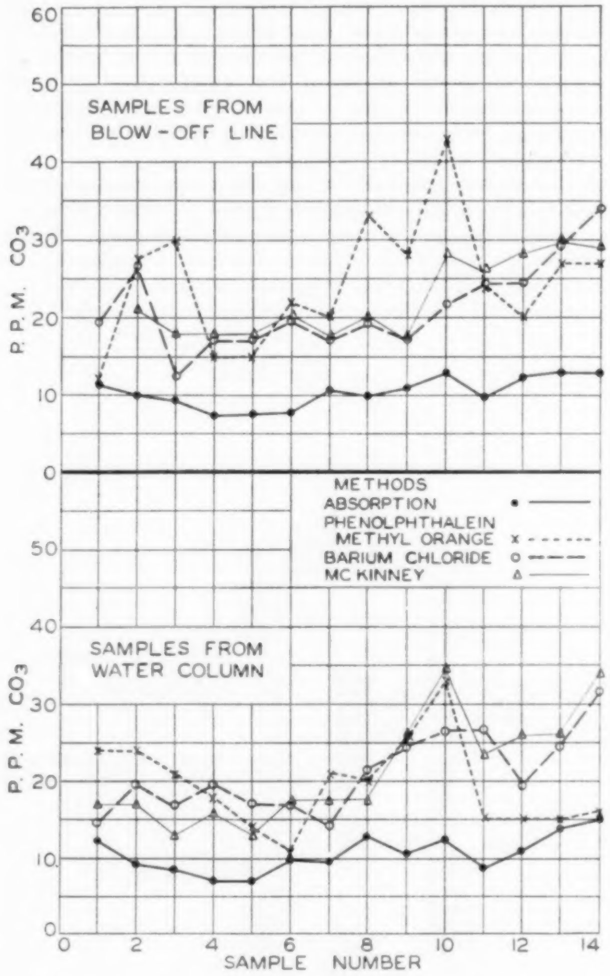

Fig. 3 Carbonate Found in Boiler by Four Methods

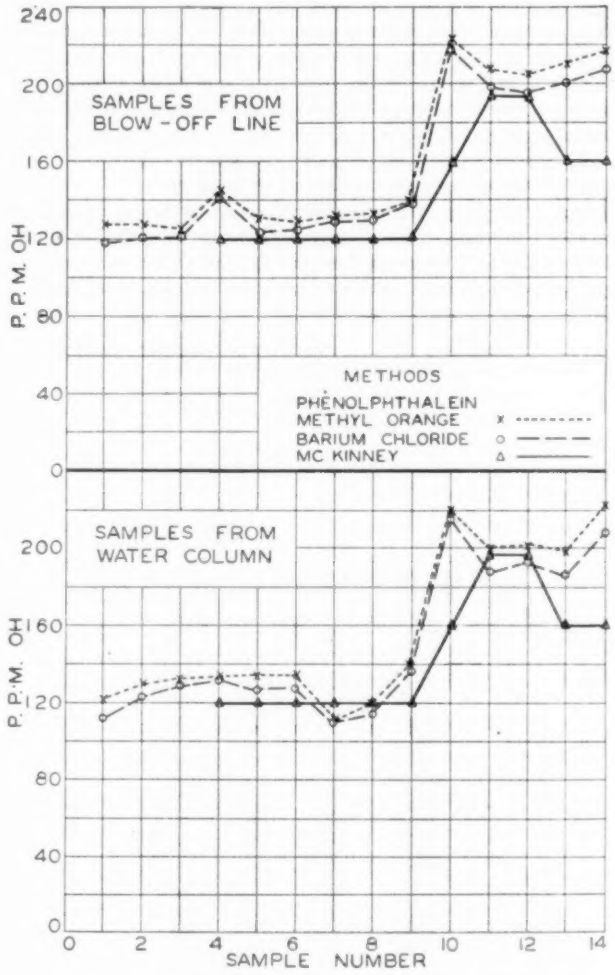

Fig. 4 Hydroxide Found in Boiler by Three Methods

The results of the tests indicate the great need for extreme care in sampling boilers, and they also indicate some of the inaccuracies in the various methods of analysis investigated.

The samples of boiler water in the plant studies were obtained by using the copper-screen type of filter, as shown in Fig. 2. In this device the sample runs through several layers of fine-mesh copper screen. The suspended particles in the boiler water are trapped between the layers and form their own filter medium. This system is fundamentally correct as long as the character of the solid phase in the boiler does not vary. This filter was inserted in the sampling line between the boiler and the cooling coil.

Comparison of Values Obtained for Carbonate and Hydroxide in the Plant Studies

Fourteen sets of boiler-water samples were taken during three successive days for a comparison of various methods for determining carbonate and hydroxide. Each set consisted of two samples, one from the mud drum and one from the steam drum. All of these samples were filtered at boiler temperature and pressure through filters such as have been described, and were not filtered in the laboratory. All samples were analyzed immediately.

Values for carbonate were determined by four different methods: first, the direct-absorption method, using the improved or simplified apparatus described in this report; second, the Winkler barium-chloride method; third, the titration with phenolphthalein and methyl orange; and fourth, the abridged equilibrium calculations of McKinney (6), based on the amount of acid required to titrate the sample between pH values of 8.5 and 5.0. The titration with phenolphthalein and methyl orange was made in flask *D* of the absorption apparatus shown in Fig. 1, prior to evolving the carbon dioxide.

The values for carbonate calculated from the results of the four methods of analysis are shown in Fig. 3. Great reliance is placed on the accuracy of the values obtained by the absorption method, which vary above and below an average value of 10 p.p.m. CO_3. The values indicated by all three of the other methods are high, deviating with no apparent consistency by amounts ranging from a few per cent up to 300 per cent. The results from the barium-chloride titration are only slightly better than those from the titration with phenolphthalein and methyl orange. McKinney's method is subject to the same difficulties encountered with the other titration methods when the end points at pH 8.5 and 5.0 are determined colorimetrically.

If it is granted that the accuracy of the absorption method is established by the tests on solutions of known composition which have been shown, it is evident that none of the other methods for carbonate investigated during the plant tests was sufficiently precise for the purpose of boiler-water control.

Values for hydroxide were calculated for the 14 sets of samples in three ways: (1) from the barium-chloride titrations; (2) from the phenolphthalein-methyl orange titrations; and (3) from the pH of the original sample, using the conveniently tabulated values of McKinney (6), for the activity of hydroxyl ion at different pH values. The activity coefficients necessary to convert the activities into concentrations were estimated as 0.89 for the samples through number 9 and 0.85 for the subsequent samples.[7]

The values for hydroxide obtained by the three methods of calculation are shown in Fig. 4. It is seen that the two titration methods agree fairly well, the barium-chloride method yielding values consistently somewhat lower than those indicated by the titration where phenolphthalein and methyl orange were used as indicators. The hydroxide concentrations calculated from pH measurements fall still lower, with considerable deviation in the case of a few samples. Since the three concentration levels on the curve (Fig. 4) representing the results obtained using this method represent pH values of 11.8, 11.9, and 12.0, respectively, it is obvious that a slight uncertainty or error of judgment in placing the pH of samples 10, 13, and 14 at 11.9 instead of 12.0 might have been responsible for the larger deviations in the case of these samples. An error of 0.1 in the estimation of pH by comparison with standards at intervals of 0.2 is quite possible with either of the sets of standards used.[8]

While the method of calculating hydroxide concentrations from pH values proposed by McKinney for boiler-water control is theoretically sound, its rigorous application necessitates a complete analysis of the water and a determination of pH by some more accurate means than is afforded by ordinary colorimetric methods. If, as in the case of the present tests, the composition of the water is estimated from incomplete data and pH is determined colorimetrically, the values may vary in rather large jumps which will increase in size as the hydroxide concentration and hence the pH increases. On the other hand, it was shown in the first part of this paper that both the phenolphthalein-methyl orange and the barium-chloride titrations yielded results which were within 10 per cent of the true value for hydroxide in solutions of known composition. Even this accuracy is better than might be expected from the calculation of the hydroxide from the pH of the sample if the pH determination is made colorimetrically.

DETERMINATION OF HYDROXIDE

Although it has been shown in the investigation of the method for determining carbonates that both the barium-chloride method and the titration using phenolphthalein and methyl orange as indicators yielded results of fair accuracy, it was felt that further study might increase that accuracy and at the same time answer some questions regarding the effect of interfering ions.

This investigation then may be divided into two parts: (1) that in which an attempt was made to use a precipitating agent to remove the interfering ions before making the titration, and (2) that in which direct titration was made.

USE OF A PRECIPITATING AGENT

The ions present in boiler waters which generally interfere with the direct titration of the hydroxide are CO_3 and PO_4. In attempting a titration of the hydroxide alone these ions should be removed from the field of action. This may be accomplished most readily by precipitation.

The carbonate, as in the Winkler method, may be precipitated as barium carbonate. The use of barium for the precipitation of carbonate in boiler-water samples has two very serious objections. First, since barium carbonate is more soluble than barium sulphate it is necessary to precipitate all of the sulphate before any of the carbonate will come down. Boiler-water samples are usually extremely high in concentration of sulphate as compared to that of the carbonate, so that such treatment results in a high degree of turbidity in the sample, which might be avoided by the selection of another precipitating agent. This makes the determination of the end point exceedingly difficult, and jeopardizes the accuracy of the titration even if the color change of the indicator used in the determination of the end point is absolutely accurate. The second objection is the great tendency of both barium carbonate and sulphate to occlude or adsorb many salts from solution. In the case of the carbonate this occlusion or adsorption would not be serious since the amount of barium carbonate present is extremely small. In the case of the sulphate, however, the seriousness of occlusion or adsorption becomes much greater. Such occlusion or adsorption of the hydroxide would result in a low indicated value for the hydroxide and a high value for the carbonate. This error may be responsible to some considerable extent for the high carbonates shown by the Winkler method.

Since strontium salts are in general very similar to barium salts the use of strontium chloride as a precipitating agent was tried. Strontium carbonate comes down in a very crystalline form and has little tendency to occlude or adsorb other salts that may be present in the solution. Strontium sulphate is much more soluble than the carbonate so that by proper adjustment of the concentration of the strontium ion in the solution it should be possible to precipitate all of the carbonate without precipitating any material amount of the sulphate. This would result in a solution for titration in which there was very little suspended matter, making the observation of the end point easy and accurate, and remove the possibility of any serious error from occlusion or adsorption.

TABLE 6 RESULTS OF THE STRONTIUM CHLORIDE METHOD FOR THE DETERMINATION OF HYDROXIDE

Sample number	Known concentrations in p.p.m. SO_4	CO_3	OH	Conc. found p.p.m. OH
10	..	88	134	133
13	..	175	134	133
15	..	8.8	268	267
20	..	88	268	267
21	2000	8.8	134	133
28	2000	88	134	134
29	2000	8.8	268	269
33	2000	44	268	267
40	4000	17	403	403
46	4000	350	403	402

A large number of samples containing hydroxide, carbonate, and sulphate were treated with strontium chloride and the hydroxide determined by titration to the phenolphthalein end point with standard acid. A few of the results are shown in Table 6. The accuracy as is shown is extremely high. These samples are representative of all of the determinations made. In no case did any error occur which was greater than 1 per cent.

The method of treating the sample for these determinations was somewhat different from that used in the case of the Winkler method. In the first place, the amount of strontium chloride added should not be greatly in excess of that needed for the precipitation of the carbonate. After the addition of the strontium chloride the sample is heated to boiling, allowed to stand for a few minutes to permit the precipitation of the carbonate, and then cooled and titrated. These differences in treatment arise mainly from the fact that it is not advisable to use a large excess of the strontium chloride in precipitation.

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[7] While a complete analysis of the boiler water was not made during the present tests, it was possible to estimate $\Sigma c\nu^2$ rather closely from the determined values for CO_3, OH, and SO_4 by assuming concentrations of Na, Cl, Ca, and Mg in line with those shown in previous complete analyses of the water. Since the feedwater is quite constant in composition, the error in determining $\Sigma c\nu^2$ in this manner is probably less than that in the colorimetric determination of pH or the application of the Debye-Hückel equation to the calculation of the activity coefficient.

[8] Both Hellige-Klett permanent glass standards and LaMotte solution standards were employed in this work.

Due to the fact that the determination of hydroxide by this method gave such extremely accurate results some further investigation of the determination of carbonate by the use of strontium chloride was carried out. In order to make use of this method for the determination of carbonate as well as hydroxide, it was necessary, of course, to combine the titration of the sample from which the carbonate had been precipitated with a second titration which would determine the hydroxide plus some fraction of the total carbonate. Such a determination might be made by titrating the sample directly to some definite pH value, but in general this did not seem to offer great possibilities since the titration would be carried out in the presence of carbonates which act to some extent as buffer salts. The result would be an end point which might not be extremely sharp. In order to remedy this difficulty of sliding end points, the addition of an excess amount of acid, boiling off the carbon dioxide, and back titration was resorted to. The actual details of the titration need not be considered here, but the results for the determination of carbonate were a great deal more accurate than those shown by either the method using phenolphthalein and methyl orange as indicators in the titration or the Winkler method. For a method involving direct titration of the sample the results at low carbonate values were extremely good, although not quite as accurate as the absorption or evolution method.

Due to the fact that the determination of carbonate by this method would be susceptible to error from various impurities in the sample, as are all of the direct titration methods, and to the fact that it is not a great deal shorter than the evolution method, its use cannot be recommended for routine control work; at least not without checking against the evolution method. Its most promising field of use seems to lie in the determination of hydroxide alone or for the determination of hydroxide and carbonate where the equipment to carry out an evolution determination is not available.

The samples so far investigated using strontium chloride as a precipitating agent have contained, besides the hydroxide, only carbonate and sulphate. Since strontium phosphate as well as the carbonate is quite insoluble, it would seem that the addition of strontium chloride to a sample would remove the phosphate from solution as well as the carbonate.

A series of hydroxide determinations was made on solutions containing sulphate, phosphate, carbonate, and hydroxide. The results indicate that errors no greater than 2 per cent were obtained. The method is not quite as accurate as in the presence of the carbonate alone.

The actual precipitation of the carbonate and phosphate is carried out exactly as described for the carbonate alone. After the sample is cooled it cannot be titrated very well to the phenolphthalein end point since strontium phosphate is appreciably soluble in a solution whose pH is about 8. This solubility causes a fading end point. In order to correct this the sample was titrated to the end point of LaMotte Purple, which has a pH range of from 9.6 to 11.2.

It may be noted here that in precipitating strontium phosphate from solution there is more danger of occlusion or adsorption of the hydroxide than in the precipitation of the carbonate alone. The strontium phosphate may under certain conditions come down as a basic salt. This in general would not be serious in boiler-water samples since the concentration of phosphate is extremely low. In the present investigation no appreciable error was found although phosphate concentrations up to 100 p.p.m. of PO_4 were investigated.

The use of strontium chloride makes it possible to determine the hydroxide content of a boiler-water sample directly in the presence of both phosphate and carbonate. It is not necessary to know the concentration of either of these interfering ions.

Titration to the Phenolphthalein End Point

In spite of the fact that by the use of strontium chloride a satisfactory determination of the hydroxide could be secured, it was felt that the method was exceedingly cumbersome for use in regular plant practice. Other precipitating agents might be used, but in general it will be found that practically all of them introduce difficulties characteristic of the particular ion used.

Electrometric titration curves for both carbonate and phosphate show a fairly rapid change in pH with each addition of acid or alkali to the sample at a pH of 8 or 9. This rapid change, as well as the low concentrations of phosphate and carbonate generally present in boiler-water samples, makes it seem possible that a fairly accurate determination of hydroxide concentration could be obtained by titrating to the end point of phenolphthalein alone since this indicator becomes colorless at about the desired pH. When such a titration to one end point is used it is necessary to know the concentrations of phosphate and carbonate, and to apply a correction to the value obtained in the titration in order to obtain the hydroxide concentration.

TABLE 7 DETERMINATION OF HYDROXIDE IN THE PRESENCE OF PHOSPHATE AND CARBONATE BY TITRATION WITH PHENOLPHTHALEIN

	Known concentrations						
Sample number	P.p.m., SO_4	P.p.m., PO_4	P.p.m., CO_3	P.p.m., OH	Apparent p.p.m., OH	Deduct p.p.m., OH	Conc. found p.p.m., OH
1	2000	20	10	69	74	6.4	68
2	2000	20	10	69	75	6.4	69
3	2000	20	19	69	77	9.0	68
4	2000	20	19	69	77	9.0	68
5	2000	20	37	69	82	14.0	68
6	2000	40	37	69	86	17.0	69
7	2000	0	91	69	95	26.0	69
8	4000	0	91	69	95	26.0	69
9	2000	20	20	137	146	9.2	137
10	2000	40	20	137	150	13.0	137
11	2000	20	47	137	154	17.0	137
12	2000	40	47	137	158	21.0	137
13	2000	20	93	137	167	29.0	138
14	2000	40	93	137	170	32.0	138
15	4000	0	20	274	280	6.0	274
16	4000	80	20	274	293	20.0	273
17	4000	80	20	274	293	20.0	273
18	4000	0	56	274	292	16.0	276
19	4000	80	56	274	305	30.0	275
20	4000	0	183	274	329	52.0	277
21	4000	80	94	274	317	41.0	276

The results for a series of samples containing known concentrations of hydroxide, carbonate, phosphate, and sulphate, which were titrated in this manner, are shown in Table 7. The accuracy is extremely good. No difficulty was experienced in these titrations with sliding end points. The apparent hydroxide concentration was calculated directly from the total acid used to the phenolphthalein end point. The deduction or correction was calculated from the known concentration of carbonate and phosphate.

In the case of a boiler water which is treated with sodium carbonate, the pH of the sample will be above 11 and the total carbonate content of the sample may be assumed to be present as CO_3 and the correction may be calculated on this basis. Each 10 p.p.m. of CO_3 calls for a correction of only 2.83 p.p.m. of OH. In the case of the phosphate it cannot ordinarily be assumed that it is all present as PO_4. By a rough determination of the pH, however, the fraction that is present in the form of PO_4 may be determined from the curves compiled by McKinney (6). Ten parts per million of PO_4 calls for a correction of 1.79 p.p.m. on the OH concentration.

Since, as will be noticed, the corrections on the hydroxide concentrations are rather small when compared to the concentrations of the ions causing this correction, small errors in the determination of the carbonate and phosphate will not create any large error in the indicated hydroxide concentration. This is, of course, merely due to the fact that the equivalents of phosphate and carbonate present in a boiler water are usually extremely low as compared to the equivalents of hydroxide present. For the same reason it will not be necessary to know the pH of the sample with an accuracy greater than 0.7 to find the fraction of phosphate that is present as PO_4.

This titration to the phenolphthalein end point for the determination of hydroxide requires that the sample be free from the presence of any considerable amount of either silicates or aluminates. These salts will interfere in practically all direct titrations, and it is necessary to take special steps to remove them or correct for their presence. It might be noted here that the use of strontium chloride offers some possibilities in the removal of silicate since strontium silicate is quite insoluble. Barium cannot be used for this purpose as its silicate is fairly soluble. The removal of silicates by precipitation with strontium chloride will be investigated further, however, in order to determine the possibility of the use of this precipitating agent in the determination of the hydroxide concentration in boiler water.

DETERMINATION OF PHOSPHATE

The phosphate determination like the carbonate has been the subject of much discussion and work in various fields of pure and applied chemistry. Several distinct and different methods have been developed for the determination of phosphate in its various forms, and an almost endless discussion of details, modifications, and revisions of each of the methods has been given.

Methods for the determination of phosphate are found which may be classified as gravimetric, volumetric, colorimetric, or nephelometric, or combinations of these classes. The gravimetric determination or a gravimetric separation combined with a volumetric determination, and purely colorimetric determinations are most generally used in the determination of basic phosphates.

In boiler-water chemistry the purely gravimetric determination with its necessity for accurate weighing is out of the question for routine work even if the method were rapid and simple, which it is not. The purely gravimetric determination has been modified to a gravimetric separation of the phosphate and a volumetric determination of the amount of the phosphate salt so separated. This does away with the necessity of weighing, but the time required for such a determination puts it way beyond the bounds of any possible use for routine determination of phosphates in boiler waters in its present form. Even if the method were rapid, a great number of difficulties surround the separation of the phosphate as ammoniumphosphomolybdate, the form in which it is separated in the gravimetric method. It is difficult to secure a precipitate which contains all of the phosphate and which is not contaminated by impurities. The acidity of the sample, the concentration of all the salts present, the concentrations of reacting substances, the temperature, and the time allowed for precipitation must be very carefully controlled to secure an accurate determination. Even with all these precautions, however, the ideal composition of the precipitate, as represented by the formula, is seldom realized and a correction is applied. The difficulties of the gravimetric separation are so manifold that some investigators (5) after extensive work with the method have stated that it is simply a method of balancing errors.

A method which is essentially gravimetric and which has been used in boiler-water chemistry involves the precipitation of the phosphate and a rough measurement of the volume of the precipitate as a means of estimating the amount of phosphate present. The measurement of the precipitate is often carried out by simply allowing it to settle in a tube and noting the height of the settled precipitate. This modification obviously sacrifices much of the accuracy of the determination in order to make it rapid. The accuracy of the measurement of the precipitate might be increased by the use of the centrifuge, but such a method would be rather complex for use in routine work. Any increase in accuracy of measurement of the precipitate would not remove the difficulties inherent in the original precipitation, and would still leave the determination open to considerable error from this source. These errors might become very serious in control work where the tendency toward quick determinations and slighting of all the precautions necessary to secure accurate determination is always present.

As the phosphate concentrations in boiler water seldom run as high as 100 p.p.m. of PO_4 it would seem that some of the colorimetric methods might be used for this determination. The low concentration ranges are those which are generally best adapted for colorimetric determination.

Phosphate concentrations in boiler samples are often determined colorimetrically, using hydroquinone (2) as the reducing agent to develop the color. This method works with entire satisfaction in some samples. It has two disadvantages, however: first, the color development with hydroquinone is rather slow, and, second, the presence of silicates in the solution interferes with the determination. In searching for a means of overcoming these difficulties aminonaphtholsulphonic acid was substituted for hydroquinone. The results were exceedingly satisfactory.

USE OF AMINONAPHTHOLSULPHONIC ACID IN THE DETERMINATION OF PHOSPHATE

The hydroquinone method, as well as the use of 1, 2, 4-aminonaphtholsulphonic acid in the determination of phosphate, was originally worked out and used in the field of biological chemistry. The originators of the method using the amino acid were primarily interested in finding a reagent that developed its color more rapidly than hydroquinone (4). They also noted that the color development of this acid is not interfered with by low concentrations of silicates. The application of this method to boiler-water analysis did not involve any radical departure from the method used in the biological field.

The details of the method will not be considered here, but a general outline which will serve to indicate its simplicity and rapidity will be given. The boiler-water sample is treated first with a solution of ammonium molybdate and then with an aminonaphtholsulphonic acid in a sodium-bisulphite, sodium-sulphite solution. After the addition of these reagents the sample is allowed to stand about 5 min. and then is compared in a colorimeter with a standard solution containing a known amount of phosphate, and which was treated with the same reagents as the boiler-water sample. This comparison gives the amount of phosphate directly. The whole determination, from the time the boiler-water sample is measured out to the time the phosphate concentration is known, requires about 10 min.

One of the disadvantages of the amino-acid method, when this method was first used, arose from the fact that the aminonaphtholsulphonic acid had to be carefully purified before it could be used. The Eastman Kodak Company now manufactures a purified grade of the amino acid, which has been compared with the acid purified in the laboratory and has been found to be entirely satisfactory.

RESULTS OF THE AMINO-ACID METHOD FOR THE DETERMINATION OF PHOSPHATES

A very large number of determinations of phosphate have been made by the amino-acid method on solutions containing known concentrations of phosphate, carbonate, hydroxide, sulphate, chloride, and silicate. The results of a few of these determinations are shown in Table 8. In practically all cases the error is less than 0.3 p.p.m. of PO_4. None of the ions which were introduced into the sample caused any appreciable error.

It should be noted that all of the results shown were secured by comparison of the solutions in a colorimeter. A Bausch & Lomb Duboscq type was used. Some investigation of the

TABLE 8 RESULTS OF THE ANALYSIS OF SOLUTIONS OF KNOWN PHOSPHATE CONCENTRATION

Sample number	Known conc. p.p.m. PO_4	Conc. found p.p.m. PO_4
On solutions containing approximately 1600 p.p.m. SO_4, 32 p.p.m. CO_3, and 1000 p.p.m. Cl		
45	4.0	3.9
46	16.0	16.0
47	28.0	28.1
48	40.0	39.8
On solutions containing approximately 2500 p.p.m. SO_4, 50 p.p.m. CO_3, 50 p.p.m. OH, 300 p.p.m. SiO_3, and 625 p.p.m. Cl		
49	4.0	3.8
50	16.0	16.0
51	28.0	28.0
52	40.0	39.8
On solutions containing approximately 5000 p.p.m. SO_4, 100 p.p.m. CO_3, 100 p.p.m. OH, 600 p.p.m. SiO_3, and 1250 p.p.m. Cl		
32	4.0	4.1
33	12.0	12.3
34	24.0	23.9
35	32.0	32.0
36	40.0	40.1

phosphate determination by the amino-acid method in which the solutions are compared in comparison tubes is to be carried out. This modification of the method is certainly not to be recommended for referee work. It must be recognized that the results of such comparisons will not be as good as those secured in the colorimeter.

A good colorimeter is, of course, rather expensive, but for routine control work where a large number of determinations are run and where much time and effort are spent in making these determinations it would seem that the use of poor equipment, which jeopardizes the accuracy of all of the determinations, could not be justified.

One of the chief advantages resulting both from the use of the amino-acid method, which develops its color rapidly, and from the use of a colorimeter is the fact that the intensity of the color of the sample will not remain constant if there are any ions present in the solution which are interfering with the color development. This materially cuts down the danger of whole series of determinations being in error due to the presence of some unknown ions in the solution which interfere with the determination. If such ions are present the method warns the operator by the fact that the intensity of color continues to change. The use of color tubes would make the possibilities of the detection of such errors very slight since the eye is not sensitive enough to detect small differences in intensity unaided.

Conclusions

In concluding this report it is the desire of Subcommittee No. 8 of the Boiler-Feedwater Studies Committee to again call attention to the fact that the aim has been to present herein data as briefly as possible and yet of a sufficiently convincing nature that it will be clear that, first, exhaustive comparisons have been made of the known methods for the determination of carbonates in boiler water, second, that the studies so far carried out have pointed out the characteristics of the commonly used methods for hydroxides, and have indicated the course of further study pertaining to the determination of this ion and such other ions as are known to influence its apparent concentration and the method of test, and, third, that three methods for the determination of the phosphate ion have been considered.

From the considerations presented it is possible to summarize the essential facts as follows:

1 It is apparent that, no matter what analytical methods are used, a representative sample must be secured or the results will be meaningless. A satisfactory way to accomplish this, in the case of obtaining samples from an operating boiler, is to filter the sample at boiler temperature and pressure and cool it at boiler pressure before it is passed into the sampling bottle.

2 The determination of carbonate by either of the standard methods, the A.P.H.A. or the Winkler method, does not give accurate results in solutions containing low concentrations of carbonate and high concentrations of hydroxide. An evolution method for the determination of carbonate has been worked out which will give accurate results, and which can be used in routine control work as well as for referee work. The accuracy of this method is not affected by any ions which are encountered in boiler waters.

3 The hydroxide concentration of a sample may be determined by precipitation of the interfering ions, phosphate and carbonate with strontium chloride, and direct titration of the hydroxide; or if the concentrations of both phosphate and carbonate are known it may be determined by titration to the phenolphthalein end point alone. These methods will give results more accurate than those obtained by either the A.P.H.A. or the Winkler method, and can be considered as suitable for referee work.

4 By the use of aminonaphtholsulphonic-acid method for the determination of phosphate and a colorimeter, an accuracy of 0.3 p.p.m. of PO_4 may be secured on samples containing less than 40 p.p.m. of PO_4. On samples containing up to 80 p.p.m. the error should not be more than twice this value. The determination is extremely rapid and simple. It is not interfered with by any of the ions commonly found in boiler waters. This method can be considered as suitable for referee work as well as plant control work.

It is the hope of the subcommittee that many will try these methods, if possible in comparison with the more commonly used procedures for testing, and submit their comments to the chairman.

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(4) Fisk, C. H., and Subbarow, Y., *Jour. Biological Chem.*, vol. 66, pp. 375–450, 1925.

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Discussion

Joseph D. Yoder.[9] The equipment described for determining the carbonate alkalinity by the evolution and absorption of CO_2 is ingenious. Where exceedingly accurate determinations are to be made as required for determining chemical equilibria with varying amounts of carbonate and sulphate radicals in the presence of calcium, it is evident from the data submitted that neither the A.P.H.A. nor Winkler methods would be sufficiently accurate. It is interesting to note that the A.P.H.A. method, Table 1, was found consistently more accurate than the Winkler method, Table 2.

From the standpoint of protecting boilers from scale, of either the sulphate, carbonate, or silicate variety, the writer has not found the refinements described in this report to be necessary. Granting that when carrying exceedingly low carbonate alkalinity in the boiler water the A.P.H.A. and Winkler tests give relatively large inaccuracies, it is noticeable that if the carbonate alkalinities in the boiler water are in the range that showed large percentage errors (Table 1), as substantially less than 30 p.p.m. CO_3, the alkalinity is too low to generally prevent calcium silicate and carbonate scales. When the carbonate alkalinity is carried high enough to prevent the calcium-carbonate-silicate scales, the percentage errors reported when making the determinations with methyl orange as an indicator are not sufficient to cause any serious difficulty.

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[9] Cochrane Corporation, Philadelphia, Pa. Mem. A.S.M.E.

If a minimum concentration of 30 to 50 p.p.m. of PO_4 radicals is maintained within the boiler, it is then possible to prevent the formation of scale with the lowest carbonate alkalinities reported in Table 1; but when a suitable concentration of phosphate radicals is maintained, it is of little or no value to make exacting determinations of the carbonate radicals present. It is possibly for these reasons that the writer has never observed a need for greater accuracy than the methyl-orange indicator for determining the proper alkalinity of boiler water in the absence of interfering organic matter. If, as a result of some particular contamination of the feedwater, there is interfering organic matter present, then the Winkler method would seem to fulfil all practical requirements.

The advantage of the method proposed for determining carbonate alkalinity therefore seems applicable to a study of the theory of scale formation rather than to actual practice in the protection of boilers from scale.

Authors' Closure

Before considering directly the discussion submitted by Mr. Yoder, it will be well to reiterate briefly the object of the present report. Its primary purpose is to place before the Boiler-Feedwater Studies Committee and those members of the A.S.M.E. who are interested such data as have been secured concerning the relative accuracy of certain procedures for the analysis of boiler waters. The recommendations in this report are those of the authors and are not those of Subcommittee Number 8. It is hoped that the results of this investigation will lead to methods which may be recommended by the subcommittee as standard, but the present progress report does not attempt to do this.

In the case of the determination of carbonate, the laboratory investigations indicated that in certain ranges of concentration either the phenolphthalein-methyl orange method or the Winkler method gave very inaccurate results. Since these laboratory samples were made up of distilled water and contained no unknown impurities, it would be expected that even larger errors than found in the laboratory samples might occur in boiler-water samples which may be quite highly contaminated. In a boiler water containing appreciable concentrations of such ions as aluminate or silicate, the usual interpretation of the titrations for alkalinity in terms of OH and CO_3 would give a highly inaccurate idea of actual boiler-water conditions.

It has been found in work with an experimental boiler at the university that both the phenolphthalein-methyl orange and the Winkler methods can show inaccurate results for carbonate in the range in which they might be expected to be accurate. In one particular series of samples these methods showed carbonates from 60 to 80 parts per million of CO_3 when the concentration found by the evolution method was from 9 to 12 p.p.m. This points to the necessity of checking the phenolphthalein-methyl orange method or Winkler method from low to fairly high concentrations or carbonate.

The discussion has noted the fact that the phenolphthalein-methyl orange method is seriously interfered with by some kinds of organic matter, but goes on to state further that the modified Winkler method might not be subject to errors from this source. The assumption that the modified Winkler is the all-magical method which removes most of the errors in the analysis of boiler-water samples for carbonate seems to be very general. While it is true that it would not be expected to be as susceptible to error from buffer salts (which may be organic salts) as is the phenolphthalein-methyl orange method, it is certainly a long way from being free from error from this source. This is clear if it is remembered that, with these buffer salts present, it may require the addition of a large amount of acid to change the pH value even slightly. Practically this may result in very indefinite end points, and if the carbonate is calculated in the usual way for the Winkler method, the resulting values may be quite meaningless. All titration methods for the determination of carbonate, depending on measurements of the pH of the original sample for the end points, either electrometrically or by means of indicators, are open to this objection. This is one of the reasons that makes the authors believe that the evolution method for carbonate will be found more satisfactory in many cases.

The statement was also made in the discussion that "the A.P.H.A. method, Table 1, was found consistently more accurate than the Winkler method, Table 2." The authors would like to correct this statement; the Winkler method may be quite accurate in the case of samples free from sulphate, as seen in Table 2 under reference numbers 7 and 19, analyzed by procedure (3). The Winkler method of titration is inaccurate in the presence of sulphate.

If a boiler water is conditioned with phosphate, there is really little necessity for determination of its carbonate content. Determination of the CO_3 is, however, necessary as a control when soda ash is being used as a conditioning chemical for scale prevention. When a boiler is fed with a naturally alkaline or zeolite-softened water, an accurate knowledge of the CO_3 concentration may also be important from the standpoint of embrittlement prevention.

Hydrolysis of carbonate in a boiler water proceeds rapidly. When a boiler is conditioned with soda ash, this results in low carbonate concentrations and high hydroxide concentrations. The attempt to add an excess of carbonate in order to prevent scale may result in hydroxide concentrations sufficiently high to cause danger from embrittlement. The result will be that a balance will have to be struck between the amount of carbonate which must be present to prevent scale and the maximum amount which can be fed without causing danger from embrittlement. In this case the necessity of controlling the carbonate is apparent, and since in general it must be low, it would seem that the evolution method would best satisfy the requirements.

The plant studies described in the report were made on a boiler water nearly free from organic or inorganic constituents which would affect the alkalinity titrations. Nevertheless, Fig. 3, showing the results of these plant tests, indicates that both the phenolphthalein-methyl orange and the Winkler methods gave values for CO_3 which were consistently about 100 per cent too high, and at times 300 to 400 per cent too high as compared with the results for the evolution method. An error of 100 to almost 400 per cent is certainly not negligible. In fact, the authors are tempted to believe that if one evolution analysis were made on a sample of this boiler water, the carbonate could be guessed from then on, with considerably more accuracy than would be secured by titration using either the phenolphthalein-methyl orange or the Winkler method. Since the evolution method has shown itself accurate in every case in which it was tested, and since by the very nature of the method it is not susceptible to interference by such ions as might exist in boiler-water samples, the difference between the results obtained by the use of the evolution method and those obtained by the use of the other two methods examined must be ascribed to inaccuracies in these other two methods.

It may of course be contended that the particular boiler from which these data were secured contained low carbonates, and that in this range the ordinary methods of analysis show comparatively large errors, whereas if the concentrations become higher, the comparative errors are smaller. This is true, but in how many actual boilers does the true CO_3 concentration exceed 30 p.p.m.? Mr. Yoder speaks from a wide experience in the practical control of boiler waters, but this experience has been based upon methods of analysis which are now known to give consistently high values for CO_3.

The point that Mr. Yoder strives to make is that refined methods of analysis are all very well for the research laboratory, but that the practical engineer has no need for them. The trend of engineering development is, however, always in the direction of more accurate control. The engineer must base his conclusions upon data, and his conclusions can rarely be more sound than the data from which they are derived. For example, Mr. Yoder states that if the carbonate alkalinities in the boiler water are substantially less than 30 p.p.m. CO_3, "the alkalinity is too low to generally prevent calcium silicate and carbonate scales." Is it not probable that this conclusion is based upon numberless analyses by the standard titration methods, and that these analyses, even though made with scrupulous care, were inaccurate by from one hundred to several hundred per cent?

The discussion as to whether 30 p.p.m. of carbonate is too low to prevent calcium carbonate and calcium silicate scale is perhaps beside the point in a report which attempts to deal with analytical methods, yet with the few data available and a large share of these based on analytical methods which may be very greatly in error, it would seem that the establishment of this figure at any particular value would be rather premature. This problem would no doubt be worth considerable investigation, and it is hoped that when such an investigation is undertaken care will be exercised to use accurate analytical methods.

RP-54-14

Tungsten-Carbide and Other Hard Cutting Materials

Progress Report No. 3 of the Sub-Committee on Metal-Cutting Materials[1] of the A.S.M.E. Special Research Committee on the Cutting of Metals

By COLEMAN SELLERS, 3d,[2] PHILADELPHIA, PA.

Shortly after this sub-committee's Progress Report No. 2 in June, 1930, the activities were broadened to include not only tungsten-carbide but other hard cutting materials. In April, 1931, a second questionnaire was sent out to the leading manufacturing establishments of the country, with a letter asking all that had had experience with tungsten- and tantalum-carbide or other new alloys to answer the questions as far as possible. It was thought that the information thus obtained might help to determine the present status of the use of these materials and perhaps lead to the enlightenment of some as to their possibilities and limitations.

This questionnaire was sent out thirteen months after the first one. It is interesting to note that the answers show that considerable progress has been made, not only in the number of users, but in that many of those that had been using a little have found it worth while to extend the use. Furthermore, much has been learned in the methods of use.

Over 100 replies were received, of which 85 gave the information on which this report is based, and 2 more answered the first question only. The remainder of the letters stated they did not use the materials or did not feel they had had sufficient experience to be able to give reliable information. The first questionnaire brought in 75 replies, of which 50 could be used in the report. This increase of 70 per cent is very gratifying, particularly since many firms have discontinued all experimental work on account of the condition of business.

The questionnaire is divided into two parts; the first deals with general questions, and the second asks for information on typical applications. This report covers a digest of the first part, including the comments of some of the users. The second part cannot be handled in the same manner, but some of the important points are mentioned, and a few of the more complete applications are given in tabular form.

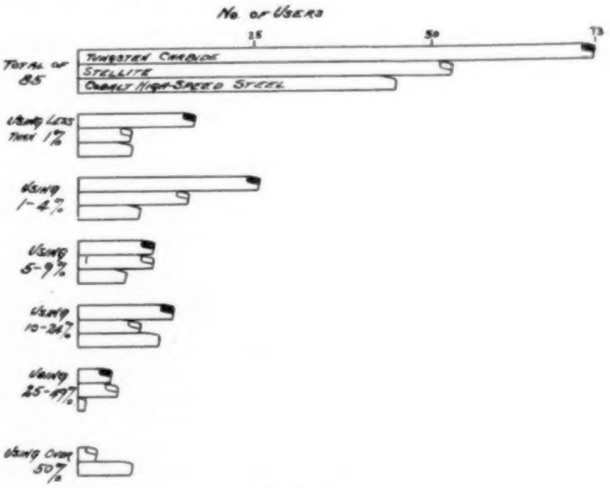

Fig. 1 Relative Use of Tungsten Carbide, Stellite, and Cobalt High-Speed Steel

A copy of the questionnaire, with the letter that accompanied it, is attached to this report.

1 Extent of Application

There were 73 firms that used tungsten-carbide tools. The percentages varied all the way up to 48 per cent, but only 20 used over 5 per cent.

Cobalt high-speed steel was not used by as many, but those that did, used a higher percentage. The figure was 45 users, and 22 used over 5 per cent. The maximum percentage used was 85, and 8 used more than 50 per cent.

Only 7 reported the use of tantalum carbide; 2 used 3 per cent, 1 less than 1 per cent, and 4 were experimenting with this material.

Stellite was used by 53, of which 32 used 5 per cent or under.

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[1] The personnel of this sub-committee is as follows:

F. C. Spencer, *Chairman*, Assistant Superintendent of Manufacturing Development, Western Electric Company, Kearny, N. J.

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A. H. Bainton, Works Manager, Brown & Sharpe Mfg. Co., Providence, R. I.

O. W. Boston, Professor, Department of Engineering Research, University of Michigan, Ann Arbor, Mich.

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M. Judkins, Firth-Sterling Steel Company, McKeesport, Pa.

R. D. Prosser, General Manager, Widia Department, Thomas Prosser & Son, New York, N. Y.

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[2] Executive Engineer, William Sellers & Co., Inc. Mem. A.S.M.E. Mr. Sellers was graduated from the University of Pennsylvania, Mechanical Engineering Department, in 1915, and entered the shops of William Sellers & Co., Inc., immediately thereafter. He worked in various parts of the shop and finally became foreman of a fitter gang. After two years he became associated with the Midvale Steel Company, in charge of repairs to machine tools in all eight machine shops. On Jan. 1, 1919, he returned to William Sellers & Co. as assistant superintendent, and has been there since, his present position being executive engineer in charge of the engineering and production departments.

Contributed by the Sub-Committee on Metal-Cutting Materials of the A.S.M.E. Special Research Committee on the Cutting of Metals and presented at the Machine Shop Practice Session at the Annual Meeting, New York, N. Y., Nov. 30 to Dec. 4, 1931, of The American Society of Mechanical Engineers.

The maximum percentage used was 81, and this was the only firm reporting the use of over 50 per cent.

The diamond as a cutting tool was not popular, but 9 reported its use—3 at 2 per cent, 3 at 1 per cent, and 3 at less than 1 per cent.

The chart in Fig. 1 was made up to show the relative use of cobalt high-speed steel, tungsten carbide, and stellite among the firms which answered the questionnaire. The chart has no particular value, but is interesting as it is based on information given by a number of the leading machine shops of the country.

2 Types of Operations Found Successful With Tungsten or Tantalum Carbide

There are 80 that report turning, 62 report boring, 11 planing, 14 milling, 6 drilling, 1 sawing, and 1 shaping slots.

At the time of the last questionnaire there were practically nothing but lathe and planer tools. Evidently milling cutters, saws, and drills are now coming into use.

3 Types of Material on Which Tungsten Carbide Is Used Successfully

The replies to this question show a wide variety of materials. There were 69 that found success on cast iron, 46 on bronze, 23 on aluminum alloys, 14 on soft steel, and 12 on hard steel. Fig. 2 shows these figures in chart form. Among the other materials were malleable iron, vanadium iron, babbitt, forged brass, experimental bearing metal, nickel silver, 50-per cent semi-steel, phenol plastic, asbestos, bakelite, hard rubber, resinous products, hardwoods, stellite, high-speed steel, stainless steel, 18-8 nickel alloy, and heat-resisting alloys.

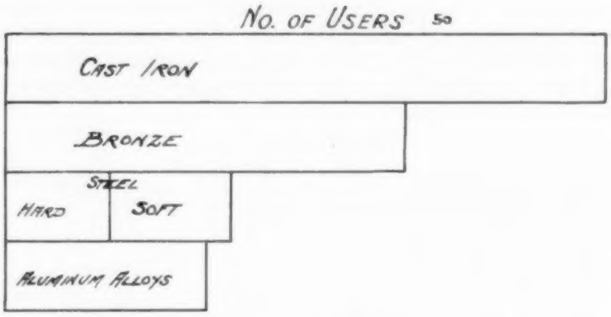

Fig. 2 Relative Successful Use of Tungsten Carbide on Different Materials

4 Types of Material on Which Tantalum Carbide Is Used Successfully

Not many replied to this question, and it is evident that the use of tantalum carbide is not widespread. There were 14 that reported success on steel (8 of these on soft steel and 5 on hard), 4 reported success on bronze, 3 on aluminum alloys, 5 on cast iron, and 1 on stellite; 4 others reported carrying on experiments, and 2 reported no success.

5 Grinding of Hard Cutting Material Tools

(*a*) As to the type of machine, 54 reported they used a pedestal grinder and the tool is held by hand; 15 of these used no other method. There were 28 that used a pedestal grinder with the tool clamped on angle-setting slide, 45 used a universal tool grinder, 30 used a surface grinder, 17 used a face-mill grinder, and 1 used a drill grinder.

Evidently there is no uniformity in the method used in grinding. However, it is apparent that the importance of machine grinding is being realized, and the methods have improved since the time of the sub-committee's first report, when only 13 shops reported machine-ground tools.

(*b*) In answer to the question as to who ground the tools, 41 reported tools ground by machine operator, 21 by tool crib attendant, 35 by grinding-machine operator, and 28 by a specialist. In 9 cases the grinding-machine operator was the specialist. Several replied that the machine operator ground high-speed tools and the tungsten-carbide tools were ground by the grinding-machine operator. In one case the machine-tool operator ground all but the form tools.

It is interesting to note that of the 30 firms using 5 per cent or more tungsten carbide, only 4 reported tools ground only by machine-tool operator, and 1 of these was in tool crib. In other words, where they have had sufficient success to warrant the purchase of a number of tools, they realize the importance of careful grinding.

(*c*) As to the place where the tools are ground, 44 reported the tools were ground in the maintenance tool room, 34 in the tool crib, and 1 ground by supplier. There were 45, including many of the foregoing, that had tools ground in the shop; in some cases they specified this was only the high-speed tools.

(*d*) There were 38 firms that used a straight wheel only in grinding their tools, and of these all but 1 ground on the periphery; 12 ground on the side of a cup wheel, and 37 used both types. Only 1 used a disk.

(*e*) The size of the wheels used varied greatly, the minimum being $3^{1}/_{4}$ in. in diameter and the maximum 26 in. in diameter. The face varied from $^{1}/_{4}$ in. to 4 in. The most popular was 10 in. in diameter, as 27 were equipped with that size, 24 used 8 in. in diameter, and 15 used 6 in. in diameter. The remainder were scattered among the various sizes. There were 10 firms that reported a large variety, but gave no sizes.

(*f*) As to specifications the grain, grit, and bond varied almost as much as the size of the wheel. There were 5 that specified that they used the same grain and grade for roughing and finishing; 2 of these used universal tool grinders, 1 used a surface grinder, and the 2 others used a pedestal grinder. There were 4 of the 5 that used a 60-grain wheel, and the other used 80. There were 39 that used a different wheel for finishing than they did for roughing; 11 specified the wheel for roughing only, and 20 the wheel for finishing only.

For roughing, the grain used varied from 20 to 200 and for finishing from 24 to 220. By far the most popular were 60 to 80 roughing and 100 to 120 finishing. The actual figures were 47 using grain 60 and 17 grain 80, 17 grain 100, and 21 grain 120.

There seems to be little doubt that silicon carbide is the abrasive to use, but the hardness varies. The majority favored G and H on the scale used by some manufacturers and S and R on the other scale. These four letters represent very little difference in hardness.

(*g*) On the method used to test cutting edge, 11 reported inspection made with a magnifying glass, 9 others judged by appearance, 2 used a microscope, 3 used a protractor, 2 used dial indicators on milling cutters, 2 used a gage, and 1 a surface plate. There were 9 that tested only by use, or by cutting action and life, and 8 specified no method.

(*h*) On the question of grinding wet or dry, 67 answered that they used the dry method only, 9 ground wet only, and 14 used both methods. Evidently either is acceptable.

(*i*) The time of grinding varied from 1 minute to 2 hours for lathe and planer tools, but the majority ran between 3 and 20 minutes. The time does not mean much unless the size and condition of the edge are known. One prominent manufacturer using a fairly high percentage of tungsten-carbide tools stated that a $1^{1}/_{4}$ in. square tool would take 5 minutes to resharpen if dull. If it was chipped, the time would be 15 minutes, and for a new tool the first sharpening would take 25 minutes. There were 8 other firms that stated times from 4 to 40 minutes for this size of tool, with the average about 15 minutes.

There were 4 answers that gave the time for a $^3/_8$ in. square bit as an average of 8 minutes, and 7 reported on $^5/_8$ in. by $1^1/_4$ in. as an average of 11 minutes, although the times varied from 3 to 30 minutes.

Milling cutters required from 15 minutes to 10 hours, depending upon the size. Some of these were: 6 in. diameter by $2^1/_2$ in. face, 27 minutes; 8 in. diameter by 1 in. face (20 teeth), 45 minutes; 8 in. diameter by $2^1/_4$ in., 25 minutes; 9 in. diameter (24 teeth), $1^1/_2$ hours; 12 in. diameter (36 teeth), 3 hours.

Drills required from 1 minute to 5 hours. One firm reported 1-in. drill, 3 minutes; another, 2-in. drill, 3 minutes; another $1^1/_2$-in. drill, 5 hours, and one stated that a $^1/_2$-in. drill required $2^1/_2$ minutes to grind and lap.

Among the special tools was a form tool $3^5/_8$ by $^1/_2$ by $^1/_2$ in., which required 30 minutes, and a circular form tool requiring 10 minutes.

(*j*) This question was "Do you have any difficulty with tools cracking from grinding?" Most of the answers were negative, but 3 answered "yes," 2 answered "some," 2 answered "not much," and 1 stated "a little, with tantalum carbide."

6 Lapping and Honing

There were 2 that answered that they lapped all tools, 3 lapped 75 per cent, and 3 lapped 50 per cent of the tools, 7 more lapped 10 per cent or less, and 31 did no lapping. Of the 15 that lapped, 9 lapped finished tools only.

Honing is much more generally practiced. There were 15 that reported that they honed all tools, and 8 more honed 15 per cent or more. Only 7 reported no honing.

Of 37 shops reporting honing, 22 honed both roughing and finishing tools; 1 firm honed all roughing tools and lapped all finishing tools; 1 reported honing tungsten-carbide tips frequently while in the machine to retain a sharp edge, claiming that this practice increases the number of pieces machined between grinds.

The lapping disk in the majority of cases was cast iron, but silicon carbide, bakelite, swedish iron, wood, and both iron and steel with copper insets were also used.

(*d*) and (*f*) The lapping disk in 70 per cent of the answers was charged with diamond dust or the latter was used as lapping compound, but flour of emery, carborundum, crystollon powder, boron carbide, and tripoli were also mentioned.

(*e*) All but one used oil for a vehicle with the lapping compound, and the majority reported it was olive oil. The one exception used water.

(*g*) The amount removed by lapping averaged 0.0003 in., although the answers varied from 0.0001 to 0.005 in. One firm merely removed the tool marks.

(*h*) The time for lapping ran from $^1/_2$ minute to 3 hours, but the majority required between 5 to 15 minutes without regard to size.

The specifications of the hone used varied from 150 to 500 grain crystollon or carborundum stones. All but 3 would be included in a range between 180 and 300; 4 reported using India oil stone, giving no other data.

7 Question of Leaving Less Metal on Work

It is interesting to note that on the question of being able to leave less metal on the work for finish because the new cutting materials can cut scale more readily, 37 found this to be the case and 35 did not. Of those that answered negatively, several admitted that they had no experience or had not attempted to change the amount of finish.

8 Detecting Hard Spots

There were 10 that reported difficulty because hard cutting materials cannot detect hard spots in finished surfaces; 65 answered no difficulty.

9 Handling Chips Commercially

There were 8 that reported difficulty in producing chips that could be commercially handled, and 67 answered "no" or that they do not use tungsten carbide on steel.

10 Protection From Flying Chips

On the question of protection against chips, 30 reported using goggles or glasses, 24 guards or screens, and 5 others did not mention the kind of protection. There were 15 that reported having no protection and no difficulty; 1 reported that if the tools were ground with chip breaker, there was no protection required.

11 Obtaining Desired Finish

There were 62 that had no difficulty in obtaining the desired finish on their work with the hard cutting materials and 3 reported a better finish. However, 14 reported that they did have trouble, and 1 had difficulty with reaming operations only; 1 stated that it was possible to obtain a smoother cut with stellite than with tungsten carbide.

12 Suggestions or Additional Information

Among the miscellaneous comments and additional information the following is interesting to note:

A manufacturer of gear-cutting machinery states that he finds cobalt–high-speed-steel tools work best in their shop as they have short runs and the tungsten-carbide tools are too "fussy." Also on heat-treated S.A.E. 3140 steel, the cobalt tools stood up better.

Another machine-tool builder who makes his own tools reports that the tips are formed before the tool is made up, which reduces the grinding to a minimum.

A prominent manufacturer states that very good speeds to use with tungsten carbide are 160 to 200 ft. per. min. for continuous turning and 100 to 140 for intermittent cutting.

An automobile manufacturer reports that he uses tungsten carbide for all operations on aluminum pistons and for finish cuts on cast iron. For roughing cast iron he uses tungsten carbide on new set-ups and where increased production is required, but in the majority of cases he obtains the required production using stellite and high-speed steel. The excessive price of tungsten carbide does not warrant its use until a higher production is required.

A tool manufacturer states that the use of tungsten-carbide tips on precision jig-boring machines has not proved profitable. He was unable to finish-bore with the tool as the cutting edge lost its keeness due to chatter in performing the roughing operation.

An automobile manufacturer reports that tungsten carbide can be used on steel where the job and tool can be held rigid and a light cut taken with a fast cutting speed.

Actual Examples

The second part of the questionnaire dealt with typical successful operations. There were 37 companies that gave definite figures, including savings in money, hours, or percentage and several gave more than one example.

Among these were 11 machine-tool builders, 6 builders of other types of machinery, 4 automotive plants, 2 electric companies, 2 manufacturers of plumbing equipment, 2 ball-bearing and roller-bearing makers, 2 chain manufacturers, and 7 miscellaneous, including manufacturers of ships, hoists, instruments, and tele-

TABLE 1 TYPICAL EXAMPLES SHOWING SAVINGS ON ENGINE LATHES

Application No.	1		2		3		4	
Material cut	Steel		Hard bronze		Cast iron		Cast iron	
Specifications	S.A.E. 3250		....		Sil 2.75, Mn 0.50, Brinell 35		Sand-blasted	
Cutting quality	Good		Good		Hard		Hard	
Condition of machine tool	Good		Good		Good		Good	
Size of tool, in.	1 1/4 × 1 1/4 × 3		5/8 × 1 1/4 × 8		3 5/8 × 1/2 × 1/2		1/2 × 1 × 7	
Size of tip, in.	3/16 × 3/4 × 5/8		5/16 × 1/4 × 25/32		9/16 × 1/4 × 5/32		3/8 × 9/16 × 3/16	
Shape of nose	Square		Round		Round		Round	
Clearance angle, deg.	6		6		5		3	
Top side rake, deg.	12		6		6		1	
Top front rake, deg.	..		4		10		7	
Method of mounting	Special holder		Plain tool post		Box tool		Plain tool post	
Operation	Rough turning		Finish turning		Rough and finish turning		Rough turning	
Tool material	T-C	H.S.S.	T-C	H.S.S.	T-C	Cob. H.S.	T-C	H.S.S.
Cutting speed, f.p.m.	150	60	519	362	75 3/4	32 3/4	230	120
Feed (per revolution), in.	0.031	0.012	0.056	0.007	0.01	0.01	1/64	1/64
Depth of cut, in.	1/4	1/4	..	..	0.05	0.05	1/8	1/8
Cu. in. removed per min.	14	2.1	..	..	0.43	0.142	2.3	1.2
	Continuous		Continuous		Continuous		Intermittent	
Number of pieces per grind	18	6	2000	50	1500	4–6	50	10
Number of grinds per tool	..	..	150	50	12	12	32	40
Cost of tools	$27.00	$8.00	$28.25	$2.50	(2) $18.00	(2) $0.52	$32.50	$2.00
Time per piece, hours	0.54	0.90	0.05	0.10	0.009	0.02	0.065	0.112
Savings in labor cost, per 100 pieces	$20		$4.55		$0.03		$2.91	
Decrease in cost, per 100 pieces, including tool expense			4.55 hr. and $0.36 decrease in tool expense		$10.015		15 per cent	

TABLE 2 TYPICAL EXAMPLES SHOWING SAVINGS ON TURRET LATHES

Application No.	5		6		7		8	
Material cut	Cast-steel alloy		Cast bronze		Vanadium iron		Cast iron	
Specifications	C 0.35, Ni 2, Cr, 0.75, Mo 0.25		C 80, Zn 3, Sn 11, Brinell 50		C 2.6, Si 3, Mn 0.47, Cr 0.032, V 0.092		Commercial gray iron 140–150 Brinell	
Cutting quality of material	Hard scale sand and holes, hard and tough, Brinell 250		Sand		Sand and blow holes; very hard		Hard	
Condition of machine tool	Good		Good		Good		Good	
Size of tool, in.	1 1/4 × 1 1/4 × 7		3/8 × 3/8		7/8 × 7/8 × 4		1 × 1 1/4	
Size of tip, in.	3/8 × 9/16 × 1 3/8		1/8 thick		(See Fig. 5)		1/4 × 1/2 × 1	
Shape of nose	Round		Square		Square		Round	
Clearance angle, deg.	5		6		6		7	
Top side rake, deg.	4–8		0		0		11	
Top front rake, deg.	12		0		12		4	
Operation	Turning and facing		Turning		Turning and facing		Turning	
Tool material	T-C	Cob. H.S.S.	T-C	Cob. H.S.S.	T-C	Stellite	T-C	Stellite
Cutting speed, ft. per min.:								
Roughing	120–150	30–50	670	183	350	110	270	125
Finishing	..	..	670	183	65	55	340	125
Feed, in. per revolution:								
Roughing	0.030	0.065	Hand	Hand	1/32	1/16	1/32	1/32
Finishing	..	..	Hand	Hand	0.028	0.028	0.020	1/32
Depth of roughing cut, in.	1/4 5/16	5/16 3/8	3/32	3/32	3/16	3/16	1/4	1/4
Cu. in. removed per minute	12.6	10.25	..	..	26	14		
	Intermittent		Continuous		Continuous		Continuous	
Number of pieces per grind	12	1/2–2	6000	70	15	3	15	15
Number of grinds per tool	..	..	22	67	..	..	20	20
Cost of tool	..	..	$1120	$1.85	$41.30	$5.00	$48.00	$3.31
Time per piece, min.	60	75	2.603	3.614	1.07	2.00	26	50
Savings in labor cost, per 100 pieces	$15.00		$1.76		$1.10		$24.00	
Decrease in cost per 100 pieces	$35.00		..		..		$9.10	

TABLE 3 TYPICAL EXAMPLES SHOWING SAVINGS ON AUTOMATICS

Application No.	9		10		11	
Material cut	Cast iron		Steel		Malleable iron	
Specifications	GMC 13M, 165–217 Brinell, normalized and cooled slowly		S.A.E. 1020		C 2.5, Si 1.05, Mn 0.43, 114 Brinell	
Cutting quality	Hard scale		Hard scale		Good	
Condition of machine tool	Good		Good		Good	
Size of tool, in.	5/8 × 3/4		2 × 2 × 2 1/2		1/2 × 1/2 × 3	
Size of tip, in.	1/4 × 3/16 × 3/4		3/4 × 3/4 × 1		..	
Shape of nose	Round		Forming tool		Diamond point	
Clearance angle, deg.	5		12		6	
Top side rake, deg.	6		0		3	
Top front rake, deg.	6		0		6	
Operation	Turning		Forming		Turning	
Tool material	T-C	Stellite	T-C	H.S.S.	T-C	Stellite
Cutting speed, f.p.m.	166	80	211	105	220	150
Feed, in. per revolution	..	..	0.002	0.002	..	..
Depth of cut, in.	3/16	1/8	..	..	0.03	0.03
Number of pieces per grind	1100 up	200	27,000	7,200	1800	800
Cost of tool	$12.85	$3.75	..	..	$15.00	$1.15
Time per piece, min.	0.534	0.876	..	..	0.50	0.66
Savings in labor cost, per 100 pieces	$0.34				$0.17	

TABLE 4 TYPICAL EXAMPLES SHOWING SAVINGS IN VERTICAL BORING MILLS

Application No.	12		13		14		15		16		17	
Material cut	C. I.		C. I.		C. I.		C. I.		C. I.		C. I.	
Specifications	Semi-steel				Si 1.58, Mn 0.76, S 0.075, P 0.26, 23 Rockwell		180–210 Brinell		20 Rockwell, annealed			
Cutting quality of material	Sand and hard scale				Hard scale		Hard scale, sand		Hard scale, blowholes		Hard scale, sand, blowholes	
Condition of machine tool	Fair		Good		Good		Poor		Good		Fair	
Size of tool, in.	$1\frac{1}{4} \times 1\frac{1}{4}$		$1 \times 1\frac{1}{4} \times 8$		$1\frac{1}{2} \times 1\frac{1}{2} \times 8$		$1\frac{1}{2} \times 1\frac{1}{2} \times 9$		$1 \times 1\frac{1}{4}$		$1\frac{1}{4} \times 1\frac{1}{4}$	
Size of tip, in.	..		$\frac{3}{8} \times \frac{1}{8} \times 1\frac{1}{8}$		$\frac{1}{2} \times \frac{1}{2} \times 1\frac{1}{4}$		$\frac{1}{2} \times \frac{13}{16} \times 1\frac{1}{2}$		$\frac{3}{8} \times \frac{1}{2} \times 1$		$\frac{3}{8} \times \frac{3}{8} \times 1\frac{1}{4}$	
Shape of nose	Style 13		..		..		Round nose		Round nose		Style 13	
Clearance angle, deg.	4		6		4		6		..		6	
Top front rake, deg.	12		12		6		12		..		4	
Top side rake, deg.	0		0		8		0		..		12	
Operation	Turning and facing		Turning		Turning and facing		Turning and facing		Turning and facing		Turning and facing	
Tool material	T-C	Cob. H.S.S.	T-C	H.S.S.	T-C	H.S.S.	T-C	H.S.S.	T-C	H.S.S.	T-C	H.S.S.
Cutting speed, f.p.m.:												
Roughing	160	75	176	60	100	65	70	30	81	60	180	90
Finishing	240	90	..	..	..	..	..	..	..	..	230	110
Feed, in. per revolution:												
Roughing	$\frac{1}{16}$	$\frac{1}{16}$	0.03	0.03	0.061	0.061	$\frac{1}{16}$	$\frac{1}{16}$	0.062	0.062	0.031	0.031
Finishing	$\frac{1}{64}$	$\frac{1}{64}$	..	..	..	..	..	..	..	..	0.010	0.010
Depth of cut roughing, in.	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{16}$	$\frac{3}{16}$	0.625	0.625	$\frac{1}{4}$–$\frac{3}{8}$	$\frac{1}{4}$–$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{16}$	$\frac{3}{16}$
Cu. in. removed per min.	..	..	..	..	..	..	..	..	..	..	12.5	6.3
Number of pieces per grind	2	1	100	8	3	1	1	$\frac{1}{2}$	20	3	600	30
Cost of tool	$73	$6	$74	$2	$48	$8.65	..	..	$59	$4	$200	$40
Time per piece, min.	90	210	96	116	240	360	114	240	162	225	11.3	17.4
Savings in labor cost			8.25 hr. per 25 pieces		$2.80 per piece		22 per cent		$73 per 100 pieces		$24 per 100 pieces	
Decrease in net cost	$3.50 per 10 pieces		$2.18 per 100 pieces						$20.50 per 100 pieces		20 per cent	

phones. This shows quite a diversification in product from which the examples were selected.

Many others gave part of the information, but did not have the figures available to give the complete story.

It is difficult to summarize all the answers as in the first part of this report because the information is such that part of it is practically meaningless unless connected with all of the other data. It seems best therefore to tabulate some of the typical examples in full, spreading them over the various operations and materials. All were with tungsten carbide.

Before presenting these, however, there are several questions which can be discussed generally.

METHOD OF TIPPING

All tips were brazed, and in addition 12 were dovetailed.

ANGLES OF CLEARANCE AND RAKE

The reports show that the clearance angle varied from 2 to 14 deg. There were 15 that reported 4 deg., 10 reported 6 deg., 11 reported 7 deg., and 4 reported 8 deg., although only 2 of these were successful, 1 on aluminum and 1 on cast iron; 1 used a clearance from 10 to 20 deg. for cutting babbitt.

The top front rake varied considerably. There were 10 that reported 0 deg., 4 reported 4 deg., 8 reported 6 deg., 4 reported 12 deg. The others were scattered among the numbers between 1 and 20 deg.

The top side rake varied from 0 to 18 deg., but the average was approximately 6 deg.; 6 reported 0 deg., 10 said 4 deg., 12 reported 6 deg., and 5 stated 12 deg.

The radius at the point of the tool is evidently not important, as successful tools were ground with a radius from $\frac{1}{32}$ to $\frac{3}{8}$ in. The most popular was $\frac{1}{16}$ in., to which 13 of the tools were ground; 8 other tools were ground to $\frac{1}{8}$ in., 3 to $\frac{3}{16}$ in., and 4 to $\frac{1}{4}$ in.

The condition of the machine tools was good for 28 of the successful applications, fair for 10, and poor in 1 case.

There were 24 applications that were on machine tools motor-driven and 17 driven from a countershaft.

In motor drives the drive was through silent chain, V-belts, flat belts, gears and worm and wheel so that practically all types are represented.

METHOD OF MOUNTING TOOL

A square turret type of tool holder was used in 16 cases and plain tool post in 7 cases. The remainder used boring bars, drill spindles, special tool holders, box tools, etc.

All tools were clamped by setscrew except that 3 had clamps, 1 a wedge, and 1 used a bolt through the shank.

Table 1 shows 4 applications on engine lathes—1 on steel, 1 on hard bronze, and 2 on cast iron.

The increases in cutting speeds and increases in number of tools per grind are extremely good.

Table 2 shows 4 applications on turret lathes with 4 different materials. The tool used in application 7 is shown in Fig. 3, as it is unique.

Application 8 is interesting in that no more pieces per grind were obtained than with stellite. However, the difference in cutting speeds permitted a net saving of $9.10 per 100 pieces.

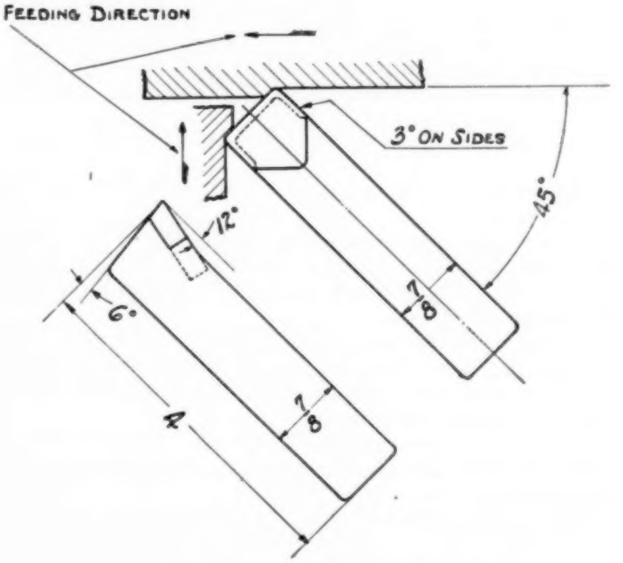

FIG. 3 TOOL USED IN APPLICATION NO. 7

Table 3 covers 3 applications on automatic lathes.

Application 9 is interesting, as it shows the elimination of the finishing cut.

Table 4 covers applications on vertical boring mills. These are nearly all large jobs.

Table 5 outlines milling-machine applications. One of the cutters used in application 20 is shown in Fig. 4. It milled on an

TABLE 5 TYPICAL EXAMPLES SHOWING SAVINGS ON MILLING MACHINES

Application No.	18		19		20	
Material cut	Cast iron		Bakelite		Rubber strips	
Specification	Brinell 210, hard scale		Hard			
Cutting quality of material	Hard		Abrasive		Abrasive	
Condition of machine tool	New		...		Good	
Size of cutter, in.	13 × 2		$3^{23}/_{32} \times ^{1}/_{2}$		$3^{1}/_{4} \times 3$	
Number of teeth	32		18		18	
Size of tip, in.	$^{5}/_{8} \times ^{1}/_{2} \times ^{1}/_{8}$		..		$^{3}/_{32} \times ^{1}/_{4} \times 3^{1}/_{8}$	
Clearance angle, deg.	7		..		..	
Top side rake, deg.	5		..		..	
Top front rake, deg.	5		..		..	
Method of holding teeth	Wedge		...		...	
Tool material	T-C	Stellite	T-C	Stellite	T-C	H.S.S.
Cutting speed, f.p.m.	120	75	2000	2000	600	600
Feed, in. per min.	18	12	252	252	85.8	85.8
Depth of cut, in.	$^{1}/_{8}$	$^{1}/_{8}$	$^{3}/_{32}$	$^{3}/_{32}$	..	..
Cu. in. removed per min.	7	4.4	...	...	...	...
Number of pieces per grind	250–350	75–100	160,413	14,583	20,000	200
Number of grinds per tool	...	...	..	..	40	20
Cost of tool	$252	$40	..	..		
Time per piece, min.	7.97	12.73				
Savings in labor cost, per 100 pieces	$6.50		$0.0225		$0.03	
Decrease in cost per 100 pieces, including tool cost	$2.50				$0.24	

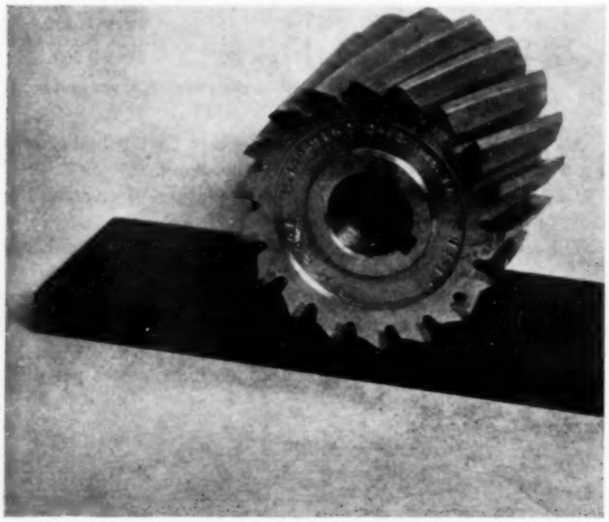

FIG. 4 CUTTER USED ON APPLICATION NO. 20

average of 20,000 pieces per grind, and there were 40 grinds in the life of the cutter.

Table 6 shows typical examples on drill-press work. There were few data available, and only one shows the figure of savings. The other is interesting, however, in that 10,000 holes per grind were obtained.

Table 7 shows two examples on horizontal boring mills. The cutting speed is not high, but the life of the tungsten carbide shows up much better than the cobalt high-speed steel and the regular high-speed steel.

There was but one application of die work. The material was 0.035 nickel-silver sheet drawn into a tube 0.232 in. inside diameter approximately $^{5}/_{8}$ in. deep.

The tungsten carbide $^{11}/_{16}$ in. o.d. by $^{5}/_{16}$ in. i.d. by $^{11}/_{32}$ in.

TABLE 6 TYPICAL EXAMPLES SHOWING SAVINGS ON DRILL PRESSES

Application No.	21		22	
Material	Cast iron		Cast iron	
Specification	Hard scale		200 Brinell, hard scale and sand	
Cutting quality of material	Hard		Hard	
Condition of machine tool	Fair		Fair	
Diameter of drill, in.	$1^{1}/_{2}$		$^{11}/_{16}$ and $1^{1}/_{4}$	
Tool material	T-C	H.S.S.	T-C	H.S.S.
Cutting speed, f.p.m.	60	20	160	60
Feed, in. per revolution	0.006	0.006	0.011	0.011
Number of holes per grind	1000–1500	50–75	10,000	200
Number of grinds per tool	30	20	...	...
Cost of tool	$60	$15	...	...
Savings in labor cost, per 100 pieces	$0.53		...	...
Decrease in net cost per 100 pieces, including cost of tool	$1.833		...	...

TABLE 7 TYPICAL EXAMPLES SHOWING SAVINGS ON HORIZONTAL BORING MILLS

Application No.	23		24	
Material	Cast iron		Cast steel	
Specifications	196 Brinell		0.40–0.50 carbon, sand and blowholes	
Cutting quality of material	Good		Soft, stringy, and abrasive	
Condition of machine tool	Good		Fair	
Size of tip, in.	$^{3}/_{8} \times ^{1}/_{4} \times ^{7}/_{8}$			
Clearance angle, deg.	...		4	
Top side rake, deg.	...		4	
Top front rake, deg.	...		6 to 8	
Operation	Boring		Boring	
Tool material	T. C.	Cob. H.S.S.	T.C.	H.S.S.
Cutting speed, f.p.m.:				
Roughing	20–36	18	85–100	40–50
Finishing	...	..	40–50	30
Feed, in. per revolution:				
Roughing	$^{1}/_{16}$–$^{3}/_{32}$	$^{1}/_{16}$–$^{3}/_{32}$	0.053	0.056
Finishing			0.40	0.29
Depth of cut, in.	$^{3}/_{16}$	$^{3}/_{16}$	0.250	0.187
Number of pieces per grind	8	1 or 2	10–15	1
Number of tools used	3	3	...	..
Cost of tool	$45	..	$48	$14
Time per piece, hr.	16	28	0.30	0.60
Savings in labor cost	$8.64 per piece		$18 per 100 pieces	
Decrease in cost of work, including tool cost	After 5 pieces, 43%		$65 per 100 pieces	

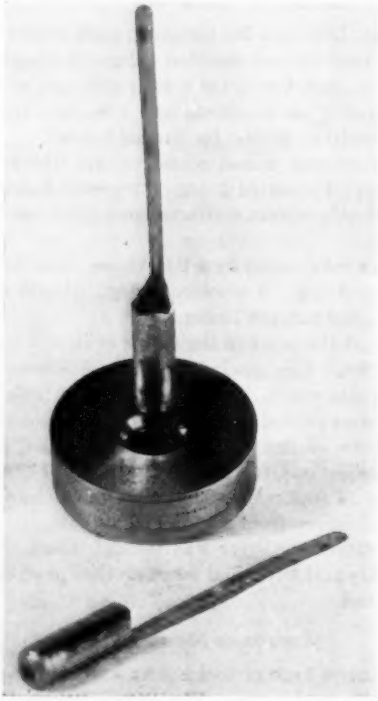

FIG. 5 DIE WITH TUNGSTEN-CARBIDE INSERT

thick was supported on all sides by a hardened shell. The surface produced was a considerable improvement over the high-speed-steel die. The number of pieces per grind was 600,000 against 15,000 with the previous die. The die is shown in Fig. 5.

One machine-tool manufacturer states that the management is a strong believer in tungsten carbide and that the concern is able to create remarkable savings. There are more than 30 milling cutters from 3 to 14 in. in diameter in use in the shop.

One company reports experimenting with tipping small air-gun chisels with stellite, but without success so far. It was about to try another grade that is designed to withstand shock and hopes to have better results. This same concern has experimented in resurfacing copper commutators for generators, cutting the time in half as compared to high-speed-steel tool or grinding. The cutting speed used was 5000 ft. per min.

One firm reports great success sawing insulating material with a tungsten-carbide-tipped saw. A tool manufacturer states that the application of tungsten carbide to saw teeth and knife edges is a proved fact.

This same company has great success cutting steel files with tungsten-carbide-tipped chisels. The chisels are machine driven, making 900 to 1900 blows per minute and from 30 to 40 impressions per inch on the file. The hardness of the file is from 8 to 28 on the Rockwell C scale. The tungsten-carbide-tipped chisels cut from 40 to 60 dozen files before becoming dull, while the high-speed-steel chisels could only cut about 1 dozen.

One manufacturer of machine tools finds the angles of the tools have to be much greater than the tungsten-carbide manufacturers recommend. Quite often a 20-deg. hook in the tool is found. This concern finds 350 ft. per min. on cast iron the proper cutting speed, but the lathes used with this material must be kept in practically perfect shape, and the size of the driving belts has been doubled to provide sufficient power.

This same concern finds the first cost of the tool relatively unimportant. It says that a $40 tool, used properly, will pay for itself in 20 hours. If the men in the shop could be induced to use the tungsten carbide on all possible jobs, the cost of the machine work could be cut to about one-third the cost of the same work with the best cobalt high-speed steel.

UNFAVORABLE COMMENT

One concern using about 20 per cent tungsten carbide states that it does not use more for the reason that the foreman and the machinists are against it because it is new. It was the management's experience that it always takes from four to five years to introduce any new cutting material in a shop.

Another company that has made several successful applications states that it is not sold on anything even approaching a general application of this material because it does not definitely show economy in operation, if unbiased accounting is applied to the problem and all factors are taken into account. This concern finds the reasons for the failures are lack of machine rigidity, poor bonding of the tips, excessive overhanging of the tool, intermittent cuts, etc. Tantalum carbide was not successful in any case. It believes that the conditions in the average shop are not favorable to the use of carbide material and that not much progress will be made until existing machine-tool equipment is replaced by tools properly designed to accommodate the material.

Several companies think that the present price level is one of the most important factors in retarding the use of tungsten carbide. They feel that it should be brought to the level that obtains in Germany and England and suggest that the committee renders its opinion of the present prices.

One manufacturer says he cannot take advantage of the material because the set-up time is so great compared to machining. He also feels that there is insufficient knowledge available as to feed, speed, and depth of cut. He suggests that a series of experiments be conducted such as Frederick W. Taylor conducted with high-speed steel. All the information now available is believed to be unreliable. He also claims that the material is being placed on the market with a minimum amount of information regarding its correct use and only in isolated cases are the results being obtained which are inherent in the material.

SUB-COMMITTEE'S ACTIVITIES

This, of course, is well known by this sub-committee, and it is one of the objects of this second questionnaire to obtain some definite data that are authentic. The sub-committee also has had in mind for two years the experiments suggested, but it is a large task and one that is difficult to get any one to undertake or to finance. If all of those interested to this extent would be willing to contribute to a fund for such a purpose, the proposition could be put across, and this would be the best means of obtaining the desired information in the shortest time.

The members of the sub-committee have been reporting progress of the companies with which they are connected, and a number of interesting developments are under way. For instance, the Union Carbide and Carbon Research Laboratory, Inc., is conducting a series of experiments correlating red hardness and cutting quality. Dr. Boston, of the University of Michigan, is at present conducting a series of tests with cobalt high-speed steel. The General Electric Company has purchased for Schenectady and Lynn especially designed lathes for the purpose of determining accurate and conclusive data pertaining to the use of tungsten carbide and tantalum carbide as a cutting medium. The lathes are about ten times more powerful than standard makes of the same nominal size, and will give an opportunity such as was never before afforded to learn the possibilities of these cutting materials. At the time this report is being prepared they are negotiating with vertical-boring-mill manufacturers with the same object in mind.

CONCLUSION

It is evident that considerable has been learned about tungsten carbide in the past year and a half. There is a great deal more to be learned, and scientific data are needed most. It seems that machine-tool manufacturers have not made much progress, probably because of the lack of definite information as to how far they should go and along what lines they should proceed. The pioneer work being done by the General Electric Company should help much, and others should follow this lead.

The sub-committee wishes to take this opportunity to thank the many companies for their ready response to the letter sent out and for their carefully prepared replies.

Appendix

LETTER ACCOMPANYING QUESTIONNAIRE

Dear Sir:

In March, 1930, the Sub-Committee on Tungsten-Carbide Cutting Materials of the A.S.M.E. Special Research Committee on the Cutting of Metals sent out a questionnaire to the leaders of the metal-cutting industry. This questionnaire dealt with the use of tungsten carbide in actual practice and enabled the committee to prepare a report which was of great interest to the users of this material as well as to machine-tool builders.

Since that time much has been learned about tungsten carbide, tantalum carbide, and other alloys, and it was felt that the sub-committee, whose activities have now been broadened to cover all hard cutting materials, should undertake the compilation of a second report. By this it is hoped to determine the present status of the use of these materials.

Accordingly, a second questionnaire is enclosed, and you are earnestly requested to have the man in your organization in charge of the introduction of these tools fill out the blanks and return them before June 1, 1931.

Please note that the form of this questionnaire is such that it will be much easier to fill out than the first. Many questions may be answered by checking the proper item, eliminating the necessity of writing out an answer. The questionnaire is divided into two parts. The first is general and is intended to determine the scope of application, etc. The second part covers several specific uses and will help to determine satisfactory practice for various operations on different materials.

The response to the first questionnaire was very gratifying, and it is hoped that the second will be as well received. Copies of the report will be mailed to all those who answer and who desire to study the data prepared.

The names of the concerns furnishing this information will not be published.
Kindly address communications to the Sub-committee on Metal Cutting Materials, American Society of Mechanical Engineers, 29 West 39th Street, New York, N. Y.

Chairman, Sub-Committee on Metal Cutting Materials,
F. C. Spencer, *Chairman.*

QUESTIONNAIRE ON THE USE OF NEW CUTTING ALLOYS

1 Extent of Application. State percentage of your total operations that are carried on with:

- *a* High-speed steel
- *b* High-cobalt steel
- *c* Stellite
- *d* Diamond
- *e* Cemented tungsten carbide
- *f* Cemented tantalum carbide
- *g* Other high-speed cutting material

2 Types of Operations Found Successful With Tungsten or Tantalum Carbide

- *a* Turning
- *b* Boring
- *c* Planing
- *d* Milling
- *e* Drilling
- *f* Sawing

Note: Please give details of at least six typical applications on sheets which follow. More sheets will be forwarded upon request.

3 Types of Material on Which Tungsten Carbide Is Used Successfully

- *a* Cast iron
- *b* Soft steel
- *c* Hard steel
- *d* Bronze
- *e* Aluminum alloys
- *f* Other material (state nature)

4 Types of Material on Which Tantalum Carbide Is Used Successfully

- *a* Cast iron
- *b* Soft steel
- *c* Hard steel
- *d* Bronze
- *e* Aluminum alloys
- *f* Other material (state nature)

5 Grinding of Hard Cutting Material Tools

- *a* Type of machine
  - 1 Pedestal (work held by hand)
  - 2 Pedestal (work clamped to angle slides)
  - 3 Universal tool-grinding machine
  - 4 Surface-grinding machine
  - 5 Face-mill-grinding machine
- *b* Who grinds your lathe and planer tools?
  - 1 Machine operator
  - 2 Tool-crib attendant
  - 3 Grinding-machine operator
  - 4 Specialist
- *c* Where are tools ground?
  - 1 Tool crib
  - 2 Maintenance tool room
  - 3 Shop
- *d* What kind of wheel used?
  - 1 Cup (grinding on side)
  - 2 Straight (grinding on periphery)
- *e* State size of wheel used
  - 1 Diameter
  - 2 Face
- State specifications of wheel
  - 1 Roughing
    - *a* Grain
    - *b* Grade
    - *c* Bond
  - 2 Finishing
    - *a* Grain
    - *b* Grade
    - *c* Bond
- State method used for testing cutting edge
- Do you grind?
  - 1 Wet
  - 2 Dry
- *i* What is average time required for grinding?
  - 1 Lathe or planer tool
  - 2 Milling cutter
  - 3 Drill
- *j* Do you have any difficulty with tools cracking from grinding?

6 Lapping and Honing

- *a* What percentage of tools do you
  - 1 Lap
  - 2 Hone
- *b* Do you lap (or hone) both roughing and finishing tools?
- *c* Of what material is lapping disk made?
- *d* What material is lap charged with, if any?
- *e* What vehicle (oil, etc.) do you use with lapping compound?
- *f* What lapping compound do you use?
- *g* How much metal do you remove by lapping?
- *h* How long does it take to lap a tool?
- *i* State specifications of hone:
  - 1 Grain
  - 2 Grade
  - 3 Bond

7 Do You Find That It Is Possible to Leave Less Metal on Your Work for Finish Because the Hard Metals Can Cut Scale More Readily?

8 Do You Encounter Difficulty Because Hard Cutting Materials Cannot Detect Hard Spots in Finished Surfaces?

9 Have You Had Difficulty in Producing Chips That Could Be Commercially Handled?

10 How Do You Protect Workmen From Flying Chips?

11 Do You Have Difficulty in Obtaining the Desired Finish on Your Work With the Hard Cutting Materials?

12 Suggestions or Additional Information

Please fill in these blanks for typical applications of tungsten or tantalum carbide tools. Application No.

A Kind of Material Cut

- *a* Type
  - Cast iron
  - Steel
  - Brass
  - Aluminum alloy
  - Other material
- *b* Analysis (if steel, give S.A.E. specification)
- *c* Rockwell or Brinell hardness
- *d* Heat treatment, if any
- *e* Condition of surface
  - Hard scale
  - Sand
  - Blowholes
  - Pickled
  - Sand-blasted
- *f* Cutting quality
  - Soft
  - Stringy
  - Good
  - Hard
  - Very hard
  - Abrasive

B Machine Equipment Used

- *a* Type of machine tool
- *b* Size (capacity)
- *c* Condition
  - Good
  - Fair
  - Poor
- *d* Nature of drive
  - Countershaft
  - Motor
  - Flat belt
  - Multiple V-belt
  - Silent chain
- *e* Horsepower
  - 1 Capacity of motor drive
  - 2 Actually used on job

C Tool Used

- *a* Material
  - Tungsten carbide
  - Tantalum carbide
  - Other material
- *b* Trade name
- *c* Dimensions of shank
- *d* Method of tipping
  - Dovetailed
  - Brazed
  - Other (state nature)
- *e* Dimensions of tip
- *f* Shape of nose
- *g* Angles of tool (state amount)
- Clearance angle
  - Top front rake
  - Top side rake
  - Radius at corner
- *h* If milling cutter, state
  - Diameter
  - Width of face
  - Number of teeth
  - How held

D How Tool Was Used

- *a* Method of mounting tool
  - Plain tool post
  - Square turret
  - Boring bar
  - Other
- *b* Method of clamping
  - Setscrew
  - Clamp
  - Wedge
- *c* Operation
  - Turning
  - Facing
  - Parting
  - Forming
  - Planing
  - Milling
  - Slotting
  - Shaping

E Grinding of Tool (method covered under general questions)

- *a* Time consumed for grinding
- *b* Time consumed for lapping or honing

F Roughing Cuts. Comparison of this tool with tool previously used (state cutting material)

- *a* Cutting speed (ft. per min.)
- *b* Feed (in. per rev.)
  - Feed (in. per stroke)
  - Feed (in. per min.)
- *c* Depth of cut (inches)
- *d* Metal removed (cu. in. per min.)
- *e* Was cut continuous or intermittent

G Finishing Cuts. Comparison of this tool with tool previously used (state cutting material)

- *a* Cutting speed (ft. per min.)
- *b* Feed (in. per rev.)
  - Feed (in. per stroke)
  - Feed (in. per min.)
- *c* Quality of finish produced

H Economies Shown. (Comparison of this tool with tool previously used)

- *a* Number of pieces per grind
- *b* Number of grinds per tool
- *c* Purchase price (of tool or cost)
- *d* Time per piece
- *e* Savings in labor cost over previous method
  - Per piece
  - Per 100 pieces
- Net increase or decrease
- In cost per piece or 100 pieces

(Taking into account increased cost of tool and grinding)

Discussion

W. S. Huson.[3] On going over the progress report on metal-cutting materials there is much that invites consideration and discussion, for it shows continued endeavor to find a cutting edge that will stand up satisfactorily under the stress of separating a chip from the stock. There is no reason to feel that, because experience broadens and the breakdown point is reached, an ultimate has been finally found, although some of the answers to the questionnaire seem to indicate this. That interest centers about tungsten carbide, Stellite, tantalum, or in fact any other hard substance that will excel high-speed steel, shows there is determined effort not to sidetrack the information and data already available. It may well be that out of it all may come a material or alloy that will help solve the limitations of cemented compounds; and although the high speeds claimed now may be modified, the ultimate result will be more profitable—for as long as a cutting edge has to depend upon a relatively weak bond, there will be a breaking away of the cutting grain, no matter how efficient it may be in hardness and durability.

The item on the construction of super-powerful machine tools leads to the thought that the mere making of heavier tools will not solve the question of the life of a cutting tool, for some seem heavy and powerful enough now, until the cutting tool in itself will prove it can stand up in continuity of acceptable machining. There is, however, this thought—that with heavier machines and the elimination of chatter and other features that have been accountable for cutting-edge failures, there will come tools in which the high-speed steel will give a higher percentage of output. A way may be found to incorporate in a homogeneous mass, say as in high-speed steel, the cutting features of the more or less segregated cutting points as now held in a cementing bond. When this comes, such tools as milling cutters, for instance, will have an ability that may tax the strength of the heaviest tools that can be devised. It is not to be wondered at that machine-tool builders hesitate in trying to devise tools that will be within the range of reasonable purchase and give a profitable return on the investment.

Roger D. Prosser.[4] It is interesting to note the greatly increased use and improved methods of application of tungsten carbide during the last year.

One point that stands out from the paper is the great length of time required to grind tungsten-carbide tools in the average shop. This emphasizes the fact that proper grinding methods are not being employed by most shops, and it shows the need of a greater study in this respect by manufacturers who wish to effect the greatest possible savings with tungsten carbide or other hard-metal-composition tools.

It is apparent that when all tungsten-carbide tools are ground in a special department, where a study of the proper grinding methods has been made, the results obtained are far superior to those in shops where each machine operator is allowed to grind his own tools. It is difficult to instruct the general run of machine operators in the proper grinding methods, and at this time it seems that it is only through careful instruction of a few expert men that the desired results can be obtained.

The grinding of intricate tools, such as milling cutters, forming tools, etc., of course requires somewhat more time than for similar high-speed-steel tools, due to the great hardness of the cemented carbide. On the other hand, simple lathe tools, such as compose the majority of the cemented-carbide tools in use at this time, should require only very little longer time than required for similar high-speed-steel tools. The average time required to grind such a tool at the plant of the Krupp Works, where thousands of these tools are ground daily, is less than 3 min., and this is accounted for by the fact that large wheels are used, with ample power, and wet grinding. The wet grinding permits much faster grinding, and the use of a large roughing wheel for taking down to size, with only a light touching up of the edge on the finishing wheel, is also a factor in attaining the desired results.

The grinding of the average cemented-carbide tool is very simple if a little study is given to proper methods and equipment.

Author's Closure

Mr. Huson's comments are very timely. Cemented carbides have improved and are likely to continue to do so. Heavier machine tools may not give greater possibilities, but who can say so definitely, as the machine development for these materials has just started? Very high speeds are now used successfully with light feeds. If the latter be increased, the amount of metal removed in a given time will increase tremendously. More heat, of course, will be generated, tending to limit the production. On the other hand, tantalum carbide shows the peculiar property of conducting heat away from the cutting edge very rapidly, and this tendency may lead to remarkable advances in the near future.

Mr. Prosser hits the nail on the head when he says that grinding is a serious problem with the carbides. This is the reason that so many questions on the subject were included in the questionnaire. It is true that the times reported in many cases are too high. With further experience will come improved methods, but the important points to note are that more care is being taken in grinding carbide tools and that machine grinding is being more generally adopted.

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Development of Wood Adhesives and Gluing Technique

By T. R. Truax,[1] Madison, Wis.

Study of adhesives used in furniture and other relics of earlier civilizations yields little of value to present-day craftsmanship. This paper therefore discusses only the more recent developments in glues and the technique of using them. While some glues were of ancient origin, the modern large-scale production and the more precise controlled methods have resulted in glues of higher and more uniform quality than the ancient glues from the same sources. Standardization of tests and testing methods is important. Joint tests constitute the principal means for determining the suitability of glues for woodworking. Tests that can be made directly upon the glues rather than upon glued joints may be developed.

Furniture and other relics of earlier civilizations attest the gluing of wood (9)[2] as an art of long standing, but they furnish little authentic information that is of practical value to the craftsman of today as to kinds and quality of adhesives used or of methods or processes. Because of mechanical and chemical progress, it is doubtful that even reliable and complete historical records would add much of practical value to present-day gluing operations. Consequently, this paper will discuss only the more recent developments in glues and in the technique of using them.

Development of Glues

It is a common assumption that the joints in chests and other articles recovered in recent times from ancient tombs were glued with adhesives that were superior to modern glues. The fact that the joints have endured for many centuries is, however, no positive evidence that the adhesives were superior to present glues. On the other hand, the fact that the wood in these particular articles was not damaged leads us to believe that the surrounding conditions were not extremely severe. In other articles recovered there has been more or less damage, presumably to both glue and wood. Adhesives are available today that might reasonably be expected to withstand permanent exposure to conditions where wood is not deteriorated.

Animal and fish glues were undoubtedly of ancient origin and may have been used by the craftsmen of 20 or 30 centuries ago. Certainly they were the best known and most extensively used a few centuries ago. Blood-albumin glues are reputed by some to have had an early origin and perhaps casein glues were also known to the ancients. Large-scale production of glues under efficient and refined methods, however, is a development of modern times. Manufacturing under more precise, controlled methods has resulted in glues that are probably of higher and more uniform quality than the ancient glues from the same sources. Coincident with better manufacture, improvements have occurred in the testing and grading of adhesives and better technique has been developed in their use.

In the United States it has been only within the last 20 years that animal and fish glues have had serious competitors in the woodworking field. Starch glues, suitable for wood joints, were developed about 1912, and during the World War casein glues came into use on a commercial scale in the woodworking trade. Both starch and casein glues have grown in importance during intervening years. Vegetable protein glues were developed commercially subsequent to the World War, and they, too, have grown rapidly in importance. Though the protein glues have a vegetable source, they are more analogous to the casein glues than to the starch glues in properties. Animal, casein, starch, and vegetable protein glues have been quite fully discussed elsewhere; (13)[3] hence only adhesives that have not yet come into general use in woodworking will be discussed here.

Miscellaneous Adhesives

There are many kinds of adhesives, such as blood-albumin, phenol-condensation products, rubber cements, cellulose esters, silicate of soda, waxes, asphalts, and gums, which have been used to some extent or proposed as glues for wood. Blood glue has been used considerably, and others may eventually become important. So far as is known no extensive tests ever have been made on most of the foregoing miscellaneous adhesives. Samples of many of them, however, have been tested at the Forest Products Laboratory in order to obtain some idea of their value and suitability for making joints in wood. The results of these preliminary tests indicate that some have outstanding properties.

Blood-albumin glues are the best known of the miscellaneous group. They have been used more extensively in European and Asiatic countries than in the United States, but their use appears to be diminishing rather than increasing. They have been used principally in plywood, but casein glues at present appear to be taking their place in part, both here and abroad. High water resistance is the most valuable property of blood-albumin glues; the necessity for hot-pressing is the chief disadvantage. Because the cheaper cold-press method of gluing prevails in the United States, the blood-albumin glues have been used here only in a few plants. Attempts to produce a cold-press blood-albumin glue have met with only partial success.[4]

Phenol-Aldehyde Adhesives. By heating a phenol with an aldehyde it is possible to produce hard, strong adhesive substances that appear to be unaffected by moisture in any form. Adhesives of this type that have been produced commercially have yielded joints in wood of high strength when tested both dry and wet. Phenol-aldehyde adhesives are apparently more water

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[2] Numbers in parentheses apply to references cited at the end of the paper.

Presented at the Sixth Annual Wood-Industries Meeting, Winston-Salem, N. C., Oct. 15 and 16, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] See also papers on these four classes of glues presented at the Sixth Annual Meeting of the Wood Industries Division of The American Society of Mechanical Engineers.

[4] A blood-albumin glue, which can be cold pressed and which is recommended only for light-weight woods, has been developed at the Forest Products Laboratory. See U. S. Department of Agriculture Technical Bulletin No. 205, page 54.

resistant than any of the glues now in common use. They are, however, several times as costly as the common woodworking glues, and the necessity for hot-pressing in order to produce the characteristic bakelite reaction is a disadvantage particularly under prevailing methods of manufacture in American factories. Unless the phenol-aldehyde adhesives can be produced more cheaply, they will be restricted to special uses. Tests made in Germany indicated that similar adhesives of this class (8) became brittle in wood joints when dry, but this characteristic was not found to be objectionable in the tests at the Forest Products Laboratory.

Adhesives From Rubber. Joints can also be made in wood by hot-pressing with adhesives made from rubber. Two commercial products of this type have been tested at the Forest Products Laboratory. The joints were strong both dry and wet, but the adhesives are expensive. They may find limited uses for special purposes.

Cellulose Esters. Cellulose cements have been suggested at various times as adhesives for wood, and tests have been made on several products. Most of them are too thin and require more than one application to make good joints in wood. When so used, they are slow drying, and they are also relatively expensive. It is reported that joints of good strength and high water resistance have been made with them in Germany. (8) Cellulose cements can be cold-pressed, but their cost and difficulty of use appear to make them impractical at present for general woodworking purposes.

Silicate of Soda. Silicate of soda makes joints in wood of fairly high dry strength. It is cheap, easy to apply, and finds a large use in fiber containers, but the joints are weak when wet. Silicate of soda changes in properties with age and is therefore not considered a permanent binder.

Waxes, Gums, and Asphalts. Waxes, gums, and asphalts have been proposed as adhesives for wood. So far none of them has proved to be satisfactory.

Dry Glue Films

Attempts have been made from time to time to produce a dry film of glue that could be used in place of a liquid adhesive. The advantages of such a process are obvious, especially in laying thin veneer, but as yet a material of this kind has not come into commercial production in the United States. Some work was done in 1918 at the Forest Products Laboratory on a dry-glue process, and a film was produced experimentally using blood albumin as a base. (1, 11) So far as is known, this material is not being produced commercially. Recently published accounts of tests in Germany indicate that attempts are being made in the same direction abroad. (8) Films made from cresol-formaldehyde and from cellulose esters are reported to give good results. The dry-glue films, so far produced, require the use of a hot press. No information is available as to cost, but the dry films of these materials are undoubtedly more expensive than common glue mixtures.

Testing Glues

Progress and development in the manufacture of glue are necessarily related to its testing. Some means of measuring quality is essential to determine the value of new products and the improvement of old ones. Not only is glue testing necessary to the manufacturer, but it is also important to the user. It is his means of determining whether he gets value received for his money, and, still more important, it helps to insure the value and quality of his own products. A table, a chair, a door, or an airplane wing beam is no better than the joints it contains.

Manufacturers commonly have their own methods of checking the quality of their raw materials and their manufactured glues. Consumers and others, likewise, use one form or another of test in an effort to check the quality of glues and to determine their suitability for a particular job. This has led to many forms and kinds of tests, all of which are aimed at essentially the same purpose. Standardization of tests and testing methods is therefore important to a common understanding. A considerable amount of work has been done upon standardization, but much remains yet to be done.

The testing of animal glues has reached a higher degree of perfection than the testing of other types of adhesives. A number of workers have investigated the physical and chemical properties of animal glues and methods of measuring them, and, as a result, clearly defined and reasonably accurate tests of the glues themselves are available. The National Association of Glue Manufacturers, comprising a considerable number of animal-glue makers, has adopted uniform testing methods (4), and the U. S. Federal Specifications Board has recently incorporated these methods in a specification for Government use in the purchase of animal glue for woodworking. (6) In Germany a committee on glue testing of the (German) Society for Testing Materials has recently carried out investigations with the object of setting up a uniform procedure in that country for testing animal glues. (7) Investigators in Russia and other countries are also studying the testing of animal glue.

There is general agreement among research workers that viscosity and jelly strength are the most reliable criteria for judging the quality of an animal glue, and one or the other or both are usually the basis for grading. In Germany viscosity is regarded as the more reliable of the two properties (7), while in the United States both are considered essential for defining the grade of a glue. (4) Other characteristics commonly considered of some importance in an animal glue for woodworking are moisture content, reaction (acidity or alkalinity), tendency to foam, and odor or keeping quality. Joint tests in glued wood blocks have been a part of most all glue-testing investigations, but they are less accurate and less satisfactory for testing the quality of animal glues than are the physical properties of the glues themselves. (14)

There are at present no generally accepted physical and chemical tests for blood-albumin, casein, starch, and vegetable-protein glues. In the absence of more refined testing methods, the strength of joints of all of these adhesives and the water resistance of joints of blood-albumin, casein, and vegetable-protein glues are the properties most often tested. Joint tests, therefore, constitute the principal means thus far available for determining the suitability of these glues for woodworking.

Forest Products Laboratory Joint Tests. The Forest Products Laboratory during the World War developed two types of joint tests for aircraft glues. (1) The tests were later used on starch and other kinds of adhesives. They are known as the block-shear and plywood-shear tests and have been described in several publications. (13) The block-shear test is adapted to thick pieces and the plywood-shear test to veneer. Both tests have since been used in the same or slightly modified forms in several other countries. (2, 5, 9, 10, 15) Both tests are simple, and with proper equipment and care can be quickly made with a moderate degree of accuracy.

All forms of joint tests involve a gluing operation, and this may have a most important effect upon the results. Poor gluing may result in weak joints with high-quality adhesives, while careful gluing may result in fairly strong joints with medium- to low-quality glues. In addition to the gluing operation, the variable properties of the wood are factors that may lead to erroneous conclusions unless careful selection of material is made. With proper care in selecting and making joints, however, the block and plywood tests will show fairly consistent differences between weak and strong glues and will enable the selection of a glue or glues satisfactory for a particular job. Other tests on blood-albumin, casein, starch, or vegetable glues, such as the viscosity and working life of the mixed glue, may give supplementary information of much value. It is to be hoped, however, that in the course of time complete tests will be developed for each of these glues that can be made directly upon the glue rather than upon glued joints.

Developments in Gluing Technique

The third phase of the development of glues involves the details of their use and the technique of joint making. Only the principles of handling glues and making joints that have undergone important changes will be described here.

Preparation of Glue for Use. In preparing glue for use the necessity of producing a glue mixture of uniform and proper consistency, in which the raw materials are completely dissolved or dispersed is of primary importance. The animal glue must be all softened and melted, the starch all "converted," the casein, vegetable protein, or albumin all dissolved in order to produce a uniform mixture. It is also important that the mixing or conversion be done without the inclusion of large amounts of air. Outside the character of the glue itself, the speed of stirring and the type of mixer govern the amount of air, or foam, included. Suitable types of equipment are available for preparing the different kinds of glues, and with reasonable care in the operation good glue mixes can readily be prepared.

An old tradition has been that an animal glue must be boiled to prepare it for use. Furthermore, it has been common practice to keep prepared animal glue boiling or near the boiling point during use. The injury to the glue and the wastefulness of such practice have been clearly shown by many investigators. (13) It is unnecessary to go any further into this subject than to point out that the most desirable temperature conditions are known and that commercial practice has been largely changed to conform to improved practice. Equipment manufacturers have been very helpful in bringing about this change by providing glue converters and pots in which the temperature is easily or automatically controlled.

Temperature control is somewhat less imperative for blood-albumin, casein, starch, and vegetable glues than for animal glue, but is nevertheless important. The quality of a starch glue and the working life of casein and vegetable-protein glues depend to some extent upon temperature. Manufacturers of starch adhesives usually recommend the temperature conditions most satisfactory in mixing, and temperature recommendations are also made by some manufacturers of casein and vegetable-protein glues. Heating blood-albumin glues and certain other adhesives during preparation above their coagulating temperatures alters their adhesive properties.

Spreading. To make a satisfactory joint it is important to spread an adequate amount of glue evenly and quickly over the surfaces to be joined. Machine spreading is quicker, more uniform, cheaper, and more economical of glue than hand spreading and is recommended on work adapted to it. Speed is essential in many operations in order to get uniform results. It is important to spread the right amount of glue—that is, a sufficient quantity to make a strong joint, but not enough to add excessive water to the glued members. The required amount of glue to spread has been worked out experimentally (13) and is fairly well established in commercial practice. Spreading both sides of a joint uses more glue than single spreading and is ordinarily not justified.

An important development in gluing technique has to do with the consistency of the glue mixture at the time of pressing. Here again, with respect to animal glue, an old tradition has been inherited to the effect that the glue mixture must be relatively thin when pressed, that the wood must be hot, and that the solution must penetrate the cells to make a strong joint. Gluing methods and practices for several decades and perhaps centuries had been built upon this principle. It is not surprising, therefore, that the results of investigations showing that glue need not penetrate deeply into the wood cells, that relatively thick mixtures give better results than thin mixtures, and that strong joints can be made with animal glue on unheated wood should be regarded as revolutionary and accepted very slowly by the craftsmen. Although these principles have been checked and verified many times in commercial practice, there are still some who regard them with suspicion.

The Forest Products Laboratory undertook, somewhat more than a decade ago, a detailed study of the factors that affect the strength of a glued joint. Two general principles were formulated as a result of these investigations: First, that glue adheres tenaciously to wood substance and that mechanical adhesion, such as exhibited by the holding power of nails, is of secondary importance; second, that the making of strong joints involves a close correlation of the amount of pressure applied and the consistency (viscosity) of the glue mixture at the time of pressing. (3, 12, 13) Since the publication of the results of these studies, similar investigations have been undertaken in other countries, including Germany, England, and Russia, and the results recently appearing in foreign publications indicate that our conclusions are being verified by the workers of other countries. (2, 8, 10, 15)

Working out the interrelation of factors for different gluing operations and different species of wood required the making of many thousands of tests and several years of work. The kind of glue and proportion of water in the mixture, the quantity of glue spread, the temperature of the glue and the room and the wood, the lapse of time between spreading and pressing, the amount of pressure, the length of the pressure period, and the species of wood were all investigated. The interrelation of these factors is complex, each depending upon the others, resulting in a large number of combinations of conditions that give good results and a like number that give poor results. However, from these investigations it is now possible to set up recommendations for gluing different species of wood with different kinds of glue that are applicable to different commercial gluing operations. (13)

In general the most satisfactory joints are made by the use of relatively thick mixtures of glue and high gluing pressures. These two conditions are necessary to bring the contact surfaces of wood together, to force the air and excess glue from the joint, to spread the glue uniformly throughout the joint in a thin continuous film, to force the glue into intimate contact with the wood substance, but not excessively into the cell cavities, and to hold the joint in correct position while the glue sets. Excessive warming of the wood keeps an animal glue too thin for immediate pressing and favors starved joints, a condition in which there is not a continuous film of glue in the joint.

In quick clamping, such as in edge-joint gluing and certain assembly operations, the glue must be mixed relatively thick or, as in the case of animal glue, permitted to come to the proper consistency by cooling. Wood at 70 deg. fahr. normally requires no further warming in operations where the pressure is applied within one minute after spreading. Gluing pressures of 100 to 200 lb. per sq. in. of joint are best; the lower pressures for lighter woods and the higher pressures for heavy woods. These recommended pressures are higher than those that still generally prevail in many operations.

Time-saving machinery and devices also have brought about important changes in the gluing room, as related to the technique of gluing. Mechanical spreaders, rotary clamp carriers, and hydraulic presses have had a marked effect on the time element in gluing—a most important factor in joint production. They have tended toward higher gluing pressures, have made it inadvisable in many operations to warm the wood, have brought about shorter clamping periods and the need for quick-setting glues, and have made it necessary to mix glues relatively thick. These developments have generally contributed to a more uniform and higher standard in glue joints, but to take full advantage of them the gluing operator must make certain that he controls the details of the operation in harmony with the changed conditions.

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Merits and Applications of Animal Glue to Veneer Work

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The growing use of crotch, burl, and stump veneers has added to the technical difficulties of fine-furniture manufacturing. Animal glue, for manufacturing fancy veneered work, has properties commending it to the plywood maker, such as (1) its fluidity, giving a moderately large spread in highly concentrated solution; (2) its neutral reaction and chemical inertness, which make it incapable of staining delicate face veneers; and (3) its quick-setting and fast-drying tendencies, which permit prompt pressing. Modern processes of sizing and gluing are described. The veneering operation is analyzed from the standpoint of (1) the large area of wood surface to be covered with glue, (2) the large proportionate amount of moisture absorbed by the wood from the glue, (3) the short distance from the glue line to the surface to be finished, (4) the comparatively long assembly time, (5) the emphasis on beauty of appearance in the fabricated article, (6) the need for speed of production, and (7) the importance of quick and proper drying. The conclusion is reached that animal glue has kept pace with the industry and is well qualified for the highly decorative, if somewhat difficult, veneering processes of today.

MANUFACTURE of veneered work makes rigorous demands on the adhesive used. The glue requirements of veneering are more exacting than those of heavy joint work. Even in such relatively difficult branches of heavy joint work as chair manufacture and automobile-body building the two chief qualities sought in glue are strength and durability. The plywood maker, however, is interested not only in these, but in many other properties as well. The so-called lower-grade glues are used more often in veneering than in heavy joint work, but the adhesive requirements of veneering are the more difficult to fulfil.

Some people have felt this so keenly that they have expressed the opinion that the ideal veneer glue has not yet been invented, and perhaps they are right. What they probably have in mind is that a glue, in order to be perfectly satisfactory for veneering, ought to be strong, elastic, fluid, moderately quick setting, fast drying, water resistant, non-staining, foam free, easy to prepare, lump free, non-injurious to cutting tools, unaffected by heat, reasonably priced, usable at a very low water-glue ratio, chemically inert, thoroughly reliable, and absolutely uniform as to quality. Considered by this list of requirements, animal glue makes a surprisingly good showing.

The principal technical differences between veneering and heavy joint work are (1) the larger area of wood surface to be glued; (2) the larger proportionate amount of water absorbed by the wood from the glue; (3) the shorter distance from the glue line to the surface which is to be finished; (4) the generally longer assembly time; (5) the greater emphasis on beauty of appearance in the fabricated article; (6) the need for speed of production; and (7) the importance of quick and proper drying. Let some of these points be considered further.

1 LARGE AREAS TO BE COVERED

The area of wood surface to be covered with glue is likely to be but a few square inches per unit in heavy joint work, whereas it may easily run up to 25 or 50 sq. ft. per unit in veneering. Hence there is need for mechanical equipment, such as glue spreaders and hydraulic presses, that the work may be handled as expeditiously as possible. The glue is practically always applied by corrugated roll spreaders, the contents of which are kept warm when animal glue is the adhesive.

There are many advantages in having a glue which is perfectly fluid. Veneer sheets are often slightly wrinkled and wavy. This is particularly true of crotch, burl, and stump veneers, which are almost always thin and therefore too fragile to be flattened forcibly between the rollers. Since the spreading must be accomplished in haste, there is no time to correct the inequalities of the machine spreading by hand. Therefore a glue that will flow a little and by its very fluidity cover up the dry spots on the sheet is less likely to encourage the formation of blisters and loose edges than a glue which is so thick that it stays exactly where it is put. Furthermore, sliced and rotary-cut veneers have an "open" side—that is to say, a side on which there are innumerable tiny checks, caused by the bending of the sheet away from the cutting knife. Since this "open" side is usually laid toward the glue, it is desirable to have a solution that is thin enough to enter these cracks and hold the face firmly to the crossband or core.

A degree of fluidity is desirable in the press, too, for the glue there is subject to redistribution under the influence of pressure. Certain types of glue, if they are too stiff at this point, may crumble rather than "give," and thus lead to the production of areas of poor adhesion.

Mechanical spreaders have a tendency to whip air into glue if it is a potential foam former. The causes of foaminess are not well understood, but fortunately the cure is. Certain sulphonated oils and greases and other agents are effective in reducing the foaminess of animal glue to a point at which it is quite negligible. No well-manufactured animal glue will give any trouble on this score.

On account of the large areas of wood to be covered, the cost of the glue is a factor. When veneers are laid with animal glue, the so-called lower grades are generally used. These are not only moderately priced, but are desirable for other reasons which will be explained later.

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The lower grades of animal glue are made from the same kind of stock as the higher grades. Sometimes they are simply the later runnings from stock that has yielded the very choicest glues. The processing which they receive results in a lowered viscosity, a lowered jelly strength, and a greater fluidity, which in turn makes it possible to use them at a very high concentration—a great advantage in veneering, where it is essential that the minimum amount of water be transmitted to the veneer. The processing by which the so-called low-grade animal glues are made does not affect their potential adhesive strength to the same extent that it affects their fluidity. In fact, any reduction in adhesive strength that may have been brought about by the processing can safely be ignored in veneering, because the shearing strength of even the lowest grades exceeds that of the wood by a comfortable margin.

High-grade animal glue has the highest tensile strength. The strength of the low grades, though not quite as great, is such that strength values of 3500 lb. to the sq. in. have repeatedly been obtained in the standard shear-block test whenever the wood was able to stand so high a load. It is hardly ever possible to go higher than this, because the shearing strength of wood is the limiting factor in the test, just as it is in use.

The National Association of Glue Manufacturers has in recent years devised a system of evaluation whereby hide and bone glues alike are graded according to their viscosity and jelly strength.[2] Thus the purchaser who specifies some particular grade of animal glue is assured of getting an adhesive uniform in properties and constant in effect. With a graded animal glue it is unnecessary for the veneer manufacturer to experiment with every new lot of glue to determine its water-taking requirement. The uniformity of graded animal glue banishes the danger of too much water on the one hand or of too stiff a glue on the other hand.

2 Absorption of Water by Veneers

The amount of water added to wood in the course of the gluing process varies. Although it is usually less than 1 per cent in heavy joint work, it may rise to 30 or 40 per cent in plywood manufacture. Veneers are sheets of wood which seldom exceed $^1/_4$ in. in thickness. In fabricating them it is customary to coat at least one of their two surfaces with glue. In the case of crossbands and cores both surfaces are generally spread with glue.

It is considered good practice to redry veneers just prior to laying them, and this of course tends to keep the final moisture content low. A moisture content of about 5 per cent is the one sought in the redrying treatment when the work is most exacting. It is not advisable, however, to dry fancy veneers lower than this on account of their fragility.

Because most of the veneers used in furniture manufacturing today are quite thin, the weight of glue relative to the weight of wood is exceedingly high. Since the glues most commonly used in veneering are solutions varying in concentration between 28.5 and 40 per cent, it can readily be seen that water is their major ingredient. Consequently, when veneers are laid, a large amount of water is enclosed in the panel. Most of this water must make its escape by migration through the wood. In doing so it causes swelling, which is the root of nearly all evil in plywood manufacture.

The amount of moisture added to wood in gluing can be estimated by a simple calculation, the formula being as follows:

$$M = \frac{12(n-1)}{62.3} \times \frac{r}{s} \times \frac{1}{n_a t_a d_a + n_b t_b d_b + n_c t_c d_c} \quad \ldots\ [1]$$

where

M = per cent of moisture added

n = total number of plies

r = number of pounds of water per pound of dry glue in the solution used

s = spread, in square feet of single glue line per pound of dry glue

n_a, n_b, n_c = number of plies of species a, b, and c

t_a, t_b, t_c = ply thickness, in inches

d_a, d_b, d_c = specific gravity[3] of species a, b, and c.

When the panel is homogeneous (that is, consists of plies of only one species and one thickness), formula [1] reduces to:

$$M = \frac{12(n-1)}{62.3\,n} \times \frac{r}{s} \times \frac{1}{dt} \quad \ldots\ldots\ldots\ldots\ [2]$$

Given a certain type of panel to make, it is obvious that the thickness, the specific gravity, and the number of plies are predetermined. The only chance of controlling the amount of moisture added lies in the factor r/s. Inspection of formulas (1) and (2) shows that the added moisture varies directly as r and inversely as s. In keeping this factor as low as possible lies the hope of obtaining minimal swelling, which is the aim of every good worker in veneers.

These quantities cannot be varied entirely independently of each other. The nature of the glue itself sets limits to the changes which can be effected in the factor r/s. That is to say, if the water-glue ratio is materially reduced, the glue becomes heavy and tends to pile up on the sheet so that it cannot be spread as far as before. Thus r and s decrease at the same time and the factor r/s does not decline to a minimum. On the other hand, when the denominator s is increased to make the fraction r/s small, r has to be increased in order to give the glue the necessary fluidity. A highly fluid glue will permit a large spread at a low water-glue ratio, thus giving rise to minimal moisture change and minimal swelling. But the reverse is true of a non-fluid glue, for it will tend to give a moderate spread, even at a high water-glue ratio.

Animal-glue solutions by reason of their being used at an elevated temperature are the most fluid of all the common veneer glues. Hence it is possible to use them at an exceedingly low water-glue ratio without loss of spreading power. From this standpoint alone animal glues are worthy of consideration by those alive to the danger of face checks and the other consequences of a high moisture content.

Sometimes ill-advised attempts are made to decrease the amount of water-absorption by making use of a very scanty spread. These attempts, undertaken to further the interests of economy, often have the contrary effect, because thin spreads of glue dry rapidly, may become non-adhesive in spots, and result in the formation of blisters, which lead to costly repairs, if not to rejections.

Furthermore, when certain other types of glue are made up for scanty spreading, they are prepared at comparatively high dilution. Thus it happens that when the glue solution is thin enough to cover a large area it is also thin enough to strike through the face veneer and cause staining. It is advised, therefore, to have a moderate spread of glue and a fairly concentrated, though fluid, solution, both of which are obtainable in animal glue.

Very briefly, the newer method of cementing fancy veneers is as follows: The cores are prepared in the usual manner, with of course careful drying before planing in order to avoid sunken joints. Then the cores are crossbanded on both sides, taking the precaution to have the "closed" or "right" side of the crossband uppermost. A moderately long assembly time may be allotted to this operation, but the crossbands should be in prime condition and the covered cores well dried before proceeding.

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[2] *Industrial and Engineering Chemistry*, vol. 16, no. 3, p. 310, 1924.

[3] In the absence of exact data on the specific gravity of the wood at its moisture content at the time of gluing, the specific gravity (oven dry, based on volume when air dry), which is given in U. S. Department of Agriculture Bulletin No. 556, table 2, can be used.

The next step is carried out according to either of two preferred techniques. In one of them the face veneer is glue-sized by dipping it in a dilute animal glue solution or else by double-spreading it with the same. After the dipping process any unabsorbed solution on the surface is removed by wiping or passing the veneer through a sort of wringer. Then the faces which have been sized are dried in racks and finally between warm, flat basswood boards, or else between spare sheets of basswood or other suitable veneer. When the glue-sized faces are down to 5 per cent moisture content, they are ready for attachment to the covered cores.

An alternative method of strengthening the faces and sealing them against glue penetration is to back them with a thin sheet of some suitable veneer, which is glued to the face parallelwise. For backing fancy faces in this way, animal glue is preferred because of its fluidity and because it is non-staining.

The final step in the making of the panels consists in attaching the glue-sized or veneer-backed faces to the crossbanded cores. For this purpose a fairly concentrated animal glue solution is best. The glue is spread on the crossbands, and then when the glue has chilled slightly or has become "tacky," the faces are laid on and the whole panel pressed between warm caul boards. The panel backs, of course, are laid at the same time; but when veneer-backed faces are used, it has been found possible to leave off the conventional panel back. This is done for reasons of economy; and although it violates the time-honored principles of balanced plywood construction, some firms have found it not to be a cause of warping when properly performed.

3 Effect of Thin Faces

The distance from the glue line to the surface which is to be finished is very small, being of course the thickness of the face veneer. Face thicknesses are rarely more than $^1/_{16}$ in. or less than $^1/_{40}$ in. It is extremely difficult to prevent glue solutions from coming through the thin faces which modern furniture styles have decreed by reason of their insistence on the use of highly figured wood surfaces.

The texture of the wood has an influence, in that the more porous species allow glue solutions to seep through to the surface with greater facility than the more compact species. In veneers thinner than $^1/_{20}$ in. there is always danger of some seepage. The difficulties arising from the seepage of glue through face veneers are staining, clogged surface pores, and the sticking of paper and wooden cauls.

Sizing the faces with animal glue serves to lessen the trouble from this source. Glue-sizing is particularly effective in the case of fancy veneers, because the grain in them does not run in orderly fashion, as it does in many plain veneers, but wanders about in every conceivable direction. Oftentimes there is a certain amount of end grain in fancy veneers. This is likely to offer an unobstructed passage for the glue from one side of the sheet to the other.

Glue size is a 1-to-8 or a 1-to-10 solution of animal glue. Applied either by dipping or by roller-spreading, it impregnates the sheets thoroughly, strengthens them so that they will be less fragile, and closes the pores, sealing them against subsequent penetration by whatever glue may be used in cementing the panel. Indeed, sizing sets up a bar against moisture as well as against glue and prevents them both from reaching the surface of the panel.

Wood is subject to staining by certain glues. A glue does not need to be fluid in order to stain, since it is not absolutely essential that the glue gets to the surface in order to discolor it. In some instances the glue remains in the joint approximately where it was put, but the moisture strikes through to the surface, carrying with it chemicals extracted from the glue. If these chemicals are alkaline, they are almost certain to discolor the surface. Some species of wood are quite seriously affected by alkalis. Therefore if the veneer faces are thin, cross-grained, and easily marred, it is advisable to choose animal glue for the adhesive, because animal glue is practically neutral in reaction and inert chemically.

Animal glue solutions, used at about 140 deg. fahr., have an appreciable vapor tension. Consequently they lose water and increase steadily in concentration. A glue that was, say, a $33^1/_3$ per cent solution to begin with will become somewhat more concentrated by the time it is spread on the panel. There it continues to lose moisture both to the air and to the wood. As it rises in concentration its chilling point rises. In consequence of this it may gelatinize very soon after spreading. Its initial warmth enables it to penetrate and secure a foothold; its increasing concentration as it loses moisture to the wood enables it to solidify before reaching the surface of the panel. Cold glues, on the other hand, remain more nearly constant in concentration because their evaporation loss is negligible in comparison. They gelatinize less promptly, since with them setting occurs primarily as the result of drying. With animal glues, however, chilling checks penetration, thereby lessening the danger of plugging the surface pores of the veneer.

4 Assembly Time

Speed is one of the distinguishing marks of gluing operations in thick lumber. Plywood is handled in large sheets and pressed in bundles. Naturally the time interval is much longer. As was pointed out in section 2, considerable amounts of moisture are added to plywood in the course of spreading and considerable swelling results. This swelling does not occur all at once. A piece of glue-coated veneer will swell at a gradually decreasing rate for more than an hour if not confined. Putting it under pressure, though, will reduce both the rate and the total amount of its expansion. Since the swelling during the gluing process accounts for checking during the drying process, it is evident that fine plywood should be made in the briefest possible assembly time. Assembly time of 20 min. is not uncommon in making ordinary plywood, but it would be well to reduce this to 10 min. or less in making fine plywood. One firm strives to get the panels under pressure within 4 min. after spreading. This means of course that the panels are pressed in small bundles, and there is a great advantage in so doing.

It has often been noticed that when severe checking occurred it was the panels on the bottom of the press that suffered the most. Such checking is correctly attributed to an overlong assembly time for the first panels in the lot, since they obviously were absorbing water and expanding while the later panels were being spread. High bundles are apt to cause checking, especially if they prolong the assembly time of any of the units in them. Sometimes the bundle size is limited to 12 panels for the sake of rapidity and minimal expansion.

Long assembly times (that is to say, long delays before going to press) operate to the disadvantage of the veneer maker, no matter what the glue. There are four types of defect which they tend to cause: namely, face checks, open veneer joints, blisters, and dislocation of the set glue, with consequent weakness. Animal glue is not immune to the difficulties resulting from overlong assembly times, but the knowledge that animal glue will set soon probably serves to put the veneering crew on their mettle and furnishes an incentive to get the panels under pressure before the glue has become too stiff.

It often happens, however, that the spread glue solidifies before the panels are pressed. This has led to the use of hot cauls. The misuse of hot cauls has occasionally made animal glue the object of unmerited criticism. The glue in the panel is apt to be a jelly at the time of pressing. Heat from the cauls reliquefies the glue and makes it subject once again to evaporation, extrusion, and seepage. If the heat is too intense, these processes will go too far. Cases have been known where cauls that were too hot drove the glue out of the joint in spots or dried it prematurely, giving rise to blisters.

The function of the hot caul is simply to melt the glue so that it will make contact with the dry face ply. The glue will probably be fluid enough for this at 100 deg. or thereabouts. Undoubtedly there is some loss of heat in transmission through the face ply. The thicker the face, the greater the heat loss. The caul temperature ought to be dependent on the face thickness, being lower for thin faces than for thick ones. If provision is made for this, and the further precaution is taken to heat the cauls to a measured temperature (and a low one), hot cauls should be a help to the veneer maker and not a hazard.

5 Proper Surfaces for Finishing

The primary purpose of veneers is to enhance the beauty of the object to which they are added. The increasing use of rare and highly figured woods and advances in the art of finishing are weather vanes indicating the tremendous importance which attaches nowadays to fineness of appearance. There is no object in facing an article with a decorative layer of wood if blemishes are left as a result of the veneering process. Faulty methods of construction result in marred surfaces because of sunken corestock joints; ripples due to poor planing of the core; checked crossbanding, resulting from the use of moist, poorly cut, or wrong-sided veneer; defective cauls; moist faces; bad taping; lapped joints, due to long assembly time, resulting in expansion in the press; blisters; wrinkles; clouded finishes, on account of glue penetration; uncemented edges, etc. In addition to these there are defacements that are due primarily to the glue itself. These have been touched upon in the preceding sections. They are attributable, mostly, to the presence of a highly alkaline glue, although the danger and the damage can be lessened materially by measures designed to keep the glue extractives from reaching the surface of the sheet. But in order to insure the product against any chance of discoloration from this source it is best to use a strictly non-alkaline binder, of which animal glue is the outstanding example. The old-fashioned cabinetmakers used to demand an opaque glue (that is, an animal glue containing whiting, zinc oxide, or the like) in order to get an invisible glueline in their joints. How much more important, then, it is for their modern successors to place in proximity to the surface of the still more beautiful products of today a glue that is all but invisible and entirely without injurious effect.

6 Production Rate

Since veneering represents additional handling, labor costs mount, and there is need for a high rate of production in the plywood department. The greatest speed consistent with sound workmanship should be the goal of every maker of veneered furniture. Adequate mechanical equipment, planned operations, and a quick-setting, fast-drying glue are the driving wheels of the efficient veneer room. A glue that sets by virtue of cooling from 140 deg. fahr. to room temperature is bound to be swift in action. It grips whatever surfaces it may be in contact with when it chills and holds them with an ever-increasing firmness until dry, when its binding force exceeds the strength of the wood itself.

7 Panel Drying

The selection of an adhesive that will stand drying is advisable, because the use of panel kilns is becoming more and more common in the industry. Animal-glue panels come from the press with the lowest moisture content. The water-glue ratio of the solution as prepared is low to begin with; it decreases steadily by reason of evaporation on the spreader and on the sheet. The use of hot cauls induces a gentle warming and drives out still further amounts of moisture. The glue at the time of leaving the press has a comparatively low moisture content and needs but little drying. Therefore it can stand mild heating at once and the full temperature of the kiln in a few hours. Good animal glue is not harmed by dry heat. Glue in solution must of course not be heated above 140 deg., but dry animal glue can be heated to at least 200 deg., without injury.

Conclusion

The increasing use of fancy veneers and the growing appreciation of the fundamental principles of veneering construction have established the place of animal glue in the furniture factory. It is not a one-purpose glue, but a many-purpose glue, and as technical difficulties multiply, the many-sided properties of animal glue give it wider usefulness.

Although effective at any task of holding wood to wood, there are places where animal glue compares favorably with other materials which are low priced because of their abundance in nature; and there are branches of plywood manufacture where there is little need for as fine and versatile an adhesive as animal glue.

But the veneering of fine furniture demands a more delicate touch in coping with its gluing problems. Animal glue is perfectly adapted to the glue-sizing of crotch, burl, and stump veneers. Its chemical inertness and its freedom from staining effect, its perfect fluidity at the relatively low concentrations used in sizing, its speed of setting and drying, and its strengthening and moisture-excluding effects make it an ideal glue for the purpose.

Its fluidity at low moisture content, its quickness of setting and drying, its freedom from alkaline extractives and staining effect, its graded uniformity, and its dependability are the chief advantages which commend it to the manufacturer of fine veneered work. Oldest of the glues, it is as up to date as the newest of them.

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Vegetable Glues

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This paper gives essential information about vegetable glues and the gluing of wood with them, considering veneer glue, joint glue, and liquid glue. There have been recent developments in vegetable joint and liquid glues that are not generally known, and the paper covers these in a practical rather than in a technical manner, intending to be of assistance in commercial operations. Vegetable glue is any adhesive made from a base or substance of vegetable matter. The paper describes vegetable glue that has a carbohydrate as its base and is a practical vegetable glue for use in the woodworking industries.

PLYWOOD operations reveal numerous advantages to be derived from the use of vegetable glue. It can produce joints as strong as or stronger than those produced by other glues. Vegetable glue has a great spread, and as it is applied cold, the warming of stock and the heating of cauls are eliminated, and thereby the hot glue room necessary when certain other glues are used. There is no deterioration in the dry state nor after it is converted.

Vegetable glue is practically odorless. It is easily mixed, and then requires no watching to see that is is not too thick or too thin for use. Being a glue rather than a cement, it is not as hard on cutting tools as some adhesives.

Though water resistance is not claimed for vegetable glue, it is sufficiently moisture resistant for all practical purposes in the making of furniture, pianos, and other high-grade work.

Staining in glue joints and face veneers in panel work is frequently objectionable. There are vegetable glues which may be used to the complete elimination of any stain even in the most delicate cabinet-wood veneers.

Preparing Vegetable Glues

Most vegetable glues, as received by the buyer, appear in a dry powdered form resembling wheat flour. They are prepared by three different methods. First, by mixing the flour with cold water, stirring the mixture until free from lumps and of a uniform consistency, and then adding a predetermined amount of caustic soda in solution. The mixture is then heated while being constantly stirred. A second method is the combining of flour with water and then adding a predetermined amount of caustic soda. In this case a sufficient amount of caustic soda is used to heat and agglutinate the mixture to the desired consistency, and no artificial heat is employed. The third method is to combine the flour and water without caustic soda and to convert the glue by the action of external heat.

The third method does not make a glue capable of producing the highest quality joints and is not much used. The second method produces a glue high in alkaline properties with a strong tendency to stain and is mostly used in making box shooks, mirror backs, or similar work. It is used in high-grade plywood construction where the face veneers are thicker than commonly used in furniture and like work. The first method is most generally used and produces the most satisfactory joints.

The proportions of water and dry glue used in all three methods vary from $1^{1}/_{4}$ parts of water to 1 part by weight of dry glue to $4^{1}/_{2}$ parts of water to 1 part by weight of dry glue depending on the particular kind of glue and the work on which it is to be used. The amount of caustic soda commonly used is equal to 3 per cent by weight of the dry glue. There are, however, some special-purpose vegetable glues where the amount of caustic soda is varied depending on whether one is gluing plywood where no stains must show or fire-resistant wood or some other special work.

Using Vegetable Glues

The types of vegetable glues used in plywood construction are very viscous and stringy and so are not readily applied by hand or brush methods. Most plywood making is on reasonably large-scale production which makes mechanical spreading advantageous, and the vegetable glues are handled economically on roller spreaders. This is accounted for by the ease in regulating a light spread, and also by the fact that the glue may remain in the spreader indefinitely without spoiling in any way.

The liquid vegetable glues are intended for brushwork in general cabinet work and repairing, but they may be used in any gluing machine. The joint vegetable glues are intended for building up lumber joints and are used on plate and roll joint glue spreaders.

The making of a good joint requires the even and quick spread of an adequate amount of glue on the surfaces to be joined. Hand spreading with liquid vegetable glues is practical on any job, and particularly on small or irregular surfaces. The veneer and joint vegetable glues require machine spreading, which is the method used for applying most of the glues used.

Pressure is required to bring the surfaces of the joint into contact, to force excess glue and air from the joint, to force glue into the wood cells, to hold the joint in position while glue sets, and to prevent warping. Hand and power presses are used, but the best results are obtained where power presses, on which exact pressure is recorded, are used. The amount of pressure required to make satisfactory joints has been the subject of many debates. In practical work for all-around use in building panels, 100 lb. per sq in. has been found most satisfactory, but there are conditions in the use of different kinds of woods and construction where better results would be obtained by using more or less than 100 lb. pressure. Conditions surrounding the particular work involved determine the amount of pressure to be used, and the experienced operator knows what to do.

Uniform pressure is absolutely necessary, and to that end the pieces to be glued must be true, must fit accurately, and in plywood construction each piece must be of uniform thickness in all its parts. A thick caul should be used at the top and bottom of glued plywood. Before the introduction of vegetable glue it was customary to use a hot caul between each panel. With the use of vegetable glue the hot caul is eliminated and fewer cauls are used in the stack.

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[1] Sales Manager, Perkins Glue Company. Mr. Maine has been with this company for 17 years, the last 10 years of which he has served in his present position.

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The length of time that glue joints made with vegetable glue should be left under pressure depends on the work being done and on the type of glue used. For safety purposes when the regular veneer vegetable glues are used the panels are left under pressure overnight. Such a long time is seldom necessary, but was adopted as a standard precautionary time. Quick-setting glue makes it possible to remove stock from pressure in two hours, and in the case of joint glue even a shorter period is possible.

Preparation of Wood for Gluing

Too much emphasis cannot be laid upon the importance of the proper preparation of wood for gluing. Drying, conditioning, and machining may be more responsible for the successful building of glued-up stock than the glue used. These facts are assumed to be generally known. It is believed, however, that the question of moisture should be given consideration. Moisture changes in wood make stresses which favor warping, checking, and even separation of plies, and should therefore be avoided as far as possible.

The range of moisture content of dry wood in service is covered in "The Gluing of Wood,"[2] by T. R. Truax.[3]

The builder of panels having highly figured face veneers seriously considers the amount of water used in his glue as well as the spread and the quick-setting qualities of the glue used, because he desires above all things to avoid the reactionary stresses. Some figures on the amount of moisture absorbed by veneers serve to show the necessity of careful handling of all veneered work.

These calculations assume that all water in the glue is absorbed by the wood and that none evaporates into the atmosphere during assembly. The percentages of moisture added are based on the weight of air-dry wood.

Since the percentage of moisture added depends very considerably on the density of the wood, the calculations have been worked out for birch, which is an average dense wood weighing about 44 lb. per cu. ft., and for poplar, which is an average light wood weighing about 27 lb. per cu. ft. Both weights given represent the average for lumber in an air-dry condition.

Moisture absorption has been calculated for three glue spreads for each glue having a certain water ratio. The spreads used would correspond to a moderately light, a medium, and a moderately heavy application. The coverage for such spreads is given in gross square feet of single veneer covered by glue made from 1 pound of dry-glue base.

The spreads of vegetable glues are approximately proportional to the weight of the liquid glue made from one unit of glue base. If the spreader is set to spread a $2^1/_8$ to 1 glue at the rate of 45 sq. ft. per pound of glue base, and then without changing the spreader adjustment another glue having approximately the same consistency but with a different water ratio is put on the spreader, the spreads would be roughly as follows: For a 1.2 to 1 glue, 31.6 sq. ft.; 1.5 to 1, 36 sq. ft.; 2.12 to 1, 45 sq. ft.; 2.5 to 1, 50.5 sq. ft.; 3.5 to 1, 65 sq. ft.

Tables 1 to 5 show the moisture added by the glue. To obtain the total amount of moisture in the finished plywood the amount of moisture in the wood before gluing would have to be added.

Laying Linoleum

Linoleum tops, etc. are laid with vegetable glue as an adhesive in what is termed two operations. The first operation is the ordinary building of three-ply construction using the regular core lumber and cross-banding veneer. These cross-banded cores are handled through gluing, drying, and sanding operations like commercial three-ply work. The second operation consists in laying the linoleum face and a suitable back veneer. This is done in quantity production by running the cross-banded core through the spreader and building up a bale for pressure as usual. No cauls are used, the work being laid face to face and back to back. Pressure of 100 lb. per sq. in. is sufficient. When removed from pressure, the tops, or panels, are carefully piled with $^1/_8$-in. stickers between linoleum faces and $^3/_4$-in. stickers between backs. As the linoleum faces are impervious to moisture, the drying action must come through the edges and back. So it is best not to try to hasten the drying through use of a panel dry

TABLE 1 WATER RATIO 1.2 TO 1 (54.6% WATER, 45.4% SOLIDS)

Spread in sq. ft. per lb. of glue base...	29	33	37
	—Moisture added, %—		
Birch (3-ply $^1/_{32}$-in. veneer)	24.4	21.6	18.9
Birch (5-ply $^1/_{32}$-in. veneer)	29.5	26.0	22.7
Birch (3-ply $^1/_{16}$-in. veneer)	12.2	10.8	9.4
Birch (5-ply $^1/_{16}$-in. veneer)	14.7	13.0	11.3
Birch (3-ply $^1/_8$-in. veneer)	6.1	5.4	4.7
Birch (5-ply $^1/_8$-in. veneer)	7.3	6.5	5.7
Poplar (3-ply $^1/_{32}$-in. veneer)	35.3	31.1	26.9
Poplar (5-ply $^1/_{32}$-in. veneer)	42.2	37.2	32.2
Poplar (3-ply $^1/_{16}$-in. veneer)	17.6	15.5	13.4
Poplar (5-ply $^1/_{16}$-in. veneer)	21.1	18.6	16.1
Poplar (3-ply $^1/_8$-in. veneer)	8.8	7.7	6.7
Poplar (5-ply $^1/_8$-in. veneer)	10.5	9.3	8.0

TABLE 2 WATER RATIO 1.5 TO 1 (60% WATER, 40% SOLIDS)

Spread in sq. ft. per lb. of glue base...	33	37	41
	—Moisture added, %—		
Birch (3-ply $^1/_{32}$-in. veneer)	26.2	23.3	21.0
Birch (5-ply $^1/_{32}$-in. veneer)	31.5	28.0	25.2
Birch (3-ply $^1/_{16}$-in. veneer)	13.1	11.7	10.5
Birch (5-ply $^1/_{16}$-in. veneer)	15.8	14.0	12.6
Birch (3-ply $^1/_8$-in. veneer)	6.5	5.8	5.2
Birch (5-ply $^1/_8$-in. veneer)	7.4	7.0	6.3
Poplar (3-ply $^1/_{32}$-in. veneer)	42.6	37.9	34.2
Poplar (5-ply $^1/_{32}$-in. veneer)	51.2	45.5	40.8
Poplar (3-ply $^1/_{16}$-in. veneer)	21.3	18.9	17.1
Poplar (5-ply $^1/_{16}$-in. veneer)	25.6	22.7	20.4
Poplar (3-ply $^1/_8$-in. veneer)	10.6	9.4	8.6
Poplar (5-ply $^1/_8$-in. veneer)	12.8	11.3	10.2

TABLE 3 WATER RATIO 2.12 TO 1 (67.95% WATER, 32.05% SOLIDS)

Spread in sq. ft. per lb. of glue base...	40	45	50
	—Moisture added, %—		
Birch (3-ply $^1/_{32}$-in. veneer)	31.5	27.4	24.5
Birch (5-ply $^1/_{32}$-in. veneer)	37.8	32.9	29.3
Birch (3-ply $^1/_{16}$-in. veneer)	15.7	13.7	12.3
Birch (5-ply $^1/_{16}$-in. veneer)	18.9	16.5	14.7
Birch (3-ply $^1/_8$-in. veneer)	7.8	6.8	6.1
Birch (5-ply $^1/_8$-in. veneer)	9.4	8.2	7.3
Poplar (3-ply $^1/_{32}$-in. veneer)	51.2	44.6	39.8
Poplar (5-ply $^1/_{32}$-in. veneer)	61.3	53.3	47.8
Poplar (3-ply $^1/_{16}$-in. veneer)	25.6	22.3	19.9
Poplar (5-ply $^1/_{16}$-in. veneer)	30.7	26.7	23.9
Poplar (3-ply $^1/_8$-in. veneer)	12.8	11.1	9.9
Poplar (5-ply $^1/_8$-in. veneer)	15.3	13.3	11.9

TABLE 4 WATER RATIO 2.5 TO 1 (71.4% WATER, 28.6% SOLIDS)

Spread in sq. ft. per lb. of glue base...	45	52	60
	—Moisture added, %—		
Birch (3-ply $^1/_{32}$-in. veneer)	32.0	28.0	24.5
Birch (5-ply $^1/_{32}$-in. veneer)	38.5	33.4	29.4
Birch (3-ply $^1/_{16}$-in. veneer)	16.0	14.0	12.2
Birch (5-ply $^1/_{16}$-in. veneer)	19.2	16.7	14.7
Birch (3-ply $^1/_8$-in. veneer)	8.0	7.0	6.1
Birch (5-ply $^1/_8$-in. veneer)	9.6	8.3	7.3
Poplar (3-ply $^1/_{32}$-in. veneer)	52.2	45.7	39.8
Poplar (5-ply $^1/_{32}$-in. veneer)	62.5	53.1	47.8
Poplar (3-ply $^1/_{16}$-in. veneer)	26.1	22.8	19.9
Poplar (5-ply $^1/_{16}$-in. veneer)	31.7	26.5	23.9
Poplar (3-ply $^1/_8$-in. veneer)	13.0	11.4	9.9
Poplar (5-ply $^1/_8$-in. veneer)	15.8	13.3	11.9

TABLE 5 WATER RATIO 3.5 TO 1 (77.8% WATER, 22.2% SOLIDS)

Spread in sq. ft. per lb. of glue base...	60	70	80
	—Moisture added, %—		
Birch (3-ply $^1/_{32}$-in. veneer)	34.4	29.2	23.9
Birch (5-ply $^1/_{32}$-in. veneer)	41.3	35.0	28.7
Birch (3-ply $^1/_{16}$-in. veneer)	17.2	14.6	12.0
Birch (5-ply $^1/_{16}$-in. veneer)	20.7	17.5	14.3
Birch (3-ply $^1/_8$-in. veneer)	8.6	7.3	6.0
Birch (5-ply $^1/_8$-in. veneer)	10.3	8.7	7.2
Poplar (3-ply $^1/_{32}$-in. veneer)	56.0	47.4	38.9
Poplar (5-ply $^1/_{32}$-in. veneer)	67.0	56.8	46.6
Poplar (3-ply $^1/_{16}$-in. veneer)	28.0	23.7	19.4
Poplar (5-ply $^1/_{16}$-in. veneer)	33.5	28.4	23.3
Poplar (3-ply $^1/_8$-in. veneer)	14.0	11.8	10.0
Poplar (5-ply $^1/_8$-in. veneer)	16.7	14.2	11.6

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[2] Department of Agriculture Bulletin No. 1500.

[3] Senior Wood Technologist, Forest Products Laboratory, Madison, Wis.

kiln, but dry with air circulation only, at factory-room temperature. To complete the building of the tops they are trimmed after drying and banded all around with a wood veneer band equal in width to the full thickness of the top. While there is a decided advantage in using a low-water-content quick-setting glue on such work, it is done successfully with the regular glues.

Laying micarta, formica, and similar materials is done with vegetable glues in the same manner as laying linoleum.

Gluing Fire-Resistant Wood

The question of gluing fire-resistant wood will no doubt become more and more important because of the constantly growing interest in and use of such wood in building construction in the large cities. The gluing of this wood requires a special technique. The chemicals which are used for impregnating the wood and making it fire resistant render it difficult to get a satisfactory joint with any type of glue. Special types of vegetable glues will glue this wood as good as any glue. The best results are obtained from the quick-setting, low-water-content vegetable glue.

Joint Vegetable Glue

In lumber gluing, particularly on edges, the desired speed of production usually requires a quick-setting glue. Such a vegetable glue has been introduced within the last two years, and is increasing in use very rapidly. It is of special value at the glue-clamp carrier where production in core construction is required. While developed especially for lumber joint work, it can be used for veneering. Advantages in the use of joint vegetable glue over other joint glues are the same as in the use of veneer glues. Actual first cost and final cost receive chief consideration, but the elimination of continuous heat in gluing operations makes more contented workers and is a factor not to be overlooked.

The joint vegetable glue is mixed in the same manner as veneer vegetable glue. When converted it has a creamy color. Its working life is indefinite if kept covered when not in use. It is best applied with a roll-joint glue spreader with the roll operating at 45 r.p.m. It can be used on the plate spreader, but not on the ordinary plate as applied for use of hot glue. A special plate is made for vegetable joint glue work.

The water ratio used in this type of glue is from 1.3 to 1.6 parts of water to 1 part of glue by weight, depending on the time of setting required, strength desired, and drying conditions. Removal of stock from clamps in less than 30 minutes is possible, but not generally recommended. Conditions in the plant determine that point. The spread for joint glue on edge gluing is from 24 sq. ft. per lb. to 30 sq. ft. per lb., depending on the amount of water used. It would be possible to spread more or less, but the spread mentioned will give a light bead for the squeeze-out. An excessive squeeze-out shows a waste of glue and retards the set.

For pressure on joint work the average clamping methods are used, but as in the use of other glues caution must be exercised against the use of excessive pressure. Enough pressure should be used to hold the stock firmly together. A starved joint is just as possible and perhaps more dangerous in a lumber joint than in a veneer joint.

The time under pressure required for lumber gluing depends entirely on the conditions. The most important of these affecting the time required for pressure are, listed approximately in the order of their importance, as follows:

(1) Degree of contact at glue joint; edges to be joined must be cut square and true.

(2) Moisture content of lumber; not over 6 per cent if speed in setting is desired.

(3) Tendency of lumber to twist on removal from clamps or in drying. Such so-called soft woods as bass, poplar, and chestnut require a shorter pressure period than some of the harder woods, because of their lesser tendency to twist.

(4) Quantity of glue spread; use only enough to give a small, beady squeeze-out with the pressure used.

(5) The water content of the glue; the lower the water content the quicker the setting.

(6) Clamp-carrier drying conditions; artificial heating and ventilating aid in shortening the time needed for pressure.

Lumber Preparation for Joint Gluing

Owing to warpage and uneven surfaces it is sometimes difficult to obtain strong lumber joints. This is especially true in edge gluing where there is more strain and less glue contact surface. With lumber gluing it is extremely important to obtain the best possible degree of contact between the surfaces to be glued. The full strength of a glue is developed only by lumber cut by the best commercial methods; if not, the joint may show a variation in strength of from only a few pounds to more than 1000 lb. per sq. in., depending entirely on the degree of contact.

It is not good practice to joint lumber and allow it to stand for any length of time before it is glued. Lumber continually works and twists under the strains produced by changes in moisture, and for this reason better contacts and consequently better joints are produced when the surfaces of the lumber to be glued have been freshly prepared.

Based on comparison of glue strength made with both sawed and jointed lumber, it has been found that there is not much difference between smooth-sawed joints and those made on the jointing machine; but if sawed joints are rough, irregular, and out of square, it is impossible to make good joints. The ordinary circular rip saw will rarely make a joint good enough to glue. With a miter or planer saw properly filed and fitted, smooth sawed joints can be made.

Obtaining Maximum Joint Strength

Joint glue when compared with any other vegetable glue having the same body will show a superiority in both strength and set. The most important variables affecting the strength of the joint are, listed in the order of importance, as follows:

(1) Degree of contact; the better the contact, the more strength.

(2) Body and water ratio of glue; the heavier bodied and lower water ratio glues develop greater strength.

(3) Moisture content of lumber; preferably not over 6 per cent.

(4) Time under pressure; average for safety 35 min., minimum under good conditions 25 min.

(5) Degree of pressure; use only enough pressure to hold stock firmly together.

Vegetable Liquid Glues

While the vegetable liquid glues have been under laboratory and practical observation for several years, they have come to the attention of the woodworking trade only recently. These glues are used on the cabinet makers' bench and in repairing, and displace the hot-glue pots and steam coils in cabinet and assembling work. Also they are used in making joint work on equipment such as the Linderman machine and replace hot glue on that operation. These glues are in liquid form and do not deteriorate. They are readily spread with a brush by hand methods. No special technique is required in their use.

Correcting Defects in Gluing Operations

It is possible to misuse any glue or to have defective work in gluing operations, regardless of what glue is used. By a knowledge of the cause of such defects the intelligent operator either will avoid trouble or will discover the reasons of his particular trouble when it occurs. It would be well if every plant had a competent personnel to check up on gluing defects, which seldom result from one condition alone, but from a combination of circumstances. The inspection of a defective sample in itself cannot always determine what caused the trouble, but such inspection, together with a full knowledge of the manufacturing process, should enable one to solve the problem accurately. It is not possible to present a complete key to the causes and remedies for all gluing defects. A number of the more common causes are given in "The Gluing of Wood," by T. R. Truax, Department of Agriculture Bulletin No. 1500.

WDI-54-4

Casein Glue

By W. F. LEICESTER,[1] NEW YORK, N. Y.

Casein glue, made from soured skim milk, then dried, ground, and blended, has been in use in Germany for 50 years and in the United States for 30 years, but it was not until the World War period that it came into general commercial use. It was then that easy-to-use formulas of standard type were developed for the production of waterproof plywood in airplane construction. It now has general use in the woodworking industry. Casein glue in the past has relied upon harsh alkalis as a waterproofing agent. The casein glue of the future for veneering will be relatively only slightly alkaline, but will be high in water resistance through the addition to the dissolved glue of a chemical solution. The smaller user will have a glue that will carry overnight without loss of strength, will be quick setting and strong, and that will have sufficient water resistance for ordinary needs.

USE of casein glue in industry is familiar to all in a general way. Casein is a dairy product. It is the same as cottage cheese in that it is the curd formed in skim milk when it sours. Commercially, casein is made in centralized creameries by natural or acid souring of skim milk. When dried, ground, and blended, it is the raw material from which is made the casein glues used by the woodworking industry. Centralization and conservation in the milk industry of the United States have resulted in a large increase in the production of casein, freeing this country from dependence upon imported supplies. Forty to fifty million pounds of casein are used industrially in the United States each year. The chief use is in the coating of paper; the next is as adhesives for wood, fiber, paper, etc.; other uses are for sizing veneers, walls, and wallboard and as the binding and waterproofing ingredient in plastic paint, calcimine, and whitewash. Solid casein in various colors is fabricated in rods, tubes, and sheets, from which it is remanufactured into buttons, drawer pulls, pens, and various novelties.

While casein glue has been used in Germany for upward of 50 years and in the United States for 30 years, its general commercial use dates from the World War period, when easy-to-use formulas of standard type were developed for the production of waterproof plywood in airplane construction. When as a result of the wartime record of casein glue the woodworking industry seized upon this new product as the solution of gluing problems, it was not quite ready for such commercial development. The formula in use at that time was highly alkaline, so that the glue affected the skin of workmen, and it stained acid woods, such as oak, mahogany, walnut, etc. When the glue set, it was very hard on machine knives. In general, it was difficult to handle, and a prejudice which existed for several years was developed against casein glue. Today, with wider chemical knowledge and with greater facilities for laboratory and factory experiments, the sphere of usefulness of casein glue has greatly widened, the old-time prejudice has disappeared, and the product is generally accepted by the woodworking industry. One now thinks of casein glue as not merely a glue or cement for exposed work where water resistance is paramount, but as an all-around adhesive to be used wherever great strength combined with high or low resistance is required. It has become possible to fit the formula to the job; to produce not just one casein glue, but a wide range of glues; to meet all wood-gluing needs from the veneer fuselage of the globe circling "Winnie Mae" airplane to the gluing of $^{1}/_{32}$-in. imported walnut, hardwood, or other modern veneers; from the laminated bowling pin to leg blocking.

A modern casein glue is easy to mix and easy to use. It comes to the user as a dry powder which has only to be mixed in cold water to form a liquid glue. It is quickly prepared for use without the addition of chemicals or the application of heat. It finds favor with both large and small factories because it meets the needs of both. The plant that uses one or two pails of liquid glue daily mixes a pailful at a time, either with a hand-operated mixer or merely in a pail with a stick. The larger factory uses a power-driven mixer of from 8 to 75 gallons capacity. In each case the mixing procedure is the same. The cold water is placed in the mixer, and the glue powder is sifted in while agitation proceeds. Then the mixing stops for 15 minutes, during which period the casein and chemicals soak up the water, thicken, and begin to dissolve. Mixing for 5 minutes finishes the dissolving process, so that at the end of about 20 minutes the liquid glue is ready for use. The consistency of dissolved casein glue may be compared to that of thick cream. It spreads readily by brush or by machine spreader. It can be used safely for the gluing of softwood or hardwood joints and for gluing any type of veneer. Summing up, the modern casein glue is easily mixed and spread, is strong enough for any gluing job, is usually highly water resistant, is quick setting, is reasonably free from stain, and offers no special tool problems in the machining operation. As to cost in the glue line, casein glue lies between that of vegetable and animal glue. It is difficult to give a specific cost figure because of the wide formula range which depends upon the work to be done and the results required.

The present use of casein glue in the woodworking industry is as wide as the industry itself. The original and most obvious use is in the fabrication of airplanes, not a large-volume business today. This is one place where metal is displacing wood, and not necessarily to the improvement of the airplane, but chiefly to the lowering of cost. Casein glue does a good job in the laminating of hardwoods for auto-body parts. Laminated and block flooring, glued with casein glue, is finding a widening field of use, and in this work it has been proved over a period of more than ten years that a casein-glued joint will stand up perfectly under conditions of hard usage and more than usual exposure. Church furniture and seating, especially plywood seating, is widely done with casein glue. This type of furniture leads a fairly strenuous life; plywood seats must stay tight right up to the unprotected edge. Doors and millwork constitute a fairly obvious field for water-resistant glue. Used at first only for outside doors, casein glue is today generally used for all veneered doors, for the core sections, the panels, and for the veneering of stiles and rails. One of the

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

latest developments is a glue for the dowel assembly operation which works in the automatic dowel gluers.

Coming to the consideration of casein glue in the furniture industry, at first it may be thought that here is no field for water-resistant glue because furniture stays within the home and lives a quiet life. There is an increasing use of casein glue in this important branch of woodworking, not only in Grand Rapids, Rockford, and other Northern points, but also in the South. The manufacturer of high-grade furniture uses casein glue for veneering in order to safeguard the quality of the goods, as an insurance against loose veneer which might be caused by conditions of use or climate beyond his control. Furniture manufacturers generally use casein glue for overlay and for core and block gluing, for reasons of convenience and economy. Throughout the industry there is a trend toward using water-resistant glue for important veneering such as tops and fronts, the parts that have unprotected edges. Leaving the matter of whether it is good policy to use water-resistant glue, look into its use in the furniture factory and see if it functions smoothly in production and if it removes any of the troubles that are so often inseparable from the veneering department. To begin with, casein glue promotes quick assembly of panels between the spreader and the press; and this is a basically good thing, because it gives the trouble maker, glue moisture, less time to get in its work. A load of casein-glued panels should be under pressure 15 minutes after the load is started; 10 minutes is better. The shorter the period, with any glue, the better. Here is what short assembly with casein glue means: a thin spread of a heavy-bodied glue; a minimum of glue moisture going into the veneer; minimum expansion of the crossbanding due to thin spread, and quick pressure, thus a lessening of internal stresses in the panel and consequently less tendency to checking and warping. Some day there may be realized the ideal of a dry gluing process, but with modern casein glue the industry is moving in that direction. Other advantages are quick setting (pressure can usually be safely released in three hours), quick redrying, and elimination of glue stain through thin veneers due to the thin spread of heavy glue not penetrating to the face, and to short period of pressure. An average size load of panels can be built in 10 to 15 minutes with casein glue because the glue, being free flowing and easily spread, the glue-spreader rolls can be run relatively fast, thus speeding up the entire veneering operation. As to glue spreaders, spiral corrugations rather than the conventional lateral corrugations are recommended for the spreading of casein glue. The spiral corrugations, cut 12 to the inch and 0.03 in. deep, spread a very fine and economical film of glue, which on the veneer almost has the appearance of having been brushed out by hand. The spreading of a fine, even film of glue is more important today in the furniture industry than ever before, because the light or natural finishes now used do not allow for the covering up with stain or shellac of veneer imperfections due to glue stain or blisters. If the spread of glue consists of alternate hills and hollows, and the hollows dry out before pressure is applied, the result is blisters in the water-staining operation, which become visible when the finish is applied and which necessitate expensive repairs. Water stain is a real test of the veneer-gluing job, and it is of course obvious that a well-built casein-glued panel should come through this operation without blistering or loose edges. Casein-glued panels can safely be treated with a water-bleaching solution, if required. Some manufacturers that use casein glue remove veneer tape from the surface of glued panels with a steam jet, thus cutting down the drum-sanding operation and preserving a greater thickness of face veneer. Another operation, well done with casein glue, is overlay gluing. A thin film of heavy-bodied glue is applied, resulting in minimum squeeze-out and speedy setting. Turning to joint gluing, casein glue is being extensively used in blocking, core gluing, and banding. It is applied by brush or single-roll spreader, can be removed from pressure quickly, and soon thereafter machined. The fact of its wide use in blocking work which has to be turned proves the statement that modern casein glues are not hard on knives. Other furniture uses are in gluing formica and other bakelite veneers to wood. The use of these durable materials is increasing; they are hard to glue, but casein glue holds them permanently. In bending and gluing veneers in one operation, the quick set of casein glue permits frequent reuse of forms, and the stiffness of the casein-glued joint when set gives bent work desirable rigidity. Another use is in the sizing of thin, delicate veneers such as crotches and burls. By a special treatment these veneers are given strength combined with flexibility, which makes them glue and finish better and helps to prevent checking.

A final word might well deal with present and future developments in the manufacture and use of casein glues. In the field of use it is to be noted that mixing and spreading machines are being improved. There are the spiral corrugations, already successful. The latest veneer spreader has the lower glue box adjustable so that the last of a batch of glue can be brought right up to the lower roll and so used up entirely without waste. Casein-glue-mixing machines are now available with a sifter built in, which is driven off the shaft, and beats the glue powder through a sieve, thus preventing lumping and producing an extremely smooth, varnish-like glue. A two-speed mixer is being developed which will allow rapid mixing while the glue powder is being added, and which can thereafter be run at slow speed while the batch is dissolving, thus speeding and improving the mixing operation. Chemically, the tendency is toward more complicated glues for large-scale use and toward simpler glues for smaller use. The casein glue of the past has, generally speaking, relied upon harsh alkalis as a waterproofing agent. This has caused stain, hardness on tools, etc. The casein glue of the future for veneering will be relatively only slightly alkaline, but will be high in water resistance through the addition to the dissolved glue of a chemical solution. The smaller user will have a glue which need not be used up the same day as mixed, which will carry overnight without loss of strength, and which with this conveniently long life will be quick setting and strong, and will have sufficient water resistance for ordinary needs.

WDI-54-5

Blood Albumin and the Woodworking Industry

By C. B. SMITH,[1] CHICAGO, ILL.

Blood-albumin glue is probably the latest commercial adhesive. Its first use apparently was in the making of plywood canoes. The World War created an intensive demand for water-resistant plywood glues, and much of it was used in airplane and boat construction. Its source is beef blood as a by-product of the packing-house industry. It is essentially a hot-press glue. There are several types, all a mixture of blood albumin, water, and an alkali.

COMMERCIAL blood albumin, as its name signifies, is a material obtained from beef blood as a by-product of the packing-house industry. It is a complicated mixture composed mainly of hemoglobin, serum albumin, and serum globulin, together with small amounts of sugars, fats, salts, coloring matter, and other substances normally found in blood. The main characteristic of blood albumin is its ability to form an irreversible gel when its water solution is heated to 160 deg. fahr.

The manufacture of blood albumin will be briefly described, and more time will be given to a discussion of its uses in the woodworking industry. Beef blood from the killing floor of a modern packing house is collected in tanks, in which it clots. The clot-forming protein, or fibrin as it is called, is removed from the liquid blood, and the latter is pumped to the drying plant to be dried in shallow pans in tunnel driers. The temperature of the drying air is kept below 130 deg. fahr. at all times in order to avoid any possibility of coagulating the blood albumins. The dried flakes contain 5 to 7 per cent of moisture and are deep red, almost black in color. They easily dissolve in water, after a preliminary soaking of about one hour, to form a dark red solution with properties similar to the original blood. In the flake form they may be kept for a reasonable length of time, although if kept in a moist and warm place they gradually lose some of their solubility. In water solution, they are food for bacteria and will decompose in time unless properly preserved.

It is difficult to determine when blood albumin was first used as an adhesive, although it probably was much later than many other glues. It is known that the early Chinese used a mixture of blood and lime as a protective covering for wood, but it is doubtful whether it was used as a glue at that time. About 1910, reports were prevalent in this country that a water-resistant blood glue had been discovered in Russia, but early experiments met with little success, and much plywood was spoiled in attempts to use blood as an adhesive.

Several years later Messrs. Haskell and Beiger together did the first recorded work with blood-albumin glues in this country. After a successful formula had been evolved, their first project was the making of plywood canoes. Shortly thereafter, the World War created an intensive demand for a water-resistant glue, and blood albumin came forward as an answer to the problem. Several large plants worked day and night turning out water-resistant plywood for the Government and for many private concerns. Practically all of this plywood went into airplane and boat construction, and it can be safely said that it was the most water-resistant glue obtainable for the purpose. Since then this unusual war demand has fallen off, and blood-albumin glue is not used for plywood to as large an extent in this country today, although it is admirably suited for many purposes and conditions of service and is still used by several large manufacturers. It has been a source of much pride to one such manufacturer that his plywood was used exclusively in many noted planes, such as Col. Charles Lindbergh's "Spirit of St. Louis," and the planes of Byrd, Chamberlain, and many others equally famous. In many countries of Europe, blood albumin is still extensively used in water-resistant plywood, although it is commonly mixed with other albuminous adhesives, such as casein or soy-bean flour. It is probable that such mixtures will soon be favored in this country, as they tend to lower glue costs, and if properly used give results comparable to those obtained with straight blood-albumin glues.

Blood-albumin glue is essentially a hot-press glue. Several years ago wood technologists at the Forest Products Laboratory discovered a cold-process formula which may be found in several of their publications, such as Bulletin No. 1500. As this formula, to the best of knowledge, is not used on a commercial scale, discussion will be limited to blood-albumin glues requiring the application of heat.

In searching through the patent literature one finds several types of blood-albumin glues. One of these early patents refers o a mixture of blood albumin, water, and silicate of soda, while later patents use lime and ammonia as the required alkali. Repeated tests have demonstrated that maximum water resistance is acquired through using a mixture of blood albumin, water, and a very small amount of a weak alkali such as ammonia, borax, or soda ash. In other words, a heat-coagulated albumin is more insoluble and water-resistant than one coagulated by means of a chemical, such as lime. However, such a glue requires less water in its make-up than others using a stronger alkali, and this fact results in an increased glue-line cost. For this reason, alkalis which act as extending agents are commonly used with blood albumin in order to obtain a lower glue-line cost, even though the water resistance of the resulting plywood is slightly lowered. As an example, in making a straight blood-albumin glue, only about 125 parts of water are required for each 100 parts of albumin. Other formulas, using various alkalis, increase the water content to 200 parts, and in admixture with other proteins, such as casein, the water content is still greater per hundred pounds of dry glue. Following the trend of present-day gluing practices and demands, all manufacturers of water-

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resistant glues are continually endeavoring to work out better glue formulas which will give maximum dry and wet strengths at the lowest possible glue-line cost.

Applications of some of the glue formulas herein briefly mentioned are ably discussed in publications of the Forest Product Laboratory. There are many factors—such as details of glue spread, assembly time before pressing, temperature, amount and duration of pressing, and the like—which are so dependent on plant conditions and the nature of the wood used that it is not advisable to discuss such variables in a paper of this kind. As with other glues, such factors must necessarily be worked out in each plant for each set of conditions encountered. However, there are several characteristics of plywood made up with blood-albumin glue that are noteworthy and of interest.

As previously stated, plywood made up with blood albumin is of primary importance because of its water resistance. As an example of the practical importance of such a trait, an article by Mr. J. W. Beiger in the March, 1930, issue of *Veneers Magazine*, says: "Our first molded canoe was made early in the fall of 1916. It was made from one sheet of plywood approximately 57 in. wide by 17 ft. long and $^{3}/_{16}$ in. thick. This panel was put over a vat of water and boiled until it was pliable (about 3 hours), then placed on the mold and dried under pressure. Oak stems were placed at each end inside and out, and birch was used for gunwales and thwarts. Seats were hung from the gunwales and the canoe was ready for the water."

Since that time many similar canoes have been made and successfully used under conditions which demonstrate the value of blood albumin as a water-resistant adhesive.

In recent years, one has seen plywood panels made with blood-albumin glue, then soaked in hot water until pliable, and placed in an embossing press and allowed to dry under pressure. Very striking deformations of the wood fibers are made possible by this procedure without affecting in any way the strength of the finished embossing. Imitations of carvings are thus made possible in which chisel marks and other details are faithfully reproduced. It is this property of the coagulated albumin becoming plastic under application of steam and pressure and returning to its original condition when dry that is one of the most striking characteristics of blood albumin.

Another valuable feature of plywood made up with blood-albumin glue is the absence of dulling action on cutting tools, such as saws. This is due to the fact that the glue is organic in composition. In the past, it has been found that such joints had a tendency to become brittle on aging due to the glue becoming too dry, this resulting in a lower joint strength, but the inclusion of hygroscopic materials in the glue mixture has aided in overcoming this trouble.

By this same inclusion of a hygroscopic material, such as glucose, in blood-albumin glue, the Forest Products Laboratory has been able to devise a process for making up plywood from very thin veneers. Strips of absorbent paper are impregnated with glue mixture and dried at a low heat to avoid coagulation of the albumins. These strips are placed between plies of veneer, and the panels are pressed in a hot-plate press in the usual manner. By this method, in which a minimum of water is introduced into the glue line, it is possible to glue up exceedingly thin veneer without checking, warping, or staining.

Discussion up to this point has dealt with glues made from flake-blood albumin. It is also possible to obtain blood albumin as a spray-dried product. In this form, it is a dark red powder with properties similar to the flake form. However, it is interesting to note one use of this product in the wood industry concerning which little has been published. This is in the gluing of green veneer, or wet veneer direct from the lathe. It has long been a dream of woodworking engineers that some time it might be possible to work out a straight-line production of plywood, taking veneer from the cutting lathes directly through a glue spreader, press, and drier in some manner that would be commercially practical. Much plywood has already been made on the Pacific Coast from soft woods by spreading the veneer with a powdered blood-albumin glue and combining the pressing and drying operation in multiple-plate hot presses having grooved plates to allow the escape of moisture as steam. When one considers the savings made possible by a straight-line production of plywood, it serves as an impetus to further research. Even though such production is limited to water-resistant plywood of the cheaper grades, such as box plywood, it seems entirely probable that blood albumin will be the glue responsible for solution of the problem.

Spray-dried blood albumin is also used extensively as a thermoplastic in the molding of small objects such as buttons. In connection with wood products, it is used with cork, sawdust, paper pulp, and the like in the molding of many articles.

The two forms of blood albumin which have been discussed are dark colored. If a light colored blood albumin is desired, it is necessary to use a third form, the so-called light albumin. This is made from beef-blood serum, or blood liquid, from which the corpuscles have been removed in passing through a centrifugal separator. Its chemical and physical properties are quite similar to those of whole blood products. In the woodworking industry, it is used in the edge-gluing of veneer for pieced faces for water-resistant plywood. The exposed glue line in a pieced face thus treated is water-resistant and not discolored by stains.

Besides the uses of various forms of blood albumin in applications of interest to engineers, there are uses of interest to all. Indeed, there are few industries which are not confronted with problems in which the use of blood albumin might be of service.

It is desired to acknowledge indebtedness to the wood technologists of the Forest Products Laboratory and to Mr. Haskell and associates for items taken from their publications.

Oil-Seed-Residue Glues

By I. F. LAUCKS[1] and GLENN DAVIDSON,[2] BLOOMINGTON, ILL.

I. F. Laucks

The paper describes the history, characteristics, uses, and limitations of soya-bean glue, or generically of seed-residue glue. This is the newcomer in the glue field, its use practically dating from 1923. The adhesive base of the seed-residue glues is the cake or meal resulting from the oil-pressing of soya beans, peanuts, cotton seed, etc. In the manufacture of plywood it is stated that the tonnage of the oil-seed-residue glues equals that of any other type of glue. In connection with the soya-bean glues there has been developed the use of carbon bisulphide as an agent to increase water resistance.

Glenn Davidson

Oil-seed-residue glues are glues the adhesive base of which is the residue cake or meal resulting from the oil pressing of soya beans, peanuts, cotton seeds, and the like. These oil-seed-residue glues are newcomers in the plywood field, for prior to 1923 they were unheard of commercially, but by 1929 they had reached such importance that, in the manufacture of plywood, the total tonnage of oil-seed-residue glues used approximated, if not exceeded, that of any other type of glue.

History of the Oil-Seed-Residue Glues

The Pacific Coast served not only as the birthplace but also as the cradle in which oil-seed-residue glues developed into a real member of the glue family.

From the beginning in 1923 until 1926, soya-bean glue faced the typical discouraging and disheartening uphill fight that is faced by most new products until all production problems are solved and the products themselves win the full confidence of the users. One of the most serious problems encountered in the production of commercial soya-bean glue was that of the chemical condition of the soya-bean meal available. The chemical state of the meal under suitable conditions may be rapidly modified by such factors as oxygen, moisture, bacterial and chemical action. This meant that but few lots of the soya-bean meal reached the Pacific Coast with the chemical state of the meal altered only slightly from the state in which it existed in the beans as they were harvested from the fields in China, but in most of the lots there had been considerable alteration, and in some of them this alteration was extremely great. Thus it was found that the lots of soya-bean meal which were presented in the early days as possible sources of soya-bean glue differed among themselves through about the same range of characteristics that a raw egg differs from a fried egg. The standardization of the chemical state of the meal constituted three major steps. First, a working arrangement was perfected with one of the largest Oriental mills producing soya-bean cake which permitted the standardization of their product within certain limits as it left their mill. Second, the meal intended for glue was transported to the United States by the fastest steamships, instead of the old method of shipping by tramp steamer or schooner. Third, a method of chemical treatment to be used in this country was developed which insured the uniformity and stability of the resultant glue. These are extremely necessary characteristics of a base material to be used in the manufacture of a commercial veneer glue.

Aside from manufacturing and standardization difficulties, soya-bean glue had other difficulties to surmount in the form of the natural sales resistance to a new and different product. Soya-bean glue does not possess the typical physical characteristics ordinarily described as "glue like;" that is, it is not sticky, tacky, or sirupy; on the contrary, it is rather "soupy" or "mushy." These physical characteristics were really a blessing in disguise, for today they represent one of the strongest arguments for the use of soya-bean glue. Practical glue men looked at the soupy, mushy liquid in the glue spreader, tested it between their thumb and forefinger for tackiness, which was conspicuous by its complete absence, and then shook their heads, saying such a thing could never stick to plywood This first impression was so strong that many refused to believe that the plywood glued with this glue could really be stuck even after the packages had been opened and the panels found to exhibit very good adhesion.

Another handicap was that no one had ever heard of a water-resistant glue from a vegetable source. The name "vegetable glue" connoted starch glues, and it was well known that vegetable glue was not water resistant.

The year 1926 proved to be the turning point in the life history of a soya-bean glue. Prior to this time there had been an insistent demand for plywood of greater water resistance on the part of automobile manufacturers, who were beginning to buy large amounts of plywood. Finally, the Pacific Coast plywood manufacturers announced through their association that they wished to arrange a series of competitive demonstrations of all water-resisting plywood glues. These demonstrations were made during April, 1926, with nearly all glue manufacturers participating. The results in these tests obtained with soya-bean glues were so outstanding that these glues were almost immediately adopted by all the Pacific Coast plywood manufacturers and have been in continuous use there.

Present Uses of Soya-Bean Glues

The present uses of soya-bean glues embrace the entire fir and pine plywood industry, together with a large share of the plywood

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box industry and the furniture plywood industry so far as glass backs, drawer bottoms, and similar panels are concerned, but not tops, drawer fronts, and other plywood where fancy veneers are used extensively. A considerable tonnage is also used in the lamination of insulating board to be used in refrigerators and for other purposes.

So far as fir and pine plywood is concerned, soya-bean glues offer at the same time a most satisfactory, strong, water-resistant, easily used, and low-cost glue. The fir plywood manufacturers will not consider any other than a water-resistant glue. They were the first to use water-resistant glue generally, and now the term fir plywood has come to imply water resistance, whether so stated or not. This is one of the reasons why fir plywood finds such a large use for industrial purposes. Some of these uses are in automobiles, shipbuilding, railroad cars, and refrigeration, besides many uses in buildings and construction. An interesting use which has been developed only lately is for concrete forms. Builders are finding that plywood can be substituted for the old-style concrete form construction, with a very decided advantage in favor of plywood. There are 17 plants in the Pacific Northwest producing fir plywood. The smallest of these will produce about a hundred thousand square feet of plywood per day and the largest about three hundred thousand square feet.

All users of glue are familiar with the fact that certain woods are more easily glued than others with a given glue. In fact, there is no glue known that gives equally good results on all woods. In the case of fir and soya-bean glue it turned out that these two seemed to have a natural affinity—a very fortunate thing for a new glue introduced in the Pacific Northwest. Not only were these advantages in adhesion and water resistance, but also the use of soya-bean glue enabled the fir plants to speed up production, because of the ability of spreader crews to handle stock faster on account of the peculiar consistency of the new glue and also because spreaders could be run at a higher speed without danger of foam.

When one is using glue by hand methods, one can choose the type of glue to be used, with no regard for any of its characteristics other than adhesion. It is quite different with a plywood glue. The latter must fit into a mechanical process, and the production and cost of the process depend to a large extent on how well the glue fits. Properties of the glue such as uniformity, life, foaming tendency, time of setting, consistency, abrasive power, and so on may become nearly as important as adhesion in a plywood glue.

The cost of the glue line is only one item in making plywood, and one of the smallest items at that. If by spending a dollar more on the glue line, the plywood manufacturer can save two dollars elsewhere, he has a net gain of one dollar.

A good example of this is in the plywood box shook field. Here, cost is the prime consideration, and all glues except starch and soya-bean glues are eliminated on this score. Even soya-bean glue would be out on this point were it not for a number of manufacturing advantages which it possesses. These advantages are so great that the total manufacturing cost of the finished plywood shooks is less with soya-bean glue than with starch. In addition, the soya-bean glue produces moisture-resistant plywood.

When soya-bean glue is used, the veneer is glued up at a relatively high moisture content, say 20 to 35 per cent. Single-ply veneer at this moisture content is tough and accordingly does not break easily as is the case with dry veneer such as must be used with starch glue. When single-ply veneer is glued at 20 to 35 per cent moisture content, it is necessary to redry the panels. Experience has shown, however, that single-ply veneer can be dried to 20 to 35 per cent moisture and the panels redried, in most cases, at about the same drying cost for both operations that is required for the single operation of drying the single-ply dry enough to glue with starch glue.

There are many other manufacturing advantages resulting from the use of soya-bean glue. For example, it is usually possible for a given glue crew to "lay up" more stock with soya-bean glue.

The absence of extreme stickiness and gumminess in the soya-bean glue makes the handling of center stock coated with it very similar to the handling of veneer without glue at all. A further advantage of soya-bean glue lies in the almost complete elimination of glue-spreader breakage of thin center stock. Thus it is seen that the very characteristics that were the basis of the early predjudice against soya-bean glues became one of their greatest selling advantages.

In the manufacture of furniture plywood, soya-bean glue is finding considerable use in the manufacture of glass backs, drawer bottoms, and similar panels. The reason for this is its comparatively low cost and its water resistance. Water resistance is more essential for this class of furniture plywood than it is for tops, etc., since these have an extremely protective coating against moisture of high-grade lacquer, while the drawer bottoms, glass backs, etc., enjoy little or no protection of this kind and are so left exposed to ravages of moisture when the furniture is stored in a basement or under similar conditions. Further, many backs and drawer bottoms are finished by dipping in a water stain, and in such cases moisture resistance is particularly desirable.

Water-resistant glues in the United States plywood industry are comparatively a recent development, only dating from the time of the war. Much plywood is still made today with non-water-resistant glues, and the manufacturer of such plywood will generally say that his product does not need water resistance. However, water in some form or other, whether as liquid or vapor, is everywhere present, and there are very few glued products that do not come in contact with it, and frequently to their detriment.

Limitations of Soya-Bean Glues

Practically all of the fancy veneers used in the manufacture of high-grade cabinet plywood are easily stained by alkali. Unfortunately, all of the present commercial soya-bean glues require considerable alkalinity to properly disperse them and to develop good adhesion. This results in an almost, if not quite, prohibitive amount of stain in the case of most fancy veneers. On the other hand, animal glue requires no alkalinity whatever, and accordingly it stains fancy veneers to only a negligible extent. Starch glues can be made up with very low alkalinity and still give fairly satisfactory adhesion. The low alkalinity possible with starch glue does not result in more than a permissible amount of stain in the manufacture of most plywood faced with fancy veneers. Water-resistant casein glues share with soya-bean glues the necessity for higher alkalinity and the resultant stain difficulties. Accordingly, to date, neither casein nor soya-bean glues have been able to make any great progress in the fields where fancy veneers are used and where glue stain is a matter of prime consideration.

The authors have considerable experimental work in progress to the point of developing a stainless soya-bean glue for fancy veneers, and it is hoped that this will become a commercial reality in the next few years.

Miscellaneous

In 1927 the manufacture of commercial veneer glue from American-grown soya beans was begun. Prior to this time, the production of domestic soya beans had been on too limited a scale to render such a venture advisable. However, since 1927, the production of soya beans in the Middle Western states has been rapidly increasing. It is believed that the use of soya-bean meal as a glue base has been a notable factor in encouraging farmers to increase their soya-bean acreage.

A matter of incidental technical interest that has been developed in connection with soya-bean glues is the use of carbon bisulphide as an agent to increase water resistance. This chemical materially increases the water resistance of protein glues and is particularly useful with soya bean and casein glues.

CONCLUSION

Soya-bean glue, although a newcomer in the plywood field, has found many uses in the last eight years, and its tonnage approximates that of all other plywood glues.

It is used for many forms of plywood, with the chief exception of gluing fancy veneers where stain is an important factor.

The chief uses at present are for Pacific Coast fir and pine plywood, where it is used exclusively; box shooks in Eastern and Southern states; drawer bottoms, glass backs, etc., in the furniture field, and for laminating fiber boards.

Manufacture and Uses of Fish Glue

By A. G. BROOKS,[1] GLOUCESTER, MASS.

This paper briefly describes the process used in the manufacture of liquid fish glue. It also mentions the field of application of this glue and the technique for using it.

THERE always have been arguments as to the relative merits of the various kinds of adhesives.

The Greeks and Romans knew that a strong adhesive could be extracted from the skins of certain fish, and used this adhesive for such delicate purposes as fastening precious stones in their setting. This product, however, was made in very small quantities and had to be used immediately. It was not until the last century, some time between 1870 and 1875, that liquid glue was made a commercial possibility by the discovery of means whereby it might be permanently preserved in liquid form.

For many years it was the custom of the producers of salt fish to sell their product whole, stretched out flat and dried hard with the skins and bones all attached. Some years ago, however, the development of the boneless-fish industry, in which the skins were stripped from the dried fish and the bones cut away from the flesh, opened the way for the commercial development of those by-products for which the fish producers had no further use. It was found that liquid glue could be manufactured from all of this material, varying in quality and quantity with the kind of fish and the part of the fish from which the glue was extracted. The development of the liquid-glue industry has utilized all of these salt-fish by-products and has made a firm place for itself in the commercial world.

The heads of certain fresh fish, as well as the backbones, fins, and tails of all salted fish, yield a glue of very good quality which is largely used in such industries as paper coating, envelope manufacturing, shoe manufacturing, bookbinding, box manufacturing, etc. Where the very highest grade of adhesive power is needed, however, nothing ever has been found to equal glue properly made from the salted skins of the cod and cusk. It is this raw material which goes into the fish glue known to the carpenter and woodworker.

The process of manufacturing liquid fish glue, while not complex, requires much skill and experience to result in a uniformly good product. The salt skins as received from the producers of food fish are first washed to remove the salt and any foreign matter which may have become mixed with the stock. Then the skins are cooked very gently in specially constructed apparatus where the gelatin contents are extracted in their purest form.

After cooking and settling in the cookers, this thin gelatin liquor is pumped to evaporating apparatus, where it is concentrated under vacuum to remove the excess water which was used in extracting it from the original raw material.

When the glue has reached the proper concentration, preservatives and scent are added in accordance with a strictly kept formula so that the resulting glue will retain its qualities indefinitely.

Liquid fish glue is claimed to be the strongest adhesive known. A scientific test has recorded the strength of 2375 lb. per sq. in., equal to the estimated holding power of eight nails in the same area. Another test shows a strength of 3015 lb. per sq. in. before breaking.

The chief advantage which fish glue seems to have is its flexibility in operation. Where a joint made with a hot glue must be immediately placed in position without any further motion, a fish-glue joint can be moved back and forth until it reaches its perfect adjustment, without detriment to the strength of the joint. Furthermore, fish glue has a much slower set-up. This means that the glue has a greater chance to penetrate before it reaches its set. It is considered that this is an advantage in a wood joint, for it is the penetration of the fibers on either side of the joint that gives it its strength, and the adhesive which penetrates best is, generally speaking, the best adhesive for the particular condition.

Whereas a user of either animal glue or casein glue is handling something which requires to be used up immediately, or at least in a day if he wishes to avoid the risk of deterioration, the user of fish glue can open his can or tube or bottle day after day and find it substantially the same as it was when he first opened it.

The technique of gluing with fish glue is not essentially different than that of using other glues. It is recommended that a sizing coat be applied to each side of the joint to be made up and that this sizing coat be allowed to become tacky before a final coat is applied and the two elements of the joint are clamped together. In this way maximum penetration is permitted, and it is assured that there will be plenty but not too much glue in the joint as finally built up. We guard against excessive glue by rubbing the elements of the joint together, after the sizing coat has been applied, which brings a uniform film of adhesive over the surface.

Fish glue is to be recommended (1) where its permanently liquid form is a convenience, such as in the journeyman carpenter's tool kit or in the household; (2) where the nature of the joining is such that it is desirable to have the power to move it after the elements of the joint have been brought into contact; (3) where the nature of the wood to be joined is such that penetration would be difficult were an adhesive used which dried immediately.

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[1] Treasurer, Russia Cement Company.

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WDI-54-8

How Dry Should Wood Be for Various Industrial Uses?

By H. L. HENDERSON,[1] SYRACUSE, N. Y.

The testing of wood for industrial uses is stated to be best made on its moisture content. Much wastage in manufacture is declared to be due to lack of knowledge of this fact, and the consequent antiquated test methods in use in the wood-goods manufacturing industries. The effect of climate on the ideal moisture content is said to count for little. Furniture, flooring, and other products are dried to about 6 per cent, wherever manufactured. A table is given of the equilibrium moisture content of wood at various atmospheric conditions and also a table of the dryness requirements for a wide variety of wood products.

MOISTURE content of wood is the most reliable known criterion of its fitness for use. Lack of knowledge of this important factor has been the direct cause of much wastage in manufacture, of the breakage of wooden articles after being put to service, and of the unwarranted replacement of wood by other materials thought to be more stable in shape and size.

Progressive as these times are said to be, a fairly large proportion of wood is still manufactured and used without any method of testing its fitness for the work in hand, or if any tests are made, they are of the most primitive type. Archaic methods are practiced in many woodworking factories, and conditions are found at their worst in some of the large industries. Such industries, where the manufacture of major products requires many skilled engineers, chemists, metallurgists, or technicians, often have overlooked the wood end of their business entirely. Even sawmills, where the principal output is lumber and finished wood products, often neglect the drying of wood.

Occasionally one can find in practice such methods of testing wood for dryness as cutting the edge of a board with a knife and breaking out a sliver. If it shows a brittle fracture, it is considered dry; otherwise it is left in the kiln a few days longer. Some still employ the senses of taste and smell to determine dryness.

Many factories do not have any means of testing wood for dryness, depending entirely on excessively long periods of drying in the kiln and factory to insure its fitness for use. Some firms, especially in the handle and turning industries, process their wood parts very slowly through their plants, a practice which permits the moisture to evaporate gradually and the wood to shrink to its final stable condition before the articles are finished.

These wasteful methods are slowly disappearing, giving way to modern methods that are more precise and reliable. Free technical information is now available on the best methods of testing wood and on the dryness required for specific purposes. The best method of testing the dryness or moisture content of wood is by the weight method. This is done by cutting a small piece (see Fig. 1) from the board to be tested, weighing it carefully, drying it in an oven until all water is evaporated, and reweighing. The loss in weight will represent the water evaporated.

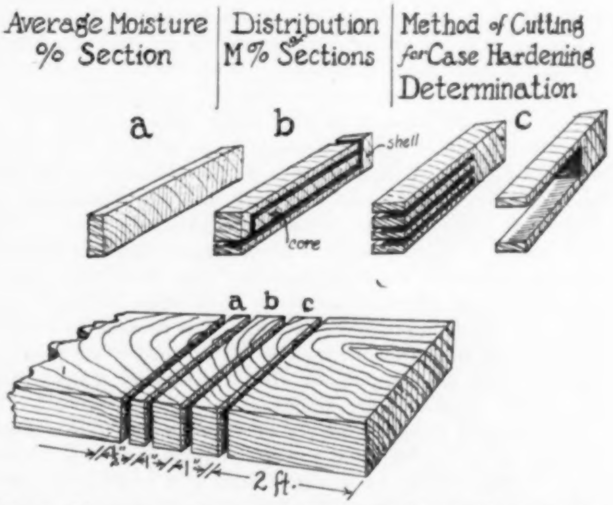
Average Moisture % Section
Distribution of M% Sections
Method of Cutting for Case Hardening Determination
a
b
shell
core
c
a b c
½"
1"
1"
2 ft.

FIG. 1 METHOD OF CUTTING A BOARD TO MAKE MOISTURE AND CASE-HARDENING TESTS

Since the dry weight of wood is a constant factor, all references to percentage of moisture are based on it. By dividing the weight of the evaporated water by the oven-dry weight of the wood and multiplying by 100, the percentage of moisture is obtained.

The temperature of the oven should not be so hot that the test pieces will be scorched. Between 215 and 225 deg. fahr. is a safe condition. If the test pieces are cut thin (from $^1/_8$ to $^1/_4$ in. thick), they will lose all their moisture in two to three hours' time.

The balance scales to use should be graduated into grams and with a sensitiveness to 0.01 gram. There are several computing scales for this purpose which do not require the use of arithmetic. More recently, electric machines have been developed for testing moisture in wood. They are based on electrical conductivity and resistance. Wet wood is a good conductor of electricity, and of course dry wood is not; therefore, by measuring the resistance to a current passed through the wood, the moisture content in percentage is shown. Since

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 EQUILIBRIUM MOISTURE CONTENT PERCENTAGE OF WOOD AT VARIOUS ATMOSPHERIC CONDITIONS[a]

	Temperature, deg. fahr.										
Wet-bulb depression, deg. fahr....	50	55	60	65	70	75	80	85	90	95	100
	Moisture percentage of wood										
4	15.7	16.2	17.0	17.3	17.6	17.9	18.1	18.1	18.2	18.2	18.3
5	13.9	14.5	15.0	15.3	15.6	15.9	16.2	16.4	16.6	16.8	17.0
6	12.5	13.0	13.5	13.9	14.2	14.5	14.8	15.0	15.1	15.3	15.5
7	11.3	11.9	12.4	12.8	13.1	13.4	13.7	13.9	14.1	14.3	14.4
8	10,2	10.8	11.4	11.8	12.2	12.5	12.8	13.1	13.3	13.5	13.7
9	9.2	9.9	10.4	10.9	11.3	11.7	12.0	12.2	12.4	12.6	12.7
10	8.3	9.0	9.6	10.1	10.6	10.9	11.2	11.5	11.8	12.0	12.2
11	7.3	8.1	8.7	9.3	9.9	10.3	10.6	10.8	11.0	11.2	11.3
12	6.4	7.3	8.1	8.6	9.1	9.6	10.0	10.2	10.4	10.6	10.8
13	5.5	6.4	7.2	7.8	8.4	8.8	9.2	9.6	9.9	10.1	10.2
14	4.6	5.7	6.6	7.3	7.9	8.3	8.7	9.1	9.4	9.7	9.9
15	..	5.0	5.8	6.6	7.3	7.8	8.2	8.5	8.8	9.1	9.3
16	..	..	5.2	6.0	6.7	7.2	7.7	8.0	8.3	8.6	8.8
17	..	..	..	5.4	6.1	6.7	7.2	7.5	7.8	8.1	8.3
18	..	..	..	4.8	5.6	6.2	6.7	7.1	7.4	7.7	7.9
19	..	..	..	..	5.0	5.6	6.2	6.6	7.0	7.3	7.5
20	..	..	..	..	..	5.2	5.7	6.2	6.6	6.9	7.1
21	..	..	..	..	..	..	5.3	5.8	6.2	6.5	6.7
22	..	..	..	..	..	..	..	5.3	5.8	6.1	6.3
23	..	..	..	..	..	..	..	..	5.4	5.7	6.0
24	..	..	..	..	..	..	..	..	5.1	5.4	5.7
25	..	..	..	..	..	..	..	..	..	5.1	5.5

[a] By permission of the U. S. Forest Products Laboratory.

wood having more than 25 per cent of water still has free water in the cell cavities, these electric instruments will not measure beyond that point, nor are they effective below 7 or 8 per cent, as the resistance is too great with wood so dry.

Wood is a hygroscopic substance, and no effective treatments have been devised to change it. If wood is placed in a dry atmosphere, it gives up its moisture to the air, and this keeps up until it reaches an equilibrium. With the rise in temperature or the reduction in relative humidity, wood has a tendency to give up its moisture, or in other words, loses its ability to retain water. Air, of course, increases its capacity to hold water with the increase in temperature. As its relative humidity increases, however, its power to absorb moisture diminishes, as is shown in Table 1. Many experiments have been made in this field, and it is found that for a given temperature and relative humidity of the air, wood will gradually come to a definite moisture percentage, which is termed the equilibrium moisture content. With this information available, it is possible to predict the exact amount of moisture to be left in wood which is to be used for some specific purpose when the average temperature and relative humidity are known.

For instance, a heated factory in winter might have a temperature of 75 deg. fahr. and a relative humidity of 15 to 20 per cent. Lumber would be at equilibrium at 4 to $4^{1}/_{2}$ per cent. During the spring, summer, and fall, when there is no artificial heat in the building, the humidity will vary from 35 to 75 per cent, with an average of about 45 per cent. This condition would allow 8 to 9 per cent of water to remain in the wood. Since it requires a long period of time for wood that is 1 in. or thicker to reach an equilibrium moisture content, and since its progress through the factory is often rapid, it is usually kiln-dried to some point between these extremes, such as 6 per cent. Before it leaves the factory it is probably filled and given several coats of varnish, paint, or lacquer, all of which tends to make it partially impervious to moisture, and so it never has a chance to dry too low or to absorb too much if air conditions are moist. Some factories do dry their stock to 5 per cent during the winter months, and only to 6 or 7 per cent in summer. This helps to keep the stock flat during manufacture. When a piece of cabinet work is finished and put into service, it has to stand the change in air humidity and temperature from season to season. There usually is a gradual taking on of moisture during the summer months, and then a loss throughout the winter. The varnish coatings retard this greatly, so that under normal conditions the swelling or shrinking will be gradual and not sufficient to warp the wood or to have it crack open at the joints.

While it is easy to determine the optimum final moisture content by a study of the factory air conditions during processing and the air conditions where the wooden article is put into service, it is not so easy to actually get the wood down to this condition. The ends, edges, and surface areas of a board always dry faster and to a lower moisture per cent than the central portions. There might be 4 to 7 per cent difference between them. Sapwood usually dries quicker than heartwood, and quarter-sawn pieces slower than plain-sawn. Specific gravity has something to do with rate of drying, as the lighter pieces dry faster. All of these conditions usually are present in a load of lumber, and thus complicate the process.

Added to these difficulties are the uneven temperature and humidity usually found in the dry kiln and the rate and direction of air circulation. These conditions are bound to produce a variety of moisture conditions in the lumber. It is not difficult to find the moisture content varying during the process as much as 20 to 30 per cent, and at the end of the process varying from 1 per cent in some boards to 18 per cent in others.

With good drying apparatus and intelligent operation, however, it is possible to produce kiln-dried stock that will not vary more than 2 per cent in moisture content. It is also possible to stop the drying at 7, 6, or 5 per cent, depending on which point is desired. It is not so easy to have stock uniformly dried if only 10 per cent is desired. The reason for this is the individual characteristics of boards and variations in rates of evaporation, as well as variables in kiln-drying conditions. There is very little difference in the final equilibrium moisture percentage of different species or textures of boards, but they do vary considerably in the time necessary to bring them to this equilibrium.

The foregoing discussion, then, should suggest a practice which is in more or less use in this industry—namely, holding lumber in dry storage for a sufficient period to permit it to equalize its moisture. There is considerable divergence of opinion as to the length of time necessary to obtain effective results, running all the way from a few days to six weeks. It is a subject that needs further study, and yet any factory might work it out satisfactorily for its own particular conditions.

Another aspect of the situation which needs explanation is that a wooden article might be used in a damp climate or in a position where the relative humidity averages a high percentage, and yet it must be dried to a much lower equilibrium moisture percentage. Numerous instances can readily be cited. Automobile wheel spokes are always dried to a very low percentage so that a slight swelling after assembly into the wheel will keep them always tight. Stock that is to be glued must not change its shape during manufacture, or the joints will be broken or the pieces warped. Caskets must be made with great care so that they will remain in shape with no fracture of joints or warped tops while they are in storage or on display. Wooden dowels are expected to swell a slight amount after use to help to keep them tight. Certain types of brushes, mirror backs, turned handles, etc. have to be dried to 1 or 2 per cent moisture because they are to have enamel baked on under high temperatures. Some cabinet work, automobile bodies, furniture, etc. which have to go through several kiln-drying periods to dry the various coats of paints and varnishes are very carefully dried before assembly.

On the contrary, some articles are not dried as low as they will come to in use, because of certain methods of manufacture.

TABLE 2 MOISTURE PERCENTAGES OF WOOD FOR VARIOUS PURPOSES

Use	—Moisture percentage—		
	Minimum	Maximum	Average
Airplanes	6	10	6
Agricultural implements	10	18	12
Artificial limbs	5	9	6
Automobile bodies	5	10	6
Baskets and fruit packages	8	45	20
Baseball bats	7	10	8
Boot and shoe findings:			
Last blocks	4	6	5
Heels	3	9	6
Boxes and crates	6	18	12
Boxes, cigar and tobacco	6	9	7
Brooms and carpet sweepers	6	9	7
Brushes	2	10	5
Bungs and faucets	8	12	10
Butchers' blocks and skewers	8	12	10
Car construction and repair:			
Coach trim	5	6	6
Sheathing, roofs, steps	10	20	12
Caskets and coffins	5	6	5½
Rough boxes	6	14	10
Chairs and chair stock	5	12	6
Clocks	5	6	6
Cooperage:			
Slack staves	6	12	7
Tight staves	5	7	6
Heading	5	7	6
Dairymen's, poulterers', and apiarists' supplies	6	11	9
Dowels	3	7	6
Elevators	6	10	6
Equipment, playground	8	12	10
Firearms	6	12	7
Fixtures	5	8	6
Flooring	6	10	6
Frames and molding, picture	5	8	6
Furniture	4	10	6
Gates and fencing	8	12	10
Handles	2	10	7
Instruments, professional and scientific	5	8	7
Instruments, musical, radio, piano cases, actions	3	6	5
Laundry appliances	6	12	9
Machine construction	6	18	10
Machinery and apparatus, electrical	2	6	5
Matches	10	12	10
Motion picture and theatrical scenery	6	15	10
Patterns	5	6	5
Flasks	5	9	7
Paving materials and conduits for treatment	18	40	25
Pencils and penholders	3	8	6
Pipes, tobacco	5	7	6
Planing-mill products	6	18	7
Plumbers' woodwork	4	7	6
Printing material, electrotype blocks, engraving blocks	3	5	5
Pulleys and conveyors	5	10	7
Pumps and wood pipe	6	10	8
Refrigerators and kitchen cabinets	4½	6	5
Rollers, shade and map	7	8	7
Screens	10	13	10
Sash, doors, blinds, and general millwork	4	8	6
Sewing machines	5	7	6
Ship and boat building:			
Frames	12	15	12
Planking	12	15	12
Decks	12	12	12
Spars	.	..	10
Cabin trim	6	11	8
Shuttles and bobbins	4	6	5
Spools	4	12	8
Signs and supplies	6	14	10
Switchboards	4	6	5
Sporting and athletic goods	7	10	8
Tanks and silos	8	16	12
Toothpicks	..	Green	6
Turned stock	4	16	7
Toys	6	7	6
Trunks and valises	4	9	6
Vehicles (non-motor)	8	12	10
Whips, canes, and umbrella sticks	6	14	9
Wheel spokes	2	5	4
Woodenware and novelties	5	8	6
Veneers:			
Face	2	7	4
Crossband	2	10	5
Cores	4	6	5
Plywood	2	9	6
Shingles	10	12	10
Lumber—rough construction	10	24	15
Lumber—joists, studs, sub-flooring	6	20	8
Stock to be bent	15	20	15
Poles	Air dry		
Ties	Air dry		
Posts	Air dry		

As one example, the veneer used in bushel fruit baskets is often very wet so that it will bend easily. The same is true of grape baskets. They must be dried after assembly to prevent stain and to reduce weight.

Clothespins and some rough-type turnings and legs for cheap furniture are turned in a green or air-dry condition, mainly because the wet wood machines more easily than dry wood. High-grade turning, however, is done at 6 or 7 per cent moisture, and with the proper knives and turning speeds, good results are obtained.

Stock which is to be bent is usually thoroughly air-dry before it is steamed and bent. There is less tendency to fracture in bending, but this has some drawbacks. Surface checking and honeycombing, as well as some warping, are likely to develop in the final drying process, which is often done with less care and attention to temperature and humidity than in the regular kiln process. In the construction of houses and factories, most joists, studding, and sub-flooring are used at moistures ranging from 15 to 25 per cent. After a year's service in a heated building they will dry to 6 or 8 per cent and shrink proportionately. Cracked plaster, sagging floors, door frames out of plumb, etc. are a result of such practices.

Stock such as shingles is often kiln-dried to a low moisture percentage simply to reduce the freight costs. They will reabsorb more moisture after being put into use. This practice is common with rough softwood lumber.

The data on wooden articles and wood uses in Table 2 were gathered by many personal contacts and experiences in the wood-using industries and by replies to several hundred questionnaires sent out early this year. The minimum and maximum moisture percentages represent the extremes found in those industries, while the average represents a composite picture of the more normal working conditions found, and in most cases can be taken as the ideal moisture condition desired. The percentages of moisture are for conditions just prior to manufacture.

Although one hears much in reference to climate and its effect on the ideal moisture content for any given location, this canvass shows that for any given article geographical location counts very little. Furniture made in New York City, Jamestown, Chicago, Grand Rapids, High Point, or Los Angeles is dried to about 6 per cent. This is also true of flooring, whether made in Shreveport, La., Williamsport, Pa., Tupper Lake, N. Y., Seattle, Wash., or Traverse City, Mich. Extreme cases can be cited against this statement in most any wood-using industry, but it must be kept in mind that there are enormous differences in quality in manufactured articles and that the average to better qualities were the only ones given consideration in this study.

Although air-dry lumber seldom gets below 12 per cent moisture, even after years of exposure, many articles shown in Table 2 as having 6 to 8 per cent have never been in a dry kiln. This is occasioned by the practice of taking well air-seasoned stock into the factory building and storing it in locations that are always warm and dry. After months and sometimes years of such exposure, the wood gradually dries down to an equilibrium moisture content of 5 or 6 per cent, or even lower. This condition of dryness is comparable to kiln-dried stock. Some firms still believe that kiln-drying injures wood structurally, and this accounts for these antiquated methods.

Discussion

Thomas D. Perry.[2] Professor Henderson has made a distinct contribution to the industry, but it is wished that he had dwelt more on the fact that wood is one of nature's growths, the character of which cannot be changed except through processing and conditioning. In other words, we may saw lumber radially or tangentially, i.e., quarter sawn or plain sawn, and have quite different characteristics in the respective boards.

Further, we may vary the moisture-removal treatments all the way from intensive drying green from the saw or a leisurely air drying, with final high-temperature drying before using.

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[2] Chief Engineer, United Plywood Sales Corp., New Albany, Ind. Mem. A.S.M.E.

Hence it seems that it would be entirely proper to place even more emphasis on using our knowledge of the processing and conditioning of lumber preparatory to manufacturing, since there are so many other factors in its characteristics that we have to take as nature supplies them.

It is the writer's observation and experience that woodworkers are notoriously careless about taking the necessary steps to maintain lumber in prime utilizable condition all the way through the remanufacturing process. It will not be long before we shall be obliged to have manufactured "weather" in all woodworking departments where precision in operation is necessary. This is already extensively practiced in textile operations, paper making, and cigar manufacturing.

This paper does much to hasten the day when such attention will be given to keeping lumber in the condition it ought to be for the ultimate stable product.

Fred F. Bahnson.[3] In the consideration of any engineering problem, it is usual to assume, among a number of variable factors, certain ones as being practically uncontrollable, and try to change the others to suit. As a manufacturer of humidifying equipment, with many years' experience in the control of air conditions in manufacturing plants, my point of view is quite different from the author's. His observations and experience seem to lead him to the assumption that humidities in factories will be very different in summer from those in winter—that is, subject to wide seasonal variations. The writer's experience over many years shows that it is not only possible, but also quite practical, to produce and to maintain, within very close limits, the same humidity the year round in any reasonably well-built factory room.

Excepting woodworkers, practically every industry whose material in process is affected by humidity changes now uses air-conditioning equipment as a matter of course. Textiles, printing, tobacco, ceramics, flour mills, and a host of others do not even try to operate without controlled humidity. Of all the industries now generally using artificial humidification, the printing industry most nearly parallels the woodworking industry. The stock used, paper, is largely composed of wood fibers, and consequently is affected by changes in relative humidity just as wood is affected. It is as important to the printer to maintain his paper uniform in size as it is to the furniture manufacturer to maintain his dimensional stock of uniform size. The printer has found that the only satisfactory way to do this is by maintaining a constant relative humidity. This is so well known throughout the industry that humidification is no longer considered merely as a desirable refinement, but is recognized as an absolute necessity for quality printing. Within a comparatively short time the same attitude will be manifested in the furniture manufacturing industry.

Instead of trying to dry lumber to fit seasonal variations due to humidity variations, that lumber should be dried to approximately the condition of use, or to the best working moisture content, and then the factory should be humidified to keep the predetermined and desired moisture content in the lumber the year round. Why manufacture shuttles, for instance, in a relative humidity of around 20 per cent (moisture content 5 per cent) when the average weave room where these shuttles are used is held at humidities ranging from 75 per cent to as high as 92 per cent (moisture content about 18 per cent).

It would seem that the sensible thing would be to dry lumber to the moisture content corresponding to the average relative humidity of 55 to 60 per cent which prevails over the United States, and then hold lumber storage rooms and factories, from start to finish (except perhaps the rooms where enamels are dried), at that humidity, so that if there are any changes, every piece in a built-up table top, for example, will expand and contract alike. What will happen if a built-up table top is made from several pieces, some of which have been stored outdoors or in an unheated room with humidity around 60 per cent, and from other pieces which have been stored in a factory in winter with a humidity of 30 per cent or less? One will swell in use, the other will shrink, and what about the veneer?

It is the writer's engineering opinion that if woodworking factories would use properly conditioned air the humidifying system would pay for itself very quickly in the saving of culls, checks, and cracks, to say nothing of the better health of operatives and the certainty of having dimension stock stay put.

The time element in the response of fairly thick wood stock to changes of humidity means that woodworking plants will not need as close regulation of humidity as textile mills, where the fibers are small and respond rapidly to variations in humidity, but the general principle is the same; and since stock is usually stored for some time under cover after leaving the kilns, the storage rooms as well as the factory should be held at practically constant humidity, and all these storage rooms and the factory should be at the same humidity.

Someone has raised the point that in the mill room of a woodworking plant there is so much air blown out through the dust-collecting system that it will be difficult to humidify such rooms. The number of complete air changes will be found to be around eight to twelve per hour, and that kind of problem is not at all unusual in other industries. Of course, such rooms require much more humidifying equipment than rubbing rooms, for example, but there is no inherently impractical condition to be met in any room in woodworking plants. There is no more reason to think a mill room cannot be humidified than to think it cannot be heated. The two problems are identical in principle, and the air changes affect heating just as much as humidifying.

Some woodworking plants use steam jets to produce a certain amount of humidity in the air of the plant. This practice is of course better than no humidifying at all, but with any type of steam jet there is always objectionable drip, and the vapor is of necessity warmer than the room air, which means condensation and eventually leads to rotting timbers and other unsatisfactory results. With a properly designed humidifying equipment, cold or slightly warm water is finely pulverized so as to be easily converted to true vapor, and a free circulation of the room air is produced which really improves working conditions and the general health of employees.

Kenneth Redman.[4] There are three comments on the paper that may be of interest. The first is that, in describing how to compute the moisture content of lumber, why is it always stated to multiply the quotient by 100. If a piece of wood weighs 12 grams, or 12 anything for that matter, and when bone dry it weighs 10, the loss is 2 parts of water. Obviously, 2 divided by 10 is 0.2 or 20 per cent. There is no increasing of any value by 100 times, and the writer believes it would simplify matters considerably for beginners if the Forest Service Laboratory simplified its instructions by omitting the phrase, "multiply by 100." The writer appreciates that the instructions actually provide for multiplying 0.20 by 100, and by adding the words "per cent" the product is thereby immediately divided by 100. But for the sake of brevity why introduce the figure 100 at all?

The majority of furniture manufacturers seem to consider 5 per cent moisture content as the necessary point to dry to, with some not particular if 6 per cent stock enters production. This of course refers to the samples tested. The writer believes these figures have become customary due to past uneven drying, and to prevent 9 to 12 per cent from entering production. Propeller stock during the war was not dried below 7 per cent, and it took 7 to 9 laminations to make a propeller, which must withstand enormous external stresses. Furthermore, tests of furniture several years old reveals a range of a fraction below 8 per cent in the winter to some tests a fraction above 9 per cent in the summer. The usual variation is slight. If the manufacturer took care to eliminate slow-drying pockets by building ample flues in the kiln load, or by mechanical circulation eliminated this hazard, would he not be justified in stopping his drying at, say, 7 per cent? He would not only save kiln time, but his finished product would be nearer to the equilibrium moisture content at which it is to remain during its useful life.

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[3] Chief Engineer and Vice-President, The Bahnson Company, Winston-Salem, N. C. Mem. A.S.M.E.

[4] Lumber Seasoning Service, High Point, N. C.

Lastly, many manufacturers carefully dry their stock, then place several protective coats of filler and finish on the exposed portions, and then leave the insides, under sides, and backs practically exposed to climatic changes. At most, one coat of stain or filler is given these portions. If these places were adequately protected against the infiltration of moisture, the annoyance of stuck bureau and desk drawers and immovable file cases, etc., would soon be but a memory.

WDI-54-9

Some Observations on Raised Grain

By ARTHUR KOEHLER,[1] MADISON, WIS.

Woodworkers and finishers are confronted with certain difficulties classed under the category of "raised grain" which, so far, have resisted satisfactory solution. This paper describes some observations on raised grain that may lead to a better understanding of the problems and which may suggest approaches toward overcoming them. Casual investigations made of raised grain suggest some remedial measures, and indicate that there is a large field for research connected with the surfacing of lumber, both as to the cause of the objectionable features encountered and methods of overcoming them. A microscopical study of the way that present-day woodworking tools and machinery cut wood and the development of new equipment which would give more nearly ideal results seem to offer possibilities which would go a long way in improving wood products or reducing their costs.

TWO TYPES of raised grain are commonly recognized and spoken of as such: (1) That in which fibers or parts of fibers project above the surface, and (2) that in which the whole summerwood (or springwood) of the annual rings extends above the surface, giving it a corrugated appearance. A third type in which the tips or edges of annual rings become loose and rise up also occurs. It is more commonly known as "loosened grain," which probably is a better descriptive term, and, in fact, has been accepted as a standard term under American Lumber Standards.[2]

Under the first of these types of raised grain there is one form in which individual fibers, groups of fibers, or even fragments have been loosened at one end, and these ends project like fuzz or minute slivers from the surface after planing or sanding. (Fig. 1.) This phenomenon seems to be a characteristic of certain kinds or types of wood rather than of the method of surfacing them. For example, in some species, such as cottonwood and elm, it occurs commonly, whereas in mahogany, oak, and certain other species of hardwood, it is troublesome in only certain types of lumber. Whether in such cases the fibers are not equally well attached by growth along their full length or whether they are more interlaced, so as to make it difficult for the end which remains fastened to pull out, has not been determined. Possibly in the wood which gives little trouble in this respect the fibers split lengthwise more easily, and, thereby, projecting portions are more easily torn off independent of their attachment to neighboring fibers. A detailed microscopical study of the behavior of the fibers would make an interesting contribution to the subject of raised grain. It has already been found that individual fibers have a natural tendency to twist as they dry. Therefore, any loose ends or fragments of fibers on dry wood would be expected to protrude and assume any position but that which they had in the greenwood. The fact that this kind of raised grain occurs more commonly in some than in other pieces of the same species also raises the question of how the conditions under which a tree grows affect the quality of the wood in it, a subject about which little is known, but one that is important in any kind of timber-growing program.

Another form of raised grain observed is that in which groups of fibers and pore walls overlying the pores are pressed down into the pores in sanding, leaving the surface fairly smooth for the time being, but eventually resulting in the protrusion of these depressed portions, especially if the surface is moistened with any liquid that has a swelling effect on wood, such as water or spirit stains.

Fig. 2*a* shows a surface of mahogany which had been sanded shortly before photographing and Fig. 2*b* shows the same after moistening. The fragments of fibers and pore walls projecting over the pores can readily be seen in Fig. 2*b*. The same is shown even more strikingly for a sample of red oak in Figs. 3*a* and 3*b*. The photographs from which these illustrations were made were taken with oblique light so that all projections cast distinct shadows, thereby making them more conspicuous.

The mahogany and oak specimens were sanded lightly by hand with fine sandpaper, but even under those conditions the "caving in" of the pores could not be prevented. It is obvious that under heavier pressure this condition would be even more serious.

After the microscope had revealed the origin of the raised grain, planing with a sharp hand plane without sanding was tried to see if a smoother surface could be obtained in that way. The results are shown in Figs. 2*c*, 2*d*, 3*c*, and 3*d* for both mahogany and oak. It can be seen that the pores lying at the surface were cut open smoothly without pressing any fibers into them. On moistening the surface there was practically no raising of the grain.

Furthermore, the hand-planed surface of mahogany had a distinctly more brilliant luster than the sanded surface, on which the raised grain had a "frosting" effect. How much of this superior brilliance would be retained after varnish or other transparent finish is applied was not determined. There is a possibility that it might even be enhanced, since transparent finishes usually enhance whatever natural luster wood has.

The second type of raised grain, namely, that in which the summerwood or springwood of the annual rings stands out above the surface after planing, is a common annoyance in the finishing of softwood lumber in particular, although occasionally it also gives trouble with hardwoods. One case in which yellow poplar, which is a wood of exceptional homogeneity, gave trouble in the manufacture of clock cases came to the author's attention. Any irregularity of the surface of finished parts shows up, especially when a glossy coating is applied, on account of the distortion produced in the reflection of light.

Flat-grain lumber in particular gives trouble in this respect, the trouble arising from the planer pounding the harder summerwood into the softer springwood underneath, and both later regaining their former relative positions, thereby raising the sum-

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[1] In charge of the Section of Silvicultural Relations, Forest Products Laboratory, Forest Service, U. S. Department of Agriculture, maintained in cooperation with the University of Wisconsin. Mr. Koehler has been engaged since 1911 in the study of the cellular structure of wood for identification purposes and to explain the variations in the behavior of wood.

[2] For a description of loosened grain see "Raised Grain: Its Cause and Prevention," by Arthur Koehler, *Southern Lumberman*, Dec. 15, 1929; and *Lumber Trade Journal*, Dec. 15, 1929.

Presented at the Sixth National Wood-Industries Meeting, Winston-Salem, N. C., Oct. 15 and 16, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

merwood above the plane of the surface. (Fig. 4.) This regain of shape may occur gradually with the lapse of time or rapidly if the surface is moistened. Here, as in the previously mentioned type of raised grain in which the fibers are pressed into the pores, a peculiar characteristic of wood comes into play—namely, that wood if crushed will partly resume its original dimension, particularly so if it is moistened with hot water. It is an old woodworking trick to take dents out of wood by applying a wet cloth and hot flat iron over the dent.

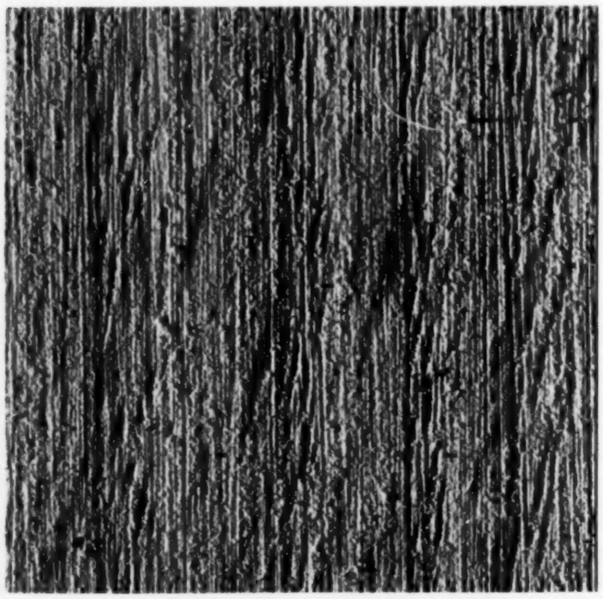

Fig. 1 Raised Grain on Surface of Oak After Sanding but Not Sponged. Magnified 4.1 Times

This type of raised grain occurs principally in lumber that is flat grained or nearly so because the hard summerwood is superimposed over the softer springwood, whereas in edge-grained lumber both springwood and summerwood layers are more or less at right angles to the surface and do not offer opportunity for such deformation in the planer.

In flat-grained softwood lumber it has been found that this type of grain raising is much more pronounced on the pith side than on the bark side of a board. This is due to the fact that on the pith side the outer and harder part of the summerwood immediately overlies the inner and softer part of the springwood underneath. (Fig. 4.) The hard summerwood resists the cutting of the planer most, and therefore is pounded into the immediately underlying springwood. On the bark side of a board the inner and softer portion of the summerwood runs out to a thin edge at the surface. It does not resist cutting so much, and for that reason and because the springwood immediately below the summerwood is a little harder than at the beginning of the annual ring, the summerwood is not pressed into the springwood so readily as on the other side of the board.

A somewhat similar appearance is sometimes produced by dull planer knives that tear away more of the softer springwood, leaving the summerwood projecting above the surface; or in the sanding operation more of the springwood may be taken away if care is not taken. Ordinary wear, such as occurs in unprotected flooring, rustic furniture, drain boards, and lumber exposed to the weathering action of the elements, may also produce a similar effect.

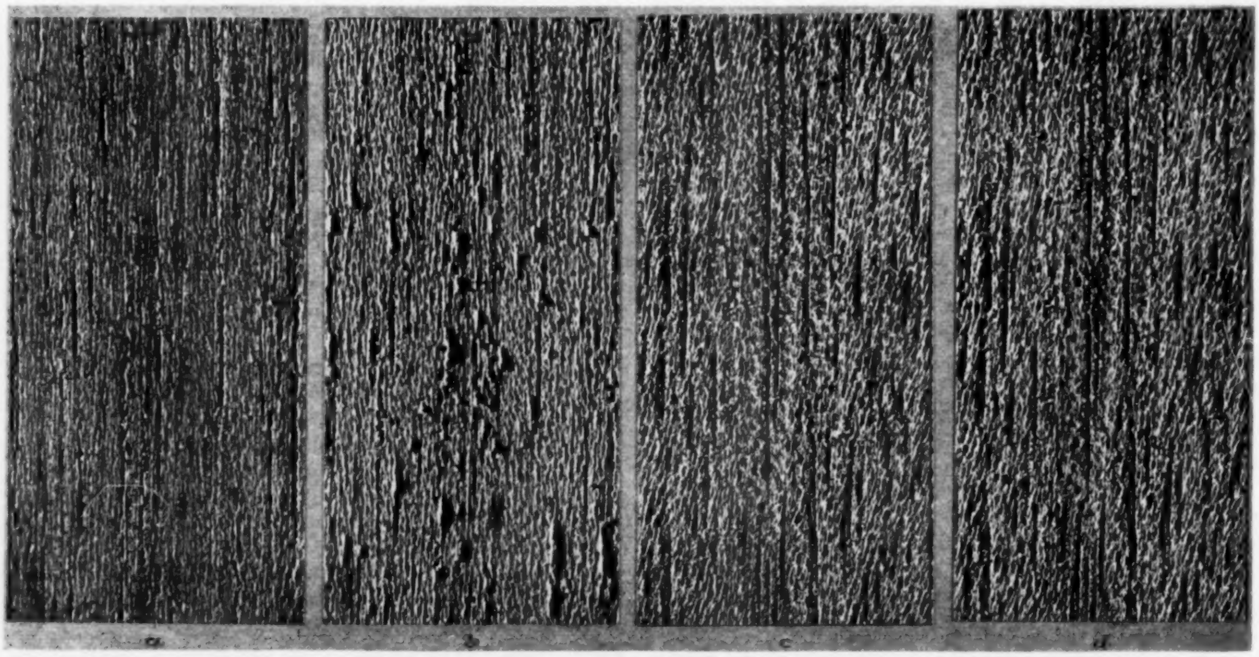

Fig. 2 Plain-Sawed Mahogany. Magnified 4.1 Times
(*a*, Surface sanded but not sponged; *b*, same after sponging; *c*, surface hand planed but not sponged; *d*, same after sponging.)

Another cause of corrugation of lumber is the greater swelling and shrinking of summerwood as compared with springwood. If surfaced lumber dries out considerably after surfacing, the summerwood will become depressed below the springwood, or if it takes on moisture the summerwood will protrude beyond the springwood, in either case leaving the surface corrugated. This condition usually occurs to a more pronounced degree in edge-grained than in flat-grained stock, because in the former the lateral shrinking and swelling of the springwood and summerwood takes place at right angles to the surface and therefore differences are more pronounced.

Occasionally a similar type of raised grain may be produced by uneven pressure on the surface of lumber before it enters the planer. Fig. 5 shows a piece of Western yellow-pine window sash which was very uneven on one surfaced side although smooth on the other. An examination with a magnifying glass showed that the cells in and below the ridges were partly compressed. Evidently they had been compressed even more, but had partly resumed their original shape, thereby rising above the rest of the planed surface. The fact that these ridges were of irregular length and had no relation to the grain of the wood indicates that they were not due to imperfections in the knives or irregularities in the grain, but to uneven pressure on the wood before it reached the knives. Such a condition might arise from sliding the rough lumber over uneven surfaces during handling or on the feeding side of the planer.

A coarser type of corrugation sometimes occurs on the surface of sawed lumber, as illustrated in Fig. 6, which may be mistaken for a type of raised grain. In this case, however, the elevations and depressions have no relation to the grain of the wood. Neither are they the result of shrinking or swelling, since it is evident on close examination that the hollows are actually gouged out. The extreme regularity of these corrugations attests that they are of mechanical origin and undoubtedly are due to vibration of the band saw, the vibrations passing up the saw a little slower or faster than the saw travels downward, thereby making diagonal marks as the saw passes through the log.

REMEDIAL MEASURES

In the foregoing it has been pointed out that there are various types of raised grain and that they are more prevalent in some kinds or cuts of lumber than in others. So far no extensive experiments have been carried on at the Forest Products Laboratory to determine how raising of grain may be prevented or reduced, or how to select wood without this objectionable feature. Observations on the cause of raised grain, however, have automatically suggested some remedial measures.

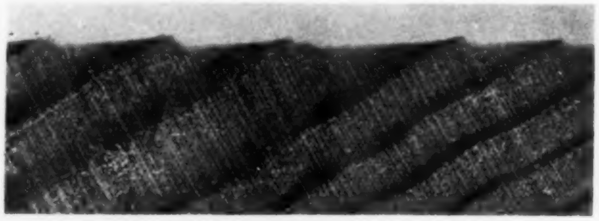

FIG. 4 CROSS-SECTION OF WHITE-FIR PARTITION STOCK, SHOWING RAISED GRAIN DUE TO PROJECTION OF SUMMERWOOD. MAGNIFIED 4.4 TIMES

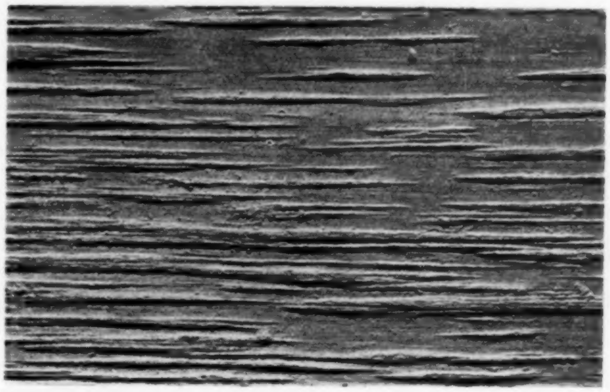

FIG. 5 SURFACE OF SASH RAIL SHOWING RAISED GRAIN DUE TO COMPRESSION OF WOOD BY EXTRANEOUS OBJECTS BEFORE PLANING

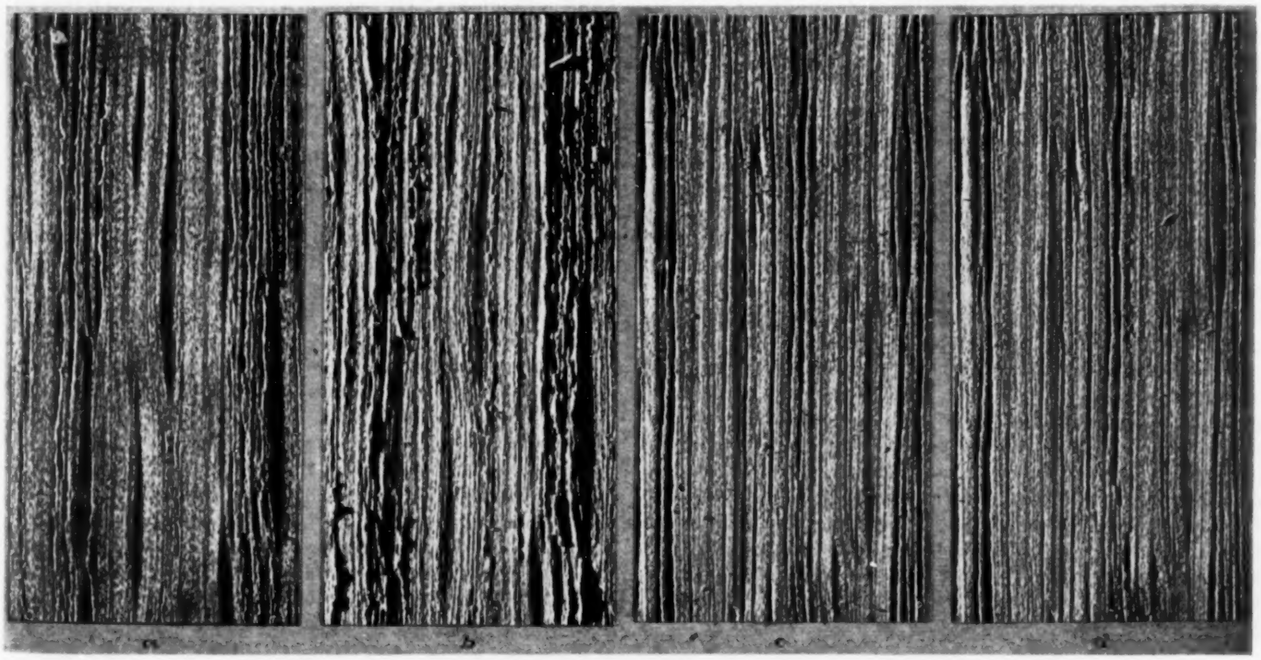

FIG. 3 PLAIN-SAWED OAK. MAGNIFIED 4.1 TIMES

(*a*, Surface sanded but not sponged; *b*, same after sponging; *c*, surface hand planed but not sponged; *d*, same after sponging.)

So far as reducing the projection of fibers or fragments of fibers is concerned, it appears that sanding the surface is not so helpful as planing the wood with a sharp hand plane. In the first place the granules on sandpaper have a tearing action which tends to

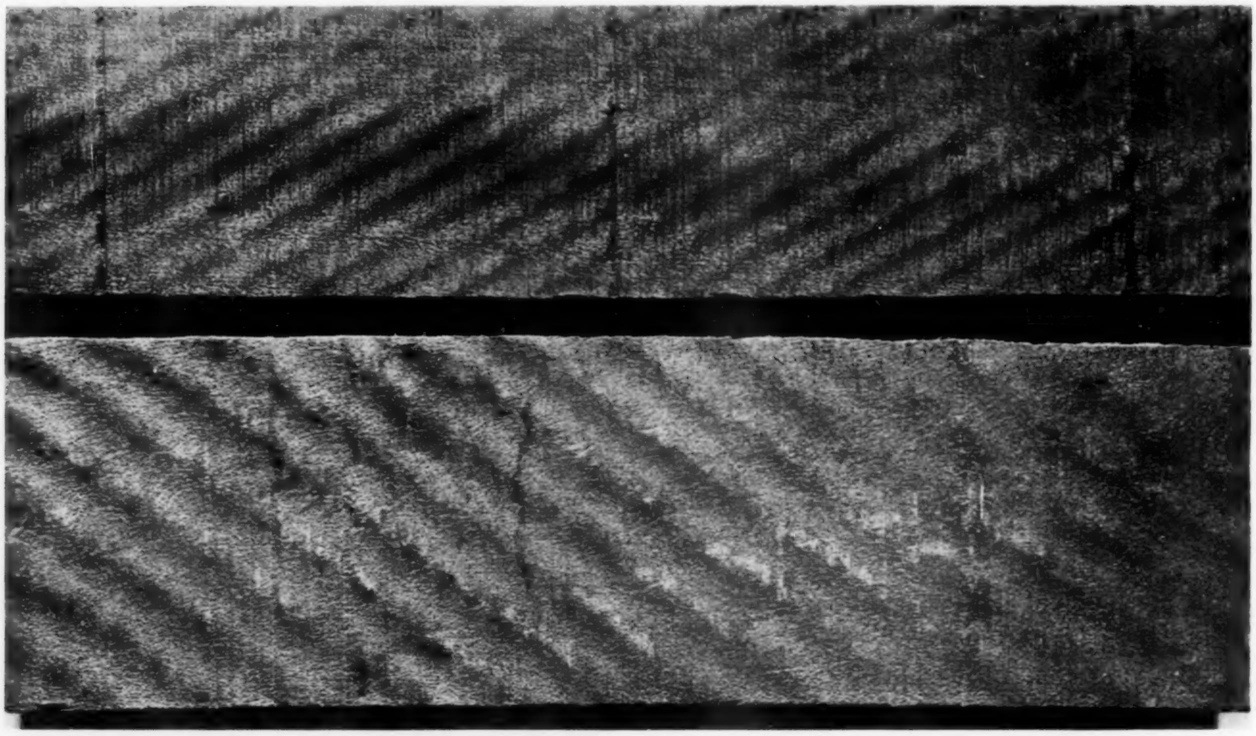

Fig. 6 Corrugated Surfaces of Tupelo Gum at Top and Red Gum at Bottom Due to Vibration of Band Saw

loosen some fiber fragments, and in the second place the pressure which must necessarily be applied in sanding forces fibers and other tissues into the surface pores only to bulge out later, especially if a water stain is applied. Sponging and a second light sanding eliminates a large part of the raised grain, but involves additional expense and is not entirely satisfactory, especially with refractory woods.

Obviously hand planing also is not commercially feasible on account of the expense involved and the difficulty of getting a perfectly smooth finish on flat surfaces. Machines with rotary cutters as designed at present do not solve the problem on account of the "planer marks" which they are apt to leave. Furthermore, such machines could not be used on panels overlaid with thin veneer since it would be impossible to keep the planer from going too deep in places. Possibly a redesign of present machinery or the invention of entirely new methods of wood surfacing are required for the solution of this perplexing problem.

The type of raised grain in which the summerwood projects above the surface also presents a serious problem. As already pointed out it is much more pronounced on the pith side than on the bark side of a board, and edge-grained lumber does not show it so much as flat-grained stock. Lumber therefore can be selected to some extent to avoid excessive grain raising of this type. Undoubtedly the sharpness of the planer and the bevel of the knives also are important factors. Lumber surfaced with a hand plane or sharp rotary planer does not show so much of this type of raised grain as that surfaced with the usual commercial rotary planers.

Discussion

J. C. **Duke.**[3] The author's observation on the non-effectiveness of "sanding" raised the question concerning the care and technique adopted in his sanding practice. Due to the generalness of the statement that "specimens were sanded lightly by hand with fine sandpaper," the question was asked what was meant by "fine sandpaper."

"Sandpaper" is far too broad a description, as it encompasses a field of five distinctly different types of mineral, each with specific properties to meet definite requirements. "Fine sandpaper" includes a vast range of grit sizes, a variety of backing weights, types of mineral coating, and different bond formulas.

It was developed that the author in his term "fine sandpaper" referred to $^1/_0$ and $^2/_0$ flint paper. Flint mineral, by reason of its points of hardness, toughness, and mineral fracture, is one of the least efficient abrasives for woodworking purposes. Due to the relative dullness and lack of hardness of flint mineral, it was thought that some of the observations in the panels described by the author might be accounted for.

There was also the question of how the author's results could compare in measure with those obtained from mechanical sanding procedure in modern woodworking plants. There is a long step from results obtained in surfacing by hand application and by machine. In hand application the abrasive paper passes in the direction of the wood grain and back against the grain, while modern mechanical sanding practice sands or cuts in one direction. It was felt that hand sanding would be analogous with planing where the bit of the plane would cut the surface on the forward stroke with the grain and then reverse the stroke against the grain. Mechanical sanding can be compared with planing where the work is surfaced, the cut being made with the grain. In addition, modern sanding equipment is designed to obtain certain specific results where sanding pressures are controlled and where speed of cut, coupled with selection of type of abrasive grain, delivers the result needed to fit specified requirements.

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[3] Minnesota Mining and Manufacturing Co., Philadelphia, Pa.

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WDI-54-10

Studies in the Design of Local Exhaust Hoods

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J. M. Dallavalle

T. Hatch

The failure of many local exhaust systems properly to control industrial dust hazards may be attributed, in part, to the lack of sufficient knowledge of the factors governing the collection efficiency of various types of hoods. The present study was undertaken in an effort to obtain data on the aerodynamic characteristics of simple geometric suction openings which may serve as a basis for future studies of the problem of design of local exhaust systems. Round, square, and rectangular openings were investigated and their aerodynamic characteristics are expressed by means of velocity contours or lines of equal velocity.

Contour lines were found not to be influenced by the total airflow through the hood. They did not differ in their chief characteristics with the geometric shape of the opening, although rectangular openings exhibited a flattening of their contours that varied with the ratio of the sides. An increase in the area was found to cause a displacement of contour lines at right angles to the axis of the opening. Along the axis, however, velocities coincided at distances of 6 in. or more from the face of the hood.

Gross characteristics of suction openings, it was found, could be expressed by means of center-line velocity curves. These curves, which are obtained with less labor than are the velocity contours, may be compared graphically and also mathematically. Simple charts are presented and mathematical equations were obtained for the three types of openings that may be used within the zone of hood influence.

LOCAL exhaust ventilation for the removal of dusts and fumes has been applied with varying degrees of success in almost every industry. Its general application has been recognized by most states, and legal specifications for design have been incorporated in many industrial codes.

Although the use of local exhausts has undoubtedly reduced the health hazards associated with industrial dusts and fumes, recent dust surveys in certain industrial plants have demonstrated that many systems have been badly designed and do not have the required efficiency of dust removal. (1) (2) (3)[3]

The low efficiency of local exhaust systems may be attributed, in most cases, to the improper design and placing of collecting hoods and failure to provide sufficient airflow through them. As a matter of fact, but little fundamental information on the design of hoods is available at the present time. The common basis of design is the rate of airflow through the hood as measured by the negative static pressure at the hood throat. Tables of suction values, based upon experience, have been compiled for various dust-producing processes. (4) The need for wide experience and good judgment in the general use of these factors is stressed, although state codes require rigid adherence to legal specifications. Without attempting to minimize the importance of experience in engineering design, it seems proper to point out that most of the past experience in the design of local exhaust hoods has not been associated with quantitative studies of the actual efficiency of dust removal. Little consideration has been given to the characteristics of different types of hoods or to their relative efficiencies when applied to the various dust-producing processes. This is clearly demonstrated by the fact that the various state codes do not agree one with another. A more complete knowledge of the function and characteristics of various types of suction openings is therefore needed. The final test of efficiency of dust removal should, of course, be based upon actual dust determinations.

The authors have studied the aerodynamic characteristics of a number of simple geometric openings under suction. Results of their investigations are presented herewith, and the data presented should serve as a starting point for more elaborate studies and the collection of detailed information on the design of local exhaust hoods.

In operation, an exhaust hood creates an air velocity at the point of dust generation which counteracts the tendency of the dust to escape into the surrounding space and, if sufficiently great, carries it into the collecting system. The minimum air velocity required for the efficient removal of a particular dust depends upon a number of factors associated with the dust-producing process such as the size and weight of the dust particles and the force with which they are thrown off. The air velocity to be created at the point of dust generation, therefore, constitutes the primary specification upon which the design of exhaust hoods must be based.

The air velocity obtaining at a given point in the zone of influence of a suction opening depends upon a number of factors which must be considered in the design. Chief among these are the shape and size of the opening, the distance from the hood, the position of the hood, and the total airflow through the opening. These factors constitute what may be termed the aerodynamic characteristics of suction openings, and their influence upon the air velocity at the point of dust generation should determine the choice of hoods and minimum airflow requirements.

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[1] With the Department of Trade Waste, City of Cleveland, Ohio. Received a Bachelor's degree in electrical engineering from Harvard University in 1927 and did graduate work in industrial hygiene and air conditioning. In 1927 he was a member of the O. L. Austin Ornithological Expedition to Labrador. His work for the City of Cleveland, since 1930, has been on the control of dusts, smokes, and fumes with special reference to the design of air-filtration apparatus.

[2] Instructor in sanitary engineering, Harvard Engineering School. Mr. Hatch received the degree of B.S. in 1924 from University of Maine and of S.M. in 1925 from Harvard University. He has made investigations of the size and properties of non-uniform substances, and of the methods and instruments for the determination of air-suspended concentrations. He has designed dust-control systems for use with pneumatic granite-cutting tools and for the control of dust generated by pneumatic rock drills. Before taking his present position, he was assistant sanitary engineer, State Department of Public Health, Nashville, Tenn.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Numbers in parentheses apply to references at the end of the paper.

TABLE 1 TYPES AND DIMENSIONS OF HOODS INVESTIGATED

Type	Diameter, inches	Ratio of sides	Dimensions of opening, inches	Area of opening, sq. in.
Round	4	...	....	12.6
Round	6	...	....	28.3
Round	8	...	....	50.2
Round	11.3	...	....	100
Round	16	...	....	200
Square	..	1.000	4 × 4	16
Square	..	1.000	6 × 6	36
Square	..	1.000	8 × 8	64
Square	..	1.000	10 × 10	100
Rectangular	..	0.750	4.1 × 5.5	22.6
Rectangular	..	0.750	6 × 8	48
Rectangular	..	0.750	7.5 × 10	75
Rectangular	..	0.750	12 × 16	192
Rectangular	..	0.667	4 × 6	24
Rectangular	..	0.667	5 × 7.5	37.5
Rectangular	..	0.667	6 × 9	54
Rectangular	..	0.667	12 × 18	216
Rectangular	..	0.667	18 × 27	486
Rectangular	..	0.500	4 × 8	32
Rectangular	..	0.500	5 × 10	50
Rectangular	..	0.500	6 × 12	72
Rectangular	..	0.500	8 × 16	128
Rectangular	..	0.500	15 × 30	450
Rectangular	..	0.333	3 × 9	27
Rectangular	..	0.333	4 × 12	48
Rectangular	..	0.333	5 × 15	75
Rectangular	..	0.333	8 × 24	192

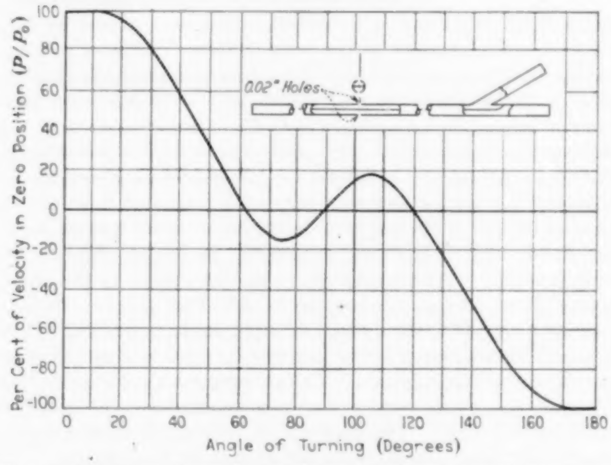

FIG. 1 MODIFIED PITOT-STATIC TUBE AND EFFECT OF TURNING ON VELOCITY-HEAD READING

The value of this method of approach has been demonstrated in a study of the design of exhaust hoods for use with pneumatic granite-cutting tools. (5)

In the present study, the characteristics of round, square, and rectangular openings were investigated. The rectangular hoods were classified according to the ratio of the sides of the openings. These ratios were four in number; viz., $^3/_4$, $^2/_3$, $^1/_2$, and $^1/_3$. The number and sizes of the various openings examined are given in Table 1.

LABORATORY SET-UP

The exhaust system consisted of 40 ft. of 8-in. round duct connected to the intake of a 30-in. steel-plate planing-mill exhauster. Air was discharged from the fan through an outlet chimney which reduced its velocity. Owing to the lowered outlet velocity and the long exhaust duct, the circulation of air in the room caused no disturbance within the zone of influence of the experimental hoods inserted in the open end of the duct. Varying rates of airflow through the hoods up to 2000 cu. ft. per min. were obtained by means of a variable-speed motor, and measurements of airflow through the system were determined by means of a standard 6-in. thin-plate orifice inserted in the exhaust duct. The static-pressure taps of the orifice were located as specified in the report on fluid meters published by The American Society of Mechanical Engineers. (6) During the latter part of the study the orifice was replaced by a venturi meter.

THE MODIFIED PITOT-STATIC TUBE

The instrument used to measure point velocities in the region of hood influence is shown in Fig. 1. It consisted of a straight brass tube 26 in. in length and $^1/_8$ in. in diameter, and was separated into two chambers throughout its length by means of a thin partition. Two holes, 0.02 in. in diameter, were drilled in the mid-section diametrically opposite each other. Each chamber was carefully sealed off from the other and constituted a separate pressure chamber with its own terminus. Because of its small diameter, this instrument probably caused little disturbance of air motion and served to determine the velocity at a given point. In this respect, it was superior to other instruments available.

In operation, the instrument was placed in the air stream with the axis through the pressure openings parallel to the direction of flow. Thus, the total pressure of the air stream was produced in the chamber with its opening facing the stream,[4] whereas the pressure produced in the second chamber was a function of the static pressure of the stream. The modified tube was calibrated against a standard pitot-static tube and it was found that the modified tube gave readings from 1.75 to 2.00 times higher than those obtained with the standard tube; the coefficient was constant at 1.75 for velocities above 950 ft. per min. The calibration curve for the instrument is shown in Fig. 2 for air velocities from 70 to 2000 ft. per. min. Pressure readings are in inches of alcohol and for an air temperature of 80 deg. fahr.

The rotational characteristics of the tube are shown by the graph in Fig. 1. The per cent of maximum pressure (obtained when the openings are parallel to the airflow) is plotted against the angle through which the tube has been rotated. The maximum reading of the tube occurs over a range of ±15 deg., and shows that the instrument is not particularly sensitive to direction in this region. On the other hand, zero pressure readings were recorded at 63 deg., 90 deg., and 117 deg. from the direction of airflow. Therefore, by careful location of any one of these zero positions, the direction of airflow could be determined quickly.

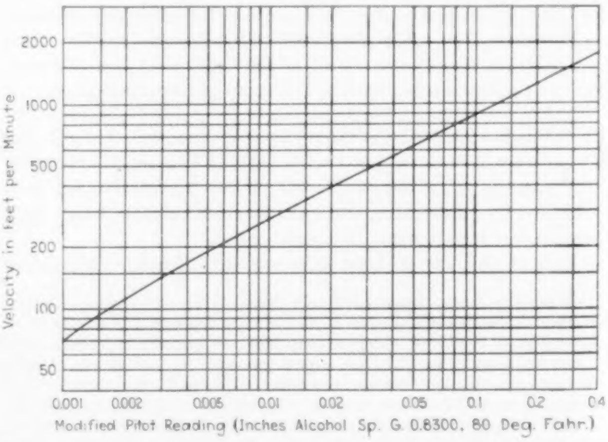

FIG. 2 CALIBRATION CURVE FOR MODIFIED PITOT-STATIC TUBE

Pressure readings were made with a Wahlen gage (7) which was sensitive to 0.001 in. or less of alcohol. Thus, air velocities as low as 100 ft. per min. were determined with considerable accuracy.

VELOCITY DETERMINATIONS

The hood to be studied was connected to the exhaust duct and air was drawn through it at a measured rate. The modified pitot-static tube was then brought to those points within the region of influence where velocity determinations were to be made. Careful location of the tube with reference to the hood was obtained by means of a suitable stand capable of movement in horizontal and vertical directions. Velocity measurements were made at coordinate points in a vertical plane through the axis of symmetry of the hood as indicated in Fig. 3-A. Readings were taken as far out from the face of the hood as possible, the limiting factor being the sensitivity of the Wahlen gage.

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[4] Since the total pressure of the air stream in space is zero, this chamber was not absolutely essential. Because of fluctuating flow, however, it was felt that the pressure readings would be less subject to errors than with one leg of the manometer directly open to the atmosphere.

Owing to the symmetry of a round opening, velocity measurements in one plane served to determine the characteristics of the opening throughout its sphere of influence. Such was not the case, however, for the square and rectangular openings. In the latter, the velocity conditions were not the same in planes at right angles to each other, and in both cases the corners disturbed the flow considerably. A study of air motion at the corner of a suction opening necessitates the determination of three-dimensional flow and involves a vast amount of labor. The mapping of the whole field of influence of square and rectangular openings was therefore considered impractical. For the square openings, velocity measurements were made in one plane only, through the axis and bisecting one side. For the rectangular openings, studies were made in two center-planes at right angles to each other and bisecting the sides of the opening. Velocity determinations were made at two or more different rates of airflow.

VELOCITY CONTOURS

The velocity measurements were plotted in the manner shown in Fig. 3-B. From the resulting curves velocity contours or lines of equal velocity were obtained. For example, to obtain the contours of velocities equivalent to 20 per cent and 70 per cent of the average velocity at the face of the opening, the values of the abscissas corresponding to the intercepts of these ordinates were obtained for the various velocity curves, as in Fig. 3-B, and plotted as in Fig. 3-C. Velocity contours were prepared in this manner for the several openings investigated.

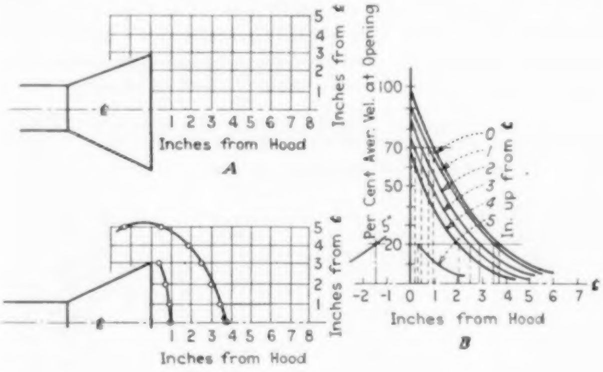

FIG. 3 PROCEDURE IN MAKING VELOCITY SURVEY IN THE ZONE OF INFLUENCE

(A, Coordinate points at which velocities were determined. B, Change in velocity, with distance from hood parallel to axis. C, Velocity contours, plotted from curves in B.)

CENTER-LINE VELOCITIES

It was found, during preliminary studies, that the characteristics of suction openings could be measured grossly by means of the change in air velocity along the axis of the hoods. Center-line velocities were therefore obtained for the various openings studied.

VELOCITY CONTOURS

The aerodynamic characteristics of suction openings, expressed in terms of the velocity contours have been examined with reference to the following factors: (*a*) Volume of airflow through the opening, (*b*) geometric shape of opening, (*c*) area of opening, and (*d*) shape of transition piece.

Effect of Volume of Airflow on Shape of Velocity Contours. An examination of the velocity contours for a given opening shows that they have the same shape regardless of the quantity of flow. This is clearly demonstrated by the fact that the ratio of the velocity at any point to the average velocity across the suction opening remains constant for all flows. Thus, an 8-in. round opening gave approximately constant ratios of point velocity to the average velocity at the hood opening at corresponding points for flows of 1000, 8000, and 450 cu. ft. per min. (Table 2.) This constant ratio was found to hold true for all the openings studied in the present investigation.[5]

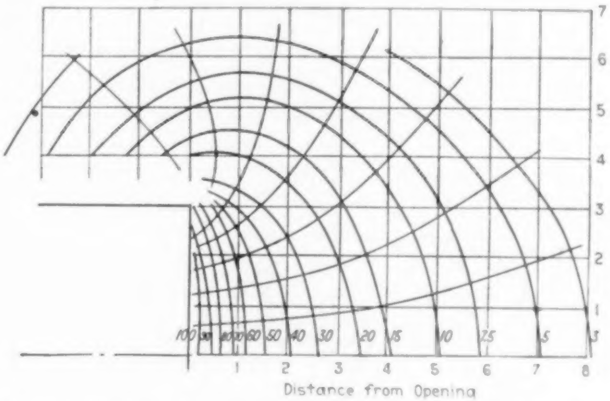

FIG. 4 VELOCITY CONTOURS FOR 6-INCH DIAMETER OPENING. CONTOURS EXPRESSED AS PERCENTAGES OF AVERAGE VELOCITY AT FACE OF OPENING

The average velocity at the face of the hood was calculated from the rate of airflow, measured by the orifice or venturi and the area of the opening. It was observed, however, that the 100-per cent contour did not coincide with the face of the hood, but was located at a distance outward which varied with the size and shape of the opening. This is shown by the velocity contours for a 6-in. diameter opening in Fig. 4. Owing to the disturbance at the edge of the opening, where a sudden change in direction of flow took place, the effective area decreased and hence produced a velocity across the central region higher than the average velocity for the opening. The amount of restriction appears to be a function of the area and perimeter of the hood.

TABLE 2 RELATION BETWEEN VOLUME OF FLOW AND RATIO OF POINT VELOCITY TO AVERAGE VELOCITY FOR 8-INCH DIAMETER DUCT END

Distance from hood, inches	Per cent average velocity Q = 1000 c.f.m.	 Q = 800 c.f.m.	 Q = 450 c.f.m.
1	82.0	83.0	82.2
2	56.6	56.0	56.0
3	35.7	37.6	37.4
4	26.6	26.2	24.8
5	19.4	18.4	17.0
6	12.1	12.9	12.4
7	9.0	10.4	8.4
8	7.99	7.52	7.6
9	6.02	6.55	..
10	4.75	5.35	..

Effect of Geometric Shape of Opening. The velocity contours of all openings studied do not differ greatly in their chief character-

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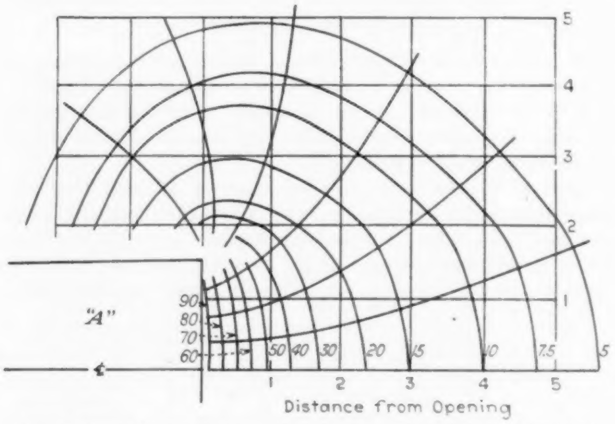
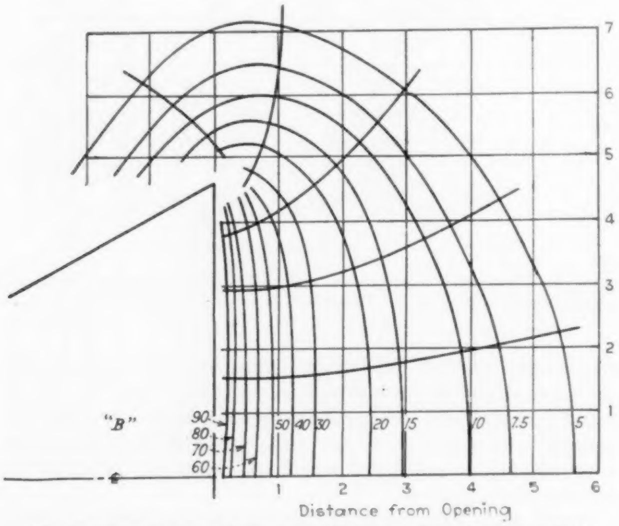
[5] This observation is in accordance with the mathematical theory of contour lines. (8)



(*A*, Contours in center-plane, perpendicular to long side of opening.)



(*B*, Contours in center-plane, perpendicular to short side of opening.)

FIG. 5 VELOCITY CONTOURS FOR 3-INCH BY 9-INCH OPENING

istics. The contours are nearly parallel to the face of the hood over the region projected by it, and bending generally takes place in the region corresponding to the edge. As the distance from the opening increases, the bending of the contours becomes more gradual and the velocity lines terminate on the surface of the transition piece behind the opening.

Contours in one center-plane determine the shape of contour surfaces of round openings, owing to their geometric symmetry. In the case of square and rectangular openings, however, the distribution of airflow at the corners differs from the distribution in center-planes. The effect of the corners is to reduce the effective area of the opening which results in a general outward displacement of contour lines as compared with those for round openings of the same area. In the case of rectangular openings, moreover, the flow distribution differs in the two center-planes. The difference is proportional to the ratio of short to long side. There is a flattening of contours along the long side and a corresponding shortening in the plane bisecting the short side. Along the axis there is an inward displacement of contour lines as compared with round openings having the same area. Contour lines for the two sides of a rectangular opening 3 in. by 9 in. in dimension are shown in Fig. 5. They may be compared with the contours of the round opening having approximately the same area shown in Fig. 4.

Effect of Area of Opening. An increase in the area of the opening, with the geometric shape remaining the same, causes an outward displacement of the contour lines at right angles to the axis of the hood. The rate of decrease of velocity along the axis is reduced, but at distances of 6 in. or more from the face the velocities tend to coincide, regardless of the area of the opening. This is shown by the velocity contours for the same flow for 4-in. and 8-in. square openings in Fig. 6. Thus, there is considerable advantage to be gained in using a large opening for the collection of dust at some distance from the hood. Large openings not only give a wider and better contour distribution for the same quantity of flow, but also reduce the loss of energy at the hood entrance.

It should be remembered that in choosing a hood for a particular operation, consideration must be given to the area and distribution of dust generation; i.e., the hood chosen should create maximum flow within the "effective area" of dust generation and minimum flow from areas in which no dust is produced (5, p. 88).

Shape of Transition Piece. The extent to which abruptness of transition enters into the distribution and shape of the velocity contours is of importance since the location of hoods often limits the amount of transition space available. This effect has been examined for three types of 4 in. by 8 in. openings. The transition pieces for the three openings included a simple duct end (no transition), a gradual transition to a 4-in. round duct in a distance of 7.75 in., and an abrupt transition to a 4-in. round duct in a distance of 3.5 in. Point velocities along the center lines of the three hoods are compared in Table 3. The results show no change in velocity with abruptness of the transition piece. Complete velocity surveys were also made for these openings and no change in position of the contour lines was observed. A comparison of contours of a plain 8-in. diameter duct end with a similar opening having a conical transition to a 4-in. duct in a distance of 6 in. also showed that the shape of the transition piece did not materially change the distribution of airflow in the region of influence. Thus it may be said that within the limits of this study the shape of the transition piece has no significant effect upon the distribution of airflow about the suction opening.

TABLE 3 EFFECT OF FLARING ON THE CENTER-LINE VELOCITIES FOR THREE FORMS OF 4-INCH AND 8-INCH HOOD

(Velocities in per cent of average velocity at opening)

Distance from opening, inches	Type of hood		
	Rect. duct end	Gradual opening	Abrupt opening
1	66.0	65.5	65.0
2	37.5	37.5	36.2
3	23.4	23.0	22.6
4	15.1	15.2	14.6
5	10.7	9.20	9.60
6	7.61	6.90	7.00
7	5.50	4.40	4.54
8	3.84	3.72	3.08

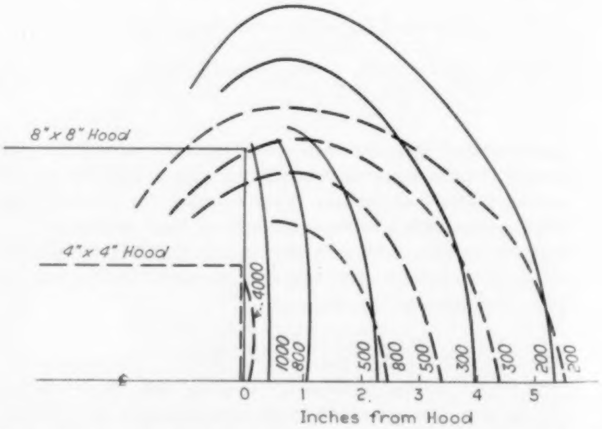


FIG. 6 VELOCITY CONTOURS FOR 4-INCH BY 4-INCH AND FOR 8-INCH BY 8-INCH OPENINGS FOR THE SAME TOTAL FLOW

(Q = 445 cu. ft. per min.)

The transition distance may be decreased until the sides of the hood are perpendicular to the duct to which it is connected. In this case, the hood acts simply as a flange on the duct itself. Under this condition, it has been found that a significant change in velocity distribution takes place. This is a special case, however.

CENTER-LINE VELOCITIES

Representation of aerodynamic characteristics of exhaust hoods by means of velocity contours offers a fairly simple means of comparing graphically various types of hoods. The determination of contour lines, however, involves considerable time and labor. Moreover, the velocity contours do not lend themselves easily to mathematical analysis.

A simpler and more direct means of comparison may be had by the analysis of velocities along the projected axis of the openings. Since exhaust hoods ordinarily draw dust from the area directly in front, the chief factor governing their efficiencies is the distribution of airflow in this region, which may be expressed grossly in terms of center-line velocity curves. They may be compared mathematically as well as graphically. A series of curves for several round openings are shown in Fig. 7. The point velocities are expressed as percentages of the average velocity at the opening.

Graphical Representation of Center-Line Velocity Curves. Center-line velocity curves (Fig. 7) may be presented graphically in the more convenient form shown in Fig. 8, which gives the relation between point-velocity and size of round openings for various distances out along the center-line. By means of the diagonal lines, the velocity at any point may be obtained directly in terms of Q, the total flow through the hood. The logarithmic scale is used for convenience in reading. As an example in the use of the chart: Assume a round opening, 6 in. in diameter, and a total flow through it of 800 cu. ft. per min. The center-line velocity at a distance of 8 in. from the hood is found, and from the chart, the velocity is 4.9 per cent of the average velocity at the face or, from the diagonal lines, 0.25 Q per min. Since $Q = 800$ cu. ft. per min., the actual velocity will be 200 ft. per min. In designing, the limiting factors will ordinarily be the velocity requirement at the point of dust generation and the distance to the hood. Assuming these two factors to be 300 ft. per min. and 7 in. respectively, the chart indicates that a flow of 940 cu. ft. per min. will be necessary through round openings 8 in. in diameter or less. For larger openings, the total flow must be greater, 1250 cu. ft. per min. being required in the case of a 16-in. opening.

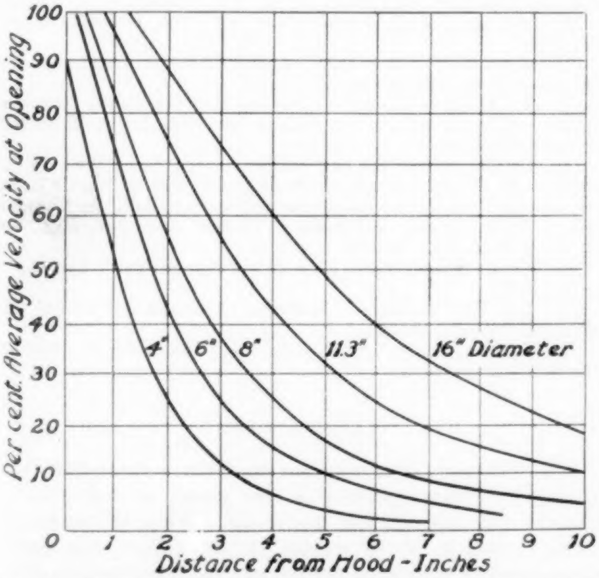


FIG. 7 CENTER-LINE VELOCITY CURVES FOR SEVERAL ROUND OPENINGS—VELOCITIES EXPRESSED AS PERCENTAGES OF AVERAGE VELOCITY AT FACE OF OPENINGS

From the equations presented later similar charts may be prepared for square openings or rectangular shapes having various ratios of sides.

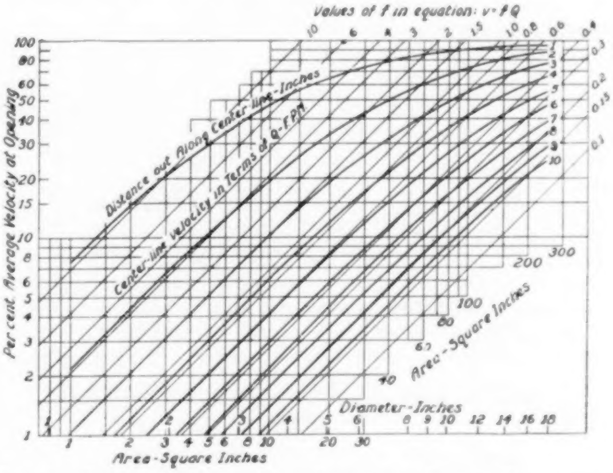


FIG. 8 CENTER-LINE VELOCITY CHART FOR ROUND OPENINGS—VELOCITIES EXPRESSED AS PERCENTAGES OF AVERAGE VELOCITY AT FACE OF OPENING AND ALSO IN TERMS OF Q, THE FLOW THROUGH THE HOOD

Mathematical Analysis of Center-Line Velocity Curves. The center-line velocity curves are limited in one direction and appear to be approximately hyperbolic in shape. An analysis of the data indicates that the curves may be defined by the equation,

$$f(Y) = \frac{m}{x^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

when Y is the point velocity expressed as a percentage of the average velocity at the opening, x the distance from the opening, and m and n constants. When the function $f(Y)$ has the form,

$$f(Y) = \frac{Y}{100 - Y}$$

a linear relationship is given when the equation is written in its logarithmic form, thus:

$$\log \frac{Y}{100 - Y} = \log m - n \log x \quad \ldots\ldots\ldots\ldots [2]$$

Hence, the center-line velocity curves may be rectified to straight lines by plotting the function $\frac{Y}{100 - Y}$ against x on double-logarithmic paper and thus the constants m and n may be obtained.

The center-line velocities for several round openings, plotted in this manner, are shown in Fig. 9. The data follow the straight lines very closely and are representative of the plotted curves obtained for the other openings.[6]

It will be observed that the lines are parallel. The square and rectangular, openings also gave lines parallel to those shown in Fig. 9, thus indicating that the value of n does not change with size or shape of the opening. Its value was found to be —1.91.

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[6] There was a slight divergence from the straight line at extreme values of x.

The value of m was found to be a function of the area of the opening, according to the relation

$$m = bA^k$$

when b and k are constants. For round openings, the constants had the values 0.0825 and 1.04, respectively. Thus, the final equation for round openings took the form,

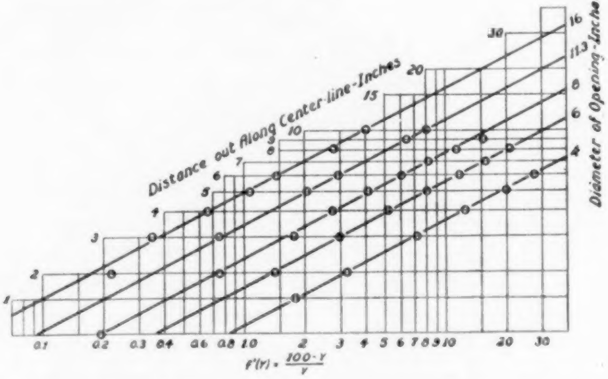

FIG. 9 CENTER-LINE VELOCITY CURVES FOR ROUND OPENINGS RECTIFIED TO STRAIGHT LINES

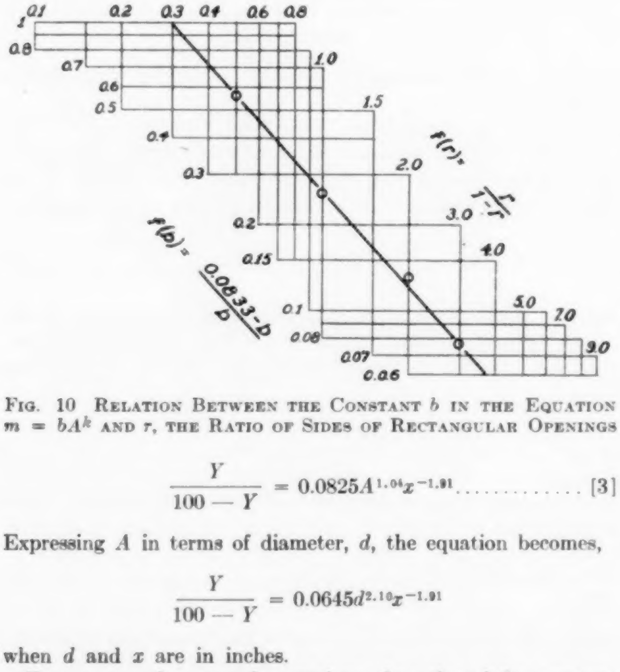

FIG. 10 RELATION BETWEEN THE CONSTANT b IN THE EQUATION $m = bA^k$ AND r, THE RATIO OF SIDES OF RECTANGULAR OPENINGS

$$\frac{Y}{100 - Y} = 0.0825A^{1.04}x^{-1.91} \ldots\ldots\ldots\ldots [3]$$

Expressing A in terms of diameter, d, the equation becomes,

$$\frac{Y}{100 - Y} = 0.0645d^{2.10}x^{-1.91}$$

when d and x are in inches.

For square and rectangular openings, the value of the constant k was found to be the same as for round openings, namely, 1.04. The value of b, however, was found to vary with the ratio of sides, as indicated in Table 4. The relationship is definite and has the form (see Fig. 10),

$$\frac{0.0833 - b}{b} = 0.259\left(\frac{r}{1 - r}\right)^{-1.104}$$

TABLE 4 VALUE OF b IN THE EQUATION, $m = bA^k$, FOR VARIOUS RATIOS OF SIDES OF RECTANGULAR OPENINGS

Ratio of sides	Value of b
1.000	0.0833
0.750	0.0774
0.667	0.0735
0.500	0.0663
0.333	0.0535

Substituting this value of b in the general equation, we obtain finally for square and rectangular openings:

$$\frac{Y}{100 - Y} = 0.0833\left[\frac{1}{1 + 0.259\left(\frac{r}{1 - r}\right)^{-1.104}}\right]A^{1.04}x^{-1.91} \ldots [4]$$

Thus, the center-line velocity curves may be defined completely in terms of the geometric shape and size of the suction openings. The equations may be used to calculate point-velocities along the axis or as a basis for the construction of general charts such as that shown in Fig. 8.

It is interesting to note that the exponents of A and x in the general equations are close to 1 and 2. This suggests that the true form of the general equation may be

$$f(Y) = bAx^{-2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

For, it may be argued that the experimental determinations of the constants are in error or that the function

$$f(Y) = \frac{Y}{100 - Y}$$

is not a correct expression of the conditions. It will be recalled that the 100-per cent contours did not coincide with the face of the openings, thus indicating that the upper limit of Y in the center-line velocity curves is greater than 100 per cent. Whatever the form of the true expression, the inverse-square relationship in Equation [5] may be used for approximate calculations.

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(3) Department of Industrial Hygiene, Harvard School of Public Health, Boston. (Unpublished.)

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(6) "Fluid Meters, Their Theory and Application," publication of A.S.M.E.

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Discussion

J. L. ALDEN.[7] It is apparent that the authors have been guided by their experience in the design of dust-collecting systems for stone-cutting equipment. While the statement, "The air velocity to be created at the point of dust generation, therefore, constitutes the primary specification upon which the design of exhaust hoods must be based," is correct for the design of hoods for the collection of dust particles with random trajectories, it does not apply to the case where the dust particles leave their source with predictable trajectories. Some examples of the latter condition are the dusts and chips from woodworking knives and cutters and from abrasive wheels and buffs. In these instances the dust is thrown from the point of generation in a definite direction and with definite velocity. When designing a hood for such a tool it should be so shaped and positioned that the dust is thrown from its source into the point of highest air velocity in the hood and that its direction of motion at that point coincides so far as possible with that of the air stream. When the hood has been so proportioned, it will be found that the required air velocity and hence the suction may be considerably reduced without loss of effectiveness. In other words, contrary to the practice as outlined in the quotation from the author's paper, the hood should first be designed and located to take advantage of the natural velocity and direction of the dust particles, after which the air velocity to keep the dust in suspension may be determined.

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[7] Assistant Superintendent of Manufacturing Development of the Western Electric Company, Hawthorne Works, Chicago, Ill. Mem. A.S.M.E.

The charts of this paper effectively demonstrate the futility of attempting to set in motion dust particles which have come to rest several inches from the hood mouth or to change the direction of particles which are already flying away from the hood. As a matter of fact, the principle of hood design stated in the preceding paragraph is a necessary conclusion to be drawn from the information presented by the authors. If this principle is ignored, either the hood will be ineffective or the power cost will be excessive because of the necessity for abnormally high suction. The charts also illustrate the non-directional nature of an exhaust hood as contrasted with the strong directional characteristics of a pressure air nozzle, thereby explaining why the vacuum cleaning of machines except in unusual situations is ineffective.

H. A. COLVIN.[8] If every one, machine designers and owners, will remember one point brought out so clearly, it will prevent a great many mistakes. It is shown that suction will not reach out. It is altogether different from blowing. The suction disappears only a short distance from the opening.

There is another point mentioned that is generally misunderstood; namely, that conveying is done by velocity—of course, velocity and volume, but not by static. A high static or suction reading merely means resistance. Many people think that if they can get strong suction readings, they have a wonderful dust-collecting system; whereas, as a matter of fact, it is the easiest thing in the world to get strong suction readings by adding resistance that actually cuts down the effectiveness of the pipe in handling dust.

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[8] Secretary, Allington & Curtis Mfg. Co., Saginaw, Mich.

WDI-54-11

Double-End Tenoners and Their Use

By J. H. MANSFIELD,[1] ROCKFORD, ILL.

The paper describes the construction, development, and use of the double-end tenoner from the original belt-driven machine with babbitted bearings, with its limited range of cutting stock to length and making tenons, to the present motorized ball-bearing machine, with its broad range of sizing, tenoning, shaping, dadoing, relishing, notching, and dovetailing all rectangular parts in two passes through the machine, and making certain styles of lock joints. It is by far the nearest to a universal use machine yet developed for a production woodworking plant.

DOUBLE-END TENONERS have been in use for about 75 years. The early machines were belt-driven and usually of the lathe-bed type, similar to Fig. 1. This machine was patented in 1866. A later design of machine, in which the present type of floor base or bed piece was used, is shown in Fig. 2. The method used in driving the copes consisted of a vertical countershaft for each housing, driven by a quarter-turn belt from the main countershaft and driving each of the copes through straight belts over pulleys of sufficient width of face to permit vertical adjustment of the cope spindles. Trouble was experienced in obtaining sufficient power for heavy cope cuts, such as occur in sash and heavy door work, because of the comparatively short, narrow belts. This, together with the inconvenience and danger of the high-speed shaft and belt near the out-feed position of the machine, led to the development of a different method of drive.

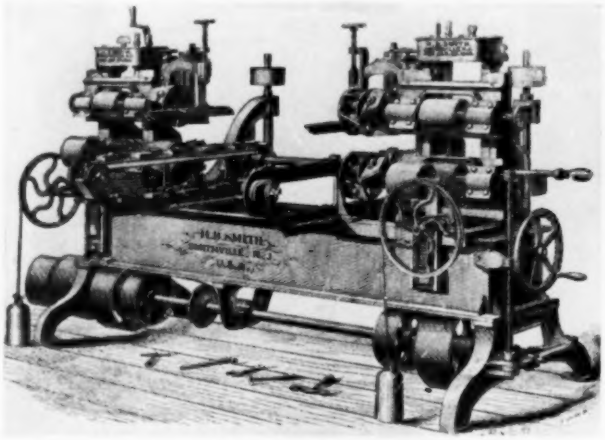

FIG. 1 ORIGINAL DOUBLE-END TENONER, PATENTED 1866

Fig. 3 is about the next step in the development, showing a machine having only lower copes driven by enclosed bevel gears from a secondary countershaft running lengthwise of the machine and gear driven from the main countershaft. The bevel gears consisted of a metal pinion driven by a rawhide gear. This drive delivers much more power to the spindles than the old belted type, takes up less space, and is less dangerous.

These machines were used in sash, door, and furniture work almost entirely and did not have sufficient capacity to handle cuts required on heavy timber framing used in freight-car and passenger-car construction.

Fig. 4 shows a car tenoner suitable for handling timbers up to 12 in. square and 12 ft. long, making single, double, or triple tenons for the heavy sills of cars and locomotive pilots. These were first built in 1898. This machine was sufficiently fast and adjustable to permit handling also the cutting off and grooving of floor boards and other miscellaneous material for freight cars. The characteristics of all these machines are similar in that the right-hand housing is adjustable and in most cases carried the mechanism for controlling the power device for moving the housing. No provision for accurate, quick set-up was provided, such as gages, dials, or graduations on the base, it being necessary for the operator to measure the distance between the various cutters and then check the first pieces for accuracy. The hold-

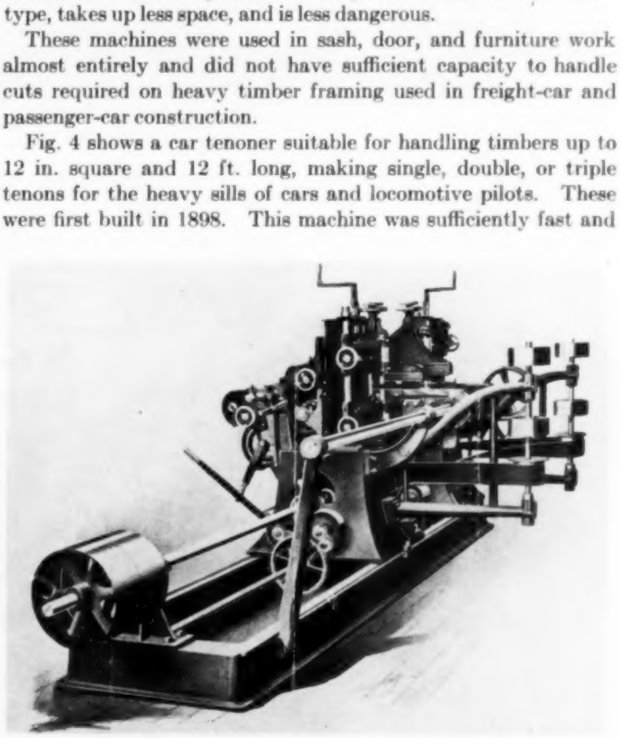

FIG. 2 LATER DESIGN, IN WHICH PRESENT TYPE OF FLOOR BASE IS USED

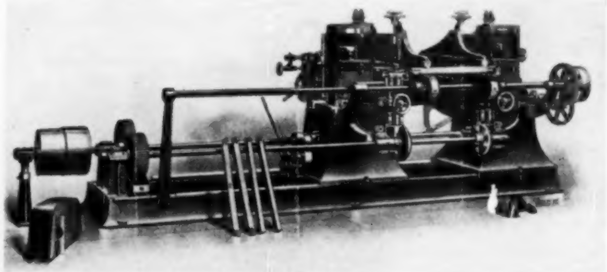

FIG. 3 TYPE HAVING LOWER COPES DRIVEN BY ENCLOSED GEARS

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[1] Chief Engineer in charge of design, Greenlee Bros. & Co. The writer has been with Greenlee Bros. & Co. as an engineer for about 20 years, having been in charge of all engineering work since 1918 and conducting the research and development work in bringing out the double-end tenoner.

Presented at the Sixth Annual Wood-Industries Meeting, Winston-Salem, N. C., Oct. 15 and 16, 1931, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

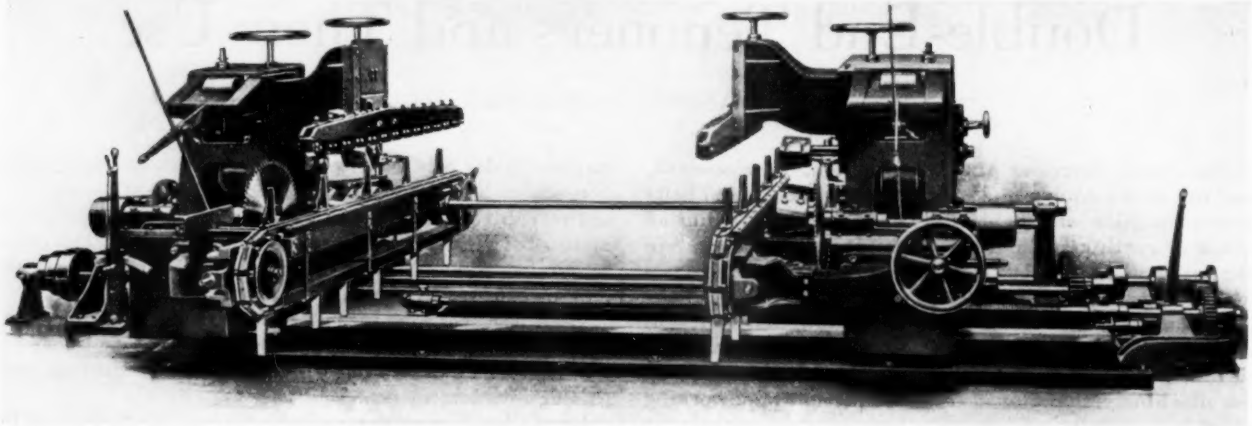


Fig. 4 Car Tenoner for Handling Large Timbers

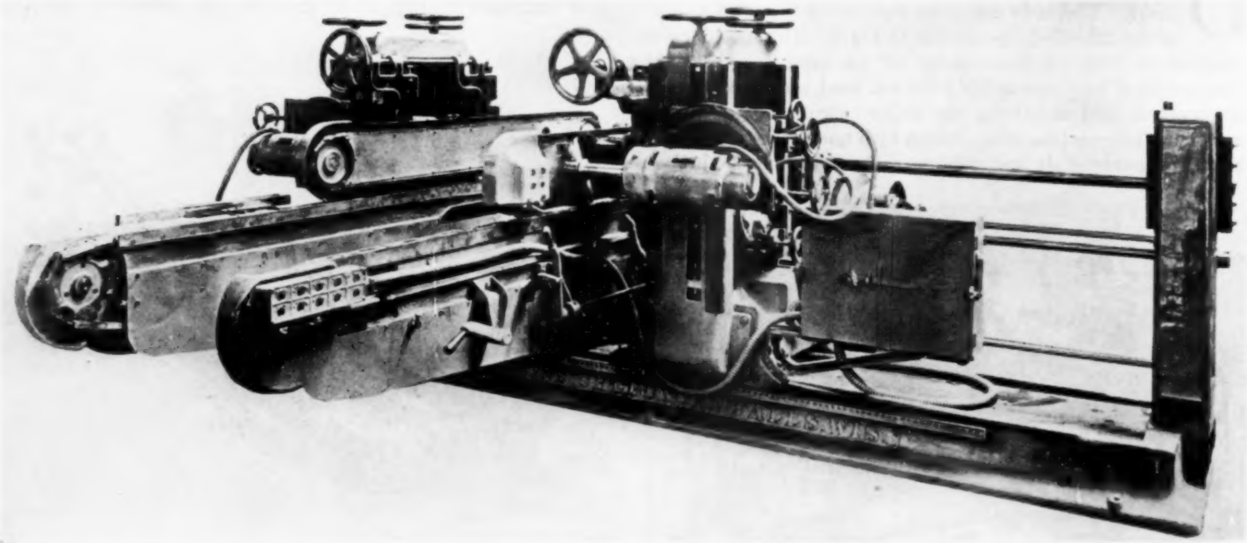
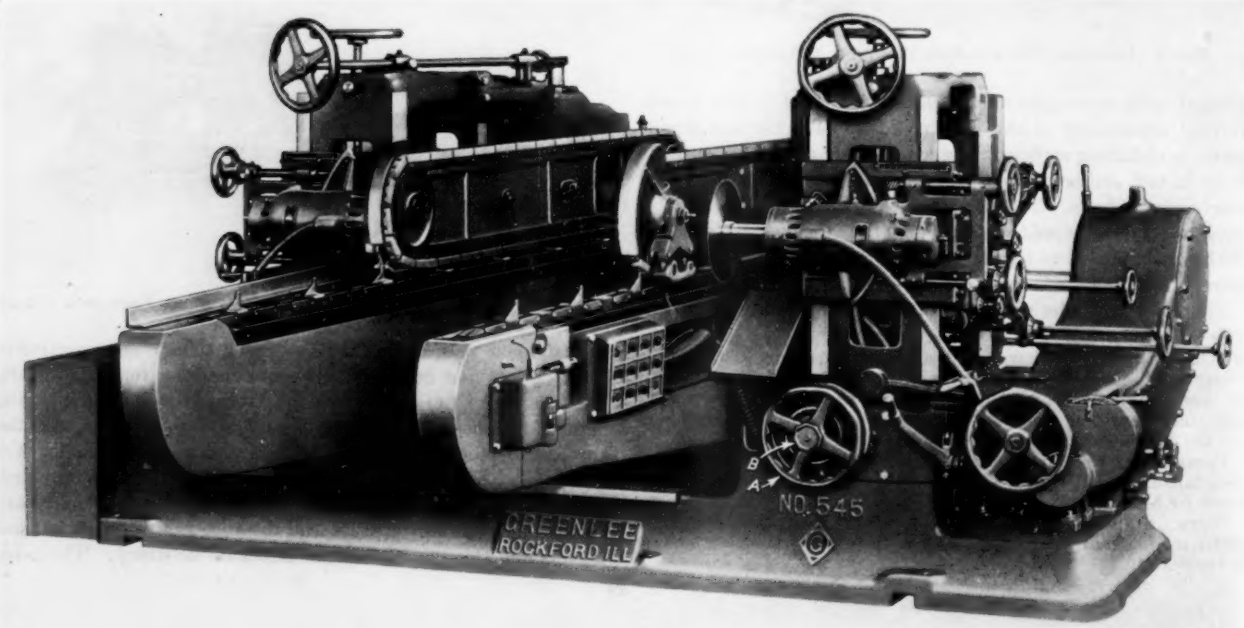


Fig. 5 Late Design of Motor-Driven Tenoner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NO. 545
GREENLEE
ROCKFORD ILL

Fig. 6 Recent Motor-Driven Tenoner, With Right-Hand Housing Carrying Controls

downs were either of a spring or shoe type in one piece or with short sections, such as is shown in Fig. 2. There was no possibility of angle adjustment of any of the heads. In case angle cuts were necessary, they had to be accomplished through the use of special shaped heads.

The next improvements were undoubtedly the adaptation of ball bearings to the spindle housings and the present type of hold-down chains carrying rubber pads. The hold-down chain beams were hinged to swing up and be locked out of the way to save time in making set-ups. In the machine with these improvements the controls for the feed and changing the position of the right-hand housing are at the left-hand end.

Shortly after the introduction of ball bearings in woodworking machinery, direct motor drive to cutter-head spindles came into use. This was accomplished in many cases by users of machines changing over their own tenoners, in some cases putting motor-driven copes only on belt-driven machines, and in others both the motor-driven copes and cut-off saws. With this direct motor drive, angle cuts with the saws and the copes were possible. The drives for the top and bottom tenon heads were not changed, since there was not sufficient space for motors and the belt drive was more satisfactory for them than for either the cope or saws.

An early motor-driven machine was one on which magnetic controls for the motors operated by push buttons were used, with both the push-button and mechanical controls mounted at the left-hand end of the machine. Another development was the addition of the graduated bar or scale over the top of the machine to show the setting for length of stock. These scales were graduated to $^1/_{16}$ in. One of the saws was tilted and the other straight, but it is not known whether the tenon heads and copes were similarly adjusted.

On these earlier machines, attachments such as are now available on all double-end tenoners could not be had, since it was impossible to drive them until direct built-in motors had been developed. The use of the old-style tenoners was thus limited to straight cuts and to such work as cutting off or cutting off and tenoning in straight work. The belt- or gear-driven copes were alleged to be not free enough from vibration to permit their use for shaping.

Fig. 5 shows a late design of motor-driven tenoner which can be had with practically all the various attachments, including jump or stationary dado, relishing attachments, notching attachment, scoring saws, and drop dogs.

In this design the right-hand housing is adjustable and carries push-button motor controls and feed-control levers in a location convenient for the operator. The scale for indicating the length of pieces is carried on the front of the base instead of overhead, and the top of the base is enclosed to keep the dirt out.

Fig. 6 is another recent design of motor-driven tenoner in which the right-hand housing is stationary and carries all of the electrical and mechanical controls. This design permits of the operator standing in one position next to the controls at all times.

Fig. 7 is a double-end tenoner manufactured in Europe. This is of interest because of the novel type of stock hold-downs and the large number of cutter-head motors available. It is claimed that from 8 to 16 motors can be used. The hold-down arrangement appears to be complicated, and sawdust and shavings might cause trouble. No angle adjustment for the top and bottom heads or cut-off saw is indicated, but there are gages indicating the location of the saws.

Present designs of tenoners are capable of handling an almost endless variety of work as now encountered in the woodworking industry. The range varies from ordinary straight or angle cut-offs or tenoning work to sizing large or small panels, table and cabinet tops, shaping and molding edges, and grooving, forming lock joints of various kinds, single or jump multiple dado work in desk or cabinets for shelving, relishing cuts, miters, corner notches, etc.

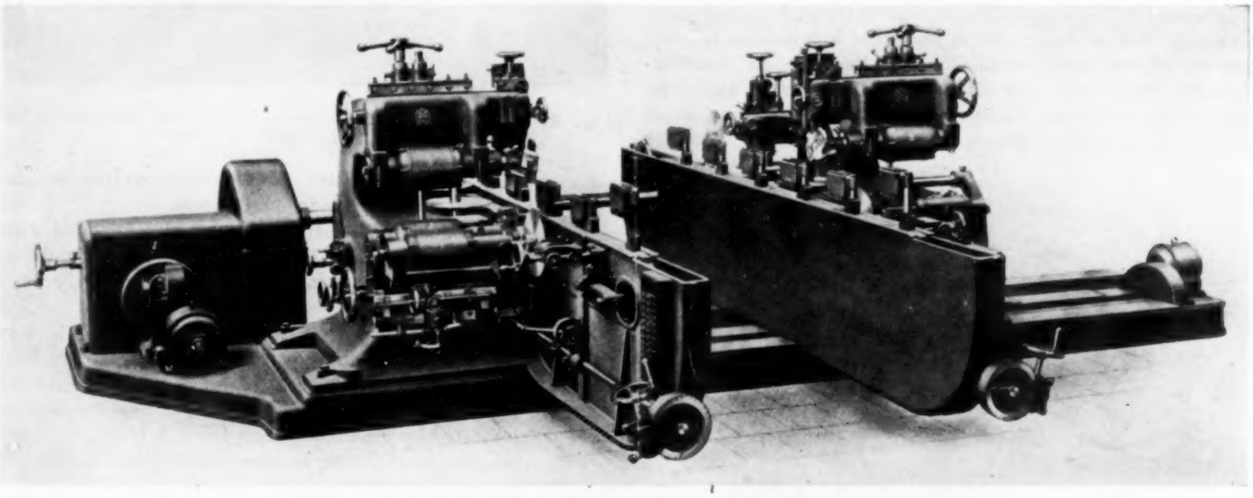

FIG. 7 DOUBLE-END TENONER OF FOREIGN MAKE

As possibilities of the motorized tenoners have been realized, greater accuracy and means for quicker set-ups have been demanded. In woodworking plants, quantities that will keep a double-end tenoner running three or four hours are becoming unusual. Even in automobile-body plants the day's requirements for certain pieces are not sufficient to keep a tenoner running on one piece of work alone, thus making numerous set-ups necessary during the day's run. On the machine shown in Fig. 6 every effort was made while the machine was being designed to facilitate easy set-ups. One of the results of this study is the hinged pressure beam as shown in Fig. 8. To use this to advantage the rate of adjustment of the movable housing had to be faster than had been considered advisable, and it is now possible to open the machine wide from a closed position in a small fraction of the time it used to take. The slow adjustment was quite necessary on early machines as no quick hand adjustment for final setting was provided.

Fig. 9 shows the controls and indicating dials for controlling the movement of the movable housing. The hand lever *A* when pushed either to the right or left controls frictions which move the housing in either direction. As the housing is moved, the dials *B* and *C* are rotated by gears from the adjusting screw and indicate the exact location of the movable housing at all times. Dial *B* is graduated in inches and in feet and inches, and it moves one notch for each inch that the housing travels. Dial *C* has 64 graduations about $^{5}/_{16}$ in. apart, each graduation indicating $^{1}/_{64}$-in. movement of the housing. This is used for the final setting by hand and is accomplished by pushing in handwheel *D* to engage a clutch for rotating the adjusting screw either way. It is thus possible to adjust the movable housing and locate it accurately to within one-quarter of $^{1}/_{64}$ in. without the operator moving from his working position.

This dial arrangement has been used for years on all machine tools such as milling machines, lathes, planers, etc., for measuring closer than mechanics could read direct on their scales. Most of them are graduated to the thousandths of an inch, and on a precision machine manufactured in Switzerland, and used extensively in toolroom work, jigs, dies, etc., the graduations read to 0.0001 in.

In some furniture plants this tenoner is used for cutting parts for sample sets, as the amount of handwork is materially reduced and the various parts are machined in less time and more accurately. One foreign user of this machine has steel go and no-go gages with a 0.003-in. variation for checking the thickness in mahogany and walnut tenons on pieces 18 in. wide. Cabinet makers in that plant have no hand tools whatever and are discharged if found bringing any in, as hand fitting of parts will not be tolerated on radio and phonograph cabinets.

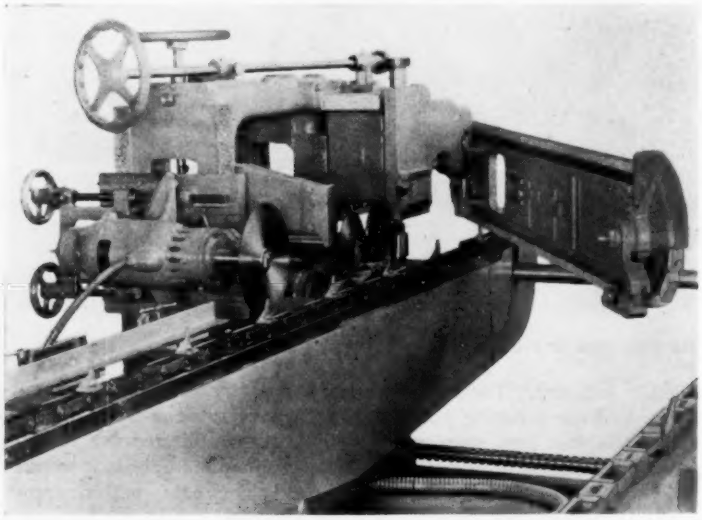

FIG. 8 HINGED PRESSURE BEAM TO FACILITATE EASY SET-UPS

There are domestic concerns also using steel gages to insure interchangeability in manufacturing, a common practice being to send all parts to stock bins before they go to the cabinet department for assembly.

The screws for both vertical and horizontal adjustments of all the cutter heads and saw housings are cut the same pitch and fitted with a graduated collar indicating sixty-fourths of an inch, the graduations being about an inch apart. These graduated collars, in addition to scales, give the exact position of the cut-off saws and tenon and cope heads to within a fraction of $^{1}/_{64}$ in. over the full range of adjustment. The end of each of the screws is fitted with a square for a detachable hand crank for quickly making the larger adjustments.

Locks for each of the screws are provided, so that after being once set, they will hold their position.

For adjusting the top and bottom tenon heads, either with or without scoring saws, the two handwheels mounted on the same center, at *A* in Fig. 6, are fitted with a friction-clamp arrangement, so that when the proper space between the top and bottom heads governing the thickness of the tenon has been reached, the two handwheels can be securely locked together by the hand-clamp wheel *B*, and the raising and lowering can then be done without changing the tenon thickness.

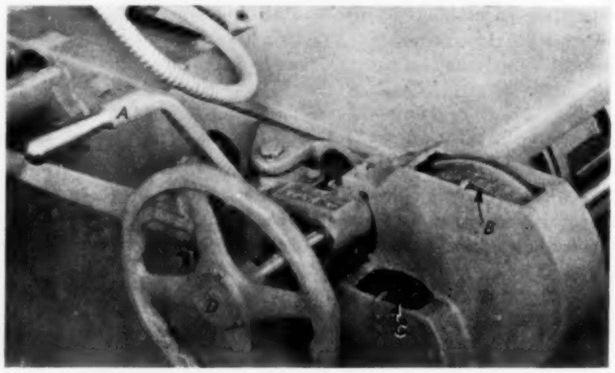

FIG. 9 CONTROLS AND INDICATING DIALS FOR MOVEMENT OF HOUSING

The hold-down chain beams are also provided with scales to show their height above the feed chains, so that this adjustment can be quickly and accurately made without guesswork.

Work that requires parallel angle cuts at both ends can be run on double-end tenoners if it is possible to advance one chain ahead of the other. It is possible to adjust one chain out of square the amount required as indicated by graduations and bring it back to square relative to the other chain accurately and quickly on present-day machines.

With any double-end-tenoner feed chains there are pulsations or variations in the rate of feed for each sprocket tooth in the drive sprocket. When one chain is advanced one-half of the pitch or center distance from chain pin to chain pin, they do not travel together, and the pulsations amount to about $^{5}/_{32}$ in. with an eight-tooth sprocket and 4-in. pitch chains such as are now used. If the angle cut is on short stock, this variation will cause such inaccurate work that it cannot be used; but where the parts are over 2 ft. in length and under 6 in. in width, very satisfactory results can be accomplished. Short angular work can be handled accurately by using special dogs of suitable lengths to give the required angle when the chains are left square. The accuracy of work so far as square cuts are concerned is controlled almost entirely by the feed chains, sprockets, and feed shaft. The length of each of the links must be held within very close limits or the feed will not be uniform. The sprockets must be very accurately cut and of material which will show very little wear over a long period and must be mounted on a shaft of sufficient strength to eliminate any tendency to spring when the machine is set for long stock. Play in bearings for the feed sprocket and shaft also contribute to lack of accuracy, and for that reason ball or roller bearings of hardened steel are used at these points.

Most feed chains are flat on the bottom and guided by gibs on each side. There must be some running clearance between the chain links and gibs, which wear increases, in some cases allowing the stock to shift endwise when passing through the tenon heads and copes on heavy cuts. To eliminate this disadvantage, chain links with a V-bottom running on V-tracks have been developed. This construction insures straight travel of the chains through the machine without any side motion. This straight-line motion is not affected by wear.

Feed dogs are made up of several types—non-adjustable, adjustable, drop dogs, and wood faced for backing up stock when shaping. The author believes firmly in non-adjustable dogs stamped to correspond with the links on which they fit, as this method saves considerable time when changing dogs. A tongue on the dog fits into a slot in the chain link so that the dogs will always go back in the same position and no time is lost squaring up as with adjustable dogs. Greater accuracy in the manufacture of the chains and dogs is required, but the saving in set-up time is well worth the additional cost. Drop dogs are used in some plants where the work varies greatly in width, as the dogs can be left on the machine and work of varying widths put through. When stock wider than the space between two dogs is run, it pushes down one or more dogs below the chain level while the piece passes through the machine, and the dogs are returned by a counterweight or spring when the stock leaves the chains. Drop dogs are more complicated and of weaker construction than the solid ones, and so are naturally less dependable.

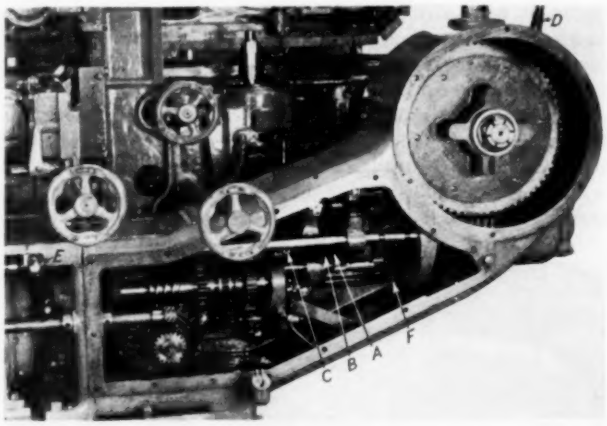

Fig. 10 Feed-Gear Box Showing Drive Mechanism

For large panels and other work, where the cuts are very heavy, the top of the feed chains should be corrugated to hold the stock more securely. Automatic lubricators driven by the feed chains are most essential for insuring lubrication of the tracks. With this method the track and chain receive oil only when the chains are running and do not become flooded, so as to leave oil on the work to cause trouble in the finishing room.

Fig. 10 shows the feed-gear box with both covers removed presenting a clear view of the drive mechanism. The feed motor is geared direct to a shaft having a sliding cluster of three change gears which engage with gears on the worm shaft for changing the rates of feed. Standard feeds are 14, 20, and 27 ft. per min., but these can be increased or decreased for special needs. With a positive drive of this type, it is necessary to build in some form of overload protection, so that in case the stock becomes jammed, something will slip to avoid serious breakages. There are several different methods of accomplishing this, and the one selected as the most suitable consists of a rattle clutch which is used as a safety, and also a start and stop clutch. Part of the clutch is shown at *A* on the side of the large gear of the nest of change gears and is driven by the motor through the three cluster gears. At *B* is the other member of the clutch mounted on the worm shaft and held in contact with the other half *A* by a coil spring *C*, except when disengaged by means of the hand lever *D* or its companion lever at the front of the machine. The tension on the spring *C* can be adjusted to suit requirements and should be just a little more than enough to feed the machine when running on the heaviest work. In case of a jam, the half *B* moves endwise on the worm shaft, permitting its teeth to ride over the top of the teeth on the half *A*. The feed is thus instantly stopped and the noise caused by the clutch teeth sliding over each other attracts the operator's attention, so the operator will shift the feed lever to entirely disengage the feed and locate the trouble.

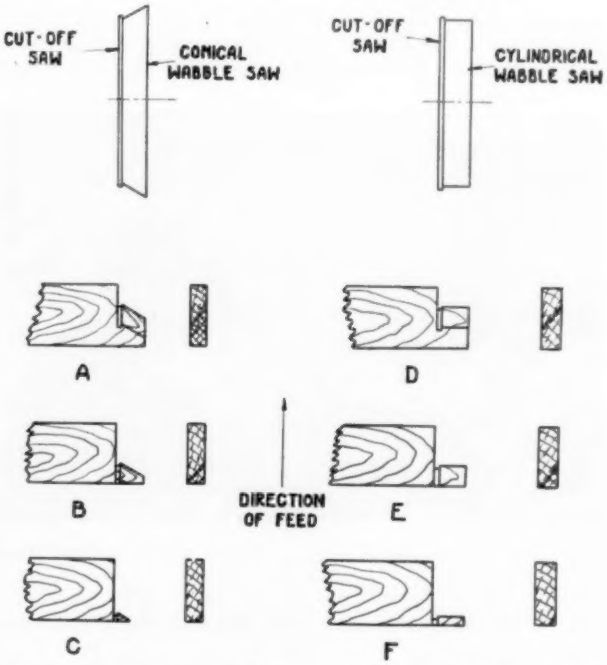

Fig. 11 Cuts Nearing Completion, With Both Conical and Cylindrical Wabble Saw and Cut-Off Saw

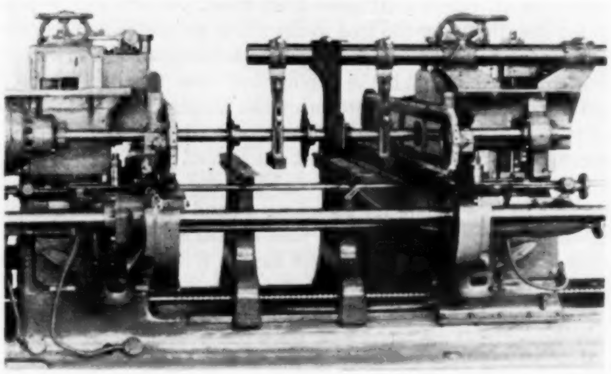

Fig. 12 Rear View of Tenoner Equipped With Jump Dado

In this design the leverage of the clutch over the feed chain is constant regardless of the rate of feed being used.

Other methods of overload protection have been tried with varying degrees of success, among them the use of a belt with spring-loaded or weighted idler, friction clutches, and electrical protection for the motor. Friction clutches and belts depend upon friction for their driving power, and with most surfaces a variation in temperature, atmospheric conditions, or a little oil will affect the power transmitted. With an electrical overload protection for the feed motor an adjustment is necessary for each change of feed, as the ratio between the r.p.m. of the motor and the speed of the feed chains varies with the rate of feed. The pull on the chain delivered by the motor at 27 ft. per min. is about three-fifths of that at 15 ft. Most electrical devices depend either upon a magnetic circuit or heating elements for their operation, and in case of the latter too much time is required for its operation to make it effective as an overload protection on any feed mechanism. The magnetic type can be adjusted to very close ranges, but must be set for change of feed, which the average operator will not do.

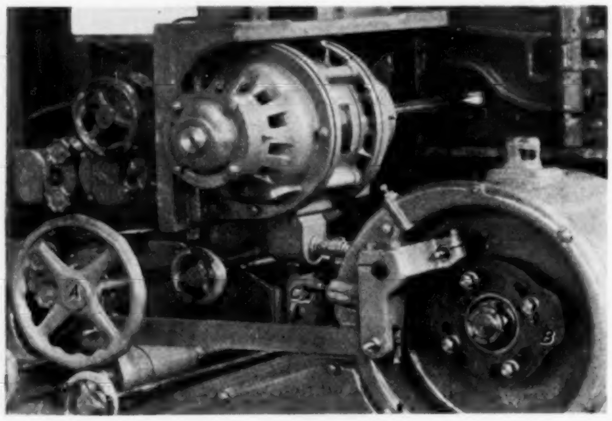

Fig. 13 End View Showing Cam, Motor, and Adjusting Hand Wheels

In the feed box shown a gear-type lubricating pump positively driven by gearing from the motor delivers oil to an indicator at the top of the housing, from where it is carried through suitable piping to all points that need lubrication. A reservoir is provided directly under the feed worm which maintains oil at a constant level, so that the worm is partly submerged. After the oil has circulated through the piping and over the bearings and gears it returns to the bottom of the housing and is then recirculated. All of the gearing is of hardened steel, and the bearings are anti-friction type, either ball or roller.

The worm shaft projects through the end of the housing and terminates in a square for attaching a hand crank to be used in making set-ups. The rate of feed is changed by moving the cluster gears *F* by means of the handle *E* which indicates the desired feed.

Veneered stock, especially when built up with casein glue and face veneer of $^1/_{28}$ in. thickness, tears badly while being tenoned unless scoring saws are used or the direction of rotation of the tenon head is reversed to cut with the feed instead of against it. Of the two methods the scoring saw is much more satisfactory. These saws are motor driven, and the housings are mounted in some machines on ways in front of those for the top and bottom tenon heads and adjusted in a similar manner. On one of the latest machines they are mounted on the housings of each of the top and bottom tenon-head motors and provided with vertical and horizontal adjustment so that after once being set to cut with their tenon head, they require no further adjustment as they move with the tenon head unit whenever the set-up is changed. With either type of scoring saw, motors can be shut off and the saws adjusted clear of the stock when not needed.

Refuse cutters for hogging up the off-fall have been used with varying success to eliminate the blocks falling on the floor when cutting off narrow pieces and in the more difficult job of handling edgings when sizing panels and case-goods tops. The usual type is a cylindrical wabble saw mounted next to a cut-off saw of slightly larger diameter. This cuts up the scrap in line with the cut-off saw so that as the cut-off saw progresses in the work the scrap is disposed of by the refuse cutter. The difficulty is that the pressure of the wabble saw against the work is sufficient to cause not only tearing up of the stock at the end of the cut, but also splintering back into the stock as the cut is completed. One type of hogging attachment, consisting of a conical wabble saw and cut-off saw combination, has been developed that eliminates this condition. Fig. 11, at *A*, *B*, *C*, *D*, *E*, and *F*, indicates the condition with both types as the cuts near completion. Referring to view *C* a very small portion of scrap is left in the shape of a triangle with its base next to the stock. As the cut progresses this triangle decreases in area faster than the amount of stock connecting it to the board. This reduces the pressure and leverage brought to bear by the wabble saw and avoids breaking off. Highly satisfactory results have been experienced with this on a large variety of work. Desk tops of built-up veneered up to 2 in. thick have been handled. The off-fall is chewed up and carried away by the exhaust system instead of falling on the floor as edgings. With a successful hogging attachment, both edging and shaping can be accomplished on tops which are of greater length than the distance between the cut-off saws and the cope spindles, as there are no edgings carried through to the cope cutters.

Figs. 12 and 13 show the rear view of the machine equipped with a jump dado and an end view of the same machine showing the cam, motor, and adjusting hand wheels. The dado shaft is mounted in a ball bearing at the adjustable end of the machine, and this bearing is housed in a slide carried on ways on the vertical housing. At the stationary end of the machine the shaft is supported in a sleeve that carries the rotor of the motor and is mounted on ball bearings. The whole motor assembly travels on a slide gibbed to ways on the vertical housing. On long machines over-arms carrying stock hold-downs and steady bearings for the dado shaft may be mounted on either end of the machine. When not in use, the arm can be removed or slid back through its supporting bearings out of the way.

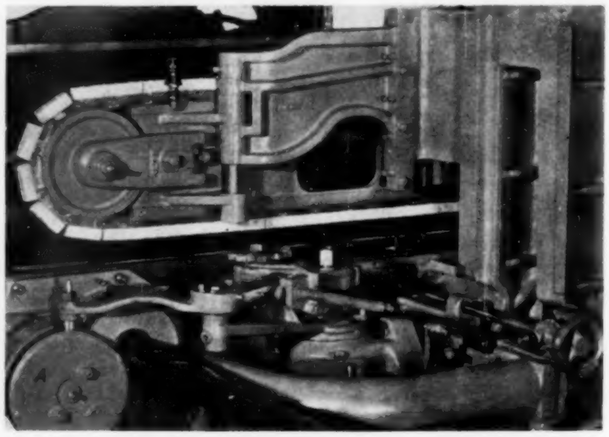

Fig. 14 Relishing Attachment Driven by Cope Motor

The slides carrying the end bearing and motor are vertically adjustable by means of a handwheel, at *A* in Fig. 13, connected to the vertical adjusting screws by means of worm and worm gears in each housing. This vertical adjustment is used either with the jump or stationary dado shaft. The jump movement is controlled by the cams *B*, which can be furnished with either single or double lobes for a jump every 16 or 32 in. of travel of the feed chains. Two cams are used, one outside the other, so that the duration or length of jump can be adjusted to suit requirements. The amount of raise is set by the adjusting screw *C* varying the movement from zero to $^3/_4$ in. The cams *B* are mounted on an extension of the hub of the feed-worm gear, insuring a very steady positive movement. Cuts as wide as 14 in., such as are used in lock boards for automobile doors, down to slotting desk-pigeonhole stock, are handled with great savings in time.

Fig. 14 shows a relishing attachment mounted on and driven by one of the cope motors and controlled by a cam *A*, gear driven from the main feed shaft. The cam *A* is made in two pieces, similar to the one for controlling the jump dado, to permit adjustment, instead of making up a special cam for each job. The cutter is driven by the cope motor through enclosed hardened-steel gearing and is held into the cut by heavy coil springs which are positively pulled out at the right time by the cams. In case of an accident, such as forgetting to start the cutters, the unit will be pushed back by the stock compressing the spring. Cams can be furnished for relishing one or both ends of a tenon.

A single-notching attachment, mounted on the horizontal ways for the cope-spindle housing, is used for notching the tops of telephone stands and similar novelty furniture. It is controlled by a cam geared to the main feed shaft. Another type is for notching the front and rear edges of the stock as it passes through the machine so that at one pass all four corners are notched out for legs.

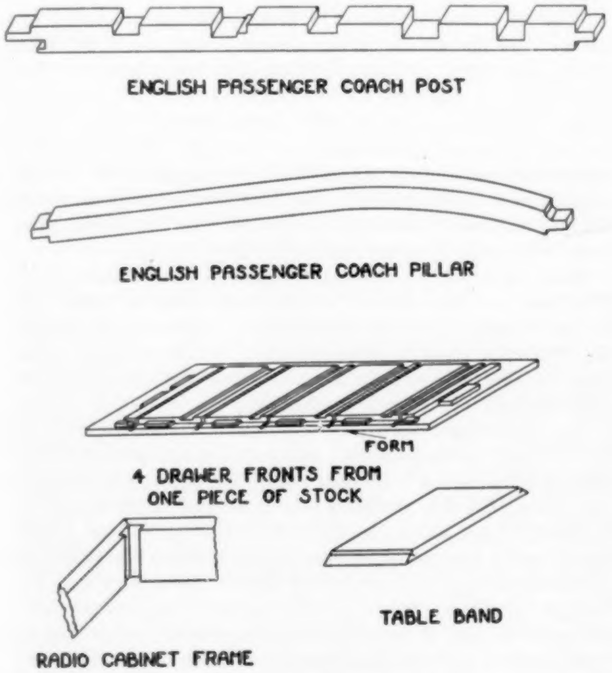
ENGLISH PASSENGER COACH POST
ENGLISH PASSENGER COACH PILLAR
FORM
4 DRAWER FRONTS FROM ONE PIECE OF STOCK
TABLE BAND
RADIO CABINET FRAME

Fig. 15 Unusual Pieces of Work

The standard double-end tenoner used to consist of cut-off saws, top and bottom tenon heads, and cope spindles with a feed mechanism. On modern electrified machines, various modifications or attachments have become almost as standard in certain lines of work as the saws and tenon heads were on the old belt-driven machines.

Other features, such as sanding attachments, dovetailing attachments, serpentine dovetailing attachments for seats and tops of school furniture, corner-rounding attachments, and transfer tables, have been developed.

The amount of stock cut at one setting determines to a large degree the economic success or failure of complicated set-ups on any machine, whether for metal or woodwork. In woodworking plants this is especially true, as the rates of feed are very high.

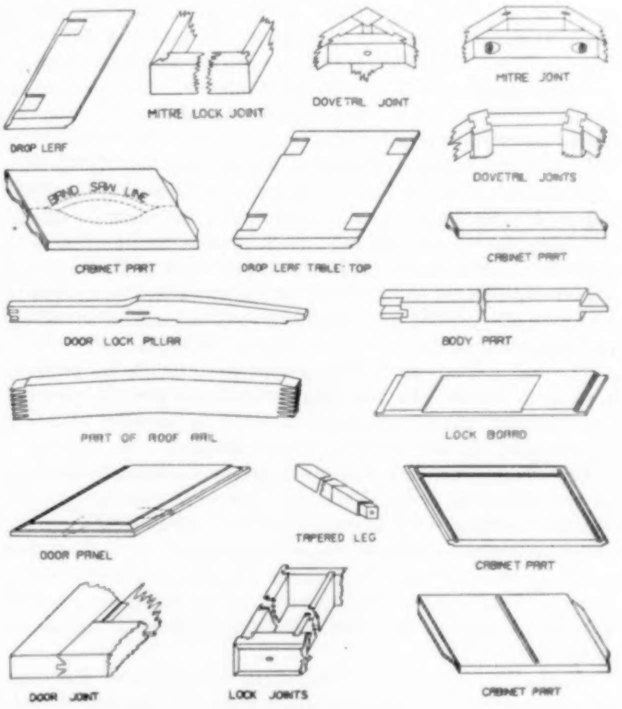

Fig. 16 Range of Standard Work Handled on Tenoner

Reports of as many as 46,000 pieces being put through a double-end tenoner in eight hours are on record, whereas in metal work, other than punched parts, 200 to 300 a day is reasonable output.

A great many attachments for a tenoner which are mechanically feasible are not successful because the difficulty of quick and accurate setting for different jobs makes it desirable to break up the operations and run them on simpler hand-fed machines at lower costs.

The double-end tenoner can be equipped with the exhaust piping and hoods assembled on the machine. Formerly the piping and hoods were made up by local tinners or blowpipe companies especially for each machine to the purchaser's order. This was objectionable because the piping might not work satisfactorily.

Maximum production of a double-end tenoner is determined by the spacing of the dogs, the rate of feed, and the number of pieces between the dogs. With the dogs located on 8-in. centers on alternate links, the numbers passing through the machine per minute are, for 15 ft., $22\,^1/_2$ dogs, for 20 ft., 30 dogs, and for 27 ft., $40^1/_2$ dogs. How well the space between dogs is filled depends on the operator's ability to pick up several pieces quickly. The way stock is piled on trucks has considerable influence on the feeder's ability to pick up a large number of small parts. Piling successive layers in alternate directions or sticking them similar to lumber in a kiln is a great help. Two men can advantageously feed small stock, although in most cases one man can keep the chains filled up.

Set-up time varies according to the number of heads that have to be changed and the skill of the set-up man. Some plants allow 25 minutes for each set-up, considering that it is an average for all cases. Where jump dado, relishing, and other attachments are used, the time for setting-up will be much greater.

A canvass of users of the most modern tenoners shows that work is changed at intervals of from 30 minutes to 2 hours and that average runs are less than an hour, thus showing that set-up time has a marked effect on production. There are cases where 24 distinct set-ups are made on one machine, with a total output of 10,000 pieces for an average day's run. In another case, a machine running continuously tapered and tenoned 23,000 desk legs, requiring two passes through the machine, making a total of 46,000 pieces handled in 8 hours and 3 minutes. For production as high as this it is evident that the operator kept the space on the chains between the dogs full of legs.

Further examples of production are the running of 300 desk tops through the machine twice, sizing and shaping the front edge and both ends in 1 hour and 20 minutes, including the set-up, and 250 radio-cabinet banded tops being sized and shaped in 45 minutes. The time for these operations when performed on a shaper was 9 hours, and the work produced on the tenoner was of higher quality, owing to the steady feed, more secure holding, and the perfect straight-line travel due to the V-tracks.

The ease of set-up and a large production is further indicated at a plant in which, after running a double-end tenoner for some time, it was found that 118 jobs could be handled at a decided saving that were formerly run on shapers.

The extreme accuracy of an up-to-date tenoner is demonstrated in the following work. Veneered tops were run through the machine, putting a $^1/_4$-in. groove in the front and both ends to receive solid bands and mitered corners glued in place. The tops are then brought back for shaping the edges to meet the top veneer. This requires that the first job of sizing must be absolutely perfect and the stock accurately located for the second run.

Another concern has similar work on executive-desk tops which range from $1^3/_4$ to $2^1/_4$ in. thick and up to 10 and 12 ft. long. The shaping lines match perfectly on the sides and corners. It is claimed that ability to maintain this accuracy is due entirely to V-chains and V-tracks.

Multiple operations such as cutting and grooving desk-drawer fronts can be handled by placing the panel in a form supported on the feed chains, making the first cuts with the trim saws and the center cuts and grooves with saws and grooving cutters on the stationary dado shaft. The forms can be returned to the in-feed side of the machine, either by truck or a chute between the chain ways.

Curved English passenger-coach pillars are successfully handled in forms, clamping several pieces at a time. This particular work is performed in teak, which is extremely difficult to cut because of the large amount of silica or sand embedded in the wood. It is necessary to regrind the knives after 2 to $2^1/_2$ hours, and they then have to be whetted up three or four times between grinds.

Fig. 15 shows a few rather unusual pieces of work, especially the English passenger-coach pillar and the four drawer fronts cut from one piece of stock. Both of these are handled in special forms. The English passenger-coach post is a good example of combined tenon and dado work.

Fig. 16 shows some of the standard work in furniture, sash and door, and auto-body plants and gives a good idea of the variety and range that can be economically handled on up-to-date tenoners.

Discussion

Peter J. Schloemer.[2] The author has devoted the major part of the paper to the construction and detail of only one particular make of tenoner, rather than conveying the many features available and methods of operation possible on various modern-type tenoners.

It is true that great developments have been made since the introduction of direct motor-driven tenoners, but the writer does not agree with the author when he states that the use of the old-style or belt-driven tenoner was limited to only such work as cutting off and tenoning straight work and that attachments such as are now available could not be had.

It is a fact that attachments such as automatic jump dado arbor, automatic jump relishers, combination groover and trim saw cuts, scoring saws, and many other features were all used with great success on belt-driven tenoners long before the direct motor-driven tenoner was first brought out. Shaping of edges with cope heads was also accomplished successfully through gear-driven cope spindles, as many tenoners of this type are still in successful operation today.

The author calls special attention to the hinged pressure beam shown in Fig. 8, but does not explain what particular benefit is derived from this feature in relation to the use of or the production possible on the tenoner.

In the writer's opinion, the methods of adjustment of the pressure beams as used on other makes of tenoners have a decided advantage, because with those methods the operator is enabled to do certain grooving operations directly over the feed chains by moving the pressure beams forward or just outside the feed chains.

It is true that accuracy in production operations on any tenoner depends entirely on the accuracy of the feed chains and driving sprockets, as these are the most vital and important features on the machine.

The writer does not agree with the statements relative to the feed chains, as the author endeavors to convey that perfect and accurate work can be done only on tenoners equipped with V-chains running on V-tracks. Work of the finest nature and accuracy has been accomplished on tenoners equipped with flat chains many years before the V-chains were introduced.

Referring to Fig. 10, the paper goes into detail relative to the construction of the feed-gear box, but here again the author does not state what benefit this feature has in relation to the use of or the production possible on the tenoner. Neither does the author state whether it is possible to make quick changes of feed rates with this type of feed-gear mechanism.

The writer would like to add to the author's reference to the feed drive as used on modern direct motor-driven tenoners the method used on a tenoner of a different make than the one illustrated in the paper. On this tenoner a four-speed feed motor is used and operates through a drum control. With this method the operator can instantly increase or decrease the feed rates without stopping the feed chains or without loss of time in the production of the tenoner. Further, the operator is enabled to instantly increase or decrease the rate of travel of the movable base when changing from a long to short distance between shoulder lengths or reverse, without stopping the feed motor. In addition, he can also reverse the direction of travel of the feed chains in case of necessity. As the production of any tenoner is based to a great extent on the feed rates, it is readily seen where this method has a decided advantage.

With reference to the work that can be accomplished on a tenoner, there is no limit, as new and additional uses for it are being found constantly. In addition to the many varieties illustrated in this paper, the writer wishes to add the following:

Grooving operations are done with the scoring saws in conjunction with the trim saw, tenon, and cope operations.

Drawer-front panels can now be glued up in one piece, enabling the better matching of figured or fine face veneers properly and with a saving of core stock material. These can then be ripped to proper widths on the tenoner in one pass through the machine.

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[2] Wilkin-Challoner Company, Oshkosh, Wis.

Door fronts with matched veneers are built up in one panel, then trimmed to width, edges jointed or shaped, and ripped through center in one pass through the machine.

Milled-to-pattern cutter heads are now used on the dado arbor to cut special shapes and grooves across grain and through the finest of face veneers without any sign of tearing out at the edges of cuts.

End panels of kitchen cabinets can now be trimmed to length with a shelf groove cut with groover saw on same spindle with trim saw, tenoned on one or both ends, three jump dado grooves across center, one or both ends rounded with cope cutters, and either one, two, three, or all four corners notched complete in one pass through the machine.

Many other operations of equally interesting nature are now being done regularly, and there is no limit to which the modern tenoner can be adapted and to the savings that it can bring about in the production of almost any plant.

Special saws and cutters are now available, enabling one to work the hardest kinds of materials successfully.

Perfect work is being done today in many plants on old-type tenoners with capable and intelligent operators, but it would be amazing to many plant managers could they realize how much greater production with less cost these same operators could produce on a modern double-end tenoner.

Arthur D. Smith, Jr.[3] The paper has brought out the history and evolution of the double-end tenoner. However, the type of cuts that can be made on the double-end tenoner is important enough to bear further stressing.

We have line drawings covering the equipment and cuts that we can make. Blueprints have been made of these for the draftsmen (or detailers) and the machine operators. The draftsmen have a clear understanding of our equipment and the type of cut possible, also the capacity of the machine as to length, width, thickness, depth of cuts, etc., and they make their detail drawings accordingly.

The author did not mention the saw, head, and knife equipment required and it might be interesting if the writer describes the drawings. Starting at the infeed end of the machine, the cut-off saws bring the piece to the desired overall length or width. This 14-in. or 16-in. combination saw is filed to give a smooth cut when ripping or cutting. The 14-in. saw will cut off in multiple lengths stock up to 12 in. wide. By removing the stationary dogs and using drop dogs, stock of unlimited width may be fed, because as soon as the material is engaged between the chain and traveling rubber hold-downs, the stock will be pulled through without the assistance of the dogs.

The cutting saw arbor may be tipped 45 deg. from horizontal in a clockwise or an anti-clockwise direction. There is a notation that when cutting fireproof material or plywood glued with an abrasive casein glue, the operator is to use a $^3/_{16}$-in. Carboloy-tipped saw, as also on plywood covered with linoleum, rubber, bakelite, or Formica.

The lower half of our drawing shows a hogging head, which is a standard Huther Bros. dado set, made up with $^1/_8$-in. spur saws at the outside and two-point cutters in between with a total thickness of $1^1/_2$ in. Whereas a single saw will leave a waste strip with is apt to cause trouble at the tenon or cope heads, the hogging heads reduce the off-fall to shavings, which are taken up by the exhaust hood. By changing the arrangement of the hogging head, the rabbeting may be done at the cut-off saw, provided it is not convenient to do these operations with tenon heads or copes.

Another drawing shows the scoring-saw mandrels with standard equipment—the $^1/_{16}$-in. scoring saws used to score down through face veneer and crossband, preliminary to tenoning. The scoring saws may be tilted to 45 deg. in either direction. The maximum and minimum length between shoulders and the maximum and minimum thickness of stock are given.

Grooving saws mounted over a special head may replace the scoring saws. This machine is equipped with these groovers for both $^3/_{16}$-in. and $^1/_4$-in. grooving. When grooving fireproof wood or other abrasive materials, the operator is instructed to use a groover with Carboloy teeth. This rule is standard, but is shown on each drawing for emphasis.

The scoring-saw mandrels may also be equipped with a small dado set. This is mounted on the same type of head as are the grooving saws. Each dado set consists of two $^1/_8$-in. spur saws, two $^1/_4$-in., one $^1/_8$-in., and one $^1/_{16}$-in. two-point cutters. This enables cutting a dado varying in width from $^1/_4$ to $^{15}/_{16}$-in. The depth of cut depends mainly upon the power of the scoring-saw motors. As delivered, these motors could not drive a $^{15}/_{16}$-in. cut deep into hardwood. Accordingly they have been changed for operation from a frequency changer, speeding them up to 6000 r.p.m., and practically doubling the horsepower. At this higher speed, there is no difficulty in pulling any cut desired.

The scoring-saw mandrels may also be equipped with special dovetail cutters for making a long male dovetail on the end of the stock, as used in store-fixture and cabinet work, on drawer sides, divisions, etc.

On the tenon heads, with the usual arrangement of both heads cutting a tenon spaced in the center of the piece, the maximum width of tenon is 3 in., but by double-passing through the machine, a 6-in. tenon could be cut.

The top tenon head can also cut a rabbet from the top face of the stock. Similarly, the bottom head could be used to cut a rabbet from the bottom face.

The cope arbors are the most versatile of all the arbors of the standard double-end tenoner. This arbor may be tilted 45 deg. in one direction and 10 deg. in the other. The maximum and minimum length of stock and the maximum and minimum thickness of stock are shown. These arbors are equipped with 3-in. joining heads. This is a solid steel four-knife round head, used for jointing or facing the edge of the stock where a smoother cut than given by the trim saws is required.

The cope arbor may also be equipped with a grooving saw. The machine is provided with $^3/_{16}$-in., $^3/_4$-in., and $^3/_8$-in. groovers, each 9 in. diameter. The operator is instructed to use Carboloy groovers when cutting fireproof wood or other abrasive materials.

The cope arbor is equipped with a panel raiser for scooping out backs of panels. This is a two-point steel cutter, especially ground. The equipment is placed on the cope arbor and secured with collars and lock nut on top of shaft. This reduces to a minimum the set-up time for these operations.

By employing the standard Wilkin-Challoner cope heads, but with a cutter ground with 45 deg. face, one is able to make both male and female members of the lock miter. Other cuts necessary to the lock miter have been made by the tenon or scoring-saw mandrels previous to the material reaching the cope head.

Another type of cope head used is the shaper disk. High-speed steel knives are inserted in grooves and held with bolts which go through the disks and firmly lock the knives. Most any type of molding can be ground in the face of the knife, which enables the double-end tenoner to do all types of shaping.

The cope spindle may also be equipped with a dado set. This machine is equipped with dado sets both 9 in. and 10 in. in diameter, each set consisting of two $^1/_8$-in. spur saws, eight $^1/_4$-in., two $^1/_8$-in., and one $^1/_{16}$-in. two-point cutters. The operator can assemble a dado set to give any desired width from $^1/_4$ to $2^7/_{16}$ in. He can cut to a depth of 3 in. with the 9-in.-diameter sets and $3^1/_2$ in. with the 10-in.-diameter sets.

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[3] President, George W. Smith Woodworking Company, Philadelphia, Pa. Assoc-Mem. A.S.M.E.

It is easily seen that a great variety of work can be done on a standard double-end tenoner. If special equipment, such as fixed and jump dado arbors and relishing and notching attachment, be added to the standard machine, the breadth of work which can be done on a double-end tenoner is greatly increased.

It has been asked why we do not run the grooving saws on the tenon arbor instead of using the scoring-saw arbor. Because of the importance of reducing the set-up time to a minimum, we never disturb the tenon heads. It is much more simple to use the scoring-saw arbors, as we can get at them more readily.

Author's Closure

In Mr. Schloemer's discussion he asks the particular benefits of the hinged pressure beam as shown on Fig. 8. The design of beam was worked out for the purpose of making the space around the top and bottom tenon heads more accessible by swinging open like a gate. It is supported by a vertical hinge on a bracket at the outfeed end, fits in between finished surfaces at the infeed end of the bracket, and is further supported by a clamp on a bracket just at the rear of the cut-off saws when in use. Savings of 5 to 10 minutes are made with this type of pressure beam over others each time either the top or bottom tenon head has to be removed.

The objection raised, lack of adjustability of the pressure bar relative to the feed chains, is not serious, as the question has never been raised by the users of these machines. There is space between the hinged support bracket and the pressure bar for grooving cutters.

Many will argue the merits of V-chains and tracks as against flat chains and tracks. A study of machine tools for producing accurate work, such as grinders, planers, boring machines, and lathes, indicates the wide use of V-guides where extreme close work is demanded. In the woodworking field the saws known as the straight-line rippers have chain feeds using V-tracks.

In connection with feed-chain drives on tenoners or any other machine, either single-speed or multi-speed motors may be used. With either, the same reduction from the motor to the feed sprockets or rolls must be made, requiring about the same number of gears, pulleys, or sprockets. With a single-speed motor in addition to the reducing drive, it is necessary to provide some means of speed changing such as a cone pulley or change gears. In the machine described, sliding gears similar to those used in automobile transmissions are used, and they are controlled by a lever conveniently located, providing a feed-changing method which is quick and can be accomplished without stopping the feed or its motor.

The electrical control for the single-speed motor may be of the simplest type.

With the four-speed motor the only saving in the mechanical hook-up possible is the gears or pulleys for speed changes. The electrical control must be more complicated and usually calls for a drum controller in connection with some type of overload protection.

It is almost impossible to provide a simple and effective electrical overload protection device for a multi-speed motor without going to great expense. Overload protection in case of jams is very necessary on machines having positive feeds by means of chains and dogs where there is no chance for the work to slip as with feed rolls.

Another objection to four-speed motors is that the range of speeds available is in a ratio of 600, 900, 1200, 1800, having a range of 3 to 1 from high to low, or in a ratio of 900, 1200, 1800, 3600 having a range of 4 to 1 from high to low. Either of these ratios is greater than is required in average tenoner work.

Production on a double-end tenoner is not controlled by the ability to change from one rate of feed to another quickly, but by utilizing the highest rate of feed possible and keeping the chains full of work. With either type of feed, higher rates of feed than standard can be provided and are used in plants on small work that can be handled with hopper feeds.

Mr. A. D. Smith brings out clearly the savings that can be made in tenoner operations through close coordination of the engineering and production departments and indicates that a great deal of study has been given this phase of work in his plant.

WDI-54-12

Developing a New Wood Product Through Research

By THOMAS D. PERRY,[1] NEW ALBANY, IND.

Many appeals have been made before the Wood Industries Division for research, and several papers have been presented, particularly that of Dr. J. W. Lawrie, in October, 1930. The development of a new prefinished plywood flooring is covered in detail, beginning with the product selection and discussing competitive products of both solid wood and non-wood. Considerable attention is given to the principles of market analysis, although it is admitted that the present depression renders such investigation of less than usual value. Detailed consideration is given to product analysis, emphasizing the limitations and shortcomings of available wood floorings. This analysis is followed by the respective and corresponding steps required to overcome these shortcomings. Practical or applied research is the basis for all the solutions. Emphasis is placed on the strength and rigidity qualities of plywood as well as on the manifest advantages of prefinishing. Several unanticipated advantages appeared from time to time among the research steps. In conclusion it is pointed out that the development of this plywood flooring, from its inception to its acceptance by the market, has been accomplished in an unusually short time, and therefore offers encouragement to other woodworkers to do likewise in their field.

IT IS a well-established manufacturing axiom that new products almost invariably come as the result of aggressive research methods. To many woodworkers, however, research is a relatively unknown term, with somewhat vague possibilities. Wood products have probably benefited less by research than other major branches of manufacturing, while electrical manufacturing, communication engineering, mechanical refrigeration, the radio, the auto, and many modern conveniences could not have come without research. Hence, a brief outline of an actual problem and answer in the wood-industries field may offer information and inspiration to other woodworkers.

Research is recognized as of two distinct varieties, i.e., fundamental and applied. Fundamental, or pure, research is the searching out of scientific facts with regard to the nature of raw materials, as well as the principles controlling their conversion into useful products, but seldom concerns itself with the gainful character or individual application of these facts. Applied research, on the other hand, is that type of research that converts fundamental facts into profitable industrial applications. The former requires a scientific, investigative turn of mind that can concentrate on the search after truth wherever it may lead. The latter type of research requires an entirely different type of mind—one that can concentrate on a definite practical objective, that can sense and grasp the dollar value, that can sell the idea to industrial leaders, and that can explore and select the necessary immediate steps between the fundamental idea and its profitable development into a practicable product.

In the woodworking field fundamental research is well exemplified by the activities of the Forest Products Laboratory at Madison, Wis., as well as by such schools as the New York State College of Forestry at Syracuse and the Forestry Department of the University of Washington at Seattle. Outstanding examples of applied research in this field are "Balsam Wool," an insulating blanket, developed by the Weyerhauser Lumber interests, "Plymetl," a laminated wood-and-metal product originated by the Haskelite Company, and "Masonite," a composite board or sheet of disrupted wood fiber.

The new product "Uplyco" prefinished flooring, the development of which is outlined in this paper, was distinctly an applied-research program. Laboratory activity had a relatively small part in the program, unless the building construction industry be termed a widely extended field laboratory. The practical research work was about evenly divided between field investigation, as well as intensive customer demonstration, to determine consumer requirements, and factory experimentation, to develop economic and efficient manufacturing processes.

Excess Capacity From Consolidation

Upon the completion of the legal steps in the consolidation of this corporation in July, 1930, we found ourselves with three plywood-manufacturing units admirably located and adequately equipped, but serving an industry that was suffering acutely from excess productive capacity. To close down one or more of the plants was easy, but to preserve and increase industrial profits in a period of depression was a challenging task. The first step, obviously, was to concentrate all the available general plywood business into the one plant best equipped for this purpose, and thus reasonably insure its profitability. The next step was to foresee so far as possible the probable trend of industry and select known or develop new products to keep the other plants both busy and profitable. One of the difficulties encountered in making this choice was the tendency of former factory users of plywood to make their own plywood rather than to purchase it. While this trend was evident during busy years, it was most conspicuous during such lean years as the present. Not even the most optimistic executive could visualize a return of the normal demand for plywood that had existed in past years. This definitely barred the path against any known plywood product, since excess capacity of two kinds already existed. This background made imperative the selection of a new product that would inevitably

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

require research, development, and promotion in order to preserve and enhance corporation profits.

PRODUCT SELECTION

There are two principal reasons influencing the selection of a new product: First, that of having the manufacturing facilities, and, second, that of possessing the necessary distributing organization. It is a well-established fact that it is economically sound, if possible, to select new products that can be marketed advantageously through the existing distributing organization rather than to choose a product that may be suited to manufacturing facilities. Manufacturing problems are more concentrated and tangible, permitting easier readjustment, than are the marketing problems, where the organization has a broad geographical spread and many intangibles such as manufacturing reputation, recognized publicity, salability, etc. In the case in question, however, the distributing organization had not been extensive, since the previous output had been sold as a semi-raw material (plywood) to a relatively few large manufacturing organizations that would complete the processing and convert the purchased plywood into furniture, radios, doors, interior equipment, showcases, and the like. Plywood, therefore, was in a somewhat difficult situation, since its value and even its very existence were seldom recognized by the general buying public that bought a wood product, not because it was plywood, but because its ultimate form was either useful or attractive. Since the existing distribution facilities were not extensive or elaborate, it seemed justifiable to select and develop a product more in keeping with existing manufacturing facilities even though it might require the creation of a new and intensive distribution organization.

The plywood factory chosen for the development of the new product had already done some experimenting, in previous lean seasons, with a unit or sheet flooring that would appeal to the public because it could be purchased and laid in larger sections than the conventional strip flooring. The experimenting on this floor had extended over a period of several years, but at no time had it been regarded as a serious problem nor had it ever been considered as an outlet that promised large volume.

While many other products were considered, hardwood flooring was ultimately selected as the most promising of the various suggested products that were reasonably available. The reason for this decision was partly the substantial experimental work that had been done, partly its adaptability to manufacturing facilities, but chiefly that the home-building program of the United States promises greater volume in the next few years than any other field in which plywood might be used. Not only is the country behind in its home-building program, but the lure of the roads created by the automobile is yielding quite definitely to the attractiveness of the home with its radio. The automobile caused people to go away from home for their pleasure, but the radio has brought them back and has emphasized the need and desire for comfort and luxury in the home. Consequently, not only is there a shortage of existing homes, but we are facing the renovating and remodeling of a multitude of existing homes to make them more comfortable, more pleasing, and more livable.

Flooring, in the main, was found to have been selected chiefly for utility and low price; as a decorative factor it had been given relatively little thought or attention. Hence, surrounding conditions appeared clearly to justify the development of a new and better hardwood floor, and plywood construction seemed to offer unusual opportunities for improvement over present solid-wood products.

COMPETITIVE PRODUCTS

While no ardent proponent of a new product will admit it has any actual competition, it is absolutely necessary to recognize that there are many allied competitive products that must be carefully considered in their relation to a proposed new product. Hardwood flooring had been used for generations, and a wide variety of types and species was available. The wood flooring in the United States was divided into four principal groups, as follows:

Strip, laid in relatively narrow units ($1^1/_2$ to $2^1/_4$ in.), tongued and grooved, blind nailed, reasonable in cost, and adapted to a wide variety of uses for homes, offices, factories, public buildings, and the like. This had wearing qualities, but a minimum of artistic value.

Parquetry, consisting of various symmetrical and artistic unit blocks and assembled patterns, used principally where cost was not a consideration and where the appearance was all-important. Combinations of strip and parquetry were seldom utilized.

Plank, or colonial flooring, used in random widths and lengths to reproduce the intrinsic charm and historic background of the colonial periods of home building.

Factory, in the main built for sturdiness to resist hard service, but without regard to appearance. Some types were unusually thick, with either edge or flat grain exposed, and in some instances with end-grain wearing surface (these grades of factory flooring were not considered in the development program).

In addition to the competitive wood floorings enumerated, there was found to be a wide range of other materials which might be termed substitutes for wood flooring, such as concrete, stone, tile, terrazo, rubber, fiber, composition, and linoleum. It was found that many of these products had been aggressively promoted, and their success had been due partly to intrinsic merit, but more especially to promotional zeal and merchandising skill. In other words, a large amount of substitute materials had been installed where general wood flooring would have been better had it been merchandised with equal skill and vision. It was therefore necessary to develop a promotional and distribution program that was at least the equivalent of such substitute flooring.

It is not intended to convey the impression that wood flooring is always suitable for every location. It is well recognized that non-wood flooring materials have a legitimate place, but the investigation led to the conclusion that in many instances skilful merchandising had extended some of these products beyond their normal field of application.

MARKET ANALYSIS

Market analysis is a broad term that has been used to cover a wide variety of surveys to determine the past and potential marketability of a new product. In the judgment of the author, its scope should be limited to these four principal functions:

(1) Volume of similar products sold during the five years preceding, together with a prediction of reasonable future sales volume.

(2) Effective sales channels for the new product.

(3) Probable customer demand, or the proportion of the available business that may be anticipated for a new product.

(4) Pricing problems, including standard costs and operating budgets.

The peculiar current business conditions made this market analysis particularly difficult and compelled the use of methods that would ordinarily be considered too inexact.

(1) *Volume of Flooring.* Complete market statistics of manufacture and sales for the whole flooring industry were nowhere available, since some trade organizations in the field were found to be demoralized and hardwood flooring also had been made by many small non-reporting manufacturing units. It was possible, however, to obtain partial statistics from two trade organizations representing the principal manufacturers of oak and

maple flooring, as given in Table 1 and 2. It was also possible to ascertain with accuracy the floor area of school buildings erected year by year, since this branch of public service is thoroughly covered by reporting agencies; this is shown in Table 3.

TABLE 1 OAK FLOORING SHIPMENTS (INCOMPLETE)

(Compiled by Oak Flooring Manufacturers Association)

	Thousand Square Feet, Board Measure				
	1926	1927	1928	1929	1930
January	...	...	30,080	31,999	26,317
February	...	...	40,232	28,551	28,272
March	...	...	47,599	40,902	34,829
April	...	...	46,105	41,837	...
May	...	...	50,256	41,558	...
June	...	...	49,475	39,979	...
July	...	...	47,832	37,002	...
August	...	...	51,860	37,631	...
September	...	44,471	45,020	31,464	...
October	...	45,091	45,652	29,704	...
November	...	37,343	37,232	21,202	...
December	...	35,074	31,665	47,506	...
Yearly total	...	...	531,008	429,335	...
Monthly average	45,024	42,756	44,251	35,778	...

TABLE 2 MAPLE FLOORING SHIPMENTS, INCLUDING BIRCH AND BEECH (INCOMPLETE)

(Compiled by Maple Flooring Manufacturers Association)

	Thousand Square Feet, Board Measure				
	1926	1927	1928	1929	1930
January	...	...	7,252	5,114	4,625
February	...	...	6,889	6,053	3,771
March	...	...	8,184	6,141	4,761
April	...	...	7,545	7,583	4,768
May	...	...	8,256	6,848	5,357
June	...	...	8,686	6,420	4,469
July	...	...	8,311	7,883	...
August	...	...	9,515	8,435	...
September	...	9,532	7,929	7,239	...
October	...	7,913	8,543	6,896	...
November	...	6,884	6,230	4,379	...
December	...	6,545	5,784	3,060	...
Yearly total	...	...	93,124	76,051	...
Monthly average	9,983	9,163	7,760	6,338	...

TABLE 3 AREA OF SCHOOLHOUSES, ACTUAL CONTRACTS AWARDED

(Compiled by American School Board Journal, Milwaukee)

	Square Feet of Floor Area				
	1927	1928	1929	1930	1931
January	2,376,000	3,498,100	2,729,100	2,886,700	2,800,100
February	3,009,800	3,157,400	3,848,200	4,068,200	2,272,300
March	5,997,800	5,316,300	6,348,400	6,135,000	4,021,100
April	4,808,300	5,124,500	4,782,400	5,841,500	3,823,400
May	5,346,900	7,160,500	6,360,400	6,213,000	3,491,000
June	5,675,600	7,390,400	6,662,600	6,729,700	3,558,200
July	5,466,300	5,904,700	6,944,300	6,513,900	......
August	5,937,800	5,761,700	5,427,900	4,140,700	......
September	4,263,700	6,019,400	5,075,400	4,150,000	......
October	4,171,200	4,965,100	5,668,900	3,583,900	......
November	4,684,900	4,490,300	3,928,200	4,275,400	......
December	3,736,200	4,194,500	3,232,200	2,162,800	......
Yearly total	55,474,500	62,982,900	60,644,000	56,700,800	......
Monthly average	4,622,892	5,248,575	5,053,667	4,725,067	3,327,683

School-house area allocation: Instructional purposes, 50%; stairs and corridors, 20%; administration rooms, 16%; walls and partitions, 10%; flues and ducts, 3%; accessories, 1%—100%.

It did not seem wise to extend the period of investigation further than five years, since the trend of industrial events would make earlier statistics of relatively little current value.

The slump in volume during the years 1929 and 1930 rendered these statistics of relatively little actual value and precluded any reasonable estimates of probable volume for 1931 and 1932. The investigation of trade volume is, however, an essential step in market analysis and would have been of marked importance in any period except that of a world-wide depression.

(2) *Effective Sales Channels.* The selection of the most advantageous sales channel was found to be a decidedly complex problem, although not seriously affected by the current business situation. If hardwood flooring was to be distributed, was it best to adopt a policy of selling direct to the consumer, endeavoring to contact a multitude of prospects, either by mail or commission men, or of selling through the wholesaler or jobber, who had a large number of products and could not well concentrate on a few, or of going direct from the factory to the retail dealer, who might be willing to undertake a campaign on a speciality, but who would require from the manufacturer an extensive demonstration and sales organization? A careful study was made of the methods adopted by other specialty manufacturers, such as of rock-faced shingles, fiber boards, insulating and acoustic panels, plaster materials, builders' hardware, and other construction specialties. The result of this investigation was to conclude that a direct customer distribution was not advisable, since this would alienate all the recognized and existing distribution channels. Where a jobber gave evidence of merchandising ability, it was more advantageous to work through such a jobber than to approach his trade direct. There were also found many locations where jobber distribution was inefficient and undesirable. Price differentials were therefore set up to provide for a reasonable pricing of product direct from the factory to the retail local dealer or from the factory through the jobber to the retail local dealer. Experience has demonstrated the wisdom of this course.

(3) *Probable Customer Demand.* An intensive investigation into local market conditions in various vicinities should yield reasonably definite data as to local absorptive capacity, especially in the case of a well-established product. In the case of a new and relatively unknown product, the data would be much less satisfactory and dependable.

The current flooring demand was found to be characterized by distinct style and period trends, with particular emphasis on colonial types and on styles harmonizing with the novel and somewhat startling modernistic tendencies.

A new product, with individual characteristics of its own, could not afford to ignore these contemporaneous conditions, and therefore should partly meet them. At the same time it should not entirely subordinate its own outstanding qualities and ignore opportunities that only required concentrated development.

Depressed business conditions also complicated the determination of a volume objective. It was therefore decided to start the sales campaign rather modestly in a zone around the factory extending some 200 miles in each direction. With the information as to customer requirement and absorptive capacity in this area, it then became practicable to enlarge this zone and gradually cover more and more states. Such a field demonstration may be slow, but its results are reasonably certain, and it is much less costly than unintelligently covering too much territory. The prompt response of the consuming public in the first zone permitted a more rapid expansion than had been anticipated.

(4) *Pricing Standards.* A large mass of evidence was collected showing purchase price and installation cost of the various available solid-wood floorings, as well as substitute floorings. Cost comparisons vary widely in different communities, according to size, social status, labor costs, and many other circumstances. While such comparative costs have a definite bearing on the price set-up of a new product and on the manufacturing-cost allocations that can be made, yet they are likely to vary too much to be conclusive. It should be kept in mind that any new product will have its own inherent advantages that influence comparisons of competitive cost. The final determination of logical and obtainable prices was made only after several extensive field demonstrations in selected localities, as outlined.

Much study was given to price schedules, standard factory costs, and budgetary control, not omitting a sincere application of the "trial and error" methods to some problems that might appear otherwise unsolvable.

The reproduction of actual tabulations is inadvisable, partly because accounting problems are too extensive for engineering discussion, but more especially since it is difficult to illustrate principles without divulging facts that might be wrongly used.

PRODUCT ANALYSIS

In order to create and develop a new product that would deserve and win consumer attention, it was necessary to examine with care the advantageous qualities and definite limitations, in various directions, of the existing styles and grades of flooring. It would have been unwise to go directly into a competitive field with only a normal type of product and to enter a difficult competitive situation that promised little profit and recklessly increased productive capacity in an already overproduced field. It was therefore the aim to overcome, so far as was feasible, the principal shortcomings of existing types of flooring. The principal limitations and shortcomings were found to be as follows:

(*A*) The shrink, swell, warp, and twist of ordinary solid lumber flooring required that it be laid in relatively narrow strips to avoid unpleasant curling of the individual board and insanitary cracks between the boards when atmospheric conditions varied, as in a dry heated home in the winter and in a damp unheated room in the summer.

(*B*) The tendency of solid wood flooring to split around nails either when damaged in laying or when strained to hold the flooring down. These splits usually developed in the tongues and grooves and were found to be the principal cause of loose and squeaky floors.

(*C*) The unreasonable cost of laying narrow strip flooring in random lengths with excessive amount of nailing. Not only did the cost element enter, but also the time required to complete the job.

(*D*) Parquet floors, in both wood and other materials, were found to be nearly prohibitive in price, because of the manufacturing cost of small units and the abnormally high installation cost when each piece had to be individually laid in place and fastened by the various available methods. It was found that practically all parquet was so manufactured and laid, and that consequently its use was greatly restricted.

(*E*) The high cost of scraping, sanding, and applying finish materials as well as the poor quality of finish on floors installed under modern construction conditions with unfavorable temperature and humidity and unavoidable dust and damage.

(*F*) The entire absence of eye appeal, characteristic of all unfinished wood products. The feminine buyer, and she is in the big majority, is particularly susceptible to color and harmony values that can be developed in prefinished flooring.

(*G*) The lack of protective packaging and standard sized units for convenient and efficient warehousing and delivery.

(*H*) The laying of wood floors in reinforced concrete (fireproof) buildings was done in two ways; either over a subfloor placed on strips or screeds anchored to the concrete slab, sacrificing some 3 to 4 in. per story height, or laid direct on concrete with an adhesive mixture or mastic, producing a bond of uncertain strength and problematical life.

PRODUCT DEVELOPMENT

The overcoming of these various limitations of solid wood flooring became a series of more or less overlapping research problems. Each answer was a combination of past experience, field demonstration, economical factory processing, and customer acceptance. While in no instance was a complete solution reached, it was extremely stimulating to find that intensive and correlated research eventually led to reasonably complete results. Using the same designations as in the foregoing list of limitations, an outline of these results is as follows:

(*A*) *Constant Dimensions of Plywood.* The shrink, swell, and twist of solid lumber have long been successfully overcome in the making of furniture, cabinets, desks, phonographs, pianos, doors, and the like through the use of plywood construction, and plywood manufacturers had conclusive experience in that direction. The hard usage and wear to which all floors are normally subjected put standard plywood with its thin veneer faces out of the question, but the fundamental principle of plywood cross-laminations had been adequately demonstrated to neutralize the normal shrink and swell of wood, or at least to reduce these variations to a practicably neglible minimum.

Government laboratories have repeatedly demonstrated that, pound for pound, well-designed plywood is stronger than steel. It was necessary, therefore, to develop a type of flooring in plywood that would have a thick enough face to withstand hard wear, as well as cross-plies of sufficient stiffness to prevent side warping, and still have adequate lengthwise strength in the face and back.

The result of extensive experimenting and rigorous tests under all conditions proved that a segmental sawn lumber face, made of narrow strips and frequent butted ends, would destroy the stresses and strains of long, wide boards and thus eliminate the twisting tendency from the face, as well as utilize the face lumber economically; that the single or duplex cross-plies must have a thickness of not less than an aggregate of $^1/_4$ in., with equally thick longitudinal members in one or two units; and that the face must approach $^1/_4$ in. thickness to provide adequate wearing surface. This resulted in a balanced five-ply construction with three lengthwise members, the normal direction of lumber strength, and two crosswise members to neutralize shrink and swell, all firmly glued together, to prevent changing widths and to keep the boards from warping sidewise.

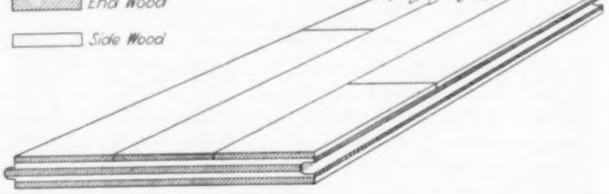

FIG. 1 PLYWOOD FLOORING, FIVE-PLY $^5/_8$-IN. BY 6-IN. BY 6-FT. 8-IN. STANDARD STRIP

(Showing balancing effect of alternating crosswise layers. The laminations in tongue and groove prevent splitting.)

Thus five-ply laminated flooring, with faces of close-textured dense oak or maple and backs and cores of chestnut or poplar (with their well-known inert properties and gluing qualities) resulted in a plywood construction of sufficient strength and adequate balance.

A water-resistant glue, like casein (designated by the U. S. Government for airplane plywood), produced an adhesive joint stronger than the natural bond of the wood fibers that will tear apart in the wood rather than separate at the glue joint. The machining operations will slightly reduce the thickness of face and back, but still leave a sturdy strip of five-ply flooring $^5/_8$ in. thick. This type of plywood floor fabrication and the resulting floorboards are shown in Fig. 1.

(*B*) *Non-Splitting Quality of Plywood.* Reference to Fig. 1 will show that the tongue in plywood floor is actually three-ply and that the upper and lower grooves are both two-ply. It is well known that plywood cannot be split across a glue joint. Tests proved that when lifting plywood flooring from a pine subfloor each sloping nail held up to 75 lb. direct pull and the nail was drawn out with the floor; when nailed to a hardwood plug in concrete, the individual nail power was 150 lb. In no instance did either tongue or groove show any sign of splitting. A challenging test of the non-splitting quality of plywood floor is shown in Fig. 2, in which 40d spikes were driven into 2 in. lengths of floor without predrilling of holes, resulting in neither splitting of wood nor breaking of glue joint.

It is evident that this non-splitting quality permits the use of larger-bodied nails. Most floor nails were discovered to have been so slender (to prevent splitting) that they bent easily, resulting in bruised floors and installation delays.

A special large-bodied ($12^{1}/_{2}$-gage), high-carbon steel, coated fivepenny ($1^{3}/_{4}$ in.) nail was an accessory product, developed and sold with the plywood flooring.

(*C*) *Reducing Nailing Expense.* In order to overcome the excessive laying and nailing cost, it was demonstrated by a series of tests that plywood flooring, as described, could be made up to 6 in. wide and blind-nailed through the tongue, thereby reducing the actual laying cost to less than half that of the normal $2^{1}/_{4}$ in. wide strip flooring and to approximately a quarter that of the $1^{1}/_{2}$ in. width. The relative widths are shown in Fig. 3.

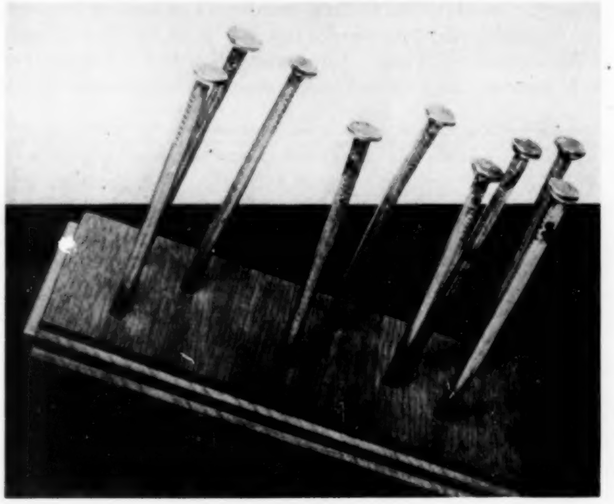

FIG. 2 NON-SPLITTING QUALITY OF PLYWOOD
(These 40d spikes were driven in a section of plywood floor 2 in. long, without predrilling holes for nails.)

Solid flooring was necessarily laid in $2^{1}/_{4}$, 2, and $1^{1}/_{2}$ in. widths to minimize (and not with entire success) the shrink, swell, and warp of untreated solid wood. Treated or impregnated wood changed dimension less than untreated, but did not allow any great increase in practicable width of flooring strips. The "come and go" of untreated wood is between 5 and 10 per cent, which in a room 20 ft. wide means a minimum aggregate of 12 in. if concentrated at one point, but also results in a minimum average of 0.1 in. between 2-in. strips in the same area. Plywood variations are less than one-half of 1 per cent, and consequently negligible.

The 6-in. plywood floor boards, blind-nailed every 16 in. ($1^{1}/_{2}$ nails per sq. ft.) with specially designed nails, as previously described, provided adequate floor attachment, and the more frequent nailing of narrow solid boards (6 to 10 nails per square foot) was found to be excessive in time, labor, and material costs.

(*D*) *Popularzing Parquet by Reducing Its Cost.* The problem of reducing the cost of parquetry flooring to promote its use and develop its popularity required another series of demonstrations to develop an application of plywood construction principles that would give a similar stability and laying economy to this more attractive type of flooring. Parquetry is especially designed to add beauty to the decorative scheme, either to increase the appeal of unoccupied rooms or to supplement the attractiveness of furnished quarters. The rectangular parquetry faces of artistic and colorful woods were non-continuous and required balancing cross-bands for the crosswise as well as the lengthwise square faces. The answer proved to be a three-ply balanced foundation with a superimposed parquetry face, thus providing the necessary counterbalancing for the transverse face squares and still preserving the stiffness of the cross-core used in the strip flooring already described. This four-ply construction is shown in Fig. 4 and obviously brings an even greater advantage to parquetry flooring than does plywood construction to the strip flooring. Parquetry flooring, which can be inexpensively laid in long boards, is a far-reaching improvement that opens the entire field of moderate-priced homes to parquetry floors.

(*E*) *Prefinishing Flooring at the Factory.* The type and quality of floor finish achieved under normal construction and contracting procedure left much to be desired. The scraping and sanding was laborious, much of it inefficient, and neither the finishing materials nor the mode of application were on a par with modern knowledge and mechanical application equipment.

This emphasized the importance of prefinishing the floor at the factory before laying. Furniture has long been so prefinished before sale and installation, and why not flooring? Plywood flooring, made in standard widths and lengths as described, tongued and grooved on edges and ends, could be laid with practically no cutting except at the walls of each room. The only nailing required was the blind-nailing through the plywood tongue, and that in the hands of skilful workmen, with "unbendable" nails, need not harm the finished surface. Hence the problem of prefinishing became one of determining the best finishing materials under a series of standard tests and of devising a progressive chain conveyor in the factory to carry flooring boards past the various mechanical application stations and through a series of controlled and regulated driers to the final rubbing and waxing station.

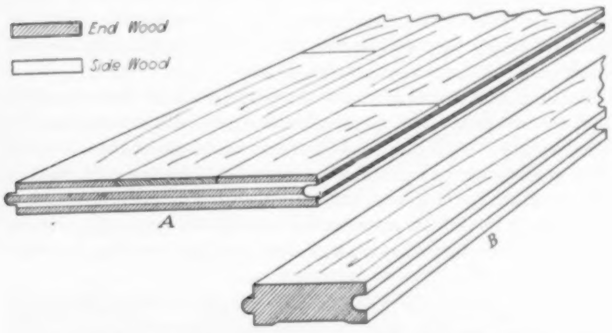

FIG. 3 GRAPHIC WIDTH AND THICKNESS COMPARISON
(*A*, standard plywood flooring, 6 in. wide by $^{5}/_{8}$ in. thick. *B*, standard solid flooring, $2^{1}/_{4}$ in. wide by $^{13}/_{16}$ in. thick. The $^{5}/_{8}$-in. plywood exceeds the $^{13}/_{16}$-in. solid flooring in aggregate strength.)

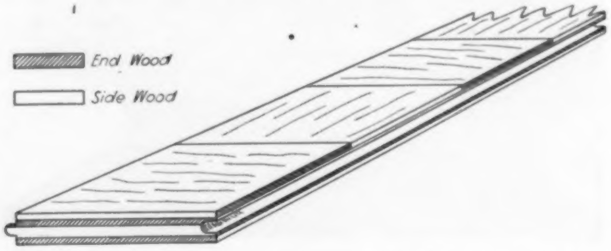

FIG. 4 PARQUET PLYWOOD FLOORING FOUR-PLY, $^{5}/_{8}$-IN. BY 4-IN. BY 6-FT. STANDARD BOARD
(Showing balanced three-ply base, on which is superimposed the non-continuous parquetry face.)

Prefinishing, however, is not a necessary feature of plywood flooring, but it is a distinct step in advance over the customary hand methods and is equally applicable to solid types of wood floorings and to other flooring materials, if the floor is manufactured with sufficient care and precision.

(*F*) *Expressing Beauty in Wood.* Rough lumber, as well as planed but unfinished lumber "in the white," arouses only a minimum amount of artistic vision and imagination, having a very low "eye appeal" value, and consequently has little real attractiveness to the homemaker, especially in this day of strong color contrasts and of brilliantly hued linoleum and tile floorings. Parquetry squares of oak, maple, walnut, mahogany, and other species, in various combinations, artistically stained and prefinished, were found to add greatly to customer appreciation and to facilitate sales approach as well as to insure buying interest.

Woodworkers in general, as well as retail distributors of lumber, were found to be singularly backward in intelligently developing this color value of their raw material.

The value of beautiful wood had long been recognized by makers of sheet-metal products, who had consistently imitated the beauty of wood in steel passenger vehicles, sheet-metal furniture, and metal office partitions.

(*G*) *Standard Packaging.* It is only within a comparatively short time that producers of wood products have given any serious thought to the manifest advantages of efficient packaging. The day of unpackaged distribution, except in the case of bulky low-grade products, is rapidly passing, and successful industries are discovering that convenient, protective, and attractive containers are a distinct merchandising aid.

In the case of flooring, an investigation revealed that no manufacturer had progressed beyond the open bundling, strapping, or wiring stage. Random lengths were one fundamental reason, thus confirming our program of standard lengths and widths. Open-bundle flooring was found to absorb a harmful amount of moisture in transit, in warehouses, and on construction. Hence, a complete corrugated container was adopted (see Fig. 5), overlapping at sides and ends, with steel strapping, affording adequate protection from rough handling, reasonable protection to normal moisture content, and convenient size for one-man handling and piling.

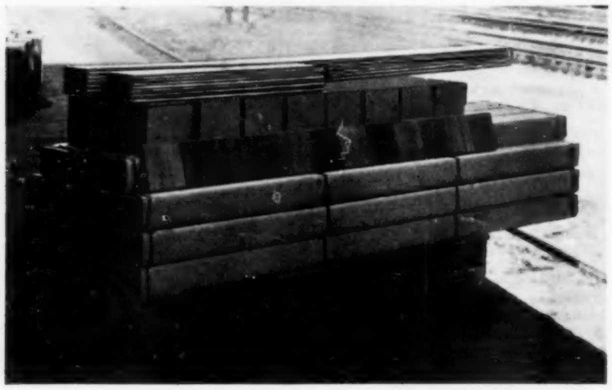

Fig. 5 Standard Carton
(Plywood flooring in standard packages, both for adequate protection and convenient handling.)

(*H*) *Attaching Wood Floors to Concrete.* Whenever reinforced-concrete construction depends on T-section calculations for its strength, it prevents bedding wood strips or screeds in the slab and requires that they be laid on top of the slab and grouted. This, with its accompanying sub-floor, sacrifices some 3 to 4 in. per story height, or nearly 3 per cent of building height.

Hot or cold mastic adhesives have rather serious objections, since slight variations in temperature or humidity may seriously weaken the bond. If a mastic floor once "lets go," there is no remedy but entire replacement. The ultimate holding life of mastics is not yet known. At best, these mastics are untidy and likely to deface finished floors.

Careful study and experimenting resulted in an innovation in anchoring hardwood floors to concrete, as indicated in Fig. 6. This process is a distinct improvement over laying in mastic, since it is cleaner and makes a stronger mechanical attachment. Holes are drilled in seasoned concrete with a 1-in. electric star drill, and these holes are plugged with $^{15}/_{16}$ in. square walnut plugs driven down flush. The concrete and plugs are then thoroughly coated with a waterproof asphaltum paint or are covered with an inexpensive roofing on which the plug locations are clearly marked. Of course the concrete sub-floor is laid out accurately in 6-in. zones (the width of the flooring boards) before drilling for plugs. The flooring is then blind-nailed into the plugs with a nail angle of approximately 45 deg. Actual tests show that a pull of 1000 lb. is necessary to withdraw these plugs, and approximately 200 lb. to withdraw a flooring nail driven at the 45-deg. angle into the plug. This strength greatly exceeds nailing into a pine sub-floor.

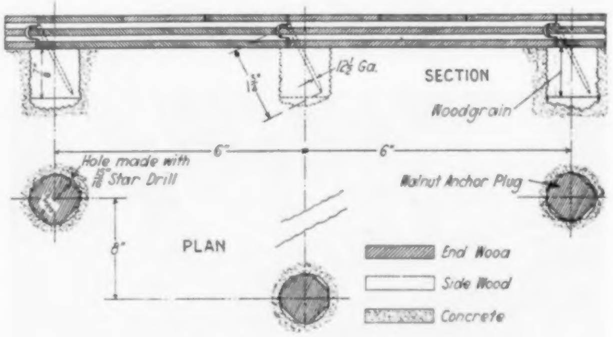

Fig. 6 Anchor Plugs in Concrete
(Mechanical method of attaching plywood flooring to a concrete sub-floor more dependable than mastic adhesive.)

General Problems

Space will not permit more than passing mention of the difficulties involved in remodeling and equipping a factory for straight-line plywood-flooring production, particularly when it is realized that straight-line production was a novelty to a woodworker. The problems encountered in promoting, marketing, and distributing the product were not only challenging, but withal decidedly encouraging. The objections to be met in persuading floor layers and customers to abandon obsolete customs and careless procedure, as well as to provide level and substantial sub-floors, involved a distinct change in standard specifications and education toward a better flooring consciousness for an improved type of floor.

These and many other problems added zest and interest to the task. It became increasingly evident that the consuming public was in a distinctly receptive mood for the new type of prefinished plywood flooring.

Pattern Parquetry

Many distinct advantages made their unexpected appearance from stage to stage of the development, adding many intermediate incentives to the research problem. One of these was the wide range of combination parquetry and strip pattern panels that can be developed from short lengths with artistic and pleasing results. An actual installation in colorful walnut and oak is shown in Fig. 7.

Overlaying Old Floors

Overlaying existing floors in occupied edifices opened a tremendous field not formerly available to wood flooring of any type.

While this overlaying was first undertaken with standard $^5/_8$-in. flooring, it was soon discovered that for such purposes a $^3/_8$-in. three-ply (see Fig. 8) was adequate, more reasonable in cost, and required less change in floor levels. The original worn floor was usually left in place, thus affording unusually sturdy support for the $^3/_8$-in. overlay.

PATENT PROTECTION

The development of adequate patent protection was another phase of the major problem. The ease of obtaining patents in this country, with no assurance of their probable legal validation in court, complicated the drawing of the patent specifications. Time only can prove the wisdom and ability expressed in the group of patents as actually filed. No new product should be developed without careful patent coverage.

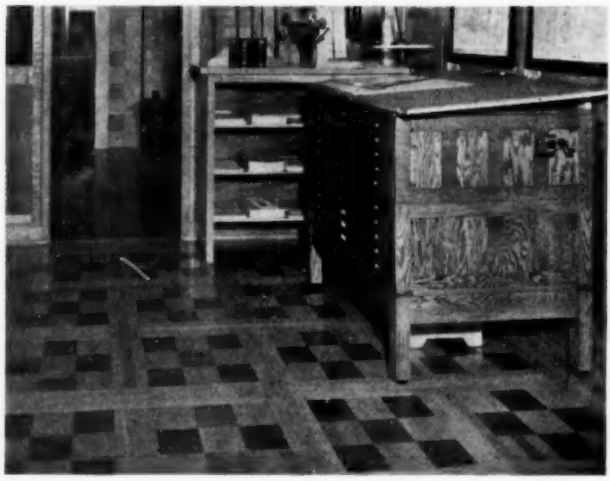

FIG. 7 PATTERN PARQUETRY
(Photograph of floor in author's office.)

PLYWOOD SUB-FLOORS

The advantages of a smooth, level sub-floor were undeniable, since ordinary sub-floors were found to be laid usually from almost any grade of rough or planed lumber, and in many instances from lumber previously used for concrete forms. Obviously no top floor could be expected to overcome the faults of a defective sub-floor. Investigation revealed that a five-ply $^5/_8$-in. subfloor, of fir or pine in 4-ft. by 8-ft. sheets, cost little if any more than the inferior sub-floors, added much to the bracing of the building, and also provided a level and continuous surface for any type of pattern parquetry.

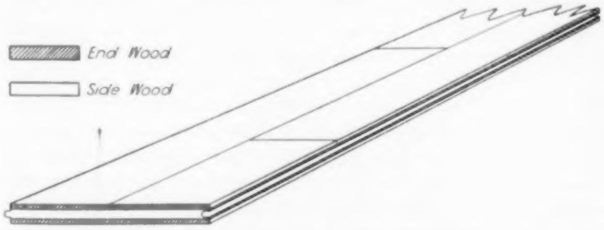

FIG. 8 OVERLAY PLYWOOD FLOORING
(Made in $^3/_8$-in. by 4-in. by 6-ft. 8-in. standard strip, three ply, with plywood grooves and solid tongue prevented from splitting by adjacent plies, above and below.)

CONCLUSION

The actual experimenting extended over a period of some five years, although the intensive research work was limited to less than a year. Mr. Charles F. Kettering, of the General Motors Research Laboratories, states that research projects take an average of seven years for fruition and that only a few out of many projects prove to be worth while. Not only has the past year of constructive development seen the product brought to successful marketability, but during this time the factory has been tooled up and a promotion and distribution organization has been created. The trade—wholesale, retail, architectural, and consumer—has enthusiastically accepted the product as the most outstanding flooring improvement of a decade.

Research does not always bring results so quickly nor does every idea prove so fertile. A consistent and persistent research program, however, will always reward men of vision and initiative who drive ahead vigorously and judiciously along the highway of progress.

Discussion

CARLE M. BIGELOW.[2] This paper could well be used as a standard-practice instruction by a great many manufacturers of woodworking products in developing new products. There is probably hardly an industry that has accomplished less research along the lines of product development than the woodworking industry. Furthermore, whenever they have approached this problem they usually considered it solely from the manufacturing rather than the distribution angle.

As the author brings out, it is more important with most products that satisfactory distribution is assured than it is that they can be manufactured readily. Almost any product can today be manufactured rather readily. We have an immense technique of manufacturing knowledge. On the other hand, we are woefully ignorant as to how to distribute products.

For several decades the fundamental theory of most manufacturers has been to give their principal consideration to the manufacture of a product and secondary consideration to its disposal. Under present conditions and the conditions which will probably be met for several decades to come, it is essential that we must first determine as to whether we can distribute a product, where it can be distributed, in what amounts, and at what expense, and then proceed to build up manufacturing facilities to meet the selling conditions, rather than expecting that selling conditions will meet manufacturing conditions.

The development of prefinishing and proper packaging and application also is of particular interest. Too often, wooden products which are to be used in subsequent fabrication or construction application are turned out in bulk amounts and the methods of application and finish left to the user. Very often, wood-fabricated materials are criticized for failure which really has been caused by method of application or finish.

The author has covered these points thoroughly in his development. For the short period of time in which this development has been accomplished he and his organization deserve great credit.

ARTHUR KOEHLER.[3] This is a good example of the possibilities of research in the development of wood-using industries. Unfortunately wood use does not have as much of this kind of work to its credit as would be desirable from the standpoint of the manufacturer and user of wooden products, and even from the standpoint of the owner of timber land. New and better uses of wood have not developed like those for steel, rubber, petroleum, and coal-tar products. There are several good and, to a large extent, hitherto unavoidable reasons why such has been the case, but the time has come when the wood-using industries must pay as much attention to making new and better products as they have in the past to reducing production costs, if they are to stay in the picture of national industries and if we are to find a ready market for all the wood that can and should be grown on millions of acres fit for nothing but growing timber.

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[2] President, Bigelow, Kent, Willard & Co., Inc., Boston, Mass. Mem. A.S.M.E.

[3] In charge of Section of Silvicultural Relations, Forest Products Laboratory, Madison, Wis.

Aluminum Oxide Versus Garnet as a Wood Abrasive

By J. F. Traendly,[1] St. Paul, Minn.

A comparison is made of aluminum oxide and garnet as surface-coated abrasives in woodworking. Garnet as found in the native state breaks into wedge-shaped pieces that when cemented to a surface of paper or cloth continue to fracture so as to produce further sharp edges. Aluminum oxide is a manufactured product, made in the electrochemical furnace by fusing bauxite and adding coke and iron. The hardness, toughness, and fracture of the two abrasive materials are compared. A table is given that covers time-study tests of belts used in a woodworking plant, with the labor and material costs.

Though the subject under discussion is a comparison of aluminum oxide with garnet as a wood abrasive, a more complete understanding of the properties of these two abrasives will be gained by knowing the early history of surface-coated abrasives, commonly but incorrectly known as "sandpaper." The idea of the present form of surface abrasives was gained probably from the twelfth century through their use of dried sharkskin and splintered whalebone for scouring purposes. In the next century, the Chinese attached crushed seashells to parchment with natural tree gum. Later, Swiss artizans used ground glass coated on a parchment or hide. The gumming of abrasive grains to flexible backings appears to have been started on a commercial scale in this country during the year 1828 by the Baeder Adamson Company.[2] The manufacturing process was the hand coating of individual sheets, but after a few years it was made in long strips, power being provided by dog-driven treadmills.

Of further interest is the reference to a leaf found in tropical seacoast areas that exudes a resinous gum, upon which the windstorms had deposited small particles of beach sand. When the gum dried and hardened, the result afforded nature's first conception of a surface-coated abrasive product.

Confining the discussion to commercially made products, one of the earliest and most common abrasives used was "silica," which occurs in nature as quartz, flint, and chalk flint. Crushed or ground glass was an accepted abradant in earlier days for sandpaper manufacture, and is still in use in England and continental Europe. Natural aluminum oxide, known as "corundum," has been used, but not to any great extent because its properties vary.

Garnet was introduced in 1880, but was then used more extensively in the shoe-manufacturing industry for polishing soles and heels. The introduction of garnet represented the first constructive step in the improvement of woodworking abrasives.

Aluminum-oxide grain, while generally and favorably known for many years as a metal-cutting abrasive, was not introduced to the woodworking industry for machine sanding until about 1923. Prior to that time its principal function as a wood abrasive was in mechanical floor sanding and surfacing.

Garnet

Garnet is as old as time. It was formed in the cooling and hardening of the earth's crust. It is found in many sections of the United States and in Canada, as well as in Spain. There are six well-defined species, of which one alone has proved of value as a wood abrasive, and this is almandite.

Almandite, so valuable as a wood abrasive, is obtained in the northern section of New York State. The deposits center largely in Warren County in the Adirondack mountain range.

Extraction of garnet is from quarries, which consist of a series of open pits that are developed into regular benches. Holes are drilled 10 ft deep, and these are shot with either 40 per cent or 60 per cent gelatin. Boulders are block-holed with small charges of dynamite. The broken ore is loaded into 3-ton cars with a steam shovel and hauled to the mill. The garnet-bearing ore passes through a milling operation which liberates the garnet from the surrounding rock by crushing. The separation of garnet from the crushed rock is accomplished by wet jigging. The garnet, being heavier, separates from the crushed rock under the influence of the motion and water. The yield is approximately 8 per cent, meaning that $12^1/_2$ tons of rock must be handled to secure 1 ton of garnet concentrate. The production o abrasive garnet in the United States in 1924 was 8290 short tons. In all likelihood this figure represents the maximum yearly output because during that year aluminum-oxide abrasive grain was beginning to receive attention as a woodworking abrasive having unusual value in certain operations.

The physical characteristics of almandite garnet which make it a desirable abrasive to use in the manufacture of garnet paper and cloth and combinations are its properties of hardness, toughness, and fracture.

Hardness. This is a property not easily defined. In the Mohs scale of hardness, wherein the diamond, the hardest known mineral, rates as 10 and talc rates as 1, garnet ranges from 6.5 to 7.5. Hardness in an abrasive mineral is the factor that determines which materials the point of the abrasive can penetrate. Obviously, if the mineral cannot penetrate the surface of the object being abraded, the only effect will be frictional attrition. Garnet has therefore not been used successfully as a metal-cutting abrasive.

Toughness. This property is important, since the penetration of a mineral into an object is only part of the problem. The mineral must have sufficient resistance to shear to prevent a point breaking off prematurely, instead of performing like a cutting tool. The toughness of minerals is not known. It cannot yet be accurately measured by scientific methods. All data are relative. According to some tests, garnet is about 65 per cent as tough as aluminum oxide, and to others 30 per cent.

Fracture. Garnet naturally breaks into ideal shape for coated-abrasive use. Its shape is a wedge having a triangular cross-section whose base varies from 0.3 to 0.7 the length of its sides. There are some particles whose cross-section is a parallelogram. It is important to note that these predominant shapes have edges caused by the intersection of sides at less than right angles, and

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[1] Sales Manager, Minnesota Mining and Manufacturing Company.

[2] Now the Baeder Adamson Division of the Minnesota Mining and Manufacturing Company.

Presented at the Sixth National Wood Industries Meeting, Winston-Salem, N. C., October 15 and 16, 1931, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

it is the length and nature of these edges that make garnet an effective wood abrasive. The sides of garnet particles are often conchoidal (dish shaped like a shell, or having a hollow-ground effect). This is of secondary importance to the nature and extent of edges. As the grains are broken in use, further sharp edges are produced, as garnet appears to have just sufficient brittleness to break under the strain of ordinary use rather than to polish down to a smooth surface.

ALUMINUM OXIDE

Aluminum-oxide abrasive grain differs from garnet in many ways. Garnet is a mineral, used about as found in nature. Aluminum-oxide is manufactured, being a product of the electrochemical furnace. It is made by fusing bauxite under carefully controlled conditions, adding coke and iron. The heat melts the ore, and a slag containing impurities comes to the top. The iron combines with the silica and forms a metallic button on the bottom. The molten aluminum-oxide liquid is permitted to cool slowly to allow proper crystalline structure. The best mineral has a good distribution of the remaining impurities, and has a consistently large and tough crystalline structure. Extremely high heats are used in the formation of crystalline aluminum oxide, exceeding 3600 deg fahr. It takes several days to cool a pig of grain. The desirable properties that make this type of mineral acceptable for woodworking grain are hardness, toughness, and fracture.

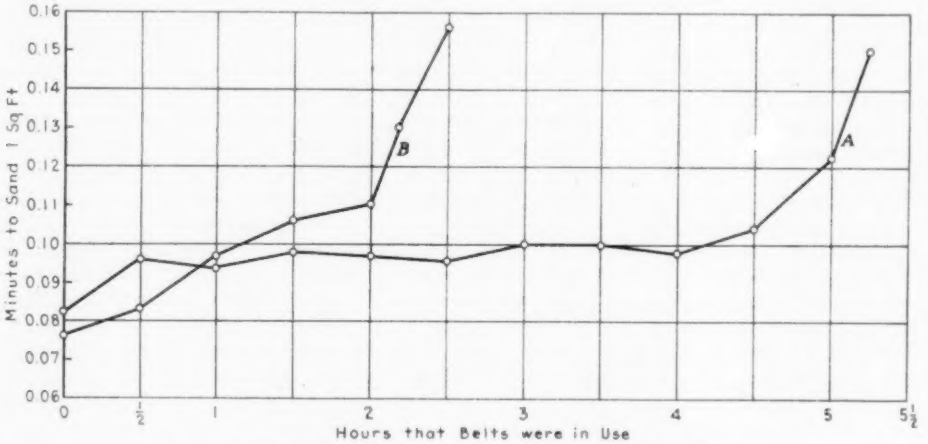

FIG. 1 TIME-STUDY TESTS OF BELTS USED IN SANDING WALNUT
(Curve *A*, grade 3/0 aluminum-oxide paper; curve *B*, grade 3/0 garnet paper.)

COMPARATIVE COSTS OF ALUMINUM OXIDE (*A*) AND GARNET (*B*), IN DOLLARS

	Cost of belt	Sq ft sanded	Time of sanding, min	Labor cost of sq ft sanded	Labor costs per min	Labor costs per sq ft	Sandpaper cost per sq ft	Added costs of belt changes	Total costs per sq ft	Total costs per M sq ft
A........	0.62	1277	130.91	1.4182	0.01833	0.001111	0.000485	....	0.001596	1.60
B........	0.49	435	51.95	0.5628	0.01833	0.001290	0.001130	0.000340	0.002760	2.76

Hardness. In the Mohs scale of hardness it has a rating of 9.5 as compared with garnet's 6.5 to 7.5. This hardness is ample to abrade any surface except that of a diamond.

Toughness. As indicated under the heading of garnet, toughness is a most important factor. It is this property, as well as hardness, that makes aluminum oxide preeminent in the field of wood abrasives, especially for machine sanding. The resistance to shear or premature rupture is the property that adds to its longevity as a wood abrasive. In addition, this toughness permits the retention of a cutting edge longer than is possible with the more friable, relatively brittle garnet.

Fracture. Initially, crystalline aluminum oxide for woodworking-abrasive purposes had a poorer fracture than garnet. Its toughness made it difficult to break the crystals to produce edges. The normal shape was the crystal just as it developed while cooling in the pig, and crushing tended to preserve this shape except where small particles were broken off, giving a faceted appearance. Considerable improvement has been made in the crushing of this type of grain, and the trend has been in the direction of producing a close approximation to the garnet shape. The grain manufacturer today classes aluminum oxide in three broad steps, starting with the cube-like structure, which is converted into the manufacture of grinding wheels, and extending to the more wedge-like crystals, which are selected for use in surface-coated abrasives. The objective is a grain having the garnet shape, combined with all the physical properties of hardness and toughness that may be found in aluminum oxide.

Comparison of these two minerals is not complete without a careful study of relative performance in coated-abrasive form under conditions as alike as it is possible to obtain in a woodworking plant. From the viewpoint of the manufacturer of coated abrasives, it was necessary to make an early determination of the practical utility of aluminum-oxide grain in coated form as a wood abrasive so as to avoid needless duplication of manufacture in a line as varied as sandpaper proves to be.

For the purpose of illustration, the Department of Commerce, cooperating with the surface-coated abrasive manufacturers, and functioning through the Bureau of Standards, to determine recommendations for simplified practice on the elimination of waste, reduced the number of these items, without reducing the number of minerals or the backings, down to a point where they now are as follows:

1 Five distinctly different types of abrasive mineral in which there are 17 different grits ranging from fine to coarse.
2 Two kinds of coating, both open and closed, with the exception of two types of mineral
3 Five different weights of paper backing
4 Three weights of cloth backing multiplied by three distinctly different colors of cloth backing
5 Two weights of combination backing
6 Twelve to 15 standard widths of roll material, with a tremendous variety of products cut to special and standard sizes and shapes.

The variety of manufacture resultant under these conditions gives a range of products totaling many thousands of individual items. Accordingly, the question of adding a new line to be manufactured as well as stocked in many parts of the country was a problem of no insignificant proportions, especially if the economic justification was not well established.

Before aluminum-oxide grain in coated form was offered to the woodworking industry it was tried in a preliminary way in the company's testing laboratories on belts, disks, and drum sanders. The immediate indication was that this new application of aluminum-oxide grain had very decided possibilities. This was followed by field work in the plants of many large users of coated abrasives, and finally the finished result was turned over to a limited number of users for actual test under average operating conditions. Plants were selected that would give a wide variety of experience from the angle of different types of wood and veneers commonly in use. The graph (shown in Fig. 1) depicts the comparative results obtained by the time-study department of one large user, and is representative of the data recorded in other plants under a wide variety of operating conditions.

The use of aluminum-oxide grain, particularly in the field of abrasive-belt sanding, opened some interesting studies as to the advantage of increasing belt speeds by reason of the greater relative toughness and hardness of aluminum-oxide grain as compared with garnet. Some plants advanced their belt speed from between 2000 and 3000 ft per min to between 4000 and 5000 ft per min. Under this advance of speed, garnet, because of its relatively more brittle nature, would shatter and lose its cutting edge in a relatively short time.

Finish was a question that was given considerable attention, with some quite controversial angles from the start. This was settled conclusively by making a careful comparison of panels surfaced, under like conditions, by both types of coated-abrasive belts, with results in favor of aluminum-oxide grain.

In addition to the utilization of higher sanding speeds, due to the greater toughness of aluminum oxide, which permitted retention of the cutting edges longer than garnet, it also developed that the inherently greater hardness of aluminum oxide enabled the use of lighter sanding contact pressures, particularly for belt sanders, than was possible with garnet except in the initial stages of its use. Cutting-efficiency curves showing the relative performances of the two types of belts generally started with a rate of cut initially higher with garnet, but when the first "bite" was diminished through early fracture of the needlelike cutting points of garnet, the relatively higher rate of cut diminished, and gradual dulling took place, requiring increased pressure to maintain an economical cutting rate.

In the field of drum and disk sanding, a similarity of problems existed. The desirable properties of aluminum oxide, in greater hardness and toughness, are called into use to equally great advantage except in the instance of excessively gummy and pitchy woods.

In conclusion it may be said that aluminum oxide has found a definite place in the woodworking industry. Garnet continues to find favor in operations where it is not possible to use it to the maximum capacity potentially available in aluminum-oxide grain.

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A.S.M.E. Record and Index, 1932

THIS section of the Transactions contains a record of the activities of The American Society of Mechanical Engineers for the year 1932, together with an index of its publications. It is issued in pamphlet form to every member of the Society who requests it and when bound with the technical papers of the Professional Divisions for the use of libraries and official depositories throughout the world, it completes the permanent record of the Society's activities.

The Record and Index Section of Transactions includes all the reference material likely to be of permanent value and of interest to future searchers. As an index it provides a means of locating the technical information in the Society's publications. It includes the separate indexes of the Transactions and *Mechanical Engineering,* together with a miscellaneous index to reports or other technical publications of the Society that have appeared during the year and a list of papers issued in mimeographed or similar form for meetings in 1932, but not published.

A considerable portion of the Record and Index is devoted to memorial notices of members who have died during the year. Special care has been given to the preparation of these brief biographies with the idea in mind that fitting attention should be given to significant events in the lives of deceased members, even though they may fall outside the immediate field of mechanical engineering.

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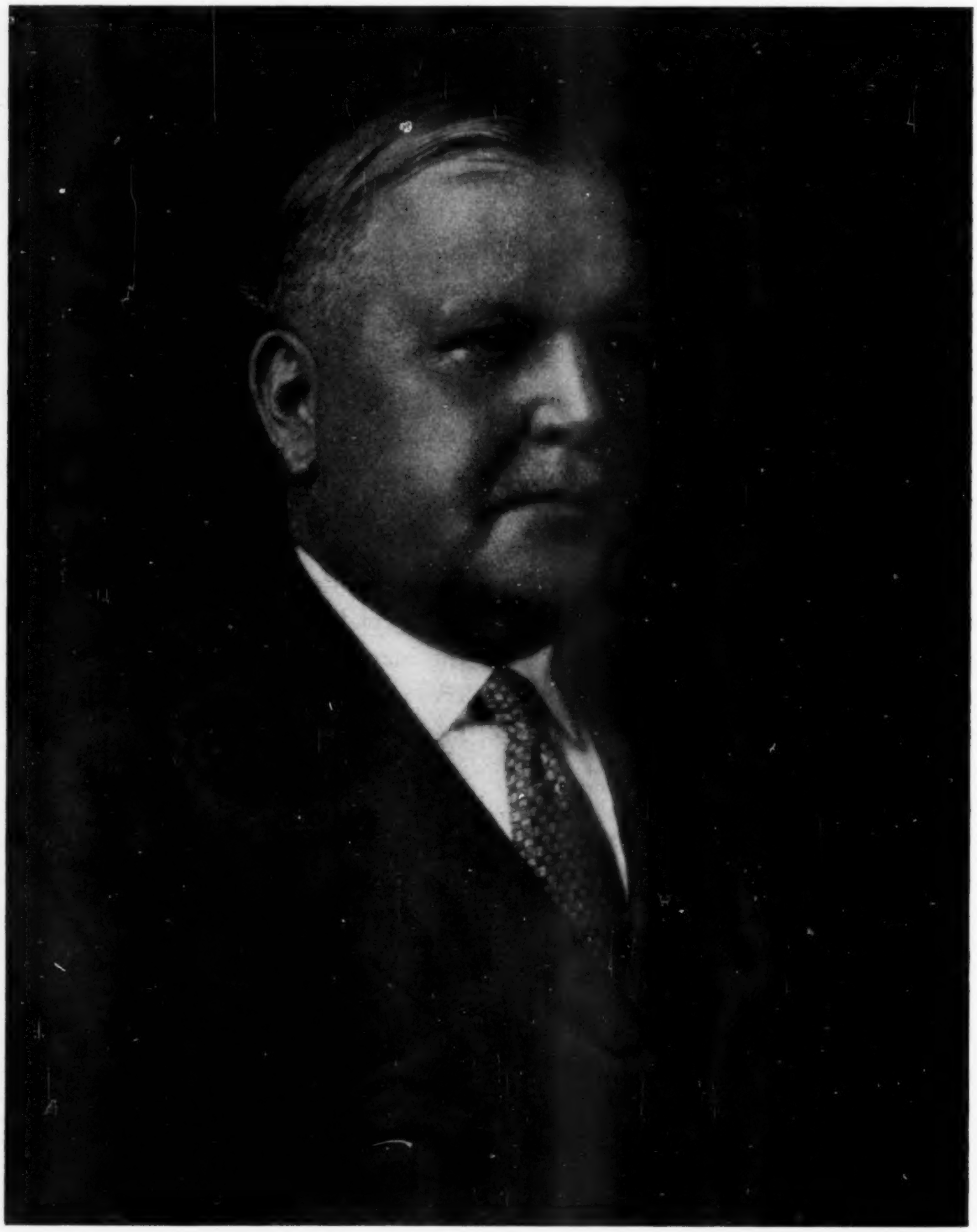

CONRAD NEWTON LAUER

President of The American Society of Mechanical Engineers
1931–1932

Conrad Newton Lauer

CONRAD NEWTON LAUER, president of The American Society of Mechanical Engineers for the term 1931–1932, was born on November 25, 1869, at Three Tuns, Montgomery County, Pennsylvania. He received his education in public and private schools in Montgomery County and through special instructors. From 1893 to 1902 Mr. Lauer was with the Link-Belt Company, serving in various capacities ranging from time clerk to acting superintendent of the plant. During this time he was sent to the Bethlehem Steel Company to secure data, and later assisted in introducing scientific management methods in the Link-Belt plant.

In 1903 Mr. Lauer became associated with Dodge & Day, engineers, and he remained with their successors, Day & Zimmermann, Inc. During the first ten years of his service he dealt with a large variety of problems incidental to the selection of industrial equipment, the layout of equipment, routing of materials, and the determination of types of building structure best suited to house such equipment and processes. His work also embraced investigations for banking interests. During the World War he was in charge of large construction projects under Government contracts. In the latter years of his connection with the Day & Zimmermann organization he was its vice-president and director. Since 1929 he has been president of the Philadelphia Gas Works.

In 1923 Mr. Lauer was the Cyrus Fogg Brackett lecturer before Princeton University. He is the author of "Engineering in American Industry," and is a contributor to the technical press. He has also been a special lecturer before the University of Pennsylvania and is a trustee of the Corporation of Worcester Polytechnic Institute. He received the honorary degree of Mechanical Engineer from Stevens Institute of Technology in 1929.

Mr. Lauer became an associate of the A.S.M.E. in 1903, and a member in 1923. He was a member of the Committee on Meetings and Program of the Society from 1922 to 1926, serving as its chairman in 1926. He was a manager during the next three years and vice-president from 1929 until he became president of the Society. In 1919 he was chairman of the Philadelphia Local Section. He is also a member of the Committee on Revenues of the Society appointed December 5, 1927, and of the Committee on Economic Status of the Engineer on which he served as chairman from May 14, 1928, to January, 1932.

In 1929, Mr. Lauer, desiring to commemorate the civic and humanitarian achievements of engineers, established a trust fund of $15,000 for the institution of a gold medal, named, with Mr. Hoover's consent, the Hoover Gold Medal. The first award was made to Herbert Hoover in April, 1930, on the occasion of the Fiftieth Anniversary of the A.S.M.E., and other awards will be made from time to time for distinguished public service. The trust fund is held by the A.S.M.E. and administered by a Board of Award consisting of representatives of the four national engineering societies.

Mr. Lauer is active in numerous engineering and civic undertakings and a member and director of many social and professional clubs. He is vice-president and a member of the executive committee of both the Philadelphia Chamber of Commerce and the Regional Planning Federation of the Philadelphia Tri-State District. He is an honorary member of the Princeton Engineering Association of New York, past-president of the Philadelphia Engineers' Club, and a member of the New York Engineers' Club, the Taylor Society, the Newcomen Society of London, The Franklin Institute, the Executive Board of the American Gas Association, and the Pennsylvania Gas Association.

Unifying the Engineering Profession

By CONRAD N. LAUER

MY ILLUSTRIOUS predecessor, Robert Henry Thurston, first President of The American Society of Mechanical Engineers, delivered an address before this Society in November, 1880, in which he set up for all who were to follow him in this office an obligation to deliver a presidential address and a standard of excellence for that address that it is difficult to attain. Those who, in recent years, have read Dr. Thurston's words express amazement at his wide knowledge of contemporary engineering, his prophetic vision of future developments, and his clear understanding of the relations of engineers to their profession and of their profession to questions of public interest.

The concluding section of Dr. Thurston's address is entitled "The Ethics of Cooperation." He had affirmed that in the work of the engineering society there was a need for men of the world and men of science, "observers, discoverers, and inventors, explorers in all fields of research, students of nature's facts, and codifiers of nature's laws and teachers of all applied sciences, as well as workers in field, in workshop, and in office." He added, "We shall find that the promotion of mutual welfare is consistent with the most perfect system of mutual cooperation and mutual aid in all the truest and highest aims of life." Tonight I wish to direct your attention to some cooperative efforts that affect not only our professional welfare but the public interest, and to indicate the desirability of finding a simple and effective way of coordinating them.

Obviously an effective profession must be a united one, and for this reason this Society may well concern itself with the organization of the engineering profession as a whole. To that end let me review briefly the steps that have been taken over a period of years by which united action by the profession has been secured, and let me offer some suggestions as to future steps for the unification of society organization.

These projects illustrate convincingly the benefits of cooperation in accordance with the unwritten principles which have always actuated this Society in such relationships: First, unselfish cooperation with other related societies; second, planning actively for the future so that development may not be haphazard; third, giving the most effective service to members of the engineering profession; and fourth, inspiring the engineering profession to render the most valuable service that it can to the public.

The Development of the Spirit of Cooperation in Engineering Societies

The earliest engineering societies were formed, as was our own, by groups of individuals as agencies for the self-improvement of members through the presentation and publication of papers, the development of the professional spirit, and the pooling of individual library resources. In addition these societies provided certain standards of attainment in engineering as expressed in the qualifications set up for various grades of membership. These were worthy objectives in days when the engineering profession was young and its members were poorly organized. They will always remain the essential objectives of such organizations. They satisfied our needs as specialists in engineering and absorbed practically all of the activities of engineering societies until the turn of the century, when the impact of engineering was making itself felt in the social and economic changes in civilization. At that time engineers were becoming conscious of their growing importance in the changing life of the world, and sought by working together with other men practising in the same fields to give direction and effectiveness to their professional work. Engineering had emerged as an important cultural factor that transcends the narrow limits within which its numerous branches are traditionally confined. An insistent urge for a united profession as a means of increasing the effectiveness and prestige of the individual engineer in private practice and of professional bodies in the public service, with the same recognition of the profession by the public generally that is accorded to older professions, had made itself known in various movements looking toward greater unity of objectives and actions. This seemed in many ways to be counteracting the centrifugal forces that had been at work for some time and had resulted in a galaxy of individual societies of specialists, and to lead logically to the coalescence of these numerous groups into a single national engineering society. Before discussing the desirability of such a coalescence, let me review briefly some of the major activities that have been the result of cooperation for the common good.

Some Examples of Engineering-Society Cooperation

An early evidence of this desire for team work was the foundation, in 1902, of the John Fritz Medal for the recognition of leaders in engineering. Later, Andrew Carnegie, a man of rare and far-seeing vision, provided an exceptional opportunity for cooperation by his gift of the Engineering Societies Building in New York, the present home of many of the major professional engineering organizations, where the library facilities of the four Founder Societies are consolidated in one of the best and most widely used engineering libraries in the entire world. This building, dedicated in 1907, brought the engineering profession together on an intimate working basis, and for that reason has been of incalculable value in advancing the common objectives of all branches of the profession. Its affairs and those of the library are administered by the United Engineering Trustees, Inc., and the Engineering Societies Library Board, respectively.

In 1914, actuated by a desire to promote engineering knowledge, our own Past-President, Ambrose Swasey, established The Engineering Foundation as an instrument of several engineering societies to aid in the research activities of these bodies. His proposal was accompanied by substantial gifts that have reached a total of $750,000. With characteristic modesty and vision Mr. Swasey declined to have his name attached to the Foundation. In all his statements in connection with the Foundation he has expressed the hope that others will assist the progress of research by adding to the funds he has provided. I am happy to state that Mr. Henry R. Towne, another past-president, and Mr. Edward D. Adams have been inspired by Mr. Swasey's example to make substantial additions to the Foundation's endowment. The Engineering Foundation is now carrying on its work of aiding research in science and engineering, and of advancing in many other ways the profession of engineering and the welfare of mankind.

Cooperation for the Public Welfare

Turning now to the relations of engineering societies with the public, it will be recalled that the World War provided a multitude of opportunities for engineers to serve the Government and

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Presidential Address delivered at the Annual Meeting, New York, N.Y., December 5 to 9, 1932, of The American Society of Mechanical Engineers.

the public without regard to their professional society affiliations on questions which required the competent advice of technically trained men. In order to make the services of individual experts of the engineering profession available and to provide an organization of authority and professional standing, the four Founder Societies formed the Engineers' Council in 1917. The services rendered by this Council proved to be valuable and effective, and demonstrated that in peacetime, as well as in war, there is a need by government bureaus, legislative and administrative departments, and other national bodies for unbiased and informed opinion on the engineering factors involved in questions of public interest. In 1920, therefore, the Council was reorganized as the Federated American Engineering Societies, subsequently renamed the American Engineering Council.

The formation of this body represented a significant change in the attitude of the profession toward the public. It calls together periodically in Washington representatives of many national and local engineering societies for the purpose of focusing the thinking of engineers for group service on national problems in which engineering principles and interests are involved. The American Engineering Council has been a powerful factor in developing a spirit of cooperation in the engineering profession. Its committees and representatives are highly respected by legislative and administrative agencies in Washington because they provide competent and disinterested advice on matters of public interest. It has sponsored a series of important studies on the relationship of engineering to industrial operations, such as those on waste in industry, the relation of safety to production, and the twelve-hour shift in industry. A committee of the Council is now preparing the groundwork for a study of the balancing of the forces of production, consumption, and distribution, a significant contribution to economic and sociological thinking.

State councils to perform locally services similar to those that the American Engineering Council renders to the nation at large have been formed in a few states, such as Minnesota, Colorado, and Utah, and offer further examples of the effectiveness of cooperation by engineering organizations.

Cooperation for Industry and the Engineering Profession

The effectiveness of the cooperation of engineering societies with industry is demonstrated by developments in industrial and engineering standardization. Up to 1918 many bodies were engaged in the formulation of such standards. In 1918 the American Engineering Standards Committee, now known as the American Standards Association, was formed through the joint efforts of five engineering societies, and ever since has grown in membership and in influence. Forty-five national organizations are now affiliated as member-bodies, and the Association is the official channel for international cooperation with similar national standardizing bodies in each of twenty foreign countries; it acts also as a clearing house for the distribution of information about standardization in the United States and elsewhere.

The registration and licensing of engineers, touching as they do the legality of the engineer's right to practice, are also questions of common interest. While one state, California, had enacted a law in 1891 requiring registration and while there were altogether eleven states with licensing laws on their books in 1920, most of the discussion and interest in these questions on the part of engineers has developed since 1920. Fifteen states have put on their statute books some form of requirement for legal recognition of the engineering profession. Now, under the leadership of the American Society of Civil Engineers, a model law is being developed. There are at present twenty-six states with licensing and registration laws, and a body, known as the National Council of State Boards of Engineering Examiners, has been organized to provide a working arrangement for the coordination of procedure in the administration of these laws in the various states. This is an activity of increasing importance and interest.

The study of engineering education, undertaken in 1923 by the Society for the Promotion of Engineering Education, brought about the cooperation of engineering societies, industry, governmental agencies, and individuals throughout the entire range of the field of study. The elaborate report issued by that organization in 1930 gave a clear picture of the conditions in the schools and of the relationship between the education and training of engineers and the requirements of professional, industrial, and economic phases of modern life. It has provided the basis for sound advances in engineering education and in the profession itself, and demonstrated once again the value of cooperation.

In addition to the John Fritz Medal, already mentioned, other awards are made by several societies acting jointly so as to represent more completely the engineering profession of the United States in doing honor to men of attainment. The most recently established of these is the Herbert Hoover Medal, founded, in 1930, to signalize the public service of men who have gone outside of their strictly professional work to interest themselves in civic and human affairs. The medal is awarded by a joint board of the Founder Societies, and was first bestowed, in 1930, on the engineer-statesman whose name it bears.

A Recent Outstanding Example of Cooperation

During the past two months seven engineering bodies, acting jointly, have formed the Engineers' Council for Professional Development. The purposes of the Council are to coordinate and promote efforts and aspirations directed toward higher professional standards in engineering education and practice, to enhance the prestige and solidarity of the engineering profession, and to secure greater effectiveness in dealing with technical, social, and economic problems. It will immediately concern itself with the development of a scheme whereby the progress of the young engineer toward professional standing can be recognized by the public, by the profession, and by the individual himself through the development of technical and other qualifications which will enable him to meet high professional standards. This is a conspicuous example of what extensive and comprehensive objectives may be set up by engineering societies working together in harmony and with intelligence.

The American Society of Mechanical Engineers is proud of the fact that the formation of this new body resulted from the work of its Committee on the Economic Status of the Engineer. But without the enthusiastic cooperation of the American Society of Civil Engineers, the American Institute of Mining and Metallurgical Engineers, the American Institute of Electrical Engineers, the American Institute of Chemical Engineers, the Society for the Promotion of Engineering Education, and the National Council of State Boards of Engineering Examiners, who joined with the A.S.M.E. in forming the Council, it could never have been hoped that this would become the truly representative and authoritative body that it now is.

The Problem of Unification

These examples of engineers working together toward common objectives are concrete expressions of some of the ideals that are held for the advancement of the engineering profession. The objectives are worthy of the efforts expended for their attainment, and these latter have been organized with the idea of bringing about certain results useful either to the profession itself, to industry, which it serves, or to the public. However, it has been noted that many of these cooperative enterprises have necessitated the formation of organizations to carry out their projects. These organizations have grown both in number and in size. They have set up for themselves, or have been asked to set up.

additional objectives. Their functions are not always clearly understood, with the result that there is danger of overlapping and a lack of clearly defined relationships among themselves and the engineering societies that have created them. The conclusion seems inescapable that the engineering profession is tending toward overorganization. It would seem that one of two alternatives must be chosen as a way out of our perplexing position. If we are to assume that these cooperative activities are more important than the numerous specific objectives for which individual engineering societies were formed, then the profession and the public would be well served by the consolidation of all engineering societies and related organizations into a single body that could act and speak for the engineering profession. If these joint activities are merely those in which all engineering bodies can work most effectively through cooperation, then, by proper organization their usefulness can be assured and their procedure and relationship clarified and simplified without losing the undoubted benefits of the existence of long-established and vigorous societies, held together by common professional interests, whose very strength and progressiveness have given rise to these related bodies engaged in tasks of mutual concern and profit.

It is my opinion that the time is not ripe, and indeed it may never be ripe, to advocate the first of these alternatives, that is, the formation of a single engineering society to serve a profession which, in spite of its many common objectives, has so many specialized interests. The effectiveness of the existing major national engineering societies in dealing with the problems, practices, and procedures in the fields they represent is unquestioned, and the vigorous individual development of these societies must continue. Their joint activities must also continue and expand, as the individual societies collectively represent the profession and because common interests and objectives do exist. In general, the common objectives are:

1 The development of the individual engineer by selection, education, guidance, and recognition of his attainments
2 The focusing of the intelligence and knowledge that engineers possess on problems of public interest
3 The aiding of research and standardization in engineering and industry.

Joint agencies, therefore, may be in need of coordination and simplification, but not of consolidation. A committee representing the four Founder Societies is engaged in a study of this question. I believe that if the members of the engineering profession will support this committee, its findings may lead to such coordination and simplification. I do not believe that there exists a sentiment in favor of a single engineering society replacing not only the numerous organizations created by the present engineering societies, but those societies themselves as well. Rather I believe that sentiment is in favor of some form of my second alternative.

CONCLUSION

The numerous examples of successful cooperative endeavor which I have quoted demonstrate the practicability of achieving noteworthy results without destroying the autonomy of existing societies or diverting them from the purposes for which they were formed. It is an impressive indication of the vigor and progressiveness of these bodies that in a period of economic distress they have been able to bring about the formation of such a potentially useful and comprehensive organization as the Engineers' Council for Professional Development. This great accomplishment, which marks the beginning of an important epoch in the history of engineering, was made possible by the desire and ability of organized groups representing professional, educational, and governmental points of view to work together constructively and harmoniously in matters of common concern. No single engineering society embracing the entire profession could do more, and might, because of the inertia of its ponderous machinery, do even less.

Today engineers may look with renewed hope and enthusiasm to the unfolding of this new epoch. Every member of the profession, through the society to which he belongs because of his personal interests, has an active concern in a program of professional development that starts with the secondary school and carries on through the engineering college and apprenticeship in the profession to a final recognition of competency and attainment. Never has so comprehensive a program been laid before engineers with the machinery for its accomplishment ready for operation and the combined energies of long-established professional societies guaranteeing effective action. In attaining the objectives of this program, each engineering society will develop greater strength and prestige, and realize the fruits of former cooperative endeavors. All will find, I feel sure, as Dr. Thurston affirmed more than fifty years ago, that "the promotion of mutual welfare is consistent with the most perfect system of mutual cooperation and mutual aid in all the truest and highest aims of life."

The American Society of Mechanical Engineers

THE personnel records and other information presented in this first section of the Record and Index refer, except where otherwise noted, as in the case of Local Section and Student Branch chairmen, to the official year beginning in December, 1931. Such information is purely of historical nature and is included here as a permanent record. Further information may be secured from the Headquarters of the Society.

Officers and Council[1]

Dates in parentheses denote expiration of terms

PRESIDENT

Conrad N. Lauer

PAST-PRESIDENTS

Terms expire in December

William L. Abbott (1932)
Alex Dow (1934)
Charles M. Schwab (1933)
Charles Piez (1935)
Roy V. Wright (1936)

VICE-PRESIDENTS

Terms expire December, 1932

Harvey N. Davis
William A. Hanley
Thomas R. Weymouth

Terms expire December, 1933

Robert M. Gates
William B. Gregory
Frederick H. Dorner
Charles M. Allen

MANAGERS

Terms expire December, 1932

Harold V. Coes
James D. Cunningham
C. F. Hirshfeld

Terms expire December, 1933

William L. Batt
Harold L. Doolittle
Herbert L. Whittemore

Terms expire December, 1934

Eugene W. O'Brien
Harry R. Westcott
Alexander J. Dickie

TREASURER

Erik Oberg

SECRETARY

Calvin W. Rice

EXECUTIVE COMMITTEE OF COUNCIL

Conrad N. Lauer, *Chairman*
Harvey N. Davis, *Vice-Chairman*
Calvin W. Rice, *Secretary*
Robert M. Gates
Harold V. Coes
Harry R. Westcott

Advisory Members: Chairmen of the Finance Committee, the Local Sections Committee, and the Professional Divisions Committee.

CHAIRMEN OF STANDING COMMITTEES

Representatives on Council but without vote

Finance, David Lofts, *Chairman*
K. L. Martin, *Vice-Chairman*
Meetings and Program, F. M. Feiker
Publications, W. H. Winterrowd
Membership, H. W. Butler
Professional Divisions, W. F. Dixon
Local Sections, J. M. Todd
Constitution and By-Laws, T. C. McBride
Awards, A. M. Greene, Jr.
Relations With Colleges, M. C. Maxwell
Education and Training for Industries, R. L. Sackett
Library, A. S. Miller
Standardization, L. K. Sillcox
Research, W. H. Fulweiler
Power Test Codes, F. R. Low
Safety, H. W. Mowery
Professional Conduct, A. G. Christie

EXECUTIVE SECRETARY

C. E. Davies

EDITOR

G. A. Stetson

ASSISTANT SECRETARIES

Ernest Hartford
C. B. LePage

Standing Committees

Dates in parentheses denote expiration of terms

FINANCE

David Lofts, *Chairman and Representative on Council* (1932)
K. L. Martin, *Vice-Chairman* (1933)
E. A. Muller (1935)
J. H. Lawrence (1934)
Walter Rautenstrauch (1936)

Council Representatives:

R. V. Wright
W. L. Batt

MEETINGS AND PROGRAM

F. M. Feiker, *Chairman and Representative on Council* (1932)
J. W. Parker, *Vice-Chairman* (1933)
R. I. Rees (1935)
C. P. Bliss (1934)
E. C. Hutchinson (1936)

PUBLICATIONS

W. H. Winterrowd, *Chairman and Representative on Council* (1932)
L. C. Morrow, *Vice-Chairman* (1933)
S. W. Dudley (1935)
S. F. Voorhees (1934)
W. F. Ryan (1936)

(Personnel of Special Committee, p. 11)

MEMBERSHIP

H. W. Butler, *Chairman and Representative on Council* (1932)
Hosea Webster (1933)
H. A. Lardner (1935)
O. E. Goldschmidt (1934)
R. H. McLain (1936)

PROFESSIONAL DIVISIONS

W. F. Dixon, *Chairman and Representative on Council* (1932)
P. T. Sowden (1933)
W. A. Shoudy (1935)
C. B. Peck (1934)
Walter Samans (1936)

(Chairmen of Professional Divisions' Executive Committees and Personnel of Special Committees, p. 11)

LOCAL SECTIONS

J. M. Todd, *Chairman and Representative on Council* (1933)
H. R. Westcott (1932)
W. L. Dudley (1935)
J. W. Haney (1934)
W. W. Macon (1936)

(Chairmen of Local Sections' Executive Committees, p. 11)

CONSTITUTION AND BY-LAWS

T. C. McBride, *Chairman and Representative on Council* (1932)
R. S. Neal (1933)
P. R. Faymonville (1935)
H. H. Snelling (1934)
J. A. Hall (1936)

AWARDS

A. M. Greene, Jr., *Chairman and Representative on Council* (1932)
K. H. Condit (1933)
W. L. Batt (1935)
F. L. Eidmann (1934)
D. B. Rushmore (1936)

RELATIONS WITH COLLEGES

M. C. Maxwell, *Chairman and Representative on Council* (1932)
D. B. Prentice (1933)
W. L. Abbott (1935)
E. F. Church, Jr. (1934)
L. F. Gordon (1936)

(Student Branches and Honorary Chairmen, p. 12)

EDUCATION AND TRAINING FOR THE INDUSTRIES

R. L. Sackett, *Chairman and Representative on Council* (1932)
J. T. Faig (1933)
C. F. Bailey (1935)
H. S. Falk (1934)
S. S. Edmands (1936)

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[1] Special Committees of Council, page 10.

LIBRARY

A. S. Miller, *Chairman and Representative on Council* (1932)

G. F. Bateman (1933)
W. M. Keenan (1934)
E. P. Worden (1935)
The Secretary, Calvin W. Rice, *Ex-Officio*

RESEARCH

W. H. Fulweiler, *Chairman and Representative on Council* (1932)

A. D. Bailey (1933)
A. A. Potter (1934)
G. M. Eaton (1935)
D. B. Bullard (1936)

STANDARDIZATION

L. K. Sillcox, *Chairman and Representative on Council* (1932)

W. S. Monroe (1933)
Earle Buckingham (1934)
C. W. Spicer (1935)
Alfred Iddles (1936)

POWER TEST CODES

F. R. Low, *Chairman and Representative on Council* (1935)

Term expires 1932

Harte Cooke
E. R. Fish
O. P. Hood
H. B. Oatley
W. J. Wohlenberg

Term expires 1933

E. H. Brown
I. E. Moultrop
G. A. Horne
L. S. Marks
E. N. Trump

Term expires 1934

C. Harold Berry
Francis Hodgkinson
D. S. Jacobus
L. F. Moody
E. B. Ricketts

Term expires 1935

F. R. Low
A. T. Brown
R. H. Fernald
C. F. Hirshfeld
R. J. S. Pigott

Term expires 1936

A. G. Christie
E. C. Hutchinson
Paul Diserens
G. A. Orrok
Wm. Monroe White

SAFETY

H. W. Mowery, *Chairman and Representative on Council* (1933)

Lewis Price (1932)
T. A. Walsh, Jr. (1934)
M. H. Christopherson (1935)
W. M. Graff (1936)

PROFESSIONAL CONDUCT

A. G. Christie, *Chairman and Representative on Council* (1933)

J. H. Herron (1932)
H. S. Philbrick (1934)
D. Robert Yarnall (1935)
E. R. Fish (1936)

Special Technical Committee

BOILER CODE

F. R. Low, *Chairman*
D. S. Jacobus, *Vice-Chairman*
C. W. Obert, *Honorary Secretary*
M. Jurist, *Acting Secretary*
H. E. Aldrich
W. H. Boehm
R. E. Cecil
F. S. Clark
F. W. Dean
W. F. Durand
E. R. Fish
V. M. Frost
C. E. Gorton
A. M. Greene, Jr.
F. B. Howell
J. O. Leech
E. F. Miller
M. F. Moore
I. E. Moultrop
C. O. Myers
H. B. Oatley
James Partington
C. L. Warwick
A. C. Weigel
H. LeRoy Whitney

Honorary Members

T. E. Durban
C. L. Huston
W. F. Kiesel, Jr.
H. H. Vaughan

Special Administrative Committees

TELLERS OF ELECTION FOR OFFICERS

J. C. Hardigg, *Chairman*

C. J. Beckwith
G. B. McGovern, Jr.

REGULAR NOMINATING COMMITTEE

GROUP	REPRESENTATIVE	ALTERNATE
I	R. E. Newcomb	P. N. Kistler
II	W. A. Shoudy, *Chairman*	A. A. Adler
III	Harte Cooke, *Secretary*	F. A. Allner V. M. Palmer
IV	D. E. Kehoe	C. S. Hammatt J. M. Foster
V	W. M. Taylor	E. G. DeCoriolis James Burke
VI	Max Toltz[2]	G. C. Shaad
VII	U. B. Hough	C. I. Carpenter

LOCAL SECTIONS IN NOMINATING COMMITTEE GROUPS

GROUP I

Boston
Bridgeport
Green Mountain
Hartford
Meriden
New Britain
New Haven
Norwich
Providence
Waterbury
Western Massachusetts
Worcester

GROUP II

Metropolitan (N. Y.) and Foreign Members

GROUP III

Anthracite-Lehigh Valley
Baltimore
Central Pennsylvania
Ontario
Philadelphia
Plainfield
Rochester
Schenectady
Susquehanna
Syracuse
Utica
Washington, D. C.

GROUP IV

Atlanta
Birmingham
Charlotte
Chattanooga
Florida
Greenville
Houston
Knoxville
Memphis
New Orleans
North Texas
Raleigh
Savannah
Virginia

GROUP V

Akron-Canton
Buffalo
Cincinnati
Cleveland
Columbus
Dayton
Detroit
Erie
Indianapolis
Louisville
Peninsula
Pittsburgh
Toledo
West Virginia
Youngstown

GROUP VI

Chicago
Kansas City
Mid-Continent
Milwaukee
Minneapolis
Nebraska
Rock River Valley
St. Joseph Valley
St. Louis
St. Paul
Tri-Cities

GROUP VII

Colorado
Inland Empire
Los Angeles
Oregon
San Francisco
Utah
Western Washington

Special Council Committees

BUDGETING POLICY

R. E. Flanders, *Chairman*
J. D. Cunningham, *Vice-Chairman*
W. L. Abbott
H. V. Coes
C. F. Hirshfeld
Leo Loeb
H. R. Westcott

Erik Oberg, *Treasurer*
F. A. Schaff, *Chairman, Finance Committee, 1931*
W. L. Batt, *Chairman, Revenues Committee*
} *Ex-Officio*

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[2] Deceased January 11, 1932.

ECONOMIC STATUS OF THE ENGINEER

C. F. Hirshfeld, *Chairman*
D. S. Kimball
C. N. Lauer
H. B. Oatley
W. A. Starrett[3]
H. L. Whittemore
W. E. Wickenden

J. M. Todd, *Chairman, Committee on Local Sections*
M. C. Maxwell, *Chairman, Committee on Relations With Colleges*
} *Ex-Officio*

MEMBERSHIP CERTIFICATES

L. K. Comstock, *Chairman*
P. R. Faymonville
J. W. Roe

REGISTRATION OF ENGINEERS

J. H. Lawrence, *Chairman*
D. S. Kimball
Paul Doty
C. F. Hirshfeld
V. M. Palmer
J. M. Todd

REVENUES

W. L. Batt, *Chairman*
J. D. Cunningham
D. S. Kimball
R. E. Flanders
C. N. Lauer
Erik Oberg
J. L. Walsh

Special Committees of Standing Committees

Publications Committee

BIOGRAPHY ADVISORY COMMITTEE

F. R. Low, *Chairman*
R. V. Wright, *Vice-Chairman*
G. A. Orrok
J. R. Freeman[4]
R. E. Flanders

Professional Divisions Committees

PROCESS INDUSTRIES COMMITTEE

C. E. Harrington, *Chairman*
Victor Wichum, *Secretary*
Crosby Field
H. D. Munson
C. P. Tolman

PURE AIR COMMITTEE

(Dates in parentheses denote expiration of terms)

E. C. Hutchinson, *Chairman* (1936)
E. H. Whitlock, *Secretary* (1934)
G. W. Bach (1932)
J. W. Armour (1933)
O. P. Hood (1935)

Professional Divisions

Division	Executive Committee Chairmen
Aeronautic	E. A. Sperry, Jr.
Applied Mechanics	G. B. Pegram
Fuels	R. A. Sherman
Hydraulic	B. R. Van Leer
Iron and Steel	A. G. McKee
Machine-Shop Practice	F. C. Spencer
Management	F. E. Raymond
Materials Handling	J. B. Webb
National Defense	Calvin W. Rice (*Secretary*)
Oil and Gas Power	L. R. Ford
Petroleum	W. G. Heltzel
Power	Alfred Iddles
Printing Industries	G. C. Van Vechten
Railroad	T. C. McBride
Textile	H. M. Burke
Wood Industries	C. L. Babcock

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[3] Deceased March 25, 1932.
[4] Deceased October 6, 1932.

Local Sections

Section	Executive Committee Chairmen[5]
Akron-Canton	J. T. Johnson
Anthracite-Lehigh Valley	W. H. Lesser
Atlanta	W. V. Dunkin
Baltimore	F. T. Leilich
Birmingham	J. E. Fries[6]
Boston	C. Harold Berry
Bridgeport	D. B. Bullard
Buffalo	H. L. Smith
Central Pennsylvania	C. L. Allen
Charlotte	Earle Mauldin
Chattanooga	Newell Sanders
Chicago	D. H. Skeen
Cincinnati	R. E. W. Harrison
Cleveland	H. K. Kugel
Colorado	Arthur Halliwell
Columbus	H. M. Bush
Dayton	H. B. Canby
Detroit	W. P. Thomas
Erie	H. E. Goetz
Florida	Ernest Kreher
Green Mountain	J. B. Johnson
Greenville	B. E. Fernow
Hartford	L. W. Stevens
Houston	J. H. Pound
Indianapolis	Homer Rupard
Inland Empire	U. B. Hough
Kansas City	J. A. Keeth
Knoxville	T. I. Stephenson
Los Angeles	R. R. Robertson
Louisville	B. M. Brigman
Memphis	W. D. Edwards
Meriden	J. D. Tierney
Metropolitan	W. M. Keenan
Mid-Continent	A. N. Horne
Milwaukee	W. D. Bliss
Minneapolis	D. M. Forfar
Nebraska	C. F. Moulton
New Britain	R. H. Barlow
New Haven	R. L. Parsell
New Orleans	G. R. Hammett
North Texas	M. M. Stetler
Norwich	Clarence Barber
Ontario	S. L. Fear
Oregon	L. E. Knerr
Peninsula	G. A. Whinery
Philadelphia	J. M. Barnes
Pittsburgh	J. A. Dent
Plainfield	G. W. Rink
Providence	W. S. Brown
Raleigh	L. L. Vaughan
Rochester	A. E. Schell
Rock River Valley	H. T. Hanitz
St. Joseph Valley	C. C. Wilcox
St. Louis	L. C. Farquhar
St. Paul	E. B. Curry
San Francisco	R. S. Danforth
Savannah	F. M. Exley
Schenectady	P. H. Knowlton, Jr.
Susquehanna	C. L. Berger
Syracuse	E. G. White
Toledo	E. G. DeCoriolis
Tri-Cities	W. P. Hunt
Utah	P. W. Belcher
Utica	Walter Clement
Virginia	A. F. Macconochie
Washington, D. C.	Harvey Boltwood
Waterbury	F. G. Purinton
Western Massachusetts	C. F. Dupee
West Virginia	C. E. Bliven
Western Washington	R. H. G. E[illegible]onds
Worcester	W. C. Seal[illegible]e
Youngstown	M. H. Mawhinney

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[5] Term of office, July, 1931–July, 1932.
[6] Deceased January 23, 1932. S. D. Moxley appointed to complete term.

Student Branches

Name and Location	Honorary Chairmen[7]
Akron, Univ. of, Akron, Ohio	F. S. Griffin
Alabama Polytechnic Inst., Auburn, Ala.	Solon Dixon
Alabama, Univ. of, University, Ala.	J. M. Gallalee
Arkansas, Univ. of, Fayetteville, Ark.	
Armour Inst. of Technology, Chicago, Ill.	J. C. Peebles
Brooklyn, Polytechnic Inst. of, Brooklyn	Gerardo Immediato
Brown Univ., Providence, R. I.	J. A. Hall
Bucknell Univ., Lewisburg, Pa.	F. E. Burpee
California Inst. of Technology, Pasadena	R. L. Daugherty
California, Univ. of, Berkeley, Calif.	H. B. Langille
Carnegie Inst. of Tech., Pittsburgh, Pa.	S. B. Ely
Case School of Applied Science, Cleveland	C. L. Tuve
Catholic Univ. of America, Washington	G. H. Heine
Cincinnati, Univ. of, Cincinnati, Ohio	J. W. Bunting
Clarkson College of Tech., Potsdam, N. Y.	J. H. Davis
Clemson College, Clemson College, S. C.	C. P. Philpot
Colorado Agricultural College, Fort Collins	L. D. Crain
Colorado, Univ. of, Boulder, Colo.	W. F. Mallory
Columbia Univ., New York, N. Y.	F. L. Eidmann
Cooper Union, New York, N. Y.	G. F. Bateman
Cornell Univ., Ithaca, N. Y.	K. D. Wood
Delaware, Univ. of, Newark, Del.	J. L. Coyle
Detroit, Univ. of, Detroit, Mich.	F. J. Linsenmeyer
Drexel Inst., Philadelphia, Pa.	C. V. Hahn
Florida, Univ. of, Gainesville, Fla.	C. H. Janes
George Washington Univ., Washington, D. C.	M. A. Lett
Georgia School of Technology, Atlanta, Ga.	R. M. Matson
Harvard Univ., Cambridge, Mass.	L. S. Marks
Idaho, Univ. of, Moscow, Idaho	H. F. Gauss
Illinois, Univ. of, Urbana, Ill.	C. W. Ham
Iowa State College, Ames, Ia.	R. A. Norman
Iowa, State Univ. of, Iowa City, Ia.	R. M. Barnes
Johns Hopkins Univ., Baltimore, Md.	A. G. Christie
Kan. State Agricultural College, Manhattan	J. P. Calderwood
Kansas, Univ. of, Lawrence, Kan.	J. A. King
Kentucky, Univ. of, Lexington, Ky.	C. C. Jett
Lafayette College, Easton, Pa.	P. B. Eaton
Lehigh Univ., Bethlehem, Pa.	F. V. Larkin
Louisiana State Univ., Baton Rouge, La.	William Whipple
Louisville, Univ. of, Louisville, Ky.	H. H. Fenwick
Lowell Textile Inst., Lowell, Mass.	H. J. Ball
Maine, Univ. of, Orono, Me.	Harry Watson
Marquette Univ., Milwaukee, Wis.	J. E. Schoen
Mass. Inst. of Technology, Cambridge	C. E. Fuller
Michigan College of Mining and Technology, Houghton, Mich.	H. W. Risteen
Michigan State College, East Lansing, Mich	L. N. Field
Michigan, Univ. of, Ann Arbor, Mich.	R. S. Hawley
Minnesota, Univ. of, Minneapolis, Minn.	B. J. Robertson
Mississippi A. & M. College, A. & M. College	O. D. M. Varnado
Missouri School of Mines and Metallurgy, Rolla, Mo.	R. O. Jackson
Missouri, Univ. of, Columbia, Mo.	J. R. Wharton
Montana State College, Bozeman, Mont.	R. T. Challender
Nebraska, Univ. of, Lincoln, Neb.	C. A. Sjogren
Nevada, Univ. of, Reno, Nev.	F. H. Sibley
Newark College of Engineering, Newark, N. J.	J. Ansel Brooks
New Hampshire, Univ. of, Durham, N. H.	T. J. Laton
New York, College of City of, New York	G. J. Bischof
New York Univ., New York, N. Y.	J. W. Roe
North Carolina State College, Raleigh, N. C.	F. B. Turner
North Carolina, Univ. of, Chapel Hill, N. C.	N. P. Bailey
North Dakota Agricultural College, Fargo, N. D.	J. R. Van Dyke
North Dakota, Univ. of, Grand Forks, N. D.	N. T. Bourke
Northeastern Univ., Boston, Mass.	J. W. Zeller
Notre Dame, Univ. of, Notre Dame, Ind.	C. C. Wilcox
Ohio Northern Univ., Ada, Ohio	J. A. Needy
Ohio State Univ., Columbus, Ohio	W. T. Magruder
Oklahoma A. & M. College, Stillwater, Okla.	V. W. Young
Oklahoma, Univ. of, Norman, Okla.	Joseph Liston
Oregon State Agricultural College, Corvallis.	R. E. Summers
Pennsylvania State College, State College	C. L. Allen
Pennsylvania, Univ. of, Philadelphia, Pa.	W. A. Sloan
Pittsburgh, Univ. of, Pittsburgh, Pa.	H. C. Pavian
Porto Rico, Univ. of, Mayaguez, P. R.	Luis Stefani
Pratt Inst., Brooklyn, N. Y.	R. B. Dale
Princeton Univ., Princeton, N. J.	A. E. Sorenson
Purdue Univ., W. Lafayette, Ind.	H. L. Solberg
Rensselaer Polytechnic Inst., Troy, N. Y.	T. F. FitzGerald
Rhode Island State College, Kingston, R. I.	R. L. Wales
Rice Inst., Houston, Tex.	J. V. Pennington
Rose Polytechnic Inst., Terre Haute, Ind.	H. C. Gray
Rutgers Univ., New Brunswick, N. J.	R. C. H. Heck
Santa Clara, Univ. of, Santa Clara, Calif.	G. L. Sullivan
South Dakota, Univ. of, Vermilion, S. D.	M. W. Davidson
Southern California, Univ. of, Los Angeles	S. F. Duncan
Stanford Univ., Stanford University, Calif.	B. M. Green
Stevens Institute of Technology, Hoboken, N. J.	E. H. Fezandie
Swarthmore College, Swarthmore, Pa.	G. A. Bourdelais
Syracuse Univ., Syracuse, N. Y.	S. T. Hart
Tennessee, Univ. of, Knoxville, Tenn.	D. A. White
Texas, A. & M. College of, College Station	V. M. Faires
Texas Technological College, Lubbock, Tex.	V. L. Doughtie
Texas, Univ. of, Austin, Tex.	J. L. Bruns
Tufts College, Tufts College, Mass.	Edgar MacNaughton
U. S. Naval Academy, Post Graduate School, Annapolis	P. J. Kiefer
Utah, Univ. of, Salt Lake City, Utah	M. B. Hogan
Vanderbilt Univ., Nashville, Tenn.	F. J. Reed
Vermont, Univ. of, Burlington, Vt.	E. L. Sussdorff
Villanova College, Villanova, Pa.	J. S. Morehouse
Virginia Polytechnic Inst., Blacksburg, Va.	N. W. Conner
Virginia, Univ. of, Charlottesville, Va.	H. C. Hesse
Washington, State College of, Pullman, Wash	F. W. Candee
Washington Univ., St. Louis, Mo.	E. H. Sager
Washington, Univ. of, Seattle, Wash.	B. T. McMinn
West Virginia Univ., Morgantown, W. Va.	C. R. Jones
Wisconsin, Univ. of, Madison, Wis.	B. G. Elliott
Worcester Polytechnic Inst., Worcester	M. L. Price
Wyoming, Univ. of, Laramie, Wyo.	Robert Rhoads
Yale Univ., New Haven, Conn.	S. W. Dudley

A.S.M.E. Representatives on Other Activities[8]

Dates in parentheses denote expiration of terms

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

SECTION M, ENGINEERING

H. N. Davis (1932)
Walter Rautenstrauch (1932)

Representatives appointed annually from locality in which meeting is to be held.

AMERICAN ENGINEERING COUNCIL

Terms expire January 1, 1933	*Terms expire January 1, 1934*
W. S. Conant	L. P. Alford
E. O. Eastwood	Paul Doty
B. E. Hull	C. E. Ferris
J. W. Roe	R. E. Flanders
Warner Seely	E. C. Hutchinson
W. Trinks	C. N. Lauer, *Chairman*
E. N. Trump	J. H. Lawrence
W. R. Webster	R. C. Marshall, Jr.
R. V. Wright	A. A. Potter
D. Robert Yarnall, *Vice-Chairman*	W. H. Trask, Jr.

AMERICAN SOCIETY OF SAFETY ENGINEERS

ENGINEERING SECTION, NATIONAL SAFETY COUNCIL

H. W. Mowery
Ernest Hartford

AMERICAN STANDARDS ASSOCIATION

K. H. Condit (1934)[9]	C. B. LePage	*Alternates* (*Serve one year*)
L. K. Sillcox (1932)	C. W. Spicer	
C. M. Chapman (1933)	V. R. Willoughby	

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[7] In office December 31, 1932.

[8] The A.S.M.E. is also represented on a number of special boards and on many committees of other organizations.

[9] Represents the A.S.M.E. on the Executive Committee of the A.S.A.

THE ENGINEERING FOUNDATION

A. E. White (1933) D. Robert Yarnall (1934)
E. R. Fish (1935)[10]

ENGINEERING SOCIETIES EMPLOYMENT SERVICE

NATIONAL BOARD

Calvin W. Rice, *Secretary*

ENGINEERS' COUNCIL FOR PROFESSIONAL DEVELOPMENT

C. F. Hirshfeld, *Chairman* J. H. Lawrence
W. E. Wickenden

NATIONAL RESEARCH COUNCIL

DIVISION OF ENGINEERING AND INDUSTRIAL RESEARCH

A. E. White (1932) Herman Diederichs (1934)
R. L. Streeter (1933)[11] Calvin W. Rice, *Ex-Officio*

UNITED ENGINEERING TRUSTEES, INC.

H. Hobart Porter (1932) H. V. Coes (1933)
E. R. Fish (1934)

Exchange of Courtesies

Special arrangements have been made between the A.S.M.E. and the following organizations whereby their members will be mutually accorded such courtesies as the use of reading and writing rooms, assistance in securing general information and in arranging to visit industrial plants, receipt of mail, etc.

OUTSIDE THE UNITED STATES

Austria—Vienna: Osterreichischer Ingenieur und Architekten Verein.
Belgium—Bruxelles: Federation des Associations Belges d'Ingenieurs; Société Belge des Ingenieurs et des Industriels.
Canada—Montreal: The Engineering Institute of Canada.
Cuba—Havana: Sociedad Cubana de Ingenieros.
Czechoslovakia—Prague: Association of Czechoslovakian Engineers and Architects in Praha; Masarykova Akademie Prace; Spolek Ceskoslovenskych Inzenyru.
Denmark—Copenhagen: Dansk Ingeniorforening.
England—London: British Association for the Advancement of Science; British Engineering Standards Association; British Engineers Association; Chemical Society; The Institute of Fuel; The Institute of Marine Engineers; The Institute of Metals; The Institution of Automobile Engineers; The Institution of Civil Engineers; The Institution of Electrical Engineers; The Institution of Engineering Inspection; The Institution of Mechanical Engineers; The Institution of Mining Engineers; Institution of Naval Architects; The Institution of Structural Engineers; International Electrotechnical Commission; The Iron and Steel Institute; Junior Institution of Engineers; The Royal Aeronautical Society; The Society of Engineers.
Newcastle-on-Tyne: North East Coast Institution of Engineers and Shipbuilders; North of England Institute of Mining and Mechanical Engineers.
South Kensington: Newcomen Society.
France—Paris: l'Association Technique Maritime et Aeronautique; Société des Ingènieurs Civils de France; Société Francaise des Electriciens.
Germany—Berlin: Verein deutscher Ingenieure.
Italy—Rome: Sindacato Nazionale Fascista delgi Ingegneri.
Japan—Tokyo: Kogakkai.
Mexico—Mexico City: Asociacion de Ingenieros y Arquitectos de Mexico.
Netherlands—The Hague: Koninklijk Instituut van Ingenieurs.
Norway—Oslo: Den Norske Ingeniorforening.
Scotland—Glasgow: Institution of Engineers and Shipbuilders in Scotland.
South Africa—Johannesburg: South African Institution of Engineers.
South America—Colombia: Sociedad Colombiana de Ingenieros.
Sweden—Stockholm: Svenska Teknologforeningen.
Switzerland—Winterthur: Verein Schweizerischer Maschinen-Industrieller.

UNITED STATES

California—Los Angeles: Engineering and Architects Association.
District of Columbia—Washington: Society of American Military Engineers.
Florida—Gainesville: Florida Engineering Society.
Illinois—Chicago: Western Society of Engineers.
Louisiana—New Orleans: Louisiana Engineering Society.
Maryland—Baltimore: Baltimore Engineers Club.
Massachusetts—Boston: Boston Society of Civil Engineers.
Michigan—Detroit: Detroit Engineering Society.
Missouri—St. Louis: Engineers Club of St. Louis.
New York—New York: American Gas Association; American Institute of Electrical Engineers; American Institute of Mining and Metallurgical Engineers; American Society of Civil Engineers; American Society of Refrigerating Engineers; American Society of Safety Engineers; Chemists Club Library; Columbia University Library; New York Railroad Club.
Rochester: Rochester Engineering Society.
Syracuse: Technology Club of Syracuse.
Utica: Mohawk Valley Engineers' Club.
Ohio—Cleveland: Cleveland Engineering Society.
Dayton: Engineers' Club of Dayton.
Pennsylvania—Philadelphia: Engineers Club of Philadelphia.
Pittsburgh: Engineers Society of Western Pennsylvania.
Scranton: Engineers Society of Northeastern Pennsylvania; Scranton Engineers Club.
Rhode Island—Providence: Providence Engineering Society.
Tennessee—Nashville: Engineering Association of Nashville.
Washington—Seattle: Pacific Northwest Society of Engineers.
Spokane: Associated Engineers of Spokane.

Past Presidents[12]

Alexander Lyman Holley, *Chairman of the Preliminary Meeting for Organization of The American Society of Mechanical Engineers* (1882)[13]

1880–1882	Robert Henry Thurston (1903)
1883	Erasmus Darwin Leavitt (1916)
1884	John Edson Sweet (1916)
1885	Josephus Flavius Holloway (1896)
1886	Coleman Sellers (1907)
1887	George H. Babcock (1893)
1888	Horace See (1909)
1889	Henry Robinson Towne (1924)
1890	Oberlin Smith (1926)
1891	Robert Woolston Hunt (1923)
1892	Charles Harding Loring (1907)
1893–1894	Eckley Brixton Coxe (1895)
1895	Edward F. C. Davis (1895)
1895	Charles Ethan Billings (1920)
1896	John Fritz (1913)
1897	Worcester Reed Warner (1929)
1898	Charles Wallace Hunt (1911)
1899	George Wallace Melville (1912)
1900	Charles Hill Morgan (1911)
1901	Samuel T. Wellman (1919)
1902	Edwin Reynolds (1909)
1903	James Mapes Dodge (1915)
1904	Ambrose Swasey
1905	John Ripley Freeman (1932)
1906	Frederick Winslow Taylor (1915)
1907	Frederick Remsen Hutton (1918)
1908	Minard Lafever Holman (1925)
1909	Jesse Merrick Smith (1927)
1910	George Westinghouse (1914)
1911	Edward Daniel Meier (1914)
1912	Alexander Crombie Humphreys (1927)
1913	William Freeman Myrick Goss (1928)
1914	James Hartness
1915	John Alfred Brashear (1920)
1916	David Schenck Jacobus
1917	Ira Nelson Hollis (1930)
1918	Charles Thomas Main
1919	Mortimer Elwyn Cooley
1920	Fred J. Miller
1921	Edwin S. Carman
1922	Dexter Simpson Kimball

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[10] A.S.M.E. representative from Board of United Engineering Trustees, Inc.

[11] Deceased December 10, 1932.

[12] A list of past vice-presidents, managers, treasurers, and secretaries will be found in the 1930 Record and Index, pages 10–12.

[13] Dates in parentheses denote year of death.

1923	John Lyle Harrington
1924	Frederick Rollins Low
1925	William Frederick Durand
1926	William Lamont Abbott
1927	Charles M. Schwab
1928	Alex Dow
1929	Elmer Ambrose Sperry (1930)
1930	Charles Piez
1931	Roy V. Wright
1932	Conrad N. Lauer

Honorary Members

HONORARY MEMBERS IN PERPETUITY

Alexander Lyman Holley, Founder of the Society. Died 1882.

John Edson Sweet, Founder of the Society. Died 1916.

Henry Rossiter Worthington, Founder of the Society. Died 1880.

DECEASED HONORARY MEMBERS

	ELECTED	DIED
Horatio Allen	1880	1889
Sir William Arrol	1905	1913
Sir Benjamin Baker	1886	1907
Johann Bauschinger	1884	1893
Sir Henry Bessemer	1891	1898
Sir Frederick Joseph Bramwell	1884	1903
John Alfred Brashear	1908	1920
Gustave Canet	1900	1908
Andrew Carnegie	1907	1919
Daniel Kinnear Clark	1882	1896
Rudolph Julius Emmanuel Clausius	1882	1888
Sir John Coode	1889	1892
Peter Cooper	1882	1883
Carl Gustaf Patrik de Laval	1912	1913
Rudolph Diesel	1912	1913
James Dredge	1886	1906
Victor Dwelshauvers-Dery	1886	1913
Thomas Alva Edison	1904	1931
Alexandre Gustave Eiffel	1889	1923
Marshal Ferdinand Foch	1921	1929
Sir Charles Douglas Fox	1900	1921
John Ripley Freeman	1932	1932
John Fritz	1900	1913
Major-General George Washington Goethals	1917	1928
Franz Grashof	1884	1893
Otto Hallauer	1882	1883
Charles Haynes Haswell	1905	1907
Friedrich Gustav Herrmann	1884	1907
Gustav Adolph Hirn	1882	1890
Joseph Hirsch	1889	1901
Ira N. Hollis	1928	1930
Robert Woolston Hunt	1920	1923
Benjamin Franklin Isherwood	1894	1915
Henri Léauté	1891	1916
Erasmus Darwin Leavitt	1915	1916
Anatole Mallet	1912	1919
Charles H. Manning	1913	1919
Rear-Admiral George Wallace Melville	1910	1912
The Honorable Sir Charles Algernon Parsons	1920	1931
Charles Talbot Porter	1890	1910
Auguste C. E. Rateau	1919	1930
Sir Edward J. Reed	1882	1906
Franz Reuleaux	1882	1905
Henri Adolphe-Eugene Schneider	1882	1898
C. William Siemans	1882	1883
Viscount Ei-ichi Shibusawa	1929	1931
Henry Robinson Towne	1921	1924
Henri Tresca	1882	1885
Francis A. Walker	1886	1897
Worcester Reed Warner	1925	1929
Sir William Henry White	1900	1913
George Westinghouse	1897	1914
Sir Alfred Fernandez Yarrow	1914	1932

LIVING HONORARY MEMBERS

	ELECTED
Sir John Audley Frederick Aspinall	1911
William Wallace Atterbury	1925
Mortimer Elwyn Cooley	1928
Charles de Fréminville	1919
Rear-Admiral Robert Stanislaus Griffin	1920
Nathanael Greene Herreshoff	1921
Honorable Herbert Clark Hoover	1925
Masawo Kamo	1929
Henri Le Chatelier	1927
Grande Ufficiale Ing. Pio Perrone	1920
Calvin Winsor Rice	1931
Palmer C. Ricketts	1931
Charles M. Schwab	1918
Ambrose Swasey	1916
Elihu Thomson	1930
William Cawthorne Unwin	1898
Samuel Matthews Vauclain	1920
Oskar von Miller	1912
Right Honorable Lord Weir	1920
Orville Wright	1918

Report of the Council

THE close of the fiscal year 1931–1932 finds the Society in a strong position. Notable advances have been made and the work of the Society has been highly fruitful despite the curtailment of expenditures made necessary by the reduction of income from dues and advertising.

Mechanical engineers have suffered from the lack of industrial and engineering activity. The Employment Service has striven to meet a situation which was beyond its control. Members have contributed time and money through local agencies which have been organized to provide emergency aid, in some cases solely for engineers. The Council, in sympathy with the times, when requested has made possible quarterly payments on dues. The first payment assures the sending of *Mechanical Engineering* and the *A.S.M.E. News* throughout the year. Further, rather than having a member withdraw from the Society, the Council has also provided for members in arrears to retain membership status without receiving publications and to be exempt from payment of dues until further notice during the period of the business depression.

MEMBERSHIP CHANGES—OCTOBER 1, 1931, TO SEPTEMBER 30, 1932

	Membership		Losses				Additions			Totals		
	Oct. 1, 1931	Oct. 1, 1932	Transferred from	Resigned	Dropped	Died	Transferred to	Elected	Reinstated	Loss	Gain	Net Changes
Honorary Members....	25	25	0	0	0	3	3	0	0	3	3	0
Life Members	70	68	1	0	0	2	0	1	0	3	1	— 2
Members............	8,968	8,878	31	145	53	116	101	149	5	345	255	— 90
Associates..........	602	564	9	29	5	5	1	9	0	48	10	— 38
Associate-Members....	4,127	4,150	33	70	35	29	82	104	4	167	190	+ 23
Juniors (20)..........	1,668	2,076	65	55	46	1	575	0	0	167	575	+408
Juniors (10)..........	4,549	4,318	623	87	72	7	0	556	2	789	558	—231
	20,009	20,079	762	386	211	163	762	819	11	1522	1592	+ 70

The Council trusts that as a result of this liberal policy all members may keep their connection with the Society until such time as they can meet their obligations.

The opportunity offered to individual members for participation in the activities of the Society has been large. Two meetings of the Society, eight national meetings and six special meetings of Professional Divisions, including the First National Process Meeting, and 434 Local Section gatherings indicate that this opportunity was not materially less than that offered in the previous year.

Because of the exigencies of the times, the Council held only two meetings during the year and placed upon the Executive Committee the responsibility of carrying on between Council meetings. That Committee formulated a definite program of Society activity and met monthly to forward it and to transact routine business. The results of this program and of the Standing Committee activities are summarized in the following paragraphs.

The success of the new Student Branch plan in the 15 schools in the South was demonstrated at the First Southern Student Branch Conference at Chattanooga in April. The Council authorized the installation of the plan in three additional areas during 1932–1933.

The recommendation of the Committee on the Economic Status of the Engineer resulted in the establishment of the Engineers' Council for Professional Development, a joint organization whose general objective is the enhancement of the professional status of the engineer. The American Society of Civil Engineers, American Institute of Mining and Metallurgical Engineers, The American Society of Mechanical Engineers, American Institute of Electrical Engineers, American Institute of Chemical Engineers, Society for the Promotion of Engineering Education, and National Council of State Boards of Engineering Examiners are participating in this program.

As further steps in the broad program of improving the status of the engineer, the Council approved the general principle of State Councils of Engineers, and authorized the Committee on Registration of Engineers to represent the Society on all joint agencies dealing with registration.

Mechanical Engineering was developed into a magazine which touches more broadly the work and interest of every member. The publication of articles on economics and social problems and on biography and history has met with a cordial response and has demonstrated the engineer's interest in these subjects.

The balancing of income and expenditures has been of continuous concern during the year. The income from dues and advertising decreased more rapidly than expenditures could be decreased, and a draft upon reserve of $67,998.38 was necessary. Fortunately, the Society is in a sound position financially, and with a balanced budget in 1932–1933, will be able to carry on its activities without embarrassment.

The Administrative Office has been reorganized by direction of the Council on the basis of a report of an outside firm engaged for that purpose, with a resultant smaller personnel. This reduction, with lowered salary rates and vacations without pay, decreased the payroll expenditure by 22 per cent. The Council providéd an unemployment relief fund available on a made-work basis for members of the staff who have been released from service.

The reports of the Standing Committees of Council follow.

CALVIN W. RICE, *Secretary*

FINANCE

Vigorous efforts were made during the past year, 1931–1932, to reduce the expenditures of the Society to meet the greatly curtailed income. Through the splendid cooperation of standing committees, the appropriations under their supervision were reduced by 17 per cent. Printing economies and the reduced number of advertising pages made possible a reduction of 32 per cent in the cost of printing and distributing publications. Drastic economies in the Secretary's Office, accomplished by reorganizing the Office, by reducing the staff, and by salary reductions, resulted in a decrease of 19 per cent in the administrative expense. A reduction of 23 per cent in the expenses of the Engineering Index was brought about largely by a change in the method of printing the cards. These reductions have been achieved without affecting to any great extent the usual activities of the Society or its service to members.

The income for the past year, excluding that received from initiation fees, which is not used for operating purposes, was 32 per cent less than the income for 1930–1931. This is largely accounted for by the decreased revenue from advertising and the sale of publications and from a probable loss in dues. Gross income from advertising this year was 48 per cent less than that in 1930–1931; gross income from the sale of publications was 31 per cent less than in the previous year.

Dues in arrears on September 30, 1932, amounted to $110,876.36. In the belief that a considerable part of this amount will not be paid, a reserve of $55,000.00 was set aside to cover a probable loss, thereby reducing the dues receivable to $55,876.36, amounting to a 17 per cent reduction in dues income for the fiscal year.

TOTAL EXPENDITURES

For control purposes, the expenditures of the Society are segregated into four classes: first, Committee Expenditures, that is, monies spent directly by Committees or Council; second, Printing and Distribution Expenses of Publications (Editorial and Advertising Expenses appear under the heading of Administrative Expense); third, the Administrative Expenses, including all salaries and office operating expenses under the supervision of the Secretary and in addition the Editorial Department and Advertising Department expenses; fourth, Engineering Index Expense, including all salaries and other expenses of preparation, printing, selling, and distribution.

A comparison of the total expenditures for the fiscal years 1930–1931 and 1931–1932 is given in the following table:

	1931–1932	1930–1931
Committee Expense	$ 96,523.86	$116,073.62
Publications Expense (Printing and Distribution Only)	156,606.36	230,932.77
Administrative Office Expense	306,905.89	380,295.57
Engineering Index Expense	84,552.41	113,379.71
	$644,588.52	$840,681.67
Relief Fund	2,978.75	
	$647,567.27	

The use to which these expenditures is put in carrying on the work of the Society is shown in the following tabulation:

	1931–1932	1930–1931
General	$ 35,493.33	$ 44,630.18
Society Activities	150,769.97	175,241.50
Technical Committees	55,356.48	64,883.97
Text Publications (Printing, Editorial, and Distribution)	142,573.54	179,390.56
Advertising and Sales (Printing, Soliciting, and Distribution)	175,842.79	263,155.75
Engineering Index (Printing, Editorial, Soliciting, and Distribution)	84,552.41	113,379.71
	$644,588.52	$840,681.67

General Expenses include those for Council, the Library, American Engineering Council, the Nominating Committee, and Awards. The decrease of $9863.15 in General Expenses was due in part to the discontinuation of assessments for the study of the Economic Status of the Engineer; and to reductions in appropriations of 31 per cent to Council, of 11 per cent to American Engineering Council, and 15 per cent to the Nominating Committee. Assessments for the Library and for Awards were slightly increased.

Expenses for Society Activities include those for Meetings, Local Sections, Professional Divisions, Student Branches, Employment Service, and for the staff expense of handling these activities and the routine procedure for the Membership Committee. These expenses were decreased by $24,471.53, a reduction of 14 per cent.

Technical Committee expenses include the direct appropriation to the American Standards Association ($1500.00), grants in aid of research ($300.00), and staff expenses to support the Boiler Code, Research, Standardization, Power Test Codes, and Safety Committees. The decrease of $9527.49, or 15 per cent, was brought about by discontinuing the appropriation to the International Electrotechnical Commission ($1320 in 1930–1931) and by savings made in the staff expenses.

Text Publications Expense include the expense of printing, distributing, and editing Transactions, the *A.S.M.E. News*, and the text pages of *Mechanical Engineering*. The decrease of $36,817.02, or 20 per cent, was due to suspending publication of the membership list, to a reduction in the number of pages in *Mechanical Engineering*, to the adoption of a less expensive method of printing advance papers, to a reduction in the number of papers included in Transactions, and to savings made in the expenses for the editorial staff.

Expenses of Advertising and Sales include (1) printing and distributing the Mechanical Catalog and the advertising pages of *Mechanical Engineering*, (2) cost of securing advertising income, (3) printing and distributing publications sold, and (4) the expenses of the publication sales department. The decrease of $87,312.96, or 33 per cent, in this item is due to the reduced number of advertising pages sold and printed.

TOTAL INCOME

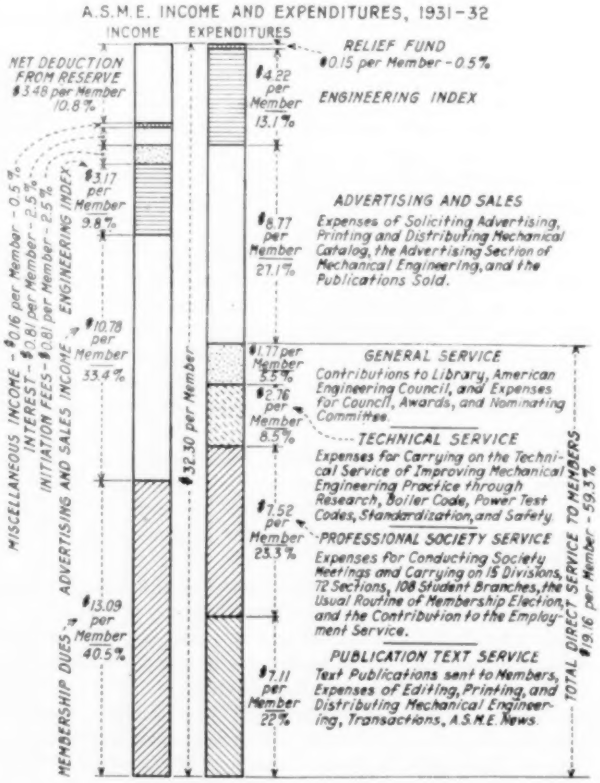
A comparison of income for the past two years is given in the following tabulation:

	1931–1932	1930–1931
Initiation Fees	$ 16,288.33	$ 25,863.27
Student Fees	$1,158.00	....
Membership Dues (Including Dues Receivable)	316,137.28	$314,828.31
Interest and Discount	16,168.57	22,080.10
Mechanical Engineering Advertising	103,757.32	213,771.23
Mechanical Catalog Advertising	60,807.47	102,262.77
A.S.M.E. News	802.00	889.00
Publication Sales	53,134.03	61,768.71
Engineering Index	63,552.17	108,508.64
Miscellaneous Sales	2,212.07	3,364.35
	$617,728.91	$827,473.11
Less: Bad Debts	2,427.10	1,558.99
	$615,301.81	$825,914.12
Less: Probable Loss of Dues Receivable	55,000.00	
	$560,301.81	
Unexpended Commitments, 1930–1931	1,030.50	
	$561,332.31	
From Reserve	86,234.96	
	$647,567.27	

The net income available for operating purposes, after deducting the $55,000.00 reserve for probable loss of dues, was $560,301.81, or $84,286.71 less than the total expenditures of $644,588.52, excluding the relief fund expenditure from reserve. The Council authorized that this amount be taken from the accumulated surplus. Inasmuch as $16,288.33 received from initiation fees was placed in the initiation fee reserve fund, the net deduction from surplus was $67,998.38, of which $55,000.00 was the reserve for probable loss of dues and $12,998.38 was a net operating loss.

INCOME AND EXPENSES PER MEMBER

An analysis of the income and expenditures for 1931–1932 on a per-member basis is shown in the accompanying chart.



NOTE: The average membership during 1931–1932 was 20,053.

BALANCE SHEET

A statement of the assets and liabilities of the Society is given in the accompanying balance sheet in the simplified form adopted by the Council.

The accounting procedure for the Engineering Index has been a matter of continuous discussion by the Council and the Finance Committee. The Index was carried as a separate project and advances to it were recorded as assets. The Index operation has faced serious financial difficulties during the past year. The Council, believing that carrying this item as an asset was unwise, voted to write it off and to combine the Index accounting with the accounts of the Society.

Because of the reduced income, the Council decided not to make a deduction this year from the deferred charges for the Fiftieth Anniversary, nor to set aside the usual $5,000.00 reserve for retirement allowance. The Council made an appropriation from working surplus of $7,000.00, or as much thereof as might be necessary to provide aid on a make-work basis to members of the staff released from service. Expenditures this year amounted to $2,978.75, leaving an unexpended balance of $4,021.25.

In order to continue its reputation for the prompt payment of its obligations, the Society's borrowing requirements amounted to $185,000.00 at the end of the fiscal year. Substantial reductions from this amount will be made in the first part of next year by the receipt of dues and by collections for Mechanical Catalog advertising.

Wm. J. Struss & Co., certified public accountants, give the results of their examination of the books of the Society for the fiscal year ended September 30, 1932, in the accompanying statement of assets and liabilities.

Respectfully submitted,

David Lofts, *Chairman* **E. A. Muller**
Kingsley L. Martin, *Vice-Chairman* **Walter Rautenstrauch**
John H. Lawrence
R. V. Wright } *Council Representatives.*
W. L. Batt }

BALANCE SHEET, SEPTEMBER 30, 1932

ASSETS

CURRENT ASSETS			
Cash		$ 21,848.49	
Accounts Receivable			
Dues	$110,867.36		
Less: Probable Loss	55,000.00		
	$ 55,867.36		
Publications and Advertising	85,233.96		
Engineering Index	23,410.94	164,512.26	
Inventories			
Supplies	11,378.34		
Publications in Process	12,870.77		
Publications for Sale	124,403.94	148,653.05	
Investments (at Cost)		321,457.88	
TOTAL CURRENT ASSETS			$ 656,471.68
INVESTMENTS FOR TRUST FUNDS (at Cost)			168,833.70
FIXED ASSETS (Book Value)			
Building (one-quarter interest)		496,948.48	
Furniture and Equipment		33,847.20	
Library Books		1.00	
Engineering Index		1.00	530,797.68
ADVANCES			
Loan to Committee on Thermal Properties of Steam			15,000.00
DEFERRED CHARGES			
Fiftieth Anniversary		24,399.66	
Society Activities Next Year		2,276.68	26,676.34
TOTAL ASSETS			$1,397,779.40

LIABILITIES

CURRENT LIABILITIES			
Notes Payable	$185,000.00		
Accounts Payable	20,171.32		
Unfilled Commitments	13,393.30		
Gift for Thurston Biography	460.82		
Custodian Funds (30)	24,175.93		
Dues Paid in Advance	38,229.03		
TOTAL CURRENT LIABILITIES		$281,430.40	
RESERVE AGAINST DEPRECIATION OF SECURITIES		15,000.00	
RESERVE FOR RETIREMENT ALLOWANCE		25,011.61	
TRUST FUNDS		168,833.70	
TOTAL LIABILITIES			$ 490,275.71
SURPLUS ACCOUNT			
In Fixed Assets		$530,797.68	
In Working Surplus[1]		360,041.28	
Additional		16,664.73	907,503.69
TOTAL			$1,397,779.40

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[1] Working Surplus consists of the following three items:

Initiation Fee Reserve	$211,440.38
Previous Surplus Investment	70,005.89
Other Working Surplus	78,595.01
	$360,041.28

MEETINGS AND PROGRAM

During the fiscal year, October 1, 1931, through September 30, 1932, two meetings of the Society were held, the Annual Meeting in New York, November 30 to December 4, 1931, and the Semi-Annual Meeting at Bigwin Inn, Lake of Bays, Ontario, Canada, June 27 to July 1, 1932. The program for the former meeting appears in the Record and Index section of the Transactions for 1931 (see pages 45–47) and the latter program will be found in the 1932 Record and Index.

ANNUAL MEETING

Planning the Annual Meeting program consists of (1) selecting a series of events of broad interest and of inspiring forward-looking nature, (2) coordinating the programs of the Professional Divisions, and (3) planning the entertainment and excursion events.

In the Annual Meeting program for 1931 the broad-interest program events were generally excellent: The Towne Lecture by Dean Wallace B. Donham on "The Temporary Emergency and Twenty-Year Planning," the Thurston Lecture by Dr. E. L. Thorndike on "Psychology and Engineering," the address by Commissioner Frances Perkins on "State Labor Departments and Professional Engineers—Cooperation Essential to Progress in Accident Prevention," the Presidential Address of Dr. Roy V. Wright on "The Engineer Militant," and the award of Honorary Membership with suitable exercises to Dr. Calvin W. Rice, Secretary of the Society for twenty-five years, and to Dr. Palmer C. Ricketts.

In addition, a Public Speaking Contest was initiated as a means of interesting individual engineers in the importance of a better manner of presenting engineering material before audiences, the series of lectures by Dr. S. Marion Tucker on "Talking With an Audience" were continued before enthusiastic audiences, and the Second Art Exhibit showed work of engineers during 1931.

The number of papers presented was equal to that at the previous Annual Meeting but the attendance was about ten per cent smaller.

BIGWIN MEETING

The Bigwin Meeting presented an international program arranged with the cooperation of the Engineering Institute of Canada and individual members of the Institution of Mechanical Engineers of Great Britain. This program of excellent papers, combined with the fine entertainments arranged by the Ontario Section and the beautiful setting for the meeting, made the gathering a most successful one.

IMPROVEMENT OF ENGLISH

The Meetings and Program Committee thoroughly supports efforts to improve spoken and written English. Following the address at the June, 1931, meeting of the Society for the Promotion of Engineering Education by J. W. Parker on "The Need for Speech Training by Engineers" the Committee cooperated in the plans for the 1932 Summer Session for teachers of English to engineers, and two members of the Committee, J. W. Parker and R. I. Rees, delivered lectures at the Session.

Respectfully submitted,

F. M. Feiker, *Chairman*
J. W. Parker, *Vice-Chairman* **R. I. Rees**
C. P. Bliss **E. C. Hutchinson.**

PUBLICATIONS

The effects of a reduced income were seriously felt by the Publications Committee in carrying out its function of supervising the Society's publications, but the emergency was met and was not without beneficial compensations. Transactions for the calendar year

1931 appeared in a single volume instead of the two which have been necessary since the new method of publishing was adopted in 1926. The original plan of quarterly publications for every division has been so modified, except for the Aeronautical Division, that semi-monthly issues, making a total of 24 (including the Record and Index) divided among all of the remaining divisions, are now published. Definite publication dates have been set by the requirements of the Post Office Department and have greatly facilitated the work of the staff.

Among the benefits resulting from hard times has been the necessity of scrutinizing every paper with more than usual care to appraise its value in printed form. The Committee on Meetings and Program, the Divisions, and other paper-procuring agencies have aided wholeheartedly in this effort to reduce the volume of papers in Transactions. In cooperation with the Committee on Meetings and Program, the Committee on Professional Divisions, and the Research Committee, a procedure looking toward the improvement of the quality of papers has been worked out and approved by the Council.

The problem of better and cheaper publication of papers not destined for Transactions has been worked out with the result that all preprints and reports, not to be set in type, are reproduced by the photo-offset process in accordance with a standard style. Such papers are generally available for meetings and for distribution on request. Synopses of them, with request blank, appear in *Mechanical Engineering*, and copies are preserved, bound together, and filed with the Engineering Societies Library. There they are appropriately cataloged. They are also indexed for the Society's Record and Index. Their availability is thus assured.

The printing of the Membership List was omitted this year as an economy measure.

The *A.S.M.E. News* appeared regularly twice monthly, but in much abbreviated form, set in very small type to conserve space, and lacking its usual illustrations. Only most pressing matters could be treated, and then inadequately. Such a situation obviously cannot be justified except in view of the low state of Society finances. A comprehensive study of the *News* is in progress by the Committee.

Mechanical Engineering, while reduced in size, showed a healthy development in reader interest, by a more attractive make-up and articles of broader scope and more general nature. Comment has been favorable and encouraging.

The Mechanical Catalog reflected the general lack of business throughout the country with its reduced number of pages and entries.

One of the encouraging aspects of the publication situation is to be found in the reduced costs of publication. A careful check-up of costs was made and bids on the printing of Transactions and *Mechanical Engineering* were solicited from a number of printing houses. In view of the reductions granted by the Society's printer it was found impossible to save money by making a change. The committee was satisfied that its printing costs are as low as can be obtained anywhere for such quality and volume as exist. Other economies in printing have greatly reduced costs of Transactions papers. The newly adopted photo-offset process should result in further savings.

A major portion of the Committee's attention was directed to a review, made at the request of President Lauer, of the Society's publications and of the policy under which they are administered. This review includes a thorough analysis of the advertising problem and resulted in recommendations that the policy of accepting advertising in *Mechanical Engineering* and other Society publications be continued, and that all of the present publications of the Society be continued. A proviso with respect to the *News*, still under study, made the report subject to further recommendations on this publication.

At the 1931 Annual Meeting the Publications Committee attempted to interest the delegates to the Local Sections Conference in publication matters, particularly in *Mechanical Engineering*. Such efforts must be continued, and with a magazine that touches more broadly the work and interest of every member, attractively put together, the Committee feels that its enthusiasm for this publication and the part it plays in the profession of engineering is justified. It is significant that the publication of articles on economic and social problems, and on biography and history, have met with a cordial response and have demonstrated the engineer's interest in these subjects. New writers have been discovered and readers who too long have been silent are writing their comments to the editor. There is ample evidence that *Mechanical Engineering* is becoming more widely read and discussed as every month goes by.

The Engineering Index entered its fourth year of operation by the Society. Rigorous economies in the operation of the Index were effective in reducing the expenses by $29,000.00. Unfortunately the income dropped $45,000.00 and the Engineering Index is compelled to report a deficit. Further economies have been put into effect to balance the budget during 1932–1933.

The report of the Finance Committee shows figures from income of publications. The Committee on Publications attempted to bring the expenses for the printing and distributing of publications in line with this decreased income. The following figures show the amount spent for printing and distributing Society publications during the past fiscal year in comparison with the preceding year.

Publications Income

	1931–1932	1930–1931
Mechanical Engineering Advertising	$103,757.32	$213,771.23
Mechanical Catalog Advertising	60,807.47	102,262.77
A.S.M.E. News	802.00	889.00
Publication Sales	53,134.03	61,768.71
	$218,500.82	$378,691.71
Less: Bad Debts	2,427.10	1,558.99
	$216,073.72	$377,132.72

Publications Expense
(Printing and Distribution)

Mechanical Engineering Text Pages	$ 31,676.16	$ 36,566.61
A.S.M.E. News	9,133.48	10,543.39
Transactions	45,591.20	59,026.34
Membership List	197.63	7,127.35
Mechanical Engineering Advertising Pages	22,836.73	55,565.59
Mechanical Catalog	21,783.92	38,702.45
Publications for Sale	25,387.24	23,400.94
	$156,606.36	$230,932.77

Engineering Index

Income from Card Service and Annual Volumes	$63,552.17	$108,508.64
Expense (Editorial, Printing, Selling, and Distributing)	$84,552.41	$113,379.71

Respectfully submitted,

W. H. Winterrowd, *Chairman*
L. C. Morrow, *Vice-Chairman*
S. F. Voorhees
S. W. Dudley
W. F. Ryan.

PROFESSIONAL DIVISIONS

During the past year Professional Division activities have been slightly curtailed, chiefly in respect to national meetings, several Divisions feeling that the time was not opportune for such meetings. Efforts were concentrated in fields where favorable conditions prevailed, so that some of the Divisions have had very successful national meetings. Probably the most outstanding of these from the point of attendance and interest was the national meeting of the Printing Industries Division, held in New York, March 14 and 15. This meeting had a total attendance of nearly 1,000 and surpassed any previous meeting held by the Division. The Aeronautic Division sponsored a special Pacific Coast Aeronautic Meeting at Berkeley, Calif., in June, in addition to its national meeting in Buffalo.

The Process Industries Committee held the first national Process Meeting of the Society at Buffalo in June, assisted by the Petroleum Refining Committee of the Petroleum Division.

In planning the activities of the Divisions, an effort was made to combine meetings, and at the same time to present all the valuable material in the way of papers that was offered; so that, in spite of the depression, the Divisions were able to provide almost the usual amount of technical material, which was made available in either printed or mimeographed form.

During the last year the other activities of the Divisions have progressed. The Divisions have continued their excellent work in arranging sessions at the Society's Annual and Semi-Annual Meetings, and have assisted Local Sections in meetings. They have prepared their annual progress reports covering developments in their fields. In research and standardization work they have been active either through their own subcommittees or in initiating committees under the A.S.M.E. Standing Committees on Research, Standardization, and Safety.

All activities of the Divisions for the year can be summarized as follows:

8 National Meetings: Printing, Management, Materials Handling, Aeronautic, Oil and Gas Power, Process, Applied Mechanics, and Wood Industries
7 Special Meetings (larger than local): Pacific Aeronautic, Buffalo Machine Shop, Fuels, two Tulsa Petroleum, and two Textile
25 Local Section Meetings aided
44 Special Division subcommittees working
1 New research project suggested
34 Sessions arranged at Annual and Semi-Annual Meetings
15 Progress reports prepared.

As to additional activities for the coming year there seems to be an opportunity for the Divisions to assist the Society in improving the status of the engineer. Some years ago this work was really started by the Railroad Division's analysis of earnings and opportunities of engineers in the railroad industry. The Machine Shop Practice Division later conducted a similar survey. The splendid work which the Society is now doing in cooperation with other societies in the matters of certification and registration seems to point the way for the Professional Divisions to give active assistance in their own fields in advising and assisting especially the younger engineer in improving his status in accordance with whatever plans are developed by the main societies. It was recently pointed out by one of the Divisions that the Society was doing very little to assist the young engineer after he leaves college, as most of the material that was published in the way of technical papers was for the more advanced engineer. It was suggested that something might be done to assist the young engineer after he leaves college to further prepare himself in his profession. This matter is now being given consideration by the Divisions, and they will be prepared to assist the Society in its general plans for improving the status of the engineer.

A brief review of each Division's activities during the year follows.

SUMMARY OF DIVISION ACTIVITIES

Aeronautic Division, during the present year, held its national meeting in Buffalo, June 6–8, presenting 33 papers, and a Pacific Coast Aeronautic Meeting at Berkeley, Calif., June 9–10, presenting 12 papers; assisted Local Sections in holding aeronautic meetings in New York, Cleveland, Tulsa, Cincinnati, and St. Louis; and held a session at the Annual Meeting in cooperation with the American Institute of Physics.

The Spirit of St. Louis Medal, awarded for meritorious services in behalf of aeronautics, which was founded by the St. Louis Section, was presented for the second time at the national meeting in Buffalo, the recipient being Paul W. Litchfield, for his work in encouraging and sponsoring airship design and construction in this country.

A Special Subcommittee on Wage Incentives in Aviation has been active during the year and presented a report at Buffalo on Wage Incentives in the Aeronautic Industry. Another activity is the Special Committee on Aircraft Safety and Inspection, under the A.S.M.E. Standing Committee on Safety, which reported at the Buffalo meeting on an "Inspection Manual for Maintenance of Conventional Aircraft." A draft of the Manual met with approval from the industry.

During the coming year the Aeronautic Division will have a session at the Annual Meeting and is planning to hold its national meeting as part of the Semi-Annual Meeting in Chicago in June, 1933, with an international program.

During the last year the Society appointed a representative on the American Committee on Marking of Obstructions to Air Navigation, of the Department of Commerce, and the Division is cooperating in this work.

Applied Mechanics Division held its national meeting in New Haven, Conn., June 23–25, presenting 27 papers. This meeting was coordinated with the American Physical Society's summer meeting, and one session was held jointly. The Division also sponsored three sessions during the last Annual Meeting and arranged a symposium of two sessions at the Semi-Annual Meeting at Bigwin on June 27–30. It will also sponsor, at the 1932 Annual Meeting, a special symposium of three sessions on Working Stresses. Its 1933 national meeting will be held as part of the Semi-Annual Meeting in Chicago, for which joint sessions will be arranged with other Divisions and with other societies.

The Division has three special subcommittees as follows: Vibration, which is sponsoring presentation of papers on this subject and helping to coordinate the research work being done; Plasticity, which sponsored sessions at the New Haven meeting and is planning additional sessions in cooperation with the Iron and Steel Division for the Semi-Annual Meeting in Chicago; and a Survey Committee on Impact Testing, which will participate in the Working Stresses symposium and is planning special activities during the coming year.

Fuels Division cooperated in the Mid-West Bituminous Conference held at Purdue University, April 14–15, 1932, and had a session at the last Annual Meeting. It is sponsoring sessions for the 1932 Annual Meeting and is planning to hold its national meeting as part of the Semi-Annual Meeting in Chicago. The Division has a recently appointed Special Advisory Committee on Boiler Furnace Explosions. The Division also assists the Pure Air Committee which grew out of its former Subcommittee on Smoke Abatement. This Pure Air Committee has a special Subcommittee on Standards for Setting Heights which will hold a conference during the 1932 Annual Meeting.

Hydraulic Division arranged sessions at the meeting at Bigwin in June and at the last Annual Meeting. The Division also has an active Subcommittee on Translation of Foreign Publications. The Division plans to have a symposium on water hammer at the Semi-Annual Meeting in Chicago and is sponsoring two sessions for the 1932 Annual Meeting.

Iron and Steel Division held a meeting in April, 1932, in cooperation with the Pittsburgh Section and the Engineering Society of Western Pennsylvania, and held two sessions at the last Annual Meeting. The Division held no national meeting, but instead is sponsoring three sessions for the Annual Meeting in 1932, and is planning to have three sessions at the Semi-Annual Meeting in Chicago. The Division initiated a Special Research Committee on Heavy Duty Anti-Friction Bearings, which has been reporting progress at its meetings for the last few years. The Division is also cooperating with the Applied Mechanics Division on the question of plastic deformation of metal, so as to aid in developing a research project that can secure the support of the iron and steel industry.

Machine Shop Practice Division held a special meeting of two sessions and a luncheon in Buffalo on October 3, in cooperation with the Buffalo Section during the Metal Industries Exposition. Sessions were arranged at the Bigwin Meeting and at the last Annual Meeting. The Division has two subcommittees, one on Machine Design and another on Foundry Practice. The latter committee has sponsored sessions at the last two Annual Meetings. Through this Division papers are presented by the Special Research Committee on Cutting of Metals, which has regularly sponsored sessions. During the coming year, the Machine Shop Practice Division will hold three sessions at the Annual Meeting and plans to hold several sessions at the Semi-Annual Meeting in Chicago. If conditions should warrant it, possibly two technical sessions will be held at any exposition of interest to their field that may be held next fall.

Management Division, in cooperation with the Materials Handling Division, held a Management, Maintenance, and Materials Handling Congress in Detroit, May 4–5, under the auspices of the Detroit Section. It also sponsored sessions at the Bigwin and last Annual Meetings. The Division has several active subcommittees. That on waste elimination has been active for some years in stimulating factory elimination-of-waste campaigns, and sponsored again this year an elimination-of-waste campaign during the month of April and also a session on the elimination of waste at the Detroit meeting. They again issued this year a small waste unit to assist small companies in putting on a factory waste campaign. The Subcommittee on Maintenance participated in the Detroit Meeting. Its Subcommittee on Economics and Business Stabilization has been very active. The Special Research Committee on Measures of Management has recently taken over as one of its activities the further investigation of the application of the replacement-of-equipment formula which was developed by the Materials Handling Division. In the Metropolitan Section the Division sponsors informal evening conferences on management. The Division is represented on the International Management Institute. This year it has formed a special Subcommittee on Organization and Operating Machine Shops and it is planning subcommittees on marketing and human relations. During the coming year the Division is planning sessions for the Chicago Meeting, in addition to three sessions at the Annual Meeting.

Materials Handling Division held its national meeting in Detroit, May 4–5, as part of the Three-M Congress. It sponsored a session at the last Annual Meeting and will hold one this year jointly with the Railroad Division. During the last year there has been formed a Materials Handling Institute, and the Division is cooperating with this Institute to encourage materials-handling progress. The Division has a special subcommittee known as the Materials Handling Manufacturer's Committee which during the last year developed and reported on standards on cranes and on the capacity of containers. The Division originally had a Replacement of Equipment Committee, which has been reorganized as a Subcommittee of the Special Research Committee on Measures of Management, under the sponsorship of this Division and the Management Division.

Oil and Gas Power Division held its national meeting at Pennsylvania State College on June 8–11, with 11 papers presented. It also had a session at the last Annual Meeting and at the Semi-Annual Meeting. It has one special Subcommittee on Oil Engine Power Costs which each year for the last four years has prepared a report on oil power costs. This report has received a good deal of attention and favorable comment. More than a hundred plants report in detail their cost of oil power, and these data are assembled in extensive tables and charts. The Division also initiated a Special Research Committee on Diesel Fuel Oil Specifications, which usually reports at Division meetings. The Division is planning to hold a national meeting next year and will have a session at this annual meeting.

Petroleum Division has a number of special subcommittees. Several of these are located in Tulsa, Oklahoma, and are assisted by the Field Secretary there. In March the Tulsa subcommittees held a special conference of one day, presenting a number of technical papers on production and pipe-line problems, and in October a conference on metallurgical problems. In the East the Division has special subcommittees, on Lubrication Engineering, Petroleum Refining, and Petroleum Utilization. The Lubrication Engineering Committee held a session at the last annual meeting and two special conferences, one on railroad lubrication in cooperation with the Railroad Division and the other on mechanism of lubrication. This committee will have a session at the Annual Meeting and it is also preparing an Industrial Lubrication Manual, which is being reviewed and criticized by some sixty representatives in different fields. During the coming year the committee is planning activities in railroad lubrication and wire rope lubrication in cooperation with the Special Research Committee on Wire Rope. It is also planning to hold a conference on lubrication problems in cooperation with the Pennsylvania State College, there next May. The Petroleum Refining Committee has been active during the past year in preparing recommendations on the testing of heat-exchange equipment for oil-refining service which is now available, and it also sponsored a session at the Process Meeting held in Buffalo. The Subcommittee on Petroleum Utilization sponsored a Metropolitan Section Meeting on the problems of firing Bunker C Oil.

The Petroleum Division is planning to hold a national Meeting in Tulsa next May at the time of the International Petroleum Exposition.

Power Division held sessions at the Annual and Semi-Annual Meetings. It has two subcommittees, one on Central Station Power and the other on Industrial Power. The Industrial Power Committee has three special subcommittees, which will report at a luncheon meeting during the Annual Meeting. Several of the Society's special research committees sponsor sessions jointly with the Division.

Printing Industries Division held its national meeting in New York on March 14 to 15, presenting 11 papers. Several group conferences were held at that time, as a result of which special committees are being considered. The Division will hold a session at the Annual Meeting and plans to hold its next national meeting during the Semi-Annual Meeting in Chicago. The Division has affiliated with the Inter-Society Color Council to assist in encouraging progress in color measurement and standards.

Process Industries Committee held the first national Process Meeting in Buffalo, June 6–8, at which 11 papers were presented. At the Buffalo Meeting a Special Subcommittee on Cotton-Seed-Oil Processing made a report, as a result of which the Process group is now recommending that this subcommittee become a special research committee of the Society. At the Buffalo Meeting there were also sessions on pulverizing and dust problems. As a result of the interest shown, it was decided to organize a special committee on pulverizing and grinding to stimulate activities in that field with a possibility of later forming a special research committee. A special Subcommittee on Unit Costs has been organized for the purpose of analyzing unit cost operations in processing and also a Committee on Air Conditioning was formed. During the last year both the New York Section and the Buffalo Section have sponsored local informal process meetings. A request for the formation of a sanitary subcommittee is being considered. There is also a joint committee with the American Institute of Chemical Engineers to coordinate activities.

Railroad Division held sessions at the Semi-Annual and Annual Meetings and was joint sponsor with the Lubrication Engineering Committee of the Petroleum Division of a conference on railroad lubrication on February 19 in New York. A session is being arranged for the Chicago Semi-Annual Meeting. The Railroad Division is continuing the work of developing its organization so that the railway-mechanical engineer can utilize the information and data developed by the other professional divisions and also by the various technical committees of the Society. The railroad mechanical-department officer is primarily concerned with the maintenance of locomotives and cars. There are also railway-mechanical engineers who are engaged in the design and manufacture of rolling stock and equipment for railroad transportation. The registration in the Railroad Division is made up of mechanical engineers from both these groups. The mechanical engineer engaged in the maintenance of railroad rolling equipment has under his supervision machine shops, power plants, and forge and heat-treating shops, as well as work which pertains to practically every branch of mechanical engineering served by the other professional divisions. Thus, the Executive and General Committees of the Railroad Division have found it necessary to so organize their work and activities as to coordinate with and utilize the work of the Society as a whole.

Textile Division held a Southern Textile Meeting in Greenville, S. C., on October 19, with the Greenville Section and a New England Textile Meeting in Boston with the Boston Local Section on October 27. The meetings each consisted of one session and a dinner or luncheon. The Greenville meeting was held during the Greenville Southern Power Exposition. The Division held a session at the last Annual Meeting and will have one at this Annual Meeting.

Wood Industries Division will hold its national meeting in Jamestown, N.Y., on November 15–16, at which 14 papers are to be presented. This meeting will be held in cooperation with the National Association of Furniture Manufacturers. The Division has several subcommittees, one of which is a Survey Committee for research projects. This Committee will sponsor a session at the Jamestown meeting. Another subcommittee, on the Manufacture of Doors, will report at the Jamestown Meeting. The Wood Industries Division has reported at its meetings two special research committees, one on Saws and Knives and the other on Existing Supplies of Hard Woods.

Respectfully submitted,

Walter F. Dixon, *Chairman*
William A. Shoudy
Park T. Sowden
Clair B. Peck
Walter Samans.

LOCAL SECTIONS

Scope of Service

Local Sections have been urged to economize in every possible way with regard to their activities, but the use of some of the Society's funds has been necessary to meet the obligation of the Society through its Sections to participate in matters relating to the welfare of the membership. Especially is this true with regard to the collection of funds for relief, and the organization of committees to handle unemployment and of other groups to cooperate with Governmental agencies in the furtherance of construction projects which would require the employment of engineers. Likewise, the Sections have been urged to develop activity in the matter of uniformity in laws for the registration of engineers, so that the respective states in which they are located would adopt laws in accordance with the recommendations contained in the Model Law prepared by the Committee representing most of the leading national engineering societies.

Economic Operation of Local Sections to Meet Existing Conditions

When the budget for 1931–1932 was originally adopted in the spring of 1931, a tentative provision was made for financing Local Sections on the same basis as during the previous year. Subsequently, however, it was found necessary to reduce the appropriations to meet the decreased income. This resulted in a reduction for the Sections of $12^1/_2$ per cent of the original allotment. Great credit is due the Sections for the fine cooperation which they gave in effecting economies without impairing the quality of their meetings.

Meetings and Membership Chart

Collectively 434 meetings were held by the Local Sections during 1931–1932. The number of meetings held by each individual Section is shown in the accompanying chart (Table 1). This table also details the membership of the respective Sections on October 1, 1931, and October 1, 1932, the net increase, the percentage of membership increase, etc.

New Membership

The potential membership of the Society continues to be enormous. However, with industrial conditions as they exist, many, of course, who are interested are unable to afford the cost. Therefore, it has seemed unwise to make any drive for members. The membership development work has been maintained, however, through consistent quiet effort on the part of the individual Sections and through special attention to Professional Divisions, especially when any of them held a national meeting. The Committee believes that as soon as there is a noticeable upturn in business it will be wise to inaugurate a membership drive.

Prospective Sections

Few industrial areas of the United States remain to be provided with Local Sections. Ft. Wayne, Ind., Central Illinois (headquarters probably at the University of Illinois), Nashville, Tenn., and Central Wisconsin are interested.

In view of the need for pursuing economy, the Committee on Local Sections has felt that it was unwise to undertake the establishment of any new Sections during the past year.

Local Sections Visited by the Committee

Thirty-two Local Sections were visited during the past year by a member of the Local Sections Committee. (See Table 2.)

In 1930, after consulting with delegates from Group VII, the Committee decided to recommend the appointment of William Lyle Dudley of Seattle. This decision was based on two points: First, that it would give representation on the Standing Committee from the Far West, and second, it was agreed that in lieu of Mr. Dudley's attendance at three of the four annual meetings of the Committee he would travel within Group VII, so as to keep contact with the Local Sections and to some extent also with the Student Branches in that area. The Committee regrets that illness made it impossible for him to travel during 1932 but is glad that he has recovered sufficiently to plan future visits.

TABLE 1 MEMBERSHIP IN LOCAL SECTIONS, OCTOBER 1, 1932

Section	Membership, Oct. 1, 1931	Membership, October 1, 1932: Good standing	Suspended dues	Dues unpaid 9/30/32	Total	Increase or decrease	Per cent	Meetings, 10/31–10/32
Akron-Canton	170	111	18	41	170	...	....	8
Anthracite-Lehigh Valley	296	212	10	73	295	−1	−0.3	8
Atlanta	107	65	1	37	103	−4	−3.7	4
Baltimore	228	175	6	51	232	4	1.8	11
Birmingham	92	53	8	28	89	−3	−3.1	9
Boston	776	575	37	163	775	−1	−0.1	3
Bridgeport	198	131	7	57	195	−3	−1.5	11
Buffalo	240	180	9	65	254	14	5.9	5
Central Pennsylvania	112	88	3	19	110	−2	−1.8	1
Charlotte	40	34	2	8	44	4	10.0	2
Chattanooga	22	14	1	10	25	3	13.7	2
Chicago	1160	775	30	290	1095	−65	−5.6	9
Cincinnati	327	221	9	67	297	−30	−9.1	11
Cleveland	416	283	14	105	402	−14	−3.4	8
Colorado	97	79	5	16	100	3	3.0	9
Columbus	89	66	2	19	87	−2	−2.3	9
Dayton	119	91	4	29	124	5	4.2	4
Detroit	557	382	22	135	539	−18	−3.2	9
Erie	100	67	4	25	96	−4	−4.0	1
Florida	67	54	7	18	79	12	18.0	3
Green Mountain	48	38	1	10	49	1	2.1	1
Greenville	39	29	1	6	36	−3	−7.7	3
Hartford	142	108	...	33	141	−1	−1.7	12
Houston	148	106	10	37	153	5	3.4	6
Indianapolis	149	123	5	35	163	14	9.4	2
Inland Empire	23	19	1	5	25	2	8.6	7
Kansas City	143	115	7	31	153	10	7.9	7
Knoxville	52	30	1	19	50	−2	−3.8	4
Los Angeles	452	331	22	114	467	15	3.3	7
Louisville	39	35	...	11	46	7	19.0	...
Memphis	32	23	...	14	37	5	15.7	...
Meriden	32	23	3	6	32	...	....	3
Metropolitan	5060	3748	172	1150	5070	10	0.2	31
Mid-Continent	184	138	5	55	198	14	7.7	8
Milwaukee	297	218	8	54	280	−17	−5.4	6
Minneapolis	75	55	6	17	78	3	4.0	4
Nebraska	40	33	1	10	44	4	10.0	3
New Britain	46	37	3	10	50	4	8.7	1
New Haven	134	99	5	23	127	−7	−5.2	9
New Orleans	101	77	4	21	102	1	0.9	2
North Texas	72	61	3	23	87	15	20.8	3
Norwich	39	33	3	9	45	6	15.6	2
Ontario	191	134	13	36	183	−8	−4.1	10
Oregon	71	57	3	21	81	10	14.1	7
Peninsula	85	58	2	28	88	3	3.5	1
Philadelphia	1276	964	43	257	1264	−12	−9.4	10
Pittsburgh	608	426	22	123	571	−37	−6.1	1
Plainfield	321	241	17	40	298	−23	−7.2	12
Providence	212	168	7	29	204	−8	−3.8	11
Raleigh	27	22	3	8	33	6	22.2	4
Rock River Valley	79	54	6	22	82	3	3.8	8
Rochester	140	109	6	24	139	−1	−0.7	6
St. Joseph Valley	64	34	3	20	57	−7	−10.9	6
St. Louis	299	217	14	64	295	−4	−1.4	7
St. Paul	48	34	4	10	48	...	....	2
San Francisco	438	352	9	93	454	16	3.6	9
Savannah	21	15	...	6	21	...	....	1
Schenectady	244	184	10	42	236	−8	−3.3	7
Susquehanna	70	59	4	15	78	8	11.4	4
Syracuse	135	99	8	30	137	2	1.5	2
Toledo	95	65	4	21	90	−5	−5.3	12
Tri-Cities	78	60	7	12	79	1	1.2	9
Utah	37	23	4	9	36	−1	−2.7	7
Utica	34	25	...	9	34	...	....	2
Virginia	183	134	2	49	185	2	1.1	2
Washington, D. C.	202	183	8	31	222	20	9.9	7
Waterbury	95	75	1	16	92	−3	−3.2	9
Western Massachusetts	153	102	4	39	145	−8	−5.2	5
Western Washington	107	69	11	34	114	7	6.5	10
West Virginia	73	57	1	15	73	...	....	2
Worcester	191	155	10	22	187	−4	−2.1	7
Youngstown	82	63	2	20	85	3	3.7	6
	18,219	13,373	688	4094	18,155	−64	−0.3	434

Total membership in Society, Oct. 1, 1931, 19,979; Oct. 1, 1932, 20,057. Total number of Student Branches, 108.

EMPLOYMENT SERVICE

The Engineering Societies Employment Service maintained its full efficiency in spite of the falling off of income from contributions and subscriptions to the Employment Bulletin. The Finance Committee and Council, with full appreciation of the need to maintain the Employment Service, increased the appropriation for next year by approximately 25 per cent. There have been constant requests made to the Local Sections to join with the other engineering groups of their respective communities to foster the Employment Service. Partly as a result of this there have been developed in a number of cities special committees which are dealing with the unemployment problem. Some idea of the demands made upon the Service is indicated by the fact that in the New York office alone an average of 700 unemployed were interviewed each week during the month of August, 1932. It is the hope of the Committee that members of the Society will think first of its Employment Service when in need of technical assistance.

MID-WEST AND MID-CONTINENT OFFICES

The Mid-West Office has continued to assist the Local Sections and Student Branches within its territory in every possible way. Marked savings in operating expenses have been effected by cooperative speaking tours for Local Section and Student Branch meetings.

Efforts have been made to induce groups of members in districts where Local Sections do not exist to effect some sort of organization, if it is nothing more than a luncheon committee, for the purpose of developing professional consciousness and acquaintanceship with the problems of other engineers. Two cities in the Middle West offer excellent possibilities for the development of such committees during the coming season.

Considerable assistance has also been rendered by the Mid-West Office in unemployment relief work, and closer contacts established with industry and individuals in the interest of technical committee activity. The Mid-West Office is being approached to an ever greater extent for A.S.M.E. information and assistance.

The Mid-Continent Office has devoted special effort to organizing research committees for investigating and reporting on problems and developments in the oil industry, increasing public and society interest in A.S.M.E. undertakings, rendering assistance in outlining programs for general meetings and in securing men of national recognition for the presentation of papers at the meetings, tying in the four Student Branches with Sectional activities, providing prominent speakers for student meetings, and organizing a club for junior engineers in Tulsa.

At the present time three major research investigations are being conducted by the A.S.M.E. in this section at no extra cost to the Society; all equipment, materials, and investigators' time being furnished by manufacturers, companies in the industry, and the universities.

This office has also rendered assistance in unemployment relief, and has established itself as a valuable point of contact for the Society in that territory.

COOPERATION WITH 1931–1932 BUDGET COMMITTEE

The Committee on Local Sections has done everything possible to maintain the Sections at their present efficiency, despite curtailed funds. The Local Sections Department has reduced its expenditures without seriously restricting constructive work in the Sections. The Committee thoroughly appreciates the fact that the Sections form the links of the Society's chain, and is confident that adequate financial assistance will be tendered them as soon as conditions improve.

COOPERATION WITH STUDENT BRANCHES

Many Local Sections, realizing an important obligation to the Student Branches, have entered actively into the work and have arranged for an interchange of attendance at the Sections and Student Meetings. Many of the Sections have offered prizes to win the interest and active participation of undergraduates. Through the splendid cooperation of one of the Sections, the first District Student Branch Conference was held this year and proved to be one of the outstanding successes of the new Student Branch plan.

COOPERATION WITH THE PROFESSIONAL DIVISIONS

The Local Sections have availed themselves of every opportunity to cooperate with the Professional Divisions. The larger Sections, such as the Metropolitan and Philadelphia, have worked with the Professional Divisions in planning Section meetings, and other Sections have supported the Professional Divisions in their meetings and in other Division activities.

Elimination of Waste. Seven Sections cooperated with the Management Division in organizing campaigns and meetings on the subject of Elimination of Waste, as listed below.

Section	Date of Meeting	Subject
Detroit	May 4–5, 1932	National Meeting, Management, Materials Handling, Maintenance Divisions
Greenville	Apr. 7, 1932	Elimination of Waste in Industry
Hartford	June 8, 1932	Management of Soviet Plants
Los Angeles	Feb. 17, 1932	Some New Ideas on Economic Theory, with Reflections on the Old
Metropolitan	Jan. 6, 1932	Planning and Scheduling
	Jan. 21, 1932	Planning the Merchandise Line
	Feb. 3, 1932	Glorifying Manufactured Products with Color
	Feb. 17, 1932	Purchasing Power and Its Relation to Marketing
	Mar. 2, 1932	Factory Waste Elimination Campaign
	Mar. 16, 1932	Consumer Expenditures
	Apr. 6, 1932	Value of Graphic Charts in Production
	Apr. 20, 1932	Knowledge of Markets and Distribution and Its Relation to Production
	May 4, 1932	Effect of Motion Study on Plant Layout
St. Louis	Feb. 26, 1932	Management
Western Washington	Oct. 23, 1931	Industrial Management

Professional Division Meetings. Local Sections cooperated in the following Division Meetings held between September 1, 1931, and September 1, 1932.

Cleveland Aeronautic Meeting, September 4, 1931

Iron and Steel Division and Machine Shop Practice Division, cooperating in Metal Congress, Boston, Mass., week of September 21, 1931

New England Textile Meeting, Providence, R. I., October 6, 1931

National Wood Industries Meeting, Winston-Salem, N. C., October 15–16, 1931

National Printing Industries Conference, New York, March 14–15, 1932

Iron and Steel Meeting, Pittsburgh, Pa., April 28, 1932

Materials Handling, Management, and Maintenance Congress, Detroit, Mich., May 4–5, 1932

National Aeronautic Meeting, Buffalo, N. Y., June 6–7, 1932

National Process Meeting, Buffalo, N. Y., June 6–7, 1932

National Oil and Gas Power Meeting, Pennsylvania State College, June 8–11, 1932

Pacific Coast Aeronautic Meeting, University of California, Berkeley, Calif., June 9–10, 1932

Applied Mechanics National Meeting, New Haven, Conn., June 23–25, 1932.

TOURS AND TRANSPORTATION SERVICE

The Committee on Local Sections arranged for reduced fares to the Annual, Semi-Annual, and other National Meetings of the Society during the year, thereby saving thousands of dollars to the members of the Society.

In connection with the Semi-Annual Meeting held at Bigwin, Ontario, Canada, a tour of the St. Lawrence-Saguenay Rivers was planned but because business conditions resulted in only a few reservations for the trip, it had to be canceled. Several of those interested, however, took part in sections of this tour and their transportation was arranged for by the Headquarters Office.

TOUR OF THE INSTITUTION OF MECHANICAL ENGINEERS

Although the visit of about 125 members and ladies of the Institution of Mechanical Engineers of Great Britain was not handled entirely by the Local Sections, they cooperated, especially at Rochester and Schenectady, in the entertainment of the party. Also, in a number of individual cases they provided opportunities for these visitors to inspect plants in which they were especially interested.

FOREIGN RELATIONS

In addition to the cooperation given by the Local Sections in connection with the I.M.E. tour, 42 other visitors were received from nine foreign countries, as follows:

Australia	1	France	4
Austria	1	Germany	10
Belgium	1	Sweden	2
England	20	Turkey	1
	Switzerland	2	

Letters of introduction to various companies and plants throughout the United States were prepared for each visitor.

Of the 20 visitors from England 13 were students of the Liverpool University, Liverpool, England. These students were interested in

TABLE 2 LOCAL SECTIONS VISITED BY MEMBERS OF THE COMMITTEE ON LOCAL SECTIONS, 1931–1932

Section	J. M. TODD, *Chairman*	H. R. WESTCOTT	J. W. HANEY	W. L. DUDLEY	W. M. MACON	ERNEST HARTFORD, *Secretary*
Atlanta	Apr. 1932	....	....	....	Apr. 1932	Oct. 1931
Birmingham	Apr. '32, Oct. '32	....	....	....	....	Oct. 1931
Bridgeport	....	Dec. 1931	....	....	....	....
Buffalo	....	....	....	....	....	Apr. 1932
Charlotte	....	....	....	....	....	Oct. 1931
Chattanooga	Apr. 1932	....	Apr. 1932	....	Apr. 1932	Apr. 1932
Chicago	....	....	June 1932	....	....	Oct. 1932
Cincinnati	....	....	....	....	....	Oct. 1931
Cleveland	....	....	....	....	Sept. 1932	....
Dayton	....	....	....	....	....	Apr. 1932
Greenville	....	....	....	....	Apr. 1932	Oct. 1931
Indianapolis	Oct. 1932	....	....	....	....	....
Louisville	Oct. 1932	....	....	....	....	Oct. 1931
Memphis	Oct. 1932	....	....	....	....	....
Metropolitan	Dec. 1931*	Dec. 1931*	Dec. 1931*	Dec. 1931*	(Resident)	(Resident)
Milwaukee	....	....	Sept. 1932	....	....	Oct. 1932
Minneapolis	....	....	Sept. 1932	....	....	....
Nebraska	....	....	(Resident)	....	....	....
New Haven	....	(Resident)	....	....	....	....
New Orleans	(Resident)	....	....	....	....	Oct. 1931
Ontario	June 1932	Oct. 1932	June 1932	....	June 1932	June 1932
Rochester	....	Sept. 1932	....	....	....	Apr. 1932
Rock River Valley	Oct. 1932	Oct. 1932	Oct. 1932	....	Oct. 1932	Oct. 1932
St. Joseph Valley	....	....	....	....	....	Oct. 1932
St. Louis	Oct. 1932	June 1932	....	....	....	....
St. Paul	....	....	Sept. 1932	....	....	....
Syracuse	....	Sept. 1932	....	....	....	Apr. 1932
Toledo	....	....	....	....	Sept. 1932	....
Utica	....	....	....	....	....	Apr. 1932
Virginia	....	....	....	....	....	Oct. 1931
Waterbury	....	May 1932	....	....	....	May 1932
Western Washington	....	....	....	(Resident)	....	....

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* Annual Meeting.

the following phases of engineering, and letters to plants were given accordingly:

Civil Engineering	2	Mechanical Engineering	4
Electrical Engineering	4	Mining Engineering	1
General Engineering	2		

There has also been an opportunity to serve more of our members than heretofore by giving them letters of introduction which aided them in their visits to plants in the United States and to European countries.

During the past year letters of introduction have been written for 33 members of the Society. Thirteen of these were members going abroad and letters were given to foreign plants, companies, and societies according to what countries were to be visited. Letters for these members were issued to sixteen foreign countries.

The other 20 members were furnished with introductions from headquarters, to plants in the United States.

In each instance a copy of the letter of introduction was mailed in advance to the person addressed in the letter.

Respectfully submitted,

James M. Todd, *Chairman*
Harry R. Westcott — William Lyle Dudley
Jiles W. Haney — W. W. Macon.

MEMBERSHIP

The Committee on Membership held 13 regular meetings and 12 by proxy during the fiscal year 1931–1932.

The number of applications considered in the transaction of its work and a summary showing the action taken follow:

Applications pending, October 1, 1931	218
Applications received during fiscal year 1931–1932	1379
Total applications handled during year 1931–1932	1597
Recommended for membership	1302
Transfers denied	3
Deferred	3
Withdrawn	34
Applications pending, October 1, 1932	255
Total applications handled during year 1931–1932	1597

Those recommended for membership were divided into the following grades:

Members	166
Transfers to Member	80
Associates	11
Transfer to Associate	1
Associate-Members	112
Transfers to Associate-Member	98
Juniors	190
Juniors R5-Rule 1 (Southern Group)	184
Juniors R5-Rule 1	460
Total recommended	1302
Transfers	179
Total new members recommended	1123

During the fiscal year 1931–1932 the Membership Committee made the following recommendations:

Elections declared void	98
Resignations accepted	380

Respectfully submitted,

H. W. Butler, *Chairman*
H. Webster — H. A. Lardner
O. E. Goldschmidt — R. H. McLain.

AWARDS

During the past year the Committee has procured the dies for the Warner Medal, executed by Julio Kilenyi. The Committee has had the hearty cooperation of Mrs. Worcester R. Warner in this work and has her approval of the final medal.

The Committee recommended that the Council approve the Charles T. Main Award to Marshall Anderson of the University of Michigan for his paper on "Apprenticeship and Vocational Training." This was deemed the best paper of the seven submitted for the award.

The Committee recommended that the student award be given to H. E. Foster, Jr., of the University of Tennessee, for his paper on "Factors Affecting Spray Pond Design." Mr. Foster is an undergraduate student.

The Committee also recommended for the student award William A. Mason of Stanford University for his paper on "An Experimental Investigation of the Flame Propagation in Internal-Combustion Engines." Mr. Mason is a graduate student.

Altogether, there were 35 student papers presented by graduates and undergraduates, seven of which were by joint authors.

The Committee recommended for the Junior Award Edward M. Wagner of Stanford University for the thesis on "Frictional Resistance of a Cylinder Rotating in a Viscous Fluid Within a Coaxial Cylinder." Five papers were presented to the Society for the Junior Award.

The Committee recommended for the Melville Award Alexey Stepanoff for his paper on "Leakage Loss and Axial Thrust in Centrifugal Pumps." This was selected from three papers offered for the Melville Award.

Respectfully submitted,

Arthur M. Greene, Jr., *Chairman*
Kenneth H. Condit — William L. Batt
Frank L. Eidmann — David B. Rushmore.

CONSTITUTION AND BY-LAWS

The Committee on Constitution and By-Laws is now engaged in the study of the By-Laws and Rules with a view to their revision and possible condensation and expects to make a definite report on its findings later in the year.

Respectfully submitted,

Thos. C. McBride, *Chairman*
R. S. Neal — P. R. Faymonville
H. H. Snelling — Jas. A. Hall.

RELATIONS WITH COLLEGES

The Committee on Relations with Colleges inaugurated the new Student Branch policy in the 15 colleges in Group IV, in the Southeastern States, in October, 1931. Messrs. Maxwell or Gordon of the Committee and also its secretary visited the Branches at the University of Virginia, Virginia Polytechnic Institute, University of North Carolina, North Carolina State College, Clemson College, Georgia School of Technology, University of Florida, Alabama Polytechnic Institute, Mississippi State College, Louisiana State College, University of Alabama, University of Tennessee, Vanderbilt University, University of Louisville, and University of Kentucky. A full day was spent at Chattanooga with the officers of the Local Section, arranging plans for the first Student Conference to be held in April, 1932.

In the majority of the colleges both the faculty and student body enthusiastically approved the adoption of the new policy, but there were a few cases where local conditions made necessary somewhat extended conferences to adjust the procedure to meet existing regulations. Successful adoption was finally achieved in every instance with the result that the trip was considered to have been well worth the time, effort, and money expended on it.

Wherever possible throughout the trip, the representatives of the Committee on Relations with Colleges met with the Local Sections groups and enlisted their aid in the installation and functioning of the new Student Branch plan.

Southern Student Branch Conference

The first Southern Student Branch Conference was held at Chattanooga, Tenn., April 3–5, 1932, with headquarters at the Hotel Patten. Mileage was paid for a student delegate from each Branch. More than 100 students and 31 engineers registered, and in addition 40 students from the University of Tennessee participated in the plant visits but did not attend the meetings. There were meetings of two Standing Committees, the Committee on Relations with Colleges and the Committee on Local Sections, and also a joint meeting of these Committees.

Two splendid technical sessions were held, at which papers were presented by Student Members from each of the fifteen southern Branches. The George Hillyer, Jr., Award (fifty dollars in gold), inaugurated through the generous gift of his widow, was awarded for the first time to J. C. Whitehurst of North Carolina State College for his paper entitled "Practical Economics, a Necessity for the Engineer." Two other prizes were made possible through contributions from several Southern Local Sections, augmented by funds available for the meeting. Twenty-five dollars in gold was awarded to R. H. Hughes of Clemson College for his paper on "The Usefulness of Dynamite to the Engineer" and ten dollars in gold went to W. W. Knight of Georgia Institute of Technology for his contribution, "Light for Radiation End." Other papers presented by Student Members were as follows:

"Progress of Safety in Aviation," J. C. Barrett, Jr., Alabama Polytechnic Institute

"Progress in Industrial Management," E. R. Owens, University of Alabama

"Engineering Safety in Transportation," R. S. Thompson, University of Florida

"Diesel Engines," W. L. Albert, University of Kentucky

"Steam Distillation of Wood Naval Stores Products," R. M. Dreyfus, Louisiana State University

"The Modern Diesel Engine," F. S. Cornell, University of Louisville

"Procedure Control of Pipe Line Welding With Acetylene Torch," T. H. Suttle, Mississippi State College

"The Deceleration Method of Testing for Losses and Moment of Inertia of a Turbo-Generator," E. L. Midgett, University of South Carolina

"Design and Construction of Spray Ponds," H. E. Foster, University of Tennessee

"Carrier Centrifugal Refrigeration," J. P. Turner, Vanderbilt University

"Fuels for Internal-Combustion Engines," S. C. Smith, Virginia Polytechnic Institute

"Bremo Bluff—an Outstanding Central Station," W. D. Thornhill, University of Virginia.

Two formal luncheons and a banquet were also part of the meeting, as well as a breakfast attended by Honorary Chairmen and faculty members of the Student Branches.

Nine plants in Chattanooga and vicinity opened their doors for inspection trips and local engineers were generous in providing private cars for these excursions, as well as for a fifty-mile trip to points of historic and scenic interest.

At the meeting of the Committee on Relations with Colleges held during the Chattanooga Student Conference, the many details regarding the operation of the new plan were discussed and a course of action decided upon. The Committee voted to recommend to the Council that the plan be continued in the fifteen southeast Student Branches, and be extended during the 1932-1933 school year to include Student Branches in the following geographical groups:

Group I, New England and New York State outside of the Metropolitan area; Group II, Metropolitan areas of New York and Philadelphia, also Bethlehem and Easton, Pennsylvania; Group V, Indiana, Illinois, Michigan, Wisconsin, Iowa, Minnesota, and St. Louis, Missouri.

EXTENSION OF NEW STUDENT PLAN

Chairman Maxwell presented to the Council at the Bigwin Meeting a report on the success of the new Student Branch plan and its recommendation for the extension of the plan next year. The Council approved this recommendation and the Committee began (in September, 1932) to visit the Student Branches in the new territory of Group V. The Student Branches visited up to the end of the fiscal year received and accepted the new plan in the same fine spirit which had been evidenced in the Southeast at the beginning of the year.

There are at present 16 Student Branches in Group I, 17 in Group II, and 15 in Group V. Messrs. Maxwell and Church volunteered to visit the Student Branches in Groups I and II and Messrs. Abbott and Prentice those in Group V. Mr. Gordon will continue his special interest in Group IV.

Membership Under Old and New Policies. The following table gives the actual enrolment in Student Branches under the old plan and the new. The figures given for Group IV are actual, but those for the other groups are actual figures only for before the establishment of the new policy, the figures under "after" being estimated on the basis of the Group IV figures. This basis seems justified, inasmuch as the average student attending college in the South is, without doubt, no better off than the average student attending institutions in the other three groups.

	Before	After
Group IV—Alabama Polytechnic Institute	16	7
University of Alabama	19	28
Clemson College	19	34
University of Florida	20	19
Georgia Institute of Technology	83	43
University of Kentucky	39	37
Louisiana State College	33	19
University of Louisville	13	17
Mississippi State College	24	32
North Carolina State College	31	46
University of North Carolina	16	16
University of Tennessee	15	19
Vanderbilt University	31	23
Virginia Polytechnic Institute	35	21
University of Virginia	11	16
Group IV—Total	405	377
Group I—	737	679
Group II—	924	840
Group V—	927	853
Total	2993	2749

Expenditures Under New Policy. The original budget granted by the Council to take care of the inauguration of the new policy among the Student Branches in Group II was $2850. This was later reduced to $2500 by the Finance Committee. Up to June 1 $2007.84 had been expended. This expenditure would have been higher but for the fact that the Branches called for only $262.50 of the $375 which was allotted to them for local operation. The net amount expended by the Society outside of headquarters administration was $469.84. Administration expenses were decreased, in spite of the additional work required because of the installation of the new policy in Group IV.

The estimated enrolment involving 63 Student Branches in the new policy in the next year will be approximately 3000. Therefore the Committee has made a survey upon which it bases its recommendation that *Mechanical Engineering* be included in the new policy to the student paying $3.00 annual dues. The Committee found that this could be done within the funds received on the basis of 3000 Student Members if the advertising pages were omitted. It was the Committee's opinion that the advertising section could be omitted without detriment to the Society's income from advertising, providing a special cover was used on the copies distributed to Student Members. However, the Committee welcomed the cooperation of the Publications Committee, which desires to include the advertising pages. The Committee wishes to point out that if this is done, it should of course build for good-will among our advertisers; and even if it does not warrant an immediate increase in advertising rates, it will at least help to postpone the necessity for reducing advertising rates. Likewise it will accelerate the date when an increase in advertising rates may be made effective, consistent with improvement in the general industrial conditions of the country.

The Committee on Relations with Colleges likewise recommended that there be established a special two-page Student Branch supplement for insertion in the *A.S.M.E. News*. It suggests that this college news insert be printed in sufficient quantity to take care of the student enrolment, plus members of Council, faculty members, and other members of the Society who are especially interested in the progress of the Student Branches and who request it. It is recommended that the supplement be shipped in bulk with the copies of the *News* to each Student Branch Honorary Chairman. In this way no complications will have to be overcome regarding second-class postal regulations.

The Committee was granted a special budget of $9000 for the year 1932–1933 with which to promote the new policy in Groups I, II, and V.

General Conditions. In the 108 Student Branches there were enrolled a total of 4754 students during 1931–1932. These Branches reported a total of 896 meetings, and all meetings held may not have been reported. Local Sections have increased their interest in and support of Student Branch activities and many of them have held special meetings for Student Branch Members and offered prizes for competition within the Student Branches in their respective territories. The increased number of Student Members being transferred to Junior Membership from the colleges in Group IV was such as to nearly balance the total number of applications acted upon by the Membership Committee as compared with last year, as shown in the report of that Committee.

The many benefits accruing to the Student Branch movement because of the extensive visits made during the previous year by President Roy V. Wright, were clearly indicated in the fine manner in which the Student Branches functioned throughout the country.

The Committee wishes to record its appreciation of the intimate interest and cordial support which members of the Council and officers of the Society have given to this very important phase of the Society's activities.

Respectfully submitted,

Maxwell Carson Maxwell, *Chairman*
Donald B. Prentice William L. Abbott
E. F. Church, Jr. Lewis F. Gordon.

EDUCATION AND TRAINING FOR THE INDUSTRIES

Objective. The Committee on Education and Training for the Industries emphasizes the importance of organized instruction within the industry, for foremen and workmen, and the coordination of that training with existing public and private agencies for improving the skill of industrial workers.

Methods. By answering inquiries from individuals and firms, by sending literature, by programs suggested to Local Sections, and by organized discussion at meetings of the Society, the Committee has disseminated information and provoked discussion of means and methods by which better training for workers can be provided. Correct methods save time, materials, and machines.

The papers presented at meetings have had a wide discussion by and circulation to the membership of the Society and to those outside.

Programs. The program presented at the Annual Meeting, December 3, 1931, jointly with the Management Division, included opening remarks by Erik Oberg, representing Roy V. Wright, President of the Society, and two papers, one on "Foreman Training," by U. J. Lupein, of Cheney Brothers, and the other on "Modern Industrial Training," by George Saylor, of the Lunkenheimer Company.

The following program has been prepared for the Annual Meeting, 1932: "Foreman Training in the United States," by E. S. Cowdrick, Standard Oil Company of New Jersey; "The Fundamental Principles of Training in Industry," by G. Guy Via, supervisor of training, Newport News Shipbuilding & Dry Dock Co.; "Survey of Adult Technical Education in the Greater New York Industrial Area," by O. W. Eshbach, Personnel Department, American Telephone & Telegraph Co.

There has been a very satisfactory response to a letter which was sent to each Local Section chairman, giving him information concerning the purposes of the Committee and offering assistance in preparing programs on subjects in the field of training for the industries which were appropriate to the industries in his vicinity.

Respectfully submitted,

R. L. Sackett, *Chairman*
C. F. Bailey John T. Faig
S. S. Edmands Harold S. Falk.

LIBRARY

The Library Committee represents the Society upon the directorate of the Engineering Societies Library, which is owned jointly by this Society, the American Society of Civil Engineers, the American Institute of Mining and Metallurgical Engineers, and the American Institute of Electrical Engineers.

The statistical and fiscal year of the library is the calendar year. This report, therefore, is based upon the operations of 1931.

During that year more than 42,000 members made use of the library. Thirty-six per cent of these were non-visitors, who relied upon the staff to supply the desired information. The number of members visiting the library, and the total number assisted, both exceeded all previous records.

The current year shows continued increase in visitors. It is obvious that many members are employing enforced leisure in professional study and that the professional library is finding, as public libraries are, that it has a useful function in counteracting the depressing influence of unemployment.

Almost eight thousand books were added during 1931, and the collection, now containing more than 145,000 books and pamphlets, is one of the largest engineering libraries in existence.

The income of the Library Board has been barely sufficient to carry on with efficiency the lines of work that have been developed in previous years. New developments which seem desirable must await more favorable times.

The Chairman of the Library Committee, who served as Chairman of the Library Board during 1931, was reelected to that position for 1932.

Respectfully submitted,

Alten S. Miller, *Chairman*
Geo. F. Bateman E. P. Worden
W. M. Keenan Calvin W. Rice.

RESEARCH

Relying, as it does to a great extent, on the support of industry, it was only natural to expect that the Society's research activities should fall off during the past year owing to the business depression. Nevertheless, the receipts and disbursements of the special and joint research committees were only 20 per cent below those of last year, a far better record than was enjoyed by industry as a whole. This record is an indication also of industry's belief in the value of research in poor times as well as in good times.

Financial Record

Receipts from industry and other sources outside the Society have totaled $40,500 during the year, while disbursements were approximately $44,000. The difference was made up from accumulated balances. Of the total contributions received, however, $5,000 came from The Engineering Foundation, the joint agency of the Founder Societies which provides funds for the aid of their research projects. The Society itself spent about $10,000 in administrative and secretarial support of this cooperative research work. The ratio between outside contributions and Society contributions was, therefore, four to one—better than in the previous year.

Some 25 technical societies, trade associations, and government bureaus cooperated with the committees as joint sponsors, financial supporters, and collectors of technical data.

Committee Activity

The special and joint research committees totaled 28 at the end of the year. Two, the Joint Research Committee on Welding of Pressure Vessels and the Special Research Committee on Airplane Vibration with Special Reference to Instruments, completed their work and were discharged. One new committee was set up, with the Society of Automotive Engineers and the A.S.M.E. as joint sponsors, to study combustion in Diesel engine cylinders. A total of 460 men, representative of manufacturers, users, and independent interests, are serving on these committees, 50 per cent of whom are non-members of the Society. It seems fair to say that the value of time spent and traveling expenses incurred by these men in volunteer committee work far exceeds the direct money contributions received by the committees. Forty-four committee meetings were held during the year.

The pressure of business duties unfortunately made it necessary for F. C. Biggert, vice-president, United Engineering & Foundry Co., to resign his membership from the main Research Committee, which supervises the Society's research activities for the Council. G. M. Eaton, director of research, Spang-Chalfant & Co., Inc., and long an active member of the Society, was appointed by President Lauer in Mr. Biggert's place.

Experimental Work

The contributed funds are spent chiefly on experimental work in various institutional laboratories throughout the country. It was necessary to drop only five men from the payrolls during the year, a total of twenty men being maintained on whole or part time at nine laboratories. These are: Bureau of Standards, Washington, D. C., and Columbus, Ohio; Battelle Memorial Institute; Massachusetts Institute of Technology; University of Michigan; University of Illinois; Ohio State University; Union College; and Carnegie Institute of Technology. The additional number of laboratories—university, institutional, and industrial—cooperating with the committees without receiving financial support from them are too numerous to list here. According to a list recently published by the National Research Council, the A.S.M.E. is one of the principal supporters of research fellowships in the research institutions of the United States.

Publications

The progress reports and related papers growing out of the committees' activities covered some 320 pages of *Mechanical Engineering* and the Transactions; a list of these will be found in the appendix to this Report. This material is being bound into a new section of the Transactions called "Research Papers," the publication of which was authorized by the Council during the year. These "Research Papers" will not be distributed to the members of the Society but will be included in the bound volume of Transactions. This collection of papers will, however, be offered for sale under a separate cover as an A.S.M.E. Research Publication. It will take the place of the "A.S.M.E. Research Reports and Papers" which have been published for sale over the past few years.

In addition to this series of Research Papers another research publication was produced under the title "Report of Tests on Steam Equipment for Drilling Rotary-Drilled Oil Wells." This pamphlet reports the result of one committee's work in the oil fields. In addition, the Society published, jointly with the American Society for Testing Materials, the "Symposium on the Effect of Temperature on the Properties of Metals." This volume of 800 pages contains the papers with discussion presented at the Symposium held in Chicago in June, 1931, under the auspices of the Joint A.S.M.E.-A.S.T.M. Research Committee dealing with this problem.

The Bibliography on Riveted Joints has been completed by A. E. R. de Jonge, who has been making an exhaustive search of the literature on the subject during the past three years for the main Research Committee. Some 1350 references have been prepared, most of which have been fully abstracted. Publication is being delayed owing to lack of funds.

Items of current interest on the activities of the various committees have appeared from time to time in the *A.S.M.E. News* and *Mechanical Engineering*.

INTERNATIONAL COOPERATION

Contacts with foreign groups working on similar problems have been maintained and developed further by our committees during the year. This has been accomplished through the exchange of reports, correspondence, and personal visits by individual representatives. The Institution of Mechanical Engineers in Great Britain and the Verein deutscher Ingenieure in Germany act as the coordinators in their respective countries. Recent developments give promise for the setting up of similar contacts with French workers.

SPECIAL AND JOINT COMMITTEE REPORTS

Lubrication. (1)* A. E. Flowers, Chairman. The experimental work on journal bearing problems at the Bureau of Standards was terminated about the middle of January owing to lack of funds. A report on this investigation was presented at the 1931 Annual Meeting by S. A. and T. R. McKee, entitled "Pressure Distribution in Oil Films of Journal Bearings." A paper, "Optimum Conditions in Journal Bearings," by Albert Kingsbury, a member of the Committee, was presented at the same meeting.

The mathematical studies on viscosity at high pressure were completed and the results were published in the *Journal of Rheology*, July, 1932, under the title "High-Pressure Capillary Flow," by M. D. Hersey and G. H. S. Snyder.

Fourteen original papers on lubrication, many of which are out of print or buried in various publications, have been assembled and edited preparatory to republication.

Fluid Meters. (2) R. J. S. Pigott, Chairman. The Committee has progressed with the revision of the fourth and final edition of "Fluid Meters, Part 1, Theory and Application" and the manuscript of Part 3, on "Influence of Installation" is now receiving final review before publication.

Early in the year, a joint committee, composed of members of the Fluid Meters Committee and of a similar committee of the American Gas Association, was organized to harmonize existing values of orifice coefficients. This joint committee sponsored a series of tests at the Ohio State University under the supervision of Prof. S. R. Beitler. The object of these tests was to determine the variation of the coefficient of discharge of sharp-edged concentric orifices with pipe size and for the various types of pressure taps. These tests have been completed and the results are expected to clear up existing discrepancies in the coefficients.

The subcommittee formed in the Mid-Continent area last year to determine the accuracy of existing coefficients and correction factors for the flow of oil through orifice plates, flow nozzles, and venturi tubes at varying temperatures and viscosities has been active during the present year. Equipment amounting in value to approximately $7,000 has been contributed by industry for these tests. This apparatus has been set up at the University of Oklahoma and the experimental investigation will be started in October under the supervision of Prof. W. H. Carson.

Eight committee meetings were held during the year, and the Committee contributed two papers at the Fluid Meters Session which it sponsored at the 1931 Annual Meeting of the Society. These were "A Study of Primary Metering Elements in 3-In. Pipe," by S. R. Beitler, P. Bucher, and T. C. Barnes, Jr., and "Flow of Steam Through Square-Edged Orifices in a 4-In. Line," by W. W. Frymoyer and A. Herschel.

Thermal Properties of Steam. (3) Alex Dow, Chairman. The Third International Steam Tables Conference scheduled to be held in America in September, 1932, was postponed because the economic conditions made it impossible for the foreign cooperating organizations to participate.

Dr. F. G. Keyes, at Massachusetts Institute of Technology, reports that his experimental work on specific volumes is completed and that correlation of results is proceeding satisfactorily.

At the Bureau of Standards, tests have been going on, under the direction of Dr. N. S. Osborne, with a new steel calorimeter in the range between 800 lb per sq in. and the critical point. He is measuring the total heat of the liquid, the latent heat, and the pressure-temperature relationship for saturated steam.

Doctor Osborne has also completed a series of measurements on vapor pressures practically to the critical point and has prepared a report for presentation at the 1932 Annual Meeting. He has correlated his results with the data of Doctor Keyes and those which have been recently reported by the British and German workers, and has paved the way for a further revision of the international skeleton tables.

Five reports were presented by the Committee at the Steam Tables Session which it sponsored at the A.S.M.E. 1931 Annual Meeting. A similar session will be conducted next December.

Owing to the effective appeals of Chairman Dow, industry has continued financial support of this Committee, thus enabling it to carry forward its program without interruption. Nearly $150,000 has been spent on this project to date.

Strength of Gear Teeth. R. E. Flanders, Temporary Chairman. A series of tests on the Lewis Gear Testing Machine, designed by the committee and temporarily located at Massachusetts Institute of Technology, of the surface fatigue limits of gear materials have been conducted throughout the year by Prof. Earle Buckingham. A machine to facilitate the measurement of surface endurance by the use of small rolls as samples instead of gears is now being constructed in one of the cooperating commercial laboratories.

Professor Buckingham is ready to proceed with studies of dynamic loading on helical gears when samples are available. It is contemplated that these tests on helical gears will contribute further data to those recently obtained by a study of spur gears. He is continuing also a further study of the mechanical phases of wear.

Cutting of Metals. (4) F. C. Spencer, Chairman. During the year this Special Research Committee has been reorganized, and is now composed of the chairman, the secretary, and the chairmen of the four subcommittees.

The experimental program on cutting fluids under Prof. O. W. Boston at the University of Michigan has been stopped for lack of funds. A progress report, entitled "Performance of Cutting Fluids in Drilling Various Metals," by O. W. Boston and C. J. Oxford, will be presented at the 1932 Annual Meeting.

The Subcommittee on Metal Cutting Materials has been continuing its study and correlation of data regarding the new cutting alloys secured through answers to a questionnaire. A report on the types of these alloys available and the extent of their application was presented at the 1931 Annual Meeting. The following three papers, dealing with various phases of the subcommittee's program, have been prepared for presentation at the 1932 Annual Meeting: "Effect of Lathe Cutting Conditions on the Hardness of Carbon and Alloy Steels," by T. G. Digges; "Grinding Cemented Tungsten and Tantalum Carbide Tipped Tools Efficiently and Economically," by J. M. Highducheck; and "What Can Be Accomplished With Modern Machine Tools and Cemented Carbide Cutting Tools," by A. A. Merry.

The manuscript of the "Bibliography on Milling," compiled for the Committee at Purdue University by Prof. J. D. Hoffman, has been completed and is now in the hands of the Society for publication. It contains 200 selected and abstracted references covering the period 1904–1931.

A Subcommittee on Metal Cutting Data has been organized, with King Hathaway as chairman, to make accessible to men engaged in machine shop operation reliable data in conveniently usable form as to the proper feeds and speeds of machine tools in the cutting of various metals. This subcommittee, with the assistance of Carl

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*** See Appendix to this Report.**

Barth, plans to develop a series of practical handbooks covering drilling, milling, engine lathe, planer work, etc. These books will be based upon the ideas advanced by F. W. Taylor in his "Art of Cutting Metals."

Two papers, "The Elements of Milling," by O. W. Boston and C. E. Kraus, and "Tungsten-Carbide and Other Hard Cutting Materials," by Coleman Sellers, 3d, were contributed by the Committee for the Metal Cutting Session sponsored jointly by the Committee and the Machine Shop Practice Division at the 1931 Annual Meeting. The Committee will sponsor, jointly with the same Division, two sessions at the 1932 Annual Meeting; the four papers already mentioned have been prepared for these sessions.

Mechanical Springs. (5) J. R. Townsend, Chairman. The experimental program of the Committee on Mechanical Springs which has been supervised by Prof. M. F. Sayre at Union College has been completed. This study covered the elastic behavior of spring materials. He will submit a proposed new code of design for springs, based on this work, at the 1932 Annual Meeting.

At the Bureau of Standards, Dr. D. J. McAdam, with the assistance of a young engineer whose services were contributed by one of the steel companies, is completing a survey and digest of all available information, published and unpublished, on the fatigue and other mechanical properties of all known spring materials. This work, which was begun for the Committee by Doctor McAdam while at the U. S. Naval Experiment Station at Annapolis, has recently contributed much useful data.

During the year, Prof. J. B. Reynolds, at Lehigh University, has been conducting an investigation to determine the eccentricity effects and stress effects on coiled springs resulting from different types of end restraint.

Several industrial laboratories are cooperating with the Committee in tests of the fatigue life of large size helical springs as affected by the grade or analysis of material, the heat treatment applied, the proportions of the springs, and other critical variables. A similar program on wire springs is under way.

From month to month Dr. W. G. Brombacher has continued to compile and distribute to committee members and financial supporters a bibliography on the current literature on springs.

Reports on the Committee's work were presented at an open meeting of the Committee held during the 1931 Annual Meeting, and there will be further reports at a Mechanical Springs Session during the 1932 Annual Meeting. Four committee meetings have been held during the past year.

Effect of Temperature on the Properties of Metals. (6) (Joint Committee with American Society for Testing Materials.) H. J. French, Chairman. This Committee has been successful in raising 60 per cent of the $20,000 fund needed to support certain special experimental work in institutional laboratories. Accordingly, a research was started at the University of Illinois under the direction of Prof. H. F. Moore to determine the endurance properties of austenitic chromium nickel steels at elevated temperatures. These tests will be coordinated with creep tests being made at the Battelle Memorial Institute under Dr. H. W. Gillette. The purpose of this investigation is to determine whether the limiting stresses, for design purposes, are the fatigue stresses or creep stresses, and to determine the temperature ranges within which these considerations are important. The amount of work being done in the industrial laboratories which are cooperating with the committees on similar studies, has been somewhat reduced because of the business depression. Several useful reports on the results of this cooperative work have, however, been made and were presented at the June meeting of the Committee held in connection with the Annual Meeting of the American Society for Testing Materials.

A plan is in operation by which important articles relating to the effect of temperature on the properties of metals are abstracted regularly and published monthly under a separate heading in the abstract section of *Metals and Alloys.* This provides continuous contact with the progress made in the United States and foreign countries as recorded in the technical literature.

The sponsor societies published in March an 800-page volume containing the papers and discussion presented at the Symposium on Effect of Temperature on the Properties of Metals which was conducted by the Committee during the June, 1931, Annual Meeting of the A.S.T.M.

Condenser Tubes. (7) A. E. White, Chairman. Progress Report No. 5 of the Committee was prepared by Bert Houghton, Vice-Chairman of the Committee, and was presented at the 1931 Annual Meeting. It included results of a comparison of the service life of certain alloys; a report on marine practice embodying the experience of a large steamship company as to the effect of air and other entrained gases on tube corrosion; a discussion of end corrosion; condenser water box and pump design; and some interesting cases of tube failures.

Owing to lack of funds to defray the expenses of an investigator to institute and supervise a field study of condenser tube corrosion, the Committee is attempting to carry on its work by correspondence. To this end a questionnaire has been circulated among the larger power plants throughout the country requesting information regarding the effect of entrained air and other gases in the circulating water, the hydrogen-ion concentration of the water, velocity of the water through the tubes, and the kind of tube material used, so that the effect of these factors upon condenser tube corrosion may be coordinated.

Boiler Feedwater Studies. (8) (Joint Committee with the American Boiler Manufacturers' Association, the American Railway Engineering Association, the American Society for Testing Materials, the American Water Works Association, and the National Electric Light Association.) S. T. Powell, Chairman. The experimental programs initiated last year at the Ohio State University and the University of Michigan have been continued by this Committee with the financial aid of The Engineering Foundation. Prof. C. W. Foulk is conducting the investigation at Ohio State University on priming and foaming of boiler waters, and Prof. A. H. White at the University of Michigan is directing a careful study of all known methods of boiler-water analysis with a view to recommending tests suitable for referee and control purposes. Both investigations are being carried on as research fellowships. Progress reports were presented at the A.S.M.E. 1931 Annual Meeting and also at meetings of some of the other sponsor societies during the year.

The Committee is now laying plans for an experimental study of sodium sulphate solubility equilibria in an effort to throw more light on the general problem of embrittlement.

Boiler Furnace Refractories. W. A. Carter, Chairman. Close to $50,000 has been raised and spent by this Committee during the past seven years. Experimental work has been financed throughout the present year at the U. S. Bureau of Standards Ceramic Station, Columbus, Ohio. The University of Illinois and Battelle Memorial Institute have continued their cooperative programs.

At the University of Illinois laboratory slag tests on a series of eight representative fireclay bricks and six representative coal ashes have been completed under Prof. R. K. Hursh's direction. A new design of the slag testing furnace was developed and the furnace is now being rebuilt.

At the Bureau of Standards laboratory slag tests on a special series of fireclay bricks and two coal ashes to check the results obtained at the University of Illinois have been carried out. Work has steadily progressed also in the development of the equilibrium diagrams for silica, alumina, lime, and iron oxide. T. A. Klinefelter directs the work at this laboratory.

Battelle Memorial Institute brought its laboratory slag test program to a close during the year. Dr. H. E. Simpson reported the results in a paper read before the American Ceramic Society in February, 1932.

Elevators. M. H. Christopherson, Chairman. Tests on elevator safeties were continued at the Bureau of Standards under the Research Fellowship established by the Committee.

Buffer tests, as required by the American Standard Safety Code for Elevators, are now being carried out at the Bureau by the Committee's Research Fellows. The Committee held eight meetings during the year to direct this work. The elevator industry has contributed to date $80,000 to cover the expenses of this investigation.

Worm Gears. Earle Buckingham, Chairman. This study of the nature and influence of contact conditions on capacity and efficiency of worm gear drives has been carried on in cooperating industrial laboratories. Copies of the resulting data are sent the Chairman for study and analysis. Business conditions, however, have retarded the work materially. A special testing machine for worm gears, similar in principle to the Lewis Spur Gear Testing Machine, has been designed by the Chairman and built by a cooperating company, in whose plant it is now in use. This machine will be loaned to the Committee for use by Professor Buckingham at the Massachusetts Institute of Technology when conditions are such that it can be used there to better advantage than in its present location. A mathematical analysis of contact on Hindley worm gears was made during the summer.

Existing Supplies of Hardwoods. The test program at the University of Michigan on sample logs of tropical hardwoods has progressed slowly during the year. There are on hand at the University samples of a total of 46 species of tropical woods obtained from 10 different countries. Work on five of the species has been completed; on seventeen species it is well advanced; while work has been begun on several more. In all, twelve thousand tests and determinations—mechanical and related physical tests and determinations, and moisture content determinations—have been made to date in addition to the extensive task of preparing the test specimens, making and checking the mathematical computations involved, etc.

Prof. W. Kynoch, who is conducting the program under the direction of Prof. D. M. Matthews, estimates that it will take at least two years more to complete the tests with the present limited equipment and personnel.

Strength of Vessels Under External Pressure. W. D. Halsey, Chairman. Proposed methods of computation have been submitted by both T. McL. Jasper of A. O. Smith Corporation, and Commander H. E. Saunders of the U. S. Navy Model Basin, based on their recent experimental work. Comparison is now being made of these two methods and a meeting of the Committee has been called for December, 1932, at which time it is hoped a decision will be reached as to an acceptable method of design for plain cylindrical vessels subject to external pressure.

Absorption of Radiant Heat in Boiler Furnaces. W. J. Wohlenberg, Chairman. The committee is continuing to collect and analyze data available from installed furnaces in the power industry. Sufficient material, however, has not as yet been accumulated to warrant a published report.

Velocity Measurement of Fluid Flow. W. F. Durand, Chairman. An accurate and compact device for adjusting and reading the phase relationship of sound waves in two tubes was constructed by Professors H. E. Hartig and H. B. Wilcox at the University of Minnesota. This will be set up and tested during the next few months.

Measures of Management. (9) W. E. Freeland, Chairman. A report on the Committee's investigation of the problem of selection and replacement of equipment was presented by H. O. Vorlander and F. E. Raymond at the 1931 Annual Meeting under the title "Economic Life of Equipment."

The Committee is cooperating with and sponsoring research projects of merit in the management field, all for the purpose of placing management on a more scientific basis. Some of these projects are (1) cooperation with Prof. R. M. Barnes, University of Iowa, in his study of the ability of experienced time study observers to judge skill and effort of workers, (2) cooperation with Prof. T. H. Brown, Harvard Graduate School of Business Administration, who is undertaking to discover the character of the statistical summaries needed to control various parts of the business enterprise, and to indicate in connection with such summaries how standards of performance might be built up for a particular business, (3) cooperation with Prof. R. C. Davis, Ohio State University, on determination of fundamental considerations underlying the successful development of operating organization and control in any field of business activity; and (4) cooperation with Prof. E. D. Smith, Yale University, in the determination of causes of human problems and other effects of technological developments in industry.

Diesel Fuel Oil Specifications. L. H. Morrison, Chairman. Arrangements have been made with the Submarine Base at New London to carry out a series of tests in an effort to discover the effect of variations in the viscosity and Conradson carbon of fuel oils on the operation of Diesel engines. The work is now under way.

The laboratories of several of the large oil companies have continued their studies of the physical and chemical properties of Diesel fuels in an attempt to discover the characteristics of motor fuel unsatisfactory for engine operation.

C. LeMaistre, director of the British Standards Institution, visited the United States in May and discussed with the Committee the plans of a similar research work under way in Great Britain. Arrangements were made for the exchange of data between the two groups, which it is hoped will lead to international agreement on specifications for fuel oil.

Diesel Fuel Combustion. (Joint Committee with the Society of Automotive Engineers.) O. D. Treiber, Chairman. This Committee is an outgrowth of the Subcommittee on Combustion of the foregoing A.S.M.E. Special Research Committee on Diesel Fuel Oil Specifications. Its purpose is to conduct a study of combustion in a Diesel engine cylinder. Because of business conditions it has not been possible to raise funds from industry to support the experimental work at the Bureau of Standards as originally planned. Several engine builders having suitable experimental equipment are, therefore, undertaking these studies according to a revised plan laid out by the Joint Committee.

Wire Rope. W. H. Fulweiler, Chairman. More than 100 samples of worn rope removed from service have been sent in from the field to the National Bureau of Standards for examination and test according to the program laid out by this Committee. Rope users in all the important fields of wire rope application are cooperating in the collection of samples. The associations of insurance companies, elevator manufacturers, and building owners have been especially active in this direction.

The details of a test program have been decided upon after a careful review of all the suggestions presented by representative rope manufacturers and users, and the tests are now in progress at the Bureau under the direction of H. L. Whittemore. The program calls for a very careful physical examination and tension tests of the rope samples, and an attempt will be made to relate the breaking strength results to the amount of wear and the number of broken wires observed in the samples before the tests. It is hoped by the Committee that a definite relation can thus be established between the number of broken wires and the wear of wire rope and its remaining useful life.

Heavy Duty Anti-Friction Bearings. (10) W. Trinks, Chairman. Experimental work on the strength of roll necks and a study of power requirements of rolling mills, as affected by the use of different types of roll neck bearings, were completed during the year. Progress reports describing this work were distributed to contributors.

A study of the relative suitability of the various types of roll neck bearings for different rolling mill applications, considering load-carrying capacity, rate of wear, coefficient of friction, requirements of lubrication and maintenance, and probable length of life, is under way. A progress report, which is a comparative study of the common types of roll neck bearings with respect to total costs of operation, is nearing completion.

A technical session on Steel Mill Bearings, jointly sponsored by the Committee and the Iron and Steel Division, was held during the 1931 Annual Meeting. At the Annual Meeting this year, a paper, "The Strength of Roll Necks," which has been written by Chairman Trinks and J. H. Hitchcock, Research Associate, will be presented during one of the sessions.

A campaign for funds to permit the completion of the program early in 1933 was successful. Some $7,500 has been contributed by industry to this project, which has been carried on at the Carnegie Institute of Technology.

Removal of Ash as Molten Slag From Powdered-Coal Furnaces. (11) K. M. Irwin, Chairman. Mr. Irwin, mechanical engineer, Philadelphia Electric Company, was elected chairman of this Committee to replace P. B. Rice, who had resigned.

Fluxing tests have been carried on in a large gas furnace at the Bureau of Mines Experimental Station at Pittsburgh, under the direction of P. Nicholls, using ash with iron ore, iron ore and limestone, and salt cake and limestone, respectively, as fluxes. Studies of the fluxing effect of silica, magnesium oxide, and calcium chloride were also made. Further work was undertaken for the Fe_2O_3-CaO diagram to check and better establish the field in the region of the 2650 to 2550 lines. In addition, a study was made of the relationship between fluid temperature and ash cone-fusion temperature.

This year also an extensive investigation was started on the effect of the state of the iron in slag on the flow temperature.

A progress report, "Fluxing of Ashes and Slags as Related to the Slagging Type Furnace," was presented at the session sponsored jointly by the Committee and the Fuels Division at the 1931 Annual Meeting.

Automatic Oil Pipe Line Pumping Stations. W. G. Heltzel, Chairman. Reports have been completed on the relative cost of using Diesel, electric, and gas prime movers in driving an automatic 40,000-barrel, 750-lb pressure, 10-in. main line pumping station. The pioneer station built by the Committee on the Petroleum Exposition Grounds at Tulsa, Okla., in 1930, has been improved upon so much by the oil producers, who became interested in its possibilities, that it is now practically obsolete. This station will be rebuilt on the basis of the experience gained in the field and will be in operation at the time of the next Petroleum Exposition in May, 1933.

Methods and Apparatus for Noise Measurement. W. B. White, Chairman. The activity of this Committee and of several similar groups in other societies developed the need for coordination of these efforts, as far as their standardization aspects were concerned. A Sectional Committee was, therefore, organized during the year under the procedure of the American Standards Association, and will undertake the preparation of standards of terminology, units, scales, and methods of measurements in the field of acoustics. This A.S.M.E. Research Committee is ably represented on the Sectional Committee and will contribute its data through them.

Rotary Drilling of Oil Wells. (12) D. L. Trax, Chairman. The Committee has completed studies and prepared reports on the operating efficiency of both steam equipment and electrical equipment when used in the rotary drilling of oil wells. The steam equipment report has been published by the Society as one of its series of A.S.M.E. Research Publications. Publication of the electric drilling report has been delayed pending a study of the reception accorded the steam drilling report. The Committee is now engaged in the collection of efficiency data on the use of Diesel, gas, and gasoline engines as prime movers.

Critical Pressure Steam Boiler. (13) A. A. Potter, Chairman. Experimental work has been under way throughout the year on the 3500-lb forced circulation steam generator at Purdue University. The papers, "Characteristics of a High-Pressure Series Steam Generator," by A. A. Potter, H. L. Solberg, and G. A. Hawkins, and "Once-Through Series Boiler for 1500 to 5000 Lb Pressure," by H. J. Kerr, presented at the December, 1931, Annual Meeting, describe this boiler and the results of studies completed on the effect upon operating conditions of changes in pressures, temperatures, and air supply with automatic and hand control. A condensing calorimeter designed for the determination of the enthalpy of high-pressure superheated steam has been constructed and calibrated, and fifty tests have been made on steam at pressures between 1900 and 3500 lb per sq in. Studies of flow in parallel circuits and of the friction of steam in pipes at pressures up to 3500 lb and 830 F are also under way. A progress report on this investigation will be presented at the 1932 Annual Meeting.

Respectfully submitted,

W. H. Fulweiler, *Chairman*
A. D. Bailey G. M. Eaton
A. A. Potter D. B. Bullard.

APPENDIX

The papers, reports, and special publications resulting from the work of the committees during the year are listed below. The numbers refer to the committees responsible for them.

(1) "Pressure Distribution in Oil Films of Journal Bearings," by S. A. McKee and T. R. McKee. Research Papers (A.S.M.E. Trans.), RP-54-8
"Optimum Conditions in Journal Bearings," by Albert Kingsbury. Research Papers (A.S.M.E. Trans.), RP-54-7
"High-Pressure Capillary Flow," by M. D. Hersey and G. H. S. Snyder. *Journal of Rheology*, July, 1932
"Lubrication Research of the A.S.M.E.," by A. E. Flowers and M. D. Hersey. *Mechanical Engineering*, April, 1932

(2) "A Study of Primary Metering Elements in 3-In. Pipe," by S. R. Beitler, Paul Bucher, and T. C. Barnes, Jr. Research Papers (A.S.M.E. Trans.), RP-54-3
"Flow of Steam Through Square-Edged Orifices in a 4-In. Line," by W. W. Frymoyer and A. Herschel. Research Papers (A.S.M.E. Trans.), RP-54-6

(3) "Progress in Steam Research:" "Work at the U. S. Bureau of Standards," by N. S. Osborne and H. F. Stimson; "Work at Massachusetts Institute of Technology," Parts 1 and 2, by F. G. Keyes and L. B. Smith; "Other Current Steam Research," by Harvey N. Davis. *Mechanical Engineering*, February, 1932
"A Steam Chart for Second-Law Analysis," by J. H. Keenan. *Mechanical Engineering*, March, 1932

(4) "The Elements of Milling," by O. W. Boston and Charles E. Kraus. Research Papers (A.S.M.E. Trans.), RP-54-4
"Tungsten-Carbide and Other Hard Cutting Materials," by Coleman Sellers, 3d. Research Papers (A.S.M.E. Trans.), RP-54-14

(5) "Design and Investigation of a Spring in Which All Coils Nest Simultaneously," by J. B. Reynolds and O. B. Schier. Research Papers (A.S.M.E. Trans.), RP-54-11
"Design and Investigation of Conical Springs With Coils of a Constant Slope," by J. B. Reynolds and O. B. Schier. Research Papers (A.S.M.E. Trans.), RP-54-12
"The Active Portion of Large Helical Springs," by Alan N. Lukens. Mimeographed
"Effect of Low Temperature Heat Treatment on Music Wire," by L. E. Abbott. Mimeographed

(6) "Symposium on Effect of Temperature on the Properties of Metals." A.S.M.E.-A.S.T.M., 800 pages
Report of A.S.M.E.-A.S.T.M. Joint Research Committee on Effect of Temperature on Properties of Metals. A.S.T.M. Proceedings, June, 1932

(7) "Corrosion Tests on Condenser Tubes." Progress Report No. 5 of the Special Research Committee on Condenser Tubes. Abstract will appear in *Mechanical Engineering*. Photo-offset

(8) "Foaming and Priming of Boiler Water," by C. W. Foulk. Research Papers (A.S.M.E. Trans.), RP-54-5
"Determination of Carbonate, Hydroxide, and Phosphate in Boiler Waters," by W. C. Schroeder and C. H. Fellows. Research Papers (A.S.M.E. Trans.), RP-54-13
"The SO_4–CO_3 Ratio for the Prevention of Sulphate Boiler Scale," by Everett P. Partridge, W. C. Schroeder, and R. C. Adams, Jr. Research Papers (A.S.M.E. Trans.), RP-54-10

(9) "Economic Life of Equipment," by H. O. Vorlander and F. E. Raymond. Research Papers (A.S.M.E. Trans.), RP-54-2 and Management (A.S.M.E. Trans.), July, 1932

(10) "Application of Tapered Roller Bearings in Merchant-Bar Mills," by S. M. Weckstein. Iron and Steel (A.S.M.E Trans.), April, 1932
"Application of Celoron Bearings to Roll Necks," by Arthur J. Schmitt. Iron and Steel (A.S.M.E. Trans.), April, 1932

(11) "Fluxing of Ashes and Slags as Related to the Slagging Type Furnace," by P. Nicholls and W. T. Reid. Research Papers (A.S.M.E. Trans.), RP-54-9

(12) "Reports of Tests on Steam Equipment for Drilling Rotary-Drilled Oil Wells." Report of the Special Research Committee on Rotary Drilling of Oil Wells. A.S.M.E. Research Publication

(13) "Characteristics of a High-Pressure Series Steam Generator," by A. A. Potter, H. L. Solberg, and G. A. Hawkins. Research Papers (A.S.M.E. Trans.), RP-54-1b
"Once-Through Series Boiler for 1500 to 5000 Lb Pressure," by H. J. Kerr. Research Papers (A.S.M.E. Trans.), RP-54-1a.

STANDARDIZATION

The A.S.M.E. Standardization Committee presents the following as its report for the year October 1, 1931, to October 1, 1932.

Organization

During the past year the Society accepted one joint sponsorship with the Society of Automotive Engineers under the procedure of the American Standards Association for the Standardization of Classification and Designation of Surface Qualities, in March, 1932. The work of organizing this Sectional Committee has been carried forward during the summer in anticipation of a meeting in October. Sixty organizations were invited to appoint official representatives on this Committee and so far twenty-three have made favorable replies.

To date the Society has accepted sponsorship or joint sponsorship for a total of 30 sectional committees under the procedure of the ASA. These sectional committees have subdivided themselves into 253 subcommittees and subgroups in order that the work assigned to them may be carried forward more promptly and effectively. This number includes the organization of three new Technical Committees of the Sectional Committee on the Standardization of Small Tools and Machine Tool Elements. The total number of members of these committees is now 1118, of which 403 are members of the A.S.M.E.

In addition to the 30 sectional committees for which the Society holds sponsorship or joint sponsorship, the A.S.M.E. was represented this year on 29 other committees by 39 of its members.

Eighty-one standards committee meetings were held during this year.

Standards Approved

Four standards were completed under the ASA procedure and published by the Society in pamphlet form within the dates covered by this report. These were:

800-Lb Hydraulic Cast-Iron Pipe Flanges and Flanged Fittings (B16b1-1931)
Rolled Threads for Screw Shells of Electric Sockets and Lamp Bases (C44-1931)
Rotating Air Cylinders and Adapters (B5.5-1932)
Symbols for Mechanics, Structural Engineering, and Testing Materials (Z10a-1932).

In addition to these the following ten standards were completed by the several committees for which the A.S.M.E. holds sponsorship or joint sponsorship. Of these all but the standard for Wrench

Head Bolts and Nuts and Wrench Openings have been approved and submitted to the ASA.

Engineering and Scientific Charts for Lantern Slides
Abbreviations for Scientific and Engineering Terms
Shaft Couplings
Spur Gear Tooth Form (including Composite, Full Depth, and Stub Involute Systems for $14^{1}/_{2}$ deg and 20 deg)
Steel Flanged Fittings and Companion Flanges (revision of 1927 edition with addition of Steel Companion Flanges and Base Fittings)
Wrench Head Bolts and Nuts and Wrench Openings (revision of 1927 edition)
Ball and Roller Bearings (revision of 1930 edition)
Graphical Symbols Used for Electric Power and Wiring
Graphical Symbols Used for Electric Traction including Railway Signaling
Graphical Symbols Used in Radio.

PUBLICITY

The Committee's record for publicity has been maintained. During the year 23 pages of text pertaining to the Society's work in standardization were published in *Mechanical Engineering.* Part of this material was published in the Engineering and Industrial Standardization section of that journal, and the remainder consisted of abstracts of preliminary proposals issued from time to time by the committees. Three thousand and twenty-seven copies of preliminary drafts of proposed standards were distributed within the twelve months to consumer and producer interests and to the technical press.

In an attempt to meet the recommendations made by those who attended the "Standards Luncheon" in New York in December, 1931, who urged in no uncertain terms greater publicity on the accomplishments of their committees, the A.S.M.E. Standardization Committee developed three special display advertisements of three natural groupings of American Standards now available. These advertisements were printed in 2 issues of *Civil Engineering,* 2 issues of *Mining and Metallurgy,* 1 issue of *Industrial and Engineering Chemistry,* 1 issue of the *Journal* of the American Ceramic Society, 2 issues of *Science News Letter,* 4 issues of *Refrigerating Engineering,* 1 issue of *Machinery,* 5 issues of the *Bulletin* of the Heating and Piping Contractors National Association, and 5 issues of *Mechanical Engineering.*

Another effort to increase the knowledge of American Standards has been directed this year toward the vocational schools and high schools having vocational education departments. The Committee was greatly assisted in this by Frank Cushman, chief of the Industrial Education Service of the Federal Board for Vocational Education. In all, something over 400 letters were written to the directors, supervisors, teacher-trainers, principals, and instructors of these schools situated in practically every State of the Union. The responses to these letters so far received have been most enthusiastic and cordial. So it is expected that the coming year will show interesting developments in this direction.

During the brief visit of Charles le Maistre, director of the British Standards Institution, to the United States in May, 1932, the Committee arranged an informal luncheon in his honor which was attended by a number of members of the Society and others interested in national and international standardization. Mr. le Maistre was spending a few weeks in our country following a seven months' visit to Australia, New Zealand, and Canada in connection with an effort to bring about closer cooperation between the activities of the national standardizing bodies in these British Dominions and with the B.S.I. He described in a vivid way some of the experiences of his trip and conveyed the greetings of his Institution.

SECTIONAL COMMITTEE AND SUBCOMMITTEE REPORTS

Shafting. C. M. Chapman, Chairman. The Sectional Committee on Standardization of Shafting has been active during this past year in completing a revision of the several standards on shafting and keys developed by it and approved some years ago. The Committee held two meetings in October and November, 1931, at which the work of revision was concluded with the exception of a few controversial points. Through a questionnaire sent out to industry these questions have been settled and the revision is now ready for final approval by the Committee membership. Immediately upon its release the proposal will be submitted to the sponsor and the American Standards Association for approval and adoption.

Allowances and Tolerances for Cylindrical Parts and Limit Gages. E. J. Kearney, Chairman. The past year has been one of preparation for the Sectional Committee on Allowances and Tolerances for Cylindrical Parts and Limit Gages. At its second meeting in December, 1931, the Chairman was instructed to appoint a subcommittee to consider the basic hole, basic shaft, and G M systems of allowances and tolerances and to prepare a preliminary draft proposal for review and study by the members of the Sectional Committee. The Chairman of this Subcommittee is R. E. W. Harrison, and his colleagues are F. E. Banfield, F. O. Hoagland, J. O. Johnson, P. V. Miller, D. W. Ovaitt, E. Pugsley, and H. L. Van Keuren.

At this meeting the proposal submitted by the International Standards Association's Technical Committee on Fits, copies of which had been mailed to the membership in July, 1931, was referred to the Subcommittee for study. The paper, in which this proposal was described and interpreted, had been prepared by John Gaillard of the staff of the American Standards Association. It was printed in the July, 1931, issue of the ASA Bulletin and the August, 1931, issue of *Mechanical Engineering.*

In the December, 1931, issue of the ASA Bulletin, H. W. Bearce, a member of the Sectional Committee, published an article bearing on a phase of the Committee's work. It is entitled "Unilateral and Bilateral Tolerances for Machine Parts and Gages."

During the year Chairman Harrison has organized the work of his Subcommittee by correspondence and completed a definite outline of its scope. This outline, including a statement of policy set up by the Subcommittee, was published in the October issue of *Mechanical Engineering* and the October issue of the ASA Bulletin.

Ball and Roller Bearings. F. W. Gurney, Chairman. Slight revisions and extensions of the American Standard for Ball and Roller Bearings (B3-1930) were completed by the Sectional Committee and finally approved at its meeting held in New York on May 11, 1932. These proposed changes and additions were approved by the two sponsor organizations, the S.A.E. in June, 1932, and the A.S.M.E. in August. The revised standard is now before the ASA for review.

At its May meeting the Committee gave consideration also to a proposal for a series of standard taper roller bearings (dated March, 1932) which had been submitted as the American proposal for an international standard. It was decided to take no further action toward the development of an American Standard until the results of the International Conference on Ball and Roller Bearings held in Milan, Italy, in June, 1932, were known. The Committee was represented at the Conference by E. Wooler, one of its members.

Some progress has been made also during the year by the Committee in the development of standard nomenclature for ball and roller bearings.

Gears. B. F. Waterman, Chairman. In July, 1932, the Sectional Committee on the Standardization of Gears gave its approval to the proposed American Standard for Spur Gear Tooth Form, $14^{1}/_{2}$-deg and 20-deg Full Depth Involute System. This new standard then received the approval of the A.G.M.A. and the A.S.M.E. in their capacities as joint sponsors. Consequently, the standard is now before the ASA for approval. When published the Committee plans to combine this standard with the present American Standard for Spur Gear Tooth Form, $14^{1}/_{2}$-deg Composite System and 20-deg Stub Involute System (B6b-1927).

The proposed American Recommended Practice on Gear Materials and Blanks of Alloy Steel, Cast Steel, Forged and Rolled Carbon Steel, also Bronze and Brass, has just been released by the Subcommittee and is now ready for vote on approval by the Sectional Committee.

Copies of an A.G.M.A. Recommended Practice on Inspection of Gears were sent in August to the members of the Subcommittee at work on this phase of the project as a basis for its preliminary report.

Pipe Flanges and Fittings. C. P. Bliss, Chairman. The Sectional Committee on Pipe Flanges and Fittings held a meeting in April, 1932, to pass upon the proposed American Standard for Steel Flanged Fittings and Companion Flanges. This proposal is a revision and amplification of the present American Tentative Standard for Steel Pipe Flanges and Flanged Fittings (B16e-1927) and is the result of a great deal of painstaking work and cooperation on the part of the membership of Subcommittee No. 3 on Steel Flanges and Flanged Fittings. In July this revised standard was approved by letter ballot of the Sectional Committee. It has been since approved by the three sponsor organizations and transmitted to the American Standards Association.

In addition, the proposed American Standard for 800-lb (Hydraulic) Cast-Iron Flanged Fittings developed by Subcommittee No. 1, A. M. Houser, Chairman, and approved by the Sectional Committee last year, was granted the designation of American Standard in December, 1931, by the American Standards Association.

At the April meeting of the Sectional Committee the proposed standard for Screwed Pipe Plugs developed by Subcommittee No. 2 on Screwed Fittings, Stanley G. Flagg, Chairman, was also discussed and it also was approved subsequently by letter ballot.

Three other projects have been advanced considerably during the past year but are not yet ready for final approval. These are (1) the proposed standard for welding flanges under development by Subcommittee No. 3, (2) a proposed standard series of face-to-face dimensions for ferrous flanged valves being developed by Subcommittee No. 5, of which F. H. Morehead is Chairman, and (3) a revision of the port openings of the 125-lb Cast-Iron Flanged Fittings in sizes 14 to 24 inches, which was assigned to Subcommittee No. 9, with W. W. Hubbard as Chairman.

Bolt, Nut, and Rivet Proportions. A. E. Norton, Chairman. An extensive revision of the American Tentative Standard for Wrench Head Bolts and Nuts and Wrench Openings was approved by letter ballot of the Sectional Committee on Bolt, Nut, and Rivet Proportions in July, 1932. This revision was undertaken by Subcommittee No. 2, H. A. Spanagel, Chairman, and includes the addition of a heavy series of bolt heads and nuts and the extension and revision of the existing regular and light series. Subcommittee No. 2 held two meetings during the year, one in December, 1931, and the second in April, 1932. The revised standard, as approved by the Sectional Committee, was submitted to the sponsor organizations in September, 1932, and upon release will be transmitted to the ASA for approval and designation as an American Standard.

Subcommittee No. 1 on Large and Small Rivets, J. E. Kiernan, Chairman, met in April of this year to review the tentative draft of the standard for Large Rivets in the light of a number of objections which had been received from the manufacturers. At this meeting certain changes in the proposed head designs were agreed upon and the tolerances were modified to meet commercial requirements. Agreement on all outstanding points was reached at that time and a revised draft has been prepared and will be distributed shortly to the Subcommittee members for their final approval prior to a redistribution to industry for comment.

The proposed standard for Socket Type Set and Cap Screws prepared by Subcommittee No. 9, of which H. Koester is Chairman, should be available in October, 1932, for distribution to industry for comment. While this Committee has held no meetings during the year, its work has progressed very satisfactorily by correspondence. In January, 1932, a tentative draft of the standard based on the Committee's previous deliberations was distributed to the membership for criticism. The suggestions received were summarized by Chairman Koester and a second draft prepared.

Small Tools and Machine Tool Elements. C. W. Spicer, Chairman. At its December 2, 1931, meeting the Sectional Committee on Small Tools and Machine Tool Elements approved the appointment of three new technical committees to develop standards for rotating tool shanks, nomenclature for small tools and machine tool elements, and multiple spindle drilling heads. These committees have since been organized and the progress made in each case is detailed later. The proposed Standard for Rotating Air Cylinders and Adapters was approved by the Sectional Committee in January, 1932.

Technical Committee on T-Slots. E. Oberg, Chairman. Technical Committee No. 1 had its Standard for T-Slots, Their Bolts, Nuts, Tongues, and Cutters approved in March, 1927. Early in this year one of the large manufacturers of milling machines submitted to the Committee a suggestion that the depth of engagement of the present standard tongue be increased. The Committee held a meeting in December, 1931, to discuss this point and J. B. Armitage was appointed a committee of one to make some tests and to submit drawings to other members of the committee for their consideration with the view of making any necessary changes in the present standard.

Technical Committee on Machine Tapers. F. S. Blackall, Jr., Chairman. Mr. Blackall was appointed Chairman of this Technical Committee in February, 1932, upon the resignation of E. F. DuBrul. The Committee held a meeting in April, 1932, at which the report of its Subgroup on Dimensions, Tolerances, and Gages was received by the Technical Committee. After a full discussion, however, the proposed standard tables of dimensions of a series of small angle tapers was laid on the table for the present, and it was moved that this subgroup be relieved from further consideration of this subject and discharged. Then a subgroup was appointed to develop a proposal for an American Standard steep taper with a hold-back device.

Technical Committee on Spindle Noses and Collets. J. E. Lovely, Chairman. Up to December, 1931, E. F. DuBrul served as Chairman of this Technical Committee. Upon his resignation a new chairman in the person of J. E. Lovely was appointed. The Technical Committee's progress is dependent upon the work of Subgroup No. 8 which has held several meetings during the past year and has called a meeting for October 9, 1932. This Subgroup has been testing out a newly designed spindle nose before releasing it to industry as a proposed standard for criticism and comment.

Technical Committee on Designation and Working Ranges of Machine Tools. B. S. Dayton, Chairman. In May, 1932, B. S. Dayton was appointed chairman of the Technical Committee upon the resignation of E. F. DuBrul. No progress has been reported by this committee since its meeting in October, 1930.

Technical Committee on Twist Drill Sizes. C. J. Oxford, Chairman. While this Committee has held no meetings during the current year, through the efforts of C. W. Spicer, Chairman of the Sectional Committee, considerable progress has been made toward the acceptance of the principles on which the Technical Committee based its proposal last year.

Technical Committee on Drill Bushings. C. E. Rundorff, Chairman. The Committee's efforts to complete its proposal for an American Standard for Drill Bushings have been unsuccessful because of the absence of certain authentic data. It has been decided, accordingly, to distribute a tentative proposal to industry for comment on these points, the principal one of which is the determination of the proper values for the hole tolerances.

Technical Committee on Punch and Die Sets. S. Diamant, Chairman. A meeting of Technical Committee No. 9 on Punch and Die Sets was held in November, 1931. At this meeting the Committee authorized the consolidation of the data before it into a tentative proposal in the form of tables for distribution to industry for criticism and comment. Under the direction of the Chairman such a draft proposal was prepared. It includes introductory notes, cuts, and tabulated dimensions for the five types of punch and die sets and details of the holding clamps. In June, 1932, copies of this proposed standard, together with a questionnaire, were mailed to approximately 600 manufacturers and users of this type of equipment for study and comment. Replies of a constructive nature have been received from more than 35 per cent of those to whom the proposal was sent. The comments and suggestions contained in these replies will form the basis of the revision of the first tentative draft which will be reviewed by the Committee at its next meeting.

Technical Committee on Circular Forming Tools and Holders. W. C. Mueller, Chairman. The replies received to the questionnaire sent out in July, 1931, were analyzed and thoroughly discussed at the December, 1931, meeting of the Committee held in New York. During the remaining months of the year this project has been further developed by correspondence among the members of the Technical Committee.

Technical Committee on Chucks and Chuck Jaws. J. E. Lovely, Chairman. Two proposed standards have been developed by Technical Committee No. 11, one for Rotating Air Cylinders and Adapters and the other for Chucks and Chuck Jaws. The first of these was approved in September, 1932, as American Standard by the ASA. A meeting of the Committee was held in December, 1931. The progress toward the completion of the proposal for chucks and chuck jaws has been dependent upon the studies and decisions of Technical Committee No. 4 on Spindle Noses and Collets. While considerable progress has been made toward the development of standard designs for spindle noses and chucks, certain refinements and adjustments are still necessary.

Technical Committee on Cut and Ground Thread Taps. C. M. Pond, Chairman. Since the approval of the American Standard for Cut and Ground Thread Taps in April, 1930, and its adoption by industry, the desirability of reducing the overall length and thread length of hand taps has been raised. Accordingly, in January, 1932, a canvass was made, the results of which have been sent to the members of the Committee by the Chairman for study.

During the year the Committee has developed also standard dimensions for pulley taps, tapper taps (machine screw size), bent shank tapper taps (cut and ground thread), and taper and straight pipe taps which will be submitted for approval as American Standards. These tables constitute a supplement to the 1930 standard.

Technical Committee on Splines and Splined Shafts. C. W. Spicer, Chairman. The type of splines which is preferred by industry in general has been undergoing a change during the past few years. For this reason the Committee has decided to mark time for the present until this situation has cleared.

Technical Committee on Milling Machine Tables. G. A. Bouvier, Chairman. This Committee decided at its meeting in December, 1931, to concentrate its efforts upon standards for slot spacing for the five different ranges of table widths. A tentative draft proposal for standard knee type milling machine tables in line with the discussion at the December meeting are now before the Committee members for study and comment.

Technical Committee on Rotating Tool Shanks. E. J. Bryant, Chairman. The first meeting of this Committee was held in April, 1932, at which three subgroups were appointed to develop the sections on (*a*) drill reamers and tap shanks, diameters and squares; (*b*) bit brace shanks; and (*c*) shanks on socket wrenches with driven nuts and screw driver shanks for rotating drivers. To carry this program forward adequately an increase in the size of the Committee seemed necessary. This has been accomplished during the year and these subgroups are at work drafting the three sections of the tentative proposal.

Technical Committee on Nomenclature for Small Tools and Machine Tool Elements. H. E. Harris, Chairman. This Committee, authorized at the December, 1931, meeting of the Sectional Committee, has held two meetings, one in February and the other in April, at which two subgroups were appointed to develop standard nomenclature for all types of small tools and machine tool elements. The Subgroup for Lathe and Planer Tools has already submitted a tentative list of terms and definitions.

Technical Committee on Multiple Spindle Drilling Heads. G. A. Bouvier, Chairman. One of the three technical committees authorized at the December, 1931, meeting of the Sectional Committee was the Technical Committee on Multiple Spindle Drilling Heads. Its first meeting was held in April, 1932, at which time a tentative program for work was proposed and adopted.

Scientific and Engineering Symbols and Abbreviations. J. F. Meyer, Chairman. During the year five standards have been completed and approved by the Sectional Committee on Scientific and Engineering Symbols and Abbreviations.

The first of these, Symbols for Mechanics, Structural Engineering and Testing Materials, was approved by the ASA in January, 1932, and was subsequently published in pamphlet form.

The proposed American Tentative Standard, Abbreviations for Engineering and Scientific Terms, was transmitted to the ASA by the five sponsor bodies in June, 1932. However, action by that Association has been delayed pending the approval of some minor changes and additions to the standard which the Committee has been requested to make.

The remaining three standards, Graphical Symbols Used for Electric Power and Wiring, Graphical Symbols Used in Radio, and Graphical Symbols Used for Electric Traction Including Railway Signaling, were presented in July, 1932, to the ASA for final approval. It is expected that approval by this body will be completed in a few weeks.

Plain and Lock Washers. C. W. Squier, Chairman. A meeting of the Sectional Committee on the Standardization of Plain and Lock Washers was held in October, 1931, at which time the revision of the personnel of the Committee was discussed and the addition of six members representing the manufacturers was approved. The tentative proposal of the Subcommittee on Lock Washers developed under the chairmanship of C. H. Loutrel was approved for distribution to industry for criticism and comment. This was done in December, 1931, and the Subcommittee met in April, 1932, to consider the criticisms and comments which had been received. At this meeting it was decided to include with the new series recommended in December the three series of the S.A.E. standard. A redraft of the proposal on this basis is now in progress.

In order to further the work of drafting a preliminary proposal for a standard on plain washers Chairman Squier was authorized at the October, 1931, meeting of the Sectional Committee to appoint a temporary chairman of Subcommittee No. 1 with instructions to develop a proposal with all possible speed. Some progress in this direction has been made.

Code for Pressure Piping. E. B. Ricketts, Chairman. The Editing Committee has reviewed the parts of the proposed American Standard Code for Pressure Piping dealing with (*a*) power piping, (*b*) district heating piping, and (*c*) fabrication details, including hangers, welded joints, and joints other than welded. The Sectional Committee met in December, 1931, and again in April, 1932. On both of these occasions several of the subcommittees also held meetings. A. Iddles, Chairman of the Subcommittee on Gas and Air Piping, has been developing this section of the Code in close coöperation with the group in the American Gas Association specially interested in the transportation of natural gas. It is now anticipated that this part of the Code will be completed very shortly. The part dealing with oil piping, which was prepared by a Subcommittee of which A. D. Sanderson is Chairman, was reported completed at the April meeting.

Drawings and Drafting Room Practice. F. DeR. Furman, Chairman. A meeting of the Editing Committee of the Sectional Committee on Drawings and Drafting Room Practice was held in New York in December, 1931. This Editing Committee is now at work on the unification of the reports of the several subcommittees covering specifications for paper and cloth, method of indicating dimensions, lettering, layout, line work, and graphical symbols.

Subcommittee No. 6, of which T. E. French is Chairman, distributed to industry for criticism and comment a proposal covering Graphical Symbols for Drawings in July, 1932. This proposal includes sets of symbols for plumbing fixtures, conventional rivet layouts, heat power apparatus, electric power apparatus, pipe and pipe fittings, and heating and ventilating layouts. Letters of criticism and comment are still being received as a result of the distribution of approximately 500 copies of this preliminary report.

Graphic Presentation. E. F. DuBrul, Chairman. The most tangible result of the work of the Sectional Committee on Graphic Presentation during the past year was the completion of the monograph on Engineering and Scientific Charts for Lantern Slides. This proposal originated in the Subcommittee on Engineering and Scientific Charts, of which W. A. Shewhart is Chairman. The original draft was formulated by a subgroup consisting of H. F. Dodge, C. D. Hanscom, and L. Stone. One of the early drafts was discussed at an open meeting of the Committee held in New York on December 3, 1931. The final draft after approval by the members of the Sectional Committee and the sponsor society was transmitted to the ASA on August 24, 1932.

The joint subcommittee, of which A. H. Richardson is chairman, has continued its work during the year and now expects to submit a draft of the proposal for preferred practice in graphic presentation in the very near future.

Pipe Threads. E. M. Herr, Chairman. The Sectional Committee on Standardization of Pipe Threads has been engaged for the past few years in a revision of the American Standard for Pipe Threads (B2-1919). Subcommittee No. 2 on Taper Pipe Threads, under the chairmanship of S. B. Terry, has, with the exception of one point, completed its revision of the taper thread specifications. This year, of the pipe thread group of committees, Subcommittee No. 2 met in December, 1931, and agreed upon the few points then in question, but decided to postpone final approval of the revised proposal until the proposed inclusion of limit gages had received the vote of the entire Subcommittee membership. This vote was taken and indicates unanimous approval of the addition of the alternative design of gage. Since completion of the revision of the Taper Pipe Thread Standard as authorized at the December meeting, however, the Chairman has made a further effort to bring the A.P.I. and American Standard thread lengths into closer agreement. Release of the revised standard for review by industry and the members of the Sectional Committee will be withheld in consequence of this action until a definite agreement on the question of thread lengths can be reached within the Subcommittee membership.

Wrought Iron and Wrought Steel Pipe and Tubing. H. H. Morgan, Chairman. In June, 1932, a meeting of the Sectional Committee on Wrought Iron and Wrought Steel Pipe and Tubing was held during the A.S.T.M. Annual Convention at Atlantic City. This meeting received a joint report prepared by Subcommittee No. 2 on Pipe and Tubing for Low Temperature Service, J. J. Shuman, Chairman, and Subcommittee No. 3 on Pipe and Tubing for High Temperature Service, J. R. Tanner, Chairman. This report, dated June, 1932, was in the nature of a tentative proposal which had resulted from the agreements reached in the discussion during the December, 1931, joint meeting of these committees in New York.

This report was thoroughly discussed by those present at the June, 1932, meeting and the Subcommittees were requested to prepare and distribute for general criticism and comment a proposed standard schedule of pipe sizes and thicknesses. This proposal will be ready for review shortly.

Speeds of Machinery. A. E. Hall, Temporary Chairman. The Subcommittee on Questionnaire and Canvass to Industry under the Chairmanship of F. S. English reviewed and correlated the replies received to its inquiry sent out in March, 1931, and met in October to discuss the content of its first tentative proposal. This proposal in revised form was reviewed by Subcommittee No. 1 at a meeting held in December, 1931, and a new series of speed numbers was developed which has since undergone slight modifications as the result of correspondence.

Screw Threads for Hose Couplings. H. W. Bearce, Chairman. Two meetings of the Sectional Committee on Screw Threads for Hose Couplings were held during the year in New York, the first on December 2, 1931, and the second on April 22, 1932. The Committee is now preparing a revision of its draft standard, dated March, 1932, on which it expects to take final action at a meeting scheduled for the week of October 17, 1932.

Plumbing Equipment. W. C. Groeniger, Chairman. While the Sectional Committee on Plumbing Equipment has held no meetings this year, its eight subcommittees have been at work and have made some progress.

A report has been received from A. R. McGonegal, Chairman of the Subcommittee on Traps, to the effect that the work on the proposed standard for brass lavatory and sink traps, brass traps, lead traps, vitreous china traps and interceptors, catch basins, and miscellaneous traps has been completed. The work on a draft proposal for cast-iron traps has yet to be formulated.

At the October, 1931, meeting of the Subcommittee on Brass Plumbing Products, of which J. L. Murphy is Chairman, it was voted to drop the work on water works brass goods and brass valves. Two new subgroups were appointed at this meeting to develop proposals for (*a*) minimum requirements for brass alloys used in the manufacture of brass plumbing products and (*b*) roughing-in dimensions. The proposal of the Subgroup on Compression Cocks, Washer Screws, Faucets and Tail Piece Parts, L. A. Cornelius, Chairman, was approved for distribution to industry for criticism. R. H. Zinkil, Chairman of the Subgroup on Fixture Connections and Fixture Valves, Waste Holes, Plugs and Connections, Overflows, reported completion of this group's recommendations and a progress report was received from R. L. Ottke on Nomenclature.

In January, 1932, Subcommittee No. 7 on Copper Water Tube Fittings, F. L. Riggin, Chairman, met to discuss the comments received on the October, 1930, draft of its proposed Standard for Brass Fittings for Flared Copper Tubes distributed to industry in February, 1931. Certain changes were made in the proposal and new printer's proofs were prepared for final review by the members of the Subcommittee prior to submittal to the Sectional Committee for approval.

Four meetings have been held by Subcommittee No. 8 on Cast Iron Soil Pipe and Fittings, J. J. Crotty, Chairman, during the year. The discussion at the first three meetings related to the standard for cast-iron fittings. At the September, 1932, meeting the Committee made a careful study of wall thicknesses of soil pipe. In order that it may complete its work without further delay the committee plans to meet every two weeks this fall.

Standardization and Unification of Screw Threads. R. E. Flanders, Chairman. The Sectional Committee on Screw Threads held a meeting in April, 1932, at which the progress made by the several subcommittees to date was carefully reviewed. One of the principal items on the agenda was the report of the Subgroup on Screw Thread Survey of the United States. At the request of this Subgroup D. R. Miller and I. H. Fullmer of the Gage Section of the Bureau of Standards had been invited to attend this meeting and made a complete report on the measurement of the screws and a preliminary report on the nuts. A summary of this report prepared by Earle Buckingham, a member of the Subgroup, was published in the September, 1932, issue of *Mechanical Engineering* and the October issue of the S.A.E. *Journal*. A complete and detailed report is in preparation which will be published in pamphlet form for general distribution. The Sectional Committee plans a revision of the present American National Standard for Screw Threads based on the final results of the survey as set forth in this report. All fundamental changes will, of course, be made with the full knowledge and cooperation of the National Screw Thread Commission.

At this meeting, also, recommendations were received regarding the adoption of three special pitch series, 8, 12, and 16 threads per inch, to replace the fine thread series above 1$^1/_2$ inch diameter.

Subcommittees Nos. 2, 3, and 5 held a joint meeting with R. E. Flanders as Acting Chairman the day following that of the Sectional Committee. At this meeting it was agreed to include the 8, 12, and 16 pitch series in their recommendations to meet certain well defined industrial needs. Certain other phases of the subject, including a report on minor diameters of nuts by E. H. Ehrman and the possibility of revising to some extent the tolerances prescribed by the four classes of fit, were considered. It is expected that definite action will be taken on these points as soon as the several Subcommittees concerned have had an opportunity to review the final report of the Screw Thread Survey. Subcommittee No. 4, of which Earle Buckingham is Chairman, has been active during the year in preparing a preliminary proposal on acme threads. At a meeting held in April a report prepared by C. W. Bettcher was adopted as the basis for discussion of this proposal and it was decided to include specifications for four classes of acme threads, namely, general purpose, fine threads, 29-deg stub threads, and 60-deg stub threads. The immediate activities of the Committee are being directed toward the development of the general purpose type and a subgroup of three members has been appointed to expedite this department of the work.

Rolled Threads for Screw Shells of Electric Sockets and Lamp Bases. R. E. Myers, Chairman. The American Standard for Rolled Threads for Screw Shells of Electric Sockets and Lamp Bases was approved by the ASA and published in final pamphlet form in December, 1931. The personnel of the Sectional Committee is identical with that of the U. S. National Committee, Advisory Committee No. 6, Lamp Caps and Holders, of the International Electrotechnical Commission. Close touch with developments in the international field is accordingly maintained.

Stock Sizes, Shapes and Lengths for Hot and Cold Finished Iron and Steel Bars. F. H. Frankland, Temporary Chairman. There has been no occasion for activity on the part of this Sectional Committee, as the work of none of its three Subcommittees is as yet ready for release. However, Subcommittee No. 2 on Cold Finished Steels, L. E. Creighton, Chairman, held a meeting in New York in October, 1931. This Committee is cooperating closely with the Cold Finished Steel Bar Institute in the preparation of this part of the proposal for an American Standard and, although nothing of a tangible nature has been released, it is expected that a preliminary report will be presented within a reasonable time.

Pressure and Vacuum Gages. M. D. Engle, Chairman. The Subcommittee on Definitions, C. H. Schwep, Chairman, issued a questionnaire in December, 1931, and from the replies received a list of definitions was developed which the Subcommittee revised at its meeting held in New York on April 22, 1932. Copies of the revised list of definitions have been sent out to industry for criticism and comment.

During the month of August this Subcommittee also distributed to a number of manufacturers for critical review copies of a proposed specification form and proposed rules for installation and use of pressure and vacuum gages.

The Subcommittee at work on setting up gage and mounting dimensions, H. B. Reynolds, Chairman, held a meeting in New York on October 23, 1931, and the Subcommittee on Accuracy and Test Methods, O. J. Hodge, Chairman, met on December 3, 1931. Both of these Committees are making progress in the development of their preliminary proposals.

Foundry Equipment and Supplies. E. S. Carman, Chairman. The rounding out of the personnel of the Sectional Committee on Foundry Equipment and Supplies entailed a considerable amount of correspondence during the early part of the year. Good progress was made in this direction, however, with the result that the personnel of three subcommittees were named in April, 1932. Efforts were made to hold organization meetings of these subcommittees during the convention of the American Foundrymen's Association, joint sponsor for this project, in Detroit, in May, 1932, but this proved inadvisable because of prevailing economic conditions. Subcommittee No. 1, with LeR. M. Sherwin as Chairman, reports that a considerable amount of progress has been made in the development of its work by correspondence.

An A.F.A. recommended practice for colors of foundry patterns in wood was presented in August, 1932, by that association to the ASA for approval and recognition as an American Recommended Practice. This designation was granted the following month. Pattern markings are included within the scope of this Sectional Committee, so future revisions of this Recommended Practice will be undertaken by it.

Leather Belting. The Chairmanship of the Sectional Committee on Leather Belting is temporarily vacant due to the death of the former Chairman, J. C. Mattern, which is sincerely regretted by the members of the Committee. At the next meeting a successor to this office will be elected. Subcommittee No. 1 on Specifications, with R. C. Bowker, Chairman, held a meeting in November, 1931, at which a general outline of the specifications for leather belting was agreed upon. In accordance with this action the Chairman prepared a preliminary draft of the proposed standard specifications which, on May 26, 1932, was distributed to the Committee members for criticism and comment. On the basis of these comments the draft is being revised prior to distribution to industry for review. During the past year Subcommittee No. 2 on Recommendations for Selection, Care, and Installation, G. A. Schieren, Chairman, has been active in compiling data for inclusion in its recommended practice. This part of the work, it is anticipated, will be ready for submission to the members of the Sectional Committee at an early date.

Classification and Designation of Surface Qualities. The sponsorship for this project was accepted by the Society of Automotive Engineers and the A.S.M.E. in March, 1932. The scope of this project is intended to include standards of quality of finish for machined surfaces, ground surfaces, methods of inspection for the quality grades standardized, and designations of quality grades for use in specifications and on drawings.

A.S.M.E. Committee on Shaft Couplings. D. J. McCormick, Chairman. The proposed American Standard for Shaft Couplings, which had been distributed to industry for comment in the summer of 1931 and published in the October, 1931, issue of *Mechanical Engineering*, was revised in line with the comments received and finally approved by the Committee late in November. In February, 1932, this proposal received the approval of the A.S.M.E. Standardization Committee and in the following April it was approved and adopted by the A.S.M.E. Council as an A.S.M.E. Standard. Immediately upon ratification by the Council the proposal was transmitted to the ASA for approval and designation as an American Standard under the "proprietary standards" method. It was found, however, upon investigation by the ASA, that certain interested organizations were opposed to approval under the "proprietary standards" method in this instance, and the "existing standards" method was offered as a substitute. After carefully weighing all considerations bearing on the case, the Society indicated its willingness to concede this point. As no objections of a technical nature are anticipated, it is expected that the standard for Shaft Couplings will receive the approval of the ASA and designation as an American Standard within the coming month.

Development of Applications for Statistics in Engineering and Manufacturing. W. A. Shewhart, Chairman. In the early part of this year the Chairman of this Joint Committee was invited to give a series of three lectures at the University of London on the role of statistical method in economic industrial standardization. These lectures were delivered in London in May, 1932. While abroad Dr. Shewhart visited several of the industrial laboratories and plants in Great Britain and Germany where work of this character is under way. Arrangements were made also through the sponsor societies and the American Standards Association for him to discuss the work of the committee with the officers of some of the engineering societies in these countries. While in England he had the privilege of attending a round table conference called under the auspices of the British Standards Institution and similar in character to the one held in New York in December, 1929, by the sponsor societies of the U. S. Committee. At the British conference a resolution was passed organizing a committee under the chairmanship of B. H. Wilsdon, of the British Government Building Research Laboratories, to consider undertaking the development of applications of statistical method in engineering and to cooperate with our committee in this effort. While he was in Germany a similar round table conference was called by the president of the Deutscher Normenausschuss which resulted in the formation of a committee under the chairmanship of Dr. W. Hellmich, Managing Director of the Normenausschuss and Co-Director of the Verein deutscher Ingenieure, as chairman, with much the same objective as the British and American committees.

The United States Joint Committee plans to serve in every way possible as a clearing house of information and in particular to issue, from time to time, reports in the columns of the journals of the A.M.S., the ASA, the A.S.T.M., and the A.S.M.E., the sponsor societies, calling attention to the progress that has been made in this particular field. It plans also to cooperate with (*a*) other committees representing societies and industries in America, such as that of the American Society for Testing Materials, on the interpretation and presentation of data, and (*b*) the general committees referred to in this report sponsored by the British Standards Institution and the Deutscher Normenausschuss.

Respectfully submitted,

LEWIS K. SILLCOX, *Chairman*
WILLIAM S. MONROE CLARENCE W. SPICER
EARLE BUCKINGHAM ALFRED IDDLES.

SAFETY

Throughout the year the standing committee on Safety has continued to arrange for joint safety sessions with the A.S.M.E. Professional Divisions when desired, to promote the development of safety codes, and to continue cooperation with the American Society of Safety Engineers, the engineering division of the National Safety Council.

Safety Education. During the December, 1931, meeting of the Committee instruction in industrial safety in trade schools and colleges was discussed at length. This discussion resulted in the appointment of W. M. Graff as a committee of one to cooperate with the committee of six, Prof. J. W. Hallock, Chairman, appointed by the Society for the Promotion of Engineering Education and W. Dean Keefer, Secretary of the National Safety Council. Contact was made with the S.P.E.E. Committee through one of its members, J. P. Jackson, a former chairman of the A.S.M.E. Standing Committee. As a direct result of these conferences a group of suggestions to college deans and professors concerning courses of study was incorporated in the report of the S.P.E.E. Committee.

Aircraft Safety. A special committee appointed to develop recommendations for aircraft safety and inspection distributed in May, 1932, some three hundred copies of a tentative draft of the proposed "Inspection Manual for Maintenance of Conventional Aircraft." With the exception of the reports on airport and hangar inspection, this material is a compilation and rearrangement of a number of inspection forms used at the present time by leading aircraft operators. It is the intention of the committee to revise the Manual from time to time.

Of the 44 safety code committees now functioning under the procedure of the American Standards Association five are sponsored solely or jointly by the A.S.M.E. These are:

Sectional Committee on a Safety Code for Mechanical Power Transmission Apparatus
Sectional Committee on a Safety Code for Elevators
Sectional Committee on a Safety Code for Machinery for Compressing Air
Sectional Committee on a Safety Code for Conveyors and Conveying Machinery
Sectional Committee on a Safety Code for Cranes, Derricks, and Hoists.

Personnel. The Sectional Committees on safety codes sponsored by the A.S.M.E. now have a membership of 210, 84 of which are members of the Society. These five Committees have subdivided their work among 22 subcommittees and subgroups, making a total of 27 committees at work on this safety program.

Publicity. During February, 1932, Ernest Hartford, Assistant Secretary, A.S.M.E., on behalf of the A.S.M.E. participated in the planning for the Greater New York Safety Conference held in the Pennsylvania Hotel the week of the 21st of February. One evening that week he gave a radio address over WNYC outlining the part that safety codes and standards play in promoting accident prevention work.

Elevators. S. W. Jones, Chairman. Following the approval, in July, 1931, of the American Standard Safety Code for Elevators, Dumbwaiters, and Escalators (a revision of the 1925 Code), it became evident that the Committee would be called on for interpretations of the rules forming this code. After a full discussion of all aspects of the problem, it was decided that requests for interpretations should be sent to John Dickinson, Secretary of the Committee. They would then be considered at the next meeting of the Subcommittee on Research, Interpretations, and Recommendations, M. H. Christopherson, Chairman, and a tentative reply mailed to the inquirer. At annual or semi-annual intervals these interpretations will be sent to the members of the entire Sectional Committee for approval after which they will be published in *Mechanical Engineering* and issued as an addenda to the Code. Copies of these interpretations will then be sent to each jurisdiction using the code. The Subcommittee held eight meetings during the year.

With the adoption of the Safety Code for Elevators by many states and cities of the country, has grown a demand for an arrangement whereby elevator safeties and buffers may be tested and certified by a recognized and impartial testing laboratory. The National Bureau of Standards enjoys this distinction and, since it had been cooperating with this Committee in the research side of its activities, the necessary facilities and personnel were available for the conduct of these certification tests. Following conferences with the Director of the Bureau of Standards, the Committee announced its willingness to accept the responsibility of arranging for and supervising such tests for the manufacturers. This action received hearty approval on the part of the manufacturers and a number of them have already submitted their buffers for test and certification.

Work on the Elevator Inspectors' Handbook has been advanced considerably during the year by the Subcommittee of which K. A. Colahan is Chairman. A tentative draft (August, 1932) of this handbook is now being reviewed by the members of this Subcommittee. It consists of Part I, "Routine Inspection," and Part II, "Initial or Data Inspection," with an appendix covering an inspection form and tables in which are set forth safety factors and clearances.

Cranes, Derricks, and Hoists. J. C. Wheat, Chairman. During the year the Subcommittee on Editing completed its work of editing and correlating the sections of the code and released the first draft to Chairman Wheat. All agreed, however, that this editing process had revealed certain weaknesses and gaps in the Code as it stands. On August 29, therefore, a letter was written to all members of the Sectional Committee submitting a copy of the edited code and asking them to send their comments direct to Lieut. Commdr. P. J. Searles, Chairman of the Editing Committee. The Chairman hoped that by bringing certain detailed points still in doubt to the attention of the members of the entire Sectional Committee by correspondence, the work of the Committee would be completed before the end of the year without the calling of a meeting.

Boiler Code. For many years the Society has sponsored the work on the "A.S.M.E. Boiler Code" in its eight subdivisions. Although this part of the technical committee work is handled by the Special Boiler Code Committee, it is mentioned here so as to complete this report of the Society's activity in the field of industrial safety.

In addition to these Committees, the Society has representation on 26 other safety code committees. Seventeen of them have completed their codes. These committees are as follows:

Sectional Committee on Safety Code for—

- Abrasive Wheels
- Amusement Parks
- Correlating Committee
- Aeronautics
- Floor Wall Openings, Railings, and Toe Boards
- Industrial Sanitation
- Lighting Factories, Mills, and Other Work Places
- Forging and Hot Metal Stamping
- Ladders
- Logging and Sawmill Machinery
- Textiles
- Underground Power Transmission and Power Equipment for Metal Mines
- Machine Tools
- Paper and Pulp Mills
- Power Presses, and Foot and Hand Presses
- Rubber Machinery
- Walkway Surfaces
- Laundry Machinery and Operation
- Window Washing
- Color for the Identification of Gas Mask Canisters
- Construction Work
- Protection of Industrial Workers in Foundries
- Prevention of Dust Explosion
- Ventilation
- Mechanical Refrigeration

Special ASA Committee on Low Voltage Electrical Hazard.

Respectfully submitted,

Harold W. Mowery, *Chairman*
Martin H. Christopherson   Lewis Price
Thomas A. Walsh, Jr.   Wesley M. Graff.

POWER TEST CODES

The Committee on Power Test Codes presents the following report for the year ending September 30, 1932.

Test Codes Completed

During the year just closed three sections of Instruments and Apparatus were completed and issued in pamphlet form, namely, Part 9, Heat of Combustion; Part 12, Measurement of Time; and Part 18, Humidity Determinations. This means that 20 of the proposed 23 test and supplementary codes and 15 of the proposed 35 sections of Instruments and Apparatus have been completed and published during the period 1922–1932.

Personnel Changes

The Committee records with deep regret the deaths during the year of two of its members who have been active in the work of the Individual Committees: Max E. R. Toltz, who was a member of Committee No. 15 on Steam Locomotives; and William B. Chapman, who was a member of Committee No. 16 on Gas Producers.

A reorganization of Committee No. 5 on Reciprocating Steam Engines was undertaken during the year and Prof. Alexander G. Christie was appointed to succeed William C. Brown as Chairman. The personnel of this Committee now includes Harte Cooke, Alexander G. Christie, Kenneth Davidson, Herman Diederichs, Henrik Greger, Thomas Hall, John A. Hunter, Herman G. Mueller, Bruno V. E. Nordberg, Jr., J. F. Max Patitz, Alex V. Saharoff, A. G. Witting, and J. C. Workman.

Due to a contemplated revision of the Test Code for Steam Turbines, the enlargement of the personnel of Committee No. 6 was reviewed by the Main Committee this year and the following experts in the field of turbine testing have been added: Messrs. Oliver D. H. Bentley, Alexander G. Christie, Vincent M. Frost, Arthur E. Grunert, James N. Landis, Richard O. Muller, and T. E. Purcell. E. H. Brown has relinquished his membership on Committee No. 6 since his present work is foreign to this subject.

G. T. Felbeck has been appointed to membership on Committee No. 9 on Displacement Compressors and Blowers, while J. F. G. Miller has been released from membership.

At the time of the December, 1931, meeting of the Main Committee, the November, 1931, draft of the proposed Test Code for Complete Steam-Electric Power Plants was presented but not approved for a number of reasons. It seemed desirable, also, to dissolve Committee No. 11 with thanks for the work it had done and to organize a new Committee. The personnel of the reorganized Committee No. 11 is as follows: Ward F. Davidson, G. H. Fellows, R. A. Forseman, Vincent M. Frost, W. W. Johnson, Edw. W. Norris, C. U. Savoye, David S. Wegg, and Herbert S. Whiton, with Frank M. Van Deventer as Chairman.

The following appointments were made to the recently reorganized Committee No. 18 on Hydraulic Power Plants: Charles M. Allen, Harold L. Doolittle, William F. Durand, Norman R. Gibson, James P. Growdon, Thomas H. Hogg, Daniel J. McCormack, Lewis F. Moody, R. V. Terry, and William M. White, with Ely C. Hutchinson as Chairman.

It was with considerable regret that the Committee accepted the resignation of R. J. S. Pigott from Committees No. 2 on Definitions and Values and No. 19 on Instruments and Apparatus.

The Main Committee on Power Test Codes and the 20 Individual Committees now include in their personnel 118 members of the Society and 24 non-members.

Committee Activity

As usual the four quarterly meetings of the Main Committee were held in October, December, March, and June.

Regularly called committee meetings were held during the year by Committee No. 10 on Centrifugal and Turbo-Compressors and Blowers; Committee No. 12 on Condensers, Water-Heating and Cooling Equipment; and Committee No. 18 on Hydraulic Power Plants and Their Equipment. Several of the other committees have carried on extensive correspondence relative to work on their codes.

The preliminary draft of the Test Code for Gaseous Fuels was completed within the year. In the preparation of this Code, due to the rather wide variety of constituents encountered in the fuel, it has seemed desirable to avoid as much as possible the more or less empirical formulas for volume-weight conversion, air requirements, and products of combustion, as well as similar equations for heat losses. There has been an endeavor to develop these items on the pound-mole basis, referring all calculations to international atomic weights, thus developing a system applicable to the combustion of any fuel wherein the results are secured in a logical manner from primary data.

A preliminary draft of a proposed revision of the Test Code for Reciprocating Steam Engines has been completed and is being worked over for final review by the members of Committee No. 5 before it is submitted to the Main Committee for approval.

During the year a preliminary draft of a revised Test Code for Steam Turbines has been in preparation, since recent developments in the art and practice warranted a revision and extension of the scope and nature of the Code. In the preparation of the present Code the Committee deliberately avoided treatment of extraction and mixed-pressure turbines because their use was not sufficiently general and had not covered sufficient time for the development and crystallization of the testing practice. Further, the Committee had refused, up to that time, to recognize methods of measurement which it believed were still in the experimental or research stage of development, such, for example, as the use of thermocouples for measuring steam temperature, the use of nozzles for measuring steam flow, and the use of absolute-pressure manometers for measuring exhaust pressure. However, a number of these methods have now received general acceptance and are to be recognized in the revised draft now in preparation. With the inclusion of bleeder and mixed-pressure turbines in the Code, it becomes imperative that rules be given for making corrections of departure of test conditions from the operating conditions specified in the guarantees.

The preliminary draft of the Test Code for Centrifugal Compressors, Exhausters, and Fans has been distributed for criticism and comment to a group of individuals who are known to be interested in the subject. The present Code has been formulated in two parts, the first dealing with compressors and exhausters and the second with fans. It is expected that this Code will be in shape for a public hearing at the coming Annual Meeting of the Society in December, 1932. Prompted by the activities of Committee No. 10,

H. F. Hagen, one of its members, has prepared a paper on "Pulsation in Air Flow From Fans and Its Effects on Test Procedure," and Prof. L. S. Marks, a member of the Main Committee, also has developed a paper, entitled "Influence of Bends in Inlet Ducts on the Performance of Induced Draft Fans." Since it is likely that the presentation of these papers will prompt discussion having a bearing on the proposed new test code, the Main Committee recommended that the papers be presented to the members of the Society in conjunction with this Public Hearing.

While the Test Code for Condensing Apparatus provides for the conduct of performance tests for all types of condensing apparatus with their auxiliaries, it covers only a test of the condenser with the surface condition of the tubes as it happens to be at the time of the test. Since compliance with the contract and acceptance should be based on new or clean tubes it often becomes desirable to determine the degree of cleanliness of the transfer surfaces. During the past year, at the request of the Main Committee, Committee No. 12 on Condensers, Water-Heating, and Cooling Equipment has been giving the problem careful consideration and study. It is recognized, however, that the subject is a difficult one and some little time will elapse before an accepted method will be developed for inclusion in the Test Code.

Committee No. 18 on Hydraulic Power Plants has been developing a first draft of a proposed revised Test Code for Hydraulic Power Plants and Their Equipment. It is the Committee's hope to set up a test code which can be bound in with a contract so that the contract, specification, and test code will form a complete document. In order to meet the present demands of the industry, the Committee plans to develop the Test Code in two separate parts, one covering the rules for acceptance tests, and the other, rules for operating tests after acceptance.

A résumé of the progress which has been made during the past year in the preparation of the various parts and chapters of Instruments and Apparatus is as follows: Part 4, Head Measuring Apparatus, appeared in the June, 1932, issue of *Mechanical Engineering;* Part 12, Measurement of Time, in the October, 1931, issue; Part 18, Humidity Determinations, in the November, 1931, issue; and Part 21, Leakage Measurement—Chapter 2 on Boiler and Piping Leakage and Chapter 3 on Steam Engine Leakage, in the January, 1932, issue. The development of the following parts and chapters has progressed appreciably within the past year: Part 3, Temperature Measurement—Chapter 2 on Radiation Pyrometers, Chapter 4 on Resistance Thermometers, and Chapter 8 on Optical Pyrometers; Part 4 on Head Measuring Apparatus; Part 5 on Measurement of Quantity of Material; Part 6 on Electrical Measurements; Part 10 on Flue Gas Analysis; Part 14 on Mechanical Measurements; Part 15 on Measurement of Surface Areas; and Part 19 on Concentration of Dilute Solutions.

Some time ago Committee No. 19 on Instruments and Apparatus informed the Main Committee that it had been greatly handicapped in certain parts of its work by the lack of information which would enable it to state the degree of precision and accuracy of the various types of measuring instruments employed in the measurement of the quantities involved in acceptance tests. As one means of relieving this situation the Committee suggested that the determination of these data would furnish excellent material for research work for senior students in technical colleges and universities. The Main Committee was in accord with the suggestion and requested that a list of short-time assignments for individuals be prepared for distribution to the interested colleges and universities for their review.

Abstracts of preliminary drafts of the Power Test Codes and other related material have been prepared and published in *Mechanical Engineering* during the year. This material has filled ten pages.

Special Problems

One of the two problems brought to the attention of the Power Test Codes Committee this year was the desirability of its taking up the formulation of a code for the determination of solid matter and undesirable chemical substances in flue gases. This proposal was carefully considered and the Committee decided that preparatory to the development of such a code, a thorough investigation of the subject should be made by the A.S.M.E. Pure Air Committee.

Consideration was given also to a communication received from the A.S.M.E. Boiler Code Committee relative to the present method of calculating boiler heating surface. The sections of the Code on Definitions and Values and the Test Code for Stationary Steam-Generating Units deal with this quantity and the Committee decided to make no changes in the methods there described.

Principles Underlying the A.S.M.E. Power Test Codes

During the past two years the members of the Committee have given considerable thought to the revision of the statement of principles which underlie the formulation and use of the A.S.M.E. Power Test Codes. At the May 25, 1931, meeting the Main Committee appointed a special committee consisting of F. Hodgkinson, Chairman, C. Harold Berry, C. F. Hirshfeld, E. B. Ricketts, and William Monroe White to study this subject in a broad way and to prepare a revised statement if that was found to be desirable. The report of this Special Committee was approved and adopted by the Main Committee at its October 5, 1931, meeting and copies were distributed to the entire membership of the Committee on Power Test Codes for their future guidance. At that time the Main Committee also reaffirmed the fundamental objectives and purposes of the Power Test Codes as adopted in 1918.

International Electrotechnical Commission

Through membership on the U. S. National Committee of the I.E.C., the Society and the Main Committee on Power Test Codes have been actively engaged in assisting to bring about international agreements on acceptance tests for steam turbines and internal-combustion engines. The Secretariat for the Advisory Committees on Steam Turbines, Internal-Combustion Engines, and Hydraulic Turbines is held by the United States. The Society was represented this year on the U. S. National Committee by Harvey N. Davis, Francis Hodgkinson, and Ely C. Hutchinson, with C. Harold Berry and Paul Diserens as alternates.

I.E.C. Steam Turbines

The Secretariat reported last year that complete documents for Part I, "Specification," and Part II, "Rules for Acceptance Tests," were agreed to at the Stockholm meeting in July, 1930. These publications were issued by the Central Office in July, 1931. As reported last year, also, the following subjects still are before Advisory Committee No. 5: (1) the Supplementary Notes on Instruments and Methods of Measurement; (2) recommendations of I.S.A. on Velocity Measurements of Fluids; (3) consideration of proposals contained in a paper by Mr. Darrieus on "A Rational Definition of Steam Turbine Efficiencies;" and (4) consideration of increasing the Scope of the Rules beyond that which had been previously agreed to. It is believed that, with the recommendations which have been received thus far from the member countries on these several subjects, the work of the Secretariat has progressed as far as is possible without a meeting of Advisory Committee No. 5 and it is thought that such a meeting might advantageously be called during the year 1933.

I.E.C. Internal-Combustion Engines

During the past year the Secretariat has prepared a document for Part I, "Specification" for the I.E.C. Publication on Internal-Combustion Engines based on the Recommendations received from the several member countries since the Stockholm, 1930, meeting of Advisory Committee No. 19.

Proposals by the U. S. National Committee for Part II, "Rules for Acceptance Tests" were submitted at Stockholm but were not discussed. It was agreed, however, that the member countries would send their recommendations and comments thereon to the Secretariat to the end that amendments may be made to the proposals, reconciling as far as possible the views of the respective nations. France, Germany, and Sweden have requested an extension of time, which the Secretariat has granted to September 1, 1932, following which the Secretariat will prepare a document reconciling as far as possible the views of the member countries. With the completion of this task after that date, the work of the Secretariat will have gone as far as possible without a meeting of Advisory Committee No. 19 and it is thought that such a meeting might be called advantageously during the year 1933.

Organization of Electrical Standards Committee

From time to time during the past six years a number of efforts have been made to unify and simplify the organization for standardization in the electrical industry. This unification involved the joining of the present U. S. National Committee of the International Electrotechnical Commission with the other groups which formed the Electrical Advisory Committee affiliated with the American Standards Association. At its October 5, 1931, meeting the Main Committee on Power Test Codes carefully reviewed the steps in this development which had been taken and considered the possible effect of the newly organized Electrical Standards Committee on the work of the A.S.M.E. Committees on Power Test Codes and Boiler Construction Codes.

The Main Committee then addressed a communication to the A.S.M.E. Council relative to the full meaning of the "Scope" of the Constitution of the Electrical Standards Committee. The recommendations offered were accepted at the October 9, 1931, meeting of the Executive Committee of the Council. The Executive Committee approved also the recommendation of the A.S.M.E. representatives on the U.S. National Committee, I.E.C., that the Constitution of the E.S.C. provide that the A.S.M.E. be designated as a member of the U.S.N.C. with three representatives.

An official letter of interpretation from the American Standards Association has been received giving assurance that the Electrical Standards Committee will not concern itself with projects dealing with apparatus and operation on the prime mover side of the generator coupling. Likewise unanimous approval was given by the E.S.C. and the ASA to the amendment to the Constitution of the former providing for the addition of three representatives of the A.S.M.E.

Respectfully submitted,

Fred R. Low, *Chairman*
C. Harold Berry
Albert T. Brown
Edwin H. Brown
Alexander G. Christie
Harte Cooke
Paul Diserens
Robert H. Fernald
Edwards R. Fish
C. F. Hirshfeld
Francis Hodgkinson
Ozni P. Hood
George A. Horne
Ely C. Hutchinson
David S. Jacobus
Lionel S. Marks
Lewis F. Moody
Irving E. Moultrop
Henry B. Oatley
Geo. A. Orrok
Reginald J. S. Pigott
Edwin B. Ricketts
Edward N. Trump
William Monroe White
Walter J. Wohlenberg.

PROFESSIONAL CONDUCT

The Committee has had only routine matters to consider during the past year, none of which involved serious misconduct on the part of any of our members.

The Committee has received criticisms of the form of the present Code of Ethics and has under consideration certain modifications which may be submitted to Council in due course. These changes are intended to further dignify and add to the appeal of the Code.

Respectfully submitted,

A. G. Christie, *Chairman*
D. Robert Yarnall
James H. Herron
H. S. Philbrick
E. R. Fish.

BOILER CODE

The Boiler Code Committee submits the following report for the fiscal year ending September 30, 1932.

During this period the Committee held nine regular meetings, one of which was a three-day meeting, and several Executive Committee meetings, which were devoted to interpretations of the various sections of the Code and to the formulation of revisions and addenda thereto. Forty-four interpretations were added during the past fiscal year, making a total of 731 interpretations now in force.

The Committee has been requested to formulate rules for the design and bolting of flanges and has given consideration to the problem. In view of the work being carried on by the Sectional Committee on Standardization of Pipe Flanges and Fittings, further consideration by the Boiler Code Committee has been postponed until the report of the Sectional Committee is available.

The Committee has devoted careful study to a problem in regard to conformity to the Code of two mercury boiler equipments for use in power generating stations. As this was a new and distinct step in power plant development, the boilers and parts could not be made to meet the Code requirements in all respects, and as the boilers were to be installed in states requiring compliance with the Code rules, special action was necessary by the authorities in charge of the enforcement of the Code. It was explained that in dealing with new types of apparatus not covered by its Codes, the Boiler Code Committee had on several occasions taken the position that it favors having trials made by responsible parties in order to extend the knowledge of the art, and that the Committee had recommended to the authorities in charge of enforcement of the Code that permission be given to install new types of apparatus as special cases with the understanding that those making an installation assume any risk that might be involved through its being of a special design not fully covered by the Codes. Recommendation was made to the chief boiler inspectors of the states in which the boilers were to be installed that each of the units be accepted as a special case.

In all of the Codes for Unfired Pressure Vessels that have been issued a statement is made that the rules are not applicable to certain vessels, some of which are used in the petroleum industry. The American Petroleum Institute is now cooperating with The American Society of Mechanical Engineers and a Joint Committee is preparing rules for the construction of vessels for flammable liquids and gases.

F. A. Page, chief boiler inspector of the State of California, submitted proposed revisions of the California Air Tank Safety Orders to the Boiler Code Committee for comments and suggestions. The Boiler Code Committee made a number of recommendations and explained certain provisions of its Code for Unfired Pressure Vessels. It advised Mr. Page that the Joint Committee of the American Petroleum Institute and The American Society of Mechanical Engineers is working on rules to meet the special requirements of the petroleum field and suggested that the State of California wait until the results of the work of this Joint Committee are available before endeavoring to cover the entire field of unfired pressure vessels.

Revised rules covering the reinforcement of openings have been embodied in the Code. A great amount of painstaking work was done in preparing these rules by the Subcommittee on Reinforcements, as well as by the Boiler Code Committee itself, and there is a general agreement that the revised rules are much better than those previously used.

In addition to the foregoing decisions many revisions have been made which either amplify or clarify the new welding rules which were adopted and incorporated in the Code last year.

A number of revisions have been made in the Materials Specifications Section of the Code. Many of the specifications appearing in this section have been adopted from the American Society for Testing Materials. Therefore, as a result of the continued cooperation with that society, when changes are made by the A.S.T.M. in these identical specifications, corresponding changes are made in the A.S.M.E. Code to bring them into conformity.

The Committee has also made numerous revisions and additions to the various sections of its Code as a result of suggestions from manufacturers, inspectors, users, and others. These revisions were formally adopted by the Council on August 17, 1932, and distributed in pink-colored addenda sheets.

At the request of the Boiler Code Committee, the Council made the following appointments: R. E. Cecil and A. C. Weigel as members of the Main Committee; J. W. Lockett as a member of the Subcommittee on Heating Boilers to succeed W. A. Nevin; C. A. Adams, A. W. Limont, Jr., H. S. Smith, and D. B. Wesstrom as members of the Subcommittee on Unfired Pressure Vessels. J. A. Dixon resigned from the Subcommittee on Welding.

Respectfully submitted,

F. R. Low, *Chairman*
D. S. Jacobus, *Vice-Chairman*
C. W. Obert, *Hon. Secretary*
H. E. Aldrich
W. H. Boehm
R. E. Cecil
F. S. Clark
F. W. Dean
W. F. Durand
E. R. Fish
V. M. Frost
C. E. Gorton
A. M. Greene, Jr.
F. B. Howell
J. O. Leech
E. F. Miller
M. F. Moore
I. E. Moultrop
C. O. Myers
H. B. Oatley
James Partington
C. L. Warwick
A. C. Weigel
H. LeRoy Whitney.

REGISTRATION OF ENGINEERS

The Committee on Registration of Engineers has been cooperating with the joint committee of the various engineering societies which has been formulating a Model Law for the registration of professional engineers and architects, and since the last Annual Meeting a member of the Committee has been officially representing our Society at the joint conferences.

The Model Law has been printed and distributed to the various organizations interested, and one of the major societies has officially approved it. The Committee will recommend to the Council at its Annual Meeting this year that the A.S.M.E. also approve the Model Law. This action will not commit the Society to a policy of advocating registration laws but only of cooperating to the extent that legislation shall be carried along rational lines.

The Committee desires again to call the attention of the membership to the fact that a majority of the states have registration laws and the movement is being extended. Each member should study the laws in his state and ascertain whether his work is of such a nature that he should be registered. The Committee cannot issue blanket recommendations which will cover all the members living in the United States, and, in fact, would not attempt to interpret some of the laws as now written.

We have found many mechanical engineers inclined to disregard the law. This is a hazardous thing to do. Even though failure to register may result in no immediate difficulty, the lack of a license may be a bar to promotion to a position of greater responsibility. A license cannot be obtained quickly, but may require several months or even several years, as it is becoming more difficult each year to acquire a license in some states.

Respectfully submitted,

John H. Lawrence, *Chairman* C. F. Hirshfeld
D. S. Kimball V. M. Palmer
Paul Doty J. M. Todd.

ECONOMIC STATUS OF THE ENGINEER

At the Annual Meeting in 1931 this Committee rendered two reports, one the report on 1930 Earnings of Mechanical Engineers, the other a recommendation for the establishment of a joint agency of engineering bodies through which the general status of the profession could be improved.

The Earnings Report, presented at the 1931 Annual Meeting business session, has had a profound effect on thought about the status of the engineer, the process of educating engineers, and the steps that must be taken to advance the broad status of the profession. An authoritative interpretation was presented at the Corvallis meeting of the Society for the Promotion of Engineering Education. A paper dealing with the fundamental philosophy of the engineer as an administrator was presented at the Cleveland Meeting of the American Institute of Electrical Engineers. Both of these papers dealt very largely with the conclusions of the Report. The significance and portent of some of the results of the studies have been brought to the attention of deans of engineering schools by correspondence. The Local Sections of the Society have held a score or more of meetings at which the findings have been discussed and the comments indicate that the investigation has been very much worthwhile. The underlying statistics for the Report have been consulted by several agencies which are engaged in similar studies for other professions. Slides of the curves are available for the use of any who wish to present the findings of the Committee before audiences. In following up this phase of its activity the Committee is developing a plan for selecting career consultants who will be available to younger members of the Society wishing advice and guidance.

The Committee's finding that the task of improving the earnings of the engineer involves improving his professional and social status, led to the conclusion that the united profession must further education, subsequent training, and some form of professional recognition. The Council approved this view and authorized the necessary conferences. As a result, the Engineers' Council for Professional Development organized on October 3, 1932.

During the year the Committee has arranged for the appointment of an ex-officio member from the Committee on Relations with Colleges, inasmuch as a great deal of the work of the Committee will have a relationship to the educational work of the Society.

The death of Col. W. A. Starrett was a great loss to the Committee. His high ideals for the status of the engineer, and his vigor of thought and expression were of tremendous value.

Respectfully submitted,

C. F. Hirshfeld, *Chairman* H. L. Whittemore
Dexter S. Kimball W. E. Wickenden
H. B. Oatley C. N. Lauer
J. M. Todd } *Ex-Officio.*
M. C. Maxwell }

Reports of Meetings

THE following pages present brief reports of meetings of the Society and of its Professional Divisions during 1932. More detailed accounts were published in *Mechanical Engineering* and the *A.S.M.E. News*. The meetings reported here are:

MEETINGS OF THE SOCIETY

Semi-Annual, Bigwin Inn, Lake of Bays, Ontario, Canada, June 27–July 1, 1932
Annual, New York, N. Y., December 5–9, 1932

MEETINGS OF THE PROFESSIONAL DIVISIONS

Aeronautic, Berkeley, Calif., June 9–10, 1932
Aeronautic, Buffalo, N. Y., June 6–7, 1932
Applied Mechanics, New Haven, Conn., June 23–25, 1932
Iron and Steel, Pittsburgh, Pa., April 28, 1932
Machine Shop Practice, Buffalo, N. Y., October 3, 1932
Management and Material Handling (M-M-M Congress), Detroit, Mich., May 4–5, 1932
Oil and Gas Power, State College, Pa., June 8–11, 1932
Printing Industries, New York, N. Y., March 14–15, 1932
Process Industries, Buffalo, N. Y., June 6–7, 1932
Textile, Boston, Mass., October 27, 1932
Wood Industries, Jamestown, N. Y., November 15–16, 1932.

Semi-Annual Meeting

BIGWIN INN, LAKE OF BAYS, ONTARIO, CANADA, JUNE 27–JULY 1, 1932

The majority of those attending the 1932 Semi-Annual Meeting reached Toronto on the morning of June 27 and were given an opportunity to inspect the harbor developments there before going on to Bigwin Inn in the afternoon. The first evening was spent in an informal get-together, the technical sessions not beginning until the second day.

A Business Meeting was held on the morning of June 28 at which an announcement was made that the 1933 Semi-Annual Meeting would be held in Chicago during the week of June 26, 1933, at the time of the Century of Progress. At the close of the Business Meeting the Council went into its first session since December, 1931. After a résumé of the activities of the executive committee, which had met monthly during the intervening time, the Council took up the subject of the budget for the fiscal year starting October 1, 1932. The new budget provided for a reduction in expenditures of 34 per cent over the year 1929–1930, for the development in three new areas of the Student Branch program tried out in the South during the past year, and for writing off the loan to The Engineering Index except for items actually in inventory. The Council also approved the formation of a joint council, to be called the Engineers' Council for Professional Development, to establish standards of admission into the profession. The proposal to establish such a council grew out of the work of the Committee on the Economic Status of the Engineer.

The Nominating Committee, meeting on Tuesday and Wednesday, selected the following candidates for new officers of the Society in 1932–1933:

President, A. A. Potter
Vice-Presidents, Harold V. Coes, James D. Cunningham, C. F. Hirshfeld
Managers, Alex D. Bailey, John A. Hunter, R. L. Sackett

The following delegates to the American Engineering Council were nominated:

B. M. Brigman, W. S. Conant, Hugo Diemer, W. A. Hanley, Ernest Kreher, G. E. Pellissier, J. W. Roe, B. R. Van Leer, T. R. Weymouth, D. Robert Yarnall

Dr. J. M. R. Fairbairn, chief engineer of the Canadian Pacific Railway, was toastmaster at the main entertainment event, the banquet and dance on Thursday evening. The speakers were Past-President Wm. L. Abbott, Dr. Lillian M. Gilbreth, and George D. Y. Leacock. Special recognition was made of the services to the Society of Prof. Robert W. Angus, and prizes were awarded for the sports events to which the afternoons of the meeting were largely devoted. Other social events included a masquerade dance.

A conference of Local Sections Delegates took place on Wednesday morning, and honorary chairmen of Student Branches held a luncheon conference on Thursday.

As the following list of papers indicates, the sessions reflected engineering practices in Canada and Great Britain, as well as in the United States.

Tuesday Morning, June 28

APPLIED MECHANICS (I)

Symposium on Use of Models in Engineering

Use of Models in Aerodynamics and Hydrodynamics, O. G. Tietjens
Determination of the Speed and Power of Ships From Model Experiments, H. C. Sadler
Use of Models Under Centrifugal Force, Philip B. Bucky

RAILROAD

Roller Bearings for Railroad Service, T. V. Buckwalter
The C. P. R. Multipressure Locomotive No. 8000—
  Its Construction and Operation, H. B. Bowen*
  Its Machinery, Locomotive Chassis, and Tender, J. B. Ennis
  Its Steam Generating System, F. A. Schaff

MACHINE SHOP PRACTICE (I)

Oxygen Machine Cutting in Great Britain, C. G. Bainbridge**
Developments in the Design and Application of Gearing, Henry E. Merritt**

Wednesday Morning, June 29

APPLIED MECHANICS (II)

Symposium on Use of Models in Engineering

Model Testing as Applied to Strength of Materials, R. E. Peterson
Use of Models in Vibration Research, J. P. Den Hartog
Use of Models in Studies of Stresses in Structures, Geo. Beggs
Use of Models in Determining the Strength of Thin Wall Structures, Harold E. Saunders and Dwight F. Windenburg

HYDRAULIC (I)

Hydraulic Practice in Europe, R. W. Angus*
Hydro-Electric Development in Canada, T. H. Hogg*

MANAGEMENT

Current Problems of Industrial Management, Jas. D. Mooney
Is There Any Basis Left for the Taxable Value of Manufacturing Properties? Chas. T. Main

OIL AND GAS POWER AND RAILROAD

British High-Speed Oil Engines, J. L. Chaloner**
Surface-Volume Ratio as a Critical Factor in Automotive Diesel Combustion Chambers, Julius Kuttner

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* Contributions from Canada.
** Contributions from Great Britain.

Wednesday Evening, June 29

Standards of Qualifications for Engineers

Present Status of Professional Engineering in Canada, Col. Ibbotson Leonard*

Thursday Morning, June 30

Power (I)

British Practice in Steam Turbine Design, F. W. Gardner**
Steam Turbine Plant Practice in the United States, Vern E. Alden and W. H. Balcke

Machine Shop Practice (II)

Research on Steels and Forgings for Greater Density, Machinability, and Durability, Fred W. Cederleaf and W. E. Sanders
Life of Tools Machining Free Cutting Brass, L. D. Spence and Jas. A. Hall

Hydraulic (II)

Hydraulic Jump in Sloping and Horizontal Flumes, R. W. Ellms
Investigation of Flow Conditions in a Centrifugal Pump, Karl Fischer and D. Thoma

Friday Morning, July 1

Power (II)

Economic Balance Between Steam and Hydro Capacity, K. M. Irwin and Joel D. Justin

Annual Meeting, New York, N. Y.

DECEMBER 5–9, 1932

The Fifty-Third Annual Meeting brought to the Engineering Societies Building more than 2100 members of the A.S.M.E. and their friends during the week of December 4, 1932. The Local Sections delegates met in conference on Sunday, December 4, and continued throughout the day Monday and on Tuesday morning. Among the subjects discussed was the Junior Division of the Los Angeles Section, organized in January, 1932, to promote greater activity among the Junior and Student members in that Section and reported as providing a forum where these younger engineers may secure experience and confidence helpful to them when participating in meetings with older members. The delegates passed a resolution asking for a revision of the Constitution, By-Laws, and Rules to give constitutional status to the Local Sections Delegates Conference, and voted for the discontinuance of the *A.S.M.E. News* in view of the reduced publications budget. The regular Nominating Committee for officers for 1933–1934 was selected and a Committee of the Conference was appointed to meet with the Revenues Committee to consider the financial problems of the Society. A more complete report of the conferences was published in the *A.S.M.E. News* for December 22, 1932.

This issue also contained a summary of the meeting as a whole, particularly the non-technical events, and the January, 1933, issue of *Mechanical Engineering* will include details of the technical sessions, the program for which was as follows:

Monday Morning, December 5

Working Stress Symposium (I)

(Auspices of Applied Mechanics Division)

Theories of Strength, A. Nadai
Working Stresses for Columns and Thin-Walled Structures, S. Timoshenko
The Fatigue of Metals—Its Nature and Significance, H. F. Moore
Stress-Concentration Phenomena in Fatigue of Metals, R. E. Peterson

Locomotives

(Auspices of Railroad Division)

Horsepower and Tractive Effort of Steam Locomotives (Locomotive Ratios), A. I. Lipetz
Progress Report of Railroad Division, presented by L. K. Sillcox

Soaking Pits and Radiant Heat

(Auspices of Iron and Steel and Fuels Divisions)

Use of Blast-Furnace Gas in Soaking Pits, G. T. Hollett
Effect of Reradiation on Heat Transmission in Furnaces and Through Openings, H. C. Hottel and J. D. Keller

Monday Afternoon, December 5

Working Stress Symposium (II)

(Auspices of Applied Mechanics Division)

Applications of Creep Tests, Gleason H. MacCullough
Factors Affecting Choice of Working Stresses for High-Temperature Service, P. G. McVetty
Metals at High Temperature—Test Procedure and Analysis of Test Data, Ernest L. Robinson

Special Design of Cars

(Auspices of Railroad Division)

Car Design of the Future, C. E. Barba
Development of Special Rail Equipment, R. C. Pierce

Metals

(Auspices of Iron and Steel Division)

The Rolling and Extrusion of Aluminum and Its Alloys, Robert L. Streeter
Carburizing With Butane Gas, C. W. Spicer
Progress Report of Iron and Steel Division

Power Test Code Public Hearing

(American Society of Heating and Ventilating Engineers Cooperating with the A.S.M.E. in Sponsoring this Session)

Influence of Bends in Inlet Ducts on the Performance of Induced-Draft Fans, L. S. Marks, John Lomax, and Randolph Ashton
Pulsation of Air Flow From Fans and Its Effect on Test Procedure, Harold F. Hagen
Public Hearing on Test Code for Centrifugal Compressors, Exhausters, and Fans

Tuesday Morning, December 6

Industrial Power

(Auspices of Power Division)

Observations of Liquid Water in Steam Passages and Means for Effecting Separation, Carl C. Thomas
High-Pressure Steam-Generator Research (Progress Report of Investigation at Purdue Univ. by A.S.M.E. Special Research Comm. on Critical-Pressure Steam Boilers), H. L. Solberg, G. A. Hawkins, and P. A. Willis
Progress Report of Power Division, presented by Alfred Iddles

Working Stress Symposium (III)

(Auspices of Applied Mechanics Division)

Working Stresses—A General Review of the Fundamental Aspects of the Problem, C. Richard Soderberg
Notched-Bar Tests, Max Moser
Allowable Working Stresses Under Impact, N. N. Davidenkoff
Progress Report of Applied Mechanics Division, presented by George B. Pegram

Materials Handling on Railroads

(Auspices of Materials Handling Division)

Factors Affecting Shipping Other Than the Cost of Transportation, Matthew W. Potts and John A. Cronin
Effect of Transportation Requirements on the Evolution of Freight Cars, C. B. Peck
Progress Report of Materials Handling Division, presented by G. E. Hagemann

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* Contribution from Canada.
** Contribution from Great Britain.

Cutting Metals (I)

(Auspices of Machine Shop Practice Division and Special Research Committee on Cutting of Metals)

Performance of Cutting Fluids in Drilling Various Metals (Progress Report No. 4 of Subcomm. on Cutting Fluids of Comm. on Cutting of Metals), O. W. Boston and Carl J. Oxford
Effect of Lathe Cutting Conditions on the Hardness of Carbon and Alloy Steels, T. G. Digges

Tuesday Afternoon, December 6

Heat Transmission and Stokers

(Auspices of Fuels Division)

The Measurement of Metal Temperatures on the Heat-Receiving Side of Heat-Exchanging Apparatus, Arthur Williams
Stoker Developments at The Detroit Edison Company's Delray Power House No. 3, Paul W. Thompson and Fred J. Chatel
Discussion of the Present Status of Fuel-Burning Equipment, Geo. A. Orrok, A. G. Christie, and James N. Landis
Progress Report of Fuels Division, presented by Ralph A. Sherman

Cutting Metals (II)

(Auspices of Machine Shop Practice Division and Special Research Committee on Cutting of Metals)

What Can Be Accomplished With Modern Machine Tools and Cemented Carbide Cutting Tools, Arthur A. Merry
Grinding Cemented Tungsten- and Tantalum-Carbide-Tipped Tools Economically, J. M. Highducheck

Mechanical Springs

(Auspices of Special Research Committee on Mechanical Springs)

Number of Active Coils in Helical Springs, R. F. Vogt
Fatigue Tests of Helical Springs, and Number of Inactive Coils in Compression (Progress Report of Subcomm. on Heavy Railroad Springs), C. T. Edgerton
Progress Reports of Other Committee Studies, D. J. McAdam, Jr., and M. F. Sayre

Roll Neck Bearings

(Auspices of Iron and Steel Division and Special Research Committee on Heavy Duty Anti-Friction Bearings)

The Morgoil Roll-Neck Bearing, F. P. Dahlstrom
Strength of Roll Necks (Report of Special Research Comm. on Heavy Duty Anti-Friction Bearings), W. Trinks and J. H. Hitchcock

Wednesday Morning, December 7

Symposium on Long-Time Planning

(Auspices of Management Division)

The Problem of Business Stability, Sumner H. Slichter
Trade Associations—Their Part in Coordinate Planning, Edgar L. Heermance
Planning in Industry—Successful Plans Adopted and Results, Thomas B. McCabe, Howard E. Blood, Frank D. Geraghty, W. B. Flanders, W. T. Conlon, E. E. Yake, and Paul Diserens

Foundry

(Auspices of Machine Shop Practice Division)

Special Steels for Castings, R. A. Bull
Malleable Iron as a Component Part of Machines and Structures, Enrique Touceda
Progress Report of Machine Shop Practice Division, presented by R. E. W. Harrison

Flow Measurement and Hydraulic Design

(Auspices of Hydraulic Division)

Problems of Modern Pump and Turbine Design, Wilhelm Spannhake
A Symposium on Water Measurement—Discussion of "Research Institute for Hydraulic Engineering and Water Power" (Trans. A.S.M.E., HYD-54-3, May 15, 1932), by Hunter Rouse, led by Chas. M. Allen

Dyeing

(Auspices of Textile Division)

Peroxide Bleaching of Textiles, and Outlines of Its Recent History, Present Applications, and Immediate Prospects, Harry G. Smolens
Elements of Cost in Dyeing Cotton Piece Goods, J. Andrew Clark
Progress Report of Textile Division, presented by W. L. Conrad

Wednesday Afternoon, December 7

Industrial Management Today

(Auspices of Management Division)

The Economic Characteristics of the Manufacturing Industries, Walter Rautenstrauch
Dissolving of Concentrated Industries, Harold V. Coes
Management Essentials for Recovery, Carle M. Bigelow

Small-Boat Diesels

(Auspices of Oil and Gas Power Division)

Diesels for Small Boats, A. B. Newell
Progress Report of Oil and Gas Power Division, presented by P. H. Schweitzer

Steam Tables Research

(Auspices of Special Research Committee on Thermal Properties of Steam)

Report of the Progress at the Massachusetts Institute of Technology, F. G. Keyes and L. B. Smith
Report of the Progress in Steam Research at the Bureau of Standards, N. S. Osborne
Progress in Steam Table Experiments, J. H. Keenan
High-Pressure Steam Research at the Masaryk Academy of Work in Prague, Czechoslovakia (up to Nov. 1, 1932), Jaroslav M. Havlicek
The Pressure of Saturated Water Vapor in the Range 100 C to 374 C, N. S. Osborne, H. F. Stimson, E. F. Fiock, and D. C. Ginnings

Flow of Fluids

(Auspices of Hydraulic Division)

A Study of the Data on the Flow of Fluids in Pipes, Emory Kemler
A Tribute to John Ripley Freeman and His Hydraulic Accomplishments, Blake R. Van Leer
Progress Report of Hydraulic Division (presented by title)

Thursday Morning, December 8

Central Station Power

(Auspices of Power Division)

Stresses in Boiler Tubes Subjected to High Rates of Heat Absorption, Wm. L. DeBaufre
A System for the Measurement of Steam With Flow Nozzles for Turbine Performance Tests, Sanford A. Moss and Wistar Johnson
Performance of Modern Steam-Generating Units, C. F. Hirshfeld and G. U. Moran

Progress in Management

(Auspices of Management Division)

Ten Years' Progress in Management, 1923–1932, Leon P. Alford
Applications of the Kmh Method of Analyzing Manufacturing Operation, Leon P. Alford and J. E. Hannum
Informal Discussion of Engineering and Its Effect on Economic Conditions, Dexter S. Kimball
Progress Report of Management Division (presented by title)

Aeronautics

(Auspices of Aeronautic Division)

Airfoil Theory, With Special Reference to Aileron Rolling and Hinge Moments, Albert Toussaint
The Relation of Weather to Ocean Flying, James H. Kimball
Progress Report of Aeronautic Division

Thursday Afternoon, December 8

LUBRICATION TESTING

(Auspices of Petroleum Division)

Some Problems on the Lubrication of Vertical Journal Bearings, A. I. PONOMAREFF and E. D. HOWE
Chemical Mechanism of Lubrication, WILLIAM F. PARISH and LEON CAMMEN
Progress Report of Petroleum Division (presented by title)

EDUCATION AND TRAINING FOR THE INDUSTRIES

(Auspices of Committee on Education and Training for the Industries)

The Engineer's Interest in Foreman Training, EDWARD S. COWDRICK
Fundamentals of Training, G. GUY VIA
Adult Technical Education, O. W. ESHBACH

Preceding the sessions on Tuesday, Wednesday, and Thursday mornings, Dr. S. Marion Tucker, Head of the Department of English of The Polytechnic Institute, Brooklyn, N. Y., gave a series of lectures on "Talking With an Audience."

Both the Towne and Thurston Lectures were on the program, the former being presented by A. W. Robertson, Chairman of the Board of Directors of the Westinghouse Elec. & Mfg. Co., New York, and the latter by Dr. W. F. G. Swann, Director of the Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa. Mr. Robertson took for his subject "The Scientific Approach to Human Affairs," and Dr. Swann spoke on "Recent Advances in Physics." Another special lecture was that by H. W. Heinrich, Assistant Superintendent, Engineering and Inspection Division, Travelers Insurance Company, Hartford, Conn., on "Conservation an Essential in Industrial Recovery."

The annual Business Meeting was held on Monday afternoon, preceding the sessions, and adjourned to Tuesday evening, when the report of the Local Sections Delegates naming the 1933–1934 Nominating Committee was presented and approved.

The 1932 Council held sessions for the transaction of regular business on the first day of the meeting and at an adjourned meeting on Friday morning received a resolution from the Hydraulic Division proposing joint action with the American Society of Civil Engineers and the Boston Society of Civil Engineers in regard to traveling scholarships in hydraulics, as a memorial to John R. Freeman, past-president and honorary member of the Society. The new Council for 1933 organized on Friday.

Wednesday was Student Branch Day, the program for which included participation in the morning excursion, a luncheon, and an afternoon session at which Student Branch activities were discussed, the 1932 Charles T. Main paper presented, and addresses made by Dr. Lillian M. Gilbreth, consulting management engineer, Montclair, N. J., and William A. Hanley, director of engineering, Eli Lilly & Co., Indianapolis, Ind.

The social events of the meeting began with a card party on Monday evening, sponsored by the Woman's Auxiliary to the A.S.M.E., the proceeds to be used for unemployed engineers. On Tuesday evening Conrad N. Lauer presented his presidential address, the subject of which was "Unifying the Engineering Profession," honorary membership was conferred posthumously on John R. Freeman, and the Melville, Charles T. Main, Junior, and Student Awards were announced. A reception and dance followed.

The Annual Dinner was held at the Hotel Astor on Wednesday evening. General James G. Harbord, Chairman of the Board of Directors, Radio Corporation of America, New York, gave an address on "Industrial Rehabilitation." James D. Cunningham, Manager, A.S.M.E., was the toastmaster. The address to new members was delivered by Roy V. Wright, past-president, and the president-elect, Dean A. A. Potter, was presented and spoke briefly.

A large number of committee meetings were held during the week, and an interesting program of excursions included trips to several generating stations, and the S.S. *Europa*, Seatrain Lines, Inc., and S.S. *Conte di Savoia*. Thursday evening, as usual, was reserved for college reunions. The events for ladies included trips to the Morgan Library, Wanamaker's, and the Museum of the City of New York. An informal tea was given on Thursday afternoon at the Engineering Woman's Club, and the Annual Business Meeting and Luncheon of the Auxiliary was held on Tuesday at the Pan-Hellenic Hotel.

Meetings of the Professional Divisions

Aeronautic Division

BERKELEY, CALIF., JUNE 9–10, 1932

More than 200 attended the Pacific Coast Aeronautic Meeting held under the auspices of the Aeronautic Division on June 9 and 10 at the University of California.

The afternoon of the first day was devoted to an excursion to the San Francisco Bay Cities Airdrome, Oakland Airport, Boeing School, and Stanford University. At a dinner held in San Francisco that evening, E. T. Allen, formerly test pilot of the Boeing Airplane Company, was the principal speaker.

The *A.S.M.E. News* of July 22, 1932, comments upon the three technical sessions, at which the following papers were presented:

Thursday Morning, June 9

AERODYNAMICS

Description and Calibration of 10-Foot Wind Tunnel at California Institute of Technology, C. B. MILLIKAN and A. L. KLEIN
The Effects of Fillets on Wing-Fuselage Interference, A. L. KLEIN
The Present State of the Turbulence Problem, WALTER TOLLMEIN
Wind Tunnel Tests of a 1/75 Scale Model of the Goodyear Zeppelin Airship "Akron" Z.R.S. 4 With Normal and Ring Tail Surfaces, REINHOLD SEIFERTH (presented by title)

Friday Morning, June 10

STRUCTURES

Rules for the High and Low Angle of Attach Loading Conditions, A. S. NILES
Survey of the Problems of Thin Walled Structures, E. E. SECHLER, L. H. DONNELL, TH. VON KARMAN
Principle of Similitude as Applied to Research on Thin Sheet Structures, J. E. YOUNGER
The Application of the Hardy-Cross Method of Moment Distribution, H. A. WILLIAMS

Friday Afternoon, June 10

TRANSPORTATION AND MANAGEMENT

Spin Flight Testing of Three Similar Airplanes, E. T. ALLEN
Development of Transport Airplanes, C. N. MONTEITH
Safety in Air Transportation, C. E. JOHNSON
The Technic of Mechanical Engineering Laboratory Instruction at the University of California, L. M. K. BOELTER

Aeronautic Division

BUFFALO, N. Y., JUNE 6–7, 1932

The Sixth National Meeting of the Aeronautic Division and the First National Process Meeting of the Process Industries Committee (see page 11) were held under the auspices of the Buffalo Section, with joint entertainment features. Several hundred were registered for the meetings, which were reported in the *A.S.M.E. News* of June 22, 1932.

The morning of June 7 was devoted to inspection trips, the aeronautic group visiting the plants of the Irving Air Chute Company, Consolidated Aircraft Company, Hall-Aluminum Aircraft Corporation, and Curtiss Aeroplane & Motor Co., where lunch was served.

The banquet on the evening of the 7th was the occasion for the presentation of the Spirit of St. Louis Medal to Paul W. Litchfield, president of the Goodyear Tire & Rubber Co., Akron, Ohio, and of the Goodyear-Zeppelin Corporation, for meritorious services in the advancement of airship design and construction in America. The speakers at the banquet were the Honorable W. Irving Glover, second assistant postmaster general, on "Future Transportation," and Dr. A. Stuart Hunter, secretary of the Society of Rheology, on "The Scientist in Industry." Theodore P. Wright presented the medal to Mr. Litchfield.

The third day of the meeting was devoted to a trip to Niagara Falls, with visits to a number of plants along the way.

Another feature of the meeting was an exhibit of engineering aeronautic problems, including photographs, structural forms, and other interesting specimens arranged by the Buffalo Committee.

The program for the technical sessions follows. It will be noted that the Aero Club of Buffalo joined in the session on airship and airplane carriers on Monday evening.

Monday Morning, June 6

Aerodynamics

Photogrammetric Measurement of Airplane Landing Speeds, Volney C. Finch
Some Observations on Spins, J. M. Gwinn, Jr.
A Study of Airplane Performance Parameters, Frederick R. Weymouth
Wind Loads on Airship Hangars, H. MacT. Sylvester

Navigation

Recent Advances in Weather Service in Relation to Aerial Navigation, Willis Ray Gregg
A Sonic Marker Beacon for Fog Aviation, Chester W. Rice
Aircraft Sextant, P. V. H. Weems
Parachutes in Relation to Passenger Air Transport, Guy M. Ball

Monday Afternoon, June 6

Metal Construction

Steel Airplane Construction, E. J. W. Ragsdale
Developments in Aluminum Alloys in Relation to Economies in Aircraft Construction, C. F. Nagel, Jr., and G. O. Hoglund
Evolution of Metal Construction, Gardner W. Carr (presented by title)
A Comparative Study of Welded and Riveted Fuselages, Philadelphia Aeronautic Society. Presented by Louis C. Rosenstein and C. Bennett
Monocoque Aircraft Construction in Theory and Practice, Richard L. Schleicher

Engines

Shaft Drives for Airship and Airplane Propellers, J. L. Goldthwaithe
Torsional Vibration of Aircraft Engine Crankshafts, W. G. Lundquist
Air Compression With Temperatures Above Adiabatic—With Special Reference to Airplane Superchargers, Sanford A. Moss

Monday Evening, June 6

Airship and Airplane Carriers

(Joint auspices Aero Club of Buffalo and Aeronautic Division)
Airplane Carriers, H. B. Miller
Mechanical Equipment for Handling Large Rigid Airships, C. M. Bolster
Artificial Payload for Coming Commercial Airships, T. L. Blakemore

Maintenance

Aviation Maintenance Manual, Jerome Lederer
Relation of Maintenance to Design, R. G. Lockwood
Effect of Plane Impact on Airport Surfacing, C. N. Conner
Report on Wage Incentives in Aeronautic Industry, W. T. Tiffin

Tuesday Afternoon, June 7

Design

Cost of Air-Line Operation, W. Irving Glover
The Ideal Transport Airplane, Ernest W. Dichman
Standardizing Location and Direction of Movement of Stabilizer Control, Clifford Swartz
Heating and Ventilating of Cabin Airplanes, M. Gould Beard
The Possibilities of Cabin Type Autogiros, A. E. Larsen
Aeronautical Research in Canada, J. J. Green, George J. Klein, and K. F. Tupper

Fuels and Engines

Aircraft Engine Steels, N. L. Deuble
Combustion Processes in Aircraft Engines, H. K. Cummings
Recent Developments in Grading of Fuels for Aeronautic Use, W. W. White
Relation of Quality to Economies in Manufacture of Aviation Gasoline, G. G. Oberfell

Applied Mechanics Division

NEW HAVEN, CONN., JUNE 23–25, 1932

The Second National Meeting of the Applied Mechanics Division was coordinated with the summer meeting of the American Physical Society at Yale University, under the auspices of the New Haven Section, with a combined attendance of nearly three hundred. The meeting was reported in the *A.S.M.E. News* of July 22, 1932.

The Applied Mechanics Division held five sessions of its own and one jointly with the American Physical Society. The program for these sessions follows.

Thursday Morning, June 23

Hydro- and Aerodynamics

Resistance of Slender Bodies Moving With Supersonic Velocities, Th. von Karman and N. B. Moore
On the Life Distribution for a Wing of Arbitrary Plan Form in a Circular Wind Tunnel, C. B. Millikan
On the Buckling Strength of Columns With Elastic Intermediate Supports, W. Klemperer (presented by title)
A New Kind of High-Speed Water Craft (Film), O. Tietjens

Thursday Afternoon, June 23

Vibration

Vibration During Acceleration Through a Critical Speed, F. M. Lewis
Self-Induced Vibrations, J. G. Baker
Critical Torsional Oscillations of a Rotating Accelerating Shaft, M. Biot
Vibration With Velocity Damping, M. Stone
Vibrations of a System With Two Degrees of Freedom Analyzed by Vector Diagrams, J. L. Mansa (presented by title)
Calculation of the Stresses Occurring in a Building During an Earthquake, M. Biot (presented by title)

Friday Morning, June 24

Bending of Long Rectangular Plates With Lateral Loading, S. WAY
Strength of Semi-Circular Plates and Rings Under Uniform External Pressure, A. M. WAHL
Shearing Stresses in Torsion and Bending by Membrane Analogy, P. ALLERTON CUSHMAN
Concentration of Stress Around Spherical and Cylindrical Inclusions and Flaws, N. GOODIER
The Design of Columns of Varying Cross-Sections, A. N. DINNIK (presented by title)

Friday Afternoon, June 24

ELASTICITY AND PLASTICITY

Creep of Metals, A. NADAI
The Elastic Behavior of Vulcanized Rubber, H. HENCKY
Creep in Bending, G. MACCULLOUGH
Stress Distribution in a Re-entrant Corner, J. H. A. BRAHTZ
Distribution of Stress Due to a Single Force Acting on the Edge of a Circular Hole, etc., W. O. RICHMOND (presented by title)
The Distribution of Pressure Concentrated on a Small Surface, M. A. SADOWSKY (presented by title)
Solution of Problem of Plane Strain in Bending, J. M. KLITCHIEFF (presented by title)

Saturday Morning, June 25

JOINT SESSION WITH AMERICAN PHYSICAL SOCIETY

Mosaic Crystals Structure, F. ZWICKY
Plasticity of Metals, F. BITTER
Dissipational Resistance to Distortion in Solid Bodies and Its Correlation With Other Physical Properties, R. L. WEGEL and H. WALTHER

Saturday Afternoon, June 25

THERMAL STRESSES

Thermal Stresses in Spheres and Cylinders Produced by Temperatures Varying With Time, C. H. KENT
Some Changes of Shape Characteristics of a Smooth, a Corrugated, and a Creased Pipe Bend Under Load, E. T. COPE and E. A. WERT

The entertainment features included an inspection of the Sterling Memorial Library, where tea was served on Thursday afternoon, a dinner at the Lawn Club that evening, followed by a performance at the University Theater by the Yale Dramatic Society, and a joint outing of the Applied Mechanics Division and American Physical Society on Friday afternoon. Another feature of the meeting was an exhibit of special testing devices arranged by the local committee.

Iron and Steel Division

PITTSBURGH, PA., APRIL 28, 1932

The Iron and Steel Division, in cooperation with the Pittsburgh Section of the Society and the Engineer's Society of Western Pennsylvania, held a joint meeting on Thursday afternoon, April 28, 1932. Arthur G. McKee, Chairman of the Division, presided. The meeting was held at the William Penn Hotel in Pittsburgh, and was preceded by a luncheon program.

Machine Shop Practice Division

BUFFALO, N. Y., OCTOBER 3, 1932

As part of the National Metal Congress held in Buffalo the week of October 3, 1932, the Buffalo Section and Machine Shop Practice Division of the Society held two technical sessions on Monday, October 3, at the Hotel Statler. The Society of Automotive Engineers also held a session on the same day, and both societies joined in a luncheon meeting at which Kenneth Condit, editor, *American Machinist*, discussed "Equipment Policies."

The following papers were presented at the A.S.M.E. sessions:

Monday Afternoon, October 3

The Influence of Oil Compressibility on Speed Characteristics of Hydraulic High-Speed Presses, WALTER ERNST
The Torque Required to Tap Cast Iron Using National Coarse-Thread Series and National Fine-Thread Series Taps, J. E. MILLER
Recent Trends in Machine-Tool Design, GUY HUBBARD

Monday Evening, October 3

Technique of Size Control in Precision Grinding, R. E. W. HARRISON
Design of Products to Utilize Die Castings of Machine Parts, L. H. MORIN
Materials for Modern Cutting Tools, J. V. EMMONS

As part of the Congress there was the annual exhibit of metal-treating equipment.

During the week the American Society for Steel Treating presented a program of thirty papers, the American Welding Society held its semi-annual meeting, and the American Institute of Mining and Metallurgical Engineers, through its Iron and Steel and Institute of Metals Divisions, sponsored five sessions.

A somewhat more complete report of this meeting was published in the *A.S.M.E. News* for October 22, 1932.

M-M-M Congress

DETROIT, MICH., MAY 4–5, 1932

The Detroit Section of the Society was host to the Management, Maintenance, and Materials Handling Congress held on May 4 and 5 in connection with the annual meeting of the Ameri-Foundrymen's Association.

The attendance at the M-M-M Congress was more than two hundred. The meeting was recorded in the *A.S.M.E. News* of June 7, 1932.

The Management Division and its Subcommittee on the Elimination of Waste and the Materials Handling Division and its Manufacturers' Committee joined in the following technical program, with the Materials Handling Institute cooperating in one of the sessions.

Wednesday Morning, May 4

Maintenance of Materials-Handling Foundry Equipment, THOS. A. BISSELL
Relative Wear of Metals by Abrasion, C. R. WEISS
Management Policies and Practices in a Continuous Production Gray-Iron Foundry, W. G. REICHERT

Thursday Morning, May 5

MANAGEMENT

Waste Elimination Symposium

Elimination of Waste in Industry, E. S. CARMAN
Savings by Substitution of Other Materials, GEO. BANGS
Common Savings in Shop Wastes, C. B. HALL
Weekly Accounting of Waste, S. K. COOPER (presented by title)
Maintaining Interest in Suggestion Systems, W. E. FISHER (presented by title)

MATERIALS HANDLING

(Auspices of Manufacturer's Committee, Materials Handling Division, Materials Handling Institute cooperating)

Report on Performance Standards of Locomotive and Crawler Type Cranes and Shovels, F. A. SMYTHE
Report on Capacity of Containers, W. E. FARRELL
Report on Nomenclature and Classification of Materials-Handling Equipment, GEO. L. MOREHEAD
Report on Materials-Handling Lectures and Meetings in Colleges, MATTHEW W. POTTS
Report on Standardization of Materials-Handling Equipment, J. M. ENDERS

Thursday Afternoon, May 5

Management

Knowledge of Markets and Its Effect on Product Design, R. F. Elder

International Trade of the United States, George Quisenberry

Materials Handling

(Auspices American Foundrymen's Association, Materials Handling Division cooperating)

Tramrail for Materials Handling in the Foundry

Materials-Handling Problems in the Foundry Department of the Western Electric Company

Materials Handling as Related to Miscellaneous Jobbing Foundries

The program also included an inspection trip to the plant of the Ford Motor Company and a dinner at which the speakers were R. L. Lockwood, of the U. S. Department of Commerce; John Van Deventer, editor of *Iron Age;* and representatives of the Divisions.

Oil and Gas Power Division

STATE COLLEGE, PA., JUNE 8–11, 1932

The Fifth National Meeting of the Oil and Gas Power Division was held at Pennsylvania State College concurrent with the Fifth Oil Power Conference of the college. As in previous years there was an interesting exhibit of Diesel engine parts and accessories. The program of technical papers was as follows:

Wednesday Morning, June 8

Heating Fuels for Injection Engines, Edward Adams Richardson

Spray Characteristics of a Jerk-Pump Injection System, O. F. Zahn, Jr.

Wednesday Afternoon, June 8

Surface-Volume Ratio as a Critical Factor in Automotive Diesel Combustion Chambers, Julius Kuttner, with discussion of Fuel Injection Engines With Subdivided Combustion Chamber and Heat Accumulator, by Kurt Neumann

Fuel Spray Formation, Dana W. Lee

Combustion Knock in Diesel Engines, P. H. Schweitzer

Thursday Morning, June 9

Metallurgical Problems of Diesel Engines, Russell J. Allen

Fuel Injection in Gas Engines, E. G. Beardsley and J. N. MacKendrick

Friday Morning, June 10

Diesel Power Cost Committee Report, presented by M. J. Reed

Burning Heavy Oil in Diesel Engines, H. W. Stalnaker

Saturday Morning, June 11

Inertia Supercharging of Engine Cylinders, E. S. Dennison

Ignition Quality of Diesel Fuels; Cetane Figures as Compared With Octane Figures, G. D. Boerlagd

The *A.S.M.E. News* of June 22, 1932, includes comments on the discussion of these papers.

The new Nittany Lion Inn on the campus served as headquarters and was the scene of several entertainment features, including an informal dance, card party, and banquet. Prof. C. M. Allen, vice-president of the Society, was the chief speaker at the banquet. Other events were a golf tournament, and a picnic and baseball game.

Printing Industries Division

NEW YORK, N. Y., MARCH 14–15, 1932

The Third Conference of the Technical Experts in the Printing Industry, sponsored by the Printing Industries Division, with the New York Club of Printing House Craftsmen assisting, and the McGraw-Hill Publishing Company as official host, drew an attendance of nearly one thousand. The technical sessions were held in the new building of the McGraw-Hill Publishing Company on West 42nd Street, and a considerable number of those attending the conference took advantage of the opportunity to be shown through the building.

In addition to the technical sessions there were four group conferences, dealing with the following subjects: Thickness of Curved Plates and the Depth of the Recess for Such Plates, Humidity as It Affects Printing, Standards on Paper, and Closer Cooperation Between the Different Printing Crafts.

At the opening session of the meeting Calvin W. Rice, secretary of the A.S.M.E., gave a talk on "The Art of Printing as a Source of Special Satisfactions." The speaker at the banquet on the evening of March 14 was Joseph F. Gillick, president of the American Type Founders Company, whose subject was "What Is Ahead of the Printing Industry?" Arthur C. Jewett, director of the College of Industries of the Carnegie Institute of Technology, was toastmaster, and Hon. George H. Carter, Public Printer of the United States, gave a brief talk. The telephotograph service was demonstrated at the banquet by Dr. J. O. Perrine, and the Walter Howey photoelectric engraver process by B. W. Woodward. There was also a cabinet exhibit, "Edison as a Printer." Following the morning session on Monday Dr. Perrine demonstrated television, and after Tuesday's morning session Robert E. Naumburg demonstrated the automatic printing visagraph.

The program of technical papers for the conference, which was reported in the *A.S.M.E. News* of April 7, 1932, follows. The papers presented at the conference were published with discussion in a special pamphlet form.

Monday Morning, March 14

Application of Engineering to the Printing Industry, George C. Van Vechten

Developments in New Printing Processes, Clarence L. Johnston

Monday Afternoon, March 14

Advanced Stereotype Methods, Arnold A. Schwartz

Advanced Electrotype Methods, John H. Davis

Premakeready Developments, George F. Bryant

Tuesday Morning, March 15

Newspaper Multi-Color Printing, George E. Pancoast

Progress in Offset Printing, Otto R. Trampusch

What Can Science Do for Practical Determination of Color Standards for Printing, Loyd A. Jones

Tuesday Afternoon, March 15

Relation of Press Speeds to Production, John J. Maloney

Condition of Paper as It Affects Printing, William G. MacNaughton

Printing Rollers, Arthur H. Burns

Process Industries Committee

BUFFALO, N. Y., JUNE 6–7, 1932

As already noted in the report of the Aeronautic Meeting at Buffalo, the Process Industries Committee held its First National Process Meeting on the same dates, the two groups having joint entertainment features. The meetings were reported in the *A.S.M.E. News* of June 22, 1932.

The plants visited by the process group on the morning of June 7 were those of Pratt & Lambert, Inc., the Buffalo Foundry & Machine Co., and the Buffalo Pottery Company (Larkin Company), the last-named being luncheon hosts.

An exhibit of process equipment arranged by the Buffalo Committee attracted considerable interest. Four technical sessions were held as follows:

Monday Morning, June 6

Grinding Symposium

An Analysis of Crushing and Pulverizing Processes as Related to Industrial Operation, Lincoln T. Work

Monday Afternoon, June 6

Heat Transmission

The Flow of Fluids in Closed Conduits, R. J. S. Pigott
Design Alignment Charts, T. H. Chilton, A. P. Colburn, R. P. Generaux, and H. C. Vernon
Fluid Friction at Parallel and Right Angles to Tubes and Tube Bundles, E. N. Seider and N. A. Scott, Jr.

Dust Problems

Silicosis, A. E. Russell
Dust Determinations, J. J. Bloomfield
Improved Methods of Dust Collecting, George Kelley
Shop Methods of Controlling Dust Hazards, F. Willson

Tuesday Afternoon, June 7

General Process

The Manufacture of Rubber-Covered Wire for Telephone Installations, S. E. Brillhart
Industrial Air Conditioning, Geo. B. Bailey
Developments in Cotton-Seed-Oil Processing, W. R. Woolrich

Textile Division

BOSTON, MASS., OCTOBER 27, 1932

A New England Textile Meeting was held on October 27, 1932, at the Massachusetts Institute of Technology under the auspices of the Boston Section and the Textile Division. After a morning inspection trip through the Boston Woven Hose & Rubber Co. and the laboratories of the Institute, luncheon was served in the Walker Memorial Building of the Institute.

At an afternoon session four papers were presented, as follows:

Textiles as They Are, E. R. Schwarz
Recent Developments in Spinning, James L. Truslow
The All-Electric Dye Jig, C. E. Robinson
Variable Speed Spinning, C. L. Irwin.

Wood Industries Division

JAMESTOWN, N. Y., NOVEMBER 15–16, 1932

The Seventh National Meeting of the Wood Industries Division, with the National Association of Furniture Manufacturers cooperating, was held at Jamestown, N. Y., on November 15 and 16, 1932, with two hundred in attendance. The Jamestown Hotel was headquarters for the meeting.

The meeting was opened at a luncheon with a talk by Dr. A. P. Haake, secretary of the National Association of Furniture Manufacturers, in which he outlined the economic problems facing the furniture industry and the need for engineering in their solution.

There were four technical sessions at which papers were presented as follows:

Tuesday Afternoon, November 15

Research

Timber Research and Wood Industries, W. Kynoch
Faith in Research, Arthur Koehler
Why Research in Furniture Manufacturing, G. R. Petrie and A. C. Saunders
Furniture Distribution, D. D. Fennell

Tuesday Evening, November 15

Machinery

Different Methods of Taking Lumber Out of Wind, A. S. Kurkjian
Different Methods of Making Glue Joints, G. M. Diehl
The Electric Moulder and Its Efficiency, Anton Vonnegut

Wednesday Afternoon, November 16

Manufacturing

Door Construction, Design, and Manufacture, Sern Madsen
Modern Developments in Wood Finishing Materials, M. J. Pearce
Accelerated Varnish Drying, R. H. McCarthy and L. A. Lefcort

Wednesday Evening, November 16

Veneer and Plywood

Fundamentals of Knife Cut Veneers, J. R. and J. N. Roberts
Manufacture of Custom-Built Plywood, E. R. Morrison and E. J. Maroney
Some Practical and Theoretical Considerations Involved in Laminating With Phenolic Resin, E. H. Merritt
The Moisture Content, Specific Gravity, and Air Space in Woods, J. D. MacLean

Tuesday, Wednesday, and Thursday mornings were devoted to inspection trips to plants in Jamestown and vicinity manufacturing furniture and wooden toys and novelties, plywood, and woodworking equipment. Among the plants open for inspection were those of the Jamestown Lounge Company, Maddox Table Company, Union Furniture Company, Jamestown Panel Company, Curtis Machine Company, and Merritt Engineering & Sales Co. Considerable interest was displayed in the Jamestown Furniture Market during which the meeting was held. A more detailed report will be found in the *A.S.M.E. News* for December 7, 1932.

Necrology

WILLIAM AHLÉN (1857–1932)

William Ahlén, whose death occurred on July 17, 1932, was born at Arla, Sköllersta, Sweden, on April 7, 1857. He was graduated from the technical school at Örebro, Sweden, at the age of 21 and served a two-year apprenticeship at the Söderbloms Foundry & Machine Works, Eskilstuna, Sweden, following his graduation. He then came to the United States, where he was employed until 1887 by the Pennsylvania Railroad Company, at Altoona, Pa., being promoted from the position of machinist and draftsman to that of assistant chief draftsman.

After leaving the Pennsylvania he spent three years with the Norfolk & Western R.R. at Roanoke, Va., as draftsman for locomotives, cars, and machinery. From that time until his retirement in 1919 he was connected with the Carnegie Steel Company. He was located at the Homestead Steel Works of the company from 1890 to 1894 as assistant civil engineer, working on the construction of a pumping station, electric power station, armor plate plant, and cogging and beam mills. During the remainder of the time he was stationed at the Duquesne Steel Works and Blast Furnaces, for four years as civil engineer and thereafter as chief engineer. A large number of furnaces and mills were designed and built under his supervision, many novel features being introduced.

Mr. Ahlén became a member of the A.S.M.E. in 1912. Since his retirement he had made his home in Stockholm.

HARRY ALEXANDER (1871–1932)

Harry Alexander, who retired from active business in 1929, died in New York, N.Y., on October 18, 1932. He was born in that city on August 6, 1871, and after leaving school served an apprenticeship in shop work and machine construction with the Daft Electric Light Company, Greenville and Marion, N.J., from 1886 to 1888. Subsequently he worked as draftsman for that company and the Thomson-Houston Electric Company, Lynn, Mass. He was superintendent of construction for the Thomson-Van Depoele Mining Company from 1889 to 1891, and supervised the building of special and mining machinery for the company at the Somersworth Machine Shops at Dover, N.H.

During the latter part of 1891 Mr. Alexander organized the Alexander-Chamberlain Electric Company, Inc., New York, N.Y., and became its vice-president and general manager, in charge of electrical and mechanical engineering work. He designed and installed power and electric plants for factories, hotels, and other buildings. The firm was reorganized in 1899 as Harry Alexander and was incorporated as Harry Alexander, Inc., in 1914, and he continued general mechanical and electrical engineering contracting, testing, and consulting work until his retirement in 1929.

Mr. Alexander became a junior member of the A.S.M.E. in 1895, an associate four years later, and a member in 1901.

MAGNUS WASHINGTON ALEXANDER (1870–1932)

Magnus Washington Alexander, a founder and president of the National Industrial Conference Board, Inc., died suddenly of heart disease at his apartment in New York, N.Y., on September 10, 1932.

Mr. Alexander was a native of New York, having been born there on February 17, 1870, the son of Alexander M. Alexander and the former M. Jelenkiewicz. He was educated abroad, studying mechanical, metallurgical, and electrical engineering at Vienna, Leoben, and Gratz in Austria. Trinity College later gave him an honorary M.S. degree.

After a few years in iron and steel work with the Oesterr. Alpine Montan. Gesellschaft, Mr. Alexander returned to the United States, in 1893, and entered the employ of the Weston Electrical Instrument Company as a designer. The following year he took a position with the Westinghouse Electric & Manufacturing Co., with which he remained until 1899. He then spent several months as chief designer for the Siemens & Halske Electric Co. of America, after which, in 1900, he became associated with the General Electric Company at Lynn, Mass. For 18 years he was engineer in charge of design for that company and then was made "consulting engineer on economic issues."

In 1916 Mr. Alexander helped to organize the National Industrial Conference Board, a body devoted to industrial planning and research, and since 1919 he had given the greater part of his time to its work. The offices of the board were moved from Boston to New York in 1920. Mr. Alexander was intensely interested in the industrial and economic problems of the country and each year made one or more trips to Europe to study conditions there. He was the author of many magazine articles and monographs upon economic subjects and an ardent spokesman in support of the policies of the board. He published a book on "Safety in the Foundry" and in other ways aided in the movement to further safety in industry. He was chairman from 1908 to 1910 of the Massachusetts Commission on Old Age Pensions, Annuities, and Insurance, a member from 1910 to 1912 of the Massachusetts Commission on Industrial Accidents, and chairman of the 1914 Commission on Pensions.

Mr. Alexander became a member of the A.S.M.E. in 1906 and also belonged to the American Institute of Electrical Engineers, the Verein deutscher Ingenieure, and the American Academy of Political and Social Science. His only club affiliation was the Engineers' Club, New York.

Surviving Mr. Alexander are his widow, the former Baroness Irmgard von Rothenthal, whom he married in 1927, and a son, Herbert.

SAMUEL SAUL AMDURSKY (1892–1932)

Samuel Saul Amdursky, consulting engineer and head of the mechanical department of the Taylor Instrument Companies, Rochester, N.Y., died on October 6, 1932, after a short illness.

Mr. Amdursky was born in Rochester on August 15, 1892. He prepared for college at the Rochester Athenium and Mechanics Institute, and received an M.E. degree from Syracuse University in 1916. During his college years he had considerable practical experience at the Rochester Municipal Laboratory, testing building materials such as cement, asphalt, and paving brick, and with the Rochester Gas & Elec. Corp., for which he designed gas distribution lines, made efficiency studies, and supervised construction work. He continued to work for this company for about two years after his graduation, designing and installing industrial gas furnaces and ovens.

During part of 1918 Mr. Amdursky was in the Mechanical Research Division of the Chemical Warfare Service, designing and perfecting various devices. In January, 1919, he entered the sales engineering department of the Taylor Instrument Companies, his work having to do with the design and application of temperature and pressure indicating, recording, and controlling devices. In this connection he also assisted in the layout of plants and equipment and in the design of heating systems required by various industrial processes.

Mr. Amdursky became a member of the A.S.M.E. in 1928 and belonged also to the Rochester Engineering Society and Society of American Military Engineers.

CARL ANGSTRÖM (1856–1932)

Carl Angström, a leading figure in the development of mechanical engineering in Sweden, died at his home in Stockholm on September 18, 1932. Mr. Angström was born in Taberg, Sweden, on April 22, 1856, the son of Carl Arendt Angström, a professor at the Technological Institute in Stockholm. After his graduation from the Institute Mr. Angström came to the United States. He was employed by the Pennsylvania Railroad as machinist and draftsman in 1877 and 1878 and subsequently by the Washburn & Moen Mfg. Co., Worcester, Mass., on the design of rolling mills and wire machinery.

He returned to Sweden in 1884, taking a position as mechanical engineer at the iron and steel works in Domnarfvet, where he remained until 1890. During the next two years he was government inspector of factories in the Southern District, Jonköping, Sweden. He then became managing director of the Motala Works, a position he retained until 1910. Since then he had engaged in consulting work in Stockholm.

Mr. Angström was a member of the Swedish Society of Engineers, which in 1882 awarded him the Polhem medal for his treatise on hydraulic turbines. He was a well known contributor to the technical press. He had been a member of the Swedish Society of Inventors since 1893 and held patents on a slide valve control for double cylinder or compound engines, an arrangement with two-cycle explosion or internal-combustion engines, and a device for control of water injection in explosion or internal-combustion engines. He was a director of the association of industrial owners in Sweden (Verkstadsföreningens överstyrelse) and had served as a judge at several industrial exhibitions in that country. He belonged to several other Swedish technical organizations and was a Knight of the Royal Order of Wasa and the Order of the North Star. He had been a member of the A.S.M.E. since 1883 and also belonged to the Iron and Steel Institute (London).

WILLIAM HOGG BAKER (1879–1932)

William Hogg Baker, whose death occurred on May 13, 1932, at his home in Montclair, N.J., was born in Philadelphia, Pa., on July 25, 1879, the son of Franklin and Sarah J. (Hogg) Baker. After securing his M.E. degree from Cornell University in 1901 Mr. Baker worked as draftsman for a year each with the Pencoyd Iron Works, Philadelphia, and the Atlas Portland Cement Company, New York, N.Y. He then went to Louisville, Ky., as chief engineer on the construction of the Kosmos Portland Cement Company plant, where he remained for two years. From 1905 to 1907 he served as manager of the Colorado Consolidated Mining & Electric Co., Breckenridge, Colo., and during the next four years engaged in private practice in engineering and construction work with headquarters in New York. In 1911 he returned to the employ of the Atlas Portland Cement Company, being assistant general superintendent for a time and later manager of the Western Manufacturing Department, at Hannibal, Mo.

About 1920 Mr. Baker accepted the vice-presidency of the Hardinge Conical Mill Company, New York. After several years in that office he was manager for a short time of the Fuller-Lehigh Company, New York, and president of the Quigley Fuel Systems, Inc. He became president in 1925 of the Acme Cement Corporation, Catskill, N.Y. He returned to private engineering practice in 1927 as president of Baker and Spencer, Inc., New York. Since 1928 he had been president of the Merritt-Chapman & Scott Corporation, New York. He was also director of the North America Cement Corporation and United Dry Docks, Inc., New York.

Mr. Baker became a member of the A.S.M.E. in 1915. His clubs included the Engineers', Whitehall, Down Town Athletic, and Cornell, in New York, and several other social and athletic clubs, among them being the Montclair Golf, The Knoll, Seaview Golf, and Boca Raton, Florida. He is survived by his widow, formerly Miss Catherine E. Brady of Denver, Colo., whom he married in 1906, and by four children, William H., Jr., Charles E., Edith L., and Sarah J. (Mrs. David Lindsay III).

THURMAN HARRISON BANE (1884–1932)

Colonel Thurman H. Bane, executive vice-president of the American Airways, Inc., New York, N.Y., a subsidiary of the Aviation Corporation, died on February 22, 1932, at the Neurological Institute of New York. Surviving him are his widow, Bessie Louise (Dichman) Bane, whom he married in 1909, and one daughter, Louise Bane.

Colonel Bane was born at San Jose, Calif., on June 12, 1884, the son of the Rev. Adam C. and Florence C. (Harrison) Bane. He was graduated from the United States Military Academy at West Point with the class of 1907, of which he was president. He served as a lieutenant of cavalry in the United States and the Philippines until 1912, when he was transferred to the Ordnance Department as first lieutenant. He was stationed at the Sandy Hook Proving Ground, New Jersey, for two years, first as a student officer in the School of Application and then as a proof officer, testing ordnance materiel. For a year, beginning in June, 1914, he was located at the Rock Island Arsenal, as assistant to the officer in charge of the armory and the harness shop.

In July, 1915, he returned to duty with his regiment, the 6th Cavalry, on the Mexican Border and with Pershing in Mexico. During the year spent there he became interested in aviation, and was transferred to the Signal Corps Aviation School, at San Diego, toward the end of 1916. He completed his training in May of the following year and was appointed secretary of the school and officer in charge of the experimental and repair shops. As a student flyer he worked out a course in aerodynamics and design which was accepted as standard.

In October, 1917, he was promoted to lieutenant-colonel, Air Service, and transferred to Washington, D.C. He was made chief of the technical section of the Bureau of Military Aeronautics early the following year, and advanced to the rank of colonel later in 1918. Following the Armistice he was appointed chief of the Engineering Division and commanding officer of the Aeronautical Experimental and Research Station of the Air Service at McCook Field, Dayton, Ohio, where he was stationed until being retired, at his request, at the end of 1922. He also served on the National Advisory Committee for Aeronautics from 1918 to 1922. While he was at McCook Field a parachute which is now standard equipment was developed by him. He also was a leader in aerial photography, and designed air-cooled engines and earth-induction compasses.

After his retirement in 1922 Colonel Bane spent six years in California. He turned his attention to commercial aviation in 1928, when he became chief of the technical section of the Aviation Corporation of Delaware, and was active in laying out some nine thousand miles of airways, which were lighted for night flying and equipped with landing fields and ground facilities under his direction. Colonel Bane also assisted in the development of a scientific staff for forecasting weather conditions for pilots which was complementary to the Government system. Radio telephony as adapted to airplanes occupied his attention and it was due to his efforts that the planes of the American Airways were equipped with telephones.

The King of Italy appointed Colonel Bane Commander of the Order of the Crown of Italy for technical service during the World War. He had been an associate member of the A.S.M.E. since 1918 and also belonged to the Society of Automotive Engineers and the Army and Navy clubs in New York and Washington, D.C.

LOUIS BARBER (1870–1932)

Louis Barber, mechanical engineer for The National Screw & Mfg. Co., Cleveland, Ohio, died of heart failure at his home in that city on July 9, 1932.

Mr. Barber was born at Ottawa, Canada, on June 17, 1870, the son of Alfred and Caroline (Cooper) Barber. He was educated by a private tutor and courses of the International Correspondence Schools. After about ten years in wood and metal pattern making he took a position as draftsman with The National Screw & Tack Co., which later became The National Screw & Mfg. Co. He was connected with this company during the remainder of his life, except from 1902 to 1904, when he was draftsman for the Brown Hoisting Machinery Company and the Royal Motor Car Company. He served the company for many years as machine designer and assigned to it two patents for improvements in a nut castellating machine.

Mr. Barber became a member of the A.S.M.E. in 1920 and was also a member of the Masonic fraternity. Surviving him are his widow, formerly Miss Eleanor Galloway, and two children, Mrs. C. G. Fox, Hudson, Ohio, and W. Donald Barber, of Cleveland.

FRANK GEORGE BAUM (1870–1932)

Frank George Baum, internationally known hydroelectric engineer, died at Redding, Calif., where he made his home, on March 14, 1932. As an inventor, writer, and consultant he made many notable contributions to the development of hydroelectric engineering.

Mr. Baum was born at Sainte Genevieve, Mo., on July 18, 1870, the son of Christen and Caroline Klein Baum. He was graduated from Stanford University with an A.B. degree in electrical engineering in 1898 and was an assistant instructor in that subject at the University during the following year, while working for his E.E. degree, which he received in 1899.

It is interesting to note that Mr. Baum's first position after leaving the University, with the Standard Electric Company of California, San Francisco, involved high-tension transmission work, for it was the transmission phase of the industry which he furthered to the greatest extent. After a few months in San Francisco he went to Pittsfield, Mass., where for about a year he was employed at the works of the Stanley Electric Company. He returned to Stanford University in 1900 as an instructor in electrical engineering, and during two years so spent also did special work in electrical transmission for the Bay Counties Power Company and others.

In March, 1902, Mr. Baum entered the employ of the California Gas & Electric Corp., remaining with this company and its successor, the Pacific Gas & Electric Co., for five years, in charge of hydraulic and electrical construction and operation. His work as a consultant was begun in 1907 and in this capacity and later as its chief engineer he continued for many years to assist the Pacific Gas & Electric Co. in the development of its system, and as consultant for many other companies played a large part in building up hydroelectric power on the Pacific Coast.

In the Piedmont, California, station he originated a method of compounding synchronous motor-generator sets, whereby the compounding of the exciters was taken from the shunt on the busbar of the railway load, changing the power factor of the motors and keeping the potential of the substation nearly normal. This was in line with his much later invention, the synchronizing condenser and regulator, announced in 1927, a device which solved the problem of economical transmission of large quantities of electrical power over distances of several hundred miles, dividing the lines into sections of from 100 to 200 miles linked by rotary regulators or condensers, each transmitting the current onward with small losses. By means of this device it became possible to materially increase the capacity of a line at a comparatively low cost, thus bringing about the economical transmission of power from distant points and permitting sparsely settled districts and farms to be supplied with electric power at a reasonable price.

The conception as well as the plans for the Pit River, California, project of the Pacific Gas & Electric Co. may be traced to Mr. Baum's appreciation of the possibilities of the basin. He designed the Hat Creek stations Nos. 1 and 2, Pit River No. 1, the 220,000-volt Pit transmission line, and the great Vaca substation at the receiving end.

He also put into practical form a type of high-tension switch and invented an outdoor high-tension switch which were quite generally introduced in western transmission systems.

In 1900, in conjunction with Dr. F. A. C. Perrine, Mr. Baum presented the first paper published on the calculation of the charging current in three-phase transmission lines, and developed his method of calculating the regulation of transmission systems, which forms the basis for his "Alternating-Current Calculating Device," published in 1902. His paper on "Surges in Transmission Systems," presented in 1902 before the Pacific Coast Transmission Association, first pointed out in a practical way a simple method of calculating the rise in pressure due to surges in transmission systems. In 1904 he presented a notable paper before the International Electric Congress, St. Louis, on high-tension, long-distance transmission and control. He was the author of a book entitled "The Alternating-Current Transformer," and in 1923 published an "Atlas of U.S.A. Electric Power Industry," outlining a plan for nation-wide electric service. His paper on "Voltage Regulation and Insulation of Large-Power Long-Distance Transmission System," won for him the annual transmission prize of the American Institute of Electrical Engineers in 1921 and was the basis for later work on the development of insulation and regulation systems. Many other articles dealing with different phases of hydroelectric power development and transmission were contributed to the technical press by Mr. Baum.

His work as a consultant took Mr. Baum to France and Germany, and he spent several years studying the European electrical power industry and compiled a voluminous report on the enlargement and extension of one of Germany's largest electric utilities. He had recently established extensive laboratories in the mountains in Northern California where he planned further study of hydroelectric problems.

Mr. Baum became a member of the A.S.M.E. in 1906. He served as vice-president of the American Institute of Electrical Engineers from 1906 to 1908 and was also a member of the American Society of Civil Engineers, several clubs in New York and San Francisco, and the Sigma Xi fraternity.

Surviving Mr. Baum are his widow, Mary Elizabeth (Dawson) Baum, formerly of Butte, Mont., whom he married in 1900, and three daughters.

BERNARD ARTHUR BEHREND (1875–1932)

Bernard Arthur Behrend, widely known as a consulting engineer in the electrical field and who had recently been in ill health, died on March 25, 1932, at his home in Wellesley Hills, Mass. Mrs. Behrend, formerly Miss Margaret Chase, of Boston, Mass., whom he married in 1926, survives her husband.

Dr. Behrend was an authority on the design of heavy alternating-current machinery, and his long and distinguished career placed him in the front rank of contributors to theory and practice. In recognition of his achievements, the honorary degree of doctor of engineering was conferred upon him in 1931 by the University of Darmstadt, Germany. The John Scott Medal was awarded him by the City of Philadelphia in 1912, upon the recommendation of The Franklin Institute.

He was born at Villeneuve, Switzerland, on May 9, 1875, the son of Moritz and Rebecca (Wolf) Behrend. His early education was secured by private tutelage in Switzerland, Paris, and in England, where part of his boyhood was spent. He studied civil and mechanical engineering at the Polytechnic Institute of Charlottenburg, Germany, and at the University of Berlin, from which he was graduated in 1894 with a C.E. degree. During the year following he was assistant to Gisbert Kapp as designer of power plants, and for the next three years assistant to the chief engineer of the Oerlikon Company, Switzerland, in charge of the Experimental and Testing Departments, working on the development and design of electrical machinery and the design of the Jungfrau Railway.

Dr. Behrend came to the United States in 1899, applied for citizenship at once, and was granted naturalization in 1903. After some consulting work and the delivery of a course of lectures at the University of Wisconsin on the theory and design of induction motors, he became connected in 1900 with the Bullock Electric Manufacturing Company, Cincinnati, Ohio, not long afterward being appointed its chief engineer. In 1904 this company was absorbed by the Allis-Chalmers Manufacturing Company, Dr. Behrend continuing in the office of chief engineer of the electrical departments in the United States and serving as consulting engineer to Allis-Chalmers Bullock, Ltd., Montreal, until 1908, with headquarters at the main plant at West Allis, near Milwaukee, Wis., during the latter part of the time. In 1908 he was invited to join the engineering force of the Westinghouse Electric & Manufacturing Co. at East Pittsburgh, Pa., and assumed his duties early the following year, shortly afterward taking charge of the Power Division. About two years later he opened an office in Boston as consulting engineer and was retained in that capacity by the Westinghouse company for many years. He gave up the Boston office in 1926 and since then had continued his practice at Wellesley Hills.

Dr. Behrend was a frequent contributor to technical literature. An early article of first-rate importance, written in the summer of 1895 and published in *Elektrotechnische Zeitschrift* in February, 1896, dealt with the theory of the circle diagram which he had originated for calculating the performance of induction motors in a simple and direct manner. The substance of his lectures at the University of Wisconsin in 1899, covering single-phase as well as polyphase motors, was published in 1901 under the title, "The Induction Motor, Its Theory and Design," translations appearing in French in 1902, German, 1903, and some sections later even in Japanese. A new edition, considerably enlarged and revised, appeared under the title, "The Induction Motor and Other Alternating-Current Motors," in 1921.

Another of Dr. Behrend's early achievements was the successful design of alternating-current machinery, including a.-c. generators and induction motors, many of them of difficult ratings for those days. In a paper which he presented before the American Institute of Electrical Engineers in 1903, entitled "The Experimental Basis for the Theory of the Regulation of Alternators," he suggested improved methods for determining regulation which were later made the basis of the rule adopted by the Standards Committee of the Institute.

The following year he presented a paper on "The Testing of Alternating-Current Generators" at the International Electrical Congress in St. Louis, Mo., dealing with his "split-field" method. This method enabled him to test alternators at nearly zero power factor, adequate load being obtained by one part of the machine having lagging, and the other leading, current.

The first turbo-alternator built by the Bullock Electric Manufacturing Company, jointly with Hoovens, Owen & Rentschler under Dr. Behrend's supervision, was exhibited at the Louisiana Purchase Exposition at St. Louis, in 1904. This unit demonstrated the radial-slot cylindrical turbo-rotor with chrome-nickel-steel end rings now generally standard construction with all manufacturers of turbo-generators. It received a grand prize at the exposition and Dr. Behrend was awarded a gold medal for its design. The largest power unit of the exposition, a 3500-kw. generator driven by an Allis-Chalmers engine, was also designed by him and secured for itself the name of "Old Reliable," as it was always functioning when other units were out of commission.

While Dr. Behrend was still with the Bullock company he made a study of the distribution of stresses in rotating disks and tested the permanent deformation of lead disks driven at high speed. Some of his results were published in a paper entitled "A New Large Generator for Niagara Falls," presented to the A.I.E.E. in 1908, in which he showed how stresses are distributed, both radially and tangentially, in a high-speed disk.

Among many other technical papers contributed to the A.I.E.E. by him were those dealing with the mechanical forces in dynamos caused by magnetic attraction, the elementary theory of surges on long lines, the winding of electric generators for 22,000 or more volts, and the feasibility of this voltage, even for small outputs. He was also the author of miscellaneous publications, including papers on the achievements of Oliver Heaviside and C. E. L. Brown, engineering education, and machine tools.

As an educator he had been non-resident lecturer at Leland Stanford, Jr., University, McGill University, and the Massachusetts Institute of Technology, in addition to the University of Wisconsin. In 1901 he organized the first engineering training classes at Cincinnati, Ohio, under A. G. Wessling.

His work between 1900 and 1908 included several other achievements of particular note. In collaboration with Ralph D. Mershon he brought about the first successful paralleling of several large power stations connected through transmission systems and synchronous frequency changers. This was the linkage, at Montreal, of power from the Lachine Rapids, the Montreal Light, Heat & Power Co., and Shawinigan Falls, 80 miles distant.

In an important instance of the parallel operation of large gas-engine-driven alternators dealt with by him and his assistant, Mr. A. B. Field, the somewhat bold course was successfully adopted of using very light flywheels, with the natural frequency between two forced frequencies, rather than lower than the lowest frequency; the fifteen 3000-kva sets of the Indiana Steel Company, at Gary, were so designed. Other groups of large gas-engine-driven alternators were built for the power houses of the United States Steel Corporation at Rankin, Pa., the Illinois Steel Company, South Chicago, Ill., and the American Steel & Wire Co., Cleveland, Ohio. Dr. Behrend also designed the electric generating units for the Kern River Power Company, the Pacific Electric Company, the Denver Gas & Electric Co., a large group of units for Niagara Falls, and the steam-turbine units of the Brooklyn Edison Company and Brooklyn Rapid Transit Company, at that time the largest and fastest of their type.

The plate-rotor construction which the Westinghouse Company has used for many years in its large turbine generators of more than two poles was another product of Dr. Behrend's inventive ability. It overcame the danger of failure occurring in high-speed turbine-generator rotors from lack of homogeneity, from internal shrinkage stresses, or from pipes, cracks, etc., which are liable to appear in large steel castings or forgings. In all, he took out more than 80 patents, the most of which were assigned to the Allis-Chalmers or Westinghouse companies.

Dr. Behrend was a senior vice-president of the American Institute of Electrical Engineers, which he had also served as manager. He was a Fellow of the Institute and had been particularly active in its committee work. He had served as chairman of its Electrical Machinery Committee and Committee on Professional Conduct and had represented the Institute on the Edison Medal Committee, Standards Committee, U.S. National Committee of the International Electrotechnical Commission, and Engineering Societies Library Board. He was the first chairman of the Cincinnati Section of the Institute.

He was also a Fellow of the American Physical Society, the American Association for the Advancement of Science, and the American Academy of Arts and Sciences. He had been a member of the A.S.M.E. since 1909 and also belonged to the American Society of Civil Engineers, The Franklin Institute, the Institution of Electrical Engineers (Great Britain), and the Society for the Preservation of New England Antiquities, as well as the Engineers' Clubs in New York and Boston and the Pittsburgh Athletic Club.

WILLIAM DIETRICH BERGMANN (1892–1932)

William Dietrich Bergmann, who died on December 15, 1932, was born on March 20, 1892, in Schenectady, N.Y., the son of William Dietrich and Henrietta (Fischer) Bergmann. He attended the New York State Institute for the Deaf until he was fourteen and then the Barnard School for Boys of Columbia University for several years. From 1916 to 1922 he had a varied experience in designing, making detailed drawings and layouts for different companies and individuals in New York and vicinity. He worked on engines for the Aeronautical Motor Corporation, Long Island City, N.Y., and the Duesenburg Motor Corporation, Elizabeth, N.J.; a motorcycle and light automobile for the Militor Corporation, Jersey City, N.J.; and an electric furnace for a foundry in Brazil, an asphalt plant in Japan, and a U.S. Army tank tractor, as draftsman for consulting engineers. For a time he was assistant in charge of a film laboratory in the New York district for Movette, Inc., of Rochester, N.Y., and had made a special study of moving picture photography and also of television. He also was employed by the Electric Car & Locomotive Corp., Petroleum Power Devices Corporation, and Dickinson Cord Tire Corporation. Subsequently he was on the staff of the Borough president of Queens, in Long Island City, as mechanical designer and engineer for some years.

Mr. Bergmann became a junior member of the Society in 1920 and an associate member six years later. He is survived by his widow, Lillian (Berg) Bergmann and two daughters, Etta L. and Marie E. Bergmann, and by his mother.

JAMES LORD BERNARD (1870–1932)

James Lord Bernard, who died of pneumonia at Beaver, Pa., on August 20, 1932, was born on November 10, 1870, in Philadelphia. His parents were William and Sarah (Lord) Bernard.

After completing his public school work Mr. Bernard attended the Episcopal Academy in Philadelphia and was graduated from the University of Pennsylvania in 1892 with a B.S. degree. From then until 1901 he was connected with the A. & P. Roberts Co., Pencoyd, Pa., for three years in the office of the superintendent of motive power, as draftsman, for about five years in the repairs department, working on engines, boilers, etc., and the remainder of the time as inspector and engineer of tests.

Mr. Bernard became assistant mechanical engineer of the American Bridge Company in Philadelphia in 1901 and three years later was made chief mechanical engineer, with office at Ambridge, Pa., the position which he held at the time of his death. He held a number of patents in the United States and Canada relating to power house, rolling mill, and bridge shop operations. He was an authority on matters pertaining to power generation and utilization and outstanding in machinery invention and design.

Mr. Bernard became a member of the A.S.M.E. in 1902. He is survived by his widow, Sophia (Lehman) Bernard, whom he married in 1894.

CHARLES HAROLD BEST (1898–1932)

Charles Harold Best, technical engineer of the Marysville Power House of The Detroit Edison Company, Port Huron, Mich., died on March 28, 1932, in Detroit. He was born at St. Helens, Lancashire, England, on August 2, 1898, the son of Thomas Thompson and Ada (Fisk) Best. He was educated in England, receiving a Bachelor of Engineering degree from Liverpool University in 1922.

Mr. Best worked in the shops of the Metropolitan Vickers Company in Manchester during his university vacations. His first position following his graduation was in Canada, for the Canadian Salt Co., Ltd., Windsor, Ontario. He came to the United States late in 1922 and had since been connected with The Detroit Edison Company, first as a student engineer and since 1925 as technical engineer at the Marysville plant.

Mr. Best became a junior member of the A.S.M.E. in 1927 and belonged to the Congregational Church and several local clubs. He is survived by his widow, Elsie (Degen) Best, whom he married in 1930.

HUGO BILGRAM (1847–1932)

Hugo Bilgram, inventor of the bevel gear generator and an economist of note, died at his home in Moylan, Pa., on August 27, 1932, at the age of 85. Until two years before his death he had been active in the Bilgram Machine Works, Inc., of Philadelphia, of which he was president and owner.

Mr. Bilgram was born in Memmingen, Bavaria, on January 13, 1847, the son of George David and Rosina (Wiedemann) Bilgram. He was graduated from the Augsburg Maschinenbau Schule, where he studied mechanical engineering, in 1865, and worked as a draftsman and machinist for several firms in Germany before coming to the United States four years later.

His first business connections in this country were with the L. B. Flanders Company and the Southwark Foundry & Machine Co., in Philadelphia, for which he designed instruments and machines. During this period, in 1876, he also assisted in an evening class in drawing at The Franklin Institute. In connection with the drawing of accurate outlines of the teeth of gear wheels, he developed a method which led to the design in 1883 of a machine for cutting the teeth of gear wheels, particularly of bevel wheels. Later it was improved and adapted to production work, especially in the automotive industry, greatly facilitating the manufacture of bevel gears.

The Bilgram Machine Works grew out of the Brehmer Brothers Company, for which Mr. Bilgram first worked as a machinist. In 1879 the main works were transferred to Germany, and Mr. Bilgram was put in charge of the branch in this country. He became sole owner in 1884 and later incorporated the company under his own name.

Mr. Bilgram is also credited with several other inventions in connection with gear manufacturing, and wrote several books on mechanical engineering and on economics, including "Slide Valve Gears," "The Remedy for Overproduction and Unemployment," "Involuntary Idleness," and "The Cause of Business Depressions." His achievements won him a number of medals, including the Elliott Cresson Gold Medal of The Franklin Institute, the John Scott Medal of the City of Philadelphia, and awards of various world expositions. He was also an amateur botanist and was keenly interested in microscopic work.

Mr. Bilgram's was the oldest membership in The Franklin Institute and he was one of the early members of the A.S.M.E., which he joined in 1885. He also belonged to the American Academy of Political and Social Science, the Philadelphia Technische Verein, and the Engineer's Club of that city.

Surviving Mr. Bilgram are a son, Oscar Bilgram, and a daughter, Miss Bertha Bilgram, both of Moylan. His wife was Mary Fisher, of Philadelphia, whom he married in 1872.

GEORGE HUTCHINSON BLACK (1872–1932)

George Hutchinson Black, mechanical superintendent of the Berkshire Fine Spinning Associates, Adams, Mass., died on November 9, 1932, after a few weeks' illness. He was an associate member of the A.S.M.E., which he joined in 1919, and belonged to the Masonic fraternity.

Mr. Black was born in Dunfermline, Fifeshire, Scotland, on March 26, 1872, the son of David and Janet (Hutchinson) Black. He was educated in Scotland and served a four-year apprenticeship as draftsman, patternmaker, erecting engineer, and lathe hand at the foundry of Messrs. Bennett & Sons, Dunfermline.

During the next two years, 1891–1893, he was erecting and repair engineer for Hendersons Anchor Line, Govan, Glasgow. He then went to Hyde, near Manchester, England, where he was connected for about seven years with Messrs. G. B. Goodfellows & Sons, stationary engineers, experimenting on and building cold-air refrigerating machines for ship cargoes. After two more years in Hyde as mechanical engineer for the Newton Bank Calico Printing Company, he came to the United States.

Mr. Black's first position here was with the American & British Mfg. Co., Providence, R.I., as erecting engineer and on general work in connection with oil and steam engines. He was also employed for a time by the George H. Corliss Co. in Providence. In 1908 he became mechanical superintendent at the Kilburn Mills, New Bedford, Mass., with which he was connected until the latter part of 1917. After about six months with the Acushnett & Hathaway Corp., also in New Bedford, he became mechanical superintendent for the Berkshire Cotton Manufacturing Company, Adams, Mass., which later merged into the Berkshire Fine Spinning Associates.

Surviving Mr. Black is his widow, Elizabeth (MacLaren) Black, whom he married in 1928.

HOWARD EDWARD BOARDMAN (1879–1932)

Howard Edward Boardman, recently appointed Dudley professor of railroad engineering in the Graduate School of Yale University, died of pneumonia on February 28, 1932, at his home in New Haven, Conn. Professor Boardman was the first to hold this chair, the purpose of which is to develop, in cooperation with railroads, practical and scientific research in the general field of railroad engineering, and to guide graduate students in this field. Although he had but recently begun the work, he had already achieved marked success in outlining a program of research and in interesting leading railroad executives and promising students in his plans.

Professor Boardman was born in Meriden, Conn., on March 17, 1879, the son of Edward Allen and Nellie (Savage) Boardman. His early education was secured in Meriden and he was graduated from the Civil Engineering Department of the Sheffield Scientific School of Yale with a Ph.B. degree in 1899. The honorary degree of Master of Arts was conferred upon him by Yale in 1931.

Following a year as instrumentman for the Pennsylvania Railroad making preliminary surveys for branch-line construction in Central Pennsylvania and New Jersey and another year as assistant in the Engineer Corps on lines west of Pittsburgh, during the latter part of the time in charge of construction at a large freight interchange yard at Conway, Pa., he was transferred to work on the Pennsylvania tunnels under the Hudson River. He was engaged in this work from 1902 to 1909, serving successively as chief of party, assistant engineer, and senior assistant engineer. Surveys, contract drawings and specifications, estimates, cost records, and reports were made by him, and he was in charge of the alignment of the tunnels and inspection of excavations and tunnel linings.

In 1909 Professor Boardman entered the employ of the Missouri Pacific Railway as senior assistant engineer in its Maintenance of Way Department. He had charge of the construction carried on by the office of the chief engineer, entailing drafting, estimating, investigations and reports, and varied construction work. Later, as division engineer for that road, he was in charge of maintenance of more than five-hundred miles of line.

Leaving the Missouri Pacific in 1912, Professor Boardman went to Buenos Aires, Argentina, taking the position of chief assistant engineer of the Buenos Aires Western Railway. He remained there for about four years, in charge of layout and construction of four miles of concrete-lined electrified railway tunnel under the city of Buenos Aires, with a joint underground passenger station.

After his return to the United States in 1916 he was appointed engineering assistant to the General Valuation Counsel, New York Central Lines, New York, N.Y. Special reports and cost analyses, the preparation of data for the Interstate Commerce Commission, and general valuation work on the several lines of the road, engaged his attention during his seven years with it.

During the latter part of 1922 Professor Boardman accepted the office of president and treasurer of the Foster Merriam & Co., Meriden, Conn., hardware and metal specialty manufacturers and gray-iron and brass founders. He entirely reorganized the financial structure of the company, greatly reduced overhead expenses, designed and brought out new products, realigned the sales policy, and increased sales about seventy-five per cent during the next four years. In 1927 and 1928 he was sales manager, in charge of sales, sales engineering, and experimental work for the Divine Brothers Company, Utica, N.Y., constructing, designing, demonstrating, and selling automatic and other machinery for the use of the metal finishing industry. Following this he was for one year manager of research and assistant to the president of the National Industrial Conference Board in charge of the organization engaged in collecting and interpreting authentic industrial economic information and statistics. From then until his appointment at Yale he had been employed as construction engineer for the Boston & Maine R.R.

Professor Boardman had been a member of the A.S.M.E. since 1927 and also belonged to the American Society of Civil Engineers. Surviving him are his widow, Enna M. (Jenkins) Boardman, whom he married in 1903, and two sons, Edward T., and Howard Foster Boardman.

FREDERICK BAYLIS BROWN (1881–1932)

Frederick Baylis Brown, consulting engineer, Montreal, Canada, was instantly killed on August 6, 1932, when struck by the propeller of a seaplane at Lac Ouareau, his summer home.

Mr. Brown was born in Montreal on December 27, 1881, the son of William Forbes and Annie C. (Baylis) Brown. He received his early training at the Montreal Collegiate Institute and the Montreal High School, and then enrolled at McGill University, from which he received the degree of Bachelor of Science in 1903, with honors in mechanical engineering. The following year he took honors in electrical engineering and in 1905 he was awarded the degree of Master of Science for research work in civil, hydraulic, and mechanical engineering subjects.

Mr. Brown's first practical experience in engineering was as an apprentice in the machine shop of the firm of Pillow and Hersey in Montreal during his college vacations. In 1904 he was employed in the locomotive shops of the Grand Trunk Railway System at Point St. Charles, and for the next five years was a member of the staff of Messrs. Ross and Holgate, consulting engineers, Montreal. He then became a partner in the consulting firm of Walter J. Francis and Company, and in 1924, upon the death of Mr. Francis, became the senior partner.

Among Dominion engineers Mr. Brown took first rank. His work embraced civil, hydraulic, mechanical, and electrical problems, and was carried on in every province of the Dominion. He had taken part in arbitration and legal cases, made appraisals and estimates, determined rates for public utilities, and designed and built hydroelectric and steam power plants, waterworks, and various other structures. Some of the outstanding accomplishments with which he was associated were the power plant for the Angus Shops of the Canadian Pacific Railway; among several assignments for the City of Westmount, the first successful installation in America of high-temperature refuse destructors and the first installation of magnetite arc lamps in Canada; a complete water supply system, costing about $600,000, for Moose Jaw, Saskatchewan; surveys and designs for the Carillon hydroelectric plant on the Ottawa River, an eighteen million dollar project covering several hundred miles; building problems in connection with the Mount Royal Tunnel; supervision, with other members of special board, of the design and construction of the Montreal Aqueduct; Gatineau River hydroelectric development for the Hull Electric Company; and hydroelectric plant and street railway for the Sherbrooke Railway, Light & Power Co., Ltd. He served as consulting engineer with the Royal Commission of Inquiry in 1922 and 1923, investigating the engineering and economic aspects of the Ontario Hydro Electric Power Commission, and was called upon in a consulting capacity to assist many municipalities and corporations in connection with engineering problems. Just before his death he had attended the official opening of a new street lighting system in the town of Mount Royal. During the World War he was engaged in munition plant work, including power plant problems and steel treatment.

The Beauharnois power development is a monument to his engineering skill, for he was associated with the enterprise from the start, was responsible for all the preliminary plans, and was a member of the board of consulting engineers at the time of his death.

Mr. Brown's investigations of industrial and manufacturing problems included shop organization and management and he had given special lectures on engineering economics at McGill University.

He was a contributor to the publications of the technical organizations to which he belonged. These included the A.S.M.E., which he joined in 1913, the American Institute of Electrical Engineers, and The Engineering Institute of Canada, of which he was a life member and past council member. He was a charter member of the Corporation of Professional Engineers of Quebec and served as honorary-secretary-treasurer of this Corporation from 1920 to 1930. He was a Graduates' Representative Fellow in Applied Science and a member of the Corporation of McGill University. For many years previous to his death he was president of his graduating class, Science '03. He was elected a life member of the University Club of Montreal in 1920, was a member of the Homeopathic Hospital Association and a director from 1914 to 1925, and belonged to a number of social and athletic organizations. In 1924 Mr. Brown was an official Canadian delegate at the First World Power Conference held in London, England. His Masonic affiliations included University Lodge at Montreal, Royal Albert Chapter of Royal Archmasons, and in the Scottish Rite the Hochelaga Grand Lodge of Perfection, the Sovereign Chapter of Rose Croix, and the Montreal Sovereign Consistory 32d degree.

Surviving Mr. Brown are his widow, Alice (Lech) Brown, whom he married in 1907, one son, Frederick William Lech Brown, two daughters, Mrs. John A. MacLennan, of McMasterville, and Alison Hamilton Brown, and a grandson, Frederick F. MacLennan.

LUTHER D. BURLINGAME (1865–1932)

Luther D. Burlingame, industrial superintendent of the Brown & Sharpe Manufacturing Co., Providence, R.I., and for many years a leading figure in Providence civic affairs, died at his home in that city on June 2, 1932, after a short illness.

Mr. Burlingame was born in Whitestown (now Whitesboro), N.Y., on January 12, 1865, the son of Luther R. and Emeline (Aldrich) Burlingame. He attended the public schools of Dover, N.H., and Providence, being graduated from the English Department of the Providence High School in 1884 as valedictorian. He entered the employ of the Brown & Sharpe Manufacturing Co., as an apprentice in the drafting department, in the fall of the same year, and had training both in that department and in the machine shop.

He was made chief draftsman in 1888, when there were 14 employees in the department, and continued in that position for more than 25 years, seeing the personnel of the department increase in number to more than 100. He then became industrial superintendent, chairman of the safety committee, and patent specialist.

For many years Mr. Burlingame had been a leader in work for the improvement of Rhode Island's park and recreational facilities and the beautification of State roads. He served as secretary and president of the League of Improvement Societies in Rhode Island, and in 1922 helped to effect the consolidation of the League with the Public Parks Association and the Citizens' City Planning Association, forming a new organization called the Civic Improvement and Park Association. He was elected the first president of the association and served continuously in that capacity until his death.

Recently he had been made chairman of the committee on subsistence gardens of the Governor's Conference on Employment and Relief and had been working tirelessly to organize the state in a program of gardening for the benefit of the unemployed. During the World War he gained much experience in this line of work by organizing and directing a large garden for employees of the Brown & Sharpe Manufacturing Co.

Mr. Burlingame had long been active in the Providence Chamber of Commerce; he was a member of its Civic Affairs Committee from 1915 to 1919, chairman of the committee from 1922 to 1927, and a member of the Chamber's Board of Directors from 1921 to 1928. He was president of the Providence Safety Council and an organizer and president, in 1897, of the Providence Association of Mechanical Engineers, later reorganized to embrace all branches of engineering and now known as the Providence Engineering Society. He was president and a director of the Pleasant Valley Land Company and a member of the corporation of the Providence Institution for Savings.

Mr. Burlingame joined the A.S.M.E. in 1903 and during his 29 years of membership was a frequent contributor to its publications and an active member of its technical committees. In 1920 when the Sectional Committee on Plain Limit Gages for General Engineering Work was organized he was recognized as a pioneer in this field and was appointed to membership on this committee as an official representative of the A.S.M.E. He made important contributions to its report. Then, when a new committee was formed to revise and amplify the report published in 1925, he was reappointed as a representative of the Drill and Reamer Society, the Tap and Die Institute, and the Milling Cutter Society.

Screw thread fits was another of his interests, so he was a member of all the committees on this subject organized since he became a member of the Society. With the reorganization of the Sectional Committee on the Standardization and Unification of Screw Threads he was appointed as one of the Society's official representatives. In 1924, as a direct result of his activities in this field, the A.S.M.E. nominated him for appointment to membership on the National Screw Thread Commission.

In addition, he has represented the Society's interests on the Technical Committees on T-slots and Machine Tapers, both subdivisions of the Sectional Committee on the Standardization of Small Tools and Machine Tool Elements.

He enjoyed traveling, so rarely missed attendance at the Spring Meetings of the Society. He and Mrs. Burlingame were always present at the Annual Meeting in New York.

Mr. Burlingame was a member of the Executive Committee of the American Institute of Weights and Measures, a member of the Board of Directors of the National Safety Council, and a member of The Franklin Institute, the National Economic League, the Economic Club of Rhode Island, the corporation of the Rhode Island School of Design, the Rhode Island Field Naturalists' Club, the Grand Central Art Galleries (New York), and a number of clubs, including the Players and Turks Head in Providence and the Engineers' in New York. He was a prominent member of the Fourth Baptist Church, Providence, for many years, serving as a leader of the Sunday School and the teacher of the Men's Burlingame Bible Class. He had been an active worker in the cause of prohibition.

Mr. Burlingame had made a noteworthy collection of mounted butterflies and moths. He used friezes of the mounted butterflies for screens and other decorations in his home.

Surviving him are his widow, formerly Miss Christine Ward, four children, Mrs. Grace C. (Ernest L.) Lockwood, Warwick, R.I., Harold Luther, residing in California, and Mrs. Ethel B. (Henry J., 3d) Spooner, and Stanley W. Burlingame, both of Providence, and 12 grandchildren.

CARLETON STRONG CAIROLI (1886–1932)

Carleton Strong Cairoli, mechanical engineer for the Western Electric Company, New York, died on June 14, 1932. He had been on the Western Electric staff since 1924, and was also connected with the company in 1918.

Mr. Cairoli was born at Milford, Conn., on February 12, 1886. He received a Ph.B. degree from the Sheffield Scientific School of Yale University in 1908. His first position was with the B. & R. Rubber Co., North Brookfield, Mass. From there he went to Stamford, Conn., where he was employed by the Yale & Towne Mfg. Co., from 1909 to 1915, as instructor and foreman in the Apprentice Department and as designing draftsman. During the next three years he was connected successively with the Remington Arms & Ammunition Co., Bridgeport, Conn., in designing and installing power transmission, heating, and other equipment; the Kathodion Bronze Works, Nyack, N.Y., since discontinued, in similar work; Wm. S. Gilbert Clock Co., Winsted, Conn., in analyzing and standardizing operation and production; and the August Buerman Manufacturing Company, Newark, N.J., in charge of the design and installation of exhaust, ventilating, and heating systems. From 1918 to 1923 Mr. Cairoli was maintenance engineer for the National Conduit & Cable Co., Inc., Hastings-on-Hudson, N.Y. Since then he had been in charge of heating and ventilating in maintenance and construction work for the Western Electric Company.

Mr. Cairoli became an associate member of the A.S.M.E. in 1919. He is survived by his widow, Florence (Maeder) Cairoli, whom he married in 1920, two daughters, Emily Maeder and Myrtis Strong Cairoli, and his mother, Susan Strong Cairoli.

ALEXANDER WINCHESTER CARROLL (1867–1931)

Alexander Winchester Carroll, consulting engineer, Elizabeth, N.J., whose death occurred on December 17, 1931, was born at Baltimore, Md., on May 31, 1867. His parents were Henry Hill and Mary (Winchester) Carroll. He attended private schools and the Baltimore Manual Training School and served an apprenticeship as a machinist with the Chesapeake Screw Company and Bartlett Haywood & Co., in Baltimore, from 1885 to 1888.

Before beginning his consulting work in 1908 Mr. Carroll was connected successively with the Maryland Steel Company, Sparrows Point, Md., the Homestead Steel Works of the Carnegie Steel Company, steel, wire, and tube works in Beaver Falls, Pa., the De Lamar Copper Refining Company, Carteret, N.J., and the Chrome Steel Works, Chrome, N.J. In 1911 he was mechanical engineer for J. G. White & Co., New York, and for the next five years held a similar position with Barclay, Parsons & Klapp, New York. For about ten years, beginning in 1916, he was assistant superintendent of the Raritan Copper Works of the Anaconda Copper Mining Company, Perth Amboy, N.J., and from 1926 to 1928 superintendent of the American Brass Company, Hastings-on-Hudson, N.Y. Since then he had devoted himself to consulting work in Elizabeth. He was a member of the Home Guard in Elizabeth during the World War.

Mr. Carroll became a member of the A.S.M.E. in 1905, and also belonged to several clubs in Elizabeth.

ROY B. CHAMPION (1881–1932)

Roy B. Champion, superintendent of Public Works at Holland, Mich., died in that city on April 21, 1932, of apoplexy. He had been in ill health for about two years.

Mr. Champion became a member of the A.S.M.E. in 1928 and was serving on the Executive Committee of the Peninsula Section at the time of his death. He also belonged to the American Water Works Association and the Michigan Engineering Society. Locally he was affiliated with the Chamber of Commerce, Knights of Pythias, Exchange and Social Progress Clubs, and Sons of the American Revolution. He was a member of the Third Reformed Church and superintendent of the church Bible School.

Mr. Champion was born at Loveland, Iowa, on October 21, 1881. His parents, James A. and Addie (Waters) Champion, removed to Cherokee, Iowa, and later to Iowa City, where he was graduated from high school. He then attended the University of Iowa, which gave him a B.S. degree in 1906 and an E.E. degree in 1907. While at the University he became major of the University Battalion of the Iowa National Guard. For a short time after graduation he was superintendent of construction with the North Shore Electric Company in Chicago (since become the Public Service Company of Northern Illinois). He then went to Holland as city electrician in charge of the electric distribution system and station switchboard. He was promoted to the office of superintendent of public works in 1910 and retained the position for eight years. He was in direct charge of the electric, water, and sanitary sewer systems, supervising both construction and operation. He rebuilt part of the electric plant, putting in new switchboard and outgoing feeders, and developed an additional well-water supply.

In 1918 Mr. Champion resigned to become superintendent of the new Chester Water Power Company, Chester, Pa., where he remained two years. He returned to his former position in 1920, since when he had designed and built a motor-operated pumping station, rebuilt the boiler plant, and made many other improvements. Under his management the city of Holland has won a national reputation as an example of the successful operation of public utilities. With rates much lower than those of near-by public utilities, the municipal plants have returned to the city nearly $300,000 during the past eight years, representing a reduction in the annual tax rate on an average of $2.28 per thousand dollars of assessed valuation. In addition a splendid new municipal hospital was opened in 1929 at a cost of $184,000, financed entirely from the earnings of the utilities.

Surviving Mr. Champion, besides his parents and two sisters, are his widow, formerly Miss Oma G. Schaeffer of Iowa City, whom he married in 1907, and one daughter, Eula.

WILLIAM BREWSTER CHAPMAN (1873–1931)

William Brewster Chapman, whose death occurred in Chicago, Ill., on December 17, 1931, was born at Elyria, Ohio, on October 11, 1873, the son of James W., and Margaret (Darling) Chapman. He attended Oberlin College, 1892–1895, and then engaged for several years in civil engineering for the City of Cleveland and the Big Four Railroad.

From 1899 to 1901 Mr. Chapman was secretary and treasurer of the International Chemical Company and from then until 1905 was engaged in designing and building chemical works in Michigan for the Wood Alcohol Chemical Works. Subsequently he was connected for about a year with the Gas Producer Department of the Morgan Construction Company, Worcester, Mass. He then established the Chapman Engineering Company, New York, N.Y., manufacturers of producer-gas equipment. Mr. Chapman solved a number of difficult problems involved in the mechanical handling of coal, fire, and ash in mechanically operated gas producers during his years as president of the company, and earlier invented a process for manufacturing wood alcohol.

Mr. Chapman became a member of the A.S.M.E. in 1915. His clubs included the Machinery and Columbia Yacht, of New York, and the Mt. Vernon Country Club. He was twice married. His first wife was Rose Woodallen, of Ann Arbor, Mich., whom he married in 1902. A son, Bruce Chapman, of New York, N.Y., survives. His second marriage was to Ida Bailey Allen, of New York, in 1921.

LYLE GLENTWORTH CHASE (1886–1932)

Lyle Glentworth Chase, New York district sales manager for the Yarnall-Waring Company of Philadelphia, Pa., died on June 10, 1932, at Ithaca, N.Y., as the result of an automobile accident.

Mr. Chase was born at Chadron, Neb., on May 8, 1886, the son of Charles G. and Belle (Whitehill) Chase. His early education was secured at Kings Ferry and Union Springs, N.Y., and he was graduated from Cornell University with an M.E. degree in 1908. In the fall of that year he took a position as draftsman and designer in the Engineering Department of Thos. A. Edison, Inc., in Orange, N.J. After a year and a half there he became test engineer for the Parson Manufacturing Company, Elizabeth, N.J., later being advanced to the position of assistant superintendent. He remained there for about four years, working on forced-draft systems, vacuum ash ejectors, and fuel distributors. His next connection was with the Eastern Coal & Export Corp., Richmond, Va., first as assistant engineer and later as an expert on fuel economy and general power-plant consulting work.

Mr. Chase did some special work for the Yarnall-Waring Company in the summer of 1918, and in 1919, after serving for a few months as efficiency engineer for the Rosemary (N.C.) Manufacturing Company, in charge of power plants, he returned to the employ of the former company. He engaged in experimental and service work for about three years and then was transferred to New York as sales manager for that district.

Mr. Chase became a member of the A.S.M.E. in 1921. He was an executive member of the Watchung (N.J.) Council of the Boy Scouts of America. He belonged to the First Presbyterian Church and the Cornell Club in Plainfield, N.J., where he made his home, and was a Mason. Surviving him are his widow, Alice (Miller) Chase, whom he married in 1909, and three children, Nancy G., Margaret C., and Glentworth C. Chase.

ROBERT INGHAM CLEGG (1866–1931)

Robert Ingham Clegg, who died at Cleveland, Ohio, on December 3, 1931, was born at Acerington, Lancashire, England, on May 13, 1866. He secured his early education and served an apprenticeship as a machinist in Acerington. After coming to the United States in 1886 he worked in the shops of several Rhode Island companies, and was for some time associated with A. J. Cross, of Providence, in handling Shipman engines and as consultant to inventors.

Mr. Clegg entered the publishing field in 1898 as mechanical editor of *Horseless Age*, New York. From there he went to Cleveland as editor of the Machinery Department of the *Iron Trade Review* and subsequently was editor of *Woodcraft and Castings*, managing editor, *Blast Furnace and Steel Plant* and *Automobile Builder*, and vice-president of the Gardner Printing Company, Cleveland. He was a regular contributor of articles to many scientific publications, as well as to Masonic publications.

Mr. Clegg, a 33d degree Mason, had been a member of the fraternity since 1905 and for some years had been vice-president and editor-in-chief of The Masonic History Company, Chicago, Ill. He had received the most of the degrees conferred by Freemasons in Europe and the United States. For a number of years he was president of the Masonic Temple Association of Cleveland and had served on building committees for various masonic structures. Perhaps his greatest work for the Order was the revision of Mackey's History, Encyclopedia, Symbolism, and Jurisprudence of Freemasonry.

He was active in civic affairs, having served as a member and president of the Cleveland School Board and given both time and money to the development of the Cleveland Public Library. He had belonged to the A.S.M.E. since 1902 and continued to have a keen interest in its activities after entering upon the work of the Masonic fraternity. He served on the Constitution and By-Laws Committee from 1921 to 1923, as chairman during 1923. He was also a member of the Cleveland Engineering Society and had served a term as its vice-president.

Surviving Mr. Clegg are three children, Mrs. Ruth C. Hahn, and Norman V. and Lee M. Clegg. Mrs. Clegg, formerly Miss Emma Poole, of Providence, R.I., whom he married in 1892, died in 1914.

WILLIAM FLORENCE COCHRANE (1886–1932)

William Florence Cochrane, whose death occurred on January 5, 1932, was born at Steele, N.D., on May 5, 1886, the son of William Florence and Alma (Fridley) Cochrane. He was graduated from the United States Naval Academy at Annapolis in 1907 and spent seven years in the Navy, serving as chief engineer on several ships until 1913, when he was appointed assistant inspection machinist, U.S.N., at the Wm. Cramp & Sons Ship & Engine Bldg. Co., Philadelphia, Pa.

Following his resignation from the Navy in 1914, Mr. Cochrane became chief engineer for the U.S. Industrial Alcohol Company and its subsidiaries, the U.S. Industrial Chemical Co., Cuba Distilling Company, and Curtis Bay Copper & Iron Works, South Baltimore, Md.

During the World War he served as a lieutenant in the Ordnance Department, working on airplane fuel development. He was a director of the Cochrane Sales Company, Industrial Bank (Baltimore), and National Safety Fenders Corporation. He became a member of the A.S.M.E. in 1920 and also belonged to the Society of Automotive Engineers, American Society of Naval Engineers, the Army and Navy Club, Washington, D.C., Chemists Club, New York, and several other clubs.

Mr. Cochrane was married in Troy, N.Y., in 1909, to Gretchen Schneider, who, together with a son, William Henry Cochrane, survive him.

EUGENE LAMBERT COOKE (1894–1932)

Eugene Lambert Cooke, treasurer and general manager of the Cooke Boiler Works, Inc., New Haven, Conn., died of pneumonia at his home in that city on January 6, 1932.

Mr. Cooke was born in Philadelphia, Pa., on April 17, 1894, the son of John and Anna L. (Blakely) Cooke. He had two years' of study at the Erasmus Hall High School and a year at the Mechanics Institute. He then entered the employ of the Babcock & Wilcox Boiler Co., New York, working for two years on the erection of boilers in the field. From 1914 to 1916 he was assistant to the engineer of tests of the New York Edison Company at the Waterside Station. During the next two years he engaged in sales engineering work for the Elliott Company, New York, handling boiler equipment such as oil and water filtration systems, steam specialties and mechanical cleaners, and for the Vincent-Gilson Engineering Company, New York, selling and servicing combustion recorders and other instruments for controlling and regulating combustion in boiler furnaces. He returned to the employ of this company early in 1919, after several months in the Coast Artillery School for Enlisted Specialists, at Fort Monroe, Va., where he was enrolled in the electrical course.

Mr. Cooke went to New Haven in 1922 and spent two years with the New Haven Boiler Works before forming his own company. He enjoyed trap-shooting, hunted big game in Maine every fall, and was an excellent pianist. He is survived by his widow, Pauline (Berk) Cooke, whom he married in 1923, and by one son, Eugene L. Cooke, Jr.

Mr. Cooke became a junior member of the A.S.M.E. in 1917 and an associate member four years later.

RICHARD TELLER CRANE, JR. (1873–1931)

Richard Teller Crane, Jr., was born in Chicago, Ill., on November 7, 1873, fourth son of Richard Teller Crane, founder in 1855 of the Crane Co., manufacturers of valves, fittings, fabricated piping, plumbing, heating, and air-conditioning equipment. His mother was Mary Josephine (Prentice) Crane.

His early education was received at the Tilden School in Chicago, where his father organized and supported at his own expense the first public grade manual training school in Chicago. Later he attended Hill School, and in 1895 was graduated from Yale University with the degree of Bachelor of Science in the Sheffield Scientific School. Shortly thereafter he entered actively into the affairs of the rapidly growing business his father had founded. He held various official positions with the company until August 1, 1914, when, upon the retirement of his elder brother, Charles R. Crane, from the presidency, he was elected to that office, retaining it actively until the time of his death.

During the 17 years that Mr. Crane was president, Crane Co. saw the greatest expansion in its 76 years of growth. Under his leadership it developed into an organization with factories, branch houses, sales offices, and exhibit rooms in 200 cities throughout the world and with more than 20,000 employees. In administering its affairs Mr. Crane continued the traditions and policies of his father, which included a deep concern for the welfare of the personnel. The Crane Veteran League, which he organized in 1915 as a memorial to his father, is supported by an endowment fund provided by Mr. Cran and, at the time of his death, had a membership of 1401. Membership in the League is based on a minimum continuous service with the company of 25 years. During his presidency Mr. Crane distributed from his private fortune more than $13,000,000 worth of common stock in the company, and in his will supplemented this with a further large allotment of stock to those workers who had been with the company ten years and longer.

Mr. Crane's philanthropies were not confined to the company. To the City of Chicago he contributed the major portion of the cost of Iroquois Memorial Hospital, as a memorial to those who perished in the Iroquois Theatre fire in 1903. He also presented a hospital to the town of Ipswich, Mass., in memory of his friend, Benjamin Stickney Cable, and, jointly with Mrs. Crane, the completely equipped eye, ear, nose, and throat Florence Crane Hospital to the Hartford (Conn.) Hospital group and $100,000 toward the endowment of the Department of Therapeutics of New York University and Bellevue Hospital Medical College. He memorialized his parents in the gift of a carillon of 43 bells and the tower to house them, to St. Chrysostom's Episcopal Church, in Chicago, together with an endowment fund for maintenance. Buffalo Rock, a picturesque 50-acre spot on the Illinois River, which previously had served as a recreational and recuperative place for Crane employees and their families, was presented in 1928 to the State of Illinois, to be maintained perpetually by the State as a public park. "Crane Farms," near Lake Geneva, Wis., is now used by Crane employees. Among other donations are a valuable collection of Washingtoniana to the United States Naval Academy, the Library Building to Morristown (N.J.) School, and an aquamarine gem weighing 34.5 carats, said to be the largest perfect gem of its kind in the United States, to the Field Museum of Natural History in Chicago.

In addition to the manifold responsibilities shouldered by Mr. Crane as executive head of a large corporation, he found time to devote to public activities. From 1901 to 1912 he was consul general of Persia in Chicago; he served on the Yale Committee for Participation in the Restoration of the Library of the University of Louvain; he was a life member of the Art Institute, Chicago; life member and director of the Field Museum of Natural History, Chicago; and vice-president (1921–1923) of the Civic Opera Association of Chicago and a guarantor for ten years. On several occasions he joined in syndicates of members of the New York Yacht Club, of which he was a life member, to finance the construction and operation of defenders of the America's Cup, the last time in 1917, when he participated in building the yacht *Wetamoe*, one of the candidates against Sir Thomas Lipton's challenger.

Mr. Crane became a member of the A.S.M.E. in 1916. Among the clubs to which he belonged were the Chicago Club, New York Yacht Club, The Brook (New York), Jekyll Island Club (Brunswick, Ga.), Old Elm (Chicago), Piping Rock Club, and Mount Royal Club (Montreal).

The honorary degree of Doctor of Laws was conferred upon Mr. Crane by Hobart College, Geneva, N.Y., in 1931. The following citation accompanied the degree:

"Richard Teller Crane, Jr., promotor of human relationships in American industry, philanthropist, worthy citizen of a great city, Bachelor of Philosophy of Yale University. A loyal and devoted son of his Alma Mater, he has translated its ideal of 'Light and Truth' into the complex mechanism of a great industrial organization, ever emphasizing men rather than things, so that it stands as a conspicuous example of mutual and cooperative work and benefit. Recognizing his wide interest in educational and social welfare, Hobart is glad to welcome him to fellowship with her sons."

Mr. Crane died of heart disease on his fifty-eighth birthday, November 7, 1931, at the Doctors Hospital in New York, N.Y. He is survived by his widow, Florence (Higinbotham) Crane, of Chicago, whom he married in 1904, and by a son, Cornelius, and a daughter, Florence.

ROBERT EDWARD CRANSTON (1873–1932)

Robert Edward Cranston, who died in San Francisco, Calif., on July 30, 1932, was one of the outstanding authorities on gold and platinum dredging. He was known to the mining profession chiefly through his activities in gold dredge design and operation and his sampling and valuation of alluvial deposits, but he also had had extensive experience in the sampling and operation of lode properties in Colorado, California, Montana, and others of the western states. His work on alluvial properties took him to Brazil, Colombia, New Zealand, Australia, and other parts of the world.

Mr. Cranston was born at St. Johns, Mich., on June 14, 1873. He prepared for college at the Michigan Military Academy, Orchard Lake, and then attended the Michigan College of Mines for four years. During the early years of his career, from 1893 to 1898, Mr. Cranston was associated with mining and consulting engineers in several states, assisting in placer and lode examinations and operations, patent and underground surveying, assaying and sampling. He served in 1895 and 1896 as U.S. Deputy Mineral Surveyor in the Cripple Creek District in Colorado and later examined and acted as consulting engineer for several mines, notably the Smuggler Union at Telluride, Colo. He also was active in the Leadville, Colo., district.

In 1898 Mr. Cranston took charge of the Pacific Coast examinations of the New England Exploration Company, Boston, and also became connected with the Ashburton Mining Company; he developed the property of this company on the American River near Folsom, Calif., and operated the mine for many years, as manager of the company. He also organized the Marysville Dredging Company, which began operations in 1905 on the Yuba River. Both these companies were supplied with gold dredges designed by Mr. Cranston. From 1907 to 1910 he was secretary and consulting engineer to the Greater Nevada Mining Company, Rhyolite, Nev., and did other consulting work. Subsequently he devoted himself largely to consulting practice, which included the investigation, equipment, and operation of many mining properties, both placer and lode, in nearly every mining state in the United States and in Canada, Mexico, Brazil, and Colombia.

During the World War Mr. Cranston was Chief of Division of Collateral Commodities, U.S. Food Administration, Washington, D.C. After his return to private life he developed a gold dredging property in New Zealand and formed the Rimu Gold Dredging Company, Ltd., Hokitika, to operate it. He was still serving as managing director of this company and was also director and general manager of the South American Gold & Platinum Co., New York, at the time of his death, and held similar positions with various companies related to these organizations.

Mr. Cranston became a member of the A.S.M.E. in 1907 and belonged to several other societies, including the American Institute of Mining and Metallurgical Engineers, the Mining and Metallurgical Society of America, and the Engineer's Club of San Francisco. Other club memberships included the Olympic and Bohemian, San Francisco, Marin Golf and Country Club, New Rochelle (N.Y.) Yacht, and Lawyers' (N.Y.).

Surviving Mr. Cranston are his widow, Mrs. Mary P. Cranston, and one son, Graham.

JOHN C. CROMWELL (1866–1932)

John C. Cromwell, whose death occurred on May 31, 1932, at Columbus, Ohio, was born on September 26, 1866, in Frankfort, Ky., the son of Alvin W. and Sidney A. (Jett) Cromwell. He secured his early education in Frankfort and was graduated from the University of Illinois in 1886 with a B.S. degree in mechanical engineering, receiving his M.E. degree in 1911.

During the ten years following his graduation Mr. Cromwell did drafting for the Ellithorpe Air-Brake Company, Chicago, on steam and hydraulic elevators, the Bates Machine Company, Joliet, Ill., on Corliss engines and wire machinery, and the Joliet Works of the Illinois Steel Company, as chief draftsman.

For a period of twenty years, beginning in 1897, Mr. Cromwell was associated with the Garrett-Cromwell Engineering Company, of Cleveland, designing and constructing rolling mills.

During the early days of the World War Mr. Cromwell purchased some 80,000 tons of steel car axles, for the Allies, for the manufacture of shrapnel. The Cromwell Steel Company, which he organized at Cleveland in 1917, was discontinued after the close of the War. Since 1921 he had been connected with the Groeniger Associates, consulting engineers, Columbus. In 1927 he organized the Steel Sanitary Company, at Alliance, Ohio, and served as its general manager for three years. He was also a former president of the Alliance Machine Company.

Among a number of inventions patented by Mr. Cromwell were processes for rolling steel, pressing steel into bathtubs, etc.

Mr. Cromwell is survived by a sister, Florence (Cromwell) Denton, of Columbus. His wife died in 1909.

EDWARD HARRIS CROSBY (1868–1932)

Edward Harris Crosby, vice-president of Veeder-Root, Inc., Hartford, Conn., died at his home in that city on April 30, 1932, after a short illness. Mr. Crosby was born at Hebron, in Yarmouth County, Nova Scotia, on February 23, 1868. After leaving school he served an apprenticeship with the Hinkley Locomotive Works, Boston, Mass., and worked for the American Watch & Tool Co. and American Waltham Watch Company in Waltham, Mass., as machinist and toolmaker, and the E. W. Bliss Co., Brooklyn, N.Y., as a model maker for several years before going to Hartford.

Mr. Crosby's first position in Hartford was with the Pope Manufacturing Company, as toolmaker. He was also employed for a time by the Pratt & Whitney Co. as a model maker.

The Veeder Manufacturing Company was organized by Curtis H. Veeder in 1895 to manufacture a cyclometer invented by him. Mr. Crosby entered the employ of the company shortly after its organization and later was made superintendent and had a large part in the development of Mr. Veeder's many inventions in counting and registering machines. When the company was merged with the Root Company of Bristol in 1928 Mr. Crosby was made factory manager. He was elected vice-president and made consulting engineer in 1931.

Mr. Crosby became a member of the A.S.M.E. in 1913. He belonged to the Hartford Get-Together Club and the Masonic fraternity and was active in the Central Baptist Church. He is survived by his widow, Mrs. Belle (Backadar) Crosby, a son, Dr. Edward H. Crosby, of Cleveland, Ohio, and two daughters, Miss Martha B. Crosby, of Hartford, and Mrs. A. E. Anderson, of Jamestown, N.Y.

CHARLES NORMAN CROSS (1879–1932)

Charles Norman Cross, a member of the faculty of the Mechanical Engineering Department of Stanford University since 1906, died on January 9, 1932, of heart trouble. He was honorary chairman of the A.S.M.E. Student Branch at the University, and had been a member of the Society since 1911.

Professor Cross was born on April 9, 1879, at Warsaw, Wyoming County, N.Y., the son of Norman D. and Harriet (Beckwith) Cross. He prepared for college at the Pike (N.Y.) Seminary, and then worked for about three years in his father's machine shop before entering Cornell University. During one vacation he was employed as machinist's helper by the Lake Shore & Michigan Southern Railroad at Collinwood, Ohio, and Elkhart, Ind., and from February 1, 1905, to February 1, 1906, worked at the Long Island State Hospital as assistant electrical engineer and at the Waterside Station No. 1 of the New York Edison Company. He received his M.E. degree at Cornell in June, 1906, and during the intervening time between then and the beginning of his work at Stanford was electrician for the Lackawanna Steel Company at Buffalo, N.Y.

In addition to his duties on the staff of instruction in the Department of Mechanical Engineering at Stanford, he was frequently called in as a consultant for the study of various problems in the domains of construction, power, and oil-well engineering. Thus he acted as engineering expert for the Kern Trading & Oil Co. on special testing and investigations of power used in the drilling and pumping of oil wells. At different periods he served as consultant for several construction companies in California and for the Pacific Gas & Electric Co. of San Francisco and the Palo Alto Municipal Gas Service. During 1914–1915 he spent a sabbatical year in the Research Department of the General Electric Company at Lynn, Mass. He had been, for some time, conducting research work on the viscosity of steam at high pressures and temperatures. He held a number of patents on gas burners and a gas furnace. He was the author of a textbook on Heat Engines, published in 1930.

During the World War, Professor Cross was efficiency engineer for the Federal Fuel Administration in California, and in 1921 and 1922 he was a member of the Committee on the Revision of Boiler Safety Orders of the Industrial Accident Commission of California. He was a member of the Pacific Coast Gas Association, American Association of University Professors, honorary engineering fraternity, Sigma Xi, Stanford Faculty Club, and Engineers' Club of San Francisco.

Professor Cross is survived by his widow, Laura Parke (Mumaw) Cross, whom he married at Scottdale, Pa., in 1908, and by two children, Charles Mumaw and Eleanor Parke Cross.

EDWARD NEWTON CUNNINGHAM (1878–1932)

Edward Newton Cunningham, of Birmingham, Ala., representative of the Kewanee & Fitzgibbons Boiler Company in that district, was fatally injured in an automobile accident near Selma, Ala., on February 15, 1932. Driving with him was Raymond Newcomb, vice-president of the company and likewise a member of the A.S.M.E., who also died as the result of injuries.

Mr. Cunningham was born at Pinckneyville, Ill., on February 23, 1878, the son of William T. and Martha A. Cunningham. After completing his high-school education he became an apprentice in the Pinckneyville Electric Company. During the ten years from 1896 to 1906 he was employed in Brookhaven, Poplarville, and Hattiesburg, Miss., handling operations, repairs, new installations, and improvements in electric light and power plants and advancing to the position of superintendent. The next two years were spent as designing, contracting, and supervising engineer with the Orleans Engineering Company, New Orleans, La. In 1909 and 1910 he was associated with R. C. Houston, of Memphis, Tenn., in contracting work, making estimates and serving as field engineer on several projthe United States Naval Academy at Annapolis in 1907 and spent seven years in the Navy, serving as chief engineer on several ships until 1913, when he was appointed assistant inspection machinist, U.S.N., at the Wm. Cramp & Sons Ship & Engine Bldg. Co., Philadelphia, Pa.

Following his resignation from the Navy in 1914, Mr. Cochrane became chief engineer for the U.S. Industrial Alcohol Company and its subsidiaries, the U.S. Industrial Chemical Co., Cuba Distilling Company, and Curtis Bay Copper & Iron Works, South Baltimore, Md.

During the World War he served as a lieutenant in the Ordnance Department, working on airplane fuel development. He was a director of the Cochrane Sales Company, Industrial Bank (Baltimore), and National Safety Fenders Corporation. He became a member of the A.S.M.E. in 1920 and also belonged to the Society of Automotive Engineers, American Society of Naval Engineers, the Army and Navy Club, Washington, D.C., Chemists Club, New York, and several other clubs.

Mr. Cochrane was married in Troy, N.Y., in 1909, to Gretchen Schneider, who, together with a son, William Henry Cochrane, survive him.

EUGENE LAMBERT COOKE (1894–1932)

Eugene Lambert Cooke, treasurer and general manager of the Cooke Boiler Works, Inc., New Haven, Conn., died of pneumonia at his home in that city on January 6, 1932.

Mr. Cooke was born in Philadelphia, Pa., on April 17, 1894, the son of John and Anna L. (Blakely) Cooke. He had two years' of study at the Erasmus Hall High School and a year at the Mechanics Institute. He then entered the employ of the Babcock & Wilcox Boiler Co., New York, working for two years on the erection of boilers in the field. From 1914 to 1916 he was assistant to the engineer of tests of the New York Edison Company at the Waterside Station. During the next two years he engaged in sales engineering work for the Elliott Company, New York, handling boiler equipment such as oil and water filtration systems, steam specialties and mechanical cleaners, and for the Vincent-Gilson Engineering Company, New York, selling and servicing combustion recorders and other instruments for controlling and regulating combustion in boiler furnaces. He returned to the employ of this company early in 1919, after several months in the Coast Artillery School for Enlisted Specialists, at Fort Monroe, Va., where he was enrolled in the electrical course.

Mr. Cooke went to New Haven in 1922 and spent two years with the New Haven Boiler Works before forming his own company. He enjoyed trap-shooting, hunted big game in Maine every fall, and was an excellent pianist. He is survived by his widow, Pauline (Berk) Cooke, whom he married in 1923, and by one son, Eugene L. Cooke, Jr.

Mr. Cooke became a junior member of the A.S.M.E. in 1917 and an associate member four years later.

RICHARD TELLER CRANE, JR. (1873–1931)

Richard Teller Crane, Jr., was born in Chicago, Ill., on November 7, 1873, fourth son of Richard Teller Crane, founder in 1855 of the Crane Co., manufacturers of valves, fittings, fabricated piping, plumbing, heating, and air-conditioning equipment. His mother was Mary Josephine (Prentice) Crane.

His early education was received at the Tilden School in Chicago, where his father organized and supported at his own expense the first public grade manual training school in Chicago. Later he attended Hill School, and in 1895 was graduated from Yale University with the degree of Bachelor of Science in the Sheffield Scientific School. Shortly thereafter he entered actively into the affairs of the rapidly growing business his father had founded. He held various official positions with the company until August 1, 1914, when, upon the retirement of his elder brother, Charles R. Crane, from the presidency, he was elected to that office, retaining it actively until the time of his death.

During the 17 years that Mr. Crane was president, Crane Co. saw the greatest expansion in its 76 years of growth. Under his leadership it developed into an organization with factories, branch houses, sales offices, and exhibit rooms in 200 cities throughout the world and with more than 20,000 employees. In administering its affairs Mr. Crane continued the traditions and policies of his father, which included a deep concern for the welfare of the personnel. The Crane Veteran League, which he organized in 1915 as a memorial to his father, is supported by an endowment fund provided by Mr. Cran and, at the time of his death, had a membership of 1401. Membership in the League is based on a minimum continuous service with the company of 25 years. During his presidency Mr. Crane distributed from his private fortune more than $13,000,000 worth of common stock in the company, and in his will supplemented this with a further large allotment of stock to those workers who had been with the company ten years and longer.

Mr. Crane's philanthropies were not confined to the company. To the City of Chicago he contributed the major portion of the cost of Iroquois Memorial Hospital, as a memorial to those who perished in the Iroquois Theatre fire in 1903. He also presented a hospital to the town of Ipswich, Mass., in memory of his friend, Benjamin Stickney Cable, and, jointly with Mrs. Crane, the completely equipped eye, ear, nose, and throat Florence Crane Hospital to the Hartford (Conn.) Hospital group and $100,000 toward the endowment of the Department of Therapeutics of New York University and Bellevue Hospital Medical College. He memorialized his parents in the gift of a carillon of 43 bells and the tower to house them, to St. Chrysostom's Episcopal Church, in Chicago, together with an endowment fund for maintenance. Buffalo Rock, a picturesque 50-acre spot on the Illinois River, which previously had served as a recreational and recuperative place for Crane employees and their families, was presented in 1928 to the State of Illinois, to be maintained perpetually by the State as a public park. "Crane Farms," near Lake Geneva, Wis., is now used by Crane employees. Among other donations are a valuable collection of Washingtoniana to the United States Naval Academy, the Library Building to Morristown (N.J.) School, and an aquamarine gem weighing 34.5 carats, said to be the largest perfect gem of its kind in the United States, to the Field Museum of Natural History in Chicago.

In addition to the manifold responsibilities shouldered by Mr. Crane as executive head of a large corporation, he found time to devote to public activities. From 1901 to 1912 he was consul general of Persia in Chicago; he served on the Yale Committee for Participation in the Restoration of the Library of the University of Louvain; he was a life member of the Art Institute, Chicago; life member and director of the Field Museum of Natural History, Chicago; and vice-president (1921–1923) of the Civic Opera Association of Chicago and a guarantor for ten years. On several occasions he joined in syndicates of members of the New York Yacht Club, of which he was a life member, to finance the construction and operation of defenders of the America's Cup, the last time in 1917, when he participated in building the yacht *Weetamoe*, one of the candidates against Sir Thomas Lipton's challenger.

Mr. Crane became a member of the A.S.M.E. in 1916. Among the clubs to which he belonged were the Chicago Club, New York Yacht Club, The Brook (New York), Jekyll Island Club (Brunswick, Ga.), Old Elm (Chicago), Piping Rock Club, and Mount Royal Club (Montreal).

The honorary degree of Doctor of Laws was conferred upon Mr. Crane by Hobart College, Geneva, N.Y., in 1931. The following citation accompanied the degree:

"Richard Teller Crane, Jr., promotor of human relationships in American industry, philanthropist, worthy citizen of a great city, Bachelor of Philosophy of Yale University. A loyal and devoted son of his Alma Mater, he has translated its ideal of 'Light and Truth' into the complex mechanism of a great industrial organization, ever emphasizing men rather than things, so that it stands as a conspicuous example of mutual and cooperative work and benefit. Recognizing his wide interest in educational and social welfare, Hobart is glad to welcome him to fellowship with her sons."

Mr. Crane died of heart disease on his fifty-eighth birthday, November 7, 1931, at the Doctors Hospital in New York, N.Y. He is survived by his widow, Florence (Higinbotham) Crane, of Chicago, whom he married in 1904, and by a son, Cornelius, and a daughter, Florence.

ROBERT EDWARD CRANSTON (1873–1932)

Robert Edward Cranston, who died in San Francisco, Calif., on July 30, 1932, was one of the outstanding authorities on gold and platinum dredging. He was known to the mining profession chiefly through his activities in gold dredge design and operation and his sampling and valuation of alluvial deposits, but he also had had extensive experience in the sampling and operation of lode properties in Colorado, California, Montana, and others of the western states. His work on alluvial properties took him to Brazil, Colombia, New Zealand, Australia, and other parts of the world.

Mr. Cranston was born at St. Johns, Mich., on June 14, 1873. He prepared for college at the Michigan Military Academy, Orchard Lake, and then attended the Michigan College of Mines for four years. During the early years of his career, from 1893 to 1898, Mr. Cranston was associated with mining and consulting engineers in several states, assisting in placer and lode examinations and operations, patent and underground surveying, assaying and sampling. He served in 1895 and 1896 as U.S. Deputy Mineral Surveyor in the Cripple Creek District in Colorado and later examined and acted as consulting engineer for several mines, notably the Smuggler Union at Telluride, Colo. He also was active in the Leadville, Colo., district.

In 1898 Mr. Cranston took charge of the Pacific Coast examinations of the New England Exploration Company, Boston, and also became connected with the Ashburton Mining Company; he developed the property of this company on the American River near Folsom, Calif., and operated the mine for many years, as manager of the company. He also organized the Marysville Dredging Company, which began operations in 1905 on the Yuba River. Both these companies were supplied with gold dredges designed by Mr. Cranston. From 1907 to 1910 he was secretary and consulting engineer to the Greater Nevada Mining Company, Rhyolite, Nev., and did other consulting work. Subsequently he devoted himself largely to consulting practice, which included the investigation, equipment, and operation of many mining properties, both placer and lode, in nearly every mining state in the United States and in Canada, Mexico, Brazil, and Colombia.

During the World War Mr. Cranston was Chief of Division of Collateral Commodities, U.S. Food Administration, Washington, D.C. After his return to private life he developed a gold dredging property in New Zealand and formed the Rimu Gold Dredging Company, Ltd., Hokitika, to operate it. He was still serving as managing director of this company and was also director and general manager of the South American Gold & Platinum Co., New York, at the time of his death, and held similar positions with various companies related to these organizations.

Mr. Cranston became a member of the A.S.M.E. in 1907 and belonged to several other societies, including the American Institute of Mining and Metallurgical Engineers, the Mining and Metallurgical Society of America, and the Engineer's Club of San Francisco. Other club memberships included the Olympic and Bohemian, San Francisco, Marin Golf and Country Club, New Rochelle (N.Y.) Yacht, and Lawyers' (N.Y.).

Surviving Mr. Cranston are his widow, Mrs. Mary P. Cranston, and one son, Graham.

JOHN C. CROMWELL (1866–1932)

John C. Cromwell, whose death occurred on May 31, 1932, at Columbus, Ohio, was born on September 26, 1866, in Frankfort, Ky., the son of Alvin W. and Sidney A. (Jett) Cromwell. He secured his early education in Frankfort and was graduated from the University of Illinois in 1886 with a B.S. degree in mechanical engineering, receiving his M.E. degree in 1911.

During the ten years following his graduation Mr. Cromwell did drafting for the Ellithorpe Air-Brake Company, Chicago, on steam and hydraulic elevators, the Bates Machine Company, Joliet, Ill., on Corliss engines and wire machinery, and the Joliet Works of the Illinois Steel Company, as chief draftsman.

For a period of twenty years, beginning in 1897, Mr. Cromwell was associated with the Garrett-Cromwell Engineering Company, of Cleveland, designing and constructing rolling mills.

During the early days of the World War Mr. Cromwell purchased some 80,000 tons of steel car axles, for the Allies, for the manufacture of shrapnel. The Cromwell Steel Company, which he organized at Cleveland in 1917, was discontinued after the close of the War. Since 1921 he had been connected with the Groeniger Associates, consulting engineers, Columbus. In 1927 he organized the Steel Sanitary Company, at Alliance, Ohio, and served as its general manager for three years. He was also a former president of the Alliance Machine Company.

Among a number of inventions patented by Mr. Cromwell were processes for rolling steel, pressing steel into bathtubs, etc.

Mr. Cromwell is survived by a sister, Florence (Cromwell) Denton, of Columbus. His wife died in 1909.

EDWARD HARRIS CROSBY (1868–1932)

Edward Harris Crosby, vice-president of Veeder-Root, Inc., Hartford, Conn., died at his home in that city on April 30, 1932, after a short illness. Mr. Crosby was born at Hebron, in Yarmouth County, Nova Scotia, on February 23, 1868. After leaving school he served an apprenticeship with the Hinkley Locomotive Works, Boston, Mass., and worked for the American Watch & Tool Co. and American Waltham Watch Company in Waltham, Mass., as machinist and toolmaker, and the E. W. Bliss Co., Brooklyn, N.Y., as a model maker for several years before going to Hartford.

Mr. Crosby's first position in Hartford was with the Pope Manufacturing Company, as toolmaker. He was also employed for a time by the Pratt & Whitney Co. as a model maker.

The Veeder Manufacturing Company was organized by Curtis H. Veeder in 1895 to manufacture a cyclometer invented by him. Mr. Crosby entered the employ of the company shortly after its organization and later was made superintendent and had a large part in the development of Mr. Veeder's many inventions in counting and registering machines. When the company was merged with the Root Company of Bristol in 1928 Mr. Crosby was made factory manager. He was elected vice-president and made consulting engineer in 1931.

Mr. Crosby became a member of the A.S.M.E. in 1913. He belonged to the Hartford Get-Together Club and the Masonic fraternity and was active in the Central Baptist Church. He is survived by his widow, Mrs. Belle (Backadar) Crosby, a son, Dr. Edward H. Crosby, of Cleveland, Ohio, and two daughters, Miss Martha B. Crosby, of Hartford, and Mrs. A. E. Anderson, of Jamestown, N.Y.

CHARLES NORMAN CROSS (1879–1932)

Charles Norman Cross, a member of the faculty of the Mechanical Engineering Department of Stanford University since 1906, died on January 9, 1932, of heart trouble. He was honorary chairman of the A.S.M.E. Student Branch at the University, and had been a member of the Society since 1911.

Professor Cross was born on April 9, 1879, at Warsaw, Wyoming County, N.Y., the son of Norman D. and Harriet (Beckwith) Cross. He prepared for college at the Pike (N.Y.) Seminary, and then worked for about three years in his father's machine shop before entering Cornell University. During one vacation he was employed as machinist's helper by the Lake Shore & Michigan Southern Railroad at Collinwood, Ohio, and Elkhart, Ind., and from February 1, 1905, to February 1, 1906, worked at the Long Island State Hospital as assistant electrical engineer and at the Waterside Station No. 1 of the New York Edison Company. He received his M.E. degree at Cornell in June, 1906, and during the intervening time between then and the beginning of his work at Stanford was electrician for the Lackawanna Steel Company at Buffalo, N.Y.

In addition to his duties on the staff of instruction in the Department of Mechanical Engineering at Stanford, he was frequently called in as a consultant for the study of various problems in the domains of construction, power, and oil-well engineering. Thus he acted as engineering expert for the Kern Trading & Oil Co. on special testing and investigations of power used in the drilling and pumping of oil wells. At different periods he served as consultant for several construction companies in California and for the Pacific Gas & Electric Co. of San Francisco and the Palo Alto Municipal Gas Service. During 1914–1915 he spent a sabbatical year in the Research Department of the General Electric Company at Lynn, Mass. He had been, for some time, conducting research work on the viscosity of steam at high pressures and temperatures. He held a number of patents on gas burners and a gas furnace. He was the author of a textbook on Heat Engines, published in 1930.

During the World War, Professor Cross was efficiency engineer for the Federal Fuel Administration in California, and in 1921 and 1922 he was a member of the Committee on the Revision of Boiler Safety Orders of the Industrial Accident Commission of California. He was a member of the Pacific Coast Gas Association, American Association of University Professors, honorary engineering fraternity, Sigma Xi, Stanford Faculty Club, and Engineers' Club of San Francisco.

Professor Cross is survived by his widow, Laura Parke (Mumaw) Cross, whom he married at Scottdale, Pa., in 1908, and by two children, Charles Mumaw and Eleanor Parke Cross.

EDWARD NEWTON CUNNINGHAM (1878–1932)

Edward Newton Cunningham, of Birmingham, Ala., representative of the Kewanee & Fitzgibbons Boiler Company in that district, was fatally injured in an automobile accident near Selma, Ala., on February 15, 1932. Driving with him was Raymond Newcomb, vice-president of the company and likewise a member of the A.S.M.E., who also died as the result of injuries.

Mr. Cunningham was born at Pinckneyville, Ill., on February 23, 1878, the son of William T. and Martha A. Cunningham. After completing his high-school education he became an apprentice in the Pinckneyville Electric Company. During the ten years from 1896 to 1906 he was employed in Brookhaven, Poplarville, and Hattiesburg, Miss., handling operations, repairs, new installations, and improvements in electric light and power plants and advancing to the position of superintendent. The next two years were spent as designing, contracting, and supervising engineer with the Orleans Engineering Company, New Orleans, La. In 1909 and 1910 he was associated with R. C. Houston, of Memphis, Tenn., in contracting work, making estimates and serving as field engineer on several projects, and in 1912 and 1913 he was operating engineer for the R. C. Houston Co. The intervening period was spent as salesman for the Gulf Refining Company.

Mr. Cunningham leased the Mt. Pleasant (Tenn.) Electric Company in 1913 and devoted three years to rebuilding the system and operating it on a profit-sharing basis. The Greensboro (Ala.) Light & Water Plant was operated by him in 1916 on the same basis.

In 1917 and 1918 and also in 1920 Mr. Cunningham was associated as a steam power engineer with Howard M. Gassman of Birmingham, consulting engineer for mining and industrial plants. During the interim he was superintendent of operations at the Alabama Power Company's Warrior Steam Plant at Gorgas, Ala.

Since 1921 Mr. Cunningham had been engaged in sales and consulting work for different companies, chiefly in Birmingham. He became a member of the A.S.M.E. in 1926. His mother and a brother, Eugene R. Cunningham, of Pensacola, Fla., survive him.

LOUIS CHRISTIAN DATZ (1880–1932)

Louis Christian Datz, chief engineer of the St. Louis Public Service Co., died as the result of a cerebral hemorrhage on May 20, 1932. Mr. Datz went to St. Louis in 1925 as advisory engineer in connection with the reorganization of the United Railways Company of St. Louis. Two years later, when the Public Service Company took over the street railways, he became chief engineer.

Mr. Datz was born in New Orleans, La., on April 27, 1880, the son of Henry and Charlotte (Caesar) Datz. At the age of 21 he secured his B.E. degree in mechanical and electrical engineering from Tulane University. He remained in New Orleans until 1914, the major portion of the time being spent with the New Orleans Railway & Light Co., which he served as assistant engineer from 1903 to 1910 and later as engineer of maintenance of way and superintendent of track and overhead. His first position after graduation was with a New Orleans contractor in charge of designing and constructing dredges, derricks, etc. He was also secretary for a time at Tulane University, and spent a year as engineer with Ford, Bacon & Davis, Inc.

After leaving New Orleans in 1914 Mr. Datz spent three years in Birmingham, Ala., with the Southern Properties, United Gas & Electric Engineering Corp. Subsequently he was connected for three years with the American Cities Company, Birmingham, as chief engineer, and from 1922 to 1924 he held a similar position with the Memphis Power & Light Co., Memphis, Tenn.

Mr. Datz became a member of the A.S.M.E. in 1916. He was president in 1915 and again in 1922–1923 of the Louisiana Engineering Society and was also a member of the American Electric Railway Engineering Association and the American Society of Civil Engineers. He was a member of the St. Louis Electric Board of Trade, the Missouri Athletic Association, and the Engineers and Rotary Clubs in St. Louis and chairman of the Finance Committee of the Westminster Presbyterian Church there.

Surviving Mr. Datz are his widow, formerly Miss May C. Eblen, whom he married in 1905, and three children, Warren E., Elsie C., and Rosemary Datz.

EDWIN KENT DAVIS (1893–1931)

Edwin Kent Davis, chief engineer of the Fulton Bag & Cotton Mills, Atlanta, Ga., was born in Baltimore, Md., on June 17, 1893, the son of Edwin and Mary Register (Pope) Davis. He was graduated from the Technological High School, Atlanta, in 1911, and for the next two years was employed by the Mackler-Crawford Construction Company (later known as the Mackler Construction Company). He was successively draftsman, timekeeper, carpenter-foreman, and building superintendent for the company, working on business and public buildings in different parts of the South.

With the exception of about fifteen months during the World War Mr. Davis had been connected with the Fulton Bag & Cotton Mills since September, 1913. From that date until his enlistment in the Aviation Section of the Signal Corps, U.S.A., in November, 1917, he held different positions in the purchasing and mechanical departments of the company. When he returned in March, 1919, he was made assistant purchasing agent and mechanical engineer, and six years later chief engineer of the company. He was in charge of power plants both at Atlanta and at the branch factories, and of the maintenance of equipment and buildings. Machine design and development work was carried on under his direction, and bag-cleaning machinery and salvage systems, as well as fire protection and insurance programs, were introduced by him.

Mr. Davis trained as a pilot and was commissioned a second lieutenant in May, 1918. After flying at different fields and serving as commanding officer of the Student Officers Squadron at Love Field, Dallas, he was sent to France, in August, 1918, and given further training in testing combat planes. He was discharged in February, 1919, with a citation for excellent service as a test pilot. He retained his interest in aviation, serving as a first lieutenant in the Reserve Air Corps, and was second in command of the unit at Candler Field.

Mr. Davis became a junior member of the A.S.M.E. in 1919, an associate-member three years later, and a full member in 1926. He had served as chairman of the Atlanta Section. He also belonged to the Aero Club of America and held its regular and expert pilot's licenses.

Mr. Davis died at Atlanta on November 18, 1931. He is survived by his widow, Katherine (Du Bose) Davis, whom he married in 1925, and by two daughters, Katherine and Mary Pope Davis.

WILLIAM L. DEMPSEY (1866–1932)

William L. Dempsey, patent attorney and consulting engineer, St. Louis, Mo., died on May 17, 1932. He was born at Louisville, Ky., on October 7, 1866. He attended the Burnett Academy, Auburn, Ky., and Beck's College, Smith Grove, Ky., before entering Vanderbilt University, Nashville, Tenn., from which he was graduated in 1888. During the next year he was president of Beech Grove College, Beech Grove, Ky., and then he opened a law office in Fayette, Mo.

In 1897 Mr. Dempsey closed his Fayette office and took a position in the Law, Engineering, and Sales Departments of the International Harvester Company, Chicago, where he remained for twenty years. In 1917 he became president of the Dempsey Cycle Company, Inc., of Philadelphia, Pa., a company organized to handle the Dempsey cycle supercharger, which greatly increases brake horsepower and torque, especially at low speeds. Mr. Dempsey established his St. Louis office in 1923.

Mr. Dempsey was a fellow of the Royal Society of Great Britain, and a member of the Engineers' Club of St. Louis and The Franklin Institute. He became a member of the A.S.M.E. in 1928.

JOHN DEVRIES (1853–1932)

John DeVries, retired mechanical engineer and manufacturer, died at his home in Clifton, N.J., on April 4, 1932. He was born in the Netherlands on November 30, 1853, emigrating to the United States with his parents in 1865. He supplemented his early education in the Netherlands with courses in mechanical drawing in evening school in Newark, N.J., and by private tuition.

At the age of nineteen Mr. DeVries began a three-year apprenticeship in the manufacture of machine tools with the New York Steam Engine works, Passaic, N.J. He then spent a year in repair work in the bleaching and printing business at the Rockland Print Works, Haverstraw, N.Y. From that time until 1892 he was connected with the Hewes & Phillips Iron Works at Newark, N.J., in the capacity of erecting and testing engineer. For several years he erected and tested refrigerating plants and during the remainder of the time he erected and ran tests on various types of engines, particularly Corliss cross compound condensing and non-condensing and Boyden automatic cut-off engines. He designed and applied the first adjustable cut-off for low and intermediate pressure cylinders on compound and triple expansion engines, and also designed an adjusting screw for taking up lateral motion on the valves of both high and low pressure cylinders which was adopted as universal equipment.

In 1892 Mr. DeVries and his brother founded The Novelty Iron Works in Passaic, for general machine and engineering work. The company built and tested both steam and belt driven condensers and repaired all classes of steam and gas engines. It also built the DeVries patent feedwater heater and purifier, a continuous circulation feedwater heater, and various devices used in manufacturing textiles, artificial leather, chemicals, rubber goods, matches, tin cans, and other products. In addition to the feedwater heater Mr. DeVries devised and patented an all-metal device for feeding the uncombed wool to the points of a comber, replacing the so-called dabbing brushes and greatly increasing the operating speed of the combs. Mr. DeVries retired from active business in 1925.

Mr. DeVries became a member of the A.S.M.E. in 1921. He was a charter member of the Passaic Branch No. 11 of the National Association of Power Engineers. He was also active in the civic life of Passaic, having served for two terms as a member of the Board of Education. He was one of the founders and a director of the Merchants Bank, Passaic, and a director in the Service Trust Company, Hobart Trust Company, and The Peoples Bank & Trust Co. until illness necessitated his retirement about a year before his death. He was also vice-president of the Mutual Loan & Building Association, an auditor of the Home Loan & Building Association, and treasurer of the Lodi Cemetery Company. He belonged to the Clifton Reformed Church.

Surviving Mr. DeVries are his widow, Mrs. Jemima (Lokker) DeVries, four sons, and one daughter.

ROBERT CONRAD DOERR (1883–1932)

Robert Conrad Doerr, sales and consulting engineer and power plant analyst for the Sarco Company, New York, N.Y., since 1925, died at St. Mary's Hospital in Hoboken, N.J., following an operation, on November 6, 1932.

Mr. Doerr was born in Chicago, Ill., on September 26, 1883, the son of Adam and Elizabeth (Becker) Doerr. He attended Lewis Institute, Chicago, for two years and in 1901 entered the employ of the Daimler Manufacturing Company, Steinway, L.I., N.Y. After two years there he went to Coronado Beach, Calif., where he was superintendent of construction of the Dupee estate until 1909.

In 1909 Mr. Doerr went into business for himself in the hydraulic engineering field. Later he designed and developed a portable coal elevator for bunkering express passenger ships and a system for cleaning condenser tubes and express type boilers, called bullet tube cleaners. He was commissioned a lieutenant in the U.S. Naval Reserve in 1918 and placed in charge of all bunkering equipment of the Navy transport force. Subsequently he was appointed director of fuel in the Fifth Naval District. He was made a lieutenant-commander in the U.S. Naval Reserve in 1928.

After the World War Mr. Doerr became sales engineer for the M. K. Bowman-Edson Company, New York, with which he was connected until he joined the Sarco Company. He became a member of the A.S.M.E. in 1921. He was a Royal Arch Mason and Knight Templar and a member of the Elks organization and several military groups. His wife died in 1927.

Early in 1932 Mr. Doerr was granted a patent for chemically treated felt pads for use on windshields to repel rain and snow.

CHARLES VINCENT EYRE DOVE (1868–1932)

Charles Vincent Eyre Dove, whose death occurred in Chicago, Ill., on March 26, 1932, was born in London, England, on March 10, 1868. His early education was obtained in London and at schools in Germany and Switzerland. He was graduated from the Hammond Technical College in London in 1886 with an E.E. degree, and qualified as a civil engineer in Australia during the next four years.

Between 1890 and 1908 Mr. Dove practiced as a consulting engineer, maintaining an office in Honolulu, Hawaii. In 1908 he took charge of the development of the University Park Addition in Seattle, Wash. Not long after entering upon this work he was appointed Park Engineer for the City of Seattle, and later opened offices as consulting engineer in that city. Since 1926 he had been located in Chicago and vicinity, where for five years he was associated with James Walker in consulting work.

Mr. Dove became a member of the A.S.M.E. in 1921. He also belonged to the Pacific Northwest Society of Engineers.

Surviving Mr. Dove are his widow, formerly Miss Kathleen Monsarrot, whom he married in 1908, and a son, John Forbes Dove.

HAROLD GEORGE DRYSDALE (1891–1932)

Harold George Drysdale, consulting engineer, Brooklyn, N.Y., who died on March 12, 1932, was born in Birmingham, England, on March 18, 1891, the son of Robert and Ruth (Butler) Drysdale. He studied engineering at the Lowell Institute, securing M.E. and E.E. degrees, and served a four-year apprenticeship as machinist and tool and die maker at the General Electric Company, West Lynn, Mass. He remained with this company until October, 1915, working as a journeyman for the first year and subsequently on the design of tools and dies for railway motors, steam turbines, and special automatic devices. He supplemented his experience during the next two years as tool and machine designer for the United Shoe Machinery Company, Beverly, Mass., the Hamel Machinery Company, Lynn, and E. B. Badger & Sons Co., Gray & Davis, and the National Packaging Machinery Company, all of Boston.

In the fall of 1917 Mr. Drysdale entered the employ of the Curtiss Aeroplane & Motor Corp., Buffalo, N.Y., soon becoming chief of the tool design section and later assistant aeronautical mechanical engineer. He left this post in November, 1918, to sell a mechanical device for the New Era Manufacturing Company of New York, for about a year. Next, he directed the design and construction of a special automatic machine for the Pull-Man Engineering Service, Brooklyn. When this was completed, in May, 1920, he took a position as tool and die designer with the Automatic Straight Air Brake Company, New York, later becoming production engineer. For a short time he was connected with the Safeguard Sales Company, selling mechanical devices, and then spent a few months on the design of a special automatic machine for the Lancaster Engineering Corporation. Early in 1922 he became factory manager and chief engineer of the Twisted Wire & Steel Co., New York, his duties including the supervision of manufacturing, inspection, personnel, and purchasing.

From 1923 to 1925 and from 1926 to 1928 Mr. Drysdale was connected with the American Machine & Foundry Co., Brooklyn, working on the design of cigar and cigarette machines and other machinery, in the capacity of assistant to the chief of the cigarette machine design department during the second period. A brief intervening period was spent as chief draftsman for the F. L. Smith Machine Co., New York. He engaged again in airplane manufacture in 1929 as assistant to the production chief of the Fairchild Airplane Company, Farmingdale, L.I. During the next two years he was consulting engineer for Gibson Arnoldi & Co., New York, in charge of all mechanical work as well as plant surveys, estimates, and valuations. Since late in 1931 he had been on the engineering staff of the Hoffman Beverage Company, Newark, N.J.

Two of Mr. Drysdale's many designs were patented by him, one a guard for circular saws and the other an automatic egg boiler.

Mr. Drysdale became an associate member of the A.S.M.E. in 1923. He belonged to the Masonic fraternity, Odd Fellows, and the Sheepshead Bay Yacht Club. He is survived by a brother, Leslie W. Drysdale, Brooklyn, N.Y., and four sisters, Mrs. A. J. Summers, Mrs. H. D. Ricker, Mrs. Prescott W. Whittier, and Mrs. E. V. Holmes, all of Lynn, Mass.

FRANCIS HOSKINS EASBY (1861–1932)

Francis Hoskins Easby, a member of the A.S.M.E. since 1888, died at his residence in Philadelphia, Pa., his birthplace, on March 1, 1932. Mr. Easby was born on August 18, 1861, the son of William and Mary Anne (Hoskins) Easby. He attended the public school in Media, Pa., and the Episcopal Academy and Lauderbach's Preparatory School in Philadelphia before taking a four-year course in mechanical engineering at the University of Pennsylvania, with the class of 1881. He had some early experience in the repair shop of his father's sugar refinery and after completing his work at the university took a position as draftsman for the Baldwin Locomotive Works. After a year there he was similarly employed at the machine tool works of F. B. Miles for a year and by the Pratt & Whitney Co., Hartford, Conn., for two years.

In 1885 Mr. Easby opened a machine shop in Philadelphia, later moving to Bristol, Pa. He gave this up the following year and went to Johnstown, Pa., where he was engaged for a time in designing a cold saw for the Johnson Steel Street Railway Company. When that piece of work was done he returned to Philadelphia as draftsman and designer for Bement, Miles & Co., with which he was connected for two years. He next spent four years with the Betts Machine Company, of Wilmington, Del., as designer and superintendent. For 17 years, beginning in 1891, he was superintendent for the Detrick & Harvey Co., Baltimore, Md., then for three years engineer for Ladd & Baker, Philadelphia, and from 1913 to 1917 engineer for the Richard DeCou Company.

Mr. Easby entered the United States Navy as senior inspector of engineering materials in 1917 and continued in that work until illness necessitated his retirement in 1930.

Mr. Easby was also a member of the Academy of Natural Sciences in Philadelphia and of Alpha Tau Omega fraternity. He had patented an open-side planer feed device of his invention. He was a student of American political history. Surviving him are his widow, Gertrude K. (Peirce) Easby, whom he married in 1884, and two children, Mrs. Charlotte Easby (William C.) Grave of Philadelphia, and John Peirce Easby, of Merion, Pa.

CHARLES LEAVITT EDGAR (1860–1932)

Charles Leavitt Edgar, president of The Edison Electric Illuminating Company of Boston, with which he had been associated for nearly forty-five years, died of pneumonia on April 14, 1932, at Atlantic City, N.J., where he had gone for a brief relaxation. The tremendous growth of the company which he headed from the small beginnings of a pioneer time to the all-inclusive service now given reflects his courage to try new things, his thorough study of conditions to be met, his perseverance in the face of difficulties, and his grasp of administrative, legal, and engineering problems.

Charles Leavitt Edgar was born on December 23, 1860, at Griggstown, near Princeton, N.J., the son of Thomas and Annie (Veghte)

Edgar. He attended the Rutgers Preparatory School, New Brunswick, N.J., for three years and received an A.B. degree from Rutgers College in 1882. He then took up post-graduate work there in electricity and for this and subsequent practical experience he received from Rutgers an A.M. degree in 1885 and an E.E. degree two years later. In 1927, at the 161st Commencement of Rutgers, he was awarded the honorary degree of Doctor of Science, and in the same year Tufts College conferred upon him the honorary degree of Doctor of Laws. The words of the president of the latter institution upon that occasion were particularly well chosen:

"In this age of science it is natural that every result of research in the realm of pure science should be hailed with acclamation. He who reveals new truth deserves and receives the plaudits of his fellows. But the history of human achievements indicates very clearly that in general, before any scientific discovery may be turned to the advantage of mankind, genius of a special kind must be brought to bear upon it. There must be imagination which visualizes human needs. There must be idealism which transcends mere venture for gain. There must be practical sense of values which avoids the shallows of romantic optimism, there must be skilfulness to manage men and manipulate materials, there must be steadfastness of purpose which never falters. Mr. Edgar, you share with the inventor credit for the development of the electric light. Your years of service to the community demonstrate of what I say."

Mr. Edgar's decision to enter the field of electricity came from his interest in the work of Edison at Menlo Park. He sought employment of Mr. Edison in January, 1883, and was sent to the old Edison Machine Works in New York, where he remained for about a year. He was next sent to the factory of the Bergmann Company, manufacturers of central-station equipment. Subsequently he was located at the original New York factory of the Edison Electric Light Company, where he stayed until 1887, rising to the position of chief engineer. His work there was of a general engineering character, particularly relating to the installation of small central stations in New York and neighboring states.

Mr. Edgar was transferred to Boston in 1887 to serve as superintendent of The Edison Electric Illuminating Company of Boston, which had recently been organized. Two years later he was made general manager, in 1894 advanced to the post of vice-president, and in 1900, upon the death of Mr. Jacob C. Rogers, elected president of the company.

Mr. Edgar was a pioneer in the electrical industry and he advanced with it, and while he advanced with the industry the industry advanced because of his initiative and progressiveness. He studied closely the practice of the principal stations both in the United States and abroad and adopted the most up-to-date equipment and methods adaptable to the needs of the territory served by his company. He also originated engineering practices and operations which later were adopted by the industry. Ideas which he gathered during several weeks spent abroad in 1890 were incorporated in the Atlantic Avenue generating station. The first storage battery used in this country as a central-station auxiliary was a Tudor battery which he imported from Germany. The first vertical engines used in America for central-station work were installed by him. Following a study of its use in England in 1898 he introduced the Wright demand system of charging in the Boston Edison service. The adoption of the steam turbine as a prime mover of large capacity, undertaken by the company under his direction in 1904, was the outstanding event of that year in the electrical industry. And his introduction of the 1200-lb boiler and turbine in the Edgar Station was an innovation in engineering practice which created wide interest.

Under such leadership the district supplied by the company increased from eleven-hundredths of a square mile, serving only 260 customers in the downtown district, in 1887, to an area of nearly 600 square miles, serving some 43,000 customers, twenty-five years later. The first notable expansion of the company was in 1902, when the Boston Electric Light Company and the Suburban Light & Power Company became part of the Edison system. Two years later the service began to spread into the cities and towns of Greater Boston. The system today supplies more than 400,000 customers. The development of district steam-heating service and the interconnection of New England central-station companies were among the many results of his endeavors. The Charles Leavitt Edgar generating station in Weymouth, Mass., one of the largest and most modern electric steam-generating plants in the United States, is a monument to his achievements.

Mr. Edgar was equally progressive in regard to the welfare of the employees of the company. The Edison Employes' Club, which he helped to form, served as a model for similar clubs in other cities. He provided comfortable, attractive, and healthy working conditions for the employees of the company, with very many of whom he was personally acquainted. The company also has established recreational and educational facilities, liberal sickness, accident, and death benefits, and a generous pension system. In 1912, on the 25th anniversary of his connection with the company, the employees presented him a loving cup in appreciation of his personal interest in their behalf. A message from Thomas Alva Edison on that occasion read as follows:

"When you took hold of the Boston Edison Company a quarter of a century ago, I expected you would give a good account of yourself, but I must confess I did not anticipate the magnificent showing that has been made by you and your company in these later days.

"Allow me to compliment and congratulate you on your success and to wish you many more years of useful endeavor."

On his 40th anniversary with the company, Owen D. Young, who wrote a testimonial article, spoke particularly of his service to his fellowmen. "Edison Service," in Boston, stood for what Mr. Edgar stood.

The electrical industry through several associations in that field also knew Mr. Edgar as a leader. He was president of the Association of Edison Illuminating Companies from 1893 to 1895; chairman of the board of directors of the New England Power Association; and three times vice-president and in 1903 president of the National Electric Light Association. He had served as president of the Massachusetts Gas and Electric Association and as chairman of the Boston Section of the American Institute of Electrical Engineers, of which he was a Fellow. He had also served the Institute as a member of the Edison Medal Committee. He was a past-president of the Edison Pioneers and a director and member of the executive committee of the Employers' Liability Assurance Corporation, Ltd.; director of the Employers' Fire Insurance Company, American Employers' Insurance Company of Boston, and the Electrical Testing Laboratories, New York; and a trustee of the Massachusetts Utilities Associates and of Rutgers College.

Mr. Edgar became a member of the A.S.M.E. in 1910. He also belonged to the Illuminating Engineering Society, the Society for Electrical Development, the Institution of Electrical Engineers (Great Britain), the Boston Society of Civil Engineers, the National Geographic Society, and the American Academy of Political and Social Science.

When the United States entered the World War Mr. Edgar was appointed an associate member of the Naval Consulting Board. He was chairman of the board of directors for the State of Massachusetts Organization for Industrial Preparedness, served on the national committee on gas and electric service, and was also connected with the fuel administration work.

During recent years Mr. Edgar had devoted much of his time to the affairs of the National Civic Federation, serving as a member of its committee of twenty-four, investigating the question of municipal ownership of public utilities in other countries. He also was chairman of the industrial welfare department of the federation for several years, in which work he made several surveys of the problem of old-age dependency and pensions.

Mr. Edgar lived quietly at his home in Brookline, Mass. He liked to attend first nights at the theater and was a prolific reader, finding relaxation in fiction. He was a member of the Brookline Country Club and played golf frequently when business permitted. His other clubs included the Engineers' and University in New York and Boston; Exchange and Algonquin in Boston; and Kingswood and Bald Peak in Wolfeboro, N.H., where he had a country home.

He is survived by his widow, Annette (Duclos) Edgar, whom he married in 1896, a son, Leavitt L. Edgar, vice-president and director of the Boston Edison Company, one brother, and two grandchildren.

JOHN EDGAR ELDER (1873–1932)

John Edgar Elder, building manager for the N.Y. Stock Exchange Building Co., New York, since 1922, died in that city on October 16, 1932.

Mr. Elder was born at Bells Valley, Va., on October 30, 1873, the son of Robert C. and Nancy Jane Elder. He worked as fireman, oiler, and assistant engineer for the Virginia Iron & Railway Co., and for several hotels in that state before going to New York in 1898.

During the remainder of his life he had charge of power plants and other equipment in a number of hotels and other buildings in New York. He was assistant engineer at the Mills Hotel for five years, in full charge of operation and maintenance. From 1903 to 1910 he was located at power stations, first the 74th Street Station of the Interborough Rapid Transit Company and then at the Port Morris Station of the New York Central Railroad. Since then he had been chief engineer for Saks & Company and the Washington Life Building (1910–1912), assistant chief engineer, Hotel McAlpin (1912–1917), chief engineer, American Express Building (1917–1918), and chief engineer and superintendent of building, in full charge of the mechanical, electrical, and building equipment, Hotel Pennsylvania (1918–1922). He also acted in an advisory capacity in Statler plants in Buffalo, Detroit, Cleveland, and St. Louis, in connection with changes and new work. The N.Y. Stock Exchange Building Co., with which he had been associated since 1922, controls four buildings.

Mr. Elder became a member of the A.S.M.E. in 1920. He had belonged to the National Association of Power Engineers for 25 years and was its president in 1913. He was a 32d degree Mason. Mr. Elder was predeceased by his wife, Emma L. (Clark) Elder.

IRVING STANILAUS FELLNER (1891–1932)

Irving Stanilaus Fellner, who had been connected with the McGraw-Hill Publishing Company for some years, died suddenly at his home in Chappaqua, N.Y., on October 30, 1932.

Mr. Fellner was born at Ft. Smith, Ark., on March 5, 1891, the son of Samuel and Lena (Berck) Fellner. Before going to New York he spent six years with the Kawneer Manufacturing Company, in charge of the sales staff and construction in Southern and Central California. During his last two years there he also was assistant sales manager for the entire United States.

In 1919 Mr. Fellner became business manager for *Chemical and Metallurgical Engineering*, in charge of the sales staff and marketing counselor for the McGraw-Hill Publishing Company. Later he was also made publishing director of *Power* and at the time of his death was with *Management Methods*.

Mr. Fellner was the author of many articles on design and construction and a lecturer before the architectural departments of eastern universities. He was active in the evolution of the philosophy of the sales of materials in industries and closely identified with the design and sales departments of many large industrial organizations.

During the World War Mr. Fellner served overseas as first lieutenant of infantry with the 26th and later the 30th Division. He became an associate of the A.S.M.E. in 1927.

CHARLES H. FISH (1861–1932)

Charles Henry Fish died on December 20, 1932, at the home of his niece, Miss Margaret Fish, in Brookline, Mass.

Mr. Fish was born in Taunton, Mass., on April 4, 1861, one of four sons of Captain Frederick and Mary Jarvis (Perry) Fish. He secured his first engineering experience with the Amoskeag Manufacturing Company, Manchester, N.H., working in the various shops and mills for four years and serving as assistant superintendent for two years. The next two years (1886–1888) were spent as agent for the Chicopee Manufacturing Company, Chicopee Falls, Mass. He then became treasurer and constructing engineer for the P. C. Cheney Paper Co., Manchester, N.H., and Excelsior Paper Stock Company, Goffstown Centre, N.H.

From 1895 to 1909 Mr. Fish was agent of the Cocheco Manufacturing & Print Works, Dover, N.H., the following year general manager and construction engineer for B.B. & R. Knight, Providence, R.I., and from then until 1913 general manager at the Garner Print Works & Bleachery in Garnerville and Wappingers Falls, N.Y. Since then he had engaged in consulting work as a textile engineer in Boston.

Mr. Fish became a member of the A.S.M.E. in 1891. He was a past-president of the National Association of Cotton Manufacturers, a member of the Engineers Clubs of Boston and New York and the California Club of Los Angeles, and belonged to several local clubs, including the Union Club in Boston. He enjoyed sailing and had a collection of old prints of ships. He also made sketches and water colors.

Mr. Fish is survived by his widow, Agnes (Cheney) Fish, whom he married in 1888, and three daughters, Mrs. Charles P. Woodworth of Weston, Mass., Mrs. Clarence F. Graham of Albany, N.Y., and Mrs. Philip F. Broughton of Brooklyn Heights, N.Y.

WALTER EVERETT FOSTER (1877–1932)

Walter Everett Foster, district sales manager for Reading-Pratt & Cady Co., Inc., Chicago, Ill., died suddenly at Decatur, Ill., on December 22, 1932.

Mr. Foster was born at Harvard, Mass., on January 29, 1877, the son of Joseph Ansil and Angelia B. (Hatch) Foster. He attended public school in Rochester, N.Y., and studied at St. Lawrence University before entering Worcester Polytechnic Institute. After his graduation in 1899 with a B.S. degree in the electrical engineering course, he worked for a short time as factory superintendent for the Sargent Automatic Railway Signal Company, Rochester. From 1900 to 1904 he was signal inspector for the Pennsylvania lines west of Pittsburgh, in full charge of all new signal work. During the next six years he advanced from construction engineer to western manager for the Union Switch & Signal Co., Swissvale, Pa. He directed the design and installation of signal apparatus aggregating several million dollars and patented several devices in the field. From 1908 to 1910 he was also president and treasurer of the Signal Supply Company, Chicago, Ill.

In 1910 Mr. Foster became sales engineer for the Ingalls-Shepbard Forging Company, Harvey, Ill., with which he remained until 1918. He was next associated with the Pfaudler Company, Rochester, N.Y., as manager of the Eastern District for three years, and subsequently became sales engineer for The Superheater Company, New York. In 1925 he took a position as sales engineer for the Reading Steel Casting Company, Bridgeport, Conn., now known as Reading-Pratt & Cady Co., Inc. He was transferred to Chicago as district sales manager in 1927.

Mr. Foster was technical secretary of the Western Division for the Naval Consulting Board in 1917 and 1918 and also technical secretary to the Committee on Engineering of the Illinois Council of Defense. He was in charge of an industrial inventory survey for Illinois and a member of the Illinois Home Guard at Oak Park, Ill.

Mr. Foster became a member of the A.S.M.E. in 1919 and also belonged to the Railway Signal Association, the Western Society of Engineers, and the Chicago and New York Engineers Clubs.

Surviving Mr. Foster are his widow, Harriett Combs Foster, whom he married in 1906, and two children, Josephine C. and Robert C. Foster.

CHARLES COLMAN FREDERICKS (1882–1932)

Charles Colman Fredericks, president of the St. Louis Meter & Pump Co., St. Louis, Mo., died on June 15, 1932. Mr. Fredericks became a member of the A.S.M.E. in 1921. At that time the following statement concerning him was submitted by the St. Louis Pump & Equipment Co.

"Mr. Fredericks has devoted practically his entire life to liquid handling engineering, giving special attention to the problems involving the safe and economical storage, distribution, and reclamation of oils and similar liquids.

"At the present time he is vice-president and general manager of the St. Louis Pump & Equipment Co., engineers and manufacturers of oil-handling equipment. Prior to this association he was connected with S. F. Bowser & Co., Ft. Wayne, Ind., in both executive and engineering capacities. He gave particular attention to liquid handling problems in connection with railroads, and has made personal surveys of oil and similar liquid handling facilities at practically every important railway terminal in the United States.

"During the past year he has been in close collaboration with his associated engineers perfecting, and is now manufacturing, the 'St. Louis' rotary piston pump, and is giving special attention to the improvement in design and construction of pumps, both hand and power, measuring and non-measuring."

Mr. Fredericks was born in Richmond, Va., on August 18, 1882, and held an LL.B. degree from Valparaiso University.

JOHN RIPLEY FREEMAN (1855–1932)

John Ripley Freeman was born on a farm at West Bridgton, Me., July 27, 1855. Early education in the public schools of Portland, Me., and Lawrence, Mass., led to studies in science and engineering at the Massachusetts Institute of Technology, from which he was graduated in 1876, with the degree of bachelor of science from the department of civil engineering.

In accordance with a custom which is as sound today as it was then, Mr. Freeman spent his summer vacations in gaining profitable experience in industry, working as assistant engineer with the water-power company at Lawrence, Mass. Returning to this work after graduation, he soon became principal assistant engineer to the company and to its chief engineer, Mr. Hiram F. Mills, in whose extensive consulting practice in water power, foundations, and factory construction he participated. It was here also that he came in contact with James B. Francis, of Lowell, Mass., recognized as the foremost hydraulic engineer of his day in this country, with John C. Hoadley, the eminent mechanical engineer, and with Charles S. Storrow, one of the best educated engineers of his day, who planned the new city of Lawrence, designed and developed its water-power plant, and wrote the earliest treatise on hydraulics of water supply in the English language.

Ten years of this valuable apprenticeship led, in 1886, to Mr. Freeman's resignation to become engineer and special inspector for the Associated Factory Mutual Fire Insurance Companies of Boston, Mass. These companies were associations of manufacturers established for the double purpose of insurance and the development of means for preventing diastrous fires. Because of Freeman's ability and technical training, he was soon asked to undertake the reorganization of the corps of inspectors maintained by the fire-insurance companies. This work he supervised for about ten years, conducting also many researches on the improvement and standardization of fire-prevention apparatus, including automatic sprinklers, fire pumps, fire hose, and the causes of fire. It was during this period that he presented to the A.S.C.E. his well-known papers, "Experiments Relating to the Hydraulics of Fire Streams," which received that society's gold medal in 1889, and "The Nozzle as an Accurate Water Meter," the gold-medal paper in 1891.

While carrying on the work of supervising the inspectors of the insurance companies, Mr. Freeman engaged also in consulting practice in Boston, Mass. This twofold interest was maintained throughout the remainder of his active life. Or perhaps it would be better to say threefold—or many fold, for the researches in which he was constantly engaged, or in which he displayed a lively personal interest, also persisted throughout his entire career. During recent years he was engaged in preparing a book on researches on the flow of water in pipes, elbows, and tees.

In 1896 Mr. Freeman was called to Providence, R.I., to become president and treasurer of what is now the Manufacturers Mutual Fire Insurance Company, the position which he held at the time of his death.

Space will not permit a complete recital of Mr. Freeman's achievements in the many fields in which he worked. While a definite chronology pertains to each, his work lay in several of them simultaneously. For convenience, therefore, a general classification of them has been attempted.

Of his business and insurance interests mention has already been made, and the success of the company he served seems adequately to express Mr. Freeman's competence and accomplishments in its non-engineering aspects. Its principal success lying in adequate fire prevention, it is only natural that a great many of Mr. Freeman's engineering achievements were also in this field. To it he brought a technical training and a valuable knowledge of hydraulics, the subject in which most of his engineering practice lay. He wrote on hydraulic and piping arrangements for the protection of cities in 1892, and of the protection of cities from fire in 1915. In 1905 he presented to the A.S.M.E. in his presidential address a comprehensive study of theater fires, their causes and means of prevention, and in 1923 this Society awarded him its gold medal "for eminent service rendered to industry in fire prevention."

In his consulting practice, as has already been indicated, Mr. Freeman made a speciality of hydraulics. It is possible to mention only briefly the principal projects that engaged his attention.

In 1895 to 1896 he was engineer-member of the Massachusetts Metropolitan Water Board.

In 1903–1909 he was chief engineer in charge of investigations for damming the Charles River between Boston and Cambridge, Mass., creating a fresh-water "basin." The project and its result is well known to all visitors to these cities.

He served, at various times, the Metropolitan Park Commission of Massachusetts on matters of river improvements and drainage.

In 1899 he was engaged by the Finance Department of the City of New York to investigate additional sources of water supply—the first step in works since built. Noteworthy researches on water waste in municipalities were conducted as part of this work.

Later, in 1904, he again served the New York Metropolitan District as a member of the Burr-Hering-Freeman Commission for further study of additional water supply, and, in 1905, became consulting engineer to the New York Board of Water Supply (a post which he filled for the remainder of his life) after having refused, because of his consulting practice and insurance business, to become its chief engineer.

Other communities came to him for advice on their waterworks problems. In 1907–1908 it was the New York State Water Supply Commission, later known as the State Water Conservation Commission, on studies for conserving water resources. He was one of the engineering commission of three reporting on the feasibility and location of the Los Angeles aqueduct, 240 miles long, from the Owens River (1906). Baltimore, Md., secured from him in 1909 designs for a new source of water supply. San Francisco employed his services from 1909 to 1912 in planning the Hetch Hetchy water-supply system substantially as it is now being constructed. He made electric-power development an important feature of this project, greatly adding to the income of the city. Nashua, N.H., Denver, Colo., Seattle, Wash., San Diego, Calif., and Mexico City also sought out his services.

In the field of water-power development he was employed by the Aluminum Company of America at Massena, N.Y., at the Long Sault, Niagara, and on several southern rivers. He planned power developments on the Feather River in California for the Great Western Power Company; made investigations for the Canadian Government on water-power development in the provinces of Alberta, Manitoba, and British Columbia; designed high masonry dams for the Mexican Northern Power Company at La Boquilla de Conchos, for the Medina Irrigation Works in Texas, the Pacific Gas & Electric Company at Lake Spaulding, the Spring Valley Water Company at Calaveras, the Great Western Power Company at Big Bend, Calif., the Aluminum Co. at Badin, N.C., etc.; and assisted engineers of the U.S. Reclamation Service on construction methods for the Arrow Rock Dam near Boise, Idaho. He also designed and supervised the construction of a high masonry dam on peculiarly difficult foundations on the Missouri River at Holter, Mont.; prepared designs (not executed) for developing the Great Falls of the Potomac with a great reservoir by a single dam 200 feet high; designed extensions and to some extent supervised construction of high earth dams by hydraulic fill at Coquitlam, British Columbia, and San Pablo, Calif.; and made outline plans (not executed) for a new subterranean power development for the Ontario Power Company at Niagara and for the development of 800,000 continuous horsepower on the St. Lawrence River, near Montreal, from the Lachine Rapids, with its backwater raising Lake St. Louis nearly to its flood level.

In the field of drainage and flood control Mr. Freeman acted as consulting engineer, 1917 to 1920, to the Chinese Government on the improvement of the Grand Canal, including preliminary studies of flood prevention on the Yellow River and the Hwai River. In 1922 he prepared designs for a large railroad bridge across the Yellow River. In his plan the bridge was reduced to about one-fifth of its previous length by training and confining the river to a narrow channel. From 1924 to 1926 he served on the Engineering Board of Review of the Sanitary District of Chicago on the controversy over the lowering of the level of the Great Lakes.

Space will permit only the bare mention of his services to the United States during the World War on the National Advisory Committee for Aeronautics, his report on the Hog Island Shipyard, his studies, in 1905, 1908, and 1915, of various special problems of the Panama Canal; of his connection with the Providence, Rhode Island, gas company, and his studies in city planning, highways, and parks for the City of Providence.

One of Mr. Freeman's most patriotic services was his untiring efforts to secure for the United States a National Hydraulic Laboratory. The bill authorizing the laboratory was passed by the Congress under the leadership of Senators Ransdell and Hebert, and the laboratory, constructed largely in conformity to designs supervised by Mr. Freeman, was dedicated in the spring of 1932. A description of it will be found in *Mechanical Engineering*, May, 1932. Many hydraulic laboratories exist abroad, and in a book, "Hydraulic Laboratory Practice," published by the A.S.M.E. in 1929 but financed and edited by Mr. Freeman, these laboratories and their work are described.

Geologists and seismologists have felt the stimulus of Mr. Freeman's enthusiasm and breadth of interest. Studies in water supply and flood control naturally presupposed a comprehension of geological formations. He was a pioneer in calling upon geologists and seismologists to aid in clearing up questions that arise in engineering works. He studied the continental depression of the New England shore line in his investigation for the Charles River Basin, and summarized new and old data and drew valuable conclusions as to continental movement in connection with his report on the regulation of the Great Lakes.

In the words of the Reverend John Joseph Lynch, S. J., representing the Seismological Society of America at a testimonial dinner to Mr. Freeman in Providence in April, 1931, "Divine Providence has decreed that there will be earthquakes, but the City of Providence through one of its honored sons and engineers has arranged that the effects of these earthquakes shall not be so disastrous." Presenting a paper before this society in 1930, Mr. Freeman was (quoting Father Lynch again) "embarrassingly insistent with the seismologists that we must make better measurements regarding the basic facts of earth motion and seismology before we can put up buildings that are going to withstand earthquake shock, and he was so insistent that one felt one had to go out and find these data." To the work of this society Mr. Freeman made financial contributions; and within the past year he brought out his 600-page book on a "Study of a Rational Basis for Earthquake Insurance."

Not only in science, in engineering, and in business did Mr. Freeman's talents exercise themselves, but in the field of education and engineering societies as well. A graduate of Massachusetts Institute of Technology, twice president of its Alumni Association, and life member of its corporation, on which he served for more than forty years, he was offered the presidency of the institution when Dr. Pritchett resigned. Twice President Eliot offered him the chair of Civil Engineering at Harvard University. These honors he declined, feeling that his services to mankind were better engaged in the practice of his profession. He spoke and wrote on educational matters. He held honorary degrees from Brown, Tufts, University of Pennsylvania, Yale, and the Sächsische Technische Hochschule, Dresden.

In 1923 Mr. Freeman gave $25,000 each to the American Society of Civil Engineers, The American Society of Mechanical Engineers, and the Boston Society of Civil Engineers to provide for traveling scholarships in hydraulics. The first Freeman Traveling Scholarships were awarded in 1927. They enable young engineers or junior professors between the ages of 25 and 35 years to profit by a year or more of study in the best engineering laboratories of Europe.

Mr. Freeman was a loyal and active member of many engineering societies. He was an honorary member of the Providence Engineering Society, the Boston Society of Civil Engineers, the American Society of Civil Engineers, and the American Water Works Association. He was elected to honorary membership in the A.S.M.E. in 1932. He was president of the Boston Society of Civil Engineers in 1893, of The American Society of Mechanical Engineers (of which he became a member in 1887) in 1905, and of the American Society of Civil Engineers in 1922.

He was a member of the National Academy of Arts and Sciences, the Visiting Committee of the Bureau of Standards, honorary member of the Badische Technische Hochschule, Karlsruhe, and Mitgleid des Wissenschaftlichen Beirats des Forschungsinstituts in Munich and Walchensee, Bavaria.

Mr. Freeman died in his sleep on the night of October 6, 1932, following a brief illness of two days. He is survived by his widow; four sons, Clarke F., Hovey T., John R., Jr., and Evert W. Freeman; and a daughter, Miss Mary Elizabeth Freeman.

JOENS ELIAS FRIES (1876–1932)

Joens Elias Fries, chief engineer of the Tennessee Coal, Iron & Railroad Co., Birmingham, Ala., died at the Fairfield Hospital in that city on January 23, 1932, after a long illness.

Mr. Fries was born in Frinnaryd, Sweden, on April 6, 1876, the son of Andreas Peter and Maria (Castenson) Fries. He was graduated from the College of Uppsala in 1894 and from the Royal Technical Academy (Kongl. Tekniska Hogskolan), Stockholm, in 1898, with degrees in electrical and mechanical engineering.

Mr. Fries worked in Sweden until 1903. For the first two years following his graduation he was employed by a Stockholm firm manufacturing telephone apparatus. During the remainder of the time he was connected with the General Swedish Electric Company (Allmanna Svenska Electriska A.B.) as estimator for plants of every kind, and with the Holmia Electric Company, Stockholm, as superintendent.

In 1903 Mr. Fries came to the United States, his first position in this country being with the New York Edison Company. After a short time in that connection, he was employed by the Westinghouse, Church, Kerr & Co. for drafting work for the electrification of the Long Island Railroad. He next took a position, in 1905, with the Allis-Chalmers Company, Milwaukee, Wis., as engineer in the contract department, where he remained for two years. He then went to Toronto in the capacity of engineer of power plants and power transmission for the Canadian General Electric Company, but returned to the United States in 1908. From then until 1916 he was identified with the Crocker-Wheeler Company, spending a year and a half in charge of the estimating department, a like period as industrial engineer, two years as Pacific Coast engineer, and the remainder of the time as assistant chief engineer. His work for that company included the construction of the 6000-kw power plant on Bishops Creek for the Southern Sierras Power Company, and the electrification of a large sugar mill in Cuba. He was responsible for new designs entailed in all this work.

Mr. Fries had been connected with the Tennessee Coal, Iron & Railroad Co., a subsidiary of the United States Steel Corporation, since September, 1916. At first he was in charge of the Electrical Department, as chief electrical engineer. About 1920 he succeeded Mr. R. E. Brakeman, who resigned to enter the employ of the Otis Steel Company, as chief engineer. At that time the blooming mill, structural mill, and plate mill for the company's fabricating shop at Fairfield and the assembling plant at Mobile had been completed, and the Engineering and Drafting Departments of the company were consolidated and Mr. Fries placed in charge of the Ensley and Fairfield Works. He designed the power-house equipment and crane equipment for the Mobile plant of the Chickasaw Shipbuilding & Car Co. This plant was dismantled not long afterward, but freight cars continued to be manufactured at the Fairfield Steel Works, for which Mr. Fries had designed much of the equipment.

Mr. Fries was a past director of the Association of Iron and Steel Electrical Engineers and a Fellow of the American Institute of Electrical Engineers. He became a member of the A.S.M.E. in 1921 and also belonged to the Svenska Teknologforeningen, American Iron and Steel Institute, American Association for the Advancement of Science, American Society for Steel Treating, the Engineers' Club of New York, Engineers' Club of San Francisco, and Birmingham Country Club.

Since 1908 Mr. Fries had frequently presented technical papers before different organizations and contributed articles to the technical press. Among the subjects with which he dealt were distribution voltage for central stations, electric drive of turbine pumps, power for steel mills, current surge in an inductive circuit, influence of gear ratio on speed of operation, motor heating and contactor wear in auxiliary steel mill drives, and parallel operation of turbine-driven central stations. He was also an expounder in lectures and articles of Einstein's theory of relativity, and made a number of translations, including "Death and Resurrection," by Bjorklund, from the Swedish, and several works by Arrhenius, from the Swedish and German. These dealt with the transmission of life through the universe, the infinity of the universe, and the destinies of the stars.

Surviving Mr. Fries are his widow, Annerika (Bjerkander) Fries, and a son, Dr. Erik F. B. Fries.

WILLIAM SPENCER GEOHEGAN (1892–1932)

William Spencer Goehegan, who since 1930 had owned and operated a garage and machine shop in Los Angeles, Calif., died of pneumonia at his home in Pasadena, Calif., on December 13, 1932. He is survived by his widow, Elsie G. (Adney) Geohegan, whom he married in 1917, and by a daughter, Elsie.

Mr. Geohegan was born in Chicago, Ill., on November 24, 1892, the son of William and Elizabeth Geohegan. He attended public school in Chicago and later took courses in management from LaSalle Extension University and studied four years at the Throop Polytechnic Institute (now California Institute of Technology), Pasadena, Calif.

He was employed by the Crown Hardware Company in Pasadena from 1912 to 1916 and during the next three years was corporation salesman for the Union Hardware & Metal Co., of Los Angeles. In 1919 he joined the staff of the Southern California Edison Company as a buyer for the Kern River No. 3 hydroelectric project, stationed at Bakersfield, Calif. In this capacity he took care of emergency orders for construction materials for both the power house and transmission lines. After two years in that work he was transferred to the main office of the company in Los Angeles, in charge of purchasing construction materials for hydroelectric and steam generating projects. He was connected with the company until 1928, when he became sales manager for the Junior-Norris Galvanizing Company, Los Angeles, with which he remained until he went into business for himself in 1930.

Mr. Geohegan became an associate of the A.S.M.E. in 1928. He also belonged to the Society of American Military Engineers, the Electric Club of Los Angeles, and the Masonic fraternity. He was an Episcopalian.

ERNEST T. P. GREENIDGE (1881–1932)

Ernest T. P. Greenidge, superintendent of the Indiana Pipe Line Company, Huntington, Ind., died at the Grant County Hospital, Marion, Ind., on October 25, 1932, from injuries sustained in an automobile accident.

Mr. Greenidge was born on Barbados Island, British West Indies, on September 12, 1881. His technical education was secured at Stevens Institute of Technology, from which he was graduated with an M.E. degree in 1910, having spent some years in travel before entering the Institute. His first position was with the Southern Pipe Line Company, Lancaster, Pa., where he was engaged in testing pumping stations for economical operation, installing pumping machinery, boilers, etc., and general plant construction work for two years following his graduation. With the exception of two years during the World War he had been associated with the Indiana Pipe Line Company since 1912, entering the employ of the company as assistant to the superintendent and being promoted to the position of division engineer in 1914 and superintendent in 1919.

Mr. Greenidge was one of the first men in Huntington to attend the first officers' training camp at Fort Benjamin Harrison. He entered army service in May, 1917, and was commissioned a first lieutenant in the Corps of Engineers in August of that year. After further training at Camp Funston, Kan., he was sent overseas and served on the administrative staff at the main engineering depot in Paris, where he was promoted to the rank of captain. He was a charter member of Huntington Post No. 7 of the American Legion.

Surviving Mr. Greenidge are four brothers, Arthur, Sydney, and Frank, of New York, N.Y., and Philip, of Barbados. He had been a member of the A.S.M.E. since 1921, and was also a member of the honorary engineering fraternity of Tau Beta Pi, the Elks, and the LaFontaine Country Club.

OLAF NICOLAUS GULDLIN (1858–1932)

Olaf Nicolaus Guldlin, one of the founders of the Western Gas Construction Company, Fort Wayne, Ind., died at his home in that city on September 6, 1932. He was born on December 6, 1858, in Christiania (now Oslo), Norway, the only son of a family of seven children of Lars O. and Maren Sander Guldlin. After he had completed his preparatory schooling at Christiania, he entered the technical college at Bergen, Norway, from which he was graduated in 1878. He then took a year of special studies at the Polytechnikum at Munich, Bavaria, and a year of practical training in the iron works of A. L. Thune, in Christiania.

He came to the United States in 1880 and found his first employment in this country in the Baldwin Locomotive Works in Philadelphia, Pa., as a draftsman. He returned to Norway in 1882 to induce his family to come with him to America.

Upon returning to the United States, he was employed by James A. Smedberg, a gas engineer of Lancaster, Pa. At a convention in Washington, D.C., in 1884, he met A. D. Cressler, of Fort Wayne, who urged him to go to Fort Wayne. He did so not long afterward and entered the employ of the Kerr Murray Manufacturing Company.

In 1888, he decided to work independently and formed a partnership with W. A. Croxton and Frank D. Moses as consulting gas engineers. Two years later, when the original partners of Mr. Guldlin withdrew, he and Gordon W. Lloyd and Ernest F. Lloyd, of Detroit, incorporated the firm under the name of the Western Gas Construction Company. Mr. Guldlin became president and general manager.

In 1902, Mr. Guldlin, Samuel M. Foster, Charles McCulloch, and J. Ross McCulloch, all of Fort Wayne, purchased the interests of the Lloyds in the Western Gas Construction Company. Mr. Guldlin continued as president and general manager.

The Koppers Company, of Pittsburgh, Pa., in 1917 took a five-year lease on the Western Gas Construction Company with the option of purchasing the company. Mr. Guldlin retired as president but continued as a member of the board of directors until the Koppers Company made the purchase in 1922.

Mr. Guldlin had designed and patented about fifty devices, including a gas condenser, coal-treating apparatus, ammonia washer, and various governors and valves for use in the gas industry.

The Western Gas Association sent Mr. Guldlin to the International Gas Congress in Paris in 1900. He visited the principal gas plants in France, Austria, Switzerland, Germany, England, and Scotland during 1900 and 1901.

He received the grand prize at the Louisiana Purchase Exposition in St. Louis, in 1904, in recognition of his inventions and developments in the gas industry, and was appointed to the International Jury of Awards in the Department of Liberal Arts and unanimously elected to honorary membership in the Indiana Commission.

Mr. Guldlin was appointed on the American Gas Institute committee in charge of the gas industries exhibits at the Panama-Pacific Exposition at San Francisco, Calif., in 1917. His company's exhibit was awarded the highest distinction, the "Medal of Honor," two gold medals, and two silver medals. Again Mr. Guldlin was personally honored with the Exposition's Gold Medal on recommendation of the American Gas Institute, which was followed by the award to him by the International Jury of the "Medal of Honor."

Mr. Guldlin was prominent in many business enterprises and civic endeavors in Fort Wayne, and was a member of fraternal and social organizations in that city and elsewhere. He was president of the old Fort Wayne Commercial Club, which became the present Chamber of Commerce. He and Mrs. Guldlin had made contributions to the equipment for playgrounds in Fort Wayne, and he helped to plan and build the Anthony Hotel and Fairfield Manor there.

He was a charter member of the Allen County-Fort Wayne Historical Society and ranked high in the Masonic fraternity. He became a member of the A.S.M.E. in 1905 and also belonged to the Society of American Military Engineers.

Previous to his retirement in 1922 he was a member of the American Gas Institute, the Pacific Gas Association and similar industrial organizations in Michigan, Indiana, Wisconsin, and Illinois, the American Academy of Political and Social Science, the American Political Science Association, and the American Economic Association.

He had also been a member of the Cosmos Club, of San Francisco, Calif., the Lions Club, of New York, and the Missouri Athletic Club at St. Louis.

He married Addie L. Bleekman at Rarden, Ohio, in 1889, and is survived by her.

Mr. and Mrs. Guldlin returned to Europe in May, 1930, where they visited the college from which he had been graduated, his birthplace, and other places in Norway and Sweden. His health failed shortly after his return to his home here.

CLARENCE SHERMAN HAMMATT (1858–1932)

Clarence Sherman Hammatt, past-president of the Florida Engineering Society, died at a hospital in Jacksonville, Fla., on December 6, 1932. He had been in failing health for some time.

Mr. Hammatt was born in Geneseo, N.Y., on November 11, 1858, the son of Edward Rumney and Eliza H. (Pheips) Hammatt. He was graduated from the Mount Pleasant Military Academy in New York in 1877 and later took courses from Johns Hopkins University and the Bryant and Stratton Business College.

From 1880 to 1883 he was employed by the Detroit Machine Screw Works. He then learned the plumbing trade and in 1886 took charge of the plumbing department of the Drew Hardware Company, Jacksonville, Fla. The following year he was connected with the Jacksonville Water Works and in 1888 became superintendent and consulting engineer for the Gas and Electric Companies of Jacksonville, which incorporated the Citizen's Gas and Electric Company, American Illuminating Company, and Jacksonville Light Company. After ten years with this organization he accepted the position of consulting engineer and manager of the Jacksonville Electric Light Company and during the years 1898–1890 was engaged in planning and building a new plant to furnish all electric current for Jacksonville. About this time he also organized and was made vice-president of the Florida Electric Company, dealers in electrical work and supplies, with which he continued to be associated until he sold out his interests in 1907.

For nearly ten years, beginning in 1901, Mr. Hammatt engaged in general contracting, covering the erection of machinery and structural iron work, under the name of the Hammatt Contracting Company. He was granted a process patent for fractional distillation of wood products in 1907. In 1910 and 1911 he was superintendent of the Ware County Gas, Electric and Ice Company, Waycross, Ga., and during the next two years part owner and operator of the Milldale Ice Works, a small ice plant in a suburb of Jacksonville. For a number of years, from 1915 on, he was president of the Consolidated Engineering Company, Jacksonville, engaging in both mechanical and electrical engineering.

From 1912 until his death Mr. Hammatt was consulting engineer to the Board of Commissioners of State Institutions in Florida and with his assistance a great many improvements in various state institutions had been made in recent years, including new buildings at the State Prison Farm. During the World War he was also a member of the Board of National Preparedness and an associate member of the Naval Consulting Board.

Mr. Hammatt was one of the organizers, in 1916, of the Florida Engineering Society and served as its fifth president. He became chairman of the society's Legislative Committee upon its formation and when the state enacted its Professional Engineering Bill, in 1917, he was appointed by the governor to the first State Board of Engineering Examiners. He served on this board continuously until his death, and had held the offices of secretary, treasurer, and president. He was also a member of the National Council of State Boards of Engineering Examiners and had been its president. In recognition of his services to the engineering profession and to the state, the C. S. Hammatt Engineering Library was established in 1926 at the University of Florida by the Florida Engineering Society.

Mr. Hammatt joined the A.S.M.E. in 1922. He was instrumental in forming the Florida Local Section in 1925, had been a member of its executive committee, and was a member of its nominating committee for 1932–1933. He also belonged to the American Institute of Electrical Engineers and the Society of American Military Engineers. He was a 33d degree Mason and a Shriner, and a member of the St. John's Episcopal Church in Jacksonville and of a number of social organizations there.

In 1885 Mr. Hammatt married Elizabeth Pratt, of Buffalo, N.Y., who survives him, as do also two sons, Sherman and Edward, and a daughter, Mrs. Gertrude Pratt, all of Jacksonville.

FRANCIS M. HARTMANN (1870–1932)

Francis M. Hartmann, professor of electrical engineering and dean of the Schools of Engineering of Cooper Union, New York, died on March 28, 1932, at his home in that city after a month's illness. A graduate of Cooper Union, Dean Hartmann had been a member of the engineering faculty for more than thirty years.

Dean Hartmann was born at Cochecton, N.Y., on August 31, 1870, the son of Franz Eugen and Paulina (Hoenninger) Hartmann. He attended Damascus Academy, in Pennsylvania, and evening high school in New York prior to entering upon his studies at Cooper Union. At the latter institution he took a five-year course in general science, a three-year course in chemistry, and post-graduate work in physics and electrical engineering, receiving an A.B. degree in 1895 and an E.E. degree eight years later. He also did graduate work in mathematics at New York University.

From 1891 to 1894 he served as city surveyor, New York, and from then until 1901 as computer of the department of public improvements. He was an instructor in physics and electrical engineering in the Cooper Union Night School of Engineering in 1899, that being the beginning of his long association with the Cooper Union day and night schools in a teaching capacity. He taught in both the day and night schools from 1901 to 1905, when he was appointed assistant professor of physics and electrical engineering. He was made a full professor two years later, and from 1908 to 1921 was professor of electrical and mechanical engineering in charge of the department. He was named dean of the Schools of Engineering in 1921.

In 1912 Dean Hartmann designed and equipped the new dynamo laboratories and thermodynamic laboratory for the Hewitt Memorial Building of Cooper Union. He also helped to develop a number of courses, including the five-year night-school course in electrical engineering, a five-year night-school course in general science, and a day-school technical science course.

Dean Hartmann also did some consulting work, in conjunction with A. G. Koenig and the E. E. Carey Company, New York, the latter relating particularly to transformer iron. He was a contributor to *Electrical Review* and was the author of "Elementary Mechanics," published in 1910, "Heat and Thermodynamics," 1911, "Biography of George W. Plympton," 1920, and "Alternating Currents and Transmission," 1923.

He had been a member of the A.S.M.E. since 1910, and also belonged to the American Institute of Electrical Engineers, the American Association for the Advancement of Science, the American Physical Society, the American Mathematical Society, and the Society for the Promotion of Engineering Education.

Surviving Dean Hartmann are his widow, Phoebe (Koenig) Hartmann, two brothers, and three sisters.

HUGO ADELBERT HECKER (1866–1932)

Hugo Adelbert Hecker, a member of the A.S.M.E. since 1906, died at his home in Norwood, Ohio, on July 24, 1932, after an illness of seven months.

Mr. Hecker was born at Stepstone, Ky., on February 10, 1866. He attended public school in Cincinnati and took night courses in engineering for three years at the Ohio Mechanics Institute there. He served a four-year apprenticeship in milling machine work and subsequently was employed by a number of Cincinnati engineers and firms, including the Lodge & Davis Machine Tool Co.; Lane & Bodley Co., working on Corliss engines; Bert L. Baldwin, as assistant draftsman on street railroad, incline planes, and power house work; F. M. Watkins Co., as chief draftsman, designing and testing gas engines; and the Watkins Laundry Machinery Company, as chief draftsman, designing and constructing both machinery and plants. For ten years, from 1891 to 1901, he was also instructor in mechanical drawing in evening classes at the Ohio Mechanics Institute.

After leaving the employ of the Watkins Laundry Machinery Company Mr. Hecker designed machinery for the American Laundry Machinery Company, Cincinnati, for a time and spent six years in similar work at the American Tool Works Company, Cincinnati, with which he was connected at the time of his death.

Mr. Hecker is survived by his widow, Nellie (Kottmyer) Hecker, whom he married in 1892, two children, Charles H. Hecker and Mrs. Dorothy E. (Hecker) Zinker, and two grandchildren, Stanley and Woodward Zinker.

JOHN BROWN FRANCIS HERRESHOFF (1850–1932)

John Brown Francis Herreshoff, first American recipient of the Perkin Gold Medal, inventor of a contact process for manufacturing sulphuric acid, died at his home in Atlanta, Ga., on January 30, 1932.

Dr. Herreshoff, the son of Charles Frederick and Julia Ann (Lewis) Herreshoff, was born on February 7, 1850, at Bristol, R.I. His father, manager of the Point Pleasant estate on Popasquash Neck, Bristol, was very fond of sailing and built a few boats for his own pleasure. His grandfather, Charles Frederick Herreshoff, came from Prussia in 1790, settling in Providence the following year and marrying Sarah Brown, the daughter of one of the largest shipowners of Rhode Island. Dr. Herreshoff's mother was the daughter of a well-known sea captain of Boston, Joseph W. Lewis.

This was the background for the Herreshoff Manufacturing Company, builders of many pleasure and commercial vessels as well as the yachts which for more than twenty years defended the United States in the international races. The business was founded in 1863 by Dr. Herreshoff's brother, John B. Herreshoff, famous blind designer, and two other brothers, Nathanael Greene, whose talents as a designer and builder are equally well known, and Charles F., Jr., joined the firm in 1878, although the latter remained with it only a year.

Dr. Herreshoff himself had comparatively little interest in the family business. He was graduated from Brown University in 1870 with the degree of Ph.B. In his sophomore year he had been appointed laboratory assistant in the department of chemistry and in his junior year was made assistant professor of analytical chemistry. He retained this post after his graduation until 1872.

After leaving Brown University Dr. Herreshoff worked as a chemist at the Silver Spring Bleachery in Providence for about a year. He then went to New York and secured employment as assistant to Professor Habirshaw, who carried on a business as general chemical expert. About 1874 he entered the chemical-metallurgical field as manager of the Laurel Hill Chemical Works, Newton Creek, L.I., then the largest concern of its kind in the United States. Out of this organization grew the Nichols Chemical Company, for which Dr. Herreshoff became designer and consulting engineer in November, 1875, and later superintendent and vice-president. In 1899 one branch of the business was organized as the Nichols Copper Company, which Dr. Herreshoff served as superintendent and vice-president, and the remaining interests, together with the business of 24 major chemical concerns in the United States, were incorporated into the General Chemical Corporation. Dr. Herreshoff was consulting engineer to the corporation. Upon his retirement in 1924 he retained the vice-presidency of the Nichols Copper Company and was made an honorary vice-president of the corporation. He was also a director of the Granby Consolidated Mining, Smelting & Power Co.

The invention of the process which revolutionized the manufacture of sulphuric acid came early in Dr. Herreshoff's career, while he was superintendent of the Laurel Hill Chemical Works.

Dr. Herreshoff was honored by Brown University in 1905 with an honorary Master of Arts degree and in 1909 the degree of Doctor of Science was conferred upon him. His contributions to the "improvement of the science and manufacture of chemical and metallurgical products" were recognized in the award of the Perkin Medal, established as a memorial to Sir William A. Perkin, English scientist.

Dr. Herreshoff was married four times. His first marriage was to Miss Grace Eugenia Dyer, of Providence, in 1876. Of this marriage a daughter, Mrs. Louis H. Eaton, of Providence, survives him. His second wife was Miss Emily Duval Lee, of Philadelphia, whom he married in 1882. They were divorced in 1919. Of their three children a son, Francis Lee Herreshoff, of New York, and a daughter, Mrs. Rudolph Borgianni, of Florence, Italy, are living. Dr. Herreshoff's third marriage was to Mrs. Carrie Ridley Enslow, who died in 1924, and his fourth to her sister, Irma Grey Ridley, who survives him.

Dr. Herreshoff became a member of the A.S.M.E. in 1894. He was past-president of the American Chemical Society and a member of the Society of Chemical Industry, the Scientific Alliance, and the Brown University Alumni Association. His clubs were the University and Chemists', of Manhattan, and the Hamilton, of Brooklyn.

F. ADOLF HEYMANN (1875–1932)

F. Adolf Heymann, machine designer for the Mergenthaler Linotype Company, Brooklyn, N.Y., died on March 25, 1932, at his home in Waterbury, Conn., after a short illness. He is survived by his widow, Mrs. Helen Heymann, and his mother, Mrs. Margaret Heymann, as well as by one brother, Otto, of Austria.

Mr. Heymann was born on September 27, 1875, at Stettin, Germany. He served a three-year apprenticeship as a machinist with Hespe & Co., manufacturers of woodworking machinery in Hamburg, and worked for a number of years as machinist, toolmaker, and shop foreman in the United States, to which he came for the first time in 1898, and in Germany. He supplemented this practical experience with courses at technical schools in Hamburg and Neustadt, securing an M.E. degree in 1903.

In 1905 Mr. Heymann was connected with the H. B. Smith Machine Co., Smithville, N.J., as designer. He left in April of the following year to take the position of chief draftsman and mechanical engineer with the Marlin Fire Arms Company, New Haven, Conn., where he remained for five years. He designed all the tools and special machines necessary for the manufacture of a new line of sporting guns. During the next two years he was located in Munich, Germany, in the position of industrial and process engineer for the Isaria Meter Works. He reorganized the manufacturing department and introduced a tool designing department for developing labor-saving devices. As the result of his efforts, production was doubled, with only about a fifty per cent increase in the working force.

Mr. Heymann returned to the United States in 1913 and took the position of tool and machine designer with the Winchester Repeating Arms Company, New Haven, subsequently being promoted to overseer and assistant foreman. After two years there he became chief draftsman and process engineer at the Marlin-Rockwell Corporation in the same city. This company was just beginning to build the Colt machine gun. Mr. Heymann supervised the design of tools and special machines for three different machine guns, being in charge of about fifty draftsmen. Later he was transferred to a branch factory and took charge of a manufacturing job until the factory was shut down in January, 1919.

From this time until 1924 Mr. Heymann was employed as process engineer and chief draftsman by the Premier & Potter Printing Press Co., Derby, Conn., for which he organized an engineering department and laid out a more economical manufacturing system.

After a short time in production and research work for the Marlin Fire Arms Company, Mr. Heymann became associated with the Sargent Company, New Haven, in similar work.

In October, 1926, Mr. Heymann took a position with the Scovill Manufacturing Company at Waterbury, Conn., where he remained until the first of April, 1931. He developed and improved various types of automatic buffing machines, continuous spray lacquer machines, automatic plating machinery, and other equipment used in the application of the results of his research on metal finishes to the fabricated products of the firm.

After leaving the Scovill Manufacturing Company Mr. Heymann engaged in special engineering work for the Mergenthaler Linotype Company until ill health necessitated his retirement.

Mr. Heymann was greatly interested in amateur photography and was also an excellent shot, winning numerous prizes in various state and local contests. He was vice-president of the Scovill Recreation Rod and Gun Club, and wore its "Monogram," which meant an average of 95 per cent or more. He was also a member of the New Haven Gun Club. He joined the A.S.M.E. with the grade of member in 1919.

MICHAEL SULLIVANT HOPKINS (1872–1931)

Michael Sullivant Hopkins, who died of pneumonia at his home in Columbus, Ohio, on November 9, 1931, was born at Henderson, Ky., on May 21, 1872, the son of William Allison and Lucy Jane (Sullivant) Hopkins. He attended high school in Henderson and served an apprenticeship in a foundry and machine shop there.

Mr. Hopkins went to Columbus in 1890 and was employed during the next two years by the Minott Elec. Machine Co. and the Thompson & Houston Elec. Co. He became foreman of shops for the Columbus Consolidated Street Railway Company, in 1892 and was connected with the railway and lighting properties of Columbus until ill health caused his retirement from business several years prior to his death. He was mechanical engineer for the Columbus Street Railway Company from 1894 to 1898; general superintendent of the Columbus Railway Company from 1899 to 1902 and of the Columbus Railway & Light Co. during the next two years; and chief engineer and subsequently vice-president of the properties operated by E. W. Clark & Co. He was also vice-president of the Portland Railway Light & Power Co., Chattanooga Railway & Light Co., East St. Louis & Suburban Railway Co., Tennessee Power Company, and Nashville Railway & Light Co.

After giving up active work Mr. Hopkins became interested in securing exact data on automobile performance and developed a system of testing for gasoline consumption, performance of spark plugs, engine compression, and all points of efficiency, etc. He had the testing apparatus built and in practical use, intending to develop it commercially, when his failing health caused him to drop this and all other commercial interests.

Mr. Hopkins was fuel administrator for Ohio during the World War. He became a member of the A.S.M.E. in 1913 and belonged to a number of clubs in Columbus and Chattanooga. He was a member of the Presbyterian Church.

Surviving Mr. Hopkins are his divorced wife, Matie Atkinson (Lockett) Hopkins, whom he married in 1899, and a married daughter, Mrs. Sidney D. L. Jackson, of Toledo, Ohio.

JAMES ARTHUR INGALLS (1889–1932)

James Arthur Ingalls, chief of the Testing Bureau of the Brooklyn-Manhattan Transit Corporation, Brooklyn, N.Y., died on March 24, 1932. Mr. Ingalls had been connected with the Bureau since 1910, with the exception of about nine months' work as instructor in drawing at the New York Vocational School.

Mr. Ingalls was a native of Brooklyn, where he was born on May 22, 1889. He became a member of the Bureau in 1910 following his graduation with a Ph.B. degree from the Sheffield Scientific School of Yale University. The Testing Bureau is connected with the Williamsburgh Power Plant Corporation, a subsidiary of the Brooklyn-Manhattan Transit Corporation, and is divided into three sections, a chemical laboratory, an electrical division, and a mechanical division.

Mr. Ingalls was appointed mechanical assistant in charge of all mechanical work in 1916 and made chief of the Bureau three years later. In addition to varied tests of fuels, equipment, and materials, the Bureau handles considerable routine work, such as the analysis of lubricants, feedwater, and boiler water, the calculation of daily water rates and efficiencies, etc., as well as analytical and consultation work for various divisions of the parent company.

Mr. Ingalls claimed credit for the design of a smoke density indicator and recorder in successful operation for several years and several remote reading level gages, designed particularly to meet special conditions in different parts of the plant, and also for the development of an indirect method of checking the accuracy of water flowmeters using a proportionate salinity method combined with electrical conductivity measurements. He collaborated in the design of a very effective type of cinder catcher used in the Williamsburgh Power Station.

Mr. Ingalls became an associate member of the A.S.M.E. in 1916 and attained full membership in 1931.

Surviving Mr. Ingalls are his widow, Lillian Grace (Williamson) Ingalls, whom he married in Brooklyn in 1914, two sons, James Arthur and George Williamson, and a daughter, Ruth Elizabeth Ingalls.

GILTNER RUSSELL INGELS (1897–1932)

Giltner Russell Ingels, whose death occurred at Cleveland, Ohio, on October 16, 1932, following an operation for appendicitis, was born at Pleasant Hill, Mo., on March 22, 1897. His parents were George and Rosa (Russell) Ingels. He prepared for college at the high school in Columbia, Mo., and was graduated from the University of Missouri in 1914. He did post-graduate work in mechanical engineering there during the next two years, and in 1917 went into the 139th Infantry of the U. S. Army with the rank of sergeant. He was commissioned a lieutenant in the Air Service after having completed a special training course and appointed an instructor in aerial navigation. He continued his studies at the University of Missouri in 1919 and obtained his M.E. degree in 1921.

Mr. Ingels spent the summer of 1921 in design work for a new power plant at the University and during the next year was an instructor in mechanical drawing and descriptive geometry there. He entered the business field at the close of the school year as chief engineer for the Peet Bros. Mfg. Co., of Kansas City, Mo., and Berkeley, Calif., and for about three years had charge of design and maintenance work at a large soap and glycerine plant.

In the fall of 1925, after a few months in mechanical design work for the California Packing Corporation, San Francisco, Calif., he went to Aurora, Ill., to become chief engineer for the Lyon Metallic Mfg. Co., in charge of all plant layout, construction, and maintenance work. Early in 1926 he began his association with the Charles E. Bedaux Co. of Illinois, Inc., in Chicago, of which he was vice-president and chief engineer until February, 1932. As field engineer during the early days of his connection with the company, he installed the Bedaux system of human power measurement for wage payment and labor control and cost purposes. Since leaving the company he had been engaged in design work for himself, and was collecting data for a book on aeronautics.

Mr. Ingels became an associate member of the A.S.M.E. in 1927. He belonged to the Masonic and several Greek letter fraternities. He is survived by his widow, Helen (Bradford) Ingels, whom he married in 1922.

GEORGE E. IRELAND (1884–1932)

George E. Ireland was asphyxiated on October 28, 1932, by carbon monoxide gas while adjusting the motor of his automobile in the garage of his home in Toledo, Ohio.

Mr. Ireland was born in Greenfield, Ohio, on February 23, 1884, the son of Mr. and Mrs. Matthew O. Ireland. He was educated in the public schools of Washington Courthouse, Ohio, later moving to Dayton, where he was first employed by the Davis Sewing Machine Company as assistant foreman of its woodworking and millwrighting departments. Later he was connected with the Recording & Computing Machines Co., for which he designed street car registers, models for naval telescopes, and special machinery for accounting and banking. He also acquired a practice as a consulting engineer and was president and general manager of the City Engineering Company in Dayton, designers and builders of tools and machinery for special requirements.

In 1917 Mr. Ireland went to Springfield, Ohio, where he became assistant manager of the Steel Products Engineering Company and the Gem City Machine Company. In 1918 he was made general manager of the former organization, with which he remained until 1925, working on special tools for quantity production and developing an aeronautical motor. During the next two years he was connected with the Michigan Stamping Company, Detroit. He had been in Toledo since then, as assistant general manager for the City Machine & Tool Co. until January 1, 1931. Mr. Ireland had designed several types of radiator shells adopted by automobile companies.

Surviving Mr. Ireland are his widow, Edith Ireland, two daughters by a former marriage, Georgie E. and Mary S. Ireland, of Detroit, and his parents. He was a member of the Presbyterian Church and the Masonic fraternity and had been an associate member of the A.S.M.E. since 1919.

GEORGE HENRY ISLEY (1882–1932)

George Henry Isley, whose death occurred on November 30, 1932, had been associated with the Morgan Construction Company since 1906. He had served as chief draftsman and designer, and at the time of his death was manager of the Combustion Control Department of the company.

Mr. Isley was born in Jersey City, N.J., on November 25, 1882. After two years at Cooper Union in New York he did drafting for a number of firms before entering the employ of the Morgan Construction Company. Among these were the A. B. See Elevator Co., Rockwell Engineering Company, M. H. Treadwell Co., Rider Conley Manufacturing Company, United Coke & Gas Co., and Power Mining Machinery Company.

Mr. Isley made important contributions to the development of the Morgan producer gas machine, and invented the Isley reversing valve, the Isley system of furnace control, and other devices. He became a member of the A.S.M.E. in 1918.

WALTER GILBERT JAMESON (1883–1931)

Walter Gilbert Jameson, district superintendent of the Philadelphia District for the Babcock & Wilcox Co. died in Philadelphia on November 12, 1931, of pneumonia. He is survived by his widow, Laura (Nesbit) Jameson, whom he married in 1916.

Mr. Jameson was born at Norwalk, Ohio, on December 2, 1883. His early engineering work was that of fireman and engineer with dredging companies in Toledo and Cleveland, Ohio, and the Wheeling & Lake Erie and Pere Marquette Railroads. In 1912 he entered the employ of the J. B. McCrory of Atlanta, Ga., in charge of sewer and water works construction in Washington, N.C., and he continued as a construction engineer for a number of years, directing the erection of a post-office at Rocky Mount, N.C., the Hopewell, Va., plant of the DuPont Companies, a temporary trestle and cofferdam for the Philadelphia & Reading R.R. at Milton, Pa., and a sand blast plant for the American Car & Foundry Co., also at Milton.

In 1916 Mr. Jameson became erection engineer on power plant work for the American Engineering Company of Philadelphia, Pa. Four years later he was made district construction engineer for the company, in charge of all work in the Philadelphia District. He was transferred to the Western Division, with office at Chicago, in 1924, and remained with the company until 1929. Between then and his connection with the Babcock & Wilcox Co. in 1930 as field engineer, stationed at Barberton, Ohio, he served as field superintendent for the Combustion Engineering Corporation, with office at Detroit, Mich., and for the Fuller-Lehigh Company, at Fullerton, Pa.

Mr. Jameson became an associate-member of the A.S.M.E. in 1924.

WILLIAM EDWARD JERAULD (1873–1931)

William Edward Jerauld, works manager of the Consolidated Ashcroft Hancock Co., Inc., Boston, Mass., died at the Newton (Mass.) Hospital on October 30, 1931.

Mr. Jerauld was born at Clinton, Mass., on July 14, 1873, the son of Albert Edward and Mary J. (Earl) Jerauld. Employment by the American Steam Gauge & Valve Mfg. Co., Boston, Mass., followed the completion of his public school education, and at the age of twenty-six he had become superintendent of works for the company. In 1923 he took a position with Manning, Maxwell & Moore, Inc., and in 1927 with its subsidiary, organized in that year, the Consolidated Ashcroft Hancock Co., Inc., which is successor to the American Steam Gauge & Valve Co. and several other organizations.

Two United States patents on safety valves were held by Mr. Jerauld. He became a member of the A.S.M.E. in 1915 and also belonged to the Boston Engineers' Club and the Masonic fraternity, in which he was a Knight Templar.

Surviving Mr. Jerauld are his widow, Beatrice C. (Silman) Jerauld, whom he married in 1899, two sons, Albert E., Waban, Mass., and Ralph E. of Newtonville, Mass., and two daughters, Mrs. Gladys M. Bruce, Newtonville, and Mrs. Doris B. Hadley, Sterling, Mass.

ARTHUR ROBERT RICHARD JOHN (1896–1932)

Arthur Robert Richard John was born at Fürstenwalde, near Berlin, Germany, on January 28, 1896. He attended the Lycee of Haguenau, France, for six years and took three years of night school work at the Arts et Metiers in Paris. In August, 1910, he entered the firm of Dietrich & Co., at Reichshoffen, Bas Rhin, France, as apprentice engineering draftsman. After the completion of his apprenticeship he remained with the company until 1925, securing practical experience in the design of machine parts, timber construction, and other work, and taking technical courses.

In 1925, Mr. John went to Paris and found work as a draftsman with the Société Rateau, where he remained for about two years, working for part of the time as detailer in the pump section and later, on the building of steam turbines. His next position was with the Constructions Électrique de France, Paris, as a designer in the construction of electric locomotives, erection of electrical apparatus, and similar work.

Mr. John came to the United States in the latter part of 1928 and was employed for nearly two years by the Carrier Engineering Corporation, Newark, N.J., in layouts for heating, ventilating, and refrigerating systems. He returned to Paris early in 1931 to assist in the design of subway cars for the firm of Berboom & Dureuchard, but was again living in the United States when his death occurred on April 30, 1932. He had been an associate member of the A.S.M.E. since 1929.

JOHN PACKARD JORDAN (1877–1932)

John Packard Jordan, a member of the firm of Stevenson, Jordan & Harrison, management engineers, New York, N.Y., died suddenly at his home in that city on July 21, 1932.

Mr. Jordan was born in Auburn, Me., on November 1, 1877, the son of the Rev. John B. and Marietta Frances Jordan. He took a high school course in Pawtucket, R.I., at the same time studying mechanical drafting in evening school classes, and in 1893 was awarded prizes for his drawings exhibited at the Chicago World Fair. Practical experience in drafting was secured during summer vacations in the office of the Pawtucket Manufacturing Company, where he worked on bolt and nut machinery. He entered Brown University, Providence, R.I., with the class of 1899 but was obliged to leave after about a year and half because of eyestrain and other reasons.

In the fall of 1901 Mr. Jordan entered the employ of the Lackawanna Steel Company, Buffalo, for which he successively inspected materials, assisted in the supervision of repair and construction shops, reported progress of work on all building operations, and directed the erection of Power House No. 1, a 725-ft. building built in a record time of 19 days. Upon the completion of this plant Mr. Jordan was appointed supervisor of buildings, in charge of all steel erection, outside the blast furnaces, but not long afterward was stricken with infantile paralysis. After an illness of eight months he returned to the company, working in the Cost Department until August, 1903. He then resigned to take up industrial engineering, studying management and manufacturing problems in different plants in a consulting capacity.

Feeling the need of a knowledge of cost accounting work, Mr. Jordan associated himself from 1905 to 1908 with Mr. W. J. Gunnell, of Buffalo, in whose employ he had practical plant experience in this field. He resumed his private practice in 1908, continuing reorganization work begun at the Waterous Engine Works, Brantford, Canada, while he was associated with Mr. Gunnell, and conducting similar work in other plants. Among these was the Gould Coupler Company, Depew, N.Y., where for two years he worked on problems of reorganization. The company then desired him to remain in its employ as assistant to the general superintendent. He greatly decreased costs through changes in methods and equipment, a particular instance being the development of a variable speed grinder for grinding steel castings which resulted in an annual saving of from $25,000 to $30,000. He terminated his connection with the company in 1917 and during the World War was associated with a friend in the organization of a small aniline plant.

In July, 1918, Mr. Jordan became vice-president and director of operation for C. E. Knoeppel & Co., New York, N.Y., with which he remained for about five years. Shortly afterward the firm of Stevenson, Jordan & Harrison was organized.

Mr. Jordan lectured on industrial engineering subjects at New York University, the Harvard Graduate School of Business Administration, The Pennsylvania State College, and Massachusetts Institute of Technology. His "Cost Accounting Principles and Practice" was published by the Jordan-Harris-Ronald Press in 1921, and articles by him on industrial management have appeared in the technical press from time to time.

Mr. Jordan became a member of the A.S.M.E. in 1920 and also belonged to the American Management Association, Society of Industrial Engineers, National Association of Cost Accountants, of which he was a past-president, as well as of the Institute of Management, and a number of clubs in New York and vicinity and in Buffalo.

Surviving Mr. Jordan are his widow, Edith (Nagel) Jordan, whom he married in 1909, a son, John Alden Jordan, and two daughters, Marietta Jordan and Mrs. Benjamin F. Cutler.

FREDERICK WILLIAMS KELLEY (1870–1932)

Frederick Williams Kelley, president of the North American Cement Corporation, died on September 19, 1932, at his summer home in Altamont, N.Y., after an illness of four years. Though confined to his home, he had been able to continue in charge of the company and only a few days before his death had presided at a meeting of its directors.

Mr. Kelley was born at Albany, N.Y., on December 15, 1870, the son of James B. and Alice (Williams) Kelley. He prepared for college at the Albany Academy and Toledo (Ohio) High School, where he studied manual training. He received an M.E. degree from Cornell University in 1893. He was elected to membership in the Sigma Xi and Psi Upsilon fraternities while in college and was a member of the crew for four years.

Following his graduation Mr. Kelley entered the employ of the Consolidated Car Heating Company, Albany, working for the first two years in the drafting room and developing and experimental department. He was made assistant to the general manager and assistant treasurer in 1895 and four years later general manager and treasurer. Prior to and during the Spanish-American War he served in the infantry and signal corps of the New York National Guard.

In 1900 Mr. Kelley became vice-president and general manager of the Helderberg Cement Company and for the next 14 years made his home at Howes Cave, N.Y., where the plant of the company was located. He returned to Albany as president of the company and retained that position until the Helderberg Cement Company and Security Cement & Lime Company of Maryland were merged into the North American Cement Corporation in 1925, with Mr. Kelley as its president.

Mr. Kelley also served as president of the Consolidated Car Heating Company from 1912 to 1915, and was later made a director of the company and continued in that capacity until he died. He was vice-president of the National Commercial Bank & Trust Co., trustee of the City Savings Bank, and a director of the Commercial Safe Deposit Corporation, all of Albany; also president of the Albany Hospital, trustee of the Albany Medical College, The Albany Academy, Albany Institute of History and Art, and Dudley Observatory, and a member of the Board of Governors of Union University. He became a member of the A.S.M.E. in 1905 and also belonged to the American Institute of Electrical Engineers, American Society for Testing Materials, American Concrete Institute, Albany Society of Engineers, Portland Cement Association of America, International Cement Manufacturers Association, and the Society of Engineers of Eastern New York, of which he was a former president, as well as of the Albany Chamber of Commerce. He belonged to the Fort Orange and Albany Country clubs, and the University Club in New York.

Besides his mother, now Mrs. Timothy S. Williams, of Huntington, L.I., N.Y., and his widow, formerly Miss Lydia L. Hun, whom he married in 1903, Mr. Kelley is survived by a daughter Alice (Mrs. Arthur Brooks Harlow, Noroton, Conn.) and a son, Frederick Williams Kelley, Jr., Albany, N.Y.

JEREMIAH JOSEPH KENNEDY (1864–1932)

Jeremiah Joseph Kennedy, who died on February 15, 1932, was born in Philadelphia, Pa., on October 22, 1864. As a child Mr. Kennedy was delicate and was unable to attend school after the age of ten years. This condition, however, did not prevent him from studying at home, and being ambitious, he embraced every opportunity to learn. He was particularly fond of mathematics and borrowed from the Apprentices' Library, in Philadelphia, all books relating to this subject. Later in life, he was instrumental in this same library in helping poor boys to further their education.

Mr. Kennedy had a wide and varied career in the engineering profession. In January, 1882, he started as a rodman with the Norfolk and Western Railroad, and after a year became principal assistant to General Russell Thayer, engaging in miscellaneous engineering activities. In 1884 and 1885, as engineer for the Grubb Iron Works, Blue Ridge Springs, Va., he constructed railway extensions, operated mines, and planned and constructed improvements for the old Lynchburg Iron Company.

Then followed varied engineering work with the Henry Disston and Sons Steel and Saw Works, with Morris Tasker & Co., and with others. From 1892 to 1894, he was engaged in designing and constructing gas works, gas holders, retorts, and accessories, and in this period he owned and operated the National Iron Works at Linwood, Pa.

In May, 1894, he became chief engineer of the White, Crosby & Company of Baltimore, which later became the J. G. White & Co. of New York. He planned, constructed, and operated, among the many engineering projects of this company under his supervision, the Atlantic Highlands, Red Bank, and Long Branch Electric Railway.

In February, 1901, Mr. Kennedy severed his connection with the J. G. White & Co., to engage in private practice as consulting engineer, in which he continued until his death.

He designed and constructed for the Whitney Reduction Company in Rowan and Stanly Counties, in North Carolina, a masonry dam 38 feet high, together with a $4\frac{1}{2}$-mile-long hydraulic canal to develop 20,000 horsepower. He designed and constructed hydroelectric developments for the Schuylkill Valley Illuminating Company at Phoenixville, Pa.; for the Lynchburg Traction & Light Company at Lynchburg, Va.; and for the American Gas Company at Winooski and Essex Junction, Vt. He also planned and constructed improvements in the power plants of the Detroit and Lake Orion Railway at Rochester, Mich., and for the Cortland County Traction Company at Cortland and Homer, N.Y. For the Baldt Steel Company he designed and constructed the buildings, furnaces, and equipment for its steel plant at New Castle, Del.

In this period, from 1901 to 1932, Mr. Kennedy was also engaged in consultations, investigations, and reorganizations of many engineering projects in various parts of the country.

From 1904 to 1907, he was in charge of all operations and extensions to the underground pneumatic-mail-tube systems in New York, Chicago, St. Louis, and Boston.

In 1908, Mr. Kennedy became financially interested in the Biograph Company, a pioneer company in the motion-picture industry. He acquired an interest in and control of a number of patents relating to the industry, and for many years helped to guide the motion-picture industry through its early difficulties and developments.

Mr. Kennedy was a member of the American Society of Civil Engineers, the American Institute of Electrical Engineers, the American Institute of Mining and Metallurgical Engineers, the New York Electrical Society, the American Society for Testing Materials, the American Association for the Advancement of Science, the Illuminating Engineering Society, and The Franklin Institute. He had been a resident of New York since 1896, and was a member of the Lotos Club, the Engineers' Club, and the Bankers Club of America. He became a member of the A.S.M.E. in 1902.—Robert H. Hammer.[1]

JULIAN KENNEDY (1852–1932)

Julian Kennedy, who for his services and contributions to the iron and steel industry was awarded the A.S.M.E. medal in 1928 and the Gary Memorial Medal at the Spring Meeting of the American Iron and Steel Institute on May 19, 1932, died suddenly of heart failure on May 28, 1932, at his home in Pittsburgh, Pa. Mr. Kennedy was also the recipient, in 1904, of the Grand Prize for blast-furnace and steel-plant construction granted by the International Jury of Awards at the St. Louis Exposition.

Mr. Kennedy was born at Poland, Ohio, on March 15, 1852, the son of Thomas Walker and Margaret (Truesdale) Kennedy. After attending public school and the Poland Union Seminary he became a draftsman, under his father's supervision, in the construction of the Struthers blast furnace at Struthers, Ohio. He entered the Sheffield Scientific School of Yale University in 1872 and was graduated three years later with a Ph.B. degree. After graduation he remained at the school for a year, giving instruction in physics and taking post-graduate work in the chemistry of iron and steel. He received the honorary degree of Master of Arts from Yale in 1900, and in 1909 the

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[1] Assistant to Mr. Kennedy since 1899.

Stevens Institute of Technology conferred upon him the honorary degree of Doctor of Engineering.

In 1876 Mr. Kennedy became superintendent of blast furnaces of the Brier Hill Steel Company, Youngstown, Ohio. The following year he went to the Struthers Iron Company in the same capacity, and in subsequent years he served successively as superintendent of the Morse Bridge Works, Youngstown; superintendent of blast furnaces at the Edgar Thomson Works of Carnegie Steel Company, Braddock, Pa.; superintendent of the Lucy Furnaces, Pittsburgh; and general superintendent for Carnegie, Phipps & Co., with headquarters at Homestead, Pa. In all of these positions he was responsible for construction and operation.

Mr. Kennedy was appointed chief engineer of the Latrobe Steel Works, Latrobe, Pa., in 1888, and was in charge of the construction of that plant. Two years later, while continuing as chief engineer of the Latrobe Works, he opened an office in Pittsburgh for private practice as consulting engineer in blast-furnace and steel-plant construction, subsequently gaining world-wide recognition in this field. His principal projects abroad were the design and construction of the steel works and blast furnaces for both the Tata Iron and Steel Works, Sakchi, India, and the Nicopol-Maripol Mining & Metallurgical Co., Sartana, Russia.

Mr. Kennedy patented many improvements in blowing engines, reversing engines, blooming mills, hot-blast stoves, blast-furnace filling devices, and other equipment for steel works. He was president of a number of coal and coke companies, including the Ontario Gas Coal Company, Lowber Gas Coal Company, Poland Coal Company, and Orient Coke Company. He was a director of the Toledo Furnace Company.

Mr. Kennedy became a member of the A.S.M.E. in 1912. He served as vice-president of the Society during the term of 1915–1917 and was selected as the A.S.M.E. medallist in 1928. Other technical organizations to which he belonged included the Iron and Steel Institute (Great Britain), American Institute of Mining and Metallurgical Engineers, American Iron and Steel Institute, Iron and Steel Electrical Engineers, and Engineers' Society of Western Pennsylvania. He belonged to a number of social and athletic clubs in Pittsburgh and New York.

Surviving Mr. Kennedy are two sons, Julian, Jr., and Joseph Walker, both associated with their father's engineering organization, and two daughters, Mrs. Lucy Belle (J.O.) Miller and Mrs. Eliza Jane (R.T.) Smith, of Pittsburgh. Their mother was Jennie E. Brenneman, whom Mr. Kennedy married in Poland, Ohio, in 1878.

MORRIS KNOWLES (1869–1932)

Morris Knowles, sanitary engineer and city planner, died suddenly of a heart attack at his home in Pittsburgh, Pa., on November 8, 1932.

Mr. Knowles was born in Lawrence, Mass., on October 13, 1869, the son of Charles E. and Ellen B. (Richardson) Knowles. As a young man, he was a neighbor of Hiram F. Mills, who was at that time conducting the historic studies at the Lawrence Experiment Station, in the field of water and sewage treatment. In part, at least, as a result of his contacts with Mr. Mills, he elected to study engineering at the Massachusetts Institute of Technology, where he was graduated in 1891 with an S.B. degree.

From his early acquaintance and friendship with Mr. Mills and from his up-bringing in the atmosphere of sanitary engineering that prevailed in Lawrence at that time, Mr. Knowles went on to practice his profession under men who were then or later to become outstanding leaders in the field of sanitary engineering. During his student days at M.I.T., he served an apprenticeship under Richard A. Hale, then principal assistant engineer, later chief engineer of the Essex Company, Lawrence. Upon graduation from M.I.T., Mr. Knowles became assistant engineer for the East Jersey Water Company, Montclair, N.J., under Clemens Herschel, chief engineer, and J. Waldo Smith, principal assistant engineer. In 1893, he became assistant engineer on the Massachusetts State Board of Health under F. P. Stearns. Mr. Knowles was personal assistant to Mr. Stearns in the latter's investigation of a water supply for Boston and vicinity, leading to the formation of the Massachusetts Metropolitan Water Board, and the development of the Wachusett Reservoir and Aqueduct to supply Boston and 28 surrounding cities and towns.

At the same time, Mr. Knowles was a member of the Water Board at Lawrence, his home town, which marked the beginning of an intimate contact with the municipal water supply problems of Lawrence, which he always maintained, returning there many years later to build additions to the slow sand filter plant and to report on new sources of supply.

In 1897, Mr. Knowles became resident engineer of the Pittsburgh Filtration Commission, and established a laboratory and experimental filters looking to the improvement of the Pittsburgh water supply. Following completion of the report of the Pittsburgh Filtration Commission, and prior to the actual construction of the Pittsburgh filters, Mr. Knowles went to Philadelphia in 1899 to perform a similar service for that city in experimental work on filters and an investigation of a proposed new gravity water supply for Philadelphia. In 1900, Mr. Knowles became assistant engineer under John R. Freeman, aiding him in the preparation of his comprehensive report on additional water supply for the City of New York.

Mr. Knowles returned to Pittsburgh in 1901, to become chief engineer of the Pittsburgh Bureau of Filtration and to design and supervise the construction of the slow sand filters for the Pittsburgh water supply on the Allegheny River at Aspinwall.

Upon the completion of the Pittsburgh Filtration Plant, Mr. Knowles devoted himself actively to the consulting engineering practice which he had formed in partnership with L. E. Chapin in Pittsburgh a few years before, and which he continued, first as a partnership with Mr. Chapin, and later as a corporation under his own name until his death.

In private practice, Mr. Knowles designed an impounding dam for the development of an industrial water supply for the Tennessee Coal, Iron & Railroad Co., on Village Creek in the Birmingham, Ala., district. He was a member of the engineering committee of the Flood Commission of Pittsburgh, which compiled an exhaustive report upon flood protection and prevention, and following the completion of the report in 1911, Mr. Knowles continued as a member of the executive committee of the Pittsburgh Flood Commission. He was a member of the Board of Advisory Engineers of the Miami Conservancy District, 1914, and chief engineer of the Essex Border Utilities Commission, Ontario, 1916–1921.

During the World War, he was supervising engineer in charge of the construction of Camps Meade and McClellan, and in 1918 and 1919 was chief engineer of the Housing Division of the United States Shipping Board. He was a member of the Engineering Board of Review of the Chicago Sanitary District in 1924, and served this Board as chairman of its committee on construction program.

Mr. Knowles early sensed the eventual participation of the civil engineer in the field of city planning and zoning, and as a consequence of his pioneering activities in this direction, he became a member of the Advisory Committee on Zoning appointed by Herbert Hoover when Secretary of Commerce. Mr. Knowles was Chairman of the City Planning Commission of Pittsburgh from 1922 to 1929, and Chairman of the Zoning Board of Appeals of Pittsburgh from 1923 to 1927.

Mr. Knowles designed and supervised the construction of sewage disposal and harbor improvements for Lynn, Mass., costing $2,500,000. He was consulting engineer to the Attorney General of the State of Connecticut, in conducting comprehensive investigations in support of the latter's unsuccessful suit to restrain Massachusetts from the diversion of interstate waters in the extension of the Boston Metropolitan District water supply.

Mr. Knowles was director of the Department of Sanitary Engineering, University of Pittsburgh, from 1911 to 1920 and in 1929 was awarded the honorary degree of Doctor of Engineering by the University. He was vice-chairman of the Executive Committee of the Commission appointed by the Governor of Pennsylvania to study municipal consolidation in counties of the second-class in Pennsylvania, and drew up a proposed charter for a federated city to amalgamate more than 100 minor political subdivisions in Allegheny County into a Greater Pittsburgh. Mr. Knowles was chairman of the Committee on Utilities in President Hoover's Conference on Home Building and Home Ownership in 1931. He was a term member of the Corporation of the Massachusetts Institute of Technology from 1924 to 1929. He was a director in the Pittsburgh Chamber of Commerce from 1922 to 1930.

Mr. Knowles was a director in the American Society of Civil Engineers from 1928 to 1930, a former director of the National Conference on City Planning, and a member of the Institution of Civil Engineers, Great Britain, and the Verein deutscher Ingenieure, Germany. He became a member of the A.S.M.E. in 1907, was a past-president of the Engineers' Society of Western Pennsylvania, and a member of the American, New England, and Pennsylvania Water Works Association, the American Public Health Association, the American Academy of Political and Social Science, the National Institute of Social Science, the Boston Society of Civil Engineers, the National Municipal League, and the International Housing and Town-Planning Association.

He belonged to the Duquesne Club and the University Club at Pittsburgh, the Engineers' Club in New York, the Cosmos Club of Washington, D.C., and other clubs. He was the author of "Industrial Housing," 1920, and many professional papers and contributions to the technical press.

Mr. Knowles is survived by his widow, Mina (McDavitt) Knowles, whom he married in 1893, one son, Morris, Jr., of Pittsburgh, and a daughter, Helen, Mrs. H. N. Schreuder, of New York, N.Y.

WALTER CHARLES LANGE (1889–1932)

Walter Charles Lange, consulting engineer, New York, N.Y., died on February 16, 1932, in West Englewood, N.J. He was born in New York on July 24, 1889, the son of Mr. and Mrs. Gustav Lange. He attended the High School of Commerce in New York and was graduated from the Stevens Institute of Technology, Hoboken, N.J., with an M.E. degree in 1912. During the remainder of that year he worked as draftsman in the Construction Department of the Otis Elevator Company, New York, and the Engineering Department of the Nichols Copper Company, Laurel Hill, L.I., N.Y. He then became assistant engineer in the Department of Utilization of the Consolidated Gas Company of New York, where his work consisted of conducting investigations of various types of gas appliances, designing of gas burners, and other work incidental to the utilization of gas as a fuel. For part of the time he was in charge of the Hotel and Restaurant Bureau, concentrating on appliances for hotel kitchens.

Early in 1916 Mr. Lange went to Homestead, N.J., to act as superintendent of the shop of the Schmitt Manufacturing Company, manufacturers of gas-burning appliances. He spent only a few months there, returning to New York to become technical assistant to the superintendent of Duparquet, Huot & Moneuse Co. For about three years, beginning in 1917, he was in sales work for the Doehler Die Casting Company, Brooklyn, and the Hammel Oil Burning Equipment Company, New York, being in charge of the office of the latter for some months.

Mr. Lange accepted the position of combustion and sales engineer for the Uehling Instrument Company, Paterson, N.J., in September, 1920, and in January of the following year was made manager of its Chicago office. He left in November, 1922, to open an office of his own in Chicago as combustion engineer. He represented several companies as manufacturers agent, including the Uehling company and the Hammel. In 1925 he went to Memphis, Tenn., to do some work for the Southern Coal Company, upon the completion of which he opened his New York office. He specialized in fuel economy, furnace design and control, boiler-room efficiency, etc.

Mr. Lange became a junior member of the A.S.M.E. in 1914, an associate member in 1921, and a member in full in 1925. He also belonged to the American Chemical Society and the American Association for the Advancement of Science.

Surviving Mr. Lange are his widow, formerly Miss Katherine G. Clark, of Memphis, Tenn., whom he married in 1916, and four children, Clark, Katherine, Helen, and Gustav.

JAMES GAVIN LARMER (1869–1932)

James Gavin Larmer was born in Glasgow, Scotland, on September 19, 1869, the son of David and Margaret (Gavin) Larmer. He served an apprenticeship at the Dalmellington Iron Works, near Glasgow, from 1883 to 1890, and also studied at the technical college in Glasgow. During the next three years he was engaged in marine engineering with Robt. Napier & Sons, Lancefield, Glasgow, and the Fairfield Ship Building & Engineering Co., Govan, Glasgow.

Mr. Larmer came to the United States in 1893 and was employed for a year in the mechanical department of Thomas A. Edison & Co., Schenectady, N.Y., and another year in the shops of the Burlington Railroad at Havelock, Neb. He then went to Lead, S.D., where until 1915 he was assistant master mechanic for the Homestead Mining Company. His next connection was with the Standard Oil Company in Sioux City, Ia., as lubricating engineer from 1918 to 1927. Since then, until his death on April 8, 1932, in Glendale, Calif., he had engaged in consulting work in lubrication in Los Angeles.

Mr. Larmer became a member of the A.S.M.E. in 1914. He was unmarried.

WILLIAM STEVENS LECKIE (1904–1931)

William Stevens Leckie, outside plant engineer for the New Jersey Bell Telephone Company, Newark, N.J., died at his home in Elizabeth, N.J., on November 19, 1931. He was born at Lake Placid, N.Y., on August 9, 1904, the son of William John and Bernice V. (Stevens) Leckie. He attended the Battin High School in Elizabeth and took night courses in drafting at the Newark Technical School, before entering Pratt Institute, Brooklyn, N.Y., from which he was graduated in 1927. He was draftsman for the New Jersey Bell Telephone Company for a time before going to Pratt Institute, and since his graduation had been in the employ of that company.

Mr. Leckie became a junior member of the A.S.M.E. in 1928. He was secretary of the Omega Gamma Delta fraternity and a member of the Third Presbyterian Church in Elizabeth.

HENRY MARTYN LELAND (1843–1932)

Henry Martyn Leland died on March 26, 1932, in the Grace Hospital, Detroit, Mich., after a month's illness. He was 89 years old. At the age of 61 he organized the Cadillac Automobile Company; at 74 he formed the Lincoln Motor Company; at 80, when he demonstrated his vitality by walking up to his office on the 22d floor, 433 steps, he declared that he had no intention of retiring. He remained active in civic affairs until his illness in February.

Leland manufactured tools for rifle making during the Civil War and produced airplane engines for the use of the United States Government in the World War. Educated, as he said, "in the little red schoolhouses of Vermont and Massachusetts," he became an inventor and manufacturer of such ability as to win him the honorary degree of Doctor of Engineering from the University of Michigan (1920) and from the University of Vermont (1923).

Mr. Leland was born at Danville, Vt., on February 16, 1843, a son of Leander Barton and Zilpha (Tifft) Leland. He attended the public schools in Millbury, Mass., and then learned the machinist's trade at the Crompton Loom Works in Worcester, Mass. During the Civil War he worked at the United States Armory in Springfield, Mass., Colt's Armory, Hartford, Conn., and a rifle factory in Worcester.

From 1866 to 1871 Mr. Leland built card-setting machines by contract. He then began an association of nearly twenty years with the Brown & Sharpe Manufacturing Co., Providence, R.I. He was foreman for a number of years, and from 1878 to 1890, when he resigned, he was superintendent of the sewing-machine department. Two of his inventions, the barbers' hair clippers and his grinding machine, were turned over to the company. This led Mr. Leland to realize the advantages of being able to manufacture his products himself. Accordingly he went to Detroit and organized the Leland, Faulconer & Norton Co. to manufacture machine tools. This company was later reorganized as the Leland & Faulconer Manufacturing Co., of which Mr. Leland was vice-president and superintendent until it was merged into the Cadillac Automobile Company in 1905. He entered the automotive field in 1900 with the manufacture of transmission gears for the one-cylinder motors for the car designed by Ransom E. Olds. He had previously had some early experience in making engines for "naphtha" launches, and predicted an almost unlimited future for internal-combustion engines.

Mr. Leland was general manager of the Cadillac Automobile Company during the first five years of its existence and then president and advisory manager. This company developed his electric starting device, thermostatic control of engine temperatures, and other inventions, as well as the V-type engine which his son, associated with him in the company, had worked out.

In 1917, Mr. Leland and his son resigned from the Cadillac Automobile Company to organize the Lincoln Motor Company, which manufactured thousands of Liberty motors for airplanes for the United States Government during the World War. He was an ardent admirer of Lincoln and had a notable collection of Lincoln literature and relics.

Mr. Leland became a member of the A.S.M.E. in 1895 and served as manager of the Society from 1912 to 1915. He was president of the Society of Automotive Engineers from 1909 to 1914, a founder of the National Foundrymen's Association and of the National Metal Trades Association, and prominent in the work of the National Association of Manufacturers. He was also a founder of the Detroit Citizens League and for some years its president. He belonged to a number of clubs and fraternal organizations in Detroit and vicinity and to the Jefferson Avenue Presbyterian Church. Reading, in which the study of the Bible was prominent, was Mr. Leland's main recreation.

Surviving Mr. Leland are two children, Mrs. M. Gertrude (Angus C.) Woodbridge and Wilfred Chester Leland, both of Detroit. Mrs. Leland, formerly Miss Ellen R. Hull, of Millbury, Mass., whom he married in 1867, died in 1914.

WILLIAM EDWARD LINDSAY (1864–1931)

William Edward Lindsay was born at Eldersburgh, Carroll County, Md., on February 29, 1864, and died in New York, N.Y., on December 26, 1931. Mr. Lindsay was the son of William C. and Julia (Allen) Lindsay. He served an apprenticeship as a machinist with Murrill and Keizer, Baltimore, Md., from 1882 to 1885, at the same time studying at the Maryland Institute Night School of Design. He attended high school in Ithaca, N.Y., the following year and then worked as journeyman machinist for Murrill and Keizer for a year, before entering Sibley College, Cornell University, from which he received an M.E. degree in 1891.

After his graduation Mr. Lindsay took a position at the Akron Rubber Works of The B. F. Goodrich Co., in the capacity of mechanical and designing engineer. He severed the connection late in 1892 to become chief electrician at the packing-house plant of Swift & Co. at the Union Stock Yards, Chicago, and had complete charge of the electric lighting and power system of this plant until April of the following year. He was then transferred to East St. Louis to install an electric lighting plant at a new packing house there, and directed the operation of the entire mechanical and electrical equipment, including the refrigerating and ice-making departments.

From 1922 until his death Mr. Lindsay was head of the Department of Machine Design of the Mechanics Institute, New York. He also was a Y.M.C.A. home study instructor from 1921 to 1928, and an instructor in the mechanical laboratory of New York University in 1929–1930.

Mr. Lindsay became a member of the A.S.M.E. in 1895 and also belonged to the Masonic fraternity. He is survived by his widow, Emily (Schirm) Lindsay, whom he married in 1891.

OTTO LUHR (1860–1932)

Otto Luhr, a pioneer in the refrigerating industry, died on May 5, 1932, following several years' illness. He was well known both as a consulting engineer in his field and as engineering instructor at the Wahl-Henius Institute of Fermentology in Chicago and associate editor of the Wahl-Henius Mechanical and Refrigerating Engineers Handy Book. He was an early contributor to *Ice and Refrigeration* on subjects relating to practical operation of refrigerating machinery. He had designed and installed a number of successful ice making and refrigerating plants and had recently invented a mechanical refrigerator car known as the Frigicar.

Mr. Luhr was born in Magdeburg, Germany, on March 21, 1860, the son of Adolph and Augusta Rieke Luhr. His early training was secured in Germany in the Magdeburg schools and as an apprentice in general shop practice and design in Backau. Prior to his association with the Wahl-Henius Institute he had been designer for the Keystone Bridge Works, Pittsburgh, Pa., and chief engineer for the Chicago Brewery & Malting Co., Ltd.

Mr. Luhr became a member of the A.S.M.E. in 1907. He was one of the organizers of the Linde Association, a Chicago branch of the National Association of Stationary Engineers, and belonged to the American Society of Refrigerating Engineers, National Association of Practical Refrigerating Engineers, and National Association of Steam Engineers. He was examining engineer of all civil service engineers in Chicago for some time.

A son, Henry Luhr, of Chicago, survives Mr. Luhr. His wife died in 1910.

BURT MACKENZIE (1869–1930)

Burt MacKenzie, owner of the contracting firm of B. MacKenzie, Inc., Greensboro, N.C., died in that city on November 12, 1930. He is survived by his widow, Ethel (Tucker) MacKenzie, whom he married in 1920, and by a son, Burt C. MacKenzie, and two daughters, Marie and Jeannette, all of Greensboro.

Mr. MacKenzie was born at Louisville, Ky., on March 28, 1869. After finishing his high-school course he served a four-year apprenticeship and was employed for some years by Joseph McWilliams' Steam Heating Company, of Louisville. For several years he was located in Philadelphia and Atlantic City, in similar work. He moved to Greensboro about the year 1900 and opened an office as a heating, ventilating, and engineering contractor. He designed and installed some of the largest plants in the South, including the central heating plant for about eighteen buildings on the campus of the North Carolina College for Women, at Greensboro, and plants for the A. & T. College, Guilford County Court House, and O'Henry Hotel, in Greensboro, State Hospital for the Insane, Goldsboro, N.C., Sir Walter Hotel, Raleigh, N.C., Forsyth County Court House, Winston-Salem, N.C., and the Cullowhee (N.C.) Normal and Industrial School and Salisbury (N.C.) High School.

Mr. MacKenzie became a member of the A.S.M.E. in 1923. He also belonged to the American Society of Heating and Ventilating Engineers and was a Shriner, an honorary life member of the Elks, and a member of several local commercial and social organizations.

WILHELM LUDWIG FERDINAND MARTENS (1847–1931)

Wilhelm Ludwig Ferdinand Martens, generally known as Ferdinand Martens, died on September 12, 1931, at Berkeley, Calif., after a prolonged illness.

Mr. Martens was born in Germany on October 30, 1847. He served an apprenticeship in machine shop work and studied engineering in Hamburg and Karlsruhe. After some experience in drafting and erecting machinery he emigrated to America in 1871, and found employment at the Williamsburgh Gas Company in New York and subsequently at the India Rubber Comb Company in College Point, N.Y. In 1874 he established himself as a surveyor in College Point. He was appointed superintendent of water works there the following year and about five years later became drainage commissioner. He designed a number of buildings in College Point, taught freehand and mechanical drawing at the Poppenhusen Institute from 1872 to 1882, was superintendent for a time of the College Point Gas Light Company, and served as a member of the board of trustees of the College Point Savings Bank.

In 1881 Mr. Martens returned to the employ of the India Rubber Company (later known as the American Hard Rubber Company) as superintendent of its India Rubber Department. He retired from active business in 1910 and made his home in College Point until 1918, when he went to Berkeley. He had been a member of the A.S.M.E. since 1885.

JACOB C. MATTERN (1866–1932)

Jacob C. Mattern, assistant superintendent in charge of equipment at the Elizabethport, N.J., plant of The Singer Manufacturing Company since 1920, died on February 7, 1932, of injuries suffered in an automobile accident.

Mr. Mattern was born at Pomeroy, Ohio, on January 21, 1866, the son of Conrad and Elizabeth Mattern. He worked as a machinist for a number of years after finishing his high-school education. In 1889 he also took up the study of law, receiving an LL.B. degree from the University of Cincinnati Law School in 1892. He practised law in Omaha, Neb., for four years and then went back to the machine-tool industry. He was connected with the Laidlaw-Dunn-Gordon Company, Cincinnati, as machinist and later foreman, until 1900. During the next seven years he held positions with several firms, the latter part of the time that of superintendent for the National Automatic Tool Company, Dayton, Ohio.

In 1907 Mr. Mattern went East, becoming superintendent for the Cummings Machine Company, New York, N.Y. While connected with this company he designed a large number of jigs and fixtures, among which were many of those used by Sloan and Chace in building tabulating machines for the Government. The company also did considerable work for the Singer company under Mr. Mattern's supervision, and he contributed to the design of other special machinery.

Mr. Mattern had been connected with the Singer company since 1911. In addition to serving as manager of the Oil Milling Department during 1911 and 1912, he spent a large amount of time on the design of jigs, fixtures, special machines, and labor-saving devices for other departments. Later he took charge of the company's tool room, supervising the building of many very intricate automatic machines. He went to Haverhill, Mass., for a time, planning the factory of the Hamel Shoe Machinery Company, purchasing and installing the machinery, and supervising the design of jigs and fixtures for manufacturing the product. During the World War he was superintendent of the factory erected by the Singer company to manufacture the recoil mechanism for the French 75-mm gun and had a large part in the development of the special machines required. He helped to organize the manufacturing and inspection forces and supervised the final assembly of the mechanism.

Mr. Mattern became a member of the A.S.M.E. in 1920 and at the time of his death was chairman of the Sectional Committee on Standardization of Specifications for Leather Belting. He was a member of the Plainfield Engineer's Club, and had served as recorder and was otherwise active in civic affairs in Roselle, N.J., where he made his home.

In 1896 Mr. Mattern married Margaret Chashatt. He is survived by her and by two sons, Charles and Oliver, and two daughters, Mrs. Kenneth Perkins and Mrs. H. C. Beadle.

HENRY E. McGOWAN (1872–1932)

Henry E. McGowan, for nearly thirty-five years associated with the Brooklyn Union Gas Company, was born in Brooklyn on October 3, 1872. His early education was secured in private schools, Holbrook's Military Academy, and Stevens Preparatory School. He was graduated from the Stevens Institute of Technology, Hoboken, N.J., as a mechanical engineer in 1894 and spent the next three years in the employ of Church & Co., soda manufacturers, in Trenton, Mich. After he had acquired a knowledge of the processes in the laboratory he was put in charge of an eight-hour shift as process chemist, being responsible for about 100 men engaged in the manufacture of soda by an ammonia process.

In 1898 Mr. McGowan became electrical engineer for the Brooklyn Union Gas Company, being retained particularly to investigate electrolytic damage to a subsurface plant, on which subject he contributed several articles to the technical press. His work also included following up prospects for gas-engine-driven electric installations. In 1903, in addition to these duties, he was elected general manager of the Flatbush Gas Company, a subsidiary of the Brooklyn Union Gas Company, the capacity and service of which were greatly expanded under Mr. McGowan's direction. In 1914 he was elected assistant secretary of the Brooklyn Union Gas Company, retaining also his position of general manager of the Flatbush Gas Company. He had been secretary of the former since 1918.

Mr. McGowan became a member of the A.S.M.E. in 1915. He also belonged to the American Gas Association, the American Institute of Electrical Engineers, the Empire State Gas and Electric Association, the Brooklyn Chamber of Commerce, the Merchants Association of New York, and the Crescent Athletic-Hamilton Club, Brooklyn. His death occurred on May 7, 1932. He is survived by his mother, Mrs. Henry D. McGowan.

HOWARD SMITH McILVAIN (1889–1932)

Howard Smith McIlvain, who died on March 25, 1932, after a brief illness, was born in New York, N.Y., on July 10, 1889, the son of David H. and Charlotte (Hart) McIlvain. He prepared for Stevens Institute of Technology at the Montclair (N.J.) High School and Stevens Preparatory School, and received an M.E. degree in 1911. Following his graduation he spent a year each with the Western Union Telegraph Company, for which he travelled as far as the Pacific Coast as inspector of materials and construction, and with the New York Fire Insurance Exchange, New York, as inspector of automatic sprinkler equipment.

For a short time in 1914 Mr. McIlvain worked on the design and cost estimates of sprinkler equipment for the Raisler Sprinkler Company, with headquarters in New York. The following year he was inspection and production engineer for Hildreth & Co., manufacturers of aeroplane parts. He then took the position of production engineer (later being promoted to assistant chief engineer) with the Agcy Canadian Car & Foundry Co., New York and Toronto, Canada, engaging in the development of production methods and costs and the design of shells and gages in connection with the manufacture of shrapnel and high explosive ammunition.

During part of 1917 he was equipment engineer at the Watertown plant of the Maxim Munitions Corporation, supervising the design, manufacture, and installation of production equipment for small arms ammunition manufacture. He resigned this position to enter service in the Ordnance Department. He was first stationed at Frankford Arsenal with the rank of Captain, organizing and directing the manufacture and inspection of small arms ammunition, and later was made head of the Small Arms Ammunition Branch, Engineering Division, Office of the Chief Ordnance Officer, A.E.F.

In 1919 Mr. McIlvain returned to private life as engineer for the Russell Sales Company, Newark, N.J., working on the redesign and manufacture of an automobile lock. Subsequently he did similar work for the Titan Automatic Tool Company. He then entered the employ of R. H. Macy & Co., New York, in the capacity of production engineer, developing methods and cost accounting, and supervising the operation of the service and manufacturing departments. In 1924 and 1925 he was research engineer for L. Bamberger & Co., Newark, N.J., and for one year engineer with the Ware Radio Company, New York.

In 1927 he joined the pioneer solid carbon dioxide company, The Dry Ice Corporation of America, and devoted the last five years of his life to research and engineering work important in developing the manufacture and use of dry ice. He was in charge of the construction of the first large plant built exclusively for the production of dry ice, located in Elizabeth, N.J. His headquarters were in New York but he traveled considerably, spending many weeks in Europe. At the time of his death he was in charge of all research and development activities of the corporation.

Mr. McIlvain became a member of the A.S.M.E. in 1923. He is survived by his widow, Amy (Chapman) McIlvain, whom he married in 1912, and by four children, Howard Hyland, David Hart, Amy, and Marilyn McIlvain.

CHARLES JARRETT McPHERSON (1886–1932)

Charles Jarrett McPherson, sales manager, Automotive Car Division, The J. G. Brill Company, died suddenly of heart failure on June 20, 1932, while in Chicago on business.

Mr. McPherson was born on July 5, 1886, in Hopkinsville, Ky., the son of J. E. and Sarah (Glass) McPherson. He prepared for college in the schools of Hopkinsville and secured a B.S. degree in mechanical engineering from the University of Kentucky in 1906. An M.E. degree was conferred upon him by that institution three years later.

After serving a two-year apprenticeship with the Baldwin Locomotive Works Mr. McPherson spent several years as draftsman with the American Car & Foundry Co., and the National Steel Car Company, Hamilton, Ontario, before joining the Brill engineering staff in 1913, as assistant chief draftsman. During the early days of the World War he was sent abroad to represent the company's interests, and spent some time in Russia. Except for a short time in 1920 as works manager for The Hopkins Manufacturing Company, Hanover, Pa., Mr. McPherson remained with the Brill company the remainder of his life. After serving successively as assistant superintendent he was transferred to the Sales Department, in charge of sales of the Automotive Car Division, handling rail motor cars for steam railroads.

Mr. McPherson became an associate member of the A.S.M.E. in 1914 and a member in 1925. He also belonged to the American Railway Car Institute, Society of Automotive Engineers, Army Ordnance Association, Railroad Club of New York, and the Art and Penn Athletic Clubs of Philadelphia. He is survived by his widow, Mrs. Anne (Melhorn) McPherson, whom he married in 1920.

MERLE ARTHUR MESTA (1895–1932)

Merle Arthur Mesta, of Pittsburgh, Pa., died at Atlantic City, N.J., on August 28, 1932. He was born at Leechburgh, Pa., on July 6, 1895, the son of Charles J. and Hattie (Rambler) Mesta. He prepared for college at the Kiskiminitas Springs School and took a special machine design course at the Carnegie Institute of Technology. He also attended the Bordentown Military Institute for a time.

Mr. Mesta spent all of his vacations in the engineering department of the Mesta Machine Company during his school years and was subsequently connected with that and other departments, studying shop methods and practice. He also was associated for short periods with the National Tube Company, Ellwood City, Pa., working on tools, dies, and repairs on mills, and the Production Engineering Department of the Worthington Pump & Machinery Corp., Harrison, N.J.

During the World War Mr. Mesta was with the Bureau of Aircraft Production of the War Department, inspecting and directing production at different plants. After the War he was appointed assistant production engineer for the Mesta Machine Company and subsequently, until within about a year of his death, had worked on sales.

Mr. Mesta became a junior member of the A.S.M.E. in 1920. He was active in the Masonic fraternity and a member of several Pittsburgh clubs, and was also vice-president of the Bank of America Trust Company, Pittsburgh.

HENRY JOSEPH MISTELE (1865–1932)

Henry Joseph Mistele, chief engineer of the Power Department, The Falk Corporation, Milwaukee, Wis., died in that city on May 4, 1932. Surviving him are his widow, formerly Miss Nell Richards, whom he married in 1892, and four children, two sons and two daughters, all of Milwaukee.

Mr. Mistele was born at Sullivan, Wis., on January 8, 1865, and attended the Milwaukee schools. He took courses in mechanical engineering through the International Correspondence Schools and prior to his connection with The Falk Corporation in 1909 worked for the U. S. Gypsum Company as station engineer in Milwaukee and plant superintendent at Granville, Mich., and for the Maryland Casualty Company as boiler inspector. He also engaged for two years as a private consultant in steam plant efficiency.

Mr. Mistele had made a study of the causes of boiler explosions and had been called upon for civil testimony on the subject. He had also made indicator studies of Corliss engine efficiency. These studies and power plant costs were among the subjects about which he had written for the technical press.

The A.S.M.E. elected Mr. Mistele to membership in 1916 and he was also a member of the National Association of Power Engineers, which he served as national trustee from 1912 to 1930; he was a local past-president of the association.

ROBERT CHARLES MONTEAGLE (1859–1932)

Robert Charles Monteagle, secretary and treasurer of the Transportation Guarantee Company, San Francisco, Calif., died at Burlingame, Calif., on January 19, 1932. He is survived by his widow, Mariett E. (Berg) Monteagle, of Carmel by the Sea, Calif., whom he married in 1898, and by two daughters and a son, Mrs. R. Eugene

McClung, Portland, Ore., Mrs. W. F. Stearns, Jr., Carmel, Calif., and F. James Monteagle, Palo Alto, Calif.

Mr. Monteagle was born in Glasgow, Scotland, on October 14, 1859, the son of James and Jane (Findlay) Monteagle. He went from the Glasgow high school to Andersonian University, of that city. After serving an apprenticeship in the pattern shop and machine shop of William King & Co., he worked for about two years in the drafting room of the Fairfield Shipbuilding Works belonging to James and George Thomson, Glasgow.

His shop experience in the United States was secured at the Fulton Iron Works, in San Francisco, from 1882 to 1884, and he was chief draftsman for the company during the next three years. He held the position of chief draftsman successively with the Honolulu Iron Works, Honolulu, from 1889 to 1892; the Charles Hillman Ship & Engine Building Co., Philadelphia, Pa., from 1892 to 1895; and the Burke Dry Dock, New York, N.Y., from 1895 to 1898. After two years as chief engineer at the Crescent Shipyard, New Jersey, he was engaged for four years as general superintendent for the Gas Engine & Power Co. and Charles L. Seabury & Co., Morris Heights, New York, N.Y., in building launches, yachts, and other vessels and engines and boilers of all kinds, including watertube boilers.

In 1905 Mr. Monteagle went to East Boston, where he remained for ten years, the first five as chief engineer of the Atlantic Works and the remainder of the time assistant manager and engineer of the Lockwood Manufacturing Company. Since 1915 he had been located on the Pacific Coast. He was assistant to the president of the Seattle Construction & Dry Dock Co., Seattle, Wash., for two years, and then helped to found The Pacific Construction & Engineering Co., in Seattle, of which he was president and general manager. This company furnished shipyards on the Coast with heavy forgings which previously had to come from the East and cooperated in other ways to speed up construction during the War. Mr. Monteagle was a member of the American Defense Society of the U.S.A., the Aerial League of America, and other local and patriotic organizations.

Mr. Monteagle held patents on packing and piston rings and a valve and had developed a watertube boiler. He was interested in amateur photography and sketching outdoor scenes. He belonged to the Episcopal Church, the Rainier Club in Seattle, and the Masonic fraternity. The technical organizations to which he belonged were the American Society of Naval Engineers, Society of Naval Architects and Marine Engineers, and the A.S.M.E., which he joined in 1904.

ALBERT BRIDGES MOORE (1868–1932)

Albert Bridges Moore, whose death occurred in Roselle, N.J., on May 19, 1932, was born at Elizabeth, N.J., on April 4, 1868, the son of Miller F. and Helen S. (Brown) Moore. He attended the Stevens Preparatory School and was graduated from the Stevens Institute of Technology in 1890 with an M.E. degree. Following his graduation he entered the drawing room of the Samuel L. Moore & Sons Co., Elizabeth. He was made foreman iron worker in 1892 and in 1893 vice-president and assistant superintendent of the machine shop and foundries.

In 1899 Mr. Moore went to Harrison, N.J., to work as superintendent of the Marine Engine & Machine Co., where during the next three years he had charge of the design and erection of a new plant and redesigned the Alco-Vapor launch motors built by the company. He did some special engineering work with the Tirrell Gas Machine Lighting Company, of Newark, N.J., in 1903 and in that year he was also issued a patent for his internally packed plunger pump. He was also the inventor of a brush holder for electric motors and generators.

During 1904 and 1905 Mr. Moore was mechanical engineer with the Crane Company in Chicago and Bridgeport, Conn., and subsequently was connected with the Griffen Wheel Company, Chicago, as chief engineer, and the Shedd Electric Company, Roselle, as vice-president and treasurer. He retired in 1926.

Mr. Moore became a member of the A.S.M.E. in 1903 and also belonged to the Western Society of Engineers and the Roselle Golf Club. Surviving him are his widow, Ethel G. (Field) Moore, whom he married in 1896, and two daughters, Margaret and Helen.

WENSEL MORAVA (1853–1932)

Wensel Morava, consulting engineer of Chicago, Ill., who died on October 13, 1932, had been a member of the A.S.M.E. since 1886.

Mr. Morava was born in Bohemia on September 12, 1853. He entered the University of Illinois, at Champaign, in 1874 and received the degree of B.S. from the College of Engineering four years later. He ran a stationary engine for a few months following his graduation and then worked as mechanical draftsman for the U.S. Rolling Stock Co. at Urbana, Ohio, until February, 1879. He then took a position with the Excelsior Iron Works, Chicago, as chief draftsman. He designed special machinery for rolling mills and the brick industry, hoisting and stationary engines, and mining machinery. In 1880 he had charge of the machinery department of an interstate machinery exposition in Chicago. About two years later he invented and patented an improved dynamo and electric arc lamp. A plant built to manufacture these lamps was built and sold to the Excelsior Iron Works.

In the summer of 1885, after making the plans for a large stamp mill to be erected in the Black Hills, Mr. Morava secured a position as assistant to the contractor. Upon the completion of the mill he returned to Chicago and worked for several years for the Union Foundry & Pullman Car Wheel Works, developing a high-speed engine and superintending the erection of coal gas machinery. Subsequently he was employed by the Bouton Foundry Company and Link Belt Machinery Company, both of Chicago, before going into business for himself.

In 1900 he founded the Morava Construction Company, a steel fabricating and erecting concern, of which he was president until it was purchased by the McClintic-Marshall Corporation in 1924. Among the structures which were erected by this concern are the Park Row (12th St.) Railroad Station, the Boston Store, the Marshall Field Annex, all in Chicago, the Armory of the University of Illinois at Champaign, and the Mt. Wilson Observatory at Pasadena, Calif.

After his retirement from active business in 1924 he devoted himself to the development of a mechanical cotton-picking machine which he had very nearly perfected at the time of his death. He was greatly interested in travel and has written several short monographs on his travels. He was also greatly interested in assisting young engineers attain a higher education; he provided funds at the University of Illinois and at the Technical High School in Prague, Czechoslovakia, for this purpose.

RAYMOND NEWCOMB (1897–1932)

Raymond Newcomb, who, together with Edward N. Cunningham, also an A.S.M.E. member, was fatally injured in an automobile accident near Selma, Ala., on February 15, 1932, was born at Boston, Mass., on July 12, 1897. His parents were Thomas and Clara S. Newcomb. From the Boston English High School he went to Worcester Polytechnic Institute in 1914. He transferred to the Massachusetts Institute of Technology in his junior year, specializing in engineering administration.

For three months in 1917 and from June 1918, to April, 1919, he served in the United States Naval Reserve. After a time as fireman and oiler on different ships he was transferred to the U.S.N. Steam Engineering School with an ensign's commission, to train for junior engineer officer for transport service. He was graduated from M.I.T. with the class of 1919, receiving a B.S. degree in mechanical engineering.

Since leaving the Navy in 1919 Mr. Newcomb had been associated with the Kewanee Boiler Company, Inc., and its affiliated company, the Fitzgibbons Boiler Company, Inc. He spent the first six months in the engineering department at the factory at Kewanee, Ill., conducting a series of efficiency tests on tubular boilers equipped with the Hawley down-draft furnace. He was then transferred as service engineer to the New York office, where for two years his work consisted of putting new installations into operation. He had been sales engineer for the companies since 1921, representing them in Brooklyn, Long Island, and Boston, prior to his appointment in 1928 as vice-president in charge of branch sales. He assisted in the design, installation, and operation of the boilers.

Mr. Newcomb was elected to junior membership in the A.S.M.E. in 1920, promoted to the grade of associate-member in 1924, and made a member in full in 1931. He was a member of the Boiler Code Subcommittee on Heating Boilers and had served on the boiler rating committee of the American Society of Heating and Ventilating Engineers and technical committees of the Steel Heating Boiler Institute.

Mr. Newcomb is survived by his widow, Hope (Nichols) Newcomb, whom he married in 1922, and by two children, Joan and Nancy, all of White Plains, N.Y.

JOSEPH NEWMAN, JR. (1896–1932)

Joseph Newman, Jr., field supervisor of the Apparatus Repair Shops of the Air Reduction Sales Company, Philadelphia, Pa., died in that city on February 28, 1932.

Mr. Newman was born in Philadelphia on March 17, 1896, the son of Joseph and Mary (Carbaugh) Newman. He was graduated from the West Philadelphia High School and studied for a time at Temple University. He was first employed as machinist and assistant to the foreman at the Lanston Monotype Company, Philadelphia, in 1914 and in the winter of 1914–1915 assisted in Board of Health analyses at the Bryn Mawr laboratory.

During the World War Mr. Newman worked for the Remington Arms Corporation as assistant to the foreman of the manufacturing department, and at the Eddystone Ammunition Corporation, where he served successively as foreman of the repair department, foreman of the scraping department, and assistant production manager. He was assistant to the shop superintendent at the Philadelphia Navy Yard from 1917 to 1920, designing dies for structural work, making layouts and supervising construction of shops, installing toolroom operating systems, etc. During the next two years he was engaged in similar work at the League Island Navy Yard, near Philadelphia.

In 1922 Mr. Newman established the Newman Engineering Company in Philadelphia and for two years engaged in miscellaneous machine and experimental work, oxyacetylene welding and cutting, and general work for mills not large enough to maintain their own jobbing shops.

Mr. Newman had been with the Air Reduction Sales Company since 1924, working first as assistant to the manager of the Apparatus Manufacturing Plant and later being appointed to the position he held at the time of his death.

Mr. Newman joined the A.S.M.E. as a junior member in 1921 and in 1931 was promoted to the grade of member. He is survived by his widow, Marie E. (Kristel) Newman, whom he married in 1920, and by two children, Joseph and Edna M. Newman.

PAUL OEHMIG, JR. (1898–1932)

Paul Oehmig, Jr., son of the founder of Paul Oehmig & Co., Chicago, Ill., manufacturers of mixing and grinding machinery, died on September 15, 1932. He was born on March 18, 1898, in Chicago, and became associated with his father in the business immediately following his graduation from Lewis Institute in 1915.

At that time the company was making a special line of food mixing and grinding machinery, such as mustard grinders, horseradish graters, peanut butter mills, etc. During his first four years in the business Mr. Oehmig worked in the positions of machinist, shop foreman, draftsman, designer, and erection engineer, installing the company's equipment in different cities.

In 1919 he and another son formed a partnership to take over the business, Paul Oehmig becoming shop superintendent as well as erection engineer. The company purchased the paint machinery division of the Kaestner & Hecht Co., of Chicago, and Mr. Oehmig took on the responsibility of developing and producing new lines of equipment. He became sole owner in 1927 and continued to augment the business, acquiring the paint machinery division of the American Oven & Machine Co. of Chicago, and adding other special lines from time to time.

Mr. Oehmig became a junior member of the A.S.M.E. in 1923 and was transferred to the grade of a member in 1930.

HARVEY ROBINSON OGDEN (1899–1932)

Harvey Robinson Ogden, chief pilot for the Curtiss Aeroplane & Motor Co., Inc., stationed at Buffalo, N.Y., was killed on May 20, 1932, when the parachute with which he attempted to land from a disabled plane became entangled in the airplane structure and was rendered useless. Mr. Ogden attempted to jump from an Army observation plane which he was testing, when it lost a wing at an altitude of approximately 15,000 ft.

Mr. Ogden was born on January 13, 1899, in Minneapolis, Minn., the son of Ida S. and James K. Ogden. After completing his preparation for college at the Shattuck School in Faribault, Minn., he spent two years at the University of Minnesota and a year and a half at the West Point Military Academy. He entered the employ of the Curtiss company late in 1921 as draftsman. About a year later he went to Seattle, Wash., to do drafting for the American Can Company, and later was connected in a similar capacity with the Boeing Airplane Company there. For several years, beginning in 1925, he served in the United States Air Corps, successively as flying cadet, second lieutenant, flying instructor, and aide-de-camp to Brigadier-General Frank P. Lahm. He returned to the Curtiss company in 1929 and was regarded as an exceptionally able pilot.

Mr. Ogden became an associate member of the A.S.M.E. early in 1932. He is survived by his widow, Ruth (Stroud) Ogden, and by a son, Harvey Robinson Ogden, Jr.

ENOCH OHNSTRAND (1862–1932)

Enoch Ohnstrand, consulting engineer at the Tonawanda, N.Y., plant of Remington Rand, Inc., N.Y., died of heart disease at his home in Kenmore, N.Y., on September 20, 1932.

Mr. Ohnstrand was born at Norrköping, Sweden, on January 9, 1862. At the age of nineteen he entered the employ of the National Tube Company, and worked in the sheet rolling and tube mills for two years. In 1888, after several years as a carpenter and builder, he became connected with the Art Metal Construction Company, Jamestown, N.Y. During nearly twenty years spent with this company he rose to the position of general superintendent. He helped to develop ornamental works for banks, libraries, and other public buildings; designed and constructed steel furniture approved by the naval constructor in 1909 for use on United States battleships; and invented a process of manufacturing the cold-drawn steel molding used in steel cars and fireproof buildings by various manufacturers.

Mr. Ohnstrand held a number of patents on steel furniture and under his supervision the graining of steel furniture to imitate different kinds of wood was accomplished. In 1906 he organized the Ohnstrand Metal Furniture Company, Falconer, N.Y., which later was reorganized as the Jamestown Metal Furniture Company and finally became Plant No. 2 of the Art Metal Construction Company.

In 1909 Mr. Ohnstrand served as expert engineer for the Art Metal Company and during the next three years was connected with the O. M. Edwards Company of Syracuse, N.Y., in the capacity of general superintendent, developing a metal furniture and office equipment business. He next organized the U.S. Steel Furniture Co., Syracuse, for the manufacture of steel desks and tables according to his designs and patents. In 1914 he became chief engineer of the U.S. Metal Products Co., New York, N.Y., and in 1915 chief engineer of the Library Bureau Division of Remington Rand, Inc., Ilion, N.Y., the position he held until shortly before his death.

Mr. Ohnstrand had been a member of the A.S.M.E. since 1918.

PETER L. O'NEILL (1874–1932)

Peter L. O'Neill, whose death occurred on February 26, 1932, was born in Jersey City, N.J., on May 17, 1874, the son of John and Catherine (Lawless) O'Neill. After completing his high-school course he became head of the Unisol Manufacturing Company, Jersey City, whose product is a preparation for the correction of undesirable boiler feedwater conditions.

Mr. O'Neill became an associate of the A.S.M.E. in 1923. He is survived by his widow, Catherine (Langton) O'Neill, and by three children, John I. and Catherine O'Neill and Mrs. Frederick Yeck, all of Jersey City.

WILLIAM FREDERICK OTTO (1864–1932)

William Frederick Otto, vice-president of the Wilmot Engineering Company, Hazleton and White Haven, Pa., and manager and treasurer of the Weatherly Foundry & Manufacturing Co., Weatherly, Pa., died on January 25, 1932.

Mr. Otto was born in Germany on March 13, 1864. After being graduated in 1879 from the Hoehre Bürger Schule, Hersfeld, Germany, he came to the United States and served a four-year apprenticeship at the Jeanesville (Pa.) Iron Works. Additional experience was secured during the next two years in various sections of New York and Pennsylvania. In 1886 he became erecting engineer of refrigerating machinery for the Consolidated Ice Machinery Company of New York, where he remained for three years. He received his full citizenship papers at Wilkes-Barre, Pa., in 1888.

From 1889 to 1895 he was engineer for A. Pardee & Co., Hazleton, Pa., and during the next seven years was connected with the Lehigh Valley Coal Company, of Wilkes-Barre, as superintendent of outside operations and master mechanic. His duties included redesigning and reconstructing a large pumping plant at Hazleton, the installation of a power plant at the Maltby Colliery, and the design and installation of air haulage systems at two other collieries.

Mr. Otto accepted a position in 1902 with the Eynon-Evans Manufacturing Company, in Philadelphia, designing and selling steam and water specialties for power and pumping plants in anthracite mines. He was connected with the company until 1920. As vice-president of the Wilmot Engineering Company from 1907 until his death, he assisted in the building up of that organization, designing and perfecting coal machinery. He had been associated with the Weatherly Foundry & Manufacturing Co. since 1921.

Mr. Otto became a member of the A.S.M.E. in 1918. He also belonged to the American Society of Military Engineers, the Masonic fraternity, and a number of clubs, including the Engineers and Manufacturers of Philadelphia, Franklyn and Westmoreland of Wilkes-Barre, South Wilkes-Barre Rod and Gun Club, and Izaak Walton League of America. He was affiliated with the Lutheran Church, and served as a school director for the City of Hazleton for eleven years.

Surviving him are his widow, Anna E. Otto, of Hazleton, a daughter, Mrs. Elmo D. French, Miami, Fla., and two sons, George W. Otto, Port Carbon, Pa., and H. H. Otto, Scranton, Pa.

GEORGE ALLEN PAGE (1890–1931)

George Allen Page, consulting engineer and patent attorney of Freehold, N.J., died on December 27, 1931, after a year's illness.

Mr. Page was born in New York, N.Y., the son of Allen H. and Emma (Hauselman) Page, on January 15, 1890. The early training which he received in his father's business, supplemented by extension courses in engineering subjects, enabled him to secure the position of general draftsman and foreman of machine-shop work and manufacturing in the factory of the Tube Bending Machine Company, of Harrison, N.J. in 1908. As superintendent's assistant at the Thos. A. Mack Co., Marion, N.J., in 1909, he had some experience in estimating and cost accounting. During the early part of 1910 he was employed by the Emil Grossman Company, New York, N.Y., at its factory in Newark. Here he had his first contact with patent work, as assistant to the firm's patent attorney, an experience which led him before the close of the year to establish himself in private practice at Hartford, Conn., in partnership with a patent attorney. During the next two years, in addition to the making of patent drawings and serving inventors as engineering counsel on general machine design, he was employed by the J. H. Sessions & Sons, Bristol, Conn., in design and installation work involved in a factory extension; by the Page Brass Company, Elmwood, Conn., in fitting up a brass foundry and manufacturing plant; by the Hart Manufacturing Company, Hartford, to develop experience in the manufacture of electrical goods and in practical electrical laboratory work; and by the Hartford Auto Parts Company as assistant foreman.

From 1912 to 1914 Mr. Page was associated with the Merrow Machine Company, Hartford, as patent attorney and general engineering consultant. He was next employed in a similar capacity by the Bausch & Lomb Optical Co., Rochester, N.Y., for which he established a patent bureau, reorganized the library, and took over certain special engineering work, leading to his appointment as auxiliary engineer of the mechanical division of the company. He resigned this position in 1919 and worked for a short time on machinery improvements for the United States Machinery Company, manufacturers of pressing machines for tailors and factories, with a factory in Syracuse, N.Y., and offices in New York.

In the fall of 1920 Mr. Page severed this connection and went to Detroit, Mich., obtaining a position there as patent attorney with the firm of Barthel, Flanders, and Barthel. Ill health forced him to leave Detroit late in 1921 and he went to Portland, Ore., where he took up private practice again. He helped to develop a fruit grader and did some other experimental work, in addition to handling patents. He returned to Rochester in 1923 and during a year's stay there spent considerable time on the development of an optical invention of his own, for which a patent application had been filed at the time of his death.

Late in 1924 Mr. Page left Rochester and went to Freehold, N.J., where in addition to consulting work, he pursued the development of his own inventions, although ill health restricted his activities. He suffered a complete breakdown early in 1931, while located temporarily in Philadelphia, and was unable to do any work from that time.

Among his inventions which he had not been able to bring to perfection were a special camera film and shutter combination for avoiding double exposures, optical instruments for motorists, and machinery for the tanning industry. He had assisted in the design of a great variety of machinery, including automatic tube polishing and bending machinery, automatic cam-cutting machines, and semi-automatic machines for drilling field glass parts and for finishing special surfaces, as well as automobile wind shields and attachments, annealing furnaces, overseam sewing machine stitching mechanism, and many other inventions.

Mr. Page became an associate member of the A.S.M.E. in 1919. He is survived by his widow, Nanon (Ashcraft) Page, whom he married in 1912, and a daughter, Virginia Nanon Page.

ALFRED BASSETT PATTERSON (1887–1931)

Alfred Bassett Patterson, president and general manager of the Patterson Steel Company, Tulsa, Okla., died on December 12, 1931, at a hospital in Kirksville, Mo., where he was being treated for paralysis caused by the third stroke which he had suffered within two months. He was widely known in Tulsa for his support of a weekday Bible school during the last four years. He made generous donations to the work and interested others in it, building up a school with an enrolment of some 10,000 children in 1931. He was an elder in the First Presbyterian Church, a charter member of the Rotary Club, and active in the work of the Chamber of Commerce. He also was a member and past-president of the Tulsa Traffic Association and of the Manufacturers Bureau, and belonged to the Masonic fraternity. He was associated with the Tulsa Chapter of the American Association of Engineers, past-president of the Mid-West Structural Society, and a member of the American Institute of Steel Construction. He became a member of the A.S.M.E. in 1926.

Mr. Patterson was born at Winslow, Me., on October 1, 1887, the son of Mr. and Mrs. George W. Patterson. He was graduated from the Winslow High School in 1905 and spent the next four years at the University of Maine, securing his B.S. degree in mechanical engineering in 1909. After a few months in machine shops and drafting rooms in New York he went to Houston, Texas, where he worked for a year as draftsman for building internal oil combustion engines for Layne and Bowler. He was next engaged by Halloway and Cathey, patent attorneys, to make Patent Office drawings and to contract drafting in Houston. In January, 1911, he went to Sour Lake, Texas, to do drafting work for the Texas Company, and worked in the Production Department of the company until August of that year. During the remainder of 1911 he was engaged in contracting miscellaneous drawings.

For four years, 1912–1915, Mr. Patterson was chief engineer for the Oklahoma Iron Works, Tulsa, in charge of shops, design of new machinery, and general contracting. He then went into contracting in oil fields, constructing oil refineries, plants, pipe lines, and other heavy work, including gasoline plants for the Standard Oil of Indiana at Drumright, Okla., and work for different oil companies at Ranger and Breckenridge, Texas, and Minden and Homer, La., as well as many other places. The Patterson Steel Company was organized by him and his brother, Nathan R. Patterson, in August, 1920. It handled heavy steel construction for buildings, bridges, refineries, oil-field work, and equipment. Mr. Patterson was responsible for the design of many oil field tools and a great deal of oil-refinery equipment. Shortly before his death he had applied for patents covering steel frame construction for residences and for sectional steel buildings.

Mr. Patterson is survived by his parents, two brothers and a sister, his widow, Myrtle Mae (Reynolds) Patterson, and two children by a former marriage, George W. and Elizabeth May Patterson.

THORLEIF BJERKE PAULSON (1886–1932)

Thorleif Bjerke Paulson, whose death occurred in San Francisco, Calif., on October 12, 1932, was born at Grimstad, Norway, on June 14, 1886. His early education was secured from private tutors and schools in Vestheim. He studied mechanical engineering at the Christiania Technical School from 1901 to 1905. After a year's apprenticeship in the shop of Akers Mekaniske Verksted, Christiania, Mr. Paulson came to the United States. He worked in the East, testing pumps for Henry R. Worthington, Harrison, N.J., and as mechanical draftsman for the Standard Steel Car Company, Butler, Pa., for a short time. The latter company sent him to Hammond, Ind., in 1907 and later in the year he went to San Francisco. For four years he was mechanical draftsman, designing hydraulic machinery and boilers, for the Risdon Iron Works, and then spent about six months on centrifugal pump design for the Yuba Construction Company, Marysville, Calif.

In July, 1910, Mr. Paulson began an association with Chas. C. Moore & Co. of San Francisco which continued until his death. He was chief draftsman for the first fifteen years and since then had been designing engineer, in charge of the design of power stations. Mr. Paulson was responsible for the design of numerous steam electric power and pumping stations on the Pacific Coast and tested a number of them. He directed all test work of the company after 1917. Some of his outstanding work included the design and testing of power plants for the Long Bell Lumber Company, Longview, Wash., Power River Company, in British Columbia, Consolidated Mining Company, Trail, B.C., Calumet & Arizona Copper Mining Co., Ajo, Ariz., and United States Potash Company, Carlsbad, N.M.

Mr. Paulson is survived by his widow, Edith (Erickson) Paulson, whom he married in 1912, and by two daughters, Edith Lillian and Betty Constance Paulson. He became a member of the A.S.M.E. in 1925, and was a past-president of the Norwegian Club of San Francisco, and a member of the Masonic fraternity.

HENRY PILLING (1867–1931)

Henry Pilling, director of Galloways Limited, Manchester, England, died on February 20, 1931, at the age of sixty-four. He was born at Bolton, England, on July 13, 1867, and after completing his early education at the Manchester Grammar School, served an apprenticeship in the erecting shop of John and Edward Wood, of Bolton, makers of slow-speed steam engines. From 1888 to 1895 he worked as draftsman for engineers, boilermakers, and millwrights in Bolton, being with John Musgrave & Sons Limited for five years and Hick, Hargreaves & Co., Limited, for two years. He then became head of the Consulting Engineering and Engine Insurance Departments of the National Boiler & General Insurance Co., Ltd., Manchester, with which he remained until 1904. For the next five years he was technical director of Yates & Thom, Ltd., Blackburn, manufacturers of slow-speed steam engines and boilers. He was appointed general manager of Galloways Limited in 1909 and later elected director of the company. This company, which absorbed the business of John Musgrave & Sons Limited and owns the drawings of John and Edward Wood, with which firms Mr. Pilling had secured his earliest experience, deals in steam and gas engines and repairs, valves and fittings, castings, etc.

Mr. Pilling had an extensive experience in all phases of steam-engine development, and particularly with engines for the tinplate industry, in reversing engines for heavy steel rolling, in blast-furnace engines, both steam and gas, and the like. He possessed a creative and facile mind, and among his many inventions are a single-lever control gear by which the operations of reversing, adjusting the cut-off, and controlling the handling valves of rolling-mill engines were all performed by means of one lever, while at the same time permitting full steam for the full stroke at the beginning of rolling operations. He was also the inventor of a device for reversing winding engines by means of which the stroke of the valves remains constant throughout. He did pioneer work in developing the manufacture of the uniflow steam engine, and invented a hydraulically operated valve gear, which has been successful. He introduced the construction of large gas engines of the continental type and effected various improvements, including a piston with internal diagonal bracing ribs.

Mr. Pilling had presented papers before various technical societies, including the Manchester Association of Engineers, of which he was a past-president. For many years he was a member of the executive committee of the Manchester and District Engineering Trades Employers' Association. He became a member of the A.S.M.E. in 1923, and also belonged to the Institution of Mechanical Engineers and the Order of the British Empire.

WILLIAM DUTTON POMEROY (1874–1932)

William Dutton Pomeroy, general manager, vice-president, and director of Goulds Pumps, Inc., Seneca Falls, N.Y., was born on July 30, 1874, in Utica, N.Y. His parents were George D. and Julia B. (Burritt) Pomeroy. After completing his preparatory education at the Utica Free Academy, he entered Cornell University with the class of 1895 and remained there five years, earning both an M.E. and an E.E. degree. He was elected a member of Sigma Phi fraternity.

Mr. Pomeroy's first position following his graduation was that of draftsman for The Bossert Electrical Contracting Company, in Utica. In January, 1897, he was made shop foreman, and remained with the company in that capacity for another six months. He then worked for a few months as foreman of the wiring gang of the Utica Belt Line Street Railroad. In October he went to Akron, Ohio, where he was connected until 1901 with the Akron Electric Manufacturing Company, first as draftsman and later engaged in construction work and electric mine haulage; during the last two years he was superintendent of the company.

From 1901 to 1907 Mr. Pomeroy was employed by the Bullock Electric Manufacturing Company, of Cincinnati, a subsidiary of the Allis-Chalmers Manufacturing Company. He was advanced from the position of chief draftsman to that of general superintendent, from 1903 to 1907 being in entire charge of the manufacturing department and general plant layout, and supervising the design of plant extensions and special machinery. During this time the number of employees was nearly tripled, increasing from 650 to about 1800.

Mr. Pomeroy went to Seneca Falls as works manager of The Goulds Manufacturing Company, predecessor of Goulds Pumps, Inc., in 1907. He was made general manager in 1915 and for a number of years prior to his death had also served as vice-president and director. Curtailments in the shops necessitated by business conditions and their effects upon employees weighed heavily upon his mind. Depressed by these conditions and suffering from ill health for more than a year, he committed suicide on January 6, 1932.

Civic affairs also claimed much of Mr. Pomeroy's time and interest. He had served as chairman of the Seneca Falls Water Commission and of the Building Committee of Mynderse Academy, and had been a member of the Board of Education of Seneca Falls until shortly before his death. He was director of the Military Census in Seneca County and chairman of the Seneca County War Chest. He served on the executive committee of the Red Cross during the World War. In 1919 he directed the building of the Gould Hotel and for a number of years had been a director and vice-president of the Seneca County Trust Company, resigning in the summer of 1931. In these and many other community activities he gave generously of his time and ability. He belonged to the Masonic fraternity, Elks, and Trinity Church. He was one of the early members of the Seneca Falls Rotary Club, and fostered the formation of the Seneca Falls Country Club. He also belonged to the Owasco Country Club, Auburn, N.Y.

Mr. Pomeroy was likewise an active member of the engineering fraternity. He became a member of the A.S.M.E. in 1908. At the 1930 Annual Meeting of the Society the Melville Award was conferred jointly upon him and Herman Diederichs, of Cornell University, for their paper on "The Occurrence and Elimination of Surge or Oscillating Pressures in Discharge Lines from Reciprocating Pumps." Mr. Pomeroy also belonged to the American Institute of Electrical Engineers, the American Society for Testing Materials, the Institute of Metals, and a number of technical clubs, and had been elected to the honorary engineering fraternity of Sigma Xi about a year before his death.

Automobiling and motor-boating absorbed a good deal of Mr. Pomeroy's leisure. He operated one of the first automobiles in Seneca Falls and launched seven speed-boats on Cayuga Lake.

Surviving Mr. Pomeroy are his widow, Fanny C. (McIntosh) Pomeroy, whom he married in 1901, a daughter Katherine, of New York, N.Y., and a son, William D. Pomeroy, Jr., of Baltimore, Md.

CHARLES TALBOT PORTER (1885–1932)

Charles Talbot Porter died on January 22, 1932, following a very short illness. He is survived by his widow, Anna M. (Newton) Porter, formerly of Norwich, N.Y., whom he married in 1913, and by four sons, Charles Talbot, Howard Newton, John, and Stephen Winthrop Porter.

Mr. Porter was born in Montclair, N.J., on November 13, 1885, the son of John and Elizabeth Keeler (Baker) Porter. He was the grandson and namesake of Charles Talbot Porter, pioneer in the development of the high-speed steam engine and a recipient of the John Fritz Medal. Mr. Porter was graduated from the Sheffield Scientific School of Yale University in 1907 with a Ph.B. degree in mechanical engineering, and received his M.E. degree there in 1913.

During Mr. Porter's early career, from 1907 to 1910, he was in charge of the department of mechanical engineering at the Imperial Polytechnic College at Shanghai, China. Upon his return he did a year of graduate work at Columbia University. In 1911 he took a position as instructor in mechanical engineering at Yale, where he remained until 1918, in charge of work in applied mechanics during the last three years.

From 1918 to 1920 Mr. Porter was an aeronautical engineer at the Naval Aircraft Factory, Philadelphia, Pa. He later was one of the incorporators of the Huff Daland Aircraft Company in Ogdensburg, N.Y., holding the offices of treasurer, vice-president, and chief engineer. This company developed many experimental airplane types for the United States Government.

During 1926 the Huff Daland Company was absorbed by the Keystone Aircraft Corporation, Briston, Pa., and Mr. Porter became vice-president and chief engineer of the Keystone Company, in which capacity he served for more than five years. During his association with the Huff Daland Company and the Keystone Company Mr. Porter became probably the foremost authority in the design and construction of bombardment airplanes. The outstanding achievement of his professional life was the development of the long series of bombers, beginning with the XLB-1 and terminating with the B-4A and B-6A machines, in use at the time of his death as the standard heavy bombers of the Army Air Corps.

Mr. Porter served as a member of the Aeronautical Standards Committee of the Aircraft Division of the Society of Automotive Engineers. He became a junior member of the A.S.M.E. in 1913 and a member six years later. He also belonged to the Society for the Promotion of Engineering Education, the honorary society of Sigma Xi, Engineers Club of Dayton, and Century Club of Ogdensburg, N.Y.

WILLIAM PRELLWITZ (1869–1932)

William Prellwitz, chief engineer of the Ingersoll-Rand Company, died on April 12, 1932, at Easton, Pa., following an operation. He was 63 years old, having been born in New York on February 7,

1869. After completing grammar school he served a four-year apprenticeship in patternmaking, molding, and machine work at the DeLamater Iron Works, where he was subsequently engaged as a draftsman. He also did drafting for the United States Lighthouse Board. He spent his spare time in securing a technical education at the College of the City of New York and Cooper Union, receiving a B.S. degree from the latter in 1893.

It was in 1890, after the death of Mr. DeLamater, that he became an employee of the Sergeant Drill Company, which was located in New York. When the company was merged into the Ingersoll-Sergeant Drill Company, and plans were made to move the manufacturing plant to Easton, Pa., Mr. Prellwitz helped to lay out and to construct the new factory, which was placed in operation in November, 1893. At that time he became chief draftsman and served in that capacity until 1900, when he became chief engineer. He retained that post when the present Ingersoll-Rand Company was formed and held it continuously until his death. In 1905 Mr. Prellwitz had full charge of the design and construction of the Phillipsburg, N.J., plant of the Ingersoll-Rand Company.

Mr. Prellwitz contributed much to the advancement of compressed-air machinery in general, but more especially to the development of the products of the company with which he spent his entire adult life. While he was thoroughly familiar with all Ingersoll-Rand machines, he gave the most of his attention to compressors and rock drills, with the result that many developments in these products are attributable to him. Numerous inventions stand to his credit; and along with his bent for mechanical design he possessed a broadmindedness which allowed him to see merit in the creations of others. Over a span of years he made periodic trips to Europe, and out of his contacts and studies there came the application of various European ideas and methods to American practice. Recognizing the value of plate valves in compressors, he investigated their development abroad and obtained for his company the rights to manufacture the Rogler valve in this country. Similarly, he foresaw the place that turbo-blowers were destined to take in the blast-furnace industry and was quick to improve upon and to help commercialize here the European types of this equipment.

Mr. Prellwitz designed some of the first air compressors for discharge pressures above 1000 lb. These early units served to furnish power for operating street cars in cities and locomotives in mines. He developed the first torpedo-charger compressor used in the United States Navy. It proved so efficient that the Government requested Ingersoll-Rand not to patent it or to supply foreign countries with it. These compressors were originally built for battleships, but were later applied to both cruisers and submarines. The units were designed for 3000 lb pressure. Subsequently, Mr. Prellwitz was closely identified with the development of high-pressure air and gas compressors for industrial purposes.

He was instrumental in perfecting the Sergeant drills which established his company as one of the foremost makers of rock-drilling machinery in the United States. In 1898 he designed the first vertical, 4-stage, 2400-lb-discharge compressor for the Metropolitan Street Railway Company of New York for operating air engines of cross-town street cars.

He was a vital factor in the development of the hammer type of rock drill, of turbo-blowers, and of direct-connected electric-driven compressors. He was responsible for the clearance-control system now used on Ingersoll-Rand compressors. He had a flair for taking hold of new ideas and of making them commercially practicable. When the company was small, he frequently would design and set a price upon a piece of equipment to meet the requirements set forth in an inquiry. He would then call upon the customer, sell the machine, return to the plant, and supervise its construction.

Among his co-workers Mr. Prellwitz was affectionately known as "Pop" Prellwitz. His capacity for teamwork was outstanding and he did much to promote harmony in the organization. He was active in welfare and social work both in the company and in Easton. He was a director in the Phillipsburg Development Corporation, a subsidiary company formed to build homes for employees. As a trustee and member of the building committee, he aided in the construction of the new Easton Hospital.

He became a member of the A.S.M.E. in 1900, belonged to numerous clubs in his home city, and was an ardent fisherman. He was a director in the First National Bank & Trust Co. of Easton.

Mr. Prellwitz is survived by his widow, Mrs. Clara Prellwitz, one daughter, a step-son, a step-daughter, and a brother, Henry Prellwitz, of New York.

REXFORD M. PRUITT (1893–1932)

Rexford M. Pruitt, formerly treasurer of the Indiana Pump & Compressor Co., Indianapolis, Ind., died at his home in that city on February 18, 1932, after a long illness.

Mr. Pruitt was born in Indianapolis on May 25, 1893, the son of Mr. and Mrs. O. M. Pruitt. After four years at the Indianapolis Manual Training High School he entered Butler College, Inadianpolis, from which he received an A.B. degree in 1915. He was then given the position of secretary and sales engineer with the Indiana Air Pump Company, for which he had worked during vacations while in high school and college. He continued with the company throughout his life, becoming treasurer when it was reorganized in 1931 as the Indiana Pump & Compressor Co.

Mr. Pruitt handled a large part of the correspondence, particularly the technical, of the firm. He planned the equipment for hundreds of municipal and industrial water supply systems and supervised many installations. Part of his time was spent in actual selling. His work required familiarity with the fundamentals of hydraulic and pneumatic practice in order to select and install the varied equipment suitable for the needs of each installation. Among the jobs for which he was responsible were equipment for wells at the Cedar Street, Riverside, and Pennsylvania Street stations for Lansing, Mich.; air-lift installations in Rockford, Ill., Winona, Minn., and at the Norton Iron Works, Ashland, Ky., which pumped as much as 6000 gpm from the Ohio River; and many other installations involving a knowledge of air compressors, electric motors, centrifugal pumps, oil engines, etc.

In addition to such work Mr. Pruitt gave air-lift instruction in the sales schools of the Chicago Pneumatic Tool Company at Pittsburgh, Chicago, Detroit, Los Angeles, and San Francisco, the Harris Pump & Supply Co. at Pittsburgh, the Charleston Electrical Supply Company at Charleston, W. Va., and others.

Mr. Pruitt became a member of the A.S.M.E. in 1926. He also belonged to the Wisconsin Engineering Society.

JAMES K. REED (1886–1932)

James K. Reed, design engineer for mechanical and electrical equipment for the Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa., died at his home in Pittsburgh on April 23, 1932. Mr. Reed had been associated with the Westinghouse company since 1929, when he took a position as mechanical engineer in its Switchboard Engineering Department.

Prior to 1929 he spent 13 years as factory superintendent with the S. Keighley Metal Ceiling & Manufacturing Co. at Follansbee, W. Va., and its successor, the Keighley Manufacturing Company, at Carnegie, Pa. He developed and installed a cost system and new routing schedule at the former plant and designed the equipment necessary for the manufacture of steel lockers, steel ceilings, and other specialties.

Mr. Reed's earlier experience included short periods in drafting work for the Columbia Bridge Company, Carnegie, and the National Tube Company, Pittsburgh, two years in drafting and layouts for firebrick linings of all kinds and for brickyard machinery for the Harbison-Walker Refractories Company, Pittsburgh, and three years designing and testing gas producers, gas engines, steam turbines, thrust bearings, and power plants for the Westinghouse Machine Company, East Pittsburgh.

Mr. Reed was born in Pittsburgh on November 26, 1886, and secured his education there, taking technical subjects in night school for several years and securing a B.S. degree in mechanical engineering at the Carnegie Institute of Technology in 1912. He became a junior member of the A.S.M.E. in 1916, resigned in 1924, and was reinstated as a member in 1930. He is survived by his widow, formerly Miss Blanche Crosbie, whom he married in 1914, and by three daughters, Helen, Margaret, and Mary.

SAMUEL AUSTIN REILLY, JR. (1906–1932)

Samuel Austin Reilly, Jr., whose death occurred at the Presbyterian Hospital in New York, N.Y., on August 20, 1932, was born at Little Falls, N.J., on March 4, 1906, the son of Samuel Austin and Cora (Keefe) Reilly. He attended high school in Newark, N.J., and took a mechanical engineering course at the Stevens Institute of Technology. Following his graduation in 1929 Mr. Reilly taught at the Institute for a year, in the Electrical Engineering Department, and then took a postgraduate course at the Massachusetts Institute of Technology.

At the time of his death Mr. Reilly was a member of the staff of the New York laboratories of the Bell Telephone Company, where he had worked during several vacations while attending Stevens Institute.

Mr. Reilly was a star lacrosse player at Stevens and a member of the track team. He belonged to the Montclair Athletic Club, Tau Beta Pi, honorary engineering fraternity, and Theta Upsilon Omega, and also to Gear and Triangle, a local honorary society at Stevens. He was a teacher in the Sunday School of the Irvington (N.J.)

Methodist Episcopal Church. He became a junior member of the A.S.M.E. in 1929.

Surviving Mr. Reilly are his parents, with whom he made his home in Irvington, and two brothers.

CLAUDE BRYANT ROBERTS (1884–1932)

Claude Bryant Roberts, president and treasurer of the C. B. Roberts Engineering Company, New York, N.Y., died suddenly at his summer residence in Northport, Me., on September 3, 1932.

Mr. Roberts was born at Vinal Haven, Me., on April 10, 1884. He attended the Camden (Me.) High School and took courses in mechanical engineering through the International Correspondence Schools. At the age of nineteen he entered the employ of the Belfast (Me.) Gas & Electric Co., with which he remained for five years, working up to the position of superintendent of operations.

From 1909 to 1919, when he established his own company, Mr. Roberts was connected with Stone & Webster of Boston, Mass. During the first three years he was located at Ponce, Porto Rico, beginning as assistant chief engineer and being promoted to chief engineer in about six months' time. In 1912 he entered the Engineering Department, Power Plant Betterment Division, of the company. For the first three years he was in full charge of betterment work in power plants of several railway, gas, and electric companies in New England. During the remainder of his association with Stone & Webster he directed in an advisory capacity all operations, maintenance, and personnel for about fifteen power plants in Texas, Louisiana, Florida, and Georgia. His last assignment for the company was to assist in a report on the power plants of the Interborough Rapid Transit Company of New York.

Mr. Roberts became an associate of the A.S.M.E. in 1920. He was also a member of the Masonic fraternity and Downtown Athletic Club, New York. He is survived by his widow, Dorothea (Sargent) Roberts, whom he married in 1917, and by a daughter, Emily Ann Roberts.

RALPH LEONARD ROBINSON (1890–1932)

Ralph Leonard Robinson, sales executive for The Babcock & Wilcox Co., New York, N.Y., was killed in an airplane accident on March 19, 1932, when the plane in which he and six other passengers, flying from Phoenix, Ariz., to Los Angeles, Calif., in a fog, struck a high-tension transmission line, near San Bernardino, Calif.

Mr. Robinson was born at Wollaston, Mass., on June 21, 1890. After his graduation in 1912 from Leland Stanford Junior University with an A.B. degree in electrical engineering, he entered the employ of the General Electric Company at Schenectady, N.Y. He did testing work on railway, industrial, and power-plant equipment, assisted in the design of power-plant equipment, and was sales engineer for the power and mining department during the two years he spent with the company.

In 1914 Mr. Robinson was appointed to the New York Public Service Commission, in the First District, New York, and for two years engaged in the inspection of new construction at the 59th and 74th Streets Power Stations of the Interborough Rapid Transit Company and in the appraisal of public utilities. He then became associated with the Griscom-Russell Company, New York, in the design and sale of auxiliary power-plant equipment and heat transfer equipment for ships. He supervised marine engineering and sales on the Pacific Coast. Mr. Robinson took charge of sales of special process equipment for The Babcock & Wilcox Co. on December 1, 1930.

Mr. Robinson became an associate member of the A.S.M.E. in 1919 and also belonged to the National Electric Light Association, American Society of Naval Engineers, and the American Society for Steel Treating. He is survived by his widow Faith (Swift) Robinson.

ROY WINTHROP ROUNSEVILLE (1885–1931)

Roy Winthrop Rounseville, son of Robert and Nellie Rounseville, was born at Neponset, Ill., on February 5, 1885. With his parents he later moved to Kewanee, Ill., and attended the public schools there. After his graduation from the Kewanee High School he studied mechanical engineering at the University of Illinois for three years with the class of 1909.

For a number of years Mr. Rounseville had spent his school vacations in the employ of the National Tube Company, predecessor of the Walworth Manufacturing Company, at the Kewanee Works. In June, 1909, he was engaged as a draftsman and until the first of 1912 worked on the design of products, machinery, and structural-steel and reinforced-concrete construction. The following year was devoted to special work on the design and development of molding and core-making machines. Early in 1913 he was made general foreman of the Iron Valve and Cock Departments, supervising finishing, assembling, and testing operations. After six years in this capacity Mr. Rounseville was promoted to the superintendency of the Finishing Division in charge of the following ten departments: Iron Valve, Brass Valve, Union, Tapping, Blank Stock, Inspection, Forge, Pipe Fabricating, Nipple and Well Point, and Tool Making. Shortly afterward he was also made general superintendent for the company's works at Kewanee, which entailed the supervision of all production and maintenance at this Works, including the Foundry Division, consisting of Malleable, Grey Iron, Steel, and Brass Foundries, and Annealing, Galvanizing, and Pattern Departments; the Finishing Division; and the Maintenance Division with its Machine Repair Shop, Boiler and Tin Shop, Carpenter Shop, Electrical Repair Shop, Power Generating and Distributing Plant, and a number of small maintenance and service departments. At the time of his death he held the title of general manager of the Works.

Mr. Rounseville planned to terminate his connection with the Walworth Company on the first of January, 1932, and to join in the reorganization of the Peters Pump Company, of Kewanee. He died suddenly following a cerebral hemorrhage early in the morning of December 26, 1931. He is survived by his widow, Mrs. Helen (Gunther) Rounseville, and by his mother and a sister, Mrs. Lawrence (Sarah) Bradbury, of Philadelphia.

Mr. Rounseville had been a member of the A.S.M.E. since 1923. He was active in the community work of Kewanee, being a member of the chamber of Commerce, the Rotary Club, Elks, Knights Templar, and First Congregational Church. He was a director and vice-president of the Kewanee Airports, Inc., and a member of the board of governors of the Midland Country Club.

GEORGE ROSS ROWLAND (1867–1932)

George Ross Rowland, supervising engineer of The Texas Company's sales department and an authority on lubrication, died of heart disease at his home in Elizabeth, N.J., on February 10, 1932. He is survived by his widow, Katherine L. (Hirtzel) Rowland, whom he married in 1900, and by a son, Ross.

Mr. Rowland was born at Hampton Junction, N.J., on August 3, 1867, the son of George W. and Mary E. (Johnson) Rowland. He began work for the Central Railroad of New Jersey in 1883 as rivet heater and worked his way up to the position of locomotive engineer, remaining with the railroad until 1910. During the latter part of the time he served as the official representative of the locomotive firemen and engineers on their Board of Adjustment with the officials of the road, and represented both organizations at a number of their conventions.

In November, 1910, Mr. Rowland became demonstrating engineer for The Texas Company, New York, N.Y., and for the next two years was engaged in running comparative tests and demonstrating the products of the company. He was promoted to the position of chief engineer of the Lubricating Division of the Sales Department in January, 1912, superintendent of Street Railway Sales ten years later, and superintendent of Lubricating Sales in April, 1925. He had been supervising engineer since September, 1927. Special lubricants required for modern mechanism were developed, tested, and demonstrated under his direction. He served in an advisory capacity to engine and machine builders in regard to changes in construction to secure more efficient and economical lubrication, as well as to the company's engineers on points relating to the production, characteristics, and application of Texaco products. He wrote many articles pertaining to lubricants and lubrication which have been presented before engineering societies, and others which are in general use by the company for the instruction of employees and for advertising purposes. He served as president of The Lubrication Engineers' Association of the company, which had as its object the standardization of methods and policy concerning lubrication of various types of mechanism with Texaco products, from the time of its organization in April, 1929, until his death.

Mr. Rowland became a member of the A.S.M.E. in 1917, and at the time of his death was serving on the Committee on Lubrication Engineering of the Petroleum Division of the Society. He also belonged to the Society of Automotive Engineers, the American Electric Railway Association, National Association of Stationary Engineers, the Brotherhood of Locomotive Engineers, the Masonic fraternity, and the Knights of Pythias.

GILBERT E. RYDER (1880–1932)

Gilbert E. Ryder, vice-president of The Superheater Company, New York and Chicago, died suddenly of heart failure on May 17, 1932, at his home in Larchmont, N.Y. Mr. Ryder joined The Superheater Company in 1911 as a service engineer and was elected vice-president, in charge of sales and service, in 1921. Since that time the company had broadened its locomotive services and extended its activities to other fields, important among which is the sale of superheaters for stationary power plants, inaugurated under Mr. Ryder's direction.

Mr. Ryder was born on May 29, 1880, in Minneapolis, Minn., the son of Charles Gilbert and Mary L. (Gray) Ryder. His elementary schooling was received in the public schools of Sandwich, Ill., and at Tabor Academy, Marion, Mass., and he studied engineering at the University of Wisconsin and University of Illinois. He served an apprenticeship on the Chicago, Milwaukee and St. Paul Railway (now the Chicago, Milwaukee, St. Paul and Pacific) and was employed for several years as a machinist and draftsman in the Mechanical Department of that road at Dubuque and Ottumwa, Ia., and West Milwaukee, Wis.

In 1906 and 1907, as assistant mechanical engineer in the Fuel Testing Bureau (Briquette Division) of the United States Geological Survey, he had charge of a series of locomotive tests made with briquetted fuel on several railroads. He next served the city of Chicago as deputy smoke inspector in charge of the Railway Department, where his work included investigating and recommending methods and devices for the operation of locomotives without smoke. Following this experience he became associate editor of *Railway and Engineering Review*, where he remained until he joined The Superheater Company.

Mr. Ryder became a member of the A.S.M.E. in 1913. He was a past-president of the Railway Supply Manufacturers' Association and had been active in the work of several other railway associations and clubs. He was well known for his articles and speeches on fuel conservation and increase in power of locomotives through the use of superheated steam. He also belonged to the Engineers Clubs in New York and Boston, Western Universities in New York, and yacht and golf clubs in the vicinity of his home. Surviving him are his widow, the former Miss Lottie [illegible] Cole, whom he married in 1908, a daughter, Mary Evaline, and a son, Charles Gilbert.

MALCOLM P. RYDER (1874–1932)

Malcolm P. Ryder, an associate of the A.S.M.E. since 1901, died on May 24, 1932, at his home in Westfield, Mass. He is survived by his widow, Mrs. Lanette (Corsa) Ryder, whom he married in 1895, a daughter, Mrs. Howard Kling, and two grandchildren.

Mr. Ryder had resided in Westfield for sixteen years and had taken a prominent part in its affairs. He was a member of the Westfield Fire Commission for nearly six years, resigning as chairman early in 1932 to accept an appointment on the Municipal Light Board. He belonged to a number of Masonic groups in Westfield and Springfield, to the Westfield Post of the American Legion, and to the Engineering Society of Western Massachusetts.

Mr. Ryder was born in February 22, 1874, in the township of Ossining, N.Y., the son of Malcolm and Eleanor (Reynolds) Ryder. He began work at the age of fourteen with the Hydromaze Manufacturing Company, New York, N.Y., in toolmaking and brass finishing, and from 1890 to 1892 served an apprenticeship as a machinist at the Edison General Electric Company, Schenectady, N.Y. During the next six years he had a varied experience, doing electrical testing and experimental work at the Lynn, Mass., plant of the General Electric Company, in charge of magnetic ore separators and the lighting plant for Mr. Edison at his concentrating works at Edison, N.J.; in charge of an electric light station at Maracaibo, Venezuela; as superintendent of the North River Electric Light & Power Co., New York; and as foreman of the winding and testing departments of the C. &. C. Electric Co., Garwood, N.J.

In 1898 Mr. Ryder began a connection of about twelve years' duration with the New York Edison Company, in the position of superintendent of the Bronx district. Subsequently for several years he engaged in business for himself as a wiring contractor in New York. He then went to Springfield, Mass., where he was employed by the Wico Electric Company and the Witherbee Igniter Company for some years prior to his retirement from business.

Mr. Ryder perfected numerous inventions and held several patents on radio and electrical-mechanical inventions.

During the World War Mr. Ryder served for a short time in the Air Corps, being stationed at Hempstead, L.I. He also saw service in the Spanish-American War.

One of Mr. Ryder's hobbies was amateur photography, especially motion pictures, and for a number of years he had recorded various events in Westfield and vicinity, including sports and special construction work. He also enjoyed yachting and during the summer spent considerable time on Long Island Sound. He was a member of the Springfield Yacht Club.

JULIAN SALENTINE SCHOLL (1867–1932)

Julian Salentine Scholl, general manager and treasurer of the Julian Scholl Company, Englewood, N.J., died at his home at Tenafly, N.J., on July 27, 1932. Mr. Scholl was born at Henryville, Ind., on April 28, 1867, the son of Charles and Elizabeth (Sulzer) Scholl. He attended high school in Indianapolis and was graduated from Rose Polytechnic Institute in 1888 with a B.S. degree in mechanical engineering. During his vacations he worked for the Sulzer-Vogt Machine Company, Louisville, Ky., and during the three years following his graduation for the Harrisburg Foundry and Machine Works and the Weston Engine Company, Painted Post, N.Y.

In 1891 he established the firm of Julian Scholl & Co., in New York, for designing and erecting complete power plants, designing and perfecting machinery and promoting its manufacture and sale. The name was changed to the Julian Scholl Company in 1910 and the firm has had headquarters in Englewood, N.J., since 1930. During recent years it has dealt in gasoline shovels, sand-washing plants, and contractors' equipment.

Mr. Scholl became a junior member of the A.S.M.E. in 1889 and a member in 1900. He is survived by his widow, Katharine B. (Hargest) Scholl, whom he married in 1893, and by a daughter, Ione S. Eckersen, and three sons, Julian S., Jr., Charles C., and Robert H. Scholl.

CHARLES HENRY SERGEANT (1864–1932)

Charles Henry Sergeant, consulting engineer, Rutherford, N.J., died suddenly of heart failure on January 23, 1932. He is survived by his widow, Dorothy (Bicks) Sergeant, whom he married in 1922.

Mr. Sergeant was born at Columbus, Ohio, on August 25, 1864, the son of Henry Clark Sergeant, a partner in several early firms which later grew into the present Ingersoll-Rand Company. His mother was Caroline (Luckhaupt) Sergeant.

After attending public school in New York, Mr. Sergeant took technical courses at the University of Denver, trained as a machinist with Sergeant & Cullingworth Co., New York, worked as a draftsman with J. W. Jackson, of Denver, and with his father at the Bridgeport Machine Tool Works, secured shop experience as superintendent of a small testing and erecting shop of the Sergeant Drill Company, New York, and spent a season in surveying and making drawings for the Denver and California Short Line Railroad.

Following this varied experience he took a position as draftsman with the Ingersoll Sergeant Drill Company, New York, in 1886. After a short time he was put in charge of the drawing room and occasionally during the six years he remained with the company served temporarily as foreman of the Drill and Compressor Departments. He standardized the company's rock drills and designed special air compressors and tools and supervised drill tests in mines, tunnels, etc.

From 1892 to 1898 Mr. Sergeant was draftsman for the National Meter Company, manufacturers of power and pumping gas engines and water meters of several kinds. He worked on the design of several engines for the Canda Manufacturing Company, Carteret, N.J., a wire opening envelope machine for Munn & Co., New York, air compressors for the McKiernan Drill Company, New York, and other machinery as a consulting engineer. The Merrill-Sergeant Pneumatic Company which he helped to form, dealt in water air lifts and displacement pumps, oil engine compressors, etc. For a time he was engineer for the Fuel Oil Power Company, demonstrating a 12 by 12 in. two-stroke vertical engine. For a year and a half he was superintendent and engineer for the Acme Oil Engine Company, Bridgeport, Conn., engaged in making small-power stationary oil engines, and for a similar period demonstrated his own 2-cycle 6 by 7 in. marine oil engine, running generator and light to secure data. Subsequently he designed a plate air valve of various sizes, a heavy oil engine, convertible to natural gas, and vacuum and air-pressure cylinders for gasoline extraction arranged in powers for one frame. Shortly after the War he went to Bradford, Pa., to work on internal-combustion engines and air compressors for the U.S. & Cuban Allied Works Engineering Corporation, whose main offices are located in New York. He had recently patented a high-speed engine, economical to manufacture and adaptable to automotive, aeronautic, or marine use. He had also recently designed an air-cooled 8-cylinder automobile engine, a radial, air-cooled oil engine, and an automatic automobile clutch.

Mr. Sergeant first became a member of the A.S.M.E. in 1895. He resigned in 1914, not being at the time actively engaged in engineering, but was reinstated in 1921.

JAMES SHEAHAN (1868–1932)

James Sheahan, general superintendent of the Memphis (Tenn.) Artesian Water Department, died of heart failure at his home in that city on June 9, 1932. His widow, formerly Miss Mary Collins of Batavia, Ill., whom he married in 1890, survives him.

Mr. Sheahan was born in Riverton, Va., on December 16, 1868, the son of Mr. and Mrs. John Sheahan. Early in his life the family moved to Vandalia, Ill., and later to Batavia, near Chicago, where he secured the most of his education. At the age of fifteen he began work with the Vandalia Paper Company and was employed by it and the Vannortwick Paper Company, Batavia, until 1887. During the next eight years he was with the United States Wind Engine & Pump Co., Batavia, being in charge of one of the manufacturing departments after completing an apprenticeship as a machinist.

Mr. Sheahan went to Memphis in 1895 and for three years was general superintendent of the Southern Bag & Paper Co., manufacturers of wooden ware. He erected machinery and did some general construction work for the Memphis Artesian Water Company in 1898 and 1899, and from then until 1912 was general superintendent of Scatchard & Son, lumber dealers.

During the first ten years of his connection with the Memphis Water Department Mr. Sheahan held the position of chief mechanical engineer. Under his direction in that capacity and later as general superintendent, the water system was greatly augmented. A recently constructed pumping station was named in Mr. Sheahan's honor.

Mr. Sheahan had been a member of the A.S.M.E. since 1925. He also belonged to the Memphis Engineers Club.

HARRY EALES SMALLBONE (1866–1932)

Harry Eales Smallbone, chief engineer for the Willcox & Gibbs Sewing Machine Co., New York, N.Y., died at St. Luke's Hospital in that city on July 28, 1932, after a brief illness.

Mr. Smallbone was born in England on December 8, 1866, and was educated there. He served a seven-year apprenticeship as an engineer and machinist, and studied drafting and designing at the Science and Art Schools, South Kensington, London. He came to the United States in 1889 and was connected with the Wheeler & Wilson Manufacturing Co., Chicago, Ill., as master mechanic until that company was purchased by the Singer Manufacturing Company in 1905. During the next two years he worked on inventions for the Singer company in Elizabeth, N.J. He returned to Chicago to become president and superintendent of the Smallbone Manufacturing Company, manufacturers of the Smallbone buttonhole attachment and other sewing machine parts, and designers and builders of dies, tools, models, and special machinery.

After ten years in this company Mr. Smallbone went to Buffalo to become assistant superintendent of the King Sewing Machine Company, with which he remained for an equal period. In 1928 he was engaged as consulting engineer by the New Home Sewing Machine Company, Orange, Mass. He entered the employ of the Willcox & Gibbs Sewing Machine Co. late in 1928 in the capacity of assistant superintendent, becoming chief engineer in 1929.

Mr. Smallbone had been a member of the A.S.M.E. since 1914. He also belonged to the Knights of Pythias and the American Order of Sons of St. George. He was a widower.

AUGUSTUS SMITH (1868–1932)

Augustus Smith, vice-president and for many years a member of the New Jersey State Commission of Commerce and Navigation, president of the Bergen Point Iron Works, Bayonne, N.J., died suddenly of heart disease at his home in Roselle, N.J., on February 29, 1932. He had been active in civic affairs in Roselle for many years, and during three years as a member of the Borough Council served as head of the Public Works Department. He was first appointed to the New Jersey State Commission in 1924. He had visited many cities in this country and Europe to increase his knowledge of municipal government, particularly that phase of it which deals with engineering and construction problems, in which he was very keenly interested.

Mr. Smith was born in New York, N.Y., on July 30, 1868, and attended the public schools there, the College of the City of New York, and the School of Mines of Columbia University, from which he received an A.B. degree in 1887 and a C.E. degree two years later. From then until 1893, when he established his own business as engineer and contractor, he was employed by the Link Belt Engineering Company, of New York and Philadelphia, except for a year spent as manager of a saw mill in South Carolina.

The Bergen Point Iron Works was founded by Mr. Smith in 1906. His engineering work included many public projects, among them the construction of naval coaling stations along the Atlantic Coast and at the Balboa and Cristobal terminals of the Panama Canal. He also had a number of inventions to his credit, one of his most important being a type of houseboat on pontoons. He designed and built coal-handling machinery, movable bridges, derricks, cranes, etc.

Mr. Smith had been a member of the A.S.M.E. since 1902 and also belonged to the American Society of Civil Engineers and The Franklin Institute. He is survived by a son, Henry Smith, a student at Princeton University. His wife died several years ago.

CAMERON C. SMITH (1861–1932)

Colonel Cameron C. Smith, founder of the Union Steel Casting Company, Pittsburgh, Pa., died on May 2, 1932, on his way north from Eustis, Fla. He was born on April 24, 1861, at Culmerville, Butler County, Pa., the son of Joseph and Mary (Watson) Smith. He prepared for college at the Sharpsburg (Pa.) Academy and spent three years at Waynesburg College and Lebanon University, where he studied civil engineering.

After leaving college Colonel Smith spent about nine years with Wilson-Walker & Co. and their successor, the Carnegie Steel Company. Here he had experience in the manufacture of railroad forged links, pins, couplers, axles, etc., as well as steel plates, beams, and armor plates.

The Union Steel Casting Company was founded by Colonel Smith in 1899 to manufacture open-hearth and crucible steel and steel castings, and he served as its president and general manager until 1917, when he became chairman of the board. He personally superintended the making of steel castings and took out a number of patents. He also contributed to the perfecting of cast-steel locomotive frames. He was widely consulted on the manufacture of steel.

During the World War he was business manager of the New York Ordnance District and a member of the War Claims Board, and was commissioned Lieutenant-Colonel. He had been chairman of the board of the Pittsburgh Roll Corporation since 1922. He was a director of the Potter Title & Trust Co. and the Potter Mortgage & Guarantee Co., and treasurer of the Duncan Land & Improvement Co. in Pittsburgh. He became a member of the A.S.M.E. in 1906 and held membership in the American Iron and Steel Institute, American Society for Testing Materials, Engineers Society of Western Pennsylvania, Army Ordnance Association, Highland Presbyterian Church, and several clubs in Pittsburgh and vicinity. He was a member of the Masonic fraternity and trustee of Waynesburg College. Colonel Smith had contributed articles to several periodicals, including *Iron Trade Review*, *Better Homes and Gardens*, and the *Literary Digest*.

Surviving Colonel Smith are his widow, formerly Maude Forsaith, of Etna, Pa., whom he married in 1902, and a married daughter, Dorothy (Mrs. J. W.) Badgley, of Dunkirk, N.Y.

HAROLD BABBITT SMITH (1869–1932)

Harold Babbitt Smith, whose study of dielectric phenomena and stress distribution was of inestimable importance in the development of long-distance power transmission, died in Worcester, Mass., on February 9, 1932, after being in ill health for several years. Direct-current generators and motors, alternating-current transformers, and insulators of his design contributed largely to the practical application of electricity in industry and everyday life. In recognition of such achievements, as well as of his long services as an educator, he was honored in 1929 by both Purdue University and Worcester Polytechnic Institute with the degree of doctor of engineering. He was head of the Department of Electrical Engineering at the latter institution for thirty-five years.

Dr. Smith was born at Barre, Mass., on May 23, 1869, the son of Samuel Francis and Julia Asenath (Babbitt) Smith. He prepared for college at the Barre High School and was graduated from Cornell University in June, 1891, with an M.E. degree in electrical engineering. After engaging in post-graduate study there for the remainder of the year, he entered the teaching field as professor of electrical engineering, in charge of the department, at the University of Kansas. In 1893 he became head designer and electrical engineer for the Elektron Manufacturing Company of Springfield, Mass. In the fall of that year he took up the duties of professor and director in the School of Electrical Engineering at Purdue University, where he remained until he went to Worcester in 1896. He was retained as consultant for the Elektron Manufacturing Company until 1902 and served a number of other organizations in a similar capacity. From 1904 to 1911 he was treasurer of the Smith & Rice Co., wholesale lumber manufacturers. In 1905 he became engineer and designer for the Westinghouse Electric & Manufacturing Co., of Pittsburgh, Pa., and since 1913 had been consulting engineer for that company. He retired from the Worcester faculty in 1931. His summer home was at Princeton, Mass.

At the St. Louis Exposition in 1904 Dr. Smith was chairman of the

International Group, Jury of Awards in Electrical Engineering. In 1917–1918 he was an associate member of the Naval Consulting Board and consultant of the special board of the Navy on anti-submarine research. He had traveled extensively in the interest of his profession in Europe, Africa, Japan, China, and India, and had written many papers for engineering organizations and the technical press.

Dr. Smith had belonged to the American Institute of Electrical Engineers since 1891, and had risen through various offices to the presidency of the Institute in 1929–1930. He had been prominent in much of the committee work of the Institute and had represented it on such joint-activity organizations as the American Engineering Council, John Fritz Medal Board of Award, and the United States National Committee of the International Electrotechnical Commission. He became a member of the A.S.M.E. in 1894 and belonged also to the Institution of Electrical Engineers (Great Britain), Society for the Promotion of Engineering Education, American Association for the Advancement of Science, and a number of clubs, including the Worcester Economic Club, Cornell Club of New England, and Cornell Club of New York. He was a member of several fraternities, including the Sigma Xi, Tau Beta Pi, and Beta Theta Pi.

Dr. Smith was twice married. His first wife, the former Miss Laura Bertha Smith, of Ithaca, N.Y., whom he married in 1894, died some years ago. In 1911 he married Miss Persis H. Smith of New York. They were divorced in 1930. He is survived by three children, Miss P. Nancy, Miss Margaret S., and Richard S. Smith, all of New York.

H. COLLIER SMITH (1866–1932)

H. Collier Smith, sole owner of The Quickwork Company, St. Marys, Ohio, died in that city on September 16, 1932.

Mr. Smith was born at Raleigh, N.C., on May 27, 1866, the son of Henry Collier and Laura Ann (Nowell) Smith. Mr. Smith learned the sheet metal business in his father's shop at Raleigh and later as a general mechanic, principally in connection with sheet and plate metal work and as draftsman in the Hull Department of the Newport News Shipbldg. & Dry Dock Co., Newport News, Va. He became president, in 1900, of the Smith-Warren Company, Boston, Mass. This company promoted his system of fireproof, hollow, sheet-metal window frames and sashes, for use in exposed portions of buildings, instead of fire shutters; he secured more than twenty patents on this system and was the pioneer sheet metal window promoter.

In 1906 Mr. Smith went to Brooklyn, N.Y., where for two years he was general assistant to Mr. Grace of the Herman & Grace Co., sheet and metal contractors. Subsequently he held a similar position for about a year with the Moore & Munger Co., in New York, his work relating to automobile bodies and sheet metal work. During this period he designed and built a rotary shear, in addition to a number of other tools and devices.

He designed and built other shears and sheet metal working tools while connected during the next two years with the Springfield (Mass.) Metal Body Company and the Racine (Wis.) Mfg. Co. In 1911 and 1912 he was superintendent of the Sheet Metal Department of the Cadillac Motor Car Company, Detroit, and designed many machines, tools, dies, jigs, and fixtures.

About this time Mr. Smith established his own business in Detroit for building plate and sheet metal working machinery, later removing to St. Marys. He designed and manufactured rotary shears for straight, irregular, and circular cutting, beveling, flanging, and joggling operations in building boilers, ships, cars, tanks, stacks, etc., and in plate mills, as well as many machines for sheet metal work. He contributed articles in his field to the technical press.

Mr. Smith became a member of the A.S.M.E. in 1921. He was also a member of the Society of Automotive Engineers, a 32d degree Mason and Shriner, and a member of the Episcopal Church.

In 1903 Mr. Smith married Miss Agnes T. Flatley, Newark, N.J. He is survived by her and by one son, H. Collier Smith, Jr., under whose direction The Quickwork Company is being continued.

WILLIAM AIKEN STARRETT (1877–1932)

William Aiken Starrett, whose achievements as a builder of skyscrapers were outstanding, died at his home in Madison, N.J., on March 25, 1932, after an illness of three months. The Empire State Building in New York, at the time of its construction the highest building in the world, and the Union Station in Washington were among the many structures for which he and his associates were responsible.

Colonel Starrett was president of The Starrett Corporation, New York, holding company of the Starrett realty and building enterprises, in all of which he was an officer or a director, or both. He was vice-president and director of Starrett Brothers & Eken, Inc., the building affiliate of the varied interests, which erected the Empire State Building and other large buildings of note, including the seventy-story Forty Wall Street Building and the nineteen-story Starrett-Lehigh Terminal Building, in New York, the Carew Tower in Cincinnati, the tallest building in the Ohio Valley, and the forty-story Ramsey Tower, tallest structure in Oklahoma City. Colonel Starrett was also president of the Starrett Investing Corporation, the investing and realty affiliate; the Forty Wall Street Corporation; and the Wall and Hanover Street Realty Corporation; and a director of several contingent interests.

William Starrett was born at Lawrence, Kan., on June 14, 1877, a son of the Reverend William A. and Helen (Ekin) Starrett. He attended the Chicago Preparatory Schools and spent two years at the University of Michigan as a member of the Class of 1897, leaving in 1895. He received his B.S. degree in civil engineering in 1917, as of the Class of 1897, and in 1931 the university conferred the honorary degree of Doctor of Engineering upon him.

After leaving the University, he worked for a time in a large wholesale grocery concern in Chicago and in 1897 went to New York and entered the employ of the Thompson Starrett Company, which his brothers Theodore and Ralph and Henry Thompson had recently organized. He spent about three years with that company, working as timekeeper, field engineer, and building superintendent. After several years as building superintendent on various projects in New York, he took charge of the erection of the Union Station in Washington, and on its completion he was made general superintendent and vice-president of his company. In 1913, Colonel Starrett left the Thompson Starrett Company to become a member of the firm of Starrett & Van Vleck, architects, a connection which continued until 1918.

In 1916 Colonel Starrett spent some time in training work at the Plattsburg Camp and with the Engineers Corps. When the United States entered the World War, he was called to Washington to take charge of emergency construction work for the Council of National Defense and was appointed Chairman of the Emergency Construction Division of the War Industries Board, serving throughout the War. This Division had charge of the building of all cantonments, hospitals, and army bases in the United States. He was commissioned a Captain in the Engineers Reserve Corps of the National Army in May, 1917, and a Major the following month; he was promoted to the rank of Colonel in the Quartermaster Corps in March, 1918, and, when discharged a year later, retained that rank in the Reserve Corps.

After the War, Colonel Starrett was made vice-president of the George A. Fuller Company, an office which he retained until 1921. During this period, he was connected with the erection of several large buildings in Tokyo, Japan. These steel frame buildings which he built then withstood the 1927 quake.

In 1922, together with his brothers Paul and Ralph and Andrew J. Eken, he founded Starrett Brothers, Inc., the firm later becoming Starrett Brothers & Eken, Inc., with Paul Starrett as president and the other members vice-presidents. The Starrett Corporation was formed in 1929 and Colonel Starrett was active in its work up to the time of the illness leading to his death.

Colonel Starrett was called into consultation by President Hoover in August, 1931, in connection with a nation-wide program for enlivening the building trades. He spent the next several months in intensive work as a member of the President's Emergency Committee for Employment and as a director of the Gibson Emergency Committee in New York. He also engaged in public service in Madison, N.J., as its Mayor, on the Republican ticket, from 1920 to 1922. He declined renomination on account of increased building activities at that time.

Colonel Starrett was the author of "Skyscrapers and the Men Who Build Them," published in 1928, as well as a frequent contributor to magazines. He had made several addresses over national broadcasting chains in regard to the utilization in home building of experience secured in the erection of skyscrapers.

He became a member of the A.S.M.E. in 1919 and was a member of its Committee on the Economic Status of the Engineer. He also belonged to the American Society of Civil Engineers, the Society of American Military Engineers, and the Engineers' Club in New York. He was a former member of the American Institute of Architects.

Surviving Colonel Starrett are his widow, formerly Miss Eloise Gedney, of East Orange, N.J., whom he married in 1900, a son David, and a daughter Helen Ruth, as well as his brother Paul and two sisters.

LEWIS STARR STRENG (1876–1931)

Following an illness of several months, Lewis Starr Streng, vice-president in charge of operation for the Louisville Gas & Electric Co., Louisville, Ky., died at his home in that city on December 24, 1931. He is survived by his widow, Mrs. Fannie S. (Henly) Streng, whom he married in 1910, and by two children, John, a student at the Massachusetts Institute of Technology, and Ida Henly Streng, a student at the University of Louisville.

Mr. Streng was a native of Louisville, the son of Manuel J. Streng and Frances (Starr) Streng. He was born on September 19, 1876. He attended the public schools of Louisville, and was a member of the class of 1894 at the du Pont Manual Training School there, the first graduating class of that institution. He was graduated from the Massachusetts Institute of Technology with an S.B. degree in 1898.

His first business connection was in the Testing Department of the General Electric Company at Schenectady, N.Y., for the latter part of 1918. From there he went into the employ of the Public Service Corporation of New Jersey and was with that corporation and its subsidiary organizations until 1906, during the latter part of the time as assistant chief engineer of the Electric Department and in charge of electric design and construction of generating and substations and transmission lines.

In 1906 Mr. Streng returned to Kentucky as chief engineer for the Kentucky Electric Company, designing and building the entire station and lines. Seven years later, when that firm and the Louisville Lighting Company were merged into the Louisville Gas & Electric Co., he was appointed chief engineer and general superintendent. He was elected to the office of vice-president in 1923 and continued in that capacity until his death.

His career at Louisville was marked with a number of engineering achievements. In 1914 he supervised the piping of the first natural gas to the city. He was supervising engineer in the construction of the Bear Grass Gas Plant on the Upper River Road, and of the Ohio Falls Hydro Electric Station, and superintended the enlargement of the Waterside Steam Station at Second Street and the Ohio River.

Mr. Streng was in charge of the purchase of additional power-plant facilities when his company, in 1929, began to furnish power to the Louisville Railway. He also supervised the construction of the transmission line from the hydro plant to the Union Gas & Electric Co. at Cincinnati.

In addition to his membership in the A.S.M.E., which he joined in 1917, Mr. Streng was a Fellow of the American Institute of Electrical Engineers, and was a member of the National Electric Light Association, American Gas Association, and of a number of clubs, including the Engineers and Architects, Electric, Quindecim, and Standard Country clubs of Louisville.

DAVID LONG SUMMEY (1875–1932)

David Long Summey, consulting engineer associated with the United States Metals Refining Company, New York, N.Y., died at his home in that city on November 14, 1932. He was born in Davison, Mich., on March 10, 1875, the son of George E. and Mary (Long) Summey. After studying for a time at the University of Illinois, he secured employment in the Engineers Corps of the Erie Railroad, Buffalo Division. Subsequently he was assistant engineer in the Distribution Department of the Peoples Gas, Light & Coke Co., Buffalo; in charge of a street main construction for the Jenkintown & Cheltenham Gas Co., Jenkintown, Pa., and for the Philadelphia Suburban Gas Company, Darby, Pa.; and superintendent of the River Shore Gas Company, Riverton, N.J.

About 1900 Mr. Summey became associated with the United Gas Improvement Company, Philadelphia, Pa., as erecting engineer and draftsman. In 1903 he was sent to Waterbury, Conn., as assistant superintendent in charge of the operating departments. From 1905 to 1913 he was a designing mechanical engineer, specializing in extrusion processes at the Chase Rolling Mill Co., Inc., of Waterbury, and from 1913 he continued his work on extrusion at the Scovill Manufacturing Company of the same city. Later he became chief engineer of that company.

About five years before his death he was engaged as consultant to the United States Metals Refining Company to design an electric melting furnace as the first step in the development of a process for producing high conductivity oxygen-free copper. This metal is now being produced on a commercial scale at the Carteret, N.J., plant of that company.

Mr. Summey became a member of the A.S.M.E. in 1907 and served on the executive committee of the Waterbury Section in 1923. He also belonged to the Institute of Metals.

Surviving Mr. Summey are his widow, Virginia (Weeks) Summey, whom he married in 1899, and three children, Mrs. Elizabeth (Summey) Roudebush, Virginia Piers Summey, and Richard W. Summey, all of New York.

KARL JOHAN SUNSTRÖM (1851–1932)

Karl Johan Sunström, who died in Stockholm, Sweden, on April 22, 1932, was born in Sunne, Sweden, on November 18, 1851, the son of Jan Henrik and Johanna (Fryxell) Sunström. He attended high school in Karlstad and secured his technical training in Stockholm. He began work in 1870 as draftsman for the Munkfors Iron Works, where he remained for about a year. The next three years were spent in designing and building peat machinery in Stockholm, and from 1874 to 1881 he was connected successively with the Lesjöfors Iron Works, as draftsman, Aker Ordnance Works, as superintendent and mechanical engineer, and Hagfors Iron Works, as chief draftsman, all of these plants being located in Sweden.

Mr. Sunström came to the United States in 1881 and took a position as draftsman with the Washburn & Moen Manufacturing Co. in Worcester, Mass. The next year he went to Providence, R.I., where he worked as draftsman for the Brown & Sharpe Manufacturing Co. until 1886. After another year with the Washburn company he was associated with Charles H. Morgan in developing the continuous rolling mill.

Returning to Sweden in 1890 Mr. Sunström engaged in consulting work for a year and subsequently was associated with the Kopparsbergs-Berglagets Works, Falun, as mechanical engineer, Berglagets Works, as chief engineer, and Degerfors Steel Works, also as chief engineer, until 1906. From then until his retirement he engaged in consulting work in Stockholm.

Mr. Sunström became a member of the A.S.M.E. in 1884 and belonged also to the Iron and Steel Institute, the Verein deutscher Eisenhüttenleute, and other technical organizations. He contributed several articles to *Järnkontorets Annaler* and other periodicals on the manufacture of welded pipes and tubes, the horseshoe nail and its manufacture, old-time rolling mills, and other subjects. The King of Sweden bestowed the Royal Vasaorden upon him.

Since his retirement Mr. Sunström had interested himself in studying and collecting engineering relics. He is survived by a married daughter, Mrs. Agnes Westerberg, of Degerfors, and a son, Rudolf Sunström, of Gunnebobruk, Sweden.

HANS OTTO SWOBODA (1870–1932)

Hans Otto Swoboda, consulting electrical and mechanical engineer, Pittsburgh, Pa., died on June 30, 1932, in that city after a short illness. Mr. Swoboda was born on February 4, 1870, in Buchholz, Saxony, Germany, the son of Herman and Thekla (Eisenstuck) Swoboda. He was educated in Germany, attending technical schools in Dresden, Hanover, and Darmstadt. After two years as electrical engineer with Siemens & Halske, in Berlin, he came to the United States in 1892. Here he was associated successively with the General Incandescent Arc Light Company, absorbed in 1899 by the General Electric Company, the Falcon Electric & Mfg. Co., and the H.O.S. Engineering Company, all of New York, and the Amboy Manufacturing Company, Perth Amboy, N.J. He went to Pittsburgh in 1910 and was connected for two years with the Westinghouse Elec. & Mfg. Co. as superintendent of the electrical department. He then organized the firm of H. O. Swoboda, Inc., consulting electrical and mechanical engineers, of which he was president and treasurer until his death.

Mr. Swoboda was a recognized authority on electric lighting and power plant work and had devoted considerable time to electric heating research. He designed and developed numerous electrical devices for which he received United States and foreign patents, and made frequent contributions to scientific journals. The Falcon electric heat-treating furnaces and bare electric immersion heaters for asphalt and other viscous materials are products of his inventive ingenuity.

Mr. Swoboda became a member of the A.S.M.E. in 1921. He also belonged to the American Institute of Electrical Engineers, American Society for Steel Treating, Technischer Verein, and the Lincoln Club of Pittsburgh and Pittsburgh Athletic Association. He is survived by his widow, Olga (Schmelz) Swoboda, whom he married in 1897, and by two sons, Herman and Walter.

WILLIAM WOLCOTT TEFFT (1882–1932)

William Wolcott Tefft, prominent in the middle-western field of hydroelectric power, died on June 24, 1932, at the Cowie Private Hospital in Ann Arbor, Mich., after an illness of several weeks. His home was in Jackson, Mich., where he was active in church and other local activities. He is survived by his widow, Mrs. Ethel (Foote) Tefft, whom he married in 1910, and by a son, Robert, as well as by one brother, Leo C. Tefft, of Jackson.

Mr. Tefft was born at Milbank, S.D., on February 28, 1882, the son of Levant C. and Ada M. (Wolcott) Tefft. He prepared for college at the Jackson High School and received a B.S. degree in civil engineering from the University of Michigan in 1907. The degree of C.E. was conferred upon him by the University in 1930. During summer vacations while he was in college Mr. Tefft had experience as a rodman in surveying for electric railway construction; as dredging inspector for the Federal Government at the Lime Kiln crossing of the Detroit River at Amherstburg, Ontario, Canada; as chief of party in waterpower surveys on the Muskegon River; as assistant engineer to Gardner S. Williams, consulting engineer of Ann Arbor; and as resident engineer in the construction of a hydroelectric plant at Superior, Mich.

Following his graduation Mr. Tefft spent three years with the Commonwealth Power Company, supervising work on dams, designing reinforced-concrete power houses, and serving for a time as resident engineer at the Cooke Dam on the Au Sable River, Michigan. From 1911 to 1919 he was a member of the firm of the Fargo Engineering Company, Jackson. His work during these eight years included cost estimates and appraisals for hydroelectric developments, in several states. He made a special investigation for his father-in-law, W. A. Foote, founder of the Commonwealth Power Corporation, of power possibilities on the rivers of the Southern Peninsula of Michigan, a survey for the Federal Government during the World War on water-power possibilities in Michigan and Wisconsin, a study of general conditions for the construction of dams along the Chippewa and Menomonie Rivers in Wisconsin, and a survey of the feasibility of substituting water for steam power in Michigan, thereby effecting large savings in coal and railroad haulage. During this period he also invented the Tefft conduit spillway, which reduced hydroelectric construction costs as much as 42 per cent in some instances.

In 1919 Mr. Tefft joined the staff of Hodenpyl, Hardy & Company, of New York, with an office in Jackson. He covered general engineering of large steam electric generating plants, hydroelectric power plants and dams, electric transmission lines, steam generating and electric generating plants, etc. He also became consulting hydraulic and civil engineer to the Consumers Power Company, a subsidiary of the Commonwealth Power Corporation. He was made vice-president of the former in 1922 and of the parent organization in 1924. He served as chief engineer of the Central Illinois Light Company, Southern Indiana Gas & Electric Co., Ohio Edison Company, Illinois Power Company, Illinois Electric Power Company, Northern Ohio Power & Light Co., and Tennessee Electric Power Company from 1924 to 1928, when he entered private practice as a consulting engineer. The following year he rejoined the Fargo Engineering Company as vice-president and since 1930 had also been president of the General Power Corporation.

Among the noteworthy construction achievements of Mr. Tefft were the Alcona Dam on the Au Sable River, built on a quicksand foundation, and the Ox Bow Dam on the Muskegon River, said to be the highest dam in the world on an earth foundation with a spillway through it.

Mr. Tefft was president of the board of trustees of Albion College, Albion, Mich., president of the Methodist Foundation of Michigan, and president of the board of trustees of the First Methodist Church in Jackson. He had served as president of the Jackson Council of the Boy Scouts and was a director of the Y.M.C.A. He had recently accepted the chairmanship of the finance committee of the Michigan George Washington Bicentennial Commission. He was a director of the Union & Peoples National Bank in Jackson and belonged to the Rotary Club there and to a number of local recreational clubs.

Mr. Tefft became a member of the A.S.M.E. in 1928. Other technical organizations to which he belonged were the American Society of Civil Engineers, American Institute of Electrical Engineers, American Association for the Advancement of Science, American Society of Military Engineers, and the National Electric Light Association. He was a director of the Michigan Engineering Society and a member of engineering societies in several other mid-western states. In 1929 he was elected to membership in Sigma Xi for his accomplishments in hydraulic engineering. He was also a member of the American Committee of the World Power Conference.

EDWARD G. THOMAS (1865–1932)

Edward G. Thomas, who had belonged to the A.S.M.E. since 1890, was born at Waltham, Mass., on November 1, 1865, the son of Reuben and Mary Galbraith Thomas. He was graduated in 1887 from the Massachusetts Institute of Technology with an S.B. degree in mechanical engineering. After two months as draftsman with the Ames Manufacturing Company, Chicopee, Mass., he spent a year in drafting and experimental work at the Edison Laboratory at Orange, N.J., and four months with the Thomson Houston Electric Company, San Francisco, Calif. He then became a member of the firm of Thomas, Shepard, and Searing, mechanical and electrical engineers of Denver, Colo., where he remained for two years. Subsequently he was connected for about a year each with the Edison General Electric Company at Butte, Mont., as agent and engineer, and with the Butte General Electric Company, as superintendent of a 1200-hp plant.

In 1894 Mr. Thomas located in Boston, Mass., where for seven years he engaged in general mechanical engineering and the development of automatic weighing machinery. For three years, beginning in 1902, he was engineer of the New York office of the Lamson Consulting Stove Service Company and engineer in the Special Machinery Department at Boston. From 1905 to 1909 he was superintendent of the Ariston Marble Company, Brooklyn, and the Aero Pulverizer Company, New York. During the next two years he was engineer for the Choralcelo Manufacturing Company, Boston.

Mr. Thomas left the East in 1911 to accept a position as mechanical engineer with the Boss Manufacturing Company, Kewanee, Ill., with which he remained until 1915. He spent the following two years with the Wilson Tire & Rubber Co., Springfield, Ill. In 1917 he took a position as mechanical engineer with the Toledo (Ohio) Scale Company, and was in the employ of that company until about two years before his death. Out of work since that time and in ill health he committed suicide on January 19, 1932.

About seventy-five patents were issued to Mr. Thomas, covering weighing and packaging apparatus, conveying machinery, agricultural implements, and other inventions.

Mr. Thomas became a junior member of the A.S.M.E. in 1890 and a member in 1907. He also belonged to the Engineers' Club, New York. He is survived by his widow, Mable Odiorne Thomas, whom he married in 1913, and by one daughter, Mary Galbraith Thomas.

FRED HEGLER THOMAS (1884–1932)

Fred Hegler Thomas died of heart failure on March 27, 1932, at his home in Mt. Vernon, Ohio, following an illness of four months. He was a native of Ohio, having been born at London on July 24, 1884, the son of Madison M. and Cordelia (Hegler) Thomas. After two years in preparatory work at the University School of Columbus, Ohio, he entered the mechanical engineering course at the Case School of Applied Science, Cleveland, Ohio, from which he received a B.S. degree in 1905. For about a year following his graduation he was engaged in drafting and testing work for the Jeffrey Manufacturing Company in Columbus. He then accepted a position in the Engineering Department of the American Steel & Wire Co., at Cleveland, in charge of the layout of all power equipment and the operation of all engine and boiler plants. He also carried on some experimental work for the company.

He resigned in 1907 to become sales agent for The C. & G. Cooper Co., at Mt. Vernon. Subsequently he was advanced through the offices of assistant treasurer and treasurer to that of vice-president and general sales manager of the company, which in 1929 merged with the Bessemer Gas Engine Company of Grove City, Pa., to form The Cooper-Bessemer Corporation. He helped to develop large gas-engine units for oil and gas field service, designed to compress natural gas for pipe-line distribution and to make gasoline from casinghead gas. His company was one of the early builders in this particular field, developing a complete line of engines in different sizes. Mr. Thomas was also associated in the development of mill and uniflow engines. During the World War the company built a large number of special marine engines for the Emergency Fleet Corporation, as well as hydraulic forging presses, etc., and also equipped a complete shop for the manufacture of shells, and built cutting-off machines, etc., used in shell manufacture.

Mr. Thomas became a junior member of the A.S.M.E. in 1909 and was promoted to the grade of member in 1921. He belonged to the Masonic fraternity, in which he was a Shriner, and was a member of the Congregational Church in Mt. Vernon. He is survived by his widow, Irene (Cooper) Thomas, whom he married in 1926, a daughter, Louise, and one brother, Clyde, of London, Ohio.

MAX E. R. TOLTZ (1857–1932)

Max E. R. Toltz, whose death occurred at his home in St. Paul, Minn., on January 11, 1932, was born at Koeslin, Germany, on September 2, 1857, the son of Hermann and Malvine (Beilfuss) Toltz. He attended the Royal Gymnasium at Koeslin and the Royal Polytechnikum in Berlin, receiving a civil engineering degree in 1878. Ramsey Institute of Technology, St. Paul, conferred the degree of Doctor of Engineering upon him in 1924.

Mr. Toltz served a short apprenticeship with a Berlin firm dealing in pumps and water-works equipment, and was connected with the

Prussian government as assistant engineer until he came to the United States in 1882. He became a citizen in 1889.

During the first twenty-five years in the United States Major Toltz was active in the development of railroads, especially in the Northwest. He was connected with the St. Paul, Minneapolis & Manitoba, later the Great Northern Railway, from 1882 to 1904, being promoted through various positions from draftsman to mechanical engineer in charge of motive power. During the next three years he was consulting mechanical engineer at the Angus Shops in Montreal and at the Winnipeg Shops of the Canadian Pacific Railway, and at the Jersey City Shops of the Erie Railroad. He also held the position of vice-president and general manager of the Manistee & Grand Rapids Railroad.

Major Toltz had devoted himself entirely to consulting work since 1907. During the first three years of that time he was engaged in special work on electrification for the Great Northern, Northern Pacific, Butte, Anaconda & Pacific, and Chicago, Milwaukee & St. Paul Railroads, and on ore docks for the Great Northern, Northern Pacific & Soo Lines. In 1910 he organized the Toltz Engineering Company, St. Paul, and was president of it and of its successor, Toltz, King & Day, Inc., formed in 1918, until his retirement from active business in 1928. He specialized in labor-saving machinery, hydroelectric and steam power, grain elevators, and docks, and held patents on steam superheaters, car lighting, and iron-ore docks. He had contributed a number of papers to the technical societies of which he was a member and to the technical press. These dealt with highly superheated steam and its applications, the distillation of peat coke, compressed acetylene gas, and other subjects in his field.

During the World War Major Toltz, who had been a captain in the Minnesota National Guard for many years, received his commission as a major in the Construction Division of the Quartermaster Corps. He supervised the construction of General Hospital No. 2 at Ft. McHenry, Baltimore, Md.

Major Toltz gave much of his time and ability to the advancement of the engineering profession. He was a valued member of the St. Paul Chapter of the American Interprofessional Institute, also a director of its national board. He was a past vice-president of the American Society of Civil Engineers and a member of the American Institute of Electrical Engineers, the American Railway Master Mechanics Association, and the American Society of Military Engineers. He had served as president of both the Engineering Society of St. Paul and the Federation of Architectural and Engineering Societies of Minnesota. He was a member of the Verein deutscher Ingenieure.

Major Toltz became a member of the A.S.M.E. in 1904. He was a manager of the Society from 1914 to 1917 and a vice-president during the next two years. He was one of the organizers and first chairman of the Local Section at St. Paul, and was a member of its executive committee at the time of his death. He was also serving on the 1932 Nominating Committee of the Society and on the Committee on Awards. In 1925 he established a fund of $15,000, the income of which was to be used for loans to students of engineering, under the administration of the A.S.M.E. In his will he has provided similar funds to be administered by the American Society of Civil Engineers and the American Institute of Electrical Engineers.

Among other organizations to which Major Toltz belonged were the St. Paul Engineers' Club, the Minnesota and University Clubs, and the Masonic fraternity. He was an honorary member of Pi Tau Sigma, honorary mechanical engineering fraternity. He was a Christian Scientist, and made bequests to several of its churches and other organizations.

Major Toltz is survived by his widow, Mrs. Elizabeth (Argue) Toltz, whom he married in 1919, and by three grandchildren through a former marriage.

JOHN TRIX (1848–1932)

John Trix, founder and president of the American Injector Company, Detroit, Mich., died at his summer home in Algonac, Mich., on September 26, 1932, after an illness of about nine months.

Mr. Trix was born in New Orleans, La., on December 13, 1848, the son of John and Catherine (Cook) Trix. His father died when he was ten years old and later the family moved north to Buffalo, N.Y., and then to Sandusky, Ohio. After completing work in the public schools there, Mr. Trix served an apprenticeship as machinist, toolmaker, and die worker, and for a time manufactured tobacco in Ohio. In 1870 he went to Detroit, where for sixteen years he was superintendent of the John J. Bagley Tobacco Co. During his leisure hours he worked on several mechanical devices and became interested in a patent for an injector, which led to the establishment of the American Injector Company, manufacturers of injectors, oil and grease cups of all kinds, and many other devices.

Mr. Trix was also president of the Maplewood Land Company, the Dover Court Land Company, the Baldwin Park Land Company, and the Blackstone Land Company; vice-president of the Steere Engineering Company; and a director of the McCleary-Harmon Electric Company, during various periods of his active life. He was one of sixteen men who organized the Detroit Employers' Association, and served the Michigan Manufacturers Association as a director and the National Manufacturers Association as vice-president. In politics he was a Republican and he served as a school inspector in Detroit for a time.

He became a member of the A.S.M.E. in 1913, was a member of the Detroit Athletic Club and the Detroit Board of Commerce, and was very active in the Masonic fraternity, in which he had attained the 33d degree.

Mr. Trix married Harriet Phelps, of Detroit, in 1883, and is survived by her and by four children, John J., Ralph, and Herbert B. Trix, and Caroline (Trix) Bryant.

AMBROSE GILMORE WARREN (1873–1932)

Ambrose Gilmore Warren, who died of heart disease on March 16, 1932, was born at Hubbardstown, Mass., on April 1, 1873, the son of Elijah and Eliza (Wood) Warren. After four years at the Gardner (Mass.) High School he entered Worcester Polytechnic Institute, from which he was graduated in 1895 with an S.B. degree.

Mr. Warren's first position was with the Deane Steam Pump Company, at Holyoke, Mass., where he spent three years in general work and the design of jigs and fixtures and another year in laying out castings. After he left the Deane company he was employed for six months by the Farrell Foundry & Machine Co., Ansonia, Conn. He located in Philadelphia in 1900 and for nearly twenty-five years served the J. W. Paxson Co. as mechanical engineer, designing, manufacturing, and selling foundry machinery, in charge of the drawing room and machine shop. Since 1924 he had been vice-president of the Pennsylvania Bronze & Brass Co., and vice-president and treasurer of the Aetna Foundry Company, both of Philadelphia.

Mr. Warren developed sand-blast machinery and a dust arrester now widely used in foundries.

Mr. Warren became a member of the A.S.M.E. in 1907. He also belonged to the Manufacturers' Club of Philadelphia and the Masonic fraternity, and was a Presbyterian. He is survived by his widow, Mabel P. Warren.

FRANK W. WEARIN (1867–1930)

Frank W. Wearin, engaged in appraisal engineering for the firm of Murrie & Co., Inc., New York, N.Y., since 1925, died on September 20, 1930.

Mr. Wearin was born at Maryborough, Queensland, Australia, on November 7, 1867, and received practical training as machinist and engineer on marine work during five years with Walker's Limited there. He also took some courses at Cooper Institute, New York. He spent three years, 1892–1895, in the United States Navy, and an equal period in the construction and repair of marine engines for the American Line. In 1898 he was appointed chief engineer to the New York Edison Company, to handle the construction and repair of all mechanical apparatus. He returned to naval service in 1906 as master mechanic in the steam engineering department at the Portsmouth (N.H.) Navy Yard, and remained there seven years. From 1915 to 1925 he was engineer for the Selson Engineering Company of London in its New York offices, working on the design and manufacture of machine tools for completely equipping machine shops.

Mr. Wearin became an associate-member of the A.S.M.E. in 1919.

HENRY WEAVER (1879–1932)

Henry Weaver, designer for Warren Webster & Company, Camden, N.J., died suddenly on March 19, 1932, at his home in Maple Shade, N.J. He is survived by his widow, formerly Miss Kate Ellen Colley, of Lincoln, England, whom he married in 1904, and by his father, Edward Weaver, a retired civil engineer, residing in England.

Mr. Weaver was born on June 21, 1879, at Grimsby, England. He spent six years at King Edward the Sixth College, Louth, England, completing his studies there in 1897. He then served a six-year apprenticeship in the machine shops, foundry, and drafting room of Robey & Co., Lincoln, England, at the same time taking night courses in drafting, mathematics, steam, light, and heat, and magnetism and electricity at the Lincoln Technical School. During the next five years he was draftsman at the Engineering Works of Ruston, Proctor & Co., at Lincoln, working chiefly on boilers of different types.

Mr. Weaver came to the United States in 1910 and secured employment at the Baldwin Locomotive Works in Philadelphia. He began work as a detailer in the drafting room, soon being advanced to the position of assistant to the designer on electric locomotives. After two years he was put in charge of the extra order work of the electric locomotive department and in 1914 was selected to check the original drawings with the finished locomotives in the shops, to make sure that any alterations or additions which had been found necessary were recorded on the drawings.

He was connected with the Baldwin Locomotive Works until 1918, with the exception of a few months in 1914 when scarcity of work there caused him to do drafting for several other companies. Since leaving the Baldwin Locomotive Works he had been with Warren Webster & Company, specialists in steam heating, except for a few months during 1920 when he was assistant to the chief in the power, light, and heat department of the Victor Talking Machine Company, designing a new plant to supply steam and water necessary for manufacturing records. His work for Warren Webster & Company was largely designing, though he also had charge of considerable tool work during the first few years.

Mr. Weaver became an associate member of the A.S.M.E. in 1924.

WILLIAM PATTON WESCOTT (1864–1932)

William Patton Wescott, who died at Highland Mills, N.Y., on July 22, 1932, was a native of Philadelphia, Pa., where he was born on November 29, 1864. His parents were William Patton and Sarah P. (Baker) Wescott. His early education was secured in the public schools of Jersey City, N.J., and later he took courses in mechanical and mining engineering through the International Correspondence Schools.

Mr. Wescott was connected with the Galena Signal Oil Company from 1896 until his retirement in 1927. He was located in New York until 1911, in the position of mechanical engineer for the firm. From then until the close of the World War he was managing director of the London branch. He was appointed treasurer in 1922 and during his last three years with the company served both as treasurer and as vice-president.

A mine car invented by Mr. Wescott was adopted as standard equipment by the coal mining department of the D. L. & W. R.R. and was also used in other mines.

Mr. Wescott was a member of both the A.S.M.E., which he joined in 1910, and of the A.I.M.E. He belonged to the Engineers' Club in New York and was a 32d degree Mason. He was an active member of the Protestant Episcopal Church, belonging to St. Johns in Jersey City, where he resided for some years, and more recently to St. Davids in Highland Mills. In 1914, during his residence in London, he served as a member of Ambassador Walter Hines Page's Committee of One Hundred for Relief.

Surviving Mr. Wescott are his widow, Carrie A. (Lothridge) Wescott, of Highland Mills, whom he married in 1887, and two children, Albert L. Wescott and Helen W. Van Winkle, both of Leonia, N.J.

HAROLD SAYRE WHEELER (1884–1932)

Harold Sayre Wheeler, who died in Pasadena, Calif., on November 19, 1932, was born at Manistee, Mich., on July 23, 1884. His parents were Edward D. and Emma A. (Sayre) Wheeler.

Mr. Wheeler received a B.S. degree in mechanical engineering from the University of Michigan in 1906 and was assistant to the head of the mechanical engineering department there during the following year, while working for his M.Sc. degree. He then became head of the designing department of the E. P. Thomas Automobile Co. and in 1908 was consulting engineer to the Chalmers Motor Car Company in Detroit. Because of his work in the automotive field the University of Michigan chapter of Sigma Xi elected him to membership in 1910.

In 1909, following the death of his father, Mr. Wheeler went to Tacoma, Wash., to become associated with a brother in handling clay and timber property which his father had owned there. He took the office of vice-president in both the Far West Lumber Company, a timber-holding corporation, and the Far West Clay Company, of which he also became general manager. A clay plant and machinery for it were designed and built, and the products of the plant steadily were increased and perfected under his supervision. Not only the engineering work but also sales and administration were directed by him.

Mr. Wheeler invented and patented the Wheeler interlocking paving block, a process for the lubrication of bearings, and a means of cooling combustion engines. During the World War he served for a time as a lieutenant in the Engineers Corps, stationed at Camp A. A. Humphries, Va. Prior to his enlistment he was vice-chairman of a Committee of Military Defense at Tacoma to promote the development of Camp Lewis.

Mr. Wheeler became a member of the A.S.M.E. in 1920 and belonged to the Christ Church, the Masonic fraternity, and several clubs in Tacoma and Pasadena. He had built several large homes for sale in Pasadena, where he had a winter residence. He is survived by his widow, Ruth (Anderson) Wheeler, whom he married in 1915, and by two children, Laura and Edward Anderson Wheeler.

JOHN CHARLES FREMONT WHITE (1862–1932)

John Charles Fremont White was born on September 15, 1862, at Wilmington, N.Y., the son of George Clinton and E. A. (Comstock) White. He was graduated from high school in Granville, N.Y., in June, 1879, and during the next three years served a machinist's apprenticeship at the Schenectady Locomotive Works. He then secured employment as assistant master mechanic of the Railroad Repair Shop, at Plattsburg, N.Y. After a year there he opened a general repair shop in Plattsburg, but gave this up in 1887. During the next two years he worked as a machinist at the Watts Campbell Engine Works, Newark, N.J., and Armington & Simms Engine Works, Providence, R.I.

For more than twenty years, beginning in 1890, Mr. White was associated with the Hersey Manufacturing Company, South Boston, Mass., five years as machinist, ten as assistant mechanical engineer, designing and supervising the construction of special sugar, soap, and malting machinery, and seven as efficiency engineer, working on special single purpose duplicating tools, both automatic and semi-automatic, for the rapid production of water meter parts.

In 1912 Mr. White became factory superintendent of the Standard Meter Company, Watervleit, N.Y., where he remained for three years. For part of 1915 he was superintendent of an ammunition factory, later called the Liberty Arms & Fuse Co., in Long Island City, N.Y. The following year he took the position of superintendent of machinery for the A. Kimball Co., New York. He had been foreman of the machine shop of the New York, New Haven & Hartford Railroad Co., Van Nest, N.Y., since 1923. He died on October 9, 1932, following an operation for sinus trouble.

Mr. White became an associate member of the A.S.M.E. in 1918. He also belonged to the Masonic fraternity, the Cable Club, a railroad organization, and the League of American Wheelmen. He was a charter member of the Savin Hill Yacht Club (New Haven, Conn.), when it was reorganized in 1907, and had served on its house committee and as its chairman. He was also active in the Isham Park Yacht Club at Spuyten Duyvil. His own yacht, the *Whitecap*, was well known along the New England Coast.

Mr. White is survived by his widow, Florence Editha (Macdonald) White, whom he married in 1895 in Boston, Mass.

GARDNER STEWART WILLIAMS (1866–1931)

Gardner Stewart Williams, vice-president of the American Engineering Council since 1923, and consulting hydraulic engineer of Ann Arbor, Mich., died in that city on December 12, 1931, after a long illness.

Major Williams had represented the Detroit Engineering Society on the F.A.E.S. and the American Engineering Council since 1921. He was the leader of the group which organized that society, under the name of the Detroit Association of Graduate Engineers of the University of Michigan, in 1895. It is one of a number of technical organizations in which he had taken an active interest. These included the A.S.M.E., which he joined in 1921, the A.S.C.E., of which he was a past vice-president and past director, the American Water Works Association, New England Water Works Association, Western Society of Engineers, Michigan Engineering Society, and Grand Rapids Engineering Society.

The son of Stewart B. and Juliet M. (Ripley) Williams, Gardner S. Williams was born at Saginaw City, Mich., on October 22, 1866. His early education was obtained in the Saginaw schools, from which he entered the Engineering College of the University of Michigan. He received a B.S. degree in civil engineering in 1889, and a C.E. degree was conferred upon him ten years later.

The summer vacations of 1887, 1888, and 1889 were spent by Major Williams in waterworks construction at Bismarck, N.D., and Greenville and Owosso, Mich. For three years following his graduation he was draftsman, designer, and superintendent of erection for the Russel Wheel & Foundry Co. in Detroit. He was civil engineer to the Board of Water Commissioners of Detroit from 1893 to 1898 and during the latter part of that period joined with Messrs. Hubbell and Fenkell in an investigation of the flow of water in pipes, for which they were presented the Norman Medal of the A.S.C.E. in 1901.

In 1899 Major Williams accepted the chair of experimental hydraulics at Cornell University, and took charge of the Hydraulic Laboratory there until 1904. He also conducted a study of the flow of water over dams for the International Waterways Commission, of which he was a member. He designed and built the first dome dam in this country, at Ithaca, in 1902. The Department of the Interior adopted the features of this design in the Coolidge Dam. He also did pioneer work in building multiple-arch dams on soft foundations in 1909, and in 1910 designed underdrained earthen embankments for dams.

After leaving Cornell in 1904, Major Williams taught civil, hydraulic, and sanitary engineering at the University of Michigan until 1911. Since that time he had been engaged in general consulting work, with headquarters at Ann Arbor. He had practised in twenty-one states and in Ontario and Quebec, and his latest multiple arch dam was the Magnitogorsky on the Ural River. Major Williams had also studied water turbines and designed a scroll pit and elbow draft tube for vertical shaft turbines. He was recognized as an authority on the hydraulics of the Great Lakes and was an early advocate of the St. Lawrence Waterway. The last two or three years of his life were devoted mainly to the hydraulics of the Mississippi River, leading him to propose straightening and deepening, as an alternate to levee construction. He was co-author with Allen Hazen of Hydraulic Tables (1905), and he also wrote the chapter on hydraulics in the American Civil Engineers' Handbook.

Major Williams rendered special service to the public and the Government in a number of cases. About 1896 he succeeded in tracing typhoid fever infection from Port Huron to Detroit through the waters of the St. Clair and Detroit Rivers. He appeared before the United States Supreme Court as an expert in the Drainage Canal Case of the State of Missouri vs. the State of Illinois and the Sanitary District of Chicago. He was also an expert witness in the United States Circuit Court of Illinois in the case of the United States vs. the Sanitary District of Chicago. Early in 1927 he appeared in this same case before Charles Evans Hughes, special master for the Supreme Court. Major Williams served in the Officers Reserve Corps in 1917 in connection with camp construction at Alexandria, La.

Surviving Major Williams are his widow, Jessie B. (Wright) Williams, whom he married in 1893, a son, William Wright Williams, and a daughter, Harriet R. (Williams) Waite.

WILLIAM AUGUST TJEENK WILLINK (1859-1932)

William A. T. Willink, who died on April 24, 1932, was born at Zutphen, Netherlands, on July 14, 1859. He studied engineering at the Royal Technical High School and University at Aix-la-Chapelle, Germany, receiving his M.E. degree in 1884.

After a short apprenticeship with a manufacturer of pumps and engines in Burtscheid, Germany, he went to England, where he worked for Butterworth & Dickinson, Burnley, and Woolsume, Rye & Co., Oldham, in 1885. From 1886 to 1890 he was connected with the Zwolle Machine Works, Zwolle, Netherlands, as technical manager. The next two years were spent in the capacity of chief engineer for the Peat Utilization Company and Continental Phosphate & Chemical Manure Co., in London. He then returned to Germany where he worked at the Grevenbroich Machine Works in 1892.

During the years 1893 to 1898 Mr. Willink was in Cuba, holding the position of chief engineer, first at the Santa Lucia Sugar Factory and then at the Vicini Sugar Factories in Santo Domingo. From there he went to Brooklyn, N.Y., spending the next five years at the Pioneer Iron Works. In 1904 he became manager of the Augustin Fuller Company, New York, exporters of sugar machinery. He returned to Cuba in 1905, acting as agent for the Pioneer Iron Works, with office at Havana, and practicing consulting engineering in that city until 1910. He also became a registered patent attorney in 1904.

From 1910 to 1914 Mr. Willink was associated with Lebedjeff & Co., New York, contractors for sugar factories and machinery for Cuba and Porto Rico. He was mechanical engineer in charge of the drafting office of the Arbuckle Brothers Sugar Refinery in Brooklyn for three years, 1914-1916; sugar engineer for the M. H. Treadwell Co., New York, 1917-1921, designing sugar machinery and in 1920-1921 surveying the needs for machinery in Argentina and Brazil; also sugar engineer for the Southwark Foundry & Machine Co. of Philadelphia, 1920-1921; and had practiced consulting engineering in the sugar industry, with an office in Philadelphia, since then.

Mr. Willink became a member of the A.S.M.E. in 1912. Surviving him are his widow, Freda Willink, of Philadelphia, and a son, Lieutenant A. Willink, stationed at Frankford Arsenal.

JOHN P. WINTERS (1876-1932)

John P. Winters was born at Shippenville, Pa., on August 28, 1876, the son of Peter and Sarah L. Winters. He attended Grove City (Pa.) College and in 1916 became superintendent of production in District No. 1, State of Kansas, for the Empire Gas & Fuel Co. From 1919 to 1921 he held a similar position with the Kansas & Gulf Co. and during the next two years with the Skelly Oil Company.

Since 1923 Mr. Winters had been production superintendent for The National Refining Company in the Mid-Continent field, with headquarters at El Dorado, Kan. He was in complete charge of production and of construction of all oil-field equipment. He designed and built casinghead plants, booster stations, compressor plants, and all types of pipe lines. He was a joint patentee of an oil filter and a true-bore-forming guide for rotary drill stems.

Mr. Winters served as a private in the infantry during the Spanish-American War and was a member of the Lawton Camp of Spanish-American War Veterans. He also was postmaster at Pittsville, Pa., for eight years. He became a member of the A.S.M.E. in 1931 and belonged to the American Petroleum Institute. He was a Shriner in the Masonic fraternity and a member of the Presbyterian Church.

Surviving Mr. Winters, whose death occurred on September 26, 1932, at Tulsa, Okla., are his widow, Clementine (Stover) Winters, whom he married in 1899, and two daughters, Miss Jean Winters and Mrs. John E. Howell, both of Bartlesville, Okla.

ALEXANDER WINTON (1860-1932)

Alexander Winton, who in 1898 sold his first "horseless carriage," died in Cleveland, Ohio, on June 21, 1932, after a two weeks' illness. His first "one-lunger" gasoline automobile was repurchased by Mr. Winton after a few years and is now in the Smithsonian Institution at Washington.

Mr. Winton was born in Grangemouth, Scotland, on June 20, 1860, the son of Alexander Winton, a manufacturer of farm implements, and Helen (Fea) Winton. He received a public-school education in Scotland and served an apprenticeship in the shipyards on the Clyde River. He came to the United States in 1880 and for four years was employed by the DeLamater Iron Works, New York. He then went to Cleveland, where he took the position of superintendent of the Phoenix Iron Works.

Mr. Winton's first invention was a bicycle, which was manufactured by the Winton Bicycle Company, established in Cleveland in 1890. He then began experimental work, in the cellar of his home, on a gasoline vehicle. By 1897 he was ready to organize the Winton Motor Carriage Company, later the Winton Motor Car Company, and the first car was sold for $1000 the following year. In the beginning the various parts for his cars were manufactured by hand, but Mr. Winton soon designed special machinery for turning out many pieces. In later years he built the first two-braking system and a Winton car was first to be equipped with internal and external brakes on the one drum. An important phase of his work was done in the development of the storage battery. The difficulty with glass-jar cells in use at the time was that the liquid contents spilled whenever the car struck a rut in the road.

Mr. Winton was the first to devise a successful speed governor and also a completely accessible motor, as well as the first to manufacture four and eight cylinder motors. Recently he had announced the perfection of a light oil-burning motor which he planned to use in airplanes.

Like other pioneer automotive manufacturers, Mr. Winton tested his cars and attracted public attention to them on the race tracks. For several years his cars were winners. In 1903 he established a speed record of 68 miles an hour at Daytona Beach, Florida. In 1905 he built an eight-cylinder racing car, which he drove that year in the James Gordon Bennett Cup race in Ireland. He lost the race, he said later, because someone put candles in the gasoline tanks to "keep away the devils." In those days some people still regarded an automobile as an evil contraption.

Mr. Winton was also the organizer and president of the Winton Gas Engine & Manufacturing Co., established in 1897, and the Winton Marine Oil Engine Works, founded in 1912. The declining popularity of the Winton car caused him to suspend its manufacture in 1924.

Mr. Winton became a member of the A.S.M.E. in 1913. He also belonged to the Cleveland Engineering Society and the Masonic fraternity. He was ex-commander of the Inter-Lake Yachting Association and his club memberships included the Boston Yacht Club, the Civil Engineers Club (Cleveland), and the Chicago, Clifton, and Cleveland Automobile Clubs. He is survived by his fourth wife, Mrs. Mary E. (Avery) Winton, whom he married in 1930, and by six children by his first marriage and one by his second.

GEORGE MONROE WISNER (1870-1932)

George Monroe Wisner was born on February 9, 1870, in Detroit, Mich., the son of George Y. and Carrie (Palmer) Wisner. He attended the public schools of Detroit and the University of Michigan, from which he was graduated in 1892, with a B.S. degree in Civil Engineering. In the same year he accepted a position in the Engineering Department of The Sanitary District of Chicago, and throughout his engineering life of about 40 years he was connected with the work of the District, first with construction and later with operation and maintenance, and the various problems related thereto. He passed through the various grades; as instrument man he had charge of the construction of four rock sections between Lemont and Lockport, Ill.; as assistant engineer he was in charge of cement testing, but was employed chiefly on hydraulic and design work from 1895 to 1897 and during the next four years was in charge of the Chicago River Improvement, consisting of the dredging and docking of the Chicago River and the construction of a "by-pass" between Adams and Van Buren Streets along the west side of the river, the construction of the Des Plaines River Improvement through Joliet, and the construction of the Water Power Development at Lockport. As assistant chief engineer from 1901 to 1907, he directed all construction work, including the building of docks, coffer dams, bridges, both substructures and the superstructures, and the water power development.

Under his direction as chief engineer, to which position he was elected in 1907, the North Shore Channel and the Calumet-Sag Channel, adjuncts to the Main Channel, were planned and built. When Chicago was experiencing its rapid growth both in population and industry, between 1900 and 1910, Mr. Wisner foresaw the necessity of sewage treatment to supplement the existing dilution system. He convinced the Board of Trustees of the need of a thorough study of the sanitary program and early in 1909 secured an appropriation for a testing station on domestic sewage at the 39th Street pumping Station, where the then known methods of sewage treatment were tried. He also began the organization of a sanitary engineering staff, equipped with a laboratory adequate to handle the biological, bacterial, and chemical problems, as well as coal, oil, and other routine analyses. Following this, he directed an intensive study of the Packingtown and Stock Yards wastes of Chicago, the tannery wastes and the corn products wastes, whereby for the first time the magnitude of the industrial waste problem in the Sanitary District was revealed. He developed a keen interest in sewage treatment as then practised in the United States and abroad, visiting England, France, and Germany in 1910 to study foreign practice. In 1911, he made a report to the Board of Trustees on sewage disposal works for the Sanitary District, laying the foundation for later developments. Following these and other investigations, the Des Plaines River Sewage Treatment Works and the Calumet Sewage Treatment works were designed and constructed, the first units of the sewage treatment system. The Morton Grove Sewage Treatment Works, the 32nd Avenue Outlet Sewer, the West 39th Street Conduit, the Stock Yards Intercepting Sewer, the North Shore Intercepting Sewer, the Summit Outfall Sewer, the Evanston Intercepting Sewers and Pumping Station, the Des Plaines Intercepting Sewer and the Calumet Intercepting Sewer and Pumping Station were all designed and built under his direction during his term of office. At the same time, he was following the negotiations and litigation relating to the diversion from Lake Michigan and was one of the principal witnesses in the case of the United States vs. the Sanitary District and later in the hearings before the Rivers and Harbors Committee of Congress, and became an expert in advising the lawyers in such matters.

Mr. Wisner resigned as chief engineer in June, 1920, to enter private consulting practice, but was retained by the District as consulting engineer, a relationship which continued until his death. As a consultant, he proved to be a valuable advisor during the development of the sewage treatment program and the recent Lake level litigation (Wisconsin et al vs. Illinois et al), and was also instrumental in making great savings for the Sanitary District.

Mr. Wisner's private consulting work comprised harbor works in Labrador, dock work and shore protection in Florida, and drainage and road work in Cook County, Illinois. He served one year as Chairman of the Advisory Subway Engineering Commission of the City of Chicago. In spite of total blindness during the last six years of his life he kept up his interest in engineering works, giving particular thought to the transportation problem of the City of Chicago.

In 1902, Mr. Wisner married Edith K. Young, of Chicago, by whom he is survived.

Mr. Wisner became a member of the A.S.M.E. in 1907. He also belonged to the American Society of Civil Engineers and the Western Society of Engineers. His club affiliations included the Beta Theta Pi, Chicago Athletic Association, Chicago Engineers, South Shore Country, and Beverly Country Clubs. He took a great deal of interest in athletic events, playing football when attending the University of Michigan. Throughout his working life, he kept up his skill in swimming. He enjoyed yachting and travel. He died suddenly at his home on August 26, 1932, after a short illness.

FRANK WARREN WOOD (1870–1932)

Frank Warren Wood, whose death occurred on May 18, 1932, was born at Ancora, N.J., on November 9, 1870, the son of Nathan H. and Emmaline (Spaulding) Wood.

In 1903 he became general manager of the Bessemer Coal & Coke Co. of Pittsburgh, Pa. He resigned in 1906 to organized the Cleveland Castings Pattern Company, manufacturers of wood and metal patterns. In 1909 he organized The Wood & Spencer Co., also in Cleveland, manufacturers of taps and automobile and airplane parts. He served as president and general manager in these last two companies until the time of his death.

He was a lieutenant in the Spanish American War, and a major in the Ordnance Department in the World War, in charge of the Ordnance Department at Limoges, France.

He became an associate of the A.S.M.E. in 1924 and also belonged to the Cleveland Engineering Society and the Army Ordnance Association.

SIR ALFRED YARROW (1842–1932)

Sir Alfred Yarrow, shipbuilder, naval engineer, philanthropist, and honorary member, A.S.M.E., who died at the age of ninety on January 24, 1932, was well known for the boiler that bears his name and for the high-speed and shallow-draft vessels made at his works on the River Clyde. He was born into the age of steam that had developed out of the introduction of the engine, the locomotive, and the steamship. Endowed, as his schoolmaster said, with "a talent for engineering and a thirst for affection—to give and to take," and, as he himself once said, with a good constitution and a necessity of working, he made more profitable use of his talents and his opportunities than do most men. He inherited from his mother an ability for affairs unusual in an engineer, good health, and longevity (she lived to be 95), and from both his parents a warmth of human sympathy and understanding which directed the abundant philanthropies made possible by his well-earned prosperity. Starting his apprenticeship to an engineering firm at the age of fifteen, he was more fortunate than most men in being able to crowd into an active three-quarters of a century an extraordinary list of good works. His approach to engineering was a strictly practical one; his school was the shop and drafting room; and no small measure of his success lay in the intrepid pioneering spirit that gave him courage to try new and apparently impossible tasks, such as constructing a steamship that could be taken apart and carried in 50-lb pieces through 60 miles of African jungle, and vision to seek through research the necessary data upon which to base new developments. His specialties, shallow-draft steam yachts, mostly used on rivers in undeveloped countries, and high-speed vessels for the British and other navies, demanded pioneering and research methods, for almost every ship was constructed for an unusual purpose, or to travel at a hitherto unattained speed. It was characteristic of him that he should have been the first to set up and operate a privately owned telegraph in London; that, when less than twenty, he should have constructed a steam-propelled road vehicle which, on its nocturnal travels between Greenwich and Bromley at twenty-five miles an hour, had the misfortune to cause a horse to shy, a mounted policeman to break his leg, and an aroused parliament to pass a bill prohibiting the use of steam upon the road, unless the carriage were preceded by a man carrying a flag; and it was equally characteristic of this same spirit of progress that he should donate £20,000 to the Royal Society in 1911 for an experimental tank for shipbuilders at the National Physical Laboratory at Teddington; and later, in 1923, to offer the same society £100,000 to be used at the discretion of its council in promoting scientific research.

Alfred Fernandez Yarrow was born in London on January 13, 1842, the son of Edgar and Esther (Lindo) Yarrow. His mother was the daughter of Moses Lindo, a Jewish merchant who had been in business on the island of St. Thomas, West Indies, and for whom Edgar Yarrow worked as a clerk. After some schooling, in which he showed an aptitude for mathematics and science, Alfred Yarrow, at the age of 15, was apprenticed for a period of 5 years to Messrs. Ravenhill, marine engineers, builders of marine engines. During his apprenticeship his spare time was spent in shops, in attendance at engineering meetings and lectures (Faraday was demonstrating new-found scientific principles before large audiences during these years), and in working at home. Inventive ability, which had manifested itself in childhood and sometimes led to mischievous pranks, found expression before the lad was twenty in two devices that were developed in cooperation with his chum, James Hilditch: an improved method for steam plowing, and a steam-propelled road carriage, both of which which were taken over by others on a royalty basis; and also gave rise to the building of the steam yacht *Isis*, the first of the long line of vessels that the famous shipbuilder was to produce.

It was the success that attended the building of the *Isis* that finally encouraged Yarrow to undertake the building of small yachts. Ravenhill had held out small inducement for the ambitious boy, and finding a partner and a site for a shop at Poplar, on the Isle of Dogs in the River Thames, the first works with a sign bearing the name Yarrow & Hedley, Engineers, was started. In 1868, after advertising his ability to build steam launches, Yarrow received his first order, a 24-ft steam launch for Colonel Halpin, and thereby established a profitable business. Differences between Yarrow and Hedley terminated the partnership in1875. Up to that time 350 steam launches had been built. Having bought out Hedley, Yarrow continued the shipbuilding business with rapidly growing success to the day of his death, moving his works to Scotstoun, on the Clyde, in 1906.

An impressive list of steam launches have come out of the Yarrow Works. An unusual problem in shipbuilding was successfully solved in the construction of the 55-ft *Ilala*, launched in 1875 on Lake Nyassa, Africa. It had been designed so that no piece weighed more than 50 lb, and was bolted rather than riveted together. After a trip up the Zambesi and Shiré rivers, it was taken apart and carried 60 miles through the jungle and reassembled without loss or injury of a single part.

Between 1877 and 1879 Yarrow, who was specializing in the newly conceived torpedo boat, arranged first for spar torpedoes and subsequently for self-propelled Whiteheads, built torpedo launches for the Argentine, Dutch, French, Russian, Spanish, Austrian, Italian, and Chilean governments. The British Admiralty Torpedo Boat No. 425, built to a guarantee of 18 knots, actually realized 21.9 knots, and led Yarrow into some interesting and valuable experiments on 25 propellers. His first ocean-going torpedo boat, the *Batoom*, built in 1880, made 22 knots. By 1892 these smaller craft had developed into the destroyer class, of which the *Havock* and *Hornet*, 180 ft long and developing 4000 hp, were conspicuous examples. The *Havock* had the usual locomotive type of boiler and made 26.1 knots, but in the *Hornet* had been placed the new Yarrow water-tube boiler. With 11 tons less weight, the *Hornet* made 27.3 knots. Speed was raised again by the Russian yacht *Sokol*, holding a record of 30 knots, and, in 1899, by the Japanese destroyer *Sazanami*, 220 ft long, 6000 hp, which made 31 to 32 knots.

In 1910 Yarrow convinced Lord Jellicoe and subsequently Lord Churchill that, in view of developments elsewhere, the 27-knot destroyer class should be replaced by ships of greater speed, and the *Firedrake*, *Lurcher*, and *Oak* were built, the *Lurcher* attaining a speed of 35.345 knots.

The Yarrow boiler, familiar to all engineers, was developed between 1877 and 1887, and when finally perfected, was used for years with practically no changes in fundamental design. A demand for greater steaming capacity, for which increased speed was responsible, led to the development in 1905 of the double-ended boiler for the Spanish Navy, and in 1908–1910 to two boilers developing 9000 hp for a Brazilian ship. In 1911 with the introduction of superheated steam, the superheated damper was installed in the boilers for the *Archer* and the *Attack*.

With the coming of the World War, Yarrow's speedy destroyers gave valiant aid to the British Navy. Twenty-nine of these ships were built from August 4, 1914, to November 11, 1918, and with the success of the Tigris flotilla, brought Yarrow the baronetcy shortly after. Of these vessles the *Tyrian* made 40 knots.

During a trip to the United States Sir Alfred made an experimental study of ship vibration and subsequently developed the Yarrow-Schlick-Tweedy system of balancing reciprocating engines.

No less famous than the torpedo boats and destroyers were numerous shallow-draft steamers built by Yarrow. In addition to the *Ilala*, a ship of a special sectional construction was built in which Stanley explored the Congo in 1883. The sections could be carried on a steamer and lowered over the ship's side and assembled afloat, and could be carried overland on specially provided carriages. For navigating the upper Nile four boats were built in 1874. Of these no part weighed more than a camel could carry. They were driven by means of a stern wheel. In a fruitless attempt to save Gordon at Khartoum, two shallow-draft gunboats, the *Lotus* and the *Waterlily*, were built in 1884. In 1897 a Nile expedition under Lord Kitchener used the *Sheik*, a 24-in.-draft gunboat with its screws in a tunnel and provided with the Yarrow hinged flap that subsequently became a feature of the ships of the Tigris flotilla. For use on the Yangtze Kiang, in 1903, the gunboat *Widgeon*, 160 ft long and of 27 in. draft, was designed.

The Tigris flotilla consisted of river gunboats built in two sizes; one 250 ft long with 4 ft draft, equipped with twin screws, and the other of the "fly" class, 120 ft long and with 2 ft 6 in. draft and a single screw. They were shipped in pieces to the mouth of the Tigris, assembled at Abadan by native labor, and used in connection with General Townshend's campaign in Mesopotamia.

Aside from his engineering achievements, Sir Alfred's many philanthropies made him a well-known Briton. He established at Broadstairs in 1894 a children's convalescent home, the trusteeship of which passed to the Institution of Civil Engineers in 1903. Among other benefactions he provided funds for the building of the outpatient department of the London Hospital, established a fund for medical research, a nurses' training home in Scotland, as well as a fund to provide nurses in the highlands and islands of Scotland, and a studentship in Assyriology at Cambridge in honor of his son killed during the war, in addition to the two funds for research at the Royal Society already referred to. He was deeply interested in education and in his workmen. During the strike of 1897 he suggested that a delegation of workmen visit the United States at his expense to see for themselves the results that workmen in this country had attained socially and financially, but the suggestion was never carried out.

Sir Alfred was married in 1875 to Minnie Franklin, who died in 1922. Six children were the result of this union: Harold Edgar, now managing director of the Yarrow & Company, Glasgow, who succeeds to the baronetcy; Norman Alfred, managing director, Yarrows, Ltd., Victoria, B.C.; and Eric, killed at Ypres; and three daughters; of whom the eldest is wife of Vice-Admiral P. M. Royds, the second, wife of Alfred Earnest Yarrow of Pinner, and the youngest, wife of Lord Dawson of Penn. Sir Alfred's second wife, who survives him, is Eleanor Barnes, author of "Alfred Yarrow, His Life and Work." Both of Sir Alfred's sons are members of the A.S.M.E.

Sir Alfred was elected an honorary member of the A.S.M.E. in 1914. He had been a member of the Institution of Civil Engineers since 1869 and a member of its council, of the Institution of Mechanical Engineers since 1889, and of the Naval Architects and Marine Engineers, which latter society he served as vice-president in 1896.

WILLIAM FRANKLIN YEAGER (1879–1932)

William Franklin Yeager, consulting engineer for the United Engineers & Constructors, Inc., Philadelphia, Pa., died on February 13, 1932. Mr. Yeager was a native of Philadelphia, having been born there on September 18, 1879. After leaving grammar school he spent two years at the Philadelphia Manual Training School and then three at the Williamson (Pa.) Free School of Mechanical Trades, from which he was graduated in 1899 as an electrical machinist. He followed up this training with practical experience as machinist, electrician, and draftsman in the employ of ten or twelve different Philadelphia companies, including the Brush Electric Company, predecessor of the Philadelphia Electric Company. He also spent about a year with the General Electric Company in Schenectady, engaged in general structural design and drawings for substations, power houses, and other buildings, and pipe lines.

Mr. Yeager was connected with Dodge & Day, an early predecessor of the United Engineers & Constructors, Inc., from 1905 to 1908, making calculations and detail drawings for electrical work in various plants, during the latter part of the time in the capacity of chief draftsman. Subsequently he was associated for five or six years with Frank C. Roberts & Co., and Edgar V. Seeler, Philadelphia, largely in the design, construction, and testing of electrical equipment for the Curtis Publishing Company.

Since early in 1916, when he accepted the position of chief draftsman with Day & Zimmermann, Inc., Mr. Yeager had been in charge of a large volume and variety of design and construction work. Among the contracts to which he was assigned were a factory and auxiliary structures for the Philadelphia Textile Company; several bridge and pipe line structures for the New Process Gear Corporation, Syracuse, N.Y.; a brick, concrete, and steel substation for the Erie (Pa.) Lighting Company; a brick and wood factory for the Chicago Automatic Machine Screw Company; foundry and other buildings for the Toledo Machine Tool Company; and power plant additions for the Penn Central Power Company and other power companies.

In 1917 he was placed in charge of the design of the gun plant at Erie, for which Day & Zimmermann acted as supervising engineers for the Government. Later he handled the general structural design of a plant for the Baltimore Shipbuilding Company and two Government projects, the Quartermaster Terminal at Oregon Avenue, Philadelphia, and the Delaware River, and Motor Transport headquarters in Philadelphia. The latter assignments involved pier construction, warehouses, railroad track and yard, power houses, as well as temporary structures used while the work was in progress. During his last six months on this work his position was that of supervising engineer directing both design and construction.

Since returning to the main offices of Day & Zimmermann in 1920 Mr. Yeager had spent the major portion of his time in surveys and reports on development projects for many clients of the company.

Mr. Yeager became a member of the A.S.M.E. in 1927 and was also an associate of the American Institute of Electrical Engineers.

Mr. Yeager is survived by his widow, Lillie M. (Mandeville) Yeager, whom he married in 1905, and by two sons, William M. and Edward E. Yeager.

WILLIAM ALFRED YOUNG (1870–1931)

William Alfred Young, consulting engineer for the Harnischfeger Corporation, Milwaukee, Wis., died in that city on December 23, 1931.

Mr. Young was born at Cuyahoga Falls, Summit County, Ohio, on October 16, 1870, the son of John and Anne Elizabeth (Davis) Young. He served an apprenticeship as a machinist with the Hughes Steam Pump Company, Cleveland, and studied civil and mechanical engineering through the International Correspondence Schools and Ohio Northern University, from which he received a B.S. degree in civil and mechanical engineering in 1897. His early experience included shop training with the Turner, Vaughn & Taylor Co., Cuyahoga Falls, and the Otis Steel Company, Cleveland. He worked as draftsman for several years for the Whitman & Barnes Manufacturing Co., Akron and Chicago, and the Grant Ball & Machine Co., Cleveland. He was machine-shop foreman for a year for the A. P. Dickey Manufacturing Co., of Racine, Wis., and later chief draftsman for the Falls Rivet & Machinery Co., Cuyahoga Falls, and assistant to the chief draftsman of the Shaw Electric Crane Company, Muskegon, Mich. About the year 1908 he became chief draftsman for the Pawling & Harnischfeger Co., which was reorganized as the Harnischfeger Corporation following the death of Mr. Pawling some fifteen years ago.

A fully enclosed, spur-geared, crane-type trolley was developed by Mr. Young, and one of his most recent contributions to the metal-working industry was a special apparatus for handling material in welderies.

Mr. Young became a junior member of the A.S.M.E. in 1901, an associate in 1905, and a member the following year. He belonged to the Masonic fraternity and the Congregational Church. His widow, Anna Belle (Damp) Young, whom he married in 1899, survives him.

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Terre Haute	Rose Polytechnic Institute
West Lafayette	Library, Purdue University

Iowa

Ames	Iowa State College
Des Moines	Public Library
Iowa City	State University of Iowa

Kansas

Kansas City	Public Library, Huron Park
Lawrence	Library, University of Kansas
Manhattan	Kansas State Agricultural College
Wichita	Wichita City Library

Kentucky

Lexington	University of Kentucky
Louisville	Speed Scientific School University of Louisville

Louisiana

Baton Rouge	Louisiana State University
New Orleans	Howard Memorial Library Louisiana Engineering Society Public Library

Maine

Orono	University of Maine

Maryland

Annapolis	United States Naval Academy
Baltimore	Johns Hopkins University Engineers Club of Baltimore Public Library

Massachusetts

Boston	Northeastern University Boston Public Library
Cambridge	Harvard University (Engineering Library) Massachusetts Institute of Technology
Fall River	Public Library
Lowell	Lowell Textile Institute
Lynn	Free Public Library
New Bedford	Free Public Library
Springfield	Springfield City Library
Tufts College	Tufts College
Worcester	Worcester Polytechnic Institute Free Public Library

Michigan

Ann Arbor	University of Michigan
Detroit	Public Library Cass Technical High School Highland Park Public Library University of Detroit
East Lansing	Michigan State College
Flint	Public Library
Grand Rapids	Public Library
Houghton	Michigan College of Mining & Technology
Jackson	Public Library

Minnesota

Duluth	Public Library
Minneapolis	University of Minnesota Minneapolis Public Library (Engineering and Circulating Libraries)
St. Paul	James Jerome Hill Reference Library

Mississippi

State College	Mississippi State College

Missouri

Columbia	University of Missouri
Kansas City	Public Library
Rolla	Missouri School of Mines and Metallurgy
St. Louis	Engineers Club of St. Louis Public Library Washington University Mercantile Library

Montana

Bozeman	Montana State College

Nebraska

Lincoln	University of Nebraska
Omaha	Public Library

Nevada

Reno	University of Nevada Library

New Hampshire

Durham	University of New Hampshire

New Jersey

Bayonne	Free Public Library
Camden	Free Public Library
Elizabeth	Free Public Library
Hoboken	Stevens Institute of Technology
Jersey City	Free Public Library
Newark	Newark College of Engineering Free Public Library
New Brunswick	Rutgers University
Paterson	Free Public Library
Princeton	Princeton University
Trenton	Free Public Library

New York

Albany	New York State Library
Brooklyn	Polytechnic Institute Pratt Institute Brooklyn Public Library
Buffalo	The Grosvenor Library Engineering Society of Buffalo Buffalo Public Library
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Jamaica, L. I.	Queens Borough Public Library
New York	Engineering Societies Library Public Library College of the City of New York Cooper Union Columbia University New York Museum of Science and Industry New York University Library
Potsdam	Clarkson College of Technology
Rochester	Rochester Engineering Society
Schenectady	Union College
Syracuse	Syracuse University Public Library
Troy	Rensselaer Polytechnic Institute
Utica	Public Library
Yonkers	Public Library

North Carolina

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Raleigh	North Carolina State College

North Dakota

Fargo	North Dakota State College
Grand Forks	University of North Dakota

Ohio

Ada	Ohio Northern University
Akron	Public Library University of Akron
Canton	Public Library
Cincinnati	University of Cincinnati Public Library Engineers Club of Cincinnati
Cleveland	Public Library Case School of Applied Science Cleveland Engineering Society
Columbus	State of Ohio Library Public Library Ohio State University
Dayton	Engineers Club of Dayton
Toledo	Public Library University of Toledo
Youngstown	Public Library

Oklahoma

Norman	Oklahoma University
Oklahoma City	Public Library
Stillwater	Oklahoma Agricultural and Mechanical College
Tulsa	Public Library

Oregon

Corvallis	Oregon State Agricultural College
Portland	Portland Library Association

Pennsylvania

Allentown	Free Library
Bethlehem	Lehigh University
Easton	Public Library Lafayette College
Erie	Public Library
Lewisburg	Bucknell University
Philadelphia	Engineers Club Drexel Institute University of Pennsylvania Franklin Institute
Pittsburgh	University of Pittsburgh Engineers' Society of Western Pennsylvania Carnegie Institute of Technology Carnegie Library (Schenley Park) Carnegie Free Library of Allegheny
Reading	Public Library
Scranton	Public Library
State College	Pennsylvania State College
Swarthmore	Swarthmore College
Villanova	Villanova College
Wilkes-Barre	Public Library

Rhode Island

Kingston	Rhode Island State College
Providence	Brown University Providence Engineering Society Public Library

South Carolina

Clemson College	Library, Clemson College

Tennessee

Kingsport	Public Library
Knoxville	University of Tennessee
Memphis	Goodwin Institute
Nashville	Vanderbilt University

Texas

Austin	University of Texas
College Station	Agricultural & Mechanical College of Texas
Dallas	Public Library
El Paso	Public Library
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San Antonio	Carnegie Library

Utah

Salt Lake City	University of Utah Public Library

Vermont

Burlington	University of Vermont

Virginia

Blacksburg	Virginia Polytechnic Institute
Charlottesville	University of Virginia
Norfolk	Public Library
Richmond	Virginia State Library

Washington

Pullman	State College of Washington
Seattle	Public Library Engineers Club University of Washington
Spokane	Public Library
Tacoma	Public Library

West Virginia

Morgantown	West Virginia University

Wisconsin

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Milwaukee	Public Library Board of Industrial Education, Vocational School Library Marquette University

Wyoming

Laramie	Wyoming University

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Perth	University of Western Australia Library
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Brazil	
Rio de Janeiro	Bibliotheca da Escola Polytechnica Bibliotheca Nacional
Sao Paulo	Bibliotheca da Escola Polytechnica
Canada	
Montreal	McGill University Engineering Institute of Canada
Toronto	University of Toronto, Library
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Czechoslovakia	
Prague	Masarykova Akademie Prace Society of Czechoslovak Engineers
Danzig Free City	Bibliothek der Technischen Hochschule
Denmark	
Copenhagen	The Royal Technical College
England	
Birmingham	Birmingham Public Libraries
Bristol	University of Bristol
Cambridge	University of Cambridge
Leeds	University of Leeds
Liverpool	Public Library of Liverpool Liverpool Engineering Society
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Sheffield	Sheffield Public Libraries
Wales	
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Germany	
Berlin	Verein deutscher Ingenieure Bibliothek der Technischen Hochschule
Breslau	Bibliothek der Technischen Hochschule
Cologne (Köln)	Universitäts- und Stadtbibliothek
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Düsseldorf	Bücherei des Vereines deutscher Eisenhüttenleute
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Hamburg	Bibliothek der Technischen Staatslehranstalten
Hanover	Bibliothek der Technischen Hochschule
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Leipsic	Stadtbibliothek
Munich	Bibliothek der Technischen Hochschule Bibliothek des Deutschen Museums
Stuttgart	Bibliothek der Technischen Hochschule
Hawaii	
Honolulu	University of Hawaii Library
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Delft	Bibliotheek der Technische Hoogeschool
The Hague	Koninklijk Instituut van Ingenieurs
Rotterdam	National Technisch - Schlep - Caartkundig Museum
India	
Bangalore	Mysore Engineers Association
Calcutta	Bengal Engineering College
Poona	Poona College of Engineering
Rangoon	University of Rangoon
Ireland	
Belfast	Queen's University of Belfast
Italy	
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Turin	Biblioteca della R. Scuola d'Ingegneria
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Yokohama	Library of Yokohama
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Poland	
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Mayaguez	University of Porto Rico
Portugal	
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Roumania	
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